2.RELATIONS AND FUNCTIONS

Single Correct Answer Type

1. Let $f: [-10, 10] \to R$, where $f(x) = \sin x + [x^2/a]$ be an odd function. Then the set of values of parameter a is/are
a) $(-10, 10) \sim \{0\}$ b) $(0, 10)$ c) $[100, \infty)$ d) $(100, \infty)$

2. If the graph of the function $f(x) = \frac{a^x - 1}{x^n(a^x + 1)}$ is symmetrical about y-axis, then n equals
a) 2 b) $\frac{2}{3}$ c) $\frac{1}{4}$ d) $-\frac{1}{3}$

3. The domain of the function $f(x) = \sqrt{\log\left(\frac{1}{|sin\, x|}\right)}$ is
a) $R - \{-\pi, \pi\}$ b) $R - \{n\pi | n \in Z\}$ c) $R - \{2n\pi | n \in z\}$ d) $(-\infty, \infty)$

4. Let $f(x) = \frac{\alpha x}{x+1}, x \neq -1$. Then for what value of α is $f(f(x)) = x$?
a) $\sqrt{2}$ b) $-\sqrt{2}$ c) 1 d) -1

5. If f: $[1, \infty) \to [2, \infty)$ is given by $f(x) = x + \frac{1}{x}$, then $f^{-1}(x)$ equals
a) $\frac{(x + \sqrt{x^2 - 4})}{2}$ b) $\frac{x}{1 + x^2}$ c) $\frac{(x - \sqrt{x^2 - 4})}{2}$ d) $1 + \sqrt{x^2 - 4}$

6. The domain of $f(x) = \sin^{-1}[2x^2 - 3]$, where [.] denotes the greatest integer function, is
a) $\left(-\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}}\right)$ b) $\left(-\sqrt{\frac{3}{2}}, -1\right] \cup \left(-\sqrt{\frac{5}{2}}, \sqrt{\frac{5}{2}}\right)$
c) $\left(-\sqrt{\frac{5}{2}}, \sqrt{\frac{5}{2}}\right)$ d) $\left(-\sqrt{\frac{5}{2}}, -1\right] \cup \left[1, \sqrt{\frac{5}{2}}\right)$

7. Domain of definition of the function $f(x) = \sqrt{\sin^{-}(2x) + \frac{\pi}{6}}$ for real valued x, is
a) $\left[-\frac{1}{4}, \frac{1}{2}\right]$ b) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ c) $\left(-\frac{1}{2}, \frac{1}{9}\right)$ d) $\left[-\frac{1}{4}, \frac{1}{4}\right]$

8. Let $f: R \to R$, g: $R \to R$ be two given functions such that f is injective and g is surjective, then which of the following is injective?
a) $g_0 f$ b) $f_0 g$ c) $g_0 g$ d) None of these

9. Let $X = \{a_1, a_2, \dots, a_6\}$ and $Y = \{b_1, b_2, b_3\}$. The number of functions f from x to y such that it is onto and there are exactly three elements x in X such that $f(x) = b_1$ is
a) 75 b) 90 c) 100 d) 120

10. $f: N \to N$ where $f(x) = x - (-1)^x$ then f is
a) One-one and into b) Many-one and into c) One-one and onto d) Many-one and onto

11. If f is a function such that $f(0) = 2, f(1) = 3$ and $f(x + 2) = 2f(x) - f(x + 1)$ for every real x, then $f(5)$ is
a) 7 b) 13 c) 1 d) 5

12. The domain of the function $f(x) = \log_2\left(-\log_{1/2}\left(1 + \frac{1}{x^{1/4}}\right) - 1\right)$ is
a) $(0, 1)$ b) $(0, 1]$ c) $[1, \infty)$ d) $(1, \infty)$

13. The domain of $f(x) = \sqrt{2\{x\}^2 - 3\{x\} + 1}$, where {.} denotes the fractional part in $[-1, 1]$, is
a) $[-1, 1] \sim \left(\frac{1}{2}, 1\right)$ b) $\left[-1, -\frac{1}{2}\right] \cup \left[0, \frac{1}{2}\right] \cup \{1\}$
c) $\left[-1, \frac{1}{2}\right]$ d) $\left[-\frac{1}{2}, 1\right]$

14. Let $f: R \to R$ and g: $R \to R$ be two one-one and onto functions such that they are the mirror images if each other about the line $y = a$. If $h(x) = f(x) + g(x)$, then $h(x)$ is

a) One-one and onto.
b) Only one-one and not onto.
c) Only onto but not one-one.
d) Neither one-one nor onto.

15. The range of $f(x) = \sec^{-1}(\log_3 \tan x + \log_{\tan x} 3)$ is
a) $\left[\frac{\pi}{3}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{2\pi}{3}\right]$
b) $\left[0, \frac{\pi}{2}\right)$
c) $\left(\frac{2\pi}{3}, \pi\right]$
d) None of these

16. The domain of definition of the function $f(x)$ given by the equation $2^x + 2^y = 2$ is
a) $0 < x \le 1$
b) $0 \le x \le 1$
c) $-\infty < x \le 0$
d) $-\infty < x < 1$

17. The domain of $f(x)$ is $(0, 1)$, then, domain of $f(e^x) + f(\ln|x|)$ is
a) $(-1, e)$
b) $(1, e)$
c) $(-e, -1)$
d) $(-e, 1)$

18. If $f(x) = \begin{cases} x^2, & \text{for } x \ge 0 \\ x, & \text{for } x < 0 \end{cases}$ then $fof(x)$ is given by
a) x^2 for $x \ge 0$, x for $x < 0$
b) x^4 for $x \ge 0$, x^2 for $x < 0$
c) x^4 for $x \ge 0$, $-x^2$ for $x < 0$
d) x^4 for $x \ge 0$, x for $x < 0$

19. The function $f(x) = \frac{\sec^{-1} x}{\sqrt{x - [x]}}$, where $[x]$ denotes the greatest integer less than or equal to x, is defined for all $x \in$
a) R
b) $R - \{(-1, 1) \cup \{n | n \in Z\}\}$
c) $R^+ - (0, 1)$
d) $R^+ - \{n | n \in N\}$

20. Let $f: \left[-\frac{\pi}{3}, \frac{2\pi}{3}\right] \to [0, 4]$ be a function defined as $f(x) = \sqrt{3} \sin x - \cos x + 2$. Then $f^{-1}(x)$ is given by
a) $\sin^{-1}\left(\frac{x-2}{2}\right) - \frac{ð}{6}$
b) $\sin^{-1}\left(\frac{x-2}{2}\right) + \frac{\pi}{6}$
c) $\frac{2\pi}{3} + \cos^{-1}\left(\frac{x-2}{2}\right)$
d) None of these

21. Let $f: N \to N$ defined by $f(x) = x^2 + x + 1, x \in N$, then f is
a) One-one onto
b) Many-one onto
c) One-one but not onto
d) None of these

22. If $f(3x + 2) + f(3x + 29) = 0 \ \forall \ x \in R$, then the period of $f(x)$ is
a) 7
b) 8
c) 10
d) None of these

23. Which of the following functions is periodic?
a) $f(x) = x - [x]$ where $[x]$ denotes the largest integer less than or equal to the real number x
b) $f(x) = \sin \frac{1}{x}$ for $x \ne 0, f(0) = 0$
c) $f(x) = x \cos x$
d) None of these

24. If $f(x) = \begin{cases} x^2 \sin \frac{\pi x}{2}, & |x| < 1 \\ x|x|, & |x| \ge 1 \end{cases}$ then $f(x)$ is
a) An even function
b) An odd function
c) A periodic function
d) None of these

25. The domain of the function $f(x) = \frac{\sin^{-1}(3-x)}{\text{In}(|x|-2)}$ Is
a) $[2, 4]$
b) $(2, 3) \cup (3, 4]$
c) $[2, \infty)$
d) $(-\infty, -3) \cup [2, \infty)$

26. If $f(x)$ is an even function and satisfies the relation $x^2 f(x) - 2f\left(\frac{1}{x}\right) = \text{g}(x)$ where $\text{g}(x)$ is an odd function, then $f(5)$ equals
a) 0
b) $\frac{50}{75}$
c) $\frac{49}{75}$
d) None of these

27. If $f(x) = \frac{\sin([x]\pi)}{x^2 + x + 1}$, where [.] denotes the greatest integer function, then
a) f is one-one
b) f is not one-one and non-constant
c) f is a constant function
d) None of these

28. If x satisfies $|x - 1| + |x - 2| + |x - 3| \ge 6$, then
a) $0 \le x \le 4$
b) $x \le -2$ or $x \ge 4$
c) $x \le 0$ or $x \ge 4$
d) None of these

29. The period of function $2^{\{x\}} + \sin \pi x + 3^{\{x/2\}} + \cos 2\pi x$ (where $\{x\}$ denotes the fractional part of x) is

a) 2 b) 1 c) 3 d) None of these

30. If $f(x) = (-1)^{\left[\frac{2x}{\pi}\right]}$, $g(x) = |\sin x| - |\cos x|$ and $\emptyset(x) = f(x)g(x)$ (where [.] denotes the greatest integer function) then the respective fundamental periods of $f(x)$, $g(x)$ and $f(x)$, $g(x)$and $\emptyset(x)$ are

a) π, π, π b) $\pi, 2\pi, \pi$ c) $\pi, \pi, \frac{\pi}{2}$ d) $\pi, \frac{\pi}{2}, \pi$

31. The range of $\sin^{-1}\left[x^2 + \frac{1}{2}\right] + \cos^{-1}\left[x^2 - \frac{1}{2}\right]$, where [.] denotes the greatest integer function, is

a) $\left\{\frac{\pi}{2}, \pi\right\}$ b) $\{\pi\}$ c) $\left\{\frac{\pi}{2}\right\}$ d) None of these

32. If $f(x)$ and $g(x)$ are periodic functions with period 7 and 11, respectively. Then the period of $F(x) = f(x)g\left(\frac{x}{5}\right) - g(x)f\left(\frac{x}{3}\right)$ is

a) 177 b) 222 c) 433 d) 1155

33. The exhaustive domain of $f(x) = \sqrt{x^{12} - x^9 + x^4 - x + 1}$ is

a) $[0, 1]$ b) $[1, \infty)$ c) $(-\infty, 1]$ d) R

34. Let $E = \{1, 2, 3, 4\}$ and $F = \{1, 2\}$. Then the number of onto functions from E to F is

a) 14 b) 16 c) 12 d) 8

35. If $f(x) = \sin x + \cos x$, $g(x) = x^2 - 1$, then $g(f(x))$ is invertible in the domain

a) $\left[0, \frac{\pi}{2}\right]$ b) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ d) $[0, \pi]$

36. The range of the function $f(x) = \frac{e^x - e^{|x|}}{e^x + e^{|x|}}$

a) $(-\infty, \infty)$ b) $[0,1)$ c) $(-1, 0]$ d) $(-1, 1)$

37. Let f be a function satisfying$g$of x then $f(xy) = \frac{f(x)}{y}$ for all positive real numbers x and y if $f(30) = 20$, then the value of $f(40)$ is

a) 15 b) 20 c) 40 d) 60

38. The range of $f(x) = \sin^{-1}\left(\frac{x^2+1}{x^2+2}\right)$ Is

a) $[0, \pi/2]$ b) $(0, \pi/6)$ c) $[\pi/6, \pi/2]$ d) None of these

39. Domain (D) and range (R) of $f(x) = \sin^{-1}(\cos^{-1}[x])$ where [.] denotes the greatest integer function is

a) $D \equiv x \in [1, 2), R \in \{0\}$

b) $D \equiv x \in [0, 1], R \equiv \{-1, 0, 1\}$

c) $D \equiv x \in [-1, 1], R \equiv \left\{0, \sin^{-1}\left(\frac{\pi}{2}\right), \sin^{-1}(\pi)\right\}$

d) $D \equiv x \in [-1, 1], R \equiv \left\{-\frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$

40. The range of the function $f(x) = {}^{7-x}P_{x-3}$ is

a) {1, 2, 3} b) {1, 2, 3, 4, 5, 6} c) {1, 2, 3, 4} d) {1, 2, 3, 4, 5}

41. If x is real, then the value of the expression $\frac{x^2+14x+9}{x^2+2x+3}$ lies between

a) 5 and 4 b) 5 and -4 c) -5 and 4 d) None of these

42. The domain of the function $f(x) = \frac{x}{\sqrt{\sin(\ln x) - \cos(\ln x)}}$ $(n \in Z)$ Is

a) $(e^{2n\pi}, e^{(3n+1/2)\pi})$ b) $\left(e^{(2n+1/4)\pi}, e^{(2n+5/4)\pi}\right)$

c) $\left(e^{(2n+1/4)\pi}, e^{(3n-3/4)\pi}\right)$ d) None of these

43. If $af(x+1) + bf\left(\frac{1}{x+1}\right) = x, x \neq -1, a \neq b$, then $f(2)$ is equal to

a) $\frac{2a+b}{2(a^2-b^2)}$ b) $\frac{a}{a^2-b^2}$ c) $\frac{a+2b}{a^2-b^2}$ d) None of these

44. The range of $f(x) = [|\sin x| + |\cos x|]$, where [.] denotes the greatest integer function, is

a) {0} b) {0, 1} c) {1} d) None of these

45. The number of roots of the equation $x \sin x = 1, x \in [-2\pi, 0) \cup (0, 2\pi]$, is

a) 2 b) 3 c) 4 d) 0

46. If $f(2x + 3y, 2x - 7y) = 20x$, then $f(x, y)$ equals

a) $7x-3y$ b) $7x+3y$ c) $3x-7y$ d) $x-ky$

47. The range of $f(x)=\sin^{-1}(\sqrt{x^2+x+1})$ is

a) $\left(0,\frac{\pi}{2}\right]$ b) $\left(0,\frac{\pi}{3}\right]$ c) $\left[\frac{\pi}{3},\frac{\pi}{2}\right]$ d) $\left[\frac{\pi}{6},\frac{\pi}{3}\right]$

48. The function $f(x)=\sin\left(\log\left(x+\sqrt{1+x^2}\right)\right)$ is

a) Even function b) Odd function c) Neither even nor odd d) Periodic function

49. Let $f(x)=\begin{cases}\sin x+\cos x, & 0<x<\frac{\pi}{2}\\ a, & x=\pi/2\\ \tan^2 x+\operatorname{cosec} x, & \pi/2<x<\pi\end{cases}$ then its odd extension is

a) $\begin{cases}-\tan^2 x-\operatorname{cosec} x, & -\pi<x<-\frac{\pi}{2}\\ -a, & x=-\frac{\pi}{2}\\ -\sin x+\cos x, & -\frac{\pi}{2}<x<0\end{cases}$ b) $\begin{cases}-\tan^2 x+\operatorname{cosec} x, & -\pi<x<-\frac{\pi}{2}\\ -a, & x=-\frac{\pi}{2}\\ \sin x-\cos x, & -\frac{\pi}{2}<x<0\end{cases}$

c) $\begin{cases}-\tan^2 x+\operatorname{cosec} x, & -\pi<x<-\frac{\pi}{2}\\ a, & x=-\frac{\pi}{2}\\ \sin x-\cos x, & -\frac{\pi}{2}<x<0\end{cases}$ d) $\begin{cases}\tan^2 x+\cos x, & -\pi<x<-\frac{\pi}{2}\\ -a, & x=-\frac{\pi}{2}\\ \sin x+\cos x, & -\frac{\pi}{2}<x<0\end{cases}$

50. Let $f(x)=(x+1)^2-1, x\geq 1$, Then the set $\{x: f(x)=f^{-1}(x)\}$ is

a) $\left\{0,-1,\frac{-3+i\sqrt{3}}{2},\frac{-3-i\sqrt{3}}{2}\right\}$ b) $\{0,1,-1\}$

c) $\{0,-1\}$ d) empty

51. The domain of the function $f(x)=\sqrt{x^2-[x]^2}$, where $[x]=$ the greatest integer less than or equal to x, is

a) R b) $[0,+\infty)$ c) $(-\infty,0]$ d) None of these

52. If the period of $\frac{\cos(\sin(nx))}{\tan(x/n)}$, $n\in N$, is 6π, then n is equal to

a) 3 b) 2 c) 6 d) 1

53. Let R be the set o real numbers. If $R\to R$ is a function defined by $f(x)=x^2$, then f is

a) Injective but not surjective b) Surjective but not injective

c) Bijective d) Nine of these

54. The range of $f(x)=\left[\sin x+\left[\cos x+\left[\tan x+[\sec x]\right]\right]\right]$, $x\in(0,\pi/4)$, where [.] denotes the greatest integer function $\leq x$, is

a) {0, 1} b) {−1, 0, 1} c) {1} d) None of these

55. Let $f: R\to\left[0,\frac{\pi}{2}\right)$ defined by $f(x)=\tan^{-1}(x^2+x+a)$, then the set of values of a for which f is onto is

a) $[0,\infty)$ b) $[2,1]$ c) $\left[\frac{1}{4},\infty\right)$ d) None of these

56. The domain of the function $f(x)=\frac{1}{\sqrt{4x-|x^2-10x+9|}}$ is

a) $(7-\sqrt{40},7+\sqrt{40})$ b) $(0,7+\sqrt{40})$ c) $(7-\sqrt{40},\infty)$ d) None of these

57. Range of the function $f(x)=\frac{x^2+x+2}{x^2+x+1}$; $x\in R$ is

a) $(1,\infty)$ b) $(1,11/7)$ c) $[1,7/3]$ d) $(1,7/5)$

58. The function $f: R\to R$ is defined by $f(x)=\cos^2 x+\sin^4 x$ for $x\in R$, then the range of $f(x)$ is

a) $\left(\frac{3}{4},1\right]$ b) $\left[\frac{3}{4},1\right)$ c) $\left[\frac{3}{4},1\right]$ d) $\left(\frac{3}{4},1\right)$

59. $f(x)=\frac{\cos x}{\left[\frac{2x}{\pi}\right]+\frac{1}{2}}$, where x is not an integral multiple of π and [.] denotes the greatest integer function is

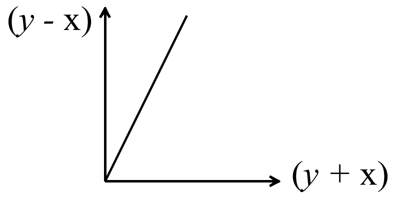
a) An odd function b) Even function c) Neither odd nor even d) None of these

60. The number of solutions of the equation $[y+[y]]=2\cos x$, where $y=\frac{1}{3}\left[\sin x+\left[\sin x+[\sin x]\right]\right]$ (where

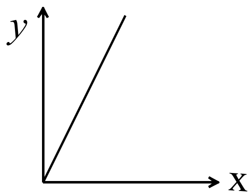
[.] denotes the greatest integer function) is

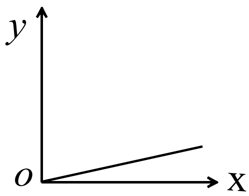
a) 4 b) 2 c) 3 d) 53

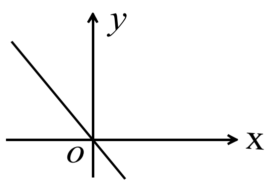
61. The graph of $(y-x)$ against $(y+x)$ is shown

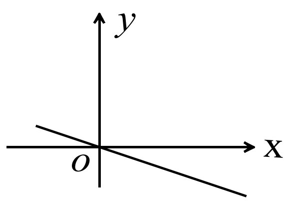


Which one of the following shows the graph of y against x?

a) 

b) 

c) 

d) 

62. Let $g(x)=1+x-[x]$ and $f(x)=\begin{cases}-1, & x<0\\ 0, & x=0\\ 1, & x>0\end{cases}$ Then for all x, $f(g(x))$ is equal to (where [.] represents greatest integer function)

a) x b) 1 c) $f(x)$ d) $g(x)$

63. Let $f(x)=x+2|x+1|+2|x-1|$. If $f(x)=k$ has exactly one real solution, then the value of k is

a) 3 b) 0 c) 1 d) 2

64. If $f(x)=ax^7+bx^3+cx-5$, a, b, c are real constants and $f(-7)=7$, then the range of $f(7)+17\cos x$ is

a) $[-34, 0]$ b) $[0, 34]$ c) $[-34, 34]$ d) None of these

65. If $[\cos^{-1} x]+[\cos^{-1} x]=0$, where [.] denotes the greatest integer function, then the complete set of values of x is

a) $(\cos 1, 1]$ b) $(\cos 1, \cot 1)$ c) $(\cot 1, 1]$ d) $[0, \cot 1)$

66. The range of $f(x)=\sqrt{(1-\cos x)\sqrt{(1-\cos x)\sqrt{(1-\cos x)\sqrt{\ldots\infty}}}}$ is

a) $[0, 1]$ b) $[0, 1/2]$ c) $[0, 2]$ d) None of these

67. The values of b and c for which the identity $f(x+1)-f(x)=8x+3$ is satisfied, where $f(x)=bx^2+cx+d$, are

a) $b=2, c=1$ b) $b=4, c=-1$ c) $b=-1, c=4$ d) $b=-1, c=1$

68. If the graph of $y=f(x)$ is symmetrical about lines $x=1$ and $x=2$, then which of the following is true?

a) $f(x+1)=f(x)$ b) $f(x+3)=f(x)$ c) $f(x+2)=f(x)$ d) None of these

69. If the function $f:[1,\infty)\to:[1,\infty)$ is defined by $f(x)=2^{x(x-1)}$, then $f^{-1}(x)$ is

a) $\left(\frac{1}{2}\right)^{x(x-1)}$ b) $\frac{1}{2}\left(1+\sqrt{1+4\log_2 x}\right)$

c) $\frac{1}{2}\left(1-\sqrt{1+4\log_2 x}\right)$ d) Not defined

70. Which of the following functions is inverse to itself ?

a) $f(x)=\frac{1-x}{1+x}$ b) $f(x)=5^{\log x}$ c) $f(x)=2^{x(x-1)}$ d) None of these

71. The domain of the function $f(x)=\log_{3+x}(x^2-1)$ is

a) $(-3,-1)\cup(1,\infty)$ b) $[-3,-1)\cup[1,\infty)$

c) $(-3,-2) \cup (-2,-1) \cup (1,\infty)$ d) $[-3,-2) \cup (-2,-1) \cup [1,\infty)$

72. If $f: X \to Y$, where X and Y are sets containing natural numbers, $f(x) = \frac{x+5}{x+2}$ then the number of elements in the domain and range of $f(x)$ are respectively

a) 1 and 1 b) 2 and 1 c) 2 and 2 d) 1 and 2

73. Given the function $f(x) = \frac{a^x + a^{-x}}{2}$ (where $a > 2$). Then $f(x+y) + f(x-y) =$

a) $2f(x).f(y)$ b) $f(x).f(y)$ c) $\frac{f(x)}{f(y)}$ d) None of these

74. Let $f(n) = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$, then $f(1) + f(2) + f(3) + \dots + f(n)$ is equal to

a) $nf(n) - 1$ b) $(n+1)f(n) - n$ c) $(n+1)f(n) + n$ d) $nf(n) + n$

75. The domain of $f(x) = \cos^{-1}\left(\frac{2-|x|}{4}\right) + [\log(3-x)]^{-1}$ Is

a) $[-2, 6]$ b) $[-6, 2) \cup (2, 3)$ c) $[-6, 2]$ d) $[-2, 2] \cup (2, 3)$

76. If the function $f: [1,\infty) \to [1,\infty)$ is defined by $f(x) = 2^{x(x-1)}$, then $f^{-1}(x)$ Is

a) $\left(\frac{1}{2}\right)^{x(x-1)}$ b) $\frac{1}{2}\left(1 + \sqrt{1 + 4\log_2 x}\right)$

c) $\frac{1}{2}\left(1 - \sqrt{1 + 4\log_2 x}\right)$ d) Not defined

77. If $f\left(x + \frac{1}{2}\right) + f\left(x - \frac{1}{2}\right) = f(x)$ for all $x \in R$, then the period of $f(x)$ is

a) 1 b) 2 c) 3 d) 4

78. The domain of the function $f(x) = \frac{1}{\sqrt{{}^{10}C_{x-1} - 3 \times {}^{10}C_x}}$ contains the points

a) 9, 10, 11 b) 9, 10, 12 c) All natural numbers d) None of these

79. The function $f: N \to N$ (N is the set of natural numbers) defined by $f(n) = 2n + 3$ is

a) Surjective only b) Injective only c) Bijective d) None of these

80. If $f: R^+ \to R, f(x) + 3xf\left(\frac{1}{x}\right) = 2(x+1)$, then $f(99)$ is equal to

a) 40 b) 30 c) 50 d) 60

81. If $[x]$ and $\{x\}$ represent the integral and fractional parts of x, respectively, then the value of $\sum_{r=1}^{2000} \frac{\{x+r\}}{2000}$ is

a) x b) $[x]$ c) $\{x\}$ d) $x + 2001$

82. If $f: [0,\infty] \to [0,\infty]$ and $f(x) = \frac{x}{1+x}$, then f is

a) One-one and onto b) One-one but not onto

c) Onto but not one-one d) Neither one-one nor onto

83. If $f(x) = \begin{cases} x, & x \text{ is rational} \\ 1-x, & x \text{ is irrational} \end{cases}$ then $f(f(x))$ is

a) $x \forall x \in R$ b) $= \begin{cases} x, & x \text{ is irrational} \\ 1-x, & x \text{ is rational} \end{cases}$

c) $\begin{cases} x, & x \text{ is rational} \\ 1-x, & x \text{ is irrational} \end{cases}$ d) None of these

84. Let $h(x) = |kx + 5|$, the domain of $f(x)$ is $[-5, 7]$, the domain of $f(h(x))$ is $[-6, 1]$ and the range of $h(x)$ is the same as the domain of $f(x)$, then the value of k is

a) 1 b) 2 c) 3 d) 4

85. $f(x) = \begin{cases} x, \text{if } x \text{ is rational} \\ 0, \text{if } x \text{ is irrational} \end{cases}$ and

$f(x) = \begin{cases} 0, \text{if } x \text{ is rational} \\ x, \text{if } x \text{ is irrational} \end{cases}$. Then, $f - g$ is

a) One-one and into b) Neither one-one nor onto

c) Many one and onto d) One-one and onto

86. The domain of $f(x) = \log|\log x|$ Is

a) $(0,\infty)$ b) $(1,\infty)$ c) $(0,1) \cup (1,\infty)$ d) $(-\infty, 1)$

87. If $f: R \to R$ is an invertible function such that $f(x)$ and $f^{-1}(x)$ are symmetric about the line $y = -x$, then

a) $f(x)$ is odd

b) $f(x)$ and $f^{-1}(x)$ may not be symmetric about the line $y = x$
c) $f(x)$ may not be odd
d) None of these

88. If $f\left(2x + \frac{y}{8}, 2x - \frac{y}{8}\right) = xy$, then $f(m, n) + (n, m) = 0$
a) Only when $m = n$ b) Only when $m \neq n$ c) Only when $m = -n$ d) For all m and n

89. The period of the function $f(x) = c^{\sin^2 x + \sin^2\left(x+\frac{\pi}{3}\right) + \cos x \cos\left(x+\frac{\pi}{3}\right)}$ is (where c is constant)
a) 1 b) $\frac{\pi}{2}$
c) π d) Cannot be determined

90. The range of $f(x) = [1 + \sin x] + \left[2 + \sin\frac{x}{2}\right] + \left[3 + \sin\frac{x}{3}\right] + \cdots + \left[n + \sin\frac{x}{n}\right], \forall\, x \in [0, \pi]$, where [.] denotes the greatest integer function, is
a) $\left\{\frac{n^2+n-2}{2}, \frac{n(n+1)}{2}\right\}$ b) $\left\{\frac{n(n+1)}{2}\right\}$
c) $\left\{\frac{n^2+n-2}{2}, \frac{n(n+1)}{2}, \frac{n^2+n+2}{2}\right\}$ d) $\left\{\frac{n(n+1)}{2}, \frac{n^2+n+2}{2}\right\}$

91. The range of $f(x) = (x+1)(x+2)(x+3)(x+4) + 5$ for $x \in [-6, 6]$ is
a) [4,5045] b) [0, 5045] c) [−20, 5045] d) None of these

92. If $f(x) = \log_e\left(\frac{x^2+e}{x^2+1}\right)$, then the range of $f(x)$ is
a) (0, 1) b) [0, 1] c) [0, 1) d) (0, 1]

93. Let $f(x) = \sqrt{|x| - \{x\}}$ (where {.} denotes the fractional part of x) and X, Y are its domain and range, respectively, then
a) $x \in \left(-\infty, \frac{1}{2}\right)$ and $Y \in \left[\frac{1}{2}, \infty\right)$ b) $x \in \left(-\infty, -\frac{1}{2}\right] \cup [0, \infty)$ and $Y \in \left[\frac{1}{2}, \infty\right)$
c) $X \in \left(-\infty, -\frac{1}{2}\right] \cup [0, \infty)$ and $Y \in [0, \infty)$ d) None of these

94. If X and Y are two non-empty sets where $f: X \to Y$ is function is defined such that
$f(C) = \{f(x): x \in C\}$for $C \subseteq X$
And $f^{-1}(D) = \{x: f(x) \in D\} for\ D \subseteq Y$,
For any $A \subseteq X$ and $B \subseteq Y$, then
a) $f^{-1}(f(A)) = A$ b) $f^{-1}(f(A)) = A$ only if $f(X) = Y$
c) $f(f^{-1}(B)) = B$ only if $B \subseteq f(x)$ d) $f(f^{-1}(B)) = B$

95. Possible values of a such that the equation $x^2 + 2ax + a = \sqrt{a^2 + x - \frac{1}{16}} - \frac{1}{16}, x \geq -a$, has two distinct real roots are given by
a) [0, 1] b) [−∞, 0) c) [0, ∞) d) $\left(\frac{3}{4}, \infty\right)$

96. If $f(x)$ is a polynomial satisfying $f(x)f(1/x) = f(x) + f(1/x)$ and $f(3) = 28$, then $f(4)$ is equal to
a) 63 b) 65 c) 17 d) None of these

97. If $f(x + f(y)) = f(x) + y\ \forall\, x, y \in R$ and $f(0) = 1$, then the value of $f(7)$ is
a) 1 b) 7 c) 6 d) 8

98. If $f(x) = \cos(\log_e x)$, then $f(x)f(y) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(xy)\right]$ has the value
a) -1 b) 1/2 c) -2 d) None of these

99. If f is periodic, g is polynomial function and $f(\mathrm{g}(x))$ is periodic and g(2)=3, g(4)=7 then g(6) is
a) 13 b) 15 c) 11 d) None of these

100. A function $F(x)$ satisfies the functional equation $x^2F(x) + F(1-x) = 2x - x^4$ for all real x. $F(x)$ must be
a) x^2 b) $1 - x^2$ c) $1 + x^2$ d) $x^2 + x + 1$

101. The total number of solutions of $[x]^2 = x + 2\{x\}$, where [.] and {.} denote the greatest integer function and fractional part, respectively, is equal to
a) 2 b) 4 c) 6 d) None of these

102. The domain of definition of the function $y = \frac{1}{\log_{10}(1-x)} + \sqrt{x+2} =$

a) (-3, -2) excluding -2.5 b) [0, 1] excluding 0.5 c) $[-2, 1)$ excluding 0 d) None of these

103. Let $f(x) = \sin x$ and $g(x) = \log_e|x|$. If the ranges of the composition function fog and gof are R_1 and R_2, respectively, then

a) $R_1 = \{u: -1 \le u < 1\}, R_2 = \{v: -\infty < v < 0\}$

b) $R_1 = \{u: -\infty < u < 0\}, R_2 = \{v: -\infty < v < 0\}$

c) $R_1 = \{u: -1 < u < 1\}, R_2 = \{v: -\infty < v < 0\}$

d) $R_1 = \{u: -1 \le u \le 1\}, R_2 = \{v: -\infty < v \le 0\}$

104. Let $f(x) = ([a]^2 - 5[a] + 4)x^3 - (6\{a\}^2 - 5\{a\} + 1)x - (\tan x) \times \text{sgn}\, x$ be an even function for all $x \in R$, then the sum of all possible values of $'a'$ is (where $[\cdot]$ and $\{\cdot\}$ denote greatest integer function and fractional part functions, respectively)

a) $\frac{17}{6}$ b) $\frac{53}{6}$ c) $\frac{31}{3}$ d) $\frac{35}{3}$

105. The range of the function f defined by $f(x) = \left[\frac{1}{\sin\{x\}}\right]$ (where [.] and {.} respectively denote the greatest integer and the fractional part functions) is

a) I, the set of integers b) N, the set of natural numbers

c) W, the set of whole numbers d) {1, 2, 3, 4,...}

106. The period of the function $f(x) = [6x + 7] + \cos \pi x - 6x$, where [.] denotes the greatest integer function, is

a) 3 b) 2π c) 2 d) None of these

107. If $f(x + y) = f(x).f(y)$ for all real x, y and $f(0) \neq 0$, then the function $g(x) = \frac{f(x)}{1+\{f(x)\}^2}$ is

a) Even function b) Odd function c) Odd if $f(x) > 0$ d) Neither even nor odd

108. A real-valued function $f(x)$ satisfies the functional equation $f(x - y) = f(x)f(y) - f(a - x)f(a + y)$, where a is a given constant and $f(0) = 1$. $f(2a - x)$ is equal to

a) $f(x)$ b) $-f(x)$ c) $f(-x)$ d) $f(a) + f(a - x)$

109. If f and g are one-one function, then

a) f +g is one-one b) f g is one-one c) fog is one-one d) None of these

110. Let $f_1(x) \begin{cases} x, 0 \le x \le 1 \\ 1, x > 1 \\ 0, \text{otherwise} \end{cases}$ and $f_2(x) = f_1(-x)$ for all x

$f_3(x) = -f_2(x)$ for all x

$f_4(x) = f_3(-x)$ for all x

Which of the following is necessarily true?

a) $f_4(x) = f_1(x)$ for all x

b) $f_1(x) = -f_3(-x)$ for all x

c) $f_2(-x) = f_4(x)$ for all x

d) $f_1(x) + f_3(x) = 0$ for all x

111. The range of the function $f(x) = |x - 1| + |x - 2|, -1 \le x \le 3$, is

a) [1, 3] b) [1, 5] c) [3, 5] d) None of these

112. Let $f(x) = |x - 1|$. Then

a) $f(x^2) = (f(x))^2$ b) $f(x + y) = f(x) + f(y)$

c) $f(|x|) = |f(x)|$ d) None of these

113. The number of real solutions of the equation $\log_{0.5}|x| = 2|x|$ is

a) 1 b) 2 c) 0 d) None of these

114. The function f satisfies the functional equation $3f(x) + 2f\left(\frac{x+59}{x-1}\right) = 10x + 30$ for all real $x \neq 1$. The value of $f(7)$ is

a) 8 b) 4 c) -8 d) 11

115. Let $g(x) = f(x) - 1$. If $f(x) + f(1 - x) = 2\ \forall\ x \in R$, then $g(x)$ is symmetrical about

a) Origin b) The line $x = \frac{1}{2}$ c) The point $(1, 0)$ d) The point $\left(\frac{1}{2}, 0\right)$

116. The range of $f(x) = \cos^{-1}\left(\frac{1+x^2}{2x}\right) + \sqrt{2 - x^2}$ is

a) $\left\{0, 1 + \frac{\pi}{2}\right\}$ b) $\{0, 1 + \pi\}$ c) $\left\{1, 1 + \frac{\pi}{2}\right\}$ d) $\{1, 1 + \pi\}$

117. If $f(x + 1) + f(x - 1) = 2f(x)$ and $f(0) = 0$, then $f(n), n \in N$, is

a) $nf(1)$ b) $\{f(1)\}^n$ c) 0 d) None of these

118. The domain of the function $f(x) = \frac{1}{\sqrt{\{\sin x\} + \{\sin(\pi + x)\}}}$ where $\{\cdot\}$ denotes the fractional part, is

a) $[0, \pi]$ b) $(2n + 1)\pi/2, n \in Z$ c) $(0, \pi)$ d) None of these

119. The domain of the function $f(x) = \left[\log_{10}\left(\frac{5x - x^2}{4}\right)\right]^{1/2}$ Is

a) $-\infty < x < \infty$ b) $1 \le x \le 4$ c) $4 \le x \le 16$ d) $-1 \le x \le 1$

120. The period of the function $\left|\sin^3 \frac{x}{2}\right| + \left|\cos^5 \frac{x}{5}\right|$ is

a) 2π b) 10π c) 8π d) 5π

121. The entire graph of the equation $y = x^2 + kx - x + 9$ is strictly above the x-axis if and only if

a) $k < 7$ b) $-5 < k < 7$ c) $k > -5$ d) None of these

122. The domain of $f(x) = \frac{1}{\sqrt{|\cos x| + \cos x}}$ is

a) $[-2n\pi, 2n\pi], n \in Z$ b) $(2n\pi, \overline{2n + 1}\pi), n \in Z$

c) $\left(\frac{(4n + 1)\pi}{2}, \frac{(4n + 3)\pi}{2}\right), n \in Z$ d) $\left(\frac{(4n - 1)\pi}{2}, \frac{(4n + 1)\pi}{2}\right), n \in Z$

123. Let $f: X \to y f(x) = \sin x + \cos x + 2\sqrt{2}$ is invertible. Then which $X \to Y$ is not possible?

a) $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right] \to [\sqrt{2}, 3\sqrt{2}]$ b) $\left[-\frac{3\pi}{4}, \frac{\pi}{4}\right] \to [\sqrt{2}, 3\sqrt{2}]$

c) $\left[-\frac{3\pi}{4}, \frac{3\pi}{4}\right] \to [\sqrt{2}, 3\sqrt{2}]$ d) None of these

124. If $f(x) = \frac{1}{x}, g(x) = \frac{1}{x^2}$ and $h(x) = x^2$

a) $fog(x) = x^2 \quad x \ne 0, h(g(x)) = \frac{1}{x^2}$

b) $h(g(x)) = \frac{1}{x^2} \quad x \ne 0, fog(x) = x^2$

c) $fog(x) = x^2, x \ne 0, h(g(x)) = (g(x))^2, x \ne 0$

d) None of these

125. If $f: R \to R$ is a function satisfying the property $f(2x + 3) + f(2x + 7) = 2, \forall\, x \in R$, then the fundamental period of $f(x)$ is

a) 2 b) 4 c) 8 d) 12

126. The domain of $f(x) = \frac{\log_2(x+3)}{x^2+3x+2}$ Is

a) $R - \{-1, -2\}$ b) $(-2, \infty)$ c) $R - \{-1, -2, -3\}$ d) $(-3, \infty) - \{-1, -2\}$

127. The sum of roots of the equation $\cos^{-1}(\cos x) = [x]$, [.] denotes the greatest integer function is

a) $2\pi + 3$ b) $\pi + 3$ c) $\pi - 3$ d) $2\pi - 3$

128. The domain of definition of $f(x) = \frac{\log_2(x+3)}{x^2+3x+2}$ is

a) $R—\{-1, -2\}$ b) $(-2, \infty)$ c) $R—\{-1, -2, -3\}$ d) $(-3, \infty) - \{-1, -2\}$

129. The period of $f(x) = [x] + [2x] + [3x] + [4x] + \cdots [nx] - \frac{n(n+1)}{2}x$, where $n \in N$, is (where $[\cdot]$ represents greatest integer function)

a) n b) 1 c) $\frac{1}{n}$ d) None of these

130. The number of solutions of $\tan x - mx = 0,\ m > 1$ in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is

a) 1 b) 2 c) 3 d) m

131. The domain of $f(x) = \ln(ax^3 + (a + b)x^2 + (b + c)x + c)$, where $a > 0, b^2 - 4ac = 0$, is (where $[\cdot]$

represents greatest integer function).

a) $(-1,\infty)\sim\left\{-\frac{b}{2a}\right\}$ b) $(1,\infty)\sim\left\{-\frac{b}{2a}\right\}$ c) $(-1,1)\sim\left\{-\frac{b}{2a}\right\}$ d) None of these

132. If $f(x) = \text{maximum}\left\{x^3, x^2, \frac{1}{64}\right\} \forall x \in [0,\infty)$, then

a) $f(x) = \begin{cases} x^2, & 0 \le x \le 1 \\ x^3, & x > 1 \end{cases}$

b) $f(x) = \begin{cases} \frac{1}{64}, & 0 \le x \le \frac{1}{4} \\ x^2, & \frac{1}{4} < x \le 1 \\ x^3, & x > 1 \end{cases}$

c) $f(x) = \begin{cases} \frac{1}{64}, & 0 \le x \le \frac{1}{8} \\ x^2, & \frac{1}{8} < x \le 1 \\ x^3, & x > 1 \end{cases}$

d) $f(x) = \begin{cases} \frac{1}{64}, & 0 \le x \le \frac{1}{8} \\ x^3, & x > 1/8 \end{cases}$

133. If the period of $\frac{\cos(\sin(nx))}{\tan\left(\frac{x}{n}\right)}, n \in N$ is 6π then $n =$

a) 3 b) 2 c) 6 d) 1

134. The domain of the function $f(x) = \sqrt{\text{In}_{(|x|-1)}(x^2+4x+4)}$ is

a) $[-3,-1] \cup [1,2]$ b) $(-2,-1) \cup [2,\infty)$
c) $(-\infty,-3] \cup (-2,-1) \cup (2,\infty)$ d) None of these

135. The number of solutions of $2\cos x = |\sin x|, 0 \le x \le 4\pi$, is

a) 0 b) 2 c) 4 d) Infinite

136. Function $f: (-\infty,-1) \to (0, e^5]$ defined by $f(x) = e^{x^3-3x+2}$ is

a) Many-one and onto b) Many-one and into c) One-one and onto d) One-one and into

137. If $F(n+1) = \frac{2F(n)+1}{2} n = 1,2,\dots$ and $F(1) = 2$, then $F(101)$ equals

a) 52 b) 49 c) 48 d) 51

138. The equation $||x-2|+a| = 4$ can have four distinct real solution for x if a belongs to the interval

a) $(-\infty,-4)$ b) $(-\infty,0]$ c) $[4,\infty)$ d) None of these

139. Let $f(x) = e^{\{e^{|x|}\text{sgn } x\}}$ and $g(x) = e^{\{e^{|x|}\text{sgn } x\}}, x \in R$ where {} and [ ] denotes the fractional and integral part functions, respectively. Also $h(x) = \log(f(x)) + \log(g(x))$ then for real $x, h(x)$ is

a) An odd function. b) An even function.
c) Neither an odd nor an even function. d) Both odd as well as even function.

140. If $f(x) = \sqrt[n]{x^m}, n \in N$, is an even function, then m is

a) Even integer b) Odd integer
c) Any integer d) $f(x)$-even is not possible

141. If $g(x) = x^2 + x - 2$ and $\frac{1}{2} gof(x) = 2x^2 - 5x + 2$, then which is not a possible $f(x)$?

a) $2x-3$ b) $-2x+2$ c) $x-3$ d) None of these

142. If $\log_3(x^2-6x+11) \le 1$, then exhaustive range of values of x is

a) $(-\infty,2) \cup (4,\infty)$ b) $(2,4)$
c) $(-\infty,1) \cup (1,3) \cup (4,\infty)$ d) None of these

143. If $f(x+y) = f(x) + f(y) - xy - 1 \;\forall\; x, y \in R$ and $f(1) = 1$, then the number of solutions of $f(n) = n, n \in N$ is

a) 0 b) 1 c) 2 d) More than 2

144. If $g: [-2,2] \to R$ where $f(x) = x^3 + \tan x + \left[\frac{x^2+1}{P}\right]$ is a odd function, then the value of parametric P where

[.] denotes the greatest integer function is

a) $-5 < P < 5$ b) $P < 5$ c) $P > 5$ d) None of these

145. Let S be the set of all triangles and R^+ be the set of positive real numbers. Then the function $f: S \to R^+, f(\Delta) =$ area of Δ, where $\Delta \in S$ is

a) Injective but not surjective b) Surjective but not injective
c) Injective as well as surjective d) Neither injective nor surjective

146. The second degree polynomial $f(x)$, satisfying $f(0) = 0, f(1) = 1, f'(x) > 0$ for all $x \in (0, 1)$

a) $f(x) = \phi$ b) $f(x) = ax + (1-a)x^2; \forall a \in (0, \infty)$
c) $f(x) = ax + (1-a)x^2, a \in (0, 2)$ d) No such polynomial

147. Let $f(x)$ be defined for all $x > 0$ and be continuous. Let $f(x)$ satisfy $f\left(\frac{x}{y}\right) = f(x) - f(y)$ for all x, y and $f(e) = 1$. Then

a) $f(x)$ is bounded b) $f\left(\frac{1}{x}\right) \to 0$ as $x \to 0$ c) $xf(x) \to 1$ as $x \to 0$ d) $f(x) = \log_e x$

Multiple Correct Answers Type

148. Let $f(x) + f(y) = f\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$($f(x)$ is not identically zero). Then

a) $f(4x^3 - 3x) + 3f(x) = 0$ b) $f(4x^3 - 3x) + 3f(x)$
c) $f\left(2x\sqrt{1-x^2}\right) + 2f(x) = 0$ d) $f\left(2x\sqrt{1-x^2}\right) + 2f(x)$

149. If $f(x) = \cos[\pi^2]x + \cos[-\pi^2]x$, where $[x]$ stands for the greatest integer function , then

a) $f\left(\frac{\pi}{2}\right) = -1$ b) $f(\pi) = 1$ c) $f(-\pi) = 0$ d) $f\left(\frac{\pi}{4}\right) = 1$

150. Which of the following pairs of functions is/are identical?

a) $f(x) = \tan(\tan^{-1} x)$ and $g(x) = \cot(\cot^{-1} x)$
b) $f(x) = \text{sgn}(x)$ and $g(x) = \text{sgn}\left(\text{sgn}(x)\right)$
c) $f(x) = \cot^2 x . \cos^2 x$ and $g(x) = \cot^2 x - \cos^2 x$
d) $f(x) = e^{\ln \sec^{-1} x}$ and $g(x) = \sec^{-1} x$

151. Which of the following is/are not a function ([.] and {.} denotes the greatest integer and fractional part functions respectively)?

a) $\frac{1}{\ln[1-|x|]}$ b) $\frac{x!}{\{x\}}$ c) $x!\{x\}$ d) $\frac{\ln(x-1)}{\sqrt{(1-x^2)}}$

152. If $f(x) = 3x - 5$, then $f^{-1}(x)$

a) Is given by $\frac{1}{3x-5}$ b) Is given by $\frac{x+5}{3}$
c) Does not exist because f is not one-one d) Does not exist because f is not onto

153. If $f(x)$ is a polynomial of degree n such that $f(0) = 0, f(1) = \frac{1}{2}, \dots, f(n) = \frac{n}{n+1}$, then the value of $f(n+1)$ is

a) 1 when n is odd b) $\frac{n}{n+2}$ when n is even c) $-\frac{n}{n+1}$ when n is odd d) -1 when n is even

154. If $f: R \to N \cup \{0\}$, where f (area of triangle joining points $P(5, 0), Q(8, 4)$ and $R(x, y)$ such that the angle PRQ is a right)=number of triangle. Then, which of the following is true?

a) $f(5) = 4$ b) $f(7) = 0$ c) $f(6.25) = 2$ d) $f(x)$ is into

155. Let $f: R \to R$ be a function defined by $f(x+1) = \frac{f(x)-5}{f(x)-3} \forall x \in R$. Then which of the following statement(s) is/are true

a) $f(2008) = f(2004)$ b) $f(2006) = f(2010)$ c) $f(2006) = f(2002)$ d) $f(2006) = f(2018)$

156. The domain of the function $f(x) = \log_e \left\{\log_{|\sin x|}(x^2 - 8x + 23) - \frac{3}{\log_2|\sin x|}\right\}$ contains which of the following interval/intervals.

a) $(3, \pi)$ b) $\left(\pi, \frac{3}{2}\right)$ c) $\left(\frac{3\pi}{2}, 5\right)$ d) None of these

157. Let $f(x)=\sec^{-1}[1+\cos^2 x]$ where [.] denotes the greatest integer function. Then

a) The domain of f is R
b) The domain of f is [1, 2]
c) The domain of f is [1, 2]
d) The range of f is $\{\sec^{-1}1, \sec^{-1}2\}$

158. $f(x)=x^2-2ax+a(a+1), f:[a,\infty)\to[a,\infty)$. If one of the solutions of the equation $f(x)=f^{-1}(x)$ is 5049, then the other may be

a) 5051
b) 5048
c) 5052
d) 5050

159. $f: R\to[-1,\infty)$ and $f(x)=\ln([|\sin 2x|+|\cos 2x|])$(where [.] is the greatest integer function).

a) $f(x)$ has range Z
b) $f(x)$ is periodic with fundamental period $\pi/4$
c) $f(x)$ is invertible in $\left[0,\frac{\pi}{4}\right]$
d) $f(x)$ is into function

160. If $f(x)$ satisfies the relation $f(x+y)=f(x)+f(y)$ for all $x,y\in R$ and $f(1)=5$, then

a) $f(x)$ is an odd function
b) $f(x)$ is an even function
c) $\sum_{r=1}^{m} f(r)=5\,{}^{m+1}C_2$
d) $\sum_{r=1}^{m} f(r)=\frac{5m(m+2)}{3}$

161. Let $f(x)=\max\{1+\sin x, 1, 1-\cos x\}, x\in[0,2\pi]$ and $g(x)=\max\{1, |x-1|\} x\in R$, then

a) $g(f(0))=1$
b) $g(f(1))=1$
c) $f(f(1))=1$
d) $f(g(0))=1+\sin 1$

162. If $y=f(x)=\frac{x+2}{x-1}$ then

a) $x=f(y)$
b) $f(1)=3$
c) y increases with x for $x<1$
d) f is a rational function of x

163. If the function f satisfies the relation $f(x+y)+f(x-y)=2f(x)f(y)\forall x,y\in R$ and $f(0)\neq 0$, then

a) $f(x)$ is an even function
b) $f(x)$ is an odd function
c) If $f(2)=a$ then $f(-2)=a$
d) If $f(4)=b$ then $f(-4)=-b$

164. If $g(f(x))=|\sin x|$ and $f(g(x))=\left(\sin\sqrt{x}\right)^2$, then

a) $f(x)=\sin^2 x, g(x)=\sqrt{x}$
b) $f(x)=\sin x, g(x)=|x|$
c) $f(x)=x^2, g(x)=\sin\sqrt{x}$
d) f and g cannot be determined

165. Which of the following function is/are periodic

a) $f(x)=\begin{cases}1, & x \text{ is rational}\\ 0, & x \text{ is irrational}\end{cases}$
b) $f(x)=\begin{cases}x-[x]; & 2n\le x<2n+1\\ \frac{1}{2}; & 2n+1\le x<2n+2\end{cases}$, where [.] denotes the greatest integer function, $n\in Z$
c) $f(x)=(-1)^{\left[\frac{2x}{\pi}\right]}$, where [.] denotes the greatest integer function
d) $f(x)=x-[x+3]+\tan\left(\frac{\pi x}{2}\right)$, where [.] denotes the greatest integer function, and a is a rational number

166. Let $g(x)$ be a function defined on $[-1,1]$. If the area of the equilateral triangle with two of its vertices as $(0,0)$ and $(x, g(x))$ is $\sqrt{3}/4$ then the function $g(x) is$

a) $g(x)=\pm\sqrt{1-x^2}$
b) $g(x)=\sqrt{1-x^2}$
c) $g(x)=-\sqrt{1-x^2}$
d) $g(x)=\sqrt{1+x^2}$

167. Let $f(x)=\frac{3}{4}x+1$, and $f^n(x)$ be defined as $f^2(x)=f(f(x))$, and for $n\ge 2, f^{n+1}(x)=f(f^n(x))$. If $\lambda=\lim_{n\to\infty}f^n(x)$, then

a) λ is independent of x
b) λ is a linear polynomial in x
c) The line $y=\lambda$ has slope 0
d) The line $4y=\lambda$ touches the unit circle with centre at the origin

168. If the following functions are defined from $[-1,1]$ to $[-1,1]$, select those which are not objective

a) $\sin(\sin^{-1}x)$
b) $\frac{2}{\pi}\sin^{-1}(\sin x)$
c) $(\text{sgn}(x))\ln(e^x)$
d) $x^3(\text{sgn}(x))$

169. Consider the real-valued function satisfying $2f(\sin x)+f(\cos x)=x$. then

a) Domain of $f(x)$ is R

b) Domain of $f(x)$ is $[-1, 1]$

c) Range of $f(x)$ is $\left[-\frac{2\pi}{3}, \frac{\pi}{3}\right]$

d) Range of $f(x)$ is R

170. Let $f(x) = \text{sgn}(\cot^{-1} x) + \tan\left(\frac{\pi}{2}[x]\right)$, where $[x]$ is the greatest integer function less than or equal to x. Then which of the following alternatives is/are true?

a) $f(x)$ is many one but not even function

b) $f(x)$ is periodic function

c) $f(x)$ is bounded function

d) Graph of $f(x)$ remains above the x-axis

171. Which of the following function/ functions have the graph symmetrical about the origin?

a) $f(x)$ given by $f(x) + f(y) = f\left(\frac{x+y}{1-xy}\right)$

b) $f(x)$ given by $f(x) + f(y) = f\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$

c) $f(x)$ given by $f(x+y) = f(x) + f(y) \forall x, y \in R$

d) None of these

172. If $f: R^+ \to R^+$ is a polynomial function satisfying the functional equation $f(f(x)) = 6x - f(x)$, then $f(17)$ is equal to

a) 17

b) -51

c) 34

d) -34

173. Which of the following functions are identical?

a) $f(x) = \ln x^2$ and $g(x) = 2 \ln x$

b) $f(x) = \log_x e$ and $g(x) = \frac{1}{\log_e x}$

c) $f(x) = \sin(\cos^{-1} x)$ and $g(x) = \cos(\sin^{-1} x)$

d) None of these

174. Consider the function $y = f(x)$ satisfying the condition $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} (x \neq 0)$, then

a) Domain of $f(x)$ is R

b) Domain of $f(x)$ is $R - (-2, 2)$

c) Range of $f(x)$ is $[-2, \infty)$

d) Range of $f(x)$ is $[2, \infty)$

175. Let $f(x) = \begin{cases} x^2 - 4x + 3, & x < 3 \\ x - 4, & x \geq 3 \end{cases}$ and $g(x) = \begin{cases} x - 3, & x < 4 \\ x^2 + 2x + 2, & x \geq 4 \end{cases}$ then, which of the following is/are true?

a) $(f + g)(3.5) = 0$

b) $f(g(3)) = 3$

c) $(fg)(2) = 1$

d) $(f - g)(4) = 0$

Assertion - Reasoning Type

This section contain(s) 0 questions numbered 176 to 175. Each question contains STATEMENT 1(Assertion) and STATEMENT 2(Reason). Each question has the 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

a) Statement 1 is True, Statement 2 is True; Statement 2 **is** correct explanation for Statement 1

b) Statement 1 is True, Statement 2 is True; Statement 2 **is not** correct explanation for Statement 1

c) Statement 1 is True, Statement 2 is False

d) Statement 1 is False, Statement 2 is True

176 Consider the functions $f: R \to R, f(x) = x^3$ and $g: R \to R, g(x) = 3x + 4$.

Statement 1: $f(g(x))$ is an onto an function.

Statement 2: $g(x)$ is an onto function.

177

Statement 1: $f(x) = \log_e x$ cannot be expressed as a sum of odd and even function

Statement 2: $f(x) = \log_e x$ is neither odd nor even function.

178

Statement 1: Function $f(x) = x^2 + \tan^{-1} x$ is a non-periodic function.

Statement 2: The sum of two non-periodic functions is always non-periodic.

179 Consider f and g be real-valued functions such that $f(x+y) + f(x-y) = 2f(x).g(y) \forall x, y \in R$.

Statement 1: If $f(x)$ is not identically zero and $|f(x)| \leq 1 \forall x \in R$, then $|g(y)| \leq 1 \forall y \in R$.

Statement 2: For any two real numbers x and y, $|x+y| \leq |x| + |y|$.

180

Statement 1: If $f: R \to R, y = f(x)$ is periodic and continuous function, then $y = f(x)$ cannot be onto.

Statement 2: A continuous periodic function is bounded.

181

Statement 1: If $x \in [1, \sqrt{3}]$, then the range of $f(x) = \tan^{-1} x$ is $[\pi/4, \pi/3]$.

Statement 2: If $x \in [a, b]$, then the range of $f(x)$ is $[f(a), f(b)]$.

182 Consider the function if $f(x) = \sin(kx) + \{x\}$, where $\{x\}$ represents the fractional part function.

Statement 1: $f(x)$ is periodic for $k = m\pi$ where m is a rational number.

Statement 2: The sum of two periodic functions is always periodic.

183

Statement 1: If $f(x) = \cos x$ and $g(x) = x^2$, then $f(g(x))$ is an even function.

Statement 2: If $f(g(x))$ is an even function, then both $f(x)$ and $g(x)$ must be even function.

184 Consider the function satisfying the relation if $f\left(\frac{2\tan x}{1+\tan^2 x}\right) = \frac{(1+\cos 2x)(\sin^2 x + 2\tan x)}{2}$

Statement 1: Range of $y = f(x)$ is R.

Statement 2: Linear function has range R if domain is R.

185 Let $f(x) = (x+1)^2 - 1, x \geq -1$

Statement 1: The set $\{x: f(x) = f^{-1}(x)\} = \{0, -1\}$

Statement 2: f is a bijection.

186

Statement 1: $f(x) = \cos(x^2 - \tan x)$ is a non-periodic function.

Statement 2: $x^2 - \tan x$ is a non-periodic function.

187

Statement 1: The graph of $y = \sec^2 x$ is symmetrical about y-axis.

Statement 2: The graph of $y = \tan x$ is symmetrical about origin.

188 Consider the functions $f(x) = \log_e x$ and $g(x) = 2x + 3$.

Statement 1: $f(g(x))$ is a one-one function.

Statement 2: $g(x)$ is a one-one function.

189

Statement 1: The solution of equation $||x^2 - 5x + 4| - |2x - 3|| = |x^2 - 3x + 1|$ is $x \in (-\infty, 1] \cup \left[\frac{3}{2}, 4\right]$.

Statement 2: If $|x + y| = |x| + |y|$, then $x.y \geq 0$.

190

Statement 1: The period of function $f(x) = \sin\{x\}$ is 1, where $\{.\}$ represents fractional part function.

Statement 2: $g(x) = \{x\}$ has period 1.

191

Statement 1: If $g(x) = f(x) - 1$. If $f(x) + f(1 - x) = 2 \forall\ x \in R$, then $g(x)$ is symmetrical about the point $(1/2, 0)$.

Statement 2: If $g(a - x) = -g(a + x) \forall x \in R$, then $g(x)$ is symmetrical about the point (a, 0).

192

Statement 1: A continuous surjective function $f: R \to R, f(x)$ can never be a periodic function.

Statement 2: For a surjective function $f: R \to R, f(x)$ to be periodic, it should necessarily be a discontinuous function.

193

Statement 1: The period of $f(x) = \sin x$ is $2\pi \Rightarrow$ the period of $g(x) = |\sin x|$ is π.

Statement 2: The period of $f(x) = \cos x$ is $2\pi \Rightarrow$ the period of $g(x) = |\cos x|$ is π.

194

Statement 1: $f(x) = \sin x$ and $g(x) = \cos x$ are identical functions.

Statement 2: Both the functions have the same domain and range.

195

Statement 1: $f: N \to R, f(x) = \sin x$ is a one-one function.

Statement 2: The period of $\sin x$ is 2π and 2π is an irrational number.

196

Statement 1: $f(x) = \sqrt{ax^2 + bx + c}$ has a range $[0, \infty)$ if $b^2 - 4ac > 0$.

Statement 2: $ax^2 + bx + c = 0$ has real roots if $b^2 - 4ac = 0$.

Matrix-Match Type

This section contain(s) 0 question(s). Each question contains Statements given in 2 columns which have to be matched. Statements (A, B, C, D) in **columns I** have to be matched with Statements (p, q, r, s) in **columns II**.

197.

	Column-I		Column- II
(A)	$f(x) = \log_3(5 + 4x - x^2)$	(p)	Function not defined
(B)	$f(x) = \log_3(x^2 - 4x - 5)$	(q)	$[0, \infty)$
(C)	$f(x) = \log_3(x^2 - 4x + 5)$	(r)	$(-\infty, 2]$
(D)	$f(x) = \log_3(4x - 5 - x^2)$	(s)	R

CODES :

	A	**B**	**C**	**D**
a)	r	s	q	p
b)	q	p	r	s
c)	p	r	q	s
d)	q	s	p	r

198. {.} denotes the fractional part function and [.] denotes the greatest integer function:

	Column-I		Column- II
(A)	$f(x) = e^{\cos^4 \pi x + x - [x] + \cos^2 \pi x}$	(p)	1/3
(B)	$f(x) = \cos 2\pi \{2x\} + \sin 2\pi \{2x\}$	(q)	1/4
(C)	$f(x) = \sin 3\pi \{x\} + \tan \pi [x]$	(r)	1/2
(D)	$f(x) = 3x - [3x + a] - b$, where $a, b \in R^+$	(s)	1

CODES :

	A	**B**	**C**	**D**
a)	s	r	s	p
b)	s	p	s	r
c)	p	s	s	r
d)	s	r	p	s

199. Let $f: R \to R$ and g: $R \to R$ be functions such that $f(g(x))$ is a one-one function.

	Column-I		Column- II
(A)	Then g(x)	(p)	Must be one-one

(B) Then $f(x)$	(q)	May not be one-one
(C) If $g(x)$ is onto then $f(x)$	(r)	May be many-one
(D) If $g(x)$ is into then $f(x)$	(s)	Must be many-one

CODES :

	A	B	C	D
a)	p	q	p	q, r
b)	q	p	q, r	p
c)	q	p	p	q, r
d)	q, r	p	q	p

200.

Column-I		Column- II
(A) $x^2 \tan x = 1, x \in [0, 2\pi]$	(p)	5
(B) $2^{\cos x} = \lvert\sin x\rvert, x \in [0, 2\pi]$	(q)	2
(C) If $f(x)$ is a polynomial of degree 5 with real coefficients such that $f(\lvert x\rvert) = 0$ has 8 real roots, then the number of roots of $f(x) = 0$	(r)	3
(D) $7^{\lvert x\rvert}(\lvert 5 - \lvert x\rvert\rvert) = 1$	(s)	4

CODES :

	A	B	C	D
a)	q	s	p	s
b)	p	s	q	s
c)	s	q	p	s
d)	q	p	s	s

201. The function $f(x)$ is defined on the interval [0, 1]
Then match the following columns

Column-I		Column- II
(A) $f(\tan x)$	(p)	$\left[2n\pi - \frac{\pi}{2}, 2n\pi + \frac{\pi}{2}\right], n \in Z$
(B) $f(\sin x)$	(q)	$\left[2n\pi, 2n\pi + \frac{\pi}{6}\right] \cup \left[2n\pi + \frac{5\pi}{6}, (2n+1)\pi\right], n \in Z$
(C) $f(\cos x)$	(r)	$[2n\pi, (2n+1)\pi], n \in Z$
(D) $f(2\sin x)$	(s)	$\left[n\pi, n\pi + \frac{\pi}{4}\right], n \in Z$

CODES :

	A	B	C	D
a)	s	r	p	q
b)	q	p	s	r
c)	q	s	r	p
d)	p	s	r	q

202.

Column-I	Column- II
(A) $f(x) = \cos(\lvert\sin x\rvert - \lvert\cos x\rvert)$	(p) π
(B) $f(x) = \cos(\tan x + \cot x)\cos(\tan x - \cot x)$	(q) $\pi/2$
(C) $f(x) = \sin^{-1}(\sin x) + e^{\tan x}$	(r) 4π
(D) $f(x) = \sin^3 x \sin 3x$	(s) 2π

CODES :

	A	B	C	D
a)	q	q	s	p
b)	q	s	q	p
c)	q	p	q	s
d)	p	q	q	s

203. If $f: R \to R$ is defined by

$$f(x) = \begin{cases} x + 4 & \text{for} \quad x < -4 \\ 3x + 2 & \text{for} \quad -4 \le x < 4, \\ x - 4 & \text{for} \quad x \ge 4 \end{cases}$$

Then the correct matching of List I from List II is

Column-I	Column- II
(A) $f(-5) + f(-4)$	(1) 14
(B) $f(\lvert f(-8)\rvert)$	(2) 4
(C) $f(f-7) + f(3)$	(3) -11
(D) $f(f\left(f\left(f(0)\right)\right)) + 1$	(4) -1
	(5) 1
	(6) 0

CODES :

	A	B	C	D
a)	3	6	2	5

b)	3	4	2	5
c)	4	3	2	1
d)	3	6	5	2

204.

	Column-I		Column- II
(A)	$f: R \to \left[\frac{3\pi}{4}, \pi\right)$ and $f(x) = \cot^{-1}(2x - x^2 - 2)$, then $f(x)$ is	(p)	One-one
(B)	$f: R \to R$ and $f(x) = e^{px} \sin q\, x$ where $p, q \in R^+$, then $f(x)$ is	(q)	Into
(C)	$f: R^+ \to [4, \infty]$ and $f(x) = 4 + 3x^2$, then $f(x)$ is'	(r)	Many-one
(D)	$f: X \to X$ and $f(f(x)) = x \forall x \in X$, then $f(x)$ is	(s)	Onto

CODES :

	A	B	C	D
a)	r, s	r, s	p, q	p, s
b)	p, q	p, s	r, s	r, s
c)	r, s	p, q	r, s	p, s
d)	r, s	p, s	r, s	p, q

205.

	Column-I		Column- II
(A)	$f(x) = \{(\text{sgn}\, x)^{\text{sgn}\, x}\}^n; x \neq 0, n$ is an odd integer	(p)	Odd function
(B)	$f(x) = \frac{x}{e^x - 1} + \frac{x}{2} + 1$	(q)	Even function
(C)	$f(x) = \begin{cases} 0, & \text{If } x \text{ is rational} \\ 1, & \text{If } x \text{ is irrational} \end{cases}$	(r)	Neither odd nor even function
(D)	$f(x) = \max\{\tan x, \cot x\}$	(s)	Periodic

CODES :

	A	B	C	D
a)	p	q	q,s	p,s
b)	q,s	p,s	p	q
c)	p	q	p,s	q,s
d)	p	q,s	q	p,s

206.

Column-I	Column- II

(A)	$f(x)=\tan^{-1}\left(\frac{2x}{1-x^2}\right), g(x)=2\tan^{-1}x$	(p) $x\in\{-1,1\}$
(B)	$f(x)=\sin^{-1}(\sin x)$ and $g(x)=\sin(\sin^{-1}x)$	(q) $x\in[-1,1]$
(C)	$f(x)=\log_{x^2}25$ and $g(x)=\log_x 5$	(r) $x\in(-1,1)$
(D)	$f(x)=\sec^{-1}x+\text{cosec}^{-1}x, g(x)=\sin^{-1}x+\cos^{-1}x$	(s) $x\in(0,1)$

CODES :

	A	**B**	**C**	**D**
a)	r,s	p,q,r,s	s	p
b)	p,q,r,s	r,s	p	s
c)	p	p,q,r,s	s	r,s
d)	p,q,r,s	p	r,s	s

Linked Comprehension Type

This section contain(s) 21 paragraph(s) and based upon each paragraph, multiple choice questions have to be answered. Each question has atleast 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 207 to -207

Consider the functions

$f(x)=\begin{cases} x+1, & x\le 1 \\ 2x+1, & 1<x\le 2\end{cases}$ and $g(x)=\begin{cases} x^2, & -1\le x<2 \\ x+2, & 2\le x\le 3\end{cases}$.

207. The domain of the function $f(g(x))$ is

a) $[0,\sqrt{2}]$ b) $[-1,2]$ c) $[-1,\sqrt{2}]$ d) None of these

Paragraph for Question Nos. 208 to - 208

Consider the function $f(x)$ satisfying the identity $f(x)+f\left(\frac{x-1}{x}\right)=1+x, \forall x\in R-\{0,1\}$ and $g(x)=2f(x)-x+1$.

208. The domain of $y=\sqrt{g(x)}$ is

a) $\left(-\infty,\frac{1-\sqrt{5}}{2}\right]\cup\left[1,\frac{1+\sqrt{5}}{2}\right]$

b) $\left(-\infty,\frac{1-\sqrt{5}}{2}\right]\cup(0,1)\cup\left[\frac{1+\sqrt{5}}{2},\infty\right)$

c) $\left[\frac{1-\sqrt{5}}{2},0\right]\cup\left[\frac{-1+\sqrt{5}}{2},1\right]$

d) None of these

Paragraph for Question Nos. 209 to - 209

Let $f: N\to R$ be a function satisfying the following conditions, $f(1)=1/2$ and $f(1)+2, f(2)+3, f(3)+\cdots+nf(n)=n(n+1), f(n)$ for $n\ge 2$.

209. The value of $f(1003)=\frac{1}{K}$, where K equals

a) 1003 b) 2003 c) 2005 d) 2006

Paragraph for Question Nos. 210 to - 210

If $\left(f(x)\right)^2 \times f\left(\frac{1-x}{1+x}\right) = 64x, \forall x \in Df$, then

210. $f(x)$ is equal to

a) $4x^{2/3}\left(\frac{1+x}{1-x}\right)^{1/3}$ b) $x^{1/3}\left(\frac{1-x}{1+x}\right)^{1/3}$ c) $x^{2/3}\left(\frac{1-x}{1+x}\right)^{1/3}$ d) $x\left(\frac{1+x}{1-x}\right)^{1/3}$

Paragraph for Question Nos. 211 to - 211

$f(x)=\begin{cases} x-1, -1 \le x \le 0 \\ x^2, \ 0 \le x \le 1 \end{cases}$ and $g(x)=\sin x$. Consider the functions $h_1(x)=f(|g(x)|)$ and $h_2(x)=|f(g(x))|$

211. Which of the following is not true about $h_1(x)$?

a) It is periodic function with period π
b) Range is [0, 1]
c) Domain is R
d) None of these

Paragraph for Question Nos. 212 to - 212

If $a_0 = x, a_{n+1} = f(a_n)$, where $n = 0, 1, 2, \ldots$, then answer the following questions.

212. If $f(x)=\sqrt[m]{(a-x^m)}, x>0, m \ge 2, m \in N$. Then

a) $a_n = x, n = 2k+1$, where k is integer
b) $a_n = f(x)$ if $n = 2k$, where k is integer
c) Inverse of a_n exists for any value of n and m
d) None of these

Paragraph for Question Nos. 213 to - 213

Let $f(x)=f_1(x)-2f_2(x)$

Where $f_1(x)=\begin{cases}\min\{x^2,|x|\}, & |x|\le 1 \\ \max\{x^2,|x|\}, & |x|>1\end{cases}$

And $f_2(x)=\begin{cases}\min\{x^2,|x|\}, & |x|>1 \\ \max\{x^2,|x|\}, & |x|\le 1\end{cases}$

And $g(x)=\begin{cases}\min\{f(t): -3\le t\le x, -3\le x<0\} \\ \max\{f(t): 0\le t\le x, 0\le x\le 3\}\end{cases}$.

213. For $-3 \le x \le -1$, the range of $g(x)$ is

a) $[-1, 3]$ b) $[-1, -15]$ c) $[-1, 9]$ d) None of these

Paragraph for Question Nos. 214 to - 214

Let $f(x)=\begin{cases}2x+a, & x\ge -1\\ bx^2+3, & x<-1\end{cases}$

And $g(x)=\begin{cases}x+4, & 0\le x\le 4\\ -3x-2, & -2<x<0\end{cases}$

214. $g(f(x))$ is not defined if

a) $a\in(10,\infty), b\in(5,\infty)$
b) $a\in(4,10), b\in(5,\infty)$
c) $a\in(10,\infty), b\in(0,1)$
d) $a\in(4,10), b\in(1,5)$

Paragraph for Question Nos. 215 to - 215

Let $f:R\to R$ is a function satisfying $f(2-x)=f(2+x)$ and $f(20-x)=f(x), \forall x\in R$. For this function f, answer the following.

215. If $f(0)=5$, then the minimum possible number of values of x satisfying $f(x)=5$, for $x\in[0,170]$, is

a) 21
b) 12
c) 11
d) 22

Paragraph for Question Nos. 216 to - 216

Consider two functions $f(x)=\begin{cases}[x], -2\le x\le -1\\ |x|+1, -1<x\le 2\end{cases}$ and $g(x)=\begin{cases}[x], -\pi\le x<0\\ \sin x, \ 0\le x\le\pi\end{cases}$, where [.] denotes the greatest integer function.

216. The exhaustive domain of $g(f(x))$ is

a) $[0,2]$
b) $[-2,0]$
c) $[-2,2]$
d) $[-1,2]$

Integer Answer Type

217. Let f be a real – valued invertible function such that $f\left(\frac{2x-3}{x-2}\right)=5x-2, x\ne 2$.Then the value of $f^{-1}(13)$ is

218. $f:R\to R f(x^2+x+3)+\ 2f(x^2-3x+5)=6x^2-10x+17\ \forall\ x\ \epsilon\ R$, then the value of $f(5)$ is

219. A continuous function $f(x)$ on $R\to R$ satisfies the relation $f(x)+f(2x+y)+5xy=f(3x-y)+2x^2+1$ for $\forall x,y\ \epsilon\ R$, Then the value of $|f(4)|$ is

220. Let $E=\{1,2,3,4\}$ and $F=\{1,2\}$. If N is number of onto function from E to F, then the value of $N/2$ is

221. Number of integral values of x satisfying the inequality $\left(\frac{3}{4}\right)^{6x+10-x^2}<\frac{27}{64}$

222. If $4^x-2^{x+2}+5+\big||b-1|-3\big|=|\sin y|, x,y,b\ \epsilon\ R$, then the possible value of b is

223. The function of f is continuous and has the property $f(f(x))=1-x$, then the value of $f\left(\frac{1}{4}\right)+f\left(\frac{3}{4}\right)$ is

224. If a, b and c are non-zero rational numbers, the sum of all the possible values of $\frac{|a|}{a}+\frac{|b|}{b}+\frac{|c|}{c}$ is

225. Let $f(x)=\sin^{23}x-\cos^{22}x$ and $g(x)1+\frac{1}{2}\tan^{-1}|x|$, then the number of values of x in interval $[-10\pi, 8\pi]$ satisfying the equation $f(x)=\text{sgn}(g(x))$ is

226. If $f(x)=\sin^2 x+\sin^2\left(x+\frac{\pi}{3}\right)+\cos x\cos\left(x+\frac{\pi}{3}\right)$ and $g\left(\frac{5}{4}\right)=1$ then $(gof)(x)$ is

227. An even polynomial function $f(x)$ satisfies a relation $f(2x)\left(1-f\left(\frac{1}{2x}\right)\right)+f(16x^2y)=f(-2)-f(4xy)\ \forall\ x,y\ \epsilon\ R-\{0\}$ and $f(4)=-255, f(0)=1$, then the value of $|(f(2)+1)/2|$ is

228. Number of integral values of x for which $\frac{2^{\frac{\pi}{\tan^{-1}x}}-4(x-4)(x-10)}{x!-(x-1)!}<0$

229. Number of integers in the domain of function, satisfying $f(x)+f(x^{-1})=\frac{x^2+1}{x}$, is

230. If θ be the fundamental period of function $f(x)=\sin^{99}x+\sin^{99}\left(x+\frac{2\pi}{3}\right)+\sin^{99}\left(x+\frac{4\pi}{3}\right)$, then complex number $z=|z|(\cos\theta+i\sin\theta)$ lies in the quadrant number.

231. Number of integral values of a for which $f(x)=\log(\log_{1/3}(\log_7(\sin x+a)))$ be defined for every real values of x

232. If $f: N\to N$, and $x_2>x_1\Rightarrow f(x_2)>f(x_1), \forall\, x_1, x_2\in N$ and $f(f(n))=3n, \forall\, n\in N$, then $f(2)=$

233. Suppose that $f(x)$ is a function of the form $f(x)=\frac{ax^8+bx^6+cx^4+dx^2+15x+1}{x}\,(x\neq 0)$. If $f(5)=2$, then the value of $|f(-5)/4|$ is

234. Suppose that f is an even, periodic function with period 2, and that $f(x)=x$ for all x in the interval $[0,1]$. The value of $[10f(3.14)]$ is (where [.] represents the greatest integer function)

235. Number of values of x for which $\left|\left||x^2-x+4|-2\right|-3\right|=x^2-x-12$ is

236. Let $f: R\to R$ be a continuous onto function satisfying $f(x)+f(-x)=0, \forall\, x\in R$. If $f(-3)=2$ and $f(5)=4$ in $[-5,5]$, then the minimum number of roots of the equation $f(x)=0$ is

237. Let $f(x)=3x^2-7x+c$, where $'c'$ is a variable coefficient and $x>\frac{7}{6}$.Then the value of $[c]$ such that $f(x)$ touches $f^{-1}(x)$ is (where [.]represents greatest integer function

238. If $f(x)=\sqrt{4-x^2}+\sqrt{x^2-1}$, then the maximum value of $(f(x))^2$ is

239. Let $a>2$ be a constant. If there are just 18 positive integers satisfying the inequality $(x-a)(x-2a)(x-a^2)<0$, then the value of a is

240. The function $f(x)=\frac{x+1}{x^3+1}$ can be written as the sum of an even function $g(x)$ and an odd function $h(x)$.then the value of $|g(0)|$ is

241. Let $f: R^+\to R$ be a function which satisfies $f(x).f(y)=f(xy)+2\left(\frac{1}{x}+\frac{1}{y}+1\right)$ for $x,y>0$, then possible value of $f(1/2)$ is

242. If $f(x)$ is an odd function and $f(1)=3$, and $f(x+2)=f(x)+f(2)$, then the value of $f(3)$ is

243. Number of integral values of x for which the function $\sqrt{\sin x+\cos x}+\sqrt{7x-x^2-6}$ is defined is

244. If $x=\frac{4}{9}$ satisfy the equation $\log_a(x^2-x+2)>\log_a(-x^2+2x+3)$, then sum of all possible distinct values of $[x]$ is (where [.]represents greatest integer function)

245. A function f from integers to integers is defined as $(x)=\begin{cases} n+3, & n\in \text{odd} \\ n/2, & n\in \text{even}\end{cases}$. Suppose $k\in$ odd and $f\left(f(f(k))\right)=27$, then the sum of digits of k is

246. If T is the period of the function $f(x)=[8x+7]+|\tan 2\pi x+\cot 2\pi x|-8x$ (where [.] denotes the greatest integer function), then the value of $1/T$ is

: ANSWER KEY :

1)	d	2)	d	3)	b	4)	d
5)	a	6)	d	7)	a	8)	d
9)	d	10)	c	11)	b	12)	a
13)	b	14)	d	15)	a	16)	d
17)	c	18)	d	19)	b	20)	b
21)	c	22)	d	23)	a	24)	b
25)	b	26)	a	27)	c	28)	c
29)	a	30)	c	31)	b	32)	d
33)	d	34)	a	35)	b	36)	c
37)	a	38)	c	39)	a	40)	a
41)	c	42)	b	43)	a	44)	c
45)	c	46)	b	47)	c	48)	b
49)	b	50)	c	51)	d	52)	c
53)	d	54)	c	55)	c	56)	d
57)	c	58)	c	59)	a	60)	d
61)	c	62)	b	63)	a	64)	a
65)	c	66)	c	67)	b	68)	c
69)	b	70)	a	71)	c	72)	a
73)	a	74)	b	75)	b	76)	b
77)	c	78)	d	79)	b	80)	c
81)	c	82)	b	83)	a	84)	b
85)	d	86)	c	87)	a	88)	d
89)	d	90)	d	91)	a	92)	d
93)	c	94)	c	95)	d	96)	b
97)	a	98)	d	99)	c	100)	b
101)	b	102)	c	103)	d	104)	d
105)	d	106)	c	107)	a	108)	b
109)	c	110)	b	111)	b	112)	d
113)	b	114)	b	115)	d	116)	c
117)	a	118)	d	119)	b	120)	b
121)	b	122)	d	123)	c	124)	c
125)	c	126)	d	127)	a	128)	d
129)	b	130)	c	131)	a	132)	c
133)	c	134)	c	135)	c	136)	d
137)	a	138)	a	139)	a	140)	a
141)	c	142)	d	143)	b	144)	c

145) b 146) c 147) d 1) a, d 2) a, c 3) a, b, c 4) a, b, d

5) b 6) a, b 7) a, b, c, d 8) a, b, c, d

9) a, b, c 10) a, d 11) b, d 12) b, d

13) a, c 14) a,b,d 15) a,d 16) a, c

17) a 18) a, b, c, d 19) b,c 20) a, c, d

21) b, c, d 22) b, c 23) a, b, c, d 24) a, b, c

25) b, c 26) b, c 27) b, d 28) a, b, c

1) b 2) b 3) c 4) a

5) a 6) c 7) c 8) c

9) d 10) c 11) b 12) a

13) b 14) b 15) b 16) a

17) a 18) a 19) d 20) a

21) d 1) a 2) a 3) a 4) a

5) a 6) a 7) a 8) a

9) a 10) a 1) c 2) b 3) d 4) a

5) d 6) d 7) a 8) a

9) c 10) c 1) 3 2) 7 3) 7 4) 7

5) 7 6) 4 7) 1 8) 0

9) 9 10) 1 11) 7 12) 5

13) 2 14) 3 15) 3 16) 3

17) 7 18) 8 19) 1 20) 3

21) 5 22) 6 23) 5 24) 0

25) 4 26) 9 27) 3 28) 1

29) 6 30) 4

: HINTS AND SOLUTIONS :

1 **(d)**

Since $f\ (x)$ is an odd function, $\left[\frac{x^2}{a}\right] = 0$ for all $x \in [-10, 10]$

$\Rightarrow 0 \le \frac{x^2}{a} < 1$ for all $x \in [-10, 10] \Rightarrow a > 100$

2 **(d)**

$f(x) = \frac{a^x - 1}{x^n(a^x + 1)}$

$f(x)$ is symmetrical about y-axis

$\Rightarrow\ f(x) = f(-x)$

$\Rightarrow \frac{a^x - 1}{x^n(a^x + 1)} = \frac{a^{-x} - 1}{(-x)^n(a^{-x} + 1)}$

$\Rightarrow \frac{a^x - 1}{x^n(a^x + 1)} = \frac{1 - a^x}{(-x)^n(1 + a^x)} \Rightarrow x^n = -(-x)^n$

$\Rightarrow$ the value of n which satisfy this relation is $-\frac{1}{3}$

3 **(b)**

$f(x)$ is defined for $\log\left(\frac{1}{|\sin x|}\right) \ge 0$

$\Rightarrow \frac{1}{|\sin x|} \ge 1$ and $|\sin x| \ne 0$

$\Rightarrow |\sin x| \ne 0 \quad \left[\because \frac{1}{|\sin x|} \ge 1 \text{ for all } x\right]$

$\Rightarrow x \ne n\pi, n \in Z$

Hence, the domain of $f(x) = R - \{n\pi : n \in Z\}$

4 **(d)**

$f(x) = \frac{\alpha x}{x+1}, x \ne -1$

$f(f(x)) = x \Rightarrow \frac{\alpha\left(\frac{\alpha x}{x+1}\right)}{\frac{\alpha x}{x+1} + 1} = x$

$\Rightarrow \frac{\alpha^2 x}{(\alpha + 1)\,x + 1} = x$

$\Rightarrow (\alpha + 1)x^2 + (1 - \alpha^2)x = 0$

$\Rightarrow \alpha + 1 = 0$ and $1 - \alpha^2 = 0$

[As true $\forall\ x \ne 1\ \therefore$ Eq. (1) is an identity]

$\Rightarrow \alpha = -1$

5 **(a)**

$f: [1, \infty) \to [2, \infty)$

$f(x) = x + \frac{1}{x} = y$

$\Rightarrow x^2 - yx + 1 = 0$

$\Rightarrow x = \frac{y \pm \sqrt{y^2 - 4}}{2}$

But given $f: [1, \infty) \to [2, \infty)$

$\therefore x = \frac{y + \sqrt{y^2 - 4}}{2}$

6 **(d)**

We must have $-1 \le [2x^2 - 3] \le 1$

$\Rightarrow -1 \le 2x^2 - 3 < 2 \Rightarrow 1 \le x^2 < \frac{5}{2}$

$\Rightarrow x \in \left(-\sqrt{\frac{5}{2}}, -1\right] \cup \left[1, \sqrt{\frac{5}{2}}\right)$

7 **(a)**

Here, $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$, to find domain we must have,

$\sin^{-1}(2x) + \frac{\pi}{6} \ge 0$

$\left(\text{but } -\frac{\pi}{2} \le \sin^{-1}\theta \le \frac{\pi}{2}\right)$

$\therefore \qquad -\frac{\pi}{6} \le \sin^{-1}(2x) \le \frac{\pi}{2}$

$\Rightarrow \sin\left(-\frac{\pi}{6}\right) \le 2x \le \sin\left(\frac{\pi}{2}\right)$

$\Rightarrow\ -\frac{1}{2} \le 2x \le 1\ \Rightarrow x \in \left[-\frac{1}{4}, \frac{1}{2}\right]$

8 **(d)**

If f is injective and g is surjective

$\Rightarrow f o$g is injective

$\Rightarrow fof$ is injective

9 **(d)**

Image b_1 is assigned to any three of the six pre-images in $^6C_3$

ways

Rest two images can be assigned to remaining three pre-images in $2^3 - 2$ ways (as function is onto)

Hence number of functions are $^6C_3 \times (2^3 - 2) = 20 \times 6 = 120$

10 **(c)**

$f(x) = \begin{cases} x - 1, & x \text{ is even} \\ x + 1, & x \text{ is odd} \end{cases}$, where is clearly are one-one and onto

11 **(b)**

Put $x = 0 \Rightarrow f(2) = 2f(0) - f(1) = 2 \times 2 - 3 = 1$

Put $x = 1 \Rightarrow f(3) = 6 - 1 = 5$

Put $x = 2 \Rightarrow f(4) = 2f(2) - f(3) = 2 \times 1 - 5 = -3$

Put $x = 3 \Rightarrow f(5) = 2f(3) - f(4) = 2(5) - (-3) = 13$

12 **(a)**

$f(x)$ is defined if $-\log_{1/2}\left(1 + \frac{1}{x^{1/4}}\right) - 1 > 0$

$\Rightarrow \log_{1/2}\left(1 + \frac{1}{x^{1/4}}\right) < -1$

$\Rightarrow 1+\frac{1}{x^{1/4}}>\left(\frac{1}{2}\right)^{-1}$

$\Rightarrow \frac{1}{x^{1/4}}>1$

$\Rightarrow 0<x<1$

13 **(b)**

We must have

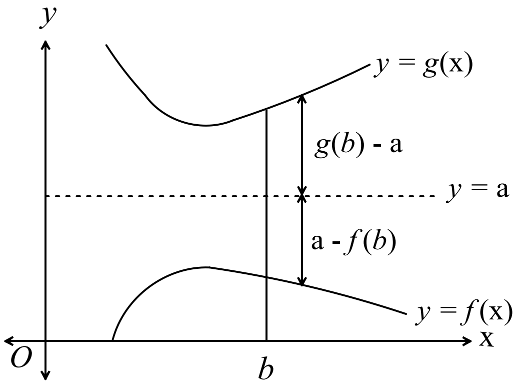
$2\{x\}^2-3\{x\}+1\geq 0 \Rightarrow \{x\}\geq 1$ or $\{x\}\leq 1/2$

Thus, we have $0\leq\{x\}\leq 1/2 \Rightarrow x\in \Big[n, n+$

12, n∈I

14 **(d)**

$y=f(x)$ and $y=\mathrm{g}(x)$ are mirror image of each other about line $y=a$



$\Rightarrow$ for some $x=b, \mathrm{g}(b)-a=a-f(b)$

$\Rightarrow f(b)+\mathrm{g}(b)=2a$

$\Rightarrow h(b)f(b)+\mathrm{g}(b)=2a$ (constant)

Hence $h(x)$ is constant function Thus it is neither one-one nor onto

15 **(a)**

$f(x)=\sec^{-1}(\log_3\tan x+\log_{\tan x}3)$

$f(x)=\sec^{-1}\left(\log_3\tan x+\frac{1}{\log_3\tan x}\right)$

Now for $\log_3\tan x$ to get defined, $\tan x\in(0,\infty)$

$\Rightarrow \log_3\tan x\in(-\infty,\infty)$ or $\log_3\tan x\in R$

Also $x+\frac{1}{x}\leq -2$ or $x+\frac{1}{x}\geq 2$

$\Rightarrow \log_3\tan x+\frac{1}{\log_3\tan x}\leq -2$ or $\log_3\tan x+\frac{1}{\log_3\tan x}\geq 2$

$\Rightarrow \sec^{-1}\left(\log_3\tan x+\frac{1}{\log_3\tan x}\right)\leq\sec^{-1}(-2)$ or $\sec^{-1}\left(\log_3\tan x+\frac{1}{\log_3\tan x}\right)\geq\sec^{-1}2$

$\Rightarrow\ f(x)\leq\frac{2\pi}{3}$ or $f(x)\geq\frac{\pi}{3}$

$\Rightarrow\ f(x)\in\left[\frac{\pi}{3},\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\frac{2\pi}{3}\right]$

16 **(d)**

It is given that $2^x+2^y=2\ \forall\ x,y\ \epsilon\ R$

$\Rightarrow 2^y=2-2^x$

$\Rightarrow y=\log_2(2-2^x)$

$\Rightarrow$ function is defined only when $2-2^x>0$ or $2^x<2$

Or $x<1$

17 **(c)**

$f(x)$ is defined for $x\in(0,,1)$

$\Rightarrow f(e^x)+f(\ln|x|)$ is defined for,

$0<e^x<1$ and $0<\ln|x|<1$

$\Rightarrow -\infty<x<0$ and $1<|x|<e$

$\Rightarrow x\in(-\infty,0)$ and $x\in(-e,-1)\cup(1,e)$

$\Rightarrow\ x\in(-e,-1)$

18 **(d)**

$$f(f(x))=\begin{cases}(f(x))^2, & \text{for } f(x)\geq 0\\ f(x), & \text{for } f(x)<0\end{cases}$$

$$=\begin{cases}(x^2)^2, & x^2\geq 0,\ x\geq 0\\ x^2, & x\geq 0,\quad x<0\\ x^2, & x^2<0,\quad x\geq 0\\ x, & x<0,\quad x<0\end{cases}=\begin{cases}x^4, & x\geq 0\\ x, & x<0\end{cases}$$

19 **(b)**

The function $\sec^{-1}x$ is defined for all $x\in R-(-1,1)$ and the function $\frac{1}{\sqrt{x-[x]}}$ is defined for all $x\in R-Z$ So the given function is defined for all $x\in R-\{(-1,1)\cup\{n|n\in Z\}\}$

20 **(b)**

$y=f(x)=\sqrt{3}\sin x-\cos x+2=2\sin\left(x-\frac{\pi}{6}\right)+$ *2* (1)

Since $f(x)$ is one-one and onto, f is invertible.

From (1) $\sin\left(x-\frac{\pi}{6}\right)=\frac{y-2}{2}$

$\Rightarrow x=\sin^{-1}\frac{y-2}{2}+\frac{\pi}{6}$

$\Rightarrow f^{-1}(x)=\sin^{-1}\left(\frac{x-2}{2}\right)+\frac{\pi}{6}$

21 **(c)**

Let $x,y\in N$ such that $f(x)=f(y)$

Then $f(x)=f(y)$

$\Rightarrow x^2+x+1=y^2+y+1$

$\Rightarrow (x-y)(x+y+1)=0$

$\Rightarrow x=y$ or $x=(-y-1)\notin N$

$\therefore\ f$ is one-one

Also, $f(x)$ does not take all positive integral values. Hence f is into

22 **(d)**

$f(3x+2)+f(3x+29)=0$ (1)

Replacing x by $x+9$, we get

$f(3(x+9)+2)+f(3(x+9)+29=0$

$\Rightarrow\ f(3x+29)+f(3x+56)=0$ (2)

From (1) and (2), we get

$f(3x+2)=f(3x+56)$

$\Rightarrow\ f(3x+2)=f(3(x+18)+2)$

$\Rightarrow\ f(x)$ is periodic with period 54

23 **(a)**

$f(x)=\{x\}$ is periodic with period 1

$f(x)=\sin\frac{1}{x}$ for $x\neq 0, f(0)=0$ is non-periodic as

$g(x)=\frac{1}{x}$ is non-periodic

Also $f(x)=x\cos x$ is non-periodic as $g(x)=x$ is non-periodic

24 **(b)**

$$f(-x)=\begin{cases}(-x)^2\sin\dfrac{\pi(-x)}{2}, & |-x|<1\\ (-x)|-x|, & |-x|\geq 1\end{cases}$$

$$=\begin{cases}-x^2\sin\dfrac{\pi x}{2}, & |x|<1\\ -x|x|, & |x|\geq 1\end{cases}$$

$=-f(x)$

25 **(b)**

$$f(x)=\frac{\sin^{-1}(3-x)}{\log(|x|-2)}$$

Let $g(x)=\sin^{-1}(3-x)$

$\Rightarrow -1\leq 3-x\leq 1$

The domain of $g(x)$ is [2, 4]

And let $h(x)=\log(|x|-2)$

$\Rightarrow |x|-2>0$ or $|x|>2$

$\Rightarrow x<-2$ or $x>2$

$\Rightarrow (-\infty,-2)\cup(2,\infty)$

We know that

$(f/g)(x)\dfrac{f(x)}{g(x)}\forall x\in D_1\cap D_2-\{x\in R: g(x)=0\}$

$\therefore$ the domain of $f(x)=(2,4]-\{3\}=(2,3)\cup(3,4]$

26 **(a)**

$x^2f(x)-2f\left(\frac{1}{x}\right)=g(x)$ and $2f\left(\frac{1}{x}\right)-4x^2f(x)$

$=2x^2g\left(\frac{1}{x}\right)$

(Replacing$x$ by $\frac{1}{x}$)

$\Rightarrow\ -3x^2f(x)=g(x)+2x^2g\left(\frac{1}{x}\right)$

(Eliminating$f\left(\frac{1}{x}\right)$)

$$\Rightarrow\ f(x)=-\left(\frac{g(x)=2x^2g\left(\frac{1}{x}\right)}{3x^2}\right)$$

$\because$ $g(x)$ and x^2 are odd and even functions, respectively

So, $f(x)$ is an odd function But $f(x)$ is given even

$\Rightarrow\ f(x)=0\forall x$ Hence, $f(5)=0$

27 **(c)**

$$f(x)=\frac{\sin[x]\pi}{x^2+x+1}$$

Let $[x]=n\in$ integer

$\Rightarrow\sin[x]\pi=0$

$\Rightarrow\ f(x)=0$

$\Rightarrow f(x)$ is constant function

28 **(c)**

Let $|x-1|+|x-2|+|x-3|<6$

$\Rightarrow|(x-1)+(x-2)+(x-3)|$

$<|x-1|+|x-2|+|x-3|<6$

$\Rightarrow|3x-6|<6$

$\Rightarrow|x-2|<2$

$\Rightarrow-2<x-2<2$

$\Rightarrow 0<x<4$

Hence, for $|x-1|+|x-2|+|x-3|\geq 6, x\leq 0$ or $x\geq 4$.

29 **(a)**

The period of $\sin\pi x$ and $\cos 2\pi x$ is 2 and 1, respectively

The period of $2^{\{x\}}$ is 1

The period of $3^{\{x/2\}}$ is 2

Hence, the period of $f(x)$ is LCM of 1 and 2=2

30 **(c)**

Clearly $f(x+\pi)=f(x), g(x+\pi)=g(x)$ and $\emptyset\left(x+\frac{\pi}{2}\right)$

$=\{(-1)f(x)\}\{(-1)g(x)\}=\emptyset(x)$

31 **(b)**

$$\left[x^2+\frac{1}{2}\right]=\left[x^2-\frac{1}{2}+1\right]=1+\left[x^2-\frac{1}{2}\right]$$

Thus, from domain point of view,

$\left[x^2-\frac{1}{2}\right]=0,-1\Rightarrow\left[x^2+\frac{1}{2}\right]=1,0$

$\Rightarrow\ f(x)=\sin^{-1}(1)+\cos^{-1}(0)$ or $\sin^{-1}(0)+\cos^{-1}(-1)$

$\Rightarrow\ f(x)=\{\pi\}$

32 **(d)**

The period of $f(x)$ is 7 $\Rightarrow$ The period of $f\left(\frac{x}{3}\right)$ is $\frac{7}{1/3}=21$

The period of $g(x)$ is 11 $\Rightarrow$ The period of $g\left(\frac{x}{5}\right)$ is $\frac{11}{1/5}=55$

Hence, $T_1=$ period of $f(x)g\left(\frac{x}{5}\right)=7\times 55=385$

and

$T_2=$ period of $g(x)f\left(\frac{x}{3}\right)=11\times 21=231$

$\therefore$ period of $F(x)=\text{LCM}\{T_1,T_2\}$

$=\text{LCM}\{385,231\}$

$=7\times 11\times 3\times 5$

$=1155$

33 **(d)**

$f(x)=\sqrt{x^{12}-x^9+x^4-x+1}$

We must have $x^{12}-x^9+x^4-x+1\geq 0$

Obviously (1) is satisfied by $x\in(-\infty,0]$

Also, $x^9(x^3-1)+x(x^3-1)+1\geq 0\forall x\in[1,\infty)$

Further, $x^{12}-x^9+x^4-x+1=(1-x)+x^4(1-x^5)+x^{12}$ is also satisfied by $x\in(0,1)$

Hence, the domain is R

34 **(a)**

From E to F we can define, in all, $2\times2\times2\times2=16$ functions (2 options for each elements of E) out of which 2 are into, when all the elements of E map to either 1 or 2.

∴ No. of onto function = 16 – 2 =14

35 **(b)**

$\because g(f(x))=(\sin x+\cos x)^2-1$, is invertible (*ie*, bijective)

$\Rightarrow g(f(x))=\sin 2x$, is bijective

We know $\sin x$ is bijective only when $x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

Thus, $g(f(x))$ is bijective if, $-\frac{\pi}{2}\leq 2x\leq\frac{\pi}{2}$

$\Rightarrow \quad -\frac{\pi}{4}\leq x\leq\frac{\pi}{4}$

36 **(c)**

$$f(x)=\frac{e^x-e^{|x|}}{e^x+e^{|x|}}=\begin{cases}0, & x\geq 0\\ \frac{e^x-e^{-x}}{e^x+e^{-x}}, & x<0\end{cases}$$

Clearly, $f(x)$ is identically zero if $x\geq 0$ (1)

If $x<0$, let $y=f(x)=\frac{e^x-e^{-x}}{e^x+e^{-x}}\Rightarrow e^{2x}=\frac{1+y}{1-y}$

$\because x<0\Rightarrow e^{2x}<1\Rightarrow 0<e^{2x}<1$

$\because 0<\frac{1+y}{1-y}<1$

$\Rightarrow \frac{1+y}{1-y}>0$ and $\frac{1+y}{1-y}<1$

$\Rightarrow (y+1)(y-1)<0$ and $\frac{2y}{1-y}<0$

$\Rightarrow -1<y<1$ and $y<0$ or $y>1$

$\Rightarrow -1<y<0$ (2)

Combining(1) and (2), we get $-1<y\leq 0\Rightarrow$ Range $=(-1,0]$

37 **(a)**

$f(xy)=\frac{f(x)}{y}$

$\Rightarrow f(y)=\frac{f(1)}{y}$ (putting $x=1$)

$\Rightarrow f(30)=\frac{f(1)}{30}$ or $f(1)=30\times f(30)=30\times 20=600$

Now $f(40)=\frac{f(1)}{40}=\frac{600}{40}=15$

38 **(c)**

Here, $\frac{x^2+1}{x^2+2}=1-\frac{1}{x^2+2}$

Now, $2\leq x^2+2<\infty$ for all $x\in R$

$\Rightarrow \frac{1}{2}\geq\frac{1}{x^2+2}>0$

$\Rightarrow -\frac{1}{2}\leq\frac{-1}{x^2+2}<0$

$\Rightarrow \frac{1}{2}\leq 1-\frac{1}{x^2+2}<1$

$\Rightarrow \frac{\pi}{6}\leq\sin^{-1}\left(1-\frac{1}{x^2+2}\right)<\frac{\pi}{2}$

39 **(a)**

When $[x]=0$ we have $\sin^{-1}(\cos^{-1}0)=\sin-1(\pi 2)$, not defined

When $[x]=-1$ we have $\sin^{-1}(\cos^{-1}-1)=\sin-1(\pi)$, not defined

When $[x]=1$ we have $\sin^{-1}(\cos^{-1}1)=\sin-10=0$

Hence, $x\in[1,2)$ and the range of function is $\{0\}$

40 **(a)**

We have $f(x)={}^{7-x}P_{x-3}=\frac{(7-x)!}{(10-2x)!}$

We must have $7-x>0, x\geq 3$ and $7-x\geq x-3$

$\Rightarrow x>7, x\geq 3$ and $x\leq 5$

$\Rightarrow 3\leq x\leq 5$

$\Rightarrow x=3,4,5$

Now $f(3)=\frac{4!}{4!}=1, f(4)=\frac{3!}{2!}=3, f(5)=\frac{2!}{0!}=2$

Hence, $R_f=\{1,2,3\}$

41 **(c)**

$\frac{x^2+14x+9}{x^2+2x+3}=y$

$\Rightarrow x^2+14x+9=x^2y+2xy+3y$

$\Rightarrow x^2(y-1)+2x(y-7)+(3y-9)=0$

Since x is real,

$\therefore 4(y-7)^2-4(3y-9)(y-1)>0$

$\Rightarrow 4(y^2+49-14y)-4(3y^2+9-12y)>0$

$\Rightarrow (y+5)(y-4)<0$;

∴ y lies between -5 and 4

42 **(b)**

For the domain $\sin(\ln x)>\cos(\ln x)$ and $x>0$

$2n\pi+\frac{\pi}{4}<\ln x<2n\pi+\frac{5\pi}{4}, n\in N\cup\{0\}$

43 **(a)**

$af(x+1)+bf\left(\frac{1}{x+1}\right)=(x+1)-1$ (1)

Replacing $x+1$ by $\frac{1}{x+1}$, we get

$\therefore af\left(\frac{1}{x+1}\right)+bf(x+1)=\frac{1}{x+1}-1$ (2)

$(1)\times a-(2)\times b\Rightarrow(a^2-b^2)f(x+1)$

$$=a(x+1)-a-\frac{b}{x+1}+b$$

Putting $x=1$, $(a^2-b^2)f(2)=2a-a-\frac{b}{2}+b=a+\frac{b}{2}$

$= \dfrac{2a+b}{2}$

44 **(c)**

$y = |\sin x| + |\cos x|$

$\Rightarrow y^2 = 1 + |\sin 2x|$

$\Rightarrow 1 \le y^2 \le 2$

$\Rightarrow y \in [1, \sqrt{2}]$

$\Rightarrow\ f(x) = 1 \forall x \in R$

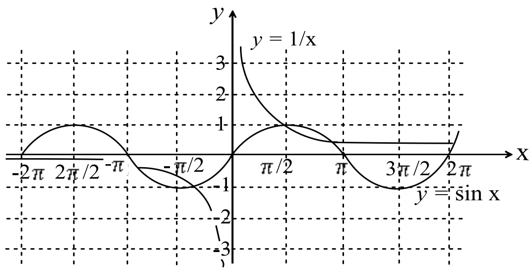
45 **(c)**

$x \sin x = 1\ \ (1)$

$\Rightarrow y = \sin x = \dfrac{1}{x}$

Root of equation (1) will be given by the point(s) of intersection of the graphs $y = \sin x$ and $y = \frac{1}{x}$.

Graphically, it is clear that we get four roots.



46 **(b)**

Let $2x + 3y = m$ and $2x - 7y = n$

$\Rightarrow y = \frac{m-n}{10}$ and $x = \frac{7m-3n}{20}$

$\Rightarrow f(m, n) = 7m + 3n$

$\Rightarrow\ f(x, y) = 7x + 3y$

47 **(c)**

For the function to get defined $0 \le x^2 + x + 1 \le 1$,

But $x^2 + x + 1 \ge \frac{3}{4} \Rightarrow \frac{\sqrt{3}}{2} \le \sqrt{x^2 + x + 1} \le 1$

$\Rightarrow \dfrac{\pi}{3} \le \sin^{-1}\left(\sqrt{x^2 + x + 1}\right) \le \dfrac{\pi}{2}$

48 **(b)**

$f(x) = \sin\left(\log\left(x + \sqrt{1 + x^2}\right)\right)$

$\Rightarrow f(-x) = \sin\left[\log\left(-x + \sqrt{1 + x^2}\right)\right]$

$$\Rightarrow\ f(-x) = \sin\log\left(\left(\sqrt{1+x^2} - x\right)\frac{\left(\sqrt{1+x^2}+x\right)}{\left(\sqrt{1+x^2}+x\right)}\right)$$

$$\Rightarrow\ f(-x) = \sin\log\left[\frac{1}{\left(x + \sqrt{1+x^2}\right)}\right]$$

$\Rightarrow\ f(-x) = \sin\left[-\log\left(x + \sqrt{1 + x^2}\right)\right]$

$\Rightarrow\ f(-x) = -\sin\left[\log\left(x + \sqrt{1 + x^2}\right)\right]$

$\Rightarrow\ f(-x) = -f(x)$

$\Rightarrow\ f(x)$ is an odd function

49 **(b)**

For odd function

$f(x) = -f(-x)$

$$= -\begin{cases} \sin(-x) + \cos(-x), & 0 \le -x < \pi/2 \\ a, & -x = \pi/2 \\ \tan^2(-x) + \operatorname{cosec}(-x), & \pi/2 < -x < \pi \end{cases}$$

$$= \begin{cases} \sin x - \cos x, & -\pi/2 < x \le 0 \\ -a, & x = -\pi/2 \\ \tan^2 x + \operatorname{cosec} x, & -\pi < x < -\pi/2 \end{cases}$$

50 **(c)**

Since $f(x) = (x+1)^2 - 1$ is continuous function, solution of

$f(x) = f^{-1}(x)$ lies on the line $y = x$

$\Rightarrow f(x) = f^{-1}(x) = x$

$\Rightarrow (x+1)^2 - 1 = x$

$\Rightarrow x^2 + x = 0$

$\Rightarrow x = 0$ or -1

$\Rightarrow$ The required set is $\{0, -1\}$

51 **(d)**

$x^2 - [x]^2 \ge 0 \Rightarrow x^2 \ge [x]^2$

This is true for all positive values of x and all negative integer x

52 **(c)**

The period of $\cos(\sin nx)$ is $\frac{\pi}{n}$ and the period of $\tan\left(\frac{x}{n}\right)$ is πn

Thus, $6\pi = \text{LCM}\left(\frac{\pi}{n}, \pi n\right)$

By checking for the different values of $n, n = 6$

53 **(d)**

$f(x) = x^2$ is many-one as $f(1) = f(-1) = 1$.

Also f is into, as the range of function is $[0, \infty)$ which is subset of R (co-domin).

$\therefore f$ is neither injective nor surjective.

54 **(c)**

Given $f(x) = \left[\sin x + \left[\cos x + \left[\tan x + [\sec x]\right]\right]\right]$

$= [\sin + p]$, where $p = \left[\cos x + \left[\tan x + [\sec x]\right]\right]$

$= [\sin x] + p$, (as p is integer)

$= [\sin x] + \left[\cos x + \left[\tan x + [\sec x]\right]\right]$

$= [\sin x] + [\cos x] + [\tan x] + [\sec x]$

Now, for $x \in (0, \pi/4)$, $\sin x \in \left(0, \frac{1}{\sqrt{2}}\right)$, $\cos x \in \left(\frac{1}{\sqrt{2}}, 1\right)$, $\tan x \in (0, 1)$, $\sec x \in (1, \sqrt{2})$

$\Rightarrow [\sin x] = 0, [\cos x] = 0, [\tan x] = 0$ and $[\sec x] = 1$

$\Rightarrow$ The range of $f(x)$ is 1

55 **(c)**

Since co-domain $= \left[0, \frac{\pi}{2}\right)$

$\therefore$ for f to be onto, the range $= \left[0, \frac{\pi}{2}\right)$

This is possible only when $x^2 + x + a \geq 0 \forall x \in R$

$\therefore 1^2 - 4a \leq 0 \Rightarrow a \geq \frac{1}{4}$

56 **(d)**

$f(x) = \frac{1}{\sqrt{4x - |x^2 - 10x + 9|}}$

For $f(x)$ to be defined $|x^2 - 10x + 9| < 4x$

$\Rightarrow x^2 - 10x + 9 < 4x$ and $x^2 - 10x + 9 > -4x$

$\Rightarrow x^2 - 14x + 9 < 0$ and $x^2 - 6x + 9 > 0$

$\Rightarrow x \in (7 - \sqrt{40}, 7 + \sqrt{40})$ and $x \in R - \{-3\}$

$\Rightarrow x \in (7 - \sqrt{40}, -3) \cup (-3, 7 + \sqrt{40})$

57 **(c)**

Let $y = \frac{x^2+x+2}{x^2+x+1}$

$\Rightarrow x^2(y-1) + x(y-1) + (y-2) = 0, \forall x \in R$

Now, $D \geq 0 \Rightarrow (y-1)^2 - 4(y-1)(y-2) \geq 0$

$\Rightarrow (y-1)\{(y-1) - 4(y-2)\} \geq 0$

$\Rightarrow (y-1)(-3y+7) \geq 0$

- + -

1 $\frac{7}{3}$

$\Rightarrow 1 \leq y \leq \frac{7}{3}$

58 **(c)**

$y = f(x) = \cos^2 x + \sin^4 x$

$\Rightarrow y = f(x) = \cos^2 x + \sin^2 x(1 - \cos^2 x)$

$\Rightarrow y = \cos^2 x + \sin^2 x - \sin^2 x \cos^2 x$

$\Rightarrow y = 1 - \sin^2 x \cos^2 x$

$\Rightarrow y = 1 - \frac{1}{4}\sin^2 2x$

$\therefore \frac{3}{4} \leq f(x) \leq 1 \ (\because 0 \leq \sin^2 2x \leq 1)$

$\Rightarrow f(x) \in [3/4, 1]$

59 **(a)**

$f(-x) = \frac{\cos(-x)}{\left[-\frac{2x}{\pi}\right] + \frac{1}{2}} = \frac{\cos x}{-1 - \left[\frac{2x}{\pi}\right] + \frac{1}{2}}$

(as x is not an integral multiple of π)

$\Rightarrow f(-x) = -\frac{\cos x}{\left[\frac{2x}{\pi}\right] + \frac{1}{2}} = -f(x)$

$\Rightarrow f(x)$ is an odd function.

60 **(d)**

$[y + [y]] = 2\cos x$

$\Rightarrow [y] + [y] = 2\cos x \quad (\because [x+n] = [x] + n$ if $n \in I)$

$\Rightarrow 2[y] = 2\cos x \Rightarrow [y] = \cos x$ (1)

Also $y = \frac{1}{3}\left[\sin x + [\sin x + [\sin x]]\right]$

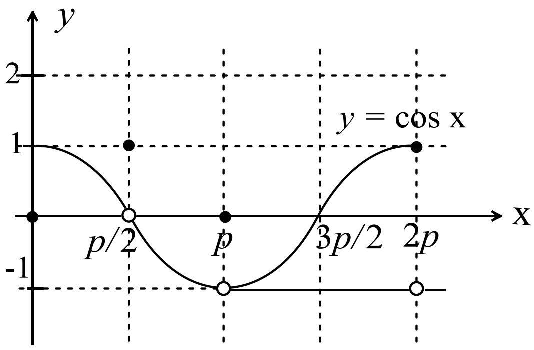
$= \frac{1}{3}(3[\sin x])$

$= [\sin x]$ (2)

From (1) and (2)

$[[\sin x]] = \cos x$

$\Rightarrow [\sin x] = \cos x$



The number of solutions is 0

61 **(c)**

$\frac{y-x}{y+x} = k(k > 1); y - x = k(y + x)$

$\Rightarrow y(1+k) = x(1+k)$

$\Rightarrow y = \left(\frac{1+k}{1-k}\right)x$, where $\frac{1+k}{1-k} < -1$

62 **(b)**

$g(x) = 1 + \{x\}, f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$ where $\{x\}$ represents the fractional part function.

$\Rightarrow f(g(x)) = \begin{cases} -1, & 1 + \{x\} < 0 \\ 0, & 1 + \{x\} = 0 \\ 1 & 1 + \{x\} > 0 \end{cases}$

$\Rightarrow f(g(x)) = 1, 1 + \{x\} > 0 (\because 0 \leq \{x\} < 1)$

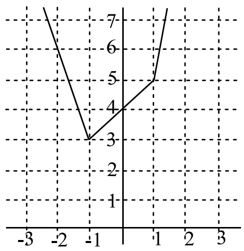
$\Rightarrow f(g(x)) = 1 \ \forall x \in R$

63 **(a)**

Let $f(x) = x + 2|x+1| + 2|x-1|$

$\Rightarrow f(x) = \begin{cases} x - 2(x+1) - 2(x-1), & x < -1 \\ x + 2(x+1) - 2(x-1), & -1 \leq x \leq 1 \\ x + 2(x+1) + 2(x-1), & x > 1 \end{cases}$

Or $f(x) = \begin{cases} -3x, & x < -1 \\ x + 4, & -1 \leq x \leq 1 \\ 5x, & x > 1 \end{cases}$



Graph of $y = f(x)$ is as shown. Clearly $y = k$ can intersect $y = f(x)$ at exactly one point only if $k = 3$

64 **(a)**

$f(7) + f(-7) = -10$

$\Rightarrow f(7) = -17$

$\Rightarrow f(7) + 17\cos x = -17 + 17\cos x$ which has the range $[-34, 0]$

65 **(c)**

We have $[\cos^{-1} x] \geq 0 \forall x \in [-1, 1]$

And $[\cot^{-1} x] \geq 0 \ \forall x \in R$

Hence, $[\cot^{-1} x] + [\cot^{-1} x] = 0$

$\Rightarrow [\cot^{-1} x] = [\cot^{-1} x] = 0$
If $[\cos^{-1} x] = 0 \Rightarrow x \in (\cos 1, 1]$
If $[\cot^{-1} x] = 0 \Rightarrow x \in (\cot 1, \infty)$
$\Rightarrow x \in (\cot 1, 1]$

66 **(c)**
Given
$f(x) = \sqrt{(1-\cos x)\sqrt{(1-\cos x)\sqrt{(1-\cos x)\sqrt{\ldots\infty}}}}$
$\Rightarrow f(x) = (1-\cos x)^{\frac{1}{2}}(1-\cos x)^{\frac{1}{4}}(1-\cos x)^{\frac{1}{8}}\ldots\infty$
$\Rightarrow f(x) = (1-\cos x)^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots\infty}$
$\Rightarrow f(x) = (1-\cos x)^{\frac{\frac{1}{2}}{1-\frac{1}{2}}}$
$\Rightarrow f(x) = 1-\cos x$
$\Rightarrow$ The range of $f(x)$ is $[0, 2)$

67 **(b)**
$\because f(x+1) - f(x) = 8x + 3$
$\Rightarrow \{b(x+1)^2 + c(x+1) + d\} - \{bx^2 + cx + d\} = 8x + 3$
$\Rightarrow b\{(x+1)^2 - x^2\} + c = 8x + 3$
$\Rightarrow b(2x+1) + c = 8x + 3$
On comparing co-efficient of x and constant term, we get $2b = 8$ and $b + c = 3$
Then $b = 4$ and $c = -1$

68 **(c)**
From the given data
$f(1-x) = f(1+x)$ (1)
And $f(2-x) = f(2+x)$ (2)
In (2) replacing x by $1+x$, we have
$f(1-x) = f(3+x)$
$\Rightarrow f(1+x) = f(3+x)$ [From (1)]
$\Rightarrow f(x) = f(2+x)$

69 **(b)**
$y = 2^{x(x-1)} \Rightarrow x^2 - x - \log_2 y = 0;$
$\Rightarrow x = \frac{1}{2}(1 \pm \sqrt{1 + 4\log_2 y})$
Since $x \epsilon [1, \infty)$, we choose
$x = \frac{1}{2}\left(1 + \sqrt{1 + 4\log_2 y}\right)$
Or $f^{-1}(x) = \frac{1}{2}\left(1 + \sqrt{1 + 4\log_2 x}\right)$

70 **(a)**
By checking for different function, we find that for $f(x) = \frac{1-x}{1+x}, f^{-1}(x) = f(x)$

71 **(c)**
$f(x)$ is to be defined when $x^2 - 1 > 0$ and $3 + x > 0$ and $3 + x \neq 1$
$\Rightarrow x^2 > 1$ and $x > -3$ and $x \neq -2$
$\Rightarrow x < -1$ or $x > 1$ and $x > -3$ and $x \neq -2$
$\therefore D_f = (-3, -2) \cup (-2, -1) \cup (1, \infty)$

72 **(a)**
Let $y = \frac{x+5}{x+2} = 1 + \frac{3}{x+2} \Rightarrow x = 1$
Also, $y - 1 = \frac{3}{x+2} \Rightarrow x + 2 = \frac{3}{y-1}$
$\Rightarrow x = -2 + \frac{3}{y-1}$
$\Rightarrow y = 2$ only as x and y are natural numbers

73 **(a)**
We have $f(x+y) + f(x-y)$
$= \frac{1}{2}[a^{x+y} + a^{-x-y} + a^{x-y} + a^{-x+y}]$
$= \frac{1}{2}[a^x(a^y + a^{-y}) + a^{-x}(a^y + a^{-y})]$
$= \frac{1}{2}(a^x + a^{-x})(a^y + a^{-y}) = 2f(x)f(y)$

74 **(b)**
In the sum, $f(1) + f(2) + f(3) + \cdots + f(n)$, 1 occurs n times, $\frac{1}{2}$ occurs $(n-1)$ times, $\frac{1}{3}$ occurs $(n-2)$ times and so on $\therefore f(1) + f(2) + f(3) + \cdots + f(n)$
$= n.1 + (n-1).\frac{1}{2} + (n-2).\frac{1}{3} + \cdots + 1.\frac{1}{n}$
$= n\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right) - \left(\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots + \frac{n-1}{n}\right)$
$= nf(n) - \left[\left(1 - \frac{1}{2}\right) + \left(1 - \frac{1}{3}\right) + \left(1 - \frac{1}{4}\right) + \cdots + \left(1 - \frac{1}{n}\right)\right]$
$= nf(n) - [n - f(n)]$
$= (n+1)f(n) - n$

75 **(b)**
$\cos^{-1}\left(\frac{2-|x|}{4}\right)$ exists if $-1 \leq \frac{2-|x|}{4} \leq 1$
$\Rightarrow -6 \leq -|x| \leq 2$
$\Rightarrow -2 \leq |x| \leq 6$
$\Rightarrow |x| \leq 6$
$\Rightarrow -6 \leq x \leq 6$
The function $[\log(3-x)]^{-1} = \frac{1}{\log(3-x)}$ is defined if $3 - x > 0$ and $x \neq 2$, i.e., if $x \neq 2$ and $x < 3$
Thus, the domain of the given function is $\{x | -6 \leq x \leq 6\} \cap \{x | x \neq 2, x < 3\} = [-6, 2) \cup (2, 3)$

76 **(b)**
Given $y = 2^{x(x-1)}$
$\Rightarrow x(x-1) = \log_2 y$
$\Rightarrow x^2 - x - \log_2 y = 0$
$\Rightarrow x = \frac{1 \pm \sqrt{1 + 4\log_2 y}}{2}$

Only $x = \frac{1 \pm \sqrt{1+4\log_2 y}}{2}$ lies in the domain

$\Rightarrow f^{-1}(x) = \frac{1}{2}\left[1 + \sqrt{1 + 4\log_2 x}\right]$

77 **(c)**

$f\left(x + \frac{1}{2}\right) + f\left(x - \frac{1}{2}\right) = f(x)$

$\Rightarrow\ f(x+1) + f(x) = f\left(x + \frac{1}{2}\right)$

$\Rightarrow\ f(x+1) + f\left(x - \frac{1}{2}\right) = 0$

$\Rightarrow\ f\left(x + \frac{3}{2}\right) = -f(x)$

$\Rightarrow\ f(x+3) = -f\left(x + \frac{3}{2}\right) = f(x)$

$\therefore\ f(x)$ is periodic with period 3

78 **(d)**

Given function is defined if ${}^{10}C_{x-1} > 3\,{}^{10}C_x$

$\Rightarrow \frac{1}{11-x} > \frac{3}{x} \Rightarrow 4x > 33$

$\Rightarrow x \geq 9$ but $x \leq 10 \Rightarrow x = 9, 10$

79 **(b)**

$f: N \to N, f(n) = 2n + 3$

Here, the range of the function is $\{5, 6, 7, \ldots\}$ or $N - \{1, 2, 3, 4\}$

Which is a subset of N (co-domain).

Hence, function is into.

Also, it is clear that $f(n)$ is one-one or injective.

Hence, $f(n)$ is injective only

80 **(c)**

$f(x) + 3xf\left(\frac{1}{x}\right) = 2(x+1)$ (1)

Replacing$x$ by $\frac{1}{x}$, we get

$f\left(\frac{1}{x}\right) + 3\frac{1}{x}f(x) = 2\left(\frac{1}{x} + 1\right)$

$\Rightarrow\ xf\left(\frac{1}{x}\right) + 3f(x) = 2(x+1)$ (2)

From (1) and (2), we have $f(x) = \frac{x+1}{2}$

$\Rightarrow f(99) = 50$

81 **(c)**

$$\sum_{r=1}^{2000} \frac{\{x+r\}}{2000} = \sum_{r=1}^{2000} \frac{\{x\}}{2000} = 2000\frac{\{x\}}{2000} = \{x\}$$

82 **(b)**

Here, $f: [0, \infty] \to [0, \infty)$ *ie*, domain is $[0, \infty)$ and codomain is $[0, \infty)$.

For one-one $f(x) = \frac{x}{1+x}$

$\Rightarrow \qquad f'(x) = \frac{1}{(1+x)^2} > 0, \forall\, x \in [0, \infty)$

$\therefore f(x)$ is increasing in its domain. Thus, $f(x)$ is one-one in its domain.

For onto (we find range)

$f(x) = \frac{x}{1+x}$ *ie*, $y = \frac{x}{1+x} \Rightarrow y + yx = x$

$\Rightarrow x = \frac{y}{1-y} \Rightarrow \frac{y}{1-y} \geq 0$ as $x \geq 0 \therefore 0 \leq y \neq 1$

ie, Range $\neq$ Codomain

$\therefore f(x)$ is one-one but not onto.

83 **(a)**

$f(f(x)) = \begin{cases} f(x), & f(x) \text{ is rational} \\ 1 - f(x), & f(x) \text{ is irrational} \end{cases}$

$\Rightarrow\ f(f(x)) = \begin{cases} x, & x \text{ is rational} \\ 1 - (1 - x) = x, & x \text{ is irrational} \end{cases}$

84 **(b)**

$-5 \leq |kx + 5| \leq 7$

$\Rightarrow -12 \leq kx \leq 2$ where $-6 \leq x \leq 1$

$\Rightarrow -6 \leq \frac{k}{2}x \leq 1$ where $-6 \leq x \leq 1$

$\therefore k = 2$ $\quad$ [$\because$ the range of $h(x)$ = the domain of $f(x)$]

85 **(d)**

Let $\phi(x) = f(x) - g(x)$

$= \begin{cases} x, x \in Q \\ -x, x \notin Q \end{cases}$

For one-one

Take any straight line parallel to x-axis which will intersect $\phi(x)$ only at one point.

$\Rightarrow \phi(x)$ is one-one.

Foe onto

As, $\phi(x) = \begin{cases} x, x \in Q \\ -x, x \notin Q \end{cases}$, which shows

$y = x$ and $y = -x$ for irrational values $\Rightarrow y \notin$ real numbers.

$\therefore$ Range=Codomain

$\Rightarrow \quad \phi(x)$ is onto.

Thus, $f - g$ is one-one and onto.

86 **(c)**

$f(x) = \log|\log x|, f(x)$ is defined if $|\log x| > 0$ and $x > 0$ i.e., if $x > 0$ and $x \neq 1$ ($\because |\log x| > 0$ if $x \neq 1$)

$\Rightarrow x \in (0, 1) \cup (1, \infty)$

87 **(a)**

Since $f(x)$ and $f^{-1}(x)$ are symmetric about the line $y = -x$

If (α, β) lies on $y = f(x)$ then $(-\beta, -\alpha)$ on $y = f^{-1}(x)$

$\Rightarrow (-\alpha, -\beta)$ lies on $y = f(x)$

$\Rightarrow y = f(x)$ is odd

88 **(d)**

Let $2x + \frac{y}{8} = \alpha$ and $2x - \frac{y}{8} = \beta$, then $x = \frac{\alpha+\beta}{4}$ and $y = 4(\alpha - \beta)$

Given, $f\left(2x + \frac{y}{8}, 2x - \frac{y}{8}\right) = xy$

$\Rightarrow f(\alpha, \beta) = \alpha^2 - \beta^2$

$\Rightarrow f(m, n) + f(n, m) = m^2 - n^2 + n^2 - m^2 = 0$

for all m, n

89 **(d)**

$\sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right)$

$= \sin^2 x + \left(\frac{\sin x}{2} + \frac{\sqrt{3}\cos x}{2}\right)^2 + \cos x\left(\frac{\cos x}{2} - \frac{\sqrt{3}\sin x}{2}\right)$

$= \sin^2 x + \frac{\sin^2 x}{4} + \frac{3\cos^2 x}{4} + \frac{\cos^2 x}{2}$

$= \frac{5\sin^2 x}{4} + \frac{5\cos^2 x}{4} = 5/4$

Hence, $f(x) = c^{5/4}$ = constant, which is periodic whose period cannot be determined

90 **(d)**

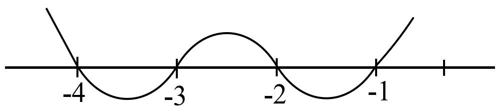
$f(x) = \frac{n(n+1)}{2} + [\sin x] + \left[\sin\frac{x}{2}\right] + \cdots + \left[\sin\frac{x}{n}\right]$

Thus, the range of $f(x) = \left\{\frac{n(n+1)}{2}, \frac{n(n+1)}{2} + 1\right\}$ as $x \in [0, \pi]$

91 **(a)**

Let $g(x) = (x+1)(x+2)(x+3)(x+4)$

The rough graph of $g(x)$ is given as



$\therefore g(x) = (x+1)(x+2)(x+3)(x+4)$

$= (x+1)(x+4)(x+2)(x+3)$

$= (x^2+5x+4)(x^2+5x+6)$

$= t(t+2) = (t+1)^2 - 1,$

Where $t = x^2 + 5x$

Now $g_{min} = -1$, for which $x^2 + 5x - 1$ has real roots in $[-6, 6]$

Also $g(6) = 7 \times 8 \times 9 \times 10 = 5040$

Hence, the range of $g(x)$ is $[-1, 5040]$ for $x \in [-6, 6]$

Then, the range of $f(x)$ is $[4, 5045]$

92 **(d)**

$f(x) = \ln\left(\frac{x^2+e}{x^2+1}\right) = \ln\left(\frac{x^2+1+e-1}{x^2+1}\right)$

$= \ln\left(1 + \frac{e-1}{x^2+1}\right)$

Now, $1 \le x^2 + 1 < \infty$

$\Rightarrow 0 < \frac{1}{x^2+1} \le 1 \Rightarrow 0 < \frac{e-1}{x^2+1} \le e - 1$

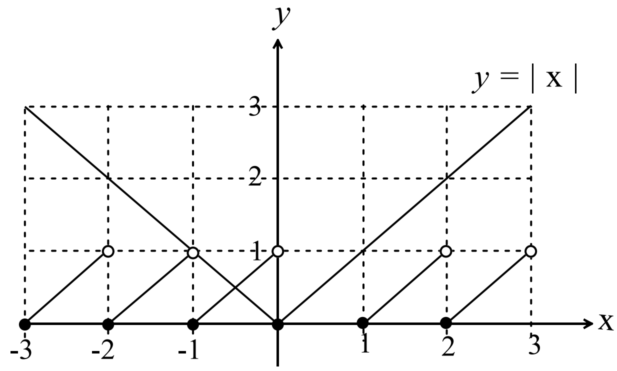
$\Rightarrow 1 < 1 + \frac{e-1}{x^2+1} \le e \Rightarrow 0 < \ln\left(1 + \frac{e-1}{x^2+1}\right) \le 1$

Hence, the range is $(0, 1]$

93 **(c)**

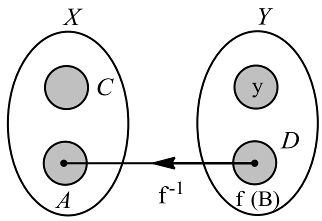
$f(x) = \sqrt{|x| - \{x\}}$ is defined if $|x| \ge \{x\}$

$\Rightarrow x \in \left(-\infty - \frac{1}{2}\right] \cup [0, \infty) \Rightarrow Y \in [0, \infty)$



94 **(c)**

The given data is shown in the figure below



Since, $f^{-1}(D) = x$

$\Rightarrow f(x) = D$

Now, if $B \subset X, f(B) \subset D$

$\Rightarrow f^{-1}(f(B)) = B$

95 **(d)**

The equation is $x^2 + 2ax + \frac{1}{16} = -a + \sqrt{a^2 + x - \frac{1}{16}}$

$\Rightarrow f(x) = f^{-1}(x)$

Which has the solution if $x^2 + 2ax + \frac{1}{16} = x$

$\Rightarrow x^2 + (2a-1)x + \frac{1}{16} = 0$

For real and distinct roots $(2a-1)^2 - 4\frac{1}{16} \ge 0$

$\Rightarrow 2a - 1 \le \frac{-1}{2}$ or $2a - 1 \ge \frac{1}{2} \Rightarrow a \le \frac{1}{4}$ or $a \ge \frac{3}{4}$

96 **(b)**

$f(x) = x^n + 1$

$\Rightarrow f(3) = 3^n + 1 = 28$

$\Rightarrow 3^n = 27$

$\therefore n = 3$

$\Rightarrow f(4) = 4^3 + 1 = 65$

97 **(a)**

$f(x + f(y)) = f(x) + y, f(0) = 1$

Putting $y = 0$, we get $f(x + f(0)) = f(x) + 0$

$\Rightarrow f(x+1) = f(x) \forall x \in R$

Thus, $f(x)$ is the period with 1 as one of its period

$\Rightarrow f(7) = f(6) = f(5) = \cdots = f(1) = (0) = 1$

98 **(d)**

$f(x) = \cos(\log x)$

$$\Rightarrow f(x)f(y)-\frac{1}{2}\left[f\left(\frac{x}{y}\right)+f(xy)\right]$$

$$=\cos(\log x)\cos(\log y)-\frac{1}{2}[\cos(\log x-\log y)]$$
$$+\cos(\log x+\log y)$$
$$=\cos(\log x)\cos(\log y)$$
$$-\frac{1}{2}[2\cos(\log x)\cos(\log y)]$$
$$=0$$

99 **(c)**
From the given data $g(x)$ must be linear function
Hence, $g(x)=ax+b$
Also $g(2)=2a+b=3$ and $g(4)=4a+b=7$
Solving, we get $a=2$ and $b=-1$
Hence, $g(x)=2x-1$
Then, $g(6)=11$

100 **(b)**
$x^2F(x)+F(1-x)=2x-x^4$ (1)

Replacing x by $1-x$, we get
$\Rightarrow (1-x)^2F(1-x)+F(x)=2(1-x)-(1-x)^4$ (2)
Eliminating $F(1-x)$ from (1) and (2), we get
$F(x)=1-x^2$

101 **(b)**
$[x]^2=x+2\{x\}$
$\Rightarrow [x]^2=x+3\{x\}$
$$\Rightarrow \{x\}=\frac{[x]^2-[x]}{3}$$
$$\Rightarrow 0\le\frac{[x]^2-[x]}{3}<1$$
$\Rightarrow 0\le[x]^2-[x]<3$
$$\Rightarrow [x]\in\left(\frac{1-\sqrt{3}}{2},0\right]\cup\left[1,\frac{1+\sqrt{3}}{2}\right)$$
$\Rightarrow [x]=-1,0,1,2$
$\Rightarrow \{x\}=\frac{2}{3},0,0,\frac{2}{3},$ (respectively)
$$\Rightarrow x=-\frac{1}{3},0,1,\frac{8}{3}$$

102 **(c)**
$$y=\frac{1}{\log_{10}(1-x)}+\sqrt{x-2}$$
$y=f(x)+g(x)$
Then, the domain of given function is $D_f\cap D_g$
Now, for the domain of $f(x)=\frac{1}{\log_{10}(1-x)}$,
We know it is defined only when $1-x>0$ and $1-x\ne 1$
$\Rightarrow x<1$ and $x\ne 0 \therefore D_f=(-\infty,1)-\{0\}$
For the domain of $g(x)=\sqrt{x+2}$
$x+2\ge 0\Rightarrow x\ge -2$
$\therefore D_g=[-2,1)-\{0\}$

103 **(d)**
We have $fog(x)=f(g(x))=\sin(\log_e|x|)$
$\log_e|x|$ has range R, for which
$\sin(\log_e|x|)\,\epsilon[-1,1]$
$\therefore R_1=\{u:-1\le u\le 1\}$
Also $gof(x)=g(f(x))=\log_e|\sin x|$
$\because 0\le|\sin x|\le 1$
$\Rightarrow -\infty<\log_e|\sin x|\le 0$
$\Rightarrow R_2=\{v:-\infty<v\le 0\}$

104 **(d)**
$f(x)=\alpha x^3-\beta x-(\tan x)\text{sgn } x$
$f(-x)=f(x)$
$\Rightarrow -\alpha x^3+\beta x-\tan x\text{ sgn } x=\alpha x^3-\beta x-(\tan x)(\text{sgn } x)$
$\Rightarrow 2(-\alpha x^2-\beta)x=0\forall x\in R$
$\Rightarrow \alpha=0$ and $\beta=0$
$\therefore [a]^2-5[a]+4=0$ and $6\{a\}^2-5\{a\}+1=0$
$\Rightarrow (3\{x\}-1)(2\{x\}-1)=0$
$$\therefore a=1+\frac{1}{3},1+\frac{1}{2},4+\frac{1}{3},4+\frac{1}{2}$$
Sum of values of $a=\frac{35}{3}$

105 **(d)**
$\because \{x\}\in[0,1)$
$\sin x\in(0,\sin 1)$ as $f(x)$ is defined if $\sin\{x\}\ne 0$
$$\Rightarrow \frac{1}{\sin\{x\}}\in\left(\frac{1}{\sin 1},\infty\right)\Rightarrow\left[\frac{1}{\sin\{x\}}\right]\in\{1,2,3,\dots\}$$
Note that $1<\frac{\pi}{3}\Rightarrow\sin 1<\sin\frac{\pi}{3}=0.866\Rightarrow\frac{1}{\sin 1}>1.155$.

106 **(c)**
$f(x)=[6x+7]+\cos\pi x-6x$
$=[6x]+7+\cos\pi x-6x$
$=7+\cos\pi x-\{6x\}$
$\{6x\}$ has the period 1/6 and $\cos\pi x$ has the period 2, then the period of $f(x)=$ LCM of 2 and 1/6 which is 2
Hence, the period is 2

107 **(a)**
Given $f(x+y)=f(x)f(y)$ Put $x=y=0$, then $f(0)=1$
Put $y=-x$, then $f(0)=f(x)f(-x)\Rightarrow f(-x)=\frac{1}{f(x)}$
Now, $g(x)=\frac{f(x)}{1+\{f(x)\}^2}$
$$\Rightarrow g(-x)\frac{f(-x)}{1+\{f(-x)\}^2}=\frac{\frac{1}{f(x)}}{1+\frac{1}{\{f(x)\}^2}}$$
$$=\frac{f(x)}{1+\{f(x)\}^2}=g(x)$$

108 **(b)**
We have $f(x-y)=f(x)f(y)-f(a-x)f(a+$

$y)$
Putting $x = a$ and $y = a - x$, we get
$f(a-(x-a) = f(a)f(x-a) - f(0)f(x)$ (1)
Putting $x = 0, y = 0$, we get
$f(0) = f(0)(f(0)) - f(a)f(a)$
$\Rightarrow f(0) = (f(0))^2 - (f(a))^2$
$\Rightarrow 1 = (1)^2 - (f(a))^2$
$\Rightarrow f(a) = 0$
$\Rightarrow f(2a - x) = -f(x)$

109 **(c)**
(a) $f(x) = \sin x$ and $g(x) = \cos x, x \in [0, \pi/2]$

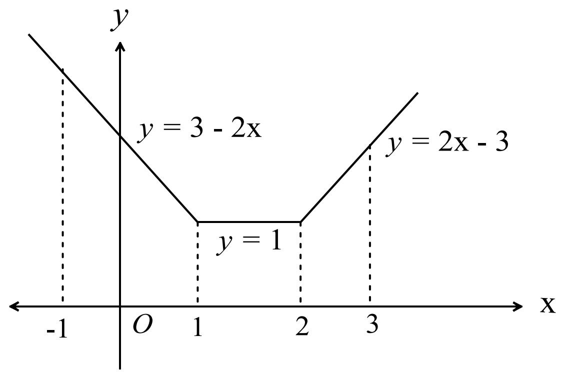
Here, both $f(x)$ and $g(x)$ are one-one functions

But $h(x) = f(x) + g(x) = \sin x + \cos x$ is many-one as $h(0) = h(\pi/2) = 1$

(b) $h(x) = f(x)g(x) = \sin x \cos x = \frac{\sin 2x}{2}$ is many-one, as $h(0) = h(\pi/2) = 0$

(c) It is a fundamental property

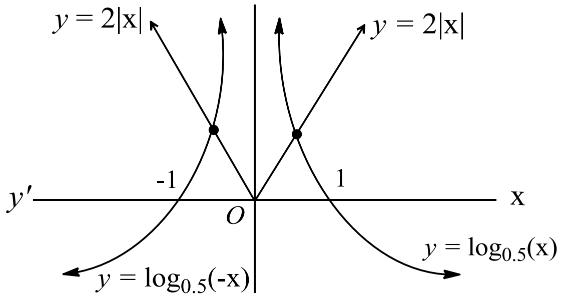
111 **(b)**



Clearly, from the graph, the range is $[1, f(-1)] \equiv [1, 5]$
If $x < 1, f(x) = -(x-1) - (x-2) = -2x + 3$.
In this interval, $f(x)$ is decreasing
If $1 \le x < 2, f(x) = x - 1 - (x-2) = 1$
In this interval, $f(x)$ is constant
If $2 \le x \le 3, f(x) = x - 1 + x - 2 = 2x - 3$
In this interval, $f(x)$ is increasing
$\therefore \max f(x) =$ the greatest among $f(-1)$ and $f(3) = 5$, $\min f(x) = f(1) = 1$
So, the range $= [1, 5]$

112 **(d)**
$f(x) = |x - 1|$
$\Rightarrow f(x)^2 = |x^2 - 1|$ and $(f(x))^2 = |x-1|^2 = x^2 - 2x + 1$
$\Rightarrow f(x^2) \ne (f(x))^2$
Hence, option **a** is not true.
$f(x+y) = f(x) + f(y) \Rightarrow |x + y - 1| = |x - 1| + |y - 1|$, Which is absurd. Put $x = 2, y = 3$ and verify.
Hence, option **b** is true.
Consider $f(|x|) = |f(x)|$
Put $x = -5$, then $f(|-5|) = f(5) = 4$ and $|f(-5)| = |-5 - 1| = 6$.
$\therefore$ **c** is not correct.

113 **(b)**
Draw the graph of $y = \log_{0.5}|x|$ and $y = 2|x|$



Clearly, from the graph, there are two solutions

114 **(b)**
$$3f(x) + 2f\left(\frac{x+59}{x-1}\right) = 10x + 30$$
For $x = 7, 3f(7) + 2f(11) = 70 + 30 = 100$
For $x = 11, 3f(11) + 2f(7) = 140$
$\frac{f(7)}{-20} = \frac{f(11)}{-220} = \frac{-1}{9-4} \Rightarrow f(7) = 4$

115 **(d)**
$f(x) - 1 + f(1-x) - 1 = 0$ so $g(x) + g(1-x) = 0$
Replacing x by $x + \frac{1}{2}$, we get $g\left(\frac{1}{2} + x\right) + g\left(\frac{1}{2} - x\right) = 0$
So, it is symmetrical about $\left(\frac{1}{2}, 0\right)$

116 **(c)**
$\cos^{-1}\left(\frac{1+x^2}{2x}\right)$ is defined if $\left|\frac{1+x^2}{2x}\right| \le 1$ and $x \ne 0$
$\Rightarrow 1 + x^2 - 2|x| \le 0$
$\Rightarrow (|x| - 1)^2 \le 0$
$\Rightarrow x = 1, -1$
Thus, the domain of $f(x)$ is $\{1, -1\}$ Hence, the range is $\{1, 1 + \pi\}$

117 **(a)**
Putting $x = 1, f(2) + f(0) = 2f(1) \Rightarrow f(2) = 2f(1)$
Putting $x = 2, f(3) + f(1) = 2f(2)$
$\Rightarrow f(3) = 2 \times 2f(1) - f(1) = 3f(1)$, and so on
$\therefore f(n) = nf(1)$, for $n = 1, 2, \dots, n$
$f(n+1) + f(n-1) = 2f(n)$
$\Rightarrow f(n+1) + (n-1)f(1) = 2nf(1)$
$\Rightarrow f(n+1) = (n+1)f(1)$

118 **(d)**

$f(x)=\frac{1}{\sqrt{\{\sin x\}+\{\sin(\pi+x)\}}}=\frac{1}{\sqrt{\{\sin x\}+\{-\sin x\}}}$

Now

$\{\sin x\}+\{-\sin x\}=\begin{cases}0, & \sin x \text{ is an integer}\\ 1, & \sin x \text{ is not an integer}\end{cases}$

For $f(x)$ to get defined $\{\sin x\}+\{-\sin x\}\neq 0$

$\Rightarrow \sin x \neq$ integer

$\Rightarrow \sin x \neq \pm 1, 0$

$\Rightarrow x \neq \frac{n\pi}{2}, n \in I$

Hence, the domain is $R-\left\{\frac{n\pi}{2}/n \in I\right\}$

119 **(b)**

We have $f(x)=\left[\log_{10}\left(\frac{5x-x^2}{4}\right)\right]^{1/2}$ (1)

From (1), clearly $f(x)$ is defined for those values of x for which $\log_{10}\left[\frac{5x-x^2}{4}\right]\geq 0$

$\Rightarrow \left(\frac{5x-x^2}{4}\right)\geq 10^0$

$\Rightarrow \left(\frac{5x-x^2}{4}\right)\geq 1$

$\Rightarrow x^2-5x+4\leq 0$

$\Rightarrow (x-1)(x-4)\leq 0$

Hence, the domain of the function is $[1,4]$

120 **(b)**

$f(x)=\left|\sin^3\frac{x}{2}\right|+\left|\cos^5\frac{x}{5}\right|$

The period of $\sin^3 x$ is 2π

$\Rightarrow$The period of $\sin^3\frac{x}{2}$ is $\frac{2\pi}{1/2}=4\pi$

$\Rightarrow$ The period of $\sin^3\frac{x}{2}$ is 2π

The period of $\cos^5 x$ is 2π

$\Rightarrow$ The period of $\cos^5\frac{x}{5}$ is $\frac{2\pi}{\left(\frac{1}{5}\right)}=10\pi$

$\Rightarrow$ The period of $\left|\cos^5\frac{x}{2}\right|$ is 5π

Now the period of $f(x)=$ LCM of $\{2\pi,10\pi\}=10\pi$

121 **(b)**

$y=x^2+(k-1)x+9$

$=\left(x+\frac{k-1}{2}\right)^2+9-\left(\frac{k-1}{2}\right)^2$

For entire graph to be above x-axis we should have $9-\left(\frac{k-1}{2}\right)^2>0$

$\Rightarrow k^2-2k-35<0 \Rightarrow (k-7)(k+5)<0$

$\Rightarrow -5<k<7$

122 **(d)**

$|\cos x|+\cos x=\begin{cases}0, & \cos x\leq 0\\ 2\cos x, & \cos x>0\end{cases}$

For $f(x)$ to defined $\cos x>0$

$\Rightarrow x\in\left(\frac{(4n-1)\pi}{2},\frac{(4n+1)\pi}{2}\right) n\in Z$ (1^{st} and 4^{th} quadrant)

123 **(c)**

$f(x)=\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)+2\sqrt{2}$

Or $f(x)=\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)+2\sqrt{2}$

$\Rightarrow Y=[\sqrt{2},3\sqrt{2}]$ and $X=\left[-\frac{3\pi}{4},\frac{\pi}{4}\right]$ or $\left[\frac{\pi}{4},\frac{5\pi}{4}\right]$

124 **(c)**

$f(x)=\frac{1}{x}, g(x)=\frac{1}{x^2}$ and $h(x)=x^2$

$f(g(x))=x^2, x\neq 0$

$h(g(x))=\frac{1}{x^4}=(g(x))^2, x\neq 0$

125 **(c)**

$f(2x+3)+f(2x+7)=2$ (1)

Replace x by $x+2$, $f(2x+7)+f(2x+11)=2$ (2)

From (1) − (2) we get $f(2x+3)-f(2x+11)=0$

$\Rightarrow f(2x+3)=f(2x+11)$

$\Rightarrow f(2x+3)=f(2(x+4)+3)$

$\Rightarrow$ Period of $f(x)$ is 8

126 **(d)**

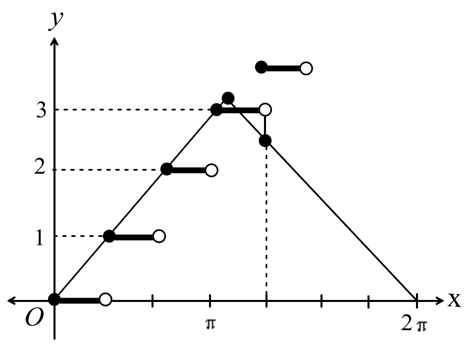
Here $x+3>0$ and $x^2+3x+2\neq 0$

$\therefore x>-3$ and $(x+1)(x+2)\neq 0$, i.e., $x\neq -1,-2$

$\therefore$ The domain $=(-3,\infty)-\{-1,-2\}$

127 **(a)**

$\cos^{-1}(\cos x)=[x]$



The solutions are clearly 0, 1, 2, 3, and $3=2\pi-x$ or $x=2\pi-3$

128 **(d)**

$\Rightarrow f^{-1}(x)=\frac{x+\sqrt{x^2-4}}{2}$

For domain of $f(x)=\frac{\log_2(x+3)}{x^2+3x+2}$

$x^2+3x+2\neq 0$ and $x+3>0$

$\Rightarrow x\neq -1,-2$ and $x>-3$

$\therefore D_f=(-3,\infty)-\{-1,-2\}$

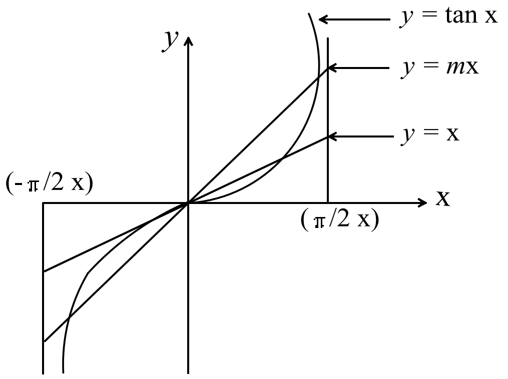
129 **(b)**

$f(x)=[x]+[2x]+[3x]+\cdots+[nx]-(x+2x+3x+\cdots nx)$

$=-(\{x\}+\{2x\}+\{3x\}+\cdots+\{nx\})$

The period of $f(x)=LCM\left(1,\frac{1}{2},\frac{1}{3},\ldots,\frac{1}{n}\right)=1$

130 **(c)**



In $\left(-\frac{\pi}{2},0\right)$, the graph of $y=\tan x$ lies below the line $y=x$ which is the tangent at $x=0$ and in $\left(0,\frac{\pi}{2}\right)$ it lies above the lies $y=x$

For $m>1$, the line $y=mx$ lies below $y=x$ in $\left(-\frac{\pi}{2},0\right)$ and above $y=x$ in $\left(0,\frac{\pi}{2}\right)$ Thus graphs of $y=\tan x$ and $y=mx, m>1$, meet at three points including $x=0$ in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ independent of m

131 **(a)**

We must have $ax^3+(a+b)x^2+(b+c)x+c>0$

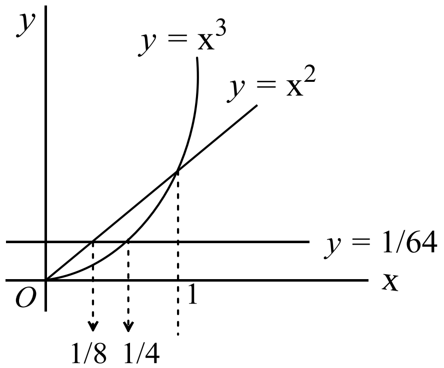
$\Rightarrow ax^2+(x+1)+bx(x+1)+c(x+1>0)$

$\Rightarrow (x+1)(ax^2+bx+c)>0$

$\Rightarrow a(x+1)\left(x+\frac{b}{2a}\right)^2>0$ as $b^2=4ac$

$\Rightarrow x>-1$ and $\neq \frac{b}{2a}$

132 **(c)**



Clearly, from the graph $f(x)=\begin{cases}\frac{1}{64}, & 0\le x\le\frac{1}{8}\\ x^2, & \frac{1}{8}<x\le 1\\ x^3, & x>1\end{cases}$

133 **(c)**

The period of $\cos(\sin nx)$ is $\frac{\pi}{n}$ and the period of $\tan\left(\frac{x}{n}\right)$ is πn

Thus, $6\pi=\text{LCM}\left(\frac{\pi}{n},\pi n\right)$

$\Rightarrow 6\pi=\frac{\pi}{n}\lambda_1 \Rightarrow n=\frac{\lambda_1}{6}, and\ 6\pi=\lambda_2\pi n\Rightarrow n$

$=\frac{6}{\lambda_2}, \lambda_1,\lambda_2\in I^+$

From $n=\frac{6}{\lambda_2}\Rightarrow n=6,3,2,1$

Clearly, for $n=6$, we get the period of $f(x)$ to be 6π

134 **(c)**

Case I

$0<|x|-1<1\Rightarrow 1<|x|<2$, then

$x^2+4x+4\le 1$

$\Rightarrow x^2+4x+3\le 0$

$\Rightarrow -3\le x\le -1$

So $x\in(-2,-1)$ (1)

Case II

$|x|-1>1\Rightarrow |x|>2$, then $x^2+4x+4\ge 1$

$\Rightarrow x^2+4x+3\ge 0$

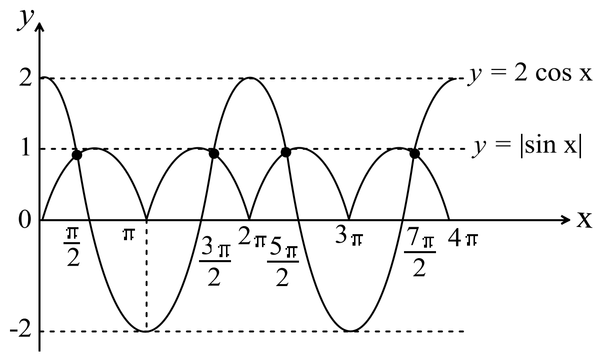
$\Rightarrow x\ge -1$ or $x\le -3$

So, $x\in(-\infty,-3]\cup(2,\infty)$ (2)

From (1) and (2), $x\in(-\infty,-3]\cup(-2,-1)\cup(2,\infty)$

135 **(c)**

See the graph of $y=2\cos x$ and $y=|\sin x|$, their points of intersection represent the solution of the given equation



We find that the graphs intersect at four points. Hence, the equation has four solutions

136 **(d)**

$f(x)=e^{x^3-3x+2}$

Let $g(x)=x^3-3x+2; g'(x)$

$=3x^2-3=3(x^2-1)$

$g'(x)\ge 0$ for $x\in(-\infty,-1]$

$\therefore f(x)=$ is increasing function $\therefore f(x)$ is one-one

Now, the range of $f(x)=(0,e^4]$

But co-domain is $(0,e^5]$ $\therefore f(x)$ is an into function

137 **(a)**

$F(n+1)=\frac{2F(n)+1}{2}\Rightarrow F(n+1)-F(n)=\frac{1}{2}$

Put $n=1,2,3,\dots,100$ and add, we get

$F(101)-F(1)=100\times\frac{1}{2}$

$\Rightarrow F(101)=52$ $[\because F(1)=2]$

138 **(a)**

$|x-2|+a=\pm 4$

$\Rightarrow |x-2| = \pm 4 - a$
For 4 real roots, $4 - a > 0$ and $-4 - a > 0$
$\Rightarrow a \in (-\infty, -4)$

139 **(a)**
$h(x) = \log(f(x).\mathrm{g}(x)) = \log e^{\{y\}+[y]} = \{y\} + [y] = e^{|x|}\operatorname{sgn} x$

$\therefore h(x) = e^{|x|}\operatorname{sgn} x = \begin{cases} e^x, & x > 0 \\ 0, & x = 0 \\ -e^{-x}, & x < 0 \end{cases}$

$\Rightarrow h(-x) = \begin{cases} e^{-x}, & x < 0 \\ 0, & x = 0 \\ -e^{x}, & x > 0 \end{cases} \Rightarrow h(x) = h(-x) = 0$ for all x

140 **(a)**
Given $f(x) = \sqrt[n]{x^m}, n \in N$ is an even function where $m \in I$
$\Rightarrow f(x) = f(-x)$
$\Rightarrow \sqrt[n]{x^m} = \sqrt[n]{(-x)^m}$
$\Rightarrow x^m = (-x)^m$
$\Rightarrow m$ is an even integer
$\Rightarrow m = 2k, k \in I$

141 **(c)**
$\frac{1}{2}(\mathrm{g}of)(x) = 2x^2 - 5x + 2$ or $\frac{1}{2}\mathrm{g}[f(x)]$
$= 2x^2 - 5x + 2$
$\therefore [\{f(x)\}^2 + \{f(x)\} - 2] = 2[2x^2 - 5x + 2]$
$\Rightarrow f(x)^2 + f(x) - (4x^2 - 10x + 6) = 0$
$\therefore f(x) = \frac{-1 \pm \sqrt{1 + 4(4x^2 - 10x + 6)}}{2}$
$= \frac{-1 \pm \sqrt{(16x^2 - 40x + 25)}}{2} = \frac{-1 \pm (4x-5)}{2} = 2x - 3$ or $-2x + 2$

142 **(d)**
$\log_3(x^2 - 6x + 11) \le 1$
$\Rightarrow 0 < x^2 - 6x + 11 \le 3$
$\Rightarrow x \in [2, 4]$

143 **(b)**
Given $f(x+y) = f(x) + f(y) - xy - 1 \forall x, y \in R$
$f(1) = 1$
$f(2) = f(1+1) = f(1) + f(1) - 1 - 1 = 0$
$f(3) = f(2+1) = f(2) + f(1) - 2 \quad 1 - 1 = -2$
$f(n+1) = f(n) + f(1) - n - 1 = f(n) - n < f(n)$
Thus, $f(1) > f(2) > f(3) >$... and $f(1) = 1$
$\therefore f(1) = 1$ and $f(n) < 1$, for $n > 1$
Hence, $f(n) = n, n \in N$ has only one solution $n = 1$

144 **(c)**
$\mathrm{g}(x) = x^3 + \tan x + \left[\frac{x^2+1}{P}\right]$

$\Rightarrow \mathrm{g}(-x) = (-x)^3 + \tan(-x) + \left[\frac{(-x)^2+1}{P}\right]$

$\Rightarrow \mathrm{g}(-x) = -x^3 - \tan x + \left[\frac{x^2+1}{P}\right]$

$\Rightarrow \mathrm{g}(x) + \mathrm{g}(-x) = 0$
Because $\mathrm{g}(x)$ is a odd function

$\therefore \left(-x^3 - \tan x + \left[\frac{x^2+1}{P}\right]\right) + \left(-x^3 - \tan x + \left[\frac{x^2+1}{P}\right]\right) = 0$

$\Rightarrow 2\left[\frac{(x^2+1)}{P}\right] = 0 \Rightarrow 0 \le \frac{x^2+1}{P} < 1$

Now $x \in [-2, 2]$

$\Rightarrow 0 \le \frac{5}{P} < 1 \Rightarrow P > 5$

145 **(b)**
Two triangles may have equal areas
$\therefore f$ is not one-one
Since each positive real number can represent area of a triangle
$\therefore f$ is onto

146 **(c)**
Let $f(x) = bx^2 + ax + c$
Since, $f(0) = 0 \Rightarrow c = 0$
And $f(1) = 0 \Rightarrow a + b = 1$
$\therefore \quad f(x) == ax + (1-a)x^2$
Also, $f'(x) > 0$ for $x \in (0, 1)$
$\Rightarrow \quad a + 2(1-a)x > 0 \quad \Rightarrow \quad a(1-2x) + 2x > 0$
$\Rightarrow \quad a > \frac{2x}{2x-1} \quad \Rightarrow \quad 0 < a < 2$
Since, $x \in (0, 1)$
$\therefore \quad f(x) = ax + (1-a)x^2; 0 < a < 2$

147 **(d)**
$f(x)$ is continuous for all $x > 0$ and $f\left(\frac{x}{y}\right) = f(x) - f(y)$
Also $f(e) = 1$
$\Rightarrow$ Clearly, $f(x) = \log_e x$ satisfies all these properties.
$\therefore f(x) = \log_e x$, which is an unbounded function.

148 **(a, d)**
Given $f(x) + f(y) = \left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$ (1)
Replace y by $x \Rightarrow 2f(x) = f\left(2x\sqrt{1-x^2}\right)$
$3f(x) = f(x) + 2f(x)$
$= f(x) + f\left(2x\sqrt{1-x^2}\right)$
$= f\left(x\sqrt{1-4x^2(1-x^2)} + 2x\sqrt{1-x^2}\sqrt{1-x^2}\right)$
$= f\left(x\sqrt{(2x^2-1)^2} + 2x(1-x^2)\right)$
$= f(x|2x^2-1| + 2x - 2x^3)$
$= f(2x^3 - x + 2x - 2x^3)$ or $f(x - 2x^3 + 2x -$

$2x^3)$

$= f(x)$ or $f(3x - 4x^3)$

$\Rightarrow f(x) = 0$ or $3f(x) = f(3x - 4x^3)$

Consider $3f(x) = f(3x - 4x^3)$

Replace x $by -x$, we get

$3f(-x) = f(4x^3 - 3x)$ (2)

Also from (1), $f(x) + f(-x) = f(0)$

Put $x = y = 0$ in (1), we have $f(0) = 0 \Rightarrow f(x) + f(-x) = 0$, thus $f(x)$ is an odd function

Now from $(2) - 3f(x) = f(4x^3 - 3x)$

$\Rightarrow f(4x^3 - 3x) + 3f(x) = 0$

149 **(a, c)**

$f(x) = \cos[\pi^2]x + \cos[-\pi^2]x$,

We know $9 < \pi^2 < 10$ and $-10 < -\pi^2 < -9$

$\Rightarrow [\pi^2] = 9$ and $[-\pi^2] = -10$

$\Rightarrow f(x) = \cos 9x + \cos(-10x)$

$\Rightarrow f(x) = \cos 9x + \cos 10x$

a. $f\left(\frac{\pi}{2}\right) = \cos\frac{9\pi}{2} + \cos 5\pi = -1$ (true)

b. $f(x) = \cos 9\pi + \cos 10\pi = -1 + 1 = 0$ (false).

c. $f(-\pi) = \cos(-9\pi) + \cos(-10\pi) = \cos 9\pi + \cos 10\pi$

$= -1 + 1 = 0$ (true)

d. $f\left(\frac{\pi}{4}\right) = \cos\frac{9\pi}{4} + \cos\frac{5\pi}{4} = \cos\left(2\pi + \frac{\pi}{4} + 0\right)$ (false)

Thus, **a** and **c** are correct options

150 **(a, b, c)**

$f(x) = \tan(\tan^{-1} x) = x$ for all x and

$g(x) = \cot(\cot^{-1} x) = x$ for all x

Hence, this pair is identical functions

$f(x) = \text{sgn}(x)$ and $g(x) = \text{sgn}(\text{sgn}(x))$ have domain R

$f(x)$ has range $\{-1, 0, 1\}$ and $g(x) = \text{sgn}(\text{sgn}(x))$ has range $\{-1, 0, 1\}$

Also $f(x) = g(x)$ for any x, then this pair is identical functions

$g(x) = \cot^2 x - \cos^2 x = \cos^2 x(\text{cosec}^2 x - 1)$

$= \cos^2 x \cot^2 x = f(x)$

$f(x) = e^{\log_e \sec^{-1} x}$ has the domain $[1, \infty)$, whereas $g(x) = \sec^{-1} x$ has the domain $(-\infty, -1] \cup [1, \infty)$

Hence, this pair is not identical functions

151 **(a, b, d)**

$f(x) = \frac{1}{\ln[1-|x|]}$ is defined if $[1 - |x|] > 0$ and $1 - [x] \neq 1$

$\Rightarrow [1 - |x|] \geq 2 \Rightarrow 1 - |x| \geq 2 \Rightarrow |x| \leq -1$ which is not possible

$f(x) = \frac{x!}{\{x\}}$ Hence $x!$ is defined only when x is natural number, but $\{x\}$ becomes zero for these values of x Hence, $f(x)$ is not defined in this case

$f(x) = x!\{x\}$ is defined for x being a natural number Hence, $f(x)$ is a function whose domain $x \in N$

$f(x) = \frac{\ln(x-1)}{\sqrt{(1-x^2)}}$ Here $\ln(x-1)$ is defined only when $x - 1 > 0 \Rightarrow x > 1$ Also $1 - x^2 > 0$ for denominator, i.e. $-1 < x < 1$ Hence, $f(x)$ is not defined for any value of x

152 **(b)**

$f(x) = 3x - 5$ (given)

Let $y = f(x) = 3x - 5$

$\Rightarrow y + 5 = 3x \Rightarrow x = \frac{y+5}{3}$

$\Rightarrow f^{-1}(x) = \frac{x+5}{3}$

153 **(a, b)**

$(x + 1)f(x) - x$ is a polynomial degree $n + 1$

$\Rightarrow (x + 1)f(x) - x = k(x)[x - 1][x - 2] \dots [x - n]$ (i)

$\Rightarrow [n + 2]f(n + 1) - (n + 1) = k[(n + 1)!]$

Also, $1 = k(-1)(-2) \dots ((-n - 1))$ (Putting $x = -1$ in (i)]

$\Rightarrow 1 = k(-1)^{n+1}(n + 1)!$

$\Rightarrow (n + 2)f(n + 1) - (n + 1) = (-1)^{n+1}$

$\Rightarrow f(n + 1) = 1$, if n is odd and $\frac{n}{n+2}$, if n is even

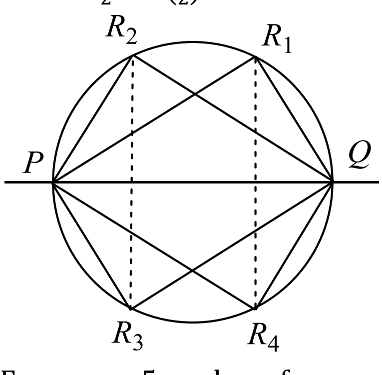
154 **(a, b, c, d)**

Since $\angle PRQ = \pi/2$ and points P, Q, R lie on the circle with PQ as diameter

Also $PQ = 5$

Now, the maximum area of the triangle is

$\Delta_{\max} = \frac{1}{2}(5)\left(\frac{5}{2}\right) = 6.25$



For area $= 5$, we have four symmetrical positions of point R (shown as R_1, R_2, R_3, R_4)

For area $= 6.25$ we have exactly two points

For area $= 7$, no such points exist

155 **(a, b, c, d)**

$f(x + 1) = \frac{f(x)-5}{f(x)-3}$ (1)

$\Rightarrow f(x)f(x + 1) - 3f(x + 1) = f(x) - 5$

$\Rightarrow f(x) = \frac{3f(x+1) - 5}{f(x+1) - 1}$

Replacing x by $(x - 1)$, we get

$f(x-1)=\frac{3f(x)-5}{f(x)-1}$ (2)

Using (1), $f(x+2)=\frac{f(x+1)-5}{f(x+1)-3}=\frac{\frac{f(x)-5}{f(x)-3}-5}{\frac{f(x)-5}{f(x)-3}-3}=\frac{2f(x)-5}{f(x)-2}$ (3)

Using (2), $f(x-2)=\frac{3f(x-1)-5}{f(x-1)-1}=\frac{3\left(\frac{3f(x)-5}{f(x)-1}\right)-5}{\frac{3f(x)-5}{f(x)-1}-1}=\frac{2f(x)-5}{f(x)-2}$ (4)

Using (3) and (4), we have $f(x+2)=f(x-2)$

$\Rightarrow f(x+4)=f(x) \Rightarrow f(x)$ is periodic with period 4

156 **(a, b, c)**

$f(x)$ is defined if $\log_{|\sin x|}(x^2-8x+23)-3\log_2 \sin x>0$

$$\Rightarrow \log_{|\sin x|}\left(\frac{x^2-8x+23}{8}\right)>0$$

This is true if $|\sin x|\neq 0,1$ and $\frac{x^2-8x+23}{8}<1$

Now, $\frac{x^2-8x+23}{8}<1 \Rightarrow x^2-8x+15<0$

$\Rightarrow x\in(3,5)-\left\{\pi,\frac{3\pi}{2}\right\}$

Domain $=(3,\pi)\cup\left(\pi,\frac{3}{2}\right)\cup\left(\frac{3\pi}{2},5\right)$

157 **(a, d)**

$f(x)=\sec^{-1}[1+\cos^2 x]$

$f(x)$ is defined if $[1+\cos^2 x]\leq -1$ or $[1+\cos^2 x]\geq 1$

$\Rightarrow [\cos^2 x]\leq -2$ (not possible) or $[\cos^2 x]\geq 0$

$\Rightarrow \cos^2\geq 0 \Rightarrow x\in R$

Now $0\leq\cos^2 x\leq 1\Rightarrow 1\leq 1+\cos^2 x\leq 2$

$\Rightarrow [1+\cos^2 x]=1,2$

$\Rightarrow \sec^{-1}[1+\cos^2 x]=\sec^{-1}1,\sec^{-1}2$

Hence, the range is $\{\sec^{-1}1,\sec^{-1}2\}$

158 **(b, d)**

$f(x)=x^2-2ax+a(a+1)$

$f(x)=(x-a)^2+a, x\in[a,\infty)$

Let $y=(x-a)^2+a$ clearly $y\geq a$

$\Rightarrow (x-a)^2=y-a$

$\Rightarrow x=a+\sqrt{y-a}$

$\therefore f^{-1}(x)=a+\sqrt{x-a}$

Now $f(x)=f^{-1}(x)$

$\Rightarrow (x-a)^2+a=a+\sqrt{x-a}$

$(x-a)^2=\sqrt{x-a}$

$\Rightarrow (x-a)^4=(x-a)$

$\Rightarrow x=a$ or $(x-a)^3=1$

$\Rightarrow x=a$ or $a+1$

If $a=5049$, then $a+1=5050$

If $a+1=5049$, then $a=5048$

159 **(b, d)**

The period of $f(x)=|\sin 2x|+|\cos 2x|$ is $\pi/4$

$\Rightarrow [f(x)]$ is also periodic with period $\pi/4$

Also $1\leq f(x)\leq\sqrt{2}$

$\Rightarrow [f(x)]=1 f(x)$ is a many-one and into function

160 **(a, c)**

$f(2)=f(1+1)=2f(1)=10$

$f(3)=f(2+1)=f(2)+f(1)=10+5=15$

Then, $f(n)=5n$

$$\Rightarrow \sum_{r=1}^{m} f(r)=5\sum_{r=1}^{m} r=\frac{5m(m+1)}{2}$$

Replace y by $-x$, $\Rightarrow f(0)=f(x)+f(-x)$

Also put $x=y=0\Rightarrow f(0)=f(0)+f(0)\Rightarrow f(0)=0$

$\Rightarrow f(x)+f(-x)=0$, hence, the function is odd

161 **(a,b,d)**

$f(0)=\max\{1+\sin 0,1,1-\cos 0\}=1$

$g(0)=\max\{1,|0-1|\}=1$

$f(1)=\max\{1+\sin 1,1,1-\cos 1\}=1+\sin 1$

$g(f(0))=g(1)=\max\{1,|1-1|\}=1$

$f(g(0))=f(1)=1+\sin 1$

$g(f(1))=g(1+\sin 1)=\max\{1,|1+\sin 1-1|\}=1$

162 **(a,d)**

Given that $f(x)=y=\frac{x+2}{x-1}$

a. Let $f(x)=\frac{x+2}{x-1}=y\Rightarrow x+2=xy-y$

$\Rightarrow x=\frac{2+y}{y-1}\Rightarrow x=f(y)$

$\therefore$ **a** is correct.

b. $f(1)\neq 3$ $\therefore$ **b** is not correct.

c. $f'(x)=\frac{x-1-x-2}{(x-1)^2}=\frac{-3}{(x-1)^2}<0$ for $\forall\, x\in R-\{1\}$

$\Rightarrow f(x)$ is decreasing $\forall\, x\neq 1$

$\therefore$ **c** is not correct

d. $f(x)=\frac{x+2}{x-1}$ is a rational function of x

$\therefore$ **d** is the correct answer

Thus, we get that **a,** and **d** are correct answer

163 **(a, c)**

$f(x+y)+f(x-y)=2f(x).f(y)$ (1)

Put $x=0\Rightarrow f(y)+f(-y)=2f(0)f(y)$ (2)

Put $x=y=0\Rightarrow f(0)+f(0)=2f(0)f(0)$

$\Rightarrow f(0)=1$ (as $f(0)\neq 0$)

$\Rightarrow f(-y)=f(y)$ (from (2))

Hence the function is even then $f(-2)=f(2)=a$

164 **(a)**

If $f(x)=\sin^2 x$ and $g(x)=\sqrt{x}$

Now, $fog=f(g(x))=f(\sqrt{x})=\sin^2\sqrt{x}$

and $gof(x)=g(f(x))=g(\sin^2 x)=\sqrt{\sin^2 x}=|\sin x|$

again if $f(x) = \sin x, g(x) = |x|$

$fog(x) = f(g(x)) = f(|x|) = \sin|x| \neq (\sin\sqrt{x})^2$

When $f(x) = x^2, g(x) = \sin\sqrt{x}$

$fog(x) = f[g(x)] = f(\sin\sqrt{x}) = (\sin\sqrt{x})^2$

and $(gof)(x) = g[f(x)] = g(x^2) = \sin\sqrt{x^2} = \sin|x| \neq |\sin x|$

$\therefore$ **a** is the correct option.

165 **(a, b, c, d)**

$f(x) = \begin{cases} 1, & x \text{ is rational} \\ 0, & x \text{ is irrational} \end{cases}$

$\Rightarrow f(x+k) = \begin{cases} 1, & x+k \text{ is rational} \\ 0, & x+k \text{ is irrational} \end{cases}$

Where k is any rational number

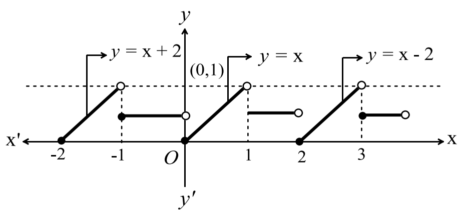
$\Rightarrow f(x+k) = \begin{cases} 1, & x \text{ is rational} \\ 0, & x \text{ is irrational} \end{cases}$

$\Rightarrow f(x+k) = f(x)$

$\Rightarrow f(x)$ is periodic function, but its fundamental period cannot be determined

$f(x) = \begin{cases} x - [x], & 2n \leq x < 2n+1 \\ 1/2, & 2n+1 \leq x < 2n+2 \end{cases}$

Draw the graph from which it can be verified that period is 2



$f(x) = (-1)^{\left[\frac{2x}{\pi}\right]}$

$\Rightarrow f(x+\pi) = (-1)^{\left[\frac{2(\pi+x)}{\pi}\right]} = (-1)^{\left[\frac{2x}{\pi}\right]+2} = (-1)^{\left[\frac{2x}{\pi}\right]}$

Hence, the period is π

$f(x) = x - [x+3] + \tan\left(\frac{\pi x}{2}\right)$

$= \{x\} - 3 + \tan\left(\frac{\pi x}{2}\right)$

$\{x\}$ is periodic with period 1, $\tan\left(\frac{\pi x}{2}\right) x$ is periodic with period 2

Now, the LCM of 1 and 2 is 2 Hence, the period of $f(x)$ is 2

166 **(b,c)**

As $(0,0)$ and $(x, g(x))$ are two vertices of an equilateral triangle; therefore, length of the side of Δ is

$\sqrt{(x-0)^2 + (g(x)-0)^2} = \sqrt{x^2 + (g(x))^2}$

$\therefore$ The area of equilateral $\Delta = \frac{\sqrt{3}}{4}\left(x^2 + (g(x))^2\right) = \frac{\sqrt{3}}{4}$

$\Rightarrow g(x)^2 = 1 - x^2$

$\Rightarrow g(x) = \pm\sqrt{1-x^2}$

$\therefore$ **b, c** are the correct answers as **a** is not a function ($\because$image of x is not unique)

167 **(a, c, d)**

$f^2(x) = f\left(\frac{3}{4}x + 1\right) = \frac{3}{4}\left(\frac{3}{4}x + 1\right) + 1$

$= \left(\frac{3}{4}\right)^2 x + \frac{3}{4} + 1 \quad (1)$

$f^3(x) = f\{f^2(x)\} = \frac{3}{4}\{f^2(x) + 1\}$

$= \frac{3}{4}\left\{\left(\frac{3}{4}\right)^2 x + \frac{3}{4} + 1\right\} + 1$

$= \left(\frac{3}{4}\right)^3 x + \left(\frac{3}{4}\right)^2 + \frac{3}{4} + 1$

$\therefore f^n(x) = \left(\frac{3}{4}\right)^n x + \left(\frac{3}{4}\right)^{n-1} + \left(\frac{3}{4}\right)^{n-2} + \cdots + \left(\frac{3}{4}\right) + 1$

$= \left(\frac{3}{4}\right)^n x + \frac{1 - \left(\frac{3}{4}\right)^n}{1 - \frac{3}{4}}$

$\therefore \lambda = \lim_{n\to\infty} f^n(x) = 0 + 4 = 4$

168 **(b, c, d)**

$f(x) = \sin(\sin^{-1} x) = x \forall x \in [-1, 1]$ which is one-one and onto

$f(x) = \frac{2}{\pi}\sin^{-1}(\sin x) = \frac{2}{\pi}x$

The range of the function for $x \in [-1, 1]$ is $\left[-\frac{2}{\pi}, \frac{2}{\pi}\right]$ which is a subset of $[-1, 1]$

Hence, the function is one-one but not onto, hence not bijective

$f(x) = (\text{sgn}(x))\ln(e^x) = (\text{sgn}(x))x$

$= \begin{cases} x, & x > 0 \\ -x, & x < 0 \\ 0, & x = 0 \end{cases}$

This function has the range $[0, 1]$ which is a subset of $[-1, 1]$

Hence, the function is into Also, the function is many-one

$f(x) = x^3\text{sgn}(x) = \begin{cases} x^3, & x > 0 \\ -x^3, & x < 0 \\ 0, & x = 0 \end{cases}$

Which is many-one and into

169 **(b, c)**

Given $2f(\sin x) + f(\cos x) = x \ (1)$

Replace x by $\frac{\pi}{2} - x$

$\Rightarrow 2f(\cos x) + f(\sin x) = \frac{\pi}{2} - x \ (2)$

Eliminating $f(\cos x)$ from (1) and (2), we get

$\Rightarrow 3f(\sin x) = 3x - \frac{\pi}{2}$

$\Rightarrow f(\sin x) = x - \frac{\pi}{6}$

$\Rightarrow f(x) = \sin^{-1} x - \frac{\pi}{6}$

$f(x)$ has the domain $[-1, 1]$

Also, $\sin^{-1} x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \Rightarrow \sin^{-1} x - \frac{\pi}{6} \in \left[-\frac{2\pi}{3}, \frac{\pi}{3}\right]$

170 **(a, b, c, d)**

$f(x) = \text{sgn}(\cot^{-1} x) + \tan\left(\frac{\pi}{2}[x]\right)$

$\text{sgn}(\cot^{-1} x)$ is defined when $\cot^{-1} x$ is defined, which is for $\forall x \in R$

$\tan\left(\frac{\pi}{2}[x]\right)$ is defined when $\frac{\pi}{2}[x] \neq \frac{(2n+1)}{2}\pi$, where $n \in Z$

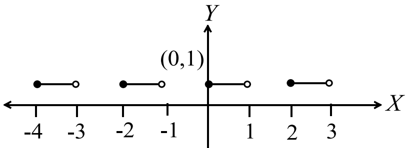
$\Rightarrow [x] \neq 2n+1 \Rightarrow x \notin [2n+1, 2x+2)$

Hence domain of $f(x)$ is $\cup_{n \in Z}[2n, 2n+1)$

Also $\cot^{-1} x > 0, \forall x \in R$,

Then $f(x) = 1 + \tan\left(\frac{\pi}{2}[x]\right) = 1$

$\Rightarrow f(x) = 1, x \in D_f$



Graph of $f(x) = \text{sgn}(\cot{-1}x) + \tan\frac{\pi}{2}[x]$

From graph $f(x)$ is periodic with period 2

171 **(a, b, c)**

$f(x) + f(y) = f\left(\frac{x+y}{1-xy}\right)$

Replace y by $-x \Rightarrow f(x) + f(-x) = f(0)$ (1)

Put $x = y = 0 \Rightarrow f(0) + f(0) = f(0) \Rightarrow f(0) = 0$

$\Rightarrow f(x) + f(-x) = 0$ (from (1))

Hence, $f(x)$ is an odd function

$f(x) + f(y) = f\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$

Replace y by $-x \Rightarrow f(x) + f(-x) = f(0)$ (2)

Put $x = y = 0 \Rightarrow f(0) + f(0) = f(0)$

$\Rightarrow f(0) = 0 \Rightarrow f(x) + f(-x) = 0$ (from (2))

Hence, $f(x)$ is an odd function

$f(x+y) = f(x) + f(y)$

Replace y by $-x \Rightarrow f(0) = f(x) + f(-x)$ (3)

Put $x = y = 0 \Rightarrow f(0+0) = f(0) + f(0) \Rightarrow$
$f(0) = 0 \Rightarrow f(x) + f(-x) = 0$ (from(3))

Hence, $f(x)$ is an odd function

172 **(b, c)**

$f(x)$ must be a linear function, let $f(x) = ax + b$

$\Rightarrow f(ax+b) = 6x - ax - b$

$\Rightarrow a(ax+b) + b = 6x - ax - b$

$\Rightarrow a^2 = 6 - a$ and $ab + b = -b$

$\Rightarrow a = 2$ or $-3 \Rightarrow b = 0$

$\Rightarrow f(x) = 2x$ or $-3x \Rightarrow f(17) = 34$ or -51

173 **(b, c)**

1. For $f(x) = \log x^2, x^2 > 0 \Rightarrow x \in R - \{0\}$

For $g(x) = 2\log x, x > 0$

Hence, $f(x)$ and $g(x)$ are not identical

2. $f(x) = \log_x e = \frac{1}{\log_e x} = g(x)$

Hence, the functions are identical

3. $f(x) = \sin(\cos^{-1} x) = \sin\left(\frac{\pi}{2} - \sin^{-1} x\right) =$ cossin$-1x$=gx

Hence, the functions are identical

174 **(b, d)**

$f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$

$\Rightarrow f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$

$\Rightarrow f(y) = y^2 - 2$

Now $y = x + \frac{1}{x} \geq 2$ or ≤ -2

Hence, the domain of the function is $(-\infty, -2] \cup [2, \infty)$

Also for these values of $y, y^2 \geq 4 \Rightarrow y^2 - 2 \geq 2$

Hence, the range of the function is $[2, \infty)$

175 **(a, b, c)**

$(f + g)(3.5) = f(3.5) + g(3.5) = (-0.5) + 0.5$
$= 0$

$f(g(3)) = f(0) = 3$

$(fg)(2) = f(2)g(2) = (-1) \times (-1) = 1$

$(f - g)(4) = f(4) - g(4) = 0 - 26 = -26$

176 **(b)**

Both the statements are true, but statement 2 is not a correct explanation of statement 1, as for $f(g(x))$ is onto it is necessary that $f(x)$ is onto, but there is no restriction on $g(x)$.

177 **(b)**

A function which can be expressed as a sum of odd and even function need not to be odd or even

But $f(x) = \log e^x$ is not defined for $x < 0$, hence statement 2 is true but not correct explanation of statement 1

178 **(c)**

Obviously, $f(x) = x^2 + \tan^{-1} x$ is non-periodic, but sum of two non-periodic function is not always non-periodic, as $f(x) = x$ and $g(x) = -[x]$, where [.] represents the greatest integer function.

$f(x) + g(x) = x - [x] = \{x\}$ is a periodic function

({.} represents the fractional part function)

179 **(a)**
Let $\max|f(x)| = M$ where $0 < M \leq 1$ (since f is not identically zero and $|f(x)| \leq 1 \forall x \in R$)

Now, $f(x+y) + f(x-y) = 2f(x).\mathrm{g}(y)$

$\Rightarrow |2f(x).\mathrm{g}(y)| = |f(x+y) + f(x-y)|$

$\Rightarrow 2|f(x)||\mathrm{g}(y)| \leq |f(x+y)| + |f(x-y)|$
$\leq M + M$

$\Rightarrow |\mathrm{g}(y)| \leq 1$ for $y \in R$

180 **(a)**
It is a fundamental concept.

181 **(c)**
$f(x)$ $\tan^{-1} x$ is an increasing function, then the range of function is $[\tan^{-1} 1, \tan^{-1}\sqrt{3}] \equiv [\pi/4, \pi/3]$.

Hence, statement 1 is true. But statement 2 is not true in general. For non-monotonic function, statement 2 is false

182 **(c)**
$\sin(kx)$ has period $\frac{\pi}{k}$ and period of $\{x\}$ is 1

Now LCM of $\frac{\pi}{k}$ and 1 exists only if k is a rational multiple of π (as LCM of rational and irrational number does not exist). Hence, statement 1 is true.

But statement 2 is false as sum of two periodic function is not necessarily periodic. Consider $f(x) = \sin x + \{x\}$

183 **(c)**
$fo\mathrm{g}(x)$ can be even also when one of them is even and other is odd

184 **(d)**
$$f\left(\frac{2\tan x}{1+\tan^2 x}\right) = \frac{(1+\cos 2x)(\sin^2 x + 2\tan x)}{2}$$

$$\Rightarrow f(\tan 2x) = \frac{2\cos^2 x\,(\sec^2 x + 2\tan x)}{2}$$

$= 1 + 2\sin x\cos x = 1 + \sin 2x$

$\Rightarrow f(y) = 1 + y$ where $y = \sin 2x$, now $\sin 2x \in [-1, 1]$

$\Rightarrow f(y) \in [0, 2]$

Hence, statement 1 is false but statement 2 is true

185 **(c)**
Given $f(x) = (x+1)^2 - 1, x \geq -1$

$\Rightarrow f'(x) = 2(x+1) \geq 0$ for $x \geq -1$

$\Rightarrow$ $f(x)$ is one-one

Since, codomain of the given function is not given, hence it can be considered as R, the set of reals and consequently R is not onto.

Hence, f is not bijective. Statement II is false.

Also, $f(x) = (x+1)^2 - 1 \geq -1$ for $x \geq -1$

$\Rightarrow \quad R_f = [-1, \infty)$

$f^{-1}(x) = \sqrt{x+1} - 1$

Clearly, $f(x) = f^{-1}(x)$ at $x = 0$ and $x = -1$

Statement I is true.

186 **(b)**
Obviously, both the statements are true but statement 2 is not a correct explanation of statement 1, as function $f(x) = \cos(2x+3)$ which is periodic though $\mathrm{g}(x) = 2x + 3$ is non-periodic

187 **(a)**
Obviously, the graph of $y = \tan x$ is symmetrical about origin, as it is an odd function.

Also derivative of an odd function is an even function, and $\sec^2 x$ is derivative of $\tan x$, hence both the statements are true, and statement 2 is a correct explanation of statement 1

188 **(b)**
Both the statements are true, but statement 2 is

not a correct explanation of statement 1 as $f(g(x))$ is one-one when $g(x)$ is one-one and $f(x)$ is many-one

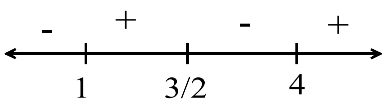
189 **(b)**
$||x^2 - 5x + 4| - |2x - 3|| = |x^2 - 3x + 1|$

$\Rightarrow ||x^2 - 5x + 4| - |2x - 3||$
$= |(x^2 - 5x + 4) + (2x - 3)|$

$\Rightarrow (x^2 - 5x + 4) + (2x - 3) \le 0$

$\Rightarrow (x - 1)(2x - 3)(x - 4) \le 0$

$\Rightarrow x \in (-\infty, 1] \cup \left[\frac{3}{2}, 4\right]$



Hence, statement 1 is true.

Statement 2 is true as it is the property of modulus function but is not a correct explanation of statement 1

190 **(b)**
Obviously, both the statements are true but statement 2 is not a correct explanation of statement 1, as for $f(x) = \cos(\sin x)$ the period is π, where $\sin x$ has period 2π. Thus, the period of $f(g(x))$ is not always same as that of $g(x)$

191 **(a)**
$f(x) - 1 + f(1 - x) - 1 = 0$; so $g(x) + g(1 - x) = 0$

Replacing x by $x + \frac{1}{2}$, we get $g\left(\frac{1}{2} + x\right) + g\left(\frac{1}{2} - x\right) = 0$

So it is symmetrical about $\left(\frac{1}{2}, 0\right)$

192 **(a)**
Consider $f(x) = \tan x$, which is surjective, periodic but discontinuous

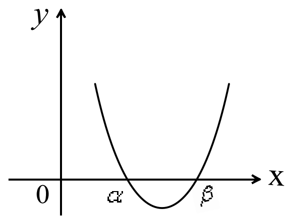
194 **(d)**
Statement 1 is false, though $f(x) = \sin x$ and $g(x) = \cos x$ have same domain and range, $\cos x = \sin x$ does not hold for all $x \in R$.

However, the statement 2 is true

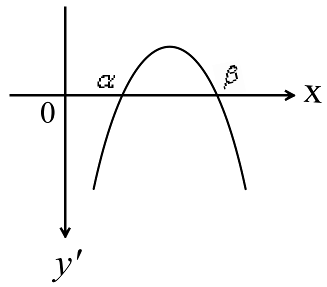
195 **(a)**
For any integer k, we have $f(k) = f(2n\pi + k)$ where $n \in Z$, but $2n\pi + k$ is not integer, hence $f(x)$ is one-one

196 **(d)**
If $b^2 - 4ac > 0$ then $ax^2 + bx + c = 0$ has real distinct roots α, β.

If $a > 0$, then for $f(x) = \sqrt{ax^2 + bx + c}$ to get defined, $ax^2 + bx + c \ge 0$, then the range of $f(x)$ is $[0, \infty)$ (as $b^2 - 4ac > 0$)



If $a < 0$, then for $f(x)$ to get defined, $ax^2 + bx + c \ge 0$, then the range of $f(x)$ is $\left[0, -\frac{b}{2a}\right]$.(as $b^2 - 4ac > 0$)



Hence, statement 1 is false, but statement 2 is true

197 **(a)**
a. $f(x) = \log_3(5 + 4x - x^2)$
$= \log_3(9 - (x - 2)^2)$
Now $-\infty < 9(x - 2)^2 \le 9$
But for $f(x)$to get defined, $0 < 9 - (x - 2)^2 \le 9$
$\Rightarrow -\infty < \log_3(9 - (x - 2)^2) \le \log_3 9$
$\Rightarrow\Rightarrow -\infty < \log_3(9 - (x - 2)^2) \le 2$
Hence the range is $(-\infty, 2)$
b. $f(x) = \log_3(x^2 - 4x - 5)$
$= \log((x - 2)^2 - 9)$
For $f(x)$to get defined , $0 < (x - 2)^2 - 9 < \infty$
$\Rightarrow \lim_{x \to 0} \log x < \log_e (x - 2)^2 - 9 < \lim_{x \to \infty} \log x$
$\Rightarrow -\infty < f(x) < \infty$
Hence the range is R
c. $f(x) = \log_3(x^2 - 4x + 5)$
$= \log_3((x - 2)^2 + 1)$
$(x - 2)^2 + 1 \,\epsilon [1, \infty)$

$\Rightarrow \log_3(x^2-4x+5)\,\epsilon\,[0,\infty)$
d. $x=\log_3(4x-5-x^2)$
$=\log_3(-5-(x^2-4x))$
$=\log_3(-1-(x-2)^2)$
Now, $-1-(x-2)^2<0$ for all x
Hence, the function is not defined

198 **(a)**

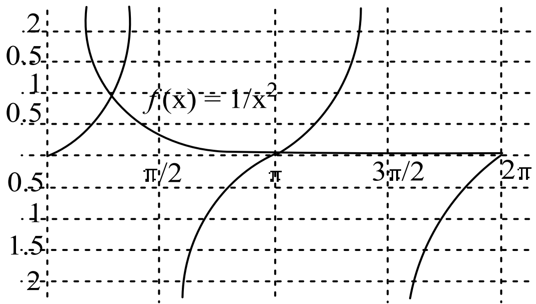
a. $f(x)=e^{\cos^4\pi x+x-[x]+\cos^2\pi x}$
$\cos^2\pi x+\cos^4\pi x$ has period 1
$x-[x]=\{x\}$has period 1
Then the period of $f(x)$is 1
b. $f(x)=\cos 2\pi\{2x\}+\sin 2\pi\{2x\}$
the period {2} is 1/2 then the period of $f(x)$ is 1/2
c. Clearly,
$\tan\pi[x]=0\ \forall\ x\,\epsilon\,R$ and the period of $\sin 3\pi\{x\}$ is equal to 1
d. $f(x)=3x-[3x+a]-b=3x+a-[3x+a]-(a+b)$
$=\{3x+a\}-(a+b)$
Thus the period f $f(x)$ is 1

199 **(a)**

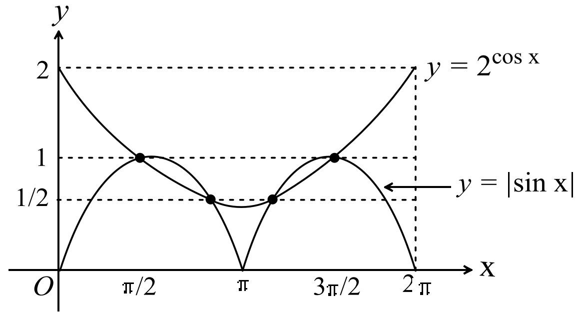
Since, $f(g(x))$is a one – one function
$\Rightarrow f(g(x_1))\neq f(g(x_2))$whenever $g(x_1)=g(x_2)$
$\Rightarrow (g(x_1))\neq(g(x_2))$ whenever $x_1\neq x_2$
$\Rightarrow g(x)$ is one – one
If
$f(x)$ is not one – one, then $f(x)=y$ is satisfied by $x=x_1,x_2$
$\Rightarrow f(x_1)=f(x_2)=y$ also *if* $g(x)$ is onto, then
Let $g(x_1)=x_1$ and $g(x_2)=x_2$
$\Rightarrow f(g(x_1))=f(g(x_2))$
$\Rightarrow f(g(x))$ can not be one - one.

200 **(a)**

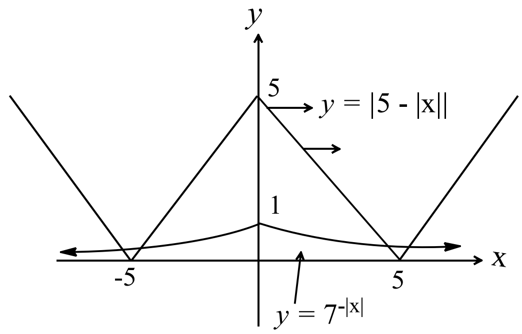
p. $y=\tan x=\frac{1}{x^2}$
From the graph, it is clear that it will have two real roots.



q. See the graph of
$y=2^{\cos x}$and $y=|\sin x|$. Two curves meet at four points for $\epsilon[0,2\pi]$



So, the equation $2^{\cos x}=|\sin x|$ has our solutions
r. Given that $f(|x|)=0$ has real roots $\Rightarrow f(x)=0$ has four positive roots.
Since $f(x)$ is a polynomial of degee 5, $f(x)$ cannot have even number of real roots.
$\Rightarrow f(x)$ has all the five roots and one root is negative
s. $7^{|x|}(|5-|x||)=1$
$\Rightarrow |5-|x||=7^{-|x|}$
Draw the graph of $y=7^{-|x|}$ and $y=|5-|x||$



From the graph, the number of roots is four

201 **(a)**

$f\ (\tan x)$ is defined if $0\le\tan x\le 1$
$\Rightarrow x\epsilon\left[n\pi,n\pi+\frac{\pi}{4}\right],n\,\epsilon\,I$
$f(\sin x)$ is defined if $0\le\sin x\le 1$
$\Rightarrow x\epsilon[2n\pi,(2n+1)],n\,\epsilon\,I$
$f(\cos x)$ is defined if $0\le\cos x\le 1$
$\Rightarrow x\epsilon\left[2n\pi-\frac{\pi}{2},2n\pi+\frac{\pi}{2}\right],n\,\epsilon\,I$
$f(2\sin x)$ is defined if $0\le 2\sin x\le 1\Rightarrow 0\le\sin x\le 1/2$
$\Rightarrow\left[2n\pi,2n\pi+\frac{\pi}{6}\right]\cup\left[2n\pi+\frac{5\pi}{6},(2n+1)\pi\right],n\,\epsilon\,I$

202 **(a)**

a. $f(x+\pi/2)=\cos(|\sin(x+\pi/2)|-\cos(x+\pi/2)$
$=\cos(|\cos x|-|-\sin x|)$
$=\cos(|\cos x|-|\sin x|)$
$=\cos(|\sin x|-|\cos x|)$
$=f(x)$
b. $f(x+\pi/2)=\cos[\tan(x+\pi/2)+\cot(x+\pi/2.\cos\tan x+\pi/2-\cot x+\pi/2$
$=\cos[-\cot x-\tan x].\cos[-\cot x+\tan x]$

$= \cos(\tan x + \cot x).\cos(\tan x - \cot x)$
$= f(x)$
c. The period of $\sin^{-1}(\sin x)$ is 2π. The period of e^t
$= LCM(2\pi, \pi) = 2\pi$
d. the given function is $f(x) = \sin^3 x \sin 3x$
$\Rightarrow f(x) = \left(\frac{3\sin x - \sin 3x}{4}\right)\sin 3x$
$\Rightarrow f(x) = \frac{3}{8}(\cos 2x - \cos 4x) - \frac{1}{8}(1 - \cos 6x)$
$\Rightarrow$ The period of $f(x)$ is π

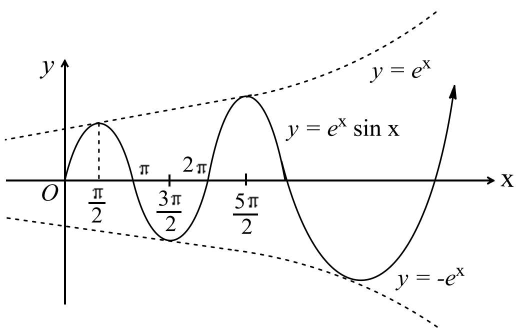
203 **(a)**

Given, $f(x) = \begin{cases} x+4, & \text{for } x < -4 \\ 3x+2, & \text{for } -4 \le x < 4 \\ x-4, & \text{for } x \ge 4 \end{cases}$

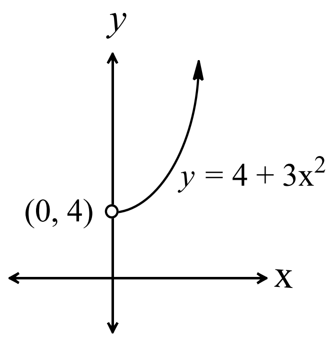
(A) $f(-5) + f(-4) = (-5+4) + (3(-4)+2) = -11$
(B) $f(|f(-8)|) = f(|-8+4|) = f(4) = 4 - 4 = 0$
(C) $f\big(f(-7) + f(3)\big) = f(-7+4+9+2)$
$= f(8) = 8 - 4 = 4$
(D) $f\left(f\left(f\big(f(0)\big)\right)\right) + 1 = f\left(f\big(f(2)\big)\right) + 1$
$= f\big(f(6+2)\big) + 1$
$= f\big(f(6+2)\big) + 1$
$= f\big(f(8)\big) + 1$
$= f(8-4) + 1$
$= f(4) + 1$
$= 4 - 4 + 1 = 1$

204 **(a)**

a. $f(x) = \cot^{-1}(2x - x^2 - 2)$
$= \cot^{-1}(-1 - (x-1)^2) - 1 - (x-1)^2 \le -1$
$\Rightarrow f(0) = f(2)$. Hence, $f(x)$ is many $-$ one
$\Rightarrow \cot^{-1}(2x - x^2 - 2) \, \epsilon \left[\frac{3\pi}{4}, \pi\right.$
Hence, $f(x)$ is onto
Also, $f(3) = f(-1)$, hence function is many $-$ one
$-1 - (x-1)^2 = -5$
b.



Clearly, from the graph that $f(x)$ is many $-$ one and onto
c.



d. Let $X = \{x_1, x_2, \dots, x\_n\}$
Let $f(x_1) = x_2$
$\Rightarrow f\big(f(x_1)\big) = f(x_2) \Rightarrow x_1$
Thus $f(x)$ is one-one and onto.

205 **(a)**

a. $f(x) = \{(sgn\, x)^{sgn\, x}\}^n = \begin{cases} [(1)^1]^n, & x > 0 \\ [(-1)^{-1}]^n, & x < 0 \end{cases}$
$= \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$
Hence , $f(x)$ is an odd fundtion
b. $f(x) = \frac{x}{e^x - 1} + \frac{x}{2} + 1$
$\Rightarrow f(-x) = \frac{-x}{e^{-x} - 1} - \frac{x}{2} + 1 = \frac{xe^x}{e^x - 1} - \frac{x}{2} + 1$
$= \frac{xe^x - x + x}{e^x - 1} - \frac{x}{2} + 1$
$= x + \frac{x}{e^x - 1} - \frac{x}{2} + 1 = \frac{x}{e^x - 1} + \frac{x}{2} + 1$
$= f(x)$
c. $f(x) \begin{cases} 0, & \text{If } x \text{ is rational} \\ 1, & \text{If } x \text{ is irrational} \end{cases}$
$\Rightarrow f(-x) = \begin{cases} 0, & \text{If} - x \text{ is rational} \\ 1, & \text{If} - x \text{ is irrational} \end{cases}$
$= \begin{cases} 0, & \text{If } x \text{ is rational} \\ 1, & \text{If } x \text{ is irrational} \end{cases}$
$= f(x)$
d. $f(x) = \max\{\tan x, \cot x$
$\Rightarrow f(-x) = \max\{\tan(-x), \cot(-x)\}$
$= \max\{-\tan(x), -\cot(x)\}$
$= -\max\{\tan(x), \cot(x)\}$
$= -f(x)$
Hence, $f(x)$ is an odd function
Also $f(x + \pi) = \max\{\tan(x + \pi), \cot(x + \pi)\}$
$= \max\{\tan x, \cot x\}$
Hence, $f(x)$ is periodic with period π

206 **(a)**

a. $\tan^{-1}\left(\frac{2x}{1-x^2}\right) \epsilon \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\Rightarrow 2\tan^{-1} x \epsilon \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\Rightarrow \tan^{-1} x \epsilon \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$
$\Rightarrow \tan^{-1} x \, \epsilon \, (-1, 1)$
b. $f(x) = \sin^{-1}(\sin x)$ and $g(x) = \sin(\sin^{-1} x)$

$f(x)$ is defined if $\sin x \in [-1,1]$ which is true for all $x \in R$
But $g(x)$ is defined for only $x \in [-1,1]$
Hence, $f(x)$ and $g(x)$are identical if $x \in [-1,1]$
c. $f(x) = \log_{x^2} 25$ and $g(x) = \log_x 5$
$f(x)$ is defined for $\forall\ x \in R$
$-\{0,1\}$and $g(x)$is defined for $(0,\infty) - \{1\}$
Hence, $f(x)$ and $g(x)$ are identical if $x \in (0,1) \cup (1,\infty)$
d. $f(x) = \sec^{-1} x + \operatorname{cosec}^{-1} x, g(x) =$ $\sin -1x+\cos -1x$
$f(x)$has domain R
$-(-1,1)$ and $g(x)$has domain $[-1,1]$
Hence, both the functions are identical only if $x = -1, 1$

207 **(c)**

$$f(x) = \begin{cases} x+1, & x \le 1 \\ 2x+1, & 1 < x \le 2 \end{cases}$$

$$g(x) = \begin{cases} x^2, & -1 \le x < 2 \\ x+2, & 2 \le x \le 3 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} g(x)+1, & g(x) \le 1 \\ 2g(x)+1, & 1 < g(x) \le 2 \end{cases}$$

$\Rightarrow f(g(x))$

$$= \begin{cases} x^2+1, \ x^2 \le 1, -1 \le x < 2 \\ x+2+1, \ x+2 \le 1, 2 \le x \le 3 \\ 2x^2+1, 1 < x^2 \le 2, -1 \le x < 2 \\ 2(x+2)+1, 1 < x+2 \le 2, 2 \le x \le 3 \end{cases}$$

$$\Rightarrow f(g(x)) = \begin{cases} x^2+1, & -1 \le x \le 1 \\ 2x^2+1, & 1 < x \le \sqrt{2} \end{cases}$$

Hence, the domain of $f(x)$ is $[-1, \sqrt{2}]$

208 **(b)**

$f(x) + f\left(\frac{x-1}{x}\right) = 1 + x$ (1)

In (1) replace x by $\frac{x-1}{x}$, we have $f\left(\frac{x-1}{x}\right) + f\left(\frac{\frac{x-1}{x} - 1}{\frac{x-1}{x}}\right)$

$= 1 + \frac{x-1}{x}$

$\Rightarrow f\left(\frac{x-1}{x}\right) + f\left(\frac{1}{1-x}\right) = 1 + \frac{x-1}{x}$ (2)

Now from (1) and (2), we have $f(x) - f\left(\frac{1}{1-x}\right) = x - \frac{x-1}{x}$ (3)

In (3) replace x by $\frac{1}{1-x}$, we have $f\left(\frac{1}{1-x}\right) - f\left(\frac{x-1}{x}\right)$

$= \frac{1}{1-x} - \frac{\frac{1}{1-x} - 1}{\frac{1}{1-x}}$

Or $f\left(\frac{1}{1-x}\right) - f\left(\frac{x-1}{x}\right) = \frac{1}{1-x} - x$ (4)

Now from (1) + (3) + (4), we have $2f(x) = 1 + x + x - \frac{x-1}{x} + \frac{1}{1-x} - x$

$$\Rightarrow f(x) = \frac{x^3 - x^2 - 1}{2x(x-1)}$$

$$f(x) = \frac{x^3 - x^2 - 1}{2x(x-1)}$$

$$\Rightarrow g(x) = \frac{x^3 - x^2 - 1}{x(x-1)} - x + 1$$

$$= \frac{x^2 - x - 1}{x(x-1)}$$

Now for $y = \sqrt{g(x)}$, we must have $\frac{x^2-x-1}{x(x-1)} \ge 0$ or $\frac{\left(x - \frac{1-\sqrt{5}}{2}\right)\left(x - \frac{1+\sqrt{5}}{2}\right)}{x(x-1)} \ge 0$

$$\Rightarrow x \in \left(-\infty, \frac{1-\sqrt{5}}{2}\right] \cup (0,1) \cup \left[\frac{1+\sqrt{5}}{2}, \infty\right)$$

209 **(d)**

Here,
$f(1) + 2f(2) + 3f(3) + \cdots nf(n) = n(n+$ $1fn,$for $n\ge 2$ (1)
Replacing n by n+1, we get
$f(1) + 2f(2) + 3f(3) + \cdots + (n+1)f(n+1)$
$= (n+1)(n+2)f(n+1)$ (2)
From (2) – (1), we get
$(n+1)f(n+1)$
$= (n+1)\{(n+2)f(n+1) - nf(n)\}$
$\Rightarrow f(n+1) = (n+2)f(n+1) - nf(n)$
$\Rightarrow nf(n) = (n+2)f(n+1) - f(n+1)$
$\Rightarrow nf(n) = (n+1)f(n+1)$
Putting n=2,3,4,…, we get
$2f(2) = 3f(3) = 4f(4) = \cdots = nf(n)$
From (1), $f(1) + 2f(2) + 3f(3) + \cdots + nf(n) = n(n+1)f(n)$
$\Rightarrow f(1) + (n-1).nf(n)$
$= n(n+1)f(n)$
$\Rightarrow f(1)$
$= 2nf(n)$

$$\Rightarrow f(n) = \frac{f(1)}{2n}$$

$$= \frac{1}{2n}$$

$$f(1003) = \frac{1}{2(1003)} = \frac{1}{2006}$$

210 **(a)**

$$\left(f(x)\right)^2 f\left(\frac{1-x}{1+x}\right) = 64x \ \ (1)$$

Putting $\frac{1-x}{1+x}=y$, or $x=\frac{1-y}{1+y}$, we get

$$\left\{f\left(\frac{1-y}{1+y}\right)\right\}.f(y)=64\left(\frac{1-y}{1+y}\right)$$

$$\Rightarrow f(x).\left\{f\left(\frac{1-x}{1+x}\right)\right\}^2=64\left(\frac{1-x}{1+x}\right) \quad (2)$$

From $(1)^2/(2)$, we get

$$\frac{f(x)^4\left\{f\left(\frac{1-x}{1+x}\right)\right\}^2}{f(x)\ \left\{f\left(\frac{1-x}{1+x}\right)\right\}^2}=\frac{(64)^2}{64\left(\frac{1-x}{1+x}\right)}$$

$$\Rightarrow \{f(x)\}^3=64x^2\left(\frac{1+x}{1-x}\right)$$

$$\Rightarrow f(x)=4x^{2/3}\left(\frac{1+x}{1-x}\right)^{1/3}$$

$x=f(9/7)=-4(9/7)^{2/3}(2)$

211 **(d)**

$|g(x)|=|\sin x| x\,\epsilon\,R$

$$f(|g(x)|)=\begin{cases}|\sin x|-1, & -1\le|\sin x|<0\\ (|\sin x|)^2, & 0\le(|\sin x|)\le 1\end{cases}$$

$=\sin^2 x, x\,\epsilon\,R$

$$f(g(x)=\begin{cases}\sin x-1, & -1\le \sin x<0\\ \sin^2 x, & 0\le \sin x\le 1\end{cases}$$

$$=\begin{cases}\sin x-1, & (2n+1)\pi<x<2n\pi\\ \sin^2 x, & 2n\pi\le x\le(2n+1)\pi\end{cases} n\,\epsilon\,Z$$

$\Rightarrow f(|g(x)|)$

$$=\begin{cases}1-\sin x, & (2n+1)\pi<x<2n\pi\\ \sin^2 x, & 2n\pi\le x\le(2n+1)\pi\end{cases} n\,\epsilon\,Z$$

Clearly $h_1(x)=f(|g(x)|)=\sin^2 x$ has period π, range [0, 1] and domain R

212 **(d)**

Given $a_{n+1}=f(a_n)$

Now $a_1=f(a_0)=f(x)$

$\Rightarrow a_2=f(a_1)=f(f(a_0))=fof(x)$

$$\Rightarrow a_n=\frac{fofofof\ \dots f(x)}{n\ times}$$

$a_1=f(x)=(a-x^m)^{1/m}$

$$\Rightarrow a_2=f(f(x))=\left[a-\{(a-x^m)^{1/m}\}^m\right]^{1/m}=x$$

$$\Rightarrow a_3=f\left(f(f(x))\right)=f(x)$$

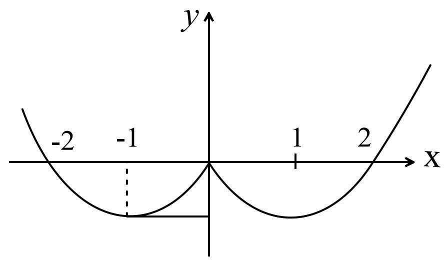
Obviously, the inverse does not exist when m is even and n is odd

213 **(a)**

$f_1(x)=x^2$ and $f_2(x)=|x|$

$\Rightarrow f(x)=f_1(x)-2f_2(x)=x^2-2|x|$

Graph of $f(x)$



$$g(x)=\begin{cases}f(x), -3\le x<-1\\ -1, -1\le x<0\\ 0,\ 0\le x\le 2\\ f(x),\ 2<x\le 3\end{cases}$$

$$=\begin{cases}x^2+2x, -3\le x<-1\\ -1, -1\le x<0\\ 0,\ 0\le x\le 2\\ x^2-2x,\ 2<x\le 3\end{cases}$$

The range of $g(x)$ for $[-3,-1]$ is $[-1,3]$

214 **(a)**

$g(f(x))$ is not defined if

(i)$-2+a>8$ and (ii)$b+3>8$

$a>10$ and $b>5$

215 **(c)**

$f(2-x)=f(2+x) \quad (1)$

Replace x by $2-x, \Rightarrow f(x)=f(4-x)$ (2)

Also given $f(20-x)=f(x)$ (3)

From (1) and (2), $f(4-x)=f(20-x)$

Replace $x\ by\ 4-x, \Rightarrow f(x)=f(x+16)$

Hence the period of $f(x)$ is 16.

Given $f(0)=5$.

216 **(c)**

$$g(f(x)=\begin{cases}[f(x)] & -\pi\le f(x)<0\\ \sin f(x), & 0\le f(x)<\pi\end{cases}$$

$$=\begin{cases}[[x]], & -\pi\le[x]<0, & -2\le x\le -1\\ [|x|+1], & \pi\le|x|+1<0, & -1<x\le 2\\ \sin x, & 0\le[x]<\pi, & -2\le x\le -1\\ \sin(|x|+1), & 0\le|x|+1\le\pi, & -1<x\le 2\end{cases}$$

$$=\begin{cases}[x], & -2\le x\le -1\\ \sin(|x|+1), & -1<x\le 2\end{cases}$$

Hence, the range domain is $[-2,2]$

Also for $-2\le x\le -1, [x]=-2,-1$

And for $-1<x\le 2, |x|+1\,\epsilon\,[1,3]$

$\Rightarrow \sin(|x|+1)\,\epsilon\,[\sin 3, 1]$

Hence, the number of integral points in the range is 4

217 **(3)**

We have $f\left(\frac{2x-3}{x-2}\right)=5x-2 \Rightarrow f^{-1}(5x-2)\Rightarrow \frac{2x-3}{x-2}$

Let $5x-2=13$, then $x=3$

Hence, $f^{-1}(13)=\frac{2(3)-3}{3-2}=3$

218 **(7)**

Obviously f is a linear polynomial

Let $f(x) = ax + b$ hence $f(x^2 + x + 3) +$
$2f(x^2 - 3x + 5) = 6x^2 - 10x + 17$
$\Rightarrow [a(x^2 + x + 3) + b] + 2[a(x^2 - 3x + 5) + b]$
$\equiv 6x^2 - 10x + 17$
$\Rightarrow a + 2a = 6$ (1)
$\Rightarrow a - 6a = -10$ (2)
(comparing coeff. of x^2 and coeff. of x on both sides)
$a \Rightarrow 2$
Again, $3a + b + 10a + 2b = 17$ (comparing constant term)
$\Rightarrow 6 + b + 20 + 2b = 17$
$\therefore f(x) = 2x - 3$
$\Rightarrow f(5) = 7$

219 **(7)**
Let $2x + y = 3x - y \Rightarrow 2y = x \Rightarrow y = \frac{x}{2}$
$\therefore$ Put $y = \frac{x}{2}$
$$\Rightarrow f(x) + f\left(\frac{5x}{2}\right) + \frac{5x^2}{2} = f\left(\frac{5x}{2}\right) + 2x^2 + 1$$
$$\Rightarrow f(x) = 1 - \frac{x^2}{2}$$
$\Rightarrow f(4) = -7$

220 **(7)**
From E to Fwe can define, in all, $2 \times 2 \times 2 \times 2 = 16$ functions (2 options for each elements of E either map to 1 or to 2
$\therefore$ Number of onto function $= 16 - 2 = 14$

221 **(7)**
$$\left(\frac{3}{4}\right)^{6x+10-x^2} < \frac{27}{64}$$
$\Rightarrow 6x + 10 - x^2 > 3$
$\therefore x^2 - 6x - 7 < 0$
$\therefore (+1)(x - 7) < 0$
$\Rightarrow 0, 1, 2, 3, 4, 5, 6$

222 **(4)**
$(2x^2 - 4.2^x + 4) + 1 + ||b - 1| - 3| = |\sin y|$
$\Rightarrow (2^x - 2)^2 + 1 + ||b - 1| - 3| = |\sin y|$
$\Rightarrow (2^x - 2)^2 + 1 + ||b - 1| - 3| = |\sin y|$
LHS ≥ 1 and RHS ≤ 1
$\therefore 2^x = 2, |b - 1| - 3 = 0$
$\Rightarrow (b - 1) = \pm 3$
$\Rightarrow b = 4, -2$

223 **(1)**
Given $f\big(f(x)\big) = -x + 1$
Replacing $x \to f(x)$
$f\left(f\big(f(x)\big)\right) = -f(x) + 1$
$f(1 - x) = -f(x) + 1$
$f(x) + f(1 - x) = 1$
$$\Rightarrow f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right) = 1$$

224 **(0)**
Let $x = \frac{|a|}{a} + \frac{|b|}{b} + \frac{|c|}{c}$
If exactly one – ve, then $x = 1$
Exactly two – ve, then $x = -1$
All three – ve , then $x = -3$
All three +ve , then $x = 3$
Then the required sum is 0

225 **(9)**
$$g(x) + \frac{1}{2}\tan^{-1}|x| + 1$$
$\Rightarrow sgn\big(g(x)\big) = 1$
$\Rightarrow \sin^{23} x - \cos^{22} x = 1$
$\Rightarrow \sin^{23} x = 1 + \cos^{22} x$ which is possible if $\sin x = 1$ and $\cos x = 0$
$$\Rightarrow \sin x = 1, x = 2n\pi + \frac{\pi}{2}$$
hence $-10\pi \leq 2n\pi + \frac{\pi}{2} \leq 8\pi \Rightarrow -\frac{21}{4} \leq n \leq \frac{15}{4}$
$\Rightarrow -5 \leq n \leq 3$
Hence, number of values of $x = 9$.

226 **(1)**
$$f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right)$$
$$= \sin^2 x + \frac{1}{4}\left(\sin x + \sqrt{3}\cos x\right)^2 + \frac{1}{2}\cos x\left(\cos x - \sqrt{3}\sin x\right)$$
$$= \frac{5}{4}(\sin^2 x + \cos^2 x) = \frac{5}{4}$$
$(gof)x = g[f(x)]g(5/4) = 1$

227 **(7)**
We have $f(2x) - f(2x)f\left(\frac{1}{2x}\right) + f(16x^2y) = f(-2) - f(4xy)$
Replacing y by $\frac{1}{8x^2}$, We get
$$f(2x) - f(2x)\left(\frac{1}{2x}\right) + f(2) = f(-2) - f\left(\frac{1}{2x}\right)$$
$\therefore f(2x) + f\left(\frac{1}{2x}\right) = f(2x)f\left(\frac{1}{2x}\right)$ [as $f(x)$ is even]
$\therefore f(2x) = 1 \pm (2x)^n$
$\Rightarrow f(x) = 1 \pm x^n$
Now $f(4) = 1 \pm 4^n = -255$ (Given)
Taking negative sign, we get $256 = 4^n \Rightarrow n = 4$
Hence $f(x) = 1 - x^4$, which is an even function.
$\Rightarrow f(2) = -15$

228 **(5)**
$x! - (x - 1)! \neq 0 \Rightarrow x \epsilon I^+ - \{1\}$
$2^{\frac{\pi}{\tan^{-1} x}} > 4$ as $\tan^{-1} x < \frac{\pi}{2}$
$$\Rightarrow \frac{(x - 4)(x - 10)}{(x - 1)!\,(x - 1)} < 0$$

$\Rightarrow x \epsilon\{5,6,\ldots,9\}$

229 **(2)**

$f(x)+f\left(\frac{1}{x}\right)=x^2+\frac{1}{x}$

Replacing $x \to \frac{1}{x}$; $f\left(\frac{1}{x}\right)+f(x)=\frac{1}{x^2}+x$

$\Rightarrow \frac{1}{x^2}+x=x^2-\frac{1}{x^2}$

$\Rightarrow x-\frac{1}{x}=x^2-\frac{1}{x^2}$

$\Rightarrow \left(x-\frac{1}{x}\right)=\left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)$

$\Rightarrow \left(x-\frac{1}{x}\right)=\left(x+\frac{1}{x}-1\right)=0$

$x=\frac{1}{x}$; $x+\frac{1}{x}=1$ (rejected)

Hence $x=1$ or -1

230 **(3)**

Clearly fundamental period is $\frac{4\pi}{3}$, then z lies in the third quadrant.

231 **(3)**

$\log_{1/3}(\log_7(\sin x+a)>0$

$\Rightarrow 0<\log_7(\sin x+a)<1$

$1<(\sin x+a)<7\ \forall x \epsilon R$[‘$a$’should be less than the minimum value of $7-\sin x$ and ‘a’ must be greater than maximum value of $1-\sin x$]

$\Rightarrow 1-\sin x<a<7-\sin x\ \forall x \epsilon R$

$2<a<6$

232 **(3)**

$f(3n)=f\left(f\left(f(n)\right)\right)=3f(n), \forall n \epsilon N$

Put $n=1, f(3)=3f(1)$

If $f(1)=1$, then $f\left(f(1)\right)=f(1)=1$, but $f\left(f(n)\right)=3n$

$\Rightarrow f\left(f(1)\right)=3$, giving $1=3$ which is absurd.

$\therefore f(1)\neq 1$

$\therefore 3=f\left(f(1)\right)>f(1)>1$

So $f(1)=2$

$f(2)=f\left(f(1)\right)=3$

233 **(7)**

$f(x)=\frac{ax^8+bx^6+cx^4+dx^2+15x+1}{x}$

$=\underbrace{ax^7+bx^5+cx^3+dx+\frac{1}{x}}_{\text{odd function}}+15$

Now $f(x)+f(-x)=30$

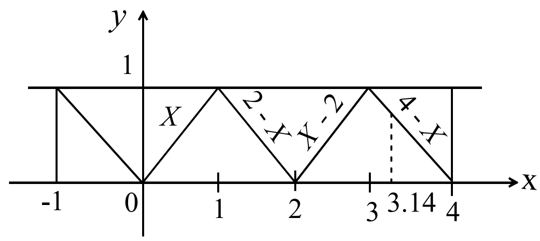
$\Rightarrow f(-5)=30-f(5)=28$

234 **(8)**

Since f is periodic with period 2 and $f(x)=x\ \forall x \epsilon[0,1]$ also $f(x)$ is even

$\Rightarrow$ symmetry about y-axis

$\therefore$ graph of $f(x)$ is as shown



$f(3.14)=4-3.14=0.86$

235 **(1)**

$\left|\left||x^2-x+4|-2\right|-3\right|=x^2+x-12$

$\Rightarrow \left||x^2-x+2|-3\right|=x^2+x-12$

$\Rightarrow |x^2-x-1|=x^2+x-12$

$\Rightarrow 2x=11$

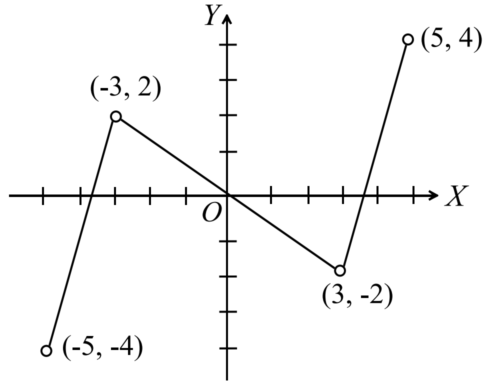
$\Rightarrow x=11/2$

236 **(3)**

$f(x)+f(-x)=0$

$\Rightarrow f(x)$ is an odd function.

Since point $(-3,2)$ and $(5,4)$ lie on the curve, therefore $(3,-2)$ and $(-5,-4)$ will also lie on the curve. For minimum number of roots, graph of continuous function $f(x)$ is as follows.



From the above graph of $f(x)$, it is clear that equation $f(x)$=0 has at least three real roots.

237 **(5)**

$f(x)$ and $f^{-1}(x)$ can only intersect on the line $y=x$ and therefore $y=x$ must be tangent at the common point of tangency

$\therefore 3x^2-7x+c=x$

$\Rightarrow 3x^2-8x+c=0$ ...(1)

This equation must have equal roots

$\Rightarrow 64-12c=0$

$\Rightarrow c=\frac{64}{12}=\frac{16}{3}$

238 **(6)**

Let $x^2=4\cos^2\theta+\sin^2\theta$

Then $(4-x^2)=3\sin^2\theta$ and $(x^2-1)=3\cos^2\theta$

$\therefore f(x)=\sqrt{3}|\sin\theta|+\sqrt{3}|\cos\theta|$

$\Rightarrow y_{\min}=\sqrt{3}$ and

$y_{\max}=\sqrt{3}\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)=\sqrt{6}$

Hence range of $f(x)$ is $[\sqrt{3}, \sqrt{6}]$

Hence maximum value of $(f(x))^2$ is 6

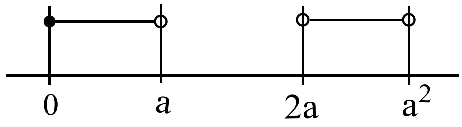
239 **(5)**

As $a > 2$, hence

$a^2 > 2a > a > 2$

Now $(x-a)(x-2a)(x-a^2) < 0$

$\Rightarrow$ the solution set is as shown



Between $(0, a)$ there are $(a-1)$ positive integers and between $(2a, a^2)$ there are $a^2 - 2a - 1 + a - 1 = 18 \Rightarrow a^2 - a - 20 = 0$

$(a-5)(a+4) = 0$

$\therefore a = 5$

240 **(0)**

$$g(x) = \frac{f(x) + f(-x)}{2}$$

$$= \frac{1}{2}\left[\frac{x+1}{x^3+1} + \frac{1-x}{1-x^3}\right]$$

$$= \frac{1}{2}\left[\frac{1}{x^2-x+1} + \frac{1}{1+x+x^2}\right]$$

$$= \frac{1}{2}\left[\frac{2(x^2+1)}{(x^2+1)^2 - x^2}\right]$$

$$= \frac{x^2+1}{x^4+x^2+1}$$

$$= \frac{x^4-1}{x^6+1} \Rightarrow g(0) = 1$$

241 **(4)**

Put $x = 1$ and $y = 1$,

$f^2(1) - f(1) - 6 = 0$

$\Rightarrow f(1) = 3$ or $f(1) = -2$

Now put $y = 1$

$$\Rightarrow f(x).f(1) = f(x) + 2\left(\frac{1}{x} + 2\right)$$

$$= f(x) + 2\left(\frac{2x+1}{x}\right)$$

$$\Rightarrow f(x)[f(1) - 1] = \frac{2(2x+1)}{2}$$

$$\Rightarrow f(x) = \frac{2(2x+1)}{x[f(1)-1]}$$

For $f(1) = 3$ $f(x) = \frac{2x+1}{x}$ (1)

and for $x = -2 f(x) = \frac{2(2x+1)}{-3x}$ (2)

$\Rightarrow f(1/2) = 4$

242 **(9)**

Given $f(x+2) = f(x) + f(2)$

Put $x = -1$, we have $f(1) = f(-1) + f(2)$

$\Rightarrow f(1) = -f(1) + f(2)$ (as $f(x)$ is an odd function)

$\Rightarrow f(2) = 2f(1) = 6$

Now put $x = 1$,

We have $f(3) = f(1) + f(2) = 3 + 6 = 9$

243 **(3)**

$$f(x) = \sqrt{\sin x + \cos x} + \sqrt{7x - x^2 - 6}$$

$$= \sqrt{\sqrt{2}\sin\left(x + \frac{\pi}{4}\right)} + \sqrt{(x-6)(1-x)}$$

Now $f(x)$ is defined if $\sin\left(x + \frac{\pi}{4}\right) \geq 0$ and $(x-6)(1-x) \geq 0$

$\Rightarrow 0 \leq x + \frac{\pi}{4} \leq \pi$ or $2\pi \leq x + \frac{\pi}{4} \leq 3\pi$ and $1 \leq x \leq 6$

$\Rightarrow -\frac{\pi}{4} \leq x \leq \frac{3\pi}{4}$ or $\frac{7\pi}{4} \leq x \leq \frac{11\pi}{4}$ and $1 \leq x \leq 6$

$$\Rightarrow x \epsilon \left[1, \frac{3\pi}{4}\right] \cup \left[\frac{7\pi}{4}, 6\right]$$

Integral values of x are $x = 1$, 2 and 6

244 **(1)**

$\log_a(x^2 - x + 2) > \log_a(-x^2 + 2x + 3)$

Put $x = \frac{4}{9}$, $\log_a\left(\frac{142}{81}\right) > \log_a\left(\frac{299}{81}\right)$

$$\because \frac{142}{81} < \frac{299}{81} \Rightarrow 0 < a < 1$$

$\Rightarrow \log_a(x^2 - x + 2) > \log_2(-x^2 + 2x + 3)$

Gives $0 < x^2 - x + 2 < -x^2 + 2x + 3$

$x^2 - x + 2 > 0$ and $2x^2 - 3x - 1 < 0$

$$\Rightarrow \frac{3 - \sqrt{17}}{4} < x < \frac{3 + \sqrt{17}}{4}$$

245 **(6)**

$\because k \,\epsilon$ odd

$f(k) = k + 3$

$$f(f(k)) = \frac{k+3}{2}$$

If $\frac{k+3}{2}$ is odd $\Rightarrow 27 = \frac{k+3}{2} + 3 \Rightarrow 45$ not possible

$\Rightarrow \frac{k+3}{2}$ is even

$$\therefore 27 = f\left(f(f(k))\right) = f\left(\frac{k+3}{2}\right) = \frac{k+3}{4}$$

$\therefore k = 105$

Verifying $f\left(f(f(105))\right) = f(f(108)) = f(54) = 27$

$\therefore k = 105$

246 **(4)**

$f(x) = [8+7] + |\tan 2\pi x + \cot 2\pi x| - 8x$

$= [8x] - 8x - 7 + |\tan 2\pi x + \cot 2\pi x|$

$= -\{8x\} + |\tan 2\pi x + \cot 2\pi x| + 7$

Period of $\{8x\}$ is 1/8

Also, $|\tan 2\pi x + \cot 2\pi x|$

$$= \left|\frac{\sin 2\pi x}{\cos 2\pi x} + \frac{\cos 2\pi x}{\sin 2\pi x}\right| = \left|\frac{1}{\sin 2\pi x \cos 2\pi x}\right|$$

$$= |2 \operatorname{cosec} 4\pi x|$$

Now period of 2 cosec $4\pi x$ is 1/2, then period of 2 cosec $4\pi x$ is 1/4,

∴ Period is L.C.M. of $\frac{1}{8}$ and $\frac{1}{4}$ which is $\frac{1}{4}$