Explanations

1. Speed decreases due to decrease in wavelength of wave but energy carried by the light wave depends on the amplitude of the wave. Thus, energy carried by the wave remains unchanged. **(1)**

2. Refer to text given on pages 300 and 301. (Wavefront and law of refraction on the basis of Huygens' wave theory). **(2)**

3. **Wavefront** Refer to text on page 300. **(1)**

Law of reflection from Huygens' wave theory Refer to text on page 301. **(2)**

4. **Law of refraction from Huygens' wave theory** Refer to text on pages 301 and 302. **(3)**

5. (i) Frequency is the characteristic of the sources while wavelength is the characteristic of the medium. When monochromatic light travels from one medium to another, its speed changes, so its wavelength changes but frequency remains same. Reflection and refraction arise through interaction of incident light with atomic constituents of matter which vibrate with the same frequency as that of the incident light. Hence, frequency remains unchanged. **(1)**

(ii) Refer to Sol. 1 on page 304. **(1)**

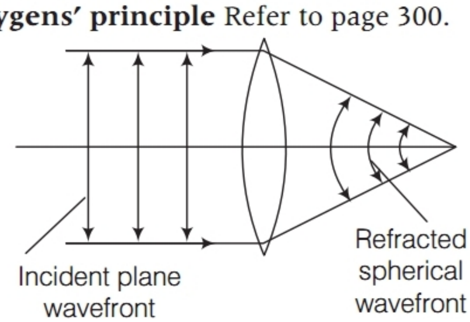
(iii) In the photon picture of light, intensity of a light is determined by the number of photons incident per unit area.

For a given frequency, intensity of light in the photon picture is determined by

$$I = \frac{\text{Energy of photons}}{\text{area} \times \text{time}} = \frac{n \times h\nu}{A \times t}$$ **(1)**

6. **Wavefront** Refer to text on page 300. **(1)**

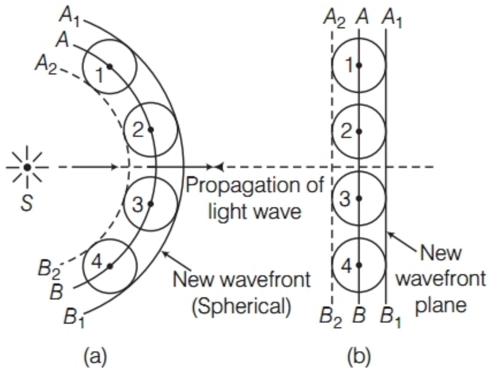
Huygens' principle Refer to page 300.

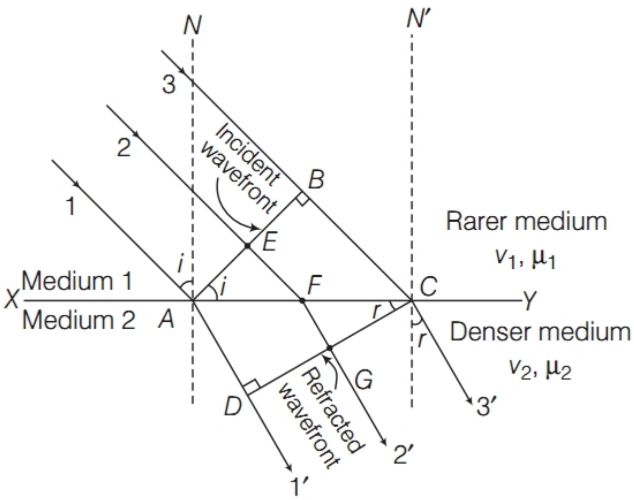


When a plane wavefront (parallel rays) is incident on a thin convex lens, the emergent rays are focused on the focal point of the lens. Thus, the shape of emerging wavefront is spherical. **(2)**

7. According to Huygens' principle, each point on the given wavefront (called primary wavefront) is the source of a secondary disturbance (called secondary wavelets) and the wavelets emanating from these point spread out in all the directions with the speed of the wave. **(1)**

A surface touching these secondary wavelets, tangentially in the forward direction at any instant gives the new wavefront at that instant. This is called secondary wavefront. **(1)**





If v_1, v_2 are the speeds of light into two media and t is the time taken by light to go from B to C or A to D or E to G through F, then $t = EF/v_1 + FG/v_2$

In ΔAFE , $\sin i = EF/AF$

In ΔFGC, $\sin r = FG/FC$

$\Rightarrow \quad t = AF \sin i/v_1 + FC \sin r/v_2$

$\Rightarrow \quad t = AC \sin r/v_2 + AF \ (\sin i/v_1 - \sin r/v_2)$

For rays of light from the different parts on the incident wavefront, the values of AF are different. But light from different points of the incident wavefront should take the same time to reach the corresponding points on the refracted wavefront.

So, t should not depend on F. This is possible only, if $\quad \sin i/v_1 - \sin r/v_2 = 0$

or $\quad \sin i/\sin r = v_1/v_2 = \mu$

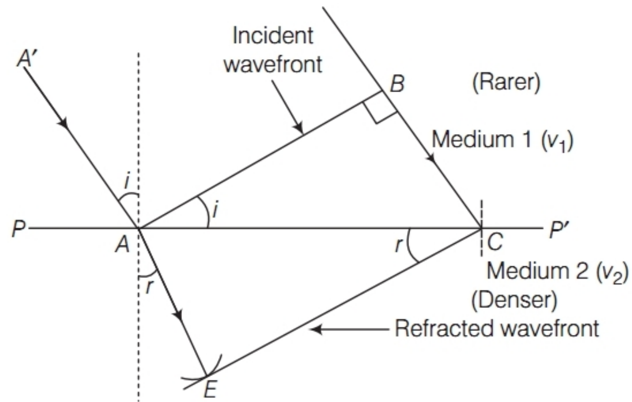
Now, if c represents the speed of light in vacuum, then $\mu_1 = c/v_1$ and $\mu_2 = c/v_2$ are known as the refractive index of medium 1 and medium 2 respectively.

Then, $\quad \mu_1 \sin i = \mu_2 \sin r$

$\Rightarrow \quad \mu = \sin i/\sin r$

This is known as Snell's law of refraction. **(1)**

8. Snell's law of refraction Let PP' represents the surface separating medium 1 and medium 2 as shown in figure.



Let v_1 and v_2 represent the speed of light in medium 1 and medium 2 respectively. We assume a plane wavefront AB propagating in the direction $A'A$ incident on the interface at an angle i. Let t be the time taken by the wavefront to travel the distance BC. **(1)**

$\therefore BC = v_1 t$ [$\because$ distance = speed × time]

In order to determine the shape of the refracted wavefront, we draw a sphere, radius $v_2 t$ from the point A in the second medium (the speed of the wave in second medium is v_2). Let CE represents a tangent plane drawn from the point C.

Then $\quad AE = v_2 t$

$\therefore CE$ would represents the refracted wavefront.

In ΔABC and ΔAEC, we have

$$\sin i = \frac{BC}{AC} = \frac{v_1 t}{AC} \text{ and } \sin r = \frac{v_2 t}{AC}$$

where, i and r are the angles of incident and refraction respectively.

$$\frac{\sin i}{\sin r} = \frac{v_1 t}{AC} \cdot \frac{AC}{v_2 t}$$

$$\therefore \quad \frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$ **(1)**

If c represents the speed of light in vacuum, then

$$\mu_1 = \frac{c}{v_1} \text{ and } \mu_2 = \frac{c}{v_2}$$

$$\Rightarrow \quad v_1 = \frac{c}{\mu_1} \text{ and } v_2 = \frac{c}{\mu_2}$$

where, μ_1 and μ_2 are the refractive indices of medium 1 and medium 2.

$$\therefore \quad \frac{\sin i}{\sin r} = \frac{c/\mu_1}{c/\mu_2}$$

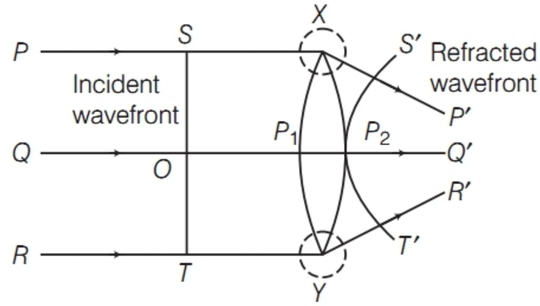
$$\Rightarrow \quad \frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \Rightarrow \mu_1 \sin i = \mu_2 \sin r$$

This is the Snell's law of refraction. **(1)**

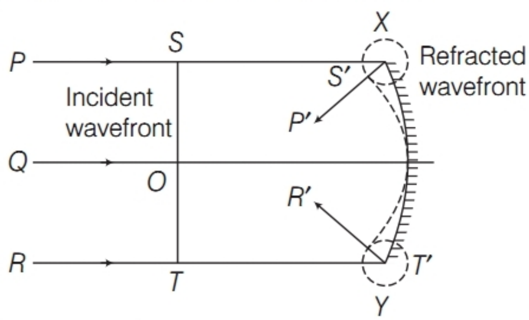
9. Refer to Sol. 7 on pages 304 and 305. **(3)**

10. Refer to Sol. 8 on pages 305 and 306.

11. (i) (a) **Behaviour of a converging lens**



(b) **Behaviour of a concave mirror**



(2)

(ii) Refer to Sol. 5 (i) on page 304. **(1)**

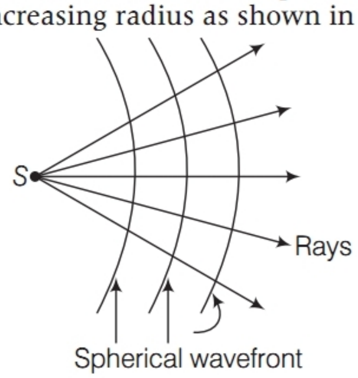
12. Laws of reflection Refer to text on page 301. **(3)**

13. Law of refraction Refer to text on pages 301 and 302. **(3)**

14. (i) When light is emitted from a source, then the particles present around it begins to vibrate. The locus of all such particles which are vibrating in the same phase is termed as wavefront. **(1)**

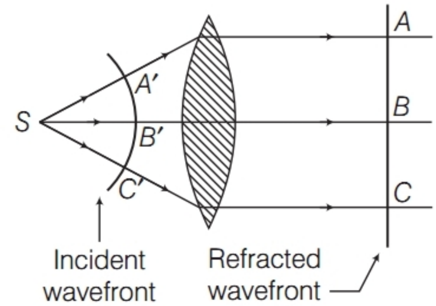
A line perpendicular to a wavefront is called a ray, it is the path along which light travels. **(1)**

(ii) (a) The wavefront will be spherical of increasing radius as shown in figure.



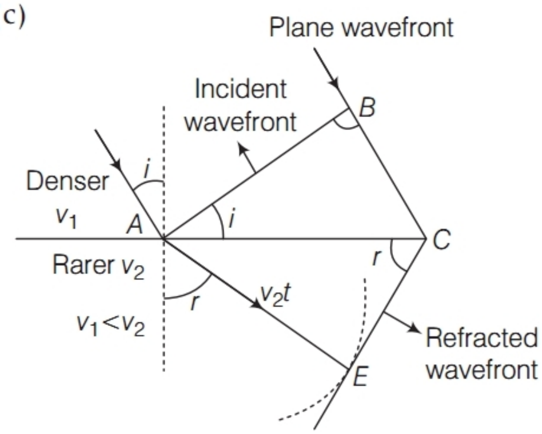
(1)

(b) When sources is at the focus, the rays coming out of the convex lens are parallel, so wavefront is plane as shown in figure.



(1)

(c)



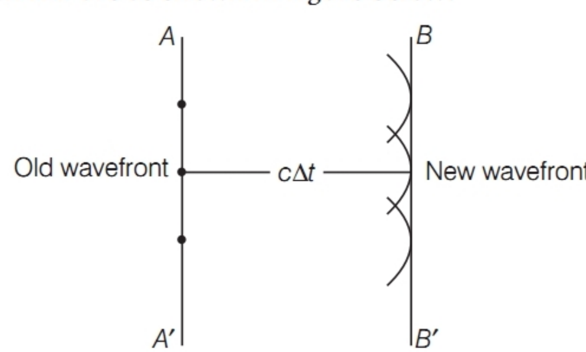
(1)

15. Huygens' Principle Each point on the primary wavefront acts as a source of secondary wavelets, sending out disturbance in all directions in a similar manner as the original source of light does. **(1)**

The new position of the wavefront at any instant (called secondary wavefront) is the envelope of the secondary wavelets at that instant. **(1)**

Law of Refraction Refer to text on pages 301 and 302. **(3)**

16. (i) Consider a plane wave moving through free space as shown in figure. At $t = 0$, the wavefront is indicated by the plane labelled AA'. According to Huygens' principle, each point on this wavefront is considered a point source. For clearity, only three point sources on AA' are as shown in figure below.

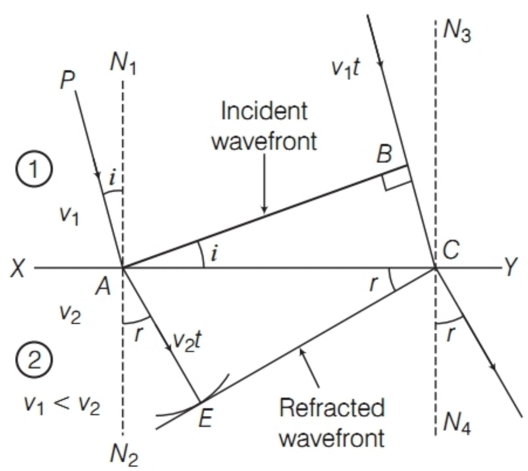


With these sources for the wavelets, we draw circular arcs, each of radius $c\Delta t$, where c is the speed of light in vacuum and Δt is sometime interval during which the wave propagates. The surface drawn tangent to these wavelets is the plane BB', which is the wavefront at a later time and is parallel to AA'. **(1)**

(ii) **Verification of Snell's Law** Refer to text on pages 301 and 302. **(2)**

(iii) The reflection and refraction phenomenon occur due to interaction of corpuscles of incident light and the atoms of matter on receiving light energy, the atoms are forced to oscillate about their mean positions with the same frequency as incident light. According to Maxwell's classical theory, the frequency of light emitted by a charged oscillator is same as its frequency of oscillation. Thus, the frequency of reflected and refracted light is same as the incident frequency. **(2)**

17. (i) Let a plane wavefront AB is incident at the interface XY separating two media such that medium 1 is optically denser than medium 2. Let time t is taken by the wave to reach from B to C,



then $BC = v_1 t$ ...(i)

where, v_1 is the velocity of light in medium 1.
In the duration of time t, the secondary wavelets emitted from point A gets spread over a hemisphere of radius, $AE = v_2 t$ ...(ii)
in the medium 2 and $v_2 > v_1$.

The tangent plane CE from C over this hemisphere of radius $v_2 t$ will be the new refracted wavefront of AB.

It is evident that angle of refraction r is greater than angle of incidence i.

By geometry, $\angle N_2AE = \angle ECA = r$

(angle of refraction)

Also, $\angle PAN_1 = \angle BAC = i$

(angle of incidence) **(2)**

(ii) Now, in ΔABC,

$\sin i = BC/AC = v_1 t/AC$ [from Eq. (i)]...(iii)

In ΔAEC, $\sin r = \frac{AE}{AC} = \frac{v_2 t}{AC}$ from Eq. (ii)] ...(iv)

Dividing Eq. (iii) by Eq. (iv), we get

$$\frac{\sin i}{\sin r} = \frac{v_1 t/AC}{v_2 t/AC}$$

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$

$$= \text{constant} = {}_1\mu_2$$

where, ${}_1\mu_2$ = refractive index of second medium w.r.t. first medium. **(2)**

Hence, Snell's law of refraction is verified.

(iii) No, energy carried by the wave does not depend on its speed instead, it depends on the frequency of wave. **(1)**

Explanations

1. Path difference in YDSE,

For (a) **constructive interference**

path difference $= \frac{dy}{D} = n\lambda$, where $n = 0, 1, 2, 3, \ldots$.

For (b) **destructive interference**

path difference $= \frac{dy}{D} = (2n-1)\frac{\lambda}{2}$, where

$n = 1, 2, 3, 4, \ldots\ldots$. **(1)**

2. For a single slit of width a, the first minima of the interference pattern of a monochromatic light of wavelength λ occurs at an angle of (λ / a) because the light from centre of the slit differs by a half of wavelength.

Whereas a double slit experiment at the same angle of (λ / a) and slits separation a produces maxima because one wavelength difference in path length from these two slits is produced. **(1)**

3. Two monochromatic sources of light having a constant phase difference are known as coherent sources. **(1)**

4. Fringe width, $\beta = \frac{D\lambda}{d}$

For given λ and d, $\beta \propto D$

Fringe width becomes double to that of original one. **(1)**

5. (i) The fringe width of interference pattern increases with the decrease in separation between $S_1 S_2$ as $\beta \propto \frac{1}{d}$ **(1/2)**

(ii) The fringe width decrease as wavelength gets reduced when interference set up is taken from air to water. **(1/2)**

6. Given, $\beta_1 = 7.2 \times 10^{-3}$ m,

$\beta_2 = 8.1 \times 10^{-3}$ m

and $\lambda_1 = 630 \times 10^{-9}$ m

$\because$ Fringe width, $\beta = \frac{D\lambda}{d}$

where, $\lambda =$ wavelength, $D =$ separation between slits and screen and $d =$ separation between two slits.

$\Rightarrow \quad \beta_1/\beta_2 = \lambda_1/\lambda_2$ ($\because D$ and d are same) **(1/2)**

Wavelength of another source of laser light

$$\Rightarrow \quad \lambda_2 = \frac{\beta_2}{\beta_1} \times \lambda_1$$

$$= \frac{8.1 \times 10^{-3}}{7.2 \times 10^{-3}} \times 630 \times 10^{-9} \text{ m}$$

or $\lambda_2 = 708.75 \times 10^{-9}$ m

$\therefore \quad \lambda_2 = 708.75$ nm **(1/2)**

7. Two independent monochromatic sources cannot produce sustained interference pattern because the phase difference between the light waves from two independent sources keeps on changing continuously. **(2)**

8. (i) The essential condition, which must be satisfied for the sources to be coherent are:

(a) the two light waves should be of same wavelength.

(b) the two light waves should either be in phase or should have a constant phase difference. **(1½)**

(ii) Because coherent sources emit light waves of same frequency or wavelength and a stable phase difference. **(½)**

9. Given, $\lambda = 600\text{ nm} = 600\times10^{-9}\text{ m}$

$d = 0.8\text{ mm} = 0.8\times10^{-3}\text{ m}$

$D = 1.6\text{ m}$

(i) Fringe width, $\beta = \dfrac{D\lambda}{d} = \dfrac{1.6\times600\times10^{-9}}{0.8\times10^{-3}}$

$= 1.2\times10^{-3}\text{ m or } 1.2\text{ mm}$ **(1)**

(ii) (a) For minima, $y_n = (2n-1)\dfrac{D\lambda}{2d}$

For third minimum, $n = 3$

$$y_3 = [2(3)-1]\times\frac{1.6\times600\times10^{-9}}{2\times0.8\times10^{-3}}$$

$$= \frac{5\times1.6\times600\times10^{-6}}{2\times0.8} = 3\text{ mm}$$

(b) Similarly, for fifth maximum,

$$y_n = \frac{nD\lambda}{d}$$

$$\Rightarrow y_5 = \frac{5\times1.6\times600\times10^{-9}}{0.8\times10^{-3}} = 6\times10^{-3}\text{ m or } 6\text{ mm}$$ **(2)**

10. Given, intensity, $I = 5\times10^{-2}\text{ Wm}^{-2}$

path difference, $\Delta x = \dfrac{\lambda}{6}$

(i) As, path difference between the interfering waves is given as, $\Delta x = \dfrac{\lambda}{2\pi}\phi$

where, ϕ = phase difference.

$$\Rightarrow \frac{\lambda}{2\pi}\phi = \frac{\lambda}{6} \Rightarrow \phi = \frac{\pi}{3}$$ **(1)**

(ii) Resultant intensity,

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

Here, $I_1 = I_2 = I = 5\times10^{-2}\text{ Wm}^{-2}$

$$\Rightarrow I_R = 2I + 2I\cos\frac{\pi}{3} = 3I = 3\times5\times10^{-2}$$

$$= 15\times10^{-2}\text{ Wm}^{-2}$$

(iii) As, $I_R = 3I$ ...(i)

Maximum intensity is obtained when phase difference is zero or even integral multiple of 2π.

$$\Rightarrow I_{max} = I + I + 2\sqrt{I\times I}\cos 2n\pi$$

$= 2I + 2I$ $[\because \cos 2n\pi = 1]$

$= 4I$ ...(ii)

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{I_R}{I_{max}} = \frac{3I}{4I}$$

or $I_R = \dfrac{3}{4}I_{max}$ **(1)**

11. (i) Given, $I_1 = I_0$ and $I_2 = 50\%$ of I_1 i.e., $I_2 = \dfrac{I_0}{2}$

Now, ratio of maximum and minimum intensity is given as, $\dfrac{I_{max}}{I_{min}} = \left(\dfrac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2$

$$= \left(\frac{\sqrt{I_0}+\sqrt{I_0/2}}{\sqrt{I_0}-\sqrt{I_0/2}}\right)^2 = \frac{\left(\sqrt{I_0}+\sqrt{I_0/2}\right)^2}{\left(\sqrt{I_0}-\sqrt{I_0/2}\right)^2}$$

$$= \frac{\left(1+\frac{1}{\sqrt2}\right)^2}{\left(1-\frac{1}{\sqrt2}\right)^2} = \left(\frac{\sqrt2+1}{\sqrt2-1}\right)^2 = \frac{3+2\sqrt2}{3-2\sqrt2}$$

$$= \left(\frac{3+2\sqrt2}{3-2\sqrt2}\right)\times\left(\frac{3+2\sqrt2}{3+2\sqrt2}\right)$$

$$= \frac{(3+2\sqrt2)^2}{(3)^2-(2\sqrt2)^2} = 17+12\sqrt2$$ **(1½)**

(ii) When a white light source is used, the interference patterns due to different component of white light overlap incoherently. The central bright fringe for different colours is at centre. So, central bright fringe is white. As $\lambda_{blue} < \lambda_{red}$, fringe closest on either side of central bright fringe is blue and the farthest is red. After few fringes, no clear pattern of fringes will be visible. **(1½)**

12. (i) Angular width, $\theta = \dfrac{\lambda}{d}$ or $d = \dfrac{\lambda}{\theta}$

Here, $\lambda = 600\text{ nm} = 6\times10^{-7}\text{ m}$

$$\theta = \frac{0.1\pi}{180}\text{ rad} = \frac{\pi}{1800}\text{ rad},$$

$$\therefore \quad d = \frac{6\times10^{-7}\times1800}{\pi} = 3.44\times10^{-4}\text{ m}$$ **(1½)**

(ii) Frequency of a light depends on its source only. So, the frequencies of reflected and refracted light will be same as that of incident light.

Reflected light is in the same medium (air) so its wavelength remains same as 500 Å.

Wavelength of refracted light, $\lambda_r = \lambda/\mu_w$

μ_w = refractive index of water.

So, wavelength of refracted wave will be decreased. **(1½)**

13. The intensity of light due to slit is directly proportional to width of slit.

$$\therefore \quad \frac{I_1}{I_2} = \frac{W_1}{W_2} = \frac{4}{1}$$

$$\Rightarrow \quad \frac{a_1^2}{a_2^2} = \frac{4}{1} \Rightarrow \frac{a_1}{a_2} = \frac{2}{1} \Rightarrow a_1 = 2a_2$$

$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$$

$$= \frac{(2a_2 + a_2)^2}{(2a_2 - a_2)^2} = \frac{9a_2^2}{a_2^2} = 9:1$$ **(3)**

14. (i) Given, $y_1 = a\cos\omega t$

$y_2 = a\cos(\omega t + \phi)$

The resultant displacement is given by

$$y = y_1 + y_2$$

$$= a\cos\omega t + a\cos(\omega t + \phi)$$

$$= a\cos\omega t + a\cos\omega t\cos\phi - a\sin\omega t\sin\phi$$

$$= a\cos\omega t\,(1 + \cos\phi) - a\sin\omega t\sin\phi$$

Put, $R\cos\theta = a(1 + \cos\phi)$ ...(i)

$R\sin\theta = a\sin\phi$ ...(ii)

By squaring and adding Eqs. (i) and (ii), we get

$$R^2 = a^2(1 + \cos^2\phi + 2\cos\phi) + a^2\sin^2\phi$$

$$= 2a^2(1 + \cos\phi) = 4a^2\cos^2\frac{\phi}{2}$$

$\therefore$ $$I = R^2 = 4a^2\cos^2\frac{\phi}{2} = 4I_0\cos^2\frac{\phi}{2}$$ **(1½)**

(ii) For constructive interference, $\cos\frac{\phi}{2} = \pm 1$

$\Rightarrow$ $\frac{\phi}{2} = n\pi$

$\Rightarrow$ $\phi = 2n\pi$

For destructive interference,

$$\cos\frac{\phi}{2} = 0 \Rightarrow \frac{\phi}{2} = (2n+1)\frac{\pi}{2}$$

$\Rightarrow$ $\phi = (2n+1)\pi$ **(1½)**

15. Given, $\frac{I_{min}}{I_{max}} = \frac{9}{25}$

But $$\left[\frac{\sqrt{I_1} - \sqrt{I_2}}{\sqrt{I_1} + \sqrt{I_2}}\right]^2 = \frac{9}{25}$$

$\Rightarrow$ $$\frac{\sqrt{I_1} - \sqrt{I_2}}{\sqrt{I_1} + \sqrt{I_2}} = \frac{3}{5}$$

$\Rightarrow$ $$5\sqrt{I_1} - 5\sqrt{I_2} = 3\sqrt{I_1} + 3\sqrt{I_2}$$

$$2\sqrt{I_1} = 8\sqrt{I_2}$$

$\Rightarrow$ $$\sqrt{I_1/I_2} = 4$$

Ratio of intensities $I_1/I_2 = 16/1$

Ratio of widths of the slits $d_1/d_2 = I_1/I_2 = 16/1$ **(2)**

16. Given, $OP = y_n$

The distance OP equals one-third of fringe width of the pattern

i.e. $$y_n = \frac{\beta}{3} = \frac{1}{3}\left(\frac{D\lambda}{d}\right) = \frac{D\lambda}{3d}$$

$\Rightarrow$ $$\frac{dy_n}{D} = \frac{\lambda}{3}$$ **(1)**

Path difference,

$$S_2P - S_1P = \frac{dy_n}{D} = \frac{\lambda}{3}$$

Now for phase difference corresponding to path difference.

Phase difference $= \frac{2\pi}{\lambda} \times$ Path difference $= \frac{2\pi}{\lambda} \times \frac{\lambda}{3}$

$\therefore$ Phase difference $= 2\pi/3$. **(1)**

If intensity at central fringe is I_0, then intensity at a point P, where phase difference is ϕ, is given by $I = I_0\cos^2\phi$

$\Rightarrow$ $$I = I_0\left(\cos\frac{2\pi}{3}\right)^2$$

$$= I_0\left(-\cos\frac{\pi}{3}\right)^2$$

$$= I_0\left(-\frac{1}{2}\right)^2 = \frac{I_0}{4}$$

Hence, the intensity at point P would be $I_0/4$. **(1)**

17. Distance between the two sources

$$d = 0.15\text{ mm} = 1.5 \times 10^{-4}\text{ m}$$

Wavelength, $\lambda = 450\text{ nm} = 4.5 \times 10^{-7}$ m

Distance of screen from source, $D = 1$ m

(i) (a) The distance of nth order bright fringe from central fringe is given by

$$y_n = Dn\lambda/d$$

For second bright fringe, $y_2 = \frac{2D\lambda}{d}$

$$= \frac{2 \times 1 \times 4.5 \times 10^{-7}}{1.5 \times 10^{-4}}$$

$$y_2 = 6 \times 10^{-3}\text{ m}$$

The distance of the second bright fringe

$$y_2 = 6\text{ mm}$$ **(1)**

(b) The distance of nth order dark fringe from central fringe is given by $y'_n = (2n-1)\frac{D\lambda}{2d}$

For second dark fringe, $n = 2$

$$y'_n = (2 \times 2 - 1)\frac{D\lambda}{2d} = \frac{3D\lambda}{2d}$$

$$y'_n = \frac{3}{2} \times \frac{1 \times 4.5 \times 10^{-7}}{1.5 \times 10^{-4}}$$

The distance of the second dark fringe,

$$y'_n = 4.5\text{ mm}$$ **(1)**

(ii) With increase of D, fringe width increases as

$$\beta = \frac{D\lambda}{d} \text{ or } \beta \propto D$$ **(1)**

18. Given, $D = 1$ m, $d = 4 \times 10^{-3}$m, $\lambda_1 = 560$ nm, and $\lambda_2 = 420$ nm

Let n_1th order bright fringe of λ_1 coincides with n_2th order bright fringe of λ_2.

$$\Rightarrow \quad \frac{Dn_1\lambda_1}{d} = \frac{Dn_2\lambda_2}{d}$$

$$\Rightarrow \quad n_1\lambda_1 = n_2\lambda_2$$ **(1)**

$$\Rightarrow \quad \frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2} = \frac{560 \times 10^{-9}}{420 \times 10^{-9}}$$

$$\Rightarrow \quad = \frac{4}{3}$$ **(1)**

Thus, 4th order bright fringe of λ_2 coincides with 3rd order bright fringe of λ_1.

Hence, distance of 3rd order bright fringe of λ_1, is given as

$$\Rightarrow \quad y_n = \frac{3D\lambda_1}{d} = \frac{3 \times 1 \times 560 \times 10^{-9}}{4 \times 10^{-3}}$$

$$y_n = 420 \times 10^{-6} \text{ m}$$

$$= 0.42 \times 10^{-3} \text{ m}$$

$$\therefore \quad y_n = 0.42 \text{ mm}$$ **(1)**

19. (i) (b) The fringe width in YDSE is given by

$$\beta = \frac{D\lambda}{d}$$

$$\Rightarrow \quad \beta \propto D$$

If screen is moved closer to slits plane, then the fringe width will increase, but the intensity of bright fringe decreases. **(1)**

(ii) (d) When two slits are replaced by two independent but identical sources, then no pattern will be observed on the screen as two identical sources cannot be coherent. **(1)**

(iii) (b) Two sources of light are said to be coherent, when both emit light waves of same wavelength, same frequency and have a constant phase difference.

(iv) (c) When the YDSE set-up is immersed in a liquid of refractive index μ, then the new fringe width will be $\frac{\beta}{\mu}$. **(1)**

(v) (*d*) Since at P_1, the path difference is of form $(2n-1)\frac{D\lambda}{2d}$, so a dark fringe is formed at P_1.

Similarly, at P_2, the path difference is form $\frac{nD\lambda}{d}$, so a bright fringe is formed at P_2. **(1)**

20. (i) Refer to Sol. 14 on page 315. **(1)**

(ii) (a) For fringes to be seen, $s/S \le \lambda/d$ Condition should be satisfied.

where, s = size of the source,

d = distance between the two slits and

S = distance of the source from the plane of two slits.

As, the source slit width increase, the fringe pattern get less and less sharp. **(1)**

When the source slit is so wide, then above condition is not satisfied and the interference pattern disappears. **(1)**

(b) The interference pattern due to different colour components of white light overlap. The central bright fringes for different colours are at the same position. Therefore, the centre fringes are white. And on the either side of the central fringe i.e. central maxima, coloured bands will appear. **(1)**

The fringe closed to either side of central white fringe is red and the farthest will be blue. After a few fringes, no clear fringes pattern is seen. **(1)**

21. (i) (a) Two independent monochromatic sources of light cannot produce a sustained interference pattern because their relative phases are changing randomly. When d is negligibly small fringe width β is proportional to $1/d$ may become too large. Even a single fringe may occupy the screen.

Hence, the pattern cannot be detected.

(b) Refer to Sol. 14 (i) on page 315. **(2)**

(ii) Intensity, $I = 4I_0 \cos^2 \frac{\phi}{2}$ ...(i)

where, I_0 is incident intensity and I is resultant intensity.

At a point where path difference is λ

Phase difference, $\phi = 2\pi/\lambda \times \lambda = 2\pi$

Substituting the value of ϕ in Eq.(i), we get

$$I = 4I_0 \cos^2 \frac{2\pi}{2} = 4I_0 \cos^2 \pi = 4I_0 = K$$

At a point, where path difference is $\lambda/3$,

$$\text{Phase difference, } \phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{3} = \frac{2\pi}{3}$$

$$I_2 = 4I_0 \cos^2 \frac{\phi}{2} = 4\left(\frac{K}{4}\right)\cos^2 \frac{\pi}{3}$$

$$= 4\frac{K}{4} \times \frac{1}{4} = \frac{K}{4}$$ **(3)**

22. (i) Refer to Sol. 14 (ii) on page 315. **(3)**

(ii) Refer to Sol 18 on page 316.

23. (i) (a) From the fringe width expression,

$$\beta = \frac{\lambda D}{d}$$

With the decrease in separation between two slits, the fringe width β increases. **(1)**

(b) For interference fringes to be seen, $\frac{s}{S} < \frac{\lambda}{d}$

Condition should be satisfied where, s = size of the source,

S = distance of the source from the plane of two slits. As, the source slit width increases, fringe pattern gets less and less sharp.

When the source slit is so wide, the above condition does not satisfied and the interference pattern disappears. **(1)**

(c) Refer to Sol. 20 (ii) (b) on page 316. **(1)**

(ii) Intensity at a point is given by

$$I = 4I' \cos^2 \phi/2$$

where, ϕ = phase difference

I' = intensity produced by each one of the individual sources.

At central maxima, $\phi = 0$, the intensity at the central maxima, $I = I_0 = 4I'$

or $$I' = I_0/4 \quad \text{...(i)}$$

As, path difference = $\lambda/3$

Phase difference, $\phi' = \frac{2\pi}{\lambda} \times$path difference

$$= \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3}$$

Now, intensity at this point

$$I'' = 4I' \cos^2 \frac{1}{2}\left(\frac{2\pi}{3}\right)$$

$$= 4I' \cos^2 \frac{\pi}{3}$$

$$= 4I' \times \frac{1}{4} = I'$$

or $I'' = \frac{I_0}{4}$ [from Eq. (i)]

24. (i) **For coherent sources** Refer to Sol. 3 on page 313. **(1)**

For necessary condition of interference Refer to Sol. 14 (ii) on page 315.

The two coherent sources are derived from the single source by division of wavefront. Two slits, namely S_1 and S_2 are kept at equidistant from source slit S, then S_1 and S_2 are lying on the same wavefront emitting secondary wavelets of constant phase difference. **(1)**

(ii) Given, the displacement of two coherent sources, Refer to Sol. 14 (i) on page 315.

Condition for bright fringe or constructive interference

$\cos(\phi/2) = \pm 1$, then, $I = I_0 = 4a^2$

$$\Rightarrow \quad \phi/2 = n\pi$$

$$\phi = 2n\pi$$

Also path difference = $\frac{\lambda}{2\pi} \times$ Phase difference

or $$x = \frac{\lambda}{2\pi} \times 2n\pi$$

Path difference, $x = n\lambda$

Bright fringe obtained when path difference of interfering wave is $n\lambda$ and phase difference is $2n\pi$. **(1)**

Condition for dark fringe or destructive interference

$$I = 0 \Rightarrow \cos\frac{\phi}{2} = 0$$

or $$\cos\frac{\phi}{2} = 0 = \cos(2n+1)\frac{\pi}{2}$$

$$\Rightarrow \quad \frac{\phi}{2} = (2n+1)\frac{\pi}{2}, \text{ where, } n = 0, 1, 2 \ldots$$

$$\Rightarrow \quad \phi = (2n+1)\pi, \text{ where, } n = 0, 1, 2, \ldots$$

Path difference,

$$x = \frac{\lambda}{2\pi} \times \phi = \frac{\lambda}{2\pi} \times (2n+1)\pi \Rightarrow x = (2n+1)\frac{\lambda}{2}$$

Path difference, $x = (2n+1)\frac{\lambda}{2}$

Dark fringes obtained when interfering wave have path difference is odd multiple of $\frac{\lambda}{2}$ and phase difference is odd multiple of π. **(2)**

Explanations

1. As, angular width, $\theta = \frac{\lambda}{d}$. Here, θ is independent of separation between slit and screen. Angular separation remains the same. **(1)**

2. When a wavefront strikes to the corner of an obstacle, light wave bends around the corner because every point on wavefront again behaves like a light source and emit secondary wavelets in all directions (Huygens' wave theory) including the region of geometrical shadow.This explains diffraction. **(1)**

3. Angular width of central maxima is given by $2\theta = \frac{2\lambda}{d}$, where d is slit width. Since, $\lambda'_r > \lambda_b$.

Therefore, width of central maxima of red light is greater than the width of central maxima of blue light. **(1)**

4. We know that, width of the central maximum (bright band) of a single slit diffraction experiment is given by $2y = \frac{2D\lambda}{a}$

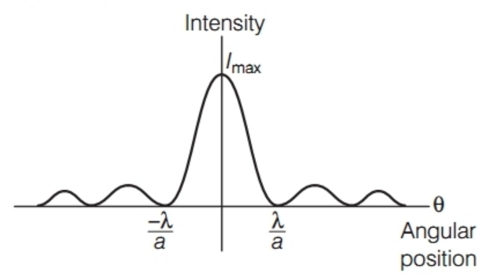
where, a is the width of the slit.

$\Rightarrow$ Width of central maximum $\propto \frac{1}{a}$

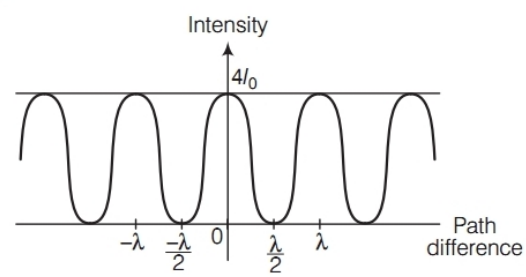
(i) So, with the decrease in the width of slit, the size of the central bright band increases.

(ii) But in the case of diffraction, intensity of central bright band does not change with the width size. Thus, the intensity remains the same. **(1)**

5. Intensity pattern for single slit diffraction is shown below



Intensity pattern for double slit interference pattern is shown.



(1)

Difference between diffraction and interference patterns are

(i) In interference pattern, all maxima and all minima are of same width but in diffraction pattern, width of central maxima is maximum and for successive maxima, it goes on decreasing.

(ii) In interference pattern, each maxima have same intensity while in diffraction pattern, intensity of central maxima is largest and it decreases rapidly for successive maxima. **(1)**

6. According to given condition, path difference

$$n\lambda = a\sin\theta \quad \ldots(i)$$

Since, θ is very small,

$$\therefore \quad \sin\theta \simeq \theta$$

Also, for first order diffraction, $n = 1$

$\therefore$ From Eq.(i), we get

$$1 \cdot \lambda = a\theta$$

$$\Rightarrow \quad \theta = \frac{\lambda}{a}$$

We know that θ must be very small ($\theta \simeq 0$) because of which the diffraction pattern is minimum.

In the case of interference, for two interfering waves of intensity I_1 and I_2, we must have two slits separated by a distance a.

$\therefore$ Resultant intensity,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2 \cos\theta}$$

Since, $\theta \simeq 0 \Rightarrow \cos 0° = 1$

$$\therefore \quad I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

We see that, we get a maximum for two narrow slits separated by a distance a. **(2)**

7. The distance of the nth minimum from the centre of the screen is

$$x_n = \frac{nD\lambda}{d} \quad \ldots(i)$$

where, D = distance of slit from screen

λ = wavelength of the light

d = width of the slit for first minimum, $n = 1$

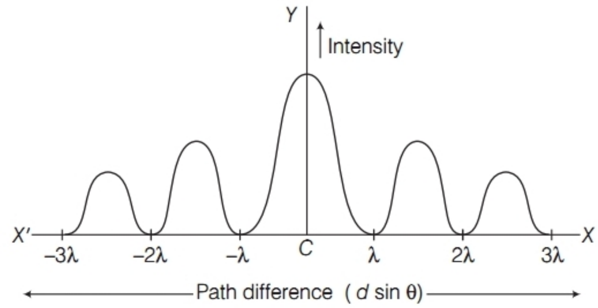
$$x_n = 2.5 \text{ mm (given)}$$
$$= 2.5 \times 10^{-3}\text{m}, D = 1 \text{ m}$$
$$\lambda = 500 \text{ nm}$$
$$= 500 \times 10^{-9}\text{m}$$

Putting this values in Eq. (i), we get

$$2.5 \times 10^{-3} = \frac{1(1)(500 \times 10^{-9})}{d}$$

$$\Rightarrow \quad d = 2 \times 10^{-4} \text{ m}$$
$$= 0.2 \text{ mm}$$ **(2)**

8. In case of single slit The diffraction pattern obtained on the screen consists of a central bright band having alternate dark and weak bright band of decreasing intensity on both sides. The diffraction pattern can be graphically represented as



Comparison of intensity distribution between interference and diffraction is given below.

(a) In interference, all bright fringes have same intensity, but in diffraction all the bright fringes are not of same intensity.

(b) In interference, the widths of all the fringes are same but in diffraction fringes are of different widths.

The point C corresponds to the position of central maxima and the position $-3\lambda, -2\lambda, -\lambda, \lambda, 2\lambda, 3\lambda \ldots$ are secondary minima. The above conditions for diffraction maxima and minima are exactly reverse of mathematical conditions for interference maxima and minima. **(1)**

9. Given, $\lambda = 6000 \text{ Å} = 6 \times 10^{-7}$ m and $d = 1 \times 10^{-4}$ m

Separation between slit and screen, $D = 1.5$ m.

The separation between two dark lines on either side of the central maxima = fringe width of central maxima $= \frac{2D\lambda}{d}$

$$= \frac{2 \times 1.5 \times 6 \times 10^{-7}}{1 \times 10^{-4}} = 18 \times 10^{-3} \text{ m} = 18 \text{ mm}$$ **(1)**

Angular Spread $= \lambda / d = \frac{6000 \times 10^{-10}}{1 \times 10^{-4}}$

$$= 6 \times 10^{-3}\text{m} = 6 \text{ mm}$$ **(1)**

10. (i) Refer to Sol. 8(i) on page 313. **(1)**

(ii) Refer to text on page 319

(Difference between the interference pattern and the diffraction pattern). **(2)**

11. (i) Refer to text on page 318 **(1)**

(Intensity of central maxima).

(ii) For nth secondary maxima, $y_n = (2n + 1)\frac{D\lambda}{2a}$

For first maximum, $n = 1$

$$\Rightarrow \quad y_1 = (2 + 1)\frac{D\lambda}{2a} = \frac{3D\lambda}{2a}$$ **(2)**

12. The angular width of central maxima is given by

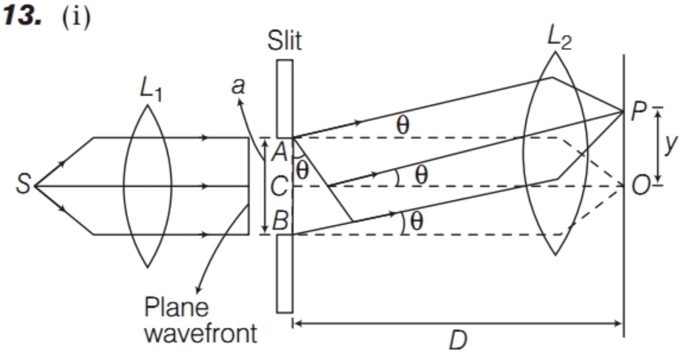
$$2\theta = \frac{2\lambda}{d}$$

(i) Since, $\theta \propto \lambda$ and $\lambda_o > \lambda_g$.
So, the angular width of central maxima for orange light is greater than that for green light.
(ii) Since, θ is independent of the distance of screen from the slit (D). So, the angular width remain same, when screen is moved closer to the slit. **(1)**

(iii) Since $\theta \propto \frac{1}{d}$.

So, when the slit width is decreased, the angular width of the central maximum will increase.

13. (i)

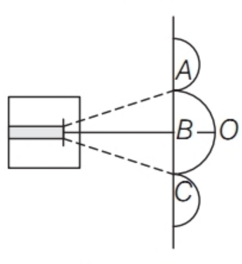


Consider a parallel beam of light from a lens falling on a slit AB. As, diffraction occurs, the pattern is focused on the screen with the help of lens L_2. We will obtain a diffraction pattern that is central maximum at the centre O, flanked by a number of dark and bright fringes called secondary maxima and minima.

Each point on the plane wavefront AB sends out the secondary wavelets in all directions.

The waves from points equidistant from the centre C, lying on the upper and lower half, reach point O with zero path difference and hence, reinforce each other producing maximum intensity at O. **(1)**

(ii) Let λ and d be the wavelength and slit width of diffracting system, respectively. Let O be the position of central maximum.



Condition for the first minimum is given by $$d\sin\theta = m\lambda \quad \text{...(i)}$$

Let θ be the angle of diffraction.

As, diffraction angle is small

$\therefore \quad \sin\theta \approx \theta$

For first diffraction minimum,

$$\theta = \theta_1 \quad \text{(let)}$$

For the first minimum, take $m = 1$

$$d\theta_1 = \lambda \Rightarrow \theta_1 = \lambda/d$$

Now, angular width, $AB = \theta_1$

Angular width, $BC = \theta_1$

Angular width, $AC = 2\theta_1$ **(1)**

(iii) On increasing the value of n, the part of slit contributing to the maximum decreases. **(1)**
Hence, the maximum becomes weaker.

14. (i) Refer to Sol 13 (ii) on page 324.

(ii) The intensity of maxima decreases as the order (n) or diffraction maxima increases. This is because, on dividing the slit into odd number of parts, the contributions of the corresponding (outermost) pairs cancel each other, leaving behind the contribution of only the innermost segment. e.g. for first maxima, slit is divided into three parts; out of these three parts of the slit, the contribution from first two parts cancel each other, only $\frac{1}{3}$rd position of slit contributes to the maxima of intensity. Similarly, for second maxima, slit is divided into five parts, contribution of first four parts will be zero. The remaining $\frac{1}{5}$th portion will only contribute for maxima and so on. **(1½)**

15. (i) If the width of each slit is comparable to the wavelength of light used, the interference pattern thus obtained in the double slit experiment is modified by diffraction from each of the two slits. **(1)**

(ii) Given that, wavelength of the light beam,

$$\lambda_1 = 590 \text{ nm} = 5.9 \times 10^{-7} \text{m}$$

Wavelength of another light beam,

$$\lambda_2 = 596 \text{ nm} = 5.96 \times 10^{-7} \text{m}$$

Distance of the slit from the screen,

$$D = 1.5 \text{ m}$$

Aperture of the slit $= d = 2 \times 10^{-4}$ m

For the first secondary maxima,

$$\sin\theta = \frac{3\lambda_1}{2d} = \frac{x_1}{D} \text{ or } x_1 = \frac{3\lambda_1 D}{2d}$$

and $x_2 = \frac{3\lambda_2 D}{2d}$

$\therefore$ Spacing between the positions of first secondary maxima of two sodium lines.

$$x_2 - x_1 = \frac{3D}{2d}(\lambda_2 - \lambda_1)$$

Substituting the value of all elements

$$= \frac{3 \times 1.5}{2 \times 2 \times 10^{-4}}(5.96 - 5.9) \times 10^{-7}$$

$$= 6.75 \times 10^{-5} \text{m}$$ **(2)**

16. (i) Two pin holes with two sodium lamps cannot produce coherent sources of light, so the phenomenon of interference cannot be observed. **(1)**

(ii) Refer to Sol. 14 on page 315. **(2)**

(iii) Given, $\lambda_1 = 590\text{ nm}, \lambda_2 = 596\text{ nm},$

$$d = 2 \times 10^{-6}\text{ m}, D = 1.5\text{ m}$$

Distance of secondary maxima from centre,

$$x = \frac{3}{2}\frac{D\lambda}{d}.$$

Spacing between the first two maxima of sodium light

$$\Rightarrow x_2 - x_1 = \frac{3D}{2d}(\lambda_2 - \lambda_1)$$

$$= \frac{3 \times 1.5}{2 \times 2 \times 10^{-6}}(596 - 590) \times 10^{-9}$$

$$= 6.75 \times 10^{-3}\text{ m}$$

$$= 6.75\text{ mm}$$ **(2)**

17. (i) **Difference between Interference and Diffraction**

(a) The interference pattern has a number of equally spaced bright and dark bands. Where as the diffraction pattern has a central bright maximum, which is twice as wide as the other maxima. The intensity falls as we go to successive maxima away from the centre on either side.

(b) We calculate the interference pattern by superposing two waves originating from the two narrow slites. The diffraction patter is a superposition of a continuous family of waves originating from each point on a single slit. **(2)**

(ii) Given, $\lambda = 620\text{ nm} = 620 \times 10^{-9}\text{ m}$

Aperture of slit, $b = 3\text{mm} = 3 \times 10^{-3}\text{ m}$

Distance between source and screen, $D = 1.5\text{ m}$

The distance of first order minima from central maxima, $x_1 = \frac{n\lambda D}{b} = \frac{\lambda D}{b}\ (n = 1)$

$$= \frac{620 \times 10^{-9} \times 1.5}{3 \times 10^{-3}} = 310 \times 10^{-6}\text{ m}$$

The distance of third order maxima from central maxima,

$$x_3 = \frac{(2n + 1)\lambda D}{2b}$$

$$= \frac{7\lambda D}{2b} = \frac{7 \times 620 \times 10^{-9} \times 1.5}{2 \times 3 \times 10^{-3}}$$

$$= 1085 \times 10^{-6}\text{ m}$$

Thus, distance between x_1 and x_3 is

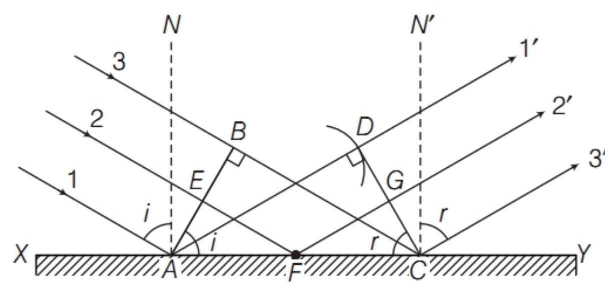
$$x = x_3 - x_1 = (1085 - 310) \times 10^{-6}$$

$$= 775 \times 10^{-6}\text{ m}$$ **(3)**

18. (i) **Wavefront** It is the locus of points (wavelets) having the same phase (a surface of constant phase) of oscillations. **(1)**

Laws of reflection at a plane surface (On Huygens' principle)

Let 1, 2, 3 be the incident rays and 1′, 2′, 3′ be the corresponding reflected rays.



Laws of reflection by Huygens' principle

If c is the speed of the light, t is the time taken by light to go from B to C or A to D or E to G through F, then

$$t = \frac{EF}{c} + \frac{FG}{c} \quad \text{...(i)}$$

In ΔAEF, $\sin i = \frac{EF}{AF}$

In ΔFGC, $\sin r = \frac{FG}{FC}$

or $t = \frac{AF \sin i}{c} + \frac{FC \sin r}{c}$

or $t = \frac{AC \sin r + AF(\sin i - \sin r)}{c}$

$(\because FC = AC - AF)$

For rays of light from different parts on the incident wavefront, the values of AF are different. But light from different points of the incident wavefront should take the same time to reach the corresponding points on the reflected wavefront.

So, t should not depend upon AF. This is possible only if $\sin i - \sin r = 0$

i.e. $\sin i = \sin r$ or

$$\angle i = \angle r \quad \ldots(ii)$$

which is the **first law of reflection.**

Further, the incident wavefront AB, the reflecting surface XY and the reflected wavefront CD are all perpendicular to the plane of the paper.

Therefore, incident ray, normal to the mirror XY and reflected ray all lie in the plane of the paper. This is **second law of reflection**. **(2)**

(ii) We know that, width of central maximum is given as $2y = 2D\lambda/a$

where, a = width of slit. When $a = 2a$

$\therefore$ Width of central maximum

$$= \frac{2D\lambda}{2a} = \frac{\lambda D}{a}$$

Thus, the width of central maximum became half when slit width is doubled. But in case of diffraction, intensity of central maxima does not changes with slit width. Thus, the intensity remains same in both cases. **(1)**

(iii) When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the obstacle because the waves diffracted from the edge of circular obstacle interfere constructively at the centre of the shadow resulting in the formation of a bright spot. **(1)**

19. (i) The features to distinguish is given as

(a) In Young's experiment, width of all the fringes are equal but in diffraction fringes, width of central fringe is twice the other fringes.

(b) The intensity of all the fringes are equal in interference fringe but intensity of fringes go on decreasing in diffraction as we go away from the central fringe. **(2½)**

(ii) Given, wavelength $(\lambda) = 500 \text{ nm} = 500 \times 10^{-9}$ m

Width of single slit $(d) = 0.2 \text{ mm} = 0.2 \times 10^{-3}$ m

Angular width of central fringe

$$= 2 \times \frac{\lambda}{d}$$

$$= \frac{2 \times 500 \times 10^{-9}}{0.2 \times 10^{-3}} = \frac{10^{-6}}{2 \times 10^{-4}}$$

$$= \frac{1}{200} = 5 \times 10^{-3} \text{ rad}$$

Let distance between the slit and screen be 1m. (which is not given in the problem but this data is necessary to solve the problem).

Linear width of central fringe of single slit

$$= 5 \times 10^{-3} \times 10^{3} \text{ mm}$$

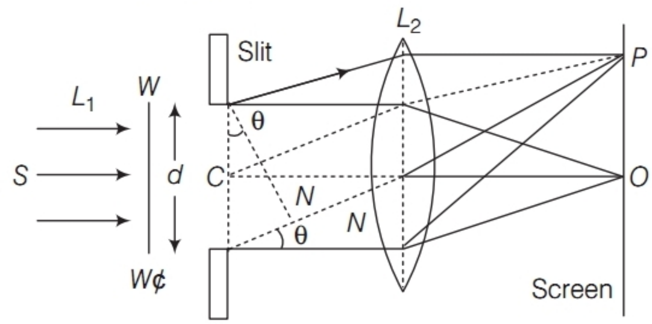
$$= 5 \text{ mm}$$

Number of double slit fringe accommodated in central fringe $= \frac{50}{5} = 10$ fringes. **(2½)**

20. **Phenomenon is diffraction** All the secondary wavelets going straight across the slit are focussed at the central point of the screen.

The wavelets from any two corresponding points of the two halves of the slit reach the point O in the same phase, they add constructively to produce a central bright fringe.

(i) The condition for first dark fringe is $d\sin\theta_1 = \lambda$. Similarly, the condition for second dark fringe will be $d\sin\theta_2 = 2\lambda$



Hence, the condition for nth dark fringe can be written as $d \sin\theta_n = n\lambda$, where $n = 1, 2, 3, \ldots$.

The directions of various minima are given by

$$\theta_n \approx \sin\theta_n = n\frac{\lambda}{d} \quad [\text{As } \lambda << d, \text{ so } \sin\theta_n = \theta_n]$$

Suppose the point P is so located that $\theta = \frac{3\lambda}{2}$

When $\theta = \theta_1'$, then $a\sin\theta_1' = \frac{3}{2}\lambda$

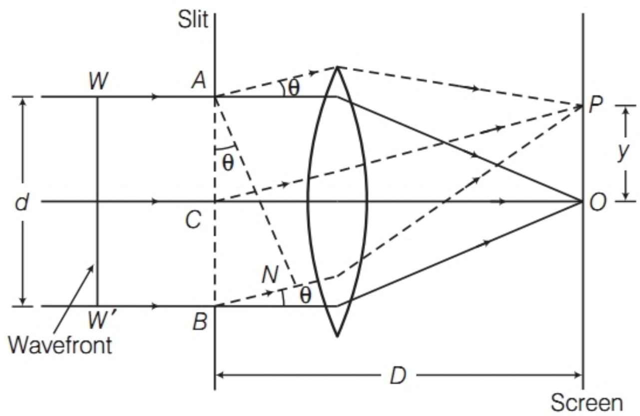
We can divide the slit into three equal parts. The path difference between two corresponding points of the first two parts will be $\frac{\lambda}{2}$. The wavelets from these points will interfere destructively.

However, the wavelets from the third part of the slit will contribute to some intensity forming a secondary maximum. The intensity of this maximum is much less than that of the central maximum. Thus, the condition for the first secondary maximum is $a\sin\theta_1' = \frac{3}{2}\lambda$ **(2½)**

(ii) The reason is that the intensity of the central maximum is due to the constructive interference of wavelets from all parts of the slit, the first secondary maximum is due to the contribution of wavelets from one-third part of the slit (wavelets from remaining two parts interfere destructively), the second secondary maximum is due to the contribution of wavelets from the one fifth part only (the remaining four parts interfere destructively) and so on. Hence, the intensity of secondary maximum decreases with the increases with the order n of the maximum. **(2)**

(iii) Width of central band halves. **(1/2)**

21. (i) **Diffraction of light at a single slit** A parallel beam of light with a plane wavefront WW' is made to fall on a single slit AB. As width of the slit $AB = d$ is of the order of wavelength of light, therefore, diffraction occurs on passing through the slit.



The wavelets from the single wavefront reach the centre O on the screen in same phase and hence, interfere constructively to give central maximum (bright fringe). **(1)**

The diffraction pattern obtained on the screen consists of a central bright band having alternate dark and weak bright band of decreasing intensity on both sides.

Consider a point P on the screen at which wavelets travelling in a direction making an angle θ with CO are brought to focus by the lens. The wavelets from points A and B will have a path difference equal to BN.

From the right angled ΔANB, we have

$$BN = AB\sin\theta \text{ or } BN = d\sin\theta$$

To establish the condition for secondary minima, the slit is divided into 2,4,6... equal parts such that corresponding wavelets from parts such that corresponding wavelets from successive regions interfere with path difference of $\lambda/2$ **(2)**

or for ***n*th secondary minimum**, the slit can be divided into $2n$ equal parts.

Hence, for nth secondary minimum, path difference $= d\sin\theta_n = n\lambda$

or $$\sin\theta_n = \frac{n\lambda}{d} \quad (n = 1,2,3,...)$$

To establish the condition for secondary maxima, the slit is divided into 3,5,7... equal parts such that corresponding wavelets from alternate regions interfere with path difference of $\lambda/2$

or for ***n*th secondary maximum**, the slit can be divided into $(2n+1)$ equal parts.

Hence, for nth secondary maximum

$$d\sin\theta_n = (2n+1)\frac{\lambda}{2} \quad (n = 1, 2, 3,...)$$

or $$\sin\theta_n = (2n+1)\frac{\lambda}{2d}$$ **(1)**

(ii) Refer to Sol. 16 (iii) on page 325

22. (i) (a) Interference is the result of interaction of light coming from two different wavefronts originating from two coherent sources, whereas diffraction pattern is the result of interaction of light coming from different parts of the same wavefront. **(1)**

(b) In interference,the fringes may or may not be the same width, while in diffraction, the fringes are always of varying widths. **(1)**

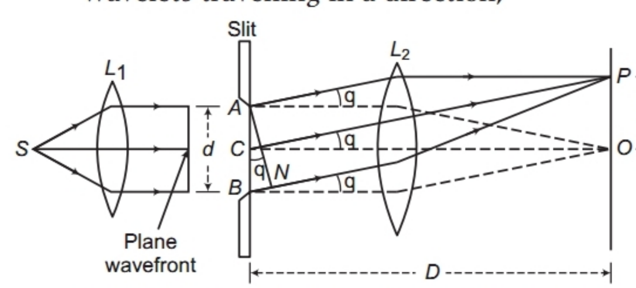
(c) In interference,the bright fringes are of the same intensity, while in diffraction, the bright fringes are of varying intensities. **(1)**

(ii) Distance of first minimum, $y = \frac{\lambda}{d}$

Slit width, $d = \frac{\lambda}{y}$

$$\therefore \quad a = \frac{500\times10^{-9}}{2.5\times10^{-4}} = 0.002\,\text{m}$$ **(2)**

23. (i) Consider a point P on the screen at which wavelets travelling in a direction,



(1)

making an angle θ with CO, are brought to focus by the lens. The wavelets from points A and B will have a path difference equal to BN.

From the right angled ΔANB, we have

$$BN = AB \sin\theta$$
$$BN = d\sin\theta \quad \ldots(\text{i})$$

Suppose, $BN = \lambda$ and $\theta = \theta_1$

Then, the above equation gives

[slit width $d = a$]

$$\lambda = d\sin\theta_1$$
$$\Rightarrow \quad \sin\theta_1 = \frac{\lambda}{d} \quad \ldots(\text{ii})$$

Such a point on the screen will be the position of first secondary minimum.

If $BN = 2\lambda$ and $\theta = \theta_2$, then

$$2\lambda = d\sin\theta_2$$
$$\sin\theta_2 = \frac{2\lambda}{d} \quad \ldots(\text{iii})$$

Such a point on the screen will be the position of second secondary minimum.

In general, for nth minimum at point P,

$$\sin\theta_n = \frac{n\lambda}{d} \quad \ldots(\text{iv})$$

If y_n is the distance of the nth minimum from the centre of the screen, then from right angled ΔCOP, we have

$$\tan\theta_n = \frac{OP}{CO} = \frac{y_n}{D} \quad \ldots(\text{v})$$

In case of θ_n is small, $\sin\theta_n \approx \tan\theta_n$

There Eqs. (iv) and (v) are given as

$$\frac{y_n}{D} = \frac{n\lambda}{d} \Rightarrow y_n = \frac{n\lambda D}{d}$$ **(2)**

If $BN = \frac{3\lambda}{2}$ and $\theta = \theta_1'$, then from Eq. (i), we have

$$\sin\theta_1' = \frac{3\lambda}{2d}$$

Such a point on the screen will be the position of the first secondary maximum.

Corresponding to path difference, if, $BN = \frac{5\lambda}{2}$ and $\theta = \theta_2'$, the second secondary maximum is produced.

In general, for the nth maximum at point P,

$$\sin\theta_n' = \frac{(2n+1)\lambda}{2d} \quad \ldots(\text{vi})$$

where, $n = 1, 2, 3, \ldots$ an integer.

If y_n is the distance of nth maximum from the centre of the screen, then the angular position of the nth maximum is given by,

$$\tan\theta_n' = \frac{y_n}{D} \quad \ldots(\text{vii})$$

In case of θ_n' is small, $\sin\theta_n' \approx \tan\theta_n'$

$$\Rightarrow \quad y_n' = \frac{(2n+1)\lambda D}{2d}$$

For $n = 1$, $\theta' = \frac{3\lambda}{2d}$

[From Eq. (vi), small angle approximation,

$$\sin\theta' = \theta' = \frac{(2n+1)\lambda}{2d}$$ **(1)**

This angle is midway between the two dark fringes. Divide the slit into three equal parts. If we take the first two-third part of the slit, then path difference between the two ends would be

$$\frac{2}{3}d \times \theta' = \lambda$$

The first two-third is divided into two halves which have path difference $\lambda/2$. The contribution due to these two halves is 180° out of phase and gets cancel.

Only the remaining one-third part of the slit contributes to the intensity at a point between the two minima which will be much weaker than the intensity of central maxima. Thus, with increase in the intensity, the maxima gets weaker.

(ii) As, the number of point sources increases, their contribution towards intensity also increases. Intensity varies as square of the slit width.

Thus, when the width of the slit is made double the original width, intensity will get four times of its original value.

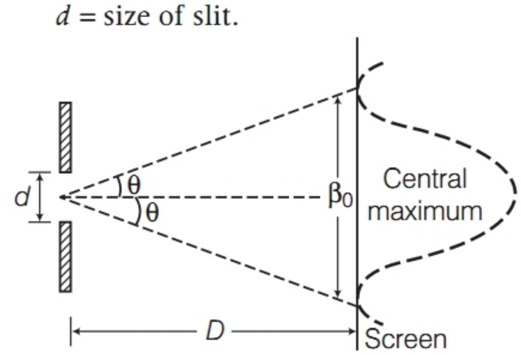
Width of central maximum is given by

$$\beta = 2D\lambda/d$$

where, D = distance between screen and slit

λ = wavelength of the light

d = size of slit.



So, with the increase in size of slit, the width of central maxima decreases. Hence, double the size of the slit would results as half the width of the central maxima. **(1)**

24. (i) **For Huygens' theory** Refer to text on page 300. **(1)**

Diffraction pattern Refer to Sol. 13 (i) on page 324. **(1)**

(ii) The angular width of central maximum is the angular separation between the directions of the first minima on the two sides of the central maxima.

The angular width of first minima on either side of central maximum is given by

$$\theta = \lambda / d \quad \textbf{(1)}$$

$\therefore$ The angular width of central maxima

$$= 2\lambda / d \quad \ldots(\text{i})$$

$\therefore$ Angular separation of nth secondary minimum, $\theta_n = n\lambda / d$

Angular separation of $(n+1)$th secondary minima, $\theta_{n+1} = (n+1)\dfrac{\lambda}{d}$

Therefore, the angular width of secondary maxima of nth order is equal to the difference of angular separation of $(n+1)$th and nth order secondary minima.

$\therefore$ Angular width of secondary maxima

$$= \theta_{n+1} - \theta_n = (n+1)\frac{\lambda}{d} - n\frac{\lambda}{d} = \frac{\lambda}{d} \quad \ldots(\text{ii})$$

From Eqs. (i) and (ii), we get

Angular width of first diffraction fringe is half of that of the central fringe. **(1)**

(iii) If a monochromatic source of light is replaced by white light, then coloured fringe pattern is obtained on the screen.

The central maximum will be white but other bands will be coloured. As bandwidth $\propto \lambda$, therefore red bandwidth will be wider than the violet bandwidth. **(1)**

25. (i) Refer to Sol. 13 on page 324. **(3)**

(ii) Size of central diffraction band is inversely proportional to the slit width i.e. size of central diffraction band $= \dfrac{2\lambda}{d}$ **(2)**