14.OSCILLATIONS

Single Correct Answer Type

1. A spring balance has a scale that can read from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance when displaced and released oscillates harmonically with a time period of 0.6 s. The mass of the body is (take $d = 10 \text{ m/s}^2$)
a) 10 kg b) 25 kg c) 18 kg d) 22.8 kg

2. Two particles P and Q describe SHM of same amplitude a and frequency v along the same straight line. The maximum distance between the two particles is $\sqrt{2}a$. The initial phase difference between them is
a) Zero b) $\pi/2$ c) $\pi/6$ d) $\pi/3$

3. A particle of mass m moving along the x-axis has a potential energy $U(x) = a + bx^2$ where a and b are positive constants. It will execute simple harmonic motion with a frequency determined by the value of
a) b alone b) b and a alone c) b and m alone d) b, a and m alone

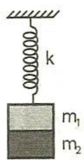
4. The time taken by a particle executing simple harmonic motion to pass from point A to B where its velocities are same, is 2 s. After another 2 s, it returns to (b). The time period of oscillation is
a) 2 s b) 4 s c) 6 s d) 8 s

5. A block is resting on a piston which executes simple harmonic motion in vertical plain with a period of 2.0 s in vertical plane at an amplitude just sufficient for the block to separate from the piston. The maximum velocity of the piston is
a) $\frac{5}{\pi}$ m/s b) $\frac{10}{\pi}$ m/s c) $\frac{\pi}{2}$ m/s d) $\frac{20}{\pi}$ m/s

6. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A \sin \omega t$ and $x_2(t) = A \sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B \sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ
a) $\sqrt{2}A, \frac{3\pi}{4}$ b) $A, \frac{4\pi}{3}$ c) $\sqrt{3}A, \frac{5\pi}{6}$ d) $A, \frac{\pi}{3}$

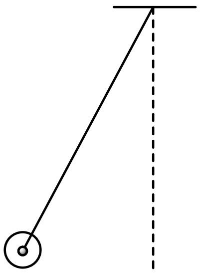
7. A particle executing SHM has velocities u and v and accelerations a and b in two of its positions. Find the distance between these two positions
a) $\frac{u^2 - v^2}{a + b}$ b) $\frac{v^2 - u^2}{a - b}$ c) $\frac{v^2 + u^2}{a + b}$ d) $\frac{v^2 - u^2}{a - b}$

8. Two masses m_1 and m_2 are suspended together by a massless spring of constant k. When the masses are in equilibrium, m_1 is removes without disturbing the system; the amplitude of vibration is:



a) $m_1 g/k$ b) $m_2 g/k$ c) $\frac{(m_1 + m_2)g}{k}$ d) $\frac{(m_2 - m_1)g}{k}$

9. A metal rod of length L and mass m is pivoted at one end. A thin disk of mass M and radius $R(< L)$ is attached at its centre to the free end of the rod. Consider two ways the disc is attached **case *A***- the disc is not free to rotate about its centre and **case *B*** – the disc is free to rotate about its centre. The rod-disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is/are true?



a) Restoring torque in case A=Restoring torque in case B

b) Restoring torque in case A<Restoring torque in case B

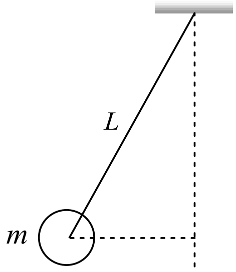
c) Angular frequency for case A<Angular frequency for case B

d) Angular frequency for case A<Angular frequency for case B

10. A particle, free to move along the x-axis, has potential energy given by $U_{(x)} = K[1 - \exp(-x^2)$ for $-\infty < x < +\propto$ where k is a positive constant of appropriate dimensions. Then
    a) For small displacement from $x = 0$, the motion is simple harmonic
    b) If its total mechanical energy is $k/2$, it has its minimum kinetic energy at the origin
    c) For any finite non-zero value of x, there is a force directed away from the origin
    d) At points away from the origin, the particle is in unstable equilibrium

11. A particle is performing SHM according to the equation $x = (3\text{ cm}) \sin\left(\frac{2\pi}{18} + \frac{\pi}{6}\right)$, where t is in second. The distance travelled by the particle in 39 s is
    a) 24 cm b) 1.5 cm c) 25.5 cm d) None of these

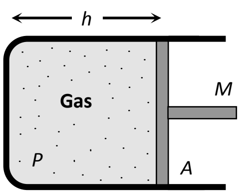
12. A ball of mass $(m)0.5\ kg$ is attached to the end of a string having length $(L)0.5\ m$. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in radian/s) is



    a) 9 b) 18 c) 27 d) 36

13. A particle performs simple harmonic motion with amplitude A and time period T. The mean velocity of the particle over the time interval during which it travels a distance of $A/2$ starting from executing position is
    a) $\frac{A}{T}$ b) $\frac{2A}{T}$ c) $\frac{3A}{T}$ d) $\frac{A}{2T}$

14. A cylindrical piston of mass M slides smoothly inside a long cylinder closed at one end, enclosing a certain mass of gas. The cylinder is kept with its axis horizontal. If the piston is disturbed from its equilibrium position, it oscillates simple harmonically. The period of oscillation will be



    a) $T = 2\pi\sqrt{\left(\frac{Mh}{PA}\right)}$ b) $T = 2\pi\sqrt{\left(\frac{MA}{Ph}\right)}$ c) $T = 2\pi\sqrt{\left(\frac{M}{PAh}\right)}$ d) $T = 2\pi\sqrt{MPhA}$

15. A thin uniform rod of mass 1kg and length 12 cm is suspended by a wire that passes through its centre and is perpendicular to its length. The wire is twisted and the rod is set oscillating. Time period of oscillation is found to be 3 s. when a flat triangular plate is suspended in same way through its centre of mass, the time period is found to be 6 s. The moment of inertia of the triangular plate about this axis is

a) $0.12\text{kg-}m^2$ b) $0.24\text{ kg-}m^2$
c) $0.48\text{ kg-}m^2$ d) Information insufficient

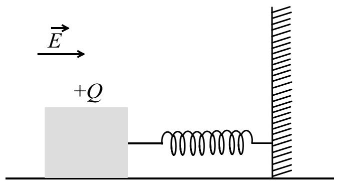
16. The potential energy of a particle of mass 1 kg in motion along the x-axis is given by: $U = 4(1 - \cos 2x)$, where x is in metres. The period of small oscillation (in seconds) is
a) 2π b) π c) $\pi/2$ d) $\sqrt{2}\pi$

17. A block of mass 1 kg hangs without vibrating at the end of a spring whose force constant is 200 N/m and which is attached to the ceiling of an elevator. The elevator is rising with an upward acceleration of g/3 when the acceleration suddenly ceases. The angular frequency of the block after the acceleration cases is
a) 13 rad/s b) 14 rad/s c) 15 rad/s d) None of these

18. A simple harmonic motion along the x-axis has the following properties: amplitude $= 0.5$ m, the time to go from one extreme position to other is 2 s and $x = 0.3$ m at $t = 0.5$ s. The general equation of the simple harmonic motion is
a) $x = (0.5\text{ m})\sin\left(\frac{\pi t}{2} + 8°\right)$ b) $x = (0.5\text{ m})\sin\left(\frac{\pi t}{2} - 8°\right)$
c) $x = (0.5\text{ m})\cos\left(\frac{\pi t}{2} + 8°\right)$ d) $x = (0.5\text{ m})\cos\left(\frac{\pi t}{2} - 8°\right)$

19. A horizontal spring-block system of mass 2 kg executes SHM. When the block is passing through its equilibrium position, an object of mass 1 kg is put it and the two move together. The new amplitude of vibration is (A being its initial amplitude)
a) $\sqrt{\frac{2}{3}}A$ b) $\sqrt{\frac{3}{2}}A$ c) $\sqrt{2}A$ d) $\frac{A}{\sqrt{2}}$

20. A wooden block performs SHM on a frictionless surface with frequency, v_0. The block carries a change +Q on its surface. If now a uniform electric field $\vec{E}$ is switched-on as shown, then SHM of the block will be



a) Of the same frequency and with shifted mean position
b) Of the same frequency and with the same mean position
c) Of changed frequency and with shifted mean position
d) Of changed frequency and with the same mean position

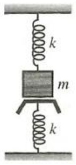
21. A particle executing SHM of amplitude 'a' has a displacement $a/2$ at $t = T/4$ and a negative velocity. The epoch of the particle is
a) $\frac{\pi}{3}$ b) $\frac{2\pi}{3}$ c) π d) $\frac{5\pi}{3}$

22. A physical pendulum is positioned so that its centre of gravity is above the suspension point. When the a pendulum is released it passes the point of stable equilibrium with an angular velocity ω. The period of small oscillations of the pendulum is
a) $\frac{4\pi}{\omega}$ b) $\frac{2\pi}{\omega}$ c) $\frac{\pi}{\omega}$ d) $\frac{\pi}{2\omega}$

23. A particle of mass m is present in a region where the potential energy of the particle depends on the x-coordinate according to the expression $U = \frac{a}{x^2} - \frac{b}{x}$, where a and b are positive constants. The particle will perform
a) Oscillatory motion but not simple harmonic motion about its mean position for small displacements
b) Simple harmonic motion with time period $2\pi\sqrt{\frac{8a^2m}{b^2}}$ about its mean position for small displacements
c) Neither simple harmonic motion nor oscillatory about its mean position for small displacements
d) None of the above

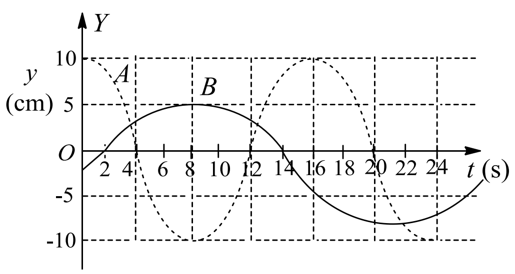
24. A mass m is suspended from a spring of force constant k and just touches another identical spring fixed to

the floor as shown in the figure. The time period of small oscillations is



a) $2\pi\sqrt{\frac{m}{k}}$ b) $\pi\sqrt{\frac{m}{k}}+\pi\sqrt{\frac{m}{k/2}}$ c) $\pi\sqrt{\frac{m}{3k/2}}$ d) $\pi\sqrt{\frac{m}{k}}+\pi\sqrt{\frac{m}{2k}}$

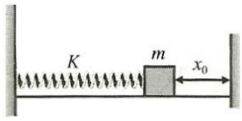
25. A block of mass m is suspended from the ceiling of an elevator (at rest) through a light spring of spring constant k. Suddenly, the elevator starts falling down with acceleration g. Then
  a) The block executes simple harmonic motion with time period $2\pi\sqrt{\frac{m}{k}}$
  b) The block executes simple harmonic motion with amplitude $\frac{mg}{k}$
  c) The block executes simple harmonic motion about its mean position and the mean position is the position when the spring acquires its natural length
  d) All of the above
26. The string of a simple pendulum is replaced by a uniform rod of length L and mass M while the bob has a mass m. It is allowed to make small oscillations. Its time period is
  a) $2\pi\sqrt{\left(\frac{2M}{3m}\right)\frac{L}{g}}$
  b) $2\pi\sqrt{\frac{2(M+3m)L}{3(M+2m)g}}$
  c) $2\pi\sqrt{\left(\frac{M+m}{M+3m}\right)\frac{L}{g}}$
  d) $2\pi\sqrt{\left(\frac{2m+M}{3(M+2m)}\right)\frac{L}{g}}$
27. Two particles move parallel to the x-axis about the origin with same amplitude 'a' and frequency ω. At a certain instant they are found at a distance $a/3$ from the origin on opposite sides but their velocities are in the same direction. What is the phase difference between the two?
  a) $\cos^{-1}\frac{7}{9}$ b) $\cos^{-1}\frac{5}{9}$ c) $\cos^{-1}\frac{4}{9}$ d) $\cos^{-1}\frac{1}{9}$
28. A particle performs SHM of amplitude A along a straight line. When it is at a distance $\sqrt{3}/2\ A$ from mean position, its kinetic energy gets increased by an amount $1/2m\omega^2A^2$ due to an impulsive force. Then its new amplitude becomes
  a) $\frac{\sqrt{5}}{2}A$ b) $\frac{\sqrt{3}}{2}A$ c) $\sqrt{2}A$ d) $\sqrt{5}A$
29. The following figure shows the displacement versus time graph for two particles A and B executing simple harmonic motions. The ratio of their maximum velocities is



  a) 3:1 b) 1:3 c) 1:9 d) 9:1
30. In problem 9, the maximum displacement and acceleration of the particle are respectively:

a) $\dfrac{10}{\pi}$ m and 5π m/s^2 b) $\dfrac{5}{\pi}$ m and $\dfrac{5\pi}{5}$ m/s^2 c) $\dfrac{10}{\pi}$ m and $\dfrac{5\pi}{2}$ m/s^2 d) $\dfrac{5}{\pi}$ m and $\dfrac{5\pi}{4}$ m/s^2

31. One end and a spring of force constant K is fixed to a vertical cal wall and the other to a body of mass m resting on a smooth horizontal surface. There is another wall at a distance x_0 from the s body. The spring is then compressed by $3x_0$ and released. The time taken to strick the wall from the instant of release is (given $\sin^{-1}(1/3) = (\pi/9)$)

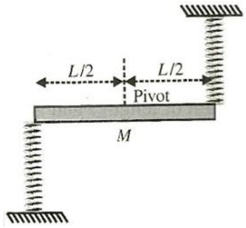


a) $\dfrac{\pi}{6}\sqrt{\dfrac{m}{K}}$ b) $\dfrac{2\pi}{3}\sqrt{\dfrac{m}{K}}$ c) $\dfrac{\pi}{4}\sqrt{\dfrac{m}{K}}$ d) $\dfrac{11\pi}{9}\sqrt{\dfrac{m}{K}}$

32. A block of mass 'm' is suspended from a spring and executes vertical SHM of time period T as shown in figure. The amplitude to the SHM is A and spring is never in compressed state during the oscillation. The magnitude of minimum force exerted by spring on the block is

a) $mg - \dfrac{4\pi^2}{T^2}mA$ b) $mg + \dfrac{4\pi^2}{T^2}mA$ c) $mg - \dfrac{\pi^2}{T^2}mA$ d) $mg + \dfrac{\pi^2}{T^2}mA$

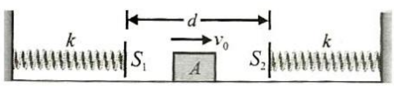
33. A uniform stick of mass M and length L is pivoted at its centre. Its ends are tied to two springs each of force constant K. In the position shown in figure, the strings are in their natural length. When the string is displaced through a small angle θ and released, the stick



a) Executes non-periodic motion
b) Executes periodic motion which is not simple harmonic
c) Executes SHM of frequency $\dfrac{1}{2\pi}\sqrt{\dfrac{6K}{M}}$
d) Executes SHM of frequency $\dfrac{1}{2\pi}\sqrt{\dfrac{K}{2M}}$

34. A particle of mass m is executing oscillations about the origin on the x-axis. Its potential energy is $U(x) = k[x]^3$, where k is a positive constant. If the amplitude of oscillation is a, then its time period T is
a) Proportional to $\dfrac{1}{\sqrt{a}}$ b) Independent to a c) Proportional to $\sqrt{a}$ d) Proportional to $a^{3/2}$

35. A block 'A' of mass m is placed on a smooth horizontal platform P and between two elastic massless spring S_1 and S_2 fixed horizontally to two fixed vertical walls. The elastic constants of the two springs are equal to k and the equilibrium distance between the two springs both in relaxed states is d. The block is given a velocity v_0 initially towards one of the springs and it then oscillations and minimum separation d_m of the springs will be



a) $T = 2\left(\dfrac{d}{v} + \pi\sqrt{\dfrac{m}{k}}\right), d_m = d$
b) $T = 2\left(\dfrac{d}{v} + 2\pi\sqrt{\dfrac{m}{k}}\right), d_m = d - v\sqrt{\dfrac{m}{k}}$

c) $T = 2\left(\frac{d}{v} + 2\pi\sqrt{\frac{m}{k}}\right), d_m = d - 2v\sqrt{\frac{m}{k}}$

d) $T = 2\pi\sqrt{\frac{m}{k}}, d_m = d$

36. While a particle executes linear simple harmonic motion

a) Its linear velocity and acceleration pass through their maximum and minimum values once in each oscillation

b) Its linear velocity and acceleration pass through their maximum and minimum values twice in each oscillation

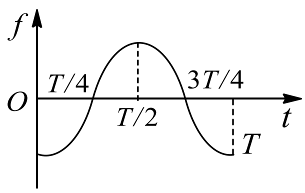
c) Its linear velocity and acceleration pass through their maximum and minimum values four times in each oscillation

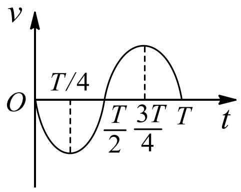
d) Its linear velocity and acceleration always attain their peak values simultaneously

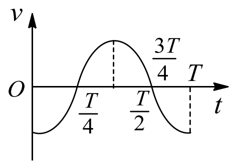
37. A particle performing simple harmonic motion having time period 3 s is in phase with another particle which also undergoes simple harmonic motion at $t = 0$. The time period of second particle is T (less than 3 s). If they are again in the same phase for the third time after 45 s, then the value of T will be

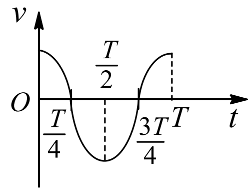
a) 2.8 s b) 2.7 s c) 2.5 s d) None of these

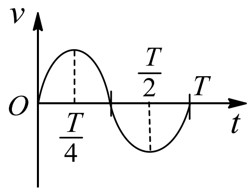
38. A body is performing simple harmonic motion with amplitude a and time period T. Variation of its acceleration (f) with time (t) is shown in figure. If at time t, velocity of the body is v, which of the following graphs is correct?



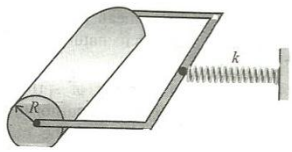
a) 

b) 

c) 

d) 

39. A soil cylinder of mass M and radius R is connected to a spring as shown in figure. The cylinder is placed on a rough horizontal surface. All the parts except the cylinder shown in the figure are alight. If the cylinder is displaced slightly from its mean position and released, so that it performs pure rolling back and forth about its equilibrium position, determine the time period of oscillation?

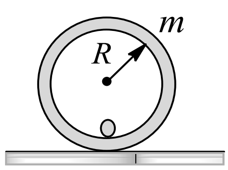


a) $2\pi\sqrt{\frac{M}{k}}$ b) $2\pi\sqrt{\frac{3M}{2k}}$ c) $2\pi\sqrt{\frac{3M}{k}}$ d) None of these

40. A particle executes SHM with time period 8 s. Initially, it is at its mean position. The ratio of distance travelled by it in the 1st second to that in the 2nd second is

a) $\sqrt{2}:1$ b) $1:(\sqrt{2}-1)$ c) $(\sqrt{2}+1):\sqrt{2}$ d) $(\sqrt{2}-1):1$

41. A thin–walled tube of mass m and radius R has a rod of mass m and very small cross section soldered on its inner surface. The side-view of the arrangement is as shown in the following figure



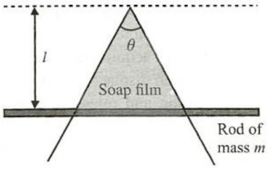
The entire arrangement is placed on a rough horizontal surface. The system is given a small angular displacement from its equilibrium position, as a result, the system performs oscillations. The time period of resulting oscillations if the tube rolls without slipping is

a) $2\pi\sqrt{\frac{4R}{g}}$ b) $2\pi\sqrt{\frac{2R}{g}}$ c) $2\pi\sqrt{\frac{R}{g}}$ d) None of these

42. A particle executed S.H.M. starting from its mean position at $t = 0$. If its velocity is $\sqrt{3}b\omega$, when $\omega = 2\pi/T$, the time taken by the particle to move from b to the extreme position on the same side is

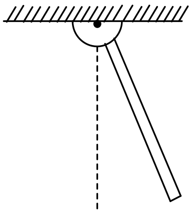
a) $\frac{5\pi}{6\omega}$ b) $\frac{\pi}{3\omega}$ c) $\frac{\pi}{2\omega}$ d) $\frac{\pi}{4\omega}$

43. A wire is bent at an angle θ. A rod of mass m can slide along the bended wire without friction as shown in figure. A soap film is maintained in the frame kept in a vertical position and the rod is in equilibrium as shown in the figure. If rod is displaced slightly in vertical direction, then the time period of small oscillation of the rod is



a) $2\pi\sqrt{\frac{l}{g}}$ b) $2\pi\sqrt{\frac{l\cos\theta}{g}}$ c) $2\pi\sqrt{\frac{l}{g\cos\theta}}$ d) $2\pi\sqrt{\frac{l}{g\tan\theta}}$

44. A metre stick swinging in vertical plane about a fixed horizontal axis passing through its one end undergoes small oscillation of frequency f_0. If the bottom half of the stick were cut off, then its new frequency of small oscillation would become

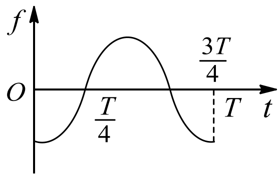


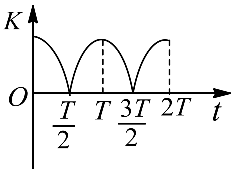
a) f_0 b) $\sqrt{2}f_0$ c) $2f_0$ d) $2\sqrt{2}f_0$

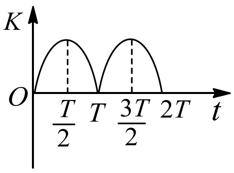
45. An object of mass 0.2 kg executes simple harmonic motion along the x-axis with a frequency of $25/\pi$ Hz. At the position $x = 0.04$ m, the object has a kinetic energy of 0.5 J and potential energy of 0.4 J. The amplitude of oscillation is

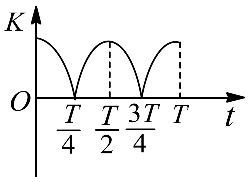
a) 0.05 m b) 0.06 m c) 0.01 m d) None of these

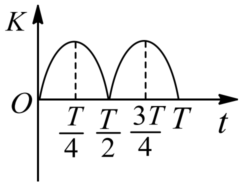
46. A particle is performing SHM with amplitude a and time period T. Its acceleration f varies with time as shown in figure. If at time t, kinetic energy of the particle is K, which of the following graphs is correct?



a) 

b) 

c) 

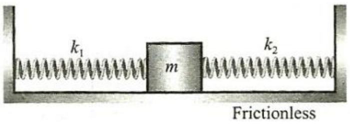
d) 

47. A certain simple harmonic vibrator of mass 0.1 kg has a total energy of 10 J. Its displacement from the mean position is 1 cm when it has equal kinetic and potential energies. The amplitude A and frequency n of vibration of the vibrator are

a) $A = \sqrt{2}$ cm, $n = \frac{500}{\pi}$ Hz b) $A = \sqrt{2}$ cm, $n = \frac{1000}{\pi}$ Hz

c) $A = \frac{1}{\sqrt{2}}$ cm, $n = \frac{500}{\pi}$ Hz d) $A = \frac{1}{\sqrt{2}}$ cm, $n = \frac{1000}{\pi}$ Hz

48. Two spring with negligible masses and force constants $k_1 = 200$N/m are attached to the block of mass $m = 10$ kg as shown in the figure. Initially the block is at rest, at the equilibrium position in which both springs are neither stretched nor compressed. At time $t = 0$, sharp impulse of 50 Ns is given to the block with a hammer along the spring

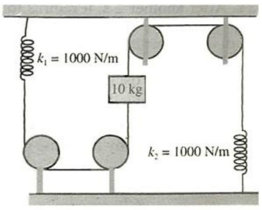


a) Period of oscillations for the mass m is $\pi/6$ s
b) Maximum velocity of the mass m during its oscillation is 10 m/s
c) Data are insufficient to determine maximum velocity
d) Amplitude of oscillation is 0.83 m

49. The instantaneous displacement x of a particle executing simple harmonic motion is given by $x = a_1 \sin \omega t + a_2 \cos(\omega t + \pi/6)$. The amplitude A of oscillation is given by

a) $\sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos\frac{\pi}{6}}$ b) $\sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos\frac{\pi}{3}}$
c) $\sqrt{a_1^2 + a_2^2 - 2a_1a_2 \cos\frac{\pi}{6}}$ d) $\sqrt{a_1^2 + a_2^2 - 2a_1a_2 \cos\frac{\pi}{3}}$

50. A block of mass 10 kg is in equilibrium as shown in figure. Initially the springs have same stretch. If the block is displaced in vertical direction by a small amount, then the angular frequency of resulting motion is (assume strings are never slack)



a) $10\sqrt{2}$ b) $10\sqrt{5}$ c) $5\sqrt{2}$ d) $5\sqrt{5}$

51. A spring is placed in vertical position by suspending it from a hook at its top. A similar hook on the bottom of the spring is at 11 cm above a table top. A mass of 75 g and of negligible size is then suspended from the bottom hook, which is measured to be 4.5 cm above the table top. The mass is then pulled down a distance of 4 cm and released. Find the approximate position of the bottom hook after 5 s? Take g = 10 m/s ^{2} and hook's mass to be negligible

a) 5 cm above the table top b) 4.5 cm above the table top
c) 9 cm above the table top d) 0.5 cm above the table top

52. A vertical spring carries a 5 kg body and is hanging in equilibrium, an additional force is applied so that the spring is further stretched. When released from this position, it performs 50 complete oscillations in 25 s, with an amplitude of 5 cm. The additional force applied is

a) 80 N b) $80\pi^2$N c) $4\pi^2$N d) 4 N

53. A mass m attached to a spring of spring constant k is stretched a distance x_0 from its equilibrium position and released with no initial velocity. The maximum speed attained by mass in its subsequent motion and the time at which this speed would be attained are, respectively,

a) $\sqrt{\frac{k}{m}}x_0, \pi\sqrt{\frac{m}{k}}$ b) $\sqrt{\frac{k}{m}}\frac{x_0}{2}, \frac{\pi}{2}\sqrt{\frac{m}{k}}$ c) $\sqrt{\frac{k}{m}}x_0, \frac{\pi}{2}\sqrt{\frac{m}{k}}$ d) $\sqrt{\frac{k}{m}}\frac{x_0}{2}, \pi\sqrt{\frac{m}{k}}$

54. A particle performs simple harmonic motion about O with amplitude A and time period T. The magnitude of its acceleration at $t = T/8$ s after the particle reaches the extreme position would be

a) $\frac{4\pi^2 A}{\sqrt{2T^2}}$ b) $\frac{4\pi^2 A}{T^2}$ c) $\frac{2\pi^2 A}{\sqrt{2T^2}}$ d) None of these

55. The K.E. and P.E. of a particle executing SHM with amplitude A will be equal when its displacement is:

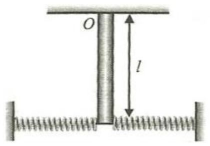
a) $A\sqrt{2}$ b) $A/2$ c) $A/\sqrt{2}$ d) $A\sqrt{2/3}$

56. A particle executing harmonic motion is having velocities v_1 and v_2 at distances is x_1 and x_2 from the

equilibrium position. The amplitude of the motion is

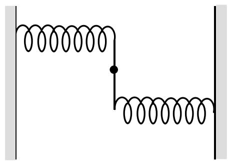
a) $\sqrt{\dfrac{v_1^2 x_2 - v_2^2 x_1}{v_1^2 + v_2^2}}$ b) $\sqrt{\dfrac{v_1^2 x^2{}_1 - v_2^2 x_2^2}{v_1^2 + v_2^2}}$ c) $\sqrt{\dfrac{v_1^2 c_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}}$ d) $\sqrt{\dfrac{v_1^2 x_2^2 + v_2^2 x_1^2}{v_1^2 + v_2^2}}$

57. A thin uniform vertical rod of mass m and length l pivoted at point O is shown in figure. The combined stiffness to the springs is equal to k. The mass of the spring is negligible. The frequency of small oscillation is



a) $\sqrt{\dfrac{3k}{2m} + \dfrac{g}{l}}$ b) $\sqrt{\dfrac{3k}{2m} + \dfrac{3g}{l}}$ c) $\sqrt{\dfrac{3k}{m} + \dfrac{3g}{2l}}$ d) $\sqrt{\dfrac{3k}{m} + \dfrac{2g}{3l}}$

58. A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constant k. The springs are fixed to rigid supports as shown in the figure, and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is



a) $\dfrac{1}{2\pi}\sqrt{\dfrac{2k}{M}}$ b) $\dfrac{1}{2\pi}\sqrt{\dfrac{k}{M}}$ c) $\dfrac{1}{2\pi}\sqrt{\dfrac{6k}{M}}$ d) $\dfrac{1}{2\pi}\sqrt{\dfrac{24k}{M}}$

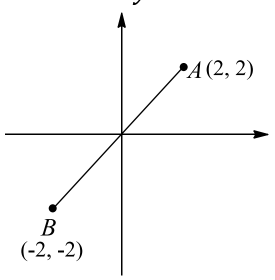
59. In problem 9, the acceleration of the particle is

a) $\dfrac{5\sqrt{3}\pi}{2}\,\text{m/s}^2$ b) $\dfrac{5\pi^2}{2}\,\text{m/s}^2$ c) $\dfrac{5\sqrt{3}\pi}{4}\,\text{m/s}^2$ d) $5\sqrt{3}\pi\ \text{m/s}^2$

60. A particle moves with a simple harmonic motion in a straight line. In the first second starting from rest it travels a distance a and in the next second it travels a distance b in the same direction. The amplitude of the motion is

a) $\dfrac{2a^2}{3b-a}$ b) $\dfrac{3a^2}{3a-b}$ c) $\dfrac{2a^2}{3a-b}$ d) $\dfrac{3a^2}{3b-a}$

61. A particle of mass $m = 2$ kg executes SHM in xy plane between points A and B under the action of force $\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath}$. Minimum time taken by the particle to move from A to B is 1 s. At $t = 0$ the particle is at $x = 2$ and $y = 2$. Then F_x, as function of time t is



a) $-4\pi^2 \sin \pi t$ b) $-4\pi^2 \cos \pi t$ c) $4\pi^2 \cos \pi t$ d) None of these

62. The potential energy of a particle executing SHM along the x-axis is given by $U = U_0 - U_0 \cos ax$. What is the period of oscillation

a) $2\pi\sqrt{\dfrac{ma}{U_0}}$ b) $2\pi\sqrt{\dfrac{U_0}{ma}}$ c) $\dfrac{2\pi}{a}\sqrt{\dfrac{m}{U_0}}$ d) $2\pi\sqrt{\dfrac{m}{aU_0}}$

63. The number of independent constituent simple harmonic motions yielding a resultant displacement equation of the periodic motion as $y = 8\sin^2(t/2)\sin(10t)$ is

a) 8 b) 6 c) 4 d) 3

64. A cork floating on the pond water executes a single harmonic motion, moving up and down over a range of 4 cm. The time period of the motion is 1 s. At $t = 0$, the cork is at its lowest position of oscillation, the position and velocity of the cork at $t = 10.5$ s, would be

a) 2 cm above the mean position, 0 m/s
b) 2 cm below the mean position, m/s
c) 1 cm above the mean position, $2\sqrt{3}\pi$ m/s up
d) 1 cm below the mean position, $2\sqrt{3}\pi$ m/s up

65. If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then, which of the following does not change with time?

a) $a^2T^2 + 4\pi^2v^2$ b) $\frac{aT}{x}$ c) $aT + 2\pi v$ d) $\frac{aT}{v}$

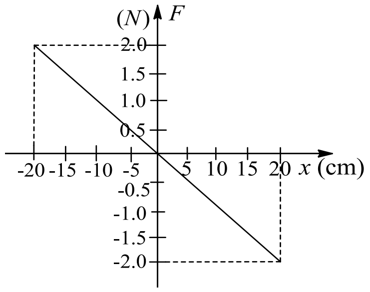
66. A body of mass 100 g attached to a spring executes SHM of period 2 s and amplitude 10 cm. How long a time is required for it to move from a point 5 cm below its equilibrium position to a point 5 cm above it, when it makes simple harmonic vertical oscillations (take g=10 m/s^2)?

a) 0.6 s b) 1/3 s c) 1.5 s d) 2.2 s

67. A simple pendulum oscillates slightly above a large horizontal metal plate. The bob is given a charge. The time period

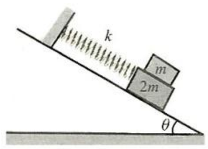
a) Has no effect, whatever be the nature of charge
b) Always decreases, whatever be the nature of charge
c) Always increases, whatever be the nature of charge
d) A increases or decreases depending upon the nature of charge

68. Figure shows the variation of force acting on a particle of mass 400 g executing simple harmonic motion the frequency of oscillation of the particle is



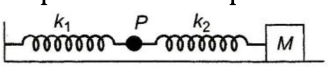
a) 4s^{-1} b) (5/2π) s^{-1} c) (1/8π) s^{-1} d) (1/2π) s^{-1}

69. The coefficient of friction between block of mass m and $2m$ is $\mu = 2\tan\theta$. There is no friction between block of mass $2m$ and inclined plane. The maximum amplitude of the two block system for which there is no relative motion between both the blocks is



a) $g\sin\theta\sqrt{\frac{k}{m}}$ b) $\frac{mg\sin\theta}{k}$ c) $\frac{3\,mg\sin\theta}{k}$ d) None of these

70. The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is



a) $\frac{k_1A}{k_2}$ b) $\frac{k_2A}{k_1}$ c) $\frac{k_1A}{k_1+k_2}$ d) $\frac{k_2A}{k_1+k_2}$

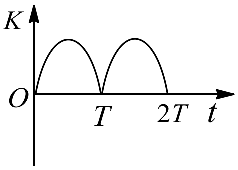
71. Two springs are made to oscillate simple harmonically due to the same mass individually. The time periods obtained are T_1 and T_2. If both the springs are connected in series and then made to oscillate by the same mass, the resulting time period will be

a) $T_1 + T_2$ b) $\frac{T_1 T_2}{T_1 + T_2}$ c) $\sqrt{T_1^2 + T_2^2}$ d) $\frac{T_1 + T_2}{2}$

72. A hallow sphere is filled with water. It is hung by a long thread to make it a simple pendulum. As the water flows out of a hole at the bottom of the sphere, the frequency of oscillation will

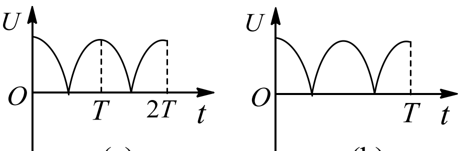
a) Go on increasing
b) Go on decreasing
c) First increases and then decreases
d) First decreases and then increases

73. A particle is performing SHM. Its kinetic energy K varies with time t as shown in the figure. Then



Period of oscillations of the particle is equal to T

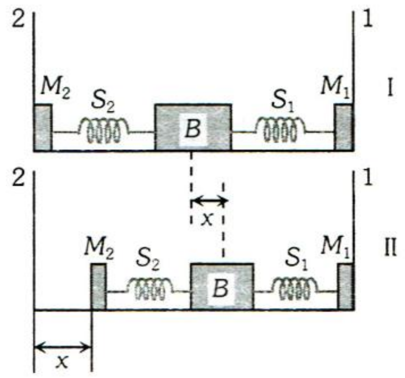
a)



(a) (b)

b) Excess potential energy U of the particle varies with time t as shown in figure (a)
c) Excess potential energy U of the particle varies with time t as shown in figure (a)
d) None of these

74. A block (B) is attached to two unstretched springs S_1 and S_2 with spring constants k and 4 k, respectively (see figure I). The other ends are attached to identical supports M_1 and M_2 not attached to the walls. The springs and supports have negligible mass. There is no friction anywhere. The block B is displaced towards wall 1 by a small distance x (figure II) and released. The block returns and moves a maximum distance y towards wall 2. Displacements x and y are measured with respect to the equilibrium position of the block B. The ratio $\frac{y}{x}$ is

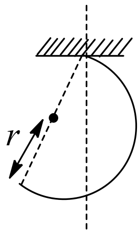


a) 4 b) 2 c) $\frac{1}{2}$ d) $\frac{1}{4}$

75. A body of mass m is released from a height h to a scale pan hung from a spring. The spring constant of the spring is k, the mass of the scale pan is negligible and the body does not bounce relative to the pan; then the amplitude of vibration is

a) $\frac{mg}{k}\sqrt{1 - \frac{2hk}{mg}}$ b) $\frac{mg}{k}$ c) $\frac{mg}{k} + \frac{mg}{k}\sqrt{1 + \frac{2hk}{mg}}$ d) $\frac{mg}{k} - \frac{mg}{k}\sqrt{1 - \frac{2hk}{mg}}$

76. A uniform semicircular ring having mass m and radius r is hanging at one of its ends freely as shown in figure. The ring is slightly disturbed so that it oscillates in its own plane. The time period of oscillation of the ring is



a) $2\pi\sqrt{\frac{r}{g\left(1+{}^{1}/_{\pi^2}\right)}}$

b) $2\pi\sqrt{\frac{r}{g\left(1-{}^{4}/_{\pi^2}\right)^{1/2}}}$

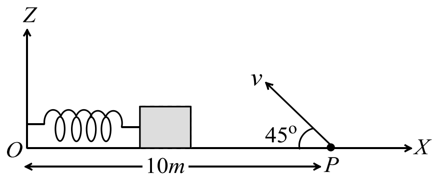
c) $2\pi\sqrt{\frac{r}{g\left(1-{}^{2}/_{\pi^2}\right)^{1/2}}}$

d) $2\pi\sqrt{\frac{2r}{g\left(1+{}^{4}/_{\pi}\right)^{1/2}}}$

77. Frequency of a particle executing SHM is 10 Hz. The particle is suspended from a vertical spring. At the height point of its oscillation the spring is unstretched. Maximum speed of the particle is ($g = 10\ m/s^2$)

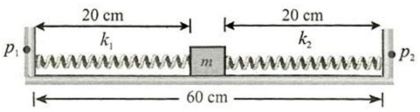
a) 2π m/s b) π m/s c) $1/\pi$ m/s d) $1/2\pi$ m/s

78. A small block is connected to one end of a massless spring of un-stretched length $4.9m$. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by $0.2m$ and released from rest at $t = 0$. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3} rad/s$. Simultaneously at $t = 0$, a small pebble is projected with speed v from point P is at angle of 45° as shown in the figure. Point P is at a horizontal distance of 10 m from O. If the pebble hits the block at $t = 1s$, the value of v is (take $g = 10m/s^2$)



a) $\sqrt{50}m/s$ b) $\sqrt{51}m/s$ c) $\sqrt{52}m/s$ d) $\sqrt{53}m/s$

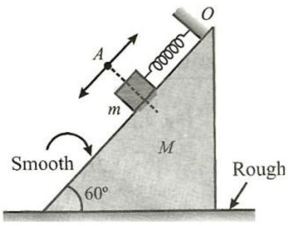
79. Two spring, each of unstretched length 20 cm but having different spring constants k_1 =1000 N/m and k_2=3000 N/m, are attached to two opposite faces of a small block of mass $m = 100$ g kept on a smooth horizontal surface as shown in the figure



The outer ends of the two springs are now attached to two attached to two pins P_1 and P_2 whose locations are shown in the figure. As a result of this, the block acquires a new equilibrium position. The block has been displaced by small amount from its equilibrium position and released to perform simple harmonic motion; then

a) New equilibrium position is at 35 cm from P_1 and time period of simple harmonic motion is $\pi/100$ s
b) New equilibrium position is at 20 cm from P_1 and time period of simple harmonic motion is $\pi/100$ s
c) New equilibrium position is at 35 cm from P_1 and time period of simple harmonic motion is $\pi/26$ s
d) New equilibrium position is at 30 cm from P_1 and time period of simple harmonic motion is $\pi/26$ s

80. A block of mass m, attached to a fixed position O on a smooth inclined wedge of mass M, oscillates with amplitude A and linear frequency f. The wedge is located on a rough horizontal surface. If the angle of the wedge is 60°, then the force of friction acting on the wedge is given by (coefficient of static friction $= \mu$)



a) $\mu(M+m)g$
b) $\frac{1}{2}m\omega^2 A\sin\omega t$
c) $\mu\left[(M+m)g+\frac{\sqrt{3}}{2}m\omega^2 A\sin\omega t\right]$
d) $\mu(M+m)\omega^2 Asin\,\omega t$

81. A block of mass 4 kg hangs from a spring of spring constant $k = 400$ N/m. The block is pulled down through 15 cm below and released. What is its kinetic energy when the block is 10 cm above the equilibrium position?
a) 5 J
b) 2.5 J
c) 1 J
d) 1.9 J

82. The displacement y of a particle executing periodic motion is given by $y = 4\cos^2(t/2)\sin(1000t)$. This expression may be considered to be a result of the superposition of........ independent harmonic motions
a) Two
b) Three
c) Four
d) Five

83. A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation $y = k\,t^2, (k = 1\text{ ms}^{-2})$ where y is the vertical displacement. The time period now becomes T_2. The ratio of $\frac{T_1^2}{T_2^2}$ is $(g = 10\text{ ms}^{-2})$
a) $6/5$
b) $5/6$
c) 1
d) $4/5$

84. A simple pendulum is making oscillations with its bob immersed in a liquid of density n times less than the density of the bob. What is its period?
a) $2\pi\sqrt{\frac{l}{ng}}$
b) $2\pi\sqrt{\frac{l}{\left(1-\frac{1}{n}\right)g}}$
c) $2\pi\sqrt{\frac{ln}{g}}$
d) $2\pi\sqrt{\frac{l}{(n-1)g}}$

85. One end of a long metallic wire of length L is tied to the ceiling. The other end is tied to massless spring of spring constant K. A mass m hangs freely from the free end of the spring. The area of cross-section and Young's modulus of the wire are A and Y respectively. If the mass is slightly pulled down and released, it will oscillate with a time period T equal to
a) $2\pi\left(\frac{m}{K}\right)$
b) $2\pi\left\{\frac{(YA+KL)m}{YAK}\right\}^{1/2}$
c) $2\pi\frac{mYA}{KL}$
d) $2\pi\frac{mL}{YA}$

86. An object of mass 4 kg is attached to a spring having spring constant 100 N/m. It performs simple harmonic motion on a smooth horizontal surface with an amplitude of 2m. A 6 kg object is dropped vertically onto the 4 kg object when it crosses the mean position, and sticks to it. The change in amplitude of oscillation due to collision is
a) 1 m
b) Zero
c) $2\left[1-\sqrt{\frac{2}{5}}\right]$
d) $2\left[1-\frac{1}{\sqrt{5}}\right]$

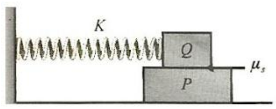
87. Two particles are executing identical simple harmonic motions described by the equations, $x_1 = a\cos(\omega t+(\pi/6))$ and $x_2 = a\cos(\omega t+\pi/3)$. The minimum interval of time between the particles crossing the respective mean positions is
a) $\frac{\pi}{2\omega}$
b) $\frac{\pi}{3\omega}$
c) $\frac{\pi}{4\omega}$
d) $\frac{\pi}{6\omega}$

88. Two simple harmonic motions are represented by equations
$y_1 = 4\sin(10t+\phi)$
$y_2 = 5\cos 10t$
What is the phase difference between their velocities?
a) ϕ
b) $-\phi$
c) $\left(\phi+\frac{\pi}{2}\right)$
d) $\left(\phi-\frac{\pi}{2}\right)$

89. The mass and diameter of a planet are twice those of earth. The period of oscillation of pendulum on this planet will be
(If it is a second's pendulum on earth)
a) $\frac{1}{\sqrt{2}}s$ b) $2\sqrt{2}\,s$ c) $2\,s$ d) $\frac{1}{2}s$

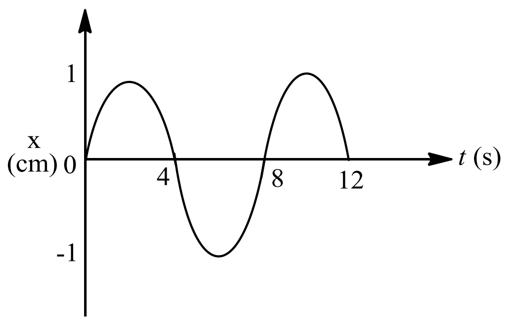
90. A simple pendulum of length l and a mass m of the bob is suspended in a car that is travelling with a constant speed v around a circle of radius R. If the pendulum undergoes small oscillations about its equilibrium position, the frequency of its oscillation will be
a) $\frac{1}{2\pi}\sqrt{\frac{g}{l}}$ b) $\frac{1}{2\pi}\sqrt{\frac{g}{R}}$ c) $\frac{1}{2\pi}\sqrt{\frac{\left(g^2+\frac{v^4}{R^2}\right)^{1/2}}{l}}$ d) $\frac{1}{2\pi}\sqrt{\frac{v^2}{Rl}}$

91. A particle executes simple harmonic motion with a frequency f. The frequency with which its kinetic energy oscillates is
a) $f/2$ b) f c) $2f$ d) $4f$

92. A block P of mass m is placed on a smooth horizontal surface. A block Q of same mass is placed over the block P and the coefficient of static friction between them is μ_S. A spring of spring constant K is attached to block Q. The blocks are displaced together to a distance A and released. The upper block oscillates without slipping over the lower block. The maximum frictional force between the block is


a) Zero b) K c) $KA/2$ d) μg

93. In the previous problem, the displacement of the particle from the mean position corresponding to the instant mentioned is
a) $\frac{5}{\pi}$ m b) $\frac{5\sqrt{3}}{\pi}$ m c) $\frac{10\sqrt{3}}{\pi}$ m d) $\frac{5\sqrt{3}}{2\pi}$ m

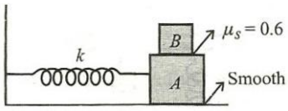
94. The x-t graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at $t=\frac{4}{3}$ s is


a) $\frac{\sqrt{3}}{32}\pi^2\text{cms}^{-2}$ b) $-\frac{\pi^2}{32}\text{ cms}^{-2}$ c) $\frac{\pi^2}{32}\text{ cms}^{-2}$ d) $-\frac{\sqrt{3}}{32}\pi^2\text{cms}^{-1}$

95. While a particle executes simple harmonic motion, the rate of change of acceleration is maximum and minimum, respectively at
a) The mean position and extreme positions
b) The extreme positions and mean position
c) The mean position alternatively
d) The extreme positions alternatively

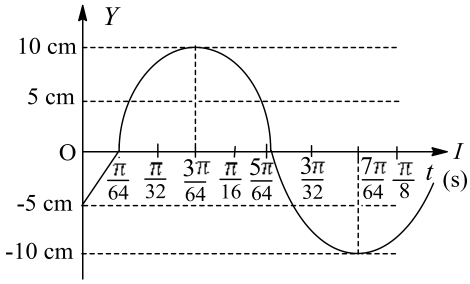
96. In the previous question, the magnitude of velocity of particle at the mentioned instant is
a) $\frac{\pi A}{T}$ b) $\frac{\sqrt{2}\pi A}{R}$ c) Zero d) $\sqrt{\frac{7}{8}}\times\frac{2\pi A}{T}$

97. A block A is connected to spring and performs simple harmonic motion with a time period of 2 s. Another block B rests on a. The coefficient of static friction between A and B is $\mu_s = 0.6$. The maximum amplitude of oscillation which the system can have so that there is no relative motion between A and B is (take $\pi^2 = g = 10$)



a) 0.3 m b) 0.6 m c) 0.4 m d) 0.52 m

98. The diagram below shows a sinusoidal curve. The equation of the curve will be



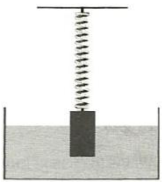
a) $y = 10 \sin\left(16t + \frac{\pi}{4}\right)$ cm

b) $y = 10 \sin\left(16t + \frac{\pi}{3}\right)$ cm

c) $y = 10 \sin\left(16t - \frac{\pi}{4}\right)$ cm

d) $y = 10 \cos\left(16t + \frac{\pi}{4}\right)$ cm

99. A solid right circular cylinder of weight 10 kg and cross-sectional area 100 cm^2 is suspended by a spring, where $k = 1$ kg/m^3 as shown in figure. What is its period when it makes simple harmonic vertical oscillations? (Take $g = 10$ m/s^2)

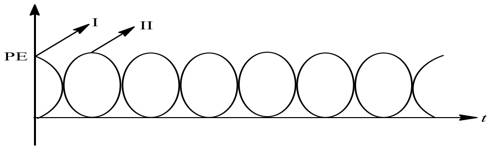


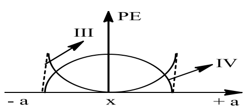
a) 0.6 s b) 1 s c) 1.5 s d) 2.2 s

100. In a certain oscillatory system (particle is performing SHM), the amplitude of motion is 5 m and the time period is 4 s. Time minimum time taken by the particle for passing between point, which are at distance of 4 m and 3 m from the centre and on the same side of it will approximately be

a) $\frac{16}{45}$ s b) $\frac{7}{45}$ s c) $\frac{8}{45}$ s d) $\frac{13}{45}$ s

101. For a particle executing SHM the displacement x is given by $x = A \cos \omega t$. Identify the graph which represents the variation of potential energy (PE) as a function of time t and displacement x.



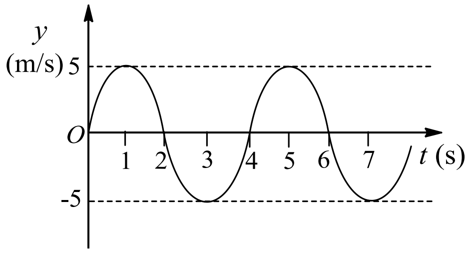


a) I, III b) II, IV c) II, III d) I, IV

102. A particle performs SHM on the x-axis with amplitude A and time period T. The time taken by the particle to travel a distance $A/5$ starting from rest is

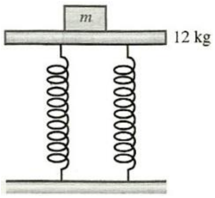
a) $\frac{T}{20}$ b) $\frac{T}{2\pi}\cos^{-1}\left(\frac{4}{5}\right)$ c) $\frac{T}{2\pi}\cos^{-1}\left(\frac{1}{5}\right)$ d) $\frac{T}{2\pi}\sin^{-1}\left(\frac{1}{5}\right)$

103. The variation of velocity of a particle executing SHM with time is shown in figure. The velocity of the particle when a phase change of $\pi/6$ takes place from the instant it is at one of the extreme positions will be



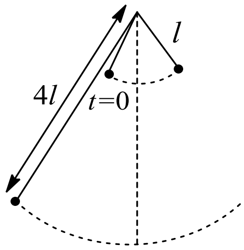
a) 3.53 m/s b) 2.5 m/s c) 4.330 m/s d) None of these

104. A plank of mass 12 kg is supported by two identical springs as shown in figure. The plank always remains horizontal. When the plank is pressed down and released, it performs simple harmonic motion with time period 3 s. When a block of mass m is attached to the plank the time period changes to 6 s. The mass of the block is



a) 48 kg b) 36 kg c) 24 kg d) 12 kg

105. Two simple pendulums of lengths l and $4l$ are suspended from same point and brought aside together and released at the same time. If the time period of smaller pendulum is T then after how much time will they be together again and moving in the same direction?

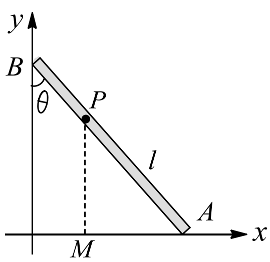


a) $T/2$ b) T c) $2T$ d) None of these

106. A particle performs SHM about $x = 0$ such that at $t = 0$ it is at $x = 0$ and moving towards positive extreme.The time taken to go from $x = 0$ to $x = A/2$ is .... times the time taken to go from $x = A/2$ to a. The most suitable option for the blank space is

a) 2 b) $\frac{1}{2}$ c) $\frac{11}{12}$ d) $\frac{12}{11}$

107. A rod length l is in motion such that its ends A and B are moving along the x-axis and the y-axis, respectively. It is given that $d\theta/dt = 2$ rad/s always. P is a fixed point on the rod. Let M be the projection of P on the x-axis. For the time interval in which θ changes from 0 to $\pi/2$, choose the correct statement



a) The acceleration of M is always direction towards right
b) M executes SHM
c) M moves with constant speed

d) M moves with constant acceleration

108. A particle of mass 4 kg moves between two points A and B on a smooth horizontal surface under the action of two forces such that when it is at a point P, the forces are $\vec{F}_1 = 2(\overrightarrow{PA})N$ and $\vec{F}_2 = 2(\overrightarrow{PB})N$. If the particle is released from rest from A, the time period of motion is

a) πs b) $2\pi s$ c) $3\pi s$ d) $\sqrt{2}\pi s$

109. Two bodies M and N of equal masses are suspended from two separate massless springs of force constants k_1 and k_2 respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude M to that of N is

a) k_1/k_2 b) $\sqrt{k_1/k_2}$ c) k_2/k_1 d) $\sqrt{k_2/k_1}$

Multiple Correct Answers Type

110. A spring of spring constant k stores 5 J of energy when stretched by 25 cm. It is vertical with one end fixed. A mass m is attached to the other end. It makes 5 oscillations per second. Then

a) $m = 0.16$ kg b) $m = 0.32$ kg c) $k = 160$ N/m d) $k = 320$ N/m

111. A coin is placed on a horizontal platform AB which undergoes SHM about mean point O in a vertical plane. The coin does not slip on the platform. The forces of friction acting on the coin is F. Then

a) F is always directed towards one end of platform

b) F is directed towards A when the coin tends to move away from A and away from B when the coin tends to move towards (b)

c) F is maximum when the coin and the platform come to rest momentarily at the one extreme position of SHM

d) F is maximum when the coin and platform come to rest momentarily at the other extreme position of SHM

112. A particle of mass m moves on the x-axis as follows:
It start from rest at $t = 0$ from point $x = 0$ and comes to rest at $t = 1$ at the point $x = 1$. No other information is available about its motion at intermediate time $(0 < t < 1)$. If α denotes the instantaneous acceleration of the particle. Then which of the following statements are true?

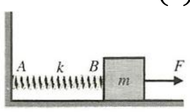
a) α cannot remain positive for all t in the interval $0 \le t \le 1$.

b) $|\alpha|$ Cannot exceed 2 at any point in its path.

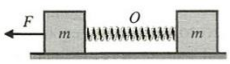
c) $|\alpha|$ must be ≥ 4 at some point or points in its path.

d) α must change sing during the motion but no other assertion can be made with the information given.

113. Figure shows a spring of force constant k fixed at one end carrying a mass m at the other end placed on a horizontal frictionless surface. The spring is stretched by a force F. Figure shows the same spring with both ends free and a mass m fixed at each free end. Each of the spring is stretched by the same force F. The mass in case (a) and the masses in case (b) are then released. Which of the following statements are true?



(a)



(b)

a) While oscillating, the maximum extension of the spring is more in case (a) than in case (b)

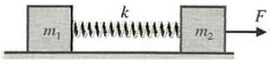
b) The maximum extension of the spring is same in both cases

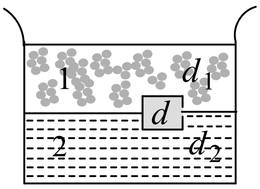
c) The time period of oscillation is the same in both cases

d) The time period of oscillation in case (a) is $\sqrt{2}$ times that in case (b)

114. Which of the following expression represent simple harmonic motion

a) $x = A\sin(\omega t + \delta)$ b) $x = B\cos(\omega t + \phi)$ c) $x = A\tan(\omega t + \phi)$ d) $x = A\sin\omega t\cos\omega t$

115. A mass of 0.2 kg is attached to the lower end of a massless spring of force constant 200 N/m, the upper end of which is fixed to a rigid support. Study the following statements
a) In equilibrium the spring will be stretched by 1 cm
b) If the mass is raised till the spring becomes unstretched and then released, it will go down by 2 cm before moving upwards
c) The frequency of oscillation will be nearly 5 Hz
d) If the system is taken to the moon, the frequency of oscillation will be the same as that on the earth
116. The phase of particle executing simple harmonic motion is $\frac{\pi}{2}$ when it is
a) Maximum velocity
b) Maximum acceleration
c) Maximum energy
d) Maximum displacement
117. A vertical mass-spring system executes simple harmonic oscillations with a period of 2 s. A quantity of this system which exhibits simple harmonic variation with a period of 1 s is
a) Velocity
b) Potential energy
c) Phase difference between acceleration and displacement
d) Difference between kinetic energy and potential energy
118. Two blocks connected by a spring rest on a smooth horizontal plane as shown in figure. A constant force F starts acting on block m_2 as shown in the figure. Which of the following statements are not correct?


a) Length of the spring increases continuously if $m_1 > m_2$
b) Blocks start performing SHM about centre of mass of the system, which moves rectilinearly with constant acceleration
c) Blocks start performing oscillations about centre of mass of the system with increasing amplitude
d) Acceleration of m_2 is maximum at initial moment of time only
119. A cylindrical block of density d stays fully immersed in a beaker filled with two immiscible liquids of different densities d_1 and d_2. The block is in equilibrium with half of it in liquid 1 and the other half in liquid 2 as shown in the figure. If the block is given a displacement downwards and released, then neglecting friction study the following statements


a) It executes simple harmonic motion
b) Its motion is periodic but not simple harmonic
c) The frequency of oscillation is independent of the size of the cylinder
d) The displacement of the centre of the cylinder is symmetric about its equilibrium position
120. A coin is placed on a horizontal platform, which undergoes vertical simple harmonic motion of angular frequency ω. The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time
a) At the highest position of the platform
b) At the mean position of the platform
c) For an amplitude of g/ω^2
d) For an amplitude of $\sqrt{g/\omega}$
121. A particle is subjected to SHM as given by equations $x_1 = A_1 \sin \omega t$ and $x_2 = A_2 \sin(\omega t + \pi/3)$. The maximum acceleration and amplitude of the resultant motion are $a_{\max}$ A, respectively. Then
a) $a_{\max} = \omega^2 \sqrt{A_1^2 + A_2^2 + A_1 A_2}$
b) $a_{\max} = \omega^2 \sqrt{A_1 A_2}$
c) $A = A_1 + A_2$
d) $A = \sqrt{A_1^2 + A_2^2 + A_1 A_2}$
122. Three SHMs in the same direction having the same amplitude a and same period are superposed. If each

differs in phase from the next by 45^0, then

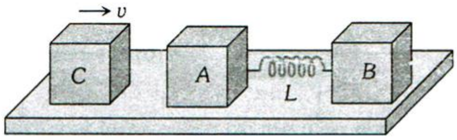
a) the resultant amplitude is $a(1+\sqrt{2})$

b) the phase of the resultant motion relative to the first is 90^0

c) the energy associated with the resulting motion is $(\sqrt{3}+2\sqrt{2})$ times the energy associated with any single motion

d) the resultant motion is not simple harmonic

123. Two blocks A and B each of mass m are connected by a massless spring of natural length L and spring constant K. The blocks are initially resting on a smooth horizontal floor with the spring at its natural length as shown in figure. A third identical block C also of mass m moves on the floor with a speed v along the line joining A and B and collides with A. Then



a) The kinetic energy of the A-B system at maximum compression of the spring is zero
b) The kinetic energy of the A-B system at maximum compression of the spring is $mv^2/4$
c) The maximum compression of the spring is $v\sqrt{m/K}$
d) The maximum compression of the spring is $v\sqrt{m/2K}$

124. The potential energy U of a body of unit mass moving in one dimensional conservative force field is given by $U = x^2 - 4x + 3$. All units are in SI. For this situation mark out the correct statement(s)
a) The body will perform simple harmonic motion about $x = 2$ units
b) The body will perform oscillatory motion but not simple harmonic motion
c) The body will perform simple harmonic motion with time period $\sqrt{2\pi}$ s
d) If speed of the body at equilibrium position is 4 m/s, then the amplitude of oscillation would be $2\sqrt{2}$ m

125. The time period of a particle in simple harmonic motion is T. Assume potential energy at mean position to be zero. After a time of $T/6$ it passes its mean position, its
a) Velocity will be half its maximum velocity
b) Displacement will be half its amplitude
c) Acceleration will be nearly 86% of its maximum acceleration
d) KE = PE

126. When the point of suspension of pendulum is moved, its period of oscillation
a) Decreased when it moves vertically upwards with an acceleration a
b) Decreases when it moves vertically downwards with acceleration greater than 2 g
c) Increases when it moves horizontally with acceleration a
d) All of the above

127. A simple pendulum is oscillating between extreme positions P and Q about the mean position O. Which of the following statements are true about the motion of pendulum?
a) At point O, the acceleration of the bob is different from zero
b) The acceleration of the bob is constant throughout the oscillation
c) The tension in the string is constant throughout the oscillation
d) The tension is maximum at O and minimum at P or Q

128. A linear harmonic oscillator of force constant 2×10^6 N/m times and amplitude 0.01 m has a total mechanical energy of 160 $joules$. Its

a) Maximum potential energy is 100 J

b) Maximum K.E. is 100 J

c) Maximum P.E. is 160 J

d) Minimum P.E. is zero

129. The potential energy of a particle of mass 0.1 kg moving along the x-axis, is given by $U = 5x(x-4)$ J, where x is in meter. It can be concluded that
a) The particle is acted upon by a constant force
b) The speed of the particle is maximum at $x = 2$ m
c) The particle executes SHM

d) The period of oscillation of the particle is $(\pi/5)$ s

130. A 20 g particle is subjected to two simple harmonic motions

$x_1 = 2\sin 10t, x_2 = 4\sin\left(10t + \frac{\pi}{3}\right)$, where x_1 and x_2 are in metres and t is in seconds

a) The displacement of the particle at $t = 0$ will be $2\sqrt{3}$ m

b) Maximum speed of the particle will be $20\sqrt{7}$ m/s

c) Magnitude of maximum acceleration of the particle will be $200\sqrt{7}$ m/s^2

d) Energy of the resultant motion will be 28 J

131. At two particular closest instant of time t_1 and t_2 the displacement of a particle performing SHM are equal. At these instants

a) Instantaneous speeds are equal
b) Instantaneous accelerations are equal
c) Phase of the motion are unequal
d) Kinetic energies are equal

132. The displacement-time relation for a particle can be expressed as $= 0.5\,[\cos^2(n\pi t) - \sin^2(n\pi t)]$. This relation shows that

a) The particle is executing a SHM with amplitude 0.5 m

b) The particle is executing a SHM with a frequency n times that of a second's pendulum

c) The particle is executing a SHM and the velocity in its mean position is $(n\pi)$ m/s

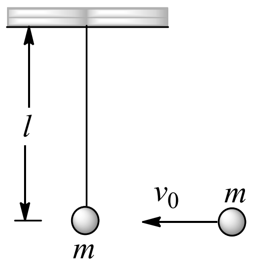
d) The particle is not executing a SHM at all

133. The displacement of a particle of mass 0.1 kg from its mean position is given by

$y = 0.05\sin 4\pi(5t + 0.4)$. then

a) The period of motion is 0.1 s
b) The maximum acceleration of the particle is $10\pi^2\ ms^{-2}$
c) The total mechanical energy of the particle is $0.05\pi^2$J
d) The force acting on the particle is zero corresponding to $y = 0.4$ m

134. A simple pendulum consists of a bob of mass m and a light string of length l as shown in the figure. Another identical ball moving with the small velocity v_0 collides with the pendulum's bob and sticks to it. For this new pendulum of mass $2m$, mark out the correct statement(s)



a) Time period of the pendulum is $2\pi\sqrt{\frac{l}{g}}$

b) The equation of motion for this pendulum is $\theta = \frac{v_0}{2\sqrt{gl}}\sin\left[\sqrt{\frac{g}{l}}t\right]$

c) The equation of motion for this pendulum is $\theta = \frac{v_0}{2\sqrt{gl}}\cos\left[\sqrt{\frac{g}{l}}t\right]$

d) Time period of the pendulum is $2\pi\sqrt{\frac{2l}{g}}$

135. For the spring pendulum shown in figure, the value of spring constant is 3×10^4 N/m and amplitude of oscillation of 0.1 m. The total mechanical energy of oscillating system is 200 J. Mark out the correct option(s)

a) Minimum PE of the oscillating system is 50 J
b) Minimum PE of the oscillating system is 200 J
c) Minimum KE of the oscillating system is 200 J
d) Minimum KE of the oscillating system is 150 J

136. Three simple harmonic motions in the same direction, each of amplitude 'a' and periodic time 'T', are superposed. The first and second, and the second and third differ in phase from each other by $\pi/4$, with the first and third not being identical. Then
a) The resultant motion is not simple harmonic
b) The resultant amplitude is $(\sqrt{2}+1)a$
c) The phase difference between the second SHM and the resultant motion is zero
d) The energy in the resultant motion is three times the energy in each separate SHM

137. For a simple harmonic motion with given angular frequency ω, two arbitrary initial conditions are necessary and sufficient to determine the motion completely. These initial conditions may be
a) Initial position and initial velocity
b) Amplitude and initial phase
c) Total energy of oscillation and amplitude
d) Total energy of oscillation and initial phase

138. A block of mass m is suspended by a rubber cord of natural length $l = mg/k$, where k is force constant of the cord. The block is lifted upwards so that the cord becomes just tight and then block is released suddenly. Which of the following will not be true?
a) Block performs periodic motion with amplitude greater than l
b) Block performs SHM with amplitude equal to l
c) Block will never return to the position from where it was released
d) Angular frequency ω is equal to 1 rad/s

139. For a particle executing SHM, select the correct statements (s) out of the following.
a) Total mechanical energy of the particle remains constant
b) The restoring force is maximum at extreme position of the motion
c) The restoring force is always directed towards a fixed point
d) The velocity is minimum at equilibrium position

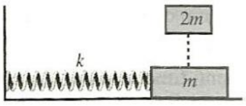
140. A horizontal plank has a rectangular block placed on it. The plank starts oscillating vertically and simple harmonically with an amplitude of 40 cm. The block just loses contact with the plank when the latter is at momentary rest. Then
a) The period of oscillation is $\left(\frac{2\pi}{5}\right)$
b) The block weighs 0.5 times its weight on the plank halfway up
c) The block weighs 1.5 times its weight on the plank halfway down
d) The block weight its true weight on the plank when the later moves fastest

141. A particle is subjected to two simple harmonic motions along x and y directions according to $x = 3$ sin $100\pi t$; $y = 4\sin 100\,\pi t$
a) Motion of particle will be on ellipse travelling in clockwise direction
b) Motion of particle will be on a straight line with slope 4/3
c) Motion will be simple harmonic motion with amplitude 5
d) Phase difference between two motions is $\pi/2$

142. The speed v of a particle moving along a straight line, when it is at a distance (x) from a fixed point of the line is given by $v^2 = 108 - 9x^2$ (all equations are in CGC units):
a) The motion is uniformly accelerated along the straight line

b) The magnitude of the acceleration at a distance 3 cm from the point is 27 cm/s^2
c) The motion is simple harmonic about the given fixed point
d) The maximum displacement from fixed point is 4 cm

143. An object of mass m is performing simple harmonic motion on a smooth horizontal surface as shown in figure. Just as the oscillating object reaches its extreme position, another object of mass $2m$ is dropped on to oscillating object, which sticks to it. For this situation mark out the correct statement(s)



a) Amplitude of oscillation remains unchanged
b) Time period of oscillation remains unchanged
c) The total mechanical energy of the system does not change
d) The maximum speed of the oscillating object changes

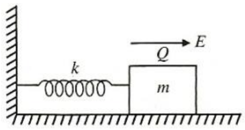
144. The displacement (x) of a particle as a function of time (t) is given by
$x = a\sin(bt + c)$
Where a, b and c are constants of motion. Choose the correct statements from the following
a) The motion repeats itself in a time interval of $2\pi/b$
b) The energy of the particle remains constant
c) The velocity of the particle is zero at $x = \pm a$
d) The acceleration of the particle is zero at $x = \pm a$

145. In a simple harmonic motion, which of the following pairs of quantities are always oriented in mutually opposite directions?
a) Restoring force and acceleration
b) Restoring force and displacement
c) Velocity and displacement
d) Acceleration and displacement

146. Function $x = A\sin^2 \omega t + B\cos^2 \omega t + C \sin \omega t \cos \omega t$ represents SHM
a) For any value of A, B and C (except $C = 0$)
b) If $A = -B; C = 2B$, amplitude $= |B\sqrt{2}|$
c) If $A = B; C = 0$
d) If $A = B; C = 2B$, amplitude $= |B|$

147. A particle performing simple harmonic motion undergoes initial displacement of $A/2$ (where A is the amplitude of simple harmonic motion) in 1 s. At $t = 0$, the particle may be at the extreme position or mean position. The time period of the simple harmonic motion can be
a) 6 s
b) 2.4 s
c) 12 s
d) 1.2 s

148. A spring block system undergoes SHM on a smooth horizontal surface, the block is now given some charge and a uniform horizontal electric field E is switched on as shown in figure. As a result



a) Time period of oscillation will increase
b) Time period of oscillation will decrease
c) The period of oscillation will remain unaffected
d) The mean position of SHM will shift to the right

Assertion - Reasoning Type

This section contain(s) 0 questions numbered 149 to 148. Each question contains STATEMENT 1(Assertion) and STATEMENT 2(Reason). Each question has the 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

a) Statement 1 is True, Statement 2 is True; Statement 2 **is** correct explanation for Statement 1

b) Statement 1 is True, Statement 2 is True; Statement 2 **is not** correct explanation for Statement 1

c) Statement 1 is True, Statement 2 is False

d) Statement 1 is False, Statement 2 is True

149 A circular metal hoop is suspended on the edge by a hook. The hoop can oscillate side to side in the plane of the hoop, or it can oscillate back and forth in a direction perpendicular to the plane of the hoop. The time period of oscillation would be more when oscillations are carried out in the plane of hoop

Statement 1: Time period of physical pendulum is more if moment of inertia of the right body about corresponding axis passing through the pivoted point is more

Statement 2: Time period of physical pendulum is more if moment of inertia of the rigid body about corresponding axis passing through the pivoted point is more

150

Statement 1: A particle is moving along the x-axis. The resultant force F acting on it is given by $F = -ax - b$, where a and b are both positive constants. The motion of this particle is not SHM

Statement 2: In SHM resultant force must be proportional to the displacement from mean position

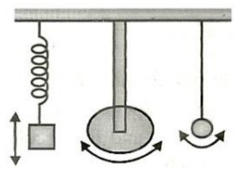
151

Statement 1: When a simple pendulum is made to oscillate on the surface of moon, its time period increases

Statement 2: Moon is much smaller as compared to earth

152

Statement 1: There pendulums are suspended from ceiling as shown in figure



These three pendulums are set to oscillate as shown by arrows, and it is found that all three have same time period. Now, all three are taken to a place where acceleration due to gravity changes to 4/9th of its value at the first place. If spring pendulum makes 60 cycles in a given time at this place, then torsion pendulum and simple pendulum will also make 60 oscillations in same (given) time interval

Statement 2: Time period of torsion pendulum is independent of acceleration due to gravity

153

Statement 1: Sine and cosine functions are periodic functions

Statement 2: Sinusoidal functions repeats it values after a definite interval of time

154

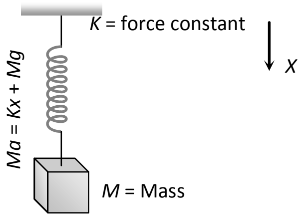
Statement 1: For an oscillating simple pendulum, the tension in the string is maximum at the mean position and minimum at the extreme position

Statement 2: The velocity of oscillating bob in simple harmonic motion is maximum at the mean position

155

Statement 1: Consider motion for mass spring system under gravity, motion of M is not a simple harmonic motion unless Mg is negligibly small

Statement 2: For simple harmonic motion acceleration must be proportional to displacement and is directed towards the mean position



156

Statement 1: In S.H.M., the motion is 'to and fro' and periodic

Statement 2: Velocity of the particle $(v) = \omega\sqrt{k^2 - x^2}$ (where x is the displacement and k is amplitude)

157

Statement 1: All oscillatory motions are necessarily periodic motion but all periodic motion are not oscillatory

Statement 2: Simple pendulum is an example of oscillatory motion

158

Statement 1: The periodic time of a hard spring is less as compared to that of a soft spring

Statement 2: The periodic time depends upon the spring constant, and spring constant is large for hard spring

159

Statement 1: During the oscillations of simple pendulum, the direction of its acceleration at the mean position is directed towards the point of suspension and at extreme position it is directed towards the mean position

Statement 2: The direction of acceleration of a simple pendulum at the mean position or at the extreme position is decided by the tangential and radial components of force by gravity

160

Statement 1: If the length of a spring is made n times, the spring factor of the spring becomes $1/n$ th of its original value.

Statement 2: Time of oscillation of a spring pendulum is $T = 2\pi\sqrt{\frac{m}{k}}$

161

Statement 1: The percentage change in time period is 1.5%, if the length of simple pendulum increases by 3%

Statement 2: Time period is directly proportional to length of pendulum

162

Statement 1: If the amplitude of a simple harmonic oscillator is doubled, its total energy becomes four times

Statement 2: The total energy is directly proportional to the square of the amplitude of vibration of the harmonic oscillator

163

Statement 1: Acceleration is proportional to the displacement. This condition is not sufficient for motion in simple harmonic

Statement 2: In simple harmonic motion direction of displacement is also considered

164

Statement 1: The time period of a simple pendulum is of infinite length is infinite

Statement 2: The time period of simple pendulum is directly proportional to the square root of length.

165

Statement 1: Water in a U-tube executes SHM, the time period of mercury filled up to the same height in the U-tube be greater than that in case of water

Statement 2: The amplitude of an oscillating pendulum goes on increasing

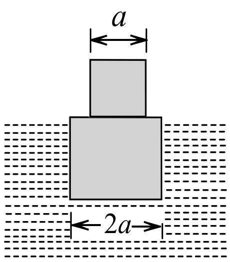
166

Statement 1: Damped oscillation indicates loss of energy

Statement 2: The energy loss is damped oscillation may be due to friction, air resistance etc.

167

Statement 1: Two cubical blocks of same material and of sides a and $2a$, respectively are attached rigidly and symmetrically to each other as shown. The system of two blocks is floating in water in such a way that upper surface of bigger block is just submerged in the water. If the system of blocks is displaced slightly in vertical directions, then the amplitude of oscillation on either side of equilibrium position would be different



Statement 2: The force constant on two sides of equilibrium position in the above-described situation is different

168

Statement 1: The total energy of a particle performing simple harmonic motion could be negative

Statement 2: Potential energy of a system could be magnetic

169

Statement 1: The amplitude of a particle of a particle executing SHM with frequency of 60 Hz is 0.01 m. The maximum value of acceleration of the particle is $\pm 144\pi^2 ms^{-2}$

Statement 2: Acceleration amplitude= $\omega^2 A$, where A is displacement amplitude

170

Statement 1: In simple harmonic motion, the velocity is maximum when acceleration is minimum

Statement 2: Displacement and velocity of S.H.M. differ is phase by $\pi/2$

171

Statement 1: For a particle of mass 1 kg executing simple harmonic motion, if slope of restoring force vs. displacement graph is -1, then the time period of oscillation will be 6.28 s

Statement 2: If 1 kg mass is replaced by 2 kg mass and rest of the information remains same as in Statement I, then the time period of oscillation will remain 6.28 s

172

Statement 1: In a simple pendulum performing SHM, net acceleration is always between tangential and radial acceleration except at lowest point

Statement 2: At lowest point tangential acceleration is zero

173

Statement 1: In a S.H.M., kinetic and potential energies become equal when the displacement is $1/\sqrt{2}$ times the amplitude

Statement 2: In SHM, kinetic energy is zero when potential energy is maximum

174

Statement 1: The spring constant of a spring is K. When it is divided into n equal parts, then spring constant of one piece is K/n

Statement 2: The spring constant is independent of material used for the spring

175

Statement 1: Ocean waves hitting a beach are always found to be nearly normal to the shore

Statement 2: Ocean waves hitting a beach are assumed as plane waves

176

Statement 1: In extreme position of a particle executing S.H.M., both velocity and acceleration are zero

Statement 2: In S.H.M., acceleration always acts towards mean position

177

Statement 1: Resonance is special case of forced vibration in which the natural frequency of vibration of the body is the same as the impressed frequency of external periodic force and the amplitude of forced vibration is maximum

Statement 2: The amplitude of forced vibrations of a body increases with an increase in the frequency of the externally impressed periodic force

178

Statement 1: The amplitude of an oscillation pendulum decreases gradually with time

Statement 2: The frequency of the pendulum decreases with time

179

Statement 1: The bob of simple pendulum is a ball full of water. If a fine hole is made at the bottom of the ball. Then the time period will no more remain constant.

Statement 2: The time period of a simple pendulum does not depend upon mass.

180

Statement 1: Soldiers are asked to break steps while crossing the bridge

Statement 2: The frequency of marching may be equal to the natural frequency of bridge and may lead to resonance which can break the bridge

181

Statement 1: If stretched by the same amount, work done on S_1, will be more than that on S_2

Statement 2: $k_1 < k_2$

182

Statement 1: When a girl sitting on a swing stands up, the periodic time of the swing decreases.

Statement 2: When the girl stands up more weight is felt on the string of swing.

183

Statement 1: When a simple pendulum is made to oscillate on the surface of moon, its time period increases

Statement 2: Moon is much smaller as compared to earth

184

Statement 1: A spring of force constant k is cut into two pieces whose lengths are in the ratio 1:2. The force constant of series combination of the two parts is $\frac{\sqrt{3k}}{2}$

Statement 2: The spring connected in series are represented by $k=k_1+k_2$.

185

Statement 1: The graph of total energy of a particle in SHM $w.r.t.$, position is a straight line with zero slope

Statement 2: Total energy of particle in SHM remains constant throughout its motion

186

Statement 1: The amplitude of oscillation can never be infinite

Statement 2: The energy of oscillator is continuously dissipated

187

Statement 1: The height of a liquid column in a u tube is 0.3m. if the liquid in one of the limbs is depressed and then released the time period of a liquid column will be 1.1s.

Statement 2: This follows from, the relation $T=2\pi\frac{\sqrt{h}}{g}$

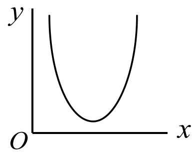
Matrix-Match Type

This section contain(s) 0 question(s). Each question contains Statements given in 2 columns which have to be matched. Statements (A, B, C, D) in **columns I** have to be matched with Statements (p, q, r, s) in **columns II**.

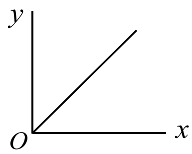
188. Column I gives a list of possible set of parameters measured in some experiments. The variation of the parameters in the form of graphs are shown in Column II. Match the set of parameters given in Column I with the graphs given in Column II

Column-I **Column- II**

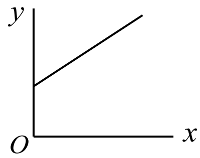
(A) Potential energy of a simple pendulum (y-axis) as a function of displacement (x-axis)

(p)


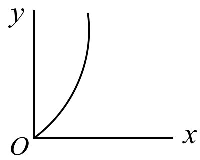
(B) Displacement (y-axis) as a function of time (x-axis) for a one dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction

(q)


(C) Range of a projectile (y-axis) as a function of its velocity (x-axis) when projected at a fixed angle

(r)


(D) The square of the time period (y-axis) of a simple pendulum as a function of its length (x-axis)

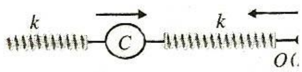
(s)


CODES :

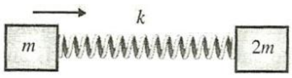
	A	B	C	D
a)	p	q,s	s	q
b)	q,s	p	q	s
c)	s	q	q,s	p
d)	q	s	p	q,s

189. Column I lists the various modes of oscillations of masses connected to springs. Column II lists the corresponding frequencies of oscillations when executing SHM

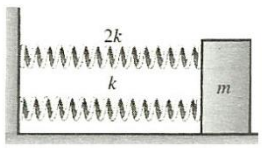
Column-I **Column- II**

(A)


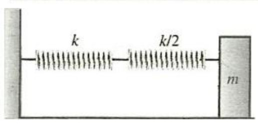
(p) $\frac{1}{2\pi}\sqrt{\frac{3k}{2m}}$

(B)


(q) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

(C)


(r) $\frac{1}{2\pi}\sqrt{\frac{k}{3m}}$

(D)


(s) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

CODES :

	A	B	C	D
a)	B	a	d	c
b)	d	c	b	a

c)	b	c	d	a
d)	d	b	a	c

190. Match the following

	Column-I		**Column- II**
(A)	Linear combination of two SHMs	(p)	$T = \sqrt{\frac{R}{g}}$ (R is radius of the earth)
(B)	$y = A\sin\omega_1 t + A\sin(\omega_2 t + \phi)$	(q)	SHM for equal frequencies and amplitude
(C)	Time period of a pendulum of infinite length	(r)	Superposition may not always be an SHM
(D)	Maximum value of time period of an oscillating pendulum	(s)	Amplitude will be $\sqrt{2}A$ for $\omega_1 = \omega_2$ and phase difference of $\pi/2$

CODES :

	A	**B**	**C**	**D**
a)	B,c	b,c,d	a	a
b)	c	d	a	b
c)	b,c	d	a,d	c
d)	c,d,a	a,b,c	d	b,a

191. Two particles 'A' and 'B' start SHM at $t = 0$. Their positions as function of time are given by
$X_A = A\sin\omega t$
$X_B = A\sin(\omega t + \pi/3)$

	Column-I		**Column- II**
(A)	Minimum time when x is same	(p)	$\frac{5\pi}{6\omega}$
(B)	Minimum time when velocity is same	(q)	$\frac{\pi}{3\omega}$
(C)	Minimum time after which $v_A < 0$ and $v_B < 0$	(r)	$\frac{\pi}{\omega}$
(D)	Minimum time after which $x_A < 0$ and $x_B < 0$	(s)	$\frac{\pi}{2\omega}$

CODES :

	A	**B**	**C**	**D**
a)	A	b	c	d
b)	d	c	b	a
c)	b	a	d	c
d)	d	b	a	c

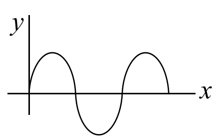
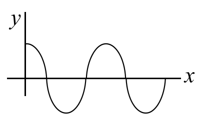
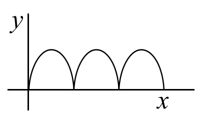
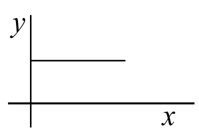
192. Column I describes some situations in which a small object moves. Column II describes some characteristics of these motions. Match the situation in Column I with the characteristics in Column II

Column-I	Column- II
(A) The object moves on the x-axis under a conservative force in such a way that its speed and position satisfy $v = c_1\sqrt{c_2 - x^2}$, where c_1 and c_2 are positive constants	(p) The object executes a simple harmonic motion
(B) The object moves on the x-axis in such a way that its velocity and its displacement from the origin satisfy $v = -kx$, where k is a positive constant	(q) The object does not change its direction
(C) The object is attached to one end of a massless spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration a. The motion of the object is observed from the elevator during the period it maintains this acceleration	(r) The kinetic energy of the objects keeps on decreasing
(D) The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{GM_e/R_e}$, where M_e is the mass of the earth and R_e is the radius of the earth Neglect forces from objects other than the earth	(s) The object can change its direction only once

CODES :

	A	**B**	**C**	**D**
a)	r,q	p	q,r	p
b)	p	q,r	p	r,q
c)	q,r	p	r,q	p
d)	p	r,q	p	q,r

193. In simple harmonic, match the following graphs:

Column-I	Column- II
(A) Position (y) vs. time (x)	(p) 
(B) Velocity (y) vs. time (x)	(q) 
(C) Potential energy (y) vs. time (x)	(r) 
(D) Total energy (y) vs. time (x)	(s) 

CODES :

	A	B	C	D
a)	B,c	d	a,d	c
b)	c,d,a	a,b,c	d	b,a
c)	a,b	a,b	c	d
d)	c	d	a	b

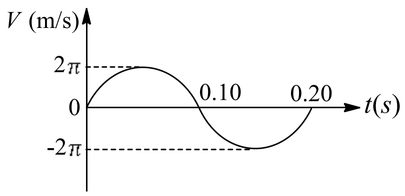
194. In Column I equations describing the motion of a particle are given and in column II possible nature of the motions. Match the entries of Column I with the entries of Column II

Column-I	Column- II
(A) $y = Ae^{i(\omega t+\phi)}$	(p) SHM
(B) $y = Bsin\ \omega t + C\cos\omega t$	(q) Periodic
(C) $y = A\sin(\omega i + kx)$	(r) Oscillatory
(D) $y = kx$	(s) Rectilinear

CODES :

	A	B	C	D
a)	A,b,c	a,b,c	a	d
b)	c	d	a	b
c)	b,c	d	a,d	c
d)	c,d,a	a,b,c	d	b,a

195. A simple harmonic oscillator consists of a block attached to a spring with $k = 200$ N/m. The block slides on a frictionless horizontal surface, with equilibrium point $x = 0$. A graph of the block's velocity v as a function of time t is shown. Correctly match the required information in Column I with the values given in Column II (use $\pi^2 = 10$):



Column-I	Column- II
(A) The block's mass in kg	(p) -0.20
(B) The block's displacement at $t = 0$ in metres	(q) -200
(C) The block's acceleration at $t = 0.10$ s in m/s^2	(r) 0.20
(D) The block's maximum kinetic energy in joules	(s) 4.0

CODES :

	A	B	C	D
a)	A	b	c	d
b)	d	c	b	a
c)	b	a	d	c
d)	c	a	b	d

196. A spring pendulum executes SHM in such a way that the block is having velocity v when it crosses the mean position. Now the changes have been made in such a way that the velocity while crossing the mean position gets doubled without changing mass of the block. In Column I some statements (incomplete) are given and corresponding completions are given in Column II. Match the entries of Column I with the entries of Column II. Assume the horizontal configuration of pendulum

	Column-I		Column- II
(A)	The frequency of oscillation will change by a factor of	(p)	2
(B)	The amplitude of oscillation will change by a factor of	(q)	$\sqrt{2}$
(C)	The magnitude of maximum acceleration will change by a factor of	(r)	1
(D)	Maximum PE increases by a factor of	(s)	4

CODES :

	A	B	C	D
a)	C	d	a	b
b)	b,c	d	a,d	c
c)	c,d,a	a,b,c	d	b,a
d)	c	a	a	d

197. Match the following

	Column-I		Column- II
(A)	A constant force acting along the line of SHM affects	(p)	The time period
(B)	A constant torque acting along the arc of angular SHM affects	(q)	The frequency
(C)	A particle falling on the block executing SHM when the later crosses the mean position affects	(r)	The mean position
(D)	A particle executing SHM when kept on a uniformly accelerated car affects	(s)	The amplitude

CODES :

	A	B	C	D
a)	B,c	b,c,d	a	a

b)	c,d	c,d	a,b,d	c,d
c)	b,c	d	a,d	c
d)	c,d,a	a,b,c	d	b,a

198. A particle of mass 2 kg is moving on a straight line under the action of force $F = (8 - 2x)$ N. The particle is released from rest at $x = 6$ m. For the subsequent motion, match the following (all the values in the Column II in are in their SI units):

	Column-I		**Column- II**
(A)	Equilibrium position is at x	(p)	$\pi/4$
(B)	Amplitude of SHM is	(q)	$\pi/4$
(C)	Time taken to go directly from $x = 2$ to $x = 4$	(r)	4
(D)	Energy of SHM is	(s)	6
(E)	Phase constant of SHM assuming equation of the form $A\sin(\omega t + \phi)$	(t)	2

CODES :

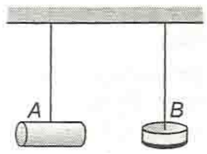
	A	**B**	**C**	**D**	**E**
a)	C	e	b	c	b
b)	d	c	b	a	b
c)	b	a	d	c	b
d)	d	b	a	c	b

Linked Comprehension Type

This section contain(s) 28 paragraph(s) and based upon each paragraph, multiple choice questions have to be answered. Each question has atleast 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 199 to -199

A uniform cylindrical metal rod A of length L and radius R is suspended at its mid-point from a rigid support through a strong metal wire of length$l$. The rod is given a small angular twist and released so that it oscillates to and fro about its mean position with a time periodT_1. Another body B of an irregular shape is suspended from the same rigid support using the same length of given suspension wire and its time period is found to be T_2



199. The rotational inertia of metal rod about the wire as an axis is

a) $\frac{ML^2}{12}$ b) $\frac{MR^2}{2}$ c) $M\left[\frac{L^2}{12} + \frac{R^2}{2}\right]$ d) $M\left[\frac{L^2}{12} + \frac{R^2}{4}\right]$

Paragraph for Question Nos. 200 to - 200

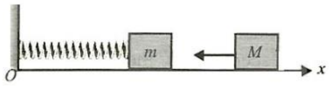
A particle performs harmonic oscillation along the x-axis about the equilibrium position$x = 0$. The oscillation frequency is$\omega = 4.00\ \text{s}^{-1}$. At a certain moment of time the particle has a coordinate $x_0 = 25.0$ cm and its velocity is equal to
$v_{x_0} = 100\ \text{cms}^{-2}$

200. Find the amplitude of oscillation

a) $13\sqrt{3}$ cm b) $25\sqrt{2}$ cm c) $27\sqrt{5}$ cm d) $2\sqrt{3}$ cm

Paragraph for Question Nos. 201 to - 201

One end of an ideal spring is fixed to a wall at origin O and the axis of spring is parallel to the x-axis. A block of mass $m = 1$ kg is attached to free end of the spring and it is performing SHM. Equation of position of the block in coordinate system shown in figure is $x = 10 + 3\sin(10\,t)$, where t is in second and x in cm.
Another block of mass $M = 3$ kg, moving towards the origin with velocity 30 cm/s collides with the block performing SHM at $t = 0$ and gets stuck to it

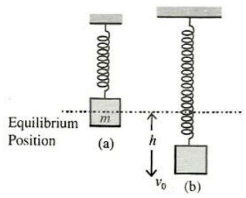


201. Angular frequency of oscillation after collision is

a) 20 rad/s b) 5 rad/s c) 100 rad/s d) 50rad/s

Paragraph for Question Nos. 202 to - 202

A block of mass m is connected to a spring of spring constant k and is at rest in equilibrium as shown. Now, the block is displaced by h below its equilibrium position and imparted a speed v_0 towards down as shown in figure. As a result of the jerk, the block executes simple harmonic motion about its equilibrium position. Based on this information, answer the following questions:

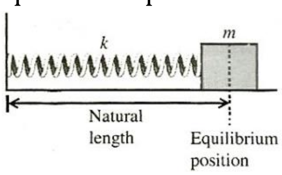


202. The amplitude of oscillation is

a) h b) $\sqrt{\dfrac{mv_0^2}{k} + h^2}$ c) $\sqrt{\dfrac{m}{k}}v_0 + h$ d) None of these

Paragraph for Question Nos. 203 to - 203

A block of mass m is connected to a spring of spring constant k as shown in figure. The block is found at its equilibrium position at $t = 1$ s and it has a velocity of $+0.25$ m/s at $t = 2$ s. The time period of oscillation is 6 s



Based on the information, answer the following questions:

203. The amplitude of oscillation is

a) $\frac{3}{2\pi}$ m
b) 3 m
c) $\frac{1}{\pi}$ m
d) 1.5 m

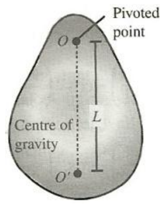
Paragraph for Question Nos. 204 to - 204

In a physical pendulum, the time period for small oscillation is given by, $T = 2\pi\sqrt{I/Mgd}$ where I is the moment of inertia of the body about an axis passing through a pivoted point O and perpendicular to the plane of oscillation and d is the separation point between centre of gravity and the pivoted point.
In the physical pendulum, a special point exists where if we concentrate the entire mass of body the resulting simple pendulum (w.r.t. pivot point O) will have the same time period as that of physical pendulum. This point is termed centre of oscillation.

$$T = 2\pi\sqrt{\frac{I}{Mgd}} = 2\pi\sqrt{\frac{L}{g}}$$

Moreover, this point possesses two other important remarkable properties:
Property I: Time period of physical pendulum about the centre of oscillation (if it would be pivoted) is same as in the original case
Property II: If an impulse is applied at the centre of oscillation in the plane of oscillation, the effect of this impulse at pivoted point is zero. Because of this property, this point is also known as the centre of percussion.
From the given information answer the following questions:

204. A uniform rod of mass M and length L is pivoted about point O as shown in figure. It is slightly rotated from its mean position so that it performs angular simple harmonic motion. For this physical pendulum, determine the time period of oscillation



a) $2\pi\sqrt{\frac{L}{g}}$
b) $\pi\sqrt{\frac{7L}{3g}}$
c) $2\pi\sqrt{\frac{2L}{3g}}$
d) None of these

Paragraph for Question Nos. 205 to - 205

A block of mass m is suspended from one end of a light spring as shown. The origin O is considered at distance equal to natural length of the spring from the ceiling from the ceiling and vertical downward direction as positive y-axis. When the system is in equilibrium, a bullet of mass $m/3$ moving in vertical upward direction with velocity v_0 strikes the block and embeds into it. As a result, the block (with bullet embedded into it) moves up and starts oscillating.
Based on the given information, answer the following question:

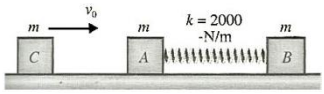
205. Mark out the correct statement(s)

a) The block-bullet system performs SHM about $y = mg/k$
b) The block-bullet system performs oscillatory motion but not SHM about $y = mg/k$
c) The block –bullet system performs SHM about $y = 4mg/3k$

d) The block-bullet system performs oscillatory motion but not SHM about $y = 4mg/3k$

Paragraph for Question Nos. 206 to - 206

Two identical blocks A and B, each of mass $m = 3$ kg, are connected with the help of an ideal spring and placed on a smooth horizontal surface as shown in figure. Another identical block C moving with velocity $v_0 = 0.6$ m/s collides with A and sticks to it, as a result, the motion of system takes place in some way

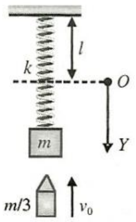


Based on this information, answer the following questions:

206. After the collision of $C$and A, the combined body and block B would
   a) Oscillate about centre of mass of system and centre of mass is at rest
   b) Oscillate about centre of mass of system and centre of mass is moving
   c) Oscillate but about different locations other than the centre of mass
   d) Not oscillate

Paragraph for Question Nos. 207 to - 207

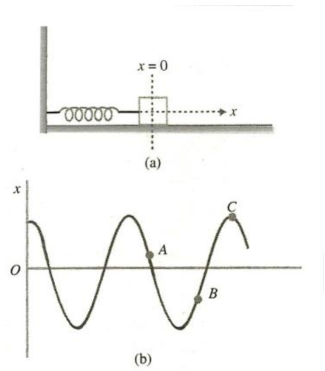
A small of block of mass m is fixed at upper end a massive vertical spring of spring constant $k = 4mg/L$ and natural length '10 L'. The lower end of spring is free and is at a height L from fixed horizontal floor as shown. The spring is initially unstressed and the spring-block system is released from rest in the shown position



207. At the instant the speed of block is maximum, the magnitude of force exerted by the spring on the block is
   a) $\frac{mg}{2}$ b) mg c) Zero   d) None of these

Paragraph for Question Nos. 208 to - 208

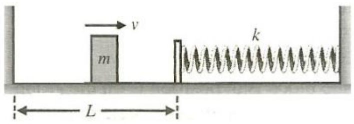
A 100 g block is connected to a horizontal massless spring of force constant 25.6 N/m. as shown in figure (a), the block is free to oscillate on a horizontal frictionless surface. The block is displaced 3 cm from the equilibrium position and, at $t = 0$, it is released from from rest at $x = 0$. It executes simple harmonic motion with the positive x-direction indicated in figure (a)
The position-time $(x - t)$ graph of motion of the block is as shown in figure



208. When the block is at position A on the graph, its
   a) Position and velocity both are negative
   b) Position is positive and velocity is negative
   c) Position is negative and velocity is positive
   d) Position and velocity both the positive

Paragraph for Question Nos. 209 to - 209

A spring having a spring constant k is fixed to a vertical wall as shown in figure. A block of mass m moves with velocity v towards the spring from a parallel wall opposite to this wall. The mass hits the end of the spring compressing it and is decelerated by the spring force and comes to rest and then turns back till the spring acquires its natural length and contact with the spring acquires its natural length and contact with the spring is broken. In this process, it regains its angular speed in the opposite direction and makes a perfect elastic collision on the opposite left wall and starts moving with same speed as before towards right. The above processes are repeated and there is periodic oscillations

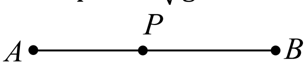


209. What is the maximum compression produced in the spring?

a) $v\sqrt{\frac{m}{k}}$ b) $\sqrt{\frac{m}{k}}$ c) $v\sqrt{mk}$ d) $v\sqrt{\frac{k}{m}}$

Paragraph for Question Nos. 210 to - 210

A and B are two fixed point at a distance $3l$ apart. A particle of mass m placed at a point P experiences the force $2(mg/l)PA$ and the force $(mg/l)PB$ simultaneously. Initially at $t = 0$, the particle is projected from A towards B with speed $3\sqrt{gl}$



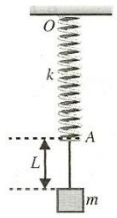
210. The particle moves simple harmonically with period T and amplitude A

a) $A = 2l, T = 2\pi\sqrt{\frac{L}{g}}$ b) $A = 3l, T = 2\pi\sqrt{\frac{l}{2g}}$ c) $A = 2l, T = 2\pi\sqrt{\frac{l}{3g}}$ d) $A = l, T = 2\pi\sqrt{\frac{l}{g}}$

Paragraph for Question Nos. 211 to - 211

A body of mass m is attached by an inelastic string to a suspended spring of spring constant k. Both the string and the spring have negligible mass and the string is inextensible and of length L. Initially, the mass m is at rest

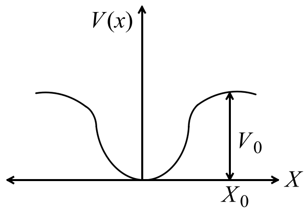
211. If the mass m is now raised up to point A (the top end of the string, see figure and allowed to fall from rest, the maximum extension of the spring in the subsequent motion will be



a) L b) $\frac{mg}{k}$ c) $\frac{mg}{k}\sqrt{1+\frac{2kL}{mg}}$ d) $\frac{mg}{k}\left[1+\frac{2kL}{mg}\right]$

Paragraph for Question Nos. 212 to - 212

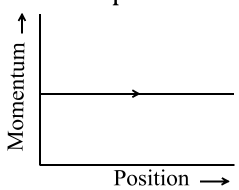
When a particle of mass m moves on the x-$axis$ in a potential of the form $V(x) = kx^2$, it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of $x = 0$ in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x-$axis$. Its potential energy is $V(x) = ax^4, (a > 0)$ for $|x|$ near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see figure)



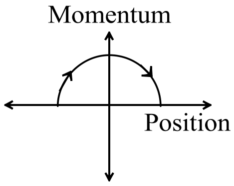
212. If the total energy of the particle is E, it will perform periodic motion only if
a) $E < 0$ b) $E > 0$ c) $V_0 > E > 0$ d) $E > V_0$

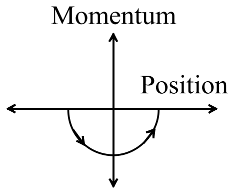
Paragraph for Question Nos. 213 to - 213

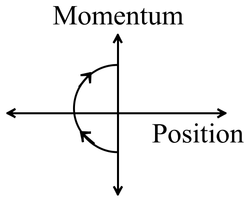
Phase space diagrams are useful tools is analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one-dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is $x(t)$ vs. p(t) curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which position or momentum upwards (or to right) is positive and downwards (or to left) is negative

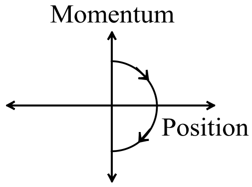


213. The phase space diagram for a ball thrown vertically up from ground is

a) 

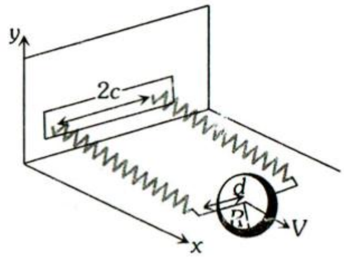
b) 

c) 

d) 

Paragraph for Question Nos. 214 to - 214

A uniform thin cylindrical disk of mass M and radius R is attached to two identical massless springs of spring constant k which are fixed to the wall as shown in the figure. The springs are attached to the axle of the disk symmetrically on either side at a distance d from its centre. The axle is massless and both the springs and axle are in a horizontal plane. The unstretched length of each spring is L. The disk is initially at its equilibrium position with its centre of mass (CM) at a distance L from the wall. The disk rolls without slipping with velocity $\vec{V}_0 = V_0\hat{\imath}$. The coefficient of friction is μ

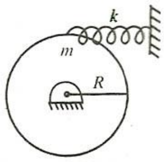


214. The net external force acting on the disk when its centre of mass is at displacement x with respect to its equilibrium position is

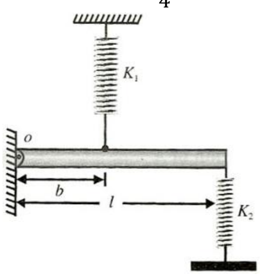
a) $-kx$　　b) $-2kx$　　c) $-\frac{2kx}{3}$　　d) $-\frac{4kx}{3}$

Integer Answer Type

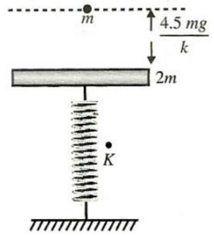
215. A uniform disc of mass m and radius R is pivoted smoothly at its centre of mass. A light spring of stiffness k is attached with the disc tangentially as shown in the figure. Find the angular frequency in rad/s of torsional oscillations of the disc. (Take $m = 5$kg and $K = 10$ N/m)



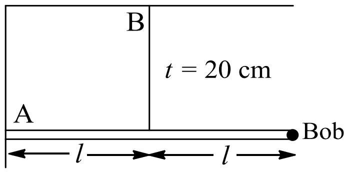
216. A rod of mass m and length l hinged at one end is connected by two springs of spring constants k_1, and k_2, so that it is horizontal at equilibrium. What is the angular frequency of the system? (in rad/s) (Take $\ell = 1\text{m}, b = \frac{1}{4}\text{m}, K_1 = 16\frac{\text{N}}{\text{m}}, K_2 = 61$ N/m)



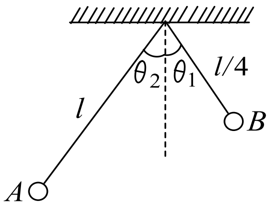
217. In the figure shown a plate of mass 60 g is at rest and in equilibrium. A particle of mass $m = 30$ g is released from height 4.5 mg/k from the plate. The particle sticks to the plate. Neglecting the duration of collision find the time from the collision of the particle and plate to the moment when the spring has maximum compression. Spring has force constant 1 N/m. Calculate the value of time in the form π/x and find the value of X



218. A weightless rigid rod with a small iron bob at the end is hinged at point A to the wall so that it can rotate in all directions. The rod is kept in the horizontal position by a vertical inextensible string of length 20 cm, fixed at its midpoint. The bob is displaced slightly perpendicular to the plane of the rod and string. Find period of small oscillations of the system in the form $\frac{\pi X}{10}$ s and fill the value of X.

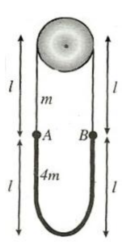


219. Two simple pendulums A and B having lengths l and $l/4$, respectively are released from the position as shown in figure. Calculate the time (in seconds) after which the two strings become parallel for the first time. (Take $\ell = \frac{90}{\pi^2}$ m and g $= 10\text{m/s}^2$)

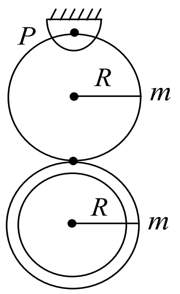


220. A small body of mass m is connected to two horizontal springs of elastic constant k. natural length $3d/4$. In the equilibrium position both springs are stretched to length d, as shown figure. What will be the ratio of period of the motion (T_b/T_a) if the body is displaced horizontally by a small distance where T_a is the time period when the particle oscillates along the line of springs and T_b is time period when the particle oscillates perpendicular to the plane of the figure? Neglect effects of gravity

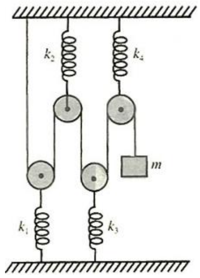
221. Two uniform ropes having linear mass densities m and $4m$ are joined to form a closed loop. The loop is hanging over a fixed frictionless small pulley with the lighter rope above as shown in the figure (in the figure equilibrium position is shown). Now if point A (joint) is slightly displaced in downward direction and released, It is found that the loop performs SHM with the period of oscillation equal to N. Find the value of N (take $l = \frac{150m}{4\pi^2}$, g $= 10\text{m/s}^2$)



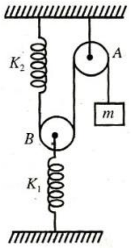
222. A uniform disc of mass m and radius $R = \frac{80}{23\pi^2} m$ is pivoted smoothly at P. If a uniform disc of mass m and radius R is welded at the lowest point of the disc, find the period of SHM of the system (disc + ring).(in seconds)



223. In the arrangement shown in figure, pulleys are small and light and springs are ideal and $K_1 = 25\pi^2$ N/m, $K_2 = 2K_1, K_3 = 3K_1$ and $K_4 = 4K_1$ are the force constants of the springs. Calculate the period of small vertical oscillations of block of mass $m = 3$ kg



224. A block of mass m is tied to one end of a spring which passes over a smooth fixed pylley A and under a light smooth movable pulley B. The other end of the string is attached to the lower end of a spring of spring constant K_2. Find the period of small oscillation of mass m about its equilibrium position (in second). (Take $m = \pi^2$ kg, $K_2 = 4K_1$ and $K_1 = 17$ N/m)



: ANSWER KEY :

1)	d	2)	b	3)	c	4)	d
5)	b	6)	b	7)	a	8)	a
9)	d	10)	a	11)	c	12)	d
13)	c	14)	a	15)	c	16)	c
17)	b	18)	b	19)	a	20)	a
21)	a	22)	a	23)	b	24)	d
25)	d	26)	b	27)	a	28)	c
29)	a	30)	c	31)	d	32)	a
33)	c	34)	a	35)	a	36)	b
37)	c	38)	a	39)	b	40)	b
41)	a	42)	b	43)	a	44)	b
45)	b	46)	d	47)	a	48)	d
49)	d	50)	a	51)	d	52)	c
53)	c	54)	a	55)	c	56)	c
57)	c	58)	c	59)	c	60)	c
61)	b	62)	c	63)	d	64)	a
65)	b	66)	b	67)	b	68)	b
69)	c	70)	d	71)	c	72)	d
73)	b	74)	c	75)	c	76)	d
77)	d	78)	a	79)	a	80)	b
81)	b	82)	b	83)	a	84)	b
85)	b	86)	c	87)	d	88)	d
89)	b	90)	c	91)	c	92)	c
93)	b	94)	d	95)	a	96)	b
97)	b	98)	c	99)	a	100)	c
101)	a	102)	b	103)	b	104)	b
105)	c	106)	b	107)	b	108)	b
109)	d	1)	a,c	2)	b,c,d	3)	
	a,d	4)	b,d				
5)	a,b,d	6)	c,d	7)	b,d	8)	
	b,d						
9)	a,c,d	10)	a,d	11)	a,c	12)	
	a,d						
13)	a,c	14)	b,d	15)	a,c,d	16)	
	a,c						
17)	a,b	18)	a,d	19)	b,c	20)	
	b,c,d						
21)	a,b,c,d	22)	a,b,c,d	23)	a,c	24)	
	a,c						
25)	a, b	26)	a,b,d	27)	b,c	28)	
	a,b,d						
29)	a,c,d	30)	a,b,c	31)	a,b,c,d	32)	
	b,c						
33)	b,c	34)	a,b,c,d	35)	a,b,c	36)	
	b,d						
37)	a,b,d	38)	a,c	39)	c,d	1)	a
	2)	d	3)	b	4)	d	

5)	a	6)	b	7)	e	8)	b
9)	b	10)	a	11)	a	12)	a
13)	c	14)	a	15)	a	16)	b
17)	d	18)	b	19)	a	20)	a
21)	b	22)	b	23)	c	24)	d
25)	b	26)	d	27)	a	28)	e
29)	c	30)	c	31)	b	32)	a
33)	d	34)	c	35)	b	36)	d
37)	a	38)	a	39)	a	1)	a
	2)	a	3)	a	4)	c	
5)	b	6)	c	7)	a	8)	d
9)	d	10)	b	11)	a	1)	d
	2)	b	3)	b	4)	b	
5)	a	6)	b	7)	c	8)	b
9)	b	10)	b	11)	a	12)	c
13)	d	14)	c	15)	d	16)	d
1)	2	2)	8	3)	5	4)	4
5)	1	6)	2	7)	5	8)	2
9)	2	10)	1				

: HINTS AND SOLUTIONS :

1 **(d)**

As the scale can read maximum 50 kg, for a length of 20 cm, let the spring constant be k, then

$kx_0 = mg$[for $m = 50$ kg, $x_0 = 20$ cm]

$\Rightarrow k \Rightarrow 0.2 = 50 \Rightarrow 10 = 2500$ N/m

Let mass of the body be m_0, then from $T = 2\pi\sqrt{\frac{m_0}{k}}$

$\Rightarrow 0.6 = 2\pi\sqrt{\dfrac{m_0}{2500}} \Rightarrow m_0 = 22.8$ kg

2 **(b)**

$y_1 = a\sin\omega t$ and $y_2 = \sin(\omega t + \phi)$

$y_2 - y_1 = a\sqrt{2} = a\sin(\omega t + \phi) - a\sin\omega t)$

or $\sqrt{2}a = 2a\cos\left(\frac{\omega t+\phi+\omega t}{2}\right) = \sin\left(\frac{\omega t+\phi-\omega t}{2}\right)$

$= 2a\cos\left(\omega t + \dfrac{\phi}{2}\right)\sin\dfrac{\phi}{2}$

For maximum value, $\cos(\omega t + \phi) = 1$, therefore

$2\sin\dfrac{\phi}{2} = \sqrt{2} \Rightarrow \sin\dfrac{\phi}{2} = \dfrac{1}{\sqrt{2}}$

or $\frac{\phi}{2} = \frac{\pi}{4}$ or $\phi = \frac{\pi}{2}$

3 **(c)**

$U = a + bx^2$

$F = -\dfrac{dU}{dx} = -[0 + 2bx]$

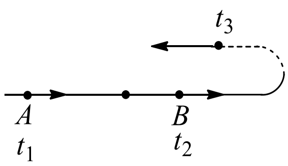
$F = ma = -2bx \Rightarrow a = -\left(\dfrac{2b}{m}\right)x$

$\omega = 2\pi f = \left(\dfrac{2b}{m}\right)^{1/2}$

Hence frequency depends upon b and m

4 **(d)**

The situation is illustrated in the following figure,



$t_2 - t_1 = 2$ s, $t_3 - t_1 = 2$ s

I.e., $t_3 - t_1 = 4$ s which is half of the time period

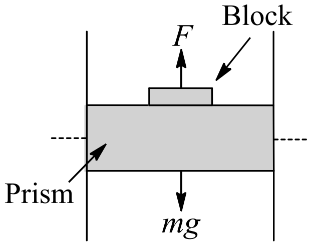
So, $T = 8$ s

5 **(b)**

The situation when the block is just below the mean position is shown in figure, the restoring forces acting on the piston cause a normal reaction F to act on the block. For the block to separate $F \geq mg$

i.e., $m\omega^2 A \geq mg$

(where ω = angular frequency and A = amplitude)

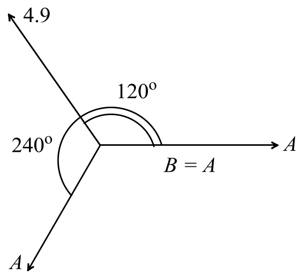


or $A \geq \frac{g}{\omega^2} = \frac{g}{\left(\frac{2\pi}{T}\right)^2}$ or $A \geq \frac{gT^2}{4\pi^2}$

Now, the maximum velocity v_{max} at that instant $= \omega A$

$v_{max} = \left(\dfrac{2\pi}{T}\right)\left(\dfrac{gT^2}{4\pi^2}\right) = \dfrac{10}{\pi}$ m/s^2

6 **(b)**



So $B = A, \phi = 240° = \frac{4\pi}{3}$

7 **(a)**

Let x_1 and x_2 be the distances of the two positions from centre. Then with usual notations

$u^2 = \omega^2(A^2 - x_1^2)$ (i)

$v^2 = \omega^2(A^2 - x_2^2)$ (ii)

$a = \omega^2 x_1$ (iii)

$b = \omega^2 x_2$ (iv)

Subtracting Eq. (ii) from Eq.(i),

$u^2 - v^2 = \omega^2(x_2^2 - x_1^2)$ (v)

Adding Eqs. (iii) and (iv),

$a + b = \omega^2(x_1 + x_2)$ (vi)

Dividing Eq. (v) by Eq. (vi) $\frac{u^2-v^2}{a+b} = x_2 - x_1$

8 **(a)**

With mass m_2 alone, the extension of the spring l is given as

$M_2\,g = kl$ (i)

With mass $(m_1 + m_2)$, the extension l is given by

$(m_1 + m_2)g = kl' = k(l + \Delta l)$ (ii)

Hence Δl is the amplitude of vibration

Subtracting Eq.(i) from Eq. (ii), we get $m_1 g = k\Delta l$

or $\Delta l = \frac{m_1 g}{k}$

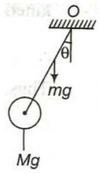
9 **(d)**

$\tau_A = \tau_B = (mg\frac{L}{2}\sin\theta + M\,gL\sin\theta)$

$= $ Restoring torque about point O.

In case A, moment of inertia will be more. Hence, angular acceleration ($\alpha = \tau/I$) will be less. Therefore angular frequency will be less.

Note Question is difficult because this type of SHM is rarely.



10 **(a)**

Since $F = -\frac{dU}{dx} = 2kx \exp(-x^2)$

$F = 0$ (at equilibrium as $x = 0$)

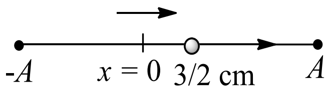
U is minimum at $x = 0$ and $U_{\min} = 0$

U is maximum at $x \longrightarrow \pm\infty$ and $U_{\max} = k$

The particle would oscillate about $x = 0$ for small displacement from the origin and it is in stable equilibrium at the origin

11 **(c)**

The time period of simple harmonic motion is $T = 18$ s,



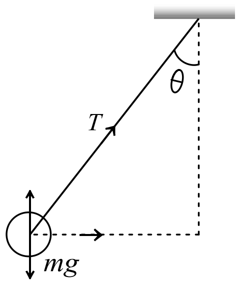
At $t = 0$, the particular is at $x = 3/2$ cm and approaching positive extreme

$39\text{ s} = 2 \times 18 + 3 = 2T + 3 = 2T + T/6$

Distance travelled by the particle in $2T$ is $8A = 24$ cm. Distance travelled by the particle in further $T/6$ is $A/2 = 1.5$ cm

Total distance travelled $= 25.5$ cm

12 **(d)**



$T \sin\theta = mL \sin\theta\omega^2$

$324 = 0.5 \times 0.5 \times \omega^2$

$$\Rightarrow \omega^2 = \frac{324}{0.5 \times 0.5}$$

$$\Rightarrow \omega = \sqrt{\frac{324}{0.5 \times 0.5}}$$

$$\Rightarrow \omega = \frac{18}{0.5} = 36\ rad/sec$$

13 **(c)**

Time interval is $T/6$

Let the equation of simple harmonic motion be $x = A\cos\omega t, v = -A\sin\omega t,$ where $\omega = 2\pi/T$

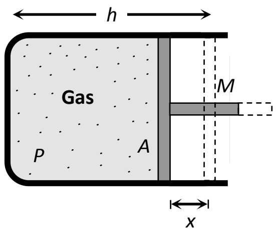
Mean velocity over the required interval is

$$< v > = \frac{\int_0^{T/6} v dt}{T/6}$$

$$= \frac{\int_0^{T/6} -A\omega \sin\omega t\, dt}{T/6} = \frac{6A}{T}\cos\omega t\Big|_0^{T/6} = \frac{-3A}{T}$$

14 **(a)**

Let the piston be displaced through distance x towards left, then volume decreases, pressure increases. If ΔP is increase in pressure and ΔV is decreases in volume, then considering the process to take place gradually (i.e. isothermal)



$P_1V_1 = P_2V_2$

$\Rightarrow PV = (P + \Delta P)(V - \Delta V)$

$\Rightarrow PV = PV + \Delta PV - P\Delta V - \Delta P\Delta V$

$\Rightarrow \Delta P.V - P.\Delta V = 0$ (neglecting $\Delta P.\Delta V$)

$$\Delta P(Ah) = P(Ax) \Rightarrow \Delta P = \frac{P.x}{h}$$

This excess pressure is responsible for providing the restoring force (F) to the piston of mass M

Hence $F = \Delta P.A = \frac{PAx}{h}$

Comparing it with $|F| = kx \Rightarrow k = M\omega^2 = \frac{PA}{h}$

$$\Rightarrow \omega = \sqrt{\frac{PA}{Mh}} \Rightarrow T = 2\pi\sqrt{\frac{Mh}{PA}}$$

Short trick: by checking the options dimensionally. Option (a) is correct

15 **(c)**

For torsional pendulum, time period is given by

$$T = 2\pi\sqrt{\frac{I}{k}}$$

Where k is torsional constant of string and it is same for both the cases

$$I_{\text{rod}} = \frac{mL^2}{12} = \frac{1 \times 1.2^2}{12} = 0.12\text{ kg} - \text{m}^2$$

$$\frac{T_{\text{rod}}}{T_{\text{plate}}} = \left(\frac{I_{\text{rod}}}{I_{\text{plate}}}\right)^{1/2} \Rightarrow \left(\frac{3}{6}\right)^2 = \frac{0.12}{I_{\text{plate}}}$$

$I_{\text{plate}} = 0.48\text{ kg} - \text{m}^2$

16 **(c)**

$$F = -\frac{dU}{dx} = -8\sin 2x$$

For small oscillations, $\sin 2x = 2x$
i.e., $a = -16x$
since $a\mu - x$, the oscillations are simple harmonic in nature

$$T = 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2\pi\sqrt{\frac{1}{16}} = \frac{\pi}{2}\text{ s}$$

17 **(b)**
The angular frequency is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{T}} \approx 14 \text{ rad/s}$$

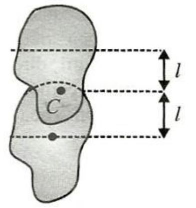
18 **(b)**
Let equation of simple harmonic motion is
$x = A\sin(\omega t + \delta)$
It is given, $A = 0.5$ m and $\omega = \frac{2\pi}{T} = \frac{2\pi}{4}\text{s}^{-1} = \frac{\pi}{2}\text{s}^{-1}$
At $t = 0.5$ s, $x = 0.3$ m, so $0.3 = 0.5\sin(\omega t + \delta)$
$\Rightarrow \sin\left(\frac{\pi}{2}\times\frac{1}{2} + \delta\right) = \frac{3}{5} \Rightarrow \frac{\pi}{4} + \delta = 37°$
$\Rightarrow \delta = 37° - 45° = -8°$
So, equation of motion is $x = (0.5\text{ m})\sin\left[\frac{\pi t}{2} - 8°\right]$

19 **(a)**
Conserving momentum, $2V = 3V'$
$V' = \frac{2}{3}V$
$E_i = \frac{1}{2}m_1v_1^2 = \frac{1}{2}\times 2V^2 = V^2 = \frac{1}{2}KA^2$
$E_f = \frac{1}{2}\times m_2V_2^2 = \frac{1}{2}\times 3\times\frac{2}{3}\times\frac{2}{3}V^2 = \frac{2}{3}V^2$
$\frac{1}{2}KA^2 = \frac{2}{3}V^2 = \frac{2}{3}E_i$ $\quad(\because E_i = V^2$ from above)
$\frac{1}{2}KA^2 = \frac{2}{3}\left(\frac{1}{2}KA^2\right) \Rightarrow A' = \sqrt{\frac{2}{3}}A$

20 **(a)**
The frequency will be same $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ but due to the constant qE force, the equilibrium position gets shifted by $\frac{qE}{K}$ in forward direction. So Sol. will be (a)

21 **(a)**
Equation of SHM $= y\sin(\omega t + \phi)$
When $y = \frac{a}{2}, t = \frac{T}{4} = \frac{2\pi}{4\omega} = \frac{\pi}{2\omega}$
$v = a\omega\cos(\omega t + \phi)$, velocity is negative
$\frac{\pi}{2} < (\omega t + \phi) < \frac{3\pi}{2}$
$\frac{a}{2} = a\sin(\omega t + \phi) \Rightarrow \sin(\omega t + \phi) = \frac{1}{2}$
$\left(\frac{\pi}{2} + \phi\right) = \frac{5\pi}{6}$
Substituting in the above equation, we get
$\phi = \pi/3$

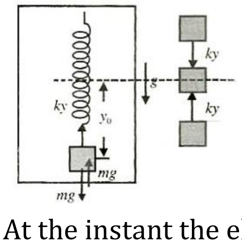
22 **(a)**
$\frac{1}{2}I\omega^2 = mg\,(2l) \Rightarrow \frac{I}{mgl} = \frac{4}{\omega^2}$
$T = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{4}{\omega^2}} = \frac{4\pi}{\omega}$



23 **(b)**
$U = \frac{a}{x^2} - \frac{b}{x}; F = -\frac{dU}{dx} = -\left[\frac{-2a}{x^3}.\frac{b}{x^2}\right]$
At equilibrium position, $F = 0$
$x = x_0 = \frac{2a}{b}$
At $x = x_0 + \Delta$, i.e., at a displacement of $\Delta x(\ll x_0)$ from mean position,
$F = -\left[\frac{bx - 2a}{x^3}\right] = -\left[\frac{b(x_0 + \Delta x) - 2a}{(x_0 + \Delta x)^3}\right] = \frac{b\Delta x}{x_0^3}$
$= -\frac{b^4}{8a^3}\Delta x$
As $F\ \mu\Delta x$, so particle performs simple harmonic motion with time period
$T = 2\pi\sqrt{\frac{8a^3m}{b^4}}$

24 **(d)**
When the spring undergoes displacement in the downward direction it completes one-half oscillation while it completes another half oscillation in the upward direction. The total time period is
$T = \pi\sqrt{\frac{m}{k}} + \pi\sqrt{\frac{m}{2k}}$

25 **(d)**
When the elevator is at rest, the elongation in spring is given by $ky_0 = mg$
or $y_0 = \frac{mg}{k}$



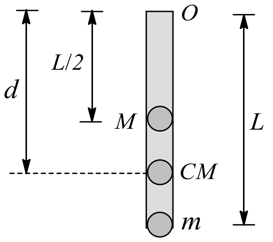
At the instant the elevator starts falling down with acceleration g, the block is at rest w.r.t. elevator and the net force acting on it is ky_0 in the ward direction w.r.t. lift frame of reference. Due to thin the block moves up and as a result elongation in the spring decreases, and the force experienced by the block becomes zero when spring and stops momentarily till compression in the spring becomes mg/k

Hence the block will always have net force towards relaxed position of spring and the block will perform simple harmonic notion with time period $T = 2\pi\sqrt{\frac{m}{k}}$ and with amplitude mg/k

26 **(b)**

When the rod is replaced by the string the simple pendulum will act as compound pendulum for which time period is given by

$$T = 2\pi\sqrt{\frac{I_0}{Mgd}} \quad \text{(i)}$$



I_0 =moment of inertia about point of suspension

$= \frac{ML^2}{3} + mL^2 = \frac{(M+3m)L^2}{3}$

M'=total mass $= (M+m)$

d = seperation between point of suspension and centre of mass of the pendulum

$= \frac{M(L/2)+m(L)}{(M+m)} = \frac{(M+2m)L}{2(M+m)}$

Substituting the value in Eqs (i), we get

$$\Rightarrow\ T = 2\pi\sqrt{\frac{2(M+3m)L}{3(M+2m)g}}$$

27 **(a)**

Let $x_1 = a\sin\omega t$ and $x_2 = a = a\sin(\omega t+\delta)$ be two SHMs

$\frac{a}{3} = a\sin\omega t$ and $-\frac{a}{3} = a\sin(\omega t+\delta) = -1/3$

$\sin\omega t = 1/3$ and $\sin(\omega t+\delta) = -1/3$

Eliminating $t, \frac{1}{3}\cos\delta + \sqrt{1+\frac{1}{9}}\sin\delta = -\frac{1}{3}$

$9\cos^2\delta + 2\cos\delta\text{-}7\text{=}0$

$\cos\delta = -1$ or $\frac{7}{9}$

i.e., $\delta = 180°$ or $\cos^{-1}\left(\frac{7}{9}\right)$

if we have 180°, we find that v_1 and v_2 are of opposite sings hence $\delta = 180°$ is not applicable

$\therefore \delta = \cos^{-1}\left(\frac{7}{9}\right)$

28 **(c)**

Due to impulse, the total energy of the particle becomes

$\frac{1}{2}m\omega^2A^2 + \frac{1}{2}m\omega^2A^2 = m\omega^2A^2$

Let A' be the new amplitude,

$\frac{1}{2}m\omega^2(A')^2 = m\omega^2A^2 \Rightarrow A' = \sqrt{2}A$

29 **(a)**

For A, time period $T_A = 16$ s (distance between two adjacent crests)

For B, time period $T_B = 2(20-8) = 24$ s (length between the crest and through shown $= 20\text{ s} - 8\text{ s} = 12$ s)

Also, amplitudes $a_A = 10$ cm, $a_B = 5$ cm

Now, $\frac{(V_{\max})_A}{(V_{\min})_B} = \frac{\omega_A a_A}{\omega_B a_B}$

$= \frac{\left(\frac{2\pi}{T_A}\right)a_A}{\left(\frac{2\pi}{T_B}\right)a_B} = \frac{T_B a_A}{T_A a_B} = \frac{24\times 10}{16\times 5} = \frac{3}{1}$

30 **(c)**

From the graph $T = (5-1) = 4$ s

(distance between the two adjacent crests shown in the figure)

And $v_{\max} = 5$ m/s; $\omega A = 5$ m/s

$\left(\frac{2\pi}{T}\right)A = 5 \Rightarrow A = \frac{5T}{2\pi} = \frac{5\times 4}{2\pi} = \frac{10}{\pi}$ m

Also, $\omega = \frac{2\pi}{4} = \frac{\pi}{2}$ rad/s

The equation of velocity can be written as

$V = 5\sin\left(\frac{\pi}{2}t\right)$ m/s

At extreme position, $v = 0; \sin\left(\frac{\pi}{2}t\right) = 0$ or $t = 2$s

Phase of the particle velocity at that instant corresponding to the above equation $= \pi$

Therefore, when a phase change of $\pi/6$ takes place, the resulting phase $= \pi + \pi/6$

$v = 5\sin\left(\pi + \frac{\pi}{6}\right) = -5\sin\frac{\pi}{6} = -5\left(\frac{1}{2}\right)$

$= 2.5$ m/s (numerically)

$\frac{dy}{dt} = v \Rightarrow dy = vdt$

$dy = \int 5\sin\left(\frac{\pi t}{2}\right)dt = \frac{10}{\pi}\left[-\cos\frac{\pi t}{2}\right] + C$

Since at $t = 0$, the particle is at the extreme position, therefore at $t = 0; y = -\frac{10}{\pi}$

$-\frac{10}{\pi} = -\frac{10}{\pi}\cos\theta$

$y = -\frac{10}{\pi}\cos\frac{\pi t}{2}$

Clearly a phase change of $\pi/6$ corresponds to a time difference of

$\frac{T}{2\pi}\left(\frac{\pi}{6}\right) = \frac{T}{12} = \frac{4}{12} = \frac{1}{3}$s

$$y=-\frac{10}{\pi}\cos\frac{\pi}{6}=-\frac{10}{\pi}\left(\frac{\sqrt{3}}{2}\right)$$

$=\frac{5\sqrt{3}}{\pi}$ m (numerically)

Acceleration, $a=\frac{dv}{dt}=\frac{d}{dt}\left(5\sin\frac{\pi t}{2}\right)=\frac{5\pi}{2}\frac{\cos\pi t}{2}$

a at $t=\frac{1}{3}\text{ s}=\frac{5\pi}{2}\cos\frac{\pi}{6}=\frac{5\pi\sqrt{3}}{4}\text{ m/s}^2$

Maximum displacement, $x_{\max}=A=\frac{10}{\pi}$ m

and maximum acceleration, $a_{\max}=\omega^2 A$

$$=\left(\frac{\pi}{2}\right)^2\times\frac{10}{\pi}=\frac{5\pi}{2}\text{ m/s}^2$$

31 **(d)**

When spring is compressed by $3x_0$. Amplitude, $A=3x_0$. The time taken from extreme compressed position to mean position, $t_1=T/4$

If time taken (t_2) from mean position to $x=x_0$ is given by

$$x=A\sin\frac{2\pi t_2}{T}\Rightarrow x_0=3x_0\sin\frac{2\pi t_2}{T}$$

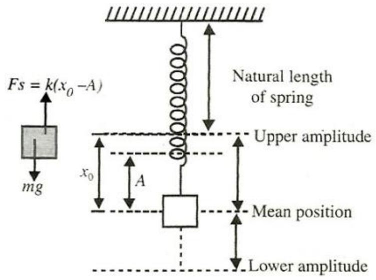
$$\sin\frac{2\pi t_2}{T}=\frac{1}{3}\Rightarrow\frac{2\pi t_2}{T}=\frac{\pi}{9}\Rightarrow\ t_2=\frac{T}{18}$$

$$t_1+t_2=\frac{T}{4}+\frac{T}{18}=\frac{11}{18}T\ =\frac{11}{18}2\pi\sqrt{\frac{m}{K}}=\frac{11}{9}\pi\sqrt{\frac{m}{K}}$$

32 **(a)**

The spring is never compressed. Hence spring shall exert least force on the block when the block is at topmost position

At equilibrium position $kx_0=mg\Rightarrow x_0=mg/k$



$$F_{\max}=k(x_0-A)=k\left[\frac{mg}{k}-A\right]$$

$$=mg-4\frac{\pi^2}{T^2}mA=mg-kA=mg-m\omega^2 A$$

33 **(c)**

If the stick is rotated through a small angle θ, the spring is stretched by a distance $L\theta/2$. Therefore restoring force in spring $=(KL\theta/2)$. Each spring cause a torque

$$\tau(=Fd)=\left(\frac{KL\theta}{2}\right)\frac{L}{2}$$

In the same direction. Therefore equation of rotational motion is $\tau=I\alpha$

$$-2\left(\frac{KL\theta}{2}\right)\frac{L}{2}=I\alpha\ \text{ with }\ I=\frac{ML^2}{12}$$

$$\alpha=-\left(\frac{6K}{M}\right)\theta$$

Standard equation of angular SHM is $\alpha=-\omega^2\theta$

$$\omega^2=\frac{6K}{M}\Rightarrow f=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{6K}{M}}$$

34 **(a)**

$U=k|x|^3\Rightarrow F=-\frac{dU}{dx}=-3k|x|^2$ ...(i)

Also, for SHM $x=a\sin\omega t$ and $\frac{d^2x}{dt^2}+\omega^2 x=0$

$\Rightarrow$ acceleration $a=\frac{d^2x}{dt^2}=-\omega^2 x\Rightarrow F=ma$

$=m\frac{d^2x}{dt^2}=-m\omega^2 x$ ...(ii)

From equation (i) & (ii) we get $\omega=\sqrt{\frac{3kx}{m}}$

$$\Rightarrow T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{m}{3kx}}=2\pi\sqrt{\frac{m}{3k(a\sin\omega t)}}\Rightarrow T$$

$$\propto\frac{1}{\sqrt{a}}$$

35 **(a)**

The oscillation of the block will be periodic but not simple harmonic. The springs are only compressed but not extended since the block loses contact with either spring just in its relaxed state. The minimum distance between the springs will therefore be when both the springs are relaxed, i.e., during the interval when the block moves between the springs

Time taken by the block to move from one spring to the other is $t_1=d/v$

Time taken by the block in contact with either spring is

$$t_2=\frac{1}{2}\left[2\pi\sqrt{\frac{m}{K}}\right]=\pi\sqrt{\frac{m}{K}}$$

Hence the periodic time T of the oscillation of the block is $T=2(t_1+t_2)$

$$=2\left[\frac{d}{v}+\pi\sqrt{\frac{m}{K}}\right]$$

Also minimum distance between the springs $=d$

36 **(b)**

The velocity is minimum (zero) at the extreme position and maximum $(\pm\omega A)$ at the mean position

The acceleration is maximum (zero) at the mean position. Since the particle crosses the mean and extreme positions twice, during each oscillation, hence the result

37 **(c)**

Let ω_1 and ω_2 be the angular frequencies of first and second particle, respectively. Then, the phase by which they will proceed in time t is $\omega_1 t$ and $\omega_2 t$, respectively. According to the given situation,

$\omega_2 t - \omega_1 t = 3 \times 2\pi$ for $t = 45$ s

$$\frac{2\pi}{T} - \frac{2\pi}{3} = \frac{3 \times 2\pi}{45}$$

$$\frac{1}{T} = \frac{1}{3} + \frac{1}{15} = \frac{6}{15} \Rightarrow T = 2.5 \text{ s}$$

38 **(a)**

From graph of acceleration and time graph

$f = -f_{\max} \cos \omega t = -(\omega^2 A) \cos \omega t$

And we have relationship between acceleration and position as $f = -\omega^2 x \Rightarrow \quad x = A \cos \omega t$

Which gives $v = \frac{dx}{dt} = -A\omega \sin \omega t = -v_{\max} \sin \omega t$

Hence option (a) is correct

39 **(b)**

Let us say in displaced position, the axis of cylinder is at a distance x from its mean position and its velocity of centre of mass is v and angular velocity is ω. Then, as cylinder is not slipping, $v = R\omega$. In this position, the spring elongates by x

Using energy method we can find frequency of oscillation very easily

Total energy of oscillation is

$$E = \frac{I\omega^2}{2} + \frac{Mv^2}{2} + \frac{kx^2}{2}$$

We have $I = \frac{MR^2}{2}$, so $E = \frac{3}{4} M v^2 + \frac{kx^2}{2}$

$$E = v^2 + \left(\frac{2}{3}\frac{k}{M}\right) x^2 = \text{constant}$$

Comparing with $v^2 + \omega^2 x^2 = \text{constant}$

So, $\omega = \sqrt{\frac{2k}{3M}} \Rightarrow T = 2\pi \sqrt{\frac{3M}{2k}}$

40 **(b)**

$$x = a \sin \omega t \text{ with } \omega = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4} \text{rad/s}$$

$$x = a \sin \frac{\pi t}{4}$$

In 2 s (which is equal to $T/4$), one amplitude will be covered . In 1st second

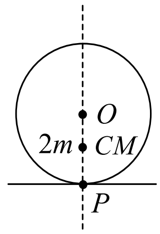
$$x = a \sin \frac{\pi}{4} = \frac{a}{\sqrt{2}}$$

Required ratio $= \frac{\frac{a}{\sqrt{2}}}{a - \frac{a}{\sqrt{2}}} = \frac{1}{\sqrt{2} - 1}$

41 **(a)**

Rolling can be considered as pure rotation about point of contact. We can consider this system as a compound pendulum oscillating about P.

Time period $T = 2\pi \sqrt{\frac{I_P}{mgd}}$



I_P =moment of inertia about rotation axis

$= I_0 + mR^2$

$= mR^2 + mR^2 = 2mR^2$

The centre of mass of tube and rod will be at a height $R/2$ from P, hence $d = R/2$

$$\Rightarrow T = 2\pi \sqrt{\frac{2mR^2}{mgR/2}} = 2\pi \sqrt{\frac{4R}{g}}$$

42 **(b)**

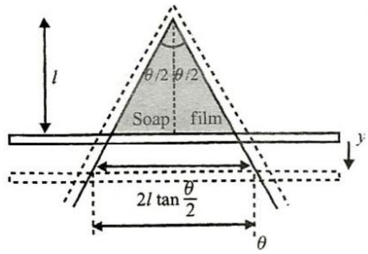
$v^2 = \omega^2(A^2 - b^2) = 3\omega^2 b^2 \Rightarrow A^2 = 4b^2$

$$b = \frac{A}{2} = A \sin \omega t \ \Rightarrow t = \frac{\pi}{6\omega}$$

Required time, $t_1 = \frac{T}{4} - t = \frac{2\pi}{4\omega} - \frac{\pi}{6\omega} = \frac{\pi}{3\omega}$

43 **(a)**

Let S be the surface tension of the soap film. For equilibrium of rod



$$2(l + y) \tan \frac{\theta}{2}$$

$$mg = (F_{\text{surface}})_1$$

$$mg = \left(2l \tan \frac{\theta}{2}\right) S \times 2; \quad mg = 4Sl \tan \frac{\theta}{2}$$

If the rod is displaced from its mean position by small displacement y, then restoring force on the is

$$F_{\text{rest}} = -[(F_{\text{surface}})_2 - mg] - (F_{\text{surface}})_1$$

$$F_{\text{rest}} = -\left[4S(l + y) \tan \frac{\theta}{2} - mg\right] - \left[4S \tan \frac{\theta}{2} y\right]$$

$$a = -\frac{4S \tan \frac{\theta}{2}}{m} y = \frac{-4S \tan \frac{\theta}{2}}{\left(\frac{4Sl \tan \frac{\theta}{2}}{g}\right)} y$$

$$\frac{d^2 y}{dt^2} = -\left(\frac{g}{l}\right) y \quad \therefore T = 2\pi \sqrt{\frac{l}{g}}$$

44 **(b)**

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{mgl}{I}}$$

Where l is the distance between point of suspension and centre of mass of the body. Thus, for the stick of length L and mass m

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{mg\frac{L}{4}}{\frac{m}{2}\frac{(L/2)^2}{12}}} = \frac{1}{2\pi}\sqrt{\frac{12g}{L}} = \sqrt{2}f_0$$

45 **(b)**

$$E = \frac{1}{2}m\omega^2A^2 = \frac{1}{2}m(2\pi f)^2A^2$$

$$A = \frac{1}{2\pi f}\sqrt{\frac{2E}{m}}$$

Putting $E = K + U$, we get

$$A = \frac{1}{2\pi(25/\pi)}\sqrt{\frac{2\times(0.5+0.4)}{0.2}} = 0.06\text{ m}$$

46 **(d)**

Since acceleration of the particle at initial moment is maximum possible and is negative; therefore, the particle is at right extreme position at this moment. When the particle is released, it starts to move to the left. It means, velocity starts to increase from zero initial value to negative value and its magnitude becomes maximum possible at mean position (at $t = T/4$). It means at $t = T/4$, kinetic energy is equal to maximum possible.

At $t = T/2$, the particle comes to instantaneous rest at left extreme position. It means at $t = T/2, v$ is equal to zero. Hence kinetic energy is equal to zero. At $t = 3T/4$, particle comes back to mean position and now moves to the right. Therefore, velocity is positive and has maximum possible magnitude. Therefore. Kinetic energy is maximum possible.

At $t = T$, particle comes back to initial position (extreme right position). Velocity and kinetic energy become equal to zero

47 **(a)**

For a displacement x, the kinetic and potential energies

are KE $= \frac{1}{2}m(A^2 - x^2)\omega^2$ and PE $= \frac{1}{2}mx^2\omega^2$

Each of these $= \frac{10}{2} = 5$ J when $x = 1$ cm

Hence $\frac{1}{2}mx^2\omega^2 = \frac{1}{2}\times 0.1\times(1\times10^{-2})^2\omega^2 = 5$

This gives $\omega^2 = \frac{10}{0.1\times10^{-4}} = 10^6$

Giving $\omega = 10^3 = 1000$ rad/s

Hence $T = \frac{2\pi}{\omega} = \frac{\pi}{500}$ s

And frequency $f = \frac{500}{\pi}$ s

Also, KE $= 5 = \frac{1}{2}\times 0.1[A^2 - (1\times10^{-2})^2](10^6)$

$A^2 - 10^{-4} = 10^{-4}$

$A^2 = 2\times10^{-4} \Rightarrow A = \sqrt{2}\times10^2\text{ m} = \sqrt{2}$ cm

48 **(d)**

$$T = 2\pi\sqrt{\frac{m}{K_1+K_2}} = 2\pi\sqrt{\frac{10}{360}} = \frac{\pi}{3}\text{s}$$

The maximum velocity is always at equilibrium position since at any other point there will be a restoring force attempting to slow the mass

$$\therefore V_{\max} = \frac{\text{impulse}}{\text{mass}} = \frac{50}{10} = 5\text{ m/s} \Rightarrow \omega = \frac{2\pi}{T} = 6\text{ rad/s}$$

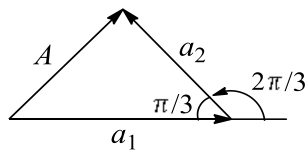
$\Rightarrow A =$amplitude $= \frac{V_{\max}}{\omega} = \frac{5}{6} = 0.83$ m

49 **(d)**

$$x = a_1\sin\omega t + a_2\cos\left(\omega t + \frac{\pi}{6}\right)$$

$$x = a_1\sin\omega t + a_2\sin\left(\omega t + \frac{\pi}{6} + \frac{\pi}{2}\right)$$

or $x = a_1\sin\omega t + a_2\sin\left(\omega t + \frac{2\pi}{3}\right)$



$$\therefore\ A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\frac{2\pi}{3}$$

$$= a_1^2 + a_2^2 - 2a_1a_2\cos\frac{\pi}{3}$$

$$\therefore\ A = \sqrt{a_1^2 + a_2^2 - 2a_1a_2\cos\frac{\pi}{3}}$$

50 **(a)**

In equilibrium, let us say deformations (elongation) in spring are x_{01} and x_{02}. Then

$mg + k_1x_{01} = k_2x_{02}$

Let the block be displaced down by x; then elongation in spring 1 reduce by x and in spring 2 it increases by x. In this situation, the net force acting on the block towards equilibrium position is

$F = k_2(x + x_{02}) - mg - k_1(x_{01} - x)$

$= (k_2 + k_1)x$ (using equilibrium equation)

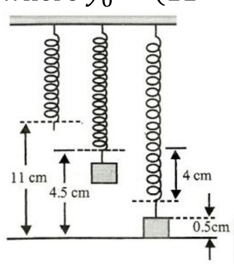
So, the angular frequency of SHM is

$$\sqrt{\frac{k_1+k_2}{m}} = 10\sqrt{2}\text{ rad/s}$$

51 **(d)**

From equilibrium position of mass, $mg = ky_0$

where $y_0 = (11 - 4.5)$ cm $= 6.5$ cm



So, time period of simple harmonic motion is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{y_0}{g}}$$

$$T = 2\pi\sqrt{\frac{5.6\times 10^{-2}}{10}} = 0.5065\text{ s} = 0.5\text{ s}$$

5 s is equivalent to 10 complete time period; so the mass is at its initial position at $t = 5$ s, i.e., it is at 0.5 cm above the table top at $t = 5$ s

52 **(c)**

$$T = \frac{25}{30} = \frac{1}{2}\text{s} \Rightarrow \omega = \frac{2\pi}{T} = 4\pi\text{ rad/s}$$

Spring constant $k = m\omega^2 = 5\times(4\pi)^2 = 80\pi^2\text{N/m}$

Force required to stretch the spring by 5 cm is

$F = kx = 80\pi^2\times 0.05\text{ N} = 4\pi^2$

53 **(c)**

At mean position, the speed will be maximum

$$\frac{kx_0^2}{2} = \frac{mv^2}{2} \Rightarrow v_{\max} = \sqrt{\frac{k}{m}}x_0$$

and this is attained at $t = T/4$

Time period of motion is $T = 2\pi\sqrt{\frac{m}{k}}$

So required time is $t = \frac{\pi}{2}\sqrt{\frac{m}{k}}$

54 **(a)**

Let at $t = 0$, the particle is at extreme position, then the equation of SHM can be written as

$$x = A\cos(\omega t) = A\cos\left(\frac{2\pi}{T}t\right)$$

At $t = T/8$,

$$x = A\cos\frac{\pi}{4} = \frac{A}{\sqrt{2}}$$

Acceleration $= -\omega^2 x = -\left(\frac{2\pi}{T}\right)^2\times\frac{A}{\sqrt{2}}$

Magnitude of acceleration $= \frac{4\pi^2 A}{\sqrt{2}T^2}$

55 **(c)**

$$K = \frac{1}{2}m\omega^2(A^2 - y^2)$$

$$U = \frac{1}{2}m\omega^2 y^2$$

$$K + U = \frac{1}{2}m\omega^2(A^2 - y^2) + \frac{1}{2}m\omega^2 y^2$$

i.e., $2y^2 = A^2$ or $y = \frac{A}{\sqrt{2}}$

56 **(c)**

$$v_1 = \omega\sqrt{a^2 - x_1^2}, v_2 = \omega\sqrt{a^2 - x_2^2}$$

We get $a = \sqrt{\frac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}}$

57 **(c)**

Let the bar be rotated through a small angle θ. The restoring torque of the forces mg, $k_1 x$ and $k_2 x$ about O can be given as

$$\tau = -\left[mg\left(\frac{l}{2}\right)\sin\theta + k_1 x(l\cos\theta) + k_2 x(l\cos\theta)\right]$$

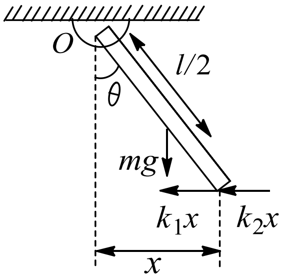
Since θ is small, $\sin\theta \cong \theta, x = l\theta$ and $\cos\theta \cong 1$

Putting $k_1 + k_2 = k$, we obtain

$$\tau = -\left[kl^2 + mg\left(\frac{l}{2}\right)\right]\theta$$

or $I\alpha = -\left[kl^2 + mg\left(\frac{l}{2}\right)\right]\theta$

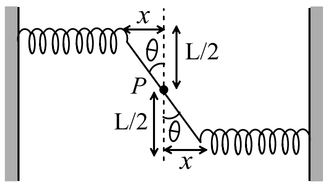
$$\Rightarrow \omega_{\text{osc.}} = \sqrt{\left(\frac{[kl^2 + mg(l/2)]}{(ml^2/3)}\right)} = \sqrt{\frac{3k}{m} + \frac{3g}{2l}}$$



58 **(c)**

Torque about $P = -(kx)\frac{L}{2} + \left(-kx\frac{L}{2}\right) = -kxL = -k\frac{L^2}{2}\theta$

For small angle $\theta, x = \frac{L}{2}\theta;\ \tau = -I\alpha$



$$\Rightarrow -\frac{KL^2}{2}\theta = \frac{ML^2}{12}\alpha$$

$$\Rightarrow \frac{-6K\theta}{M} = \alpha$$

$\Rightarrow \omega = \sqrt{\frac{6K}{M}}$ and $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{6K}{M}}$

59 **(c)**

From the graph $T = (5-1) = 4$ s

(distance between the two adjacent crests shown in the figure)

And $v_{\max} = 5$ m/s; $\omega A = 5$ m/s

$$\left(\frac{2\pi}{T}\right)A = 5 \Rightarrow A = \frac{5T}{2\pi} = \frac{5\times 4}{2\pi} = \frac{10}{\pi}\text{m}$$

Also, $\omega = \frac{2\pi}{4} = \frac{\pi}{2}$ rad/s

The equation of velocity can be written as

$$V = 5\sin\left(\frac{\pi}{2}t\right)\text{ m/s}$$

At extreme position, $v = 0$; $\sin\left(\frac{\pi}{2}t\right) = 0$ or $t = 2$s

Phase of the particle velocity at that instant corresponding to the above equation $= \pi$

Therefore, when a phase change of $\pi/6$ takes place, the resulting phase $= \pi + \pi/6$

$v = 5\sin\left(\pi + \frac{\pi}{6}\right) = -5\sin\frac{\pi}{6} = -5\left(\frac{1}{2}\right)$

$= 2.5$ m/s (numerically)

$\frac{dy}{dt} = v \Rightarrow dy = vdt$

$dy = \int 5\sin\left(\frac{\pi t}{2}\right)dt = \frac{10}{\pi}\left[-\cos\frac{\pi t}{2}\right] + C$

Since at $t = 0$, the particle is at the extreme position, therefore at $t = 0; y = -\frac{10}{\pi}$

$-\frac{10}{\pi} = -\frac{10}{\pi}\cos\theta$

$y = -\frac{10}{\pi}\cos\frac{\pi t}{2}$

Clearly a phase change of $\pi/6$ corresponds to a time difference of

$\frac{T}{2\pi}\left(\frac{\pi}{6}\right) = \frac{T}{12} = \frac{4}{12} = \frac{1}{3}$s

$y = -\frac{10}{\pi}\cos\frac{\pi}{6} = -\frac{10}{\pi}\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{5\sqrt{3}}{\pi}$ m (numerically)

Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}\left(5\sin\frac{\pi t}{2}\right) = \frac{5\pi}{2}\frac{\cos\pi t}{2}$

a at $t = \frac{1}{3}\text{s} = \frac{5\pi}{2}\cos\frac{\pi}{6} = \frac{5\pi\sqrt{3}}{4}\text{m/s}^2$

Maximum displacement, $x_{\max} = A = \frac{10}{\pi}$m

and maximum acceleration, $a_{\max} = \omega^2 A$

$= \left(\frac{\pi}{2}\right)^2 \times \frac{10}{\pi} = \frac{5\pi}{2}\text{m/s}^2$

60 **(c)**

Let the acceleration be $f, f = -\omega^2 x$

Therefore, distance of the particle from the centre at any time t is given by

$x = r\cos(\omega t)$, where r is the amplitude

When $t = 1$ s, $x = r - a$

$\therefore (r - a) = r\cos\omega$

$\cos\omega = \frac{r - a}{r}$ (i)

When $t = 2$ s, $x = r - a - b$,

Therefore $r - a - b = \cos 2\omega$

$\therefore\ r - a - b = r(2\cos^2\omega - 1)$ (ii)

Substituting the value of $\cos\omega$ from Eq.(i) in Eq.(ii), we get

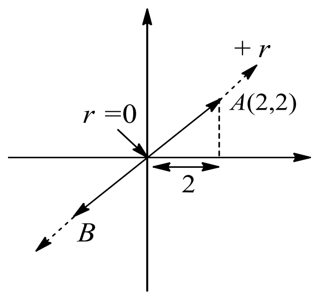
$r - a - b = r\left[2\frac{(r-a)^2}{r^2} - 1\right]$

$r - a - b = \frac{2(r - a)^2}{r} - r$

$\therefore r(3a - b) = 2a^2 \Rightarrow r = \frac{2a^2}{3a - b}$

61 **(b)**

Let the line joining AB represents axis 'r'. By the conditions given 'r'coordinate of the particle at time t is



$\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi \Rightarrow r = 2\sqrt{2}\cos\omega t$

$x = r\cos 45° = \frac{r}{\sqrt{2}} = 2\cos\pi t$

$a_x = -\omega^2 x = -\pi^2 2\cos\pi t$

$F_x = ma_x = -4\pi^2\cos\pi t$

62 **(c)**

$U = U_0 - U_0\cos ax$

$f = -\frac{dU}{dx} = -U_0\sin ax = m\frac{d^2x}{dt^2}$

This is the equation of SHM $\Rightarrow T = \frac{2\pi}{a}\sqrt{\frac{m}{U_0}}$

63 **(d)**

$y = 8\sin^2\left(\frac{t}{2}\right)\sin(10t)$

$= 4[1 - \cos t]\sin(10\,t)$(using $2\sin^2\frac{\theta}{2}$
$= 1 - \cos\theta$)

$= 4\sin(10t) - 4\sin(10t)\cos t$

$= 4\sin(10t) - 2[\sin 11t + \sin 9t]$

(using $2\ \sin C\cos D = \sin(C + D) + \sin(C - D)$)

$= 4\sin(10\,t) - 2\sin(11t) - 2\sin(9t)$

Evidently, y is obtained as the superimposition of three independent (i.e., having different angular frequency ω) SHMs

64 **(a)**

As the range of motion is 4 cm, the amplitude of motion is +2 cm. 10.5 s is equal to 10.5 time period of simple harmonic motion; so we have to find the height and position of cork after one-half of time period as $T/2 = 0.5$ s. As at $t = 10$ s, the particle is at its lowest position, after half a time period the cork would be at its maximum height and velocity of cork at extreme position is zero

65 **(b)**

$\frac{aT}{x} = \frac{\omega^2 xT}{x} = \frac{4\pi^2}{T} \times T = \frac{4\pi^2}{T} = \text{constant}$

66 **(b)**

Period $= 2\frac{\pi}{\omega} = 2s\ \ (or)\omega = \pi$ rad/s,

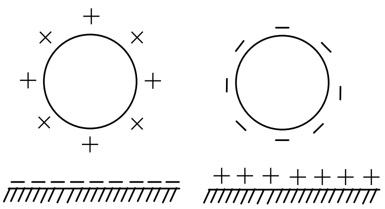
Amplitude $= 10$ cm

If the system is released, the equation of motion is

$y = A\cos\omega t$

$5 = 10\cos \pi t_1$ when $y = 5$ cm
$-5 = 10\cos \pi t_2$ when $y = -5$ cm
$\pi t_1 = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$ or $t_1 = \frac{1}{3}$ s
$\pi t_2 = \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$ or $t_2 = \frac{2}{3}$ s
$\therefore$ Time interval $= t_2 - t_1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$ s

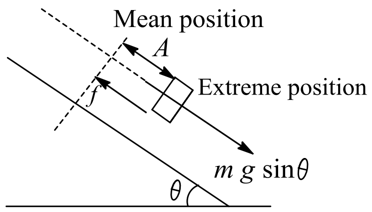
67 **(b)**
When any charge is given to the bob of the pendulum, it induces opposite charge on the metal plate, and hence a net force of attraction acts on the bob. Thus, the effective value of g increases



So, from $T = 2\pi\sqrt{l/g_{\text{eff}}}$; the value of T decreases

68 **(b)**
The slope of the length
$= \frac{F}{x} = -\frac{0.5}{5} = -0.1$ N/cm $= -10$ N/m
But $F = -m\omega^2 x$ or, $F/x = -m\omega^2$
So, $-m\omega^2 = -10$ or, $m\omega^2 = 10$
or, $\omega^2 = 10/m$
$\therefore \omega^2 = \frac{10}{4 \times 10^{-1}} \Rightarrow \omega = \frac{10}{2} = 5$
$\therefore f = \frac{\omega}{2\pi} = \frac{5}{2\pi}$/s

69 **(c)**
Here $\omega = \sqrt{\frac{l}{3m}}$. The maximum static frictional force is
$f_{\max} = \mu\, mg\cos\theta = 2\tan\theta\, mg\cos\theta = 2\, mg\sin\theta$

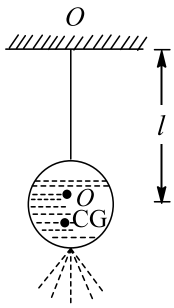


Applying Newton's second law on the block at lower extreme position
$f - mg\sin\theta = m\omega^2 A \Rightarrow f = m\omega^2 A + mg\sin\theta$
As $f \le f_{\max}$ $\omega^2 A = g\sin\theta$ or $A = \frac{3mg\sin\theta}{k}$

70 **(d)**
$x_1 + x_2 = A$ and $k_1 x_1 = k_2 x_2$ or $\frac{x_1}{x_2} = \frac{k_2}{k_1}$
Solving these equations, we get
$$x_1 = \left(\frac{k_2}{k_1 + k_2}\right)A$$

71 **(c)**
Let spring constant of two spring are k_1 and k_2, respectively, then
$T_1 = 2\pi\sqrt{\frac{m}{k_1}}$ and $T_2 = 2\pi\sqrt{\frac{m}{k_2}}$
$k_1 = \frac{4\pi^2 m}{T_1^2}$ and $k_2 = \frac{4\pi^2 m}{T_2^2}$
When the two springs are connected in series , then
$T = 2\pi\sqrt{\frac{m}{k_{eq}}}$ where $k_{\text{eq}} = \frac{k_1 k_2}{k_1 + k_2}$
$T = \sqrt{T_1^2 + T_2^2}$

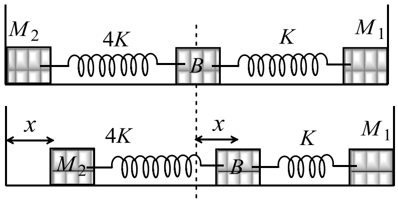
72 **(d)**
Frequency of a simple pendulum
$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$
Evidently, $f \propto \frac{1}{\sqrt{l}}$ at any place
The effective length l is the distance of the point of suspension O' from the centre of gravity (CG) of the bob. As, water flows out of the hole at the bottom, the CG descends from centre towards the bottom, increasing the effective length, and consequently f decreases. However, when all the water has flows out, the CG of a hollow sphere is once again at its centre and hence the effective length would decrease, thereby increasing the frequency



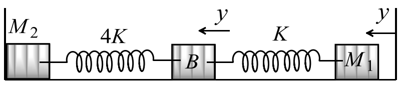
73 **(b)**
When a particle performs SHM, its total energy remains constant. It means, kinetic energy plus excess potential energy is a constant, which is equal to maximum possible kinetic energy. Hence, excess potential energy will be maximum when KE is equal to zero and zero when KE is maximum possible. Hence option (b) is correct
If frequency of oscillations of a particle is equal to n, then frequency of variation of its KE is equal to $2n$. It means, if time period of variation of KE is equal to T, then time period of oscillation of the particle will be equal to $2T$. Hence option (a) is

wrong

74 **(c)**



As springs and supports (M_1 and M_2) are having negligible mass. Whenever springs pull the massless supports, springs will be in natural length. At maximum compression, velocity of B will be zero



And by energy conservation

$$\frac{1}{2}(4K)y^2 = \frac{1}{2}Kx^2 \Rightarrow \frac{y}{x} = \frac{1}{2}$$

75 **(c)**

Decrease in potential energy of the mass when the pan gets lowered by distance y (due to mass hitting on the pan) $= mg(h + y)$, where h is the height through which the mass falls on the pan.

Increases in elastic potential of the spring $= 1/2\ ky^2$ (according to low of conservation of energy)

or $mg(h + y) = \frac{1}{2}ky^2$

or $ky^2 - 2mgy - 2\ mgh = 0$

$$\therefore y = \frac{2mg \pm \sqrt{4m^2g^2 + 8mghk}}{2k}$$

$$= \frac{mg}{k} \pm \frac{mg}{k}\sqrt{\left(1 + \frac{2hk}{mg}\right)}$$

Velocity of the pan will be maximum at the time of collision and will be zero at the lowest position. Hence y should be the amplitude of oscillation

So, amplitude of vibration $= \left[\frac{mg}{k} + \frac{mg}{k}\sqrt{\left(1 + \frac{2hk}{mg}\right)}\right]$

76 **(d)**

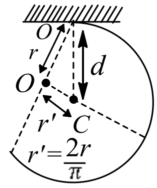
$$T = 2\pi\sqrt{\frac{I_0}{mgd}}; I_0 = I_C + mr'^2 \Rightarrow mr^2 = I_C + mr'^2$$

$$\Rightarrow I_C = m(r^2 - r^2)$$

$$I_0 = I_C + md^2m(r^2 - r'^2) + m(r^2 + r'^2) = 2mr^2$$

$$d = \sqrt{r^2 + \left(\frac{2r}{\pi}\right)^2} = r\sqrt{1 + \frac{4}{\pi^2}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{2r}{g\left(1 + \frac{4}{\pi}\right)^{1/2}}}$$



77 **(d)**

Mean position of the particle is mg/k distance below the unstretched position of spring.

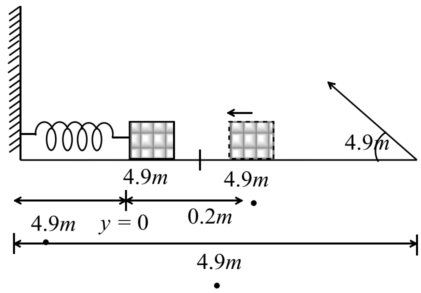
Therefore, amplitude of oscillation is $A = \frac{mg}{k}$

$$\omega = \sqrt{\frac{k}{m}} = 2\pi f = 20\pi (f = 10\text{Hz})$$

$$\frac{m}{k} = \frac{1}{400\pi^2}$$

$$v_{\max} = A\omega = \frac{g}{400\pi^2} \times 20\pi = \frac{1}{2\pi}\ \text{m/s}$$

78 **(a)**



The block is released from A

$$x = 4.9m + (0.2m)\sin\left(\omega t + \frac{\pi}{2}\right)$$

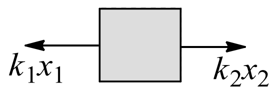
at $t = 1s; x = 5m$

so range of projectile will be $5m$

Now $5 = \frac{v^2 \sin 90°}{g} \Rightarrow v^2 = 50 \Rightarrow v = \sqrt{50}$

79 **(a)**

In equilibrium position, net force acting on the object (block of mass m) is zero. Let spring of spring constant k_1 is stretched by x_1 and spring of spring constant k_2 is stretched by x_2, then free body diagram of the clock is as shown in figure



Now , $x_1 + x_2 = 20$ cm and $k_1x_1 = k_2x_2$

$x_1 = 15$ cm and $x_2 = 5$ cm

So, new equilibrium position is at $x = (20 + 15)$ cm from P_1, and time period of oscillation of the block is given by

$$T = 2\pi\sqrt{\frac{m}{k_1 + k_2}} = 2\pi\sqrt{\frac{0.1}{4000}} = \frac{\pi}{100}\text{s}$$

80 **(b)**

The small block oscillates along the inclined plane with an amplitude A. As a result the centre of mass of the system undergoes SHM along the horizontal direction

$$x_{\text{CM}} = \frac{[m(A\sin\omega t.\cos 60°) + MO]}{m+M}$$
$$= \frac{1}{2}\frac{m}{m+M}A\sin\omega t$$

The acceleration of CM is $a_{\text{CM}} = -\omega^2 x_{\text{CM}}$, along the horizontal while the net horizontal force is $= (M+m)a_{\text{CM}}$, which is equal to the force of friction acting on it

81 **(b)**

Amplitude $= 0.15$ m

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{400}{4}} = 10 \text{ rad/s} \Rightarrow f = \frac{\omega}{2\pi} = \frac{5}{\pi}\text{Hz}$$

Energy of the particle executing SHM = KE + PE

$$= \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

Therefore, $\frac{1}{2}mv^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2 = \frac{1}{2}k(A^2 - x^2)$

$$= \frac{1}{2}\times 400(0.15^2 - 0.1^2) = 2.5 \text{ J}$$

82 **(b)**

$$y = 4\cos^2\left(\frac{t}{2}\right)\sin 1000\,t$$
$$\Rightarrow y = 2(1+\cos t)\sin 1000\,t$$
$$\Rightarrow y = 2\sin 1000\,t + 2\cos t\sin 1000\,t$$
$$\Rightarrow y = 2\sin 1000\,t + \sin 999\,t + \sin 1001\,t$$

It is a sum of three S.H.M.

83 **(a)**

$$y = kt^2$$
$$\frac{d^2y}{dt^2} = 2k$$

or $a_y = 2 \text{ ms}^{-2}$

(as $k = 1 \text{ ms}^{-2}$)

$$T_1 = 2\pi\sqrt{\frac{l}{g}}$$

and $T_2 = 2\pi\sqrt{\frac{l}{g+a_y}}$

$\therefore \quad \frac{T_1^2}{T_2^2} = \frac{g+a_y}{g} = \frac{10+2}{10} = \frac{6}{5}$

84 **(b)**

As discussed in theory part

$$T = 2\pi\sqrt{\frac{\ell}{g\left(1-\frac{\rho}{\sigma}\right)}};$$

$\rho =$ density of liquid
$\sigma =$ density of bob material

Given $\rho = \frac{\sigma}{n}$

Hence period is $2\pi\sqrt{\frac{\ell}{(1-(1/n))g}}$

85 **(b)**

The wire may be treated as a spring for which force constant

$$k_1 = \frac{\text{Force}}{\text{Extension}} = \frac{YA}{L}\left(\because Y = \frac{F}{A}\times\frac{L}{\Delta L}\right)$$

Spring constant of the spring $k_2 = K$

Hence spring constant of the combination (series)

$$k_{eq} = \frac{k_1k_2}{k_1+k_2} = \frac{(YA/L)K}{(YA/L)+K} = \frac{YAK}{YA+KL}$$

$\therefore$ Time period $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\left[\frac{(YA+KL)m}{YAK}\right]^{1/2}$

86 **(c)**

Time period of the system (object of mass 4 kg) before collision is $T_1 = 2\pi\sqrt{\frac{4}{100}}$

After collision, time period of the combined mass is

$$T_2 = 2\pi\sqrt{\frac{10}{100}}$$

We can apply momentum conservation for just before the collision and just after the collision in the horizontal direction

$$4A_1\omega_1 = 10A_2\omega_2 \Rightarrow A\frac{4\times 2\times\sqrt{\frac{100}{4}}}{10\times\sqrt{\frac{100}{10}}} = 2\sqrt{\frac{2}{5}}\text{m}$$

So, change in amplitude, $\Delta A = A_1 - A_2 =$

$2\left[1-\sqrt{\frac{2}{5}}\right]$ m

87 **(d)**

Equation are $x_1 = a\cos\left(\omega t + \frac{\pi}{6}\right)$

and $x_2 = a\cos\left(\omega t + \frac{\pi}{3}\right)$

The first will fast through the mean position when $x_1 = 0$

i.e., for instants t for which $\left(\omega t + \frac{\pi}{6}\right) = \frac{n\pi}{2}$, where n is an

integer

The smallest value of t is $n = 1, \omega t_1 = (\pi/2) = (\pi/6) = \pi/3$

The second will pass through the mean position when $x_2 = 0$,i.e., for instants t for which $\left(\omega t + \frac{\pi}{3}\right) = \frac{m\pi}{2}$ where m is an integer

The smallest value of t is $m = 1, = (\pi/2) - (\pi/3) = \pi/6$

The smallest interval between the instants $x_1 = 0$ and $x_2 = 0$ is therefore

$$\omega(t_1 : t_2) = \left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \frac{\pi}{6} \Rightarrow t_1 : t_2 = \frac{\pi}{6\omega}$$

88 **(d)**

$y_1 = 4\sin(10t + \phi), y_2 = 5\cos 10t$

$v_1 = \frac{dy_1}{dt} = 40\cos(10t + \phi)$

$v_2 = \frac{dy_2}{dt} = -50\sin 10t = 50\cos\left(10t + \frac{\pi}{2}\right)$

Phase difference between v_1 and $v_2 = \left(\phi - \frac{\pi}{2}\right)$

89 **(b)**

As we know $g = \frac{GM}{R^2}$

$\Rightarrow \frac{g_{\text{earth}}}{g_{\text{planet}}} = \frac{M_e}{M_p} \times \frac{R_\rho^2}{R_e^2} \Rightarrow \frac{g_e}{g_p} = \frac{2}{1}$

Also $T \propto \frac{1}{\sqrt{g}} \Rightarrow \frac{T_e}{T_p} = \sqrt{\frac{g_p}{g_e}} \Rightarrow \frac{2}{T_p} = \sqrt{\frac{1}{2}}$

$\Rightarrow T_p = 2\sqrt{2}\ s$

90 **(c)**

The centripetal acceleration on the bob as it oscillates (acting along the radius of circle) $= v^2/R$. This will act horizontally towards the centre of circular path

The total acceleration acting on the pendulum bob is therefore $a = \sqrt{\text{g}^2 + \left(\frac{v^2}{R}\right)^2}$

The frequency of oscillation will therefore be

$$n = \frac{1}{2\pi}\sqrt{\frac{a}{l}} = \frac{1}{2\pi}\sqrt{\frac{\left(\text{g}^2 \frac{v^4}{R^2}\right)^{1/2}}{l}}$$

91 **(c)**

Kinetic energy $K = \frac{1}{2}mv^2 = \frac{1}{2}ma^2\omega^2\cos^2\omega t$

$= \frac{1}{2}m\omega^2a^2(1 + \cos 2\omega t)$ hence kinetic energy varies periodically with double the frequency of S.H.M. $i.e., 2\omega$

92 **(c)**

Angular frequency of system,

$$\omega = \sqrt{\frac{K}{m+m}} = \sqrt{\frac{K}{2m}}$$

Maximum acceleration, $a_{\max} = \omega^2 A$

Friction force between P and Q

=force exerted on lower block

$= m\omega^2 A = m\left(\frac{K}{2m}\right)A = \frac{KA}{2}$

93 **(b)**

From the graph $T = (5-1) = 4$ s

(distance between the two adjacent crests shown in the figure)

And $v_{\max} = 5$ m/s; $\omega A = 5$ m/s

$\left(\frac{2\pi}{T}\right)A = 5 \Rightarrow A = \frac{5T}{2\pi} = \frac{5\times 4}{2\pi} = \frac{10}{\pi}$ m

Also, $\omega = \frac{2\pi}{4} = \frac{\pi}{2}$ rad/s

The equation of velocity can be written as

$V = 5\sin\left(\frac{\pi}{2}t\right)$ m/s

At extreme position, $v = 0; \sin\left(\frac{\pi}{2}t\right) = 0$ or $t = 2$s

Phase of the particle velocity at that instant corresponding to the above equation $= \pi$

Therefore, when a phase change of $\pi/6$ takes place, the resulting phase $= \pi + \pi/6$

$v = 5\sin\left(\pi + \frac{\pi}{6}\right) = -5\sin\frac{\pi}{6} = -5\left(\frac{1}{2}\right)$

$= 2.5$ m/s (numerically)

$\frac{dy}{dt} = v \Rightarrow dy = vdt$

$dy = \int 5\sin\left(\frac{\pi t}{2}\right)dt = \frac{10}{\pi}\left[-\cos\frac{\pi t}{2}\right] + C$

Since at $t = 0$, the particle is at the extreme position, therefore at $t = 0; y = -\frac{10}{\pi}$

$-\frac{10}{\pi} = -\frac{10}{\pi}\cos\theta$

$y = -\frac{10}{\pi}\cos\frac{\pi t}{2}$

Clearly a phase change of $\pi/6$ corresponds to a time difference of

$\frac{T}{2\pi}\left(\frac{\pi}{6}\right) = \frac{T}{12} = \frac{4}{12} = \frac{1}{3}$s

$y = -\frac{10}{\pi}\cos\frac{\pi}{6} = -\frac{10}{\pi}\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{5\sqrt{3}}{\pi}$ m (numerically)

Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}\left(5\sin\frac{\pi t}{2}\right) = \frac{5\pi}{2}\frac{\cos\pi t}{2}$

a at $t = \frac{1}{3}$s $= \frac{5\pi}{2}\cos\frac{\pi}{6} = \frac{5\pi\sqrt{3}}{4}$ m/s^2

Maximum displacement, $x_{\max} = A = \frac{10}{\pi}$ m

and maximum acceleration, $a_{\max} = \omega^2 A$

$= \left(\frac{\pi}{2}\right)^2 \times \frac{10}{\pi} = \frac{5\pi}{2}$ m/s^2

94 **(d)**

$T = 8\text{s}, \omega = \frac{2\pi}{T} = \left(\frac{\pi}{4}\right)\text{rads}^{-1}$

$x = A\sin\omega t$

$\therefore \quad a = -\omega^2 x = -\left(\frac{\pi^2}{16}\right)\sin\left(\frac{\pi}{4}t\right)$

Substituting $t = \frac{4}{3}$s, we get

$a = -\left(\frac{\sqrt{3}}{32}\pi^2\right)\text{cms}^{-2}$

95 **(a)**

$a = -\omega^2 x = \omega^2 x$ (numerically)

Rate of change of acceleration

$\frac{da}{dt} = \omega^2\frac{dx}{dt} = \omega^2 v$

$$\frac{da}{dt} = \omega^3\sqrt{A^2 - x^2}$$
For da/dt to be maximum, x^2 should the minimum i.e., $x = 0$ and for da/dt to be minimum x^2 should be maximum i.e., $x = \pm a$

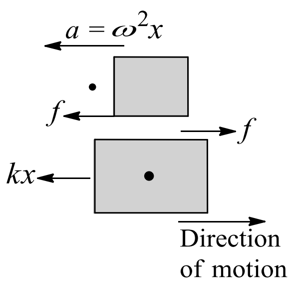
96 **(b)**

Velocity at any displacement x is given by
$v = \omega\sqrt{A^2 - x^2}$

So, the required velocity $= \frac{2\pi}{T}\sqrt{A^2 - \frac{A^2}{2}} = \frac{\sqrt{2}\pi A}{T}$

97 **(b)**

Let A be the maximum amplitude of oscillation for the required situation. Then $f = m\omega^2 x$ (from free body diagram of blocks)



For required conduction, $f = m\omega^2 A$
For no slipping $f < f_L$
i.e., $m\omega^2 A < \mu_s mg$
$$\Rightarrow A < \frac{0.6 \times \mathrm{g}}{\frac{4\pi^2}{4}} = 0.6\text{ m} \Rightarrow A_{\max} = 0.6\text{ m}$$

98 **(c)**

Let T be the time period then $\frac{T}{2} = \frac{5\pi}{64} - \frac{\pi}{64} = \frac{4\pi}{64}$
$$\Rightarrow T = \frac{\pi}{8}\text{s}$$
$$\therefore \omega = \frac{2\pi}{T} = \frac{2\pi}{(\pi/8)} = 16\text{ rad/s}$$
Also, $A = 10$ cm (from the graph)
The equation of the sinusoidal wave can be written as $y = 10\sin(16t + \phi)$ cm, where ϕ is the initial phase. From the graph, corresponding to the crest = 10 cm; when $t = 3\pi/64$
$$10\text{ cm} = 10\sin\left[16\left(\frac{3\pi}{64}\right) + \phi\right]\text{cm}$$
$$\sin\left[\frac{3\pi}{4} + \phi\right] = 1 \text{ or } \frac{3\pi}{4} + \phi = \frac{\pi}{2} \Rightarrow \phi = -\frac{\pi}{4}$$
$$\therefore\ y = 10\sin\left(16t - \frac{\pi}{4}\right)$$

99 **(a)**

Let the cylinder be depressed a distance x metres into water from the position of rest.
Increases in tension of spring = $100x$ metres into water from the position of rest
Increase in tension of spring = $100x$ kg − wt
$(A = 100\text{ cm}^2 = 10^{-2}\text{m}^2)$
Increases in buoyancy $= \frac{100}{100^2}x \times 1000$ kg − wt
$K = 1$ kg − wt/cm = 100 kg − wt/m
$= 10x$ kg − wt
Total unbalanced vertical force on the cylinder $= 100x + 10x = 110x$ kg − wt
Acceleration of cylinder $= \frac{110x \times 10}{10}\text{ m/s}^2 = 110x\text{ m/s}^2$
Period of one oscillation $= \frac{2\pi}{\sqrt{110}} = 0.6$ s

100 **(c)**

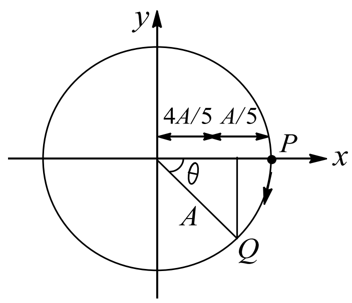
$x = A\sin\omega t$, where $\omega = \dfrac{2\pi}{4} = \dfrac{\pi}{2}$ rad/s
$$3 = 5\sin\omega t_1 \Rightarrow t_1 = \frac{1}{\omega}\sin^{-1}\left(\frac{3}{5}\right) = \frac{2 \times 37}{180}$$
$$4 = 5\sin\omega t_2 \Rightarrow t_2 = \frac{1}{\omega}\sin^{-1}\left(\frac{4}{5}\right) = \frac{2 \times 53}{180}$$
Hence time interval $(t_2 - t_1) = \frac{1}{90}(53 - 37) + \frac{8}{45}$s

101 **(a)**

Potential energy is minimum (in the case zero) at mean position ($x = 0$) and maximum at extreme positions ($x = \pm A$). At time $t = 0, x = A$. Hence, PE should be maximum. Therefore, graph I is correct. Further in graph III, PE is minimum at $x = 0$. Hence, this is also correct.

102 **(b)**

Method 1:
Particle is starting form rest i.e., from one of its extreme position.
As particle moves a distance $A/5$, we can represent it on a circle as shown



$$\cos\theta = \frac{4A/5}{A} = \frac{4}{5} \Rightarrow \theta = \cos^{-1}\left(\frac{4}{5}\right)$$
$$\omega t = \cos^{-1}\left(\frac{4}{5}\right) \Rightarrow t$$
$$= \frac{1}{\omega}\cos^{-1}\left(\frac{4}{5}\right) = \frac{T}{2\pi}\cos^{-1}\left(\frac{4}{5}\right)$$
Method 2: As the particle starts from rest, i.e., from extreme position $x = A\sin(\omega t - \phi)$
At $t = 0; x = A \Rightarrow \phi = \frac{\pi}{2}$
$$A - \frac{A}{5} = A\cos\omega t$$

$$\frac{4}{5} = \cos\omega t \Rightarrow \omega t = \cos^{-1}\frac{4}{5}$$

$$t = \frac{T}{2\pi}\cos^{-1}\left(\frac{4}{5}\right)$$

103 **(b)**

From the graph $T = (5-1) = 4$ s

(distance between the two adjacent crests shown in the figure)

And $v_{\max} = 5$ m/s; $\omega A = 5$ m/s

$$\left(\frac{2\pi}{T}\right)A = 5 \Rightarrow A = \frac{5T}{2\pi} = \frac{5\times 4}{2\pi} = \frac{10}{\pi}\text{m}$$

Also, $\omega = \frac{2\pi}{4} = \frac{\pi}{2}$ rad/s

The equation of velocity can be written as

$$V = 5\sin\left(\frac{\pi}{2}t\right)\text{m/s}$$

At extreme position, $v = 0$; $\sin\left(\frac{\pi}{2}t\right) = 0$ or $t = 2$s

Phase of the particle velocity at that instant corresponding to the above equation $= \pi$

Therefore, when a phase change of $\pi/6$ takes place, the resulting phase $= \pi + \pi/6$

$$v = 5\sin\left(\pi + \frac{\pi}{6}\right) = -5\sin\frac{\pi}{6} = -5\left(\frac{1}{2}\right)$$

$= 2.5$ m/s (numerically)

$$\frac{dy}{dt} = v \Rightarrow dy = vdt$$

$$dy = \int 5\sin\left(\frac{\pi t}{2}\right)dt = \frac{10}{\pi}\left[-\cos\frac{\pi t}{2}\right] + C$$

Since at $t = 0$, the particle is at the extreme position, therefore at $t = 0$; $y = -\frac{10}{\pi}$

$$-\frac{10}{\pi} = -\frac{10}{\pi}\cos\theta$$

$$y = -\frac{10}{\pi}\cos\frac{\pi t}{2}$$

Clearly a phase change of $\pi/6$ corresponds to a time difference of

$$\frac{T}{2\pi}\left(\frac{\pi}{6}\right) = \frac{T}{12} = \frac{4}{12} = \frac{1}{3}\text{s}$$

$$y = -\frac{10}{\pi}\cos\frac{\pi}{6} = -\frac{10}{\pi}\left(\frac{\sqrt{3}}{2}\right)$$

$= \frac{5\sqrt{3}}{\pi}$ m (numerically)

Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}\left(5\sin\frac{\pi t}{2}\right) = \frac{5\pi}{2}\frac{\cos\pi t}{2}$

a at $t = \frac{1}{3}$s $= \frac{5\pi}{2}\cos\frac{\pi}{6} = \frac{5\pi\sqrt{3}}{4}$ m/s^2

Maximum displacement, $x_{\max} = A = \frac{10}{\pi}$ m

and maximum acceleration, $a_{\max} = \omega^2 A$

$$= \left(\frac{\pi}{2}\right)^2 \times \frac{10}{\pi} = \frac{5\pi}{2}\text{m/s}^2$$

104 **(b)**

Let spring constant of each spring be k, then the equivalent spring constant of the two spring system in parallel is $2k$

Without mass, $T = 3\text{ s} = 2\pi\sqrt{\frac{12}{2k}}$

With mass, $T_1 = 65 = 2\pi\sqrt{\frac{12+m}{2k}}$

$$\frac{T}{T_1} = \frac{1}{2} = \sqrt{\frac{12}{12+m}}$$

$m = 36$ kg

105 **(c)**

Time period $T = 2\pi\sqrt{\frac{l}{g}}$

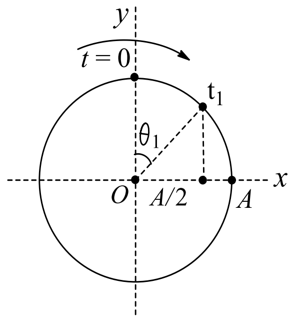
$$T_1 = 2\pi\sqrt{\frac{l}{g}} = T \Rightarrow T_2 = 2\pi\sqrt{\frac{4l}{g}} = 2T_1 = 2T$$

Hence, in time $T_2(2T)$ small pendulum will perform two oscillations and againu at initial position

106 **(b)**

From circular motion representation, we can represent SHM by uniform circular motion

Let t_1 is the time taken by particle to go from $x = 0$ to $x = A/2$



$$\sin\theta_1 = \frac{A/2}{A} = \frac{1}{2}$$

$$\theta_1 = \frac{\pi}{6} \Rightarrow \omega t_1 = \frac{\pi}{6} \Rightarrow t_1 = \frac{\pi\times T}{6\times 2\pi} = \frac{T}{12}$$

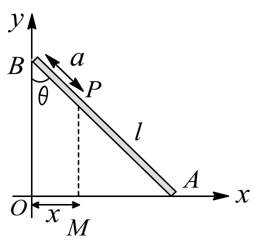
So, time taken to go from $x = A/2$ to A is

$$t_2 = \frac{T}{4} - t_1 = \frac{T}{4} - \frac{T}{12} = \frac{T}{6}$$

Hence, $\frac{t_1}{t_2} = \frac{T/12}{T/6} = \frac{1}{2}$

107 **(b)**

$$\frac{d\theta}{dt} = 2 \therefore \theta = 2t$$



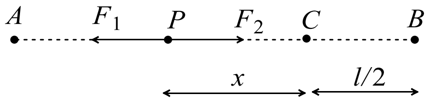
Let $BP = a$; therefore, $x = OM = a\sin\theta = a\sin(2t)$

Hence M executes SHM within the given time period and its acceleration is opposite to 'x' that means towards left

108 **(b)**

$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2, \vec{F}_{\text{net}}$ is zero when $PA = PB$

Let $AB = 1$



$F_1 = 2\left(\frac{1}{2} \times x\right)$ and $F_2 = 2\left(\frac{1}{2} \times x\right)$

$F_{\text{net}} = -4x$

Therefore, motion in SHM will have $k = 4$

$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow T = 2\pi$

109 **(d)**

Maximum velocity $= a\omega = a\sqrt{\frac{K}{m}}$

Given that $a_1\sqrt{\frac{K_1}{m}} = a_2\sqrt{\frac{K_2}{m}} \Rightarrow \frac{a_1}{a_2} = \sqrt{\frac{K_2}{K_1}}$

110 **(a,c)**

Energy stored $= \frac{1}{2}kx^2 = \frac{1}{2}k \times (0.25)^2 = 5$ J

$k = 160$ N/m

Period $T = 2\pi\sqrt{\frac{m}{k}}$

$T = \frac{1}{5}$s $\quad \therefore T^2 = \frac{4\pi^2 m}{k}$

$\frac{1}{25} = \frac{4\pi^2 m}{160}$

$\therefore m = \frac{4}{25}$kg (As $\pi^2 = 10$), hence $m = 0.16$ kg

111 **(b,c,d)**

The only horizontal force acting on the coin is the force of friction F. Hence its horizontal acceleration is always in the direction of F its magnitude is F/m. The magnitude and direction of F can thus be obtained from the magnitude and direction of acceleration. At the highest point, the normal reaction has the minimum value,

$N_H = mg - m\omega^2 A \Rightarrow F_{\min} = \mu N_H$

At the lowest point, the normal reaction has the maximum value,

$N_L = mg + m\omega^2 A \Rightarrow F_{\max} = \mu N_L$

112 **(a,d)**

Since, the particle starts from rest and finally again comes to rest, hence, it is clear that acceleration α cannot remain positive for all values of time. Hence, statement (a) is correct

As no other information is available about its motion, hence we cannot find value of $|\alpha|$ at a particular instant. Hence, (b) and (c) are not correct. Due to this very reason (d) is correct

113 **(b,d)**

The maximum extension x produced in the spring in case (a) is given by

$F = kx$ or $x = \frac{F}{k}$

The time period of oscillation is

$T = 2\pi\sqrt{\frac{\text{mass}}{\text{force constant}}} = 2\pi\sqrt{\frac{m}{k}}$

In case (a) one end A of the spring is fixed to the wall. When a force F is applied to the free end B in the direction shown in figure (a), the spring is stretched exerting a force on the wall which in turn exerts an equal and opposite reaction force on the spring, as a result of which every coil of the spring is elongated producing a total extension x. In case (b), shown in figure(b), both ends of the spring are free equal forces are applied at ends. By application of forces both the cases are same. Thus, the maximum extension produced in the spring in cases (a) and (b) is the same. In case (b) the mid-point of spring will not move, we can say the blocks are connected with the springs whose lengths are half the original length of spring. Now, the force constant of half the spring is twice of complete spring. In case (b) the force constant= $2k$. Hence, the time period of oscillation will be

$T' = 2\pi\frac{m}{2k}$

$\frac{T'}{T} = \sqrt{2}$

Hence, the correct choice are (b) and (d)

114 **(a,b,d)**

$x = a\sin\omega t\cos\omega t = \frac{a}{2}\sin 2\omega t$

115 **(c,d)**

If in equilibrium position, elongation of the spring is equal to x_0, then

$Kx_0 = mg$ or $x_0 = \frac{mg}{K} = 1$ cm

If the block is raised till spring becomes unstretched and then released, then during subsequent motion maximum elongation of the spring (y) will be calculated by energy conservation law

Loss of PE of block (mgy) =strain energy $\left(\frac{1}{2}Ky^2\right)$

$y = \frac{2\,mg}{K} = 2$ cm

Hence option (b) is correct

Frequency of oscillations will be, $f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} =$ 5 Hz Hence option (c) is correct

Since frequency f, does not depend upon gravitational acceleration, therefore frequency will remain unchanged, even if the system is taken to moon. Hence option (d) is also correct

116 **(b,d)**

For S.H.M. displacement $y = a \sin \omega t$ and acceleration $A = -\omega^2 y \sin \omega t$ these are maximum at $\omega t = \frac{\pi}{2}$

117 **(b,d)**

The time period of potential energy and kinetic energy is half that of SHM

118 **(a,c,d)**

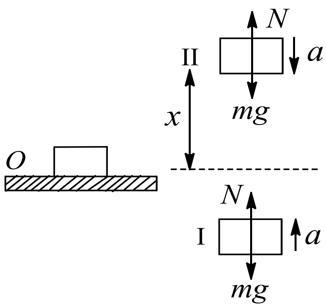
The only external horizontal force acting on the system of the two blocks and the spring if F.
Therefore, acceleration of the centre of mass of the system is equal to $Flm_1 + m^2$.
Hence, centre of mass of the system moves with a constant acceleration. Initially, there is no tension in the spring, therefore at initial moment m_2 has an acceleration Flm_2 and it starts to move to the right. Due to its motion, the spring elongates and a tension is developed. Therefore, acceleration of m_2 decreases while that m_1 increases from zero initial value.
The blocks starts to perform SHM about their centre of mass and the centre of mass moves with the acceleration calculated above. Hence, option (b) is correct
Since the blocks perform SHM about centre of mass, therefore the length of the spring varies periodically. Hence, option (a) is wrong
Since magnitude of the force F remains constant, therefore amplitude of oscillations also remains constant. So, option (c) is also wrong
Acceleration of m_2 is maximum at the instant when the spring is in its minimum possible length, which is equal to its natural length. Hence, at initial moments, acceleration of m_2 is maximum possible
The spring is in its natural length, not only at initial moment but at time $t = T, 2T, 3T, \ldots$ also, where T is the period of oscillation. Hence, option (d) is wrong

119 **(a,d)**

Since liquid 2 is below liquid 1, liquid 2 is denser than liquid 1. Let area of cross section of the cylindrical block be A and it be displaced downward by y. Then volume of liquid 2 displaced will get increased by Ay and that of liquid 1 will get decreased by the same amount Ay. Hence, net increase in upthrust on the block will be equal to $(Ayd_2\text{g} - Ayd_1\text{g})$. This additional upthrust tries to restore the block in original position
It means, the block experiences a restoring force $Ay(d_2 - d_1)\text{g}$. Since this force is restoring and directly proportional to displacement y, it will execute SHM along a vertical line
Hence, option (a) is correct and option (b) is wrong. If mass of the block is equal to m, then its acceleration will be equal to $\frac{Ay\text{g}(d_2-d_1)}{m}$
Since its acceleration depends on mass m, frequency of oscillations will depend on size of the cylinder. Hence option (c) is wrong
If the cylinder is displacement upward through y from equilibrium position, then it will experience a net downward force, equal to calculated above. This shows that its motion will be symmetric about its equilibrium position

120 **(a,c)**

Let O be the mean position and a be the acceleration at a displacement x from O
At position $l, N - mg = ma$
$\therefore\ N \neq O$
At position II, $mg - N = ma$
For $N = 0$ (loss of contact), $\text{g} = a = \omega^2 x$
Loss of contact will occur for amplitude
$x_{\max} = \text{g}/\omega^2$ at the highest point of the motion



121 **(a,d)**

The resultant of two motion is simple harmonic of same angular frequency ω
The amplitude of the resultant motion is

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\frac{\pi}{3}} = \sqrt{A_1^2 + A_2^2 + A_1A_2}$$

Maximum acceleration

$$= \omega^2 A = \omega^2\sqrt{A_1^2 + A_2^2 + A_1A_2}$$

122 **(a,c)**

Here,

$$y = y_1 + y_2 + y_3$$

$$= \alpha \sin \omega t + \alpha \sin\left(\omega t + \frac{\pi}{4}\right) + \alpha \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= a\left[\left\{\sin \omega t + \sin\left(\omega t + \frac{\pi}{2}\right)\right\} + \sin\left(\omega t + \frac{\pi}{2}\right)\right]$$

$= a\left[2\sin\left(\omega t+\frac{\pi}{4}\right)\cos\frac{\pi}{4}+\sin\left(\omega t+\frac{\pi}{4}\right)\right]$

$= a\left[\frac{2}{\sqrt{3}}\sin\left(\omega t+\frac{\pi}{4}\right)+\sin\left(\omega t+\frac{\pi}{4}\right)\right]$

$= a(\sqrt{2}+1)\sin\left(\omega t+\frac{\pi}{4}\right)$

Thus, resultant motion is SHM with an amplitude $a(\sqrt{2}+1)$

Again energy of resultant motion

$=\frac{1}{2}m\omega^2\left[a(\sqrt{2}+1)^2\right]=\frac{1}{2}m\omega^2a^2(3+2\sqrt{2})$

$=(3+2\sqrt{2})$ times energy of any one motion

123 **(b,d)**

Let the velocity acquired by A and B be V, then

$mv = mV + mV \Rightarrow V = \frac{v}{2}$

Also $\frac{1}{2}mv^2=\frac{1}{2}mV^2+\frac{1}{2}mV^2+\frac{1}{2}kx^2$

Where x is the maximum compression of the spring

On solving the above equations, we get

$x = v\left(\frac{m}{2k}\right)^{1/2}$

At maximum compression, kinetic energy of the $A-B$ system $=\frac{1}{2}mV^2+\frac{1}{2}mV^2=mV^2=\frac{mv^2}{4}$

124 **(a,c,d)**

$U = x^2-4x+3$ and $F=-\frac{dU}{dx}=-(2x-4)$

At equilibrium position $F=0$, so $x=2$ m

Let the particle is displaced by Δx from equilibrium position, i.e., from $x=2$, then restoring force on body is,

$F=-2(2+\Delta x)+4=-2\Delta x$

$i.e., F\alpha-\Delta x$, so performs simple harmonic motion about $x=2$ m

Time period, $T=2\pi\sqrt{\frac{m}{k}}=2\pi\sqrt{\frac{1}{2}}=\sqrt{2}\pi$ s

From energy conservation, $\frac{mv_{\max}^2}{2}+U_{\min}=U_{\max}$

$\frac{1\times 4^2}{2}+(2^2-4\times 2+3)$

$=(A+2)^2-4(A+2)+3$

Where A is amplitude. Solving the above equation, we get $A=2\sqrt{2}$ m

125 **(a,c)**

$y = a\sin\omega t = a\sin\frac{2\pi t}{T}$

$v=\frac{dy}{dt}=\omega a\cos\frac{2\pi t}{T}$

At $t=\frac{T}{6}$, $v=\omega a\cos\left(\frac{2\pi}{T}\frac{T}{6}\right)=\frac{1}{2}\omega a$

or, $v=(v_{\max}/2)$

$y=a\sin\frac{2\pi}{T}\times\frac{T}{6}=a\sin\frac{\pi}{3}$

It is not half of a

Acceleration $=\frac{d^2y}{dt^2}=\frac{dv}{dt}=\omega^2a\sin\frac{2\pi t}{T}$

$=\omega^2a\sin\frac{\pi}{3}=0.86\,(AC)_{\max}$

At this instant,

$\text{KE}=\frac{1}{2}mv^2=\frac{1}{2}m\left(\frac{v_{\max}}{2}\right)^2=\frac{(\text{KE})_{\max}}{4}=\frac{1}{4}(\text{TE})$

$\therefore \text{PE}=\text{TE}-\text{KE}=\frac{3}{4}(\text{TE})$

i.e., KE $\neq$ PE

126 **(a,b)**

When point of suspension of pendulum is moved upwards, $g_{\text{eff}}=g+a, g_{\text{eff}}>g$ and as $T\propto 1/\sqrt{g_{\text{eff}}}$, hence T decreases, i.e., choice (a) is correct

When point of suspension of pendulum is moved downwards and $a>2g$, then T decreases, i.e., choice (b) is also correct

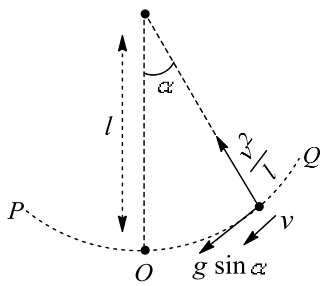
In case of horizontal acceleration

$g_{\text{eff}}=\sqrt{g^2+a^2}$, i.e., $g_{\text{eff}}>g$

i.e., again T decreases

127 **(a,d)**

Statement (a) is correct. At any position O and P or between O and Q, there are two accelerations-a tangential acceleration g $\sin\alpha$ and a centripetal acceleration v^2/l (because the pendulum moves along the arc of a circle or radius l), where l is the length of the pendulum and v its speed at that position. When the bob is at the mean position O, the angle $\alpha=0$, therefore $\sin\alpha=0$; hence, the tangential acceleration is zero. But at O, speed v is maximum and the centripetal acceleration v^2/l is disrected radially towards the point of support. When the bob is at the end points P and Q, the speed v is zero, hence the centripetal acceleration is zero at the end points, but tangential acceleration is maximum and is directed along the tangent to the curve at P and Q. The tension in the string is not constant throughout the oscillation. At any position between O and end point P or Q, the tension in the string is given by $T=mg\cos\alpha$



At the end point P and Q, the value of α is the greatest, hence the tension is the least. At the mean position O, $\alpha=0$ and $\alpha=1$ which is the greatest; hence tension is greatest at the mean position

128 **(b,c)**

Harmonic oscillator has some initial elastic potential energy and amplitude of harmonic variation of energy is $\frac{1}{2}Ka^2 = \frac{1}{2} \times 2 \times 10^6 \times (0.01)^2 = 100J$

This is the maximum kinetic energy of the oscillator.

Thus $K_{\max} = 100J$

This energy is added to initial elastic potential energy may give maximum mechanical energy to have value $160J$

129 **(b,c,d)**

$U = 5x(x-4) = 5x^2 - 20x$

$F = -\frac{dU}{dx} = -10x + 20$

i.e., force is not constant

KE or speed of the particle will be maximum at the mean position where force becomes zero

$F = 0$ or $x = 2$m

Acceleration experienced by the particle is

$a = \frac{F}{m} = \frac{-10x + 20}{0.1} = -(100x - 200)$

i.e., particle executes SHM

As $\omega^2 = 100$

$\therefore\ \omega = 10$

Hence $T = \frac{2\pi}{\omega} = \frac{\pi}{5}$ s

130 **(a,b,c,d)**

At $t = 0$

Displacement, $x = x_1 + x_2 = 4\sin\frac{\pi}{3} = 2\sqrt{3}$ m

Resulting amplitude,

$A = \sqrt{2^2 + 4^2 + 2(2)(4)\cos\pi/3}$

$= 2\sqrt{7}$m

Maximum speed, $A\omega = 20\sqrt{7}$m/s

Maximum acceleration, $A\omega^2 = 200\sqrt{7}$m/s^2

Energy of the motion $= \frac{1}{2}m\omega^2A^2 = 28$ J

131 **(a,b,c,d)**

If a particle is performing SHM with amplitude A and angular frequency ω and if its initial phase is equal to zero, then its displacement from the mean position will be given by $x = A\sin\omega t$

Value of $\sin\omega t$ is same at two different values of the phase. If one is ωt, then the other is $(\pi - \omega t)$. Hence at there two instants, the phase are unequal. Therefore option (c) is correct

Velocity of the particle will be equal to $\omega\sqrt{(A^2 - x^2)}$. Since x is same at these two instants, therefore the magnitude of velocity will be same or the speeds are equal. However, velocities are unequal because at one instant direction of motion of the particle will be towards the extreme position or away from mean position and at the other instant, it will be towards the mean position. Hence (a) is also correct.

Since speeds are equal, kinetic energies are also equal. Hence, option (d) is also correct.

Acceleration of the particle is $a = -\omega^2x$. Since x is same at these two instants, accelerations are also equal. Therefore option (b) is also correct

132 **(a,c)**

$y = 0.5\,[\cos^2(n\pi t) - \sin^2(n\pi t)] = 0.5\cos\ 2\,n\pi t$

$\frac{dy}{dt} = -0.5 \times 2n\pi \times \sin 2n\pi t$

$\frac{d^2y}{dt^2} = -0.5(2n\pi^2)\cos 2n\pi t = -4n^2\pi^2y \quad$ (i)

i.e., $\frac{d^2y}{dt^2} \propto -y$

i.e., particle is executing SHM with amplitude 0.5 m, i.e., choice (a) is correct

As standard equation of SHM is

$\frac{d^2y}{dt^2} = -\omega^2y \quad$ (ii)

Hence, $\omega = 2n\pi$

For a second's pendulum, $T = 2$ s

Hence, $\omega' = \frac{2\pi}{T} = \pi = 2n$

i.e., choice (b) is not correct

$\left(\frac{dy}{dt}\right)_{\max} = n\pi$

i.e., choice (c) is also correct

133 **(a,c)**

Here amplitude, $A = 0.05$ m, $\omega = 20\pi$

and $\phi = 4\pi \times 0.4 = 1.6\,\pi$

$\therefore\quad T = \frac{2\pi}{\omega} = \frac{2\pi}{20\pi} = 0.1\,s$

Maximum acceleration$= A\omega^2 = 0.05 \times (20\pi)^2 = 20\pi^2$,

Hence (b) is not correct

Total energy

$E = \frac{1}{2}\ m\omega^2A^2 = \frac{1}{2} \times 0.1 \times (20\pi)^2 \times (0.05)^2$

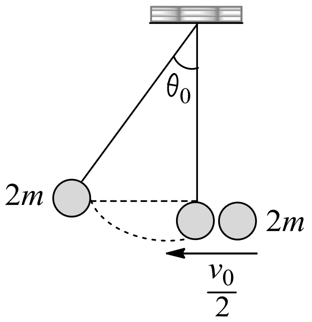
$= 0.05\pi^2$J

Hence, relation (c) is also correct. Force is zero only at mean position corresponding to $y = 0$

134 **(a, b)**

The time period of simple harmonic pendulum is independent of mass, so it would be same as that $T = 2\pi\sqrt{1/g}$. After collision, the combined mass acquires a velocity of $v_0/2$, as a result of this velocity, the mass $(2m)$ moves up and at an angel

θ_0(say) with vertical, it stops, this is the extreme position of bob



From work-energy theorem, $\Delta K = W_{\text{total}}$

$$0 - \frac{2m}{2}\left(\frac{v_0}{2}\right)^2 = -2\,mgl(1-\cos\theta_0)$$

$$\frac{v_0^2}{8gl} = 1-\cos\theta = 2\sin^2\frac{\theta_0}{2}$$

$$\sin\frac{\theta_0}{2} = \frac{v_0}{4\sqrt{gl}}$$

If θ_0 is small, $\sin\frac{\theta_0}{2} \approx \frac{\theta_0}{2} \Rightarrow \theta_0 = \frac{v_0}{2\sqrt{g\ell}}$

So, the equation of simple harmonic motion is

$\theta = \theta_0\sin(\omega t)$

135 **(a,b,d)**

Total mechanical energy of the oscillating system is, $E = K_{\max} + U_{\min} + K_{\min,} K_{\min} = 0$ at extreme position

So, $U_{\max} = E = 200$ J

$$K_{\max} = \frac{mv_{\max}^2}{2} = \frac{m\times A^2\omega^2}{2} = \frac{KA^2}{2} = 150 \text{ J}$$

So, $U_{\min} = 50$ J

136 **(b,c)**

Let the simple harmonic motion be given by

$$x_1 = a\sin\left(2\pi\frac{t}{T}\right) \quad \text{(i)}$$

$$x_2 = a\sin\left(2\pi\frac{t}{T}+\frac{\pi}{4}\right) \quad \text{(ii)}$$

and $x_3 = a\sin\left(2\pi\frac{t}{T}+\frac{\pi}{2}\right)$ (iii)

Then the resultant periodic motion, by the principle of superposition is given by

$x = x_1 + x_2 + x_3$

$$x = a\left(\sin\frac{2\pi t}{T}\right) + a\sin\left(\frac{2\pi t}{T}+\frac{\pi}{4}\right) + a\sin\left(\frac{2\pi t}{T}+\frac{\pi}{2}\right)$$

$$x = a\left[\sin\left(\frac{2\pi t}{T}\right) + a\sin\left(\frac{2\pi t}{T}+\frac{\pi}{2}\right)\right] + a\sin\left(\frac{2\pi t}{T}+\frac{\pi}{4}\right)$$

$$x = 2a\sin\left(\frac{2\pi t}{T}+\frac{\pi}{4}\right)\cos\frac{\pi}{4} + a\sin\left(\frac{2\pi t}{T}+\frac{\pi}{4}\right)$$

$$= a\left(\sqrt{2}+1\right)\sin\left(\frac{2\pi t}{T}+\frac{\pi}{4}\right)$$

Which is a simple harmonic motion with an amplitude $a\left(\sqrt{2}+1\right)$ and phase angle $\pi/4$ and the same period; it has the same phase as second SHM

The energy of resultant motion is proportional to $\left[a\left(\sqrt{2}+1\right)\right]^2$ (i.e.) $\left[a^2\left(2+1+2\sqrt{2}\right)\right] = \left(3+2\sqrt{2}\right)a^2$ which is greater than three times the energy of each separate SHM

137 **(a,b,d)**

Description of motion is completely specified if we know the variation of x as a function of time. For simple harmonic motion, the general equation of motion is $x = A(\omega t + \delta)$. As ω is given, to describe the motion completely, we need the values of A and δ

From options (b) and (d), we can have the values of A and δ directly.

For option (a), we can find A and δ if we know initial velocity and initial position. Option (c) cannot give the values of A and δ so it is not the correct condition

138 **(a,c,d)**

When the block is released suddenly, it starts to move down. During its downward motion the rubber cord elongates. Hence, a tension is developed in it but the block contains to accelerate downwards till tension becomes equal to weight mg of the block

After this moment, the block continues to move down due to its velocity and rubber cord further elongates. Therefore, tension becomes greater than the weight; hence, the block now retards and comes to an instantaneous rest. At lowest position of the block, strain energy in the cord equals loss of potential energy of the block. Suppose the block comes to an instantaneous rest when elongation of the rubber cord is equal to y. Then

$$\frac{1}{2}ky^2 = mgy \Rightarrow y = \frac{2mg}{k} \text{ and } 0$$

Hence, block will be instantaneously at rest, at $y = 0$ and at $y = 2\,mg/k$

In fact, the block oscillates between these two values. Since the rubber cord is elastic, tension in it is directly proportional to elongation.

Therefore, the block will perform SHM

Its amplitude will be equal to half of the distance between these extreme positions of the block or amplitude

$$= \frac{1}{2}\times\frac{2mg}{k} = \frac{mg}{k} = l$$

Hence, option (b) is correct

The angular frequency of its SHM will be equal to

$\omega = \sqrt{\dfrac{k}{m}}$

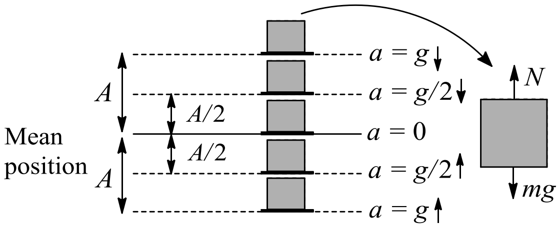
Since k and m are not given in the question, it cannot be calculated. Hence option (d) is not correct

139 **(a,b,c)**

As per basic theory of SHM statements (a), (b), (c) are true. Statement (d) is not correct because at the equilibrium position the velocity is maximum

140 **(a,b,c,d)**

The position of momentary rest is SHM is the extreme position where velocity of particle is zero



As the block loses contact with the plank at this position, i.e., normal force becomes zero, it has to be the upper extreme where acceleration of the block will be g downwards

$\omega^2 A = \text{g} \;\; \Rightarrow \omega^2 = \dfrac{10}{0.4} = 25$

$\omega = 5\text{ rad/s} \Rightarrow T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{5}\text{s}$

Acceleration in SHM is given as $a = \omega^2 x$

From the figure, we can see that at lower extreme, acceleration is g upwards

$N - mg = ma$

or $N = m(a + \text{g}) = mg$

At halfway up acceleration is g/2 downwards

$mg - N = ma \;\;$ or $N = m(\text{g} - \text{g}/2) = \dfrac{1}{2}\text{mg}$

At halfway down, acceleration is g/2 upwards

$N - mg - ma$ or $N = m(\text{g} + \text{g}/2) = \dfrac{3}{2}mg$

At mean position, velocity is maximum and acceleration is zero

$\therefore \; N = mg$

141 **(b,c)**

$x = 3\sin 100\,\pi t$

$y = 4\sin 100\pi t$

Equation of path is $\frac{y}{x} = \frac{4}{3}$

i.e., $t = \frac{4}{3}x$

which is equation of a straight line having slope 4/3. Equation of resulting motion is

$\vec{r} = x\hat{\imath} + y\hat{\jmath} = (3\hat{\imath} + 4\hat{\jmath})\sin 100\pi t$

Amplitude is $\sqrt{3^2 + 4^2} = 5$

142 **(b,c)**

$v^2 = 108 - 9x^2$ or $v^2 = 9(12 - x^2)$

We can compare the above expression with $v = \omega\sqrt{A^2 - x^2}$, which is the expression of velocity for SHM

From this, we will get

$\omega = 3$ and $A = \sqrt{12}$

SHM is not a uniformly accelerated motion

Acceleration at a distance 3 cm from the mean position,

$a = \omega^2(3\text{ cm}) = 27\text{ cm/s}^2$

Maximum displacement from mean position

$= A = \sqrt{12}$ cm

143 **(a,b,c,d)**

Period of oscillation changes as it depends on mass and becomes three times. The amplitude of oscillation does not change, because the new object is attached when the original object is at rest. Total energy does not change as at extreme position the energy is in the form of potential energy stored in spring which is independent of mass, and hence maximum; KE also does not change but as mass changes the maximum speed changes

144 **(a,b,c)**

The motion of the particles simple harmonic. The displacement at time $t =$ is: $x = a\sin(bt + c)$

Therefore, displacement at time $(t + (2\pi/b))$ is

x at $\left(t + \dfrac{2\pi}{b}\right) = a\sin\;\left[b\left(t + \dfrac{2\pi}{b}\right) + c\right]$

$= a\sin[bt + c + 2\pi]$

$\qquad = a\sin(bt + c) = x\;$ (at time t)

Hence, statement (a) is correct

Statement (b) is also correct since the same displacement is recovered after a time interval of $(2\pi/b)$. Statement (c) is correct because the velocity is zero when the displacement $= \pm$ the amplitude, i.e., at the extreme ends of the motion. Statement (d) is incorrect, the acceleration is maximum (in magnitude) at $x = \pm A$

145 **(b,d)**

In SHM acceleration $a = -\omega^2 y$ and

Force $F = ma = -m\omega^2 y = -ky$

Hence, relations (b) and (d) are true

146 **(a,b,d)**

$x = \dfrac{A}{2}(1 - \cos 2\omega t) + \dfrac{B}{2}(1 + \cos 2\omega t)$

$\qquad + \dfrac{C}{2}\sin 2\omega t$

(a) For A = 0, B = 0; $\left(x = \frac{C}{2}\sin 2\omega t\right)$

(b) For $A = -B$ and $C = 2B$

$x = B\cos 2\omega t + B\sin 2\omega t$; Amplitude $= |B\sqrt{2}|$

(c) For $A = B; C = 0; x = A$

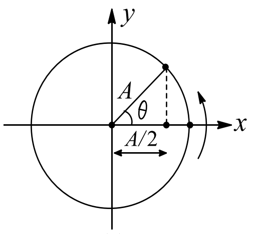
Hence this is not correct option

(d) For $A = B, C = 2B; x = B + B\sin 2\omega t$

It also represents SHM

147 **(a,c)**

At $t = 0$ when particle is at extreme position, the situation is as shown in figure

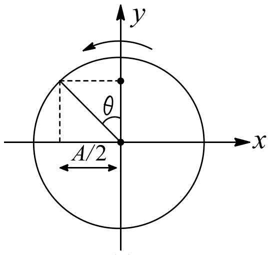


(1)

From the figure ,$\cos\theta = \frac{A/2}{A} = \frac{1}{2}$

$\theta = \frac{\pi}{3} \Rightarrow \frac{\pi}{3} = \frac{2\pi}{T} \times 1 \Rightarrow T = 6$ s

At $t = 0$ when particle is at mean position, the situation is as shown in figure



(2)

$\theta = \omega t$

From the figure, $\sin\theta = \frac{A/2}{A}$

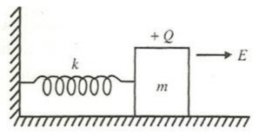
$\theta = \frac{\pi}{6}$, but $\theta = \omega t \Rightarrow \frac{\pi}{6} = \frac{2\pi}{T} \times T = 12$ s

If initially the particle is located somewhere else, then time period comes out to be different. A reverse question can also be formed on the same concept

148 **(c,d)**

When a constant force is superimposed on a system which undergoes SHM along the line of SHM, the time period does not change as it depends on mass of the block and force constant of spring

The mean position changes as this is the point where net force on the particle is zero



149 **(a)**

When the hoop oscillates in its plane, moment of inertia is $I_1 = mR^2 + mR^2$, i. e., $I_1 = 2mR^2$

While when hoop oscillates in a direction perpendicular to plane of hoop, moment of inertia is

$$I_2 = \frac{mR^2}{2} + mR^2 = \frac{3mR^2}{2}$$

The time period of physical pendulum is,

$T = 2\pi\sqrt{\frac{I}{mgd}}$, d is same in both the cases

150 **(d)**

The mean position of the particle in Statement I is $x = -b/a$ and the force is always proportional to displacement from this mean position. The particle executes SHM about this mean position. Hence, Statement I is false

151 **(b)**

At the moon the value of g is less than on plane.

As$T = 2\pi\sqrt{\frac{l}{g}}$ or $T \propto \frac{l}{g}$, so T increases. It is also true that moon; is smaller than the earth, but this statement is not explaining the assertion

152 **(d)**

Time period of the spring pendulum is

$T = 2\pi\sqrt{m/k}$,

Where k is the force constant

Time period of the simple pendulum is

$T = 2\pi\sqrt{l/g}$

Time period of the torsional pendulum is

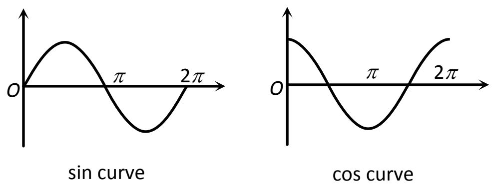
$T = 2\pi\sqrt{I/k}$, where k is the torsion constant

If frequencies of all three are same at one place, then at some other place frequency of torsional and spring pendulums would be same and that of simple pendulum may differ (dependent upon g)

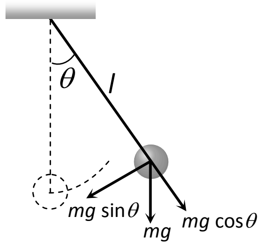
So, Statement I is wrong and Statement II is correct

153 **(a)**

A periodic function is one whose value repeats after a definite interval of time. $\sin\theta$ and $\cos\theta$ are periodic functions because they repeat itself after 2π interval of time



154 **(b)**
In simple pendulum, when bob is in deflection position, the tension in the spring is $T = mg\cos\theta + \frac{mv^2}{l}$. Since the value of θ is different at different positions, hence tension in the string is not constant throughout the oscillation



At end points θ is maximum; the value of $\cos\theta$ is least, hence the value of tension in the string is least. At the mean position, the value of $\theta = 0°$ and $\cos 0° = 1$, so the value of tension is largest

Also velocity is given by $v = \omega\sqrt{a^2 - y^2}$ which is maximum when $y = 0$, at mean position

158 **(a)**
The time period of a oscillating spring is given by,

$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow T \propto \frac{1}{\sqrt{k}}$. Since the spring constant is large for hard spring, therefore hard spring has a less periodic time as compared to soft spring

159 **(a)**
Tangent compound of weight $= mg\sin\theta$ radial compound of weight $= mg\cos\theta$ at mean position, $\theta = 0, \Rightarrow$ tangent compound $= 0$

Therefore direction of acceleration is along radial component of weight.

At extreme position, tangent compound is maximum. Hence direction of acceleration is along tangent component

160 **(a)**
Correct explanation of the assertion is that spring constant of a given material and thickness is inversely proportional to its length, $ie, k \propto \frac{1}{l}$

161 **(c)**
Time period of simple pendulum of length l is

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T \propto \sqrt{l}$$

$$\sqrt{\frac{\Delta T}{T}} = \frac{1}{2}\frac{\Delta l}{l}$$

$$\therefore \quad \frac{\Delta T}{T} = \frac{1}{2} \times 3 = 1.5\%$$

162 **(a)**
Total energy, $E = \frac{1}{2}m\omega^2 a^2$

i.e., $E \propto a^2$

$$\frac{E'}{E} = \left(\frac{2a}{a}\right)^2 \Rightarrow E' = 4E$$

163 **(a)**
In SHM, the acceleration is always in a direction opposite to that of the displacement $i.e.$, proportional to $(-y)$

164 **(b)**
If length of the pendulum is large, g no longer remains vertical but will be directed towards the center of the earth and expression of the time period is given by

$$T = 2\pi\sqrt{\frac{1}{\left(\frac{1}{l} + \frac{1}{R}\right)}}$$

Here, R is the radius of earth.

If $\quad l \to \infty, \frac{1}{1} \to 0$

And $\quad T = 2\pi\sqrt{\frac{R}{g}} = 84.6\ min$

In general, time period of a simple pendulum

$$T = 2\pi\sqrt{\frac{1}{g}}$$

Or $\quad T \propto \sqrt{1}$

165 **(d)**
The period of the liquid executing SHM in a U-

tube does not depend upon the density of the liquid. Therefore, time period will be the same, when mercury is filled up to the same height as the water in the U-tube.

Now, as the pendulum oscillates, it drags air along with it. Therefore, its kinetic energy is dissipated in overcoming viscous drag due to air and hence, its amplitude goes on decreasing

166 **(b)**
Energy of damped oscillator at an any instant t is given by

$E = E_0 e^{-bt/m}$ [where $E_0 = \frac{1}{2}kx^2$ =maximum energy]

Due to damping forces the amplitude of oscillator will go on decreasing with time whose energy is expressed by above question

167 **(a)**
Let us assume that density of material of cubes is ρ_0 and density of liquid is ρ. Then from equilibrium condition, $(8a^3 + a^3)\rho_0 g = (8a^3)\rho g$

$\Rightarrow 9\rho_0 = 8\rho$
When the block is displaced down by x, the restoring force is $F_1 = (8a^3 + a^2x)\rho g - 9a_0^3 g = \rho a^2 g \times x$

When the block is displacement by x above the mean position, the restoring force is

$F_2 = 9a^3\rho_0 g - (8a^3 - 4a^2x)\rho g = 4a^2\rho g x$

As force constants on two sides of equilibrium position are not same, amplitudes of oscillations on the two sides are different

168 **(a)**
Total energy of the particle performing simple harmonic motion is $E = K + U = k_{\max} + U_{\min}$. K is always positive, which U could be +ve, −ve or zero. If $U_{\min}$ is-ve and its value is greater than $K_{\max}$, then E would be –ve

169 **(b)**
Maximum acceleration

$= \omega^2 A = 4\pi^2 v^2 A = 4\pi^2 \times (60)^2 \times 0.01$
$= 144\pi^2 \text{ ms}^{-1}$

As during SHM the direction of deflection is opposite of displacement. It may be $+ve$ or $-ve$.

Hence, maximum acceleration $= \pm 144\pi^2 \text{ms}^{-2}$

170 **(b)**
$x = a \sin \omega t$ and $v = \frac{dx}{dt} = a\omega \cos \omega t$

It is clear phase difference between $'x'$ and $'a'$ is $\pi/2$

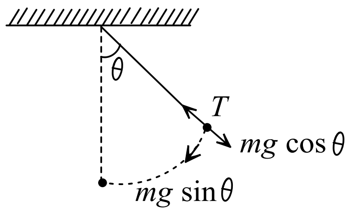
171 **(c)**
As $F = -m\omega^2 y \Rightarrow$ slope of $F - y$ graph is $-m\omega^2$

$-1 = -m\omega^2 = -\omega^2 \Rightarrow \omega = 1$

$T = 2\pi = 6.28$ s

If mass is changed but slope remains same, the time period will change

172 **(d)**



$T - mg\cos\theta = \frac{mv^2}{r} \Rightarrow mg\sin\theta = F_T$

173 **(b)**
In SHM $K.E. = \frac{1}{2}m\omega^2(a^2 - y^2)$ and $P.E. = \frac{1}{2}m\omega^2 y^2$

For $K.E. = P.E. \Rightarrow 2y^2 = a^2 \Rightarrow y = a/\sqrt{2}$. Since total energy remains constant through out the motion, which is $E = K.E. + P.E.$ So, when $P.E.$ is maximum then $K.E.$ is zero and viceversa

174 **(d)**
Spring constant $\propto \frac{1}{\text{length of spring}} \Rightarrow K' = \frac{K}{n}$

Also, spring constant depends on material properties of the spring

175 **(a)**
Waves produced on the surface of water are transverse in nature. When such waves are produced in water they spread out. Till the ocean waves reach the beach-shore, they acquire such a large radius of curvature that they may be assumed as plane waves. Hence, ocean waves hit

the beach normal to the shore

176 **(e)**
In simple harmonic motion the velocity is given by,

$v = \omega\sqrt{a^2 - y^2}$ at extreme position, $y = a$

$\therefore v = 0$

But acceleration $A = -\omega^2 a$, which is maximum at extreme position

177 **(c)**
Amplitude of oscillation for a forced, damped oscillator is $A = \frac{F_0/m}{\sqrt{(\omega^2-\omega_0^2)+(b\omega/m)^2}}$, where b is constant related to the strength of the resistive force, $\omega_0 = \sqrt{k/m}$ is natural frequency of undamped oscillator ($b = 0$)

When the frequency of driving force $(\omega) \approx \omega_0$, then amplitude A is very larger.

For $\omega < \omega_0$ or $\omega > \omega_0$, the amplitude decreases

178 **(c)**
The amplitude of an oscillating pendulum decreases with time because of friction due to air. Frequency of pendulum is independent $\left(T = \frac{1}{2\pi}\sqrt{\frac{g}{l}}\right)$ of amplitude

180 **(a)**
If the soldiers while crossing a suspended bridge march in steps, the frequency of marching steps of soldiers may match with the natural frequency of oscillations of the suspended bridge. In that situation resonance will take place, then the amplitude of oscillation of the suspended bridge will increase enormously, which may cause the collapsing of the bridge. To avoid such situations the soldiers are advised to break steps on suspended bridge

181 **(d)**
$k_1x_1 = k_2x_2 = F$

$W_1 = \frac{1}{2}k_1x_1^2 = \frac{(k_1x_1)^2}{2k_1} = \frac{F^2}{2k_1}$

Similarly $W_2 = \frac{F^2}{2k_2} \Rightarrow W \propto \frac{1}{k}$

$W_1 > W_2 \Rightarrow k_1 < k_2$ statement 2 is true

Statement 1 $W_1 = \frac{1}{2}k_1x^2$

$w_2 = \frac{1}{2}k_2x^2$

So, $W_2 > W_1$

Statement 1 is false

182 **(c)**
Correct reason is that when the girl stands up, the centre of mass of swing-girl system moves vertically upward and hence, effective length of swing pendulum (distance of centre of mass from fixed point at the top of swing) decreases and consequently, its time period decreases

183 **(b)**
$T = 2\pi\sqrt{\frac{l}{g}}$. On moon, g is much smaller compared to g on earth. Therefore, T increases

It is also true that moon is smaller than the earth, but this statement is not explaining the assertion

184 **(d)**
$k = \frac{F}{1}$

Or $k \propto \frac{1}{l}$

Or $\frac{K_1}{K_2} = \frac{l_2}{L_1} = \frac{2}{1}$

$\because$ $K_1 = 2K, K_2 = K$

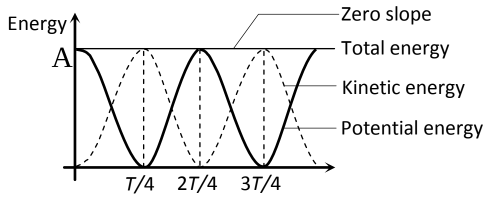
In series, $\frac{1}{K'} = \frac{1}{K_1} + \frac{1}{K_2}$

$= \frac{1}{2K} + \frac{1}{K} = \frac{3}{2K}$

$\because K' \frac{2}{3}k$

185 **(a)**
The total energy of S.H.M = Kinetic energy of particle + potential energy of particle

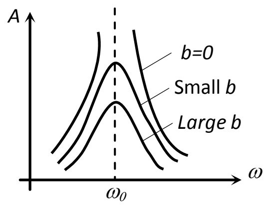
The variation of total energy of the particle in SHM with time is shown in a graph



186 **(a)**
From equation, amplitude of oscillation

$$A=\frac{F_0/m}{\sqrt{(\omega^2-\omega_0^2)^2+(b\omega/m)^2}}$$

In absence of damping force ($b=0$), that the steady state amplitude approaches infinity as $\omega\to\omega_0$



That is, if there is no resistive force in the system and then it is possible to drive an oscillator with sinusoidal force at the resonance frequency, the amplitude of motion will build up without limit. This does not occur in practice because some damping is always present in real oscillation

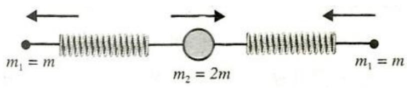
187 **(a)**
Time period of a liquid column

$$T=2\pi\frac{\sqrt{h}}{g}$$

$$=2\times\frac{22}{7}\times\sqrt{\frac{0.3}{9.8}}=1.1\text{s}$$

188 **(a)**
(A) From nature of SHM, the graph of potential energy as function of displacement will be parabolic graph as given in option P. Hence
(A) → (p)
(B) $a=0$ or $a=$constant. [As per given condition]
$V>0$ moving along positive x-axis
$y-$ displacement
$y=ut\pm\frac{1}{2}at^2$ for $a=$ constant
$y=ut$ for $a=0$
These two conditions are satisfied by (q) and (s)
(B) → (q, s)
(p) is rejected because at t = 0 the displacement is not zero and velocity has negative values
(C) $R=\frac{u^2\sin(2\theta)}{g}$ and $R\propto u^2$ for a fixed angle of projection and $u=0, R=0$
(C) → (s)
(D) $T=2\pi\sqrt{\frac{l}{g}}\Rightarrow T^2=4\pi^2\frac{l}{g}\Rightarrow y=\frac{4\pi^2}{g}x$ (D) → (q)

189 **(a)**
1. Motion in simple harmonic



$$\frac{d^2x}{dt^2}=\frac{-k}{m_1m_2}(m_2+2m_1)x$$

$$\omega^2=k\left(\frac{1}{m_1}+\frac{2}{m_2}\right)=k\left(\frac{1}{m}+\frac{2}{2m}\right)x$$

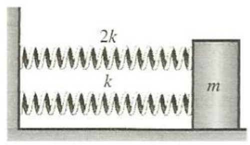
$$\omega=\sqrt{\frac{2k}{m}}$$

∴ Frequency $=\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

ii. Angular frequency $\omega=\sqrt{\frac{k(m+2m)}{2m^2}}=\sqrt{\frac{3k}{2m}}$

∴ Frequency $=\frac{1}{2\pi}\sqrt{\frac{3k}{2m}}$

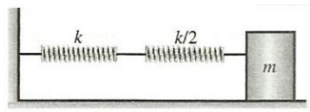
iii. Here effective spring constant $=3k$



∴ Frequency $=\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

iv. effective spring constant (K) is given by

$$\frac{1}{k}=\frac{1}{k}+\frac{1}{\frac{k}{2}}=\frac{3}{k}$$



$k=\frac{k}{3}$ ∴ Frequency $=\frac{1}{2\pi}\sqrt{\frac{k}{3m}}$

190 **(a)**
i. Linear combination of two SHMs will be an SHM if the individual SHMs have equal frequencies, their magnitudes may be different
ii. $y=A\sin\omega_1t+A\sin(\omega_2t+\phi)$
If $\omega_1=\omega=\omega, \phi=17/2$
$y=A\sin\omega t+A\cos\omega t$
Its amplitude will be $\sqrt{2}A$

iii. Time period of a pendulum of infinite length is

$$T = 2\pi\sqrt{\frac{R}{g}}$$

iv. The above will be the maximum value of time period of an oscillating pendulum

193 **(c)**

Position and velocity both can be positive and negative. If at mean position, PE is zero, then at any other position, PE is positive. Total energy is always constant

194 **(a)**

For $A, y = Ae^{i(\omega t+\phi)}$

$$\frac{dy}{dt} = A \times i\omega e^{i(\omega t+\phi)}$$

$$\frac{d^2y}{dt^2} = Ai\omega \times i\omega e^{i(\omega t+\phi)} = -\omega^2 \times Ae^{i(\omega t+\phi)}$$

$$\frac{d^2y}{dt^2} = -\omega^2 y$$

Which represents the standard equation of motion of SHM

For $B, y = B\sin\omega t + C\cos\omega t$ can be written as

$y = \sqrt{B^2 + C^2}\sin(\omega t + \delta)$

Where $\tan\delta = \frac{C}{B}$

So, B also represents an SHM

For C, it is a standard equation of harmonic travelling wave in which the particle performs an SHM

For $D, y/x$ =constant, it represents the equation of the translatory motion

195 **(d)**

$V_m = A\omega$

$$A = \frac{V_m}{\omega} = \frac{2\pi}{2\pi} \times (0.2) = 0.20 \text{ m}$$

$$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow m = \frac{T^2k}{4\pi^2} = 0.2 \text{ kg}$$

At $t = 0.1$ s, acceleration is maximum

$= -\omega^2 A = -200 \text{ m/s}^2$

Maximum energy $= \frac{1}{2}mv_m^2 = 4$ J

$$\frac{1}{2}kA^2 = E_{\max} = \frac{1}{2} \times 200 \times 0.04 = 4 \text{ J}$$

196 **(d)**

Frequency depends only on the mass and the spring constant, so it will change. Due to increases in KE by a factor of 4, the amplitude gets doubled and hence the magnitude of maximum KE increases by a factor of 4, the maximum PE also increases by the same factor

197 **(b)**

A constant force and a constant torque affect only the mean position. In third case, as the block falls on mean position, the mean position is not affected. In a car a constant pseudo force will act which will affect only the mean position

198 **(a)**

$F = 8 - 2x = -2(x - 4)$

At equilibrium position, $F = 0$

$x = 4$ m

As particle is released at rest from $x = 6$ m, i.e., it is one of the extreme position, amplitude $A = 2$ m

Hence, force constant $k = 2$ N/m

$m\omega^2 = 2$ or $\omega = 1$

Time period $T = \frac{2\pi}{\omega} = 2\pi$

Time taken to go from $x = 2$ m to $x = 4$ m (i.e., from extreme position to mean position)

$$= \frac{T}{4} = \frac{\pi}{2}$$

Energy of SHM $= \frac{1}{2}kA^2 = \frac{1}{2} \times 2 \times 4 \text{ N} - \text{m} = 4$ J

As the particle has started its motion from positive extreme

Phase constant $= \frac{\pi}{2}$

199 **(d)**

The moment of inertia of a cylindrical rod about axis of wire (*ie*, an axis passing through the centre of rode and perpendicular to its length) is

$$I = M\left[\frac{L^2}{12} + \frac{R^2}{4}\right]$$

200 **(b)**

$v_{x_0}^2 = \omega^2(A^2 - x_0^2)$

or $A^2 = \frac{v_{x_0}^2}{\omega^2} + x_0^2 \quad [\therefore w = 4]$

or $A = 25\sqrt{2}$ cm

201 **(b)**

$$\omega = \sqrt{\frac{K}{m}},\ K = \omega^2 m = (10)^2 = 100 \text{ N/m}$$

Angular frequency of oscillation of combined body is

$$\omega' = \sqrt{\frac{K}{m+M}} = \sqrt{\frac{100}{4}} = 5 \text{ rad/s}$$

202 **(b)**

The angular frequency of simple harmonic motion is given by

$$\omega = \sqrt{\frac{k}{m}}$$

The velocity of block when it is a displacement of y from men position is given by $v = \omega\sqrt{A^2 - y^2}$, From given initial condition, $v_0 = \sqrt{\frac{k}{m}}\sqrt{A^2 - h^2}$

$\Rightarrow A^2 = \frac{mv_0^2}{k} + h^2 \Rightarrow A = \sqrt{\frac{mv_0^2}{k} + h^2}$

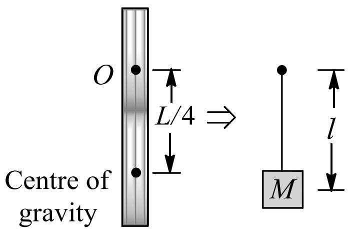
203 **(a)**

Let the equation of simple harmonic motion be
$x = A\sin(\omega t + \delta)$
Then, at $t = 1$s, $x = 0$, so
$0 = A\sin(\delta + \omega) = \sin(\omega + \delta) = 0$
At $t = 2$s, $v = +0.25$ m/s,
So $0.25 = A\omega\cos(2\omega + \delta)$
Solving above equation, we get, $A = \frac{3}{2\pi} m$ and $\delta = -\frac{\pi}{3}$
Hence equation of SHM will be $x = \frac{3}{2\pi}\sin\left(\frac{\pi}{3}t - \frac{\pi}{3}\right)$
For velocity at $t = 5$ s, i.e., after half a period of $t = 2$ s, the velocity is having same magnitude but opposite direction

204 **(b)**

In the present case, $d = \frac{L}{4}$

$I = \frac{ML^2}{12} + M\left(\frac{L}{4}\right)^2 = \frac{7}{48}ML^2$



So, time period of the physical pendulum is

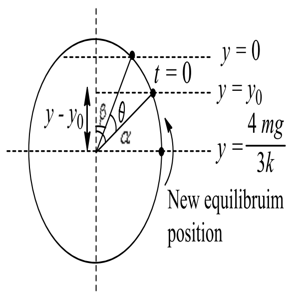
$T = 2\pi\sqrt{\frac{I}{Mgd}}$

$= 2\pi\sqrt{\frac{7/48ML^2}{Mg \times L/4}} = \pi\sqrt{\frac{7L}{3g}}$

205 **(c)**

Initially in equilibrium let the elongation in spring be y_0, then $mg = ky_0$

$y_0 = \frac{mg}{k}$



As the bullet strikes the block with velocity v_0 and gets embedded into it, the velocity of the combined mass can be computed by using the principle of moment conservation

$\frac{m}{3}v_0 = \frac{4m}{3}v \Rightarrow v = \frac{v_0}{4}$

Let new mean position is at distance y from the origin, then

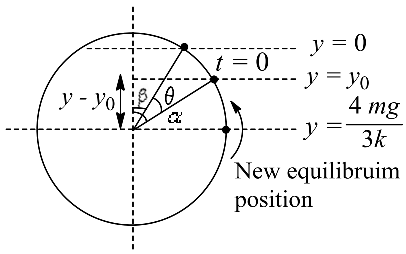
$ky = \frac{4m}{3}g \Rightarrow y = \frac{4mg}{3k}$

Now, the block executes SHM about mean position defined by $y = 4\,mg/3k$ with time period, $T = 2\pi\sqrt{4m/3k}$. At $t = 0$, the combined mass is at a displacement of $(y - y_0)$ from mean position and is moving with velocity v, then by using $v = \omega\sqrt{A^2 - x^2}$, we can find the amplitude of motion

$\left(\frac{v_0}{4}\right)^2 = \frac{3k}{4m}[A^2 - (y - y_0)^2] = \frac{3k}{4m}\left[A^2 - \left(\frac{mg}{3k}\right)^2\right]$

$\Rightarrow A = \sqrt{\frac{mv_0^2}{12k} + \left(\frac{mg}{3k}\right)^2}$

To compute the time taken by the combined mass from $y = mg/k$ to $y = 0$, we can either go for equation method or circular motion projection method



Required time, $t = \frac{\theta}{\omega} = \frac{\alpha - \beta}{\omega}$

$\cos a = \frac{y - y_0}{A} = \left(\frac{\frac{4mg}{3k} - \frac{4mg}{k}}{A}\right) = \frac{mg}{3kA}$

$\cos\beta = \frac{y}{A} = \frac{4mg}{3kA}$

So, $t = \frac{\cos^{-1}\left(\frac{mg}{3kA}\right) - \cos^{-1}\left(\frac{4mg}{3kA}\right)}{\omega}$

$= \sqrt{\frac{4m}{3k}}\left[\cos^{-1}\left(\frac{mg}{3kA}\right) - \cos^{-1}\left(\frac{4mg}{3kA}\right)\right]$

206 **(b)**

As C collides with A and sticks to it, the combined mass moves rightward to compress the spring and hence B moves rightwards due to the spring forces .i.e., B acceleration and the combined mass decelerates. The deformation in the spring is changing and the centre of mass of the system continues to move rightwards with constant speed, while both the blocks oscillates about centre of mass of the system
The velocity of the combined mass just after collision is $mv_0 = 2mv$

$v = \frac{v_0}{2} = 0.3$ m/s

Velocity of energy of mass of system

$v_{CM} = \frac{2mv + 0}{3m} = \frac{2v}{3} = \frac{v_0}{3} = 0.2$ m/s

Time period with which block B and the combined mass oscillate about centre of mass could be computed by using the reduced mass concept

$T = 2\pi\sqrt{\frac{2m}{3k}} = \frac{\pi}{5\sqrt{10}}$ s

Oscillation energy of the system is

$E = \frac{2mv^2}{2} = 0.27$ J

The translation kinetic energy of the centre of mass of system is

$E_{CM} = \frac{3mv_{CM}^2}{2} = 0.18$ J

The remaining energy is oscillating between kinetic and potential energy during the motion of blocks.

For maximum compression in spring either we can use centre of mass approach or energy approach, here we are using the second method

Oscillation energy=Maximum elastic potential energy

$0.09 = \frac{1}{2}kx_m^2 \Rightarrow x_m = 3\sqrt{10}$ mm

207 **(b)**

When speed of block is maximum, net force on block is zero. Hence at that instant spring exerts a force of magnitude 'mg' on the block

208 **(b)**

At position A, block is in positive region and slope is negative, so velocity is negative

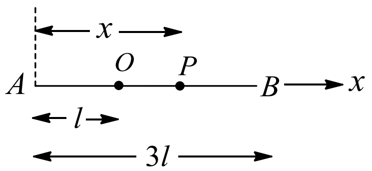
209 **(a)**

Using conservation of mechanical energy

$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \Rightarrow x^2 = \frac{m}{k} \Rightarrow x^2 = v\sqrt{\frac{m}{k}}$

210 **(c)**

Take x axis along AB and origin at A. Let $AP = x$, then $PB = (3l - x)$



The force on the particle of mass m at P is $\vec{F}$ given by

$\vec{F} = 2\left(\frac{mg}{l}\right)(-\vec{x}) + \frac{mg}{l}\left(\frac{3l - x}{x}\right)\vec{x}$

$\vec{F} = \left(3mg - \frac{3mg}{l}x\right)$ directed towards A

$= \frac{3mg}{l}(l - x)$ directed towards A

Hence the equation of motion of mass m is

$m\frac{d^2x}{dt^2} = -\frac{3mg}{l}(l - x) = m\omega^2(l - x)$

Where $\omega^2 = 3$ g$/l$

This is a simple harmonic motion of period

$T = 2\pi\sqrt{\frac{l}{2g}}$

The mean position is at the point $x = l$(i. e.)at O, where $AO = l$

Now for point A, displacement $= -l$ and hence

$v_A^2 = \omega^2(a^2 - (-l)^2) = \omega^2(a^2 - l^2)$

Where $v_A^2 = \omega^2(a^2 - (-l)^2) = \omega^2(a^2 - l^2)$ and a is the amplitude

Now $v_A = 3\sqrt{gl}$ (give) and hence $v_A^2 = 9gl = \frac{3g}{l}(a^2 - l^2)$

$a^2 = 4l^2 \Rightarrow a = 2l$

211 **(d)**

Let x be the maximum extension of the spring. The mass falls from rest through the vertical distance $(L + x)$ and so the energy given by it to the spring is $E_k = mg(L + x)$. This must be stored in the spring as elastic energy at its maximum extension. Hence $E_k = \frac{1}{2}kx^2 = mg(L + x)$

This is quadratic, $x^2 - \frac{2mg}{k}x - \frac{2mgL}{k} = 0$

Hence, $x = \frac{\frac{2mg}{k} \pm \sqrt{\left(\frac{2mg}{k}\right)^2 + \frac{8mgL}{k}}}{2}$

Maximum value of $x = \frac{mg}{k} + \frac{mg}{k}\sqrt{1 + \frac{2kL}{mg}}$

$= \frac{mg}{k}\left(1 + \sqrt{1 + \frac{2kL}{mg}}\right)$

212 **(c)**

When $0 < E < V_0$ there will be acting a restoring force to perform oscillation because in this case particle will be in the region $|x| \leq x_0$

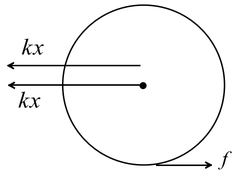
214 **(d)**

Applying equation of torque about lowest point

$(2\,kx)R = \left(\frac{3}{2}MR^2\right)\alpha \Rightarrow \alpha R = \frac{4kx}{3M}$

As there is no slipping

$a = \alpha R = \frac{4\,kx}{3M}$



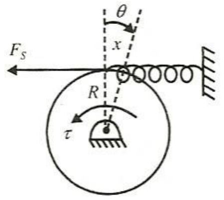
Net force $= Ma = \frac{4\,kx}{3}$

Which is directed opposite to displacement

$F_{net} = \frac{-4kx}{3}$

215 **(2)**

If we twist (rotate) the disc through a small clockwise angle θ, the spring will be deformed (compressed) by a distance $x = R\theta$. Hence, the spring force $F_s = kx = k(R\theta)$ will produce a restoring torque



Restoring torque: $\tau = -F_s R$ where $F_s = kR\theta$

This gives $\tau = -kR^2\theta$

It means after removing the external (applied) torque, the restoring torque rotates the disc with an angular acceleration α which will bring the spring–disc system back to its original state

Newton's law of rotation (or torque equation): Applying Newton's second law of rotation, we have

$\tau = I_C\alpha$

Where $\tau = -kR^2\theta$

This gives $\alpha = -\frac{kR^2\theta}{I_c}$ where $I_C = \frac{mR^2}{2}$

Then $a = -\frac{2k}{m}\theta$

Comparing the above equation with $\alpha = -\omega^2\theta$, we have

$$\omega = \sqrt{\frac{2k}{m}}$$

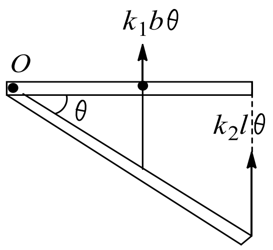
After substituting the values we get $\omega = 2$ rad/s

216 **(8)**

Applying torque equation about

$\tau_0 = \tau_0\alpha$

$$k_1 b\theta \times b\cos\theta + \frac{k_2 l_\theta}{\theta} \times l\cos\theta = -\frac{Id^2\theta}{dt^2}$$



Here, $I = \frac{ml^2}{3}$, and as θ is small, $\cos\theta = 1$

$$\frac{ml^2 d^2\theta}{3dt^2} + (k_1 b^2 + k_2 l^2)\theta = 0$$

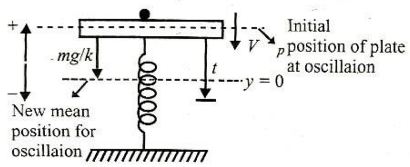
Hence, $\omega = \sqrt{\frac{3k_1b^2 + k_2l^2}{ml}}$

On substituting the values we get $\omega = 8$ rad/s

217 **(5)**

Velocity of the particle before collision

$$u = \sqrt{2g \times \frac{4.5\,mg}{K}} \Rightarrow u = 3g\sqrt{\frac{m}{K}}$$



Now it collides with the plate

Now just after collision velocity (V) of the system of 'plate+ particle'

$$mu = 3mV \Rightarrow V = \frac{u}{3} = g\sqrt{\frac{m}{K}}$$

Now the system performs SHM with time period $T = 2\pi\sqrt{3m/K}\left(\omega = \sqrt{\frac{K}{3m}}\right)$ and mean position as mg/K

Distance below the point of oscillation

Let the equation of motion is

$y = A\sin(\omega t + \phi)$ (i)

$v = \frac{dy}{dt} = A\omega\cos(\omega t + \phi)$ (ii)

At $t = 0, y = \frac{mg}{K}$ and $v = g\sqrt{\frac{m}{K}}$

From Eqs. (i) and (ii) $\frac{mg}{K} = A\sin\phi$ (iii)

$A = \frac{2mg}{K}$ (iv)

From Eqs. (iii) and (iv) $\Rightarrow \phi = \frac{5\pi}{6}$

$$y = \frac{2mg}{K}\sin\left(\sqrt{\frac{K}{3m}}t + \frac{5\pi}{6}\right)$$

Hence equation of SHM should be

$$y = -A = -\frac{2\,mg}{k} = \frac{2mg}{K}\sin\left(\sqrt{\frac{K}{3m}}t + \frac{5\pi}{6}\right)$$

The plate will be at rest again when

$$y = -A = -\frac{2mg}{K} = \frac{2mg}{K}\sin\left(\sqrt{\frac{K}{2m}}t + \frac{5\pi}{6}\right)$$

$$\Rightarrow \sin\left(\sqrt{\frac{K}{3m}}t + \frac{5\pi}{6}\right) = -1 = \sin\frac{3\pi}{2}$$

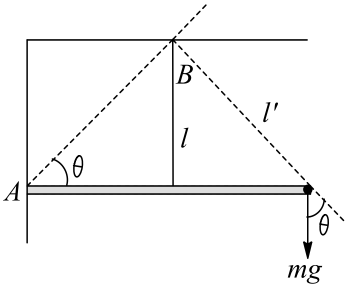
$$\Rightarrow \sqrt{\frac{K}{3m}}t + \frac{5\pi}{6} = \frac{3\pi}{2} \Rightarrow \quad t = \frac{2\pi}{3}\sqrt{\frac{3m}{K}}$$

Using value, $t = \pi/5$ s

218 **(4)**

The bob will execute SHM about the stationary axis passing through AB. If its effective length is l'then

$$T = 2\pi\sqrt{\frac{l'}{g'}}$$



$$l' = \frac{l}{\sin\theta} = \sqrt{2}l \quad \text{[because } \theta = 45°\text{]}$$

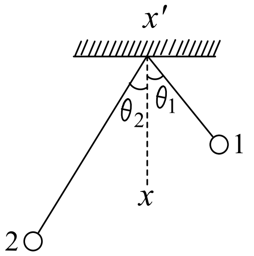
$$g' = g\cos\theta = \frac{g}{\sqrt{2}}$$

$$T = 2\pi\sqrt{\frac{2l}{g}} = 2\pi\sqrt{\frac{2\times 0.2}{10}} = \frac{2\pi}{5}\text{s}$$

$X = 4$

219 **(1)**

The angular position of pendulums 1 and 2 are (taking angles to the right of reference line xx' to be positive)



$$\theta_1 = \theta\cos\left(\frac{4\pi}{T}t\right)\left(\text{where } T = 2\pi\sqrt{\frac{l}{g}}\right)$$

$$\theta_2 = -\theta\left(\frac{2\pi}{T}t\right) = \cos\left(\frac{2\pi}{T}t + \pi\right)$$

For the straight to be parallel for the first time

$\theta_1 = \theta_2$

$$\cos\left(\frac{4\pi}{T}t\right) = \cos\left(\frac{2\pi}{T}t + \pi\right)$$

$$\therefore \frac{4\pi}{T}t = 2n\pi \pm \left(\frac{2\pi}{T}t + \pi\right)$$

For $n = 0, t = T/2$

For $n = 1, t = T/6, 3T/2$

Both the pendulums are parallel to each other for the first time after

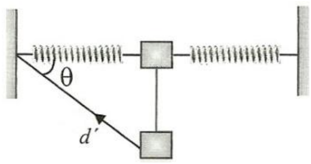
$$t = \frac{T}{6} = \frac{\pi}{3}\sqrt{\frac{l}{g}} = 1\text{ s}$$

220 **(2)**

Initial stretch in both springs $= d - \frac{3d}{4} = \frac{d}{4}$

$$F_{\text{restering}} = k\left(\frac{d}{4} + x\right) - k\left(\frac{d}{4} - x\right) = 2kx$$

$$\Rightarrow T_a = 2\pi\sqrt{\frac{m}{2k}}$$



$d' = d\sec\theta$

$$x' = d\sec\theta - \frac{3d}{4} = d\left(\frac{1}{\cos\theta} - \frac{3}{4}\right)$$

Force towards equilibrium position $(kx'\sin\theta)$

$= dk\left(\tan\theta - \frac{3\sin\theta}{4}\right)$ due to one spring and

Net $= 2dk\left(\tan\theta - \frac{3\sin\theta}{4}\right)$ for small θ, force

$$= 2\,dk\left[\theta - \frac{3\theta}{4}\right] = k\left(\frac{d\theta}{2}\right)$$

$d\theta$ =displacement from mean position

$$\Rightarrow F - \frac{kx}{2} \Rightarrow T_B = 2\pi\sqrt{\frac{2m}{k}}$$

$$\Rightarrow \frac{T_B}{T_A} = 2\pi\sqrt{\frac{m}{2k}}\Big/2\pi\sqrt{\frac{2m}{k}}$$

$$\Rightarrow \frac{T_B}{T_A} = 2$$

221 **(5)**

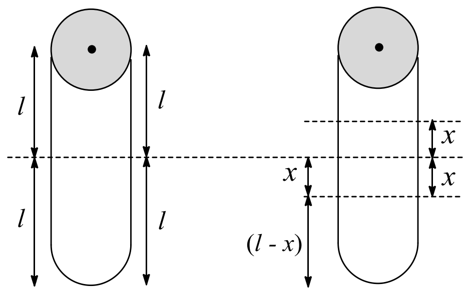
Let point B is displace in the downward direction by x. Net imbalance forces $= m_{\text{total}}.a$

$$[4mg(\ell + x) + mg(\ell - x) - \{4m(\ell - x)g + m(\ell + x)g\}]$$

$$= (4m \times 2l + m \times 2l)a$$

$$a = -\left(\frac{3g}{5l}\right)x$$

$$\omega = \sqrt{\frac{3g}{5l}} \Rightarrow T = 2\pi\sqrt{\frac{5l}{3g}} = 5\text{ s}$$

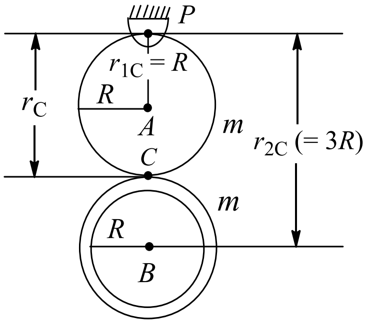


222 **(2)**

The time period of a physical pendulum is

$$T = 2\pi\sqrt{\frac{I_P}{Mgr}}$$

Here we have three quantities I_P, m and r



Let us calculate the quantities one by one as follows:

Finding I_P: $I_P = (I_P)_{\text{disc}} + (I_P)_{\text{ring}}$

Where $(I_P)_{\text{disc}} = I_A + m(PA)^2 = \frac{mR^2}{2} + mR^2 = \frac{3mR^2}{2}$

And $(I_P)_{\text{ring}} = I_B + m(PB)^2 = mR^2 + m(3R)^2 = 10\ mR^2$

Then, we have $I_P = \frac{3}{2}mR^2 = 10\ mR^2 = \frac{23}{2}mR^2$

Finding r:

$$r = \overrightarrow{r_C} = \frac{m_1\vec{r}_{iC} + m_2\vec{r}_{2C}}{m_1 + m_2},$$

Where $m_1 = m, m_2 = m$

$\overrightarrow{r_{1C}} = -R$ and $r_{2C} = -3R$

This gives $r_C = 2R$

Finding M:

$M = (m)_{\text{disc}} + (m)_{\text{ring}} = m + m = 2m$

Substituting $I_P = 23/2\ mR^2, M = 2m$ and $r = 2R$ in the expression

$T = 2\pi\sqrt{\frac{I_p}{Mgr}}$ we have $T = \pi\sqrt{\frac{23R}{2g}}$

After substituting the values we get $t = 2$ s

223 **(2)**

In static equilibrium of block, tension in the string is exactly equal to its weight. Let a vertically downward force F be applied on the block to pull it downward. Equilibrium is again restored when tension in the string is increased by the same amount F. Hence, total tension in the string becomes equal to $(mg + F)$. Strings are further elongated due to the extra tension F. Due to this extra tension F in strings, tension in each spring increases by $2F$. Hence increases in elongation of springs is $2F/K_1$ $2F/K_2$ $2F/K_3$ and $2F/K_4$, respectively

According to geometry of the arrangement, downward displacement of the block from its equilibrium position is

$$y = 2\left(\frac{2F}{K_1} + \frac{2F}{K_2} + \frac{2F}{K_3} + \frac{2F}{K_4}\right) \quad \text{(i)}$$

If the block is released now, it starts to accelerate upwards due to extra tension F in the strings. It means restoring force on the block is equal to F.

From Eq.(i),

$$F = \frac{y}{4\left(\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \frac{1}{K_4}\right)}$$

Restoring acceleration of block

$$\frac{F}{m} = \frac{y}{4m\left(\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \frac{1}{K_4}\right)}$$

Since acceleration of block is restoring and is directly proportional to displacement y, the block performs SHM

Its period $T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$

$$T = 2\pi\sqrt{4m\left(\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \frac{1}{K_4}\right)}$$

$$T = 4\pi\sqrt{m\left(\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \frac{1}{K_4}\right)}$$

After substitution the values we get $T = 2$ s

224 **(1)**

Net increment in the tension in string connecting the block will provide acceleration to the block.

We can write

$\Delta T = ma$ (i)

When the block m is displaced by a distance x beyond equilibrium position, the addition stretch of springs 1 and 2 are x_1 and x_2 respectively

We can write

$x_1 = \frac{x - x_2}{2} \Rightarrow x = 2x_1 + x_2$ (ii)

As $\Delta T = k_2x_2$ and $2\Delta T = k_1x_1$ then $2(k_2x_2) = k_1x_1$

$\Rightarrow x_1 = \frac{2k_2x_2}{k_1}$ (iii)

From Eqs. (ii) and (iii) $x = 2\left[\frac{2k_2x_2}{k_1}\right] + x_2 = \frac{(4k_2 + k_1)}{k_1}x_2$

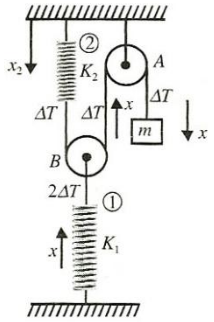
$$\Rightarrow x_2 = \frac{k_1x}{(k_1 + 4k_2)}$$

and $\vec{a} = \frac{-k_1k_2}{(k_1+4k_2)m}.x \Rightarrow \vec{a} = \frac{-k_1k_2}{(k_1+4k_2)m}.x$

$$\omega^2 = \frac{k_1k_2}{(K_1 + 4k_2)m}$$

Hence $R = \frac{80}{23\pi^2}m$

After substituting the values, we get $T = 1$ s


x_2
②
K_2
A
ΔT
ΔT
x
ΔT
m
x
B
$2\Delta T$
①
x
K_1