9.SEQUENCES AND SERIES

Single Correct Answer Type

1. If $x, 2y, 3z$ are in A.P., where the distinct numbers x, y, z are in G.P., then the common ratio of the G.P. is
a) 3 b) $\frac{1}{3}$ c) 2 d) $\frac{1}{2}$

2. If $b_i = 1 - a_i, na = \sum_{i=1}^{n} a_i, nb = \sum_{i=1}^{n} b_i$, then $\sum_{i=1}^{n} a_i b_i + \sum_{i=1}^{n}(a_i - a)^2 =$
a) ab b) $-nab$ c) $(n+1)ab$ d) nab

3. If $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$ to $\infty = \frac{\pi^2}{6}$, then $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ equals
a) $\pi^2/8$ b) $\pi^2/12$ c) $\pi^2/3$ d) $\pi^2/2$

4. Consider the sequence 1, 2, 2, 4, 4, 4, 4, 8, 8, 8, 8, 8, 8, 8, 8, … Then 1025^{th} term will be
a) 2^9 b) 2^{11} c) 2^{10} d) 2^{12}

5. If x, y, z are in G.P. and $a^x = b^y = c^z$, then
a) $\log_b a = \log_a c$ b) $\log_c b = \log_a c$ c) $\log_b a = \log_c b$ d) None of these

6. The sum $\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \cdots$ to 16 terms is
a) 246 b) 646 c) 446 d) 746

7. If $H_1, H_2, \ldots, H_{20}$ be 20 harmonic means between 2 and 3, then $\frac{H_1+2}{H_1-2} + \frac{H_{20}+3}{H_{20}-3} =$
a) 20 b) 21 c) 40 d) 38

8. The value of $\sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{k=1}^{j} 1 = 220$, then the value of n equals
a) 11 b) 12 c) 10 d) 9

9. The value of $0.2^{\log_{\sqrt{5}}\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots\right)}$ is
a) 4 b) $\log 4$ c) $\log 2$ d) None of these

10. The sum $1 + 3 + 7 + 15 + 31 + \cdots$ to 100 terms is
a) $2^{100} - 102$ b) $2^{99} - 101$ c) $2^{101} - 102$ d) None of these

11. The positive integer n for which $2 \times 2^2 + 3 \times 2^3 + 4 \times 2^4 + \cdots + n \times 2^n = 2^{n+10}$ is
a) 510 b) 511 c) 512 d) 513

12. If $x_1, x_2, \ldots, x_{20}$ are in H.P. and $x_1, 2, x_{20}$ are in G.P., then $\sum_{r=1}^{19} x_r x_{r+1} =$
a) 76 b) 80 c) 84 d) None of these

13. Let $n \in N, n > 25$. Let A, G, H denote the arithmetic mean, geometric mean and harmonic mean of 25 and n. The least value of n for which $A, G, H \in \{25, 269, \ldots, n\}$ is
a) 49 b) 81 c) 169 d) 225

14. The sum of 20 terms of the series whose r^{th} term is given by $T(n) = (-1)^n \frac{n^2+n+1}{n!}$ is
a) $\frac{20}{19!} - 2$ b) $\frac{21}{20!} - 1$ c) $\frac{21}{20!}$ d) None of these

15. If $a^2 + b^2, ab + bc$ and $b^2 + c^2$ are in G.P. , then a, b, c are in
a) A.P. b) G.P. c) H.P. d) None of these

16. If S_p denotes the sum of the series $1 - r^p + r^{2p} - r^{3p} + \cdots$ to ∞ and S_p the sum of the series $1 - r^p + r^{2p} - r^{3p} + \cdots$ to ∞, $|r| < 1$, then $S_p + s_p$ in terms of S_{2p} is
a) $2S_{2p}$ b) 0 c) $\frac{1}{2} S_{2p}$ d) $-\frac{1}{2} S_{2p}$

17. Find the sum $(x+2)^{n-1} + (x+2)^{n-2}(x+1) + (x+2)^{n-3}(x+1)^2 + \cdots + (x+1)^{n-1}$
a) $(x+2)^{n-2} - (x+1)^n$ b) $(x+2)^{n-1} - (x+1)^{n-1}$
c) $(x+2)^n - (x+1)^n$ d) None of these

18. If the sum of n terms of an A.P. is $cn(n-1)$, where $c \neq 0$, then sum of the squares of these terms is
a) $c^2 n(n+1)^2$ b) $\frac{2}{3} c^2 n(n-1)(2n-1)$ c) $\frac{2c^2}{3} n(n+1)(2n+1)$ d) None of these

19. If S_n denotes the sum of first n terms of an A.P. whose first term is a and $\frac{S_{nx}}{S_x}$ is independent of x, then $S_p =$

a) p^3 b) $p^2 a$ c) pa^2 d) a^3

20. If $a_1, a_2, a_3, \dots, a_{2n+1}$ are in A.P., then $\frac{a_{2n+1}-a_1}{a_{2n+1}+a_1} + \frac{a_{2n}-a_2}{a_{2n}+a_2} + \cdots + \frac{a_{n+2}-a_n}{a_{n+2}+a_n}$ is equal to

a) $\frac{n(n+1)}{2} \times \frac{a_2 - a_1}{a_{n+1}}$ b) $\frac{n(n+1)}{2}$ c) $(n+1)(a_2 - a_1)$ d) None of these

21. If the sides of a right angled triangle are in A.P., then the sines of the acute angles are

a) $\frac{3}{5}, \frac{4}{5}$ b) $\frac{1}{\sqrt{3}}, \sqrt{\frac{2}{3}}$ c) $\frac{1}{2}, \frac{\sqrt{3}}{2}$ d) None of these

22. The geometric mean between -9 and -16 is

a) 12 b) -12 c) -13 d) None of these

23. Concentric circles of radii 1, 2, 3,…,100 cm are drawn. The interior of the smallest circle is coloured red and the angular regions are coloured alternately green and red, so that no two adjacent regions are of the same colour. Then, the total area of the green regions in sq. cm is equal to

a) 1000π b) 5050π c) 4950π d) 5151π

24. The third term of a geometric progression is 4. The product of the first five terms is

a) 4^3 b) 4^5 c) 4^4 d) None of these

25. If a, b, c, d are in G.P., then $(b-c)^2 + (c-a)^2 + (d-b)^2$ in equal to

a) $(a-d)^2$ b) $(ad)^2$ c) $(a+d)^2$ d) $(a/d)^2$

26. The sum of the series $\frac{x}{1-x^2} + \frac{x^2}{1-x^4} + \frac{x^4}{1-x^8} + \cdots$ to infinite terms, if $|x| < 1$, is

a) $\frac{x}{1-x}$ b) $\frac{1}{1-x}$ c) $\frac{1+x}{1-x}$ d) 1

27. If $\text{In}(a+c), \text{In}(a-c)$, and $\text{In}(a-2b+c)$ are in A.P., then

a) a, b, c are in A.P. b) a^2, b^2, c^2 are in A.P. c) a, b, c are in G.P. d) a, b, c are in H.P.

28. The sum of $0.2 + 0.004 + 0.00006 + 0.0000008 + \cdots$ to ∞ is

a) $\frac{200}{891}$ b) $\frac{2000}{9801}$ c) $\frac{1000}{9801}$ d) None of these

29. In a sequence of $(4n+1)$ terms the frist $(2n+1)$ terms are in AP whose common difference is 2, and the last $(2n+1)$ terms are in GP whose common ratio is 0.5 if the middle terms of the AP and GP are equal then the middle term of the sequence is

a) $\frac{n \cdot 2^{n+1}}{2^n - 1}$ b) $\frac{n \cdot 2^{n+1}}{2^{2n} - 1}$ c) $n \cdot 2^n$ d) None of these

30. If $a_1, a_2, \dots, a_n$ are in A.P. with common difference $d \neq 0$, then sum of the series $\sin d\,[\sec a_1 \sec a_2 + \sec a_2 \sec a_3 + \cdots + \sec a_{n-1} \sec a_n]$ is

a) $\text{cosec}\, a_n - \text{cosec}\, a$ b) $\cot a_n - \cot a$ c) $\sec a_n - \sec a_1$ d) $\tan a_n - \tan a_1$

31. $a, b, c, d \in R^+$ such that a, b, and c are in A.P. and b, c and d are in H.P., then

a) $ab = cd$ b) $ac = bd$ c) $bc = ad$ d) None of these

32. $ABCD$ is a square of length $a, a \in N, a > 1$. Let $L_1, L_2, L_3, \dots$ be points on BC such that $BL_1 = L_1L_2 = L_2L_3 = \cdots = 1$ and $M_1, M_2, M_3, \dots$ be points on CD such that $CM_1 = M_1M_2 = M_2M_3 = \cdots = 1$. Then $\sum_{n=1}^{a-1}(AL_n^2 + LnMn2)$ is equal to

a) $\frac{1}{2}a(a-1)^2$ b) $\frac{1}{2}(a-1)(2a-1)(4a-1)$

c) $\frac{1}{2}a(a-1)(4a-1)$ d) None of these

33. Value of $\left(1+\frac{1}{3}\right)\left(1+\frac{1}{3^2}\right)\left(1+\frac{1}{3^4}\right)\left(1+\frac{1}{3^8}\right)\cdots\infty$ is equal to

a) 3 b) $\frac{6}{5}$ c) $\frac{3}{2}$ d) None of these

34. If S denotes the sum top infinity and S_n the sum of n terms of the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$, such that $S - S_n < \frac{1}{1000}$, then the least value of n is

a) 8 b) 9 c) 10 d) 11

35. If $(1+x)(1+x^2)(1+x^4)\cdots(1+x^{128}) = \sum_{r=0}^{n} x^r$, then n is equal to
a) 256 b) 255 c) 254 d) None of these

36. Let $S = \frac{4}{19} + \frac{44}{19^2} + \frac{444}{19^3} + \cdots$ up to ∞. Then S is equal to
a) 40/9 b) 38/81 c) 36/171 d) None of these

37. The first term of an infinite geometric series is 21. The second term and the sum of the series are both positive integers. Then which of the following is not the possible value of the second term
a) 12 b) 14 c) 18 d) None of these

38. Let $S \subset (0, \pi)$ denote the set of values of x satisfying the equation $8^{1+|\cos x|+\cos^2 x+|\cos^3 x|+\cdots \text{to } \infty} = 4^3$. Then, $S =$
a) $\{\pi/3\}$ b) $\{\pi/3, -2\pi/3\}$ c) $\{-\pi/3, 2\pi/3\}$ d) $\{\pi/3, 2\pi/3\}$

39. The coefficient of x^{19} in the polynomial $(x-1)(x-2)(x-2^2)\cdots(x-2^{19})$ is
a) $2^{20}-2^{19}$ b) $1-2^{20}$ c) 2^{20} d) None of these

40. In a G.P. the first, third and fifth terms may be considered as the first, fourth and sixteenth terms of an A.P. Then the forth term of the A.P., knowing that its first term is 5 is
a) 10 b) 12 c) 16 d) 20

41. Consider an A.P. $a_1, a_2, a_3, \ldots$ such that $a_3 + a_5 + a_8 = 11$ and $a_4 + a_2 = -2$, then the value of $a_1 + a_6 + a_7$ is
a) -8 b) 5 c) 7 d) 9

42. If a, b, c are in A.P., then $\frac{a}{bc}, \frac{1}{c}, \frac{2}{b}$ will be in
a) A.P. b) G.P. c) H.P. d) None of these

43. If the sum to infinity of the series $1 + 2r + 3r^2 + 4r^3 + \cdots$ is 9/4, then value of r is
a) 1/2 b) 1/3 c) 1/4 d) None of these

44. If $1^2 + 2^2 + 3^2 + \cdots + 2003^2 = (2003)(4007)(334)$ and $(1)(2003) + (2)(2002) + (3)(2001) + \cdots + (2003)(1) = (2003)(334)(x)$, then x equals
a) 2005 b) 2004 c) 2003 d) 2001

45. If $(1^2 - t_1) + (2^2 - t_2) + \cdots + (n^2 - t_n) = \frac{n(n^2-1)}{3}$, then t_n is equal to
a) n^2 b) $2n$ c) $n^2 - 2n$ d) None of these

46. Let the positive numbers a, b, c, and d be in A.P. Then abc, abd, acd, and bcd are
a) Not in A.P./G.P./H.P. b) In A.P. c) In G.P. d) In H.P.

47. If the sum of first n terms of an AP is cn^2, then the sum of squares of these n terms is
a) $\frac{n(4n^2-1)c^2}{6}$ b) $\frac{n(4n^2+1)c^2}{3}$ c) $\frac{n(4n^2-1)c^2}{3}$ d) $\frac{n(4n^2+1)c^2}{6}$

48. If S_n denotes the sum of first 'n' terms of an A.P. and $\frac{S_{3n}-S_{n-1}}{S_{2n}-S_{2n-1}} = 31$, then the value of n is
a) 21 b) 15 c) 16 d) 19

49. The sum of an infinite geometric series is 162 and the sum of its first n terms is 160. If the inverse of its common ratio is an integer, then which of the following is not a possible first term?
a) 108 b) 144 c) 160 d) None of these

50. If the sides of a triangle are in G.P., and its largest angle is twice the smallest, then the common ratio r satisfies ratio r satisfies the inequality
a) $0 < r < \sqrt{2}$ b) $1 < r < \sqrt{2}$ c) $1 < r < 2$ d) None of these

51. If $x, 2x+2$, and $3x+3$ are first three terms of a G.P., then the fourth term is
a) 27 b) -27 c) 13.5 d) -13.5

52. The sum of $i - 2 - 3i + 4 \cdots$ up to 100 terms, where $i = \sqrt{-1}$ is
a) $50(1-i)$ b) $25\, i$ c) $25(1+i)$ d) $100(1-i)$

53. Let $f(x) = 2x + 1$. Then the number of real number of real values of x for which the three unequal numbers $f(x), f(2x), f(4x)$ are in G.P. is
a) 1 b) 2 c) 0 d) None of these

54. Let a_n be the n^{th} term of a G.P. of positive numbers. Let $\sum_{n=1}^{100} a_{2n} = \alpha$ and $\sum_{n=1}^{100} a_{2n-1} = \beta$, such that $\alpha \neq \beta$, then the common ratio is

a) α/β b) β/α c) $\sqrt{\alpha/\beta}$ d) $\sqrt{\beta/\alpha}$

55. 150 workers were engaged to finish a piece of work in a certain number of days. Four workers stopped working on the second day, four more workers stopped their work on the third day and so on. It took 8 more days to finish the work. Then the number of days in which the work was completed is

a) 29 days b) 24 days c) 25 days d) None of these

56. If a, b, c are digits, then the rational number represented by $0.cababab$ ... is

a) $cab/990$ b) $(99c + ba)/990$
c) $(99c + 10a + b)/99$ d) $(99c + 10a + b)/990$

57. Sum of the first n terms of the series $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \cdots$ is equal to

a) $2^n - n - 1$ b) $1 - 2^{-n}$ c) $n + 2^{-n} - 1$ d) $2^n + 1$

58. Suppose that $F(n+1) = \frac{2F(n)+1}{2}$ for $n = 1, 2, 3, \ldots$ and $F(1) = 2$. Then, $F(101)$ equals

a) 50 b) 52 c) 54 d) None of these

59. If $(1-p)(1 + 3x + 9x^2 + 27x^3 + 81x^4 + 243x^5) = 1 - p^6, p \neq 1$, then the value of $\frac{p}{x}$ is

a) $\frac{1}{3}$ b) 3 c) $\frac{1}{2}$ d) 2

60. Sum of three numbers in G.P. be 14. If one is added to first and second and 1 is subtracted from the third, the new numbers are in A.P. The smallest of them is

a) 2 b) 4 c) 6 d) 10

61. If $(p+q)^{\text{th}}$ term of a G.P. is 'a' and its $(p-q)^{\text{th}}$ term is 'b' where $a, b \in R^+$, then its p^{th} term is

a) $\sqrt{\frac{a^3}{b}}$ b) $\sqrt{\frac{b^3}{a}}$ c) $\sqrt{ab}$ d) None of these

62. If $a_1, a_2, a_3 (a_1 > 0)$ are three successive terms of a G.P. with common ratio r, the value of r for which $a_3 > 4a_2 - 3a_1$ holds is given by

a) $1 < r < 3$ b) $-3 < r < -1$ c) $r > 3$ or $r < 1$ d) None of these

63. If $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots = \frac{\pi}{4}$, then value of $\frac{1}{1\times3} + \frac{1}{5\times7} + \frac{1}{9\times11} + \cdots$ is

a) $\pi/8$ b) $\pi/6$ c) $\pi/4$ d) $\pi/36$

64. If $a_1, a_2, a_3, \ldots$ are in A.P., then a_p, a_q, a_r are in A.P. if p, q, r are in

a) A.P. b) G.P. c) H.P. d) None of these

65. If $\sum_{r=1}^{n} r^4 = I(n)$, then $\sum_{r=1}^{n} (2r-1)^4$ is equal to

a) $I(2n) - I(n)$ b) $I(2n) - 16I(n)$ c) $I(2n) - 8I(n)$ d) $I(2n) - 4I(n)$

66. Let T_r and S_r be the r^{th} term and sum up to r^{th} term of a series respectively. If for an odd number $n, S_n = n$ and $T_n = \frac{T_{n-1}}{n^2}$, then T_m (m being even) is

a) $\frac{2}{1+m^2}$ b) $\frac{2m^2}{1+m^2}$ c) $\frac{(m+1)^2}{2+(m+1)^2}$ d) $\frac{2(m+1)^2}{1+(m+1)^2}$

67. If $H_n = 1 + \frac{1}{2} + \cdots + \frac{1}{n}$, then value of $S_n = 1 + \frac{3}{2} + \frac{5}{3} + \cdots + \frac{99}{50}$ is

a) $H_{50} + 50$ b) $100 - H_{50}$ c) $49 + H_{50}$ d) $H_{50} + 100$

68. Three numbers form an increasing G.P. If the middle number is doubled, then the new numbers are in A.P. The common ratio of the G.P. is

a) $2 - \sqrt{3}$ b) $2 + \sqrt{3}$ c) $\sqrt{3} - 2$ d) $3 + \sqrt{2}$

69. If x, y, and z are in G.P., and $x+3, y+3$, and $z+3$ are in H.P., then

a) $y = 2$ b) $y = 3$ c) $y = 1$ d) $y = 0$

70. Let α, and β be the roots of $x^2 - x + p = 0$ and γ and δ be the root of $x^2 - 4x + q = 0$. If α, β, and γ, δ are in G.P., then the integral values of p and q, respectively, are

a) $-2, -32$ b) $-2, 3$ c) $-6, 3$ d) $-6, -32$

71. If the $p^{\text{th}}, q^{\text{th}}, r^{\text{th}}$ and s^{th} terms of an A.P. are in G.P., then $p-q, q-r, r-s$ are in

a) A.P. b) G.P. c) H.P. d) None of these

72. If the ratio of the sum to n terms of two A.P.'s is $(5n+3):(3n+4)$, then the ratio of their 17th terms is
a) 172:99 b) 168:103 c) 175:99 d) 171:103

73. If a, b, and c are in A.P. and p, p' are, respectively, A.M. and G.M. between a and b while q, q' are, respectively, the A.M. and G.M. between b and c, then
a) $p^2+q^2=p'^2+q'^2$ b) $pq=p'q'$ c) $p^2-q^2=p'^2-q'^2$ d) None of these

74. Greatest integer by which $1+\sum_{r=1}^{30} r \times r!$ is divisible is
a) Composite number b) Odd number c) Divisible by 3 d) None of these

75. After striking the floor, a certain ball redounds $(4/5)^{\text{th}}$ of height from which it has fallen. Then the total distance that it travels before coming to rest, if it is gently dropped from a height of 120 m is
a) 1260 m b) 600 m c) 1080 m d) None of these

76. Consider the ten numbers $ar, ar^2, ar^3, \dots, ar^{10}$. If their sum is 18 and the sum of their reciprocals is 6 then the product of these ten numbers, is
a) 81 b) 243 c) 343 d) 324

77. Let $a_1, a_2, a_3, \dots$ be terms of an A.P. If $\frac{a_1+a_2+\cdots+a_p}{a_1+a_2+\cdots+a_q}=\frac{p^2}{q^2}, p \neq q$, then $\frac{a_6}{a_{21}}$ equals
a) 41/11 b) 7/2 c) 2/7 d) 11/41

78. In a geometric series, the first term is a and common ratio is r. If S_n denotes the sum of the n terms and $U_n=\sum_{n=1}^{n} S_n$ then $rS_n+(1-r)U_n$ equals
a) 0 b) n c) na d) nar

79. If x, y, and z are distinct prime numbers, then
a) x, y, and z may be in A.P. but not in G.P. b) x, y, and z may be in G.P. but not in A.P.
c) x, y, and z can neither be in A.P. nor in G.P. d) None of these

80. If $(1+3+5+\cdots+p)+(1+3+5+\cdots+q)=(1+3+5+\cdots+r)$ where each set of parentheses contains the sum of of $p+q+r$ (where $p>6$) is
a) 12 b) 21 c) 45 d) 54

81. If $a_1, a_2, \dots, a_n$ are in H.P., then
$\frac{a_1}{a_2+a_3+\cdots+a_n}, \frac{a_2}{a_1+a_3+\cdots+a_n}, \dots, \frac{a_n}{a_1+a_2+\cdots+a_{n-1}}$ are in
a) A.P. b) G.P. c) H.P. d) None of these

82. If $\log_2(5 \times 2^x+1), \log_4(2^{1-x}+1)$ and 1 are in A.P., then x equals
a) $\log_2 5$ b) $1-\log_5 2$ c) $\log_5 2$ d) None of these

83. The 15th term of the series $2\frac{1}{2}+1\frac{7}{13}+1\frac{1}{9}+\frac{20}{23}+\cdots$ is
a) $\frac{10}{39}$ b) $\frac{10}{21}$ c) $\frac{10}{23}$ d) None of these

84. Let a_1, a_2, a_3, a_4 and a_5 be such that a_1, a_2 and a_3 are in A.P., a_2, a_3 and a_4 are in G.P., and a_3, a_4 and a_5 are in H.P. Then $\log_e a_1, \log_e a_3$ and $\log_e a_5$ are in
a) G.P. b) A.P. c) H.P. d) None of these

85. Let $a=111\dots1$ (55 digits), $b=1+10+10^2+\cdots+10^4, c=1+10^5+10^{10}+10^{15}+\cdots+10^{50}$, then
a) $a=b+c$ b) $a=bc$ c) $b=ac$ d) $c=ab$

86. The number of terms common between the series $1+2+4+8+\cdots$ to 100 terms and $1+4+7+10+\cdots$ to 100 terms is
a) 6 b) 4 c) 5 d) None of these

87. If $\alpha \epsilon \left(0, \frac{\pi}{2}\right)$, then $\sqrt{x^2+x}+\frac{\tan^2\alpha}{\sqrt{x^2+x}}$ is always greater than or equal to
a) $2\tan\alpha$ b) 1 c) 2 d) $\sec^2\alpha$

88. If ax^3+bx^2+cx+d is divisible by ax^2+c, then a, b, c, d are in
a) A.P. b) G.P. c) H.P. d) None of these

89. If the $p^{\text{th}}, q^{\text{th}}$, and r^{th} terms of an A.P. are in G.P., then common ratio of the G.P. is
a) $\frac{pr}{q^2}$ b) $\frac{r}{p}$ c) $\frac{q+r}{p+q}$ d) $\frac{q-r}{p-q}$

90. The largest term common to the sequences 1, 11, 21, 31, ... to 100 terms and 31, 36, 41, ... to 100 terms is
a) 381 b) 471 c) 281 d) None of these

91. The number of terms of an A.P. is even; the sum of the odd terms is 24, and of the even terms is 30, and the last term exceeds the first by 10/2, then the number of terms in the series is
a) 8 b) 4 c) 6 d) 10

92. Let $\{t_n\}$ be a sequence of integers in G.P. in which $t_4 : t_6 = 1 : 4$ and $t_2 + t_5 = 216$. Then t_1 is
a) 12 b) 14 c) 16 d) None of these

93. If $b_{n+1} = \frac{1}{1-b_n}$ for $n \geq 1$ and $b_1 = b_3$, then $\sum_{r=1}^{2001} b_r^{2001}$ is equal to
a) 2001 b) -2001 c) 0 d) None of these

94. The rational number which equals the number $2.\overline{357}$ with recurring decimal is
a) $\frac{2355}{1001}$ b) $\frac{2379}{997}$ c) $\frac{2355}{999}$ d) None of these

95. If a, b, and c are in G.P. and x, y, respectively, be arithmetic means between a, b and b, c, then the value of $\frac{a}{x} + \frac{c}{y}$ is
a) 1 b) 2 c) 1/2 d) None of these

96. If the sum of the first $2n$ terms of the A.P. 2, 5, 8, ..., is equal to the sum of the first n terms of A.P. 57, 59, 61, ..., then n equals
a) 10 b) 12 c) 11 d) 13

97. If x, y, and z are $p^{\text{th}}, q^{\text{th}}$ and r^{th} terms respectively of an A.P. and also of a G.P., then $x^{y-z}y^{z-x}z^{x-y}$ is equal to
a) xyz b) 0 c) 1 d) None of these

98. In an A.P. of which a is the first term, if the sum of the first p terms is zero, then the sum of the next q terms is
a) $-\frac{a(p+q)p}{q+1}$ b) $\frac{a(p+q)p}{p+1}$ c) $-\frac{a(p+q)q}{p-1}$ d) None of these

99. If three positive real numbers a, b, c are in A.P., such that $abc = 4$, then the minimum value of b is
a) $2^{1/3}$ b) $2^{2/3}$ c) $2^{1/2}$ d) $2^{3/2}$

100. The sum of the series $a - (a + d) + (a + 2d) - (a + 3d) + \cdots$ up to $(2n + 1)$ terms is
a) $-nd$ b) $a + 2nd$ c) $a + nd$ d) $2nd$

101. The sum of 20 terms of a series of which every even term is 2 times the term before it, and every odd term is 3 times the term before it, the first term being unity is
a) $\left(\frac{2}{7}\right)(6^{10} - 1)$ b) $\left(\frac{3}{7}\right)(6^{10} - 1)$ c) $\left(\frac{3}{5}\right)(6^{10} - 1)$ d) None of these

102. If the sum of m terms of an A.P. is the same as the sum of its n terms, then the sum of its $(m + n)$ terms is
a) mn b) $-mn$ c) $1/mn$ d) 0

103. If a, b, and c are in A.P., p, q and r are in H.P. and ap, bq, and cr are in G.P., then $\frac{p}{r} + \frac{r}{p}$ is equal to
a) $\frac{a}{c} - \frac{c}{a}$ b) $\frac{a}{c} + \frac{c}{a}$ c) $\frac{b}{q} + \frac{q}{b}$ d) $\frac{b}{q} - \frac{q}{b}$

104. Coefficients of x^{18} in $(1 + x + 2x^2 + 3x^3 + \cdots + 18x^{18})^2$ is equal to
a) 995 b) 1005 c) 1235 d) None of these

105. If a, b, and c are in G.P., then the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in
a) A.P. b) G.P. c) H.P. d) None of these

106. If a, b, and c are in G.P., then $a + b, 2b$, and $b + c$ are in
a) A.P. b) G.P. c) H.P. d) None of these

107. If in a progression $a_1, a_2, \ldots$, etc., $(a_r - a_{r+1})$ bears a constant ratio with $a_r \times a_{r+1}$, then the terms of the progression are in
a) A.P. b) G.P. c) H.P. d) None of these

108. $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{r}{1\times3\times5\times7\times9\times\cdots\times(2r+1)}$ is equal to

a) $\frac{1}{3}$ b) $\frac{3}{2}$ c) $\frac{1}{2}$ d) None of these

109. Consider an infinite geometric series with first term a and common ratio r. If its sum is 4 and the second term is 3/4, then

a) $a=\frac{4}{7}, r=\frac{3}{7}$ b) $a=2, r=\frac{3}{8}$ c) $a=\frac{3}{2}, r=\frac{1}{2}$ d) $a=3, r=\frac{1}{4}$

110. ABC is a right-angled triangle in which $\angle B=90°$ and $BC=a$. If n points $L_1, L_2, \dots, L_n$ on AB is divided in $n+1$ equals parts and $L_1M_1, L_2M_2, \dots, L_nM_n$ are line segments parallel to BC and $M_1, M_2, \dots, M_n$ are on AC, then the sum of the lengths of $L_1M_1, L_2M_2, \dots, L_nM_n$ is

a) $\frac{a(n+1)}{2}$ b) $\frac{a(n-1)}{2}$ c) $\frac{an}{2}$ d) None of these

111. If a, b and c are in A.P. then $a^3+c^3-8b^3$ is equal to

a) $2\ abc$ b) $6\ abc$ c) $4\ abc$ d) None of these

112. An infinite GP has first term x and sum 5, then

a) $x<-10$ b) $-10<x<0$ c) $0<x<10$ d) $x>10$

113. If t_n denotes the n^{th} term of the series $2+3+6+11+18+\cdots$ then t_{50} is

a) 49^2-1 b) 49^2 c) 50^2+1 d) 49^2+2

114. The coefficient of x^{49} in the product $(x-1)(x-3)\cdots(x-99)$ is

a) -99^2 b) 1 c) -2500 d) None of these

115. If a, b, and c are in A.P. and $b-a, c-b$ and a are in G.P., then $a:b:c$ is

a) 1:2:3 b) 1:3:5 c) 2:3:4 d) 1:2:4

116. The sum of an infinite G.P. is 57 and the sum of their cubes is 9747, then common ratio of the G.P. is

a) 1/3 b) 2/3 c) 1/6 d) None of these

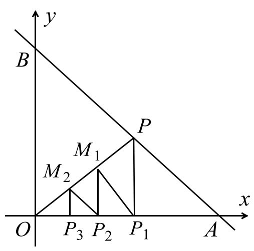
117. The maximum sum of the series $20+19\frac{1}{3}+18\frac{2}{3}+\cdots$ is

a) 310 b) 300 c) 320 d) None of these

118. Let $a\in(0,1]$ satisfies the equation $a^{2008}-2a+1=0$ and $S=1+a+a^2+\dots+a^{2007}$. Sum of all possible value(s) of S, is

a) 2010 b) 2009 c) 2008 d) 2

119. The line $x+y=1$ meets x-axis at A and y-axis at B, P is the mid-point of AB;
P_1 is the foot of the perpendicular from P to OA:
M_1 is that of P_1 from OP; P_2 is that of M_1 from OA; M_2 is that of P_2 from OP; P_3 is that of M_2 from OA; and so on.
If P_n denotes the n^{th} foot of the perpendicular on OA: then OP_n is



a) $\left(\frac{1}{2}\right)^{n-1}$ b) $\left(\frac{1}{2}\right)^{n}$ c) $\left(\frac{1}{2}\right)^{n+1}$ d) None of these

120. The sum to 50 terms of the series $1+2\left(1+\frac{1}{50}\right)+3\left(1+\frac{1}{50}\right)^2+\cdots$ is given by

a) 2500 b) 2550 c) 2450 d) None of these

121. If $t_n=\frac{1}{4}(n+2)(n+3)$ for $n=1,2,3,\dots,$ then $\frac{1}{t_1}+\frac{1}{t_2}+\frac{1}{t_3}+\cdots+\frac{1}{t_{2003}}=$

a) $\frac{4006}{3006}$ b) $\frac{4003}{3007}$ c) $\frac{4006}{3008}$ d) $\frac{4006}{3009}$

122. The sum of series $1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+\cdots\infty$ is

a) 7/16 b) 5/16 c) 105/64 d) 35/16

123. The value of $\sum_{r=0}^{n}(a+r+ar)(-a)^r$ is equal to

a) $(-1)^n[(n+1)a^{n+1}-a]$ b) $(-1)^n(n+1)a^{n+1}$

c) $(-1)^n\frac{(n+2)a^{n+1}}{2}$ d) $(-1)^n\frac{na^n}{2}$

124. If $|a|<1$ and $|b|<1$, then the sum of the series $1+(1+a)b+(1+a+a^2)b^2+(1+a+a^2+a^3)b^3+\cdots$ is

a) $\frac{1}{(1-a)(1-b)}$ b) $\frac{1}{(1-a)(1-ab)}$

c) $\frac{1}{(1-b)(1-ab)}$ d) $\frac{1}{(1-a)(1-b)(1-ab)}$

125. If $a,\frac{1}{b},c$ and $\frac{1}{p},q,\frac{1}{r}$ form two arithmetic progressions of the same common difference, then a,q,c are in A.P. if

a) p,b,r are in A.P. b) $\frac{1}{p},\frac{1}{b},\frac{1}{r}$ are in A.P. c) p,b,r are in G.P. d) None of these

126. Let $\alpha,\beta\in R$. If α,β^2 be the roots of quadratic equation $x^2-px+1=0$ and α^2,β be the roots of quadratic equation $x^2-qx+8=0$, then the value of 'r' if $\frac{r}{8}$ be arithmetic mean of p and q is

a) $\frac{83}{2}$ b) 83 c) $\frac{83}{8}$ d) $\frac{83}{4}$

127. If x,y,z are real and $4x^2+9y^2+16z^2-6xy-12yz-8zx=0$, then x,y,z are in

a) A.P. b) G.P. c) H.P. d) None of these

128. Let $a_1,a_2,\dots,a_{10}$ be in A.P. and $h_1,h_2,\dots,h_{10}$ be in H.P. If $a_1=h_1=2$ and $a_{10}=h_{10}=3$, then a_4h_7 is

a) 2 b) 3 c) 5 d) 6

129. If a,x and b are in A.P., $a,y,$ and b are in G.P. and a,z,b are in H.P. such that $x=9z$ and $a>0,b>0$, then

a) $|y|=3z$ b) $x=3|y|$ c) $2y=x+z$ d) None of these

130. The sum to 50 terms of the series $\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+\cdots$ is

a) $\frac{100}{17}$ b) $\frac{150}{17}$ c) $\frac{200}{51}$ d) $\frac{50}{17}$

131. The harmonic mean of the roots of the equation $(5+\sqrt{2})x^2-(4+\sqrt{5})x+8+2\sqrt{5}=0$ is

a) 2 b) 4 c) 6 d) 8

Multiple Correct Answers Type

132. The consecutive digits of a three digit number are in G.P. If the middle digit be increased by 2, then they form an A.P. If 792 is subtracted from this, then we get the number constituting of same three digits but in reverse order. Then number is divisible by

a) 7 b) 49 c) 19 d) None of these

133. In the 20^{th} row of the triangle

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 2 & & 3 & & \\ & 4 & & 5 & & 6 & \\ 7 & & 8 & & 9 & & 10 \\ \vdots & & \vdots & & \vdots & & \vdots \end{array}$$

a) Last term =210 b) First term =191 c) Sum = 4010 d) Sum = 4200

134. If the first and the $(2n-1)^{st}$ terms of an A.P., a G.P. and a H.P. are equal and their n^{th} terms are a,b and c respectively, then

a) $a=b=c$ b) $a\geq b\geq c$ c) $a+b=b$ d) $ac-b^2=0$

135. If A_1,A_2: G_1,G_2; and H_1,H_2 are two arithmetic, geometric and harmonic means respectively, between two quantities a and b then ab is equal to

a) A_1H_2 b) A_2H_1 c) G_1G_2 d) None of these

136. Let $E=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots$. Then,

a) $E<3$ b) $E>3/2$ c) $E>2$ d) $E<2$

137. If a,b, and c, are in G.P. and x and y, respectively, be arithmetic means between a,b and b,c, then

a) $\frac{a}{x}+\frac{c}{y}=2$ b) $\frac{a}{x}+\frac{c}{y}=\frac{c}{a}$ c) $\frac{1}{x}+\frac{1}{y}=\frac{2}{b}$ d) $\frac{1}{x}+\frac{1}{y}=\frac{2}{ac}$

138. The terms of an infinitely decreasing G.P. in which all the terms are positive, the first term is 4, and the difference between the third and fifth term is 32/81, then

a) $r=1/3$ b) $r=2\sqrt{2}/3$ c) $S_\infty=6$ d) None of these

139. If the first and the $(2n-1)$th term of an AP, GP and HP are positive and equal and their nth terms are a,b,c respectively, then

a) $a=b=c$ b) $a\geq b\geq c$ c) $a+c=b$ d) $ac-b^2=0$

140. Let T_r be the r^{th} term of an A.P., for $r=1,2,3,\ldots$. If for some positive integers m,n, we have $T_m=\frac{1}{n}$ and $T_n=\frac{1}{m}$, then T_{mn} equals

a) $\frac{1}{mn}$ b) $\frac{1}{m}+\frac{1}{n}$ c) 1 d) 0

141. If a,b,c are in HP, then the value of $\left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)\left(\frac{1}{c}+\frac{1}{a}-\frac{1}{b}\right)$ is

a) $\frac{2}{bc}-\frac{1}{b^2}$ b) $\frac{1}{4}\left(\frac{3}{c^2}+\frac{2}{ca}-\frac{1}{a^2}\right)$

c) $\frac{3}{b^2}-\frac{2}{ab}$ d) None of these

142. If $S_n=1^2-2^2+3^2-4^2+5^2-6^2+\cdots$, then

a) $S_{40}=-820$ b) $S_{2n}>S_{2n+2}$ c) $S_{51}=1326$ d) $S_{2n+1}>S_{2n-1}$

143. If $x>1, y>1$, and $z>1$ are in G.P., then $\frac{1}{1+\text{In}\,x}, \frac{1}{1+\text{In}\,y}$, and $\frac{1}{1+\text{In}\,z}$ are in

a) A.P. b) H.P. c) G.P. d) None of these

144. If sum of an infinite G.P. $p, 1, 1/p, 1/p^2, \ldots$ is 9/2, then value of p is

a) 2 b) 3/2 c) 3 d) 9/2

145. For the series, $S=1+\frac{1}{(1+3)}(1+2)^2+\frac{1}{(1+3+5)}(1+2+3)^2+\frac{1}{(1+3+5+7)}(1+2+3+4)^2+\cdots$

a) 7th term is 16 b) 7th term is 18

c) Sum of first 10 terms is $\frac{505}{4}$ d) Sum of first 10 terms is $\frac{405}{4}$

146. $\frac{1}{\sqrt{2}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{11}}+\cdots n$ terms, is equal to

a) $\frac{\sqrt{3n+2}-\sqrt{2}}{3}$ b) $\frac{n}{\sqrt{2+3n}+\sqrt{2}}$ c) Less than n d) Less than $\sqrt{\frac{n}{3}}$

147. If $x^2+9y^2+25z^2=xyz\left(\frac{15}{x}+\frac{5}{y}+\frac{3}{z}\right)$, then

a) x,y, and z are in H.P. b) $\frac{1}{x},\frac{1}{y},\frac{1}{z}$ are in A.P. c) x,y,z are in G.P. d) $\frac{1}{x},\frac{1}{y},\frac{1}{z}$ are in G.P.

148. Consider a sequence $\{a_n\}$ with $a_1=2$ and $a_n=\frac{a_{n-1}^2}{a_{n-2}}$ for all $n\geq 3$, terms of the sequence being distinct. Given that a_2 and a_5 are positive integers and $a_5\leq 162$ then the possible values(s) of a_5 can be

a) 162 b) 64 c) 32 d) 2

149. If p,q, and r are in A.P. then which of the following is/are true?

a) $p^{\text{th}},q^{\text{th}}$ and r^{th} terms of A.P, are in A.P. b) $p^{\text{th}},q^{\text{th}}$ and r^{th} terms of G.P. are in G.P.

c) $p^{\text{th}},q^{\text{th}}$ and r^{th} terms of H.P, are in H.P. d) None of these

150. If a,b,c, and d are four unequal positive numbers which are in A.P., then

a) $\frac{1}{a}+\frac{1}{d}>\frac{1}{b}+\frac{1}{c}$ b) $\frac{1}{a}+\frac{1}{d}<\frac{1}{b}+\frac{1}{c}$ c) $\frac{1}{b}+\frac{1}{c}>\frac{4}{a+d}$ d) $\frac{1}{a}+\frac{1}{d}=\frac{1}{b}+\frac{1}{c}$

151. If $n>1$, the values of the positive integer m for which n^m+1 divides $a=1+n+n^2+\cdots+n^{63}$ is/are

a) 8 b) 16 c) 32 d) 64

152. If $\sum_{r=1}^{n} r(r+1)(2r+3)=an^4+bn^3+cn^2+dn+e$, then

a) $a-b=d-c$
b) $e=0$
c) $a, b-2/3, c-1$ are in A.P.
d) $(b+d)/a$ is an integer

153. For an increasing A.P. $a_1, a_2, \dots, a_n$ if $a_1+a_3+a_5=-12$ and $a_1a_3a_5=80$, then which of the following is/are true?
a) $a_1=-10$
b) $a_2=-1$
c) $a_3=-4$
d) $a_5=+2$

154. If $\frac{1}{b-a}+\frac{1}{b-c}=\frac{1}{a}+\frac{1}{c}$, then
a) a, b, and c are in H.P.
b) a, b, and c are in A.P.
c) $b=a+c$
d) $3a=b+c$

155. Given that $x+y+z=15$ when a, x, y, z, b are in A.P. and $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{5}{3}$ when a, x, y, z, b are in H.P. Then
a) G.M. of a and b is 3
b) One possible value of $a+2b$ is 11
c) A.M. of a and b is 6
d) Greatest value of $a-b$ is 8

156. If $1+2x+3x^2+4x^3+\cdots\infty \geq 4$, then
a) Least value of x is 1/2
b) Greatest value of x is 4/3
c) Least value of is x 2/3
d) Greatest value of x does not exists

157. For a positive integer n, let $a(n)=1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\cdots+\frac{1}{(2^n)-1}$. Then,
a) $a(100) \leq 100$
b) $a(100)>100$
c) $a(200) \leq 100$
d) $a(200)>100$

158. The next term of the G.P. x, x^2+2, and x^3+10 is
a) $\frac{729}{16}$
b) 6
c) 0
d) 54

159. If the non-zero numbers x, y, z are in AP and $\tan^{-1} x, \tan^{-1} y, \tan^{-1} z$ are in AP, then
a) $x=y=z$
b) $y^2=zx$
c) $x^2=yz$
d) $z^2=xy$

160. For a positive integer n, let $a(n)=1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\dots\frac{1}{(2^n)-1}$. Then,
a) $a(100)<100$
b) $a(100)>100$
c) $a(200) \leq 100$
d) $a(200)>100$

161. Let $a_1, a_2, a_3, \dots, a_n$ be in G.P. such that $3a_1+7a_2+3a_3-4a_5=0$. Then common ratio of G.P. can be
a) 2
b) $\frac{3}{2}$
c) $\frac{5}{2}$
d) $-\frac{1}{2}$

162. If the sum of n terms of an A.P. is given by $S_n=a+bn+cn^2$, where a, b, c are independent of n, then
a) $a=0$
b) Common difference of A.P. must be $2b$
c) Common difference of A.P. must be $2c$
d) First term of A.P. is $b+c$

163. The numbers 1, 4, 16 can be three terms (not necessarily consecutive) of
a) No AP
b) Only one GP
c) Infinite number of APs
d) Infinite number of GPs

164. If a, b, and c are in H.P. then the value of $\frac{(ac+ab-bc)(ab+bc-ac)}{(abc)^2}$ is
a) $\frac{(a+c)(3a-c)}{4a^2c^2}$
b) $\frac{2}{bc}-\frac{1}{b^2}$
c) $\frac{2}{bc}-\frac{1}{a^2}$
d) $\frac{(a-c)(3a+c)}{4a^2c^2}$

165. For $0<\phi<\pi/2$, if
$x=\sum_{n=0}^{\infty} \cos^{2n}\phi, y=\sum_{n=0}^{\infty} \sin^{2n}\phi$, and $z=\sum_{n=0}^{\infty} \cos^{2n}\phi \sin^{2n}\phi$, then
a) $xyz=xz+y$
b) $xyz=xy+z$
c) $xyz=x+y+z$
d) $xyz=yz+x$

166. If $p(x)=\frac{1+x^2+x^4+\cdots+x^{2n-2}}{1+x+x^2+\cdots+x^{n-1}}$ is a polynomial in x, then n can be
a) 5
b) 10
c) 20
d) 17

167. Which of the following can be terms (not necessarily consecutive) of any A.P.
a) 1, 6, 19
b) $\sqrt{2}, \sqrt{50}, \sqrt{98}$
c) $\log 2, \log 16, \log 128$
d) $\sqrt{2}, \sqrt{3}, \sqrt{7}$

168. Let $S_1, S_2, \dots$ be squares such that for each $n \geq 1$, the length of a side of S_n equals the length of a diagonal of S_{n+1}. If the length of a side of S_1 is 10 cm, then for which of the following values of n is the area of S_n less than 1 sq. cm?
a) 7
b) 8
c) 9
d) 10

169. Let n be an odd integers. If $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r \theta$, for every value of θ, then

a) $b_0 = 1, b_1 = 3$
b) $b_0 = 0, b_1 = n$
c) $b_0 = -1, b_1 = n$
d) $b_0 = 0, b_1 = n^2 - 3n + 3$

Assertion - Reasoning Type

This section contain(s) 0 questions numbered 170 to 169. Each question contains STATEMENT 1(Assertion) and STATEMENT 2(Reason). Each question has the 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

a) Statement 1 is True, Statement 2 is True; Statement 2 **is** correct explanation for Statement 1

b) Statement 1 is True, Statement 2 is True; Statement 2 **is not** correct explanation for Statement 1

c) Statement 1 is True, Statement 2 is False

d) Statement 1 is False, Statement 2 is True

170

Statement 1: If $x^2 + 9y^2 + 25z^2 = xyz\left(\frac{15}{x} + \frac{5}{y} + \frac{3}{z}\right)$, then x, y, z are in H.P.

Statement 2: If $a_1^2 + a_2^2 + \cdots + a_n^2 = 0$, then $a_1 = a_2 = a_3 = \cdots a_n = 0$

171 Let a, b, c be three positive real numbers which are in HP.

Statement 1: $\dfrac{a+b}{2a-b} + \dfrac{c+b}{2c-b} \geq 4.$

Statement 2: If $x > 0$, then $x + \frac{1}{x} \geq 4.$

172

Statement 1: If sum f n terms of a series $2n^2 + 3n + 1$, then series is an AP.

Statement 2: Sum of n terms of an AP is always of the form $pn^2 + qn$.

173

Statement 1: If $|x-1|, |x-3|$ are first three terms of an AP, then its sixth term is 7< third terms.

Statement 2: $a, a+d, a+2d, \ldots$ are in AP $(d \neq 0)$, then sixth term is $(a + 5d)$.

174

Statement 1: In a G.P. if the $(m+n)^{\text{th}}$ term be p and $(m-n)^{\text{th}}$ term be q, then its m^{th} term is $\sqrt{pq}$

Statement 2: T_{m+n}, T_m, T_{m-n} are in G.P.

175

Statement 1: Sum of the series $1^3 - 2^3 + 3^3 - 4^3 + \cdots + 11^3 = 378$

Statement 2: For any odd integer $n \geq 1, n^3 - (n-1)^3 + \cdots + (-1)^{n-1}\, 1^3 = \frac{1}{4}(2n-1)(n+1)^2$

176

Statement 1: There are infinite geometric progressions for which 27, 8 and 12 are three of its terms (not necessarily consecutive)

Statement 2: Given terms are integers

177

Statement 1: If $3x+4y=5$, then the greatest value of x^2y^3 is $\frac{3}{16}$.

Statement 2: Greatest value occurs when $9x=8y$.

178

Statement 1: Let $p_1, p_2, \dots, p_n$ and x be distinct real number such that $(\sum_{r=1}^{n-1} p_r^2)x^2 + 2(\sum_{r=1}^{n-1} p_r\, p_{r+1})x + \sum_{r=2}^{n} p_r^2 \leq 0$, then $p_1, p_2, \dots, p_n$ are in G.P. and when $a_1^2 + a_2^2 + a_3^2 + \cdots + a_n^2 = 0, a_1 = a_2 = a_3 = \cdots = a_n = 0$

Statement 2: If $\frac{p_2}{p_1} = \frac{p_3}{p_2} = \cdots = \frac{p_n}{p_{n-1}}$, then $p_1, p_2, \dots, p_n$ are in G.P.

179 Let $a, r \in R - \{0, 1, -1\}$ and n be an even number

Statement 1: $a \times ar \times ar^2 \cdots ar^{n-1} = (a^2r^{n-1})^{n/2}$

Statement 2: Product of i^{th} term from the beginning and from the end in a G.P. is independent of i

180

Statement 1: The sum of n terms of two arithmetic progressions are in the ratio $(7n+1):(4n+17)$, then the ratio of their nth terms is 7: 4.

Statement 2: If $S_n = ax^2+bx+c$, then $T_n = S_n - S_{n-1}$

181

Statement 1: If the arithmetic mean of two numbers is $5/2$, geometric mean of the numbers is 2, then the harmonic mean will be $8/5$

Statement 2: For a group of positive numbers $(\text{G.M.})^2 = (\text{A.M.}) \times (\text{H.M.})$

182

Statement 1: If sum of n terms of a series is $6n^2+3n+1$ then the series is in AP.

Statement 2: Sum of n terms of an AP is always of the form an^2+bn.

183

Statement 1: 3, 6, 12 are in GP, then 9, 12, 18 are in HP.

Statement 2: If middle term is added in three consecutive terms of a GP, resultant will be in HP.

184

Statement 1: The numbers $\sqrt{2}, \sqrt{3}, \sqrt{5}$ cannot be the terms of a single A.P. with non-zero common difference

Statement 2: If $p, q, r (p \neq q)$ are terms (not necessarily consecutive) of an A.P., then there exists a rational number k such that $(r-q)/(q-p) = k$

185

Statement 1: $x = 1111 \cdots 91$ times is composite number

Statement 2: 91 is composite number

186

Statement 1: Coefficient of x^{14} in $(1+2x+3x^2+\cdots+16x^{15})^2$ is 560

Statement 2: $\sum_{r=1}^{n} r(n-r) = \frac{n(n^2-1)}{6}$

187

Statement 1: If an infinite G.P. has 2nd term x and its sum is 4, then x belongs to $(-8, 1)$

Statement 2: Sum of an infinite G.P. is finite if for its common ratio $r, 0 < |r| < 1$

188

Statement 1: Let $F_1(n) = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$, then $\sum_{r=1}^{n} F_1(r) = (n+1)F_1(n) - n$.

Statement 2: $\frac{1^{-1} + 2^{-1} + 3^{-1} + \ldots + n^{-1}}{n}$

$> \left(\frac{1+2+3+\ldots+n}{n}\right)^{-1}$

or $\left(1 + \frac{1}{2} + \frac{1}{3} + + \ldots \frac{1}{n}\right) > \frac{n^2}{\sum n}$

or $\left(1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}\right) > \frac{2n}{(n+1)}$

189

Statement 1: $1^{99} + 2^{99} + \cdots + 100^{99}$ is divisible by 10100

Statement 2: $a^n + b^n$ is divisible by $a + b$ if n is odd

Matrix-Match Type

This section contain(s) 0 question(s). Each question contains Statements given in 2 columns which have to be matched. Statements (A, B, C, D) in **columns I** have to be matched with Statements (p, q, r, s) in **columns II**.

190.

	Column-I		Column- II
(A)	If $\sum n = 210$, then $\sum n^2$ is divisible by the greatest prime number which is greater than	(p)	16
(B)	Between 4 and 2916 is inserted odd number $(2n+1)$ G.M's. Then the $(n+1)$ the G.M. is divisible by greatest odd integer which is less than	(q)	10
(C)	In a certain progression, four consecutive terms are 40, 30, 24, 20. Then the integral part of the next term of the progression is more than	(r)	34
(D)	$1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \cdots$ to $\infty = \frac{a}{b}$, where H.C.F. $(a, b) = 1$, then $a - b$ is less than	(s)	30

CODES :

	A	B	C	D
a)	R,s	p,q	r,s	p,q,r,s
b)	p,q,r,s	r,s	p,q	r,s
c)	p,q	r,s	p,q,r,s	r,s
d)	r,s	p,q,r,s	r,s	p,q

191.

	Column-I		Column- II
(A)	If a, b, c are in G.P., then $\log_a 10, \log_b 10, \log_c 10$ are in	(p)	A.P.
(B)	If $\frac{a+be^x}{a-be^x} = \frac{b+ce^x}{b-ce^x} = \frac{c+de^x}{c-de^x}$, then a, b, c, d are in	(q)	H.P.
(C)	If a, b, c are in A.P.; a, x, b are in G.P. and b, y, c are in G.P., then x^2, b^2, y^2 are in	(r)	G.P.
(D)	If x, y, x are in G.P., $a^x = b^y = c^z$, then $\log a, \log b, \log c$ are in	(s)	None of these

CODES :

	A	B	C	D
a)	q	r	p	r
b)	r	p	q	s
c)	s	r	p	q
d)	p	s	r	q

Linked Comprehension Type

This section contain(s) 16 paragraph(s) and based upon each paragraph, multiple choice questions have to be answered. Each question has atleast 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 192 to -192

Directions (Q.No.27 and 28) For first n natural numbers

$$1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

If $a_1, a_2, a_3, \ldots a_n \in$ AP, then sum of n terms of the sequence $\frac{1}{a_1 a_2}, \frac{1}{a_2 a_3}, \ldots \frac{1}{a_{n-1} a_n}$ equals $\frac{n-1}{a_1 a_n}$. The sum of n terms of a GP with first term a and common ratio r is given by $S_n = \frac{lr-a}{r-1}$ for $r \neq 1$ and na for $r = 1$. The sum of infinite term of GP is the limiting value of $\frac{lr-a}{r-1}$ when $n \to \infty$ and $-1 < r < 1$ where l is the last term of the sequence.

On the basis of above information, answer the following questions.

192. The sum of n terms of the series

$\frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + \ldots$ is

a) $\frac{6n}{n+1}$ b) $\frac{n}{n+1}$ c) $\frac{6n}{(n+1)^2}$ d) None of these

Paragraph for Question Nos. 193 to - 193

If A, G and H are respectively arithmetic, geometric and harmonic means between a and b both being unequal and positive, then

$A = \frac{a+b}{2} \Rightarrow a + b = 2A$

$G = \sqrt{ab} \Rightarrow G^2 = ab, H = \frac{2ab}{a+b}$

$\Rightarrow G^2 = AH$

On the basis of above information, answer the following question.

193. If the geometric and harmonic means of two numbers are 16 $12\frac{4}{5}$, then the ratio of one number to the other is

a) 1 : 4 b) 2 : 3 c) 1 : 2 d) 2 : 1

Paragraph for Question Nos. 194 to - 194

Sum of certain consecutive odd positive integers is $57^2 - 13^2$

194. Number of integers are

a) 40 b) 37 c) 44 d) 51

Paragraph for Question Nos. 195 to - 195

Consider three distinct real numbers a, b, c in a G.P. with $a^2 + b^2 + c^2 = t^2$ and $a + b + c = \alpha t$. Sum of the common ratio and its reciprocal is denoted by S

195. Complete set of α^2 is

a) $\left(\frac{1}{3}, 3\right)$ b) $\left[\frac{1}{3}, 3\right]$ c) $\left(\frac{1}{3}, 3\right) - \{1\}$ d) $\left(-\infty, \frac{1}{3}\right) \cup (3, \infty)$

Paragraph for Question Nos. 196 to - 196

In a G.P., the sum of the first and last term is 66, the product of the second and the last but one is 128 and the sum of the terms is 126

196. If an increasing G.P. is considered, then the number of terms in G.P. is

a) 9 b) 8 c) 12 d) 6

Paragraph for Question Nos. 197 to - 197

Four different integers form an increasing A.P. One of these numbers is equal to the sum of the squares of the other three numbers. Then

197. The product of all numbers is

a) -2 b) 1 c) 0 d) 2

Paragraph for Question Nos. 198 to - 198

Consider the sequence in the form of groups (1), (2, 2), (3, 3, 3), (4, 4, 4, 4), (5, 5, 5, 5), …

198. The 2000th term of the sequence is not divisible by

a) 3 b) 9 c) 7 d) None of these

Paragraph for Question Nos. 199 to - 199

There are two sets A and B each of which consists of three numbers in A.P. whose sum is 15 and where D and d are the common differences such that $D - d = 1$. If $\frac{p}{q} = \frac{7}{8}$, where p and q are the product of the numbers, respectively, and $d > 0$ in the two sets

199. Sum of the product of the numbers in set A taken two at a time is

a) 51 b) 71 c) 74 d) 86

Paragraph for Question Nos. 200 to - 200

Let $A_1, A_2, A_3, \dots, A_m$ be the arithmetic means between -2 and 1027 and $G_1, G_2, G_3, \dots, G_n$ be the geometric means between 1 and 1024. The product of geometric means is 2^{45} and sum of arithmetic means is 1025×171

200. The value of $\sum_{r=1}^{n} G_r$ is

a) 512 b) 2046 c) 1022 d) None of these

Paragraph for Question Nos. 201 to - 201

Two consecutive numbers from $1, 2, 3, \dots, n$ are removed. The arithmetic mean of the remaining numbers is 105/4

201. The value of n lies in

a) [45, 55] b) [52, 60] c) [41, 49] d) None of these

Paragraph for Question Nos. 202 to - 202

Two arithmetic progressions have the same numbers. The ratio of the last term of the first progression to first term of the second progression is equal to the ratio of the last term of the second progression to the first term of the first progression and is equal to 4, the ratio of the sum of the n terms of the first progression to the sum of the n terms of the second progression is equal to 2

202. The ratio of their common difference is

a) 12 b) 24 c) 26 d) 9

Paragraph for Question Nos. 203 to - 203

The numbers a, b, and c are between 2 and 18, such that

1. Their sum is 25
2. The numbers $2, a,$ and b are consecutive terms of an A.P.
3. The numbers $b, c, 18$ are consecutive terms of a G.P.

203. The value of abc is
 a) 500 b) 450 c) 720 d) None of these

Paragraph for Question Nos. 204 to - 204

Let $T_1, T_2, T_3, \dots, T_n$ be the terms of a sequence and let $(T_2 - T_1) = T_1', (T_3 - T_2) = T_2', \dots, (T_n - T_{n-1}) = T_{n-1}'$

Case I:

If $T_1', T_2', \dots, T_{n-1}'$ are in A.P., then T_n is quadratic in 'n'. If $T_1' - T_2', T_2' - T_3', \dots$, are in A.P., then T_n is cubic in n

Case II:

If $T_1', T_2', \dots, T_{n-1}'$ are not in A.P., but in G.P., then $T_n = ar^n + b$, where r is the common ratio of the G.P. $T_1', T_2', T_3', \dots$ and $a, b \in R$. Again, if $T_1', T_2', \dots, T_{n-1}'$ are not in G.P. but $T_2' - T_1', T_3' - T_2', \dots, T_{n-2}'$ are in G.P., then T_n is of form $ar^n + bn + c$ and r is the common ratio of the G.P. $T_2' - T_1', T_3' - T_2', T_4' - T_3, \dots$ and $a, b, c \in R$

204. The sum of 20 terms of the series $3 + 7 + 14 + 24 + 37 + \cdots$ is
 a) 4010 b) 3860 c) 4240 d) None of these

Integer Answer Type

205. Let a, b, c, d be four distinct real numbers in A.P. Then half of the smallest positive value of k satisfying $2(a-b) + k(b-c)^2 + (c-a)^3 = 2(a-d) + (b-d)^2 + (c-d)^3$ is
206. Let sum of first three terms of G.P. with real terms is $\frac{13}{12}$ and their product is -1. If the absolute value of the sum of their infinite terms is S, then the value of $7S$ is
207. Let S denote sum of the series $\frac{3}{2^3} + \frac{4}{2^4 \cdot 3} + \frac{5}{2^6 \cdot 3} + \frac{6}{2^7 \cdot 5} + \cdots \infty$. Then the value of S^{-1} is
208. Let $S = \sum_{n=1}^{9999} \frac{1}{(\sqrt{n} + \sqrt{n+1})(\sqrt[4]{n} + \sqrt[4]{n+1})}$, then S equals
209. The 5^{th} and 8^{th} terms of a geometric sequence of real numbers are $7!$ and $8!$ respectively. If the sum to first n terms of the G.P. is 2205, then n equals
210. The coefficient of the quadratic equation $ax^2 + (a+d)x + (a+2d) = 0$ are consecutive terms of a positively valued, increasing arithmetic sequence. Then the least integral value of $\frac{d}{a}$ such that the equation has real solutions is
211. Let $a_1, a_2, a_3, \dots a_{101}$ are in G.P. with $a_{101} = 25$ and $\sum_{i=1}^{201} a_i = 625$. Then the value of $\sum_{i=1}^{201} \frac{1}{a_i}$ equals
212. If the roots of $10x^3 - nx^2 - 54x - 27 = 0$ are in harmonic progression, then 'n' equals
213. The terms a_1, a_2, a_3 form an arithmetic sequence whose sum is 18. The terms $a_1 + 1, a_2\ a_3 + 2$, in that order, form a geometric sequence. Then the absolute value of the sum of all possible common difference of the A.P. is
214. The difference between the sum of the first k terms of the series $1^3 + 2^3 + 3^3 + \dots + n^3$ and the sum of the first k terms of $1 + 2 + 3 + \dots + n$ is 1980. The value of k is
215. Number of positive integral ordered pairs of (a, b) such that $6, a, b$ are in harmonic progression is
216. Let $a_n = 16, 4, 1, \dots$ be a geometric sequence. Define P_n as the product of the first n terms. Then the value of

$\frac{1}{4}\sum_{n=1}^{\infty} \sqrt[n]{P_n}$ is

217. If the equation $x^3 + ax^2 + bx + 216 = 0$ has three real roots in G.P. then b/a has the equal to
218. Given a, b, c are in A.P., b, c, d are in G.P. and c, d, e are in H.P. If $a = 2$ and $e = 18$, then the sum of all possible value of 'c' is
219. Let $a + ar_1 + ar_1^2 + \ldots + \infty$ and $a + ar_2 + ar_2^2 + \ldots + \infty$ be two infinite series of positive numbers with the same first term. The sum of the first series is r_1, and the sum of the second series is r_2. Then the value of $(r_1 + r_2)$ is
220. For $a, b, > 0$, let $5a - b, 2a + b, a + 2b$ be in A.P. and $(b + 1)^2, ab + 1, (a - 1)^2$ are in G.P., then the value of $(a^{-1} + b^{-1})$ is
221. The first term of an arithmetic progression is 1 and the sum of the first nine terms equal to 369. The first and the ninth term of a geometric progression coincide with the first and the ninth term of the arithmetic progression. If the seventh term of the geometric progression is T_7, then the value of $T_2/9$ is
222. The value of the $\sum_{n=0}^{\infty} \frac{2n+3}{3^n}$ is equal to

: ANSWER KEY :

1)	b	2)	d	3)	a	4)	c
5)	c	6)	c	7)	c	8)	c
9)	a	10)	c	11)	d	12)	a
13)	d	14)	b	15)	b	16)	a
17)	c	18)	b	19)	b	20)	a
21)	a	22)	b	23)	b	24)	b
25)	a	26)	a	27)	d	28)	d
29)	a	30)	d	31)	c	32)	c
33)	c	34)	d	35)	b	36)	b
37)	d	38)	d	39)	b	40)	d
41)	c	42)	d	43)	b	44)	a
45)	d	46)	d	47)	c	48)	b
49)	c	50)	b	51)	d	52)	a
53)	d	54)	a	55)	c	56)	d
57)	c	58)	b	59)	b	60)	a
61)	c	62)	c	63)	a	64)	a
65)	b	66)	d	67)	b	68)	b
69)	b	70)	a	71)	b	72)	b
73)	c	74)	d	75)	c	76)	b
77)	d	78)	c	79)	a	80)	b
81)	c	82)	d	83)	a	84)	b
85)	b	86)	c	87)	a	88)	d
89)	d	90)	d	91)	b	92)	a
93)	b	94)	c	95)	b	96)	c
97)	c	98)	c	99)	b	100)	c
101)	c	102)	d	103)	b	104)	b
105)	a	106)	c	107)	c	108)	c
109)	d	110)	c	111)	d	112)	c
113)	d	114)	c	115)	a	116)	b
117)	a	118)	a	119)	b	120)	a
121)	d	122)	d	123)	b	124)	c
125)	b	126)	b	127)	c	128)	d
129)	b	130)	a	131)	b	1)	
	a,b,c	2)	a,b,c	3)	b,d	4)	
	a,b,c						
5)	a,b,d	6)	a,c	7)	a,b,c	8)	
	b,d						
9)	c	10)	a,b,c	11)	a,b,c,d	12)	b
13)	b,c	14)	a,c	15)	a,b,c	16)	
	a,b						
17)	a,c	18)	a,b,c	19)	a,c	20)	
	a,b,c						
21)	a,b,c,d	22)	a,c,d	23)	a,c	24)	
	a,b,d						
25)	a,d	26)	a,d	27)	a,d	28)	
	a,b						
29)	a,d	30)	b,d	31)	a,c,d	32)	

	c,d						
33)	a,b	34)	b,c	35)	a,d	36)	
	a,b,c						
37)	b,c,d	38)	b	1)	a	2)	c
	3)	d	4)	d			
5)	a	6)	d	7)	b	8)	a
9)	b	10)	a	11)	d	12)	a
13)	d	14)	a	15)	a	16)	b
17)	a	18)	d	19)	b	20)	a
1)	b	2)	a	1)	a	2)	a
	3)	c	4)	c			
5)	d	6)	c	7)	d	8)	b
9)	c	10)	a	11)	c	12)	d
13)	c	1)	8	2)	4	3)	2
	4)	9					
5)	3	6)	7	7)	1	8)	0
9)	1	10)	9	11)	7	12)	8
13)	6	14)	0	15)	1	16)	6
17)	3	18)	6				

: HINTS AND SOLUTIONS :

1 **(b)**

x, y, and z are in G.P. Hence,

$y = xr, z = xr^2$

Also, $x, 2y$, and $3z$ are in A.P. Hence,

$4y = x + 3z$

$\Rightarrow 4xr = x + 3xr^2$

$\Rightarrow 3r^2 - 4r + 1 = 0$

$\Rightarrow (3r-1)(r-1) = 0$

$\Rightarrow r = 1/3$ ($r \neq 1$ is not possible as x, y, z are distinct)

2 **(d)**

$\sum a_i b_i = \sum a_i(1 - a_i)$

$= na - \sum a_i^2$

$= na - \sum (a_i - a + a)^2$

$= na - \sum [(a_i - a)^2 + a^2 + 2a(a_i - a)]$

$= na - \sum (a_i - a)^2 - \sum a^2 - 2a \sum (a_i - a)$

$\Rightarrow \sum a_i b_i + \sum (a_i - a)^2$

$= na - na^2 - 2a(na - na)$ $\left[\because \sum b_i = \sum 1 - \sum a_i \;\; \therefore \;\; nb = n - na \;\; \text{or} \;\; a + b = 1\right]$

$= na(1-a) = nab$

3 **(a)**

$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$

$= \left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \frac{1}{7^2} + \cdots\right) - \left(\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \cdots\right)$

$= \frac{\pi^2}{6} - \frac{1}{4}\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right)$

$= \frac{\pi^2}{6} - \frac{1}{4}\left(\frac{\pi^2}{6}\right)$

$= \frac{\pi^2}{8}$

4 **(c)**

Let the 1025th term fall is in the n^{th} group. Then

$1 + 2 + 4 + \cdots + 2^{n-1} < 1025 \leq 1 + 2 + 4 + \cdots + 2^n$

$\Rightarrow 2^{n-1} < 1026 \leq 2^{n+1}$

$\Rightarrow n = 10$

$\Rightarrow$1025th term 2^{10}

5 **(c)**

x, y, and z are in G.P. Hence,

$y^2 = -xz$ (1)

We have,

$a^x = b^y = c^z = \lambda$ (say)

$\Rightarrow x \log a = y \log b = z \log c = \log \lambda$

$\Rightarrow x = \frac{\log \lambda}{\log a}, y = \frac{\log \lambda}{\log b}, z = \frac{\log \lambda}{\log c}$

Putting the values of x, y, and z in (1), we get

$\left(\frac{\log \lambda}{\log b}\right)^2 = \frac{\log \lambda}{\log a} \frac{\log \lambda}{\log c}$

$\Rightarrow (\log b)^2 = \log a \log c$

$\Rightarrow \log_b a = \log_c b$

6 **(c)**

$T_n = \frac{1^3 + 2^3 + 3^3 + \cdots + n^3}{1 + 3 + 5 + \cdots n \text{ terms}}$

$= \frac{\sum n^3}{\frac{n}{2}[2 \times 1 + (n-1)2]}$

$= \frac{1}{4} \times \frac{n^2(n+1)^2}{n^2} = \frac{1}{4}(n^2 + 2n + 1)$ (1)

Now,

$S_n = \frac{1}{4}\left(\sum n^2 + 2\sum n + n\right)$

$= \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + 2 \times \frac{n(n+1)}{2} + n\right]$

$= \frac{n}{24}[2n^2 + 3n + 1 + 6n + 6 + 6]$

$= \frac{n}{24}[2n^2 + 9n + 13]$

Putting $n = 16$, we get

$S_{16} = \frac{16}{24}[2(256) + 144 + 13]$

$= \frac{2}{3}(669) = 446$

7 **(c)**

$\frac{H_1 + 2}{H_1 - 2} + \frac{H_{20} + 3}{H_{20} - 3} = \frac{\frac{1}{2} + \frac{1}{H_1}}{\frac{1}{2} - \frac{1}{H_1}} + \frac{\frac{1}{3} + \frac{1}{H_{20}}}{\frac{1}{3} - \frac{1}{H_{20}}}$

$= \frac{\frac{1}{2} + \frac{1}{2} + d}{\frac{1}{2} - d - \frac{1}{2}} + \frac{\frac{1}{3} + \frac{1}{3} - d}{\frac{1}{3} + d - \frac{1}{3}}$

$= \frac{\frac{2}{2} + d}{-d} + \frac{\frac{2}{3} - d}{d}$

$= \frac{\frac{2}{3} - 1}{d} - 2$

$= 2 \times 21 - 2$ [as also, $\frac{1}{3} = \frac{1}{2} + 21d$]

$= 40$

8 **(c)**

The sum equals $\frac{n(n+1)(n+2)}{6} = 220$

Which is true for $n = 10$

9 **(a)**

We have,

$$\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots = \frac{1/4}{1-1/2} = \frac{1}{2}$$

Hence,

$$0.2^{\log_{\sqrt{5}}\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots\right)} = 0.2^{\log_{\sqrt{5}}\frac{1}{2}}$$

$$= \left(\frac{1}{5}\right)^{\log_{\sqrt{5}}\frac{1}{2}}$$

$$= (5^{-1})^{2\log_5\frac{1}{2}}$$

$$= (5)^{\log_5 4}$$

$$= 4$$

10 **(c)**

Here the successive differences are 2, 4, 8, 16,… which are in G.P.

$$S = 1+3+7+15+31+\cdots+T_{100}$$

$$S = (2^1-1)+(2^2-1)+(2^3-1)+\cdots+(2^{100}-1)$$

$$= (2+2^2+2^3+\cdots+2^{100})-100$$

$$= 2\left(\frac{2^{100}-1}{2-1}\right)-100$$

$$= 2^{101}-102$$

11 **(d)**

We have,

$$2^{n+10} = 2\times2^2+3\times2^3+4\times2^4+\cdots+n\times2^n$$

$$\Rightarrow 2(2^{n+10}) = 2\times2^3+3\times2^4+\cdots+(n-1)\times2^n+n\times2^{n+1}$$

Subtracting, we get

$$-2^{n+10} = 2\times2^2+2^3+2^4+\cdots+2^n-n\times2^{n+1}$$

$$= 8+\frac{8(2^{n-2}-1)}{2-1}-n\cdot2^{n+1}$$

$$= 8+2^{n+1}-8-n\times2^{n+1} = 2^{n+1}-(n)2^{n+1}$$

$$\Rightarrow 2^{10} = 2n-2 \Rightarrow n = 513$$

12 **(a)**

Clearly, $\frac{1}{x_1},\frac{1}{x_2},\ldots,\frac{1}{x_{20}}$ will be in A.P. Hence,

$$\frac{1}{x_2}-\frac{1}{x_1} = \frac{1}{x_3}-\frac{1}{x_2} = \cdots = \frac{1}{x_{r+1}}-\frac{1}{x_r} = \cdots = \lambda \text{ (say)}$$

$$\Rightarrow \frac{x_r-x_{r+1}}{x_rx_{r+1}} = \lambda$$

$$\Rightarrow x_rx_{r+1} = -\frac{1}{\lambda}(x_{r+1}-x_r)$$

$$\Rightarrow \sum_{r=1}^{19}x_rx_{r+1} = -\frac{1}{\lambda}\sum_{r=1}^{19}(x_{r+1}-x_r)$$

$$= -\frac{1}{\lambda}(x_{20}-x_1)$$

Now, $\frac{1}{x_{20}} = \frac{1}{x_1}+19\lambda$

$$\Rightarrow \frac{x_1-x_{20}}{x_1x_{20}} = 19\lambda$$

$$\Rightarrow \sum_{r=1}^{19}x_rx_{r+1} = 19x_1x_{20} = 19\times4 = 76$$

($\because x_1, 2, x_{20}$ are in G.P., then $x_1x_{20} = 4$)

13 **(d)**

$$A = \frac{25+n}{2}, G = 5\sqrt{n}, H = \frac{50n}{25+n}$$

As A, G, H are natural numbers, n must be odd perfect square. Now, H will be a natural number, if we take $n = 225$

14 **(b)**

$$T_r = (-1)^r\frac{r^2+r+1}{r!}$$

$$= (-1)^r\left[\frac{r}{(r-1)!}+\frac{1}{(r-1)!}+\frac{1}{r!}\right]$$

$$= (-1)^r\left[\frac{1}{(r-2)!}+\frac{1}{(r-1)!}+\frac{1}{(r-1)!}+\frac{1}{r!}\right]$$

$$= \left[\frac{(-1)^r}{r!}+\frac{(-1)^r}{(r-1)!}\right]+\left[\frac{(-1)^r}{(r-1)!}+\frac{(-1)^r}{(r-2)!}\right]$$

$$= \left[\frac{(-1)^r}{r!}-\frac{(-1)^{r-1}}{(r-1)!}\right]+\left[\frac{(-1)^{r-1}}{(r-1)!}-\frac{(-1)^{r-2}}{(r-2)!}\right]$$

$$= V(r)-V(r-1)$$

$$\therefore \sum_{r=1}^{n}T_r = V(n)-V(0)$$

$$= \left[\frac{(-1)^n}{n!}-\frac{(-1)^{n-1}}{(n-1)!}\right]-1$$

Therefore the sum of 20 terms is

$$\left[\frac{1}{20!}-\frac{-1}{19!}\right]-1 = \frac{21}{20!}-1$$

15 **(b)**

$a^2+b^2, ab+bc, b^2+c^2$ are in G.P.

$$\Rightarrow (ab+bc)^2 = (a^2+b^2)(b^2+c^2)$$

$$\Rightarrow a^2b^2+b^2c^2+2ab^2c = a^2b^2+a^2c^2+b^2c^2+b^4$$

$$\Rightarrow b^4+a^2c^2-2ab^2c = 0$$

$$\Rightarrow (b^2-ac)^2 = 0$$

$$\Rightarrow b^2 = ac$$

$\Rightarrow a, b,$ and c are in G.P.

16 **(a)**

$$S_p = \frac{1}{1-r^p}, s_p = \frac{1}{1+r^p}, S_{2p} = \frac{1}{1-r^{2p}}$$

Clearly, $S_p+s_p = \frac{2}{1-r^{2p}} = 2S_{2p}$

17 **(c)**

We have,

$$\frac{(x+2)^n-(x+1)^n}{(x+2)-(x+1)}$$

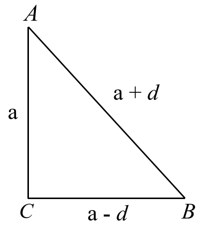
$= (x+2)^{n-1} + (x+2)^{n-2}(x+1)$
$+ (x+2)^{n-3}(x+1)^2$
$+ \cdots (x-1)^{n-1}$
Hence, the required sum is
$(x+2)^n - (x+1)^n$ [$\because$ $(x+2)-(x+1)=1$]

18 **(b)**
If t_r be the r^{th} term of the A.P., then
$t_r = S_r - S_{r-1}$
$= cr\,(r-1) - c(r-1)(r-2)$
$= c(r-1)(r-r+2) = 2c(r-1)$
We have,
$t_1^2 + t_2^2 + \cdots + t_n^2$
$= 4c^2(0^2+1^2+2^2+\cdots$
$+(n-1)^2)$
$= 4c^2 \dfrac{(n-1)n(2n-1)}{6}$
$= \dfrac{2}{3}\, c^2 n\,(n-1)(2n-1)$

19 **(b)**
$\dfrac{S_{nx}}{S_x} = \dfrac{\frac{nx}{2}[2a+(nx-1)d]}{\frac{x}{2}[2a+(x-1)d]}$
$= \dfrac{n[(2a-d)+nxd]}{(2a-d)+xd}$
For $\frac{S_{nx}}{S_x}$ to be independent of x,
$2a-d=0 \Rightarrow 2a=d$
Now, $S_P = \frac{p}{2}[2a+(p-1)d] = p^2a$

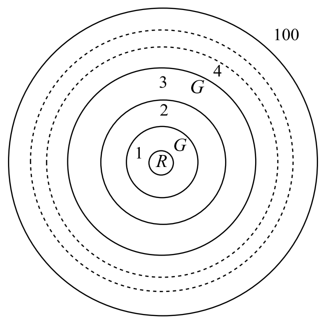
20 **(a)**
The general term can be given by
$t_{r+1} = \dfrac{a_{2n+1-r} - a_{r+1}}{a_{2n+1-r} + a_{r+1}}, r = 0,1,2,\dots,n-1$
$= \dfrac{a_1 + (2n-r)d - \{a_1 + rd\}}{a_1 + (2n-r)d + \{a_1 + rd\}}$
$= \dfrac{(n-r)d}{a_1 + nd}$
Therefore, the required sum is
$S_n = \sum_{r=0}^{n-1} t_{r+1}$
$= \sum_{r=0}^{n-1} \dfrac{(n-r)d}{a_1+nd}$
$= \left[\dfrac{n+(n-1)+(n-2)+\cdots+1}{a_1+nd}\right] d$
$= \dfrac{n(n+1)d}{2a_{n+1}}$
$= \dfrac{n(n+1)}{2}\,\dfrac{a_2-a_1}{a_{n+1}}$ [$\because d = a_2 - a_1$]

21 **(a)**
Let $\angle C = 90°$ being greatest and $B = 90° - A$
The sides are $a-d, a$ and $a+d$
We have $(a+d)^2 = (a-d)^2 + a^2$
(using Pythagoras Theorem)
$\therefore$ $4ad - a^2 = 0 \Rightarrow a = 4d$
Hence the sides are $3d, 4d, 5d$
Clearly, $\sin A = \frac{BC}{AB} = \frac{a-d}{a+d} = \frac{3d}{5d} = \frac{3}{5}$
$\sin B = \dfrac{AC}{AB} = \dfrac{a}{a+d} = \dfrac{4d}{5d} = \dfrac{4}{5}$



22 **(b)**
Required G.M. is $\sqrt{-9 \times -16} = -12$

23 **(b)**



$\pi[(r_2^2 - r_1^2) + (r_4^2 - r_3^2) + \cdots + (r_{100}^2 - r_{99}^2)]$
$= \pi[r_1 + r_2 + r_3 + r_4 + \cdots + r_{100}]$ $(\because r_2 - r_1$
$= r_4 - r_3 = \cdots = r_{100} - r_{99} = 1)$
$= \pi[1+2+3+\cdots+r_{100}]$
$= 5050\pi$ sq. cm

24 **(b)**
Given,
$ar^2 = 4$
$\Rightarrow a \times ar \times ar^2 \times ar^3 \times ar^4 = a^5r^{10} = (ar^2)^5$
$= 4^5$

25 **(a)**
Let r be the common ratio of the G.P., a, b, c, d.
Then,
$b = ar, c = ar^2$ and $d = ar^3$
$\therefore$ $(b-c)^2 + (c-a)^2 + (d-b)^2$
$= (ar - ar^2)^2 + (ar^2 - a)^2 + (ar^3 - ar)^2$
$= a^2r^2(1-r)^2 + a^2(r^2-1)^2 + a^2r^2(r^2-1)^2$
$= a^2(r^6 - 2r^3 + 1)$
$= a^2(1-r^3)^2$
$= (a - ar^3)^2$
$= (a-d)^2$

26 **(a)**
The general term of the given series is
$t_n = \dfrac{x^{2^{n-1}}}{1 - x^{2^n}} = \dfrac{1 + x^{2^{n-1}} - 1}{\left(1 + x^{2^{n-1}}\right)\left(1 - x^{2^{n-1}}\right)}$

$\Rightarrow t_n = \dfrac{1}{1-x^{2^{n-1}}} - \dfrac{1}{1-x^{2^n}}$

Now,

$S_n = \sum_{n=1}^{n} t_n$

$= \left[\left\{\dfrac{1}{1-x} - \dfrac{1}{1-x^2}\right\} + \left\{\dfrac{1}{1-x^2} - \dfrac{1}{1-x^4}\right\} + \cdots + \left\{\dfrac{1}{1-x^{2^{n-1}}} - \dfrac{1}{1-x^{2^n}}\right\}\right]$

$= \dfrac{1}{1-x} - \dfrac{1}{1-x^{2^n}}$

Therefore, the sum to infinite terms is

$\lim_{n\to\infty} S_n = \dfrac{1}{1-x} - 1$

$= \frac{x}{1-x}$ $[\because \lim_{n\to\infty} x^{2n} = 0, \text{as } |x| < 1]$

27 **(d)**

In $(a+c)$, In $(c-a)$, In $(a-2b+c)$ are in A.P.

Hence, $a+c, c-a, a-2b+c$ are in G.P.

Therefore,

$(c-a)^2 = (a+c)(a-2b+c)$

$\Rightarrow (c-a)^2 = (a+c)^2 - 2b(a+c)$

$\Rightarrow 2b(a+c) = (a+c)^2 - (c-a)^2$

$\Rightarrow b = \dfrac{2ac}{a+c}$

Hence, a, b, and c are in H.P.

28 **(d)**

$S = \frac{2}{10} + \frac{4}{10^3} + \frac{6}{10^5} + \frac{8}{10^7} + \cdots \text{ to } \infty \quad (1)$

$= \dfrac{\frac{2}{10}}{1-\frac{1}{10^2}} + \dfrac{2\times\left(\frac{1}{10^2}\right)}{\left(1-\frac{1}{10^2}\right)^2}$

$= \dfrac{20}{99} + \dfrac{200}{9801}$

$= \dfrac{2180}{9801}$

29 **(a)**

Series is $a, a+2, a+4, \ldots + a+4n, (a+$
4n0.5, a+4n0.52, …a+4n0.52n−1

The middle term of A.P. and G.P. are equal

$\Rightarrow a + 2n = (a+4n)(0.5)^n$

$\Rightarrow a.2^n + 2^{n+1}n = a + 4n$

$\Rightarrow a = \dfrac{4n - n2^{n+1}}{2^n - 1}$

$\Rightarrow$ The middle term of entire sequence

$= (a+4n)0.5 = \left(\dfrac{4n - n2^{n+1}}{2^n-1} + 4n\right)\dfrac{1}{2} = \dfrac{n.2^{n+1}}{2^n-1}$

30 **(d)**

As $a_1, a_2, a_3, \ldots, a_{n-1}, a_n$ are in A.P., hence

$d = a_2 - a_1 = a_3 - a_2 = \cdots = a_n - a_{n-1}$

$\sin d\,[\sec a_1 \sec a_2 + \sec a_2 \sec a_3 + \cdots + \sec a_{n-1}\sec a_n]$

$= \dfrac{\sin(a_2-a_1)}{\cos a_1 \cos a_2} + \dfrac{\sin(a_3-a_2)}{\cos a_2\cos a_3} + \cdots + \dfrac{\sin(a_n - a_{n-1})}{\cos a_{n-1}\cos a_n}$

$= (\tan a_2 - \tan a_1) + (\tan a_3 - \tan a_2) + \cdots + (\tan a_n - \tan a_{n-1})$

$= \tan a_n - \tan a_1$

31 **(c)**

$2b = a + c, c = \dfrac{2bd}{b+d}$

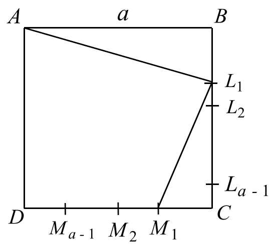
$\Rightarrow\ 2bd = c(b+d)$

$\Rightarrow (a+c)d = c(b+d)$ [as $2b = a+c$]

$\Rightarrow ad + cd = bc + cd$

$\Rightarrow bc = ad$

32 **(c)**



$(AL_1)^2 + (L_1M_1)^2 = (a^2 + 1^2) + \{(a-1)^2 + 1^2\}$

$(AL_2)^2 + (L_2M_2)^2 = (a^2 + 2^2) + \{(a-2)^2 + 2^2\}$

$\vdots$

$(AL_{a-1})^2 + (L_{a-1}M_{a-1})^2 = a^2 + (a-1^2) + \{1^2 + (a-1)^2\}$

Therefore, the required sum is

$(a-1)a^2 + \{1^2 + 2^2 + \cdots + (a-1)^2\} + 2\{1^2 + 2^2 + \cdots + (a-1)^2\}$

$= (a-1)a^2 + 3\dfrac{(a-1)a(2a-1)}{6}$

$= a(a-1)\left(a + \dfrac{2a-1}{2}\right)$

$= \dfrac{1}{2}(a-1)(4a-1)$

33 **(c)**

Consider the first product,

$P = \left(1+\dfrac{1}{3}\right)\left(1+\dfrac{1}{3^2}\right)\left(1+\dfrac{1}{3^4}\right)\left(1+\dfrac{1}{3^8}\right) \ldots \left(1+\dfrac{1}{3^{2^n}}\right)$

$= \dfrac{\left(1-\frac{1}{3}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{3^2}\right)\left(1+\frac{1}{3^4}\right)\left(1+\frac{1}{3^8}\right)\ldots\left(1+\frac{1}{3^{2^n}}\right)}{\left(1-\frac{1}{3}\right)}$

$= \dfrac{\left(1-\frac{1}{3^2}\right)\left(1+\frac{1}{3^2}\right)\left(1+\frac{1}{3^4}\right)\left(1+\frac{1}{3^8}\right)\ldots\left(1+\frac{1}{3^{2^n}}\right)}{\left(1-\frac{1}{3}\right)}$

$$\frac{\left(1-\frac{1}{3^4}\right)\left(1+\frac{1}{3^4}\right)\left(1+\frac{1}{3^8}\right)\ldots\left(1+\frac{1}{3^{2^n}}\right)}{\left(1-\frac{1}{3}\right)}$$

$$=\frac{1}{\left(1-\frac{1}{3}\right)}\left(1-\left(\frac{1}{3}\right)^{2^{n+1}}\right)$$

$$=\frac{3}{2}\left(1-\left(\frac{1}{3}\right)^{2^{n+1}}\right)$$

$$\Rightarrow \left(1+\frac{1}{3}\right)\left(1+\frac{1}{3^2}\right)\left(1+\frac{1}{3^4}\right)\left(1+\frac{1}{3^8}\right)\ldots \text{infinity}$$

$$=\lim_{n\to\infty}\frac{3}{2}\left(1-\left(\frac{1}{3}\right)^{2^{n+1}}\right)$$

$$=\frac{3}{2}$$

34 **(d)**

We have,

$$S=\frac{1}{1-\frac{1}{2}}=2$$

$$S_n=\frac{(1-1/2^n)}{(1-1/2)}=2\left(1-\frac{1}{2^n}\right)=2-\frac{1}{2^{n-1}}$$

$$\therefore\ S-S_n<\frac{1}{1000}\ \Rightarrow \frac{1}{2^{n-1}}<\frac{1}{1000}$$

$\Rightarrow 2^{n-1}>1000$

$\Rightarrow n-1\geq 10$

$\Rightarrow n\geq 11$

Hence, the least value of n is 11

35 **(b)**

Degree of x on L.H.S. is

$1+2+4+\cdots+128$

$=1+2+2^2+\cdots+2^7$

$$=\frac{2^8-1}{2-1}$$

$=255$

36 **(b)**

$S=\frac{4}{19}+\frac{44}{19^2}+\frac{444}{19^3}+\cdots \quad (1)$

$\Rightarrow \frac{1}{19}S=\frac{4}{19^2}+\frac{44}{19^3}+\cdots \quad (2)$

Subtracting (2) from (1), we get

$$\frac{18}{19}S=\frac{4}{19}+\frac{40}{19^2}+\frac{400}{19^3}+\cdots$$

$$=\frac{\frac{4}{19}}{1-\frac{10}{19}}$$

$=4/9$

$\Rightarrow S=38/81$

37 **(d)**

Let the series be $21, 21r, 21r^2, \ldots$

Sum $=\frac{21}{1-r}$ is a positive integer

Also $21r$ is a positive integer

$S=\frac{(21)(21)}{21-21r}$ as $21r\in N$ hence $21-21r$ must be an integer

Also $21r<21$

Hence $21-21r$ may be equal to 1, 3, 7 or 9

i.e., must be a divisor of $(21)(21)$

hence $21-21r=1$ or 3 or 7 or 9

$21r=20, 18, 14$ or 12

38 **(d)**

We have that

$-1\leq \cos x\leq 1$

$\Rightarrow |\cos x|\leq 1$

But, $x\in S\ \Rightarrow x\in(0,\pi)\Rightarrow |\cos x|<1$

Now, $8^{1+|\cos x+\cos^2 x+|\cos^3 x|+\cdots \text{ to } \infty}=4^3$

$\Rightarrow 8^{1/(1-|\cos x|)}=8^2$

$$\Rightarrow \frac{1}{1-|\cos x|}=2$$

$$\Rightarrow |\cos x|=\frac{1}{2}$$

$$\Rightarrow \cos x=\pm\frac{1}{2}$$

$\Rightarrow x=\pi/3, 2\pi/3$

$\Rightarrow S=\{\pi/3, 2\pi/3\}$

39 **(b)**

The coefficient of x^{19} in the polynomial

$(x-1)(x-2)(x-2^2)\ldots(x-2^{19})$ is

$$-(1+2+2^2+\cdots+2^{19})=-1\left(\frac{2^{20}-1}{2-1}\right)$$

$=1-2^{20}$

40 **(d)**

$a=5, ar^2=a+3d, ar^4=a+15d$

$\therefore\ \ 5r^2=5+3d, 5r^4=5+15d$

$\Rightarrow\ \ r^4=1+3d$

$\Rightarrow 25r^4=25+75d$

$\Rightarrow (5+3d)^2=25+75d$

$\Rightarrow 25+30d+9d^2=25+75d$

$\Rightarrow 9d^2-45d=0$

$\Rightarrow d=5, 0$

$\Rightarrow T_4=a+3d=5+15=20$

41 **(c)**

Given that

$a_3+a_5+a_8=11$

$\Rightarrow a+2d+a+4d+a+7d=11$

$\Rightarrow 3a+13d=11 \quad (1)$

Given,

$a_4+a_2=-2$

$\Rightarrow a+3d+a+d=-2$

$\Rightarrow\ a=-1-2d \quad (2)$

Putting value of a from (2) in (1), we get

$3(-1-2d)+13d=11\ \Rightarrow 7d=14\ \Rightarrow d=2$

and $a=-5$

$\Rightarrow a_1 + a_6 + a_7 = 7$

42 **(d)**

a, b, and c are in A.P. Hence,

$2b = a + c \quad (1)$

$\frac{a}{bc} + \frac{2}{b} = \frac{a+2c}{bc} \neq \frac{2}{c}$

$\Rightarrow \frac{a}{bc}, \frac{1}{c}, \frac{2}{b}$ are not in A.P.

$\frac{bc}{a} + \frac{b}{2} = \frac{2bc+ab}{2a} \neq c$

Hence, the given numbers are not in H.P. Again,

$\frac{a}{bc}\frac{2}{b} = \frac{2a}{b^2c} \neq \frac{1}{c^2}$

Therefore, the given numbers are not in G.P.

43 **(b)**

$S = 1 + 2r + 3r^2 + 4r^3 + \cdots$

$rS = r + 2r^2 + 3r^3 + 4r^4 + \cdots$

$\Rightarrow (1-r)S = 1 + r + r^2 + r^3 + \cdots$

$= \frac{1}{1-r}$

$\Rightarrow S = \frac{1}{(1-r)^2}$

Given, $S = 9/4 \Rightarrow \frac{1}{(1-r)^2} = 9/4$

$\Rightarrow 1 - r = \pm\frac{2}{3}$

$\Rightarrow r = 1/3$ or $5/3$

Hence, $r = 1/3$ as $0 < |r| < 1$

44 **(a)**

$S = (1)(2003) + (2)(2002) + (3)(2001) + \cdots + (2003)(1)$

$= \sum_{r=1}^{2003} r\big(2003 - (r-1)\big)$

$= \sum_{r=1}^{2003} r(2004 - r)$

$= \sum_{r=1}^{2003} 2004r - \sum_{r=1}^{2003} r^2$

$= \frac{2004 \times 2003 \times 2004}{2} - 2003 \times 4007 \times 334$

$= 2003 \times 334 \times (6012 - 4007)$

$= 2003 \times 334 \times 2005$

Hence, $x = 2005$

45 **(d)**

$(1^2 - t_1) + (2^2 - t_2) + \cdots + (n^2 - t_n) = \frac{1}{3}n(n^2-1)$

$\Rightarrow 1^2 + 2^2 + 3^2 + \cdots + n^2$

$-\{t_1 + t_2 + \cdots + t_n\} = \frac{1}{3}n(n^2-1)$

$\Rightarrow \frac{n(n+1)(2n+1)}{6} - S_n = \frac{1}{3}n(n^2-1)$

$\Rightarrow S_n = \frac{n(n+1)}{6}[2n + 1 - 2(n-1)]$

$= \frac{n(n+1)}{6}[2n + 1 - 2n + 2]$

$= \frac{n(n+1)}{2}$

$\Rightarrow S_{n-1} = \frac{n(n-1)}{2}$

$\Rightarrow T_n = S_n - S_{n-1} = n$

46 **(d)**

a, b, and c, d are in A.P. Therefore, d, c, b and a are also in A.P. Hence,

$\frac{d}{abcd}, \frac{c}{abcd}, \frac{b}{abcd}, \frac{a}{abcd}$ are also in A.P.

$\Rightarrow \frac{1}{abc}, \frac{1}{abd}, \frac{1}{acd}, \frac{1}{bcd}$ are in A.P.

$\Rightarrow abc, abd, acd, bcd$ are in H.P.

47 **(c)**

Let $S_n = cn^2$, then

$S_{n-1} = c(n-1)^2 = cn^2 + c - 2cn$

$\therefore\ T_n = 2\,cn - c \quad (\because T_n = S_n - S_{n-1})$

$T_n^2 = (2cn - c)^2 = 4c^2n^2 + c^2 - 4c^2n$

$\therefore$ Sum $= \sum T_n^2$

$= \frac{4c^2.n(n+1)(2n+1)}{6} + nc^2 - 2c^2n(n+1)$

$= \frac{2c^2n(n+1)(2n+1) + 3nc^2 - 6c^2n(n+1)}{3}$

$= \frac{nc^2(4n^2 + 6n + 2 + 3 - 6n - 6)}{3}$

$= \frac{nc^2(4n^2-1)}{3}$

48 **(b)**

$S_{3n} = \frac{3n}{2}[2a + (3n-1)d]$

$S_{n-1} = \frac{n-1}{2}[2a + (n-2)d]$

$\Rightarrow S_{3n} - S_{n-1} = \frac{1}{2}[2a(3n - n + 1)] + \frac{d}{2}[3n(3n-1) - (n-1)(n-2)]$

$= \frac{1}{2}[2a(2n+1) + d(8n^2 - 2)]$

$= a(2n+1) + d(4n^2 - 1)$

$= (2n+1)[a + (2n-1)d]$

$S_{2n} - S_{2n-1} = T_{2n} = a + (2n-1)d$

$\Rightarrow \frac{S_{3n} - S_{n-1}}{S_{2n} - S_{2n-1}} = (2n+1)$

Given,

$\frac{S_{3n} - S_{n-1}}{S_{2n} - S_{2n-1}} = 31 \Rightarrow n = 15$

49 **(c)**

$S_\infty = \frac{a}{1-r} = 162$

$S_n = \frac{a(1-r^n)}{1-r} = 160$

Dividing,

$1 - r^n = \frac{160}{162} = \frac{80}{81}$

$\Rightarrow 1 - \frac{80}{81} = r^n$

$\Rightarrow r^n = \frac{1}{81}$ or $\left(\frac{1}{r}\right)^n = 81$

Now, it is given that $1/r$ is an integer and n is also an integer

Hence, the relation (1) implies that $1/r = 3, 9$ or 81 so that $n = 4, 2$ or 1

$\therefore\ a = 162\left(1 - \frac{1}{3}\right)$ or $162\left(1 - \frac{1}{9}\right)$ or $162\left(1 - \frac{1}{81}\right)$

$= 108$ or 144 or 160

50 **(b)**

Let the sides of the triangle be $a/2$, a and ar, with $a > 0$ and $r > 1$. Let α be the smallest angle, so that the largest angle is 2α. Then α is opposite to the side a/r, and 2α is positive to the side ar. Applying sine rule, we get

$\frac{a/r}{\sin\alpha} = \frac{ar}{\sin 2\alpha}$

$\Rightarrow \frac{\sin 2\alpha}{\sin\alpha} = r^2$

$\Rightarrow 2\cos\alpha = r^2$

$\Rightarrow r^2 < 2$

$\Rightarrow r < \sqrt{2}$

Hence, $1 < r < \sqrt{2}$

51 **(d)**

$x, 2x+2, 3x+3$ are in G.P. Hence,

$(2x+2)^2 = x(3x+3)$

$\Rightarrow 4x^2 + 8x + 4 = 3x^2 + 3x$

$\Rightarrow x^2 + 5x + 4 = 0$

$\Rightarrow x = -1, -4$

So, the G.P. is $-4, -6, -9, \dots$ (considering $x = -4$, as for $x = -1, 2x + 2 = 0$). Hence, the fourth term is $-9 \times 1.5 = -13.5$

52 **(a)**

Let,

$S = i - 2 - 3i + 4 + 5i + \cdots + 100i^{100}$

$= i + 2i^2 + 3i^3 + 4i^4 + 5i^5 + \cdots + 100i^{100}$

$\Rightarrow iS = i^2 + 2i^3 + 3i^4 + \cdots + 99i^{100} + 100i^{101}$

$\Rightarrow S - iS = [i + i^2 + i^3 + i^4 + \cdots + i^{100}] - 100i^{101}$

$\Rightarrow S(1-i) = \frac{i(i^{100} - 1)}{i - 1} - 100\, i^{101}$

$= -100i^{101}$

$\Rightarrow S = \frac{-100i}{1-i} = -50i(1+i) = -50(i-1)$

$= 50(1-i)$

53 **(d)**

$f(x) = 2x + 1$

$\Rightarrow f(2x) = 2(2x) + 1 = 4x + 1$ and $f(4x) = 2(4x) + 1 = 8x + 1$

Now, $f(x), f(2x), f(4x)$ are in G.P. Hence,

$(4x+1)^2 = (2x+1)(8x+1)$

$\Rightarrow 2x = 0$

Hence, $f(x), f(2x)$, and $f(4x)$ is equal to 1 which contradicts the given condition. Hence no such x exists

54 **(a)**

Let a be the first term and r be the common ratio of the given G.P. Then,

$\alpha = \sum_{n=1}^{100} a_{2n} \Rightarrow\ \alpha = a_2 + a_4 + \cdots + a_{200}$

$= ar + ar^3 + \cdots + ar^{199}$

$= ar(1 + r^2 + r^4 + \cdots + r^{198})$

$\beta = \sum_{n=1}^{100} a_{2n-1} \ \Rightarrow\ \beta = a_1 + a_3 + \cdots + a_{199}$

$= a + ar^2 + \cdots + ar^{198}$

$= a(1 + r^2 + \cdots + r^{198})$

Clearly, $\alpha/\beta = r$

55 **(c)**

Suppose the work is completed in n days when the workers stopped working. Since four workers stopped working every day except the first day. Therefore, the total number of workers who worked all the n days is the sum of n terms of an A.P. with first term 150 and common difference -4 i.e.,

$\frac{n}{2}[2 \times 150 + (n-1) \times -4] = n(152 - 2n)$

Had the workers not stopped working, then the work would have finished in $(n-8)$ days with 150 workers working on each day. Therefore, the total number of workers who would have worked all the n days is $150(n-8)$

$\therefore\ n(152 - 2n) = 150(n-8)$

$\Rightarrow n^2 - n - 600 = 0$

$\Rightarrow (n-25)(n+24) = 0$

$\Rightarrow n = 25$

Thus, the work is completed in 25 days

56 **(d)**

Let $P = 0.cababab \dots$

$\Rightarrow 10P = c.ababab \dots$ (1)

and $1000P = cab.ababab \dots$ (2)

$990P = cab - c$

or $P=\frac{100c+10a+b-c}{990}=\frac{99c+10a+b}{990}$

57 **(c)**

Let,

$S=\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+\cdots n$ terms

$=\left(1-\frac{1}{2}\right)+\left(1-\frac{1}{4}\right)+\left(1-\frac{1}{8}\right)+\left(1-\frac{1}{16}\right)+\ldots n$ terms

$=(1+1+1+\cdots n$ times$)-\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\cdots+\frac{1}{2^n}\right)$

$=n-\left[\frac{\frac{1}{2}\left(1-\frac{1}{2^n}\right)}{1-\frac{1}{2}}\right]=n-1+2^{-n}$

58 **(b)**

Given,

$F(n+1)=\frac{2F(n)+1}{2}$

$\Rightarrow F(n+1)-F(n)=1/2$

Hence, the given series is an A.P. with common difference 1/2 and first term being 2. $F(101)$ is 101^{st} term of A.P. given by $2+(101-1)(1/2)=52$

59 **(b)**

$(1-p)(1+3x+9x^2+27x^3+81x^4+243x^5)=1-p^6$

$\Rightarrow 1+3x+9x^2+27x^3+81x^4+243x^5=\frac{1-p^6}{1-p}$

$\Rightarrow 1+3x+9x^2+27x^3+81x^4+243x^5=1+p+p^2+p^3+p^4+p^5$

Comparing, we get $p=3x$ or $p/x=3$

60 **(a)**

Let the numbers be a, ar, ar^2. Then,

$a+ar+ar^2=14$ (given) (1)

Now, $a+1, ar+1, ar^2-1$ are in A.P.

$\Rightarrow 2(ar+1)=a+1+ar^2-1$

$\Rightarrow 2ar+2=a+ar^2$ (2)

From (1) and (2),

$2ar+2=14-ar$

$\Rightarrow 3ar=12$

$\Rightarrow ar=4$ (3)

From (1),

$a+4+4r=14$

$\Rightarrow a+4r=10$ (4)

From (3) and (4),

$a+\frac{16}{a}=10\Rightarrow a=2,8$

Hence, the smallest number is 2

61 **(c)**

Let 'A' be first term and 'r' be the common ratio

We have,

$a=Ar^{p+q-1}, b=Ar^{p-q-1}$

$\Rightarrow ab=A^2\times r^{2p-2}$

$\Rightarrow \sqrt{ab}=Ar^{p-1}=p^{\text{th}}$ term

62 **(c)**

Let a_1, a_2, and a_3 be first three consecutive terms of G.P. with common ratio r. Then,

$a_2=a_1r$ and $a_3=a_1r^2$

Now, $a_3>4a_2-3a_1$

$\Rightarrow a_1r^2>4a_1r-3a_1$

$\Rightarrow r^2>4r-3$

$\Rightarrow r^2-4r+3>0$

$\Rightarrow (r-1)(r-3)>0$

$\Rightarrow r<1$ or $r>3$

63 **(a)**

We have,

$\frac{\pi}{4}=\left(1-\frac{1}{3}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)+\left(\frac{1}{9}-\frac{1}{11}\right)+\cdots$

$=\frac{2}{1\times 3}+\frac{2}{5\times 7}+\frac{2}{9\times 1}+\cdots$

$\Rightarrow \frac{1}{1\times 3}+\frac{1}{5\times 7}+\frac{1}{9\times 11}+\cdots=\frac{\pi}{8}$

64 **(a)**

If p, q, r are in A.P., then in an A.P. or G.P. or an H.P. $a_1, a_2, a_3, \ldots$ etc, the terms a_p, a_q, a_r are in A.P., G.P. or H.P., respectively

65 **(b)**

$I(2n)=1^4+2^4+3^4+\cdots+(2n-1)^4+(2n)^4$

$=[(1^4+3^4+5^4+\cdots+(2n-1)^4]+2^4(1^4+2^4+3^4+4^4+\cdots n^4)$

$=\sum_{r=1}^{n}(2r-1)^4+16\times I(n)$

$\Rightarrow \sum_{r=1}^{n}(2r-1)^4=I(2n)-16I(n)$

66 **(d)**

$S_n-S_{n-2}=2$

$\Rightarrow T_n+T_{n-1}=2$

Also, $T_n+T_{n-1}=\left(\frac{1}{n^2}+1\right)T_{n-1}=2$

$\Rightarrow T_{n-1}=\frac{2}{1+\frac{1}{n^2}}=\frac{2n^2}{1+n^2}$

So, $T_m=\frac{2(m+1)^2}{1+(m+1)^2}$

67 **(b)**

$S_n=1+\frac{3}{2}+\frac{5}{3}+\cdots+\frac{99}{50}$

$=(2-1)+\left(2-\frac{1}{2}\right)+\left(2-\frac{1}{3}\right)+\cdots+\left(2-\frac{1}{50}\right)$

$=100-H_{50}$

68 **(b)**

Let the three numbers be $a/r, a, ar$. As the numbers form an increasing G.P., so $r > 1$. It is given that $a/r, 2a, ar$ are in A.P.

Hence,

$4a = \frac{a}{r} + ar$

$\Rightarrow r^2 - 4r + 1 = 0$

$\Rightarrow r = 2 \pm \sqrt{3}$

$= 2 + \sqrt{3}$ $[\because r > 1]$

69 **(b)**

x, y, z are in G.P. Hence,

$y = xz$

Now, $x + 3, y + 3, z + 3$ are in H.P. Hence,

$y + 3 = \frac{2(x+3)(z+3)}{(x+3)+(z+3)}$

$= \frac{2[xz + 3(x+z) + 9]}{[(x+z)+6]}$

$= \frac{2[y^2 + 3(x+z) + 9]}{[x+z+6]}$

Obviously, $y = 3$ satisfies it

70 **(a)**

α, β are the roots of $x^2 - x + p = 0$. Hence,

$\alpha + \beta = 1$

$\alpha\beta = p$

γ, δ are the roots of $x^2 - 4xd + q = 0$. Hence,

$\therefore \gamma + \delta = 4$ (3)

$\alpha, \beta, \gamma, \delta$ are in G.P. Let $\alpha = a, \beta = ar, \gamma = ar^2, \delta = ar^3$, Substituting these values in Eqs. (1), (2), (3) and (4), we get

$\alpha + ar = 1$ (5)

$a^2 r = p$ (6)

$ar^2 + ar^3 = 4$ (7)

$a^2 r^5 = q$ (8)

Dividing (7) by (5), we get

$\frac{ar^2(1+r)}{a(1+r)} = \frac{4}{1} \Rightarrow r^2 = 4 \Rightarrow r = 2, -2$

(5) $\Rightarrow a = \frac{1}{1+r} = \frac{1}{1+2}$ or $\frac{1}{1-2} = \frac{1}{3}$ or -1

As p is an integer (given), r is also an integer (2 or -2). Therefore, from (6), $a \neq 1/3$. Hence, $a = -1$ and $r = -2$

$\therefore p = (-1)^2 \times (-2) = -2$

$q = (-1)^2 \times (-2)^5 = -32$

71 **(b)**

Given that

$a + (p-1)d = A$

$a + (q-1)d = AR$

$a + (r-1)d = AR^2$

$a + (s-1)d = AR^3$

Where R is common ratio of G.P. Now,

$p - q = \frac{A - AR}{d}, q - r = R\left(\frac{A - AR}{d}\right)$

$r - s = R^2\left(\frac{A - AR}{d}\right)$

Clearly, $p - q, q - r, r - s$ are in G.P.

72 **(b)**

$\frac{\frac{n}{2}(2a + (n-1)d)}{\frac{n}{2}(2a' + (n-1)d')} = \frac{5n+3}{3n+4}$

$\Rightarrow \frac{(2a + (2n-2)d)}{(2a' + (2n-2)d')}$

$= \frac{5(2n-1)+3}{3(2n-1)+4}$ (replace n by $2n - 1$)

$\Rightarrow \frac{(a + (n-1)d)}{(a' + (n-1)d')} = \frac{10n-2}{6n+1}$

$\Rightarrow \frac{(a + (17-1)d)}{(a' + (17-1)d')} = \frac{168}{103}$

73 **(c)**

$2b = a + c$

a, p, b, q, c are in A.P. Hence,

$p = \frac{a+b}{2}$ and $q = \frac{b+c}{2}$

Again, a, p', b, q', and c are in G.P. Hence,

$p' = \sqrt{ab}$ and $q' = \sqrt{bc}$

$\Rightarrow p^2 - q^2 = \frac{(a-c)(a+c+2b)}{4}$

$= (a-c)b$

$= ab - bc$

$= p'^2 - q'^2$

74 **(d)**

$r \times r! = (r + 1 - 1) \times r!$

$= (r+1)! - r!$

$= V(r) - V(r-1)$

$\Rightarrow \sum_{r=1}^{30} r(r!) = V(31) - V(0)$

$\Rightarrow 1 + \sum_{r=1}^{30} r(r!) = 31!$

Which is divisible by 31 consecutive integers which is a prime number

75 **(c)**

Initially the ball falls from a height of 120 m. After striking the floor, it rebounds and goes to a height of $\frac{4}{5} \times (120)$ m. Now, if falls from a height of $\frac{4}{5} \times (120)$ m and after rebounding goes to a height of $\frac{4}{5}\left(\frac{4}{5}(120)\right)$ m. This process is continued till the ball comes to rest

Hence, the total distance travelled is

$$120+2\left[\frac{4}{5}(120)+\left(\frac{4}{5}\right)^2(120)+\cdots\infty\right]$$

$$=120+2\left[\frac{\frac{4}{5}(120)}{1-\frac{4}{5}}\right]=1080\text{ m}$$

76 **(b)**

Given $\frac{ar(r^{10}-1)}{r-1}=18$ (1)

Also $\frac{\frac{1}{ar}\left(1-\frac{1}{r^{10}}\right)}{1-\frac{1}{r}}=6$

$\Rightarrow \frac{1}{ar^{11}}\cdot\frac{(r^{10}-1)r}{r-1}=6$

$\Rightarrow \frac{1}{a^2 r^{11}}\cdot\frac{ar(r^{10}-1)}{r-1}=6$ (2)

From (1) and (2),

$\frac{1}{a^2 r^{11}}\cdot 18=6$

$\Rightarrow a^2r^{11}=3$

Now $P=a^{10}r^{55}=(a^2r^{11})^5=3^5=243$

77 **(d)**

Given, $a_1, a_2, a_3, \dots$ are terms of A.P.

$\therefore \frac{a_1+a_2+\cdots a_p}{a_1+a_2+\cdots a_q}=\frac{p^2}{q^2}$

$\Rightarrow \frac{\frac{p}{2}[2a_1+(p-1)d]}{\frac{q}{2}[2a_1+(q-1)d]}=\frac{p^2}{q^2}$

$\Rightarrow \frac{2a_1+(p-1)d}{2a_1+(q-1)d}=\frac{p}{q}$

$\Rightarrow [2a_1+(p-1)d]q=[2a_1+(q-1)d]p$

$\Rightarrow 2a_1(q-p)=d[(q-1)p-(p-1)q]$

$\Rightarrow 2a_1(q-p)=d[(q-p)$

$\Rightarrow 2a_1=d$

$\therefore \frac{a_6}{a_{21}}=\frac{a_1+5d}{a_1+20d}=\frac{a_1+10a_1}{a_1+40a_1}=\frac{11}{41}$

78 **(c)**

$S_n=\frac{a(r^n-1)}{r-1}$

$\Rightarrow U_n=\sum_{n=1}^{n}\frac{a(r^n-1)}{r-1}=\frac{a}{r-1}\sum_{n=1}^{n}(r^n-1)$

$\Rightarrow U_n=\frac{a}{r-1}\{r+r^2+\cdots+r^n-n\}$

$=\frac{a}{r-1}\left\{\frac{r(r^n-1)}{r-1}-n\right\}$

$\Rightarrow (r-1)U_n=\frac{ar(r^n-1)}{r-1}-an$

$\Rightarrow (r-1)U_n=rS_n-an$

$\Rightarrow rS_n+(1-r)U_n=an$

79 **(a)**

x, y, z are in G.P.

$\Leftrightarrow y^2=xz$

$\Leftrightarrow x$ is factor of y (not possible)

Taking $x=3, y=5, z=7$, we have x, y, z are in A.P. Thus x, y, z may be in A.P. but not in G.P.

80 **(b)**

We know that $1+3+5+\cdots+(2k-1)=k^2$.

Thus, the given equation can be written as

$\left(\frac{p+1}{2}\right)^2+\left(\frac{q+1}{2}\right)^2=\left(\frac{r+1}{2}\right)^2$

$\Rightarrow (p+1)^2+(q+1)^2=(r+1)^2$

As $p>6, p+1>7$, we may take $p+1=8, q+1=6, r+1=10$

Hence, $p+q+r=21$

81 **(c)**

$a_1, a_2, \dots, a_n$ are in H.P.

$\Rightarrow \frac{1}{a_1}, \frac{1}{a_2}, \dots, \frac{1}{a_n}$ are in A.P.

$\Rightarrow \frac{a_1+a_2+a_3+\cdots+a_n}{a_1}$,

$\frac{a_1+a_2+a_3+\cdots+a_n}{a_2}$,

$\dots, \frac{a_1+a_2+a_3+\cdots+a_n}{a_n}$ are in A.P.

$\Rightarrow 1+\frac{a_2+a_3+\cdots+a_n}{a_1}$,

$1+\frac{a_1+a_3+\cdots+a_n}{a_2}$,

$\dots, 1+\frac{a_1+a_2+\cdots+a_{n-1}}{a_n}$ are in A.P.

$\Rightarrow \frac{a_2+a_3+\cdots+a_n}{a_1}$,

$\frac{a_1+a_3+\cdots+a_n}{a_2}$,

$\dots, \frac{a_1+a_2+\cdots+a_{n-1}}{a_n}$ are in A. P

$\Rightarrow \frac{a_1}{a_2+a_3+\cdots+a_n}$,

$\frac{a_2}{a_1+a_3+\cdots+a_n}$,

$\dots, \frac{a_n}{a_1+a_2+\cdots a_{n-1}}$ are in H. P.

82 **(d)**

The given numbers are in A.P. Therefore,

$2\log_4(2^{1-x}+1)=\log_2(5\times 2^x+1)+1$

$\Rightarrow 2\log_{2^2}\left(\frac{2}{2^x}+1\right)=\log_2(5\times 2^x+1)+\log_2 2$

$\Rightarrow \frac{2}{2}\log_2\left(\frac{2}{2^x}+1\right)=\log_2(5\times 2^x+1)\,2$

$\Rightarrow \log_2\left(\frac{2}{2^x}+1\right)=\log_2(10\times 2^x+2)$

$\Rightarrow \frac{2}{2^x}+1=10\times 2^x+2$

$\Rightarrow \frac{2}{y}+1=10y+2$, where $2^x=y$

$\Rightarrow 10y^2+y-2=0$

$\Rightarrow (5y-2)(2y+1)=0$

$\Rightarrow y=2/5$ or $y=-1/2$

$\Rightarrow 2^x=2/5$ or $2^x=-1/2$

$\Rightarrow x = \log_2(2/5)$ [$\because 2^x$ cannot be negative]
$\Rightarrow x = \log_2 2 - \log_2 5$
$\Rightarrow x = 1 - \log_2 5$

83 **(a)**
Reciprocals of the terms of the series are 2/5, 13/20, 9/10, 23/20, ... or 8/20, 13/20, 18/20, 23/20, ... Its n^{th} term is
$$\frac{8+(n-1)5}{20} = \frac{5n+3}{20}$$
Therefore, the 15^{th} term is $\frac{20}{78} = \frac{10}{39}$

84 **(b)**
We have,
a_1, a_2, a_3 are in A.P. $\Rightarrow 2a_2 = a_1 + a_3$ (1)
a_2, a_3, a_4 are in G.P. $\Rightarrow a_3^2 = a_2 a_4$ (2)
a_3, a_4, a_5 are in H.P. $\Rightarrow a_4 = \frac{2a_3a_5}{a_3+a_5}$ (3)
Putting $a_2 = \frac{a_1+a_3}{2}$ and $a_4 = \frac{2a_3a_5}{a_3+a_5}$ in (2), we get
$$a_3^2 = \frac{a_1+a_3}{2} \times \frac{2a_3a_5}{a_3+a_5}$$
$\Rightarrow a_3^2 = a_1a_5$
Hence, a_1, a_3, and a_5 are in G.P. So, $\log_e a_1$, $\log_e a_3$ and $\log_e a_5$ are in A.P.

85 **(b)**
$a = 1 + 10 + 10^2 + \cdots + 10^{54}$
$$= \frac{10^{55}-1}{10-1} = \frac{10^{55}-1}{10^5-1} \times \frac{10^5-1}{10-1} = bc$$

86 **(c)**
For G.P., $t_n = 2^{n-1}$; for A.P. $T_m = 1 + (m-1)3 = 3m - 2$
They are common if $2^{n-1} = 3m - 2$. For G.P. 100^{th} term is 2^{99}. For A.P. 100^{th} term is $1 + (100-1)3 = 298$. Now we must choose value of m such that $3m-2$ is of type 2^{n-1}. Hence, $3m - 2 = 1, 2, 4, 8, 16, 32, 64, 128, 256$ for which $m = 1, 4/3, 2, 10/3, 6, 34/2, 22, 130/3, 86$. Hence, possible values of m are 1, 2, 6, 22, 86. Hence, there are five common terms

87 **(a)**
Here, $\alpha \in \left(0, \frac{\pi}{2}\right) \Rightarrow \tan\alpha \; is \; (+ve)$
[as, we know if
$a, b > 0 \Rightarrow \frac{a+b}{2} \geq \sqrt{ab} \; ie, AM \geq GM$]
$$\frac{\sqrt{x^2+x} + \frac{\tan^2\alpha}{\sqrt{x^2+x}}}{2}$$
$$\geq \sqrt{\sqrt{x^2+x}.\frac{\tan^2\alpha}{\sqrt{x^2+x}}} [using \; AM \; \geq GM]$$
$$\Rightarrow \sqrt{x^2+x} + \frac{\tan^2\alpha}{\sqrt{x^2+x}} \geq 2\tan\alpha$$

88 **(d)**
Since $ax^3 + bx^2 + cx + d$ is divisible by $ax^2 + c$, therefore, when $ax^3 + bx^2 + cx + d$ is divided by $ax^2 + c$ the remainder should be zero. Now when $ax^3 + bx^2 + cx + d$ is divided by $ax^2 + c$, then the remainder is $(bc/a) - d$
$$\therefore \frac{bc}{a} - d = 0$$
$\Rightarrow bc = ad$
$$\Rightarrow \frac{b}{a} = \frac{d}{c}$$
Hence, from this, a, b, c, d are not necessarily in G.P.

89 **(d)**
$p^{\text{th}}, q^{\text{th}}, r^{\text{th}}$ terms of A.P. are
$a + (p-1)d = x$ (1)
$a + (q-1)d = xR$ (2)
$a + (r-1)d = xR^2$ (3)
Where R is common ratio of G.P.
Subtracting (2) from (3) and (1) from (2) and then dividing the former by the later, we have
$$\frac{q-r}{p-q} = \frac{xR^2 - xR}{xR - x} = R$$

90 **(d)**
100^{th} term of 1, 11, 21, 31, ... is $1 + (100-1)10 = 991$
100^{th} term of 31, 36, 41, 46, ... is $31 + (100-1)5 = 526$
Let the largest common term be 526. Then,
$526 = 31 + (n-1)10$
$\Rightarrow n = 50.5$
But n is an integer; hence $n = 50$. Hence, the largest common term in $31 + (50-1)10 = 521$

91 **(b)**
Let the series have $2n$ terms and the series is
$a, a+d, a+2d, \ldots, a+(2n-1)d$
According to the given conditions, we have
$[a + (a+2d) + (a+4d) + \cdots + (a+(2n-2)d)] = 24$
$$\Rightarrow \frac{n}{2}[2a + (n-1)2d] = 24$$
$\Rightarrow n[a + (n-1)d] = 24$ (1)
Also, $[(a+d) + (a+3d) + \cdots + (a+$
$2n-1d=30$
$$\Rightarrow \frac{n}{2}[2(a+d) + (n-1)2d] = 30$$
$\Rightarrow n[(a+d) + (n-1)d] = 30$ (2)
Also, the last term exceeds the first by 21/2.
Therefore,
$a + (2n-1)d - a = 21/2$
$\Rightarrow (2n-1)d = 21/2$ (3)

Now, subtracting (1) from (2),
$nd = 6 \quad (4)$
Dividing (3) by (4), we get
$\frac{2n-1}{n} = \frac{21}{12}$
$\Rightarrow n = 4$

92 **(a)**
$\frac{t_4}{t_6} = \frac{1}{4} \Rightarrow \frac{ar^3}{ar^5} = \frac{1}{4} \Rightarrow \; r^2 = 4 \Rightarrow r = \pm 2$
Also, $t_2 + t_5 = 216$
$\Rightarrow ar + \mathrm{a}r^4 = 216$
$\Rightarrow a + 8\mathrm{a} = 108$
$\Rightarrow a = 12$ (where $r = 2$)

93 **(b)**
$b_2 = \frac{1}{1-b_1}$
$b_3 = \frac{1}{1-b_2} = \frac{1}{1-\frac{1}{1-b_1}} = \frac{1-b_1}{-b_1} = \frac{b_1-1}{b_1}$
$b_1 = b_3 \Rightarrow \; b_1^2 - b_1 + 1 = 0$
$\Rightarrow b_1 = -\omega$ or $\omega^2 \Rightarrow b_2 = \frac{1}{1+\omega} = -\omega$ or ω^2
$\sum_{r=1}^{2001} b_r^{2001} = \sum_{r=1}^{2001} (-\omega)^{2001}$
$= -\sum_{r=1}^{2001} 1$
$= -2001$

94 **(c)**
$2.\overline{357} = 2 + 0.357 + 0.000357 + \cdots \infty$
$= 2 + \frac{357}{10^3} + \frac{357}{10^6} + \ldots \infty$
$= 2 + \frac{\frac{357}{10^3}}{1-\frac{1}{10^3}}$
$= 2 + \frac{357}{999} = \frac{2355}{999}$
Alternative solution:
Let,
$x = 2.\overline{357}$
$\Rightarrow 1000x = 2357.\overline{357}$
On subtracting, we get
$999x = 2355 \;\Rightarrow\; x = \frac{2355}{999}$

95 **(b)**
Given, $b^2 = ac$ and $x = \frac{a+b}{2}, y = \frac{b+c}{2}$. Therefore,
$\frac{a}{x} + \frac{c}{y} = \frac{2a}{a+b} + \frac{2c}{b+c}$
$= \frac{2a(b+c) + 2c(a+b)}{(a+b)(b+c)}$
$= 2\frac{2ac+ab+bc}{ab+ac+b^2+bc}$
$= 2\frac{2ac+ab+bc}{2ac+ab+bc}$
$=2$

96 **(c)**
Given,
$2 + 5 + 8 + \ldots 2n$ terms $= 57 + 59 + 61 + \ldots n$ terms
$\Rightarrow \frac{2n}{2}[4(2n-1)3] = \frac{n}{2}[114 + (n-1)2]$
$\Rightarrow 6n + 1 = n + 56$
$\Rightarrow 5n = 55$
$\Rightarrow n = 11$

97 **(c)**
Given that x, y, and z are $p^{\text{th}}, q^{\text{th}}$ and r^{th} terms of an A.P.
$\therefore \; x = A + (p-1)D$
$y = A + (q-1)D$
$z = A + (r-1)D$
$\Rightarrow x - y = (p-q)D$
$y - z = (q-r)D$
$z - x = (r-p)D$
Where A is the first term and D is the common difference. Also x, y, z are the $p^{\text{th}}, q^{\text{th}}$ and r^{th} terms of a G.P.
$\therefore x = aR^{p-1}, y = aR^{q-1}, z = aR^{r-1}$
$\therefore x^{x-z}\, y^{z-x}\, z^{x-y}$
$= (aR^{p-1})^{y-z}\, (aR^{q-1})^{z-x} (aR^{r-1})^{x-y}$
$= a^{y-z+z-x+x-y} R^{(p-1)(y-z)+(q-1)(z-x)+(r-1)(x-y)}$
$= A^0 R^{(p-1)(q-r)D+(q-1)(r-p)D+(r-1)(p-q)D}$
$= A^0 R^0 = 1$

98 **(c)**
Given, $S_p = 0$. Therefore,
$\frac{p}{2}[2a + (p-1)d] = 0 \;\Rightarrow d = \frac{-2a}{p-1} \quad (1)$
Sum of next q terms is sum of an A.P. whose first term will be
$T_{p+1} = a + pd$
$\therefore \; S = \frac{q}{2}[2(a+pd) + (q-1)d]$
$= \frac{q}{2}[2a + (p-1)d + (p+q)d]$
$= \frac{q}{2}\left[0 - (p+q)\frac{2a}{p-1}\right]$
$= -a\frac{(p+q)q}{p-1}$ [Using (1)]

99 **(b)**
Since a, b, c are in A.P., therefore, $b - a = d$ and $c - b = d$, where d is the common difference of the A.P.
$\therefore \; a = b - d$ and $c = b + d$

Now, $abc = 4$
$\Rightarrow \ (b-d)b(b+d) = 4$
$\Rightarrow b(b^2-d^2) = 4$
But, $b(b^2-d^2) < b \times b^2$
$\Rightarrow b(b^2-d^2) < b^3$
$\Rightarrow 4 < b^3$
$\Rightarrow \ b^3 > 4$
$\Rightarrow b > 2^{2/3}$
Hence, the minimum value of b is $2^{2/3}$

100 **(c)**
$S = [a-(a+d)]+[(a+2d)-(a+3d)]+\cdots+$
$[(a+(2n-2)d)] - \text{a} + (2n-1)d] + (\text{a}+2nd)$
$= [(-d)+(-d)+\cdots+n \text{ times}] + a + 2nd$
$= -nd + a + 2nd$
$= a + nd$

101 **(c)**
The series is
$1+2+2\times 3+2^2\times 3+2^2\times 3^2+2^3\times 3^2+\cdots$ to 20 terms
$= (1+2\times 3+2^2\times 3^2+\cdots \text{to 10 terms})$
$+(2+2^2\times 3+2^3\times 3^2+\cdots \text{to 10 terms})$
$= \dfrac{1(2^{10}3^{10}-1)}{6-1} + \dfrac{2(2^{10}3^{10}-1)}{6-1}$
$= \left(\dfrac{3}{5}\right)(6^{10}-1)$

102 **(d)**
Let a be the first term and d be the common difference of the given A.P. Then,
$S_m = S_n \ \Rightarrow \ \dfrac{m}{2}[2a+(m-1)d]$
$= \dfrac{n}{2}[2a+(n-1)d]$
$\Rightarrow 2a(m-n) + \{m(m-1)-n(n-1)\}d = 0$
$\Rightarrow 2a(m-n) + \{(m^2-n^2)-(m-n)\}d = 0$
$\Rightarrow (m-n)[2a+(m+n-1)d] = 0$
$\Rightarrow 2a+(m+n-1)d = 0 \quad [\because m-n \neq 0] \qquad (1)$
Now, $S_{m+n} = \frac{m+n}{2}[2a+(m+n-1)d] = \frac{m+n}{2} \times 0 = 0$ [Using (1)]

103 **(b)**
$\dfrac{p}{r}+\dfrac{r}{p} = \dfrac{p^2+r^2}{pr} = \dfrac{(p+r)^2-2pr}{pr}$
$= \dfrac{\frac{4p^2r^2}{q^2}-2pr}{pr} \quad \left[\begin{array}{c}\because p,q,r \text{ are in H. P.} \\ \therefore q = \dfrac{2pr}{p+r}\end{array}\right]$
$= \dfrac{4pr}{q^2} - 2 = \dfrac{4b^2}{ac}$
$-2 \quad [\because ap, bq, cr$ are in A. P.
$\Rightarrow b^2q^2 = acpr]$
$= \dfrac{(a+c)^2}{ac} - 2 \quad [a,b,c,$ are in A. P. $\Rightarrow 2b = a+c]$
$= \dfrac{a}{c} + \dfrac{c}{a}$

104 **(b)**
Coefficient of x^{18} in $(1+x+2x^2+3x^3+\cdots+$
18x182
=Coefficient of x^{18} in $(1+x+2x^2+3x^3+\cdots+$
18x18×1+x+2x2+3x3+…+18x18
$= 1\times 18 + 1\times 17 + 2\times 16 + \cdots + 17\times 1 + 18 \times 1$
$= 36 + \sum_{r=1}^{17} r(18-r)$
$= 36 + 18\sum_{r=1}^{17} r - \sum_{r=1}^{17} r^2$
$= 1005$

105 **(a)**
For first equation $D = 4b^2 - 4ac = 0$ (as given a, b, c are in G.P.)
$\Rightarrow$ equation has equal roots which are equal to $-\frac{b}{a}$ each
Thus it should also be the root of the second equation
Thus, $d\left(\frac{-b}{a}\right)^2 + 2e\left(\frac{-b}{c}\right) + f = 0$
$\Rightarrow d\dfrac{b^2}{a^2} - 2\dfrac{be}{a} + f = 0$
$\Rightarrow d\frac{ac}{a^2} - 2\frac{be}{a} + f = 0$ (as $b^2 = ac$)
$\Rightarrow \dfrac{d}{a} + \dfrac{f}{c} = 2\dfrac{eb}{ac} = 2\dfrac{e}{b}$

106 **(c)**
Let $a = 1, b = 2, c = 4$ Then,
$a+b = 3, 2b = 4, b+c = 6$
$\Rightarrow \dfrac{1}{4} - \dfrac{1}{3} = -\dfrac{1}{12}$ and $\dfrac{1}{6} - \dfrac{1}{4} = -\dfrac{1}{12}$
Hence, $a+b, 2b, b+c$ are in H.P.

107 **(c)**
$\frac{a_r - a_{r+1}}{a_r\, a_{r+1}} = k$ (constant)
$\Rightarrow \dfrac{1}{a_{r+1}} - \dfrac{1}{a_r} = k$
$\Rightarrow \frac{1}{a_1}, \frac{1}{a_2}, \ldots, \frac{1}{a_n}$ are in A.P.
$\Rightarrow a_1, a_2, a_3, \ldots,$ are in H.P.

108 **(c)**
$T(r) = \dfrac{r}{1\times 3\times 5\times\cdots\times(2r+1)}$
$= \dfrac{2r+1-1}{2(1\times 3\times 5\cdots(2r+1))}$
$= \dfrac{1}{2}\left(\dfrac{1}{1\times 3\times 5\cdots(2r-1)} - \dfrac{1}{1\times 3\times 5\cdots(2r+1)}\right)$

$= -\frac{1}{2}[V(r) - V(r-1)]$

$\Rightarrow \sum_{r=1}^{n} T(r) = -\frac{1}{2}(V(n) - V(0))$

$= \frac{1}{2}\left(1 - \frac{1}{1 \times 3 \times 5 \times \cdots \times (2n+1)}\right)$

$\Rightarrow \lim_{n\to\infty} \sum_{r=1}^{n} \frac{r}{1 \times 3 \times 5 \times 7 \times 9 \times \cdots \times (2r+1)}$

$= \lim_{n\to\infty} \frac{1}{2}\left(1 - \frac{1}{1 \times 3 \times 5 \times \cdots \times (2n+1)}\right) = \frac{1}{2}$

109 **(d)**

Sum is 4 and second term is ¾. It is given that first term is a and common ratio is r. Hence,

$\frac{a}{1-r} = 4$ and $ar = 3/4 \Rightarrow r = \frac{3}{4a}$

Therefore,

$\frac{a}{1-\frac{3}{4a}} = 4 \Rightarrow \frac{4a^2}{4a-3} = 4$

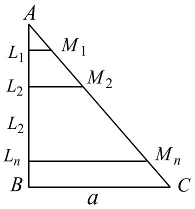
$\Rightarrow a^2 - 4a + 3 = 0$

$\Rightarrow (a-1)(a-3) = 0$

$\Rightarrow a = 1$ or 3

When $a = 1, r = 3/4$ and when $a = 3, r = 1/4$

110 **(c)**



$\frac{AL_1}{AB} = \frac{L_1M_1}{BC}$

$\Rightarrow \frac{1}{n+1} = \frac{L_1M_1}{a}$

$\Rightarrow L_1M_1 = \frac{a}{n+1}$

$\frac{AL_2}{AB} = \frac{L_2M_2}{BC}$

$\Rightarrow \frac{2}{n+1} = \frac{L_2M_2}{a} \Rightarrow L_2M_2 = \frac{2a}{n+1}$, etc

Hence, the required sum is

$\frac{a}{n+1} + \frac{2a}{n+1} + \frac{3a}{n+1} + \cdots + \frac{na}{n+1}$

$= \frac{a}{n+1}\frac{n(n+1)}{2} = \frac{an}{2}$

111 **(d)**

$2b = a + c$

$\Rightarrow 8b^3 = (a+c)^3 = a^3 + c^3 + 3ac(a+c)$

$\Rightarrow 8b^3 = a^3 + c^3 + 3ac(2b)$

$\Rightarrow a^3 + c^3 - 8b^3 = -6abc$

112 **(c)**

Since, $S_\infty = \frac{x}{1-r} = 5 \quad \Rightarrow r = \frac{5-x}{x}$ [thus $|r| < 1$]

$\Rightarrow -1 < \frac{5-x}{5} < 1 \quad \Rightarrow 0 < x < 10$

113 **(d)**

$2 + 3 + 6 + 11 + 18 + \cdots$

$= (0^2 + 2) + (1^2 + 2) + (2^2 + 2) + (3^2 + 2) + \cdots$

Hence, $t_{50} = 49^2 + 2$

114 **(c)**

Here, number of factors is 50. Therefore, the coefficient of x^{49} is

$-1 - 3 - 5 - \cdots - 99 = -\frac{50}{2}(1 + 99) = -2500$

115 **(a)**

We have,

$2b = a + c$

$(c-b)^2 = (b-a)a$

$\Rightarrow (b-a)^2 = (b-a)a$ $[2b = a + c \Rightarrow b - a = c - b]$

$\Rightarrow b = 2a$

$\Rightarrow c = 3a$ [Using $2b = a + c$]

$\Rightarrow a : b : c = 1 : 2 : 3$

116 **(b)**

Let a be the first term and r the common ratio of the G.P. Then, the sum is given by

$\frac{a}{1-r} = 57 \quad (1)$

Sum of the cubes is 9747. Hence,

$a^3 + a^3r^3 + a^3r^6 + \cdots = 9747$

$\Rightarrow \frac{a^3}{1-r^3} = 9747 \quad (2)$

Dividing the cube of (1) by (2), we get

$\frac{a^3}{(1-r)^3}\frac{(1-r^3)}{a^3} = \frac{(57)^3}{9747}$

$\Rightarrow \frac{1-r^3}{(1-r)^3} = 19$

$\Rightarrow \frac{1+r+r^2}{(1-r)^2} = 19$

$\Rightarrow 18r^2 - 39r + 18 = 0$

$\Rightarrow (3r-2)(6r-9) = 0$

$\Rightarrow r = 2/3$ or $r = 3/2$

$= 2/3$ [$\because r \neq 3/2$, because $0 < |r| < 1$ for an infinite G.P.]

117 **(a)**

n^{th} term of the series is $20 + (n-1)(-2/3)$

For the sum to be maximum,

n^{th} term ≥ 0

$\Rightarrow 20 + (n-1)\left(-\frac{2}{3}\right) \geq 0$

$\Rightarrow n \leq 31$

Thus, the sum of 31 terms is maximum and is

$$\frac{31}{2}\left[40+30\times\left(-\frac{2}{3}\right)\right]=310$$

118 **(a)**

Let $a=1$, then $S_1=2008$

If $a\neq 1$ then $S=\frac{a^{2008}-1}{a-1}$

But $a^{2008}=2a-1$, therefore, $S_2=\frac{2(a-1)}{a-1}=2$

$\therefore\ S=S_1+S_2=2010$

119 **(b)**

We have,

$(OM_{n-1})^2=(OP_n)^2+(P_nM_{n-1})^2$

$=2(OP_n)^2$

$=2\alpha_n^2$ (say)

Also,

$(OP_{n-1})^2=(OM_{n-1})^2+(P_{n-1}M_{n-1})^2$

$\Rightarrow \alpha_{n-1}^2=2\alpha_n^2+\frac{1}{2}\alpha_{n-1}^2$

$\Rightarrow \alpha_n=\frac{1}{2}\alpha_{n-1}$

$\Rightarrow OP_n=\alpha_n=\frac{1}{2}\alpha_{n-1}=\frac{1}{2^2}\alpha_{n-1}=\cdots=\frac{1}{2^n}$

$=\left(\frac{1}{2}\right)^n$

120 **(a)**

Let $1+1/50=x$. Let S be the sum of 50 terms of the given series. Then,

$S=1+2x+3x^2+4x^3+\cdots+49x^{48}+50x^{49}$ (1)

$xS=x+2x^2+3x^3+\cdots+49x^{49}+50x^{50}$ (2)

$(1-x)S=1+x+x^2+x^3+\cdots+x^{49}-50x^{50}$

[Subtracting (2) from (1)]

$\Rightarrow S(1-x)=\frac{1-x^{50}}{1-x}-50x^{50}$

$\Rightarrow S(-1/50)=-50(1-x^{50})-50x^n$

$\Rightarrow \frac{1}{50}S=50$

$\Rightarrow S=2500$

121 **(d)**

Let $t_n=\frac{1}{4(n+2)(n+3)}$. Then,

$\frac{1}{t_1}+\frac{1}{t_2}+\frac{1}{t_3}+\cdots+\frac{1}{t_{2003}}$

$=4\left[\frac{1}{3\times 4}+\frac{1}{4\times 5}+\frac{1}{5\times 6}+\cdots+\frac{1}{2005\times 2006}\right]$

$=4\left[\frac{1}{3}-\frac{1}{2006}\right]$

$=4\times\frac{2003}{3(2006)}=\frac{4006}{3009}$

122 **(d)**

Let,

$S=1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+\cdots\infty$

Then,

$\frac{1}{5}S=\frac{1}{5}+\frac{4}{5^2}+\frac{7}{5^3}+\cdots\infty$

$\Rightarrow S\left(1-\frac{1}{5}\right)=1+3\left[\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+\cdots\infty\right]$

$\Rightarrow \frac{4}{5}S=1+3\times\frac{1/5}{1-(1/5)}=1+\frac{3}{4}=\frac{7}{4}$

$\Rightarrow S=\frac{35}{16}$

123 **(b)**

$T_r=r(-a)^r+(r+1)a(-a)^r$

$=r(-a)^r-(r+1)(-a)^{r+1}$

$=v_r-v_{r+1}$ (say)

So,

$\sum_{r=0}^{n}T_r=\sum_{r=0}^{n}(v_r-v_{r+1})$

$=v_0-v_{n+1}$

$=-(n+1)(-a)^{n+1}$

124 **(c)**

We have,

$1+(1+a)b+(1+a+a^2)b^2$
$+(1+a+a^2+a^3)b^3+\cdots\infty$

$=\sum_{n=1}^{\infty}(1+a+a^2+\cdots+a^{n-1})\,b^{n-1}$

$=\sum_{n=1}^{\infty}\left(\frac{1-a^n}{1-a}\right)b^{n-1}$

$=\sum_{n=1}^{\infty}\frac{b^{n-1}}{1-a}-\sum_{n=1}^{\infty}\frac{a^nb^{n-1}}{1-a}$

$=\frac{1}{1-a}\sum_{n=1}^{\infty}b^{n-1}-\frac{a}{1-a}\sum_{n=1}^{\infty}(ab)^{n-1}$

$=\frac{1}{1-a}[1+b+b^2+\cdots\infty]-\frac{a}{1-a}[1+ab$
$+(ab)^2+\cdots\infty]$

$=\frac{1}{1-a}\times\frac{1}{1-b}-\frac{a}{(1-a)(1-ab)}$

$=\frac{1}{(1-ab)(1-b)}$

125 **(b)**

Since a,q and c are in A.P., so

$2q=a+c$

$\Rightarrow \frac{1}{p}+\frac{1}{r}=\frac{2}{b}$

$\Rightarrow \frac{1}{p},\frac{1}{b},\frac{1}{r}$ are in A.P.

126 **(b)**

For the equation $x^2-px+1=0$,

The product of roots, $\alpha\beta^2=1$

And for the equation $x^2 - qx + 8 = 0$,

The product of roots $\alpha^2\beta = 8$

Hence, $(\alpha\beta^2)(\alpha^2\beta) = 8$

$\Rightarrow \alpha^3\beta^3 = 8 \Rightarrow \alpha\beta = 2$

$\therefore$ From $\alpha\beta^2 = 1$, we have $\beta = \frac{1}{2}$ and from $\alpha^2 \cdot \beta = 8$, we have $\alpha = 4$

Hence, from sum of roots $= -\frac{b}{a}$, we have

$p = \alpha + \beta^2 = 4 + \frac{1}{4} = \frac{17}{4}$ and $q = \alpha^2 + \beta = 16 + \frac{1}{2} = \frac{33}{2}$

$\frac{r}{8}$ is arithmetic mean of p and q

$$\therefore \frac{r}{8} = \frac{p+q}{2}$$

$$\Rightarrow r = 4(p+q) = 4\left(\frac{17}{4} + \frac{33}{2}\right) = 17 + 66 = 83$$

127 **(c)**

Multiplying the given expression by 2 and rewriting it, we have

$\Rightarrow (2x - 3y)^2 + (3y - 4z)^2 + (4z - 2x)^2 = 0$

$\Rightarrow 2x = 3y = 4z$

$\Rightarrow \frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

$\Rightarrow x, y, z$ are in H.P.

128 **(d)**

$a = h_1 = 2, a_{10} = h_{10} = 3$

$3 = a_{10} = 2 + 9d \Rightarrow d = 1/9$

$\therefore a_4 = 2 + 3d = 7/3$

Also,

$$3 = h_{10} \Rightarrow \frac{1}{3} = \frac{1}{h_{10}} = \frac{1}{2} + 9D$$

$$\Rightarrow D = -\frac{1}{54}$$

$$\Rightarrow \frac{1}{h_7} = \frac{1}{2} + 6D = \frac{1}{2} - \frac{1}{9} = \frac{7}{18}$$

$$\therefore a_4 h_7 = \frac{7}{3} \times \frac{18}{7} = 6$$

129 **(b)**

x is A.M. of a and b, y is G.M. of a, and b, z is H.M. of a and b, z is H.M. of a and b

$y^2 = xz$

Also given,

$x = 9z$

$\Rightarrow x = 9y^2/x \Rightarrow 9y^2 = x^2 \Rightarrow x = 3|y|$

130 **(a)**

Let T_r be the r^{th} term of the given series. Then,

$$T_r = \frac{2r+1}{1^2 + 2^2 + \cdots + r^2}$$

$$= \frac{6(2r+1)}{(r)(r+1)(2r+1)}$$

$$= 6\left(\frac{1}{r} - \frac{1}{r+1}\right)$$

So, sum is given by

$$\sum_{r=1}^{50} T_r = 6\sum_{r=1}^{50}\left(\frac{1}{r} - \frac{1}{r+1}\right)$$

$$= 6\left[\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots + \left(\frac{1}{50} - \frac{1}{51}\right)\right]$$

$$= 6\left[1 - \frac{1}{51}\right]$$

$$= \frac{100}{17}$$

131 **(b)**

Harmonic mean H of roots α and β is

$$H = \frac{2\alpha\beta}{\alpha + \beta} = \frac{2\frac{5+2\sqrt{5}}{5+\sqrt{2}}}{\frac{4+\sqrt{5}}{5+\sqrt{2}}} = 4$$

132 **(a,b,c)**

Let the three digit number be xyz. According to given condition, we have

$y^2 = xz$ (1)

$2(y+2) = x + z$ (2)

$100x + 10y + z - 792 = 100z + 10y + x$

$\Rightarrow x - z = 8$ (3)

Squaring (2) and (3), and subtracting, we have

$4xz = 4(y+2)^2 - 64$ (4)

$\Rightarrow y^2 = (y+2)^2 - 16$ [Using (1)]

$\Rightarrow y = 3$

$\Rightarrow x + z = 10$ [Using (2)]

$\Rightarrow x = 9, z = 1$

Hence, the number is $931 = 7^2 \times 19$

133 **(a,b,c)**

Last term in n^{th} row is

$1 + 2 + 3 + \cdots + n = \frac{1}{2}n(n+1)$ (1)

As terms in the n^{th} row forms an A.P. with common difference 1, so

First term = Last term – $(n-1)(1)$

$= \frac{1}{2}n(n+1) - n + 1$

$= \frac{1}{2}(n^2 - n + 2)$ (2)

Sum of terms $= \frac{1}{2}n \left[\frac{1}{2}(n^2 - n + 2) + \frac{1}{2}(n^2 + n)\right]$

$= \frac{1}{2}n(n^2 + 1)$ (3)

Now, put $n = 20$ in (1), (2), (3) to get required answers

134 **(b,d)**

Let x be the first and y be the $(2n-1)^{\text{th}}$ term of A.P., G.P. and H.P. whose n^{th} terms are a, b, c, respectively. Now according to the property of A.P., G.P. and H.P., x, a, y ar ein A.P.; x, b, y are in G.P. and x, c, y are in H.P. Hence,

$a = \frac{x+y}{2} = \text{A.M.}$
$b = \sqrt{xy} = \text{G.M.}$
$c = \frac{2xy}{x+y} = \text{H.M.}$
Now, A.M., G.M. and H.M. are in G.P. Hence,
$b^2 = ac$
Also, $\text{A.M.} \geq \text{G.M.} \geq \text{H.M.}$ Hence,
$a \geq b \geq c$

135 **(a,b,c)**
Since A_1, A_2 are two arithmetic means between a and b, therefore, $a, A_1, A_2\ b$ are in A.P. with common difference d given by

$$d = \frac{b-a}{2+1} = \frac{b-a}{3} \left[\text{Using } d = \frac{b-a}{n+1}\right]$$

Now,

$$A_1 = a + d = a + \frac{b-a}{3} = \frac{2a+b}{3}$$

and

$$A_2 = a + 2d = a + 2\left(\frac{b-a}{3}\right) = \frac{a+2b}{3}$$

It is given that $G_1 G_2$ are two geometric means between a and b. Therefore, a, G_1, G_2, b are in G.P. with common ratio r given by

$$r = \left(\frac{b}{a}\right)^{\frac{1}{2+1}} = \left(\frac{b}{a}\right)^{1/3} \quad \left[\because\ r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}\right]$$

Now,

$$G_1 = ar = a\left(\frac{b}{a}\right)^{1/3} = a^{2/3}b^{1/3}$$

and

$$G_2 = ar^2 = a\left(\frac{b}{a}\right)^{2/3} = a^{1/3}b^{2/3}$$

It is also given that H_1, H_2 are two harmonic means between a and b, therefore, a, H_1, H_2, b are in H.P. Hence, $1/a, 1/H_1, 1/H_2, 1/b$, are in A.P. with common difference D given by

$$D = \frac{a-b}{(2+1)ab} = \frac{a-b}{3ab} \quad \left[\because D = \frac{a-b}{(n+1)ab}\right]$$

Now,

$$\frac{1}{H_1} = \frac{1}{a} + D = \frac{1}{a} + \frac{a-b}{3ab} = \frac{a+2b}{3ab}$$

$$\Rightarrow\ H_1 = \frac{3ab}{a+2b}$$

$$\frac{1}{H_2} = \frac{1}{a} + 2D$$
$$= \frac{1}{a} + \frac{2(a-b)}{3ab}$$
$$= \frac{2a+b}{3ab}$$
$$\Rightarrow\ H_2 = \frac{3ab}{2a+b}$$

We have,

$$A_1 H_2 = \frac{2a+b}{3} \times \frac{3ab}{2a+b} = ab,$$
$$A_2 H_1 = \frac{a+2b}{3} \times \frac{3ab}{a+2b} = ab,$$
$$G_1 G_2 = (a^{2/3}\, b^{1/3})(a^{21/3}\, b^{2/3}) = ab$$
$$\therefore\ A_1 H_2 = A_2 H_1 = G_1 G_2 = ab$$

136 **(a,b,d)**

$$E < 1 + \frac{1}{(1)(2)} + \frac{1}{(2)(3)} + \cdots$$
$$= 1 + \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \cdots = 2$$
$$E > 1 + \frac{1}{(2)(3)} + \frac{1}{(3)(4)} + \cdots$$
$$= 1 + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots = \frac{3}{2}$$

137 **(a,c)**
a, b, c are in G.P. Hence,
$b^2 = ac \quad (1)$
x is A.M. of a and b. Hence,
$2x = a + b \quad (2)$
y is A.M. of b and c. Hence,
$2y = b + c \quad (3)$

$$\therefore \frac{a}{x} + \frac{c}{y} = a \times \frac{2}{a+b} + c \times \frac{2}{b+c} \quad \text{[Using (2) and (3)]}$$
$$= 2\left[\frac{ab+ac+ac+bc}{ab+ac+b^2+bc}\right]$$
$$= 2 \quad \text{[Using (i)]}$$

Again,

$$\frac{1}{x} + \frac{1}{y} = \frac{2}{a+b} + \frac{2}{b+c}$$
$$= \frac{2(a+c+2b)}{ab+ac+b^2+bc}$$
$$= \frac{2(a+c+2b)}{ab+2b^2+bc} \quad (\because b^2 = ac)$$
$$= \frac{2(a+c+2b)}{b(a+c+2b)}$$
$$= \frac{2}{b}$$

138 **(a,b,c)**
Given that $a = 4, T_3 - T_5 = 32/81$. Hence,
$a(r^2 - r^4) = 32/81$
or $r^4 - r^2 + 8/81 = 0$
or $81r^4 - 81r^2 + 8 = 0$
or $(9r^2 - 8)(9r^2 - 1) = 0$
$\therefore r^2 = 8/9, 1/9$
Therefore, the value of r is to be +ve since all the terms are +ve
For $r = 1/3$

$S_\infty = \dfrac{a}{1-r} = \dfrac{4}{1-\frac{1}{3}} = \dfrac{4\times 3}{2} = 6$

Similarly, we can find S_∞ when $r = 2\sqrt{2}/3$

139 **(b,d)**

Since, nth term of the first $(2n-1)$ terms is the middle term. Therefore, a is the AM(A); b is the GM(G) and c is the HM(H) of the series, whose first term and the last term are equal. We know that

$A \ge G \ge H$ and $AH = G^2$

Therefore, $a \ge b \ge c$ and $ac - b^2 = 0$

140 **(c)**

$T_m = a + (m-1)d = 1/n$

$T_n = a + (n-1)d = 1/m$

$\Rightarrow (m-n)d = 1/n - 1/m = (m-n)/mn$

$\Rightarrow d = 1/mn$

$\Rightarrow a = \dfrac{1}{mn}$

$\therefore\ T_{mn} = a + (mn-1)d$

141 **(a,b,c)**

If a, b, c are in HP, then $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

Let $E = \left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)\left(\frac{1}{c}+\frac{1}{a}-\frac{1}{b}\right)$

$= \left(\dfrac{3}{b} - \dfrac{2}{a}\right)\dfrac{1}{b} = \dfrac{3}{b^2} - \dfrac{2}{ab}$

Again, $E = \left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)\left(\frac{1}{c}+\frac{1}{a}-\frac{1}{b}\right)$

$= \left(\dfrac{2}{c} - \dfrac{1}{b}\right)\dfrac{1}{b} = \dfrac{2}{bc} - \dfrac{1}{b^2}$

Also, $E = \left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)\left(\frac{1}{c}+\frac{1}{a}-\frac{1}{b}\right)$

$= \dfrac{1}{4}\left(\dfrac{1}{a}+\dfrac{1}{c}\right)^2 + \dfrac{1}{2}\left(\dfrac{1}{c^2} - \dfrac{1}{a^2}\right)$

$= \dfrac{1}{4}\left(\dfrac{3}{c^2} + \dfrac{2}{ca} - \dfrac{1}{a^2}\right)$

142 **(a,b,c,d)**

Clearly, n^{th} term of the given series is negative or positive according as n is even or odd, respectively

Case I: When n is even: In this case, the given series is

$S_n = 1^2 - 2^2 + 3^2 - 4^2 + \cdots + (n-1)^2 - n^2$

$= (1^2 - 2^2) + (3^2 - 4^2) + \cdots + ((n-1)^2 - n^2)$

$= (1-2)(1+2) + (3-4)(3+4) + \cdots + \big((n-1) - (n)\big)(n-1+n)$

$= -(1+2+3+4+\cdots+(n-1)+n)$

$= -\frac{n(n+1)}{2}$ (1)

Case II: When n is odd: In this case, the given series is

$S_n = (1^2 - 2^2) + (3^2 - 4^2) + \cdots + \{(n-2)^2 - (n-1)^2\} + n^2$

$= (1-2)(1+2) + (3-4)(3+4) + \cdots + \big((n-2)-(n-1)\big) \times \big((n-2)+(n-1)\big) + n^2$

$= -\big(1+2+3+4+\cdots+(n-2)+(n-1)\big) + n^2$

$= -\frac{(n-1)(n-1+1)}{2} + n^2 = \frac{n(n+1)}{2}$ (2)

$\Rightarrow S_{40} = -820$ [Using (1)]

$S_{51} = 1326$ [Using (2)]

Also,

$S_{2n} > S_{2n+2}$ [From (1)]

$S_{2n+1} > S_{2n-1}$ [From (2)]

143 **(b)**

If $x, y,$ and z are in G.P. $(x, y, z > 1)$, then $\log x, \log y, \log z$ are in A.P. Hence, $1+\log x, 1+\log y, 1+\log z$ will also be in A.P.

$\Rightarrow \frac{1}{1+\log x}, \frac{1}{1+\log y}, \frac{1}{1+\log z}$ will be in H.P.

144 **(b,c)**

We have,

$\dfrac{p}{1-1/p} = \dfrac{9}{2}$

$\Rightarrow 2p^2 - 9p + 9 = 0$

$\Rightarrow p = 3/2, 3$

145 **(a,c)**

$$S = 1 + \frac{1}{(1+3)}(1+2)^2 + \frac{1}{(1+3+5)}(1+2+3)^2 + \frac{1}{(1+3+5+7)}(1+2+3+4)^2 + \cdots$$

The r^{th} term is given by

$T_r = \dfrac{1}{r^2}(1+2+\cdots+r)^2$

$= \dfrac{1}{r^2}\left\{\dfrac{r(r+1)}{2}\right\}^2$

$= \dfrac{r^2+2r+1}{4}$

$\therefore\ T_7 = 16$ and $S_{10} = \sum_{r=1}^{10} T_r$

$= \dfrac{1}{4}\left\{\dfrac{(10)(10+1)(20+1)}{6} + (10)(10+1) + 10\right\}$

$= \dfrac{505}{4}$

146 **(a,b,c)**

$$\frac{1}{\sqrt{2}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{11}}+\cdots n \text{ terms}$$

$$=\frac{\sqrt{5}-\sqrt{2}}{3}+\frac{\sqrt{8}-\sqrt{5}}{3}+\cdots+\frac{\sqrt{5+(n-1)3}-\sqrt{2+(n-1)3}}{3}$$

$$=\frac{\sqrt{3n+2}-\sqrt{2}}{3}$$

$$=\frac{3n+2-2}{3(\sqrt{3n+2}+\sqrt{2})}$$

$$=\frac{n}{\sqrt{3n+2}+\sqrt{2}}$$

$$=\frac{n}{\sqrt{2+3n}+\sqrt{2}}<\frac{n}{\sqrt{3n}}<n$$

147 **(a,b)**

$x^2+9y^2+25z^2=15yz+5zx+3xy$

$\Rightarrow (x)^2+(3y)^2+(5z)^2-(x)(3y)-(3y)(5z)-(x)(5z)=0$

$\Rightarrow \frac{1}{2}[(x-3y)^2+(3y-5z)^2+(x-5z)^2]=0$

$x=34=5z$

$\Rightarrow x:y:z=\frac{1}{1}:\frac{1}{3}:\frac{1}{5}$

Therefore, $1/x, 1/y$, and $1/z$ are in A.P. and x, y, and z are in H.P.

148 **(a,c)**

Given $a_1=2; \frac{a_n}{a_{n-1}}=\frac{a_{n-1}}{a_{n-2}}$

$\Rightarrow a_1, a_2, a_3, a_4, a_5, \ldots$ in G.P.

Let $a_2=x$ then for $n=3$ we have

$$\frac{a_3}{a_2}=\frac{a_2}{a_1}=\frac{x^2}{2}$$

$\Rightarrow a_1^2=a_1a_3$

$\Rightarrow a_3=\frac{x^2}{2}$

i.e. $2, x, \frac{x^2}{2}, \frac{x^3}{4}, \frac{x^4}{8}, \ldots$ with common ratio $r=\frac{x}{2}$

given $\frac{x^4}{8}\le 162$

$\Rightarrow x^4\le 1296\le x\le 6$

Also x $\frac{x^4}{8}$ and are integers

$\Rightarrow x$ must be even then only $\frac{x^4}{8}$ will be an integer

Hence possible values of x is 4 and 6. ($x\ne 2$ as terms are distinct)

Hence possible value of $a_5=\frac{a^4}{8}$ is $\frac{4^4}{8}, \frac{6^4}{8}$

149 **(a,b,c)**

If p, q, r are in A.P., then $p^{\text{th}}, q^{\text{th}}$ and r^{th} terms are equal distant terms which are always in the same series of which they are terms

150 **(a,c)**

Let $b=a+p, c=a+2p, d=a+3p$ (where p is common difference). Then,

$$\frac{\frac{1}{a}+\frac{1}{d}}{\frac{1}{b}+\frac{1}{c}}=\frac{\frac{1}{a}+\frac{1}{a+3p}}{\frac{1}{a+p}+\frac{1}{a+2p}}$$

$$=\frac{(a+p)(a+2p)}{a(a+3p)}$$

$$=\frac{a^2+3ap+2p^2}{a^2+3ap}>1$$

$$\therefore \frac{1}{a}+\frac{1}{d}>\frac{1}{b}+\frac{1}{c}$$

$$\left(\frac{1}{b}+\frac{1}{c}\right)(a+d)=\left(\frac{1}{a+p}+\frac{1}{a+2p}\right)(a+a+3p)$$

$$=\frac{(2a+3p)^2}{a^2+3ap+2p^2}$$

$$=4+\frac{p^2}{a^2+3ap+2p^2}>4$$

151 **(a,b,c)**

$$a=\frac{n^{64}-1}{n-1}$$

$$=(n+1)(n^2+1)(n^4+1)(n^8+1)(n^{16}+1)(n^{32}+1)$$

152 **(a,b,c,d)**

$an^4+bn^3+cn^2+dn+c$

$$=2\sum_{r=1}^{n} r(r+1)(r+2)-\sum_{r=1}^{n} r(r+1)$$

$$=\frac{2}{4}n(n+1)(n+2)(n+3)-\frac{1}{3}n(n+1)(n+2)$$

$$=\frac{1}{6}(3n^4+16n^3+27n^2+14n)$$

153 **(a,c,d)**

$a_1+a_3+a_5=-12$

$a+a+2d+a+4d=-12 \quad (d>0)$

$a+2d=-4 \quad (1)$

$a_1a_3a_5=80$

$a(a+2d)(a+4d)=80$

or $(-4-2d)(-4+2d)=-20 \Rightarrow d=\pm 3$

Since A.P. is increasing, so $d=+3; a=-10$.

Hence,

$$\left.\begin{aligned} a_1=-10; a_2=-7\\ a_3=a+2d=-10+6=-4\\ a_5=a+4d=-10+12=2\end{aligned}\right\}$$

154 **(a,c)**

$$\frac{1}{b-a}+\frac{1}{b-c}=\frac{1}{a}+\frac{1}{c}$$

$$\Rightarrow \frac{1}{b-a}-\frac{1}{c}=\frac{1}{a}-\frac{1}{b-c}$$

$$\Rightarrow \frac{c-b+a}{c(b-a)}=\frac{b-c-a}{a(b-c)}$$

$\Rightarrow c-b+a=0$ or $\frac{1}{c(b-a)}=\frac{1}{a(c-b)}$

$\Rightarrow b=a+c$ or $bc-ac=ac-ab$

$\Rightarrow b = a + c$ or $b = \frac{2ac}{a+c}$

155 **(a,b,d)**

$x + y + z = 3\left(\frac{a+b}{2}\right)$

$\Rightarrow 15 = 3\frac{(a+b)}{2}$

$\Rightarrow a + b = 10$ (1)

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{3\left(\frac{1}{a}+\frac{1}{b}\right)}{2}$

$\Rightarrow \frac{5}{3} = \frac{3(a+b)}{2ab} = \frac{3 \times 10}{2ab}$

$\Rightarrow ab = 9$ (2)

From (1) and (2), $a = 9, b = 1$ or $a = 1$ and $b = 9$. Hence, G.M.

$= \sqrt{ab} = 3, a + 2b = 11$ or 19

156 **(a,d)**

$S = 1 + 2x + 3x^2 + 4x^3 + \cdots \infty$

$\Rightarrow xS = x + 2x^2 + 3x^3 + 4x^4 + \cdots \infty$

$\Rightarrow (1-x)S = 1 + x + x^2 + \cdots \infty = \frac{1}{1-x}$

$\Rightarrow S = \frac{1}{(1-x)^2}$

Now,

$S \geq 4 \Rightarrow \frac{1}{(1-x)^2} > 4$

$\Rightarrow (x-1)^2 \leq \frac{1}{4}$

$\Rightarrow -\frac{1}{2} \leq x - 1 \leq \frac{1}{2}$

$\Rightarrow \frac{1}{2} \leq x \leq \frac{3}{2}$. Also $0 < |x| < 1$

$\Rightarrow \frac{1}{2} \leq x < 1$

157 **(a,d)**

We have

$a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots + \frac{1}{2^n - 1}$

$= 1 + \left(\frac{1}{2} + \frac{1}{3}\right) + \left(\frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}\right) + \left(\frac{1}{8} + \ldots + \frac{1}{15}\right) + \ldots + \frac{1}{2^n - 1}$

$= 1 + \left(\frac{1}{2} + \frac{1}{2^2 - 1}\right) + \left(\frac{1}{2^2} + \frac{1}{5} + \frac{1}{6} + \frac{1}{2^3 - 1}\right) + \left(\frac{1}{2^3} + \ldots + \frac{1}{2^4 \mp 1}\right) + \ldots$

$< 1 + 1 + \ldots + 1 = n$

Thus,

$a(100) < 100$

Also,

$a(n) = 1 + \frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right) + \ldots \frac{1}{2^n - 1}$

$= 1 + \frac{1}{2} + \left(\frac{1}{2^1 + 1} + \frac{1}{2^n}\right) + \left(\frac{1}{2^n} + \frac{1}{2^3}\right) + \cdots + \left(\frac{1}{2^{n-1}} + 1\right)$

$> 1 + \frac{1}{2} + \frac{2}{4} + \frac{4}{8} + \cdots + \frac{2^{n-1}}{2^n} - \frac{1}{2^n}$

$> 1 + \frac{1}{2} + \frac{2}{4} + \frac{4}{8} + \cdots \frac{2^{n-1}}{2^n} - \frac{1}{2^n}$

$= 1 + \left(\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \cdots + \frac{1}{2}\right) - \frac{1}{2^n}$

$= 1 + \frac{n}{2} - \frac{1}{2^n} = \left(1 - \frac{1}{2^n}\right) + \frac{n}{2}$

Thus,

$a > \left(1 - \frac{1}{2^{200}}\right) + \frac{200}{2} > 100$

i.e.,

$a(200) > 100$

158 **(a,d)**

$x, x^2 + 2, x^3 + 10$ are in G.P. Hence,

$x(x^3 + 10) = (x^2 + 2)^2 = x^4 + 4x^2 + 4$

$\Rightarrow 4x^2 - 10x + 4 = 0$

$\Rightarrow 2x^2 - 5x + 2 = 0$

$\Rightarrow x = 2, \frac{1}{2}$

The 4th term of G.P. is

$(x^3 + 10)r = (x^3 + 10)\left(\frac{x^2 + 2}{x}\right)$

$= \begin{cases} 54 \text{ when } x = 2 \\ \frac{729}{16} \text{ when } x = \frac{1}{2} \end{cases}$

159 **(a,b)**

We have, $2y = x + z$ and $2\tan^{-1} y =$ tan−*1x+*tan−*1z.*

$\Rightarrow \frac{2y}{1 - y^2} = \frac{x + z}{1 - xz}$

$\Rightarrow y^2 = xz$

S0, x, y, z are in GP which is possible, if $x = y = z$

160 **(a,d)**

$a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{2^n - 1}$

$= 1 + \left(\frac{1}{2} + \frac{1}{3}\right) + \left(\frac{1}{4} + \ldots + \frac{1}{7}\right)$

$+ \left(\frac{1}{8} + \ldots + \frac{1}{15}\right) + \ldots \left(\frac{1}{2^{n-1} + 1} \cdots \frac{1}{2^n - 1}\right) < 1$

$+ \left(\frac{1}{2} + \frac{1}{2}\right) + \left(\frac{1}{4} + \frac{1}{4} + \ldots \frac{1}{4}\right) + \left(\frac{1}{8} + \frac{1}{8} + \ldots + \frac{1}{8}\right)$

$+ \ldots + \left(\frac{1}{2^{n-1} + 1} + \frac{1}{2^{n-1} + 1} + \ldots + \frac{1}{2^{n-1} + 1}\right)$

$$= 1+\frac{2}{2}+\frac{4}{4}+\frac{8}{8}+\ldots\frac{2^{n-1}}{2^{n-1}}\underbrace{1+1+1+\ldots+1}_{(n-1)\text{times}}$$

$= n$

Thus, $a(100) < 100$.

Next, $a(n) = 1+\frac{1}{2}+\left(\frac{1}{3}+\frac{1}{4}\right)$

$$+\left(\frac{1}{5}+\ldots+\frac{1}{8}\right)+\ldots+\frac{1}{2^{n-1}+1}+\ldots+\frac{1}{2^n-1}$$

$$> 1+\frac{1}{2}+\left(\frac{1}{4}+\frac{1}{4}\right)$$

$$+\left(\frac{1}{8}+\frac{1}{8}\ldots+\frac{1}{8}\right)+\ldots+\left(\frac{1}{2^n-1}+\frac{1}{2^n-1}+\ldots+\frac{1}{2^n-1}\right)$$

$$= 1+\frac{1}{2}+\frac{2}{4}+\frac{4}{8}+\ldots+\frac{2^n-1}{2^n}-\frac{1}{2^n}$$

$$= \underbrace{1+\tfrac{1}{2}+\tfrac{1}{2}+\tfrac{1}{2}+\ldots+\tfrac{1}{2}-\tfrac{1}{2^n}}_{n\text{ times}}$$

$$= \left(1-\frac{1}{2^n}\right)+\frac{n}{2}$$

Therefore, $a(200) > \left(1-\frac{1}{2^n}\right)+\frac{200}{2} > 100$

161 **(b,d)**

Given $3a_1+7a_2+3a_3-4a_5=0$

$\Rightarrow 7(a_1+a_2+a_3) = 4(a_1+a_3+a_5)$

$\Rightarrow 7(1+r+r^2) = 4(1+r^2+r^4)$

$\Rightarrow 7 = 4(r^2-r+1)$

$\Rightarrow 4r^2-4r+1 = 4$

$\Rightarrow (2r-1)^2 = 4$

$\Rightarrow 2r-1 = \pm 2$

$\Rightarrow r = 3/2, -1/2$

162 **(a,c,d)**

$$S_n = \frac{n}{2}[2a'+(n-1)d] = a+bn+cn^2$$

$$\Rightarrow na'+\frac{n(n-1)}{2}d = a+bn+cn^2$$

$$\Rightarrow \left(a'-\frac{d}{2}\right)n+\frac{n^2d}{2} = a+bn+cn^2$$

On comparing,

$$a=0, b=a'-\frac{d}{2}, c=\frac{d}{2} \Rightarrow d=2c$$

163 **(c,d)**

$4 = 1+(n-1)d, 16 = 1+(m-1)d \Rightarrow \frac{15}{3} = \frac{m-1}{n-1}$

or $\frac{n-1}{1} = \frac{m-1}{5} = p =$ positive integer

$\therefore\ n = p+1, m = 5p+1$. So, $n\ m$ have infinite pairs of values

Also, $4 = 1.r^n, 16 = 1,.r^m \Rightarrow rm^{-n} = 4 = r^n$. So, $m-n=n$

$\therefore \frac{m}{2} = \frac{n}{1} = q =$ positive integer. So, m, m have infinite pairs of values

164 **(a,b)**

$$\left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)\left(\frac{1}{c}+\frac{1}{a}-\frac{1}{b}\right)$$

$$= \left(\frac{1}{b}+\frac{1}{c}-\frac{2}{b}+\frac{1}{c}\right)\left(\frac{1}{c}+\frac{1}{b}-\frac{1}{c}\right)$$

$$= \left(\frac{2}{c}-\frac{1}{b}\right)\frac{1}{b} = \frac{2}{bc}-\frac{1}{b^2}$$

Also by eliminating b, we get the given expression $\frac{(a+c)(3a-c)}{4a^2c^2}$

165 **(b,c)**

We have, for $0 < \phi < \pi/2$

$$x = \sum_{n=0}^{\infty}\cos^{2n}\phi$$

$$= 1+\cos^2\phi+\cos^4\phi+\ldots\infty$$

$$= \frac{1}{1-\cos^2\phi}$$

$$= \frac{1}{\sin^2\phi}$$

$$y = \sum_{n=0}^{\infty}\sin^{2n}\phi$$

$$= 1+\sin^2\phi+\sin^4\phi+\ldots\infty$$

$$= \frac{1}{1-\sin^2\phi}$$

$$= \frac{1}{\cos^2\phi}$$

$$z = \sum_{n=0}^{\infty}\cos^{2n}\phi\sin^{2n}\phi$$

$$= 1+\cos^2\phi\sin^2\phi+\cos^4\phi\sin^4\phi+\ldots\infty$$

$$= \frac{1}{1-\cos^2\phi\sin^2\phi} \quad (3)$$

Subtracting the values of $\cos^2\phi$ and $\sin^2\phi$ in (3), from (1) and (2), we get

$$z = \frac{1}{1-\frac{1}{x}\frac{1}{y}}$$

$$\Rightarrow\ z = \frac{xy}{xy-1}$$

$\Rightarrow xyz - z = xy$

$\Rightarrow xyz = xy+9z$

Also $x+y+z = \frac{1}{\cos^2\phi}+\frac{1}{\sin^2\phi}+\frac{1}{1-\cos^2\phi\sin^2\phi}$

$$= \frac{\sin^2\phi(1-\cos^2\phi\sin^2\phi)+\cos^2\phi(1-\cos^2\phi\sin^2\phi)+\cos^2\phi\sin^2\phi}{\cos^2\phi\sin^2\phi(1-\cos^2\phi\sin^2\phi)}$$

$$= \frac{(\sin^2\phi+\cos^2\phi)(1-\cos^2\phi\sin^2\phi)+\cos^2\phi\sin^2\phi}{\cos^2\phi\sin^2\phi(1-\cos^2\phi\sin^2\phi)}$$

$$= \frac{1}{\cos^2\phi\sin^2\phi(1-\cos^2\phi\sin^2\phi)} = xyz$$

Thus, (b) and (c) both are correct

166 **(a,d)**

$p(x) = \left(\frac{1-x^{2n}}{1-x^2}\right)\left(\frac{1-x}{1-x^n}\right) = \frac{1+x^n}{1+x}$

As $p(x)$ is a polynomial, $x = -1$ must be a zero of $1 + x^n$, Hence, $1 + (-1)^n = 0$. So, n must be odd

167 **(a,b,c)**

Let a, b, c are pth, qth and rth terms of A.P. then $a = A + (p-1)D, b = A + (q-1)D, c = A + (r-1)D$

$\Rightarrow \frac{r-q}{q-p} = \frac{c-b}{b-a}$ is rational number

Now for, 1, 6, 19 $\frac{r-q}{q-p} = \frac{19-6}{6-1}$ is rational number

For $\sqrt{2}, \sqrt{50}, \sqrt{98}$, $\frac{r-q}{q-p} = \frac{\sqrt{98}-\sqrt{50}}{\sqrt{50}-\sqrt{2}} = \frac{7\sqrt{2}-5\sqrt{2}}{5\sqrt{2}-\sqrt{2}}$

$= \frac{1}{2}$ is rational number

For $\log 2, \log 16, \log 128$

$$\frac{r-q}{q-p} = \frac{\log 128 - \log 16}{\log 16 - \log 2} = \frac{7\log 2 - 4\log 2}{4\log 2 - \log 2}$$

$= 1$ is rational number

But for $\sqrt{2}, \sqrt{3}, \sqrt{7}$, $\frac{r-q}{q-p}$ is not rational number

168 **(b,c,d)**

We have, length of side of S_n is equal to the length of a diagonal of S_{n+1}. Hence,

Length of a side of $S_n = \sqrt{2}$ (Length of a side of S_{n+1})

$\Rightarrow \frac{\text{Length of a side of } S_{n+1}}{\text{Length of side of } S_n} = \frac{1}{\sqrt{2}}$, for all $n \geq 1$

Hence, sides of $S_1, S_2, \ldots, S_n$ form a G.P. with common ratio $1/\sqrt{2}$ and first term 10

$\therefore$ Side of $S_n = 10\left(\frac{1}{\sqrt{2}}\right)^{n-1} = \frac{10}{2^{\frac{n-1}{2}}}$

$\Rightarrow$ Area of $S_n = (\text{side})^2 = \left(\frac{10}{2^{\frac{n-1}{2}}}\right) = \frac{100}{2^{n-1}}$

Now, area of $S_n < 1 \Rightarrow n = b, c, d$

169 **(b)**

Putting $\theta = 0$, we get $b_0 = 0$

$\therefore \sin n\theta = \sum_{r=1}^{n} b_r \sin^r \theta$

$\Rightarrow \frac{\sin n\theta}{\sin\theta} = \sum_{r=1}^{n} b_r (\sin\theta)^{r-1}$

$= b_1 + b_2 \sin\theta + b_3 \sin^2\theta + \ldots b_{hn} \sin^{n-1}\theta$

Taking limit as $\theta \to 0$, we obtain

$\lim_{\theta\to 0} \frac{\sin n\theta}{\sin\theta} = b_1 \Rightarrow b_1 = n$

170 **(a)**

$x^2 + 9y^2 + 25z^2 = xyz\left(\frac{15}{x} + \frac{5}{y} + \frac{3}{z}\right)$

$\Rightarrow x^2 + 9y^2 + 25z^2 - 15yz - 5xz - 3xy = 0$

$\Rightarrow 2x^2 + 18y^2 + 50z^2 - 30yz - 10xz - 6xy = 0$

$\Rightarrow (x-3y)^2 + (3y-5z)^2 + (5z-x)^2 = 0$

$\Rightarrow x - 3y = 0, 3y - 5z = 0, 5z - x = 0$

$\Rightarrow x = 3y = 5z = k$ (say)

$\Rightarrow x = k, y = k/3, z = k/5$

Hence, x, y, z are in H.P. Hence option (a) is correct

171 **(c)**

$\because b = \frac{2ac}{a+c}$

$\Rightarrow \frac{b}{a} = \frac{2c}{a+c}$ and $\frac{b}{c} = \frac{2a}{a+c}$

Now, $\frac{a+b}{2a-b} = \frac{1+b/a}{2-b/a} = \frac{a+3c}{2a}$

And $\frac{c+b}{2c-b} = \frac{1+b/c}{2-b/c} = \frac{3a+c}{2c}$

Thus, $\frac{a+b}{2a-b} + \frac{c+b}{2c-b} \geq 1 + \frac{3}{2}\left(\frac{c}{a} + \frac{a}{c}\right) \geq 1 + \frac{3}{2}(2) = 4$

172 **(d)**

Sum of n terms of AP is given by $S_n = \frac{n}{2}[2a + (n-1)d]$.

Hence, sum of n terms of an AP is always of the form $pn^2 + qn$

Hence, option (d) is correct.

173 **(d)**

Case I : $x < 1$

Then, $1 - x, 3, 3 - x$ are in AP.

$6 = 4 - 2x \Rightarrow x = -1$

$\therefore$ Terms are 2, 3, 4

$\therefore$ Sixth term = 7

Case II : $1 < x < 3$

Then, $x - 1, 3, x - 3$ are in AP.

$6 = 2$ (impossible)

Case III : $x > 3$

Then, $x - 1, 3, x - 3$ are in AP.

$6 = 2x - 4 \Rightarrow x = 5$

Then term are 4, 3, 2

$\therefore$ Sixth term is -1

174 **(a)**

Statement 2 is true as it is a property of sequence in G.P.

Now T_{m-n}, T_m and T_{m+n} are in G.P> ($\because$ Tm from $Tm-n$ and $Tm+n$ from T are at same distance

$\therefore T_m^2 = T_{m-n}T_{m+n}$

$\Rightarrow T_m = \sqrt{pq}$

175 **(d)**

For odd integer n, we have

$S_n = n^3 - (n-1)^3 + \cdots + (-1)^{n-1}1^3$

$= [1^3 + 2^3 + 3^3 + \cdots + (n-1)^3 + n^3] - 2[2^3 + 4^3 + 6^3 + \cdots + (n-1)^3]$

$= \frac{n^2(n+1)^2}{4} - 2 \times 2^3 \left[1^3 + 2^3 + \cdots \left(\frac{n-1}{2}\right)^3\right]$

$= \frac{n^2(n+1)^2}{4} - 2^4 \frac{\left(\frac{n-1}{2}\right)^2 \left(\frac{n-1}{2} + 1\right)^2}{4}$

$= \frac{n^2(n+1)^2}{4} - \frac{(n-1)^2(n+1)^2}{4}$

$= \frac{(n+1)^2}{4}[n^2 - (n-1)^2]$

$= \frac{1}{4}(2n-1)(n+1)^2$

Now, putting $n = 11$ in above formula, $S_{11} = 756$. Hence statement 1 is false and statement 2 is correct

176 **(b)**

Let, if possible, 8 be the first term and 12 and 27 be n^{th} and n^{th} terms, respectively. Then,

$12 = ar^{m-1} = 8r^{m-1}, 27 = 8r^{n-1}$

$\Rightarrow \frac{3}{2} = r^{m-1}, \left(\frac{3}{2}\right)^3 = r^{n-1} = r^{3(m-1)}$

$\Rightarrow n - 1 = 3m - 3$ or $3m + n + 2$

$\Rightarrow \frac{m}{1} = \frac{n+2}{3} = k$ (say)

$\therefore m = k, n = 3k - 2$

By giving k different values, we get the integral value of m and n. Hence there can be infinite number of G.P.'s whose any the three terms will be 8, 12, 27 (not consecutive). Obviously, statement 2 is not a correct explanation of statement 1

177 **(a)**

We know, if $ax + by = k$ and the expression $x^m y^m (m, n \geq 1)$ will be maximum when $\left(\frac{ax}{m}\right)^m \left(\frac{by}{n}\right)^n$ is maximum and this is maximum at $\frac{ax}{m} = \frac{by}{n} = \frac{ax+by}{m+n} = \frac{k}{m+n}$.

Since, x^2y^3 will be maximum at

$\frac{3x}{2} = \frac{4y}{3} = \frac{5}{5}$

$\Rightarrow x = \frac{2}{3}, y = \frac{3}{4}$

$\therefore \frac{x}{y} = \frac{8}{9}$ or $9x = 8y$

$\therefore$ Maximum value of $x^2y^3 = \left(\frac{2}{3}\right)^2 \left(\frac{3}{4}\right)^3 = \frac{3}{16}$

178 **(b)**

The given inequality is

$(p_1^2 + p_2^2 + \ldots + p_{n-1}^2)x^2 + 2(p_1p_2 + p_2p_3 + \ldots p_{n-1}p_n)x + (p_2^2 + \ldots + p_n^2) \leq 0$

$\Rightarrow (p_1x + p_2)^2 + (p_2x + p_3)^2 + \ldots + (p_{n-1}x +$ $pn2 \leq 0$ (1)

But each one of the terms on the L.H.S. is a perfect square and hence is positive or zero

Therefore (1) holds only if

$$p_1x + p_2 = 0 = p_2x + p_3 = p_3x + p_4 = \cdots$$
$$= p_{n-1}x + p_n$$

$$\Rightarrow\ -x = \frac{p_2}{p_1} = \frac{p_3}{p_2} = \cdots = \frac{p_n}{p_{n-1}}$$

Hence, $p_1, p_2, \dots, p_n$ are in G.P.

179 **(a)**
We have,

$$a \times ar \times ar^2 \times \dots \times ar^{n-1} = a^n \times r^{1+2+\dots+(n-1)}$$
$$= a^n r^{\frac{n(n-1)}{2}} (a^2 r^{n-1})^{n/2}$$

Hence, statement 1 is true

Also, $(a \times r^{i-1})(a \times r^{n-ik}) = a^2 \times r^{n-1}$, which is independent of k. Hence, statement 2 is a correct explanation for statement 1, as in the product of $a, ar, ar^2, \dots, ar^{n-1}$, there are $n/2$ groups of numbers, whose product is $a^2 r^{n-1}$. Hence (a) is the correct option

180 **(d)**
$$\because\ \frac{S_n}{S_n{}'} = \frac{(7n+1)}{(4n+17)} = \frac{n(7n+1)}{n(4n+17)}$$

$$\therefore\ S_n = (7n^2 + n)\lambda,\ S'_n = (4n^2 + 17n)\lambda$$

Then, $\frac{T_n}{T'_n} = \frac{S_n - S_{n-1}}{S'_n - S'_{n-1}}$

$$= \frac{7(2n-1)+1}{4(2n-1)+17}$$

$$= \frac{14n-6}{8n+13}$$

$$\Rightarrow\ T_n : T'_n = (14n-6) : (8n+13)$$

181 **(a)**
For two positive numbers $(\text{G.M.})^2 = (\text{A.M.}) \times (\text{H.M.})$

182 **(d)**
$\because$ Sum of n terms of an AP is $S_n = \frac{n}{2}[2A +$ *n−1D.*

Where A and D are first term and common difference.

Hence, sum always of the form $an^2 + bn$.

183 **(a)**
If a, b, c are in GP, then $a+b, b+b, c+b$ are in HP.

$$\Rightarrow (2b) = \frac{2(a+b)(b+c)}{(a+b)+(c+b)}$$

$$\Rightarrow b(a+2b+c) = (a+b)(b+c)$$

$$\Rightarrow b(a+c) + 2b^2 = ab + ac + b^2 + bc$$

$$\Rightarrow\ b^2 = ac \quad (\because a, b, c \text{ are in GP})$$

184 **(a)**
Let p, q, r be the $l^{\text{th}}, m^{\text{th}}$ and n^{th} terms of an A.P. Then

$p = (a + (l-1)d, q = a + (m-1)d$ and
$r = a + (n-1)d$

Hence, $r - q = (n-m)d$ and $q - p = (m-l)d$, so that

$$\frac{r-q}{p-q} = \frac{(n-m)d}{(m-l)d} = \frac{n-m}{m-l} \quad (\because d \neq 0)$$

Since, l, m, n are positive integers and $m \neq l, (n-m)/(m-l)$ is a rational number.
From (1), using $p = \sqrt{2}, q = \sqrt{3}, r = \sqrt{5}$, we have

$$\frac{\sqrt{5}-\sqrt{3}}{\sqrt{3}-\sqrt{2}} = \frac{n-m}{m-l} \text{ (which is not possible)}$$

Hence, $\sqrt{2}, \sqrt{3}, \sqrt{5}$ cannot be the terms of an A.P.

185 **(b)**
$x = 1111 \dots 91$ times

$$= 1 + 10 + 10^2 + 10^3 + \cdots + 10^{90}$$

$$= \frac{1(10^{91}-1)}{10-1}$$

$$= \frac{(10^{13\times7}-1)}{10-1}$$

$$= \frac{((10^{13})^7 - 1)}{10^{13}-1} \times \frac{(10^{13}-1)}{10-1}$$

$$= (1 + 10^{13} + 10^{26} + \dots 10^{78}) \times (1 + 10 + 10^2 + \dots + 10^{12})$$

=composite numbers

But statement 2 is not a correct explanation of statement 1 as 111 has 1 digit 3 times, and 3 is a prime number but $111 = 3 \times 37$ is a composite number. Hence (b) is the correct option

186 **(a)**

Coefficient of x^{14} in $(1+2x+3x^2+\ldots+16x^{15})^2$

Coefficient of x^{14} in $(1+2x+3x^2+\ldots+16x^{15})^2(1+2x+3x^2+$ *...16x15*

$= 1\times 15 + 2\times 14 + \cdots + 15\times 1$

$$\sum_{r=1}^{15} r(16-r)$$

Also,

$$\sum_{r=1}^{n-1} r(n-r) = \sum_{r=1}^{n-1} nr - \sum_{r=1}^{n-1} r^2$$

$$= n\frac{n(n-1)}{2} - \frac{n(n-1)(2n-1)}{6}$$

$$= \frac{n(n-1)}{6}(3n-(2n-1))$$

$$= \frac{n(n^2-1)}{6}$$

$$\Rightarrow \sum_{r=1}^{15} r(16-r) = \frac{15(15^2-1)}{6} = 560$$

Hence option (a) is correct

187 **(d)**

Sum $= \frac{x/r}{1-r} = 4$ (where r is common ratio)

$x = 4r(1-r) = 4(r-r^2)$

For $r \in (-1,-1) - \{10\}$

$r - r^2 \in \left(-2, \frac{1}{4}\right) - \{0\}$

$\Rightarrow\ x \in (-8, 1) - \{0\}$

188 **(b)**

$$\therefore \sum_{r=1}^{n} F_1(r) = \sum_{r=1}^{n}\left\{1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{r}\right\}$$

$$= 1\cdot n + \frac{1}{2}(n-1) + \frac{1}{3}(n-2) + \ldots + 1\cdot\frac{1}{n}$$

$$= n\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}\right) - \left\{\frac{1}{2}+\frac{2}{3}+\ldots+\left(\frac{n-1}{n}\right)\right\}$$

$$= nF_1(n) - \left\{\left(1-\frac{1}{2}\right)+\left(1-\frac{1}{3}\right) + \left(1-\frac{1}{4}\right)+\ldots+\left(1-\frac{1}{n}\right)\right\}$$

$$= nF_1(n) - \{n - F_1(n)\} = (n+1)F_1(n) - n$$

189 **(a)**

Statement 2 is true as

$$\frac{a^n+b^n}{a+b} = \frac{a^n-(-b)^n}{a-(-b)} = a^{n-1} - a^{n-2}b + a^{n-3}b^2 - \cdots (-1)^{n-1}b^{n-1}$$

Now,

$$1^{99}+2^{99}+\cdots 100^{99} = (1^{99}+100^{99}) + (2^{99}+99^{99}) + \cdots + (50^{99}+51^{99})$$

Each bracket is divisible by 101; hence the sum is divided by 101. Also,

$$1^{99}+2^{99}+\text{L}\ldots 100^{99} = (1^{99}+99^{99}) + (2^{99}+98^{99})$$

$$+\cdots+(49^{99}+51^{99}) + 50^{99} + 100^{99}$$

Here, each bracket and 50^{99} and 100^{99} are divisible by 100. Hence sum is divisible by 100. Hence sum is divisible by $101\times 100 = 10100$

190 **(b)**

1. $\sum n = 210$

$\Rightarrow n(n-1) = 420$

$\Rightarrow (n-20)(n+21) = 0$

$\Rightarrow n = 20$

Hence,

$$\sum n^2 = \frac{n}{6}(n+1)(2n+1)$$

$$= \frac{20}{6}(21)(41)$$

$$= (10)(7)(41)$$

Hence, the greatest prime number by which $\sum n^2$ which is divisible is 41

2. $4, G_1, G_2, \ldots G_{n+1}, \ldots G_{2n}, G_{2n+1}, 2916$

G_{n+1} will be the middle mean of $(2n+1)$ odd means and it will be equidistant from the first and last terms. Hence,

$4, G_{n+1} 2916$ will also be in G.P. So,

$\Rightarrow G_{n+1}^2 = 4 \times 2916$

$= 4 \times 9 \times 324$

$= 4 \times 9 \times 4 \times 81$

$\Rightarrow G_{n+1} = 2 \times 3 \times 2 \times 9 = 108$

Hence, the greatest odd number by which G_{n+1} is divisible is 27

3. Terms are 40, 30, 24, 20. Now,

$$\frac{1}{30} - \frac{1}{40} = \frac{1}{120}$$

$$\Rightarrow \frac{1}{24} - \frac{1}{30} = \frac{6}{24 \times 30} = \frac{1}{120}$$

and

$$\frac{1}{20} - \frac{1}{24} = \frac{4}{20} = \frac{1}{120}$$

Hence, 1/30, 1/24, 1/20 are in A.P. with common difference $d = 1/120$. Hence, the next term is 1/20+1/120=7/120. Therefore, the next term of given series is $\frac{120}{7} = 17\frac{1}{7}$. Hence, the integral part of $17\frac{1}{7}$ is 17

4. $S = 1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \ldots \infty$

$$\Rightarrow \frac{1}{5}S = \frac{1}{5} + \frac{4}{5^2} + \frac{7}{5^3} + \cdots$$

$$\Rightarrow S\left(1 - \frac{1}{5}\right) = 1 + 3\left[\frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \cdots \infty\right]$$

$$\Rightarrow \frac{4}{3}S = 1 + 3\left[\frac{1/5}{1 - 1/5}\right] = 1 + \frac{3}{4} = \frac{7}{4}$$

$$\Rightarrow S = \frac{35}{16}$$

$\Rightarrow a = 35$ and $b = 16$

$\Rightarrow a - b = 19$

191 **(a)**

1. $a, b, c,$ are in G.P. Hence,

$b^2 = ac$

$\Rightarrow 2\log_{10} b = \log_{10} a + \log_{10} c$

$$\Rightarrow \frac{2}{\log_b 10} = \frac{1}{\log_a 10} + \frac{1}{\log_c 10}$$

$$\Rightarrow \frac{2}{y} = \frac{1}{x} + \frac{1}{z}$$

Hence, x, y, z are in H.P.

2. $\frac{a+be^x}{a-be^x} = \frac{b+ce^x}{b-ce^x} = \frac{c+de^x}{c-de^x}$

$$\Rightarrow \frac{2a}{a - be^x} - 14 = \frac{2b}{b - ce^x} - 1 = \frac{2c}{c - de^x} - 1$$

$$\Rightarrow \frac{a - be^x}{a} = \frac{b - ce^x}{b} = \frac{c - de^x}{c}$$

$$\Rightarrow 1 - \frac{b}{a}e^x = 1 - \frac{c}{b}e^x = 1 - \frac{d}{e}e^x$$

$$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$$

Hence, a, b, c, d are in G.P.

3. Given, $2b = a + c, x^2 = ab, y^2 = bc$. Now,

$x^2 + y^2 = b(a + c) = b\,2b = 2b^2$

$\Rightarrow x^2 + y^2 = 2b^2$

Hence, x^2, b^2, y^2 are in A.P.

4. $x \log a = y \log b = z \log c = k$ (say)

Also,

$y^2 = xz$

$$\Rightarrow \frac{k^2}{(\log b)^2} = \frac{k^2}{\log a \log c}$$

Hence, $\log a, \log b, \log c$ are in G.P.

192 **(a)**

Let $S = \frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + \ldots$

$$\therefore T_r = \frac{2r + 1}{1^2 + 2^2 + \ldots + r^2} = 6\left(\frac{1}{r(r + 1)}\right)$$

$$= 6\left(\frac{1}{r} - \frac{1}{r + 1}\right)$$

$$\therefore S_n = \frac{6n}{n + 1}$$

193 **(a)**

Since, $\sqrt{ab} = 16$ and $\frac{2ab}{a+b} = 12\frac{4}{5}$

$$\therefore 2 \times \frac{256}{a + b} = \frac{64}{5}$$

$\Rightarrow a + b = 40$

$= 8 + 32$

$\Rightarrow \dfrac{a}{b} = \dfrac{1}{4}$

194 **(c)**

Let the odd integers be $2m+1, 2m+3, 2m+5, \ldots$ and let their number be n. Then,

$57^2 - 13^2 = (n/2)[2(2m+1) + (n-1) \times 2]$

$= n(2m+1)$

$= 2mn + n^2$

$\Rightarrow 57^2 - 13^2 = (n+m)^2 - m^2$

$\Rightarrow m = 13$ and $n + m = 57$

$\Rightarrow n = 57 - 13 = 44$

Hence, the required odd integers are 27, 29, 31, ...,113

195 **(c)**

a, b, c are in G.P. Hence, a, ar, ar^2 are in G.P. So,

$\dfrac{a^2 + a^2r^2 + a^2r^4}{(a + ar + ar^2)^2} = \dfrac{t^2}{\alpha^2 t^2} = \dfrac{1}{\alpha^2}$

$\alpha^2 = \dfrac{r^2 + r + 1}{r^2 - r + 1}$

Let $\alpha^2 = y$

$\therefore \ y = \dfrac{r^2 + r + 1}{r^2 - r + 1}$

$(y-1)r^2 - r(y+1) + (y-1) = 0$

For real r,

$(y+1)^2 - 4(y-1)^2 \geq 0$

$\Rightarrow \dfrac{1}{3} \leq y \leq 3$

But $y \neq 1/3, 1, 3$ ($\because r \neq 1, -1, 0$)

$\therefore \dfrac{1}{3} < y < 3$ and $y \neq 1$

$\alpha^2 \in \left(\dfrac{1}{3}, 3\right) - \{1\}$

196 **(d)**

Let a be the first term and r the common ratio of the given G.P.

Further, let there be n terms in the given G.P. Then,

$a_1 + a_n = 66 \ \Rightarrow a + ar^{n-1} = 66 \quad$ (i)

$a_2 \times a_{n-1} = 128$

$\Rightarrow ar \times ar^{n-2} = 128$

$\Rightarrow a \times (ar^{n-1}) = 128 \ \Rightarrow ar^{n-1} = \dfrac{128}{a}$

Putting this value of ar^{n-1} in (i), we get

$a + \dfrac{128}{a} = 66$

$\Rightarrow a^2 - 66a + 128 = 0$

$\Rightarrow (a-2)(a-64) = 0$

$\Rightarrow a + 2, 64$

Putting $a = 2$ in (1), we get

$2 + 2 \times r^{n-1} = 66 \ \Rightarrow r^{n-1} = 32$

Putting $a = 64$ in (1), we get

$64 + 64r^{n-1} = 66 \ \Rightarrow r^{n-1} = \dfrac{1}{32}$

for an increasing G.P., $r > 1$. Now,

$S_n = 126$

$\Rightarrow 2\left(\dfrac{r^n - 1}{r - 1}\right) = 126$

$\Rightarrow \dfrac{r^n - 1}{r - 1} = 63$

$\Rightarrow \dfrac{r^{n-1} \times r - 1}{r - 1} = 63$

$\Rightarrow \dfrac{32r - 1}{r} = 63$

$\Rightarrow r = 2$

$\therefore r^{n-1} = 32 \ \Rightarrow 2^{n-1} = 32 = 2^5 \Rightarrow n - 1 = 5$

$\Rightarrow n = 6$

For decreasing G.P., $a = 64$ and $r = 1/2$. Hence, the sum of infinite terms is $64/\{1 - (1/2)\} = 128$

For $a = 2, r = 2$ terms are 2, 4, 8, 16, 32, 64. For $a = 64, r = 1/2$ terms are 64, 32, 16, 8, 4, 2.

Hence difference is 62

197 **(c)**

Let the four integers be $a - d, a + d$ and $a + 2d$, where a and d are integers and $d > 0$. Now,

$a + 2d = (a-d)^2 + a^2 + (a + d^2)$

$\Rightarrow 2d^2 - 25d + 3a^2 - a = 0 \quad (1)$

$\therefore d = \frac{1}{2}\left[1 \pm \sqrt{1 + 2a - 6a^2}\right] \quad (2)$

Since d is a positive integer, so

$1 + 2a - 6a^2 > 0$

$\Rightarrow 6a^2 - 2a - 1 < 0$

$\Rightarrow \frac{1-\sqrt{7}}{6} < a < \frac{1+\sqrt{7}}{6} \quad$ ($\because a$ is an integer)

$\Rightarrow a = 0$

Hence from (2),

$d = 1$ or 0

But since $d > 0$

$\therefore d = 1$

Hence, the four numbers are $-1, 0, 1, 2$

198 **(d)**

1, 2, 2, 3, 3, 3, 4, 4, 4, 4, ...

Let us write the terms in the groups as follows: 1, (2, 2), (3, 3, 3), (4, 4, 4, 4), ... consisting of 1, 2, 3, 4, ... terms. Let 2000th term fall in n^{th} group. Then,

$\dfrac{(n-1)n}{2} < 2000 \leq \dfrac{n(n+1)}{2}$

$\Rightarrow n(n-1) < 4000 \leq n(n+1)$

Let us consider,

$n(n-1) < 4000$

$\Rightarrow n^2 - n - 4000 < 0$

$\Rightarrow n < \dfrac{1 + \sqrt{16001}}{2} \ \Rightarrow n < 64$

We have,

$n(n+1) \geq 4000 \Rightarrow n^2+n-4000 \geq 0 \Rightarrow n \geq 63$

That means 2000th term falls is 63rd group. That also means that the 2000th term is 63. Now, total number of terms up to 62nd group is $(62 \times 63)/2 = 1953$. Hence, sum of first 2000 terms is

$1^2+2^2+\ldots 62^2+63(2000-1953)$

$$= \frac{62(63)125}{6} + 63 \times 47 = 84336$$

Sum of the remaining terms is $63 \times 16 = 1008$

199 **(b)**

Let numbers is set A be $a-D$ and these in set B be $b-d, b, b+d$. Now,

$3a = 3b = 15$

$\Rightarrow a = b = 5$

Set $A = \{5-D, 5, 5+D\}$

Set $B = \{5-d, 5, 5+d\}$

Where $D = d+1$

Also,

$$\frac{p}{q} = \frac{5(25-D^2)}{5(25-d^2)} = \frac{7}{8}$$

$\Rightarrow 25(8-7) = 8(d+1)^2 - 7d^2$

$\Rightarrow s = -17, 1$ but $d > 0 \Rightarrow d = 1$

So, the numbers in set A are 3, 5, 7 and the numbers in set B are 4, 5, 6

Now, sum of product of numbers in set A taken two at a time is $3 \times 5 + 3 \times 7 + 5 \times 7 = 71$. The sum of product of numbers in set B taken two at a time is $4 \times 5 + 5 \times 6 + 6 \times 4 = 74$. Also,

$p = 3 \times 5 \times 7 = 105$ and $q = 4 \times 5 \times 6 = 120$

$\Rightarrow q - p = 10$

200 **(c)**

$G_1, G_2, \ldots G_n = \left(\sqrt{1 \times 1024}\right)^n = 2^{5n}$

Given,

$2^{5n} = 2^{45} \Rightarrow n = 9$

Hence,

$r = (1024)^{\frac{1}{9+1}} = 2$

$\Rightarrow G_1 = 2, r = 2$

$$\Rightarrow G_1 + G_2 + \ldots + G_9 = \frac{2 \times (2^9 - 1)}{2-1} = 1024 - 2 = 1022$$

201 **(a)**

Let m and $(m+1)$ be the removed numbers from $1, 2, \ldots, n$

Then, sum of the remaining numbers is

$n(n+1)/2 - (2m+1)$

From given condition,

$$\frac{105}{4} = \frac{\frac{n(n+1)}{2} - (2m+1)}{(n-2)}$$

$\Rightarrow 2n^2 - 103n - 8m + 206 = 0$

Since n and m are integers, so n must be even. Let $n = 2k$. Then,

$$m = \frac{4k^2 + 103(1-k)}{4}$$

Since m is an integer, then $1-k$ must be divisible by 4. Let $k = 1 + 4t$. Then we get $n = 8t + 2$ and $m = 16t^2 - 95t + 1$. Now,

$1 \leq m < n$

$\Rightarrow 1 \leq 16T^2 - 95\,t + 12 < 8t + 2$

Solving, we get $t = 6$. Hence,

$n = 50$ and $m = 7$

Hence, the removed numbers are 7 and 8. Also, sum of all numbers is $50(50+1)/2 = 1275$

202 **(c)**

Let the first term a and common difference d of the first A.P. and the first6 term b and commo0n difference e of the second A.P. and let the number of terms be n. Then,

$\frac{a+(n-1)d}{b} = \frac{b+(n-1)e}{a} = 47 \quad (1)$

$\frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2b+(n-1)e]} \quad (2)$

From (1) and (2), we get

$a - 4b + (n-1)d = 0 \quad (3)$

$b - 4a + (n-1)e = 0 \quad (4)$

$2a - 4b + (n-1)d - 2(n-1)e = 0 \quad (5)$

$4 \times (3) + (4)$ gives

$-15b + 4(n-1)d + (n-1)e = 0 \quad (6)$

$(4) \times 2 + (5)$ gives

$-7b + 2(n-1)d - 3(n-1)e = 0 \quad (7)$

Further, $15 \times (7) - 7 \times (6)$ gives

$2(n-1)d - 52(n-1)e = 0$

Or $de = 26e$ $(\because n > 1)$

$\therefore d/e = 26$

Putting $d = 26e$ in (3) and solving it with (4), we get

$a = 2(n-1)e, b = 7(n-1)e$

Then, the ratio of their n^{th} terms is

$$\frac{2(n-1)e + (n-1)26e}{7(n-1)e + (n-1)e} = \frac{7}{2}$$

203 **(d)**

We have,

$a+b+c=25$ (1)

$2a=b+2$ (2)

$c^2=18b$ (3)

Eliminating a from (1) and (2), we have

$b=16-\frac{2c}{3}$

Then from (3),

$c^2=18\left(16-\frac{2c}{3}\right)$

$\Rightarrow c^2+12c-18\times 16=0$

$\Rightarrow (c-12)(c+24)=0$

Now, $c=-24$ is not possible since it does not lie between 2 and 18. Hence, $c=12$. Then from (3), $b=8$ and finally from (2), $a=5$

Thus, $a=5, b=8$ and $c=12$. Hence, $abc=5\times 8\times 12=480$

Also, equation $ax^2+bc+c=0$ is $5x^2+8x+12=0$, which has imaginary roots

If a,b,c are roots of the equation $x^3+qx^2+rx+s=0$, then sum of product of roots taken two at a time is $r=5\times 8+5\times 12+8\times 12=196$

204 **(c)**

Clearly here the differences between the successive terms are

$7-2, 14-7, 24, 14$, …. i.e…4, 7, 10, … which are in A.P.

$\therefore\ T_n=an^2+bn+c$

Thus, we have

$3=a+b+c$

$7=4a+2b+c$

$14=9a+3b+c$

Solving, we get $a=3/2, b=-1/2, c=2$. Hence,

$T_n=\frac{1}{2}(3n^2-n+4)$

$\therefore S_n=\frac{1}{2}\left[3\sum n^2-\sum n+4n\right]$

$=\frac{1}{2}\left[3\frac{n(n+1)(2n+1)}{6}-\frac{n(n+1)}{2}+4n\right]$

$=\frac{n}{2}(n^2+n+4)$

$\Rightarrow S_{20}$=4240

205 **(8)**

Since a,b,c,d are in A..P.

$\therefore b-a=c-b=d-c=D$ (let common difference)

$\Rightarrow d=a+3D$

$\Rightarrow a-d=-3D$ and $d=b+2D$

$\Rightarrow b-d=-2D$

Also $c=a+2D \ \Rightarrow c-a=2D$

$\therefore$ Given equation $2(a-b)+k(b-c)^2+(c-a)^3=2(a-d)+(b-d)^2+(c-d)^3$

Becomes $-2D+kD^2+(2D)^3=-6D+4D^2-D^3$

$\Rightarrow 9D^2+(k-4)D+4=0$

Since D is real $\Rightarrow (k-4)^2-4(4)(9)\geq 0$

$\Rightarrow k^2-8k-128\geq 0 \ \Rightarrow (k-16)(k+8)\geq 0$

$\therefore k\in(-\infty,-8]\cup[16,\infty)$

Hence, the smallest positive value of $k=16$

206 **(4)**

Let $\frac{a}{r}, a, ar$ be the three terms in G.P>

$\therefore$ Product of terms $=a^3-1$ (Given)

$\Rightarrow a=-1$

Now, sum of terms $=\frac{a}{r}+a+ar=\frac{13}{12}$ (Given)

$\Rightarrow \frac{-1}{r}-1-r=\frac{13}{12}$

$\Rightarrow 12r^2+25r+12=0$

$\therefore\ (3r+4)(4r+3)=0$

$\Rightarrow r=\frac{-4}{3},\frac{-3}{4}$

But $r\neq\frac{-4}{5}$

$\therefore\ |S|=\left|\frac{a}{1-r}\right|=\left|\frac{-1}{1-\left(\frac{-3}{4}\right)}\right|=\left|\frac{-1}{1+\frac{3}{4}}\right|=\left|\frac{-4}{7}\right|$

$=\frac{4}{7}$

207 **(2)**

Let $S=\sum_{r=1}^{\infty}\frac{r+2}{2^{r+1}.r.(r+1)}$

$=\sum_{r=1}^{\infty}\frac{2(r+1)-r}{2^{r+1}.r.(r+1)}$

$=\sum_{r=1}^{\infty}\frac{1}{2^{r+1}}\left(\frac{2}{r}-\frac{1}{r+1}\right)$

$=\sum_{r=1}^{\infty}\left(\frac{1}{2^r.r}-\frac{1}{2^{r+1}(r+1)}\right)$

$=\lim_{n\to\infty}\left[\left(\frac{1}{2^1.1}-\frac{1}{2^2.2}\right)+\left(\frac{1}{2^2.2}-\frac{1}{2^3.3}\right)+\left(\frac{1}{2^3.3}-\frac{1}{2^4.4}\right)\right]$

$=+\ldots+\left(\frac{1}{2^n.n}-\frac{1}{2^{n+1}.(n+1)}\right)$

$=\lim_{n\to\infty}\left(\frac{1}{2}-\frac{1}{2^{n+1}.(n+1)}\right)$

$\therefore\ S=\frac{1}{2}$

Hence, $S^{-1}=2$

208 **(9)**

Given $S=\sum_{n=1}^{9999}\frac{1}{(\sqrt{n}+\sqrt{n+1})(\sqrt[4]{n}+\sqrt[4]{n+1})}$

$$= \sum_{n=1}^{9999} \frac{1}{(\sqrt{n}+\sqrt{n+1})(\sqrt[4]{n}+\sqrt[4]{n+1})}$$
$$\left(\frac{\sqrt[4]{n}-\sqrt[4]{n+1}}{\sqrt[4]{n}-\sqrt[4]{n+1}}\right)$$
$$= \sum_{n=1}^{9999} \left((n+1)^{1/4} - n^{1/4}\right)$$
$$= \left(\begin{matrix}\left(2^{\frac{1}{4}}-1\right)+\left(3^{\frac{1}{4}}-2^{\frac{1}{4}}\right)+\left(4^{\frac{1}{4}}-3^{\frac{1}{4}}\right)+\cdots+ \\ \left((9999+1)^{\frac{1}{4}}-(9999)^{\frac{1}{4}}\right)\end{matrix}\right)$$
$$= (10^4)^{\frac{1}{4}} - 1 = 9$$

209 **(3)**

Let $a, ar, ar^2, ar^3, \ldots$ are in G.P.

Now $ar^4 = 7!$ and $ar^7 = 8!$

$\therefore$ On dividing, we get $r^3 = 8 \Rightarrow r = 2$

Hence, $a.2^4 = 5040$

$$\therefore a = \frac{5040}{16} = 315$$

So 315, 630, 1260, … are in G.P.

$\therefore S_3 = 2205 \Rightarrow n = 3$

210 **(7)**

$ax^2 + (a+d)x + (a+2d) = 0$

$a, a+d, a+2d$ are in increasing A.P. $(d > 0)$

For real roots $D \geq 0$

$\Rightarrow (a+d)^2 - 4a(a+2d) \geq 0$

$\Rightarrow a^2 - 3a^2 - 6ad \geq 0$

$\Rightarrow (d-3a)^2 - 12a^2 \geq 0$

$\Rightarrow (d-3a)^2 - 12a^2 \geq 0$

$\Rightarrow \left(d - 3a - \sqrt{12}a\right)\left(d - 3a + \sqrt{12}a\right) \geq 0$

$$\Rightarrow \left[\frac{d}{a} - \left(3+2\sqrt{3}\right)\right]\left[\frac{d}{a} - \left(3-2\sqrt{3}\right)\right] \geq 0$$

$$\therefore \left.\frac{d}{a}\right|_{\text{Min}} = 3 + 2\sqrt{3}$$

$\Rightarrow$ least integral value = 7

211 **(1)**

Let a be the first term r be the common ratio of G.P.

$$\therefore \frac{a(1-r^{201})}{1-r} = 625$$

Now $\sum_{i=1}^{201} \frac{1}{a_i} = \left(\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_{201}}\right)$

$$= \frac{1}{a} + \frac{1}{ar} + \cdots + \frac{1}{ar^{200}}$$

$$= \frac{\frac{1}{a}\left(\left(\frac{1}{r}\right)^{201} - 1\right)}{\left(\frac{1}{r} - 1\right)}$$

$$= \frac{1}{a}\left(\frac{1-r^{201}}{1-r}\right)\frac{1}{r^{200}}$$

$= \frac{1}{a} \times \frac{625}{a} \times \frac{1}{r^{200}}$ [from (1)]

$$= \frac{625}{(ar^{100})^2} = \frac{625}{(a_{101})^2} = \frac{625}{625} = 1$$

212 **(0)**

$10x^2 - nx^2 - 54x - 27 = 0$ has roots in H.P.

Put $x = 1/t$

$27t^2 + 54t^2 + nt - 10 = 0$

This equation ahs roots in A.P., let the roots are $a-d, a$ and $a+d$

$$\therefore 3a = -\frac{54}{27} \Rightarrow a = -\frac{2}{3}$$

Also $(a-d)a(a+d) = \frac{10}{27}$

$$\therefore \frac{2}{3}\left(\frac{4}{9} - d^2\right) = -\frac{10}{27} \Rightarrow \left(\frac{4}{9} - d^2\right) = -\frac{5}{9}$$

$\therefore d^2 = 1 \Rightarrow d = \pm 1$

For $d = 1$, roots are $-\frac{2}{3}+1, -\frac{2}{3}, -\frac{2}{3}-1 \Rightarrow \frac{1}{3}, -\frac{2}{3}, -\frac{5}{3}$

For $d = -\frac{2}{3}, -1, -\frac{2}{3}, -\frac{2}{3}+1 \Rightarrow -\frac{5}{3}, -\frac{2}{3}, \frac{1}{3}$

$$\therefore \frac{n}{27} = \frac{10}{9} - \frac{5}{9} - \frac{2}{9} \Rightarrow \frac{n}{27} = \frac{3}{9}$$

$\Rightarrow n = 9$

213 **(1)**

Let $a_1 = a - d;\ a_2 = a; a_3 = a + d$

$\therefore 3a = 18 \Rightarrow a = 6$

Hence, the number in A.P.

$6-d, d, 6+d$

$a_1 + 1, a_2, a_3 + 2$ in G.P.

i.e. $7-, 68+d$ in G.P>

$\therefore 36 = (7-d)(8+d)$

$36 = 56 - d - d^2$

$d^2 + d - 20 = 0$

Hence, the sum of all possible common different is -1

214 **(9)**

$$\left[\frac{k(k+1)}{2}\right]^2 - \frac{k(k+1)}{2} = 1980$$

$$\Rightarrow \frac{k(k+1)}{2}\left[\frac{k(k+1)}{2} - 1\right] = 1980$$

$\Rightarrow k(k+1)(k^2+k-2) = 1980 \times 4$

$\Rightarrow (k-1)k(k+1)(k+2) = 8.9.10.11$

$\therefore k - 1 = 8 \Rightarrow k = 9$

215 **(7)**

$6, a, b$ in H.P>

$\Rightarrow \frac{1}{6}, \frac{1}{a}, \frac{1}{b}$ are in A.P.

$$\Rightarrow \frac{2}{a} = \frac{1}{6} + \frac{1}{b}$$

$$\Rightarrow \frac{1}{b} = \frac{2}{a} - \frac{1}{6}$$

$$\Rightarrow \frac{1}{b} = \frac{12-a}{6a}$$

$\Rightarrow b = \dfrac{6a}{12-a}$

$a \in \{3, 4, 6, 8, 9, 10, 11\}$

216 **(8)**

For the G.P. $a, ar, ar^2, \ldots$

$P_n = a(ar)(ar^2)\ldots(ar^{n-1}) = a^n . r^{n(n-1)/2}$

$\therefore S = \sum_{n=1}^{\infty} \sqrt[n]{P_n} = \sum_{n=1}^{\infty} ar^{(n-1)/2}$

Now, $\sum_{n=1}^{\infty} ar^{(n-1)/2} = a[1 + \sqrt{r} + r +$

$rr+...+\infty=a1-r$

Given $a = 16$ and $r = 1/4$

$\therefore S = \dfrac{16}{1-(1/2)} = 32$

217 **(6)**

Let $\frac{\alpha}{r}, \alpha, \alpha r$ be the roots

$\therefore \ \alpha^3 = -216$

Again $\frac{\alpha^2}{r} + \alpha^2 r + \alpha^2 = b$

$\alpha^2\left(1 + r + \frac{1}{r}\right) = b \quad (2)$

And $\left(1 + r + \frac{1}{r}\right) = -a \quad (3)$

On dividing (2) by (3), we get

$\Rightarrow \alpha = -\dfrac{b}{a}$

$\Rightarrow \ \alpha^3 = -\frac{b^3}{a^3} \quad (4)$

From (1) and (4), $\left(\frac{b}{a}\right)^3 = 216$

$\Rightarrow \dfrac{b}{a} = 6$

218 **(0)**

a, b, c are in A.P> $\Rightarrow b = \frac{a+c}{2} \quad (1)$

b, c, d are in G.P. $\Rightarrow c^2 = bd \quad (2)$

And c, d, e are in H.P. $\Rightarrow d = \frac{2ce}{c+e} \quad (3)$

Now $c^2 = bd \ \Rightarrow c^2 = \left(\frac{a+c}{2}\right)\left(\frac{2ce}{c+e}\right)$ [using (1) and (3]

$\therefore c^2 + ce = ae + ce$

$\Rightarrow c^2 = ae$

Now given $a = 2$ and $e = 18$

$\therefore c^2 = ae \ \Rightarrow c^2 = 2 \times 18 = 36 \ \Rightarrow c = 6$ or -6

219 **(1)**

$\frac{a}{1-r_1} = r_1$ and $\frac{a}{1-r_2} = r_2$

Hence, r_1 and r_2 are the roots of $\frac{a}{1-r} = r$

$\Rightarrow r^2 - r + a = 0$

$\Rightarrow r_1 + r_2 = 1$

220 **(6)**

10 for the given A.P., we have $2(2a + b) = (5a - b) + (a + 2b)$

$\Rightarrow b = 2a \quad$ (i)

Also for the given G.P. , we have $(ab+1)^2 = (a-1)^2(b+1)^2 \quad$ (ii)

$\therefore$ Putting $b = 2a$ from (i) in (ii), we get $a = 0, -2$ or $\frac{1}{4}$

But $a > 0$, so $a = \frac{1}{4}$ and $b = 2a = \frac{1}{2}$

Hence, $(a^{-1} + b^{-1}) = 2 + 4 = 6$

221 **(3)**

$369 = \dfrac{9}{2}[2 + (9-1)d]$

$\Rightarrow 82 = 2 + 8d$

$\Rightarrow d = 10$

Now $ar^8 = a + 8d = 1 + 8 \times 80 = 81$

$\Rightarrow r^8 = 81$

$\Rightarrow r = \sqrt{3}$

$\Rightarrow ar^{(7-1)} = 1 \times \left(\sqrt{3}\right)^6 = 27$

222 **(6)**

We have $S = 3 + \sum_{n=1}^{\infty} \frac{2n+3}{3^n} = 3 + \underbrace{\sum_{n=1}^{\infty} \frac{1}{3^{n-1}}}_{S_1} + \underbrace{\sum_{n=1}^{\infty}}_{S_2} \frac{2n}{3^n}$

Now $S_1 = \sum_{n=1}^{\infty} \frac{1}{3^{n-1}} = 1 + \frac{1}{3} + \frac{1}{3^2} + \cdots \infty$

$\therefore \ S_1 = \dfrac{1}{1-\left(\frac{1}{3}\right)} = \dfrac{3}{2}$

$S_2 = \sum_{n=1}^{\infty} \dfrac{2n}{3^n} = \dfrac{2}{3} + \dfrac{4}{3^2} + \dfrac{6}{3^3} + \dfrac{8}{3^4} + \cdots \infty$

$S_2 = \dfrac{2}{3} + \dfrac{4}{3^2} + \dfrac{6}{3^3} + \dfrac{8}{3^4} + \cdots \infty$

Now, $\frac{S_2}{3} = +\frac{2}{3} + \frac{4}{3^2} + \frac{6}{3^3} + \cdots \infty$

$\frac{2S_2}{3} = \frac{2}{3}\left[1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots \infty\right]$ [On subtracting]

$\therefore S_2 = \dfrac{1}{1-\left(\frac{1}{3}\right)} = \dfrac{3}{2}$

Hence, $S = 3 + \frac{3}{2} + \frac{3}{2} = 6$