Single Correct Answer Type

1. The probability that a marksman will hit a target is given as 1/5. Then the probability that at least once hit in 10 shots is
a) $1-(4/5)^{10}$ b) $1/5^{10}$ c) $1-(1/5)^{10}$ d) $(4/5)^{10}$
2. In a certain town, 40% of the people have brown hair, 25% have brown eyes and 15% have both brown hair and brown eyes. If a person selected at random from the town, has brown hair, the probability that he also has brown eyes is
a) 1/5 b) 3/8 c) 1/3 d) 2/3
3. Let A and B be two events such that $P(\overline{A\cup B})=1/6. P(A\cap B)=1/4$ and $P(\overline{A})=1/4$where $\overline{A}$ stands for complement of event A. Then events A and B are
a) Equally likely but not independent b) Equally likely and mutually exclusive
c) Mutually exclusive and independent d) Independent but not equally likely
4. There are 10 prizes, five A's, there B's and two C's, placed in identical sealed envelopers for the top 10 contestants in a mathematics contest. The prizes are awarded by allowing winners to select an envelope at random from those remaining. When the 8^{th} contestant goes to select the prize, the probability that the remaining three prizes are one A, one B and one C is
a) 1/4 b) 1/3 c) 1/12 d) 1/10
5. One mapping is selected at random from all mappings of the set $S=\{1,2,3,\dots,n\}$ into itself. If the probability that the mapping is one-one is 3/32, then the value of n is
a) 2 b) 3 c) 4 d) None of these
6. In a n-sided regular polygon, the probability that the two diagonal chosen at random will intersect inside the polygon is
a) $\frac{2\,{}^nC_2}{{}^{({}^nC_2-2)}C_2}$ b) $\frac{{}^{n(n-1)}C_2}{{}^{({}^nC_2-n)}C_2}$ c) $\frac{{}^nC_4}{{}^{({}^nC_2-2)}C_2}$ d) None of these
7. A bag contains 20 coins. If the probability that thebag contains exactly 4 biased coin is 1/3 and that of exactly 5 biased coin is 2/3, then the probability that all the biased coin are sorted out from the bag in exactly 10 draws is
a) $\frac{5}{10}\frac{{}^{16}C_6}{{}^{20}C_9}+\frac{1}{10}\frac{{}^{15}C_5}{{}^{20}C_9}$ b) $\frac{2}{33}\left[\frac{{}^{16}C_6+5\,{}^{15}C_5}{{}^{20}C_9}\right]$ c) $\frac{5}{33}\frac{{}^{16}C_6}{{}^{20}C_9}+\frac{1}{11}\frac{{}^{15}C_5}{{}^{20}C_9}$ d) None of these
8. An unbiased coin is tossed 6 times. The probability that third head appears on the sixth trial is
a) 5/16 b) 5/32 c) 5/8 d) 5/64
9. If n integers taken at random are multiplied together, then the probability that the last digit of the product is 1, 3, 7 or 9 is
a) $2^n/5^n$ b) $4^n-2^n/5^n$ c) $4^n/5^n$ d) None of these
10. A four figure number is formed of the figures 1, 2, 3, 5 with no repetitions. The probability that the number is divided by 5 is
a) 3/4 b) 1/4 c) 1/8 d) None of these
11. An event X can take place in conjuction with any one of the mutually exclusive and exhaustive events A,B and$C$. If A,B,C are equiprobable and the probability of X is 5/12, and the probability of X taking place when A has happened is 3/8 while it is 1/4 when B has taken place, then the probability of X taking place in conjuction with C is
a) 5/8 b) 3/8 c) 5/24 d) None of these
12. If $\overline{E}$ and $\overline{F}$ are the complementary events of events E and F, respectively, and if $0<P(F)<1$, then
a) $P(E/F)+P(\overline{E}/F)=1/2$ b) $P(E/F)+P(E/\overline{F})=1$
c) $P(\overline{E}/F)+P(E/\overline{F})=1$ d) $P(E/\overline{F})+P(\overline{E}/\overline{F})=1$
13. A father has 3 children with at least one boy. The probability that he has 2 boys and 1 girl is
a) 1/4 b) 1/3 c) 2/3 d) None of these

14. A pair of unbiased dice is rolled together till a sum of either 5 or 7 is obtained. The probability that 5 comes before 7 is

a) 2/5 b) 3/5 c) 4/5 d) None of these

15. A $2n$ digit number starts with 2 and all its digits are prime, then the probability that the sum of all 2 consecutive digits of the number is prime is

a) 4×2^{3n} b) 4×2^{-3n} c) 2^{3n} d) None of these

16. A coin is tossed 7 times. Then the probability that at least 4 consecutive heads appear is

a) 3/16 b) 5/32 c) 3/16 d) 1/8

17. A speaks truth in 60% cases and B speaks truth in 70% cases. The probability that they will say the same thing while describing a single even is

a) 0.56 b) 0.54 c) 0.38 d) 0.94

18. A box contains 2 block, 4 white and 3 red balls. One ball is drawn at random from the box and kept aside. From the remaining balls in the box, another ball is drawn at random and kept aside the first. This process is repeated till all the balls are drawn from the box. The probability that the balls drawn are in the sequence of 2 black, 4 white and 3 red is

a) 1/1260 b) 1/7560 c) 1/126 d) None of these

19. If a is an integer lying in $[-5, 30]$, then the probability that the graph of $y = x^2 + 2(a+4)x - 5a + 64$ is strictly above the x-axis is

a) 1/6 b) 7/36 c) 2/9 d) 3/5

20. Twelve balls are distributed among three boxes. The probability that the first box contains three balls is

a) $\frac{110}{9}\left(\frac{2}{3}\right)^{10}$ b) $\frac{9}{100}\left(\frac{2}{3}\right)^{10}$ c) $\frac{^{12}C_3}{12^3} \times 2^9$ d) $\frac{^{12}C_3}{3^{12}}$

21. There are only two women among 20 persons taking part in a pleasure trip. The 20 persons are divided into two group, each group consisting of 10 persons. Then the probability that the two women will be in the same group is

a) 9/19 b) 9/38 c) 9/35 d) None

22. A box contains tickets numbered from 1 to 20. Three tickets are drawn from the box with replacement. The probability that the largest number on the tickets is 7 is

a) 2/19 b) 7/20 c) $1 - (7/20)^3$ d) None of these

23. South African cricket captain lost the loss of coin 13 times out of 14. The chance of this happening was

a) $7/2^{13}$ b) $1/2^{13}$ c) $13/2^{14}$ d) $13/2^{11}$

24. Five different marbles are placed in 5 different boxes randomly. Then the probability that exactly two boxes remain empty is (each box can any number of marbles)

a) 2/5 b) 12/25 c) 3/5 d) None of these

25. A signal which can be green or red with probability$\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station$B$. The probability of each station receiving the signal correctly is $\frac{3}{4}$.If the signal received at station B is green, then the probability that the original signal green is

a) $\frac{3}{5}$ b) $\frac{6}{7}$ c) $\frac{20}{23}$ d) $\frac{9}{20}$

26. Let A, B, C be three mutually independent events. Consider the two statements S_1 and S_2

S_1: A and $B \cup C$ are independent

S_2: A and $B \cap C$ are independent

Then

a) Both S_1 and S_2 are true b) Only S_1 is true

c) Only S_2 is true d) Neither S_1 nor S_2 is true

27. A six-faced dice is so biased that it is twice as likely to show an even number as an odd number when thrown. It is throe twice, the probability that the sum of two numbers thrown is even is

a) 1/12 b) 1/6 c) 1/3 d) 5/9

28. A man alternately tosses a coin and thrown a die beginning with the coin. The probability that he gets a head in the coin before he gets a 5 or 6 in the dice is

a) 3/4 b) 1/2 c) 1/3 d) None of these

29. A student appears for tests I, II and III. The students is successful if he passes either in tests I and II or tests I and III. The probabilities of the student passing in tests I, II and III are4, respectively, p, q are ½. If the probability that the student is successful is 1/2 , then $p(1+q)=$
a) 1/2 b) 1 c) 3/2 d) 3/4

30. The probability that a random chosen three-digit number has exactly 3 factors is
a) 2/225 b) 7/900 c) 1/800 d) None of these

31. A and B play a game of tennis. The situation of the game is as follows; if one scores two consecutive points after a deuce, he wins, if loss of a points is followed by win of a point, it is deuce. The chance of a server to win a point is 2/3. The game is at deuce and $A$is serving. Probability that A will win the match is (serves are changed after each game)
a) 3/5 b) 2/5 c) 1/2 d) 4/5

32. Words from the letters of the word PROBABILITY are formed by taking all letters at a time. The probability that both B's are not together and both l's are not together is
a) 52/55 b) 53/55 c) 54/55 d) None of these

33. Events A and C are independent. If the probabilities relating A, B and C are$P(A)=1/5, P(B)=1/6, P(A\cap C)=1/20; P(B\cup C)=3/8$. Then
a) Events B and C are independent
b) Events B and C mutually exclusive
c) Events B and C bare neither independent nor mutually exclusive
d) Events B and C areequiprobable

34. Whenever horses a, b, c race together, their respective probabilities of winning the race are 0.3, 0.5 and 0.2, respectively. If they race three times the probability that the same horse wins all the three races, and the probability that a, b, c each wins one race are, respectively
a) 8/50, 9/50 b) 16/100, 3/100 c) 12/50, 15/50 d) 10/50, 8/50

35. Given two events$B$. If odds against A are as 2:1 and those in favour of $A\cup B$ areas 3:1, then
a) $1/2\leq P(B)\leq 3/4$ b) $5/12\leq P(B)\leq 3/4$ c) $1/4\leq P(B)\leq 3/5$ d) None of those

36. Two players toss 4 coins each. The probability that they both obtain the same number of heads is
a) 5/256 b) 1/16 c) 35/128 d) None of these

37. Three integers are chosen at random from the first 20 integers. The probability that their product is even is
a) 2/19 b) 3/29 c) 17/19 d) 4/29

38. A die is rolled 4 times. The probability of getting a larger number than the previous number each time is
a) 17/216 b) 5/432 c) 15/432 d) None of these

39. A die is thrown a fixed number of times. If probability of getting number 3 times is same as the probability of getting even number 4 times, then probability of getting even number exactly once is
a) 1/6 b) 1/9 c) 5/36 d) 7/128

40. An unbiased cubic die marked with 1, 2, 2, 3, 3, 3 is rolled 3 times. The probability of getting a total score of 4 or 6 is
a) 16/216 b) 50/216 c) 60/216 d) None of these

41. An unbiased coin is tossed n times. If the probability that head occurs 6 times is equal to the probability that head occurs 8 times, then $n=$
a) 7 b) 14 c) 16 d) 19

42. In a game called 'odd man out' $m(m>2)$ persons toss a coin to determine who will buy refreshments for the entire group. A person who gets an outcome different from that of the rest of the members of the group is called the odd man out. The probability that there is a loser in any game is
a) $1/2m$ b) $m/2^{m-1}$ c) $2/m$ d) None of these

43. The probability that a teacher will give an unannounced test during any class meeting is 1/5. If a student is absent twice, then the probability that the student will miss at least one test is
a) 4/5 b) 2/5 c) 7/25 d) 9/25

44. Let A and B be two events such that $P(A \cap B') = 0.20, P(A' \cap B) = 0.15, P(A' \cap B') = 0.1$, then $P(A/B)$ is equal to
a) 11/14 b) 2/11 c) 2/7 d) 1/7
45. Five different games are to be distributed among 4 children randomly. The probability that each child get atleast one game is
a) 1/4 b) 15/64 c) 21/64 d) None of these
46. Let A and B are events of an experiment and $P(A) = 1/4, P(A \cup B) = 1/2$, then value of $P(B/A')$ is
a) 2/3 b) 1/3 c) 5/6 d) 1/2
47. Two numbers are selected randomly from the set $S = \{1, 2, 3, 4, 5, 6\}$ without replacement one by one. The probability that minimum of the two numbers is less than, 4 is
a) 1/15 b) 14/15 c) 1/5 d) 4/5
48. A sample space consists of 3 sample points with associated probabilities given as $2p, p^2, 4p - 1$. Then the value of p is
a) $p = \sqrt{11} - 3$ b) $\sqrt{10} - 3$ c) $\frac{1}{4} < p < \frac{1}{2}$ d) None
49. The sum of two positive quantities is equal to $2n$. The probability that their product is not less than 3/4 times their greatest product is
a) 3/4 b) 1/2 c) 1/4 d) None of these
50. If odds against solving a question by three students are 2:1, 5:2 and 5:3, respectively, then probability that the question is solved only by one student is
a) 31/56 b) 24/56 c) 25/56 d) None of these
51. If a party of n persons sit at a round table, then the odds against two specified individual sitting next to each other are
a) $2:(n-3)$ b) $(n-3):2$ c) $(n-2):2$ d) $2:(n-2)$
52. The probability that a bulb produced by a factory will fuse after 150 days if used is 0.50. What is the probability that out of 5 such bulbs none will fuse after 150 days of use
a) $1 - (19/20)^5$ b) $(19/20)^2$ c) $(3/4)^5$ d) $90(1/4)^5$
53. A bag contains an assortment of blue and red balls. If two balls are drawn at random, the probability of drawing two red balls if five times the probability of drawing two blue balls. Furthermore, the probability of drawing one ball of each colour is six times the probability of drawing two blue balls. The number of red and blue balls in the bag is
a) 6, 3 b) 3, 6 c) 2, 7 d) None of these
54. Two dices are rolled one after the other. The probability that the number on the first is smaller than the number on the second is
a) 1/2 b) 7/18 c) 3/4 d) 5/12
55. A fair die is tossed repeatedly. A wins if it is 1 or 2 on two consecutive tosses and B wins if it is 3, 4, 5 or 6 on two consecutive tosses. The probability that A wins if the die is tossed indefinitely is
a) 1/3 b) 5/21 c) 1/4 d) 2/5
56. Two numbers x and y are chosen at random (without replacement) from amongst the numbers 1, 2, 3, …, 2004. The probability that $x^3 + y^3$ is divisible by 3 is
a) 1/3 b) 2/3 c) 1/6 d) 1/4
57. A pair of numbers is picked up randomly (without replacement) from the set {1,2, 3,5, 7, 11, 12, 13, 17, 19}. The probability that the number 11 was picked given that the sum of the numbers was even is nearly
a) 0.1 b) 0.125 c) 0.24 d) 0.18
58. Cards are drawn one-by-one at random from a well-shuffled pack of 52 playing cards until 2 aces are obtained from the first time. The probability that 18 draws are required for this is
a) 3/34 b) 17/455 c) 561/15925 d) None of these
59. Two numbers a, b are chosen from the set of integers 1,2, 3, …., 39. Then probability that the equation $7a - 9b = 0$ is satisfied is
a) 1/247 b) 2/247 c) 4/741 d) 5/741
60. Words from the letters of the word PROBABILITY are formed by taking all letters at a time. The probability

that both B's are not together and both I's are not together is

a) 52/55 b) 53/55 c) 54/55 d) None of these

61. Four numbers are multiplied together. Then, the probability that the product will be divisible by 5 or 10 is

a) 369/625 b) 399/625 c) 123/625 d) 133/625

62. One Indian and four American men and their wifes are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife, is

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{2}{5}$ d) $\frac{1}{5}$

63. If p is the probability that a man aged x will die in a year, then the probability that out of n men $A_1, A_2, \dots A_n$ each aged x, A_1 will die in an year and be the first to die is

a) $1-(1-p)^n$ b) $(1-p)^n$ c) $1/n[1-(1-p)^n]$ d) $1/n(1-p)^n$

64. A hat contains a number of cards with 30% white on both sides, 50% black on one side and white on the other side, 20% black on both sides. The cards are mixed up, and a single card is drawn at random and placed on the table. Its upper side shows up black. The probability that its other side is also black is

a) 2/9 b) 4/9 c) 2/3 d) 2/7

65. A pair of four dice is thrown independently three times. The probability of getting a score of exactly 9 twice is

a) 8/9 b) 8/729 c) 8/243 d) 1/729

66. The numbers $1, 2, 3, \dots n$ are arrange in random order. The probability that the digits $1,2,3,\dots,k (k < n)$ appears as neighbours in that order is

a) $1/n!$ b) $k!/n!$ c) $(n-k)!\, n!$ d) $(n-k+1)!\, n!$

67. A and B toss a fair coin each simultaneously 50 times. The probability that both of them will not get tail at the same toss is

a) $(3/4)^{50}$ b) $(2/7)^{50}$ c) $(1/8)^{50}$ d) $(7/8)^{50}$

68. A fair coin is tossed 5 times, then the probability that no two consecutive heads occur is

a) 11/32 b) 15/32 c) 13/32 d) None of these

69. Three ships A, B and C sail from England to India. If the ratio of their arriving safely are 2:5, 3:7 and 6:11, respectively, then the probability of all the ships for arriving safely is

a) 18/595 b) 6/17 c) 3/10 d) 2/7

70. A drawer contains 5 brown socks and 4 blue socks well mixed. A man reaches the drawer and pulls out socks at random. What is the probability that they match?

a) 4/9 b) 5/8 c) 5/9 d) 7/12

71. Five different games are to be distributed among 4 children randomly. The probability that each child get atleast one game is

a) 1/4 b) 16/64 c) 21/64 d) None of these

72. Let A be a set containing n elemnts. A subset P of the set A is chosen at random. The set A is reconstructed by replacing the elements of P, and another subset Q of A is chosen at random. The probability that $P \cap Q$ contains exactly $m (m < n)$ elements is

a) $3^{n-m}/4^n$ b) ${}^nC_m \times 3^m/4^n$ c) ${}^nC_m \times 3^{n-m}/4^4$ d) None of these

73. A bag has 10 balls. Six balls are drawn in an attempt and replaced. Then another draw of 5 balls is made from the bag. The probability that exactly two balls are common to both the draw is

a) 5/21 b) 2/21 c) 7/21 d) 3/21

74. If any four numbers are selected and they are multiplied, then the probability that the last digit will be 1, 3, 5 or 7 is

a) 4/625 b) 18/625 c) 16/625 d) None of these

75. An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, the number of outcomes that B must have so that A and B are independent, is

a) 2,4 or 8 b) 3,6 or 9 c) 4 or 8 d) 5 or 10

76. The probability that in a family of 5 members, exactly two members have birthday on Sunday is

a) $(12 \times 5^3)7^5$ b) $(10 \times 6^2)7^5$ c) 2/5 d) $(10 \times 6^2)7^5$

77. A man has 3 pairs of block socks and 2 pairs of brown socks kept together in a box. If he dressed hurriedly in the dark, the probability that after he has put on a block sock, he will, then put on another black sock is
a) 1/3 b) 2/3 c) 3/5 d) 2/15
78. A three-digit number is selected at random from the set of all three-digit numbers. The probability that the number selected has all the three digits same is
a) 1/9 b) 1/10 c) 1/50 d) 1/100
79. A fair die is rolled. The probability that the first time 1 occurs at the even throw is
a) $\frac{1}{6}$ b) $\frac{5}{11}$ c) $\frac{6}{11}$ d) $\frac{5}{36}$
80. The probability that an automobile will be stolen and found within one week is 0.0006. The probability that an automobile will be stolen is 0.0015. The probability that a stolen automobile will be found in one week is
a) 0.3 b) 0.4 c) 0.5 d) 0.6
81. A cricket club has 15 members, of whom only 2 can bowl. If the names of 15 members are put into a box and 11 are drawn at random, then the probability of getting an eleven containing at least 3 bowlers is
a) 7/13 b) 6/13 c) 11/15 d) 12/13
82. One ticket is selected at random from 100 tickets numbered 00, 01,02, …, 98, 99. If x_1 and x_2 denotes the sum and product of the digits on the tickets, then $P(x_1 = 9/x_2 = 0)$ is equal to
a) 2/19 b) 19/100 c) 1/50 d) None of these
83. The numbers (a, b, c) are selected by throwing a dice thrice, then the probability that (a, b, c) are in A.P.is
a) 1/12 b) 1/6 c) 1/4 d) None of these
84. A letter is known to have come either from LONDON or CLIFTON; on the postmark only the two consecutive letters ON are legible. The probability that if came from LONDON is
a) 1/17 b) 12/17 c) 17/30 d) 3/5
85. A dice is thrown six times, it being known that each time a different digit is shown. The probability that a sum of 12 will be obtained in the first three throws is
a) 5/24 b) 25/216 c) 3/20 d) 1/12
86. If the papers of 4 students can be checked by any one of the 7 teachers, then the probability that all the 4 papers are checked by exactly 2 teachers is
a) 2/7 b) 12/49 c) 32/343 d) None of these
87. On a Saturday night, 20% of all drivers in U.S.A. are under the influence of alcohol. The probability that a driver under the influence of alcohol will have an accident is 0.001. The probability that a sober driver will have an accident is 0.0001.If a car on a Saturday night smashed into a tree, the probability that the driver was under the influence of alcohol is
a) 3/7 b) 4/7 c) 5/7 d) 6/7
88. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse is
a) 3/5 b) 1/5 c) 2/5 d) 4/5
89. Let p, q be chosen one by one from the set $\{1, \sqrt{2}, \sqrt{3}, 2, e, \pi\}$ with replacement. Now a circle is drawn taking (p, q) as its centre. Then the probability that at the most two rational points exist on the circle is (rational points are those points whose both the coordinates are rational)
a) 2/3 b) 7/8 c) 8/9 d) None of these
90. There are 3 bags. Bag 1 contains 2 red and $a^2 - 4a + 8$ black balls, bag 2 contains 1 red and $a^2 - 4a + 9$ black balls and bag 3 contains 3 red and $a^2 - 4a + 7$ black balls. A ball is drawn at random from at random chosen bag. Then the maximum value of probability that is a red ball is
a) 1/3 b) 1/2 c) 2/9 d) 4/9
91. Three integers are chosen at random from the set of first 20 natural numbers. The chance that their product is a multiple of 3 is
a) 194/285 b) 1/57 c) 13/19 d) 3/4
92. A purse contains 2 six-sided dice. One is a normal fair die, while the other has two 1's , and two 5's. A die is picked up and rolled. Because of some secret magnetic attraction of the unfair die, there is 75% chance of

picking the unfair die and a 25% chance of picking a fair die. The die is rolled and shows up the face 3.The probability that a fair die was picked up is

a) 1/7 b) 1/4 c) 1/6 d) 1/24

93. A problem in mathematics is given to three students A, B, C and their respective probability of solving the problem is 1/2, 1/3 and 1/4. Probability that the problem is solved is

a) 3/4 b) 1/2 c) 2/3 d) 1/3

94. A fair die is thrown 20 times. The probability that on the 10th throw, the fourth six appears is

a) ${}^{20}C_{10} \times 5^6/6^{20}$ b) $120 \times 5^7/6^{10}$ c) $84 \times 5^6/6^{10}$ d) None of these

95. Mr. A lives at origin on the Cartesian plane and has his office at (4, 5). His friend lives at (2,3) on the same plane. Mr. A can go to his office travelling one black at a time either in the $+y$ or $+x$ direction. If all possible paths are equally likely then the probability that Mr. A passed his friends house is (shortest path for any event must be considered)

a) 1/2 b) 10/21 c) 1/4 d) 11/21

96. There are two urns A and B. Urn A contains 5 red, 3 blue and 2 white balls, urn B contains 4 red, 3 blue and 3 white balls. An urn is choosen at random and a ball is drawn. Probability that the ball drawn is red is

a) 9/10 b) 1/2 c) 11/20 d) 9/20

97. In a game a coin is tossed $2n + m$ times and a player wins if he does not get any two consecutive outcomes same for atleast $2n$ times in a row. The probability that player wins the game is

a) $\frac{m+2}{2^{2n}+1}$ b) $\frac{2n+2}{2^{2n}}$ c) $\frac{2n+2}{2^{2n+1}}$ d) $\frac{m+2}{2^{2n}}$

98. Let A and B be events. Suppose $P(A) = 0.4, P(B) = p$ and $P(P \cup B) = 0.7$. The value of p for which A and B are independent is

a) 1/3 b) 1/4 c) 1/2 d) 1/5

99. If a and b are chosen randomly from the set consisting of numbers 1, 2, 3, 4, 5, 6 with replacement. Then the probability that $\lim_{x\to 0}[(a^x + b^x)/2]^{2/x} = 6$ is

a) 1/3 b) 1/4 c) 1/9 d) 2/9

100. Four die are thrown simultaneously. The probability that 4 and 3 appear on two of the die given that 5 and 6 have appeared on other two die is

a) 1/6 b) 1/36 c) 12/151 d) None of these

101. Cards are drawn one by one without replacement from a pack of 52 cards. The probability that 10 cards will precede the first ace is

a) 241/1456 b) 164/4165 c) 451/884 d) None of these

102. Forty teams play a tournament. Each team plays every other team just once. Each game result in a win for one team. If each team has a 50% chance of winning each game, the probability that at the end of the tournament, every team has won a different number of games is

a) 1/780 b) $40!/2^{780}$ c) $40!/3^{780}$ d) None of these

103. $2n$ boys are randomally divided into two subgroups containing n boys each. The probability that the two tallest boys are in different groups is

a) $n/(2n-1)$ b) $(n-1)/(2n-1)$ c) $(n-1)/4n^2$ d) None of these

104. The probability of solving a question by three students are 1/2, 1/4, 1/6 respectively. Probability of question being solved will be

a) 33/48 b) 35/48 c) 31/48 d) 37/48

105. A fair coin is tossed 10 times. Then the probability that two heads do not occurs consecutively is

a) 7/64 b) 1/8 c) 9/16 d) 9/64

106. If A and B each toss three coins. The probability that both get the same number of heads is

a) 1/9 b) 3/16 c) 5/16 d) 3/8

107. A draws a card from a pack of n cards marked $1, 2, \dots n$. The card is replaced in the pack and B draws a card. Then the probability that A draws a higher card than B is

a) $(n+1)2n$ b) 1/2 c) $(n-1)/2n$ d) None of these

108. All the jacks, queens, kings and aces of a regular 52 cards deck are taken out. The 16 cards are thoroughly shuffled and my opponent, a person who always tells the truth, simultaneously draws two cards at random

and says, ‘I hold at least one ace’. The probability that he holds two aces is

a) 2/8 b) 4/9 c) 2/3 d) 1/9

109. The probability of winning a race by three persons A, B and C are 1/2, 1/4, and 1/4, respectively. They run two races. The probability of A winning the second race when B wins the first race is

a) 1/3 b) 1/2 c) 1/4 d) 2/3

110. A composite number is selected at random from the first 30 natural numbers and it is divided by 5. The probability that there will be a remainder is

a) 14/19 b) 5/19 c) 5/6 d) 7/15

111. A car is parked among N cars standing in a row, but not at either end. On his return, the owner finds that exactly ‘r’ of the N palces are still occupied. The probability that the places neighouring his car are empty is

a) $\frac{(r-1)!}{(N-1)!}$ b) $\frac{(r-1)!\,(N-r)!}{(N-1)!}$ c) $\frac{(N-r)(N-r-1)}{(N+1)(N+2)}$ d) $\frac{{}^{N-r}C_2}{{}^{N-1}C_2}$

112. If three square are selected at random from chessboard, then the probability that they form the letter ‘L’ is

a) $196/\,{}^{64}C_3$ b) $49/\,{}^{64}C_3$ c) $36/\,{}^{64}C_3$ d) $98/\,{}^{64}C_3$

113. Three houses are available in a locality. Three persons apply for the houses. Each applies for one houses without consulting others. The probability that all three apply for the same houses is

a) 1/9 b) 2/9 c) 7/9 d) 8/9

114. One ticket is selected at random from 100 tickets numbered 00, 01, 02, …, 99. Suppose A and B are the sum and product of the digit found on the ticket. Then $P((A=7)/(B=0))$ is given by

a) 2/13 b) 2/19 c) 1/50 d) None of these

115. A doctor is called to see a sick child. The doctor knows (prior to the visit) that 90% of the sick children in that neighborhood are sick with the flu, denoted by F, while 10% are sick with the measles, denoted by M. A well-known symptom of measles is a rash, denoted by R. The probability of having a rash for a child sick with the measles is 0.95. however, occasionally children with the flu also develop a rash, with conditional children with the flu also develop a rash, with conditional probability 0.08. Upon examination the child, the doctor finds a rash. Then what is the probability that the child has the measles?

a) 91/165 b) 90/163 c) 82/161 d) 95/167

116. An artillery may be either at point I with probability 8/9 or at point II with probability 1/9. We have 55 shells, each of which can be fired either rat point I or II. Each shell may hit the target, independent of the other shells, with probability 1/2. Maximum number of shells must be fired at point I to have maximum probability is

a) 20 b) 25 c) 29 d) 35

117. An urn contains 3 red balls and n white balls. Mr. A draws two balls together from the urn. The probability that they have the same colour is 1/2.Mr. B draws one balls form the urn, notes its colour and replaces it. He then draws a second ball from the urn and finds that both balls have the same colour is 5/8. The possible value of n is

a) 9 b) 6 c) 5 d) 1

118. Let E be an event which is neither a certainty nor an impossibility. If probability is such that $P(E)=1+\lambda+\lambda^2$ and $P(E')=(1+\lambda)^2$ in terms of an unknown λ. Then $P(E)$ is equal to

a) 1 b) 3/4 c) 1/4 d) None of these

119. There are 3 bags which are known to contain 2 white and 3 black, 4 white and 1 black, and 3 white and 7 black balls, respectively. A ball is drawn at random from one of the bags and found to be a black ball. Then the probability that it was drawn from the bag containing the most black ball is

a) 7/15 b) 5/19 c) 3/4 d) None of these

120. Thirty-two players ranked 1 to 32 are playing in a knockout tournament. Assume that in every match between any two players, the better-ranked player wins, the probability the ranked 2 players are winner and runner up, respectively, is

a) 16/31 b) 1/2 c) 17/31 d) None of these

121. A class consists of 80 students, 25 of them are girls and 55 are boys. If 10 of them are rich and the

remaining are poor and also 20 of them are intelligent, then the probability of selecting an intelligent rich girl is

a) 5/128 b) 25/128 c) 5/512 d) None of these

122. Three critics review a book. Odds in favour of the book are 5:2, 4:3 and 3:4, respectively, for the three cities. The probability that majority are in favour of the book is

a) 35/49 b) 125/343 c) 164/343 d) 209/343

123. A bag contains n white and n black balls. Pairs of balls are drawn without replacement until the bag is empty. The probability that each pair consists of one white and one black ball is

a) $1/\ ^{2n}C_n$ b) $2n/\ ^{2n}C_n$ c) $2n/n!$ d) $2n/(2n!)$

124. Consider $f(x) = x^3 + ax^2 + bx + c$.Parameters a, b, c are chosen, respectively, by throwing a die throwing a die three times. Then the probability that $f(x)$ is an increasing function is

a) 5/36 b) 8/36 c) 4/9 d) 1/3

125. Dialing a telephone number an old man forgets the last two digits remembering only that these are different dialed at random. The probability that the number is dialed correctly is

a) 1/45 b) 1/90 c) 1/100 d) None of these

126. A bag contains some white and some black balls, all combinations of balls being equally likely. The total number of balls in the bag is 10. If three balls are drawn at random without replacement and all of them are found to be black, the probability that the bag contains 1 white and 9 black balls is

a) 14/55 b) 12/55 c) 2/11 d) 8/55

127. A student can solve 2 out of 4 problem of mathematics, 3 out of 5problem of physics and 4 out of 5 problems of chemistry. There are equal number of books of math, physics and chemistry in his shelf. He selects one book randomly and attempts 10 problems from it. If he solves the first problem, then the probability that he will be able to solve the second problem is

a) 2/3 b) 25/38 c) 13/21 d) 14/23

128. There are 20 cards. Ten of these cards have the letter 'l' printed on them and the other 10 have the letter. 'T' printed on them. If three cards are picked up at random and kept in the same order, the probability of making word IIT is

a) 4/27 b) 5/38 c) 1/8 d) 9/80

129. A natural number is chosen at random from the first 100 natural numbers. The probability that $x + \frac{100}{x} > 50$ is

a) 1/10 b) 11/50 c) 11/20 d) None of these

130. A bag contains 3 red and 3 green balls and a person draws out 3 at random. He then drops 3 blue balls into the bag and again out 3 at random. The chance that the 3 later balls being all of different colours is

a) 15% b) 20% c) 27% d) 40%

131. The chance of an event happening is the square of the chance of a second event but the odds against the first are the cube of the odds against the second. The chances of the events are

a) 1/9, 1/3 b) 1/16, 1/4 c) 1/4, 1/2 d) None of these

132. Let A, B, C, D be independent events such that $P(A) = 1/2, P(B) = 1/3, P(C) = 1/5$ and $P(D) = 1/6$. Then the probability that none of A, B, C and D occurs

a) 1/180 b) 1/45 c) 1/18 d) None of these

133. A coin is tossed $2n$ times. The chance that the number of times one gets head is not equal to the number of times one gets tails is

a) $\frac{(2n!)}{(n!)^2}\left(\frac{1}{2}\right)^{2n}$ b) $1 - \frac{(2n!)}{(n!)^2}$ c) $1 - \frac{(2n!)}{(n!)^2}\frac{1}{4^n}$ d) None of these

134. A bag contains 20 coins. If the probability that bag contains exactly 4 biased coin is 1/3 and that of exactly 5 biased coin is 2/3, then the probability that all the biased coin are sorted out from the bag in exactly 10 draws is

a) $\frac{5}{33}\frac{^{16}C_6}{^{20}C_9} + \frac{1}{11}\frac{^{15}C_5}{^{20}C_9}$ b) $\frac{2}{33}\left[\frac{^{16}C_6 + {^{15}C_5}}{^{20}C_9}\right]$ c) $\frac{5}{33}\frac{^{16}C_7}{^{20}C_9} + \frac{1}{11}\frac{^{15}C_6}{^{20}C_9}$ d) None of these

135. Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1, r_2 and r_3 are the

numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ is

a) $\frac{1}{18}$ b) $\frac{1}{9}$ c) $\frac{2}{9}$ d) $\frac{1}{36}$

136. A fair coin is tossed 100 times. The probability of getting tails 1,3, ..., 49 times is

a) 1/2 b) 1/4 c) 1/8 d) 1/16

Multiple Correct Answers Type

137. A and B are two events defined as follows:

A: It rains today with $P(A) = 40\%$

B: It rains tomorrow with $P(B) = 50\%$

Also, P(it rains today and tomorrow) $= 30\%$

Also, $E_1: P((A \cap B)/(A \cup B))$ and $E_2: P(\{(A \cap \overline{B}) or\ (B \cup \overline{A})\}/(A \cup B))$. Then which of following is/are true?

a) A and B are independent b) $P(A/B) < P(B/A)$

c) E_1 and E_2 are equiprobable d) $P(A/(A \cup B)) = P(B/(A \cup B))$

138. Two events A and B have probability 0.25 and 0.50, respectively. The probability that both A and B occur simultaneously is 0.14. then the probability that neither A nor B occurs is

a) 0.39 b) 0.25 c) 0.11 d) None of these

139. For two given events A and B, $P(A \cap B)$ is

a) Not less than $P(A) + P(B) - 1$ b) Not greater than $P(A) + P(B)$

c) Equal to $P(A) + P(B) - P(A \cup B)$ d) Equal to $P(A) + P(B) + P(A \cup B)$

140. If $\overline{E}$ and $\overline{F}$ are the complementary events of events E and F, respectively, and if $0 < P(F) < 1$, then

a) $P(E/F) + P(\overline{E}/F) = 1$ b) $P(E/F) + P(E/\overline{F}) = 1$

c) $P(\overline{E}/F) + P(E/\overline{F}) = 1$ d) $P(E/\overline{F}) + P(\overline{E}/\overline{F}) = 1$

141. If A and B are two events, the probability that exactly one of them occurs is given by

a) $P(A) + P(B) - 2P(A \cap B)$ b) $P(A \cap \overline{B}) + P(\overline{A} \cap B)$

c) $P(A \cup B) - P(A \cap B)$ d) $P(\overline{A}) + P(\overline{B}) - 2P(\overline{A} \cap \overline{B})$

142. The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p and c, respectively. Of these subjects, the students has a 75% chance of passing in at least one, a 50% chance of passing in at least two and a 40% chance of passing in exactly two. Which of the following relations are true?

a) $p + m + c = 19/20$ b) $p + m + c = 27/20$ c) $pmc = 1/10$ d) $pmc = 1/4$

143. The probability that an event A happens in one trial of an experiment is 0.4. three independent trials of the experiment are performed. The probability that the event A happens at least once is

a) 0.936 b) 0.784 c) 0.904 d) None of these

144. India plays two matches each with West Indies and Australia. In any match, the probabilities of India getting points 0, 1 and 2 are 0.45, 0.05 and 0.50, respectively. Assuming that the outcomes are independent, the probability of India getting at least 7 points is

a) 0.8750 b) 0.0875 c) 0.0625 d) 0.0250

145. If from each of the three boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, one ball is drawn at random, then the probability that 2 white and 1 black ball will be drawn is

a) 13/32 b) ¼ c) 1/32 d) 3/16

146. A bag contains b blue balls and r red balls. If two balls are drawn at random, the probability of drawing two red balls is five times the probability of drawing two blue balls. Furthermore, the probability of drawing one ball of each colour is six times the probability of drawing two blue balls. Then

a) $b + r = 9$ b) $br = 18$ c) $|b - r| = 4$ d) $b/r = 2$

147. If A and B are two events such that $P(A) > 0$ and $P(B) \neq 1$, then $P(\overline{A}/\overline{B})$is equal to

(Here $\overline{A}$ and $\overline{B}$ are complements of A and B, respectively)

a) $1-P\left(\frac{A}{B}\right)$ b) $1-P\left(\frac{\overline{A}}{B}\right)$ c) $\frac{1-P(A\cup B)}{P(\overline{B})}$ d) $\frac{P(\overline{A})}{P(B)}$

148. If A and B are two independent events such that $P(A)=1/2$ and $P(B)=1/5$, then

a) $P(A\cup B)=3/5$ b) $P(A/B)=1/4$ c) $P(A/A\cup B)=5/6$ d) $P(A\cap B/\overline{A}\cup\overline{B})=0$

149. The probability of India winning a test match against West Indies is 1/2. Assuming independence from match to match, the probability that in a five match series India's second win occurs at third test is

a) 1/8 b) ¼ c) ½ d) 2/3

150. A fair is tossed 99 times. Let X be the number of times heads occurs. Then $P(X=r)$ is maximum when r is

a) 79 b) 52 c) 51 d) 50

151. An unbiased die with faced marked 1, 2, 3, 4,5 and 6 is rolled four times. Out of four face values obtained, the probability that the minimum face value is not less than 2 and the maximum face value is not greater than five is then

a) 16/81 b) 1/81 c) 80/81 d) 65/81

152. $P(A)=3/8, P(B)=1/2;\ P(A\cup B)=5/8$, which of the following do/does hold good?

a) $P(A^C/B)=2P(A/B^C)$ b) $P(B)=P(A/B)$
c) $15P(A^C/B^C)=8P(B/A^C)$ d) $P(A/B^C)=(A\cap B)$

153. If A and B are two events, then which one of the following is/are always true?

a) $P(A\cap B)\geq P(A)+P(B)-1$ b) $P(A\cap B)\leq P(A)$
c) $P(A'\cap B')\geq P(A')+PB')-1$ d) $P(A\cap B)=P(A)P(B)$

154. If E and F are independent events such that $0<P(E)<1$ and $0<P(F)<1$, then

a) B and F are mutually exclusive
b) E and F^C (the complement of the event F) are independent
c) $E^C$and F^C are independent
d) $P(E/F)+P(E^C|F)=1$

155. A student appears for tests I, II and III. The students is successful if he passes either in tests I and II or tests I and III. The probabilities of the students passing in tests I, II and III are p, q and 1/2 respectively. The probability that the student is successful is then

a) $p=q=1$ b) $p=q=1/2$ c) $p=1, q=0$ d) None of these

156. Let A and B be two events such that $P(A\cap B')=0.20, P(A'\cap B)=0.15$ and $P(A$ and B both fail$)=$ 0.10.Then

a) $P(A/B)=2/7$ b) $P(A)=0.3$ c) $P(A\cup B)=0.55$ d) $P(B/A)=1/2$

157. Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle with three vertices is equilateral is

a) ½ b) 1/5 c) 1/10 d) 1/20

158. Two fair dice are tossed. Let $x$be the event that the first die shows an even number and y be the event that the second die shows an odd number. The two events x and y are

a) Mutually exclusive b) Independent and mutually exclusive
c) Dependent d) None of these

159. Two numbers are chosen from {1, 2, 3, 4, 5, 6, 7, 8} one after another without replacement. Then the probability that

a) The smaller value of two is less than 3 is 13/28
b) The bigger value of two is more than 5 is 9/14
c) Product of two number is even is 11/14
d) None of these

160. If M and N are any two events, the probability that exactly one of them occurs is

a) $P(M)+P(N)-2P(M\cap N)$ b) $P(M)+P(N)-P(M\cap N)$
c) $P(M^C)+P(N^C)-2P(M^C\cap N^C)$ d) $P(M\cap N^C)+P(M^C\cap N)$

161. There are four machines and it is known that exactly two of them are faulty. They are tested, one by one, in a random order till both the faulty machines are identified. Then the probability that only two tests are needed is

a) 1/3 b) 1/6 c) ½ d) ¼

162. If A and B are two mutually exclusive events, then

a) $P(A) \leq P(\overline{B})$ b) $P(A) > P(B)$ c) $P(B) > P(\overline{A})$ d) $P(A) > P(B)$

163. If P and q are chosen randomly from the set {1, 2, 3, 4, 5,6, 7,8, 9,10} with replacement, then the probability that the roots of the equation $x^2 + px + q = 0$

a) Are real is 33/50 b) Are imaginary is 19/50

c) Are real and equal is 3/50 d) Are real and distinct is 3/5

164. In a precision bombing attack, there is a 50% chance that any one bomb will strike the target. Two direct hits are required to destroy the target completely. The number of bombs which should be dropped to give a 99% chance or better of completely destroying the target can be

a) 12 b) 11 c) 10 d) 13

165. A fair coin is tossed repeatedly. If the tail appears on first four tosses, then the probability of the head appearing on the fifth toss equals

a) ½ b) 1/32 c) 31/32 d) 1/5

166. Which of the following statement is/are correct?

a) Three coins are tossed once. At least two them must land the same way. No mater they land heads or tails, the third coin is equally likely to land either the same way or oppositely. So, the chance that all the three coins land the same way is ½

b) Let $0 < P(B) < 1$ and $P(A/B) = P(A/B^C)$. Then A and B are independent

c) Suppose an urn contains 'w' white and 'b' black balls and a ball is drawn from it and is replaced along with 'd' additional balls of the same colour. Now a second ball is drawn from it. The probability that the second drawn ball is white is independent of the value of 'd'

d) A, B, C simultaneously satisfy
$P(ABC) = P(A)P(B)P(C)$
$P(AB\overline{C}) = P(A)P(B)P(\overline{C})$
$P(A\overline{B}C) = P(A)P(\overline{B})P(C)$
$P(A - BC) = P(\overline{A})P(B)P(C)$
Then A, B, C are independent

167. Suppose m boys and m girls take their seats randomly round a circle. The probability of their sitting is $(\,^{2m-1}C_m\,)^{-1}$ when

a) No two boys sit together b) No two girls sit together

c) Boys and girls sit alternatively d) All the boys sit together

168. Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently equals

a) ½ b) 7/15 c) 2/15 d) 1/3

169. The probabilities that a student in Mathematics. Physics and Chemistry are α, β and γ respectively. Of these subjects, a student has 75% chance of passing in atleast one, 50% chance of passing in atleast two and 40% chance of passing in exactly two subjects. Which of the following relations are true?

a) $\alpha + \beta + \gamma = 19/20$ b) $\alpha + \beta + \gamma = 27/20$ c) $\alpha\beta\gamma = 1/10$ d) $\alpha\beta\gamma = 1/4$

170. Fifteen coupons are numbered 1, 2, 3, …,15. Seven coupons are selected at random one at a time with replacement. The probability that the largest number appearing on selected coupon is 9 is

a) $(9/16)^6$ b) $(8/15)^7$ c) $(3/5)^7$ d) None of these

171. The probability that a 50-year-old man will be alive at 60 is 0.83 and the probability that a 45-year-old woman will be alive at 55 is 0.87. Then

a) The probability that both will be alive is 0.7221

b) At least one of them will alive is 0.9779

c) At least one of them will alive is 0.8230

d) The probability that both will be alive is 0.6320

172. If A and B are two events such that $P(A) = 3/4$ and $P(B) = 5/8$, then

a) $P(A \cup B) \geq 3/4$ b) $P(A' \cap B) \leq 1/4$

c) $3/8 \leq P(A \cap B) \leq 5/8$ d) $3/8 \leq P(A \cap B) \leq 5/8$

173. One-hundred identical coins, each with probability, p, of showing up heads are tossed once. If $0 < p < 1$ and the probability of heads showing on 50 coins is equal to that of heads showing on 51 coins, then the value of p is

a) ½ b) 49/101 c) 50/101 d) 51/101

174. A bag initially contains 1 red and 2 blue balls. An experiment consisting of selecting a ball at random, nothing its colour and replacing it together with an additional ball of the same colour. If three such trials are made, then

a) Probability that at least one blue ball is drawn is 0.9
b) Probability that exactly one blue ball is drawn is 0.2
c) Probability that all the drawn balls are red given that all the drawn balls are of same colour is 0.2
d) Probability that atleast one red ball is drawn is 0.6

175. E and F are two independent events. The probability that both E and F happen is 1/12 and the probability that neither E nor F happens is 1/2. Then,

a) $P(E) = 1/3, P(E) = 1/4$ b) $P(E) = 1/4, P(F) = 1/3$
c) $P(E) = 1/6, P(E) = 1/2$ d) $P(E) = 1/2, P(F) = 1/6$

176. The probability that a married man watches a certain TV show is 0.4 and the probability that a married woman watches the show is 0.5. the probability that a man watches the show, given that his wife does, is 0.7. Then

a) The probability that married couple watches the show is 0.35
b) The probability that a wife watches the show given that her husband does is 7/8
c) The probability that atleat one person of a married couple will watch the show is 0.55
d) None of these

177. Two buses A and B are scheduled to arrive at a town central bus station at noon. The probability that bus A will be late is 1/5. The probability that bus B will be late is 7/25. The probability that the bus B is late given that bus A is late is 9/10. Then,

a) Probability that neither bus will be late on particle day is 7/10
b) Probability that bus A is late given that bus B is late us 18/28
c) Probability that at least one bus is late is 3/10
d) Probability that at least one bus is in time is 4/5

178. If the integers m and n are chosen at random between 1 and 100, then the probability that a number of the from $7^m + 7^n$ is divisible by 5 equals

a) ¼ b) 1/7 c) 1/8 d) 1/49

179. Probability if n heads in $2n$ tosses of a fair coin can be given by

a) $\prod_{r=1}^{n}\left(\frac{2r-1}{2r}\right)$ b) $\prod_{r=1}^{n}\left(\frac{n+r}{2r}\right)$ c) $\sum_{r=0}^{n}\left(\frac{{}^nC_r}{2^n}\right)^2$ d) $\frac{\sum_{r=0}^{n}({}^nC_r)^2}{(\sum_{r=0}^{2n}{}^{2n}C_r)}$

180. For two events A and $B, P(A \cap B)$ is

a) Not less than $P(A) + P(B) - 1$ b) Not greater than $P(A) + P(B)$
c) Equal to $P(A) + P(B) - P(A \cup B)$ d) Equal to $P(A) + P(B) + P(A \cup B)$

181. For the three events A, B and C, P(exactly one of the events A or B occurs)$=P$(exactly one of the two events B or C occurs)$=P$(exactly one of the events C or A occurs)$=p$ and p (all the three events occur simultaneously)$=p^2$, where $0 < p < 1/2$. Then the probability of at least one of the three events A, B and occurring is

a) $\frac{3p+2p^2}{2}$ b) $\frac{p+3p^2}{4}$ c) $\frac{p+3p^2}{2}$ d) $\frac{3p+2p^2}{4}$

182. The probability that at least one of the events A and B occurs is 0.6. If Aa nd B occur simultaneously with probability 0.2, then $P(\overline{A}) + P(\overline{B})$ is

(Here $\overline{A}$ and $\overline{B}$ are complements of A and B, respectively)

a) 0.4 b) 0.8 c) 1.2 d) None

183. Let $0 < P(A) < 1,\ 0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A)P(B)$, then

a) $P(B/A)=P(B)-P(A)$ b) $P(\overline{A}-\overline{B})=P(\overline{A})-P(\overline{B})$
c) $P(\overline{A\cup B})=P(\overline{A})P(\overline{B})$ d) $P(A/B)=P(A)$

184. If A and B are two independent events such that $P(A)=1/2, P(B)=1/5$, then
a) $P(A/B)=1/2$ b) $P\left(\frac{A}{A\cup B}\right)=\frac{5}{6}$ c) $P\left(\frac{A\cap B}{A'\cup B'}\right)=0$ d) None of these

185. For any two events $A$and B in a sample space,
a) $P(A/B)\geq\frac{P(A)+P(B)-1}{P(B)}(P(B)\neq 0)$ is always true
b) $P(A\cap\overline{B})=P(A)-P(A\cap B)$ does not hold
c) $P(A\cup B)=1-P(\overline{A})P(\overline{B})$, if A and B are independent
d) $P(A\cup B)=1-P(\overline{A})P(\overline{B})$, if A and B are disjoint

186. Two numbers are randomly selected and multiplied. Consider two events E_1 and E_2 defined as
E_1: Their product is divisible by 5
E_2: Unit's places in their product is 5
Which of the following statement is/are correct?
a) E_1 is twice as likely to occurs as E_2 b) E_1 and E_2 are disjoint
c) $P(E_2/E_1)=1/4$ d) $P(E_1/E_2)=1$

187. If E and F are events with $P(E)\leq P(F)$ and $P(E\cap F)>0$. Then
a) Occurrence of $E\Rightarrow$occurrence of F b) Occurrence of $F\Rightarrow$occurrence of E
c) Non-occurrence of $E\Rightarrow$occurrence of F d) None of the above implication holds

188. The chance of an event happening is the square of the chance of a second event but the odds against the first are the cube of the odds against the second. The chances of the events are
a) $p_1=1/9$ b) $p_1=1/16$ c) $p_2=1/3$ d) $p_2=1/4$

189. If the probability of choosing an integer 'k' out of $2m$ integers $1, 2, 3, \dots 2m$ is inversely proportional to $k^4(1\leq k\leq m)$. If x_1 is the probability that chosen number is odd and x_2 is the probability that chosen number is even, then
a) $x_1>1/2$ b) $x_1>2/3$ c) $x_2<1/2$ d) $x_2<2/3$

190. Let $0<P(A)<1, 0<P(B)<1$ and $P(A\cup B)=P(A)+P(B)-P(A)P(B)$. Then
a) $P(B/A)=P(B)-P(A)$ b) $P(A'-B')=P(A')-P(B')$
c) $P(A\cup B)=P(A')+P(B')$ d) $P(A/B)=P(A)$

191. If A and B are two independent events such that $P(\overline{A}\cap B)=2/15$ and $P(A\cap\overline{B})=1/6$, then $P(B)$ is
a) 1/5 b) 1/6 c) 4/5 d) 5/6

192. A card is selected at random from cards numbered as 00, 01, 02,….,99. An event is said to have occurred. If product of digits of the card number is 16. If card is selected 5 times with replacement each time, then the probability that the event occurs exactly three time is
a) ${}^5C_3\left(\frac{3}{100}\right)^2\left(\frac{97}{100}\right)^3$ b) ${}^5C_3\left(\frac{3}{100}\right)^3\left(\frac{97}{100}\right)^2$ c) ${}^5C_3\left(\frac{0.3}{100}\right)^3\left(\frac{9.7}{100}\right)^3$ d) $10(0.03)^3(0.97)^2$

Assertion - Reasoning Type

This section contain(s) 0 questions numbered 193 to 192. Each question contains STATEMENT 1(Assertion) and STATEMENT 2(Reason). Each question has the 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

a) Statement 1 is True, Statement 2 is True; Statement 2 **is** correct explanation for Statement 1

b) Statement 1 is True, Statement 2 is True; Statement 2 **is not** correct explanation for Statement 1

c) Statement 1 is True, Statement 2 is False

d) Statement 1 is False, Statement 2 is True

193 Let $H_1, H_2, \dots, H_n$ be mutually exclusive events with $P(H_i) > 0, i = 1,2, \dots, n$. Let E be any other event with $0 < P(E) < 1$.

Statement 1: $P(H_i/E) > P(E/H_i) \cdot P(H_i)$ for $i = 1, 2, \dots, n$

Statement 2: $\sum_{i=1}^{n} P(H_i) = 1$

194

Statement 1: If $\frac{1}{5}(1+5p), \frac{1}{3}(1+2p), \frac{1}{3}(1-p)$ and $\frac{1}{5}(1-3p)$ are probabilities of four mutually exclusive events, then p can take infinite number of values

Statement 2: If A, B, C and D are four mutually exclusive events, then $P(A), P(B), P(C), P(D) \geq 0$ and $P(A) + P(B) + P(C) + P(D) \leq 1$

195

Statement 1: 20 persons are sitting in a row. Two of these persons are selected at random. The probability that two selected persons are not together is 0.7.

Statement 2: If A is an event, then $P(\text{not } A) = 1 - P(A)$.

196 Let A and B be two independent events

Statement 1: If $P(A) = 0.4$ and then $P(A \cup \overline{B}) = 0.9$, then $P(B)$ is 1/6

Statement 2: If $A$and B are independent, then $P(A \cap B) = P(A)P(B)$

197

Statement 1: If A, B, C be three mutually independent events, then A and $B \cup C$ are also independent events

Statement 2: Two events $A$and B are independent if and only if $P(A \cap B) = P(A)P(B)$

198

Statement 1: There are 4 addressed envelopes and 4 letters for each one of them. The probability that no letter is mailed in its correct envelopes is 3/8

Statement 2: The probability that all letters are not mailed in their correct envelope is 23/24

199 Let $m \in N$ and suppose three numbers are chosen at random from the numbers $1,2,3, \dots . m$

Statement 1: If $m = 2n$ for some $n \in N$, then the chosen numbers are in A.P. with probability $\frac{3}{2(2n-1)}$

Statement 2: If $m = 2n + 1$, then the chosen numbers are in A.P. with probability $\frac{3n}{4n^2-1}$

200 From an urn containing a white and b black balls $k(<, b)$ balls are drawn and laid aside, their colour is un noticed. Then another ball, that is $(k+1)^{th}$ ball, is drawn

Statement 1: Probability that $(k+1)^{th}$ ball drawn is white is $\frac{a}{a+b}$

Statement 2: Probability that $(k+1)^{th}$ ball drawn is black is $\frac{a}{a+b}$

201

Statement 1: For events A and B of sample space if $P\left(\frac{A}{B}\right) \geq P(A)$, then $P\left(\frac{B}{A}\right) \geq P(B)$

Statement 2: $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} (P(B) \neq 0)$

202 Let A and B be two independent events

Statement 1: If $P(A) = 0.3$ and $P(A \cup \overline{B}) = 0.8$, then $P(B)$ is 2/7

Statement 2: $P(\overline{E}) = 1 - P(E)$, where E is any event

203

Statement 1: If A and B are two events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{2}{3}$, then $\frac{1}{6} \leq P(A \cap B) \leq \frac{1}{2}$.

Statement 2: $P(A \cup B) \leq \max\{P(A), P(B)\}$ and $P(A \cap B) \geq \min\{P(A), P(B)\}$.

204

Statement 1: A natural number is chosen at random. The probability that the sum of the digits of its square is 93, is 0

Statement 2: A natural is divisible by 31 iff sum of its digits is divisible by 31

205 Let A and B be two event such that $P(A \cup B) \geq 3/4$ and $1/8 \leq P(A \cap B) \leq 3/8$

Statement 1: $P(A) + P(B) \geq 7/8$

Statement 2: $P(A) + P(B) \leq 11/8$

206

Statement 1: The probability of drawing either an ace or a king from a pack of card in a single draw is 2/13

Statement 2: For two events A and B which are not mutually exclusive, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

207 In the random experiment of tossing two unbiased dice let E be the event of getting the sum 8 and F be the event of getting even numbers on both the dice. Then,

Statement 1: $P(E) = \frac{7}{36}$

Statement 2: $P(F) = \frac{1}{3}$

208 Four numbers are chosen at random (without replacement) from the set {1, 2, 3,………, 20}

Statement 1: The probability that the chosen numbers when arranged in some order will form an AP, is $\frac{1}{85}$

Statement 2: If the four chosen numbers form an AP, then the set of all possible values of common difference is $\{\pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$

209

Statement 1: If A and B are two events such that $0 < P(A), P(B) < 1$, then $P(A/\overline{A}) + P(\overline{A}/\overline{B}) = 3/2$

Statement 2: If A and B are two events such that $0 < P(A), P(B) < 1$, then $P(A/B) = P(A \cap B)/P(B)$ and $P(\overline{B}) = P(A \cap \overline{B}) + P(\overline{A} \cap \overline{B})$

210 A set P contains n elements. Two subsets A and B of P are chosen independently

Statement 1: Probability that $A \cap B = A$ is $(3/4)^n$

Statement 2: Probability that $A \cup B = P$ is $(1/2)^n$

211 Consider an event for which probability of success is 1/2

Statement 1: Probability that in n trials, there are r success where $r = 4k$ and k is an integer is $\frac{1}{4} + \frac{1}{2^{n/2+1}} \cos\left(\frac{n\pi}{4}\right)$

Statement 2: ${}^nC_0 + {}^nC_4 + {}^nC_8 + \ldots = 2^{n/2} \sin\left(\frac{n\pi}{4}\right)$

212 Let A and B be two events such that $P(A) = 3/5$and $P(B) = 2/3$. Then

Statement 1: $4/15 \leq P(A \cap B) \leq 3/5$

Statement 2: $2/5 \leq P(A/\cap B) \leq 9/10$

213

Statement 1: A natural number x is chosen at random from the first 100 natural numbers. The probability that $\frac{(x-10)(x-50)}{x-30} > 0$ is 0.69.

Statement 2: If A is an event, then $O < P(A) < 1$.

214

Statement 1: If 12 coins are thrown simultaneously, then probability of appearing exactly five head is equal to probability of appearing exactly 7 heads.

Statement 2: ${}^nC_r = {}^nC_s \Rightarrow$ either $r = s$ or $r + s = n$ and $P(H) = P(T)$ in a single trial.

215

Statement 1: If $A = \{2,4,6\}, B = \{1,2,3\}$ where A and B are the events of numbers occurring on a dice, then $P(A) + P(B) = 1$

Statement 2: If $A_1, A_2, A_3, \ldots, A_n$ are all mutually exclusive events, then $P(A_1) + P(A_2) + \ldots + P(A_n) = 1$

216

Statement 1: Out of 5 tickets consecutively numbered three are drawn at random. The chance that the numbers on them are in A.P. is 2/15

Statement 2: Out of $2n + 1$ tickets consecutively numbered, three are drawn at random, the chance that the numbers on them are in A.P. is $3n/(4n^2 - 1)$

217 Let A, B and C be three events associated to a random experiment

Statement 1: If $A \cap B \subseteq C$, then $P(C) \geq P(A) + P(B) - 1$

Statement 2: If $P\{(A \cap B) \cup (B \cap C) \cup (C \cap A)\}$
$\leq \min\{P(A \cup B), P(B \cup C), P(C \cup A)\}$

218

Statement 1: If $P(A) = 0.25, P(B) = 0.50$ and $P(A \cap B) = 0.14$, then the probability that neither A nor B occurs is 0.39

Statement 2: $(\overline{A \cup B}) = \overline{A} \cup \overline{B}$

219 Let A and B be two events such that $P(A) > 0$

Statement 1: If $P(A) + P(B) > 1$, then
$P(B/A) \geq 1 - (\overline{B})/P(A)$

Statement 2: If $P(A/\overline{B}) \geq P(A)$, then $P(A) \geq P(A/B)$

220

Statement 1: If a fair coin is tossed 15 times, then the probability of getting head as many times in the first ten throws as in the last five is 3003/32768

Statement 2: Sum of the series ${}^{m}C_r\,{}^{n}C_0 + {}^{m}C_{r-1}\,{}^{n}C_1 + \ldots + {}^{m}C_0\,{}^{n}C_r = {}^{m+n}C_r$

221 Let $H_1, H_2, \ldots, H_n$ be mutually exclusive events with $P(H_1) > 0, i = 1,2, \ldots, n$.Let E be any event with $0 < P(E) < 1$

Statement 1: $P(H_i/E) > P(E/H_i)P(H_i)$ for $i = 1,2, \ldots, n$

Statement 2: $\sum_{i=1}^{n} P(H_i) = 1$

222 A fair die is rolled once

Statement 1: The probability of getting a composite number is 1/3

Statement 2: There are three possibilities for the obtained number: (i) the number is a prime number, (ii) the number is a composite number and (iii) the number is 1. Hence probability of getting a prime number is 1/3

Matrix-Match Type

This section contain(s) 0 question(s). Each question contains Statements given in 2 columns which have to be matched. Statements (A, B, C, D) in **columns I** have to be matched with Statements (p, q, r, s) in **columns II**.

223. Suppose that E_1 and E_2 are two events of a random experiment such that $P(E_1) = \frac{1}{4}, P\left(\frac{E_2}{E_1}\right) = \frac{1}{2}$ and $P\left(\frac{E_1}{E_2}\right) = \frac{1}{4}$, observe the lists given below

Column-I	Column- II
(A) $P(E_2)$	(p) 1/4
(B) $P(E_1 \cup E_2)$	(q) 5/8
(C) $P(\bar{E}_1/\bar{E}_2)$	(r) 1/8
(D) $P(E_1/\bar{E}_2)$	(s) 1/2
	(t) 3/8
	(u) 3/4

CODES :

	A	**B**	**C**	**D**
a)	b	c	d	a
b)	d	e	d	a
c)	d	b	d	a
d)	a	b	c	d

224.

Column-I	Column- II

(A)	The probability of a bomb hitting a bridge is 1/2. Two direct hits are needed to destroy it. The number of bombs required so that the probability of the bridge being destroyed is greater than 0.9 can be	(p)	4
(B)	A bag contains 2 red, 3 white and 5 black balls, a ball is drawn its colour is noted and replaced. The number of times, a ball can be drawn so that the probability of getting a red ball for the first time is at least 1/2	(q)	6
(C)	A drawer contains a mixture of red socks and blue socks, at most 17 in all. It so happens that when two socks are selected randomly without replacement, there is a probability of exactly 1/2 that both are red or both are blue. Then number of red socks in the drawer can be	(r)	7
(D)	There are two red, two blue, two white and certain number (greater than 0) of green socks in a drawer. If two socks are taken at random from the drawer without replacement, the probability that they are of the same colour is 1/5, then the number of green socks are	(s)	10

CODES :

	A	**B**	**C**	**D**
a)	P,s	q	r,p	s
b)	c,d	a,c	b,d	a,b
c)	r,s	p,q,r,s	p,q,r,s	p
d)	p,s	t,r,p	q	p,q

225. An urn contains r red balls and b black balls

	Column-I		**Column- II**
(A)	If the probability of getting two red balls in first two draws (without replacement) is ½, then value of r can be	(p)	10
(B)	If the probability of getting two red balls in first two draws (without replacement) is ½ and b is an even number, then r can be	(q)	3
(C)	If the probability of getting exactly two red balls in four draws (with replacement) from the urn is 3/8 and $b = 10$, then r can be	(r)	8
(D)	If the probability of getting exactly n red balls in $2n$ draw (with replacement) is equal to probability of getting exactly n black balls in $2n$ draws (with replacement), then the ratio	(s)	2

r/b can be

CODES :

	A	B	C	D
a)	S	r	p,q	t
b)	q,r	r	p	a,b,c,d
c)	q	p,q,r	s,t	r
d)	p,q,r,s	p	q	r,s

226. A bag contains some white and some black balls, all combinations being equally likely. The total number of balls in the bag is 12.four balls are drawn at random from the bag at random without replacement

	Column-I		Column- II
(A)	Probability that all the four balls are black is equal to	(p)	14/33
(B)	If the bag contains 10 black and 2 white balls then the probability that all four balls are black is equal to	(q)	1/3
(C)	If all the four balls are black, then the probability that the bag contains 10 black balls is equal to	(r)	70/429
(D)	Probability that two balls are black and two are white is	(s)	13/165

CODES :

	A	B	C	D
a)	q	p	r	q
b)	p	q	r	s
c)	t	s	p	r
d)	s	q	p	r

227.

	Column-I		Column- II
(A)	If the probability of getting at least one head is at least 0.8 is n trails then value of n can be	(p)	2
(B)	One mapping is selected at random from all mappings of the set $s = \{1, 2, 3, \dots, n\}$ into itself. If the probability that the mapping being one-one is 3/32, then find the value of n is	(q)	3
(C)	If m is selected at random from set {1,2, …,10} and the probability that the real roots is k, then value of $5k$ is more than	(r)	4
(D)	A man firing at a distant target as 20% chance of hitting the target in one shoot. If P be the probability of hitting the target in 'n' attempts,	(s)	5

where $20P^2 - 13P + 2 \leq 0$, then the ratio of maximum and minimum value of n is less than

CODES :

	A	B	C	D
a)	P	q	r	s
b)	q,r,s	r	p,q	p,q,r,s
c)	s	t	s,r	t
d)	p,q,r,s	p,q	s,r	t,q

228. 'n' whole numbers are randomly chosen and multiplied

	Column-I		**Column- II**
(A)	The probability that the last digit is 1,3, 7 or 9 is	(p)	$\frac{8^n - 4^n}{10^n}$
(B)	The probability that the last digit is 2,4, 6, 8 is	(q)	$\frac{5^n - 4^n}{10^n}$
(C)	The probability that the last digit is 5 is	(r)	$\frac{4^n}{10^n}$
(D)	The probability that the last digit is zero is	(s)	$\frac{10^n - 8^n - 5^n + 4^n}{10^n}$

CODES :

	A	B	C	D
a)	P	q	r	s
b)	t	s	r	q
c)	r	p	q	s
d)	s	p	r	q

229. An urn contains four black and eight white balls. Three balls are drawn from the urn without replacement. Three events are defined on this experiment
A: Exactly one black ball is drawn
B: All balls are drawn are of the same colour
C: Third drawn ball is black
Match the entries of column I with none, one or more entries of column II

	Column-I		**Column- II**
(A)	The events A and B are	(p)	Mutually exclusive
(B)	The events B and C are	(q)	Independent
(C)	The events C and A are	(r)	Neither independent nor mutually exclusive
(D)	The events A, B and C are	(s)	Exhaustive

CODES :

	A	B	C	D

a)	P	r	q	p
b)	p	q	r	s
c)	q	p	r	t
d)	s	p	r	q

230.

	Column-I		Column- II
(A)	Six different balls are put in three different boxes, none being empty. The probability of putting the balls equal number is	(p)	20/27
(B)	Six letters are posted in three letter boxes. The probability that no letter box remains empty is	(q)	1/6
(C)	Two persons A and B throw two dice each. If A throw a sum of 9, then the probability of B throwing a sum greater than A is	(r)	1/3
(D)	If A and B are independent and $P(A) = 0.3$ and $P\left(A \cup \overline{B}\right) = 0.8$, the $P(B)$ is equal to	(s)	2/7

CODES :

	A	**B**	**C**	**D**
a)	P	q	r	s
b)	t	s	p	r
c)	s	q	p	r
d)	q	p	q	s

231. Let A and B are two independent events such that $P(A) = 1/3$ and $P(B) = 1/4$

	Column-I		Column- II
(A)	$P(A \cup B)$ is equal to	(p)	1/12
(B)	$P(A/A \cup B)$ is equal to	(q)	1/2
(C)	$P(B/A' \cap B')$ is equal to	(r)	2/3
(D)	$P(A'/B')$ is equal to	(s)	0

CODES :

	A	**B**	**C**	**D**
a)	P	q	r	s
b)	t	s	p	r
c)	s	q	p	r
d)	q	r	s	r

Linked Comprehension Type

This section contain(s) 26 paragraph(s) and based upon each paragraph, multiple choice questions have to be answered. Each question has atleast 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 232 to -232

If a particular experiment be given n(a finite) independent trials. If the probability of success in one trial (say) p

$\therefore$ We get probability of failure, $q = (1 - p)$

The probability of r success in n trials $= {}^nC_r p^r q^{n-r}$

On the basis of above information, answer the following questions :

232. The probability of man hitting a target in one fire is 1/4. At least n times he must fire at the target that the chances of hitting the target at least once will exceed 2/3, then n is

a) 2 b) 4 c) 6 d) 8

Paragraph for Question Nos. 233 to - 233

There are n urns each containing $(n+1)$ balls such that the ith urn contains `i' white balls and $(n+1-i)$ red balls. Let u_i be the event of selecting ith urn, $i = 1, 2, 3, \dots, n$ and W deontes the event of getting white balls.

On the basis of above information, answer the following questions :

233. If $P(u_i) \propto i$, where $i = 1, 2, 3 \dots, n$, then $\lim_{n\to\infty} P(W)$ is equal to

a) 1 b) 2/3 c) 1/4 d) 3/4

Paragraph for Question Nos. 234 to - 234

A is set containing 10 elements. A subset P of A is chosen at random and the set A is reconstructed by replacing the elements of P. Another subset Q of A is now chosen at random. Then, the probability if.

On the basis of the above information, answer the following questions :

234. $P \cup Q = A$, is

a) $\left(\frac{1}{2}\right)^{10}$ b) $\left(\frac{2}{3}\right)^{10}$ c) $\left(\frac{3}{4}\right)^{10}$ d) $\left(\frac{2}{5}\right)^{10}$

Paragraph for Question Nos. 235 to - 235

In a class of 10 students, probability of exactly i students passing an examination is directly proportional to i^2. Then answer the following questions:

235. The probability that exactly 5 students passing an examination is

a) 1/11 b) 5/77 c) 25/77 d) 10/77

Paragraph for Question Nos. 236 to - 236

A shopping mall is running a scheme: 'Each packet of detergent "SURE" contains a coupon which bears letter of the word, "SURF", if a person buys at least four packets of detergent "SURF", and produce all the letters of the word “SURF”, then he gets one free packet of detergent

236. If a person buys 8 such packets at a time, then number of different combinations of coupon he has

a) 4^8 b) 8^4 c) ${}^{11}C_3$ d) ${}^{12}C_4$

Paragraph for Question Nos. 237 to - 237

In an objective paper, there are two sections of 10 questions each. For ‘section 1’, each question has 5 options and only one option is correct and ‘section 2’ has 4 options with multiple answers and marks for a question in this section is awarded only if he ticks all correct answers. Marks for each questions in ‘section 1’ is and in ‘section 2’ is 3. (There is no negative marking)

237. If a candidate attempts only tw3o questions by guessing, one from ‘section 1’ and one from ‘section 2’, the probability that he scores in both questions is

a) 74/75 b) 1/25 c) 1/15 d) 1/75

Paragraph for Question Nos. 238 to - 238

There are two die A and B both having six faces. Die A has three faces marked with 1, two faces marked with 2 and one face marked with 3. Die B has one face marked with 1, two faces marked with 2 and three faces marked with 3. Both dices are thrown randomly once. If E be the event of getting sum of the numbers appearing on top faces equal to x and let $P(E)$ be the probability of even E, then

238. $P(E)$ is maximum when x equal to

a) 5 b) 3 c) 4 d) 6

Paragraph for Question Nos. 239 to - 239

A JEE aspirant estimates that she will be successful with an 80% chance if she studies 10 hours per day, with a 60% chance if she studies 7 hours per day and with a 40% chance if she studies 4 hours per day. She further believes that she will study 10 hours, 7 hours and 4 hours per day with probabilities 0.1, 0.2 and 0.7, respectively

239. The chance she will be successful is

a) 0.28 b) 0.38 c) 0.48 d) 0.58

Paragraph for Question Nos. 240 to - 240

Let S and T are two events defined on a sample space with probabilities $P(S) = 0.5, P(T) = 0.69, P(S/T) = 0.5$

240. Events S and T are

a) Mutually exclusive b) Independent

c) Mutually exclusive and independent d) Neither mutually exclusive nor independent

Paragraph for Question Nos. 241 to - 241

An amoeba either splits into two or remains the same or eventually dies out immediately after completion of every second with probabilities, respectively,1/2, 1/4 and ¼. Let the initial amoeba if it is distinct from the previous one, be called as 2^{nd}, 3^{rd}, … generations

241. The probability that immediately after completion of 2s all the amoeba population dies out is
  a) 9/32 b) 11/32 c) 1/2 d) 3/32

Paragraph for Question Nos. 242 to - 242

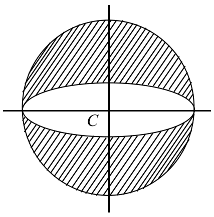
A cube having all of its sides painted is cut by two horizontal, two vertical and other two planes so as to form 27 cubes all having the same dimensions. Of these cubes, a cube is selected at random

242. The probability that the cube selected has none of its sides painted is
  a) 1/9 b) 1/27 c) 1/18 d) 5/54

Paragraph for Question Nos. 243 to - 243

There are some experiments in which the outcomes cannot be identified discretely. For example, an ellipse of eccentricity $2\sqrt{2}/3$ is inscribed in a circle and a point within the circle is chosen at random. Now, we want to find the probability that this point lies outside the ellipse. Then, the point must lie in the shaded region shown in figure. Let the radius of the circle be a and length of minor axis of the ellipse be $2b$. Given that

$$1-\frac{b^2}{a^2}=\frac{8}{9}\Rightarrow\frac{b^2}{a^2}=\frac{1}{9}$$



Then, the area of circle serves as sample space and area of the shaded region represents the area for favourable cases. Then, required probability is

$$p=\frac{\text{area of shaded region}}{\text{area of circle}}$$
$$=\frac{\pi a^2-\pi ab}{\pi a^2}$$
$$=1-\frac{b}{a}$$
$$=1-\frac{b}{a}$$
$$=1-\frac{1}{3}$$
$$=\frac{2}{3}$$

Now answer the following questions

243. A point is selected at random inside a circle. The probability that the point is closer to the centre of the circle than to its circumference is

a) 1/4 b) 1/2 c) 1/3 d) $1/\sqrt{2}$

Paragraph for Question Nos. 244 to - 244

If the squares of a 8×8 chessboard are painted either red or black at random

244. The probability that not all the squares in any column are alternating in colour is
a) $(1 - 1/2^7)^8$ b) $1/2^{56}$ c) $1 - 1/2^7$ d) None of these

Paragraph for Question Nos. 245 to - 245

Two fair dice are rolled. Let $P(A_i) > 0$ denote the event that the sum of the faces of the dice is divided by i

245. Which one of the following events is most probable?
a) A_3 b) A_4 c) A_5 d) A_6

Paragraph for Question Nos. 246 to - 246

A player tosses a coin and scores one point for every head and two points for every tail that turns up. He plays on until his score reaches or passes $n.P_n$ denotes the probability of getting a score of exactly n

246. The value of P_n is equal to
a) $(1/2)[P_{n-1} + P_{n-2}]$ b) $(1/2)[2P_{n-1} + P_{n-2}]$ c) $(1/2)[P_{n-1} + 2P_{n-2}]$ d) None of these

Paragraph for Question Nos. 247 to - 247

The probability that a family has exactly n children is $ap^n, n \geq 1$. All sex distributions of n children in a family have the same probability

247. The probability that a family contains exactly k boys is (where $k \geq 1$)
a) $\alpha p^k(1-p)^{-k-1}$ b) $2\alpha p^k(2-p)^{-k-1}$ c) $2\alpha p^k(2-p)^{-k}$ d) $2\alpha p^{k-1}(2-p)^{-k-1}$

Integer Answer Type

248. Two cards are drawn from a well shuffled pack of 52 cards. The probability that one is a heart card and the other is a king is p, then the value of $104p$ is
249. If A and B are events such that $P(A) = 0.6$ and $P(B) = 0.8$, if the greatest value that $P(A/B)$ can have is p, then the value of $8p$ is
250. A bag contains 10 different balls. Five balls are drawn simultaneously and then replaced and then seven balls are drawn. The probability that exactly three balls are common to the two drawn is p, then the value of $12p$ is
251. A die is weighted such that the probability of rolling the face numbered n is proportional to $n^2 (n = 1, 2, 3, 4, 5, 6)$. The die is rolled twice, yielding the numbers a and b. The probability that $a < b$ is p, then the value of $[2/p]$ is, where [.] represents the greatest integer function
252. If two loaded dice each have the property that 2 or 4 is three times as likely to appear as 1,3, 5 or 6 on each roll. When two such dice are rolled, the probability of obtaining a total of 7 is p, then the value of $[1/p]$ is,

where $[x]$ represents the greatest integer less than or equal to x

253. A die is thrown three times. The chance that the highest number shown on the die is 4 is p, then the value of $[1/p]$ is where [.] represents greatest integer function is
254. Suppose A and $B$are two events with $P(A) = 0.5$ and $P(A \cup B) = 0.8$. let $P(B) = p$ if A and B are mutually exclusive and $P(B) = q$ if A and B are independent events, then the value of q/p is
255. Five different games are to be distributed among four children randomly. The probability that each child get at least one game is p, then the value of $[1/p]$ is, where [.] represents the greatest integer function
256. Thirty-two players ranked 1 to 32 are playing in a knockout tournament. Assume that in every match between any two players the better ranked player wins, the probability that ranked 1 and ranked 2 players are winner and runner up respectively is p, then the value of $[2/p]$ is, where [.] represents the greatest integer function
257. An urn contains three red balls and n white balls. Mr. A draws two balls together from the urn. The probability that they have the same colour is $1/2$. Mr. B draws one balls from the urn, note its colour and replaces it. He then draws a second ball from the urn and finds that both balls have the same colour is: $5/8$.The possible value of n is
258. A drawer contains a mixture of red and blue socks, at most 17 in all. It so happens that when two socks are selected randomly without replacement, there is a probability of exactly $1/2$ that both are red or both are blue. The largest possible number of red socks in the drawer that is consistent with this data is
259. Two different numbers are taken from the set (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10). The probability that their sum and positive difference are both multiple of 4 is $x/55$, then x equals
260. There are two red, two blue, two white and certain number (greater than 0) of green socks in a drawer. If two socks are taken at random from the drawer without replacement, the probability that they are of the same colour is $1/5$, then the number of green socks are
261. If the probability that the product of the outcomes of three rolls of a fair dice is a prime number is p, then the value of $1/(4p)$ is
262. If the probability of a six-digit number N whose six digit are 1, 2, 3, 4, 5, 6 written as random order is divisible by 6 is p, then the value of $1/p$ is

: ANSWER KEY :

1)	a	2)	b	3)	d	4)	a
5)	c	6)	c	7)	b	8)	b
9)	a	10)	b	11)	a	12)	d
13)	b	14)	a	15)	b	16)	b
17)	b	18)	a	19)	c	20)	a
21)	a	22)	d	23)	a	24)	c
25)	c	26)	a	27)	d	28)	a
29)	b	30)	b	31)	c	32)	b
33)	a	34)	a	35)	b	36)	c
37)	c	38)	b	39)	d	40)	b
41)	b	42)	b	43)	d	44)	a
45)	b	46)	b	47)	d	48)	a
49)	b	50)	c	51)	b	52)	b
53)	a	54)	d	55)	b	56)	a
57)	c	58)	c	59)	c	60)	b
61)	a	62)	c	63)	c	64)	b
65)	c	66)	d	67)	a	68)	c
69)	a	70)	a	71)	b	72)	c
73)	a	74)	c	75)	d	76)	d
77)	a	78)	d	79)	b	80)	b
81)	d	82)	a	83)	d	84)	b
85)	c	86)	d	87)	c	88)	c
89)	c	90)	a	91)	a	92)	a
93)	a	94)	c	95)	b	96)	d
97)	d	98)	c	99)	c	100)	c
101)	b	102)	b	103)	a	104)	a
105)	d	106)	c	107)	c	108)	d
109)	b	110)	a	111)	d	112)	a
113)	a	114)	b	115)	d	116)	c
117)	d	118)	b	119)	a	120)	a
121)	c	122)	d	123)	b	124)	c
125)	b	126)	a	127)	b	128)	b
129)	d	130)	c	131)	a	132)	b
133)	c	134)	c	135)	c	136)	b
1)	b,c	2)	a	3)	a,b,c	4)	
	a,d						
5)	a,b,c,d	6)	b,c	7)	b	8)	b
9)	a	10)	a,b	11)	c	12)	
	a,c,d						
13)	b	14)	a,d	15)	a	16)	
	a,b,c,d						
17)	a,b,c	18)	b,c,d	19)	c	20)	
	a,b,c						
21)	b	22)	d	23)	a,b,c	24)	
	a,c,d						
25)	a	26)	a,c	27)	b,c,d	28)	
	a,b,d						
29)	a	30)	b,c,d	31)	a,b,c	32)	b
33)	b,c	34)	d	35)	a,b	36)	
	a,b,c,d						
37)	d	38)	a,b,c,d	39)	a,b	40)	
	a,b,c						
41)	a,b,c	42)	a	43)	a,c,d	44)	
	a,b,c						
45)	a	46)	c	47)	c,d	48)	
	a,b,c						
49)	a,c	50)	c,d	51)	d	52)	
	a,c						
53)	a,c	54)	c,d	55)	b,c	56)	
	b,d						
1)	d	2)	a	3)	d	4)	b
5)	a	6)	b	7)	b	8)	b
9)	a	10)	a	11)	c	12)	c
13)	d	14)	b	15)	b	16)	c
17)	d	18)	c	19)	c	20)	a
21)	b	22)	a	23)	c	24)	a
25)	b	26)	c	27)	c	28)	a
29)	d	30)	c	1)	c	2)	c
	3)	b	4)	a			
5)	b	6)	c	7)	a	8)	d
9)	d	1)	b	2)	b	3)	c
	4)	b					
5)	c	6)	d	7)	c	8)	c
9)	b	10)	d	11)	b	12)	a
13)	a	14)	a	15)	a	16)	b
1)	4	2)	6	3)	5	4)	5
5)	7	6)	5	7)	2	8)	4
9)	3	10)	1	11)	7	12)	6
13)	4	14)	6	15)	2		

: HINTS AND SOLUTIONS :

1 **(a)**
The probability of hitting a target is $p = 1/5$. Therefore, the probability of not hitting a target is $q = 1 - 1/5 = 4/5$. Hence, the required probability is $1 - (4/5)^{10}$

2 **(b)**
We have,
$P(A) = \frac{40}{100}, P(B) = \frac{25}{100}$ and $P(A \cap B) = \frac{15}{100}$
So, $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{15/100}{40/100} = \frac{3}{8}$

3 **(d)**
We have,
$P(\overline{A \cup B}) = \frac{1}{4}, P(A \cap B) = \frac{1}{4}$
$P(\overline{A}) = \frac{1}{4} \Rightarrow P(A) = 1 - \frac{1}{4} = \frac{3}{4}$
$\therefore P(\overline{A \cup B}) = 1 - P(A \cup B)$
$= 1 - [P(A) + P(B) - P(A \cap B)]$
$\Rightarrow \frac{1}{6} = 1 - \frac{3}{4} - P(B) + \frac{1}{4}$
$\Rightarrow P(B) = 1 - \frac{1}{2} - \frac{1}{6} = \frac{6-3-1}{6} = \frac{2}{6} = \frac{1}{3}$
Since $P(A \cap B) = P(A)P(B)$ and $P(A) \neq P(B)$, therefore $A$and B are independent but not equally likely

4 **(a)**
$n(S) = {}^{10}C_7 = 120$
$n(A) = {}^5C_4 \times {}^3C_2 \times {}^2C_1$
$P(E) = \frac{5 \times 3 \times 2}{120} = \frac{1}{4}$

5 **(c)**
The total number of mapping is n^n. The number of one-one mapping is ${}^nC_1\,{}^{n-1}C_1 \ldots {}^1C_1 = n!$ Hence, the probability is
$\frac{n!}{n^n} = \frac{3}{32} = \frac{4!}{4^4}$
Comparing, we get $n = 4$

6 **(c)**
When 4 points are selected, we get one intersecting point. So, probability is
$\frac{{}^nC_4}{{}^{({}^nC_2 - n)}C_2}$

7 **(b)**
$P(4 \text{ biased coin}) = \frac{1}{3}$
$P(5 \text{ biased coin}) = \frac{1}{4}$
Hence, the required probability is
$\frac{1}{3}\frac{{}^4C_3\;{}^{16}C_6}{{}^{20}C_9} + \frac{2}{3}\frac{{}^5C_4\;{}^{15}C_5}{{}^{20}C_9}\frac{1}{{}^{11}C_1}$
$= \frac{2}{33}\left[\frac{{}^{16}C_6 + 5\,{}^{15}C_5}{{}^{20}C_9}\right]$

8 **(b)**
Probability of getting 2 heads in the first 5 trials is
${}^5C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^3 = \frac{10}{32} = \frac{5}{16}$
Therefore, the probability that third head appears on the sixth trial is $5/16 \times 1/2 = 5/32$

9 **(a)**
In any number the last digit can be 0, 1, 2, 3,4, 5, 6, 7, 8, 9. Therefore, last digit of each number can be chosen in 10 ways. Thus, exhaustive number of ways is 10^n. If the last digit be 1, 3, 7 or 9, then none of the numbers can be even or end in 0 or 5. Thus, we have a choice of 4 digits, viz, 1,3, 7 or 9 with which each of n numbers should end. So favourable number of ways is 4^n. Hence, the required probability is
$\frac{4^n}{10^n} = \left(\frac{2}{5}\right)^n$

10 **(b)**
The total number of ways in which four-figure numbers can be formed is $4! = 24$. A number is divisible by 5 if at unit's place we have 5. Therefore, unit's place can be filled in one way and the remaining 3 places can be filled with the other digits in 3! ways. Hence, total number of numbers divisible by 5 is $3! = 6$. So, the required probability is 6/24=1/4

11 **(a)**
$P(A) = P(B) = P(C)$ and $P(A) + P(B) + P(C) = 1$
$\therefore\; P(A) = P(B) = P(C) = \frac{1}{3}$
Also, $P(X) = \frac{5}{12}, P(X/A) = 3/8, P(X/B) = \frac{1}{4}$
We have,
$P(X) = P(A)P(X/A) + P(B)P(X/B) + P(C)P(X/C)$
$\therefore\; \frac{5}{12} = \frac{1}{3}\left\{\frac{3}{8} + \frac{1}{4} + P\left(\frac{X}{C}\right)\right\}$
$\Rightarrow P(X/C) = \frac{5}{8}$

12 **(d)**

$$P(E/F)+P(\overline{E}/F)=\frac{P(E\cap F)+P(\overline{E}\cap F)}{P(F)}$$

$$=\frac{P\{(E\cap F)\cup(\overline{E}\cap F)\}}{P(F)}$$

[$\because$ $E\cap F$ and $\overline{E}\cap F$ are disjoint]

$$=\frac{P\{(E\cup\overline{E})\cap F\}}{P(F)}=\frac{P(F)}{P(F)}=1$$

Similarly, we can show that (b) and (c) are not true while (d) is true

$$P\left(\frac{E}{\overline{F}}\right)+P\left(\frac{\overline{E}}{\overline{F}}\right)=\frac{P(E\cap\overline{F})}{P(\overline{F})}+\frac{P(\overline{E}\cap\overline{F})}{P(\overline{F})}=\frac{P(\overline{F})}{P(\overline{F})}$$

$$=1$$

13 **(b)**

Consider the following events:

A: Father has at least one boy

B: father has 2 boys and one girl

Then,

$A=$ one boy and 2 girls, 2 boys and one girl, 3 boys and no girl

$A\cap B=$ 2 boys and one girl

Now, the required probability is

$$P(A/B)=\frac{P(A\cap B)}{P(A)}=\frac{1}{3}$$

14 **(a)**

Let A denote the event that a sum of 5 occurs, B the event that a sum of 7 occurs and C the event that either a sum of 5 nor a sum of 7 occurs. We have,

$$P(A)=\frac{4}{36}=\frac{1}{9},P(B)=\frac{6}{36}=\frac{1}{6}$$

$$P(C)=\frac{26}{36}=\frac{13}{18}$$

Thus, probability that A occurs before B is

$P[A$ or $(C\cap A)$ or $(C\cap C\cap A)$ or ...]

$$=P(A)+P(C\cap A)+P(C\cap C\cap A)+\cdots$$

$$=P(A)+P(C)P(A)+P(C)^2P(A)+\cdots$$

$$=\frac{1}{9}+\left(\frac{13}{18}\right)\times\frac{1}{9}+\left(\frac{13}{18}\right)^3\frac{1}{9}+\ldots$$

$$=\frac{1/9}{1-13/18}=\frac{2}{5}$$

15 **(b)**

2							

The prime digits are 2, 3, 5, 7. If we fix 2 at first place, then other $2n-1$ places are filled by all four digits. So the total number of cases is 4^{2n-1}

Now, sum of 2 consecutive digits is prime when consecutive digits are (2,3) or (2, 5) Then 2 will be fixed at all alternative places

2		2		2		2	

So favourable number of cases is 2^n. Therefore, probability is

$$\frac{2^n}{4^{2n-1}}=2^n2^{-4n+2}=2^22^{-3n}=\frac{4}{2^{3n}}$$

16 **(b)**

Let H denote the head, T the tail and * any of the head or tail. Then, $P(H)=1/2, P(T)=1/2$ and $P(*)=1$. For at least four consecutive heads, we should have any of the following patterns:

	Probability
(i)$H\ H\ H\ H\ ***$	$(1/2)^4\times 1=1/16$
(ii)$T\ H\ H\ H\ H\ **$	$(1/2)^5=1/32$
(iii)$*\ T\ H\ H\ H\ H\ *$	$(1/2)^5=1/32$
(iv)$**\ T\ H\ H\ H\ H$	$(1/2)^5=1/32$

Since all the above cases are mutually exclusive, the probability of getting at least four consecutive heads (on adding) is $1/16+3/32=5/32$

17 **(b)**

Consider the following events:

A_1: A speaks truth

A_2: $=B$ speaks truth

Then, $P(A_1)=60/100=3/5, P(A_2)=70/100=7/10$

For the required event, either both of them should speak the truth or both of them should tell a lie.

Thus, the required probability is

$$P((A_1\cap A_2)\cup\overline{A}_1\cap\overline{A}_2))$$

$$=P(A_1\cap A_2)+P(\overline{A}_1\cap\overline{A}_2)$$

$$=P(A_1)P(A_2)+P(\overline{A}_1)(\overline{A}_2)$$

$$=\frac{3}{5}\times\frac{7}{10}+\left(1-\frac{3}{5}\right)\left(1-\frac{7}{10}\right)=0.54$$

18 **(a)**

The required probability is

$$\frac{2}{9}\times\frac{1}{8}\times\frac{4}{7}\times\frac{3}{6}\times\frac{2}{5}\times\frac{1}{4}\times 1\times 1\times 1=\frac{1}{1260}$$

19 **(c)**

$$x^2+2(a+4)x-5a+64\geq 0$$

If $D\leq 0$, then

$$(a+4)^2-(-5a+64)<0$$

$$\Rightarrow a^2+13a-48<0$$

$$\Rightarrow (a+16)(a-3)<0$$

$$\Rightarrow\ -16<a<3\ \Leftrightarrow\ -5\leq a\leq 2$$

Then, the favourable cases is equal to the number

of integers in the interval $[-5, 2]$, i.e., 8
Total number of cases is equal to the number of integers in the interval $[-5, 30]$, i.e., 36
Hence, the required probability is 8/36=2/9

20 **(a)**
Since each ball can be placed in any one of the 3 boxes, therefore there are 3 ways in which a ball can be placed in any one therefore there are 3 ways in which a ball can be placed in any one of the three boxes. Thus, there are 3^{12} ways in which 12 balls can be placed in 3 boxes. The number of ways in which 3 balls out of 12 can be put in the box is ${}^{12}C_3$. The remaining 9 balls can be placed in 2 boxes in 2^9 ways. So, required probability is
$$\frac{{}^{12}C_3}{3^{12}} 2^9 = \frac{110}{9} \left(\frac{2}{3}\right)^{10}$$

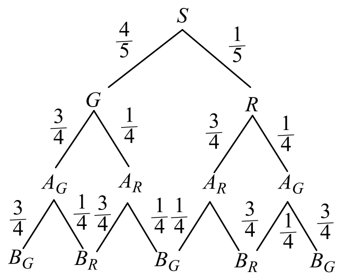
21 **(a)**
The number of ways in which 20 peoples can be divided into two equal groups is
$$n(s) = \frac{20!}{10!\,10!\,2!}$$
The number of ways in which 18 peoples can be divided into groups of 10 and 8 is
$$n(A) = \frac{18!}{10!\;\;8!}$$
$$\therefore P(E) = \frac{18!}{10!\,8!} \frac{10!\,10!\,2}{20!} = \frac{10 \times 9 \times 2}{20 \times 19} = \frac{9}{19}$$

22 **(d)**
Let X denote the largest number on the 3 tickets drawn
Then, $P(X \leq 7) = (7/20)^3$ and $P(X \leq 6) = (6/20)^3$. Then, the required probability is
$$P(X = 7) = \left(\frac{7}{10}\right)^3 - \left(\frac{6}{20}\right)^3$$

23 **(a)**
L and W can be filled at 14 places in 2^{14} ways
$\therefore n(S) = 2^{14}$
Now 13 L's and 1 W can be arranged at 14 places in 14 ways
Hence, $n(A) = 14$
$$\therefore p = \frac{14}{2^{14}} = \frac{7}{2^{13}}$$

24 **(c)**
We have,
$n(S) = 5^5$
For computing favourable outcomes, 2 boxes which are to remain empty, can be selected in ${}^5C_2$ ways and 5 marbles can be placed in the remaining 3 boxes in groups of 221 or 311 in
$$3!\left[\frac{5!}{2!\,2!\,2!} + \frac{5!}{3!\,2!}\right] = 150 \text{ ways } \Rightarrow n(A)$$
$$= {}^5C_2 \times 150$$
Hence,
$$P(E) = {}^5C_2 \times \frac{150}{5^5} = \frac{60}{125} = \frac{12}{25}$$

25 **(c)**
From the three diagram it follows that



$$P(B_G) = \frac{46}{80}$$
$$P(B_G \middle| G) = \frac{10}{16} = \frac{5}{8}$$
$$P(B_G \cap G) = \frac{5}{8} \times \frac{4}{5} = \frac{1}{2}$$
$$P(G|B_G) = \frac{P(B_G \cap G)}{P(B_G)} = \frac{1}{2} \times \frac{80}{46} = \frac{20}{23}$$

26 **(a)**
We are given that
$P(A \cap B) = P(A)P(B)$
$P(B \cap C) = P(B)P(C)$
$P(C \cap A) = P(C)P(A)$
$P(A \cap B \cap C) = P(A)(B)P(C)$
We have,
$P(A \cap B \cap C) = P(A \cap B \cap C) = P(A)P(B)P(C)$
$= P(A)P(B \cap C)$
$\Rightarrow A$ and $B \cap C$ are independent
Therefore, S_2 is true. Also,
$P[(A \cap (B \cap C))] = P[(A \cap B) \cup (A \cap C)]$
$= P(A \cap B) + P(A \cap C) - P[(A \cap B) \cap (A \cap C)]$
$= P(A \cap B) + P(A \cap C) - P[(A \cap B \cap C)]$
$= P(A)P(B) + P(A)P(C) - P(A)P(B)P(C)$
$= P(A)[P(B) + P(C) - P(B)P(C)]$
$= P(A)[P(B) + P(C - P(B \cap C)]$
$= P(A)P(B \cup C)$
Therefore, $A$and $B \cup C$ are independent

27 **(d)**

Let the probability for getting an odd number be p. Therefore, the probability for getting an even number is $2p$

$$\therefore\ p + 2p = 1\ \Rightarrow 3p = 1\ \Rightarrow p = \frac{1}{3}$$

Sum of two numbers is even means either both are odd or both are even. Therefore, the required probability is

$$\frac{1}{3}\times\frac{1}{3}+\frac{2}{3}\times\frac{2}{3}=\frac{1}{9}+\frac{4}{9}=\frac{5}{9}$$

28 **(a)**

The probability of getting a head in a single toss of a coin is $p = 1/2$ (say). The probability of getting 5 or 6 in a single throw of a die is $q = 2/6 = 1/3$ (say). Therefore, the required probability is

$$p + (1-p)(1-q)p + (1-p)(1-q)(1-p)(1-q)+\ldots$$

$$= p + (1-p)(1-q)p + (1-p)^2(1-q)^2p+\ldots$$

$$= \frac{p}{1-(1-p)(1-q)}$$

$$= \frac{1/2}{1-1/2\times 2/3} = \frac{3}{4}$$

29 **(b)**

If A, B, C represent events that the student is successful in tests I, II, III, respectively. Then the probability the student is successful is

$$P[(A\cap B\cap C')\cup(A\cap B'\cap C)\cup(A\cap B\cap C)]$$

$$= P(A\cap B\cap C) + P(A\cap B\cap C) + P(A\cap B\cap C)$$

$$= P(A)P(B)P(C') + P(A)P(B')P(C) + P(A)P(B)P(C)$$

[$\therefore A, B, C$ are independent events]

$$= pq\left(1-\frac{1}{2}\right) + \frac{p(1-q)1}{2} + pq\frac{1}{2}$$

$$= pq + \frac{1}{2}p - \frac{1}{2}pq$$

$$= \frac{1}{2}(pq+p)$$

$$\therefore \frac{1}{2}p(1+q) = \frac{1}{2}$$

$$\Rightarrow p(1+q) = 1$$

30 **(b)**

A number has exactly 3 factors if the number is squares of a prime number. Squares of 11, 13, 17, 19, 23, 39, 31 are 3-digit numbers. Hence, the required probability is 7/900

31 **(c)**

Let us assume that A wins after n deuces, $n = 0, 1, 2, 3, \ldots$ The probability of deuce is $(2/3)\times(2/3) + (1/3)\times(1/3) = (5/9)$. [A wins his serve, then B wins his serve or A loses his serve]. So, the probability that 'A' wins game after n denuces is $(5/9)^n \times (2/3)\times(1/3)$. [After n^{th} deuce, A serves and wins, then B serves and loses]. Therefore, the required probability of 'A' winning the game is

$$\sum_{n=0}^{\infty}\left(\frac{5}{9}\right)^n \times\frac{2}{3}\times\frac{1}{3} = \frac{1}{1-\frac{5}{9}}\times\frac{2}{9} = \frac{1}{2}$$

32 **(b)**

The total number of cases is $11!/2!\times 2!$. the number of favorable cases is $[11!\,(2!\times 2!)] - 9!$. Therefore the required probability is

$$1 - \frac{9!\times 4}{11!} = \frac{53}{5}$$

33 **(a)**

$$P(A\cap C) = P(A)P(C)$$

$$\Rightarrow \frac{1}{20} = \frac{1}{5}P(C)$$

$$\Rightarrow P(C) = \frac{1}{4}$$

Now, $P(B\cup C) = \frac{1}{6}+\frac{1}{4} - P(B\cap C)$

$$\Rightarrow P(B\cup) = \frac{3}{8}-\frac{1}{3} = \frac{1}{24} = \frac{1}{24} = P(B)P(C)$$

Therefore, B and C are independent

34 **(a)**

$P(a) = 0.3, P(b) = 0.5, P(c) = 0.2$. hence, a, b, c are exhaustive

P(same horse wins all the three races)$= P(aaa$ or bbb or $ccc)$

$$= (0.3)^3 + (0.5)^3 + (0.2)^3$$

$$= \frac{27+125+8}{1000} = \frac{160}{1000}$$

$$= \frac{4}{25}$$

P(each horse wins exactly one race)

$= P(abc$ or acb or bca or bac or cab or $cba)$

$$= 0.3\times 0.5\times 0.2\times 6 = 0.18 = \frac{9}{50}$$

35 **(b)**

$$P(A) = \frac{1}{3}, P(A\cup B) = \frac{3}{4}$$

Now, $P(A\cup B) = P(A) + P(B) - P(A\cap B \le P(A) + P(B)$

$$\Rightarrow \frac{3}{4} \le \frac{1}{3} + P(B)$$

$$\Rightarrow \frac{5}{12} \le P(B)$$

Again we have $B \subseteq A\cup B$

$$\therefore P(B) \le P(A\cup B) = \frac{3}{4}$$

Hence, $5/12 \le P(B) \le 3/4$

36 **(c)**

The required probability is

$$\left[{}^4C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4\right]^2+\left[{}^4C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^3\right]^2$$
$$+\left[{}^4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2\right]^2$$
$$+\left[{}^4C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1\right]^2$$
$$+\left[{}^4C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0\right]^2=\frac{35}{128}$$

37 **(c)**

The total number of ways in which 3 integers can be chosen from first 20 integers is ${}^{20}C_3$. The product of three integers will be even if at least of the integers is even. Therefore, the required probability is

1 −probability that none of the three integers is even

$$=1-\frac{{}^{10}C_3}{{}^{20}C_3}=1-\frac{2}{19}=\frac{17}{19}$$

38 **(b)**

Given that $n(S)=6\times 6\times 6\times 6=6^4$. The number of favourable ways is ${}^6C_4=6\times 5/2=15$. Therefore, the required probability is

$$\frac{15}{6\times 216}-\frac{5}{2\times 216}=\frac{5}{432}$$

39 **(d)**

According to the given condition

$${}^nC_3\left(\frac{1}{2}\right)^n={}^nC_4\left(\frac{1}{2}\right)^n$$

Where n is the number of times die is thrown

$\therefore\ {}^nC_3={}^nC_1\ \Rightarrow n=7$

Thus, the required probability is

$${}^7C_1\left(\frac{1}{2}\right)^7=\frac{7}{2^7}=\frac{7}{128}$$

40 **(b)**

Die marked with 1, 2, 2, 3, 3, 3 is throw 3 times

$$P(1)=\frac{1}{6},P(2)=\frac{2}{6},P(3)=\frac{3}{6}$$

$P(S)=P(4\ or\ 6)$

$=P(112(3\text{ cases})\text{or }123\ (6\text{ cases})\text{or }222)$

$$=3\times\frac{1}{6}\times\frac{2}{6}\times\frac{2}{6}+6\frac{1}{6}\times\frac{2}{6}\times\frac{3}{6}+\frac{2}{6}\times\frac{2}{6}\times\frac{2}{6}$$

$$=\frac{6+36+8}{216}=\frac{50}{216}=\frac{25}{108}$$

41 **(b)**

Let X denote the number of heads in n trials. Then X is a binomial variant with $p=q=1/2$. Therefore,

$$P(X=r)={}^nC_r\left(\frac{1}{2}\right)^n$$

Now, $P(X=6)=P(X=8)$

$$\Rightarrow {}^nC_6\left(\frac{1}{2}\right)^n={}^nC_8\left(\frac{1}{2}\right)^n$$

$\Rightarrow {}^nC_6={}^nC_8\Rightarrow n=14$

42 **(b)**

Let A denote the event that there is an odd man out in a game. The total number of possible cases is2^m. A person is odd man out if he is alone in getting a head or a tail.

The number of ways in which there is exactly one tail (head) and the rest are heads (tails) is ${}^mC_1=m$. Thus, the number of favourable ways is $m+m=2m$. Therefore

$$P(A)=\frac{2m}{2^m}=\frac{m}{2^{m-1}}$$

43 **(d)**

The probability that one test is held is$2\times(1\times 5)\times(4\times 5)=8/25$. The probability that test is held on both days is $(1\times 5)\times(1\times 5)=1/25$. Therefore, the probability that the student misses at least one test is 8/25+1/25=9/25

44 **(a)**

$P(A\cap B')=P(A)-P(A\cap B)=0.20$

Also, $P(A'\cap B)=P(B)-P(A\cap B)=0.15$

$\Rightarrow P(A)+P(B)-2P(A-B)=0.35$

Now,

$P(A'\cap B')=1-P(A\cup B)$

$\Rightarrow 0.1=1-P(A)-P(B)+P(A\cap B)$

$\Rightarrow P(A)+P(B)-P(A\cap B)=0.9$

$\Rightarrow P(A\cap B)=0.9-0.35=0.55$

And

$P(A)=0.75, P(B)=0.70$

Now, $P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{0.55}{0.70}$

45 **(b)**

The total number of ways of distribution is 4^5

$\therefore n(S)=4^5$

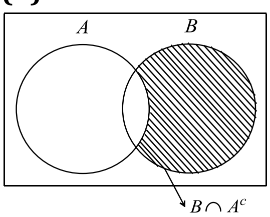
The total number of ways of distribution so that each child gets at least one game is $4^5-{}^4C_1 3^5+{}^4C_2 2^5-{}^4C_3$

$\therefore n(E)=240$

Hence, the required probability is

$$\frac{n(E)}{n(S)}=\frac{240}{4^5}=\frac{15}{64}$$

46 **(b)**



$P(A) = \frac{1}{4}, P(A \cup B) = \frac{1}{2}$

$P\left(\frac{B}{A^C}\right) = \frac{P(B \cap A^C)}{P(A^C)}$

$= \frac{P(A \cup B) - P(A)}{1 - P(A)}$ [

$\because P(A \cup B) + P(B) - P(A \cap B)]$

$= \frac{\frac{1}{2} - \frac{1}{4}}{1 - \frac{1}{4}} = \frac{1}{4} \times \frac{4}{3} = \frac{1}{3}$

47 **(d)**

Here, two numbers are selected from {1,2,3,4,5,6,}

$\Rightarrow n(S) = 6 \times 5$ {as one by one without replacement}

Favourable cases,

First number	Possible value for second number
1	2, 3, 4, 5, 6
2	3, 4, 5, 6
3	4, 5, 6

There are 12 ways but the numbers may be interchanged

$\therefore\ n(E) = 2 \times 12 = 24$

$\therefore$ Required probability$= \frac{n(E)}{n(S)} = \frac{24}{30} = \frac{4}{5}$

48 **(a)**

$p^2 + 2p + 4p - 1 = 1$ (Exhaustive)

$p^2 + 6p - 2 = 0$

$\Rightarrow p = -3 \pm \sqrt{11}$

$\Rightarrow p = \sqrt{11} - 3$

49 **(b)**

Let one of the quantities be x. Then the other is$2n - x$. Their product will be greatest when they are equal, i.e., each is n in which case the product is n^2. According to the propositions

$x(2n - x) \geq \frac{3}{4}n^2$

$\Rightarrow 4x^2 - 8nx + 3n^2 \leq 0$

$\Rightarrow (2x - 3n)(2x - n) \leq 0$

$\Rightarrow \frac{n}{2} \leq x \leq \frac{3}{2}n$

So, favourable number of cases is $3/2n - n - 2 = n$. Hence, the required probability is $n/2n = 1/2$

50 **(c)**

The probability of solving the question by these three students are 1/3, 2/7 and 3/8, respectively

$\therefore\ P(A) = \frac{1}{3}; P(B) = \frac{2}{7}; P(C) = \frac{3}{8}$

Then probability of question solved by only one student is

$P(A\overline{BC}$ or $\overline{A}B\overline{C}$ or $\overline{AB}C) = P(A)P(\overline{B})P(\overline{C}) + P(\overline{A}) + P(B)P(\overline{C}) + P(\overline{A})P(\overline{B})P(C)$

$= \frac{1}{3} \times \frac{5}{7} \times \frac{5}{8} + \frac{2}{3} \times \frac{2}{7} \times \frac{5}{8} + \frac{2}{3} \times \frac{5}{7} \times \frac{3}{8}$

$= \frac{25 + 20 + 30}{168} = \frac{25}{56}$

51 **(b)**

The total number of ways in which n persons can sit at a round table is $(n-1)!$. So, total number of cases is $(n-1)!$

Let A and B be two specified persons. Considering these two as one person, the total number of ways in which $n - 1$ persons, $n - 2$ other persons and one AB can sit at a round table is $(n-2)!$. So, favourable number of cases is $2!\,(n-2)!$ Thus, the required probability is

$p = \frac{2!\,(n-2)!}{(n-1)!} = \frac{2}{n-1}$

Hence, the required odds are $(1 - p): p$ or $(n - 3): 2$

52 **(b)**

Here $p = 19/20, q = 1/20, n = 5, r = 5$. The required probability is

${}^5C_5 \left(\frac{19}{20}\right)^5 \left(\frac{1}{20}\right)^6 = \left(\frac{19}{20}\right)^5$

53 **(a)**

Let the number of red and blue balls be r and b, respectively. Then, the probability of drawing two red balls is

$p_1 = \frac{{}^rC_2}{{}^{r+b}C_2} = \frac{r(r-1)}{(r+b)(r+b-1)}$

The probability of drawing two blue balls is

$p_2 = \frac{{}^bC_2}{{}^{r+b}C_2} = \frac{b(b-1)}{(r+b)(r+b-1)}$

The probability of drawing one red and one blue ball is

$p_3 = \frac{{}^rC_1\,{}^bC_1}{{}^{r+b}C_2} = \frac{2br}{(r+b)(r+b-1)}$

By hypothesis, $p_1 = 5p_2$ and $p_3 = 6p_2$

$\therefore r(r-1) = 5b(b-1)$ and $2br = 6b(b-1)$

$\Rightarrow r = 6, b = 3$

54 **(d)**

Consider two events as follows:

A_1: getting number i on first die

B_1: getting a number more than i on second die

The required probability is

$P(A_1 \cap B_1) + P(A_2 \cap B_2) + P(A_3 \cap B_3) + P(A_4 \cap B_4) + P(A_5 \cap B_5)$

$= \sum_{i=1}^{5} P(A_i \cap B_i) = \sum_{i=1}^{5} P(A_i)P(B_i)$

[$\because A_i, B_i$ are independent]

$= \frac{1}{6}\ [P(B_1) + P(B_2) + \ldots + P(B_5)]$

$= \frac{1}{6}\left(\frac{5}{6} + \frac{4}{6} + \frac{3}{6} + \frac{2}{6} + \frac{1}{6}\right) = \frac{5}{12}$

55 **(b)**

Let,

$P(S) = P(1 \text{ or } 2) = 1/3$

$P(F) = P(3 \text{ or } 4 \text{ or } 5 \text{ or } 6) = 2/3$

$P(A \text{ wins})$

$= P[(S\,S \text{ or } S\,F\,S\,S \text{ or } S\,F\,S\,S\,F\,S\,S \text{ or} \ldots)$

Or $(F\,S\,S \text{ or } F\,S\,F\,S\,S \text{ or} \ldots)]$

$= \frac{\frac{1}{9}}{1-\frac{2}{9}} + \frac{\frac{2}{27}}{1-\frac{2}{9}}$

$= \frac{1}{9} \times \frac{9}{7} + \frac{2}{27} \times \frac{9}{7}$

$= \frac{1}{7} + \frac{2}{21} = \frac{3+2}{21} = \frac{5}{21}$

$P(A \text{ winning}) = \frac{5}{21}, P(B \text{ winning}) = \frac{16}{21}$

56 **(a)**

Let $E_1 = 1, 4, 7, \ldots$ (n each)

$E_2 = 2, 5, 8, \ldots$ (n each)

$E_3 = 3, 6, 9, \ldots$ (n each)

x and y belong to $(E_1, E_2), (E_2, E_1)$ or (E_3, E_3). So, the required probability is

$\frac{n^2 + {}^nC_2}{{}^{3n}C_2} = \frac{1}{3}$

57 **(c)**

Let A be the event that 11 is picked and B be the event that sum is even. The number of ways of selecting 11 along with one more-odd number is

$n(A \cap B) = {}^7C_1$

The number of ways of selecting either two even numbers or selecting two odd numbers is

$n(B) = 1 + {}^8C_2$

$\therefore P(A/B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{7}{29} = 0.24$

58 **(c)**

18 draws are required for 2 aces means in the first 17 draws, there is one ace and 16 other cards and 18th draw produces an ace. So, the required probability is

$\frac{{}^{48}C_{16} \times {}^4C!}{{}^{52}C_{17}} \times \frac{3}{35} = \frac{561}{15925}$

59 **(c)**

Given,

$7a - 9b = 0 \Rightarrow b = \frac{7}{9}a$

Hence, number of pairs (a, b) can be (9, 7); (18, 14); (27, 21); (36, 28). Hence, the required probability is $4/{}^{39}C_2 = 4/741$

60 **(b)**

The total number of cases is $11!/2! \times 2!$ The number of favourable cases is $11!/(2! \times 2!) - 9!$ Therefore, required probability is

$1 - \frac{9! \times 4}{11!} = \frac{53}{55}$

61 **(a)**

The divisibility of the product of four numbers depends upon the value of the last digit of each number. The last digit of a number can be any one of the 10 digits 0,1, 2, …9. So, the total number of ways of selecting last digits of four numbers is $10 \times 10 \times 10 \times 10 = 10^4$. If the product of the 4 numbers is not divisible by 5 or 10, then the number of choices for the last digit of each number is 8 (excluding 0 or 5). So, favourable number of ways is 8^4. Therefore, the probability that the product is not divisible by 5 or 10 is $(8/10)^4$. Hence, the required probability is $1 - (8/10)^4 = 369/625$

62 **(c)**

Let E =event when each American man is seated adjacent to his wife

and A =event when Indian man is seated adjacent to his wife.

Now, $n(A \cap E) = (4!) \times (2!)^5$

Even when each American man is seated adjacent to his wife.

Again, $n(E) = (5!) \times (2!)^4$

$\therefore P\left(\frac{A}{E}\right) = \frac{n(A \cap E)}{n(E)}$

$= \frac{(4!) \times (2!)^3}{(5!) \times (2!)^4} = \frac{2}{5}$

63 **(c)**

Let the probability that a man aged x dies in a year p. Thus the probability that a man aged x does not die in a year $= 1 - p$. The probability that all n men aged $x$do not die in a year is $(1-p)^n$. Therefore, the probability that at least one man dies in a year is $1 - (1-p)^n$. The probability that out of n men, A_1 dies first is $1/n$. Since this event is independent of the event that at least one man dies in a year, hence, the probability that A_1 dies in the year and he is first one to die is $1/n[1-(1-p)^n]$

64 **(b)**

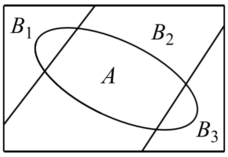
Let us consider the following events

A: card shows up black

B_1: card with both sides black

B_2: card with both sides white

B_1: card with one side white and one black



$$P(B_1)=\frac{2}{10}, P(B_2)=\frac{3}{10}, P(B_3)=\frac{5}{10}$$

$$P(A/B_1)=1,\ P(A/B_2)=0\ \ P(A/B_3)=\frac{1}{2}$$

$$P(B_1/A)=\frac{\frac{2}{10}\times 1}{\frac{2}{10}\times 1+\frac{3}{10}\times(0)+\frac{5}{10}\times\frac{1}{2}}=\frac{4}{4+5}=\frac{4}{9}$$

65 **(c)**

Possibilities of getting 9 are (5, 4), (4, 5), (6, 3), (3, 6)

$$\therefore p=\frac{4}{36}=\frac{1}{9} \text{ and } q=1-\frac{1}{9}=\frac{8}{9}$$

Therefore, the required probability is

$$^3C_2 q^1 p^2=(3)\left(\frac{8}{9}\right)\left(\frac{1}{9}\right)^2=\frac{8}{243}$$

66 **(d)**

The number of ways of arranging n numbers is $n!$ In each order obtained, we must now arrange the digits 1, 2, ... k as group and the $n-k$ remaining digits. This can be done in $(n-k+1)!$ ways. Therefore, the probability for the required event is $(n-k+1)!/n!$

67 **(a)**

For each toss, there are four choices:

1. A gets head, B gets head
2. A gets tail, B gets head
3. A gets head, B gets tail
4. A gets tail, B gets tail

Thus, exhaustive number of ways is 4^{50}. Out of the four choices listed above, (iv) is not favourable to the required event in a toss. Therefore, favourable number of cases is 3^{50}. Hence, the required probability is $(3/4)^{50}$

68 **(c)**

Let a_n be the number5 of strings of H and T of length n with no two adjacent H's. Then $a_1=2, a_2=3$. Also,

$a_{n+2}=a_{n+1}+a_n$ (since the string must with T or HT)

So, $a_3=5, a_4=8, a_5=8+5=13$

Therefore, the required probability is

$13/2^5=13/52$

69 **(a)**

We have ratio of the ships A, B and C for arriving safely are 2:5,3:7 and 6:11, respectively.

Therefore, the probability of ship A for arriving safely is 2/(2+5)=2/7

Similarly, for B the probability is 3/(3+7)=3/10 and for C the probability is $C=6/(6+11)=6/17$

Therefore, the probability of all the ships for arriving safely is $(2/7)\times(3/10)\times(6/17)$ $18/595$

70 **(a)**

Out of 9 socks, 2 can be drawn in $^9C_2$ ways. Therefore, the total number of cases is $^9C_2$. Two socks drawn from the drawer will match if either both are brown or both are blue. Therefore, favourable number of cases is $^5C_2+\,^4C_2$. Hence, the required probability is

$$\frac{^5C_2+\,^4C_2}{^9C_2}=\frac{4}{9}$$

71 **(b)**

Total number of ways of distribution is 4^5

$\therefore n(S)=4^5$

Total number of ways of distribution so that each child gets at least one game is

$4^5-\,^4C_1\,3^5+\,^4C_2\,2^5-\,^4C_3=1024-4\times 243+6\times 32-4=240$

$\therefore\ \ n(E)=240$

Therefore, the required probability is

$$\frac{n(E)}{n(S)}=\frac{240}{4^5}=\frac{15}{64}$$

72 **(c)**

We know that the number of subsets of a set containing n elements is 2^n. Therefore, the number of ways of choosing P and Q is

$^{2^n}C_1\times\,^{2^n}C_1=2^n\times 2^n=4^n$

Out of n elements, m elements are chosen and then from the remaining $n-m$ elements either an element belongs to P or$Q$. But not both P and$Q$. Suppose P contains r elements from the remaining $n-m$ elements. Then, Q may contain any number of elements from the remaining $(n-m)-r$ elemts. Therefore, P and Q can be chosen in $^{n-m}C_r 2^{(n-m)-r}$ ways

But r can vary from 0 to $n-m$. So, in general the number of ways in which P and Q can be chosen is

$$\left(\sum_{r=0}^{n-m} {}^{n-m}C_r 2^{(n-m)-r}\right) {}^nC_m = (1+2)^{n-m}\, {}^nC_m$$

$$= {}^nC_m 3^{n-m}$$

Hence, the required probability is ${}^nC_m\ 3^{n-m}/4^n$

73 **(a)**

The total number of ways of making the second draw is ${}^{10}C_5$

The number of draw of 5 balls containing 2 balls common with first draw of 6 balls is ${}^6C_2\ {}^4C_3$.

Therefore, the probability is

$$\frac{{}^6C_2\ {}^4C_3}{{}^{10}C_5} = \frac{5}{21}$$

74 **(c)**

The total number of digits in any number at the unit's place is 10

$\therefore\ n(S) = 10$

If the last digit in product is 1,3, 5 or 7, then it is necessary that the last digit in each number must be 1,3,5 or 7

$\therefore n(A) = 4$

$\therefore P(A) = \frac{4}{10} = \frac{2}{5}$

Hence, the required probability is $(2/5)^4 = 16/625$

75 **(d)**

$P(A) = \frac{2}{5}$

For independent events,

$P(A \cap B) = P(A)P(B)$

$\Rightarrow P(A \cap B) \le \frac{2}{5}$

$\Rightarrow P(A \cap B) = \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}$

[Maximum 4 outcomes may be in $P(A \cap B)$]

1. When$P(A \cap B) = \frac{1}{10}$

$\Rightarrow P(A).P(B) = \frac{1}{10}$

$\Rightarrow P(B) = \frac{1}{10} \times \frac{5}{2} = \frac{1}{4}$,not possible

2. When $P(A \cap B) = \frac{2}{10} \Rightarrow \frac{2}{5} \times P(B) = \frac{2}{10}$

$\Rightarrow P(B) = \frac{5}{10}$,outcomes of $B = 5$

3. When $P(A \cap B) = \frac{3}{10}$

$\Rightarrow P(A)P(B) = \frac{3}{10}$

$\Rightarrow \frac{2}{5} \times P(B) = \frac{3}{10}$

$P(B) = \frac{3}{4}$,not possible

4. When $P(A \cap B) = \frac{4}{10}$

$\Rightarrow P(A).P(A) = \frac{4}{10}$

$\Rightarrow P(B) = 1$, outcomes of $B = 10$

76 **(d)**

A person can have his/her birthday on any one of the seven days of the week. So 5 persons can have their birthdays in 7^5 ways. Out of 5, three persons can have their birthday on days other than Sunday in 6^3 ways and other 2 on Sundays. Hence, the required probability is

$$\frac{{}^5C_2 \times 6^3}{7^5} = \frac{10 \times 6^3}{7^5}$$

(Note that 2 persons can be selected out of 5 in ${}^5C_2$ ways)

77 **(a)**

$P(B_1) = \frac{{}^6C_1}{{}^{10}C_1} = \frac{6}{10} = \frac{3}{5}$

$P(B_2/B_1) = \frac{5}{9} (B_2 = \text{black})$

$\therefore P(B_1 \cap B_2) = P(B_1)P(B_2/B_1) = \frac{3}{5} \times \frac{5}{9} = \frac{1}{3}$

78 **(d)**

Three-digit numbers are 100,101, ...999. Total number of such numbers is 900. The three-digit numbers (which have all same digits) are 111,222, 333, ..., 999. Favourable number of cases is 9. Therefore, the required probability is 9/900=1/100

79 **(b)**

Let E be the event of getting 1 on a die

$\Rightarrow P(E) = \frac{1}{6}$ and $P(\bar{E}) = \frac{5}{6}$

$\therefore P(\text{first time 1 occurs at the even throw})$

$= t_2$ or t_4 or t_6 or t_8 ... and so on.

$= \{P(\bar{E}_1).P(E_2)\}$

$+ \{P(\bar{E}_1)P(\bar{E}_2)P(\overline{E_3})P(E_4)\} + \ldots\infty$

$= \left(\frac{5}{6}.\frac{1}{6}\right) + \left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right) + \ldots\infty$

$$= \frac{\frac{5}{36}}{1 - \frac{25}{36}} = \frac{5}{11}$$

80 **(b)**

$P(S \cap F) = 0.006$, where S is the event that the motor cycle is stolen and F is the event that the motor cycle is found. Therefore,

$P(S) = 0.0015$

$$P(F/S) = \frac{P(F \cap S)}{P(S)} = \frac{6 \times 10^{-4}}{15 \times 10^{-4}} = \frac{2}{5}$$

81 **(d)**

The total number of ways of choosing 11 players out of 15 is ${}^{15}C_{11}$. A team of 11 players containing at least 3 bowlers can be chosen in the following mutually exclusive ways:

5. Three bowlers out of 5 bo0wlers and 8 other players out of the remaining 10 players

6. Four bowlers out of 5 bowlers and 7 other players out of the remaining 10 players

7. Five bowlers out of 5 bowlers and 6 other players out of the remaining 10 players

So, required probability is

$$P(I) + P(II) + P(III) = \frac{{}^5C_3 \times {}^{10}C_8}{{}^{15}C_{11}} + \frac{{}^5C_4 \times {}^{10}C_7}{{}^{15}C_{11}} + \frac{{}^5C_5 \times {}^{10}C_6}{{}^{15}C_{11}}$$

$$= \frac{1260}{1365} = \frac{12}{13}$$

82 **(a)**

Let the number selected by xy. Then

$x + y = 9, 0 < x, y \leq 9$

And $xy = 0 \Rightarrow x = 0, y = 9$

Or $y = 0, x = 9$

$$P(x_1 = 9/x_2 = 0) = \frac{P(x_1 = 9 \cap x_2 = 0)}{P(x_2 = 0)}$$

Now, $P(x_2 = 0) = \frac{19}{100}$

And $P(x_1 = 9 \cap x_2 = 0) = \frac{2}{100}$

$$\Rightarrow P(x_1 = 9/x_2 = 0) = \frac{2/100}{19/100} = \frac{2}{19}$$

83 **(d)**

Since a, b, c are in A.P>, therefore, $2b = a + c$.

The possible cases are tabulated as fallows

b	a	c	Number of ways
1	1	1	1
2	2	2	1
2	1	3	6
3	3	3	1
3	1	5	6
3	2	4	6

Total number of ways is 21. So, required probability is 21/216=7/72

84 **(b)**

We define the following events:

A_1: Selecting a pair of consecutive letters from the word LONDON

A_2: Selecting a pair of consecutive letters from the word CLIFTON

E: Selecting a pair of letters 'ON'

Then, $P(A_1E) = 2/5$ as there are 5 pairs of consecutive letters out of which 2 are ON and $P(A_2E) = 1/6$ as there are 6 pairs of consecutive letters of which 1 is ON. Therefore, the required probability is

$$P\left(\frac{A_1}{E}\right) = \frac{P(A_1 \cap E)}{P(A_1 \cap E) + P(A_2 \cap E)} = \frac{\frac{2}{5}}{\frac{2}{5} + \frac{1}{6}} = \frac{12}{17}$$

85 **(c)**

The sum is 12 in first three throws if they are (1,5, 6) in any order or (2, 4, 6) in any order or (3,4, 5) in any order. Therefore, the required probability is

$$\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times 3! + \frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times 3! + \frac{1}{6} \times \frac{1}{5} \times \frac{1}{4} \times 3! = \frac{3}{20}$$

(because after throwing 1, in the next throw 1 cannot come, etc.)

86 **(d)**

The total number of ways in which papers of 4 students can be checked by seven teachers is 7^4. The number of ways of choosing two teachers out of 7 is ${}^7C_2$. The number of ways in which they can check four papers is 2^4. But this includes two ways in which all the papers will be checked by a single teacher. Therefore, the number of ways in which 4 papers can be checked by exactly two teachers is $2^4 - 2 = 14$. Therefore, the number of favourable ways is $({}^7C_2)(14) = (21)(14)$. Thus, the required probability is $(21)(14)/7^4 = 6/49$

87 **(c)**

A: car met with an accident

B_1: driver was alcoholic, $P(B_1) = 1/5$

B_2: driver was sober,$P(B_2) = 4/5$

$P(A/B_1) = 0.001$; $P(A/B_2) = 0.0001$

$$P\left(\frac{B_1}{A}\right)=\frac{(0.2)(0.001)}{(0.2)(0.001)+(0.8)(0.0001)}=5/7$$

88 **(c)**

Out of 5 horses, only one is the wining horse. The probability that Mr. A selected that losing horse is $4/5\times 3/4$. Therefore, the required probability is

$$1-\frac{4}{5}\times\frac{3}{4}=1-\frac{3}{5}=\frac{2}{5}$$

89 **(c)**

Suppose, there exist three rational points or more on the circle $x^2+y^2+2gx+2fy+c=0$. Therefore, if $(x_1,y_1),(x_2,x_2)$ and (x_3,y_3) be those three points, then

$x_1^2+y_1^2+2gx_1+2fy_1+c=0$ (1)

$x_2^2+y_2^2+2gx_2+2fy_1+c=0$ (2)

$x_3^2+y_3^2+2gx_3+2fy_3+c=0$ (3)

Solving Eqs. (1), (2) and (3), we will get g,f,c as rational. Thus, centre of the circle $(-g.-f)$ is a rational point. Therefore, both the coordinates of the centre are rational numbers. Obviously, the possible values of p are 1, 2. Similarly, the possible values of q are 1, 2. Thus fro this case (p,q) may be chosen in 2×2, i.e., 4 ways. Now, (p,q) can be, without restriction, chosen in 6×6, i.e., 36 ways

Hence, the probability that at the most two rational points exist on the circle is $(36-4)/36=32/36=8/9$

90 **(a)**

The required probability is

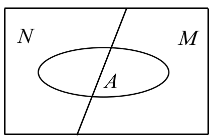
$$P(A)=\frac{1}{3}\frac{6}{a^2-4a+10}$$

$$(P(A))_{\max}=\frac{1}{3}\times\frac{6}{6}=\frac{1}{3}$$

91 **(a)**

The total number of ways of selecting 3 integers from 20 natural numbers is ${}^{20}C_3=1140$. Their product is a multiple of 3 means at least one number is divisible by 3. The number which are divisible by 3 are 3, 6, 9,12, 15,18 and the number of ways of selecting at least one of them is ${}^{6}C_1\times{}^{14}C_2+{}^{6}C_2\times{}^{14}C_1+{}^{6}C_3=776$. Hence, the required probability is 776/1140=194/285

92 **(a)**



Let N be the event of picking up a normal die: $P(N)=1/4$. Let M be the event of picking up a magnetic die : $P(M)=3/4$. Let A be the event that die shows up 3

$\therefore P(A)=P(A\cap N)+P(A\cap M)$

$=P(N)P(A/N)+P(M)P(A/M)$

$=\frac{1}{4}\times\frac{1}{6}+\frac{3}{4}\times\frac{7}{24}$

$$P(N/A)=\frac{P(N\cap A)}{P(A)}=\frac{(1/4)(1/6)}{7/24}=\frac{1}{7}$$

93 **(a)**

$$P(A)=\frac{1}{2},P(B)=\frac{1}{3},P(C)=\frac{1}{4}$$

$$\therefore\ P(\overline{A})=1-\frac{1}{2}-\frac{1}{2},P(\overline{B})=1-\frac{1}{3}=\frac{2}{3}$$

$$P(\overline{C})=1-\frac{1}{4}=\frac{3}{4}$$

Therefore, the required probability is

$$1-P(\overline{A})P(\overline{B})P(\overline{C})=1-\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}$$

$$=1-\frac{1}{4}=\frac{3}{4}$$

94 **(c)**

In the first 9 throws, we should have three sixes and six non-sixes; and a six in the 10th throw, and thereafter it does not matter whatever face appears. So, the required probability is

$${}^{9}C_3\left(\frac{1}{6}\right)^3\times\left(\frac{5}{6}\right)^6=\frac{1}{6}\times 1\times 1\times 1\times\ldots\times 1$$

10 times

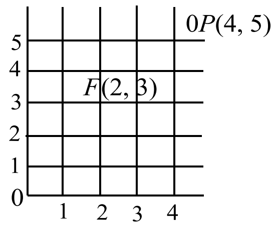
$$=\frac{84\times 5^6}{6^{10}}$$

95 **(b)**

$$n(S)=\frac{9!}{4!\,5!}=126$$

$n(A)=0$ to F and F to P

$$=\frac{5!}{2!\times 3!}\times\frac{4}{2!\times 2!}=60$$



$$\Rightarrow P(A)=\frac{60}{126}=\frac{10}{21}$$

96 **(d)**

Let A and B, respectively, be the events that urn A and urn B are selected. Let R be the event that the selected ball is red. Since the urn is chosen at random. Therefore,

$$P(A)=P(B)=\frac{1}{2}$$

And $P(R)=P(A)P(R/A)+P(B)P(R/B)$

$$=\frac{1}{2}\times\frac{5}{10}+\frac{1}{2}\times\frac{4}{10}$$

$= \frac{9}{20}$

97 **(d)**

Player should get $(HT, HT, HT, \dots)$ or $(TH, TH, \dots)$ at least $2n$ times. If the sequence starts from first place, then the probability is $1/2^{2n}$ and if starts from any other place, then the probability is $1/22^{2n+1}$. Hence, required probability is

$2\left(\frac{1}{2^{2n}} + \frac{m}{2^{2n+1}}\right) = \frac{m+2}{2^{2n}}$

98 **(c)**

$P(A \cup B) = P(A) + P(B) - P(A) \times P(B)$

$\Rightarrow 0.7 = 0.4 + p - 0.4p$

$\therefore 0.6p = 0.3 \ \Rightarrow p = \frac{1}{2}$

99 **(c)**

Given limit,

$$\lim_{x\to 0}\left(\frac{a^x + b^x}{2}\right)^{\frac{2}{x}}$$

$$= \lim_{x\to 0}\left(1 - \frac{a^x + b^x - 2}{2}\right)^{\frac{2}{a^x+b^x-2}\lim_{x\to 0}\left(\frac{a^x-1+b^x-1}{x}\right)}$$

$= e^{\log ab} = ab = 6$

Total number of possible ways in which a, b can take values is $6 \times 6 = 36$. Total possible ways are (1, 6), (6, 1), (2, 3), (3, 2). The total number of possible ways is 4. Hence, the4 required probability is 4/36=1/9

100 **(c)**

Given that 5 and 6 have appeared on two of the dice, the sample space reduces to $6^4 - 2 \times 5^4 + 4^4$ (inclusion-exclusion principle). Also, the number of favourable cases are $4! = 24$. So, the required probability is 24/302=12/151

101 **(b)**

Let event A be drawing 9 cards which are not ace and B be drawing an ace card. Therefore, the required probability is

$P(A \cap B) = P(A) \times P(B)$

Now, there are four aces and 48 other cards. Hence,

$P(A) = \frac{^{48}C_9}{^{52}C_9}$

After having drawn 9 non-ace cards, the 10th card must be ace. Hence,

$P(B) = \frac{^4C_1}{^{42}C_1} = \frac{4}{42}$

Hence,

$P(A \cap B) = \frac{^{48}C_9}{^{52}C_9}\frac{4}{42}$

102 **(b)**

Team totals must be 0, 1, 2, ..., 39. Let the teams be $T_1, T_2, \dots, T_{40}$, so that T_1 loses to T_1 for $i < j$. In other words, this order uniquely determines the result of every game. There are 40! Such orders and 780 games, so 2^{780} possible outcomes for the games. Hence, the probability is $40!/2^{780}$

103 **(a)**

The total number of ways in which $2n$ boys can be divided into two equal groups is

$\frac{(2n)!}{(n!)^2 2!}$

Now, the number of ways in which $2n - 2$ boys other than the two tallest boys can be divided into equal group is

$\frac{(2n-2)!}{(n-1!)^2 2!}$

Two tallest boys can be put in different groups in $^2C_1$ ways. Hence, the required probability is

$$\frac{2\,\frac{(2n-2)!}{((n-1)!)^2 2!}}{\frac{(2n)!}{(n!)^2 2!}} = \frac{n}{2n-1}$$

104 **(a)**

1. This question can also be solved by one student
2. This question can be solved by two students simultaneously
3. This question can be solved by three students all together

$P(A) = \frac{1}{2}, P(B) = \frac{1}{4}, P(C) = \frac{1}{6}$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - [P(A)P(B) + P(B)P(C) + P(C)P(A)] + [P(A)P(B)P(C)]$$

$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{6} - \left[\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{2}\right] + \left[\frac{1}{2} \times \frac{1}{4} \times \frac{1}{6}\right]$$

$= \frac{33}{48}$

Alternative solution:

We have,

$P(\overline{A} = \frac{1}{2}, P(\overline{B}) = \frac{3}{4}, P(\overline{C}) = \frac{5}{6}$

Then the probability that the problem is not solved is

$$P(\overline{A})P(\overline{B})P(\overline{C}) = \left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{5}{6}\right) = \frac{5}{16}$$

Hence probability that problem is solved is
$1 - 5/16 = 11/16$

105 **(d)**
Let p_1 denote the probability that out of 10 tosses, head occurs i times and no two heads occurs consecutively. It is clear that $i > 5$
For $i = 0$, i.e., no head, $p_0 = 1/2^{10}$
For $i = 1$, i.e., one head $p_1 = {}^{10}C_1(1/2)^1(1/2)^9 = 10/2^{10}$
Now for $i = 2$, we have 2 heads and 8 tails. Then, we have 9 possible places for heads. For example, see the constructive:
$xTxTxTxTxTxTxTxTx$
Here x represents possible places for heads
$\therefore p_2 = {}^9C_2\left(\frac{1}{2}\right)^2 (1/2)^8 = 36/2^{10}$
Similarly,
$p_3 = {}^8C_3/2^{10} = 56/2^{10}$
$p_4 = {}^7C_2/2^{10} = 35/2^{10}$
$p_5 = {}^6C_5/2^{10} = 6/2^{10}$
$\therefore p = p_0 + p_1 + p_3 + p_4 + p_5$
$= \dfrac{1 + 10 + 36 + 56 + 35 + 6}{2^{10}} = \dfrac{144}{2^{10}} = \dfrac{9}{64}$

106 **(c)**
The probability that A gets r heads in three tosses of a coin is

$$P(X = r) = {}^3C_r\left(\frac{1}{2}\right)^r\left(\frac{1}{2}\right)^{3-r} = {}^3C_r\left(\frac{1}{2}\right)^3$$

The probability that A and B both get r heads in three tosses of a coin is

$${}^3C_r\left(\frac{1}{2}\right)^3 {}^3C_r\left(\frac{1}{2}\right)^3 = ({}^3C_r)^2\left(\frac{1}{2}\right)^6$$

Hence, the required probability is

$$\sum_{r=0}^{3}({}^3C_r)^2\left(\frac{1}{2}\right)^6 = \left(\frac{1}{2}\right)^6 = \{1 + 9 + 9 + 1\} = \frac{20}{64}$$
$$= \frac{5}{16}$$

107 **(c)**
If A draws card higher than B, then number of favourable cases is $(n-1) + (n-2) + \ldots + 3 + 2 + 1$ (as when B draws card from 2 to n and so on). Therefore, the required probability is

$$\frac{\frac{n(n-1)}{2}}{n^2} = \frac{n-1}{2n}$$

108 **(d)**
A: exactly one ace
B: both aces
E: $A \cup B$

$$P(B/A \cup B)) = \frac{{}^4C_2}{{}^4C_1\ {}^{12}C_1 + {}^4C_2} = \frac{6}{54} = \frac{1}{9}$$

109 **(b)**
The probability of winning of A the second race is 1/2 (since both events are independent)

110 **(a)**
The number of composite numbers in 1 to 30 is $n(S) = 19$
The number of composite number when divided by 5 leaves a remainder is (E) =14. Therefore, the required probability is 14/19

111 **(d)**
Since there are r cars in N places, total number of selection of places out of $N-1$ places for $r-1$ cars (excepting the owner's car) is,

$${}^{N-1}C_{r-1} = \frac{(N-1)!}{(r-1)!\,(N-r)!}$$

If neighboring places are empty, then $r-1$ cars must be parked in $N-3$ places. So, the favourable number of cases is

$${}^{N-3}C_{r-1} = \frac{(n-3)!}{(r-1)!\,(N-r-2)!}$$

Therefore, the required probability is

$$\frac{(N-3)!}{(r-1)!\,(N-r-2)!} \times \frac{(r-1)!\,(N-r)!}{(N-1)!}$$
$$= \frac{(N-1)(N-r-1)}{(N-1)(N-2)} = \frac{{}^{N-r}C_2}{{}^{N-1}C_2}$$

112 **(a)**
We have,
$n(S) = {}^{64}C_3$
Let 'E' be the event selecting 3 squares which form the letter 'L'.
The number of ways of selecting squares consisting of 4 unit squares is $7 \times 7 = 49$
Each square with 4 unit squares form 4 L-shapes consisting of 3 unit squares
$\therefore n(E) = 7 \times 7 \times 4 = 196$
$\therefore\ P(E) = \dfrac{196}{{}^{64}C_3}$

113 **(a)**
Required probability $= \dfrac{\text{No.of favourite cases}}{\text{Total no.of exhaustive cases}}$
$= \dfrac{3}{3 \times 3 \times 3} - \dfrac{1}{9}$

114 **(b)**
The sum of the digits can be 7 in the following ways: 07, 16, 25, 34, 43, 52, 61, 70
$\therefore (A = 7) = \{07, 16, 25, 34, 43, 52, 61, 70\}$

Similarly,

$(B = 0) = \{00, 01, 02, \dots 10, 20, 30, \dots, 90\}$

Thus,

$(A = 7) \cap (B = 0) = \{09, 70\}$

$\therefore\ P((A = 7) \cap (B = 0)) = \frac{2}{100}, P((B = 0))$

$= \frac{19}{100}$

Hence,

$P(A = 7|B = 0) = \frac{P((A = 7) \cap (B = 0))}{P(B = 0)}$

$= \frac{\frac{2}{100}}{\frac{19}{100}} = \frac{2}{19}$

115 **(d)**

A: Doctor finds a rash

B_1: Child has measles

S: Sick children

$P(S/F) = 0.9$

B_2: Child has flu $\Rightarrow P(B_2) = 9/10$

$P(S/M) = 0.10$

$P(A/B_1) = 0.95$

$P(R/M) = 0.95$

$P(A/B_2) = 0.08$

$P(R/F) = 0.08$

$P(B_1/A) = \frac{0.1 \times 0.95}{0.1 \times 0.95 + 0.9 \times 0.08}$

$= \frac{0.095}{0.095 + 0.072}$

$= \frac{0.095}{0.167} = \frac{95}{167}$

116 **(c)**

Let A denote the event that target is hit when x shells are filled at point I. Let P_1 and P_2 denote the event that the target is at point I and II, respectively. We have $P(P_1) = 8/9, P(P_2) = 1/9, P(A/P_1) = 1 - (1/2)^x, P(A/P_2) = 1 - (1/2)^{55-x}$

Now from total probability theorem

$P(A) = P(P_1)P(A/P) + P(P_2)P(A/P_2)$

$= \frac{1}{9}\left(8 - 8\left(\frac{1}{2}\right)^x + 1 - \left(\frac{1}{2}\right)^{55-x}\right)$

$= \frac{1}{9}\left(9 - 8\left(\frac{1}{2}\right)^x - \left(\frac{1}{2}\right)^{55-x}\right)$

Now,

$\frac{dP(A)}{dx} = \frac{1}{9}\left(-8\left(\frac{1}{2}\right)^x \text{In}\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^{55-x} \text{In}\left(\frac{1}{2}\right)\right)$

(Note that in this step, it is being assumed that $x \in R^+$)

$= \frac{1}{9} In\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{55-x}\left(1 - \left(\frac{1}{2}\right)^{2x-58}\right)$

>0 if $x < 29$

<0 if $x > 29$

Therefore, $P(A)$ is maximum at $x = 29$. Thus, '29' shells must be fired at point I

117 **(d)**

In the first case, the urn constains 3 red and n white balls. The probability that colour of both the balls matches is

$\frac{{}^3C_2\,{}^nC_2}{{}^{n+3}C_2} = \frac{1}{2}$

$\Rightarrow \frac{6 + n(n-1)}{(n+3)(n+2)} = \frac{1}{2}$

$\Rightarrow 2(n^2 - n + 6) = n^2 + 5n + 6$

$\Rightarrow vn^2 - 7n + 6 = 0$

$\Rightarrow n = 1$ or 6 (1)

In the second case,

$\frac{3}{n+3}\frac{3}{n+3} + \frac{n}{n+3}\frac{n}{n+3} = \frac{5}{8}$

Solving, we get

$n^2 - 10n + 9 = 0$

$\Rightarrow n = 9$ or 1 (2)

From Eqs.(1) and (2), we have $n = 1$

118 **(b)**

$P(E) + P(E') = 1 = 1 + \lambda + \lambda^2 + (1 + \lambda)^2$

$\Rightarrow 2\lambda^2 + 3\lambda + 1 = 0$

$\Rightarrow (2\lambda + 1) + (\lambda + 1) = 0$

$\Rightarrow \lambda = -1, -\frac{1}{2}$

Then, $P(E) = 1 + (-1) + (-1)^2 = 1$ (not possible)

$\Rightarrow P(E) = 1 + \left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)^2 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

119 **(a)**

Consider the following events:

A: ball drawn is black

E_1:bag I is chosen

E_2: bag II is chosen

E_3: bag III is chosen

Then,

$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

$P(A/E_1) = \frac{3}{5}, P(A/E_2) = \frac{1}{5}, \quad P(A/E_3) = \frac{7}{10}$

Therefore, the required probability is

$P(E_3/A)$

$= \frac{P(E_3)P(A/E_3)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P(A/E_3)} = \frac{7}{15}$

120 **(a)**

For ranked 1 and 2 players to be winners and runners up, respectively, they should not be paired with each other in any round. Therefore, the required probability is $30/31 \times 14/15 \times 6/7 \times 2/3 = 16/31$

121 **(c)**

Total number of the students is 80. Total number of girls is 25. Total number of boys is 55. There are 10 rich, 70 poor, 20 intelligent students in the class. Therefore, required probability is

$$\frac{1}{4} \times \frac{1}{8} \times \frac{25}{80} = \frac{5}{512}$$

(I) (R) (G)

122 **(d)**

The probability that the first critic favours the book is

$$P(E_1) = \frac{5}{5+2} - \frac{5}{7}$$

The probability that the second critic favours the book is

$$P(E_2) = \frac{4}{4+3} = \frac{4}{7}$$

The probability that the third critic favours the book is

$$P(E_3) = \frac{3}{3+4} = \frac{3}{7}$$

Majority will be in favour of the book if at least two crities favour the book. Hence, the probability is

$$P(E_1 \cap E_2 \cap \overline{E_3}) + P(E_1 \cap \overline{E_2} \cap E_3) + P(\overline{E_1} \cap E_2 \cap E_3) + P(E_1 \cap E_2 \cap E_3)$$

$$= P(E_1)P(E_2)P(\overline{E_3}) + P(E_1)P(\overline{E_2})P(E_3) + P(\overline{E_1} \cap E_2 \cap E_3) + P(E_1)P(E_2)P(E_3)$$

$$= \frac{5}{7} \times \frac{4}{7} \times \left(1 - \frac{3}{7}\right) + \frac{5}{7} \times \left(1 - \frac{4}{7}\right) \times \frac{3}{7} + \left(1 - \frac{5}{7}\right) \times \frac{4}{7} \times \frac{3}{7} + \frac{5}{7} \times \frac{4}{7} \times \frac{3}{7}$$

$$= \frac{209}{343}$$

123 **(b)**

The required probability is

$$\frac{n^2}{^{2n}C_2} \frac{(n-1)^2}{^{2n-2}C_2} \frac{(n-2)^2}{^{2n-4}C_2} \cdots \frac{2^2}{^4C_2} \frac{1^2}{^2C_2}$$

$$= \frac{(1 \times 2 \times 3 \times 4 \times \ldots \times (n-1)n^2)}{\frac{(2n)!}{2^n}} = \frac{2^n (n)^2}{(2n)!}$$

$$= \frac{2^n}{^{2n}C_n}$$

124 **(c)**

$f'(x) = 3x^2 + 2ax + 9$

$y = f(x)$ is increasing

$\Rightarrow f'(x) \geq 0, \forall\, x$ and for $f'(x) = 0$ should not form an interval

$\Rightarrow (2a)^2 - 4 \times 3 \times b \leq 0 \;\Rightarrow a^2 - 3b \leq 0$

This is true for exactly 16 ordered pairs $(a, b) \leq a, b \leq 6$, namely (1,1), (1, 2), (1, 3)(1, 4), (1, 5), (1, 6), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 6 and (4, 6). Thus, the required probability is 16/36=4/9

125 **(b)**

There are 10 digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The last two digits can be dialed in $^{10}P_2 = 90$ ways out of which only one way is favourable, thus, the required probability is 1/90

126 **(a)**

Let E_i denote the event that the bag contains i black and $(10 - i)$ white balls $(i = 0, 1, 2, \ldots, 10)$. Let A denote the event that the three balls drawn at random from the bag are black. We have,

$$P(E_i) = \frac{1}{11} (i = 0,1,2, \ldots, 10)$$

$P(A/E_i) = 0$ for $i = 0, 1, 2$ and $P(A/E_i) = iC_3/\, ^{10}C_3$ for $i \geq 3$

$$\Rightarrow P(A) = \frac{1}{11} \times \frac{1}{^{10}C_3} [\,^3C_3 + \,^4C_3 + \ldots + \,^{10}C_3]$$

But $^3C_3 + \,^4C_3 + \,^5C_3 + \ldots + \,^{10}C_3 = \,^4C_4 + \,^4C_3 + \,^5C_3 + \ldots + \,^{10}C_3$

$= \,^5C_4 + \,^5C_3 + \,^6C_3 + \ldots + \,^{10}C_3$

$\vdots$

$= \,^{11}C_4$

$$\Rightarrow p(A) = \frac{1}{11} \times \frac{1}{^{10}C_3} \times \,^{11}C_4$$

$$= \frac{\frac{11 \times 10 \times 9 \times 8}{4!}}{11 \times \frac{10 \times 9 \times 8}{3!}} = \frac{1}{4}$$

$$\therefore P\left(\frac{E_9}{A}\right) = \frac{P(E_9)P(A/E_9)}{P(A)}$$

$$= \frac{\frac{1}{11} \times \frac{^9C_3}{^{10}C_3}}{\frac{1}{4}}$$

$$= \frac{14}{55}$$

127 **(b)**

Let $P(m), P(p), P(c)$ be the probability of selecting a book of maths, physics and chemistry, respectively, clearly,

$$P(m) = P(p) = P(c) = \frac{1}{3}$$

Again let $P(s_1)$ and $P(s_2)$ be the probability that he solves the first as well as second problem, respectively. Then,

$$P(s_1) = P(m) \times p\left(\frac{S_1}{m}\right) + p(p) \times p\left(\frac{S_1}{p}\right) + P(c) \times P\left(\frac{S_1}{c}\right)$$

$$\Rightarrow P(s_1) = \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{3}{5} + \frac{1}{3} \times \frac{4}{5} = \frac{19}{30}$$

Similarly,

$$P(S_2) = \frac{1}{3}\left(\frac{1}{2}\right)^2 + \frac{1}{3} \times \left(\frac{3}{5}\right)^2 + \frac{1}{3} \times \left(\frac{4}{5}\right)^2 = \frac{125}{300}$$

$$\Rightarrow P\left(\frac{S_2}{S_1}\right) = \frac{\frac{125}{300}}{\frac{19}{30}} = \frac{25}{38}$$

128 **(b)**

Consider the following events:

A: getting a card with mark I in first draw

B:getting card with mark I in second draw

C: getting a card with mark T in this draw

Then, the required probability is

$$P(A \cap B \cap C) = P(A)P(B/A)P(C/A \cap B)$$

$$= \frac{10}{20} \times \frac{9}{19} \times \frac{10}{18} = \frac{5}{38}$$

129 **(d)**

We have,

$$x + \frac{100}{x} > 50$$

$$\Rightarrow x^2 + 100 > 50\,x$$

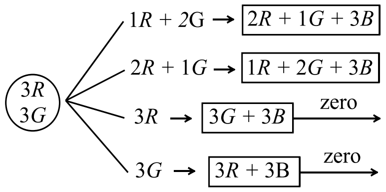
$$\Rightarrow (x-25)^2 > 525$$

$$\Rightarrow x - 25 < \sqrt{525} \ \ or \ \ x - 25 > \sqrt{525}$$

$$\Rightarrow x < 25 - \sqrt{525} \ or \ 25 + \sqrt{525}$$

As x is a positive integer and $\sqrt{525} = 22.91$, we must have $x \le 2$ or $x \ge 48$. Thus, the favourable number of cases is $2 + 53 = 55$. Hence, the required probability is 55/100=11/20

130 **(c)**



The required probability is

$$\frac{{}^3C_1\ {}^3C_2}{{}^6C_3}\frac{{}^2C_1\ {}^1C_1\ {}^3C_1}{{}^6C_3} + \frac{{}^3C_2\ {}^3C_1}{{}^6C_3}\frac{{}^1C_1\ {}^2C_1\ {}^3C_1}{{}^6C_3}$$

$$= 2\frac{9}{20} \times \frac{6}{20}$$

$$= \frac{27}{100}$$

131 **(a)**

Let p_1 and p_2 be the chances of happening of the first and second events, respectively, then according to the given conditions, we have

$$p_1 = p_2^2 \text{ and } \frac{1-p_1}{p_1} = \left(\frac{1-p_2}{p_2}\right)^3$$

$$\Rightarrow \frac{1-p_2^2}{p_2^2} = \left(\frac{1-p_2}{p_2}\right)^3$$

$$\Rightarrow p_2(1+p_2) = (1-p_2)^2$$

$$\Rightarrow p_2 = \frac{1}{3}$$

And so

$$p_1 = \frac{1}{9}$$

132 **(b)**

$$P(A' \cap B \cap C' \cap D) = P(A')P(B)P(C')P(D)$$

$$= \left(1 - \frac{1}{2}\right)\frac{1}{3}\left(1 - \frac{1}{5}\right)\left(\frac{1}{6}\right)$$

$$= \frac{1}{2} \times \frac{1}{3} \times \frac{4}{5} \times \frac{1}{6} = \frac{1}{45}$$

133 **(c)**

The required is

1 −Probability of getting equal number of heads and tails

$$= 1 - {}^{2n}C_n\left(\frac{1}{2}\right)^n\left(\frac{1}{2}\right)^{2n-n}$$

$$= 1 - \frac{(2n)!}{n!\,n!}\left(\frac{1}{4}\right)^n$$

$$= 1 - \frac{(2n)!}{(2!)^2} \times \frac{1}{4^n}$$

134 **(c)**

$P(4 \text{ biased coins}) = \frac{1}{4}$

$P(5 \text{ biased coins}) = \frac{1}{4}$

The required probability is

$$\frac{1}{3}\frac{{}^4C_3\ {}^{16}C_6}{{}^{20}C_9}\frac{1}{{}^{11}C_1} + \frac{2}{3}\frac{{}^5C_4\ {}^{15}C_5}{{}^{20}C_9}\frac{1}{{}^{11}C_1}$$

$$= \frac{2}{33}\left[\frac{{}^{16}C_6 + {}^{15}C_5}{{}^{20}C_9}\right]$$

135 **(c)**

A dice is thrown thrice, $n(S) = 6 \times 6 \times 6$

Favorable events of $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$

ie, (r_1, r_2, r_3) are ordered triplets which can take values,

$$\left.\begin{matrix}(1,2,3),(1,5,3),(4,2,3),(4,5,3)\\(1,2,6),(1,5,6),(4,2,6),(4,5,6)\end{matrix}\right\}$$

ie, 8 ordered triplets and each can be arranged in 3! ways = 6

$$\therefore n(E) = 8 \times 6$$

$$\Rightarrow P(E) = \frac{8 \times 6}{6 \times 6 \times 6}$$

$$= \frac{2}{9}$$

136 **(b)**

Let the probability of getting a tail in a single trail be $p = 1/2$,. The number of trails be $n = 100$ and the number of trails in 100 trials be X. We have,

$P(X=r)= {}^{100}C_r p^r q^{n-r}$

$= {}^{100}C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{100-r}$

$= {}^{100}C_r \left(\frac{1}{2}\right)^{100}$

Now,

$P(X=1)+P(X=3)+\cdots+P(X=49)$

$= {}^{100}C_1 \left(\frac{1}{2}\right)^{100} + {}^{100}C_3 \left(\frac{1}{2}\right)^{100} + \cdots$

$+ {}^{100}C_{49} \left(\frac{1}{2}\right)^{100}$

$= ({}^{100}C_1 + {}^{100}C_3 + \ldots + {}^{100}C_{49}) \left(\frac{1}{2}\right)^{100}$

But ${}^{100}C_1 + {}^{100}C_3 + \cdots + {}^{100}C_{99} = 2^{99}$

Also,

${}^{100}C_{99} = {}^{100}C_1$

${}^{100}C_{97} = {}^{100}C_3, \ldots {}^{100}C_{51} = {}^{100}C_{49}$

Thus,

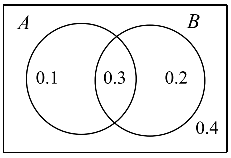
$2({}^{100}C_1 + {}^{100}C_3 + \cdots + {}^{100}C_{49}) = 2^{99}$

$\Rightarrow {}^{100}C_1 + {}^{100}C_3 + \cdots + {}^{100}C_{49} = 2^{99}$

Therefore, probability of required event is

$\frac{2^{98}}{2^{100}} = \frac{1}{4} 2^{98}/2^{100} = 1/4$

137 **(b,c)**



$P(A)=0.4, P(B)=0.5, P(A\cap B)=0.3$

$\Rightarrow P(A\cap B)=P(A)+P(B)-P(A\cap B)=0.6$

Now,

$P(E_1)=\frac{P(A\cap B)}{P(A\cup B)}=\frac{0.3}{0.6}=\frac{1}{2}$

$P(E_2)=\frac{P(A\cap \overline{B})+P(\overline{A}\cap B)}{P(A\cup B)}=\frac{0.1+0.2}{0.6}=\frac{1}{2}$

$P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{0.3}{0.5}=\frac{3}{5}=0.60;$

$P(B/A)=\frac{0.3}{0.4}=\frac{3}{4}=0.75=0.75$

$P(A/(A\cup B))=\frac{P(A)}{P(A\cup B)}=\frac{0.4}{0.6}=\frac{2}{3}$

$\frac{P(B)}{P(A\cup B)}=\frac{0.5}{0.6}=\frac{5}{6}$

138 **(a)**

$P(P\cup B)=P(A)+P(B)-P(A\cap B)$

$=0.25+0.50-0.14$

$=0.61$

$\therefore P(A'\cap B')=P((A\cup B)')=1-P(A\cup B)$

$=1-0.61=0.39$

139 **(a,b,c)**

We know that

$P(A\cap B)=P(A)+P(B)-P(A\cup B)$ (1)

Also, $P(A\cup B)\le 1$

$\Rightarrow -P(A\cup B)\ge -1$ (2)

$\therefore P(A\cap B)\ge P(A)+P(B)-1$ [Using Eqs. (1) and (2)]

Therefore, option (a) is correct. Again,

$P(A\cup B)\ge 0$

$\Rightarrow -P(A\cup B)\le 0$ (3)

$\Rightarrow P(A\cap B)\le P(A)+P(B)$ [Using Eqs. (1) and (3)]

Therefore, option (b) is also correct

From Eq. (1), option (c) is correct and (d) is not correct

140 **(a,d)**

We have,

$P(E/F)+P(\overline{E}/F)=\frac{P(E\cap F)}{P(E)}+\frac{P(\overline{E}\cup F)}{P(E)}$

$=\frac{P(E\cap F)+P(\overline{E}\cap F)}{P(E)}$

$\frac{P(F)}{P(F)}=1$

Therefore, option (a) holds. Also,

$P(E/F)+P(\overline{E}/F)=\frac{P(E\cap F)+P(\overline{F}\cap E)}{P(F)}$

$=\frac{P(E)}{P(F)}\ne 1$

Therefore option (b) does not hold

Similarly, we can show that option (c) does not hold but option (d) holds

141 **(a,b,c,d)**

We have,

P(exactly one of A,B occurs)

$=P[(A\cap\overline{B})\cup\overline{A}\cap B)]$

$=P(A\cap\overline{B})+P(\overline{A}\cap B)$

$=P(A)-P(A\cap B)+P(B)-P(A\cap B)$

$=P(A)+P(B)-2P(A\cap B)$

$=P(A\cup B)-P(A\cap B)$

Also,

P(exactly one of A,B occurs)

$=[1-P(\overline{A}\cap\overline{B})]-[1-P(\overline{A}\cup\overline{B})]$

$=P(\overline{A}\cup\overline{B})-P(\overline{A}\cap\overline{B})$

$=P(\overline{A})+P(\overline{B})-2P(\overline{A}\cap\overline{B})$

142 **(b,c)**

According to the problem,

$m+p+c-mp-mc-pc+mpc=3/4$ (1)

$mp(1-c)+mc(1-p)+pc(1-m)=2/5$ (2)

Also,

$mp+pc+mc-2mpc=1/2$ (3)

From Eqs. (2) and (3),

$mpc = \frac{1}{2} - \frac{2}{5} = \frac{1}{10}$

$\therefore\ mp + mc + pc = \frac{2}{5} + \frac{3}{10} = \frac{7}{10}$

$\therefore\ m + p + c = \frac{3}{4} + \frac{7}{10} - \frac{1}{10} = \frac{15 + 14 - 2}{20} = \frac{27}{20}$

143 **(b)**

$p = 0.4, q = 0.6$

$\therefore\ P(X \geq 1) = 1 - P(X = 0)$

$= 1 - {}^3C_0(0.4)^0(0.6)^3$

$= 1 - 0.216 = 0.784$

144 **(b)**

P(at least 7 points)= P(7 points)+P(8 points)

[∵ at most 8 points can be scored]

Now, 7 points can be scored by scoring 2 points in 3 matches and 1 point in one match. Similarly, 8 points can be scored by scoring 2 points in each of 4 matches. Therefore, the required probability is

${}^4C_1 \times [P(2\text{ points})]^3 P(1\text{ point}) + [P(2\text{ points})]^4$

$= 4(0.5)^3 \times 0.05 + (0.50)^4$

$= 0.0250 + 0.0625 = 0.0875$

145 **(a)**

P(2 white and 1 black)$=P(W_1W_2B_3$ or $W_1B_2W_3$ or $B_1W_2W_3)$

$= P(W_1W_2B_3) + P(W_1B_2W_3) + P(B_1W_2W_3)$

$= P(W_1)P(W_2)P(B_3) + P(W_1)P(B_2)P(W_3) + P(B_1)(W_2)(W_3)$

$= \frac{3}{4} \times \frac{2}{4} \times \frac{3}{4} + \frac{3}{4} \times \frac{2}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{2}{4} \times \frac{1}{4}$

$= \frac{1}{32}(9 + 3 + 1)$

$= \frac{13}{32}$

146 **(a,b)**

Let the number of red and blue balls be r and b, respectively

Then, the probability of drawing two red balls is

$p_1 = \frac{{}^rC_2}{{}^{r+b}C_2} = \frac{r(r-1)}{(r+b)(r+b-1)}$

The probability of drawing two blue balls is

$p_2 = \frac{{}^bC_2}{{}^{r+b}C_2} = \frac{b(b-1)}{(r+b)(r+b-1)}$

The probability of drawing one red and one blue ball is

$p_3 = \frac{{}^rC_1 \times {}^bC_1}{{}^{r+b}C_2} = \frac{2br}{(r+b)(r+b-1)}$

By hypothesis $p_1 = 5p_2$ and $p_5 = 6p_2$

$\therefore r(r-1) = 5b(b-1)$ and $2br = 6b(b-1)$

$\Rightarrow r = 6, b = 3$

147 **(c)**

$P(\overline{A}/\overline{B}) = \frac{P(\overline{A} \cap \overline{B})}{P(\overline{B})}$

$= \frac{P\overline{(A \cup B)}}{P(\overline{B})}$

$= \frac{1 - (A \cup B)}{P(\overline{B})}$

148 **(a,c,d)**

Since A and B are independent events, therefore,

$P(A \cap B) = P(A)P(B) = \frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$

$P(A/B) = P(A) = \frac{1}{2}$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{1}{5} - \frac{1}{10} = \frac{3}{5}$

Now, $P(A/A \cup B) = \frac{P[A \cap (A \cup B)]}{P(A \cup B)} = \frac{P(A)}{P(A \cup B)} = \frac{1/2}{3/5} = \frac{5}{6}$

$P[(A \cap B)/(\overline{A} \cup \overline{B})] = P(A \cap B)/(\overline{A \cap B}) = 0$

149 **(b)**

Given that

P(India wins)$= p = 1/2$

$\therefore P$(India loses)$= p' = 1/2$

Out of 5 matches. India's second is win occurs at third test. Hence, India wins third test and simultaneously it has won one match from first two and lost the other. Therefore, the required probability is

$P(LWW) + P(WLW) = \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3$

$= \frac{1}{4}$

150 **(a,d)**

The probability that head appears r times is

${}^{99}C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{99-r}$

Which is maximum when $r = 49$ or 50

151 **(a)**

The minimum face value is not less than 2 and maximum face value is not greater than 5 if we get any of the members 2, 3,4, 5, while total possible outcomes are 1, 2,3, 4, 5 and 6. Therefore, in one throw of die, probability of getting any number out of 2, 3, 4 and 5 is 4/6=2/3

If the die is rolled four times, then all these events being independent, the required probability is $(2/3)^4 = 16/81$

152 **(a,b,c,d)**

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{5}{8} = \frac{3}{8} + \frac{4}{8} - P(A \cap B)$

$\Rightarrow P(A\cap B)=\frac{2}{8}=\frac{1}{4}$

Now,

$P(A^c/B)=\frac{P(A^C\cap B)}{P(B)}$

$=\frac{P(B)-P(A\cap B)}{P(B)}$

$=1-2\left(\frac{1}{4}\right)$

$=\frac{1}{2}$

$2P(A/B^C)=\frac{2P(A\cap B^C)}{P(B^C)}$

$=\frac{2(P(A)-P(A\cap B))}{1-P(B)}$

$=4\left(\frac{3}{8}-\frac{2}{8}\right)=\frac{1}{2}$

Hence option (a) is correct

$P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{1}{4}\times\frac{2}{1}=\frac{1}{2}=P(B)$

Hence (b) is correct. Again,

$P(A^C/B^C)=\frac{P(A^C\cap B^C)}{P(B^C)}$

$=\frac{1-P(A\cup B)}{1-P(B)}$

$=2\left(1-\frac{5}{8}\right)=\frac{3}{4}$

$P(B/A^C)=\frac{P(B\cap A^C)}{1-P(A)}$

$=\frac{P(B)-P(A\cap B)}{5/8}$

$=\frac{1/2-1/4}{5/8}$

$=\frac{1}{4}\times\frac{8}{5}$

$=\frac{2}{5}$

Hence,

$8P(A^C/B^C)=15P(B/A^C)$

Hence, (c)is not correct. Again,

$2P(A/B^C)=\frac{1}{2}$

$\Rightarrow P\left(\frac{A}{B^C}\right)=\frac{1}{4}=P(A\cap B)$

Hence (d) is correct

153 **(a,b,c)**

Option (d) is true if and only if A and B are independent

154 **(b,c,d)**

$P(E\cap F)=P(E)P(F)$

Now, $P(E\cap F)=P(E)-P(E\cap F)=P(E)[1-P(F)]$

$=P(E)P(F)$

And $P(E^c\cap F^c)=1-P(E\cup F)=1-[P(E)+P(F)-P(E\cap F)]$

$=[1-P(E)][1-P(F)]=P(E^c)P(F^c)$

Also,

$P(E/F)=P(E)$ and $P(E^c/F^c)=P(E^c)$

$\Rightarrow P(E/F)+P(E^c/F^c)=1$

155 **(c)**

Let A,B,C be the events that the student passes tests I, II, III respectively. Then, according to question, $P(A)=p, P(B)=q, P(C)=1/2$

Now the student is successful if A and B happen or A and C happen or A,B and C happen

$\therefore P(AB\overline{C})+P(AC\overline{B})+P(ABC)=\frac{1}{2}$

$\Rightarrow pq\left(1-\frac{1}{2}\right)+p\frac{1}{2}(1-q)+pq\frac{1}{2}=\frac{1}{2}$

$\Rightarrow \frac{1}{2}pq+\frac{1}{2}p-\frac{1}{2}pq+\frac{1}{2}pq=\frac{1}{2}$

$\Rightarrow p+pq=1$

$\Rightarrow p(1+q)=1$

Which holds for $p=1$ and $q=0$

156 **(a,b,c)**

We are given

$P(A\cap B')=0.20, P(A'\cap B)=0.15, P(A\cap B)=0.10$

Now,

$P(B)=P(A'\cap B)+P(A\cap B)=0.15+0.20=0.35$

$\therefore P(A/B)=\frac{P(A\cap B)}{P(A)}=\frac{0.10}{0.35}=\frac{2}{7}$

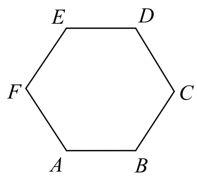
$P(A)=P(A\cap B')+P(A\cap B)=0.3$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)=0.55$

$P(B/A)=\frac{P(B\cap A)}{P(A)}=\frac{1}{3}$

157 **(b)**

Out of 6 vertices, 3 can chosen in ${}^6C_3$ ways. The triangle will be equilateral if it is ΔACE or ΔBDF (2 ways)



Therefore, the required probability is

$\frac{2}{{}^6C_3}=\frac{2}{20}=\frac{1}{10}$

158 **(d)**

The two events can happen simultaneously, e.g., (2, 3). Therefore, they are not mutually exclusive. Also, the two event are not dependent on each other

159 **(a,b,c)**

Here total number of cases is ${}^8C_2 = 28$

a. Favourable number of case is 13

For 2 → 6 choices

For 1 → 7 choice

b. For 7 → 6 choices

For 8 → 7 choices

c. For 1 → 4 choices (2, 4, 6, 8)

For 2 → 6 choices (3, 4, 5, 6, 7, 8)

For 3 → 3 choices (4, 6, 8)

For 4 → 4 choices (5, 6, 7, 8)

For 5 → 2 choices (6, 8)

For 6 → 2 choices (7, 8)

For 7 → 1 choices (8)

Alternative solution:

1. $\frac{{}^8C_2 - {}^6C_2}{{}^8C_2} = \frac{13}{28}$

2. $\frac{{}^8C_2 - {}^5C_2}{{}^8C_2} = \frac{9}{14}$

3. $\frac{{}^8C_2 - {}^4C_2}{{}^8C_2} = \frac{11}{14}$

160 **(a,c,d)**

$P(M) + P(N) - 2P(M \cap N)$

$= [P(M) + P(N) - P(M \cap N)] - P(M \cap N)$

$= P(M \cup N) - P(M \cap N)$

=Probability that exactly one of M and N occurs

$P(M) + P(N) - P(M \cap N)$

$= P(M \cup N)$

=Probability that at least of M and N occurs

$P(M^c) + P(N^c) - 2P(M^c \cap N^c)$

$= 1 - P(M) + 1 - P(N) - 2p(M \cup N)^c$

$= 2 - P(M) - P(N) - 2[1 - P(M \cup N)]$

$= P(M \cup N) + P(M \cup N) - P(M) - P(M)$

$= P(M \cup N) - P(M \cap N)$

=Probability that exactly one of M and N occurs

$P(M \cap N^c) + P(M^c \cap N)$

=Probability that M occurs but N does not or probability that M does not occurs but N occurs

=Probability that exactly one of M and N occurs

Thus, (a), (c) and (d) are the correct options

161 **(a)**

the probability that only two tests are needed is (probability that the first machine tested is faulty)×(probability that the second machine tested is faulty given the first machine tested is faulty), which is given by $(2/4) \times (1/3) = 1/6$

162 **(a,c)**

Given that A and B are mutually exclusive events

$\therefore A \cap B = \phi$

$\Rightarrow A \subseteq \overline{B}$ and $B \subseteq \overline{A}$

$\Rightarrow P(A) \leq P(\overline{B})$ and $P(B) \leq P(\overline{A})$

163 **(b,c,d)**

Roots of $x^2 + px + q = 0$ will be real if $p^2 \geq 4q$

The possible selections are as follows

p	q
1	–
2	1
3	1, 2
4	1, 2, 3, 4
5	1, 2, 3, 4, 5, 6
6	1, 2, ..., 9
7	1, 2, ...,10
8	1, 2, ...,10
9	1, 2, ...,10
10	1, 2, ...,10
Total	62

Therefore, number of favourable ways is 62 and total number of ways is $10^2 = 100$. Hence, the required probability is 62/100=31/50. The probability that the roots are imaginary is $1 - 31/50 = 19/50$

Roots are equal when $(p, q) \equiv (2,1), (4,4), (9,6)$. The probability that the roots are real and equal is 3/50. Hence, probability that the roots are real and distinct is 3/5

164 **(a,b,d)**

We have, the probability that the bomb strikes the target is $p = 1/2$. Let n be the number of bombs which should be dropped to ensure 99% chance or better of completely destroying the target. Then, the probability that out of n bombs at least two bombs strike the target is greater than 0.99. Let X denote the number of bombs striking the target. Then

$$P(X = r) = {}^{n}C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{n-r} = {}^{n}C_r \left(\frac{1}{2}\right)^n, r = 0, 1, 2, \dots, n$$

We should have

$P(X \geq 2) \geq 0.99$

$\Rightarrow \{1 - P(X < 2)\} \geq 0.99$

$\Rightarrow 1 - \{P(X = 0) + P(X = 1)\} \geq 0.99$

$\Rightarrow 1 - \left\{(1 + n)\frac{1}{2^n}\right\} \geq 0.99$

$\Rightarrow 0.001 \geq \frac{1+n}{2^n}$

$\Rightarrow 2^n > 100100n \ \Rightarrow n \geq 11$

Thus, the minimum number of bombs is 11

165 **(a)**

The event that the fifth toss result in a head is independent of the event that the first four tosses result in tails. Therefore, the probability of the required event is 1/2

166 **(b,c,d)**

False

$$P(TTT \text{ or } HHH) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

$$\frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B^C)}{1 - P(B)} = \frac{P(A) - P(A \cap B)}{1 - P(B)}$$

$$P(A \cap B)[1 - P(B)] = P(B)P(A) - P(B)P(A \cap B)$$

$$P(A \cap B) = P(A)P(B)$$

Hence, the given statement is true

Let E_1 be the white ball is drawn in first draw,; E_2 be the event that black ball is drawn in second draw; E be the event that white ball is drawn in second draw

$$\therefore P(E) = P(E/E_1)P(E_1) + P(E/E_2)P(E_2)$$

$$= \frac{d + w}{w + b + d}\left(\frac{w}{w + b}\right) + \frac{w}{w + b + d}\left(\frac{b}{w + d}\right)$$

$$= \left(\frac{w}{w + b}\right)\left(\frac{d + w}{w + b + d} + \frac{b}{w + b + d}\right)$$

$$= \left(\frac{w}{w + b}\right)$$

which is independent of d

To prove that A, B, C are pair wise independent only. Now,

$$P(A \cap B = P((A \cap B) \cap C) \cup (A \cap B \cap C))$$

$$= P(A \cap B \cap C) + P(A \cap B \cap C)$$

$$= P(A)P(B)P(\overline{C}) + P(A)P(B)P(C) \quad \text{(given)}$$

$$= P(A) \times P(B)[P(C') + P(C)]$$

$$= P(A) \times P(B)$$

Similarly, for the other two. Hence, this statement is correct

167 **(a,b,c)**

The number of ways in which m boys and m girls can take their seats around a circle

$= \big((m + m) - 1\big)! = (2m - 1)!$

Option (a) No two boys sit together

We make the girls sit first around the table. This can be done in $(m - 1)!$ ways. After this boys can take their seats in $m!$ ways.

$\therefore$ The probability no two boys sit together

$= \frac{m!\,(m - 1)!}{(2m - 1)!} = \frac{1}{{}^{2m-1}C_m}$

$= ({}^{2m-1}C_m)^{-1}$

Option (b) No two girls sit together.

We make the boys sit first around the table. This can be done in $(m - 1)!$ ways, after this girls can take their seats in $(m)!$ ways.

$\therefore$ The probability no two girls sit together

$= \frac{m!\,(m - 1)!}{(2m - 1)!}$

$= \frac{1}{{}^{2m-1}C_m} = ({}^{2m-1}C_m)^{-1}$

Option (c) Boys and girls sit alternatively *ie*, no two boys (girls) sit together

$\therefore$ Required probability $= ({}^{2m-1}C_m)^{-1}$

Option (d) All the boys sit together.

$\because$ All the boys sit together then treat them as a single boy.

Here, $(m + 1)$ objects (m girls + 1 boy)

$\therefore$ We can put $(m + 1)$ objects around a circle in $m!$ ways. But boys can be tied in $m!$ ways.

$\therefore$ Required probability $= \frac{m!m!}{(2m-1)!} \neq ({}^{2m-1}C_m)^{-1}$

168 **(b)**

The number of ways of arranging 10 balls without any restriction is 10!. As for no two black balls are placed adjacently, first arrange 7 white balls is 7! ways

$-W-W-W-W-W-W-W-$

Now white balls must be placed in three of eight gaps created in ${}^8C_3 3!$ ways. Hence, number of favourable ways is ${}^8C_3 3!\,7!$

Therefore, the required probability is

$$\frac{{}^8C_3 3!\,7!}{10!}=\frac{7}{15}$$

169 **(b,c)**

Here, $P(M)=\alpha, P(P)=\beta$ and $P(C)=\gamma$

$\therefore$ The probability of passing in atleast one subject $=0.75$ (given)

$\Rightarrow 1-P(\overline{M}\,\overline{P}\,\overline{C})=0.75$

$\Rightarrow 1-P(\overline{M})P(\overline{P})P(\overline{C})=0.75$

$\Rightarrow 1-(1-\alpha)(1-\beta)(1-\gamma)=0.75$

or $\alpha+\beta+\gamma-\alpha\beta-\beta\gamma-\gamma\alpha+\alpha\beta\gamma=\frac{3}{4}$ ...(i)

The probability of passing in atleast two subjects $=0.50$ (given)

or $P(M\,P\,\overline{C})+P(M\,\overline{P}\,C)+P(\overline{M}\,P\,C)+P(M\,P\,C)=0.50$

$\Rightarrow P(M)\,P(P)\,P(\overline{C})+P(M)\,P(\overline{P})\,P(C)+P(\overline{M})P(P)\,P(C)+P(M)\,P(P)P(C)=0.50$

$\Rightarrow \alpha\beta(1-\gamma)+\alpha(1-\beta)\gamma+(1-\alpha)\beta\gamma+\alpha\beta\gamma=\frac{1}{2}$

$\Rightarrow 2\alpha\beta\gamma=\alpha\beta+\beta\gamma+\gamma\alpha-\frac{1}{2}$ ....(ii)

and the probability of passing in exactly two subject $=0.40$ (given)

$\Rightarrow P(M\,P\,\overline{C})+P(M\,\overline{P}C)+P(\overline{M}\,P\,C)=\frac{2}{5}$

$\Rightarrow P(M)P(P)P(\overline{C})+P(M)P(\overline{P})P(C)+P(\overline{M})P(P)P(C)=\frac{2}{5}$

$\Rightarrow \alpha\beta(1-\gamma)+\alpha(1-\beta)\gamma+(1-\alpha)\beta\gamma=\frac{2}{5}$

$\Rightarrow \alpha\beta+\beta\gamma+\gamma\alpha-3\alpha\beta\gamma=\frac{2}{5}$ ....(iii)

From Eqs. (ii) and (iii), we get

$2\alpha\beta\gamma+\frac{1}{2}-3\alpha\beta\gamma=\frac{2}{5}$

$\Rightarrow \alpha\beta\gamma=\frac{1}{2}-\frac{2}{5}=\frac{1}{10}$

From Eq. (ii),

$\frac{1}{5}=\alpha\beta+\beta\gamma+\gamma\alpha-\frac{1}{2}$

$\Rightarrow \alpha\beta+\beta\gamma+\gamma\alpha=\frac{7}{10}$ ...(iv)

On substituting the values of $\alpha\beta\gamma$ and $\alpha\beta+\beta\gamma+\gamma\alpha$ in Eq. (i), we get

$\alpha+\beta+\gamma-\frac{7}{10}+\frac{1}{10}=\frac{3}{4}$

$\Rightarrow \alpha+\beta+\gamma=\frac{6}{10}+\frac{3}{4}$

$=\frac{12+15}{20}=\frac{27}{20}$

170 **(d)**

Since there are 15 possible cases for selecting a coupon and seven coupons are selected, the total number of cases of selecting seven coupons is 15^7. It is given that the largest number on the selected coupon is 9. Therefore the selection is to be made from the coupons numbered 1 to 9. This can be made in 9^7 ways. Out of these 9^7 cases, 8^7 cases do not contain the number 9. Thus, the favourable number of cases is 9^7-8^7. Hence, the required probability is $(9^7-8^7)/(15^7)$

171 **(a,b)**

The probability that both will be alive for 10 years, hence, i.e., the probability that the man and his wife both will be alive 10 years hence is $0.83\times0.87=0.7221$. The probability that at least one of them will be alive is

$1-P$\{That none of them remains alive 10 years hence\}

$=1-(1-0.83)(1-0.87)=1-0.17\times0.13=0.9779$

172 **(a,b,c,d)**

$A\subseteq A\cup B$

$\Rightarrow P(A)\le P(A\cup B)\Rightarrow P(A\cup B)\ge\frac{3}{4}$

Also, $P(A\cap B)=P(A)+P(B)-P(A\cup B)$

$\ge P(A)+P(B)-1$

$=\frac{3}{4}+\frac{5}{8}-1=\frac{3}{8}$

Now,

$A\cap B\subseteq B$

$\Rightarrow P(A\cap B)\le P(B)=\frac{5}{8}$

$\therefore \frac{3}{8}\le P(A\cap B)\le\frac{5}{8}$

And $P(A\cap B')=P(A)-P(A\cap B)$

$\Rightarrow \frac{3}{4}-\frac{5}{8}\le P(A\cap B')\le\frac{3}{4}-\frac{3}{8}$

$\Rightarrow \frac{1}{8}\le P(A\cap B')\le\frac{3}{8}$

$\therefore P(A\cap B)=P(B)-P(A'\cap B)$ [Using Eq. (1)]

$\Rightarrow \frac{3}{8}\le P(B)-P(A'\cap B)\le\frac{5}{8}$

$\Rightarrow 0\le P(A'\cap B)\le\frac{1}{4}$

173 **(d)**

Let p be the probability of one coin showing head.

Then the probability of one coin showing tail is $1-p$. According to question, the coin is tossed 100 times and probability of 50 coins showing head is equal to the probability of 51 coins showing head.

Using binomial probability distribution

$P(X=r)=C_r p^r q^{n-r}$. We get

$^{100}C_5 p^{50}(1-p)^{50} = {}^{100}C_{51}p^{51}(1-p)^{49}$

$\Rightarrow \dfrac{1-p}{p}=\dfrac{^{100}c_{51}}{^{100}c_{50}}=\dfrac{50!\,50!}{51!\,49!}=\dfrac{50}{51}$

$\Rightarrow 51-51p=50p$

$\Rightarrow 101p=51 \;\Rightarrow p=\dfrac{51}{101}$

174 **(a,b,c,d)**

1. $P(E_1)=1-P(RRR)$

$=1-\left[\dfrac{1}{3}\times\dfrac{2}{4}\times\dfrac{3}{5}\right]=0.9$

2. $P(E_2)=3P(BRR)=3\times\frac{2}{3}\times\frac{1}{4}\times\frac{2}{5}=0.2$

3. $P(E_3)=P(RRR/RRR\cup BBB)$

$=\dfrac{P(RRR)}{P(RRR)+P(BBB)}$

$=\dfrac{0.1}{0.1+\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}}$

$=\dfrac{0.1}{0.1+0.4}=0.2$

4. $P(E_4)=1-P(BBB)=1-\frac{2}{5}=0.6$

175 **(a,b)**

Let $P(E)=x$ and $P(F)=y$. According to the question,

$P(E\cap F)=\dfrac{1}{12}$

As E and F are independent events, we have

$P(E\cap F)=P(E)P(F)$

$\Rightarrow \frac{1}{12}=xy$ (1)

Also,

$P(\overline{E}\cap\overline{F})=P(\overline{E\cup F})$

$=1-P(E\cup F)$

$\Rightarrow \dfrac{1}{2}=1-[P(E)+P(F)-P(E)P(F)]$

$\Rightarrow x+y=\frac{7}{12}$ (2)

Solving Eqs. (1) and (2), we get either $x=1/3$ and $y=1/4$ or $x=1/4$ and $y=1/3$

Therefore, options (a) and (b) are correct

176 **(a,b,c)**

Let 'H' be the event that married man watches the show and 'W' be the probability that married woman watches the show

$\therefore P(H)=0.4, P(W)=0.5, P(H/W)=0.7$

1. $P(H\cap W)=P(W)P(H/W)=0.5\times 0.7=0.35$

2. $P(E/H)=\frac{P(H\cap W)}{P(H)}=\frac{0.35}{0.4}=\frac{7}{8}$

3. $P(H\cup W)=P(H)+P(W)-P(H\cap W)$

$=0.4+0.5-0.35=0.55$

177 **(a,b,c)**

$P(A)=\frac{1}{5}, P(B)=\frac{7}{25}, P(B/A)=\frac{9}{10}$

$P(\overline{A}\cap\overline{B})=1-P(A\cup B)$

$=1-[P(A)+P(B)-P(A\cap B)]$

$=1-\left[\frac{1}{5}+\frac{7}{25}-P(A)P(B/A)\right]$

$=1-\left[\frac{1}{5}+\frac{7}{25}-\frac{1}{5}\times\frac{7}{25}\right]=\frac{7}{10}$

$P(A/B)=\frac{P(A\cap B)}{P(b)}$

$=\frac{P(A)P(B/A)}{P(B)}$

$=\frac{\frac{1}{5}\times\frac{9}{10}}{\frac{7}{25}}$

$=\frac{9}{50}\times\frac{25}{7}=\frac{9}{14}=\frac{18}{28}$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=P(A)+P(B)-P(A)P(B/A)$

$=\frac{1}{5}+\frac{7}{25}-\frac{1}{5}\times\frac{9}{10}$

$=\frac{10+14-9}{50}$

$=\frac{3}{10}$

$P(A'\cup B)=1-P(A\cap B)$

$=1-P(A)P(B/A)$

$=1-\frac{1}{5}\times\frac{9}{10}$

$=\frac{41}{50}$

178 **(a)**

We know that $7^1=7, 7^2=49, 7^3=343, 7^4=2401, 7^5=16807$

Therefore, 7^k (where $k\in Z$) results in number whose unit's digit is 7 or 9 or 3 or 1

Now, 7^m+7^n will be divisible by 5 if unit's place digit in the resulting number is 5 or 0. Clearly, it can never be 5. But it can be 0 if we consider values of m and n such that the sum of unit's place digits become 0. And this can be done by choosing

$\left.\begin{matrix} m=1,5,9,\dots,97 \\ n=3,7,11,\dots,99\end{matrix}\right\}$ (25 options each)[7+3=10]

or

$\left.\begin{matrix} m=2,6,10,\dots,98 \\ n=4,8,12,\dots,100\end{matrix}\right\}$ (25 options each)[9+3=13]

Therefore, the total number of selections of m, n such that 7^m+7^n is divisible by 5 is $(25\times25+25\times25)\times2$ (since we can interchange value of m and n)

Also the number of total possible selections of m and n out of 100 is 100×100. Therefore, the required probability is

$\frac{2(25\times25+25\times25)}{100\times100}=\frac{1}{4}$

179 **(a,c,d)**

$P(E)=\frac{^{2n}C_n}{2^{2n}}=\frac{(2n)!}{n!\,n!\,2^n2^n}$

$=\frac{1\times2\times3\times\dots\times(2n)}{n!\,n!\,2^n2^n}$

$=\frac{1\times3\times5\dots\times(2n-1)}{n!\,2^n}$

Now,

$\prod_{r=1}^{n}\left(\frac{2r-1}{2r}\right)=\frac{1\times3\times5\times\dots\times(2n-1)}{2\times4\times6\times\dots\times(2n)}$

$=\frac{1\times3\times5\times\dots\times(2n-1)}{(1\times2\times3\times\dots\times n\times)2^n}$

$\sum_{r=0}^{n}\left(\frac{^nC_r}{2^n}\right)^2=\frac{1}{2^n2^n}\sum_{r=0}^{n}(^nC_r)^2$

$=\frac{1}{2^n2^n}\ {}^{2n}C_n$

Also,

$\frac{\sum_{r=0}^{n}(^nC_r)^2}{(\sum_{r=0}^{2n}\ {}^{2n}C_r)}=\frac{^{2n}C_n}{2^{2n}}$

180 **(a,b,c)**

We know that, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow P(A\cap B)=P(A)+P(B)-P(A\cup B)$ [Option (c)]

$\because\ P(A\cup B)\le 1$

$\Rightarrow P(A)+P(B)-P(A\cap B)\le1$

or $P(A\cap B)\ge P(A)+P(B)-1$

$\therefore\ P(A\cap B)$ is not less than $P(A)+P(B)-1$ [Option (a)]

Also, $P(A\cap B)\le P(A\cup B)\le P(A)+P(B)$

$\Rightarrow\ P(A\cap B)\le P(A)+P(B)$

$ie, P(A\cap B)$ is not greater than $P(A)+P(B)$ [Option (b)]

181 **(a)**

We know that

P(exactly one of A or B occurs)$=P(A)+P(B)-$

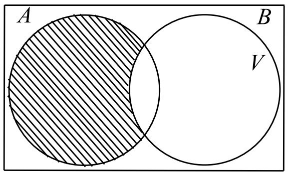
$2P(A\cap B)$
Therefore,
$P(A)+P(B)-2P(A\cap B)=p$ (1)
Similarly,
$P(B)+P(C)-2P(B\cap A)=p$ (2)
And $P(C)+P(A)-2P(C\cap A)=p$ (3)
Adding Eqs. (1), (2) and (3) we get
$2[P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)]=3p$
$\Rightarrow P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)=3p/2$ (4)
It is also given that
$P(A\cap B\cap C)=p^2$
Now,
P(at least one of A,B and C)$=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)$
$=\frac{3p}{2}+p^2$ [Using Eqs. (4) and (5)]
$=\dfrac{3p+2p^2}{2}$

182 **(c)**
Given that
$P(A\cup B)=0.6; P(A\cap B)=0.2$
$\therefore P(\bar{A})+P(\bar{B})=1-P(A)\pm 1-P(B)$
$=2-(P(A)+P(B)$
$=2-[P(A\cup B)+P(A\cap B)]$
$=2-[0.6+0.2]$
$=2.08$
$=1.2$

183 **(c,d)**
Given $P(A\cup B)=P(A)+P(B)-P(A)\cdot P(B)$
$\therefore\ P(A\cap B)=P(A)\cdot P(B)$
Then, $P(A/B)=\frac{P(A\cap B)}{P(B)}=P(A)$
and $P(\overline{A\cup B})=1-\ P(A\cup B)$
$=1-\{P(A)+P(B)-P(A)\cdot P(B)\}$
$=\{1-P(A)\}\{1-P(B)\}$
$=P(\overline{A})P(\overline{B})$

184 **(a,b,c)**
$P(A/B)=P(A)=\dfrac{1}{2}$
$P[A/(A\cup B)]=\dfrac{P[A\cap(A\cup B)]}{P(A\cup B)}$
$[\because A\cap(A\cup B)=A\cap(A-B-A\cap B)]$
$=A-A\cap B-A\cap B=a\}$
$\Rightarrow P\left(\dfrac{A}{A\cup B}\right)=\dfrac{P(A)}{P(A\cup B)}$
$=\dfrac{\frac{1}{2}}{\frac{1}{2}-\frac{1}{5}-\frac{1}{10}}=\dfrac{\frac{1}{2}}{\frac{6}{10}}=\dfrac{5}{6}$
and similarly,
$P\left(\dfrac{A\cap B}{A'\cap B'}\right)=0$

185 **(a,c)**
For any two events A and B,
$P(A/B)=\dfrac{P(A\cap B)}{P(B)}$
Now we know that
$P(A\cup B)\le 1$
$\Rightarrow P(A)+P(B)-P(A\cap B)\le 1$
$\Rightarrow P(A\cap B)\ge P(A)+P(B)-1$
$\Rightarrow \frac{P(A\cap B)}{P(B)}\ge\frac{P(A)+P(B)-1}{P(B)}$ [as $P(B)\ne 0 \therefore P(B)>0$]
$\Rightarrow\ P(A/B)\ge\dfrac{P(A)+P(B)-1}{P(B)}$
Therefore, option (a) is correct
From Venn's diagram, we can clearly conclude the



$P(A\cup\overline{B})=P(A)-P(A\cap B)$
Therefore, option (b) is incorrect
$P(A\cap B)=P(A)+P(B)-P(A\cap B)$
$=1-P(\overline{A})+1-P(\overline{B})-P(A)P(B)$
[$\because A$ and B are independent event]
$=2-P(\overline{A})-P(\overline{B})-[1-P(\overline{A})][1-P(\overline{B})]$
$=2-P(\overline{A})-P(\overline{B})-1+P(\overline{A})+P(\overline{B})-P(\overline{A})P(\overline{B})$
$=1-P(\overline{A}\cap\overline{B})$ [$\because$ if A and B are independent $\overline{A}$ and $\overline{B}$ are also independent]
Therefore, option (c) is the correct statement
For disjoint events,
$P(A\cup B)=P(A)+P(B)$
Therefore, option (d) is incorrect

186 **(c,d)**
$P(E_1)=1-P$ (unit's place in both is 1,2, 3,4, 6, 7, 8, 9)

$P(E_1 = 0 \text{ or } 5) = 1 - \left(\frac{4}{5}\right)^2 = \frac{9}{25}$

$P(E_2 : 5) = P(1, 3, 5\ 7\ 9) - P(1\ 3\ 7\ 9)$

$= \frac{1}{4} - \frac{4}{25}$

$= \frac{25 - 16}{100} = \frac{9}{100}$

$\frac{P(E_2)}{P(E_1)} = \frac{9}{100} \times \frac{25}{9} = \frac{1}{4}$

$P(E_1) = 4P(E_2)$

$P(E_2/E_1) = \frac{P(E_2 \cap E_1)}{P(E_1)} = \frac{P(E_2)}{P(E_1)} = \frac{1}{4}$

$P(E_1/E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{P(E_2)}{P(E_2)} = 1$

187 **(d)**

Given that $P(E) \leq P(F)$ and $P(E \cap F) > 0$. It does not necessary mean that E is the subset of F. Therefore, the choices (a), (b), (c) do not hold in general. Hence, option (d) is the right choice here

188 **(a,c)**

Let p_1, p_2 be the chances of happenings of the first and second events, respectively. Then according to the given conditions, we have

$p_1 = p_2^2$

And $\frac{1-p_1}{p_1} = \left(\frac{1-p_2}{p_2}\right)^3$

Hence,

$\frac{1 - p_2^2}{p_2^2} = \left(\frac{1 - p_2}{p_2}\right)^3 \Rightarrow p_2(1 + p_2) = (1 - p_2)^2$

$\Rightarrow 3p_2 = 1 \Rightarrow p_2 = \frac{1}{3}$

and so

$p_1 = \frac{1}{9}$

189 **(a,c)**

Let one probability of choosing one integer k be $P(k) = \lambda/k^4$. (λ is one constnat of probability). Then

$$\sum_{k=1}^{2m} \frac{\lambda}{k^4} = 1$$

$$\Rightarrow \lambda \sum_{k=1}^{2m} \frac{1}{k^4} = 1$$

Let x_1 be the probability of choosing the odd number. Then,

$$x_1 = \sum_{k=1}^{m} P(2k - 1) = \lambda \sum_{k=1}^{m} \frac{1}{(2k - 1)^4}$$

Also,

$$1 - x_1 = \sum_{k=1}^{m} P(2k)$$

$$= \lambda \sum_{k=1}^{m} \frac{1}{(2k)^4}$$

$$< \lambda \sum_{k=1}^{m} \frac{1}{(2k - 1)^4}$$

$\Rightarrow 1 - x_1 < x_1$

$\Rightarrow x_1 > 1/2$

$\Rightarrow x_2 > 1/2$

190 **(c,d)**

$P(A \cup B') = 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - P(A) - P(B) + P(A)P(B)$

$= [1 - P(A)][1 - P(B)]$

$= P(A')P(B')$

Also,

$P(A \cup B) = P(A) + P(B) - P(A)P(B)$

$\Rightarrow P(A \cap B) = P(A)P(B)$

$\therefore P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)\,P(B)}{P(B)} = P(A)$

Also,

$P(B/A) = P(B)$

$P(A' - B') = P(A') - P(A' \cap B')$

$= P(A') - P(A')P(B')$

$= P(A')\big(1 - P(B')\big)$

$= P(A')P(B)$

191 **(b,c)**

Let $P(A) = x$ and $P(B) = y$. Since A and B are independent events, therefore,

$P(\overline{A} \cap B) = 2/15 \Rightarrow P(\overline{A})P(B) = 2/15$

$\Rightarrow \big(1 - P(A)\big)P(B) = 2/15$

$\Rightarrow (1 - x)y = 2/15 \quad (1)$

And $P(A \cap \bar{B}) = \frac{1}{6} \Rightarrow P(A)P(\bar{B}) = \frac{1}{6}$

$\Rightarrow x(1 - y) = \frac{1}{6}$

$\Rightarrow x - xy = \frac{1}{6} \quad (2)$

Subtracting Eq. (1) from Eq. (2), we get

$x - y = \frac{1}{30} \Rightarrow x = \frac{1}{30} + y$

Putting this value of x in Eq. (1), we get

$y - y\left(\frac{1}{30} + y\right) = \frac{2}{15}$

$\Rightarrow 30y - y - 30y^2 = 2/5$

$\Rightarrow 30y^2 - 29y + 4 = 0$

$\Rightarrow (6y - 1)(5y - 4) = 0$

$\Rightarrow y = 1/6 \text{ or } y = 4/5$

$\Rightarrow P(B) = 1/6 \text{ or } P(B) = 4/5$

192 **(b,d)**

Out of the numbers 00, 01, 02, 03, …., 99 those numbers the product of whose digits is 16 are 28, 44, 82 *ie*, only 3

$\therefore P = P(E) = \frac{3}{100}$

and $q = P(\overline{E}) = 1 - P(E) = 1 - p = 1 - \frac{3}{100} = \frac{97}{100}$

$\therefore$ The binomial distribution is $\left(\frac{97}{100} + \frac{3}{100}\right)^5$

$\therefore$ The probability that the event occurs exactly three times

$= {}^5C_3 \left(\frac{3}{100}\right)^3 \left(\frac{97}{100}\right)^2$

$= 10(0.03)^3(0.97)^2$

193 **(d)**

If $P(H_i \cap E) = 0$ for some i, then

$$P\left(\frac{H_i}{E}\right) = P\left(\frac{E}{H_i}\right) = 0$$

If $P(H_i \cap E) = 0$ for $i = 1, 2, \dots, n$, then

$$P\left(\frac{H_i}{E}\right) = \frac{P(H_i \cap E)}{P(H_i)} \times \frac{P(H_i)}{P(E)}$$

$= \frac{P\left(\frac{E}{H_i}\right) \times P(H_i)}{P(E)} > P\left(\frac{E}{H_i}\right), P(H_i)$ [as $0 < P(E) < 1$]

195 **(d)**

The number of ways of selecting two persons out of 20 is ${}^{20}C_2 = 190$.

The number of ways in which two selected persons together is 19.

Required probability $= 1 - \frac{19}{190} = 0.9$

196 **(b)**

$$P(A \cup \overline{B}) = 1 - \left(\overline{A \cup \overline{B}}\right) = 1 - (\overline{A} \cap B)$$
$$= 1 - P(\overline{A})P(B)$$

$\Rightarrow 0.9 = 1 - 0.6 \times P(B)$

$\Rightarrow P(B) = \frac{1}{6}$

Clearly, statement 2 is nor correct explanation of statement 1

197 **(a)**

$P\{A \cap (B \cap C)\} = P(A \cap B \cap C) = P(A)P(B)P(C)$

$\therefore P[A \cap (B \cup C)]$

$= P[(A \cap B) \cup (AQ \cap C)]$

$= P[(A \cap B) + (A \cap C) - P[(A \cap B) \cap (A \cap C)]$

$= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$

$= P(A)P(B) + P(A)P(C) - P(A)P(B)P(C)$

$= P(A)[P(B) + P(C) - P(B)P(C)]$

$= P(A)P(B \cup C)$

Therefore, A and $B \cup C$ are independent events

198 **(b)**

The total number of cases, $n(S) = 4!$. Let E be the event that no letter is mailed in its correct envelop. Then the favourable number of cases is

$$4!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right) = 9$$

Hence, the required probability is

$$P(E) = \frac{9}{24} = \frac{3}{8}$$

Also, the probability that all the letters are placed in the correct envelope is 1/24.

Hence, the probability that all the letters are not placed in the correct envelope is 23/24

Hence, statement 2 is correct but does not explain statement 1

201 **(a)**

$P(A/B) \geq P(A)$

$\Rightarrow \frac{P(A \cap B)}{P(B)} \geq P(A)$

$\Rightarrow \frac{P(A \cap B)}{P(A)} \geq P(B)$

$\Rightarrow P(B/A) \geq P(B)$

202 **(a)**

We have,

$$P(A \cap \overline{B}) = 1 - (\overline{A \cup \overline{B}}) = 1 - (\overline{A} \cap B)$$
$$= 1 - P(\overline{A})P(B)$$

$= 0.8 = 1 - 0.7 \times P(B)$

$\Rightarrow P(B) = \frac{2}{7}$

203 **(c)**

We have, $P(A \cup B) \geq \max\{P(A), P(B)\} = \frac{2}{3}$

or $P(A \cup B) \geq \frac{2}{3}$

Now, $P(A \cap B) = P(A) + P(B) - P(A \cup B) \geq P(A) + P(B) - 1$

$= \frac{1}{2} + \frac{2}{3} - 1 = \frac{1}{6}$

or $P(A \cap B) \geq \frac{1}{6}$ ....(i)

and $P(A \cap B) \leq \min\{P(A), P(B)\} = \frac{1}{2}$

$P(A \cap B) \leq \frac{1}{2}$ ...(ii)

From relations (i) and (ii), we get

$\frac{1}{6} \leq P(A \cap B) \leq \frac{1}{2}$

205 **(d)**

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore 1 \geq P(A) + P(B) - P(A \cap B) \geq 3/4$

$\Rightarrow P(A) + P(B) - 1/8 \geq 3/4$

[since minimum value of $P(A \cap B)$ is 1/8]

$\Rightarrow P(A) + P(B) \leq 1/8 + 3/4 = 7/8$

As the maximum value of $P(A \cap B)$ is 3/8, we get

$1 \geq P(A) + P(B) - 3/8$

$\Rightarrow P(A) + P(B) \leq 1 + 3/8 = 11/8$

206 **(b)**

Clearly both are correct but statement 2 is not the correct explanation for statement 1

207 **(b)**

E =Event of getting the sum 8
$= \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$
and F =Event of getting the even numbers on both the dice
$= \{(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)\}$
$\therefore P(E) = 5$ and $P(F) = 9$

Also, $n(S) = 36$

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{5}{36}$

and $P(F) = \frac{n(F)}{n(S)} = \frac{9}{36} = \frac{1}{4}$

$\therefore$ Neither I nor II is true

208 **(c)**

$n(S) = {}^{20}C_4$

Statement I

Common difference is 1; total number of cases $= 17$
Common difference is 2; total number of cases $= 14$
Common difference is 3; total number of cases $= 11$
Common difference is 4; total number of cases $= 8$
Common difference is 5; total number of cases $= 5$
Common difference is 6; total number of cases $= 2$
Hence, required probability

$$= \frac{17 + 14 + 11 + 8 + 5 + 2}{{}^{20}C_4} = \frac{1}{85}$$

209 **(d)**

$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$ (by definition)

$\Rightarrow P(\overline{B}) = P\left(\left(A \cup \overline{A}\right) \cap \overline{B}\right)$
$= P(\left(A \cap \overline{B}\right) \cup \left(\overline{A} \cap \overline{B}\right))$

Hence, statement 2 is true. Now,

$$P(A/\bar{B}) + P(\bar{A}/\bar{B}) = \frac{P(A \cap \overline{B})}{P(\overline{B})} + \frac{P(\overline{A} \cap \overline{B})}{P(\overline{B})}$$

$$= \frac{P\left(A \cap \overline{B}\right) + P\left(\overline{A} \cap \overline{B}\right)}{P\left(\overline{B}\right)}$$

$$= \frac{P\left(\overline{B}\right)}{P\left(\overline{B}\right)} = 1$$

Therefore, statement 1 is false

211 **(c)**

According to statement 1, the required probability is

$${}^nC_0\left(\frac{1}{2}\right)^n + {}^nC_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{n-4} + {}^nC_8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^{n-8} + \cdots$$

$$= ({}^nC_0 + {}^nC_4 + {}^nC_8 + \cdots)\left(\frac{1}{2}\right)^n$$

Now consider the binomial expansion,

$$(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \ldots$$

Putting $x + i$, where $i = \sqrt{-1}$, we get

$$(1+i)^n = ({}^nC_0 - {}^nC_2 + {}^nC_4 - \cdots) + i({}^nC_1 - {}^nC_3 + {}^nC_5 - \ldots)$$

$$\Rightarrow \left[\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]^n = ({}^nC_0 - {}^nC_2 + {}^nC_4 - \cdots) + i({}^nC_1 - {}^nC_3 + {}^nC_5 - \ldots)$$

$$\Rightarrow {}^nC_0 - {}^nC_2 + {}^nC_4 - \cdots = 2^{n/2}\cos\frac{n\pi}{4}$$

Also we know that

$${}^nC_0 + {}^nC_2 + {}^nC_4 - \cdots = 2^{n-1}$$

$$\Rightarrow 2({}^nC_0 + {}^nC_4 + {}^nC_8 + \cdots) = 2^{n-1} + 2^{n/2}\cos\frac{n\pi}{4}$$

Hence, the required probability is

$$\frac{1}{4} + \frac{1}{2^{n/2+1}}\cos\left(\frac{n\pi}{4}\right)$$

212 **(a)**

$\because P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$\geq P(A) + P(B) - 1$

$\therefore P(P \cap B) \geq \frac{3}{5} + \frac{2}{3} - 1$

$\Rightarrow P(A \cap B) \geq \frac{4}{15}$ (1)

$P(A \cap B) \leq P(A)$

$\Rightarrow P(A \cap B) \leq \frac{3}{5}$ (2)

From Eqs. (1) and (2),

$\frac{4}{15} \leq P(A \cap B) \leq \frac{3}{5}$ (3)

From (3),

$$\frac{4}{15P(B)} \leq \frac{P(A \cap B)}{P(B)} \leq \frac{3}{5P(B)}$$

$$\Rightarrow \frac{2}{5} \leq P\left(\frac{A}{B}\right) \leq \frac{9}{10}$$

213 **(b)**

Since, $\frac{(x-10)(x-50)}{x-30} > 0$

$\Rightarrow 10 < x < 30$ or $x > 50$

$\therefore x = 11, \ldots, 29$ or $x = 51, 52, \ldots, 100$

$n(x) = 69$

$\therefore$ Required probability $= \frac{69}{10} = 0.69$

214 **(a)**

Here, $p = q = \frac{1}{2}$

Probability of appearing exactly five heads

$$= {}^{12}C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^7$$

$$= {}^{12}C_{12-5}\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^5$$

$$= {}^{12}C_7 = \left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^5$$

= Probability of appearing exactly seven heads.

215 **(c)**

$P(A) + P(B) = 1$ is true as A and B are mutually exclusive and exhaustive events, but statement 2 is false as it is not given that he events are exhaustive

216 **(a)**
$2n+1=5, n=2$

$$P(E)=\frac{3n}{4n^2-1}=\frac{6}{15}=\frac{2}{5}$$

As a, b, c are in A.P., so

$a+c=2b$

$\Rightarrow a+c$ is even

Therefore, a and c are both even or both odd. So, the number of ways of choosing a and c is ${}^nC_2+{}^{n+1}C_2=n^2$

$$\therefore P(E)=\frac{n^2}{{}^{2n+1}C_3}=\frac{3n}{4n^2-1}$$

218 **(c)**
The required probability is

$P(\overline{A}\cap\overline{B})=1-P(A\cup B)$

$=1-[P(A)+P(B)-P(A\cap B)]$

$=0.36$

220 **(a)**
In binomial theorem, we have proved statement 2. Now, there may be 0,1,2, 3,4 or 5 heads in the last five throws and the same can be for the first 10 throws. The number of cases thus may be given by

$$m={}^5C_0\,{}^{10}C_0+{}^5C_1\,{}^{10}C_1+{}^5C_2\,{}^{10}C_2+{}^5C_3\,{}^{10}C_3+{}^5C_4\,{}^{10}C_4+{}^5C_5\,{}^{10}C_5$$

$$={}^5C_0\,{}^{10}C_{10}+{}^5C_1\,{}^{10}C_9+{}^5C_2\,{}^{10}C_8+{}^5C_3\,{}^{10}C_7+{}^5C_4\,{}^{10}C_6+{}^5C_5\,{}^{10}C_5$$

$={}^{10+5}C_{10}={}^{15}C_{10}$

$=3003$

The total number of ways (N) is $2^{15}=32768$. Hence, the required probability is $m/N=3003/32768$

221 **(d)**
Statement I If $P(H_1\cap E)=0$for some i, then

$$P\left(\frac{H_i}{E}\right)=P\left(\frac{E}{H_i}\right)=0$$

If $P(H_i\cap E)\neq 0$ for $\forall\ i=1,2\ldots.,n$, then

$$P\left(\frac{H_i}{E}\right)=\frac{P(H_i\cap E)}{P(H_i)}\times\frac{P(H_i)}{P(E)}$$

$$=\frac{P\left(\frac{E}{H_i}\right)\times P(H_i)}{P(E)}>P\left(\frac{E}{H_i}\right).P(H_i)$$

[as$0<P(E)<1$]

Hence, statement I may not always be true.

Statement II

Clearly, $H_1\cup H_2\cup\ldots.\cup H_n=S$

[sample space]

$\Rightarrow P(H_1)+P(H_2)+\ldots+P(H_n)=1$

222 **(c)**
Statement 1 is the true as there are six equally likely possibilities of which only two are favourable (4 and 6). Hence, probability that the obtained number is complete is 2/6=1/3

Statement 2 is not true, as the three possibilities are not equally likely

223 **(c)**
(A)Given $P\left(\frac{E_2}{E_1}\right)=\frac{1}{2}$

$$\Rightarrow\frac{P(E_2\cap E_1)}{P(E_1)O}=\frac{1}{2}$$

$$\Rightarrow P(E_2\cap E_1)=\frac{1}{8}$$

Also, $P\left(\frac{E_1}{E_2}\right)=\frac{1}{4}$

$$\Rightarrow\frac{P(E_1\cap E_2)}{P(E_2)}=\frac{1}{4}$$

$$\Rightarrow\frac{1}{8P(E_2)}=\frac{1}{4}$$

$$\Rightarrow P(E_2)=\frac{1}{2}$$

(B) $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{5}{8}$

(C) $P\left(\frac{\overline{E_1}}{\overline{E_2}}\right) = \frac{P(\overline{E_1} \cap \overline{E_2})}{P(E_2)}$

$= \frac{1 - P(E_1 \cup E_2)}{1 - P(E_2)} = \frac{1 - \frac{5}{8}}{1 - \frac{1}{2}}$

$= \frac{3}{4}$

(D) $P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cap \overline{E_2})}{P(\overline{E_2})}$

$= \frac{P(E_1) - P(E_1 \cap E_2)}{1 - P(E_2)}$

$= \frac{\frac{1}{4} - \frac{1}{8}}{1 - \frac{1}{2}} = \frac{\frac{1}{8}}{\frac{1}{2}} = \frac{1}{4}$

224 **(c)**

1. $P(\text{success}) = 1/2; P(\text{failure}) = 1/2$

Suppose 'n' bombs are to dropped. Let E be the event that the bridge is destroyed. Then,
$P(E) = 1 - P(0 \text{ or } 1 \text{ succes})$
$= 1 - \left(\left(\frac{1}{2}\right)^n + {}^nC_1 \frac{1}{2}\left(\frac{1}{2}\right)^{n-1}\right) = 1 - \left(\frac{1}{2^n} + \frac{n}{2^n}\right)$
≥ 0.9
$\Rightarrow \frac{1}{10} \geq \frac{n+1}{2^n}$ or $\frac{2^n}{10(n+1)} \geq 1$

2. The bag contains 2 red, 3 white and 5 black balls. Hence

$P(S) = 1/5; P(F) = 4/5$; Let E be the even of getting a red ball

$P(E) = P(S \text{ or } FS \text{ or } FFS \text{ or } \dots) \geq \frac{1}{2}$

$\therefore P(F)^n \leq \frac{1}{2}; \left(\frac{4}{5}\right)^n \leq \frac{1}{2}$

The value of n consistent is 4

3. Let there be x red socks and y blue socks and $x > y$. Then

$\frac{{}^xC_2 + {}^yC_2}{{}^{x+y}C_2} = \frac{1}{2}$

or $\frac{x(x-1) + y(y-1)}{(x+y)(x+y-1)} = \frac{1}{2}$

Multiplying both sides by $2(x+y)(x+y-1)$ and expanding, we get

$2x^2 - 2x + 2y^2 - 2y = x^2 + 2xy + y^2 - x - y$

Rearranging, we have

$x^2 - 2xy + y^2 = x + y$

$\Rightarrow (x-y)^2 = x + y$

$\Rightarrow |x - y| = x + y$

Now, $x + y \leq 17$

$x - y \leq \sqrt{17}$

As $x - y$ must be an integer, so

$x - y = 4$

$\therefore x + y = 16$

Adding both together and dividing by 2 yields
$x \leq 10$

4. Let the number of green socks be $x > 0$. Let E: be the event that two socks drawn are of the same colour

$P(E) = P(RR \text{ or } BB \text{ or } WW \text{ or } GG)$

$= \frac{3}{{}^{6+x}C_2} + \frac{{}^xC_2}{{}^{6+x}C_2}$

$= \frac{6}{(x+6)(x+5)} + \frac{x(x-1)}{(x+6)(x+5)}$

$= \frac{1}{5}$

$\Rightarrow 5(x^2 - x + 6) = x^2 + 11x + 30$

$\Rightarrow 4x^2 - 16x = 0$

$\Rightarrow x = 4$

225 **(b)**

1. $\frac{^rC_2}{^{r+b}C_2} = \frac{1}{2} 2r(r-1) = (r-b)(r+b-1)$

$= 2r(r-1) = (r+b)(r+b-1)$

$\Rightarrow\ 2r^2 - 2r = r^2 + (2b-1)r + b^2 - 1$

$\Rightarrow r^2 - (1+2b)r + 1 - b^2 = 0$

$\Rightarrow b^2 + 2br + r - r^2 - 1 = 0$

$\Rightarrow b = \frac{-2r \pm \sqrt{4r^2 - 4(r - r^2 - 1)}}{2}$

$= -r \pm \sqrt{2r^2 - r + 1}$

Since b is integer, possible values of r are 3 and 8

2. $^4C_2 \left(\frac{r}{r+10}\right)^2 \left(\frac{10}{r+10}\right)^2 = \frac{3}{8}$

3. $\left(\frac{r}{r+10}\right)^2 \left(\frac{10}{r+10}\right)^2 = \frac{1}{16}$

$\Rightarrow r = 10$

4. Probability of getting exactly n red balls in $2n$ draws is always equal to probability of getting exactly n black balls in $2n$ draws for any value of r and b, hence the ratio r/b can be 10, 3, 8, 2

226 **(a)**

Let E_i denote the event that the bag contains i black and $(12-i)$ white balls $(i = 0,1,2,\dots,12)$ and A denote the event that the four balls drawn are all black. Then

$P(E_i) = \frac{1}{13} (i = 0,1,2,\dots,12)$

$P\left(\frac{A}{E_i}\right) = 0$ for $i = 0,1,2,3$

$P\left(\frac{A}{E_i}\right) = \frac{^iC_4}{^{12}C_4}$ for $i \geq 4$

1. $P(A) = \sum_{i=0}^{12} P(E_i)\, P\left(\frac{A}{E_i}\right)$

$= \frac{1}{13} \times \frac{1}{^{12}C_4} [\,^4C_4 + \,^5C_4 + \dots + \,^{12}C_4]$

$= \frac{^{13}C_5}{13 \times \,^{12}C_4} = \frac{1}{5}$

2. Clearly,

$P\left(\frac{A}{E_{10}}\right) = \frac{^{10}C_4}{^{12}C_4} = \frac{14}{33}$

3. By Bayer's theorem,

$P\left(\frac{E_{10}}{A}\right) = \frac{P(E_{10})\left(\frac{A}{E_{10}}\right)}{P(A)}$

$= \frac{\frac{1}{13} \times \frac{14}{33}}{\frac{1}{5}} = \frac{70}{429}$

4. Let B denote the probability of drawing 2 white and 2 black balls. Then

$P\left(\frac{B}{E_i}\right) = 0$ if $i = 0, 1$ or $11, 12$

$P\left(\frac{B}{E_i}\right) = \frac{^iC_2 \times \,^{12-i}C_2}{^{12}C_4}$ for $i = 2, 3, \dots, 10$

$\therefore\ P(B) = \sum_{i=0}^{12} P(E_i)\, P\left(\frac{B}{E_i}\right)$

$= \frac{1}{13} \times \frac{1}{^{12}C_4} [2\{\,^2C_2 \times \,^{10}C_2 + \,^3C_2 + \dots + \,^{10}C_2 \times \,^2C_2\}]$

$= \frac{1}{13} \times \frac{1}{^{12}C_4} [2\{\,^2C_2\,^{10}C_2 + \,^3C_2\,^9C_2 + \dots + \,^5C_2 \times \,^7C_2\} + \,^6C_2 \times \,^6C_2]$

$= \frac{1}{13} \times \frac{1}{495} (1287)$

$= \frac{1}{5}$

227 **(b)**

1. Suppose the coin is tossed n times. The probability of getting head or tail is 1/2. The probability of not getting any head in n tosses is $(1/2)^n$. The probability of getting at least one head is $1 - (1/2)^n$.

Now given that

$1 - (1/2)^n \geq 0.8$

$\Rightarrow \left(\frac{1}{2}\right)^n \leq 0.2$

$\Rightarrow 2^n \geq 5$

Therefore, the least value of n is3

2. The total number of mappings is n^n. The number of one-one mappings is $n!$ Hence the probability is

$\frac{n!}{n^n} = \frac{3}{32} = \frac{6}{64} = \frac{3!}{4^3} = \frac{4!}{4^4}$

Comparing. We get $n = 4$

3. Given equation is

$2x^2 + 2mx + m + 1 = 0$

$D = 4m^2 - 8(m + 1) \geq 0$

$m^2 - 2m - 2 \geq 0$

$(m - 1)^2 - 3 \geq 0$

$\Rightarrow m = 3, 4, 5, 6, 7, 8, 9, 10$

Also, the number of ways of choosing m is 10. Therefore, the required probability is 4/5

$\therefore\ 5k = 4$

4. $20P^2 - 13p + 2 \leq 0$

$\Rightarrow (4P - 1)(5P - 2) \leq 0$

$\Rightarrow \frac{1}{4} \leq P \leq \frac{2}{5}$

$\Rightarrow \frac{1}{4} \leq \frac{1}{5} + \frac{1}{5}\left(\frac{4}{5}\right) + \frac{1}{5}\left(\frac{4}{5}\right)^2 + \ldots + \frac{1}{5}\left(\frac{4}{5}\right)^{n-1} \leq \frac{2}{5}$

$\Rightarrow n = 2$

Hence, maximum as well as minimum value of n is 2

228 **(c)**

The required event will occur if last digit in all the chosen numbers is 1, 3, 7 or 9. Therefore, the required probability is $(4/10)^n$

The required probability is equal to the probability that the last digit is 2, 4, 6, 8 and is given by P(last digit is 1, 2, 3, 4, 6, 7, 8, 9)$-P$ (last digit is 1, 3, 7, 9)$= \frac{8^n - 4^n}{10^n}$

$P(1, 3, 5, 7, 9) - P(1, 3, 7, 9) = \frac{5^n - 4^n}{10^n}$

The required probability is

$$P(0, 5) - P(5) = \frac{(10^n - 8^n) - (5^n - 4^n)}{10^n}$$

$$= \frac{10^n - 8^n - 5^n + 4^n}{10^n}$$

229 **(a)**

$$P(A) = \frac{{}^4C_1\,{}^8C_2}{{}^{12}C_3} = \frac{428}{220} = \frac{112}{220} = \frac{28}{55}$$

$$P(B) = \frac{{}^4C_3\,{}^8C_3}{{}^{12}C_3} = \frac{4 + 56}{220} = \frac{60}{220} = \frac{3}{11}$$

$P(C) = P(WBB\ or\ BWB\ \ or\ WWB\ or\ BBB)$

$$= \frac{8}{12} \times \frac{4}{11} \times \frac{3}{10} \times \frac{4}{12} \times \frac{8}{11} \times \frac{3}{10} \times \frac{8}{12} \times \frac{7}{11} \times \frac{4}{10} \times \frac{4}{12} \times \frac{3}{11} \times \frac{2}{10}$$

$$= \frac{96 + 96 + 224 + 24}{12 \times 110} = \frac{440}{12 \times 110} = \frac{1}{3}$$

$A \cap B = \phi\ \Rightarrow A$ and B are mutually exclusive

$$P(B \cap C) = P(BBB) = \frac{4 \times 3 \times 2}{12 \times 110} = \frac{1}{55}$$

Also,

$$P(B)P(C) = \frac{3}{11} \times \frac{1}{3} = \frac{1}{11}$$

Hence, B and $C$are neither independent nor mutually exclusive

$$P(C \cap A) + P(WWB)\frac{8 \times 7 \times 4}{12 \times 11 \times 10} = \frac{28}{3 \times 55}$$

$P(C)P(A) = \frac{1}{3} \times \frac{112}{220} = \frac{28}{3 \times 55}\ \Rightarrow C$ and A are independent

Also, A, B, C are mutually exclusive as A and B are mutually exclusive

230 **(d)**

1. When no box remain empty, then

$n(S) = 3^6 - {}^3C_1 2^6 + {}^3C_2 = 3(243 - 64 + 1)$

$= 540$

When each box contains equal number of balls, then

$n(E) = \dfrac{6!}{2!\,3!} 3! = 90$

Therefore, the required probability is 90/540=1/6

2. The required probability is

$\dfrac{3^6 - {}^3C_1 2^6 + {}^3C_2}{3^6} = \dfrac{20}{27}$

3. Let A be the event that A is throwing sum of 9 and B be the event that B throws a number greater than that thrown by A. We have to find $P(B/A) = P(A \cap B)/P(A) = P(B)$ (as A and B are independent). The probability that is throwing dice so that sum is higher than 9 is

$P(B) = P((4,6)$ or $(6,4)$ or $(5,5)$

or $(6,5)$ or $(5,6)$ or $(6,6))$

$= \dfrac{6}{36} = \dfrac{1}{6}$

4. $P(A \cup \overline{B}) = P(A) + P(\overline{B}) - P(A \cap \overline{B})$

$= P(A) + P(\overline{B}) - P(A)P(\overline{B})$

$\Rightarrow\ 0.8 = 0.3 + P(\overline{B}) - 0.3P(\overline{B})$

$\Rightarrow\ 0.5 = 0.7\,P(\overline{B})$

$\Rightarrow P(\overline{B}) = \dfrac{5}{7}$

$\Rightarrow P(B) = 1 - \dfrac{5}{7} = \dfrac{2}{7}$

231 **(d)**

We have,

$P(A \cap B) = P(A)P(B) = \dfrac{1}{12}$

1. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{3} + \frac{1}{4} - \frac{1}{12} = \frac{1}{2}$

2. $P\left(\frac{A}{A \cup B}\right) = \frac{P(A)}{P(A \cup B)} = \frac{2}{3}$

3. $(C)P\left(\frac{B}{A' \cap B'}\right) = \frac{P(B \cap (A' \cap B'))}{P(A' \cap B')} = \frac{P(\phi)}{P(A' \cap B')} = 0$

4. $P\left(\frac{A'}{B}\right) = \frac{P(A' \cap B)}{P(B)} = P(A') = \frac{2}{3}$

232 **(b)**

Here, $p = \frac{1}{4}, q = 1 - \frac{1}{4} = \frac{3}{4}$

Since, 1−(Probability of not hitting the target) $> \frac{2}{3}$

$\Rightarrow 1 - {}^nC_n \left(\dfrac{3}{4}\right)^n > \dfrac{2}{3}$

or $\left(\frac{1}{3}\right) > \left(\frac{3}{4}\right)^n$

Hence, minimum value of n is 4.

233 **(b)**

Here, $P(u_i) = K_i, \sum P(u_i) = 1$

$\Rightarrow K = \dfrac{2}{n(n+1)}$

$\therefore \lim\limits_{n \to \infty} P(W) = \lim\limits_{n \to \infty} \sum\limits_{i=1}^{n} \dfrac{2i^2}{n(n+1)^2}$

$= \lim\limits_{n \to \infty} \dfrac{2n(n+1)(2n+1)}{6n(n+1)^2} = \dfrac{2}{3}$

234 **(c)**

Let $A = \{a_1, a_2, a_3, \dots a_{10}\}$

For each $a_i \in A (1 \le i \le 10)$ we have the following four choices

(i) $a_i \in P$ and $a_i \in Q$

(ii) $a_i \in P$ and $a_i \notin Q$

(iii) $a_i \notin P$ and $a_i \in Q$

(iv) $a_i \notin P$ and $a_i \notin Q$

Let S be the sample space and E be the favourable event.

$\therefore n(S) = 4^{10}$

$\because P \cup Q = A$

$\therefore n(E) = 3^{10}$ [$\because$ (iv) $\notin A$]

$\Rightarrow P(E) = \dfrac{n(E)}{n(S)} = \dfrac{3^{10}}{4^{10}} = \left(\dfrac{3}{4}\right)^{10}$

235 **(b)**

Let $P(i)$ be the probability that exactly i students are passing an examination. Now given that $P(A_i) = \lambda i^2$ (where λ is constant)

$\Rightarrow \sum\limits_{i=1}^{10} P(A_i) = \sum\limits_{i=1}^{10} \lambda i^2 = \lambda \dfrac{10 \times 11 \times 21}{6} = \lambda 385$

$= 1 \Rightarrow \lambda = 1/385$

Now, $P(5) = 25/385 = 5/77$

Let A represent the event that selected students have passes the examination

$$\therefore P(A)=\sum_{i=1}^{10} P(A/A_i)\,P(A_i)$$
$$=\sum_{i=1}^{10}\frac{i}{10}\frac{i^2}{385}$$
$$=\frac{1}{3850}\sum_{i=1}^{10} i^3$$
$$=\frac{10^2 11^2}{4\times 3850}=\frac{11}{14}$$
Now,
$$P(A_1/A)=\frac{P(A/A_1)P(A_1)}{P(A)}$$
$$=\frac{\frac{1}{385}\frac{1}{10}}{\frac{11}{14}}$$
$$=\frac{1}{11\times 55}\frac{1}{5}$$
$$=\frac{1}{3025}$$

236 **(c)**

Let in 8 coupons S, U, R, F appears x_1, x_2, x_3, x_4 times. Then $x_1+x_2+x_3+x_4=8$, where $x_1, x_2, x_3, x_4 > 0$

We have to find non-negative integral solutions of the equation. The total number of such solutions is ${}^{8+4-1}C_{4-1} = {}^{11}C_3 = 165$

If a person gets at least one free packet, then he must get each coupon at least once, which is equal to number of positive integral solutions of the equation. The number of such solutions is ${}^{8-1}C_{4-1} = {}^{7}C_3 = 35$. Then, the probability that he gets exactly one free packet is

$(35-1)/165 = 102/495$

The probability that he gets two free packets is $1/{}^{11}C_3 = 1/165$

237 **(d)**

Let p_1 be the probability of being an answer correct from section 1. Then $p_1 = 1/5$. Let p_2 be the probability of being an answer correct from section 2. Then $p_2 = 1/15$. Hence, the required probability is $1/5 \times 1/15 = 1/75$

238 **(c)**

x can be 2, 3, 4, 5, 6. The number of ways in which sum of 2, 3, 4, 5, 6 can occurs is given by the coefficients of x^2, x^3, x^4, x^5, x^6 in

$$(3x+2x^2+x^3)(x+2x^2+3x^3)$$
$$=3x^2+8x^3+14x^4+8x^5+3x^6$$

This shows that sum that occurs most often is 4

239 **(c)**

A: She gets a success

T: She studies 10 h : $P(T)=0.1$

S: She studies 7 h; $P(S)=0.2$

F: She studies 4 h ; $P(F)=0.7$

$P(A/T)=0.8,\ P(A/S)=0.6, P(A/F)=0.4$

$$P(A)=P(A\cap T)+P(A\cap S)+P(A\cap F)$$
$$=P(T)P(A/T)+P(S)P(A/S)+P(F)P(A/F)$$
$$=(0.1)(0.8)+(0.2)(0.6)+(0.7)(0.4)$$
$$=0.08+0.12+0.28=0.48$$
$$=P(F/A)=\frac{P(F\cap A)}{P(A)}$$
$$=\frac{(0.7)(0.4)}{0.48}$$
$$=\frac{0.28}{0.48}=\frac{7}{12}$$
$$P(F/\overline{A})=\frac{P(F\cap\overline{A})}{P(\overline{A})}$$
$$=\frac{P(F)-P(F\cap A)}{0.52}$$
$$=\frac{(0.7)-0.28}{0.52}$$
$$=\frac{0.42}{0.52}=\frac{21}{26}$$

240 **(b)**

$$P(S/T)=\frac{P(S\cap T)}{P(T)}$$
$$\Rightarrow\ 0.5=\frac{P(S\cap T)}{0.69}$$
$$\Rightarrow P(S\cap T)=0.5\times 0.69=P(S)P(T)$$

Therefore, S and T are independent

$\therefore\ P(S \text{ and } T)=P(S)P(T)$

$=0.69\times 0.5=0.345$

$P(S \text{ or } T)=P(S)+P(T)-P(S\cap T)$

$=0.5+0.69-0.345$

$=0.8450$

241 **(d)**

Let E be the event that all the amoeba population dies out:

E_1 be the event that after first second amoeba splits into two:

E_2 be the event that after first second amoeba remains the same. Then,

$$P(E)=P(E_1)P(E/E_1)+P(E_2)P(E/E_2)$$
$$=\frac{1}{2}\frac{1}{4}\frac{1}{4}+\frac{1}{4}\frac{1}{4}$$
$$=\frac{3}{32}$$

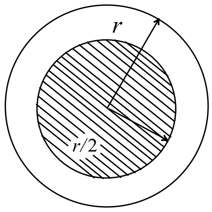
242 **(b)**

The number of cubes having at least one side painted is $9+9+3+3+1+1=26$. The number of cubes having two sides painted

$4+4+1+1+1=12$

243 **(a)**

For the favourable cases, the points should lie inside the concentric circle of radius $r/2$. So the desired probability is given by



$$\frac{\text{Area of smaller circle}}{\text{Area of larger circle}} = \frac{\pi\left(\frac{r}{2}\right)^2}{\pi r^2} = \frac{1}{4}$$

244 **(a)**

The total number of ways of painting first column when colours are not alternating is $2^8 - 2$. The total number of ways when no column has alternating colours is $(2^8 - 2)^8/2^{64}$

245 **(a)**

$P(A_2) = \frac{18}{36}$

$P(A_3) = \frac{12}{36} = \frac{1}{3}$

$P(A_4) = \frac{9}{36} = \frac{1}{4}$

$P(A_5) = \frac{7}{36}$

$P(A_6) = \frac{6}{36} = \frac{1}{6}$

Hence, A_3 is most probable

246 **(a)**

The scores of n can be reached in the following two mutually exclusive events:

1. By throwing a head when the score is $(n-1)$
2. By throwing a tail when the score is $(n-2)$

Hence,

$P_n = P_{n-1} \times \frac{1}{2} + P_{n-2} \times \frac{1}{2}$ $[\because P(\text{head}) = P(\text{tail}) =$ *1/2*

$\Rightarrow P_n = \frac{1}{2}[P_{n-1} + P_{n-2}]$ (1)

$$\Rightarrow P_0 + \frac{1}{2}P_{n-1} = P_{n-1} + \frac{1}{2}P_{n-2}$$

(adding $(1/2)P_{n-1}$ on both sides)

$$= P_{n-2} + \frac{1}{2}P_{n-3}$$

$\vdots$

$= P_2 + \frac{1}{2}P_1$ (2)

Now, a score of 1 can be obtained by throwing a head at a single toss

$$\therefore P_1 = \frac{1}{2}$$

And a score of 2 can be obtained by throwing either a tail at a single toss or a head at the first toss as well as second toss

$$\therefore P_2 = \frac{1}{2} + \left(\frac{1}{2} \times \frac{1}{2}\right) = \frac{3}{4}$$

From Eq. (2), we have

$$P_n = \frac{1}{2}P_{n-1} = \frac{3}{4} + \frac{1}{2}\left(\frac{1}{2}\right) = 1$$

$$\Rightarrow P_n = 1 - \frac{1}{2}P_{n-1}$$

$$\Rightarrow P_n - \frac{2}{3} = 1 - \frac{1}{2}P_{n-1} - \frac{2}{3}$$

$$\Rightarrow P_n - \frac{2}{3} = -\frac{1}{2}\left(P_{n-1} - \frac{2}{3}\right)$$

$$= \left(-\frac{1}{2}\right)^2\left(P_{n-2} - \frac{2}{3}\right)$$

$$= \left(-\frac{1}{2}\right)^3\left(P_{n-3} - \frac{2}{3}\right)$$

$$= \left(-\frac{1}{2}\right)^{n-1}\left(P_1 - \frac{2}{3}\right)$$

$$= \left(-\frac{1}{2}\right)^{n-1}\left(\frac{1}{2} - \frac{2}{3}\right)$$

$$= \left(-\frac{1}{2}\right)^{n-1}\left(-\frac{1}{6}\right)$$

$$= \left(-\frac{1}{2}\right)^{n}\frac{1}{3}$$

$$\Rightarrow P_n = \frac{2}{3} + \frac{(-1)^n}{2^n}\frac{1}{3} = \frac{1}{3}\left\{2 + \frac{(-1)^n}{2^n}\right\}$$

Now,

$$P_{100} = \frac{2}{3} + \frac{1}{3 \times 2^{100}} > \frac{2}{3} \text{ and } P_{101} = \frac{2}{3} - \frac{1}{3 \times 2^{101}} < \frac{2}{3}$$

$$\Rightarrow P_{101} < \frac{2}{3} < P_{100}$$

247 **(b)**

If a family of n children contains exactly k boys, then, by binomial distribution, its probability is

$${}^nC_k\left(\frac{1}{2}\right)^k\left(\frac{1}{2}\right)^{n-k}$$

Hence, by total probability law, the probability of a family of n children having exactly k boys is given by

$$\alpha p^n\, {}^nC_k\left(\frac{1}{2}\right)^k\left(\frac{1}{2}\right)^{n-k} \quad (\text{where } n \geq k)$$

Therefore, the required probability is

$$= \sum_{n=k}^{\infty} \alpha p^n\ {}^nC_k\left(\frac{1}{2}\right)^k\left(\frac{1}{2}\right)^{n-k}$$

$$= \alpha\left(\frac{1}{2}\right)^k p^k \sum_{n=k}^{\infty} {}^nC_k\left(\frac{1}{2}\right)^{n-k} (P^{n-k})$$

$$= \alpha\left(\frac{1}{2}\right)^k p^k\left[1 + {}^{(k+1)}C_1\left(\frac{p}{2}\right) + {}^{(k+2)}C_2\left(\frac{p}{2}\right)^2 + \ldots\right]$$

$$= \alpha\left(\frac{1}{2}\right)^k p^k\left(1 - \frac{p}{2}\right)^{-(k+1)}$$

$$= \alpha p^k(2-p)^{-(k+1)}$$

$$= \frac{2\alpha}{2-p}\left(\frac{p}{2-p}\right)^k, k \geq 1$$

248 **(4)**

Let event A:Card is of heart but not king (12 cards)

Event B: King but not heart (3 cards)

Event C: Heart and king (1 card)

∴ required probability

$$p = P(E) = \frac{{}^{12}C_1.\ {}^3C_1 +\ {}^3C_1.\ {}^1C_1 +\ {}^1C_1.\ {}^{12}C_1}{C_2}$$

$$= \frac{2}{52}$$

$\therefore^{104}\ p = 4$

249 **(6)**

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.6}{0.8} = \frac{3}{4}$$

(Maximum value of $P(A \cap B) = P(A) = 0.6$)

250 **(5)**

The number of ways of drawing 7 balls (second draws)= ${}^{10}C_7$

For each set of 7 balls of the second draw, 3 must be common to the set of 5 balls of the first draw, i.e., 2 other balls can be drawn in ${}^3C_2$ ways

Thus, for each set of 7 balls of the second draw, there are ${}^7C_3 \times {}^3C_2$ ways of making the first draw so that there are 3 balls common

Hence, the probability of having three balls in common is

$$\frac{{}^7C_3\ {}^3C_2}{{}^{10}C_7} = \frac{5}{12}$$

251 **(5)**

$P(n) = Kn^2$

Given $P(1) = K; P(2) = 2^2K; P(3) = 3^2K; P(4) = 4^2K; P(5) = 5^2K; P(6) = 6^2K$

∴ Total= $91\ K = 1$

$$\Rightarrow K = \frac{1}{91}$$

$\therefore P(1) = \frac{1}{91}; P(2) = \frac{4}{91}$ and so on

Let three events A, B, C are defined as

$A: a < b$

$B: a = b$

$C: a > b$

Bu symmetry, $P(A) = P(C)$. Also $P(A) + P(B) + P(C) = 1$

Since $P(B) = \sum_{i=1}^{6}[P(i)]^2$

$$= \left[\frac{1 + 16 + 81 + 256 + 625 + 1296}{91 \times 91}\right]$$

$$= \frac{2275}{91.91} = \frac{25}{91}$$

Now $2P(A) + P(B) = 1$

$$\Rightarrow P(A) = \frac{1}{2}[1 - P(B)] = \frac{33}{91}$$

252 **(7)**

Let the probability of the faces 1, 3, 5 or 6 be p for each face

Hence, probability of each of the faces 2 or 4 is $3p$

Now according to the question $4p + 9p = 1$

$$\Rightarrow p = \frac{1}{10}$$

$\therefore P(1) = P(3) = P(5) = P(6) = \frac{1}{10}$

And $P(2) = P(4) = \frac{3}{10}$

⇒ Required probability

$p = P$(total of 7 with a draw of dice)

$= P(16, 61, 25, 52, 43, 34)$

$$= 2\left(\frac{1}{10}.\frac{1}{10}\right) + 2\left(\frac{3}{10}.\frac{1}{10}\right) + 2\left(\frac{3}{10}.\frac{1}{10}\right)$$

$$\frac{2+6+6}{100} = \frac{14}{100} = \frac{7}{50}$$

253 **(5)**

Highest number in three throws 4

⇒ At least one of the throws must be equal to 4

Number of ways when three blocks are filled from $\{1, 2, 3, 4\} = 4^3$

∴ number of ways when filled from $\{1, 2, 3\} = 3^3$

∴ required number of ways $= 4^3 - 3^3$

∴ Probability $p = \frac{4^3-3^3}{6^3} = \frac{37}{216}$

254 **(2)**

When A and B are mutually exclusive,
$P(A \cap B) = 0$
$\therefore P(A \cup B) = P(A) + P(B)$ (1)
$\Rightarrow 0.8 = 0.5 + p \Rightarrow p = 0.3$ (2)
$P(A \cup B) = P(A) + P(B)$
$= P(A) + P(B) - P(A \cap B)$
$= P(A) + P(B) - P(A)P(B)$
$\Rightarrow 0.8 = 0.5 + q - (0.5)q$
$\Rightarrow 0.3 = q/2$
$\Rightarrow q = 0.6$
$\Rightarrow p/q = 2$ (3)

255 **(4)**
Total ways of distribution $= n(S) = 4^5$
Total ways of distribution so that each child get at least one game
$n(E) = 4^5 - {}^4C_1 3^5 + {}^4C_2 2^5 - {}^4C_3$
$= 1024 - 4 \times 243 + 6 \times 32 - 4 = 240$
Required probability $p = \frac{n(E)}{n(S)} = \frac{240}{4^5} = \frac{15}{64}$

256 **(3)**
For ranked 1 and 2 players to be winners and runners up, respectively, they should not be paired with each other in any round
$\Rightarrow p = \frac{30}{31} \times \frac{14}{15} \times \frac{6}{7} \times \frac{2}{3} = \frac{16}{31}$

257 **(1)**
There are n white balls in the turn
$\Rightarrow$ Probability of Mr. A to draw two balls of same colour is
$\frac{{}^3C_2 + {}^nC_2}{{}^{n+3}C_2} = \frac{1}{2}$ (given)
$\Rightarrow \frac{6 + n(n-1)}{(n+3)(n+2)} = \frac{1}{2}$
$\Rightarrow n^2 - 7n + 6 = 0$
$\Rightarrow n = 1$ or 6 (1)
Also required probability for Mr. B according to the question is
$\frac{3}{n+3}\frac{3}{n+3} + \frac{n}{n+3}\frac{n}{n+3} = \frac{5}{8}$ (given)
Solving, we get $n^2 - 10n + 9 = 0, n = 1$ or 9 (2)
From (1) and (2), $n = 1$

258 **(7)**
Let there be x red socks and y blue socks. Let $x > y$
Then, $\frac{{}^xC_2 + {}^yC_2}{{}^{x+y}C_2} = \frac{1}{2}$
$\Rightarrow \frac{x(x-1) + y(y-1)}{(x+y)(x+y-1)} = \frac{1}{2}$
$\Rightarrow 2x^2 - 2x + 2y^2 - 2y = x^2 + 2xy + y^2 - x - y$
$\Rightarrow x^2 - 2xy + y^2 = x + y$
$\Rightarrow (x-y)^2 = x + y$
$\Rightarrow |x - y| = (x+y)^{1/2}$
Since $x + y \le 17$, $x - y \le \sqrt{17}$
As $x - y$ must be an integer $\Rightarrow x - y = 4$
$\therefore x + y = 16$
Solving, we get $x = 7$

259 **(6)**
Let the two numbers are 'a' and 'b'
According to the question $a + b = 4p$ and $a - b = 4q$ where $p, q \in I$
$\Rightarrow 2a = 4(p+q)$ and $2b = 4(p-q)$
$\Rightarrow a = 2I_1$ and $b = 2I_2$
Hence, both $a$and b must be even
Also if $(a-b)$ is a multiple of 4 then $(a+b)$ will also be a multiple of 4
Hence, $n(S) = {}^{11}C_2$
$n(A) = (0,4), (0,8), (2,6), (2,10), (4,8), (6,10)$
$= 6$
$\therefore P(A) = \frac{6}{{}^{11}C_2} = \frac{6}{55}$

260 **(4)**
Let the number of green socks be $x > 0$
E: Two socks drawn are of the same color
$\Rightarrow P(E) = P(R\,R$ or $B\,B$ or $W\,W$ or $G\,G)$
$= \frac{3}{{}^{6+x}C_2} + \frac{{}^xC_2}{{}^{6+x}C_2}$
$= \frac{6}{(x+6)(x+5)} + \frac{x(x-1)}{(x+6)(x+5)} = \frac{1}{5}$ (given)
$\Rightarrow 5(x^2 - x + 6) = x^2 + 11x + 30$
$\Rightarrow 4x^2 - 16x = 0$
$\Rightarrow x = 4$

261 **(6)**
Total number of cases $n(S) = 6^3 = 216$
Product is prime only when two outcomes are 1 and the third is prime i.e., 2, 3, 5
If it is 2, 1, 1, then the number of cases is 3
Similarly, 3 cases for 3, 1, 1 and 5, 1, 1 each
Hence, favourable cases =9
Hence, required probability $p = \frac{1}{24}$
$\Rightarrow \frac{1}{4p} = 6$

262 **(2)**
Total number of cases $= n(S) = 6!$
Now sum the given digits is 1+2+3+4+5+6=21, which is divisible by 3
Now we have to form the number which is divisible by 6, then we have to ensure that the digit in unit place is even
$\Rightarrow$ Favourable cases $= n(A) = 3.5!$
Hence, $P(A) = \frac{3.5!}{6!} = \frac{1}{2}$