DCAM classes

Dynamic Classes for Academic Mastery

Explanations

1. (a) Given, $V = 3x^2$

Since, electric field, $E = -\frac{dV}{dx} = -\frac{d}{dx}(3x^2) = -6x$

At point (1, 0, 2), $E = -6(1) = -6$ V/m

$\therefore$ Electric field at (1, 0, 2) is 6 V/m along $-X$-axis.

2. (d) Given, $Q = +3\text{nC} = 3\times10^{-9}$ C

$$q = +5\text{nC} = 5\times10^{-9}\text{ C}$$

$$r_1 = 10\text{ cm} = 10\times10^{-2}\text{ m}$$

$$r_2 = 15\text{ cm} = 15\times10^{-2}\text{ m}$$

Initial potential energy of system is

$$U_i = K\frac{Q\cdot q}{r_1} = \frac{9\times10^9\times3\times10^{-9}\times5\times10^{-9}}{10\times10^{-2}}$$

$$= 13.5\times10^{-7}\text{ J}$$

Final potential energy of system is

$$U_f = K\frac{Q\cdot q}{r_2} = \frac{9\times10^9\times3\times10^{-9}\times5\times10^{-9}}{15\times10^{-2}}$$

$$= 9\times10^{-7}\text{ J}$$

$\therefore$ Work done, $W = -$ (change in potential energy)

$$= U_i - U_f$$

$$= (13.5-9)\times10^{-7} = 4.5\times10^{-7}\text{ J}$$

3. (a) When a system of charges is placed in an external electric field, the electrostatic potential energy of the system will be

$$U = q_1V(r_1) + q_2V(r_2) + K\frac{q_1q_2}{r}$$

$$= q_1E_1r_1 + q_2E_2r_2 + K\frac{q_1q_2}{r}$$

$$= q_1\frac{B}{r_1^2}r_1 + q_2\frac{B}{r_2^2}r_2 + K\frac{q_1q_2}{r}$$

Here, $B = 1.2\times10^6\text{N/cm}^2$, $q_1 = 14\times10^{-6}$C,

$$q_2 = -4\times10^{-6}\text{C}$$

$$r_1 = r_2 = 12\times10^{-2}\text{ m}$$

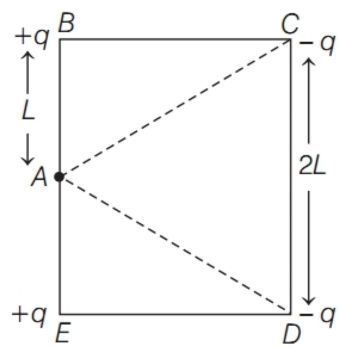
and $r = r_1 + r_2 = 24\times10^{-2}$m

$$\therefore \quad U = \frac{14\times10^{-6}\times1.2\times10^6}{12\times10^{-2}} + \frac{(-4\times10^{-6})\times1.2\times10^6}{12\times10^{-2}} + \frac{9\times10^9\times14\times10^{-6}\times(-4\times10^{-6})}{24\times10^{-2}}$$

$$= 140 - 40 - 2.1 = 97.9\text{ J}$$

4. (a) The equipotentials at a large distance from a collection of charges whose total sum is not zero are spheres with centre at the collection of changes.

5. (a) The given system of charges can be shown as



Here, $AC = AD = \sqrt{AB^2 + BC^2} = \sqrt{L^2 + (2L)^2} = \sqrt{5}L$

Electric potential at A,

$$V_A = K\left[\frac{q}{AB} - \frac{q}{AC} - \frac{q}{AD} + \frac{q}{AE}\right]$$

$$= K\left[\frac{q}{L} - \frac{q}{\sqrt{5}L} - \frac{q}{\sqrt{5}L} + \frac{q}{L}\right] = \frac{1}{4\pi\varepsilon_0}\frac{2q}{L}\left(1 - \frac{1}{\sqrt{5}}\right)$$

6. (a) Given, voltage, $V = 200$V

initial capacitance, $C_1 = 2\mu$F

and final capacitance, $C_2 = X\mu$F

Energy stored in a capacitor is given by $E = \frac{1}{2}CV^2$

$\therefore$ Decrease in energy, $\Delta E = E_2 - E_1$

$$= \frac{1}{2}(C_2 - C_1)V^2$$

$$\Rightarrow \quad -2\times10^{-2} = \frac{1}{2}(X-2)10^{-6}\times(200)^2$$

$$\Rightarrow \quad (X-2)10^{-6} = -1\times10^{-6}$$

$$\Rightarrow \quad 10^{-6}\times X = 2\times10^{-6} - 1\times10^{-6}$$

$$10^{-6} \times X = 1 \times 10^{6}$$

$$X = 1$$

$$\therefore \quad C_2 = 1\,\mu F$$

7. (a) The potential at the centre of the sphere will be 80 V, because it remains same at each point inside the metallic hollow sphere.

8. (c) Option (c) is the only incorrect statement, because for electric dipole the potential depends on position vector and dipole moment vector and expressed as

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p \cdot \hat{\mathbf{r}}}{r^2}$$

Electric dipole potential varies as $\frac{1}{r^2}$ at large distances also.

Hence, option (b) is correct, but option (c) is incorrect.

Whereas, for point charge electric potential is

$$V = \frac{q}{4\pi\varepsilon_0 r}$$

i.e., $\quad V \propto \frac{1}{r}$

and electric field, $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$

i.e., $\quad E \propto \frac{1}{r^2}$

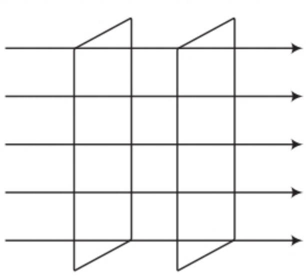
Hence, both options (a) and (d) are correct.

9. (d) The potential of electric dipole at a distance r from the centre of the dipole is

$$V = \frac{1}{4\pi\varepsilon_0} \frac{p\cos\theta}{r^2} \Rightarrow V \propto \frac{1}{r^2}$$

10. (b) Equipotential surfaces for uniform electric field are perpendicular planes to the field as shown in figure.

Option (b) is not true as two equipotential surfaces cannot cross each other.



11. (d) On giving a charge q to a conductor the electric potential of the conductor becomes V. Then, the capacitance of the conductor is

$$C = \frac{q}{V}$$

Potential on sphere of radius R is

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{R}$$

Now, $\quad C = \frac{q}{V}$

$$\Rightarrow \quad C = \frac{q \times 4\pi\varepsilon_0 R}{q} = 4\pi\varepsilon_0 R$$

$$\Rightarrow \quad C \propto R$$

12. (b) Here, C_1 and C_2 are in series. Hence, their effective capacitance C' is given by

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\Rightarrow \quad \frac{1}{C'} = \frac{1}{20} + \frac{1}{20}$$

$$\Rightarrow \quad C' = 10\,\mu F$$

Similarly, $\quad \frac{1}{C''} = \frac{1}{C_3} + \frac{1}{C_4}$

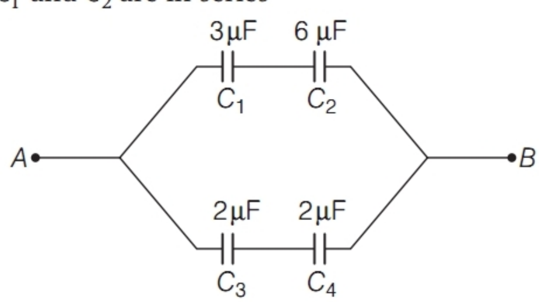
$$\Rightarrow \quad \frac{1}{C''} = \frac{1}{10} + \frac{1}{10}$$

$$\Rightarrow \quad C'' = 5\,\mu F$$

Now, C' and C'' are in parallel. Hence, resultant capacitance C will be

$$C = C' + C'' = 10 + 5 = 15\,\mu F$$

13. (d) C_1 and C_2 are in series



$$\therefore \quad \frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{3} + \frac{1}{6}$$

$$\Rightarrow \quad C' = 2\,\mu F$$

Similarly, C_3 and C_4 are in series

$$\frac{1}{C''} = \frac{1}{C_3} + \frac{1}{C_4} = \frac{1}{2} + \frac{1}{2} \Rightarrow C'' = 1\,\mu F$$

Now, C' and C'' are in parallel

$$\therefore \quad C_{eq} = C' + C'' = 2 + 1 = 3\,\mu F$$

14. (d) From the formula,

$$W = \frac{Q^2}{2C} \qquad \text{...(i)}$$

$$W' = \frac{(2Q)^2}{2C} \qquad (\because Q' = 2Q)$$

$\Rightarrow \quad W' = 4\frac{Q^2}{2C}$

$\Rightarrow \quad W' = 4W$ [from Eq. (i)]

15. (d) Electric potential of a charge conductor depends not only on the amount of charge and volume but also on the shape of the conductor. Hence, if their shapes are different, they may have different electric potential.

16. (b) Potential energy of a system of two charges,

$$U = K\frac{q_1 q_2}{r}$$

$\therefore$ When two positive point charges move away from other, then their potential energy decreases and work done by force can always be expressed in terms of a potential energy, when the particle moves from a point.

17. (c) Five forces of equal magnitude are acting on $-Q$. When they are added as per polygon law, their vector sum is zero.

18. (a) Among the given figure under consideration, it can be said that for $Q < 0$, the force on unit positive charge is attractive, so that the electrostatic force and displacement are in same direction.

So, due to this reason, work done by the electrostatic force in bringing the unit positive charge from infinity to point P is positive.

19. (a) Polar dielectric molecules are the one in which centres of positive and negative charges are separated even when there is no external field. It means, they have permanent dipoles.

So, in the absence of any external electric field, the different permanent dipoles are oriented randomly due to thermal agition. So, the total dipole moment is zero.

20. (a) The maximum amount of charge, a capacitor can have depends on the shape and size of capacitor and also on the surrounding medium.

Thus, a capacitor can be given only a limited quantity of charge.

21. (i) (b) Potential energy of the proton increases as it moves in opposite direction of electric field.

(ii) (b) $\Delta V = -E \cdot \Delta x = -4 \times 10^4 \times 0.5$

$= -2 \times 10^4 \text{ V}$

(iii) (c) $\Delta U = q_0 \Delta V = 1.6 \times 10^{-19} \times (-2 \times 10^4)$

$= -3.2 \times 10^{-15} \text{ J}$

(iv) (b) As, $\Delta K = -\Delta U = 3.2 \times 10^{-15} \text{ J}$

$\therefore \quad \Delta K = \frac{1}{2} m v_B^2$

$\Rightarrow \quad v_B = \sqrt{\frac{2\Delta K}{m}} = \sqrt{\frac{2 \times 3.2 \times 10^{-15}}{1.66 \times 10^{-27}}}$

$= 1.96 \times 10^6 \text{ m/s}$

(v) (c) Electrostatic potential energy of the system

$U = 9 \times 10^9 \cdot \frac{q_1 q_2}{r}$

$= 9 \times 10^9 \times 2 \times 10^{-6} \times 2 \times 10^{-6}$

$= 36 \times 10^{-3} = 3.6 \times 10^{-2} \text{ J}$

22. (i) (a) Charged surface of sphere behaves like a equipotential surface, hence work done to move a charge particle from one point to another point on the surface of sphere will be always zero.

(ii) (c) The positively charged particle experiences electrostatic force along the direction of electric field, i.e. from high electrostatic potential to low electrostatic potential. Thus, the work is done by the electric field on the positive charge, hence electrostatic potential energy of the positive charge decreases.

(iii) (b) Potential energy of the system,

$$U = \frac{k(-Q)q}{l} + \frac{kq^2}{l} + \frac{kq(-Q)}{l} = 0$$

$\Rightarrow \quad \frac{kq}{l}[-Q + q - Q] = 0$

$q \neq 0$

$\therefore \quad -Q + q - Q = 0$

$\Rightarrow \quad Q = \frac{q}{2}$

(iv) (b) Since, electric force on electron in electric field is opposite direction to the direction of electric field, hence work done on electron by electric field is positive.

(v) (c) Work done, $W = Vq$

$= 5 \times 2 \times 10^{-6} = 10^{-5} \text{ J}$