Explanations

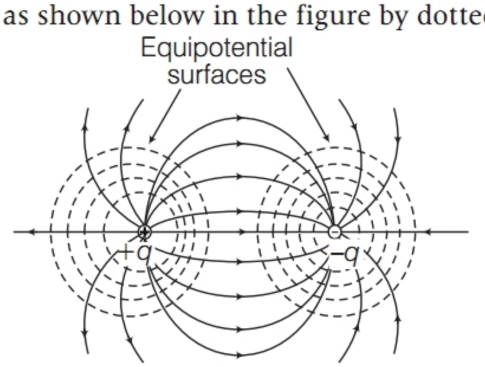
1. electric potential

2. decreasing

As, electric field lines starts from higher potential and ends at lower potential. So, when a proton is released from rest in electric field, then it moves towards the region of decreasing potential in that field. **(1)**

3. The work done in moving a charge particle between two points in a uniform electric field depends only on the position of initial point and final point. Hence, it is independent of the path followed by the particle. **(1)**

4. The equipotential surfaces for an electric dipole are as shown below in the figure by dotted lines.

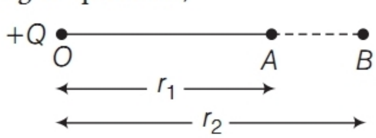


(1)

5. Refer to text and diagram on page 37 (Equipotential surface). **(1)**

6. In case of metallic sphere, charge given to it is mostly resides on its surface. Therefore, there is no difference whether the sphere is hollow or solid. As in both the cases, the charge that will reside will be the same. **(1)**

7. According to question,



$r_2 > r_1$ (given)

Potential at point A due to charge $+Q, (V_A) = KQ/r_1$
Potential at point B due to charge $+Q, (V_B) = KQ/r_2$ **(1/2)**

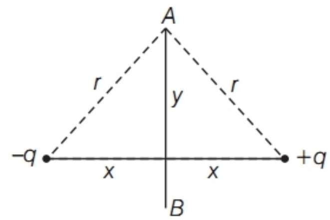
As, $V \propto \frac{1}{r}$ and $r_2 > r_1$. So, $V_A > V_B$.

Thus, $(V_A - V_B)$ is positive. **(1/2)**

8. According to question,

Total potential at point A due to $+q$ charge,

$$V_A = \frac{Kq}{r} = \frac{Kq}{(x^2 + y^2)^{\frac{1}{2}}}$$



Total potential at point A due to $-q$ charge,

$$V'_A = \frac{-Kq}{r} = \frac{-Kq}{(x^2 + y^2)^{1/2}}$$

So, net potential at $A = V_A + V'_A = 0$

Similarly at point B, potential will be 0.

So, net work done, $W = \Delta V \times q = 0$. **(1)**

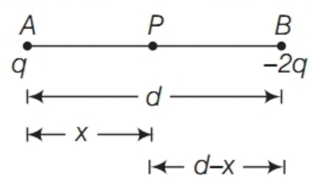
9. Electric field is always normal to the equipotential surface, because there is no work is done on the equipotential surface.

As $\quad W = q_0(V_A - V_B) = 0$

$\Rightarrow \quad V_A - V_B = 0$

Hence, $\quad -\int \mathbf{E} \cdot d\mathbf{l} = 0 \Rightarrow \mathbf{E} \perp d\mathbf{l}$ **(1)**

10. According to the question, $q_A = q$ and $q_B = -2q$



Let $PA = x$, $PB = d - x$

Hence, potential between P and A,

$$V_{PA} = \frac{Kq}{x}$$

Potential between P and B, $V_{PB} = \frac{K(-2q)}{(d-x)}$

As, the potential due to system of charges is zero.

Hence,

$$V_{PA} + V_{PB} = 0$$

$$\Rightarrow \quad \frac{Kq}{x} = \frac{2Kq}{(d-x)}$$

$$d - x = 2x \Rightarrow 3x = d$$

$$\Rightarrow \quad x = d/3$$ **(1)**

11. Work done by charge is given by

$W = q$ (potential at $Q -$ potential at P).

where, q = small positive charge **(1/2)**

The electric potential at a point of distant r due to the field created by a positive charge q is given by

$$V = q/4\pi\varepsilon_0 r \Rightarrow \quad V \propto \frac{1}{r}$$

$\because$ As, $r_P < r_Q \Rightarrow V_P > V_Q$

So, work done will be negative. **(1/2)**

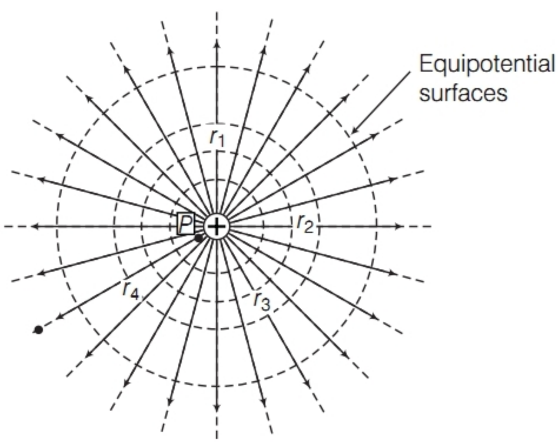
12. No work is done in moving the test charge from one point of an equipotential surface to the other. **(1/2)**

$\therefore \quad W_B - W_A = 0 = -\int \mathbf{E}\cdot dl$

$\Rightarrow \quad \mathbf{E}\cdot dl = 0$

Hence, $\quad \mathbf{E} \perp dl$ **(1/2)**

13. The equipotential surfaces produced by a single point charge are concentric spheres. In these concentric spheres shown below in the figure, the lines of force point radially outwards, so they are perpendicular to the equipotential surfaces at all points.

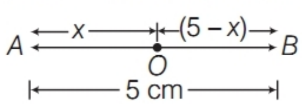


(1)

14. The amount of work done in carrying a charge on equipotential surface is always zero (potential difference is zero). **(1)**

15. Given, $q_A = 2\,\mu\text{C} = 2\times10^{-6}\text{C}$

$q_B = -2\,\mu\text{C} = -2\times10^{-6}\text{C}$ and $r = 5$ cm



As, potential at point O (equipotential surface) is zero.

$$V = \frac{2\times10^{-6}}{4\pi\varepsilon_0\, x\times10^{-2}} + \frac{-2\times10^{-6}}{4\pi\varepsilon_0 (5-x)\times10^{-2}} = 0$$

or $$\frac{2\times10^{-6}}{4\pi\varepsilon_0 x\times10^{-2}} = \frac{2\times10^{-6}}{4\pi\varepsilon_0 (5-x)\times10^{-2}}$$

or $\quad x = 5 - x$

$\Rightarrow \quad x = 2.5\,\text{cm}$ **(1)**

16. Since, electric field intensity inside the conductor is zero. So, electrostatic potential is a constant.

As, $\quad E = -\Delta V/\Delta r$

$\because \quad E = 0$ (inside conductor)

$\therefore \quad \Delta V = 0$

or $\quad V_2 - V_1 = 0, V_2 = V_1$

The potential at every point inside the conductor remains same and equal to the potential of the surface of the conductor. **(1)**

17. Electric field inside the hollow spherical charged conductor is zero. So, potential difference between two points inside the shell is zero. Thus, potential is constant and therefore, equal to its value at the surface, i.e. $V = (1/4\pi\varepsilon_0)(q/R)$ **(1)**

18. An equipotential surface is a surface at which electric potential at each point is same.

As, work done in moving a charged particle from one point to another is defined as

$$\Delta W = q(\Delta V)$$

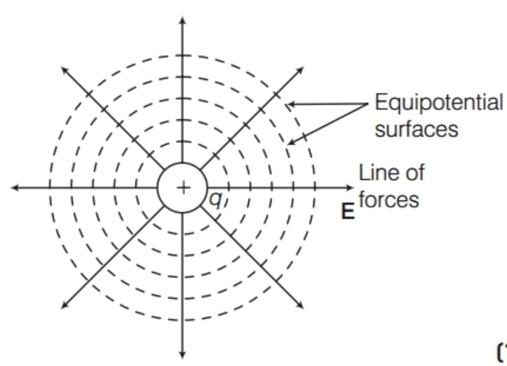
On an equipotential surface, the potential remains constant. So, ΔV = zero

$\Rightarrow$ Work done, $\Delta W = 0$ **(1)**

19. Electric field inside the hollow spherical charged conductor is zero. Thus, the potential is constant and equal to the value at the surface, i.e. equal to its value at the surface = 10V. **(1)**

20. No, two equipotential surfaces cannot intersect each other, because two normals can be drawn at intersecting point on two surfaces which give two directions of **E** at the same point which is impossible. **(1)**

21. Equipotential surfaces due to a single point charge are concentric sphere having charge at the centre are shown below.



(1)

22. JC^{-1} is the SI unit of electric potential. It is a scalar quantity. **(1)**

23. Refer to text on page 38 (Equipotential surface)

24. As we know, $\Delta V = -\int_{r_1}^{r_2} E \cdot dr$

Here, $E = 10r + 5$, $r_1 = 1$ m and $r_2 = 10$ m

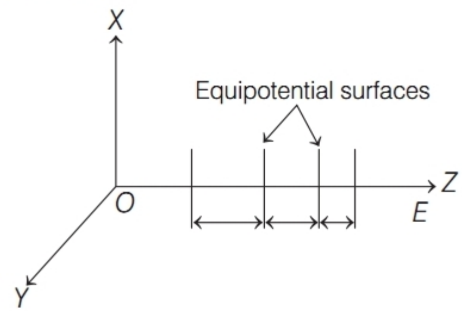
$$\Rightarrow V_1 - V_2 = \int_1^{10} (10r + 5)\, dr = \left[10\frac{r^2}{2} + 5r\right]_1^{10}$$

$$= \left[\frac{10(100-1)}{2} + 5(10-1)\right] = 495 + 45 = 540 \text{ V}$$

$$\Rightarrow \quad V_1 > V_2$$

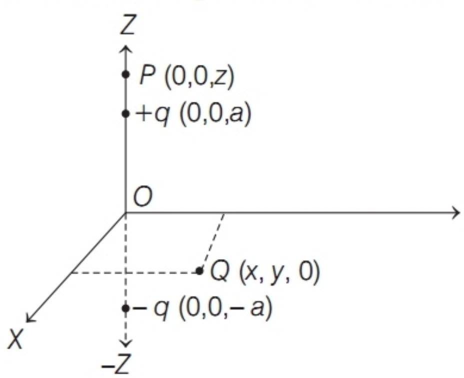
So, the electric potential decreases in moving from point at $r = 1$ m to a point at $r = 10$ m is 540 V.

25. (i) The equipotential surface are plane parallel to XY-plane. As, the field is increasing in magnitude. So, the spacing between surfaces decreases.



(1)

(ii) Let $P(0, 0, z)$ and $Q(x, y, 0)$ are two points on which electric potential are to be calculated.



Then, electrostatic potential at P

$$V_P = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(z-a)} - \frac{q}{(z+a)}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q \times 2a}{(z^2 - a^2)}\right] = \frac{1}{4\pi\varepsilon_0} \frac{p}{(z^2 - a^2)}$$

$[\because p = q \times 2a]$

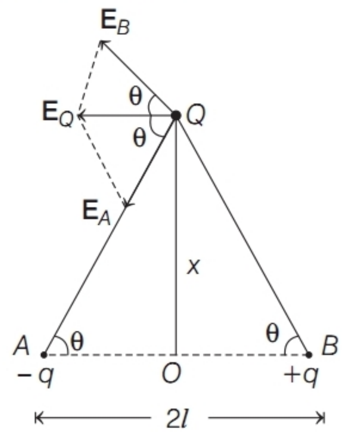
The electrostatic potential at Q is

$$V_Q = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{x^2 + y^2 - a^2}} - \frac{q}{\sqrt{x^2 + y^2 - a^2}}\right] = 0$$

(1)

26. (i) Refer to Sol. 4 on page 42. **(1/2)**

(ii) Electric field due to a dipole of consisting of two point charges $+q$ and $-q$ separated by a small distance $AB = 2l$ with its centre at O has a dipole moment **p**. According to the question, electric field of the dipole located on the perpendicular bisector of the line joining the two charges i.e. along its equatorial axis can be calculated as follow



According to the figure, dipole moment is

$$p = q\,(2l)$$

Hence, resultant field intensity at the point Q,

$$\mathbf{E}_Q = \mathbf{E}_A + \mathbf{E}_B$$

($\mathbf{E}_A$ and $\mathbf{E}_B$ are acting at an angle 2θ)

Here, $E_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(x^2 + l^2)}$

and $E_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(x^2 + l^2)}$

On resolving, vectors $\mathbf{E}_A \sin\theta$ and $\mathbf{E}_B \sin\theta$ are equal in magnitude and opposite to each other and hence cancel out.

The vectors $\mathbf{E}_A \cos\theta$ and $\mathbf{E}_B \cos\theta$ are acting along the same direction and hence add up.

$$\therefore E_Q = E_A \cos\theta + E_B \cos\theta \qquad [\because E_A = E_B]$$

$$= 2E_A \cos\theta$$

$$= \frac{2}{4\pi\varepsilon_0} \cdot \frac{q}{(x^2 + l^2)} \cdot \frac{l}{(x^2 + l^2)^{1/2}}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{2ql}{(x^2 + l^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{|\mathbf{p}|}{(x^2 + l^2)^{3/2}}$$

$(\because |\mathbf{p}| = q \times 2l)$

As the direction of E is along $QE \parallel BA$, i.e. opposite to **AB**, we can rewrite the expression as, $E_Q = -\frac{p}{4\pi\varepsilon_0 (x^2 + l^2)^{3/2}}$

If the dipole is short $2l >> x$, then

$$E_Q = \frac{1}{4\pi\varepsilon_0} \cdot \frac{|\mathbf{p}|}{x^3}$$ **(1½)**

27. Work done in bringing the charge q_1 from infinity to position r_1, $W_1 = q_1 V(r_1)$ ...(i) **(1/2)**

Work done in bringing charge q_2 to the position r_2

$$W_2 = q_2 V(r_2) + (q_1 q_2)/(4\pi\varepsilon_0 r_{12}) \quad \text{... (ii)}$$ **(1/2)**

Hence, total work done in assembling the two charges.

$$W = W_1 + W_2$$

From Eqs. (i) and (ii),we get

$$W = q_1 V(r_1) + q_2 V(r_2) + q_1 q_2 / 4\pi\varepsilon_0 r_{12}$$ **(1)**

28. Work done in moving a unit positive charge along distance δl,

$$|E_l|\delta l = V_A - V_B = V - (V + \delta V) = -\delta V$$

or $E = -\delta V/\delta l$ **(1)**

(i) Electric field is in the direction in which the potential decreases steepest. **(1/2)**

(ii) Magnitude of electric field is given by the change in the magnitude of potential per unit displacement normal to the equipotential surface at that point. **(1/2)**

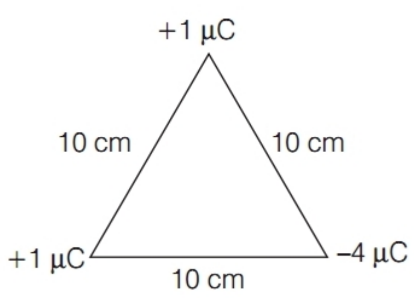
29. Given, charges, $q_1 = 1\mu C$, $q_2 = 1\mu C$ and $q_3 = -4\mu C$

Each side of equilateral triangle, $r = 10$ cm

Potential energy, U = Total work done to assemble the three charges.

$$U = W_1 + W_2 + W_3$$

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{1\times10^{-6}(-4\times10^{-6})}{0.10} + \frac{1\times10^{-6}(1\times10^{-6})}{0.10} + \frac{-4\times10^{-6}(1\times10^{-6})}{0.10}\right]$$ **(1)**



$$U = (1/4\pi\varepsilon_0)\times10^{-12}[-4\times10 + 10 - 4\times10]$$

$$U = -9\times10^9\times10^{-12}\times70$$

$$U = -0.630\text{ J}$$

Work done to dissociate the system of charges

$W = -U = 0.630$ J **(1)**

30. (i) $\because$ Electric field intensity and potential difference are related as

$$E = -\Delta V/\Delta r$$

$$\Delta V = -\mathbf{E}\cdot\Delta\mathbf{r} = -\mathbf{E}\cdot\Delta\mathbf{r}_{AB} - \mathbf{E}\cdot\Delta\mathbf{r}_{BC}$$

$$\Rightarrow \quad V_C - V_A = -4E + 0$$ **(1)**

$$\left[\begin{array}{l}\because \text{By pythagoras theorem}, AC^2 = AB^2 + BC^2 \\ \qquad \Rightarrow AB^2 = 5^2 - 3^2 \Rightarrow \Delta r = \sqrt{16} \Rightarrow \Delta r = 4\end{array}\right]$$

(ii) As $V_C - V_A = -4E$, which is negative

$\therefore \quad V_C < V_A$

Potential at point A is greater than potential at point C, as potential decreases along the direction of electric field. **(1)**

31. (i) Electric field due to a point charge,

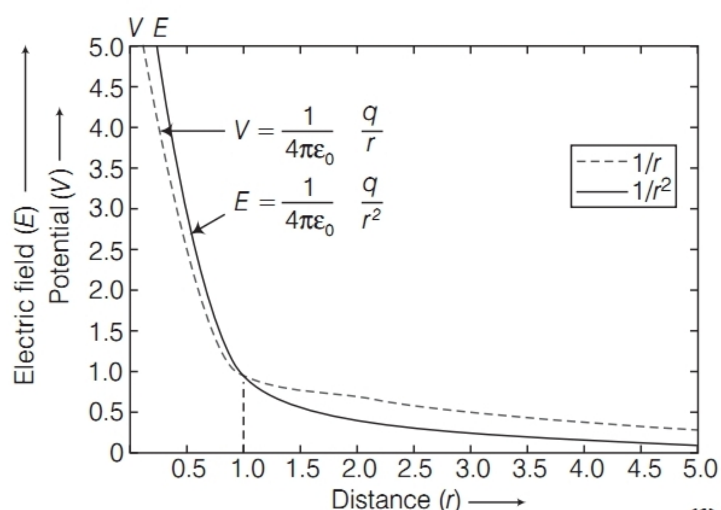
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}, E \propto \frac{1}{r^2}$$

(ii) Potential due to a point charge,

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} \Rightarrow V \propto \frac{1}{r}$$

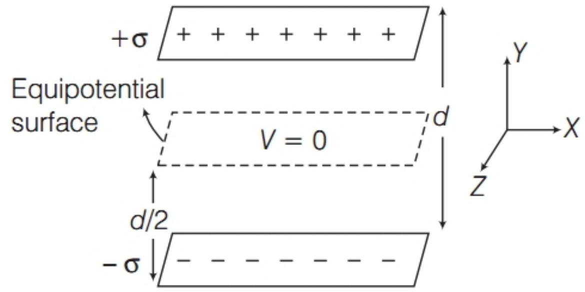
The variation of electrostatic potential with distance, i.e. $V \propto \frac{1}{r}$ and also the variation of electrostatic field with distance, i.e. $E \propto \frac{1}{r^2}$.

A graph showing variation of electric field (E) and electric potential (V) with distance (r) is shown below. **(1)**



(1)

32.



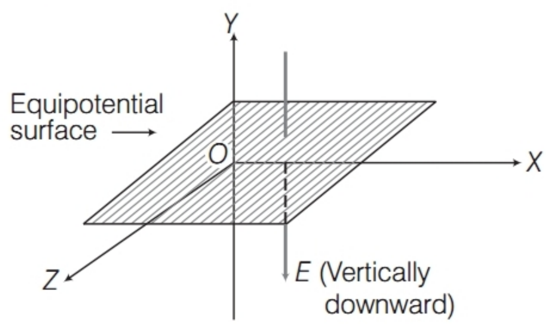
The equipotential surface is at a distance $d/2$ from either plate in XZ-plane. $-q$ charge experiences a force in a direction opposite to the direction of electric field.

$\therefore$ If $-q$ charge remains stationary in the field, then

$$qE = mg$$

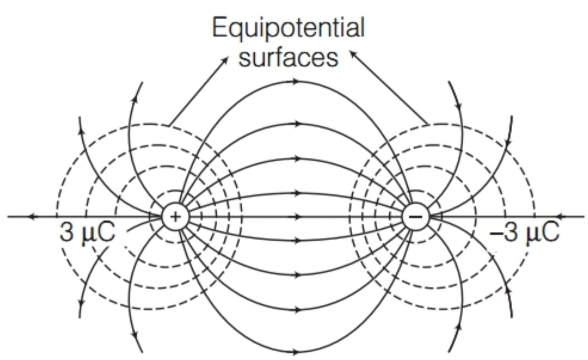
$$\Rightarrow \quad E = \frac{mg}{q}$$

The direction of electric field is along vertically downward direction in order to balance force due to field and its weight. The XZ-plane is so chosen that the direction of electric field due to two plates is along vertically downward direction, otherwise weight (mg) of charge particle could not be balanced. **(1)**



(1)

33. (i) Equipotential surfaces of the system



(1)

(ii) Equipotential surfaces get closer to each other near the point charges, because strong electric field is produced there.

$$E = -\frac{\Delta V}{\Delta r} \text{ and } E \propto -\frac{1}{\Delta r}$$

For a given equipotential surface, small Δr represents strong electric field and *vice-versa*. **(1)**

34. When two charged conducting spheres are connected, then charge flows between the two spheres till their potentials become same.

Electric potential on the surface of connected charged conducting spheres would be equal.

i.e. $$V_1 = V_2$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$$

[Assuming q_1 and q_2 are charges on the spheres connected to each other and r_1, r_2 are their radii.]

$$\frac{q_1}{r_1} = \frac{q_2}{r_2}$$

$$\Rightarrow \quad \frac{q_1}{q_2} = \frac{r_1}{r_2} \quad \ldots\text{(i)}$$

(1)

Now, ratio of electric field intensities

$$\frac{E_1}{E_2} = \frac{1/4\pi\varepsilon_0 \cdot q_1/r_1^2}{1/4\pi\varepsilon_0 \cdot q_2/r_2^2} = \frac{q_1}{q_2} \times \frac{r_2^2}{r_1^2}$$

$$\frac{E_1}{E_2} = \left(\frac{q_1}{q_2}\right) \times \frac{r_2^2}{r_1^2} = \frac{r_1}{r_2} \times \frac{r_2^2}{r_1^2} \text{ [From Eq. (i)]}$$

$$\frac{E_1}{E_2} = \frac{r_2}{r_1}$$

(1)

35. (i) The direction of electric field is perpendicular to the equipotential surface.

Hence, the direction of electric field is along X-axis, as it should be perpendicular to equipotential surface lying in YZ- plane.

(ii) Length of the dipole = $2b$

As dipole's axis is along the Y-axis. **(1)**

$\therefore$ Electric dipole moment, $\mathbf{p} = q(2b)\,\hat{\mathbf{j}}$

Electric field, $\mathbf{E} = E\,\hat{\mathbf{i}}$

$$\because \quad \tau = \mathbf{p} \times \mathbf{E} = q(2b)\,\hat{\mathbf{j}} \times E\,\hat{\mathbf{i}}$$

$$= +\,2qbE(\hat{\mathbf{j}} \times \hat{\mathbf{i}}) = 2qbE(-\hat{\mathbf{k}})$$

$\because$ Torque, $|\tau| = 2qbE$ **(1)**

Alternative method

$\mathbf{E}$ is directed along X-axis.

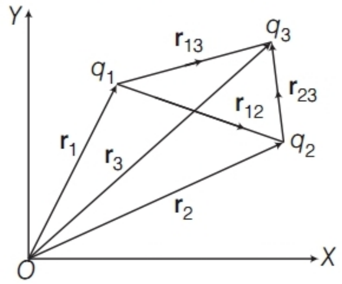
Dipole moment $p = q(2b)$ from $(0, -b, 0)$ to $(0, b, 0)$, i.e. along Y-axis. **(1)**

$\because$ Angle between $\mathbf{p}$ and $\mathbf{E}$ is 90°

$\therefore$ Torque on dipole, $\tau_{max} = pE\sin 90° = q(2b)E \times 1$

$\therefore$ Torque, $\tau = \tau_{max} = 2qbE$ **(1)**

36. Let three point charges q_1, q_2 and q_3 have position vectors $\mathbf{r}_1$, $\mathbf{r}_2$ and $\mathbf{r}_3$, respectively.



Potential energy of the charges q_1 and q_2,

$$U_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{|\mathbf{r}_{12}|} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{|\mathbf{r}_2 - \mathbf{r}_1|}$$

Similarly, $U_{23} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2 q_3}{|\mathbf{r}_3 - \mathbf{r}_2|}$

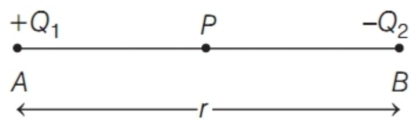
$$\Rightarrow \quad U_{31} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_3}{|\mathbf{r}_3 - \mathbf{r}_1|} \qquad \textbf{(1)}$$

$\therefore$ Net potential energy of the system,

$$U = U_{12} + U_{23} + U_{31}$$

$$U = \frac{1}{4\pi\varepsilon_0} \left[\frac{q_1 q_2}{|\mathbf{r}_2 - \mathbf{r}_1|} + \frac{q_2 q_3}{|\mathbf{r}_3 - \mathbf{r}_2|} + \frac{q_1 q_3}{|\mathbf{r}_3 - \mathbf{r}_1|} \right] \qquad \textbf{(1)}$$

37. (i) The charges $+Q_1$ and $-Q_2$ are placed at A and B respectively as shown



Let P be the mid-point of line joining A and B. The potential at P due to charge $+Q_1$ is

$$V_1 = \frac{Q_1}{4\pi\varepsilon_0 r/2}$$

and due to charge $-Q_2$ is

$$V_2 = \frac{-Q_2}{4\pi\varepsilon_0 r/2}$$

The resultant potential at P is

$$V = V_1 + V_2 = \frac{Q_1}{4\pi\varepsilon_0 r/2} - \frac{Q_2}{4\pi\varepsilon_0 r/2}$$

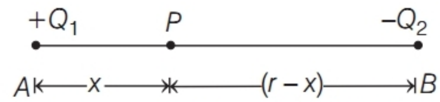
$$= \frac{2}{4\pi\varepsilon_0 r}(Q_1 - Q_2)$$

The work done to place a charge Q_3 at P is

$$W = Q_3 V = \frac{2Q_3(Q_1 - Q_2)}{4\pi\varepsilon_0 r}$$

or $$W = \frac{Q_3(Q_1 - Q_2)}{2\pi\varepsilon_0 r}$$

(ii) Let x be the distance from charge $+Q_1$ on the line joining the two charges at which the work done will be zero as shown below.



So, the potential at point P due to charge $+Q_1$ at A is equal to potential due to charge $-Q_2$ at B i.e.

$$V_{Q_1} = V_{Q_2}$$

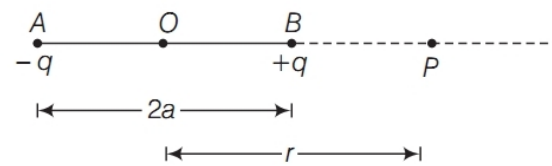
$$\Rightarrow \quad \frac{Q_1}{4\pi\varepsilon_0 x} = \frac{-Q_2}{4\pi\varepsilon_0 (r-x)} \Rightarrow \frac{Q_1}{x} = -\frac{Q_2}{r-x}$$

$$\Rightarrow rQ_1 - xQ_1 = -Q_2 x$$

or $$x = \left(\frac{Q_1}{Q_1 - Q_2}\right) r$$

38. (i) Refer to Sol. 25 (i) on page 44. **(1)**

(ii) Let us consider an electric dipole consisting of charges $+q$ and $-q$ separated by a distance $2a$. O be the centre of dipole and P be the on the axis of the dipole as shown in the figure given below



Electric potential at point P due to the dipole will be $V = V_1 + V_2$

$$= \frac{1}{4\pi\varepsilon_0} \frac{(-q)}{AP} + \frac{1}{4\pi\varepsilon_0} \frac{(+q)}{BP}$$

$$= \frac{-q}{4\pi\varepsilon_0 (r+a)} + \frac{1}{4\pi\varepsilon_0 (r-a)}$$

$$= \frac{q}{4\pi\varepsilon_0} \left(\frac{1}{r-a} - \frac{1}{r+a} \right)$$

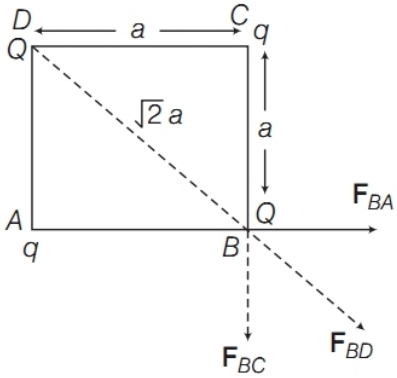
$$= \frac{q}{4\pi\varepsilon_0} \left(\frac{r+a-r+a}{r^2 - a^2} \right)$$

$$= \frac{q}{4\pi\varepsilon_0} \left(\frac{+2a}{r^2 - a^2} \right)$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^2 - a^2} \qquad [\because p = q \times 2a]$$

and for short dipole $a^2 << r^2 \Rightarrow V = \frac{1}{4\pi\varepsilon_0} \frac{p}{r^2}$ **(2)**

39. (i) Force acting on charge Q placed at point B is due to charges placed at points A, C and D.



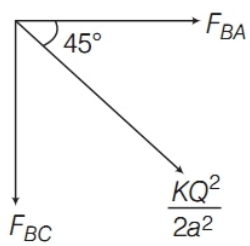
(1)

Here, magnitude of force on charge Q at point B due to charge q at point A is $F_{BA} = \frac{KQq}{a^2}$

Similarly, magnitude of force on charge Q at point B due to charge q at point C is $F_{BC} = \frac{KQq}{a^2}$

Also, the magnitude of force on charge Q at point B due charge Q at point D is

$$F_{BD} = \frac{KQ^2}{(\sqrt{2}\,a)^2} = \frac{KQ^2}{2a^2}$$



Let F is resultant of F_{BA} and F_{BC}.

$$\therefore \quad F = \sqrt{2}\cdot\frac{KQq}{a^2} \quad \left[\text{as } F_{BA} = F_{BC} = \frac{kQq}{a^2}\right]$$

$\therefore$ The resultant electric force on charge Q is

$$F_{\text{net}} = F + \frac{KQ^2}{2a^2} = \sqrt{2}\,\frac{KQq}{a^2} + \frac{KQ^2}{2a^2}$$

$$= \frac{KQ}{a^2}\left(\sqrt{2}q + \frac{Q}{2}\right)\text{N}$$

(1)

(ii) The potential energy of the system is given by

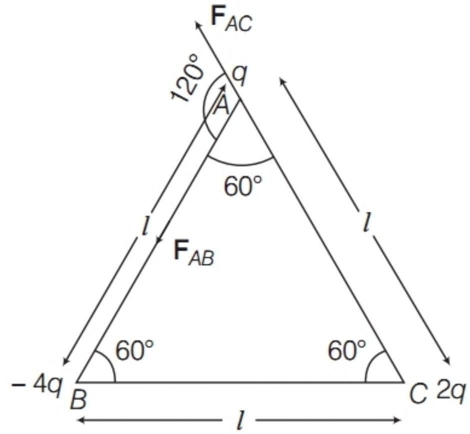
$$U = U_{AB} + U_{BC} + U_{CD} + U_{DA} + U_{AC} + U_{BD}$$

$$= \frac{KQq}{a} + \frac{KQq}{a} + \frac{KQq}{a} + \frac{KQq}{a} + \frac{Kq^2}{\sqrt{2}a} + \frac{KQ^2}{\sqrt{2}a}$$

$$= \left[4\left(\frac{KQq}{a}\right) + \frac{Kq^2}{\sqrt{2}a} + \frac{KQ^2}{\sqrt{2}a}\right]$$

(1)

40. (i) Force acting on the charge q placed at A is due to the charges placed at points B and C.



From the given figure, magnitude of the force on charge q at A due to charge $2q$ at point C is given as

$$F_{AC} = \frac{K(q)(2q)}{l^2} = F \text{ (say)}$$

Similarly, magnitude of force on charge q at point A due charge $-4q$ at point B is

$$F_{AB} = \frac{K(4q)q}{l^2}, = 2F \qquad (\because F_{AB} = 2F_{AC})$$

$$\therefore \quad F_{\text{res}} = \sqrt{F^2 + (2F)^2 + 2(F)(2F)\cos 120^\circ}$$

$$= \sqrt{F^2 + 4F^2 + 4F^2\left(-\frac{1}{2}\right)}$$

$$\left(\because \cos 120^\circ = -\frac{1}{2}\right)$$

$$= \sqrt{F^2 + 2F^2} = \sqrt{3}F$$

$$\therefore \quad F_{\text{res}} = \sqrt{3} \times \frac{2Kq^2}{l^2}\text{ N}$$

(1½)

(ii) The amount of the work done to separate the charges at infinite = Potential energy of the system

$$\therefore \quad U = U_{AB} + U_{BC} + U_{AC}$$

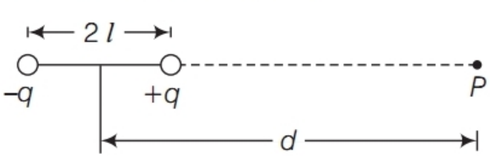
$$= \frac{K(-4q)q}{l} + \frac{K(-4q)(2q)}{l} + \frac{K(q)(2q)}{l}$$

$$= \frac{-4Kq^2}{l} - \frac{8Kq^2}{l} + \frac{2Kq^2}{l}$$

$$U = \frac{-10Kq^2}{l}\text{ J}$$

(1½)

41. (i) Let electric potential to be obtained at point P lying on the axis of dipole at distance d from the centre of the dipole.



Potential at P due to $+q$ charge
$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(d-l)}$

Potential at P due to $-q$ charge
$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0} \frac{-q}{(d+l)}$

Total potential at P due to dipole
$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(d-l)} - \frac{1}{(d+l)}\right]$
$= q \times 2l/4\pi\varepsilon_0\,(d^2 - l^2) \; = p/4\pi\varepsilon_0\,(d^2 - l^2)$

where, $p = q \cdot 2l$

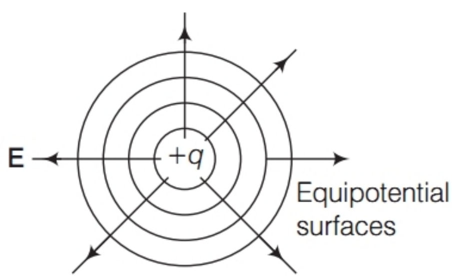
If $l << d$, then

$$V = \frac{1}{4\pi\varepsilon_0} \frac{p}{d^2}$$ **(1½)**

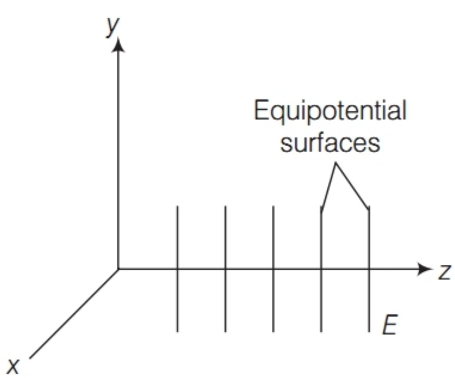
(ii) Refer to Sol. 4 on page 42. **(1½)**

42. Any surface that has same electric potential at every point on it is called **equipotential surface**.

(i) Equipotential surface in case of single point charge **(1)**



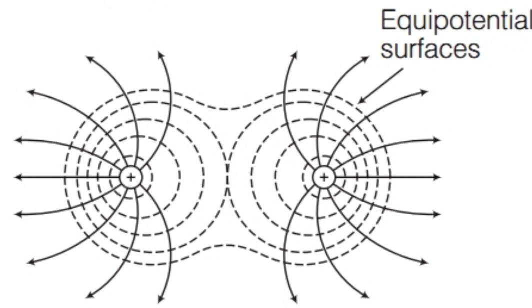
(ii) Equipotential surfaces when the electric field is in z-direction.



As electric field is constant in z-direction. So, equipotential surfaces are equispaced. The equipotential surfaces due to a single point charge is represented by concentric spherical shells of increasing radius, so they are not equidistant. **(1)**

(iii) No, the electric field does not exist tangentially to an equipotential surface because no work done in moving a charge from one point to other on equipotential surface. This indicates that the component of electric field along the equipotential surface is zero. Hence, the equipotential surface is perpendicular to field lines. **(1)**

43. (i) *The figure of equipotential surfaces of two identical positive charges is given below*



(1½)

(ii) By definition, electric potential energy of any charge q placed in the region of electric field is equal to the work done in bringing a charge q from infinity to that point and given by

$$U = qV$$ **(1½)**

$$\therefore \quad U = q_1 V(\mathbf{r}_1) + q_2 V(\mathbf{r}_2)$$

44. (i) Refer to Sol. 25(ii) on page 44.

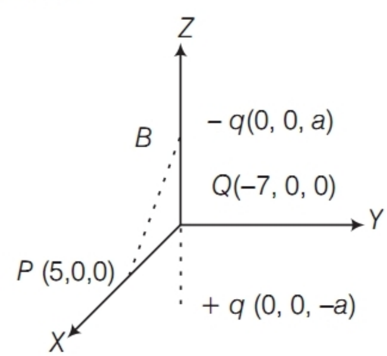
(ii) Every point on X-axis is on equatorial line of electric dipole (system of two unlike charges)

$\therefore$ Potential on it is 0.

No work is done on moving a test charge,

$$W = q\Delta V = q \times 0 = 0 \qquad [\because \Delta V = 0]$$

$\therefore$ Work done in moving a charge on equipotential surface is 0.



(2½)

(iii) The potential difference in moving a test charge between two points is independent of path followed and is only depend on its initial and final points. So, answer will not change.

(iv) When system is placed external uniform field E, then potential energy, $U = -pE\cos\theta$

In unstable equilibrium $\theta = 180°$, then

$U = -pE\cos 180° = pE = 2aqE$ **(2½)**

Explanations

1. In steady state, the electric flux between plates of a capacitor is constant.

$\therefore$ Displacement current,

$$i_d = \varepsilon_0 \frac{d\phi_E}{dt} \text{ and } \frac{d\phi_E}{dt} = 0$$

$$\Rightarrow \qquad i_d = 0$$

So, there is no current between plates when steady state is reached.

During charging, the flux between plates of capacitor is increasing.

$$\therefore \qquad \frac{d\phi_E}{dt} \neq 0$$

Hence, a displacement current exists in the capacitor which is $i_d = \varepsilon_0 \dfrac{d\phi_E}{dt}$. **(1)**

2. Line B corresponds to C_1 because slope (q *versus* V) of B is less than slope of A, i.e. $C_1 < C_2$. ($\therefore A_1 < A_2$) **(1)**

3. Dielectrics are non-conductors and do not have free electrons at all. While conductor has free electrons which make it able to pass the electricity through it. **(1)**

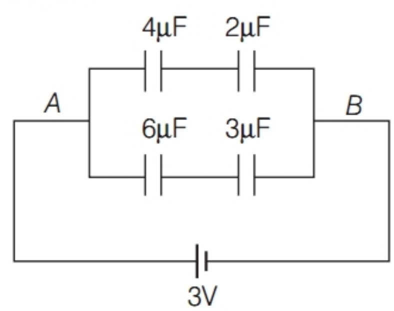
4. **Dielectric constant** It is defined as the ratio of capacity of a capacitor when a medium other than vacuum or air present between the plates of a capacitor to the capacity of a capacitor when air/vacuum is present between the plates of a capacitor.

i.e. $$K = \frac{C_{\text{dielectric}}}{C_{\text{vacuum}}}$$ **(1/2)**

It is a unitless quantity and also a dimensionless quantity. **(1/2)**

5. Refer to text given on page 53. **(2)**

6. As, the given network is like a balanced Wheatstone bridge, so no current flows through the middle wire and the network becomes as shown below



So, equivalent capacitance of upper arm (series combination) is

$$C_1 = \frac{4 \times 2}{4 + 2} = \frac{8}{6} = \frac{4}{3} \mu\text{F}$$

and of lower arm is $C_2 = \dfrac{6 \times 3}{6 + 3} = \dfrac{18}{9} = 2\mu\text{F}$

The net capacitance of the network is

$$C_{\text{net}} = C_1 + C_2 \quad \text{(for parallel combination)}$$

$$= \frac{4}{3} + 2 = \frac{10}{3} \mu F$$

$\therefore$ Total charge, $Q = C_{net} \times V = \frac{10}{3} \times 3 = 10\, \mu C$ **(2)**

7. Given $C = 100\, \mu F$, $d = 4$ mm,

$t = 4$ mm, $V = 200$ V

(i) $d' = 2d$, $K = E_r = 5$

$$C = \frac{\varepsilon_0 A}{d}$$

$$\Rightarrow 100 \times 10^{-6} = \frac{8.85 \times 10^{-12} \times A}{4 \times 10^{-3}}$$

$$\Rightarrow \quad A = 45.2 \times 10^3 \text{ m}^2$$

$$C' = \frac{E_0 A}{2d - t + \frac{t}{K}}$$

$$= \frac{8.85 \times 10^{-12} \times 45.2 \times 10^3}{\left(8 - 4 + \frac{4}{5}\right) \times 10^{-3}}$$

$$= 83.33 \mu F$$

(ii) Charge on capacitor, when 200 V is applied

$$q = C_0 V_0 = 100 \times 10^{-6} \times 200 = 2 \times 10^{-2} C$$

Even after the battery is removed, the charge of 2×10^{-2} C on the capacitor plate remains same.

So, $C_0 V_0 = C'V'$

$$\Rightarrow V' = \frac{C_0 V_0}{C'} = \frac{2 \times 10^{-2}}{83.33 \times 10^{-6}} \simeq 240 \text{ V}$$

$$E_0 = \frac{V_0}{d} = \frac{200}{4 \times 10^{-3}} = 50 \times 10^3 \text{ V/m}$$

$$E' = \frac{V'}{2d} = \frac{240}{2 \times 4 \times 10^{-3}} = 30 \times 10^3 \text{ V/m}$$ **(1)**

(iii) $\overline{U} = \frac{1}{2} \varepsilon_0 E_0^2 = \frac{1}{2} \times 8.85 \times 10^{-12} \times (50 \times 10^3)^2$

$$= 11067 \times 10^{-6} \text{ J/m}^3$$

$$(\overline{U})' = \frac{1}{2} \varepsilon_0 (E')^2 = \frac{1}{2} \times 8.85 \times 10^{-12} \times (30 \times 10^3)^2$$

$$= 3982.5 \times 10^{-6} \text{ J / m}^3$$ **(1)**

8. (i) C_1 and C_2 are in parallel combination.

$$\therefore \quad C' = C_1 + C_2 = 3 + 6 = 9\, \mu F$$

Now, C', C_3 and C_4 are in series. So, net capacitance is

$$\frac{1}{C} = \frac{1}{C'} + \frac{1}{C_3} + \frac{1}{C_4} = \frac{1}{9} + \frac{1}{4} + \frac{1}{12}$$

$$= \frac{16}{36} \Rightarrow C = \frac{9}{4} \mu F$$

(ii) (a) Given, $Q_1 = 6\, \mu C$

Now, potential across C_1

$$V = \frac{Q_1}{C_1} = \frac{6}{3} = 2 \text{ V}$$

Thus, charge on C_2

$$Q_1 = C_2 V = 6 \times 2 = 12\, \mu C$$

Total charge on C_1 and C_2, $\quad Q = 12 + 6 = 18 \mu C$

As charge is same in series combination,

$\therefore$ Charge on C_3 and C_4 is $18\, \mu C$ each. **(1)**

(b) Total capacitance,

$$\frac{1}{C''} = \frac{1}{4} + \frac{1}{12} = \frac{1}{3}$$

$$\Rightarrow \quad C'' = 3 \mu F$$

Thus, total energy stored in them is

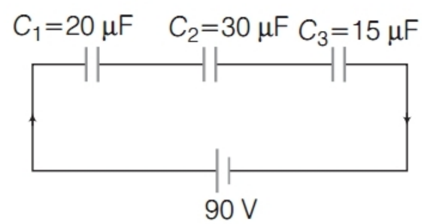
$$U = \frac{1}{2} \frac{Q^2}{C''} = \frac{1}{2} \frac{(18)^2}{3} \times 10^{-6}$$

$$= 54 \times 10^{-6} \text{ J}$$ **(1)**

9. For a series combination of three capacitors C_1, C_2 and C_3, the equivalent capacitance C_{eq} will be

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\Rightarrow \quad \frac{1}{C_{eq}} = \frac{1}{20} + \frac{1}{30} + \frac{1}{15}$$



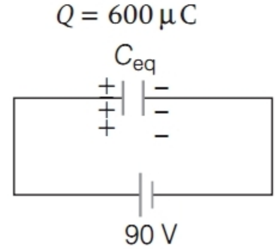
$$\Rightarrow \quad \frac{1}{C_{eq}} = \frac{3 + 2 + 4}{60}$$

$$\Rightarrow \quad C_{eq} = \frac{60}{9} \mu F = \frac{20}{3} \mu F$$ **(1)**

Charge on equivalent capacitor,

$$Q = C_{eq} V = \frac{20}{3} \times 10^{-6} \times 90$$

$$\Rightarrow \quad Q = 600\, \mu C$$



Charge on each capacitor is same as they are in series.

Now, potential drop across C_2

$$V_2 = \frac{Q}{C_2} = \frac{600 \times 10^{-6}}{30 \times 10^{-6}} = 20 \text{ V}$$

Hence, work done is stored as electric potential energy U of capacitor is

$$U = \frac{1}{2} C_2 V_2^2$$

$$U = \frac{1}{2} \times 30 \times 10^{-6} \times (20)^2 = 6 \times 10^{-3} \text{ J}$$ **(1)**

10. Let q be the charge on the charged capacitor.

$\therefore$ Energy stored in it is given by $U = q^2/2C$

When another uncharged similar capacitor is connected, then the net capacitance of the system is given by $C' = 2C$ **(1)**

The charge on the system remains constant. So, the energy stored in the system is given by

$$U' = \frac{q^2}{2C'} = \frac{q^2}{4C} \quad [\because C' = 2C]$$

Thus, the required ratio is given by

$$\frac{U'}{U} = \frac{q^2/4C}{q^2/2C} = \frac{1}{2}$$ **(1)**

11. (i) Given, $C_1 = 2C_2$ ...(i)

Net capacitance before filling the gap with dielectric slab is given by

$$C_{\text{initial}} = C_1 + C_2 \quad \text{[from Eq. (i)]}$$
$$C_{\text{initial}} = 2C_2 + C_2 = 3C_2 \quad \text{...(ii)}$$

Net capacitance after filling the gap with dielectric slab of electric constant K

$$C_{\text{initial}} = KC_1 + KC_2 = K(C_1 + C_2) \quad \text{[from Eq. (ii)]}$$
$$C_{\text{final}} = 3KC_2 \quad \text{...(iii)}$$

Ratio of net capacitance is given by

$$\frac{C_{\text{initial}}}{C_{\text{final}}} = \frac{3C_2}{3KC_2} = \frac{1}{K} \quad \text{[from Eqs. (ii) and (iii)]}$$ **(1)**

(ii) Energy stored in the combination before introduction of dielectric slab

$$U_{\text{initial}} = Q^2/3C_2 \quad \text{...(iv)}$$

Energy stored in the combination after introduction of dielectric slab.

$$U_{\text{final}} = Q^2/3KC_2 \quad \text{... (v)}$$

Ratio of energies stored,

$$\frac{U_{\text{initial}}}{U_{\text{final}}} = \frac{K}{1} \quad \text{[from Eqs. (iv) and (v)]}$$ **(1)**

12. Given, $C_1 = C_2/2$ ...(i)

Hence, $C_{\text{initial}} = C_1 + C_2 = C_2/2 + C_2 = 3C_2/2$ ...(ii)

(i) Net capacitance after filling the gap with dielectric K, $C_{\text{initial}} = \dfrac{3KC_2}{2}$ **(1)**

(ii) Energy stored, $U_i = \dfrac{q^2}{3C_2/2} = \dfrac{2q^2}{3C_2}$

and $U_f = \dfrac{q^2}{3KC_2/2} = \dfrac{2q^2}{3KC_2}$

$$\therefore \quad \frac{U_i}{U_f} = \frac{K}{1} \text{ or } U_i : U_f = K:1$$ **(1)**

13. Total current through the circuit is given by

$$I = V/R$$

Here, $V = 2\text{V}$ and $R = (10 + 20)\,\Omega = 30\,\Omega$

$$\therefore \quad I = \frac{2}{30} = \frac{1}{15} \text{ A}$$ **(1)**

Voltage across $10\,\Omega$ resistor $= I(10) = 10/15 = \dfrac{2}{3}\text{V}$

Charge on the capacitor is given by

$Q = CV = (6 \times 10^{-6}) \times 2/3 = 4\,\mu\text{C}$ **(1)**

14. Initially, when there is a vacuum between two plates, then capacitance of the plate is

$$C_0 = \frac{\varepsilon_0 A}{d}$$

where, A is the area of parallel plates.

Suppose that the capacitor is connected to a battery, an electric field E_0 is produced. If we insert the dielectric slab of thickness $t = d/2$, the electric field reduces to E.

Now, the gap between plates is divided in two parts. For distance t there is electric field E and for the remaining distance $(d - t)$ the electric field is E_0. **(1)**

If V be the potential difference between the plates of the capacitor, then

$$V = Et + E_0(d - t)$$

$$V = \frac{Ed}{2} + \frac{E_0 d}{2} = \frac{d}{2}(E + E_0) \quad \left[\because t = \frac{d}{2}\right]$$

$$\Rightarrow \quad V = \frac{d}{2}\left(\frac{E_0}{K} + E_0\right)$$

$$= \frac{dE_0}{2K}(K+1) \quad \left[\text{As, } \frac{E_0}{E} = K\right]$$

Now, $E_0 = \dfrac{\sigma}{\varepsilon_0} = \dfrac{q}{\varepsilon_0 A}$

$$\Rightarrow \quad V = \frac{d}{2K} \cdot \frac{q}{\varepsilon_0 A}(K+1)$$

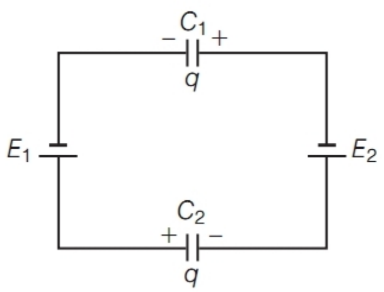
We know that,

$$C = \frac{q}{V} = \frac{2K\varepsilon_0 A}{d(K+1)}$$ **(1)**

15. Potential difference, $\frac{-q}{C_1}+E_1-\frac{q}{C_2}-E_2=0$

or $$\frac{q}{C_1}+\frac{q}{C_2}=E_1-E_2$$

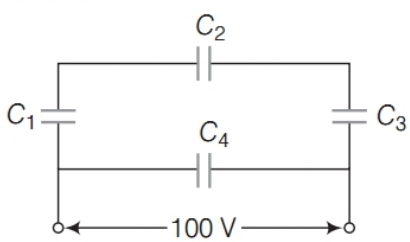
$$q=\left(\frac{C_1C_2}{C_1+C_2}\right)(E_1-E_2)$$ **(1)**



Now, $V_1=\frac{-q}{C_1}=(E_2-E_1)\left(\frac{C_2}{C_1+C_2}\right)$

and $V_2=\frac{+q}{C_2}=(E_1-E_2)\left(\frac{C_1}{C_1+C_2}\right)$ **(1)**

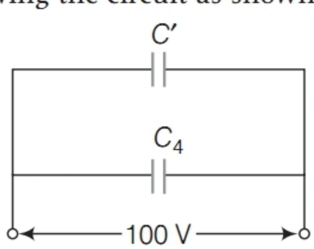
16. (i) According to the diagram given in the question,



Here C_1, C_2 and C_3 are in series. Hence, their equivalent capacitance C' is given by

$$\frac{1}{C'}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\quad [C_1=C_2=C_3=15\mu\text{F}]$$

Redrawing the circuit as shown below



$$\frac{1}{C'}=\frac{1}{15}+\frac{1}{15}+\frac{1}{15}$$

$$C'=\frac{15}{3}\mu\text{F}$$

$$\Rightarrow C'=5\ \mu\text{F}$$

Since, C' and C_4 are in parallel

$$\therefore C_{\text{net}}=C'+C_4$$
$$=5\mu\text{F}+15\mu\text{F}$$
$$=20\mu\text{F}$$ **(1)**

(ii) Since, C' and C_4 are in parallel. So, potential difference across both of them is 100 V.

$\therefore$ Charge across C_4 is $Q_4=C_4\times V=(C_4\times 100)\text{C}$
$=15\times10^{-6}\times100=15\times10^{-4}\text{C}$

$$Q_4=1.5\,\text{mC}$$ **(1)**

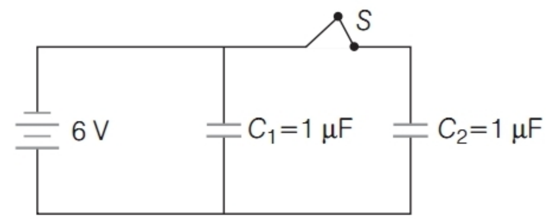
17. The energy stored in a capacitor is given by

$$U=\frac{1}{2}\cdot\frac{Q^2}{C}=\frac{1}{2}\cdot CV^2$$

(i) Energy stored will be decreased or energy stored will become $\frac{1}{K}$ times the initial energy. **(1)**

(ii) Electric field would decrease or $E'=\frac{E}{K}$ **(1)**

18. (i) According to the diagram, when the switch S is closed, the two capacitors C_1 and C_2 in parallel will be charged by the same potential difference V.



So, charge on capacitor C_1

$q_1=C_1V=1\times6=6\mu\text{C}$ ...(i)

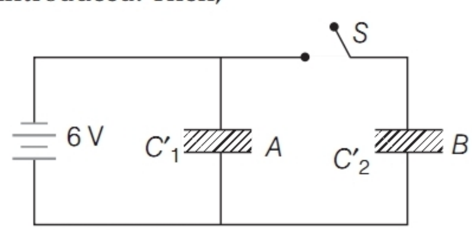
and charge on capacitor C_2

$q_2=C_2V=1\times6=6\,\mu\text{C}$ ...(ii)

Hence, total charge on capacitor,

$q=q_1+q_2=6+6=12\,\mu\text{C}$ **(1)**

(ii) When switch S is opened and dielectric is introduced. Then,



Capacity of both the capacitors becomes K times

i.e. $C_1'=C_2'=KC=3\times1=3\mu\text{F}$ (as $C_1'=C_2'$)

Capacitor A remains connected to battery

$\therefore$ $V'_1=V=6\text{V}$

$q_1'=Kq_1=3\times6\mu\text{C}=18\mu\text{C}$

Capacitor B becomes isolated

$\therefore$ $q_2'=q_2$

or $C_2'V_2'=C_2V_2$ or $(KC)V_2'=CV$

or $V_2=\left(\frac{V}{K}\right)=\frac{6}{3}=2\text{V}$ **(1)**

19. If n identical capacitors each of capacitance C are connected in series combination give equivalent capacitance, $C_s = \frac{C}{n}$ and when connected in parallel combination, then equivalent capacitance, $C_p = nC$

Also, for same voltage the energy stored in the capacitor is given by

$$U = \frac{1}{2}CV^2 \qquad \text{[for constant]}$$

$$U \propto C$$

$$C_s = 1\,\mu\text{F} \qquad [\because n = 3]$$

In series combination, $C_s = \frac{C}{n}$

In parallel combination, $C_p = nC$

According to the problem,

$$C = nC_s = 3 \times 1\,\mu\text{F} = 3\,\mu\text{F}$$

For each capacitor,

In parallel combination,

$$C_p = nC = 3 \times 3 = 9\,\mu\text{F}$$

$$C_p = 9\,\mu\text{F} \qquad \textbf{(1)}$$

For same voltage, $U \propto C$

$$\Rightarrow \frac{U_s}{U_p} = \frac{C_s}{C_p} \Rightarrow \frac{U_s}{U_p} = \frac{C/n}{nC} = \frac{1}{n^2}$$

$$\Rightarrow \frac{U_s}{U_p} = \frac{1}{(3)^2} = \frac{1}{9} \Rightarrow \frac{U_s}{U_p} = \frac{1}{9}$$

or $\quad U_s : U_p = 1 : 9$ **(1)**

20. When a dielectric medium of dielectric constant K is introduced,

(i) in an isolated (not connected with battery) capacitor, then total charge on capacitor remains same.

(ii) in a capacitor connected with battery, then potential difference across the capacitor remains same as that of potential difference across battery. **(1/2)**

Two identical capacitors C_1 and C_2 get fully charged with 5 V battery initially.

So, the charge and potential difference on both capacitors becomes, $q = CV = 2 \times 10^{-6} \times 5\text{ V} = 10\,\mu\text{C}$

and $\quad V = 5\text{ V}$

On introduction of dielectric medium of $K = 5$. **(1/2)**

For C_1 (continue to be connected with battery) potential difference of C_1, $(V') = 5\text{ V}$

Capacitance of $C_1' = KC = 5 \times 2\,\mu\text{F} = 10\,\mu\text{F}$

Charge, $q' = C'V' = (10\,\mu\text{F})\,(5\text{ V}) = 50\,\mu\text{C}$ **(1/2)**

For C_2 (disconnected with battery)

Charge, $q' = q = 10\,\mu\text{C}$

Potential difference, $V' = \frac{V}{K} = \frac{5}{5} = 1\text{ V}$ **(1/2)**

21. According to the question, $C = 2\text{F}$

$$d = 0.5\text{ cm} = 0.5 \times 10^{-2}\text{ m}$$

$$\varepsilon_0 = 8.854 \times 10^{-12}\,\text{C}^2\text{N}^{-1}\text{m}^{-2}$$

As $\quad C = \frac{\varepsilon_0 A}{d}$

$$\therefore \quad A = \frac{Cd}{\varepsilon_0} = \frac{2 \times 0.5 \times 10^{-2}}{8.854 \times 10^{-12}}$$

$$= 1.13 \times 10^9\text{ m}^2 \qquad \textbf{(2)}$$

22. The capacity of condenser is proportional to the area and inversely proportional to the distance between its plates. If a medium of dielectric constant K is filled in the space between the plates, its capacity becomes K times the capacity when there is air between the plates.

After inserting the dielectric medium, let their capacitances become C_1' and C_2'.

For C_1 $\quad C_1' = KC \qquad \ldots\text{(i)}$ **(1/2)**

For C_2 $\quad C_2' = \frac{K_1 \varepsilon_0\,(A/2)}{d} + \frac{K_2 \varepsilon_0\,(A/2)}{d}$

C_2' acts as equivalent capacitance for two capacitors each of area $A/2$ and separation d are connected in parallel combination.

$$C_2' = \frac{\varepsilon_0 A}{d}\left(\frac{K_1}{2} + \frac{K_2}{2}\right)$$

$$C_2' = C\left(\frac{K_1 + K_2}{2}\right) \qquad \ldots\text{(ii)}$$

$$\left[\because C = \frac{\varepsilon_0 A}{d}\right] \qquad \textbf{(1/2)}$$

According to the problem, $C_1' = C_2'$

$$KC = C\left(\frac{K_1 + K_2}{2}\right)$$

$$\Rightarrow \quad K = \frac{K_1 + K_2}{2} \qquad \textbf{(1)}$$

23. After introduction of dielectric medium of dielectric constants K_1 and K_2, capacitor acts as equivalent capacitor if it consists of two capacitors each having plates of area A and separation $\frac{d}{2}$ connected in series combination for

$$C_1 = \frac{\varepsilon_0 A}{d} \qquad \ldots\text{(i)}$$

$$\frac{1}{C_2} = \frac{1}{\left(\frac{K_1 \varepsilon_0 A}{d/2}\right)} + \frac{1}{\left(\frac{K_2 \varepsilon_0 A}{d/2}\right)}$$ **(1)**

$$\frac{1}{C_2} = \frac{1}{\left(\frac{\varepsilon_0 A}{d}\right)}\left(\frac{1}{2K_1} + \frac{1}{2K_2}\right)$$

$$\frac{1}{C_2} = \frac{1}{2C_1}\left(\frac{K_2 + K_1}{K_1 K_2}\right)$$

$$\Rightarrow \quad C_2 = C_1\left(\frac{2K_1 K_2}{K_1 + K_2}\right)$$

The capacitors will be in series. **(1)**

24. (i) If the foil is insulated, then the system will be equivalent to two identical capacitors connected in series combination in which two plates of each capacitor have separation half of the original separation.

Thus, new capacitance of each capacitor

$$C' = 2C \qquad \left[\because C \propto \frac{1}{d}\right]$$

$\because$ C and C' are in series

$$\Rightarrow \quad C_{net} = \frac{2C \times 2C}{2C + 2C} = C$$

$C_{net} = C$ (Original capacitor) **(1)**

(ii) If the foil is connected to the upper plate with a wire, then the system reduces to a capacitor whose separation reduces to half of original one.

$\therefore$ New capacitance, $C' = 2C$ **(1)**

25. Polar dielectrics

A polar molecule which has permanent electric dipole moment (**p**) in absence of electric field are called polar dielectrics. Polar molecules are randomly oriented. e.g. Water, alcohol, HCl, NH_3. **(1)**

Non-polar dielectrics

A non-polar molecule having zero dipole moment in its normal state are called non-polar dielectrics.

Non-polar molecules have symmetrical shapes. e.g. Any non-conducting material. **(1)**

26. Given, $C = 200\,\mu F$, $d = 5\,mm$, $t = 5\,mm$, $V = 100\,V$

(i) $C = \frac{\varepsilon_0 A}{d} \Rightarrow A = \frac{Cd}{\varepsilon_0}$

$$A = \frac{200\times10^{-6}\times5\times10^{-3}}{8.85\times10^{-12}}$$
$$= 112.99\times10^{3}\,m^2$$

When $d' = 2d$, then $C' = \frac{\varepsilon_0 A}{2d - t + \frac{t}{K}}$

$$= \frac{8.85\times10^{-12}\times112.99\times10^{3}}{\left(10 - 5 + \frac{5}{10}\right)\times10^{-3}}$$

$$= 181.8\times10^{-6} = 181.8\,\mu F$$ **(1)**

(ii) Charge on capacitor, $q = C_0V_0$

$= 200\times10^{-6}\times100$

$= 2\times10^{-2}\,C$

$\Rightarrow C_0V_0 = C'V'$

or $\quad V' = \frac{C_0V_0}{C'} = \frac{2\times10^{-2}}{181.8\times10^{-6}} = 110\,V$

$$E_0 = \frac{V_0}{d} = \frac{100}{5\times10^{-3}} = 20\times10^{3}\,V/m$$

$$E' = \frac{V'}{2d} = \frac{110}{10\times10^{-3}} = 11\times10^{3}\,V/m$$ **(1)**

(iii) $\overline{U} = \frac{1}{2}\varepsilon_0E_0^2 = \frac{1}{2}\times8.85\times10^{-12}\times(20\times10^{3})^2$

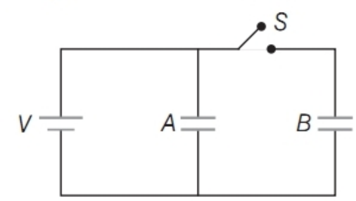
$= 1770\times10^{-6}\,J/m^3$

$\Rightarrow (\overline{U})' = \frac{1}{2}\times\varepsilon_0(E')^2$

$= \frac{1}{2}\times8.85\times10^{-12}\times(11\times10^{3})^2$

$= 535.42\times10^{-6}\,J/m^3$ **(1)**

27. The given figure is shown below.



When switch S is closed, the potential difference across capacitors A and B are same

i.e. $\quad V = \frac{Q_A}{C} = \frac{Q_B}{C}$

Initial charges on capacitors

$$Q_A = Q_B = CV$$ **(1)**

When the dielectric is introduced, the new capacitance of either capacitor

$$C' = KC$$

As switch S is opened, the potential difference across capacitor A remains same (V volts).

Let potential difference across capacitor B be V'. When dielectric is introduced with switch S open (i.e. battery disconnected), the charges on capacitor B remains unchanged, so

$$Q_B = CV = C'V'$$

$$\Rightarrow \qquad V' = \frac{C}{C'}V = \frac{V}{K} \text{ volt}$$ **(1)**

Initial energy of both capacitors

$$U_i = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2$$

Final energy of both capacitors

$$U_f = \frac{1}{2}C'V^2 + \frac{1}{2}C'V'^2 = \frac{1}{2}(KC)\,V^2 + \frac{1}{2}(KC)\left(\frac{V}{K}\right)^2$$

$$= \frac{1}{2}CV^2\left[K + \frac{1}{K}\right] = \frac{1}{2}CV^2\left(\frac{K^2+1}{K}\right)$$

$$\frac{U_i}{U_f} = \frac{CV^2}{\frac{1}{2}CV^2\left(\frac{K^2+1}{K}\right)} = \frac{2K}{K^2+1}$$ **(1)**

28. Energy stored in capacitor

$$= \frac{1}{2}C_1V^2 = \frac{1}{2}\times 12\times 10^{-12}\times(50)^2 \text{ J}$$

$$= 6\times 25\times 10^{-10}\text{ J} = 15\times 10^{-9}\text{ J}$$

With other capacitor 6 pF in series.

$$\text{Total capacitance } (C) = \frac{C_1\times C_2}{C_1+C_2} = \frac{6\times 12}{6+12}\text{ pF}$$

$$= \frac{12\times 6}{18} = 4\text{ pF}$$ **(1)**

Charge stored in each capacitor is same and is given by

$$Q = CV = 4\times 10^{-12}\times 50 = 2\times 10^{-10}\text{C}$$

Each of the capacitors will have charge equal to Q

$$= 2\times 10^{-10}\text{C}$$ **(1)**

Potential on capacitors with capacitance 12 pF is

$$= \frac{Q}{C_1} = \frac{2\times 10^{-10}}{12\times 10^{-12}}\text{ V} = 16.67\text{ V}$$

Potential on capacitor with capacitance 6 pF is

$$= \frac{2\times 10^{-10}}{6\times 10^{-12}}\text{ V} = 33.33\text{ V}$$ **(1)**

29. According to question, let the capacitance of X be C and capacitance of $Y = \varepsilon_r\, C = 4C$ $[\because \varepsilon_r = 4]$

(i) Equivalent capacitance $= \dfrac{C\times 4C}{C+4C}$ (X and Y are in series)

$$= \frac{4C^2}{5C} \Rightarrow \frac{4C}{5}$$

and it is given that $4C/5 = 4\mu\text{F}$

So, $4C = 20\mu\text{F} =$ capacitance of Y

Capacitance of $X = C = 20/4 = 5\mu\text{F}$ **(1)**

(ii) Charge stored in the capacitor is given by

$$q = CV = \frac{4C}{5}\times 15 = \frac{4\times 5}{5}\times 15 = 60\,\mu\text{C}$$

Now, let the potential difference between plates of capacitors X and Y are V_X and V_Y, respectively.

$$\text{So, } V_X = \frac{q}{C_X} = \frac{60}{5} = 12\text{ V}$$

$$\text{and } V_Y = \frac{q}{C_Y} = \frac{60}{20} = 3\text{ V}$$ **(1)**

(iii) Electrostatic energy stored in capacitance

$$X(E_X) = \frac{1}{2}CV_X^2 \qquad \ldots\text{(i)}$$

$$\text{Similarly for } Y, E_Y = \frac{1}{2}4CV_Y^2 \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\text{Ratio} = \frac{E_X}{E_Y} = \frac{\frac{1}{2}CV_X^2}{\frac{1}{2}4CV_Y^2} = \frac{V_X^2}{4V_Y^2} = \frac{12\times 12}{4\times 3\times 3} = 4:1$$ **(1)**

30. (i) As given in the question, energy of the $6\,\mu\text{F}$ capacitor is E. Let V be the potential difference along the capacitor of capacitance $6\,\mu\text{F}$. From the mathematical formula,

$$\frac{1}{2}CV^2 = E$$

$$\frac{1}{2}\times 6\times 10^{-6}\times V^2 = E$$

$$\Rightarrow \qquad V^2 = \frac{E}{3}\times 10^6 \qquad \ldots\text{(i)}$$

Since, potential is same for parallel connection. So, the potential through $12\,\mu\text{F}$ capacitor is also V. Hence, energy of $12\mu\text{F}$ capacitor is

$$E_{12} = \frac{1}{2}\times 12\times 10^{-6}\times V^2 \quad \text{[From Eq. (i)]}$$

$$= \frac{1}{2}\times 12\times 10^{-6}\times\frac{E}{3}\times 10^6 = 2E$$ **(1)**

(ii) Since, charge remains constant in series. So, the charge on $6\mu\text{F}$ and $12\,\mu\text{F}$ capacitors combined will be equal to the charge on $3\,\mu\text{F}$ capacitor. Using the formula, $Q = CV$, we can write

$$\Rightarrow (6+12)\times 10^{-6}\times V = 3\times 10^{-6}\times V'$$

$$V' = 6\text{ V}$$

Using Eq. (i) and squaring both sides, we get

$$V'^2 = 36V^2 \Rightarrow V'^2 = 12E\times 10^6$$

$$\therefore \qquad E_3 = \frac{1}{2}\times 3\times 10^{-6}\times 12E\times 10^6 = 18E$$ **(1)**

(iii) Total energy drawn from battery is

$$E_{\text{total}} = E + E_{12} + E_3$$

$$= E + 2E + 18E = 21E$$ **(1)**

31. Total energy stored in series or parallel combination of capacitors is equal to the sum of energies stored in individual capacitors. In parallel combination the energy stored in the capacitor

$$= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_1^2 \quad \text{...(i)}$$

In series combination the energy stored in the capacitor $= \frac{1}{2}\frac{C_1C_2}{(C_1 + C_2)}V_2^2$ ...(ii) **(1)**

According to the question the energy in both the cases is same.

$$\therefore \quad \left(\frac{1}{2}C_1 + \frac{1}{2}C_2\right)V_1^2 = \frac{C_1C_2}{2(C_1 + C_2)}V_2^2$$

$$\Rightarrow \quad \frac{V_1^2}{V_2^2} = \frac{C_1C_2 \times 2}{2(C_1 + C_2)(C_1 + C_2)} \Rightarrow \frac{V_1}{V_2} = \frac{\sqrt{C_1C_2}}{C_1 + C_2}$$

But, $\frac{C_1}{C_2} = \frac{1}{2} \Rightarrow C_2 = 2C_1$

So, $\frac{V_1}{V_2} = \frac{\sqrt{C_1 \times 2C_1}}{C_1 + 2C_1} = \frac{\sqrt{2}C_1}{3C_1} = \frac{\sqrt{2}}{3}$ **(2)**

32. When the capacitors are connected in parallel.
Then, equivalent capacitance, $C_P = C_1 + C_2$.
The energy stored in the combination of the capacitors, $E_P = \frac{1}{2}C_PV^2$ **(1/2)**

$$\Rightarrow \quad E_P = \frac{1}{2}(C_1 + C_2)(100)^2 = 0.25\,\text{J}$$

$$\Rightarrow \quad C_1 + C_2 = 5 \times 10^{-5} \quad \text{...(i)}$$

When the capacitors are connected in series.
Then, equivalent capacitance, $C_S = \frac{C_1C_2}{C_1 + C_2}$

The energy stored in the combination of the capacitors, $E_S = \frac{1}{2}C_SV^2$ **(1/2)**

$$\Rightarrow \quad E_S = \frac{1}{2}\cdot\frac{C_1C_2}{C_1 + C_2}(100)^2 = 0.045\,\text{J}$$

$$\Rightarrow \quad \frac{1}{2} \times \frac{C_1C_2}{5 \times 10^{-5}}(100)^2 = 0.045\,\text{J}$$

$$\Rightarrow \quad C_1C_2 = 0.045 \times 10^{-4} \times 5 \times 10^{-5} \times 2$$

$$= 4.5 \times 10^{-10}$$

$$(C_1 - C_2)^2 = (C_1 + C_2)^2 - 4C_1C_2$$

$$\Rightarrow (C_1 - C_2)^2 = 25 \times 10^{-10} - 4 \times 4.5 \times 10^{-10}$$

$$= 7 \times 10^{-10}$$

$$\Rightarrow \quad (C_1 - C_2) = \sqrt{7 \times 10^{-10}} = 2.64 \times 10^{-5}$$

$$\Rightarrow \quad C_1 - C_2 = 2.64 \times 10^{-5} \quad \text{...(ii)}$$

On solving Eqs. (i) and (ii), we get

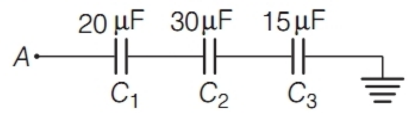
$$C_1 = 38.2\,\mu\text{F} \text{ and } C_2 = 11.8\,\mu\text{F}$$ **(1)**

$$Q_1 = C_1V = 38.2 \times 10^{-6} \times 100$$

$$= 38.2 \times 10^{-4}\,\text{C}$$ **(1/2)**

$$Q_2 = C_2V = 11.8 \times 10^{-6} \times 100 = 11.8 \times 10^{-4}\,\text{C}$$ **(1/2)**

33. Consider the given figure below



Given, $C_1 = 20\mu\text{F}, \ C_2 = 30\mu\text{F}, \ C_3 = 15\mu\text{F}$
Potential at $A = 90$ V
As, we can see that capacitor C_3 is earthed.
Therefore, potential across C_3 will be zero.
Since, capacitor C_1, C_2 and C_3 are connected in series, therefore

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \Rightarrow \frac{1}{C_{eq}} = \frac{1}{20} + \frac{1}{30} + \frac{1}{15}$$

$$\Rightarrow \quad \frac{1}{C_{eq}} = \frac{3 + 2 + 4}{60} \Rightarrow \frac{1}{C_{eq}} = \frac{9}{60}$$

$$\Rightarrow \quad C_{eq} = \frac{60}{9} \Rightarrow C_{eq} = \frac{20}{3}\mu\text{F}$$ **(1)**

Since, charge remains same in series combination,

So, $Q = C_{eq}V \Rightarrow Q = \frac{20}{3} \times 90$

$$\Rightarrow \quad Q = 600\,\mu\text{C} \Rightarrow Q = 600 \times 10^{-6}\,\text{C}$$

$$\Rightarrow \quad Q = 6 \times 10^{-4}\,\text{C}$$

$\therefore$ Potential difference across $C_2 = \frac{Q}{V_2}$

$$\Rightarrow \quad V_2 = \frac{Q}{C_2} \qquad \Rightarrow V_2 = \frac{6 \times 10^{-4}}{30 \times 10^{-6}}$$

$$\Rightarrow \quad V_2 = 0.2 \times 10^2 \Rightarrow V_2 = 20\,\text{V}$$ **(1)**

Also, energy stored in capacitor C_2 is given by

$$E = \frac{1}{2}C_2V_2^2 \Rightarrow E = \frac{1}{2} \times 30 \times (20)^2 \times 10^{-6}$$

$$\Rightarrow \quad E = \frac{1}{2} \times 30 \times 400 \times 10^{-6}$$

$$\Rightarrow \quad E = 6000 \times 10^{-6} \Rightarrow E = 6 \times 10^{-3}\,\text{J}$$ **(1)**

34. (i) The energy of a charged capacitor is measured by the total work done in charging the capacitor to a given potential.
Let us assume that initially both the plates are uncharged. Now, we have to repeatedly remove small positive charges from one plate and transfer them to other plate.

Now, when an additional small charge (dq) is transferred from one plate to another, the small work done is given by

$$dW = V'dq = \frac{q'}{C}dq$$ **(1/2)**

[Let charge on plate, when dq charge is transferred is q']

The total work done in transferring charge Q is given by

$$W = \int_0^Q \frac{q'}{C}dq = \frac{1}{C}\int_0^Q q'\,dq = \frac{1}{C}\left[\frac{(q')^2}{2}\right]_0^Q = \frac{Q^2}{2C}$$

(1/2)

This work is stored as electrostatic potential energy U in the capacitor.

$$U = \frac{Q^2}{2C} = \frac{(CV)^2}{2C} \qquad [\because Q = CV]$$

$$U = \frac{1}{2}CV^2$$ **(1)**

The energy stored per unit volume of space in a capacitor is called **energy density**.

$$U = \frac{\frac{1}{2}CV^2}{Ad} \Rightarrow U = \frac{1}{2}\frac{\varepsilon_0 AV^2}{d^2A}$$

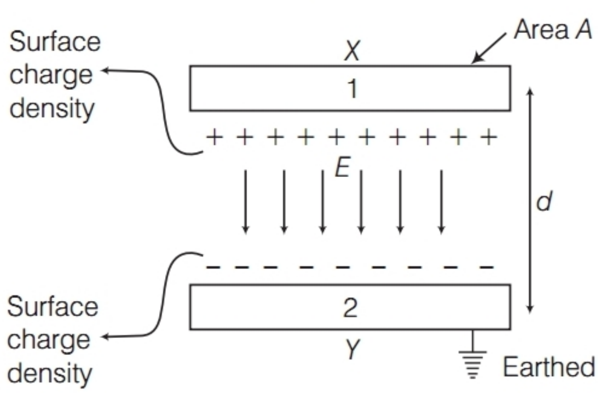
Energy density, $U = \frac{1}{2}\varepsilon_0 E^2$

Total energy stored in series combination or parallel combination of capacitors is equal to the sum of energies stored in individual capacitor.

i.e. $U = U_1 + U_2 + U_3 + \ldots$

(ii) Due to conservative nature of electric force, the work done in moving a charge in a close path in a uniform electric field is zero. **(1)**

35. (i) Parallel plate capacitor consists of two thin conducting plates each of area A held parallel to each other at a suitable distance d. One of the plates is insulated and other is earthed. There is a vacuum between the plates.



Suppose, the plate X is given a charge of $+q$ coulomb. By induction, $-q$ coulomb of charge is produced on the inner surface of the plate Y and $+q$ coulomb on the outer surface. Since, the plate Y is connected to the earth. So, the $+q$ charge on the outer surface flows to the earth. Thus, the plates X and Y have equal and opposite charges. **(1)**

Suppose, the surface density of charge on each plate is σ. We know that the intensity of electric field at a point between two plane parallel sheets of equal and opposite charges is σ/ε_0, where ε_0 is the permittivity of free space.

$$\therefore \qquad E = \frac{\sigma}{\varepsilon_0}$$

The charge on each plate is q and the area of each plate is A. Thus,

$$\sigma = \frac{q}{A} \text{ and } E = \frac{q}{\varepsilon_0 A} \qquad \ldots\text{(i)}$$

Now, let the potential difference between the two plates be V volt. Then, the electric field between the plates is given by

$$E = \frac{V}{d} \text{ or } V = Ed$$

Substituting the value of E from Eq. (i), we get

$$V = \frac{qd}{\varepsilon_0 A}$$

$\therefore$ Capacitance of the capacitor is

$$C = \frac{q}{V} = \frac{q}{qd/\varepsilon_0 A} \text{ or } C = \frac{\varepsilon_0 A}{d}$$

where, $\varepsilon_0 = 8.85\times10^{-12}\ C^2\text{-}Nm^{-2}$ **(1)**

(ii) Surface charge density is given by

$$\sigma = \frac{q}{4\pi R^2}$$

After connecting both the conductors, their potentials will become equal.

$$V_1 = V_2$$

$$\Rightarrow \qquad \frac{Kq_1}{R_1} = \frac{Kq_2}{R_2}$$

$$\left[\because \text{for spherical conductors } V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} \text{ or } V = \frac{Kq}{R}\right]$$

$$\Rightarrow \qquad \frac{q_1}{q_2} = \frac{R_1}{R_2}$$

$$\Rightarrow \qquad \frac{\sigma_1}{\sigma_2} = \frac{q_1/4\pi R_1^2}{q_2/4\pi R_2^2}$$

$$= \frac{q_1}{q_2}\left(\frac{R_2}{R_1}\right)^2 = \frac{R_2}{R_1}$$ **(1)**

36. Given, area of each plate, $A = 6\times10^{-3}\ \text{m}^2$

Distance between plates, $d = 3\ \text{mm} = 3\times10^{-3}\ \text{m}$

(i) Capacitance of parallel plate capacitor is given by

$$C = \frac{\varepsilon_0 A}{d} \Rightarrow \varepsilon_0 = 8.85\times10^{-12}\ \text{C}^2\ \text{N}^{-1}\text{m}^{-2}$$

$$C = \frac{8.85\times10^{-12}\times6\times10^{-3}}{3\times10^{-3}}$$

$$C = 1.77\times10^{-11}\ \text{F}$$ **(1)**

(ii) Charge on parallel plate capacitor is given by

$Q = CV$

Given, $V = 100\,\text{V}$

Now, $Q = 1.77\times10^{-11}\times100$

$Q = 1.77\times10^{-9}\text{C}$ ...(i) **(1)**

(iii) Given, $K = 6$

Now, $C' = KC$

$\therefore$ $Q' = KQ$ [from Eq. (i)]

$Q' = 6\times1.77\times10^{-9}$

$Q' = 10.62\times10^{-9}\text{C}$ **(1)**

37. (i) We have initial voltage, $V_1 = V$ volt and charge stored, $Q_1 = 360\mu\text{C}$.

$Q_1 = CV_1$ ...(i)

Charged potential, $V_2 = V - 120$

$Q_2 = 120\mu\text{C}$ **(1/2)**

$Q_2 = CV_2$ ...(ii)

By dividing Eq. (ii) from Eq. (i), we get

$$\frac{Q_1}{Q_2} = \frac{CV_1}{CV_2} \Rightarrow \frac{360}{120} = \frac{V}{V-120}$$

$\Rightarrow$ $V = 180\ \text{V}$ **(1/2)**

$$\therefore \quad C = \frac{Q_1}{V_1} = \frac{360\times10^{-6}}{180} = 2\times10^{-6}\ \text{F} = 2\ \mu\text{F}$$

Hence, the potential, $V = 180\,\text{V}$ and unknown capacitance is $2\mu\text{F}$. **(1½)**

(ii) Let Q be the charge stored in the capacitor

$Q = CV = 2\times10^{-6}\times(120+180)$

$Q = 6\times10^{-4}\,C$ **(1/2)**

38. Given, $C = 200\ \text{pF} = 200\times10^{-12}$ F and $V = 300$ V

The energy (initially) stored by the capacitor is

$$U_i = \frac{1}{2}CV^2 = \frac{1}{2}\times200\times10^{-12}\times300\times300$$

$$= 9\times10^{-6}\text{J}$$

The charge on the capacitor when charge through 300 V battery is

$$Q = CV = 200\times10^{-12}\times300$$
$$= 6\times10^{-8}\text{C}$$
$$= 60\times10^{-9}\text{C} = 60\,\text{nC}$$ **(1)**

When two capacitors are connected, they have their positive plates at the same potential and negative plates also at the same potential. Let V be the common potential difference. By charge conservation charge would distribute, but total charge would remain constant.

Thus, $Q = q + q'$

$$\frac{q}{C} = \frac{q'}{C'}$$

$$\frac{q}{200} = \frac{q'}{100}$$

$$q = 2q'$$

Thus, $Q = 2q' + q' = 3q'$

So, $q' = \frac{Q}{3} = \frac{60\ \text{nC}}{3} = 20\ \text{nC}$

and $q = 2q' = 40$ nC **[1]**

Thus, final energy

$$U_f = \frac{q^2}{2C} + \frac{q'^2}{2C'}$$

$$= \frac{1}{2}\times\frac{(40\times10^{-9})^2}{200\times10^{-12}} + \frac{1}{2}\times\frac{(20\times10^{-9})^2}{100\times10^{-12}}$$

$$= 4\times10^{-6} + 2\times10^{-6} = 6\times10^{-6}\text{J}$$

Difference in energy

= final energy – initial energy

$= U_f - U_i$

$= 6\times10^{-6} - 9\times10^{-6} = -3\times10^{-6}$

Thus, difference in energy is -3×10^{-6} J. **(1)**

39. In a series combination, there is no division of charge,

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

In a parallel combination, the potential difference is same.

$$C_p = C_1 + C_2 + C_3 \ldots$$

(i) Here, C_1, C_2 and C_3 are in series. Therefore, their equivalent capacitance is

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$C' = \frac{C}{3} = \frac{12}{3} = 4\,\mu\text{F}$$ **(1)**

Now, C' and C are in parallel combination.

$\therefore \quad C_{net} = C' + C$
$= 4\,\mu F + 12\,\mu F = 16\,\mu F$

$C_{net} = 16\,\mu F$ **(1)**

(ii) Being C' and C are in parallel and 500 V potential difference is applied across them.

$\therefore$ Charge on C'

$$q_1 = C'V = (4\,\mu F) \times 500 = 2000\,\mu C$$

$\therefore C_1, C_2$ and C_3 capacitors each will have same charge, i.e. $2000\,\mu C$ charge.

Charge on C_4, $q_2 = C_4 \times V$

$= 12 \times 500 = 6000\,\mu C$ **(1)**

40. On introduction of dielectric slab in an isolated charged capacitor.

(i) The capacitance (C') becomes K times of original capacitor as

$$C = \frac{\varepsilon_0 A}{d} \text{ and } C' = \frac{K\varepsilon_0 A}{d}$$ **(1)**

(ii) $\because$ Charge remains conserved in this phenomenon.

$\therefore \quad CV = C'V'$

$$V' = \frac{CV}{C'} = \frac{CV}{KC} \quad \text{[refer part (i)]}$$

$$\Rightarrow \quad V' = \frac{V}{K}$$

Potential difference decreases and become $\frac{1}{K}$ times of original value. **(1)**

(iii) Energy stored initially, $U = \frac{q^2}{2C}$

Energy stored later $U' = \frac{q^2}{2(KC)}$ $[\because C' = KC]$

where, K = dielectric constant of medium

$$\Rightarrow \quad U' = \frac{1}{K}\left(\frac{q^2}{2C}\right)$$

$$\Rightarrow \quad U' = \frac{1}{K}(U)$$

$$\Rightarrow \quad U' = \frac{1}{K} \times U$$

The energy stored in the capacitor decreases and becomes $\frac{1}{K}$ times of original energy. **(1)**

41. On introduction of dielectric slab to fill the gap between plates of capacitor completely when capacitor is connected with battery.

(i) The potential difference V between capacitors is same due to connectivity with battery and hence, charge q' becomes K times of original charge as

$$q' = C'V' = (KC)(V) = K(CV) = Kq$$

$$q' = Kq$$ **(1)**

(ii) Electric field intensity continue to be the same as potential difference and separation between two plates remain unaffected as

$$E = \frac{V}{d}.$$ **(1)**

(iii) The capacitance of capacitor becomes K times of original capacitor.

$$\therefore \quad C' = KC = \frac{K\varepsilon_0 A}{d}$$ **(1)**

42. After disconnection from battery and doubling the separation between two plates will

(i) Charge on capacitor remains same.

i.e. $\quad CV = C'V'$

$$\Rightarrow \quad CV = \left(\frac{C}{2}\right)V' \Rightarrow V' = 2V$$

$\because$ Electric field between the plates

$$E' = \frac{V'}{d'} = \frac{2V}{2d}$$

$$E' = \frac{V}{d} = E$$

$\Rightarrow$ Electric field between the two plates will remain same. **(1)**

(ii) Capacitance reduces to half of original value as

$$C \propto \frac{1}{d} \Rightarrow C' = \frac{C}{2}$$ **(1)**

(iii) Energy stored in the capacitor before disconnection from battery

$$U_1 = \frac{q^2}{2C}$$

Now, energy stored in the capacitor when distance between the plates is doubled

$$U_2 = \frac{q^2}{2(C')} = \frac{q^2}{2 \times \left(\frac{C}{2}\right)} = \frac{q^2}{C}$$

$$\Rightarrow \quad U_2 = 2\left(\frac{q^2}{2C}\right) = 2U_1$$

$$U_2 = 2U_1$$

Hence, energy stored in capacitor gets doubled to its initial value. **(1)**

43. As we know, to determine the electric field at any point at distance r from centre if we apply Gauss's theorem

$$E = \frac{q}{4\pi\varepsilon_0 \cdot r^2}$$

Hence, $V = \int \mathbf{E} \cdot d\mathbf{r} = \dfrac{q}{4\pi\varepsilon_0 \cdot r}$

where, $\varepsilon_0 = 8.854 \times 10^{-12}\ \mathrm{C^2\,N^{-1}\,m^{-1}}$

The capacitance of the spherical conductor situated in vacuum is given by

$$C = \frac{q}{V} = \frac{q}{\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}} \Rightarrow C = 4\pi\varepsilon_0 r.$$ **(2)**

Hence, the capacitance of an isolated spherical conductor situated in vacuum is $4\pi\varepsilon_0$ times of its radius. **(1)**

44. Let V_1 and V_2 are the potential differences across the series and parallel combination of two identical capacitors each of capacitance C.

Equivalent capacitance in series combination,

$$C_s = \frac{C}{2}$$ **(1)**

Equivalent capacitance in parallel combination,

$$C_p = 2C$$

According to the question, $U_s = U_p$ **(1)**

$$\frac{1}{2} C_s V_s^2 = \frac{1}{2} C_p V_p^2$$

$$\Rightarrow \quad \frac{V_s^2}{V_p^2} = \frac{C_p}{C_s} = \frac{2C}{(C/2)}$$

$$\frac{V_s^2}{V_p^2} = 4 \Rightarrow \frac{V_s}{V_p} = 2$$

$$V_s : V_p = 2:1$$ **(1)**

45. (i) The total charge on the capacitor remains conserved on introduction of dielectric slab. Also, the capacitance of capacitor increases to K times of original values.

$$\therefore \quad CV = C'V'$$

$$CV = (KC)\,V' \Rightarrow V' = \frac{V}{K}$$

$\therefore$ New electric field,

$$E' = \frac{V'}{d} = \left(\frac{V/K}{d}\right) = \left(\frac{V}{d}\right)\frac{1}{K} = \frac{E}{K}$$

$\therefore$ On introduction of dielectric medium, new electric field E' becomes $\dfrac{1}{K}$ times of its original value. **(1)**

(ii) $\because$ Capacitance of a parallel plate capacitor partially filled with dielectric medium is given by

$$C = \frac{\varepsilon_0 A}{(d - t + t/K)}$$ **(1)**

where, t is the thickness of dielectric medium.

Here, $t = \dfrac{d}{2}$

$$C = \frac{\varepsilon_0 A}{d - \frac{d}{2} + \frac{d}{2K}} = \frac{\varepsilon_0 A}{\frac{d}{2}\left(1 + \frac{1}{K}\right)}$$

$$\therefore \quad C = \frac{2\varepsilon_0 AK}{(K+1)\,d}$$ **(1)**

46. (i) The graph comparing the variation of potential V and electric field E. Refer to Sol. 31 on page 45. **(1)**

(ii) Let $C_1 = C$ and $C_2 = 2C$

$\therefore$ Equivalent capacitance

In series, $C_s = \dfrac{2C \times C}{2C + C} = \dfrac{2C^2}{3C} = \dfrac{2C}{3}$

In parallel, $C_p = 2C + C = 3C$ **(1)**

$\therefore$ V_p and V_s are potential difference across the final capacitor in parallel and series combination respectively. So, to have same potential energy in parallel and series combination.

$$U_p = U_s$$

$$\frac{1}{2} C_p V_p^2 = \frac{1}{2} C_s V_s^2$$

$$\Rightarrow \quad \frac{V_p}{V_s} = \sqrt{\frac{C_s}{C_p}}$$

$$= \sqrt{\frac{(2C/3)}{(3C)}} = \sqrt{\frac{2}{9}}$$

$$V_p : V_s = \sqrt{2} : 3$$ **(1)**

47. (i) Refer to text on pages 51 and 52 (Capacitance of parallel plate capacitor) and page 53 (Energy stored in a capacitor). **(3)**

(ii) Let q be the charge on the charged capacitor.

$\therefore$ Energy stored in it is given by $U = \dfrac{q^2}{2C}$

When another uncharged similar capacitor is connected, then the net capacitance of the system is given by

$$C' = 2C$$ **(1)**

The charge on the system remains constant. So, the energy stored in the system is given by

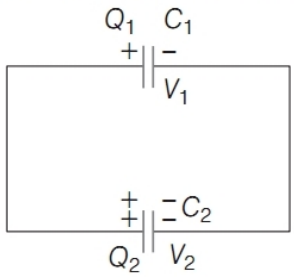
$$U' = \frac{q^2}{2C'} = \frac{q^2}{4C} \qquad [\because C' = 2C]$$

Thus, the required ratio is given by

$$\frac{U'}{U} = \frac{q^2/4C}{q^2/2C} = \frac{1}{2}$$ **(1)**

48. Let two capacitors of capacitances C_1 and C_2 of potentials V_1 and V_2 are connected in parallel.

According to the diagram given below



After connecting the charges redistribute in such a way that the potential differences across C_1 and C_2 become equal.

Hence, before connection the charges are

$$Q_1 = C_1V_1, Q_2 = C_2V_2$$

Common potential after connection, $V = \dfrac{Q_1 + Q_2}{C_1 + C_2}$

Hence, $\quad V = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}$ ...(i)

After sharing, let the charges be Q_1' and Q_2'

$\Rightarrow \quad \dfrac{Q_1'}{Q_2'} = \dfrac{C_1V}{C_2V} = \dfrac{C_1}{C_2}$ (ii)

(i) Hence, the total energy stored in the two capacitors before they are connected together will be

$$U = U_1 + U_2 = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 \quad \text{... (iii)}$$ **(1)**

(ii) When the two capacitors are connected parallel together, total charge on the capacitor

$$q = q_1 + q_2 = C_1V_1 + C_2V_2$$

Total capacitance, $C = C_1 + C_2$

Hence, total energy after they are connected parallel

$$U' = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{C_1 + C_2} \quad \text{...(iv)}$$ **(2)**

(iii) **Difference of energy**

Subtracting Eq. (iv) from Eq. (iii), we get

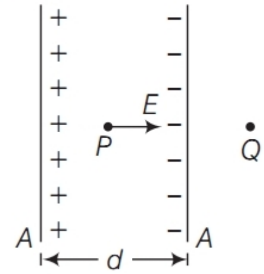
$$U - U' = \left(\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2\right) - \frac{1}{2}\frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

$\Delta U = \dfrac{C_1C_2(V_1 - V_2)^2}{2(C_1 + C_2)}$ is a positive quantity.

Where, ΔU is the loss of energy in sharing charges.

Since, ΔU is positive. So, there is always a loss of energy arises. Hence, when two charged capacitors are connected together energy loss comes in the form of heat radiations due to electric current while charging. **(2)**

49. (i) According to question,



(a) Electric field due to a plate of positive charge at point $P = \sigma/2\varepsilon_0$

Electric field due to other plate $= \sigma/2\varepsilon_0$

Since, they have same direction,

$$\therefore \quad E_{net} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

Outside the plate the electric field will be zero because of its opposite direction. **(1)**

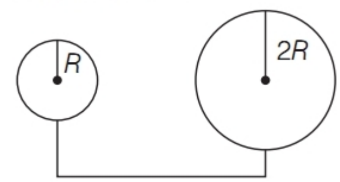
(b) Potential difference between the plates is given by

$$V = Ed = \sigma d/\varepsilon_0 \quad (\because E = \sigma/\varepsilon_0)$$ **(1)**

(c) Capacitance of the capacitor is given by

$$C = \frac{Q}{V} = \frac{\sigma A}{\sigma d}\varepsilon_0 = \frac{\varepsilon_0 A}{d} \quad (\because Q = CV)$$ **(1)**

(ii) According to question,



Potential at the surface of radius R

$$= \frac{Kq}{R} \quad [\because q = \sigma \times 4\pi R^2]$$

$$= \frac{K\sigma 4\pi R^2}{R} = \sigma K 4\pi R = 4K\sigma\pi R$$

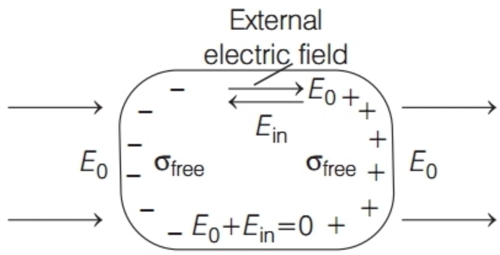
Potential at the surface of radius $2R$,

$$= \frac{Kq}{2R} \quad [\because q = \sigma \times 4\pi(2R)^2 = 16\sigma\pi R^2]$$

So, $\quad = \dfrac{K\sigma 16\pi R^2}{2R} = 8K\sigma\pi R$

Since, the potential of bigger sphere is more. So, charge will flow from sphere of radius $2R$ to sphere of radius R. **(2)**

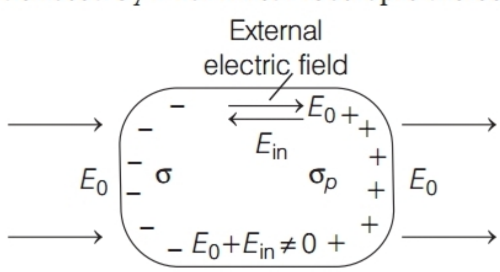
50. (i) (a) When a conductor is placed in an external electric field the free charges present inside the conductor redistribute themselves in such a manner that the electric field due to induced charges opposes the external field within the conductor. This happens until a static situation is achieved, i.e. when the two fields cancels each other, then the net electrostatic field in the conductor becomes zero. **(1)**



(b) Dielectrics are non-conducting substances i.e. they have no charge carriers. Thus, in a dielectric free movement of charges is not possible. When a dielectric is placed in an external electric field the molecules are re-oriented and thus, induces a net dipole moment in the dielectric. This produces an electric field.

However, the opposing field is so induced that it does not exactly cancel the external field. It only reduces it.

Both polar and non-polar dielectric develop net dipole moment in the presence of an external field. The dipole moment per unit volume is called polarisation and is denoted by P for linear isotropic dielectrics.



(2)

$$P = \chi E$$

(ii) (a) At point C inside the shell.

The electric field inside a spherical shell is zero. Thus , the force experienced by charge at the centre C will also be zero.

$\because$ $F_C = qE$ $(E_{\text{inside the shell}} = 0)$

$\therefore$ $F_C = 0$

At point A, $|F_A| = 2Q\left(\dfrac{1}{4\pi\varepsilon_0}\dfrac{3Q/2}{x^2}\right)$

$F = \dfrac{3Q^2}{4\pi\varepsilon_0 x^2}$, away from shell

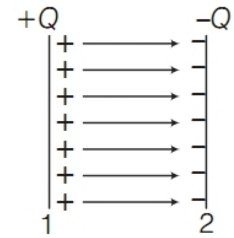
(b) Electric flux through the shell,

$\phi = \dfrac{1}{\varepsilon_0} \times$ magnitude of the charge enclosed by the shell.

$\Rightarrow$ $\phi = 1/\varepsilon_0 \times Q/2 = Q/2\varepsilon_0$ **(2)**

51. (i) To find the energy stored in the capacitor charge configuration, suppose the conductors 1 and 2 are initially uncharged.

Let positive charge be transferred from conductor 2 to conductor 1 in very small installments of each till conductor 1 get charge $+Q$. By charge conservation, conductor 2 would get charge $-Q$.



At every stage of charging conductor 1 is at higher potential than conductor 2. Therefore, work is done externally in transferring each installment of charge.

$\therefore$ Potential difference between conductors 1 and 2 is q/C.

$\therefore$ Potential of condenser $= q/C$.

Small amount of work done in giving an additional charge dq to the condenser is

$$dW = \frac{q}{C} \times dq$$

$\therefore$ Total work done in giving a charge Q to the condenser is

$$W = \int_{q=0}^{q=Q} \frac{q}{C} = \frac{1}{C}\left[\frac{q^2}{2}\right]_{q=0}^{q=Q}$$

$$\Rightarrow \quad W = \frac{1}{C}\frac{Q^2}{2}$$

As, electrostatic force is conservative. So, this work is stored in the form of potential energy (U) of the condenser.

$$U = W = \frac{1}{2}\frac{Q^2}{C}$$

$$\because \quad Q = CV$$

$$\Rightarrow \quad U = \frac{1}{2}\frac{(CV)^2}{C} = \frac{1}{2}CV^2$$

$$\because \quad CV = Q \Rightarrow U = \frac{1}{2}QV$$

Hence, $U = \dfrac{1}{2}\dfrac{Q^2}{C} = \dfrac{1}{2}CV^2 = \dfrac{1}{2}QV$

Energy density (U) is defined as the total energy per unit volume of the condenser.

i.e. $U = \frac{\text{Total energy } (U)}{\text{Volume } (V)} = \frac{\frac{1}{2}CV^2}{Ad}$

Using, $C = \frac{\varepsilon_0 A}{d}$ and $V = Ed$

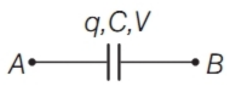
We get, $U = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)\left(\frac{E^2 d^2}{Ad}\right)$

$$= \frac{1}{2}\varepsilon_0 E^2$$

Here, E is the strength of electric field in the space between the plates of the capacitor. **(2½)**

(ii) **Initial condition**

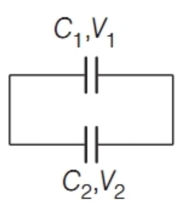
If we consider a charge capacitor, then its charge would be given by $q = CV$



and energy stored in it is given by

$$U_1 = \frac{1}{2}CV^2 \quad \text{...(i)}$$

When this charged capacitor is connected to uncharged capacitor,



Let the common potential be V_1, the charge flow from first capacitor to the other capacitor unless both the capacitor attain the common potential.

$\Rightarrow \quad Q_1 = CV_1$ and $Q_2 = CV_2$

Applying conservation of charge,

$$Q = Q_1 + Q_2$$

$\Rightarrow \quad CV = CV_1 + CV_2$

$\Rightarrow \quad V = V_1 + V_2 \quad (\because V_1 = V_2)$

$\Rightarrow \quad V_1 = V_2 = \frac{V}{2}$

Total energy stored on both the capacitors

$$U_2 = \frac{1}{2}CV_1^2 + \frac{1}{2}CV_2^2$$

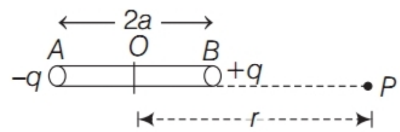
$\Rightarrow \quad U_2 = \frac{1}{2}C\left(\frac{V}{2}\right)^2 + \frac{1}{2}C\left(\frac{V}{2}\right)^2$

$$U_2 = \frac{2CV^2}{8} = \frac{1}{4}CV^2 \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get, $U_2 < U_1$

It means that the energy stored in the combination is less than that the energy stored initially in the single capacitor. **(2½)**

52. (i)



Let P be an axial point at distance r from the centre of the dipole. Electric potential at point P will be

$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-q)}{r+a} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r-a}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r-a} - \frac{1}{r+a}\right] = \frac{q}{4\pi\varepsilon_0}\cdot\frac{2a}{r^2-a^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2-a^2} \quad [\because p = q(2a)]$$

For a far away point, $r >> a$

$\therefore V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2}$ or $V \propto \frac{1}{r^2}$

Thus, due to a dipole the potential at a point is $V \propto 1/r^2$. **(2½)**

(ii) Let A be the area of each plate and C_1 and C_2 are capacitances of the slabs.

Let initially, $C_1 = C = \frac{\varepsilon_0 A}{d} = C_2$

After inserting respective dielectric slabs.

$$C_1' = KC \quad \text{...(i)}$$

and $\quad C_2' = K_1\frac{\varepsilon_0(A/2)}{d} + K_2\frac{\varepsilon_0(A/2)}{d}$

$$= \frac{\varepsilon_0 A}{2d}(K_1 + K_2)$$

$$C_2' = \frac{C}{2}(K_1 + K_2) \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

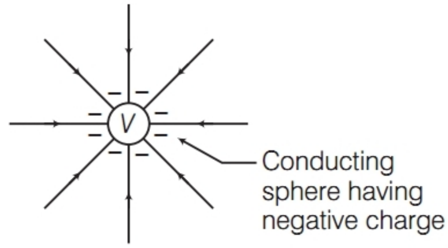
$$C_1' = C_2'$$

$\Rightarrow \quad KC = \frac{C}{2}(K_1 + K_2)$

$\Rightarrow \quad K = \frac{1}{2}(K_1 + K_2)$ **(2½)**

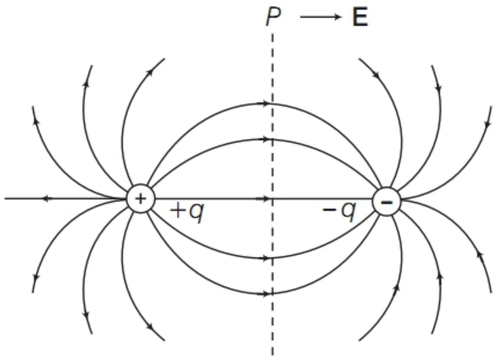
53. (i) Refer to Sol. 41 (iii),(ii) and 42 (iii) on page 70. **(3)**

(ii) (a) Electric field lines due to a conducting sphere are shown in the figure



(1)

(b) Electric field lines due to an electric dipole are shown in the figure



(1)

54. As, charge on capacitor increases. So, we have to work more against electrostatic repulsion and this amount of work done will be stored in the form of potential energy in the capacitor.

(i) We know that, $q = CV \Rightarrow V = q/C$

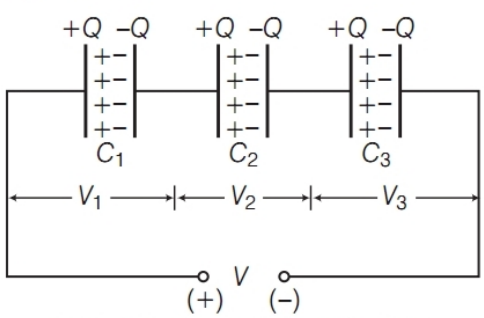
$$dW = Vdq = \frac{q}{C}dq$$

where, q = instantaneous charge
C = instantaneous capacitance and
V = instantaneous voltage

$\therefore$ Total work done in storing charge from 0 to q is given by

$$W = \int_0^q \frac{q}{C}dq = \frac{q^2}{2C}$$ **(2)**

(ii) In series combination of capacitors same charge lies on each capacitor for any value of capacitances.



Capacitors in series combination **(1)**

Also, potential difference across the combination is equal to the algebraic sum of potential differences across each capacitor.

i.e. $$V = V_1 + V_2 + V_3 \quad \ldots(i)$$

where, V_1 , V_2, V_3 and V are the potential differences across C_1 , C_2, C_3 and equivalent capacitor respectively.

$$\therefore \quad q = C_1V_1 \Rightarrow V_1 = \frac{q}{C_1}$$

Similarly, $V_2 = \frac{q}{C_2}$ and $V_3 = \frac{q}{C_3}$

$\therefore$ Total potential difference [from Eq. (i)]

$$V = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$ **(1)**

$$\frac{V}{q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\Rightarrow \quad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

[$\because \frac{V}{q} = \frac{1}{C}$ where C is equivalent capacitance of combination]

$$\text{or} \quad \frac{1}{C} = \frac{C_2\,C_3 + C_3\,C_1 + C_1\,C_2}{C_1\,C_2\,C_3}$$

$$\Rightarrow \quad C = \frac{C_1\,C_2\,C_3}{C_1\,C_2 + C_2\,C_3 + C_3\,C_1}$$ **(1)**