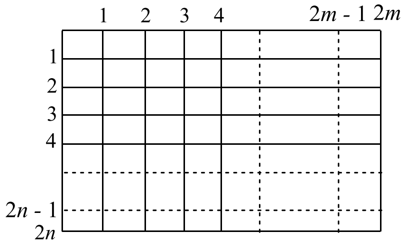
7.PERMUTATIONS AND COMBINATIONS

Single Correct Answer Type

1. The number of ways in which we can get a score of 11 by throwing three dice is
a) 18 b) 27 c) 45 d) 56
2. There are two bags each containing m balls. If a man has to select equals number of balls from both the bags the number of ways in which he can do so if he must chose at least one ball from each bag is
a) m^2 b) ${}^{2m}C_m$ c) ${}^{2m}C_m - 1$ d) none of these
3. Eight chairs are numbered 1 to 8. Two women and three men wish to occupy one chair each. First, the women choose the chairs from amongst the chairs marked 1 to 4, and then the men select the chairs from amongst the remaining. The number of possible arrangements is
a) ${}^6C_3 \times {}^4C_2$ b) ${}^4P_2 \times {}^4P_3$ c) ${}^4C_2 + {}^4P_3$ d) None of these
4. n is selected from the set $\{1, 2, 3, \dots, 10\}$ and the number $2^n + 3^n + 5^n$ is formed. Total number of ways of selecting n so that the formed number is divisible by 4 is equal to
a) 50 b) 49 c) 48 d) none of these
5. Let $A = \{x_1, x_2, x_3, \dots, x_7\}, B = \{y_1, y_2, y_3\}$. The total number of functions $f: A \to B$ that are on to and there are exactly three element x in A such that $f(x) = y_2$ is equal to
a) 490 b) 510 c) 630 d) none of these
6. Number of ways in which 25 identical things be distributed among five persons if each gets odd number of things is
a) ${}^{25}C_4$ b) ${}^{12}C_8$ c) ${}^{14}C_{10}$ d) ${}^{13}C_3$
7. The number of integral solutions of $x + y + z = 0$ with $x \geq -5, y \geq -5, z \geq -5$, is
a) 134 b) 136 c) 138 d) 140
8. The total number of five-digit numbers of different digits in which the digit in the middle is the largest is
a) $\sum_{n=4}^{9} {}^nP_4$ b) 33(3!) c) 30(3!) d) none of these
9. The total number of ways in which $2n$ persons can be divided into n couples is
a) $\frac{2n!}{n!n!}$ b) $\frac{2n!}{(2!)^n}$ c) $\frac{2n!}{n!(2!)^n}$ d) none of these
10. In how many different ways can the first 12 natural numbers be divided into three different groups such that numbers in each group are in A.P.?
a) 1 b) 5 c) 6 d) 4
11. The number of ways in which ten candidates $A_1, A_2, \dots, A_{10}$ can be ranked such that A_1 is always above A_{10} is
a) 5! b) 2(5!) c) 10! d) $\frac{1}{2}(10!)$
12. Among 10 persons, A, B, C are to speak at a function. The number of ways in which it can be done if A wants to speak before B and B wants to speak before C is
a) 10!/24 b) 9!/6 c) 10!/6 d) none of these
13. A team of four students is to be selected from a total of 12 students. The total number of ways in which the team can be selected such that two particular students refuse to be together and other two particular students which to be together only is equal to
a) 220 b) 182 c) 226 d) none of these
14. The number of nine-non-zero digits such that all the digits in the first four places are less than the digit in the middle and all the digits in the last four places are greater than that in the middle is
a) 2(4!) b) 3(7!)/2 c) 2(7!) d) ${}^4P_4 \times {}^4P_4$
15. Messages are conveyed by arranging four white, one blue and three red flags on a pole. Flags of the same colour are alike. If a message is transmitted by the order in which the colours are arranged, the total number of messages that can be transmitted if exactly six flags are used is

a) 45 b) 65 c) 125 d) 185

16. A rectangle with sides $2m-1$ and $2n-1$ is divided into squares of unit length by drawing parallel lines as shown in the diagram, then the number of rectangles possible with odd side lengths is



a) $(m+n-1)^2$ b) 4^{m+n-1} c) m^2n^2 d) $m(m+1)n(n+1)$

17. The number of triangles that can be formed with 10 points as vertices, n of them being collinear, is 110. Then n is

a) 3 b) 4 c) 5 d) 6

18. If all the permutations of the letters in the word 'OBJECT' are arranged (and numbered serially) in alphabetical order as in a dictionary, then the 717th word is

a) TOJECB b) TOEJBC c) TOCJEB d) TOJCBE

19. The maximum number of points of intersection of five lines and four circles is

a) 60 b) 72 c) 62 d) none of these

20. The number of different seven digit numbers that can be written using only the three digits 1,2 and 3 with the condition that the digit 2 occurs twice in each number is

a) ${}^7P_2\, 2^5$ b) ${}^7C_2\, 2^5$ c) ${}^7C_2\, 5^2$ d) none of these

21. $2m$ white counters and $2n$ red counters are arranged in a straight line with $(m+n)$ counters on each side of a central mark. The number of ways of arranging the counters, so that the arrangements are symmetrical with respect to the central mark, is

a) ${}^{m+n}C_m$ b) ${}^{2m+2n}C_{2m}$ c) $\frac{1}{2}\frac{(m+n)!}{m!\,n!}$ d) none of these

22. The number of even divisors of the number $N = 12600 = 2^3 3^2 5^2 7$ is

a) 72 b) 54 c) 18 d) none of these

23. The total number not more than 20 digits that are formed by using the digits 0,1,2,3 and 4 is

a) 5^{20} b) $5^{20}-1$ c) $5^{20}+1$ d) none of these

24. There are $(n+1)$ white and $(n+1)$ black balls each set numbered 1 to $n+1$. The number of ways in which the balls can be arranged in a row so that the adjacent balls are of different color is

a) $(2n+2)!$ b) $(2n+2)! \times 2$ c) $(n+1)! \times 2$ d) $2\{(n+1)!\}^2$

25. If m parallel lines in a plane are intersected by a family of n parallel lines, the number of parallelograms that can be formed is

a) $\frac{1}{4}mn(m-1)(n-1)$ b) $\frac{1}{2}mn(m-1)(n-1)$ c) $\frac{1}{4}m^2n^2$ d) none of these

26. A person predicts the outcome of 20 cricket matches of his home team. Each match can result in a either win, loss or tie for the home team. Total number of ways in which he can make the predictions so that exactly 10 predictions are correct is equal to

a) ${}^{20}C_{10} \times 2^{10}$ b) ${}^{20}C_{10} \times 3^{20}$ c) ${}^{20}C_{10} \times 3^{10}$ d) ${}^{20}C_{10} \times 2^{20}$

27. The number of ways in which we can distribute mn students equally among m sections is given by

a) $\frac{(mn)!}{n!}$ b) $\frac{(mn)!}{(n!)^m}$ c) $\frac{(mn)!}{m!\,n!}$ d) $(mn)^m$

28. The number of possible outcomes in a throw of n ordinary dice in which at least one of the dice shows an odd number is

a) 6^n-1 b) 3^n-1 c) 6^n-3^n d) none of these

29. The total number of ways of selecting six coins out of 20 one rupee coins, 10 fifty-paise coins and 7 twenty-five paise coins is

a) 28 b) 56 c) ${}^{37}C_6$ d) none of these

30. A seven-digit number without repetition and divisible by 9 is to be formed by using seven digits out of 1, 2, 3, 4, 5, 6, 7, 8, 9. The number of ways in which this can be done is

a) 9! b) 2(7!) c) 4(7!) d) none of these

31. In how many ways can 17 persons depart from railway station in 2 cars and 3 autos, given that 2 particular persons depart by same car (4 persons can sit in a car and 3 persons can sit in an auto)?

a) $\frac{15!}{2!4!(3!)^3}$ b) $\frac{16!}{(2!)^2 4!(3!)^3}$ c) $\frac{17!}{2!4!(3!)^3}$ d) $\frac{15!}{4!(3!)^3}$

32. The number of ways in which 12 books can be put in the three shelves with four on each shelf is

a) $\frac{12!}{(4!)^3}$ b) $\frac{12!}{(3!)(4!)^3}$ c) $\frac{12!}{(3!)^3 4!}$ d) none of these

33. Let A be a set of n (≥ 3) distinct elements. The number of triplets (x, y, z) of the A elements in which at least two coordinates is equal to

a) ${}^nP_3$ b) $n^3 - {}^nP_3$ c) $3n^2 - 2n$ d) $3n^2(n-1)$

34. A bag contains four one-rupee coins, two twenty-five paisa coins and five ten-paisa coins. In how many ways can an amount, not less then Re 1 be taken out from the bag? (consider coins of the same denominations to be identical)

a) 71 b) 72 c) 73 d) 80

35. The number of ways to give 16 different things to three persons A, B, C so that B gets one more than A and C gets two more than B, is

a) $\frac{16!}{4!\,5!\,7!}$ b) $4!\,5!\,7!$ c) $\frac{16!}{3!\,5!\,8!}$ d) none of these

36. A man has three friends. The number of ways he can invite one friend everyday for dinner on six successive nights so that no friend is invited more than three times is

a) 640 b) 320 c) 420 d) 510

37. There are 10 points in a plane of which no three points are collinear and four points are concyclic. The number of different circles that can be drawn through at least three points of these points is

a) 116 b) 120 c) 117 d) none of these

38. How many numbers can be made with the digits 3, 4, 5, 6, 7, 8 lying between 3000 and 4000, which are divisible by 5 while repetition of any digit is not allowed in any number?

a) 60 b) 12 c) 120 d) 24

39. The value of expression ${}^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3$ is equal to

a) ${}^{47}C_5$ b) ${}^{52}C_5$ c) ${}^{52}C_4$ d) None of these

40. The value of $\sum_{r=0}^{n-1} {}^nC_r / ({}^nC_r + {}^nC_{r+1})$ equals

a) $n + 1$ b) $n/2$ c) $n + 2$ d) none of these

41. There are three copies each of four different books. The number of ways in which they can be arranged in a shelf is

a) $\frac{12!}{(3!)^4}$ b) $\frac{12!}{(4!)^3}$ c) $\frac{21!}{(3!)^4 4!}$ d) $\frac{12!}{(4!)^3 3!}$

42. Total number of six-digit numbers that can be formed, having the property that every succeeding digit is greater than the preceding digit, is equal to

a) ${}^9C_3$ b) ${}^{10}C_3$ c) ${}^9p_3$ d) ${}^{10}p_3$

43. Total number of words that can be formed using all letters of the word 'BRIJESH' that neither begins with 'I' nor ends with 'B' is equal to

a) 3720 b) 4920 c) 3600 d) 4800

44. The number of five-digit numbers that contain 7 exactly once is

a) $(41)(9^3)$ b) $(37)(9^3)$ c) $(7)(9^4)$ d) $(41)(9^4)$

45. Total number less than 3×10^8 and can be formed using the digits 1, 2, 3 is equal to

a) $\frac{1}{2}(3^9 + 4 \times 3^8)$ b) $\frac{1}{2}(3^9 - 3)$ c) $\frac{1}{2}(7 \times 3^8 - 3)$ d) $\frac{1}{2}(3^9 - 3 + 3^8)$

46. The number of ways to fill each of the four cells of the table with a distinct natural number such that the sum of the numbers is 10 and the sums of the numbers placed diagonally are equal is

a) 4 b) 8 c) 24 d) 6

47. If $\alpha = {}^{m}C_2$, then ${}^{\alpha}C_2$ is equal to
a) ${}^{m+1}C_4$ b) ${}^{m-1}C_4$ c) $3\ {}^{m+2}C_4$ d) $3\ {}^{m+1}C_4$

48. The sum of all the numbers of four different digits that can be made by using the digits 0, 1, 2 and 3 is
a) 26664 b) 39996 c) 38664 d) none of these

49. Among the 8! Permutations of the digits $1,2,3,\ldots,8$, consider those arrangements which have the following property. If we take any five consecutive positions, the product of the digits in these positions is divisible by 5. The number of such arrangements is equal to
a) 7! b) 2. (7!) c) ${}^{7}C_4$ d) None of these

50. 20 persons are sitting in a particular arrangement around a circular table. Three persons are to be selected for leaders. The number of ways of selection of three persons such that no two were sitting adjacent to each other is
a) 600 b) 900 c) 800 d) none of these

51. The letters of word 'ZENITH' are written in all possible ways. If all these words are written in the order of a dictionary, then the rank of the word 'ZENITH' is
a) 716 b) 692 c) 698 d) 616

52. Let there be $n \geq 3$ circles in a plane. The value of n for which the number of radical centres is equal to the number of radical axes is (assume that all radical axes and radical centre exist and are different)
a) 7 b) 6 c) 5 d) None of these

53. In a class tournament, all participants were to play different games with one another. Two players fell ill after having played three games each. If the total number of games played in the tournament is equal to 84, the total number of participants in the beginning was equal to
a) 10 b) 15 c) 12 d) 14

54. There are four letters and four directed envelopes. The number of ways in which all the letters can be put in the wrong envelope is
a) 8 b) 9 c) 16 d) none of these

55. Ten IIT and 2 DCE students sit in a row. The number of ways in which exactly 3 IIT students sit between 2 DCE students is
a) ${}^{10}C_3 \times 2! \times 3! \times 8!$ b) $10! \times 2! \times 3! \times 8!$ c) $5! \times 2! \times 9! \times 8!$ d) none of these

56. A variable name in certain computer language must be either an alphabet or an alphabet followed by a decimal digit. The total number of different variable names that can exist in that language is equal to
a) 280 b) 290 c) 286 d) 296

57. A student is allowed to select at most n books from a collection of $(2n + 1)$ books. If the total number of ways in which he can select at least one book is 63, then the value of n is
a) 2 b) 3 c) 4 d) 5

58. Let T_n denote the number of triangles, which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 21$, then n equals
a) 5 b) 7 c) 6 d) 4

59. The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is
a) 55 b) 66 c) 77 d) 88

60. The number of ordered pairs of integers (x, y) satisfying the equation $x^2 + 6x + y^2 = 4$ is
a) 2 b) 8 c) 6 d) none of these

61. There were two women participating in a chess tournament. Every participant played two games with the other participates. The number of games that the men played among themselves proved to exceed by 66 number of games that the men played with women. The number of participants is
a) 6 b) 11 c) 13 d) none of these

62. A teacher takes three children from her class to the zoo at a time, but she does not take the same three

children to the zoo more than once. She finds that she went to the zoo 84 times more than a particular child has gone to the zoo. The number of children in her class is

a) 12 b) 10 c) 60 d) none of these

63. Number of ways in which a lawn-tennis mixed double be made from seven married couples if no husband and wife play in the same set is

a) 240 b) 420 c) 720 d) none of these

64. In how many ways can a team of 11 players be formed out of 25 players, if six out of them are always to be included and five always to be excluded

a) 2020 b) 2002 c) 2008 d) 8002

65. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then r is

a) 1 b) 2 c) 3 d) None of these

66. Number of ways in which Rs. 18 can be distributed amongst four persons such that nobody receives less than Rs. 3 is

a) 4^2 b) 2^4 c) 4! d) none of these

67. A person buys eight packets of TIDE detergent. Each packet contains one coupon, which bears one of the letters of the word TIDE. If he shows all the letters of the word TIDE, he gets one free packet. If he gets exactly one free packet, then the number of different possible combinations of the coupons is

a) ${}^7C_3$ b) ${}^8C_4$ c) ${}^8C_3$ d) 4^4

68. A five-digit number of divisible by 3 is to be formed using the digits 0, 1, 2, 3, 4 and 5, without repetition. The total number of ways this can be done is

a) 216 b) 240 c) 600 d) 3125

69. The total number of six-digit natural numbers that can be made with digits 1,2,3,4, if all digits are to appear in the same number at least once is

a) 1560 b) 840 c) 1080 d) 480

70. The number less than 1000 that can be formed using the digits 0,1,2,3,4,5, when repetition is not allowed is equal to

a) 130 b) 131 c) 156 d) 155

71. In a polygon, no three diagonals are concurrent. It the total number of points of intersection of diagonal interior to the polygon is 70, then the number of diagonals of the polygon is

a) 20 b) 28 c) 8 d) none of these

72. The number of words of four letters containing equal number of vowels and consonants, where repetition is allowed, is

a) 105^2 b) 210×243 c) 105×243 d) 150×21^2

73. If the difference of the number of arrangements of three things from a certain number of dissimilar things and the number of selections of the same number of things from them exceeds 100, then the least number of dissimilar things is

a) 8 b) 6 c) 5 d) 7

74. The last digit of $(1! + 2! + \cdots + 2005!)^{500}$ is

a) 9 b) 2 c) 7 d) 1

75. Two player P_1 and P_2 play a series of '$2n$' games. Each games can result in either a win or a loss for P_1. The total number of ways in which P_1 can win the series of these games is equal to

a) $\frac{1}{2}(2^{2n} - {}^{2n}C_n)$ b) $\frac{1}{2}(2^{2n} - 2 \times {}^{2n}C_n)$ c) $\frac{1}{2}(2^n - {}^{2n}C_n)$ d) $\frac{1}{2}(2^n - 2 \times {}^{2n}C_n)$

76. A class contains three girls and four boys, every Saturday, five go on a picnic (a different group of students is sent every week). During the picnic, each girl in the group is given a doll by the accompanying teacher. If all possible groups of five have gone for picnic once, the total number of dolls that the girls have got is

a) 21 b) 45 c) 27 d) 24

77. A person always prefers to eat 'parantha' and 'vegetable dish' in his meal. How many ways can he make his platter in a marriage party if there are three types of paranthas, four types of 'vegetable dish' three types of 'salads' and two types of 'sauces'?

a) 3360 b) 4096 c) 3000 d) none of these

78. Let $x_1, x_2, \ldots, x_k$ be the divisors of positive integer 'n' (including 1 and n). If $x_1 + x_2 + \ldots + x_k = 75$, then $\sum_{i=1} 1/x_i$ is equal to

a) $\frac{75}{n^2}$ b) $\frac{75}{n}$ c) $\frac{75}{k}$ d) none of these

79. Two teams are to play a series of five matches between them. A match ends in a win, loss or draw for team. A number of people forecast the result of each match and no two people make the same forecast for the series of matches. The smallest group of people in which one person forecasts correctly for all the matches will contain n people, where n is

a) 81 b) 243 c) 486 d) none of these

80. The sum of all four-digit number that can be formed by using the digits 2,4,6,8 (when repetition of digits is not allowed) is

a) 133320 b) 533280 c) 53328 d) none of these

81. In a room, there are 12 bulbs of the same wattage, each having a separate switch. The number of ways to light the room with different amount of illumination is

a) $12^2 - 1$ b) 2^{12} c) $2^{12} - 1$ d) 12^2

82. The number of ways of arranging m positive and $n(< m + 1)$ negative signs in a row so that no two negative signs are together is

a) ${}^{m+1}P_n$ b) ${}^{n+1}P_m$ c) ${}^{m+1}C_n$ d) ${}^{n+1}C_m$

83. In an election, the number of candidates is one greater than the persons to be elected. If a voter can vote in 254 ways, the number of candidates is

a) 7 b) 10 c) 8 d) 6

84. The number of three-digit numbers of the form xyz such that $x < y$ and $z \le y$ is

a) 276 b) 285 c) 240 d) 244

85. The total number of flags with three horizontal strips in order, which can be formed using 2 identical red, 2 identical green and 2 identical white strips is equal to

a) 4! b) $3 \times (4!)$ c) $2 \times (4!)$ d) none of these

86. The total number of times, the digit '3' will be written, when the integers having less than 4 digits are listed is equal to

a) 300 b) 310 c) 302 d) 306

87. The number of distinct natural numbers up to a maximum of four digits and divisible by 5, which can be formed with the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 each digit not occurring more than once in each number is

a) 1246 b) 952 c) 1106 d) none of these

88. Numbers greater than 1000 but not greater than 4000, which can be formed with the digits 0,1,2,3,4 (repetition of digits is allowed), are

a) 350 b) 375 c) 450 d) 576

89. The total number of ways of selecting two number from the set $\{1, 2, 3, 4, \ldots, 3n\}$ so that their sum is divisible by 3 is equal to

a) $\frac{2n^2-n}{2}$ b) $\frac{3n^2-n}{2}$ c) $2n^2 - n$ d) $3n^2 - n$

90. In how many ways can a team of six horses be selected out of a stud of 16, so that there shall always be three out of $A\ B\ C\ \ A'\ B'C'$, but never $A\ A'\ \ B\ B'$ or $C\ C'$ together

a) 840 b) 1260 c) 960 d) 720

91. The sum of the digits in the unit's place of all numbers formed with the help of 3,4,5,6 taken all at a time is

a) 18 b) 432 c) 108 d) 144

92. There are three coplanar parallel lines. If any p points are taken on each of the lines, the maximum number of triangles with vertices on these points is

a) $3p^2(p-1)+1$ b) $3p^2(p-1)$ c) $p^2(4p-3)$ d) none of these

93. The total number of ways in which 15 identical blankets can be distributed among four persons so that each of them gets at least two blankets is equal to

a) ${}^{10}C_3$ b) ${}^{9}C_3$ c) ${}^{11}C_3$ d) none of these

94. How many different nine-digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions?

a) 16 b) 36 c) 60 d) 180

95. The total number of three-letter words that can be formed from the letter of the word 'SAHARANPUR' is equal to

a) 210 b) 237 c) 247 d) 227

96. A is a set containing 'n' different elements. A subset P of A is chosen. The set A is reconstructed by replacing the elements of P. A subset Q of A is again chosen. The number of ways of choosing P and Q so that $P \cap Q$ contains exactly two element is

a) ${}^nC_3 \times 2^n$ b) ${}^nC_2 \times 3^{n-2}$ c) 3^{n-2} d) none of these

97. A train timetable must be compiled for various days of the week so that two trains twice a day depart for three days, one train daily for two days and three trains once a day for two days. How many different timetables can be compiled?

a) 140 b) 210 c) 133 d) 72

98. Three boys of class X, four boys of class XI and five boys of class XII sit in a row. The total number of ways in which these boys can sit so that all the boys of same class sit together is equal to

a) $(3!)^2(4!)(5!)$ b) $(3!)(4!)^2(5!)$ c) $(3!)(4!)(5!)$ d) $(3!)(4!)(5!)^2$

99. In a group of boys, two boys are brothers and six more boys are present in the group. In how many ways can they sit if the brothers are not to sit long with each other?

a) $2 \times 6!$ b) ${}^7P_2 \times 6!$ c) ${}^7C_2 \times 6!$ d) None of these

100. If n objects are arranged in a row, then the number of ways of selecting three of these objects so that no two of them are next to each other is

a) ${}^{n-2}C_3$ b) ${}^{n-3}C_2$ c) ${}^{n-3}C_3$ d) none of these

101. Total number of six-digit numbers in which all and only odd digits appear is

a) $\frac{5}{2}(6!)$ b) $6!$ c) $\frac{1}{2}(6!)$ d) none of these

102. In a three-storey building, there are four rooms on the ground floor, two on the first and two on the second floor. If the rooms are to be allotted to six persons, one person occupying one room only, the number of ways in which this can be done so that no floor remains empty is

a) ${}^8P_6 - 2(6!)$ b) ${}^8P_6$ c) ${}^8P_5(6!)$ d) none of these

103. If ${}^nC_3 + {}^nC_4 > {}^{n+1}C_3$, then

a) $n > 6$ b) $n > 7$ c) $n < 6$ d) none of these

104. If r, s, t are prime numbers and p, q are the positive integers such that LCM of p, q is $r^2s^4t^2$, then the number of ordered pairs (p, q) is

a) 252 b) 254 c) 225 d) 224

105. Let $f(n, k)$ denote the number of ways in which k identical balls can be coloured with n colours so that there is at least one ball of each color. Then $f(2n, n)$ must be equal to

a) ${}^{2n}C_n$ b) ${}^{2n-1}C_{n+1}$ c) ${}^{2n-1}C_n$ d) none of these

106. The total number of positive integral solution of $15 < x_1 + x_2 + x_3 \leq 20$ is equal to

a) 685 b) 785 c) 1125 d) none of these

107. In a certain test, there are n questions. In the test 2^{n-i} students gave wrong answers to at least i questions, where $i = 1, 2, \dots, n$. If the total number of wrong answers given is 2047, then n is equal to

a) 10 b) 11 c) 12 d) 13

108. The total number of ways in which n^2 number of identical balls can be put in n numbered boxes (1,2,3, ... n) such that i^{th} box contains at least i number of balls is

a) ${}^{n^2}C_{n-1}$ b) ${}^{n^2-1}C_{n-1}$ c) ${}^{\frac{n^2+n-2}{2}}C_{n-1}$ d) none of these

109. Fifteen identical balls have to be put in five different boxes. Each box can contain any number of balls. The total number of ways of putting the balls into the boxes so that each box contains at least two balls is equal to

a) ${}^9C_5$ b) ${}^{10}C_5$ c) ${}^6C_5$ d) ${}^{10}C_6$

110. Rajdhani express going from Bombay to Delhi stops at five intermediate stations, 10 passengers enter the train during the journey with 10 different ticket of two classes. The number of different sets of tickets they

may have is

a) ${}^{15}C_{10}$ b) ${}^{20}C_{10}$ c) ${}^{30}C_{10}$ d) none of these

111. A library has 'a' copies of one book, 'b' copies each of two books, 'c' copies each of three books, an single copy of 'd' books. The total number of ways in which these books can be arranged in a shelf is equal to

a) $\frac{(a+2b+3c+d)!}{a!\,(b!)^2\,(c!)^3}$ b) $\frac{(a+2b+3c+d)!}{a!\,(2b!)(c!)^3}$ c) $\frac{(a+b+3c+d)!}{(c!)^3}$ d) $\frac{(a+2b+3c+d)!}{a!\,(2b)!\,(3c)!}$

112. The letters of the word COCHIN are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word COCHIN is

a) 360 b) 192 c) 96 d) 48

113. To fill 12 vacancies there are 25 candidates of which five are from scheduled caste. If three of the vacancies are reserved for scheduled cast candidates while the rest are open to all, the number of ways in which the selection can be made is

a) ${}^{5}C_3 \times {}^{22}C_9$ b) ${}^{22}C_9 - {}^{5}C_3$ c) ${}^{22}C_3 + {}^{5}C_3$ d) none of these

114. The number of different ways in which five 'alike dashes' and eight 'alike dots' can be arranged using only seven of these 'dashes' and 'dots' is

a) 350 b) 120 c) 1287 d) none of these

115. $ABCD$ is a convex quadrilateral and 3, 4, 5 and 6 points are marked on the sides AB, BC, CD and DA, respectively. The number of triangles with vertices on different sides is

a) 270 b) 220 c) 282 d) 342

116. In a city no two persons have identical set of teeth and there is no person without a tooth. Also no person has more than 32 teeth. If we disregard the shape and size of tooth and consider only the positioning of the teeth, the maximum population of the city is

a) 2^{32} b) $(32)^2 - 1$ c) $2^{32} - 1$ d) 2^{32-1}

117. Two packs of 52 cards are shuffled together. The number of ways in which a man can be dealt 26 cards so that he does not get two cards of the same suit and same denomination is

a) ${}^{52}C_{26}.\, 2^{26}$ b) ${}^{104}C_{26}$ c) $2.\ {}^{52}C_{26}$ d) none of these

118. The total number of ways in which three distinct numbers in $A.P.$ can be selected from the set {1,2,3, ….24} is equal to

a) 66 b) 132 c) 198 d) none of these

119. In an examination of nine papers, a candidate has to pass in more papers than the number of papers in which he fails in order to be successful. The number of ways in which he can be unsuccessful is

a) 255 b) 256 c) 193 d) 319

120. The total number of divisors of 480, that are of the form $4n + 2, n \geq 0$, is equal to

a) 2 b) 3 c) 4 d) none of these

121. The number of ways in which we can select four numbers from 1 to 30 so as to exclude every selection of four consecutive number is

a) 27378 b) 27405 c) 27399 d) none of these

122. Ten different letters of an alphabet are given. Words with five letters are formed from these given letters. The number of words, which have at least one letter repeated is

a) 59720 b) 79260 c) 69760 d) None of these

123. In a group of 13 cricket players, four are bowlers. Find out in how many ways can they form a cricket team of 11 players in which at least 2 bowlers are included

a) 55 b) 72 c) 78 d) none of these

124. The number of words of four letters that can be formed from the letters of the word 'EXAMINATION' is

a) 1464 b) 2454 c) 1678 d) none of these

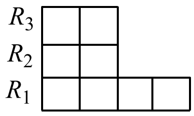
125. A candidate is required to answer six out of 10 questions, which are divided into two groups, each containing five questions. He is not permitted to attempt more than four questions from either group. The number of different ways in which the candidate can choose six questions is

a) 50 b) 150 c) 200 d) 250

126. The number of five-digit telephone numbers having at least one of their digits repeated is

a) 90000 b) 100000 c) 30240 d) 69760

127. The number of ways in which the letters of the word 'PERSON' can be placed in the squares of the given figure so that no row remains empty is



a) $24 \times 6!$ b) $26 \times 6!$ c) $26 \times 7!$ d) $27 \times 6!$

128. In the decimal system of numeration of six-digit numbers in which the sum of the digits is divisible by 5 is

a) 180000 b) 540000 c) 5×10^5 d) none of these

129. n lines are drawn in a place such that no two of them are parallel and no three of them are concurrent. The number of different points at which these lines will cut is

a) $\sum_{k=1}^{n-1} k$ b) $n(n-1)$ c) n^2 d) none of these

130. The number of ways of choosing a committee of two women and three men from give women and six men, if Mr. A refuses to serve on the committee if Mr. B is a member and Mr. B can only serve, if Miss C is the member of the committee is

a) 60 b) 84 c) 124 d) none of these

131. Straight lines are drawn by joining m points on a straight line to n points on another line. Then excluding the given points, the number of point of intersections of the lines drawn is (no two lines drawn are parallel and no three lines are concurrent)

a) $\frac{1}{4}mn(m-1)(n-1)$ b) $\frac{1}{2}mn(m-1)(n-1)$ c) $\frac{1}{2}m^2n^2$ d) $\frac{1}{4}m^2n^2$

132. An n-digit number is a positive number with exactly n digits. Nine hundred distinct n-digit numbers are to be formed using only the three digits 2, 5 and 7. The smallest value of n for which this is possible is

a) 6 b) 7 c) 8 d) 9

133. The number of four-digit numbers that can be made with the digits 1,2,3,4 and 5 in which at least two digits are identical is

a) $4^5 - 5!$ b) 505 c) 600 d) none of these

Multiple Correct Answers Type

134. If $10! = 2^p.3^q.5^r.7^s$, then

a) $2q = p$

b) $pqrs = 64$

c) Number of divisors of 10! is 280

d) Number of ways of putting 10! as a product of two natural numbers is 135

135. A is a set containing n elements. A subset P_1 of A is chosen. The set A is reconstructed by replacing the elements of P_1. Next, a subset P_2 of A is chosen and again the set is reconstructed by replacing the elements of P_2. In this way, $m(> 1)$ subsets $P_1, P_2, \dots, P_m$ of A are chosen. The number of ways of choosing $P_1, P_2, \dots, P_m$ is

a) $(2^m - 1)^n$ if $P_1 \cap P_2 \cap \dots \cap P_m = \phi$ b) 2^{mn} if $P_1 \cup P_2 \cup \dots \cup P_m = A$

c) 2^{mn} if $P_1 \cap P_2 \cap \dots \cap P_m = \phi$ d) $(2^m - 1)^n$ if $P_1 \cup P_2 \cup \dots \cup P_m = A$

136. Number of ways in which 30 identical things are distributed among six persons is

a) ${}^{17}C_5$ if each gets odd number of things

b) ${}^{16}C_{11}$ if each gets odd number of things

c) ${}^{14}C_5$ if each gets even number of things (excluding 0)

d) ${}^{15}C_{10}$ if each gets even number of things (excluding 0)

137. Number of ways of selecting three integers from $\{1, 2, 3, \dots, n\}$ if their sum is divisible by 3 is

a) $3\left({}^{n/3}C_3\right) + (n/3)^3$ if $n = 3k, k \in N$

b) $2\ ({}^{(n-1)/3}C_3) +\ ({}^{(n+2)3}C_3) + \left((n-1)/3\right)^2(n+2)$, if $n = 3k+1, k \in N$

c) $2\left({}^{(n-1)/3}C_3\right) + \left({}^{(n+2)/3}C_3\right) + \left((n-1)/3\right)^2 (n+2)$, if $n = 3k+2, k \in N$
d) Independent of n

138. Ten persons numbered 1, 2, … ,10 play a chess tournament, each player playing against every other player exactly one game. It is known that no game ends in a draw. If $w_1, w_2, \dots w_{10}$ are the number of games won by players 1, 2, 3, …, 10, respectively, and $l_1, l_2, \dots l_{10}$ are the number of games lost by the players 1, 2, …,10, respectively, then
a) $\sum w_i = \sum l_i = 45$
b) $w_i + 1_i = 9$
c) $\sum w l_1^2 = 81 + \sum l_1^2$
d) $\sum w_i^2 = \sum l_i^2$

139. If a seven-digit number made up of all distinct digits 8, 7, 6, 4, 3, x and y is divisible by 3, then
a) Maximum value of $x - y$ is 9
b) Maximum value of $x + y$ is 12
c) Minimum value of xy is 0
d) Minimum value of $x + y$ is 3

140. If n is number of necklaces which can be formed using 17 identical pearls and two identical diamonds and similarly m is number of necklaces which can be formed using 17 identical pearls and different diamonds, then
a) $n = 9$
b) $m = 18$
c) $n = 18$
d) $m = 9$

141. If N denotes the number of ways of selecting r objects out of n distinct objects $(r \geq n)$ with unlimited repetition but with each object included at least once in selection, then N is equal to
a) ${}^{r-1}C_{r-n}$
b) ${}^{r-1}C_n$
c) ${}^{r-1}C_{n-1}$
d) None of these

142. Given that the divisors of $n = 3^p.5^q.7^r$ are of the form $4\lambda + 1, \lambda \geq 0$. Then
a) $p + r$ is always even
b) $p + q + r$ is always odd
c) q can be any integer
d) If p is odd then r is even

143. A forecast is to be made of the results of five cricket matches, each of which can be a win or a draw or a loss for Indian team. Let,
p = number of forecasts with exactly 1 error
q = number of forecasts with exactly 3 errors and
r = number of forecasts with all five errors
Then the correct statement(s) is/are
a) $2q = 5r$
b) $8p = q$
c) $8p = 5r$
d) $2(p + r) > q$

144. Number of ways in which three numbers in AP. can be selected from 1,2,3, … n is
a) $\left(\frac{n-1}{2}\right)^2$ if n is even
b) $\frac{n(n-2)}{4}$ if n is even
c) $\frac{(n-1)^2}{4}$ if n is odd
d) None of these

145. Triplet (x, y, z) is chosen from the set $\{1, 2, 3, \dots n\}$ such that $x \leq y < z$. The number of such triplets is
a) n^3
b) ${}^{n+1}C_3$
c) ${}^nC_2$
d) ${}^nC_2 + {}^nC_3$

146. Let n is of four-digit integer in which all the digits are different. If x is number of odd integers and y is number of even integers, then
a) $x < y$
b) $x > y$
c) $x + y = 4500$
d) $|x - y| = 54$

147. Consider seven digit numbers $x_1, x_2, \dots . x_7$,where $x_1, x_2, \dots x_7 \neq 0$ having the property that x_4 is the greatest digits and digits towards the left and right of x_4 are in decreasing order. Then, total number of such number in which all digits are distinct, is
a) ${}^9C_7.{}^6C_3$
b) ${}^9C_6.{}^5C_3$
c) ${}^{10}C_7.{}^6C_3$
d) ${}^9C_2.{}^6C_3$

148. Number of ways in which 200 people can be divided in 100 couples is
a) $\frac{(200)!}{2^{100}(100)!}$
b) $1 \times 3 \times 5 \dots 199$
c) $\binom{101}{2}\binom{102}{2}\dots\binom{200}{2}$
d) $\frac{(200)!}{(100)!}$

149. If $P = 21(21^2 - 1^2)(21^2 - 2^2)(21^2 - 3^2) \dots (21^2 - 10^2)$, then P is divisible by
a) 22!
b) 21!
c) 19!
d) 20!

150. Two players P_1 and P_2 plays a series of $2n$ games. Each game can result in either a win or loss for P_1. Total number of ways in which P_1 can win the series of these games, is equal to
a) $\frac{1}{2}(2^{2n} - {}^{2n}C_n)$
b) $\frac{1}{2}(2^{2n} - 2.{}^{2n}C_n)$
c) $\frac{1}{2}(2^n - {}^{2n}C_n)$
d) None of these

151. Number of shortest ways in which we can reach from the point (0, 0, 0) to point (3, 7, 11) in space where the movement is possible only along the x-axis, y-axis and z-axis or parallel to them and change of axes is

permitted only at integral points (an integral point is one which has its coordinate as integer) is

a) Equivalent of number of ways of dividing 21 different objects in the three groups of size 3, 7, 11

b) Equivalent to coefficient of $y^3 z^7$ in the expansion of $(1+y+z)^{21}$

c) Equivalent to number of ways of distributing 21 different objects in three boxes of size 3, 7, 11

d) Equivalent to number of ways of arranging 21 objects of which 3 are alike one kind, 7 are alike of second type and 11 are alike of third type

152. The number of ways of arranging seven persons (having A, B, C anf D among them)in a row so that A, B, C and D are always in order $A-B-C-D$ (not necessarily together) is

a) 210 b) 5040 c) $6 \times {}^7C_3$ d) ${}^7P_3$

153. The number of ways of choosing triplet (x, y, z) such that $z \geq \max\{x, y\}$ and $x, y, z \in \{1,2, \dots n, n+1\}$ is

a) ${}^{n+1}C_3 + {}^{n+2}C_3$ b) $n(n+1)(2n+1)/6$ c) $1^2+2^2+\cdots+n^2$ d) $2({}^{n+2}C_3) - {}^{n+1}C_2$

154. Kanchan has 10 friends among whom two are married to each other. She wishes to invite five of them for a party. If the married couples refuse to attend separately, then the number of different ways in which she can invite five friends is

a) ${}^8C_5$ b) $2 \times {}^8C_3$ c) ${}^{10}C_5 - 2 \times {}^8C_4$ d) None of these

155. Total number of ways of giving at least one coin out of three 25 paise and two 50 paise coins to a beggar is

a) 32 b) 12 c) 11 d) ${}^{12}P_1 - 1$

156. Let $f(n)$ be the number of regions in which n coplanar circles can divide the plane. If it is known that each pair of circles intersect in two different point and no three of them have common point of intersection, then

a) $f(20) = 382$ b) $f(n)$is always an even number

c) $f^{-1}(92) = 10$ d) $f(n)$ can be odd

157. Number of points of intersection of n straight lines if n satisfies

${}^{n+5}P_{n+1} = \frac{11(n-1)}{2} \times {}^{n+3}P_n$ is

a) 15 b) 28 c) 21 d) 10

Assertion - Reasoning Type

This section contain(s) 0 questions numbered 158 to 157. Each question contains STATEMENT 1(Assertion) and STATEMENT 2(Reason). Each question has the 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

a) Statement 1 is True, Statement 2 is True; Statement 2 **is** correct explanation for Statement 1

b) Statement 1 is True, Statement 2 is True; Statement 2 **is not** correct explanation for Statement 1

c) Statement 1 is True, Statement 2 is False

d) Statement 1 is False, Statement 2 is True

158

Statement 1: Number of ways in which two persons A and B select objects from two different groups each having 20 different objects such that B selects always more objects than A (including the case when A selects no object) is $(2^{40} - {}^{40}C_{20})/2$

Statement 2: The sum $\sum_{0 \leq i < j \leq n} {}^nC_i \, {}^nC_j = (2^{2n} - {}^{2n}C_n)/2$

159

Statement 1: The number of positive integral solutions of $abc = 30$ is 27

Statement 2: Number of ways in which three prizes can be distributed among three persons is 3^3

160

Statement 1: The number of different car licence plates can be constructed if the licences contain three letters of the English alphabet followed by a three digit number is $(26)^3 \times (900)$(if repetitions are allowed).

Statement 2: The number of permutations of n different things taken r at a time when each things may be repeated any number of times is n^r

161

Statement 1: The sum of the digits in the tens place of all numbers formed with the help of 2, 3, 4, 5 taken all at a time is 84.

Statement 2: The sum of the digits in the units place of all numbers formed with the help of $a_1, a_2, \dots. a_n$taken all at a time is $(n-1)!\,(a_1 + a_2 + \dots + a_n)$ (repetition of digits is not allowed)

162

Statement 1: Number of rectangle on a chess board is ${}^8C_2 \times {}^8C_2$

Statement 2: To form a rectangle we have to select any two of the horizontal line and any two of the vertical line

163

Statement 1: Number of terms in the expansion of $(x + y + z + w)^{50}$ is ${}^{53}C_3$

Statement 2: Number of non-negative solution of the equation $p + q + r + s = 50$ is ${}^{53}C_3$

164

Statement 1: Number of ways in which India can win the series of 11 matches is 2^{10}.(if no match is drawn)

Statement 2: For each match there are two possibilities, either India wins or loses

165

Statement 1: Total number of five-digit numbers having all different digits and divisible by 4 can be formed using the digits $\{1, 3, 2, 6, 8, 9\}$ is 192

Statement 2: A number is divisible by 4, if the last two digits of the number are divisible by 4

166

Statement 1: A number of four different digit is formed with the help of the digits 1, 2, 3, 4, 5, 6, 7 in all possible ways. Then, number of ways which are exactly divisible by 4 is 200

Statement 2: A number divisible by 4 if unit place digit divisible by 4

167

Statement 1: If $p, q < r$, the number of different selections of $p + q$ things taking r at a time, where p things are identical and q other things are identical, is $p + q - r + 1$

Statement 2: If $p, q > r$, the number of different selections of $p + q$ things taking r at a time, where p things are identical and q other things are identical, is $r - 1$

168

Statement 1: $(n^2)!/(n!)^n$ is a natural number for all $n \in N$

Statement 2: Number of ways in which n^2objects can be distributed among n persons equally is $(n^2)!/(n!)^n$

169 In a shop there are five types of ice-creams available. A child buys six ice-creams

Statement 1: The number of different ways the child can buy the six ice-creams is $^{10}C_5$

Statement 2: The number of different ways the child can buy the six ice-creams is equals to the number of different ways of arranging 6 A's and B's in a row.

170

Statement 1: Number of ways in which 30 can be partitioned into three unequal parts, each part being a natural number is 61

Statement 2: Number of ways of distributing 30 identical objects in three different boxes is $^{30}C_2$

171

Statement 1: Number of ways in which Indian team (11 players) can bat, if Yuvraj wants to bat before Dhoni and Pathan wants to bat after Dhoni is 11!/3!

Statement 2: Yuvraj, Dhoni and Pathan can be arranged in batting order in 3! Ways

172

Statement 1: Number of zeros at the end of 50! Is equal to 12

Statement 2: Exponent of 2 in 50! is 47

173

Statement 1: The number of non-negative integral solutions of $x_1 + x_2 + x_3 + \ldots + x_n = r$ is $^{r+n-1}C_r$

Statement 2: The number of ways in which n identical things can be distributed into r different groups is $^{n+r-1}C_n$

174

Statement 1: The number of ways in which n persons can be seated at a round table, so that all shall not have the same neighbours in any two arrangements is $(n-1)!/2$

Statement 2: Number of ways of arranging n different beads in circles in which is $(n-1)!/2$

175

Statement 1: Number of ways of selecting 10 objects from 42 objects of which, 21 objects are identical and remaining objects are distinct is 2^{20}

Statement 2: $^{42}C_0 + {}^{42}C_1 + {}^{42}C_2 + \cdots + {}^{42}C_{21} = 2^{41}$

176

Statement 1: Number of ways in which 10 identical toys can be distributed among 3 students, if each receives atleast one toys is $^{9}C_2$

Statement 2: Number of positive integral solutions of $x + y + z + w = 7$ is $^{6}C_2$

177

Statement 1: When number of ways of arranging 21 objects of which r objects are identical of one type and remaining are identical of second type is maximum, then maximum value of $^{13}C_r$ is 78

Statement 2: $^{2n+1}C_r$ is maximum when $r = n$

178

Statement 1: From a group of 8 men and 4 women at team of 5, including at least one women can be formed in 736 ways

Statement 2: Number of ways of selecting at least one woman from m men and n women is ${}^{m+n}C_n - {}^{m}C_n$

179

Statement 1: If a, b, c are positive integers such that $a + b + c \leq 8$, then the number of possible values of the ordered triplets (a, b, c) is 56

Statement 2: The number of ways in which n identical things can be distributed into r different groups is ${}^{n-1}C_{r-1}$

180

Statement 1: The number of ways in which three distinct numbers can be selected from the set $\{3^1, 3^2, 3^3, \dots, 3^{100}, 3^{101}\}$ so that they form a G.P. is 2500

Statement 2: If a, b, c are in A.P., then $3^a, 3^b, 3^c$ are in G.P.

181

Statement 1: The number of ways of writing 1400 as a product of two positive integers is 12

Statement 2: 1400 is divisible by exactly three prime factors

182

Statement 1: Product of five consecutive natural numbers is divisible by 4!

Statement 2: Product of n consecutive natural numbers is divisible by $(n + 1)!$

Matrix-Match Type

This section contain(s) 0 question(s). Each question contains Statements given in 2 columns which have to be matched. Statements (A, B, C, D) in **columns I** have to be matched with Statements (p, q, r, s) in **columns II**.

183. Consider the convex polygon, which has 35 diagonals. Then match the following column

	Column-I		Column- II
(A)	Number of triangles joining the vertices of the polygon	(p)	210
(B)	Number of points of intersections of diagonal which lies inside the polygon	(q)	120
(C)	Number of triangles in which exactly one side is common with that of polygon	(r)	10
(D)	Number of triangles in which exactly two sides are common that of polygon	(s)	60

CODES :

	A	**B**	**C**	**D**
a)	p	q	r	s
b)	r	s	p	q
c)	q	p	s	r

d)	s	r	q	p

184.

	Column-I		Column- II
(A)	Four dice (six faced) are rolled The number of possible outcomes in which at least one dice shows 2 is	(p)	210
(B)	Let A be the set of 4-digit number $a_1a_2a_3a_4$ where $a_1 > a_2 > a_3 > a_4$. Then $n(A)$ is equal to	(q)	480
(C)	The total number of three-digit numbers, the sum of whose digits is even, is equal to	(r)	671
(D)	The number of four-digit numbers that can be formed from the digits $0, 1, 2, 3, 4, 5, 6, 7$ so that each number contains digit 1 is	(s)	450

CODES :

	A	**B**	**C**	**D**
a)	r	p	s	q
b)	p	s	q	r
c)	s	q	r	p
d)	q	r	p	s

185.

	Column-I		Column- II
(A)	Total number of function $f: \{1, 2, 3, 4, 5\} \rightarrow \{1, 2, 3, 4, 5\}$ that are on to and $f(i) \neq i$ is equal to	(p)	Divisible by 11
(B)	If $x_1x_2x_3 = 2 \times 5 \times 7^2$, then the number of solution set for (x_1, x_2, x_3) where $x_i \in N, x_i > 1$ is	(q)	Divisible by 7
(C)	Number of factors of 3780 are divisible by either 3 or 2 or both is	(r)	Divisible by 3
(D)	Total number of divisors of $n = 2^5 \times 3^4 \times 5^{10}$ that are of the form $4\lambda + 2, \lambda \geq 1$ is	(s)	Divisible by 4

CODES :

	A	**B**	**C**	**D**
a)	Q,r	p,s	r	p,s
b)	p,s	r	p,s	q,r
c)	r	p,s	q,r	p,s
d)	p,s	q,r	p,s	r

186. A function is defined as $f: \{a_1, a_2, a_3, a_4, a_5, a_6\} \rightarrow \{b_1, b_2 b_3\}$

	Column-I		Column- II
(A)	Number of subjective functions	(p)	Is divisible by 9
(B)	Number of functions in which $f(ai) \neq bi$	(q)	Is divisible by 5
(C)	Number of invertible functions	(r)	Is divisible by 4
(D)	Number of many one functions	(s)	Is divisible by 3
		(t)	Not possible

CODES :

	A	**B**	**C**	**D**
a)	p,r,s	q,r,s	s	p,q,r,s
b)	q,r,s	s	p,q,r,s	p,r,s
c)	s	p,q,r,s	p,r,s	q,r,s
d)	p,q,r,s	p,r,s	q,r,s	s

187.

	Column-I		Column- II
(A)	Number of straight lines joining any two of 10 points of which four points are collinear	(p)	30
(B)	Maximum number of points of intersection of 10 straight lines in the plane	(q)	60
(C)	Maximum number of points of intersection of six circles in the plane	(r)	40
(D)	Maximum number of points of intersection of six parabolas	(s)	45

CODES :

	A	**B**	**C**	**D**
a)	r	s	p	q
b)	p	q	r	s
c)	q	r	s	p
d)	s	p	q	r

188.

	Column-I		Column- II
(A)	If a denotes the number of permutations of $x + 2$ things taken all at a time, b the number of permutations of x things taken 11 at a time and c the number of permutations of $x - 11$	(p)	6

things taken all at a time such that $a = 182bc$, then the value of x is product of

(B)	The number of six-digit numbers that can be made with the digits 0, 1, 2, 3, 4 and 5 so that even digits occupy odd places is product of	(q)	5
(C)	The number of five-digit numbers that can be made using the digits 1 and 2 and in which at least one digit is different is product of	(r)	4
(D)	In a polygon the number of diagonals is 54. The number of sides of the polygon is product of	(s)	3

CODES :

	A	**B**	**C**	**D**
a)	R,s	r,s	p,r	p,q
b)	r,s	p,r	p,q	r,s
c)	p,r	p,q	r,s	r,s
d)	p,q	r,s	r,s	p,r

189.

	Column-I		**Column- II**
(A)	The number of five-digit numbers having the product of digits 20 is	(p)	(p)<70
(B)	A closest has five pairs of shoes. The number of ways in which four shoes can be drawn from it such that there will be no complete pair is	(q)	(q)<60
(C)	Three ladies have each brought their one child for admission to a school. The principal wants to interview the six persons one by one subject to the condition that no mother is interviewed before her child. The number of ways in which interviews can be arranged is	(r)	(r) $\in (50, 110)$
(D)	The figures 4, 5, 6, 7, 8 are written in every possible order. The number of numbers greater than 56000 is	(s)	(s) $\in (40, 70)$

CODES :

	A	**B**	**C**	**D**
a)	P,q	r,s	p,s	q,r
b)	r,s	p,s	q,r	p,q
c)	q,s	p,r	p,r	p,r
d)	p,s	q,r	p,r	q,s

190. Consider a 6×6 chessboard. Then match the following columns

	Column-I		Column- II
(A)	Number of rectangles	(p)	${}^{10}C_5$
(B)	Number of squares	(q)	441
(C)	Number of ways three squares can be selected if they are not in same row or column	(r)	91
(D)	In how many ways eleven '+' sign can be arranged in the squares if no row remains empty	(s)	2400

CODES :

	A	**B**	**C**	**D**
a)	r	s	p	q
b)	q	r	s	p
c)	p	q	r	s
d)	s	p	q	r

Linked Comprehension Type

This section contain(s) 18 paragraph(s) and based upon each paragraph, multiple choice questions have to be answered. Each question has atleast 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 191 to -191

If p is a prime, then exponent of p in $n!$ equal

$$E_P(n) = \left[\frac{n}{p}\right] + \left[\frac{n}{P^2}\right] + \ldots +$$

191. The number of zeros at the 100! is

a) 22 b) 23 c) 24 d) 26

Paragraph for Question Nos. 192 to - 192

Suppose a lot of n objects contains n_1 objects of one kind, n_2 objects of second kind, n_3 object of third kind,….,n_k, objects of kth kind. Such that $n_1 + n_2 + n_3 + \ldots + n_k = n$, then the number of possible arrangements/permutations of r objects out of this lot is the coefficient of x^r in the expansion of $r! \prod \left(\sum_{\lambda=0}^{n_1} \frac{x^\lambda}{\lambda!}\right)$

192. The number of permutations of the letters of the word INDIA taken three at a time must be

a) 27 b) 30 c) 33 d) 57

Paragraph for Question Nos. 193 to - 193

Different words are beging formed by arranging the letters of the word "SUCCESS". All the words obtained by written in the form of a dictionary

193. The number of words in which the two C are together but no two S are together, is

a) 120 b) 96 c) 24 d) 420

Paragraph for Question Nos. 194 to - 194

We have to choose 11 players for cricket team from eight batsmen, six bowlers, four all rounder and two wicket keepers in the following conditions

194. The number of selections when almost one all rounder and one wicket keeper will play

a) ${}^4C_1 \times {}^{14}C_{10} + {}^2C_1 \times {}^{14}C_{10} + {}^4C_1 \times {}^2C_1 \times {}^{14}C_9 + {}^{14}C_{11}$

b) ${}^4C_1 \times {}^{15}C_{11} + {}^{15}C_{11}$

c) ${}^4C_1 \times {}^{15}C_{10} + {}^{15}C_{11}$

d) None of these

Paragraph for Question Nos. 195 to - 195

Twelve 12 persons are to be arranged around two round tables such that one table can accommodate seven persons and another five persons only. Answer the following questions

195. Number of ways in which these 12 persons can be arranged is

a) ${}^{12}C_5 6!\,4!$ b) $6!\,4!$ c) ${}^{12}C_5 6!\,4!$ d) None of these

Paragraph for Question Nos. 196 to - 196

Five balls are to be placed in three boxes. Each box should hold all the five balls so that no box remains empty

196. Number of ways if balls are different but boxes are identical is

a) 30 b) 25 c) 21 d) 35

Paragraph for Question Nos. 197 to - 199

Let $f(n)$ denote the number of different ways in which the positive integer n can be expressed as the sum of 1s and 2s. For example, $f(4) = 5$, since $4 = 2 + 2 = 2 + 1 + 1 = 1 + 2 + 1 = 1 + 1 + 2 = 1 + 1 + 1 + 1$. Note that order of 1s and 2s is important

197. $f: N \to N$ is

a) One-one and onto b) One-one and into c) Many-one and onto d) Many-one and into

198. The value of $f(f(6))$ is

a) 400 b) 350 c) 377 d) None of these

199. The value of $f(6)$ is

a) 12 b) 13 c) 18 d) 21

Paragraph for Question Nos. 200 to - 202

There are m seats in the first row of a theatre, of which n are to be occupied

200. The number of ways of arranging n persons, if out of any two seats located symmetrically in the middle of the row at least one is empty is
a) $\left({}^{m/2}C_n\right)(2^n) - 1$ b) ${}^{m/2}P_n$ c) $\left({}^{m/2}P_n\right)(2^n - 1)$ d) $\left({}^{m/2}P_n\right)(2^n)$
201. If n is even, the number of ways of arranging n persons if each person has exactly one neighbor is
a) $\left({}^{n}P_{n/2}\right)\left({}^{m-n+1}P_{n/2}\right)$ b) $\left({}^{n}P_n\right)\left({}^{m-n+1}P_{n/2}\right)$ c) $\left({}^{n}P_{n/2}\right)\left({}^{m-n+1}P_n\right)$ d) None of these
202. The number of ways of arranging n persons if no two persons sit side by side is
a) $\frac{(m-n+1)!}{(m-3n+1)!}$ b) $\frac{(m-n+1)!}{(m-2n)!}$ c) $\frac{(m-n+1)!}{(m-2n+1)!}$ d) $\frac{(m-n+2)!}{(m-2n-1)!}$

Paragraph for Question Nos. 203 to - 205

Consider the letters of the word 'MATHEMATICS'

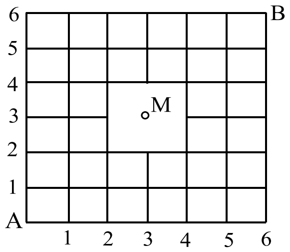
203. Possible number of words in which no two vowels are together is
a) $7!\ {}^8C_4 \frac{4!}{2!}$ b) $\frac{7!}{2!}\ {}^8C_4 \frac{4!}{2!}$ c) $\frac{7!}{2!\,2!}\ {}^8C_4 \frac{4!}{2!}$ d) $\frac{7!}{2!\,2!\,2!}\ {}^8C_4 \frac{4!}{2!}$
204. Possible number of words taking all letters at a time such that in each word both M's are together and both T's are together but both A's are not together is
a) $\frac{1\ 1!}{2!\,2!\,2!} - \frac{10!}{2!\,2!}$ b) $7!\ {}^8C_2$ c) $\frac{6!\,4!}{2!\,2!}$ d) $\frac{9!}{2!\,2!\,2!}$
205. Possible number of words taking all letters at a time such that at least one repeating letter is at odd position in each word is
a) $\frac{1\ 1!}{2!\,2!\,2!} - \frac{9!}{2!\,2!}$ b) $\frac{9!}{2!\,2!\,2!}$ c) $\frac{9!}{2!\,2!}$ d) $\frac{11!}{2!\,2!\,2!}$

Integer Answer Type

206. The number of three digit numbers having only two consecutive digits identical is N, then the value of $(N/2)^{1/2}$ is
207. A man has 3 friends. If N is number of ways he can invite one friend everyday for dinner on 6 successive nights so that no friend is invited more than 3 times then the value of $N/170$ is
208. A class has three teachers, Mr. P, Ms. Q and Mrs. R and six students A, B, C, D, E, F. Number of ways in which they can be seated in a line of 9 chairs, if between any two teachers there are exactly two students, is $k!(18)$, then the value of k is
209. The number of n digit numbers which consists of the digits 1 and 2 only if each digit is to be used at least once, is equal to 510 then n is equal to
210. Number of ways in which 7 people can occupy six seats, 3 seats on each side in a first class railway compartment if two specified persons are to be always included and occupy adjacent seats on the same side, is $(5!).k$ then k has the value equal to
211. There are 20 books on Albegra and Calculus in one library. For the greatest number of selections each of which consists of 5 books on each topic possible number of Algebra books are N then the value of $N/2$ is
212. There are n distinct white and n distinct black balls. If the number of ways of arranging them in a row so that neighboring balls are of different colors is 1152 then value of 'n' is
213. Number of ways in which 5 A's and 6 B's can be arranged in a row which reads the same backwards and

forwards, is N then value of $N/2$ is

214. If N is the number of ways in which a person can walk up a stairway which has 7 steps if he can take 1 or 2 steps up the stairs at a time, then the value of $N/3$ is
215. Let P_n denotes the number of ways in which three people can be selected out of 'n' people sitting in a row, if no two of them are consecutive. If, $P_{n+1} - P_n = 15$ then the value of 'n' is
216. If N is the number of different paths of length-12 which leads from A to B in the grid which do not pass through M, then the value of $[N/10]$, where [.] represents the greatest integer function, is



217. If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$ then the value of $n + r$ is
218. Number of 4 digit numbers of the form $N = abcd$ which satisfy following three conditions
    1. $4000 \le N < 6000$
    2. N is a multiple of 5
    3. $3 \le b < c \le 6$

    is equal to N then the value of $N/3$ is
219. There are 2 women participating in a chess tournament. Every participant played 2 games with the other participants. The number of games that the men played between themselves exceeded by 66 as compared to the number of games that the men played with the women. If the number of participants is n then the value of $n - 6$ is
220. Consider the five points comprising of the vertices of a square and the intersection point of its diagonals. How many triangles can be formed using these points?
221. A person has 'n' friends. The minimum value of 'n' so that a person can invite a different pair of friends every day for four weeks in a row is
222. Number of ways in which the letters of the word 'ABBCABBC can be arranged such that the word ABBC does not appear is any word, is N then the value of $(N^{1/2} - 10)$ is
223. If number of selections of 6 different letters that can be made from the words 'SUMAN' and 'DIVYA' so that each contains 3 letters from each word, is N^2 then the value of N is
224. There are 3 men and 7 women taking a dance class. If N is number of different ways in which each man be paired with a woman partner, and the four remaining women be paired into two pairs each of two, then the value of $N/90$ is
225. There are 720 permutations of the digits 1, 2, 3, 4, 5, 6. Suppose these permutations are arranged from smallest to largest numerical values, beginning from 1 2 3 4 5 6 and ending with 6 5 4 3 2 1. Then the digit in unit place of number at 267^{th} position is
226. If N is the number of ways in which 3 distinct numbers can be selected from the set $\{3^1, 3^2, 3^3, \dots \dots 3^{10}\}$ so that they form a G.P. then the value of $N/5$ is
227. Let $f(n) = \sum_{r=0}^{n} \sum_{k=r}^{n} \binom{k}{r}$. Find the total number of divisors of $f(9)$
228. Number of permutations of 1, 2, 3, 4, 5, 6, 7, 8 and 9 taken all at a time are such that the digit
    1 appearing somewhere to the left of 2
    3 appearing to the left of 4 and
    5 somewhere to the left of 6, is $k \times 7!$ then the value of k is
229. Numbers from 1 to 1000 are divisible by 60 but not by 24 is

: ANSWER KEY :

1)	b	2)	b	3)	d	4)	b	133)	b	1)	b,c,d	2)	a,d	3)	
5)	a	6)	c	7)	b	8)	d		a,c	4)	a,b,c				
9)	c	10)	d	11)	d	12)	c	5)	a,b,d	6)	a,b,c,d	7)	a,d	8)	
13)	c	14)	d	15)	d	16)	c		a,c						
17)	c	18)	d	19)	c	20)	b	9)	a,c	10)	a,b,d	11)	b,c	12)	
21)	a	22)	b	23)	a	24)	d		b,d						
25)	a	26)	a	27)	b	28)	c	13)	a,d	14)	a,d	15)	a,b,c	16)	
29)	a	30)	c	31)	d	32)	a		b,c,d						
33)	b	34)	c	35)	a	36)	d	17)	a	18)	a,b,d	19)	a,c,d	20)	
37)	c	38)	b	39)	c	40)	b		b,c,d						
41)	a	42)	a	43)	a	44)	a	21)	b,c	22)	c,d	23)	a,b,c	24)	
45)	c	46)	b	47)	d	48)	c		a,c						
49)	b	50)	c	51)	d	52)	c	1)	a	2)	a	3)	d	4)	a
53)	b	54)	b	55)	a	56)	c	5)	d	6)	a	7)	b	8)	a
57)	b	58)	b	59)	c	60)	b	9)	c	10)	c	11)	a	12)	d
61)	c	62)	b	63)	b	64)	b	13)	b	14)	a	15)	b	16)	a
65)	c	66)	d	67)	a	68)	a	17)	a	18)	c	19)	d	20)	d
69)	a	70)	b	71)	a	72)	d	21)	c	22)	a	23)	a	24)	b
73)	d	74)	d	75)	b	76)	b	25)	c	1)	c	2)	a	3)	d
77)	a	78)	b	79)	b	80)	a		4)	d					
81)	c	82)	c	83)	c	84)	a	5)	a	6)	b	7)	c	8)	b
85)	a	86)	a	87)	c	88)	b	1)	c	2)	c	3)	c	4)	a
89)	b	90)	c	91)	c	92)	c	5)	c	6)	b	7)	b	8)	c
93)	a	94)	c	95)	c	96)	b	9)	b	10)	d	11)	a	12)	c
97)	b	98)	a	99)	b	100)	a	13)	c	14)	b	15)	d	1)	9
101)	a	102)	a	103)	a	104)	c		2)	3	3)	6	4)	9	
105)	c	106)	a	107)	b	108)	c	5)	8	6)	5	7)	4	8)	5
109)	a	110)	c	111)	a	112)	c	9)	7	10)	8	11)	5	12)	5
113)	a	114)	b	115)	d	116)	c	13)	8	14)	7	15)	8	16)	8
117)	a	118)	b	119)	b	120)	c	17)	9	18)	8	19)	7	20)	6
121)	a	122)	c	123)	c	124)	b	21)	4	22)	8	23)	9	24)	8
125)	c	126)	d	127)	b	128)	a								
129)	a	130)	c	131)	b	132)	b								

: HINTS AND SOLUTIONS :

1 **(b)**

Dice is marked with numbers 1, 2, 3, 4, 5, 6. If the sum of dice in three throws is 11, then observations must be 1, 4, 6;…1, 5, 5;…2, 3, 6;…2, 4, 5;… 3, 3, 5; …3, 4, 4

We can get this observation in 3! + 3!/2! + 3! + 3! + 3!/2! + 3!/2! = 27 ways

2 **(b)**

The number of ways of selecting $r(0 \leq r \leq m)$ balls out of m is ${}^{m}C_r$. Therefore, the number of ways if selecting r balls from each of the bag is $({}^{m}C_r)^2$, Further the number of ways of selecting equal number of balls from each of the two bags, choosing at least one from each bag, is

$({}^{m}C_1)^2 + ({}^{m}C_2)^2 + \dots + ({}^{m}C_m)^2 = {}^{2m}C_m - 1$

$[\because ({}^{m}C_0)^2 + ({}^{m}C_1)^2 + \dots + ({}^{m}C_m)^2 = {}^{2m}C_m]$

3 **(d)**

$\overline{1}\overline{2}\overline{3}\overline{4}\overline{5}\overline{6}\overline{7}\overline{8}$

Two women can choose two chairs out of 1, 2, 3, 4 in ${}^{4}C_2$ ways, and can arrange among themselves in 2! ways. Three men can choose 3 chairs out of 6 remaining chairs in ${}^{6}C_3$ways and can arrange themselves in 3! ways

Therefore, total number of possible arrangements is ${}^{4}C_2 \times 2! \times {}^{6}C_3 \times 3! = {}^{4}P_2 \times {}^{6}P_3$

4 **(b)**

If n is odd

$3^n = 4\lambda_1 - 1, 5^n = 4\lambda_2 + 1$

$\Rightarrow 2^n + 3^n + 5^n$ is divisible by 4 if $n \geq 2$

Thus, $n = 3, 5, 7, 9, \dots, 99$, i.e. n can take 49 different values. If n is even. $3^n = 4\lambda_1 + 1, 5^n = 4\lambda_2 + 1$

$\Rightarrow 2^n + 3^n + 5^n$ is not divisible by 4

As $2^n + 3^n + 5^n$ will be in the form of $4\lambda + 2$

Thus, the total number of ways of selecting 'n' is equal to 49

5 **(a)**

Three elements from set 'A' can be selected in ${}^{7}C_3$ways. Their image has to be y_2. Remaining 2 images can be assigned to remaining 4 pre-images in 2^4 ways. But the function is onto, hence the number of ways is $2^4 - 2$. Then the total number of functions is ${}^{7}C_3 \times 14 = 490$

6 **(c)**

Let person P_i gets x_i number of things such that $x_1 + x_2 + x_3 + x_4 + x_5 = 25$

Lets $x_i = 2\lambda_i + 1$, where $\lambda_i \geq 0$. Then

$2(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5) + 5 = 25$

$\Rightarrow \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 = 10$

We have to simply obtain the number of non-negative integral solutions of this equation, which is equal to ${}^{14}C_4$

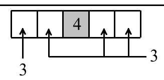
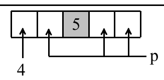
7 **(b)**

Let $x = p - 5, y = q - 5$ and $z = r - 5$, where $p, q, r \geq 0$

Then the given equation reduces to $p + q + r = 15$ (1)

Now, we have to find non-negative integral solution of Eq. (1). The total number of such solutions is ${}^{15+3-1}C_{3-1} = {}^{17}C_2 = 136$

8 **(d)**

Middle digit	Digits available for remaining four places	Pattern	Number of ways filling remaining four places
4	0, 1, 2, 3		$3 \times {}^{3}P_3$
5	0,1, … ,4		$4 \times {}^{4}P_3$
6	0,1, … ,5	⋯	$5 \times {}^{5}P_3$
7	0,1, … ,6	⋯	$6 \times {}^{6}P_3$
8	0,1, … ,7	⋯	$7 \times {}^{7}P_3$
9	0,1, … ,8	⋯	$8 \times {}^{8}P_3$

9 **(c)**

Here, we are dividing $2n$ people in n group of 2 each, and we are concerned with mere grouping. Hence, the required number of ways is $\frac{2n!}{n!(2!)^n}$

10 **(d)**

No group of four members from the first 12 natural number can have the common difference 4

If one group including 1 is selected with the common difference 1, then the other two group can have the common difference 1 or 2

If one group including 1 is selected with the common difference 2, then one of the other two groups can have the common difference 2 and the remaining group will have common difference 1

If one group including 1 is selected with common difference 3, then the other two group can have the common difference 3

Therefore, the required number of ways is

$2 + 1 + 1 = 4$

11 **(d)**

Two positions for A_1 and A_{10} can be selected in ${}^{10}C_2$ ways. Rest 8 students can be ranked in 8! ways. Hence total number of ways is ${}^{10}C_2 \times 8! = (1/2)\,(10!)$

12 **(c)**

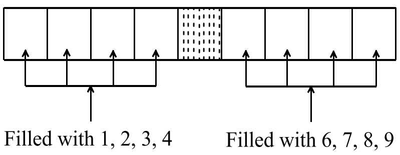
Places for A, B, C can be chosen in ${}^{10}C_3$ ways. Remaining 7 persons can speak in 7! ways. Hence, the number of ways in which they can speak is ${}^{10}C_3 \times 7! = 10!/6$

13 **(c)**

Let S_1 and S_2 refuse to be together and S_3 and S_4 want to be together only. The total number of ways when S_3 and S_4 are selected is $({}^{8}C_2 + {}^{2}C_1 \times {}^{8}C_1) = 44$. The total ways when S_3 and S_4 are not selected is $({}^{8}C_4 + {}^{2}C_1 \times {}^{8}C_3) = 182$. Thus, the total number of ways is $44 + 182 = 226$

14 **(d)**

According to given conditions, numbers can be formed by the following format:



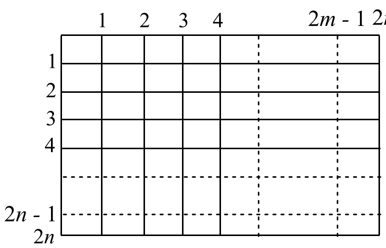
The required number of numbers is ${}^{4}P_4 \times {}^{4}P_4$

15 **(d)**

We will consider the following cases

Case	Flags	No. of signals
4 alike and 2 others alike	4 white and 2 red	$\frac{6!}{4!\,2!} = 15$
4 alike and 2 others different	4 white, 1 red and 1 blue	$\frac{6!}{4!} = 30$
3 alike and 3 others alike	3 white, 3 red	$\frac{6!}{3!\,3!} = 20$
3 alike and 2 other alike and 1 different	3 white, 1 blue, 2 red or 3red, 1 blue, 2 white	${}^{2}C_1 \times \frac{6!}{3!\,2!} = 120$
	Total	185

16 **(c)**



If we see the blocks in terms of lines, then there are $2m$ vertical lines and $2n$ horizontal lines

To form the required rectangle we must select two horizontal lines, one even numbered (out of $2, 4, \ldots, 2n$) and one odd numbered (out of $1, 3, \ldots, 2n - 1$) and similarly two vertical lines

The number of rectangles is ${}^{m}C_1 \times {}^{m}C_1 \times {}^{n}C_1 \times {}^{n}C_1 = m^2 n^2$

17 **(c)**

Out of 10 points let n points are collinear. Then the number of triangles is ${}^{10}C_3 - {}^{n}C_3 = 110$

$\Rightarrow \frac{10 \times 9 \times 8}{6} - \frac{n(n-1)(n-2)}{6} = 110$

$\Rightarrow n(n-1)(n-2) = 60$

$\Rightarrow n = 5$

18 **(d)**

The order of letters of the words 'OBJECT' is B C E J O T

Words starting with B can be formed in 5! Ways.

Words starting with C can be formed in 5! Ways.

Words starting with E can be formed in 5! Ways.

Words starting with J can be formed in 5! Ways.

Words starting with O can be formed in 5! Ways.

Words starting with TB can be formed in 4! Ways.

Words starting with TC can be formed in 4! Ways.

Words starting with TE can be formed in 4! Ways.

Words starting with TJ can be formed in 4! Ways.

Words starting with TOB can be formed in 3! Ways.

Words starting with TOC can be formed in 3! Ways.

Words starting with TOE can be formed in 3! Ways.

Words starting with TOJB can be formed in 2! Ways.

Words starting with TOJC can be formed in 2! Ways. Therefore, the total number of words is 718 words. Hence 717th word is TOJCBE

19 **(c)**

Two circles intersect at two distinct points. Two straight lines intersect at one point. One circle and one straight line intersect at two distinct points. Then the total numbers of points of intersections are as follows:

Number of ways of selection	Points of intersection
Two straight lines : ${}^{5}C_2$	${}^{5}C_2 \times 1 = 10$
Two circles : ${}^{4}C_2$	${}^{4}C_2 \times 2 = 12$
One line and one circle: ${}^{5}C_1 \times {}^{4}C_1$	${}^{5}C_1 \times {}^{4}C_1 \times 2 = 40$

Total	62

20 **(b)**
Other than 2, remaining five places can be filled by 1 and 3 for each place. The number of ways for five places is $2 \times 2 \times 2 \times 2 \times 2 = 2^5$. For 2, selecting 2 places out of 7 is ${}^7C_2$. Hence, the required number of ways is ${}^7C_2 \times 2^5$

21 **(a)**
$m + n$ counters on one side can be arranged in $\frac{(m+n)!}{m!n!}$ ways
For each arrangement on one side, corresponding arrangement on the other side is just one as arrangements are symmetrical. Hence, the total number of arrangements is $\frac{(m+n)!}{m!n!} = {}^{m+n}C_m$

22 **(b)**
Number of even divisors is equal to number of ways in which one or more '2', zero more '3', zero or more '5' and zero or more '7' can be selected, and is given by $(3)(2+1)(2+1)(1+1) = 54$

23 **(a)**
Each position can be filled in 5 ways. Hence, the total number of numbers is 5^{20}

24 **(d)**
Since the balls are to be arranged in a row so that the adjacent balls are of different colors, we can therefore begin with a white ball or a black ball. If we begin with a white ball. We find that $n+1$ white balls numbered 1 to $n+1$ can be arranged in a row in $(n+1)!$ ways. Now $n+2$ places are created among $n+1$ white balls which can be filled by $n+1$ black balls in $(n+1)!$ ways
So, the total number of arrangements in which adjacent balls are of different colors and first ball is a white ball is $(n+1)! \times (n+1)! = [(n+1)!]^2$. But we can begin with a black ball also. Hence, the required number of arrangements is $2[(n+1)!]^2$

25 **(a)**
The number of selection of two parallel lines from m lines is ${}^mC_2$
The number of selection of two parallel lines from n lines is ${}^nC_2$
Hence, the number of parallelograms lines is

$${}^mC_2 \times {}^nC_2 = \frac{1}{4}mn(m-1)(n-1)$$

26 **(a)**
Matches whose predictions are correct can be selected in ${}^{20}C_{10}$ ways. Now each wrong prediction can be made in 2 ways. Thus, the total number of ways is ${}^{20}C_{10} \times 2^{10}$

27 **(b)**
The number of ways is

$$\frac{(mn)!}{(n!)^m m!}m! = \frac{(mn)!}{(n!)^m}$$

28 **(c)**
The total number of ways is $6 \times 6 \times \ldots$ to n times= 6^n. The total number of ways to show only even numbers is $3 \times 3 \times \ldots$ to n times= 3^n. Therefore, the required number of ways is $6^n - 3^n$

29 **(a)**
Since the total number of selections of r things from n things where each thing can be repeated as many times as one can is ${}^{n+r-1}C_r$. Therefore the required number is ${}^{3+6-1}C_6 = 28$

30 **(c)**
Sum of 7 digits is a multiple of 9. Sum of numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 is 45; so two left digits should also have sum of 9. The pairs of left numbers are $(1,8), (2,7), (3,6), (4,5)$. With each pair left number of 7-digit number is 7! So with all 4 pairs, total number is $4 \times 7!$

31 **(d)**
Make 1 group of 2 persons, 1 group of 4 persons and 3 groups of 3 persons among 15 persons (except 2 particular persons). Hence the number of ways by grouping method is $\frac{15!}{2!4!(3!)^3 3!}$
Now we add that 2 persons in the group of 2 persons and thus number of arrangements of these groups into cars autos is

$$\frac{15!}{2!\,4!\,(3!)^3 3!} \times 2! \times 3! = \frac{15!}{4!\,(3!)^3}$$

32 **(a)**
Since the shelves which are to receive the books are different, therefore the required number of ways is $12!/(4!)^3$

33 **(b)**
Total number of triplets without restriction is $n \times n \times n$. The number of triplets with all different coordinates is ${}^nP_3$

34 **(c)**
When at least one one-rupee coin is selected we can select any number of twenty five coins and ten paisa coins. Then number of ways of such selection is $4(2+1)(5+1) = 72$ as we can select zero or more twenty five paisa and ten paisa coins to ensure that amount selected is Re. 1 or more
But when none of one-rupee coins is selected we have to select all twenty five paisa coins and ten paisa coins to ensure sum of Re. 1, which can be done only in one way. Then the total number of

ways is 73

35 **(a)**

Obviously, A, B and C get 4, 5 and 7 objects, respectively. Then, number of distribution ways is equal to number of division of ways, which is given by 16!/(4! 5! 7!)

36 **(d)**

Let x, y, z, be the friends and a, b, c denote the case when x is invited a times, y is invited b times and z is invited c times. Now, we have the following possibilities:

$(a, b, c) = (1, 2, 3)$ or (3 3 0) or (2 2 2)

[grouping of 6 days of week]

Hence, the total number of ways is

$$\frac{6!}{1!\,2!\,3!}3! + \frac{6!}{3!\,3!\,2!}3! + \frac{6!}{(2!\,2!\,2!)3!}3!$$

$= 360 + 60 + 90 = 510$

37 **(c)**

Number of points required for the fixed circle is 3. So, first select any three points from the 10 points in ${}^{10}C_3$ ways.

In these ways, circle with four concyclic points is selected in ${}^4C_3$ ways. But is should be taken once then total number of circles is $({}^{10}C_3 - {}^4C_3) + 1$

38 **(b)**

3 must be at thousand's place and since the number should be divisible by 5, or 5 must be at unit's place. Now, we have to fill two places (tens and hundreds), i.e., ${}^4P_2 = 12$

39 **(c)**

$${}^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3$$

$= {}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{47}C_3 + {}^{47}C_4$

[Using ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$]

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + ({}^{48}C_3 + {}^{48}C_4)$

$= {}^{51}C_3 + {}^{50}C_3 + ({}^{49}C_3 + {}^{49}C_4)$

$= {}^{51}C_3 + {}^{51}C_4$

$= {}^{52}C_4$

40 **(b)**

$$\sum_{r=0}^{n-1} \frac{{}^nC_r}{{}^nC_r + {}^nC_{r+1}}$$

$$= \sum_{r=0}^{n-1} \frac{1}{1 + \frac{{}^nC_{r+1}}{{}^nC_r}}$$

$$= \sum_{r=0}^{n-1} \frac{1}{1 + \frac{n-r}{r+1}}$$

$$= \sum_{r=0}^{n-1} \frac{r+1}{n+1} = \frac{1}{n+1}\sum_{r=0}^{n-1} (r+1)$$

$$= \frac{1}{(n+1)}[1 + 2 + \cdots + n] = \frac{n}{2}$$

41 **(a)**

Here, we have to divide 12 books into sets 3 books each. Therefore the required number of ways is

$\frac{12!}{(3!)^4 4!} \times 4!$

42 **(a)**

$x_1 < x_2 < x_3 < x_4 < x_5 < x_6$, when the number is $x_1x_2x_3x_4x_5x_6$. Clearly no digit can be zero. Also, all the digits are distinct. So, let us first select six digits from the list of digits 1, 2, 3, 4, 5, 6, 7, 8, 9 which can be done in ${}^9C_6$ ways.

After selecting these digits they can be put only in one order. Thus, total number of such numbers is ${}^9C_6 \times 1 = {}^9C_3$

43 **(a)**

Total number of words without any restriction is 7!

Total number of words beginning with I is 6!

Total number of words ending with B is 6!

Total number of words beginning with I and ending with B is 5!

Thus the total number of required words is

$7! - 6! - 6! + 5! = 7! - 2(6!) + 5!$

44 **(a)**

If 7 is used at first place, the number of numbers is 9^4 and for any other four places it is 8×9^3

45 **(c)**

Formed number can be utmost of nine digits.

Total number of such numbers is

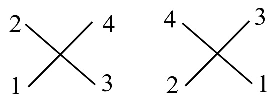
$3 + 3^2 + 3^3 + \cdots + 3^8 + 2 \times 3^8$

$$= \frac{3(3^8 - 1)}{3 - 1} + 2 \times 3^8 = \frac{3^9 - 3 + 4 \times 3^8}{2}$$

$$= \frac{7 \times 3^8 - 3}{2}$$

46 **(b)**

The natural numbers are 1, 2, 3, 4. Clearly, in one diagonal we have to place 1, 4 and in the other 2, 3



(i) (ii)

The number of ways in (i) is $2! \times 2! = 4$

The number of ways in (ii)is $2! \times 2! = 4$

Therefore, the total number of ways is 8

47 **(d)**

$\alpha = {}^mC_2 \Rightarrow \alpha = \frac{m(m-1)}{2}$

$\therefore\ {}^\alpha C_2 = \frac{\alpha(\alpha-1)}{2}$

$= \frac{1}{2}\frac{m(m-1)}{2}\left\{\frac{m(m-1)}{2} - 1\right\}$

$= \frac{1}{8}m(m-1)(m-2)(m+1)$

$= \frac{1}{8}(m+1)m(m-1)(m-2) = 3\ {}^{m+1}C_4$

48 **(c)**

The number of numbers with 0 in the unit's place is $3! - 2! = 4$. Therefore the sum of the digits in the unit's place is $6 \times 0 + 4 \times 1 + 4 \times 2 + 4 \times 3 = 24$

Similarly, for the ten's and hundred's places, the number of numbers with 1 or 2 in the thousand's place is 3! Therefore, the sum of the digits in the thousand's place is $6 \times i + 6 \times 2 + 6 \times 3 = 36$

Hence, the required sum is $36 \times 1000 + 24 \times 100 + 24 \times 10 + 24$

49 **(b)**

Let the arrangement be $x_1x_2x_3x_4x_5x_6x_7x_8$ and clearly 5 should occupy the position x_4 or x_5. Thus required number is $2(7!)$

50 **(c)**

The total number of ways of selection without restriction is ${}^{20}C_3$. The number of ways of selection when two are adjacent is $20\ \times {}^{16}C_1$. The number of ways of selection when all the three are adjacent is 20. The required number of ways is

${}^{20}C_3 - 20\ \times 16 - 20$

$= \frac{20 \times 19 \times 18}{6} - 20 \times 16 - 20$

$= 20[57 - 16 - 1]$

$= 20 \times 40 = 800$

51 **(d)**

The total number of words is $6! = 720$. Let us write the letters of word ZENITH alphabetically, i.e., EHINTZ

For ZENITH word start with	Word starting with	Number of words
Z	E	5!
	H	5!
	I	5!
	N	5!
	T	5!
ZEN	ZEH	3!
	ZEI	3!
ZENI	ZENH	2!
ZENIT	ZENIH	1
	Total number of words before ZENITH	615

Hence, there are 615 words before ZENITH, so the rank of ZENITH is 616

52 **(c)**

For a radical centre, 3 circles are required. The total number or radical centres is ${}^nC_3$.

The total number or radical axis is ${}^nC_2$. Now, ${}^nC_2 = {}^nC_3 \Rightarrow n = 5$

53 **(b)**

Suppose there 'n' players in the beginning. The total number of games to be played was equal to ${}^nC_2$ and each player would have played $n - 1$ games

Let us assume that A and B fell ill. Now the total number of games of A and B is $(n-1) + (n-1) - 1 = 2n - 3$. But they have played 3 games each. Then their remaining number of games is $2n - 3 - 6 = 2n - 3$. But they have played 3 games each. Then their remaining number of games is $2n - 3 - 6 = 2n - 9$. Given.

${}^nC_2 - (2n - 9) = 84$

$\Rightarrow n^2 - 5n - 150 = 0$

$\Rightarrow n = 15$

Alternative solution:

The number of games excluding A and B is ${}^{n-2}C_2$. But before leaving A and B played 3 games each. Then,

${}^{n-2}C_2 + 6 = 84$

Solving this equation, we get $n = 15$

54 **(b)**

There is concept of derangement. The required number is

$4!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right] = 9$

55 **(a)**

Three IIT students who will be between the IIT students can be selected in ${}^{10}C_3$ ways. Now, two DCE students having three IIT students between them can be arranged in $2! \times 3!$ ways. Finally, a group of above five students and the remaining seven students together can be arranged in 8! ways. Hence, total number of ways is

${}^{10}C_3 \times 2! \times 3! \times 8!$

56 **(c)**

Total number of variables if only alphabet is used is 286. Total number of variables if alphabets and digits both are used is 26×10. Hence, the total number of variables is $26(1+10)=286$

57 **(b)**

Since the student is allowed to select at most n books out of $(2n+1)$ books, therefore in order to select one book he has the choice to select one, two, three,… , n books. Thus if, T is the total number of ways of selecting one book, then

$T = {}^{2n+1}C_1 + {}^{2n+1}C_2 + \cdots + {}^{2n+1}C_n = 63$ (i)

Again the sum of binomial coefficients is

$$ {}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \cdots + {}^{2n+1}C_n + {}^{2n+1}C_{n+1} + {}^{2n+1}C_{n+2} + \cdots + {}^{2n+1}C_{2n+1} = (1+1)^{2n+1} = 2^{2n+1} $$

or

$$ {}^{2n+1}C_0 + 2({}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n) + {}^{2n+1}C_{2n+1} = 2^{2n+1} $$

$\Rightarrow 1 + 2(T) + 1 = 2^{2n+1}$

$\Rightarrow 1 + T = \dfrac{2^{2n+1}}{2} = 2^{2n}$

$\Rightarrow 1 + 63 = 2^{2n}$

$\Rightarrow 2^6 = 2^{2n} \Rightarrow n = 3$

58 **(b)**

A regular polygon of n sides has n vertices, no two of which are collinear. Out of these n points, ${}^nC_3$ triangles can be formed

$\therefore\ T_n = {}^nC_3; T_{n+1} = {}^{n+1}C_3$

Given,

$T_{n+1} - T_n = 21$

$\Rightarrow {}^{n+1}C_3 - {}^nC_3 = 21$

$\Rightarrow \dfrac{(n+1)n(n-1)}{3\times 2\times 1} - \dfrac{n(n-1)(n-2)}{3\times 2\times 1} = 21$

$\Rightarrow n(n-1)(n+1-n+2) = 126$

$\Rightarrow n(n-1) = 42$

$\Rightarrow n(n-1) = 7 \times 6$

$\Rightarrow n = 7$

59 **(c)**

There are two possible cases

Case I Five 1's, one 2's, one 3's

Number of numbers$= \frac{7!}{5!} = 42$

Case II Four 1's, three 2's

Number of numbers$= \frac{7!}{4!3!} = 35$

Total number of numbers $42 + 35 = 77$

60 **(b)**

$(x+3)^2 + y^2 = 13$

$\Rightarrow x+3 = \pm 2, y = \pm 3$ or $x + 3 = \pm 3, y = \pm 2$

61 **(c)**

Let there be n men participants. Then the number of games that the men play between themselves is $2 \times {}^nC_2$ and the number of games that the men played with the women is $2 \times (2n)$

$\therefore 2 \times {}^nC_2 - 2 \times 2n = 66$ (by hypothesis)

$\Rightarrow n^2 - 5n - 66 = 0 \Rightarrow n = 11$

Hence, the number of participants is 11 men +2 women =13

62 **(b)**

The number of times the teacher goes to the zoo is ${}^nC_3$. The number of times a particular child goes to the zoo is equal to number of ways two other children can be selected who accompany a particular child i.e., ${}^{n-1}C_2$. From the question,

${}^nC_3 - {}^{n-1}C_2 = 84$

or $(n-1)(n-2)(n-3) = 6 \times 84 = 9 \times 8 \times 7 \Rightarrow n - 1 = 9$

63 **(b)**

We first select 2 men out of 7 in ${}^7C_2$ ways. Now we exclude the wives of these two selected men and so select 2 ladies from remaining 5 ladies in ${}^5C_2$ ways. Let A, B be two men and X, Y be the ladies playing in one set. Then we can have

1. A and X plying against B and Y
2. A and Y playing against B and X

Then the total number of ways is ${}^7C_2 \times {}^5C_2 \times 2 = 21 \times 10 \times 2 = 420$

64 **(b)**

Since 5 players are always to be excluded and 6 players always to be included, therefore 5 players are to be chosen from 14. Hence required number of ways is ${}^{14}C_5 = 2002$

65 **(c)**

${}^nC_{r-1} = 36$, ${}^nC_r = 84$, ${}^nC_{r+1} = 126$

We know that

$\dfrac{{}^nC_{r-1}}{{}^nC_r} = \dfrac{r}{n-r+1}$

$\Rightarrow \dfrac{36}{84} = \dfrac{r}{n-r+1}$

$\Rightarrow \dfrac{r}{n-r+1} = \dfrac{3}{7}$

$\Rightarrow 3n - 10r + 3 = 0$ (1)

Also,

$\dfrac{{}^nC_r}{{}^nC_{r+1}} = \dfrac{r+1}{n-r} = \dfrac{84}{126} = \dfrac{2}{3}$

$\Rightarrow 2n - 5r - 3 = 0$ (2)

Solving (1) and (2), we get $n = 9$ and $r = 3$

66 **(d)**

Suppose i^{th} person receives Rs x_i; $i = 1, 2, 3, 4$

Then, $x_1 + x_2 + x_3 + x_4 = 18$, where $x_i \geq 4$

Let $y_i = x_i - 3$, $i = 1, 2, 3, 4$. Then,

$y_1 + y_2 + y_3 + y_4 = 6$

The total number of ways is equal to number of solutions of the above equation, which is given by ${}^{6+4-1}C_{4-1} = {}^{9}C_3 = 84$

67 **(a)**

Let x_1, x_2, x_3, x_4 be the number of times T, I, D, E appears on the coupon. Then we must have $x_1 + x_2 + x_3 + x_4 = 8$, where $1 \le x_1, x_2, x_3, x_4 \le 8$ (as each letter must appear once). Then the required number of combinations of coupons is equivalent to number of positive integral solutions of the above equation, which is further equivalent to number of ways of 8 identical objects distributed among 4 persons when each gets at least one objects, and is given by ${}^{8-1}C_{4-1} = {}^{7}C_3$

68 **(a)**

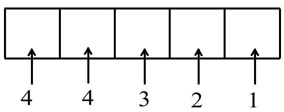
We know that a number is divisible by 3 if the sum of its digits is divisible by 3. Now out of 0, 1, 2, 3, 4, 5 if we take 1, 2, 3, 4, 5 or 0, 1, 2, 4, 5, then the 5-digit numbers will be divisible by 3

Case I:

Total number of five-digit numbers formed using the digits 1, 2, 3, 4, 5 is $5! = 120$

Case II:

Taking 0, 1, 2, 4, 5, total number is $4 \times 4! = 96$



From case I and case II, total number divisible by 3 is $120 + 96 = 216$

69 **(a)**

There can be two types of numbers.

1. Any one of the digits 1, 2, 3, 4 appears thrice and the remaining digits only once, i.e., of the type 1, 2, 3, 4, 4, 4, etc. Number of ways of selection of digit which appears thrice is ${}^4C_1$

   Then the number of numbers of this type is $(6!/3!) \times {}^4C_1 = 480$

2. Any two of the digits 1, 2, 3, 4 appears twice and the remaining two only once, i.e., of the type 1, 2, 3, 3, 4, 4, etc. The number of ways of selection of two digits which appear twice is ${}^4C_2$. Then the number of numbers of this type is $[6!\,(2!\,2!) \times {}^4C_2]$. Therefore, the required number of numbers is $480 + 1080 =$ 1560

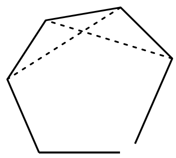
70 **(b)**

The number of one-digit numbers is $5 \times 5 = 25$

The number of three-digit numbers is $5 \times 5 \times 4 = 100$

Hence, the total number are is 131

71 **(a)**



Let the number of sides be n. A selection of four vertices of the polygon gives an interior intersection

$\therefore\ {}^nC_4 = 70$

$\Rightarrow n(n-1)(n-2)(n-3) = 24 \times 70$

$= 8 \times 7 \times 6 \times 5$

$\Rightarrow n = 8$

72 **(d)**

Let us first select two places for vowel, which can be selected from 4 places in ${}^4C_2$ ways. Now this places can be filled by vowels in $5 \times 5 = 25$ ways as repetition is allowed. The remaining two places can be filled by consonants in 21×21 ways. Then the total number of words is ${}^4C_2 \times 25 \times 21^2 = 150 \times 21^2$

73 **(d)**

Here,

${}^nP_3 - {}^nC_3 > 100$

$\Rightarrow \dfrac{n!}{(n-3)!} - \dfrac{n!}{3!\,(n-3)!} > 100$

$\Rightarrow \dfrac{5}{6} n(n-1)(n-2) > 100$

$\Rightarrow n(n-1)(n-2) > 120$

$\Rightarrow n(n-1)(n-2) > 6 \times 5 \times 4$

$\Rightarrow n = 7, 8, \ldots$

74 **(d)**

$N = 1! + 2! + \cdots + 2005!$

$= (1! + 2! + 3! + 4!) + (5! + \cdots + 2005!)$

$= 33+$ an integer having 0 in its unit's place

=an integer having 3 in its unit's place

Hence, N^{500} is an integer having 1 in its unit's place

75 **(b)**

'P_1' must win at least $n+1$ games. Let 'P_1' win $n+r$ games ($r = 1, 2, \dots, n$). Therefore, corresponding number of ways is ${}^{2n}C_{n+r}$. The total number of ways is

$$\sum_{r=1}^{n} {}^{2n}C_{n+1} + {}^{2n}C_{n+2} + \cdots + {}^{2n}C_{2n}$$

$$= \frac{2^{2n}}{2} - {}^{2n}C_n$$

$$= \frac{1}{2}(2^{2n} - 2 \times {}^{2n}C_n)$$

76 **(b)**

Number of girls	Number of boys	Number of groups going to picnic	Total number of dolls
1	4	${}^3C_1\,{}^4C_4$	$1({}^3C_1\,{}^4C_4) = 3$
2	3	${}^3C_2\,{}^4C_3$	$2({}^3C_2\,{}^4C_3) = 24$
3	2	${}^3C_3\,{}^4C_2$	$3({}^3C_3\,{}^4C_2) = 18$
		Total	45

77 **(a)**

The number of ways he can select at least one parantha is $2^3 - 1 = 7$. The number of ways he can select at least one vegetable dish is $2^4 - 1 = 15$. The number of ways he can select zero or more items from salads and sauces is 2^5 Hence, the total number of ways is $7 \times 15 \times 32 = 3360$

78 **(b)**

$$\sum_{i=1}^{k} \frac{1}{x_i} = \frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_k} = \frac{\sum x_i}{n} = \frac{75}{n}$$

(as L.C.M. of $x_1, x_2, \dots, x_n$ is n)

79 **(b)**

The smallest number of people = total number of possible forecasts

= total number of possible results

$= 3 \times 3 \times 3 \times 3 \times 3$

80 **(a)**

Total numbers ending with 2 is 3! as after fixing 2 in the unit's place other three places can be filled by 3! Ways. Thus, 2 appears in the unit's place 3! times.

Similarly, all other digits 4, 6 and 8 also appear 3! times. Then sum of the digits in the unit's place is $6(2+4+6+8) = 120$ units. Similarly, sum of digits in ten's place is 120 tens and that in hundred's place is 120 hundreds, etc Hence, sum of all the 24 numbers is $120(1 + 10 + 10^2 + 10^3) = 120 \times 1111 = 133320$

81 **(c)**

For each bulb there are two possibilities. It will be switched either on or off. Hence, total number of ways in which the room can be illuminated is $2^{32} - 1$

82 **(c)**

First arrange m positive signs. The number of ways is just 1 (as all + signs are identical). Now, $m+1$ gaps are created of which n are to be selected for placing '−' signs. Then the total number of ways of doing so is ${}^{m+1}C_n$. After selecting the gaps '−' signs can be arranged in one way only

83 **(c)**

Let there be n candidates. Then,

${}^nC_1 + {}^nC_2 + \cdots + {}^nC_{n-1} = 254$

$\Rightarrow 2^n - 2 = 254$

$\Rightarrow 2^n = 2^8 \Rightarrow n = 8$

84 **(a)**

If zero is included it will be at $z \Rightarrow {}^9C_2$

$$\text{if zero is excluded} \begin{cases} x, y, z \text{ all diff.} & \Rightarrow {}^9C_3 \times 2! \\ x = z < y & \Rightarrow {}^9C_2 \\ x < y = z & \Rightarrow {}^9C_2 \end{cases}$$

The total number of ways is 276

Alternative method:

y can be from 2 to 9; so total number of ways is

$$\sum_{r=}^{9} (r^2 - 1) = 276$$

85 **(a)**

All strips are of different colors, then number of flags is $= 3! = 5$. When two strips are of same color, then number of flags is ${}^3C_1 \times (3!/2) \times {}^2C_1 = 18$. Total number of flags is $6 + 18 = 24 = 4!$

86 **(a)**

Number of times 3 used	Pattern	Numbers of type	Number of times 3 appears
1	− − 3 −3 − 3 − −	$3 \times 9 \times 9$	$1(3 \times 9 \times 9)$
2	−33 33 − 3 − 3	3×9	$2(3 \times 9)$
3	333	1	3
		Total	300

Any place other than 3 is filled by 9 ways as '0' can appear anywhere which gives all types of numbers like single digit, two digits, etc

Alternative solution:

A three-digit block from 000 to 999 means 1000 numbers, each number constituting 3 digits. Hence, the total numbers of digits which we have to write is 3000.

Since the total number of digits is 10 (0 to 9) no digit is filled preferentially. Therefore, number of times we write 3 is $3000/10 = 300$

87 **(c)**

Number of digits	Numbers ending with 0	Numbers ending with 5	Total
×	0	1	1
××	8	9	17
×××	$9\cdot8 = 72$	$8\cdot8 = 64$	136
××××	$9\cdot8\cdot7 = 504$	$8\cdot8\cdot7 = 448$	952
		total	1106

88 **(b)**

Numbers greater than 1000 and less than or equal to 4000 will be of 4 digits and will have either 1 (except 1000) or 2 or 3 in the 1st place with 0 in each of remaining places. After fixing 1st place, the 2nd place can be filled by any of the 5 digits. Similarly the 3rd place can be filled up in 5 ways and 4th place can be filled up in 5 ways and 4th place can be filled up in 5 ways. Thus, there will be $5 \times 5 \times 5 = 125$ ways in which 1 will be in first place but this also includes 1000. Hence, there will be 124 numbers having 1 in the first place. Similarly, 125 for each 2 or 3. One number will be there in which 4 will be in the first place, i.e., 4000. Hence, the required number of ways is $124 + 125 + 125 + 1 = 375$

89 **(b)**

Given number can be rearranged as

$1, 4, 7, \dots, 3n-2 \to 3\lambda - 2$

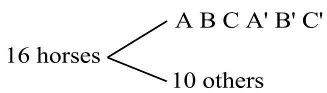
$2, 5, 8, \dots, 3n-1 \to 3\lambda - 1$

$3, 6, 9, \dots, 3n \to 3\lambda$

That means, we must take two numbers from last row or one number each from first and second rows. Therefore, the total number of ways is

$$^nC_2 + {}^nC_1 \times {}^nC_1 = \frac{n(n-1)}{2} + n^2 = \frac{3n^2 - n}{2}$$

90 **(c)**



The number of ways is $^{10}C_3$ ×number of ways of choosing out of $ABC\ A'B'C'$, so that AA', BB' or CC' are not together

$= {}^{10}C_3$(one from each of pairs AA', BB', CC')

$= {}^{10}C_3 \times 8$

$= \frac{10 \times 9 \times 8}{1 \times 2 \times 3} 8 = 960$

91 **(c)**

Required sum is $3!\,(3 + 4 + 5 + 6) = 6 \times 18 = 108$

[If we fix 3 in the unit place, other three digits can be arranged in 3! ways. Similarly for 4, 5, 6]

92 **(c)**

Select any three points from total $3p$ points, which can be done $^{3p}C_3$ ways. But this also includes selection of three collinear points. Now three collinear points from each straight line can be selected in $^pC_3$ways. Then the number of triangles is $^{3p}C_3 - 3\,^pC_3 = p^2(4p - 3)$

93 **(a)**

Let the blankets received by the persons are x_1, x_2, x_3 and x_4 we have,

$x_1 + x_2 + x_3 + x_4 = 15$ and $x_i \geq 2$

$\Rightarrow (x_1 - 2) + (x_2 - 2) + (x_3 - 2) + (x_4 - 2) = 7$

$\Rightarrow y_1 + y_2 + y_3 + y_4 = 7$ (where $y_i = x_i - 2 \geq 0$)

The required number is equal to the number of non-negative integral solutions of this equation which is equal to $^{4+7-1}C_7$

i.e., $^{10}C_7 = {}^{10}C_3$

94 **(c)**

X − X − X − X − X

The four digits 3, 3, 5, 5 can be arranged at (−) places in $\frac{4!}{2!2!} = 6$ ways. The five digits 2, 2, 8, 8, 8 can be arranged at (X) place in $\frac{5!}{2!3!} = 10$ ways

Total number of arrangements is $6 \times 10 = 60$

95 **(c)**

1 S, 3 A, 1 H, 2 R, 1 N, 1 P, 1 U

When all letters are different corresponding ways is $^7C_3 \times 3! = {}^7C_3 = 210$. When two letters are of one kind and other is different, corresponding number of ways is $^2C_1 \times ^6 C_1 \times (3!/2!) = 36$. When all letters are alike, corresponding number of ways is 1. Thus, total number of words that can be formed is $210 + 36 + 1 = 247$

96 **(b)**

The two common elements can be selected in $^nC_2$ ways. Remaining $n - 2$ elements, each can be chosen in three ways, i.e. $a \in P$ and $a \notin Q$ or $a \in Q$ and $a \notin P$ or a is neither in P nor in Q. Therefore, the total number of ways is $^nC_2 \times 3^{n-2}$

97 **(b)**

The number of trains a day (the digits 1, 2, 3) are three groups of like elements from which a same must be formed. In the time-table for a week, the number 1 is repeated twice, the number 2 is repeated 3 times and the number 3 is repeated twice.

The number of different time-table is given by

$$p(2,3,2) = \frac{7!}{2!\,3!\,2!} = 210$$

98 **(a)**

We can think of three packets. One consisting of three boys of class X, other consisting of 4 boys of class XI and last one consisting of 5 boys of class XII. These packets can be arranged in 3! ways and contents of these packets can be further arranged in 3! 4! and 5! ways, respectively. Hence, the total number of ways is 3! × 3! × 4! × 5!

99 **(b)**

$\times B_1 \times B_2 \times B_3 \times B_4 \times B_5 \times B_6 \times$

Let first six boys sit, which can be done in 6! ways. Once they have been seated, the two brothers can be made to occupy seats in between or in extreme (i.e. on crosses) in ${}^7P_2$ ways

Hence, required number of ways is ${}^7P_2 \times 6!$

100 **(a)**

Let x be the number of objects to the left of the first object chosen, y the number of objects between the first and the second, z the number of objects between the second and the third and u the number of objects to the right of the third objects. Then, $x, u \geq 0; y, z \geq 1$and $x + y + z + u = n - 3$. Let $y_1 = y - 1$ and $z_1 = z - 1$. Then, $y_1 \geq 0, z_1 \geq 0$ such that $x + y_1 + z_1 + u = n - 5$

The total number of non-negative integral solutions of this equation is ${}^{n-5+4-1}C_{4-1} = {}^{n-2}C_3$

101 **(a)**

Clearly, one of the odd digits 1, 3, 5, 7, 9 will be repeated. The number of selections of the sixth digit is ${}^5C_1 = 5$. Then the required number of numbers is 5 × (6!/2!)

102 **(a)**

The number of ways of allotment without any restriction is ${}^8P_6$. Now, it is possible that all rooms of 2nd floor or 3rd floor are not occupied. Thus, there are two ways in which one floor remains unoccupied. Hence, the number of ways of allotment in which a floor is unoccupied is 2 × 6!. Hence, number of ways in which none of the floor remains unoccupied is ${}^8P_6 - 2(6!)$

103 **(a)**

${}^nC_3 + {}^nC_4 > {}^n + {}^1C_3$

$\Rightarrow {}^{n+1}C_4 > {}^{n+1}C_3 \; (\because {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1})$

$$\Rightarrow \frac{{}^{n+1}C_4}{{}^{n+1}C_3} > 1$$

$$\Rightarrow \frac{n-2}{4} > 1$$

$\Rightarrow n > 6$

104 **(c)**

Since, r, s, t are prime numbers

∴ Selection of p and q are as under

p	q	Number of ways
r^0	r^2	1 way
r^1	r^2	1 way
r^2	r^0, r^1, r^2	3 way

∴ Total number of ways to select $r = 5$

s^0	s^4	1 way
s^1	s^4	1 way
s^2	s^4	1 way
s^3	s^4	1 way
s^4	s^0, s^1, s^2, s^3, s^4	5 ways

∴ Total number of ways to select $s = 9$

Similarly, the number of ways to select $t = 5$

∴ Total number of ways$5 \times 9 \times 5 = 225$

105 **(c)**

$f(2n, n)$ must be equal to number of positive integer solutions of $x_1 + x_2 + \cdots + x_n = 2_n$, which must be equal to ${}^{2n-1}C_{n-1} = {}^{2n-1}C_n$

106 **(a)**

$15 < x_1 + x_2 + x_3 \leq 20$

$\Longrightarrow x_1 + x_2 + x_3 = 16 + r, r = 0, 1,2,3,4$

Now the number of positive integral solution of $x_1 + x_2 + x_3 = 16 + r$ is

${}^{13+r+3-1}C_{13+r}$, i.e., ${}^{15+r}C_{13+r} = {}^{15+r}C_2$

The total number of solutions is

$$\sum_{r=0}^{4} {}^{15+r}C_2 = {}^{15}C_2 + {}^{16}C_2 + {}^{17}C_2 + {}^{18}C_2 + {}^{19}C_2$$

$$= \frac{1}{2}(15 \times 14 + 16 \times 15 + 17 \times 16 + 18 \times 17 + 19 \times 18)$$

$= 685$

107 **(b)**

Since the number of students giving wrong answers to at least i questions $(i = 1, 2, \dots, n)$ is 2^{n-i}

The number of students answering exactly $i \; (1 \leq i \leq -1)$ questions wrongly = {the number of students answering at least i questions wrongly, $i = 1,2, \dots$)}–{the number of students answering at least $(i + 1)$ questions wrongly

$(2 \le i+1 \le n)\} = 2^{n-i} - 2^{n-(i+1)}(1 \le i \le n - 1)$

Now, the number of students answering all the n questions wrongly is $2^{n+2} = 2^0$

Thus, the total number of wrong answers is

$$1(2^{n-1} - 2^{n-2}) + 2(2^{n-2} - 2^{n-3}) + 3(2^{n-3} - 2^{n-4}) + \cdots + (n-1)(2^1 - 2^0) + n(2^0)$$

$= 2^{n-1} + 2^{n-2} + 2^{n-3} + \cdots + 2^0 = 2^n - 1$ (∵ it is a G.P.)

Therefore, as given

$2^n - 1 = 2047 \Rightarrow 2^n = 2048 = 2^{11} \Rightarrow n = 11$

108 **(c)**

If we put minimum number of balls required in each box, balls left are $n(n-1)/2$ which can be put in ${}^{(n^2+n-1)/2}C_{n-1}$ ways without restriction

109 **(a)**

Let the balls put in the box are x_1, x_2, x_3, x_4 and x_5. We have,

$x_1 + x_2 + x_3 + x_4 + x_5 = 15, x_i \ge 2$

$\Rightarrow (x_1 - 2) + (x_2 - 2) + (x_3 - 2) + (x_4 - 2) + (x_5 - 2) = 5$

$\Rightarrow y_1 + y_2 + y_3 + y_4 + y_5 = 5, y_i = x_i - 2 \ge 0$

The total number of ways is equal to number of non-negative integral solutions of the last equation, which is equal to ${}^{5+5-1}C_5 = {}^9C_5$

110 **(c)**

For a particular class the total number of different tickets from first intermediate station is 5. Similarly, number of different tickets from second intermediate station is 4. So the total number of different tickets is 5+4 + 3 + 2 + 1 = 15. And same number of tickets for another class is equal to total number of different tickets, which is equal to 30 and number of selection is ${}^{30}C_{10}$

111 **(a)**

The total number of books is $a + 2b + 3c + d$. The total number of ways in which these books can be arranged in a shelf (in same row) is $\frac{(a+2b+3c+d)!}{a!(b!)^2\,(c!)^3}$

112 **(c)**

Arrange the letter of the word COCHIN as in the order of dictionary CCHINO

Which number of words with the two C's occupying first and second place= 4!

Number of words starting with CH, CI, CN is 4! each

∴ Total number of ways = 4! + 4! + 4! + 4! = 96

There are 96 words before COCHIN

113 **(a)**

The selection can be made in ${}^5C_3 \times {}^{22}C_9$ ways.

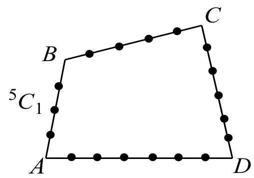
(Since 3 vacancies are filled from 5 candidates in ${}^5C_3$ ways and now remaining candidates are 22 and remaining seats are 9)

114 **(b)**

Dashes	Dots	Arrangements
5	2	${}^7C_2$
4	3	${}^7C_3$
3	4	${}^7C_4$
2	5	${}^7C_5$
1	6	${}^7C_6$
0	7	${}^7C_7$

The total number of ways is ${}^7C_2 + {}^7C_3 + {}^7C_4 + {}^7C_5 + {}^7C_6 = 2^7 - 8 = 120$

115 **(d)**



The number of triangles with vertices on sides AB, BC, CD is ${}^3C_1 \times {}^4C_1 \times {}^5C_1$

Similarly, for other cases, the total number of triangles is

$${}^3C_1 \times {}^4C_1 \times {}^5C_1 + {}^3C_1 \times {}^4C_1 \times {}^6C_1 + {}^3C_1 \times {}^5C_1 \times {}^6C_1 + {}^4C_1 \times {}^5C_1 \times {}^6C_1 = 342$$

116 **(c)**

We have 32 places for teeth. For each place, we have two choices either there is a tooth or there is no tooth. Therefore, the number of ways to fill up these places is 2^{32}. As there is no person without a tooth, the maximum population is $2^{32} - 1$

117 **(a)**

26 cards can be chosen out of 52 cards in ${}^{52}C_{26}$ ways. There are two ways in which each card can be dealt, because a card can be either from the first pack or from the second. Hence the total number of ways is ${}^{52}C_{26} \times 2^{26}$

118 **(b)**

Let the number selected be x_1, x_2, x_3. We must have $2x_2 = x_1 + x_3$

$\Rightarrow x_1 + x_3 =$ even

Therefore, x_1, x_3 both are odd or both are even. If x_1 and x_3 both are odd, we can again select them in ${}^{12}C_2$ ways. Thus, the total number of ways is $2 \times {}^{12}C_2 = 132$

119 **(b)**

The candidate is unsuccessful if he fails in 9 or 8 or 7 or 6 or 5 papers. Therefore, the number of ways to be unsuccessful is

$^9C_9 + {}^9C_8 + {}^9C_7 + {}^9C_6 + {}^9C_5$

$= {}^9C_0 + {}^9C_1 + {}^9C_2 + {}^9C_3 + {}^9C_4$

(recall the concept of half series)

$= \frac{1}{2}({}^9C_0 + {}^9C_1 + \ldots + {}^9C_9)$

$= \frac{1}{2} \times 2^9 = 2^8$

120 **(c)**

$480 = 2^5 \times 3 \times 5$

Now, $4n + 2 = 2(2n + 1) =$ odd multiple of 2.

Thus, the total number of such divisors is

$1 \times 2 \times 2 = 4$

121 **(a)**

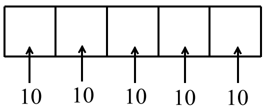
The number of ways of selecting four numbers from 1 to 30 without any restriction is ${}^{30}C_4$. The number of ways of selecting four consecutive [i.e. (1, 2, 3, 4), (2, 3, 4, 5), …, (27, 28, 29,3 0)] number is 27. Hence, the number of ways of selecting four integers which excludes consecutive four selections is

$${}^{30}C_4 - 27 = \frac{30 \times 29 \times 28 \times 27}{24} - 27 = 27378$$

122 **(c)**

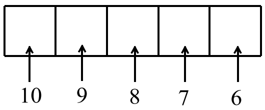
Number of words when repetition is allowed is

$10 \times 10 \times 10 \times 10 \times 10 = 10^5$



Number of words when repetition is not allowed is

$10 \times 9 \times 8 \times 7 \times 6 = 30240$



Hence, required number of words in which at least one letter is repeated is $100000 - 30240 = 69760$

123 **(c)**

The number of ways can be given as follows:

2 bowlers and 9 other players: ${}^4C_2 \times {}^9C_9$

3 bowlers and 8 other players: ${}^4C_3 \times {}^9C_8$

4 bowlers and 7 other players: ${}^4C_4 \times {}^9C_7$

Hence required number of ways is $6 \times 1 + 4 \times 9 + 1 \times 36 = 78$

124 **(b)**

There are 11 letters A, A; I, I; N, N; E, X, M, T, O. For the selection of 4 letters we have the following possibilities:

1. 2 alike, 2 alike

2. 2 alike, 2 different

3. All four different

1. There are 3 pairs of 2 letters. So, the number of ways of selection of 2 pairs is ${}^3C_2$ and permutation of these 4 letters is 4!/ 2! 2!. Therefore, the number of words in this case is ${}^3C_2 \times 4!/2!\,2! = 18$

2. We have to select one pair from 3 pairs and 2 distinct letters from remaining 7 distinct letters. For illustration, let us select both A, A; then we have I, N, E, X, M, T, O i.e., 7 as remaining distinct letters. Hence, the number of selections is ${}^3C_1 \times {}^7C_2$ and these 4 (2 same, 2 distinct) can be permuted in 4!/2! ways. Therefore, number of words is ${}^3C_1 \times {}^7C_2 \times 4!/2! = 3 \times 21 \times 12 = 756$

3. There are 8 distinct letters so number of words of 4 letters is ${}^8C_4 \times 4! = 1680$. By sum rule, the total number of words is $18 + 756 + 1680 = 2454$

125 **(c)**

The number of ways the candidate can choose questions under the given conditions is enumerated below

Group 1	Group 2	Number of ways
4	2	$({}^5C_4)({}^5C_2) = 50$
3	3	$({}^5C_3)({}^5C_3) = 100$
2	4	$({}^5C_2)({}^5C_4) = 50$
	Total number of ways	200

126 **(d)**

Using the digits 0, 1, 2, …, 9 the number of five digit telephone numbers which can be formed is 10^5 (since repetition is allowed). The number of five digit telephone, numbers which have none of the digits repeated is ${}^{10}P_5 = 30240$. Therefore, the required number of telephone numbers is $10^5 - 30240 = 69760$

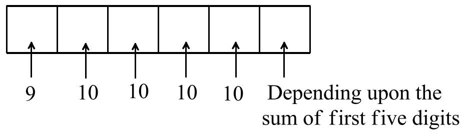
127 **(b)**

There are 6 different letters. We have to select 6 squares, taking at least one from each row and then arranging in each selection. Let us first select places in each row such that no row remains empty

R_1	R_2	R_3	Number of selections
1	1	4	${}^2C_1\,{}^2C_1\,{}^4C_4 = 4$
1	2	3	${}^2C_1\,{}^2C_2\,{}^4C_3 = 8$
2	1	3	${}^2C_2\,{}^2C_1\,{}^4C_3 = 8$
2	2	2	${}^2C_2\,{}^2C_2\,{}^4C_2 = 6$

Therefore, the total number of selections of 6 squares is $4 + 8 + 8 + 6 = 26$. For each selection of 6 squares, the number of arrangements of 6 letters is $6! = 720$. Hence, the required number of ways is $26 \times 720 = 18720$

128 **(a)**



First place from left cannot be filled with 0. Next four places can be filled with any of the 10 digits. After filling the first five places, the last place can be filled in following ways.

Sum of digits in first five places	Digit in the unit's place
$5k$	0 or 5
$5k+1$	4 or 9
$5k+2$	3 or 8
$5k+3$	2 or 7
$5k+4$	1 or 6

Thus, in any case the last place can be filled in to ways. Hence, the required number of numbers is $9 \times 10^4 \times 2$

129 **(a)**

The number of points of intersection is equal to the number of ways two lines are selected, which is given by

$$^nC_2 = \frac{n(n-1)}{2} = \sum_{k=1}^{n-1} K$$

130 **(c)**

(i) Miss C is taken

1. B included ⇒ A excluded $\Rightarrow^4 C_1 \times {}^4C_2 = 24$

2. B excluded ⇒ ${}^4C_1 \times {}^5C_3 = 40$

(ii) Miss C is not taken

⇒ B does not com : ⇒ ${}^4C_2 \times {}^5C_3 = 60$

⇒ Total = 124

Alternative method:

Case I:

Mr. 'B' is present

⇒ 'A' is excluded and 'C' included

Hence, the number of ways is ${}^4C_2\ {}^4C_1 = 24$

Case II:

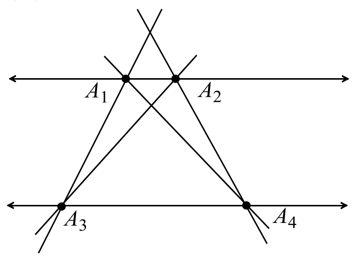
Mr. 'B' is absent

⇒ No constraint

Hence, the number of ways is ${}^5C_3\,{}^5C_2 = 100$

∴ Total = 124

131 **(b)**



For intersection point we must have two straight lines, for which 2 points from each straight line must be selected. Now selection of these points can be done in ${}^mC_2 \times {}^nC_2$ ways. Now as shown in diagram these four points can give two different sets of straight lines, which generate two distinct points of intersection

Then total number of points of intersection is ${}^mC_2 \times {}^nC_2 \times 2$

132 **(b)**

Distinct n-digit numbers which can be formed using digits 2, 5 and 8 are 3^n. We have to find n so that

$3^n \geq 900$

$\Rightarrow 3^{n-2} \geq 100$

$\Rightarrow n - 2 \geq 5$

$\Rightarrow n \geq 7$

So the least value of n is 7

133 **(b)**

The number of numbers when repetition is allowed is 5^4

The number of numbers when digits cannot be repeated is ${}^5P_5$

Therefore, the required number of numbers is $5^4 - 5!$

134 **(b,c,d)**

Exponent of 2 is

$$\left[\frac{10}{2}\right] + \left[\frac{10}{2^2}\right] + \left[\frac{10}{2^3}\right] = 5 + 2 + 1 = 8$$

Exponent of 3 is

$\left[\frac{10}{3}\right] + \left[\frac{10}{3^2}\right] = 3 + 1 = 4$

Exponent of 5 is

$\left[\frac{10}{5}\right] = 2$

Exponent of 7 is

$\left[\frac{10}{7}\right] = 1$

The number of divisors of 10! is $(8+1)(4+1)(2+1)(1+1) = 270$. The number of ways of putting N as a product of two natural numbers is $270/2 = 135$

135 **(a,d)**

Let $A = \{a_1, a_2 \ldots, a_n\}$. For each $a_i (1 \le i \le n)$, we have either $a_i \in P_j$ or $a_i \notin P_j (1 \le j \le m)$. That is, there are 2^m choices in which $a_i (1 \le i \le n)$ may belong to the P_j's. One of these, there is only one choice, in which $a_i \in P_j$ for all $j = 1,2 \ldots, m$ which is not favourable for $P_1 \cap P_2 \cap \ldots \cap P_m$ to be ϕ.

Thus, $a_i \notin P_1 \cap P_2 \cap \ldots \cap P_m$ in $2^m - 1$ ways

Since there are n elements in set A, the total number of choices is $(2^m - 1)^n$

Also, there is exactly one choice, in which $a_i \notin P_j$ for all $j = 1, 2, 1, \ldots, m$ which is not favourable for $P_1 \cup P_2 \cup \ldots \cup P_m$ to be equal to A

Thus, a_j can belong to $P_1 \cup P_2 \cup \ldots P_m$ in $(2^m - 1)$ ways

Since there are n elements in set A, the number of ways in which $P_1 \cup P_2 \cup \ldots \cup P_m$ can be equal to A is $(2^m - 1)^n$

136 **(a,c)**

Let person P_i get x_i number of things such that

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 30$

If x_i is odd or $x_i = 2\lambda_i + 1$, where $\lambda_i \ge 0$, then

$2(\lambda_i + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6) + 6 = 30$

$\Rightarrow \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 = 12$

Then number of solutions is ${}^{12+6-1}C_{6-1} = {}^{17}C_5$.

If x_1 is even or $x_1 = 2\lambda_1$, where $\lambda_i \ge 1$, then

$2(\lambda_i + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6) = 30$

$\Rightarrow \lambda_i + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 = 15$

Therefore, required number of ways is

${}^{15-1}C_{6-1} = {}^{14}C_5$

137 **(a,b,c)**

When $n = 3k$, there are exactly $n/3$ integers of each type $3p, 3p+1, 3p+2$.

Now, sum of three selected integers is divisible by 3. Then either all the integers of the same type $3p, 3p+1$ or $3p+2$ or one-one integer form each type. Then number of selection ways is

${}^{n/3}C_3 + {}^{n/3}C_3 + {}^{n/3}C_3 +$

$({}^{n/3}C_1)({}^{n/3}C_1)({}^{n/3}C_1) = 3({}^{n/3}C_3) + (n/3)^3$

If $n = 3k+1$, then there are $(n-1)/3$ integers of the type $3p, 3p+2$ and $(n+2)/3$ integers of the type $3p+1$. Then number of selection ways is $2({}^{(n-1)/3}C_3) + ({}^{(n+2)/3}C_3) + ((n-1)/3)^2(n+2)$. When $n = 3k+2$, the number of selection ways are same as in the case of $n = 3k+1$

138 **(a,b,d)**

Clearly, each player will play 9 games. And total number of games is ${}^{10}C_2 = 45$. Clearly,

$w_1 + l_i = 9$ and $\sum w_i = \sum l = 45$

$\Rightarrow w_i = 9 - l_i \Rightarrow w_i^2 = 81 + l_1^2 = 181_i$

$$\Rightarrow \sum w_i^2 = 81 \times 10 + \sum l_i^2 - 180 \sum l_i$$

$$= 180 \sum l_i^2 - 18 \times 45 = \sum l_i^2$$

139 **(a,b,c,d)**

$8,7,6,4,2, x$ and y

Any number is divisible by 3 if sum of digits by 3, i.e., $x + y + 27$ is divisible by $3, x$ and y can take values from 0, 1, 3, 5, 9. Possible pairs are (5, 1) (3, 0) (9, 0) (9, 3) and (1, 5), (0, 3) (0, 9) (3, 9)

140 **(a,d)**

Problem is same as dividing 17 identical things in two groups

$\therefore\ n = \frac{17+1}{2} = 9$

There is no effect it two diamonds are different as necklace can be flipped over. Hence, $n = m = 9$

141 **(a,c)**

Let $x_i (1 \le i \le n)$ be the number of objects selected of the i^{th} type. Since each object is to be selected at least once, we must have $x_i \ge 1$ and $x_1 + x_2 + \cdots + x_n = r$. We have to find number of positive integral solutions of the above equation. Total number of such solutions is ${}^{r-1}C_{n-1} = {}^{r-1}C_{r-n}$

142 **(a,c)**

$3^p = (4-1)^p = 4\lambda + (-1)^p$

$5^q = (4+1)^q = 4\lambda_2 + 1$

$7^r = (8-1)^r = 4\lambda_3 + (-1)^r$

Hence, any positive integer power of 5 will be in the form of $4\lambda_2 + 1$. Even power of 3 and 7 will be in the form of $4\lambda + 1$ and odd power of 3 and 7 will be in the form of $4\lambda - 1$. Hence, both p and r must be odd or both must be even. Thus $p + r$ is always even. Also, $p + q + r$ can be odd or even

143 **(a,b,d)**

$p = {}^5C_4 \times {}^2C_1 = 10$

$q = {}^5C_2({}^2C_1)^3 = 80$

$r = {}^5C_0({}^2C_1)^5 = 32$

$\Rightarrow 2q = 5r, 8p = q$ and $2(p+r) > q$

144 **(b,c)**

If a, b, c are in A.P., then a and c both are odd or both are even

Case I: n is even

The number of ways of selection of two even numbers a and c is ${}^{n/2}C_2$. Number of ways of selection of two odd numbers is ${}^{n/2}C_2$. Hence the number of A.P.'s is

$$2\,{}^{n/2}C_2 = 2\frac{\frac{n}{2}\left(\frac{n}{2}-1\right)}{2} = \frac{n(n-2)}{4}$$

Case II: n is odd

The number of ways of selection of two odd numbers a and c is ${}^{(n+1)/2}C_2$. The number of ways of selection of two even numbers a and c is ${}^{(n-1)/2}C_2$. Hence the number of A.P.'s is

$${}^{(n+1)/2}C_2 + {}^{(n-1)/2}C_2$$

$$= \frac{\left(\frac{n+1}{2}\right)\left(\frac{n+1}{2}-1\right)}{2} + \frac{\left(\frac{n-1}{2}\right)\left(\frac{n-1}{2}-1\right)}{2}$$

$$= \frac{1}{8}(n-1)\big((n+1)+(n-3)\big)$$

$$= \frac{(n-1)^2}{4}$$

145 **(b,d)**

Number of selection when $x < y < z = {}^{n}C_3$

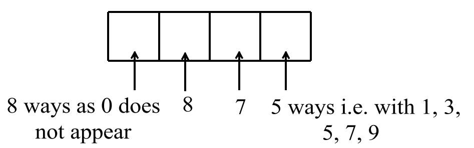
Number of selection when $x = y < z = {}^{n}C_2$

(we have to select only two numbers out of n numbers)

$\therefore$ Required number $= {}^{n}C_3 + {}^{n}C_2 = {}^{n+1}C_3$

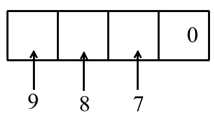
146 **(a,d)**

When n is odd:

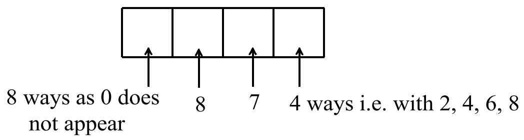


The number of such numbers is $8 \times 8 \times 7 \times 5 = 2240$

When n is enven:



If unit's place is filled with 0, then the total number is $9 \times 8 \times 7 = 504$



If unit's place is not filled with 0, then the total number is $8 \times 8 \times 7 \times 4 = 1792$

Hence, the total number of even numbers is $y = 504 + 1792 = 2296$

147 **(a,d)**

Number of selections of 7 digits out of the digit $1, 2, 3, \ldots 9 = {}^{9}C_7$

Number of digits out of these 7 selected digits excluding the greatest digit=6

These 6 digits can be divided in two groups each having 3 digits

in $\frac{6!}{3!3!2!} = {}^{6}C_3 \times \frac{1}{2!}$ ways

But the 3 digits on one side can go on the other side.

$\therefore$ Required number of ways

$$= {}^{9}C_7.\,{}^{6}C_3.\frac{1}{2!}2! = {}^{9}C_7.\,{}^{6}C_3$$

$$= {}^{9}C_2.\,{}^{6}C_3$$

148 **(a,b,c)**

$$\frac{(200)!}{\underbrace{2!\,2!\ldots2!}_{100\text{ times}}(100)!}$$

$$= \frac{(200)!}{100!\,2^{100}}$$

$$= 1 \times 3 \times 5 \ldots 199$$

Also,

$$\frac{(200)!}{100!\,2^{100}} = \left(\frac{101}{2}\right)\left(\frac{102}{2}\right)\cdots\left(\frac{200}{2}\right)$$

149 **(b,c,d)**

$$P = 21(21+1)(21-1)(21+2)(21-2)\ldots(21+10)(21-10)$$

$$= (21-10)(21-9)\ldots(21-1)21(21+1)(21+10)\ldots(21+10)$$

$$= 41 \times 40 \cdots 11$$

Which is divisible by 21!, and hence by 20! and 19!

150 **(a)**

Total number of ways $= \sum_{r=1}^{n} {}^{2n}C_{n+r}$

$$= {}^{2n}C_{n+1} \ldots + {}^{2n}C_{2n}$$

$$= \frac{1}{2}\left(2^{2n} - {}^{2n}C_n\right)$$

151 **(a,b,d)**

Total number of units to be covered is $3 + 7 + 11 = 21$. A person can choose 3 units in ${}^{21}C_3$ ways. A person can choose 7 units in ${}^{18}C_7$ ways. The rest 11 units can be chosen in 1 way.

Therefore, total number of ways is ${}^{21}C_3 \times {}^{18}C_7 \times 1 = 21!/(3!\,7!\,11!)$

152 **(a,c,d)**

Total number of arrangements $= 7!$

Number of arrangements of A, B, C, D among themselves $= 4!$

$\therefore$ Number of arrangements when A, B, C, D occur in a particular order

$= \frac{7!}{4!} = 210 = {}^7P_3 = 3! \times {}^7C_3$

153 **(b,c,d)**

When, $z = n + 1$, we can choose x, y from $\{1,2, \dots, n\}$

When $z = n + 1, x, y$ can be chosen in n^2 ways and when $z = n, x, y$ can be chosen in $(n-1)^2$ ways and so on. Therefore, the number of ways of choosing triplets is

$n^2 + (n-1)^2 + \dots + 1^2 = \frac{1}{6} n(n+1)(2n+1)$

Alternatively triplets with $x = y < z, x < y < z, y < z < z$ can be chosen in ${}^{n-1}C_2$, ${}^{n+1}C_3$, ${}^{n+1}C_3$ ways. Therefore,

${}^{n+1}C_2 + 2({}^{n+1}C_3) = {}^{n+2}C_2 + {}^{n+1}C_3$

$= 2({}^{n+2}C_3) - {}^{n+1}C_2$

154 **(b,c)**

The number of ways of inviting, with the couple not included, is ${}^8C_5$. The number of ways of inviting with the couple included is ${}^8C_3$.

Therefore the required number of ways is

${}^8C_5 + {}^8C_3 = {}^8C_3 + {}^8C_3$ $(\because {}^8C_5 = {}^8C_3)$

Also,

$C_5 - 2 \times C_4 = \frac{10!}{5!\,5!} - 2 \times \frac{8!}{4!\,4!}$

$= \frac{10 \times 9 \times 8 \times 7 \times 6}{120} - 2 \times \frac{8 \times 7 \times 6 \times 5}{24}$

$= 9 \times 4 \times 7 - 140$

$= 112 = 2 \times \frac{8!}{3!\,5!}$

155 **(c,d)**

Required number =number of selections of one or more out of three 25 paise coins and two 50 paise coins

$= 4 \times 3 - 1 = 11 = {}^{12}P_1 - 1$

156 **(a,b,c)**

The number of regions for 'n' circles be $f(n)$. clearly, $f(1) = 2$. Now,

$f(n) = f(n-1) + 2(n-1), \forall\, n \geq 2$

$\Rightarrow f(n) - f(n-1) = 2(n-1)$

Putting $n = 2,3, \dots, n$, we get

$(n) - f(1) = 2(1 + 2 + 3 + \cdots n - 1) = (n-1)n$

$\Rightarrow f(n) = n(n-1) + 2 = (n^2 - n + 2)$ (which is always even)

$\Rightarrow f(20) = 20^2 - 20 + 2 = 382$

Also,

$n^2 - n + 2 = 92$

$\Rightarrow n^2 - n - 90 = 0 \Rightarrow n = 10$

157 **(a,c)**

${}^{n+5}P_{n+1} = \frac{11(n+1)}{2} \times {}^{n+3}P_n$

$\Rightarrow P_{n+1} = \frac{(n+5)!}{4!} = \frac{11(n-1)}{2} \frac{(n+3)!}{3!}$

$\Rightarrow (n+5)(n+4) = 22(n-1)$

After solving, we get $n = 6$ or $n = 7$

The number of points of intersection of lines is ${}^6C_2$ or ${}^7C_2$

158 **(a)**

Statement 2 is true. Also in statement 1, if A selects i objects and B selects j objects then $i < j$. Hence number of ways is $\sum_{0 \leq i < j \leq 20} {}^{20}C_i\, {}^{20}C_j$

159 **(a)**

We have, $30 = 2 \times 3 \times 5$. So, 2 can be assigned to either a or b or c, i.e. 2 can be assigned in 3 ways. Similarly, each of 3 and 5 can be assigned in 3 ways. Thus, the number of solutions is

$3 \times 3 \times 3 = 27$

160 **(d)**

Total letters= $26(ie, A, B, C, \dots\dots, Y, Z)$and total digit number $= 10(ie, 0, 1, 2, 3, \dots, 9)$

$\because$ Repetition of letters is allowed.

$\therefore$ The three letters can be filled by $26 \times 26 \times 26 = (26)^3$

ways and three digit numbers on plate by 999 ways

$(ie, 001, 002, \dots .999)$

$\therefore$ Required number of ways $= (26)^3 \times 999$

161 **(a)**

Sum of the digits in the tens places

=sum of the digits in the unit's place

$= (4-1)!\,(2 + 3 + 4 + 5)$

$= 6.14 = 84$

162 **(d)**

In a chess board there are 9 horizonal and 9 vartical lines. Number of rectangles of any size are $^9C_2 \times ^9C_2$

Hence, option (d) is correct

163 **(a)**

General in the expansion of $(x+y+z+w)^{50}$ is $\frac{50!}{p!q!r!s!}x^p y^q z^r w^s$

Where $p+q+r+s=50, 0 \le p, q, , s \le 50$.

Now number of terms is equal to number of ways in which we can adjust powers of x, y, z and w such that their sum is 50, i.e., equal to the non-negative solutions of $p+q+r+s=50$, which is given by $^{50+4-1}C_{4-1}$

164 **(b)**

India must win at least 6 matches of 11 matches. Then number of ways in which India can win the series is $^{11}C_6 + {}^{11}C_7 + \cdots + {}^{11}C_{11} = 2^{10}$

Thus, both the statements are true, but statement 2 is not correct explanation of statement 1

165 **(a)**

A number is divisible by 4, if the last two digits are divisible by 4. Last two digits can be 12, 16, 28, 32, 36, 68, 92, 96. Thus last two places can be filled in 8 ways. The remaining three places can be filled with remaining 4 digits in $^4C_3 3!$ ways. Total number of such numbers is $8 \times ({}^4C_3 3!) = 192$

166 **(c)**

For the number exactly divisible by 4, then last two digit must be divisible by 4, the last two digits are viz 12, 16, 24, 32, 36, 52, 56, 64, 72, 76

Total 10 ways. Now the remaining two first places on the left of 4 digit numbers are to be filled from the remaining 5 digits and this can be done in $^5P_2 = 20$ ways.

$\therefore$ Required number of ways$= 20 \times 10 = 200$

167 **(c)**

When $p, q < r$, we have selection procedure as follows:

From p identical things	From q identical things
p	$r-p$
$p-1$	$r-(p-1)$
$p-2$	$r-(p-2)$
$\vdots$	$\vdots$
$\vdots$	$\vdots$
$r-q$	q
	Total: $p+q-r+1$

When $p, q > r$ we have selection procedure as follows:

From p identical things	From q identical things
r	0
$r-1$	1
$r-2$	2
$\vdots$	$\vdots$
$\vdots$	$\vdots$
0	r
	Total: $r+1$

Thus, statement 1 is correct, but statement 2 is false

168 **(a)**

Number of ways of dividing n^2 objects into n groups of same size is $\frac{(n^2)!}{(n!)^n n!}$

Now number of ways of distributing these n groups among n persons is $\left[\frac{(n^2)!}{(n!)^n n!}\right] n! = \frac{(n^2)!}{(n!)^n}$ which is always an integer

Also we know that product of r is divisible by $r!$

Now, $(n^2)! = 1 \times 2 \times 3 \times 4 \cdots n^2$

$$= 1 \times 2 \times 3 \cdots n \times (n+1)(n+2) \cdots 2n \times (2n+1)(2n+2) \cdots 3n \times \left(n^2 - (n^2-1)\right)\left(n^2 - (n^2-1)\right) \cdots n^2$$

Thus, in $n^2!$ there are n rows each consisting product of n integers. Each row is divisible by $n!$

Hence $(n^2)!$ is divisible by $(n!)^n$ or $\frac{(n^2)!}{(n!)^n}$ is a natural number

Hence, both statements are correct and statement 2 is correct explanation of statement 1

169 **(d)**

Since, the number of ways that child can buy the six ice-creams is equal to the number of different ways of arranging 6A's and 4B's in a row

$\therefore$ Number of ways to arrange 6A's and 4B's in a row

$= \frac{10!}{6!\,4!} = {}^{10}C_4$

And number of integral solution of the equation

$x_1 + x_2 + x_3 + x_4 + x_5 = 6$

$= {}^{6+5-1}C_{5-1}$

$= {}^{10}C_4 \neq {}^{10}C_5$

Statement I is false and Statement II is true

170 **(b)**
We have $a + b + c = 30$, and $a \neq b \neq c$. Let $a < b < c$

Now relative value of a, b, c are tabulated ad follows

a	b	c	Number of triplets (a, b, c)
1	2	27	
	3	26	
	4	25	
	⋮	⋮	
	14	15	13
2	3	25	
	4	24	
	⋮	⋮	
	13	15	11
3	4	23	
	5	22	
	⋮	⋮	
	13	14	10
4	5	21	
	6	20	
	⋮	⋮	
	12	14	8
5	6	19	
	7	18	
	⋮	⋮	
	12	13	7
6	7	17	
	⋮	⋮	
	11	13	5
7	8	15	
	⋮	⋮	
	11	12	4
8	9	13	
	10	12	2
9	10	11	1
		Total	61

Statement 2 is correct but it does not explain statement 1

171 **(a)**
The batting order of 11 players can be decided in 11! ways. Now Yuvraj, Dhoni and Pathan can be arranged in 3! ways. But the order of these three players is fixed, i.e., Yuvraj-Dhoni-Pathan. Now 11! Answer is 3! Times more, hence the required answer is 11!/3!

172 **(b)**
Exponent of 2 is 50! is

$$\left[\frac{50}{2}\right] + \left[\frac{50}{4}\right] + \left[\frac{50}{8}\right] + \left[\frac{50}{16}\right] + \left[\frac{50}{32}\right]$$
$$= 25 + 12 + 6 + 3 + 1 = 47$$

And exponent of 5 in 50! is

$$\left[\frac{50}{5}\right] + \left[\frac{50}{25}\right] = 12$$

Now number of zeros in the end of 50! is equal to exponent of 10 in 50! which is equal to exponent of 5 in 50! Therefore, number of zeros in the end depends on exponent of 5, but not on the exponent of 2

Hence both statement 1 and 2 are true; but statement 2 is not a correct explanation for statement 1

173 **(a)**
The number of non-negative integral solutions

=coefficient of x^rin $(1 + x + x^2 + \dots)^n$

=coefficient of x^r in $(1 - x)^{-n}$

$= {}^{n+r-1}C_r$ or ${}^{n+r-1}C_{r-1}$

174 **(a)**
When n persons be seated at a round table, so that all shall not have the same neighbours in any two arrangements, clock-wise arrangements are considered to be the same, which is the case when n different beads are arranged in the circle. Hence, number of ways is $(n - 1)!/2$

175 **(c)**

Number of objects from 21 different objects	Number of objects form 21 identical objects	Number of ways of selections

10	0	${}^{21}C_{10} \times 1$
9	1	${}^{21}C_9 \times 1$
⋮	⋮	⋮
0	10	${}^{21}C_0 \times 1$

Thus, total number of ways of selection is ${}^{21}C_0 + {}^{21}C_1 + {}^{21}C_2 + \cdots + {}^{21}C_{10} = 2^{20}$.

Statement 2 is false, as given series is not exact half series.

176 **(d)**

Since each student receive at least one toy. Then firstly we give each student one toy and the remaining 7 toys can be distributed in three students in ${}^{7-1}C_{3-1} = {}^6C_2$ ways.

Hence, statement I is false and statement II is true

177 **(d)**

Number of ways of arranging 21 identical objects when r is identical of one type and remaining are identical of second type is $\frac{21!}{r!(21-r)!} = {}^{21}C_r$ which maximum when $r = 10$ or 11

Therefore, ${}^{13}C_r = {}^{13}C_{10}$ or ${}^{13}C_{11}$, hence maximum value of ${}^{13}C_{11}$ is ${}^{13}C_{10} = 286$

Hence, statement 1 is false. Obviously statement 2 is true

178 **(c)**

Number of required ways

$= $ (1women, 4 men) or (2 women, 3 men)

or (3 women, 2 men) or (4 women, 1 man)

$= {}^4C_1 \times {}^8C_4 + {}^4C_2 \times {}^8C_3 + {}^4C_3 \times {}^8C_2 + {}^4C_4 \times {}^8C_1$

$= 736$

179 **(a)**

Since, $a + b + c = 3,4,5,6,7,8$

$\therefore$ Required number of triplets

$= {}^2C_2 + {}^3C_2 + {}^4C_2 + {}^5C_2 + {}^6C_2 + {}^7C_2$

$= {}^3C_3 + {}^3C_2 + {}^4C_2 + {}^5C_2 + {}^6C_2 + {}^7C_2$

$= {}^8C_3 = 56$

180 **(a)**

Statement 2 is correct as when $3^a, 3^b, 3^c$ are in G.P., we have $\left(3^b\right)^2 = (3^a)(3^c) \Rightarrow 2b = a + c \Rightarrow$ a, b, c are in A.P. Thus, selecting three numbers in G.P. from $\{3^1, 3^2, 3^3, \dots, 3^{100}, 3^{101}\}$ is equivalent to selecting 3 numbers from $\{1, 2, 3, \dots, 101\}$ which are in A.P. Now, a, b, c are in A.P. if either a and c are odd or a and c are even.

Number of selection ways of 2 odd numbers is ${}^{51}C_2$

Number of selection ways of 2 even numbers is ${}^{50}C_2$. Hence, total number of ways is ${}^{51}C_2 + {}^{50}C_2 = 1275 + 1225 = 2500$

181 **(b)**

$1400 = 2^3 5^2 7$

The number of ways in which 1400 can be expressed as a product of two positive integers is

$$\frac{(3+1)(2+1)(1+1)}{2} = 12$$

Statement 2 is correct but does not explain statement 1 as it just gives the information about the prime factor about which 1400 is divisible

182 **(c)**

Product of nj consecutive normal number

$= (m+1)(m+2)(m+3) \dots (m+n), m \in$ whole number

$$= \frac{(m+n)!}{m!} = n! \times \frac{(m+n)!}{m!\,n!}$$

$= n! \times {}^{m+n}C_m$

$\Rightarrow$ Product is divisible by $n!$, then it is always divisible by $(n-1)!$ but not by $(n+1)!$

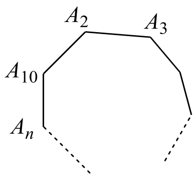
183 **(c)**

1. If polygon has n sides, then number of diagonals is ${}^nC_2 - n = 35$ (given). Solving we get $n = 10$. Thus, there are 10 vertices, from which ${}^{10}C_3 (= 120)$ triangles can be formed
2. Four vertices can be selected in ${}^{10}C_4 (= 210)$ ways. Using these four vertices two diagonals can be formed, which has exactly one point of intersection lying inside the polygon

Hence, number of points of intersections of diagonal which lies inside the polygon is

$^{10}C_4 \times 1 = 210$

c.



Suppose one of the sides of the triangle is A_1A_2. Then third vertex cannot be A_3 or A_{10}. Thus, for the third vertex six vertices are left. There are six triangles in which side A_1A_2 is common with that of polygon. Similarly, for each of the sides $A_2A_3, A_3A_4, \dots, A_9A_{10}$ there are six triangles. Then total number of triangles is 6

d. Triangles $A_1A_2A_3, A_2A_3A_4, \dots, A_8A_9A_{10}$ have two sides common with that of polygon. Hence, there are 10 such triangles

184 **(a)**

The number of possible outcomes with 2 on at least one dice

= The total number of outcomes with 2 on at least one dice

= (The total number of outcomes)−(The number of outcomes in which 2 does not appear on any dice)= $6^4 - 5^4 = 1296 - 625 = 671$

Any selection of four digits from the 10 digits 0, 1, 2, 3, ...9 gives one number. So, the required number of numbers is $^{10}C_4$

Let the number be $n = pqr$. Since $p + q + r$ is even, p can be filled in 9 ways and q can be filled in 10 ways

r can be filled in number of ways depending upon what is the sum of p and q.

If $p + q$ is odd, then r can be filled with any one of five odd digits.

If $p + q$ is even, then r can be filled with any one of five even digits.

In any case, r can be filled in five ways.

Hence, total number of numbers is $9 \times 10 \times 5 = 450$

After fixing 1 at one position out of 4 places 3 places can be filled by $^7P_3$ways. But for some numbers whose fourth digit is zero, such type of ways is $^6P_2$.

Therefore, total number ways is $^7P_3 - {}^6P_2 = 480$

185 **(d)**

1. Total number of required functions is equal to number of derangement of 5 objects, which is given by

$5!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} = 44\right)$

2. $x_1x_2x_3 = 2 \times 35 \times 7 = 2 \times 49 \times 5 = 10 \times 7 \times 7 = 14 \times 7 \times 5$

So total number of solution set is $3 \times 3! + 3!/2! = 21$

3. $3780 = 2^2 \times 3^3 \times 5 \times 7$

Number of divisors which are divisible by 2 but not by 3 is

$2 \times 2 \times 2 = 8$

Number of divisors which are divisible by 3 but not by 2 is $3 \times 2 \times 2 = 12$

Number of divisors which are divisible by 2 as well as 3 is $2 \times 3 \times 2 \times 2 = 24$

Hence, total number of divisors is 44

4. $4\lambda + 2 = 2(2\lambda + 1)$ =odd multiple of 2

Thus, total number of divisors is $1 \times 5 \times 11 - 1 = 54$. (1 is subtracted and powers of three and five are zero each and this will make $\lambda = 0$)

186 **(d)**

1. Number of subjective functions is $3^6 - {}^3C_1(3-1)^6 + {}^3C_2(3-2)^6 = 729 - 192 + 3 = 540$

2. If $f(a_i) \neq b_i$, then pre-image a_1, a_2, a_3 cannot be assigned images b_1, b_2, b_3 respectively

Hence, each of a_1, a_2, a_3 can be assigned images in 2 ways a_4, a_5, a_6 can be assigned images in 3 ways each

Hence number of functions is $2^3 3^3 = 216$

3. One-one functions are not possible are pre-images are more than images.

4. Number of many-one functions is

Total number of functions−number of one-one functions

$= 3^6 - {}^6P_3 = 729 - 120 = 609$

187 **(a)**

1. ${}^{10}C_2 - {}^4C_2 + 1 = 45 - 6 + 1 = 40$

2. $1 \times {}^{10}C_2 = 45$

3. $2 \times {}^6C_2 = 30$

4. ${}^6C_2 \times 4 = 60$

188 **(b)**

We have,

$a =^x + {}^2P_{x+2} = (x+2)!,$

$b = {}^xP_{11} = \frac{x!}{(x-11)!}$

$C = {}^{x-11}P_{x-11} = (x-11)!$

Now,

$a = 182\, bc \Rightarrow (x+2)!$
$= 182 \times \frac{x!}{(x-11)!}(x-11)!$

$\Rightarrow (x+2)! = 182x! \Rightarrow (x+2)(x+1) = 182$
$\Rightarrow x = 12$

× |×|×|

Even digits occupy odd places shown by crosses. Crosses can be filled in $2 \times 2 \times 1$ ways (∵ 0 cannot go in the first place from the left). The remaining places can be filled in 3! ways

Therefore, the required number of numbers is $2 \times 2 \times 1 \times 3! = 24$

Total number of numbers without restriction is 2^5. Two numbers have all the digits equal. So, the required number of numbers is $2^5 - 2$.

Let number of sides of polygon be n. Number of sides of polygon is equal to number of vertices of polygon. Now number of diagonals of polygon is

${}^nC_2 - n = 54$
$\Rightarrow \frac{n(n-1)}{2} - n = 54$
$\Rightarrow n^2 - 3n - 162 = 0$
$\Rightarrow (n-12)(n+9) = 0$
$\Rightarrow n = 12$

189 **(c)**

There are two case

1. 5, 4, 1, 1, 1

Number of ways of selection is $5!/3! = 20$

2. 5, 2, 2, 1, 1

Number of ways of selection is $5!/2!\,2!\,1 = 30$

Hence, total number is $20 + 30 = 50$

Select 4 pairs in ${}^5C_4 = 5$ ways. Now select exactly one shoe from each of the pairs selected in $({}^2C_1)^4$ ways. This will fulfill the condition. Hence required answer
$5 \times 16 = 80$

The first child C_1 can be chosen in 3 ways; his/her mother can be interviewed in 5 ways; the second child C_2 can be chosen in 2 ways, and his/her mother can be interviewed in 3 ways

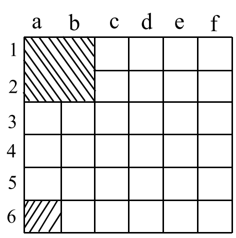
Hence total number of ways is $3 \times 5 \times 2 \times 3 = 90$

Required number of ways is $5! - 4! - 3! = 120 - 64 - 6 = 90$. (Number will be less then 56000 only if either 4 occurs on the first place or 5, 4 occurs on the first two places)

190 **(b)**

1. Number of rectangles is equal to number of ways we can select two vertical lines and two horizontal lines. Total number of ways is ${}^7C_2 \times {}^7C_2 = 441$

b.

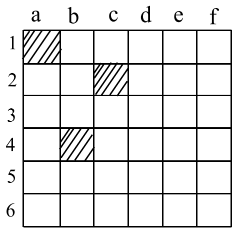


If the square is of 1 sq. units like $a6$, then we have such $6 \times 6 = 36$ squares.

If the square is of 4 sq. units like shaded region of the squares $a1, a2, b1, b2$, then we have such 5 squares in the belt formed by rows 1 and 2. Similarly we have 4 more belts $23, 34, 45$ and 56. Hence, there are $5 \times 5 = 25$ such squares.

Similarly we have $4 \times 4, 3 \times 3, 2 \times 2, 1 \times 1$ squares of increasing sizes

Hence, total number of squares is $1 + 4 + 9 + 16 + 25 + 36 = 91$

c. 

The first square can be selected in 36 ways. If one such square $a1$ is selected, we are left with 25 squares; second square cannot be selected from row 1 and column a. If second square is $c2$, we are left with 16 squares, from which third square can be selected, e.g., $b4$

Hence, number of ways of selections is $36 \times 25 \times 16$. But in this one-by-one type of selection order of selection is also consider. Hence, actual number of ways is $(36 \times 25 \times 16)/3! = 2400$

d. Given number of ways is equivalent to selecting 11 squares form 36 squares if no row remains empty

Suppose $x_1, x_2, x_3, x_4, x_5, x_6$ be the number of squares selected from the $1^{\text{st}}, 2^{\text{nd}}, 3^{\text{rd}}, 4^{\text{th}}, 5^{\text{th}}$ and 6^{th} row

Then we must have $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 11$ (where$1 \le x_i \le 6$)

The number of positive integral solutions of the above equation is ${}^{11-1}C_{6-1} = {}^{10}C_5$

191 **(c)**

$E_5(100) = \left[\frac{100}{5}\right] + \left[\frac{100}{5^2}\right]$

$= 20 + 4 = 24$

192 **(c)**

The letters in the word INDIA are (A, D, I, I, N)

Required permutation

=coefficient of x^3in $3!\left(1 + \frac{x}{1!} + \frac{x^2}{2!}\right)\left(1 + \frac{x}{1!}\right)^3$

=coefficient of x^3in $\frac{6(2+2x+x^2)(1+x)^3}{2}$

=coefficient of x^3 in $3[\{1 + (1 + x)^2\}(1 + x)^3]$

=coefficient of x^3 in $3\{(1 + x)^3 + (1 + x)^5\}$

$= 3({}^3C_3 + {}^5C_3) = 3(1 + 10) = 33$

193 **(c)**

Considering CC as single object, U, CC, E can be arranged in 3! ways

$\times$ U $\times$ CC $\times$ E $\times$

Now the three S are to be placed in four available places. Hence, required number of ways $= {}^4P_3 = 24$

194 **(a)**

When one all rounder and ten players from bowlers and batsmen play, number of ways is ${}^4C_1\,{}^{14}C_{10}$

When one wicketkeeper and 10 players from bowlers and batsmen play, number of ways is ${}^2C_1\,{}^{14}C_{10}$

When one all rounder, one wicketkeeper and nine from batsmen and bowels play, number of ways is ${}^4C_1\,{}^2C_1\,{}^{14}C_9$

When all elven players play from bowlers and batsmen then, number of ways is ${}^{14}C_{11}$

Total number of selections is ${}^4C_1\,{}^{14}C_{10} + {}^2C_1\,{}^{14}C_{10} + {}^4C_1\,{}^2C_1 + {}^{14}C_9\,{}^{14}C_{11}$

195 **(c)**

Seven persons can be selected for first table in ${}^{12}C_7$ ways. Now these seven persons can be arranged in 6! ways. The remaining five persons can be arranged on the second table in 4! ways. Hence, total number of ways is ${}^{12}C_5 6!\,4!$

196 **(b)**

If no box remains empty, then we can have (1, 1, 3) or (1, 2, 2) distribution pattern

When balls are different and boxes are identical, number of distributions is equal to number of divisions in (1, 2, 3) or (1, 2, 2) ways. Hence, total number of ways is

$$\frac{5!}{1!\cdot 2!\cdot 3!}+\frac{5!}{(2!)^2 1!\cdot 2!}=25$$

199 **(b)**

$6=0(2)+6(1)=1(2)+4(1)=2(2)+2(1)$
$=3(2)+0(1)$

Number of 2s	Number of 1s	Number of permutations
0	6	1
1	4	$\frac{5!}{4!}=5$
2	2	$\frac{4!}{2!\,2!}=6$
3	0	$\frac{3!}{3!}=1$
		Total = 13

$\therefore f(6)=13$

Now, $f(f(6))=f(13)$

Number of 1s	Number of 2s	Number of permutations
13	0	1
11	1	$\frac{12!}{11!}=12$
9	2	$\frac{11!}{9!\,2!}=55$
7	3	$\frac{10!}{7!\,3!}=120$
5	4	$\frac{9!}{5!\,4!}=126$
3	5	$\frac{8!}{3!\,5!}=56$
1	6	$\frac{7!}{6!}=7$
		Total = 377

$\therefore f(f(6))=f(13)=377$

$f(1)=1(1)$

$f(2)=2(1{,}1 \text{ or } 2)$

$f(3)=3(1,1,1 \text{ or } 2,1 \text{ or } 1,2)$

$f(4)=5$(explained in the paragraph)

By taking higher value of n in $f(n)$, we always get more value of $f(n)$. Hence, $f(x)$ is one-one. Clearly, $f(x)$ is into

200 **(d)**

m is even. Let $m=2k$, where k is some positive integer. We can choose n seats out of the k seats to the left of the middle seat in ${}^kC_n$ ways. Each chosen seat can be either empty or occupied. Thus, the number of ways of choosing seats for n persons is equal to $({}^kC_n)(2^n)$. We can arrange n persons at these seats in ${}^nP_n$ ways. Hence, the required number of arrangements is given by

$(n!)({}^kC_n)(2^n)=({}^kP_n)(2^n)=({}^{m/2}P_n)(2^n)$

201 **(a)**

Let $n=2k$, where k is some positive integer. Let x_0 denote the number of empty seats to the left of the first pair, $x_i(1\le i\le k-1)$ the number of empty seats between i^{th} and $(i+1)^{\text{th}}$ pair and x_k the number of empty seats to the right of the k^{th} pair. Note that $x_0,x_k\ge 0, x_i\ge 1(1\le i\le k-1)$ and

$x_0+x_1+\cdots+x_k=(m-2k)$ (2)

The number of integral solutions of Eq. (2) is ${}^{m-2k+1}C_k$

Since we can permute n persons in $n!$ ways, the required number of ways is

$$({}^{m-2k+1}C_k)(2k)!=\frac{(m-2k+1)!}{k!\,(m-3k+1)!}(2k)!$$

$$=\frac{(2k)!}{(k)}\frac{(m-2k+1)!}{(m-3k+1)!}$$

$$=({}^{2k}P_k)({}^{m-2k+1}P_k)$$

$$=({}^nP_{n/2})({}^{m-n+1}P_{n/2})$$

202 **(c)**

Let x_0 denote the number of empty seats to the left of the first person, $x_i(1\le i\le n-1)$ the number of empty seats between the i^{th} and $(i+1)^{\text{th}}$ persons of the n^{th} person. Then

$x_0,x_n\ge 0$ and $x_i\ge 1$ for $1\le i\le n-1$

$x_0+x_1+\cdots+x_n=(m-n)$ (1)

Putting $x_1=y_i+1$, where $y_i\ge 0$, we have

$$x_0+y_1+\cdots+y_{n-1}+x_n + (1+1+1+\cdots+(n-1)\text{times}) = (m-n)$$

$\Rightarrow x_0+y_1+\cdots+y_{n-1}+x_n=m-n-(n-1)$

$\Rightarrow x_0+y_1+\cdots+y_{n-1}+x_n=m-2n+1$

Now number of non-negative integral solutions is ${}^{n+1+(m-2n+1)-1}C_{n+1-1}={}^{m-n+1}C_n$. Since we can permute n persons in $n!$ ways, the required number of ways is

$$({}^{m-n+1}C_n)(n!)=\frac{(m-n+1)!}{n!\,(m-2n+1)!}n!$$

$$=\frac{(m-n+1)!}{(m-2n+1)!}$$

203 **(c)**

Consonants can be places in 7!/(2! 2!) ways. Then there are 8 places and 4 vowels. Therefore, number of ways is

$$\frac{7!}{2!\,2!}\ {}^8C_4\frac{4!}{2!}$$

204 **(b)**
Make a group of both M's and another group of T's. Then except A's we have 5 letters remaining. So M's, T's and the letters except A's can be arranged in 7! ways. Therefore, total number of arrangements is $7! \times {}^{8}C_{2}$

205 **(d)**
Since there are 5 even places and 3 pairs of repeated letters, therefore at least one of these must be at an odd place. Therefore, the number of ways is $11!/(2!\,2!\,2!)$

206 **(9)**

x	x	

When two consecutive digits are 11, 22, etc
$= 9 \cdot 9 = 81$

	0	0

When two consecutive digits are $0\ 0 = 9$

	x	x

When two consecutive digits are $11, 22, 33, \dots = 9 \cdot 8 = 72$
Total number of number are $N = 162$

207 **(3)**

1. He can invite 2 friends three times each

Lets select first those 2 friends in ${}^{3}C_{2}$ ways

Now these two friends each three time can be invited on 6 days in $\frac{6!}{3!3!}$

Thus total number of ways 2 friends can be invited three times $= {}^{3}C_{2}\frac{6!}{3!3!}$

2. Another possibility is that he invites all three friends 2 times each

Then number of ways $= \frac{6!}{2!2!2!}$

3. One more possibility is that he invites one friend three times, one two times and one three times

Then number of ways $= \frac{6!\times 6}{3!2!}$

Hence total number of ways $= {}^{3}C_{2} \times \frac{6!}{3!3!} + \frac{6!\times 6}{3!2!} + \frac{6!}{2!2!2!} = 510$

208 **(6)**
Let T and S denotes teacher and student respectively
Then we have following possible patterns according to question

1. T S S T S S T S S
2. S T S S T S S T S
3. S S T S S T S S T

Hence total number of arrangements are
$3 \cdot (3!)6! = 18 \times 6!$

$\Rightarrow k = 6$

209 **(9)**
We have $2^{n} - 2 = 510$;
$\Rightarrow 2^{n} = 512$
$\Rightarrow n = 9$

210 **(8)**
Including the two specified people, 4 others can be selected in ${}^{5}C_{4}$ ways
The two adjacent seats can be taken in 4 ways and the two specified people can be arranged in 2! ways, remaining 4 people can be arranged in 4! ways
$\Rightarrow 5C_{4}. 4.2!\,4! = 5!\,8 = 8!\,.5!$

211 **(5)**
Let r no. of books of algebra and $20 - r$ of calc. no. of selections $= {}^{r}C_{5} \times {}^{20-r}C_{5}$
Which has maximum value when $r = 10$

212 **(4)**
Number of arrangements are $2n!\,n!$
Given that $2n!\,n! = 1152$
$\Rightarrow (n!)^{2} = 576$
$\Rightarrow n! = 24$
$\Rightarrow n = 4$

213 **(5)**
A AAAA | B BBBBB
Since word reads the same backwards and forwards, the middle digit must be A

M
× × × × × ↓ × × × × ×

So that even number of A's and B's are available for arrangement about middle position M in the above figure
Take AABBB on one side of M (6^{th} place) and then their image about M in a unique way
∴ Number of ways N$= \frac{5!}{2!\cdot 3!} = 10$

214 **(7)**
x denotes the number of times he can take unit step and y denotes the number of times he can take 2 steps, then $x + 2y = 7$,
Then we must have $x = 1, 3, 5$,
If $x = 1$, the steps will be 1 2 2 2

$\Rightarrow$ number of ways $= \frac{4!}{3!} = 4$

If $x = 3$, the steps will be 1 1 1 2 2

$\Rightarrow$ number of ways $= \frac{5!}{2!.3!} = 10$

If $x = 5$, the steps will be 1 1 1 1 1 2

$\Rightarrow$ number of ways $= {}^6C_1 = 6$

If $x = 7$, the steps will be 1 1 1 1 1 1 1

$\Rightarrow {}^7C_0 = 1$

Hence total number of ways $= N = 21$

$\Rightarrow$ N/3 = 7

215 **(8)**

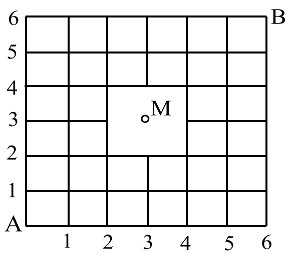
Here $P_n = {}^{n-2}C_3$ and $P_{n+1} = {}^{n-1}C_3$

hence ${}^{n-3}C_3 - {}^{n-2}C_3 = 15$

$\Rightarrow {}^{n-2}C_3 + {}^{n-2}C_2 - {}^{n-2}C_3 = 15$

$\Rightarrow {}^{n-2}C_2 = 15 \Rightarrow n = 8$

216 **(5)**



Here the path which leads from A to B is of length-12

Now without considering the constrain of passing through the point M, number of ways in which we can reach B from A is equal to number of ways we can select 6 steps from left to right and 6 from bottom to top which is equal to ${}^{12}C_6$

Now we can reach from A to M is 6 steps in ${}^6C_3$ ways and can reach from M to B in ${}^6C_3$ ways

Hence we can reach from A to B through M in ${}^6C_3 \times {}^6C_3$ ways

Hence required number of ways $= {}^{12}C_6 - [{}^6C_3 \times {}^6C_3] = 924 - 400 = 524$

217 **(5)**

${}^nP_r = {}^nP_{r+1}$

$\Rightarrow \frac{n!}{(n-r)!} = \frac{n!}{(n-r-1)!} \Rightarrow n - r = 1$ (1)

Again ${}^nC_r = {}^nC_{r-1} = \frac{n!}{(n-r)!r!} = \frac{n!}{(n-r+1)!(r-1)!}$

$\Rightarrow \frac{1}{r} = \frac{1}{n-r+1} \Rightarrow n - 2r = -1$ (2)

Solving (1) and (2), $n = 3, r = 2$

218 **(8)**

We have $N =$ | a | b | c | d |

First place a can be filled in 2 ways i.e. 4,5, $(4000 \le N < 6000)$

For b and c, total possibilities are '6' $(3 \le b < c \le 6)$

i.e. 34, 35, 36, 45, 46, 56

last place d can be filled in 2 ways i.e. 0, 5 (N is a multiple of 5)

hence, total numbers $= 2 \times 6 \times 2 = 24 = N$ then $N/3 = 8$

219 **(7)**

There are 2 women and let number of men are n

According to question

$2 \times {}^nC_2 = 66 + 2 \times {}^nC_1 \times {}^2C_1$

$\Rightarrow 2.\frac{n(n-1)}{2} = 2[33 + 2n]$

$\Rightarrow \frac{n(n-1)}{1.2} = 33 + n(2)$

$\Rightarrow n^2 - 5n - 66 = 0 \Rightarrow (n-11)(n+6) = 0$

$\therefore n = 1(\because sn > 0)$

Total participants $= 2 + 11 = 13$

220 **(8)**

To form a triangle, 3 points out of 5 can be chosen in ${}^5C_3 = 10$ ways

But of these, the three points lying on the 2 diagonals will be collinear

So $10 - 2 = 8$ triangles can be formed

221 **(8)**

We have ${}^nC_2 = 28$

$\Rightarrow n = 8$ (as there are 7 days in week)

222 **(9)**

We have A's $= 2$; B's $= 4$; C's $= 2$

Total words formed $= \frac{8!}{4!2!2!} = 420$ (1)

Let ABBC= '×'

Number of ways in which ×ABBC can be arranged $= \frac{5!}{2!} = 60$ but this includes ×ABBC and ABBC×

But this includes ABBCABBC is counted twice in 60 hence it should be 59

Hence required number of ways $= 420 - 59 = 361$

223 **(8)**

Here A is common letter in words 'SUMAN' and 'DIVYA'

Now for selecting six different letters we must select A either from word 'SUMAN' or from word 'DIVYA'

Hence for possible selections, we have

A excluded from SUMAN +A included in SUMAN

$= {}^4C_3 \cdot {}^5C_3 + {}^4C_2 \cdot {}^4C_3 = 40 + 24 = 64$

Hence $N^2 = 64 \Rightarrow N = 8$

224 **(7)**

3 women can be selected in ${}^7C_3$ ways and can be

paired with 3 men in 3! ways

Remaining 4 women can be grouped into two couples in $\frac{4!}{2!\cdot2!\cdot2!} = 3$

$\therefore$ Total $= {}^7C_3 \cdot 3! \cdot 3 = 630 = N$

Then the value of N/90 is 7

225 **(6)**

Number of numbers beginning with 1=120

1					

Number of number beginning with 2 =120

2					

Starting with 31……………….=24

3	1				

Starting with 3214……………….=2

3	2	1	4		

Finally =1

3	2	1	5	4	6

Hence unit place digit of 267th number is 6

226 **(4)**

If three numbers are in G.P., then their exponent must be in A.P.

If a, b, c are selected number in G.P., then the exponent of a and c both are either odd or even, ot otherwise exponent b will not be integer b will not be integer

Now two odd exponent (from 1, 2, 3, ..., 10) can be selected in ${}^5C_2$ ways and two even exponent can be selected in 2^5C_2 ways

Hence number of G.P.'s are $2\,{}^5C_2 = 20$

227 **(8)**

$\sum_{k=r}^{n} {}^kC_r = {}^rC_r + {}^{r+1}C_1 + {}^{r+2}C_r + \cdots + {}^nC_r$

$= 1 + {}^{r+1}C_1 + {}^{r+2}C_2 + {}^{r+3}C_3 + \cdots + {}^nC_{n-r}$

$= {}^{r+1}C_0 + {}^{r+1}C_1 + {}^{r+2}C_2 + \cdots + {}^nC_{n-r}$

$= {}^{r+2}C_1$

${}^{r+3}C_2$ and so on finally ${}^{n+1}C_{n-r}$

Now, ${}^{n+1}C_{n-r} = {}^{n+1}C_{r+1}$

$$\therefore f(n) = \sum_{r=0}^{n} {}^{n+1}C_{r+1} = {}^{n+1}C_1 + {}^{n+1}C_2 + {}^{n+1}C_3 + \cdots + {}^{n+1}C_{n+1}$$

$= {}^{n+1}C_0 + {}^{n+1}C_1 + {}^{n+1}C_2 + \cdots + {}^{n+1}C_{n+1} - 1$

$f(n) = (2^{n+1}) - 1$

$f(9) = 2^{10} - 1 = 1023 = 3 \cdot 11 \cdot 31$

Hence number of divisors are $(1+1)(1+1)(1+1) = 8$

228 **(9)**

Number of digits are 9

Select 2 places for the digit 1 and 2 in ${}^9C_2$ ways

From the remaining 7 places select any two places for 3 and 4 in ${}^7C_2$ ways

And from the remaining 5 places select any two for 5 and 6 in ${}^5C_2$ ways

Now, the remaining 3 digits can be filled in 3! Ways

$\therefore$ Total ways $= {}^9C_2 \cdot {}^7C_2 \cdot {}^5C_2 \cdot 3!$

$$= \frac{9!}{2!\cdot7!} \cdot \frac{7!}{2!\cdot5!} \cdot \frac{5!}{2!\cdot3!} \cdot 3!$$

$$= \frac{9!}{8} = \frac{9\cdot8\cdot7!}{8} = 9\cdot7!$$

229 **(8)**

Let $n(A)$ = number of divisible by 60 = $(60, 120, \ldots, 960) = 16$

$n(B)$ =number divisible by 24 $= (24, 48, \ldots, 984) = 41$

$n(A \cap B)$ =number divisible by both $= 120 + 240 + \cdots + 960 = 8$

Hence $n(A \cap B) = n(A) - n(A \cap B) = 16 - 8 = 8$