DCAM classes

Dynamic Classes for Academic Mastery

10.VECTOR ALGEBRA

Single Correct Answer Type

1. Vector $\vec{c}$ is perpendicular to vectors $\vec{a}=(2,-3,1)$ and $\vec{b}=(1,-2,3)$ and satisfies the condition $\vec{c}\cdot(\hat{\imath}+2\hat{\jmath}-7\hat{k})=10$. Then vector $\vec{c}$ is equal to

a) $(7,5,1)$ b) $(-7,-5,-1)$ c) $(1,1,-1)$ d) None of these

2. If $\vec{a},\vec{b}$ and $\vec{c}$ are unit coplanar vectors, then the scalar triple product $[2\vec{a}-\vec{b}2\vec{b}-\vec{c}2\vec{c}-\vec{a}]$ is

a) 0 b) 1 c) $-\sqrt{3}$ d) $\sqrt{3}$

3. Points $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ are coplanar and $(\sin\alpha)\vec{a}+(2\sin 2\beta)\vec{b}+(3\sin 3\gamma)\,\vec{c}-\vec{d}=0$. Then the least value of $\sin^2\alpha+\sin^2 2\beta+\sin^2 3\gamma$ is

a) $1/14$ b) 14 c) 6 d) $1/\sqrt{6}$

4. If $|\vec{a}|=2$ and $|\vec{b}|=3$ and $\vec{a}\cdot\vec{b}=0$, then$\left(\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times(\vec{a}\times\vec{b})\right)\right)\right)$is equal to

a) $48\hat{b}$ b) $-48\hat{b}$ c) $48\hat{a}$ d) $-48\hat{a}$

5. A parallelogram is constructed on $3\vec{a}+\vec{b}$ and$\vec{a}-4\vec{b}$, where $|\vec{a}|=6$ and $|\vec{b}|=8$, and $\vec{a}$and$\vec{b}$are anti-parallel. Then the length of the longer diagonal is

a) 40 b) 64 c) 32 d) 48

6. Let $\vec{a},\vec{b}$and $\vec{c}$ be three non-coplanar vector and $\vec{p},\vec{q}$and $\vec{r}$ the vector defined by the relations $\vec{p}=\frac{\vec{b}\times\vec{c}}{[\vec{a}\vec{b}\vec{c}]},\vec{q}=\frac{\vec{c}\times\vec{a}}{[\vec{a}\vec{b}\vec{c}]}$and $\vec{r}=\frac{\vec{a}\times\vec{b}}{[\vec{a}\vec{b}\vec{c}]}$. Then the value of the expression $(\vec{a}+\vec{b})\cdot\vec{p}+(\vec{b}+\vec{c})\cdot\vec{q}+(\vec{c}+\vec{a})\cdot\vec{r}$ is

a) 0 b) 1 c) 2 d) 3

7. If the vectors $\vec{a},\vec{b}$ and $\vec{c}$ from the sides BC,CA and AB, respectively, of triangle ABC, then

a) $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=0$ b) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

c) $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}$ d) $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=\vec{0}$

8. Vector $3\vec{a}-5\vec{b}$ and $2\vec{a}+\vec{b}$ are mutually perpendicular. If $\vec{a}+4\vec{b}$ and $\vec{b}-\vec{a}$ are also mutually perpendicular, then the cosine of the single between $\vec{a}$and$\vec{b}$is

a) $\frac{19}{5\sqrt{43}}$ b) $\frac{19}{3\sqrt{43}}$ c) $\frac{19}{2\sqrt{45}}$ d) $\frac{19}{6\sqrt{43}}$

9. If $\vec{a}$is parallel to$\vec{b}\times\vec{c}$,then $(\vec{a}\times\vec{b})\cdot(\vec{a}\times\vec{c})$is equal to

a) $|\vec{a}|^2(\vec{b}\cdot\vec{c})$ b) $|\vec{b}|^2(\vec{a}\cdot\vec{c})$ c) $|\vec{c}|^2(\vec{a}\cdot\vec{b})$ d) None of these

10. If G is the centroid of a triangle ABC, then $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}$ is equal to

a) $\vec{O}$ b) $3\overrightarrow{GA}$ c) $3\overrightarrow{GB}$ d) $3\overrightarrow{GC}$

11. Vector $\hat{a}$in the plane of $\vec{b}=2\hat{\imath}+\hat{\jmath}$and $\vec{c}=\hat{\imath}-\hat{\jmath}+\hat{k}$ is such it is equally inclined to $\vec{b}$and$\vec{d}$where $\vec{d}=\hat{\jmath}+2\hat{k}$. The value of $\hat{a}$ is

a) $\frac{\hat{\imath}+\hat{\jmath}+\hat{k}}{\sqrt{3}}$ b) $\frac{\hat{\imath}-\hat{\jmath}+\hat{k}}{\sqrt{3}}$ c) $\frac{2\hat{\imath}+\hat{\jmath}}{\sqrt{5}}$ d) $\frac{2\hat{\imath}+\hat{\jmath}}{\sqrt{5}}$

12. $ABCD$ is a quadrilateral. E is the point intersection of the line joining the midpoint of the opposite sides. If O is any point and $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}=\overrightarrow{xOE}$, then x is equal to

a) 3 b) 9 c) 7 d) 4

13. Let $P(3,2,6)$ be a point in space and Qbe a point on the line $\mathbf{\vec{r}}=(\mathbf{\hat{i}}-\mathbf{\hat{j}}+2\mathbf{\hat{k}})+\mu(-3\mathbf{\hat{i}}+\mathbf{\hat{j}}+5\mathbf{\hat{k}})$. Then, the value of μ for which the vector $\overrightarrow{\mathbf{PQ}}$ is parallel to the plane $x-4y+3z=1$ is

a) $\frac{1}{4}$ b) $-\frac{1}{4}$ c) $\frac{1}{8}$ d) $-\frac{1}{8}$

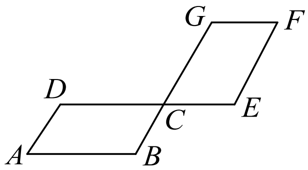
14. If $'P'$ is any arbitrary point on the circumcircle of the equilateral triangle of side length$l$ units, then $|\overrightarrow{PA}|^2+|\overrightarrow{PB}|^2+|\overrightarrow{PC}|^2$ is always equal to

a) $2l^2$ b) $2\sqrt{3}l^2$ c) l^2 d) $3l^2$

15. Let the pairs$\vec{a},\vec{b}$and$\vec{c},\vec{d}$each determine a plane. Then the planes are parallel if

a) $(\vec{a}\times\vec{c})\times(\vec{b}\times\vec{d})=\vec{0}$ b) $(\vec{a}\times\vec{c})\cdot(\vec{b}\times\vec{d})=\vec{0}$ c) $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=\vec{0}$ d) $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})=\vec{0}$

16. In the following figure,AB, DE and GF are parallel to each other and AD, BG and EF are parallel to each other. If $CD:CE=CG:CB=2:1$, then the value of area (ΔAEG): area (ΔABD) is equal to



a) 7/2 b) 3 c) 4 d) 9/2

17. Let $\vec{a}=2i+j+k, \vec{b}=i+2j-k$ and a unit vector $\vec{c}$ be coplanar.If $\vec{c}$ is perpendicular to$\vec{a}$, then $\vec{c}$ is

a) $\frac{1}{\sqrt{2}}(-j+k)$ b) $\frac{1}{\sqrt{3}}(-i-j-k)$ c) $\frac{1}{\sqrt{5}}(i-2j)$ d) $\frac{1}{\sqrt{3}}(i-j-k)$

18. $P(\vec{p})$ and $Q(\vec{q})$ are the position vectors of two fixed points and $R(\vec{r})$ is the position vector of a variable point. If R moves such that $(\vec{r}-\vec{p})\times(\vec{r}-\vec{q})=\vec{0}$, then the locus of R is

a) A plane containing the origin$O$ and parallel to two non-collinear vectors $\overrightarrow{OP}$and$\overrightarrow{OQ}$

b) The surface of a sphere described on PQ as its diameter

c) A line passing through points P and Q

d) A set of lines parallel to line PQ

19. If $\vec{a},\vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors, then the vector which is equally inclined to these vectors is

a) $\vec{a}+\vec{b}+\vec{c}$ b) $\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{b}}{|\vec{b}|}+\frac{\vec{c}}{|\vec{c}|}$ c) $\frac{\vec{a}}{|\vec{a}|^2}+\frac{\vec{b}}{|\vec{b}|^2}+\frac{\vec{c}}{|\vec{c}|^2}$ d) $|\vec{a}|\vec{a}-|\vec{a}|\vec{b}+|\vec{c}|\vec{c}$

20. A, B, C and D have position vectors $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ respectively, such that $\vec{a}-\vec{b}=2(\vec{d}-\vec{c})$. Then

a) AB and CD bisect each other b) BD and AC bisect each other

c) AB and CD trisect each other d) BD and AC trisect each other

21. If a is a real constant and A, B and C are variable angled and $\sqrt{a^2-4}\tan A+a\tan B+\sqrt{a^2+4}\tan c=6a$, then the least value of $\tan^2 A+\tan^2 B+\tan^2 C$ is

a) 6 b) 10 c) 12 d) 3

22. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three non-coplanar vectors and $\vec{r}$ be any arbitrary vector. Then $(\vec{a}\times\vec{b})\times(\vec{r}\times\vec{c})+(\vec{b}\times\vec{c})\times(\vec{r}\times\vec{a})+(\vec{c}\times\vec{a})\times(\vec{r}\times\vec{b})$ is always equal to

a) $[\vec{a}\vec{b}\vec{c}]\vec{r}$ b) $2[\vec{a}\vec{b}\vec{c}]\vec{r}$ c) $3[\vec{a}\vec{b}\vec{c}]\vec{r}$ d) None of these

23. The scalar $\vec{A}\cdot(\vec{B}+\vec{C})\times(\vec{A}+\vec{B}+\vec{C})$ equals

a) 0 b) $[\vec{A}\vec{B}\vec{C}]+[\vec{B}\vec{C}\vec{A}]$ c) $[\vec{A}\vec{B}\vec{C}]$ d) None of these

24. $\vec{a}$and $\vec{b}$ are two vectors such that $|\vec{a}|=1, |\vec{b}|=4$ and $\vec{a}\cdot\vec{b}=2$. If $\vec{c}=(2\vec{a}\times\vec{b})-3\vec{b}$, then find the angle between $\vec{b}$and $\vec{c}$

a) $\frac{\pi}{3}$ b) $\frac{\pi}{6}$ c) $\frac{3\pi}{4}$ d) $\frac{5\pi}{6}$

25. The volume of the parallelepiped whose sides are given by $\overrightarrow{OA}=2i-2j, \overrightarrow{OB}=i+j-k$ and$\overrightarrow{OC}=3i-k$ is

a) 4/13 b) 4 c) 2/7 d) 2

26. If $\vec{b}$ is a vector whose initial point divides the join of $5\hat{\imath}$ and $5\hat{\jmath}$ in the ratio $k:1$ and whose terminal point is the origin and $|\vec{b}|\le\sqrt{37}$, then k lies in the interval

a) $[-6,-1/6]$ b) $[-\infty,-6]\cup[-1/6,\infty]$ c) $[0,6]$ d) None of these

27. The number of the distinct real values of λ, for which the vectors $-\lambda^2\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$, $\hat{\mathbf{i}}-\lambda^2\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $\hat{\mathbf{i}}+\hat{\mathbf{j}}-\lambda^2\hat{\mathbf{k}}$ are coplanar, is

a) Zero b) One c) Two d) Three

28. If $4\vec{a}+5\vec{b}+9\vec{c}=0$, then $(\vec{a}\times\vec{b})\times[(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})]$is equal to

a) A vector perpendicular to the plane of $\vec{a},\vec{b}$ and $\vec{c}$ b) A scalar quantity

c) $\vec{0}$ d) None of these

29. Let $\vec{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k},\vec{b}=b_1\hat{\imath}+\hat{b}_2\hat{\jmath}+b_3\hat{k}$ and $\vec{c}=c_1\hat{\imath}+c_2\hat{\jmath}+c_3\hat{k}$be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$and$\vec{b}$ is $\pi/6$, then the value of $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}^2$ is

a) 0 b) 1

c) $\frac{1}{4}(a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2)$ d) $\frac{3}{4}(a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2)$

30. The position vectors of the point P and Q with respect to the origin O are $\vec{a}=\hat{\imath}+3\hat{\jmath}-2\hat{k}$ and $\vec{b}=3\hat{\imath}-\hat{\jmath}-2\hat{k}$,respectively. If M is a point on PQ, such that OM is the bisector of POQ, then $\overrightarrow{OM}$ is

a) $2(\hat{\imath}-\hat{\jmath}+\hat{k})$ b) $2\hat{\imath}+\hat{\jmath}-2\hat{k}$ c) $2(-\hat{\imath}+\hat{\jmath}-\hat{k})$ d) $2(\hat{\imath}+\hat{\jmath}+\hat{k})$

31. Let $\hat{a}$and $\hat{b}$ be mutually perpendicular unit vectors. Then for any arbitrary $\vec{r}$

a) $\vec{r}=(\vec{r}\cdot\hat{a})\hat{a}+(\vec{r}\cdot\hat{b})\hat{b}+\left(\vec{r}\cdot(\hat{a}\times\hat{b})\right)(\hat{a}\times\hat{b})$ b) $\vec{r}=(\vec{r}\cdot\hat{a})-(\vec{r}\cdot\hat{b})\hat{b}-\left(\vec{r}\cdot(\hat{a}\times\hat{b})\right)(\hat{a}\times\hat{b})$

c) $\vec{r}=(\vec{r}\cdot\hat{a})\hat{a}-(\vec{r}\cdot\hat{b})\hat{b}+\left(\vec{r}\cdot(\hat{a}\times\hat{b})\right)(\hat{a}\times\hat{b})$ d) None of these

32. If $\vec{p}=\frac{\vec{b}\times\vec{c}}{[\vec{a}\vec{b}\vec{c}]},\vec{q}=\frac{\vec{c}\times\vec{a}}{[\vec{a}\vec{b}\vec{c}]}$and $\vec{r}=\frac{\vec{a}\times\vec{b}}{[\vec{a}\vec{b}\vec{c}]}$, where $\vec{a},\vec{b}$ and $\vec{c}$ are three non-coplanar vectors, then the value of the expression $(\vec{a}+\vec{b}+\vec{c})\cdot(\vec{p}+\vec{q}+\vec{r})$ is

a) 3 b) 2 c) 1 d) 0

33. If $\hat{\imath}-3\hat{\jmath}+5\hat{k}$ bisects the angle between $\hat{a}$ and $-\hat{\imath}+2\hat{\jmath}+2\hat{k}$, where $\hat{a}$ is a unit vectors, then

a) $\hat{a}=\frac{1}{105}(41\hat{\imath}+88\hat{\jmath}-40\hat{k})$ b) $\hat{a}=\frac{1}{105}(41\hat{\imath}+88\hat{\jmath}+40\hat{k})$

c) $\hat{a}=\frac{1}{105}(-41\hat{\imath}+88\hat{\jmath}-40\hat{k})$ d) $\hat{a}=\frac{1}{105}(41\hat{\imath}-88\hat{\jmath}-40\hat{k})$

34. $\vec{a}$ and $\vec{b}$ are two unit vectors that are mutually perpendicular. A unit vector that is equally inclined to $\vec{a},\vec{b}$and$\vec{a}\times\vec{b}$ is

a) $\frac{1}{\sqrt{2}}(\vec{a}+\vec{b}+\vec{a}\times\vec{b})$ b) $\frac{1}{2}(\vec{a}\times\vec{b}+\vec{a}+\vec{b})$ c) $\frac{1}{\sqrt{3}}(\vec{a}+\vec{b}+\vec{a}\times\vec{b})$ d) $\frac{1}{3}(\vec{a}+\vec{b}+\vec{a}\times\vec{b})$

35. Let $\vec{r},\vec{a},\vec{b}$, and$\vec{c}$,be four non-zero vectors such that $\vec{r}\cdot\vec{a}=0,|\vec{r}\times\vec{b}|=|\vec{r}||\vec{b}|$ and $|\vec{r}\times\vec{c}|=|\vec{r}||\vec{c}|$.Then $[a,b,c]$ is equal to

a) $|a||b||c|$ b) $-|a||b||c|$ c) 0 d) None of these

36. If $\vec{a},\vec{b},\vec{c}$ are any three non-coplanar vectors, then the equation $[\vec{b}\times\vec{c}\vec{c}\times\vec{a}\vec{a}\times\vec{b}]x^2+[\vec{a}+\vec{b}\vec{b}+\vec{c}\vec{c}+\vec{a}]x+1+[\vec{b}-\vec{c}\vec{c}-\vec{a}\vec{a}-\vec{b}]=0$has roots

a) Real and distinct b) Real c) Equal d) Imaginary

37. Given that $\vec{a},\vec{b},\vec{p},\vec{q}$ are four vectors such that $\vec{a}+\vec{b}=\mu\vec{p},\vec{b}\cdot\vec{q}=0$ and $(\vec{b})^2=1$, where μ is a scalar. Then $|(\vec{a}\cdot\vec{q})\vec{p}-(\vec{p}\cdot\vec{q})\vec{a}|$is equal to

a) $2|\vec{p}\cdot\vec{q}|$ b) $(1/2)|\vec{p}\cdot\vec{q}|$ c) $|\vec{p}\times\vec{q}|$ d) $|\vec{p}\cdot\vec{q}|$

38. If $\vec{a}$ and $\vec{b}$ are any two vectors of magnitudes 1 and 2 respectively, and $(1-3\vec{a}\cdot\vec{b})^2+|2\vec{a}+\vec{b}+$ *3a×b2=47,* then the angle between $a$and$b$ is

a) $\pi/3$ b) $\pi-\cos^{-1}(1/4)$ c) $\frac{2\pi}{3}$ d) $\cos^{-1}(1/4)$

39. For non-zero vectors $\vec{a},\vec{b}$and $\vec{c}$, $|(\vec{a}\times\vec{b})\cdot\vec{c}|=|\vec{a}||\vec{b}||\vec{c}|$ holds if and only if

a) $\vec{a}\cdot\vec{b}=\vec{0},\vec{b}\cdot\vec{c}=0$ b) $\vec{b}\cdot\vec{c}=0,\vec{c}\cdot\vec{a}=0$

c) $\vec{c}\cdot\vec{a}=0,\vec{a}\cdot\vec{b}=0$ d) $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$

40. If $\vec{x}+\vec{c}\times\vec{y}=\vec{a}$ and $\vec{y}+\vec{c}\times\vec{x}=\vec{b}$, where $\vec{c}$ is a non-zero vector, then which of the following is not correct

a) $\vec{x} = \dfrac{\vec{b}\times\vec{c} + \vec{a} + (\vec{c}\cdot\vec{a})\vec{c}}{1 + \vec{c}\cdot\vec{c}}$

b) $\vec{x} = \dfrac{\vec{c}\times\vec{b} + \vec{b} + (\vec{c}\cdot\vec{a})\vec{c}}{1 + \vec{c}\cdot\vec{c}}$

c) $\vec{y} = \dfrac{\vec{a}\times\vec{c} + \vec{b} + (\vec{c}\cdot\vec{b})\vec{c}}{1 + \vec{c}\cdot\vec{c}}$

d) None of these

41. $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}, \hat{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}, \vec{c} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$. A vector coplanar with $\vec{b}$and$\vec{c}$ whose projection on $\vec{a}$ is magnitude $\sqrt{\frac{2}{3}}$ is

a) $2\hat{\imath} + 3\hat{\jmath} - 3\hat{k}$ b) $-2\hat{\imath} - \hat{\jmath} + 5\hat{k}$ c) $2\hat{\imath} + 3\hat{\jmath} + 3\hat{k}$ d) $2\hat{\imath} + \hat{\jmath} + 5\hat{k}$

42. A point O is the centre of a circle circumscribed about a triangle ABC. Then $\overline{OA}\sin 2A + \overline{OB}\sin 2B + \overline{OC}\sin 2C$ is equal to

a) $(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC})\sin 2A$

b) $3\overrightarrow{OG}$, where G is the centroid of triangle ABC

c) $\vec{0}$

d) None of thee

43. Find the value of λ so that the points P, Q, R and S on the sides OA, OB, OC and AB, respectively, of a regular tetrahedron $OABC$ are coplanar. It is given that $\frac{OP}{OA} = \frac{1}{3}, \frac{OQ}{OB} = \frac{1}{2}, \frac{OR}{OC} = \frac{1}{3}$ and $\frac{OS}{AB} = \lambda$

a) $\lambda = \dfrac{1}{2}$ b) $\lambda = -1$ c) $\lambda = 0$ d) For no value of λ

44. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three units vectors such that $3\vec{a} + 4\vec{b} + 5\vec{c} = 0$. Then which of the following statements is true?

a) $\vec{a}$ is parallel to $\vec{b}$

b) $\vec{a}$ is perpendicular to $\vec{b}$

c) $\vec{a}$ is neither parallel nor perpendicular to $\vec{b}$

d) None of these

45. A non-zero vector $\vec{a}$ is such that its projections along vectors $\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}, \frac{-\hat{\imath}+\hat{\jmath}}{\sqrt{2}}$ and $\hat{k}$ are equal, then unit vectors along $\vec{a}$ is

a) $\dfrac{\sqrt{2}\hat{\jmath} - \hat{k}}{\sqrt{3}}$ b) $\dfrac{\hat{\jmath} - \sqrt{2}\hat{k}}{\sqrt{3}}$ c) $\dfrac{\sqrt{2}}{\sqrt{3}}\hat{\jmath} + \dfrac{\hat{k}}{\sqrt{3}}$ d) $\dfrac{\hat{\jmath} - \hat{k}}{\sqrt{2}}$

46. If $\vec{a} = \hat{\imath} + \hat{\jmath}, \vec{b} = \hat{\jmath} + \hat{k}, \vec{c} = \hat{k} + \hat{\imath}$, then in the reciprocal system of vectors $\vec{a}, \vec{b}, \vec{c}$ reciprocal $\vec{a}$ of vector $\vec{a}$ is

a) $\dfrac{\hat{\imath} + \hat{\jmath} + \hat{k}}{2}$ b) $\dfrac{\hat{\imath} - \hat{\jmath} + \hat{k}}{2}$ c) $\dfrac{-\hat{\imath} - \hat{\jmath} + \hat{k}}{2}$ d) $\dfrac{\hat{\imath} + \hat{\jmath} - \hat{k}}{2}$

47. Let $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ be unit vectors such that $\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}} = 0$. Which of the following is correct?

a) $\vec{\mathbf{a}}\times\vec{\mathbf{b}} = \vec{\mathbf{b}}\times\vec{\mathbf{c}} = \vec{\mathbf{c}}\times\vec{\mathbf{a}} = \vec{\mathbf{0}}$

b) $\vec{\mathbf{a}}\times\vec{\mathbf{b}} = \vec{\mathbf{b}}\times\vec{\mathbf{c}} = \vec{\mathbf{c}}\times\vec{\mathbf{a}} \neq \vec{\mathbf{0}}$

c) $\vec{\mathbf{a}}\times\vec{\mathbf{b}} = \vec{\mathbf{b}}\times\vec{\mathbf{c}} = \vec{\mathbf{a}}\times\vec{\mathbf{c}} = \vec{\mathbf{0}}$

d) $\vec{\mathbf{a}}\times\vec{\mathbf{b}}, \vec{\mathbf{b}}\times\vec{\mathbf{c}}, \vec{\mathbf{c}}\times\vec{\mathbf{a}}$ are mutually perpendicular

48. For any two vectors $\vec{a}$ and$\vec{b}$, $(\vec{a}\times\hat{\imath})\cdot(\vec{b}\times\hat{\imath}) + (\vec{a}\times\hat{\jmath})\cdot(\vec{b}\times\hat{\jmath}) + (\vec{a}\times\hat{k})\cdot(\vec{b}\times\hat{k})$ is always equal to

a) $\vec{a}\cdot\vec{b}$ b) $2\vec{a}\cdot\vec{b}$ c) Zero d) None of these

49. If the vectors $\vec{a}$ and $\vec{b}$ are linearly independent satisfying $(\sqrt{3}\tan\theta + 1)\vec{a} + (\sqrt{3}\sec\theta - 2)\vec{b} = 0$, then the most general values of θ are

a) $n\pi - \dfrac{\pi}{6}, n \in Z$ b) $2n\pi \pm \dfrac{11\pi}{6}, n \in Z$ c) $n\pi \pm \dfrac{\pi}{6}, n \in Z$ d) $2n\pi + \dfrac{11\pi}{6}, n \in Z$

50. If $\vec{r}\cdot\vec{a} = \vec{r}\cdot\vec{b} = \vec{r}\cdot\vec{c} = 0$, where $\vec{a}, \vec{b}$and$\vec{c}$ are non-coplanar, then

a) $\vec{r} \perp (\vec{c}\times\vec{a})$ b) $\vec{r} \perp (\vec{a}\times\vec{b})$ c) $\vec{r} \perp (\vec{b}\times\vec{c})$ d) $\vec{r} = \vec{0}$

51. Let α, β and γ be distinct and real numbers. The points with position vectors $\alpha i + \beta\hat{\jmath} + \gamma\hat{k}$ $\beta\hat{\imath} + \gamma\hat{\jmath} + \alpha\hat{k}$ and $\gamma\hat{\imath} + \alpha\hat{\jmath} + \beta\hat{k}$

a) Are collinear

b) From an equilateral triangle

c) From a scalene triangle

d) Form a right-angled triangle

52. Let $\vec{a} = \vec{i} - \vec{k}, \vec{b} = x\vec{i} + \vec{j} + (1 - x)\vec{k}$ and $\vec{c} = y\vec{i} + x\vec{j} + (1 + x - y)\vec{k}$. Then $\vec{a}, \vec{b}$ and $\vec{c}$ are non- coplanar for

a) Some values of x

b) Some values of y

c) No values of x and y

d) For all values of x and y

53. $\vec{p}, \vec{q}$and $\vec{r}$ are three mutually perpendicular vectors of the same magnitude. If vector $\vec{x}$ satisfies the equation $\vec{p}\times((\vec{x} - \vec{q})\times\vec{p}) + \vec{q}\times((\vec{x} - \vec{r})\times\vec{q}) + \vec{r}\times((\vec{x} - \vec{p})\times\vec{r}) = \vec{0}$,then $\vec{x}$is given by

a) $\frac{1}{2}(\vec{p}+\vec{q}-2\vec{r})$ b) $\frac{1}{2}(\vec{p}+\vec{q}+\vec{r})$ c) $\frac{1}{3}(\vec{p}+\vec{q}+\vec{r})$ d) $\frac{1}{3}(2\vec{p}+\vec{q}-\vec{r})$

54. Let two non-collinear unit vectors $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ from and acute angle. A point P moves so that at any time t the position vector $\overrightarrow{\mathbf{OP}}$ (where O is the origin) is given by $\hat{\mathbf{a}}\cos t+\hat{\mathbf{b}}\sin t$. When P is farthest form origin O, let M be the length of $\overrightarrow{\mathbf{OP}}$ and $\hat{\mathbf{u}}$ be the unit vector along $\overrightarrow{\mathbf{OP}}$ Then,

a) $\hat{\mathbf{u}}=\frac{\hat{\mathbf{a}}+\hat{\mathbf{b}}}{|\hat{\mathbf{a}}+\hat{\mathbf{b}}|}$ and $M=(1+\hat{\mathbf{a}}\cdot\hat{\mathbf{b}})^{1/2}$ b) $\hat{\mathbf{u}}=\frac{\hat{\mathbf{a}}-\hat{\mathbf{b}}}{|\hat{\mathbf{a}}-\hat{\mathbf{b}}|}$ and $M=(1+\hat{\mathbf{a}}\cdot\hat{\mathbf{b}})^{1/2}$

c) $\hat{\mathbf{u}}=\frac{\hat{\mathbf{a}}+\hat{\mathbf{b}}}{|\hat{\mathbf{a}}+\hat{\mathbf{b}}|}$ and $M=(1+2\hat{\mathbf{a}}\cdot\hat{\mathbf{b}})^{1/2}$ d) $\hat{\mathbf{u}}=\frac{\hat{\mathbf{a}}-\hat{\mathbf{b}}}{|\hat{\mathbf{a}}-\hat{\mathbf{b}}|}$ and $M=(1+2\hat{\mathbf{a}}\cdot\hat{\mathbf{b}})^{1/2}$

55. If $\vec{a}\cdot\vec{b}=\beta$ and $\vec{a}\times\vec{b}=\vec{c}$, then $\vec{b}$ is

a) $\frac{(\beta\vec{a}-\vec{a}\times\vec{c})}{|\vec{a}|^2}$ b) $\frac{(\beta\vec{a}+\vec{a}\times\vec{c})}{|\vec{a}|^2}$ c) $\frac{(\beta\vec{c}-\vec{a}\times\vec{c})}{|\vec{a}|^2}$ d) $\frac{(\beta\vec{a}+\vec{a}\times\vec{c})}{|\vec{a}|^2}$

56. If $\vec{a}$ is a non-zero vector of modulus a and m is a non-zero scalar, then $m\vec{a}$ is a unit vector it

a) $m\pm 1$ b) $a=|m|$ c) $a=1/|m|$ d) $a=1/m$

57. Let, $\vec{\mathbf{a}}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}$, $\vec{\mathbf{b}}=\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$, $\vec{\mathbf{c}}=\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$. A vector coplanar to $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ has a projection along $\vec{\mathbf{c}}$ of magnitude $\frac{1}{\sqrt{3}}$, then the vector is

a) $4\hat{\mathbf{i}}-\hat{\mathbf{j}}+4\hat{\mathbf{k}}$ b) $4\hat{\mathbf{i}}+\hat{\mathbf{j}}-4\hat{\mathbf{k}}$ c) $2\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$ d) None of these

58. Given three non-zero, non-coplanar vectors $\vec{a},\vec{b}$ and $\vec{c}$. $\vec{r_1}=p\vec{a}+q\vec{b}+\vec{c}$ and $\vec{r_2}=\vec{a}+p\vec{b}+q\vec{c}$. if the vectors $\vec{r_1}+2\vec{r_2}$ and $2\overrightarrow{r_1+r_2}$ are collinear, then (p,q) is

a) $(0,0)$ b) $(1,-1)$ c) $(-1,1)$ d) $(1,1)$

59. Gives three vectors $\vec{a},\vec{b}$ and $\vec{c}$, two of which are non-collinear.Further if $(\vec{a}+\vec{b})$ is collinear with $\vec{c}$, $(\vec{b}+\vec{c})$ is collinear with $\vec{a}$, $|\vec{a}|=|\vec{b}|=|\vec{c}|\sqrt{2}$. Find the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$

a) 3 b) -3 c) 0 d) Cannot be evaluated

60. A uni-modular tangent vector on the curve $x=t^2+2, y=4t-5, z=2t^2-6t$ at $t=2$ is

a) $\frac{1}{3}(2\hat{\imath}+2\hat{\jmath}+\hat{k})$ b) $\frac{1}{3}(\hat{\imath}-\hat{\jmath}-\hat{k})$ c) $\frac{1}{6}(2\hat{\imath}+\hat{\jmath}+\hat{k})$ d) $\frac{2}{3}(\hat{\imath}+\hat{\jmath}+\hat{k})$

61. In a quadrilateral $ABCD$, $\overrightarrow{AC}$ is the bisector of $\overrightarrow{AB}$ and $\overrightarrow{AD}$, angle between $\overrightarrow{AB}$ and $\overrightarrow{AD}$ is $2\pi/3$, $15|\overrightarrow{AC}|=3|\overrightarrow{AB}|=5|\overrightarrow{AD}|$. Then the angle between $\overrightarrow{BA}$ and $\overrightarrow{CD}$ is

a) $\cos^{-1}\frac{\sqrt{14}}{7\sqrt{2}}$ b) $\cos^{-1}\frac{\sqrt{21}}{7\sqrt{3}}$ c) $\cos^{-1}\frac{2}{\sqrt{7}}$ d) $\cos^{-1}\frac{2\sqrt{7}}{14}$

62. Let $ABCD$ be a tetrahedron such that the edge AB, AC and AD are mutually perpendicular. Let the area of triangles ABC, ACD and ADB be 3,4 sq. units, respectively. Then the area of triangle BCD is

a) $5\sqrt{2}$ b) 5 c) $\frac{\sqrt{5}}{2}$ d) $\frac{5}{2}$

63. If $\vec{a},\vec{b}$ and $\vec{c}$ are three non-coplanar vectors, then $(\vec{a}+\vec{b}+\vec{c})\cdot[(\vec{a}+\vec{b})\times(\vec{a}+\vec{c})]$equals

a) 0 b) $[\vec{a}\vec{b}\vec{c}]$ c) $2[\vec{a}\vec{b}\vec{c}]$ d) $-[\vec{a}\vec{b}\vec{c}]$

64. If V be the volume of a tetrahedron and V' be the volume of another tetrahedran formed by the centroids of faces of the previous tetrahedron and $V=KV'$;then K is equal to

a) 9 b) 12 c) 27 d) 81

65. Resolved part of vector$\vec{a}$ and along vector $\vec{b}$ is $\vec{a}_1$ and that perpendicular to $\vec{b}$is$\vec{a}_2$, then$\vec{a}_1\times\vec{a}_2$is equal to

a) $\frac{(\vec{a}\times\vec{b})\cdot\vec{b}}{|\vec{b}|^2}$ b) $\frac{(\vec{a}\cdot\vec{b})\vec{a}}{|\vec{a}|^2}$ c) $\frac{(\vec{a}\cdot\vec{b})(\vec{b}\times\vec{a})}{|\vec{b}|^2}$ d) $\frac{(\vec{a}\cdot\vec{b})(\vec{b}\times\vec{a})}{|\vec{b}\times\vec{a}|}$

66. Three vectors $\hat{\imath}+\hat{\jmath}$, $\hat{\jmath}+\hat{k}$ and $\hat{k}+\hat{\imath}$ taken two at a time form three planes. The three unit vector drawn perpendicular to these three planes form a parallelepiped of volume

a) $1/3$ b) 4 c) $(3\sqrt{3})/4$ d) $4\sqrt{3}$

67. Let ABC be a triangle, the position vectors of whose vertices are respectively $\hat{\imath}+2\hat{\jmath}+4\hat{k}-2\hat{\jmath}+2\hat{\jmath}+\hat{k}$ and $2\hat{\imath}+4\hat{\jmath}-3\hat{k}$. Then ΔABC is

a) Isosceles b) Equilateral c) Right angled d) None of these

68. If $\vec{\alpha}+\vec{\beta}+\vec{\gamma}=a\vec{\delta}$ and $\vec{\beta}+\vec{\gamma}+\vec{\delta}=b\,\vec{\alpha}, \vec{\alpha}$ and $\vec{\delta}$ are non-collinear, then $\vec{\alpha}+\vec{\beta}+\vec{\gamma}+\vec{\delta}$ equals

a) $a\vec{\alpha}$ b) $b\vec{\delta}$ c) 0 d) $(a+b)\vec{\gamma}$

69. $A(\vec{a}), B(\vec{b})$ and $C(\vec{c})$are the vertices of triangle ABC and $R(\vec{r})$ is any point in the plane of triangle ABC, then $\vec{r}.(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})$is always equal to

a) Zero b) $[\vec{a}\vec{b}\vec{c}]$ c) $-[\vec{a}\vec{b}\vec{c}]$ d) None of these

70. If $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ and $\vec{\mathbf{d}}$ are the unit vectors such that $(\vec{\mathbf{a}}\times\vec{\mathbf{b}})\cdot(\vec{\mathbf{c}}\times\vec{\mathbf{d}})=1$ and $\vec{\mathbf{a}}\cdot\vec{\mathbf{c}}=\frac{1}{2}$, then

a) $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are non-coplanar b) $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{d}}$ are non-coplanar

c) $\vec{\mathbf{b}}, \vec{\mathbf{d}}$ are non-parallel d) $\vec{\mathbf{a}}, \vec{\mathbf{d}}$ are parallel and $\vec{\mathbf{b}}, \vec{\mathbf{c}}$ are parallel

71. If the two adjacent sides of two rectangles are represented by vectors $\vec{p}=5\vec{a}-3\vec{b}; \vec{q}=-\vec{a}-2\vec{b}$ and $\vec{r}=-4\vec{a}-\vec{b}; \vec{s}=-\vec{a}+\vec{b}$, respectively, then the angle between the vectors$\vec{x}=\frac{1}{3}(\vec{p}+\vec{r}+\vec{s})$ and$\vec{y}=\frac{1}{5}(\vec{r}+\vec{s})$is

a) $-\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$ b) $\cos^{1}\left(\frac{19}{5\sqrt{43}}\right)$ c) $\pi\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$ d) Cannot be evaluated

72. If $\vec{a}$ and$\vec{b}$ are unit vectors such that $(\vec{a}+\vec{b})\cdot(2\vec{a}+3\vec{b})\times(3\vec{a}-2\vec{b})=\vec{0}$, then angle between $\vec{a}$ and $\vec{b}$ is

a) 0 b) $\pi/2$ c) π d) indeterminate

73. If $\vec{a}, \vec{b}$ and $\vec{c}$ are such that $[\vec{a}\vec{b}\vec{c}]=1, \vec{c}=\lambda\vec{a}\times\vec{b}$, angle between $\vec{a}$and $\vec{b}$ is $2\pi/3$, $|\vec{a}|=\sqrt{2}, |\vec{b}|=\sqrt{3}$ and $|\vec{c}|=\frac{1}{\sqrt{3}}$, then the angle between $\vec{a}$ and $\vec{b}$ is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

74. Let vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ be such that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=\vec{0}$.Let $P_1$and P_2 be planes determined by the pairs of vectors $\vec{a}, \vec{b}$and $\vec{c}, \vec{d}$, respectively. Then the angle between P_1 and P_2 is

a) 0 b) $\pi/4$ c) $\pi/3$ d) $\pi/2$

75. $\vec{A}$ is a vector with direction $\cos\alpha, \cos\beta$ and $\cos\gamma$. Assuming the $y-z$ plane as a mirror, the direction cosines of the reflected image of $\vec{A}$ in the $y-z$ plane are

a) $\cos\alpha, \cos\beta, \cos\gamma$ b) $\cos\alpha, -\cos\beta, \cos\gamma$ c) $-\cos\alpha, \cos\beta, \cos\gamma$ d) $-\cos\alpha, -\cos\beta, -\cos\gamma$

76. If $\vec{r}.\vec{a}=\vec{r}.\vec{b}=\vec{r}.\vec{c}=\frac{1}{2}$ for some non-zero vectors $\vec{r}$, then the area of the triangle whose vertices are A $(\vec{a}), B(\vec{b})$and $C(\vec{c})$is ($\vec{a}, \vec{b}, \vec{c}$are non-coplanar)

a) $|[\vec{a}\vec{b}\vec{c}]|$ b) $|\vec{r}|$ c) $|[\vec{a}\vec{b}\vec{c}]\vec{r}|$ d) None of these

77. Let $\overrightarrow{r_1}, \overrightarrow{r_2}, \overrightarrow{r_3}, \dots, \overrightarrow{r_n}$ be the position vectors of points $P_1, P_2, \dots P_n$ reelative to the origin O. If the vector equation $a_1\overrightarrow{r_1}+a_2\overrightarrow{r_2}+\dots+a_n\,\vec{r}_n$=0 holds then a similar equation will also hold w.r.t. to any other origin provided

a) $a_1+a_2+\dots+a_n=n$ b) $a_1+a_2+\dots+a_n=1$

c) $a_1+a_2+\dots+a_n=0$ d) $a_1=a_2=a_3=\dots=a_n=0$

78. Let the position vectors of the points P and Q be $4\hat{\imath}+\hat{\jmath}+\lambda\hat{k}$ and $2\hat{\imath}-\hat{\jmath}+\lambda\hat{k}$, respectively. Vector $\hat{\imath}-\hat{\jmath}+6\hat{k}$ is perpendicular to the plane containing the orgin and the points P and Q. Then λ equals

a) $-1/2$ b) $1/2$ c) 1 d) None of these

79. Let $\vec{a}$and$\vec{b}$ be unit vectors that are perpendicular to each other. Then $[\vec{a}+(\vec{a}\times\vec{b})\vec{b}+(\vec{a}\times\vec{b})\vec{a}\times\vec{b}]$ will always be equal to

a) 1 b) 0 c) -1 d) None of these

80. If $\vec{a}, \vec{b}$and $\vec{c}$ are non-coplanar vectors and $\vec{a}\times\vec{c}$ is perpendicular to $\vec{a}\times(\vec{b}\times\vec{c})$, then the value of $[\vec{a}\times(\vec{b}\times\vec{c})]\times\vec{c}$is equal to

a) $[\vec{a}\vec{b}\vec{c}]\vec{c}$ b) $[\vec{a}\vec{b}\vec{c}]\vec{b}$ c) $\vec{0}$ d) $[\vec{a}\vec{b}\vec{c}]\vec{a}$

81. In a trapezium, vector $\overrightarrow{BC}=\alpha\overrightarrow{AD}$. We will then find that $\vec{p}=\overrightarrow{AC}+\overrightarrow{BD}$ is collinear with $\overrightarrow{AD}$. If $\vec{p}=$

$\mu\overrightarrow{AD}$, then which of the following is true?

a) $\mu = \alpha + 2$ b) $\mu + \alpha = 1$ c) $\alpha = \mu + 1$ d) $\mu = \alpha + 1$

82. The position vectors of points A, B and C are $\hat{\imath} + \hat{\jmath} + \hat{k}, \hat{\imath} + 5\hat{\jmath} - \hat{k}$ and $2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$, respectively. The greatest angle of triangle ABC is

a) 120° b) 90° c) $\cos^{-1}(3/4)$ d) None of these

83. Let $\vec{a} = \hat{\imath} - \hat{\jmath}, \vec{b} = \hat{\jmath} - \hat{k}$ and $\vec{c} = \hat{k} - \hat{\imath}$. If $\vec{d}$ is a unit vector such that $\vec{a} \cdot \vec{d} = 0 = [\vec{b}\vec{c}\vec{d}]$, then $\vec{d}$ equals

a) $\pm\frac{\hat{\imath} + \hat{\jmath} - 2\hat{k}}{\sqrt{6}}$ b) $\pm\frac{\hat{\imath} + \hat{\jmath} - \hat{k}}{\sqrt{3}}$ c) $\pm\frac{\hat{\imath} + \hat{\jmath} + \hat{k}}{\sqrt{3}}$ d) $\pm\hat{k}$

84. Given three vectors $\vec{b} = 6\hat{\imath} - 3\hat{\jmath}, \vec{b} = 2\hat{\imath} - 6\hat{\jmath}$ and $\vec{c} = -2\hat{\imath} + 21\hat{\jmath}$ such that $\vec{\alpha} = \vec{a} + \vec{b} + \vec{c}$. Then the resolution of the vectors $\vec{\alpha}$ into compounds with respect to $\vec{a}$ and $\vec{b}$ is given by

a) $3\vec{a} - 2\vec{b}$ b) $3\vec{b} - 2\vec{a}$ c) $2\vec{a} - 3\vec{b}$ d) $\vec{a} - 2\vec{b}$

85. The condition for equations $\vec{r} \times \vec{a} = \vec{b}$ and $\vec{r} \times \vec{c} = \vec{d}$ to be consistent is

a) $\vec{b} \cdot \vec{c} = \vec{a} \cdot \vec{d}$ b) $\vec{a} \cdot \vec{b} = \vec{c} \cdot \vec{d}$ c) $\vec{b} \cdot \vec{c} + \vec{a} \cdot \vec{d} = 0$ d) $\vec{a} \cdot \vec{b} + \vec{c} \cdot \vec{d} = 0$

86. If the diagonals of one of its faces are $6\hat{\imath} + 6\hat{k}$ and $4\hat{\jmath} + 2\hat{k}$ and of the edges not containing the given diagonals is $\vec{c} = 4\hat{\jmath} - 8\hat{k}$ then the volume of a parallelepiped is

a) 60 b) 80 c) 100 d) 120

87. The vertex A of triangle ABC is on the line $\vec{r} = \hat{\imath} + \hat{\jmath} + \lambda\hat{k}$ and the vartices B and C have respective position vectors $\hat{\imath}$ and $\hat{\jmath}$. Let Δ be the area of the triangle and $\Delta \in [3/2], \sqrt{33}/2]$. Then the range of values of λ corresponding to A is

a) $[-8, -4] \cup [4, 8]$ b) $[-4,4]$ c) $[-2,2]$ d) $[-4, -2] \cup [2,4]$

88. The value of x for which the angle between $\vec{a} = 2x^2\hat{\imath} + 4x\,\hat{\jmath} + \hat{k}$ and $\hat{b} = 7\hat{\imath} - 2\hat{\jmath} + x\hat{k}$ is obuse and the angle between $\vec{b}$ and the z-$axis$ is acute and less than $\pi/6$, is

a) $a < x < 1/2$ b) $1/2 < x < 15$ c) $x > 1/2$ or $x < 0$ d) None of these

89. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three mutually orthogonal unit vectors, then the triple product $[\vec{a} + \vec{b} + \vec{c}\vec{a} + \vec{b}\vec{b} + \vec{c}]$ equals

a) 0 b) 1 or -1 c) 1 d) 3

90. $'I'$ is the incentre of triangle ABC whose corresponding sides are a, b, c respectively. $a\overrightarrow{IA} + b\overrightarrow{IB} + c\overrightarrow{IC}$ is always equal to

a) $\vec{0}$ b) $(a + b + c)\overrightarrow{BC}$ c) $(\vec{a} + \vec{b} + \vec{c})\overrightarrow{AC}$ d) $(a + b + c)\overrightarrow{AB}$

91. Let $\vec{a} = 2i + j - 2k$ and $b = i + j$. If c is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is 30°, then $|(\vec{a} \times \vec{b}) \times \vec{c}|$ is equal to

a) 2/3 b) 3/2 c) 2 d) 3

92. If $\hat{a}, \hat{b}$ and $\hat{c}$ are unit vectors, then $|\hat{a} - \hat{b}|^2 + |\hat{b} - \hat{c}|^2 + |\hat{c} - \hat{a}|^2$ does not exceed

a) 4 b) 9 c) 8 d) 6

93. If the vector product of a constant vector $\overrightarrow{OA}$ with a variable vector $\overrightarrow{OB}$ in a fixed plane OAB be a constant vector, then the locus of B is

a) A straight line perpendicular to $\overrightarrow{OA}$ b) A circle with centre O and radius equal to $|\overrightarrow{OA}|$

c) A straight line parallel to $\overrightarrow{OA}$ d) None of these

94. The points with position vectors $60\hat{\imath} + 3\hat{\jmath}, 40\hat{\imath} - 8\hat{\jmath}, a\hat{\imath} - 52\hat{\jmath}$ are collinear if

a) $a = -40$ b) $a = 40$ c) $a = 20$ d) None of these

95. Two adjacent sides of a parallelogram $ABCD$ are given by $\overrightarrow{\mathbf{AB}} = 2\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$ and $\overrightarrow{\mathbf{AD}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angle α is given by

a) $\frac{8}{9}$ b) $\frac{\sqrt{17}}{9}$ c) $\frac{1}{9}$ d) $\frac{4\sqrt{5}}{9}$

96. $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors of equal magnitude. The angle between each pair of vectors is $\pi/3$ such that $|\vec{a} + \vec{b} + \vec{c}| = \sqrt{6}$. Then $|\vec{a}|$ is equal to

a) 2 b) -1 c) 1 d) $\sqrt{6}/3$

97. Value of $[\vec{a}\times\vec{b}\vec{a}\times\vec{c}\vec{d}]$is always equal to

a) $(\vec{a}\cdot\vec{d})[\vec{a}\vec{b}\vec{c}]$ b) $(\vec{a}\cdot\vec{c})[\vec{a}\vec{b}\vec{d}]$ c) $(\vec{a}\cdot\vec{b})[\vec{a}\vec{b}\vec{d}]$ d) None of these

98. If $(\vec{a}\times\vec{b})\times(\vec{b}\times\vec{c})=\vec{b}$, where $\vec{a},\vec{b}$ and $\vec{c}$ are non-zero vectors, then

a) $\vec{a},\vec{b}$and $\vec{c}$ can be coplanar b) $\vec{a},\vec{b}$and $\vec{c}$ must be coplanar

c) $\vec{a},\vec{b}$and $\vec{c}$ cannot be coplanar d) None of these

99. If $\vec{a}$ and $\vec{b}$ are two vectors, such that $\vec{a}\cdot\vec{b}<0$ and $|\vec{a}\cdot\vec{b}|=|\vec{a}\times\vec{b}|$, then the angle between vectors $\vec{a}$and$\vec{b}$ is

a) π b) $7\pi/4$ c) $\pi/4$ d) $3\pi/4$

100. The position vectors of the vertices A,B and C of a triangle are three units vectors $\hat{a},\hat{b}$ and $\hat{c}$, respectively. A vector $\vec{d}$ is such that $\vec{d}\cdot\hat{a}=\vec{d}\cdot\hat{b}=\vec{d}\cdot\hat{c}and\vec{d}=\lambda(\hat{b}+\hat{c})$Then triangle ABC is

a) Acute angled b) Obtuse angled c) Right angled d) None of these

101. Let $\vec{a}(x)=(\sin x)\hat{\imath}+(\cos x)\hat{\jmath}$and $\vec{b}(x)=(\cos 2x)\hat{\imath}+(\sin 2x)\hat{\jmath}$ be two variable vectors $(x\in R)$,then $\vec{a}(x)$and $\vec{b}(x)$ are

a) Collinear for unique value of x b) Perpendicular for infinite values of x

c) Zero vectors for unique value of x d) None of these

102. If $\vec{a}$ satisfies $\vec{a}\times(\hat{\imath}+2\hat{\jmath}+\hat{k})=\hat{\imath}-\hat{k}$, then $\vec{a}$ is equal to

a) $\lambda\hat{\imath}+(2\lambda-1)\hat{\jmath}+\lambda\hat{k},\lambda\in R$ b) $\lambda\hat{\imath}+(1-2\lambda)\hat{\jmath}+\lambda\hat{k},\lambda\in R$

c) $\lambda\hat{\imath}+(2\lambda+1)\hat{\jmath}+\lambda\hat{k},\lambda\in R$ d) $\lambda\hat{\imath}-(1+2\lambda)\hat{\jmath}+\lambda\hat{k},\lambda\in R$

103. If $\vec{a}'=\hat{\imath}+\hat{\jmath},\vec{b}'=\hat{\imath}-\hat{\jmath}+2\hat{k}$ and $\vec{c}'=2\hat{\imath}+\hat{\jmath}-\hat{k}$,then the altitude of the parallelepiped formed by the vectors $\vec{a},\vec{b}$ and $\vec{c}$ having base formed by $\vec{b}$and$\vec{c}$ is (where $\vec{a}'$ is reciprocal vector $\vec{a}$, etc)

a) 1 b) $3\sqrt{2}/2$ c) $1/\sqrt{6}$ d) $1/\sqrt{2}$

104. Vectors $\vec{a}=-4\hat{\imath}+3\hat{k}$; $\vec{b}=14\hat{\imath}+2\hat{\jmath}-5\hat{k}$ are laid off from one point. Vector $\vec{d}$, which is being laid off from the same point dividing the angle between vectors $\vec{a}$ and $\vec{b}$ in equal halves and having the magnitude $\sqrt{6}$, is

a) $\hat{\imath}+\hat{\jmath}+2\hat{k}$ b) $\hat{\imath}-\hat{\jmath}+2\hat{k}$ c) $\hat{\imath}+\hat{\jmath}-2\hat{k}$ d) $2\hat{\imath}-\hat{\jmath}-2\hat{k}$

105. Given $\vec{a}=x\hat{\imath}+y\hat{\jmath}+2\hat{k},\vec{b}=\hat{\imath}-\hat{\jmath}+\hat{k},\vec{c}=\hat{\imath}+2\hat{\jmath};\vec{a}\perp\vec{b},\vec{a}\cdot\vec{c}=4$.Then

a) $[\vec{a}\vec{b}\vec{c}]^2=|\vec{a}|$ b) $[\vec{a}\vec{b}\vec{c}]=|\vec{a}|$ c) $[\vec{a}\vec{b}\vec{c}]=0$ d) $[\vec{a}\vec{b}\vec{c}]=|\vec{a}|^2$

106. $\vec{a},\vec{b}$and$\vec{c}$ are unit vectors such that $|\vec{a}+\vec{b}+3\vec{c}|=4$. Angle between $\vec{a}$and$\vec{b}$ is θ_1, between$\vec{b}$ and $\vec{c}$is θ_2 and between $\vec{a}$ and $\vec{c}$ varies $[\pi/6,2\pi/3]$. Then the maximum value of $\cos\theta_1+3\cos\theta_2$ is

a) 3 b) 4 c) $2\sqrt{2}$ d) 6

107. If $\vec{a}$and$\vec{b}$are any two vectors of magnitudes 2 and 3, respectively, such that $|2(\vec{a}\times\vec{b})|+|3(\vec{a}\cdot\vec{b})|=k$, then the maximum values of $k$is

a) $\sqrt{13}$ b) $2\sqrt{13}$ c) $6\sqrt{13}$ d) $10\sqrt{13}$

108. G is the centroid of triangle ABC and A_1 and B_1 are the midpoints of sides AB and AC, respectively.If Δ_1 be the area of quadrilateral GA_1AB_1 and Δ be the area of triangle ABC, then Δ/Δ_1 is equal to

a) $\frac{3}{2}$ b) 3 c) $\frac{1}{3}$ d) None of these

109. The value of a so that the volume of parallelopiped formed by $\hat{\mathbf{i}}+a\hat{\mathbf{j}}+\hat{\mathbf{k}},\hat{\mathbf{j}}+a\hat{\mathbf{k}}$ and $a\hat{\mathbf{i}}+\hat{\mathbf{k}}$ becomes minimum is

a) -3 b) 3 c) $1/\sqrt{3}$ d) $\sqrt{3}$

110. If $\hat{a},\hat{b}$ and $\hat{c}$ are three unit vectors inclined to each other at an angle θ, then the maximum value of $\theta$is

a) $\frac{\pi}{3}$ b) $\frac{\pi}{2}$ c) $\frac{2\pi}{3}$ d) $\frac{5\pi}{6}$

111. Four non-zero vectors will always be

a) Linearly dependent b) Linearly independent c) Either a or b d) None of these

112. If $\vec{a}$and $\vec{b}$ are two unit vectors inclined at an angle $\pi/3$,then$\{\vec{a}\times(\vec{b}+\vec{a}\times\vec{b})\}\cdot\vec{b}$ is equal to

a) $\frac{-3}{4}$ b) $\frac{1}{4}$ c) $\frac{3}{4}$ d) $\frac{1}{2}$

113. P be a point interior to the acute triangle ABC. If $\vec{PA} + \vec{PB} + \vec{PC}$ is a null vector then w.r.t. triangle ABC, point P is its
a) Centroid b) Orthocentre c) Incentre d) Circumcentre

114. If $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ is a non-zero vector and $|(\vec{d} \cdot \vec{c})(\vec{a} \times \vec{b}) + (\vec{d} \cdot \vec{a})(\vec{b} \times \vec{c}) + (\vec{d} \cdot \vec{b})(\vec{c} \times \vec{a})| = 0$,then
a) $|\vec{a}| = |\vec{b}| = |\vec{c}|$
b) $|\vec{a}| + |\vec{b}| + |\vec{c}| = |\vec{d}|$
c) $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar
d) None of these

115. If $\vec{a} = 2\hat{\imath} + \hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}, \vec{c} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$and $(1+\alpha)\hat{\imath} + \beta(1+\alpha)\hat{\jmath} + \gamma(1+\alpha)(1+\beta)\hat{k} = \vec{a} \times (\vec{b} \times \vec{c})$,then $\alpha, \beta$and γ are
a) $-2, -4 - \frac{2}{3}$ b) $2, -4, \frac{2}{3}$ c) $-2, 4, \frac{2}{3}$ d) $2, 4, -\frac{2}{3}$

116. If $\vec{\alpha} || (\vec{\beta} \times \vec{\gamma})$, then $(\vec{\alpha} \times \vec{\beta}) \cdot (\vec{\alpha} \times \vec{\gamma})$ equals to
a) $|\vec{\alpha}|^2(\vec{\beta} \cdot \vec{\gamma})$ b) $|\vec{\beta}|^2(\vec{\gamma} \cdot \vec{\alpha})$ c) $|\vec{\gamma}|^2(\vec{\alpha} \cdot \vec{\beta})$ d) $|\vec{\alpha}||\vec{\beta}||\vec{\gamma}|$

117. Two vectors in space are equal only if they have equal component in
a) A given direction
b) Two given directions
c) Three given directions
d) In any arbitrary direction

118. A vector of magnitude $\sqrt{2}$coplanar with the vectors $\vec{a} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$, and perpendicular to the vector $\vec{c} = \hat{\imath} + \hat{\jmath} + \hat{k}$,is
a) $-\hat{\jmath} + \hat{k}$ b) $\hat{\imath} - \hat{k}$ c) $\hat{\imath} - \hat{\jmath}$ d) $\hat{\imath} - \hat{\jmath}$

119. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}, \vec{b} = 4\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ and $\vec{c} = \hat{\imath} + \alpha\hat{\jmath} + \beta\hat{k}$ are linearly dependent vectors and $|\vec{c}| = \sqrt{3}$, then
a) $a = 1, b = -1$ b) $a = 1, b = \pm 1$ c) $\alpha = -1, \beta = \pm 1$ d) $\alpha = \pm 1, \beta = 1$

120. If vectors $\vec{a}$ and $\vec{b}$ are two adjacent sides of a parallelogram, then the vector representing the altitude of the parallelogram which is perpendicular to $\vec{a}$ is
a) $\vec{b} + \frac{\vec{b} \times \vec{a}}{|\vec{a}|^2}$ b) $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2}$ c) $\vec{b} - \frac{\vec{b} \cdot \vec{a}}{|\vec{a}|^2}\vec{a}$ d) $\frac{\vec{a} \times (\vec{b} \times \overrightarrow{a})}{|\vec{b}|^2}$

121. If $\vec{a} \perp \vec{b}$, then vector $\vec{v}$ in terms of $\vec{a}$and$\vec{b}$ satisfying the equations $\vec{v} \cdot \vec{a} = 0$ and $\vec{v} \cdot \vec{b} = 1$ and $[\vec{v}\vec{a}\vec{b}] = 1$is
a) $\frac{\vec{b}}{|\vec{b}|^2} + \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|^2}$ b) $\frac{\vec{b}}{|\vec{b}|} + \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|^2}$ c) $\frac{\vec{b}}{|\vec{b}|^2} + \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$ d) None of these

122. Locus of the point P, for which $\overline{OP}$ represents a vector with direction cosine $\alpha = \frac{1}{2}$('O' is the origin) is
a) A circle parallel to the $y - z$ plane with centre on the x- axis
b) A cone concentric with the positive x-axis having vertex at the origin and the slant height equal to the magnitude of the vector
c) A ray emanating from the origin and making an angle of 60° with the x -axis
d) A disc parallel to the $y - z$plane with centre on the x-axis and radius equal to $|\overrightarrow{OP}| \sin 60°$

123. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{\mathbf{i}} - \hat{\mathbf{j}}$,$4\hat{\mathbf{i}}$, $3\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ and $-3\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ respectively. The quadrilateral $PQRS$ must be
a) Parallelogram, which is neither a rhombus nor a rectangle
b) Square
c) Rectangle, but not a square
d) Rhombus, but not a square

124. If $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are three non-zero, non-coplanar vectors and
$\vec{\mathbf{b}}_1 + \vec{\mathbf{b}} - \frac{\vec{\mathbf{b}} \cdot \vec{\mathbf{a}}}{|\vec{\mathbf{a}}|^2}\vec{\mathbf{a}}$,$\vec{\mathbf{b}}_2 + \vec{\mathbf{b}} - \frac{\vec{\mathbf{b}} \cdot \vec{\mathbf{a}}}{|\vec{\mathbf{a}}|^2}\vec{\mathbf{a}}$
And

$\vec{\mathbf{c}}_1 + \vec{\mathbf{b}} - \frac{\vec{\mathbf{c}} \cdot \vec{\mathbf{a}}}{|\vec{\mathbf{a}}|^2}\vec{\mathbf{a}} + \frac{\vec{\mathbf{c}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{b}}|^2}\vec{\mathbf{b}}_1$

$\vec{\mathbf{c}}_2 + \vec{\mathbf{c}} - \frac{\vec{\mathbf{c}} \cdot \vec{\mathbf{a}}}{|\vec{\mathbf{a}}|^2}\vec{\mathbf{a}} - \frac{\vec{\mathbf{c}} \cdot \vec{\mathbf{b}}_1}{|\vec{\mathbf{b}}_1|^2}\vec{\mathbf{b}}_1$,

$\vec{\mathbf{c}}_3 = \vec{\mathbf{c}} - \frac{\vec{\mathbf{c}} \cdot \vec{\mathbf{a}}}{|\vec{\mathbf{c}}|^2}\vec{\mathbf{a}} - \frac{\vec{\mathbf{c}} \cdot \vec{\mathbf{b}}_2}{|\vec{\mathbf{c}}|^2}\vec{\mathbf{b}}_1$,

$\vec{\mathbf{c}}_4 = \vec{\mathbf{c}} - \frac{\vec{\mathbf{c}} \cdot \vec{\mathbf{a}}}{|\vec{\mathbf{c}}|^2}\vec{\mathbf{a}} - \frac{\vec{\mathbf{b}} \cdot \vec{\mathbf{c}}}{|\vec{\mathbf{b}}|^2}\vec{\mathbf{b}}_1$

Then, which of the following is a set of mutually orthogonal vectors?

a) $\{\vec{\mathbf{a}}, \vec{\mathbf{b}}_1, \vec{\mathbf{c}}_1\}$ b) $\{\vec{\mathbf{a}}, \vec{\mathbf{b}}_1, \vec{\mathbf{c}}_2\}$ c) $\{\vec{\mathbf{a}}, \vec{\mathbf{b}}_2, \vec{\mathbf{c}}_3\}$ d) $\{\vec{\mathbf{a}}, \vec{\mathbf{b}}_2, \vec{\mathbf{c}}_4\}$

125. If $\vec{x}$ and $\vec{y}$ are two non-collinear vectors and ABC is a triangle is a triangle with side length a, b and c satisfying $(20a - 15b)\vec{x} + (15b - 12c)\vec{y} + (12c - 20a)(\vec{x} \times \vec{y}) = \vec{0}$, then triangle ABC is

a) An acute-angled triangle b) An obtuse-angled triangle
c) A right-angled triangle d) An isosceles triangle

126. If $a(\vec{\alpha} \times \vec{\beta}) + b(\vec{\beta} \times \vec{\gamma}) + c(\vec{\gamma} \times \vec{\alpha}) = 0$ and at least one of a, b and c is non-zero, then vectors $\vec{\alpha}, \vec{\beta}$and $\vec{\gamma}$are

a) Parallel b) Coplanar
c) Mutually perpendicular d) None of these

127. If $\vec{r}$ and $\vec{s}$ are non-zero constant vectors and the scalar b is chosen such that $|\vec{r} + b\vec{s}|$ is minimum, then the value of $|b\vec{s}|^2 + |\vec{r} + b\vec{s}|^2$ is equal to

a) $2|\vec{r}|^2$ b) $|\vec{r}|^2/2$ c) $3|\vec{r}|^2$ d) $|\vec{r}|^2$

128. If $\hat{a}, \hat{b}$and $\hat{c}$are three unit vectors, such that $\hat{a} + \hat{b} + \hat{c}$ is also a unit vector and θ_1, θ_2 and θ_3 are angles between the vectors $\hat{a}, \hat{b}$; $\hat{b}, \hat{c}$and$\hat{c}, \hat{a}$, respectively, then among θ_1, θ_2 and θ_3

a) All are acute angles b) All are right angles
c) At least one is obtuse angle d) None of these

129. Let $\vec{a} \cdot \vec{b} = 0$, where $\vec{a}$and$\vec{b}$ are unit vectors and the unit vector $\vec{c}$ is inclined at an angle θ to both $\vec{a}$and$\vec{b}$.If $\vec{c} = m\vec{a} + n\vec{b} + p(\vec{a} \times \vec{b}), (m, n, p \in R)$, then

a) $-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$ b) $\frac{\pi}{4} \le \theta \le \frac{3\pi}{4}$ c) $0 \le \theta \le \frac{\pi}{4}$ d) $0 \le \theta \le \frac{3\pi}{4}$

130. $\vec{a}$ and $\vec{c}$ are unit vectors and $|\vec{b}| = 4$. The angle between $\vec{a}$and$\vec{c}$ is $\cos^{-1}$ $(1/4)$and $\vec{b} - 2\vec{c} = \lambda\vec{a}$.The value of λ is

a) $3, -4$ b) $1/4, 3/4$ c) $-3, 4$ d) $-1/4, 3/4$

131. If $\vec{\mathbf{a}} = (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}), \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 1$ and $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \hat{\mathbf{j}} - \hat{\mathbf{k}}$, then $\vec{\mathbf{b}}$ is

a) $\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ b) $2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ c) $\hat{\mathbf{i}}$ d) $2\hat{\mathbf{i}}$

132. The volume of a tetrahedron formed by the coterminus edges $\vec{a}, \vec{b}$and $\vec{c}$ is 3. Then the volume of the parallelepiped formed by the conterminous edges $\vec{a} + \vec{b}$, $\vec{b} + \vec{c}$and$\vec{c} + \vec{a}$ is

a) 6 b) 18 c) 36 d) 9

133. If $\vec{a}$and$\vec{b}$ are non-zero non-collinear vectors, then $[\vec{a}\vec{b}\hat{\imath}]\hat{\imath} + [\vec{a}\vec{b}\hat{\jmath}]\hat{\jmath} + [\vec{a}\vec{b}\hat{k}]\hat{k}$ is equal to

a) $\vec{a} + \vec{b}$ b) $\vec{a} \times \vec{b}$ c) $\vec{a} - \vec{b}$ d) $\vec{b} \times \vec{a}$

134. If $\vec{a}$ and $\vec{y}$ are two non-collinear vectors and a, b, and c represent the sides of a ΔABC satisfying $(a - b)\vec{x} + (b - c)\vec{y} + (c - a)(\vec{x} \times \vec{y}) = 0$, then ΔABC is (where $\vec{x} \times \vec{y}$ is perpendicular to the plane of $\vec{a}$ and $\overrightarrow{y)}$

a) An acute- angled triangle b) An obtuse-angled triangle
c) A right-angled triangle d) A scalene triangle

135. $[(\vec{a} \times \vec{b}) \times (\vec{b} \times \vec{c})(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a})(\vec{c} \times \vec{a}) \times (\vec{a} \times \vec{b})]$is equal to (where $\vec{a}, \vec{b}$and $\vec{c}$ are non-zero non-coplanar vectors)

a) $[\vec{a}\vec{b}\vec{c}]^2$ b) $[\vec{a}\vec{b}\vec{c}]^3$ c) $[\vec{a}\vec{b}\vec{c}]^4$ d) $[\vec{a}\vec{b}\vec{c}]$

136. $ABCD$ a parallelogram, and A_1 and B_1 are the midpoints of sides BC and CD, respectively. If $\overrightarrow{AA_1}+\overrightarrow{AB_1}=\lambda\overrightarrow{AC}$, λ is equal to

a) $\frac{1}{2}$ b) 1 c) $\frac{3}{2}$ d) 2

137. If in a right- angled triangle ABC, the hypotenuse $AB=p$,then $\overrightarrow{AB}\cdot\overrightarrow{AC}+\overrightarrow{BC}\cdot\overrightarrow{BA}+\overrightarrow{CA}\cdot\overrightarrow{CB}$ is equal to

a) $2p^2$ b) $\frac{p^2}{2}$ c) p^2 d) None of these

138. If $\vec{a}$and $\vec{b}$ are orthogonal unit vectors, then for a vector $\vec{r}$ non-coplanar with $\vec{a}$and $\vec{b}$, vector $\vec{r}\times\vec{a}$ is equal to

a) $[\vec{r}\vec{a}\vec{b}]\vec{b}-(\vec{r}\cdot\vec{b})(\vec{b}\times\vec{a})$ b) $[\vec{r}\vec{a}\vec{b}](\vec{a}+\vec{b})$

c) $[\vec{r}\vec{a}\vec{b}]\vec{a}+(\vec{r}\cdot\vec{a})\vec{a}\times\vec{b}$ d) None of these

139. If $\vec{a},\vec{b}$ and $\vec{c}$ are non-coplanar unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c})=\frac{\vec{b}+\vec{c}}{\sqrt{2}}$, then the angle between $\vec{a}$and $\vec{b}$ is

a) $3\pi/4$ b) $\pi/4$ c) $\pi/2$ d) π

140. The edges of a parallelopiped are unit length and are parallel to non-coplanar unit vectors $\vec{\mathbf{a}},\vec{\mathbf{b}},\vec{\mathbf{c}}$ such that $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=\vec{\mathbf{b}}\cdot\vec{\mathbf{c}}=\vec{\mathbf{c}}\cdot\vec{\mathbf{a}}=\frac{1}{2}$ Then, the volume of the parallelopiped is

a) $\frac{1}{\sqrt{2}}$cu unit b) $\frac{1}{2\sqrt{2}}$ cu unit c) $\frac{\sqrt{3}}{2}$cu unit d) $\frac{1}{\sqrt{3}}$cu unit

141. Let $\vec{a}=\hat{\imath}+\hat{\jmath}$; $\vec{b}=2\hat{\imath}-\hat{k}$. Then vector $\vec{r}$ satisfying the equations $\vec{r}\times\vec{a}=\vec{b}\times\vec{a}and\vec{r}\times\vec{b}=\vec{a}\times\vec{b}$ is

a) $\hat{\imath}-\hat{\jmath}+\hat{k}$ b) $3\hat{\imath}-\hat{\jmath}+\hat{k}$ c) $3\hat{\imath}+\hat{\jmath}-\hat{k}$ d) $\hat{\imath}-\hat{\jmath}-\hat{k}$

142. If $4\hat{\imath}+7\hat{\jmath}+8\hat{k}$, $2\hat{\jmath}+3\hat{\jmath}+4\hat{k}$ and $2\hat{\imath}+5\hat{\jmath}+7\hat{k}$ are the position vectors of the vertices A,B and C respectively, of triangle ABC, the position vector of the point where the bisector of angle A meets BC, is

a) $\frac{2}{3}(-6\hat{\imath}-8\hat{\jmath}-6\hat{k})$ b) $\frac{2}{3}(6\hat{\imath}+8\hat{\jmath}+6\hat{k})$ c) $\frac{1}{3}(6\hat{\imath}+13\hat{\jmath}+18\hat{k})$ d) $\frac{1}{3}(5\hat{\jmath}+12\hat{k})$

143. Two adjacent sides of a parallelogram $ABCD$ are $2\hat{\imath}+4\hat{\jmath}-5\hat{k}and\hat{\imath}+2\hat{\jmath}+3\hat{k}$. Then the value of $|\overrightarrow{AC}\times\overrightarrow{BD}|$is

a) $20\sqrt{5}$ b) $22\sqrt{5}$ c) $24\sqrt{5}$ d) $26\sqrt{5}$

144. Let $\vec{u},\vec{v}$ and $\vec{w}$ be such that $|\vec{u}|=1,|\vec{v}|=2$ and $|\vec{w}|=3$. If the projection of $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and vectors $\vec{v}$ and $\vec{w}$ are perpendicular to each other, then $|\vec{u}-\vec{v}+\vec{w}|$ equals

a) 2 b) $\sqrt{7}$ c) $\sqrt{14}$ d) 14

145. If $\vec{a},\vec{b},\vec{c}$ are unit vectors such that $\vec{a}\cdot\vec{b}=0=\vec{a}\cdot\vec{c}$ and the angle between $\vec{b}$ and $\vec{c}$ is $\pi/3$, then the value of $|\vec{a}\times\vec{b}-\vec{a}\times\vec{c}|$ is

a) 1/2 b) 1 c) 2 d) None of these

146. $\vec{b}$and $\vec{c}$ are unit vectors. Then for any arbitrary vector $\vec{a}$, $\left(\left((\vec{a}\times\vec{b})+(\vec{a}\times\vec{c})\right)\times(\vec{b}\times\vec{c})\right)\cdot(\vec{b}-\vec{c})$is always equal to

a) $|\vec{a}|$ b) $\frac{1}{2}|\vec{a}|$ c) $\frac{1}{3}|\vec{a}|$ d) None of these

147. Let $x^2+3y^2=3$ be the equation of an ellipse in the $x-y$ plane. A and B are two points whose position vectors are $-\sqrt{3}\hat{\imath}$ and $-\sqrt{3}\hat{\imath}+2\hat{k}$. Then the position vector of a point P on the ellipse such that $\angle APB=\pi/4$ is

a) $\pm\hat{\jmath}$ b) $\pm(\hat{\imath}+\hat{\jmath})$ c) $\pm\hat{\imath}$ d) None of these

148. Position vector$\hat{k}$ is rotated about origin by angle 135° in such a way that the plane made by it bisects the angle between $\hat{\imath}$and$\hat{\jmath}$. Then its new position is

a) $\pm\frac{\hat{\imath}}{\sqrt{2}}\pm\frac{\hat{\jmath}}{\sqrt{2}}$ b) $\pm\frac{\hat{\imath}}{2}\pm\frac{\hat{\jmath}}{2}-\frac{\hat{k}}{\sqrt{2}}$ c) $\frac{\hat{\imath}}{\sqrt{2}}-\frac{\hat{k}}{\sqrt{2}}$ d) None of these

149. Vectors $\vec{a}=\hat{\imath}+2\hat{\jmath}+3\hat{k}$; $\vec{b}=2\hat{\imath}-\hat{\jmath}+\hat{k}$ and $\vec{c}=3\hat{\imath}+\hat{\jmath}+4\hat{k}$ are so placed that the end point of one vector is the starting point of the next vector. Then the vector are

a) Not coplanar b) Coplanar but cannot from a triangle

c) Coplanar and from a triangle
d) Coplanar and can from a right-angled triangle

150. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be the three vectors having magnitudes 1,5 and 3, respectively, such that the angle between $\vec{a}$ and $\vec{b}$ is θ and $\vec{a} \times (\vec{a} \times \vec{b}) = \vec{c}$. Then $\tan\theta$ is equal to

a) 0
b) 2/3
c) 3/5
d) 3/4

151. If $\vec{r} = x_1(\vec{a} \times \vec{b}) + x_2(\vec{b} \times \vec{a}) + x_3(\vec{c} \times \vec{d})$and $4[\vec{a}\vec{b}\vec{c}] = 1$,then $x_1 + x_2 + x_3$ is equal to

a) $\frac{1}{2}\vec{r} \cdot (\vec{a} + \vec{b} + \vec{c})$
b) $\frac{1}{4}\vec{r} \cdot (\vec{a} + \vec{b} + \vec{c})$
c) $2\vec{r} \cdot (\vec{a} + \vec{b} + \vec{c})$
d) $4\vec{r} \cdot (\vec{a} + \vec{b} + \vec{c})$

152. Let $\vec{u}, \vec{v}$and $\vec{w}$ be vectors such that $\vec{u} + \vec{v} + \vec{w} = 0$. If $|\vec{u}| = 3, |\vec{v}| = 4and|\vec{w}| = 5$, then $\vec{u} \cdot \vec{v} + \vec{v} \cdot \vec{w} + \vec{w} \cdot \vec{u}$ is

a) 47
b) -25
c) 0
d) 25

153. Let $\vec{f}(t) = [t]\hat{\imath} + (t - [t])\hat{\jmath} + [t+1]\hat{k}$,where [.] denotes the greatest integer function. Then the vectors$\vec{f}\left(\frac{5}{4}\right)$and $\vec{f}(t), 0 < t < 1$, are

a) Parallel to each other
b) perpendicular to each other
c) Inclined at an angle $\cos^{-1}\frac{2}{\sqrt{7(1-t^2)}}$
d) Inclined at $\cos^{-1}\frac{8+t}{9\sqrt{1+t^2}}$

154. If $\vec{a}$ and $\vec{b}$ are two unit vectors and θ is the angle between them, then the unit vector along the angular bisector of $\vec{a}$ and $\vec{b}$ will be given by

a) $\frac{\vec{a} - \vec{b}}{2\cos(\theta/2)}$
b) $\frac{\vec{a} + \vec{b}}{2\cos(\theta/2)}$
c) $\frac{\vec{a} - \vec{b}}{\cos(\theta/2)}$
d) None of these

155. In triangle $ABC, \angle A = 30°, H$ is the orthocentre and D is the midpoint of BC. Segment HD is produced to T such that HD=DT. The length AT is equal to

a) $2BC$
b) $3BC$
c) $\frac{4}{3}BC$
d) None of these

156. If vectors $\overrightarrow{AB} = -3\hat{\imath} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$ are the sides of a ΔABC, then the length of the medium through A is

a) $\sqrt{14}$
b) $\sqrt{18}$
c) $\sqrt{29}$
d) 5

157. If $|\vec{a} + \vec{b}| < |\vec{a} - \vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ can lie in the interval

a) $(-\pi/2, \pi/2)$
b) $(0, \pi)$
c) $(\pi/2, 3\pi/2)$
d) $(0, 2\pi)$

158. A vector magnitude 10 along the normal to the curve $3x^2 + 8xy + 2y^2 - 3 = 0$ at its point $P(1,0)$ can be

a) $6\hat{\imath} + 8\hat{\jmath}$
b) $-8\hat{\imath} + 3\hat{\jmath}$
c) $6\hat{\imath} - 8\hat{\jmath}$
d) $8\hat{\imath} + 6\hat{\jmath}$

159. The unit vector orthogonal to vector $-\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and making equal angles with the x-and y-axis is

a) $\pm\frac{1}{3}(2\hat{\imath} + 2\hat{\jmath} - \hat{k})$
b) $\pm\frac{1}{3}(\hat{\imath} + \hat{\jmath} - \hat{k})$
c) $\pm\frac{1}{3}(2\hat{\imath} - 2\hat{\jmath} - \hat{k})$
d) None of these

160. Let us define the length of a vector $a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ as $|a| + |b| + |c|$. This definition coincides with the usual definition of length of a vector $a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ if any only if

a) $a = b = c = 0$
b) Any two of a, b and c are zero
c) Any one of a, b and c is zero
d) $a + b + c = 0$

161. Let a, b, and c be distinct non-negative numbers. If vectors $a\hat{\imath} + a\hat{\jmath} + c\hat{k}, \hat{\imath} + \hat{k}$ and $c\hat{\imath} + c\hat{\jmath} + b\hat{k}$ are coplanar, then c is

a) The arithmetic mean of a and b
b) The geometric mean of a and b
c) The harmonic mean of a and b
d) Equal to zero

Multiple Correct Answers Type

162. If vectors $\vec{b} = (\tan\alpha, -1, 2\sqrt{\sin\alpha/2}$ and $\vec{c}$=$\left(\tan\alpha, \tan\alpha, -\frac{3}{\sqrt{\sin\alpha/2}}\right)$are orthogonal and vector$\vec{a} =$ $(1, 3, \sin 2\alpha)$ makes an obtuse angle with the z-axis, then the value of α is

a) $\alpha = (4n+1)\pi + \tan^{-1} 2$
b) $\alpha = (4n+1)\pi - \tan^{-1} 2$
c) $\alpha = (4n+2)\pi + \tan^{-1} 2$
d) $\alpha = (4n+2)\pi - \tan^{-1} 2$

163. If vectors $\vec{a}$ and $\vec{b}$ are non-collinear, then $\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{b}}{|\vec{b}|}$ is

a) A unit vector
b) In the plane of $\vec{a}$ and $\vec{b}$
c) Equally inclined to $\vec{a}$ and $\vec{b}$
d) Perpendicular to $\vec{a}\times\vec{b}$

164. The angles of a triangle, two of whose sides are represented by vectors $\sqrt{3}(\hat{a}\times\vec{b})$ and $\hat{b}-(\hat{a}\cdot\vec{b})\hat{a}$, where $\vec{b}$ is a non-zero vectors and $\hat{a}$ is a unit vector in the direction of $\vec{a}$, are

a) $\tan^{-1}(\sqrt{3})$
b) $\tan^{-1}(1/\sqrt{3})$
c) $\cot^{-1}(0)$
d) $\tan^{-1}(1)$

165. $\vec{a}$ and $\vec{b}$ are two non-collinear unit vectors, and $\vec{u}=\vec{a}-(\vec{a}\cdot\vec{b})\vec{b}$ and $\vec{v}=\vec{a}\times\vec{b}$. Then $|\vec{v}|$ is

a) $|\vec{u}|$
b) $|\vec{u}|+|\vec{u}\cdot\vec{b}|$
c) $|\vec{u}|+|\vec{u}\cdot\vec{a}|$
d) None of these

166. A, B, C and D are four points such that $\overrightarrow{AB}=m(2\hat{\imath}-6\hat{\jmath}+2\hat{k})$, $\overrightarrow{BC}=(\hat{\imath}-2\hat{\jmath})$ and $\overrightarrow{CD}=n(-6\hat{\imath}+15\hat{\jmath}-3\hat{k})$. If CD intersects AB at some point E, then

a) $m\geq 1/2$
b) $n\geq 1/3$
c) $m=n$
d) $m<n$

167. Let $\vec{a}$ and $\vec{b}$ be two non-collinear unit vectors. If $\vec{u}=\vec{a}-(\vec{a}\cdot\vec{b})\vec{b}$ and $\vec{v}=\vec{a}\times\vec{b}$, then $|\vec{v}|$ is

a) $|\vec{u}|$
b) $|\vec{u}|+|\vec{u}\cdot\vec{a}|$
c) $|\vec{u}|+|\vec{u}\cdot\vec{b}|$
d) $|\vec{u}|+\vec{u}\cdot(\vec{a}+\vec{b})$

168. Let ABC be a triangle, the position vectors of whose are $7\hat{\jmath}+10\hat{k}, -\hat{\imath}+6\hat{\jmath}+6\hat{k}$ and $-4\hat{\imath}+9\hat{\jmath}+6\hat{k}$. Then ΔABC is

a) Isosceles
b) Equilateral
c) Right angles
d) None of these

169. $\vec{b}$ and $\vec{c}$ are non-collinear if $\vec{a}\times(\vec{b}\times\vec{c})+(\vec{a}\cdot\vec{b})\vec{b}=(4-2x-\sin y)\vec{b}+(x^2-1)\vec{c}$ and $(\vec{c}\cdot\vec{c})\,\vec{a}=\vec{c}$. Then

a) $x=1$
b) $x=-1$
c) $y=(4n+1)\frac{\pi}{2}, n\in I$
d) $y=(2n+1)\frac{\pi}{2}, n\in I$

170. If non-zero vectors $\vec{a}$ and $\vec{b}$ are equally inclined to coplanar vector $\vec{c}$, then $\vec{c}$ can be

a) $\frac{|\vec{a}|}{|\vec{a}|+2|\vec{b}|}\vec{a}+\frac{|\vec{b}|}{|\vec{a}|+|\vec{b}|}\vec{b}$
b) $\frac{|\vec{b}|}{|\vec{a}|+|\vec{b}|}\vec{a}+\frac{|\vec{a}|}{|\vec{a}|+|\vec{b}|}\vec{b}$
c) $\frac{|\vec{a}|}{|\vec{a}|+2|\vec{b}|}\vec{a}+\frac{|\vec{b}|}{|\vec{a}|+2|\vec{b}|}\vec{b}$
d) $\frac{|\vec{b}|}{2|\vec{a}|+|\vec{b}|}\vec{a}+\frac{|\vec{a}|}{2|\vec{a}|+|\vec{b}|}\vec{b}$

171. The number of vectors of unit length perpendicular to vectors $\vec{a}=(1,1,0)$ and $\vec{b}=(0,1,1)$ is

a) One
b) Two
c) Three
d) infinite

172. A vector $\vec{a}$ has the compounds $2p$ and 1 w.r.t. a rectangular Cartesian system. This system is rotated through a certain angle about the origin in the counterclockwise sence. If, with respect to a new system, $\vec{a}$ has components $(p+1)$ and I, then p is equal to

a) -1
b) $-1/3$
c) 1
d) 2

173. Let $\vec{r}$ be a unit vector satisfying $\vec{r}\times\vec{a}=\vec{b}$, where $|\vec{a}|=\sqrt{3}$ and $|\vec{b}|=\sqrt{2}$. Then

a) $\vec{r}=\frac{2}{3}(\vec{a}+\vec{a}\times\vec{b})$
b) $\vec{r}=\frac{1}{3}(\vec{a}+\vec{a}\times\vec{b})$
c) $\vec{r}=\frac{2}{3}(\vec{a}-\vec{a}\times\vec{b})$
d) $\vec{r}=\frac{1}{3}(-\vec{a}+\vec{a}\times\vec{b})$

174. In a four dimensional space where unit vectors along the axes are $\hat{\imath},\hat{\jmath},\hat{k}$ and $\hat{l}$, and $\vec{a}_1,\vec{a}_2,\vec{a}_3,\vec{a}_4$ are four non-zero vectors such that no vector can be expressed as linear combination of others and $(\lambda-1)(\vec{a}_1-\vec{a}_2)+\mu(\vec{a}_2+\vec{a}_3)+\gamma(\vec{a}_3+\vec{a}_4-2\vec{a}_2)+\vec{a}_3+\delta\vec{a}_4=\vec{0}$, then

a) $\lambda=1$
b) $\mu=-2/3$
c) $\gamma=2/3$
d) $\delta=1/3$

175. If unit vectors $\vec{a}$ and $\vec{b}$ are inclined at an angle 2θ such that $|\vec{a}-\vec{b}|<1$ and $0\leq\theta\leq\pi$, then θ lies in the interval

a) $[0,\pi/6)$
b) $(5\pi/6,\pi]$
c) $[\pi/6,\pi/2)$
d) $(\pi/2,5\pi/6]$

176. The vectors $\vec{\mathbf{a}}=x\,\hat{\mathbf{i}}-2\hat{\mathbf{j}}+5\hat{\mathbf{k}}$ and $\vec{\mathbf{b}}=\hat{\mathbf{i}}+y\hat{\mathbf{j}}-z\hat{\mathbf{k}}$ are collinear, if

a) $x=1, y=-2, z=-5$
b) $x=1/2, y=-4, z=-10$
c) $x=-1/2, y=4, z=10$
d) $x=-1, y=2, z=5$

177. If $\vec{\mathbf{a}},\vec{\mathbf{b}},\vec{\mathbf{c}}$ are three non-coplanar vectors such that $\vec{\mathbf{r}}_1=\vec{\mathbf{a}}-\vec{\mathbf{b}}+\vec{\mathbf{c}}$, $\vec{\mathbf{r}}_2=\vec{\mathbf{b}}+\vec{\mathbf{c}}-\vec{\mathbf{a}}$, $\vec{\mathbf{r}}_3=\vec{\mathbf{c}}+\vec{\mathbf{a}}+\vec{\mathbf{b}}$, $\vec{\mathbf{r}}=2\vec{\mathbf{a}}-3\vec{\mathbf{b}}+4\vec{\mathbf{c}}$ is $\vec{\mathbf{r}}=\lambda_1\vec{\mathbf{r}}_1+\lambda_2\vec{\mathbf{r}}_2+\lambda_3\vec{\mathbf{r}}_3$, then

a) $\lambda_1=7/2$
b) $\lambda_1+\lambda_2=3$
c) $\lambda_1+\lambda_2+\lambda_3=4$
d) $\lambda_2+\lambda_3=2$

178. If $\vec{a}$and $\vec{b}$ are two vectors and angle between them is θ, then

a) $|\vec{a}\times\vec{b}|^2+(\vec{a}\cdot\vec{b})^2=|\vec{a}|^2|\vec{b}|^2$

b) $|\vec{a}\times\vec{b}| \quad =(\vec{a}\cdot\vec{b})$ if$\theta=\pi/4$

c) $\vec{a}\times\vec{b}=(\vec{a}\cdot\vec{b})\hat{n}$, ($\hat{n}$ is normal unit vector), if $\theta=\pi/4$

d) $(\vec{a}\times\vec{b})\cdot(\vec{a}+\vec{b})=0$

179. If $\vec{a},\vec{b}$ and $\vec{c}$ are non-coplanar vectors and λ is a real number, then the vectors $\vec{a}+2\vec{b}+3\vec{c},\lambda\vec{b}+\mu\vec{c}$ and $(2\lambda-1)\vec{c}$ are coplanar when

a) $\mu\in R$ b) $\lambda=\frac{1}{2}$ c) $\lambda=0$ d) No value of λ

180. A parallelogram is constructed on vectors $\vec{a}=3\vec{\alpha}-\vec{\beta},\vec{b}=\vec{\alpha}+3\vec{\beta}$ if $|\vec{\alpha}|=|\vec{\beta}|=2$, and angle between $\vec{\alpha}$ and $\vec{\beta}$ is $\frac{\pi}{3}$, then the length of a diagonal of parallogram is

a) $4\sqrt{5}$ b) $4\sqrt{3}$ c) $4\sqrt{7}$ d) None of these

181. Vectors perpendicular to $\hat{\imath}-\hat{\jmath}-\hat{k}$ and in the plane of $\hat{\imath}+\hat{\jmath}+\hat{k}$and $-\hat{\imath}+\hat{\jmath}+\hat{k}$ are

a) $\hat{\imath}+\hat{k}$ b) $2\hat{\imath}+\hat{\jmath}+\hat{k}$ c) $3\hat{\imath}+2\hat{\jmath}+\hat{k}$ d) $-4\hat{\imath}-2\hat{\jmath}-2\hat{k}$

182. The vectors $x\hat{\imath}+(x+1)\hat{\jmath}+(x+2)\hat{k},(x+3)\hat{\imath}+(x+4)\hat{\jmath}+(x+5)\hat{k}$ and $(x+6)\hat{\imath}+(x+7)\hat{\jmath}+(x+8)\hat{k}$ are coplanar if x is equal to

a) 1 b) -3 c) 4 d) 0

183. If $\vec{a}\times\vec{b}=\vec{c}$ and $\vec{b}\times\vec{c}=\vec{a}$, then

(a) $\vec{a},\vec{b},\vec{c}$ are orthogonal in pairs and$|\ \vec{a}|=|\ \vec{a}|,|\vec{b}|=1$

(b) $\vec{a},\vec{b},\vec{c}$ are not orthogonal to each other

(c) $\vec{a},\vec{b},\vec{c}$ are orthogonal in pairs but $|\vec{a}|\neq|\vec{c}|$

(d) $\vec{a},\vec{b},\vec{c}$ are orthogonal but $|\vec{b}|\neq 1$

Or

If $\vec{a}\times\vec{b}=\vec{c},\vec{b}\times\vec{c}=\vec{a}$, then

a) $|\vec{a}|=1,\vec{b}=\vec{c}$ b) $|\vec{c}|=1,|\vec{a}|=1$ c) $|\vec{b}|=2,\vec{c}=2\,\vec{a}$ d) $|\vec{b}|=1,|\vec{c}|=|\vec{a}|$

184. Let $\vec{a},\vec{b}$and $\vec{c}$ be three non-coplanar vectors and $\vec{d}$ be a non-zero vector, which is perpendicular to $(\vec{a}+\vec{b}+\vec{c})$.Now $\vec{d}=(\vec{a}\times\vec{b})\sin x+(\vec{b}\times\vec{c})\cos y+2(\vec{c}\times\vec{a})$.Then

a) $\frac{\vec{d}\cdot(\vec{a}+\vec{c})}{[\vec{a}\vec{b}\vec{c}]}=2$ b) $\frac{\vec{d}\cdot(\vec{a}+\vec{c})}{[\vec{a}\vec{b}\vec{c}]}=-2$

c) Minimum value of $x^2+y^2$is $\pi^2/4$ d) Minimum value of $x^2+y^2$is $5\pi^2/4$

185. If $A(-4,0,3)$ and $B(14,2,-5)$, then which one of the following points lie on the bisector of the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$ (O is the origin of reference)?

a) (2, 2, 4) b) (2, 11, 5) c) (−3, −3, −6) d) (1, 1, 2)

186. Let $\vec{a},\vec{b}$ and $\vec{c}$ be non-zero vectors and $\vec{V}_1=\vec{a}\times(\vec{b}\times\vec{c})and\vec{V}_2=(\vec{a}\times\vec{b})\times\vec{c}$

Vectors $\vec{V}_1$and $\vec{V}_2$ are equal. Then

a) $\vec{a}$ and $\vec{b}$ are orthogonal b) $\vec{a}$and $\vec{c}$ are collinear

c) $\vec{b}$and $\vec{c}$are orthogonal d) $\vec{b}=\lambda(\vec{a}\times\vec{c})$ when λ is a scalar

187. Which of the following expression are meaningful?

a) $\vec{u}\cdot(\vec{v}\times\vec{w})$ b) $(\vec{u}\cdot\vec{v})\cdot\vec{w}$ c) $(\vec{u}\cdot\vec{v})\vec{w}$ d) $\vec{u}\times(\vec{v}\cdot\vec{w})$

188. If points $\hat{\imath}+\hat{\jmath},\hat{\imath}-\hat{\jmath}$ and $p\hat{\imath}+q\hat{\jmath}+r\hat{k}$ are collinear, then

a) $p=1$ b) $r=0$ c) $q\in R$ d) $q\neq 1$

189. $\vec{a}$ and$\vec{b}$ are two given vectors. With these vectors as adjacent sides, a parallelogram is constructed. The vector which is the altitude of the parallelogram and which is perpendicular to $\vec{a}$is

a) $\frac{(\vec{a}\cdot\vec{b})}{|\vec{a}|^2}\vec{a}-\vec{b}$ b) $\frac{1}{|\vec{a}|^2}\{|\vec{a}|^2\vec{b}-(\vec{a}\cdot\vec{b})\vec{a}\}$

c) $\frac{\vec{a}\times(\vec{a}\times\vec{b})}{|\vec{a}|^2}$ d) $\frac{\vec{a}\times(\vec{b}\times\vec{a})}{|\vec{b}|^2}$

190. The sides of a parallelogram are $2\hat{\imath}+4\hat{\jmath}-5\hat{k}$ and $\hat{\imath}+2\hat{\jmath}+3\hat{k}$. The unit vector parallel to one of the diagonals is

a) $\frac{1}{7}(3\hat{\imath}+6\hat{\jmath}-2\hat{k})$ b) $\frac{1}{7}(3\hat{\imath}-6\hat{\jmath}-2\hat{k})$ c) $\frac{1}{\sqrt{69}}(\hat{\imath}+2\hat{\jmath}+8\hat{k})$ d) $\frac{1}{\sqrt{69}}(-\hat{\imath}-2\hat{\jmath}+8\hat{k})$

191. Vector $\frac{1}{3}(2\hat{\imath}-2\hat{\jmath}+\hat{k})$ is

a) a unit vector

b) Makes an angle $\pi/3$ with vector $(2\hat{\imath}-4\hat{\jmath}+3\hat{k})$

c) Parallel to vector $\left(-\hat{\imath}+\hat{\jmath}-\frac{1}{2}\hat{k}\right)$

d) Perpendicular to vector $3\hat{\imath}+2\hat{\jmath}-2\hat{k}$

192. Let $\vec{a}$and$\vec{b}$ be two non-zero perpendicular vectors. A vector $\vec{r}$ satisfying the equation $\vec{r}\times\vec{b}=\vec{a}$ can be

a) $\vec{b}-\frac{\vec{a}\times\vec{b}}{|\vec{b}|^2}$ b) $2\vec{b}-\frac{\vec{a}\times\vec{b}}{|\vec{b}|^2}$ c) $|\vec{a}|\vec{b}-\frac{\vec{a}\times\vec{b}}{|\vec{b}|^2}$ d) $|\vec{b}|\vec{b}-\frac{\vec{a}\times\vec{b}}{|\vec{b}|^2}$

193. If $\vec{a}\times\vec{b}=\vec{c}, \vec{b}\times\vec{c}=\vec{a}$,where $\vec{c}\neq\vec{0}$, then

a) $|\vec{a}|=|\vec{c}|$ b) $|\vec{a}|=|\vec{b}|$ c) $|\vec{b}|=1$ d) $|\vec{a}|=|\vec{b}|=|\vec{c}|=1$

194. Let $\vec{a}=2\hat{\imath}-\hat{\jmath}+\hat{k}, \vec{b}=\hat{\imath}+2\hat{\jmath}-\hat{k}$ and $\vec{c}=\hat{\imath}+\hat{\jmath}-2\hat{k}$ be three vectors. A vectors in the plane of $\vec{b}$ and $\vec{c}$, whose projection on $\vec{a}$ is of magnitude $\sqrt{2/3}$, is

a) $2\hat{\imath}+3\hat{\jmath}-3\hat{k}$ b) $2\hat{\imath}+3\hat{\jmath}+3\hat{k}$ c) $-2\hat{\imath}-\hat{\jmath}+5\hat{k}$ d) $2\hat{\imath}+\hat{\jmath}+5\hat{k}$

195. The scalars l and m such that $l\vec{a}+m\vec{b}=\vec{c}$, where $\vec{a},\vec{b}$and $\vec{c}$ are given vectors, are equal to

a) $l=\frac{(\vec{c}\times\vec{b}).(\vec{a}\times\vec{b})}{(\vec{a}\times\vec{b})^2}$ b) $l=\frac{(\vec{c}\times\vec{a}).(\vec{b}\times\vec{a})}{(\vec{b}\times\vec{a})}$ c) $m=\frac{(\vec{c}\times\vec{b}).(\vec{b}\times\vec{a})}{(\vec{b}\times\vec{a})^2}$ d) $m=\frac{(\vec{c}\times\vec{a}).(\vec{b}\times\vec{a})}{(\vec{b}\times\vec{a})^2}$

196. A vector $\vec{d}$ is equally inclined to three vectors $\vec{a}=\hat{\imath}-\hat{\jmath}+\hat{k}, \vec{b}=2\hat{\imath}+\hat{\jmath}$ and $\vec{c}=3\hat{\jmath}-2\hat{k}$. Let $\vec{x},\vec{y}$and $\vec{z}$ be three vectors in the plane of $\vec{a},\vec{b};\vec{b},\vec{c};\vec{c},\vec{a}$, respectively Then

a) $\vec{x}\cdot\vec{d}=-1$ b) $\vec{y}\cdot\vec{d}=1$

c) $\vec{z}\cdot\vec{d}=0$ d) $\vec{r}\cdot\vec{d}=0$,where $\vec{r}=\lambda\vec{x}+\mu\vec{y}+\delta\vec{z}$

197. $\vec{a},\vec{b}$ and $\vec{c}$ are three coplanar unit vectors such that $\vec{a}+\vec{b}+\vec{c}=0$. If three vectors $\vec{p},\vec{q}$ and $\vec{r}$ are parallel to $\vec{a},\vec{b}$ and $\vec{c}$, respectively, and have integral but different magnitudes, then among the following options, $|\vec{p}+\vec{q}+\vec{r}|$ can take a value equal to

a) 1 b) 0 c) $\sqrt{3}$ d) 2

198. If $\vec{a},\vec{b}$and $\vec{c}$ are three unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c})=\frac{1}{2}\vec{b}$,then ($\vec{b}$and $\vec{c}$being non-parallel)

a) Angle between $\vec{a}$and $\vec{b}$ is $\pi/3$ b) Angle between $\vec{a}$and $\vec{c}$ is $\pi/3$

c) Angle between $\vec{a}$and $\vec{b}$ is $\pi/2$ d) Angle between $\vec{a}$and $\vec{c}$ is $\pi/2$

199. If side $\overrightarrow{AB}$ of an equilateral triangle ABC lying in the $x-y$ plane is $3\hat{\imath}$, then side$\overrightarrow{CB}$can be

a) $-\frac{3}{2}(\hat{\imath}-\sqrt{3}\hat{\jmath})$ b) $\frac{3}{2}(\hat{\imath}-\sqrt{3}\hat{\jmath})$ c) $-\frac{3}{2}(\hat{\imath}+\sqrt{3}\hat{\jmath})$ d) $\frac{3}{2}(\hat{\imath}+\sqrt{3}\hat{\jmath})$

200. For three vectors $\vec{u},\vec{v}$and $\vec{w}$ which of the following expression is not equal to any of the remaining three?

a) $\vec{u}\cdot(\vec{v}\times\vec{w})$ b) $(\vec{v}\times\vec{w})\cdot\vec{u}$ c) $\vec{v}\cdot(\vec{u}\times\vec{w})$ d) $(\vec{u}\times\vec{v})\cdot\vec{w}$

201. Let $\vec{a},\vec{b}$ and $\vec{c}$ be vectors forming right-hand tried. Let $\vec{p}=\frac{\vec{b}\times\vec{c}}{[\vec{a}\vec{b}\vec{c}]}, \vec{q}=\frac{\vec{c}\times\vec{a}}{[\vec{a}\vec{b}\vec{c}]}$ and $\vec{r}=\frac{\vec{a}\times\vec{b}}{[\vec{a}\vec{b}\vec{c}]}$. If $x\in R^+$, then

a) $x[\vec{a}\vec{b}\vec{c}]+\frac{[\vec{p}\vec{q}\vec{r}]}{x}$ has least value 2 b) $x^4[\vec{a}\vec{b}\vec{c}]^2+\frac{[\vec{p}\vec{q}\vec{r}]}{x^2}$has least value $(3/2^{2/3})$

c) $[\vec{p}\vec{q}\vec{r}]>0$ d) None of these

202. Let $\mathbf{\vec{a}}=2\mathbf{\hat{i}}-\mathbf{\hat{j}}+\mathbf{\hat{k}}, \mathbf{\vec{b}}=\mathbf{\hat{i}}+2\mathbf{\hat{j}}-\mathbf{\hat{k}}$ and $\mathbf{\vec{c}}=\mathbf{\hat{i}}+\mathbf{\hat{j}}-2\mathbf{\hat{k}}$ be three vectors. A vector in the plane of $\mathbf{\vec{b}}$ and $\mathbf{\vec{c}}$ whose projection at $\mathbf{\vec{a}}$ is of magnitude $\sqrt{(2/3)}$, is

a) $2\hat{\mathbf{i}}+3\hat{\mathbf{j}}-3\hat{\mathbf{k}}$ b) $2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ c) $-2\hat{\mathbf{i}}-\hat{\mathbf{j}}+5\hat{\mathbf{k}}$ d) $2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+5\hat{\mathbf{k}}$

203. If vectors $\vec{a},\vec{b}$and $\vec{c}$ are non-coplanar and l,m and n are distinct scalars, then $[(l\vec{a}+m\vec{b}+n\vec{c})(l\vec{b}+m\vec{c}+n\vec{a})(l\vec{c}+m\vec{a}+n\vec{b})]=0$ implies

a) $l+m+n=0$ b) Roots of the equation $lx^2+mx+n=0$ are real

c) $l^2+m^2+n^2=0$ d) $l^3+m^3+n^3=3lmn$

204. If $\vec{a}$ and$\vec{b}$ are non zero vectors such that $|\vec{a}+\vec{b}|=|\vec{a}-2\vec{b}|$, then

a) $2\vec{a}.\vec{b}=|\vec{b}|^2$ b) $\vec{a}.\vec{b}=|\vec{b}|^2$

c) Least value of $\vec{a}\cdot\vec{b}+\frac{1}{|\vec{b}|^2+2}$is $\sqrt{2}$ d) Least value of $\vec{a}\cdot\vec{b}+\frac{1}{|\vec{b}|+2}$is $\sqrt{2}-1$

205. Let $\vec{\alpha}=a\hat{\imath}+b\hat{\jmath}+c\hat{k},\vec{\beta}=b\hat{\imath}+c\hat{\jmath}+a\hat{k}$ and $\vec{\gamma}=c\hat{\imath}+a\hat{\jmath}+b\hat{k}$ be three coplanar vectors with $a\neq b$, and $\vec{v}=\hat{\imath}+\hat{\jmath}+\hat{k}$. Then $\vec{v}$ is perpendicular to

a) $\vec{\alpha}$ b) $\vec{\beta}$ c) $\vec{\gamma}$ d) None of these

206. If $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})\cdot(\vec{a}\times\vec{d})=0$, then which of the following may be true?

a) $\vec{a},\vec{b},\vec{c}$and $\vec{d}$ are necessarily coplanar b) $\vec{a}$ lies in the plane of $\vec{c}$and $\vec{d}$

c) $\vec{b}$lies in the plane of $\vec{a}$ and$\vec{d}$ d) $\vec{c}$lies in the plane of $\vec{a}$and $\vec{d}$

207. If vectors $\vec{A}=2\hat{\imath}+3\hat{\jmath}+4\hat{k},\vec{B}=\hat{\imath}+\hat{\jmath}+5\hat{k}$ and $\vec{C}$ form a left-handed system, then $\vec{C}$ is

a) $11\hat{\imath}-6\hat{\jmath}-\hat{k}$ b) $-11\hat{\imath}+6\hat{\jmath}+\hat{k}$ c) $11\hat{\imath}-6\hat{\jmath}+\hat{k}$ d) $-11\hat{\imath}+6\hat{\jmath}-\hat{k}$

208. $\vec{a}\cdot\vec{b}$and $\vec{c}$ are unimodular and coplanar. A unit vector $\vec{d}$ is perpendicular to them. If $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=\frac{1}{6}\hat{\imath}-\frac{1}{3}\hat{\jmath}+\frac{1}{3}\hat{k}$, and the angle between $\vec{a}$and $\vec{b}$ is 30°, then $\vec{c}$ is

a) $(\hat{\imath}-2\hat{\jmath}+2\hat{k})/3$ b) $(-\hat{\imath}+2\hat{\jmath}-2\hat{k})/3$ c) $(2\hat{\imath}+2\hat{\jmath}-\hat{k})/3$ d) $(-2\hat{\imath}-2\hat{\jmath}+\hat{k})/3$

209. A parallelogram is constructed on the vectors $\mathbf{\vec{a}}=3\vec{\alpha}-\vec{\beta},\mathbf{\vec{b}}=\vec{\alpha}+3\vec{\beta}$, if $|\vec{\alpha}|=|\vec{\beta}|=2$ and angle between $\vec{\alpha}$ and $\vec{\beta}$ is $\frac{\pi}{3}$, then the length of a diagonal of the parallelogram is

a) $4\sqrt{5}$ b) $4\sqrt{3}$ c) $4\sqrt{7}$ d) None of these

210. $[\vec{a}\times\vec{b}\vec{c}\times\vec{d}\vec{e}\times\vec{f}]$is equal to

a) $[\vec{a}\vec{b}\vec{d}][\vec{c}\vec{e}\vec{f}]-[\vec{a}\vec{b}\vec{c}][\vec{d}\vec{e}\vec{f}]$ b) $[\vec{a}\vec{b}\vec{e}][\vec{f}\vec{c}\vec{d}]-[\vec{a}\vec{b}\vec{f}][\vec{e}\vec{c}\vec{d}]$

c) $[\vec{c}\vec{d}\vec{a}][\vec{b}\vec{e}\vec{f}]-[\vec{a}\vec{d}\vec{b}][\vec{a}\vec{e}\vec{f}]$ d) $[\vec{a}\vec{c}\vec{e}][\vec{b}\vec{d}\vec{f}]$

211. Vectors $\vec{A}$and $\vec{B}$ satisfying the vector equation $\vec{A}+\vec{B}=\vec{a},\vec{A}\times\vec{B}=\vec{b}$and $\vec{A}\cdot\vec{a}=1$,where $\vec{a}$and $\vec{b}$ are given vectors, are

a) $\vec{A}=\dfrac{(\vec{a}\times\vec{b})-\vec{a}}{a^2}$ b) $\vec{B}=\dfrac{(\vec{b}\times\vec{a})+\vec{a}(a^2-1)}{a^2}$

c) $\vec{A}=\dfrac{(\vec{a}\times\vec{b})+\vec{a}}{a^2}$ d) $\vec{B}=\dfrac{(\vec{b}\times\vec{a})-\vec{a}(a^2-1)}{a^2}$

212. $a_1,a_2,a_3\in R-\{0\}$ and $a_1+a_2\cos 2x+a_3\sin^2 x=0$for all $x\in R$, then

a) Vector $\vec{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$ and $\vec{b}=4\hat{\imath}+2\hat{\jmath}+\hat{k}$ are perpendicular to each other

b) Vector $\vec{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$and $\vec{b}=-\hat{\imath}+\hat{\jmath}+2\hat{k}$ are parallel to each other

c) If vector $\vec{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$is of length $\sqrt{6}$units, then one of the ordered triplet $(a_1,a_2,a_3)=(1,-1,-2)$

d) If $2a_1+3a_2+6a_3=26$,then $|a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}|$is $2\sqrt{6}$

213. If $\vec{a}$and $\vec{b}$ are unequal unit vectors such that $(\vec{a}-\vec{b})\times[(\vec{b}+\vec{a})\times(2\vec{a}+\vec{b})]=\vec{a}+\vec{b}$, then angle θbetween $\vec{a}$and $\vec{b}$ is

a) 0 b) $\pi/2$ c) $\pi/4$ d) π

214. If the resultant of three forces $\vec{F_1}=p\vec{\imath}+3\hat{\jmath}-\hat{k},\vec{F_2}=6\hat{\imath}-\hat{k}$ and $\vec{F_3}=-5\hat{\imath}+\hat{\jmath}+2\hat{k}$ acting on a particle has a magnitude equal to 5 units, then the value of p is

a) -6 b) -4 c) 2 d) 4

215. If in triangle $ABC,\overrightarrow{AB}=\frac{\vec{u}}{|\vec{u}|}-\frac{\vec{v}}{|\vec{v}|}$and $\overrightarrow{AC}=\frac{2\vec{u}}{|\vec{u}|}$, where $|\vec{u}|\neq|\vec{v}|$, then

a) $1+\cos 2A+\cos 2B+\cos 2C=0$ b) $\sin A=\cos C$
c) Projection of AC on BC is equal to BC d) Projection of AB on BC is equal to AB

216. If the vectors $\hat{\imath}-\hat{\jmath}, \hat{\jmath}+\hat{k}$ and $\vec{a}$ from a triangle, then $\vec{a}$ may be
a) $-\hat{\imath}-\hat{k}$ b) $\hat{\imath}-2\hat{\jmath}-\hat{k}$ c) $2\hat{\imath}+\hat{\jmath}+\hat{k}$ d) $\hat{\imath}+\hat{k}$

217. If $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$, then $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=$
a) $2(\vec{a}\times\vec{b})$ b) $6(\vec{b}\times\vec{c})$ c) $3(\vec{c}\times\vec{a})$ d) $\vec{0}$

218. Unit vectors $\vec{a}$ and $\vec{b}$ are perpendicular, and unit vector $\vec{c}$ is inclined at an angle θ to both $\vec{a}$ and $\vec{b}$. If $\vec{c}=\alpha\vec{a}+\beta\vec{b}+\gamma(\vec{a}\times\vec{b})$, then
a) $\alpha=\beta$ b) $\gamma^2=1-2\alpha^2$ c) $\gamma^2=-\cos 2\theta$ d) $\beta^2=\dfrac{1+\cos 2\theta}{2}$

219. If $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b})\times\vec{c}$, then
a) $(\vec{c}\times\vec{a})\times\vec{b}=\vec{0}$ b) $\vec{c}\times(\vec{a}\times\vec{b})=\vec{0}$
c) $\vec{b}\times(\vec{c}\times\vec{a})=\vec{0}$ d) $(\vec{c}\times\vec{a})\times\vec{b}=\vec{b}\times(\vec{c}\times\vec{a})=\vec{0}$

220. Let $\vec{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}, \vec{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$ and $\vec{c}=c_1\hat{\imath}+c_2\hat{\jmath}+c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both vectors $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\pi/6$, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & b_2 & c_3 \end{vmatrix}^2$ is equal to
a) 0
b) 1
c) $\frac{1}{4}(a_1^2+a_2^2+a_2^2)(b_1^2+b_2^2+b_3^2)$
d) $\frac{3}{4}(a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2)(c_1^2+c_2^2+c_3^2)$

221. The vectors $(x, x+1, x+2), (x+3, x+4, x+5)$ and $(x+6, x+7, x+8)$ are coplanar for
a) All values of x b) $x<0$ c) $x>0$ d) None of these

222. The vector $\hat{\imath}+x\hat{\jmath}+3\hat{k}$ is rotated through an angle θ and doubled in magnitude. It now becomes $4\hat{\imath}+(4x-2)\hat{\jmath}+2\hat{k}$. The values of x are
a) 1 b) $-2/3$ c) 2 d) $3/4$

223. If $a=x\hat{\imath}+y\hat{\jmath}+z\hat{k}, b=y\hat{\imath}+z\hat{\jmath}+x\hat{k}$ and $c=z\hat{\imath}+x\hat{\jmath}+y\hat{k}$, then $\vec{a}\times(\vec{b}\times\vec{c})$ is
a) Parallel to $(y-z)\hat{\imath}+(z-x)\hat{\jmath}+(x-y)\hat{k}$ b) Orthogonal to $\hat{\imath}+\hat{\jmath}+\hat{k}$
c) Orthogonal to $(y+z)\hat{\imath}+(z+x)\hat{\jmath}+(x+y)\hat{k}$ d) Orthogonal to $x\hat{\imath}+y\hat{\jmath}+z\hat{k}$

224. A vector $(\vec{d})$ is equally inclined to three vectors $\vec{a}=\hat{\imath}-\hat{\jmath}+\hat{k}, \vec{b}=2\hat{\imath}+\hat{\jmath}$ and $\vec{c}=3\hat{\jmath}-2\hat{k}$. Let $\vec{x}, \vec{y}, \vec{z}$ be three vectors in the plane of $\vec{a}, \vec{b}; \vec{b}, \vec{c}; \vec{c}, \vec{a}$, respectively. Then
a) $\vec{z}\cdot\vec{d}=0$ b) $\vec{x}\cdot\vec{d}=1$
c) $\vec{y}\cdot\vec{d}=32$ d) $\vec{r}\cdot\vec{d}=0$, where $\vec{r}=\lambda\vec{x}+\mu\vec{y}+\gamma\vec{z}$

225. If $\vec{a}$ and $\vec{b}$ are two unit vectors perpendicular to each other and $\vec{c}=\lambda_1\vec{a}+\lambda_2\vec{b}+\lambda_3(\vec{a}\times\vec{b})$, then which of the following is (are) true?
a) $\lambda_1=\vec{a}\cdot\vec{c}$ b) $\lambda_2=|\vec{b}\times\vec{c}|$
c) $\lambda_3=|(\vec{a}\times\vec{b})\times\vec{c}|$ d) $\lambda_1+\lambda_2+\lambda_3=(\vec{a}+\vec{b}+\vec{a}\times\vec{b})\cdot\vec{c}$

226. If $\vec{a}\times(\vec{b}\times\vec{c})$ is perpendicular to $(\vec{a}\times\vec{b})\times\vec{c}$, we may have
a) $(\vec{a}\cdot\vec{c})|\vec{b}|^2=(\vec{a}\cdot\vec{b})(\vec{b}\cdot\vec{c})$ b) $\vec{a}\cdot\vec{b}=0$
c) $\vec{a}\cdot\vec{c}=0$ d) $\vec{b}\cdot\vec{c}=0$

Assertion - Reasoning Type

This section contain(s) 0 questions numbered 227 to 226. Each question contains STATEMENT 1(Assertion) and STATEMENT 2(Reason). Each question has the 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

a) Statement 1 is True, Statement 2 is True; Statement 2 **is** correct explanation for Statement 1

b) Statement 1 is True, Statement 2 is True; Statement 2 **is not** correct explanation for Statement 1

c) Statement 1 is True, Statement 2 is False

d) Statement 1 is False, Statement 2 is True

227 A vector has components p and 1 with respect to a rectangular Cartesian system. The axes are rotated through an angle α about the origin in the anticlockwise sense

Statement 1: If the vector has component $p + 2$ and I with respect to the new system, then $p = -1$

Statement 2: Magnitude of the original vector and the new vector remains the same

228

Statement 1: If $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{c}} \times \vec{\mathbf{d}}$ and $\vec{\mathbf{a}} \times \vec{\mathbf{c}} = \vec{\mathbf{b}} \times \vec{\mathbf{d}}$ then $\vec{\mathbf{a}} - \vec{\mathbf{d}}$ is perpendicular to $\vec{\mathbf{b}} - \vec{\mathbf{c}}$.

Statement 2: If $\vec{\mathbf{p}}$ is perpendicular to $\vec{\mathbf{q}}$, then $\vec{\mathbf{p}} \cdot \vec{\mathbf{q}} = 0$.

229 Let the vectors $\overrightarrow{\mathbf{PQ}}, \overrightarrow{\mathbf{QR}}, \overrightarrow{\mathbf{RS}}, \overrightarrow{\mathbf{ST}}, \overrightarrow{\mathbf{TU}}$ and $\overrightarrow{\mathbf{UP}}$ represent the sides of a regular hexagon.

Statement 1: $\overrightarrow{\mathbf{PQ}} \times (\overrightarrow{\mathbf{RS}} + \overrightarrow{\mathbf{ST}}) \neq \vec{\mathbf{0}}$ Because

Statement 2: **II** $\overrightarrow{\mathbf{PQ}} \times \overrightarrow{\mathbf{RS}} = \vec{\mathbf{0}}$ and $\overrightarrow{\mathbf{PQ}} \times \overrightarrow{\mathbf{ST}} \neq \vec{\mathbf{0}}$

230

Statement 1: If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then $\vec{a}$ and $\vec{b}$ are perpendicular to each other

Statement 2: If the diagonals of a parallelogram are equal in magnitude, then the parallelogram is a rectangle

231

Statement 1: For $a = -\frac{1}{\sqrt{3}}$ the volume of the parallelepiped formed by vectors $\hat{\mathbf{i}} + a\hat{\mathbf{j}}, a\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\hat{\mathbf{j}} + a\hat{\mathbf{k}}$ is maximum

Statement 2: The volume of the parallelepiped having three coterminous edges $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ is $|[\vec{\mathbf{a}}\vec{\mathbf{b}}\vec{\mathbf{c}}]|$.

232

Statement 1: If $\vec{u}$ and $\vec{v}$ are unit vectors inclined at an angle α and $\vec{x}$ is a unit vector bisecting the angle between them, then $\vec{x} = (\vec{u} + \vec{v})/(2\sin(\alpha/2)$

Statement 2: If ΔABC is an isosceles triangle with $AB = AC = 1$, then the vector representing the bisector of angle A is given by $\overrightarrow{AD} = (\overrightarrow{AB} + \overrightarrow{AC})/2$

233

Statement 1: If $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{a} + \vec{b}| = 5$, then $|\vec{a} - \vec{b}| = 5$

Statement 2: The length of the diagonals of a rectangle is the same

234

Statement 1: Let $A(\vec{a}), B(\vec{b})$ and $C(\vec{c})$ be three points such that $\vec{a} = 2\hat{\imath} + \hat{k}, \vec{b} = 3\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{c} = -\hat{\imath} + 7\hat{\jmath} - 5\hat{k}$. Then $OABC$ is a tetrahedron

Statement 2: Let $A(\vec{a}), B(\vec{a})$ and $C(\vec{c})$ be three points such that vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar. Then $OABC$ is a tetrahedron, where O is the origin

235 Consider three vectors $\vec{a}, \vec{b}$ and $\vec{c}$

Statement 1: $\vec{a} \times \vec{b} = \left((\hat{\imath} \times \vec{a}) \cdot \vec{b}\right)\hat{\imath} + \left((\hat{\jmath} \times \vec{a}) \cdot \vec{b}\right)\hat{\jmath} + \left((\hat{k} \times \vec{a}) \cdot \vec{b}\right)\hat{k}$

Statement 2: $\vec{c} = (\hat{\imath} \cdot \vec{c})\hat{\imath} + (\hat{\jmath} \cdot \vec{c})\hat{\jmath} + (\hat{k} \cdot \vec{c})\hat{k}$

236 If $\vec{\mathbf{a}} \cdot \vec{\mathbf{c}} = 3/2, \vec{\mathbf{b}} \cdot \vec{\mathbf{d}} = 2, \vec{\mathbf{a}} \cdot \vec{\mathbf{d}} = 3$ and $\vec{\mathbf{b}} \cdot \vec{\mathbf{c}} = 1/2$

Statement 1: $\vec{\mathbf{a}} \times \vec{\mathbf{b}}, \vec{\mathbf{c}}, \vec{\mathbf{d}}$ are non-coplanar.

Statement 2: $(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \cdot (\vec{\mathbf{c}} \times \vec{\mathbf{d}}) = (\vec{\mathbf{b}} \cdot \vec{\mathbf{c}})(\vec{\mathbf{a}} \cdot \vec{\mathbf{d}}) - (\vec{\mathbf{a}} \cdot \vec{\mathbf{c}})(\vec{\mathbf{b}} \cdot \vec{\mathbf{d}})$.

237

Statement 1: Distance of point $D(1, 0, -1)$ from the plane of points $A(1, -2, 0), B(3, 1, 2)$ and $C(-1, 1, -1)$ is $\frac{8}{\sqrt{229}}$

Statement 2: Volume of tetrahedron formed by the points A, B, C and D is $\frac{\sqrt{229}}{2}$

238

Statement 1: $\vec{a}, \vec{b}$ and $\vec{c}$ are three mutually perpendicular unit vectors and $\vec{d}$ is a vector such that $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are non-coplanar. If $[\vec{d}\vec{b}\vec{c}] = [\vec{d}\vec{a}\vec{b}] = [\vec{d}\vec{c}\vec{a}] = 1$, then $\vec{d} = \vec{a} + \vec{b} + \vec{c}$

Statement 2: $[\vec{d}\vec{b}\vec{c}] = [\vec{d}\vec{a}\vec{b}] = [\vec{d}\vec{c}\vec{a}] \Rightarrow \vec{d}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$

239

Statement 1: If $\vec{A} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}, \vec{B} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\vec{C} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$, then $\left|\vec{A} \times \left(\vec{A} \times (\vec{A} \times \vec{B})\right) \cdot \vec{C}\right| = 243$

Statement 2: $\left|\vec{A} \times \left(\vec{A} \times (\vec{A} \times \vec{B})\right)\right| \cdot \vec{C}| = |\vec{A}|^2 |[\vec{A}\vec{B}\vec{C}]|$

240

Statement 1: If three point P, Q and R have position vectors $\vec{a}, \vec{b}$, and $\vec{c}$, respectively, and $2\vec{a} + 3\vec{b} - 5\vec{c} = 0$, then the points P, Q and R must be collinear

Statement 2: If for three points A, B and C; $\overrightarrow{AB} = \lambda\overrightarrow{AC}$, then points A, B and C must be collinear

241

Statement 1: If $\cos\alpha, \cos\beta$ and $\cos\gamma$ are the direction cosines of any line segment, then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Statement 2: If $\cos\alpha, \cos\beta$ and $\cos\gamma$ are the direction cosines of a line segment, $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

242

Statement 1: If in a $\Delta ABC, \overrightarrow{\mathbf{BC}} = \frac{\vec{\mathbf{e}}}{|\vec{\mathbf{e}}|} - \frac{\vec{\mathbf{f}}}{|\vec{\mathbf{f}}|}$ and $\overrightarrow{\mathbf{AC}} = \frac{2\vec{\mathbf{e}}}{|\vec{\mathbf{e}}|}$;
$|\vec{e}| \neq |\vec{\mathbf{f}}|$, then the value of $\cos 2A + \cos 2B + \cos 2C$ is -1.

Statement 2: If in $\Delta ABC, \angle ABC, \angle C = 90°$, then $\cos 2A + \cos 2B + \cos 2C = 1$

243

Statement 1: If $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are coplanar then $\vec{\mathbf{a}} \times \vec{\mathbf{b}}, \vec{\mathbf{b}} \times \vec{\mathbf{c}}$ and $\vec{\mathbf{c}} \times \vec{\mathbf{a}}$ are also coplanar.

Statement 2: $[\vec{\mathbf{a}} \times \vec{\mathbf{b}}\ \ \vec{\mathbf{b}} \times \vec{\mathbf{c}}\ \ \vec{\mathbf{c}} \times \vec{\mathbf{a}}] = 2[\ \vec{\mathbf{a}}\, \vec{\mathbf{b}}\, \vec{\mathbf{c}}\,]^2$.

244

Statement 1: $\vec{a} = 3\vec{\imath} + p\vec{\jmath} + 3\vec{k}$ and $\vec{b} = 2\vec{\imath} + 3\vec{\jmath} + q\vec{k}$ are parallel vectors if p=9/2 and q=2

Statement 2: If $\vec{a} = a_1\vec{\imath} + a_2\vec{\jmath} + a_3\vec{k}$ and $\vec{b} = b_1\vec{\imath} + b_2\vec{\jmath} + b_3\vec{k}$ are parallel $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

245

Statement 1: The direction cosines of one of the angular bisectors of two intersecting lines having direction cosines as l_1, m_1, n_1 and l_2, m_2, n_2 are proportional to $l_1 + l_2, m_1 + m_2, n_1 + n_2$

Statement 2: The angle between the two intersecting lines having direction cosines as l_1, m_1, n_1 and l_2, m_2, n_2 is given by $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

246

Statement 1: $|\vec{\mathbf{a}}| = |\vec{\mathbf{b}}|$ does not implies that $\vec{\mathbf{a}} = \vec{\mathbf{b}}$.

Statement 2: If $\vec{\mathbf{a}} = \vec{\mathbf{b}}$ then $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = |\vec{\mathbf{a}}|^2 = |\vec{\mathbf{b}}|^2$.

247 Let $\vec{r}$ be a non-zero vector satisfying $\vec{r} \cdot \vec{a} = \vec{r} \cdot \vec{b} = \vec{r} \cdot \vec{c} = 0$ for given non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$

Statement 1: $[\vec{a} - \vec{b}\vec{b} - \vec{c}\vec{c} - \vec{a}] = 0$

Statement 2: $[\vec{a}\vec{b}\vec{c}] = 0$

248

Statement 1: If $a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}, b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ and $c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$ are three mutually perpendicular unit vectors, then $a_1\hat{\imath} + b_1\hat{\jmath} + c_1\hat{k}, a_2\hat{\imath} + b_2\hat{\jmath} + c_2\hat{k}$ and $a_3\hat{\imath} + b_3\hat{\jmath} + c_3\hat{k}$ may be mutually perpendicular unit vectors

Statement 2: Value of determinant and its transpose are the same

249

Statement 1: If $\vec{\mathbf{r}} \cdot \vec{\mathbf{a}} = 0, \vec{\mathbf{r}} \cdot \vec{\mathbf{b}} = 0, \vec{\mathbf{r}} \cdot \vec{\mathbf{c}} = 0$ for some non-zero vector $\vec{\mathbf{r}}$, then $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are coplanar vectors, then $\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}} = 0$.

Statement 2: If $|\ \vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}\ | = 0$, then $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are coplanar.

250

Statement 1: Let $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ be the position vectors of four points A, B, C and D and $3\vec{a} - 2\vec{b} + 5\vec{c} - 6\vec{d} = \vec{0}$. Then points A, B, C and D are coplanar

Statement 2: Three non-zero, linearly dependent coinitial vectors $(\overrightarrow{PQ}, \overrightarrow{PR}$ and $\overrightarrow{PS})$ are coplanar then $\overrightarrow{PQ} = \lambda\overrightarrow{PR} + \mu\overrightarrow{PS}$,where λ and μ are scalars

251

Statement 1: Vector $\vec{c} = -5\hat{\imath} + 7\hat{\jmath} + 2\hat{k}$ is along the bisector of angle between $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{b} = -8\hat{\imath} + \hat{\jmath} - 4\hat{k}$

Statement 2: $\vec{c}$ is equally inclined to $\vec{a}$ and $\vec{b}$

252

Statement 1: In $\Delta ABC, \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$

Statement 2: If $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = \vec{b}$, then $\overrightarrow{AB} = \vec{a} + \vec{b}$

253

Statement 1: The identity
$|\vec{\mathbf{a}} \times \hat{\mathbf{i}}|^2 = |\vec{\mathbf{a}} \times \hat{\mathbf{j}}|^2 + |\vec{\mathbf{a}} \times \hat{\mathbf{k}}|^2 = 2|\vec{\mathbf{a}}|^2$ Holds for $\vec{\mathbf{a}}$.

Statement 2: $\vec{\mathbf{a}} \times \hat{\mathbf{i}} = a_3\, \hat{\mathbf{j}} - a_2 \hat{\mathbf{k}}$;
$\vec{\mathbf{a}} \times \hat{\mathbf{j}} = a_1 \hat{\mathbf{k}} - a_3 \hat{\mathbf{i}}, \vec{\mathbf{a}} \times \hat{\mathbf{k}} = a_2 \hat{\mathbf{i}} - a_2 \hat{\mathbf{j}}$
Which of the following is correct?

254

Statement 1: If $|\vec{\mathbf{a}}| = 2, |\vec{\mathbf{b}}| = 3, |2\vec{\mathbf{a}} - \vec{\mathbf{b}}| = 5$, then $|2\vec{\mathbf{a}} + \vec{\mathbf{b}}| = 5$

Statement 2: $|\vec{\mathbf{p}} - \vec{\mathbf{q}}| = |\vec{\mathbf{p}} + \vec{\mathbf{q}}|$

255

Statement 1: If $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are reciprocal vectors, then $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 1$.

Statement 2: If $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are reciprocal, then $\vec{\mathbf{a}} = \lambda\, \vec{\mathbf{b}}, \lambda \in \boldsymbol{R}^+$, and $|\vec{\mathbf{a}}||\vec{\mathbf{b}}| = 1$.

256

Statement 1: A component of vector $\vec{b} = 4\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ in the direction perpendicular to the direction of vector $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ is $\hat{\imath} - \hat{\jmath}$

Statement 2: A component of vector in the direction of $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ is $2\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$

Matrix-Match Type

This section contain(s) 0 question(s). Each question contains Statements given in 2 columns which have to be matched. Statements (A, B, C, D) in **columns I** have to be matched with Statements (p, q, r, s) in **columns II**.

257.

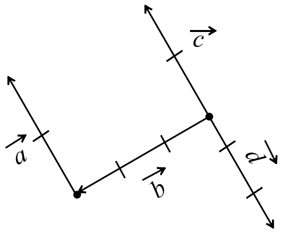
	Column-I		Column- II
(A)	If $\|\vec{a} + \vec{b}\| = \|\vec{a} + 2\vec{b}\|$, then angle between $\vec{a}$ and $\vec{b}$ is	(p)	90°
(B)	If $\|\vec{a} + \vec{b}\| = \|\vec{a} - 2\vec{b}\|$, then angle between $\vec{a}$ and $\vec{b}$ is	(q)	Obtuse
(C)	If $\|\vec{a} + \vec{b}\| = \|\vec{a} - \vec{b}\|$, then angle between $\vec{a}$ and $\vec{b}$ is	(r)	0°
(D)	Angle between $\vec{a} \times \vec{b}$ and a vector perpendicular to the vector $\vec{c} \times (\vec{a} \times \vec{b})$ is	(s)	acute

CODES :

	A	**B**	**C**	**D**
a)	s	r	q	p
b)	q	s	p	r

c)	s	p	r	q
d)	r	q	s	p

258. Refer to the following diagram:



	Column-I		**Column- II**
(A)	Collinear vectors	(p)	$\vec{a}$
(B)	Coinitial vectors	(q)	$\vec{b}$
(C)	Equals vectors	(r)	$\vec{c}$
(D)	Unlike vectors (same initial point)	(s)	$\vec{d}$

CODES :

	A	**B**	**C**	**D**
a)	P,r,s	q,r,s	p,r	r,s
b)	q,r	t,s	t,r,s	q,p
c)	s,t	r	p	s,t
d)	q,r	t	a,s	t,r

259. $\vec{a}$ and $\vec{b}$ form the consecutive sides of a regular hexagon $ABCDEF$

	Column-I		**Column- II**
(A)	If $\overrightarrow{CD} = x\vec{a} + y\vec{b}$, then	(p)	$x = -2$
(B)	If $\overrightarrow{CE} = x\vec{a} + y\vec{b}$, then	(q)	$x = -10$
(C)	If $\overrightarrow{AE} = x\vec{a} + y\vec{b}$, then	(r)	$y = 1$
(D)	$\overrightarrow{AD} = -x\vec{b}$, then	(s)	$y = 2$

CODES :

	A	**B**	**C**	**D**
a)	S,t	r	p	s,t
b)	q,r	p,r	q,s	p
c)	p,r,s	q,r,s	p,r,d	r,s
d)	q,r	t,s	t,r,s	q,p

260. Given two vectors $\vec{a} = -\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{b} = -2\hat{\imath} + \hat{\jmath} + 2\hat{k}$

Column-I		Column- II	
(A)	A vectors coplanar with $\vec{a}$and $\vec{b}$	(p)	$-3\hat{\imath}+3\hat{\jmath}+4\hat{k}$
(B)	A vector which is perpendicular to both $\vec{a}$and $\vec{b}$	(q)	$2\hat{\imath}-2\hat{\jmath}+3\hat{k}$
(C)	A vector which is equally inclined to $\vec{a}$and $\vec{b}$	(r)	$\hat{\imath}+\hat{\jmath}$
(D)	A vector which forms a triangle whit $\vec{a}$and $\vec{b}$	(s)	$\hat{\imath}-\hat{\jmath}+5\hat{k}$

CODES :

	A	B	C	D
a)	P,r	q	p,q,s	p
b)	q	p,q,s	p	p,r
c)	p	p,r	q	p,q,s
d)	p,q,s	p	p,r	p,q

261. If $\vec{\mathbf{a}}=\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$, $\vec{\mathbf{b}}=\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$, $\vec{\mathbf{c}}=\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$ and $\vec{\mathbf{d}}=\hat{\mathbf{i}}-\hat{\mathbf{j}}-\hat{\mathbf{k}}$, then observes the following lists

Column-I		Column- II	
(A)	$\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}$	(p)	$\vec{\mathbf{a}}\cdot\vec{\mathbf{d}}$
(B)	$\vec{\mathbf{b}}\cdot\vec{\mathbf{c}}$	(q)	3
(C)	$[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$	(r)	$\vec{\mathbf{b}}\cdot\vec{\mathbf{d}}$
(D)	$\vec{\mathbf{b}}\times\vec{\mathbf{c}}$	(s)	$2\,\hat{\mathbf{j}}-2\hat{\mathbf{k}}$
		(t)	$2\hat{\mathbf{j}}+2\hat{\mathbf{k}}$
		(u)	4

CODES :

	A	B	C	D
a)	c	a	b	f
b)	c	a	f	e
c)	a	c	b	f
d)	a	c	f	d

262. Volume of parallelepiped formed by vectors $\vec{a}\times\vec{b}, \vec{b}\times\vec{c}$and $\vec{c}\times\vec{a}$ is 36 sq. units

Column-I		Column- II	
(A)	Volume of parallelepiped formed by vectors $\vec{a},\vec{b}$and $\vec{c}$ is	(p)	0 sq. units
(B)	Volume of tetrahedron formed by vectors $\vec{a},\vec{b}$and $\vec{c}$ is	(q)	12 sq. units

(C)	Volume of parallelepiped formed by vectors $\vec{a}+\vec{b}, \vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ is	(r)	6sq. units
(D)	Volume of parallelepiped formed by vectors $\vec{a}-\vec{b}, \vec{b}-\vec{c}$ and $\vec{c}-\vec{a}$ is	(s)	1 sq. units

CODES :

	A	**B**	**C**	**D**
a)	r	s	q	p
b)	s	r	p	q
c)	p	q	r	s
d)	q	p	s	r

263. Given two vectors $\vec{a} = -\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\vec{b} = -\hat{\imath} - 2\hat{\jmath} - \hat{k}$

	Column-I		**Column- II**
(A)	Area of triangle formed by $\vec{a}$ and $\vec{b}$	(p)	3
(B)	Area of parallelogram having sides $\vec{a}$ and $\vec{b}$	(q)	$12\sqrt{3}$
(C)	Area of parallelogram having diagonals $2\vec{a}$ and $4\vec{b}$	(r)	$3\sqrt{3}$
(D)	Volume of parallelepiped formed by $\vec{a}, \vec{b}$ and $\vec{c} = \hat{\imath} + \hat{\jmath} + \hat{k}$	(s)	$\frac{3\sqrt{3}}{2}$

CODES :

	A	**B**	**C**	**D**
a)	p	q	r	s
b)	q	s	p	r
c)	r	p	s	q
d)	s	r	q	p

264.

	Column-I		**Column- II**
(A)	If $\|\vec{a}\| = \|\vec{b}\| = \|\vec{c}\|$, angle between each pair of vectors is $\frac{\pi}{3}$ and $\|\vec{a}+\vec{b}+\vec{c}\| = \sqrt{6}$, then $2\|\vec{a}\|$ is equal to	(p)	3
(B)	If $\vec{a}$ is perpendicular to $\vec{b}+\vec{c}, \vec{b}$ is perpendicular to $\vec{c}+\vec{a}, \vec{c}$ is perpendicular $\vec{a}+\vec{b}, \|\vec{a}\| = 2, \|\vec{b}\| = 3$ and $\|\vec{c}\| = 6$, then $\|\vec{a}+\vec{b}+\vec{c}\| - 2$ is equal to	(q)	2
(C)	$\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}, \vec{b} = -\hat{\imath} + 2\hat{\jmath} - 4\hat{k}, \vec{c} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{d} = 3\hat{\imath} + 2\hat{\jmath} + \hat{k}$, then $\frac{1}{7}(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d})$ is equal to	(r)	4

(D)	If $\|\vec{a}\| = \|\vec{b}\| = \|\vec{c}\| = 2$and $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 2$ then $[\vec{a}\vec{b}\vec{c}]\cos 45°$ is equal to	(s)	5

CODES :

	A	B	C	D
a)	p	q	r	s
b)	s	r	q	p
c)	q	s	p	r
d)	r	p	s	q

265.

	Column-I		Column- II
(A)	If $\vec{a}, \vec{b}$and $\vec{c}$ are three mutually perpendicular vectors where $\|\vec{a}\| = \|\vec{b}\| = 2, \|\vec{c}\| = 1$,then $[\vec{a}\times\vec{b}\vec{b}\times\vec{c}\vec{c}\times\vec{a}]$ is	(p)	-12
(B)	If $\vec{a}$and $\vec{b}$ are two unit vectors inclined at $\pi/3$, then $16[\vec{a}\vec{b}+\vec{a}\times\vec{b}\vec{b}]$ is	(q)	0
(C)	If $\vec{b}$and$\vec{c}$ are orthogonal unit vectors and $\vec{b}\times\vec{c} = \vec{a}$, then $[\vec{a}+\vec{b}+\vec{c}\vec{a}+\vec{b}\vec{b}+\vec{c}]$ is	(r)	16
(D)	If $[\vec{x}\vec{y}\vec{a}]$=$[\vec{x}\vec{y}\vec{b}]$=$[\vec{a}\vec{b}\vec{c}]$=0, each vector being a non-zero vector, then $[\vec{x}\vec{y}\vec{c}]$ is	(s)	1

CODES :

	A	B	C	D
a)	p	q	r	s
b)	r	p	s	q
c)	q	s	p	r
d)	s	r	q	p

266.

	Column-I		Column- II
(A)	The possible value of a if $\vec{r} = (\hat{\imath}+\hat{\jmath}) + \lambda(\hat{\imath}+2\hat{\jmath}-\hat{k})$ and $\vec{r} = (\hat{\imath}+2\hat{\jmath}) + \mu(-\hat{\imath}+\hat{\jmath}+a\hat{k})$ are not consistent, where $\lambda$and μ are scalars, is	(p)	-4
(B)	The angle between vectors $\vec{a} = \lambda\hat{\imath} - 3\hat{\jmath} - \hat{k}$ and $\vec{b} = 2\lambda\hat{\imath} + \lambda\hat{\jmath} - \hat{k}$ is acute, whereas vector $\vec{b}$ makes an obtuse angle with the axes of coordinates. Then λ may be	(q)	-2
(C)	The possible value of a such that $2\hat{\imath} - \hat{\jmath} + \hat{k}, \hat{\imath} + 2\hat{\jmath} + (1+a)\hat{k}$and $3\hat{\imath} + a\hat{\jmath} + 5\hat{k}$ are coplanar is	(r)	2

(D) If $\vec{A} = 2\hat{\imath} + \lambda\hat{\jmath} + 3\hat{k}, \vec{B} = 2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}, \vec{C} = 3\hat{\imath} +$ $\hat{\jmath}$and $\vec{A} + \lambda\vec{B}$ is perpendicular to $\vec{C}$, then $|2\lambda|$is (s) 3

CODES :

	A	B	C	D
a)	P,q,r,s	p,q	p,r	r
b)	p,q	p,r	r	p,q,r,s
c)	p,r	p,q,r,s	p,q	r
d)	r	p,r	p,q,r,s	p,q

Linked Comprehension Type

This section contain(s) 26 paragraph(s) and based upon each paragraph, multiple choice questions have to be answered. Each question has atleast 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 267 to -267

The vertices of a triangle ABC are $A \equiv (2,0,2), B \equiv (-1,1,1)$ and $C \equiv (1,-2,4)$. The points D and E divide the sides AB and CA in the ratio 1 : 2 respectively. Another point F is taken in space such that perpendicular drawn from F on ΔABC, meets the triangle at the point of intersection of the line segment CD and BE, say P. If the distance from the plane of the ΔABC is $\sqrt{2}$ unit, then

On the basis of above information, answer the following questions :

267. The position vector of P is

a) $\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$
b) $\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$
c) $2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$
d) $\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

Paragraph for Question Nos. 268 to - 268

Let $\boldsymbol{A}$ be the given point whose position vector relative to an origin O be $\vec{\mathbf{a}}$ and $\overrightarrow{\mathbf{ON}} = \vec{\mathbf{n}}$. Let $\vec{\mathbf{r}}$ be the position vector of any point P which lies on the plane and passing through A and perpendicular to ON. Then for any point P on the plane

$\overrightarrow{\mathbf{AP}} \cdot \vec{\mathbf{n}} = 0$

$\Rightarrow (\vec{\mathbf{r}} \cdot \vec{\mathbf{a}}) \cdot \vec{\mathbf{n}} = 0 \Rightarrow \vec{\mathbf{r}} \cdot \vec{\mathbf{n}} = \vec{\mathbf{a}} \cdot \vec{\mathbf{n}}$

$\Rightarrow \vec{\mathbf{r}} \cdot \vec{\mathbf{n}} = P$

Where P is perpendicular distance of the plane from origin.

On the basis of above information, answer the following questions :

268. The equation of the plane through the point $2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ and parallel to the plane $\vec{\mathbf{r}} \cdot (4\hat{\mathbf{i}} - 12\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) - 7 = 0$ is

a) $\vec{\mathbf{r}} \cdot (4\hat{\mathbf{i}} - 12\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) = 0$
b) $\vec{\mathbf{r}} \cdot (4\hat{\mathbf{i}} - 12\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) = 16$
c) $\vec{\mathbf{r}} \cdot (4\hat{\mathbf{i}} - 12\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) = 24$
d) $\vec{\mathbf{r}} \cdot (4\hat{\mathbf{i}} - 12\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) = 32$

Paragraph for Question Nos. 269 to - 269

Let $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ be three vectors such that $|\vec{\mathbf{a}}| = |\vec{\mathbf{b}}| = |\vec{\mathbf{c}}| = 4$ and angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is $\pi/3$, angle between $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ is $\pi/3$ and angle between $\vec{\mathbf{c}}$ and $\vec{\mathbf{a}}$ is $\pi/3$.

On the basis of above information, answer the following questions :

269. The volume of the parallelopiped whose adjacent edges are represented by the vectors $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$, is

a) $24\sqrt{2}$ b) $24\sqrt{3}$ c) $32\sqrt{2}$ d) $32\sqrt{3}$

Paragraph for Question Nos. 270 to - 270

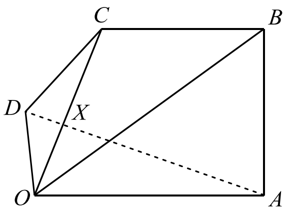
$ABCD$ is a parallelogram. L is a point on BC which divides BC in the ratio 1:2 AL intresects BD at P. M is a point on DC which divides DC in the ratio 1:2 And AM intresects BD in Q

270. Point P divides AL in the ratio

a) 1:2 b) 1:3 c) 3:1 d) 2:1

Paragraph for Question Nos. 271 to - 271

Let $OABCD$ be a pentagon in which the sides OA and CB are parallel and the sides OD and AB are parallel. Also $OA: CB = 2: 1$ and $OD: AB = 1: 3$



271. The ratio $\frac{OX}{XC}$ is

a) 3/4 b) 1/3 c) 2/5 d) 1/2

Paragraph for Question Nos. 272 to - 272

Consider the regular hexagon $ABCDEF$ with centre at O (origin)

272. $\overrightarrow{AD} + \overrightarrow{EB} + \overrightarrow{FC}$ is equal to

a) $2\overrightarrow{AB}$ b) $3\overrightarrow{AB}$ c) $4\overrightarrow{AB}$ d) None of these

Paragraph for Question Nos. 273 to - 273

Let $\vec{u}, \vec{v}$and $\vec{w}$ be three unit vectors such that $\vec{u} + \vec{v} + \vec{w} = \vec{a}, \vec{u} \times (\vec{v} \times \vec{w}) = \vec{b}, (\vec{u} \times \vec{v}) \times \vec{w} = \vec{c}, \vec{a} \cdot \vec{u} = \frac{3}{2}, \vec{a} \cdot \vec{v} = 7/4$ and $|\vec{a}| = 2$

273. Vector $\vec{u}$ is

a) $\vec{a} - \frac{2}{3}\vec{b} + \vec{c}$ b) $\vec{a} + \frac{4}{3}\vec{b} + \frac{8}{3}\vec{c}$ c) $2\vec{a} - \vec{b} + \frac{1}{3}\vec{c}$ d) $\frac{4}{3}\vec{a} - \vec{b} + \frac{2}{3}\vec{c}$

Paragraph for Question Nos. 274 to - 274

Vectors $\vec{x}, \vec{y}$ and $\vec{z}$ each of magnitude $\sqrt{2}$, make an angle of $60°$ with each other.
$\vec{x} \times (\vec{y} \times \vec{z}) = \vec{a}, \vec{y} \times (\vec{z} \times \vec{x}) = \vec{b}$ and $\vec{x} \times \vec{y} = \vec{c}$

274. Vector $\vec{x}$ is

a) $\frac{1}{2}[(\vec{a} - \vec{b}) \times \vec{c} + (\vec{a} + \vec{b})]$
b) $\frac{1}{2}[(\vec{a} + \vec{b}) \times \vec{c} + (\vec{a} - \vec{b})]$
c) $\frac{1}{2}[-(\vec{a} + \vec{b}) \times \vec{c} + (\vec{a} + \vec{b})]$
d) $\frac{1}{2}[(\vec{a} + \vec{b}) \times \vec{c} - (\vec{a} + \vec{b})]$

Paragraph for Question Nos. 275 to - 275

If $\vec{x} \times \vec{y} = \vec{a}, \vec{y} \times \vec{z} = \vec{b}, \vec{x} \cdot \vec{b} = \gamma, \vec{x} \cdot \vec{y} = 1$ and $\vec{y} \cdot \vec{z} = 1$

275. Vectors $\vec{x}$ is

a) $\frac{1}{|\vec{a} \times \vec{b}|^2}[\vec{a} \times (\vec{a} \times \vec{b})]$
b) $\frac{\gamma}{|\vec{a} \times \vec{b}|^2}[\vec{a} \times \vec{b} - \vec{a} \times (\vec{a} \times \vec{b})]$
c) $\frac{\gamma}{|\vec{a} \times \vec{b}|^2}[\vec{a} \times \vec{b} + \vec{b} \times (\vec{a} \times \vec{b})]$
d) None of these

Paragraph for Question Nos. 276 to - 276

Given two orthogonal vectors $\vec{A}$ and $\vec{B}$ each of length unity. Let $\vec{P}$ be the vector satisfying the equation $\vec{P} \times \vec{B} = \vec{A} - \vec{P}$. Then

276. $(\vec{P} \times \vec{B}) \times \vec{B}$ is equal to

a) $\vec{P}$
b) $-\vec{P}$
c) $2\vec{B}$
d) $\vec{A}$

Paragraph for Question Nos. 277 to - 277

Let $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}, \vec{b} = 2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$ and $\vec{c} = -2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$. Let $\vec{a}_1$ be the projection of $\vec{a}$ on $\vec{b}$ and $\vec{a}_2$ be the projection of $\vec{a}_1$ on $\vec{c}$. Then

277. $\vec{a}_2$ is equal to

a) $\frac{943}{49}(2\hat{\imath} - 3\hat{\jmath} - 6\hat{k})$
b) $\frac{943}{49^2}(2\hat{\imath} - 3\hat{\jmath} - 6\hat{k})$
c) $\frac{943}{49}(-2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$
d) $\frac{943}{49^2}(-2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$

Paragraph for Question Nos. 278 to - 278

Consider a triangular pyramid $ABCD$ the position vectors of whose angular point are $A(3,0,1), B(-1,4,1), C(5,2,3)$ and $D(0,-5,4)$. Let G be the point of intersection of the medians of triangle BCD

278. The length of vector $\overrightarrow{AG}$ is

a) $\sqrt{17}$
b) $\sqrt{51}/3$
c) $3/\sqrt{6}$
d) $\sqrt{59}/4$

Paragraph for Question Nos. 279 to - 279

Vertices of a parallelogram taken in order are $A(2,-1,4)$; $B(1,0-1)$; $C(1,2,3)$ and D

279. The distance between the parallel lines AB and CD is

a) $\sqrt{6}$ b) $3\sqrt{6/5}$ c) $2\sqrt{2}$ d) 3

Paragraph for Question Nos. 280 to - 280

Let $\vec{r}$ be a position vector of a variable point in Cartesion OXY plane such that $\vec{r}.(10\hat{\jmath}-8\hat{\imath}-\vec{r})=40$ and $p_1=\max\{|\vec{r}+2\hat{\imath}-3\hat{\jmath}|^2\}, p_2=\min\{|\vec{r}+2\hat{\imath}-3\hat{\jmath}|^2\}$. A tangent line is drawn to the curve $y=8/x^2$ at point A with abscissa 2. The drawn line cuts the x-axis at a point B

280. p_2 is equal to

a) 9 b) $2\sqrt{2}-1$ c) $6\sqrt{2}+3$ d) $9-4\sqrt{2}$

Paragraph for Question Nos. 281 to - 281

AB, AC and AD are three adjacent edges of a parallelepiped. The diagonal of the parallelepiped passing through A and directed away from it is vector $\vec{a}$. The vector area of the faces containing vertices A, B, C and A, B, D are $\vec{b}$ and $\vec{c}$, respectively, i.e., $\overrightarrow{AB}\times\overrightarrow{AC}=\vec{b}$ and $\overrightarrow{AD}\times\overrightarrow{AB}=\vec{c}$. The projection of each edge AB and AC on diagonal vector $\vec{a}$ is $\frac{|\vec{a}|}{3}$

281. Vector $\overrightarrow{AB}$ is

a) $\frac{1}{3}\vec{a}+\frac{\vec{a}\times(\vec{b}-\vec{c})}{|\vec{a}|^2}$

b) $\frac{1}{3}\vec{a}+\frac{\vec{a}\times(\vec{b}-\vec{c})}{|\vec{a}|^2}+\frac{3(\vec{b}\times\vec{a})}{|\vec{a}|^2}$

c) $\frac{1}{3}\vec{a}+\frac{\vec{a}\times(\vec{b}-\vec{c})}{|\vec{a}|^2}-\frac{3(\vec{c}\times\vec{a})}{|\vec{a}|^2}$

d) None of these

Integer Answer Type

282. Given that $\vec{u}=\hat{\imath}-2\hat{\jmath}+3\hat{k}$; $\vec{v}=2\hat{\imath}+\hat{\jmath}+4\hat{k}$; $\vec{w}=\hat{\imath}+3\hat{\jmath}+3\hat{k}$ and $(\vec{u}\cdot\vec{R}-15)\hat{\imath}+(\vec{v}\cdot\vec{R}-30)\hat{\jmath}+(\vec{w}\cdot\vec{R}-20)\hat{k}=\vec{0}$. Then find the greatest integer less than or equal to $|\vec{R}|$

283. Find the least positive integral value of x for the angle between vectors $\vec{a}=x\hat{\imath}-3\hat{\jmath}-\hat{k}$ and $\vec{b}=2x\hat{\imath}+x\hat{\jmath}-\hat{k}$ is acute

284. If $\vec{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$; $\vec{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$, $\vec{c}=c_1\hat{\imath}+c_2\hat{\imath}+c_3\hat{k}$ and $[3\vec{a}+\vec{b}3\vec{b}+\vec{c}3\vec{c}+\vec{a}]=\lambda\begin{vmatrix}\vec{a}.\hat{\imath} & \vec{a}.\hat{\jmath} & \vec{a}.\hat{k}\\ \vec{b}.\hat{\imath} & \vec{b}.\hat{\jmath} & \vec{b}.\hat{k}\\ \vec{c}.\hat{\imath} & \vec{c}.\hat{\jmath} & \vec{c}.\hat{k}\end{vmatrix}$, then find the value of $\frac{\lambda}{4}$

285. Let $\vec{u}$ and $\vec{v}$ are unit vectors such that $\vec{u}\times\vec{v}+\vec{u}=\vec{w}$ and $\vec{w}\times\vec{u}=\vec{v}$. Find the value of $[\vec{u}\vec{v}\vec{w}]$

286. If vectors $\vec{a}=\hat{\imath}+2\hat{\jmath}-\hat{k}$, $\vec{b}=2\hat{\imath}-\hat{\jmath}+\hat{k}$ and $\vec{c}=\lambda\hat{\imath}+\hat{\jmath}+2\hat{k}$ are coplanar, then find the value of $(\lambda-4)$

287. Find the value of λ if the volumes of a tetrahedron whose vertices are with position vectors $\hat{\imath}-6\hat{\jmath}+10\hat{k}$, $-\hat{\imath}-3\hat{\jmath}+7\hat{k}$, $5\hat{\imath}-\hat{\jmath}+\lambda\hat{k}$ and $7\hat{\imath}-4\hat{\jmath}+7\hat{k}$ is 11 cubic unit

288. Let $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} - \vec{c} = 0$. If the area of triangle formed by vectors $\vec{a}$ and $\vec{b}$ is A, then what is the value of $4A^2$?

289. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} \cdot \vec{b} = 0 = \vec{a} \cdot \vec{c}$ and the between $\vec{b}$and $\vec{c}$ is $\frac{\pi}{3}$, then find the value of $|\vec{a} \times \vec{b} - \vec{a} \times \vec{c}|$

290. Let $\overline{OA} = \vec{a}, \overline{OB} = 10\vec{a} + 2\vec{b}$and $\vec{O}C = \vec{b}$, where O, A and C are non-collinear points. Let p denote the area of quadrilateral $OACB$, and let q denote the area of parallelogram with OA and OC as adjacent sides. If $p = k\, q$, then find k

291. Let $\vec{u}$ be a vector on rectangular coordinate system with sloping angle 60°. Suppose that $|\vec{u} - \vec{\imath}|$is geometric mean of $|\vec{u}|$and $|\vec{u} - 2\hat{\imath}|$, where $\hat{\imath}$ is the unit vector along x-axis. Then find value of $(\sqrt{2} + 1)|\vec{u}|$

292. If the resultant of three forces $\vec{F}_1 = p\hat{\imath} + 3\hat{\jmath} - \hat{k}, \vec{F}_2 = -5\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\vec{F}_3 = 6\hat{\imath} - \hat{k}$ acting on a particle has a magnitude equal to 5 units, then what is difference in the values of p?

293. Let ABC be a triangle whose centroid is G. Orthocentre is H and circumcentre is the origin $'O'$.If D is any point in the plane of the triangle such that no three of O, A, C and D are collinear satisfying the relation If $\vec{A}D + \vec{B}D + \vec{C}H + 3\vec{H}G = \lambda\vec{H}D$, then what is the value of the scalar $'\lambda'$?

294. Let a three-dimensional vector $\vec{V}$ satisfies the condition, $2\vec{V} + \vec{V} \times (\hat{\imath} + 2\hat{\jmath}) = 2\hat{\imath} + \hat{k}$.If $3|\vec{V}| = \sqrt{m}$, then find the value of m

295. If $\vec{a}$and $\vec{b}$ are any two unit vectors, then find the greatest positive integer in the range of $\frac{3|\vec{a}+\vec{b}|}{2} + 2|\vec{a} - \vec{b}|$

296. Find the absolute value of parameter t for which the area of the triangle whose vertices are $A(-1,1,2); B(1,2,3)$ and$C(t, 1,1)$ is minimum

297. Let $\vec{a} = \alpha\hat{\imath} + 2\hat{\jmath} - 3\hat{k}, \vec{b} = \hat{\imath} + 2\alpha\hat{\jmath} - 2\hat{k}$ and $\vec{c} = 2\hat{\imath} - \alpha\hat{\jmath} + \hat{k}$. Find the value of 6α, such that $\{(\vec{a} \times \vec{b}) \times$ $b\times c\times c\times a=0$

298. If $\vec{x}, \vec{y}$ are two non-zero and non-collinear vectors satisfying $[(a-2)\alpha^2 + (b-3)\alpha + c]\vec{x} + [(a-2)\beta^2 +$ $b-3\beta+cy+a-2\gamma 2+b-3\gamma+cx\times y=0$,where $\alpha,\beta,\gamma$are three distinct real numbers, then find the value of $(a^2 + b^2 + c^2 - 4)$

299. Vectors along the adjacent sides of parallelogram are $\vec{a} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{b} = 2\hat{\imath} + 4\hat{\jmath} + \hat{k}$. Find the length of the longer diagonal of the parallelogram

300. Find the work done by the force $F = 3\hat{\imath} - \hat{\jmath} - 2\hat{k}$ acting on a particle such that the particle is displaced from point $A(-3, -4, 1)$ to point $B(-1, -1, -2)$

: ANSWER KEY :

1)	a	2)	a	3)	a	4)	a
5)	d	6)	d	7)	b	8)	a
9)	a	10)	a	11)	b	12)	d
13)	a	14)	a	15)	c	16)	a
17)	a	18)	c	19)	b	20)	d
21)	c	22)	b	23)	a	24)	d
25)	d	26)	b	27)	c	28)	c
29)	c	30)	b	31)	a	32)	a
33)	d	34)	c	35)	c	36)	c
37)	d	38)	c	39)	d	40)	b
41)	b	42)	c	43)	b	44)	d
45)	c	46)	d	47)	b	48)	b
49)	d	50)	d	51)	b	52)	c
53)	b	54)	a	55)	a	56)	c
57)	a	58)	d	59)	b	60)	a
61)	c	62)	a	63)	d	64)	c
65)	c	66)	d	67)	c	68)	c
69)	b	70)	c	71)	b	72)	d
73)	b	74)	a	75)	c	76)	c
77)	c	78)	a	79)	a	80)	c
81)	d	82)	b	83)	a	84)	c
85)	c	86)	d	87)	d	88)	d
89)	b	90)	a	91)	b	92)	b
93)	c	94)	a	95)	b	96)	c
97)	a	98)	c	99)	d	100)	c
101)	b	102)	c	103)	d	104)	a
105)	d	106)	b	107)	c	108)	b
109)	c	110)	c	111)	a	112)	a
113)	a	114)	c	115)	a	116)	a
117)	c	118)	a	119)	d	120)	c
121)	a	122)	b	123)	a	124)	b
125)	c	126)	b	127)	d	128)	c
129)	b	130)	a	131)	c	132)	c
133)	b	134)	a	135)	c	136)	c
137)	c	138)	a	139)	a	140)	a
141)	c	142)	c	143)	b	144)	c
145)	b	146)	d	147)	a	148)	b
149)	b	150)	d	151)	d	152)	b
153)	d	154)	b	155)	a	156)	b
157)	c	158)	a	159)	a	160)	b
161)	a	1)	b,d	2)	b,c,d	3)	
	a,b,c	4)	a,b				
5)	a,b	6)	a,c	7)	a,c	8)	
	a,c						
9)	b,d	10)	b	11)	b,c	12)	
	b,d						
13)	a,b,d	14)	a,b	15)	a,b,c,d	16)	

	a,c						
17)	a,b,c,d	18)	a,b,c	19)	b,c	20)	
	b,d						
21)	a,b,c,d	22)	a,d	23)	b,d	24)	
	a,c,d						
25)	b,d	26)	a,c	27)	a,b,d	28)	
	a,b,c						
29)	a,d	30)	a,c,d	31)	a,b,c,d	32)	
	a,c						
33)	a,c	34)	a,c	35)	c,d	36)	
	c,d						
37)	b,c	38)	b,d	39)	c	40)	
	a,c						
41)	a,c	42)	a,b,d	43)	a,d	44)	
	a,b,c						
45)	b,c,d	46)	b,d	47)	a,b	48)	
	b,c						
49)	a,b,c	50)	b,c	51)	a,b,c,d	52)	
	b,d						
53)	b,c	54)	a,b,c	55)	a,b,d	56)	
	a,b,c						
57)	a,b,c,d	58)	a,c,d	59)	c	60)	
	a,c						
61)	b,c	62)	a,b,c,d	63)	a,d	64)	
	a,d						
65)	a,c	1)	a	2)	c	3)	c
	4)	a					
5)	d	6)	d	7)	a	8)	a
9)	a	10)	c	11)	d	12)	b
13)	d	14)	a	15)	b	16)	b
17)	c	18)	a	19)	b	20)	b
21)	b	22)	a	23)	b	24)	a
25)	b	26)	c	27)	a	28)	c
29)	a	30)	c	1)	b	2)	a
	3)	b	4)	a			
5)	b	6)	a	7)	d	8)	c
9)	b	10)	a	1)	b	2)	d
	3)	c	4)	c			
5)	c	6)	c	7)	b	8)	d
9)	b	10)	b	11)	b	12)	b
13)	c	14)	d	15)	a	1)	6
	2)	2	3)	7	4)	1	
5)	9	6)	7	7)	3	8)	1
9)	6	10)	1	11)	6	12)	2
13)	6	14)	5	15)	2	16)	4
17)	9	18)	7	19)	9		

: HINTS AND SOLUTIONS :

1 **(a)**

Let $\vec{c} = \lambda(\vec{a} \times \vec{b})$

Hence $\lambda(\vec{a} \times \vec{b}) \cdot (\hat{\imath} + 2\hat{\jmath} - 7\hat{k}) = 10$

$\Rightarrow \lambda \begin{vmatrix} 2 & -3 & 1 \\ 1 & -2 & 3 \\ 1 & 2 & -7 \end{vmatrix} = 10$

$\Rightarrow \lambda = -1$

$\Rightarrow \vec{c} = -(\vec{a} \times \vec{b})$

2 **(a)**

$\vec{a}, \vec{b}$ and $\vec{c}$ are unit coplanar vectors, $2\vec{a} - \vec{b}, 2\vec{b} - 2\vec{c}$ and $2\vec{c} - \vec{a}$ are also coplanar vectors, being linear combination of $\vec{a}, \vec{b}$ and $\vec{c}$

Thus, $[2\vec{a} - \vec{b}\ 2\vec{b} - \vec{c}\ 2\vec{c} - \vec{a}] = 0$

3 **(a)**

Points $\vec{a}, \vec{b}, \vec{c}$ and$\vec{d}$ are coplanar. Therefore

$\sin\alpha + 2\sin 2\beta + 3\sin 3\gamma = 1$

Now

$|\sin\alpha + 2\sin 2\beta +$
$3\sin 3\gamma| \le 1+4+9.\sin 2\alpha + \sin 22\beta + \sin 23\gamma$

$\Rightarrow \sin^2\alpha + \sin^2 2\beta + \sin^2 3\gamma \ge \frac{1}{14}$

4 **(a)**

$(\vec{a} \times (\vec{a} \times (\vec{a} \times (\vec{a} \times \overrightarrow{b}))))$

$= (\vec{a} \times (\vec{a} \times ((\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{b})\vec{b})))$

$= (\vec{a} \times (\vec{a} \times (-4\vec{b})))$

$= -4\left(\vec{a} \times (\vec{a} \times \vec{b})\right)$

$= -4\left((\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b}\right)$

$= -4(-4\vec{b}) = 16\vec{b} = 48\,\hat{b}$

5 **(d)**

$(3\vec{a} + \vec{b}) \cdot (\vec{a} - 4\vec{b})$

$= 3|\vec{a}|^2 - 11\vec{a} \cdot \vec{b} - 4|\vec{b}|^2$

$= 3 \times 36 - 11 \times 6 \times 8\cos\pi - 4 \times 64 > 0$

Therefore, the angle between $\vec{a}$and $\vec{b}$ is acute

The longer diagonal is given by

$\vec{\alpha} = (3\vec{a} + \vec{b}) + (\vec{a} - 4\vec{b}) = 4\vec{a} - 3\vec{b}$

Now, $|\vec{\alpha}|^2 = |4\vec{a} - 3\vec{b}|^2 = 16|\vec{a}|^2 + 9|\vec{b}|^2 - 24\vec{a} \cdot \vec{b}$

$= 16 \times 36 + 9 \times 64 - 24 \times 6 \times 8\cos\pi$

$= 16 \times 144$

$\Rightarrow |4\vec{a} - 3\vec{b}| = 48$

6 **(d)**

Given that $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar. Therefore, $[a\ b, c] \neq 0$

Also $\vec{p} = \frac{\vec{b}\times\vec{c}}{[\vec{a}\vec{b}\vec{c}]}, \vec{q} = \frac{\vec{c}\times\vec{a}}{[\vec{a}\vec{b}\vec{c}]}$,

$\vec{r} = \frac{a\times b}{[\vec{a}\vec{b}\vec{c}]}$ (i)

Now, $(\vec{a} + \vec{b}) \cdot \vec{p} + (\vec{b} + \vec{c}) \cdot \vec{q} + (\vec{c} + \vec{a}) \cdot \vec{r}$

$= (\vec{a} + \vec{b}) \cdot \frac{\vec{b} \times \vec{c}}{[\vec{a}\vec{b}\vec{c}]} + (\vec{b} + \vec{c}) \cdot \frac{\vec{c} \times \vec{a}}{[\vec{a}\vec{b}\vec{c}]} + (\vec{c} + \vec{a}) \cdot \frac{\vec{a} \times \vec{b}}{[\vec{a}\vec{b}\vec{c}]}$

$= \frac{\vec{a} \cdot \vec{b} \times \vec{c}}{[\vec{a}\vec{b}\vec{c}]} \times \frac{\vec{b} \cdot \vec{c} \times \vec{a}}{[\vec{a}\vec{b}\vec{c}]} + \frac{\vec{c} \cdot \vec{a} \times \vec{b}}{[\vec{a}\vec{b}\vec{c}]}$

$[\because \vec{b} \cdot \vec{b} \times \vec{c} = \vec{c} \cdot \vec{c} \times \vec{a} = \vec{a} \cdot \vec{a} \times \vec{b} = 0]$

$= \frac{[\vec{a}\vec{b}\vec{c}]}{[\vec{a}\vec{b}\vec{c}]} + \frac{[\vec{a}\vec{b}\vec{c}]}{[\vec{a}\vec{b}\vec{c}]} + \frac{[\vec{a}\vec{b}\vec{c}]}{[\vec{a}\vec{b}\vec{c}]}$

$= 1 + 1 + 1$

$= 3$

7 **(b)**

Given $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ (by triangle law). Therefore,

$\vec{a} \times (\vec{a} + \vec{b} + \vec{c}) = \vec{a} \times \vec{0}$

$\vec{a} \times \vec{a} + \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = \vec{0}$

$\vec{a} \times \vec{b} = \vec{c} \times \vec{a}$

Similarly by taking cross product with $\vec{b}$, we get

$\vec{a} \times \vec{b} = \vec{b} \times \vec{c}$

$\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

8 **(a)**

$(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + \vec{b}) = 0$

$\Rightarrow 6|\vec{a}|^2 - 5|\vec{b}|^2 = 7\vec{a} \cdot \vec{b}$

Also, $(\vec{a} + 4\vec{b}) \cdot (\vec{b} - \vec{a}) = 0$

$\Rightarrow -|\vec{a}|^2 + 4|\vec{b}|^2 = 3\vec{a} \cdot \vec{b}$

$\Rightarrow \frac{6}{7}|\vec{a}|^2 - \frac{5}{7}|\vec{b}|^2 = -\frac{1}{3}|\vec{a}|^2 + \frac{4}{3}|\vec{b}|^2$

$\Rightarrow 25|\vec{a}|^2 = 43|\vec{b}|^2$

$\Rightarrow 3\vec{a}\cdot\vec{b} = -|\vec{a}|^2 + 4|\vec{b}|^2 = \frac{57}{25}|\vec{b}|^2$

$\Rightarrow 3|\vec{a}||\vec{b}|\cos\theta = \frac{57}{25}|\vec{b}|^2$

$\Rightarrow 3\sqrt{\frac{43}{25}}|\vec{b}|^2\cos\theta = \frac{57}{25}|\vec{b}|^2$

$\Rightarrow \cos\theta = \frac{19}{5\sqrt{43}}$

9 **(a)**

$(\vec{a}\times\vec{b})\cdot(\vec{a}\times\vec{c}) = (\vec{a}\times\vec{b})\cdot\vec{u}$, where $\vec{u} = \vec{a}\times\vec{c}$

$\Rightarrow \vec{a}\cdot(\vec{b}\times\vec{u}) = \vec{a}[\vec{b}\times(\vec{a}\times\vec{c})]$

$= \vec{a}\cdot[(\vec{b}\cdot\vec{c})\vec{a} - (\vec{a}\cdot\vec{b})\vec{c}]$

$= \vec{a}\cdot(\vec{b}\cdot\vec{c})\vec{a}$ $(\because \vec{a}\cdot\vec{b} = 0)$

$= |\vec{a}|^2(\vec{b}\cdot\vec{c})$

10 **(a)**

We have $\overrightarrow{GB} + \overrightarrow{GC} = (1+1)\overrightarrow{GD} = 2\,\overrightarrow{GD}$, where D is the midpoint of BC

$\therefore \overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = \overrightarrow{GA} + 2\overrightarrow{GD} = \overrightarrow{GA} - \overrightarrow{GA} = 0$

($\because$ G divides AC in the ratio 2:1 $\therefore 2\overrightarrow{GD} = -\overrightarrow{GA}$)

11 **(b)**

Let $\vec{a} = \lambda\vec{b} + \mu\vec{c}$

$\vec{a}$ is equally inclined to $\vec{b}$ and $\vec{d}$ where $\vec{d} = \hat{j} + 2\hat{k}$

$\Rightarrow \frac{\vec{a}\cdot\vec{b}}{ab} = \frac{\vec{a}\cdot\vec{d}}{ad}$

$\Rightarrow \frac{(\lambda\vec{b}+\mu\vec{c})\cdot\vec{b}}{b} = \frac{(\lambda\vec{b}+\mu\vec{c})\cdot\vec{d}}{d}$

$\Rightarrow \frac{[\lambda(2\hat{i}+\hat{j}) + \mu(\hat{i}-\hat{j}+\hat{k})]\cdot(2\hat{i}+\hat{j})]}{\sqrt{5}}$

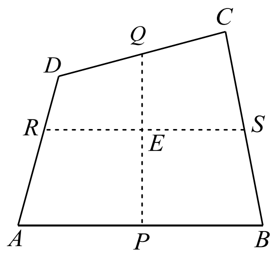
$= \frac{[\lambda(2\hat{i}+\hat{j}) + \mu(\hat{i}-\hat{j}+\hat{k})]\cdot(\hat{j}+2\hat{k})}{\sqrt{5}}$

$\Rightarrow \lambda(4+1) + \mu(2-1) = \lambda(1) + \mu(-1+2)$

$\Rightarrow 4\lambda = 0$, i.e., $\lambda = 0$

$\therefore \hat{a} = \frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$

12 **(d)**



Let $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = \vec{b}, \overrightarrow{OC} = \vec{c}$ and $\overrightarrow{OD} = \vec{d}$, therefore

$\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} = \vec{a} + \vec{b} + \vec{c} + \vec{d}$

P, the midpoint of AB, is $\frac{\vec{a}+\vec{b}}{2}$

Q, the midpoint of $\overrightarrow{CD}$, is $\frac{\vec{c}+\vec{d}}{2}$

Therefore, the midpoint of $\overrightarrow{PQ}$ is $\frac{\vec{a}+\vec{b}+\vec{c}+\vec{d}}{4}$

Similarly the midpoint of RS is $\frac{\vec{a}+\vec{b}+\vec{c}+\vec{d}}{4}$, i.e.,

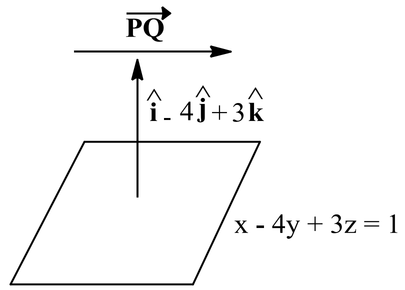
$\overrightarrow{OE} = \frac{\vec{a}+\vec{b}+\vec{c}+\vec{d}}{4} \Rightarrow x = 4$

13 **(a)**

Given,

$\mathbf{\overline{OQ}} = (1-3\mu)\hat{\mathbf{i}} + (\mu-1)\hat{\mathbf{j}} + (5\mu+2)\hat{\mathbf{k}}$

$\mathbf{\overline{OP}} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ (where O is origin)



Now,

$\overrightarrow{\mathbf{PQ}} = (1-3\mu-3)\hat{\mathbf{i}} + (\mu-1-2)\hat{\mathbf{j}}$
$+ (5\mu+2-6)\hat{\mathbf{k}}$

$= (-2-3\mu)\hat{\mathbf{i}} + (\mu-3)\hat{\mathbf{j}} + (5\mu-4)\hat{\mathbf{k}}$

$\because \overrightarrow{\mathbf{PQ}}$ is parallel to the plane $x - 4y + 3z = 1$

$\therefore -2 - 3\mu - 4\mu + 12 + 15\mu - 12 = 0$

$\Rightarrow 8\mu = 2$

$\Rightarrow \mu = \frac{1}{4}$

14 **(a)**

Let P.V. of P, A, B and C be $\vec{p}, \vec{a}, \vec{b}$ and $\vec{c}$, respectively, and $O(\vec{0})$ be the circumcentre of equilateral triangle ABC. Then

$|\vec{p}| = |\vec{b}| = |\vec{a}| = |\vec{c}| = \frac{1}{\sqrt{3}}$

Now $|\overrightarrow{PA}|^2 = |\vec{a}-\vec{p}|^2 = |\vec{a}|^2 + |\vec{p}|^2 - 2\vec{p}\cdot\vec{a}$

Similarly, $|\overrightarrow{PB}|^2 = |\vec{b}|^2 + |\vec{p}|^2 - 2\vec{p}\cdot\vec{b}$

and $|\vec{P}C|^2 = |\vec{c}|^2 + |\vec{p}|^2 - 2\vec{p}\cdot\vec{c}$

$\Rightarrow \Sigma|\overrightarrow{PA}|^2 = 6\cdot\frac{l^2}{3} - 2\vec{p}\cdot(\vec{a}+\vec{b}+\vec{c}) = 2l^2$ as $(\vec{a} +$

$\vec{b} + \vec{c}/3 = \vec{0})$

15 **(c)**

$\vec{a} \times \vec{b}$ is a vector perpendicular to the plane containing $\vec{a}$ and $\vec{b}$. Similarly, $\vec{c} \times \vec{d}$ is a vector perpendicular to the plane containing $\vec{c}$ and $\vec{d}$

Thus, the two planes will be parallel if their normals, i.e., $\vec{a} \times \vec{b}$ and $\vec{c} \times \vec{d}$, are parallel

$\Rightarrow (\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$

16 **(a)**

Let A be the origin. $\overrightarrow{AB} = \vec{a}, \overrightarrow{AD} = \vec{b}$

So, $\overrightarrow{AE} = \vec{b} + \frac{3}{2}\vec{a}, \overrightarrow{AG} = \vec{a} + 3\vec{b}$

So the required ratio $= \dfrac{\frac{1}{2}\left|(\vec{a} + 3\vec{b}) \times \left(\vec{b} + \frac{3}{2}\vec{a}\right)\right|}{\frac{1}{2}|\vec{a} \times \vec{b}|}$

$= \dfrac{7}{2}$

17 **(a)**

As $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$, we take $\vec{c} = \alpha\,\vec{a} + \beta\vec{b}$

Where α and β are scalars

As $\vec{c}$ is perpendicular to $\vec{a}$, using (i), we get,

$0 = \alpha\,\vec{a} \cdot \vec{a} + \beta\,\vec{b} \cdot \vec{a}$

$\Rightarrow 0 = \alpha(6) + \beta(2 + 2 - 1) = 3(2\alpha + \beta)$

$\Rightarrow \beta = -2\alpha$

Thus, $\vec{c} = \alpha(\vec{a} - 2\vec{b}) = \alpha(-3j + 3k) = 3\alpha(-j + k)$

$\Rightarrow |\vec{c}|^2 = 18\alpha^2$

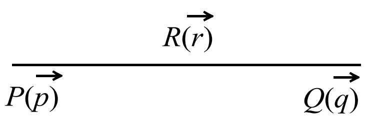
$\Rightarrow 1 = 18\alpha^2$

$\Rightarrow \alpha = \pm\dfrac{1}{3\sqrt{2}}$

$\therefore\ \vec{c} = \pm\dfrac{1}{\sqrt{2}}(-j + k)$

18 **(c)**

$R(\vec{r})$moves on PQ



19 **(b)**

Let $\vec{\alpha} = \frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|} + \frac{\vec{c}}{|\vec{c}|}$

Since $\vec{a}, \vec{b}$ and $\vec{c}$ are mutually perpendicular vectors, if $\vec{\alpha}$makes angles $\theta, \phi\ \Psi$ with $\vec{a}, \vec{b}$ and $\vec{c}$ respectively, then

$\vec{\alpha} \cdot \vec{a} = \dfrac{\vec{a} \cdot \vec{a}}{|\vec{a}|}$

$\Rightarrow |\vec{\alpha}| \cdot |\vec{a}| \cos\theta = |\vec{a}|$

$\Rightarrow \cos\theta = \dfrac{1}{|\vec{\alpha}|}$

Similarly $\cos\phi = \frac{1}{|\vec{\alpha}|}, \cos\Psi = \frac{1}{|\vec{\alpha}|}$

$\therefore\ \theta = \phi = \Psi$

20 **(d)**

$\vec{a} - \vec{b} = 2(\vec{d} - \vec{c})$

$\therefore \dfrac{\vec{a} + 2\vec{c}}{2 + 1} = \dfrac{\vec{b} + 2\vec{d}}{2 + 1}$

$\Rightarrow AC$ and BD trisect each other as L.H.S is the position vector of a point trisecting A an C, anc R.H.S. that of B and D

21 **(c)**

The given relation can be rewritten as the vector expression

$\left(\sqrt{a^2 - 4}\hat{\imath} + a\hat{\jmath} + \sqrt{a^2 + 4}\hat{k}\right)$

$\cdot (\tan A\hat{\imath} + \tan B\hat{\jmath} + \tan C\hat{k}) = 6a$

$\Rightarrow \sqrt{a^2 - 4 + a^2 + a^2 + 4}\sqrt{\tan^2\ A + \tan^2 B + \tan^2}$

$\cdot (\cos\theta) = 6a\ (\because \vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta)$

$\sqrt{3}a\sqrt{\tan^2 A + \tan^2 B + \tan^2 C} \cdot (\cos\theta) = 6a$

$\tan^2 A + \tan^2 B + \tan^2 C$

$= 12\sec^2\theta \geq 12\ (\because \sec^2\theta \geq 1)$

The least value of $\tan^2 A + \tan^2 B + \tan^2 C$ is 12

22 **(b)**

$(\vec{a} \times \vec{b}) \times (\vec{r} \times \vec{c})$

$= \left((\vec{a} \times \vec{b}) \cdot \vec{c}\right)\vec{r} - \left((\vec{a} \times \vec{b}) \cdot \vec{r}\right).\vec{c}$

$= [\vec{a}\vec{b}\vec{c}]\vec{r} - [\vec{a}\vec{b}\vec{r}]\vec{c}$

Similarly, $(\vec{b} \times \vec{c}) \times (\vec{r} \times \vec{a}) = [\vec{b}\vec{c}\vec{a}]\vec{r} - [\vec{b}\vec{c}\vec{r}]\vec{a}$

and, $(\vec{c} \times \vec{a}) \times (\vec{r} \times \vec{b}) = [\vec{c}\vec{a}\vec{b}]\vec{r} - [\vec{c}\vec{a}\vec{r}]\vec{b}$

$\Rightarrow (\vec{a} \times \vec{b}) \times (\vec{r} \times \vec{c}) + (\vec{b} \times \vec{c}) \times (\vec{r} \times \vec{a}) + (\vec{c} \times \vec{a}) \times (\vec{r} \times \vec{b})$

$= 3[\vec{a}\vec{b}\vec{c}]\vec{r} - ([\vec{b}\vec{c}\vec{r}]\vec{a} + [\vec{c}\vec{a}\vec{r}]\vec{b} + [\vec{a}\vec{b}\vec{r}]\vec{c})$

$= 3[\vec{a}\vec{b}\vec{c}]\vec{r} - [\vec{a}\vec{b}\vec{c}]\vec{r} = 2[\vec{a}\vec{b}\vec{c}]\vec{r}$

23 **(a)**

$\vec{A} \cdot (\vec{B} + \vec{C}) \times (\vec{A} + \vec{B} + \vec{C})$

$A \cdot [\vec{B} \times \vec{A} + \vec{B} \times \vec{B} + \vec{B} \times \vec{C} + \vec{C} \times \vec{A} + \vec{C} \times \vec{B} + \vec{C} \times \vec{C}]$

$= \vec{A} \cdot \vec{B} \times \vec{A} + \vec{A} \cdot \vec{B} \times \vec{C} + \vec{A} \cdot \vec{C} \times \vec{A} + \vec{A} \cdot \vec{C} \times \vec{B}$

(using $\vec{a} \times \vec{a} = 0$)

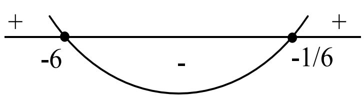
$= 0 + [\vec{A}\vec{B}\vec{C}] - [\vec{A}\vec{C}\vec{B}]$

$= [\vec{A}\vec{B}\vec{C}] - [\vec{A}\vec{B}\vec{C}]$
$= 0$

24 **(d)**
$|\vec{a}| = 1, |\vec{b}| = 4, \vec{a}\cdot\vec{b} = 2$
$\vec{c} = (2\vec{a}\times\vec{b}) - 3\vec{b}$
$\Rightarrow \vec{c} + 3\vec{b} = 2\vec{a}\times\vec{b}$
$\therefore \vec{a}\cdot\vec{b} = 2$
$\Rightarrow |\vec{a}|\cdot|\vec{b}|\cos\theta = 2$
$\Rightarrow \cos\theta = \dfrac{2}{|\vec{a}|\cdot|\vec{b}|} = \dfrac{2}{4}$
$\Rightarrow \cos\theta = \dfrac{1}{2}$
$\because \theta = \dfrac{\pi}{3}$
$\Rightarrow |\vec{c} + 3\vec{b}|^2 = |2\vec{a}\times\vec{b}|^2$
$\Rightarrow |\vec{c}|^2 + 9|\vec{b}|^2 + 2\vec{c}\cdot 3\vec{b} = 4|\vec{a}|^2|\vec{b}|^2\sin^2\theta$
$\Rightarrow |\vec{c}|^2 + 144 + 6\vec{b}\cdot\vec{c} = 48$
$\Rightarrow |\vec{c}|^2 + 96 + 6(\vec{b}\cdot\vec{c}) = 0$
$\therefore \vec{c} = 2\vec{a}\times\vec{b} - 3\vec{b}$
$\Rightarrow \vec{b}\cdot\vec{c} = 0 - 3\times 16$
$\therefore \vec{b}\cdot\vec{c} = -48$
Putting value of $\vec{b}\cdot\vec{c}$ in Eq. (i)
$|\vec{c}|^2 + 96 - 6\times 48 = 0$
$\Rightarrow |\vec{c}|^2 = 48\times 4$
$\Rightarrow |\vec{c}|^2 = 192$
Again, putting the value of $|\vec{c}|$ in Eq. (i)
$192 + 96 + 6|\vec{b}|\cdot|\vec{c}|\ \cos\alpha = 0$
$\Rightarrow 6\times 4\times 8\sqrt{3}\cos\alpha = -288$
$\Rightarrow \cos\alpha = -\dfrac{288}{6\times 4\times 8\sqrt{3}} = -\dfrac{3}{2\sqrt{3}}$
$\Rightarrow \cos\alpha = -\dfrac{\sqrt{3}}{2}$
$\therefore \alpha = \dfrac{5\pi}{6}$

25 **(d)**
Volume of parallelepiped $= [\vec{a}\vec{b}\vec{c}]$
$= \begin{vmatrix} 2 & -2 & 0 \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix} = 2(-1) + 2(-1+3) = 2$

26 **(b)**
The point that divides $5\hat{\imath}$ and $5\hat{\jmath}$ in the ratio of $k:1$ is $\frac{(5\hat{\jmath})k + (5\hat{\imath})1}{k+1}$
$\therefore\ \vec{b} = \dfrac{5\hat{\imath} + 5k\hat{\jmath}}{k+1}$
Also, $|\vec{b}| \le \sqrt{37}$



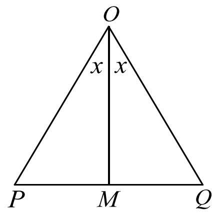
$\Rightarrow \dfrac{1}{k+1}\sqrt{25 + 25k^2} \le \sqrt{37}$
$\Rightarrow 5\sqrt{1 + k^2} \le \sqrt{37}(k+1)$
Square both sides
$25(1 + k^2) \le 37(k^2 + 2k + 1)$
or $6k^2 + 37k + 6 \ge 0 \Rightarrow (6k+1)(k+6) \ge 0$
$k \in (-\infty - 6] \cup \left[-\dfrac{1}{6}, \infty\right)$

27 **(c)**
Given vectors will be coplanar, if
$\begin{vmatrix} -\lambda^2 & 1 & 1 \\ 1 & -\lambda^2 & 1 \\ 1 & 1 & -\lambda^2 \end{vmatrix} = 0$
$\Longrightarrow \lambda^6 - 3\lambda^2 - 2 = 0$
$\Longrightarrow (1 + \lambda^2)^2(\lambda^2 - 2) = 0 \Longrightarrow \lambda = \pm\sqrt{2}$

28 **(c)**
$4\vec{a} + 5\vec{b} + 9\vec{c} = 0 \Rightarrow$ vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar
$\Rightarrow \vec{b}\times\vec{c}$ and $\vec{c}\times\vec{a}$ are collinear $\Rightarrow (\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a}) = \vec{0}$

29 **(c)**
$(\vec{a}\times\vec{b}\cdot\vec{c})^2 = \left||\vec{a}|^2|\vec{b}|^2\right||\vec{c}|^2\sin^2\theta\cos^2\phi$ (θ is the angle between $\vec{a}$and $\vec{b}, \phi = 0$)
$= \dfrac{1}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$

30 **(b)**
Since $|\overrightarrow{OP}| = |\overrightarrow{OQ}| = \sqrt{14}$, ΔOPQ is isosceles
Hence the internal bisector OM is perpendicular to PQ and M is the miodpoint of P and Q
$\therefore \overrightarrow{OM} = \dfrac{1}{2}(\overrightarrow{OP} + \overrightarrow{OQ}) = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$



31 **(a)**
Let $\vec{r} = x_1\hat{a} + x_2\hat{b} + x_3(\hat{a}\times\hat{b})$
$\Rightarrow \vec{r}.\hat{a} = x_1 + x_2\hat{a}.\hat{b} + x_3\hat{a}.(\hat{a}\times\hat{b}) = x_1$
Also ,$\vec{r}\cdot\hat{b} = x_1\hat{a}\cdot\hat{b} + x_2 + x_3\hat{b}\cdot(\hat{a} + \hat{b}) = x_2$
and $\vec{r}\cdot(\hat{a}\times\hat{b}) = x_1\hat{a}\cdot(\hat{a}\times\hat{b}) + x_2\hat{b}\cdot(\hat{a}\times\hat{b}) + x_3(\hat{a}\times\hat{b})\cdot(\hat{a}\times\hat{b}) = x_3$
$\Rightarrow \vec{r} = (\vec{r}\cdot\hat{a})\hat{a} + (\vec{r}\cdot\hat{b})\hat{b} + (\vec{r}\cdot(\hat{a}\times\hat{b}))(\hat{a}\times\hat{b})$

32 **(a)**
We have,
$\vec{a}\cdot\vec{p} = \vec{a}\cdot\dfrac{(\vec{b}\times\vec{c})}{[\vec{a}\vec{b}\vec{c}]} = \dfrac{\vec{a}\cdot(\vec{b}\times\vec{c})}{[\vec{a}\vec{b}\vec{c}]} = \dfrac{[\vec{a}\vec{b}\vec{c}]}{[\vec{a}\vec{b}\vec{c}]} = 1$

$\vec{a}\cdot\vec{q}=\vec{a}\cdot\frac{\vec{c}\times\vec{a}}{[\vec{a}\vec{b}\vec{c}]}=\frac{[\vec{a}\vec{c}\vec{a}]}{[\vec{a}\vec{b}\vec{c}]}=0$

Similarly, $\vec{a}\cdot\vec{r}=0, \vec{b}\cdot\vec{p}=0, \vec{b}\cdot\vec{q}=1, \vec{b}\cdot\vec{r}=0, \vec{c}\cdot\vec{p}=0, \vec{c}\cdot\vec{q}=0$ and $\vec{c}\cdot\vec{r}=1$

$\therefore (\vec{a}+\vec{b}+\vec{c}).(\vec{p}+\vec{q}+\vec{r})$

$=\vec{a}\cdot\vec{p}+\vec{a}\cdot\vec{q}+\vec{a}\cdot\vec{r}+\vec{b}\cdot\vec{p}+\vec{b}\cdot\vec{q}+\vec{b}\cdot\vec{r}+\vec{c}\cdot\vec{p}+\vec{c}\cdot\vec{q}+\vec{c}\cdot\vec{r}$

$=1+1+1=3$

33 **(d)**

We must have $\lambda(\hat{\imath}-3\hat{\jmath}+5\hat{k})=\hat{a}+\frac{2\hat{k}+2\hat{\jmath}-\hat{\imath}}{3}$.

Therefore,

$3\hat{a}=3\lambda(\hat{\imath}-3\hat{\jmath}+5\hat{k})-(2\hat{k}+2\hat{\jmath}-\hat{\imath})$

$=\hat{\imath}(3\lambda+1)-\hat{\jmath}(2+9\lambda)+\hat{k}(15\lambda-2)$

$\Rightarrow 3|\hat{a}|$

$=\sqrt{(3\lambda+1)^2+(2+9\lambda)^2+(15\lambda-2)^2}$

$\Rightarrow 9=(3\lambda+1)^2+(2+9\lambda)^2+(15\lambda-2)^2$

$\Rightarrow 315\lambda^2-18\lambda=0 \Rightarrow \lambda=0, \frac{2}{35}$

If $\lambda=0, \vec{a}=\hat{\imath}-2\hat{\jmath}-2\hat{k}$(not acceptable)

For $\lambda=\frac{2}{35}, \vec{a}=\frac{41}{105}\hat{\imath}-\frac{88}{105}\hat{\jmath}-\frac{40}{105}\hat{k}$

34 **(c)**

Let the required vector $\vec{r}$ be such that

$\vec{r}=x_1\vec{a}+x_2\vec{b}+x_3\vec{a}\times\vec{b}$

We must have $\vec{r}\cdot\vec{a}=\vec{r}\cdot\vec{b}=\vec{r}\cdot(\vec{a}\times\vec{b})$ (as$\vec{r},\vec{a},\vec{b}$ and $\vec{a}\times\vec{b}$ are unit vectors and $\vec{r}$ is equally inclined to $\vec{a},\vec{b}$ and $\vec{a}\times\vec{b}$)

Now $\vec{r}\cdot\vec{a}=x_1, \vec{r}\cdot\vec{b}=x_2, \vec{r}\cdot(\vec{a}\times\vec{b})=x_3$

$\Rightarrow \vec{r}=\lambda\left(\vec{a}+\vec{b}+(\vec{a}\times\vec{b})\right)$

Also , $\vec{r}\cdot\vec{r}=1$

$\Rightarrow \lambda^2(\vec{a}+\vec{b}+\vec{a}\times\vec{b})\cdot\left(\vec{a}+\vec{b}+(\vec{a}\times\vec{b})\right)=1$

$\Rightarrow \lambda^2\left(|\vec{a}|^2+|\vec{b}|^2+|\vec{a}\times\vec{b}|^2\right)=1$

$\Rightarrow \lambda^2=\frac{1}{3}$

$\Rightarrow \lambda=\pm\frac{1}{\sqrt{3}}$

$\Rightarrow \vec{r}=\pm\frac{1}{\sqrt{3}}(\vec{a}+\vec{b}+\vec{a}\times\vec{b})$

35 **(c)**

$\vec{r}\cdot\vec{a}=0, |\vec{r}\times\vec{b}|=|\vec{r}||\vec{b}|$ and $|\vec{r}\times\vec{c}|=|\vec{r}||\vec{c}|$

$\Rightarrow \vec{r}\perp\vec{a},\vec{b},\vec{c}$

$\therefore [\vec{a}\vec{b}\vec{c}]=0$

36 **(c)**

The given equation reduces to $[\vec{a}\vec{b}\vec{c}]^2x^2+2[\vec{a}\vec{b}\vec{c}]x+1=0 \Rightarrow D=0$

37 **(d)**

$\vec{a}+\vec{b}=\mu\vec{p}\vec{b}\cdot\vec{q}=0, |\vec{b}|^2=1$

$\because \vec{a}+\vec{b}=\mu\vec{p}$

$\Rightarrow (\vec{a}+\vec{b})\times\vec{a}=\mu\vec{p}\times\vec{a}, \vec{b}\times\vec{a}=\mu\vec{p}\times\vec{a}$

$\Rightarrow \vec{q}\times(\vec{b}\times\vec{a})=\mu\vec{q}\times(\vec{p}\times\vec{a})$

$\Rightarrow (\vec{q}\cdot\vec{a})\vec{b}-(\vec{q}\cdot\vec{b})\vec{a}=\mu\vec{q}\times(\vec{p}\times\vec{a}) \Rightarrow (\vec{q}\cdot\vec{a})\vec{b}=\mu\vec{q}\times(\vec{p}\times\vec{a})$

$\because \vec{a}+\vec{b}=\mu\vec{p}$

$\Rightarrow \vec{q}\cdot(\vec{a}+\vec{b})=\mu\vec{q}\cdot\vec{p}$

$\Rightarrow \vec{q}\cdot\vec{a}+\vec{q}\cdot\vec{b}=\mu\vec{p}\cdot\vec{q}$

$\Rightarrow \mu=\frac{\vec{q}\cdot\vec{a}}{\vec{p}\cdot\vec{q}}$

$\Rightarrow (\vec{q}\cdot\vec{a})\vec{b}=\frac{\vec{q}\cdot\vec{a}}{\vec{p}\cdot\vec{q}}[(\vec{q}\cdot\vec{a})\cdot\vec{p}-(\vec{q}\cdot\vec{p})\vec{a}]$

$\Rightarrow |(\vec{q}\cdot\vec{a})\vec{p}-(\vec{q}\cdot\vec{p})\vec{a}|=|(\vec{p}\cdot\vec{q})\vec{b}|=|\vec{p}\cdot\vec{q}|\cdot|\vec{b}|$

$\Rightarrow |(\vec{q}\cdot\vec{a})\vec{p}-(\vec{q}\cdot\vec{p})\vec{a}|=|\vec{p}\cdot\vec{q}|$

38 **(c)**

$1+9(\vec{a}\cdot\vec{b})^2-6(\vec{a}\cdot\vec{b})+4|\vec{a}|^2+|\vec{b}|^2+9|\vec{a}\times\vec{b}|^2=4\,\vec{a}\cdot\vec{b}=47$

$\Rightarrow 1+4+4+36-4\cos\theta=47$

$\Rightarrow \cos\theta=-\frac{1}{2}$

$\Rightarrow$ Angle between $\vec{a}$and$\vec{b}$ is $\frac{2\pi}{3}$

39 **(d)**

$|(\vec{a}\times\vec{b})\cdot\vec{c}|=|\vec{a}||\vec{b}||\vec{c}|$

Or $||\vec{a}||\vec{b}|\sin\theta\,\hat{n}\cdot\vec{c}|=|\vec{a}||\vec{b}||\vec{c}|$

Or $|\vec{a}||\vec{b}||\vec{c}||\sin\theta\cos\alpha|=|\vec{a}||\vec{b}||\vec{c}|$

Or $|\sin\theta||\cos\alpha|=1$

$\Rightarrow \theta=\pi/2$ and $\alpha=0$

$\Rightarrow \vec{a}\perp\vec{b}$ and $\vec{c}||\hat{n}$

Or perpendicular to both a and b

$\Rightarrow \vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$

40 **(b)**

$\vec{x}+\vec{c}\times\vec{y}=\vec{a}$

$\vec{y}+\vec{c}\times\vec{x}=\vec{b}$

Taking cross with $\vec{c}$

$\vec{c}\times\vec{y}+\vec{c}\times(\vec{c}\times\vec{x})=\vec{c}\times\vec{b}$

$\Rightarrow (\vec{a}-\vec{x})+(\vec{c}\cdot\vec{x})\vec{c}-(\vec{c}\cdot\vec{c})\vec{x}=\vec{c}\times\vec{b}$

Also $\vec{x}+\vec{c}\times\vec{y}=\vec{a}$

$\Rightarrow \vec{c}\cdot\vec{x}+\vec{c}\cdot(\vec{c}\times\vec{y})=\vec{c}\cdot\vec{a}$

$\Rightarrow \vec{c}\cdot\vec{x}+0=\vec{c}\cdot\vec{a}$

$\therefore \vec{c}\cdot\vec{x}=\vec{c}\cdot\vec{a}$

$\Rightarrow \vec{a}-\vec{x}+(\vec{c}\cdot\vec{a})\vec{c}-(\vec{c}\cdot\vec{c})\vec{x}=\vec{c}\times\vec{b}$

$\Rightarrow \vec{x}(1+(\vec{c}\cdot\vec{c}))=\vec{b}\times\vec{c}+\vec{a}+(\vec{c}\cdot\vec{a})\cdot\vec{c}$

$\therefore \vec{x}=\frac{\vec{b}\times\vec{c}+\vec{a}+(\vec{c}\cdot\vec{a})\vec{c}}{1+\vec{c}\cdot\vec{c}}$

Similarly on taking cross product of Eq. (i), we find

$$\vec{y} = \frac{\vec{a} \times \vec{c} + \vec{b} + (\vec{c} \cdot \vec{b})\vec{c}}{1 + \vec{c} \cdot \vec{c}}$$

41 **(b)**

Let the required vector be $\vec{r}$.Then $\vec{r} = x_1\vec{b} + x_2\vec{c}$

and $\vec{r} \cdot \vec{a} = \sqrt{\frac{2}{3}}(|\vec{a}|) = 2$

Now,$\vec{r} \cdot \vec{a} = x_1\vec{a} \cdot \vec{b} + x_2\vec{a} \cdot \vec{c} \Rightarrow 2 = x_1(2 - 2 -$

1+x22−1−2⇒x1+x2=−2

$\Rightarrow \vec{r} = x_1(\hat{\imath} + 2\hat{\jmath} - \hat{k}) + x_2(\hat{\imath} + \hat{\jmath} - 2\hat{k})$

$= \hat{\imath}(x_1 + x_2) + \hat{\jmath}(2x_1 + x_2) - \hat{k}(2x_2 + x_1)$

$= -2\hat{\imath} + \hat{\jmath}(x_1 - 2) - \hat{k}(-4 - x_1)$, where $x_1 \in R$

42 **(c)**

The position vector of the point O with respect to itself is

$$\frac{\overrightarrow{OA}\sin 2A + \overrightarrow{OB}\sin 2B + \overrightarrow{OC}\sin 2C}{\sin 2A + \sin 2B + \sin 2C}$$

$$\Rightarrow \frac{\overrightarrow{OA}\sin 2A + \overrightarrow{OB}\sin 2B + \overrightarrow{OC}\sin 2C}{\sin 2A + \sin 2B + \sin 2C} = \vec{O}$$

$\Rightarrow \overrightarrow{OA}\sin 2A + \overrightarrow{OB}\sin 2B + \overrightarrow{OC}\sin 2C = \overline{0}$

43 **(b)**

Let $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = \vec{b}$ and $\overrightarrow{OC} = \vec{c}$, then $\overrightarrow{AB} = \vec{b} - \vec{a}$

and $\overrightarrow{OP} = \frac{1}{3}\vec{a}, \overrightarrow{OQ} = \frac{1}{2}\vec{b}, \overrightarrow{OR} = \frac{1}{3}\vec{c}$

Since P, Q, R and S are coplanar, then

$\overrightarrow{PS} = \alpha\,\overrightarrow{PQ} + \beta\,\overrightarrow{PR}$ ($\overrightarrow{PS}$ can be written as a linear combination of $\overrightarrow{PQ}$ and $\overrightarrow{PR}$)

$= \alpha(\overrightarrow{OQ} - \overrightarrow{OP}) + \beta(\overrightarrow{OR} - \overrightarrow{OP})$

i.e., $\overrightarrow{OS} - \overrightarrow{OP} = -(\alpha + \beta)\frac{\vec{a}}{3} + \frac{\alpha}{2}\vec{b} + \frac{\beta}{3}\vec{c}$

$\Rightarrow \overrightarrow{OS} = (1 - \alpha - \beta)\frac{\vec{a}}{3} + \frac{\alpha}{2}\vec{b} + \frac{\beta}{3}\vec{c}$

Given $\overrightarrow{OS} = \lambda\overrightarrow{AB} = \lambda(\vec{b} - \vec{a})$

From (i) and (ii), $\beta = 0, \frac{1-\alpha}{3} = -\lambda$ and $\frac{\alpha}{2} = \lambda$

$\Rightarrow 2\lambda = 1 + 3\lambda$

$\Rightarrow \lambda = -1$

44 **(d)**

$3\vec{a} + 4\vec{b} + 5\vec{c} = 0$

$\Rightarrow \vec{a}, \vec{b}$ and $\vec{c}$ are coplanar

No other conclusion can be derives from it

45 **(c)**

Let the projection be x, then $\vec{a} = \frac{x(\hat{\imath}+\hat{\jmath})}{\sqrt{2}} + \frac{x(-\hat{\imath}+\hat{\jmath})}{\sqrt{2}} + x\hat{k}$

$\therefore \vec{a} = \frac{2x\hat{\jmath}}{\sqrt{2}} + x\hat{k} \Rightarrow \hat{a} = \frac{\sqrt{2}}{\sqrt{3}}\hat{\jmath} + \frac{\hat{k}}{\sqrt{3}}$

46 **(d)**

$$\vec{a}' = \frac{\vec{b} \times \vec{c}}{[\vec{a}\vec{b}\vec{c}]} = \frac{\hat{\imath} + \hat{\jmath} - \hat{k}}{2}$$

47 **(b)**

Given $\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}} = \vec{\mathbf{0}}$

$\Rightarrow\ \vec{\mathbf{a}} \times \vec{\mathbf{a}} + \vec{\mathbf{a}} \times \vec{\mathbf{b}} + \vec{\mathbf{a}} \times \vec{\mathbf{c}} = 0$

$\Rightarrow\ \vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{c}} \times \vec{\mathbf{a}}$

Similarly, $\vec{\mathbf{b}} \times \vec{\mathbf{c}} = \vec{\mathbf{c}} \times \vec{\mathbf{a}}$

$\therefore\ \vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{b}} \times \vec{\mathbf{c}} = \vec{\mathbf{c}} \times \vec{\mathbf{a}} \neq \vec{\mathbf{0}}$

Alternate: Since, $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are unit vectors and

$\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{a}} + \vec{\mathbf{c}} = \vec{\mathbf{0}}$,

so $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ represent an equilateral triangle.

$\therefore\ \vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{b}} \times \vec{\mathbf{c}} = \vec{\mathbf{c}} \times \vec{\mathbf{a}} \neq \vec{\mathbf{0}}$

48 **(b)**

$(\vec{a} \times \hat{\imath}).(\vec{b} \times \hat{\imath}) = \begin{vmatrix} \vec{a}.\vec{b} & \vec{a}.\hat{\imath} \\ \vec{b}.\hat{\imath} & \hat{\imath}.\hat{\imath} \end{vmatrix} = (\vec{a}.\vec{b}) - (\vec{a}.\hat{\imath})(\vec{b}.\hat{\imath})$

Similarly ,$(\vec{a} \times \hat{\jmath}) \cdot (\vec{b} \times \hat{\jmath}) = (\vec{a} \cdot \vec{b}) - (\vec{a} \cdot \hat{\jmath})(\vec{b} \cdot \hat{\jmath})$

and $(\vec{a} \times \hat{k}) \cdot (\vec{b} \times \hat{k}) = \vec{a} \cdot \vec{b} - (\vec{a} \cdot k)(\vec{b} \cdot \hat{k})$

Let $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}, \vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$.

Therefore

$(\vec{a} \cdot \hat{\imath}) = a_1, \vec{a} \cdot \hat{\jmath} = a_2, \vec{a} \cdot \hat{k} = a_3, \vec{b} \cdot \hat{\imath} = b_1, \vec{b} \cdot \hat{\jmath} = b_2, (\vec{b} \cdot \hat{k}) = b_3$

$\Rightarrow (\vec{a} \times \hat{\imath}) \cdot (\vec{b} \times \hat{\imath}) + (\vec{a} \times \hat{\jmath}) \cdot (\vec{b} \times \hat{\jmath}) + (\vec{a} \times \hat{k}) \cdot (\vec{b} \times \hat{k})$

$= 3\vec{a} \cdot \vec{b} - (a_1b_1 + a_2b_2 + a_3b_3)$

$= 3\vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{b} = 2\vec{a} \cdot \vec{b}$

49 **(d)**

$\sqrt{3}\tan\theta + 1 = 0$ and $\sqrt{3}\sec\theta - 2 = 0$

$\Rightarrow \theta = \frac{11\pi}{6}$

$\Rightarrow \theta = 2n\pi + \frac{11\pi}{6}, n \in Z$

50 **(d)**

Let $\vec{r} \neq \vec{0}$. Then $\vec{r} \cdot \vec{a} = \vec{r} \cdot \vec{b} = \vec{r} \cdot \vec{c} = 0$

$\Rightarrow \vec{a}, \vec{b}$ and$\vec{c}$ are coplanar, which is a contradiction

Therefore, $\vec{r} = \vec{0}$

51 **(b)**

Let the given position vectors be of points A, B and C, respectively. Then

$|\overrightarrow{AB}| = \sqrt{(\beta - \alpha)^2 + (\gamma - \beta)^2 + (\alpha - \gamma)^2}$

$|\overrightarrow{BC}| = \sqrt{(\gamma - \beta)^2 + (\alpha - \gamma)^2 + (\alpha - \beta)^2}$

$|\overrightarrow{CA}| = \sqrt{(\alpha - \gamma)^2(\beta - \alpha)^2 + (\gamma - \beta)^2}$

$\because |\overrightarrow{AB}| = |\overrightarrow{BC}| = |\overrightarrow{CA}|$

Hence, ΔABC is an equilateral triangle

52 **(c)**

$\vec{a} = \hat{\imath} - \hat{k}$
$\vec{b} = x\hat{\imath} + \hat{\jmath} + (1-x)\hat{k}$
$\vec{c} = y\vec{\imath} + x\vec{\jmath} + (1+x-y)\vec{k}$
$= \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix}$
$= (1+x-y-x+x^2) - 1(x^2-y)$
$= 1$

53 **(b)**
As $\vec{p}, \vec{q}$ and $\vec{r}$ are three mutually perpendicular vectors of same magnitude, so let us consider
$\vec{p} = a\,\hat{\imath}, \vec{q} = a\hat{\jmath}, \vec{r} = a\,\hat{k}$
Also let $\vec{x} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$
Given that $\vec{x}$ satisfies the equation
$\vec{p} \times [(\vec{x}-\vec{q}) \times \vec{p}] + \vec{q} \times [(\vec{x}-\vec{r}) \times \vec{q}] + \vec{r} \times [(\vec{x}-\vec{p}) \times \vec{r}] = 0$ (i)
Now $\vec{p} \times [(\vec{x}-\vec{q}) \times \vec{p}] = \vec{p} \times [\vec{x} \times \vec{p} - \vec{q} \times \vec{p}]$
$= \vec{p} \times (\vec{x} \times \vec{p}) - \vec{p} \times (\vec{q} \times \vec{p})$
$= (\vec{p}\cdot\vec{p})\vec{x} - (\vec{p}\cdot\vec{x})\vec{p} - (\vec{p}\cdot\vec{p})\vec{q} + (\vec{p}\cdot\vec{q})\vec{p}$
$= a^2\vec{x} - a^2x_1\hat{\imath} - a^3\hat{\jmath} + 0$
Similarly,
$\vec{q} \times [(\vec{x}-\vec{r}) \times \vec{q}] = a^2\vec{x} - a^2y_1\hat{\jmath} - a^3\hat{k}$
and $\vec{r} \times [(\vec{x}-\vec{p}) \times \vec{r}] = a^2\vec{x} - a^2z_1\hat{k} - a^3\hat{\imath}$
Substituting these values in the equation, we get
$3a^2\vec{x} - a^2(x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}) - a^2(a\hat{\imath} + a\hat{\jmath} + a\hat{k}) = 0$
$\Rightarrow 3a^2\vec{x} - a^2\vec{x} - a^2(\vec{p}+\vec{q}+\vec{r}) = \vec{0}$
$\Rightarrow 2a^2\vec{x} = (\vec{p}+\vec{q}+\vec{r})a^2$
$\Rightarrow \vec{x} = \frac{1}{2}(\vec{p}+\vec{q}+\vec{r})$

54 **(a)**
Given, $\overrightarrow{\mathbf{OP}} = \hat{\mathbf{a}}\cos t + \hat{\mathbf{b}}\sin t$
$\Longrightarrow |\overrightarrow{\mathbf{OP}}|$
$= \sqrt{(\hat{\mathbf{a}}\cdot\hat{\mathbf{a}}\cos^2 t + \hat{\mathbf{b}}\cdot\hat{\mathbf{b}}\sin^2 t + 2\hat{\mathbf{a}}\cdot\hat{\mathbf{b}}\sin t\cos t}$
$\Longrightarrow |\overrightarrow{\mathbf{OP}}| = \sqrt{1 + \hat{\mathbf{a}}\cdot\hat{\mathbf{b}}\sin 2t}$
$\Longrightarrow |\overrightarrow{\mathbf{OP}}|_{\text{maxx}} = \sqrt{1 + \hat{\mathbf{a}}\cdot\hat{\mathbf{b}}}$
$\left[\text{Max}(\sin 2t) = 1 \Longrightarrow t = \frac{\pi}{4}\right]$
$\Longrightarrow \overrightarrow{\mathbf{OP}}\left(\text{at } t = \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}(\hat{\mathbf{a}} + \hat{\mathbf{b}})$
$\therefore$ Unit vector along $\overrightarrow{\mathbf{OP}}$ at $\left(t = \frac{\pi}{4}\right) = \frac{\hat{\mathbf{a}}+\hat{\mathbf{b}}}{|\hat{\mathbf{a}}+\hat{\mathbf{b}}|}$

55 **(a)**
$\vec{a} \times \vec{b} = \vec{c}$
$\Rightarrow \vec{a} \times (\vec{a} \times \vec{b}) = \vec{a} \times \vec{c}$
$\Rightarrow (\vec{a}\cdot\vec{b})\vec{a} - |\vec{a}|^2\vec{b} = \vec{a} \times \vec{c}$
$\Rightarrow \vec{b} = \frac{\beta\vec{a} - \vec{a} \times \vec{c}}{|\vec{a}|^2}$ $(\because \vec{a}\cdot\vec{b} = \beta)$

56 **(c)**
$m\vec{a}$ is a unit vector if and only if
$|m\vec{a}| = 1 \Rightarrow |m||\vec{a}| = 1 \Rightarrow |m|a = 1 \Rightarrow a = \frac{1}{|m|}$

57 **(a)**
Let $\vec{\mathbf{r}} = \vec{\mathbf{a}} + t\vec{\mathbf{b}}$
$\Longrightarrow \vec{\mathbf{r}} = \hat{\mathbf{i}}(1+t) + \hat{\mathbf{j}}(2-t) + \hat{\mathbf{k}}(1+t)$
Since, The projection of $\vec{\mathbf{r}}$ on $\vec{\mathbf{c}}$,
$\frac{\vec{\mathbf{r}}\cdot\vec{\mathbf{c}}}{|\vec{\mathbf{c}}|} = \frac{|1|}{|\sqrt{3}|}$ [given]
$\Longrightarrow \frac{1\cdot(1+t) + 1\cdot(2-t) - 1\cdot(1+t)}{\sqrt{3}} = \pm\frac{1}{\sqrt{3}}$
$\Longrightarrow 2 - t = \pm 1$
$\Longrightarrow t = 1$ or 3
When, $t = 1, \vec{\mathbf{r}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$
When, $t = 3, \vec{\mathbf{r}} = 4\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

58 **(d)**
$\vec{r}_1 + 2\vec{r}_2 = (P\vec{a} + q\vec{b} + \vec{c}) + 2(\vec{a} + p\vec{b} + q\vec{c})$
$= (p+2)\vec{a} + (q+2p)\vec{b} + (1 + 2q)\vec{c}$
$2\vec{r}_1 + \vec{r}_2 = (2p+1)\vec{a} + (2q+p)\vec{b} + (2+q)\vec{c}$
$\frac{p+2}{2p+1} = \frac{q+2p}{2q+p} = \frac{1+2q}{2+q}$
$= \frac{p+q+2p+2q+3}{p+q+2p+2q+3} = 1$
$\Rightarrow p = 1$ and $q = 1$

59 **(b)**
$\vec{a} + \vec{b} = \lambda\vec{c}$ (i)
and $\vec{b} + \vec{c} = \mu\,\vec{a}$ (ii)
$\therefore (\lambda\vec{c} - \vec{a}) + \vec{c} = \mu\,\vec{a}$ (putting $\vec{b} = \lambda\vec{c} - \vec{a}$)
$\Rightarrow (\lambda+1)\vec{c} = (\mu+1)\vec{a}$
$\Rightarrow \lambda = \mu = -1$
$\Rightarrow \vec{a} + \vec{b} + \vec{c} = 0$
$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$
$\Rightarrow \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = -3$

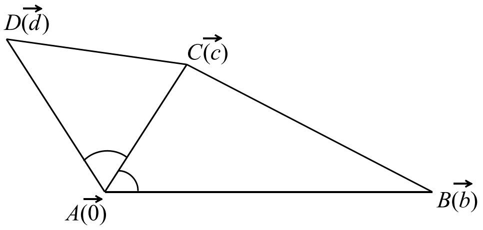
60 **(a)**
The position vector of any point at t is
$\vec{r} = (2+t^2)\hat{\imath} + (4t-5)\hat{\jmath} + (2t^2-6)\hat{k}$
$\Rightarrow \frac{d\vec{r}}{dt} = 2t\,\hat{\imath} + 4\hat{\jmath} + (4t-6)\hat{k}$

$\Rightarrow \left.\frac{d\vec{r}}{dt}\right|_{t=2} = 4\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$ and $\left|\left.\frac{d\vec{r}}{dt}\right|\right|_{t=2}$

$= \sqrt{16 + 16 + 4} = 4$

Hence, the required unit tangent vector at $t = 2$ is $\frac{1}{3}(2\hat{\imath} + 2\hat{\jmath} + \hat{k})$

61 **(c)**



Let $|\overrightarrow{AC}| = \lambda > 0$

Then from $15|\overrightarrow{AC}| = 3\,|\overrightarrow{AB}| = 5\,|\overrightarrow{AD}|$

$|\overrightarrow{AB}| = 5\lambda$

Let θ be the angle between $\overrightarrow{BA}$and $\overrightarrow{CD}$

$\Rightarrow \cos\theta = \frac{\overrightarrow{BA}\cdot\overrightarrow{CD}}{|\overrightarrow{BA}||\overrightarrow{CD}|} = \frac{-\vec{b}\cdot(\vec{d}-\vec{c})}{|\vec{b}||\vec{d}-\vec{c}|}$ (i)

Now $-\vec{b}\cdot(\vec{d}-\vec{c}) = \vec{b}\cdot\vec{c} - \vec{b}\cdot\vec{d}$

$= |\vec{b}||\vec{c}|\cos\frac{\pi}{3} - |\vec{b}||\vec{d}|\cos\frac{2\pi}{3}$

$= (5\lambda)(\lambda)\frac{1}{2} + (5\lambda)(3\lambda)\frac{1}{2}$

$= \frac{5\lambda^2 + 15\lambda^2}{2}$

$= 10\lambda^2$

Denominator of (i) $= |\vec{b}||\vec{d}-\vec{c}|$

Now $|\vec{d}-\vec{c}|^2 = |\vec{d}|^2 + |\vec{c}|^2 - 2\vec{c}\cdot\vec{d}$

$= 9\lambda^2 + \lambda^2 - 2(\lambda)(3\lambda)(1/2)$

$= 10\lambda^2 - 3\lambda^2$

$= 7\lambda^2$

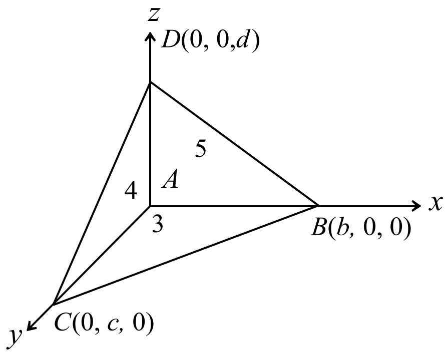
Denominator of (i)$=(5\lambda)(\sqrt{7}\lambda) = 5\sqrt{7}\lambda^2$

$\therefore \cos\theta = \frac{10\lambda^2}{5\sqrt{7}\lambda^2} = \frac{2}{\sqrt{7}}$

62 **(a)**

Area of $\Delta BCD = \frac{1}{2}\,|\overrightarrow{BC}\times\overrightarrow{BD}| = \frac{1}{2}|(b\hat{\imath} - c\hat{\jmath}) \times$ $bi{-}dk$

$= \frac{1}{2}|bd\hat{\jmath} + bc\hat{k} + dc\hat{\imath}|$



$= \frac{1}{2}\sqrt{b^2c^2 + c^2d^2 + d^2b^2}$

Now $6 = bc; 8 = cd; 10 = bd$

$b^2c^2 + c^2d^2 + d^2b^2 = 200$

Substituting the value in (i)

$A = \frac{1}{2}\sqrt{200} = 5\sqrt{2}$

63 **(d)**

$(\vec{a}+\vec{b}+\vec{c})\cdot[(\vec{a}+\vec{b})\times(\vec{a}+\vec{c})]$

$= (\vec{a}+\vec{b}+\vec{c})\cdot[\vec{a}\times\vec{a}\times\vec{c} + \vec{b}\times\vec{a} + \vec{b}\times\vec{c}]$

$= (\vec{a}+\vec{b}+\vec{c})\cdot[\vec{a}\times\vec{c} + \vec{b}\times\vec{a} + \vec{b}\times\vec{c}]$

$= \vec{a}\cdot\vec{b}\times\vec{c} + \vec{b}\cdot\vec{a}\times\vec{c} + \vec{c}\cdot\vec{b}\times\vec{a}$

$= [\vec{a}\vec{b}\vec{c}] - [\vec{a}\vec{b}\vec{c}] - [\vec{a}\vec{b}\vec{c}]$

$= -[\vec{a}\vec{b}\vec{c}]$

64 **(c)**

Consider a tetrahedron with vertices $O(0,0,0), A(a,0,0), B(0,b,0)$ and $C(0,0,c)$

Its volume $V = \frac{1}{6}[\vec{a}\vec{b}\vec{c}]$

Now centroides of the faces $OAB, OAC, OBC$and $ABC$are $G_1(a/3, b/3, 0) G_2(a/3, 0, c/3), G_3(0, b/3, c/3)$ and $G_4(a/3, b/3, c/3)$, respectively

$G_4G_1 = \vec{c}/3, \overrightarrow{G_4G_2} = \vec{b}/3, \overrightarrow{G_4G_3} = \vec{a}/3$

Volume of tetrahedron be centroids $V' = \frac{1}{6}\left[\frac{\vec{a}}{3}\frac{\vec{b}}{3}\frac{\vec{c}}{3}\right] = \frac{1}{27}V$

$\Rightarrow K = 27$

65 **(c)**

$\vec{a}_1 = (\vec{a}\cdot\hat{b})\hat{b} = \frac{(\vec{a}\cdot\vec{b})\vec{b}}{|\vec{b}|^2}$

$\Rightarrow \vec{a}_2 = \vec{a} - \vec{a}_1 = \vec{a} - \frac{(\vec{a}\cdot\vec{b})\vec{b}}{|\vec{b}|^2}$

Thus,$\vec{a}_1 \times \vec{a}_2 = \frac{(\vec{a}\cdot\vec{b})\vec{b}}{|\vec{b}|^2} \times \left(\vec{a} - \frac{(\vec{a}\cdot\vec{b})\vec{b}}{|\vec{b}|^2}\right) = \frac{(\vec{a}\cdot\vec{b})(\vec{b}\times\vec{a})}{|\vec{b}|^2}$

66 **(d)**

$(\hat{\imath}+\hat{\jmath})\times(\hat{\jmath}+\hat{k}) = \hat{\imath} - \hat{\jmath} + \hat{k}$ so that unit vector perpendicular to the plane of $\hat{\imath}+\hat{\jmath}$and $\hat{\jmath}+\hat{k}$ is $\frac{1}{\sqrt{3}}(\hat{\imath} - \hat{\jmath} + \hat{k})$

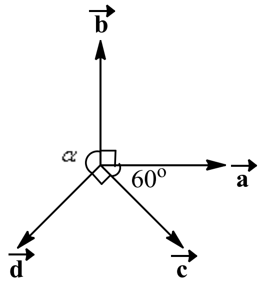
Similarly, the other two unit vectors are
$\frac{1}{\sqrt{3}}(\hat{\imath}+\hat{\jmath}-\hat{k})$and $\frac{1}{\sqrt{3}}(-\hat{\imath}+\hat{\jmath}+\hat{k})$
The required volume $=\frac{3}{\sqrt{3}}\begin{vmatrix}1 & -1 & 1\\ 1 & 1 & -1\\ -1 & 1 & 1\end{vmatrix}=4\sqrt{3}$

67 **(c)**
$\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}=4\hat{\imath}+2\hat{\jmath}-4\hat{k}$
$\overrightarrow{AB}=-3\hat{\imath}-3\hat{k}, \overrightarrow{AC}=\hat{\imath}+2\hat{\jmath}-7\hat{k}$
$BC^2=36, AB^2=18, AC^2=54$
Clearly, $AC^2=BC^2+AB^2$
$\therefore \angle B=90°$

68 **(c)**
Given $\vec{\alpha}+\vec{\beta}+\vec{\gamma}=a\vec{\delta}$
$\vec{\beta}+\vec{\gamma}+\vec{\delta}=b\vec{\alpha}$
From (i), $\vec{\alpha}+\vec{\beta}+\vec{\gamma}+\vec{\delta}=(a+1)\vec{\delta}$
From (ii), $\vec{\alpha}+\vec{\beta}+\vec{\gamma}+\vec{\delta}=(b+1)\vec{\alpha}$
From (iii) and (iv),
$(a+1)\overrightarrow{\delta}=(b+1)\vec{\alpha}$
Since $\vec{\alpha}$ is not parallel to $\vec{\delta}$
From (v), $a+1=0$ and $b+1$=0
From (iii), $\vec{\alpha}+\vec{\beta}+\vec{\gamma}+\vec{\delta}=0$

69 **(b)**
A vector perpendicular to the plane of
$A(\vec{a}), B(\vec{b})$and $C(\vec{c})$is
$(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})=\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$
Now for any point $R(\vec{r})$ in the plane of A, B and C is
$(\vec{r}-\vec{a})\cdot(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})=0$
$\vec{r}\cdot(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})$
$-\vec{a}(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})=0$
$\vec{r}\cdot(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})=\vec{0}+\vec{a}\cdot\vec{b}\times\vec{c}+\vec{0}$
$\vec{r}\cdot(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})=[\vec{a}\vec{b}\vec{c}]$

70 **(c)**
Let angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ be θ_1. $\vec{\mathbf{c}}$ and $\vec{\mathbf{d}}$ be θ_2 and $\vec{\mathbf{a}}\times\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}\times\vec{\mathbf{d}}$ beθ
Since, $(\vec{\mathbf{a}}\times\vec{\mathbf{b}})\cdot(\vec{\mathbf{c}}\times\vec{\mathbf{d}})=1$
$\Rightarrow \sin\theta_1\cdot\sin\theta_2$
$\cdot\cos\theta$
$=1 \quad (\because |\vec{\mathbf{a}}|=|\vec{\mathbf{b}}|=|\vec{\mathbf{c}}|=|\vec{\mathbf{d}}|=1)$
$\Rightarrow \theta_1=90°\cdot\theta_2=90°, \theta=0°$
$\Rightarrow \vec{\mathbf{a}}\perp\vec{\mathbf{b}}, \vec{\mathbf{c}}\perp\vec{\mathbf{d}}, (\vec{\mathbf{a}}\times\vec{\mathbf{b}})||(\vec{\mathbf{c}}\times\vec{\mathbf{d}})$
So, $\vec{\mathbf{a}}\times\vec{\mathbf{b}}=k(\vec{\mathbf{c}}\times\vec{\mathbf{d}})$ and $\vec{\mathbf{a}}\times\vec{\mathbf{b}}=k(\vec{\mathbf{c}}\times\vec{\mathbf{d}})$
$\Rightarrow (\vec{\mathbf{a}}\times\vec{\mathbf{b}})\cdot\vec{\mathbf{c}}=k(\vec{\mathbf{c}}\times\vec{\mathbf{d}})\cdot\vec{\mathbf{c}}$
and $(\vec{\mathbf{a}}\times\vec{\mathbf{b}})\cdot\vec{\mathbf{d}}=k(\vec{\mathbf{c}}\times\vec{\mathbf{d}})\cdot\vec{\mathbf{d}}$
$\Rightarrow [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]=0$ and $[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{d}}]=0$
$\Rightarrow$ $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ and $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{d}}$ are coplanar vector so option (A) and (B) are incorrect.
Let $\vec{\mathbf{b}}||\vec{\mathbf{d}} \Rightarrow \vec{\mathbf{b}}=\pm\vec{\mathbf{d}}$
As $(\vec{\mathbf{a}}\times\vec{\mathbf{b}})\cdot(\vec{\mathbf{c}}\times\vec{\mathbf{d}})=1 \Rightarrow (\vec{\mathbf{a}}\times\vec{\mathbf{b}})\cdot(\vec{\mathbf{c}}\times\vec{\mathbf{b}})=\pm1$
$\Rightarrow [\vec{\mathbf{a}}\times\vec{\mathbf{b}}\,\vec{\mathbf{c}}\,\vec{\mathbf{b}}]=\pm1$
$\Rightarrow [\vec{\mathbf{c}}\,\vec{\mathbf{b}}\,\vec{\mathbf{a}}\times\vec{\mathbf{b}}]=\pm1$
$\Rightarrow \vec{\mathbf{c}}\cdot[\vec{\mathbf{b}}\times(\vec{\mathbf{a}}\times\vec{\mathbf{b}})]=\pm1$
$\Rightarrow \vec{\mathbf{c}}\cdot[\vec{\mathbf{a}}-(\vec{\mathbf{b}}\cdot\vec{\mathbf{a}})\vec{\mathbf{b}}]=\pm1$
$\Rightarrow \vec{\mathbf{c}}\cdot\vec{\mathbf{a}}=\pm1 \quad (\because \vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=0)$
Which is a contradiction so option (c) is correct.
Let option (d) is correct



$\Rightarrow$ $\vec{\mathbf{d}}=\pm\vec{\mathbf{a}}$ and $\vec{\mathbf{c}}=\pm\vec{\mathbf{b}}$
As $(\vec{\mathbf{a}}\times\vec{\mathbf{b}})\cdot(\vec{\mathbf{c}}\times\vec{\mathbf{d}})=1$
$\Rightarrow (\vec{\mathbf{a}}\times\vec{\mathbf{b}})\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{a}})=\pm1$
Which is a contradiction so option (d) is incorrect.
Alternate Option (c) and (d) may be observed from given in figure.

71 **(b)**
We have
$\vec{p}\cdot\vec{q}=0$
$\Rightarrow (5\vec{a}-3\vec{b})\cdot(-\vec{a}-2\vec{b})=0$
$\Rightarrow 6|\vec{b}|^2-5|\vec{a}|^2-7\vec{a}\cdot\vec{b}=0$ (i)
Also $\vec{r}\cdot\vec{s}=0$
$\Rightarrow (-4\vec{a}-\vec{b})(-\vec{a}+\vec{b})=0$
$\Rightarrow 4|\vec{a}|^2-|\vec{b}|^2-3\vec{a}\cdot\vec{b}=0$ (ii)
Now $\vec{x}=\frac{1}{3}(\vec{p}+\vec{r}+\vec{s})=\frac{1}{3}(5\vec{a}-3\vec{b}-4\vec{a}-\vec{b}-$
a+b=−b
and $\vec{y}=\frac{1}{5}(\vec{r}+\vec{s})=\frac{1}{5}(-5\vec{a})=-\vec{a}$
Angle between $\vec{x}$and $\vec{y}$, i.e., $\cos\theta=\frac{\vec{x}\cdot\vec{y}}{|\vec{x}||\vec{y}|}=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$
(iii)
From (i) and (ii),$|\vec{a}|=\sqrt{\frac{25}{19}}\sqrt{\vec{a}\cdot\vec{b}}$and
$|\vec{b}|=\sqrt{\frac{43}{19}}\sqrt{\vec{a}\cdot\vec{b}}$.Therefore
$|\vec{a}||\vec{b}|=\frac{\sqrt{25\times43}}{19}\cdot\vec{a}\cdot\vec{b}$

$\theta = \cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

72 **(d)**

$\vec{0} = (\vec{a} + \vec{b}) \cdot (2\vec{a} + 3\vec{b}) \times (3\vec{a} - 2\vec{b})$

$= (\vec{a} + \vec{b}) \cdot (-4\vec{a} \times \vec{b} - 9\vec{a} \times \vec{b})$

$= -13(\vec{a} + \vec{b}) \cdot (\vec{a} \times \vec{b})$

Which is true for all values of $\vec{a}$and$\vec{b}$

73 **(b)**

$\vec{c} = \lambda(\vec{a} \times \vec{b})$

$\Rightarrow \vec{c} \cdot \vec{c} = \lambda(\vec{a} \times \vec{b}) \cdot \vec{c}$

$\Rightarrow \frac{1}{3} = \lambda$

Also $|\vec{c}|^2 = \lambda^2 |\vec{a} \times \vec{b}|^2$

$\Rightarrow \frac{1}{3} = \frac{1}{9}(a^2 b^2 \sin^2\theta) = \frac{1}{9} \times 2 \times 3 \sin^2\theta$

$\Rightarrow \sin^2\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{4}$

74 **(a)**

Given that $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are vectors such that

$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$

P_1 is the plane determined by vectors a and b.

Therefore, normal vectors $\vec{n}_1$ to P_1 will be given by $\vec{n}_1 = \vec{a} \times \vec{b}$

Similarly, P_2 is the plane determined by vectors $\vec{c}$ and $\vec{d}$. Therefore, normal vectors $\vec{n}_2$ to P_2 will be given by

$\vec{n}_2 = \vec{c} \times \vec{d}$

Substituting the values of $\vec{n}_1$ and $\vec{n}_2$ in (i), we get

$\vec{n}_1 \times \vec{n}_2 = \vec{0}$

Hence, $\vec{n}_1 || \vec{n}_2$

Hence, the planes will also be parallel to each other

Thus angle between the planes $= 0$

76 **(c)**

Any vector $\vec{r}$ can be represented in terms of three non-coplanar vectors $\vec{a}, \vec{b}$ and $\vec{c}$ as

$\vec{r} = x(\vec{a} \times \vec{b}) + y(\vec{b} \times \vec{c}) + z(\vec{c} \times \vec{a})$

Taking dot product with $\vec{a}, \vec{b}$ and $\vec{c}$, respectively, we have,

$x = \frac{\vec{r} \cdot \vec{c}}{[\vec{a}\vec{b}\vec{c}]}, y = \frac{\vec{r} \cdot \vec{a}}{[\vec{a}\vec{b}\vec{c}]}$ and $z = \frac{\vec{r} \cdot \vec{b}}{[\vec{a}\vec{b}\vec{c}]}$

From (i)

$[\vec{a}\vec{b}\vec{c}]\vec{r} = \frac{1}{2}(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a})$

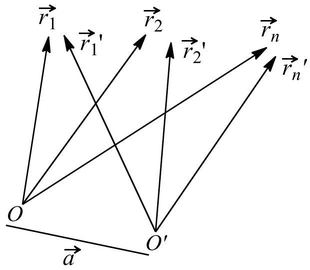
$\therefore$ Area of ΔABC

$= \frac{1}{2}|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$

$= |[\vec{a}\vec{b}\vec{c}]\vec{r}|$

77 **(c)**

Given $a_1\vec{r_1} + a_2\vec{r}_2 + \ldots + a_n\vec{r}_n = 0$



Now $\vec{a} + \vec{r}_1 = \vec{r}_1$ and so on

Hence $a_1(\vec{a} + \vec{r}_1) + a_2(\vec{a} + \vec{r}_2) + \ldots + a_n(\vec{a} + \vec{r}_n) = 0$

$a_1\vec{r}_1 + a_2\vec{r}_2 + \ldots + a_n\vec{r}_n + \vec{a}(a_1 + a_2 + \ldots + a_n) = 0$

Hence $a_1\vec{r}_1 + a_2\vec{r}_2 + \ldots + a_n\vec{r}_n = 0$ if $a_1 + a_2 + \ldots + a_n = 0$

78 **(a)**

A vector perpendicular to the plane of O, P and Q is $\overrightarrow{OP} \times \overrightarrow{OQ}$

Now, $\overrightarrow{OP} \times \overrightarrow{OQ} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & 1 & \lambda \\ 2 & -1 & \lambda \end{vmatrix} = 2\lambda\hat{\imath} - 2\lambda\hat{\jmath} - 6\hat{k}$

Therefore, $\hat{\imath} - \hat{\jmath} + 6\hat{k}$is parallel to $2\lambda\hat{\imath} - 2\lambda\hat{\jmath} - 6\hat{k}$

Hence $\frac{1}{2\lambda} = \frac{-1}{-2\lambda} = \frac{6}{-6}$

$\lambda = -\frac{1}{2}$

79 **(a)**

$[\vec{a} + (\vec{a} \times \vec{b})\vec{b} + (\vec{a} \times \vec{b})\vec{a} \times \vec{b}]$

$= (\vec{a} + (\vec{a} \times \vec{b})) \cdot ((\vec{b} + (\vec{a} \times \vec{b})) \times (\vec{a} \times \vec{b}))$

$= (\vec{a} + (\vec{a} \times \vec{b})) \cdot (\vec{b} \times (\vec{a} \times \vec{b}))$

$= (\vec{a} + (\vec{a} \times \vec{b})) \cdot (\vec{a} - (\vec{a} \cdot \vec{b})\vec{b})$

$= \vec{a} \cdot \vec{a} = 1$ (as $\vec{a} \cdot \vec{b} = 0, \vec{a} \cdot (\vec{a} \times \vec{b}) = 0$)

80 **(c)**

Given that $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar

$\Rightarrow [\vec{a}\vec{b}\vec{c}] \neq 0$

Again $\vec{a} \times (\vec{b} \times \vec{c}) \cdot (\vec{a} \times \vec{c}) = 0$

$\Rightarrow [(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}] \cdot (\vec{a} \times \vec{c}) = 0$

$\Rightarrow (\vec{a} \cdot \vec{c})[\vec{b}\vec{a}\vec{c}] = 0$

$\Rightarrow (\vec{a} \cdot \vec{c}) = 0$

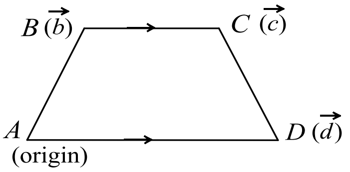
$\Rightarrow \vec{a}$ and $\vec{c}$ are perpendicular

$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

$\Rightarrow [\vec{a} \times (\vec{b} \times \vec{c})] \times \vec{c} = \vec{0}$

81 **(d)**

$\vec{c} - \vec{b} = \alpha\vec{d}$ and $\vec{p} = \overrightarrow{AC} + \overrightarrow{BD} = \mu\overrightarrow{AD}$



Hence $\vec{p} = \vec{c} + \vec{d} - \vec{b} = \mu\vec{d}$(using $\vec{c} - \vec{b} = \alpha\vec{d}$)

or $\alpha + 1 = \mu$

82 **(b)**

Since, $\overrightarrow{OA} = \hat{\imath} + \hat{\jmath} + \hat{k}$

$\overrightarrow{OB} = \hat{\imath} + 5\hat{\jmath} - \hat{k}$

$\overrightarrow{OC} = 2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$

$a = BC = |\overrightarrow{BC}| = |\overrightarrow{OC} - \overrightarrow{OB}| = |\hat{\imath} - 2\hat{\jmath} + 6\hat{k}|$

$= \sqrt{41}$

$b = CA = |\overrightarrow{CA}| = |\overrightarrow{OA} - \overrightarrow{OC}| = |-\hat{\imath} - 2\hat{\jmath} - 4\hat{k}|$

$= \sqrt{21}$

and $c = AB = |\overrightarrow{AB}| = |\overrightarrow{OB} - \overrightarrow{OA}| = |0\hat{\imath} + 4\hat{\jmath} - 2k=20$

Since $a > b > c, A$ is the greatest angle. Therefore,

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc} = \dfrac{21 + 20 - 41}{2 \cdot \sqrt{21} \cdot \sqrt{20}} = 0$

$\therefore \angle A = 90°$

83 **(a)**

Let $\vec{d} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Where $x^2 + y^2 + z^2 = 1$ (i)

($\vec{d}$ being a unit vector)

$\therefore \vec{a}.\vec{d} = 0$

$\Rightarrow x - y = 0$ or $x = y$ (ii)

$[\vec{b}\vec{c}\vec{d}] = 0$

$$\Rightarrow \begin{vmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ x & y & z \end{vmatrix} = 0$$

Or $x + y + z = 0$

Or $2x + z = 0$ [using (ii)]

Or $z = -2x$ (iii)

From (i), (ii) and (iii), we have

$x^2 + x^2 + 4x^2 = 1$

$x = \pm\dfrac{1}{\sqrt{6}}$

$\therefore \vec{d} = \pm\left(\dfrac{1}{\sqrt{6}}\hat{\imath} + \dfrac{1}{\sqrt{6}}\hat{\jmath} - \dfrac{2}{\sqrt{6}}\hat{k}\right)$

$= \pm\left(\dfrac{\hat{\imath} + \hat{\jmath} - 2\hat{k}}{\sqrt{6}}\right)$

84 **(c)**

$\vec{\alpha} = \vec{a} + \vec{b} + \vec{c} = 6\hat{\imath} + 12\hat{\jmath}$

Let $\vec{\alpha} = x\vec{a} + y\vec{b} \Rightarrow 6x + 2y = 6$

and $-3x - 6y = 12$

$\therefore x = 2, y = -3$

$\therefore \vec{\alpha} = 2\vec{a} - 3\vec{b}$

85 **(c)**

$\vec{r} \times \vec{a} = \vec{b}$

$\Rightarrow \vec{d} \times (\vec{r} \times \vec{a}) = \vec{d} \times \vec{b}$

$\Rightarrow (\vec{a} \cdot \vec{d})\vec{r} - (\vec{d} \cdot \vec{r})\vec{a} = \vec{d} \times \vec{b}$

$\vec{r} \times \vec{c} = \vec{d}$

$\Rightarrow \vec{b} \times (\vec{r} \times \vec{c}) = \vec{b} \times \vec{d}$

$\Rightarrow (\vec{b} \cdot \vec{c})\vec{r} - (\vec{b} \cdot \vec{r})\vec{c} = \vec{b} \times \vec{d}$

Adding (i) and (ii) we get

$(\vec{a} \cdot \vec{d} + \vec{b} \cdot \vec{c})\vec{r} - (\vec{d} \cdot \vec{r})\vec{a} - (\vec{b} \cdot \vec{r})\vec{c} = \vec{0}$

Now $\vec{r} \cdot \vec{d} = 0$and $\vec{b} \cdot \vec{r} = 0$ as$\vec{d}$and $\vec{r}$ as well as $\vec{b}$ and $\vec{r}$ are mutually perpendicular

Hence, $(\vec{b} \cdot \vec{c} + \vec{a} \cdot \vec{d})\vec{r} = \vec{0}$

86 **(d)**

Let $\vec{a} = 6\hat{\imath} + 6\hat{k}, \vec{b} = 4\hat{\jmath} + 2\hat{k}, \vec{c} = 4\hat{\jmath} - 8\hat{k}$

Then $\vec{a} \times \vec{b} = -24\hat{\imath} - 12\hat{\jmath} + 24\hat{k}$

$= 12(-2\hat{\imath} - \hat{\jmath} + 2\hat{k})$

$\therefore$Area of the base of the parallelepiped $= \frac{1}{2}|\vec{a} \times \vec{b}|$

$= \dfrac{1}{2}(12 \times 3)$

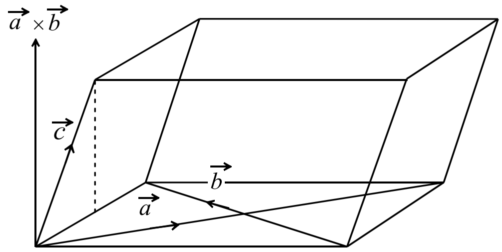
$= 18$

Height of the parallelepiped=length of projection of $\vec{c}$ on $\vec{a} \times \vec{b}$

$= \dfrac{|\vec{c} \cdot \vec{a} \times \vec{b}|}{|\vec{a} \times \vec{b}|}$

$= \dfrac{|12(-4 - 16)|}{36}$

$= \dfrac{20}{3}$



$\therefore$ Volume of the parallelepiped $= 18 \times \frac{20}{3} = 120$

87 **(d)**

$\Delta = \frac{1}{2}\left|(\hat{j}+\lambda\hat{k})\times(\hat{i}+\lambda\hat{k})\right| = \frac{1}{2}\left|-\hat{k}+\lambda\hat{i}+\lambda\hat{j}\right|$

$= \frac{1}{2}\sqrt{2\lambda^2+1}$

$\Rightarrow \frac{9}{4} \le \frac{1}{4}(2\lambda^2+1) \le \frac{33}{4}$

$\Rightarrow 4 \le \lambda^2 \le 16$

$\Rightarrow 2 \le |\lambda| \le 4$

88 **(d)**

The angle between $\vec{a}$ and $\vec{b}$ is obtuse. Therefore,

$\vec{a}\cdot\vec{b} < 0$

$\Rightarrow 14x^2 - 8x + x < 0$

$\Rightarrow 7x(2x-1) < 0$

$\Rightarrow 0 < x < 1/2$ (i)

The angle between $\vec{b}$ and the z-$axis$ is acute and less than $\pi/6$. Therefore,

$\frac{\vec{b}\cdot\vec{k}}{|\vec{b}||\vec{k}|} > \cos\pi/6$ ($\because \theta < \pi/6 \Rightarrow \cos\theta > \cos\pi/6$)

$\Rightarrow \frac{x}{\sqrt{x^2+53}} > \frac{\sqrt{3}}{2}$

$\Rightarrow 4x^2 > 3x^2 + 159$

$\Rightarrow x^2 > 159$

$\Rightarrow x > \sqrt{159}$or $x < -\sqrt{159}$ (ii)

Clearly, (i) and (ii) cannot hold together

89 **(b)**

Here $[\vec{a}\vec{b}\vec{c}] = \pm 1$

$[\vec{a}+\vec{b}+\vec{c}\vec{a}+\vec{b}\vec{b}+\vec{c}]$

$= (\vec{a}+\vec{b}+\vec{c})\times(\vec{a}+\vec{b})\cdot(\vec{b}+\vec{c})$

$= \vec{c}\times(\vec{a}+\vec{b})\cdot(\vec{b}+\vec{c})$

$= (\vec{c}\times\vec{a}+\vec{c}\times\vec{b})\cdot(\vec{b}+\vec{c})$

$= \vec{c}\times\vec{a}\cdot\vec{b} = [\vec{a}\vec{b}\vec{c}] = \pm 1$

90 **(a)**

Let the incentre be at the origin and be $A(\vec{p}), B(\vec{q})$ and $C(\vec{r})$. Then

$\overrightarrow{IA} = \vec{p}, \overrightarrow{IB} = \vec{q}$ and $\overrightarrow{IC} = \vec{r}$

Incentre I is $\frac{a\vec{p}+b\vec{q}+c\vec{r}}{a+b+c}$, where $p = BC, q = AC$ and $r = AB$

Incentre is at the origin. Therefore,

$\frac{a\vec{p}+b\vec{q}+c\vec{r}}{a+b+c} = \vec{0}$, or $a\,\vec{p}+b\vec{q}+c\vec{r} = \vec{0}$

$\Rightarrow a\,\overrightarrow{IA} + b\,\overrightarrow{IB} + c\,\overrightarrow{IC} = \vec{0}$

91 **(b)**

$|(\vec{a}\times\vec{b})\times\vec{c}| = |\vec{a}\times\vec{b}||\vec{c}|\sin 30°$

$= \frac{1}{2}|\vec{a}\times\vec{b}||\vec{c}|$ (i)

We have, $\vec{a} = 2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b} = \hat{i}+\hat{j}$

$\Rightarrow \vec{a}\times\vec{b} = 2\hat{i}-2\hat{j}+\hat{k}$

$\Rightarrow |\vec{a}\times\vec{b}| = \sqrt{9} = 3$

Also given $|\vec{c}-\vec{a}| = 2\sqrt{2}$

$\Rightarrow |\vec{c}-\vec{a}|^2 = 8$

$\Rightarrow |\vec{c}|^2 + |\vec{a}|^2 - 2\,\vec{a}\cdot\vec{c} = 8$

Given $|\vec{a}| = 3$ and $\vec{a}\cdot\vec{c} = |\vec{c}|$, using these we get

$|\vec{c}|^2 - 2|\vec{c}| + 1 = 0$

$\Rightarrow (|\vec{c}|-1)^2 = 0$

$\Rightarrow |\vec{c}| = 1$

Substituting values of $|\vec{a}\times\vec{b}|$ and $|\vec{c}|$ in (i), we get

$|(\vec{a}\times\vec{b})\times\vec{c}| = \frac{1}{2}\times 3\times 1 = \frac{3}{2}$

92 **(b)**

$\hat{a}, \hat{b}$ and $\hat{c}$ are unit vectors

Now $x = |\hat{a}-\hat{b}|^2 + |\hat{b}-\hat{c}|^2 + |\hat{c}-\hat{a}|^2$

$= \frac{1}{2}(\hat{a}\cdot\hat{a}+\hat{b}\cdot\hat{b}+\hat{c}\cdot\hat{c}) - 2\hat{a}\cdot\hat{b} - 2\hat{c} - 2\hat{c}\cdot\hat{a}$

$\Rightarrow 6 - 2(\hat{a}\cdot\hat{b}+\hat{b}\cdot\hat{c}+\hat{c}\cdot\hat{a})$

Also, $|\hat{a}+\hat{b}+\hat{c}| \ge 0$

$\Rightarrow \hat{a}\cdot\hat{a}+\hat{b}\cdot\hat{b}+\hat{c}\cdot\hat{c}+2(\hat{a}\cdot\hat{b}+\hat{b}\cdot\hat{c}+\hat{c}\cdot\hat{a}) \ge 0$

$\Rightarrow 3 + 2(\hat{a}\cdot\hat{b}+\hat{b}\cdot\hat{c}+\hat{c}\cdot\hat{a}) \ge 0$

$\Rightarrow 2(\hat{a}\cdot\hat{b}+\hat{b}\cdot\hat{c}+\hat{c}\cdot\hat{a}) \ge -3$

$\Rightarrow -2(\hat{a}\cdot\hat{b}+\hat{b}\cdot\hat{c}+\hat{c}\cdot\hat{a}) \le 3$

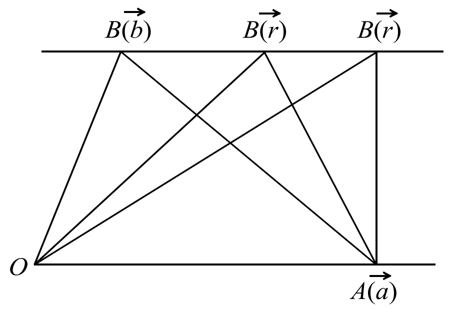
$\Rightarrow 6 - 2(\hat{a}\cdot\hat{b}+\hat{b}\cdot\hat{c}+\hat{c}\cdot\hat{a}) \le 9$

From (i) and (ii), $x \le 9$

Therefore, x does not exceed 9

93 **(c)**

$|\vec{a}\times\vec{r}| = |\vec{c}|$



Triangles on the same base and between the same parallel will have the same area

94 **(a)**

Three points $A(\vec{a}), B(\vec{b}), C(\vec{c})$ are collinear if

$\overrightarrow{AB} || \overrightarrow{AC}$

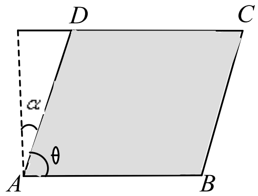
$\overrightarrow{AB} = -20\hat{i} - 11\hat{j}; \overrightarrow{AC} = (a-60)\hat{i} - 55\hat{j}$

$\Rightarrow \overrightarrow{AB}||\overrightarrow{AC} \Rightarrow \frac{a-60}{-20} = \frac{-55}{-11} \Rightarrow a = -40$

95 **(b)**

$\overrightarrow{\mathbf{AB}} = 2\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$

$\overrightarrow{\mathbf{AD}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$



$\overrightarrow{\mathbf{AB}} \cdot \overrightarrow{\mathbf{AD}} = -2 + 20 + 22 = 40$

$|\overrightarrow{\mathbf{AB}}| = \sqrt{4+100+120} = \sqrt{225} = 15$

$|\overrightarrow{\mathbf{AD}}| = \sqrt{1+4+4} = \sqrt{9} = 3$

$\therefore \cos\theta = \frac{40}{45} = \frac{8}{9}$

$\therefore \theta + \alpha = 90°$

$\Longrightarrow \alpha = 90° - \theta$

$\Longrightarrow \cos\alpha = \sin\theta = \sqrt{1 - \frac{64}{81}} = \frac{\sqrt{17}}{9}$

96 **(c)**

$|\vec{a} + \vec{b} + \vec{c}|^2 = 6$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 6$

$\Rightarrow |\vec{a}| = |\vec{b}| = |\vec{c}|$ and $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cdot\cos\frac{\pi}{3}$

i.e. $\vec{a}\cdot\vec{b} = \frac{1}{2}|\vec{a}|^2$

$\therefore 3|\vec{a}|^2 + 3|\vec{a}|^2 = 6$

$\Rightarrow |\vec{a}|^2 \Rightarrow |\vec{a}| = 1$

97 **(a)**

$[\vec{a}\times\vec{b}\,\vec{a}\times\vec{c}\,\vec{d}]$

$= (\vec{a}\times\vec{b})\cdot((\vec{a}\times\vec{c})\times\vec{d})$

$= (\vec{a}\times\vec{b})\cdot((\vec{a}\cdot\vec{d})\vec{c} - (\vec{c}\cdot\vec{d})\vec{a})$

$= (\vec{a}.\vec{d})[\vec{a}\vec{b}\vec{c}]$

98 **(c)**

$(\vec{a}\times\vec{b})\times(\vec{b}\times\vec{c}) = \vec{b}$

$\Rightarrow [\vec{a}\vec{b}\vec{c}]\vec{b} = \vec{b}$

$\Rightarrow [\vec{a}\vec{b}\vec{c}] = 1$

$\therefore \vec{a}, \vec{b}$ and $\vec{c}$ cannot be coplanar

99 **(d)**

$|\vec{a}.\vec{b}| = |\vec{a}\times\vec{b}|$

$\Rightarrow |\vec{a}||\vec{b}||\cos\theta| = |\vec{a}||\vec{b}||\sin\theta|$(where θ is the angle between $\vec{a}$ and $\vec{b}$)

$\Rightarrow |\cos\theta| = |\sin\theta|$

$\Rightarrow \theta = \frac{\pi}{4}$ or $\frac{3\pi}{4}$ (as $0 \le \theta \le \pi$)

But $\vec{a}\cdot\vec{b} < 0$,therefore $\theta = \frac{3\pi}{4}$

100 **(c)**

$\vec{d}\cdot\hat{a} = \vec{d}\cdot\hat{b} = \vec{d}\cdot\hat{c}$

$\Rightarrow \lambda(\hat{a}\cdot\hat{b} + \hat{a}\cdot\hat{c}) = \lambda(1 + \hat{b}\cdot\hat{c}) = \lambda(1 + \hat{b}\cdot\hat{c})$

$\Rightarrow 1 + \hat{b}\cdot\hat{c} = \hat{a}\cdot\hat{b} + \hat{a}\cdot\hat{c}$

$\Rightarrow 1 - \hat{a}\cdot\hat{b} + \hat{b}\cdot\hat{c} - \hat{a}\cdot\hat{c} = 0$

$\Rightarrow 1 - \hat{a}\cdot\hat{b} + (\hat{b} - \hat{a})\cdot\hat{c} = 0$

$\Rightarrow \hat{a}\cdot(\hat{a} - \hat{b}) + (\hat{b} - \hat{a})\cdot\hat{c} = 0$

$\Rightarrow (\hat{a} - \hat{c})\cdot(\hat{a} - \hat{b}) = 0 \Rightarrow \hat{a} - \hat{c}$ is perpendicular to $(\hat{a} - \hat{b})$ ⇒The triangle is right angled

101 **(b)**

If $\vec{a}(x)$and$\vec{b}(x)$are ⊥, then $\vec{a}.\vec{b} = 0$

$\Rightarrow \sin x\cos 2x + \cos x\sin 2x = 0$

$\sin(3x) = 0 = \sin 0$

$3x = n\pi \Rightarrow x = \frac{n\pi}{3}$

Therefore, the two vectors are ⊥ for infinite values of 'x'

102 **(c)**

$\vec{a}\times(\hat{\imath} + 2\hat{\jmath} + \hat{k}) = \hat{\imath} - \hat{k} = (\hat{\jmath}\times(\hat{\imath} + 2\hat{\jmath} + \hat{k}))$

$\Rightarrow (\vec{a} - \hat{\jmath})\times(\hat{\imath} + 2\hat{\jmath} + \hat{k}) = \vec{0}$

$\Rightarrow \vec{a} - \hat{\jmath} = \lambda(\hat{\imath} + 2\hat{\jmath} + \hat{k})$

$\Rightarrow \vec{a} = \lambda\hat{\imath} + (2\lambda + 1)\hat{\jmath} + \lambda\hat{k}, \lambda \in R$

103 **(d)**

Volume of the parallelepiped formed by $\vec{a}', \vec{b}'$ and $\vec{c}'$ is 4

Therefore, the volume of the parallelepiped formed by $\vec{a}, \vec{b}$and $\vec{c}$ is $\frac{1}{4}$

$\vec{b}\times\vec{c} = [\vec{a}\vec{b}\vec{c}]\vec{a}' = \frac{1}{4}\vec{a}'$

$|\vec{b}\times\vec{c}| = \frac{\sqrt{2}}{4} = \frac{1}{2\sqrt{2}}$

Length of altitude $= \frac{1}{4}\times 2\sqrt{2} = \frac{1}{\sqrt{2}}$

104 **(a)**

$\hat{a} = \frac{-4i + 3\hat{k}}{5}; \hat{b} = \frac{14\hat{\imath} + 2\hat{\jmath} - 5\hat{k}}{15}$

A vector $\vec{V}$ bisecting the angle between $\vec{a}$ and $\vec{b}$ is $\vec{V} = \hat{a} + \hat{b}$

$= \dfrac{-12\hat{\imath} + 9\hat{k} + 14\hat{\imath} + 2\hat{\jmath} - 5\hat{k}}{15}$

$= \dfrac{2\hat{\imath} + 2\hat{\jmath} + 4\hat{k}}{15}$

Required vector $\vec{d} = \sqrt{6}\hat{V} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$

105 **(d)**

$\vec{a} \perp \vec{b} \Rightarrow x - y + 2 = 0$

$\vec{a} \cdot \vec{c} = 4 \Rightarrow x + 2y = 4$

Solving we get $x = 0; y = 2$

$\Rightarrow \vec{a} = 2\hat{\jmath} + 2\hat{k}$

$\Rightarrow [\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} 0 & 2 & 2 \\ 1 & -1 & 1 \\ 1 & 2 & 0 \end{vmatrix} = 8 = |\vec{a}|^2$

106 **(b)**

$|\vec{a} + \vec{b} + 3\vec{c}|^2 = 16$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 9|\vec{c}|^2 + 2\cos\theta_1 + 6\cos\theta_2 + 6\cos\theta_3 = 16, \theta_3 \in [\pi/6, 2\pi/3]$

$\Rightarrow 2\cos\theta_1 + 6\cos\theta_2 = 5 - 6\cos\theta_3$

$\Rightarrow (\cos\theta_1 + 3\cos\theta_2)_{\max} = 4$

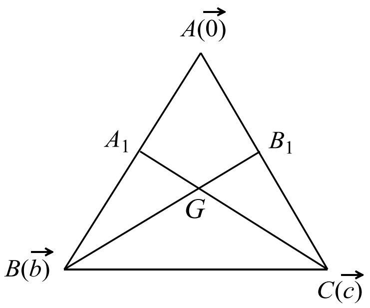
107 **(c)**

$k = |2(\vec{a} \times \vec{b})| + |3(\vec{a} \cdot \vec{b})|$

$= 12\sin\theta + 18\cos\theta$

$\Rightarrow$ maximum vlue of k is $\sqrt{12^2 + 18^2} = 6\sqrt{13}$

108 **(b)**



Let P.V. of A, B and Cbe$\vec{0}, \vec{b}$ and $\vec{c}$, respectively.Therefore

$\vec{G} = \dfrac{\vec{b} + \vec{c}}{3}$

$\vec{A}_1 = \dfrac{\vec{b}}{2}, \vec{B}_1 = \dfrac{\vec{c}}{2}$

$\Delta_{AB_1G} = \dfrac{1}{2}|\overrightarrow{AG} \times \overrightarrow{AB}_1| = \dfrac{1}{2}\left|\dfrac{\vec{b} + \vec{c}}{3} \times \left(\dfrac{\vec{c}}{2}\right)\right|$

$= \dfrac{1}{12}|\vec{b} \times \vec{c}|$

$\Delta_{AA_1G} = \dfrac{1}{2}|\overrightarrow{AG} \times \overrightarrow{AA}_1| = \dfrac{1}{2}\left|\dfrac{\vec{b} + \vec{c}}{3} \times \left(\dfrac{\vec{b}}{2}\right)\right|$

$= \dfrac{1}{12}|\vec{b} \times \vec{c}|$

$\Rightarrow \Delta_{GA_1AB_1} = \dfrac{1}{6}|\vec{b} \times \vec{c}| = \dfrac{1}{3} \cdot \dfrac{1}{2}|\vec{b} \times \vec{c}| = \dfrac{1}{3}\Delta_{ABC}$

$\Rightarrow \dfrac{\Delta}{\Delta_1} = 3$

109 **(c)**

Volume of parallelopiped,

$f(a) = \begin{vmatrix} 1 & a & 1 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 1 + a^3 - a$

Now, $f'(a) = 3a^2 - 1$

$\Longrightarrow f''(a) = 6a$

Put $f'(a) = 0$

$\Longrightarrow a \neq \pm\dfrac{1}{\sqrt{3}}$

Which shows $f(a)$ is maximum at

$a = \dfrac{1}{\sqrt{3}}$ and maximum at

$a = -\dfrac{1}{\sqrt{3}}$

110 **(c)**

$(\hat{a} + \hat{b} + \hat{c})^2 \geq 0$

$3 + 2(\hat{a} \cdot \hat{b} + \vec{b} \cdot \hat{c} + \vec{c} \cdot \vec{a}) \geq 0$

$3 + 6\cos\theta \geq 0$

$\cos\theta \geq -\dfrac{1}{2}$

$\Rightarrow \theta = \dfrac{2\pi}{3}$

111 **(a)**

Four or more than four non-zero vectors are always linearly dependent

112 **(a)**

$\{\vec{a} \times (\vec{b} + \vec{a} \times \vec{b})\} \cdot \vec{b}$

$= \{\vec{a} \times \vec{b} + \vec{a} \times (\vec{a} \times \vec{b})\} \cdot \vec{b}$

$= [\vec{a}\vec{b}\vec{b}] + \{(\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b}\} \cdot \vec{b}$

$= 0 + (\vec{a} \cdot \vec{b})^2 - (\vec{a} \cdot \vec{a})(\vec{b} \cdot \vec{b})$

$= \cos^2\dfrac{\pi}{3} - 1 = -\dfrac{3}{4}$

113 **(a)**

$\vec{a} - \vec{p} + \vec{b} - \vec{p} + \vec{c} - \vec{p} = 0$

$\Rightarrow \vec{p} = \dfrac{\vec{a} + \vec{b} + \vec{c}}{3}$

$\Rightarrow P$ is centroid

114 **(c)**

$\vec{d} \cdot \vec{c} = \vec{d} \cdot \vec{b} = \vec{d} \cdot \vec{c} = [\vec{a}\vec{b}\vec{c}]$

Then $|(\vec{d} \cdot \vec{c})(\vec{a} \times \vec{b}) + (\vec{d} \cdot \vec{a})(\vec{b} \times \vec{c}) +$

$d \cdot bc \times a = 0$

$\Rightarrow [\vec{a}\vec{b}\vec{c}]|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}| = 0$

$\Rightarrow [\vec{a}\vec{b}\vec{c}] = 0$ ($\because \vec{d}$ is non-zero)

$\Rightarrow \vec{a}, \vec{b}, \vec{c}$ are coplanar

115 **(a)**

$\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$

$= 5(\hat{\imath}+2\hat{\jmath}+2\hat{k}) - 6(\hat{\imath}+\hat{\jmath}+2\hat{k})$

$\Rightarrow (1+\alpha)\hat{\imath} + \beta(1+\alpha)\hat{\jmath} + \gamma(1+\alpha)(1+\beta)\hat{k}$

$= -\hat{\imath}+4\hat{\jmath}-2\hat{k}$

$\Rightarrow 1+\alpha = -1, \beta = -4$ and $\gamma(-1)(-3) = -2$

$\Rightarrow \gamma = -\frac{2}{3}$

116 **(a)**

$\vec{\alpha}||(\vec{\beta}\times\vec{\gamma}) \Rightarrow \vec{\alpha}\perp\vec{\beta}$ and $\vec{\alpha}\perp\vec{\gamma}$

Now, $(\vec{\alpha}\times\vec{\beta})\cdot(\vec{\alpha}\times\vec{\gamma}) = |\vec{\alpha}|^2(\vec{\beta}\cdot\vec{\gamma}) -$

$(\vec{\alpha}\cdot\vec{\beta})(\vec{\alpha}\cdot\vec{\gamma}) = |\vec{\alpha}|^2\cdot(\vec{\beta}\cdot\vec{\gamma})$

117 **(c)**

If $\vec{x} = \vec{y} \Rightarrow \hat{a}\cdot\vec{x} = \hat{a}\cdot\vec{y}$. This equality must hold for any arbitrary $\hat{a}$

118 **(a)**

A vector coplanar with $\vec{a}$ and $\vec{b}$ and perpendicular to $\vec{c}$ is $\lambda\left((\vec{a}\times\vec{b})\times\vec{c}\right)$

But $\lambda\left((\vec{a}\times\vec{b})\times\vec{c}\right) = \lambda[(\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a}]$

$= \lambda[4\vec{b} - 4\vec{a}]$

$= 4\lambda[\hat{\jmath} - \hat{k}]$

Now $4|\lambda|\sqrt{2} = \sqrt{2}$(Given)$\Rightarrow \lambda = \pm\frac{1}{4}$

Hence the required vector is $\hat{\jmath}-\hat{k}$ or $-\hat{\jmath}+\hat{k}$

119 **(d)**

Given that $\vec{a} = \hat{\imath}+\hat{\jmath}+\hat{\jmath}+\vec{k}, \vec{b} = 4\hat{\imath}+3\hat{\jmath}+4\hat{k}$ and $\vec{c} = \hat{\imath}+\alpha\hat{\jmath}+\beta\hat{k}$ are linearly dependent

$$\begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{vmatrix} = 0$$

$\Rightarrow 1-\beta = 0$

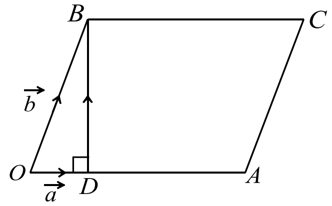
$\Rightarrow \beta = 1$

Also given that $|\vec{c}| = \sqrt{3} \Rightarrow 1+\alpha^2+\beta^2 = 3$

Substituting the value of β, we get $\alpha^2 = 1$

$\Rightarrow \alpha = \pm 1$

120 **(c)**



Let $\overrightarrow{OD} = t\vec{a}$

$\therefore \overrightarrow{DB} = \vec{b} - t\vec{a}$

$\therefore (\vec{b}-t\vec{a})\cdot\vec{a} = 0$ ($\because \overrightarrow{DB}\perp\overrightarrow{OA}$)

$\Rightarrow t = \frac{\vec{b}\cdot\vec{a}}{|\vec{a}|^2}$

$\therefore \overrightarrow{DB} = \vec{b} - \frac{(\vec{b}\cdot\vec{a})\vec{a}}{|\vec{a}|^2}$

121 **(a)**

Let $\vec{v} = x\vec{a} + y\vec{b} + z\vec{a}\times\vec{b}$

Given : $\vec{a}\cdot\vec{b} = 0, \vec{v}\cdot\vec{a} = 0, \vec{v}\cdot\vec{b} = 1, [\vec{v}\vec{a}\vec{b}] = 1$

$\Rightarrow \vec{v}\cdot\vec{a} = x\vec{a}\cdot\vec{a} = x|\vec{a}|^2$ ($\because \vec{a}\cdot\vec{b} = 0, \vec{a}\cdot\vec{a}\times\vec{b} = 0$)

$\Rightarrow x = 0$

Again, $\vec{v}\cdot\vec{b} = y|\vec{b}|^2 \Rightarrow 1 = yb^2$

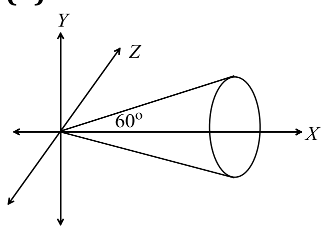
$\therefore y = \frac{1}{b^2}$

Again $\vec{v}\cdot(\vec{a}\times\vec{b}) = z(\vec{a}\times\vec{b})^2$

$\Rightarrow 1 = z(\vec{a}\times\vec{b})^2 \Rightarrow z = \frac{1}{|\vec{a}\times\vec{b}|^2}$

Hence, $\vec{v} = \frac{1}{|\vec{b}|^2}\vec{b} + \frac{1}{|\vec{a}\times\vec{b}|^2}\vec{a}\times\vec{b}$

122 **(b)**



from the diagram, it is obvious that locus is a cone concentric with the positive x-axis having vertex at the origin and the slant height equal to the magnitude of the vector

123 **(a)**

$\overrightarrow{\mathbf{PQ}} = 6\hat{\mathbf{i}} + \hat{\mathbf{j}}$l

$\overrightarrow{\mathbf{QR}} = -\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$

$\overrightarrow{\mathbf{RS}} = -6\hat{\mathbf{i}} - \hat{\mathbf{j}}$

$\overrightarrow{\mathbf{SP}} = \hat{\mathbf{i}} - 3\hat{\mathbf{j}}$

$|\overrightarrow{\mathbf{PQ}}| = \sqrt{37} = |\overrightarrow{\mathbf{RS}}|$

$|\overrightarrow{\mathbf{QR}}| = \sqrt{10} = |\overrightarrow{\mathbf{SP}}|$

$\overrightarrow{\mathbf{PQ}} \cdot \overrightarrow{\mathbf{QR}} = -6 + 3 = -3 \neq 0$

$\overrightarrow{\mathbf{PQ}}$ = is not parallel to $\overrightarrow{\mathbf{RS}}$ and their magnitude are equal.

$\Longrightarrow$ Quadrilateral $PQRS$ must be a parallelogram, which is neither a rhombus nor a rectangle.

124 **(b)**

$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}_1 + \vec{\mathbf{a}} \cdot \left(\vec{\mathbf{b}} - \frac{\vec{\mathbf{b}} \cdot \vec{\mathbf{a}}}{|\vec{\mathbf{a}}|^2}\vec{\mathbf{a}}\right)$

$= \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} - \frac{|\vec{\mathbf{a}}|^2(\vec{\mathbf{b}} \cdot \vec{\mathbf{a}})}{|\vec{\mathbf{a}}|^2}$

$=\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} - \vec{\mathbf{b}} \cdot \vec{\mathbf{a}} = 0$

Similarly, $\vec{\mathbf{a}} \cdot \vec{\mathbf{c}}_2 = \vec{\mathbf{b}}_1 \cdot \vec{\mathbf{c}}_2 = 0$

Hence, $\{\vec{\mathbf{a}}, \vec{\mathbf{b}}_1, \vec{\mathbf{c}}_2\}$ are mutually orthogonal vectors.

125 **(c)**

Since $\vec{x}, \vec{y}$ and $\vec{x} \times \vec{y}$ are linearly independent,

$20a - 15b = 15b - 12c = 12c - 20a = 0$

$\Rightarrow \frac{a}{3} = \frac{b}{4} = \frac{c}{5}$

$\Rightarrow c^2 = a^2 + b^2$

Hence, ΔABC is right angled

126 **(b)**

Taking dot product of $a(\vec{\alpha} \times \vec{\beta}) + b(\vec{\beta} \times \vec{\gamma}) + c(\vec{\gamma} \times \vec{\alpha}) = 0$ with $\vec{\gamma}, \vec{\alpha}$ and $\vec{\beta}$, respectively, we have

$a[\vec{\alpha}\vec{\beta}\vec{\gamma}] = 0$

$b[\vec{\alpha}\vec{\beta}\vec{\gamma}] = 0$

$c[\vec{\alpha}\vec{\beta}\vec{\gamma}] = 0$

$\because$ At least one of a, b and $c \neq 0$

$\therefore [\vec{\alpha}\vec{\beta}\vec{\gamma} = 0]$

Hence $\vec{\alpha}, \vec{\beta}$ and $\vec{\gamma}$ are coplanar

127 **(d)**

For minimum value $|\vec{r} + b\vec{s}| = 0$

Let $\vec{r}$ and $\vec{s}$ are anti parallel so $b\vec{s} = -\vec{r}$

So $|b\vec{s}|^2 + |\vec{r} + b\vec{s}|^2 = |-\vec{r}|^2 + |\vec{r} - \vec{r}|^2 = |\vec{r}|^2$

128 **(c)**

$|\vec{a} + \vec{b} + \vec{c}|^2 = 1$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2|\vec{a}||\vec{b}|\cos\theta_1 + 2|\vec{b}||\vec{c}|\cos\theta_2 + 2|\vec{c}||\vec{a}|\cos\theta_3 = 1$$

$\Rightarrow \cos\theta_1 + \cos\theta_2 + \cos\theta_3 = -1$

$\Rightarrow$ One of θ_1, θ_2 and θ_3 should be an obtuse angle

129 **(b)**

$\vec{c} = m\vec{a} + n\vec{b} + p(\vec{a} \times \vec{b})$

Taking dot product with $\vec{a}$ and $\vec{b}$, we have

$m = n = \cos\theta$

$\Rightarrow |\vec{c}| = |\cos\theta\,\vec{a} + \cos\theta\,\vec{b} + p(\vec{a} \times \vec{b})| = 1$

Squaring both sides, we get

$\cos^2\theta + \cos^2\theta + p^2 = 1$

$\Rightarrow \cos\theta = \pm\frac{\sqrt{1-p^2}}{\sqrt{2}}$

Now $-\frac{1}{\sqrt{2}} \leq \cos\theta \leq \frac{1}{\sqrt{2}}$ (for real value of θ)

$\therefore \frac{\pi}{4} \leq \cos\theta \leq \frac{3\pi}{4}$

130 **(a)**

$\vec{b} - 2\vec{c} = \lambda\vec{a}$

$\Rightarrow \vec{b} = 2\vec{c} + \lambda\vec{a}$

$\Rightarrow |\vec{b}|^2 = |2\vec{c} + \lambda\vec{a}|^2$

$\Rightarrow 16 = 4|\vec{c}|^2 + \lambda^2|\vec{a}|^2 + 4\lambda\vec{a} \cdot \vec{c}$

$\Rightarrow 16 = 4 + \lambda^2 + 4\lambda\frac{1}{4}$

$\Rightarrow \lambda^2 + \lambda - 12 = 0$

$\Rightarrow \lambda = 3, -4$

131 **(c)**

$\vec{\mathbf{a}} \cdot \vec{\mathbf{a}} = 1 + 1 + 1 = 3$

Using,

$\vec{\mathbf{a}} \times (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) = (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{a}} - (\vec{\mathbf{a}} \cdot \vec{\mathbf{a}})\vec{\mathbf{b}}$

$\therefore (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) \times (\hat{\mathbf{j}} - \hat{\mathbf{k}}) = (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) - 3\vec{\mathbf{b}}$

$\Longrightarrow -2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}} - 3\vec{\mathbf{b}}$

$\Longrightarrow \vec{\mathbf{b}} = \hat{\mathbf{i}}$

132 **(c)**

$3 = \frac{1}{6}[\vec{a}\vec{b}\vec{c}]$

$\Rightarrow [\vec{a}\vec{b}\vec{c}] = 18$

Volume of the required parallelepiped

$= [\vec{a} + \vec{b}\vec{b} + \vec{c}\vec{c} + \vec{a}]$

$= 2[\vec{a}\vec{b}\vec{c}] = 36$

133 **(b)**

Let $\vec{a} \times \vec{b} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$. Therefore

$[\vec{a}\vec{b}\hat{\imath}] = (\vec{a} \times \vec{b}) \cdot \hat{\imath} = x$

$[\vec{a}\vec{b}\hat{\jmath}] = (\vec{a} \times \vec{b}) \cdot \hat{\jmath} = y$

$[\vec{a}\vec{b}\hat{k}] = (\vec{a} \times \vec{b}) \cdot \hat{k} = z$

Hence, $[\vec{a}\vec{b}\hat{\imath}]\hat{\imath} + [\vec{a}\vec{b}\hat{\jmath}]\hat{\jmath} = [\vec{a}\vec{b}\hat{k}]\hat{k} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} =$

$\vec{a} \times \vec{b}$

134 **(a)**

As $\vec{x}, \vec{y}$ and $\vec{x} \times \vec{y}$ are non-collinear vectors, vectors are linearly independent

$\Rightarrow a - b = 0 = b - c = c - a$

$\Rightarrow a = b = c$

Therefore, the triangle is equilateral

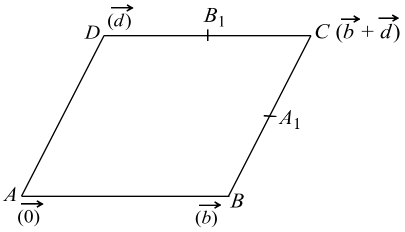
135 **(c)**

$[(\vec{a} \times \vec{b}) \times (\vec{b} \times \vec{c})(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a})(\vec{c} \times \vec{a}) \times (\vec{a} \times \vec{b})]$

$= \left[[\vec{a}\vec{b}\vec{c}]\vec{b}[\vec{a}\vec{b}\vec{c}]\vec{c}[\vec{a}\vec{b}\vec{c}] \cdot \vec{a}\right] = [\vec{a}\vec{b}\vec{c}]^3[\vec{b}\vec{c}\vec{a}]$

$= [\vec{a}\vec{b}\vec{c}]^4$

136 **(c)**



Let P. V. of A, B and D be $\vec{o}, \vec{b}$ and $\vec{d}$, respectively

Then P.V. of $C, \vec{c} = \vec{b} + \vec{d}$

Also P.V. of $A_1 = \vec{b} + \frac{d}{2}$

And P.V. of $B_1 = \vec{d} + \frac{\vec{b}}{2}$

$\Rightarrow \overrightarrow{AA_1} + \overrightarrow{AB_1} = \frac{3}{2}(\vec{b} + \vec{d}) = \frac{3}{2}\overrightarrow{AC}$

137 **(c)**

We have

$\overrightarrow{AB} \cdot \overrightarrow{AC} + \overrightarrow{BC} \cdot \overrightarrow{BA} + \overrightarrow{CA} \cdot \overrightarrow{CB}$

$= (AB)(AC)\cos\theta + (BC)(BA)\sin\theta + 0$

$= AB(AC\cos\theta + BC\sin\theta)$

$= AB\left(\frac{(AC)^2}{AB} + \frac{(BC)^2}{AB}\right)$

$= AC^2 + BC^2 = AB^2 = p^2$

138 **(a)**

$\vec{r} \times \vec{a} = \lambda\vec{a} + \mu\vec{b} + \gamma\vec{a} \times \vec{b}$

$\therefore [\vec{r}\vec{a}\vec{a}] = \lambda\vec{a} \cdot \vec{a} + \mu\vec{b} \cdot \vec{a} + \gamma[\vec{a}\vec{b}\vec{a}]$

$0 = \lambda|\vec{a}|^2 + 0 + 0$

$\lambda = 0$

Also $[\vec{r}\vec{a}\vec{b}] = \lambda\vec{a} \cdot \vec{b} + \mu\vec{b} \cdot \vec{b} + \gamma[\vec{a}\vec{b}\vec{b}] = \mu$

Also $(\vec{r} \times \vec{a}) \times \vec{b} = \gamma(\vec{a} \times \vec{b}) \times \vec{b}$

$\Rightarrow (\vec{r} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{b})\vec{r} = \gamma\{(\vec{a} \cdot \vec{b})\vec{b} - (\vec{b} \cdot \vec{b})\vec{a}\}$

$\Rightarrow (\vec{r} \cdot \vec{b})\vec{a} = -\gamma\vec{a}, \gamma = -(\vec{r} \cdot \vec{b})$

139 **(a)**

Since $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b}+\vec{c}}{\sqrt{2}}$

$\therefore (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \frac{1}{\sqrt{2}}\vec{b} + \frac{1}{\sqrt{2}}\vec{c}$

Since b and c are non-coplanar

$\Rightarrow \vec{a} \cdot \vec{c} = \frac{1}{\sqrt{2}}$ and $\vec{a} \cdot \vec{b} = -\frac{1}{\sqrt{2}}$

$\Rightarrow \cos\theta = -\frac{1}{\sqrt{2}}$

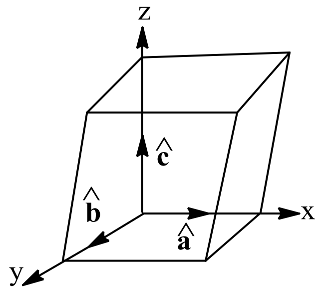
(because $\vec{a}$ and $\vec{b}$ are unit vectors)

Or $\theta = \frac{3\pi}{4}$

140 **(a)**

The volume of the parallelepiped with coterminous edges as $\hat{\mathbf{a}}, \hat{\mathbf{b}}, \hat{\mathbf{c}}$ is given by

$[\hat{\mathbf{a}}, \hat{\mathbf{b}}, \hat{\mathbf{c}}] = \hat{\mathbf{a}} \cdot (\hat{\mathbf{b}} \times \hat{\mathbf{c}})$



Now, $[\hat{\mathbf{a}}, \hat{\mathbf{b}}, \hat{\mathbf{c}}]^2 = \begin{vmatrix} \hat{\mathbf{a}} \cdot \hat{\mathbf{a}} & \hat{\mathbf{a}} \cdot \hat{\mathbf{b}} & \hat{\mathbf{a}} \cdot \hat{\mathbf{c}} \\ \hat{\mathbf{b}} \cdot \hat{\mathbf{a}} & \hat{\mathbf{b}} \cdot \hat{\mathbf{b}} & \hat{\mathbf{b}} \cdot \hat{\mathbf{c}} \\ \hat{\mathbf{c}} \cdot \hat{\mathbf{a}} & \hat{\mathbf{c}} \cdot \hat{\mathbf{b}} & \hat{\mathbf{c}} \cdot \hat{\mathbf{c}} \end{vmatrix}$

$= \begin{vmatrix} 1 & 1/2 & 1/2 \\ 1/2 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{vmatrix} = \frac{1}{2}$

$[\because |\vec{\mathbf{a}}| = |\vec{\mathbf{b}}| = |\vec{\mathbf{c}}| = 1]$

$\Rightarrow [\hat{\mathbf{a}}, \hat{\mathbf{b}}, \hat{\mathbf{c}}]^2 = \frac{1}{2}$

Thus, the required volume of the parallelepiped $= \frac{1}{\sqrt{2}}$ cu unit

141 **(c)**

$\vec{r} \times \vec{a} = \vec{b} \times \vec{a} \Rightarrow (\vec{r} - \vec{b}) \times \vec{a} = 0$

$\vec{r} \times \vec{b} = \vec{a} \times \vec{b} \Rightarrow (\vec{r} - \vec{a}) \times \vec{b} = 0$

If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then

$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x-2 & y & z+1 \\ 1 & 1 & 0 \end{vmatrix} = 0$ and $\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x-1 & y-1 & z \\ 2 & 0 & -1 \end{vmatrix} = 0$

$\Rightarrow z + 1 = 0, x - y = 2$ and $y - 1 = 0, x - 1 + 2z = 0$

$\Rightarrow x = 3, y = 1, z = -1$

142 **(c)**

Suppose the bisector of angle A meets BC at D. Then AD divides BC in the ratio $AB : AC$

So, P.V. of $D = \frac{|\overrightarrow{AB}|(2\hat{\imath}+5\hat{\jmath}+7\hat{k})+|\overrightarrow{AC}|(2\hat{\imath}+3\hat{\jmath}+4\hat{k})}{|\overrightarrow{AB}|+|\overrightarrow{AC}|}$

But $\overrightarrow{AB} = -2\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$ and $\overrightarrow{AC} = -2\hat{\imath} - 2\hat{\jmath} - \hat{k}$

$\Rightarrow |\overrightarrow{AB}| = 6$ and $|\overrightarrow{AC}| = 3$

$\therefore$ P.V. of $D = \frac{6(2\hat{\imath}+5\hat{\jmath}+7\hat{k})+3(2\hat{\imath}+3\hat{\jmath}+4\hat{k})}{6+3}$

$= \frac{1}{3}(6\hat{\imath} + 13\hat{\jmath} + 18\hat{k})$

143 **(b)**

$|\overrightarrow{AC} \times \overrightarrow{BD}| = 2|\overrightarrow{AB} \times \overrightarrow{AD}|$

$= 2\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 4 & -5 \\ 1 & 2 & 3 \end{vmatrix}$

$= |2[\hat{\imath}(12+10) - \hat{\jmath}(6+5) + \hat{k}(4-4)]|$

$= |2[22\hat{\imath} - 11\hat{\jmath}]|$

$= 22|[2\hat{\imath} - \hat{\jmath}[|$

$= 22 \times \sqrt{5}$

144 **(c)**

Given $\vec{v} \cdot \vec{u} = \vec{w} \cdot \vec{u}$

and $\vec{v} \perp \vec{w} \Rightarrow \vec{v} \cdot \vec{w} = 0$

Now, $|\vec{u} - \vec{v} + \vec{w}|^2$

$= |\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 - 2\vec{u}\cdot\vec{v} - 2\vec{w}\cdot\vec{v} + 2\vec{u}\cdot\vec{w}$

$= 1 + 4 + 9$

So $|\vec{u} - \vec{v} + \vec{w}| = \sqrt{14}$

145 **(b)**

$|\vec{a} \times \vec{b} - \vec{a} \times \vec{c}|^2 = |\vec{a} \times (\vec{b} - \vec{c})|^2$

$= |\vec{a}|^2|\vec{b} - \vec{c}|^2 - (\vec{a} \cdot (\vec{b} - \vec{c}))^2$

$= |\vec{b} - \vec{c}|^2$

$= |b|^2 + |\vec{c}|^2 - 2|\vec{b}||\vec{c}|\cos\frac{\pi}{3} = 1$

146 **(d)**

$((\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c})) \times (\vec{b} \times \vec{c})$

$= (\vec{a} \times \vec{b}) \times (\vec{b} \times \vec{c}) + (\vec{a} \times \vec{c}) \times (\vec{b} \times \vec{c})$

$= ((\vec{a} \times \vec{b}) \cdot \vec{c})\vec{b} - ((\vec{a} \times \vec{b}) \cdot \vec{b})\vec{c} + ((\vec{a} \times \vec{c}) \cdot \vec{c})\vec{b} - ((\vec{a} \times \vec{c}) \cdot \vec{b})\vec{c}$

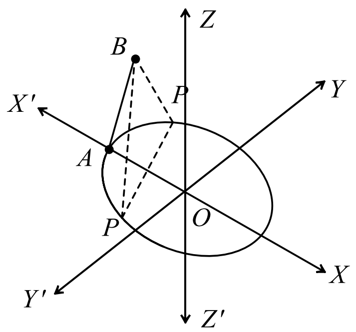
$= [\vec{a}\vec{b}\vec{c}](\vec{b} + \vec{c})$

$\Rightarrow ((\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c})) \times (\vec{b} \times \vec{c})) \cdot (\vec{b} - \vec{c})$

$= [\vec{a}\vec{b}\vec{c}](\vec{b} + \vec{c}) \cdot (\vec{b} - \vec{c})$

$= (\vec{a}\vec{b}\vec{c})(|\vec{b}|^2 - |\vec{c}|^2) = 0$

147 **(a)**



Point P lies on $x^2 + 3y^2 = 3$ (i)

Now from the diagram, according to the given conditions, $AP = AB$

or $(x+\sqrt{3})^2 + (y-0)^2 = 4$ or $(x+\sqrt{3})^2 + y^2 = 4$ (ii)

Solving (i) and (ii), we get $x = 0$ and $y = \pm 1$

Hence point P has position vector $\pm\hat{\jmath}$

148 **(b)**

Let $\vec{r}$ be the new position. Then $\vec{r} = \lambda\hat{k} + \mu(\hat{\imath} + \hat{\jmath})$

Also $\vec{r} \cdot \hat{k} = -\frac{1}{\sqrt{2}} \Rightarrow \lambda = -\frac{1}{\sqrt{2}}$

Also, $\lambda^2 + 2\mu^2 = 1 \Rightarrow 2\mu^2 = \frac{1}{2} \Rightarrow \mu = \pm\frac{1}{2}$

$\therefore \vec{r} = \pm\frac{1}{2}(\hat{\imath} + \hat{\jmath}) - \frac{\hat{k}}{\sqrt{2}}$

149 **(b)**

Note that $\vec{a} + \vec{b} = \vec{c}$

150 **(d)**

$\vec{a} \times (\vec{a} \times \vec{b}) = \vec{c} \Rightarrow |\vec{a}||\vec{a} \times \vec{b}|$

$= |\vec{c}| \left(\because \vec{a} \perp (\vec{a} \times \vec{b})\right)$

$1(1 \times 5)\sin\theta = 3 \Rightarrow \sin\theta = \frac{3}{5} \Rightarrow \tan\theta = \frac{3}{4}$

151 **(d)**

$\vec{r} = x_1(\vec{a} \times \vec{b}) + x_2(\vec{b} \times \vec{c}) + x_3(\vec{c} \times \vec{a})$

$\Rightarrow \vec{r} \cdot \vec{a} = x_2[\vec{a}\vec{b}\vec{c}], \vec{r} \cdot \vec{b} = x_3[\vec{b}\vec{c}\vec{a}]$

and $\vec{r} \cdot \vec{c} = x_1[\vec{c}\vec{a}\vec{b}] = x_1[\vec{a}\vec{b}\vec{c}]$

$\Rightarrow x_1 + x_2 + x_3 = 4\vec{r} \cdot (\vec{a} + \vec{b} + \vec{c})$

152 **(b)**

Since $\vec{u} + \vec{v} + \vec{w} = 0$, we have

$|\vec{u} + \vec{v} + \vec{w}|^2 = 0$

Or $|\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 + 2(\vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{w} + \vec{w}\cdot\vec{u}) = 0$

Or $9 + 16 + 25 + 2(\vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{w} + \vec{w}\cdot\vec{u}) = 0$

Or $\vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{w} + \vec{w}\cdot\vec{u} = -25$

153 **(d)**

$\vec{f}\left(\frac{5}{4}\right) = \left[\frac{5}{4}\right]\hat{\imath} + \left(\frac{5}{4} - \left[\frac{5}{4}\right]\right)\hat{\jmath} + \left[\frac{5}{4} + 1\right]\hat{k}$

$= \hat{\imath} + \left(\frac{5}{4} - 1\right)\hat{\jmath} + 2\hat{k}$

$= \hat{\imath} + \frac{1}{4}\hat{\jmath} + 2\hat{k}$

When $0 < t < 1, \vec{f}(t) = 0\,\vec{\imath} + \{t - 0\}\vec{\jmath} + \vec{k} = t\vec{\jmath} + \vec{k}$

$\vec{f}\left(\frac{5}{4}\right) \cdot \vec{f}(t) = 2 + \frac{t}{4}$

So $\cos\theta = \dfrac{2 + \frac{t}{4}}{\left|\vec{\imath} + \frac{1}{4}\vec{\jmath} + 2\vec{k}\right| |t\vec{\jmath} + \vec{k}|}$

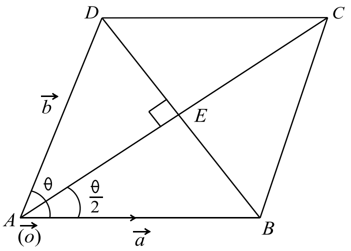
$= \dfrac{2 + \frac{t}{4}}{\sqrt{1 + \frac{1}{16} + 4}\sqrt{1 + t^2}}$

$= \dfrac{8 + t}{9\sqrt{1 + t^2}}$

154 **(b)**

Vector in the direction of angular bisector of $\vec{a}$ and $\vec{b}$ is $\frac{\vec{a}+\vec{b}}{2}$

Unit vector in this direction is $\frac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|}$



From the figure, position vector of E is $\frac{\vec{a}+\vec{b}}{2}$

Now in triangle AEB, $AE = AB\cos\frac{\theta}{2}$

$\Rightarrow \left|\frac{\vec{a} + \vec{b}}{2}\right| = \cos\frac{\theta}{2}$

Hence unit vector along the bisector is $\frac{\vec{a}+\vec{b}}{2\cos\frac{\theta}{2}}$

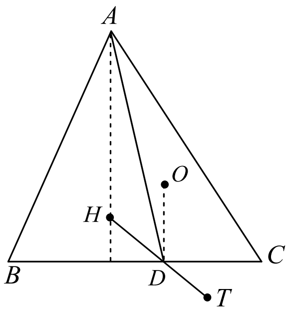
155 **(a)**

Let the origin of reference be the circumcentre of the triangle

Let $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = \vec{b}, \overrightarrow{OC} = \vec{c}$ and $\overrightarrow{OT} = \vec{t}$

Then $|\vec{a}| = |\vec{b}| = |\vec{c}| = R$ (circumeadius)

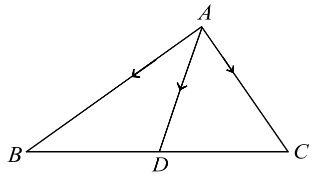
Again $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} = \overrightarrow{OA} + 2\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AH} = \overrightarrow{OH}$



Therefore, the P.V. of H is $\vec{a}, \vec{b}, \vec{c}$. Since D is the midpoint of HT, we have $\frac{\vec{a}+\vec{b}+\vec{c}+\vec{t}}{2} = \frac{\vec{b}+\vec{c}}{2} \Rightarrow \vec{t} = -\vec{a}$

$\therefore\ \overrightarrow{AT} = -2\vec{a} \Rightarrow \overrightarrow{AT} = |-2\vec{a}| = 2|\vec{a}| = 2R$. But $BC = 2R\sin A = R$, therefore $AT = 2BC$

156 **(b)**



$\overrightarrow{AB} + \overrightarrow{AC} = 2\overrightarrow{AD}$

$\therefore \overrightarrow{AD} = \frac{1}{2}\{(-3\hat{\imath} + 4\hat{k}) + (5\hat{\imath} - 2\hat{\jmath} + 4\hat{k})\}$

$= \hat{\imath} - \hat{\jmath} + 4\hat{k}$

Length of $AD = \sqrt{1 + 1 + 16} = \sqrt{18}$

157 **(c)**

$|\vec{a} + \vec{b}| < |\vec{a} - \vec{b}|$

$\Rightarrow \frac{\pi}{2} < \theta < \frac{3\pi}{2}$

158 **(a)**

Differentiate the curve

$6x + 8(xy_1 + y) + 4yy_1 = 0$

m_T at (1,0) is $6 + 8(y_1(0)) = 0$

$y_1(0) = -\frac{3}{4}$

$m_N = \frac{4}{3}$

Unit vector $= \pm\frac{(3\hat{\imath}+4\hat{\jmath})}{5}$

Again normal vector of magnitude $10 = \pm(6\hat{\imath} + 8\hat{\jmath})$

159 **(a)**

Let l, m and $n$be the direction cosines of the required vector

Then, $l = m$(given). Therefore

Required vector $\vec{r} = l\hat{\imath} + m\hat{\jmath} + n\hat{k} = l\hat{\imath} + l\hat{\jmath} + n\hat{k}$ (i)

Now, $l^2 + m^2 + n^2 = 1 \Rightarrow 2l^2 + n^2 = 1$

Since, $\hat{r}$is perpendicular to $-\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$

$\vec{r}(-\hat{\imath} + 2\hat{\jmath} + 2\hat{k}) = 0 \Rightarrow -l + 2l + 2n = 0 \Rightarrow l + 2n = 0$ (ii)

From (i)and (ii), we get: $n = \mp\frac{1}{3}, l = \pm\frac{2}{3}$

Hence, required vector $\vec{r} = \frac{1}{3}(\pm 2\hat{\imath} \pm 2\hat{\jmath} \mp \hat{k}) = \pm\frac{1}{3}(2\hat{\imath} + 2\hat{\jmath} - \hat{k})$

160 **(b)**

$|a|+|b|+|c|=\sqrt{a^2+b^2+c^2}$

$\Leftrightarrow 2\,|ab|+2|bc|+2|ca|=0$

$\Leftrightarrow ab=bc=ca=0 \Leftrightarrow$ any two of a, b and c are zero

161 **(a)**

a, b and c are distinct negative number and vectors $a\hat{\imath}+a\hat{\jmath}+c\hat{k}, \hat{\imath}+\hat{k}$ and $c\hat{\imath}+c\hat{\jmath}+b\hat{k}$ are coplanar

$$\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix}=0$$

$\Rightarrow ac+c^2-ab-ac=0$

$\Rightarrow c^2=ab$

$\Rightarrow a, c, b$ are in G.P

So c is the G.M. of a and b

162 **(b,d)**

Since $\vec{a}=(1, 3\sin 2\alpha)$ makes on obtuse angle with the z-axis, its z-component is negative

$\Rightarrow -1 \le \sin 2\alpha < 0$

But $\vec{b}\cdot\vec{c}=0$ ($\because$ orthogonal)

$\tan^2\alpha-\tan\alpha-6=0$

$\therefore (\tan\alpha-3)(\tan\alpha+2)=0$

$\Rightarrow \tan\alpha=3,-2$

Now, $\tan\alpha=3$. Therefore,

$\sin 2\alpha=\frac{2\tan\alpha}{1+\tan^2\alpha}=\frac{6}{1+9}=\frac{3}{5}$ (not possible as $\sin 2\alpha<0$)

Now, if $\tan\alpha=-2$,

$$\Rightarrow \sin 2\alpha=\frac{2\tan\alpha}{1+\tan^2\alpha}=\frac{-4}{1+4}=\frac{-4}{5}$$

$\Rightarrow \tan 2\alpha>0$

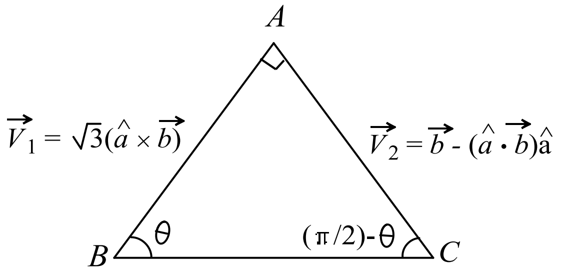
$\Rightarrow 2\alpha$ is the third quadrant. Also, $\sqrt{\sin\alpha/2}$ is meaningful. If $0<\sin\alpha/2<1$, then $\alpha=(4n+1)\pi-\tan^{-1}2$ and $\alpha=(4n+2)\pi-\tan^{-1}2$

163 **(b,c,d)**

Obviously, $\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{b}}{|\vec{b}|}$ is a vector in the plane of $\vec{a}$ and $\vec{b}$ and hence perpendicular to $\vec{a}\times\vec{b}$. It is also equally inclined to $\vec{a}$and$\vec{b}$ as it is along angle bisector

164 **(a,b,c)**

Consider $\vec{V_1}\cdot\vec{V_2}=0 \Rightarrow A=90°$



Using the sine law, $\left|\frac{\vec{b}-(\hat{a}\cdot\vec{b})\hat{a}}{\sin\theta}\right|=\frac{\sqrt{3}|\hat{a}\times\vec{b}|}{\cos\theta}$

$$\Rightarrow \tan\theta=\frac{1}{\sqrt{3}}\frac{|\vec{b}-(\hat{a}\cdot\vec{b})\hat{a}|}{|\hat{a}\times\vec{b}|}$$

$$=\frac{1}{\sqrt{3}}\frac{|(\hat{a}\times\vec{b})\times\hat{a}|}{|\hat{a}\times\vec{b}|}$$

$$=\frac{1}{\sqrt{3}}\frac{|\hat{a}\times\vec{b}||\hat{a}|\sin 90°}{|\hat{a}\times\vec{b}|}=\frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta=\frac{\pi}{6}$$

165 **(a,b)**

$\vec{u}=\vec{a}-(\vec{a}\cdot\vec{b})\vec{b}$

$=\vec{a}(\vec{b}\cdot\vec{b})-(\vec{a}\cdot\vec{b})\vec{b}$

$=\vec{b}\times(\vec{a}\times\vec{b})$

$\Rightarrow |\vec{u}|=|\vec{b}\times(\vec{a}\times\vec{b})|$

$=|\vec{b}||\vec{a}\times\vec{b}|\sin 90°$

$=|\vec{b}||\vec{a}\times\vec{b}|$

$=|\vec{v}|$

Also $\vec{u}\cdot\vec{b}=\vec{b}\cdot\vec{b}\times(\vec{a}\times\vec{b})$

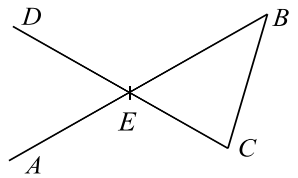
$=[\vec{b}\vec{b}\vec{a}\times\vec{b}]$

$=0$

$\Rightarrow |\vec{v}|=|\vec{u}|+|\vec{u}\cdot\vec{b}|$

166 **(a,b)**

Let $\overline{EB}=p, \overline{AB}$ and $\overline{CE}=q\overline{CD}$



Then $0<P$ and$q\le 1$

Since $\overline{EB}+\overline{BC}+\overline{CE}=\vec{0}$

$pm(2\hat{\imath}-6\hat{\jmath}+2\hat{k})+(\hat{\imath}-2\hat{\jmath})$
$+qn(-6\hat{\imath}+15\hat{\jmath}-3\hat{k})=\vec{0}$

$\Rightarrow (2\,pm+1-6qn)\hat{\imath}+(-6pm-2+15qn)\hat{\jmath}$
$+(2pm-6qn)\hat{k}=0$

$\Rightarrow 2pm-6qn+1=\vec{0}, -6pm-2+15qn$
$=\vec{0}, 2pm-6qn=\vec{0}$

Solving these, we get

$p=1/(2m)$ and$q=1/(3n)$

$\therefore 0<1/(2m)\le 1$ and $0<1/(3n)\le 1$

$\Rightarrow m\ge 1/2$ and $n\ge 1/3$

167 **(a,c)**

We have $\vec{v}=\vec{a}\times\vec{b}=|\vec{a}||\vec{b}|\sin\theta\,\hat{n}=$

$\sin\theta\,\hat{n}$, where $\vec{a}$ and $\vec{b}$ are unit vectors. Therefore, $|\vec{v}|=\sin\theta$

Now, $\vec{u}=\vec{a}-(\vec{a}\cdot\vec{b})\vec{b}$

$=\vec{a}-\vec{b}\cos\theta$ (where $\vec{a}\cdot\vec{b}=\cos\theta$)

$\therefore |\vec{u}|^2=|\vec{a}-\vec{b}\cos\theta|^2$

$=1+\cos^2\theta-2\cos\theta\cdot\cos\theta$

$= 1 - \cos^2\theta = \sin^2\theta = |v|^2$

$\Rightarrow |\vec{u}| = |\vec{v}|$

Also, $\vec{u}\cdot\vec{b} = \vec{a}\cdot\vec{b} - (\vec{a}\cdot\vec{b})(\vec{b}\cdot\vec{b})$

$= \vec{a}\cdot\vec{b} - \vec{a}\cdot\vec{b}$

$= 0$

$\therefore |\vec{u}\cdot\vec{b} = 0|$

$\therefore |\vec{v}| = |\vec{u}| + |\vec{u}\cdot\vec{b}|$ is also correct

168 **(a,c)**

We have, $\overrightarrow{AB} = -\hat{\imath} - \hat{\jmath} - 4\hat{k}, \overrightarrow{BC} = -3\hat{\imath} + 3\hat{\jmath}$ and $\overrightarrow{CA} = 4\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$. Therefore

$|\overrightarrow{AB}| = |\overrightarrow{BC}| = 3\sqrt{2}$ and $|\overrightarrow{CA}| = 6$

Clearly, $|\overrightarrow{AB}|^2 + |\overrightarrow{BC}|^2 = |\overrightarrow{AC}|^2$

Hence, the triangle is right-angled isosceles triangle

169 **(a,c)**

$\vec{a}\times(\vec{b}\times\vec{c}) + (\vec{a}\cdot\vec{b})\vec{b}$

$= (\vec{4} - 2x - \sin y)\,\vec{b} + (x^2 - 1)\vec{c}$

$\Rightarrow (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c} + (\vec{a}\cdot\vec{b})\vec{b}$

$= (4 - 2x - \sin y)\vec{b} + (x^2 - 1)\vec{c}$

Now, $(\vec{c}\cdot\vec{c})a = \vec{c}$. Therefore,

$(\vec{c}\cdot\vec{c})(\vec{a}\cdot\vec{c}) = (\vec{c}\cdot\vec{c}) \Rightarrow \vec{a}\cdot\vec{c} = 1$

$\Rightarrow 1 + \vec{a}\cdot\vec{b} = 4 - 2x - \sin y, x^2 - 1 = -(\vec{a}\cdot\vec{b})$

$\Rightarrow 1 = 4 - 2x - \sin y + x^2 - 1$

$\Rightarrow \sin y = x^2 - 2x + 2 = (x-1)^2 + 1$

But $\sin y \le 1 \Rightarrow x = 1, \sin y = 1$

$\Rightarrow y = (4n+1)\dfrac{\pi}{2}, n \in I$

170 **(b,d)**

Since $\vec{a}$ and $\vec{b}$ are equally inclined to $\vec{c}$, $\vec{c}$ must be of the from $t\left(\frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|}\right)$

Now $\frac{|\vec{b}|}{|\vec{a}|+|\vec{b}|}\vec{a} + \frac{|\vec{a}|}{|\vec{a}|+|\vec{b}|}\vec{b} = \frac{|\vec{a}||\vec{b}|}{|\vec{a}|+|\vec{b}|}\left(\frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|}\right)$

Also, $\frac{|\vec{b}|}{2|\vec{a}|+|\vec{b}|}\vec{a} + \frac{|\vec{a}|}{2|\vec{a}|+|\vec{b}|}\vec{b} = \frac{|\vec{a}||\vec{b}|}{2|\vec{a}|+|\vec{b}|}\left(\frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|}\right)$

Other two vectors cannot be written in the form $t\left(\frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|}\right)$

171 **(b)**

We know that if $\hat{n}$ is perpendicular to $\vec{a}$ as well as $\vec{b}$, then

$\hat{n} = \dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$ or $\dfrac{\vec{b}\times\vec{a}}{|\vec{b}\times\vec{a}|}$

As $\vec{a}\times\vec{b}$ and $\vec{b}\times\vec{a}$ represent two vectors in opposite directions, we have two possible values of $\hat{n}$

172 **(b,c)**

We have, $\vec{a} = 2p\hat{\imath} + \hat{\jmath}$

On rotation, let $\vec{b}$ be the vector with components $(p+1)$ and 1 so that $\vec{b} = (p+1)\hat{\imath} + \hat{\jmath}$

Now $|\vec{a}| = |\vec{b}| \Rightarrow a^2 = b^2$

$\Rightarrow 4p^2 + 1 = (p+1)^2 + 1$

$\Rightarrow 4p^2 = (p+1)^2$

$\Rightarrow 2p = \pm(p+1)$

$\Rightarrow 3p = -1$ or $p = 1$

$\therefore p = -1/3$ or $p = 1$

173 **(b,d)**

$\vec{a}\times(\vec{r}\times\vec{a}) = \vec{a}\times\vec{b}$

$3\vec{r} - (\vec{a}\cdot\vec{r})\vec{a} = \vec{a}\times\vec{b}$

Also $|\vec{r}\times\vec{a}| = |\vec{b}|$

$\Rightarrow \sin^2\theta = \dfrac{2}{3}$

$\Rightarrow (1 - \cos^2\theta) = \dfrac{2}{3}$

$\Rightarrow \dfrac{1}{3} = \cos^2\theta$

$\Rightarrow \vec{a}\cdot\vec{r} = \pm 1$

$\Rightarrow 3\vec{r} \pm \vec{a} = \vec{a}\times\vec{b}$

$\Rightarrow \vec{r} = \dfrac{1}{3}(\vec{a}\times\vec{b} \pm \vec{a})$

174 **(a,b,d)**

$(\lambda - 1)(\vec{a}_1 - \vec{a}_2) + \mu(\vec{a}_2 + \vec{a}_3)$

$+ \gamma(\vec{a}_3 + \vec{a}_4 - 2\vec{a}_2) + \vec{a}_3 + \delta\vec{a}_4$

$= \vec{0}$

i.e., $(\lambda - 1)\vec{a}_1 + (1 - \lambda + \mu - 2\gamma)\vec{a}_2 + (\mu + \gamma + 1)\vec{a}_3 + (\gamma + \delta)\vec{a}_4 = \vec{0}$

since $\vec{a}_1, \vec{a}_2, \vec{a}_3$ and $\vec{a}_4$ are linearly independent

$\lambda - 1 = 0, 1 - \lambda + \mu - 2\gamma = 0, \mu + \gamma + 1$

$= 0$ and $\gamma + \delta = 0$

i.e., $\lambda = 1, \mu = 2\gamma, \mu + \gamma + 1 = 0, \gamma + \delta = 0$

i.e., $\lambda = 1, \mu = -\frac{2}{3}, \gamma = -\frac{1}{3}, \delta = \frac{1}{3}$

175 **(a,b)**

We have, $|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2(\vec{a}\cdot\vec{b})$

$\Rightarrow |\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos 2\theta$

$\Rightarrow |\vec{a} - \vec{b}|^2 = 2 - 2\cos 2\theta \; (\because |\vec{a}| = |\vec{b}| = 1)$

$\Rightarrow |\vec{a} - \vec{b}|^2 = 4\sin^2\theta$

$\Rightarrow |\vec{a} - \vec{b}| = 2|\sin\theta|$

Now ,$|\vec{a} - \vec{b}| < |$

$\Rightarrow 2|\sin\theta| < 1$

$\Rightarrow |\sin\theta| < \dfrac{1}{2}$

$\Rightarrow \theta \in [0, \pi/6)$ or $\theta \in (5\pi/6, \pi]$

176 **(a,b,c,d)**

Since, $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are collinear.

$\therefore\ \vec{\mathbf{a}} = \lambda\vec{\mathbf{b}}$

$\Rightarrow (x\,\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) = \lambda(\hat{\mathbf{i}} + y\hat{\mathbf{j}} - z\hat{\mathbf{k}})$

On comparing

$x = \lambda, -2 = \lambda y$ and $5 = -\lambda z$

For $\lambda = 1$

$x = 1, y = -2$ and $z = -5$

Option (a) is correct.

For $\lambda = \frac{1}{2}$,

$x = \frac{1}{2}, y = -4$ and $z = -10$

Option (b) is correct.

For $\lambda = -\frac{1}{2}$,

$x = -\frac{1}{2}, y = 4$ and $z = 10$

Option (c) is correct.

and for $\lambda = -1$

$x = -1, y = 2$ and $z = 5$

Option (d) is correct.

177 **(a,c)**

$\vec{\mathbf{r}} = \lambda_1 \vec{\mathbf{r}}_1 + \lambda_2 \vec{\mathbf{r}}_2 + \lambda_3 \vec{\mathbf{r}}_3$ ....(i)

On putting the values of $\vec{\mathbf{r}}, \vec{\mathbf{r}}_1, \vec{\mathbf{r}}_2$ and $\vec{\mathbf{r}}_3$, in Eq. (i) and then compare. Then, we get

$\lambda_1, \lambda_2, \lambda_3$

178 **(a,b,c,d)**

$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta\, \hat{n}$

$\Rightarrow |\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$

$\Rightarrow \sin\theta = \dfrac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

$\Rightarrow \cos\theta = \dfrac{|\vec{a}.\vec{b}|}{|\vec{a}||\vec{b}|}$

From (i) and (ii),

$\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow |\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$

If $\theta = \pi/4$, then $\sin\theta = \cos\theta = 1/\sqrt{2}$. Therefore,

$|\vec{a} \times \vec{b}| = \frac{|\vec{a}||\vec{b}|}{\sqrt{2}}$ and $\vec{a} \cdot \vec{b} = \frac{|\vec{a}||\vec{b}|}{\sqrt{2}}$

$|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$

$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta \hat{n} = \dfrac{|\vec{a}||\vec{b}|}{\sqrt{2}} \hat{n}$

$\vec{a} \times \vec{b} = (\vec{a} \cdot \vec{b})\hat{n}$

179 **(a,b,c)**

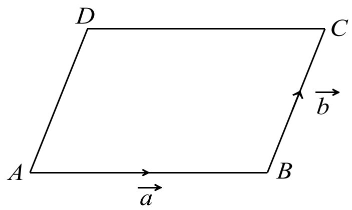
For coplanar vectors, $\begin{vmatrix} 1 & 2 & 3 \\ 0 & \lambda & \mu \\ 0 & 0 & 2\lambda - 1 \end{vmatrix} = 0$

$\Rightarrow (2\lambda - 1)\lambda = 0 \Rightarrow \lambda = 0, \dfrac{1}{2}$

180 **(b,c)**

$\overrightarrow{AC} = \vec{a} + \vec{b}$

$\therefore |\overrightarrow{AC}| = |\vec{a} + \vec{b}|$



$\therefore |\overrightarrow{AC}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b}$

$= |3\vec{\alpha} - \vec{\beta}|^2 + |\vec{\alpha} + 3\vec{\beta}|^2 + 2(3\vec{\alpha} - \vec{\beta}) \cdot (\vec{\alpha} + 3\vec{\beta})$

$= 16|\vec{\alpha}|^2 + 4|\vec{\beta}|^2 + 16\vec{\alpha} \cdot \vec{\beta}$

$= 80 + 16(2)(2)(1/2)$

$= 112$

$\therefore |\overrightarrow{AC}| = 4\sqrt{7}$

$|\overrightarrow{BD}| = |\vec{a} - \vec{b}|$

$|\overrightarrow{BD}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$

$= |3\vec{\alpha} - \vec{\beta}|^2 + |\vec{\alpha} + 3\vec{\beta}|^2 - 2(3\vec{\alpha} - \vec{\beta}) \cdot (\vec{\alpha} + 3\vec{\beta})$

$= 4|\vec{\alpha}|^2 + 4|\vec{\beta}|^2 = 16\vec{\alpha} \cdot \vec{\beta}$

$= 80 - 16(2)(2)(1/2)$

$= 48$

$\therefore |\overrightarrow{BD}| = 4\sqrt{3}$

181 **(b,d)**

Let $\vec{\alpha} = \hat{\imath} - \hat{\jmath} - \hat{k}, \vec{\beta} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{\gamma} = -\hat{\imath} + \hat{\jmath} + \hat{k}$

Let required vector $\vec{a} = x\hat{\imath} + y\hat{\jmath} + z\hat{\jmath}$

$\vec{a}, \vec{\beta}, \vec{\gamma}$ are coplanar

$\Rightarrow \begin{vmatrix} x & y & z \\ 1 & 1 & 1 \\ -1 & 1 & 1 \end{vmatrix} = 0 \Rightarrow y = z$

Also, $\vec{a}$ and $\vec{\alpha}$ are perpendicular

$\Rightarrow x - y - z = 0$

$\Rightarrow x = zy$

$\Rightarrow$ Options b and d are correct

182 **(a,b,c,d)**

$x\hat{\imath} + (x+1)\hat{\jmath} + (x+2)\hat{k}, (x+3)\hat{\imath} + (x+4)\hat{\jmath} + (x+5)\hat{k}$ and $(x+6)\hat{\imath} + (x+7)\hat{\jmath} + (x+8)\hat{k}$ are coplanar

We have determinant of their coefficients as

$\begin{vmatrix} x & x+1 & x+2 \\ x+3 & x+4 & x+5 \\ x+6 & x+7 & x+8 \end{vmatrix}$

Applying $C_2 \longrightarrow C_2 - C_1$ and $C_3 \longrightarrow C_3 - C_1$, we have

$\begin{vmatrix} x & 1 & 2 \\ x+3 & 1 & 2 \\ x+6 & 1 & 2 \end{vmatrix} = 0$

Hence $x \in R$

183 **(a,d)**

We have,

$\left.\begin{array}{l} \vec{c} = \vec{a} \times \vec{b} \Rightarrow \vec{c} \perp \vec{a} \text{ and } \vec{c} \perp \vec{b} \\ \text{and } \vec{a} = \vec{b} \times \vec{c} \Rightarrow \vec{a} \perp \vec{b} \text{ and}, \vec{a} \perp \vec{c} \end{array}\right\} \Rightarrow \vec{a} \perp \vec{b} \perp \vec{c}$

Now, $\vec{a} \times \vec{b} = \vec{c}$

$\Rightarrow (\vec{b}\times\vec{c})\times\vec{b}=\vec{c}$ $\quad [\because \vec{a}=\vec{b}\times\vec{c}]$

$\Rightarrow (\vec{b}\cdot\vec{b})\vec{c}-(\vec{b}\cdot\vec{c})\vec{b}=\vec{c}$

$\Rightarrow |\vec{b}|^2\vec{c}=\vec{c}$ $\quad [\because \vec{b}\perp\vec{c}\ \therefore \vec{b}\cdot\vec{c}=0]$

$\Rightarrow |\vec{b}|=1$

Also,

$\vec{c}=\vec{a}\times\vec{b}$

$\Rightarrow |\vec{c}|=|\vec{a}\times\vec{b}|$

$\Rightarrow |\vec{c}|=|\vec{a}|\,|\vec{a}|\sin\pi/2$

$\Rightarrow |\vec{c}|=|\vec{a}|$ $\quad [\because |\vec{b}|=1]$

184 **(b,d)**

$\vec{d}\cdot\vec{a}=[\vec{a}\vec{b}\vec{c}]\cos y=-\vec{d}\cdot(\vec{b}+\vec{c})$

$$\Rightarrow \cos y=-\frac{\vec{d}\cdot(\vec{b}+\vec{c})}{[\vec{a}\cdot\vec{b}\cdot\vec{c}]}$$

Similarly, $\sin x=-\frac{\vec{d}\cdot(\vec{a}+\vec{b})}{[\vec{a}\vec{b}\vec{c}]}$ and $\frac{\vec{d}\cdot(\vec{a}+\vec{c})}{[\vec{a}\vec{b}\vec{c}]}=-2$

$\therefore \sin x+\cos y+2=0$

$\Rightarrow \sin x+\cos y=-2$

$\Rightarrow \sin x=-1, \cos y=-1$

Since we want the minimum value of

$x^2+y^2, x=-\pi/2, y=\pi$

$\therefore$ The minimum value of x^2+y^2 is $5\pi^2/4$

185 **(a,c,d)**

$\overrightarrow{OA}=-4\hat{\imath}+3\hat{k}; \overrightarrow{OB}=14\hat{\imath}+2\hat{\jmath}-5\hat{k}$

$$\hat{a}=\frac{-4\hat{\imath}+3\hat{k}}{5}; \hat{b}=\frac{14\hat{\imath}+2\hat{\jmath}-5\hat{k}}{15}$$

$$\vec{r}=\frac{\lambda}{15}[-12\hat{\imath}+9\hat{\jmath}+14\hat{\imath}+2\hat{\jmath}-5\hat{k}]$$

$$\vec{r}=\frac{\lambda}{15}[2\hat{\imath}+2\hat{\jmath}+4\hat{k}]$$

$$\vec{r}=\frac{2\lambda}{15}[\hat{\imath}+\hat{\jmath}+2\hat{k}]$$

186 **(b,d)**

$\vec{V}_1=\vec{V}_2$

$\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b})\times\vec{c}$

$\Rightarrow (\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{b}\cdot\vec{c})\vec{a}$

$\Rightarrow (\vec{a}\cdot\vec{b})\vec{c}=(\vec{b}\cdot\vec{c})\vec{a}$

$\Rightarrow$either $\vec{c}$ and $\vec{a}$ are collinear $\vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{c}\Rightarrow\vec{b}=\lambda(\vec{a}\times\vec{c})$

187 **(a,c)**

Dot product of two vectors gives a scalar quantity

188 **(a,b,d)**

Points $A(\hat{\imath}+\hat{\jmath}), B(\hat{\imath}-\hat{\jmath})$ and $C(p\hat{\imath}+q\hat{\jmath}+r\hat{k})$ are collinear

Now $\overrightarrow{AB}=-2\hat{\jmath}$ and $\overrightarrow{BC}=(p-1)\hat{\imath}+(q-1)\hat{\jmath}+r\hat{k}$

Vector $\vec{A}B$ and $\vec{B}C$ must be collinear

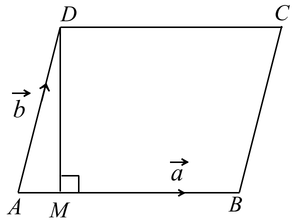
$\Rightarrow p=1, r=0$ and $q\neq 1$

189 **(a,b,c)**

We have,

AM =projection of $\vec{b}$ on $\vec{a}=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$

$$\therefore \overrightarrow{AM}=\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\right)\vec{a}$$



Now, in ΔADB

$\overrightarrow{AD}=\overrightarrow{AM}+\overrightarrow{MD}\Rightarrow\overrightarrow{DM}=\overrightarrow{AM}-\overrightarrow{AD}$

$$\Rightarrow \overrightarrow{DM}=\frac{(\vec{a}\cdot\vec{b})\vec{a}}{|\vec{a}|^2}-\vec{b}$$

Also, $\overrightarrow{DM}=\frac{1}{|\vec{a}|^2}[(\vec{a}\cdot\vec{b})\vec{a}-|\vec{a}|^2\vec{b}]$

$$\Rightarrow \overrightarrow{MD}=\frac{1}{|\vec{a}|^2}[|\vec{a}|^2\vec{b}-(\vec{a}\cdot\vec{b})\vec{a}]$$

$$\text{Now, }\frac{\vec{a}\times(\vec{a}\times\vec{b})}{|\vec{a}|^2}=\frac{1}{|\vec{a}|^2}[(\vec{a}\cdot\vec{b})\vec{a}-(\vec{a}\cdot\vec{a})\vec{b}]$$

$$=\overrightarrow{DM}$$

190 **(a,d)**

Let $\vec{a}=2\hat{\imath}+4\hat{\jmath}-5\hat{k}$ and $\vec{b}=\hat{\imath}+2\hat{\jmath}+3\hat{k}$

Then the diagonals of the parallelogram are

$\vec{p}=\vec{a}+\vec{b}$ and $\vec{q}=\vec{b}-\vec{a}$

i.e., $\vec{p}=3\hat{\imath}+6\hat{\jmath}-2\hat{k}, \hat{q}=-\hat{\imath}-2\hat{\jmath}+8\vec{k}$

So, unit vectors along the diagonals are

$\frac{1}{7}(3\hat{\imath}+6\hat{\jmath}-2\hat{k})$ and $\frac{1}{\sqrt{69}}(-\hat{\imath}-2\hat{\jmath}+8\hat{k})$

191 **(a,c,d)**

$$\vec{a}=\frac{1}{3}(2\hat{\imath}-2\hat{\jmath}+\hat{k})$$

$$|\vec{a}|^2=\frac{1}{9}(4+4+1)=1\Rightarrow|\vec{a}|=1$$

Let $\vec{b}=2\hat{\imath}-4\hat{\jmath}+3\hat{k}$. Then

$$\cos\theta=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}=\frac{5}{\sqrt{29}}\Rightarrow\theta\neq\frac{\pi}{3}$$

Let $\vec{c}=-\hat{\imath}+\hat{\jmath}-\frac{1}{2}\hat{k}=\frac{-3}{2}\hat{a}\ \Rightarrow\ \vec{c}||\vec{a}$

Let $\vec{d}=3\hat{\imath}+2\hat{\jmath}+2\hat{k}$. Then $\vec{a}\cdot\vec{d}=0\ \Rightarrow\vec{a}\perp\vec{d}$

192 **(a,b,c,d)**

Since $\vec{a},\vec{b}$ and $\vec{a}\times\vec{b}$ are non-coplnar

$\vec{r}=x\vec{a}+y\vec{b}+z(\vec{a}\times\vec{b})$

$\therefore \vec{r}\times\vec{b}=\vec{a}\Rightarrow x\vec{a}\times\vec{b}+z\{(\vec{a}\cdot\vec{b})\vec{b}-(\vec{b}\cdot\vec{b})\vec{a}\}=\vec{a}$

$\Rightarrow -(1+z|\vec{b}|^2\vec{a}+x\vec{a}\times\vec{b}=0)$(since $\vec{a}\cdot\vec{b}=0$)

$\therefore x=0$ and $x=-\frac{1}{|\vec{b}|^2}$

Thus, $\vec{r} = y\vec{b} - \frac{\vec{a}\times\vec{b}}{|\vec{b}|^2}$, where y is the parameter

193 **(a,c)**

$\vec{a}\times\vec{b} = \vec{c}, \vec{b}\times\vec{c} = \vec{a}$

Taking cross with $\vec{b}$in the first equation, we get$\vec{b}\times(\vec{a}\times\vec{b}) = \vec{b}\times\vec{c} = \vec{a}$

$\Rightarrow |\vec{b}|^2\vec{a} - (\vec{a}\cdot\vec{b})\vec{b} = \vec{a} \Rightarrow |\vec{b}| = 1$ and $\vec{a}\cdot\vec{b} = 0$

Also $|\vec{a}\times\vec{b}| = |\vec{c}| \Rightarrow |\vec{a}||\vec{b}|\sin\frac{\pi}{2} = |\vec{c}| \Rightarrow |\vec{a}| = |\vec{c}|$

194 **(a,c)**

We have $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}, \vec{c} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$

Any vector in the plane of $\vec{b}$ and $\vec{c}$ is

$\vec{u} = \mu\vec{b} + \lambda\vec{c}$

$= \mu(\hat{\imath} + 2\hat{\jmath} - \hat{k}) + \lambda(\hat{\imath} + \hat{\jmath} - 2\hat{k})$

$= (\mu + \lambda)\hat{\imath} + (2\mu + \lambda)\hat{\jmath} - (\mu + 2\lambda)\hat{k}$

Given that the magnitude of projection of $\vec{u}$ on $\vec{a}$ is $\sqrt{2/3}$

$\Rightarrow \sqrt{\frac{2}{3}} = \left|\frac{\vec{u}\cdot\vec{a}}{|\vec{a}|}\right|$

$\Rightarrow \sqrt{\frac{2}{3}} = \left|\frac{2(\mu+\lambda) - (2\mu+\lambda) - (\mu+2\lambda)}{\sqrt{6}}\right|$

$\Rightarrow |-\lambda - \mu| = 2$

$\Rightarrow \lambda + \mu = 2$ or $\lambda + \mu = -2$

Therefore, the required vector is either

$2\hat{\imath} + 3\hat{\jmath} - 3\hat{k}$ or $-2\hat{\imath} - \hat{\jmath} + 5\hat{k}$

195 **(a,c)**

Here $(l\vec{a} + m\vec{b})\times\vec{b} = \vec{c}\times\vec{b} \Rightarrow l\vec{a}\times\vec{b} = \vec{c}\times\vec{b}$

$\Rightarrow l(\vec{a}\times\vec{b})^2 = (\vec{c}\times\vec{b})\cdot(\vec{a}\times\vec{b}) \Rightarrow l = \frac{(\vec{c}\times\vec{b})\cdot(\vec{a}\times\vec{b})}{(\vec{a}\times\vec{b})^2}$

Similarly, $m = \frac{(\vec{c}\times\vec{a})\cdot(\vec{b}\times\vec{a})}{(\vec{b}\times\vec{a})^2}$

196 **(c,d)**

Since $[\vec{a}\vec{b}\vec{c}] = 0, \vec{a}, \vec{b}$and $\vec{c}$ are coplanar vectors

Further, since $\vec{d}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$

$\vec{d}\cdot\vec{a} = \vec{d}\cdot\vec{b} = \vec{d}\cdot\vec{c} = 0$

$\vec{d}\cdot\vec{x} = \vec{d}\cdot\vec{y} = \vec{d}\cdot\vec{z} = 0$

$\vec{d}\cdot\vec{r} = 0$

197 **(c,d)**

Let $\vec{a}, \vec{b}$ and $\vec{c}$ lie in the $x - y$ plane

Let $\vec{a} = \hat{\imath}, \vec{b} = -\frac{1}{2}\hat{\imath} + \frac{\sqrt{3}}{2}\hat{\imath}$ and $\vec{c} = -\frac{1}{2}\hat{\imath} - \frac{\sqrt{3}}{2}\hat{\jmath}$.

Therefore,

$|\vec{p} + \vec{q} + \vec{r}| = |\lambda\vec{a} + \mu\vec{b} + v\vec{c}|$

$= \left|\lambda\hat{\imath} + \mu\left(-\frac{1}{2}\hat{\imath} + \frac{\sqrt{3}}{2}\hat{\jmath}\right) + v\left(-\frac{1}{2}\hat{\imath} - \frac{\sqrt{3}}{2}\hat{\jmath}\right)\right|$

$= \left|\left(\lambda - \frac{\mu}{2} - \frac{v}{2}\right)\hat{\imath} + \frac{\sqrt{3}}{2}(\mu - v)\hat{\jmath}\right|$

$= \sqrt{\left(\lambda - \frac{\mu}{2} - \frac{v}{2}\right)^2 + \frac{3}{4}(\mu - v)^2}$

$= \sqrt{\lambda^2 + \mu^2 + v^2 - \lambda\mu - \lambda v - \mu v}$

$= \frac{1}{\sqrt{2}}\sqrt{(\lambda - \mu)^2 + (\mu - V)^2 + (v - \lambda)^2}$

$\geq \frac{1}{\sqrt{2}}\sqrt{1 + 1 + 4} = \sqrt{3}$

$\Rightarrow |\vec{p} + \vec{q} + \vec{r}|$ can take a value equal to $\sqrt{3}$ and 2

198 **(b,c)**

$\vec{a}\times(\vec{b}\times\vec{c}) = \frac{1}{2}\vec{b}$

$\Rightarrow (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c} = \frac{1}{2}\vec{b}$

$\Rightarrow \vec{a}\cdot\vec{c} = \frac{1}{2}$ and $\vec{a}\cdot\vec{b} = 0$

$\Rightarrow 1\cdot 1\cos\alpha = \frac{1}{2}and\vec{a}\perp\vec{b}$

$\Rightarrow \alpha = \frac{\pi}{3}$ and $\vec{a}\perp\vec{b}$

200 **(c)**

$|\vec{u}\vec{v}\vec{w}| = |\vec{v}\vec{w}\vec{u}| = |\vec{w}\vec{u}\vec{v}|$

But $|\vec{v}\vec{u}\vec{w}| = -|\vec{u}\vec{v}\vec{w}|$

201 **(a,c)**

We have$[\vec{p}\vec{q}\vec{r}] = \frac{1}{[\vec{a}\vec{b}\vec{c}]}$. Therefore,

$[\vec{p}\vec{q}\vec{r}] > 0$

a. $x > 0, x[\vec{a}\vec{b}\vec{c}] + \frac{[\vec{p}\vec{q}\vec{r}]}{x} \geq 2$ (using A.M. ≥G.M)

b. Similarly, use A.M. ≥G.M.

202 **(a,c)**

Let $\vec{\mathbf{r}} = \vec{\mathbf{b}} + t\vec{\mathbf{c}}$

Or $\vec{\mathbf{r}} = (1 + t)\hat{\mathbf{i}} + (2 + t)\hat{\mathbf{j}} - (1 + 2t)\hat{\mathbf{k}}$ ...(i)

$\because$ Projection of $\vec{\mathbf{r}}$ on $\vec{\mathbf{a}}$ is $\sqrt{(2/3)}$.

$\therefore \frac{\vec{\mathbf{r}}\cdot\vec{\mathbf{a}}}{|\vec{\mathbf{a}}|} = \sqrt{\frac{2}{3}}$

or $\frac{2(1+t) - (2+t) - (1+2t)}{\sqrt{6}} = \pm\sqrt{\frac{2}{3}}$

$\therefore -t - 1 = \pm 2$

$\therefore t = -3, 1$

Putting in Eq. (i), we get

$\vec{\mathbf{r}} = -2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

Or $\vec{\mathbf{r}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

203 **(a,b,d)**

$$\left.\begin{aligned} \vec{V}_1 &= l\vec{a} + m\vec{b} + n\vec{c} \\ \vec{V}_2 &= n\vec{a} + l\vec{b} + m\vec{c} \\ \vec{V}_3 &= m\vec{a} + n\vec{b} + l\vec{c} \end{aligned}\right\} \text{when } \vec{a}, \vec{b} \text{ and } \vec{c} \text{ are}$$

non-coplanar

Therefore,

$[\vec{V}_1\vec{V}_2\vec{V}_3] = \begin{vmatrix} l & m & n \\ n & l & m \\ m & n & l \end{vmatrix} = 0$

$\Rightarrow (l+m+n)[(l-m)^2+(m-n)^2+(n-l)^2] = 0$

$\Rightarrow l+m+n=0$

Obviously, $lx^2+mx+n=0$ is satisfied by $x=1$ due $x=1$ due to (i)

$l^3+m^3+n^3 = 3\,lmn$

$\Rightarrow (l+m+n)(l^2+m^2+n^2-lm-mn-ln) = 0$, which is true

204 **(a,d)**

$|\vec{a}+\vec{b}| = |\vec{a}-2\vec{b}|$

$\Rightarrow \vec{a}\cdot\vec{b} = \frac{|\vec{b}|^2}{2}$

Also $\vec{a}\cdot\vec{b} + \frac{1}{|\vec{b}|^2+2}$

$= \frac{|\vec{b}|+2}{2} + \frac{1}{|\vec{b}|^2+2} - 1$

$\geq \sqrt{2}-1$(using A.M. ≥G.M.)

205 **(a,b,c)**

It is given that $\vec{\alpha}, \vec{b}$ and $\vec{\gamma}$ are coplanar vectors.

Therefore,

$[\vec{\alpha}\vec{\beta}\vec{\gamma}] = 0$

$\Rightarrow \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$

$\Rightarrow 3\,abc - a^3 - b^3 - c^3 = 0$

$\Rightarrow a^3+b^3+c^3-3abc = 0$

$\Rightarrow (a+b+c)(a^2+b^2+c^2-ab-bc-ca) = 0$

$\Rightarrow a+b+c = 0$ $[\because a^2+b^2+c^2-ab-bc-ca \neq 0]$

$\Rightarrow \vec{v}\cdot\vec{\alpha} = \vec{v}\cdot\vec{\beta} = \vec{v}\cdot\vec{\gamma} = 0$

$\Rightarrow \vec{v}$ is perpendicular to $\vec{\alpha}, \vec{\beta}$ and $\vec{\gamma}$

206 **(b,c,d)**

$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})\cdot(\vec{a}\times\vec{d}) = 0$

$\Rightarrow ([\vec{a}\vec{c}\vec{d}]\vec{b} - [\vec{b}\vec{c}\vec{d}]\vec{a})\cdot(\vec{a}\times\vec{d}) = 0$

$\Rightarrow [\vec{a}\vec{c}\vec{d}][\vec{b}\vec{a}\vec{d}] = 0$

⇒Either $\vec{c}$ or $\vec{b}$ must lie in the plane of $\vec{a}$ and $\vec{d}$

207 **(b,d)**

For $\vec{A}, \vec{B}$ and $\vec{C}$ to form a left-handed system

$[\vec{A}\vec{B}\vec{C}] < 0$

$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 4 \\ 1 & 1 & 5 \end{vmatrix} = 11\hat{\imath} - 6\hat{\jmath} - \hat{k}$

(i) is satisfied by options (b) and (d)

208 **(a,b)**

Given, $\frac{1}{6}\hat{\imath} - \frac{1}{3}\hat{\jmath} + \frac{1}{3}\hat{k} = (\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})$

$= [\vec{a}\vec{b}\vec{d}]\vec{c} - [\vec{a}\vec{b}\vec{c}]\vec{d}$

$= [\vec{a}\vec{b}\vec{d}]\vec{c}$

$[\because \vec{a}, \vec{b}$ and $\vec{c}$ are coplanar$]$

$[\vec{a}\vec{b}\vec{d}] = (\vec{a}\times\vec{b})\cdot\vec{d}$

$= |\vec{a}\times\vec{b}||\vec{d}|\cos\theta$ $(\because \vec{d}\perp\vec{a}, \vec{d}\perp\vec{b}, \therefore \vec{d}||\vec{a}\times\vec{b})$

$= ab\sin 30°\cdot 1\cdot(\pm1)$ $(\because \theta = 0$ or $\pi)$

$= 1\cdot 1\cdot\frac{1}{2}\cdot 1(\pm 1) = \pm\frac{1}{2}$

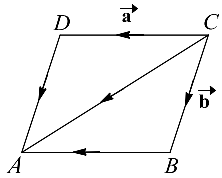
From (i),

$\vec{c} = \pm\left(\frac{1}{3}\hat{\imath} - \frac{2}{3}\hat{\jmath} + \frac{2}{3}\hat{k}\right) = \pm\frac{\hat{\imath}-2\hat{\jmath}+2\hat{k}}{3}$

209 **(b,c)**

Since, $\overline{\mathbf{AC}} = \vec{\mathbf{a}} + \vec{\mathbf{b}}$

Then, $|\overline{\mathbf{AC}}| = |\vec{\mathbf{a}} + \vec{\mathbf{b}}|$



$|\overline{\mathbf{AC}}|^2 = (\vec{\mathbf{a}})^2 + (\vec{\mathbf{b}})^2 + 2\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}$

$= \left\{(3\vec{\alpha}-\vec{\beta})^2 + (\vec{\alpha}+3\vec{\beta})^2\right\} + 2(3\vec{\alpha}-\vec{\beta})\cdot(\vec{\alpha}+3\vec{\beta})$

$= 9\,\vec{\alpha}^2 + \vec{\beta}^2 - 6\vec{\alpha}\cdot\vec{\beta} + \vec{\alpha}^2 + 9\vec{\beta}^2 + 6\vec{\alpha}\cdot\vec{\beta} + 6\alpha^2 - 6\beta^2 + 16\vec{\alpha}\cdot\vec{\beta}$

$= 16\alpha^2 + 4\beta^2 + 16\vec{\alpha}\cdot\vec{\beta}$

$= 64 + 16 + 16|\vec{\alpha}||\vec{\beta}|\cos\frac{\pi}{3}$

$= 64 + 16 + 16\times 2\times 2\times\frac{1}{2}$

$= 64 + 16 + 32 = 112$

$\therefore AC = 4\sqrt{7}$, similarly $BD = 4\sqrt{3}$

210 **(a,b,c)**

Let $\vec{A} = \vec{a}\times\vec{b}, \vec{B} = \vec{c}\times\vec{d}$ and $\vec{C} = \vec{e}\times\vec{f}$

We know that $\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$

$= (\vec{a}\times\vec{b})\cdot[(\vec{c}\times\vec{d})\times(\vec{e}\times\vec{f})]$

$= (\vec{a}\times\vec{b})\cdot[\{(\vec{c}\times\vec{d})\cdot\vec{f}\}\vec{e} - \{(\vec{c}\times\vec{d})\cdot\vec{e}\}\vec{f}\}$

$= [\vec{c}\vec{d}\vec{f}][\vec{a}\vec{b}\vec{e}] - [\vec{c}\vec{d}\vec{e}][\vec{a}\vec{b}\vec{f}]$

Similarly, other parts can be obtained

211 **(b,c)**

We have

$\vec{A} + \vec{B} = \vec{a}$

$\Rightarrow \vec{A}\cdot\vec{a} + \vec{B}\cdot\vec{a} = \vec{a}\cdot\vec{a}$

$\Rightarrow 1 + \vec{B}\cdot\vec{a} = a^2$ (given $\vec{A}\cdot\vec{a} = 1$)

$\Rightarrow \vec{B}\cdot\vec{a} = a^2 - 1$ (i)

Also $\vec{A}\times\vec{B}=\vec{b}$

$\Rightarrow \vec{a}\times(\vec{A}\times\vec{B})=\vec{a}\times\vec{b}$

$\Rightarrow (\vec{a}\cdot\vec{B})\vec{A}-(\vec{a}\cdot\vec{A})\vec{B}=\vec{a}\times\vec{b}$

$\Rightarrow (a^2-1)\vec{A}-\vec{B}=\vec{a}\times\vec{b}$(using (i) and $\vec{a}\cdot\vec{A}=1$) (ii)

and $\vec{A}+\vec{B}=\vec{a}$ (iii)

From (ii) and (iii)

$$\vec{A}=\frac{(\vec{a}\times\vec{b})+\vec{a}}{a^2}$$

$$\vec{B}=\vec{a}-\left\{\frac{(\vec{a}\times\vec{b})+\vec{a}}{a^2}\right\}$$

Or $\vec{B}=\dfrac{(\vec{b}\times\vec{a})+\vec{a}(a^2-1)}{a^2}$

Thus$\vec{A}=\dfrac{(\vec{a}\times\vec{b})+\vec{a}}{a^2}and\vec{B}=\dfrac{(\vec{b}\times\vec{a})+\vec{a}(a^2-1)}{a^2}$

212 **(a,b,c,d)**

$a_1+a_2\cos 2x+a_3\sin^2 x=0\forall x\in R$

$\Rightarrow (a_1+a_2)+\sin^2 x(a_3-2a_2)=0$

$\Rightarrow a_1+a_2=0$ and $a_3-2a_2=0$

$\dfrac{a_1}{-1}=\dfrac{a_2}{1}=\dfrac{a_3}{2}=\lambda(\neq 0)$

$\Rightarrow a_1=-\lambda, a_2=\lambda, a_3=2\lambda$

213 **(b,d)**

$(\vec{a}-\vec{b})\times[(\vec{b}+\vec{a})\times(2\vec{a}+\vec{b})]=\vec{b}+\vec{a}$

$$\Rightarrow \{(\vec{a}-\vec{b})\cdot(2\vec{a}+\vec{b})\}(\vec{b}+\vec{a}) -\{(\vec{a}-\vec{b})\cdot(\vec{b}+\vec{a})\}(2\vec{a}+\vec{b}) =\vec{b}+\vec{a}$$

$\Rightarrow (2-\vec{a}\cdot\vec{b}-1)(\vec{b}+\vec{a})=\vec{b}+\vec{a}$

$\Rightarrow$either $\vec{b}+\vec{a}=\vec{0}$ or$1-\vec{a}\cdot\vec{b}=1$

$\Rightarrow$either $\vec{b}=-\vec{a}$or $\vec{a}\cdot\vec{b}=0$

$\Rightarrow$either $\theta=\pi$ or $\theta=\pi/2$

214 **(b,c)**

Let $\vec{R}$ be the resultant

Then $\vec{R}=\vec{F}_1+\vec{F}_2+\vec{F}_3=(p+1)\hat{\imath}+4\hat{\jmath}$

Given, $|\vec{R}|=5$. Therefore,

$(p+1)^2+16=25$

$\Rightarrow p+1=\pm 3$

$\therefore p=2,-4$

215 **(a,b,c)**

$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$

$\overrightarrow{BC}=\dfrac{2\vec{u}}{|\vec{u}|}-\dfrac{\vec{u}}{|\vec{u}|}+\dfrac{\vec{v}}{|\vec{v}|}=\dfrac{\vec{u}}{|\vec{u}|}+\dfrac{\vec{v}}{|\vec{v}|}$

$$\overrightarrow{AB}\cdot\overrightarrow{BC}=\left(\frac{\vec{u}}{|\vec{u}|}-\frac{\vec{v}}{|\vec{v}|}\right)\left(\frac{\vec{u}}{|\vec{u}|}+\frac{\vec{v}}{|\vec{v}|}\right) =(\vec{u}-\vec{v})\cdot(\vec{u}+\vec{v})=1-1=0$$

$\Rightarrow \angle B=90°$

$\Rightarrow 1+\cos 2A+\cos 2B+\cos 2C=0$

216 **(a,b,d)**

$\vec{a}=[\pm(\hat{\imath}-\hat{\jmath})\pm(\hat{\jmath}+\hat{k})]$

$=\pm(\hat{\imath}+\hat{k}),\pm(\hat{\imath}-2\hat{\jmath}-\hat{k})$

217 **(a,b,c)**

We know that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then$\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

Given $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}\Rightarrow 2\vec{a}\times\vec{b}=6\vec{b}\times\vec{c}=3\vec{c}\times\vec{a}$

Hence $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=2(\vec{a}\times\vec{b})$ or $6(\vec{b}\times\vec{c})$or $3\,(\vec{c}\times\vec{a})$

218 **(a,b,c,d)**

Since $\vec{a},\vec{b}$ and $\vec{c}$ are unit vectors inclined at an angle θ

$|\vec{a}|=|\vec{b}|=1$ and $\cos\theta=\vec{a}\cdot\vec{c}=\vec{b}\cdot\vec{c}$

Now $\vec{c}=\alpha\vec{a}+\beta\vec{b}+\gamma(\vec{a}\times\vec{b})$

$\Rightarrow \vec{a}\cdot\vec{c}=\alpha(\vec{a}\cdot\vec{a})+\beta(\vec{a}\cdot\vec{b}+\gamma\{\vec{a}\cdot(\vec{a}\times\vec{b})\}$

$\Rightarrow \cos\theta=\alpha|\vec{a}|^2$ ($\because \vec{a}.\vec{b}=0, \vec{a}.(\vec{a}\times\vec{b})=0$)

$\Rightarrow \cos\theta=\alpha$

Similarly, by taking dot product on both sides of (i) by $\vec{b}$, we get $\beta=\cos\theta$

$\therefore \alpha=\beta$

Again,$\vec{c}=\alpha\vec{a}+\beta\vec{b}+\gamma(\vec{a}\times\vec{b})$

$\Rightarrow |\vec{c}|^2=|\alpha\vec{a}+\beta\vec{b}+\gamma(\vec{a}\times\vec{b})|^2$

$$=\alpha^2|\vec{a}|^2+\beta^2|\vec{b}|^2+\gamma^2|\vec{a}\times\vec{b}|^2+2\alpha\beta(\vec{a}\cdot\vec{b}) +2\alpha\gamma\{\vec{a}\cdot(\vec{a}\times\vec{b})\}+2\beta\gamma(b\cdot\{\vec{a} \times\vec{b}\})$$

$\Rightarrow 1=\alpha^2+\beta^2+\gamma^2|\vec{a}\times\vec{b}|^2$

$\Rightarrow 1=2\alpha^2+\gamma^2\{|\vec{a}|^2|b|^2\sin^2\pi/2\}$

$\Rightarrow 1=2\alpha^2+\gamma^2\Rightarrow \alpha^2=\dfrac{1-\gamma^2}{2}$

But $\alpha=\beta=\cos\theta$

$1=2\alpha^2+\gamma^2\Rightarrow\gamma^2=1-2\cos^2\theta=-\cos 2\theta$

$\therefore \beta^2=\dfrac{1-\gamma^2}{2}=\dfrac{1+\cos 2\theta}{2}$

219 **(a,c,d)**

$\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b})\times\vec{c}$

Or $(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{c}\cdot\vec{b})\vec{a}$

Or $(\vec{a}\cdot\vec{b})\vec{c}-(\vec{c}\cdot\vec{b})\vec{a}=\vec{0}$

Or $\vec{b}\times(\vec{c}\times\vec{a})=\vec{0}$

Or $(\vec{c}\times\vec{a})\times\vec{b}=\vec{0}$

Or $\vec{b}\times(\vec{c}\times\vec{a})=(\vec{c}\times\vec{a})\times\vec{b}=\vec{0}$

220 **(c)**

We are given that $\vec{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$

$\vec{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$

$\vec{c}=c_1\hat{\imath}+c_2\hat{\jmath}+c_3\hat{k}$

Then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2 = |\vec{a}\vec{b}\vec{c}|^2$

$= (\vec{a} \times \vec{b} \cdot \vec{c})^2$

$= (|\vec{a} \times \vec{b}| \cdot 1 \cos 0°)^2$ (since $\vec{c}$ is $\perp$ to $\vec{a}$ and $\vec{b}, \vec{c}$ is $\perp$ to $\vec{a} \times \vec{b}$)

$= (|\vec{a} \times \vec{b}|)^2$

$= \left(|\vec{a}||\vec{b}| \cdot \sin\frac{\pi}{6}\right)^2$

$= \left(\frac{1}{2}\sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{b_1^2 + b_2^2 + b_3^2}\right)^2$

$= \frac{1}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$

221 **(a,c)**

Since, vectors $(x, x + 1, x + 2), (x + 3, x + 4, x + 5)$ and $(x + 6, x + 7, x + 8)$ are coplanar.

$\therefore \begin{vmatrix} x & x+1 & x+2 \\ x+3 & x+4 & x+5 \\ x+6 & x+7 & x+8 \end{vmatrix} = 0$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$\begin{vmatrix} x & 1 & 2 \\ x+3 & 1 & 2 \\ x+6 & 1 & 2 \end{vmatrix} = 0$

$0 = 0 \quad (C_2 = C_3)$

$\therefore x \in R$

222 **(b,c)**

Let $\vec{\alpha} = \hat{\imath} + x\hat{\jmath} + 3\hat{k}, \vec{\beta} = 4\hat{\imath} + (4x - 2)\hat{\jmath} + 2\hat{k}$

Given, $2|\vec{\alpha}| = |\vec{\beta}|$

$\Rightarrow 2\sqrt{10 + x^2} = \sqrt{20 + 4(2x - 1)^2}$

$\Rightarrow 10 + x^2 = 5 + (4x^2 - 4x + 1)$

$\Rightarrow 3x^2 - 4x - 4 = 0$

$\Rightarrow x = 2, -\frac{2}{3}$

223 **(a,b,c,d)**

$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

$= (yz + yx + zx)\{(y - z)\hat{\imath} + (z - x)\hat{\jmath} + (x - y)\hat{k}\}$

Clearly this vector is parallel to $(y - z)\hat{\imath} + (z - x)\hat{\jmath} + (x - y)\hat{k}$

It is orthogonal to $\hat{\imath} + \hat{\jmath} + \hat{k}$ as $(y - z)(1) + (z - x)(1) + (x - y)(1) = 0$

It is orthogonal to $(y + z)\hat{\imath} + (z + x)\hat{\jmath} + (x + y)\hat{k}$

As $(y - z)(y + z) + (z - x)(z + x) + (x - y)(x + y)$

$= y^2 - z^2 + z^2 - x^2 + x^2 - y^2 = 0$

Also it is orthogonal to $x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

224 **(a,d)**

$\vec{a} = \hat{\imath} - \hat{\jmath} + \hat{k}$

$\vec{b} = 2\hat{\imath} + \hat{\jmath}$

and $\vec{c} = 3\hat{\jmath} - 2\hat{k}$

Since $[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & 0 \\ 0 & 3 & -2 \end{vmatrix} = 0$

Therefore, $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar vectors

Further since $\vec{d}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$, we have

$\therefore \vec{d} \cdot \vec{a} = \vec{d} \cdot \vec{b} = \vec{d} \cdot \vec{c} = 0$

$\therefore \vec{d} \cdot \vec{x} = \vec{d} \cdot \vec{y} = \vec{d} \cdot \vec{z} = 0$

$\therefore \vec{d} \cdot \vec{r} = 0$

225 **(a,d)**

Given $\vec{c} = \lambda_1\vec{a} + \vec{\lambda}_2\vec{b} + \lambda_3(\vec{a} \times \vec{b})$

and$\vec{a} \cdot \vec{b} = 0, |\vec{a}| = 1, |\vec{b}| = 1$

From (i), $\vec{a} \cdot \vec{c} = \lambda_1, \vec{c} \cdot \vec{b} = \lambda_2$

and$\vec{c} \cdot (\vec{a} \times \vec{b}) = |\vec{a} \times \vec{b}|^2\lambda_3$

$= (1.1 \sin 90°)^2\lambda_3 = \lambda_3$

Hence $\lambda_1 + \lambda_2 + \lambda_3 = (\vec{a} + \vec{b} + \vec{a} \times \vec{b}) \cdot \vec{c}$

226 **(a,c)**

$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$ and

$(\vec{a} \times \vec{b}) \times \vec{c} = -(\vec{c} \cdot \vec{b})\vec{a} + (\vec{a} \cdot \vec{c})\vec{b}$

We have been given $\left(\vec{a} \times (\vec{b} \times \vec{c})\right) \cdot ((\vec{a} \times \vec{b}) \times \vec{c}) = 0$. Therefore

$\left((\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}\right) \cdot \left((\vec{a} \cdot \vec{c})\vec{b} - (\vec{c} \cdot \vec{b})\vec{a}\right) = 0$

$\Rightarrow (\vec{a} \cdot \vec{c})^2|\vec{b}|^2 - (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{c})(\vec{a} \cdot \vec{b})$

$- (\vec{a} \cdot \vec{b})(\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{c})$

$+ (\vec{a} \cdot \vec{b})(\vec{b} \cdot \vec{c})(\vec{c} \cdot \vec{a}) = 0$

$\Rightarrow (\vec{a} \cdot \vec{c})^2|\vec{b}|^2 - (\vec{a} \cdot \vec{c})(\vec{a} \cdot \vec{b})(\vec{b} \cdot \vec{c})$

$\Rightarrow (\vec{a} \cdot \vec{c}) \quad \left((\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{b}) - (\vec{a} \cdot \vec{b})(\vec{b} \cdot \vec{c})\right) = 0$

$\vec{a} \cdot \vec{c} = 0$ or$(\vec{a} \cdot \vec{c})|\vec{b}|^2 = (\vec{a} \cdot \vec{b})(\vec{b} \cdot \vec{c})$

227 **(a)**

$\sqrt{(p + 2)^2 + 1} = \sqrt{p^2 + 1}$

$\Rightarrow p^2 + 4 + 4p + 1 = p^2 + 1$

$\Rightarrow 4p = -4$

$\Rightarrow p = -1$

Hence a is the correct option

228 **(c)**

We have, $\mathbf{\vec{a}} \times \mathbf{\vec{b}} = \mathbf{\vec{c}} \times \mathbf{\vec{d}}$ ...(i)

and $\mathbf{\vec{a}} \times \mathbf{\vec{c}} = \mathbf{\vec{b}} \times \mathbf{\vec{d}}$ ...(ii)

$\because (\mathbf{\vec{a}} - \mathbf{\vec{d}}) \times (\mathbf{\vec{b}} - \mathbf{\vec{c}})$

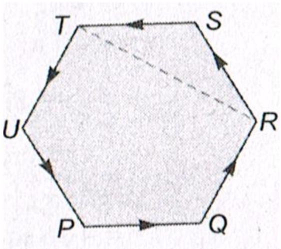
$= \mathbf{\vec{a}} \times \mathbf{\vec{b}} - \mathbf{\vec{a}} \times \mathbf{\vec{c}} - \mathbf{\vec{d}} \times \mathbf{\vec{b}} + \mathbf{\vec{d}} \times \mathbf{\vec{c}}$

$= \vec{\mathbf{c}} \times \vec{\mathbf{d}} - \vec{\mathbf{b}} \times \vec{\mathbf{d}} + \vec{\mathbf{b}} \times \vec{\mathbf{d}} - \vec{\mathbf{c}} \times \vec{\mathbf{d}}$

$= 0$ [from Eqs. (i) and (ii)]

$\therefore \vec{\mathbf{a}} - \vec{\mathbf{d}}$ and $\vec{\mathbf{b}} - \vec{\mathbf{c}}$ are parallel.

229 **(c)**
Since, $\overrightarrow{\mathbf{PQ}}$ is not parallel to, $\overrightarrow{\mathbf{TR}}$.



$\therefore$, $\overrightarrow{\mathbf{TR}}$ is resultant of , $\overrightarrow{\mathbf{RS}}$ and , $\overrightarrow{\mathbf{ST}}$ vectors.
$\Rightarrow \overrightarrow{\mathbf{PQ}} \times \left(\overrightarrow{\mathbf{RS}} + \overrightarrow{\mathbf{ST}}\right) \neq \vec{\mathbf{0}}$
But for statement II, we have $\overrightarrow{\mathbf{PQ}} \times \overrightarrow{\mathbf{RS}} = \vec{\mathbf{0}}$
Which is not possible as $\overrightarrow{\mathbf{PQ}}$ is not parallel to $\overrightarrow{\mathbf{RS}}$
Hence, statement I is true and statement II is false

230 **(a)**
$\vec{a} + \vec{b} = \vec{a} - \vec{b}$ are the diagonals of a parallelogram whose sides are $\vec{a}$ and $\vec{b}$

$|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

$\Rightarrow$ Diagonals of the parallelogram have the same length

$\Rightarrow$ The parallelogram is a rectangle $\Rightarrow \vec{a} \perp \vec{b}$

231 **(d)**

$$V = \begin{vmatrix} 1 & a & 0 \\ a & 1 & 1 \\ 0 & 1 & a \end{vmatrix} = a - 1 - a^3$$

$\therefore \frac{dV}{da} = 1 - 3a^2 = 0$ (say)

$$\therefore a = \pm \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{d^2V}{da^2} = -6a$$

$$\left(\frac{d^2V}{da^2}\right)_{\left(a=\frac{1}{\sqrt{3}}\right)} = -\frac{6}{\sqrt{3}} (-\text{ve})$$

$\therefore V$ is maximum at $a = \frac{1}{\sqrt{3}}$.

232 **(d)**
We know that the unit vector along bisector of unit vector $\vec{u}$ and $\vec{v}$ is $\frac{\vec{u}+\vec{v}}{2\cos\frac{\theta}{2}}$ where θ is the angle between vectors $\vec{u}$ and $\vec{v}$

Hence statement 1 is false, however statement 2 is true

233 **(a)**
We have adjacent sides of triangle $|\vec{a}| = 3, |\vec{b}| = 4$

The length of the diagonal is $|\vec{a} + \vec{b}| = 5$

Since it satisfies the Pythagoras theorem, $\vec{a} \perp \vec{b}$

Hence the parallelogram is a rectangle

Hence length of the other diagonal is $|\vec{a} - \vec{b}| = 5$

234 **(a)**
Given vectors are non-coplanar. Hence the answer is (A)

235 **(a)**
Statement 2 is true

Also, $(\hat{\imath} \times \vec{a}) \cdot \vec{b} = \hat{\imath} \cdot (\vec{a} \times \vec{b})$

$$\Rightarrow \vec{a} \times \vec{b} = \left(\hat{\imath} \cdot (\vec{a} \times \vec{b})\right)\hat{\imath} + \left(\hat{\jmath} \cdot (\vec{a} \times \vec{b})\right)\hat{\jmath} + \left(\hat{k} \cdot (\vec{a} \times \vec{b})\right)\hat{k}$$

236 **(c)**
Since, $(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \cdot (\vec{\mathbf{c}} \times \vec{\mathbf{d}}) = (\vec{\mathbf{a}} \cdot \vec{\mathbf{c}})(\vec{\mathbf{b}} \cdot \vec{\mathbf{d}}) - (\vec{\mathbf{b}} \cdot \vec{\mathbf{c}})(\vec{\mathbf{a}} \cdot \vec{\mathbf{d}})$

$$= \frac{3}{2} \times 2 - \frac{1}{2} \times 3$$

$$= 3 - \frac{3}{2} = \frac{3}{2}$$

237 **(d)**
$\overrightarrow{AD} = 2\hat{\imath} - \hat{k}, \overrightarrow{BD} = -2\hat{\imath} - \hat{\jmath} - 3\hat{k}$ and $\overrightarrow{CD} = 2\hat{\imath} - \hat{\jmath}$

Volume of tetrahedron is $\frac{1}{6}\left[\overrightarrow{AD}\overrightarrow{BD}\overrightarrow{CD}\right] = \frac{1}{6}\left\| \begin{matrix} 0 & 2 & -1 \\ -2 & -1 & -3 \\ 2 & -1 & 0 \end{matrix} \right\| = \frac{8}{3}$

Also, the area of the triangle ABC is $\frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \frac{1}{2}\left\| \begin{matrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 2 \\ -2 & 3 & -1 \end{matrix} \right\|$

$$= \frac{1}{2}|-9\hat{\imath} - 2\hat{\jmath} + 12\hat{k}|$$

$= \frac{\sqrt{229}}{2}$

Then $\frac{8}{3} = \frac{1}{3} \times$ (distance of D from base ABC)$\times$ (area of triangle ABC)

Distance of D from base $ABC = 16/\sqrt{229}$

238 **(b)**

Let $\vec{d} = \lambda_1\vec{a} + \lambda_2\vec{b} + \lambda_3\vec{c}$

$\Rightarrow [\vec{d}\vec{a}\vec{b}] = \lambda_3[\vec{c}\vec{a}\vec{b}] \Rightarrow \lambda_3 = 1$

$[\vec{c}\vec{a}\vec{b}] = 1$(because $\vec{a}, \vec{b}$ and $\vec{c}$ are three mutually perpendicular unit vectors)

Similarly, $\lambda_1 = \lambda_2 = 1$

$\Rightarrow \vec{d} = \vec{a} + \vec{b} + \vec{c}$

Hence statement 1 and Statement 2 are correct, but Statement 2 does not explain Statement 1 as it does not give the value of dot products

239 **(d)**

$\vec{A} \times \left((\vec{A}\cdot\vec{B})\vec{A} - (\vec{A}\cdot\vec{A})\vec{B}\right)\cdot\vec{C}$

$= \left(\underbrace{\vec{A} \times (\vec{A}\cdot\vec{B})\vec{A}}_{\text{zero}} - (\vec{A}\cdot\vec{A})\vec{A}\times\vec{B})\right)\cdot\vec{C}$

$= -|\vec{A}|^2[\vec{A}\vec{B}\vec{C}]$

Now, $|\vec{A}|^2 = 4 + 9 + 36 = 49$

$[\vec{A}\vec{B}\vec{C}] = \begin{vmatrix} 2 & 3 & 6 \\ 1 & 1 & -2 \\ 1 & 2 & 1 \end{vmatrix}$

$= 2(1+4) - 1(3-12) + 1(-6-6)$

$= 10 + 9 - 12 = 7$

$\therefore \left|-|\vec{A}|^2[\vec{A}\vec{B}\vec{C}]\right| = 49 \times 7 = 343$

240 **(a)**

$2\vec{a} + 3\vec{b} - 5\vec{c} = 0$

$\Rightarrow 3(\vec{b} - \vec{a}) = 5(\vec{c} - \vec{a}) \Rightarrow \overrightarrow{AB} = \frac{5}{3}\overrightarrow{AC}$

$\Rightarrow \overrightarrow{AB}$ and $\overrightarrow{AC}$ must be parallel since there is a common point A. The points A, B and C must be collinear

241 **(b)**

Obviously, statement 1 is true

$\cos 2\alpha + \cos 2\beta + \cos 2\gamma = 2\cos^2\alpha - 1 + 2\cos^2\beta - 1 + 2\cos^2\gamma - 1$

$= 2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) - 3 = 2 - 3 = -1$

Hence, Statement 2 is true but does not explain Statement 1 as it is result derived using the result in the statement

242 **(b)**

$\because \overrightarrow{\mathbf{BA}} = \overrightarrow{\mathbf{BC}} - \overrightarrow{\mathbf{AC}}$

$= \left(\frac{\vec{\mathbf{e}}}{|\vec{\mathbf{e}}|} + \frac{\vec{\mathbf{f}}}{|\vec{\mathbf{f}}|}\right) - \left(\frac{2\vec{\mathbf{e}}}{|\vec{\mathbf{e}}|}\right)$

$= -\left(\frac{\vec{\mathbf{e}}}{|\vec{\mathbf{e}}|} + \frac{\vec{\mathbf{f}}}{|\vec{\mathbf{f}}|}\right)$

Now, $\overrightarrow{\mathbf{BA}}\cdot\overrightarrow{\mathbf{BC}} = -\left(\frac{\vec{\mathbf{e}}}{|\vec{\mathbf{e}}|} + \frac{\vec{\mathbf{f}}}{|\vec{\mathbf{f}}|}\right)\left(\frac{\vec{\mathbf{e}}}{|\vec{\mathbf{e}}|} + \frac{\vec{\mathbf{f}}}{|\vec{\mathbf{f}}|}\right)$

$= -\left(\frac{e^2}{e^2} - \frac{f^2}{f^2}\right) = -(1-1) = 0$

$\Rightarrow \angle B = 90°$

$\therefore \cos 2A + \cos 2C = 2\cos(A+C)\cos(A-C)$

$= 2\cos(180° - B)\cos(A-C)$

$= 2\cos 90°\cos(A-C)$

$= 0$

$\therefore \cos 2A + \cos 2B + \cos 2C = -1$

Also, if $\angle C = 90°$

Then, $\cos 2A + \cos 2B = 2\cos(A+B)\cos(A-B)$

$= 2\cos(180° - C)\cos(A-B)$

$= 2\cos 90°\cos(A-B)$

$= 0$

$\therefore \cos 2A + \cos 2B + \cos 2C = -1$

243 **(c)**

$\because \left[\vec{\mathbf{a}}\times\vec{\mathbf{b}}\;\vec{\mathbf{b}}\times\vec{\mathbf{c}}\;\vec{\mathbf{c}}\times\vec{\mathbf{a}}\right]$

$= (\vec{\mathbf{a}}\times\vec{\mathbf{b}})\cdot\{(\vec{\mathbf{b}}\times\vec{\mathbf{c}})\times(\vec{\mathbf{c}}\times\vec{\mathbf{a}})\}$

$= (\vec{\mathbf{a}}\times\vec{\mathbf{b}})\cdot\{(\vec{\mathbf{b}}\cdot(\vec{\mathbf{c}}\times\vec{\mathbf{a}}))\vec{\mathbf{c}}-(\vec{\mathbf{c}}\cdot(\vec{\mathbf{c}}\times\vec{\mathbf{a}}))\vec{\mathbf{b}}\}$

$= (\vec{\mathbf{a}}\times\vec{\mathbf{b}})\cdot\{[\vec{\mathbf{b}}\;\vec{\mathbf{c}}\;\vec{\mathbf{a}}]\vec{\mathbf{c}}-0\}$

$= (\vec{\mathbf{a}}\times\vec{\mathbf{b}}\cdot\vec{\mathbf{c}})[\vec{\mathbf{b}}\;\vec{\mathbf{c}}\;\vec{\mathbf{a}}] = [\vec{\mathbf{a}}\;\vec{\mathbf{b}}\;\vec{\mathbf{c}}]^2$

$\because$ $\vec{\mathbf{a}},\vec{\mathbf{b}},\vec{\mathbf{c}}$ are coplanar.

$\therefore$ $[\vec{\mathbf{a}}\;\vec{\mathbf{b}}\;\vec{\mathbf{c}}]=0$ and then $[\vec{\mathbf{a}}\times\vec{\mathbf{b}}\;\;\vec{\mathbf{b}}\times\vec{\mathbf{c}}\;\;\vec{\mathbf{c}}\times\vec{\mathbf{a}}]=0$

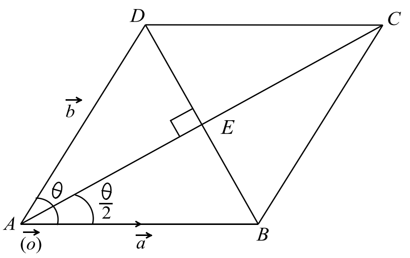
Hence, $\vec{\mathbf{a}}\times\vec{\mathbf{b}}$, $\vec{\mathbf{b}}\times\vec{\mathbf{c}}$ and $\vec{\mathbf{c}}\times\vec{\mathbf{a}}$ are also coplanar.

244 **(a)**

$\frac{3}{2}=\frac{p}{3}=\frac{3}{q}\Rightarrow p=\frac{9}{2}$ and $q=2$

Thus, both the statements are true and Statement 2 is the correct explanation for statement for Statement 1

245 **(b)**



We know that vector in the direction of angular bisector of unit vectors $\vec{a}$ and $\vec{b}$ is $\frac{\vec{a}+\vec{b}}{2\cos\frac{\theta}{2}}$

Where $\vec{a}=\overrightarrow{AB}=l_1\hat{\imath}+m_1\hat{\jmath}+n_1\hat{k}$ and $\vec{b}=\overrightarrow{AD}=l_2\hat{\imath}+m_2\hat{\jmath}+n_3\hat{k}$

Thus unit vector along the bisector is
$\frac{l_1+l_2}{\cos\frac{\theta}{2}}\hat{\imath}+\frac{m_1+m_2}{\cos\frac{\theta}{2}}\hat{\jmath}+\frac{n_1+n_2}{\cos\frac{\theta}{2}}\hat{k}$

Hence statement 1 is true

Also, in triangle ABD, by cosine rule

$$\cos\theta=\frac{AB^2+AD^2-BD^2}{2AB.AD}$$

$\Rightarrow \cos\theta$

$$=\frac{1+1-\left|(l_1-l_2)\hat{\imath}+(m_1-m_2)\hat{\jmath}+(n_1-n_2)\hat{k}\right|^2}{2}$$

$\Rightarrow \cos\theta$

$$=\frac{2-[(l_1-l_2)^2+(m_1-m_2)^2+(n_1-n_2)^2]}{2}$$

$$=\frac{2-[2-2(l_1l_2+m_1m_2+n_1n_2)]}{2}$$

$=l_1l_2+m_1m_2+n_1n_2$

Hence, Statement 2 is true but does not explain Statement 1

246 **(b)**

If $\vec{\mathbf{a}}=\vec{\mathbf{b}}$ then, $|\vec{\mathbf{a}}|=|\vec{\mathbf{b}}|$

Now, $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=\vec{\mathbf{a}}\cdot\vec{\mathbf{a}}=|\vec{\mathbf{a}}|^2$

and $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=\vec{\mathbf{b}}\cdot\vec{\mathbf{b}}=|\vec{\mathbf{b}}|^2$

$\therefore$ $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=|\vec{\mathbf{a}}|^2=|\vec{\mathbf{b}}|^2$

But it is true that if $|\vec{\mathbf{a}}|=|\vec{\mathbf{b}}|$ does not implies that $\vec{\mathbf{a}}=\vec{\mathbf{b}}$

247 **(b)**

$\vec{r}\cdot\vec{a}=\vec{r}\cdot\vec{b}=\vec{r}\cdot\vec{c}=0$ only if $\vec{a},\vec{b}$ and $\vec{c}$ are coplanar

$\Rightarrow [\vec{a}\vec{b}\vec{c}]=0$

Hence, statement 2 is true

Also, $[\vec{a}-\vec{b}\vec{b}-\vec{c}\vec{c}-\vec{a}]=0$ even if $[\vec{a}\vec{b}\vec{c}]\neq 0$

Hence, Statement 2 is not the correct explanation for Statement 1

248 **(a)**

Let the three given unit vectors be $\hat{a},\hat{b}$ and $\hat{c}$. Since they are mutually perpendicular $\hat{a}\cdot(\hat{b}\times\hat{c})=1$. Therefore,

$$\begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}=1$$

$$\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}=1$$

Hence, $a_1\hat{\imath}+b_1\hat{\jmath}+c_1\hat{k}, a_2\hat{\imath}+b_2\hat{\jmath}+c_2\hat{k}$ and $a_3\hat{\imath}+b_3\hat{\jmath}+c_3\hat{k}$ may be mutually perpendicular

249 **(b)**

We have, $\left.\begin{array}{l}\vec{\mathbf{r}}\cdot\vec{\mathbf{a}}=0 \Rightarrow \vec{\mathbf{r}} \perp \vec{\mathbf{a}}\\ \vec{\mathbf{r}}\cdot\vec{\mathbf{b}}=0 \Rightarrow \vec{\mathbf{r}} \perp \vec{\mathbf{b}}\\ \vec{\mathbf{r}}\cdot\vec{\mathbf{c}}=0 \Rightarrow \vec{\mathbf{r}} \perp \vec{\mathbf{c}}\end{array}\right\}$

$\Rightarrow$ $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are coplanar vectors.

Also, $\because \vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}=0$

$\therefore\ \vec{\mathbf{a}}\times\vec{\mathbf{b}}+\vec{\mathbf{b}}\times\vec{\mathbf{c}}+\vec{\mathbf{c}}\times\vec{\mathbf{a}}=0$

Also, $\vec{\mathbf{a}}\cdot(\vec{\mathbf{a}}\times\vec{\mathbf{b}})+\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})+\vec{\mathbf{a}}\cdot(\vec{\mathbf{c}}\times\vec{\mathbf{a}})=0$

$0+[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]+0=0$

$\therefore\ [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]=0$

Hence, $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are coplanar.

250 **(a)**

$3\vec{a}-2\vec{b}+5\vec{c}-6\vec{d}=(2\vec{a}-2\vec{b})$

$+(-5\vec{a}+5\vec{c})+(6\vec{a}-6\vec{d})$

$=-2\overrightarrow{AB}+2\overrightarrow{AC}-6\overrightarrow{AD}=\vec{0}$

Therefore $\overrightarrow{AB}, \overrightarrow{AC}$ and $\overrightarrow{AD}$ are linearly dependent. Hence by Statement 2 Statement 1 is true

251 **(b)**

A vector along the bisector is $\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{b}}{|\vec{b}|}=\frac{-5\hat{\imath}+7\hat{\jmath}+2\hat{k}}{9}$

Hence $\vec{c}=-5\hat{\imath}+7\hat{\jmath}+2\hat{k}$ is along the bisector. It is obvious that $\vec{c}$ makes an equal angle with $\vec{a}$ and $\vec{b}$. However, statement 2 does not explain Statement 1, as a vector equally inclined to given two vectors is not necessarily coplanar

252 **(c)**

In $\Delta ABC, \overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}=-\overrightarrow{CA} \Rightarrow \overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\vec{O}$

$\overrightarrow{OA}+\overrightarrow{AB}=\overrightarrow{OB}$ is the triangle law of addition

Hence, Statement 1 is true and Statement 2 is false

253 **(a)**

Given, $\vec{\mathbf{a}}=a_1\hat{\mathbf{i}}+a_2\hat{\mathbf{j}}+a_3\hat{\mathbf{k}}$
Now,
$|\vec{\mathbf{a}}\times\hat{\mathbf{i}}|^2=(\vec{\mathbf{a}}\times\hat{\mathbf{i}})\cdot(\vec{\mathbf{a}}\times\hat{\mathbf{i}})$
$=(a_3\hat{\mathbf{j}}-a_2\hat{\mathbf{k}})\cdot(a_3\hat{\mathbf{j}}-a_2\hat{\mathbf{k}})$
$=a_3^2+a_2^2$
Similarly, $|\vec{\mathbf{a}}\times\hat{\mathbf{j}}|^2=a_1^2+a_3^2$
and $|\vec{\mathbf{a}}\times\hat{\mathbf{k}}|^2=a_1^2+a_1^2$
$\therefore\ |\vec{\mathbf{a}}\times\hat{\mathbf{i}}|^2+|\vec{\mathbf{a}}\times\hat{\mathbf{j}}|^2+|\vec{\mathbf{a}}\times\hat{\mathbf{k}}|^2$
$=2(a_1^2+a_2^2+a_3^2)=2|\vec{\mathbf{a}}|^2$
Hence, both A and R are true and (R) is correct reason for (A).

254 **(c)**

$\because\ |2\vec{\mathbf{a}}-\vec{\mathbf{b}}|=5$

$\Rightarrow\ |2\vec{\mathbf{a}}-\vec{\mathbf{b}}|^2=5^2$

$\Rightarrow\ 4a^2+b^2-4\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=25$

$\Rightarrow 16+9-4\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=25$

$\therefore\ \vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=0$

$\therefore\ |2\vec{\mathbf{a}}+\vec{\mathbf{b}}|=\sqrt{|2\vec{\mathbf{a}}+\vec{\mathbf{b}}|^2}$

$=\sqrt{[4a^2+b^2+4(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}})]}$

$=\sqrt{(16+9+0)}=5$

$\therefore |\vec{\mathbf{p}}-\vec{\mathbf{q}}|=|\vec{\mathbf{p}}+\vec{\mathbf{q}}|$

Which is possible only when $\vec{\mathbf{p}}\perp\vec{\mathbf{q}}$.

255 **(a)**

If $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are reciprocal then,

$\vec{\mathbf{a}}=\lambda\vec{\mathbf{b}}, \lambda\in R^+$ and $|\vec{\mathbf{a}}||\vec{\mathbf{b}}|=1$

$\Rightarrow\ |\vec{\mathbf{a}}|=|\lambda||\vec{\mathbf{b}}|$

$\therefore |\lambda|=\frac{|\vec{\mathbf{a}}|}{|\vec{\mathbf{b}}|}=\frac{1}{|\vec{\mathbf{b}}|^2}$

$\because\ \lambda\in R^+$

$\therefore\ \lambda=\frac{1}{|\vec{\mathbf{b}}|^2}$

$\therefore \vec{\mathbf{a}}=\frac{\vec{\mathbf{b}}}{|\vec{\mathbf{b}}|^2}$

$\Rightarrow\ \vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=\frac{\vec{\mathbf{b}}}{|\vec{\mathbf{b}}|^2}\cdot\vec{\mathbf{b}}=\frac{|\vec{\mathbf{b}}|^2}{|\vec{\mathbf{b}}|^2}=1$

256 **(c)**

Component of vector $\vec{b}=4\hat{\imath}+2\hat{\jmath}+3\hat{k}$ in the

direction of $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ is $\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\frac{\vec{a}}{|\vec{a}|}$ or $3\hat{\imath} + 3\hat{\jmath} + 3\hat{k}$.
Then component in the direction perpendicular to the direction of $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ is $\vec{b} - 3\hat{\imath} + 3\hat{\jmath} + 3\hat{k} = \hat{\imath} - \hat{\jmath}$

257 **(b)**

a. $|\vec{a} + \vec{b}| = |\vec{a} + 2\vec{b}|$

$a^2 + b^2 + 2\,\vec{a}\cdot\vec{b} = a^2 + 4b^2 + 4\,\vec{a}\cdot\vec{b}$

Or $2\,\vec{a}\cdot\vec{b} = -3b^2 < 0$

Hence, angle between $\vec{a}$ and $\vec{b}$ is obtuse

b. $|\vec{a} + \vec{b}| = |\vec{a} - 2\vec{b}|$

or $a^2 + b^2 + 2\,\vec{a}\cdot\vec{b} = a^2 + 4b^2 - 4\,\vec{a}\cdot\vec{b}$

or $6\,\vec{a}\cdot\vec{b} = 3b^2$

Hence, angle between $\vec{a}$ and $\vec{b}$ is acute

c. $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

$\Rightarrow \vec{a}\cdot\vec{b}$

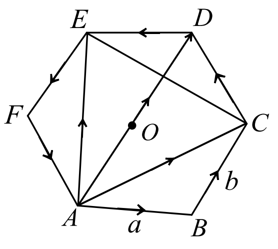
Hence, $\vec{a}$ is perpendicular to $\vec{b}$

d. $\vec{c} \times (\vec{a} \times \vec{b})$ lies in the plane of vectors $\vec{a}$ and $\vec{b}$

A vector perpendicular to this plane is parallel to $\vec{a} \times \vec{b}$

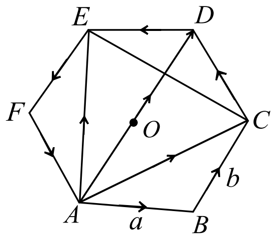
Hence, angle is $0°$

258 **(a)**



$\overrightarrow{AB} = \vec{a}, \overrightarrow{BC} = \vec{b}$
$\therefore \overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \vec{a} + \vec{a}$ (i)
$\overrightarrow{AD} = 2\overrightarrow{BC} = 2\vec{b}$ (ii)
(because AD is parallel to BC and twice its length)
$\overrightarrow{CD} = \overrightarrow{AD} - \overrightarrow{AC} = 2\vec{b} - (\vec{a} + \vec{b})$
$= \vec{b} - \vec{a}$
$\overrightarrow{FA} = -\overrightarrow{CD} = \vec{a} - \vec{b}$ (iii)
$\overrightarrow{DE} = -\overrightarrow{AB} = -\vec{a}$ (iv)
$\overrightarrow{EF} = -\overrightarrow{BC} = -\vec{b}$ (v)
$\overrightarrow{AE} = \overrightarrow{AD} + \overrightarrow{DE} = 2\vec{b} - \vec{a}$ (vi)
$\overrightarrow{CE} = \overrightarrow{CD} + \overrightarrow{DE} = \vec{b} - \vec{a} - \vec{a} = \vec{b} - 2\vec{a}$ (vii)

259 **(b)**



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260 **(a)**

a. Vector $-3\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ and $\hat{\imath} + \hat{\jmath}$ are coplanar with $\vec{a}$ and $\vec{b}$

b. $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 2 & 2 \\ -2 & 2 & 2 \end{vmatrix}$
$= 2\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

c. If $\vec{c}$ is equally inclined to $\vec{a}$ and $\vec{b}$, then we must have $\vec{a}\cdot\vec{c} = \vec{b}\cdot\vec{c}$,
which is true for vectors in options p, q, s

d. Vector is forming a triangle with a and b. Then $\vec{c} = \vec{a} + \vec{b} = -3\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$

261 **(b)**

(i)$\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})\cdot(\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 1$
$\vec{\mathbf{b}}\cdot\vec{\mathbf{d}} = (\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})\cdot(\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}) = 1$
$\therefore \vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = \vec{\mathbf{b}}\cdot\vec{\mathbf{d}}$
(ii)$\vec{\mathbf{b}}\cdot\vec{\mathbf{c}} = (\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})\cdot(\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) = -1$
$\vec{\mathbf{a}}\cdot\vec{\mathbf{d}} = (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})\cdot(\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}) = -1$
$\therefore \vec{\mathbf{b}}\cdot\vec{\mathbf{c}} = \vec{\mathbf{a}}\cdot\vec{\mathbf{d}}$
(iii)$[\vec{\mathbf{A}}\,\vec{\mathbf{B}}\,\vec{\mathbf{C}}] = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 0 + 2 + 2 = 4$
(iv)$\vec{\mathbf{b}} \times \vec{\mathbf{c}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

262 **(a)**

$[\vec{a}\times\vec{b}\vec{b}\times\vec{c}\vec{c}\times\vec{a}]=36$

Or $[\vec{a}\vec{b}\vec{c}]=6$

$\Rightarrow$ Volume of tetrahedron formed by vectors $\vec{a},\vec{b}$ and $\vec{c}$ is $\frac{1}{6}[\vec{a}\vec{b}\vec{c}]]=1$

$[\vec{a}+\vec{b}\vec{b}+\vec{c}\vec{c}+\vec{a}]=2[\vec{a}\vec{b}\vec{c}]=12$

$\vec{a}-\vec{b},\vec{b}-\vec{c}$ and $\vec{c}-\vec{a}$ are coplanar

$\Rightarrow[\vec{a}-\vec{b}\vec{b}-\vec{c}\vec{c}-\vec{a}]=0$

263 **(d)**

a. $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ -1 & 1 & 2\\ -1 & -2 & -1\end{vmatrix}=3\hat{\imath}-3\hat{\jmath}+3\hat{k}$

Hence, the area of the triangle is $\frac{3\sqrt{3}}{2}$

b. The area of the parallelogram is $3\sqrt{3}$

c. The area of a parallelogram whose diagonals are $2\vec{a}$ and $4\vec{b}$ is $\frac{1}{2}|2\vec{a}\times 4\vec{b}|=12\sqrt{3}$

d. Volume of the parallelepiped

$=|(\vec{a}\times\vec{b})\cdot\vec{c}|=\sqrt{9+36+9}=3\sqrt{6}$

264 **(c)**

a. $|\vec{a}+\vec{b}+\vec{c}|=\sqrt{6}$

$\Rightarrow\vec{a}^2+\vec{b}^2+\vec{c}^2+2\vec{a}\cdot\vec{b}+2\vec{b}\cdot\vec{c}+2\vec{c}\cdot\vec{a}=6$

$\therefore|\vec{a}|=1$

b. $\vec{a}$ is perpendicular to $\vec{b}+\vec{c}$

$\Rightarrow\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=0$ (i)

b I perpendicular to $a+c$

$\Rightarrow\vec{b}\cdot\vec{c}+\vec{b}\cdot\vec{c}=0$ (ii)

$\vec{c}$ is perpendicular to $\vec{a}+\vec{b}$

$\Rightarrow\vec{c}\cdot\vec{a}+\vec{a}\cdot\vec{c}\cdot\vec{b}=0$ (iii)

From (i), (ii) and (iii), we get

$\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$

$\therefore|\vec{a}+\vec{b}+\vec{c}|=7$

c. $(\vec{a}\cdot\vec{c})(\vec{b}\cdot\vec{d})-(\vec{b}\cdot\vec{c})(\vec{a}\cdot\vec{d})=21$

d. We know that $[\vec{a}\times\vec{b}\vec{b}\times\vec{c}\vec{c}\times\vec{a}]=[\vec{a}\vec{b}\vec{c}]^2$

and $[\vec{a}\vec{b}\vec{c}]^2=\begin{vmatrix}\vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c}\\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c}\\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c}\end{vmatrix}$

$=\begin{vmatrix}4 & 2 & 2\\ 2 & 4 & 2\\ 2 & 2 & 4\end{vmatrix}$

$=32$

$\therefore[\vec{a}\vec{b}\vec{c}]=4\sqrt{2}$

265 **(b)**

a. If $\vec{a},\vec{b}$ and $\vec{c}$ are mutually perpendicular, then

$[\vec{a}\times\vec{b}\vec{b}\times\vec{c}\vec{c}\times\vec{a}]=[\vec{a}\vec{b}\vec{c}]^2$

$=(|\vec{a}||\vec{b}||\vec{c}|)^2=16$

b. Given $\vec{a}$ and $\vec{b}$ are two unit vectors, i.e.,

$|\vec{a}|=|\vec{b}|=1$ and angle between them is $\pi/3$

$\sin\theta=\frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}\Rightarrow\sin\frac{\pi}{3}=|\vec{a}\times\vec{b}|$

$\frac{\sqrt{3}}{2}|\vec{a}\times\vec{b}|$

Now $[\vec{a}\vec{b}+\vec{a}\times\vec{b}\vec{b}]=[\vec{a}\vec{b}\vec{c}]+[\vec{a}\vec{a}\times\vec{b}\vec{b}]$

$=0+[\vec{a}\vec{a}\times\vec{b}\vec{b}]$

$=(\vec{a}\times\vec{b})\cdot(\vec{b}\times\vec{a})$

$=-(\vec{a}\times\vec{b})\cdot(\vec{a}\times\vec{b})$

$=-|\vec{a}\times\vec{b}|^2$

$=-\frac{3}{4}$

c. It $\vec{b}$ and $\vec{c}$ are orthogonal $\vec{b}\cdot\vec{c}=0$

Also, it is given that $\vec{b}\times\vec{c}=\vec{a}$. Now

$[\vec{a}\vec{a}+\vec{b}\vec{b}+\vec{c}]+[\vec{b}+\vec{c}\vec{a}+\vec{b}\vec{b}+\vec{c}]$

$=[\vec{a}\vec{b}\vec{c}]$

$=\vec{a}\cdot(\vec{b}\times\vec{c})$

$=\vec{a}\cdot\vec{a}=|\vec{a}|^2=1$ (because $\vec{a}$ is a unit vector)

d. $[\vec{x}\vec{y}\vec{a}]=0$

therefore $\vec{x},\vec{y}$ and $\vec{a}$ are coplanar (i)

$[\vec{x}\vec{y}\vec{b}]=0$

Therefore, $\vec{x},\vec{y}$ and $\vec{b}$ are coplanar (ii)

Also, $[\vec{a}\vec{b}\vec{c}]=0$

Therefore, $\vec{a},\vec{b}$ and $\vec{c}$ are coplanar (iii)

From (i), (ii) and (iii),

$\vec{x},\vec{y}$ and $\vec{c}$ are coplanar. Therefore,

$[\vec{x}\vec{y}\vec{c}]=0$

266 **(a)**

a. Given equations are consistent if

$(\hat{\imath}+\hat{\jmath})+\lambda(\hat{\imath}+2\hat{\jmath}-\hat{k})$

$=(\hat{\imath}+2\hat{\jmath})+\mu(-\hat{\imath}+\hat{\jmath}+a\hat{k})$

$\Rightarrow 1+\lambda=1-\mu, 1+2\lambda=2+\mu, -\lambda=a\mu$

$\Rightarrow\lambda=1/3$ and $\mu=-1/3$

$\Rightarrow a=1$

b. $a=\lambda\hat{\imath}-3\hat{\jmath}-\hat{k}$

$\vec{b}=2\lambda\hat{\imath}+\lambda\hat{\jmath}-\hat{k}$

Angle between $\vec{a}\cdot\vec{b}>0$

$\Rightarrow 2\lambda^2-3\lambda+1>0$

Or $(2\lambda-1)(\lambda-1)>0$

Or $\lambda\in\left(-\infty,\frac{1}{2}\right)\cup(1,\infty)$

Also $\vec{b}$ makes on obtuse angle with the axes. Therefore,

$\vec{b}\cdot\hat{\imath}<0\Rightarrow\lambda<0$

$\vec{b}\cdot\hat{\jmath}<0\Rightarrow\lambda<0$ (ii)

Combining these two, we get $\lambda=-4,-2$

c. If vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}, \hat{\imath} + 2\hat{\jmath} + (1+a)\hat{k}$ and $3\hat{\imath} + a\hat{\jmath} + 5\hat{k}$ are coplanar, then

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & 1+a \\ 3 & a & 5 \end{vmatrix}$$

Or $a^2 + 2a - 8 = 0$

Or $(a+4)(a-2) = 0$

Or $a = -4, 2$

d. $\vec{A} = 2\hat{\imath} + \lambda\hat{\jmath} + 3\hat{k}$

$B = 2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$

$C = 3\hat{\imath} + \hat{\jmath} + 0.\hat{k}$

$\vec{A} + \lambda\vec{B} = 2(1+\lambda)\hat{\imath} + (\lambda + \lambda^2)\hat{\jmath} + (3+\lambda)\hat{k}$

Now $(\vec{A} + \lambda\vec{B}) \perp \vec{C}$. Therefore,

$(\vec{A} + \lambda\vec{B}).\vec{C} = 0$

Or $6(1+\lambda) + (\lambda + \lambda^2) + 0 = 0$

Or $\lambda^2 + 7\lambda + 6 = 0$

Or $(\lambda+6)(\lambda+1) = 0$

Or $\lambda = -6, = -1$

$\Rightarrow |2\lambda| = 12, 2$

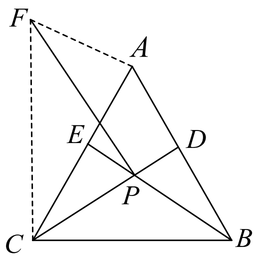
267 **(b)**

The vector equations of CD and BE are

$\vec{\mathbf{r}}_1 = (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) + \frac{\lambda}{3}(7\hat{\mathbf{j}} - 7\hat{\mathbf{k}})$ ...(i)

and $\vec{\mathbf{r}}_2 = (-\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) + \frac{\mu}{3}(7\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 7\hat{\mathbf{k}})$ ....(ii)

The intersection point of $\overrightarrow{\mathbf{CD}}$ and $\overrightarrow{\mathbf{BE}}$.



ie, $1 = -1 + \frac{7\mu}{3}, -2 + \frac{7\lambda}{3} = 1 - \frac{7\mu}{3}$

and $4 - \frac{7\lambda}{3} = 1 + \frac{7\mu}{3}$

$\Rightarrow \mu = \frac{6}{7}$ and $\lambda = \frac{3}{7}$

Substituting the value of λ in Eq. (ii), we get

$\vec{\mathbf{r}}_1 = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

268 **(d)**

The equation of the plane parallel to the given plane

$\vec{\mathbf{r}} \cdot (4\hat{\mathbf{i}} - 12\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) + \lambda = 0$ ...(i)

This plane passes through $2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 4\hat{\mathbf{k}}$.

Therefore, $(2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 4\hat{\mathbf{k}}) \cdot (4\hat{\mathbf{i}} - 12\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) + \lambda = 0$

$\Rightarrow 8 + 12 + 12 + \lambda = 0$

$\therefore \lambda = -32$

Hence, required plane is

$\vec{\mathbf{r}} \cdot (4\hat{\mathbf{i}} - 12\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) = 32$

269 **(c)**

$$\because [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}][\vec{\mathbf{u}}\,\vec{\mathbf{v}}\,\vec{\mathbf{w}}] = \begin{vmatrix} \vec{\mathbf{a}}\cdot\vec{\mathbf{u}} & \vec{\mathbf{b}}\cdot\vec{\mathbf{u}} & \vec{\mathbf{c}}\cdot\vec{\mathbf{u}} \\ \vec{\mathbf{a}}\cdot\vec{\mathbf{v}} & \vec{\mathbf{b}}\cdot\vec{\mathbf{v}} & \vec{\mathbf{c}}\cdot\vec{\mathbf{v}} \\ \vec{\mathbf{a}}\cdot\vec{\mathbf{w}} & \vec{\mathbf{b}}\cdot\vec{\mathbf{w}} & \vec{\mathbf{c}}\cdot\vec{\mathbf{w}} \end{vmatrix}$$

$$\therefore [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]^2 = [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}][\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$$

$$= \begin{vmatrix} \vec{\mathbf{a}}\cdot\vec{\mathbf{a}} & \vec{\mathbf{b}}\cdot\vec{\mathbf{a}} & \vec{\mathbf{c}}\cdot\vec{\mathbf{a}} \\ \vec{\mathbf{a}}\cdot\vec{\mathbf{b}} & \vec{\mathbf{b}}\cdot\vec{\mathbf{b}} & \vec{\mathbf{c}}\cdot\vec{\mathbf{b}} \\ \vec{\mathbf{a}}\cdot\vec{\mathbf{c}} & \vec{\mathbf{b}}\cdot\vec{\mathbf{c}} & \vec{\mathbf{c}}\cdot\vec{\mathbf{c}} \end{vmatrix} \quad .....(i)$$

Now, $\vec{\mathbf{a}}\cdot\vec{\mathbf{a}} = a^2 = |\vec{\mathbf{a}}|^2 = 16$

$\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = \vec{\mathbf{b}}\cdot\vec{\mathbf{a}} = |\vec{\mathbf{a}}|\,|\vec{\mathbf{b}}|\cos\pi/3 = 4\cdot4\cdot\frac{1}{2} = 8$

$\vec{\mathbf{a}}\cdot\vec{\mathbf{c}} = \vec{\mathbf{c}}\cdot\vec{\mathbf{a}} = |\vec{\mathbf{a}}||\vec{\mathbf{c}}|\cos\pi/3 = 4\cdot4\cdot\frac{1}{2} = 8$

$\vec{\mathbf{b}}\cdot\vec{\mathbf{b}} = b^2 = |\vec{\mathbf{b}}|^2 = 16$

$\vec{\mathbf{b}}\cdot\vec{\mathbf{c}} = \vec{\mathbf{c}}\cdot\vec{\mathbf{b}} = |\vec{\mathbf{b}}||\vec{\mathbf{c}}|\cos\pi/3 = 4\cdot4\cdot\frac{1}{2} = 8$

$\vec{\mathbf{c}}\cdot\vec{\mathbf{c}} = |\vec{\mathbf{c}}|^2 = 4^2 = 16$

From Eq. (i)

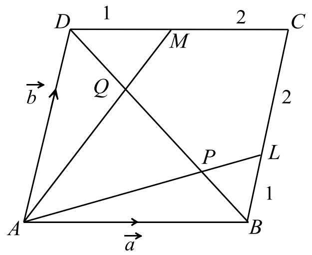
$$[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]^2 = \begin{vmatrix} 16 & 8 & 8 \\ 8 & 16 & 8 \\ 8 & 8 & 16 \end{vmatrix}$$

$$= 8^3 \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 8^3 \cdot 4 = 64 \times 32$$

$|[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]| = 32\sqrt{2}$

Volume of the parallelepiped $= |[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]| = 32\sqrt{2}$

270 **(c)**



$\overrightarrow{BL} = \frac{1}{3}\vec{b}$

$\therefore \overrightarrow{AL} = \vec{a} + \frac{1}{3}\vec{b}$

Let $\overrightarrow{AP} = \lambda\overrightarrow{AL}$ and P divides DB in the ratio $\mu : 1-\mu$

Then $\overrightarrow{AP} = \lambda\vec{a} + \frac{\lambda}{3}\vec{b}$

Also $\overrightarrow{AP} = \mu\vec{a} + (1-\mu)\vec{b}$

From (i) and (ii), $\lambda\vec{a} + \frac{\lambda}{3}\vec{b} = \mu\vec{a} + (1-\mu)\vec{b}$

$\therefore \lambda = \mu$ and $\frac{\lambda}{3} = 1 - \mu$

$\therefore \lambda = \frac{3}{4}$

Hence, P divides AL in the ratio 3:1 and P divides DB in the ratio 3:1

Similarly Q divides DB in the ratio 1:3

Thus $DQ = \frac{1}{4}DB$ and $PB = \frac{1}{4}DB$

$\therefore PQ = \frac{1}{2}DB, i.e. PQ:DB = 1:2$

271 **(c)**

Let the position vectors of A, B, C and D and be $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$, respectively

Then $OA: CB = 2:1$

$\Rightarrow \overrightarrow{OA} = 2\overrightarrow{CB}$

$\Rightarrow \vec{a} = 2(\vec{b} - \vec{c})$ (i)

and $OD: AB = 1:3$

$3\overrightarrow{OD} = \overrightarrow{AB}$

$\Rightarrow 3\vec{d} = (\vec{b} - \vec{a}) = \vec{b} - 2(\vec{b} - \vec{c})$(using (i))

$\Rightarrow 3\vec{d} = -\vec{b} + 2\vec{c}$

Let $OX: XC = \lambda: 1$ and $AX: XD = \mu: 1$

Now, X divides OC in the ratio $\lambda: 1$. Therefore,

P.V. of $X = \frac{\lambda\vec{c}}{\lambda+1}$

X also divides AD in the ratio $\mu: 1$

P.V. of $X = \frac{\mu\vec{d}+\vec{a}}{\mu+1}$ (iv)

From (iii) and (iv), we get

$$\frac{\lambda\vec{c}}{\lambda+1} = \frac{\mu\vec{d}+\vec{a}}{\mu+1}$$

$$\Rightarrow \left(\frac{\lambda}{\lambda+1}\right)\vec{c} = \left(\frac{\mu}{\mu+1}\right)\vec{d} + \left(\frac{1}{\mu+1}\right)\vec{a}$$

$$\Rightarrow \left(\frac{\lambda}{\lambda+1}\right)\vec{c} = \left(\frac{\mu}{\mu+1}\right)\left(\frac{-\vec{b}+2\vec{c}}{3}\right)$$

$$+\left(\frac{1}{\mu+1}\right)2(\vec{b} - \vec{c})\text{(using (i)and (ii))}$$

$$\Rightarrow \left(\frac{\lambda}{\lambda+1}\right)\vec{c} = \left(\frac{6-\mu}{3(\mu+1)}\right)\vec{b}$$

$$+\left(\frac{2\mu}{3(\mu+1)} - \frac{2}{\mu+1}\right)\vec{c}$$

$$\Rightarrow \left(\frac{\lambda}{\lambda+1}\right)\vec{c} = \left(\frac{6-\mu}{3(\mu+1)}\right)\vec{b}$$

$$+\left(\frac{2\mu-6}{3(\mu+1)}\right)\vec{c}$$

$$\Rightarrow \left(\frac{6-\mu}{3(\mu+1)}\right)\vec{b} + \left(\frac{2\mu-6}{3(\mu+1)} - \frac{\lambda}{\lambda+1}\right)$$

$\vec{c} = \vec{0}$

$$\Rightarrow \frac{6-\mu}{3(\mu+1)} = 0 \text{ and } \frac{2\mu-6}{3(\mu+1)} - \frac{\lambda}{\lambda+1}$$

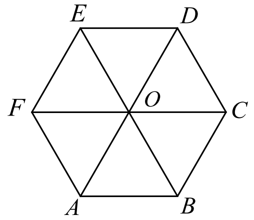
$= 0$(as $\vec{b}$ and $\vec{c}$ are non − collinear)

$\Rightarrow \mu = 6, \lambda = \frac{2}{5}$

Hence $OX: XC = 2:5$

272 **(c)**

Consider the regular hexagon $ABCDEF$ with centre at O(origin)



$\overrightarrow{AD} + \overrightarrow{EB} + \overrightarrow{FC} = 2\overrightarrow{AO} + 2\overrightarrow{OB} + 2\overrightarrow{OC}$

$= 2(\overrightarrow{AO} + \overrightarrow{OB}) + 2\overrightarrow{OC}$

$= 2\overrightarrow{AB} + 2\overrightarrow{AB}$ $(\because \overrightarrow{OC} = \overrightarrow{AB})$

$= 4\overrightarrow{AB}$

$\vec{R} = \overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF}$

$= \overrightarrow{ED} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{CD}(\because \overrightarrow{AB} = \overrightarrow{ED}$ and $\overrightarrow{AF} = \overrightarrow{CD})$

$= (\overrightarrow{AC} + \overrightarrow{CD}) + (\overrightarrow{AE} + \overrightarrow{ED}) + \overrightarrow{AD}$

$= \overrightarrow{AD} + \overrightarrow{AD} + \overrightarrow{AD} = 3\overrightarrow{AD} = 6\overrightarrow{AO}$

273 **(b)**

Taking dot product of $\vec{u} + \vec{v} + \vec{w} = \vec{a}$ with $\vec{u}$, we have

$1 + \vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{w} = \vec{a}\cdot\vec{u} = \frac{3}{2} \Rightarrow \vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{w} = \frac{1}{2}$ (i)

Similarly, taking dot product with $\vec{v}$, we have

$\vec{u}\cdot\vec{v} + \vec{w}\cdot\vec{v} = \frac{3}{4}$ (ii)

Also,$\vec{a}\cdot\vec{u} + \vec{a}\cdot\vec{v} + \vec{a}\cdot\vec{w} = \vec{a}\cdot\vec{a} = 4$

$\Rightarrow \vec{a}\cdot\vec{w} = 4 - \left(\frac{3}{2} + \frac{7}{4}\right) = \frac{3}{4}$

Again, taking dot product with $\vec{w}$, we have

$\vec{u}\cdot\vec{w} + \vec{v}\cdot\vec{w} = \frac{3}{4} - 1 = -\frac{1}{4}$ (iii)

Adding (i), (ii) and (iii), we have

$2(\vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{w} + \vec{v}\cdot\vec{w}) = 1$

$\Rightarrow \vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{w} + \vec{v}\cdot\vec{w} = \frac{1}{2}$ (iv)

Subtracting (i), (ii) and (iii) from (iv), we have

$\vec{v}\cdot\vec{w} = 0, \vec{u}\cdot\vec{w} = -\frac{1}{4}$ and $\vec{u}\cdot\vec{v} = \frac{3}{4}$

Now, the equation $\vec{u}\times(\vec{v}\times\vec{w}) = \vec{b}$ and $(\vec{u}\times\vec{v})\times\vec{w} = \vec{c}$ can be written as $(\vec{u}\cdot\vec{w})\vec{v} - (\vec{u}\cdot\vec{v})\vec{w} = \vec{b}$ and $(\vec{u}\cdot\vec{w})\vec{v} - (\vec{v}\cdot\vec{w})\vec{u} = \vec{c} \Rightarrow -\frac{1}{4}\vec{v} - \frac{3}{4}\vec{w} = \vec{b}, -\frac{1}{4}\vec{v} = \vec{c}$, i.e., $\vec{v} = -4\vec{c}$

$\Rightarrow \vec{c} - \frac{3}{4}\vec{w} = \vec{b} \Rightarrow \vec{w} = \frac{4}{3}(\vec{c} - \vec{b})$ and $\vec{u} = \vec{a} - \vec{v} - \vec{w} = \vec{a} + 4\vec{c} - \frac{4}{3}\vec{c} + \frac{4}{3}\vec{b} = \vec{a} + \frac{4}{3}\vec{b} + \frac{8}{3}\vec{c}$

274 **(d)**

Given that $|\vec{x}| = |\vec{y}| = |\vec{z}| = \sqrt{2}$ and they are inclined at an angle of 60° with each other

$\therefore \vec{x}\cdot\vec{y} = \vec{y}\cdot\vec{z} = \vec{z}\cdot\vec{x} = \sqrt{2}.\sqrt{2}\cos 60° = 1$

$\vec{x}\times(\vec{y}\times\vec{z}) = \vec{a} \Rightarrow (\vec{x}\cdot\vec{z})\vec{y} - (\vec{x}\cdot\vec{y})\vec{z} = \vec{a} \Rightarrow \vec{y} - \vec{z} = \vec{a}$ (i)

Similarly, $\vec{y}\times(\vec{z}\times\vec{x}) = \vec{b} \Rightarrow \vec{z} - \vec{x} = \vec{b}$ (ii)

$\vec{y} = \vec{a} + \vec{z}, \vec{x} = \vec{z} - \vec{b}$ (from (i)and (ii))(iii)
Now, $\vec{x} \times \vec{y} = \vec{c}$
$\Rightarrow (\vec{z} - \vec{b}) \times (\vec{z} + \vec{a}) = \vec{c}$
$\Rightarrow \vec{z} \times \vec{a} - \vec{b} \times \vec{z} - \vec{b} \times \vec{a} = \vec{c}$
$\Rightarrow \vec{z} \times (\vec{a} + \vec{b}) = \vec{c} + (\vec{b} \times \vec{a})$(iv)
$\Rightarrow (\vec{a} + \vec{b}) \times \{\vec{z} \times (\vec{a} + \vec{b})\}$
$= (\vec{a} + \vec{b}) \times \vec{c} + (\vec{a} + \vec{b}) \times (\vec{b} \times \vec{a})$
$\Rightarrow (\vec{a} + \vec{b})^2 \vec{z} - \{(\vec{a} + \vec{b}) \cdot \vec{z}\}(\vec{a} + \vec{b}) = (\vec{a} + \vec{b}) \times \vec{c} + |\vec{a}|^2 \vec{b} - |\vec{b}|^2 \vec{a} + (\vec{a} \cdot \vec{b})(\vec{b} - \vec{a})$ (v)
Now, (i) $\Rightarrow |\vec{a}|^2 = |\vec{y} - \vec{z}|^2 = 2 + 2 - 2 = 2$
Similarly, (ii) $\Rightarrow |\vec{b}|^2 = 2$
Also (i) and (ii) $\Rightarrow \vec{a} + \vec{b} = \vec{y} - \vec{x} \Rightarrow |\vec{a} + \vec{b}|^2 = 2$ (vi)
Also $(\vec{a} + \vec{b}) \cdot \vec{z} = (\vec{y} - \vec{x}) \cdot \vec{z} = \vec{y} \cdot \vec{z} - \vec{x} \cdot \vec{z} = 1 - 1 = 0$
And $\vec{a} \cdot \vec{b} = (\vec{y} - \vec{z}) \cdot (\vec{z} - \vec{x})$
$= \vec{y} \cdot \vec{z} - \vec{x} \cdot \vec{y} - |\vec{z}|^2 + \vec{x} \cdot \vec{z} = -1$
Thus from (v), we have $2\vec{z} = (\vec{a} + \vec{b}) \times \vec{c} + 2(\vec{b} - \vec{a}) - (\vec{b} - \vec{a})$ or $\vec{z} = (1/2)[(\vec{a} + \vec{b}) \times \vec{c} + b - a$
$\therefore \vec{y} = \vec{a} + \vec{z} = (1/2)[(\vec{a} + \vec{b}) \times \vec{c} + \vec{b} + \vec{a}]$ and
$\vec{x} = \vec{z} - \vec{b} = (1/2)[(\vec{a} + \vec{b}) \times \vec{c} - (\vec{a} + \vec{b})]$

275 **(b)**
Given
$\vec{x} \times \vec{y} = \vec{a}$(i)
$\vec{y} \times \vec{z} = \vec{b}$(ii)
$\vec{x} \cdot \vec{b} = \gamma$ (iii)
$\vec{x} \cdot \vec{y} = 1$ (iv)
$\vec{y} \cdot \vec{z} = 1$(v)
From (ii), $\vec{x} \cdot (\vec{y} \times \vec{z}) = \vec{x} \cdot \vec{b} = \gamma \Rightarrow [\vec{x}\vec{y}\vec{z}] = \gamma$
From (i) and (ii) $(\vec{x} \times \vec{y}) \times (\vec{y} \times \vec{z}) = \vec{a} \times \vec{b}$
$\therefore [\vec{x}\vec{y}\vec{z}]\vec{y} - [\vec{y}\vec{y}\vec{z}]\vec{x} = \vec{a} \times \vec{b} \Rightarrow \vec{y} = \frac{\vec{a} \times \vec{b}}{\gamma}$ (vi)
Also from (i), we get $(\vec{x} \times \vec{y}) \times \vec{y} = \vec{a} \times \vec{y}$
$\Rightarrow (\vec{x} \cdot \vec{y})\vec{y} - (\vec{y} \cdot \vec{y})\vec{x} = \vec{a} \times \vec{y} \Rightarrow \vec{x}$
$= (1/|\vec{y}|^2)(\vec{y} - \vec{a} \times \vec{y})$
$$= \frac{\gamma^2}{|\vec{a} \times \vec{b}|^2}\left[\frac{\vec{a} \times \vec{b}}{\gamma} - \frac{\vec{a} \times (\vec{a} \times \vec{b})}{\gamma}\right]$$
$$\Rightarrow \vec{x} \frac{\gamma}{|\vec{a} \times \vec{b}|^2}[\vec{a} \times \vec{b} - \vec{a} \times (\vec{a} \times \vec{b})]$$
Also from (ii), $(\vec{y} \times \vec{z}) \times \vec{y} = \vec{b} \times \vec{y} \Rightarrow |\vec{y}|^2 \vec{z} - (\vec{z} \cdot \vec{y})\vec{y} = \vec{b} \times \vec{y}$
$$\Rightarrow \vec{z} = \frac{1}{|\vec{y}|^2}[\vec{y} + \vec{b} \times \vec{y}]$$
$$= \frac{\gamma}{|\vec{a} \times \vec{b}|^2}[\vec{a} \times \vec{b} + \vec{b} \times (\vec{a} \times \vec{b})]$$

276 **(b)**
$\vec{P} \times \vec{B} = \vec{A} - \vec{P}$ and $|\vec{A}| = |\vec{B}| = 1$ and $\vec{A} \cdot \vec{B} = 0$ is given
Now $\vec{P} \times \vec{B} = \vec{A} - \vec{P}$
$(\vec{P} \times \vec{B}) \times \vec{B} = (\vec{A} - \vec{P}) \times \vec{B}$(taking cross product with $\vec{B}$on the sides)
$\Rightarrow (\vec{P} \cdot \vec{B})\vec{B} - (\vec{B} \cdot \vec{B})\vec{P} = \vec{A} \times \vec{B} - \vec{P} \times \vec{B}$
$\Rightarrow (\vec{P} \cdot \vec{B})\vec{B} - \vec{P} = \vec{A} \times \vec{B} - \vec{A} + \vec{P}$
$\Rightarrow 2\vec{P} = \vec{A} - \vec{A} \times \vec{B} - (\vec{P} \cdot \vec{B})\vec{B}$
$$\Rightarrow \vec{P} = \frac{\vec{A} - \vec{A} \times \vec{B} - (\vec{P} \cdot \vec{B})\vec{B}}{2}$$
Taking dot product with $\vec{B}$ on both sides of (i), we get
$\vec{P} \cdot \vec{B} = \vec{A} \cdot \vec{B} - \vec{P} \cdot \vec{B}$
$\Rightarrow \vec{P} \cdot \vec{B} = 0$
$$\Rightarrow \vec{P} = \frac{\vec{A} + \vec{B} \times \vec{A}}{2}$$
Now $(\vec{P} \times \vec{B}) \times \vec{B} = (\vec{P} \cdot \vec{B}).\vec{B} - (\vec{B} \cdot \vec{B})\vec{P} = -\vec{P}$
$\vec{P}, \vec{A}, \vec{P} \times \vec{B} (= \vec{A} - \vec{P})$ are dependent
Also $\vec{P} \cdot \vec{B} = 0$
And $|\vec{P}|^2 = \left|\frac{\vec{A} - \vec{A} \times \vec{B}}{2}\right|^2$
$$= \frac{|\vec{A}|^2 |\vec{A} \times \vec{B}|^2}{4}$$
$$= \frac{1+1}{4} = \frac{1}{2} \Rightarrow |\vec{P}| = \frac{1}{\sqrt{2}}$$

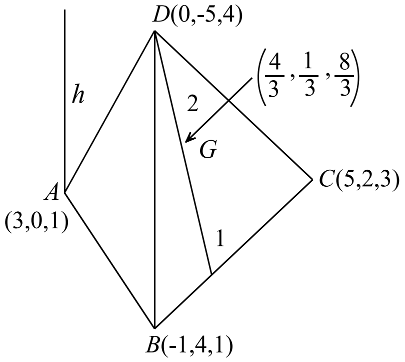
277 **(b)**
$$\vec{a} = \left[(2\hat{\imath} + 3\hat{\jmath} - 6\hat{k}).\frac{(2\hat{\imath} - 3\hat{\jmath} + 6\hat{k})}{7}\right]\frac{2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}}{7}$$
$$= \frac{-41}{49}(2\hat{\imath} - 3\hat{\jmath} + 6\hat{k})$$
$$\vec{a} = \frac{-41}{49}\left((2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}).\frac{(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})}{7}\right) \times \frac{(-2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})}{7}$$
$$= \frac{-41}{(49)^2}(-4 - 9 + 36)(-2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$$
$$= \frac{943}{49^2}(2\hat{\imath} - 3\hat{\jmath} + 6\hat{k})$$

278 **(b)**
Point G is $\left(\frac{4}{3}, \frac{1}{3}, \frac{8}{3}\right)$. Therefore,
$$|\overrightarrow{AG}|^2 = \left(\frac{5}{3}\right)^2 + \frac{1}{9} + \left(\frac{5}{3}\right)^2 = \frac{51}{9}$$

Or $|\overrightarrow{AG}| = \frac{\sqrt{51}}{3}$

$\overrightarrow{AB} = -4\hat{\imath} + 4\hat{\jmath} + 0\hat{k}$

$\overrightarrow{AC} = 2\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$



$\therefore\ \overrightarrow{AB} \times \overrightarrow{AC} = -8 - 8\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{vmatrix}$

$= 8(\hat{\imath} + \hat{\jmath} - 2\hat{k})$

Area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = 4\sqrt{6}$

$\overrightarrow{AD} = -3\hat{\imath} - 5\hat{\jmath} + 3\hat{k}$

The length of the perpendicular from the vertex D on the opposite face

$= |\text{Projection of } \overrightarrow{AD} \text{ on } \overrightarrow{AB} \times \overrightarrow{AC}|$

$= \left|\frac{(-3\hat{\imath} - 5\hat{\jmath} + 3\hat{k})(\hat{\imath} + \hat{\jmath} - 2\hat{k})}{\sqrt{6}}\right|$

$= \left|\frac{-3-5-6}{\sqrt{6}}\right| = \frac{14}{\sqrt{6}}$

279 **(c)**

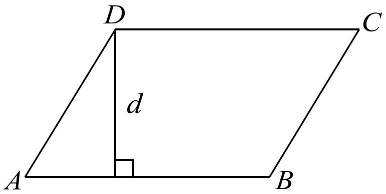
Let point D be (a_1, a_2, a_3)

$a_1 + 1 = 3$ or $a_1 = 2$

$a_2 + 0 = 1$ or $a_2 = 1$

$a_3 - 1 = 7$ or $a_3 = 8$

$\therefore D(2,1,8)$



$\vec{d} = \left|\frac{(\overrightarrow{AB}) \times (\overrightarrow{AD})}{|\overrightarrow{AB}|}\right|$

$\overrightarrow{AB} = -\hat{\imath} + \hat{\jmath} - 5\hat{k}$

$\overrightarrow{AD} = 0\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$

$\overrightarrow{AB} \times \overrightarrow{AD} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 1 & -5 \\ 0 & 2 & 4 \end{vmatrix}$

$= 14\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$

$= 2(7\hat{\imath} + 2\hat{\jmath} - \hat{k})$

$\Rightarrow d = 2\sqrt{2}$

280 **(d)**

Let $\vec{r} = x\hat{\imath} + y\hat{\jmath}$

$x^2 + y^2 + 8x - 10y + 40 = 0$, which is a circle centre $C(-4,5)$, radius $r = 1$

$p_1 = \max\{(x+2)^2 + (y-3)^2\}$

$p_2 = \min\{(x+2)^2 + (y-3)^2\}$

Let P be $(-2,3)$. Then

$CP = \sqrt{2}, r = 1$

$p_2 = (2\sqrt{2} - 1)^2$

$p_1 = (2\sqrt{2} + 1)^2$

$p_1 + p_2 = 18$

Slope $= AB = \left(\frac{dy}{dx}\right)_{(2,2)} = -2$

Equation of AB, $2x + y = 6$

$\overrightarrow{OA} = 2\hat{\imath} + 2\hat{\jmath}, \overrightarrow{OB} = 3\hat{\imath}$

$\overrightarrow{AB} = \hat{\imath} - 2\hat{\jmath}$

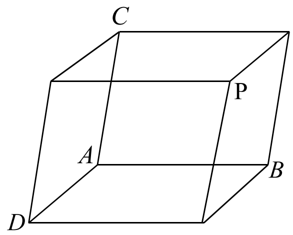
$\overrightarrow{AB}.\overrightarrow{OB} = (\hat{\imath} - 2\hat{\jmath})(3\hat{\imath}) = 3$

281 **(a)**

$\vec{a} = \overrightarrow{AP} = \overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD}$ (i)

$\overrightarrow{AB} \times \overrightarrow{AC} = \vec{b}$

$\overrightarrow{AD} \times \overrightarrow{AB} = \vec{c}$



$\overrightarrow{AB} \cdot \frac{\vec{a}}{|\vec{a}|} = \frac{|\vec{a}|}{3} \Rightarrow \overrightarrow{AB} \cdot \vec{a} = \frac{|\vec{a}|^2}{3}$

$\overrightarrow{AC} \cdot \frac{\vec{a}}{|\vec{a}|} = \frac{|\vec{a}|}{3} \Rightarrow \overrightarrow{AC} \cdot \vec{a} = \frac{|\vec{a}|^2}{3}$

$\therefore (\overrightarrow{AB} \times \overrightarrow{AC}) \times \vec{a} = \vec{b} \times \vec{a}$

$\therefore \frac{|\vec{a}|^2}{3}\overrightarrow{AC} - \frac{|a|^2}{3}\overrightarrow{AB} = \vec{b} \times \vec{a}$

$\therefore\ \overrightarrow{AC} - \overrightarrow{AB} = 3\frac{\vec{b} \times \vec{a}}{|\vec{a}|^2}$ (ii)

$|a|^2 = \overrightarrow{AB} \cdot \vec{a} + \overrightarrow{AC} \cdot \vec{a} + \overrightarrow{AD} \cdot \vec{a}$

$\therefore \frac{|\vec{a}|^2}{3} = \overrightarrow{AD} \cdot \vec{a}$

$(\overrightarrow{AD} \times \overrightarrow{AB}) \times \vec{a} = \vec{c} \times \vec{a}$

$\overrightarrow{AB} - \overrightarrow{AD} = 3\frac{\vec{c} \times \vec{a}}{|a|^2}$ (iii)

From (i), (ii) and (iii), we get

$AB = \frac{1}{3}\vec{a} + \frac{\vec{a} \times (\vec{b} - \vec{c})}{|\vec{a}|^2}$

Now from (ii) and(iii), we get $\overrightarrow{AC}$ and $\overrightarrow{AD}$ as

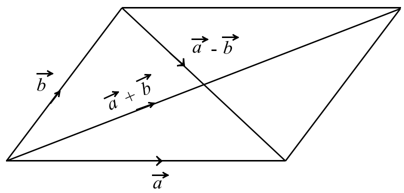
$\overrightarrow{AC} = \frac{1}{3}\vec{a} + \frac{\vec{a} \times (\vec{b} - \vec{c})}{|\vec{a}|^2} + \frac{3(\vec{b} \times \vec{a})}{|\vec{a}|^2}$

$\overrightarrow{AD} = \frac{1}{3}\vec{a} + \frac{\vec{a} \times (\vec{b} - \vec{c})}{|\vec{a}|^2} + \frac{3(\vec{c} \times \vec{a})}{|\vec{a}|^2}$

282 **(6)**

Let $\vec{R} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$
$\vec{u} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}; \vec{v} = 2\hat{\imath} + \hat{\jmath} + 4\hat{k}; \vec{w} = \hat{\imath} + 3\hat{\jmath} + 3\hat{k}$
$(\vec{u}\cdot\vec{R} - 15)\hat{\imath} + (\vec{v}\cdot\vec{R} - 30)\hat{\jmath} + (\vec{w}\cdot\vec{R} - 25)\hat{k} = \vec{0}$
(given)
So $\vec{u}\cdot\vec{R} = 15 \Rightarrow x - 2y + 3z = 15$ (i)
$\vec{v}\cdot\vec{R} = 30 \Rightarrow 2x + y + 4z = 30$ (ii)
$\vec{w}\cdot\vec{R} = 25 \Rightarrow x + 3y + 3z = 25$ (iii)
Solving, we get
$x = 4$
$y = 2$
$z = 5$

283 **(2)**
Let $\vec{a} = x\hat{\imath} - 3\hat{\jmath} - \hat{k}$ and $\vec{b} = 2x\hat{\imath} + x\hat{\jmath} - \hat{k}$ be the adjacent sides of the parallelogram now angle between $\vec{a}$ and $\vec{b}$ is acute



$\Rightarrow |\vec{a} + \vec{b}| > |\vec{a} - \vec{b}|$

$\Rightarrow |3x\hat{\imath} + (x-3)\hat{\jmath} - 2\hat{k}|^2 > |-x\hat{\imath} - (x+3)\hat{\jmath}|^2$

$\Rightarrow 9x^2 + (x-3)^2 + 4 > x^2 + (x+3)^2$

$\Rightarrow 8x^2 - 12x + 4 > 0$

$\Rightarrow 2x^2 - 3x + 1 > 0$

$\Rightarrow (2x-1)(x-1) > 0$

$\Rightarrow x < 1/2$ or $x > 1$

Hence the least positive integral value is 2

284 **(7)**
$\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$
$\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$
$\vec{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$
L.H.S. $= [3\vec{a} + \vec{b}\ \ 3\vec{b} + \vec{c}\ \ 3\vec{c} + \vec{a}]$
$= [3\vec{a}\ 3\vec{b}\ 3\vec{c}] + [\vec{b}\vec{c}\vec{a}]$
$= 3^3[\vec{a}\vec{b}\vec{c}] + [\vec{a}\vec{b}\vec{c}]$
$= 28\ [\vec{a}\vec{b}\vec{c}]$

285 **(1)**
Given, $\vec{u} \times \vec{v} + \vec{u} = \vec{w}$ and $\vec{w} \times \vec{u} = \vec{v}$
$\Rightarrow (\vec{u} \times \vec{v} + \vec{u}) \times \vec{u} = \vec{v}$
$\Rightarrow (\vec{u} \times \vec{v}) \times \vec{u} = \vec{v}$
$\Rightarrow \vec{v} - (\vec{\mu}\cdot\vec{v}) = \vec{v}$
$\Rightarrow (\vec{u}\cdot\vec{v})\vec{u} = 0 \Rightarrow (\vec{u}\vec{v}) = 0$
Now, $[\vec{u}\vec{v}\vec{w}] = \vec{u}\cdot(\vec{v}\times\vec{w})$
$= \vec{u}\cdot(\vec{v}\times(\vec{u}\times\vec{v}+\vec{u}))$
$= \vec{u}\cdot(\vec{v}\times(\vec{u}\times\vec{v}) + \vec{v}\times\vec{u})$
$= \vec{u}\left(\overrightarrow{v^2}\vec{u} - (\vec{u}\cdot\vec{v})\vec{v} + \vec{v}\times\vec{u}\right) = \overrightarrow{v^2u^2} = 1$

286 **(9)**
Vector $\vec{a} = \hat{\imath} + 2\hat{\jmath} - \hat{k}, \vec{b} = 2\hat{\imath} - \hat{\jmath} + \hat{k}, \vec{c} = \lambda\hat{\imath} + \hat{\jmath} + 2\hat{k}$ are coplanar

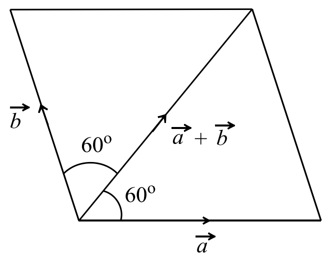
$$\Rightarrow \begin{vmatrix} 1 & 2 & -1 \\ 2 & -1 & 1 \\ \lambda & 1 & 2 \end{vmatrix} = 0$$

$\Rightarrow \lambda - 3 + 2(-5) = 0$

$\Rightarrow \lambda = 13$

287 **(7)**
Let the vertices be A, B, C, D and O be the origin
$\therefore \vec{O}A = \hat{\imath} - 6\hat{\jmath} + 10\hat{k}, OB = \hat{\imath} - 3\hat{\jmath} + 7\hat{k},$
$\vec{O}C = -5\hat{\imath} - \hat{\jmath} + \lambda\hat{k}, OD = 7\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$
$\therefore \vec{A}B = \vec{O}B - \vec{O}A = -2\hat{\imath} + 3\hat{\jmath} - 3\hat{k}$
$\vec{A}C = \vec{O}C - \vec{O}A = 4\hat{\imath} + 5\hat{\jmath} + (\lambda - 10)\hat{k}$
$\vec{A}D = \vec{O}D - \vec{O}A = 6\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$
Volume of tetrahedron
$$= \frac{1}{6}[\vec{A}B\ \vec{A}C\ \vec{A}D] = \frac{1}{6}\begin{vmatrix} -2 & 3 & -3 \\ 4 & 5 & -10 \\ 6 & 2 & -3 \end{vmatrix}$$
$$= \frac{1}{6}\{-2(-15 - 2\lambda + 20) - 3(-12 - 6\lambda + 60) - 3(8 - 30)\}$$
$$= \frac{1}{6}(4\lambda - 10 - 144 + 18\lambda + 66)$$
$$= \frac{1}{6}(22\lambda - 88) = 11$$
Or $2\lambda - 8 = 6$
Or $\lambda = 7$

288 **(3)**
Given, $\vec{a} + \vec{b} = \vec{c}$
Now vector $\vec{c}$ is along the diagonal of the parallelogram which has adjacent side vector $\vec{a}$ and $\vec{b}$. Since $\vec{c}$ is also a unit vector, triangle formed by vectors $\vec{a}$ and $\vec{b}$ is an equilateral triangle
Then, area of triangle is $\frac{\sqrt{3}}{4}$



289 **(1)**

$\vec{a}\cdot\vec{b} \Rightarrow \vec{a}\perp\vec{b}$

$\vec{a}\cdot\vec{c}=0 \Rightarrow \vec{a}\perp\vec{c}$

$\Rightarrow \vec{a}\perp\vec{b}-\vec{c}$

$|\vec{a}\times\vec{b}-\vec{a}\times\vec{c}| = |\vec{a}\times(\vec{b}-\vec{c})|$

$= |\vec{a}||\vec{b}-\vec{c}| = |\vec{b}-\vec{c}|$

Now $|\vec{b}-\vec{c}|^2 = |\vec{b}|^2 + |\vec{c}|^2 - 2|\vec{b}||\vec{c}|\cos\frac{\pi}{3}$

$= 2-2x\times\frac{1}{2} = 1$

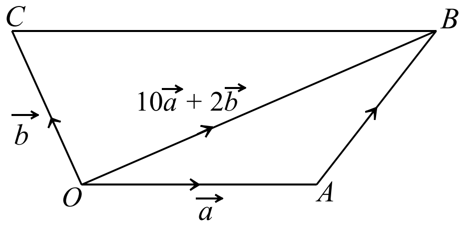
$|\vec{b}-\vec{c}| = 1$

290 **(6)**

Here $\vec{O}A=\vec{a}, \vec{O}B = 10\,\vec{a}+2\vec{b}, \vec{O}C=\vec{b}$

q = Area of parallelogram with OA and OC as adjacent sides

$\therefore q = |\vec{a}\times\vec{b}|$ (i)



p = Area of quadrilateral $OABC$

= Area of ΔOAB + Area of ΔOBC

$= \frac{1}{2}|\vec{a}\times(10\vec{a}+2\vec{b})| + \frac{1}{2}|(10\vec{a}+2\vec{b})\times\vec{b}|$

$= |\vec{a}\times\vec{b}| + 5|\vec{a}\times\vec{b}|$

$\therefore p = 6|\vec{a}\times\vec{b}|$

Or $p = 6\,q$ [From Eq. (i)]

$\therefore k = 6$

291 **(1)**

Since angle between $\vec{u}$ and $\hat{\imath}$ is 60°, we have

$\vec{u}\cdot i = |\vec{u}||\hat{\imath}|\cos 60° = \frac{|\vec{u}|}{2}$

Given that $|\vec{u}-\hat{\imath}|, |\vec{u}|, |\vec{u}-2\hat{\imath}|$ are in G.P., so

$|\vec{u}-\hat{\imath}|^2 = |\vec{u}||\vec{u}-2\hat{\imath}|$

Squaring both sides,

$[|\vec{u}|^2+|\hat{\imath}|^2-2\vec{u}\cdot\hat{\imath}]^2 = |\vec{u}|^2[|\vec{u}|^2+4|\hat{\imath}|^2-4\vec{u}\cdot\hat{\imath}]$

$\left[|\vec{u}|^2+1-\frac{2|\vec{u}|}{2}\right]^2 = |\vec{u}|^2\left[|\vec{u}|^2+4-4\frac{|\vec{u}|}{2}\right]$

Or $|\vec{u}|^2+2|\vec{u}|-1=0 \Rightarrow |\vec{u}| = -\frac{2\pm2\sqrt{2}}{2}$

Or $|\vec{u}| = \sqrt{2}-1$

292 **(6)**

Let $\vec{R}$ be the resultant

Then $\vec{R} = \vec{F_1}+\vec{F_2}+\vec{F_3} = (p+1)\hat{\imath}+4\hat{\jmath}$

Given $|\vec{R}| = 5$, therefore $R^2 = 25$

$\because (p+1)^2+16 = 25 \Rightarrow p+1 = \pm3$

$\therefore p = 2, -4$

293 **(2)**

L.H.S $= \vec{d}-\vec{a}+\vec{d}-\vec{b}+\vec{h}-\vec{c}+3(\vec{g}-\vec{h})$

$= 2\vec{d}-(\vec{a}+\vec{b}+\vec{c})+3\frac{(\vec{a}+\vec{b}+\vec{c})}{3}-2\vec{h}$

$= 2\vec{d}-2\vec{h} = 2(\vec{d}-\vec{h}) = 2\vec{H}D$

$\Rightarrow \lambda = 2$

294 **(6)**

$2\vec{V}+\vec{V}\times(\hat{\imath}+2\hat{\jmath}) = (2\hat{\imath}+\hat{k})$ (i)

Or $2\vec{V}\cdot(\hat{\imath}+2\hat{\jmath})+0 = (2\hat{\imath}+\hat{k})\cdot(\hat{\imath}+2\hat{\jmath})$

Or $2\vec{V}\cdot(\hat{\imath}+2\hat{\jmath}) = 2$

Or $|\vec{V}\cdot(\hat{\imath}+2\hat{\jmath})|^2 = 1$

Or $|\vec{V}|^2\cdot|\hat{\imath}+2\hat{\jmath}|^2\cos^2\theta = 1$

(θ is the angle between $\vec{V}$ and $\hat{\imath}+2\hat{\jmath}$)

Or $|\vec{V}|^2\,5(1-\sin^2\theta) = 1$

Or $|\vec{V}|^2\,5\sin^2\theta = 5|\vec{V}|^2-1$ (ii)

From Eq. (i), we have

$|2\vec{V}+\vec{V}\times(\hat{\imath}+2\hat{\jmath})|^2 = |2\hat{\imath}+\hat{k}|^2$

Or $4|\vec{V}|^2+|\vec{V}\times(\hat{\imath}+2\hat{\jmath})|^2 = 5$

Or $4|\vec{V}|^2+|\vec{V}|^2\cdot|\hat{\imath}+2\hat{\jmath}|^2\sin^2\theta = 5$

Or $4|\vec{V}|^2+5|\vec{V}|^2\sin^2\theta = 5$

Or $4|\vec{V}|^2+5|\vec{V}|^2-1 = 5$

Or $9|\vec{V}|^2 = 6$

Or $3|\vec{V}| = \sqrt{6}$

$= \sqrt{6} = \sqrt{m}$

$\therefore m = 6$

295 **(5)**

Let angle between $\vec{a}$ and $\vec{b}$ be θ

We have $|\vec{a}| = |\vec{b}| = 1$

Now $|\vec{a}+\vec{b}| = 2\cos\frac{\theta}{2}$ and $|\vec{a}-\vec{b}| = 2\sin\frac{\theta}{2}$

Consider $F(\theta) = \frac{3}{2}\left(2\cos\frac{\theta}{2}\right)+2\left(2\sin\frac{\theta}{2}\right)$

$\therefore F(\theta) = 3\cos\frac{\theta}{2}+4\sin\frac{\theta}{2}, \theta\in[0,\pi]$

296 **(2)**

$\overrightarrow{AB} = 2\hat{\imath}+\hat{\jmath}+\hat{k}, \overrightarrow{AC} = (t+1)\hat{\imath}+0\hat{\jmath}-\hat{k}$

$\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 2 & 1 & 1\\ t+1 & 0 & -1\end{vmatrix}$

$= -\hat{\imath}+(t+3)\hat{\jmath}-(t+1)\hat{k}$

$= \sqrt{1 + (t+3)\hat{j} - (t+1)^2}$

$= \sqrt{2t^2 + 8t + 11}$

Area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

$= \frac{1}{2}\sqrt{2t^2 + 8t + 1}$

Let $f(t) = \Delta^2 = \frac{1}{4}(2t^2 + 8t + 1)$

$f'(t) = 0 \Rightarrow t = -2$

At $t = -2, f''(t) > 0$

So Δ is minimum at $t = -2$

297 **(4)**

$a = ai + 2j - 3k, b = i + 2\alpha j - 2k,$

$c = 2i - \alpha j + k\{(a \times b) \times (b \times c)\} \times (c \times a) = 0$

Or $\{[a\ b\ c]b - [a\ b\ b]c\} \times (c \times a) = 0$

Or $[a\ b\ c]b \times (c \times a) = 0$

Or $[a\ b\ c]\big((a \cdot b)c - (b \cdot c)a\big) = 0$

Or $[a\ b\ c] = 0$ ($\because$ a and c are not collinear)

$$\Rightarrow \begin{vmatrix} \alpha & 2 & -3 \\ 1 & 2\alpha & -2 \\ 2 & -\alpha & 1 \end{vmatrix}$$

Or $\alpha(2\alpha - 2\alpha) - 2(1+4) - 3(-\alpha - 4\alpha) = 0$

Or $10 - 15\alpha = 0$

$\therefore \alpha = 2/3$

298 **(9)**

Since $\vec{x}$ and $\vec{y}$ are non-collinear vectors, therefore $\vec{x}, \vec{y}$ and $\vec{x} \times \vec{y}$ are non-coplanar vectors

$$[(a-2)\alpha^2 + (b-3)\alpha + c] + [(a-2)\beta^2 + (b-3)\beta\beta + c]y + [(a-2)\gamma^2 + (b-3)\gamma + c](\vec{x} \times \vec{y}) = 0$$

Coefficient of each vector $\vec{x}, \vec{y}$ and $\vec{x} \times \vec{y}$ is zero

$(a-2)\alpha^2 + (b-3)\alpha + c = 0$

$(a-2)\beta^2 + (b-3)\beta + c = 0$

$(a-2)\gamma^2 + (b-3)\beta + c = 0$

The above three equations will satisfy if the coefficients of α, β and γ are zero because α, β and γ are three distinct real numbers

$a - 2 = 0$ or $a = 2$,

$b - 3 = 0$ or $b = 3$ and $c = 0$

$\therefore a^2 + b^2 + c^2 = 2^2 + 3^2 + 0^2 = 4 + 9 = 13$

299 **(7)**

Vectors along the sides are $\vec{a} = \hat{\imath} + 2\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{\imath} + 4\hat{j} + \hat{k}$

Clearly the vector along the longer diagonal is $\vec{a} + \vec{b} = 3\hat{\imath} + 6\hat{j} + 2\hat{k}$

Hence length of the longer diagonal is $|\vec{a} + \vec{b}| = |3\hat{\imath} + 6\hat{j} + 2\hat{k}| = 7$

300 **(9)**

Here $\vec{F} = 3\hat{\imath} - \hat{j} - 2\hat{k}$

$\vec{AB} =$ P.V. of $B -$ P.V of A

$= (-\hat{\imath} - \hat{j} - 2\hat{k}) - (-3\hat{\imath} - 4\hat{j} + \hat{k})$

$= 2\hat{\imath} + 3\hat{j} - 3\hat{k}$

Let $\vec{s} = \vec{AB}$ be the displacement vector

Now work done $= \vec{F}.\vec{s}$

$= (3\hat{\imath} - \hat{j} - 2\hat{k}).(2\hat{\imath} + 3\hat{j} - 3\hat{k})$

$= 6 - 3 + 6 = 9$