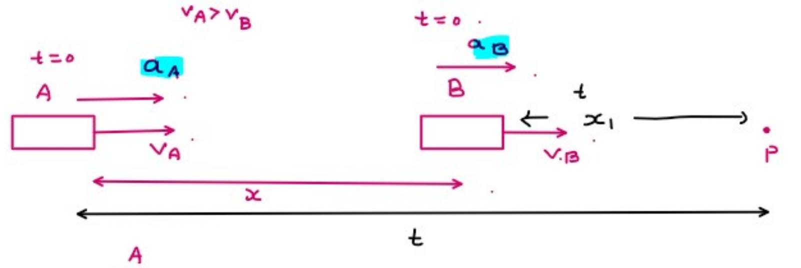
Relative motion



A

$$x + x_1 = v_A \cdot t + \frac{1}{2} a_A \cdot t^2 \quad — (1)$$

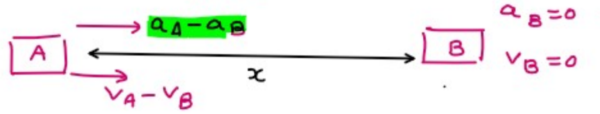
B

$$x_1 = v_B \cdot t + \frac{1}{2} a_B \cdot t^2 \quad — (2)$$

$$(1) - (2) = \quad x = (v_A - v_B) + \frac{1}{2}(a_A - a_B)\, t^2 \quad — (1)$$

motion of A relative to B / or motion of A w.r.t. B

Stopping 'B'



$$x = (v_A - v_B)\, t + \frac{1}{2}(a_A - a_B)\, t^2 \quad — (2)$$

motion of B Relative of A

−ve ⟷ +ve

$$-x = (v_B - v_A)\, t + \frac{1}{2}(a_B - a_A)\, t^2$$

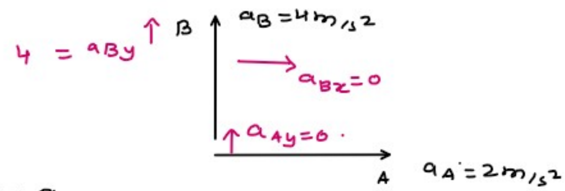
$$x = (v_A - v_B)\, t + \frac{1}{2}(a_A - a_B)\, t^2 \quad — (3)$$

Car A and car B start moving simultaneously in the same direction along the line joining them. Car A moves with a constant acceleration $a = 4\ m/s^2$, while car B moves with a constant velocity $v = 1\ m/s$. At time $t = 0$, car A is 10 m behind car B. Find the time when car A overtakes car B.

Car A has an acceleration of $2\ m/s^2$ due east and car B, $4\ m/s^2$ due north. What is the acceleration of car B with respect to car A?

$a_{BA} = 2\hat{i} + 4\hat{j}$

$a_{BA} = -2\hat{i} + 4\hat{j}$



$a_{Ax} = 2$

$a_{BA} = a_B - a_A$

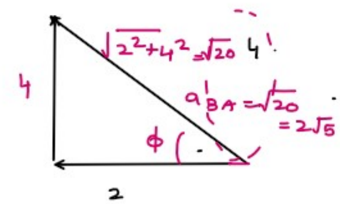
$= (0\hat{i} + 4\hat{j}) - (2\hat{i} + 0\hat{j})$

$\boxed{a_{BA} = -2\hat{i} + 4\hat{j}}$

$a_B = 0\hat{i} + 4\hat{j}$

$a_A = 2\hat{i} + 0\hat{j}$

D 6:10 5:00-6:00

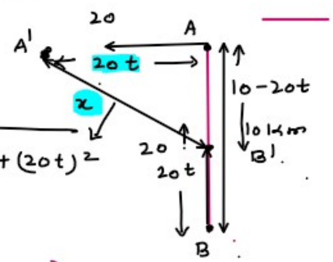


$\tan\phi = \frac{4}{2} = 2$

$\boxed{\phi = \tan^{-1}(2)}$

a_{BA} is making angle ϕ from west towards north.

Two ships A and B are 10 *km apart on a line running south to north. Ship A farther north is streaming west at* 20 *km/h and ship B is streaming north at* 20 *km/h. What is their distance of closest approach and how long do they take to reach it?*



$x = \sqrt{(10-20t)^2 + (20t)^2}$

$(10-20t)^2$

$= 2(10-20t)(-20)$

M-1 differentiation

$x = \sqrt{(10-20t)^2 + (20t)^2}$

$\frac{dx}{dt} = 0$

$\frac{1}{2\sqrt{(10-20t)^2 + (20t)^2}} \left\{ \cancel{2}(10-20t) \times (-\cancel{20}) + \cancel{2} \times \cancel{20}t \times 20 \right\} = 0$

$-10 + 20t + 20t = 0$

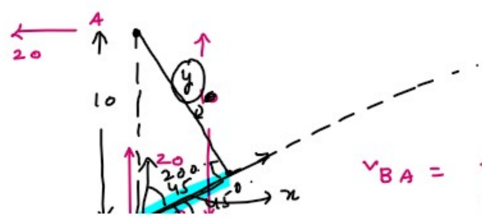
$40t = 10$

$t = 1/4$

$x = \sqrt{\left(10 - \frac{20}{4}\right)^2 + \left(\frac{20}{4}\right)^2}$

$= \sqrt{(10-5)^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$

M-2



$20\sqrt{2} \cdot t = x = \frac{10}{}$

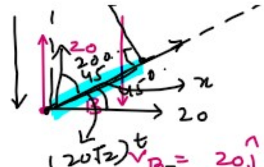
$v_{BA} = v_B - v_A$

$20\sqrt{2}\cdot t = x = \frac{10}{\sqrt{2}}$

$\frac{x}{10} = \cos 45^\circ$

$x = \frac{10}{\sqrt{2}}$

$\boxed{t = \frac{1}{4}}$



$\frac{y}{10} = \sin 45^\circ$

$y = \frac{10}{\sqrt{2}} = 5\sqrt{2}$

$v_{BA} = v_B - v_A$

$v_B = 20\hat{j} + 0\hat{i} = (0\hat{i} + 20\hat{j}) - (-20\hat{i} + 0\hat{j})$

$v_A = -20\hat{i} + 0\hat{j} = 20\hat{i} + 20\hat{j}$

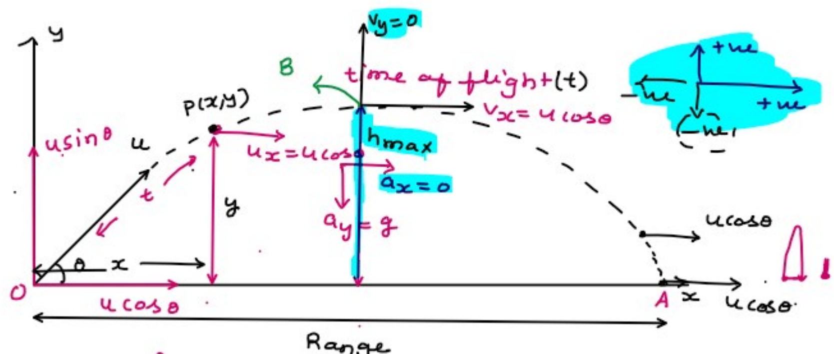
wed, Fri

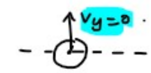
Projectile motion.

$a = 0$

$a =$ constant.

$$\boxed{\begin{aligned} S &= ut + \tfrac{1}{2}at^2 \\ v &= u + at \\ v^2 - u^2 &= 2as \end{aligned}}$$





max height

$v_y^2 - u_y^2 = 2a_y \cdot S_y$

$0^2 - (u\sin\theta)^2 = 2(-g)h_m$

$$\boxed{\frac{u^2\sin^2\theta}{2g} = h_{max}}$$

u

x-dir.

$x = ut + \frac{1}{2}at^2$

$v_x = u_x + \cancel{a_x}^{0}\cdot t$

$v_x = u_x$

$v_x = u_x = u\cos\theta$

$\text{Range} = u_x\cdot t + \frac{1}{2}(\cancel{a})^{0}t^2$

$$\boxed{\text{Range} = u_x\cdot t}$$

$= u\cos\theta\left(\frac{2u\sin\theta}{g}\right)$

$\text{Range} = \frac{u^2(2\sin\theta\cdot\cos\theta)}{g}$

$$\boxed{\text{Range} = \frac{u^2\sin 2\theta}{g}}$$

y-dir.

$0 = (u\sin\theta)t + \frac{1}{2}(-g)\cdot t^2$

$u\sin\theta\cdot\cancel{t} = \frac{1}{2}g t^{\cancel{2}}$

$\boxed{t = \frac{2u\sin\theta}{g}}$ (time of flight)

eqn. of Parabola

$y = u\sin\theta\cdot t - \frac{1}{2}gt^2$ — ①

$x = u\cos\theta\cdot t$ — ②

$t = \frac{x}{u\cos\theta}$

* $$\boxed{y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}}$$

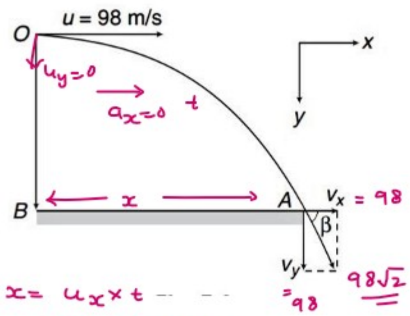
$$\boxed{y = x\tan\theta\left(1 - \frac{x}{R}\right)} \quad \boxed{R = \frac{u^2\sin 2\theta}{g}}$$

A projectile is fired horizontally with velocity of 98 m/s from the top of a hill 490 m high. Find

(a) the time taken by the projectile to reach the ground, (10 sec)

(b) the distance of the point where the particle hits the ground from foot of the hill and

(c) the velocity with which the projectile hits the ground. ($g = 9.8\ m/s^2$)



$x = u_x \times t$

$= 98 \times 10 = \underline{\underline{980}}$

H.W.

A projectile is projected at 30° from horizontal with initial velocity $40 ms^{-1}$. The velocity of the projectile at t = 2 s from the start will be : (Given $g = 10 m/s^2$)

[11-Apr-2023 shift 2]

Options:

A. Zero

B. $20\sqrt{3} ms^{-1}$

C. $40\sqrt{3} ms^{-1}$

D. $20 ms^{-1}$

y-dir

$v = u + at$

$v_y = u_y + (-g)\cdot t = 40 \sin 30^\circ - g(2) = 40 \times \frac{1}{2} - 20$

$v_y = 40 \sin 30^\circ - g(2) = 20 - 20 = 0$

$= 20 - 20 = 0$ $\boxed{v_y = 0}$

$v_y = 0$

$\sqrt{(20\sqrt{3})^2 + (0)^2} = 20\sqrt{3}$

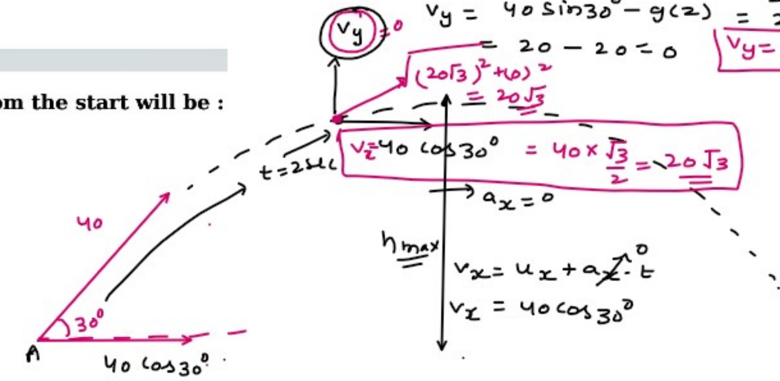
$\boxed{V_x = 40 \cos 30^\circ = 40 \times \frac{\sqrt{3}}{2} = 20\sqrt{3}}$

$a_x = 0$

h_{max}

$v_x = u_x + \cancel{a_x}^{\,0} \cdot t$

$v_x = 40 \cos 30^\circ$

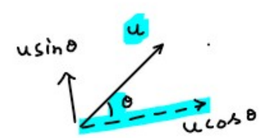


Projectile on an inclined plane

1) up the plane

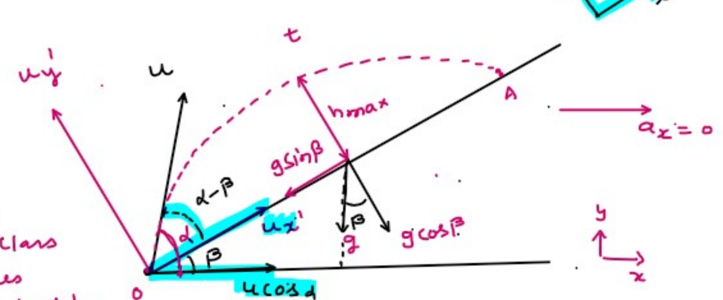
2) down the plane

a) up the plane



OA = Range

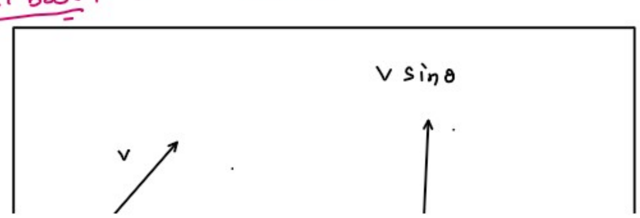
t = time of flight

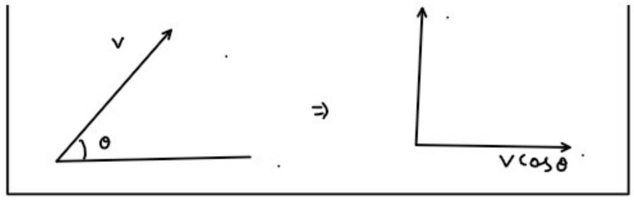


video > JEE main

> class recording / class notes

> JEE main 2027 batch



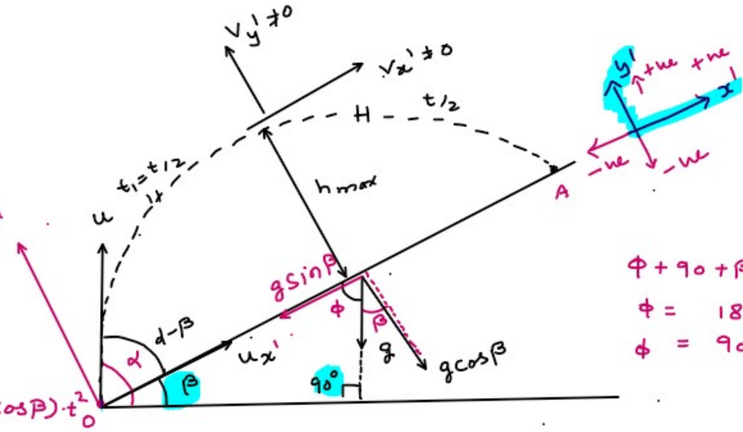


$u_{x'} = u\cos(\alpha-\beta)$

$u_{y'} = u\sin(\alpha-\beta)$

$a_{x'} = -g\sin\beta$

$a_{y'} = -g\cos\beta$



y' − dir.

$\underline{O-A}$

$S = ut + \frac{1}{2}at^2$

$y' = u_{y'}\cdot t + \frac{1}{2}a_{y'}\cdot t^2$

$0 = u\sin(\alpha-\beta)\cdot t + \frac{1}{2}(-g\cos\beta)\cdot t^2$

$$\boxed{t = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}}$$

$\phi + 90 + \beta = 180°$

$\phi = 180 - 90 - \beta$

$\phi = 90 - \beta$

Range: O − A

$OA = u_{x'}\cdot t + \frac{1}{2}a_{x'}\cdot t^2$

$= u\cos(\alpha-\beta)\cdot\frac{2u\sin(\alpha-\beta)}{g\cos\beta} + \frac{1}{\cancel{2}}(-\cancel{g}\sin\beta)\frac{\overset{2}{\cancel{4}}u^2\sin^2(\alpha-\beta)}{g^2\cos^2\beta}$

✡

$$\boxed{\begin{aligned}&\cos(A+B)\\&= \cos A\cdot\cos B - \sin A\cdot\sin B\end{aligned}}$$

$OA = \frac{2u^2}{g\cos\beta}\left\{\cos(\alpha-\beta)\sin(\alpha-\beta) + \frac{\sin^2(\alpha-\beta)(-\sin\beta)}{\cos\beta}\right\}$

$\frac{2u^2}{g\cos\beta}\cdot\sin(\alpha-\beta)\left\{\frac{\cos(\alpha-\beta)\times\cos\beta}{\cos\beta} - \frac{\sin(\alpha-\beta)\sin\beta}{\cos\beta}\right\} \Rightarrow \frac{2u^2\sin(\alpha-\beta)}{g\cos\beta}\left\{\frac{\cos(\overset{A}{\alpha-\beta})\cdot\cos(\overset{B}{\beta}) - \sin(\alpha-\beta)\cdot\sin\beta}{\cos\beta}\right\}$

$= \frac{2u^2\sin(\alpha-\beta)}{g\cos^2\beta}\left\{\cos(\alpha-\cancel{\beta}+\cancel{\beta})\right\}$

$$\boxed{OA = \frac{2u^2\sin(\alpha-\beta)\cos\alpha}{g\cos^2\beta}}$$

$\alpha - \beta = \frac{\pi}{4} - \frac{\beta}{2}$

$\underline{\underline{(OA)_{max}}}$

$$\boxed{\alpha = \pi/4 + \beta/2}$$

$$\boxed{OA_{max} = \frac{u^2}{g(1+\sin\beta)}}$$

down the plane

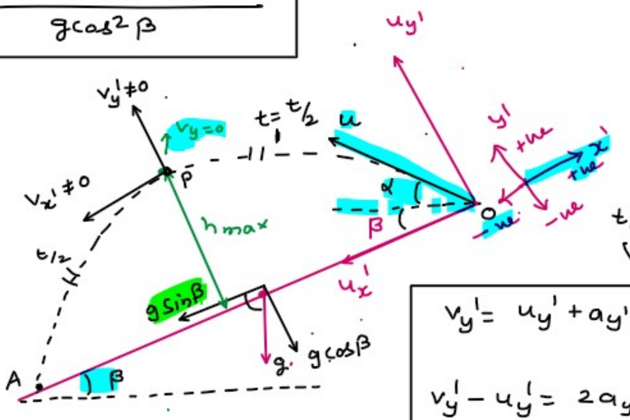
$t = \frac{2u\sin(\alpha+\beta)}{g\cos\beta}$

$u_{x'} = -u\cos(\alpha+\beta)$

$u_{y'} = u\sin(\alpha+\beta)$

$a_{x'} = -g\sin\beta$

$a_{y'} = -g\cos\beta$



$t_1 = t/2 = \frac{u\sin(\alpha+\beta)}{g\cos\beta}$

$$\boxed{\begin{aligned}&v_{y'} = u_{y'} + a_{y'}\cdot t_1\\&v_{y'}^2 - u_{y'}^2 = 2a_{y'}(h_{max})\end{aligned}}$$

y' dir.

$a_y = -g\cos\beta$

<u>y' dir.</u>

$y' = u_{y'}\cdot t + \frac{1}{2}a_{y'}\cdot t^2$

$0 = u\sin(\alpha+\beta)\cdot t + \frac{1}{2}(-g\cos\beta)\cdot t^2$

$OA = \dfrac{2u^2\sin(\alpha+\beta)\cos\alpha}{g\cos^2\beta}$

$u\sin(\alpha+\beta) = \dfrac{g\cos\beta\cdot t}{2}$

$$\boxed{t = \frac{2u\sin(\alpha+\beta)}{g\cos\beta}}$$

x'

$\cancel{-}OA = u_{x'}\cdot t + \frac{1}{2}a_{x'}\cdot t^2$

$\boxed{\cos(A-B) = \cos A\cdot\cos B + \sin A\cdot\sin B}$

$= \cancel{-}u\cos(\alpha+\beta)\cdot t + \frac{1}{2}(\cancel{-}g\sin\beta)\cdot t^2$

$$OA = u\cos(\alpha+\beta)\cdot\frac{2u\sin(\alpha+\beta)}{g\cos\beta} + \frac{\cancel{g}\sin\beta}{\cancel{2}}\cdot\frac{\overset{2}{\cancel{4}}u^2\sin^2(\alpha+\beta)}{g^{\cancel{2}}\cos^2\beta}$$

$$= \frac{2u^2}{g\cos\beta}\cdot\sin(\alpha+\beta)\left\{\cos(\alpha+\beta) + \frac{\sin\beta\cdot\sin(\alpha+\beta)}{\cos\beta}\right\}$$

$$= \frac{2u^2\sin(\alpha+\beta)}{g\cos\beta}\left\{\frac{\cos(\alpha+\beta)\cdot\cos\beta + \sin\beta\cdot\sin(\alpha+\beta)}{\cos\beta}\right\}$$

$$= \frac{2u^2\sin(\alpha+\beta)}{g\cos^2\beta}\left\{\cos(\alpha+\cancel{\beta}-\cancel{\beta})\right\}$$

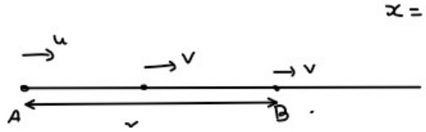
$$\boxed{OA = \frac{2u^2\sin(\alpha+\beta)\cos\alpha}{g\cos^2\beta}}$$

$$\boxed{OA_{max} = \frac{u^2}{g(1-\sin\beta)}} \Leftarrow \boxed{\alpha = \frac{\pi}{4} - \frac{\beta}{2}}$$

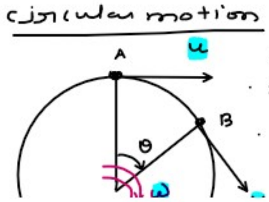
<u>circular motion.</u> <u>with constant velocity</u>

<u>Straight line motion</u>

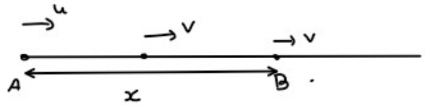
x = displacement

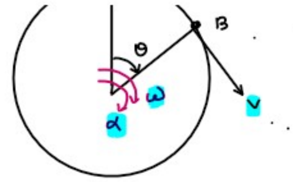


<u>circular motion</u>



u, v.
= tangential velocities at A & B





Velocities at A & B

$v = \frac{dx}{dt}$

$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

θ = angular displacement.

(angular velocity) $\omega = \frac{d\theta}{dt}$

(angular acceleration) $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

a = constant

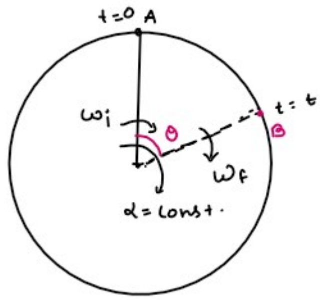
$v = u + at$

$x = ut + \frac{1}{2}at^2$

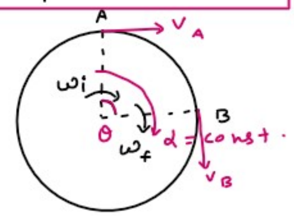
$v^2 - u^2 = 2ax$

α = constant

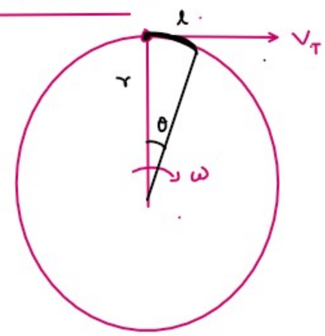
$$\boxed{\begin{array}{l} \omega_f = \omega_i + \alpha \cdot t \\ \theta = \omega_i t + \frac{1}{2}\alpha \cdot t^2 \\ \omega_f^2 - \omega_i^2 = 2\alpha\theta \end{array}}$$



$\alpha = \frac{d\omega}{dt} = 0$ (const)



totle velocities of a particle in circular motion.



$l = r\theta$

V_T = tangential velocity

ω = angular velocity

$l = r \cdot \theta$

$\frac{dl}{dt} = r \cdot \frac{d\theta}{dt}$

$\frac{d\theta}{dt} = \omega$

$\frac{dl}{dt} = v$

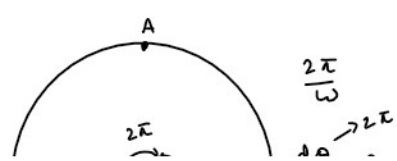
$$\boxed{V_T = r \cdot \omega}$$

A body is whirled in a horizontal circle of radius 20 cm. It has angular velocity of 10 rad/s. What is its linear velocity at any point on circular path

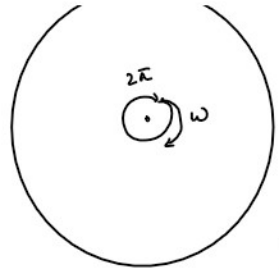
(a) 10 m/s (b) 2 m/s (c) 20 m/s (d) $\sqrt{2}$ m/s

Two particles of mass M and m are moving in a circle of radii R and r. If their time-periods are same, what will be the ratio of their linear velocities

(a) $MR : mr$ (b) $M : m$ (c) $R : r$ (d) 1 : 1



$\frac{2\pi}{\omega}$



$\frac{2\pi}{\omega}$

$\frac{d\theta}{dt} = \omega$ ($d\theta \to 2\pi$, $dt \to T$)

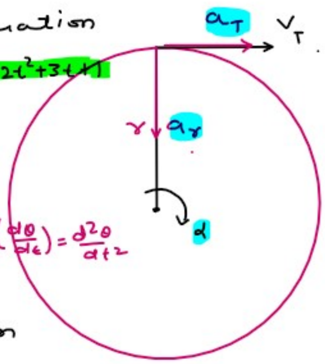
$\frac{2\pi}{T} = \omega$

$\boxed{T = \frac{2\pi}{\omega}}$

acceleration

① α = angular acceleration

$\boxed{\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}}$ — $\theta = 2t^2+3t+1$



$\alpha = \frac{d\omega}{dt}$

$\frac{d}{dt}\left(\frac{d\theta}{dt}\right) = \frac{d^2\theta}{dt^2}$

② $\boxed{a_T = \frac{dV_T}{dt}}$

a_T = tangential acceleration

$V_T = r\omega$

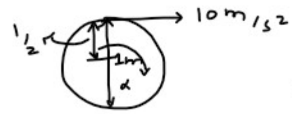
$\frac{dV_T}{dt} = r\frac{d\omega}{dt} \Rightarrow \boxed{a_T = r\alpha}$

③ centripetal (a_r) acceleration. (radial acceleration)

: always towards the center.

$a_r = \frac{V_T^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$

$\boxed{a_r = \frac{V_T^2}{r} = r\omega^2}$



$a_T = R\alpha$

$10 = \frac{1}{2}\alpha \Rightarrow \alpha = 20$

The linear acceleration of a car is $10 m/s^2$. If the wheels of the car have a diameter of 1m the angular acceleration of the wheels will be

(a) 10 rad/sec^2 (b) 20 rad/sec^2 ✓ (c) 1 rad/sec^2 (d) 2 rad/sec^2

revolution per min =

The angular speed of a motor increases from 600 rpm to 1200 rpm in 10 s. What is the angular acceleration of the motor

constant.

(a) 600 $rad\ sec^{-2}$ (b) $60\pi\ rad\ sec^{-2}$ (c) $60\ rad\ sec^{-2}$ (d) $2\pi\ rad\ sec^{-2}$ ✓

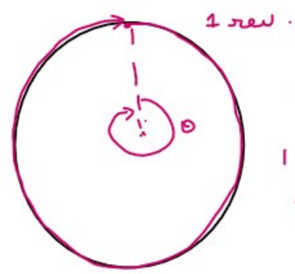
1 min = 60 sec

$\boxed{1\ rpm = \frac{2\pi}{60}\ rad/sec}$

600 rev. per min.

$\theta = 600 \times 2\pi$ $t = 60$ sec

$\frac{\theta}{t} = \omega = \frac{600 \times 2\pi}{60}$



1 rpm.

1 revolution in 60°.

1 revolution = 60 min

2π angle = 60 min.

α = const.

$\boxed{\omega_f = \omega_i + \alpha t}$

$1200 \times \frac{2\pi}{60} = 600 \times \frac{2\pi}{60} + \alpha \cdot (10)$ $\omega = \frac{d\theta}{dt}$

$\frac{600 \times 2\pi}{60}$.

$\omega = \frac{\theta}{t}$

$\alpha = 2\pi\ rad/sec^2$

. If a cycle wheel of radius 4 m completes one revolution in two seconds. Then acceleration of the cycle will be

centripetal acceleration.

[Pb. PMT 2001]

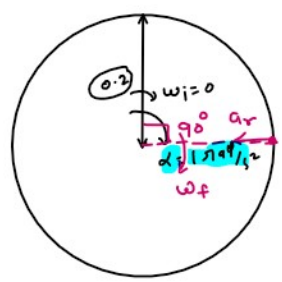
(a) $\pi^2 m/s^2$ (b) $2\pi^2 m/s^2$ (c) $4\pi^2 m/s^2$ ✓ (d) $8\pi\, m/s^2$

. Two cars going round curve with speeds one at 90 km/h and other at 15 km/h. Each car experiences same acceleration. The radii of curves are in the ratio of

(a) 4 : 1 (b) 2 : 1 (c) 16 : 1 (d) 36 : 1 ✓

. A wheel of radius $0.20 m$ is accelerated from rest with an angular acceleration of $1\ rad/s^2$. After a rotation of 90^o the radial acceleration of a particle on its rim will be

(a) $\pi\ m/s^2$ (b) $0.5\ \pi\ m/s^2$ (c) $2.0\pi\ m/s^2$ (d) $0.2\ \pi\ m/s^2$ ✓



$a_r = \omega_f^2 \cdot r = \pi \times 0.2 \quad m/s^2$

$\omega_f^2 - \cancel{\omega_i^2} = 2\alpha \cdot \theta$

$\omega_f^2 = 2 \times (1)\left(\frac{\pi}{2}\right)$

$\omega_f^2 = \pi$

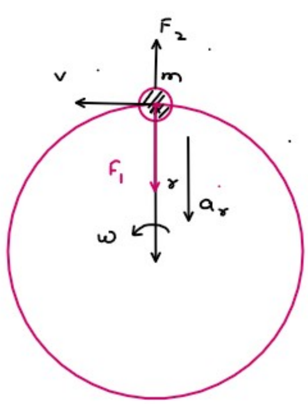
eqn. Framing.

$a_r = \frac{v^2}{r} = r\omega^2$

$F_1 - F_2 = m a_r$

$$\boxed{F_1 - F_2 = \frac{mv^2}{r}}$$

$F_1 - F_2 = m r \omega^2$



Newtons law.

a

Forces = m.a

$\boxed{+F_1 - F_2 = ma}$

Sign conv.

all Forces in the dir^n of acceleration = +ve

$F_1 - F_2 = mr\omega^2$

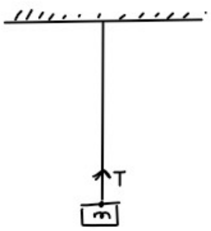
sign conv.

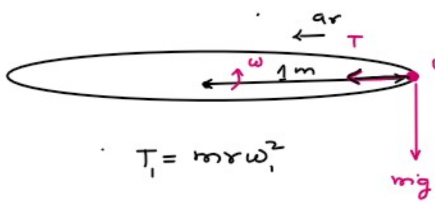
all Forces in the dir of acceleration = +ve

all forces against the dir. of acceleration = −ve

A ball of mass 0.1 *kg* is whirled in a horizontal circle of radius 1 *m* by means of a string at an initial speed of 10 *r.p.m.* Keeping the radius constant, the tension in the string is reduced to one quarter of its initial value. The new speed is

(a) 5 *r.p.m.* (b) 10 *r.p.m.* (c) 20 *r.p.m.* (d) 14 *r.p.m.*





$\omega_1 = 10\,rpm = 10 \times \frac{2\pi}{60}\ rad/s = \pi/3$

$T_1 = mr\omega_1^2$

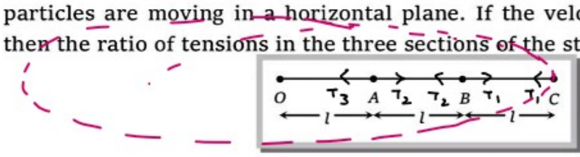
$T_2 = mr\omega_2^2$

$\frac{T_1}{T_2} = \left(\frac{\omega_1}{\omega_2}\right)^2 \quad T_2 = T_1/4 \quad 4 = \left(\frac{\omega_1}{\omega_2}\right)^2$

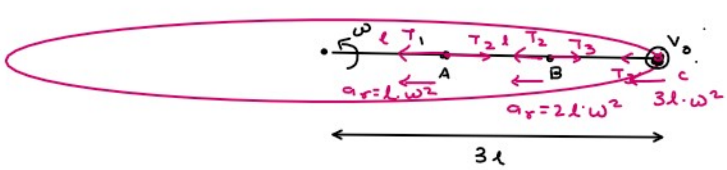
$\omega_1 = 2\omega_2 \quad \pi/3 = 2\omega_2$

$\omega_2 = \frac{\pi}{6} \times \frac{60}{2\pi} = \frac{10}{2} = 5$

Three identical particles are joined together by a thread as shown in figure. All the three particles are moving in a horizontal plane. If the velocity of the outermost particle is v_0, then the ratio of tensions in the three sections of the string is

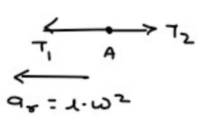


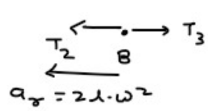
(a) 3 : 5 : 7 (b) 3 : 4 : 5 (c) 7 : 11 : 6 (d) 3 : 5 : 6



$v_0 = 3l\cdot\omega$

$\omega = \frac{v_0}{3l}$





A

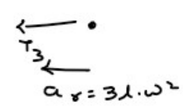
$T_1 - T_2 = l\cdot\omega^2$ — (1)

B

$T_2 - T_3 = 2l\cdot\omega^2$ — (2)

C

$T_3 = 3l\cdot\omega^2$ — (3)



$T_2 = 3l\omega^2 + 2l\omega^2$

$= 5l\omega^2$

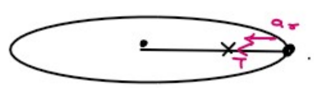
$T_1 = l\omega^2 + 5l\omega^2$

$= 6l\omega^2$

$T_3 : T_2 : T_1 = 3 : 5 : 6$

A mass is supported on a frictionless horizontal surface. It is attached to a string and rotates about a fixed centre at an angular velocity ω_0. If the length of the string and angular velocity are doubled, the tension in the string which was initially T_0 is now

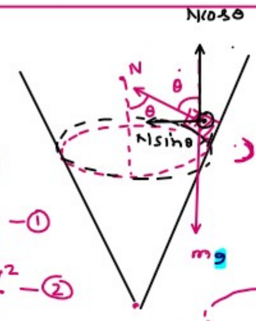
(a) T_0 (b) $T_0/2$ (c) $4T_0$ (d) $8T_0$

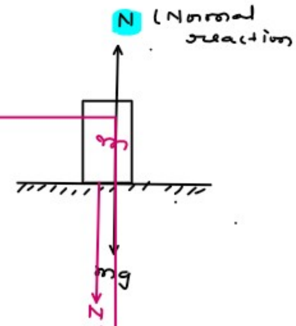


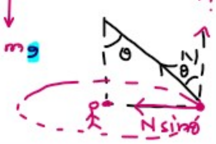
$N\cos\theta - mg = 0$

$N\cos\theta = mg$ — (1)

$N\sin\theta = \frac{mv^2}{r}$ — (2)



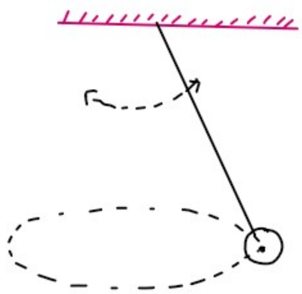


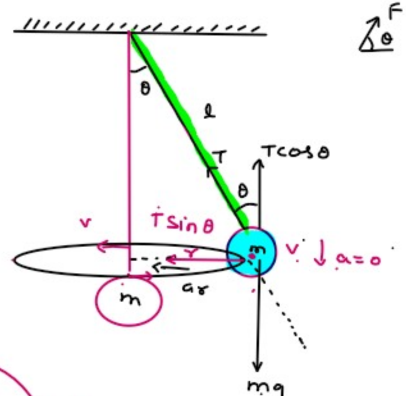


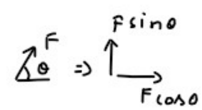
conical pendulum

$T\sin\theta = \frac{mv^2}{r}$ — (1)

$mg - T\cos\theta = 0$ — (2)





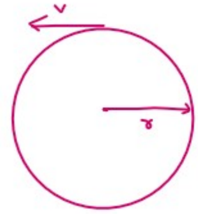


Timeperiod

$T = \frac{2\pi r}{v}$

$T\sin\theta = \frac{mv^2}{r}$ — (1)

$T\cos\theta = mg$ — (2)



$T = \frac{2\pi r}{v}$

$\frac{(1)}{(2)}$ $\tan\theta = \frac{v^2}{rg}$

$v = \sqrt{rg\tan\theta}$

$T = \frac{2\pi r}{\sqrt{rg\tan\theta}}$ = $\boxed{T = 2\pi\sqrt{\frac{r}{g\tan\theta}}}$