3.MATRICES

Single Correct Answer Type

1. If A and B are two square matrices such that $B = -A^{-1} BA$, then $(A+B)^2$ is equal to
a) $A^2 + B^2$ b) O c) $A^2 + 2AB + B^2$ d) $A + B$

2. If the system of equations $x + ay = 0$, $az + y = 0$ and$ax + z = 0$ has infinite solutions, then the value of a is
a) -1 b) 1 c) 0 d) No real values

3. If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, then $A^T A^{-1}$ is
a) $\begin{bmatrix} -\cos 2x & \sin 2x \\ -\sin 2x & \cos 2x \end{bmatrix}$ b) $\begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$ c) $\begin{bmatrix} \cos 2x & \cos 2x \\ \cos 2x & \sin 2x \end{bmatrix}$ d) None of these

4. If $P = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then $P^T\ Q^{2005}\ P$ is
a) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$
b) $\begin{bmatrix} 1 & 2005 \\ 2005 & 1 \end{bmatrix}$
c) $\begin{bmatrix} 1 & 0 \\ 2005 & 1 \end{bmatrix}$
d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

5. Let $A = \begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix}$ and$(A+1)^{50} - 50A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Then the value Of $a + b + c + d$ is
a) 2 b) 1 c) 4 d) None of these

6. The inverse of a skew-symmetric matrix of odd order is
a) A symmetric matrix b) A skew symmetric c) Diagonal matrix d) Does not exist

7. If $A = [a_{ij}]_{4\times4}$, such that $a_{ij} = \begin{cases} 2, & \text{when } i = j \\ 0, & \text{when } i \neq j \end{cases}$, then $\left\{\frac{\det(\text{adj}(\text{adj } A))}{7}\right\}$ is (where $\{\cdot\}$ represents fractional part function)
a) $1/7$ b) $2/7$ c) $3/7$ d) None of these

8. A is an involuntary matrix given by
$A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$, then the inverse of $A/2$ will be
a) $2A$ b) $\frac{A^{-1}}{2}$ c) $\frac{A}{2}$ d) A^2

9. If $A^2 - A + 1 = 0$, then the inverse of A is
a) A^{-2} b) $A + I$ c) $I - A$ d) $A - I$

10. If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix}$and $A^{-1} = \begin{bmatrix} 1/2 & 1/2 & 1/2 \\ -4 & 3 & c \\ 5/2 & 1-3/2 & 1/2 \end{bmatrix}$, then the values Of a and c are equal to
a) 1,1 b) 1,−1 c) 1, 2 d) −1, 1

11. If $\begin{bmatrix} 1/25 & 0 \\ x & 1/25 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ -a & 5 \end{bmatrix}^{-2}$, then the value of x is
a) $a/125$ b) $2\,a/125$ c) $2a/25$ d) None of these

12. Let A and B be two 2×2 matrices. Consider the statements
1. $AB = O \implies A = O$ or $B = O$
2. $AB = I_2 \implies A = B^{-1}$
3. $(A+B)^2 = A^2 + 2AB + B^2$

Then

a) (i) and (ii) are false, (iii)is true
b) (i) And (iii) are false, (i) is true
c) (i) is false, (ii) and (iii) are true
d) (i) and (iii) are false, (ii) is true

13. If A is order 3 square matrix such that $|A|$=2, then |adj(adj(adj A))| is

a) 512
b) 256
c) 64
d) None of these

14. For two unimodular complex numbers $-z_2$ andz_2,
$\begin{bmatrix} \overline{z_1} & -z_2 \\ \overline{z_2} & z_1 \end{bmatrix}^{-1} \begin{bmatrix} z_1 & z_2 \\ -\overline{z_2} & \overline{z_1} \end{bmatrix}^{-1}$ is equal to

a) $\begin{bmatrix} z_1 & z_2 \\ \overline{z_1} & \overline{z_2} \end{bmatrix}$
b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
c) $\begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$
d) None of these

15. If A is a square matrix of order n such that |adj(adjA)|= $|A|^9$then the value of n can be

a) 4
b) 2
c) Either 4 or 2
d) None of these

16. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ (where $bc \neq 0$) satisfies the equations $x^2 + k = 0$, then

a) $a + d = 0$
b) $k = -|A|$
c) $k = |A|$
d) None of these

17. Let $(x) = \frac{1+x}{1-x}$. If A is matrix for which $A^3 = 0$, then $f(A)$ is

a) $I + A + A^2$
b) $I + 2A + A^2$
c) $I - A - A^2$
d) None of these

18. The inverse of a diagonal matrix is

a) A diagonal matrix
b) a skew symmetric matrix
c) A symmetric matrix
d) None of these

19. If A and B are square matrices of order n, then $A - \lambda I$ and $B - \lambda I$ commute for every scalar λ, only if

a) $AB = BA$
b) $AB + BA = 0$
c) $A = -B$
d) None of these

20. Consider three matrices $A = \begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$ and
$C = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$. Then the value of the sum $\text{tr}(A) + \text{tr}\left(\frac{ABC}{2}\right) + tr\left(\frac{A(BC)^2}{4}\right) + \text{tr}\left(\frac{A(BC)^3}{8}\right) + \cdots + \infty$ is

a) 6
b) 9
c) 12
d) None

21. If $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, then A^8 equals

a) $4B$
b) $128B$
c) $-128B$
d) $-64B$

22. Let A be an n^{th}-order square matrix and B be its adjoint, then$|AB + KI_n|$ is (where K is a scalar quantity)

a) $(|A| + K)^{n-2}$
b) $(|A| + K)^n$
c) $(|A| + K)^{n-1}$
d) None of these

23. If $A^3 = O$, then $I + A + A^2$ eauals

a) $I - A$
b) $(I + A^1)^{-1}$
c) $(I - A)^{-1}$
d) None of these

24. If $A = \begin{bmatrix} a + ib & c + id \\ -c + id & a - ib \end{bmatrix}$ and $a^2 + b^2 + c^2 + d^2 = 1$, then A^{-1} is equal to

a) $\begin{bmatrix} a + ib & -c - id \\ -c + id & a - ib \end{bmatrix}$
b) $\begin{bmatrix} a + ib & -c + id \\ -c + id & a - ib \end{bmatrix}$
c) $\begin{bmatrix} a - ib & -c - id \\ -c - id & a + ib \end{bmatrix}$
d) None of these

25. Given that matrix$A = \begin{bmatrix} x & 3 & 2 \\ 1 & y & 4 \\ 2 & 2 & z \end{bmatrix}$. If $xyz = 60$ and $8x + 4y + 3z = 20$, then $A(\text{adj}A)$ is equal to

a) $\begin{bmatrix} 64 & 0 & 0 \\ 0 & 64 & 0 \\ 0 & 0 & 64 \end{bmatrix}$
b) $\begin{bmatrix} 88 & 0 & 0 \\ 0 & 88 & 0 \\ 0 & 0 & 88 \end{bmatrix}$
c) $\begin{bmatrix} 68 & 0 & 0 \\ 0 & 68 & 0 \\ 0 & 0 & 68 \end{bmatrix}$
d) $\begin{bmatrix} 34 & 0 & 0 \\ 0 & 34 & 0 \\ 0 & 0 & 34 \end{bmatrix}$

26. If $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$, then sum of all the elements of matrix A is

a) 0
b) 1
c) 2
d) -3

27. Let $(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, where $\alpha \in R$. Then $\left(F(\alpha)\right)^{-1}$is equal to

a) $F(\alpha^{-1})$ b) $F(-\alpha)$ c) $F(2\alpha)$ d) None of these

28. If $A(\alpha,\beta)=\begin{bmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & e^{\beta}\end{bmatrix}$then $A(\alpha,\beta)^{-1}$ is equal to

a) $A(-\alpha,-\beta)$ b) $A(-\alpha,\beta)$ c) $A(\alpha,-\beta)$ d) $A(\alpha,\beta)$

29. If $\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix} A \begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, then $A=$

a) $\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$ b) $\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ c) $\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$ d) $-\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$

30. If adj $B=A, |P|=|Q|=1$, then adj$(Q^{-1}BP^{-1})$ is

a) PQ b) QAP c) PAQ d) $PA^{-1}Q$

31. A is a 2×2 matrix such that $A\begin{bmatrix}1\\ -1\end{bmatrix}=\begin{bmatrix}-1\\ 2\end{bmatrix}$ and $A^2\begin{bmatrix}1\\ -1\end{bmatrix}=\begin{bmatrix}1\\ 0\end{bmatrix}$

The sum of the elements of A is

a) -1 b) 0 c) 2 d) 5

32. If A and B are two non-singular matrices of the same order

Such that$B^r=I$, for some positive integer$r>1$. Then $A^{-1}B^{r-1}A-A^{-1}B^{-1}A=$

a) I b) $2I$ c) O d) $-I$

33. $(-A)^{-1}$ is always equal to (where A is n^{th} -order square matrix)

a) $(-1)^nA^{-1}$ b) $-A^{-1}$ c) $(-1)^{n-1}A^{-1}$ d) None of these

34. The equation $[1\ x\ y]\begin{bmatrix}1 & 3 & 1\\ 0 & 2 & -1\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1\\ x\\ y\end{bmatrix}=[0]$ has

(i)for $y=0$(p) rational roots

(ii) for $y=-1$(q) irrational roots

(r) integral roots

Then

(i)(ii)

a) (p) (r) b) (q) (p) c) (p) (q) d) (r) (p)

35. If P is non-singular matrix, then value of adj(P^{-1}) in terms of P is

a) $P/|P|$ b) $P|P|$ c) P d) None of these

36. If A and B are two non-zero square matrices of the same order such that the product $AB=O$, then

a) Both A and B must be singular b) Exactly one of them must be singular

c) Both of them are non-singular d) None of these

37. If $A=\begin{bmatrix}0 & -\tan\alpha/2\\ \tan\alpha/2 & 0\end{bmatrix}$ and I is a 2× 2 unit matrix, then

$(I-A)\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \sin\alpha\end{bmatrix}$ is

a) $-I+A$ b) $I-A$ c) $-I-A$ d) None of these

38. If A is non-singular and $(A-2I)(A-4I)-0$, then $\frac{1}{6}A+\frac{4}{3}A^{-1}$

Is equal to

a) 0 b) I c) $2I$ d) $6I$

39. Which of the following is an orthogonal matrix?

a) $\begin{bmatrix}6/7 & 2/7 & -3/7\\ 2/7 & 3/7 & 6/7\\ 3/7 & -6/7 & 2/7\end{bmatrix}$ b) $\begin{bmatrix}6/7 & 2/7 & 3/7\\ 2/7 & -3/7 & 6/7\\ 3/7 & 6/7 & -2/7\end{bmatrix}$

c) $\begin{bmatrix}-6/7 & -2/7 & -3/7\\ 2/7 & 3/7 & 6/7\\ -3/7 & 6/7 & 2/7\end{bmatrix}$ d) $\begin{bmatrix}6/7 & -2/7 & 3/7\\ 2/7 & 2/7 & -3/7\\ -6/7 & 2/7 & 3/7\end{bmatrix}$

40. If matrix A is given by $A=\begin{bmatrix}6 & 11\\ 2 & 4\end{bmatrix}$, then the determinant of$A^{2005}-6A^{2004}$ is

a) 2^{2006} b) $(-11)^{2005}$ c) $-2^{2005}\cdot 7$ d) $(-9)^{2004}$

41. If A is a square matrix such that $A^2=A$, then $(I+A)^3-7A$ is

a) $3I$ b) O c) I d) $2I$

42. Let A, B be two matrices such that they commute, then for any positive integer n,

1. $AB^n = B^n A$ (ii) $(AB)^n = A^n B^n$

a) Only (i) is correct b) Both (i) and (ii) are correct
c) Only (ii) is correct d) None of (i) and (ii) is correct

43. Given$2x - y + 2z = 2, x - 2y + 2z = -4, x + y + \lambda z = 4$ then the value of λ such that the given system of equations has no solution, is

a) 3 b) 1 c) 0 d) -3

44. The number of solutions of the matrix equation $X^2 = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ is

a) More than 2 b) 2 c) 0 d) 1

45. If A is a skew-symmetric matrix and n is odd positive integer, then A^n is

a) A skew-symmetric matrix b) A symmetric matrix
c) A diagonal matrix d) None of these

46. If $A, B, A + I, A + B$ are idempotent matrices, then AB is equal To

a) BA b) $-BA$ c) I d) O

47. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then value of α for which $A^2 = B$ is

a) 1 b) -1 c) 4 d) No real values

48. Let a and b be two real numbers such that $a > 1$, $b > 1$.
If $A = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, then $\lim_{n\to\infty} A^{-1}$ is

a) Unit matrix b) Null matrix c) $2I$ d) None of these

49. If A and B are symmetric matrices of the same order and
$X = AB + BA$ and $Y = AB - BA$, then $(XY)^T$ is equal to

a) XY b) YX c) $-YX$ d) None of these

50. If $A = \begin{bmatrix} a & b & c \\ x & y & z \\ p & q & r \end{bmatrix}, B = \begin{bmatrix} q & -b & y \\ -p & a & -x \\ r & -c & z \end{bmatrix}$ and if A is invertible,
Then which of the following is not true?

a) $|A| = |B|$ b) $|A| = -|B|$
c) $|\text{adj}A| = -|\text{adj}B|$ d) A is invertble if and only if B is invertible

51. In which of the following type of matrix inverse does not exist always

a) Idempotent b) Orthogonal c) Involuntary d) None of these

52. The number of 3× 3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solutions, is

a) 0 b) $2^9 - 1$ c) 168 d) 2

53. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 5 \\ 0 & 2 & 1 \end{bmatrix}$ and$B = \begin{bmatrix} 0 \\ -3 \\ 1 \end{bmatrix}$. Which of the following is true?

a) $AX = B$ has a unique solution b) $AX = B$ has exactly three solutions
c) $AX = B$ has infinity many solutions d) $AX = B$ is inconsistent

54. The matrix X for which $\begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix} X = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$ is

a) $\begin{bmatrix} -2 & -4 \\ -3 & 1 \end{bmatrix}$ b) $\begin{bmatrix} -\frac{1}{5} & \frac{2}{5} \\ \frac{3}{5} & \frac{1}{5} \\ -\frac{}{10} & \frac{}{5} \end{bmatrix}$ c) $\begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$ d) $\begin{bmatrix} 6 & 2 \\ \frac{11}{2} & 2 \end{bmatrix}$

55. Let $A + 2B = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix}$ and$2A - B = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}$. Then $\text{tr}(A) - \text{tr}(B)$ has the value equal to

a) 0 b) 1 c) 2 d) None

56. Elements of matrix A of order 10×10 are defined as$a_w = w^{i+j}$(where w is cube coot of unity), then trace (A) of the matrix is

a) 0 b) 1 c) 3 d) None of these

57. The number of diagonal matrix A of order$n$ for which $A^3 = A$ is

a) 1 b) 0 c) 2^n d) 3^n

58. The product of matrices $A = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$and

$B = \begin{bmatrix} \cos^2\phi & \cos\emptyset\sin\emptyset \\ \cos\emptyset\sin\emptyset & \sin^2\emptyset \end{bmatrix}$ is a null matrix if $\theta - \phi =$

a) $2n\pi, n \in Z$ b) $n\frac{\pi}{2}, n \in Z$ c) $(2n+1)\frac{\pi}{2}, n \in Z$ d) $n\pi, n \in Z$

59. If A is an orthogonal matrix, then A^{-1}equals

a) A^T b) A c) A^2 d) None of these

60. If $A = \begin{bmatrix} a & b \\ 0 & a \end{bmatrix}$ is n^{th} root of I_2, then choose the correct statement:

1. If n is odd, $a = 1, b = 0$
2. If n is odd, $a = -1, b = 0$
3. If n is even, $a = 1, b = 0$
4. If n is even, $a = -1, b = 0$

a) i, ii, iii b) ii, iii, iv c) i, ii, iii, iv d) i, iii, iv

61. If n^{th}-order square matrix $A$is a orthogonal, then,
$|\text{adj}(\text{adj } A)|$ is

a) Always -1 if n is even b) Always 1 if n is odd
c) Always 1 d) None of these

62. For each real $x, -1 < x < 1$. Let $A(x)$be the matrix
$(1-x)^{-1}\begin{bmatrix} 1 & -x \\ -x & 1 \end{bmatrix}$ and $z = \frac{x+y}{1+xy}$. Then

a) $A(z) = A(x)A(y)$ b) $A(z) = A(x) - A(y)$ c) $A(z) = A(x) + A(y)$ d) $A(z) = A(x)[A(y)]^{-1}$

63. If A and $B$are square matrices of the same order and A is non-singular, then for a positive integer n, $(A^{-1}BA)^n$ is equal to

a) $A^{-n}B^nA^n$ b) $A^nB^nA^{-n}$ c) $A^{-1}B^nA$ d) $n(A^{-1}BA)$

64. If $k \in R_0$, then det{adj(kI_n)} is equal to

a) k^{n-1} b) $k^{n(n-1)}$ c) k^n d) k

65. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $G(y) = \begin{bmatrix} \cos y & 0 & \sin y \\ 0 & 1 & 0 \\ -\sin y & 0 & \cos y \end{bmatrix}$

Then $[F(x)G(-y)]^{-1}$ is equal to

a) $F(-x)G(-y)$ b) $G(-y)F(-x)$ c) $F(x^{-1})G(y^{-1})$ d) $G(y^{-1})F(x^{-1})$

66. If $A^2 = I$, then the value of $\det(A - I)$ is (where A has order 3)

a) 1 b) -1 c) 0 d) Cannot say anything

67. If A is a non-singular matrix such that $AA^T = A^TA$ and
$B = A^{-1}A^T$, then matrix B is

a) Involuntary b) Orthogonal c) Idempotent d) None of these

68. The matrix $A = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ is

a) Idempotent matrix b) Involutory matrix c) Nilpotent matrix d) None of these

69. If A and B are two matrices such that $AB = B$ and $BA = A$, then

a) $(A^5 - B^5)^3 = A - B$ b) $(A^5 - B^5)^3 = A^3 - B^3$
c) $A - B$ is idempotent d) $A - B$ is nilpotent

70. If A is singular matrix, then adj A is

a) Singular b) Non-singular c) Symmetric d) Not defined

71. If A is a non-diagonal involutory matrix, then

a) $A - I = O$ b) $A + I = O$
c) $A - I$ is non-zero singular d) None of these

72. If $A_1, A_3, \dots, A_{2n-1}$ are n skew symmetric matrices of same order, then $B = \sum_{r=1}^{n}(2r-1)(A_{2r-1})^{2r-1}$ will be

a) Symmetric b) Skew-symmetric
c) Neither symmetric nor skew-symmetric d) Data not adequate

73. If A and B are two non-singular matrices such that $AB = C$, then $|B|$ is equal to

a) $\frac{|C|}{|A|}$ b) $\frac{|A|}{|C|}$ c) $|C|$ d) None of these

74. If P is an orthogonal matrix and $Q = PAP^T$ and $x = P^TQ^{1000}P$,
Then x^{-1} is, where A is involutary matrix

a) A b) I c) A^{1000} d) None of these

75. If $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & 1 \end{bmatrix}$, then $A(\overline{A}^T)$ equals

a) O b) I c) $-I$ d) $2I$

76. If A is symmetric as well as skew-symmetric matrix, then A is

a) Diagonal matrix b) Null matrix c) Triangular matrix d) None of these

77. If A and B are square matrices of the same order and A is non-singular, then for a positive integer n, $(A^{-1}BA)^n$ is equal to

a) $A^{-n}B^nA^n$ b) $A^nB^nA^{-n}$ c) $A^{-1}B^nA$ d) $n(A^{-1}BA)$

78. If $A\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $f(x) = \frac{1+x}{1-x}$, then $f(A)$ is

a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ c) $\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$ d) None of these

79. If $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is to be the square root of two-rowed unit matrix,
then α, β and γ should satisfy the relation

a) $1 - \alpha^2 + \beta\gamma = 0$ b) $\alpha^2 + \beta\gamma - 1 = 0$ c) $1 + \alpha^2 + \beta\gamma = 0$ d) $1 - \alpha^2 - \beta\gamma = 0$

80. If A and B are squares matrices such that $A^{2006} = O$ and $AB = A + B$, then $\det(B)$ equals

a) 0 b) 1 c) -1 d) None of these

81. If $A(\alpha, \beta) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & e^{\beta} \end{bmatrix}$, then $A(\alpha, \beta)^{-1}$ in terms of
Function of A is

a) $A(\alpha, -\beta)$ b) $A(-\alpha, -\beta)$ c) $A(-\alpha, \beta)$ d) None of these

82. Matrix A such that $A^2 = 2A - I$, where I is the identity matrix,
Then for $n \geq 2, A^n$ is equal to

a) $2^{n-1}A - (n-1)I$ b) $2^{n-1}A - I$ c) $nA - (n-1)I$ d) $nA - I$

83. Identify the incorrect statement in respect of two square matrices A and B conformable for sum and product:

a) $t_r(A + B) = t_r(A) + t_r(B)$ b) $t_r(\alpha A) = \alpha t_r(A), \alpha \in R$
c) $t_r(A^T) = t_r(A)$ d) None of these

84. If A is a 3×3 skew-symmetric matrix, then trace of A is equal to

a) -1 b) 1 c) $|A|$ d) None of these

85. If Z is an idempotent matrix, then $(I + Z)^n$

a) $I + 2^nZ$ b) $I + (2^n - 1)Z$ c) $I - (2^n - 1)Z$ d) None of these

86. If A is a nilpotent matrix of index 2, then for any positive integer n, $A(I + A)^n$ is equal to

a) A^{-1} b) A c) A^n d) I_n

87. If both $A - \frac{1}{2}I$ and $A + \frac{1}{2}$ are orthogonal matrices, then

a) A is orthogonal b) A is skew-symmetric matrix of even order

c) $A^2 = \frac{3}{4} I$

d) None of these

Multiple Correct Answers Type

88. If $A(\theta) = \begin{bmatrix} \sin\theta & i\cos\theta \\ i\cos\theta & \sin\theta \end{bmatrix}$, then which of the following is not true?

a) $A(\theta)^{-1} = A(\pi - \theta)$

b) $A(\theta) += A(\pi + \theta)$ is a null matrix

c) $A(\theta)$ is invertible for all $\theta \in R$

d) $A(\theta)^{-1} = A(-\theta)$

89. If $S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $A = \begin{bmatrix} b+c & c+a & b-c \\ c-b & c+b & a-b \\ b-c & a-c & a+b \end{bmatrix}$ $(a, b, c \neq 0)$, then SAS^{-1} is

a) Symmetric matrix

b) Diagonal matrix

c) Invertible matrix

d) Singular matrix

90. If $A_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{bmatrix}$,

Then $A_i A_k + A_k A_i$ is equal to

a) $2I$ if $i = k$

b) O if $i \neq k$

c) 2I if $i \neq k$

d) O always

91. Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then

a) $A^2 - 4A - 5I_3 = O$

b) $A^{-1} = \frac{1}{5}(A - 4I_3)$

c) A^3 is not invertible

d) A^2 is invertible

92. If C is skew-symmetric matrix of order n and X is $n \times 1$ column matrix, then $X^T CX$ is

a) Singular

b) Non-singular

c) Invertible

d) Non-invertible

93. If $A^{-1} = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & -1/3 \end{bmatrix}$, then

a) $|A| = -1$

b) adj $A = \begin{bmatrix} -1 & 1 & -2 \\ 0 & -3 & -1 \\ 0 & 0 & 1/3 \end{bmatrix}$

c) $A = \begin{bmatrix} 1 & 1/3 & 7 \\ 0 & 1/3 & 1 \\ 0 & 0 & -3 \end{bmatrix}$

d) $A = \begin{bmatrix} 1 & 1/3 & -7 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

94. If A, B and C are three square matrices of the same order, then$AB = AC \Rightarrow B = C$. Then

a) $|A| \neq 0$

b) A is invertible

c) A may be orthogonal

d) A is symmetric

95. If A is a matrix such that $A^2 + 2A + 2I = 0$, then which of the following is/are true?

a) A is non-singular

b) A is symmetric

c) A cannot be skew-symmetric

d) $A^{-1} = -\frac{1}{2}(A + I)$

96. If $\begin{pmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{pmatrix} \begin{pmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{pmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then

a) $a = \cos 2\theta$

b) $a = 1$

c) $b = \sin 2\theta$

d) $b = -1$

97. If $A = \frac{1}{3} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is an orthogonal matrix, then

a) $a = -2$

b) $a = 2, b = 1$

c) $b = -1$

d) $b = 1$

98. If $AB = A$ and $BA = B$, then

a) $A^2 B = A^2$

b) $B^2 A = B^2$

c) $ABA = A$

d) $BAB = B$

99. If B is an idempotent matrix, and $A = 1 - B$, then

a) $A^2 = A$

b) $A^2 = I$

c) $AB = O$

d) $BA = O$

100. Let A and B are two non-singular square matrices, A^T and B^T are the transpose matrices of A and B, respectively, then which of the following are correct?

a) $B^T AB$ is symmetric matrix if A is symmetric
b) $B^T AB$is symmetric matrix if B is symmetric
c) $B^T AB$ is skew-symmetric matrix for every matrix A
d) $B^T AB$ is skew-symmetric matrix if A is skew-symmetric

101. The rank of the matrix $\begin{bmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{bmatrix}$ is
a) 1 if $a = 6$ b) 2 is $a = 1$ c) 3 if $a = 2$ d) 1 if $a = -6$

102. Suppose $a_1, a_2, \cdots$are real number, with $a_1 \neq 0$. If $a_1, a_2, a_3, \cdots$ are in AP., then
a) $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_5 & a_6 & a_7 \end{bmatrix}$ is singular (where $i = \sqrt{-1}$)
b) The system of equations $a_1x + a_2y + a_3z = 0, a_4x + a_5y + a_6z = 0$, had infinite number of solutions
c) $B = \begin{bmatrix} a_1 & ia_2 \\ ia_2 & a_1 \end{bmatrix}$ is non-singular
d) None of these

103. If $AB = A$ and $BA = B$, then which of the following is/are true?
a) A is idempotent b) B is idempotent c) A^T is idempotent d) None of these

104. Which of the following statements is/are true about square matrix A of order n?
a) $(-A)^{-1}$ is equal to $-A^{-1}$ when n is odd only.
b) If $A^n = O, then\ I + A + A^2 + \cdots + A^{n-1} = (I - A)^{-1}$.
c) IfA is skew-symmetric matrix of odd order, then its inverse does not exist.
d) $(A^T)^{-1} = (A^{-1})^T$ holds always.

105. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then
a) adj(adj$A$)=A b) |adj(adj A)=1| c) |adj A|= 1 d) None of these

106. If $A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A+B)^2 = A^2 + B^2 + 2AB$, then
a) $a = -1$ b) $a = 1$ c) $b = 2$ d) $b = -2$

107. Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$. Then
a) $A^2 - 4A - 5I_3 = 0$ b) $A^{-1} = \frac{1}{5}(A - 4I_3)$ c) $A^3$is not invertible d) A^2 is invertible

108. If A is unimodular, then which of the following is unimodular?
a) $-A$ b) A^{-1}
c) adj A d) ωA,where ω is cube root of unity

109. Let $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$. Then which of following is not true?
a) $\lim_{n\to\infty} \frac{1}{n^2} A^{-n} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$ b) $\lim_{n\to\infty} \frac{1}{n} A^{-n} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$
c) $A^{-n} = \begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix} \forall n \neq N$ d) None of these

110. If $A_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then
a) $A_\alpha \cdot A_{(-\alpha)} = I$ b) $A_\alpha \cdot A_{(-\alpha)} = 0$
c) $A_\alpha \cdot A_\beta = A_{\alpha+\beta}$ d) $A_\alpha \cdot A_\beta = A_{\alpha-\beta}$

111. Which of the following is correct?
a) $B'AB$ is symmetric if A is symmetric b) $B'AB$ is skew-symmetric if A is symmetric
c) $B'AB$ is symmetric if A is skew-symmetric d) $B'AB$ is skew-symmetric if A is skew-symmetric

112. If D_1 and D_2 are two 3× 3diagonal matrices, then which of the following is/are true?
a) D_1D_2 is diagonal matrix b) $D_1D_2 = D_2D_1$
c) $D_1^2 + D_2^2$is a diagonal matrix d) None of these

113. If A and B are two invertible matrices of the same order, then $\text{adj}(AB)$ is equal to

a) $\text{adj}(B)\,\text{adj}(A)$ b) $|B||A|B^{-1}A^{-1}$ c) $|B||A|A^{-1}B^{-1}$ d) $|A||B|(AB)^{-1}$

114. A skew-symmetric matrix A satisfies the relation $A^2 + I = 0$, where I is a unit matrix then A is

a) Idempotent b) Orthogonal c) Of even order d) Odd order

115. If A and B are symmetric and commute, then which of the following is/are symmetric?

a) $A^{-1}B$ b) AB^{-1} c) $A^{-1}B^{-1}$ d) None of these

116. If $A = (a_{ij})_{n\times n}$ and f is a function, we define $f(A) = \left(f(a_{ij})\right)_{n\times n}$

Let $A = \begin{pmatrix} \pi/2-\theta & \theta \\ -\theta & \pi/2-\theta \end{pmatrix}$. Then

a) $\sin A$ is invertible b) $\sin A = \cos A$ c) $\sin A$ is orthogonal d) $\sin(2A) = \sin A \cos A$

117. If α, β, γ are three real numbers and

$A = \begin{bmatrix} 1 & \cos(\alpha-\beta) & \cos(\alpha-\gamma) \\ \cos(\beta-\alpha) & 1 & \cos(\beta-\gamma) \\ \cos(\gamma-\alpha) & \cos(\gamma-\beta) & 1 \end{bmatrix}$, then which of following is/are true?

a) A is singular b) A is symmetric c) A is orthogonal d) A is not invertible

118. If $\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is orthogonal, then

a) $a = \pm\frac{1}{\sqrt{2}}$ b) $b = \pm\frac{1}{\sqrt{12}}$ c) $c = \pm\frac{1}{\sqrt{3}}$ d) None of these

Assertion - Reasoning Type

This section contain(s) 0 questions numbered 119 to 118. Each question contains STATEMENT 1(Assertion) and STATEMENT 2(Reason). Each question has the 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

a) Statement 1 is True, Statement 2 is True; Statement 2 **is** correct explanation for Statement 1

b) Statement 1 is True, Statement 2 is True; Statement 2 **is not** correct explanation for Statement 1

c) Statement 1 is True, Statement 2 is False

d) Statement 1 is False, Statement 2 is True

119

Statement 1: $A = \begin{bmatrix} 4 & 0 & 4 \\ 2 & 3 & 3 \\ 1 & 2 & 2 \end{bmatrix}, B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$. Then $(AB)^{-1}$ does not exist

Statement 2: Since $|A| = 0, (AB)^{-1} = B^{-1}A^{-1}$ is meaningless

120 Let Abe a 2×2 matrix with non-zero entries and let $A^2 = I$, where I is 2×2 identity matrix. Define Tr (A)=sum of diagonal elements of A and $|A|$ = determinant of matrix A.

Statement 1: Tr (A)=0

Statement 2: $|A|$=*1.*

121

Statement 1: If A is a skew-symmetric matrix of order 3×3, then $\det(A) = 0$ or $|A| = 0$

Statement 2: If A is square matrix, then $\det(A) = \det(A') = \det(-A')$

122

Statement 1: If the matrices $A, B, (A+B)$ are non-singular, then $[A(A+B)^{-1}B]^{-1} = B^{-1} + A^{-1}$

Statement 2: $[A(A+B)^{-1}B]^{-1} = [A(A^{-1}+B^{-1})B]^{-1}$
$= [(I + AB^{-1})B]^{-1}$
$= [(B + AB^{-1}B)]^{-1}$
$= [(B + AI)]^{-1}$
$= [(B + A)]^{-1}$
$= B^{-1} + A^{-1}$

123

Statement 1: Let A, B be two square matrices of the same order such that $AB = BA, A^m = 0$ and $B^n = O$ for same positive integers m, n, then there exists a positive integer r such that$(A+B)^r = O$

Statement 2: If $AB = BA$ then $(A+B)^r$ can be expanded as binomial expansion

124

Statement 1: If A is orthogonal matrix of order 2, then$|A| = \pm 1$

Statement 2: Every two-rowed real orthogonal matrix is of any one of the forms $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ or $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$

125 Let A be 2×2 matrix.

Statement 1: Adj(adj A)= A

Statement 2: $|\text{adj } A| = A$

126 Let A be a 2×2 matrix with real entries. Let I be the 2× 2 identity matrix. Denote by Tr (A), the sum of diagonal entries of A. Assume that $A^2 = I$

Statement 1: If $A \neq I$ and $A \neq -I$, then det $(A) = -1$.

Statement 2: If $A \neq I$ and $A \neq -I$, then Tr A$\neq$ 0.

127

Statement 1: If a, b, c, d are real numbers and $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $A^3 = O$, then $A^2 = O$

Statement 2: For matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ we have$A^2 - (a+d)A + (ad-bc)\mathrm{I} = 0$

128

Statement 1: If D =diag$[d_1, d_2 \ldots\ldots d_n]$, then
$D^{-1} = \text{diag}[d_1^{-1}, d_2^{-1} \ldots, d_n^{-1}]$

Statement 2: If D =diag$[d_1, d_2 \ldots d_n]$, then
D^n =diag$[d_1^n, d_2^n \ldots, d_n^n]$

129

Statement 1: If $F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then
$[F(\alpha)]^{-1} = F(-\alpha)$

Statement 2: For matrix $G(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$,
We have$[G(\beta)]^{-1} = G(-\beta)$.

130

Statement 1: The determinant of a matrix $A = [a_{ij}]_{5\times 5}$ where $a_{ij} + a_{ij} = 0$ for all i and j is zero

Statement 2: The determinant of a skew-symmetric matrix of odd order is zero

131

Statement 1: Matrix 3×3, $a_{ij} = \frac{i-j}{i+2j}$ cannot be expressed as a sum symmetric and skew-symmetric matrix

Statement 2: Matrix 3×3, $a_{ij} = \frac{i-j}{i+2j}$ is neither symmetric nor skew-symmetric

132

Statement 1: $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 7 \end{bmatrix}$ is a diagonal matrix

Statement 2: $A = [a_{ij}]$ is square matrix such that $a_{ij} = 0, \forall\, i \neq j$, then A is called diagonal matrix

133

Statement 1: For a singular square matrix A,$AB = AC \Rightarrow B = C$

Statement 2: If $|A| = 0$, then A^{-1}does not exist

134

Statement 1: The inverse of $A = \begin{bmatrix} 3 & 4 \\ 3 & 5 \end{bmatrix}$ does not exist

Statement 2: The matrix A is non-singular

135

Statement 1: If A, B, C are matrices such that $|A_{3\times 3}| = 3, |B_{3\times 3}| = -1$ and$|C_{2\times 2}| = +2$, then $|2ABC| = -12$

Statement 2: For matrices A, B, C of the same order,$|ABC| = |A||B||C|$

136

Statement 1: The inverse of the matrix $A = [a_{ij}]_{n\times n}$ where$a_{ij} = 0, i \geq j$ is $B = [a_{ij}^{-1}]_{n\times n}$

Statement 2: The inverse of singular matrix does not exist.

137

Statement 1: If a matrix of order 2×2, commutes with every matrix of order 2×2, then it is scalar matrix

Statement 2: A scalar matrix of order 2×2 commutes with every 2×2 matrix

138

Statement 1: The matrix $A = \frac{1}{3}\begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ -2 & -2 & -1 \end{bmatrix}$ is an orthogonal matrix

Statement 2: If A and B are orthogonal, then AB is also orthogonal

139

Statement 1: If $A = [a_{ij}]_{n\times n}$ is such that $a_{ij} = \bar{a}_{ji}, \forall\, i, j$ and$A^2 = O$, then matrix A null matrix.

Statement 2: $|A| = 0$

140

Statement 1: $|\text{adj}(\text{adj}(\text{adj } A))| = |A|^{(n-1)3}$

Statement 2: $|\text{adj}A| = |A|^n$

Matrix-Match Type

This section contain(s) 0 question(s). Each question contains Statements given in 2 columns which have to be matched. Statements (A, B, C, D) in **columns I** have to be matched with Statements (p, q, r, s) in **columns II**.

141.

Column-I	Column- II
(A) $(I - A)^n$ is if A is idempotent	(p) $2^{n-1}(\text{I} - \text{A})$
(B) $(I - A)^n$ is if A is involuntary	(q) $I - nA$
(C) $(I - A)^n$ is if A is nilpotent of index 2	(r) A
(D) If a is orthogonal, then $(A^T)^{-1}$	(s) $I - A$

CODES :

	A	**B**	**C**	**D**
a)	s	p	q	r
b)	p	q	r	s
c)	q	r	s	p
d)	r	s	p	q

142. Match List I with List II and select the correct answer using the codes given below the lists

Column-I	Column- II
(A) $(\text{adj } A)^{-1}$	(1) $k^{n-1}(\text{adj } A)$
(B) $(\text{adj } A^{-1})$	(2) $\dfrac{A}{|A|}$
(C) adj (kA)	(3) $|A|^{n-2}A$
(D) adj (adj A)	(4) $\dfrac{\text{adj}(\text{adj } A)}{|A|^2}$

CODES :

	A	**B**	**C**	**D**
a)	1	2	3	4

b)	3	4	2	1
c)	4	3	2	1
d)	2	4	1	3

143. Match List I with List II and select the correct answer using the codes given below the lists

	Column-I		Column- II
(A)	A is a square matrix such that $A^2 = A$	(1)	Nilpotent matrix
(B)	A is a square matrix such that $A^m = O$	(2)	Involutory matrix
(C)	A is square matrix such that $A^2 = I$	(3)	Symmetric matrix
(D)	A is square matrix such that $A^T = A$	(4)	Idempotent matrix

CODES :

	A	**B**	**C**	**D**
a)	1	3	2	4
b)	3	4	2	1
c)	4	3	2	3
d)	4	1	2	3

144.

	Column-I		Column- II
(A)	If $\|A\| = 2$, then $\|2A^{-1}\|$=(where A is of Order 3)	(p)	1
(B)	If $\|A\| = 1/8$, then $\|\text{adj}(\text{adj}(2A))\|$= (where A is of Order 3)	(q)	4
(C)	If $(A + B)^2 = A^2 + B^2$, and$\|A\| = 2$, then $\|B\|$ = (where A and B are of odd order)	(r)	24
(D)	$\|A_{2\times2}\| = 2, \|B_{3\times3}\| = 3$ and$\|C_{4\times4}\| = 4$, then $\|ABC\|$ is equal to	(s)	0

CODES :

	A	**B**	**C**	**D**
a)	P	q	r	s
b)	r	s	q	p
c)	q	p	s	r
d)	s	r	p	q

145.

Column-I	Column- II

(A)	If A is an idempotent matrix and I is an Identity matrix of the same order, then The value of n, such that $(A+I)^n = I + 127$ is	(p)	9
(B)	If $(I-A)^{-1} = I + A + A^2 + \cdots + A^7$, the $A^n = 0$ where n is	(q)	10
(C)	If A is matrix such that $a_{ij} = (i+j)(i-j)$, then A is singular if Order of matrix is	(r)	7
(D)	If a non-singular matrix A is symmetric, Show that A^{-1} is also symmetric, then Order of A can be	(s)	8

CODES :

	A	**B**	**C**	**D**
a)	S	r	r,s	r,s,q,p
b)	r	s	p,r	p,q,r,s
c)	p	q	q,s	s,p,q,r
d)	q	p	s,r	q,p,r,s

Linked Comprehension Type

This section contain(s) 16 paragraph(s) and based upon each paragraph, multiple choice questions have to be answered. Each question has atleast 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 146 to -146

A and B are two matrices of same order 3×3, where

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 5 & 6 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 & 5 \\ 2 & 3 & 8 \\ 7 & 2 & 9 \end{bmatrix}$

146. The value of adj (adj A) is equal to

a) $-A$ b) $4A$ c) $8A$ d) $16\,A$

Paragraph for Question Nos. 147 to - 147

Two $n \times n$ square matrices A and B are said to be similar, if there exists a non-singular matrix P such that $PAP^{-1} = B$

147. If A and B are two singular matrices, then

a) $\det(A) = \det(B)$ b) $\det(A) + \det(B) = 0$ c) $\det(AB) \neq 0$ d) None of these

Paragraph for Question Nos. 148 to - 148

Let A and B are two matrices of same order 3×3, where $A = \begin{bmatrix} 1 & 2 & \lambda+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 2 & 4 \\ 3 & 2 & 5 \\ 2 & 1 & 4 \end{bmatrix}$

148. If A is singular matrix, then tr $(A + B)$ is equal to

a) 6 b) 12 c) 24 d) 17

Paragraph for Question Nos. 149 to - 149

Let A is matrix of order 2×2 such that $A^2 = 0$

149. $A^2 - (a + d)A + (ad - bc)I$ is equal to

a) I b) O c) $-I$ d) None of these

Paragraph for Question Nos. 150 to - 150

If A and B are two square matrices of order 3×3 which satisfy $AB = A$ and $BA = B$, then

150. Which of the following is true?

a) If matrix A is singular then matrix B is non-singular
b) If matrix A is non-singular then matrix B is singular
c) If matrix A is singular then matrix B is also singular
d) Cannot say anything

Paragraph for Question Nos. 151 to - 151

Consider an arbitrary 3×3 matrix $A = [a_{ij}]$ a matrix $B = [b_{ij}]$ is formed
Such that b_{ij} is the sum of all the elements except a_{ij} in the i^{th} row of A
Answer the following question.

151. If there exists a matrix X with constant elements such that $AX = B$, then X is

a) Skew-symmetric b) Null matrix c) Diagonal matrix d) None of these

Paragraph for Question Nos. 152 to - 152

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ satisfies $A^n = A^{n-2} + A^2 - I$ for $n \geq 3$. And trace of a square matrix X is equal to the sum of elements in its principal diagonal.
Further consider a matrix $U_{3\times3}$ with its column as U_1, U_2, U_3 such that

$A^{50}\, U_1 = \begin{bmatrix} 1 \\ 25 \\ 25 \end{bmatrix}$, $A^{50}\, U_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $A^{50}\, U_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

Then answer the following questions

152. The value of $|A^{50}|$ equals

a) 0 b) 1 c) -1 d) 25

Paragraph for Question Nos. 153 to - 153

Let A be a square matrix of order 2 or 3 and I be the identity matrix of the same order. Then the matrix $A - \lambda I$ is called characteristic matrix of the matrix$A$, where λ is some complex number. The determinant of the characteristic matrix is called characteristic determinant of the matrix$A$ which will of course be a polynomial of degree 3 in λ. The equation $\det(A - \lambda I) = 0$ is called characteristic equation of the matrix A and its roots (the values of λ) are called characteristic roots or eigenvalues. It is also known that every square matrix has its characteristic equation

153. The eigenvalues of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{bmatrix}$ are

a) 2, 1, 1 b) 2, 3, −2 c) −1, 1, 3 d) None of these

Paragraph for Question Nos. 154 to - 154

Let A be a $m \times n$ matrix. If there exists a matrix L of type $n \times m$ such thatLAI_n, then L is called left inverse of$A$. Similarly, if there exists a matrix$R$ of type $n \times m$ such that$AR = I_m$, then R is called right inverse of A
For example, to find right inverse of matrix

$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix}$, we take $R = \begin{bmatrix} x & y & z \\ u & v & w \end{bmatrix}$

And solve $AR = I_3$, i.e.,

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x & y & z \\ u & v & w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow x - u = 1$ $y - v = 0$ $z - w = 0$

$x + u = 0$ $y + v = 1$ $z + w = 0$

$2x + 3u = 0$ $2y + 3v = 0$ $2z + 3w = 1$

As this system of equations is inconsistent, we say there is no right inverse for matrix A

154. Which of the following matrices is NOT left inverse of

Matrix $\begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix}$?

a) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ b) $\begin{bmatrix} 2 & -7 & 3 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ c) $\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 3 & -1 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$

Paragraph for Question Nos. 155 to - 155

If e^A is defined as $e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots = \frac{1}{2}\begin{bmatrix} f(x) & g(x) \\ g(x) & f(x) \end{bmatrix}$

Where $A = \begin{bmatrix} x & x \\ x & x \end{bmatrix}$ and $0 < x < 1$, then $I$is an identity matrix

155. $\int \frac{g(x)}{f(x)} dx$ is equal to

a) $\log(e^x - e^{-x}) + c$ b) $\log(e^x - e^{-x}) + c$ c) $\log(e^{2x} - 1) + c$ d) None of these

Integer Answer Type

156. If $\begin{bmatrix} a & b \\ c & 1-a \end{bmatrix}$ is an idempotent matrix and $f(x) = x - x^2$ and $bc = 1/4$ then the value of $1/f(a)$ is

157. $A = \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$ and $A^8 + A^6 + A^4 + A^2 + I)V = \begin{bmatrix} 0 \\ 11 \end{bmatrix}$ (where I is the 2×2 identity matrix), then the product of all elements of matrix V is.

158. Let $A = \begin{bmatrix} 3x^2 \\ 1 \\ 6x \end{bmatrix}$, $B = [a\ b\ c]$ and $C + \begin{bmatrix} (x+2)^2 & 5x^2 & 2x \\ 5x^2 & 2x & (x+2)^2 \\ 2x & (x+2)^2 & 5x^2 \end{bmatrix}$ be three given matrices, where a, b, c and $x \in R$, Given that $\text{tr}(AB) = \text{tr}(C)$ $x \in R$, where tr$\cdot$ (A) denotes trace of A. if $f(x) = ax^2 + bx + c$ then the value of $f(1)$ is

159. If A is an idempotent matrix satisfying, $(I - 0.4A)^{-1} = I - \alpha A$ where I is the unit matrix of the same order as that of A then the value of $|9\alpha|$ is equal to

160. The equation $\begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 4 \\ 3 & 4 & k \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ has a solution for (x, y, z) besides (0, 0, 0). Then the value of k is

161. If A is a diagonal matrix of order 3×3 is commutative with every square matrix of order 3×3 under multiplication and tr(A)=12, then the value of $|A|^{1/2}$ is

162. Let A be the set of all 3×3 skew symmetric matrices whose entries are either -1,0 or 1. If there are exactly three 0's, three 1's and three (-1)'s, then the number of such matrices, is

163. Let S be the set which contains all possible values of l, m, n, p, q, r for which $A = \begin{bmatrix} 1^2 - 3 & p & 0 \\ 0 & m^2 - 8 & q \\ r & 0 & n^2 - 15 \end{bmatrix}$ be a non-singular idempotent matrix. Then the sum of all the elements of the set S is

164. Let X be the solution set of the equation $A^x = I$, where $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ and I is the corresponding unit matrix and $x \subseteq N$ then the minimum value of $\sum(\cos^x \theta + \sin^x \theta)$, $\theta \in R$

165. $A = \begin{bmatrix} 1 & tanx \\ -tanx & 1 \end{bmatrix}$ and $f(x)$ is defined as $f(x) = det.(A^T A^{-1})$ then the value of $\underbrace{f(f(f(f \ldots\ldots\ldots f(x))))}_{n\ times}$ is $(n \geq 2)$.

166. If A is a square matrix of order3 that $|A| = 2$ then $|(\text{adj}A^{-1})^{-1}|$ is

167. Let $A = [a_{ij}]_{3\times3}$ be a matrix such that $AA^T = 4I$ and $a_{ij} + 2c_{ij} = 0$
Where c_{ij} is the cofactor of a_{ij} and I is the unit matrix of order3.
$$\begin{vmatrix} a_{11}+4 & a_{12} & a_{13} \\ a_{21} & a_{22}+4 & a_{23} \\ a_{31} & a_{32} & a_{33}+4 \end{vmatrix} + 5\lambda \begin{vmatrix} a_{11}+4 & a_{12} & a_{13} \\ a_{21} & a_{22}+4 & a_{23} \\ a_{31} & a_{32} & a_{33}+4 \end{vmatrix} = 0$$
Then the value of 10λ is

: ANSWER KEY :

1)	a	2)	a	3)	b	4)	a
5)	a	6)	d	7)	a	8)	a
9)	c	10)	b	11)	b	12)	d
13)	b	14)	c	15)	a	16)	c
17)	b	18)	a	19)	a	20)	a
21)	b	22)	b	23)	c	24)	a
25)	c	26)	b	27)	b	28)	a
29)	a	30)	c	31)	d	32)	c
33)	b	34)	c	35)	a	36)	b
37)	c	38)	b	39)	a	40)	b
41)	c	42)	b	43)	b	44)	a
45)	a	46)	b	47)	d	48)	b
49)	c	50)	a	51)	a	52)	a
53)	a	54)	d	55)	c	56)	d
57)	d	58)	c	59)	a	60)	d
61)	b	62)	a	63)	c	64)	b
65)	b	66)	d	67)	b	68)	b
69)	b	70)	a	71)	c	72)	b
73)	a	74)	b	75)	b	76)	b
77)	c	78)	c	79)	b	80)	a
81)	b	82)	c	83)	d	84)	c
85)	b	86)	b	87)	b	1)	
	a, b, c	2)	a, b, c	3)	a, b	4)	
	a,b,d						
5)	a, d	6)	a, b, c	7)	a, b, c	8)	
	a, c, d						
9)	a, c	10)	a, c	11)	a, b, c	12)	
	a, b, c						
13)	a, d	14)	b,d	15)	a, b, c	16)	
	a, b, c						
17)	b, c	18)	a, b, c	19)	a, d	20)	
	a, b, c, d						
21)	b, c	22)	b, c	23)	a,c	24)	
	a,d						
25)	a, b, c	26)	a, b, c	27)	b, c	28)	
	a, b, c						
29)	a, c	30)	a, b, c	31)	a, c	1)	a
	2)	c	3)	c	4)	c	
5)	b	6)	a	7)	b	8)	c
9)	a	10)	b	11)	b	12)	a
13)	d	14)	a	15)	d	16)	a
17)	d	18)	d	19)	a	20)	b
21)	b	22)	c	1)	a	2)	d
	3)	d	4)	c			
5)	b	1)	a	2)	a	3)	c
	4)	b					
5)	c	6)	d	7)	b	8)	c

9)	c	10)	a	1)	4	2)	0
	3)	4	4)	6			
5)	2	6)	4	7)	8	8)	0
9)	2	10)	1	11)	4	12)	4

: HINTS AND SOLUTIONS :

1 **(a)**

As $B = -A^1BA$, we get

$AB = -BA$ or $AB + BA = 0$

Now,

$(A+B)^2 = (A+B)(A+B)$

$= A^2 + BA + BA + B^2$

$= A^2 + 0 + B^2$

$= A^2 + B^2$

2 **(a)**

Given, equations $(x + ay = 0, az + y = 0, ax + z = 0$ has infinite soluations.

$\therefore$ Using Crames's rule, its determinant=0

$\Rightarrow \begin{vmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} 0$

$\Rightarrow 1 + a^3 = 0 \Rightarrow a = -1$

3 **(b)**

$|A| = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix} = 1 + \tan^2 x \neq 0$

So A is invertible. Also,

$\text{adj } A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$

Now,

$A^{-1} = \frac{1}{|A|} \text{adj } A$

$\Rightarrow A^{-1} = \frac{1}{1+\tan^2 x} \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{1+\tan^2 x} & \frac{-\tan x}{1+\tan^2 x} \\ \frac{\tan x}{1+\tan^2 x} & \frac{1}{1+\tan^2 x} \end{bmatrix}$

$\therefore A^T A^{-1}$

$= \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{1+\tan^2 x} & \frac{-\tan x}{1+\tan^2 x} \\ \frac{\tan x}{1+\tan^2 x} & \frac{1}{1+\tan^2 x} \end{bmatrix}$

$= \begin{bmatrix} \frac{1-\tan^2 x}{1+\tan^2 x} & \frac{-2\tan x}{1+\tan^2 x} \\ \frac{2\tan x}{1+\tan^2 x} & \frac{1-\tan^2 x}{1+\tan^2 x} \end{bmatrix}$

$= \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

4 **(a)**

As, $PP^T = \begin{bmatrix} \sqrt{3/2} & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow PP^T = I$ or $P^T = P^{-1}$ ...(i)

As, $Q = PAP^T$

$\therefore P^T Q^{2005} P = P^T [PAP^T)(PAP)^T) \dots 2005 \text{ times}]P$

$= \underbrace{(P^TP)A(P^TP)A(P^TP)\dots(P^TP)A(P^T P)}_{2005 \; times}$

$= IA^{2005} = A^{2005}$

$\therefore A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$...and so on

$A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow P^T Q^{2005} P = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

5 **(a)**

We have, $A^2 = 0, A^k = 0, \forall k \geq 2$

Thus,

$(A + I)^{50} = I + 50A$

$\Rightarrow (A + I)^{50} = I + 50A$

$\Rightarrow a = 1, b = 0, c = 0, d = 1$

6 **(d)**

Let A be a skew-symmetric matrix of order n. By definition,

$A' = -A$

$\Rightarrow |A'| = |-A|$

$\Rightarrow |A| = (-1)^n |A|$

$\Rightarrow |A| = -|A| [\because n \text{ is odd}]$

$\Rightarrow 2|A| = 0$

$\Rightarrow |A| = 0$

Hence, A^{-1} does not exist

7 **(a)**

From given data $|A| = 2^4$

$\Rightarrow |\text{adj (adj } A)| = (2^4)^9 = 2^{36}$

$\Rightarrow \left\{\frac{\det(\text{adj (adj } A)}{7}\right\} = \left\{\frac{2^{36}}{7}\right\} = \left\{\frac{(7+1)^{12}}{7}\right\} = \frac{1}{7}$

8 **(a)**

A is involuntary. Hence,

$A^2 = I \Rightarrow A = A^{-1}$

Also,

$(kA)^{-1} = \frac{1}{k}(A)^{-1}$

$\Rightarrow \left(\frac{1}{2}A\right)^{-1} = 2(A)^{-1} \Rightarrow 2A$

9 **(c)**

$A^2 - A + I = 0$

$\Rightarrow I = A - A^2$

$IA^{-1} = AA^{-1} - A^2A^{-1}$

$\Rightarrow A^{-1} = I - A$

10 **(b)**

We have,

$$I = \frac{1}{2}\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ -08 & 6 & 2c \\ 5 & -3 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & c+1 \\ 0 & 1 & 2(c+1) \\ 4(1-a) & 3(a-1) & 2+ac \end{bmatrix}$$
Comparing the elements of AA^{-1} with those of I, we have
$c + 1 = 0 \Rightarrow c = -1$
$\Rightarrow c = -1 \; and \; a - I = 0 \Rightarrow a = 1$

11 **(b)**
Let, $A = \begin{bmatrix} 5 & 0 \\ -a & 5 \end{bmatrix}$
$\Rightarrow \text{adj}(A) = \begin{bmatrix} 5 & 0 \\ a & 5 \end{bmatrix}$
$\Rightarrow A^{-1} = \frac{1}{|A|}\begin{bmatrix} 5 & 0 \\ a & 5 \end{bmatrix} = \frac{1}{25}\begin{bmatrix} 5 & 0 \\ a & 5 \end{bmatrix}$
$\Rightarrow A^{-2} = (A^{-1})^2 = \frac{1}{25}\begin{bmatrix} 5 & 0 \\ a & 5 \end{bmatrix}\frac{1}{25}\begin{bmatrix} 5 & 0 \\ a & 5 \end{bmatrix}$
$= \frac{1}{625}\begin{bmatrix} 25 & 0 \\ 10a & 25 \end{bmatrix}$
$= \begin{bmatrix} \frac{1}{25} & 0 \\ \frac{2a}{125} & \frac{1}{25} \end{bmatrix}$
$\begin{bmatrix} \frac{1}{25} & 0 \\ x & \frac{1}{25} \end{bmatrix}\begin{bmatrix} \frac{1}{25} & 0 \\ \frac{2a}{125} & \frac{1}{25} \end{bmatrix}$
$\Rightarrow x = 2a/125$

12 **(d)**
(i) is false

If $A = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$, then
$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

(ii) is true as the product AB is an identity matrix, if and only if$B$ is inverse of the matrix$A$

(iii) is false since matrix multiplication in not commutative

13 **(b)**
We know that $|\text{adj}(\text{adj}A))| = |A|^{(n-1)^2}$
$\Rightarrow |\text{adj}(\text{adj}(\text{adj}A))| = |\text{adj } A|^{(n-1)^2}$
$= |A|^{(n-1)^3}$
$= 2^8 = 256$

14 **(c)**
$\begin{bmatrix} \bar{z}_1 & -z_2 \\ \bar{z}_1 & z_1 \end{bmatrix}^{-1}\begin{bmatrix} z_1 & z_2 \\ -\bar{z}_2 & z_1 \end{bmatrix}^{-1}$
$= \left(\begin{bmatrix} z_1 & z_2 \\ -\bar{z}_2 & \bar{z}_1 \end{bmatrix}\begin{bmatrix} \bar{z}_1 & -z_2 \\ \bar{z}_2 & z_1 \end{bmatrix}\right)^{-1}$
$= \begin{bmatrix} z_1\bar{z}_1 & 0 \\ 0 & z_2\bar{z}_2 + z_1\bar{z}_1 \end{bmatrix}^{-1}$
$= \begin{bmatrix} |z_1|^2+|z_2|^2 & 0 \\ 0 & |z_1|^2+|z_2|^2 \end{bmatrix}^{-1}$
$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$

15 **(a)**
We know that in a square matrix of order n,
$|\text{adj } A| = |A|^{n-1}$
$\Rightarrow |\text{adj}(\text{adj } A)| = |\text{adj } A|^{n-1} = |A|^{(n-1)^2}$
$\Rightarrow n^2 - 2n - 8 = 0$
$\Rightarrow n = 4$ as $n = -2$ is not possible

16 **(c)**
We have,
$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2+bc & ab+db \\ ac+cd & bc+d^2 \end{bmatrix}$
As A satisfies $x^2 + k = 0$, therefore
$A^2 + kI = O$
$\Rightarrow \begin{bmatrix} a^2+b+k & (a+d)b \\ (a+d)c & bc+d^2+k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
$\Rightarrow a^2 + bc + k = 0, bc + d^2 + k = 0$
and $(a + d)b = (a + d)c = 0$
As $bc \neq 0, b \neq 0, c \neq 0$, so
$a + d = 0$
$\Rightarrow a = -d$
Also,
$k = -(a^2 + bc)$
$= -(d^2 + bc)$
$= -((-ad) + bc)$
$= |A|$

17 **(b)**
$(I - A)f(A) = I + A$
$\Rightarrow f(A) = (I + A)(I - A)^{-1}$
$= (I + A)(I + A + A^2)$
$= I + A + A^2 + A + A^2 + A^3$
$= I + 2A + 2A^2$

18 **(a)**
$A = \text{diag}\,(d_1, d_2, d_3, \cdots, d_n)$
$\Rightarrow |A| = (d_1 \times d_2 \times d_3 \times d_4 \times \cdots \times d_n)$
Now,
Cofactor of $d_1$is$d_2 d_3 \cdots d_n$
Cofactor of d_2 is $d_1 \times d_3 \times d_4 \cdots d_n$
Cofactor of d_3 is $d_1 \times d_2 \times d_4 \cdots d_n$
$\vdots$
Cofactor of d_n is $d_1 \times d_2 \times d_3 \cdots d_{n-1}$
$\Rightarrow A^{-1} = \frac{1}{|A|}\text{adj}(A) = \text{diag}(d_1^{-1}, d_2^{-1}, d_3^{-1}, \cdots, d_n^{-1})$
Hence, A^{-1} is also a diagonal matrix.

19 **(a)**
$(A - \lambda I)(B - \lambda I) = (B - \lambda I)(A - \lambda I)$

$\Rightarrow AB - \lambda(A+B)I + \lambda^2 I^2$

$= BA - \lambda(B+A)I + \lambda^2 I^2$

$\Rightarrow AB = BA$

20 **(a)**

$BC = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix} \Rightarrow B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \text{tr}(A) + \text{tr}\left(\frac{ABC}{2}\right) + \text{tr}\left(\frac{A(BC)^2}{4}\right) + \text{tr}\left(\frac{A(BC)^3}{8}\right) + \cdots + \infty$

$= \text{tr}(A) + \text{tr}\left(\frac{A}{2}\right) + \text{tr}\left(\frac{A}{2^2}\right) + \cdots$

$= \text{tr}(A) + \frac{1}{2}\text{tr}(A) + \frac{1}{2^2}\text{tr}(A) \ldots$

$= \frac{\text{tr}(A)}{1-(1/2)}$

$= 2\text{tr}(A) = 2(2+1) = 6$

21 **(b)**

We have,

$A = iB$

$\Rightarrow A^2 = (iB)^2 = i^2B^2 = -B^2 = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = -2B$

$\Rightarrow A^4 = (-2B)^2 = 4B^2 = 4(2B) = 8B$

$\Rightarrow (A^4)^2 = (8B)^2$

$\Rightarrow A^8 = 64B^2 = 128B$

22 **(b)**

We have, $AB = A(\text{adj}A) = |A|I_n$

$\therefore AB + KI_n = |A|I_n + KI_n$

$\Rightarrow AB + KI_n = (|A| + k)I_n$

$\Rightarrow |AB + KI_n| = |(|A| + k)I_n| (\because |\alpha I_n = \alpha^n|)$

$= (|A| + K)^n$

23 **(c)**

Given

$A^3 = 0$

Now,

$(I - A)(I + A + A^2)$

$= I^2 + IA + IA^2 - AI - A^2 - A^3$

$= I - A^3$

$= I$

$\Rightarrow (I - A)^{-1} = I + A + A^2$

24 **(a)**

We have,

$|A| = (a+ib)(a-ib) — c + id(c + id)$

$a^2 + b^2 + c^2 + d^2 = 1$

And $\text{adj}(A) = \begin{bmatrix} a-ib & -c-id \\ -c-id & a+ib \end{bmatrix}$

Then $A^{-1} = \begin{bmatrix} a-ib & -c-id \\ -c+id & a-ib \end{bmatrix}$

25 **(c)**

$A \text{ adj } A|A|I$

$|A| = xyz - 8x - 3(z-8) + 2(2-2y)$

$|A| = xyz - (8x + 3z + 4y) + 28$

$= 60 - 20 + 28$

$= 68$

$\Rightarrow A(\text{adj}A) = 68\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 68 & 0 & 0 \\ 0 & 68 & 0 \\ 0 & 0 & 68 \end{bmatrix}$

26 **(b)**

Since the product matrix is 3×3 matrix and the pre-multi-plier of A is a 3×2 matrix, therefore A is 2×3 matrix. Let,

$A = \begin{bmatrix} l & m & n \\ x & y & z \end{bmatrix}$. Then the given equation becomes

$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} l & m & n \\ x & y & z \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2l - x & 2m - y & 2n - z \\ l & m & x \\ -3l + 4x & -3m + 4y & -3n + 4z \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$

$\Rightarrow 2l - x = -1, 2m - y = -8, 2n - z = -10, l = 1, m = -2, n = -5$

$\Rightarrow x = 3, y = 4, z = 0, l = 1, m = -2, n = -5$

$\Rightarrow A = \begin{bmatrix} l & m & n \\ x & y & z \end{bmatrix} = \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$

27 **(b)**

We have,

$F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \text{adj}(F(\alpha))$

$= \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Also,

$\det(F(\alpha)) = 1$

$\Rightarrow [F(\alpha)]^{-1} = \begin{bmatrix} \cos(-\alpha) & -\sin(-\alpha) & 0 \\ \sin(-\alpha) & \cos(-\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= F(-\alpha)$

28 **(a)**

We have,

$A(\alpha,\beta)^{-1} = \frac{1}{e^\beta}\begin{bmatrix} e^\beta\cos\alpha & -e^\beta\sin\alpha & 0 \\ e^\beta\sin\alpha & e^\beta\cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= A(-\alpha, -\beta)$

29 **(a)**

$\begin{bmatrix}2 & 1\\3 & 2\end{bmatrix} A \begin{bmatrix}-3 & 2\\5 & -3\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

$A = \begin{bmatrix}2 & 1\\3 & 2\end{bmatrix}^{-1} \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \begin{bmatrix}-3 & 2\\5 & -3\end{bmatrix}^{-1}$

$\Rightarrow A = \begin{bmatrix}2 & -1\\-3 & 2\end{bmatrix} \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \begin{bmatrix}-3 & -2\\-5 & -3\end{bmatrix}$

$\Rightarrow A = \begin{bmatrix}2 & -1\\-3 & 2\end{bmatrix} \begin{bmatrix}3 & 2\\5 & 3\end{bmatrix} = \begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$

30 **(c)**

$\text{adj}(Q^{-1}BP^{-1}) = \text{adj}(P^{-1})\text{adj}(B)\text{adj}(Q^{-1})$

$= \frac{P}{|P|} A \frac{Q}{|Q|}$

$= PAQ$

31 **(d)**

$A\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}-1\\2\end{bmatrix}$ (1)

$A^2\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$ (2)

Let A be given by $A = \begin{bmatrix}a & b\\c & d\end{bmatrix}$. The first equation gives

$a - b = -1$ (3)

$c - d = 2$(4)

For second equation gives

$A^2\begin{bmatrix}1\\-1\end{bmatrix} = A\left(A\begin{bmatrix}1\\-1\end{bmatrix}\right) = A\left(\begin{bmatrix}-1\\2\end{bmatrix}\right) = \begin{bmatrix}1\\0\end{bmatrix}$

This gives

$-a + 2b = 1$ (5)

$-c + 2d = 0$ (6)

Eqs. (3)+(5) $\Rightarrow b = 0$ and $a = -1$

Eqs. (4)+(6)$\Rightarrow d = 2$ and $c = 4$

So the sum $a + b + c + d = 5$.

32 **(c)**

Given $B^r = I \Rightarrow B^rB^{-1} = IB^{-1} \Rightarrow B^{r-1} = B^{-1}$

$\Rightarrow A^{-1}B^{r-1}A - A^{-1}B^{-1}A = A^{-1}B^{-1}A - A^{-1}B^{-1}A$
$= 0$

33 **(b)**

$(-A)^{-1} = \frac{\text{adj}(-A)}{|-A|} = \frac{(-1)^{n-1}\text{adj}(A)}{(-1)^n|A|} = \frac{\text{adj}(A)}{-|A|}$
$= -A^{-1}$

34 **(c)**

$[1 \quad x \quad y]\begin{bmatrix}1 & 3 & 1\\0 & 2 & -1\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1\\x\\y\end{bmatrix} = 10$

$\Rightarrow [1 \quad 3+2x \quad 1-x+y]\begin{bmatrix}1\\x\\y\end{bmatrix} = [0]$

$\Rightarrow [1 + 3x + 2x^2 + y - xy + y^2] = [0]$

$\Rightarrow 2x^2 + y^2 + y + 3x - xy + 1 = 0$

If $y = 0, 2x^2 + 3x + 1 = 0$

$\Rightarrow (2x - 1)(x + 1) = 0$

$\Rightarrow x = -1/2, -1$(rational roots)

If $y = -1, 2x^2 + 4x + 1 = 0$

$\Rightarrow x = \frac{-4 \pm \sqrt{12}}{4} = \frac{-2 \pm \sqrt{3}}{2}$ (irrational roots)

35 **(a)**

We know that for any non-singular matrix A,

$A^{-1} = \frac{1}{|A|}\text{adj}(A)$

Now put $A = P^{-1}$. Then we have

$(P^{-1})^{-1} = \frac{1}{|P^{-1}|}\text{adj}(P^{-1})$

$\Rightarrow P = |P|\text{adj}(P^{-1})$

$\Rightarrow \text{adj}(P^{-1}) = \frac{P}{|P|}$

36 **(b)**

If possible assume that A is non-singular, then A^{-1} exists.

Thus,

$AB = 0 \Rightarrow A^{-1}(AB) = (A^{-1}A)B = 0$

$\Rightarrow IB = 0$ or $B = 0 \times$(a contradiction)

Hence, both A and B must be singular.

37 **(c)**

Since $I = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ =and

given $A = \begin{bmatrix}0 & \tan\alpha/2\\-\tan\alpha/2 & 0\end{bmatrix}$

$\therefore I - A = \begin{bmatrix}1 & -\tan\alpha/2\\\tan\alpha/2 & 1\end{bmatrix}$(1)

Now, $(I - A)\begin{bmatrix}\cos\alpha & -\sin\alpha\\\sin\alpha & \cos\alpha\end{bmatrix}$

$= \begin{bmatrix}1 & \tan\alpha/2\\-\tan\alpha/2 & 1\end{bmatrix}\begin{bmatrix}\cos\alpha & -\sin\alpha\\\sin\alpha & \cos\alpha\end{bmatrix}$

$= \begin{bmatrix}1 & \tan\alpha/2\\-\tan\alpha/2 & 1\end{bmatrix}$

$\begin{bmatrix}\frac{1-\tan^2\alpha/2}{1+\tan^2\alpha/2} & -\frac{2\tan\alpha/2}{1+\tan^2\alpha/2}\\\frac{2\tan\alpha/2}{1+\tan^2\alpha/2} & \frac{1-\tan^2\alpha/2}{1+\tan^2\alpha/2}\end{bmatrix}$

$= \begin{bmatrix}\frac{1-\tan^2\alpha/2}{1+\tan^2\alpha/2} + \frac{2\tan^2\alpha/2}{1+\tan^2\alpha/2} & -\frac{2\tan^2\alpha/2}{1+\tan^2\alpha/2} + \frac{\tan\alpha/2(1-\tan^2\alpha/2)}{1+\tan^2\alpha/2}\\\frac{-\tan\alpha/2(1-\tan^2\alpha/2)}{1+\tan^2\alpha/2} + \frac{2\tan\alpha/2}{1+\tan^2\alpha/2} & \frac{2\tan^2\alpha/2}{1+\tan^2\alpha/2} + \frac{1-\tan^2\alpha/2}{1+\tan^2\alpha/2}\end{bmatrix}$

$= \begin{bmatrix}\frac{(1+\tan^2\alpha/2)}{(1+\tan^2\alpha/2)} & -\frac{\tan\alpha/2\,(1+\tan^2\alpha/2)}{(1+\tan^2\alpha/2)}\\\frac{\tan\alpha/2\,(1+\tan^2\alpha/2)}{(1+\tan^2\alpha/2)} & \frac{(1+\tan^2\alpha/2)}{(1+\tan^2\alpha/2)}\end{bmatrix}$

$= \begin{bmatrix}1 & -\tan\alpha/2\\\tan\alpha/2 & 1\end{bmatrix}$

$= I - A$ [using (1)]

38 **(b)**

We have,

$(A-2I)(A-4I)=0$

$\Rightarrow A^2-2A-4A+8I=0$

$\Rightarrow A^2-6A+8I=0$

$\Rightarrow A^{-1}(A^2-6A+8I)=A^{-1}0$

$\Rightarrow A-6I+8A^{-1}=0$

$\Rightarrow A+8A^{-1}=6I$

$\Rightarrow \frac{1}{6}A+\frac{4}{3}A^{-1}=1$

39 **(a)**

Matrix $\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ is orthogonal if

$$\sum a_i^2=\sum b_i^2=\sum c_i^2=1;\ \sum a_ib_i=\sum b_ic_i=\sum c_ia_i=0$$

40 **(b)**

$|A^{2005}-6A^{2004}|=|A|^{2004}|A-6I|$

$2^{2004}\begin{vmatrix} 0 & 11 \\ 2 & -2 \end{vmatrix}=(-22)2^{2004}=(-11)(2)^{2005}$

41 **(c)**

Given$A^2=A$. Now,

$(I+A)^3-7A=I^3+3I^2A+3IA^2+A^3-7A$

$=I+3A+3A+A-7A$

$=I+O$

$=I$

42 **(b)**

$AB^n=ABBBBB\cdots B$

$=(AB)BBB\cdots B$

$=B(AB)BBB\cdots B$

$=BB(AB)BB\cdots B$

$\vdots$

$=B^nA$

$(AB)^n=(AB)(AB)(AB)\cdots(AB)$

$=A(BA)(BA)(BA)\cdots(BA)B$

$=A(AB)(AB)(AB)\cdots(AB)B$

$=A^2(BA)(BA)(BA)\cdots(BA)B^2$

$=A^3(BA)(BA)(BA)\cdots(BA)B^3$

$\vdots$

$=A^nB^n$

43 **(b)**

Since, given system of equations has no solution, $\Delta=0$ and any one amongst$\Delta x,\Delta y,\Delta z$ is non-zero.

Where $\Delta=\begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix}=0$

And $\Delta z=\begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & -4 \\ 1 & 1 & \lambda \end{vmatrix}=6\neq 0$

$\Rightarrow \lambda=1$

44 **(a)**

Let, $X=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\Rightarrow X^2=\begin{pmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{pmatrix}$

$\Rightarrow a^2+bc=1$ and $ab+bd=1\Rightarrow b(a+d)=1$

$ac+cd=2\Rightarrow c(a+d)=2\Rightarrow 2c=c$

Also,

$bc+d^2=3\Rightarrow d^2-a^2=2$

$\Rightarrow (d-a)(a+d)=2\Rightarrow d-a=2b$ (using $bc=1-a^2$)

$a+d=1/b$

$\Rightarrow 2d=2b+1/b,\ \ 2a=1/b-2b$

$d=b+1/b,\quad a=1/b-2b$

$c=2b$

$\Rightarrow \left(b^2+\frac{1}{4b^2}+1\right)+2b^2=3$

$\Rightarrow 3b^2+\frac{1}{4b^2}=2$

$\Rightarrow 3x+\frac{1}{4x}=2$

$\Rightarrow b=\pm\frac{1}{\sqrt{6}}$ or $b=\pm\frac{1}{\sqrt{2}}$

Therefore, matrices are

$\begin{pmatrix} 0 & 1/\sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{pmatrix}\begin{pmatrix} 0 & -1/\sqrt{2} \\ -\sqrt{2} & -\sqrt{2} \end{pmatrix}\begin{pmatrix} 2/\sqrt{6} & -1/\sqrt{6} \\ 2/\sqrt{6} & 4/\sqrt{6} \end{pmatrix}$

45 **(a)**

Given A is skew-symmetric Hence,

$A^T=-A$

$\Rightarrow A^n=(-A^T)^n=-(A^n)^T$ (given n is odd)

Hence, A^n is skew-symmetric

46 **(b)**

Given $A,B,A+I,A+B$ are idempotent. Hence,

$A^2=A, B^2=B, (A+I)^2=A+I$ and$(A+B)^2=A+B$

$\Rightarrow A^2+B^2+AB+BA=A+B$

$\Rightarrow A+B+AB+BA=A+B$

$\Rightarrow AB+BA=O$

47 **(d)**

$A^2=\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}=\begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix}$

$\therefore\ A^2=B\Rightarrow \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

$\Rightarrow\ \alpha^2=1$ and $\alpha+1=5$

Which is not possible at the same time.

$\therefore$ No real values of α exists.

48 **(b)**

$A=\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$

$\Rightarrow A^2=\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}=\begin{pmatrix} a^2 & 0 \\ 0 & b^2 \end{pmatrix}$

$\Rightarrow A^3=\begin{pmatrix} a^2 & 0 \\ 0 & b^2 \end{pmatrix}\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}=\begin{pmatrix} a^3 & 0 \\ 0 & b^3 \end{pmatrix}$

$\Rightarrow A^n=\begin{pmatrix} a^n & 0 \\ 0 & b^n \end{pmatrix}$

$\Rightarrow (A^n)^{-1} = \frac{1}{a^n b^n}\begin{pmatrix} a^n & 0 \\ 0 & b^n \end{pmatrix} = \begin{pmatrix} a^{-n} & 0 \\ 0 & b^{-n} \end{pmatrix}$

$\Rightarrow \lim_{n\to\infty}(A^n)^{-1} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ as $a > 1$ and $b > 1$

49 **(c)**

Given that

$X = AB + BA \Rightarrow X = X^T$

And

$Y = AB - BA$

$\Rightarrow Y = -Y^T$

Now,$(XY)^T = Y^T X^T = -YX$.

50 **(a)**

$|B| = \begin{vmatrix} q & -b & y \\ -p & a & -x \\ r & -c & z \end{vmatrix}$(Multiplying R_2 by -1)

$= -\begin{vmatrix} q & -b & y \\ p & -a & x \\ r & -c & z \end{vmatrix}$(Multiplying C_2 by -1)

$= \begin{vmatrix} q & b & y \\ p & a & x \\ r & c & z \end{vmatrix}$(Changing R_1 with R_2)

$= -\begin{vmatrix} q & b & y \\ p & a & x \\ r & c & z \end{vmatrix}$

$= -\begin{vmatrix} q & b & y \\ p & a & x \\ r & c & z \end{vmatrix}$

Hence $|A| = -|B|$, obviously when $|A| \neq 0, |B| \neq 0$. Also, $|\text{adj } B| = |B|^2$

$= (-|A|)^2 = |A|^2$

51 **(a)**

For involuntary matrix,

$A^2 = I$

$\Rightarrow\ |A^2| = |I| \Rightarrow |A|^2 \Rightarrow |A| = \pm 1$

For idempotent matrix,

$A^2 = A$

$\Rightarrow |A^2| = |A| \Rightarrow |A|^2 \Rightarrow |A| \Rightarrow |A| = 0$ or 1

For orthogonal matrix,

$AA^T = I$

$\Rightarrow\ |AA^T| = |I| \Rightarrow |A||A^T| = 1 \Rightarrow |A|^2 = 1 \Rightarrow |A| = \pm 1$.

Thus if matrix A is idempotent it may not be invertible.

52 **(a)**

Since, $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is linear equation in three variables and that could have only unique, no solution or infinitely many solution.

$\therefore$ It is not possible to have two solutions.

Hence, number of matrices A is zero.

53 **(a)**

$|A| = 1(0 - 10) - 2(2 - 6) + 3(4 - 0)$

$= -10 + 8 + 12 = 10$

$\Rightarrow |A| \neq 0$

$\Rightarrow$Unique solution

54 **(d)**

Let, $A = \begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$

Then the matrix equation is $AX = B$.

$\therefore |A| = \begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix} = -2 + 12 \neq 0$

So A is and invertible matrix. Also,

$\text{adj } A = \begin{bmatrix} -2 & -3 \\ 4 & 1 \end{bmatrix}^T = \begin{bmatrix} -2 & 4 \\ -3 & 1 \end{bmatrix}$

So,

$A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{10}\begin{bmatrix} -2 & 4 \\ -3 & 1 \end{bmatrix}$

Now,

$AX = B$

$\Rightarrow A^{-1}(AX) = a^{-1}B$

$\Rightarrow (A^{-1}A)X = A^{-1}B$

$\Rightarrow IX = A^{-1}B$

$\Rightarrow X = A^1 B$

$\Rightarrow X = \frac{1}{10}\begin{bmatrix} -2 & 4 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 11/2 & 2 \end{bmatrix}$

55 **(c)**

$A + 2B = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix}$ and $2A - B = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}$

$\Rightarrow \text{tr}(A) + 2\text{tr}(B) = -1$

and $2\text{tr}(A) - \text{tr}(B) = 3$

Let $\text{tr}(A) = x$ and $\text{tr}(B) = y$. Then,

$x + 2y = -1$ and $2x - y = 3$

Solving, $x = 1$ and $y = -1$. Hence,

$\text{tr}(A) - \text{tr}(B) = x - y = 2$

56 **(d)**

$\text{tr}(A) = \sum_{i=j} a_{ij}$

$= (a_{11} + a_{22} + a_{33} + \dots + a_{10\infty 10})$

$= (w^2 + w^4 + w^6 + \dots + w^{20})$

$= w^2(1 + w^2 + w^4 + \dots + w^{18})$

$= w^2[(1 + w + w^2) + \dots + (1 + w + w^2) + 1]$

$= w^2 \times 1$

$\Rightarrow \text{tr}(A) = w^2$

57 **(d)**

$A = \text{diag}\ (d_1, d_2 \dots\dots d_n)$

Given, $A^3 = A$

$\Rightarrow \text{diag}\ (d_1^3, d_2^3, \dots, d_n^3) = \text{diag}(d_1, d_2 \dots, d_n)$

$\Rightarrow d_1^3 = d_1, d_2^3 = d_2 \dots, d_n^3 = d_n$

Hence, all $d_1, d_2, d_3 \dots, d_n$ have three possible values$\pm 1, 0$. Each diagonal element can be selected in three ways. Hence, the number Of different matrices is 3^n

58 **(c)**

$AB = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$
$\begin{bmatrix} \cos^2\emptyset & \cos\emptyset\sin\emptyset \\ \cos\emptyset\sin\emptyset & \sin^2\emptyset \end{bmatrix}$

$= \begin{bmatrix} \cos^2\theta\cos^2\emptyset + \cos\theta\cos\emptyset\sin\theta\sin\emptyset & \cos^2\theta\cos\emptyset\sin\emptyset + \cos\theta\sin\theta\sin^2\emptyset \\ \cos\theta\sin\theta\cos^2\emptyset + \cos\theta\sin\theta\cos\emptyset\sin\emptyset\sin\emptyset & \cos\theta\cos\emptyset\sin\theta + \sin^2\theta\sin^2\emptyset \end{bmatrix}$

$= \begin{bmatrix} \cos\theta\cos\emptyset\,(\cos(\theta-\emptyset)) & \cos\theta\sin\emptyset\,(\cos(\theta-\emptyset)) \\ \sin\theta\cos\emptyset\,(\cos(\theta-\emptyset)) & \sin\theta\sin\emptyset\,(\cos(\theta-\emptyset)) \end{bmatrix}$

$= (\cos(\theta-\emptyset)) \begin{bmatrix} \cos\theta\cos\emptyset & \cos\theta\sin\emptyset \\ \sin\theta\cos\emptyset & \sin\theta\sin\emptyset \end{bmatrix}$

Now, $AB = O \Rightarrow \cos(\theta-\emptyset) = 0 \Rightarrow \theta - \emptyset = (2n+1)\,\pi/2, n \in Z$.

59 **(a)**
$A \times A^T = I$
$\Rightarrow |A \times A^T| = |I|$
$\Rightarrow |A|^2 = 1$
$\Rightarrow |A| = \pm 1$
$\Rightarrow A^{-1}$ exists
$\Rightarrow A^{-1} \times A \times A^T = A^{-1} \times I$
$\Rightarrow A^{-1} = A^T$

60 **(d)**
If A is n^{th} root of I_2, then $A^n = I_2$. Now,
$A^2 = \begin{bmatrix} a & b \\ 0 & a \end{bmatrix}\begin{bmatrix} a & b \\ 0 & a \end{bmatrix} = \begin{bmatrix} a^2 & 2ab \\ 0 & a^2 \end{bmatrix}$
$A^3 = A^2A = \begin{bmatrix} a^2 & 2ab \\ 0 & a^2 \end{bmatrix}\begin{bmatrix} a & b \\ 0 & a \end{bmatrix} = \begin{bmatrix} a^3 & 3a^2b \\ 0 & a^3 \end{bmatrix}$
Thus,
$A^n = \begin{bmatrix} a^n & na^{n-1}b \\ 0 & a^n \end{bmatrix}$
Now,
$A^n = I \Rightarrow \begin{bmatrix} a^n & na^{n-1}b \\ 0 & a^n \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
$\Rightarrow a^n = 1, b = 0$

61 **(b)**
Since A is orthogonal, hence
$AA^T = I$
$\Rightarrow |AA^T| = 1$
$\Rightarrow |A^2| = 1$
$\Rightarrow |A| == \pm 1$
Now, $|\text{adj}(\text{adj } A)| = |A|^{(n-1)^2}$

62 **(a)**
$A(x)A(y) = (1-x)^{-1}(1 - y)^{-1}\begin{bmatrix} 1 & -x \\ -x & 1 \end{bmatrix}\begin{bmatrix} 1 & -y \\ -y & 1 \end{bmatrix}$
$= \left(1 + xy - (x+y)\right)^{-1}\begin{bmatrix} 1+xy & -(x+y) \\ -(x+y) & 1+xy \end{bmatrix}$
$= \left(1 - \frac{(x+y)}{1+xy}\right)^{-1}\begin{bmatrix} 1 & -\frac{x+y}{1+xy} \\ -\frac{x+y}{1+xy} & 1 \end{bmatrix} = A(z)$

63 **(c)**
$(A^{-1}BA)^2 = (A^{-1}BA)(A^{-1}BA)$
$= A^{-1}B(AA^{-1})BA$
$= A^{-1}BIBA = A^{-1}B^2A$
$(A^{-1}BA)^3 = (A^{-1}B^2A)(A^{-1}BA)$
$= A^{-1}B^2(AA^{-1})BA$
$= A^{-1}B^3A$ and so on
$\therefore (A^{-1}BA)^n = A^{-1}B^nA$

64 **(b)**
$(kI_n)\text{adj}(kI_n) = |kI_n|I_n$[using $A(\text{adj}A) = |A|I$]
$\text{adj}(kI_n) = k^{n-1}I_n$
$|\text{adj}(kI_n)| = k^{n(n-1)}$

65 **(b)**
We have,
$[F(x)G(y)]^{-1} = [G(y)]^{-1}[F(x)]^{-1}$
$= G(-y)F(-x)$

66 **(d)**
$\det(A - I) = \det(A - A^2)$
$= \det A(I - A)$
$= \det A \times \det(I - A)$
$= -\det A \times \det(I - A)$
Now,
$A^2 = I$
$\Rightarrow \det(A^2) = \det(I)$
$\Rightarrow (\det A)^2 = 1$
$\Rightarrow \det(A) = \pm 1$
Thus, $\det(A)$ can be 1 or -1, which we cannot say anything about $\det(A - I)$.

67 **(b)**
Given,
$B = A^{-1}A^T$
$\Rightarrow B^T = (A^{-1}A^T) = A \times (A^{-1})^T$
$\Rightarrow B \times B^T = A^{-1}A^T \times A \times (A^{-1})^T$
$= A^{-1} \times (A^T \times A)(A^{-1})^T$
$= A^{-1}(A \times A^T)(A^{-1})^T$
$= \left(A^{-1}A\ \ \right) \times (A^{-1}A)^T = I \times I^T = I$

68 **(b)**
$A^2 = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$

$$= \begin{bmatrix} 25-24+0 & -40-40+0 & 0+0+0 \\ -15+15+0 & -24+25+0 & 0+0+0 \\ -5+6+0 & -8+10-2 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Hence, the matrix A is involutory.

69 **(b)**

Since $AB = B$ and $BA = A$, so

$BAB = B^2$

$\Rightarrow (BA)B = B^2$

$\Rightarrow AB = B^2$

$\Rightarrow B = B^2$

Hence, B is idempotent and similarly$A$.

$(A-B)^2 = A^2 - AB - BA + B^2 = A - B - A + B$
$= 0$

Therefore, $A - B$ is nilpotent

70 **(a)**

$A \text{ adj } A = |A|I$

$\Rightarrow |A \text{ adj } A| = |A|^n$ [If A is of order $n \times n$]

$\Rightarrow |A||\text{adj } A| = |A|^n$

$\Rightarrow |\text{adj } A| = |A|^{n-1}$

Now, A is singular,

$\therefore |A| = 0$

$\Rightarrow |adj\ A| = 0$

Hence adj A is singular.

71 **(c)**

$A^2 = I$

$\Rightarrow A^2 - I = O$

$\Rightarrow (A+I)(A-I) = 0$

Therefore, either $|A+I| = 0$ or $|A-I| = 0$. If $|A-I| \neq 0$, then $(A+I)(A-I) = 0 \Rightarrow A - I = 0$ which is not so.

$\therefore$ $|A-I| = 0$ and $A - I \neq 0$

72 **(b)**

$B = A_1 + 3A_3^3 + \cdots + (2n-1)(A_{2n-1})^{2n-1}$

$B^T = [A_1 + 3A_3^3 + \cdots + (2n-1)(A_{2n-1})^{2n-1}]$

$= -B$

Hence, $B$is skew-symmetric

73 **(a)**

$AB = C$

$\Rightarrow |AB| = |C|$

$\Rightarrow |A||B| = |C|$

$\Rightarrow |B| = \dfrac{|C|}{|A|}$

74 **(b)**

$P^TP = I$

$Q = PAP^T$

$\therefore x = P^TQ^{1000}P = P^T(PAP^T)^{1000}P$

$= P^TPAP^T(PAP^T)^{999}P$

$= IAP^T \cdot PAP^T(PAP^T)^{998}P$

$= AIP^T(PAP^T)^{997}P$

$= A^2P^TPAP^T(PAP^T)^{997}P$

$= A^3P^T((PAP^T)^{997}P$

$\vdots$

$= A^{1000} = I$ ($\because A$ is involuntary)

Hence, $x^{-1} = I$

75 **(b)**

Let, $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & 1 \end{bmatrix}$

$\therefore A^T = \dfrac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1-i \\ 1-i & 1 \end{bmatrix}$

$\Rightarrow (\bar{A}^T) = \dfrac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$

$\therefore A(A^{-T}) = \dfrac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix} \times \dfrac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$

$= \dfrac{1}{3}\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

76 **(b)**

Let $A = [a_{ij}]$. Since A is skew-symmetric, therefore

$a_{ij} = 0$ and $a_{ij} = -a_{ij} (i \neq j)$

A is symmetric as well, so $a_{ij} = a_{ji}$ for all i and j

$\therefore a_{ij} = 0$ for all $i \neq j$

Hence, $a_{ij} = 0$ for all i and j, i.e., A is null matrix.

77 **(c)**

$(A^{-1}BA)^2 = (A^{-1}BA)(A^{-1}BA)$

$= A^{-1}B(AA^{-1})BA$

$= A^{-1}BIBA = A^{-1}B^2A$

$\Rightarrow (A^{-1}BA)^3 = (A^{-1}B^2A)(A^{-1}BA)$

$= A^{-1}B^2(AA^{-1})BA$

$= A^{-1}B^2IBA$

$= A^{-1}B^3A$ and so on

$\Rightarrow (A^{-1}BA)^n = A^{-1}B^nA$

78 **(c)**

$f(x) = \dfrac{1+x}{1-x}$

$\Rightarrow (1-x)f(x) = 1 + x$

$\Rightarrow (I-A)f(A) = (I+A)$

$\Rightarrow f(A) = (I-A)^{-1}(I+A)$

$= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\right)^{-1}\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\right)$

$\Rightarrow f(A) = \left(\begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}^{-1}\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\right)$

$= \dfrac{\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}}{-4}$

$= \dfrac{\begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}}{-4}$

$= \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$

79 **(b)**

We have,

$$\begin{bmatrix}\alpha & \beta \\ \gamma & -\alpha\end{bmatrix}\begin{bmatrix}\alpha & \beta \\ \gamma & -\alpha\end{bmatrix}=\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$$
$$\Rightarrow \begin{bmatrix}\alpha^2+\beta\gamma & 0 \\ 0 & \alpha^2+\beta\gamma\end{bmatrix}=\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$$
$\Rightarrow \alpha^2+\beta\gamma-1=0$

80 **(a)**
$AB=A+B$
$\Rightarrow B=AB-A=A(B-I)$
$\Rightarrow \det(B)=\det(A)\det(B-I)=0$
$\Rightarrow \det(B)=0$

81 **(b)**
$|A(\alpha,\beta)|=\cos^2\alpha e^{\beta}+\sin^2\alpha e^{\beta}=e^{\beta}$
Now,
$$A(\alpha,\beta)^{-1}=\frac{1}{e^{\beta}}\text{adj}\,(A(\alpha,\beta))$$
$$=\frac{1}{e^{\beta}}\begin{bmatrix}e^{\beta}\cos\alpha & -\sin\alpha\, e^{\beta} & 0 \\ e^{\beta}\sin\alpha & \cos\alpha\, e^{\beta} & 0 \\ 0 & 0 & 1\end{bmatrix}$$
$$=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & e^{-\beta}\end{bmatrix}$$
$=A(-\alpha,-\beta)$

82 **(c)**
Given, $A^2=2A-I$
Now, $A^3=A(A^2)$
$=A(2A-I)$
$=2A^2-A$
$=2(2A-I)-A$
$=3A-2I$
$A^4=A(A^3)$
$=A(3A-2I)$
$=3A^2-2A$
$=3(2A-I)-2A$
$=4A-3I$
Following this, we can say $A^n=nA-(n-1)I$.

83 **(d)**
A is involuntary. Hence,
$A^2=I\Rightarrow A=A^{-1}$
Also,
$$(kA)^{-1}=\frac{1}{k}(A)^{-1}$$
$$\Rightarrow\left(\frac{1}{2}A\right)^{-1}=2(A)^{-1}\Rightarrow 2A$$

84 **(c)**
As A is a skew-symmetric matrix,
$A^T=-A$
$\Rightarrow a_{ij}=0,\forall\ i$
$\Rightarrow \text{tr}(A)=0$
Also,
$|A|=|A^T|=|-A|=(-1)^3|A|$
$\Rightarrow 2|A|=0$
$\Rightarrow |A|=0$

85 **(b)**
Z is idempotent, then
$Z^2=Z\Rightarrow Z^1,Z^4,\dots,Z^n=Z$
$\therefore (I+Z)^n=\ {}^nC_0I^n+\ {}^nC_1I^{n-1}Z+\ {}^nC_2I^{n-2}Z^2+\cdots+\ {}^nC_nZ^n$
$=\ {}^nC_0I+\ {}^nC_1Z+\ {}^nC_2Z+\ {}^nC_3+\cdots+\ {}^nC_nZ$
$=I+(\ {}^nC_1+\ {}^nC_2+\ {}^nC_3+\cdots+\ {}^nC_n)Z$
$=I+(2^n-1)Z$

86 **(b)**
$A^2=O,A^3=A^4=\cdots=A^n=O$
Then, $A(I+A)^n=A(I+nA)=A+nA^2=A$

87 **(b)**
$\left(A'-\frac{1}{2}I\right)\left(A-\frac{1}{2}I\right)=$I and $\left(A'+\frac{1}{2}I\right)\left(A+\frac{1}{2}I\right)=$I
$\Rightarrow A+A'=0$ (subracting the two results)
$\Rightarrow A'=-A$
$\Rightarrow A^2=-\frac{3}{4}I$
$\Rightarrow\left(\frac{-3}{4}\right)^n=(\det(A))^2$
$\Rightarrow n$ is even

88 **(a, b, c)**
We have, $|A(\theta)=1|$
Hence, A is invertible.
$$A(\pi+\theta)=\begin{bmatrix}\sin(\pi+\theta) & \cos(\pi+\theta) \\ i\cos(\pi+\theta) & \sin(\pi+\theta)\end{bmatrix}$$
$$=\begin{bmatrix}-\sin\theta & -i\cos\theta \\ -i\cos\theta & -\sin\theta\end{bmatrix}=A(\theta)$$
$$\text{adj}(A(\theta))=\begin{bmatrix}\sin\theta & \cos\theta \\ -i\cos\theta & \sin\theta\end{bmatrix}$$
$$\Rightarrow A(\theta)^{-1}=\begin{bmatrix}\sin\theta & -i\cos\theta \\ -i\cos\theta & \sin\theta\end{bmatrix}=A(\pi+\theta)$$

89 **(a, b, c)**
$$S=\begin{bmatrix}0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0\end{bmatrix}\Rightarrow S^{-1}=\frac{1}{2}\begin{bmatrix}-1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1\end{bmatrix}$$
We have,
$$SA=\begin{bmatrix}0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0\end{bmatrix}\begin{bmatrix}b+c & c+a & b-c \\ c-b & c+b & a-b \\ b-c & a-c & a+b\end{bmatrix}$$
$$=\begin{bmatrix}0 & 2a & 2a \\ 2b & 0 & 2b \\ 2c & 2c & 0\end{bmatrix}$$
$$\therefore SAS^{-1}=\begin{bmatrix}0 & 2a & 2a \\ 2b & 0 & 2b \\ 2c & 2c & 0\end{bmatrix}\frac{1}{2}\begin{bmatrix}-1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1\end{bmatrix}$$
$$=\begin{bmatrix}0 & a & a \\ b & 0 & b \\ c & c & 0\end{bmatrix}\begin{bmatrix}-1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1\end{bmatrix}$$
$$=\begin{bmatrix}2a & 0 & 0 \\ 0 & 2b & 0 \\ 0 & 0 & 2c\end{bmatrix}$$
$=\text{diag}(2a,2b,2c)$

90 **(a, b)**

Let $I = k = 1$ (say). Then,

$A_iA_k = A_iA_k = A_1A_1$

$$A_iA_k = A_1A_1 = \begin{bmatrix} 0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0 \end{bmatrix} \times \begin{bmatrix} 0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0 \end{bmatrix}$$

$$= \begin{bmatrix} 1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1 \end{bmatrix} = l$$

$$A_2A_2 = \begin{bmatrix} 0&0&0&i\\0&0&-i&0\\0&i&0&0\\-i&0&0&0 \end{bmatrix} \times \begin{bmatrix} 0&0&0&i\\0&0&-i&0\\0&i&0&0\\-i&0&0&0 \end{bmatrix}$$

$$= \begin{bmatrix} 1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1 \end{bmatrix} = l$$

$\therefore A_iA_k + A_kA_i = l + l = 2l$

If $i \neq k$ let $i = 3$ and $k = 2$, then

$$A_iA_k = A_1A_2 = \begin{bmatrix} 0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0 \end{bmatrix} \times \begin{bmatrix} 0&0&0&i\\0&0&-i&0\\0&i&0&0\\-i&0&0&0 \end{bmatrix}$$

$$= \begin{bmatrix} -i&0&0&0\\0&i&0&0\\0&0&-i&0\\0&0&0&i \end{bmatrix}$$

$$A_2A_1 = \begin{bmatrix} 0&0&0&i\\0&0&-i&0\\0&i&0&0\\-i&0&0&0 \end{bmatrix} \times \begin{bmatrix} 0&0&0&i\\0&0&-i&0\\0&i&0&0\\-i&0&0&0 \end{bmatrix}$$

$$= \begin{bmatrix} -i&0&0&0\\0&i&0&0\\0&0&-i&0\\0&0&0&-i \end{bmatrix}$$

$\Rightarrow A_1A_2 + A_2A_1 = 0$

91 **(a,b,d)**

$A^2 - 4A - 5I_3$

$$= \begin{bmatrix} 1&2&2\\2&1&2\\2&2&1 \end{bmatrix}\begin{bmatrix} 1&2&2\\2&1&2\\2&2&1 \end{bmatrix} - 4\begin{bmatrix} 1&2&2\\2&1&2\\2&2&1 \end{bmatrix} - 5\begin{bmatrix} 1&0&0\\0&1&0\\0&0&1 \end{bmatrix}$$

$$= \begin{bmatrix} 9&8&8\\8&9&8\\8&8&9 \end{bmatrix} + \begin{bmatrix} -4&-8&-8\\-8&-4&-8\\-8&-8&-4 \end{bmatrix} + \begin{bmatrix} -5&0&0\\0&-5&0\\0&0&-5 \end{bmatrix}$$

$$= \begin{bmatrix} 0&0&0\\0&0&0\\0&0&0 \end{bmatrix} = O$$

$\therefore A^2 - 4A - 5I_3 = O$

$\Rightarrow A^{-1}A^2 - 4A^{-1}A - 5A^{-1}I_3 = O$

$\Rightarrow (A^{-1}A)A - 4I_3 - 5A^{-1} = O$

$\Rightarrow IA - 4I_3 - 5A^{-1} = O$

$\Rightarrow A^{-1} = \frac{1}{5}(A - 4I_3)$

Also, $|A^2| = \begin{vmatrix} 9&8&8\\8&9&8\\8&8&9 \end{vmatrix}$

$= 9(81 - 64) - 8(72 - 64) + 8(64 - 72)$

$= 9 \times 17 - 8 \times 8 + 8 \times (-8)$

$= 153 - 128 = 25 \neq 0$

$\therefore A^2$ is invertible

And $A^3 = A \cdot A^2$

$= A \cdot (4A - 5I_3) = 4(A^2 - 5A)$

$$= \begin{bmatrix} 36&32&32\\32&36&32\\32&32&36 \end{bmatrix} + \begin{bmatrix} -5&-10&-10\\-10&-5&-10\\-10&-10&-5 \end{bmatrix}$$

$$= \begin{bmatrix} 31&22&22\\22&31&22\\22&22&31 \end{bmatrix}$$

$\therefore |A^3| \neq 0$

$\therefore A^3$ is invertible

92 **(a, d)**

Here X is a $n \times 1$ matrix, C is a $n \times n$ matrix and X^T is a $1 \times n$ matrix. Hence X^TCX is a 1×1 matrix. Let $X^TCX = k$. then,

$(X^TCX)^T = X^TC^T(X^T)^T = X^T(-C)X = -X^TCX$
$= -k$

$\Rightarrow k = -k$

$\Rightarrow k = 0$

$\Rightarrow X^TCX$ is null matrix

93 **(a, b, c)**

$|A^{-1}| = -1 \Rightarrow |A| = -1$

Now, use adj $A = |A|A^{-1}$and $A = (A^{-1})^{-1}$

94 **(a, b, c)**

If $|A| \neq 0$, then

$AB = AC$

$\Rightarrow A^{-1}AB = A^{-1}AC$

$\Rightarrow B = C$

Also if A is orthogonal matrix, then $AA^T = I$

$\Rightarrow |AA^T| = 1 \Rightarrow |A|^2 = 1 \Rightarrow A$ is invertible

95 **(a, c, d)**

Given, $A^2 + 2A + 2I = O$

$\Rightarrow A^2 + A = -2I$

$\Rightarrow |A^2 + A| = |-2I|$

$\Rightarrow |A||A + I| = (-2)^n$

$\Rightarrow |A| \neq 0$

Therefore, A is non-singular, hence its inverse exists. Also, multiplying the given equation both sides with A^{-1}, we get

$A^{-1} = -\frac{1}{2}(A + I)$

96 **(a, c)**

We have,

$$\begin{pmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{pmatrix}^{-1} = \frac{1}{1+\tan^2\theta}\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$= \cos^2\theta \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$$

$$= \cos^2\theta \begin{bmatrix} 1-\tan^2\theta & -2\tan\theta \\ 2\tan\theta & 1-\tan^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & -\sin\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$$

$\therefore a - \cos 2\theta, b = \sin 2\theta$

97 **(a, c)**

A is orthogonal matrix.

$\therefore AA^T = 1$

$$\Rightarrow \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}\frac{1}{3}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{1}{9}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9 & 0 & a+4+2b \\ 0 & 9 & 2a+2-2b \\ a+4+2b & 2a+2-2b & a^2+4+b^2 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$\Rightarrow a + 4 + 2b = 0, 2a + 2 - 2b = 0$ and$a^2 + 4 + b^2 = 9$

$\Rightarrow a + 2b + 4 = 0, a - b + 1 = 0$ and$a^2 + b^2 = 5$

$\Rightarrow a = -2, b = -1$

98 **(a, b, c)**

We have, $A^2B = A(AB) = AA = A^2, B^2A = B(BA) = BB = B$^2,

$ABA = A(BA) = AB = A$ and $BAB = B(AB) = BA = B$

99 **(a, b, c)**

B is an idempotent matrix

$\therefore B^2 = B$

Now,

$A^2 = (I - B)^2$

$= (I - B)(I - B)$

$= I - IB - IB + B^2$

$I - B - B + B^2$

$= I - 2B + B^2$

$= I - 2B + B$

$= I - B$

$= A$

Therefore, A is idempotent. Again,

$AB = (I - B)B = IB - B^2 = B - B^2 = B^2 - B^2 = O$

Similarly, $BA = B(I - B) = BI - B^2 = B - B = O$

100 **(a, d)**

$(B^TAB)^TB^TA^T(B^T)^T = B^TA^TB = B^TAB$if A is symmetric

Therefore, B^TAB is symmetric if A is symmetric

Also, $(B^TAB)^T = B^TA^TB = B^T(-A)B = -(B^TA^TB)$

Therefore, B^TAB if A is skew-symmetric if A is skew-symmetric

101 **(b,d)**

Let $A = \begin{bmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{bmatrix} \sim \begin{bmatrix} -1 & 2 & 5 \\ 0 & 0 & a+6 \\ 0 & 0 & a+6 \end{bmatrix}$

$(R_2 \to R_2 + 2R_1, R_3 \to R_3 + R_1)$

Clearly rank of A is 1, if $a = -6$

Also, for $a = 1$, $|A| = \begin{vmatrix} -1 & 2 & 5 \\ 2 & -4 & -3 \\ 1 & -2 & 2 \end{vmatrix} = 0$

and $\begin{vmatrix} 2 & 5 \\ -4 & -3 \end{vmatrix} = -6 + 20 = 14 \neq 0$

$\therefore$ Rank of A is 2, if $a = 1$

102 **(a, b, c)**

Applying $R_3 \to R_3 - R_2 \to R_2 - R_1$, we get

$|a| = 3\begin{bmatrix} a_1 & a_2 & a_3 \\ d & d & d \\ d & d & d \end{bmatrix} = 0$

Where d is the common difference of the AP.

Therefore, the given system of equation has infinite number of

Solution. Also,

$|B| = a_1^2 + a_2^2 \neq 0$

103 **(a, b, c)**

Given, $AB = A, BA = B$

$\Rightarrow B \times AB = B \times A$

$\Rightarrow (BA)B = B$

$\Rightarrow B^2 = B$

Also,

$A \times B \times A = AB$

$\Rightarrow (AB)A = A$

$\Rightarrow A^2 = A$

Now $(A^T)^2 = (A^T \times A^T) = (A \times A)^T = (A^2)^T = A^T$

Similarly, $(B^T)^2 = B^T$

$\Rightarrow A^T$andB^T are idempotent

104 **(b, c)**

$(-A)^{-1} = \frac{\text{adj}(-A)}{|-A|} = \frac{(-1)^{n-1}\text{adj}(A)}{(-1)^n|A|} = \frac{\text{adj}(A)}{-|A|} = -A^{-1}$

(for any value of n)

Given, $A^n = O$

Now,

$(I - A)(I + A + A^2 + \cdots + A^{n-1}) = I - A^n = I$

$\Rightarrow (I - A)^{-1} = I + A + A^2 + \cdots + A^{n-1}$

105 **(a, b, c)**

$\because\ |A| = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

$= 3(-3+4) + 3(2-0)$
$+ 4(-2-0) = 1$

$\therefore$ adj(adj$A$) $= |A|^{3-2} A = A$ and $|$adj(adj$A$) $=$ AA=1

Also,

$|\text{adj}A| = |A|^{3-1} = |A|^2 = I^2 = 1$

106 **(a, d)**

Given, $(A+B)^2 = A^2 + B^2 + 2AB$

$\Rightarrow (A+B)(A+B) = A^2 + B^2 + 2AB$

$\Rightarrow A^2 + AB + BA + B^2 = A^2 + B^2 + 2AB \Rightarrow BA = AB$

$\Rightarrow \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a+2 & -a+1 \\ b-2 & -b-1 \end{bmatrix} = \begin{bmatrix} a-b & 1+1 \\ 2a+b & 2-1 \end{bmatrix}$

The corresponding elements of equal matrices are equal.

$a+2 = a-b, -a+1 = 2 \Rightarrow a = -1$

$b-2 = 2a+b, -b-1 = 1 \Rightarrow b = -2$

$\Rightarrow a = -1, \quad b = -2$

107 **(a, b, c, d)**

$A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$

We have,

$A^2 - 4A - 5I_3 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$- 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$

$\Rightarrow 5I_3 = A^2 - 4A = A(A - 4I_3)$

$\Rightarrow I_3 = \frac{1}{5}(A - 4I_3) \Rightarrow A^{-1} = \frac{1}{5}(A - 4I_3)$

Note that $|A| = 5$. Since $|A^3| = |A|^3 = 5^3 \neq 0, A^3$ is invertible. Simi-larly, A^2 invertible

108 **(b, c)**

$\det(-A) = (-1)^n \det(A)$

$\det(A^{-1}) = \frac{1}{\det(A)} = 1$

$\det(\text{adj}A) = |A|^{n-1} = 1$

Hence, $|\omega A| = \omega^n |A| = 1$ only when $n = 3k, k \in Z$

109 **(b, c)**

$A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$

$\Rightarrow (A^{-1})^2 = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

Similarly,

$(A^{-1})^3 = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$

And $(A^{-1})^n = \begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix} \Rightarrow \lim_{n\to\infty} \frac{1}{n} A^{-n} =$

$\lim_{n\to\infty} \begin{bmatrix} 1/n & 0 \\ -1 & 1/n \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$

$\lim_{n\to\infty} \frac{1}{n^2} A^{-n} = \lim_{n\to\infty} \begin{bmatrix} 1/n^2 & 0 \\ -1 & 1/n^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

110 **(a,c)**

$A_\alpha \cdot A_{(-\alpha)} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & \sin\alpha\cos\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \sin\alpha\cos\alpha & \cos^2\alpha + \sin^2\alpha \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$= \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & \sin\alpha\cos\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \sin\alpha\cos\alpha & \cos^2\alpha + \sin^2\alpha \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Also, $A_\alpha \cdot A_\beta = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$

$= \begin{bmatrix} \cos\alpha\cos\beta - \sin\alpha\sin\beta & \cos\alpha\sin\beta + \sin\alpha\cos \\ \sin\alpha\cos\beta + \cos\alpha\sin\beta & -\sin\alpha\sin\beta + \cos\alpha\cos \end{bmatrix}$

$= \begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) \\ \sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix} = A_{(\alpha+\beta)}$

111 **(a,d)**

Let A be a symmetric matrix

Then, $A' = A$

Now, $(B'AB)' = B'A'(B')'$ $[\because (AB)' = B'A']$

$= B'A'B$ $[\because (B)' = B]$

$= B'AB$ $[\because A' = A]$

$\Rightarrow B'AB$ is a symmetric matrix

Now, let A be a skew-symmetric matrix

Then, $A' = -A$

$\therefore (B'AB)' = B'A'(B')'$ $[\because (AB)' = B'A']$

$= B'A'B$ $[\because (B')' = B]$

$= B'(-A)B$ $[\because A' = -A]$

$= -B'AB$

$\therefore B'AB$ is a skew-symmetric matrix

112 **(a, b, c)**

All are properties of diagonal matrix.

114 **(b, c)**

Since A is skew-symmetric, $A^T = -A$. We have,

$A^2 + I = O$

$\Rightarrow A^2 = -1$ or $AA = -I$

$\Rightarrow A(-A) = I$

$\Rightarrow AA^T = I$

Again, we know that

$|A| = |A^T|$and$|kA| = k^n|A|$

Where n is the order of A. Now,

$A^T = (-1)^n \times A$

$\Rightarrow |A^T| = (1)^n|A|$

ΦI $[1 - 1)n|A| = 0$

Hence either $|A| = 0$ *or* $1 - (-1)^n = 0$, i. e., n is even. But

$A^2 = O - 1 = -1$
$\Rightarrow |A|^2 = (-1)^n|I| = (-1)^n \neq 0$
Hence, the only possibility is that A is of even order

115 **(a, b, c)**
Given that A and B commute, we have
$AB = BA$ ($\because$ A and B are symmetric)(1)
Also,
$A^T = A, B^T = B$ (2)
$(A^{-1}B)^T = B^T(A^{-1})^T = BA^{-1}$
($\because$ if A is symmetric A^{-1} is also symmetric)
Also from Eq. (1),
$ABA^{-1} = B$ (3)
$\Rightarrow IBA^{-1} = A^{-1}B$
$\Rightarrow BA^{-1} = A^{-1}B$
Hence, from Eq. (2),
$(A^{-1}B)^T = A^{-1}B$
Thus, $A^{-1}B$ is symmetric Similarly, AB^{-1} is also symmetric Also,
$BA = AB$
$\Rightarrow (BA)^{-1} = (AB)^{-1}$
$\Rightarrow A^{-1}B^{-1} = B^{-1}A^{-1}$
$\Rightarrow (A^{-1}B^{-1})^T = (B^{-1}A^{-1})^T$
$= (A^{-1})^T(B^{-1})^T$
$= A^{-1}B^{-1}$
Hence, $A^{-1}B^{-1}$ is symmetric

116 **(a, c)**
$\sin A \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $\cos A = \begin{bmatrix} \sin\theta & \cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$
$\therefore$ $|\sin A| = \cos^2\theta + \sin^2\theta = 1.$
Hence $\sin A$ is invertible.
Also,
$(\sin A) \times (\sin A)^T =$
$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$
$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & 0 \\ 0 & \cos^2\theta + \sin^2\theta \end{bmatrix}$
$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
$= I$
Hence, $\sin A$ is orthogonal. Also,
$2\sin A \cos A = 2\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \sin\theta & \cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$
$= 2\begin{bmatrix} 2\sin\theta\cos\theta & \cos^2\theta + \sin^2\theta \\ \cos^2\theta - \sin^2\theta & 0 \end{bmatrix}$
$= 2\begin{bmatrix} \sin 2\theta & 1 \\ \cos 2\theta & 0 \end{bmatrix}$
$\neq \sin 2A$

117 **(a, b, c)**
$A = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ \cos\beta & \sin\beta & 0 \\ \cos\gamma & \sin\gamma & 0 \end{bmatrix}\begin{bmatrix} \cos\alpha & \cos\beta & \cos\gamma) \\ \sin\alpha & \sin\beta & \sin\gamma) \\ 0 & 0 & 0 \end{bmatrix}$
Thus, A is symmetric and $|A| = 0$, hence singular and not invertible. Also,
$AA^T \neq 1$

118 **(a, c)**
$A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$
Now,
$A^T = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$
Hence, A is orthogonal. Therefore,
$AA^T = I \Rightarrow \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}\begin{bmatrix} 0 & a & a \\ 2b & b & -b \\ c & -c & c \end{bmatrix}$
$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Equating the corresponding elements, we get
$4b^2 + c^2 = 1$ (1)
$2b^2 - c^2 = 0$ (2)
$a^2 + b^2 + c^2 = 1$ (3)
Solving Eqs. (1), (2) and (3), we get
$a = \pm\frac{1}{\sqrt{2}}, b = \pm\frac{1}{\sqrt{6}}, c = \pm\frac{1}{\sqrt{3}}$

119 **(a)**
Statement 1 is true as $|A| = 0$. Since $|B| \neq 0$, statement 2 is also true and correct explanation of statement 1

120 **(c)**
A satisfies $A^2 - \text{Tr}(A).A + (\det A)I = 0$

On comparing with $A^2 - I = 0$,we get

$\text{Tr}(A) = 0, |A| = -1$

Alternate

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, a, b, c, d \neq 0$

Now $A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix}$

$\Rightarrow a^2 + bc = 1, bc + d^2 = 1$

and $ab + bd = ac + cd = 0$

Also, $c \neq 0$ and $b \neq 0 \Rightarrow a + d = 0$

$\text{Tr}(A)= a + d = 0$

and $|A| = ad - bc = -a^2 - bc = -1$

121 **(c)**

Let $A = \begin{bmatrix} 0 & -c & b \\ c & 0 & a \\ -b & -a & 0 \end{bmatrix}$

And $A = -A'$

$\therefore \det(A) = \det(-A')$

$= -\det(A') = -\det A$

$\therefore \det A = 0$

$\because \det A' = \det(-A')$ is not true

$\therefore \det(-A') = (-1)^3 \det(A') = -\det A'$

122 **(c)**
$[A(A+B)^{-1}B]^{-1} = B^{-1}((A+B)^{-1})^{-1}A^{-1}$

$= B^{-1}(A+B)A^{-1} = (B^{-1}+I)A^{-1} = B^{-1}I + IA^{-1} = B^{-1} + A^{-1}$

Hence, statement 1 is true. Statement 2 is false as $(A+B)^{-1} = A^{-1} + B^{-1}$ is not true

123 **(b)**
Since $AB = BA$, we have

$(A+B)^r =$

If $r = m + n$, then

$A^{r-p}B^p = A^m B^{r-p-m} = 0$ if $p \le n$

and $A^{r-p}B^p = A^{r-p}B^n B^{p-n} = 0$ if $p > n$

Then, $(A = B)^r = 0$, for $r = m + n$

Thus, both the statements are correct but statement 2 is not currently explaining statement 1

124 **(a)**
$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a^2+b^2 & ac+bd \\ ac+bd & c^2+d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow a^2 + b^2 = 1 (1)$

$c^2 + d^2 = 1$ (2)

$ac + bd = 1 (3)$

$\Rightarrow \frac{a}{d} = \frac{-b}{c} = \pm 1$

Also, we must have $a, b, c, d \in [-1, 1]$ for Eqs. (1) and (2) to get

Defined Hence, without loss of generality, we can assume $a = \cos\theta$ and $b = \sin\theta$

So for $\frac{a}{d} = \frac{-b}{c} = 1$, we have $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and for $\frac{a}{d} = \frac{-b}{c} = -1$, we have $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$

125 **(b)**
$|\text{adj } A| = |A|^{n-1} = |A|^{2-1} = |A|$
adj (adj A) $= |A|^{n-2} A = |A|^0 A = A$

126 **(c)**
Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$,then$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $[\because A^2 = 1]$

$\Rightarrow \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow b(a + d) = 0, \; c(a+d) = 0$

and $a^2 + bc = 1, \; bc + d^2 = 1$

$\Rightarrow a = 1, d = -1, b = c = 0$

If $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, then

$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

If $A \ne I, \; A \ne -I$, then

det (A)=-1 (statement I is true)

Statement II, Tr(A) =1$-$1 $= 0$, Statement II is false.

127 **(a)**

Given

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix}$$

Hence,

$$A^2 - (a+d)A + (ad-bc)I$$

$$= \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix} - (a+d)\begin{bmatrix} a & b \\ c & d \end{bmatrix} + (ad-bc)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a^2+bc-(a^2+ad)+(ad-bc) & ab+bd- \\ ac+cd-(ac+cd) & bc+d^2-(ad+ \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= O$$

Given,

$$A^3 = O$$

$$\Rightarrow |A| = 0 \text{ or } ad - bc = 0$$

$$\Rightarrow A^2 - (a+d)A = 0 \text{ or} A^2 = (a+d)A \qquad (1)$$

Case (i)

$$a + d = 0$$

From eq. (1)

$$A^2 = O$$

Case (ii)

$$a + d \neq 0$$

Given,

$$A^3 = O$$

$$\Rightarrow A^2 A = O$$

$$\Rightarrow (A+d)A.A = O$$

$$\Rightarrow A^2 = O$$

128 **(b)**

Both the statements are true as both are standard properties of diagonal matrix. But statement 2 does not explain statement 1

129 **(b)**

$$\text{adj}(F(\alpha)) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Also,

$$|F(\alpha)| = 1$$

Then,

$$[F(\alpha)]^{-1} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(-\alpha) & \sin(-\alpha) & 0 \\ -\sin(-\alpha) & \cos(-\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= F(-\alpha)$$

Similarly, we can prove that $[G(\beta)]^{-1} = G(-\beta)$

But again given matrices $F(\alpha)$ and $G(\beta)$ are special matrices for which this type of result holds

In general, such result is not true. You can verify with any other matrix. Hence, both statements are true but statement 2 is correct explanation of statement 1

130 **(a)**

$$A = -A^T \Rightarrow |A| = -|A^T| = -|A|$$

$$\Rightarrow 2|A| = 0$$

$$\Rightarrow |A| = 0$$

131 **(d)**

Matrix $a_{ij} = \frac{i-j}{i+2j}$ is $A = \begin{bmatrix} 0 & -\frac{1}{5} & -\frac{2}{7} \\ \frac{1}{4} & 0 & -\frac{1}{8} \\ \frac{2}{5} & \frac{1}{7} & 0 \end{bmatrix}$ which is neither

Symmetric nor skew-symmetric But this is not the reason for which A cannot be expressed as sum of symmetric and skew-symmetric matrix. In fact any matrix can be expressed as a sum of symmetric and skew-symmetric matrix. Hence, statement 1 is false but statement 2 is true

132 **(a)**

$A = [a_{ij}]_{n\times n}$ is square matrix such that $a_{ij} = 0$, for $i \neq j$, then A is called diagonal matrix. Thus, the given statement is true and $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 7 \end{bmatrix}$ is a diagonal matrix

133 **(d)**
A^{-1} exists only for non-singular matrix

$\therefore\ AB = AC \Rightarrow B = C$ if A^{-1} exists

134 **(a)**
$\because |A| = \begin{vmatrix} 3 & 4 \\ 3 & 5 \end{vmatrix} = 15 - 12 = 3 \neq 0$

$\therefore A$ is non-singular matrix

$\therefore A^{-1}$ is exist

135 **(d)**
ABC is not defined, as order of A, B and C are such that theyare not conformable for multiplication

136 **(d)**
Statement 1 is false

$\because A = |A_{ij}|_{n\times n}$ where $a_{ij} = 0, i \geq j$

Therefore, $|A| = 0$ and hence A is singular. So, inverse of A is not defined

In statement 2, $|A| = 0$. Therefore, inverse of A is not defined

137 **(a)**
Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $B = \begin{bmatrix} x & y \\ z & u \end{bmatrix}$

Also, $AB = BA$ (given)

$\Rightarrow \begin{bmatrix} ax+bz & ay+bu \\ cx+dz & cy+du \end{bmatrix} = \begin{bmatrix} ax+cy & bx+dy \\ az+cu & bz+du \end{bmatrix}$

On comparing, we get

$ax + bz = ax + cy$

$\Rightarrow\ bz = cy$

$\Rightarrow \frac{z}{c} = \frac{y}{b} = \lambda$ (say)

$\therefore\ y = b\lambda,\ z = c\lambda$ ...(i)

And $ay + bu = bx + dy$

$\Rightarrow\ ab\lambda + bu = bx + bd\lambda$ [from Eq. (i)]

$\Rightarrow\ a\lambda + u = x + d\lambda = k$ (say)

For $\lambda = 0;\ y = 0, z = 0,\ u = k,\ x = k$

Then, $B = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$ = scalar matrix

Then, if $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

Then, $AB = BA = \begin{bmatrix} ak & bk \\ ck & dk \end{bmatrix} = kA$

138 **(b)**
$\because AA' = \frac{1}{3}\begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ -2 & -2 & -1 \end{bmatrix} \cdot \frac{1}{3}\begin{bmatrix} 1 & -2 & -2 \\ -2 & 1 & -2 \\ 2 & 2 & -1 \end{bmatrix}$

$= \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$\therefore A$ is orthogonal

Also, if A and B are orthogonal, then AB is orthogonal

139 **(b)**
Let, $A = \begin{bmatrix} d_1 & z_1 & z_2 \\ \overline{z}_1 & d_2 & z_3 \\ \overline{z}_2 & \overline{z}_3 & d_3 \end{bmatrix}$

$A^2 = O$

$\Rightarrow \begin{bmatrix} d_1 & z_1 & z_2 \\ \overline{z}_3 & d_2 & z_3 \\ \overline{z}_2 & \overline{z}_3 & d_3 \end{bmatrix}\begin{bmatrix} d_1 & z_1 & z_2 \\ \overline{z}_1 & d_2 & z_3 \\ \overline{z}_2 & \overline{z}_3 & d_3 \end{bmatrix}$

$= \begin{bmatrix} d_1 + |z_1|^2 + |z_2|^2 & d_1z_1 + d_2z_2 + z_2\overline{z}_3 & d_1 \\ d_1\overline{z}_1 + d_2\overline{z}_1 + z_3\overline{z}_2 & d_2^2 + |z_1|^2 + |z_3|^2 & \overline{z}_1 \\ d_1\overline{z}_2 + \overline{z}_3\overline{z}_1 + d_3\overline{z}_2 & z_1\overline{z}_2 + d_2\overline{z}_3 + d_3\overline{z}_3 & d \end{bmatrix}$

$\Rightarrow$ Diagonal elements $d_1 = d_2 = d_3 = 0$ and$|z_1| = |z_2| = |z_3| = 0$

$\Rightarrow z_1 = z_2 = z_3 = 0$

$\Rightarrow A$ =Null matrix

Thus, statement 1 is true. Also,

$A^2 = 0 \Rightarrow |A|^2 = 0$ or$|A| = 0$

Thus, statement 2 is true but it does not explain statement 1

140 **(c)**
We know that$|\text{adj}A| = |A|^{n-1}$. Hence, statement 2 is false.

Now,

$|\text{adj}(\text{adj } A)| = |\text{adj}A|^{n-1} = ||A|^{n-1}|^{n-1}$
$= |A|^{(n-1)^2}$

Then,

$\left|\text{adj}\left(\text{adj}(\text{adj } A)\right)\right| = |\text{adj}(\text{adj}A))|^{n-1}$

$= \left||A|^{(n-1)2}\right|^{n-1}$

$= |A|^{(n-1)^3}$

Hence, statement 1 is true

141 **(a)**

A is involuntary, hence,
$A^2 = I$
$\Rightarrow A^3 = A^5 = \cdots = A$ and $A^2 = A^4 = A^6 = \cdots = I$
$\Rightarrow (I - A)^n = {}^nC_0 I - {}^nC_1 A + {}^nC_2 A^2 - {}^nC_3 A^3 + \cdots$
$= {}^nC_0 I - {}^nC_1 A + {}^nC_2 I - {}^nC_3 A + {}^nC_4 I - \cdots$
$= ({}^nC_0 + {}^nC_2 + {}^nC_4 + \cdots)I - ({}^nC_1 A + {}^nC_3 + {}^nC_5 + \cdots)$
$A = 2^{n-1}(I - a)$
$\Rightarrow [(I - A)^n]A^{-1} = 2^{n-1}(I - a)A^{-1}$
$= 2^{n-1}(A^{-1} - I)$
c if a is nilpotent of index 2, then
$A^2 = A^3 = A^4 \ldots = A^n = O$
$\Rightarrow (I - A)^n = {}^nC_0 I - {}^nC_1 A + {}^nC_2 A^2 - {}^nC_3 A^3 + \cdots$
$= I - nA + O + O + \cdots$
$= I - na$
d A is orthogonal. Hence,
$AA^r = I$
$\Rightarrow (A^T)^{-1} = A$

143 **(d)**

$A^2 = A \Rightarrow A$ is idempotent matrix
$A^m = O \Rightarrow A$ is nilpotent matrix
$A^2 = I \Rightarrow A$ is involutory matrix
$A^T = A \Rightarrow A$ is symmetric matrix

144 **(c)**

$|A| = 2 \Rightarrow |2A^{-1}| = 2^3/|A| = 4$

$|\text{adj}(\text{adj}(2A))| = |2A|^4 =$
$2^{12}/|A|^4 = 2^{12}/2^{12} = 1$

$(A + B)^2 = A^2 + B^2$

$\Rightarrow AB + BA = O$

$\Rightarrow |AB| = |-BA| = -|AB|$

$\Rightarrow |AB| = 0$

$\Rightarrow |B| = 0$

Product ABC is not defined

145 **(b)**

Since A is idempotent, $A^2 = A^3 = A^4 = \cdots = A$. now,

$(A + I)^n = I + {}^nC_1 A + {}^nC_2 A^2 + \cdots + {}^nC_n A^n$

$= I + {}^nC_1 A + {}^nC_2 A + \cdots + {}^nC_n A$

$= I + {}^nC_1 A + {}^nC_2 A + \cdots + {}^nC_n A$

$= I + ({}^nC_1 A + {}^nC_2 + \cdots + {}^nC_n)A$

$= I + (2^n - 1)A$

$\Rightarrow 2^n - 1 = 127$

$\Rightarrow n = 7$

We have,

$(I - A)(I + A + A^2 + \cdots + A^7)$

$= I + A + A^2 + \cdots + A^7 + (-A - A^2 - A^3 - A^4 \ldots - A^8)$

$= I - A^8$

$= I(if\ A^8 = O)$

Here matrix A is skew-symmetric and since $|A| = |A^7| = (-1)^n|A|$

So $|A|(1 - (-1)^n) = 0$. As n is odd, hence $|A| = 0$. hence a is singular,

If A is symmetric, A^{-1} is also symmetric for matrix of any order

146 **(a)**

$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 5 & 6 & 8 \end{vmatrix}$

$= 1(24 - 24) - 2(16 - 20) + 3(12 - 15)$

$= -1$

$|B| = \begin{vmatrix} 3 & 2 & 5 \\ 2 & 3 & 8 \\ 7 & 2 & 9 \end{vmatrix}$

$= 3(27 - 16) - 2(18 - 56) + 5(4 - 21)$

$= 24$

adj (adj A) $= |A|^{n-2}A = |A|A = -A$

147 **(a)**

Since, $A = P^{-1}BP$

$\Rightarrow \det(A) = \det(P^{-1}BP)$

$= \det(P^{-1}) \det(B) \det(P)$

$= \frac{1}{\det(P)} \det(B) \det(P)$

$\Rightarrow \det(A) = \det(B)$

148 **(c)**

Since, $\begin{bmatrix} 1 & 2 & \lambda+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{bmatrix} = 0$

$\Rightarrow 1(40 - 40) - 3(20 - 24) + (\lambda + 2)(10 - 12) = 0$

$\Rightarrow \lambda = 4$

Now, $A + B = \begin{bmatrix} 4 & 5 & 10 \\ 5 & 6 & 13 \\ 5 & 6 & 14 \end{bmatrix}$

$\therefore$ tr $(A + B) = 4 + 6 + 14 = 24$

149 **(b)**

Let,

$a = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\Rightarrow A^2 - (a + d)A + (ad - bc)I$

$= \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} - (a+d)\begin{bmatrix} a & b \\ c & d \end{bmatrix} + (ad - bc)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix} - \begin{bmatrix} a^2+ad & ab+bd \\ ac+cd & ad+d^2 \end{bmatrix} + \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$

$= O$

150 **(c)**

$AB = A \Rightarrow |AB| = |A| \quad (1)$

$\Rightarrow |A| = 0$ or $|B| = 1$

$BA = B \Rightarrow |BA| = |B| \quad (2)$

$\Rightarrow |A| = 1$ or $|B| = 0$

If $|A| = 0$, then from Eq. (2), $|B| = 0$

If $|B| = 0$, then from Eq. (1), $|A| = 0$

151 **(d)**

$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$\Rightarrow B = \begin{bmatrix} a_{12}+a_{13} & a_{11}+a_{13} & a_{11}+a_{12} \\ a_{22}+a_{23} & a_{21}+a_{23} & a_{21}+a_{22} \\ a_{32}+a_{33} & a_{31}+a_{33} & a_{31}+a_{32} \end{bmatrix}$

$\Rightarrow X = A^{-1}B$

$\Rightarrow \frac{1}{|A|}\begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$

$\begin{bmatrix} a_{12}+a_{13} & a_{11}+a_{13} & a_{11}+a_{12} \\ a_{22}+a_{23} & a_{21}+a_{23} & a_{21}+a_{22} \\ a_{32}+a_{33} & a_{31}+a_{33} & a_{31}+a_{32} \end{bmatrix}$

$= \frac{1}{|A|}\begin{bmatrix} 0 & |A| & |A| \\ |A| & 0 & |A| \\ |A| & |A| & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$\Rightarrow |A_1B| = 2$

$\Rightarrow |A^1||B| = 2$

$\Rightarrow |B| = 2|A|$

152 **(b)**

$A^n - A^{n-2} = A^2 - I \Rightarrow A^{50} = A^{48} + A^2 - I$

Further,

$A^{48} = A^{46} + A^2 - I$

$A^{46} = A^{44} + A^2 - I$

$\vdots \quad \vdots \quad \vdots \quad \vdots$

$\underline{A^4 = A^2A^2 - I}$

$A^{50} = 25A^2 - 24I$

Here,

$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$\Rightarrow A^{50} = \begin{bmatrix} 25 & 0 & 0 \\ 25 & 25 & 0 \\ 25 & 0 & 25 \end{bmatrix} - 24\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$

$\therefore |A^{50}| = 1$

Also, $\text{tr}(A^{50}) = 1 + 1 + 1 = 3$, Further,

$\begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 25 \\ 25 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} \cup_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Similarly,

$\cup_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $\cup_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \cup \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, i.e., $|\cup| = 1$

153 **(c)**

$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{bmatrix}$

$\Rightarrow \lambda I = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$

$\Rightarrow A - \lambda I = \begin{bmatrix} 2-\lambda & 1 & 1 \\ 2 & 3-\lambda & 4 \\ -1 & -1 & -2-\lambda \end{bmatrix}$

$\Rightarrow \det(A - \lambda I) = -(\lambda - 1)(\lambda + 1)(\lambda - 3)$

Thus, the characteristic roots are $-1, 1$ and 3.

154 **(c)**

As second row of all the options is same, we are to look at the

Elements of the first row. Let the left inverse be $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$. Then,

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore a + b + 2c = 1$

$-a + b + 3c = 1$, i.e., $b = \frac{1-5c}{2}, a = \frac{1+c}{2}$

Thus, matrices in the options (a), (b) and (d) are the inverses and

Matrix in option (c) is not the left inverse

155 **(a)**

$$A = \begin{bmatrix} x & x \\ x & x \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 2x^2 & 2x^2 \\ 2x^2 & 2x^2 \end{bmatrix}, A^3 = \begin{bmatrix} 2^2x^2 & 2^2x^2 \\ 2^2x^2 & 2^2x^2 \end{bmatrix}$$

And so on. Then

$$e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots$$

$$= \begin{bmatrix} 1 + x + \frac{2x^2}{2!} + \frac{2^2x^3}{3!} + \cdots & x + \frac{2x^2}{2!} + \frac{2^2x^3}{3!} + \cdots \\ x + \frac{2x^2}{2!} + \frac{2^2x^3}{3!} + \cdots & 1 + x + \frac{2x^2}{2!} + \frac{2^2x^3}{3!} + \cdots \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}\left(1 + 2x + \frac{2^2x^2}{2!} + \frac{2^3x^3}{3!} + \cdots\right) + \frac{1}{2} & \frac{1}{2}\left(1 + 2x + \frac{2^2x^2}{2!} + \cdots\right) - \frac{1}{2} \\ \frac{1}{2}\left(1 + 2x + \frac{2^2x^2}{2!} + \frac{2^3x^3}{3!} + \cdots\right) - \frac{1}{2} & \frac{1}{2}\left(1 + 2x + \frac{2^2x^2}{2!} + \cdots\right) + \frac{1}{2} \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} e^{2x} + 1 & e^{2x} - 1 \\ e^{2x} - 1 & e^{2x} + 1 \end{bmatrix}$$

$\Rightarrow f(x) = e^{2x} + 1$ and $g(x) = e^{2x} - 1$

$$\int \frac{e^{2x} - 1}{e^{2x} + 1} dx = \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx = \log(e^x + e^{-x}) + c$$

156 **(4)**

$\begin{bmatrix} a & b \\ c & 1-a \end{bmatrix}$ is and idempotent matrix

$$\Rightarrow \begin{bmatrix} a & b \\ c & 1-a \end{bmatrix}^2 = \begin{bmatrix} a & b \\ c & 1-a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a & b \\ c & 1-a \end{bmatrix}\begin{bmatrix} a & b \\ c & 1-a \end{bmatrix} = \begin{bmatrix} a & b \\ c & 1-a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a^2 + bc & ab + b - ab \\ ac + c - ac & bc + (1-a)^2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & 1-a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a^2 + bc & b \\ c & bc + (1-a)^2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & 1-a \end{bmatrix}$$

$\Rightarrow a^2 + bc = a$

$a - a^2 = bc = 1/4$ (given)

$f(a) = 1/4$

157 **(0)**

$A = \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$

$\Rightarrow A^2 = A.A = \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$\Rightarrow A^4 = A^2.A^2 = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 3^2 & 0 \\ 0 & 3^2 \end{bmatrix}$

$\Rightarrow A^8 = \begin{bmatrix} 3^4 & 0 \\ 0 & 3^4 \end{bmatrix}$

And $A^6 = A^4.A^2 = \begin{bmatrix} 3^2 & 0 \\ 0 & 3^2 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 3^2 & 0 \\ 0 & 3^2 \end{bmatrix}$

Let $V = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$A^8 + A^6 + A^4 + A^2 + I$

$$\begin{bmatrix} 81 & 0 \\ 0 & 81 \end{bmatrix} + \begin{bmatrix} 27 & 0 \\ 0 & 27 \end{bmatrix} + \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 121 & 0 \\ 0 & 121 \end{bmatrix}$$

$(A^8 + A^6 + A^4 + A^2 + I)V = \begin{bmatrix} 0 \\ 11 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 121 & 0 \\ 0 & 121 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 11 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 121x \\ 121y \end{bmatrix} = \begin{bmatrix} 0 \\ 11 \end{bmatrix}$

$\Rightarrow x = 0$ and $y = 1/11$

$\Rightarrow V = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1/11 \end{bmatrix}$

158 **(4)**

We have $AB = \begin{bmatrix} 3ax^2 & 3bx^2 & 3cx^2 \\ a & b & c \\ 6ax & 6bx & 6cx \end{bmatrix}$

Now $\text{tr} \cdot (AB) = \text{tr} \cdot (C)$

$\Rightarrow\ 3ax^2 + b + 6cx = (x+2)^2 + 2x + 5x^2\ \forall x \in R$(Identity)

$\Rightarrow 3ax^2 + 6cx + b = 6x^2 + 6x + 4$

$\Rightarrow a = 2, \quad c = 1, b = 4$

159 **(6)**

Given $A^2 = A$

$\Rightarrow$ I=(I-0.4A)(I-αA)

$= I - A\alpha - 0.4A + 0.4\alpha A^2$

$= I - A\alpha - 0.4A + 0.4\alpha A$

$= I - A(0.4 + \alpha) + 0.4\alpha A$

$\Rightarrow\ 0.4\alpha = 0.4 + \alpha$

$\Rightarrow\ \alpha = -2/3$

$\Rightarrow |9\alpha| = 6$

160 **(2)**

$\begin{vmatrix} 1 & 2 & 2 \\ 1 & 3 & 4 \\ 3 & 4 & k \end{vmatrix} = 0$

$\Rightarrow\ 1(3k - 16) - 2(k - 12) + 2(4 - 9 = 0)$

$\Rightarrow \quad 3k - 16 - 2k + 24 - 10 = 0$

$\Rightarrow \quad k = 2$

161 **(4)**

A diagonal matrix is commutative with every square matrix if it is scalar matrix so every diagonal element is 4

$\therefore \ |A| = 64$

162 **(8)**

In a skew symmetric matrix, diagonal elements are zero. Also $a_{ij} + a)ij = 0$

Hence number of matric $= 2 \times 2 \times 2 = 8$

163 **(0)**

For idempotent matrix, $A^2 = A$

$\Rightarrow A^{-1}A^2 = A^{-1}A \qquad (\because \ A \text{ is non-singular})$

$\Rightarrow A + I$

Thus non-singular idempotent matrix is always a unit matrix.

$\therefore \ l^2 - 3 = 1 \ \Rightarrow \ l = \pm 2$

$m^2 - 8 = 1 \ \Rightarrow \ m = \pm 3$

$n^2 - 15 = 1 \ \Rightarrow \ n = \pm 4$

And $p = q = r = 0$

$\Rightarrow$ required sum is 0

164 **(2)**

Let X be the solution set of the equation $A^x = I$, where $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ and I is the corresponding unit matrix and $x \subseteq N$ then the minimum value of $\sum(\cos^x \theta + \sin^x \theta), \theta \in R$

165 **(1)**

$A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$

Hence, det. $A = \sec^2 x$

$\therefore \ \det A^T = \sec^2 x$

Now $f(x) = \det.(A^T A^{-1})$

$= (\det. A^T)(\det. A)^{-1}$

$= \dfrac{\det.(A^T)}{\det.(A)} = 1$

Hence, $f(x) = 1$

166 **(4)**

$|\text{adj}A^{-1}| = |A^{-1}|^2 = \dfrac{1}{|A|^2}$

$\Rightarrow \ |(\text{adj}A^{-1})^{-1}| = \dfrac{1}{|\text{adj}A^{-1}|}$

$= |A|^2 = 2^2 = 4$

167 **(4)**

Given that $AA^T = 4I$

$\Rightarrow |A|^2 = 4$

$\Rightarrow |A| = \pm 2$

So $A^T = 4A^{-1} = 4\frac{\text{adj}A}{|A|}$

$\Rightarrow \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} = \dfrac{4}{|A|}\begin{bmatrix} c_{11} & c_{21} & c_{31} \\ c_{12} & c_{22} & c_{32} \\ c_{13} & c_{23} & c_{33} \end{bmatrix}$

Now $a_{ij} = \frac{4}{|A|} c_{ij}$

$\Rightarrow \ -2c_{ij} = \dfrac{4}{|A|} c_{ij}(as a_{ij} + 2c_{ij} = 0)$

$\Rightarrow |A| = -2$

Now $|A + 4I| = |A + AA^T|$

$= |A||I + A^T|$

$= -2|(I + A)^T|$

$= -2|I + A|$

$\Rightarrow |A + 4I| + 2|A + I| = 0,$

So on comparing, we get $5\lambda = 2 \Rightarrow \lambda = \frac{2}{5}$

Hence, $10\lambda = 4$