DCAM classes

Dynamic Classes for Academic Mastery

Application of Derivatives

Objective Questions

(For Complete Chapter)

1 Mark Questions

1. A sphere increases its volume at the rate of π cm^3/ s. The rate at which its surface area increases, when the radius is 1 cm is
   (a) 2π sq cm/s (b) π sq cm/s
   (c) $\frac{3\pi}{2}$ sq cm/s (d) $\frac{\pi}{2}$ sq cm/s

2. If gas is being pumped into a spherical balloon at the rate of $30\,ft^3$/min. Then, the rate at which the radius increases, when it reaches the value 15 ft is
   (a) $\frac{1}{15\pi}$ ft/min (b) $\frac{1}{30\pi}$ ft/min
   (c) $\frac{1}{20}$ ft/min (d) $\frac{1}{25}$ ft/min

3. The radius of a cylinder is increasing at the rate of 3 m/s and its altitude is decreasing at the rate of 4 m/s. The rate of change of volume, when radius is 4 m and altitude is 6 m, is
   (a) 80π cu m/s (b) 144π cu m/s
   (c) 80 cu m/s (d) 64 cu m/s

4. The length of the longest interval, in which $f(x) = 3\sin x - 4\sin^3 x$ is increasing, is
   (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{2}$ (d) π

5. Which of the following function is decreasing on $(0, \pi/2)$?
   (a) $\sin 2x$ (b) $\cos 3x$
   (c) $\tan x$ (d) $\cos 2x$

6. For what values of x, function $f(x) = x^4 - 4x^3 + 4x^2 + 40$ is monotonic decreasing?
   (a) $0 < x < 1$ (b) $1 < x < 2$
   (c) $2 < x < 3$ (d) $4 < x < 5$

7. If $y = 2x^3 - 2x^2 + 3x - 5$, then for $x = 2$ and $\Delta x = 0.1$, value of Δy is
   (a) 2.002 (b) 1.9
   (c) 0 (d) 0.9

8. If the error committed in measuring the radius of the circle is 0.05%, then the corresponding error in calculating the area is
   (a) 0.05% (b) 0.0025%
   (c) 0.25% (d) 0.1%

9. The slope of the normal to the curve $y = x^2 - \frac{1}{x^2}$ at $(-1, 0)$ is
   (a) $\frac{1}{4}$ (b) $-\frac{1}{4}$
   (c) 4 (d) -4

10. The point of the parabola $y^2 = 64x$ which is nearest to the line $4x + 3y + 35 = 0$ has coordinates
    (a) $(9, -24)$ (b) $(1, 81)$
    (c) $(4, -16)$ (d) $-(-9, -24)$

11. The minimum radius vector of the curve $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 1$ is of length
    (a) $a - b$ (b) $a + b$
    (c) $2a + b$ (d) None of these

12. The condition that $f(x) = ax^3 + bx^2 + cx + d$ has no extreme value is

(a) $b^2 > 3ac$ (b) $b^2 = 4ac$

(c) $b^2 = 3ac$ (d) $b^2 < 3ac$

13. The maximum value of xe^{-x} is

(a) e (b) $1/e$

(c) $-e$ (d) $-1/e$

14. The least value of the function $f(x) = ax + b/x, a > 0,$ $b > 0, x > 0$ is

(a) $\sqrt{ab}$ (b) $2\sqrt{\frac{a}{b}}$ (c) $2\sqrt{\frac{b}{a}}$ (d) $2\sqrt{ab}$

2 Marks Questions

1. The total cost $C(x)$ associated with the production of x units of an item is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$.
   Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output.
2. The total revenue received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$ in rupees. Find the marginal revenue when $x = 5$, where by marginal revenue we mean the rate of change of total revenue with respect to the number of items sold at an instant.

3. The volume of a sphere is increasing at the rate of 8 cm^3/s. Find the rate at which its surface area is increasing when the radius of the sphere is 12 cm.

4. Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on R.

5. The volume of a sphere is increasing at the rate of 3 cubic centimeter per second. Find the rate of increase of its surface area, when the radius is 2 cm.

6. Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ is always increasing on R.

4 Marks Questions

7. The volume of a cube is increasing at the rate of 8 cm^3/s. How fast is the surface area increasing when the length of its edge is 12 cm?

8. Find the intervals in which the function $f(x) = \frac{x^4}{4} - x^3 - 5x^2 + 24x + 12$ is
   (i) strictly increasing
   (ii) strictly decreasing.

9. Find the intervals in which the function $f(x) = -2x^3 - 9x^2 - 12x + 1$ is
   (i) strictly increasing
   (ii) strictly decreasing.

10. The length x of a rectangle is decreasing at the rate of 5 cm/min and the width y is increasing at the rate of 4 cm/min. When $x = 8$ cm and $y = 6$ cm, find the rate of change of
    (i) the perimeter.
    (ii) area of rectangle.

11. The side of an equilateral triangle is increasing at the rate of 2 cm/s. At what rate is its area increasing, when the side of the triangle is 20 cm?

12. Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is
    (i) strictly increasing.
    (ii) strictly decreasing.

13. Find the intervals in which the function given by $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36x}{5} + 11$ is
    (i) strictly increasing.
    (ii) strictly decreasing.

14. The sides of an equilateral tria ،le are increasing at the rate of 2 cm/s. Find the rate at which the area increases, when the side is 10 cm?

15. Find the value(s) of x for which $y = [x(x-2)]^2$ is an increasing function.

16. Using differentials, find the approximate value of $(3.968)^{3/2}$.

17. Find the intervals in which the function $f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$ is
    (i) strictly increasing.
    (ii) strictly decreasing.

18. Find the approximate value of $f(3.02)$, upto 2 places of decimal, where $f(x) = 3x^2 + 15x + 3$

19. Using differentials, find approximate value of $\sqrt{49.5}$.

20. A ladder 5 m long is leaning against a wall. Bottom of ladder is pulled along the ground away from wall at the rate of 2 m/s. How fast is the height on the wall decreasing, when the foot of ladder is 4 m away from the wall?

21. Show that $y = \log(1+x) - \frac{2x}{2+x}, x > -1$ is an increasing function of x, throughout its domain.

22. Find the intervals in which the function given by $f(x) = \sin x + \cos x, 0 \le x \le 2\pi$ is
    (i) increasing. (ii)decreasing.

23. Find the intervals in which the function given by $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is

(i) increasing. (ii) decreasing.

24. Sand is pouring from the pipe at the rate of 12 cm^3/s. The falling sand forms a cone on a ground in such a way that the height of cone is always one-sixth of radius of the base. How fast is the height of sand cone increasing when the height is 4 cm?

25. If the radius of sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its surface area.

26. Find the intervals in which the function $f(x) = \sin x + \cos x, 0 \le x \le 2\pi$ is strictly increasing and strictly decreasing.

27. Show that the function $f(x) = x^3 - 3x^2 + 3x, x \in R$ is increasing on R.

28. Find the intervals in which the function $f(x) = (x-1)^3 (x-2)^2$ is

(i) increasing. (ii) decreasing.

29. Find the intervals in which the function $f(x) = 2x^3 + 9x^2 + 12x + 20$ is

(i) increasing. (ii) decreasing.

30. Find the intervals in which the function $f(x) = 2x^3 - 9x^2 + 12x - 15$ is

(i) increasing.

(ii) decreasing

31. Find the intervals in which the function $f(x) = 2x^3 - 15x^2 + 36x + 17$ is increasing or decreasing.

32. Find the intervals in which the function $f(x) = 2x^3 - 9x^2 + 12x + 15$ is

(i) increasing.

(ii) decreasing.

6 Marks Questions

33. Prove that $y = \frac{4 \sin \theta}{2 + \cos \theta} - \theta$ is an increasing function in $\left(0, \frac{\pi}{2}\right)$.

34. Find the intervals in which the function $f(x) = \sin 3x - \cos 3x, 0 < x < \pi$, is strictly increasing or strictly decreasing.

35. Prove that the function f defined by $f(x) = x^2 - x + 1$ is neither increasing nor decreasing in $(-1, 1)$. Hence, find the intervals in which $f(x)$ is

(i) strictly increasing.

(ii) strictly decreasing.

36. Find the intervals in which the function $f(x) = 20 - 9x + 6x^2 - x^3$ is

(i) strictly increasing.

(ii) strictly decreasing.

37. Find the intervals in which the function f given by $f(x) = \sin x - \cos x, 0 \le x \le 2\pi$ is strictly increasing or strictly decreasing.

4 Marks Questions

1. Find the equation of tangent to the curve $y = \sqrt{3x - 2}$ which is parallel to the line $4x - 2y + 5 = 0$. Also, write the equation of normal to the curve at the point of contact.
2. Find the equation of the normal to the curve $x^2 = 4y$, which passes through the point $(-1, 4)$.
3. Find the equations of the tangent and the normal to the curve $16x^2 + 9y^2 = 145$ at the point (x_1, y_1), where $x_1 = 2$ and $y_1 > 0$.
4. Find the angle of intersection of the curves $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$, at the point in the first quadrant.

5. Show that the equation of normal at any point t on the curves $x = 3\cos t - \cos^3 t$ and $y = 3\sin t - \sin^3 t$ is $4(y\cos^3 t - x\sin^3 t) = 3\sin 4t$.

6. Find the equation of tangents to the curve $y = x^3 + 2x - 4$ which are perpendicular to the line $x + 14y - 3 = 0$.

7. The equation of tangent at (2, 3) on the curve $y^2 = ax^3 + b$ is $y = 4x - 5$. Find the values of a and b.

8. Find the point on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts on the axes.

9. Find the equations of the tangent and normal to the curves $x = a\sin^3\theta$ and $y = a\cos^3\theta$ at $\theta = \frac{\pi}{4}$.

10. Find the equations of the tangent and normal to the curves $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(\sqrt{2}a, b)$.

11. Find the points on curve $y = x^3 - 11x + 5$ at which equation of tangent is $y = x - 11$.

12. Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which tangent is parallel to X-axis.

13. Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to y-coordinate of the point.

14. Find the equation of tangent to curves $x = \sin 3t, y = \cos 2t$ at $t = \frac{\pi}{4}$.

15. Find the equations of tangents to the curve $y = (x^2 - 1)(x - 2)$ at the points, where the curve cuts the X-axis.

16. Find the equation of tangent to the curve $4x^2 + 9y^2 = 36$ at the point $(3\cos\theta, 2\sin\theta)$.

17. Find the equation of tangent to the curve $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at point $x = 1, y = 0$.

18. Find the points on the curve $y = [x(x - 2)]^2$, where the tangent is parallel to X-axis.

19. Find the equation of tangent to the curve $y = \frac{x - 7}{x^2 - 5x + 6}$ at the point, where it cuts the X-axis.

20. Find the equations of the normal to the curve $y = x^3 + 2x + 6$, which are parallel to line $x + 14y + 4 = 0$.

6 Marks Questions

21. Find the angle of intersection of the curves $y^2 = 4ax$ and $x^2 = 4by$.

22. Find the equation of tangents to the curve $y = \cos(x + y), -2\pi \le x \le 2\pi$ that are parallel to the line $x + 2y = 0$.

23. Find the value of p for which the curves $x^2 = 9p(9 - y)$ and $x^2 = p(y + 1)$ cut each other at right angles.

24. Find the equations of the tangent to the curve $y = x^2 - 2x + 7$ which is
    (i) parallel to the line $2x - y + 9 = 0$.
    (ii) perpendicular to the line $5y - 15x = 13$.

25. Find the equation of the normal at a point on the curve $x^2 = 4y$, which passes through the point (1, 2). Also, find the equation of the corresponding tangent.

26. Find the equations of tangents to the curve $3x^2 - y^2 = 8$, which passes through the point $\left(\frac{4}{3}, 0\right)$.

27. Find all the points on the curve $y = 4x^3 - 2x^5$ at which the tangent passes through the origin.

28. Prove that the curves $x = y^2$ and $xy = k$ cut at the right angles, if $8k^2 = 1$.

29. Prove that all normals to the curves $x = a\cos t + at\sin t$ and $y = a\sin t - at\cos t$ are at a constant distance 'a' from the origin.

30. Find the equations of tangent and normal to the curve $x = 1 - \cos\theta$, $y = \theta - \sin\theta$ at $\theta = \frac{\pi}{4}$.

4 Marks Questions

1. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of material will be least when depth of the tank is half of its width. If the cost is to be borne by nearby settled lower income families, for whom water will be provided.

2. The sum of the perimeters of a circle and square is k, where k is some constant. Prove that the sum of their areas is least, when the side of the square is double the radius of the circle.

6 Marks Questions

3. A tank with rectangular base and rectangular sides, open at the top is to be constructed, so that its depth is 2 m and volume is 8 m^3. If building of tank cost ₹ 70 per sq m for the base and ₹ 45 per sq m for sides. What is the cost of least expensive tank?

4. Prove that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also, find the maximum volume.

5. Find the point on the curve $y^2 = 4x$, which is nearest to the point (2, – 8).

6. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$. Also, find the maximum volume in terms of volume of the sphere.

Or Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$. Also, show that the maximum volume of the cone is $\frac{8}{27}$ of the volume of the sphere.

7. A window is of the form of a semi-circle with a rectangle on its diameter. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

Or A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

8. Show that the surface area of a closed cuboid with square base and given volume is minimum, when it is a cube.

9. AB is the diameter of a circle and C is any point on the circle. Show that the area of ΔABC is maximum, when it is an isosceles triangle

10. If the sum of lengths of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum, when the angle between them is $\frac{\pi}{3}$.

11. A metal box with a square base and vertical sides is to contain 1024 cm^3. The material for the top and bottom costs ₹ 5 per cm^2 and the material for the sides costs ₹ 2.50 per cm^2. Find the least cost of the box.

12. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\cos^{-1} 1/\sqrt{3}$.

13. Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed, is $6\sqrt{3}r$.

14. The sum of the surface areas of a cuboid with sides x, $2x$ and $\frac{x}{3}$ and a sphere is given to be constant. Prove that the sum of their volumes is minimum, if x is equal to three times the radius of sphere. Also, find the minimum value of the sum of their volumes.

Or The sum of surface areas of a sphere and a cuboid with sides $\frac{x}{3}$, x and $2x$, is constant. Show that the sum of their volumes is minimum, if x is equal to three times the radius of sphere.

15. Find the local maxima and local minima of the function $f(x) = \sin x - \cos x$, $0 < x < 2\pi$. Also, find the local maximum and local minimum values.

16. Find the minimum value of $(ax + by)$, where $xy = c^2$.

17. Find the coordinates of a point on the parabola $y = x^2 + 7x + 2$ which is closest to the straight line $y = 3x - 3$.

18. A point on the hypotenuse of a right triangle is at distances a and b from the sides of the triangle. Show that the minimum length of the hypotenuse is $(a^{2/3} + b^{2/3})^{3/2}$.

19. If the length of three sides of a trapezium other than the base are each equal to 10 cm, then find the area of the trapezium, when it is maximum.

20. Find the point P on the curve $y^2 = 4ax$, which is nearest to the point $(11a, 0)$.

21. Prove that the semi-vertical angle of the right circular cone of given volume and least curved surface area is $\cot^{-1} \sqrt{2}$.

22. Of all the closed right circular cylindrical cans of volume 128π cm^3, find the dimensions of the can which has minimum surface area.

23. Show that a cylinder of a given volume which is open at the top has minimum total surface area, when its height is equal to the radius of its base.

24. Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

25. Show that the height of a closed right circular cylinder of given surface and maximum volume is equal to diameter of base.

26. Prove that radius of right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

27. An open box with a square base is to be made out of a given quantity of cardboard of area C^2 sq units. Show that the maximum volume of box is $\frac{C^3}{6\sqrt{3}}$ cu units.

28. Prove that the area of a right angled triangle of given hypotenuse is maximum, when the triangle is isosceles.

29. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

30. A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m, then find the dimensions of the rectangle that will produce the largest area of the window.

31. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

32. Show that of all the rectangles with a given perimeter, the square has the largest area.

33. Show that of all the rectangles of given area, the square has the smallest perimeter.

34. Show that the semi-vertical angle of a right circular cone of maximum volume and given slant height is $\tan^{-1}\sqrt{2}$.

35. Find the point on the curve $y^2 = 2x$ which is at a minimum distance from the point (1, 4).

36. A wire of length 28 m is to be cut into two pieces. One of the two pieces is to be made into a square and the other into a circle. What should be the lengths of two pieces, so that the combined area of circle and square is minimum?

37. Show that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

38. Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, with its vertex at one end of the major axis.

39. Show that the right circular cylinder, open at the top and of given surface area and maximum volume is such that its height is equal to the radius of the base.