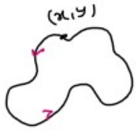
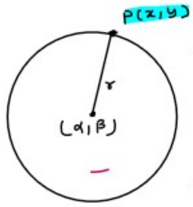
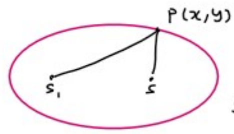
Sets ,relations and functions

Monday, May 5, 2025 5:04 PM







$SP + S_1P = 2a$

locus: movement of a point under certain conditions.

Problem-solving
M-1

$$\sqrt{(x-\alpha)^2 + (y-\beta)^2} = r$$

$$\boxed{(x-\alpha)^2 + (y-\beta)^2 = r^2}$$

M-2 $\quad P\left(\frac{t}{1+t^2}, \frac{2t}{5}\right)$

$\quad\quad\quad\quad \| \quad\quad \|$
$\quad\quad\quad\quad x \quad\quad y$

$$\frac{t}{1+t^2} = x \qquad \frac{2t}{5} = y$$

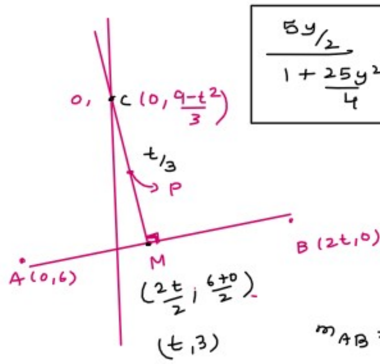
$$t = \frac{5y}{2}$$

$$\boxed{\frac{5y/2}{1+\frac{25y^2}{4}} = x}$$

Let A be a fixed point $(0, 6)$ and B be a moving point $(2t, 0)$. Let M be the mid-point of AB and the perpendicular bisector of AB meets the Y-axis at C.

The locus of the mid-point P of MC is **[2021, 27 Aug. Shift-I]**

(a) $3x^2 - 2y - 6 = 0$
(b) $3x^2 + 2y - 6 = 0$
✓(c) $2x^2 + 3y - 9 = 0$
(d) $2x^2 - 3y + 9 = 0$



$$M\left(\frac{2t}{2}, \frac{6+0}{2}\right)$$

$$(t, 3)$$

$$m_{AB} = \frac{0-6}{2t-0} = -\frac{6}{2t} = -\frac{3}{t}$$

$$m_{MC} = \frac{-1}{m_{AB}} = \frac{+1}{+3/t} = t/3$$

MC

$$y - 3 = t/3\,(x - t)$$

$$3y - 9 = tx - t^2 \quad \text{— (1)}$$

y-axis, $x = 0$, $y = \frac{9-t^2}{3}$

$$\frac{\frac{9-t^2}{3} + 3}{2}$$

$$P\left(t/2, \frac{18-t^2}{6}\right)$$

$\quad \| \quad\quad \|$
$\quad x \quad\quad y$

$$t/2 = x \Rightarrow t = 2x$$

$$\frac{18-t^2}{6} = y \Rightarrow \frac{18-(2x)^2}{3 \times 2} = y \Rightarrow 18 - 4x^2 = 6y$$

$$3y = 9 - 2x^2$$

$$2x^2 + 3y - 9 = 0$$

Sets.

collection of elements

Set = {1,2,3,4,5}

N = Natural no.
W = Whole no.
Z = integers
R = Real no's
Z^+ = +ve integers
Z^- = −ve integers
$Q^{+/-}$ = rational no.

Representation

a) Roaster form
b) Set builder form.

a) Roaster form

A = vowels in english

$\boxed{A = \{a, e, i, o, u\}}$

b) Set builder form.

$A = \{ \underset{\substack{\uparrow \\ \text{Such that}}}{x :}\ x$ is a vowel in english alphabet $\}$

NCERT :

Types of Set

1) Empty set / null set / void set : $\{\phi\}$

2) Singleton set : only one element

eg: $\{1\}$, $\{2\}$

3) finite set : has finite no. of element. eg: $\{a, e, i, o, u\}$

4) infinite set : has infinite no. of elements

eg: R

5) equivalent set : two sets having same no. of elements.

eg: $A = \{1, 2, 3, 4, 5\}$ $n(A) = 5$

$B = \{a, e, i, o, u\}$ $n(B) = 5$

6) equal sets : Sets having same no. of elements, also every element of A is also element of B

$A = \{1, 2, 3, 4, 5\}$ $B = \{1, 2, 3, 4, 5\}$

7) Subset & superset :

$A = \{1, 2, 3, 4, 5\}$ $B = \{1, 2, 3, 4, 5, 6\}$

Subset → $A \subset B$ ← Superset

Subset

8) Power Set : Set formed by all the possible subsets of

$A = \{1, 2, 3, 4\}$

$P(A) = \{\{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}\ \{2, 3\}, \{3, 4\} \ldots\ldots\}$

9) Universal set : set containing all possible elements under consideration.

$A = \{1, 2, 3, 4\}$ $B = \{3, 4, 5, 6\}$

$U = \{1, 2, 3, 4, 5, 6\}$

10) proper Subset : if $A \subset B$ & $A \neq B$

A is proper subset of B

Comparable sets / subset : $A \subseteq B$, $B \subseteq A$

11) disjoint set : when two sets have nothing in common.

$A = \{1, 2, 3, 4\}$ $B = \{5, 6, 7, 8\}$

$A \cap B = \phi$

$\cap$ → common elements

12) intersection of Sets :

$A = \{1, 2, 3, 4, 5\}$ $\quad\quad$ $B = \{3, 4, 5, 6\}$

$A \cap B = \{3, 4, 5\}$

13) Union of Sets: all the elements of A & B without repetition

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

intervals

a) closed : $x \in [a, b]$ $\quad$ $a \leq x \leq b$ $\quad$ eg: $1 \leq x \leq 2$

b) open : $x \in (a, b)$ $\quad$ $a < x < b$ $\quad$ eg $1 < x < 2$

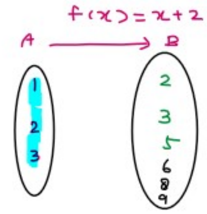
c) semi-closed / semi-open : $x \in [a, b)$ $\quad$ $a \leq x < b$ $\quad$ eg $1 \leq x < 2$

or

$x \in (a, b]$ $\quad$ $a < x \leq b$ $\quad$ eg: $1 < x \leq 2$

inequalities, domain & codomain & Ranges.

$f(x) = x + 2$



domain: all input values $= \{1, 2, 3\}$

Ranges: all output values. $= \{2, 3, 5\}$

Co-domain: Set of values including output values $= \{2, 3, 5, 6, 8, 9\}$

$A = \{1, 2, 3\}$

$f(1) = 1 + 2 = 3$

$f(2) = 2 + 2 = 4$

$f(3) = 2 + 3 = 5$

$f(x) = x + 2$

$B = \{3, 4, 5, 6, 8, 9\}$

Rules for finding domain & Ranges.

① $\frac{f(x)}{g(x)}$, $g(x) \neq 0$ $\quad$ $\frac{x+2}{x-1}$

$x - 1 \neq 0$

$x \neq 1$ $\quad\quad$ $x \in R - \{1\}$

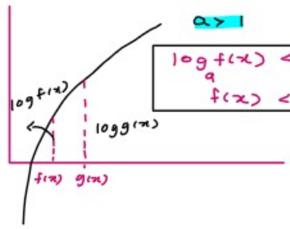
2) $\sqrt{f(x)}$ $\quad$ $f(x) \geqslant 0$ $\quad$ eg: $\sqrt{x-2}$, $x - 2 \geqslant 0$.

$x \geqslant 2$

3) $\log_{g(x)} f(x)$: $f(x) > 0$

$g(x) > 0$

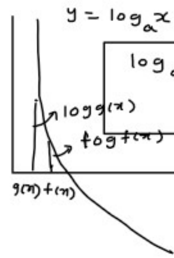
$y = \log_a x$ $\quad$ $a > 1$



$\log_a f(x) < \log_a (g(x))$

$f(x) < g(x)$

$y = \log_a x$ $\quad$ $0 < a < 1$



$\log_a f(x) < \log g(x)$ $\quad$ $f(x) > g(x)$

4) $ax^2 + bx + c = 0$ $\quad$ eg: $x \cdot t^2 + 2x \cdot t + 5 = 0$.

$D \geqslant 0$ $\quad\quad$ $(2x)^2 - 4x \cdot 5 \geqslant 0$

$x_1, x_2 = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\downarrow$

real $\boxed{b^2-4ac \geq 0}$

$4x^2 - 4x5 \geq 0$

$\boxed{4x(x-5) \geq 0}$

<u>wavy curve method.</u>

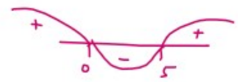
eg: $(x-\alpha)^{p\,\text{even}} (x-\beta)^{q \to \text{odd}} (x-\gamma)^{s \to \text{even}}$ ≥ 0, ≤ 0, > 0, < 0

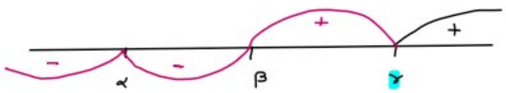
$x(x-5)^1 \geq 0$

$(x-0)^1$

$\boxed{x \in (-\infty, 0] \cup [5, \infty)}$

<u>Step-1</u> put α, β, γ on the number line $\alpha < \beta < \gamma$





<u>Step-2</u> Start wave from right most with the sign.

S = even (wave remain same) } same for

S = odd (wave changes sign) } p, q

<u>Step-3</u>

$\geq 0 \to$ take the wave regions. $x \in [\beta, \infty)$
$\searrow$ include numbers.

$> 0 \to$ take the wave region $x \in (\beta, \infty)$
$\searrow$ exclude numbers

$\leq 0 \to$ take −ve wave region $x \in (-\infty, \beta]$
$\searrow$ include no's

$< 0 \to$ take −ve wave region $x \in (-\infty, \beta)$
$\searrow$ exclude nos

5) $\dfrac{x-1}{x+2} \geq 1$ $\qquad x-1 \geq (x+2)$ ⊗

$\boxed{\dfrac{x-1}{x+2} - 1 \geq 0}$ ✓

$\dfrac{\cancel{x}-1-\cancel{x}-2}{x+2} \geq 0 \Rightarrow \boxed{\dfrac{-3}{x+2} \geq 0 \Rightarrow \dfrac{3}{x+2} \leq 0}$

$\Downarrow$

$\boxed{\dfrac{3(x+2)}{(x+2)^2} \leq 0} \Rightarrow x+2 \leq 0$

$\underline{x \leq -2}$

The domain of the definition of the function

$f(x) = \dfrac{1}{4-x^2} + \log_{10}(x^3 - x)$ is

(a) $(-1, 0) \cup (1, 2) \cup (3, \infty)$ (b) $(-2, -1) \cup (-1, 0) \cup (2, \infty)$

✓(c) $(-1, 0) \cup (1, 2) \cup (2, \infty)$ (d) $(1, 2) \cup (2, \infty)$

Q.) Solve for domain x : a) $\log_{1/5}\left(\dfrac{4x+6}{x}\right) \geq 0$ $\qquad \boxed{x \in [-2, -3/2)}$

b) $\log_{(2x+3)} x^2 < \log_{(2x+3)} (2x+3)$

<u>Case-1</u> $2x+3 > 1$ $\qquad$ <u>Case-2</u> $0 < 2x+3 < 1$

$x^2 < 2x+3$ —(1) $x^2 > 2x+3$ —(2)

Ans: $\boxed{x \in (-3/2, -1) \cup (-1, 3)}$

Functions	Curve	Domain and Range
sine	$y = \sin x$	Domain $= R$, Range $= [-1, 1]$
cosine	$y = \cos x$	Domain $= R$, Range $= [-1, 1]$
tangent	$y = \tan x$	Domain $= R - \left\{(2n+1)\frac{\pi}{2} \mid n \in Z\right\}$, Range $= R$
cosecant	$y = \operatorname{cosec} x$	Domain $= R - \{n\pi \mid n \in Z\}$ Range $= \{-\infty, -1] \cup [1, \infty)$
secant	$y = \sec x$	Domain $= R - \left\{(2n+1)\frac{\pi}{2} \mid n \in Z\right\}$, Range $= (-\infty, -1] \cup [1, \infty)$
cotangent	$y = \cot x$	Domain $= R - \{n\pi \mid n \in Z\}$, Range $= R$
Arc sine	$y = \sin^{-1} x$	Domain $= [-1, 1]$, Range $= \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
Arc cosine	$y = \cos^{-1} x$	Domain $= [-1, 1]$, Range $= [0, \pi]$
Arc tangent	$y = \tan^{-1} x$	Domain $= R$, Range $= \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
Arc cosecant	$y = \operatorname{cosec}^{-1} x$	Domain $= (-\infty, 1] \cup [1, \infty)$, Range $= \left(\frac{\pi}{2}, \frac{\pi}{2}\right) - \{0\}$
Arc secant	$y = \sec^{-1} x$	Domain $= (-\infty, -1] \cup [1, \infty)$, Range $= [0, \pi] - \left\{\frac{\pi}{2}\right\}$
Arc cotangent	$y = \cot^{-1} x$	Domain $= R$, Range $= (0, \pi)$

H.W. Domain of definition of the function

$f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$ for real valued x, is

(a) $\left[-\frac{1}{4}, \frac{1}{2}\right]$ (b) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(c) $\left(-\frac{1}{2}, \frac{1}{9}\right)$ (d) $\left[-\frac{1}{4}, \frac{1}{4}\right]$

$\sin^{-1} 2x + \pi/6 \geq 0$ —(1)

$-\pi/2 \leq \sin^{-1} 2x \leq \pi/2$ —(2) $\Rightarrow -1 \leq 2x \leq 1$

$-1/2 \leq x \leq 1/2$ —(2)

$\sin^{-1} 2x \geq -\pi/6$

$2x \geq \sin(-\pi/6)$ $x \in [-1/4, 1/2]$

$2x \geq -1/2$

$x \geq -1/4$ —(1)

The domain of definition of $f(x) = \dfrac{\log_2(x+3)}{x^2+3x+2}$ is

(a) $R / \{-1, -2\}$
(b) $(-2, \infty)$
(c) $R / \{-1, -2, -3\}$
(d) $(-3, \infty) / \{-1, -2\}$

Sol: $x^2+3x+2 \neq 0$ —(1)

$x+3 > 0$ —(2)

$\boxed{x > -3}$

$x^2+2x+x+2 \neq 0$

$x(x+2)+1(x+2) \neq 0$

$(x+1)(x+2) \neq 0$

$\boxed{x \neq -1, \; x \neq -2}$

Method to find Range

$y = ?$

find range of $y = \dfrac{x+1}{x-2}$ $\Rightarrow$ $x+1 = yx - 2y$

$x(1-y) = -1-2y$

$x - 2 \neq 0$ $x \in R - \{-2\}$ $\boxed{x = \dfrac{1+2y}{y-1}}$ $y \neq 1$

$x \neq -2$ $y \in R - \{1\}$

Range of the function $f(x) = \dfrac{x^2+x+2}{x^2+x+1}$; $x \in R$ is

(a) $(1, \infty)$ (b) $(1, 11/7)$
(c) $(1, 7/3]$ (d) $(1, 7/5)$

$y = \dfrac{x^2+x+2}{x^2+x+1}$

$yx^2 + yx + y = x^2 + x + 2$

$\underbrace{(y-1)}_{a} x^2 + \underbrace{(y-1)}_{b} x + \underbrace{y-2}_{c} = 0$

(c) (1, 7/3] (d) (1, 7/3)

$$\underbrace{(y-1)}_{a}x^2+\underbrace{(y-1)}_{b}x+\underbrace{y-2}_{c}=0$$

$ax^2+bx+c=0$

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$(y-1)^2 - 4(y-1)(y-2) \geq 0$.

$(y-1)\left((y-1)-4(y-2)\right) \geq 0$.

$(y-1)\{y-1-4y+8\} \geq 0$.

$(y-1)\{7-3y\} \geq 0$.

$(y-1)(3y-7) \leq 0$.

+ − +

1 $\quad 7/3$

$y \in [1, 7/3]$

The domain of the function $f(x)=\sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$ is $(-\infty, -a] \cup [a, \infty]$. Then a is equal to :

[Sep. 02, 2020 (I)]

(a) $\frac{\sqrt{17}}{2}$ (b) $\frac{\sqrt{17}-1}{2}$ (c) $\frac{1+\sqrt{17}}{2}$ (d) $\frac{\sqrt{17}}{2}+1$

$-1 \leq \frac{|x|+5}{x^2+1} \leq 1$

$\Leftarrow |x| \leq x^2-4$. — (2)

$\boxed{|x| \geq -x^2-6 \quad — (1)}$ ✗

$x \geq 0$, $\quad x<0$.

$x^2-x-4 \geq 0$.

$x^2-4+x \geq 0$.

$x \geq 0$.

$x^2+x+6 \geq 0$.

$(-\infty, \cancel{\frac{1-\sqrt{17}}{2}})$ <0

$\cup \left(\frac{1+\sqrt{17}}{2}, \infty\right)$ ✓

$x = \frac{1 \pm \sqrt{1+16}}{2}$

$\boxed{x = \frac{1 \pm \sqrt{17}}{2}}$

$x^2+x-4 \geq 0$.

$x = \frac{-1 \pm \sqrt{1+16}}{2} = \frac{-1-\sqrt{17}}{2}$

$x = \frac{-1 \pm \sqrt{1-24}}{2}$ ✗

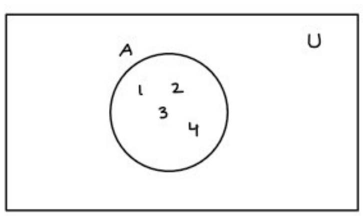
✓ $\left(-\infty, -\frac{1-\sqrt{17}}{2}\right)$

Venn diagrams.

Representation.

U = Universal set.

$A = \{1,2,3,4\}$



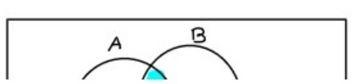
(1) Union of sets ($A \cup B$) : $A = \{1,2,3,4\}$ $\quad B = \{3,4,5,6\}$

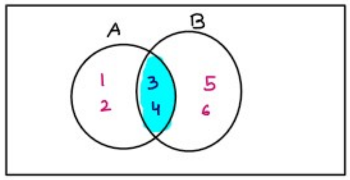
$\cup \downarrow$ Union

$A \cup B = \{1,2,3,4,5,6\}$

$A \cap B = \{3,4\}$

$\cap \downarrow$ intersection : common

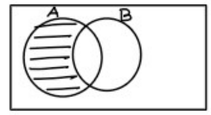




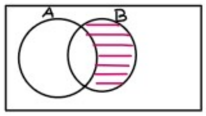
$A \cap B = \{3,4\}$
$\downarrow$
intersection : common

difference
a) $A - B = \{1,2\}$
b) $B - A = \{5,6\}$

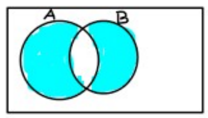
$A-B$



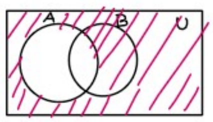
$B-A$



Symmetric difference : $(A-B) \cup (B-A) = \{1,2,5,6\}$



Complement of sets : A^c = complement of A

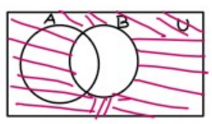


$U = R$ $A = \{1,2,3,4\}$

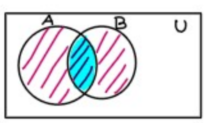
$A^c = R - \{1,2,3,4\}$

$B = \{3,4,5,6\}$

$B^c = R - \{3,4,5,6\}$



$$\boxed{n(A \cup B) = \underline{n(A)} + \underline{n(B)} - n(A \cap B)}$$



$$\boxed{\begin{aligned} n(A-B) &= n(A) - n(A \cap B) \\ n(B-A) &= n(B) - n(A \cap B) \end{aligned}}$$

$$\boxed{(A \cap B)^c = A^c \cup B^c}$$

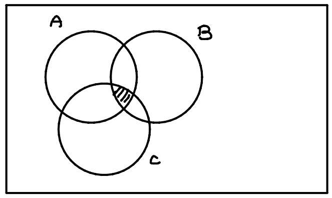
$$(A \cap B)^c = \overline{A \cap B} = \bar{A} \cap \bar{B} = A^c \cup B^c$$

$$\boxed{(A \cup B)^c = A^c \cap B^c}$$

$$n(A^c \cup B^c) = n(A \cap B)^c = n(U) - n(A \cap B)$$

$$n(A^c \cap B^c) = n(A \cup B)^c = n(U) - n(A \cup B)$$

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$



Sets A and B have 3 and 6 elements respectively. What can be the minimum number of elements in A

[MNR 1987; Karnataka CET 1996]

(a) 3 (b) 6 (c) 9 (d) 18

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

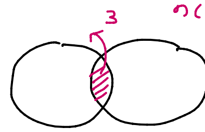
$$n(A \cup B)\big|_{min.} = 3 + 6 - n(A \cap B)\big|_{max}$$

$$= 6 + \cancel{3} - \cancel{3}$$

$$n(A \cup B)\big|_{min.} = 6$$

3

$n(A \cap B)\big|_{max} = 3$



In a town of 10000 families, it was found that 40% families buy newspaper A, 20% families buy newspaper B, 10 % families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4 % buy a and C. If 2% families buy all the three newspaper. Find

(i) the number of familiar which buy newspaper A only. = 3300

(ii) the number of familiar which buy none of A , B and C. = 4000

H.W

Two newspapers A and B are published in a city. It is known that 25% of the city population reads A and 20% reads B while 8% reads both A and B. Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then, the percentage of the population who look into advertisements is

(a) 13.5 (b) 13

(c) 12.8 (d) 13.9

Population = 100

$n(A) = 25$

$n(B) = 20$

$n(A \cap B) = 8$

$n(A \cap B^c)$ = Person who read A but not B.

$n(B \cap A^c)$ = " " " B but not A

H.W In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then, the number of students who did not opt for any of the three courses is

[2019, 10 Jan. Shift-I]

(a) 42 (b) 102 (c) 38 (d) 1

Functions : connect domain with ranges.

Types

a) inverse fn. : $\sin^{-1}x, \cos^{-1}x$

b) Logarithmic fn. : $\log x$,

c) Algebraic fn. : ax^2+bx+c, ax^3+bx^2+cx+d, etc.

d) Trigonometric fn : $\sin x, \cos x, \tan x$.

e) exponential fn : e^x, a^x

Types

a) One-one fn. (injective fn.)

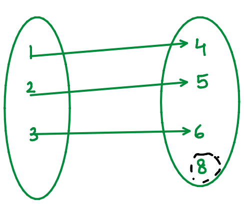
b) Many-one fn.

a) on-to function (surjective fn.)

b) in-to function.

c) one-one & on-to function.

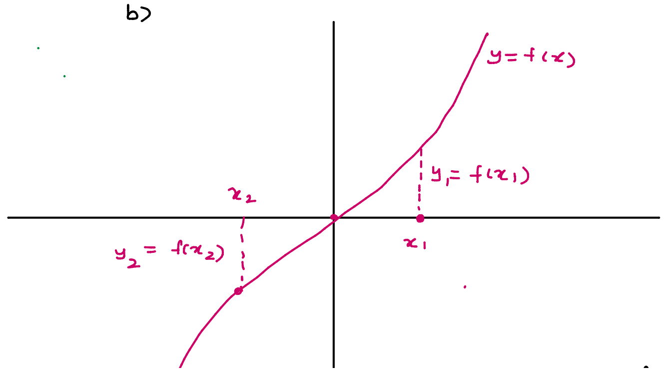
one-one fn.



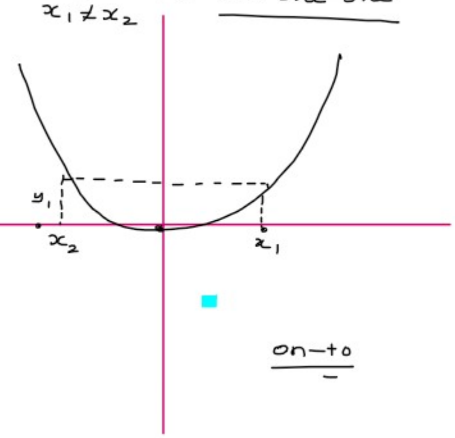
a)

$$\boxed{\begin{array}{ll} x_1 = x_2, & f(x_1) = f(x_2) \\ x_1 \neq x_2, & f(x_1) \neq f(x_2) \end{array}}$$

b)



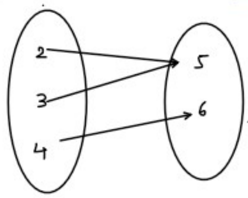
$f(x_1) = f(x_2)$ not one-one
$x_1 \neq x_2$



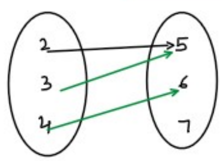
on-to

Many-one fn.

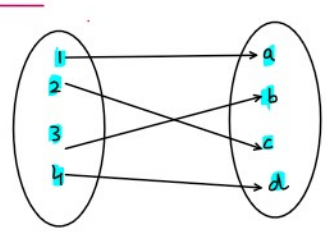
on-to fn. : condition : $\boxed{\text{Range} = \text{codomain.}}$



in-to fn: $\boxed{\text{Range} \subset \text{codomain}}$



one-one & on-to



equivalence, class

$(a \to a)$ ← a) Reflexive : $\{(a,a)\ (b,b), (c,c)\}$

$\left.\begin{matrix} a \to b, \\ b \to a \end{matrix}\right\}$ b) Symmetric : $\{(a,b), (b,a)\ (c,d), (d,c)\}$

c) transitive : $\{(a,b), (b,c), (a,c)\}$

$\left\{\begin{matrix} a \to b,\ b \to c \\ \Downarrow \\ a \to c \end{matrix}\right.$

Functions:

1) constant fn: $\boxed{y = f(x) = c}$ (constant)

$\boxed{y = c}$

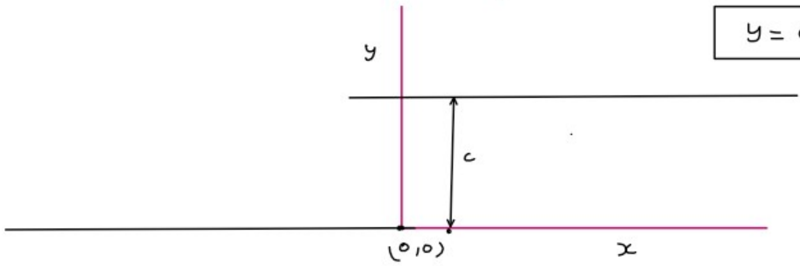
$x = 2$

$\boxed{y = f(x) = c}$ (constant)

$\boxed{y = c}$

$x = 2$
$y = 9$

$y = c$



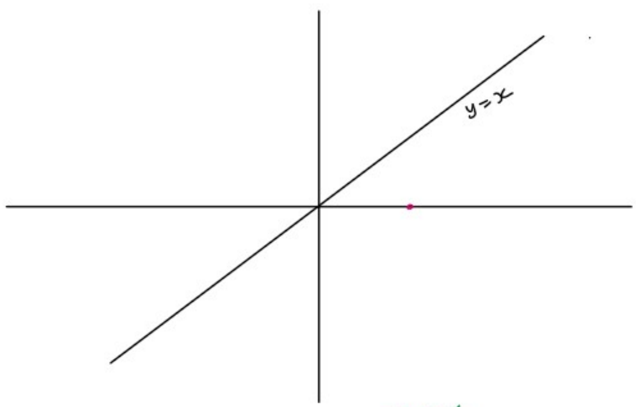
domain: $x \in R$
Range: $y = \{c\}$

2) identity function.

$y = f(x) = x$

$\boxed{y = x}$

domain: $x \in R$
Range: $y \in R$



$y = mx$

3) linear fn.

$\boxed{f(x) = y = ax + b}$

a, b = constant. $a > 0, b > 0$

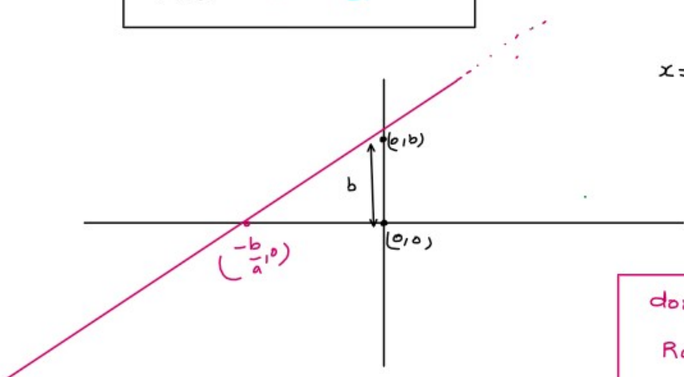
$x = 0, y = b$

$ax + b - y = 0$

$ax - y = -b$

$\frac{ax}{-b} - \frac{y}{-b} = 1$

$\frac{x}{\left(-b/a\right)} + \frac{y}{b} = 1$



domain: $x \in R$
Range: $y \in R$

4) quadratic fn.

$\boxed{(x-\alpha)^2 = 4a_1 \cdot (y - \beta)}$ (α, β) center.

$\boxed{y = ax^2 + bx + c}$

$= a\left(x^2 + \frac{b}{a} \cdot x\right) + c$

$\boxed{a > 0, b > 0, c > 0}$

$$= a\left(x^2 + 2\cdot\frac{b}{2a}\cdot x + \left(\frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right) + c$$

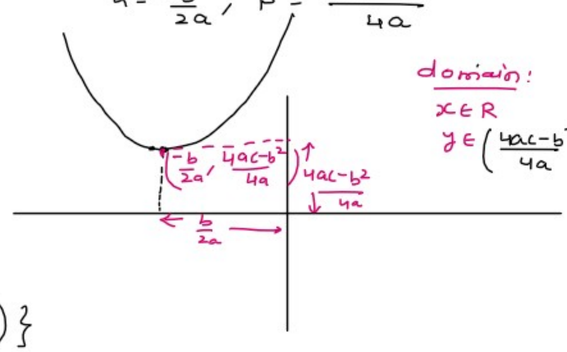
$$= a\left(\left(x+\frac{b}{2a}\right)^2 - \frac{b^2}{4a}\right) + c$$

$$y = a\left(x+\frac{b}{2a}\right)^2 + \frac{4ac-b^2}{4a}$$

$$a\left(x+\frac{b}{2a}\right)^2 = \left\{y - \left(\frac{4ac-b^2}{4a}\right)\right\}$$

$$\left\{x - \left(-\frac{b}{2a}\right)\right\}^2 = \frac{1}{a}\left\{y - \left(\frac{4ac-b^2}{4a}\right)\right\}$$

$\alpha = -\frac{b}{2a}$, $\beta = \frac{4ac-b^2}{4a}$



domain:
$x \in R$
$y \in \left(\frac{4ac-b^2}{4a}, \infty\right)$

Power fn.

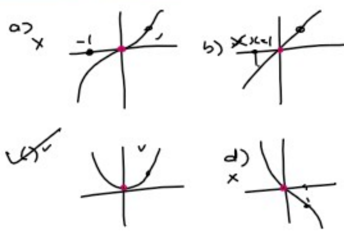
$y = x^n$

a) n = even

b) n = odd.

graph of $y = x^2$ $x=0, y=0$

$x=1, y=1$



$x=1,$

$x=-1, y=1$

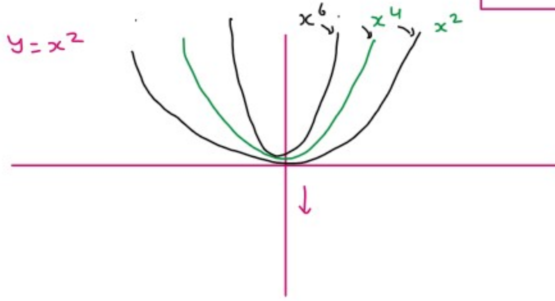
n = even

$\boxed{y = x^2}$, $x^4, x^6, \ldots\ldots$

domain: $x \in R$

Range: $y \in [0, \infty)$

$y = x^2$

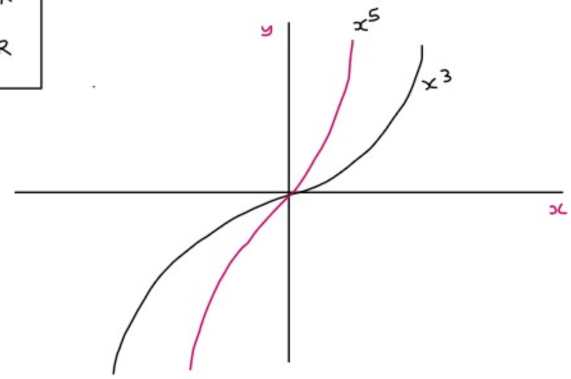


n = odd.

$y = x^3, x^5$ etc.

domain: $x \in R$

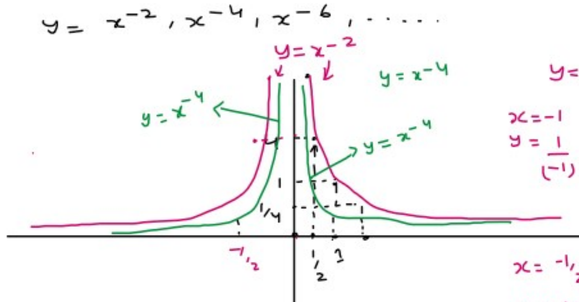
Range: $y \in R$



c) $y = x^{-2}, x^{-4}, x^{-6}, \ldots\ldots$

domain: $x \in R$

Range: $y \in (0, \infty)$



$y = x^{-2}$

$y = \frac{1}{x^2}$

$x = -1$

$y = \frac{1}{(-1)^2} = 1$

$y = \frac{1}{0} = \infty$

$y = \frac{1}{1^2} = 1$

$x = 1/2$

$x = -1/2$

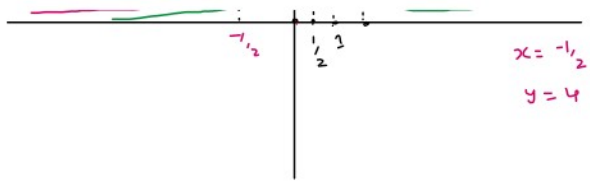
$y = 4$

$x = 2$

$y = \frac{1}{2^2} = \frac{1}{4}$

$x = \infty$

$y = \frac{1}{\infty^2} = \frac{1}{\infty} = 0$



$x = -1/2$
$y = 4$

$x = 1/2$

$y = \frac{1}{(1/2)^2} = 4$

$y = \frac{1}{\infty^2} = \frac{1}{\infty} = 0$

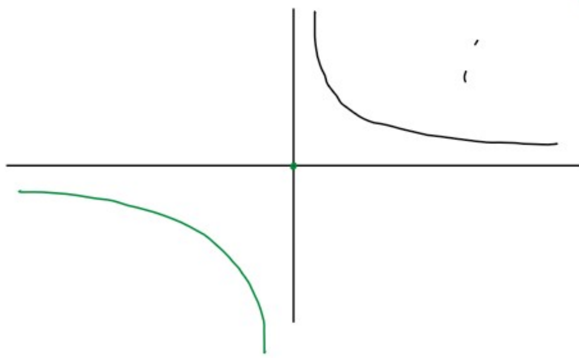
d)

$y = x^{-1}, x^{-3}, x^{-5}$

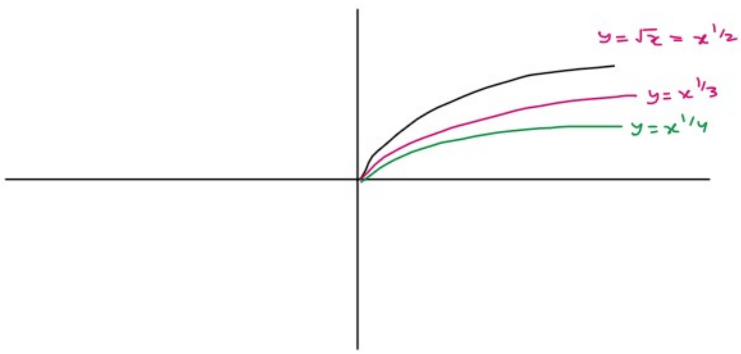
$y = x^{-1}$

domain: $x \in R - \{0\}$

Range: $y \in R - \{0\}$



d) $y = \sqrt{x}$

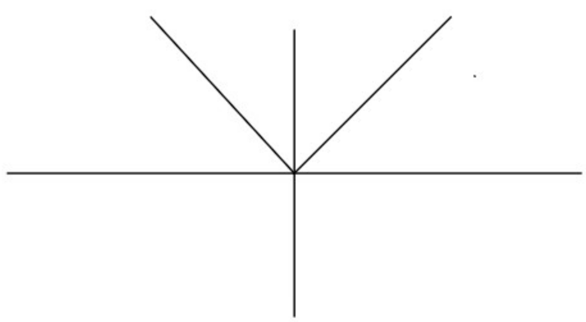


domain: $x \in [0, \infty)$

Range: $y \in [0, \infty)$

modulus fn.

$\boxed{y = |x|} \Rightarrow$ $y = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$



-1

$y = -x = -(-1) = 1$

-2

$y = -(-2) = 2$

domain: $x \in R$

Range: $y \in [0, \infty)$

Q.) if $|x-1| + |x| + |x+1| \geq 6$ then x lies in

a) $(-\infty, 2]$ b) $(-\infty, -2] \cup [2, \infty)$

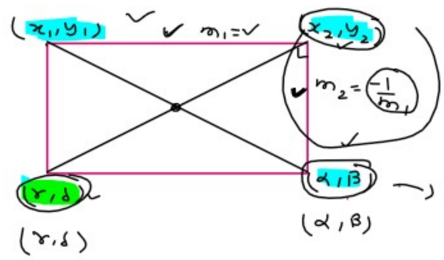
c) R D) ϕ

Signum function.

(x_1, y_1) (x_2, y_2)

Signum function.

$$\boxed{y = f(x) = \begin{cases} \frac{|x|}{x} & , \text{if } x \neq 0 \\ 0 & , \text{if } x = 0 \end{cases}}$$

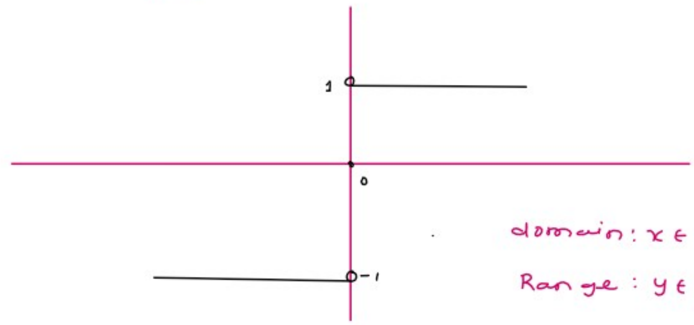


$$\frac{x_1+\alpha}{2} = \frac{\gamma + x_2}{2} \quad \text{— (3)}$$

$$\frac{y_1+\beta}{2} = \frac{\delta + y_2}{2} \quad \text{— (4)}$$

$$= \begin{cases} \frac{x}{x} = 1 & \text{if } x > 0 \\ \frac{-x}{x} = -1 & \text{if } x < 0 \\ = 0 & x = 0 \end{cases}$$

=



domain: $x \in R$.

Range: $y \in \{-1, 0, 1\}$

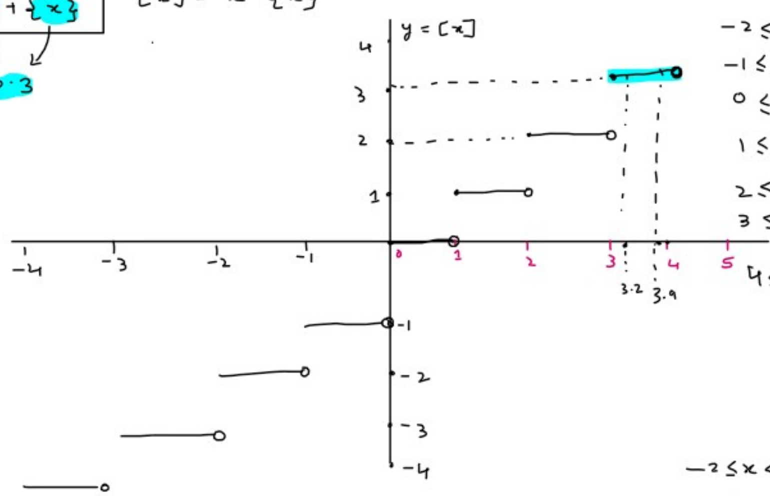
Greatest integer function.

(Step fn.)

$y = [x]$

$\boxed{x = [x] + \{x\}}$ $\quad [x] = x - \{x\}$

eg: $x = 2.3$
$= 2 + 0.3$



x	eg: $x=$	$[x]$
$-2 \le x < -1$		-2
$-1 \le x < 0$	-0.2	-1
$0 \le x < 1$	0.2	0
$1 \le x < 2$	1.2	1
$2 \le x < 3$	2.7	2
$3 \le x < 4$		3
$4 \le x < 5$		4

$-2 \le x < -1$	-2	$y = x - (-2) = x + 2$
$-1 \le x < 0$	-1	$y = x - (-1) = x + 1$

fractional Part fn. $\{x\}$

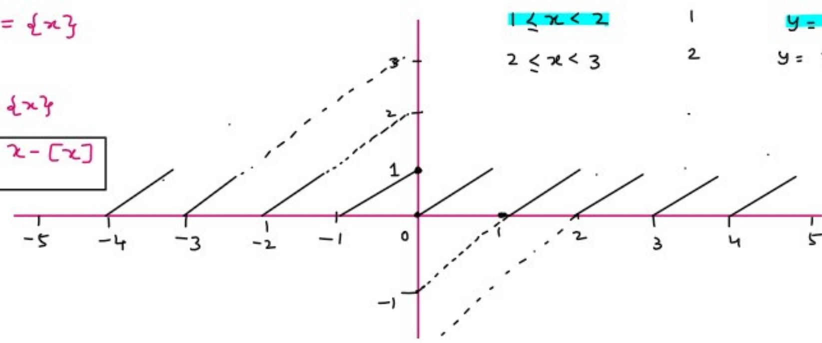
$y = f(x) = \{x\}$

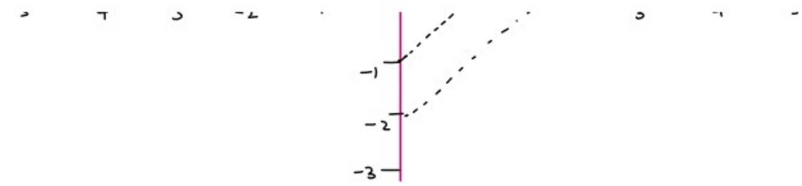
$x = [x] + \{x\}$

$y = \boxed{\{x\} = x - [x]}$

x	$[x]$	$y = \{x\} = x - [x]$
$0 \le x < 1$	0	$y = x - 0$
$1 \le x < 2$	1	$y = x - 1$
$2 \le x < 3$	2	$y = x - 2$

$y = mx + c$, $m = 1$, $C = -1$





Composit fn.

fog , gof

fog → domain: domain of $g(x)$
Range: Range of $f(x)$

$f(x) = \sin x$

$g(x) = x^2+2x+3$

gof → domain: domain of $f(x)$
Range: Range of $g(x)$

$fog = f(g(x)) = f(x^2+2x+3) = \sin(x^2+2x+3)$

$gof = g(f(x)) = g(\sin x) = (\sin x)^2 + 2(\sin x) + 3$

H.W

For $x \in \left(0, \frac{3}{2}\right)$, let $f(x) = \sqrt{x}$, $g(x) = \tan x$ and $h(x) = \frac{1-x^2}{1+x^2}$. If $\phi(x) = ((hof)og)(x)$, then $\phi\left(\frac{\pi}{3}\right)$ is equal to

(2019 Main, 12 April I)

(a) $\tan\frac{\pi}{12}$ (b) $\tan\frac{11\pi}{12}$

(c) $\tan\frac{7\pi}{12}$ (d) $\tan\frac{5\pi}{12}$