Explanations

1. As, the needle is displaced from the equilibrium position, the torque will try to bring it back in equilibrium position, hence acceleration will be related with negative of angular displacement.

When compass needle of magnetic moment **M** and moment of inertia I is slightly disturbed by an angle θ from the mean position of equilibrium.

Then, restoring torque begin to act on the needle which try to bring the needle back to its mean position which is given by

$$\tau = -MB\sin\theta$$

Since, θ is small

So, $\sin\theta \approx \theta$

$\therefore \quad \tau = -MB\theta \quad \ldots(i)$

But $\quad \tau = I\alpha \quad \ldots(ii)$

where, α = angular acceleration

and M = magnetic moment of dipole.

On comparing Eqs. (i) and (ii), we get

$\Rightarrow \quad I\alpha = -MB\theta$

$\Rightarrow \quad \alpha = -(MB/I)\theta$

$\therefore \quad \alpha \propto -\theta$ **(1)**

$\Rightarrow$ Angular acceleration $\propto$ – Angular displacement

$\Rightarrow$ Therefore, needle execute SHM.

Hence, time period,

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{MB/I}} \text{ or } T = 2\pi\sqrt{\frac{I}{MB}}$$ **(1)**

This is the required expression.

2. Given, $M = 3\text{A - m}^2$, $d = 10 \text{ cm} = 0.1 \text{ m}$

$B = 0.25 \text{ T}$ and $\theta = 30°$

The torque acting on the bar magnet,

$$\tau = MB\sin\theta\hat{\mathbf{n}} = \mathbf{M} \times \mathbf{B}$$

Also, $\tau = F\cdot d \Rightarrow F\cdot d = MB\sin\theta$

$$\Rightarrow \quad F = \frac{MB\sin\theta}{d} = \frac{3 \times 0.25 \times \sin 30°}{0.1}$$

$$= \frac{0.75}{2 \times 0.1} = 3.75 \text{ N}$$ **(1)**

If F is withdrawn from the bar magnet, then it will rotate, due to the torque (τ) and align itself along the field direction. **(1)**

3. (i) Refer to text on page 155 (Magnetism and Gauss' Law). **(1½)**

(ii) Refer to text on page 153 (Properties of Magnetic field lines). **(1½)**

4. (i) Given, magnetic moment, $M = 6 \text{ J/T}$

aligned angle, $\theta_1 = 60°$

external magnetic field,

$$B = 0.44 \text{ T}$$

(a) When the bar magnet is align normal to the magnetic field, i.e. $\theta_2 = 90°$

$\therefore$ Amount of work done in turning the magnet,

$$W = -MB(\cos\theta_2 - \cos\theta_1)$$

$$= -6 \times 0.44(\cos 90° - \cos 60°)$$

$$= +6 \times 0.44 \times \frac{1}{2} \quad \left(\begin{array}{l}\because \cos 90° = 0 \\ \text{and } \cos 60° = 1/2\end{array}\right)$$

$$= 1.32 \text{ J}$$ **(1)**

(b) When the bar magnet align opposite to the magnetic field, i.e. $\theta_2 = 180°$

$$\therefore W = -MB(\cos 180° - \cos 60°)$$

$$= -6 \times 0.44\left(-1 - \frac{1}{2}\right) \quad (\because \cos 180° = -1)$$

$$= 6 \times 0.44 \times \frac{3}{2}$$

$$= 3.96 \text{ J}$$ **(1)**

(ii) We know that, torque,

$$\tau = |\mathbf{M} \times \mathbf{B}| = MB\sin\theta$$

For case (ii), $\theta = 180°$

$\therefore \quad \tau = MB\sin 180° \qquad (\because \sin 180° = 0)$

$\quad = 0$

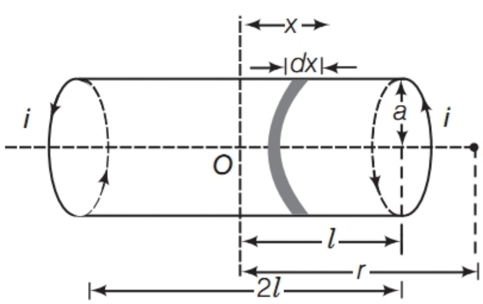
$\therefore$ Amount of torque is zero for case (b). **(1)**

5. (i) The magnetic field lines for a bar magnet and a current carrying solenoid resemble very closely. Therefore, a bar magnet can be thought as a large number of circulating currents in analogy with a solenoid.

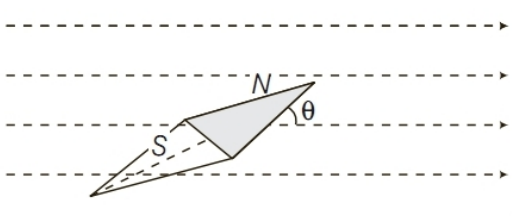
Cutting a bar magnet in half is like cutting a solenoid. We get two smaller solenoids with weaker magnetic properties. The field lines remain continuous, emerging from one face of the solenoid and entering into other face.

One can test this analogy by moving a small compass needle in the neighbourhood of a bar magnet and a current carrying finite solenoid and noting that the deflections of the needle are similar in both the cases.

To prove mathematically that magnetic field produced by a solenoid on any point on the axial line is same as that of a bar magnet. This analogy between bar magnet and solenoid can be shown by calculating the magnetic field at an axial point of solenoid which resembles to that of a bar magnet. **(2)**



Let i be the current passing through a solenoid, a be the radius of solenoid, $2l$ be the length of solenoid and n be the number of turns per unit length of solenoid.



Let P be the point at distance r from centre at which magnetic field is to be calculated. Consider a small element of thickness dx of the solenoid at a distance (x) from the centre O.

Number of turns in the element $= n\,dx$

The magnitude of the field at point P due to the circular element is given by

$$dB = \frac{\mu_0 \, i \, a^2 (n\,dx)}{2[(r-x)^2 + a^2]^{3/2}}$$

If P lies at a very large distance from O, i.e. $r >>> a$ and $r >>> x$, then $[(r-x)^2 + a^2]^{3/2} \approx r^3$

$$dB = \frac{\mu_0 \, i \, a^2 n \, dx}{2r^3}$$

Total magnetic field at point P due to current carrying solenoid.

$$B = \frac{\mu_0 n \, i \, a^2}{2r^3} \int_{-l}^{+l} dx$$

[$\because$ range of variation of x is from $-l$ to $+l$]

$$= \frac{\mu_0 n i \, a^2}{2r^3} [x]_{-l}^{+l} = \frac{\mu_0 \, n i \, a^2}{2r^3} (2l)$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{2n\,(2l)\, i\pi a^2}{r^3}$$

If M is the magnetic moment of the solenoid, then

$M =$ Total number of turns $\times$ Current $\times$ Area of cross-section

$$M = n\,(2l) \times i \times \pi \, a^2$$

$$\Rightarrow \quad \mathbf{B} = \frac{\mu_0}{4\pi} \cdot \frac{2\mathbf{M}}{r^3}$$

This is the expression for magnetic field on the axial line of a short bar magnet.

The magnetic moment of bar magnet is thus equal to the magnetic moment of an equivalent solenoid that produces the same magnetic field. **(2)**

(ii) Given, $I = 2\text{A}$ and $N = 5$

and $r = 7\text{ cm} = 0.07\text{ m}$

Magnetic dipole moment of a coil,

$M = NIA = 5 \times 2 \times \pi\,(0.07)^2 = 0.154\text{ A-m}^2$

The direction of magnetic dipole moment is perpendicular to the plane of coil, i.e. it is along Z-axis. **(1)**

Explanations

1. The susceptibility of magnetic material is inversely proportional to temperature, i.e. $\chi_m \propto \frac{1}{T}$

$$\therefore \quad \frac{\chi_m(T)}{\chi_m\,(300\text{ K})} = \frac{300}{T}$$

$$\Rightarrow \quad T = \frac{300 \times 1.2 \times 10^5}{1.44 \times 10^5} = 250\text{ K}$$

(1)

2. Substance having (small) negative value (–0.5) of magnetic susceptibility χ_m are diamagnetic. **(1)**

3. When paramagnetic materials are placed in external magnetic field, these are feebly magnetised in the direction of the applied external magnetic field whereas in case of diamagnetic materials, these are feebly magnetised opposite to that of applied external magnetic field. **(1)**

4. The nature of magnetic material is a diamagnetic. The relation between relative permeability and magnetic susceptibility is

$$\mu_r = 1 + \chi_m$$ **(1)**

5. The magnetic material is diamagnetic substance for which $\mu_r < 1$. **(1)**

6. The small and positive susceptibility of 1.9×10^{-5} represents paramagnetic substance. **(1)**

7. Negative susceptibility represents diamagnetic substance. **(1)**

8. Diamagnetic material acquires feeble magnetisation in the opposite direction of the magnetic field when they are placed in an external magnetic field. **(1)**

9. (i) The magnetic susceptibility of a magnetic material is defined as the ratio of the intensity of magnetisation (I) to the magnetic intensity (H).

i.e., $$\chi_m = \frac{I}{H}$$

Relation between magnetic susceptibility (χ_m) and relative magnetic permeability (μ_r) is given as **(1)**

$$\mu_r = 1 + \chi_m$$

(ii) For material $A, \mu_r = 0.96 < 1$

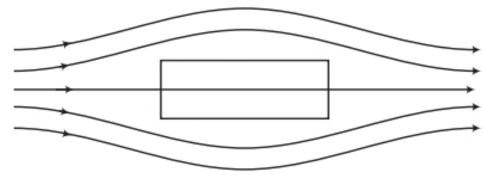
Hence, magnetic material A is diamagnetic.

For material $B, \mu_r = 500$

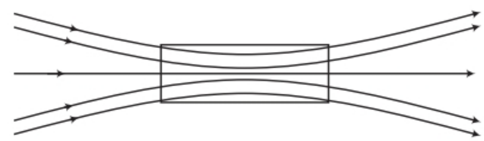
Since, μ_r is much greater than 1 for material B, therefore B is ferromagnetic material. **(1)**

10. Magnetic permeability of paramagnetic is more than air, so it allows more lines to pass through it while permeability of diamagnetic is less than air, so it does not allow lines to pass through it.

(i) Behaviour of magnetic field lines when diamagnetic substance is placed in an external field.



(ii) Behaviour of magnetic field lines when paramagnetic substance is placed in a external field.



Magnetic susceptibility distinguishes the behaviour of the field lines due to diamagnetic and paramagnetic substance. **(1)**

This difference can be explained as diamagnetic substances repel or expel the magnetic field lines while paramagnetic substance attract the magnetic field lines. **(1)**

11. The nature of the material A is paramagnetic and its susceptibility χ_m is positive.

The nature of the material B is diamagnetic and its susceptibility χ_m is negative. **(2)**

12.

Paramagnetic substance	Diamagnetic substance
A paramagnetic substance is feebly attracted by magnet.	A diamagnetic substance is feebly repelled by a magnet.
For a paramagnetic substance, the intensity of magnetisation has a small positive value.	For a diamagnetic substance, the intensity of magnetism has a small negative value. **(2)**

13. Ferromagnetic substance are those substances which have very high magnetic permeability. **(1)**

Properties (i) High retentivity

(ii) High susceptibility ($\chi_m > 1000$)

(2 × 1/2)

14. (i) For diamagnetic substances, the variation of susceptibility is very small ($0 < \chi_m < \varepsilon$), i.e. diamagnetic materials are unaffected by the change in temperature (except bismuth). **(1)**

(ii) Paramagnetic materials when cooled due to thermal agitation tendency alignment of magnetic dipoles decreases. Hence, they shows greater magnetisation. **(1)**

15. (i) Magnetic lines of force come out from North pole and enter into the South pole outside the magnet and travels from South pole to North pole inside the magnet. So, magnetic lines of force form closed loop, magnetic monopoles do not exist. **(1)**

(ii) The diamagnetic material gets slightly magnetised in a direction opposite to external field, therefore lines of force are repelled by diamagnetic material. **(1)**

NOTE When South pole of the magnet is viewed from the frame of reference, then inside the magnet, it appears as North pole and *vice-versa*. Due to this reason, magnetic field lines are traversed from South pole to North pole inside the magnet.

16. (i) **Susceptibility for diamagnetic material** It is independent of magnetic field and temperature (except for bismuth at low temperature). **(1)**

Susceptibility for ferromagnetic material The susceptibility of ferromagnetic materials decreases steadily with increase in temperature. At the Curie temperature, the ferromagnetic materials become paramagnetic.

(ii) **Behaviour in non-uniform magnetic field** Diamagnets are feebly repelled, whereas ferromagnets are strongly attracted by non-uniform field, i.e. diamagnets move in the direction of decreasing field, whereas ferromagnet feels force in the direction of increasing field intensity. **(1)**

17. (i) Refer to Sol. 10 on page 161. **(1)**

Magnetic susceptibility distinguishes the behaviour of the field lines due to diamagnetic and paramagnetic substance. **(1)**

18. Difference between para-, dia- and ferro-magnetic materials

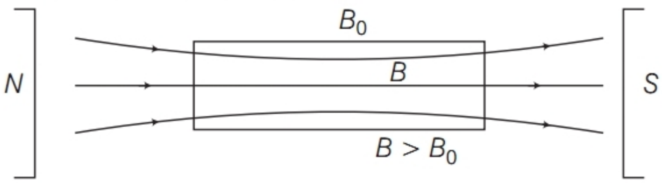
Refer to text on page 159. **(3)**

19. Given, susceptibility, $\chi_m = 0.9853$

As, the susceptibility of material is positive, but small.

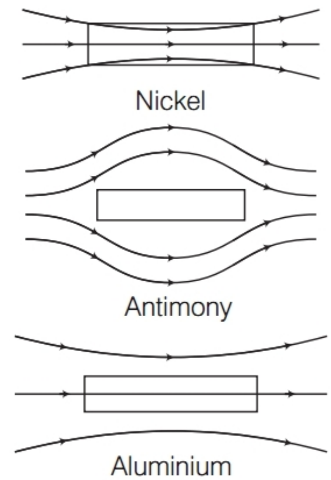
∴The material is paramagnetic in nature. For paramagnetic material, magnetic lines of external magnetic field will passes through the material without much deviation, when it is placed in between magnetic poles.

The modification of the field pattern is shown in the following figure.



(3)

20. The modifications are shown in the figure.



(½ × 3 = ½)

It happens because

(i) nickel is a ferromagnetic substance.

(ii) antimony is a diamagnetic substance.

(iii) aluminium is a paramagnetic substance.

(½ × 3 = 1 ½)