$\theta =$ Degree, radian, Grades

Degree

angles : θ degree

$$\boxed{\begin{array}{l} 1^\circ = 60' \quad , \quad 1' = 60'' \\ 1' = \text{minute} \\ 1'' = \text{seconds} \end{array}}$$

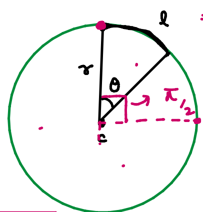
radian.

$$\boxed{l = r\theta}$$

$\theta = l/r$

radian.

$$\boxed{1 \text{ radian} = \frac{180^\circ}{\pi}}$$



$= \frac{\pi r}{2} = r \cdot \theta$

$\frac{2\pi r}{4} = \frac{\pi r}{2}$

$\theta = \pi/2$ radian.

$90^\circ = \pi/2$ radian.

π radian $= 180^\circ$.

1 radian $= \frac{180^\circ}{\pi}$

Grade (G)

$$\boxed{G = \frac{100D}{90}}$$ → degree

$D =$ Degree

$$\boxed{\frac{D}{90} = \frac{G}{100} = \frac{2R}{\pi}}$$

$\pi = \frac{22}{7} = 3.14$

$\boxed{3 = \text{radian}}$

$G = \frac{2 \times 100}{3.14} \times 3$

$= \frac{200 \times 3}{3.14}$

$= 191$ Grade

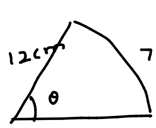
$D = \frac{2 \times 90 \times 3}{\pi}$

$D = \frac{180 \times 3}{\pi} = \frac{180 \times 3}{3.14} = \underline{171.97^\circ}$

The circular wire of radius 7 *cm* is cut and bend again into an arc of a circle of radius 12 *cm*. The angle subtended by an arc at the centre of the circle is **[Kerala (Engg.) 2002]**

(a) 50° ~~(b)~~ 210° (c) 100° (d) 60°

$r = 12$ cm



$7\text{cm} = l$

$l = r\theta$

$\theta = \frac{7}{12}$ (radian)

$\text{degree} = \frac{7}{12} \times \frac{180}{\pi}$

The degree measure corresponding to the given radian $\left[\frac{2\pi}{15}\right]^c$

(a) 21° (b) 22° (c) 23° ✓(d) 24°

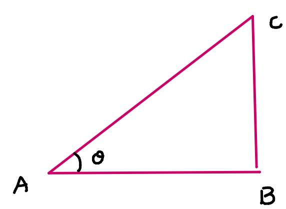
The minute hand of a clock is 10 *cm* long. How far does the tip of the hand move in 20 *minutes*

(a) $\frac{10\pi}{3}$ ✓(b) $\frac{20\pi}{3}$ (c) $\frac{30\pi}{3}$ (d) $\frac{40\pi}{3}$

The angle subtended at the centre of radius 3 *metres* by the arc of length 1 *metre* is equal to **[UPSEAT 1973]**

(a) 20° (b) 60° ✓(c) 1/3 radian (d) 3 radian

Trigonometric ratio



$\sin\theta = \frac{BC}{AC}$ $\cos\theta = \frac{AB}{AC}$

$\tan\theta = \frac{BC}{AB}$

$\frac{1}{\sin\theta} = \text{cosec}\,\theta = \frac{AC}{BC}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{AC}{AB}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{AB}{BC}$

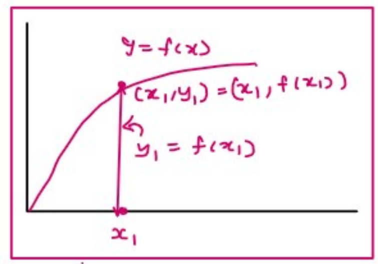
Values of some standard angle

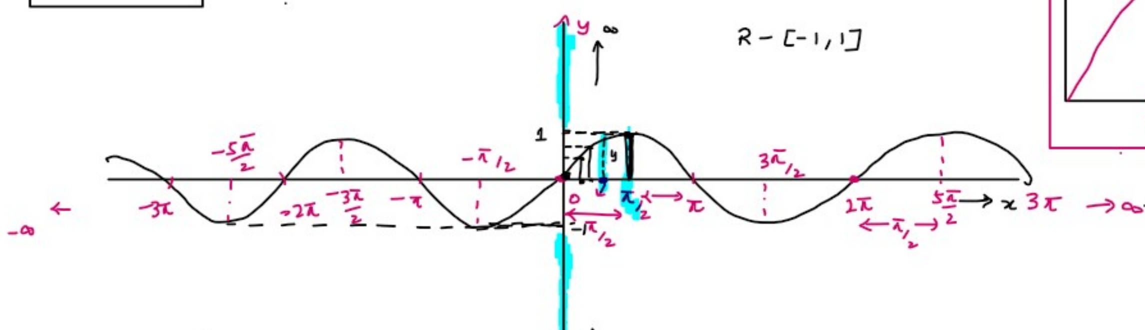
Angle	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Angle	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Graph of Trigonometric function.

$\boxed{y = \sin x}$



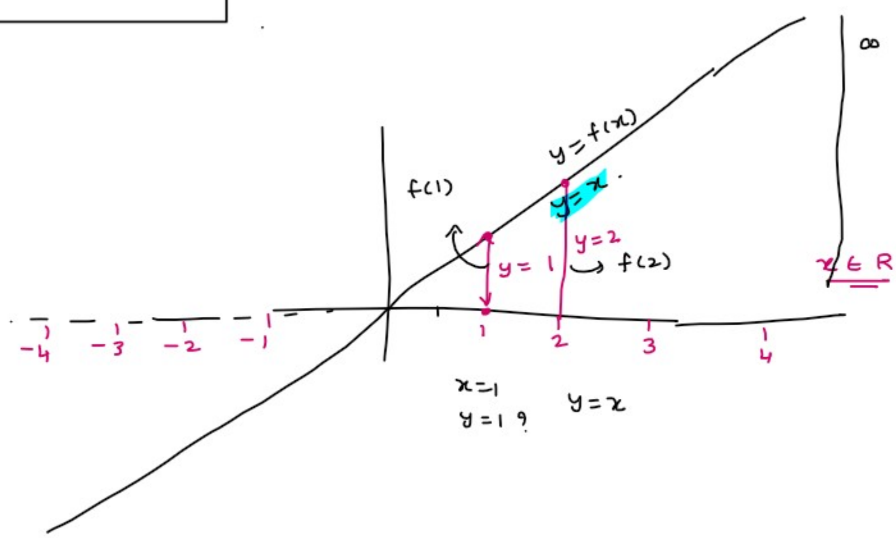


R – [–1, 1]

domain: $x \in R$

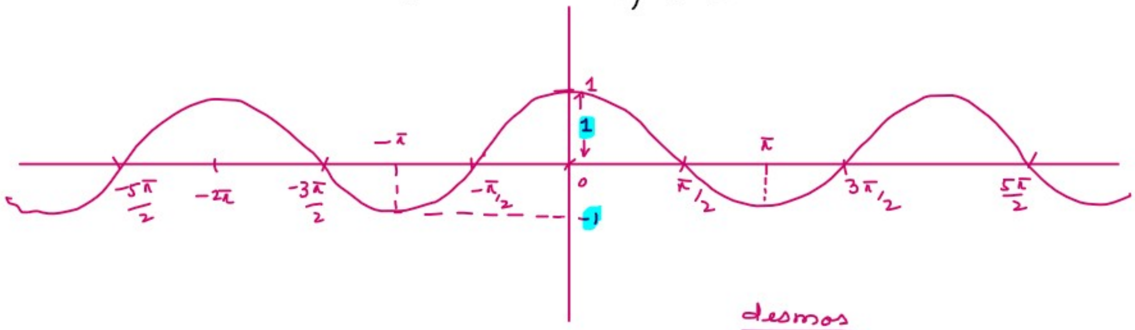
Range: $y \in [-1, 1]$

$-1 \le y \le 1$



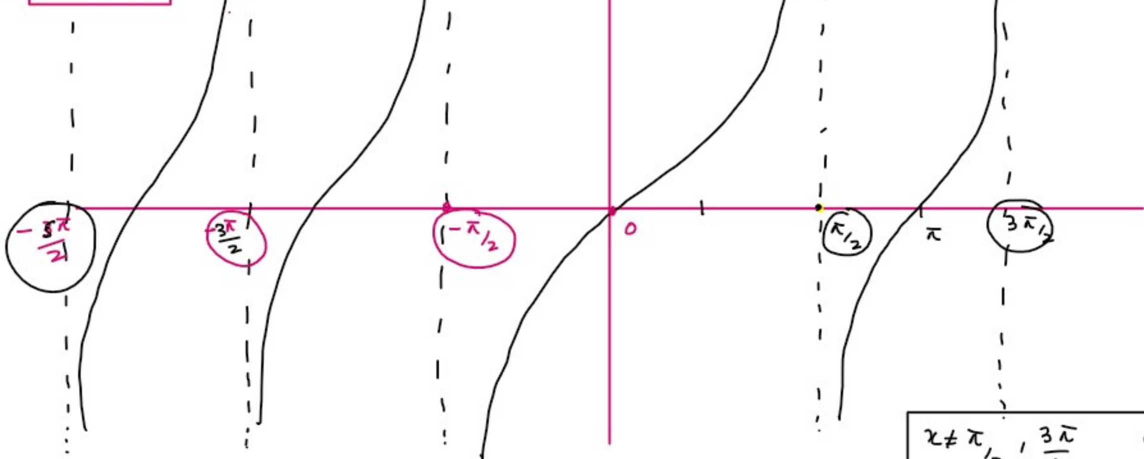
$y = \cos x$ domain: $x \in R$

Range: $y \in [-1, 1], -1 \leq y \leq 1$



desmos

3) $\boxed{y = \tan x}$



$\boxed{x \neq \pi/2,\ \frac{3\pi}{2} \qquad y \to \infty}$

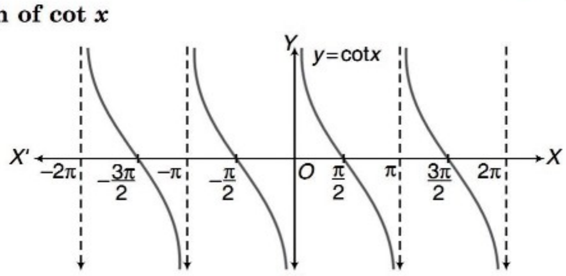
H.W. domain: x: ? Range y: ?

$\boxed{x: R - (2n+1)\pi/2 \quad n = 0, 1, 2, 3, \ldots, -1, -2, -3, \ldots}$

$(2n+1)\pi/2 = \pi/2$, $n = 1$, $\frac{3\pi}{2}$

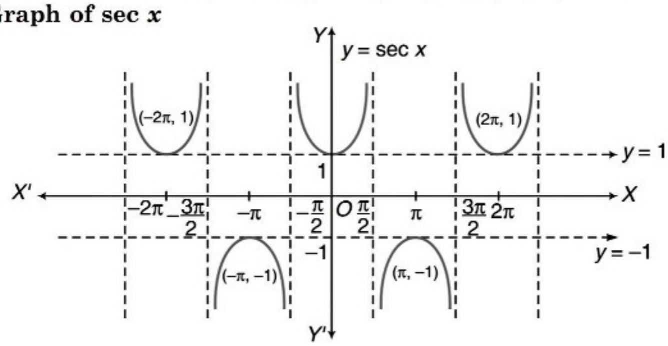
$n = 0$ $\pi/2$ $\quad n = -1$, $-\pi/2$

Graph of cot x

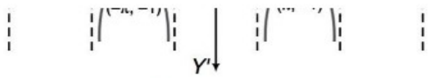


(i) Domain = $R \sim n\pi, n \in I$ (ii) Range = $(-\infty, \infty)$ (iii) Period = π

Graph of sec x



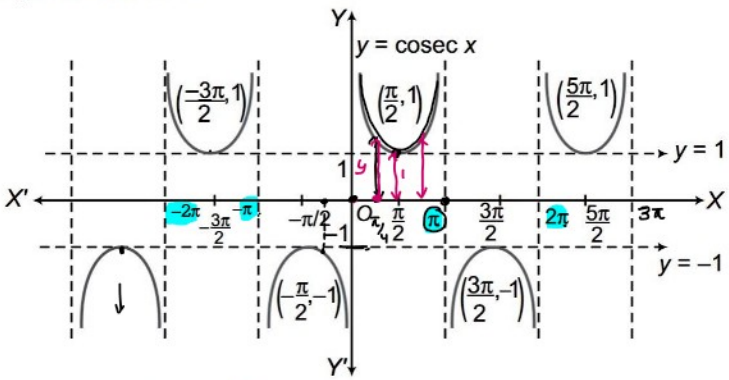
(i) Domain = $R - (2n+1)\frac{\pi}{2}, n \in I$



(i) Domain $= R \sim (2n+1)\dfrac{\pi}{2}, n \in I$

(ii) Range $= (-\infty, -1] \cup [1, \infty)$

. **Graph of cosec x**



$\text{cosec}\, x = \dfrac{1}{\sin x}$

$x = 0 \Rightarrow \dfrac{1}{0} = \infty$

$x = \pi/2 \Rightarrow \dfrac{1}{\sin \pi/2} = \dfrac{1}{1}$

$x = \pi, \quad \dfrac{1}{\sin \pi} = \dfrac{1}{0} = \infty$

(i) Domain $= R \sim n\pi, n \in I$

(ii) Range $= (-\infty, -1] \cup [1, \infty)$ (y)

Quadrant System. (already done)

$$\boxed{\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta}$$

1) $\boxed{\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B}$

2) $\sin(A-B) = \sin A \cdot \cos(-B) + \cos A \cdot \sin(-B)$

$A \to A$, $B \to -B$

$= \sin A \cdot \cos B - \cos A \cdot \sin B$

$A = 90 - B, \; B = B$

$\sin(90°) = \sin(90-B)\cos B + \cos(90-B) \cdot \sin B$

$\boxed{1 = \cos^2 B + \sin^2 B}$

$\boxed{\sin^2\theta + \cos^2\theta = 1}$

$\dfrac{1}{\cos^2\theta}$

$\tan^2\theta + 1 = \sec^2\theta$

$\boxed{\sec^2\theta - \tan^2\theta = 1}$

$\dfrac{1}{\sin^2\theta}$

$1 + \cot^2\theta = \text{cosec}^2\theta$

$\boxed{\text{cosec}^2\theta - \cot^2\theta = 1}$

3) $\cos(A+B) =$

$A = \pi/2 - A$, $B = -B$ or $A = -A$, $B = \pi/2 - B$

$\sin(\pi/2 - A + (-B)) = \sin(\pi/2 - A) \cdot \cos(-B) + \cos(\pi/2 - A) \cdot \sin(-B)$

$\sin(\pi/2 - (A+B)) = \cos A \cdot \cos B - \sin A \cdot \sin B$

$\boxed{\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B}$

4) $\cos(A-B)$ $\quad A = \pi/2 - A, \; B = B.$

$\sin(\pi/2 - A + B) = \sin(\pi/2 - A)\cos B + \cos(\pi/2 - A) \cdot \sin B$

$\sin(\pi/2 - (A-B)) = \cos A \cdot \cos B + \sin A \cdot \sin B$

$\boxed{\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \cos B}$

$\Rightarrow$ { MtG Publication
47yrs : Main + Advance

Mocks

test

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \Rightarrow$$

$$= \frac{(\sin A\cdot\cos B + \cos A\cdot\sin B)/\cos A\cdot\cos B}{(\cos A\cdot\cos B - \sin A\cdot\sin B)/\cos A\cdot\cos B} =$$

$$\boxed{\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A\cdot\tan B}}$$

$$\boxed{\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\cdot\tan B}}$$

$$\boxed{\cot(A+B) = \frac{\cot A\cdot\cot B - 1}{\cot A + \cot B}}$$

$$\boxed{\cot(A-B) = \frac{\cot A\cdot\cot B + 1}{\cot B - \cot A}}$$

$A=B$

$$\sin(A+B) = \sin A\cdot\cos B + \cos A\cdot\sin B$$

$$\boxed{\sin 2A = 2\sin A\cdot\cos A = \frac{2\tan A}{1+\tan^2 A}}$$

$\Downarrow$

$$= \frac{2\sin A\cdot\cos A}{\cos A}\cdot\cos A$$

$$= 2\tan A\cdot\cos^2 A = \frac{2\tan A}{\sec^2 A} = \frac{2\tan A}{1+\tan^2 A}$$

$$\sin(A+B) = \sin A\cdot\cos B + \cos A\cdot\sin B$$

$B = A.$

$$\sin(2A) = \sin A\cdot\cos A + \cos A\cdot\sin A$$

$$\boxed{\sin 2A = 2\sin A\cdot\cos A}$$

$$\cos(A+B) = \cos A\cdot\cos B - \sin A\cdot\sin B$$

$$\boxed{\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1-2\sin^2 A = \frac{1-\tan^2 A}{1+\tan^2 A}}$$

$$= \cos^2 A - (1-\cos^2 A) = 2\cos^2 A - 1$$

$$= (1-\sin^2 A) - \sin^2 A = 1 - 2\sin^2 A$$

$$\cos 2A = \frac{\cos^2 A - \sin^2 A}{1}$$

$$= \frac{\frac{1}{\cos^2 A}(\cos^2 A - \sin^2 A)}{\frac{1}{\cos^2 A}}$$

$$= \frac{1-\tan^2 A}{\sec^2 A} = \frac{1-\tan^2 A}{1+\tan^2 A}$$

H.W

I.T.O = In terms of

$$\tan 2A = \text{I.T.O}(\tan A) = \frac{2\tan A}{1-\tan^2 A}$$

H.W

$$\sin 3A = \text{ITO}(\sin A) = 3\sin A - 4\sin^3 A$$

$$\cos 3A = \text{ITO}(\cos A) = 4\cos^3 A - 3\cos A$$

$$\tan 3A = \text{ITO}(\tan A) = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$$

$$\sin(3A) = \sin(2A+A)$$

$$= \sin 2A\cdot\cos A + \cos 2A\cdot\sin A.$$

$\cos^2 A - \sin^2 A$

$(1-\sin^2 A - \sin^2 A)$

$$= (2\sin A\cdot\cos A)\cos A + (1-2\sin^2 A)\sin A$$

$$= 2\sin A(\cos^2 A) + \sin A - 2\sin^3 A$$

$$= 2\sin A(1-\sin^2 A) + \sin A - 2\sin^3 A$$

$$= 2\sin A - 2\sin^3 A + \sin A - 2\sin^3 A$$

$$\boxed{\sin 3A = 3\sin A - 4\sin^3 A}$$

formulas

$$\sin(A+B) = \sin A\cdot\cos B + \cos A\cdot\sin B.$$

$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$.

$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

$\sin 3A = 3\sin A - 4\sin^3 A$

Q.) If $\sin\theta + \operatorname{cosec}\theta = 2$, then $\sin^2\theta + \operatorname{cosec}^2\theta =$ **[UPSEAT 2002; MP PET 1992; MNR 1990]**

(a) 1 (b) 4 (c) 2 (d) None of these

Q.) If $\sin\theta + \cos\theta = m$ and $\sec\theta + \operatorname{cosec}\theta = n$, then $n(m+1)(m-1)$ equal to **[MP PET 1986]**

(a) m (b) n (c) $2m$ (d) $2n$

If $\tan A + \cot A = 4$, then $\tan^4 A + \cot^4 A$ is equal to **[Kerala (Engg.) 2002]**

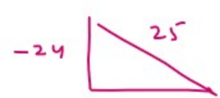
(a) 110 (b) 191 (c) 80 (d) 194

If $\sin x + \cos x = \frac{1}{5}$, then $\tan 2x$ is **[UPSEAT 2003]**

(a) $\frac{25}{17}$ (b) $\frac{7}{25}$ (c) $\frac{25}{7}$ (d) $\frac{24}{7}$

Sol:

$\sin 2A = -\frac{24}{25}$



If $\frac{2\sin\alpha}{\{1+\cos\alpha+\sin\alpha\}} = y$, then $\frac{\{1-\cos\alpha+\sin\alpha\}}{1+\sin\alpha} =$ **[BIT Ranchi 1996]**

(a) $\frac{1}{y}$ (b) y (c) $1-y$ (d) $1+y$

Sol: $\dfrac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} = y$

$\dfrac{2\times 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}} = y$

$\dfrac{\overset{2}{\cancel{4}}\sin\frac{\alpha}{2}\times\cancel{\cos\frac{\alpha}{2}}}{\cancel{2\cos\frac{\alpha}{2}}\left(\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}\right)} = y = \dfrac{2\sin\frac{\alpha}{2}}{\left(\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}\right)}$

$$\boxed{\begin{aligned}&\sin 2A = 2\sin A\cdot\cos A\\ &\sin\alpha = 2\sin\tfrac{\alpha}{2}\cos\tfrac{\alpha}{2}\end{aligned}}$$

$\dfrac{1+\sin\alpha-\cos\alpha}{1+\sin\alpha} = \dfrac{1+\sin\alpha}{1+\sin\alpha} - \dfrac{\cos\alpha}{1+\sin\alpha}$

$= 1 - \left(\dfrac{\cos\alpha}{1+\sin\alpha}\right)$

$= 1 - \dfrac{\cos\alpha}{\left(\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\right)^2}$

$= 1 - \dfrac{\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}}{\left(\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\right)^2}$

$= 1 - \dfrac{\left(\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}\right)\cancel{\left(\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}\right)}}{\left(\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\right)^{\cancel{2}}}$

$\Leftarrow$

$\dfrac{\cancel{\cos\frac{\alpha}{2}}+\sin\frac{\alpha}{2}-\cancel{\cos\frac{\alpha}{2}}+\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}} = 1 - \dfrac{\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}}$

$= \dfrac{2\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}} = y$

H.W

If $\sin\theta+\sin^2\theta+\sin^3\theta=1$, then $\cos^6\theta-4\cos^4\theta+8\cos^2\theta=$

(a) 4 (b) 2 (c) 1 (d) None of these

Sol: $\sin\theta+\sin^3\theta = 1-\sin^2\theta$

$\sin\theta(1+\sin^2\theta) = \cos^2\theta$

$\sin\theta(1+1-\cos^2\theta) = \cos^2\theta$

$\left(\sin\theta(2-\cos^2\theta)\right)^2 = \left(\cos^2\theta\right)^2$

$\sin^2\theta(4+\cos^4\theta-4\cos^2\theta) = \cos^4\theta$

$(1-\cos^2\theta)(4+\cos^4\theta-4\cos^2\theta) = \cos^4\theta$

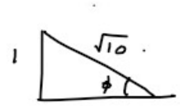
$4+\cancel{\cos^4\theta}-4\cos^2\theta-4\cos^2\theta-\cos^6\theta+4\cos^4\theta = \cancel{\cos^4\theta}$

$\cos^6\theta-4\cos^4\theta+8\cos^2\theta-4=0$

If θ and ϕ are angles in the 1st quadrant such that $\tan\theta = 1/7$ and $\sin\phi = 1/\sqrt{10}$. Then

(a) $\theta+2\phi=90^o$ (b) $\theta+2\phi=60^o$ (c) $\theta+2\phi=30^o$ (d) $\theta+2\phi=45^o$

Sol: $\tan(\theta+2\phi) = \dfrac{\tan\theta+\tan2\phi}{1-\tan\theta\cdot\tan2\phi}$



$\tan2\phi = \dfrac{2\tan\phi}{1-\tan^2\phi}$

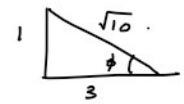
$= 2\times 1/3$

Sol:

$$\tan(\theta+2\phi) = \frac{\tan\theta + \tan 2\phi}{1-\tan\theta \cdot \tan 2\phi}$$

$$= \frac{1/7 + 3/4}{1-\frac{3}{28}} = \frac{25}{25} = 1$$

$\underline{\theta + 2\phi = 45^\circ}$



$$\tan 2\phi = \frac{2\tan\phi}{1-\tan^2\phi} = \frac{2\times 1/3}{1-1/9} = \frac{6}{8} = 3/4$$

1) $\boxed{\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)}$

2) $\boxed{\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)}$

3) $\boxed{\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cdot\cos\left(\frac{C-D}{2}\right)}$

4) $\cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)$

$= -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

Proof: $A+B = C$ — (1)

$A-B = D$ — (2)

$A = \frac{C+D}{2}$, $B = \frac{C-D}{2}$

$\sin(A+B) + \sin(A-B)$

$\sin A\cdot\cos B + \cancel{\cos A\cdot\sin B} + \sin A\cdot\cos B - \cancel{\cos A\cdot\sin B}$

$= 2\sin A\cdot\cos B = 2\sin\left(\frac{C+D}{2}\right)\cos\frac{C-D}{2}$

1) $\boxed{\sin\theta\cdot\underbrace{\sin(60-\theta)\cdot\sin(60+\theta)} = \frac{1}{4}\sin 3\theta}$

Prove $\sin\theta\{\sin^2 60 - \sin^2\theta\}$

$\sin\theta\left\{\frac{3}{4} - \sin^2\theta\right\}$

$\frac{3\sin\theta}{4} - \sin^3\theta = \frac{3\sin\theta - 4\sin^3\theta}{4}$

$= \frac{\sin 3\theta}{4}$

= Proved

R.T.P

Prove: $\boxed{\sin(A+B)\cdot\sin(A-B) = \sin^2 A - \sin^2 B}$

Prove:

$(\sin A\cdot\cos B + \cos A\cdot\sin B)(\sin A\cdot\cos B - \cos A\cdot\sin B)$

$= \sin^2 A\cdot\cos^2 B - \cos^2 A\cdot\sin^2 B$

$= \sin^2 A(1-\sin^2 B) - (1-\sin^2 A)\sin^2 B$

$= \sin^2 A - \sin^2 B - \cancel{\sin^2 A\cdot\sin^2 B} + \cancel{\sin^2 A\cdot\sin^2 B}$

$= \underline{\sin^2 A - \sin^2 B}$

Q.) H.W H.W $\boxed{\cos\theta\cdot\cos(60-\theta)\cdot\cos(60+\theta) = \frac{\cos 3\theta}{4}}$

Q.) $\boxed{\tan\theta\cdot\underbrace{\tan(60-\theta)\cdot\tan(60+\theta)} = \tan 3\theta}$

$\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta} = \tan 3\theta$

H.W $\boxed{\cos(A+B)\cdot\cos(A-B) = (\cos^2 A + \cos^2 B - 1) = \cos^2 A - \sin^2 B}$

H.W $\boxed{\tan(A+B)\cdot\tan(A-B) = \frac{\tan^2 A - \tan^2 B}{1-\tan^2 A\cdot\tan^2 B}}$

$-\frac{3}{4}\cos\theta + \cos^3\theta$

$\frac{4\cos^3\theta - 3\cos\theta}{4} = \frac{\cos 3\theta}{4}$

The value of θ lying between 0 and $\pi/2$ and satisfying the equation $\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$ $R_1: R_1 - R_2$, $\to R_2$, $\to R_3$

[IIT 1988; MNR 1992; Kurukshetra CEE 1998; DCE 1996]

(a) $\frac{7\pi}{24}$ or $\frac{11\pi}{24}$ (b) $\frac{5\pi}{24}$ (c) $\frac{\pi}{24}$ (d) None of these

Sol:

$$\begin{array}{ccc} + & - & + \end{array}$$

$$\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = x_1(y_2 z_3 - z_3 y_2) - y_1(x_2 z_3 - x_3 z_2) + z_1(x_2 y_3 - x_3 y_2)$$

$R_1: R_1 - R_2$

$$\begin{vmatrix} 1 & -1 & 0 \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} \Leftarrow \begin{vmatrix} (1+\cancel{\sin^2\theta}) - \cancel{\sin^2\theta} & \cancel{\cos^2\theta} - (1+\cancel{\cos^2\theta}) & 4\cancel{\sin 4\theta} - 4\cancel{\sin 4\theta} \\ \_ & \_ & \_ \\ \_ & \_ & \_ \end{vmatrix}$$

$R_2: R_2 - R_3$

$\Downarrow$

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

$$1(1+4\sin 4\theta + \cos^2\theta) - (-1)(0(1+4\sin 4\theta) - (-\sin^2\theta)) = 0$$

$$1+4\sin 4\theta + \cos^2\theta + \sin^2\theta = 0$$

$$4\sin 4\theta + 2 = 0$$

$$\sin 4\theta = -\tfrac{1}{2} \qquad \sin(\pi + \tfrac{\pi}{6}) = -\sin(\tfrac{\pi}{6}) = -\tfrac{1}{2}$$

$$\sin(4\theta) = \sin\left(\frac{7\pi}{6}\right), \ \sin\left(\frac{11\pi}{6}\right)$$

$$\frac{11\pi}{6} = 2\pi - \tfrac{\pi}{6} =$$

$$\sin\left(4\times\tfrac{\pi}{2} - \tfrac{\pi}{6}\right) = -\sin\tfrac{\pi}{6} = -\tfrac{1}{2}$$

$$4\theta = \frac{7\pi}{6}, \ \frac{11\pi}{6}$$

$$\theta = \frac{7\pi}{24}, \ \frac{11\pi}{24}.$$

If for all real values of x, $\frac{4x^2+1}{64x^2 - 96x.\sin\alpha + 5} < \frac{1}{32}$, then α lies in the interval [Roorkee 1998]

(a) $\left(0, \frac{\pi}{3}\right)$ ~~(b)~~ $\left(\frac{\pi}{3}, \frac{2\pi}{3}\right)$ (c) $\left(\frac{2\pi}{3}, \pi\right)$ ~~(d)~~ $\left(\frac{4\pi}{3}, \frac{5\pi}{3}\right)$

Sol:

$$\frac{4x^2+1}{64x^2 - 96x\cdot\sin\alpha + 5} - \frac{1}{32} < 0$$

$$\frac{4\pi}{3} = 240$$

$$\sin(240)$$

$$\frac{32(4x^2+1) - (64x^2 - 96x\cdot\sin\alpha + 5)}{32(64x^2 - 96x\sin\alpha + 5)} < 0$$

$\sin(270-30)$

$-\cos 30 = -\sqrt{3}/2$

$$\frac{(128x^2 + 32 - 64x^2 + 96x\cdot\sin\alpha - 5)(64x^2 - 96x\sin\alpha + 5)}{(32)(64x^2 - 96x\sin\alpha + 5)^2} < 0$$

$$\underbrace{(64x^2 + 96x\sin\alpha + 27)}\,\underbrace{(64x^2 - 96x\sin\alpha + 5)} < 0$$

$\frac{16\times 6}{96}$

$(96\sin\alpha)^2 - 4\times 27\times 64 \geq 0.$

$\boxed{64x^2 - 96x\sin\alpha + 5}$

$96\times 96.\sin^2\alpha - 4\times 64\times 27 \geq 0.$

$$= \frac{96\sin\alpha \pm \sqrt{(96\sin\alpha)^2 - 4\times 64\times 5}}{2\times 64}$$

$16(96\times 6\sin^2\alpha - 4\times 4\times 27) \geq 0$

$16\times 16(6^2\sin^2\alpha - 27) \geq 0.$

$\boxed{(96\sin\alpha)^2 - 4\times 64\times 5 \geq 0.}$

$36\sin^2\alpha - 27 \geq 0.$

$\sin^2\alpha \geq \frac{\cancel{27}\ \cancel{9}\ 3}{\cancel{36}\ \cancel{12}\ 4}$

$\sin^2\alpha - \frac{3}{4} \geq 0.$

$\left(\sin\alpha - \frac{\sqrt{3}}{2}\right)\left(\sin\alpha + \sqrt{3}/2\right) \geq 0.$

$\sin\alpha = 60.$

$(96\sin\alpha)^2 - 4\times 64\times 5 \geq 0.$

$6\times 16\times 6\times 16\sin^2\alpha - 4\times 4\times 16\times 5$

$16^2(36\sin^2\alpha - 5) \geq 0.$

$\sin^2\alpha$

$\sin\alpha = \pm\frac{\sqrt{5}}{6}$

$\alpha = \sin^{-1}(\sqrt{5}/6)$

Q.) $\cot(45+\theta).\cot(45-\theta) =$

a) -1 b) 0 c) 1 d) ∞

Q.) if $\cos\theta = \frac{1}{2}\left(a + \frac{1}{a}\right)$ then $\cos 3\theta = ?$ $4\cos^3\theta - 3\cos\theta$

Q.) if $\cos\theta = \frac{1}{2}\left(a + \frac{1}{a}\right)$ then $\cos 3\theta = ?$ $4\cos^3\theta - 3\cos\theta$

a) $\frac{1}{8}\left(a^3 + \frac{1}{a^3}\right)$ b) $\frac{3}{2}\left(a + \frac{1}{a}\right)$ ✓c) $\frac{1}{2}\left(a^3 + \frac{1}{a^3}\right)$ d) $\frac{1}{3}\left(a^3 + \frac{1}{a^3}\right)$

H.W Q.) $\tan 20^\circ \cdot \tan 40^\circ \cdot \tan 60^\circ \cdot \tan 80^\circ =$ (IIT 1974)

a) 1 b) 2 c) 3 d) $\frac{\sqrt{3}}{2}$