13.LIMITS AND DERIVATIVES

Single Correct Answer Type

1. $\lim_{x\to 0}\frac{\log(1+x+x^2)+\log(1-x+x^2)}{\sec x-\cos x}=$

a) -1 b) 1 c) 0 d) 2

2. The value of $\lim_{x\to\infty}\frac{\left(2^{x^n}\right)^{\frac{1}{e^x}}-\left(3^{x^n}\right)^{\frac{1}{e^x}}}{x^n}$ (where $n\in N$) is

a) $\log n\left(\frac{2}{3}\right)$ b) 0 c) $n\log n\left(\frac{2}{3}\right)$ d) Not defined

3. $\lim_{x\to 1}\left[\mathrm{cosec}\frac{\pi x}{2}\right]^{1/(1-x)}$ (where $[\cdot]$) represents the greatest integer function) is equal to

a) 0 b) 1 c) ∞ d) Does not exist

4. $\lim_{n\to\infty}\left(\frac{n^2-n+1}{n^2-n-1}\right)^{n(n-1)}$ is equal to

a) e b) e^2 c) e^{-1} d) 1

5. $\lim_{x\to 0}\frac{x(e^x-1)}{1-\cos x}$ is equal to

a) 0 b) ∞ c) -2 d) 2

6. $\lim_{x\to 1}\frac{1-x^2}{\sin 2\pi x}$ is equal to

a) $\frac{1}{2\pi}$ b) $\frac{-1}{\pi}$ c) $\frac{-2}{\pi}$ d) None of these

7. The value of $\lim_{x\to\pi}\frac{1+\cos^3 x}{\sin^2 x}$ is

a) 1/3 b) 2/3 c) $-1/4$ d) 3/2

8. If $f(x)=\begin{cases}\frac{\sin[x]}{x}, & \text{for } [x]\neq 0\\ 0, & \text{for } [x]=0\end{cases}$, where $[x]$ denotes the greatest integer less than or equal to x, then $\lim_{x\to 0}f(x)$ is

a) 1 b) 0 c) -1 d) None of these

9. $\lim_{x\to 0}\frac{x^4(\cot^4 x-\cot^2 x+1)}{(\tan^4 x-\tan^2 x+1)}$ is equal to

a) 1 b) 0 c) 2 d) None of these

10. $\lim_{x\to 0}\frac{x\tan 2x-2x\tan x}{(1-\cos 2x)^2}$ is equal to

a) 2 b) -2 c) 1/2 d) $-1/2$

11. $\lim_{x\to 0}\frac{\sin x^n}{(\sin x)^m}$, $(m<n)$ is equal to

a) 1 b) 0 c) n/m d) None of these

12. The value of $\lim_{x\to 2}\frac{\sqrt{1+\sqrt{2+x}}-\sqrt{3}}{x-2}$ is

a) $\frac{1}{8\sqrt{3}}$ b) $\frac{1}{4\sqrt{3}}$ c) 0 d) None of these

13. $\lim_{x\to\pi/2}\frac{\sin(x\cos x)}{\cos(x\sin x)}$ is equal to

a) 0 b) $p/2$ c) p d) $2p$

14. $\lim_{x\to\infty}\frac{\sin^4 x-\sin^2 x+1}{\cos^4 x-\cos^2 x+1}$ is equal to

a) 0 b) 1 c) $\frac{1}{3}$ d) $\frac{1}{2}$

15. If $\lim_{x\to -2^-}\frac{ae^{1/|x+2|}-1}{2-e^{1/|x+1|}}=\lim_{x\to -2^+}\sin\left(\frac{x^4-16}{x^5+32}\right)$, then a is

a) $\sin\frac{3}{5}$ b) 2 c) $\sin\frac{2}{5}$ d) $\sin\frac{1}{5}$

16. $\lim_{x\to 1} \frac{nx^{n+1}-(n+1)x^n+1}{(e^x-e)\sin \pi x}$, where $n = 100$ is equal to

a) $\frac{5050}{\pi e}$ b) $\frac{100}{\pi e}$ c) $-\frac{5050}{\pi e}$ d) $-\frac{4950}{\pi e}$

17. $\lim_{x\to 0} \left\{(1+x)^{\frac{2}{x}}\right\}$ (where $\{\cdot\}$ denotes the fractional part of x) is equal to

a) $e^2 - 7$ b) $e^2 - 8$ c) $e^2 - 6$ d) None of these

18. $\lim_{x\to\infty} \left[\sqrt{x+\sqrt{x+\sqrt{x}}} - \sqrt{x}\right]$ is equal to

a) 0 b) $\frac{1}{2}$ c) $\log 2$ d) e^4

19. The value of $\lim_{x\to a} \sqrt{a^2-x^2} \cot \frac{\pi}{2} \sqrt{\frac{a-x}{a+x}}$ is

a) $\frac{2a}{\pi}$ b) $-\frac{2a}{\pi}$ c) $\frac{4a}{\pi}$ d) $-\frac{4a}{\pi}$

20. $\lim_{x\to 1} \frac{1+\sin \pi\left(\frac{3x}{1+x^2}\right)}{1+\cos \pi x}$ is equal to

a) 0 b) 1 c) 2 d) 4

21. If $f: (1, 2) \to R$ satisfies the inequality

$\frac{\cos(2x-4)-33}{2} < f(x) < \frac{x^2|4x-8|}{x-2}, \forall\, x \in (1, 2)$, then $\lim_{x\to 2^-} f(x)$ is

a) 16 b) Cannot be determined from the given information

c) -16 d) Does not exist

22. $\lim_{x\to\infty} \frac{(2x+1)^{40}\,(4x-1)^5}{(2x+3)^{45}}$ is equal to

a) 16 b) 24 c) 32 d) 8

23. If $\lim_{x\to 0} \frac{x^n - \sin x^n}{x - \sin^n x}$ is non-zero finite, then n must be equal

a) 4 b) 1 c) 2 d) 3

24. Let $\lim_{x\to 0} \frac{[x]^2}{x^2} = l$ and $\lim_{x\to 0} \frac{[x^2]}{x^2} = m$, where $[\cdot]$ denotes greatest integer, then

a) l exists but m does not b) m exists but l does not

c) Both l and m exist d) Neither l nor m exists

25. The value of $\lim_{x\to 1} \left(\frac{p}{1-x^p} - \frac{q}{1-x^q}\right); p, q, \in N$ equals

a) $\frac{p+q}{2}$ b) $\frac{pq}{2}$ c) $\frac{p-q}{2}$ d) $\sqrt{\frac{p}{q}}$

26. If $\lim_{x\to 0} \frac{[(a-n)nx - \tan x]\sin nx}{x^2} = 0$, where n is non-zero real number, then a is equal to

a) 0 b) $\frac{n+1}{n}$ c) n d) $n + \frac{1}{n}$

27. If $\lim_{x\to\infty} \left(\frac{x^3+1}{x^2+1} - (ax+b)\right) = 2$, then

a) $a = 1, b = 1$ b) $a = 1, b = 2$ c) $a = 1, b = -2$ d) None of these

28. The value of $\lim_{x\to 2} \frac{2^x + 2^{3-x} - 6}{\sqrt{2^{-x}} - 2^{1-x}}$ is

a) 16 b) 8 c) 4 d) 2

29. The value of $\lim_{n\to\infty} \left[\frac{1}{n} + \frac{e^{1/n}}{n} + \frac{e^{2/n}}{n} + \cdots + \frac{e^{(n-1)/n}}{n}\right]$ is

a) 1 b) 0 c) $e - 1$ d) $e + 1$

30. $\lim_{y\to 0} \frac{(x+y)\sec(x+y) - x\sec x}{y}$ is equal to

a) $\sec x\,(x\tan x + 1)$ b) $x\tan x + \sec x$ c) $x\sec x + \tan x$ d) None of these

31. $\lim_{x\to\infty} \frac{\cot^{-1}(x^{-a}\log_a x)}{\sec^{-1}(a^x \log_x a)}$ $(a > 1)$ is equal to

a) 2 b) 1 c) $\log_a 2$ d) 0

32. $\lim_{h\to 0}\frac{f(2h+2+h^2)-f(2)}{f(h-h^2+1)-f(1)}$, given that $f'(2) =$ and $f'(1) = 4$

a) does not exist b) is equal to -3/2 c) is equal to 3/2 d) is equal to 3

33. If $f(x) = \begin{cases} x^n \sin(1/x^2), x \neq 0 \\ 0, x \neq 0 \end{cases}$, $(n \in I)$, then

a) $\lim\limits_{x\to 0} f(x)$ exists for $n > 1$ b) $\lim\limits_{x\to 0} f(x)$ exists for $n < 0$

c) $\lim\limits_{x\to 0} f(x)$ does not exist for any value of n d) $\lim\limits_{x\to 0} f(x)$ cannot be determined

34. If $\lim\limits_{x\to a}\left(\frac{f(x)}{g(x)}\right)$ exist, then

a) Both $\lim\limits_{x\to a} f(x)$ and $\lim\limits_{x\to a} g(x)$ must exist

b) $\lim\limits_{x\to a} f(x)$ need not exist but $\lim\limits_{x\to a} g(x)$ exists

c) Neither $\lim\limits_{x\to a} f(x)$ nor $\lim\limits_{x\to a} g(x)$ may exist

d) $\lim\limits_{x\to a} f(x)$ exists but $\lim\limits_{x\to a} g(x)$ need not exist

35. The value of $\lim\limits_{x\to 2}\left(\left(\frac{x^3-4x}{x^3-8}\right)^{-1} - \left(\frac{x+\sqrt{2x}}{x-2} - \frac{\sqrt{2}}{\sqrt{x}-\sqrt{2}}\right)^{-1}\right)$ is

a) 1/2 b) 2 c) 1 d) None of these

36. $\lim\limits_{x\to\infty}\frac{e^{1/x^2}-1}{2\tan^{-1}(x^2)-\pi}$ is equal to

a) 1 b) -1 c) $\frac{1}{2}$ d) $-\frac{1}{2}$

37. $\lim_{x\to 1}\frac{x\sin(x-[x])}{x-1}$, where $[\cdot]$ denotes the greatest integer function, is equal to

a) 0 b) -1 c) Non-existent d) None of these

38. For $x \in R$, $\lim\limits_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x$ is equal to

a) e b) e^{-1} c) e^{-5} d) e^5

39. $\lim_{x\to 0}\left[\frac{\sin(\text{sgn}(x))}{(\text{sgn}(x))}\right]$, where $[\cdot]$ denotes the greatest integer function, is equal to

a) 0 b) 1 c) -1 d) Does not exist

40. $\lim\limits_{x\to\infty}\left((x+5)\tan^{-1}(x+5) - (x+1)\tan^{-1}(x+1)\right)$ is equal to

a) π b) 2π c) $\pi/2$ d) None of these

41. $\lim\limits_{x\to\infty}\left(\frac{1}{e} - \frac{x}{1+x}\right)^x$ is equal to

a) $\frac{e}{1-e}$ b) 0 c) $e^{\frac{e}{1-e}}$ d) Does not exist

42. Let $f(x) = \begin{cases} x+1, x > 0 \\ 2-x, x \leq 0 \end{cases}$ and

$g(x) = \begin{cases} x+3, & x < 1 \\ x^2-2x-2, & 1 \leq x < 2 \\ x-5, & x \geq 2 \end{cases}$, then $\lim\limits_{x\to 0} g(f(x))$ is

a) 2 b) 1 c) -3 d) Does not exists

43. $\lim_{x\to\infty}\left(\frac{x^3}{3x^2-4} - \frac{x^2}{3x+2}\right)$ is equal to

a) Does not exist b) 1/3 c) 0 d) 2/9

44. $\lim\limits_{n\to\infty}\sum_{x=1}^{20}\cos^{2n}(x-10)$ is equal to

a) 0 b) 1 c) 19 d) 20

45. The value of the limit $\lim\limits_{x\to 0}\frac{a^{\sqrt{x}}-a^{1/\sqrt{x}}}{a^{\sqrt{x}}+a^{1/\sqrt{x}}}$, $a > 1$ is

a) 4 b) 2 c) -1 d) 0

46. If $f(x) = \sqrt{\frac{x-\sin x}{x+\cos^2 x}}$, then $\lim\limits_{x\to\infty} f(x)$ is

a) 0 b) ∞ c) 1 d) None of these

47. The value of $\lim_{x\to 0}\frac{1+\sin x-\cos x+\log(1-x)}{x^3}$ is

a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) 0 d) None of these

48. $\lim_{x\to\infty}\frac{x^2\tan\frac{1}{x}}{\sqrt{8x^2+7x+1}}$ is equal to

a) $-\frac{1}{2\sqrt{2}}$ b) $\frac{1}{2\sqrt{2}}$ c) $\frac{1}{\sqrt{2}}$ d) Does not exist

49. $\lim_{x\to 0}\left[\min(y^2-4y+11)\frac{\sin x}{x}\right]$ (where $[\cdot]$ denotes the greatest integer function) is

a) 5 b) 6 c) 7 d) Does not exist

50. $\lim_{x\to\pi/2}\left[x\tan x-\left(\frac{\pi}{2}\right)\sec x\right]$ is equal to

a) 1 b) -1 c) 0 d) None of these

51. $\lim_{x\to 0}\frac{\sin(\pi\cos^2 x)}{x^2}$ is equal to

a) $-\pi$ b) π c) $\pi/2$ d) 1

52. If $f(x)=0$ be a quadratic equation such that $f(-\pi)=f(\pi)=0$ and $f\left(\frac{\pi}{2}\right)=-\frac{3\pi^2}{4}$, then $\lim_{x\to-\pi}\frac{f(x)}{\sin(\sin x)}$ is equal to

a) 0 b) π c) 2π d) None of these

53. The value of $\lim_{m\to\infty}\left(\cos\frac{x}{m}\right)^m$ is

a) 1 b) e c) e^{-1} d) None of these

54. $\lim_{x\to\infty}\frac{(1+x+x^2)}{x(\text{In }x)^3}$ is equal to

a) 2 b) e^2 c) e^{-2} d) None of these

55. $\lim_{x\to-1}\frac{1}{\sqrt{|x|-\{-x\}}}$ (where $\{x\}$ denotes the fractional part of x) is equal to

a) Does not exist b) 1 c) ∞ d) $\frac{1}{2}$

56. If $G(x)=-\sqrt{25-x^2}$, then $\lim_{x\to 1}\frac{G(x)-G(1)}{x-1}$ is

a) $\frac{1}{24}$ b) $\frac{1}{5}$ c) $-\sqrt{24}$ d) None of these

57. $\lim_{x\to 0}\frac{(2^m+x)^{1/m}-(2^n+x)^{1/n}}{x}$ is equal to

a) $\frac{1}{m2^m}-\frac{1}{n2^n}$ b) $\frac{1}{m2^m}+\frac{1}{n2^n}$ c) $\frac{1}{m2^{m-1}}-\frac{1}{n2^{n-1}}$ d) $\frac{1}{m2^{m-1}}+\frac{1}{n2^{n-1}}$

58. The value of $\lim_{x\to 1}(2-x)^{\tan\frac{\pi x}{2}}$ is

a) $e^{-2/\pi}$ b) $e^{1/\pi}$ c) $e^{2/\pi}$ d) $e^{-1/\pi}$

59. $\lim_{n\to\infty}\left\{\frac{1}{1-n^2}+\frac{2}{1-n^2}+\cdots+\frac{n}{1-n^2}\right\}$ is equal to

a) 0 b) $-\frac{1}{2}$ c) $\frac{1}{2}$ d) None of these

60. $\lim_{x\to 0}\frac{x^a\sin^b x}{\sin(x^c)}$, where $a,b,c\in R\sim\{0\}$, exists and has non-zero value, then

a) $a+c=b$ b) $b+c=a$ c) $a+b=c$ d) None of these

61. $\lim_{x\to 0}\left(\frac{1+\tan x}{1+\sin x}\right)^{\operatorname{cosec} x}$ is equal to

a) e b) $\frac{1}{e}$ c) 1 d) None of these

62. $\lim_{x\to 0}\frac{1}{x}\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is equal to

a) 1 b) 0 c) 2 d) None of these

63. If $\lim_{n\to\infty}\frac{n.3^n}{n(x-2)^n+n.3^{n+1}-3^n}=\frac{1}{3}$, then the range, of x is (where $n\in N$)

a) $[2,5)$ b) $(1,5)$ c) $(-1,5)$ d) $(-\infty,\infty)$

64. If $\lim_{x\to 0}(x^{-3}\sin 3x + ax^{-2} + b)$ exists and is equal to 0, then

a) $a = -3$ and $b = 9/2$
b) $a = 3$ and $b = 9/2$
c) $a = -3$ and $b = -9/2$
d) $a = 3$ and $b = -9/2$

65. $\lim_{x\to 0}\left[(1-e^x)\frac{\sin x}{|x|}\right]$ is (where $[\cdot]$ represents the greatest integer function)

a) -1
b) 1
c) 0
d) Does not exist

66. $\lim_{x\to 0}\frac{\cos(\tan x)-\cos x}{x^4}$ is equal to

a) 1/6
b) $-1/3$
c) 1/2
d) 1

67. If $\lim_{x\to 0}\frac{x^n\sin^n x}{x^n-\sin^n x}$ is non-zero finite, then n is equal to

a) 1
b) 2
c) 3
d) None of these

68. If $f(x)=\frac{2}{x-3}$, $g(x)=\frac{x-3}{x+4}$ and $h(x)=-\frac{2(2x+1)}{x^2+x-12}$, then $\lim_{x\to 3}[f(x)+g(x)+h(x)]$ is

a) -2
b) -1
c) $-\frac{2}{7}$
d) 0

69. Let $f(x)=\lim_{n\to\infty}\frac{1}{\left(\frac{3}{\pi}\tan^{-1}2x\right)^{2n}+5}$. Then the set of values of x for which $f(x)=0$ is

a) $|2x|>\sqrt{3}$
b) $|2x|<\sqrt{3}$
c) $|2x|\geq\sqrt{3}$
d) $|2x|\leq\sqrt{3}$

70. The value of $\lim_{x\to 0}\left(\left[\frac{100x}{\sin x}\right]+\left[\frac{99\sin x}{x}\right]\right)$ (where $[\cdot]$ represents the greatest integral function) is

a) 199
b) 198
c) 0
d) None of these

71. $\lim_{x\to 1}\frac{(1-x)(1-x^2)\dots(1-x^{2n})}{\{(1-x)(1-x^2)\dots(1-x^n)\}^2}, n\varepsilon N$

a) ${}^{2n}P_n$
b) ${}^{2n}C_n$
c) $(2n)!$
d) None of these

72. $\lim_{x\to -1}\left(\frac{x^4+x^2+x+1}{x^2-x+1}\right)^{\frac{1-\cos(x+1)}{(x+1)^2}}$ is equal to

a) 1
b) $(2/3)^{1/2}$
c) $(3/2)^{1/2}$
d) $e^{1/2}$

73. $\lim_{x\to\infty}\frac{2\sqrt{x}+3\sqrt[3]{x}+4\sqrt[4]{x}+\dots+n\sqrt[n]{x}}{\sqrt{(2x-3)}+\sqrt[3]{(2x-3)}+\dots+\sqrt[n]{(2x-3)}}$ is equal to

a) 1
b) ∞
c) $\sqrt{2}$
d) None of these

74. For $x>0$, $\lim_{x\to 0}\left((\sin x)^{1/x}+\left(\frac{1}{x}\right)^{\sin x}\right)$ is

a) 0
b) -1
c) 1
d) 2

75. $\lim_{x\to\infty}\frac{2+2x+\sin 2x}{(2x+\sin 2x)e^{\sin x}}$ is equal to

a) 0
b) 1
c) -1
d) Does not exist

76. $\lim_{x\to 0}\left(\frac{1^x+2^x+3^x+\dots+n^x}{n}\right)^{1/x}$ is equal to

a) $(n!)^n$
b) $(n!)^{1/n}$
c) $n!$
d) $In\ (n!)$

77. The value of $\lim_{n\to\infty}\left[\frac{2n}{2n^2-1}\cos\frac{n+1}{2n-1}-\frac{n}{1-2n}\cdot\frac{n(-1)^n}{n^2+1}\right]$ is

a) 1
b) -1
c) 0
d) None of these

78. If $f(x)=\frac{\cos x}{(1-\sin x)^{1/3}}$, then

a) $\lim_{x\to\frac{\pi}{2}}f(x)=-\infty$
b) $\lim_{x\to\frac{\pi^+}{2}}f(x)=\infty$
c) $\lim_{x\to\frac{\pi}{2}}f(x)=\infty$
d) None of these

79. If $f(x)$ is differentiable and strictly increasing function, then the value of $\lim_{x\to 0}\frac{f(x^2)-f(x)}{f(x)-f(0)}$ is

a) 1
b) 0
c) -1
d) 2

80. $f(x)=\frac{\text{In}\ (x^2+e^x)}{\text{In}\ (x^4+e^{2x})}$. Then $\lim_{n\to\infty}f(x)$ is equal to

a) 1
b) 1/2
c) 2
d) None of these

81. $\lim_{x\to\infty}\frac{(x+1)^{10}+(x+2)^{10}+\dots+(x+100)^{10}}{x^{10}+10^{10}}$ is equal to

a) 0
b) 1
c) 10
d) 100

82. The value of $\lim_{x\to1^-}\frac{1-\sqrt{x}}{(\cos^{-1}x)^2}$ is

a) 4 b) 1/2 c) 2 d) 1/4

83. $\lim_{x\to\infty}\frac{n(2n+1)^2}{(n+2)(n^2+3n-1)}$ is equal to

a) 0 b) 2 c) 4 d) ∞

84. The value of $\lim_{x\to0}\frac{\sqrt{\frac{1}{2}(1-\cos 2x)}}{x}$ is

a) 1 b) -1 c) 0 d) None of these

85. $\lim_{x\to\infty}\left(\frac{x^2+2x-1}{2x^2-3x-2}\right)^{\frac{2x+1}{2x-1}}$ is equal to

a) 0 b) ∞ c) 1/2 d) None of these

86. $\lim_{x\to1}\frac{\sqrt{1-\cos 2(x-1)}}{x-1}$

a) Exists and it equals $\sqrt{2}$

b) Exits and it equals $-\sqrt{2}$

c) Does not exist because $x-1\to0$

d) Does not exist because the left-hand limit is not equal to the right-hand limit

87. $\lim_{x\to0}\frac{\sin(x^2)}{\text{In}(\cos(2x^2-x))}$ is equal to

a) 2 b) -2 c) 1 d) -1

88. $\lim_{n\to\infty}\left(\left(\frac{n}{n+1}\right)^\alpha+\sin\frac{1}{n}\right)^n$ (where $\alpha\in Q$) is equal to

a) $e^{-\alpha}$ b) $-\alpha$ c) $e^{1-\alpha}$ d) $e^{1+\alpha}$

89. If $f(x)=\lim_{n\to\infty}n(x^{1/n}-1)$, then for $x>0, y>0, f(xy)$ is equal to

a) $f(x)f(y)$ b) $f(x)+f(y)$ c) $f(x)-f(y)$ d) None of these

90. The value of $\lim_{x-0}\frac{1}{x^3}\int_0^x\frac{t\log(1+t)}{t^4+4}dt$ is

a) 0 b) $\frac{1}{12}$ c) $\frac{1}{24}$ d) $\frac{1}{64}$

91. If $x_1=3$ and $x_{n+1}=\sqrt{2+x_n}, n\ge1$, then $\lim_{n\to\infty}x_n$ is

a) -1 b) 2 c) $\sqrt{5}$ d) 3

92. $\lim_{n\to\infty}n^2(x^{1/n}-x^{1/(n+1)}), x>0$, is equal to

a) 0 b) e^x c) $\log_e x$ d) None of these

93. $\lim_{x\to\frac{\pi}{4}}\frac{\int_2^{\sec^2x}f(t)dt}{x^2-\frac{\pi^2}{16}}$ equals

a) $\frac{8}{\pi}f(2)$ b) $\frac{2}{\pi}f(2)$ c) $\frac{2}{\pi}f\left(\frac{1}{2}\right)$ d) $4f(2)$

94. Among (i) $\lim_{x\to\infty}\sec^{-1}\left(\frac{x}{\sin x}\right)$ and (ii) $\lim_{x\to\infty}\sec^{-1}\left(\frac{\sin x}{x}\right)$

a) (i) exists, (ii) does not exist b) (i) does not exist, (ii) exists

c) Both (i) and (ii) exist d) Neither (i) nor (ii) exists

95. The value of $\lim_{x\to\frac{1}{\sqrt{2}}}\frac{x-\cos(\sin^{-1}x)}{1-\tan(\sin^{-1}x)}$ is

a) $-\frac{1}{\sqrt{2}}$ b) $\frac{1}{\sqrt{2}}$ c) $\sqrt{2}$ d) $-\sqrt{2}$

Multiple Correct Answers Type

96. Given a real-valued function f such that

$f(x)=\begin{cases}\frac{\tan^2\{x\}}{(x^2-[x]^2)} & \text{for } x>0\\ 1 & \text{for } x=0\\ \sqrt{\{x\}\cot\{x\}} & \text{for } x<0\end{cases}$, where $[x]$ is the integral

Part and $\{x\}$ is the fractional part of x, then

a) $\lim_{x\to 0^+} f(x)=1$
b) $\lim_{x\to 0^-} f(x)=\cot 1$
c) $\cot^{-1}\left(\lim_{x\to 0^-} f(x)\right)^2=1$
d) $\tan^{-1}\left(\lim_{x\to 0^+} f(x)\right)=\frac{\pi}{4}$

97. Which of the following is true ($\{\cdot\}$ denotes the fractional part of the function)?

a) $\lim_{x\to\infty}\frac{\log_e x}{\{x\}}=\infty$
b) $\lim_{x\to 2^+}\frac{x}{x^2-x-2}=\infty$
c) $\lim_{x\to -1^-}\frac{x}{x^2-x-2}=-\infty$
d) $\lim_{x\to\infty}\frac{\log_{0.5} x}{\{x\}}=\infty$

98. Given $\lim_{x\to 0}\frac{f(x)}{x^2}=2$, where $[\cdot]$ denotes greatest integer function, then

a) $\lim_{x\to 0}[f(x)]=0$
b) $\lim_{x\to 0}[f(x)]=1$
c) $\lim_{x\to 0}\left[\frac{f(x)}{x}\right]$ does not exists
d) $\lim_{x\to 0}\left[\frac{f(x)}{x}\right]$ exists

99. $\lim_{n\to\infty}\frac{-3n+(-1)^n}{4n-(-1)^n}$ is equal to

a) $-\frac{3}{4}$
b) 0 if n is even
c) $-\frac{3}{4}$ if n is odd
d) None of these

100. $\lim_{n\to\infty}\frac{1}{1+n\sin^2 nx}$ is equal to

a) -1
b) 0
c) 1
d) ∞

101. If $\lim_{x\to\infty}\left(an-\frac{1+n^2}{1+n}\right)=b$, where a is finite number, then

a) $a=1$
b) $a=0$
c) $b=1$
d) $b=-1$

102. If $m,n\in N$, $\lim_{x\to 0}\frac{\sin x^n}{(\sin x)^m}$ is

a) 1, if $n=m$
b) 0, if $n>m$
c) ∞, if $n<m$
d) n/m, if $n<m$

103. If $f(x)=\frac{3x^2+ax+a+1}{x^2+x-2}$, then which of the following can be correct

a) $\lim_{x\to 1} f(x)$ exists $\Rightarrow a=-2$
b) $\lim_{x\to -2} f(x)$ exists $\Rightarrow a=13$
c) $\lim_{x\to 1} f(x)=4/3$
d) $\lim_{x\to -2} f(x)=-1/3$

104. Let $f(x)=\begin{cases}1+\frac{2x}{a}, & 0\le x<1\\ ax, & 1\le x<2\end{cases}$. If $\lim_{x\to 1} f(x)$ exists, then a is

a) 1
b) -1
c) 2
d) -2

105. If $\lim_{x\to 1}(2-x+a[x-1]+b[1+x])$ exists, then a and b can take the values (where $[\cdot]$ denotes, the greatest integer function)

a) $a=1/3, b=1$
b) $a=1, b=-1$
c) $a=9, b=-9$
d) $a=2, b=2/3$

106. If $f(x)=|x-1|-[x]$, where $[x]$ is the greatest integer less than or equal to x, then

a) $f(1+0)=-1, f(1-0)=0$
b) $f(1+0)=0=f(1-0)$
c) $\lim_{x\to 1} f(x)$ exists
d) $\lim_{x\to 1} f(x)$ does not exist

107. Let $f(x)=\frac{x^2-9x+20}{x-[x]}$ (where $[x]$ is the greatest integer not greater than x), then

a) $\lim_{x\to 5^-} f(x)=0$
b) $\lim_{x\to 5^+} f(x)=1$
c) $\lim_{x\to 5} f(x)$ does not exist
d) None of these

108. $L=\lim_{x\to a}\frac{|2\sin x-1|}{2\sin x-1}$, then

a) Limit does not exist when $a=\pi/6$
b) $L=-1$ when $a=\pi$
c) $L=1$ when $a=\pi/2$
d) $L=1$ when $a=0$

109. $f(x)=\lim_{n\to\infty}\frac{x}{x^{2n}+1}$, then

a) $f(1^+)+f(1^-)=0$

b) $f(1^+)+f(1^-)+f(1)=3/2$

c) $f(-1^+)+f(-1^-)=-1$

d) $f(1^+)+f(-1^-)=0$

Assertion - Reasoning Type

This section contain(s) 0 questions numbered 110 to 109. Each question contains STATEMENT 1(Assertion) and STATEMENT 2(Reason). Each question has the 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

a) Statement 1 is True, Statement 2 is True; Statement 2 **is** correct explanation for Statement 1

b) Statement 1 is True, Statement 2 is True; Statement 2 **is not** correct explanation for Statement 1

c) Statement 1 is True, Statement 2 is False

d) Statement 1 is False, Statement 2 is True

110

Statement 1: $\lim_{x\to\infty}\left(\frac{1^2}{x^3}+\frac{2^2}{x^3}+\frac{3^2}{x^3}+\cdots+\frac{x^2}{x^3}\right)=\lim_{x\to\infty}\frac{1^2}{x^3}+\lim_{x\to\infty}\frac{2^2}{x^3}+\cdots+\lim_{x\to\infty}\frac{x^2}{x^3}=0$

Statement 2: $\lim_{x\to a}\left(f_1(x)+f_2(x)+\cdots+f_n(x)\right)=\lim_{x\to a}f_1(x)+\lim_{x\to a}f_2(x)+\cdots+\lim_{x\to a}f_n(x)$, where $n\in N$

111

Statement 1: $\lim_{x\to 0}\frac{\sqrt{1-\cos 2x}}{x}$ does not exist

Statement 2: $f(x)=\frac{\sqrt{1-\cos 2x}}{x}$ is not defined at $x=0$

112

Statement 1: $\lim_{x\to 0}[x]\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$ (where $[.]$ represents the greatest integer function) does not exist

Statement 2: $\lim_{x\to 0}\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$ does not exist

113

Statement 1: If $< a_n >$ be a sequence such that $a_1=1$ and $a_{n+1}=\sin a_n$, then $\lim_{n\to\infty}a_n=0$

Statement 2: Since $x>\sin x\ \forall\ x>0$

114

Statement 1: If $\lim_{x\to 0}\left(f(x)+\frac{\sin x}{x}\right)$ does not exist, then $\lim_{x\to 0}f(x)$ does not exist

Statement 2: $\lim_{x\to 0}\frac{\sin x}{x}$ exists and has value 1

115

Statement 1: $\left[\lim_{x\to 0}\frac{\sin x}{x}\right]=0$

Statement 2: For $x\in(-\delta,\delta)$, where δ is positive and $\delta\to 0, \tan x>x$

116

Statement 1: $\lim_{x\to 0}\sin^{-1}\{x\}$ does not exist (where $\{\cdot\}$ denotes fractional part function)

Statement 2: $\{x\}$ is discontinuous at $x = 0$

117

Statement 1: If a and b are positive and $[x]$ denotes the greatest integer $\leq x$, then $\lim_{x\to 0^+} \frac{x}{a}\left[\frac{b}{x}\right] = \frac{b}{a}$

Statement 2: $\lim_{x\to\infty} \frac{\{x\}}{x} \to 0$, where $\{x\}$ denotes fractional part of x

118

Statement 1: $\lim_{x\to 0} \log_e \left(\frac{\sin x}{x}\right) = 0$

Statement 2: $\lim_{x\to a} f\big(g(x)\big) = f(\lim_{x\to 0} g(x))$

119

Statement 1: $\lim_{m\to\infty} \lim_{n\to\infty} \{\sin^{2m}(n!\,\pi x)\} = 0, m, n \in N$, when x is rational

Statement 2: When $n \to \infty$ and x is rational, $n!\,x$ is integer

120

Statement 1: $\lim_{x\to\alpha} \frac{\sin(f(x))}{x-\alpha}$, where $f(x) = ax^2 + bx + c$, is finite and non-zero, then $\lim_{x\to\alpha} \frac{e^{\frac{1}{f(x)}}-1}{e^{\frac{1}{f(x)}}+1}$ does not exist

Statement 2: $\lim_{x\to\alpha} \frac{\sin(f(x))}{x-\alpha}$ can take finite value only when it takes $\frac{0}{0}$ form

121

Statement 1: If $f(x) = \begin{cases} x, \text{if } x \text{ is rational} \\ 1-x, \text{if } x \text{ is rational} \end{cases}$, then $\lim_{x\to 1/2} f(x)$ does not exist

Statement 2: $x \to 1/2$ can be rational or irrational value

122

Statement 1: If $f(x) = \frac{(x-1)(x-2)}{(x-3)(x-4)}$, then $\lim_{x\to -\infty} \sin^{-1} f(x)$ exists, but $\lim_{x\to\infty} \cos^{-1} f(x)$ does not exist

Statement 2: $\sin^{-1} x$ and $\cos^{-1} x$ are defined for $x \in [-1, 1]$

Matrix-Match Type

This section contain(s) 0 question(s). Each question contains Statements given in 2 columns which have to be matched. Statements (A, B, C, D) in **columns I** have to be matched with Statements (p, q, r, s) in **columns II**.

123.

Column-I	Column- II
(A) If $L = \lim_{x\to -1} \frac{\sqrt[3]{(7-x)}-2}{(x+1)}$, then $12L =$	(p) -2
(B) If $L = \lim_{x\to \pi/4} \frac{\tan^3 x - \tan x}{\cos\left(x+\frac{\pi}{4}\right)}$, then $-L/4 =$	(q) 2
(C) If $L = +\lim_{x\to 1} \frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3}$, then $20L =$	(r) 1
(D) If $L = \lim_{x\to\infty} \frac{\log x^n - [x]}{[x]}$, where $n \in N$, ([x] denotes	(s) -1

greatest less than or equal to x), then $-2L =$

CODES :

	A	B	C	D
a)	s	r	p	q
b)	r	p	q	s
c)	p	q	s	r
d)	q	s	r	p

124.

	Column-I		Column- II
(A)	If $\lim_{x\to\infty} \left(\sqrt{x^2 - x - 1}\right) - ax - b = 0$, where $a > 0$, then there exists at least one a and b for which point $(a, 2b)$ lies on the line	(p)	$y = -3$
(B)	If $\lim_{x\to\infty} \frac{(1+a^3)+8e^{1/x}}{1+(1-b^3)e^{1/x}} = 2$, then there exist at least one a and b for which point (a, b^3) lies on the line	(q)	$3x - 2y - 5 = 0$
(C)	If $\lim_{x\to\infty} \left(\sqrt{x^4 - x^2 + 1}\right) - ax^2 - b) = 0$, then there exists at least one a and b for which point $(a, -4b)$ lies on the line	(r)	$15x - 2y - 11 = 0$
(D)	If $\lim_{x\to -a} \frac{x^7+a^7}{x+a} = 7$, where $a < 0$, then there exists at least one a for which point $(-a, 2)$ lies on the line	(s)	$y = 2$

CODES :

	A	B	C	D
a)	P,q	r,s	p,r	r,s
b)	r,	p,q	p,r	s
c)	q	p,q,r	r,s	r,s
d)	r,	p,q	q	r

125.

	Column-I		Column- II
(A)	$\lim_{x\to 0}\left(\left[100\frac{\sin x}{x}\right] + \left[100\frac{\tan x}{x}\right]\right)$	(p)	198
(B)	$\lim_{x\to 0}\left(\left[100\frac{x}{\sin x}\right] + \left[100\frac{\tan x}{x}\right]\right)$	(q)	199
(C)	$\lim_{x\to 0}\left(\left[100\frac{\sin^{-1} x}{x}\right] + \left[100\frac{\tan^{-1} x}{x}\right]\right)$	(r)	200
(D)	$\lim_{x\to 0}\left(\left[100\frac{x}{\sin^{-1} x}\right] + \left[100\frac{\tan^{-1} x}{x}\right]\right)$	(s)	201

CODES :

	A	B	C	D
a)	r	q	p	s
b)	q	r	q	p
c)	p	s	p	r
d)	q	p	r	s

Linked Comprehension Type

This section contain(s) 8 paragraph(s) and based upon each paragraph, multiple choice questions have to be answered. Each question has atleast 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 126 to -126

Let $f(x)=\frac{\sin^{-1}(1-\{x\}\times\cos^{-1}(1-\{x\})}{\sqrt{2\{x\}}\times(1-\{x\})}$, where $\{x\}$ denotes the fractional part of x

126. $R=\lim_{x\to 0+} f(x)$ is equal to

a) $\frac{p}{2}$ b) $\frac{\pi}{2\sqrt{2}}$ c) $\frac{\pi}{\sqrt{2}}$ d) $\sqrt{2}\pi$

Paragraph for Question Nos. 127 to - 127

$A_i=\frac{x-a_i}{|x-a_i|}, i=1,2,\dots,n$ and if $a_1<a_2<a_3<\ldots<a_n$

127. If $1\le m\le n, m\in N$, then the value of $L=\lim_{x\to a_m^-}(A_1\,A_2\dots A_n)$ is

a) Always 1 b) Always -1 c) $(-1)^{n-m+1}$ d) $(-1)^{n-m}$

Paragraph for Question Nos. 128 to - 128

If $L=\lim_{x\to 0}\frac{\sin x+ae^{x}+be^{-x}+c\,\mathrm{In}(1+x)}{x^3}=\infty$

128. The value of L is

a) 1/2 b) $-1/3$ c) $-1/6$ d) 3

Paragraph for Question Nos. 129 to - 129

Let $a_1>a_2>a_3\dots a_n>1$;
$p_1>p_2>p_3\dots>p_n>0$; such that $p_1+p_2+p_3+\ldots+p_n$
Also $F(x)=(p_1a_1^x+p_2a_2^x+\ldots p_na_n^x)^{1/x}$

129. $\lim_{x\to 0^+}F(x)$ equals

a) $p_1\,\mathrm{In}\,a_1+p_2\,\mathrm{In}\,a_2+\ldots+p_n\,\mathrm{In}\,a_n$ b) $a_1^{p_1}+a_2^{p_2}+\ldots+a_n^{p_n}$

c) $a_1^{p_1}.a_2^{p_2}\dots a_n^{p_n}$

d) $\sum_{r=1}^{n} a_r p_r$

Integer Answer Type

130. If $L=\lim_{x\to 0}\frac{e^{-x^2/2}-\cos x}{x^3\sin x}$, then the value of $1/(3L)$ is

131. If $\lim_{x\to\infty} f(x)$ exists and is finite and nonzero and if $\lim_{x\to\infty}\left(f(x)+\frac{3f(x)-1}{f^2(x)}\right)=3$, then the value of $\lim_{x\to\infty} f(x)$ is

132. Let $\lim_{x\to 1}\frac{x^a-ax+a-1}{(x-1)^2}=f(a)$. Then the value of $f(4)$ is

133. If $L=\lim_{x\to 2}\frac{(10-x)^{1/3}-2}{x-2}$, then the value of $|1/(4L)|$ is

134. If $\lim_{x\to 1}(1+ax+bx^2)^{\frac{c}{x-1}}=e^3$, then the value of bc is

135. The value of $\lim_{x\to\infty}\frac{\log_e(\log_e x)}{e^{\sqrt{x}}}$ is

136. The integer n, for which $\lim_{x\to 0}\frac{(\cos x-1)(\cos x-e^x)}{x^n}$ is a finite non-zero number, is

137. If $L=\lim_{n\to\infty}(2\cdot 3^2\cdot 2^3\cdot 3^4\dots 2^{n-1}\cdot 3^n)^{\frac{1}{(n^2+1)}}$, then the value of L^4 is

138. $\lim_{x\to\infty} f(x)$, where $\frac{2x-3}{x}<f(x)<\frac{2x^2+5x}{x^2}$, is

139. If $f(x)=\begin{cases}x^2+2 & x\geq 2\\ 1-x & x<2\end{cases}$ and $g(x)=\begin{cases}2x & x>1\\ 3-x & x\leq 1\end{cases}$, then the value of $\lim_{x\to 1} f(g(x))$ is

140. If $L=\lim_{x\to\infty}\left(x-x^2\log_e\left(1+\frac{1}{x}\right)\right)$, then the value of $8L$ is

141. If $f(x)=\begin{cases}x-1, x\geq 1\\ 2x^2-2\ x<1\end{cases}$, $g(x)=\begin{cases}x+1, x>0\\ -x^2+1\ x\leq 0\end{cases}$ and $h(x)=|x|$, then find $\lim_{x\to 0} f\left(g(h(x))\right)$

142. If $\lim_{x\to 0}\left[1+x+\frac{f(x)}{x}\right]^{1/x}=e^3$, then the value of $\text{In}\left(\lim_{x\to 0}\left[1+\frac{f(x)}{x}\right]^{1/x}\right)$ is

143. If $\lim_{x\to 1}\frac{a\sin(x-1)+b\cos(x-1)+4}{x^2-1}=-2$, then $|a+b|$ is

144. If $\lim_{x\to 0}\frac{1-\sqrt{\cos 2x}\cdot\sqrt[3]{\cos 3x}\cdot\sqrt[4]{\cos 4x}\dots\sqrt[n]{\cos nx}}{x^2}$ has the value equal to 10, then the value of n equals

145. $f(x)=\frac{3x^2+ax+a+1}{x^2+x-2}$ and $\lim_{x\to -2} f(x)$ exists, then the value of $(a-4)$ is

146. Let $f''(x)$ be continuous at $x=0$. If $\lim_{x\to 0}\frac{2f(x)-3a\,f(2x)+bf\,(8x)}{\sin^2 x}$ exists and $f(0)\neq 0, f'(0)\neq 0$, then the value of $3a/b$ is

147. The value of $\lim_{n\to\infty}\left[\sqrt[3]{(n+1)^2}-\sqrt[3]{(n-1)^2}\right]$ is

148. Let $S_n=1+2+3+\dots+n$ and $P_n=\frac{S_2}{S_2-1}\cdot\frac{S_3}{S_3-1}\cdot\frac{S_4}{S_4-1}\cdot\dots\cdot\frac{S_n}{S_n-1}$, where $n\in N(n\geq 2)$. Then $\lim_{n\to\infty} P_n=$

149. The reciprocal of the value of $\lim_{x\to\infty}\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\dots\left(1-\frac{1}{n^2}\right)$ is

: ANSWER KEY :

1)	b	2)	b	3)	b	4)	b	89)	b	90)	b	91)	b	92)	c
5)	d	6)	b	7)	d	8)	d	93)	a	94)	a	95)	a	1)	
9)	a	10)	c	11)	b	12)	a		a,b,c,d	2)	a,b,c	3)	a,c	4)	
13)	b	14)	b	15)	c	16)	c		a,c						
17)	a	18)	b	19)	c	20)	a	5)	b,c	6)	a,c	7)	a,b,c	8)	
21)	c	22)	c	23)	b	24)	b		a,b,c,d						
25)	c	26)	d	27)	c	28)	b	9)	b,c	10)	b,c	11)	a,d	12)	
29)	c	30)	a	31)	b	32)	d		a,b,c						
33)	a	34)	c	35)	a	36)	d	13)	a,b,c	14)	b,c,d	1)	d	2)	b
37)	c	38)	c	39)	a	40)	b		3)	b	4)	a			
41)	d	42)	c	43)	d	44)	b	5)	a	6)	b	7)	b	8)	a
45)	c	46)	c	47)	b	48)	a	9)	c	10)	a	11)	a	12)	d
49)	b	50)	b	51)	b	52)	c	13)	a	1)	a	2)	c	3)	b
53)	a	54)	d	55)	a	56)	d		1)	a	2)	c	3)	b	
57)	c	58)	c	59)	b	60)	c		4)	c					
61)	c	62)	d	63)	a	64)	a	1)	4	2)	1	3)	6	4)	3
65)	a	66)	b	67)	b	68)	c	5)	3	6)	0	7)	3	8)	6
69)	a	70)	b	71)	b	72)	b	9)	2	10)	6	11)	4	12)	0
73)	c	74)	c	75)	d	76)	b	13)	2	14)	8	15)	6	16)	9
77)	c	78)	d	79)	c	80)	b	17)	7	18)	0	19)	3	20)	2
81)	d	82)	d	83)	c	84)	d								
85)	c	86)	d	87)	b	88)	c								

: HINTS AND SOLUTIONS :

1 **(b)**

$$\lim_{x\to 0}\frac{\log(1+x+x^2)+\log(1-x+x^2)}{\sec x-\cos x}$$
$$=\lim_{x\to 0}\frac{\log[(1+x^2)^2-x^2]}{(1-\cos^2 x)/\cos x}$$
$$=\lim_{x\to 0}\frac{\log(1+x^2+x^4)}{\sin x\tan x}$$
$$=\lim_{x\to 0}\frac{\log\left(1+x^2(1+x^2)\right)}{x^2(1+x^2)}\cdot x^2\,(1+x^2)\cdot\frac{1}{\frac{\sin x}{x}\cdot\frac{\tan x}{x}\cdot x^2}$$
$$=1\cdot\left(\text{as }\lim_{x\to 0}\frac{\log(1+x)}{x}=1\right)$$

2 **(b)**

$$L=\lim_{x\to\infty}\frac{(2x^n)^{\frac{1}{e^x}}-(3x^n)^{\frac{1}{e^x}}}{x^n}$$
$$=\lim_{x\to\infty}\frac{(3)^{\frac{x^n}{e^x}}\left(\left(\frac{2}{3}\right)^{\frac{x^n}{e^x}}-1\right)}{x^n}$$

Now, $\lim_{x\to\infty}\frac{x^n}{e^x}=\lim_{x\to\infty}\frac{n!}{e^x}=0$ (differentiating numerator and denominator n times for L' Hopital's rule)

Hence $L=\lim_{x\to\infty}(3)^{\frac{x^n}{e^x}}\lim_{x\to\infty}\frac{\left(\left(\frac{2}{3}\right)^{\frac{x^n}{e^x}}-1\right)}{\frac{x^n}{e^x}}\lim_{x\to\infty}\frac{1}{e^x}$

$=1\times\log(2/3)\times 0=0$

3 **(b)**

$\operatorname{cosec}\frac{\pi x}{2}\to 1$ when $x\to 1\Rightarrow\left[\operatorname{cosec}\frac{\pi x}{2}\right]=1$

$\therefore$ limit $=1$

4 **(b)**

$$\lim_{n\to\infty}\left(\frac{n^2-n+1}{n^2-n-1}\right)^{n(n-1)}$$
$$=\lim_{n\to\infty}\left(\frac{n(n-1)+1}{n(n-1)-1}\right)^{n(n-1)}$$
$$=\lim_{n\to\infty}\frac{\left(1+\frac{1}{n(n-1)}\right)^{n(n-1)}}{\left(1-\frac{1}{n(n-1)}\right)^{n(n-1)}}=\frac{e}{e^{-1}}=e^2$$

5 **(d)**

$$\lim_{x\to 0}\frac{x(e^x-1)}{1-\cos x}=\lim_{x\to 0}\frac{2x(e^x-1)}{4\sin^2\frac{x}{2}}$$
$$=2\lim_{x\to 0}\left[\frac{(x/2)^2}{\sin^2\frac{x}{2}}\right]\left(\frac{e^x-1}{x}\right)=2$$

6 **(b)**

$$\lim_{x\to 1}\frac{1-x^2}{\sin 2\pi x}$$
$$=-\lim_{x\to 1}\frac{2\pi(1-x)(1+x)}{2\pi\sin(2\pi-2\pi x)}$$
$$=-\lim_{x\to 1}\frac{(2\pi-2\pi x)}{\sin(2\pi-2\pi x)}\;\frac{1+x}{2\pi}=\frac{-1}{\pi}$$

7 **(d)**

We have $\lim_{x\to\pi}\frac{1+\cos^3 x}{\sin^2 x}$

$$=\lim_{x\to\pi}\frac{(1+\cos x)(1-\cos x+\cos^2 x)}{(1-\cos x)(1+\cos x)}$$
$$=\lim_{x\to\pi}\frac{1-\cos x+\cos^2 x}{1-\cos x}=\frac{1+1+1}{1+1}=\frac{3}{2}$$

8 **(d)**

The given function is

$$f(x)=\begin{cases}\frac{\sin[x]}{[x]} & \text{if } x\in(-\infty,0)\cup[1,\infty)\\ 0 & \text{if } x\in[0,1)\end{cases}$$

$$\therefore\ \lim_{x\to 0^-}f(x)=\lim_{h\to 0}\frac{\sin[-h]}{[-h]}$$
$$=\lim_{h\to 0}\frac{\sin(-1)}{(-1)}=\sin 1$$
$$\lim_{x\to 0^+}f(x)=\lim_{h\to 0}0=0$$
$$\therefore\ \lim_{x\to 0^-}f(x)\neq\lim_{x\to 0^+}f(x)$$

$\therefore\ \lim_{x\to 0}f(x)$ does not exist

9 **(a)**

$$\frac{x^4(\cot^4 x-\cot^2 x+1)}{(\tan^4 x-\tan^2 x+1)}$$
$$=\frac{x^4(1-\tan^2 x+\tan^4 x)}{\tan^4 x\,(\tan^4 x-\tan^2 x+1)}=\frac{x^4}{\tan^4 x},x\neq 0$$
$$\Rightarrow\ \lim_{x\to 0}\frac{x^4(\cot^4 x-\cot^2 x+1)}{(\tan^4 x-\tan^2 x+1)}=\lim_{x\to 0}\frac{x^4}{\tan^4 x}=1$$

10 **(c)**

$$\lim_{x\to 0}\frac{x\tan 2x-2x\tan x}{4\sin^4 x}$$
$$=\lim_{x\to 0}\frac{x}{4\sin^4 x}\left[\frac{2\tan x}{1-\tan^2 x}-2\tan x\right]$$
$$=\lim_{x\to 0}\frac{x\tan^3 x}{2\sin^4 x\,(1-\tan^2 x)}$$
$$=\frac{1}{2}\lim_{x\to 0}\;\frac{x}{\sin x}\;\frac{1}{\cos^3 x}\;\frac{1}{1-\tan^2 x}$$
$$=\frac{1}{2}\times 1\times\frac{1}{1^3}\times\frac{1}{1-0}=\frac{1}{2}$$

11 **(b)**

$$\lim_{x\to 0}\frac{\sin x^n}{(\sin x)^m}=\lim_{x\to 0}\left(\frac{\sin x^n}{x^n}\right)\left(\frac{x^n}{x^m}\right)\left(\frac{x}{\sin x}\right)^m$$

$= \lim_{x\to 0} x^{n-m} = 0 \quad [\because m < n]$

12 **(a)**

$$\lim_{x\to 2} \frac{\sqrt{1+\sqrt{2+x}}-\sqrt{3}}{x-2}$$

$$= \lim_{x\to 2} \frac{1+\sqrt{2+x}-3}{\left(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right)(x-2)} \text{ (Rationalizing)}$$

$$= \lim_{x\to 2} \frac{\sqrt{2+x}-2}{\left(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right)(x-2)}$$

$$= \lim_{x\to 2} \frac{(x-2)}{\left(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right)\left(\sqrt{2+x}+2\right)(x-2)} \text{ (Rationalizing)}$$

$$= \frac{1}{(2\sqrt{3})4} = \frac{1}{8\sqrt{3}}$$

13 **(b)**

$$L = \lim_{x\to\pi/2} \frac{\sin(x\cos x)}{\sin\left(\frac{\pi}{2}-x\sin x\right)}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\sin(x\cos x)}{(x\cos x)} \frac{x\cos x}{\sin\left(\frac{\pi}{2}-x\sin x\right)} \frac{\left(\frac{\pi}{2}-x\sin x\right)}{\left(\frac{\pi}{2}-x\sin x\right)}$$

$$= 1\times 1 \lim_{x\to\pi/2} \frac{x\cos x}{\left(\frac{\pi}{2}-x\sin x\right)}$$

Put $x = \pi/2 + h$

Then, $L = \lim_{h\to 0} \frac{\left(\frac{\pi}{2}+h\right)\cos\left(\frac{\pi}{2}+h\right)}{\frac{\pi}{2}-\left(\frac{\pi}{2}+h\right)\sin\left(\frac{\pi}{2}+h\right)}$

$$= \lim_{h\to 0} \frac{-\left(\frac{\pi}{2}+h\right)\sin h}{\frac{\pi}{2}(1-\cos h)-h\cos h}$$

$$= \lim_{h\to 0} \frac{-\left(\frac{\pi}{2}+h\right)\left(\frac{\sin h}{h}\right)}{\frac{\pi}{2}\frac{(1-\cos h)}{h}-\cos h} \quad \text{(Divide } N^r \text{ and } D^r \text{ by } h)$$

$$= \frac{-\left(\frac{\pi}{2}+0\right)1}{0-1} = \frac{\pi}{2}$$

14 **(b)**

$$\lim_{x\to\infty} \frac{\sin^4 x - \sin^2 x + 1}{\cos^4 x - \cos^2 x + 1}$$

$$= \lim_{x\to\infty} \frac{(1-\cos^2 x)^2 - (1-\cos^2 x) + 1}{\cos^4 x - \cos^2 x + 1}$$

$$= \lim_{x\to\infty} \frac{\cos^4 x - \cos^2 x + 1}{\cos^4 x - \cos^2 x + 1}$$

$= 1$

15 **(c)**

$$\lim_{x\to -2^-} \frac{ae^{1/|x+2|}-1}{2-e^{1/|x+2|}} = \lim_{x\to -2^-} \frac{a-e^{-1/|x+2|}}{2e^{-1/|x+2|}-1} = -a$$

$$\lim_{x\to -2^-} \sin\left(\frac{x^4-16}{x^5+32}\right) = \lim_{x\to -2^-} \sin\left(\frac{\frac{x^4-(-2)^4}{x-(-2)}}{\frac{x^5-(-2)^5}{x-(-2)}}\right)$$

$$= \sin\left(-\frac{2}{5}\right) \Rightarrow a = \sin\frac{2}{5}$$

16 **(c)**

$$I = \lim_{x\to 1} \frac{nx^n(x-1)-(x^n-1)}{(e^x-e)\sin\pi x}$$

Put $x = 1+h$ so that as $x\to 1, h\to 0$

$$\therefore\ I = -\lim_{h\to 0} \frac{h\cdot n(1+h)^n - ((1+h)^n - 1)}{e(e^h-1)\sin\pi h}$$

$$I = -\lim_{x\to 1} \frac{n\cdot h(1+{}^nC_1h+{}^nC_2h^2+\ {}^nC_3h^3+\cdots)}{\pi e(h^2)\left(\frac{e^h-1}{h}\right)}$$

$$\frac{-(1+{}^nC_1h++{}^nC_2h^2++{}^nC_3h^3+\cdots-1)}{\left(\frac{\sin\pi h}{\pi h}\right)}$$

$$= -\frac{n^2-\ {}^nC_2}{\pi e} = -\left[\frac{2n^2-n(n-1)}{2\pi e}\right] = -\frac{n^2+n}{2(\pi e)}$$

$$= -\frac{n(n+1)}{2(\pi e)}$$

If $n = 100 \ \Rightarrow 1 = -\left(\frac{5050}{\pi e}\right)$

17 **(a)**

$(1+x)^{2/x} = (1+x)^{2/x} - [(1+x)^{2/x}]$

Now, $\lim_{x\to 0}(1+x)^{2/x} = e^2$

$\Rightarrow \lim_{x\to 0}\{(1+x)^{2/x}\} = e^2 - [e^2] = e^2 - 7$

18 **(b)**

$$\lim_{x\to\infty}\left[\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right]$$

$$= \lim_{x\to\infty} \frac{x+\sqrt{x+\sqrt{x}}-x}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}} \text{ (Rationalizing)}$$

$$\lim_{x\to\infty} \frac{\sqrt{1+x^{-1/2}}}{\sqrt{1+\sqrt{x^{-1}+x^{-3/2}}}+1} = \frac{1}{2}$$

19 **(c)**

$$\lim_{x\to a}\sqrt{a^2-x^2}\cot\frac{\pi}{2}\sqrt{\frac{a-x}{a+x}}$$

$$= \lim_{x\to a} \frac{\sqrt{a^2-x^2}}{\tan\frac{\pi}{2}\sqrt{\frac{a-x}{a+x}}}$$

$$= \frac{2}{\pi}\lim_{x\to a} \frac{\frac{\pi}{2}\sqrt{\frac{a-x}{a+x}}}{\tan\frac{\pi}{2}\sqrt{\frac{a-x}{a+x}}}(a+x) = \frac{4a}{\pi}$$

20 **(a)**

$$\lim_{x\to 1} \frac{1+\sin\pi\left(\frac{3x}{1+x^2}\right)}{1+\cos\pi x}$$

$$= \lim_{x\to 1} \frac{1-\cos\left(\frac{3\pi}{2}-\frac{3\pi x}{1+x^2}\right)}{1-\cos(\pi-\pi x)}$$

$$= \lim_{x\to 1} \frac{2\sin^2\left(\frac{3\pi}{4}-\frac{3\pi x}{2(1+x^2)}\right)}{2\sin^2\left(\frac{\pi}{2}-\frac{\pi x}{2}\right)}$$

$$= \lim_{x\to 1}\left(\frac{\frac{3\pi}{4}-\frac{3\pi x}{2(1+x^2)}}{\frac{\pi}{2}-\frac{\pi x}{2}}\right)^2$$

$= \lim_{x\to 1} 9\left(\frac{\frac{1}{2}-\frac{x}{1+x^2}}{1-x}\right)^2 = \lim_{x\to 1} 9\left(\frac{x-1}{2(1+x^2)}\right)^2 = 0$

21 **(c)**

$\frac{\cos(2x-4)-33}{2} < f(x) < \frac{x^2|4x-8|}{x-2}$

$\Rightarrow \lim_{x\to 2^-}\frac{\cos(2x-4)-33}{2} < \lim_{x\to 2^-} f(x)$

$< \lim_{x\to 2^-}\frac{x^2|4x-8|}{x-2}$

$\Rightarrow -16 < \lim_{x\to 2^-} f(x) < \lim_{x\to 2^-}\frac{x^2(8-4x)}{x-2}$

$\Rightarrow -16 < \lim_{x\to 2^-} f(x) < -16$

$\Rightarrow \lim_{x\to 2^-} f(x) = -16$ (by sandwich theorem)

22 **(c)**

$\lim_{x\to\infty}\frac{(2x+1)^{40}(4x-1)^5}{(2x+3)^{45}}$

$= \lim_{x\to\infty}\frac{\left(2+\frac{1}{x}\right)^{40}\left(4-\frac{1}{x}\right)^5}{\left(2+\frac{3}{x}\right)^{45}}$

(Dividing numerator and denominator by x^{45})

$= \frac{2^{40}4^5}{2^{45}}$

$= 2^5 = 32$

23 **(b)**

For $n = 0$, we have $\lim_{x\to 0}\frac{1-\sin 1}{x-1} = \sin 1 - 1$

For $n = 1$, $\lim_{x\to 0}\frac{x-\sin x}{x-\sin x} = 1$

For $n = 2$, $\lim_{x\to 0}\frac{x^2-\sin^2 x}{x-\sin^2 x} = \lim_{x\to 0}\frac{1-\frac{\sin^2 x}{x^2}}{\frac{1}{x}-\frac{\sin^2 x}{x^2}}$

This does not exist

For $n = 3$ also given limit does not exist

Hence $n = 0$ or 1

24 **(b)**

$\frac{[x]^2}{x^2} = \begin{bmatrix} 0 \text{ if } 0 < x < 1 \\ \frac{1}{x^2} \text{ if } -1 < x < 0 \end{bmatrix} \Rightarrow l$ does not exist

$\frac{[x^2]}{x^2} = \begin{bmatrix} 0 \text{ if } 0 < x < 1 \\ 0 \text{ if } -1 < x < 0 \end{bmatrix} \Rightarrow m$ exists and is equal to 0]

25 **(c)**

$\lim_{x\to 1}\frac{p-q+qx^p-px^q}{1-x^p-x^q+x^{p+q}}\left(\frac{0}{0}\right)$

$= \lim_{x\to 1}\frac{pqx^{p-1}-pqx^{q-1}}{-px^{p-1}-qx^{q-1}+(p+q)x^{p+q-1}}\left(\frac{0}{0}\right)$ (L' Hopital Rule)

$= \lim_{x\to 1}\frac{pq(p-1)x^{p-2}-pq(q-1)x^{q-2}}{-p(p-1)x^{p-2}-q(q-1)x^{q-2}+(p+q)(p+q-1)x^{p+q-2}}$

(L' Hopital rule)

$= \frac{p-q}{2}$

26 **(d)**

Given, $\lim_{x\to 0}\frac{\{(a-n)nx-\tan x\}\sin nx}{x^2} = 0$

$\Rightarrow \lim_{x\to 0}\left((a-n)n-\frac{\tan x}{x}\right).\frac{\sin nx}{x} = 0$

$\Rightarrow [\{a-n\}n-1]n = 0$

$\Rightarrow a = n + \frac{1}{n}$

27 **(c)**

$\lim_{x\to\infty}\left(\frac{x^3+1}{x^2+1}-(ax+b)\right) = 2$

$\Rightarrow \lim_{x\to\infty}\frac{x^3(1-a)-bx^2-ax+(1-b)}{x^2+1} = 2$

$\Rightarrow 1-a = 0$ and $-b = 2$

$\Rightarrow a = 1, b = 2$

28 **(b)**

$\lim_{x\to 2}\frac{2^x+2^{3-x}-6}{\sqrt{2^{-x}}-2^{1-x}}$

$= \lim_{x\to 2}\frac{(2^x)^2-6\times 2^x+2^3}{\sqrt{2^x}-2}$ [Multiplying N^r and D^r by 2^x]

$= \lim_{x\to 2}\frac{(2^x-4)(2^x-2)(\sqrt{2^x}+2)}{(\sqrt{2^x}-2)(\sqrt{2^x}+2)}$

$= \lim_{x\to 2}\frac{(2^x-4)(2^x-2)(\sqrt{2^x}+2)}{(2^x-4)}$

$= \lim_{x\to 2}(2^x-2)(\sqrt{2^x}+2) = (2^2-2)(2+2) = 8$

29 **(c)**

$\lim_{n\to\infty}\left[\frac{1}{n}+\frac{e^{1/n}}{n}+\frac{e^{2/n}}{n}+\dots+\frac{e^{(n-1)/n}}{n}\right]$

$= \lim_{n\to\infty}\left[\frac{1+e^{1/n}+\left(e^{1/n}\right)^2+\dots+\left(e^{1/n}\right)^{n-1}}{n}\right]$

$= \lim_{n\to\infty}\frac{1.[\left(e^{1/n}\right)^n-1]}{n(e^{1/n}-1)} = (e-1)\lim_{n\to\infty}\frac{1}{\left(\frac{e^{1/n}-1}{1/n}\right)}$

$= (e-1)\times 1 = (e-1)$

30 **(a)**

$\lim_{y\to 0}\left\{\frac{x\{\sec(x+y)-\sec x\}}{y}+\sec(x+y)\right\}$

$= \lim_{y\to 0}\left[\frac{x}{y}\left\{\frac{\cos x-\cos(x+y)}{\cos(x+y)\cos x}\right\}\right] + \lim_{y\to 0}\sec(x+y)$

$= \lim_{y\to 0}\left[\frac{x\,2\sin\left(x+\frac{y}{2}\right)\sin\left(\frac{y}{2}\right)}{y\cos(x+y)\cos x}\right] + \sec x$

$= \lim_{y\to 0}\left[\frac{x\sin\left(x+\frac{y}{2}\right)}{\cos(x+y)\cos x}\times\frac{\sin\left(\frac{y}{2}\right)}{\frac{y}{2}}\right] + \sec x$

$= x\tan x\sec x + \sec x$

$= \sec x\,(x\tan x + 1)$

31 **(b)**

$\lim_{x\to\infty}\frac{\cot^{-1}(x^{-a}\log_a x)}{\sec^{-1}(a^x\log_x a)}\,(a > 1)$

$= \lim_{x\to\infty} \dfrac{\cot^{-1}\left(\frac{\log_a x}{x^a}\right)}{\sec^{-1}\left(\frac{a^x}{\log_a x}\right)}$ as $\lim_{x\to\infty}\left(\dfrac{\log_a x}{x^a}\right) \to 0$

and $\left(\frac{a^x}{\log_a x}\right) \to \infty$ (using L' Hopital rule)

$\therefore I = \dfrac{\pi/2}{\pi/2} = 1$

32 **(d)**

$\lim_{h\to 0} \dfrac{f(2h+2+h^2)-f(2)}{f(h-h^2+1)-f(1)}$

$= \lim_{h\to 0} \dfrac{\{f'(2h+2+h^2)\}.(2+2h)-0}{\{f'(h-h^2+1)\}.(1-2h)-0}$

[using L' Hospital's rule]

$= \dfrac{f'(2).2}{f'(1).1} = \dfrac{6.2}{4.1} = 3$

33 **(a)**

For $n > 1$,

$\lim_{x\to 0} x^n \sin(1/x^2) = 0 \times$ (any value between -1 to $1) = 0$

For $n < 0$,

$\lim_{x\to 0} x^n \sin(1/x^2) = \infty \times$(any value between -1 to $1) = \infty$

34 **(c)**

If $f(x) = \sin\left(\frac{1}{x}\right)$ and $g(x) = \frac{1}{x}$, then both $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist, but $\lim_{x\to 0}\frac{f(x)}{g(x)} = 0$ exists

35 **(a)**

$\lim_{x\to 2}\left[\left(\dfrac{x^3-4x}{x^3-8}\right)^{-1} - \left(\dfrac{\sqrt{x}(\sqrt{x}+\sqrt{2})}{(\sqrt{x}-\sqrt{2})(\sqrt{x}+\sqrt{2})} - \left(\dfrac{\sqrt{2}}{\sqrt{x}-\sqrt{2}}\right)^{-1}\right]\right.$

$= \lim_{x\to 2}\left[\dfrac{x^2+2x+4}{x(x+2)} - \left(\dfrac{\sqrt{x}-\sqrt{2}}{\sqrt{x}-\sqrt{2}}\right)^{-1}\right]$

$= \lim_{x\to 2}\left[\dfrac{x^2+2x+4}{x(x+2)} - 1\right] = \dfrac{12}{8} - 1 = \dfrac{1}{2}$

36 **(d)**

$\lim_{x\to\infty} \dfrac{e^{1/x^2}-1}{2\tan^{-1}(x^2)-\pi}$

$= \lim_{t\to 0^+} \dfrac{e^{t^2}-1}{2\cot^{-1} t^2 - \pi}$

$= \lim_{t\to 0^+} \dfrac{e^{t^2}-1}{-2\tan t^2}$

$= \lim_{t\to 0^+} -\dfrac{1}{2}\dfrac{e^{t^2}-1}{t^2\frac{\tan t^2}{t^2}} = -\dfrac{1}{2}$

37 **(c)**

$\lim_{x\to 1} \dfrac{x\sin(x-[x])}{x-1}$

Now L.H.L $= \lim_{h\to 0} \frac{(1-h)\sin(1-h-[1-h])}{(1-h)-1}$

$= \lim_{h\to 0} \dfrac{(1-h)\sin(1-h)}{-h} = -\infty$

R.H.L $= \lim_{h\to 0} \frac{(1+h)\sin(1+h-[1+h])}{(1+h)-1} = \lim_{h\to 0}\frac{(1+h)\sin h}{h} = 1$

Hence, the limit does not exist

38 **(c)**

$\lim_{x\to\infty}\left(\dfrac{x-3}{x+2}\right)^x = e^{\lim_{x\to\infty}\left[\frac{x-3}{x+2}-1\right]x}$

$= e^{\lim_{x\to\infty}\left[\frac{-5x}{x+2}\right]} = e^{-5}$

39 **(a)**

$\lim_{x\to 0^+}\left[\dfrac{\sin(\text{sgn } x)}{\text{sgn }(x)}\right]$

$= \lim_{x\to 0^+}\left[\dfrac{\sin 1}{1}\right]$

$= 0$

$= \lim_{x\to 0^-}\left[\dfrac{\sin(\text{sgn } x)}{\text{sgn } x}\right]$

$= \lim_{x\to 0^-}\left[\dfrac{\sin(-1)}{-1}\right]$

$= \lim_{x\to 0^-}[\sin 1]$

$= 0$

Hence, the given limit is 0

40 **(b)**

Given limit is $\lim_{x\to\infty}(x+1)[\tan^{-1}(x+5) -$

*x+1+4*tan*−1(x+5)*

$= \lim_{x\to\infty}\left[(x+1)\tan^{-1}\dfrac{4}{1+(x+1)(x+5)} + 4\tan^{-1}(x+5)\right]$

$= \lim_{x\to\infty}\left[(x+1)\tan^{-1}\dfrac{\frac{4}{x^2+6x+6}}{\left(\frac{4}{x^2+6x+6}\right)} \times \dfrac{4}{x^2+6x+6} + 4\tan^{-1}(x+5)\right]$

$= 0 + 4 \times \dfrac{\pi}{2} = 2\pi$

41 **(d)**

$\lim_{x\to\infty}\left(\dfrac{1}{e} - \dfrac{x}{1+x}\right)^x = \lim_{x\to\infty}\left(\dfrac{1}{e} - \dfrac{1}{\frac{1}{x}+1}\right)^x$

$= \left(\dfrac{1}{e} - 1\right)^\infty$

$=$ (some negative value)$^\infty$ which is not defined as base is $-$ve

42 **(c)**

As $x \to 0^- \Rightarrow f(x) \to f(0^-) = 2^+$

$\Rightarrow \lim_{x\to 0^-} g(f(x)) = g(2^+) = -3$

Also as $x \to 0^+ \Rightarrow f(x) \to f(0^+) = 1^+$

$\Rightarrow \lim_{x\to 0^+} g(f(x)) = g(1^+) = -3$

Hence $\lim_{x\to 0} g(f(x))$ exists and is equal to -3

$\Rightarrow \lim_{x\to 0} g(f(x)) = -3$

43 **(d)**

$= \lim_{x\to\infty}\left(\frac{x^3}{3x^2-4} - \frac{x^2}{3x+2}\right)$

$= \lim_{x\to\infty}\frac{x^3(3x+2) - x^2(3x^2-4)}{(3x^2-4)(3x+2)}$

$= \lim_{x\to\infty}\frac{2x^3+4x^2}{9x^3+6x^2-12x-8}$

$= \lim_{x\to\infty}\frac{2+\frac{4}{x}}{9+\frac{6}{x}-\frac{12}{x^2}-\frac{8}{x^3}}$

$= 2/9$

44 **(b)**

$\because \lim_{n\to\infty}\cos^{2n}x = \begin{cases}1, x = r\pi, r\in I\\ 0, x\neq r\pi, r\in I\end{cases}$

Here, for $x = 10$, $\lim_{n\to\infty}\cos^{2n}(x-10) = 1$

And in all other cases it is zero

$\therefore \lim_{n\to\infty}\sum_{x=1}^{\infty}\cos^{2n}(x-10) = 1$

45 **(c)**

$\lim_{x\to 0}\frac{a^{\sqrt{x}} - a^{1/\sqrt{x}}}{a^{\sqrt{x}} + a^{1/\sqrt{x}}}, a > 1$

Put $x = t^2$

$\therefore \lim_{t\to 0}\frac{a^t - a^{1/t}}{a^t + a^{1/t}}$

$\Rightarrow \lim_{t\to 0}\frac{a^{t-1/t}-1}{a^{t-1/t}+1} = \frac{a^{-\infty}-1}{a^{-\infty}+1} = \frac{0-1}{0+1} = -1$

46 **(c)**

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty}\sqrt{\frac{1-\frac{\sin x}{x}}{1+\frac{\cos^2 x}{x}}} = \sqrt{\frac{1-0}{1+0}} = 1$

47 **(b)**

$\lim_{x\to 0}\frac{1+\sin x - \cos x + \log(1-x)}{x^3}$

$= \lim_{x\to 0}\frac{1+\left(x-\frac{x^3}{3!}+\cdots\right)-\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots\right)+\left(-x-\frac{x^2}{2}-\frac{x^3}{3}-\cdots\right)}{x^3}$

$= -\frac{1}{3!}-\frac{1}{3} = -\frac{1}{2}$

48 **(a)**

$\lim_{x\to-\infty}\frac{x^2\tan\frac{1}{x}}{\sqrt{8x^2+7x+1}} = \lim_{x\to-\infty}\frac{x^2\tan\frac{1}{x}}{-x\sqrt{8+\frac{7}{x}+\frac{1}{x^2}}}$

$= -\lim_{x\to-\infty}\frac{\tan\frac{1}{x}}{\frac{1}{x}\sqrt{8+\frac{7}{x}+\frac{1}{x^2}}} = -\frac{1}{2\sqrt{2}}$

49 **(b)**

$\min(y^2-4y+11) = \min[(y-2)^2+7] = 7$

$\Rightarrow L = \lim_{x\to 0}\left[\min(y^2-4y+11)\frac{\sin x}{x}\right]$

$= \lim_{x\to 0}\left[\frac{7\sin x}{x}\right]$

= [a value slightly lesser than 7] ($|\sin x| < |x|$, when $x \to 0$)

$\Rightarrow L = \lim_{x\to 0}\left[7\frac{\sin x}{x}\right] = 6$

50 **(b)**

$\lim_{x\to\pi/2}\left[x\tan x - \left(\frac{\pi}{2}\right)\sec x\right]$

$= \lim_{x\to\pi/2}\frac{2x\sin x-\pi}{2\cos x}$ $\left(\frac{0}{0}\text{ form}\right)$

$= \lim_{x\to\pi/2}\frac{[2\sin x + 2x\cos x]}{-2\sin x}$

$= -1$ (Applying L' Hopital's rule)

51 **(b)**

$\lim_{x\to 0}\frac{\sin(\pi\cos^2 x)}{x^2}$

$= \lim_{x\to 0}\frac{\sin(\pi-\pi\cos^2 x)}{x^2}$ $[\sin(\pi-\theta) = \sin\theta]$

$= \lim_{x\to 0}\frac{\sin(\pi\sin^2 x)}{\pi\sin^2 x}\times\frac{(\pi\sin^2 x)}{x^2} = \pi$

52 **(c)**

Given $f(x) = x^2 - \pi^2$

$\lim_{x\to-\pi}\frac{x^2-\pi^2}{\sin(\sin x)} = \lim_{h\to 0}\frac{(-\pi+h)^2-\pi^2}{\sin(\sin(-\pi+h))}$

$= \lim_{h\to 0}\frac{-2h\pi+h^2}{-\sin(\sin h)}$

$= \lim_{h\to 0}\frac{h-2\pi}{\frac{-\sin(\sin h)}{\sin h}\times\frac{\sin h}{h}} = 2\pi$

53 **(a)**

$\lim_{m\to\infty}\left(\cos\frac{x}{m}\right)^m$

$= \lim_{m\to\infty}\left[1-\left(1-\cos\frac{x}{m}\right)\right]^m$

$= \lim_{m\to\infty}\left[1-2\sin^2\frac{x}{2m}\right]^m$

$= e^{\lim_{m\to\infty}\left(-2\sin^2\frac{x}{2m}\right)m} = 1$

54 **(d)**

$\lim_{x\to\infty}\frac{1+x+x^2}{x(\text{In }x)^3} = \lim_{t\to 0^+}\frac{t^2+t+1}{t^2\frac{1}{t}\left(\text{In }\left(\frac{1}{t}\right)\right)^3}$

$= \lim_{t\to 0^+}\frac{1+t+t^2}{-t\,(\text{In }t)^3} = +\infty$

55 **(a)**

$$\text{L.H.L.} = \lim_{x\to -1^-} \frac{1}{\sqrt{|x| - \{-x\}}}$$
$$= \lim_{x\to -1^-} \frac{1}{\sqrt{-x-(x+2)}}$$
$$= \lim_{x\to -1^-} \frac{1}{\sqrt{-2x-2}} = \infty$$
$$\text{R.H.L} = \lim_{x\to -1^+} \frac{1}{\sqrt{|x| - \{-x\}}}$$
$$= \lim_{x\to -1^-} \frac{1}{\sqrt{-x-(x+1)}}$$
$$= \lim_{x\to -1^-} \frac{1}{\sqrt{-2x-1}} = 1$$

Hence, the limit does not exist

56 **(d)**

$$\lim_{x\to 1} \frac{-\sqrt{25-x^2} - (-\sqrt{24})}{x-1}$$
$$= \lim_{x\to 1} \frac{\sqrt{24} - \sqrt{25-x^2}}{x-1} \times \frac{\sqrt{24} + \sqrt{25-x^2}}{\sqrt{24} + \sqrt{25-x^2}}$$
$$= \lim_{x\to 1} \frac{x^2-1}{(x-1)\left[\sqrt{24} + \sqrt{25-x^2}\right]}$$
$$= \frac{2}{2\sqrt{24}} = \frac{1}{2\sqrt{6}}$$

57 **(c)**

$$\lim_{x\to 0} \frac{(2^m+x)^{1/m} - (2^n+x)^{1/n}}{x}$$
$$= \lim_{x\to 0} \frac{(2^m+x)^{1/m} - 2}{x} - \lim_{x\to 0} \frac{(2^n+x)^{1/n} - 2}{x}$$
$$= \lim_{a\to 2} \frac{a-2}{a^m - 2^m} - \lim_{b\to 2} \frac{b-2}{b^n - 2^n} \quad [\text{Putting } 2^m + x = a^m \text{ and } 2^n + x = b^n]$$
$$= \frac{1}{m2^{m-1}} - \frac{1}{n2^{n-1}}$$

58 **(c)**

$$\lim_{x\to 1} (2-x)^{\tan\frac{\pi x}{2}}$$
$$= \lim_{x\to 1} \{1 + (1-x)\}^{\tan\frac{\pi x}{2}}$$
$$= e^{\lim_{x\to 1} (1-x)\tan\frac{\pi x}{2}}$$
$$= e^{\lim_{x\to 1} (1-x)\cot\left(\frac{\pi}{2} - \frac{\pi x}{2}\right)}$$
$$= e^{\lim_{x\to 1} \frac{(1-x)}{\tan\left(\frac{\pi}{2} - \frac{\pi x}{2}\right)}}$$
$$= e^{\frac{2}{\pi}\lim_{x\to 1} \frac{\frac{\pi}{2}(1-x)}{\tan\left(\frac{\pi}{2}(1-x)\right)}}$$
$$= e^{2/\pi}$$

59 **(b)**

$$\lim_{n\to\infty} \left(\frac{1}{1-n^2} + \frac{2}{1-n^2} + \cdots + \frac{n}{1-n^2}\right)$$
$$= \lim_{n\to\infty} \frac{1+2+3+\cdots+n}{1-n^2}$$
$$= \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{1-n^2}$$
$$= \lim_{n\to\infty} \frac{1+\frac{1}{n}}{2\left[\frac{1}{n^2} - 1\right]} = -1/2$$

60 **(c)**

$$\lim_{x\to 0} \frac{x^a \sin^b x}{\sin x^c}$$
$$= \lim_{x\to 0} x^a \left(\frac{\sin x}{x}\right)^b \left(\frac{x^c}{\sin x^c}\right) x^{b-c} = \lim_{x\to 0} x^{a+b-c}$$

This limit will have non-zero value if $a + b = c$

61 **(c)**

The given limit is $\lim_{x\to 0} [(1+\tan x)^{\operatorname{cosec} x} / (1+\sin x)^{\operatorname{cosec} x}]$

$$= \lim_{x\to 0} [(1+\tan x)^{\cot x}\}^{\sec x} x / \{1 / (1+\sin x)^{\operatorname{cosec} x}\}]$$
$$= e^{\sec 0} \frac{1}{e} = e\frac{1}{e} = 1$$

62 **(d)**

We know that $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \begin{cases} 2\tan^{-1} x, & x \geq 0 \\ -2\tan^{-1} x, & x \leq 0 \end{cases}$

$\Rightarrow \lim_{x\to 0^+} \frac{1}{x} \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \lim_{x\to 0^+} \frac{2\tan^{-1} x}{x} = 2$, and

$$\lim_{x\to 0^-} \frac{1}{x} \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \lim_{x\to 0^+} \left[-\frac{2\tan^{-1} x}{x}\right] = -2$$

63 **(a)**

$$\lim_{n\to\infty} \frac{n \cdot 3^n}{n(x-2)^n + n \cdot 3^{n+1} - 3^n} = \frac{1}{3}$$

$\Rightarrow \lim_{n\to\infty} \frac{1}{\frac{(x-2)^n}{3^n} + 3 - \frac{1}{n}}$ (Dividing N^r and D^r by $n \times 3^n$)

For $\lim_{n\to\infty}$ to be equal to 1/3

$\lim_{n\to\infty} \frac{1}{n} \to$ (which is true) and $\lim_{n\to\infty} \left(\frac{x-2}{3}\right)^n \to 0$

$\Rightarrow 2 \leq x < 5$

64 **(a)**

$$\lim_{x\to 0} \frac{\sin 3x}{x^3} + \frac{a}{x^2} + b$$
$$= \lim_{x\to 0} \frac{\sin 3x + ax + bx^3}{x^3}$$
$$= \lim_{x\to 0} \frac{3\frac{\sin 3x}{3x} + a + bx^2}{x^2}$$

For existence, $(3 + a) = 0$

$\Rightarrow a = -3$

$$\therefore L = \lim_{x\to 0} \frac{\sin 3x - 3x + bx^3}{x^3}$$
$$= 27 \lim_{t\to 0} \frac{\sin t - t}{t^3} + b = 0 \; (3x = t)$$
$$= -\frac{27}{6} + b = 0$$
$$\Rightarrow b = \frac{9}{2}$$

65 **(a)**

$$\lim_{x\to 0^+}\left[(1-e^x)\frac{\sin x}{|x|}\right]$$

$$=\lim_{x\to 0^+}\left[(0^-)\frac{\sin x}{x}\right]=[0^-]=-1$$

$$=\lim_{x\to 0^-}\left[(1-e^x)\frac{\sin x}{|x|}\right]$$

$$=\lim_{x\to 0^-}\left[(0^+)\frac{\sin x}{-x}\right]=[0^-]=-1$$

Hence $\lim_{x\to 0}\left[(1-e^x)\frac{\sin x}{|x|}\right]=-1$

66 **(b)**

$\cos(\tan x)-\cos x$

$$=2\sin\left(\frac{x+\tan x}{2}\right)\sin\left(\frac{x-\tan x}{2}\right)$$

$$\Rightarrow \lim_{x\to 0}\frac{\cos(\tan x)-\cos x}{x^4}$$

$$=\lim_{x\to 0}\frac{2\sin\left(\frac{x+\tan x}{2}\right)\sin\left(\frac{x-\tan x}{2}\right)}{x^4}$$

$$=\lim_{x\to 0}\frac{2\sin\left(\frac{x+\tan x}{2}\right)\sin\left(\frac{x-\tan x}{2}\right)}{x^4\left(\frac{x+\tan x}{2}\right)\left(\frac{x-\tan x}{2}\right)}\left(\frac{x^2-\tan^2 x}{4}\right)$$

$$=\frac{1}{2}\lim_{x\to 0}\frac{x^2-\tan^2 x}{x^4}$$

$$=\frac{1}{2}\lim_{x\to 0}\frac{x^2-\left(x+\frac{x^3}{3}+\frac{2}{15}x^5+\cdots\right)^2}{x^4}$$

$$=\frac{1}{2}\lim_{x\to 0}\frac{1}{x^2}\left(1-\left(1+\frac{x^2}{3}+\frac{2}{15}x^4+\cdots\right)^2\right)=-\frac{1}{3}$$

67 **(b)**

$$\lim_{x\to 0}\frac{x^n\sin^n x}{x^n-\sin^n x}$$

$$\Rightarrow \lim_{x\to 0}\frac{x^n\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots\right)^n}{x^n-\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots\right)^n}$$

$$=\lim_{x\to 0}\frac{x^n\left(1-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots\right)^n}{1-\left(1-\frac{x^2}{3!}+\frac{x^4}{5!}-\cdots\right)^n}$$

$$=\lim_{x\to 0}\frac{x^n\left(1-\frac{x^2}{3!}+\frac{x^4}{5!}-\cdots\right)^n}{1-\left(1-\frac{x^2}{3!}+\frac{x^4}{5!}-\cdots\right)^n}$$

For $n=2$,

$$\lim_{x\to 0}\frac{x^2\left(1-\frac{x^2}{3!}+\frac{x^4}{5!}-\cdots\right)^2}{1-\left(1-\frac{x^2}{3!}+\frac{x^4}{5!}-\cdots\right)^2}$$

$$\Rightarrow \lim_{x\to 0}\frac{x^2\left(1-\frac{x^2}{3!}+\frac{x^4}{5!}-\cdots\right)^2}{\left(2-\frac{x^2}{3!}+\frac{x^4}{5!}-\cdots\right)\left(\frac{x^2}{3!}-\frac{x^4}{5!}+\cdots\right)}$$

$$=\frac{1(1-0+\cdots)^2}{(2-0+0)\left(\frac{1}{3!}-0+\cdots\right)}$$

$=3$

68 **(c)**

We have $f(x)+g(x)+h(x)=\frac{x^2-4x+17-4x-2}{x^2+x-12}$

$$=\frac{x^2-8x+15}{x^2+x-12}=\frac{(x-3)(x-5)}{(x-3)(x+4)}$$

$$\therefore \lim_{x\to 3}[f(x)+g(x)+h(x)]=\lim_{x\to 3}\frac{(x-3)(x-5)}{(x-3)(x+4)}$$

$$=-\frac{2}{7}$$

69 **(a)**

Given $g(x)=\lim_{n\to\infty}\frac{1}{\left(\frac{3}{\pi}\tan^{-1}2x\right)^{2n}+5}=0$

$$\Rightarrow \left[\left(\frac{3}{\pi}\tan^{-1}2x\right)^2\right]^n\to\infty$$

$$\Rightarrow \left(\frac{3}{\pi}\tan^{-1}2x\right)^2>1$$

$$\Rightarrow |\tan^{-1}2x|>\frac{\pi}{3}$$

$$\Rightarrow \tan^{-1}2x<-\frac{\pi}{3}\text{ or }\tan^{-1}2x>\frac{\pi}{3}$$

$$\Rightarrow 2x<-\sqrt{3}\text{ or }2x>\sqrt{3}\Rightarrow |2x|>\sqrt{3}$$

70 **(b)**

We know that $\lim_{x\to 0}\frac{\sin x}{x}\to 1^-$ and $\lim_{x\to 0}\frac{x}{\sin x}\to 1^+$

So, $\lim_{x\to 0}\left[100\frac{x}{\sin x}\right]+\lim_{x\to 0}\left[99\frac{\sin x}{x}\right]$

$=100+98=198$

71 **(b)**

$$\lim_{x\to 1}\frac{(1+x)(1-x^2)\dots(1-x^{2n})}{\{(1-x)(1-x^2)\dots(1-x^n)\}^2}$$

$$=\lim_{x\to 1}\frac{\left(\frac{1-x}{1-x}\right)\left(\frac{1-x^2}{1-x}\right)\dots\left(\frac{1-x^{2n}}{1-x}\right)}{\left(\left(\frac{1-x}{1-x}\right)\left(\frac{1-x^2}{1-x}\right)\dots\left(\frac{1-x^n}{1-x}\right)\right)^2}$$

$$=\frac{1\times 2\times 3\dots(2n)}{(1\times 2\times 3\dots n^2)}=\frac{(2n)!}{n!\,n!}={}^{2n}C_n$$

72 **(b)**

$$\lim_{x\to -1}\left(\frac{x^4+x^2+x+1}{x^2-x+1}\right)^{\frac{1-\cos(x+1)}{(x+1)^2}}$$

$$=\left(\lim_{x\to 1}\frac{x^4+x^2+x+1}{x^2-x+1}\right)^{\lim_{x\to 1}\frac{1-\cos(x+1)}{(x+1)^2}}$$

$$=\left(\frac{2}{3}\right)^{\lim_{x\to -1}\frac{\sin(x+1)}{2(x+1)}}=\left(\frac{2}{3}\right)^{\frac{1}{2}}$$

73 **(c)**

Since the highest degree of x is $1/2$, divide numerator and denominator by $\sqrt{x}$, then we have limit $\frac{2}{\sqrt{2}}$ or $\sqrt{2}$

74 **(c)**

Here, $\lim_{x\to 0}(\sin x)^{1/x}+\lim_{x\to 0}\left(\frac{1}{x}\right)^{\sin x}=0+$

$\lim_{x\to 0} e^{\log\left(\frac{1}{x}\right)^{\sin x}}$

$\left[\begin{array}{l}\lim\limits_{x\to 0}(\sin x)^{\frac{1}{x}} \to 0 \\ \text{as,} \quad 0 < \sin x < 1\end{array}\right]$

$= e^{\lim_{x\to 0}\frac{\log(1/x)}{\operatorname{cosec} x}} = e^{\lim_{x\to 0}\frac{x\left(-\frac{1}{x^2}\right)}{-\operatorname{cosec} x \cot x}}$

[by L'Hospital's rule]

$= e^{\lim_{x\to 0}\frac{\sin x}{x}\tan x} = e^0 = 1$

75 **(d)**

The given limit is $\lim\limits_{x\to\infty}\frac{\frac{2}{x}+2+\frac{\sin 2x}{x}}{\left(2+\frac{\sin 2x}{x}\right)e^{\sin x}}$

$$= \frac{0+2+0}{(2+0)\times\left(\text{a value between}\frac{1}{e}\text{ and } e\right)}$$

$\left[\because \lim\limits_{x\to\infty}\sin x \in (-1,1)\right]$

Hence limit does not exist

76 **(b)**

$$\lim_{x\to 0}\left(\frac{1^x+2^x+\cdots+n^x}{n}\right)^{1/x}$$

$= e^{\lim_{x\to\infty}\left(\frac{1^x-1}{n}+\frac{2^x-1}{n}+\cdots+\frac{n^x-1}{n}\right)\frac{1}{x}}$

$= e^{\lim_{x\to\infty}\frac{1}{n}\left\{\frac{1^x-1}{x}+\frac{2^x-1}{x}+\cdots+\frac{n^x-1}{x}\right\}}$

$= e^{\frac{1}{n}[\log 1+\log 2+\cdots+\log n]}$

$= e^{\frac{1}{n}(\log n!)} = e^{\log(n!)^{\frac{1}{n}}} = (n!)^{\frac{1}{n}}$

77 **(c)**

$$\lim_{n\to\infty}\left[\frac{2}{2-\frac{1}{n^2}}\cdot\frac{1}{n}\cos\left(\frac{1+1/n}{2-1/n}\right)-\frac{1}{\left(\frac{1}{n}-2\right)}\cdot\frac{(-1)^n}{\left(1+\frac{1}{n^2}\right)}\cdot\frac{1}{n}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[\frac{2}{2-\frac{1}{n^2}}\cdot\cos\left(\frac{1+\frac{1}{n}}{2-\frac{1}{n}}\right)-\frac{1}{\left(\frac{1}{n}-2\right)}\cdot\frac{(-1)^n}{\left(1+\frac{1}{n^2}\right)}\right]$$

$$= 0\times\left[\frac{2}{2}\times\cos\frac{1}{2}+\frac{1}{2}\times\frac{1}{1}\right] = 0$$

78 **(d)**

$$\lim_{x\to\frac{\pi}{2}}\frac{\cos x}{(1-\sin x)^{1/3}} = \lim_{t\to 0}\frac{-\sin t}{(1-\cos t)^{1/3}}$$

$$= -\lim_{t\to 0}\frac{2\sin\frac{t}{2}\cos\frac{t}{2}}{\left(2\sin^2\frac{t}{2}\right)^{1/3}}$$

$$= -\lim_{t\to 0}2^{2/3}\cos\frac{t}{2}\left(\sin\frac{t}{2}\right)^{1/3} = 0$$

79 **(c)**

Here, $\lim_{x\to 0}\frac{f(x^2)-f(x)}{f(x)-f(0)} = \lim_{x\to 0}\frac{f'(x^2).2x-f'(x)}{f'(x)}$

$$= \frac{-f'(0)}{f'(0)} = -1$$

80 **(b)**

$$L = \lim_{x\to\infty}\frac{\text{In }(x^2+e^x)}{\text{In }(x^4+e^{2x})} = \lim_{x\to\infty}\frac{\text{In } e^x\left(1+\frac{x^2}{e^x}\right)}{\text{In } e^{2x}\left(1+\frac{x^4}{e^{2x}}\right)}$$

$$= \lim_{x\to\infty}\frac{x+\text{In}\left(1+\frac{x^2}{e^x}\right)}{2x+\text{In}\left(1+\frac{x^4}{e^{2x}}\right)}$$

$$= \lim_{x\to\infty}\frac{1+\frac{1}{x}\text{ In}\left(1+\frac{x^2}{e^x}\right)}{2+\frac{1}{x}\text{ In}\left(1+\frac{x^4}{e^{2x}}\right)}$$

Note that $\underset{x\to\infty}{\text{as}}\frac{x^2}{e^x}\to 0$ and $\underset{x\to\infty}{\text{as}}\frac{x^2}{e^{2x}}\to 0$ (Using L' Hopital's rule)

Hence $L = \frac{1}{2}$

81 **(d)**

$$\lim_{x\to\infty}\frac{(x+1)^{10}+(x+2)^{10}+\cdots+(x+100)^{10}}{x^{10}+10^{10}}$$

$$= \lim_{x\to\infty}\frac{x^{10}\left[\left(1+\frac{1}{x}\right)^{10}+\left(1+\frac{2}{x}\right)^{10}+\ldots+\left(1+\frac{100}{x}\right)^{10}\right]}{x^{10}\left[1+\frac{10^{10}}{x^{10}}\right]}$$

$= 100$

82 **(d)**

We have $\lim\limits_{x\to 1}\frac{1-\sqrt{x}}{(\cos^{-1}x)^2}$

$$= \lim_{x\to 1}\frac{(1-\sqrt{x})(1+\sqrt{x})}{(\cos^{-1}x)^2(1+\sqrt{x})}$$

$$= \lim_{x\to 1}\frac{1-x}{(\cos^{-1}x)^2(1+\sqrt{x})}$$

$= \lim\limits_{\theta\to 0}\frac{1-\cos\theta}{\theta^2(1+\sqrt{\cos\theta})}1$, where $x = \cos\theta$ $[\because x\to 1 \Rightarrow \cos\theta\to 1\Rightarrow\theta\to 0$

$$= \lim_{\theta\to 0}\frac{1-\cos\theta}{\theta^2}\left(\frac{1}{1+\sqrt{\cos\theta}}\right)$$

$$= \lim_{\theta\to 0}\frac{2\sin^2\frac{\theta}{2}}{4\frac{\theta^2}{4}}\left(\frac{1}{1+\sqrt{\cos\theta}}\right)$$

$$= \frac{1}{2}\lim_{\theta\to 0}\left(\frac{\sin\frac{\theta}{2}}{\frac{\theta}{2}}\right)^2\frac{1}{(1+\sqrt{\cos\theta})} = \frac{1}{2}(1)^2\ \frac{1}{(1+1)}$$

$$= \frac{1}{4}$$

83 **(c)**

$$\lim_{n\to\infty}\frac{n(2n+1)^2}{(n+2)(n^2+3n-1)}$$

$$= \lim_{n\to\infty} \frac{\left(2+\frac{1}{n}\right)^2}{\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}-\frac{1}{n^2}\right)}$$

$$= \frac{(2+0)^2}{(1+0)(1+0+0)} = 4$$

84 **(d)**

$$\lim_{x\to 0} \frac{\sqrt{\frac{1}{2}(1-\cos 2x)}}{x}$$

$$= \lim_{x\to 0} \frac{\sqrt{\frac{1}{2}\cdot 2\sin^2 x}}{x} = \lim_{x\to 0} \frac{|\sin x|}{x}$$

$$\therefore \text{L.H.L.} = \lim_{h\to 0} \frac{|\sin(0-h)|}{0-h} = \lim_{h\to 0} \frac{|-\sin h|}{-h}$$

$$= \lim_{h\to 0} \frac{\sin h}{-h} = -1$$

$$\text{R.H.L.} = \lim_{h\to 0} \frac{|\sin(0+h)|}{0+h} = \lim_{h\to 0} \frac{\sin h}{h} = 1$$

As L.H.L. $\neq$ R.H.L., therefore, the given limit does not exist

85 **(c)**

$$\lim_{x\to\infty} \left(\frac{x^2+2x-1}{2x^2-3x-2}\right)^{\frac{2x+1}{2x-1}}$$

$$= \lim_{x\to\infty} \left(\frac{1+\frac{2}{x}-\frac{1}{x^2}}{2-\frac{3}{x}-\frac{2}{x^2}}\right)^{\frac{2+1/x}{2-1/x}}$$

$= 1/2$

86 **(d)**

$$\text{L.H.L} = \lim_{x\to 1^-} \frac{\sqrt{1-\cos[2(x-1)]}}{x-1}$$

$$= \lim_{x\to 1^-} \frac{\sqrt{2\sin^2(x-1)}}{x-1}$$

$$= \sqrt{2} \lim_{x\to 1^-} \frac{|\sin(x-1)|}{x-1}$$

$$= \sqrt{2}\lim_{h\to 0} \frac{|\sin(-h)|}{-h} = \sqrt{2} \lim_{h\to 0} \frac{\sin h}{-h} = -\sqrt{2}$$

$$\text{Again, R.H.L.} = \lim_{x\to 1^+} \sqrt{2}\, \frac{|\sin(x-1)|}{x-1}$$

$$= \lim_{h\to 0} \sqrt{2}\, \frac{|\sin h|}{h}$$

$$= \lim_{h\to 0} \sqrt{2} \frac{\sin h}{h} = \sqrt{2}$$

L.H.L $\neq$ R.H.L. Therefore, $\lim_{x\to 1} f(x)$ does not exist

87 **(b)**

$$\lim_{x\to 0} \frac{\sin(x^2)}{\text{In}\,(\cos(2x^2-x))}$$

$$= \lim_{x\to 0} \frac{\sin(x^2)}{\log\left(1-2\sin^2\left(\frac{2x^2-x}{2}\right)\right)}$$

$$= \lim_{x\to 0} \frac{\sin(x^2)x^2}{x^2 \frac{\log\left(1-2\sin^2\left(\frac{2x^2-x}{2}\right)\right)}{-2\sin^2\left(\frac{2x^2-x}{2}\right)}\left[-2\sin^2\left(\frac{2x^2-x}{2}\right)\right]}$$

$$= \lim_{x\to 0} -\frac{x^2}{\frac{2\sin^2\left(\frac{2x^2-x}{2}\right)}{\left(\frac{2x^2-x}{2}\right)^2}\left(\frac{2x^2-x}{2}\right)^2}$$

$$= \lim_{x\to 0} -\frac{2x^2}{(2x^2-x)^2} = \lim_{x\to 0} -\frac{2}{(2x-1)^2} = -2$$

88 **(c)**

1^∞ form

$$L = e^{\lim_{n\to\infty}\left(\left(\frac{n}{n+1}\right)^\alpha + \sin\frac{1}{n} - 1\right)}$$

$$= e^{\lim_{n\to\infty} n\sin\frac{1}{n} + \lim_{n\to\infty} n\left(\left(\frac{n}{n+1}\right)^\alpha - 1\right)}$$

Consider $\lim_{n\to\infty} n\left(\left(\frac{n}{n+1}\right)^\alpha - 1\right) = \lim_{n\to\infty} n\left(\left(\frac{1}{1+1/n}\right)^\alpha - 1\right.$

Put $n = \frac{1}{y}$

$$= \lim_{y\to 0} \frac{1}{y}\left(\left(\frac{1}{1+y}\right)^\alpha - 1\right) = \lim_{y\to 0} \frac{1-(1+y)^\alpha}{y} = -a$$

$\therefore L = e^{1-\alpha}$ (Using binomial)

89 **(b)**

$$f(x) = \lim_{n\to\infty} n(x^{1/n}-1)$$

$$= \lim_{n\to\infty} \frac{x^{1/n}-1}{1/n}$$

$= \lim_{m\to 0} \frac{x^m - 1}{m}$ (where $\frac{1}{n}$ replaced by m)

$= \text{In}\, x$

$\Rightarrow f(xy) = \text{In}\,(xy) = \text{In}\, x + \text{In}\, y = f(x) + f(y)$

90 **(b)**

Given limit

$$= \lim_{x\to 0} \frac{\int_0^x \frac{t\log(1+t)}{t^4+4}\,dt}{x^3}$$

Using L' Hospital's rule,

$$= \lim_{x\to 0} \frac{\frac{x\log(1+x)}{x^4+4}}{3x^2}$$

$$= \lim_{x\to 0} \frac{\log(1+x)}{3x} \cdot \frac{1}{x^4+4}$$

$$= \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$$

91 **(b)**

$$x_{n+1} = \sqrt{2+x_n}$$

$$\Rightarrow \lim_{n\to\infty} x_{n+1} = \sqrt{2 + \lim_{n\to\infty} x_n}$$

$$\Rightarrow t = \sqrt{2+t} \quad \left(\because \lim_{x\to\infty} x_{n+1} = \lim_{x\to\infty} x_n = t\right)$$

$$\Rightarrow t^2 - t - 2 = 0$$

$$\Rightarrow (t-2)(t+1) = 0$$

$\Rightarrow t = 2$ $(\because x_n > 0 \;\forall\, n \therefore t > 0)$

92 **(c)**

$$\lim_{n\to\infty} n^2\left(x^{1/n} - x^{\frac{1}{n+1}}\right) = \lim_{n\to\infty} n^2 \cdot x^{\frac{1}{n+1}}\left(x^{\frac{1}{n}-\frac{1}{n+1}-1}\right)$$

$$= \lim_{n\to\infty} x^{\frac{1}{n+1}}\left(x^{\frac{1}{n(n+1)}} - 1\right)n^2$$

$$= \lim_{n\to\infty} x^{\frac{1}{n+1}} \cdot \frac{x^{\frac{1}{n(n+1)}} - 1}{\frac{1}{n(n+1)}} \cdot \frac{n^2}{n(n+1)} = 1 \cdot \log_e x \cdot 1$$

$$= \log_e x$$

93 **(a)**

$$\lim_{x\to\frac{\pi}{4}} \frac{\int_2^{\sec^2 x} f(t)dt}{x^2 - \frac{\pi^2}{16}} \qquad \left[\frac{0}{0}\text{ from}\right]$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{f(\sec^2 x)2\sec x \sec x \tan x}{2x}$$

$$\therefore \quad L = \frac{2f(2)}{\pi/4} = \frac{8f(2)}{\pi}$$

94 **(a)**

i. $\lim_{x\to\infty} \sec^{-1}\left(\frac{x}{\sin x}\right)$

$= \sec^{-1}\left(\frac{\infty}{\sin\infty}\right)$

$= \sec^{-1}\left(\frac{\infty}{\text{any value between } -1 \text{ to } 1}\right)$

$= \sec^{-1}(\pm\infty) = \frac{\pi}{2}$

ii. $\lim_{x\to\infty} \sec^{-1}\left(\frac{\sin x}{x}\right) = \sec^{-1}\left(\frac{\sin\infty}{\infty}\right)$

$\sec^{-1}\left(\frac{\text{any value between } -1 \text{ to } 1}{\infty}\right)$

$\sec^{-1} 0 =$ not defined

Hence (i) exists but (ii) does not exist

95 **(a)**

Let $\sin^{-1} x = \theta$. Then, $x = \sin\theta$

Now, $x \to \frac{1}{\sqrt{2}} \Rightarrow \sin\theta \to \frac{1}{\sqrt{2}} \Rightarrow \theta \to \frac{\pi}{4}$

$$\therefore \lim_{x\to\frac{1}{\sqrt{2}}} \frac{x - \cos(\sin^{-1} x)}{1 - \tan(\sin^{-1} x)}$$

$$= \lim_{\theta\to\frac{\pi}{4}} \frac{\sin\theta - \cos\theta}{1 - \tan\theta}$$

$$= \lim_{\theta\to\frac{\pi}{4}} \frac{(\sin\theta - \cos\theta)}{(\cos\theta - \sin\theta)}\cos\theta$$

$$= \lim_{\theta\to\frac{\pi}{4}} -\cos\theta = -\frac{1}{\sqrt{2}}$$

96 **(a,b,c,d)**

We have $\lim_{x\to0^+} f(x) = \lim_{k\to0^+} \frac{\tan^2\{x\}}{(x^2-[x]^2)}$

$= \lim_{x\to0^+} \frac{\tan^2 x}{x^2} = 1 \quad (1)$

$(\because x \to 0^+; [x] = 0 \Rightarrow \{x\} = x)$

Also $\lim_{k\to0^-} f(x) = \lim_{x\to0^-} \sqrt{\{x\}\cot\{x\}} = \sqrt{\cot 1} \quad (2)$

$(\because x \to 0^-; [x] = -1 \Rightarrow \{x\} = x + 1 \Rightarrow \{x\} \to 1)$

Also, $\cot^{-1}\left(\lim_{x\to0^-} f(x)\right)^2 = \cot^{-1}(\cot 1) = 1$

97 **(a,b,c)**

$$\lim_{x\to\infty} \frac{\log_e x}{\{x\}} = \frac{\text{Positive infinity}}{\text{A value between 0 and 1}} = \infty$$

$$\lim_{x\to2^+} \frac{x}{x^2 - x - 2} = \lim_{x\to2^+} \frac{x}{(x-2)(x+1)}$$

$$= \lim_{h\to0} \frac{2+h}{h(3+h)} = \infty$$

$$\lim_{h\to-1} \frac{x}{x^2 - x - 2} = -\lim_{h\to-1^-} \frac{x}{(x-2)(x+1)}$$

$$= \lim_{h\to0} \frac{-1-h}{(-3-h)(-h)} = \lim_{h\to0} \frac{1+h}{(3+h)(h)} = -\infty$$

98 **(a,c)**

Since $x^2 > 0$ and limit equals 2, $f(x)$ must be a positive quantity. Also since $\lim_{x\to0} \frac{f(x)}{x^2} = 2$. The denominator → zero and limit is finite, therefore $f(x)$ must be approaching to zero or $\lim_{x\to0}[f(x)] = 0^+$

Hence $\lim_{x\to0}[f(x)] = 0$

$\lim_{x\to0^+}\left[\frac{f(x)}{x}\right] = \lim_{x\to0^+}\left[x\frac{f(x)}{x^2}\right] = 0$ and $\lim_{x\to0^-}\left[\frac{f(x)}{x}\right] = \lim_{x\to0^-}\left[x\frac{f(x)}{x^2}\right] = -1$

Hence $\lim_{x\to0}\left[\frac{f(x)}{x}\right]$ does not exist

99 **(a,c)**

$$\lim_{n\to\infty} \frac{-3 + \frac{(-1)^n}{n}}{4 + \frac{(-1)^n}{n}} = \frac{-3}{4}$$

100 **(b,c)**

Case I $x \neq m\pi$ (m is an integer)

$$\lim_{x\to\infty} \frac{1}{1 + n\sin^2 nx} = \frac{1}{\infty} = 0$$

Case II $x = m\pi$ (m is an integer

$$\lim_{n\to\infty} \frac{1}{1 + n\sin^2 nx} = \frac{1}{1} = 1$$

101 **(a,c)**

Limit $= \lim_{n\to\infty} \frac{an(1+n)-(1+n^2)}{1+n}$

$$= \lim_{n\to\infty} \frac{(a-1)n^2 + an - 1}{n+1}$$

$= \infty$ if $a - 1 \neq 0$

If $a - 1 = 0$, limit $= \lim_{n\to\infty} \frac{an-1}{n+1} = a = b$

$\therefore a = b = 1$

102 **(a,b,c)**

$$L=\lim_{x\to 0}\frac{\sin x^n}{(\sin x)^m}=\lim_{x\to 0}\frac{\frac{\sin x^n}{x^n}x^n}{\frac{(\sin x)^m}{x^m}x^m}=\lim_{x\to 0}x^{n-m}$$

If $n=m$, then

$L=(\text{a very small value near to zero})^{\text{exactly zero}}$
$=1$

If $n>m$, then

$L=(\text{a very small value near to zero})^{\text{positive integer}}$
$=0$

If $n<m$, then

$L=(\text{a very small value near to zero})^{\text{negative integer}}$
$=\infty$

103 **(a,b,c,d)**

$$f(x)=\frac{3x^2+ax+a+1}{(x+2)(x-1)}$$

As $x\to 1, D^r\to 0$, hence as $x\to 1, N^r\to 0$

$\therefore 3+2a+1=0\Rightarrow a=-2\Rightarrow$ (A)

As $x\to -2, D^r\to 0$, hence as $x\to -2, N^r\to 0$

$\therefore 12-2a+a+1=0\Rightarrow a=13\Rightarrow$ (B)

Now $\lim_{x\to 0}f(x)=\lim_{x\to 1}\frac{3x^2-2x-1}{(x+2)(x-1)}=\lim_{x\to 1}\frac{(3x+1)(x-1)}{(x+2)(x-1)}=\frac{4}{3}$

Now $\lim_{x\to -2}\frac{3x^2+13x+14}{(x+2)(x-1)}=\lim_{x\to -2}\frac{(3x+7)(x+2)}{(x+2)(x-1)}=-\frac{1}{3}$

104 **(b,c)**

R.H. limit $=\lim_{h\to 0}f(1+h)=\lim_{h\to 0}a(1+h)=a$

L.H. limit $=\lim_{h\to 0}f(1+h)=\lim_{h\to 0}a\left\{1+\frac{2}{a}(1+h)\right\}=1+\frac{2}{a}$

$\lim_{x\to 1}f(x)$ exists $\Rightarrow$ R.H. limit = L.H. limit

$\Rightarrow a=1+\frac{2}{a}$

$\Rightarrow a=2,-1$

105 **(b,c)**

Since the greatest integer function is discontinuous (sensitive) at integral values of x, then for a given limit to exist both left- and right-hand limit must be equal

L.H.L. $=\lim_{x\to 1^-}(2-x+a[x-1]+b[1+x])$

$=2-1+a(-1)+b(1)=1-a+b$

R.H.L. $=\lim_{x\to 1^+}(2-x+a[x-1]+b[1+x])$

$=2-1+a(0)+b(2)=1+2b$

Om comparing we have $-a=b$

106 **(a,d)**

$f(1+0)=\lim_{h\to 0}\{|1+h-1|-[1+h]\}$

$=\lim_{h\to 0}\{h-1\}=-1$

$f(1-0)=\lim_{h\to 0}\{|1-h-1|-[1-h]\}$

$=\lim_{h\to 0}\{h-0\}=0$

107 **(a,b,c)**

$$=\lim_{x\to 5^-}\frac{x^2-9x+20}{x-[x]}=\lim_{x\to 5^-}\frac{(x-5)(x-4)}{x-4}$$

$$=\lim_{x\to 5^-}(x-5)=0$$

$$=\lim_{x\to 5^+}\frac{x^2-9x+20}{x-[x]}=\lim_{x\to 5^+}\frac{(x-5)(x-4)}{x-5}$$

$$=\lim_{x\to 5^+}(x-4)=1$$

108 **(a,b,c)**

$$L=\lim_{x\to a}\frac{|2\sin x-1|}{2\sin x-1}$$

For $a=\pi/6$, L.H.L. $=\lim_{x\to\frac{\pi}{6}^-}\frac{1-2\sin x}{2\sin x-1}=-1$,

R.H.L. $=\lim_{x\to\frac{\pi}{6}^+}\frac{2\sin x-1}{2\sin x-1}=1$

Hence the limit does not exist

For $a=\pi$, $\lim_{x\to\pi}\frac{1-2\sin x}{2\sin x-1}=-1$ (as in neighbourhood of π, $\sin x$ is less than ½)

For $a=\pi$, $\lim_{x\to\pi/2}\frac{2\sin x-1}{2\sin x-1}=1$ (as in neighbourhood of $\pi/2$, $\sin x$ approaches to 1)

109 **(b,c,d)**

$$f(x)=\lim_{x\to\infty}\frac{x}{x^{2n}+1}$$

$$=\begin{cases}x, x^2<1\\ 0, x^2>1\\ 1/2, x=1\\ -1/2, x=-1\end{cases}$$

$\Rightarrow f(1^+)=f(-1^-)=0$

$f(1^-)=1, f(-1^+)=-1$

$f(1)=1/2$

110 **(d)**

$$\lim_{x\to\infty}\left(\frac{1^2}{x^3}+\frac{2^2}{x^3}+\frac{3^2}{x^3}+\cdots+\frac{x^2}{x^3}\right)$$

$$=\lim_{x\to\infty}\frac{x(x+1)(2x+1)}{6x^3}=\frac{1}{3}$$

111 **(b)**

$$L=\lim_{x\to 0}\frac{\sqrt{1-\cos 2x}}{x}=\lim_{x\to 0}\frac{\sqrt{2}|\sin x|}{x}$$

$\Rightarrow$ L.H.L. $=-\sqrt{2}$ and R.H.L. $=\sqrt{2}$

Hence, the limit of the function does not exist. Also, statement 2 is true, but it is not the correct explanation of statement 1. As for limit to exist, it is not necessary that function is defined at that point

112 **(b)**

$$\lim_{x\to 0^+}[x]\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)=\lim_{h\to 0}[h]\left(\frac{1-e^{-1/h}}{1+e^{-1/h}}\right)=0\times 1$$
$$=0$$

$$\lim_{x\to 0^-}[x]\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)=\lim_{h\to 0}[-h]\left(\frac{e^{-1/h}-1}{e^{-1/h}+1}\right)$$
$$=-1\times(-1)=1$$

Thus, given limit does not exists. Also $\lim_{x\to 0}\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$ does not exist, but this cannot be taken as only reason for non-existence of $\lim_{x\to 0}[x]\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$

113 **(a)**

$\lim_{n\to\infty} a_{n+1}=\lim_{n\to\infty}\sin a_n=\lim_{n\to\infty} a_n$

$\Rightarrow \lim_{n\to\infty}(a_n-\sin a_n)=0$ which is possible only when $\lim_{n\to\infty} a_n=0$

114 **(a)**

If $\lim_{x\to 0} f(x)$ exists, then $\lim_{x\to 0}\left(f(x)+\frac{\sin x}{x}\right)$ always exists as $\lim_{x\to 0}\frac{\sin x}{x}$ exists finitely

Hence $\lim_{x\to 0} f(x)$ must not exist

115 **(b)**

For $x\in(-\delta,\delta), \sin x<x \Rightarrow \lim_{x\to 0}\frac{\sin x}{x}=1^-$

$$\Rightarrow \left[\lim_{x\to 0}\frac{\sin x}{x}\right]=0$$

Also, $x\in(-\delta,\delta), \tan x>x$, but from this nothing can be said about the relation between $\sin x$ and x

Hence, both the statements are true but statement 2 is not the correct explanation of statement 1

116 **(b)**

Limit of function $y=f(x)$ exists at $x=a$, though it is discontinuous at $x=a$. Consider the function $f(x)=\frac{x^2-4}{x-2}$. Here, $f(x)$ is not defined at $x=2$, but limit of functions exists, as $\lim_{x\to 2^+} f(x)=\lim_{x\to 2^-}=4$

117 **(a)**

$$L=\lim_{x\to 0^+}\frac{x}{a}\left[\frac{b}{x}\right]$$

$$=\lim_{x\to 0^+}\frac{x}{a}\left(\frac{b}{x}-\left\{\frac{b}{x}\right\}\right)$$

$$=\lim_{x\to 0^+}\left(\frac{b}{a}-\frac{x}{a}\left\{\frac{b}{x}\right\}\right)$$

$$=\frac{b}{a}-\frac{b}{a}\lim_{x\to 0^+}\frac{\left\{\frac{b}{x}\right\}}{\frac{b}{x}}$$

$=\frac{b}{a}-\frac{b}{a}\lim_{y\to\infty}\frac{\{y\}}{y}$ (where $y=\frac{b}{x}$ and $b>0$) $=\frac{b}{a}$

Also, if $b<0, L=\frac{b}{a}-\frac{b}{a}\lim_{y\to\infty}\frac{\{y\}}{y}=\frac{b}{a}$

118 **(c)**

Obviously statement 1 is true, but statement 2 is not always true

Consider, $f(x)=[x]$ and $g(x)=\sin x$ (where $[\cdot]$ represents greatest integer function)

Here $\lim_{x\to\pi^+}[\sin x]=-1$

and $\lim_{x\to\pi^-}[\sin x]=0$

$\Rightarrow \lim_{x\to\pi}[\sin x]$ does not exist

119 **(a)**

When $n\to\infty$ and x is rational or $x=\frac{p}{q}$, where p and q are integers and $q\neq 0$

$n!\,x=n!\times\frac{p}{q}$ is integer as $n!$ has factor q when $n\to\infty$

Also, when $n!\,x$ is integer, $\sin(n!\,\pi x)=0\Rightarrow$ given limit is zero

120 **(a)**

For $\lim_{x\to\alpha}\frac{\sin(f(x))}{x-\alpha}$, denominator tends to 0; hence the numerator must also tena to 0 for limit to be finite. Then, α is a root of the equation $ax^2+bx+c=0$ or $f(\alpha)=0$. Also, consider $f(\alpha^+)\to 0^+$ and $f(\alpha^-)\to 0^-$

$$\Rightarrow \lim_{x\to\alpha^+}\frac{e^{1/f(x)}-1}{e^{1/f(x)}+1}=\lim_{x\to\alpha^+}\frac{1-e^{-1/f(x)}}{1+e^{-1/f(x)}}=1$$

and $\lim_{x\to\alpha^-}\frac{e^{1/f(x)}-1}{e^{1/f(x)}+1}=-1$

Thus, both the statements are true and statement 2 is the correct explanation of statement 1

121 **(d)**

Obviously, statement 2 is true, as on the number line immediate neighbourhood of 1/2 is either rational or irrational, but this does not stop $f(x)$ to have limit at $x = 1/2$. As $f(1/2) = 1/2, f(1/2^+) = \lim_{x\to 1/2^+} x = 1/2$ (if $1/2^+$ is rational) or $\lim_{x\to 1/2^+}(1-x) = 1 - 1/2 = 1/2$ (if $1/2^+$ is irrational)

Hence $\lim_{x\to 1/2^+} f(x) = 1/2$

With similar argument, we can prove that

$\lim_{x\to 1/2^-} f(x) = 1/2$. Hence, limit of function exists at $x = 1/2$

122 **(a)**

$$\lim_{x\to\infty}\frac{(x-1)(x-2)}{(x-3)(x-4)}$$

$$= \lim_{x\to\infty}\frac{x^2-3x+2}{x^2-7x+12}$$

$$= \lim_{x\to\infty}\frac{1-\frac{3}{x}+\frac{2}{x^2}}{1-\frac{7}{x}+\frac{12}{x^2}} \to 1 \text{ (from right-hand side of 1)}$$

Hence $\lim_{x\to\infty}\cos^{-1} f(x)$ does not exist as $\cos^{-1} x$ is defined for $x \in [-1,1]$

Also, $\lim_{x\to-\infty}\frac{1-\frac{3}{x}+\frac{2}{x^2}}{1-\frac{7}{x}+\frac{12}{x^2}} \to 1$ (from left-hand side of 1)

Hence $\lim_{x\to\infty}\cos^{-1} f(x)$ exists

123 **(a)**

a. Let $x+1=h$

Then, $\lim_{x\to-1}\frac{\sqrt[3]{(7-x)}-2}{(x+1)}$

$$= \lim_{h\to 0}\frac{(8-h)^{1/3}-2}{h}$$

$$= \lim_{h\to 0}\frac{2\left(1-\frac{h}{8}\right)^{1/3}-2}{h}$$

$$= 2\lim_{h\to 0}\frac{\left(1-\frac{1}{3}\frac{h}{8}\right)-1}{h}$$

$$= -\frac{1}{12}$$

b. we have $\lim_{x\to\pi/4}\frac{\tan^3 x-\tan x}{\cos(x+\pi/4)}$

$$= \lim_{x\to\pi/4}\frac{\tan x(\tan x-1)(\tan x+1)}{\cos(x+\pi/4)}$$

$$= \lim_{x\to\pi/4}\frac{\tan x\,(\sin x-\cos x)\,(\tan x+1)}{\cos x\cos(x+\pi/4)}$$

$$= -\lim_{x\to\pi/4}\frac{\tan x\,(\cos x-\sin x)(\tan x+1)}{\cos x\cos(x+\pi/4)}$$

$$= -\sqrt{2}\lim_{x\to\pi/2}\frac{\tan x\left(\frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}\sin x\right)(\tan x+1)}{\cos x\cos(x+\pi/4)}$$

$$= -\sqrt{2}\lim_{x\to\pi/4}\frac{\tan x\,(\tan x+1)}{\cos x}$$

$$= -\sqrt{2}\times 2\times\sqrt{2} = -8$$

c. $\lim_{x\to 1}\frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3}$

$$= \lim_{x\to 1}\frac{(2x-3)(\sqrt{x}-1)}{(2x+3)(x-1)}$$

$$= \lim_{x\to 1}\frac{(2x-3)(\sqrt{x}-1)}{(2x+3)(\sqrt{x}-1)(\sqrt{x}+1)}$$

$$= \lim_{x\to 1}\frac{(2x-3)}{(2x+3)(\sqrt{x}+1)}$$

$$= \frac{2-3}{(2+3)(\sqrt{1}+1)}$$

$$= -1/10$$

d. $\lim_{x\to\infty}\frac{\log x^n-[x]}{[x]}$

$$= \lim_{x\to\infty}\frac{\log x^n}{[x]} - \lim_{x\to\infty}\frac{[x]}{[x]}$$

$$= 0 - 1$$

$$= -1$$

124 **(c)**

a. Here, $a > 0$, if $a \le 0$, then limit $= \infty$

$$\therefore \lim_{x\to\infty}\frac{\left(\sqrt{(x^2-x+1)}-ax-b\right)\left(\sqrt{x^2-x+1}+ax+b\right)}{\left(\sqrt{(x^2-x+1)}+ax+b\right)}$$

$$\Rightarrow \lim_{x\to\infty}\frac{(x^2-x+1)-(ax+b)^2}{\sqrt{(x^2-x+1)}+ax+b} = 0$$

$$\Rightarrow \lim_{x\to\infty}\frac{(1-a^2)x^2-(1+2ab)x+(1-b^2)}{\sqrt{(x^2-x+1)}+ax+b} = 0$$

$$\Rightarrow \lim_{x\to\infty}\frac{(1-a^2)x-(1+2ab)+\frac{(1-b^2)}{x}}{\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a+\frac{b}{x}}$$

This is possible only when $1-a^2=0$ and $1+2ab=0$

$\therefore a = \pm 1$

$\Rightarrow a = 1 \quad (\because a > 0) \quad (1)$

$\Rightarrow b = -1/2$

$\Rightarrow (a, 2b) = (1, -1)$

b. Divide numerator and denominator by $e^{1/x}$, then

$$\lim_{x\to\infty}\frac{(1+a^3)e^{-\frac{1}{x}}+8}{e^{-\frac{1}{x}}+(1-b^3)} = 2$$

$$\Rightarrow \frac{0+8}{0+1-b^3} = 2$$

$\Rightarrow 1-b^3 = 4$

$\therefore b^3 = -3 \Rightarrow b = -3^{1/3}$

Then, $a \in R$

$\Rightarrow (a, b^3) = (a, -3)$

c. $\lim_{x\to\infty}\left(\sqrt{(x^4 - x^2 + 1)} - ax^2 - b\right) = 0$

Put $x = \frac{1}{t}$ $\therefore \lim_{t\to 0}\left(\sqrt{\left(\frac{1}{t^4} - \frac{1}{t^2} + 1\right)} - \frac{a}{t^2} - b\right) = 0$

$\Rightarrow \lim_{t\to 0}\frac{\sqrt{(1-t^2+t^4)}-a-bt^2}{t^2} = 0$ (1)

Since R.H.S. is finite, numerator must be equal to 0 at $t \to 0$

$\therefore 1 - a = 0 \therefore a = 1$

From equation (1), $\lim_{t\to 0}\frac{\sqrt{(1-t^2+t^4)}-1-bt^2}{t^2} = 0$

$\lim_{t\to 0}(-1 + t^2)\left(\frac{(1 - t^2 + t^4)^{1/2} - (1)^{1/2}}{(1 - t^2 + t^4) - 1}\right) = b$

$\Rightarrow (-1)\left(\frac{1}{2}\right) = b \Rightarrow a = 1, b = -\frac{1}{2} \Rightarrow (a, -4b)$
$= (1, 2)$

d. $\lim_{x\to -a}\frac{x^7-(-a)^7}{x-(-a)} = 7 \Rightarrow 7a^6 = 7 \Rightarrow a^6 = 1 \Rightarrow a = -1$

125 **(b)**

We know that $\lim_{x\to 0}\frac{\sin x}{x} = 1$ (but a value which is smaller than 1)

$\Rightarrow \left[\lim_{x\to 0}100\frac{\sin x}{x}\right] = 99$

and $\left[\lim_{x\to 0}100\ \frac{x}{\sin x}\right] = 100$

(Also $\lim_{x\to 0}\frac{\sin^{-1}x}{x} = 1$ (but a value which is more than 1)

$\Rightarrow \left[\lim_{x\to 0} 100\frac{\sin^{-1}x}{x}\right] = 100$

and $\left[\lim_{x\to 0}100\ \frac{x}{\sin^{-1}x}\right] = 99$

$\lim_{x\to 0}\frac{\tan x}{x} = 1$ (but a value which is bigger than 1)

$\Rightarrow \left[\lim_{x\to 0}100\frac{\tan x}{x}\right] = 100$

and $\left[\lim_{x\to 0}100\frac{\tan^{-1}x}{x}\right] = 99$

Hence

1. $\lim_{x\to 0}\left(\left[100\frac{\sin x}{x}\right] + \left[100\frac{\tan x}{x}\right]\right) = 199$
2. $\lim_{x\to 0}\left(\left[100\ \frac{x}{\sin x}\right] + \left[100\frac{\tan x}{x}\right]\right) = 200$
3. $\lim_{x\to 0}\left(\left[100\frac{\sin^{-1}x}{x}\right] + \left[100\frac{\tan^{-1}x}{x}\right]\right) = 199$
4. $\lim_{x\to 0}\left(\left[100\ \frac{x}{\sin^{-1}x}\right] + \left[100\frac{\tan^{-1}x}{x}\right]\right) = 198$

126 **(a)**

We have $f(x) = \frac{\sin^{-1}(1-\{x\})\cos^{-1}(1-\{x\})}{\sqrt{2\{x\}}(1-\{x\})}$

$\therefore \lim_{x\to 0^+}f(x) = \lim_{h\to 0}f(0 + h)$

$= \lim_{h\to 0}\frac{\sin^{-1}(1 - \{0 + h\})\cos^{-1}(1 - \{0 + h\})}{\sqrt{2\{0 + h\}}(1 - \{0 + h\})}$

$= \lim_{h\to 0}\frac{\sin^{-1}(1 - h)\cos^{-1}(1 - h)}{\sqrt{2h}(1 - h)}$

$= \lim_{h\to 0}\frac{\sin^{-1}(1 - h)}{(1 - h)}\lim_{h\to 0}\frac{\cos^{-1}(1 - h)}{\sqrt{2h}}$

In second limit put $1 - h = \cos\theta$

$= \lim_{h\to 0}\frac{\sin^{-1}(1 - h)}{(1 - h)}\lim_{\theta\to 0}\frac{\cos^{-1}(\cos\theta)}{\sqrt{2(1 - \cos\theta)}}$

$= \lim_{h\to 0}\frac{\sin^{-1}(1 - h)}{(1 - h)}\lim_{\theta\to 0}\frac{\theta}{2\sin(\theta/2)}$ $(\because\ \theta > 0)$

$= \sin^{-1}1 \times 1 = \pi/2$

and $\lim_{x\to 0^-}f(x) = \lim_{h\to 0}f(0 - h)$

$= \lim_{h\to 0}\frac{\sin^{-1}(1 + \{0 - h\})\cos^{-1}(1 - \{0 - h\})}{\sqrt{2\{0 - h\}}(1 - \{0 - h\})}$

$= \lim_{h\to 0}\frac{\sin^{-1}(1 + h - i)\cos^{-1}(1 + h - 1)}{\sqrt{2(-h + 1)}(1 + h - 1)}$

$= \lim_{h\to 0}\frac{\sin^{-1}h}{h}\lim_{h\to 0}\frac{\cos^{-1}h}{\sqrt{2(1 - h)}} = 1\frac{\pi/2}{\sqrt{2}} = \frac{\pi}{2\sqrt{2}}$

127 **(c)**

We have $A_i = \frac{x-a_i}{-(x-a_i)} = -1, i = 1, 2, \dots, n$ and

$a_1 < a_2 <\dots< a_{n-1} < a_n$

Let x be in the left neighbourhood of a_m. Then $x - a_i < 0$ for $i = m, m + 1, \dots, n$ and $x - a_i > 0$ for $i = 1, 2, \dots, m - 1$. Therefore,

$A_{\text{i}} = \frac{x-a_i}{-(x-a_i)} = -1$ for $i = m, m + 1, \dots, n$ and

$A_{\text{i}} = \frac{x-a_i}{x-a_i} = 1$ for $i = 1,2, \dots, m - 1$

Similarly, if x is in the right neighbourhood of a_m, then $x - a_i < 0$ for $i = m + 1, \dots, n$ and $x - a_i > 0$ for i $= 1, 2, \dots, m$

$\therefore A_{\text{i}} = \frac{x-a_i}{-(x-a_i)} = -1$ for $i = m + 1, \dots, n$ and

$A_{\text{i}} = \frac{x-a_i}{x-a_i} = 1$ for $i = 1,2, \dots, m$

Now, $\lim_{x\to a_m^-}(A_1\,A_2 \dots A_n) = (-1)^{n-m+1}$ and

$\lim_{x\to a_m^+}(A_1\,A_2 \dots A_n) = (-1)^{n-m}$

Hence, $\lim_{x\to a_m}(A_1A_2 \dots A_n)$ does not exist

128 **(b)**

$L = \lim_{x\to 0}\frac{\sin x + ae^x + be^{-x} + c\,\text{In}(1 + x)}{x^3}$

$= \lim_{x\to 0}\frac{\left(x - \frac{x^3}{3!}\right) + a\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!}\right) + b\left(1 - \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!}\right) + c\left(x - \frac{x^2}{2} + \frac{x^3}{3}\right)}{x^3}$

$$= \lim_{x\to 0} \frac{(a+b)+(1+a-b+c)x + \left(\frac{a}{2}+\frac{b}{2}-\frac{c}{2}\right)x^2 + \left(-\frac{1}{3!}+\frac{a}{3!}-\frac{b}{3!}+\frac{c}{3}\right)x^3}{x^3}$$

$\Rightarrow a+b=0, 1+a-b+c=0, \frac{a}{2}+\frac{b}{2}-\frac{c}{2}=0$

And $L = -\frac{1}{3!}+\frac{a}{3!}-\frac{b}{3!}+\frac{c}{3}$

Solving the first three equations, we get

$c=0, a=-1/2, b=1/2$

Then, $L=-1/3$

Equation $ax^2+bx+c=0$ reduces to

$x^2-x=0 \Rightarrow x=0,1 \big||x+c|-2a\big| < 4b$ reduces to $\big||x|+1\big| < 2$

$\Rightarrow -2 < |x|+1 < 2$

$\Rightarrow 0 \le |x| < 1$

$\Rightarrow x \in [-1,1]$

129 **(c)**

$\lim_{x\to 0^+} F(x) = \lim_{x\to 0^+} (p_1a_1^x + p_2a_2^x + \cdots + p_na_n^x)^{1/x}$

(1^∞ form)

$= e^{\lim_{x\to 0}\left(\frac{p_1a_1^x+p_2a_2^x+\cdots+p_na_n^x-1}{x}\right)}$

$= e^{\lim_{x\to 0}(p_1a_1^x \text{ In } a_1 + p_2a_2^x \text{ In } a_2 + \cdots + p_na_n^x \text{ In } a_n)}$

$= e^{(p_1 \text{ In } a_1 + p_2 \text{ In } a_2 + \cdots + p_n \text{ In } a_n)}$

$= e^{(\text{In } a_1^{p_1} + \text{In } a_2^{p_2} + \cdots + \text{In } a_n^{p_n})}$

$= e^{(\text{In } a_1^{p_1} a_2^{p_2} \ldots a_n^{p_n})}$

$= a_1^{p_1} \cdot a_2^{p_2} a_3^{p_3} \ldots a_n^{p_n}$

130 **(4)**

$$\lim_{x\to 0} \frac{e^{-x^2/2} - \cos x}{x^3 \sin x}$$

$$= \lim_{x\to 0} \frac{\left(1 - \frac{(x^2/2)}{1!} + \frac{(x^2/2)^2}{2!}\right) - \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!}\right)}{x^3\left(x - \frac{x^3}{3!}\right)}$$

$$= \lim_{x\to 0} \frac{\left(\frac{x^4}{8}\right) - \left(\frac{x^4}{24}\right)}{x^4\left(1 - \frac{x^2}{3!}\right)} = \frac{1}{12}$$

131 **(1)**

$$\lim_{x\to\infty}\left(f(x) + \frac{3f(x)-1}{f^2(x)}\right) = 3$$

$$\Rightarrow \left(\lim_{x\to\infty} f(x) + \frac{3\lim_{x\to\infty} f(x) - 1}{\left(\lim_{x\to\infty} f(x)\right)^2}\right) = 3$$

$$\Rightarrow \left(y + \frac{3y-1}{y^2}\right) = 3$$

$\Rightarrow y^3 - 3y^2 + 3y - 1 = 0$

$\Rightarrow (y-1)^3 = 0$

$\Rightarrow y = 1$

132 **(6)**

Put $x = 1+h$

Then $f(a) = \lim_{h\to 0} \frac{(1+h)^a - a(1+h) + a - 1}{h^2}$

$$= \lim_{h\to 0} \frac{\left(1 + ah + \frac{a(a-1)}{2!}h^2 + \cdots\right) - a - ah + a - 1}{h^2}$$

$$\therefore f(a) = \frac{a(a-1)}{2}$$

$\therefore f(4) = 6$

133 **(3)**

$$\lim_{x\to 2} \frac{(10-x)^{1/3} - 2}{x-2}$$

$= \lim_{h\to 0} \frac{(8-h)^{1/3}-2}{h}$ (Put $x = 2+h$)

$$= \lim_{h\to 0} \frac{2\left(1-\frac{h}{8}\right)^{1/3} - 2}{h}$$

$$= 2\lim_{h\to 0} \frac{\left(1-\frac{h}{8}\right)^{1/3} - 1}{h}$$

$$= 2\lim_{h\to 0} \frac{1 - \frac{1}{3}\frac{h}{8} - 1}{h} = -\frac{1}{12}$$

134 **(3)**

$\lim_{x\to 1} (1+ax+bx^2)^{\frac{c}{x-1}} = e^3$

$\Rightarrow e^{\lim_{x\to 1}(1+ax+bx^2-1)^{\frac{c}{x-1}}} = e^3$

$\Rightarrow e^{\lim_{x\to 1}\frac{c(ax+bx^2)}{x-1}} = e^3$

$$\Rightarrow \lim_{x\to 1} \frac{c(ax+bx^2)}{x-1} = 3$$

$$\Rightarrow \lim_{h\to 0} \frac{c(a(1+h) + b(1+h)^2)}{1+h-1} = 3$$

$$\Rightarrow \lim_{h\to 0} \frac{(ca+b) + (ac+2b)h + bh^2}{h} = 3$$

$\Rightarrow ca+b=0$ and $ac+2b=3$

$\Rightarrow b=3$ and $ac=-3$

Also the form must be 1^∞ for which $a+b=0 \Rightarrow a=-3$ and $c=1$

135 **(0)**

Let $L = \lim_{x\to\infty} \frac{\log_e(\log_e x)}{e^{\sqrt{x}}} = \left(\frac{\infty}{\infty} \text{ form}\right)$

$$= \lim_{x\to\infty} \frac{\frac{1}{x\log_e x}}{e^{\sqrt{x}}\frac{1}{2\sqrt{x}}}$$

$$= \lim_{x\to\infty} \frac{2\sqrt{x}}{e^{\sqrt{x}} x \log_e x}$$

$$= \lim_{x\to\infty} \frac{2}{e^{\sqrt{x}}\sqrt{x}\log_e x}$$

136 **(3)**

$$L = \lim_{x\to 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$$

$$= -\lim_{x\to 0} \frac{(1-\cos x)(1+\cos x)(\cos x - e^x)}{(1+\cos x)x^n}$$

$$= \lim_{x\to 0} \frac{\left(\frac{\sin x}{x}\right)^2 \left(\frac{1-\cos x}{x} + \frac{e^x - 1}{x}\right)}{x^{n-3}} \frac{1}{1+\cos x}$$

If L is finite non-zero, then $n = 3$ (as for $n = 1, 2, L = 0$ and for $n = 4, L = \infty$)

137 **(6)**

It is obvious n is even, then

$$\lim_{n\to\infty} \left(2^{1+3+5+\cdots+n/2 \text{ terms}} \cdot 3^{2+4+6+\cdots+n/2 \text{ terms}}\right)^{\frac{1}{(n^2+1)}}$$

$$= \lim_{n\to\infty} \left(2^{\frac{n^2}{4}} \cdot 3^{\frac{n(n+2)}{4}}\right)^{\frac{1}{(n^2+1)}}$$

$$= \lim_{n\to\infty} 2^{\frac{n^2}{4(n^2+1)}} \cdot 3^{\frac{n(n+2)}{4(n^2+1)}}$$

$$= 2^{\lim_{n\to\infty} \frac{1}{4\left(1+\frac{1}{n^2}\right)}} \cdot 3^{\lim_{n\to\infty} \frac{\left(1+\frac{2}{n}\right)}{4\left(1+\frac{1}{n^2}\right)}}$$

$$= 2^{\frac{1}{4}} 3^{\frac{1}{4}} = (6)^{\frac{1}{4}}$$

138 **(2)**

$$\lim_{x\to\infty} \frac{2x-3}{x} < \lim_{x\to\infty} f(x) < \lim_{x\to\infty} \frac{2x^2+5x}{x^2}$$

$$\Rightarrow \lim_{x\to\infty} \frac{2-\frac{3}{x}}{1} < \lim_{x\to\infty} f(x) < \lim_{x\to\infty} \frac{2+\frac{5}{x^2}}{1}$$

$$\Rightarrow \lim_{x\to\infty} f(x) = 2$$

139 **(6)**

$$\lim_{x\to 1^+} f(g(x)) = f(g(1^+)) = f(2^+) = 2^2 + 2 = 6$$

and $\lim_{x\to 1^-} f(g(x)) = f(g(1^-)) = f(3 - 1^-) = f(2^+) = 2^2 + 2 = 6$

Hence $\lim_{x\to 1} f(g(x)) = 6$

140 **(4)**

Let $x = 1/y$

$$\Rightarrow \lim_{x\to\infty} \left(x - x^2 \log_e\left(1 + \frac{1}{x}\right)\right)$$

$$= \lim_{y\to 0} \left(\frac{1}{y} - \frac{\log_e(1+y)}{y^2}\right)$$

$$= \lim_{y\to 0} \left(\frac{y - \log_e(1+y)}{y^2}\right)$$

$$= \lim_{y\to 0} \left(\frac{y - \left(y - \frac{y^2}{2}\right)}{y^2}\right) = 1/2$$

141 **(0)**

$$\lim_{x\to 0^+} f\left(g(h(x))\right) = f(g(0^+)) = f(1^+) = 0$$

$$\lim_{x\to 0^-} f\left(g(h(x))\right) = f(g(0^+)) = f(1^+) = 0$$

Hence $\lim_{x\to 0} f\left(g(h(x))\right) = 0$

142 **(2)**

$$\lim_{x\to 0} \left[1 + x + \frac{f(x)}{x}\right]^{1/x} = e^3$$

$$\Rightarrow \lim_{x\to 0} e^{\lim_{x\to 0}\left[1+x+\frac{f(x)}{x}-1\right]\frac{1}{x}} = e^3$$

$$\Rightarrow \lim_{x\to 0} e^{\lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right]} = e^3$$

$$\Rightarrow \lim_{x\to 0} \frac{f(x)}{x^2} = 2$$

Now $\lim_{x\to 0} \left[1 + \frac{f(x)}{x}\right]^{1/x} = e^{\lim_{x\to 0}\left[1+\frac{f(x)}{x}-1\right]\frac{1}{x}} = e^{\lim_{x\to 0} \frac{f(x)}{x^2}} = e^2$

143 **(8)**

Since RHS is finite quantity

$\therefore$ At $x \to 1$, Numerator must be $= 0$

$\therefore 0 + b + 4 = 0$

$\therefore b = -4$

Then $\lim_{x\to 1} \frac{a\sin(x-1) - 4\cos(x-1) + 4}{(x^2-1)} = -2$

Put $x = 1 + h$, Then $\lim_{h\to 0} \frac{a\sin h + 4(1-\cos h)}{h(2+h)} = -2$

$$\Rightarrow \lim_{h\to 0} \frac{\left(\frac{a\sin h}{h}\right) + 4\left(\frac{1-\cos h}{h}\right)}{2+h} = -2$$

$$\Rightarrow \frac{a(1) + 0}{2} = -2$$

$$\Rightarrow a = -4$$

$$\Rightarrow |a + b| = 8$$

144 **(6)**

$L = \lim_{x\to 0} = -\lim_{x\to 0} \frac{D\prod_{r=2}^{n}(\cos rx)^{1/r}}{2x}$ (Using L' Hospital's rule)

Let $y = \prod_{r=2}^{n}(\cos rx)^{1/r}$

$$\Rightarrow \text{In } y = \sum_{r=2}^{n} \left(\frac{1}{r} \text{ In } (\cos rx)\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -\sum_{r=2}^{n} \tan(rx)$$

$$\Rightarrow -Dy = y \sum_{r=2}^{n} \tan(rx)$$

$$\Rightarrow D \prod_{r=2}^{n} (\cos rx)^{1/r} = -y \sum_{r=2}^{n} \tan(rx)$$

$$\Rightarrow L = \lim_{x\to 0} \frac{y \cdot \sum_{r=2}^{n} \tan(rx)}{2x}$$

$$= \frac{1}{2}[2 + 3 + 4 + \cdots + n]$$

$$= \frac{1}{2}\left[\frac{n(n+1)}{2} - 1\right]$$

$$= \frac{n^2 + n - 2}{4}$$

$$\Rightarrow \frac{n^2 + n - 2}{4} = 10$$

$$\Rightarrow n^2 + n - 42 = 0$$

$$\Rightarrow (n+7)(n-6) = 0$$

$$\Rightarrow n = 6$$

145 **(9)**

$$f(x)=\frac{3x^2+ax+a+1}{(x+2)(x-1)}$$

As $x\to -2, D^r\to 0$, hence as $x\to -2, N^r\to 0$

$\therefore 12-2a+a+1=0 \Rightarrow a=13$

146 **(7)**

We have,

$$L=\lim_{x\to 0}\frac{2f(x)-3af(2x)+bf(8x)}{\sin^2 x}$$

For the limit to exist, we have $2f(0)-3af(0)+bf(0)=0$

$\Rightarrow 3a-b=2\ [\because f(0)\neq 0, \text{given}]\quad (1)$

$$\Rightarrow L=\lim_{x\to 0}\frac{2f'(x)-6a\,f'(2x)+8b\,f'(8x)}{2x}$$

For the limit to exist, we have $2f'(0)-6af'(0)+8bf'(0)=0$

$\Rightarrow 3a-4b=1\ [\because f'(0)\neq 0, \text{given}]\quad (2)$

Solving equations (1) and (2), we have $a=7/9$ and $b=1/3$

147 **(0)**

$$\lim_{n\to\infty}\left[\sqrt[3]{(n+1)^2}-\sqrt[3]{(n-1)^2}\right]$$

$$=\lim_{n\to\infty} n^{2/3}\left[\left(1+\frac{1}{n}\right)^{2/3}-\left(1-\frac{1}{n}\right)^{2/3}\right]$$

$$=\lim_{n\to\infty} n^{2/3}\left[\left(1+\frac{2}{3}.\frac{1}{n}+\frac{\frac{2}{3}\left(\frac{2}{2}-1\right)}{2!}\frac{1}{n^2}\dots\right)-\left(1-\frac{2}{3}.\frac{1}{n}+\frac{\frac{2}{3}\left(\frac{2}{3}-1\right)}{2!}\frac{1}{n^2}\dots\right)\right]$$

$$=\lim_{n\to\infty} n^{2/3}\left[\frac{4}{3}.\frac{1}{n}+\frac{8}{81}.\frac{1}{n^3}+\cdots\right]$$

$$=\lim_{n\to\infty}\left[\frac{4}{3}.\frac{1}{n^{1/3}}+\frac{8}{81}.\frac{1}{n^{7/3}}+\cdots\right]=0$$

148 **(3)**

$S_n=\frac{n(n+1)}{2}$ and $S_n-1=\frac{(n+2)(n-1)}{2}$

$$\therefore \frac{S_n}{S_n-1}=\frac{n(n+1)}{2}\cdot\frac{2}{(n+2)(n-1)}$$

$$\Rightarrow \frac{S_n}{S_n-1}=\left(\frac{n}{n-1}\right)\left(\frac{n+1}{n+2}\right)$$

$$\Rightarrow P_n=\left(\frac{2}{1}\cdot\frac{3}{2}\cdot\frac{4}{3}\cdot\frac{5}{4}\cdots\frac{n}{n-1}\right)\left(\frac{3}{4}\cdot\frac{4}{5}\cdot\frac{5}{6}\cdots\frac{n+1}{n+2}\right)$$

$$\Rightarrow P_n=\left(\frac{n}{2}\right)\left(\frac{3}{n+2}\right)$$

$$\Rightarrow \lim_{n\to\infty}P_n=3$$

149 **(2)**

We have

$$L=\lim_{n\to\infty}\prod_{n=2}^{n}\frac{n^2-1}{n^2}$$

$$=\lim_{n\to\infty}\prod_{n=2}^{n}\frac{n-1}{n}\cdot\prod_{n=2}^{n}\frac{n+1}{n}$$

$$=\lim_{n\to\infty}\left(\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdots\frac{n-1}{n}\right)\left(\frac{3}{2}\cdot\frac{4}{3}\cdot\frac{5}{4}\cdots\frac{n+1}{n}\right)$$

$$=\lim_{n\to\infty}\frac{1}{n}\cdot\frac{n+1}{2}=\frac{1}{2}$$