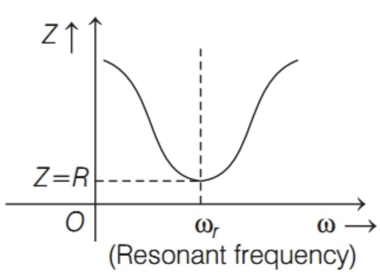
Explanations

1. Impedance of a series L-C-R circuit is given as

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \quad \textbf{(1/2)}$$

So, the graph between impedance and angular frequency (ω) of the circuit is as shown



2. The quality factor (Q) of resonance in series L-C-R circuit is defined as the ratio of voltage drop across inductor (or capacitor) to the applied voltage,

i.e. $$Q = \frac{V_L}{V_R} = \frac{I_0 X_L}{I_0 R} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

It is an indicator of sharpness of the resonance. Quality factor has no unit. **(1/2)**

3. Power factor, $\cos\phi = 0.5$

$\cos\phi = \cos 60° \Rightarrow \phi = 60°$

Phase difference between voltage and current of the circuit is 60°. **(1)**

4. Capacitor reactance is the resistance offered by a capacitor, when it is connected to an AC circuit. It is given by $X_C = 1/\omega C = 1/2\pi f C$ **(1/2)**

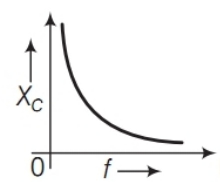
where, ω = angular frequency of the source

and C = capacitance of the capacitor.

The SI unit of capacitor reactance is Ohm (Ω). **(1/2)**

5. Variation of capacitive reactance with the change in the frequency of the AC source with graph is given below

$$X_C \propto \frac{1}{f}$$



(1)

6. AC voltage is preferred over DC voltage because of the following reasons

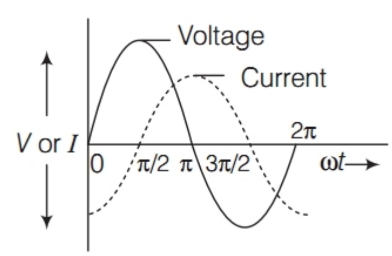
(i) The loss of energy in transmitting the AC voltage over long distances with the help of step-up transformers is negligible as compared to DC voltage. **(1/2)**

(ii) AC voltage can be stepped up and stepped down as per the requirement by using a transformer. **(1/2)**

7. For an AC circuit having inductor, the current and voltage equation are shown below

$$V = V_0 \sin\omega t \quad \textbf{(1/2)}$$
$$I = I_0 \sin(\omega t - \pi/2)$$

The graphical variation of voltage and current are given below.



(1/2)

8. E_0 = peak value of emf in a complete cycle,

(i) rms value $(E_{rms}) = E_0/\sqrt{2}$ **(1/2)**

(ii) average value (E) = zero **(1/2)**

9. Given, current flowing through the inductor,

$I = 15\cos 300t$

Comparing with $I = I_0 \sin \omega t$

Here, peak value of current,

$$I_0 = 15\text{ A}$$

(i) For complete cycle, rms value of current

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{15}{\sqrt{2}}\text{ A}$$

(ii) For complete cycle, average value of current is zero

i.e. $I_{av} = 0$ **(1/2 × 2 = 1)**

10. Wattless Current The current in an AC circuit when average power consumption in AC circuit is zero, is referred as wattless current. **(1)**

11. (i) In case of pure capacitive circuit, the current leads in phase by $\pi/2$ with respect to the applied voltage. So, the element will be a capacitor. **(1/2)**

(ii) In case of pure inductive circuit, the current lags in phase by $\pi/2$ with respect to the applied voltage. So, the element will be an inductor. **(1/2)**

12. It is defined as the value of Alternating Current (AC) over a complete cycle which would generate same amount of heat in a given resistor that is generated by steady current in the same resistor and in the same time during a complete cycle.It is also called virtual value or effective value of AC.

Let the peak value of the current be I_0

$$\therefore \quad I_{rms} = \frac{I_0}{\sqrt{2}}$$

where, I_0 = peak value of AC. **(1)**

13. Given, instantaneous voltage, $V = V_0 \sin \omega t$

As in both cases, the voltage and current differ by a phase of $\frac{\pi}{4}$. So, $\phi = \frac{\pi}{4}$.

(a) Average power dissipated,

$$P_{av} = V_{rms} I_{rms}\cos\phi = \frac{V_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} \times \cos\phi$$

$$= \frac{V_0 I_0}{2}\cos\frac{\pi}{4} = \frac{V_0 I_0}{2\sqrt{2}}$$ **(1)**

(b) As R, L and C are in series, therefore at any instant through the three elements, AC has the same amplitude and phase, i.e. instantaneous current, $I = I_0 \sin \omega t$. **(1)**

14. (i) As, the dielectric slab is introduced between the plates of the capacitor, its capacitance will increase. Hence, the potential drop across the capacitor will decrease, i.e. $V = \frac{Q}{C}$. As a result, the potential drop across the bulb will increase as they are connected in series.Thus, its brightness will increase. **(1)**

(ii) As the resistance R is increased, the potential drop across the resistor will increase. As a result, the potential drop across the bulb will decrease as they are connected in series. Thus, its brightness will decrease. **(1)**

15. Given, $L = 50\text{ mH} = 50\times10^{-3}\text{ H}$

$C = 80\,\mu\text{F} = 80\times10^{-6}\text{ F}$

$R = 40\ \Omega,\ V = 250\text{ V}$

(i) In the L-C-R, the resonant angular frequency is given by

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{50\times10^{-3}\times80\times10^{-6}}} = 500\text{ rad/s}$$

$\therefore$ The actual/source frequency is given by

$$\omega = 2\pi\nu \Rightarrow \nu = \frac{\omega}{2\pi}$$

$$\Rightarrow \nu = \frac{500}{2\pi} = \frac{250}{\pi} = 79.61 \approx 80\text{ Hz}$$ **(1½)**

(ii) Quality factor, $Q = \frac{\omega_0 L}{R} = \frac{500\times50\times10^{-3}}{40} = 0.625$ **(1/2)**

16. Refer to Sol. 15 on page 206.
[(i)500 rad/s, (ii) 0.4] **(2)**

17. Refer to Sol. 15 on page 206.
[(i) 500 rad/s, (ii) 0.6] **(2)**

18. Let us consider a capacitor C connected to an AC source as shown below

Let the AC voltage applied is

$$V = V_m \sin\omega t$$

and voltage across capacitor, $V = \frac{q}{C}$

Applying Kirchhoff's loop rule, we have

$$V_m \sin\omega t = \frac{q}{C} \Rightarrow q = CV_m \sin\omega t$$

Also, $I = \frac{dq}{dt} \Rightarrow I = \frac{d}{dt}C(V_m \sin\omega t)$

$$I = \omega C V_m \cos\omega t \quad \text{...(i)}$$

We know that, $\cos \omega t = \sin(\omega t + \pi/2)$ ...(ii)

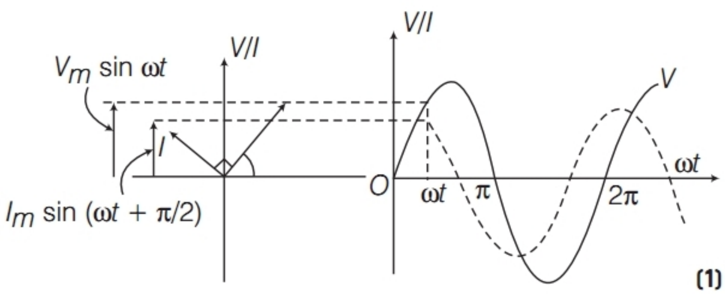
In the circuit, **(1)**

$$V_m = I_m X_C = I_m \frac{I}{\omega C} \Rightarrow I_m = V_m \omega C \quad \text{...(iii)}$$

Substituting the values of Eqs. (ii) and (iii) in Eq. (i), we get

$$I = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

The phase diagram which shows the current lead the voltage in phase by 90° is given below



(1)

19. (i) The impedance of a series *L-C-R* circuit is given by $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

Z will be minimum when $\omega L = 1/\omega C$, i.e. when the circuit is under resonance. Hence, in this condition Z will be minimum and equal to R. **(1)**

(ii) Average power dissipated through a series *L-C-R* circuit is given by

$$P_{av} = E_V I_V \cos\phi$$

where, E_V = rms value of alternating voltage

I_V = rms value of alternating current

and ϕ = phase difference between current and voltage

For wattless current, the power dissipated through the circuit should be zero.

i.e. $\cos\phi = 0$

$\Rightarrow \quad \cos\phi = \cos\frac{\pi}{2}$

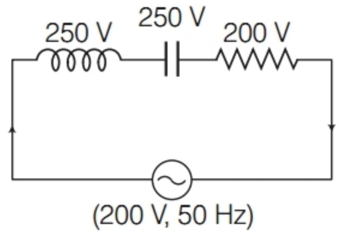
$\Rightarrow \quad \phi = \frac{\pi}{2}$

Hence, the condition for wattless current is that the circuit is purely inductive or purely capacitive in which the voltage and current differ by a phase angle of $\frac{\pi}{2}$, i.e. $\phi = \pm\frac{\pi}{2}$ **(1)**

20. (i) From given parameter $V_R = 200\,\text{V}$, $V_L = 250\,\text{V}$ and $V_C = 250\,\text{V}$. V_{eff} should be given as

$$V_{eff} = V_R + V_L + V_C$$

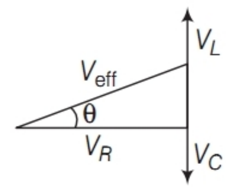
$$= 200\,\text{V} + 250\,\text{V} + 250\,\text{V} = 700\,\text{V}$$



(1)

However, $V_{eff} > 200\,\text{V}$ of the AC source. This paradox can be solved only by using phasor diagram, as given below

$$V_{eff} = \sqrt{V_R^2 + (V_L - V_C)^2}$$



Since, $V_L = V_C$

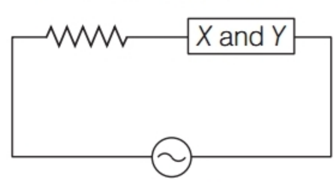
So, $V_{eff} = V_R = 200\,\text{V}$

(ii) Given, $R = 40\,\Omega$, so current in the *L-C-R* circuit.

$$I_{eff} = \frac{V_{eff}}{R} = \frac{200}{40} = 5\text{A} \quad [X_L = X_C \text{ or } Z = R]$$ **(1)**

21. (i) In *R-L* series combination, voltage leads the current by phase $\phi = \frac{\pi}{4}$. It means element X is an inductor (with reactance equal to R). In *R-C* series combination, voltage lags behind the current by phase $\phi = \frac{\pi}{4}$. So, element Y is a capacitor (with reactance equal to R). **(1)**

(ii) If both elements X and Y are connected in series with R, then power dissipation in the combination can be given as



$$P = V_{rms} \cdot I_{rms} \cdot \cos\phi$$

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Here, $X_L = X_C = R$. So, $\cos\phi = 1$

Hence, $P = V_{rms} I_{rms}$ (Maximum) **(1)**

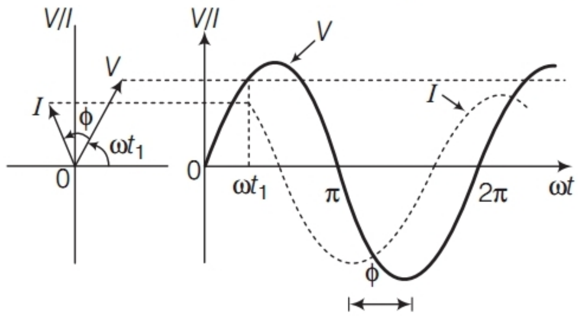
22. Impedance offered by series *L-C-R* circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

And voltage, $V = \sqrt{V_R^2 + (V_L - V_C)^2}$ **(1)**

As, V_C and V_L are the voltages applied across capacitor C and inductor L. V_C or V_L may be greater than V.

The situation may be shown in figure, where $V_C > V$.



(1)

23. (i) We know that, capacitive reactance $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$. Hence, on increasing capacitance, X_C will decrease and hence current will increase, therefore brightness of bulb will increase. As capacitance increases, capacitive reactance ($X_C = 1/\omega C$) decreases, impedance Z decreases, hence current increases i.e. brightness of bulb will increase. **(1)**

(ii) For DC source, frequency, $f = 0$.

$\therefore X_C$ becomes infinite, So, there will no flow of current and hence, bulb will not glow. **(1)**

24. Given, $L = 2.0\ \text{H}$

$$C = 2\mu\text{F} = 2\times 10^{-6}\ \text{F}, \quad R = 10\ \Omega$$

Now, $$Q\text{-factor} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{2}{2\times 10^{-6}}}$$

$$= \frac{1}{10\times 10^{-3}} = \frac{1}{10^{-2}} = 100$$ **(1)**

Quality factor is also defined as

$$Q = 2\pi f \times \frac{\text{Energy stored}}{\text{Power loss}}$$

So, higher the value of Q means the energy loss is at lower rate relative to the energy stored, i.e. the oscillations will die slowly and damping would be less. **(1)**

25. Given, $V = 140 \sin 314t$, $R = 50\ \Omega$

Comparing it with $V = V_0 \sin \omega t$

(i) Here, $\omega = 314\ \text{rad/s}$ **(1/2)**

i.e. $2\pi\nu = 314$ $[\because \omega = 2\pi\nu]$

$\Rightarrow$ $\nu = 314/2\pi$

$$\nu = \frac{314}{2\times 3.14} = 50\ \text{Hz}$$

Frequency of AC, $\nu = 50\ \text{Hz}$ **(1/2)**

(ii) As, $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{R}$ and $V_{\text{rms}} = \dfrac{V_0}{\sqrt{2}}$ **(1/2)**

Here, $V_0 = 140\ \text{V}$

$$V_{\text{rms}} = \frac{140}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = 70\sqrt{2}\ \text{V}$$

$$\therefore \quad I_{\text{rms}} = \frac{70\sqrt{2}}{R} = \frac{70\sqrt{2}}{50} = 1.9\ \text{A or } 2\ \text{A}$$ **(1/2)**

26. Refer to Sol. 25 on page 208.

(25 Hz, 4.95 A) **(2)**

27. Refer to Sol. 25 on page 208.

(50 Hz, 1.98 A) **(2)**

28. (i) $P = 150\ \text{W}$, $V = 220\ \text{V}$

Resistance of the bulb, $R = \dfrac{V^2}{P}$ **(1/2)**

$$R = \frac{220\times 220}{150} = 322.7\ \Omega$$ **(1/2)**

(ii) As, $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{R} = \dfrac{220}{322.7}$ $\quad (V_{\text{rms}} = V = 220\ \text{V})$

$\Rightarrow$ $I_{\text{rms}} = 0.68\ \text{A}$ **(1)**

29. Since, average power consumption in an AC circuit is given by

$$P_{\text{av}} = V_{\text{rms}} \times I_{\text{rms}} \times \cos\phi$$ **(1)**

But in pure capacitive circuit, phase difference between voltage and current is given by

$$\phi = \pi/2$$

$$\therefore \quad P_{\text{av}} = V_{\text{rms}} \times I_{\text{rms}} \times \cos\frac{\pi}{2}$$

$$\Rightarrow \quad P_{\text{av}} = 0 \qquad \left(\because \cos\frac{\pi}{2} = 0\right)$$

Thus, no power is consumed in pure capacitive AC circuit. **(1)**

30. (i) From graph (I), it is clear that resistance (opposition to current) is not changing with frequency, i.e. resistance does not depend on frequency of applied source, so the circuit element here is pure resistance (R).

From graph (II), it is clear that resistance increases linearly with frequency, so the circuit element here is an inductor.

Inductive resistance, $X_L = 2\pi f L$

$$\Rightarrow \quad X_L \propto f$$ **(1)**

(ii) Impedance offered by the series combination of resistance R and inductor L.

$$Z = \sqrt{R^2 + X_L^2}$$
$$= \sqrt{R^2 + (2\pi f L)^2}$$

In L-R circuit, the applied voltage leads the current in phase by $\pi/2$. **(1)**

31. (i) Let at any instant, the current and voltage in an L-C-R series AC circuit is given by

$V = V_m \sin \omega t$ and $I = I_m \sin(\omega t + \phi)$

The instantaneous power is given by

$$P = VI = I_m \sin(\omega t + \phi) V_m \sin \omega t$$
$$P = \frac{V_m I_m}{2}[2 \sin \omega t \sin(\omega t + \phi)]$$
$$P = VI = \frac{V_m I_m}{2}[\cos \phi - \cos(2\omega t + \phi)] \quad \ldots(\text{i})$$
$$[\because 2 \sin A \sin B = \cos(A - B) - \cos(A + B)]$$

Work done for a very small time interval dt is given by

$$dW = Pdt \Rightarrow dW = VIdt$$

$\therefore$ Total work done over T, a complete cycle is given by, $W = \int_0^T VIdt$ **(1/2)**

$$\text{But } P_{av} = \frac{W}{T} = \frac{\int_0^T VIdt}{T}$$
$$\Rightarrow P_{av} = \frac{1}{T}\int_0^T VIdt$$
$$= \frac{1}{T}\int_0^T \frac{V_m I_m}{2}[\cos \phi - \cos(2\omega t + \phi)]\, dt$$
$$\text{or } P_{av} = \frac{V_m I_m}{2T}\left[\int_0^T \cos \phi \, dt - \int_0^T \cos(2\omega t + \phi)\, dt\right]$$
$$= \frac{V_m I_m}{2T}[\cos \phi \, [t]_0^T - 0] \text{ (By trigonometry)}$$
$$\text{or } P_{av} = \frac{V_m I_m}{2T} \times \cos \phi \times T = \frac{V_m I_m}{2} \times \cos \phi$$
$$P_{av} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \cos \phi$$
$$\Rightarrow P_{av} = V_{rms} \times I_{rms} \times \cos \phi$$ **(1/2)**

This is the required expression.

(ii) $\because$ Power factor, $\cos \phi = \dfrac{R}{Z}$

where, R = resistance and Z = impedance.

Low power factor ($\cos \phi$) implies lower ohmic resistance and higher power loss as $P_{av} \propto \dfrac{1}{R}$ in power system (transmission line). **(1)**

32. When AC source is connected, the capacitor offers capacitive reactance $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi \nu C}$.

The current flows in the circuit and the lamp glows. **(1)**

(i) On reducing capacitance C, X_C increases so current in the circuit reduces. Therefore, the brightness of the bulb reduces.

(ii) On reducing frequency ν, X_C increases so current in the circuit reduces. Therefore, the brightness of the bulb reduces. **(1)**

33. Given, $L = 100 \text{ mH} = 100 \times 10^{-3}\text{H}$,

$C = 2\mu\text{F} = 2 \times 10^{-6} \text{ F}$, $\omega = 1000$ rad/s and $R = 400 \, \Omega$

(i) For phase difference, $\tan \phi = \dfrac{\left(\omega L - \dfrac{1}{\omega C}\right)}{R}$

[where, ϕ is phase difference between current and voltage]

$$\because \omega = 1000 \Rightarrow \omega L = 1000 \times 100 \times 10^{-3} = 100 \, \Omega$$
$$\frac{1}{\omega C} = \frac{1}{1000 \times 2 \times 10^{-6}} = \frac{1}{2 \times 10^{-3}} = 500 \, \Omega$$
$$\Rightarrow \tan \phi = \frac{100 - 500}{400} = \frac{-400}{400} = -1$$
$$\Rightarrow \phi = \tan^{-1}(-1) \Rightarrow \phi = 135°$$

Since, $\omega L < \dfrac{1}{\omega C}$ or $X_L < X_C$

Therefore, current is leading in phase by a phase angle 135°. **(1½)**

(ii) For unit power factor $\cos \phi = 1$

$$\Rightarrow \frac{R}{Z} = 1$$
$$\Rightarrow \frac{R}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C_1}\right)^2}} = 1$$

where, C_1 is the total capacitance.

$$\Rightarrow R^2 + \left(\omega L - \frac{1}{\omega C_1}\right)^2 = R^2$$
$$\Rightarrow \omega L = \frac{1}{\omega C_1} \Rightarrow 100 = \frac{1}{1000 C_1}$$
$$C_1 = \frac{1}{10^5} = 10^{-5} \text{ F} = 10 \, \mu\text{F}$$

Additional capacitance C' required in parallel

$= C_1 - C = 10 \, \mu\text{F} - 2 \, \mu\text{F} = 8 \, \mu\text{F}$ **(1½)**

34. (i) As, $P_{av} = V_{rms} I_{rms} \cos\phi$

In ideal inductor, current I_{rms} lags behind applied voltage V_{rms} by $\pi/2$.

$\therefore \quad \phi = \pi/2$

so, $P_{av} = V_{rms} I_{rms} \cos \pi/2$

or $\quad P_{av} = V_{rms} I_{rms} \times 0$

or $\quad P_{av} = 0$ **(1)**

(ii) Brightness of the lamp decreases. It is because when iron rod is inserted inside the inductor, its inductance L increases, thereby its inductive reactance X_L will also increase and hence, impedance Z of the circuit will increase. As, $I_{rms} = V_{rms}/Z$, so this decreases the current I_{rms} in the circuit and hence, the brightness of lamp will decrease. **(2)**

35. For unity power factor,

$$X_L = X_C$$
$$\omega L = 1/\omega C'$$

where, $C' = C + C''$

$$C' = \frac{1}{\omega^2 L} = \frac{1}{(1000)^2 \times 100 \times 10^{-3}}$$
$$= 10^{-5}\,\text{F} = 10\,\mu\text{F}$$

As $\quad C' = C + C''$

$$C'' = C' - C = 10 - 2 = 8\mu\text{F}$$

So, required capacitor is $8\,\mu$F which is added in parallel with the given capacitor. **(3)**

36. (i) Here, $L = 80\,\text{mH}, C = 250\,\text{mF}, \omega = 100\,\text{rad/s}$

$V_{rms} = 240\,\text{V}$

Impedance, $Z = |X_L - X_C| = \left|\omega L - \frac{1}{\omega C}\right|$ **(2)**

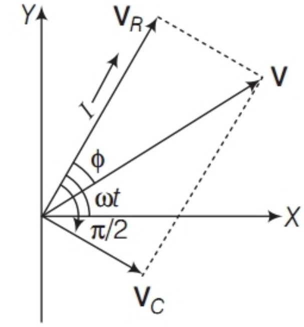
$$= \left|100 \times 80 \times 10^{-3} - \frac{1}{100 \times 250 \times 10^{-3}}\right|$$
$$= \left|8 - \frac{1}{25}\right| = 7.96$$
$$I_{rms} = \frac{V_{rms}}{\text{Reactance}} = \frac{240}{7.96} = 30.15\text{A}$$

(ii) Since, resistance of the circuit is negligible, hence given AC circuit is L-C circuit. For L-C circuit, phase difference is $\frac{\pi}{2}$, hence the total average power consumed by circuit is zero. **(1)**

37. (i) Applied AC voltage,

$$V = V_0 \sin\omega t \qquad \dots\text{(i)}$$

Phasor diagram for given R-C circuit is shown. From diagram, by parallelogram law of vector addition, $\mathbf{V}_R + \mathbf{V}_C = \mathbf{V}$ **(1)**



Using pythagorean theorem, we get

$$V^2 = V_R^2 + V_C^2 = (IR)^2 + (IX_C)^2$$
$$\Rightarrow \quad V^2 = I^2(R^2 + X_C^2)$$
$$I = V/\sqrt{R^2 + X_C^2} = V/Z$$

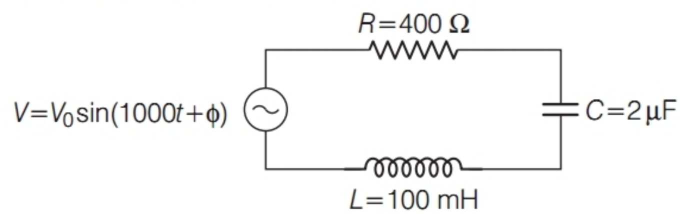
where, $Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + 1/\omega^2 C^2}$

Z = impedance.

(ii) The phase angle ϕ between resultant voltage and current is given by

$$\tan\phi = \frac{V_C}{V_R} = \frac{IX_C}{IR}$$
$$= \frac{X_C}{R} = \frac{1/\omega C}{R} = \frac{1}{\omega RC}$$ **(2)**

38. (i) Consider the given figure,



Since, the alternating emf in the above L-C-R series circuit would be represented by

$V = V_0 \sin(1000\,t + \phi) \;\Rightarrow\; \omega = 1000\,\text{Hz}$

Given, $R = 400\,\Omega$, $C = 2\,\mu\text{F}$, $L = 100\,\text{mH}$

$\because$ Capacitive reactance, $X_C = \frac{1}{\omega C}$

$$\Rightarrow \quad X_C = \frac{1}{1000 \times 2 \times 10^{-6}}$$
$$\Rightarrow \quad X_C = \frac{10^3}{2}$$
$$\Rightarrow \quad X_C = 500\,\Omega$$ **(1)**

$\because$ Inductive reactance, $X_L = \omega L$

$\Rightarrow \quad X_L = 1000 \times 100 \times 10^{-3} \Rightarrow \; X_L = 100\,\Omega$

So, we can see that $X_C > X_L$

$\Rightarrow \tan\phi$ is negative.

Hence, the voltage lags behind the current by a phase angle ϕ. The AC circuit is capacitance dominated circuit.

Phase difference, $\tan\phi = \frac{X_L - X_C}{R}$

$$\tan\phi = \frac{100-500}{400} \Rightarrow \tan\phi = \frac{-400}{400}, \tan\phi = -1$$

$$\Rightarrow \tan\phi = -\tan\left(\frac{\pi}{4}\right) \Rightarrow \phi = -\frac{\pi}{4}$$

This is the required value of the phase difference between the current and the voltage in the given series L-C-R circuit. **(1)**

(ii) Suppose, new capacitance of the circuit is C'.

Thus, to have power factor unity

$$\cos\phi' = 1 = \frac{R}{\sqrt{R^2 + (X_L - X'_C)^2}}$$

$$\Rightarrow R^2 = R^2 + (X_L - X'_C)^2$$

$$\Rightarrow X_L = X'_C = \frac{1}{\omega C'} \quad \text{or} \quad \omega L = \frac{1}{\omega C'}$$

$$\Rightarrow \omega^2 = \frac{1}{LC'} \quad \text{or} \quad (1000)^2 = \frac{1}{LC'} \quad (\because \omega = 1000)$$

$$\Rightarrow C' = \frac{1}{L \times 10^6} = \frac{1}{100 \times 10^{-3} \times 10^6}$$

$$= \frac{10}{10^6} = \frac{1}{10^5} = 10^{-5}$$

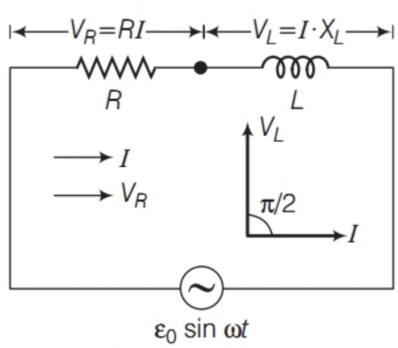
$$\Rightarrow C' = 10^{-5}\text{ F} = 10 \times 10^{-6}\text{ F} = 10\mu\text{F}$$

As, $C' > C$. Hence, we have to add an additional capacitor of capacitance $8\,\mu$F (10μF – 2μF) in parallel with previous capacitor. **(1)**

39. The inductive reactance $(X_L)\ \omega L$

where, ω = angular frequency of AC source

and L = inductance of the inductor.



Series L-R circuit

The net resistance of the circuit is given by

$$Z = \sqrt{X_L{}^2 + R^2}$$

where, R = resistance of the bulb.

(i) We known that, if the number of turns in the inductor decreases, then inductance L decreases.

So, the net resistance of the circuit decreases and hence, the current through the circuit increases, increasing the brightness of the bulb. **(1)**

(ii) If the soft iron rod is inserted in the inductor, then the inductance L increases. Therefore, the current through the bulb will decrease, decreasing the brightness of the bulb.

$$I_{\text{rms}} = \frac{\varepsilon_{\text{rms}}}{X_L}$$ **(1)**

(iii) If the capacitor of reactance, $X_C = X_L$ is connected in series with the circuit, then

$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

$$\Rightarrow \quad Z = R \quad [\because X_L = X_C]$$

This is a case of resonance. In this case, the maximum current will flow through the circuit. Hence, the brightness of the bulb will increase to maximum. **(1)**

40. Refer to Sol. 31 (i) on page 209. **(1)**

(i) If $\phi = 90°$, then no power is dissipated even though the current flows through the circuit.

$$P_{\text{av}} = 0$$

This current is called wattless current. The resistance of the circuit is zero which is shown below.

$$\because \quad \tan\phi = \left(\frac{X_L - X_C}{R}\right)$$

$$\Rightarrow \quad \tan\phi = \frac{\omega L - 1/\omega C}{R} = \infty \quad (\because \tan 90° = \infty)$$ **(1)**

$$\Rightarrow \quad R = 0$$

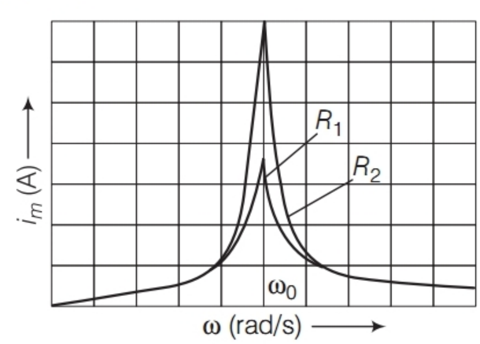
(ii) If $\phi = 0°$, then maximum power is dissipated in the circuit.

$$P_{\text{av}} = \text{maximum}$$

$$\tan\phi = \frac{X_L - X_C}{R} = 0 \quad (\because \tan 0° = 0)$$

$$\Rightarrow X_L = X_C \text{ (Resonance)}$$ **(1)**

41. Figure shows the variation of I_m with ω in a L-C-R series circuit for two values of resistance R_1 and $R_2 (R_1 > R_2)$,



(1)

The condition for resonance in the L-C-R circuit is

$$X_L = X_C$$

$$\Rightarrow \quad \omega_0 L = \frac{1}{\omega_0 C}$$

$$\Rightarrow \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

We see that the current amplitude is maximum at the resonant frequency. Since $I_m = V_m / R$ at resonance, the current amplitude for case R_2 is sharper to that for case R_1.

Quality factor or simply the Q-factor of a resonant L-C-R circuit is defined as the ratio of voltage drop across the resistance at resonance.

$$Q = V_L / V_R = \omega L / R$$

Thus finally, $Q = 1/R\sqrt{L/C}$

The Q-factor determines the sharpness at resonance as for higher value of Q-factor the tuning of the circuit and its sensitivity to accept resonating frequency signals will be much higher.

(2)

42. (i) The average power dissipated,

$$\overline{P} = (I^2 R) = (I_m^2 R \sin^2 \omega t) = I_m^2 R (\sin^2 \omega t)$$

$$\because \sin^2 \omega t = \frac{1}{2}(1 - \cos 2\omega t)$$

$$\therefore (\sin^2 \omega t) = \frac{1}{2}[1 - (\cos 2\omega t)] = \frac{1}{2}$$

$$(\because \cos 2\omega t = 0)$$

$$\therefore \quad \overline{P} = \frac{1}{2} I_m^2 R$$

(2)

(ii) Power of the bulb, $P = 100\,\text{W}$ and voltage, $V = 220$ V

The resistance of the bulb is given as

$$R = \frac{V^2}{P} = \frac{(220)^2}{100} = 484\ \Omega$$

(1)

43. (i) When a source of AC is connected to a capacitor of capacitance C_1 the charge on it grows from zero to maximum steady value Q_0.

The energy stored in a capacitor is, $E = \frac{1}{2} C V_0^2$

where, V_0 is maximum potential difference across the plates of the capacitor.

The alternating voltage applied is

$$\nu = \nu_0 \sin\omega t$$

and the current leads the emf by a phase angle of $\pi/2$

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) = I_0 \cos\omega t$$

$\therefore$ Work done over a complete cycle is

$$W = \int_0^T \nu I dt = \int_0^T (\nu_0 \sin\omega t)(I_0 \cos\omega t)\, dt$$

$$= \frac{\nu_0 I_0}{2} \int_0^T 2 \sin\omega t \cos\omega t\, dt$$

$$W = \frac{\nu_0 I_0}{2} \int_0^T \sin 2\omega t\, dt$$

$$W = \frac{\nu_0 I_0}{2} \left[-\frac{\cos 2\omega t}{2\omega} \right]_0^T = 0$$

(1½)

(ii) When DC source is connected, the condenser is charged but no current flows in the circuit. Therefore, the lamp does not glow. No change occurs even when capacitance of capacitor is reduced.

When AC source is connected, the capacitor offers capacitive reactance $X_C = 1/\omega C$. The current flows in the circuit and the lamp glows. On reducing C_1, X_C increases. Therefore, the glowing of the bulb reduces.

(1½)

44. Given, $L = 10$ H, $C = 40\,\mu$F,

$R = 60\ \Omega$, $V_{\text{rms}} = 240$ V

(i) Resonating angular frequency,

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10 \times 40 \times 10^{-6}}}$$

$$\omega_r = \frac{1}{20 \times 10^{-3}} = 50\ \text{rad/s}$$

(1)

(ii) Current at resonating frequency,

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{R} \quad (\because \text{At resonance, } Z = R)$$

$$= 240/60 = 4\ \text{A}$$

(1)

(iii) Inductive reactance, $X_L = \omega L$

At resonance, $X_L = \omega_r L = 50 \times 10 = 500\,\Omega$

Potential drop across to inductor,

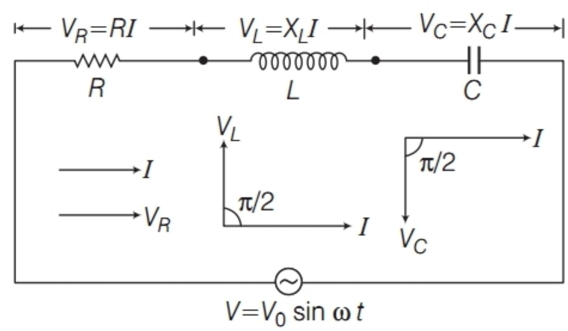
$$V_{\text{rms}} = I_{\text{rms}} \times X_L = 4 \times 500 = 2000\ \text{V}$$

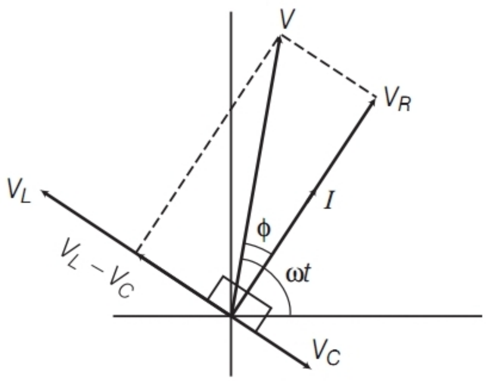
(1)

45. Assuming $X_L > X_C \Rightarrow V_L > V_C$

Net voltage, $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

where, V_L, V_C and V_R are potential difference across L, C and R respectively.





(2)

But, $V_R = IR, V_L = IX_L$,

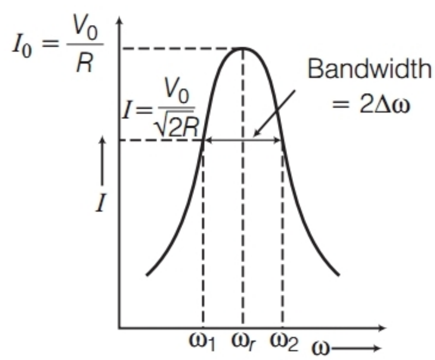
$V_C = IX_C$

$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$\frac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2}$$

Impedance of L-C-R circuit,

$$Z = \frac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2}$$



(1)

46. Given, $V_{rms} = 220$ V, $L = 20$ mH $= 2 \times 10^{-2}$ H,

$R = 110\ \Omega$,

$$C = \frac{800}{\pi^2}\ \mu F = \frac{800}{\pi^2} \times 10^{-6}\ F$$

(i) Average power observed by L-C-R series AC circuit is maximum when circuit is in resonance.

$\therefore$ Resonant frequency,

$$\omega_r = \frac{1}{\sqrt{LC}} \Rightarrow \nu_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{2 \times 10^{-2} \times \frac{800}{\pi^2} \times 10^{-6}}}$$

$$\nu_0 = \frac{1000}{2 \times 4} = 125\ s^{-1}$$

$$\nu_0 = 125\ s^{-1}$$

(1½)

(ii) As, $I_{rms} = \frac{V_{rms}}{Z} = \frac{220}{110} = 2$ A

$\because \quad Z = R = 110\ \Omega$

$\therefore$ Maximum current amplitude,

$$I_0 = I_{rms}\sqrt{2} = 2\sqrt{2}\ A$$

(1½)

47. Due to change in flux, the emf is induced in the coil. The rate of change of flux will give the value of emf.

(i) Let an alternating voltage, $V = V_0 \sin \omega t$ is applied across pure inductor of inductance L. The magnitude of induced emf is given by

$$e = L\, dI / dt$$ (1)

For the circuit,

Magnitude of induced emf = Applied voltage

i.e. $L\frac{dI}{dt} = V_0 \sin \omega t$ or $dI = \frac{V_0}{L} \sin \omega t\, dt$

On integrating both sides, we get

$$I = \frac{V_0}{L}\int \sin \omega t\, dt$$

$$= \frac{V_0}{L}\left(\frac{-\cos\omega t}{\omega}\right)$$ (1)

or $I = -\frac{V_0}{\omega L} \cos \omega t$

$$= -\frac{V_0}{\omega L} \sin\left(\frac{\pi}{2} - \omega t\right)$$

$$\therefore \quad I = \frac{V_0}{X_L} \sin\left(\omega t - \frac{\pi}{2}\right) \quad \ldots(i)$$

where, $X_L = \omega L$ = inducting reactance

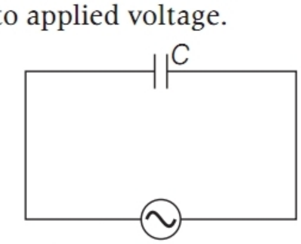
$$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right) \quad \ldots(ii)$$

where, I_0 = peak value of AC

But, $V = V_0 \sin \omega t \quad \ldots(iii)$

(ii) From Eqs. (ii) and (iii), it is clear that current lags behind the voltage by phase $\pi/2$. (1)

48. (i) Let alternating voltage, $V = V_0 \sin \omega t$ is applied across a capacitor C. At any instant, the potential difference across the capacitor is equal to applied voltage. (1)



$$V = V_0 \sin \omega t \quad \ldots(i)$$

$\therefore V$ = Potential difference across the capacitor

$$= \frac{q}{C}$$

$\Rightarrow \quad q = CV \quad \text{or} \quad q = CV_0 \sin \omega t$

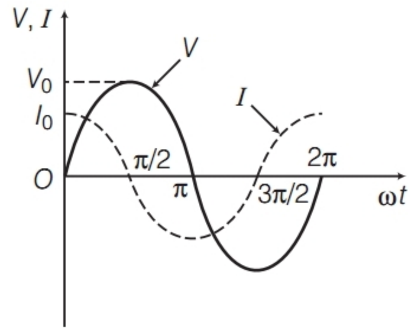
$\therefore \quad \frac{dq}{dt} = \omega C V_0 \cos \omega t \quad \text{or} \quad I = \frac{V_0}{(1/\omega C)} \cos \omega t$

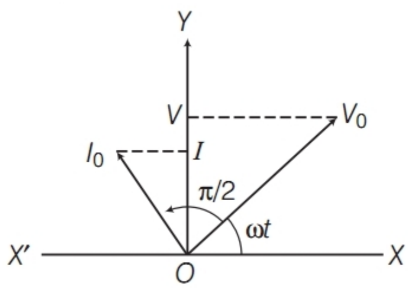
or $\quad I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \quad \ldots\text{(ii)}$

where, $I_0 = \frac{V_0}{(1/\omega C)} = \frac{V_0}{X_C}$

Capacitive reactance, $X_C = \frac{1}{\omega C}$ **(1)**

(ii) From Eqs. (i) and (ii), current leads the voltage by phase $\pi/2$.





(1)

49. (i) As average power, $P_{av} = V_{rms} I_{rms} \cos\phi$

In ideal inductor, current I_{rms} lags behind applied voltage V_{rms} by $\pi/2$.

i.e. $\quad \phi = \pi/2$

$\therefore \quad P_{av} = V_{rms} I_{rms} \cos \pi/2$

or $\quad P_{av} = V_{rms} I_{rms} \times 0$

or $\quad P_{av} = 0$

Thus, an ideal inductor does not dissipate power in an AC circuit. **(2)**

(ii) (a) As we know,

Inductive reactance, $X_L = (2\pi f)L \quad \ldots\text{(i)}$

where, L is the self-inductance of the inductor.

Since, the above equation is in the form, $y = mx$

where, $\quad y = X_L$, $x = f$ and $m = 2\pi L$.

So, from the given graph, we get

Slope of the X_L *versus* f graph, $m = \frac{\Delta y}{\Delta x}$

$= \frac{(60 - 40)}{(300 - 200)} = \frac{20}{100} = 0.2$

$\Rightarrow \quad m = 2\pi L = 0.2$

or $\quad L = \frac{0.2}{2\pi} = 0.0318\text{ H}$

(b) Let the capacitance of a capacitor be C.

Since, the given circuit will become a series L-C-R circuit. So, its power dissipation is maximum, if $\phi = 0$.

$\Rightarrow \quad \tan\phi = \frac{\omega L - 1/\omega C}{R} = 0$

$[\because \tan 0° = 0]$

$\Rightarrow \quad \omega L = \frac{1}{\omega C}$

$\Rightarrow \quad C = \frac{1}{\omega^2 L} = \frac{1}{(2\pi f)^2 L}$

Given, $f = 300\text{ s}^{-1}$

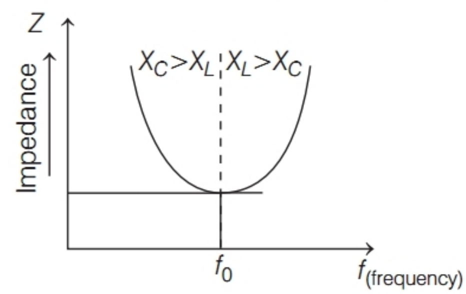
Substituting the values of f and L in the above equation, we get

$C = \frac{1}{(2\pi \times 300)^2 \times 0.0318}$

$= 8.859 \times 10^{-6}\text{ F}$

$= 8.86\ \mu\text{F}$ **(3)**

50. (i) Refer to Sol. 45 on pages 212 and 213.



Variation of Impedance with frequency

(1)

(ii) At resonance, $X_L = X_C$,

At resonance, voltage across inductor is equal to voltage across capacitor in magnitude only but both are in opposite polarities.

Hence, phase difference between V_L and V_C is 180°. **(2)**

(iii) As in case of DC supply, the current is independent of frequency. So, the value of current is 1 A but in AC supply, the current is 0.5 A as the value of impedence increases and hence value of current decreases.

For DC, $R = \frac{V}{I} = \frac{200}{1} = 200\ \Omega$

For AC, $Z = \frac{V}{I} = \frac{200}{0.5} = 400\ \Omega$

$$Z = \sqrt{R^2 + \omega^2 L^2}$$

$$\Rightarrow 400 = \sqrt{(200)^2 + 4\pi^2 (50)^2\ L^2} \quad [\because \omega = 2\pi f L]$$

$$\Rightarrow \quad 160000 = 40000 + 4\pi^2 \times 2500\ L^2$$

$$\Rightarrow \quad L^2 = \sqrt{12}$$ **(2)**

or $\quad L = 1.101$ H

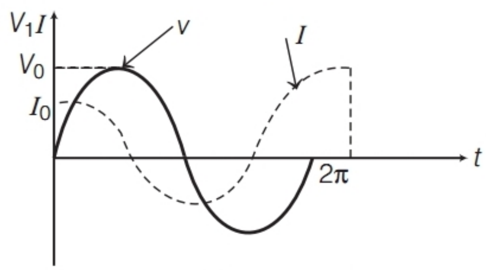
51. (i) Given, $V = V_0 \sin\omega t$

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

As it is clear that, the current leads the voltage by a phase angle $\frac{\pi}{2}$.

$\therefore$ The device X is a capacitor. **(1)**

(ii)



(1)

(iii) The reactance of the capacitance is given as

$$X_C = \frac{1}{\omega C}$$

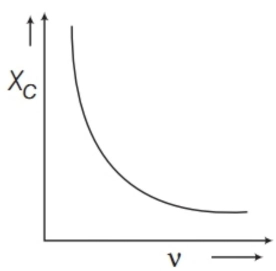
where, ω = angular frequency

and $\quad C$ = capacitance of capacitor.

$$\therefore \quad X_C = \frac{1}{2\pi \nu C}$$

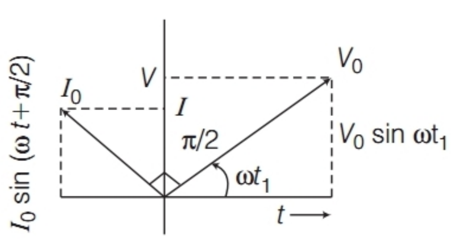
where, ν = frequency of AC or $X_C \propto \frac{1}{\nu}$

$\therefore$ The graphical representation between reactance of capacitance and frequency is given as



(2)

(iv) **Phasor diagram**



(1)

52. (i) Device X is a capacitor.

As, the current is leading voltage by $\pi/2$ rad. **(1)**

(ii) Curve A represents power,

Curve B represents voltage and

Curve C represents current.

As, $\quad V = V_0 \sin\omega t$

Current, $\quad I = I_0 \cos\omega t$

As, in the case of capacitor,

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \text{ [current is leading voltage]}$$

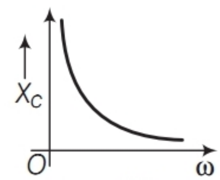
Average power, $P = V_{rms} \times I_{rms} \times \cos\phi = \frac{V_0 \cos\phi}{2}$

(2)

where, ϕ = phase difference

(iii) As, X_C = capacitive reactance $= \frac{1}{C\omega}$

where, ω is angular frequency.



So, reactance or impedance decreases with increase in frequency.

Graph of X_C *versus* ω is shown below, **[1]**

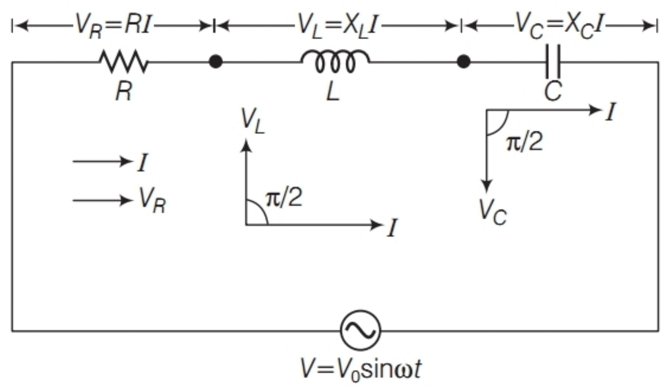
(iv) For a capacitor fed with an AC supply

$$V = \frac{q}{C} \text{ or } q = CV = CV_0 \sin\omega t$$

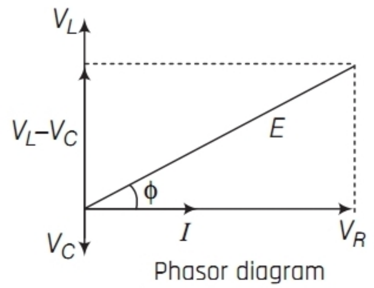
$$\therefore \quad I = \frac{dq}{dt} = \frac{V_0}{X_C} \sin\left(\omega t + \frac{\pi}{2}\right)$$ **(1)**

53. If I is the current in the circuit containing inductor of inductance L, capacitor of capacitance C and resistor of resistance R in series, then the voltage drop across the inductor is

$$V_L = I \times X_L$$



which leads current I by phase angle of $\pi/2$, and voltage drop across the capacitor is $V_2 = I \times X_C$. **(1)** which lags behind current I by phase angle of $\pi/2$, and voltage drop across the resistor is $V_R = IR$ which is in phase with current I. So, the net voltage E across the circuit is (using phasor diagram)



Phasor diagram

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\Rightarrow \quad V = I\sqrt{R^2 + (X_L - X_C)^2} \Rightarrow V = IZ$$

where, $Z = \sqrt{R^2 + (X_L - X_C)^2}$ is known as impedance. Phase angle between voltage and current is given by $\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$.

A series *L*-*C*-*R* circuit has its natural angular frequency, $\omega = \frac{1}{\sqrt{LC}}$

and natural (resonating) frequency, $\nu = \frac{1}{2\pi\sqrt{LC}}$.

When the applied AC in the circuit has this frequency, the series *L*-*C*-*R* circuit offers minimum impedance i.e. only *R* and current at this frequency flows maximum. In the case of resonance, voltage and current are in same phase.

Above mentioned condition is known as condition of resonance. In this condition **(1)**

(i) Inductive and capacitive reactances are equal

$$X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

$[\because \omega = 2\pi\nu]$

$$\Rightarrow \nu = \frac{1}{2\pi\sqrt{LC}}$$

(ii) Potential drop across inductor and capacitor are equal, $V_L = V_C$

(iii) The series resonant circuit is also called an acceptor circuit because when a number of different frequency currents are into the circuit, the circuit offers minimum impedance to natural frequency current.

For *L*-*R* circuit, $X_L = R$

Power factor, $P_1 = \cos\phi$

$$= \frac{R}{\sqrt{R^2 + X_L^2}} = \frac{R}{R^2 + R^2} = \frac{1}{\sqrt{2}}$$

For *L*-*C*-*R* circuit, as *C* is put in series with *L*-*R* circuit and $X_L = X_C$

Power factor, $P_2 = \cos\phi$

$$= \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{\sqrt{R^2 + (X_L - X_L)^2}} = \frac{R}{R} = 1$$

Required ratio $= P_1/P_2 = 1:\sqrt{2}$ **(3)**

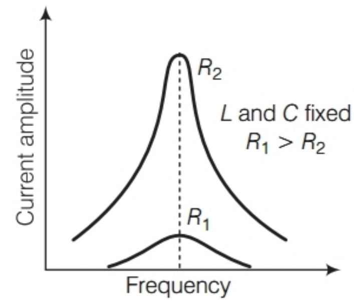
54. (i) To draw maximum current from a series *L*-*C*-*R* circuit, the circuit at particular frequency $X_L = X_C$. **(1)**

$$\nu = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\times 3.14\sqrt{8\times 2\times 10^{-6}}} = 39.80 \text{ Hz}$$

This frequency is known as the series resonance frequency.

(ii) $I_0 = \frac{V}{R} = \frac{200}{100} = 2\text{A}$ **(1)**

(iii)



(1)

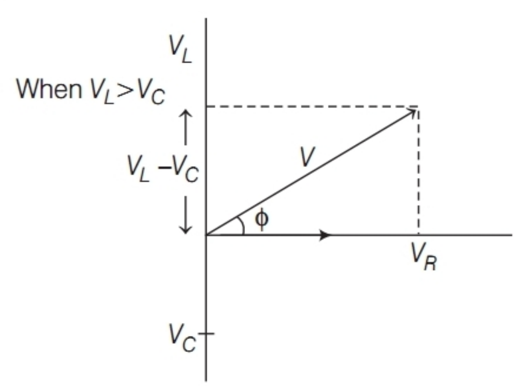
(iv) **Sharpness of resonance** It is defined as the ratio of the voltage developed across the inductance (*L*) or capacitance (*C*) at resonance to the voltage developed across the resistance (*R*).

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$ **(1)**

It may also be defined as the ratio of resonance angular frequency to the bandwidth of the circuit, $Q = \omega_r/2\Delta\omega$

Circuit become more selective if the resonance is more sharp, maximum current is more, the circuit is close to resonance for smaller range of $(2\Delta\omega)$ of frequencies. Thus, the tuning of the circuit will be good. **(1)**

55. (i) Phasor diagram for *L*-*C*-*R* series circuit is given as



Resultant volgate, $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

$$\Rightarrow IZ = \sqrt{(IR)^2 + I^2(X_L - X_C)^2}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$\therefore$ Amplitude of current,

$$I_0 = \frac{V_0}{Z} = \frac{V\sqrt{2}}{Z}$$

$$I_0 = \frac{\sqrt{2[R^2 + (X_L - X_C)^2]}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

From figure, tan ϕ

$$= \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR} = \frac{X_L - X_C}{R}$$ **(2)**

(ii) Refer to Sol. 41 on pages 211 and 212. **(3)**

56. (i) Refer to Sol. 53 on pages 215 and 216. **(2)**

(ii) $L = \frac{4}{\pi^2}\text{H}, \nu = 50\text{ Hz}, R = 100\Omega, V = 200\text{V}$

$$X_L = X_C \text{ or } \omega L = 1/\omega C$$

$$C = \frac{1}{\omega^2 L} = \frac{1}{4\pi^2\nu^2 \times L}$$

$$= \frac{1}{4\pi^2 \times 50 \times 50 \times \frac{4}{\pi^2}}$$

$$= \frac{1}{2500 \times 16} = \frac{1}{40000} = 2.5\times10^{-5}\text{F} = 25\mu\text{F}$$

$$I = \frac{V}{Z} = \frac{V}{R} = \frac{200}{100} = 2\text{ A}$$ **(3)**

57. $V = V_0 \sin\omega t$ and $I = I_0 \sin \omega t$

Work done in small dt will be **(2)**

$$dW = P\, dt = VI\, dt = V_0 I_0 \sin^2\omega t\, dt$$

$$= \frac{V_0 I_0}{2}(1 - \cos 2\omega t)\, dt$$

The average power dissipated per cycle in the resistor will be $P_{\text{av}} = \frac{W}{T} = \frac{1}{T}\int_0^T dW$ **(1)**

$$= \frac{V_0 I_0}{2T}\int_0^T (1 - \cos 2\omega t) = \frac{V_0 I_0}{2T}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{V_0 I_0}{2T}[(T-0) - 0] = \frac{V_0 I_0}{2} = \frac{V_0^2}{2R}$$

$$P_{\text{av}} = \frac{V_0 I_0}{\sqrt{2}\sqrt{2}} = V_{\text{rms}} I_{\text{rms}} = \frac{V_{\text{rms}}^2}{R} \quad \left[\because \frac{V_0}{2} = V_{\text{rms}}\right]$$

The average power is $P = V_{\text{rms}} I_{\text{rms}} \cos\phi$. If $\cos\phi$ is small, then current considerable increases when voltage is constant. Power loss, is $I^2 R$. Hence, power loss increases. **(1)**

Wattless current Refer to Sol. 10 on page 206. **(1)**

58. Refer to Sol. 45 on pages 212 and 213 **(3)**

The receiving antenna picks up the frequencies transmitted by different stations and a number of voltage appears in L-C-R circuit corresponding to different frequencies. But maximum current flows in circuit for that AC voltage which have got the frequency is equal to resonant frequency of circuit

i.e. $$\nu = \frac{1}{2\pi\sqrt{LC}}$$ **(1)**

For higher quality factor resonance, the signal received from other stations becomes weak due to sharpness of resonance. Thus, signal of desired frequency or program is tunned in. **(1)**

59. (i) Refer to Sol. 54 (iv) on page 216.

Q. factor Refer to Sol. 2 on page 205.

i.e. Quality factor, $Q = \frac{\text{Voltage across } L \text{ or } C}{\text{Applied voltage}}$

$$Q = \frac{(\omega_r L)I}{RI}$$

[$\because$ applied voltage = voltage across R]

$$Q = \omega_r L/R \text{ or } Q = \frac{(1/\omega_r C)I}{RI} = \frac{1}{RC\omega_r}$$

$$\therefore \quad Q = \frac{L}{RC \cdot \frac{1}{\sqrt{LC}}} \quad \left[\text{using } \omega_r = \frac{1}{\sqrt{LC}}\right]$$

$$= \frac{1}{R}\sqrt{\frac{L}{C}}$$

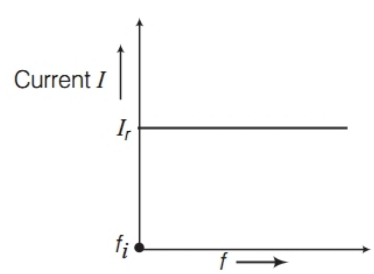
or $$Q = \frac{1\sqrt{LC}}{RC} = \frac{1}{R}\sqrt{\frac{L}{C}} \quad \left[\text{using } \omega_r = \frac{1}{\sqrt{LC}}\right]$$

Thus, $$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$ **(2)**

This is the required expression.

(ii) Let initially I_r current is flowing in all the three circuits. If frequency of applied AC source is increased, then the change in current will occur in the following manner.

(a) **Circuit containing resistance R only** There will not be any effect in the current on changing the frequency of AC source.

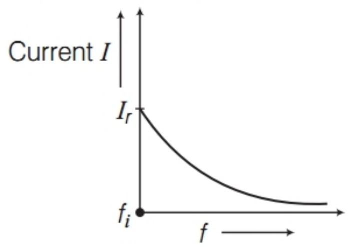


where, f_i = initial frequency of AC source. There is no effect on current with the increase in frequency. **(1)**

(b) **AC circuit containing inductance only**
With the increase of frequency of AC source inductive reactance increase as

$$I = \frac{V_{rms}}{X_L} = \frac{V_{rms}}{2\pi f L}$$

For given circuit, $I \propto 1/f$



(1)

Current decreases with the increase of frequency.

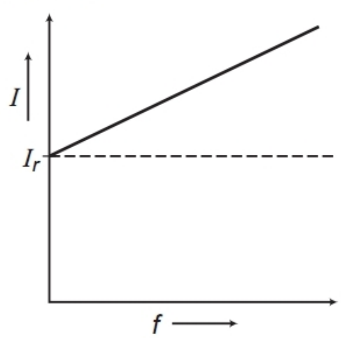
(c) **AC circuit containing capacitor only**

$$X_C = 1/\omega C = 1/2\pi f C$$

Current, $I = V_{rms}/X_C = V_{rms} / \frac{1}{2\pi f C}$

$$I = 2\pi f C V_{rms}$$

For given circuit, $I \propto f$
Current increases with the increase of frequency.



(1)

60. Phase difference between voltage and current,

$$\tan\phi = \frac{X_L - X_C}{R} \quad \ldots(i)$$

and $$I_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{(X_L - X_C)^2 + R^2}}$$

$\therefore$ Expression of AC, $I = I_0 \sin(\omega t - \phi)$ **(1)**

Conditions for resonance

(a) Inductive reactance must be equal to capacitive reactance

i.e. $X_L = X_C$

(b) As, $X_L = X_C$

$$\omega_r L = \frac{1}{\omega_r C} \Rightarrow \omega_r^2 = \frac{1}{LC} \Rightarrow \omega_r = \frac{1}{\sqrt{LC}}$$

where, ω_r = resonant angular frequency. **(1)**

(c) Impedance becomes minimum and equal to ohmic resistance

i.e. $Z = Z_{minimum} = R$

(d) AC becomes maximum,

$$\therefore \quad I_{max} = \frac{V_{max}}{Z_{min}} = \frac{V_{max}}{R}$$ **(1)**

(e) Voltage and current arrives in same phase.

Power factor

i.e. $$\cos\phi = \frac{P_{av}}{V_{rms}\, I_{rms}} = \frac{\text{True power}}{\text{Apparent power}}$$

Also, $$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$ **(1)**

(i) The power factor is maximum, i.e. $\cos\phi = +1$, in L-C-R series AC circuit when circuit is in resonance.

(ii) The power factor is minimum when phase angle between V and I is 90°, i.e. either pure inductive circuit or pure capacitive AC circuit. **(1)**

61. To find the voltage across each circuit element, steps to be followed are :

(i) To calculate the maximum current in circuit, firstly find the (Z) impedance and rms value of current.

(ii) It can be calculated with the help of formula of phase difference.

As, applied voltage ,

$$V = 140 \sin 100\,\pi t$$
$$C = \frac{50}{\pi}\,\mu F = \frac{50}{\pi} \times 10^{-6} F,$$
$$L = \frac{5}{\pi} H,$$
$$R = 400\,\Omega$$

Comparing it with $V = V_0 \sin \omega t$,

$$V_0 = 140 V, \omega = 100\,\pi$$ **(1)**

Inductive reactance, $X_L = \omega L$

$$X_L = 100\pi \times 5/\pi$$
$$= 500\,\Omega$$

Capacitive reactance,

$$X_C = \frac{1}{\omega C}$$

$$X_C = \frac{1}{100\pi \times \frac{50}{\pi} \times 10^{-6}} = 200\,\Omega$$

Impedance of the AC circuit, **(1)**

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{(400)^2 + (500 - 200)^2}$$

$$Z = \sqrt{1600 + 900} = 500\ \Omega$$ **(1)**

Maximum current in the circuit, $I_0 = \frac{V_0}{Z} = \frac{140}{500}$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{140}{500 \times \sqrt{2}} = 0.2\ \text{A}$$ **(1)**

V_{rms} across resistor R, $V_R = I_{rms}\, R$

$$= 0.2 \times 400 = 80\ \text{V}$$

V_{rms} across inductor, $V_L = I_{rms}\, X_L$

$$= 0.2 \times 500 = 100\ \text{V}$$

V_{rms} across capacitor,

$$V_C = I_{rms} \times X_C$$

$$= 0.2 \times 200 = 40\ \text{V}$$ **(1)**

Here, $V < V_R + V_L + V_C$

Because V_L and V_R are not in same phase,

$\therefore$ $$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

Explanations

1. A choke coil is needed in the use of fluorescent tubes with AC mains because it reduces the voltage across the tube without wasting much power. It is an inductor with large inductance to reduce current in AC circuits without much loss of energy. **(1)**

2. The core of transformer is laminated to reduce the energy losses due to eddy currents, for increasing the efficiency. **(1)**

3. The characteristic properties of the material suitable for making core of a transformer are as follow

(i) Low retentivity or coercivity.

(ii) Low hysteresis loss or high permeability and susceptibility. **(1)**

4. Step-up transformer converts low alternating voltage into high alternating voltage. The secondary coil of step-up transformer has a greater number of turns than the primary coil $(N_s > N_p)$. **(1)**

5. Given, input power $(V_p) = 2200$ V

Number of turns $(N_p) = 3000$

Output power $(V_s) = 220$ V

As, $\frac{V_s}{V_p} = \frac{N_s}{N_p} \Rightarrow \frac{220}{2200} = \frac{N_s}{3000}$ **[1]**

$\Rightarrow \quad N_s = \frac{220}{2200} \times 3000 = 300$

$\therefore$ Number of turns in the secondary winding, $N_s = 300$ turns. **(1)**

6. Principle of transformer A transformer is based on the principle of mutual induction, i.e. whenever the amount of magnetic flux linked with a coil changes, an emf is induced in the neighbouring coil. This changing flux sets up an induced emf in the secondary coil, also self induced emf in primary coil. **(1)**

Electric power is transmitted over long distances at high voltage. So, step-up transformers are used at power stations to increase the voltage whereas a series of step-down transformers at customer end are used to decrease the voltage upto 220 V. **(1)**

7. $V_p = 2200\text{V}, I_p = 5\text{A}, N_p = 4000$

$V_s = 220$ V, $N_s = ?$ $I_s = ?$

$$\frac{V_s}{V_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p}$$ **(1)**

$$\frac{220}{2200} = \frac{5}{I_s} = \frac{N_s}{4000}$$

$$\frac{220}{2200} = \frac{5}{I_s} \Rightarrow \frac{1}{10} = \frac{5}{I_s}$$

$$\Rightarrow \quad I_s = 50 \text{ A}$$

$$\frac{5}{I_s} = \frac{N_s}{4000}$$

$$\Rightarrow \quad \frac{5}{50} = \frac{N_s}{4000} \Rightarrow N_s = 400$$ **(1)**

8. For principle of working of transformer

Refer to Sol. 6 on page 222. **(1)**

No, transformer cannot be used to change DC voltage because DC voltage is constant and cannot change flux linked with primary or secondary coils. Due to low resistance of primary winding, a heavy DC current will flow through it, causing overheating in winding and finally transformer will burn. **(1)**

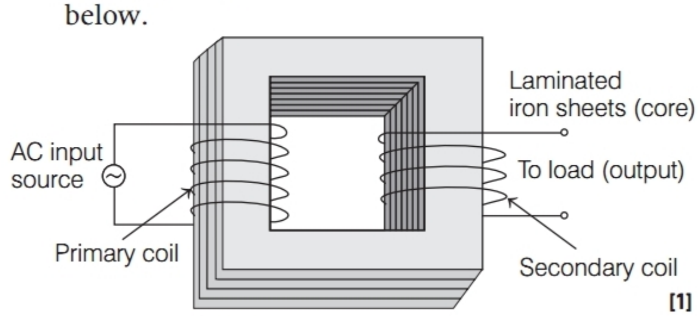
9. For energy losses in transformer are

(i) Eddy current loss (ii) Flux leakage

(iii) Copper loss (iv) Hysteresis loss

(v) Humming loss **(2)**

10. Step-down transformer convert high voltage (low current) into low voltage (high current). It works on the principle of mutual induction. It is shown below.



[1]

Step-down transformer

In step-down transformer, number of turns in secondary coil is less than number of turns in primary coil, hence voltage induced in secondary coil is less than voltage given to primary coil.

Core to transformer is laminated to reduce eddy current and thus, increase efficiency. **(1)**

11. (a) Refer to text on pages 219 and 220. (Transformer) **(2½)**

(b) Given, power = 1200 kW,

$V = 220$ V, resistance = $0.5\,\Omega$,

$V_p = 4000, V_s = 220$ V, distance = 20 km

Power = $I_p V_p$

$1200 \times 1000 = I_p \times 4000 \Rightarrow I_p = 300$ A

Power loss = $(I_p)^2 \times R$ (2 lines)

$= (300)^2 \times 0.5 \times 20 \times 2$

$= 18 \times 10^5$ W **(2½)**

12. Refer to Sol. 10 on page 222. **(2)**

(ii) $\varepsilon_s/\varepsilon_p = N_s/N_p$ **(1)**

(iii) For an ideal transformer, $P_{in} = P_{out}$ **(1)**

$\varepsilon_p I_p = \varepsilon_s I_s \Rightarrow I_p / I_s = \varepsilon_s / \varepsilon_p = N_s/N_p$

(iv) $P_{in} = P_{out} = 550$ W $\Rightarrow \varepsilon_p I_p = 550$

$220 \times I_p = 550 \Rightarrow I_p = 550/220 = 5/2 = 2.5$ A **(1)**

13. (i) Transformer is a device which converts high voltage AC into low voltage AC and *vice-versa*.

Refer to Sol. 6 on page 222.

For Diagram Refer to Sol. 15 (i) on page 223. **(2)**

There are number of energy losses in a transformer.

(a) **Copper losses** due to Joule's heating produced across the resistances of primary and secondary coils. It can be reduced by using copper wires.

(b) **Hysteresis losses** due to repeated magnetisation and demagnetization of the core of transformer. It is minimised by using soft iron core, as area of hysteresis loop for soft iron is small and hence energy loss also becomes small.

(c) **Iron losses** due to eddy currents produced in soft iron core. It is minimised by using laminated iron core.

(d) **Flux losses** due to flux leakage or incomplete flux linkage and can be minimised by proper coupling of primary and secondary coils. **(1)**

(ii) Here, $N_p = 100, N_s/N_p = 100$

$\varepsilon_p = 220$ V, $P_{in} = 1100$ W

(a) $N_p = 100$

$\therefore N_s = 10000$

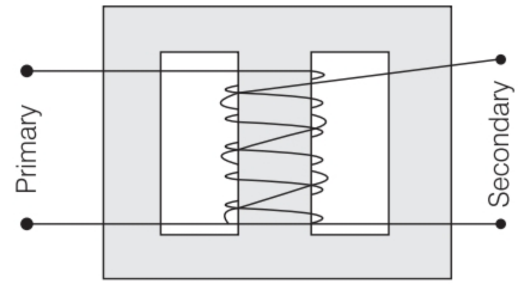
(b) $I_p = P_{in}/\varepsilon_p = 1100/220 = 5$ A

(c) $\varepsilon_s = N_s/N_p \times \varepsilon_p = 100 \times 220 = 22000$ V

(d) $I_s = \dfrac{P_{out}}{\varepsilon_s} = \dfrac{1100}{22000} = \dfrac{1}{20}$ A $\quad (\because P_{out} = P_{in})$

(e) $P_s = P_{out} = P_{in} = 1100$ W **(2)**

14. (i) The schematic arrangement of a transformer is shown as below **(1)**



(ii) **Principle of transformer**

Refer to Sol. 6 on page 222. **(2)**

Working When an alternating current is passed through the primary, the magnetic flux through the iron core changes, which does two things, produces emf in the primary and an induced emf is set up in the secondary. If we assume that the resistance of primary is negligible, then the back emf will be equal to the voltage applied to the primary.

(a) $\therefore \quad V_1 = -N_1 \dfrac{d\phi}{dt}$

and $V_2 = -N_2 \dfrac{d\phi}{dt}$

where, N_1 and N_2 are number of turns in the primary and the secondary coils respectively while V_1 and V_2 are their voltages respectively.

(b) But for ideal transformers, $V_1 I_1 = V_2 I_2$

$V_1/V_2 = I_2/I_1$

(iii) **Main assumptions**

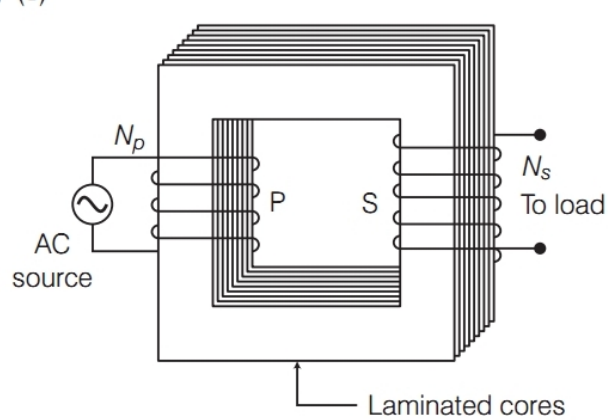
(a) The primary resistance and current are small.

(b) The flux linked with primary and secondary coil is same. There is no leakage of flux from the core.

(c) secondary current is small.

(iv) Refer to text on pages 219 and 220. (Energy losses in a transformer) **(2)**

15. (i)



(3)

$N_s > N_p$

Refer to Sol. 6 on page 222.

(ii) Refer to Sol. 13 (i) on page 223. **(2)**

16. (i) Refer to Sol. 6 on page 222. **(3)**

(ii) Refer to Sol. 13 (i) on page 223. **(1)**

(iii) No, it does not violate the law of conservation of energy because voltage increase is accompanied by decrease in current.

In step-up transformer the current decreases by the same proportion as the voltage increases. When voltage increases n times, the current reduce $1/n$ times. **(1)**

17. Refer to Sol. 13 and 14 on page 223. **(3)**

Step-up transformers are used at generating stations so as to transmit the power at high voltage to minimise the loss in the form of heat, whereas series of step-down transformers are used at receiving ends. **(2)**

18. Given, input voltages, $V_p = 2.5\times10^3$ V **(3)**

Input current, $I_p = 20\,A$

Also, $N_p/N_s = 10/1$

$\Rightarrow$ $N_s/N_p = 1/10$ ...(i) **(1/2)**

$$\text{Percentage efficiency} = \frac{\text{Output power}}{\text{Input power}}\times100$$

$$\Rightarrow \frac{90}{100} = \frac{\text{Output power}}{V_p\, I_p}$$

(i) $\text{Output power} = \dfrac{90}{100}\times(V_p\, I_p)$

$= \dfrac{90}{100}\times2.5\times10^3\times20$

$= 4.5\times10^4\, W$ **(1/2)**

(ii) $\therefore$ $V_s/V_p = N_s/N_p$

$\Rightarrow$ $V_s = N_s/N_p \times V_p$

Voltage, $V_s = \dfrac{1}{10}\times2.5\times10^3 = 250\,V$ **(1/2)**

(iii) $V_s\, I_s = 4.5\times10^4\, W$

Current, $I_s = \dfrac{4.5\times10^4}{V_s} = \dfrac{4.5\times10^4}{250}$

$\Rightarrow$ $I_s = 180$ A **(1/2)**