10.STRAIGHT LINES

Single Correct Answer Type

1. $OPQR$ is a square and M, N are the middle points of the sides PQ and QR, respectively, then the ratio of the areas of the square and the triangle OMN is
a) 4:1 b) 2:1 c) 8:3 d) 7:3
2. Vertices of a parallelogram $ABCD$ are $A(3,1), B(13,6), C(13,21)$ and $D(3,16)$. If a line passing through the origin divides the parallelogram into two congruent parts, then the slope of the line is
a) 11/12 b) 11/8 c) 25/8 d) 13/8
3. One diagonal of a square is along the line $8x - 15y = 0$ and one of its vertex is $(1, 2)$. Then the equations of the sides of the square passing through this vertex are
a) $23x + 7y = 9, 7x + 23y = 53$ b) $23x - 7y + 9 = 0, 7x + 23y + 53 = 0$
c) $23x - 7y - 9 = 0, 7x + 23y - 53 = 0$ d) None of these
4. If a straight line through origin bisects the line passing through the given points$(a\cos\alpha, a\sin\alpha)$ and $(a\cos\beta, a\sin\beta)$, then the lines
a) Are perpendicular b) Are parallel
c) Have an angle between them of $\pi/4$ d) None of these
5. If the lines represented by the equation $3y^2 - x^2 + 2\sqrt{3}x - 3 = 0$ are rotated about the point $(\sqrt{3}, 0)$ through an angle 15°, one clockwise direction and other in anti clockwise direction, so that they become perpendicular, then the equation of the pair of lines in the new position is
a) $y^2 - x^2 + 2\sqrt{3} + 3 = 0$ b) $y^2 - x^2 + 2\sqrt{3}x - 3 = 0$
c) $y^2 - x^2 - 2\sqrt{3}x + 3 = 0$ d) $y^2 - x^2 + 3 = 0$
6. If the sum of the distances of a point from two perpendicular lines a plane is 1, then its locus is
a) Square b) Circle
c) Straight line d) Two intersecting lines
7. The distance between the two lines represented by the equation $9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = 0$ is
a) 8/5 b) 6/5 c) 11/5 d) None of these
8. In $\Delta\, ABC$ if orthocentre be $(1, 2)$ and circumcentre be $(0, 0)$ centroid of ΔABC is
a) $(1/2, 2/3)$ b) $(1/3, 2/3)$ c) $(2/3, 1)$ d) None of these
9. Vertices of a triangle are $(3, 4)$, $(5\cos\theta, 5\sin\theta)$ and $(5\sin\theta, -5\cos\theta)$, where $\theta \in R$. Locus of its orthocentre is
a) $(x + y - 1)^2 + (x - y - 7)^2 = 100$ b) $(x + y - 7)^2 + (x - y - 1)^2 = 100$
c) $(x + y - 7)^2 + (x + y - 1)^2 = 100$ d) $(x + y - 7)^2 + (x - y + 1)^2 = 100$
10. If the area of the rhombus enclosed by the lines $lx \pm my \pm n = 0$ be 2 sq. units, then
a) l, m, n are in G.P. b) l, n, m are in G.P. c) $lm = n$ d) $ln = m$
11. $ABCD$ is a square $A \equiv (1, 2), B \equiv (3, -4)$. If line CD passes through $(3, 8)$ then midpoint of CD is
a) $(2, 6)$ b) $(6, 2)$ c) $(2, 5)$ d) $(24/5, 1/5)$
12. L_1 and L_2 are two lines. If the reflection of L_1 in L_2 and the reflection of L_2 in L_1 coincide, then the angle between the lines is
a) 30° b) 60° c) 45° d) 90°
13. Area of the parallelogram formed by the lines $y = mx, y = mx + 1, y = nx$ and $y = nx + 1$ equals
a) $|m + n|/(m - n)^2$ b) $2/|m + n|$ c) $1/(|m + n|)$ d) $1/(|m - n|)$
14. If the lines $ax + y + 1 = 0, x + by + 1 = 0$ and $x + y + c = 0$ (a, b, c being distinct and different from 1) are concurrent, then $\left(\frac{1}{1-a}\right) + \left(\frac{1}{1-b}\right) + \left(\frac{1}{1-c}\right) =$
a) 0 b) 1 c) $1/(a + b + c)$ d) None of these
15. The line parallel to the x-axis and passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a, b) \neq (0, 0)$ is
a) Above the x-axis at a distance of $3/2$ units from it

b) Above the x-axis at a distance of $2/3$ units from it
c) Below the x-axis at a distance of $3/2$ units from it
d) Below the x-axis at a distance of $2/3$ units from it

16. Consider the points $A(0,1)$ and $B(2,0)$, and P be a point on the line $4x+3y+9=0$. Coordinates of P such that $|PA-PB|$ is maximum are
a) $(-12/5, 17/5)$ b) $(-84/5, 13/5)$ c) $(-6/5, 17/5)$ d) $(0,-3)$

17. Two medians drawn from acute angles of a right angled triangle intersect at an angle $\pi/6$. If the length of the hypotenuse of the triangle is 3 units, the area of triangle (in sq. units) is
a) $\sqrt{3}$ b) 3 c) $\sqrt{2}$ d) 9

18. The number of integral points (x,y) (i.e., x and y both are integers) which lie in the first quadrant but not on the coordinate axes and also on the straight line $3x+5y=2007$ is equal to
a) 133 b) 135 c) 138 d) 140

19. The equation $x^2y^2-9y^2+6x^2y+54y=0$ represents
a) A pair of straight lines and a circle
b) A pair of straight lines and a parabola
c) A set of four straight lines forming a square
d) None of these

20. A triangle is formed by the lines $x+y=0, x-y=0$ and $lx+my=1$. If l and m vary subject to the condition $l^2+m^2=1$, then the locus of its circumcentre is
a) $(x^2-y^2)^2=x^2+y^2$
b) $(x^2+y^2)^2=(x^2-y^2)$
c) $(x^2+y^2)=4x^2y^2$
d) $(x^2-y^2)^2=(x^2+y^2)^2$

21. If the equations $y=mx+c$ and $x\cos\alpha+y\sin\alpha=p$ represent the same straight line, then
a) $p=c\sqrt{1+m^2}$ b) $c=p\sqrt{1+m^2}$ c) $cp=\sqrt{1+m^2}$ d) $p^2+c^2+m^2=1$

22. The graph $y^2+2xy+40|x|=400$ divides the plane into regions. Then the area of bounded region is
a) 200 sq. units b) 400 sq. units c) 800 sq. units d) 500 sq. units

23. The number of values of 'a' for which the lines
$2x+y-1=0$,
$ax+3y-3=0$, and
$3x+2y-2=0$
Are concurrent is
a) 0 b) 1 c) 2 d) Infinite

24. Locus of the image of the point (2, 3) in the line $(x-2y+3)+\lambda(2x-3y+4)=0$ is $(\lambda\in R)$
a) $x^2+y^2-3x-4y-4=0$
b) $2x^2+3y^2+2x+4y-7=0$
c) $x^2+y^2-2x-4y+4=0$
d) None of these

25. Equations of diagonals of square formed by lines $x=0, y=0, x=1$ and $y=1$ are
a) $y=x, y+x=1$ b) $y=x, x+y=2$ c) $2y=x, y+x=1/3$ d) $y=2x, y+2x=1$

26. A pair of perpendicular straight lines is drawn through the origin forming with the line $2x+3y=6$ an isosceles triangle right angled at the origin. The equation to the line pair is
a) $5x^2-24xy-5y^2=0$
b) $5x^2-26xy-5y^2=0$
c) $5x^2+24xy-5y^2=0$
d) $5x^2+26xy-5y^2=0$

27. The line PQ whose equation is $x-y=2$ cuts the x-axis at P and Q is (4, 2). The line PQ is rotated about P through 45° in the anticlockwise direction. The equation of the line PQ in the new position is
a) $y=-\sqrt{2}$ b) $y=2$ c) $x=2$ d) $x=-2$

28. If the slope of one line represented by $a^3x^2-2hxy+b^3y^2=0$ is square of the slope of another line, then
a) $h=2ab(a+b)$ b) $h=ab(a+b)$ c) $3h=2ab(a+b)$ d) $2h=ab(a+b)$

29. The equations of the sides of a triangle are $x+y-5=0, x-y+1=0$ and $y-1=0$. Then the coordinates of the circumcentre are
a) (2, 1) b) (1, 2) c) (2, −2) d) (1, −2)

30. A beam of light is sent along the line$x-y=1$, which after refraction from the x-axis enters the opposite side by running through 30° towards the normal at the point of incidence on the x-axis. Then the equation of the refracted ray is
a) $(2-\sqrt{3})x-y=2+\sqrt{3}$
b) $(2+\sqrt{3})x-y=2+\sqrt{3}$

c) $(2-\sqrt{3})x+y=(2+\sqrt{3})$ d) $y=(2-\sqrt{3})(x-1)$

31. ABC is a variable triangle such that A is $(1, 2)$ B and C lie on line $y=x+\lambda$ (where λ is a veriable), then locus of the orthocentre of triangle ABC is

a) $(x-1)^2+y^2=4$ b) $x+y=3$ c) $2x-y=0$ d) None of these

32. A line of fixed length 2 units moves so that its ends are on the positive x-axis and that part of the line $x+y=0$ which lies in the second quadrant. Then the locus of the midpoint of the line has the equation

a) $x^2+5y^2+4xy-1=0$ b) $x^2+5y^2+4xy+1=0$

c) $x^2+5y^2-4xy-1=0$ d) $4x^2+5y^2+4xy+1=0$

33. The number of integral values of m, for which the x-coordinate of the point of intersection of the lines $3x+4y=9$ and $y=mx+1$ is also n integer is

a) 2 b) 0 c) 4 d) 1

34. If $u=a_1x+b_1y+c_1=0, v=a_2x+b_2y+c_2=0$ and $a_1/a_2=b_1/b_2=c_1/c_2$, then the curve $u+kv=0$ is

a) The same straight line u b) Different straight line

c) Not a straight line d) None of these

35. If $x-2y+4=0$ and $2x+y-5=0$ are the sides of a isosceles triangle having are 10 sq. units the equation of third side is

a) $3x-y=-9$ b) $3x-y+11=0$ c) $x-3y=19$ d) $3x-y+15=0$

36. The locus of the point which moves such that its distance from the point $(4, 5)$ is equal to its distance from the line $7x-3y-13=0$ is

a) A straight line b) A circle c) A parabola d) An ellipse

37. If the vertices P and Q of a triangle PQR are given by $(2, 5)$ and $(4, -11)$, respectively, and the point R moves along the line N given by $9x+7y+4=0$, then the locus of the centriod of the triangle PQR is a straight line parallel to

a) PQ b) QR c) RP d) N

38. If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in G.P. with same common ratio, then the points $P(x_1, y_1), Q(x_2, y_2)$ and $R(x_3, y_3)$

a) Lie on a straight line b) Lie on an ellipse

c) Lie on a circle d) Are vertices of a triangle

39. The coordinates of two consecutive vertices A and B of a regular hexagon $ABCDEF$ are $(1, 0)$ and $(2, 0)$, respectively. The equation of the diagonal CE is

a) $\sqrt{3}x+y=4$ b) $x+\sqrt{3}y+4=0$ c) $x+\sqrt{3}y=4$ d) None of these

40. If sum of the distance of a point from two perpendicular lines in a plane is 1, then its locus is

a) A square b) A circle

c) A straight line d) Two intersecting lines

41. $P(m, n)$ (where m, n are natural numbers) is any point in the intertior of the quadrilateral formed by the pair of lines $xy=0$ and the lines $2x+y-2=0$ and $4x+5y=20$. The possible number of positions of the point P is

a) 7 b) 5 c) 4 d) 6

42. Equation of a straight line on which length of perpendicular from the origin is four units and the line makes an angle of 120° with the x-axis is

a) $x\sqrt{3}+y+8=0$ b) $x\sqrt{3}-y=8$ c) $x\sqrt{3}-y=8$ d) $x-\sqrt{3}y+8=0$

43. If the quadrilateral formed by the lines $ax+by+c=0, a'x+b'y+c=0, ax+by+c'=0, a'x+b'y+c'=0$ have perpendicular diagonals, then

a) $b^2+c^2=b'^2+c'^2$ b) $c^2+a^2=c'^2+a'^2$ c) $a^2+b^2=a'^2+b'^2$ d) None of these

44. Let a and b be non-zero and real numbers. Then, the equation $(ax^2+by^2+c)(x^2-5xy+6y^2)=0$ represents

a) Four straight lines, when $c=0$ and a, b are of the same sign b) Two straight lines and a circle, when $a=b$ and c is of sign opposite to that of a

c) Two straight lines and hyperbola, when a and b d) A circle and an ellipse , when a and b are of the are of the same sign and c is of sign opposite to that of a same sign and c is of sign opposite to that of a

45. If $P(1+t/\sqrt{2}, 2+t/\sqrt{2})$ be any point on a line, then the range of the values of t for which the point P lies between the parallel lines $x+2y=1$ and $2x+4y=15$ is

a) $-4\sqrt{2}/3 < t < 5\sqrt{2}/6$ b) $0 < t < 5\sqrt{2}/6$ c) $4\sqrt{2}/< t < 0$ d) None of these

46. If an triangle $ABC, A \equiv (1, 10)$, circumcentre $\equiv (-1/3, 2/3)$ and orthocentre $\equiv (11/4, 4/3)$, then the coordinates of mid-point of side opposite to A is

a) $(1, -11/3)$ b) $(1, 5)$ c) $(1, -3)$ d) $(1, 6)$

47. The area enclosed by $2|x| + 3|y| \leq 6$ is

a) 3 sq. units b) 4 sq. units c) 12 sq. units d) 24 sq. units

48. Through a point A on the x-axis a straight line is drawn parallel to y-axis so as to meet the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ in B and C. If $AB = Bc$, then

a) $h^2 = 4ab$ b) $8h^2 = 9ab$ c) $9h^2 = 8ab$ d) $4h^2 = ab$

49. A variable line $\frac{x}{a} + \frac{y}{b} = 1$ moves in such a way that harmonic mean of a and b is8. then the least area of triangle made by the line with the coordinate axes is

a) 8 sq. unit b) 16 sq. unit c) 32 sq. unit d) 64 sq. unit

50. The straight line $ax + bx + c = 0$ where $abc \neq 0$ will pass through the first quadrant if

a) $ac > 0, bc > 0$ b) $c > 0$ and $bc < 0$ c) $bc > 0$ and/or $ac > 0$ d) $ac < 0$ and/or $bc < 0$

51. If the intercept made on the line $y = mx$ by lines $y = 2$ and $y = 6$ is less than 5, then the range of values of m is

a) $(-\infty, -4/3) \cup (4/3, +\infty)$ b) $(-4/3, 4/3)$

c) $(-3/4, 4/3)$ d) None of these

52. If each of the points $(x_1, 4), (-2, y_1)$ lies on the line joining the points $(2, -1), (5, -3)$ then the point $P(x_1, y_1)$ lies on the line

a) $6(x+y) - 25 = 0$ b) $2x + 6y + 1 = 0$ c) $2x + 3y - 6 = 0$ d) $6(x+y) + 25 = 0$

53. P is a point on the line $y + 2x = 1$ and, Q and R are two points on the line $3y + 6x = 6$ such that triangle PQR is an equilateral triangle. The length of the side of the triangle is

a) $2/\sqrt{5}$ b) $3/\sqrt{5}$ c) $4/\sqrt{5}$ d) None of these

54. The equation to the straight line passing through the point $(a\cos^3\theta, a\sin^3\theta)$ and perpendicular to the line $x\sec\theta + y\,\mathrm{cosec}\,\theta = a$ is

a) $x\cos\theta - y\sin\theta = a\cos 2\theta$ b) $x\cos\theta + y\sin\theta = a\cos 2\theta$

c) $x\sin\theta + y\cos\theta = a\cos 2\theta$ d) None of these

55. A rectangular billiard table has vertices at $P(0,0), Q(0,7), R(10,7)$ and $S(10,0)$. a small billiard ball starts at $M(3,4)$ and moves in a straight line to the top of the table, bounces to the right side of the table, then comes to rest at $N(7,1)$. The y-coordinate of the point where it hits the right side, is

a) 3.7 b) 3.8 c) 3.9 d) 4

56. The equation of the straight line which passes through the point $(-4, 3)$ such that the portion of the line between the axes is divided internally by the point in the ratio 5:3 is

a) $9x - 20y + 96 = 0$ b) $9x + 20y = 24$ c) $20x + 9y + 53 = 0$ d) None of these

57. If $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$ then the two triangles with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ and $(a_1, b_1), (a_2, b_2), (a, b_3)$ are

a) Equal in area b) Similar c) Congruent d) None of these

58. The equation of the bisector of the acute angle between the lines $2x - y + 4 = 0$ and $x - 2y = 1$ is

a) $x + y + 5 = 0$ b) $x - y + 1 = 0$ c) $x - y = 5$ d) None of these

59. If one side of rhombus has end points $(4, 5)$ and $(1, 1)$ then the maximum area of the rhombus is

a) 50 sq. units b) 25 sq. units c) 30 sq. units d) 20 sq. units

60. Consider points $A(3, 4)$ and $B(7, 13)$. If P be a point on the line $y = x$ such that $PA + PB$ is minimum, then coordinates of P are

a) $(12/7, 12/7)$
b) $(13/7, 13/7)$
c) $(31/7, 31/7)$
d) $(0, 0)$

61. The number of straight lines equidistant from three non-collinear points in the plane of the points is equal to
a) 0 b) 1 c) 2 d) 3

62. The point $A(2, 1)$ is translated parallel to the line $x - y = 3$ by a distance 4 units. if the new position A' is in third quadrant, then the coordinates of A' are
a) $(2 + 2\sqrt{2}, 1 + 2\sqrt{2})$
b) $(-2 + \sqrt{2}, -1 - 2\sqrt{2})$
c) $(2 - 2\sqrt{2}, 1 - 2\sqrt{2})$
d) None of these

63. The number of possible straight lines, passing through (2, 3) and forming a triangle with coordinate axes, whose area is 12 sq. units, is
a) One b) Two c) Three d) Four

64. If the straight lines $x + y - 2 = 0, 2x - y + 1 = 0$ and $ax + by - c = 0$ are concurrent, then the family of lines $2ax + 3by + c = 0$ (a, b, c are nonzero) is concurrent at
a) $(2, 3)$ b) $(1/2, 1/3)$ c) $(-1/6, -5/9)$ d) $(2/3, -7/5)$

65. The straight lines $x + 2y - 9 = 0, 3x + 5y - 5 = 0$ and $ax + by - 1 = 0$ are concurrent, if the straight line $35x - 22y + 1 = 0$ passes through the point
a) (a, b) b) (b, a) c) $(-a, -b)$ d) None of these

66. Given $A(0, 0)$ and $B(x, y)$ with $x \in (0, 1)$ and $y > 0$. Let the slope of the line AB equal to m_1. Point C lies on the line $x = 1$ such that the slope of BC equal to m_2 where $0 < m_2 < m_1$. If the area of the triangle ABC can be expressed as $(m_1 - m_2)f(x)$, then the largest possible value of x is
a) 1 b) 1/2 c) 1/4 d) 1/8

67. If the pairs of lines $x^2 + 2xy + ay^2 = 0$ and $ax^2 + 2xy + y^2 = 0$ have exactly one line in common, then the joint equation of the other two lines is given by
a) $3x^2 + 8xy - 3y^2 = 0$
b) $3x^2 + 10xy + 3y^2 = 0$
c) $y^2 + 2xy - 3x^2 = 0$
d) $x^2 + 2xy - 3y^2 = 0$

68. If two vertices of a triangle are $(-2, 3)$ and $(5, -1)$, orthocentre lies at the origin and centroid on the line $x + y = 7$, then the third vertex lies at
a) $(7, 4)$ b) $(8, 14)$ c) $(12, 21)$ d) None of these

69. Points A and B are in the first quadrant; point 'O' is the origin. If the slope of OA is 1, slope of OB is 7 and $OA = OB$, then the slope of AB is
a) $-1/5$ b) $-1/4$ c) $-1/3$ d) $-1/2$

70. The image of $P(a, b)$ in the line $y = -x$ is Q and the image of Q in the $y = x$ is R. Then the midpoint of PR is
a) $(a + b, b + a)$
b) $((a + b)/2, (b + 2)/2)$
c) $(a - b, b - a)$
d) $(0, 0)$

71. Te area of the triangle formed by the lines $y = ax, x + y - a = 0$ and the y-axis is equal to
a) $\frac{1}{2|1 + a|}$ b) $\frac{a^2}{|1 + a|}$ c) $\frac{1}{2}\left|\frac{a}{1 + a}\right|$ d) $\frac{a^2}{2|1 + a|}$

72. The coordinates of the foot of the perpendicular from the point (2, 3) on the line $-y + 3x + 4 = 0$ are given by
a) $(37/10, -1/10)$ b) $(-1/10, 37/10)$ c) $(10/37, -10)$ d) $(2/3, -1/3)$

73. Let PS be the median of the triangle with vertices $P(2, 2), Q(6, -1)$ and $R(7, 3)$. The equation of the line passing through $(1, -1)$ and parallel to PS is
a) $2x - 9y - 7 = 0$ b) $2x - 9y - 11 = 0$ c) $2x + 9y - 11 = 0$ d) $2x + 9y + 7 = 0$

74. The pair of lines represented by $3ax^2 + 5xy + (a^2 - 2)y^2 = 0$ are perpendicular to each other for

a) Two values of a b) a c) For one value of a d) For no value of a

75. The line $L_1 = 4x + 3y - 12 = 0$ intersects the x-and the y-axes at A and$B$, respecpetively. A variable line perpendicular to L_1 intersects the x-and the y-axes at P and Q, respectively. Then the locus of the circumcentre of triangle ABQ is

a) $3x - 4y + 2 = 0$ b) $4x + 3yy + 7 = 0$ c) $6x - 8y + 7 = 0$ d) None of these

76. Two sides of a triangle are along the coordinate axes and the medians through the vertices (other than the origin) are mutually perpendicular. the number of such triangles is/are

a) Zero b) Two c) Four d) Infinite

77. The straight lines $x + y = 0, 3x + y - 4 = 0, x + 3y - 4 = 0$ from a triangle which is

a) Isosceles b) Equilateral c) Right angles d) None of these

78. Let $A = (3, -4), B = (1, 2)$, let $P = (2k - 1, 2k + 1)$ be a variable point such that $PA + PB$ is the minimum. then k is

a) 7/9 b) 0 c) 7/8 d) None of these

79. $A \equiv (-4, 0), B \equiv (4, 0)$. M and N are the variable points of y-axis such that M lies below N and $MN = 4$. Line joining AM and BN intersect at 'P'. Locus of 'P' is

a) $2xy - 16 - x^2 = 0$ b) $2xy + 16 - x^2 = 0$ c) $2xy + 16 + x^2 = 0$ d) $2xy - 16 + x^2 = 0$

80. The angle between the pair of lines whose equation is $4x^2 + 10xy + my^2 + 5x + 10y = 0$ is

a) $\tan^{-1}(3/8)$ b) $\tan^{-1}(3/4)$

c) $\tan^{-1}\left(2\sqrt{25 - 4m}/m + 4\right), m \in R$ d) None of these

81. If two the lines represented by $x^4 + x^3y + cx^2y^2 - xy^3 + y^4 = 0$ bisect the angle between the other two, then the value of c is

a) 0 b) -1 c) 1 d) -6

82. The locus of the orthocentre of the triangle formed by the lines $(1 + p)x - py + p(1 + p) = 0, (1 + q)x - qy + q(1 + q) = 0$ and $y = 0$, where $p \neq q$ is

a) A hyperbola b) A parabola c) An ellipse d) A straight line

83. The straight lines $7x - 2y + 10 = 0$ and $7x + 2y - 10 = 0$ form an isosceles triangle with the line $y = 2$. area of this triangle is equal to

a) 15/7 sq. units b) 10/7 sq. units c) 18/7 sq. units d) None of these

84. An equation of a line through the point (1, 2) whose distance from the point (3, 1) has the greatest value is

a) $y = 2x$ b) $y = x + 1$ c) $x + 2y = 5$ d) $y = 3x - 1$

85. The orthocentre of the triangle formed by the lines $xy = 0$ and $x + y = 1$ is

a) (1/2, 1/2) b) (1/3, 1/3) c) (0, 0) d) (1/4, 1/4)

86. The centroid of an equilateral triangle is (0, 0). If two vertices of the triangle lie on $x + y = 2\sqrt{2}$ then one of them will have its coordinates

a) $(\sqrt{2} + \sqrt{6}, \sqrt{2} - \sqrt{6})$ b) $(\sqrt{2} + \sqrt{3}, \sqrt{2} - \sqrt{3})$ c) $(\sqrt{2} + \sqrt{5}, \sqrt{2} - \sqrt{5})$ d) None of these

87. If the equation of base of an equilateral triangle is $2x - y = 1$ and the vertex is $(-1, 2)$, then the length of the sides of the triangle is

a) $\sqrt{\frac{20}{3}}$ b) $\frac{2}{\sqrt{15}}$ c) $\sqrt{\frac{8}{15}}$ d) $\sqrt{\frac{15}{2}}$

88. Let PQR be a right-angled isosceles triangle, right angled at $P(2, 1)$. If the equation of the line QR is $2x + y = 3$, then the equation representing the pair of lines PQ and PR is

a) $3x^2 - 3y^2 + 8xy + 20x + 10y + 25 = 0$ b) $3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$

c) $3x^2 - 3y^2 + 8xy + 10x + 15y + 20 = 0$ d) $3x^2 - 3y^2 - 8xy - 15y - 20 = 0$

89. A line is drawn perpendicular to line $y = 5x$, meeting the coordinate axes at A and B. If the area of triangle OAB is 10 sq. units where 'O' is the origin, then the equation of drawn line is

a) $3x - y - 9$ b) $x - 5y = 10$ c) $x + 4y = 10$ d) $x - 4y = 10$

90. If $a/bc = -2 = \sqrt{b/c} + \sqrt{c/b}$, where $a, b, c > 0$, then family of lines $\sqrt{a}x + \sqrt{b}y + \sqrt{c} = 0$ passes through the fixed point given by

a) $(1, 1)$ b) $(1, -2)$ c) $(-1., 2)$ d) $(-1, 1)$

91. Equation of a line which is parallel to the line common to the pair of lines given by $6x^2 - xy - 12y^2 = 0$ and $15x^2 + 14xy - 8y^2 = 0$ and at a distnace 7 from it is
a) $3x - 4y = -35$ b) $5x - 2y = 7$ c) $3x + 4y = 35$ d) $2x - 3y = 7$

92. The equation of the line segment AB is $y = x$. If A and B lie on the same side of te line mirror $2x - y = 1$, the the image of AB has the equation
a) $x + y = 2$ b) $8x + y = 9$ c) $7x - y = 6$ d) None of these

93. A light ray emerging from the point source placed at $P(2, 3)$ is reflected at a point 'Q' on the y-axis and then passes through the point $R(5, 10)$. Coordinates of 'Q' are
a) $(0, 3)$ b) $(0, 2)$ c) $(0, 5)$ d) None of these

94. If the origin is shifted to be point $(ab/(a - b, 0)$ without rotation, then the equation $(a - b)(x^2 + y^2) - 2abx = 0$ becomes
a) $(a - b)(x^2 + y^2) - (a + b)xy + abx = a^2$ b) $(a + b)(x^2 + y^2) = 2ab$
c) $(x^2 + y^2) = (a^2 + b^2)$ d) $(a - b)^2(x^2 + y^2) = a^2b^2$

95. The number of triangles that the four lines $y = x + 3, y = 2x + 3, y = 3x + 2$ and $y + x = 3$ form is
a) 4 b) 2 c) 3 d) 1

96. The point (4, 1) undergoes the following three transformation successively
i. Reflection about the line $y = x$
ii. Translation through a distance 2 units along the positive direction of x-axis
iii. Rotation through an angle $\pi/4$ about the origin in the counterclockwise direction
Then the final position of the point is given by the coordinates
a) $(1/\sqrt{2}, 7/\sqrt{2})$ b) $(-\sqrt{2}, 7\sqrt{2})$ c) $(-1/\sqrt{2}, 7/\sqrt{2})$ d) $(\sqrt{2}, 7\sqrt{2})$

97. Line L has intercepts a and b on the coordinates axes. When the axes are rotated through a given angle keeping the origin fixed, the same line L has intercepts p and q. Then,
a) $a^2 + b^2 = p^2 + q^2$ b) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$ c) $a^2 + p^2 = b^2 + q^2$ d) $\frac{1}{a^2} + \frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{q^2}$

98. In the ABC, the coordinates of B are (0, 0), $AB = 2, \angle ABC = \pi/3$ and the middle point of BC has the coordinates (2, 0. The centriod of the triangle is
a) $(1/2, \sqrt{3}/2)$ b) $(5/3, 1/\sqrt{3})$ c) $(4 + \sqrt{3}/3, 1/3)$ d) None of these

99. A triangle ABC with vertices $A(-1, 0), B(-2, 3/4)$ and $C(-3, -7/6)$ has its orthocentre H. Then the orthocentre of triangle BCH will be
a) $(-3, -2)$ b) $(1, 3)$ c) $(-1, 2)$ d) None of these

100. If the ends of the base of an isosceles triangle are at (2, 0) and (0, 1) and the equation of one side is $x = 2$, then the orthocentre of the triangle is
a) (3/2, 3/2) b) (5/4, 1) c) (3/4, 1) d) (4/3, 7/12)

101. Given $A \equiv (1, 1)$ and AB is any line through it cutting the x-axis in B. If AC is perpendicular to AB and meets the y-axis in C, then the equation of locus of midpoint P of BC is
a) $x + y = 1$ b) $x + y = 2$ c) $x + y = 2xy$ d) $2x + 2y = 1$

102. In a trinagle $ABC, A \equiv (\alpha, \beta), B \equiv (2, 3)$ and $C \equiv (1, 3)$ and point A lies on line $y = 2x + 3$ where $\alpha \in I$. Area of $\Delta ABC, \Delta$, is such that $[\Delta] = 5$. Possible coordinates of A are (where [.] represents greatest integer function)
a) $(2, 3)$ b) $(5, 13)$ c) $(-5, -7)$ d) $(-3, -5)$

103. Two vertices of a triangle are $(4, -3)$ and $(-2, 5)$. If the orthocentre of the triangle is at (1, 2), then the third vertex is
a) $(-33, -26)$ b) $(33, 26)$ c) $(26, 33)$ d) None of these

104. The line $x/3 + y/4 = 1$ meets the y-axis and x-axis at A and B, respectively. A square $ABCD$ is constructed on the line segment AB away from the origin. The coordinates of the vertex of the square farthest from the origin are
a) $(7, 3)$ b) $(4, 7)$ c) $(6, 4)$ d) $(3, 8)$

105. The distance between the two parallel lines is 1 unit. A point 'A' is chosen to lie between the lines at a

distance 'd' from one of them. Triangle ABC is equilateral with B on one line and C on the other parallel line. The length of the side of the equilateral triangle is

a) $(2/3)\sqrt{d^2+d+1}$ b) $2\sqrt{(d^2-d+1)/3}$ c) $2\sqrt{d^2-d+1}$ d) $\sqrt{d^2-d+1}$

106. The combined equation of straight lines that can be obtained by reflecting the lines $y=|x-2|$ in the y-axis is

a) $y^2+x^2+4x+4=0$ b) $y^2+x^2-4x+4=0$ c) $y^2-x^2+4x-4=0$ d) $y^2-x^2-4x-4=0$

107. In ΔABC the coordinates of the vertex A are $(4,-1)$ and lines $x-y-1=0$ and $2x-y=3$ are internal bisectors of angles B and C. Then, radius of incircle of triangle ABC is

a) $4/\sqrt{5}$ b) $3/\sqrt{5}$ c) $6/\sqrt{5}$ d) $7/\sqrt{5}$

108. θ_1 and θ_2 are the inclination of lines L_1 and L_2 with x-axis. if L_1 and L_2 pass through $p(x_1, y_1)$, then equation of one of the angle bisector of these lines is

a) $\dfrac{x-x_1}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}=\dfrac{y-y_1}{\sin\left(\frac{\theta_1-\theta_2}{2}\right)}$ b) $\dfrac{x-x_1}{-\sin\left(\frac{\theta_1-\theta_2}{2}\right)}=\dfrac{y-y_1}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}$

c) $\dfrac{x-x_1}{\sin\left(\frac{\theta_1-\theta_2}{2}\right)}=\dfrac{y-y_1}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}$ d) $\dfrac{x-x_1}{-\sin\left(\frac{\theta_1-\theta_2}{2}\right)}=\dfrac{y-y_1}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}$

109. $OPQR$ is a square and M, N are the midpoints of the sides PQ and QR, respectively. If the ratio of the areas of the square and the triangle OMN is $\lambda: 6$, then $\lambda/4$ is equal to

a) 2 b) 4 c) 2 d) 16

110. Consider 3 lines as fallows

$L_1: 5x-y+4=0$

$L_2: 3x-y+5=0$

$L_3: x+y+8=0$

If these lines enclose a triangle ABC and sum of the squares of the tangent to the interior angles can be expressed in the form p/q where p and q are relatively prime numbers, then the value of $p+q$ is

a) 500 b) 450 c) 230 d) 465

111. Let $A_r, r=1,2,3,\dots$ be points on the number line such that $OA_1, OA_2, OA_3, \dots$ are in G.P. where O is origin and the common ratio of the G.P. be a positive proper fraction. Let M_r be the middle point of the line segment A_rA_{r+1}. Then the value $\sum_{r=1}^{\infty} OM_r$ is equal to

a) $\dfrac{OA_1(OSA_1-OA_2)}{2(OA_1+OA_2)}$ b) $\dfrac{OA_1(OA_1-OA_2)}{2(OA_1+OA_2)}$ c) $\dfrac{OA_1}{2(OA_1-OA_2)}$ d) ∞

112. A is a point on either of two $y+\sqrt{3}|x|=2$ at a distance of $4\sqrt{3}$ units from their point of intersection. The coordinates of the foot of perpendicular from A on the bisector of the angle between them are

a) $(2/\sqrt{3}, 2)$ b) $(0,0)$ c) $(2\sqrt{3}, 2)$ d) $(0,4)$

113. The condition on a and b, such that the portion of the line $ax+by-1=0$, intercepted between the lines $ax+y=0$ and $x+by=0$, subtends a right angle at the origin is

a) $a=b$ b) $a+b=0$ c) $a=2b$ d) $2a=b$

114. m, n are integers with $0<n<m$. A is the point (m, n) on the Cartesian plane. B is the reflection of A in the line $y=x$. C is the reflection of B in the y-axis, D is the reflection of C in the x-axis and E is the reflection of D in the y-axis. The area of the pentagon $ABCDE$ is

a) $2m(m+n)$ b) $m(m+3n)$ c) $m(2m+3n)$ d) $2m(m+3n)$

115. The equation $a^2x^2+2h(a+b)xy+b^2y^2=0$ and $ax^2+2hxy+by^2=0$ reprsent

a) Two pairs of perpendicular straight lines

b) Two pairs of parallel straight lines

c) Two pairs of straight lines which are equally inclined to each other

d) None of these

116. The combined equation of the lines l_1, l_2 is $2x^2+6xy+y^2=0$ and that of the lines $m_1, m_2$is $4x^2+18xy+y^2=0$. If the angle between l_1 and m_2 be α, then the angle between l_2 and m_1 will be

a) $\pi/2-\alpha$ b) 2α c) $\pi/4+\alpha$ d) α

117. Let O be the origin. If $A(1,0)$ and $B(0,1)$ and $P(x,y)$ are points such that $xy>0$ and $x+y<1$, then

a) P lies either inside the triangle OAB or in the mid quadrant
b) Pcannot lie inside the triangle OAB
c) P lies inside the triangle OAB
d) P lies in the first quadrant only

118. The line $x + 3y - 2 = 0$ bisects the angle between a pair of straight line s of which one has equation $x - 7y + 5 = 0$. the equation of the other line is
a) $3x + 3y - 1 = 0$ b) $x - 3y + 2 = 0$ c) $5x + 5y - 3 = 0$ d) None of these

119. Let ABC be a triangle. Let A be the point $(1, 2)$, $y = x$ be the perpendicular bisector of AB and $x - 2y + 1 = 0$ be the angle bisector of $\angle C$. If equation of BC is given by $ax + by - 5 = 0$, then the value of $a + b$ is
a) 1 b) 2 c) 3 d) 4

120. The equation $x - y = 4$ and $x^2 + 4xy + y^2 = 0$ represent the sides of
a) An equilateral triangle b) A right angled triangle
c) An isosceles triangle d) None of these

121. If $P = (1, 0), Q = (-1, 0)$ and $R = (2, 0)$ are three given points, then locus of the point S satisfying the relation $SQ^2 + SR^2 = 2SP^2$ is
a) A straight line parallel to x-axis b) A circle passing through the origin
c) A circle with the centre at the origin d) A straight line parallel to y-axis

122. If the equation of the locus of a point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$, then the value of c is
a) $a_1^2 - a_2^2 + b_1^2 - b_2^2$ b) $\sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2}$ c) $\frac{1}{2}(a_1^2 + a_2^2 + b_1^2 + b_2^2)$ d) $\frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$

123. A rectangle $ABCD$, where $A \equiv (0, 0), B \equiv (4, 0), C \equiv (4, 2), D \equiv (0, 2)$, undergoes the following transformations successively: (i) $f_1(x, y) \to (y, x)$, (ii) $f_2(x, y) \to (x + 3y, y)$, (iii) $f_3(x, y) \to ((x - y)/2, (x + y)/2)$. The final figure will be
a) A square b) A rhombus c) A rectangle d) A parallelogram

124. The extremities of the base of an isosceles triangle are $(2, 0)$ and $(0, 2)$. If the equation of one of the equal side is $x = 2$, then equation of other equal side is
a) $x + y = 2$ b) $x - y + 2 = 0$ c) $y = 2$ d) $2x + y = 2$

125. Distance of origin from line $(1 + \sqrt{3})y + (1 - \sqrt{3})x = 10$ along the line $y = \sqrt{3}x + k$ is
a) $5/\sqrt{2}$
b) $5\sqrt{2} + k$
c) 10
d) 0

126. A light ray coming along the line $3x + 4y = 5$ gets reflected from the line $ax + by = 1$ and goes along the line $5x - 12y = 10$. Then
a) $a = 64/115, b = 112/15$ b) $a = 14/15, b = -8/115$
c) $a = 64/115, b = -8/115$ d) $a = 64/15, b = 14/15$

127. In a triangle ABC, if A is $(2, -1)$, and $7x - 10y + 1 = 0$ and $3x - 2y + 5 = 0$ are equations of an altitude and an angle bisector, respectively, drawn from B, then equation of BC is
a) $x + y + 1 = 0$ b) $5x + y + 17 = 0$ c) $4x + 9y + 30 = 0$ d) $x - 5y - 7 = 0$

128. One of the diagonals of a square is the portion of the line $x/2 + y/3 = 0$ intercepted between the axes. Then the extermities of the other diagonal are
a) $(5, 5), (-1, 1)$ b) $(0, 0), (4, 6)$ c) $(0, 0), (-1, 1)$ d) $(5, 5), (4, 6)$

129. If the extremities of the base of an isosceles triangle are the points $(2a, 0)$ and $(0, a)$ and the equation of one of the sides is $x = 2a$, then the area of the triangle is
a) $5a^2$ sq. units b) $5a^2/2$ sq. units c) $25a^2/2$ sq. units d) None of these

130. In a triangle ABC if $A \equiv (1, 2)$ and internal angle bisectors through B and C are $y = x$ and $y = -2x$, then the inradius r of the ΔABC is
a) $1/\sqrt{3}$ b) $1/\sqrt{2}$ c) $2/3$ d) None of these

131. The foot of the perpendicular on the line $3x+y=\lambda$ drawn from the origin is C. If the line cuts the x-axis and y-axis at A and B, respectively, then $BC:CA$ is

a) 1:3 b) 3:1 c) 1:9 d) 9:1

132. If it is possible to drawn a line which belongs to all the given family of lines $y-2x+1+\lambda_1(2y-x-1)=0, 3y-x-6\lambda_2(\mathrm{y}-3\mathrm{x}+6)=0, ax+y-2+\lambda_3(6x+xy-a)=0$, then

a) $a=4$ b) $a=3$ c) $a=-2$ d) $a=2$

133. The line $x+y=p$ meets the x-and y-axes at A and B, respectively. A triangle APQ is inscribed in the triangle OAB, O being the origin, with right angle at Q. P and Q lie, respectively, on OB and AB. If the area of the area of the triangle APQ is 3/8th of the area of the triangle OAB, then AQ/BQ is equal to

a) 2 b) 2/3 c) 1/3 d) 3

134. In a triangle ABC, the bisectors of angles B and C lie along the lines $x=y$ and $y=0$. if A is $(1,2)$, then the equation of line BC is

a) $2x+y=1$ b) $3x-y=5$ c) $x-2y=3$ d) $x+3y=1$

135. If $\sum_{i=1}^{4}(x_1^2+y_1^2)\le 2x_1x_3+2x_2x_4+2y_2y_3+2y_1y_4$ the points $(x_1,y_1),(x_2,y_2),(x_3,y_3),(x_4,y_4)$ are

a) The vertices of a rectangle b) Collinear
c) Trapezium d) None of these

136. The number of integral values of m, for which the x-coordinate of the point of intersection of the lines $3x+4y=9$ and $y=mx+1$ is also integer is

a) 2 b) 0 c) 4 d) 1

137. The straight lines $4ax+3by+c=0$, where $a+b+c=0$, are concurrent at the point

a) $(4,3)$ b) $(1/4,1/3)$ c) $(1/2,1/3)$ d) None of these

138. A straight line passing through $P(3,1)$ meets the coordinate axes at A and B. It is given that distance of this straight line from the origin 'O' is maximum. Area of triangle OAB is equal to

a) 50/3 sq. units b) 25/3 sq. units c) 20/3 sq. units d) 100/3 sq. units

139. A rectangle $ABCD$ has its side AB parallel to line $y=x$ and vertices A, B and D lie on $y=1, x=2$ and $x=-2$, respectively. locus of vertex 'C' is

a) $x=5$ b) $x-y=5$ c) $y=5$ d) $x+y=5$

140. Line $ax+by+p=0$ makes angle $\pi/4$ with $x\cos\alpha+y\sin\alpha=p, p\in R^+$. If these lines and the line $x\sin\alpha-y\cos\alpha=0$ are concurrent, then

a) $a^2+b^2=1$ b) $a^2+b^2=2$ c) $2(a^2+b^2)=1$ d) None of these

141. If the equation of any two diagonals of a regular pentagon belongs to family of lines $(1+2\lambda)y-(2+\lambda)x+1-\lambda=0$ and their lengths are $\sin 36°$, then locus of centre of circle circumscribing the given pentagon (the triangles formed by these diagonals with slides of pentagon have no side common) is

a) $x^2+y^2-2x-2y+1+\sin^2 72°=0$ b) $x^2+y^2-2x-2y+\cos^2 72°=0$
c) $x^2+y^2-2x-2y+1+\cos^2 72°=0$ d) $x^2+y^2-2x-2y+\sin^2 72°=0$

142. If the equation of the pair of straight lines passing through the point $(1,1)$, one making an angle θ with the positive direction of x-axis and the other making the same angle with positive direction of y-axis, is $x^2-(a+2)xy+y^2+a(x+y-1)=0, a\ne-2$, then the value of $\sin 2\theta$ is

a) $a-2$ b) $a+2$ c) $2/(a+2)$ d) $2/a$

143. The incentre of the triangle with vertices $(1,\sqrt{3})$, $(0,0)$ and $(2,0)$ is

a) $(1,\sqrt{3}/2)$ b) $(2/3,1/\sqrt{3})$ c) $(2/3,\sqrt{3}/2)$ d) $(1,1/\sqrt{3})$

144. If a pair of perpendicular straight lines drawn through the origin forms an isosceles triangle with the line $2x+3y=6$, then area of the triangle so formed is

a) 36/13 b) 12/17 c) 13/5 d) 17/13

145. A line 'L' is drawn from $P(4,3)$ to meet the lines L_1 and L_2 given by $3x+4y+5=0$ and $3x+4y+15=0$ at points A and B, respcetively. From 'A', a line perpendicular to L is drawn meeting the line L_2 at A_1. Similarly from point 'B', a line perpendicular to L is drawn meeting the line L_1 at B_1. Thus a parallelogram AA_1BB_1 is formed. Then the equation of 'L' so that the area of the parallelogram AA_1BB_1 is least is

a) $x-7y+17=0$ b) $7x+y+31=0$ c) $x-7y-17=0$ d) $x+7y-31=0$

146. The vertices of a triangle are $(pq,1/(pq),(qr,1/(qr))$ and $(rq,1/(rp))$ where p,q,r are the roots of the

equation$y^3 - 3y^2 + 6y + 1 = 0$. the coordinates of its centriod are

a) $(1, 2)$ b) $(2, -1)$ c) $(1, -1)$ d) $(2, 3)$

147. P, Q, R and S are the points of intersection with the coordinate axes of the lines $px + qy = pq$ and $qx + py = pq$, then $(P, Q > 0)$

a) P, Q, R, S from a parallelogram
b) P, Q, R, S from a rhombus
c) P, Q, R, S are concyclic
d) None of these

148. If one of the lines of $my^2 + (1 - m^2)xy - mx^2 = 0$ is a bisector of the angle between the lines $xy = 0$, then m is

a) 3 b) 2 c) $-1/2$ d) -1

149. The equation of straight line passing through $(-a, 0)$ and making the triangle with axes of area 'T' is

a) $2Tx + a^2y + 2aT = 0$ b) $2Tx - a^{2y} + 2aT = 0$ c) $2Tx - a^{2y} - 2aT = 0$ d) None of these

150. Locus of a point is equidistant from the lines $x + y - 2\sqrt{2} = 0$ and $x + y - \sqrt{2} = 0$ is

a) $x + y - 5\sqrt{2} = 0$ b) $x + y - 3\sqrt{2} = 0$ c) $2x + 2y - 3\sqrt{2} = 0$ d) $2x + 2y - 5\sqrt{2} = 0$

151. The area of a parallelogram formed by the lines $ax \pm bx \pm c = 0$ is

a) $c^2/(ab)$ b) $2c^2/(ab)$ c) $c^2/2ab$ d) None of these

152. Line L ha sintercepts a and b on the coordinate axes. When the axes are rotated through a given angle, keeping the origin fixed, the same line L has intercepts p and q then,

a) $a^2 + b^2 = p^2 + q^2$
b) $1/a^2 + 1/b^2 = 1/p^2 + 1/q^2$
c) $a^2 + p^2 = b^2 + q^2$
d) $1/a^2 + 1/p^2 = 1/b^2 + 1/q^2$

153. The lines $y = m_1x, y = m_2x$ and $y = m_3x$ make equal intercepts on the line $x + y = 1$, then

a) $2(1 + m_1)(1 + m_3) = (1 + m_2)(2 + m_1 + m_3)$
b) $(1 + m_1)(1 + m_3) = (1 + m_2)(1 + m_1 + m_3)$
c) $(1 + m_1)(1 + m_2) = (1 + m_3)(2 + m_1 + m_3)$
d) $2(1 + m_1)(1 + m_2) = (1 + m_2)(1 + m_1 + m_3)$

154. The condition that one of the straight line given by the equation $ax^2 + 2hxy + by^2 = 0$ may coincide with one of those given by the equation $a'x^2 + 2h'^{xy} + b'y'^2 = 0$ is

a) $(ab' - a'b)^2 = 4(ha' - h'a)(bh' - b'h')$
b) $(ab' - a'b)^2 = (ha' - h'a)(bh' - b'h')$
c) $(ha' - h'a)^2 = 4(ab' - a'b)(bh' - b'h')$
d) $(bh' - b'h)^2 = 4(ab' - a'b)(ha' - h'a)$

155. If the point $(x_1 + t(x_2 - x_1), y_1 + t(y_2 - y_1))$ divides the join of (x_1, y_1) and (x_2, y_2) internally, then

a) $t < 0$ b) $0 < t < 1$ c) $t > 1$ d) $t = 1$

156. The straight lines represented by $(y - mx)^2 = a^2(1 + m)$ and $(y - nx)^2 = a^2(1 + n^2)$ form a

a) Rectangle b) Rhombus c) Trapezium d) None of these

157. If the vertices of a triangle are $\left(\sqrt{5}, 0\right), (\sqrt{3}, \sqrt{2})$ and $(2, 1)$, then the orthocentre of the triangle is

a) $(\sqrt{5}, 0)$
b) $(0, 0)$
c) $(\sqrt{5} + \sqrt{3} + 2, \sqrt{2} + 1)$
d) None of these

158. If $A(1, p^2), B(0, 1)$ and $C(p, 0)$ are the coordinates of three points, then the value of p for which the area of the triangle ABC is minimum is

a) $1/\sqrt{3}$ b) $-1/\sqrt{3}$ c) $1/\sqrt{2}$ d) None of these

159. If the straight lines $2x + 3y - 1 = 0, x + 2y - 1 = 0$ and $ax + by - 1 = 0$ form a triangle with origin as orthocentre, then (a, b) is given by

a) $(6, 4)$ b) $(-3, 3)$ c) $(-8, 8)$ d) $(0, 7)$

160. A square of side a lies above the x-axis and has vertex at the origin. The side passing through the origin makes an angle $\alpha (0 < \alpha < \pi/4)$ with the positive direction of x-axis. The equation of its diagonal not passing through the origin is

a) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$
b) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$
c) $y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$
d) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$

161. If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in G.P. with same common ratio, then the points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3)

a) Lie on a straight line
b) Lie on an ellipse
c) Lie on a circle
d) Are vertices of a triangle

Multiple Correct Answers Type

162. The straight line represented by $x^2 + mxy - 2y^2 + 3y - 1 = 0$ meets at
a) $(-1/3, 2/3)$ b) $(-1/3, -2/3)$ c) $(1/3, 2/3)$ d) None of these
163. Equation (s) of the straight line(s), inclined at 30° to the x-axis such that the length of its (each of their) line segment (s) between the coordinate axes is 10 units is are
a) $x + \sqrt{3}y + 5\sqrt{3} = 0$ b) $x - \sqrt{3}y + 5\sqrt{3} = 0$ c) $x + \sqrt{3}y - 5\sqrt{3} = 0$ d) $x - \sqrt{3}y - 5\sqrt{3} = 0$
164. If $(-4, 0)$ and $(1, -1)$ are two vertices of a triangle of area 4 sq. units, then its third vertex lies on
a) $y = x$ b) $5x + y + 12 = 0$ c) $x + 5y - 4 = 0$ d) $x + 5y + 12 = 0$
165. Let $u \equiv ax + by + a\sqrt[3]{b} = 0, v \equiv bx - ay + b\sqrt[3]{a} = 0, a, b \in R$ be two straight lines. The equations of the bisectors of the angle formed by $k_1u - k_2v = 0$ and $k_1u + k_2v = 0$ for nonzero real k_1 and k_2 are
a) $u = 0$ b) $k_2u + k_1v = 0$ c) $k_2u - k_1v = 0$ d) $v = 0$
166. The points $(0, 8/3)$, $(1, 3)$ and $(82, 30)$ are vertices of
a) An abtuse-angled triangle b) And acute-angled triangle
c) A right-angled triangle d) None of these
167. Let $0 \equiv (0, 0), A \equiv (0, 4), B \equiv (6, 0)$, '$P$' be a moving point such that the area of triangle POA is two times the area of triangle POB. Locus of 'P' will be stright line whose equation can be
a) $x + 3y = 0$ b) $x + 2y = 0$ c) $2x - 3y = 0$ d) $3y - x = 0$
168. The sides of a triangle are the straight lines $x + y = 1, 7y = x$ and $\sqrt{3}y + x = 0$. Then which of the following is an interior point of the triangle?
In an obtused-angle triangle orthocentre and circumcentre are exterior to the triangle
a) Circumcentre b) Centriod c) Incentre d) Orthocentre
169. If one of the lines given by the equation $2x^2 + pxy + 3y^2 = 0$ coincide with one of those given buy $2x^2 + qxy - 3y^2 = 0$ and the other lines represented by them be perpendicular, then
a) $p = 5$ b) $p = -5$ c) $q = -1$ d) $q = 1$
170. If m_1 and m_2 are the roots of the equation $x^2 - ax - a - 1 = 0$, then the area of the triangle formed by the three straight lines $y = m_1x, y = m_2x$ and $y = a(a \neq -1)$ is
a) $\dfrac{a^2(a+2)}{2(a+1)}$ if $a > -1$ b) $\dfrac{-a^2(a+2)}{2(a+1)}$ if $a < -1$
c) $\dfrac{-a^2(a+2)}{2(a+1)}$ if $-2 < a - 1$ d) $\dfrac{a^2(a+2)}{2(a+1)}$ if $a < -2$
171. Angle made with the x-axis by two lines drown through the point $(1, 2)$ cutting the line $x + y = 4$ at a distance $\sqrt{6}/3$ from the point $(1, 2)$ are
a) $\dfrac{\pi}{12}$ and $\dfrac{5\pi}{12}$ b) $-\dfrac{7\pi}{12}$ and $-\dfrac{11\pi}{12}$ c) $\dfrac{\pi}{8}$ and $\dfrac{3\pi}{8}$ d) None of these
172. Two sides of a triangle are the lines $(a + b)x + (a - b)y - 2ab = 0$
$(a - b)x + (a + b)y - 2ab = 0$. If the triangle is isosceles and the third side passes through point $(b - a, a - b)$, then the equation of third side can be
a) $x + y = 0$ b) $x = y + 2(b - a)$ c) $x - b + a = 0$ d) $y - a + b = 0$
173. If the points $(a^3/(a-1), (a^2-3)/(a-1)), (b^3/(b-1), (b^2-3)/(b-1))$ and $(b^2-3)/(b-1)$ and $(c^3/(c-1), (c^2-3)/(c-1))$, where a, b, c are different fron 1, lie on the line $lx + my + n = 0$, then
a) $a + b + c = -\dfrac{m}{l}$ b) $ab + bc + ca = \dfrac{n}{l}$
c) $abc = \dfrac{(m+n)}{l}$ d) $abc - (bc + ca + ab) + 3(a + b + c) = 0$
174. Given three straight lines $2x + 11y - 5 = 0, 24x + 7y - 20 = 0$ and $4x - 3y - 2 = 0$. Then,
a) They form a triangle b) They are concurrent
c) One line bisects the angle between the other two d) Two of them are parallel
175. The points $A(0, 0), B(\cos\alpha, \sin\alpha)$ and $C(\cos\beta, \sin\beta)$ are the vertices of a right-angled triangle if
a) $\sin\dfrac{\alpha-\beta}{2} = \dfrac{1}{\sqrt{2}}$ b) $\cos\dfrac{\alpha-\beta}{2} = -\dfrac{1}{\sqrt{2}}$ c) $\cos\dfrac{\alpha-\beta}{2} = \dfrac{1}{\sqrt{2}}$ d) $\sin\dfrac{\alpha-\beta}{2} = -\dfrac{1}{\sqrt{2}}$

176. If (x, y) be a variable point on the line $y =\2x$ lying between the lines $2(x+1)+y=0$ and $x+3(y-1)=0$, then

a) $x \in (-1/2, 6/7)$ b) $x \in (-1/2, 3/7)$ c) $y \in (-1, 3/7)$ d) $y \in (-1, 6/7)$

177. Consider the straight lines $x+2y+4=0$ and $4x+2y-1=0$. The line $6x+6y+7=0$ is

a) Bisector of the angle including origin b) Bisector of acute angle
c) Bisector of obtuse angle d) None of the above

178. Two straight lines $u=0$ and $v=0$ pass through the origin and angle between them is $\tan^{-1}(7/9)$. If the ratio of the slope of $v=0$ and $u=0$ is $9/2$ then their equations are

a) $y+3x=0$ and $3y+2x=0$ b) $2y+3x=0$ and $3y+x=0$
c) $2y=3x$ and $3y=x$ d) $y=3x$ and $3y=2x$

179. If the straight line $ax+cy=2b$ where $a, b, c > 0$ makes a triangle of area 2 sq. units with coordinate axes, then

a) a, b, c are in G.P. b) $a, -b, c$ are in G.P. c) $a, 2b, c$ are in G.P. d) $a, -2b, c$ are in G. P.

180. The straight line $3x+4y-12=0$ meets the coordinates axes at A and B. An equilateral triangle ABC is constructed. The possible coordinates of vertex 'C' are

a) $\left(2\left(1-\frac{3\sqrt{3}}{4}\right), \frac{3}{2}\left(1-\frac{4}{\sqrt{3}}\right)\right)$ b) $\left(-2(1+\sqrt{3}), 3/2(1-\sqrt{3})\right)$

c) $\left(2(1+\sqrt{3}), 3/2(1+\sqrt{3})\right)$ d) $\left(2\left(1+\frac{3\sqrt{3}}{4}\right), \frac{3}{2}\left(1+\frac{4}{\sqrt{3}}\right)\right)$

181. If $x^2+2hxy+y^2=0$ represents the equation of the straight lines through the origin which make an angle α with the straight line $y+x=0$, then

a) $\sec 2\alpha = h$ b) $\cos\alpha = \sqrt{(1+h)/(2h)}$
c) $2\sin\alpha = \sqrt{(1+h)/h}$ d) $\cot\alpha = \sqrt{(h+1)/(h-1)}$

182. If $(a\cos\theta_1, a\sin\theta_1)$, $(a\cos\theta_2, a\sin\theta_2)$ and $(a\cos\theta_3, a\sin\theta_3)$ represents the vertices of an equilateral triangle inscribed in a circle, then

a) $\cos\theta_1+\cos\theta_2+\cos\theta_3=0$ b) $\sin\theta_1+\sin\theta_2+\sin\theta_3=0$
c) $\tan\theta_1+\tan\theta_2+\tan\theta_3=0$ d) $\cot\theta_1+\cot\theta_2+\cot\theta_3=0$

183. All points lying inside the triangle formed by the points $(1, 3)$, $(5, 0)$ and $(-1, 2)$ satisfy

a) $3x+2y \geq 0$ b) $2x+y-13 \geq 0$ c) $2x-3y-12 \leq$ d) $-2x+y \geq 0$

184. If $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ are the vertices of a triangle, then the equation

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Represents

a) The median through A b) The altitude through A
c) The perpendicular bisector of BC d) The line joining the centroid with a vertex

185. The area of triangle ABC is 20 cm^2. The coordinates of vertex A are $(-5, 0)$ and those of B are $(3, 0)$. The vertex C lies on the line $x-y=2$. The coordinates of C are

a) $(5, 3)$ b) $(-3, -5)$ c) $(-5, -7)$ d) $(7, 5)$

186. If (α, α^2) lies inside the triangle formed by the lines $2x+3y-1=0$, $x+2y-3=0$, $5x-6y-1=0$, then

a) $2\alpha+3\alpha^2-1>0$ b) $\alpha+2\alpha^2-3<0$ c) $\alpha+2\alpha^2-3<0$ d) $6\alpha^2-5\alpha+1>0$

187. If each of the vertices of a triangle has integral coordinates, then the triangle may be

a) Right angled b) Equilateral c) Isosceles d) None of these

188. If the vertices P, Q, R of a triangle PQR are rational points, which of the following points of the triangle PQR is (are) always rational points(s)? (A rational point is a point whose coordinates are rational numbers)

a) Centroid b) Incentre c) Circumcentre d) Orthocentre

189. Angles made with x-axis by a straight line drawn through $(1, 2)$ so that it intersects $x + y = 4$ at a distance $\sqrt{6}/3$ from $(1, 2)$ are

a) $105°$ b) $75°$ c) $60°$ d) $15°$

190. The equation of the lines passing through the point $(1, 0)$ and at a distance $\sqrt{3}/2$ from the origin are

a) $\sqrt{3}x + y - \sqrt{3} = 0$ b) $x + \sqrt{3}y - \sqrt{3} = 0$ c) $\sqrt{3}x - y - \sqrt{3} = 0$ d) $x - \sqrt{3}y - \sqrt{3} = 0$

191. The lines $x + 2y + 3 = 0, x + 2y - 7 = 0$ and $2x - y - 4 = 0$ are the sides of a square. Equation of the remaining side of the square can be

a) $2x - y + 6 = 0$ b) $2x - y + 8 = 0$ c) $2x - y - 10 = 0$ d) $2x - y - 14 = 0$

192. Sides of a rhombus are parallel to the lines $x + y - 1 = 0$ and $7x - y - 5 = 0$. It is given that diagonals of the rhombus intersect at $(1, 3)$ and one vertex, 'A' of the rhombus lies on the line $y = 2x$. Then the coordinates of the vertex A are

a) $(8/5, 16/5)$ b) $(7/15, 14/15)$ c) $(6/5, 12/5)$ d) $(4/15, 8/15)$

193. In a $\Delta\, ABC, A \equiv (\alpha, \beta), B \equiv (1, 2), C \equiv (2, 3)$ and point 'A' lies on the line $y = 2x + 3$ where $\alpha, \beta \in$ integer and area of the triangle is S such that $[S] = 2$ where $[.]$ denotes the greatest integer function. Then all possible coordinates of A

a) $(-7, -11)$ b) $(-6, -9)$ c) $(2, 7)$ d) $(3, 9)$

194. If $P(1, 2), Q(4, 6), R(5, 7)$ and $S(a, b)$ are the vertices of a parallelogram $PQRS$, then

a) $a = 2, b = 4$ b) $a = 3, b = 4$ c) $a = 2, b = 3$ d) $a = 1$ or $b = -1$

195. If P is a point (x, y) on the line $y = -3x$ such that P and the point $(3, 4)$ are on the opposite sides of the line $3x - 4y = 8$, then

a) $x > 8/15$ b) $x > 8/5$ c) $y < -8/5$ d) $y < -8/15$

196. Three lines $px + qy = r = 0, qx + ry + p = 0$ and $rx + py + q = 0$ are concurrent if

a) $p + q + r = 0$ b) $p^2 + q^2 + r^2 = pr + rp + pq$

c) $p^3 + q^3 + r^3 = 3pqr$ d) None of these

197. The lines joining the origin to the point of intersection of $3x^2 + mxy - 4x + 1 = 0$ and $2x + y - 1 = 0$ are at right angles. Then which of the following is not possible value of$m$?

a) -4 b) 4 c) 7 d) 3

198. If $(-6, -4), (3, 5), (-2, 1)$ are the vertices of a parallelogram, then remaining vertex can be

a) $(0, -1)$ b) $(7, 9)$ c) $(-1, 0)$ d) $(-11, -8)$

199. If $(x/a) + (y/b) = 1$ and $(x/c) + (y/d) = 1$ intersect the axes at four concyclic points and $a^2 + c^2 = b^2 + d^2$, then these lines can intersect at $(a, b, c, d > 0)$

a) $(1, 1)$ b) $(1, -1)$ c) $(2, -2)$ d) $(3, 3)$

200. If $bx + cy = a$, where a, b, c are the same sign, be a line such that the area enclosed by the line and the axes of reference is $\frac{1}{8}$ unit^2, then

a) b, a, c are in GP b) $b, 2a, c$ are in GP c) $b, \frac{a}{2}, c$ are in GP d) $b, -2a, c$ are in GP

201. If one of the lines of $my^2 + (1 - m^2)xy - mx^2 = 0$ is a bisector of the angle between the lines $xy = 0$, then m is

a) 1 b) 2 c) $-1/2$ d) -1

202. The ends of a diagonal of square are $(2, -3)$ and $(-1, 1)$. Another vertex of the square can be

a) $(-3/2, -5/2)$ b) $(5/2, 1/2)$ c) $(1/2, 5/2)$ d) One of these

203. Two sides of a rhombus $OABC$ (lying entirely in first quadrant or third quadrant) of area equal to 2 sq. units are $y = x/\sqrt{3}, y = \sqrt{3}\, x$. Then possible coordinates of $B$is/are (O being origin)

a) $(1 + \sqrt{3}, 1 + \sqrt{3})$ b) $(-1 - \sqrt{3}, -1 - \sqrt{3})$ c) $(3 + \sqrt{3}, 3 + \sqrt{3})$ d) $(\sqrt{3} - 1, \sqrt{3} - 1)$

204. Equation of a straight line passing through the point $(2, 3)$ and inclined at an angle of $\tan^{-1}(1/2)$ with the line $y + 2x = 5$ is

a) $y = 3$ b) $x = 2$ c) $3x + 4y - 18 = 0$ d) $4x + 3y - 17 = 0$

205. If the chord $y = mx + 1$ of the circle $x^2 + y^2 = 1$ subtends an angle of measure $45°$ at the major segment of the circle, then the value of m is

a) 2 b) 1 c) -1 d) None of these

206. Two roads are represented by the equation $y - x = 6$ and $x + y = 8$. An inspection bunglow has to be so constructed that it is at a distance of 100 from each of the roads. Possible location of the bunglow is given by

a) $(100\sqrt{2} + 1, 7)$ b) $(1 - 100\sqrt{2}, 7)$ c) $(1, 7 + 100\sqrt{2})$ d) $(1, 7 - 100\sqrt{2})$

207. The x-coordintes of the vertices of a square of unit area are the roots of the equation $x^2 - 3|x| + 2 = 0$ and the y-coordinates of the vertices are the roots of the equation $y^2 - 3y + 2 = 0$. Then the possible vertices of the square is/are

a) $(1, 1), (2, 1), (2, 2), (1, 2)$
b) $(-1, 1), (-2, 1), (-2, 2), (-1, 2)$
c) $(2, 1), (1, -1), (1, 2), (2, 2)$
d) $(-2, 1), (-1, -1), (-1, 2), (-2, 2)$

208. The diagonals of a parallelogram $PQRS$ are along the lines $x + 3y = 4$ and $6x - 2y = 7$. Then $PQRS$ must be a

a) Rectangle b) Square c) Cyclic quadrilateral d) Rhombus

209. The lines $x + y - 1 = 0, (m - 1)x + (m^2 - 7)y - 5 = 0$ and $(m - 2)x + (2m - 5)y = 0$ are

a) Concurrent for three values of m b) Concurrent for one value of m
c) Concurrent for no value of m d) Are parallel for $m = 3$

210. Consider the equation $y - y_1 = m(x - x_1)$. If m and x_1 are fixed and different lines are drawn for different values of y_1, then

a) The lines will pass through a fixed point b) There will be a set of parallel lines
c) All the lines intersect the line $x = x_1$ d) All the lines will be parallel to the line $y = x_1$

211. Let $P(\sin\theta, \cos\theta)$ $(0 \leq \theta \leq 2\pi)$ be a point in triangle with vertices $(0, 0)$, $(\sqrt{3/2}, 0)$ and $(0, \sqrt{3/2})$. Then,

a) $0 < \theta < \pi/12$ b) $5\pi/2 < \theta < \pi/2$ c) $0 < \theta < 5\pi/2$ d) $5\pi/2 < \theta < \pi$

212. The equation $x^3 + x^2y - xy^2 = y^3$ represents

a) Three real straight lines
b) Lines in which two of them are perpendicular to each other
c) Lines in which two of them are coincident
d) None of these

213. The equations of two equal sides AB and AC of an isosceles trinagle ABC are $x + y = 5$ and $7x - y = 3$, respectively. Then the equations of the sides BC if ar$(\Delta\, ABC) = 5$ unit^2

a) $x - 3y + 1 = 0$ b) $x - 3y - 21 = 0$ c) $3x + y + 2 = 0$ d) $3x + y - 12 = 0$

214. The equation of the lines on which the perpendiculars from the origin make $30°$ angle with x-axis and which form a triangle of area $50/\sqrt{3}$ with axes are

a) $\sqrt{3}\,x + y - 10 = 0$ b) $\sqrt{3}x + y + 10 = 0$ c) $x + \sqrt{3}y - 10 = 0$ d) $x - \sqrt{3}y - 10 = 0$

215. The combined equation of three sides of a triangle is $(x^2 - y^2)(2x + 3y - 6) = 0$. If $(-2, 0)$ is an interior point and $(b, 1)$ is an exterior point of the triangle, then

a) $2 < a < 10/3$ b) $-2 < a < 10/3$ c) $-1 < b < 9/2$ d) $-1 < b < 1$

Assertion - Reasoning Type

This section contain(s) 0 questions numbered 216 to 215. Each question contains STATEMENT 1(Assertion) and STATEMENT 2(Reason). Each question has the 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

a) Statement 1 is True, Statement 2 is True; Statement 2 **is** correct explanation for Statement 1

b) Statement 1 is True, Statement 2 is True; Statement 2 **is not** correct explanation for Statement 1

c) Statement 1 is True, Statement 2 is False

d) Statement 1 is False, Statement 2 is True

216

Statement 1: If point of intersection of the lines $4x + 3y = \lambda$ and $3x - 4y = \mu\ \forall \lambda, \mu \in R$ is(x_1, y_1), then the locus of (λ, μ) is $x + 7y = 0, \forall\ x_1 = y_1$

Statement 2: If $4\lambda + 3\mu > 0$ and $3\lambda - 4\mu > 0$, then (x_1, y_1) is in first quadrant

217

Statement 1: If a, b, c are variable such that $3a + 2b + 4c = 0$, then the family of lines given by $ax + by + c = 0$ pass through a fixed point (3, 2)

Statement 2: The equation $ax + by + c = 0$ will represent a family of straight the passing through a fixed point iff there exists a relation between a, b and c

218 Lines $L_1: y - x = 0$ and $L_2: 2x + y = 0$ intersect the line $L_3: y + 2 = 0$ at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statement 1: The ratio $PR: PQ$ equals $2\sqrt{2}: \sqrt{5}$.

Statement 2: In any triangle, bisector of an angle divides the triangle into two similar triangles.

219

Statement 1: If the lines $2x + 3y + 19 = 0$ and $9x + 6y - 17 = 0$ cut the axis in A, B and y-axis at C, D, then points A, B, C, D are concyclic

Statement 2: Since $OA \times OB = OC \times OD$, where O is origin, therefore A, B, C, D are concyclic

220

Statement 1: If the point $(2a, -5, a^2)$ is on the same side of the line $x + y - 3 = 0$ as that of the origin, then $a \in (2, 4)$

Statement 2: The points (x_1, y_1) and (x_2, y_2) lie on the same or opposite sides of the line $ax + by + c = 0$, as $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ have the same or opposite signs

221

Statement 1: If sum of algebraic distance from points $A(1, 1), B(2, 3), C(0, 2)$ is zero on the line $ax + by + c = 0$, then $a + 3b + c = 0$

Statement 2: The centroid of triangle is (1, 2)

222

Statement 1: The incentre of a triangle formed by the lines $x\cos(\pi/9) + y\sin(\pi/9) = \pi, x\cos(8\pi/9) + y\sin(8\pi/9) = \pi; x\cos(13\pi/9) + y\sin(13\pi/9) = \pi$ is (0, 0)

Statement 2: Any point equidistant from the given three non-concurrent straight lines in the plane is incentre of the triangle

223

Statement 1: If the points (1, 2) and (3, 4) be on the same side of the line $3x - 5y + \lambda = 0$, then $\lambda < 7$ or $\lambda > 11$

Statement 2: If the points $(x_1, y_1)$and (x_2, y_2) be on the same side of the line $f(x, y) \equiv ax + by + c = 0$, then

$$\frac{f(x_1, y_1)}{f(x_2, y_2)} < 0$$

224

Statement 1: Lines passing through the given point and is equally inclined to the given two lines are always perpendicular

Statement 2: Angle bisector of the given two lines are always perpendicular

225

Statement 1: The joint equation of lines $y = x$ and $y = -x$ is $y^2 = -x^2$, i.e., $x^2 + y^2 = 0$

Statement 2: The joint equation of lines $ax + by = 0$ and $cx + dy = 0$ is $(ax + by)(cx + dy) = 0$, where a, b, c, d are constant

226

Statement 1: Let the vertices of a ΔABC are $A(-5, -2), B(7, 6)$ and $C(5, -4)$. Then coordinates of circumcentre are (1, 2)

Statement 2: In a right angle triangle, midpoint of hypotenuse is the circumcenter of the triangle

227

Statement 1: Each point on the line $y - x + 12 = 0$ is equidistant from the lines $4y + 3x - 12 = 0, 3y + 4x - 24 = 0$

Statement 2: The locus of a point which is equidistant from two given lines is the angular bisector of the two lines

228

Statement 1: If $(a_1x + b_1y + c_1) + (a_2x + b_2y + c_2) + (a_3x + b_3y + c_3) = 0$, then lines $a_1x + b_1y + c_1 = 0, a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$ cannot be parallel

Statement 2: If sum of the equations for three straight lines is identically zero, then they are either concurrent or parallel

229

Statement 1: If $-2h = a + b$, then one line of the pair of lines $ax^2 + 2hxy + by^2 = 0$ bisects the angle between coordinate axes in positive quadrant

Statement 2: If $ax + y(2h + a) = 0$ is a factor of $ax^2 + 2hxy + by^2 = 0$, then $b + 2h + a = 0$

230

Statement 1: The internal angle bisector of angle C of a triangle ABC with sides AB, AC and BC as $y = 0, 3x + 2y = 0$ and $2x + 3y + 6 = 0$, respectively is $5x + 5y + 6 = 0$

Statement 2: Image of point A with respect to $5x + 5y + 6 = 0$ lies on side BC of the triangle

231

Statement 1: Each point on the line $y - x + 12 = 0$ is equidistant from the lines $4y + 3x - 12 = 0, 3y + 4x - 24 = 0$

Statement 2: The locus of a point which is equidistant from two given lines is the angular bisector of two the lines

232

Statement 1: The lines $(a + b)x + (a - b)y - 2ab = 0, (a - b)x + (a + b)y - 2ab = 0$ and $x + y = 0$ form an isosceles triangle

Statement 2: If internal bisector of any of triangle is perpendicular to the opposite side, then the given triangle is isosceles

233

Statement 1: If the vertices of a triangle are having rational coordinates then its centroid, circumcentre and orthocentre are rational

Statement 2: In any triangle, orthocentre, centroid and circumcentre are collinear and centroid divides the line joining orthocentre and circumcentre in the ratio 2:1

234

Statement 1: The area of the triangle formed by the points $A(1000, 1002), B(1001, 1004), C(1002, 1003)$ is same as the area formed by $A'(0, 0), B'(1, 2), C'(2, 1)$

Statement 2: The area of the triangle is constant with respect to translation of axes

235

Statement 1: If the diagonals of the quadrilateral formed by lines $px + qy + r = 0, p'x + q'y + r = 0$ are at right angles, then $p^2 + q^2 = p'^2 + q'^2$

Statement 2: Diagonals of a rhombus are bisected and perpendicular to each other

236

Statement 1: If joint equation of the lines $2x - y = 5$ and $x + 2y = 3$ is $2x^2 + 3xy - 2y^2 - 11x - 7y + 15 = 0$

Statement 2: Every second degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, always represents a pair of straight line

237

Statement 1: The lines $(a + b)x + (a - 2b)y = a$ are concurrent at the point (2/3, 1/3)

Statement 2: The lines $x + y - 1 = 0$ and $x - 2y = 0$ intersect at the point (2/3, 1/3)

Matrix-Match Type

This section contain(s) 0 question(s). Each question contains Statements given in 2 columns which have to be matched. Statements (A, B, C, D) in **columns I** have to be matched with Statements (p, q, r, s) in **columns II**.

238. Consider the triangle formed by the lines

Column-I	Column- II
(A) Values of α if $(0, \alpha)$ lies inside triangle	(p) $(-\infty, 7/3) \cup (13/4, \infty)$
(B) Values of α if $(\alpha, 0)$ lies inside triangle	(q) $-4/3 < \alpha < 1/2$
(C) Values of α if $(\alpha, 2)$ lies inside triangle	(r) No value of α
(D) Value of α if $(1, \alpha)$ lies outside triangle	(s) $5/3 < \alpha < 7/2$

CODES :

	A	**B**	**C**	**D**
a)	P	q	r	s
b)	s	r	q	p
c)	q	r	s	t

d)	p	q	t	s

239.

	Column-I		Column- II
(A)	The value k for which $4x^2 + 8xy + ky^2 = 9$ is the equation of a pair of straight lines is	(p)	3
(B)	If the sum of the slopes of the lines given by $x^2 - 2cxy - 7y^2 = 0$ is four times their product, then the value of c is	(q)	-3
(C)	If the gradient of one of the lines $x^2 + kxy + 2y^2 = 0$ is twice that of the other, then $h =$	(r)	2
(D)	If the lines $ax^2 + 2hxy + by^2 = 0$ are equally inclined to the lines $ax^2 + 2hxy + by^2 + \lambda(x^2 + y^2) = 0$, then the value of λ can be	(s)	4

CODES :

	A	**B**	**C**	**D**
a)	S,r	p,q	p,q,r,s	t
b)	p,q	r,s	t	q
c)	p,q,r,s	p	s	t
d)	s	r	p,q	p,q,r,s

240.

	Column-I		Column- II
(A)	The distance between the lines $(x + 7y)^2 + 4\sqrt{2}(x + 7y) - 42 = 0$ is	(p)	2
(B)	If the sum of the distance of a point from two perpendicular lines in a plane is 1, then its locus is $\lvert x\rvert + \lvert y\rvert = k$, where k is equal to	(q)	7
(C)	If $6x + 6y + m = 0$ is acute angle bisector of line $x + 2y + 4 = 0$ and $4x + 2y - 1 = 0$, then m is equal to	(r)	3
(D)	Area of the triangle formed by the lines $y^2 - 9xy + 18x^2 = 0$ and $y = 6$ is	(s)	1

CODES :

	A	**B**	**C**	**D**
a)	P	q	r	s
b)	q	p	s	r
c)	p	s	q	r
d)	t	p	r	q

241.

	Column-I		Column- II
(A)	The lines $y = 0; y = 1; x - 6y + 4 = 0$ and $x + 6y - 9 = 0$ constitute a figure which is	(p)	A cyclic quadrilateral
(B)	The points $A(a, 0), B(0, b), C(c, 0)$ and $D(0, d)$ are such that $ac = bd$ and a, b, c, d are all non-zero. The points A, B, C and D always constitute	(q)	A rhombus
(C)	The figure formed by the four lines $ax \pm by \pm c = 0$ $(a \neq b)$ is	(r)	A square
(D)	The line pairs $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$ constitute a figure which is	(s)	A trapezium

CODES :

	A	**B**	**C**	**D**
a)	Q	r	p,r	s,t
b)	p,s	p	q	p,q,r
c)	s	r	s	p,q
d)	p,q,r	s,t	r	p,q

242.

	Column-I		Column- II
(A)	A straight line with slope passing through (1, 4) meets the coordinates axes of A and B. The minimum length of $OA + OB, O$ being the origin, is	(p)	$5\sqrt{2}$
(B)	If the point P is symmetric to the point $Q(4, -1)$ with respect to the bisector of the first quadrant, then the length of PQ is	(q)	$3\sqrt{2}$
(C)	On the portion of the straight line $x + y = 2$ between the axis a square is constructed away from the origin.with this portion as one of its sides. If d denotes the perpendicular distance of a side of this square from the origin then the maximum value of d is	(r)	9/2
(D)	If the parametric equation of a line is given by $x = 4 + \lambda/\sqrt{2}$ and $y = -1 + \sqrt{2}\lambda$ where λ is a parameter, then the intercept made by the line on the x-axis is	(s)	9

CODES :

	A	**B**	**C**	**D**
a)	P	s	q	t
b)	p	p	q	s
c)	s	p	q	r

d)	q	t	r	s

243.

	Column-I		Column- II
(A)	Four lines $x+3y-10=0, x+3y-20=0, 3x-y+5=0$ and $3x-y-5=0$ form a figure which is	(p)	Quadrilateral which is neither a parallelogram nor a trapezium
(B)	The points $A(1,2), B(2,-3), C(-1,-5)$ and $D(-2,4)$ in order are the vertices of	(q)	A parallelogram
(C)	The lines $7x+3y-33=0, 3x-7y+19=0, 3x-7y-10$ and $7x+3y-4=0$ from a figure which is	(r)	A rectangle of area 10 sq. units
(D)	Four lines $4y-3x-7=0, 3y-4x+7=0, 4y-3x-21=0, 3y-4x+14=0$ form a figure which is	(s)	A square

CODES :

	A	**B**	**C**	**D**
a)	S,t	r,s	q	p
b)	q,r,s	p	q,s	q
c)	p	q	r	s,t
d)	q,s	p	r	t

244.

	Column-I		Column- II
(A)	Two vertices of a triangle are $(5,-1)$ and $(-2,3)$. If orthocentre of the third vertex are	(p)	$(-4,-7)$
(B)	A point on the line $x+y=4$ which lies at a unit distance from the line $4x+3y=10$ is	(q)	$(-7,11)$
(C)	Orthocentre of the triangle formed by the lines $x+y-1=0, x-y+3=0, 2x+y=7$ is	(r)	$(2,-2)$
(D)	If $2a, b, c$ are in A.P., then lines $ax+by=c$ are concurrent at	(s)	$(-1,2)$

CODES :

	A	**B**	**C**	**D**
a)	P	q	s	r
b)	t	r	s	p
c)	q	p	r	t
d)	s	p	q	r

245.

Column-I	Column- II

(A)	If lines $3x+y-4=0, x-2y-6=0$ and $\lambda x+4y+\lambda^2=0$ are concurrent, then value of λ is	(p)	-4
(B)	If the points $(\lambda+1,1), (2\lambda+1,3)$ and $(2\lambda+2,2\lambda)$ are collinear, then the value of λ is	(q)	$-1/2$
(C)	If line $x+y-1-\lvert\lambda/2\rvert=0$, passing through the intersection of $x-y+1=0$ and $3x+y-5=0$, is perpendicular to one of them, then the value of λ is	(r)	4
(D)	If line $y-x-1+\lambda=0$ is equidistant from the points $(1,-2)$ and $(3,4)$ then λ is	(s)	2

CODES :

	A	**B**	**C**	**D**
a)	P,q	q	s,t	r
b)	s,p	q,r	t	q
c)	s	t,s	q	p,q
d)	p,s	q,s	p,r	s

246. O is origin and B is a point on the x-axis at a distance of 2 units from the origin

	Column-I		**Column- II**
(A)	If $\Delta\, AOB$ is equilateral triangle, then the coordinates of A can be	(p)	$(-1,\sqrt{3})$
(B)	If $\Delta\, AOB$ is isosceles such that $\angle OAB$ is $30°$, then coordinates of A can be	(q)	$(-1,2-\sqrt{3})$
(C)	If OB is one side of rhombus of area $\sqrt{3}$ units, then other vertices of rhombus can be	(r)	$(-3,-\sqrt{3})$
(D)	If OB is a chord of circle with radius equal to OB, then coordinates of point A on the circumference of the circle such that ΔOAB is isosceles can be	(s)	$(1,2+\sqrt{3})$

CODES :

	A	**B**	**C**	**D**
a)	P	p,s	p,r	q,s
b)	q	p,q	s,r	t
c)	s	p	q	r
d)	s,t	r	p,s	q

247. Consider the lines repressed by equation$(x^2+xy-x)x(x-y)=0$, forming a triangle. Then match the following

	Column-I		**Column- II**
(A)	Orthocentre of triangle	(p)	$(1/6, 1/2)$

(B) Circumcentre	(q)	$(1/(2+2\sqrt{2}),1/2)$
(C) Centroid	(r)	$(0, 1/2)$
(D) Incentre	(s)	$(1/2, 1/2)$

CODES :

	A	**B**	**C**	**D**
a)	S	r	p	q
b)	s	q	r	p
c)	p	q	r	s
d)	q	r	s	t

Linked Comprehension Type

This section contain(s) 23 paragraph(s) and based upon each paragraph, multiple choice questions have to be answered. Each question has atleast 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 248 to -248

The locus of a moving point is the path traced out by that point under one or more given conditions. Technically, a locus represents the 'set of points' which lies on it.

A relation $f(x, y) = 0$ between x and y which is satisfied by each point on the locus and such that each point satisfying the equation is on the locus is called the equation of the locus.

On the basis of above information, answer the following questions :

248. The locus of the point of intersection of the lines $x\sin\theta + (1-\cos\theta)y = a\sin\theta$ and $x\sin\theta - (1+\cos\theta)\,y + a\sin\theta = 0$ is

a) $x^2 - y^2 = a^2$ b) $x^2 + y^2 = a^2$ c) $y^2 = ax$ d) None of these

Paragraph for Question Nos. 249 to - 249

Let L be the line belonging to the family of the straight lines $(a+2b)x + (a-3b)y + a - 8b = 0, a, b \in R$, which is farthest from the point (2, 2)

249. The equation of line L is

a) $x + 4y + 7 = 0$ b) $2x + 3y + 4 = 0$ c) $4x - y - 6 = 0$ d) None of these

Paragraph for Question Nos. 250 to - 250

The equation of an altitude of an equilateral triangle is $\sqrt{3}x + y = 2\sqrt{3}$ and one of the vertices is $(3, \sqrt{3})$

250. The possible number of triangle is

a) 1 b) 2 c) 3 d) 4

Paragraph for Question Nos. 251 to - 251

For points $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ of the coordinate plane, a new distance $d(P, Q)$ is defined by $d(P, Q) = |x_1 - x_2| + |y_1 - y_2|$. Let $O = (0, 0)$ and $A = (3, 2)$. Consider the set of points P in the first quadrant which are equidistant (with respect to the new distance) from O and A

251. The set of points P consists of
   a) One straight line only
   b) Union of two line segments
   c) Union of two infinite rays
   d) Union of a line segment of finite length and an infinite ray

Paragraph for Question Nos. 252 to - 252

A variable line 'L' is drawn through $O(0, 0)$ to meet the lines L_1 and L_2 given by $y - x - 10 = 0$ and $y - x - 20 = 0$ at the points A and B, respectively

252. A point P is taken on 'L' such that $2/OP = 1/OA + 1/OB$. Then the locus of 'P' is
   a) $3x + 3y = 40$ b) $3x + 3y + 40 = 0$ c) $3x - 3y = 40$ d) $3y - 3x = 40$

Paragraph for Question Nos. 253 to - 253

The line $6x + 8y = 48$ intersects the coordinates axes at A and B respectively. A line L bisects the area and the perimeter of the triangle OAB where O is the origin

253. The number of such lines possible is
   a) 1 b) 2 c) 3 d) More than 3

Paragraph for Question Nos. 254 to - 254

$A(1, 3)$ and $C(-2/5, -2/5)$ are the vertices of a triangle ABC and the equation of the internal angle bisector of $\angle ABC$ is $x + y = 2$

254. Equation of side BC is
   a) $7x + 3y - 4 = 0$ b) $7x + 3y + 4 = 0$ c) $7x - 3y + 4 = 0$ d) $7x - 3y - 4 = 0$

Paragraph for Question Nos. 255 to - 255

Let $ABCD$ be a parallelogram whose equations for the diagonals AC and BD are $x + 2y = 3$ and $2x + y = 3$, respectively. If length of diagonal $AC = 4$ units and area of parallelogram $ABCD = 8$ sq. units, then

255. The length of other diagonal BD is
   a) 10/3 b) 2 c) 20/3 d) None of these

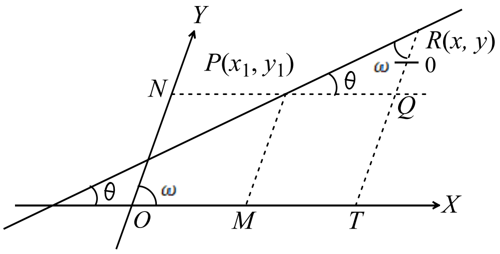
Paragraph for Question Nos. 256 to - 256

Consider a triangle PQR with coordinates of its vertices as $P(-8, 5), Q(-15, -19)$ and$R(1, -7)$. The bisector of the interior angle of P has the equation which can be written in the form $ax + 2y + c = 0$

256. The distance between the orthocentre and the circumcentre of the triangle PQR is
a) $25/2$ b) $29/2$ c) $37/2$ d) $51/2$

Paragraph for Question Nos. 257 to - 257

Let us consider the situation when axes are inclined at an angle 'ω'. If coordinates of a point P are (x_1, y_1), then $PN = x, PM = y_1$, where PM is parallel to y-axis and PN is parallel x-axis. straight line through P that makes an angle θ with x-axis is
$PQ = y - y_1, PQ = x - x_1$



From ΔPQR, we have

$$\frac{PQ}{\sin(\omega - \theta)} = \frac{RQ}{\sin\theta}$$

$$\Rightarrow \ y - y_1 = \frac{\sin\theta}{\sin(\omega - \theta)}(x - x_1)$$

Written in the form of $y - y_1 = m(x - x_1)$ where

$$m = \left(\frac{\sin\theta}{\sin(\omega - \theta)}\right)$$

Therefore, if slope of the line is m, then angle of inclination of the line with x-axis is given by

$$\tan\theta = \left(\frac{m\sin\omega}{1 + m\cos\omega}\right)$$

257. The axes being inclined at an angle of 60°, the inclination of the straight line $y = 2x + 5$ with x-axis is
a) 30° b) $\tan^{-1}(\sqrt{3}/2)$ c) $\tan^{-1} 2$ d) 60°

Paragraph for Question Nos. 258 to - 258

Consider the triangle having vertices $O(0,0), A(2,0)$ and $B(1,\sqrt{3})$. Also $b \leq \min\ (a_1, a_2, a_3, \dots a_n)$ means $b \leq a_1$ when a_1 is least; $b \leq a$ when a_2 is least and so on. From this we can say $b \leq a_1, b \leq a_2, \dots, b \leq a_n$

258. Let R be the region consisting of all those points P inside ΔOAB which satisfy $d(P, OA) \leq \min$ $[d\ (P, OB), d(P, AB)]$. where d denotes the distance from the point to the corresponding line. Then the area of the region R is
a) $\sqrt{3}$ sq. units b) $(2 + \sqrt{3})$ sq. units c) $\sqrt{3}/2$ sq. units d) $1/\sqrt{3}$ sq. units

Paragraph for Question Nos. 259 to - 259

Let $ABCD$ is a square with sides of unit length. points E and F are taken onsides AB and AD respectively so that $AE = AF$. Let P be a point inside the square $ABCD$

259. The maximum possible are a of quadrilateral $CDFE$ is
a) 1/8 b) 1/4 c) 5/8 d) 3/8

Integer Answer Type

260. Consider a $\Delta\ ABC$ whose sides AB, BC and CA are represents by the straight lines $2x + y = 0, x + py = q$ and $x - y = 3$, respectively. The point P is $(2, 3)$ is orthocentre then the value of $(p + q)/10$ is
261. Absolute value of the sum of the abscissas of the points on the line $x + y = 4$ that lie at a unit distance from the line $4x + 3y - 10 = 0$ is
262. The number of values of k for which the lines $(k + 1)x + 8y = 4k$ and $kx + (k + 3)y = 3k - 1$ are coincident
263. If the area enclosed by the graph of $x^2y^2 - 9x^2 - 25y^2 + 225 = 0$ is A if then value of $A/10$ is
264. The sides of a triangle ABC lie on the lines $3x + 4y = 0, 4x + 3y = 0$ and $x = 3$. let (h, k) be the centre of the circle inscribed in ΔABC. The value of $(h + k)$ equals
265. A man starts from the point $P(-3, 4)$ and reaches point $Q(0, 1)$ touching x-axis at $R(\alpha, 0)$such that $PR + RQ$ is minimum, then $5|\alpha| =$
266. Number of value of b fro which in an acute triangle ABC, if the coordinates of orthocentre 'H' are $(4, b)$, centroid 'G' are $(b, 2b - 8)$ and circumcentre 'S' are $(-4, 8)$ is
267. The line $x = C$ cuts the triangle with vertices $(0, 0)$, $(1, 1)$ and $(9, 1)$ into two regions. For the area of the two regions to be same, C must be equal to
268. The line $3x + 2y = 24$ meets the y-axis at A and the x-axis at B. The perpendicular bisector of AB meets the line through $(0, -1)$ parallel to x-axis at C. If the area of the triangle ABC is A then the value of $A/13$ is
269. The distance between the circumcentre and orthocentre of the triangle whose vertices are $(0, 0)$, $(6, 8)$ and $(-4, 3)$ is L, then the value of $\frac{2}{\sqrt{5}}L$ is
270. If area of the triangle formed by the line $x + y = 3$ and the angle bisectors of the pair of lines $x^2 - y^2 + 4y - 4 = 0$ is A, then the value of $16A$ is
271. For all real values of a and b, lines $(2a + b)x + (a + 3b)y + (b - 3a) = 0$ and $mx + 2y + 6 = 0$ are concurrent, then $|m|$ is equal to
272. The points (x, y) lies on the line $2x + 3y = 6$. Smallest value of the quantity $\sqrt{x^2 + y^2}$ is m then the value of $\sqrt{13}m$ is
273. Triangle ABC with $AB = 13, BC = 5$ and $AC = 12$ slides on the coordinates axis with A and B on the positive x-axis and positive y-axis respectively, the locus of vertex C is a line $12x - ky = 0$, then the value of k is
274. If the area of triangle formed by the points $(2a, b)(a + b, 2b + a)$ and $(2b, 2a)$ be 2 sq. units, then the area of the triangle whose vertices are $(a + b, a - b)$, $(3b - a, b + 3a)$ and $(3a - b, 3b - a)$ will be
275. The area of the triangular region in the first quadrant bounded on the left by the y-axis, bounded above by the line$7x + 4y = 168$ and bounded below by the line $5x + 3y = 121$ is A, then the value of $3A/10$ is
276. The sides of a triangle have the combined equation $x^2 - 3y^2 - 2xy + 8y - 4 = 0$. The third side, which is variable always passes through the point$(-5, -1)$. If the range of values of the slope of the third line is such that the origin is an interior point of the triangle is (a, b) then the value of $\left(a + \frac{1}{b}\right)$ is
277. The point A divided the join of $P(-5, 1), Q(3, 5)$ in the ratio k: 1, then the integral value of k for which the area of ΔABC where B is $(1, 5)$ and C is $(7, -2)$ is equal to 2 units in magnitude is

: ANSWER KEY :

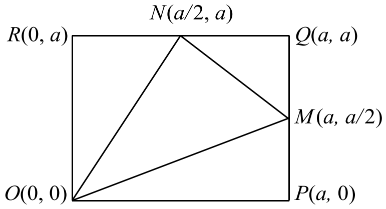
1)	c	2)	b	3)	c	4)	a
5)	b	6)	a	7)	a	8)	b
9)	d	10)	b	11)	d	12)	b
13)	d	14)	b	15)	c	16)	b
17)	a	18)	a	19)	c	20)	a
21)	b	22)	c	23)	d	24)	c
25)	a	26)	a	27)	c	28)	d
29)	a	30)	d	31)	b	32)	a
33)	a	34)	a	35)	a	36)	a
37)	d	38)	a	39)	c	40)	a
41)	b	42)	a	43)	c	44)	b
45)	a	46)	a	47)	c	48)	b
49)	c	50)	d	51)	a	52)	b
53)	a	54)	a	55)	a	56)	a
57)	a	58)	b	59)	b	60)	c
61)	d	62)	c	63)	c	64)	c
65)	a	66)	d	67)	b	68)	d
69)	d	70)	d	71)	d	72)	b
73)	d	74)	a	75)	c	76)	a
77)	a	78)	c	79)	d	80)	b
81)	d	82)	d	83)	c	84)	a
85)	c	86)	a	87)	a	88)	b
89)	a	90)	d	91)	c	92)	c
93)	c	94)	d	95)	c	96)	c
97)	b	98)	b	99)	d	100)	b
101)	a	102)	b	103)	b	104)	b
105)	b	106)	d	107)	c	108)	d
109)	b	110)	d	111)	b	112)	b
113)	b	114)	b	115)	c	116)	d
117)	a	118)	c	119)	b	120)	a
121)	d	122)	d	123)	d	124)	c
125)	d	126)	c	127)	b	128)	a
129)	b	130)	b	131)	d	132)	a
133)	d	134)	b	135)	a	136)	a
137)	d	138)	a	139)	c	140)	b
141)	a	142)	c	143)	d	144)	a
145)	a	146)	b	147)	c	148)	d
149)	b	150)	c	151)	b	152)	b
153)	a	154)	a	155)	b	156)	b
157)	c	158)	d	159)	c	160)	c
161)	a	1)	a,c	2)	b,d	3)	
	c,d	4)	a,d				
5)	d	6)	a,d	7)	b, c	8)	
	a,b,c,d						
9)	a,c,d	10)	a,d	11)	a,b	12)	
	a,b,d						
13)	b,c	14)	a,c	15)	b,d	16)	

	a,b						
17)	a,b,c,d	18)	a,b	19)	a,d	20)	
	a,b,d						
21)	a,b	22)	a,c	23)	a,d	24)	
	b,d						
25)	a,c,d	26)	a,c	27)	a,c,d	28)	
	b,d						
29)	a,c	30)	a,d	31)	a,c	32)	
	a,b,c,d						
33)	a	34)	a,c	35)	a,b,c	36)	
	a,b,c,d						
37)	b,c,d	38)	a,b,c,d	39)	b,d	40)	
	a,d						
41)	a,b	42)	a,b	43)	b,c	44)	
	b,c						
45)	a,b,c,d	46)	a,b	47)	d	48)	
	c,d						
49)	b,c	50)	a,c	51)	a,b,c	52)	
	a,b,c,d						
53)	a,b	54)	a,d	1)	b	2)	d
	3)	b	4)	a			
5)	d	6)	d	7)	c	8)	c
9)	a	10)	d	11)	a	12)	a
13)	d	14)	b	15)	b	16)	a
17)	a	18)	c	19)	a	20)	a
21)	c	22)	a	1)	b	2)	d
	3)	c	4)	b			
5)	c	6)	b	7)	a	8)	d
9)	a	10)	a	1)	b	2)	a
	3)	b	4)	d			
5)	d	6)	a	7)	b	8)	c
9)	a	10)	b	11)	d	12)	c
1)	5	2)	4	3)	1	4)	6
5)	0	6)	3	7)	0	8)	3
9)	7	10)	5	11)	8	12)	2
13)	6	14)	5	15)	8	16)	5
17)	4	18)	7				

: HINTS AND SOLUTIONS :

1 **(c)**

Let the coordinates of vertices O, P, Q, R be $(0,0), (a, 0), (a, a), (0, a)$, respectively. Then, we get the coordinates of M as $(a, a/2)$ and those of N as $(a/2, a)$

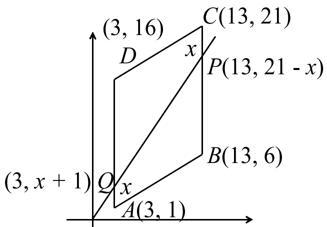


Therefore, area of ΔOMN is

$$\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ a & a/2 & 1 \\ a/2 & a & 1 \end{vmatrix} = \frac{3a^2}{8}$$

Area of the square isa^2. Hence, the required ratio is 8:3

2 **(b)**



$$m = \frac{21 - x}{13} = \frac{x + 1}{3}$$

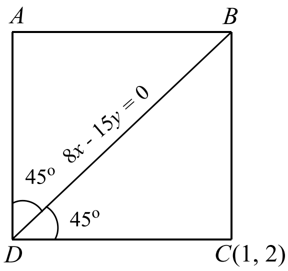
$\Rightarrow \; 63 - 3x = 13x + 13$

$\Rightarrow \; 16x = 50$

$\Rightarrow x = \frac{25}{8}$

Hence, $m = \left(\frac{25}{8} + 1\right) \times \frac{1}{3} = \frac{33}{24} = \frac{11}{8}$

3 **(c)**



Slope of BD is $8/15$ and angle made by BD with DC and BC is $45°$. So let slope of DC be$m$. Then,

$$\tan 45° = \pm \frac{m - \frac{8}{15}}{1 + \frac{8}{15}m}$$

$\Rightarrow \; (15 + 8m) = \pm(15m - 8)$

$\Rightarrow \; m = \frac{23}{7}$ and $-\frac{7}{23}$

Hence, the equations of DC and BC are

$y - 2 = \frac{23}{7}(x - 1)$

$\Rightarrow \; 23x - 7y - 9 = 0$

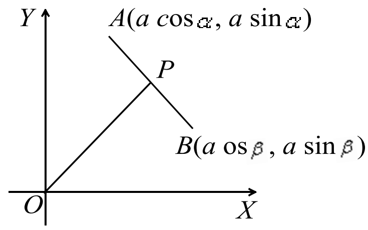
and $\; y - 2 = -\frac{7}{23}(x - 1)$

$\Rightarrow 7x + 23y - 53 = 0$

4 **(a)**

Midpoint of $(a\cos\alpha, a\sin\alpha)$ and $(a\cos\beta, a\sin\beta)$ is

$$P\left(\frac{a\,(\cos\alpha + \cos\beta)}{2}, \frac{a(\sin\alpha + \sin\beta)}{2}\right)$$



Slope of line AB is

$$\frac{a\sin\beta - a\sin\alpha}{a\cos\beta - a\cos\alpha} = \frac{\sin\beta - \sin\alpha}{\cos\beta - \cos\alpha} = m_1$$

and slope of OP is

$$\frac{\sin\alpha + \sin\beta}{\cos\alpha + \cos\beta} = m_2$$

Now, $m_1 \times m_2 = \frac{\sin^2\beta - \sin^2\alpha}{\cos^2\beta - \cos^2\alpha} = -1$

Hence, the lines are perpendicular

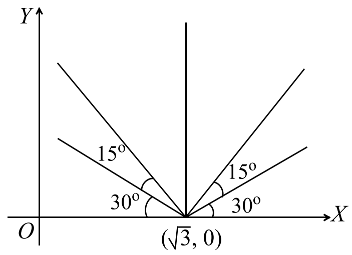
5 **(b)**

The given equation of pair of straight lines can be rewritten as $(\sqrt{3}y - x + \sqrt{3})(\sqrt{3}y + x - \sqrt{3}) = 0$

Their separate equations are $\sqrt{3}y - x + \sqrt{3} = 0$ and $\sqrt{3}y + x - \sqrt{3} = 0$

or $\; y = \frac{1}{\sqrt{3}}x - 1$ and $\; y = -\frac{1}{\sqrt{3}}x + 1$

or $y = (\tan 30°)x - 1\;$ and $\; y = (\tan 150°)x + 1$



After rotation through an angle of $15°$, the lines are $(y - 0) = \tan 45°\,(x - \sqrt{3})$ and $(y - 0) - \tan 135°\,(x - \sqrt{3})$ or $y = x - \sqrt{3}$ and $y = -x + \sqrt{3}$

Their combined equation is

$(y - x + \sqrt{3})(y + x - \sqrt{3}) = 0$ or $y^2 - x^2 + 2\sqrt{3}x - 3 = 0$

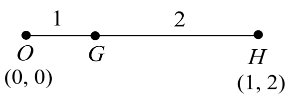
6 **(a)**

Let the two perpendicular lines be the coordinate axes. Let (x, y) be the point, sum of whose distance from two axes is 1. Then we must have
$|x| + |y| = 1$
or $\pm x \pm y = 1$
These are the four lines $x + y = 1, x - y = 1, -x + y = 1, -x - y = 1$. Any two adjacent sides are perpendicular to each other. Also, each line is equidistant from origin. Therefore, figure formed is a square

7 **(a)**

$9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = 0$
$\Rightarrow (3x - 4y + 2)(3x - 4y - 6) = 0$
Hence, distance between lines is $\frac{|6-(-2)|}{5} = 8/5$

8 **(b)**



$G \equiv \left(\frac{1 \times 1 + 2 \times 0}{3}, \frac{2 \times 1 + 2 \times 0}{3}\right)$
$= \left(\frac{1}{3}, \frac{2}{3}\right)$

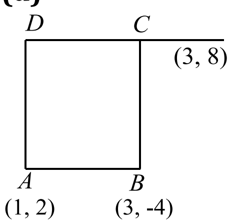
9 **(d)**

Distance of all the points from (0, 0) are 5 units. That means circumcentre of the triangle formed by the given points is (0, 0). If $G \equiv (h, k)$ be the centroid of the triangle, then $3h = 3 + 5(\cos\theta + \sin\theta), 3k = 4 + 5(\sin\theta - \cos\theta)$. If $H(\alpha, \beta)$ be the orthocenter, then
$OG: GH = 1: 2 \Rightarrow \alpha = 3h, \beta = 3k$
$\cos\theta + \sin\theta = \frac{\alpha - 3}{5}, \sin\theta - \cos\theta = \frac{\beta - 4}{5}$
$\Rightarrow \sin\theta = \frac{\alpha + \beta - 7}{10}, \cos\theta = \frac{\alpha - \beta + 1}{10}$
Thus the locus of (α, β) is
$(x + y - 7)^2 + (x - y + 1)^2 = 100$

10 **(b)**

Solving the sides of the rhombus, its vertices are $(0, -n/m), (-n/l, 0), (0, n/m)$ and $(n/l, 0)$. Hence, the area is $\frac{1}{2} \times \frac{2n}{m} \times \frac{2n}{l} = 2$
$\Rightarrow n^2 = lm$. Therefore, l, m, n are in G.P.

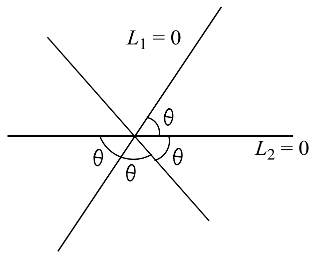
11 **(d)**



$m_{AB} = \frac{-4 - 2}{3 - 1} = -3$

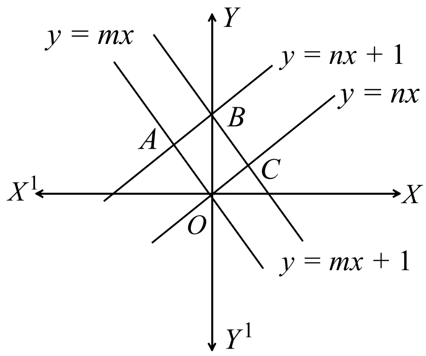
Thus equation of CD is $y - 8 = -3(x - 3)$, i.e., $y + 3x = 17$. Equation of right bisector of AB is
$y + 1 = \frac{1}{3}(x - 2)$
$\Rightarrow 3y = x - 5$
Solving it with line CD, we get
$x = 24/5, y = 1/5$. Thus midpoint of CD is (24/5, 1/5)

12 **(b)**



From the figure, $3\theta = 180 \Rightarrow \theta = 60°$

13 **(d)**



$y = mx$ is a line through (0, 0), $y = mx + 1$ is a line parallel to above line having y-intercept 1
The vertices are $O(0, 0), A(1/(m - 1), m/(m - n))$. Area of parallelogram is given by
$2 \times \text{Ar}(\Delta OAB)$
$= 2 \times \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ \frac{1}{m-n} & \frac{m}{m-n} & 1 \end{vmatrix}$
$= \frac{1}{|m - n|}$

14 **(b)**

If the given lines are concurrent, then
$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$
$\Rightarrow \begin{vmatrix} a & 1-a & 1-a \\ 1 & b-1 & 0 \\ 1 & 0 & c-1 \end{vmatrix} = 0$ (Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$)
$\Rightarrow a(b - 1)(c - 1) - (c - 1)(1 - a) - (b - 1)(1 - a) = 0$
$\Rightarrow \frac{a}{1 - a} + \frac{1}{1 - b} + \frac{1}{1 - c} = 0$
[Dividing by $(1 - a)(1 - b)(1 - c)$]
Adding 1 on both sides, we get

$$\Rightarrow \frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$$

15 **(c)**

The line passing through the intersection of the lines

$ax+2by+3b=0$ and $bx-2ay-3a=0$ is

$ax+2by+3b+\lambda(bx-2ay-3a)=0$

$\Rightarrow (a+b\lambda)x+(2b-2a\lambda)y+3b-3\lambda a=0$ (i)

Line Eq. (i) is parallel to x-axis. Therefore,

$$a+b\lambda=0 \Rightarrow \lambda=-\frac{a}{b}=0$$

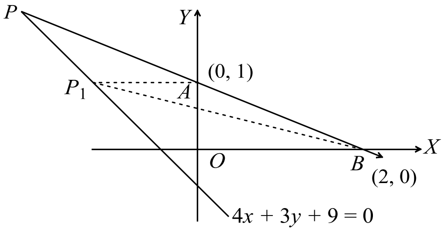
Putting the value of λ in Eq. (i), we get

$$y\left(2b+\frac{2a^2}{b}\right)+3b+\frac{3a^2}{b}=0$$

$$\Rightarrow y=-\frac{3}{2}$$

So, it is $3/2$ units below x-axis

16 **(b)**



Equation of AB is

$$y-1=\frac{0-1}{2-1}x \Rightarrow x+2y-2=0$$

$|PA-PB|\le AB$

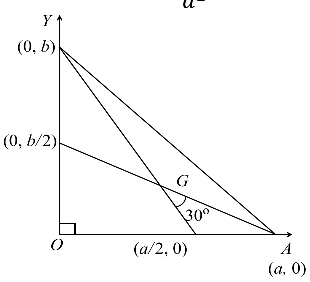
Thus, $|PA-PB|$ is maximum if points A, B and P are collinear

Hence, solving $x+2y-2=0$ and $4x+3y+9=0$, we get point $P=(-84/5, 13/5)$

17 **(a)**

Slope of is $AG=-b/(2a)$. Now,

$$\tan 30^\circ=\frac{\frac{3b}{2a}}{1+\frac{b^2}{a^2}}$$



$$\Rightarrow \frac{1}{\sqrt{3}}=\frac{3ba}{2(a^2+b^2)}$$

$$\Rightarrow \frac{1}{2}ab=\left(\frac{a^2+b^2}{3\sqrt{3}}\right)$$

$=9/3\sqrt{3}=\sqrt{3}$ (Putting $a^2+b^2=9$)

18 **(a)**

We have,

$$3x+5y=2007 \Rightarrow x+\frac{5y}{3}=669$$

Clearly, 3 must divide $5y$ and so $y=3k$, for some $k\in N$

Thus,

$x+5k=669$

$\Rightarrow 5k\le 668$

$$\Rightarrow k\le\frac{668}{5} \Rightarrow k\le 133$$

19 **(c)**

We have,

$x^2y^2-9y^2-6x^2y+54y=0$

$\Rightarrow y^2(x^2-9)-6y(x^2-9)=0$

$\Rightarrow y(y-6)(x-3)(x+3)=0$

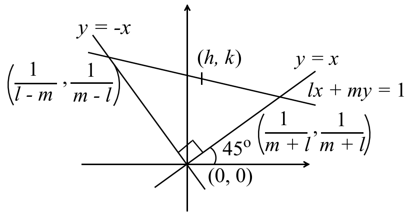
$\Rightarrow y=0, y=6, x=3, x=-3$

So, the given equation represents four straight lines which form a square

20 **(a)**

Coordinates of circumcentre are $l/(l^2-m^2), m/(m^2-l^2)$

Hence,



$h=\frac{l}{l^2-m^2}$ (i)

$k=-\frac{m}{l^2-m^2}$ (ii)

Square and adding (i)and (ii), we get

$$h^2+k^2=\frac{l^2+m^2}{(l^2-m^2)^2}=\frac{1}{(l^2-m^2)^2}$$

(putting $l^2+m^2=1$)

$$\therefore \frac{1}{(l^2-m^2)^2}=(h^2-k^2)^2$$

Therefore, the locus is $x^2+y^2=(x^2-y^2)^2$

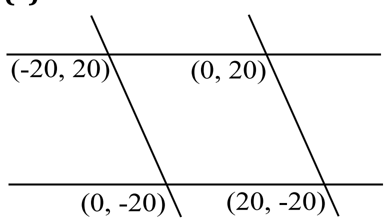
21 **(b)**

If the given lines represents the same line, then the lengths of the perpendicular from the origin to the lines are equal, so that

$$\frac{c}{\sqrt{1+m^2}}=\frac{p}{\sqrt{\cos^2\alpha+\sin^2\alpha}}$$

$$\Rightarrow c=p\sqrt{1+m^2}$$

22 **(c)**



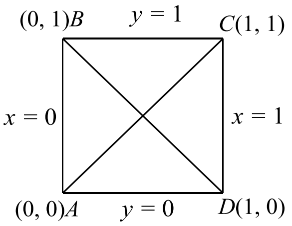
Fro $x\ge 0$, the equation is

$(y+20)(y+2x-20)=0$
For $x \le 0$, the equation is
$(y-20)(y+2x+20)=0$
Hence, the area is $20 \times 40 = 800$ sq. units

23 **(d)**
All values of 'a'

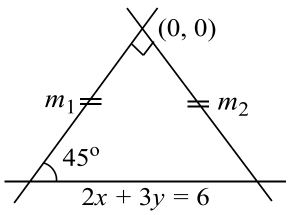
24 **(c)**
The family of lines $(x-2y+3)+\lambda(2x-3y+$ $4=0$ are concurrent at point $P(1, 2)$
If image of point $A(2,3)$ in the above variable line is $B(h,k)$ then $AP = BP$
$\Rightarrow\ (h-1)^2+(k-2)^2=(2-1)^2+(3-2)^2$
Hence, locus of point P is $x^2+y^2-2x-4y+4=0$

25 **(a)**
Coordinates of the vertices of the square are $A(0,0), B(0,1), C(1,1)$ and $D(1,0)$



Now the equation of AC is
$y = x$
And that of BD is
$y-1=-\frac{1}{1}(x-0)$
$\Rightarrow\ x+y=1$

26 **(a)**
$2x+3y=6$
$\tan 45^\circ = \left|\frac{m-\left(\frac{-2}{3}\right)}{1+m\left(\frac{-2}{3}\right)}\right|$

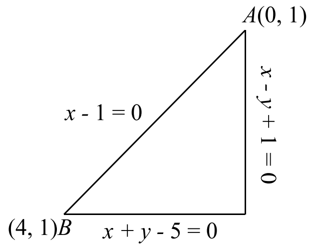


Hence, $m_1=-5,\ m_2=1/5$

27 **(c)**
Clearly, the equation of PQ in the new position is $x=2$

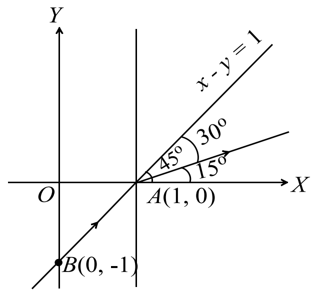
28 **(d)**
$a^3x^2-2hxy+b^3y^2=0$
Let the slope of lines be m_1 and m_2. Then,
$m_1+m_2=\frac{2h}{b^3}, m_1m_2=\frac{a^3}{b^3}$
Given $\ m_2^2=m_1 \ \ \Rightarrow\ m_2^3=\frac{a^3}{b^3}$
$\Rightarrow\ m_2=\frac{a}{b}$
Also $\ m_2^2+m_2=\frac{2h}{b^3}$
$\Rightarrow\ \frac{2h}{b^3}=\frac{a}{b}+\frac{a^2}{b^2}$
$\Rightarrow ab+a^2\frac{2h}{b}$
$\Rightarrow\ 2h=a^2b+ab^2=ab(a+b)$

29 **(a)**



Since the triangle is right angled, so the circumcentre will be the middle point of hypotenuse, i.e., (2,1)

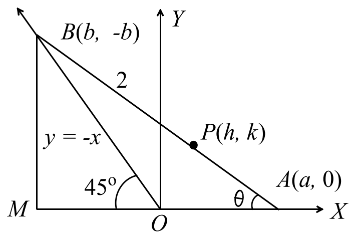
30 **(d)**



From figure refracted ray makes an angle of 15° with positive direction of x-axis and passes through the point (1,0). Its equation is
$(y-0)=\tan(45^\circ-30^\circ)\,(x-1)$
or $\ y=\left(2-\sqrt{3}\right)(x-1)$

31 **(b)**
As altitude from A is fixed and the orthocenter lies on altitude, hence $x+y=3$ is the required locus

32 **(a)**



If $\angle BAO=\theta$, then $BM=2\sin\theta$ and $MO=BM=2\sin\theta$, $MA=2\cos\theta$. Hence, $A=(2\cos\theta-2\sin\theta, 0)$ and $B=(-2\sin\theta, 2\sin\theta)$. since $P(x,y)$ is the midpoint of AB, so
$2x=(2\cos\theta)+(-4\sin\theta)$
or $\ \cos\theta-2\sin\theta=x$
$2y=(2\sin\theta)\ $ or $\ \sin\theta=y$
Eliminating θ, we have
$(x+2y)^2+y^2=1\ $ or $\ x^2+5y^2+4xy-1=0$

33 **(a)**

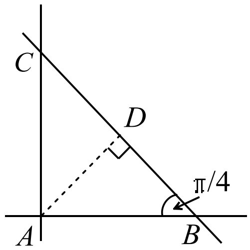
Solving $3x+4y=9, y=mx+1$, we get $x=\frac{5}{3+4m}$. Here, $x$is an integer if $3+4m=1,-1,5,-5$. Hence, $m=-2/4,-4/4,2/4,-8/4$. So, m has two integral values

34 **(a)**

Since $a_1/a_2=b_1/b_2=c_1/c_2$, then $u=0$ and $v=0$ are same straight line. Hence,$u+kv=0$ is also the same straight line

35 **(a)**

Given lines are mutually perpendicular and intersect at $(6/5, 13/5)$



Equations of angle bisectors of the given lines are $x-2y+4=\pm(2x+y-5)$,

i.e, $x+3y=9$ and $3x-y=1$

Side BC will be parallel to these bisectors. Let $AD=a$

$\Rightarrow \; AB=a\sqrt{2}$ and area of ΔABC is

$$\Delta_{ABC}=\frac{1}{2}\times\left(a\sqrt{2}\right)^2=a^2=10$$

$\Rightarrow \; a=\sqrt{10}$

Let equation of BC be $x+3y=\lambda$. Then,

$$\sqrt{10}=\frac{\frac{6}{5}-\frac{39}{5}-\lambda}{\sqrt{10}}\Rightarrow \lambda=-1,19$$

Therefore, equation of BC is $x+3y=-1$ or $x+3y=19$

If equation of BC is $3x-y=\lambda$, then

$$\sqrt{10}=\frac{\frac{18}{5}-\frac{13}{5}-\lambda}{\sqrt{10}}\Rightarrow \lambda=-9,11$$

Hence, equation of BC is $3x-y=-9$ or $3x-y=11$

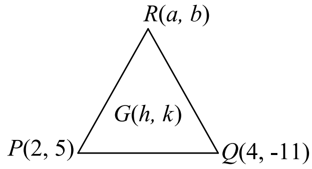
36 **(a)**

The point $(4,5)$ lies on the given line $7x-3y-13=0$. The locus of the point equidistant from the given point and the line is a line perpendicular to $7x-3y-13=0$ at $(4,5)$

37 **(d)**

$R(x,y)$ lies on $9x+7y+4=0$

Hence, $R\left(a\frac{-(4+9a)}{7}\right), a\in R$



$h=\left(\frac{2+4+a}{3}\right)=\frac{6+a}{3}$ (i)

$$k=\frac{5-11-\frac{(4+9a)}{7}}{3}$$

$=\frac{-46-9a}{7\times 3}$ (ii)

From (i) and (ii), we get

$$3h-6=\frac{-(21k-46)}{9}$$

$\Rightarrow \; 27h+21k-54+46=0$

Hence, the locus is $9x+7y-8/3=0$. This line is parallel to N

38 **(a)**

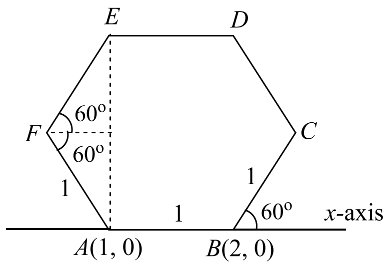
Let $x_1=a, x_2=ar$ and $x_3=ar^2; y_1=b, y_2=br$ and $y_3=br^2$. Now,

$$\frac{y_2-y_1}{x_2-x_1}=\frac{br-b}{ar-a}=\frac{b}{a}$$

And $\frac{y_3-y_2}{x_3-x_2}=\frac{br^2-br}{ar^2-ar}=\frac{b}{a}$

Therefore, slope of PQ is equal to slope of QR. Hence, points P,Q,R are collinear

39 **(c)**



$$\Rightarrow C\equiv(2+1\times\cos 60°, 1\times\sin 60°)=\left(\frac{5}{2},\frac{\sqrt{3}}{2}\right)$$

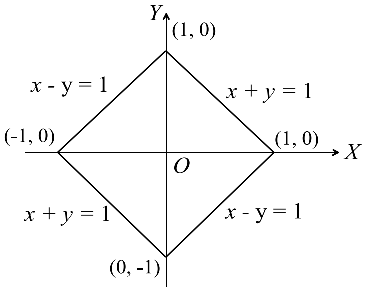
$E\equiv(1, 1\times\sin 60°+1\times\sin 60°)=\left(1,\sqrt{3}\right)$

Therefore, the equation of CE is

$$y-\sqrt{3}=\frac{\sqrt{3}-\frac{\sqrt{3}}{2}}{1-\frac{5}{2}}(x-1)$$

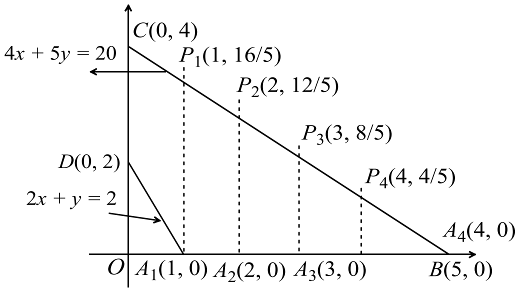
40 **(a)**

Let the two perpendicular lines be taken as the coordinate axes. If (h,k) be any point on the locus, then according to the given condition $|h|+|k|=1$. Hence, the locus of (h,k) is $|x|+|y|=1$. This consists of four line segments enclosing a square as shown in the figure below



41 **(b)**

There are clearly five points

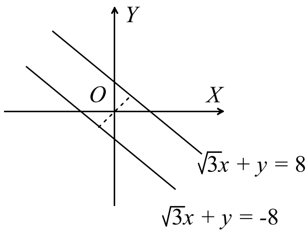


42 **(a)**

Given that slope is $-\sqrt{3}$. Therefore, the line is

$y = -\sqrt{3}x + c$

$\Rightarrow \sqrt{3}x + y = c$



Now, $\left|\frac{c}{2}\right| = 4 \Rightarrow c = \pm 8 \Rightarrow x\sqrt{3} + y = \pm 8$

43 **(c)**

Since the diagonals are perpendicular, so the given quadrilateral is a rhombus. So, the distance between two pairs of parallel sides are equal.

Hence,

$$\left|\frac{c' - c}{\sqrt{a^2 + b^2}}\right| = \left|\frac{c' - c}{\sqrt{a'^2 + b'^2}}\right|$$

$\Rightarrow\ a^2 + b^2 = a'^2 + b'^2$

44 **(b)**

Let a and b be non-zero real numbers.

Therefore, the given equation

$(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$ implies either

$x^2 - 5xy + 6y^2 = 0$

$\Rightarrow (x - 2y)(x - 3y) = 0$

$\Rightarrow x = 2y$ and $x = 3y$

Represent two straight lines passing through origin.

or $ax^2 + by^2 + c = 0$

When $c = 0$ and a and b are of same signs, then

$ax^2 + by^2 + c = 0$

$\Rightarrow x = 0$ and $y = 0$

Which is a point specified as the origin.

When $a = b$ and c is of sign opposite to that of a, then

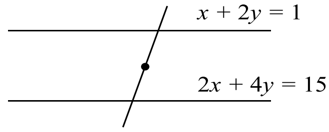
$ax^2 + by^2 + c = 0$ represents a circle.

Hence, the given equation,

$(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$

may represents two straight lines and a circle.

45 **(a)**



Let P be on $x + 2y = 1$. Then,

$$1 + \frac{t}{\sqrt{2}} + 2\left(2 + \frac{t}{\sqrt{2}}\right) = 1$$

or $t = \frac{-4\sqrt{2}}{3}$

Let P be on $2x + 4y = 15$. Then,

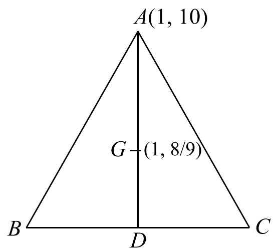
$$2\left(1 + \frac{t}{\sqrt{2}}\right) + 4\left(2 + \frac{t}{\sqrt{2}}\right) = 15$$

or $t = \frac{5\sqrt{2}}{6}$

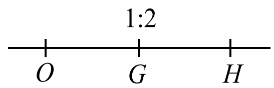
since point lies between the lines and $x = t$, then

$t \in \left(\frac{-4\sqrt{2}}{3}, \frac{5\sqrt{2}}{3}\right)$

46 **(a)**



Circumcentre $O \equiv (-1/3, 2/3)$ and orthocenter $H \equiv (11/3,\ 4/3)$



Therefore, the coordinates of G are $(1, 8/9)$, now, the point A is $(1, 10)$ as G is $(1, 8/9)$. Hence,

$AD{:}DG = 3{:}1$

$$\therefore\ D_x = \frac{3 - 1}{2} = 1, \qquad D_y = \frac{\frac{8}{3} - 10}{2} = -\frac{11}{3}$$

Hence, the coordinates of the midpoint of BC are $(1, -11/3)$

47 **(c)**

The given inequality is equivalent to the following system of inequalities

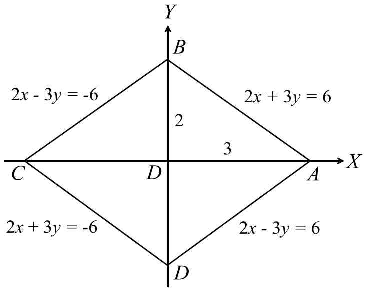
$2x + 3y \leq 6$, when $x \geq 0, y \geq 0$

$2x - 3y \leq 6$, when $x \geq 0, y \leq 0$

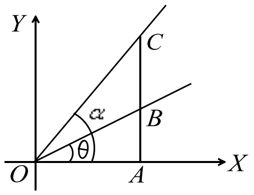
$-2x + 3y \leq 6$, when $x \leq 0, y \leq 0$

$-2x - 3y \leq 6$, when $x \leq 0, y \leq 0$

Which represents a rhombus with sides
$2x \pm 3y = 6$ and $2x \pm 3y = -6$
Length of the diagonals is 6 and 4 units along x- and y-axes. Therefore, the required area is
$1/2 \times 6 \times 4 = 12$ sq. units



48 **(b)**
Given $AB = BC$
$\tan\theta = \frac{AB}{OA} = m_1$
$\tan\alpha = \frac{2AB}{OA} = m_2$
$\frac{m_2}{m_1} = 2 \Rightarrow \frac{m_2 + m_1}{m_2 - m_1} = \frac{2+1}{2-1} = 3$



$\Rightarrow \frac{m_1 + m_2}{\sqrt{(m_1 + m_2)^2 - 4m_1 m_2}} = 3$
$\Rightarrow -\frac{\frac{2h}{b}}{\sqrt{\frac{4h}{b^2} - \frac{4a}{b}}} = 3$
$\Rightarrow \frac{4h^2}{b^2} - \frac{4a}{b} = \frac{4h^2}{9b^2}$
$\Rightarrow \frac{4h^2}{b^2} \times \frac{8}{9} = \frac{4a}{b}$
$\Rightarrow 8h^2 = 9ab$

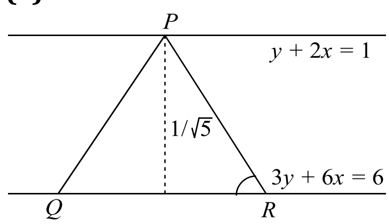
49 **(c)**
$a, 8, b$ are in $H.P$
$\Rightarrow \frac{1}{a} + \frac{1}{b} = 1/4$
$\Rightarrow b = \frac{4a}{a-4}$
$\Rightarrow$ area, $A = \frac{4a^2}{2(a-4)}$
A is minimum at $a = 8$. Hence, minimum value of A is 32 sq. units

50 **(d)**
If the line meets the x-and y-axes at A and B, then $A \equiv (-c/a, 0), B \equiv (0, -c/b)$. The line will pass through the first quadrant if
$-c/a > 0$ and/or $-c/b > 0$
$\Rightarrow ac > 0$ and/or $bc < 0$

51 **(a)**
The distance between $(2/m, 2)$ and $(6/m, 6)$ is less than 5. Hence,
$\left(\frac{2}{m} - \frac{6}{m}\right)^2 + (2-6)^2 < 25$
$\Rightarrow \frac{16}{m^2} < 9$
$\Rightarrow m^2 > \frac{16}{9}$
$\Rightarrow m > \frac{4}{3}$ or $m < \frac{-4}{3}$

52 **(b)**
The equation of the line joining the points $(2, -1)$ and $(5, -3)$ is given by
$y + 1 = \frac{-1+3}{2-5}(x-2)$
or $2x + 3y - 1 = 0$ (i)
since $(x_1, 4)$ and $(-2, y_1)$ lie on $2x + 3y - 1 = 0$, therefore
$2x_1 + 12 - 1 = 0 \quad \Rightarrow x_1 = -\frac{11}{2}$
and $-4 + 3y_1 - 1 = 0 \Rightarrow y_1 = \frac{5}{3}$
Thus (x_1, y_1) satisfies $2x + 6y + 1 = 0$

53 **(a)**



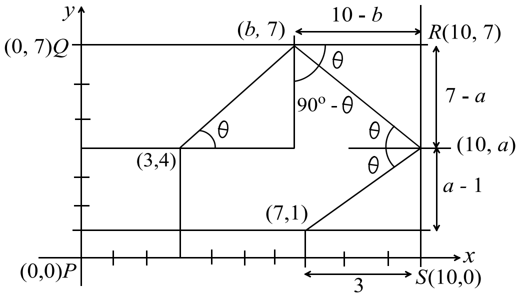
The given lines are $y + 2x = 1$ and $y + 2x = 2$.
The distance between the lines is $(2-1)/\sqrt{5} = 1/\sqrt{5}$
The side length of the triangle is
$(1/\sqrt{5})\text{cosec } 60° = 2/\sqrt{5}$

54 **(a)**
Line perpendicular to $x\sec\theta + y\,cosec\,\theta = a$ is
$x\,\text{cosec}\,\theta - y\sec\theta = \lambda$
This line passes through the point
$(a\cos^3\theta, a\sin^3\theta)$
Then,
$(a\cos^3\theta)\text{cosec}\,\theta - (a\sin^3\theta)\sec\theta = \lambda$
$\Rightarrow \lambda = a\left(\frac{\cos^3\theta}{\sin\theta} - \frac{\sin^3\theta}{\cos\theta}\right)$
$= a\frac{\cos 2\theta}{\cos\theta\sin\theta}$
Hence, the equation of line is $x\cos\theta - y\sin\theta = a\cos 2\theta$

55 **(a)**
$\tan\theta = \frac{a-1}{3} = \frac{7-a}{10-b}$

Also $\tan\theta = \frac{7-4}{b-3} = \frac{3}{b-3}$

Hence, $\frac{3}{b-3} = \frac{a-1}{3} = \frac{7-a}{10-b}$



From 1st two relations

$9 = ab - b - 3a = 3$

$3a + 6 = ab - b$ (i)

From last two

$10a - ab - 10 + b = 21 - 3a$

$13a - ab + b = 31$

or $ab - b = 13a - 31$ (2)

Hence, from (i) and (ii)

$3a + 6 = 13a - 31 \Rightarrow 10a = 37$

$\Rightarrow a = 3.7$

56 **(a)**

Let the line cut the axis at points $A(a, 0)$ and $B(0, b)$. Now given that $(-4, 3)$ divides AB in ratio 5:3. Then,

$-4 = 3a/8$ and $3 = 5b/8$. Therefore, $a = -32/3$ and$b = 24/5$. Then using the intercept from $x/a + y/b = 1$, the equation of line is

$$-\frac{3x}{32} + \frac{5y}{24} = 1$$

or $9x - 20y + 96 = 0$

57 **(a)**

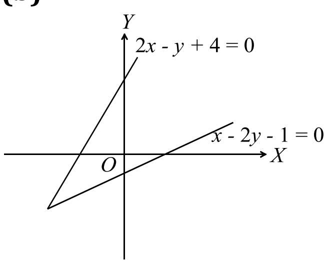
We have

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$$

Hence, the area of triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is same as the area of triangle with vertices $(a_1, b_1), (a_2, b_2), (a_3, b_3)$. Hence, the two triangle are equal in area

58 **(b)**

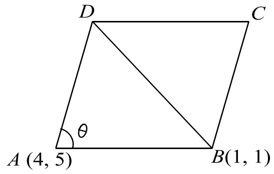


Clearly from the figure, the origin is contained in the acute angle. Writing the equations of the lines as $2x - y + 4 = 0$ and $-x + 2y + 1 = 0$, the required bisector is

$$\frac{2x - y + 4}{\sqrt{5}} = \frac{-x + 2y + 1}{\sqrt{5}}$$

59 **(b)**

From the figure



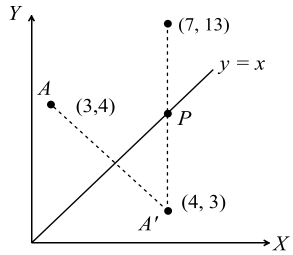
Area of rhombus $= 2 \times$ (area of ΔABD)

$= 2 \times \frac{1}{2} \times 5 \times 5 \sin\theta$

$= 25 \sin\theta$

Hence, maximum area is 25 (when $\sin\theta = 1$)

60 **(c)**



Consider a point A', the image of A in $y = x$. Therefore, the coordinates of A' are (4,3)

or (Notice that A and B lie are the same side with respect top $y = x$). Then $PA = PA'$. Thus, $PA + PB$ is minimum, if $PA' + PB$ is minimum, i.e., if P, A', B are collinear. Now, the equation of AB is

$$y - 3 = \frac{13 - 3}{7 - 4}(x - 4)$$

$\Rightarrow \ 3y - 10x + 31 = 0$

It intersects $y = x$ at $(31/7, 31/7)$, which is the required point P

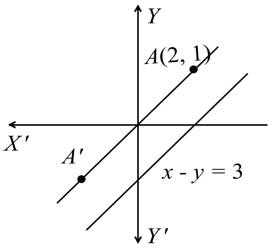
61 **(d)**

Three non-collinear points from a triangle and the line joining the midpoints of any sides is equidistant from all the three vertices

62 **(c)**

Since the point $A(2, 1)$ is translated parallel to $x - y = 3$, therefore AA' has the same slope as that of $x - y = 3$. Therefore, AA' passes through (2, 1) and has the slope of 1. Here

$\tan\theta = 1 \Rightarrow \cos\theta = 1/\sqrt{2}, \sin\theta = 1/\sqrt{2}$



Thus, the equation of AA' is

$$\frac{x-2}{\cos(\pi/4)}=\frac{y-1}{\sin(\pi/4)}$$

Since $AA'=4$, therefore the coordinates of A' are given by

$$\frac{x-2}{\cos(\pi/2)}=\frac{y-1}{\sin(\pi/4)}=-4$$

$\Rightarrow\ x=2-4\cos\frac{\pi}{4}, y=1-4\sin\frac{\pi}{4}$

$\Rightarrow\ x=2-2\sqrt{2}, y=1-2\sqrt{2}$

Hence, the coordinates of A' are $(2-2\sqrt{2}, 1-2\sqrt{2})$

63 **(c)**

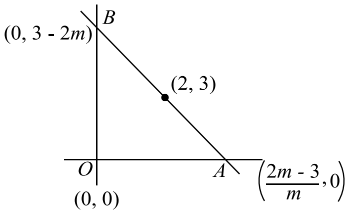
Equating of any line through (2, 3) is

$y-3=m(x-2)$

$\Rightarrow y=mx-2m+3$

From the figure, area of ΔOAB is ± 12. That is,

$$\frac{1}{2}\left(\frac{2m-3}{m}\right)(3-2m)=\pm 12$$



Taking positive sign, we get$(2m+3)^2=0$. This gives one value of m as $-3/2$. Taking negative sign, we get

$4m^2-36m+9=0\ (D>0)$

This is a quadratic in m which gives two values of m. Hence, three straight lines are possible

64 **(c)**

$$\begin{vmatrix}1 & 1 & -2\\ 2 & -1 & 1\\ a & b & -c\end{vmatrix}=0$$

$\Rightarrow a+5b-3c=0$

$\Rightarrow\ -\frac{a}{3}-\frac{5}{3}b+c=0$

Hence, $2ax+3by+c=0$ is concurrent at $2x=-1/3$ and $3y=-5/3$. So, $x=-1/6, y=-5/9$

65 **(a)**

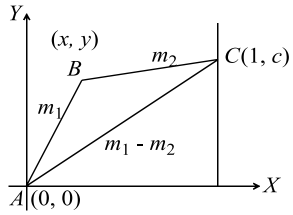
The three lines are concurrent, if

$$\begin{vmatrix}1 & 2 & -9\\ 3 & 5 & -5\\ a & b & -1\end{vmatrix}=0$$

$\Rightarrow\ 35a-22b+1=0$

Which is true if the line $35x-22y+1=0$ passes through (a,b)

66 **(d)**



Let the coordinates of C be $(1,c)$. Then,

$m_2=\frac{c-y}{1-x}$

or $m_2=\frac{c-m_1x}{1-x}$

$\Rightarrow m_2-m_2x=c-m_1x$

$\Rightarrow\ (m_1-m_2)x=c-m_2$

$\Rightarrow\ c=(m_1-m_2)x+m_2$ (i)

Now area of ΔABC is

$$\frac{1}{2}\begin{vmatrix}0 & 0 & 1\\ x & m_1x & 1\\ 1 & c & 1\end{vmatrix}=\frac{1}{2}[cx-m_1x]$$

$=\frac{1}{2}|[((m_1-m_2)x+m_2)x-m_1x]|$

$=\frac{1}{2}|[(m_1-m_2)x^2+m_2x-m_1x]|$

$=\frac{1}{2}(m_1-m_2)(x-x^2)[\because x>x^2 \text{ in } (0,1)]$

Hence, $f(x)=\frac{1}{2}(x-x^2)$

$f(x)_{\max}=\frac{1}{8}$ when $x=1/2$

67 **(b)**

Let $y=mx$ be a line common to the given pairs of lines. Then

$am^2+2m+1=0$ and $m^2+2m+a=0$

$$\Rightarrow \frac{m^2}{2(1-a)}=\frac{m}{a^2-1}=\frac{1}{2(1-a)}$$

$\Rightarrow m^2=1$ and $m=-\frac{a+1}{2}$

$\Rightarrow (a+1)^2=4$

$\Rightarrow a=1$ or -3

But for $a=1$, the two pairs have both the lines common. So $a=-3$ and the slope m of the line common to both the pairs is 1. Now,

$x^2+2xy+ay^2=x^2+2xy-3y^2$

$=(x-y)(x+3y)$

and $ax^2+2xy+y^2=-3x^2+2xy+y^2=-(x-y)(3x+y)$

so the equation of the required lines is

$(x+3y)(3x+y)=0$

$\Rightarrow 3x^2+10y+3y^2=0$

68 **(d)**

Given $O(0,0)$ is the orthocentre. Let $A(h,k)$ be the third vertex, $B(-2,3)$ and $C(5,-1)$ the other two vertices. Then the slope of the line through A and O is k/h, while the lines through B and C has the slope $-4/7$. By the property of the orthocenter, these two lines must be perpendicular, so we have

$\left(\frac{k}{h}\right)\left(-\frac{4}{7}\right) = -1 \Rightarrow k/h = \frac{7}{4}$ (i)

Also, $\frac{5-2+h}{3} + \frac{-1+3+k}{3} = 7$

$\Rightarrow\ h + k = 16$ (ii)

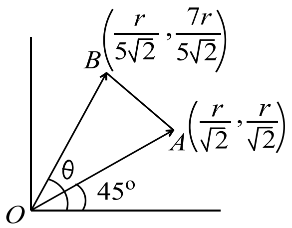
Which is not satisfied by the points given in $(a), (b)$ or (c)

69 **(d)**

$\tan\theta = 7$

$OA = OB = r$

$$\sin\theta = \frac{7}{5\sqrt{2}}, \cos\theta = \frac{1}{5\sqrt{2}}$$

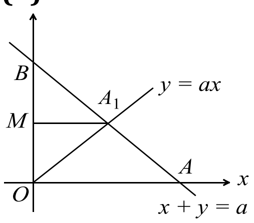


Now, $m_{AB} = -1/2$

70 **(d)**

The point Q is $(-b,-a)$ and the point R is $(-a,-b)$. Therefore, the midpoint of PR is (0, 0)

71 **(d)**



$$B \equiv (o, a), A_1 \equiv \left(\frac{a}{1+a}, \frac{a^2}{1+a}\right)$$

$$\Delta_{OA,B} = \frac{1}{2}(OB)A_1M$$

$$= \frac{1}{2}|a|\left|\frac{a}{1+a}\right| = \frac{1}{2}\frac{a^2}{|1+a|}$$

72 **(b)**

The line passing through (2, 3) and perpendicular to $-y + 3x + 4 = 0$ is

$$\frac{y-3}{x-2} = -\frac{1}{3}$$

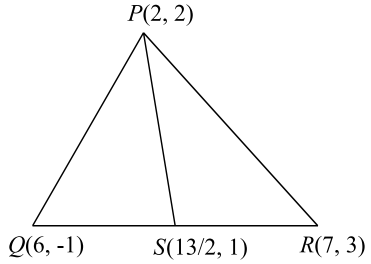
or $3y + x - 11 = 0$

Therefore, foot is $x = -1/10, y = 37/10$

73 **(d)**

S is the midpoint of Q and R. Therefore,

$S = ((7+6)/2, (3-1)/2) = (13/2, 1)$



Now slope of PS is $m = \frac{2-1}{2-13/2} = -\frac{2}{9}$

Then equation of the line passing through $(1,-1)$ and parallel to PS is

$$y + 1 = -\frac{2}{9}(x-1)$$

or $2x + 9y + 7 = 0$

74 **(a)**

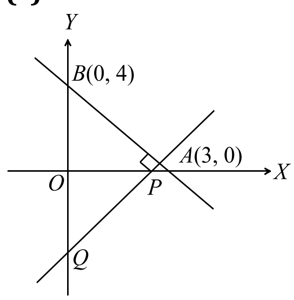
$3a + a^2 - 2 = 0$

$\Rightarrow a^2 + 3a - 2 = 0;$

$$\Rightarrow a = \frac{-3 \pm \sqrt{9+8}}{2} = \frac{-3 \pm \sqrt{17}}{2}$$

$\Rightarrow$ Two values of a

75 **(c)**



Clearly, circumcentre of triangle ABQ will lie on the perpendicular bisector of line AB. Now equation of perpendicular bisector of line AB. Now equation of perpendicular bisector of line $AB$is $3x - 4y + 7/2 = 0$. Hence, locus of circumcentre is $6x - 8y + 7 = 0$

76 **(a)**

No such triangle is possible as the medians through the vertices of a right-angled triangle (other than right angle) cannot be perpendicular to each other

77 **(a)**

Solving the given equations of lines pairwise, we get the vertices of triangle as

$A(-2,2), B(2,-2), C(1,1)$

Then,

$AB = \sqrt{16+16} = 4\sqrt{2}$

$BC = \sqrt{1+9} = \sqrt{10}$

$CA = \sqrt{9+1} = \sqrt{10}$

Hence, the triangle is isosceles

78 **(c)**

We know that $PA + PB \geq AB$ (by triangle inequality)

So, $PA + PB$ is the minimum if $PA + PB = AB$, i.e., A, P, B are collinear

$$\therefore \begin{vmatrix} 3 & -4 & 1 \\ 1 & 2 & 1 \\ 2k-1 & 2k+1 & 1 \end{vmatrix} = 0$$

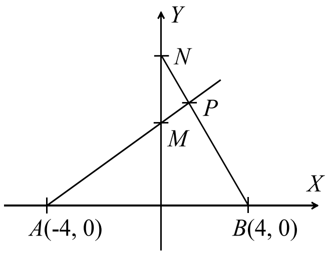
or $3(2-2k-1)+4(1-2k+1)+1(2k+1-$ *4k+2=0*

or $3-6x+8-8k+3-2k=0$

or $14-16k=0$

$\therefore k = \dfrac{7}{8}$

79 **(d)**



Let $M = (0, h)$

$\Rightarrow$ $N = (0, h+4)$. Equation of AM is

$$\frac{x}{-4} + \frac{y}{h} = 1$$

$$\Rightarrow \frac{y}{h} = \frac{4+x}{4} \Rightarrow h = \frac{4y}{4+x}$$

Equation of BN is

$$\frac{x}{4} + \frac{y}{h+4} = 1$$

$$\Rightarrow \frac{y}{h+4} = \frac{4-x}{4}$$

$$\Rightarrow h+4 = \frac{4y}{4-x}$$

$$\Rightarrow h = \frac{4y-16+4x}{4-x}$$

$$\Rightarrow \frac{4(y-4+x)}{4-x} = \frac{4y}{4+x} \text{ (eliminating } h)$$

$\Rightarrow$ $2xy - 16 + x^2 = 0$, which is a required locus

80 **(b)**

Being a pair of lines, $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

This gives $m = 4$. Now find angle between lines

81 **(d)**

Since the product of the slope of the four lines represented by the given equation is 1 and a pair of lines represents the bisectors of the angles between the other two, the product of the slopes of each pair is -1

So let the equation of one pair be $ax^2 + 2hxy - ay^2 = 0$. Then the equation of its bisectors is

$$\frac{x^2-y^2}{2a} = \frac{xy}{h}$$

By hypothesis,

$$x^4 + x^3y + cx^2y^2 - xy^3 + y^4 = (ax^2+2hxy+ay^2)(hx^2 - 2axy - hy^2)$$

$$= ah(x^4+y^4) + 2(h^2-a^2)(x^3y - xy^3) - 6ahx^2y^2$$

82 **(d)**

Given, lines are $(1+p)x - py + p(1+p) = 0$ …(i)

and $(1+q)x - qy + q(1+q) = 0$….(ii)

and $y = 0$

on solving Eqs. (i) and (ii), we get

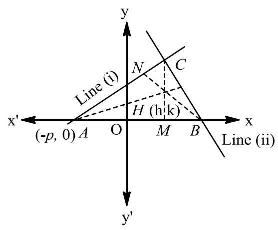
$C\{pq, (1+p)(1+q)\}$

$\therefore$ Equation of altitude CM passing through C and perpendicular to AB is

$x = pq$ …(iii)

$\because$ Slope of line (ii)is $\left(\dfrac{1+q}{q}\right)$

$\because$ Slope of altitude BN (as shown in figure)is $\dfrac{-q}{1+q}$



$\therefore$ Equation of BN is $y - 0 = \dfrac{-q}{1+q}(x+p)$

$\Rightarrow$ $y = \dfrac{-q}{(1+q)}(x+p)$ … (iv)

Let orthocentre of triangle be $H(h,k)$, which is the point of intersection of Eqs. (iii) and (iv).

$\therefore$ On solving Eqs. (iii) and (iv), we get

$x = pq$ and $y = -pq$

$\Rightarrow h = pq$ and $k = -pq$

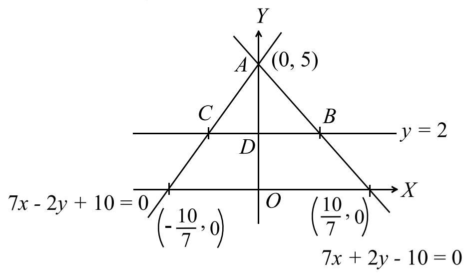
$\therefore$ $h + k = 0$

$\therefore$ Locus of $H(h,k)$ is $x + y = 0$.

83 **(c)**

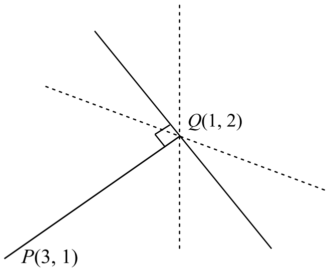
We have, $B \equiv \left(\frac{6}{7}, 2\right), C \equiv \left(-\frac{6}{7}, 2\right)$

$\Rightarrow$ $BC = \dfrac{12}{7}, AD = 3$



$\therefore$ $\Delta_{ABC} = \dfrac{1}{2} \times \dfrac{12}{7} 3 = \dfrac{18}{7}$ sq. units

84 **(a)**



Obviously line through Q is at greatest distance from point P when it is perpendicular to PQ. Now slope of line PQ is $m_{PQ} = -1/2$. Then slope of perpendicular line passing through Q is

$y - 2 = 2(x - 1)$

or $2x - y = 0$

85 **(c)**

The lines by which triangle is formed are $x = 0, y = 0$ and $x + y = 1$. Clearly, it is right triangle and we know that in a right angled triangle orthocenter coincides with the vertex at which right angle is formed. Therefore, orthocentre is (0,0)

86 **(a)**

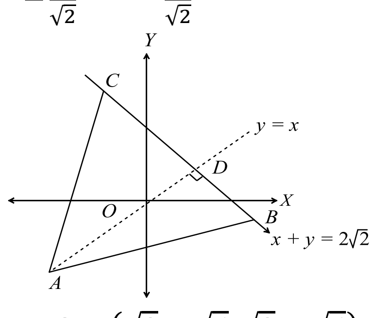
Let the vertices 'B' and 'C' lie on the given line. Then,

$OD = \frac{2\sqrt{2}}{\sqrt{2}} = 2$. Equation of OD is

$y = x \Rightarrow x = y = \sqrt{2}$ (for point D)

Also, $BD = OD \times \tan 60° = 2\sqrt{3}$ for the coordinates of B and C. Using parametric equation of line, we get

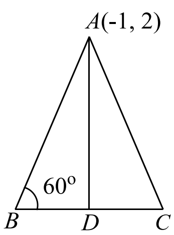
$$\frac{x - \sqrt{2}}{-\frac{1}{\sqrt{2}}} = \frac{y - \sqrt{2}}{\frac{1}{\sqrt{2}}} = \pm 2\sqrt{3}$$



$\Rightarrow\ C \equiv \left(\sqrt{2} + \sqrt{6}, \sqrt{2} - \sqrt{6}\right)$

and $B \equiv (\sqrt{2} - \sqrt{6}, \sqrt{2} + \sqrt{6})$

87 **(a)**



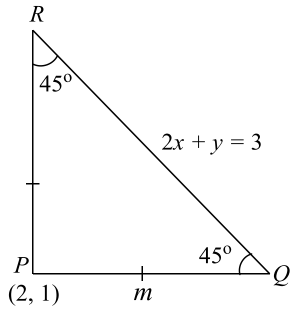
$$AD = \left|\frac{-2 - 2 - 1}{\sqrt{(2)^2 + (-1)^2}}\right| = \left|\frac{-5}{\sqrt{5}}\right| = \sqrt{5}$$

$$\because\ \tan 60° = \frac{AD}{BD} \Rightarrow \sqrt{3} = \frac{\sqrt{5}}{BD}$$

$$\Rightarrow\ BD = \sqrt{\frac{5}{3}}$$

$$\therefore\ BC = 2BD = 2\sqrt{\frac{5}{3}} = \sqrt{\frac{20}{3}}$$

88 **(b)**



Let m be the slope of PQ. Then,

$$\tan 45° = \left|\frac{m - (-2)}{1 + m(-2)}\right| = \left|\frac{m + 2}{1 - 2m}\right|$$

$\Rightarrow\ m + 2 = 1 - 2m$ or $-1 + 2m = m + 2$

$\Rightarrow m = -1/3$ or $m = 3$

Hence, equation of PQ is

$$y - 1 = -\frac{1}{3}(x - 2)$$

Or $x + 3y - 5 = 0$

And equation of PR is

$3x - y - 5 = 0$

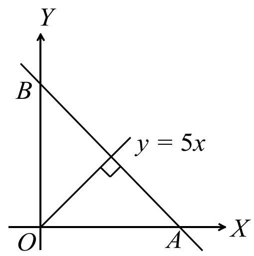
Hence, combined equation of PQ and PR is

$(x + 3y - 5)(3x - y - 5) = 0$

$\Rightarrow 3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$

89 **(a)**

Let the equation of line be $x/a + y/b = 1$



AB is perpendicular to $y = 5x$. Hence,

$-b/a \times 5 = -1 \ \Rightarrow\ 5b = a$

Area of $\Delta_{OAB} = \frac{1}{2}|ab|$

$$\Rightarrow\ 10 = \frac{1}{2}|5b^2|$$

$\Rightarrow\ b^2 = 4 \Rightarrow b = \pm 2, a = \pm 10$

The line can be $x/10 + y/2 = 1$ or $x/10 + y/2 = -1$

90 **(d)**

$$\frac{a}{\sqrt{bc}} - 2 = \sqrt{\frac{b}{c}} + \sqrt{\frac{c}{b}}$$

$\Rightarrow a = b + c + 2\sqrt{bc}$

$\Rightarrow a = (\sqrt{b} + \sqrt{c})^2$

$\Rightarrow \; (\sqrt{a} - \sqrt{b} - \sqrt{c})(\sqrt{a} + \sqrt{b} + \sqrt{c}) = 0$

$\Rightarrow \; \sqrt{a} - \sqrt{b} - \sqrt{c} = 0$

Since $\sqrt{a} + \sqrt{b} + \sqrt{c} \neq 0 \quad (\because a, b, c > 0)$

Comparing with $\sqrt{a}x + \sqrt{b}y = \sqrt{c} = 0$, we have $x = -1, y = 1$

91 **(c)**

We have, $6x^2 - xy - 12y^2 = 0$ (i)

$\Rightarrow \; (2x - 3y)(3x + 4y) = 0$

and $15x^2 + 14xy - 8y^2 = 0$

$\Rightarrow (5x - 2y)(3x + 4y) = 0 \quad$ (ii)

Equation of the lie common to (i) and (ii) is

$3x + 4y = 0 \quad$ (iii)

Equation of any line parallel to (iii) is

$3x + 4y = k$

Since its distance from (iii) is 7, so

$$\left|\frac{k}{\sqrt{3^2 + 4^2}}\right| = 7 \;\Rightarrow\; k = \pm 35$$

92 **(c)**

We have to find locus of the point (h, k) whose image in the line $2x - y - 1 = 0$ lies on the line $y = x$. Now, image of (h, k) in the line $2x - y - 1 = 0$ is given by

$$\frac{x_2 - h}{2} = \frac{y_2 - k}{-1} = \frac{2(2h - k - 1)}{5}$$

$$\Rightarrow x_2 = \frac{-3h + 4k + 4}{5}$$

and $y_2 = \frac{4h + 3k - 2}{5}$

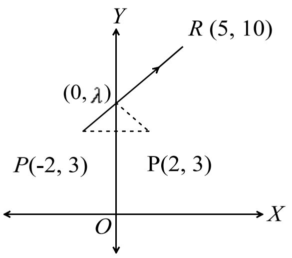
This point lies on $y = x$. Then,

$$\frac{-3h + 4k + 4}{5} = \frac{4h + 3k - 2}{5} \Rightarrow 7h - k = 6$$

93 **(c)**

If P_1 be the reflection of P in y-axis, the

$P_1 = (-2, 3)$



Equation of line P_1R is

$$(y - 3) = \frac{10 - 3}{5 + 2}(x + 2)$$

$\Rightarrow \; y = x + 5$

It meets y-axis at $(0, 5) \Rightarrow Q \equiv (0,5)$

94 **(d)**

The given equation is-b

$(a - b)^2(x^2 + y^2) - 2abx = 0 \quad$ (i)

The origin is shifted to$(ab/(a - b), 0)$. Any point (x, y) on the curve (i) must be replaced with new point (X, Y) with reference to new axes, such that $x = X + \frac{ab}{a-b}$ and $y = Y + 0$

Substituting these in (i), we get

$$(a - b)\left[\left(X + \frac{ab}{a - b}\right)^2 + Y^2\right] - 2ab\left[X + \frac{ab}{a - b}\right] = 0$$

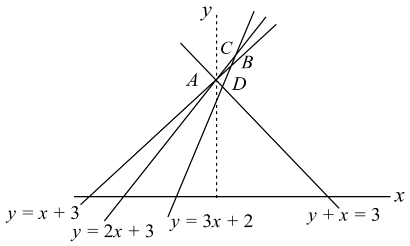
$$\Rightarrow (a - b)\left[X^2 + \frac{a^2b^2}{(a - b)^2} + Y^2 + \frac{2abX}{a - b}\right] - 2abX - \frac{2a^2b^2}{a - b} = 0$$

$$\Rightarrow \; (a - b)(X^2 + Y^2) = \frac{a^2b^2}{a - b}$$

$\Rightarrow (a - b)^2(X^2 + Y^2) = a^2b^2$

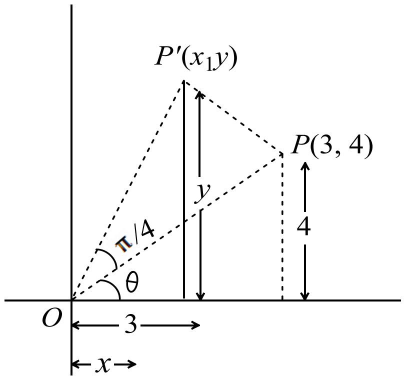
95 **(c)**

A rough sketch of the lines is given below. There are three triangles namely ABC, ABD, ACD



96 **(c)**

Reflection about the line$y = x$, changes the point (4, 1) to (1, 4). On translation of (1,4) through a distance of 2 units along $+ve$ direction of x-axis the point becomes (1+2, 4), i.e., (3, 4)



On rotation about origin through an angle$\pi/4$, the point P takes the position P' such that$OP = OP'$. Also $OP = 5 = OP'$ and $\cos\theta = 3/5, \sin\theta = 4/5$. Now,

$$x = OP' \cos\left(\frac{\pi}{4} + \theta\right)$$
$$= 5\left(\cos\frac{\pi}{4}\cos\theta - \sin\theta\right)$$
$$= 5\left(\frac{3}{5\sqrt{2}} - \frac{4}{5\sqrt{2}}\right)$$
$$= -\frac{1}{\sqrt{2}}$$
$$y = OP' \sin\left(\frac{\pi}{4} + \theta\right)$$
$$= 5\left(\sin\frac{\pi}{4}\cos\theta + \cos\frac{\pi}{4}\sin\theta\right)$$
$$= 5\frac{3}{5\sqrt{2}} + \frac{4}{5\sqrt{2}} = \frac{7}{\sqrt{2}}$$
$$\therefore\ P' \equiv \left(\frac{-1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$$

97 **(b)**

Suppose we rotate the coordinate axes in the anticlockwise direction through an angle α. The equation of the line L with respect to old axes is

$$\frac{x}{a} + \frac{y}{b} = 1$$

In this equation replacing x by $x\cos\alpha - y\sin\alpha$ and y by $x\sin\alpha + y\cos\alpha$, the equation of the line with respect to new axes is

$$\frac{x\cos\alpha - y\sin\alpha}{a} + \frac{x\sin\alpha + y\cos\alpha}{b} = 1$$
$$\Rightarrow x\left(\frac{\cos\alpha}{a} + \frac{\sin\alpha}{b}\right) + y\left(\frac{\cos\alpha}{b} - \frac{\sin\alpha}{a}\right) = 1 \quad \text{(i)}$$

The intercepts made by (i) on the coordinates axes are given as p and q

Therefore, $\frac{1}{p} = \frac{\cos\alpha}{a} + \frac{\sin\alpha}{b}$

and $\frac{1}{q} = \frac{\cos\alpha}{b} - \frac{\sin\alpha}{a}$

Squaring and adding, we get

$$\frac{1}{p^2} + \frac{1}{q^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

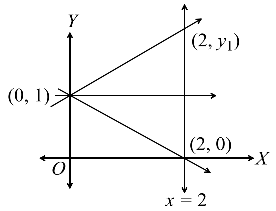
98 **(b)**

(2, 0) is midpoint of $B(0,0)$ and C, then C has coordinates (4,0). Also, A has coordinates $(0 + 2\cos\pi/3)$,$0 + 2\sin\pi/3) \equiv (1, \sqrt{3})$. Then centroid is $(5/3, 1/\sqrt{3})$

99 **(d)**

Orthocentre of triangle BCH is the vertex $A(-1, 0)$

100 **(b)**



From the figure,

$2^2 + (y_1 - 1)^2 = y_1^2$

$4 + y_1^2 + 1 - 2y_1 = y_1^2$

$5 = 2y_1$ or $y_1 = 5/2$

Equation of the line from $(2, 5/2)$ to the given base is

$y - 5/2 = 2(x - 2)$

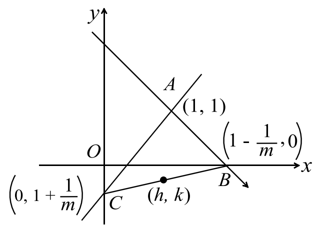
or $2y - 5 = 4(x - 2)$

at $y = 1, -3/4 = x - 2$ or $x = 5/4$

101 **(a)**

Equating of line AB is $y - 1 = m(x - 1)$

$\Rightarrow$ Equating of line AC is $y - 1 = -\frac{1}{m}(x - 1)$



$$2h = 1 - \frac{1}{m}$$
$$2k = 1 + \frac{1}{m}$$

Eliminating m we have locus $x + y = 1$

102 **(b)**

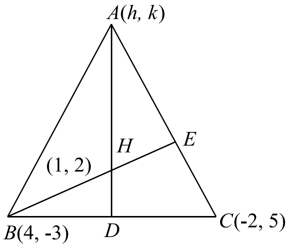
$A \equiv (\alpha, 2\alpha + 3), BC = 1$ unit. Equation of BC is $y - 3 = 0$

Distance of A from BC is $p \Rightarrow |2\alpha + 3 - 3|$

Area of $\Delta ABC = \Delta = |\alpha|; 5 \le \Delta < 6 \Rightarrow 5 \le |\alpha| < 6$

103 **(b)**

Let the third vertex be (h, k)



Now slope of AD is $(k-2)/(h-1)$ slope of BC is $(5+3)/(-2-4) = -4/3$, slope of BE is $(-3-2)/(4-1) = 5/3$ and slope of AC is $(k-5)/(h+2)$. Since $AD \perp BC$ so

$$\frac{k-2}{h-1} \times \frac{-4}{3} = -1$$

$\Rightarrow \; 3h - 4k + 5 = 0 \quad$ (i)

Again since $BE \perp AC$, so

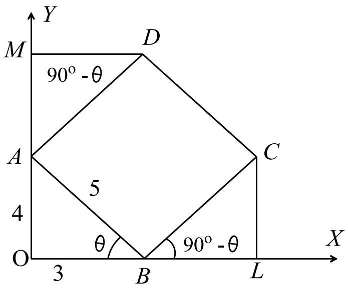
$$-\frac{5}{3} \times \frac{k-5}{h+2} = -1$$

$\Rightarrow \; 3h - 5k + 31 = 0 \quad$ (ii)

On solving (i) and (ii) we get $h = 33, k = 26$.

Hence, the third vertex is (33, 26)

104 **(b)**



The coordinates of A are (0,4) and those of B are (3,0)

$BC = AB = \sqrt{3^2 + 4^2} = 5$

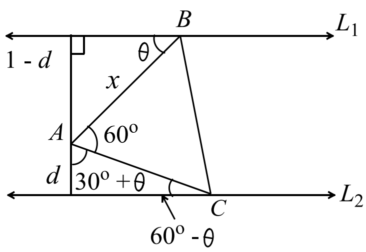
$\Rightarrow \; BL = BC \sin\theta$ and $CL = BC \cos\theta$

$\Rightarrow \; BL = 5 \times \dfrac{4}{5} = 4$ and $CL = 5 \times \dfrac{3}{5} = 3$

Similarly, $MD = 4$ and $AM = 3$. So, the coordinates of C are $(OB + BL, CL) = (7,3)$ and those of D are $(MD, OA + AM) = (4,7)$

The coordinates of the vertex farthest from the origin is (4, 7)

105 **(b)**



From the figure,

$x \cos(\theta + 30°) = d \quad$ (i)

$x \sin\theta = 1 - d \qquad$ (ii)

Dividing (i) by (ii), we have

$$\sqrt{3}\cot\theta = \frac{1+d}{1-d}$$

Squaring Eq. (ii) and putting the value of $\cot\theta$, we have

$$x^2 = \frac{1}{3}(4d^2 - 4d + 4)$$

$$\Rightarrow x = 2\sqrt{\frac{d^2 - d + 1}{3}}$$

106 **(d)**

If we reflect $y = |x-2|$ in y-axis, it will becomes $y = |-x-2| = |x+2|$. The reflected lines are $y = x+2, y = -x-2$. Their combined equation is

$(y - x - 2)(y + x + 2) = 0$

$\Rightarrow \; y^2 - (x-2)^2 = 0$

$\Rightarrow y^2 - x^2 - 4x - 4 = 0$

107 **(c)**

Let incentre be/Then/(2, 1)

$\Rightarrow \; IA = \sqrt{2^2 + 2^2} = 2\sqrt{2}$

Also, $\; AI = r \operatorname{cosec}\frac{1}{2}$

$$\angle BIC = \frac{\pi}{2} + \frac{A}{2}$$

$$\Rightarrow \; \tan\left(\frac{\pi}{2} + \frac{A}{2}\right) = \frac{1-2}{1+2}$$

$$= \frac{1}{3}\cot\frac{1}{2} = 1$$

So, $r = \frac{AI}{\operatorname{cosec}\frac{A}{2}} = \frac{2\sqrt{2}}{\sqrt{1+\frac{1}{9}}}$

$$\frac{2\sqrt{2} \times 3}{\sqrt{10}} = \frac{6}{\sqrt{5}}$$

108 **(d)**

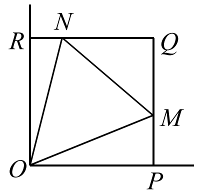
Angle bisector will make the angles $(\theta_1 + \theta_2)/2$ and $(\pi/2 + (\theta_1 + \theta_2)/2)$ with the x-axis. Hence, their equations are

$$\frac{x - x_1}{\cos\left(\frac{\theta_1 + \theta_2}{2}\right)} = \frac{y - y_1}{\sin\left(\frac{\theta_1 + \theta_2}{2}\right)}$$

or $\frac{x - x_1}{-\sin\left(\frac{\theta_1 + \theta_2}{2}\right)} = \frac{y - y_1}{\cos\left(\frac{\theta_1 + \theta_2}{2}\right)}$

109 **(b)**

We can assume that OP and OR are x-axis and y-axis, respectively. Let $OP = a$. Then area of square $OPQR$ is a^2



Coordinates of M and N are $(a, a/2)$ and $(a/2, a)$ respectively

$\therefore \text{ar}(\Delta OMN) = \frac{1}{2}\begin{vmatrix} a & a/2 \\ a/2 & a \end{vmatrix} = \frac{3a^2}{8}$

$\therefore \frac{8}{3} = \frac{\lambda}{6}$

$\lambda = 16$

110 **(d)**

Arranging the lines in descending order of slope, we have

$m_1 = 5, m_2 = 3$ and $m_3 = -1$

$\therefore \tan A = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{2}{1+15} = \frac{1}{8}$

$\tan B = \frac{m_2 - m_3}{1 + m_2 m_3} = \frac{3+1}{1-3} = -2$

$\tan C = \frac{m_3 - m_1}{1 + m_3 m_1} = \frac{-1-5}{1-5} = \frac{3}{2}$

$\sum \tan^2 A = \frac{1}{64} + 4 + \frac{9}{4} = \frac{1 + 256 + 144}{64}$

$= \frac{401}{64}$

$\Rightarrow p + q = 465$

111 **(b)**

$OM_r = OA_r + \frac{OA_{r+1} - OA_r}{2}$

$= \frac{OA_r + OA_{r+1}}{2}$

$= \frac{1}{2}\{OA_1 \times k^{r-1} + OA_1 \times k^r\}$

$= \frac{OA_1}{2}(1+k)k^{r-1}$

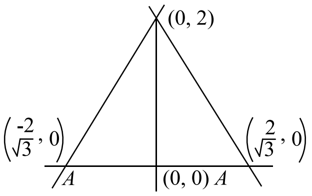
$\therefore \sum_{r=1}^{\infty} OM_r = \frac{OA_1}{2}(1+k)\sum_{r=1}^{\infty} k^{r-1}$

$= \frac{OA_1}{2}(1+k) \times \frac{1}{1-k}$

$= \frac{OA_1}{2} \times \frac{1 + \frac{OA_2}{OA_1}}{1 - \frac{OA_2}{OA_1}}$

$= \frac{OA_1(OA_1 + OA_2)}{2(OA_1 - OA_2)}$

112 **(b)**



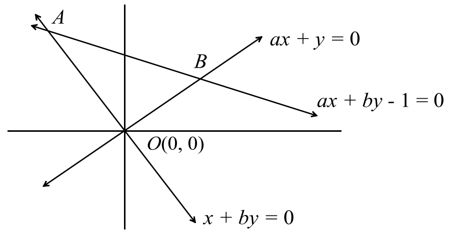
From the figure

$y + \sqrt{3}x = 2$ for $x > 0$

$y - \sqrt{3}x = 2$ for $x < 0$

113 **(b)**

Lines $ax + y = 0$ and $x + by = 0$ intersect at $O(0,0)$



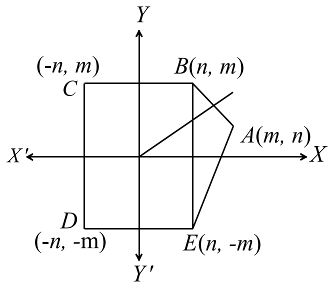
Hence, if AB subtebds right angle at $O(0,0)$, then $ax + y = 0$ and $x + by = 0$ are perpendicular to each other

So,

$(-a)\left(-\frac{1}{b}\right) = -1$

$\Rightarrow a + b = 0$

114 **(b)**



Area of rectangle $BCDE$ is $4mn$. Area of ΔABE ius $\frac{2m(m-n)}{2} = m^2 - mn$

Therefore, the area of pentagon is $4mn + m^2 - mn = m^2 + 3mn$

115 **(c)**

The given equations are

$a^2x^2 + 2h(a+b)xy + b^2y^2 = 0$ (i)

and $ax^2 + 2hxy + by^2 = 0$ (ii)

The equation of the bisectors of the angles between the lines represented by (i) is

$\frac{x^2 - y^2}{a^2 - b^2} = \frac{xy}{h(a+b)}$

or $\frac{x^2 - y^2}{a-b} = \frac{xy}{h}$

which is same as equation of the bisectors of angles between the line pair (ii). Thus, two lines pairs are equally inclined to each other

116 **(d)**

The combined equation of bisectors of angles between the lines of the first pair is

$\frac{x^2 - y^2}{2-1} = \frac{xy}{9}$

As these equations are the same, the two pairs are equally inclined to each other

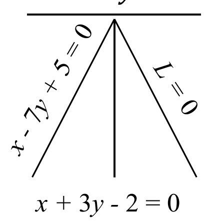
117 **(a)**

Since $xy > 0$, P lies either in the first quadrant or in the third quadrant. The inequality $x + y < 1$ represents all points below the line $x + y = 1$so that $xy > 0$ and $x + y < 1$ imply that P lies either inside the triangle OAB or in third quadrant

118 **(c)**

Family of line through the given lines is

$L \equiv x - 7y + 5 + \lambda(x + 3y - 2) = 0$ (i)



For line $L = 0$ in the diagram, distance of any point say $(2, 0)$ on the line $x + 3y - 2 = 0$ from the line $x - 7y + 5 = 0$ and the line $L = 0$ must be same

$\Rightarrow \left|\frac{2+5}{\sqrt{50}}\right| = \left|\frac{2 + 2\lambda + 5 - 2\lambda}{\sqrt{(1+\lambda)^2 + (3\lambda - 7)^2}}\right|$

$\Rightarrow 10\lambda^2 - 40\lambda = 0$

$\Rightarrow \lambda = 4$ or 0

Hence, $L = 0, \lambda = 4$

$\Rightarrow$ Required line is $5x + 5y - 3 = 0$

119 **(b)**

The point B is (2,1). The image of $A(1, 2)$ in the line $x - 2y + 1 = 0$ is given by

$\frac{x-1}{-1} = \frac{y-2}{-2} = \frac{4}{5}$

Hence, coordinates of the point are (9/5, 2/5)

Since the point lies on BC, therefore the equation of BC is $3x - y - 5 = 0$. Hence, $a + b = 2$

120 **(a)**

Acute angle between the lines $x^2 + 4xy + y^2 = 0$ is $\tan^{-}(2\sqrt{4-1})/(1+1) = \tan^{-1}\pi/3$. Angle bisector of $x^2 + 4xy + y^2 = 0$ are given by

$\frac{x^2 - y^2}{1 - 1} = \frac{xy}{2}$

$x^2 - y^2 = 0 \Rightarrow x = \pm y$

As $x + y = 0$ is perpendicular to $x - y = 4$, the given triangle is isosceles with vertical angle equal to $\pi/3$ and hence it is equilateral

121 **(d)**

We have$P = (1, 0), Q = (-1, 0), R = (2, 0)$. Let $S = (x, y)$. Now given that $SQ^2 = SR^2 = 2SP^2$. Hence,

$(x+1)^2 + y^2 + (x-2)^2 + y^2 = 2[(x-1)^2 + y^2]$

$\Rightarrow 2x^2 + 2y^2 - 2x + 5 = 2x^2 + 2y^2 - 4x + 2$

$\Rightarrow 2x + 3 = 0$

$\Rightarrow x = -3/2$

Which is a straight line parallel to y-axis

122 **(d)**

Let (h, k) be the point on the locus. Then by the given conditions,

$(h - a_1)^2 + (k - b_1)^2 = (h - a_2)^2 + (k - b_2)^2$

$\Rightarrow 2h(a_1 - a_2) + 2k(b_1 - b_2) + a_2^2 + b_2^2 - a_1^2 - b_1^2 = 0$

$\Rightarrow h(a_1 - a_2) + k(b_1 - b_2) + \frac{1}{2}(a_2^2 + b_2^2 - a_1^2 -$ *b12=0* (i)

Also, since (h, k) lies on the given locus, therefore

$(a_1 - a_2)h + (b_1 - b_2)k + c = 0$ (ii)

Comparing Eqs. (i) and (ii), we get

$c = \frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$

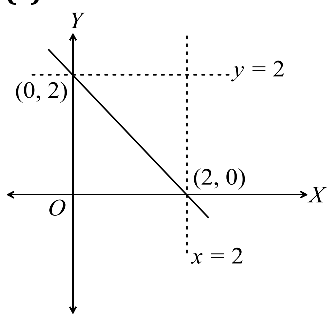
123 **(d)**

Clearly, A will remain as (0, 0); 'f_1' will make B as (0,4); 'f_2' will make it (12, 4) and 'f_3' will make it (4,8); 'f_1' will make 'C' as (2, 4); 'f_2' will make it (14,4); 'f_3' will make it (5,9). Finally 'f_1' will make 'D' as (2,0) 'f_2' will make ixt (2, 0) 'f_3' will make it (1, 1). So we finally get $A \equiv (0, 0), B \equiv (4, 8), C \equiv (5, 9), D \equiv (1, 1)$. Hence,

$m_{AB} = \frac{8}{4}, m_{BC} = \frac{9-8}{5-4} = 1, m_{CD} = \frac{9-1}{5-1} = \frac{8}{4}, m_{AD} = 1, m_{AC} = \frac{9}{5}, m_{BD} = \frac{8-1}{4-1} = 7/3$

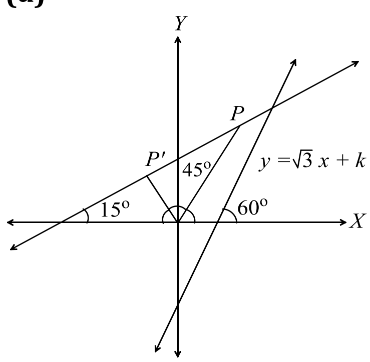
Hence, the final figure will be a parallelogram

124 **(c)**



Above figure represents the given isosceles triangle. Clearly the equation of other equal side is $y = 2$

125 **(d)**



Angle between both the lines is 45°. Hence,

$OP = OP'\sqrt{2} = (5/\sqrt{2}) \times \sqrt{2} = 5$

126 **(c)**

$ax + by = 1$ will be one of the bisector of the given line. Equation of bisectors of the given lines are

$$\frac{3x+4y-5}{5}=\pm\left(\frac{5x-12y-10}{13}\right)$$

$\Rightarrow\ 64x-8y=115$

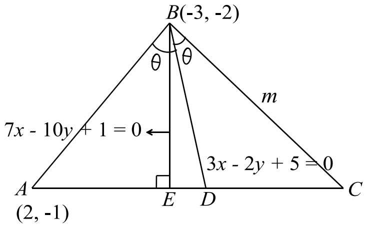
or $\ 14x+112y=15$

$\Rightarrow\ a=\frac{64}{115}, b=-\frac{8}{115}$

or $\ a=\frac{14}{15}, b=\frac{12}{115}$

127 **(b)**

BD and BE intersect at B. Coordinates of B are $(-3,-2)$



$m_{AB}=\frac{1}{5}$

$$\Rightarrow \tan\theta=\left|\frac{\frac{3}{2}-\frac{1}{5}}{1+\frac{3}{10}}\right|=\left|\frac{\frac{3}{2}-m}{1+\frac{3m}{2}}\right|$$

$$\Rightarrow 1=\left|\frac{3-2m}{2+3m}\right|$$

$$\Rightarrow\ \pm1=\frac{3-2m}{2+3m}$$

$\Rightarrow m=1/5$ (rejected) or -5

Equation of BC is

$y+2=-5(x+3)$

$\Rightarrow 5x+y+17=0$

Alternative Solution:

Take image of $(2,-1)$ in the line BD to get a point on BC

128 **(a)**

Extremities of the given diagonal are (4,0) and (0,6). Hence, slope of this diagonal is $-3/2$ and slope of other diagonal is $2/3$. The equation of the other diagonal is

$$\frac{x-2}{\frac{3}{\sqrt{13}}}=\frac{y-3}{\frac{2}{\sqrt{13}}}=r$$

For the extremities of the diagonals, $r=\pm\sqrt{13}$. Hence,

$x-2=\pm3, y-3=\pm2$

$x=5,-1$ and $y=5,1$

Therefore, the extremities of the diagonal are (5,5) and $(-1,1)$

129 **(b)**

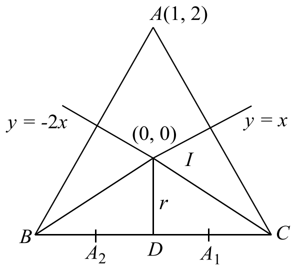
Let the coordinates of the third vertex be $(2a,t)$. Now, $AC=BC$. Hence,

$$t=\sqrt{4a^2+(a-t)^2}\ \Rightarrow t=\frac{5a}{2}$$

So the coordinates of third vertex C are $(2a, 5a/2)$. Therefore, area of the triangle is

$$\pm\frac{1}{2}\begin{vmatrix}2a & \frac{5a}{2} & 1\\ 2a & 0 & 1\\ 0 & a & 1\end{vmatrix}=\begin{vmatrix}a & \frac{5a}{2} & 1\\ 0 & -\frac{5a}{2} & 0\\ 0 & a & 1\end{vmatrix}=\frac{5a^2}{2}\text{ sq. units}$$

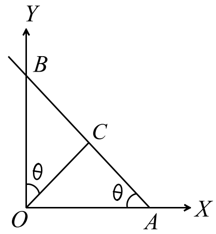
130 **(b)**



Images of A about $y=x, y=-2x$ are A_1 and A_2 which lie on BC. Now $A_1\equiv(2,1)$ and $A_2\equiv(-11/5, 2/5)$. Equation of BC is $x-7y+5=0$. Hence,

$$r=ID=\left|\frac{5}{\sqrt{1+49}}\right|=\frac{1}{\sqrt{2}}$$

131 **(d)**



$\tan(180°-\theta)=$ slope of $AB=-3$

$\therefore \tan\theta=3$

$$\therefore\ \frac{OC}{AC}=\tan\theta, \frac{OC}{BC}=\cot\theta$$

$$\Rightarrow \frac{BC}{AC}=\frac{\tan\theta}{\cot\theta}=\tan^2\theta=9$$

132 **(a)**

First two family of lines passes through (1, 1) and (3, 3), respectively. The point of intersection of lines belonging to third family of lines will lie on line $y=x$

Hence,

$\Rightarrow ax+x-2=0$ and $6x+ax-a=0$

or $\ \frac{2}{a+1}=\frac{a}{6+a}$

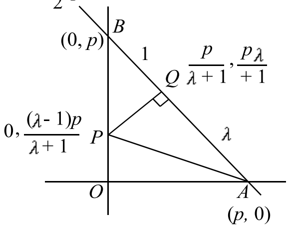
$\Rightarrow a^2-a-12=0\ \Rightarrow\ (a-4)(a+3)=0$

133 **(d)**

$$\frac{\Delta AQP}{\Delta AOB}=\frac{3}{8}$$

or $\ \frac{\frac{P^2\lambda}{(\lambda+1)^2}}{\frac{1}{2}p^2}=\frac{3}{8}$

$$\frac{\frac{P^2\lambda}{(\lambda+1)^2}}{\frac{1}{2}p^2}=\frac{3}{8}$$



$\Rightarrow \lambda = 3, \frac{1}{3}$

$\Rightarrow \frac{AQ}{BQ} = 3$ or $\frac{1}{3}$

The value 1/3 is rejected because this gives negative coordinates of P and it is given that P lies on OB

134 **(b)**

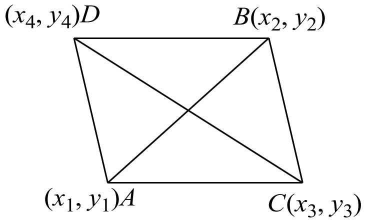
Reflection of A in the two angle bisectors will lie on the line BC, so (2,1) and $(1, -2)$ will lie on BC.

Equation of BC will be $y + 2 = \left(\frac{1+2}{2-1}\right)(x - 1)$

i.e., $3x - y = 5$

135 **(a)**

Let $A \equiv (x_1, y_1), B \equiv (x_2, y_2), C \equiv (x_3, y_3), D \equiv (x_4, y_4)$



Given,

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 + y_1^2 + y_2^2 + y_3^2 + y_4^2 - 2x_1x_3 - 2x_2x_4 - 2y_2y_3 - 2y_1y_4 \leq 0$$

$$\Rightarrow (x_1 - x_3)^2 + (x_2 - x_4)^2 + (y_2 - y_3)^2 + (y_1 - y_4)^2 \leq 0$$

$\Rightarrow x_1 = x_3, x_2 = x_4, y_2 = y_3, y_1 = y_4$

$\Rightarrow \frac{x_1 + x_2}{2} = \frac{x_3 + x_4}{2}$ and $\frac{y_1 + y_2}{2} = \frac{y_4 + y_3}{2}$

Hence, AB and CD bisect Each other

Therefore, $ACBD$ is a parallelogram

Also $AB^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$

$= (x_3 - x_4)^2 + (y_4 - y_3)^2$

$= CD^2$

Thus $ACBD$ is a parallelogram and $AB = CD$, hence is a rectangle

136 **(a)**

x-coordinate of the point of intersection is

$3x + 4(mx + 1) = 9$

$\Rightarrow (3 + 4m)x = 5 \Rightarrow x = \frac{5}{3 + 4m}$

For x to be an integer $3 + 4m$ should be a divisor of 5, i.e., $1, -1, 5$ or -5. Hence,

$3 + 4m = 1 \Rightarrow m = -1/2$ (not integer)

$3 + 4m = -1 \Rightarrow m = -1$ (integer)

$3 + 4m = 5 \Rightarrow m = 1/2$ (not an integer)

$3 + 4m = -5 \Rightarrow m = -2$ (integer)

Hence, there are two integral values of m

137 **(d)**

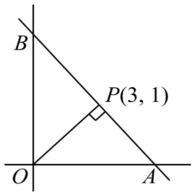
The set of lines is $4ax + 3by + c = 0$, where $a + b + c = 0$. Eliminating c, we get

$4ax + 3by - (a + b) = 0$

$\Rightarrow a(4x - 1) + b(3y - 1) = 0$

This passes through the intersection of the lines $4x - 1 = 0$ and $3y - 1 = 0$, i.e., $x = 1/4, y = 1/3$, i.e., $(1/4, 1/3)$

138 **(a)**



Line AB will be farthest from origin if OP is right angled to the line drawn. Hence,

$m_{OP} = \frac{1}{3} \Rightarrow m_{AB} = -3$

Thus equation of AB is

$(y - 1) = -3(x - 3)$

$\Rightarrow A \equiv \left(\frac{10}{3}, 0\right), B \equiv (0,10)$

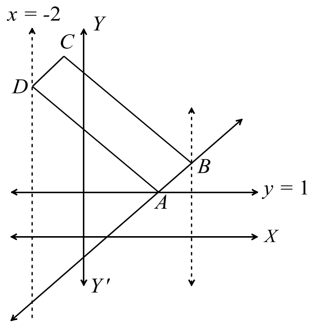
$\Rightarrow \Delta_{OAB} = \frac{1}{2}(oA)(OB) = \frac{1}{2} \times \frac{10}{3} \times 10$

$= \frac{100}{6}$ sq. units

139 **(c)**

Let the equation of side AB be $y = x + a$. Then, $A \equiv (1 - a, 1), B \equiv (2, 2 + a)$. Equation of side AD is $y - 1 = -(x - (1 - a))$. Hence, $D \equiv (-2, 4 - a)$

$x = -2$



Let $C \equiv (h, k)$. Then,

$h + 1 - a = 2 - 2$

$\Rightarrow h = a - 1$ and $k + 1 = 2 + a + 4 - a$

$\Rightarrow k = 5$

Thus locus of C is $y = 5$

140 **(b)**

Lines $x\ \cos\alpha + y\sin\alpha = p$ and $x\sin\alpha - y\cos\alpha = 0$ are mutually perpendicular. Thus $ax + by + p = 0$ will be equally inclined to these lines and would be the angle bisector of these. Now equations of angle bisectors is

$x\sin\alpha - y\cos\alpha = \pm(x\cos\alpha + y\sin\alpha - p)$

$\Rightarrow\ x(\cos\alpha - \sin\alpha) + y(\sin\alpha + \cos\alpha) = p$

or $x(\sin\alpha + \cos\alpha) - y\ (\cos\alpha - \sin\alpha) = p$

comparing these lines with $ax + by = p = 0$, we get

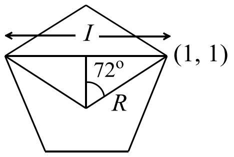
$$\frac{a}{\cos\alpha - \sin\alpha} = \frac{b}{\sin\alpha + \cos\alpha} = 1$$

$\Rightarrow\ a^2 + b^2 = 2$

or $\frac{a}{\sin\alpha+\cos\alpha} = \frac{b}{\cos\alpha-\sin\alpha} = 1$

$\Rightarrow\ a^2 + b^2 = 2$

141 **(a)**



The point of intersection of diagonals, i.e., (1, 1), lies on circumcircle. Hence

$\Rightarrow\ I = 2R\sin 72°$

$$R = \frac{\sin 36°}{2\sin 72°} = \cos 72°$$

Therefore, the locus is $(x-1)^2 + (y-1)^2 = \cos^2 72°$,

Hence, $x^2 + y^2 - 2x - 2y + 1 + \sin^2 72° = 0$

142 **(c)**

Equations of the given lines are

$y - 1 = \tan\theta\,(x-1)$ and

$y - 1 = \cot\theta\,(x-1)$

So their joint equation is

$$[(y-1) - \tan\theta\,(x-1)][(y-1) - \cot\theta\,(x-1)] = 0$$

$$\Rightarrow (y-1)^2 - (\tan\theta + \cot\theta)(x-1)(y-1) + (x-1)^2 = 0$$

$$\Rightarrow x^2 - (\tan\theta + \cot\theta)xy + y^2 + (\tan\theta + \cot\theta - 2)(x + y - 1) = 0$$

Comparing with the given equation, we get

$\tan\theta + \cot\theta = a + 2$

$$\Rightarrow \frac{1}{\sin\theta\cos\theta} = a + 2$$

$$\Rightarrow\ \sin 2\theta = \frac{2}{a+2}$$

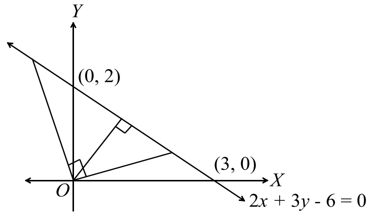
143 **(d)**

Here $AB = BC = CA = 2$. So, it is an equilateral triangle and the incentre coincides with centroid. Therefore, centroid is

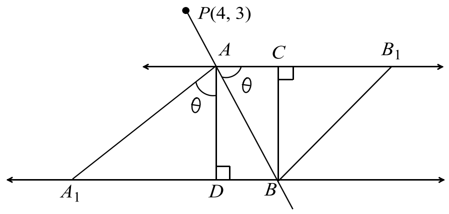
$$\left(\frac{0+1+2}{3}, \frac{0+0+\sqrt{3}}{3}\right) = \left(1, \frac{1}{\sqrt{3}}\right)$$

144 **(a)**

Distance of (0, 0) from the line $2x + 3y - 6 = 0$ is $6/\sqrt{4+9} = 6/\sqrt{13}$. The area of the triangle is $\left(6/\sqrt{13}\right)^2 = 36/13$



145 **(a)**



The given lines (L_1 and L_2) are parallel and distance between them (BC or AD) is $(15-5)/5 = 2$ units. Let $\angle BCA = \theta \Rightarrow AB = BC\ \mathrm{cosec}\,\theta$ and $AA_1 = AD\sec\theta = 2\sec\theta$. now area of parallelogram $AA_1\ BB_1$ is

$\Delta = AB \times AA_1 = 4\sec\theta\ \mathrm{cosec}\,\theta$

$$= \frac{8}{\sin 2\theta}$$

Clearly, Δ is least for $\theta = \pi/4$. Let slope AB be m

Then, $1 = \left|\frac{m+3/4}{1-\frac{3m}{4}}\right|$

$\Rightarrow\ 4m + 3 = \pm(4 - 3m) \Rightarrow m = 1/7$ or -7

Hence, the equation of 'L' is

$x - 7y + 17 = 0$

or $7x + y - 31 = 0$

146 **(b)**

p, q, r are the roots of equation $y^3 - 3y^2 + 6y + 1 = 0$. So, $p + q + r = 3, pq + qr + rp = 6$ and $pqr = -1$. Now, the centroid of the triangle is

$$\left(\frac{pq + qr + rp}{3}, \frac{\frac{1}{pq} + \frac{1}{qr} + \frac{1}{rp}}{3}\right)$$

i.e.,

$$\left(\frac{pq + qr + rp}{3}, \frac{p + q + r}{3pqr}\right) = \left(\frac{6}{3}, \frac{3}{-3}\right) \text{ or } (2, -1)$$

147 **(c)**

If the point of intersection of two lines with coordinate axes be concylic, then product of intercepts on x-axis is equal to product of intercepts on y-axis by these lines. This is a geometric property. The intercepts on x-axis are p and q and whose product is pq. Also, the intercepts on y-axis are p, and q, whose product is also pq. Hence, the four points are concylic

148 **(d)**

Here $my(y-mx)+x(y-mx)=0$, i.e.,
$(y-mx)\times(my+x)=0$. So, the lines are
$y=mx$ and $y=(-1/m)x$. Bisectors between the lines $xy=0$ are $y=x$ and $y=-x$. Therefore,
$m=1$ or -1

149 **(b)**

If the line cuts off the axes at A and B, then area of triangle is $\frac{1}{2}\times OA\times OB=T$

$\Rightarrow \frac{1}{2}\times a\times OB=T \Rightarrow OB=\frac{2T}{a}$

Hence, the equation of line is

$\frac{x}{-a}+\frac{y}{2T/a}=1$

$\Rightarrow\ 2Tx-a^2y+2aT=0$

150 **(c)**

For any point $P(x,y)$ that is equidistant from given line, we have

$x+y-\sqrt{2}=-(x+y-2\sqrt{2})$

$\Rightarrow\ 2x+2y-3\sqrt{2}=0$

151 **(b)**

The given lines

$ax\pm by\pm c=0$

$\Rightarrow \frac{x}{\pm c/a}+\frac{y}{\pm c/b}=1$

The vertex at$A(c/a,a),C(-c/a,0),B(0,c/b,D0,-c/b$. Therefore, the diagonals $AC$and BD of quadrilateral $ABCD$ are perpendicular. Hence, it is a rhombus whose area is given by

$\frac{1}{2}\times AC\times BD=\frac{1}{2}\times\frac{2c}{a}\times\frac{2c}{b}=\frac{2c^2}{ab}$

152 **(b)**

As L has intercepts a and b on the axes, equation of L is

$\frac{x}{a}+\frac{y}{b}=1$ (i)

Let x-and y-axes be rotated through an angle θ in anticlockwise direction. In new system, intercepts are p an q, therefore equation of L becomes

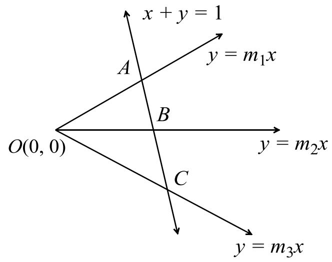
$\frac{x}{p}+\frac{y}{q}=1$ (ii)

As the origin is fixed in rotation, the distance of line from origin in both the cases should be same. Hence, we get

$$d=\left|\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}\right|=\left|\frac{1}{\sqrt{\frac{1}{p^2}+\frac{1}{q^2}}}\right|$$

$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$

153 **(a)**



Solving the equation of the lines, we get

$A=\left(\frac{1}{1+m_1},\frac{m_1}{1+m_1}\right),C=\left(\frac{1}{1+m_3},\frac{m_3}{1+m_3}\right)$

If $AB=BC$, then midpoint of AClies on $y=m_2x$

$\Rightarrow \frac{\frac{m_1}{1+m_1}+\frac{m_3}{1+m_3}}{2}=m_2\left[\frac{\frac{1}{1+m_1}+\frac{1}{1+m_3}}{2}\right]$

154 **(a)**

Let the common line be $y=mx$. Then it must satisfy both the equations. Therefore, we have

$bm^2+2hm+a=0$ (i)

$b'm^2+2h'm+a'=0$ (ii)

Solving Eqs. (i) and (ii), we get

$\frac{m^2}{2(ha'-h'a)}=\frac{m}{ab'-a'b}=\frac{a}{2(bh'-b'h)}$

Eliminating m, we get

$\left[\frac{ab'-a'b}{2(bh'-b'h)}\right]^2=\frac{ha'-h'a}{bh'-b'h}$

$\Rightarrow\ (ab'-a'b)^2=4(ha'-h'a)(bh'-b'h)$

155 **(b)**

$(x_1+t(x_2-x_1),y_1+t(y_2-y_1)=(x_1(1-t)+tx_2,y_1(1-t)+ty_2)$ is the point which divides the join of (x_1,y_1) and (x_2,y_2) in the ratio $t:(1-t)$ which is positive if $0<t<1$

156 **(b)**

The straight lines represented by $(y-mx)^2=a^2(1+m^2)$ are

$y-mx=\pm a\sqrt{1+m^2}$

i.e., $y-mx=a\sqrt{1+m^2}$ (i)

and $y-mx=-a\sqrt{1+m^2}$ (ii)

Similarly, the straight lines represented by $(y-nx)^2=a^2(1+n^2)$ are

$y-nx=a\sqrt{1+n^2}$ (iii)

and $y-nx=-a\sqrt{1+n^2}$ (iv)

Since the lines (i) and (ii) are parallel, so the distance between (i) and (ii) is

$\left|\frac{a\sqrt{1+m^2}+a\sqrt{1+m^2}}{\sqrt{1+m^2}}\right|=|2a|$

Similarly the lines (iii) and (iv) are parallel lines and the distance between them is $|2a|$. Since the distances between parallel lines are same, hence the four lines form a rhombus

157 **(c)**

The circumcentre of the triangle is (0, 0) as all the vertices lie on the circle $x^2+y^2=5$. So the orthocenter will be (sum of x coordinates, sum of y coordinates)

158 **(d)**

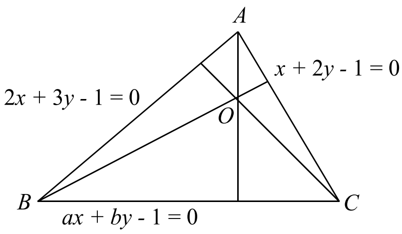
$$A=\frac{1}{2}\begin{vmatrix}1 & p^2 & 1\\ 0 & 1 & 1\\ p & 0 & 1\end{vmatrix}$$

$$=\frac{1}{2}[1(1-0)+p(p^2-1)]$$

$$=\frac{1}{2}(p^3-p+1)$$

Hence, $A=\frac{1}{2}|p^3-p+1|$. Now, minimum value of modulus is zero. Since $A(p)$ is cubic, it must vanish for some p other than given in (a), (b), (c)

159 **(c)**



Equation of AO is $2x+3y-1+\lambda(x+2y-1)=0$, where $\lambda=-1$ since the line passes through the origin. So, $x+y=0$. Since AO is perpendicular to BC, so

$$(-1)\left(-\frac{a}{b}\right)=-1$$

$\therefore a=-b$

Similarly,

$(2x+3y-1)+\mu(ax-ay-1)=0$

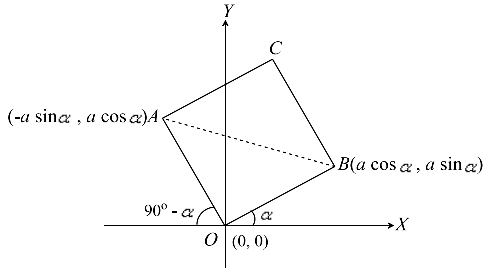
Will be the equation of BO for $\mu=-1$. Now, BO is perpendicular to AC. Hence,

$$\left\{-\frac{(2-a)}{3+a}\right\}\left(-\frac{1}{2}\right)=-1$$

$\therefore a=-8, b=8$

160 **(c)**

The coordinates of A and b are as shown in the figure



The equation of the diagonal AB is

$$y-a\sin\alpha=\frac{a\cos\alpha-a\sin\alpha}{-a\sin\alpha-a\cos\alpha}(x-a\cos\alpha)$$

$\Rightarrow\ y(\cos\alpha+\sin\alpha)-a(\sin\alpha\cos\alpha+\sin^2\alpha)$

$=-(\cos\alpha-\sin\alpha)x+a\cos\alpha(\cos\alpha-\sin\alpha)$

$\Rightarrow y(\cos\alpha+\sin\alpha)+x(\cos\alpha-\sin\alpha)=a$

161 **(a)**

Let $x_2=x_1r, x_3=x_1r^2$ and so is $y_2=y_1r, y_3=y_1r^2$

Where r is common ratio

$$\therefore\ \Delta=\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}$$

$$=\begin{vmatrix}x_1 & y_1 & 1\\ rx_1 & ry_1 & 1\\ r^2x_1 & r^2y_1 & 1\end{vmatrix}$$

$$=r\times r^2\begin{vmatrix}x_1 & y_1 & 1\\ x_1 & y_1 & 1\\ x_1 & y_1 & 1\end{vmatrix}$$

$=0$

Hence, the points are collinear

162 **(a,c)**

The equation represents a pair of straight lines. Hence,

$$1\times(-2)(-1)+2\left(\frac{3}{2}\right)\times 0\times\frac{m}{2}-1\times\left(\frac{3}{2}\right)^2-(-2)\times 0^2-(-1)\times\left(\frac{m}{2}\right)^2=0$$

$\Rightarrow m=1,-1$

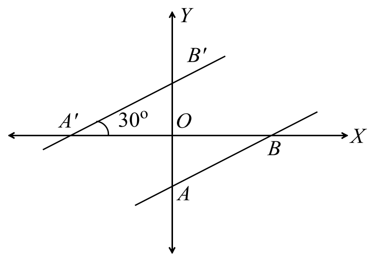
The points of intersection of the pair of lines are obtained by solving

$$\frac{\partial S}{\partial x}=2x+my=0$$

and $\frac{\partial S}{\partial y}=mx-4y+3=0$

When $m=1$, then required point is the intersection of $2x+y=0, x-4y+3=0$. When $m=-1$, the required point is the intersection of $2x-y=0,-x-4y+3=0$

163 **(b,d)**



According to question $AB = 10$. So, $OA = 10 \sin 30° = 5$. Then equation of line is

$$y = \frac{1}{\sqrt{3}} x \pm 5$$

or $x - \sqrt{3}y \pm 5\sqrt{3} = 0$

164 **(c,d)**

Let the third vertex be (x, y). Then,

$$\frac{1}{2}\begin{vmatrix} x & y & 1 \\ -4 & 0 & 1 \\ 1 & -1 & 1 \end{vmatrix} = \pm 4$$

$\Rightarrow x + 5y + 4 = \pm 4$

$\Rightarrow x + 5y + 12 = 0$ or $x + 5y - 4 = 0$

Hence, the third vertex lies on $x + 5y + 12 = 0$ or $x + 5y - 4 = 0$

165 **(a,d)**

Note that the lines are perpendicular. Assume the coordinate axes to be directed along to be directed along $u = 0$ and $v = 0$. Now the lines $k_1u - k_2v = 0$ and $k_1u + k_2v = 0$ are equally inclined with uv axes. Hence, the bisectors are $u = 0$ and $v = 0$

166 **(d)**

Let $A \equiv (0, 8/3), B \equiv (1, 3)$ and $C \equiv (82, 30)$

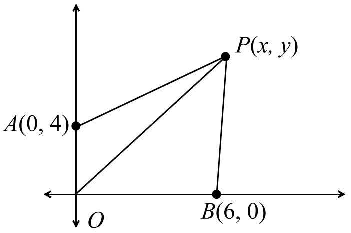
Now, slope of line AB is $(3 - 8/3)/(1 - 0) = 1/3$.

Slope of line BC is $(30 - 3)/(82 - 1) = 27/81 = 1/3$. Therefore, $AB||BC$ and B is common point.

Hence, A, B, C are collinear

167 **(a,d)**

Let point P be (x, y)



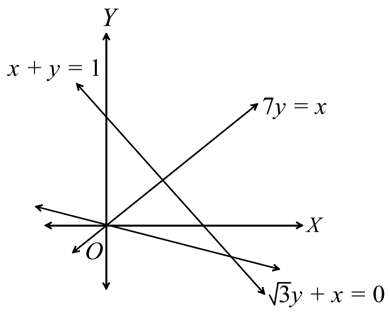
$$\Delta_{POA} = \frac{1}{2}(OA)|x| = 2|x|$$

$$\Delta_{POB} = \frac{1}{2}(OB)|y| = 2|y|$$

Where $P \equiv (x, y) \Rightarrow 2|x| = 6|y|$

$\Rightarrow |x| = 3|y| \Rightarrow 3y - x = 0$ *or* $3y + x = 0$

168 **(b, c)**



Clearly from the figure above, the triangle is obtuse angled. Hence, centroid and incentre lie inside the triangle. Orthocentre and circumcentre lie outside the triangle.

Therefore, it is an obtuse angled triangle

169 **(a,b,c,d)**

Let, $\frac{2}{3}x^2 + \frac{p}{3}xy + y^2 = (y - mx)(y - m'x)$

and $\frac{2}{-3}x^2 + \frac{q}{-3}xy + y^2 = \left(y + \frac{1}{m}x\right)(y - m'x)$

Then, $m + m' = -\frac{p}{3}, mm' = \frac{2}{3}$ (i)

$\frac{1}{m} - m' = \frac{-q}{3}, -\frac{m'}{m} = -\frac{2}{3}$ (ii)

$\Rightarrow m^2 = 1 \Rightarrow m = \pm 1$

If $m = 1, m' = 2/3$ and so $p = -5, q = -1$. If $m = -1, m' = -2/3$ and so $p = 5, q = 1$

170 **(a,c,d)**

Vertices of the given triangle are $(0, 0), (a/m, a)$ and $(a/m_2, a)$. So the area of the triangle is equal to $a^2(m_2 - m_1)/(2m_1m_2)$. Since m_1, m_2 are the roots of $x^2 - ax - a - 1 = 0$, so

$m_1 + m_2 = a; m_1m_2 = -(a + 1)$

$\Rightarrow (m_1 - m_2)^2 = (m_1 + m_2)^2 - 4m_1m_2$

$= a^2 + 4(a + 1)$

$= (a + 2)^2$

$\Rightarrow m_1 - m_2 = \pm(a + 2)$

So the required area is

$$\Delta = \pm\frac{a^2(a + 2)}{-2(a + 1)} = \pm\frac{a^2(a + 2)}{2(a + 1)}$$

Since area is a positive quantity, so

$$\Delta = \frac{a^2(a + 2)}{2(a + 1)} \text{ if } a > -1 \text{ or } a < -2$$

and $\Delta = -\frac{a^2(a+2)}{2(a+1)}$ if $-2 < a < -1$

171 **(a,d)**

$$\frac{x - 1}{\cos\theta} = \frac{y - 2}{\sin\theta} = \pm\frac{\sqrt{6}}{3}$$

$$\therefore x = 1 \pm \frac{\sqrt{6}}{3}\cos\theta, y = 2 \pm \frac{\sqrt{6}}{2}\sin\theta$$

$\because x + y = 4$

$$\Rightarrow 3 \pm \frac{\sqrt{6}}{3}(\cos\theta + \sin\theta) = 4$$

$\Rightarrow \pm(\cos\theta + \sin\theta) = \frac{3}{\sqrt{6}} = \sqrt{\frac{3}{2}}$

On squaring both sides, we get

$1 \pm \sin 2\theta = \frac{3}{2}$

$\Rightarrow \sin 2\theta = \frac{1}{2}\sin\left(\frac{\pi}{6}\right)$

$\Rightarrow 2\,\theta = n\,\pi + (-1)^n \frac{\pi}{6}, n \in I$

$\Rightarrow \theta = \frac{n\pi}{2} + (-1)^n \frac{\pi}{12}, n \in I$

For $n = 0, \theta = \frac{\pi}{12}$, for $n = 1, \theta = \frac{\pi}{2} - \frac{\pi}{12} = \frac{5\pi}{12}$

For $n = -1, \theta = -\frac{\pi}{2} - \frac{\pi}{12} = -\frac{7\pi}{12}$;

For $n = -2, \theta = -\pi + \frac{\pi}{12} = -\frac{11\pi}{12}$

172 **(a,b)**

As $(a+b)x + (a-b)y - 2ab = 0$

and $(a-b)x + (a+b)y - 2ab = 0$

the equation of the angle bisectors are

$(a+b)x + (a-b)y - 2ab$

$= \pm\{(a-b)x + (a+b)y - 2ab\}$

$\Rightarrow 2bx - 2by = 0$ $ie, x = y$ ...(i)

and $2ax + 2ay - 4ab = 0$ $ie, x + y = 2b$ ...(ii)

$\therefore$ Equation of third side as given by Eq. (i) is

$x - y = k$ satisfying the point $(b-a, a-b)$

$\therefore k = 2b - 2a$

$\therefore$ The line is $x - y = 2(b-a)$

From Eq. (ii), $x + y - 2b = k$ is passing through the point $(b-a, a-b)$

$\therefore k = -2b$

$\therefore$ The line is $x + y = 0$

173 **(a,b,d)**

Since the given point lies on the line $lx + my + n = 0$, so a, b, c are the roots of the equation

$l\left(\frac{t^3}{t-1}\right) + m\left(\frac{t^2-3}{t-1}\right) + n = 0$

or $lt^3 + mt^2 + nt - (3m+n) = 0$ (i)

Hence, $a + b + c = -\frac{m}{l}$

$ab + bc + ca = \frac{n}{l}$ (ii)

$abc = \frac{3m+n}{l}$ (iii)

So, from Eqs. (i), (ii) and (iii), we get

$abc - (bc + ca + ab) + 3(a+b+c) = 0$

174 **(b,c)**

For the two lines $24x + 7y - 20 = 0$ and $4x - 3y - 2 = 0$, the angle bisectors are given by

$\frac{24x + 7y - 20}{25} = \pm\frac{4x - 3y - 2}{5}$

Taking positive sign, we get

$2x + 11y - 5 = 0$

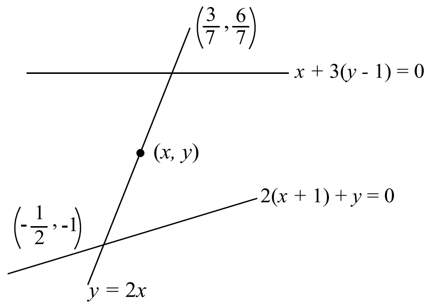
175 **(a,c)**

Since points $AB = AC = 1, \Delta$ is right angled at point A. We have

$\tan\alpha\tan\beta = 1$

$\Rightarrow \cos(\alpha - \beta) = 0 \Rightarrow \alpha - \beta = \frac{\pi}{2}$

176 **(b,d)**



Solving $y = 2x, 2(x+1) + y = 0$, we get $x = -1/2, y = -1$. Solving $y = 2x, x + 3(y-1) = 0$, we get $x = 3/7, y = 6/7$

177 **(a,b)**

Given lines are $x + 2y + 4 = 0$ and $4x + 2y - 1 = 0$

$\Rightarrow x + 2y + 4 = 0$ and $-4x - 2y + 1 = 0$

Here, $(1)(-4) + (2)(-2) = -8 < 0$

$\therefore$ Bisector of the angle is acute angle bisector including origin

Its equation is

$\frac{x + 2y + 4}{\sqrt{5}} = \frac{(-4x - 2y + 1)}{2\sqrt{5}}$

$\Rightarrow 6x + 6y + 7 = 0$

178 **(a,b,c,d)**

Let the slope of $u = 0$ be m. Then slope of $v = 0$ is $\frac{9m}{2}$

Therefore, $\frac{7}{9} = \left|\frac{m - \frac{9m}{2}}{1 + m \times \frac{9m}{2}}\right| = \left|\frac{-7m}{2 + 9m^2}\right|$

$\Rightarrow 9m^2 - 9m + 2 = 0$ or $9m^2 + 9m + 2 = 0$

$m = \frac{9 \pm \sqrt{81 - 72}}{18} = \frac{9 \pm 3}{18} = \frac{2}{3}, \frac{1}{3}$

or $m = \frac{-9 \pm 3}{18} = -\frac{2}{3}, -\frac{1}{3}$

therefore, equations of the lines are

1. $3y = x$ and $2y = 3x$
2. $3y = 2x$ and $y = 3x$
3. $x + 3y = 0$ and $3x + 2y = 0$
4. $2x + 3y = 0$ and $3x + y = 0$

179 **(a,b)**

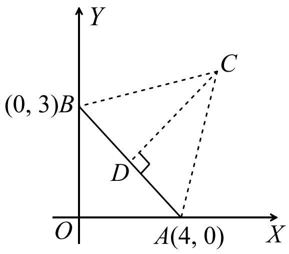
The area of the triangle is given by

$= \frac{1}{2} \times \frac{2b}{a} \times \frac{2b}{c} = \frac{2b^2}{ac} = 2$

$\Rightarrow b^2 = ac$

$\Rightarrow a, b, c$ are in G.P. So, $a, -b, c$ are in G.P.

180 **(a,d)**



$AB = 5, D \equiv \left(2, \frac{3}{2}\right)$

$CD = 5 \times \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{2}$, slope of AB is $-3/4$, slope of CD is $4/3$

If $C \equiv (h, k)$, then

$\frac{h-2}{3/5} = \frac{k-3/2}{4/5} = \pm \frac{5\sqrt{3}}{2}$

$\Rightarrow h = 2\left(1 - \frac{3\sqrt{3}}{4}\right), k = \frac{3}{2}\left(1 - \frac{4}{\sqrt{3}}\right)$

or $h = 2\left(1 + \frac{3\sqrt{3}}{4}\right), k = \frac{3}{2}\left(1 + \frac{4}{\sqrt{3}}\right)$

181 **(a,b,d)**

Equation of the lines given by $x^2 + 2hxy + y^2 = 0$ be $y = m_1 x$ and $y = m_2 x$. Since these make an angle $\alpha$with $y + x = 0$ whose slope is -1, so

$\frac{m_1 + 1}{1 - m_1} = \tan\alpha = \frac{-1 - m_2}{1 - m_2}$

$\Rightarrow m_1 + m_2 = \frac{(\tan\alpha - 1)^2 + (\tan\alpha + 1)^2}{\tan^2\alpha - 1}$

$= \frac{-2\sec^2\alpha \times \cos^2\alpha}{\cos 2\alpha}$

$\therefore -2\sec 2\alpha = -2h$

$\Rightarrow \sec 2\alpha = h$

$\Rightarrow \cos 2\alpha = \frac{1}{h} \Rightarrow 2\cos^2\alpha - 1 = \frac{1}{h}$

$\Rightarrow \cos\alpha = \sqrt{\frac{1+h}{2h}}$ and $\cot\alpha = \sqrt{\frac{h+1}{h-1}}$

182 **(a,b)**

Verteces $(a\cos\theta_1, a\sin\theta_1), (a\cos\theta_2, a\sin\theta_2)$ and $(a\cos\theta_3, a\sin\theta_3)$ are equidistant from origin $(0, 0)$. Hence, the origin is circumcentre (centroid) of circumcircle. Therefore, the coordinates of centroid are

$$\left(\frac{a(\cos\theta_1 + \cos\theta_2 + \cos\theta_3)}{3}, \frac{a(\sin\theta_1 + \sin\theta_2 + \sin\theta_3)}{3}\right)$$

But as the centroid is the origin $(0, 0)$, therefore $\cos\theta_1 + \cos\theta_2 + \cos\theta_3 = 0$ and $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 0$

183 **(a,c)**

Substituting the coordinates of the points $(1, 3)$ $(5, 0)$ and $(-1, 2)$ in $3x + 2y$, we obtain the values 9, 15 and 1 which are all +ve. Therefore, all the points lying inside the triangle formed by given points satisfy $3x + 2y \geq 0$. Substituting the co-ordinates of the given points in $2x + 3y - 13$, we find the values $-2, -3$ and -9 which are all – ve. So, (b) is not correct

Again substituting the given points in $2x - 3y - 12$ we get, $-19, -2, -20$ which are all –ve. It follows that all points lying inside the Δ formed by given point satisfy $2x - 3y - 12 \leq 0$. So (c) is the correct answer

Finally substituting the coordinates of the given point in $-2x + y$, we get $1, -10$ and 4 which are all +ve. So (d) is not correct

184 **(a,d)**

Equation of line passing through two given points (x_1, y_1) and (x_2, y_2) is

$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

Now given expression is

$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 + x_3 & y_2 + y_3 & 1 \end{vmatrix} = 0$

$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ \frac{x_2 + x_3}{2} & \frac{y_2 + y_3}{2} & 1 \end{vmatrix} = 0$

This is the equation of the line passing through the points (x_1, y_1) and $((x_2 + x_3)/2, (y_2 + y_3)/2)$. This is a equation of median through vertex A

185 **(b,d)**

Let any point on the line $x - y = 2$ be $C(h, h - 2)$

Given area of ΔABC is

$\left|\frac{1}{2}\begin{vmatrix} h & h-2 & 1 \\ -5 & 0 & 1 \\ 3 & 0 & 1 \end{vmatrix}\right| = 20$

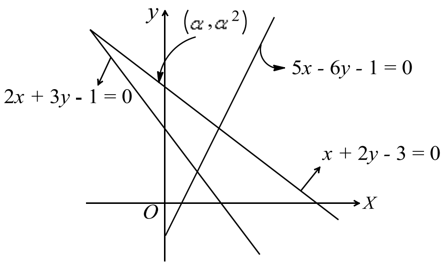
$\Rightarrow |8(h - 2)| = 40$

$\Rightarrow h - 2 = \pm 5$

$\Rightarrow h = 7, -3$

Hence, the points are $(7, 5)$ and $(-3, -5)$

186 **(a,c,d)**



O and the point (α, α^2) lie on the opposite sides w.r.t. $2x + 3y - 1 = 0$. Hence,

$\Rightarrow 2\alpha + 3\alpha^2 - 1 > 0$ (i)

O and the point (α, α^2) lie to the same side w.r.t. $x + 2y - 3 = 0$. Hence,

$\Rightarrow \ \alpha + 2\alpha^2 - 3 < 0$ (ii)

Again O and the point (α, α^2) lie on the same side w.r.t. $5x - 6y - 1 = 0$. Hence,

$5\alpha - 6\alpha^2 - 1 < 0$

$\Rightarrow 6\alpha^2 - 5\alpha + 1 > 0$

189 **(b,d)**

Let the angle be θ. then, equation of the given line is

$\frac{x-1}{\cos\theta} = \frac{y-2}{\sin\theta}$ (i)

The coordinates of a point on (i) at a distance $\sqrt{6}/3$ from (1, 2) are $(1 + \sqrt{6}/3 \cos\theta, 2 + \sqrt{6}/3 \sin\theta)$. This point lies on $x + y = 4$.

Therefore,

$$1 + \frac{\sqrt{6}}{3}\cos\theta + 2 + \frac{\sqrt{6}}{3}\sin\theta = 4$$

$$\Rightarrow \cos\theta + \sin\theta = \sqrt{\frac{3}{2}}$$

$$\Rightarrow \frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta = \frac{\sqrt{3}}{2}$$

$\Rightarrow \cos(\theta - \pi/4) = \cos(\pm\pi/6)$

$\Rightarrow \ \theta - \pi/4 = \pm\pi/6$

$\Rightarrow \ \theta = 75°$ or $\theta = 15°$

190 **(a,c)**

The equations of lines passing through (1, 0) are given by$y = m(x - 1)$. Its distance from origin is $\sqrt{3}/2$

Hence,

$$\Rightarrow \left|\frac{-m}{\sqrt{1 + m^2}}\right| = \sqrt{3}/2 \Rightarrow m = \pm\sqrt{3}$$

Hence, the lines are $\sqrt{3}x + y - \sqrt{3} = 0$ and $\sqrt{3}x - y - \sqrt{3} = 0$

191 **(a,d)**

Distance between $x + 2y + 3 = 0$ and $x + 2y - 7 = 0$ is $10/\sqrt{5}$. Let the remaining side parallel to $2x - y - 4 = 0$ be $2x - y + \lambda = 0$. We have,

$$\frac{|\lambda + 4|}{\sqrt{5}} = \frac{10}{\sqrt{5}} \Rightarrow \lambda = 6, -14$$

Thus the remaining side is $2x - y + 6 = 0$ or $2x - y - 14 = 0$

192 **(a,c)**

It is clear that diagonals of the rhombus will be parallel to the bisectors of the given lines and will pass through (1, 3). Equations of bisectors of the given lines are

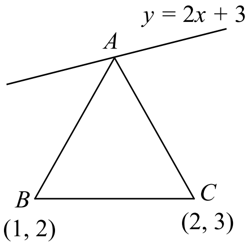
$$\frac{x + y - 1}{\sqrt{2}} = \pm\left(\frac{7x - y - 5}{5\sqrt{2}}\right)$$

or $2x - 6y = 0, 6x + 2y = 5$

Therefore, the equations of diagonals are $x - 3y + 8 = 0$ and $3x + y - 6 = 0$. Thus the required vertex will be the point where these lines meet the line$y = 2x$. Solving these lines we get possible coordinates as (8/5, 16/5) and (6/5, 12/5)

193 **(a,b,c,d)**

The point $A(\alpha, \beta)$ lies on$y = 2x + 3$. Hence,



$\beta = 2\alpha + 3$

$A \equiv (\alpha, 2\alpha + 3)$

Area of ΔABC is

$$\left|\frac{1}{2}\begin{vmatrix} \alpha & 2\alpha + 3 & 1 \\ 1 & 2 & 1 \\ 2 & 3 & 1 \end{vmatrix}\right|$$

$$= \left|\frac{1}{2}[\alpha(2 - 3) + (2\alpha + 3)(2 - 1) + 1(3 - 4)]\right|$$

$$= \frac{1}{2}|-\alpha + 2\alpha + 3 - 1| = \frac{1}{2}|\alpha + 2| = S$$

$[S] = 2 \ \Rightarrow 2 \leq S < 3$

$\therefore 2 \leq \frac{1}{2}|\alpha + 2| < 3$

$4 \leq |\alpha + 2| < 6$

$|\alpha + 2| < 6 \ \Rightarrow -6 < \alpha + 2 < 6$

$\Rightarrow \ -8 < \alpha < 4$ (i)

and $|\alpha + 2| \geq 4 \Rightarrow \ \alpha + 2 \geq 4$ or $\alpha + 2 \leq -4$

$\Rightarrow \ \alpha \geq 2$ or $\alpha \leq -6$ (ii)

From Eqs. (i) and (ii),

$-8 < \alpha \leq -6$ or $2 \leq \alpha < 4$

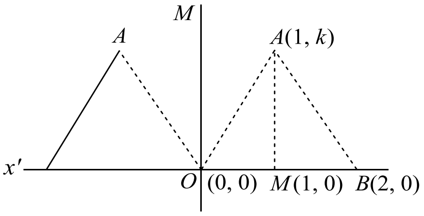
$\Rightarrow \ \alpha = -7, -6, 2, 3$

Possible coordinates of A are

$(-7, -11), (-6, -9), (2, 7), (3, 9)$

194 **(a)**

Given that points are $O(0, 0)$ and $B(2, 0)$



From figure, ΔABC is equilateral

Hence, $\tan 60° = k$

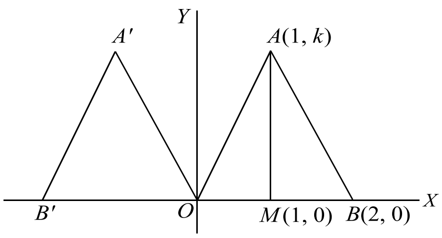
or $k = \sqrt{3}$ (for first quadrant) or $k = -\sqrt{3}$ (for fourth quadrant). Then possible coordinates are $(1, \pm\sqrt{3})$

Similarly, for second quadrant, the point is $\left(-1, -\sqrt{3}\right)$

And for third quadrant, the point is $(-1, -\sqrt{3})$

Case (i)

If $OA = AB$, then $\angle A = 30°$



$\therefore\ \angle AOB = 75°$

$\therefore \dfrac{AM}{OM} = \tan 75°$

$AM = OM \tan 75°$

$k = 1 \times (2 + \sqrt{3})$

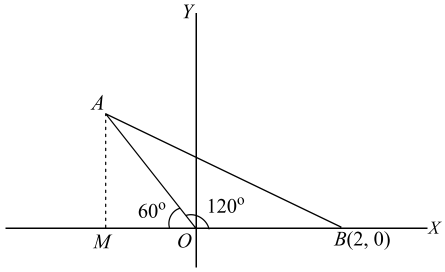
$\therefore\ \ k = 2 + \sqrt{3}$

Hence, point A is $(1, 2 + \sqrt{3})$. By symmetry, all possible points are $(\pm 1, \pm(2 + \sqrt{3})$

Case (ii)

$AO = OB$

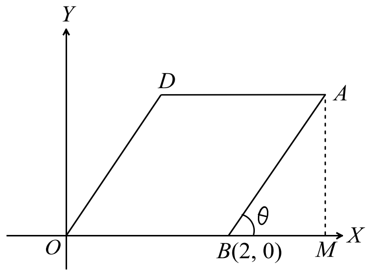
$\therefore\ \angle AOB = 120°$



$AM = 2 \sin 60° = \sqrt{3}$

And $\ OM = 2 \cos 60° = 1$

Hence, point A is $(1, -\sqrt{3})$.by symmetry, all possible points are $(\pm 1, \pm\sqrt{3})$



c.

Let$\angle DOB = \angle ABM = \theta$. Area of ΔOAB is $\frac{1}{2} \times OB \times AM = \frac{1}{2} \times \sqrt{3}$

$\Rightarrow 2 \times 2 \sin\theta = \sqrt{3}$

$\Rightarrow \sin\theta = \frac{\sqrt{3}}{4} \Rightarrow AM = \sqrt{3}$ and $BM = 1$

Hence, A has coordinates $(3, \sqrt{3})$. By symmetry, all possible coordinates are $(\pm 3, \pm\sqrt{3})$

O′$M + OA$

Hence, the coordinate A will be $(1,2 + \sqrt{3})$ in first quadrant. By symmetry, all possible coordinates of A are $(\pm 1, \pm(2 + \sqrt{3}))$

195 **(a,c)**
Let $L = 3x - 4y - 8$. Then the value of L at $(3,4)$ is $3 \times 3 - 4 \times 4 - 8 = -15 < 0$. Hence, for the point $P(x, y)$ we should have $L > 0$
$\Rightarrow 3x - 4y - 8 > 0$
$\Rightarrow 3x - 4(-3x) - 8 > 0$
$[\because P(x, y)$ lies on $y = -3x]$
$\Rightarrow x > 8/15$
and $-y - 4y - 8 > 0$
$\Rightarrow y < -8/5$

196 **(a,b,c)**
For concurrency, of three lines $px + qy + r = 0, qx + ry + p = 0, rx + py + q = 0$, we must

have,

$$\begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix} = 0$$

$\Rightarrow 3pqr - p^3 - q^3 - r^3 = 0$

$\Rightarrow (p+q+r)(p^2+q^2+r^2-pq-pr-rq) = 0$

197 **(a,b,c,d)**

Equation of the lines joining the origin to the points of intersection of the given lines is

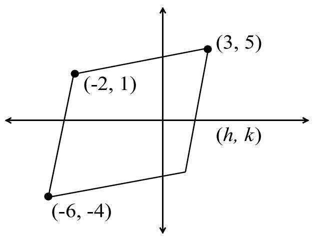
$3x^2 + mxy - 4x(2x+y) + 1(2x+y)^2 = 0$ (by homogenization)

$\Rightarrow x^2 - mxy - y^2 = 0$

Which are perpendiculars for all values of m

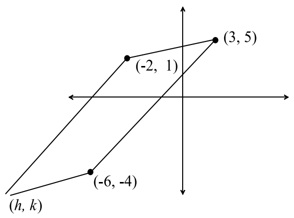
198 **(b,c,d)**

If the remaining vertex is (h, k), then



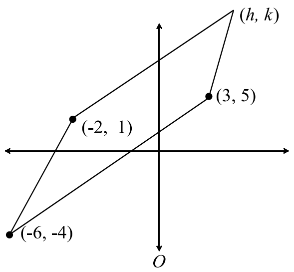
$h - 2 = -6 + 4, k + 1 = 5 - 4$

$\Rightarrow h = -1, k = 0$



$h + 3 = -6 - 2, k + 5 = -4 + 1$

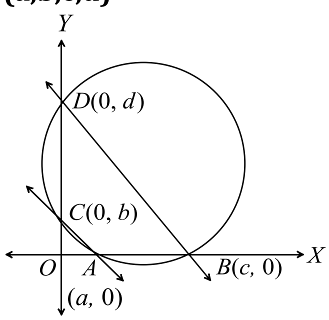
$\Rightarrow h = -11, k = -8$



$h - 6 = 3 - 2, k - 4 = 5 + 1$

$\Rightarrow h = 7, k = 9$

199 **(a,b,c,d)**



If points A, B, C, D are concyclic, then $ac = bd$. The coordinates of the points of intersection of lines are

$$\left(\frac{ac(b-d)}{bc-ad}, \frac{bd(c-a)}{bc-ad}\right)$$

Let coordinates of the point intersection be (h, k)

Then

$$h = \frac{ac(b-d)}{bc-ad}, k = \frac{bd(c-a)}{bc-ad}$$

given $c^2 + a^2 = b^2 + d^2$. Since $ac = bd$, so

$(c-a)^2 = (b-a)^2$

or $(c-a) = \pm(b-d)$

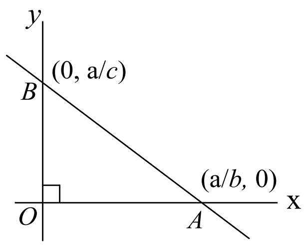
Then the locus of the points of intersection is

$y = \pm x$

200 **(b,d)**

$$bx + cy = a \Rightarrow \frac{x}{(a/b)} + \frac{y}{(a/c)} = 1$$

Area of $\Delta\, OAB = \dfrac{1}{8}$ (given)



$$\Rightarrow \frac{1}{2} \cdot \frac{a}{b} \cdot \frac{a}{c} = \pm\frac{1}{8} \Rightarrow \frac{a^2}{bc} = \pm\frac{1}{4}$$

$\Rightarrow (2a)^2 = \pm bc$

$\therefore b, \pm 2a, c$ are in GP

201 **(a,d)**

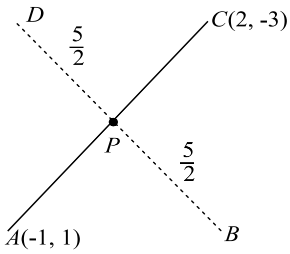
Here, $my(y - mx) + x(y - mx) = 0$

$\Rightarrow (y - mx)(my + x) = 0$

So, the lines are $y = mx$ or $y = (-1/m)x$.

Bisectors between the lines $xy = 0$ are $y = x$ and $y = -x$. Therefore, $m = 1, -1$

202 **(a,b)**



$AC = \sqrt{3^2 + 4^2} = 5$

The midpoint P of $AC = \left(\frac{1}{2}, -1\right)$

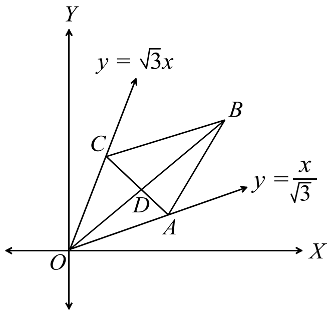
Slope of AC is $\frac{-4}{3}$

Therefore, slope of BD is $\frac{3}{4} = \tan\theta$

Therefore, coordinates of B and D are

$$\equiv \left(1/2 \pm \frac{5}{2}\cos\theta, -1 \pm \frac{5}{2}\sin\theta\right)$$

203 **(a,b)**



Here, $\angle COA = 30°$

Area of rhombus $= 2 \times \frac{1}{2} \times OA \times OC \sin 30°$

$\Rightarrow 2 = \frac{1}{2}x^2$

$\Rightarrow OA = OC = 2$

Also, $\angle OAB = 150°$

$\Rightarrow \cos 150° = \dfrac{OA^2 + AB^2 - OB^2}{2\,OA \times AB}$

$OB^2 = 8 + 4\sqrt{3} \Rightarrow OB = \sqrt{2}(\sqrt{3}+1)$

Hence, the coordinates of B are $(\pm\sqrt{2}(\sqrt{3}+$ *1*cos*45°, ±23+1*sin*45°)*

204 **(b,c)**

Let slope of line is m. Then

$\frac{1}{2} = \left|\dfrac{m-(-2)}{1+(-2)m}\right|$

$\Rightarrow m = -3/4$ and ∞

Hence, equation of line is $y - 3 = -3/4(x-2)$ and $x = 2$

205 **(b,c)**

The chord subtends 90° at the centre (0, 0). Making $x^2 + y^2 = 1$ homogenous in the second degree with the help of $y = mx + 1$, we get

$x^2 + y^2 = (y - mx)^2$

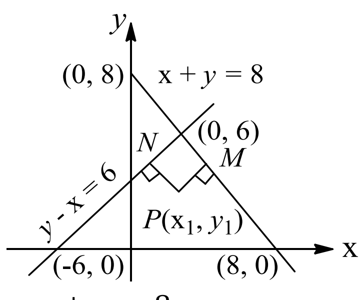
or $\quad (1 - m^2)x^2 + 2mxy = 0$

The angle between these lines is 90° if

$1 - m^2 + 0 = 0$. i.e., $m = \pm 1$

206 **(a,b,c,d)**

Let position of bunglow is $P(x_1, y_1)$, then $PM = 100$ and $PN = 100$



$\therefore \dfrac{x_1 + y_1 - 8}{\sqrt{2}} = \pm 100$

and $\dfrac{x_1 - y_1 + 6}{\sqrt{2}} = \pm 100$

After solving, we get

$x_1 = 1 \pm 100\sqrt{2}, 1$

and $y_1 = 7, 7 \pm 100\sqrt{2}$

Hence, possible location of bunglow are $(1 + 100\sqrt{2}, 7), (1 - 100\sqrt{2}, 7), (1, 7 +$ *1002,(1,7−1002)*

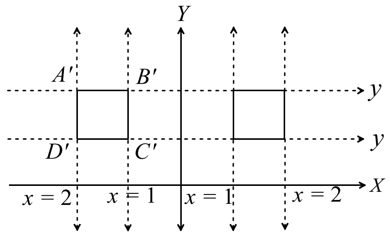
207 **(a,b)**

$x^2 - 3|x| + 2 = 0$

$\Rightarrow (|x| - 1)(|x| - 2) = 0 \Rightarrow x = \pm 1, \pm 2$

$y^2 - 3x + 2 = 0$

$\Rightarrow (y - 1)(y - 2) = 0 \Rightarrow y = 1, 2$



From the figure two such squares are possible whose coordinates are

$A(1,2), B(2,2), C(2,1), D(1,1)$ and

$A'(-2,2), B'(-1,2), C'(-1,1), D'(-2,1)$

208 **(d)**

Slope of $x + 3y = 4$ is $-1/3$ and slope of $6x - 2y = 7$ is 3. Therefore, these two lines are perpendicular which shows that both diagonals are perpendicular. Hence, $PQRS$must be a rhombus

209 **(c,d)**

If lines $x + y - 1 = 0, (m-1)x + (m^2 - 7)y - 5 = 0$ and $(m-2)x + (2m-5)y = 0$ are concurrent, then

$\Delta = 0$

$\Rightarrow \begin{vmatrix} 1 & 1 & -1 \\ m-1 & m^2-7 & -5 \\ m-2 & 2m-5 & 0 \end{vmatrix} = 0$

$\Rightarrow (m-2)(-5 + m^2 - 7)$
$\qquad - (2m-5)(-5 + m - 1) + 0 = 0$

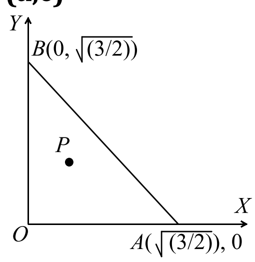
$\Rightarrow (m-2)(m^2 - 12) - (2m-5)(m-6) = 0$

$\Rightarrow m^3 - 4m^2 + 5m - 6 = 0$

$\Rightarrow (m-3)(m^2 - m + 2) = 0$

$\Rightarrow m = 3$ but $m^2 - m + 2 = 0$ has no real roots. If $m = 3$, then two lines are parallel

211 **(a,c)**



Equation of lines along OA, OB and AB are

$y = 0, x = 0$ and $x + y = \sqrt{3/2}$, respectively

Now P and B will lie on the same side of $y = 0$ if $\cos\theta > 0$. Similarly, P and A will lie on the same side of $x = 0$ if $\sin\theta > 0$ and P and O will lie on the same side of $x + y = \sqrt{3/2}$ if $\sin\theta + \cos\theta < \sqrt{3/2}$. Hence, P will lie inside the ΔABC if $\sin\theta > 0, \cos\theta > 0$ and $\sin\theta + \cos\theta < \sqrt{3/2}$.
Now,

$$\sin\theta + \cos\theta < \sqrt{\frac{3}{2}}$$

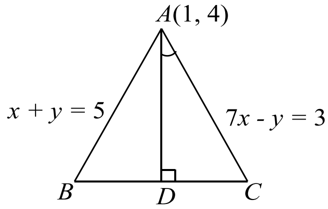
$$\Rightarrow \sin(\theta + \pi/4) < \sqrt{\frac{3}{4}}$$

Since $\sin\theta > 0$ and $\cos\theta > 0$, so $0 < \theta < \pi/12$ or $5\pi/12 < \theta < \pi/2$

212 **(a,b,c)**

The equation is
$x^2(x+y) - y^2(x+y) = 0$
or $(x+y)^2(x-y) = 0$
It represents the lines $x + y = 0, x + y = 0, x - y = 0$

213 **(a,b,c,d)**



Side BC will be perpendicular to the bisector of the angleBAC. Now equations of the bisectors of lines AB and AC are

$$\frac{(x+y-5)}{\sqrt{2}} = \pm\frac{(7x-y-3)}{5\sqrt{2}}$$

$\Rightarrow x - 3y + 11 = 0$ or $3x + y - 7 = 0$
Let equation of side BC be $x - 3y + \lambda = 0$ and altitude through vertex A b eAD. Then equation of AD is$3x + y - 7 = 0$. If$AD = \lambda$, then$\Delta ABC = \frac{1}{2} \times |BC| = \frac{1}{2} \times \lambda \times 2\lambda\,|\tan\theta| = I^2|\tan\theta\,|$. Hence,

$$\lambda \times \frac{1}{2} = 5 \Rightarrow \lambda^2 = 10$$

$\Rightarrow (11 - \lambda)^2 = 100$
$\Rightarrow 11 - \lambda = \pm 10 \Rightarrow \lambda = 1, -21$
Hence, equation of BC is $x - 3y + 1 = 0$ or $x - 3y - 21 = 0$. Similarly, if equation of BC is $3x + y + \lambda = 0$, then equation of AD will be $x - 3y + 11 = 0$. Therefore,

$$|\tan\theta| = \left|\frac{7 - \frac{1}{3}}{1 + \frac{7}{3}}\right| = 2$$

$$\Rightarrow \lambda^2|\tan\theta| = 2\lambda^2 = 5 \Rightarrow \lambda^2 = \frac{5}{2}$$

$$\frac{5}{2} = \frac{(3 + 4 + \lambda)^2}{10}$$

$\Rightarrow 7 + \lambda = \pm 5 \Rightarrow \lambda = 2, -12$
Hence, equation of BC is $3x + y + 2 = 0$ or $3x + y - 12 = 0$. Finally, there are four possible equations of side BC, viz, $x - 3y + 1 = 0, x - 3y - 21 = 0, 3x + y + 2 = 0$ or $3x + y - 12 = 0$

214 **(a,b)**

Let p be the length of the perpendicular from the origin on the given line. Then its equation in normal form is
$x\cos 30° + y\sin 30° = p$
or $\sqrt{3}x + y = 2p$
This meets the coordinate axes at $A\left(\frac{2p}{\sqrt{3}}, 0\right)$ and $B(0,2p)$. Therefore, area of ΔAOB is

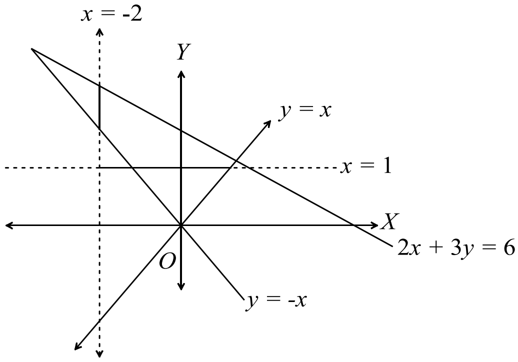
$$\frac{1}{2}\left(\frac{2p}{\sqrt{3}}\right)2p = \frac{2p^2}{\sqrt{3}}$$

By hypothesis,

$$\frac{2p^2}{\sqrt{3}} = \frac{50}{\sqrt{3}} \Rightarrow p = \pm 5$$

Hence, the lines are $\sqrt{3}x + y \pm 10 = 0$

215 **(a,d)**



The separate equations of the sides are $x - y = 0, x + y = 0$ and $2x + 3y - 6 = 0$. The point $(-2, a)$ moves on the line $x = -2$ and $(b, 1)$ moves on the line $y = 1$. From the figure, the y-coordinates of points of intersection of $x = -2$ with $y = -x$ and $2x + 3y = 6$ give the range of values of a. The x-coordinates of the points of intersection of $y = 1$ with $y = -x$ and $y = x$ give the range of values of b

216 **(b)**

On solving $4x + 3y = \lambda$ and $3x - 4y = \mu$

We get $x_1 = \frac{4\lambda+3\mu}{25}$ and $y_1 = \frac{3\lambda-4\mu}{25}$

$\because x_1 = y_1$

$\Rightarrow \dfrac{4\lambda + 3\mu}{25} = \dfrac{3\lambda - 4\mu}{25}$

$\Rightarrow \lambda + 7\mu = 0$

$\therefore$ locus of (λ, μ) is $x + 7y = 0$

For first quadrant $x_1 > 0$ and $y_1 > 0$

ie, $\dfrac{4\lambda + 3\mu}{25} > 0$ and $\dfrac{3\lambda - 4\mu}{25} > 0$

or $4\lambda + 3\mu > 0$ and $3\lambda - 4\mu > 0$

217 **(d)**

First, let the equation $ax + by + c = 0$ represent a family of straight lines passing through (a, b) for different values of a, b and c

Then, we have to show that there is a linear relation between a, b and c and have to prove that the equation $ax + by + c = 0$ represent a family of lines passing through a fixed point. Let the linear relation be

$la + mb + nc = 0$

$\Rightarrow \left(\frac{l}{n}\right) a + \left(\frac{m}{n}\right) b + c = 0$

$\Rightarrow ax + by + c = 0$ always passes through a fixed point $\left(\frac{l}{n}, \frac{m}{n}\right)$

$\therefore 3a + 2b + 4c = 0$

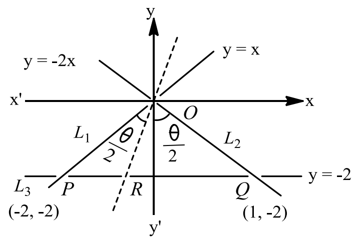
$\Rightarrow \left(\frac{3}{4}\right) a + \left(\frac{2}{4}\right) b + c = 0$

and $ax + by + c = 0$ represents a system of concurrent lines passing through $\left(\frac{3}{4}, \frac{1}{2}\right)$

Thus, statement I is false and II is true

218 **(b)**

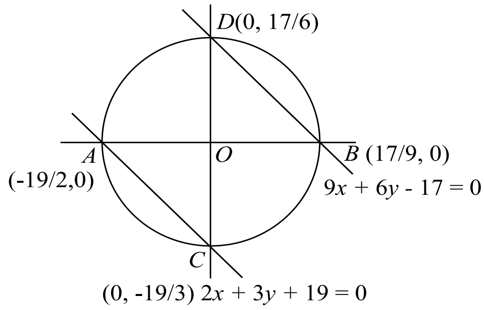
In ΔOPQ, OR is the internal bisector of $\angle POQ$



$\therefore \dfrac{PR}{RQ} = \dfrac{OP}{OQ} = \dfrac{\sqrt{2^2 + 2^2}}{\sqrt{1^2 + 2^2}} = \dfrac{2\sqrt{2}}{\sqrt{5}}$

219 **(a)**

From the figure, both the statements are true and statement 2 correctly explains statement 1



220 **(d)**

According to given data $2a - 5 + a^2 - 3 < 0$ or $a^2 + 2a - 8 < 0$ or $(a - 2)(a + 4) < 0$ or $a \in (-4, 2)$

221 **(d)**

We know that if sum of algebraic distances from three points on the variable line is zero, then the line always passes through the mean of the given point, which is centroid of triangle formed by given three points. But centroid of triangle is (1, 2). Hence, the line must pass through it, for which $a + 2b + c = 0$. Therefore, statement 1 is false and statement 2 is true

222 **(c)**

Statement 2 is false as point satisfying such property can be ex-centre of the triangle. However, statement 1 is true as (0,0) is at distance π from all the lines and it lies inside the triangle

223 **(c)**

Let $f(x, y) \equiv 3x - 5y + \lambda$

$\because$ Points (1, 2) and (3, 4) be on the same side of the line $3x - 5y + \lambda = 0$, then

$$\frac{f(1,2)}{f(3,4)} > 0$$
$$\Rightarrow \frac{3-10+\lambda}{9-20+\lambda} > 0$$
$$\Rightarrow \frac{\lambda-7}{\lambda-11} > 0$$
$\therefore\ \lambda < 7$ or $\lambda > 11$

224 **(a)**
Any line equally inclined to given lines is always parallel to angle bisectors

225 **(d)**
The joint equation of $y = x$ and $y = -x$ is $(x-y)(x+y) = 0$, i.e., $x^2 - y^2 = 0$

226 **(a)**
We have, $(m_{AC})(m_{BC}) = \left(\frac{-4+2}{5+5}\right)\left(\frac{-4-6}{5-7}\right) = -1$

Therefore, ABC is right-angled triangle with C as the right angle. Hence, circumcentre is midpoint of AB, i.e., (1, 2)

227 **(a)**
Bisectors of the given lines are$(3x + 4y - 12)/5 = \pm(4x + 3y - 24)/5$, of which one the bisectors is$y - x + 12 = 0$. Also any point on the bisector is always equidistant from the given lines

228 **(d)**
Statement 1 is false since $(x-2) + (2x-4) + (6-3x) = 0$ but the lines $x - 2 = 0, 2x - 4 = 0$ and $6 - 3x = 0$ are parallel. Statement 2 is a standard result whose more general form as follows. Let $L_1 = 0, L_2 = 0, L_3 = 0$ be three lines. Now, if we can find λ, μ, v (not all zero) such that $\lambda L_1 + \mu L_2 + vL_3 = 0$, then the three line $L_1 = 0, L_2 = 0, L_3 = 0$ are either concurrent or parallel

229 **(b)**
Put $2h = -(a+b)$ in $ax^2 + 2hxy + by^2 = 0$. Then,

$$ax^2 - (a+b)xy + by^2 = 0$$

$$\Rightarrow (x-y)(ax-by) = 0$$

Therefore, one of the lines bisects the angle between coordinates axes in positive quadrant. Also putting $b = -2h - 1$ in $ax - by$, we have $ax - by = ax - (-2h - a)y = ax + (2h + a)y$. Hence, $ax + (2h + a)$ is a factor of $ax^2 + (2h + a)$. However, statement 2 is not correct explanation of statement 1

230 **(b)**
Bisectors of angle C are

$$\frac{3x+2y}{\sqrt{13}} = \pm\frac{2x+3y+6}{\sqrt{13}}$$

or $x - y - 6 = 0$ and $5x + 5y + 6 = 0$

According to given equations of sides, internal angle bisector at C will have negative slope. Also, image of A will lie on BC respect to both bisectors, from which we can conclude that $5x + 5y + 6 = 0$ is internal angle bisector. Hence, statement 2 is not correct explanation of statement 1

231 **(a)**
Equation of bisector of $4y + 3x - 12 = 0$ and $3y + 4x - 24 = 0$ is
$$\frac{4y+3x-12}{\sqrt{16+9}} = \pm\frac{3y+4x-24}{\sqrt{9+16}}$$
$\Rightarrow y - x + 12 = 0$ and $7y + 7x - 36 = 0$
$\therefore$ The line $y - x + 12 = 0$ is the angular bisector

232 **(a)**
The given lines are

$(a+b)x + (a-b)y - 2ab = 0$ (i)

$(a-b)x + (a+b)y - 2ab = 0$ (ii)

$x + y = 0$ (iii)

The triangle formed by the lines (i), (ii) and (iii) is an isosceles triangle if the internal bisector of the vertical angle is perpendicular to the third side. Now equations of bisectors of the angle between lines (i) and (ii) are

$$\frac{(a+b)x+(a-b)y-2ab}{\sqrt{[(a+b)^2+(a-b)^2]}} = \pm\frac{(a-b)x+(a+b)y-2ab}{\sqrt{[(a-b)^2+(a+b)^2]}}$$

or $x - y = 0$ (iv)

and $x + y = 2b$ (v)

Obviously the bisector (iv) is perpendicular to the third side of the triangle. Hence, the given lines form an isosceles triangle

233 **(c)**
Centroid $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$ is a rational point.

Orthocentre is intersection point of two altitudes which will bear rational coefficients when expressed as a straight line. So, orthocentre is also rational. Circumcentre is intersection point of two perpendicular bisectors which will bear rational coefficient when expressed as a straight line. So, circumcentre is also rational. But statement 2 is not true as in equilateral triangle all the centres coincide

234 **(a)**
Area of triangles is unaltered by shifting origin to any point. If origin is shifted to (1000, 1002), A, B, C become $P(0,0)Q(1,2), R(2,1)$.both are true

235 **(a)**
The quadrilateral is obviously a parallelogram and if the diagonals are at right angles, it must be a rhombus. Hence, the distance between the pairs of opposite sides must be the same, i.e.,

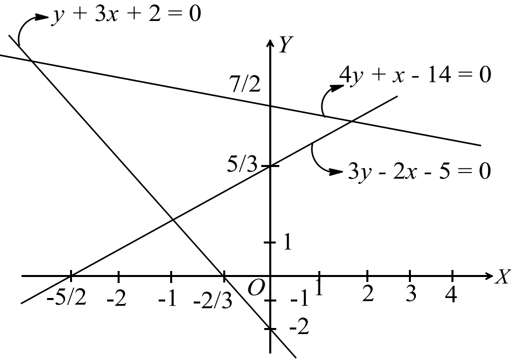
$$\frac{|r-r'|}{\sqrt{p^2+q^2}}=\frac{r-r'}{\sqrt{{p'}^2+{q'}^2}}$$

$$\Rightarrow p^2+q^2={p'}^2+{q'}^2$$

236 **(c)**
$ax^2+2bxy+by^2+2gx+2fy+c=0$
Represents the general equation of second of second degree. It represents a pair of straight lines, if
$\Delta\equiv abc+2fgh-af^2-bg^2-ch^2=0$
Thus, statement II is false
And the equation of the pair of straight lines formed by
$2x-y=5$ and $x+2y=3$ is given by
$(2x-y-5)(x+2y-3)=0$
$\Rightarrow\ 2x^2+3xy-2y^2-11x-7y+15=0$
Thus, statement I is true

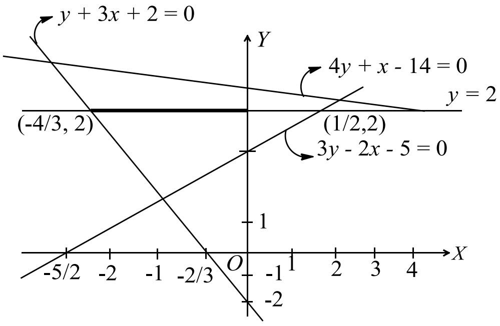
237 **(a)**
Statement 1 is true and follows from statement 2 as the family of lines can be written as
$a(x+y-1)+b(x-2y)=0$

238 **(b)**

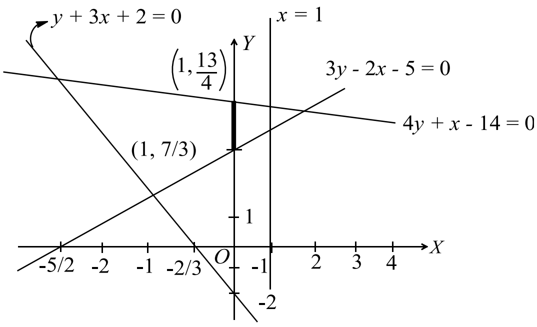


1.

2. Clearly, point $(\alpha, 0)$ lies on the x-axis, which is not intersecting any side of triangle, hence no such α exists



3.



4.

239 **(d)**
The equation $4x^2+8xy+ky^2-9=0$ represents a pair of straight lines if
$(4)(k)(-a)-(-a)(4)^2=0$

$\Rightarrow k=4$

$m_1+m_2=4m_1m_2$

$$\Rightarrow\ -\frac{2h}{b}=\frac{4a}{b}$$

$$\Rightarrow\ -\frac{2(-c)}{-7}=\frac{4\times 1}{-7}$$

$\Rightarrow 2c=4\ \ \Rightarrow c=2$

Let m_1, m_2 be the slopes of the lines

$x^2 + hxy + 2y^2 = 0$. Then,

$$m_1 + m_2 = -\frac{h}{2}, m_1 m_2 = \frac{1}{2}$$

But $m_1 = 2m_2$ (given). Therefore,

$3m_2 = -h/2$ and $2m_2^2 = 1/2$,

i.e.,$m_2^2 = \frac{1}{4}$. Also, $m_2 = -h/6$

$$\therefore \frac{h^2}{36} = \frac{1}{4} \Rightarrow h^2 = 9 \quad \Rightarrow h = \pm 3$$

Equation of the bisectors of the angle between the lines $ax^2 + 2hxy + by^2 + \lambda(x^2 + y^2) = 0$ is

$$\frac{x^2 - y^2}{(a + \lambda) - (b + \lambda)} = \frac{xy}{h}$$

or $\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$

Which is same as the equation of the bisector of angles between the lines $ax^2 + 2hxy + by^2 = 0$

Thus, the two line pairs are equally inclined to each other for any value of λ

240 **(c)**

$$(x + 7y)^2 + 4\sqrt{2}(x + 7y) - 42 = 0$$

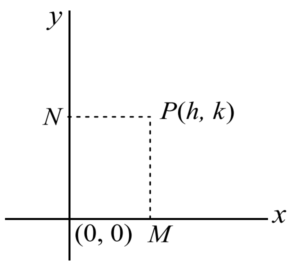
$$\Rightarrow (x + 7y)^2 + 7\sqrt{2}(x + y) - 3\sqrt{2}(x + y) - 42 = 0$$

$$\Rightarrow (x + y)[x + 7y + 7\sqrt{2}] - 3\sqrt{2}(x - 7y + 7\sqrt{2}) = 0$$

$$\Rightarrow (x + 7y + 7\sqrt{2})(x + 7y - 3\sqrt{2}) = 0$$

$$\Rightarrow x + 7y + 7\sqrt{2} = 0 \text{ and } x + 7y - 3\sqrt{2} = 0$$

$$\Rightarrow \quad d = \left|\frac{7\sqrt{2} + 3\sqrt{2}}{\sqrt{1 + 49}}\right| = \frac{10\sqrt{2}}{\sqrt{50}} = 2$$



6.

Let two perpendicular lines are coordinate axes.

Then,

$PM + PN = 1$

$\Rightarrow h + k = 1$

Hence, the locus is $x + y = 1$

But if the point lies in other quadrants also, then $|x| + |y| = 1$. Hence, value of k is

Angle bisector between the lines $x + 2y + 4 = 0$ and

$4x + 2y - 1 = 0$ is

$$\frac{x + 2y + 4}{\sqrt{1 + 4}} = \pm \frac{(-4x + 2y + 1)}{\sqrt{16 + 4}}$$

$$\Rightarrow x + 2y + 4 = \pm \frac{(-4x - 2y + 1)}{2}$$

$$\Rightarrow 2(x + 2y + 4) = \pm(-4x - 2y + 1)$$

Since $AA' + BB' < 0$ so +ve sign gives acute angle bisector. Hence,

$2x + 4y + 8 = -4x - 2y + 1$

$\Rightarrow 6x + 6y + 7 = 0$

$\Rightarrow m = 7$

We have,

$y^2 - 9xy + 18x^2 = 0$

Or $y^2 - 6xy - 3xy + 18x^2 = 0$

$\Rightarrow y(y - 6x) - 3x(y - 6x) = 0$

$\Rightarrow (y - 3x) = 0$ and $y - 6x = 0$

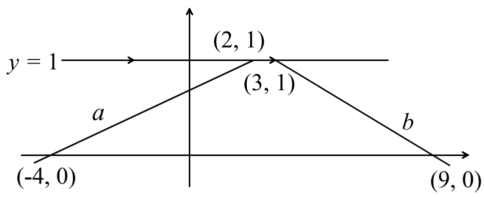
The third line is $y = 6$. Therefore, area of the triangle formed by these lines,

$$= \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 1 & 6 & 1 \\ 2 & 6 & 1 \end{vmatrix}$$

$$= \frac{1}{2}|6 - 12|$$

$= 3$ units^2

241 **(b)**

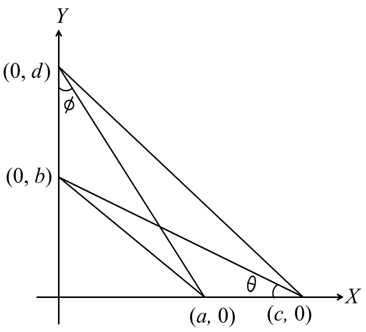
a. 

Obviously, trapezium

$$\left.\begin{aligned} a &= \sqrt{37} \\ b &= \sqrt{37} \end{aligned}\right\} \Rightarrow a = b$$

Hence, isosceles trapezium

⇒ a cyclic quadrilateral

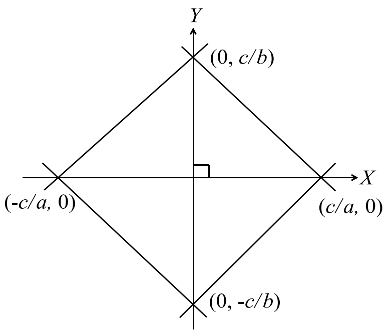
b. 

$ac = bd$

$\Rightarrow \frac{b}{c} = \frac{a}{d}$

$$\left.\begin{aligned} \tan\theta &= \frac{b}{c} \\ \tan\phi &= \frac{a}{d} \end{aligned}\right\} \Rightarrow \theta = \phi$$

Hence, cyclic quadrilateral

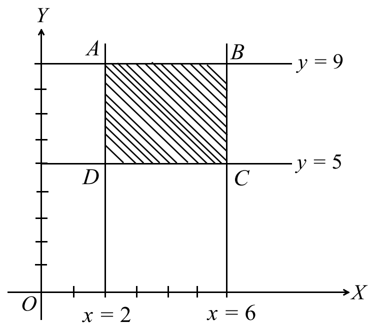
c. 

$ax \pm by \pm c = 0$

If $y = 0, y = \pm\frac{c}{a}$

If $x = 0, y = \pm\frac{c}{d}$

⇒ rhombus

d. 

$(x - 6)(x - 2) = 0$

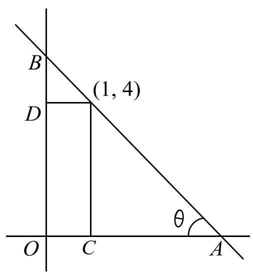
$x = 6$ and $x = 2$

$y^2 - 14y + 45 = 0$

$(y - 9)(y - 5) = 0$

⇒ a square

242 **(c)**

1. 

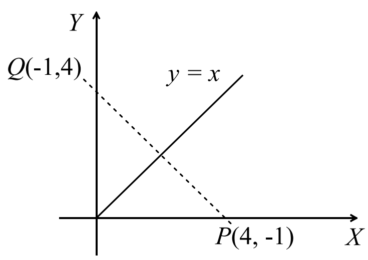
$OA = 1 + 4\cot\theta$

$OB = 4 + \tan\theta$

$OA + OB = 5 + 4\cot\theta + \tan\theta$

$\geq 5 + 2\sqrt{4\cot\theta\tan\theta}$

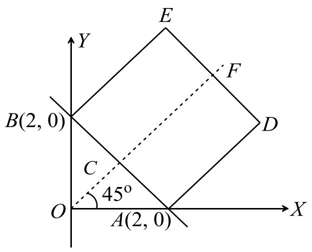
$= 5 + (2 \times 2) = 9$

2. 

Reflection of $P(4, -1)$ in $y = x$ in $Q(-1,4)$. Hence,

$PQ = \sqrt{(4+1)^2 + (-1-4)^2}$

$= \sqrt{50} = 5\sqrt{2}$

c. 

$AB = 2\sqrt{2}$

$OC = \sqrt{2}$

Maximum value of $d$is

$OF = \sqrt{2} + 2\sqrt{2}$

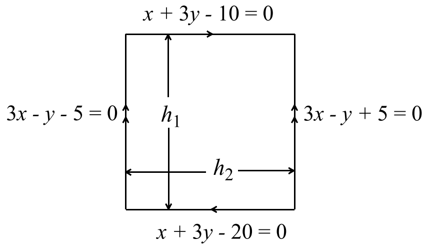
$= 3\sqrt{2}$

d. The given is

$$x = 4 + \frac{1}{\sqrt{2}}\left(\frac{y+1}{\sqrt{2}}\right) \Rightarrow y = 2x - 9$$

Hence, the intercept made by x-axis is $9/2$

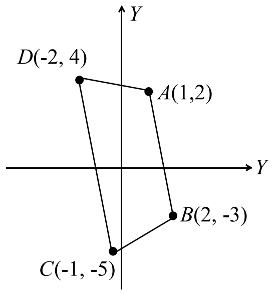
243 **(b)**

1. 

$$h_1 = \left|\frac{10}{\sqrt{10}}\right| = \sqrt{10}$$

$$h_2 = \frac{10}{\sqrt{10}} = \sqrt{10}$$

Hence, the given lines form a square of side $\sqrt{10}$. Therefore, the area 10 sq. units

2. 

$$m_{AB} = \frac{5}{-1} = -5$$

$$m_{DC} = \frac{9}{-1} = -9$$

Hence, the figure is not a parallelogram

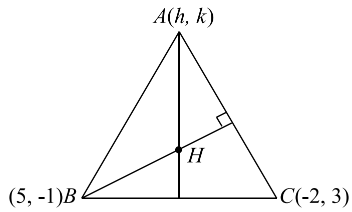
3. Lines $7x + 3y - 33 = 0, 7x + 3y - 4 = 0$ are parallel and distance between them is $|29/\sqrt{58}|$. Lines $3x - 7y + 19 = 0, 3x - 7y - 10$ are parallel and instance between them $|29/\sqrt{58}|$. Also, lines $7x + 3y - 33 = 0$ and $3x - 7y + 19 = 0$ are perpendicular. Hence, given lines form a square

d. Lines $4y - 3x - 7 = 0$ and $4y - 3x - 21 = 0$ are parallel. Lines $3y - 4x + 7 = 0, 3y - 4x + 14 = 0$ are parallel. Also, lines $4y - 3x - 7 = 0$ and $3y - 4x + 14 = 0$ are not perpendicular. Hence, given lines form parallelogram

244 **(a)**

$AH \perp BC \Rightarrow \left(\frac{k}{h}\right)\left(\frac{3+1}{-2-5}\right) = -1$

$\therefore\ 4k = 7h$ (i)



$BH \perp AC \Rightarrow \left(\frac{0+1}{0-5}\right)\left(\frac{k-3}{h+2}\right) = -1$

$\therefore k - 3 = 5(h+2)$ (ii)

$\Rightarrow 7h - 12 = 20h + 40$

$\Rightarrow 13h = -52$

$\Rightarrow h = -4$

$\therefore k = -7$

Hence, point, A is $(-4, -7)$

$x + y - 4 = 0$ (i)

$4x + 3y - 10 = 0$ (ii)

Let $(h, 4-h)$ be the point on (i). Then,

$\left|\frac{4h + 3(4-h) - 10}{5}\right| = 1$

$\Rightarrow h + 2 = \pm 5$

$\Rightarrow h = 3, h = -7$

Hence, the required point is either (3, 1) or $(-7, 11)$

Since lines $x + y - 1 = 0$ and $x - y + 3 = 0$ are perpendicular, orthocentre of the triangle is the point of intersection of these lines, i.e, $(-1,2)$

Since, $2a, b, c$ are in A.P., so

$b = \frac{2a + c}{2}$

$\Rightarrow 2a - 2b + c = 0$

Comparing with the line $ax + by + c = 0$, we have $x = 2$ and $y = -2$. Hence, lines are concurrent at $(2, -2)$

245 **(d)**

Given lines are concurrent. So,

$\begin{vmatrix} 3 & 1 & -4 \\ 1 & -2 & -6 \\ \lambda & 4 & \lambda^2 \end{vmatrix} = 0$

$\Rightarrow \lambda^2 + 2\lambda - 8 = 0$

$\Rightarrow \lambda = 2, -4$

Points are collinear. Hence,

$\begin{vmatrix} \lambda + 1 & 1 & 1 \\ 2\lambda + 1 & 3 & 1 \\ 2\lambda + 2 & 2\lambda & 1 \end{vmatrix} = 0$

$\Rightarrow 2\lambda^2 - 3\lambda - 2 = 0 \Rightarrow \lambda = 2, -1/2$

The point of intersection of $x - y + 1 = 0$ and $3x + y - 5 = 0$ is (1,2). It lies on the line $x + y - 1 - |\lambda/2| = 0$

$\Rightarrow \lambda = \pm 4$

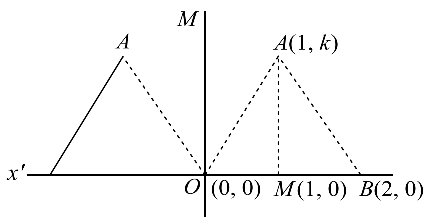
The midpoint of $(1, -2)$ and (3,4) will satisfy

$y - x - 1 + \lambda = 0$

$\Rightarrow \lambda = 2$

246 **(a)**

Given that points are $O(0, 0)$ and $B(2, 0)$



From figure, ΔABC is equilateral

Hence, $\tan 60° = k$

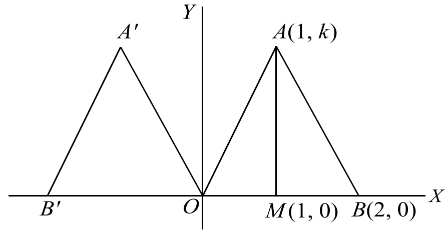
or $k = \sqrt{3}$ (for first quadrant) or $k = -\sqrt{3}$ (for fourth quadrant). Then possible coordinates are $(1, \pm\sqrt{3})$

Similarly, for second quadrant, the point is $(-1, -\sqrt{3})$

And for third quadrant, the point is $(-1, -\sqrt{3})$

Case (i)

If $OA = AB$, then $\angle A = 30°$



$\therefore\ \angle AOB = 75°$

$\therefore \dfrac{AM}{OM} = \tan 75°$

$AM = OM \tan 75°$

$k = 1 \times (2 + \sqrt{3})$

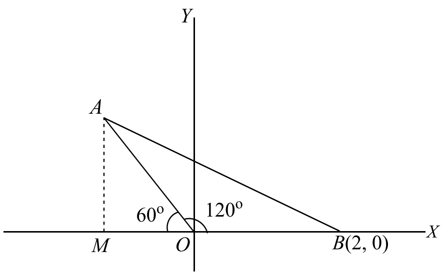
$\therefore\ \ k = 2 + \sqrt{3}$

Hence, point A is $(1, 2 + \sqrt{3})$. By symmetry, all possible points are $(\pm 1, \pm(2 + \sqrt{3})$

Case (ii)

$AO = OB$

$\therefore\ \angle AOB = 120°$

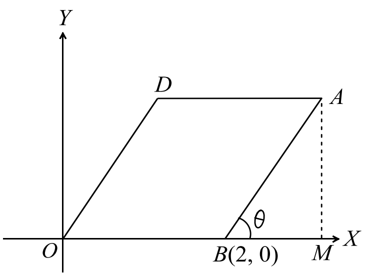


$AM = 2 \sin 60° = \sqrt{3}$

And $OM = 2 \cos 60° = 1$

Hence, point A is $(1, -\sqrt{3})$.by symmetry, all possible points are $(\pm 1, \pm\sqrt{3})$

c.

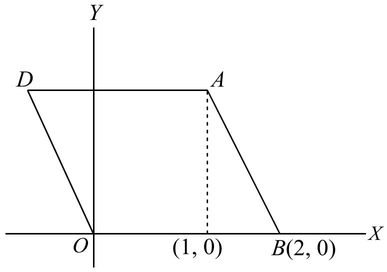


Let$\angle DOB = \angle ABM = \theta$. Area of ΔOAB is $\frac{1}{2} \times OB \times AM = \frac{1}{2} \times \sqrt{3}$

$\Rightarrow 2 \times 2 \sin\theta = \sqrt{3}$

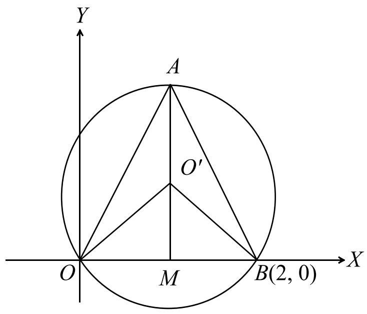
$\Rightarrow \sin\theta = \frac{\sqrt{3}}{4} \Rightarrow AM = \sqrt{3}$ and $BM = 1$

Hence, A has coordinates $(3, \sqrt{3})$. By symmetry, all possible coordinates are $(\pm 3, \pm\sqrt{3})$



From the above figure A has coordinates $(1, \sqrt{3})$

By symmetry, all possible coordinates are $(\pm 1, \pm\sqrt{3})$



d.

$OB = 2$ units $= OO' =$ radius

$\Rightarrow OM = \frac{2}{2} = 1$ unit

In $\Delta\, OO'\, M$,

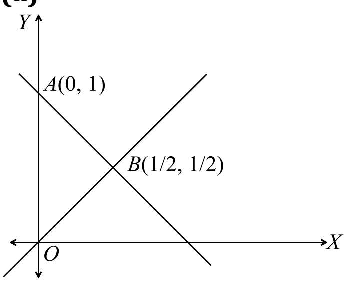
$O'M = \sqrt{4 - 1} = \sqrt{3}$

Since ΔOAB is isosceles hence point A lies on perpendicular bisector of OB

$\therefore AM = \sqrt{3} + 2 = O'M + OA$

Hence, the coordinate A will be $(1{,}2 + \sqrt{3})$ in first quadrant. By symmetry, all possible coordinates of A are $(\pm 1, \pm(2 + \sqrt{3}))$

247 **(a)**



The given lines are$x(x + y - 1)(x - y) = 0$. In other words, lines $x = 0, x + y - 1 = 0$ and $x - y = 0$ form triangle OAB as shown in the above diagram

The triangle is right angled at point B, hence orthocentre is $(1/2, 1/2)$. Also, circumcentre is midpoint of OA which is $(0, 1/2)$. The centroid is

$$\left(\frac{0+\frac{1}{2}+0}{3}, \frac{0+\frac{1}{2}+1}{3}\right) \text{ or } \left(\frac{1}{6}, \frac{1}{2}\right)$$

Also, $OA = 1, OB = OC = 1/\sqrt{2}$. Hence, the incentre is

$$\left(\frac{0\left(\frac{1}{\sqrt{2}}\right)+\frac{1}{2}(1)+0\left(\frac{1}{\sqrt{2}}\right)}{\frac{1}{\sqrt{2}}+1+\frac{1}{\sqrt{2}}}, \frac{0\left(\frac{1}{\sqrt{2}}\right)+\frac{1}{2}(1)+1\left(\frac{1}{\sqrt{2}}\right)}{\frac{1}{\sqrt{2}}+1+\frac{1}{\sqrt{2}}}\right)$$

$$\equiv \left(\frac{1}{2+2\sqrt{2}}, \frac{1}{2}\right)$$

248 **(b)**

From the given equations we have,

$$\frac{1-\cos\theta}{\sin\theta} = \frac{a-x}{y}$$

and $\frac{1+\cos\theta}{\sin\theta} = \frac{a+x}{y}$

On multiplying, we get

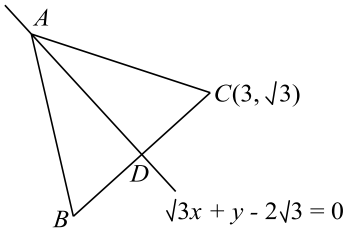
$$\frac{1-\cos^2\theta}{\sin^2\theta} = \frac{a^2-x^2}{y^2}$$

$\Rightarrow x^2 + y^2 = a^2$

249 **(a)**

Given lines $(x + y + 1) + b(2x - 3y - 8) = 0$ are concurrent at point of intersection of the lines $x + y + 1 = 0$ and $2x - 3y - 8 = 0$, which is farthest from the point $B(2, 2)$, is perpendicular to AB. Now, slope of AB is 4. Then the required line is $y + 2 = -(1/4)(x - 1)$ or $x + 4y + 7 = 0$

250 **(b)**



Let the triangle be ABC with $C \equiv (3, \sqrt{3})$ and altitude drawn through vertex (meeting BC at D) be $\sqrt{3}x + y - 2\sqrt{3} = 0$. If B is (x_b, y_b), then we have

$$\frac{2(x_b-3)}{\sqrt{3}} = \frac{y_b-\sqrt{3}}{2} = -\frac{2(3\sqrt{3}+\sqrt{3}-2\sqrt{3})}{2}$$

$$= -2\sqrt{3}$$

$\Rightarrow x_b = 0, y_b = 0$

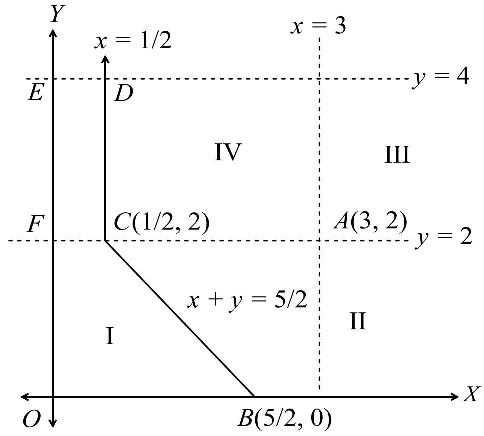
And coordinates of D is $(3/2, \sqrt{3}/2)$. Let coordinates of vertex A be (x_a, y_b). Then,

$$\frac{x_a - \frac{3}{2}}{-1/2} = \frac{y_a - \sqrt{3}/2}{\sqrt{3}/2} = \pm 3$$

$\Rightarrow (x_a, y_b) \equiv (0, 2\sqrt{3})$ or $(3, -\sqrt{3})$

Hence, the remaining vertices are $(0, 0)$ and $(0, 2\sqrt{3})$ or $(0,0)$ and $(3, -\sqrt{3})$. Also, the orthocenter is $(1, \sqrt{3})$ or $(2, 0)$

251 **(d)**



Let $P = (h, k)$ be a general point in the first quadrant such that

$d(P, A) = d(P, O)$

$|h - 3| + |k - 2| = |h| + |k| = h + k$ (i)

[h and k are +ve, point $P(h, k)$ being in first quadrant]

If $h < 3, k > 2$, then (h, k) lies in region I. Then,

$3 - h + 2 - k = h + k \Rightarrow h + k = 5/2$

If $h > 3, k < 2$, then (h, k) lies in region II. Then,

$h - 3 + 2 - k = h + k$

$\Rightarrow k = -1/2$ (not possible)

If $h > 3, k > 2$ then (h, k) lies in region III. Then,

$h - 3 + k - 2 = h + k \Rightarrow -5 = 0$ (not possible)

if $h < 3, k > 2$, then (h, k) lies in region IV. Then,

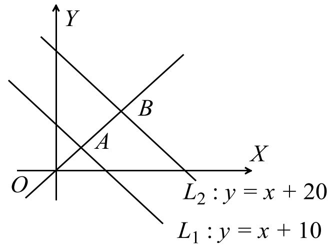
$3 - h + k - 2 = h + k \Rightarrow h = 1/2$

Hence, the required set consists of line segment $x + y = 5/2$ of finite lengths as shown in the first region and the ray

$x = 1/2$ in the fourth region

Obviously locus of P is union of line segment and one infinite ray

252 **(d)**

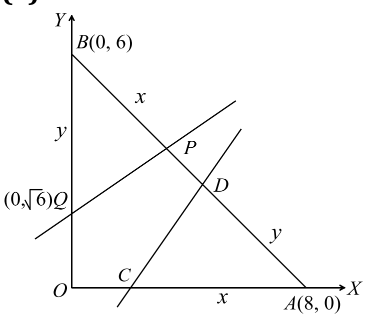


Let the parametric equation of drawn line be

$$\frac{x}{\cos\theta} = \frac{y}{\sin\theta} = r$$

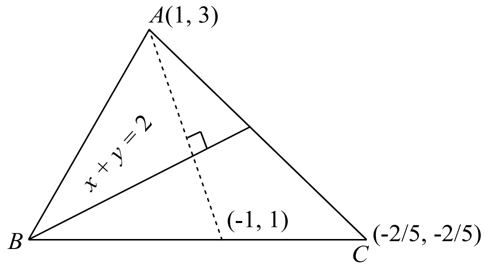
$\Rightarrow x = r\cos\theta, y = r\sin\theta$
Putting it in 'L_1', we get
$r\sin\theta = r\cos\theta + 10$
$\Rightarrow \dfrac{1}{OA} = \dfrac{\sin\theta - \cos\theta}{10}$
Similarly, putting the general point of drawn line in the equation of L_2, we get
$\dfrac{1}{OB} = \dfrac{\sin\theta - \cos\theta}{20}$
Let $P = (h, k)$ and $OP = r \Rightarrow r\cos\theta = h, r\sin\theta = k$, we have
$\dfrac{2}{r} = \dfrac{\sin\theta - \cos\theta}{10} + \dfrac{\sin\theta - \cos\theta}{20}$
$\Rightarrow \;\; 40 = 3r\sin\theta - 3r\cos\theta$
$\Rightarrow 3y - 3x = 40$

253 **(a)**



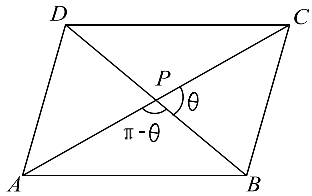
Case I: Let the line L cut AO and AB at distance x and y from A. Then, the area of the triangle with sides x and y is
$\dfrac{1}{2}xy\sin(\angle CAD) = \dfrac{1}{2}xy\dfrac{3}{5} = \dfrac{3xy}{10} = 12$
$\Rightarrow xy = 40$
Also, $x + y = 12$ (from perimeter bisection). Then x and y are roots of $r^2 - 12x + 40 = 0$ which has imaginary roots
Case II: If the line L cuts OB and BA at distance y and x from B, then we have $xy = 30$ and
$x + y = 12$
Solving, we get $x = 6 + \sqrt{6}$ and $y = 6 - \sqrt{6}$
Case III: If the line L cuts the sides OA and OB at distances x and y from O, then
$x + y = 12$ and $xy = 24$
$\therefore x, y = 6 \pm 2\sqrt{3}$ (not possible)
So there is a unique line possible. Let point P be (α, β). Using parametric equation of AB, we have
$\beta = 6 - \dfrac{3}{5}(6 + \sqrt{6})$
and $\alpha = \frac{4}{5}(6 + \sqrt{6})$
Hence, slope of PQ is
$\dfrac{\beta - \sqrt{6}}{\alpha - 0} = \dfrac{10 - 5\sqrt{6}}{10}$

254 **(b)**
Image of $A(1, 3)$ in line $x + y = 2$ is $(1 - 2(2)/2, 3 - 2(2)/2) \equiv (-1,1)$



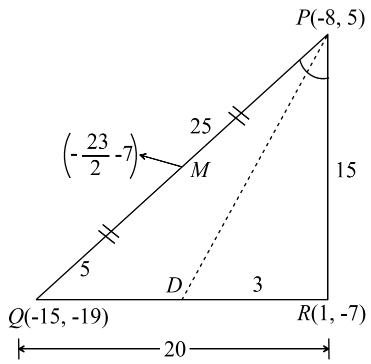
So line BC passes through $(-1, 1)$ and $(-2/5, -2/5)$. The equation of line BC is
$y - 1 = \dfrac{-2/5 - 1}{-2/5 + 1}(x + 1)$
$\Rightarrow 7x + 3y + 4 = 0$

255 **(c)**



Angle between the diagonals is given by
$\tan\theta = \left|\dfrac{-\frac{1}{2} + 2}{1 + 1}\right| = \dfrac{3}{4}$
$\Rightarrow \sin\theta = \dfrac{3}{5}$
Area of ΔCPB is
$\dfrac{1}{2} \times PC \times PB\sin\theta = 2 \;\Rightarrow PB = \dfrac{10}{3}$
$\Rightarrow BD = \dfrac{20}{3}$

256 **(a)**



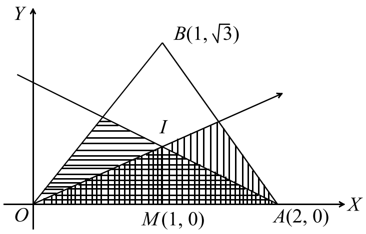
Since triangle is right angled, circumcentre is the midpoint of PQ and orthocenter is $R(1, -7)$. Hence,
$RM = \left|\sqrt{\left(\dfrac{23}{2} + 1\right)^2}\right| = 12\dfrac{1}{2}$

257 **(b)**
$\theta = 60°, m = 2$
$\tan\theta = \dfrac{m\sin\omega}{1 + m\cos\omega} = \dfrac{2\sin 60°}{1 + 2\cos 60°}$

$$= \frac{2 \times \sqrt{3}/2}{1 + 2 \times 1/2} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \ \theta = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

258 **(d)**



$d(P, OA) \le \min[d\,(P, OB), d(P, AB)]$

$\Rightarrow d(P, OA) \le d(P, OB)$

And $\ d(P, OA) \le d(P, AB)$

When $d(P, OA) = d(P < OB)$, P is equidistant from OA and OB, or P lies on angle bisector of lines OA and OB. Hence, when $d(P < OA) \le d(P, OB)$, point P is nearer to OA than OB or lies below bisector of OA and OB. Similarly, when $d(P, OA) \le d(P, AB)$, P is nearer to OA than AB, or lies below bisector of OA and AB. Therefore, the required area is equal to the area of ΔOIA

Now,

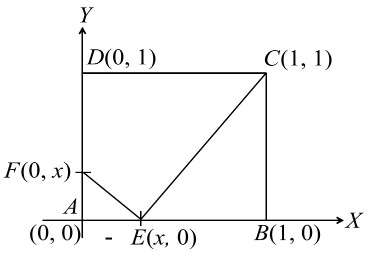
$$\tan \angle BOA = \frac{\sqrt{3}}{1} = \sqrt{3}$$

$\Rightarrow \ \angle BOA = 60°$

Hence, triangle is equilateral. Then I coincides with centroid, which is $(1, 1/\sqrt{3})$

Therefore, area of ΔOIA is $\frac{1}{2} OA \times IM = (1/2) \times 2 \times (1/\sqrt{3}) = 1/\sqrt{3}$ sq. units

259 **(c)**



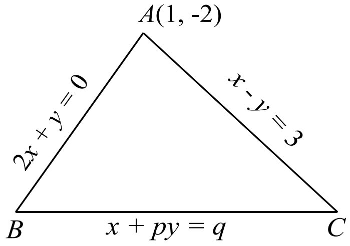
Area of $CDFE$

$$A = 1 - \frac{1}{2}x^2 - \frac{1}{2}(1 - x)$$

$$= \frac{2 - x^2 - 1 + x}{2} = \frac{1 + x - x^2}{2}$$

$$A_{\max} = \frac{1 + \frac{1}{2} - \frac{1}{4}}{2} = \frac{5}{8} \ \text{ at } \ x = \frac{1}{2}$$

260 **(5)**



$$\left(\frac{-q}{2p - 1}, \frac{2q}{2p - 1}\right) \qquad \left(\frac{3p + q}{p + 1}, \frac{q - 3}{p + 1}\right)$$

P is orthocenter

$\Rightarrow \ AP \perp BC$

$$\Rightarrow \ \left(-\frac{1}{p}\right)\left(\frac{3 + 2}{2 - 1}\right) = -1$$

$$\Rightarrow \frac{5}{p} \Rightarrow p = 5$$

$\because BP \perp AC$

$$\Rightarrow \frac{27 - 2q}{18 + q} = -1 \ \Rightarrow q = 27 + 18$$

$\Rightarrow q = 45$

$\because p + q = 5 + 45 = 50$

261 **(4)**

Any point on the line $x + y = 4$ is $(t, 4 - 4)$ where $t \in R$

Now distance of this point from the line $4x + 3y - 10 = 0$ is1

$$\Rightarrow \frac{|4t + 3(4 - t) - 10|}{5} = 1$$

$\Rightarrow |t + 2| = 5$

$\Rightarrow t = 3 \ \text{ or } \ t = -7$

$\Rightarrow$ sum of values is -4

262 **(1)**

Lines $(k + 1)x + 8y = 4k$ and $kx + (k + 3)y = 3k - 1$ are coincident then we can compare ratio of coefficients

$$\Rightarrow \frac{k + 1}{k} = \frac{8}{k + 3} = \frac{4k}{3k - 1}$$

$\Rightarrow k^2 + 4k + 3 = 8k \ \text{ and } \ 24k - 8 = 4k^2 + 12k$

$\Rightarrow (k - 3)(k - 1) = 0$ and $(k - 2)(k - 1) = 0$

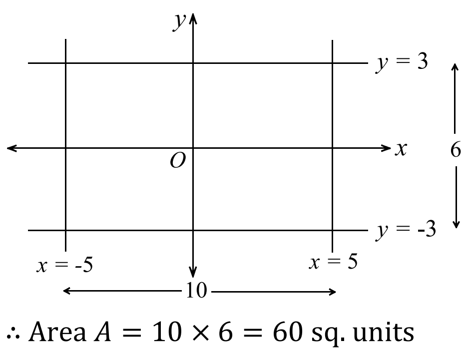
$\Rightarrow k = 1$

263 **(6)**

$x^2y^2 - 9x^2 - 25y^2 + 225 = 0$

$\Rightarrow x^2(y^2 - 9) - 25(y^2 - 9) = 0$

$\Rightarrow (y^2 - 9)(x^2 - 25) = 0$



$\therefore$ Area $A = 10 \times 6 = 60$ sq. units

264 **(0)**

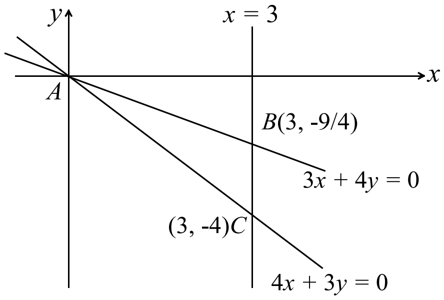
Equation of angle bisector of angle A

$$\frac{3x+4y}{5} = \pm\frac{4x+3y}{5} \Rightarrow x = \pm y$$

Equation of internal bisector is $x = -y$

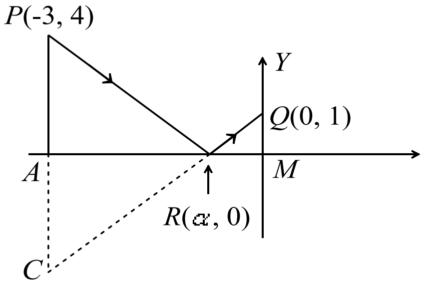
Since h and k lie on the line $x = -y$

$\Rightarrow h + k = 0$



265 **(3)**

For $PR = RQ$ to be minimum it should be the path of light



$\therefore \ \angle PRA = \angle QRM$

From similar ΔPAR and $\Delta\ QMR$

$$\frac{AR}{RM} = \frac{PA}{QM}$$

$$\Rightarrow \frac{\alpha+3}{0-\alpha} = \frac{4}{1} \Rightarrow \ \alpha = -\frac{3}{5}$$

266 **(0)**

As H, G and S are collinear

$$\therefore \begin{vmatrix} 4 & b & 1 \\ b & 2b-8 & 1 \\ -4 & 8 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 4 & b & 1 \\ b-4 & b-8 & 0 \\ -(b+4) & 16-2b & 0 \end{vmatrix} = 0$$

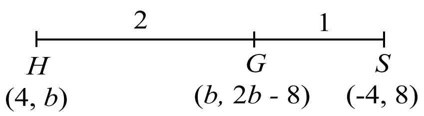
$\Rightarrow (b-4)(16-2b) + (b+4)(b-8) = 0$

$\Rightarrow 2(b-4)(8-b) + (b+4)(b-8) = 0$

$\Rightarrow (8-b)[(2b-8)-(b+4)] = 0$

$\Rightarrow (8-b)(b-12) = 0$

Also



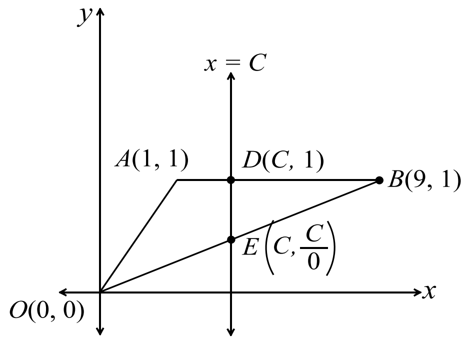
$$\therefore \frac{-8+4}{3} = b \quad \Rightarrow \quad b = \frac{-4}{3}$$

And $\frac{16+b}{3} = 2b - 8 \Rightarrow b = 8$

But no common value of 'b' is possible

267 **(3)**

Area of $\Delta OAB = \frac{1}{2}(1)(8) =$ sq units



Equation of OB is $y = \frac{1}{9}x$

Hence point E is $\left(C, \frac{C}{9}\right)$

Now area of ΔBDE is 2 square units

$$\Rightarrow \frac{1}{2}\left(1 - \frac{C}{9}\right)(9 - C) = 2$$

$\Rightarrow (9-C)^2 = 36$

$\Rightarrow 9 - C = \pm 6$

$\Rightarrow C = 3$

268 **(7)**

Line $3x + 2y = 24$ meets the axis at $B(8,0)$ and $A(0,12)$. Midpoint of AB is $D(4,6)$

Equation of perpendicular bisector of AB is

$2x - 3y + 10 = 0 \qquad (1)$

Now line through $(0,-1)$ and parallel to $x-$ axis is $y = -1$

Co-ordinates of C where line (1) meets $y = -1$ is $C\left(-\frac{13}{2}, -1\right)$

Now the area of triangle ABC

$$\Delta = \frac{1}{2}\begin{vmatrix} 0 & 12 & 1 \\ 8 & 0 & 1 \\ -\frac{13}{2} & -1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[0 - 12\left(8 + \frac{13}{2}\right) + 1(-8)\right]$$

$$= \frac{1}{2}[-6(29) - 8] = 91$$

269 **(5)**

Given vertices of triangle are $O(0,0), B(6,8)$ and $C(-4,3)$

Slope of $OB = \frac{8}{6}$

Slope of $OC = -\frac{3}{4}$

$\therefore \ \angle BOC = \frac{\pi}{2}$

Δ OBC is right angled at O

Circumcentre=midpoint of hypotenuse

$BC = \left(1, \frac{11}{2}\right)$

Orthocenter =vertex $O(0,0)$

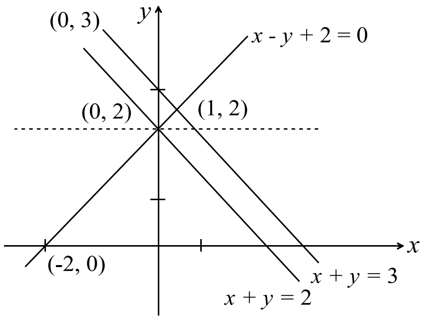
Required distance $= \sqrt{\left(1+\frac{121}{4}\right)} = \frac{5\sqrt{5}}{2}$ unit

270 **(8)**

Given pair of lines $x^2-(y^2-4y+4)=0$

$\Rightarrow x^2-(y-2)=0$

$\Rightarrow (x+y-2)(x-y+2)=0$



Required area is $A=\frac{1.1}{2}=\frac{1}{2}$

271 **(2)**

Lines $(2a+b)x+(a+3b)y+(b-3a)=0$ or $a(2x+y-3)+b(x+3y+1)=0$ are concurrent at point of intersection of lines $2x+y-3=0$ and $x+3y+1=0$ which is $(2,-1)$. Now line$mx+2y+6=0$ must pass through this point

$\Rightarrow 2m-2+6=0$ or $m=-2$

272 **(6)**

Let $x=r\cos\theta\,; y=r\sin\theta$

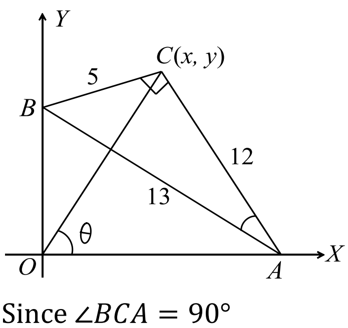
$\Rightarrow 2r\cos\theta+3r\sin\theta=6$

$\Rightarrow r=\frac{6}{2\cos\theta+3\sin\theta}$; and $r=\sqrt{x^2+y^2}$

for r to be minimum $2\cos\theta+3\sin\theta$ must be maximum i.e., $\sqrt{13}$

$\therefore\ r_{\text{rim}}=\frac{6}{\sqrt{13}}$

273 **(5)**



Since $\angle BCA=90°$

Points A, O, B, C are concyclic

Let $\angle AOC=\theta$

$\angle BOC=\angle BAC$

$\tan\left(\frac{\pi}{2}-\theta\right)=\frac{5}{12}$

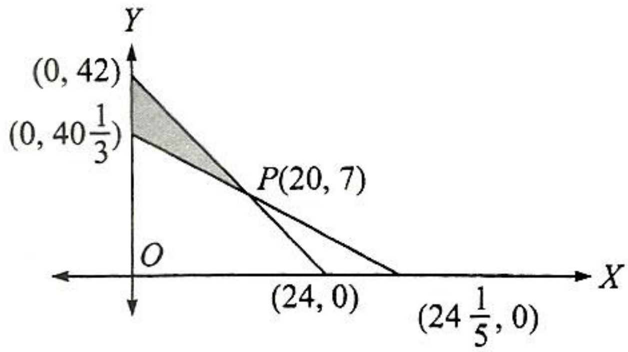
$\frac{x}{y}=\frac{5}{12}\angle 12x-5y=0$

274 **(8)**

We know that the area of the triangle formed by joining the mid points of any triangle is one fourth of that triangle. Therefore required area is 8

275 **(5)**

The given lines $7x+4y=168$ and $5x+3y=121$ intersect $P(20,7)$



$\therefore$ Area of shared region

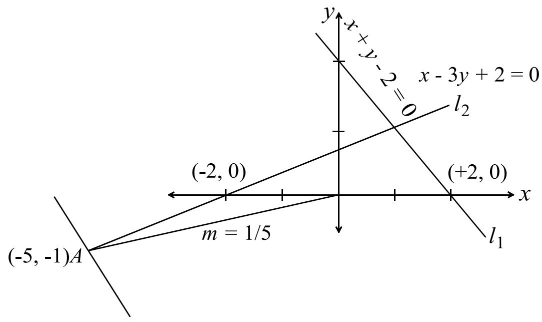
$A=\frac{1}{2}\left(42-40\frac{1}{3}\right)20$

$=\frac{1}{2}\left(\frac{5}{3}\right)20=\frac{50}{3}$ (square units)

276 **(4)**

$x^2-3y^2-2xy+8y-4$

$\equiv(x-3y+2)(x+y-2)$



Now $(-5,-1)$ lies on $x-3y+2=0$

In limiting case line passing through $(-5,-1)$ can be parallel to $x+y-2=0$

i.e. $m>-1$

and maximum slope can occur if it passes through $(0,0)$

i.e. $m<\frac{1}{5}\Rightarrow m\in\left(-1,\frac{1}{5}\right)$

$\Rightarrow a=-1$ and $b=\frac{1}{5}$

$\Rightarrow\left(a+\frac{1}{b}\right)=-1+5=4$

277 **(7)**

Using section formula $A\left(\frac{3k-5}{k+1},\frac{5k+1}{k+1}\right)$

Area of triangle ABC is 2 sq. units

$\Rightarrow\frac{1}{2}\begin{vmatrix}1 & 5 & 1\\ 7 & -2 & 1\\ \frac{3k-5}{k+1} & \frac{5k+1}{k+1} & 1\end{vmatrix}=\pm 2$

Operating $R_2\to R_2-R_1; R_3\to R_3-R_1$

$$\begin{vmatrix} 1 & 5 & 1 \\ 6 & -7 & 0 \\ \frac{3k-5}{k+1}-1 & \frac{5k+1}{k+1}-5 & 0 \end{vmatrix} = \pm 4$$

$$\Rightarrow 6\left(\frac{5k+1-5k-5}{k+1}\right)+7\left(\frac{3k-5-k-1}{k+1}\right) = \pm 4$$

$$\Rightarrow\ -24+7(2k-6)=\pm 4(k+1)$$

$$\Rightarrow k=7 \text{ or } k=\frac{31}{9}$$