1 Mark Questions

1. If $R = \{(a, a^3): a$ is a prime number less than 5$\}$ be a relation. Find the range of R.

2. If $f: \{1, 3, 4\} \to \{1, 2, 5\}$ and $g: \{1, 2, 5\} \to \{1, 3\}$ given by $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(1, 3), (2, 3), (5, 1)\}$. Write down gof.

3. Let R is the equivalence relation in the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $R = \{(a, b): 2$ divides $(a - b)\}$. Write the equivalence class $[0]$.

4. If $R = \{(x, y): x + 2y = 8\}$ is a relation on N, then write the range of R

5. If $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and $f = \{(1, 4), (2, 5), (3, 6)\}$ is a function from A to B. State whether f is one-one or not.

6. If $f: R \to R$ is defined by $f(x) = 3x + 2$, then define $f[f(x)]$.

7. Write fog, if $f: R \to R$ and $g: R \to R$ are given by $f(x) = |x|$ and $g(x) = |5x - 2|$.

8. Write fog, if $f: R \to R$ and $g: R \to R$ are given by $f(x) = 8x^3$ and $g(x) = x^{1/3}$.

9. State the reason for the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ not to be transitive.

10. What is the range of the function
$$f(x) = \frac{|x - 1|}{x - 1}, x \neq 1?$$

11. If $f: R \to R$ is defined by $f(x) = (3 - x^3)^{1/3}$, then find $fof(x)$

12. If f is an invertible function, defined as $f(x) = \frac{3x - 4}{5}$, then write $f^{-1}(x)$.

13. If $f: R \to R$ and $g: R \to R$ are given by $f(x) = \sin x$ and $g(x) = 5x^2$, then find $gof(x)$.

14. If $f(x) = 27x^3$ and $g(x) = x^{1/3}$, then find $gof(x)$.

15. If the function $f: R \to R$ defined by $f(x) = 3x - 4$ is invertible, then find f^{-1}.

4 Marks Questions

16. Check whether the relation R defined on the set $A = \{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b): b = a + 1\}$ is reflexive, symmetric or transitive.

17. Let $f: N \to Y$ be a function defined as $f(x) = 4x + 3$, where, $Y = \{y \in N : y = 4x + 3$, for some $x \in N\}$. Show that f is invertible. Find its inverse.

18. Show that the relation R on IR defined as $R = \{(a, b): (a \leq b)\}$, is reflexive and transitive but not symmetric.

19. Prove that the function, $f: N \to N$ is defined by $f(x) = x^2 + x + 1$ is one-one but not onto. Find inverse of $f: N \to S$, where S is range of f.

20. If $f: W \to W$ is defined as $f(x) = x - 1$, if x is odd and $f(x) = x + 1$, if x is even. Show that f is invertible. Find the inverse of f, where W is the set of all whole numbers.

21. If $f, g: R \to R$ are two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x, \forall x \in R$. Then, find fog and gof.

22. If R is a relation defined on the set of natural numbers N as follows: $R = \{(x, y): x \in N, y \in N$ and $2x + y = 24\}$, then find the domain and range of the relation R. Also, find whether R is an equivalence relation or not.

23. If $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f : A \to B$ defined by $f(x) = \frac{x-2}{x-3}$, for all $x \in A$. Then, show that f is bijective. Find $f^{-1}(x)$.

24. If $A = \{1, 2, 3, \dots, 9\}$ and R is the relation in $A \times A$ defined by $(a, b)\, R\, (c, d)$, if $a + d = b + c$ for $(a, b), (c, d)$ in $A \times A$. Prove that R is an equivalence relation. Also, obtain the equivalence class $[(2, 5)]$.

25. If the function $f : R \to R$ is given by $f(x) = x^2 + 2$ and $g : R \to R$ is given by $g(x) = \frac{x}{x-1}$; $x \neq 1$, then find fog and gof, and hence find fog (2) and gof (– 3).

26. If $A = R - \{2\}$, $B = R - \{1\}$ and $f : A \to B$ is a function defined by $f(x) = \frac{x-1}{x-2}$, then show that f is one-one and onto. Hence, find f^{-1}.

27. Show that the function f in $A = R - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto. Hence, find f^{-1}.

28. Consider $f : R^+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse f^{-1} of f given by $f^{-1}(y) = \sqrt{y-4}$, where R^+ is the set of all non-negative real numbers.

29. Show that $f : N \to N$, given by

$$f(x) = \begin{cases} x+1, & \text{if } x \text{ is odd} \\ x-1, & \text{if } x \text{ is even} \end{cases}$$

is bijective (both one-one and onto).

30. If $f : R \to R$ is defined as $f(x) = 10x + 7$. Find the function $g : R \to R$, such that $gof = fog = I_R$.

31. If $f : R \to R$ is the function defined by $f(x) = 4x^3 + 7$, then show that f is a bijection

32. If Z is the set of all integers and R is the relation on Z defined as $R = \{(a, b) : a, b \in Z$ and $a - b$ is divisible by 5$\}$. Prove that R is an equivalence relation.

33. Show that the relation S in the set R of real numbers defined as $S = \{(a, b) : a, b \in R$ and $a \leq b^3\}$ is neither reflexive nor symmetric nor transitive.

34. Show that the relation S in set $A = \{x \in Z : 0 \leq x \leq 12\}$ given by $S = \{(a, b) : a, b \in A, |a - b|$ is divisible by 4$\}$ is an equivalence relation. Find the set of all elements related to 1.

35. Show that the relation S defined on set $N \times N$ by $(a, b)\, S\, (c, d) \Rightarrow a + d = b + c$ is an equivalence relation.

36. If $f : X \to Y$ is a function. Define a relation R on X given by $R = \{(a, b) : f(a) = f(b)\}$. Show that R is an equivalence relation on X.

37. Show that a function $f : R \to R$ given by $f(x) = ax + b$, $a, b \in R$, $a \neq 0$ is a bijective.

6 Marks Questions

38. Let $A = \{x \in Z : 0 \leq x \leq 12\}$. Show that $R = \{(a, b) : a, b \in A, |a - b|$ is divisible by 4$\}$ is an equivalence relation. Find the set of all elements related to 1. Also, write the equivalence class [2].

39. Show that the function $f : R \to R$ defined by $f(x) = \frac{x}{x^2+1}$, $\forall\, x \in R$ is neither one-one nor onto. Also, if $g : R \to R$ is defined as $g(x) = 2x - 1$, find $fog\,(x)$.

40. Show that the relation R on the set Z of all integers defined by $(x, y) \in R \Leftrightarrow (x - y)$ is divisible by 3 is an equivalence relation.

41. Consider $f: R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{y+6} - 1}{3}\right)$

Hence find

(i) $f^{-1}(10)$

(ii) y if $f^{-1}(y) = \frac{4}{3}$

where R_+ is the set of all non-negative real numbers.

42. Consider $f: R - \left\{-\frac{4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x+3}{3x+4}$. Show that f is bijective. Find the inverse of f and hence find $f^{-1}(0)$ and x such that $f^{-1}(x) = 2$.

43. Let $f: N \to N$ be a function defined as $f(x) = 9x^2 + 6x - 5$. Show that $f: N \to S$, where S is the range of f, is invertible. Find the inverse of f and hence find $f^{-1}(43)$ and $f^{-1}(163)$.

44. If $f, g: R \to R$ be two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x, \forall x \in R$. Then, find fog and gof. Hence find $fog(-3)$, $fog(5)$ and $gof(-2)$

45. If N denotes the set of all natural numbers and R be the relation on $N \times N$ defined by $(a, b)\, R\, (c, d)$, if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation.

46. Consider $f: R^+ \to [-9, \infty)$ given by $f(x) = 5x^2 + 6x - 9$. Prove that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{54 + 5y} - 3}{5}\right)$.

[where, R^+ is the set of all non-negative real numbers.]

47. Let $f: N \to R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f: N \to S$, where S is the range of f, is invertible. Also, find the inverse of f.

48. Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b): |a - b|$ is divisible by 2$\}$, is an equivalence relation. Write all the equivalence classes of R.

Objective Questions

(For Complete Chapter)

1 Mark Questions

1. If $R = \{(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), , (3, 9), (3, 12), (3, 6)\}$ is a relation on the set $A = \{3, 6, 9, 12\}$. Then, the relation is

(a) an equivalence relation
(b) reflexive and symmetric
(c) reflexive and transitive
(d) only reflexive

2. If R is a relation on the set N, defined by $\{(x, y): 2x - y = 10\}$, then R is

(a) reflexive
(b) symmetric
(c) transitive
(d) None of the above

3. If A and B are two equivalence relations defined on set C, then

(a) $A \cap B$ is an equivalence relation
(b) $A \cap B$ is not an equivalence relation
(c) $A \cup B$ is an equivalence relation
(d) $A \cup B$ is not an equivalence relation

4. If $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$, then which of the following relations is a function from A to B?

(a) $\{(1, 2), (2, 3), (3, 4), (2, 2)\}$
(b) $\{(1, 2), (2, 3), (1, 3)\}$
(c) $\{(1, 3), (2, 3), (3, 3)\}$
(d) $\{(1, 1), (2, 3), (3, 4)\}$

5. If $A = \{1, 3, 5, 7\}$ and $B = \{1, 2, 3, 4, 5, 6, 7, 8\}$, then the number of one-one function from A into B is

(a) 1340
(b) 1860
(c) 1430
(d) 1680

6. The function $f(x) = x^2 + bx + c$, where b and c are real constants, describes

(a) one-one mapping
(b) onto mapping
(c) not one-one but onto mapping
(d) neither one-one nor onto mapping

7. If $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6\}$ are two sets and function $f: A \to B$ is defined by $f(x) = x + 2, \forall x \in A$, then the function f is

(a) bijective (b) onto
(c) one-one (d) many-one

8. A mapping $f: n \to N$, where N is the set of natural numbers is defined as

$$f(n) = \begin{cases} n^2, & \text{for } n \text{ odd} \\ 2n + 1, & \text{for } n \text{ even} \end{cases}$$

for $n \in N$. Then, f is

(a) surjective but not injective
(b) injective but not surjective
(c) bijective
(d) neither injective nor surjective

9. The domain of the function $f(x) = \sqrt{\cos x}$ is

(a) $\left[\frac{3\pi}{2}, 2\pi\right]$ (b) $\left[0, \frac{\pi}{2}\right]$
(c) $[-\pi, \pi]$ (d) $\left[0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right]$

10. The range of the function $f(x) = x^2 + 2x + 2$ is

(a) $(1, \infty)$ (b) $(2, \infty)$
(c) $(0, \infty)$ (d) $[1, \infty)$

11. Range of the function $f(x) = \frac{x}{1 + x^2}$ is

(a) $(-\infty, \infty)$ (b) $[-1, 1]$
(c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (d) $[-\sqrt{2}, \sqrt{2}]$

12. If $f(x) = \sqrt{x}$ and $g(x) = 2x - 3$, then domain of $(fog)(x)$ is

(a) $(-\infty, -3)$ (b) $\left(-\infty, -\frac{3}{2}\right)$
(c) $\left[-\frac{3}{2}, 0\right]$ (d) $\left[\frac{3}{2}, \infty\right)$

13. If $f(x) = \frac{x + 2}{3x - 1}$, then $f\{f(x)\}$ is equal to

(a) x (b) $-x$
(c) $\frac{1}{x}$ (d) $-\frac{1}{x}$

14. If $f: R \to R$ is defined by $f(x) = x^3$, then $f^{-1}(8)$ is equal to

(a) $\{2\}$ (b) $\{2, 2\omega, 2\omega^2\}$
(c) $\{2, -2\}$ (d) $\{2, 2\}$