Single Correct Answer Type

1. $\sec^2(\tan^{-1} 2) + \text{cosec}^2\ (\cot^{-1} 3)$ is equal to
a) 5 b) 13 c) 15 d) 6

2. The value of $\lim_{|x|\to\infty} \cos(\tan^{-1}(\sin(\tan^{-1} x)))$ is equal to
a) -1 b) $\sqrt{2}$ c) $-\dfrac{1}{\sqrt{2}}$ d) $\dfrac{1}{\sqrt{2}}$

3. The value of $\sin^{-1}(\sin 12) + \cos^{-1}(\cos 12)$ is equal to
a) Zero b) $24 - 2\pi$ c) $4\pi - 24$ d) None of these

4. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$, then $x^4 + y^4 + z^4 + 4x^2y^2z^2 = K(x^2y^2 + y^2z^2 + z^2x^2)$, where K is equal to
a) 1 b) 2 c) 4 d) None of these

5. $\cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right]\left(\text{ where } x \in \left[0,\frac{\pi}{2}\right]\right)$ is equal to
a) $\pi - x$ b) $2\pi - x$ c) $\dfrac{x}{2}$ d) $\pi - \dfrac{x}{2}$

6. The product of all values of x satisfying the equation
$\sin^{-1}\cos\left(\frac{2x^2+10|x|+4}{x^2+5|x|+3}\right) = \cos\left(\cot^{-1}\left(\frac{2-18|x|}{9|x|}\right)\right) + \frac{\pi}{2}$ is
a) 9 b) -9 c) -3 d) -1

7. The value of $\sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right]$ is equal to
a) $\sin^{-1} x + \sin^{-1}\sqrt{x}$ b) $\sin^{-1} x - \sin^{-1}\sqrt{x}$ c) $\sin^{-1}\sqrt{x} - \sin^{-1} x$ d) None of these

8. If $x^2 + y^2 + z^2 = r^2$, then $\tan^{-1}\left(\frac{xy}{zr}\right) + \tan^{-1}\left(\frac{yz}{xr}\right) + \tan^{-1}\left(\frac{xz}{yr}\right)$ is equal to
a) π b) $\dfrac{\pi}{2}$ c) 0 d) None of these

9. If $\tan^{-1} x + 2\cot^{-1} x = \frac{2\pi}{3}$, then x is equal to
a) $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$ b) 3 c) $\sqrt{3}$ d) $\sqrt{2}$

10. The value of $\sin^{-1}(x^2 - 4x + 6) + \cos^{-1}(x^2 - 4x + 6)$ for all $x \in R$ is
a) $\dfrac{\pi}{2}$ b) π c) 0 d) None of these

11. Let $\begin{vmatrix} \tan^{-1} x & \tan^{-1} 2x & \tan^{-1} 3x \\ \tan^{-1} 3x & \tan^{-1} x & \tan^{-1} 2x \\ \tan^{-1} 2x & \tan^{-1} 3x & \tan^{-1} x \end{vmatrix} = 0$, then the number of values of x satisfying the equation is
a) 1 b) 2 c) 3 d) 4

12. The value of $\cos^{-1}\sqrt{\frac{2}{3}} - \cos^{-1}\frac{\sqrt{6}+1}{2\sqrt{3}}$ is equal to
a) $\dfrac{\pi}{3}$ b) $\dfrac{\pi}{4}$ c) $\dfrac{\pi}{2}$ d) $\dfrac{\pi}{6}$

13. If $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - \cdots\right) + \cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{4}\cdots\right) = \frac{\pi}{2}$ for $0 < |x| < \sqrt{2}$, then x equals
a) $1/2$ b) 1 c) $-1/2$ d) -1

14. The trigonometric equation $\sin^{-1} x = 2\sin^{-1} a$ has a solution for
a) All real values b) $|a| < \dfrac{1}{2}$ c) $|a| \le \dfrac{1}{\sqrt{2}}$ d) $\dfrac{1}{2} < |a| < \dfrac{1}{\sqrt{2}}$

15. The value of $'a'$, for which $ax^2 + \sin^{-1}(x^2 - 2x + 2) + \cos^{-1}(x^2 - 2x + 2) = 0$ has a real solution is
a) $\dfrac{\pi}{2}$ b) $-\dfrac{\pi}{2}$ c) $\dfrac{2}{\pi}$ d) $-\dfrac{2}{\pi}$

16. Of $\tan\left[\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right] + \tan\left[\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right]$ is equal to

a) $\frac{2a}{b}$ b) $\frac{2b}{a}$ c) $\frac{a}{b}$ d) $\frac{b}{a}$

17. $\sin^{-1}(\sin 5) > x^2 - 4x$ holds if

a) $x = 2 - \sqrt{9 - 2\pi}$ b) $x = 2 + \sqrt{9 - 2\pi}$
c) $x > 2 + \sqrt{9 - 2\pi}$ d) $x \in (2 - \sqrt{9 - 2\pi}, 2 + \sqrt{9 - 2\pi})$

18. The sum of the solutions of the equation $2\sin^{-1}\sqrt{x^2 + x + 1} + \cos^{-1}\sqrt{x^2 + x} = \frac{3\pi}{2}$ is

a) 0 b) -1 c) 1 d) 2

19. The value of $\tan^{-1}\left(\frac{x\cos\theta}{1 - x\sin\theta}\right) - \cot^{-1}\left(\frac{\cos\theta}{x - \sin\theta}\right)$ is

a) 2θ b) θ c) $\theta/2$ d) Independent of θ

20. If $a\sin^{-1}x - b\cos^{-1}x = c$, then $a\sin^{-1}x + b\cos^{-1}x$ is equal to

a) 0 b) $\frac{\pi ab + c(b - a)}{a + b}$ c) $\frac{\pi}{2}$ d) $\frac{\pi ab + c(a - b)}{a + b}$

21. If $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then the value of $\tan^{-1}\left(\frac{\tan x}{4}\right) + \tan^{-1}\left(\frac{3\sin 2x}{5 + 3\cos 2x}\right)$ is

a) $x/2$ b) $2x$ c) $3x$ d) x

22. The value of $\sin^{-1}\left(\cot\left(\sin^{-1}\sqrt{\frac{2 - \sqrt{3}}{4}} + \cos^{-1}\frac{\sqrt{12}}{4} + \sec^{-1}\sqrt{2}\right)\right)$ is

a) 0 b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) None of these

23. The value of x which satisfies equation $2\tan^{-1}2x = \sin^{-1}\frac{4x}{1 + 4x}$ is valid in the interval

a) $\left[\frac{1}{2}, \infty\right)$ b) $\left(-\infty, -\frac{1}{2}\right]$ c) $[-1, 1]$ d) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

24. If $\left|\cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right)\right| < \frac{\pi}{2}$, then

a) $x \in \left[-\frac{1}{3}, \frac{1}{\sqrt{3}}\right]$ b) $x \in \left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ c) $x \in \left(0, \frac{1}{\sqrt{3}}\right)$ d) None of these

25. $\sum_{r=1}^{n} \sin^{-1}\left(\frac{\sqrt{r} - \sqrt{r - 1}}{\sqrt{r(r + 1)}}\right)$ is equal to

a) $\tan^{-1}(\sqrt{n}) - \frac{\pi}{4}$ b) $\tan^{-1}(\sqrt{n + 1}) - \frac{\pi}{4}$ c) $\tan^{-1}(\sqrt{n})$ d) $\tan^{-1}(\sqrt{n + 1})$

26. $\cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right)$ is given by

a) $\frac{5\pi}{4}$ b) $\frac{3\pi}{4}$ c) $\frac{-\pi}{4}$ d) None of these

27. If $\tan^{-1}\frac{a + x}{a} + \tan^{-1}\frac{a - x}{a} = \frac{\pi}{6}$, then $x^2 =$

a) $2\sqrt{3}a$ b) $\sqrt{3}a$ c) $2\sqrt{3}a^2$ d) None of these

28. The value $2\tan^{-1}\left[\sqrt{\frac{a - b}{a + b}}\tan\frac{\theta}{2}\right]$ is equal to

a) $\cos^{-1}\left(\frac{a\cos\theta + b}{a + b\cos\theta}\right)$ b) $\cos^{-1}\left(\frac{a + b\cos\theta}{a\cos\theta + b}\right)$ c) $\cos^{-1}\left(\frac{a\cos\theta}{a + b\cos\theta}\right)$ d) $\cos^{-1}\left(\frac{b\cos\theta}{a\cos\theta + b}\right)$

29. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$, then

a) $x + y + z - xyz = 0$ b) $x + y + z + xyz = 0$ c) $xy + yz + zx + 1 = 0$ d) $xy + yz + zx - 1 = 0$

30. If $2\tan^{-1}x = \pi + \tan^{-1}\left(\frac{2x}{1 - x^2}\right)$, then

a) $x > 1$ b) $x < 1$ c) $x > -1$ d) $-1 < x < 1$

31. If x takes negative permissible value, then $\sin^{-1}x$ is equal to

a) $\cos^{-1}\sqrt{1 - x^2}$ b) $-\cos^{-1}\sqrt{1 - x^2}$ c) $\cos^{-1}\sqrt{x^2 - 1}$ d) $\pi - \cos^{-1}\sqrt{1 - x^2}$

32. The solution set of the equation $\sin^{-1}\sqrt{1 - x^2} + \cos^{-1}x = \cot^{-1}\frac{\sqrt{1 - x^2}}{x} - \sin^{-1}x$ is

a) $[-1, 1] - \{0\}$ b) $(0, 1] \cup \{-1\}$ c) $[-1, 0) \cup \{1\}$ d) $[-1, 1]$

33. If $\sin^{-1}a + \sin^{-1}b + \sin^{-1}c = \pi$, then the value of $a\sqrt{(1 - a^2)} + b\sqrt{(1 - b^2)} + c\sqrt{(1 - c^2)}$ will be

a) $2abc$ b) abc c) $\frac{1}{2}abc$ d) $\frac{1}{3}abc$

34. If $2^{2\pi/\sin^{-1}x} - 2(a+2)2^{\pi/\sin^{-1}x} + 8a < 0$ for at least one real x, then

a) $\frac{1}{8} \leq a < 2$ b) $a < 2$ c) $a \in R - \{2\}$ d) $a \in \left[0, \frac{1}{8}\right) \cup (2, \infty)$

35. The number of integral values of k for which the equation $\sin^{-1} x + \tan^{-1} x = 2k + 1$ has a solution is

a) 1 b) 2 c) 3 d) 4

36. The value of $\tan(\sin^{-1}(\cos(\sin^{-1} x))) \tan(\cos^{-1}(\sin(\cos^{-1} x)))$, where $x \in (0, 1)$, is equal to

a) 0 b) 1 c) -1 d) None of these

37. If $\cot^{-1}\frac{n}{\pi} > \frac{\pi}{6}, n \in N$, then the maximum value of n is

a) 6 b) 7 c) 5 d) None of these

38. $\sum_{m=1}^{n} \tan^{-1}\left(\frac{2m}{m^4+m^2+2}\right)$ is equal to

a) $\tan^{-1}\left(\frac{n^2+n}{n^2+n+2}\right)$ b) $\tan^{-1}\left(\frac{n^2-n}{n^2-n+2}\right)$ c) $\tan^{-1}\left(\frac{n^2+n+2}{n^2+n}\right)$ d) None of these

39. $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1} x\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1} x\right), x \neq 0$, is equal to

a) x b) $2x$ c) $\frac{2}{x}$ d) None of these

40. The number of solution of the equation $\sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1} x$ is

a) 1 b) 0 c) 2 d) None of these

41. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, then

a) $x^2 + y^2 + z^2 + xyz = 0$ b) $x^2 + y^2 + z^2 + 2xyz = 0$

c) $x^2 + y^2 + z^2 + xyz = 1$ d) $x^2 + y^2 + z^2 + 2xyz = 1$

42. If $[\cot^{-1} x] + [\cos^{-1} x] = 0$, where $[\cdot]$ denotes the greatest integer function, then the complete set of values of x is

a) $(\cos 1, 1]$ b) $(\cos 1, \cos 1)$ c) $(\cos 1, 1]$ d) None of these

43. If $\sin^{-1}(x-1) + \cos^{-1}(x-3) + \tan^{-1}\left(\frac{x}{2-x^2}\right) = \cos^{-1} k + \pi$, then the value of k is

a) 1 b) $-\frac{1}{\sqrt{2}}$ c) $\frac{1}{\sqrt{2}}$ d) None of these

44. $\sin^{-1}(3x - 2 - x^2) + \cos^{-1}(x^2 - 4x + 3) = \frac{\pi}{4}$ can have a solution for $x \in$

a) $[1, 2]$ b) $\left(\frac{3+\sqrt{5}}{2}, 2+\sqrt{2}\right)$ c) $\left(\frac{3-\sqrt{5}}{2}, 2-\sqrt{2}\right)$ d) None of these

45. $\cos^{-1}\left(\cos\left(2\cot^{-1}\left(\sqrt{2}-1\right)\right)\right)$ is equal to

a) $\sqrt{2} - 1$ b) $\frac{\pi}{4}$ c) $\frac{3\pi}{4}$ d) None of these

46. If $2\sin^{-1} x = \sin^{-1}(2x\sqrt{1-x^2})$, then

a) $[-1, 1]$ b) $\left[-\frac{1}{\sqrt{2}}, 1\right]$ c) $\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$ d) None of these

47. Range of $f(x) = \sin^{-1} x + \tan^{-1} x + \sec^{-1} x$ is

a) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ b) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ c) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$ d) None of these

48. The value of x for which $\sin[\cot^{-1}(1+x)] = \cos(\tan^{-1} x)$ is

a) $\frac{1}{2}$ b) 1 c) 0 d) $-\frac{1}{2}$

49. Range of $\tan^{-1}\left(\frac{2x}{1+x^2}\right)$ is

a) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ c) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right]$ d) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

50. If $\sin^{-1}\left(\frac{5}{x}\right) + \sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2}$, then x is equal to

a) $\frac{7}{13}$ b) $\frac{4}{3}$ c) 13 d) $\frac{13}{7}$

51. If $\cos^{-1} x - \cos^{-1} \frac{y}{2} = \alpha$, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to

a) 4 b) $2\sin^2\alpha$ c) $-4\sin^2\alpha$ d) $4\sin^2\alpha$

52. The number of solution of the equation $\cos^{-1}\left(\frac{1+x^2}{2x}\right) - \cos^{-1} x = \frac{\pi}{2} + \sin^{-1} x$ is given by

a) 0 b) 1 c) 2 d) 3

53. The value of $\sin^{-1}(\cos(\cos^{-1}(\cos x) + \sin^{-1}(\sin x)))$, where $x \in \left(\frac{\pi}{2}, \pi\right)$, is equal to

a) $\frac{\pi}{2}$ b) $-\pi$ c) π d) $-\frac{\pi}{2}$

54. If $x_1 = 2\tan^{-1}\left(\frac{1+x}{1-x}\right), x_2 = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, where $x \in (0, 1)$, then $x_1 + x_2$ is equal to

a) 0 b) 2π c) π d) None of these

55. If $\cot^{-1} x + \cot^{-1} y + \cot^{-1} z = \frac{\pi}{2}, x, y, z > 0$ and $xy < 1$, then $x + y + z$ is also equal to

a) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ b) xyz c) $xy + yz + zx$ d) None of these

56. $\tan^{-1}\left[\frac{\cos x}{1+\sin x}\right]$ is equal to

a) $\frac{\pi}{4} - \frac{x}{2}$, for $x \in \left(-\frac{\pi}{2}, \frac{3\pi}{2}\right)$

b) $\frac{\pi}{4} - \frac{x}{2}$, for $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

c) $\frac{\pi}{4} - \frac{x}{2}$, for $x \in \left(\frac{3\pi}{2}, \frac{5\pi}{2}\right)$

d) $\frac{\pi}{4} - \frac{x}{2}$, for $x \in \left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right)$

57. If $\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \sin^{-1}\left(\frac{2b}{1+b^2}\right) = 2\tan^{-1} x$, then x is equal to $[a, b \in (0, 1)]$

a) $\frac{a-b}{1+ab}$ b) $\frac{b}{1+ab}$ c) $\frac{b}{1-ab}$ d) $\frac{a+b}{1-ab}$

58. If $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$, then $\sin x$ is

a) $\tan^2\frac{\alpha}{2}$ b) $\cot^2\frac{\alpha}{2}$ c) $\tan a$ d) $\cot\frac{\alpha}{2}$

59. $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right)$ is

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{4}$ or $-\frac{3\pi}{4}$

60. If $\sin^{-1} x = \theta + \beta$ and $\sin^{-1} y = \theta - \beta$, then $1 + xy$ is equal to

a) $\sin^2\theta + \sin^2\beta$ b) $\sin^2\theta + \cos^2\beta$ c) $\cos^2\theta + \cos^2\theta$ d) $\cos^2\theta + \sin^2\beta$

61. The value of $2\tan^{-1}(\operatorname{cosec}\tan^{-1} x - \tan\cot^{-1} x)$ is equal to

a) $\cot^{-1} x$ b) $\cot^{-1}\frac{1}{x}$ c) $\tan^{-1} x$ d) None of these

62. The value of $\tan\left(\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right)$ is

a) $\frac{3+\sqrt{5}}{2}$ b) $3+\sqrt{5}$ c) $\frac{1}{2}(3-\sqrt{5})$ d) None of these

63. If $A = \tan^{-1}\left(\frac{x\sqrt{3}}{2K-x}\right)$ and $B = \tan^{-1}\left(\frac{2x-K}{K\sqrt{3}}\right)$, then the value of $A - B$ is

a) 0° b) 45° c) 60° d) 30°

64. The value of the expression $\sin^{-1}\left(\sin\frac{22\pi}{7}\right) + \cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \tan^{-1}\left(\tan\frac{5\pi}{7}\right) + \sin^{-1}(\cos 2)$ is

a) $\frac{17\pi}{42} - 2$ b) -2 c) $\frac{-\pi}{21} - 2$ d) None of these

65. The value of $\sum_{r=0}^{\infty}\tan^{-1}\left(\frac{1}{1+r+r^2}\right)$ is equal to

a) $\frac{\pi}{2}$ b) $\frac{3\pi}{4}$ c) $\frac{\pi}{4}$ d) None of these

66. The value of $\cos\left(\frac{1}{2}\cos^{-1}\frac{1}{8}\right)$ is

a) $\frac{3}{4}$ b) $-\frac{3}{4}$ c) $\frac{1}{16}$ d) $\frac{1}{4}$

67. If $x \in [-1, 0)$, then $\cos^{-1}(2x^2 - 1) - 2\sin^{-1}x$ is equal to

a) $-\frac{\pi}{2}$ b) π c) $\frac{3\pi}{2}$ d) -2π

68. If $x \in (0, 1)$, then the value of $\tan^{-1}\left(\frac{1-x^2}{2x}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is equal to

a) $-\frac{\pi}{2}$ b) Zero c) $\frac{\pi}{2}$ d) π

69. If $f(x) = \sin^{-1}\left(\frac{\sqrt{3}}{2}x - \frac{1}{2}\sqrt{1-x^2}\right), -\frac{1}{2} \le x \le 1$, then $f(x)$ is equal to

a) $\sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(x)$ b) $\sin^{-1}x - \frac{\pi}{6}$ c) $\sin^{-1}x + \frac{\pi}{6}$ d) None of these

70. The number of solutions of the equation $\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$ is

a) 2 b) 3 c) 1 d) 0

71. If we consider only the principal values of the inverse trigonometric functions, then the value of $\tan\left(\cos^{-1}\frac{1}{5\sqrt{2}} - \sin^{-1}\frac{4}{\sqrt{17}}\right)$ is

a) $\frac{\sqrt{29}}{3}$ b) $\frac{29}{3}$ c) $\frac{\sqrt{3}}{29}$ d) $\frac{3}{29}$

72. If $3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$, where $|x| < 1$, then x is equal to

a) $\frac{1}{\sqrt{3}}$ b) $-\frac{1}{\sqrt{3}}$ c) $\sqrt{3}$ d) $-\frac{\sqrt{3}}{4}$

73. Complete solution set of $[\cot^{-1}x] + 2[\tan^{-1}x] = 0$, where $[\cdot]$ denotes the greatest integer function, is equal to

a) $(0, \cot 1)$ b) $(0, \tan 1)$ c) $(\tan 1, \infty)$ d) $(\cot 1, \tan 1)$

74. There exists a positive real number x satisfying $\cos(\tan^{-1}x) = x$. Then the value of $\cos^{-1}\left(\frac{x^2}{2}\right)$ is

a) $\frac{\pi}{10}$ b) $\frac{\pi}{5}$ c) $\frac{2\pi}{5}$ d) $\frac{4\pi}{5}$

75. $\sum_{r=1}^{n}\tan^{-1}\left(\frac{2^{r-1}}{1+2^{2r-1}}\right)$ is equal to

a) $\tan^{-1}(2^n)$ b) $\tan^{-1}(2^n) - \frac{\pi}{4}$ c) $\tan^{-1}(2^{n+1})$ d) $\tan^{-1}(2^{n+1}) - \frac{\pi}{4}$

76. The number of real solutions of the equation $\tan^{-1}\sqrt{x^2 - 3x + 2} + \cos^{-1}\sqrt{4x - x^2 - 3} = \pi$ is

a) One b) Two c) Zero d) Infinite

77. The number of real solutions of $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2 + x + 1} = \pi/2$ is

a) Zero b) One c) Two d) Infinite

78. If $\tan^{-1}(\sin^2\theta - 2\sin\theta + 3) + \cot^{-1}(5^{\sec^2 y} + 1) = \frac{\pi}{2}$, then the value of $\cos^2\theta - \sin\theta$ is equal to

a) 0 b) -1 c) 1 d) None of these

79. If $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$, then $\frac{1+x^4+y^4}{x^2 - x^2y^2 + y^2}$ is equal to

a) 1 b) 2 c) $\frac{1}{2}$ d) None of these

80. The value of $\tan\left[\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$ is

a) $\frac{6}{17}$ b) $\frac{7}{16}$ c) $\frac{16}{7}$ d) None of these

81. For the equation $\cos^{-1}x + \cos^{-1}2x + \pi = 0$, the number of real solution is

a) 1 b) 2 c) 0 d) ∞

82. The principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is

a) $-\frac{2\pi}{3}$ b) $\frac{2\pi}{3}$ c) $\frac{4\pi}{3}$ d) None of these

83. The range of values of p for which the equation $\sin\cos^{-1}(\cos(\tan^{-1}x)) = p$ has a solution is

a) $\left(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ b) $[0,1)$ c) $\left(\frac{1}{\sqrt{2}},1\right)$ d) $(-1,1)$

84. If $u = \cot^{-1}\sqrt{\tan\alpha} - \tan^{-1}\sqrt{\tan\alpha}$, then $\tan\left(\frac{\pi}{4}-\frac{u}{2}\right)$ is equal to

a) $\sqrt{\tan\alpha}$ b) $\sqrt{\cot\alpha}$ c) $\tan\alpha$ d) $\cot\alpha$

85. If $3\tan^{-1}\left(\frac{1}{2+\sqrt{3}}\right) - \tan^{-1}\frac{1}{x} = \tan^{-1}\frac{1}{3}$, then x is equal to

a) 1 b) 2 c) 3 d) $\sqrt{2}$

86. The maximum value of $f(x) = \tan^{-1}\left(\frac{(\sqrt{12}-2)x^2}{x^4+2x^2+3}\right)$ is

a) 18° b) 36° c) 22.5° d) 15°

87. The value of $\sin(2\sin^{-1}(0.8))$ is equal to

a) $\sin 1.2°$ b) $\sin 1.6°$ c) 0.48 d) 0.96

88. The values of x satisfying the equation $\sin(\tan^{-1}x) = \cos(\cot^{-1}(x+1))$ is

a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) $\sqrt{2}-1$ d) No finite value

89. If $\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1}x$, then x is equal to

a) 1 b) $\sqrt{3}$ c) $\frac{1}{\sqrt{3}}$ d) None of these

90. If $f(x) = x^{11}+x^9-x^7+x^3+1$ and $f(\sin^{-1}(\sin 8)) = \alpha$, α is a constant, then $f(\tan^{-1}(\tan 8))$ is equal to

a) α b) $\alpha-2$ c) $\alpha+2$ d) $2-\alpha$

91. If $\cos^{-1}\sqrt{p} + \cos^{-1}\sqrt{1-p} + \cos^{-1}\sqrt{1-q} = \frac{3\pi}{4}$, then the value of q is

a) 1 b) $\frac{1}{\sqrt{2}}$ c) $\frac{1}{3}$ d) $\frac{1}{2}$

92. The principal value of $\sin^{-1}(\sin 10)$ is

a) 10 b) $10-3\pi$ c) $3\pi-10$ d) None of these

93. The value of $\frac{\alpha^3}{2}\operatorname{cosec}^2\left(\frac{1}{2}\tan^{-1}\frac{\alpha}{\beta}\right) + \frac{\beta^3}{2}\sec^2\left(\frac{1}{2}\tan^{-1}\left(\frac{\beta}{\alpha}\right)\right)$ is equal to

a) $(\alpha-\beta)(\alpha^2+\beta^2)$ b) $(\alpha+\beta)(\alpha^2-\beta^2)$ c) $(\alpha+\beta)(\alpha^2+\beta^2)$ d) None of these

94. The least and the greatest values of $(\sin^{-1}x)^3 + (\cos^{-1}x)^3$ are

a) $\frac{-\pi}{2},\frac{\pi}{2}$ b) $\frac{-\pi^3}{8},\frac{\pi^3}{8}$ c) $\frac{\pi^3}{32},\frac{7\pi^3}{8}$ d) None of these

95. The number of real solutions of the equation $\sqrt{1+\cos 2x} = \sqrt{2}\sin^{-1}(\sin x)$, $-\pi \le x \le \pi$, is

a) 0 b) 1 c) 2 d) Infinite

96. Which of the following is the solution set of the equation $2\cos^{-1}x = \cot^{-1}\left(\frac{2x^2-1}{2x\sqrt{1-x^2}}\right)$?

a) $(0,1)$ b) $(-1,1)-\{0\}$ c) $(-1,0)$ d) $[-1;1]$

97. The value of $\sec\left[\tan^{-1}\frac{b+a}{b-a} - \tan^{-1}\frac{a}{b}\right]$ is

a) 2 b) $\sqrt{2}$ c) 4 d) 1

98. Sum of roots of the equation $\sin^{-1}x - \cos^{-1}x = \sin^{-1}(3x-2)$ is

a) 3/2 b) 1 c) 1/2 d) 2

99. If $0 < x < 1$, then
$\sqrt{1+x^2}[\{x\cos(\cot^{-1}x) + \sin(\cot^{-1}x)\}^2 - 1]^{1/2}$ is equal to

a) $\frac{x}{\sqrt{1+x^2}}$ b) x c) $x\sqrt{1+x^2}$ d) $\sqrt{1+x^2}$

100. The equation $3\cos^{-1}x - \pi x - \frac{\pi}{2} = 0$ has

a) One negative solution
b) One positive solution
c) No solution
d) More than one solution

101. If $\tan(x+y)=33$ and $x=\tan^{-1}3$, then y will be

a) 0.3
b) $\tan^{-1}(1.3)$
c) $\tan^{-1}(0.3)$
d) $\tan^{-1}\left(\frac{1}{18}\right)$

Multiple Correct Answers Type

102. If the equation $\sin^{-1}(x^2+x+1)+\cos^{-1}(\lambda x+1)=\frac{\pi}{2}$ has exactly two solutions, then λ cannot have the integral value

a) -1
b) 0
c) 1
d) 2

103. The value (s) of x satisfying the equation $\sin^{-1}|\sin x|=\sqrt{\sin^{-1}|\sin x|}$ is/are given by (n is any integer)

a) $n\pi-1$
b) $n\pi$
c) $n\pi+1$
d) $2n\pi+1$

104. If $2\tan^{-1}x+\sin^{-1}\frac{2x}{1+x^2}$ is independent of x, then

a) $x>1$
b) $x<-1$
c) $0<x<1$
d) $-1<x<0$

105. If $a\le\tan^{-1}x+\cot^{-1}x+\sin^{-1}x\le b$, then

a) $a=0$
b) $b=\frac{\pi}{2}$
c) $a=\frac{\pi}{4}$
d) $b=\pi$

106. Which one of the following quantities is/are positive?

a) $\cos(\tan^{-1}(\tan 4))$
b) $\sin(\cot^{-1}(\cot 4))$
c) $\tan(\cos^{-1}(\cos 5))$
d) $\cot(\sin^{-1}(\sin 4))$

107. Equation $1+x^2+2x\sin(\cos^{-1}y)=0$ is satisfied by

a) Exactly one value of x
b) Exactly two values of x
c) Exactly one value of y
d) Exactly two values of y

108. If $\sin^{-1}x+\sin^{-1}y=\frac{\pi}{2}$ and $\sin 2x=\cos 2y$, then

a) $x=\frac{\pi}{8}+\sqrt{\frac{1}{2}-\frac{\pi^2}{64}}$
b) $y=\sqrt{\frac{1}{2}-\frac{\pi^2}{64}}-\frac{\pi}{12}$
c) $x=\frac{\pi}{12}+\sqrt{\frac{1}{2}-\frac{\pi^2}{64}}$
d) $y=\sqrt{\frac{1}{2}-\frac{\pi^2}{64}}-\frac{\pi}{8}$

109. $2\cot^{-1}7+\cos^{-1}\left(\frac{3}{5}\right)$ is equal to

a) $\cot^{-1}\left(\frac{44}{117}\right)$
b) $\text{cosec}^{-1}\left(\frac{125}{117}\right)$
c) $\tan^{-1}\left(\frac{4}{117}\right)$
d) $\cos^{-1}\left(\frac{44}{125}\right)$

110. If the equation $\sin^{-1}(x^2+x+1)+\cos^{-1}(\lambda x+1)=\frac{\pi}{2}$ has exactly two solutions, then λ cannot have the integral value

a) -1
b) 0
c) 1
d) 2

111. If $\alpha,\beta(\alpha<\beta)$ are the roots of the equation $6x^2+11x+3=0$, then which of the following are real?

a) $\cos^{-1}\alpha$
b) $\sin^{-1}\beta$
c) $\text{cosec}^{-1}\alpha$
d) Both $\cot^{-1}\alpha$ and $\cot^{-1}\beta$

112. If $a\le\tan^{-1}x+\cot^{-1}x+\sin^{-1}x\le b$, then

a) $a=0$
b) $b=\frac{\pi}{2}$
c) $a=\frac{\pi}{4}$
d) $b=\pi$

113. If $f(x)=(\sin^{-1}x)^2+(\cos^{-1}x)^2$, then

a) $f(x)$ has the least value of $\frac{\pi^2}{8}$
b) $f(x)$ has the greatest value of $\frac{5\pi^2}{8}$
c) $f(x)$ has the least value of $\frac{\pi^2}{16}$
d) $f(x)$ has the greatest value of $\frac{5\pi^2}{4}$

114. The value of $k(k>0)$ such that the length of the longest interval in which the function $f(x)=\sin^{-1}|\sin kx|+\cos^{-1}(\cos kx)$ is constant is $\pi/4$ is/are

a) 8
b) 4
c) 12
d) 16

115. If $z=\sec^{-1}\left(x+\frac{1}{x}\right)+\sec^{-1}\left(y+\frac{1}{y}\right)$, where $xy<0$, then the possible values of z is (are)

a) $\frac{8\pi}{10}$
b) $\frac{7\pi}{10}$
c) $\frac{9\pi}{10}$
d) $\frac{21\pi}{20}$

116. If $\sin^{-1}\left(a-\frac{a^2}{3}+\frac{a^3}{9}+\cdots\right)+\cos^{-1}(1+b+b^2+\cdots)=\frac{\pi}{2}$, then

a) $b=\dfrac{2a-3}{3a}$ b) $b=\dfrac{3a-2}{2a}$ c) $a=\dfrac{3}{2-3b}$ d) $a=\dfrac{2}{3-2b}$

117. Which of the following is a rational number?

a) $\sin\left(\tan^{-1}3+\tan^{-1}\frac{1}{3}\right)$ b) $\cos\left(\frac{\pi}{2}-\sin^{-1}\frac{3}{4}\right)$

c) $\log_2\left(\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)\right)$ d) $\tan\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right)$

118. Which of the following quantities is/are positive?

a) $\cos(\tan^{-1}(\tan 4))$ b) $\sin(\cot^{-1}(\cot 4))$ c) $\tan(\cos^{-1}(\cos 5))$ d) $\cot(\sin^{-1}(\sin 4))$

119. If α,β and γ are the roots of $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$, then

a) $\alpha+\beta+\gamma=0$ b) $\alpha\beta+\beta\gamma+\gamma\alpha=-1/4$

c) $\alpha\beta\gamma=1$ d) $|\alpha-\beta|_{\max}=1$

120. If $\cot^{-1}\left(\frac{n^2-10n+21.6}{\pi}\right)>\frac{\pi}{6}, n\in N$, then n can be

a) 3 b) 2 c) 4 d) 8

121. If $\tan^{-1}y=4\tan^{-1}x$, then y is infinite, if

a) $x^2=3+2\sqrt{2}$ b) $x^2=3-2\sqrt{2}$ c) $x^4=6x^2-1$ d) $x^4=6x^2+1$

122. Which of the following is/are the value of $\cos\left[\frac{1}{2}\cos^{-1}\left(\cos\left(-\frac{14\pi}{5}\right)\right)\right]$?

a) $\cos\left(-\frac{7\pi}{5}\right)$ b) $\sin\left(\frac{\pi}{10}\right)$ c) $\cos\left(\frac{2\pi}{5}\right)$ d) $-\cos\left(\frac{3\pi}{5}\right)$

123. The value (s) of x satisfying the equation $\sin^{-1}|\sin x|=\sqrt{\sin^{-1}|\sin x|}$ is/are given by (n is any integer)

a) $n\pi-1$ b) $n\pi$ c) $n\pi+1$ d) $2n\pi+1$

124. If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$, then

a) $x^2+y^2+z^2+2xyz=1$

b) $2(\sin^{-1}x+\sin^{-1}y+\sin^{-1}z)=\cos^{-1}x+\cos^{-1}y+\cos^{-1}z$

c) $xy+yz+zx=x+y+z-1$

d) $\left(x+\frac{1}{x}\right)+\left(y+\frac{1}{y}\right)+\left(z+\frac{1}{z}\right)\geq 6$

125. If α is a real number for which $f(x)=\log_e\cos^{-1}x$ is defined, then a possible value of $[\alpha]$ (where $[\cdot]$ denotes the greatest integer function) is

a) 0 b) 1 c) -1 d) -2

126. If $f(x)=\sin^{-1}x+\sec^{-1}x$ is defined, then which of the following value/values is/are in its range?

a) $-\pi/2$ b) $\pi/2$ c) π d) $3\pi/2$

127. Indicate the relation which can hold in their respective domain for infinite values of x

a) $\tan|\tan^{-1}x|=|x|$ b) $\cot|\cot^{-1}x|=|x|$ c) $\tan^{-1}|\tan x|=|x|$ d) $\sin|\sin^{-1}x|=|x|$

128. $\cos^{-1}x+\cos^{-1}\left(\frac{x}{2}+\frac{1}{2}\sqrt{3-3x^2}\right)$ is equal to

a) $\frac{\pi}{3}$ for $x\in\left[\frac{1}{2},1\right]$ b) $\frac{\pi}{3}$ for $x\in\left[0,\frac{1}{2}\right]$

c) $2\cos^{-1}x-\cos^{-1}\frac{1}{2}$ for $x\in\left[\frac{1}{2},1\right]$ d) $2\cos^{-1}x-\cos^{-1}\frac{1}{2}$ for $x\in\left[0,\frac{1}{2}\right]$

129. To the equation $2^{2x/\cos^{-1}x}-\left(a+\frac{1}{2}\right)2^{\pi/\cos^{-1}x}-a^2=0$ has only one real root, then

a) $1\leq a\leq 3$ b) $a\geq 1$ c) $a\leq -3$ d) $a\geq 3$

130. $2\cot^{-1}7+\cos^{-1}\left(\frac{3}{5}\right)$ is equal to

a) $\cot^{-1}\left(\frac{44}{117}\right)$ b) $\operatorname{cosec}^{-1}\left(\frac{125}{117}\right)$ c) $\tan^{-1}\left(\frac{4}{117}\right)$ d) $\cos^{-1}\left(\frac{44}{125}\right)$

131. If $S_n=\cot^{-1}(3)+\cot^{-1}(7)+\cot^{-1}(13)+\cot^{-1}(21)+\cdots n$ terms, then

a) $S_{10}=\tan^{-1}\frac{5}{6}$ b) $S_\infty=\frac{\pi}{4}$ c) $S_6=\sin^{-1}\frac{4}{5}$ d) $S_{20}=\cot^{-1}1.1$

132. If $\tan^{-1} y = 4\tan^{-1} x$, then y is infinite, if

a) $x^2 = 3 + 2\sqrt{2}$ b) $x^2 = 3 - 2\sqrt{2}$ c) $x^4 = 6x^2 - 1$ d) $x^4 = 6x^2 + 1$

133. If $(\sin^{-1} x + \sin^{-1} w)(\sin^{-1} y + \sin^{-1} z) = \pi^2$, then $D = \begin{vmatrix} x^{N_1} & y^{N_2} \\ z^{N_3} & w^{N_4} \end{vmatrix} (N_1, N_2, N_3, N_4 \in N)$

a) Has a maximum value of 2 b) Has a minimum value of 0

c) 16 different D are possible d) Has a minimum value of -2

134. $2\tan^{-1}(-2)$ is equal to

a) $-\cos^{-1}\left(\frac{-3}{5}\right)$ b) $-\pi + \cos^{-1}\frac{3}{5}$ c) $-\frac{\pi}{2} + \tan^{-1}\left(-\frac{3}{4}\right)$ d) $-\pi + \cot^{-1}\left(-\frac{3}{4}\right)$

Assertion - Reasoning Type

This section contain(s) 0 questions numbered 135 to 134. Each question contains STATEMENT 1(Assertion) and STATEMENT 2(Reason). Each question has the 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

a) Statement 1 is True, Statement 2 is True; Statement 2 **is** correct explanation for Statement 1

b) Statement 1 is True, Statement 2 is True; Statement 2 **is not** correct explanation for Statement 1

c) Statement 1 is True, Statement 2 is False

d) Statement 1 is False, Statement 2 is True

135

Statement 1: $\tan\left[\cos^{-1}\left(\frac{1}{\sqrt{82}}\right) - \sin^{-1}\left(\frac{5}{\sqrt{26}}\right)\right]$ is equal to $\frac{29}{3}$

Statement 2: $\{x\cos(\cot^{-1} x) + \sin(\cot^{-1} x)\}^2 = \frac{51}{50},$

if $x = \frac{1}{5\sqrt{2}}$

136

Statement 1: The equation $\sin^{-1} x - \cos^{-1} x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ has unique solution

Statement 2: $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}; \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

137

Statement 1: $\operatorname{cosec}^{-1}\left(\frac{3}{2}\right) + \cos^{-1}\left(\frac{2}{3}\right) - 2\cot^{-1}\left(\frac{1}{7}\right) - \cot^{-1}(7)$ is equal to $\cot^{-1} 7$.

Statement 2: $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$,

$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$,

$\operatorname{cosec}^{-1} x = \sin^{-1}\left(\frac{1}{x}\right)$

$\cot^{-1}(x) = \tan^{-1}\left(\frac{1}{x}\right)$

138

Statement 1: The solution of system of equation $\cos^{-1} x + (\sin^{-1} y)^2 = \frac{p\pi^2}{4}$ and $(\cos^{-1} x)(\sin^{-1} y)^2 = \frac{\pi^4}{16}$ is $x = \cos\ \frac{\pi^2}{4}$ and $y = \pm 1, \forall p \in I$

Statement 2: $AM \geq GM$

139

Statement 1: The solution of system of equation $\cos^{-1} x + (\sin^{-1} y)^2 = \frac{p\pi^2}{4}$ and $(\cos^{-1} x)(\sin^{-1} y)^2 = \frac{\pi^4}{16}$ is $x = \cos \frac{\pi^2}{4}$ and $y = \pm 1, \forall p \in I$

Statement 2: $AM \geq GM$

140

Statement 1: Principal value of $\sin^{-1}(\sin 3)$ can be 3 if we restrict the domain of $f(x) = \sin x$ to $[\pi/2, 3\pi/2]$

Statement 2: The restriction that the principal values of $\sin^{-1}(\sin x)$ is $[-\pi/2, -\pi/2]$ is a matter of convention. We could have allowed principal values $[\pi/2, 3\pi/2]$ without affection the condition required for definition of inverse function

141

Statement 1: The value of $\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right)$ is $\frac{\pi}{4}$

Statement 2: If $x > 0, y > 0$, then $\tan^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y - x}{y + x}\right) = \frac{\pi}{4}$

142

Statement 1: Range of $f(x) = \tan^{-1} x + \sin^{-1} x + \cos^{-1} x$ is $(0, \pi)$

Statement 2: $f(x) = \tan^{-1} x + \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} + \tan^{-1} x$, for $x \in [-1, 1]$

143

Statement 1: $\sin^{-1}\left(\frac{1}{\sqrt{e}}\right) > \tan^{-1}\left(\frac{1}{\sqrt{\pi}}\right)$

Statement 2: $\sin^{-1} x > \tan^{-1} y$ for $x > y, \forall\, x, y \in (0, 1)$

144

Statement 1: $\operatorname{cosec}^{-1}\left(\frac{1}{2} + \frac{1}{\sqrt{2}}\right) > \sec^{-1}\left(\frac{1}{2} + \frac{1}{\sqrt{2}}\right)$

Statement 2: $\operatorname{cosec}^{-1} x < \sec^{-1} x$ if $1 \leq x < \sqrt{2}$

145

Statement 1: If $x < 0, \tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2}$

Statement 2: $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, \forall x \in R$

146

Statement 1: If $x < 0, \tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2}$

Statement 2: $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, \forall x \in R$

147

Statement 1: Number of roots of the equation $\cot^{-1} x + \cos^{-1} 2x + \pi = 0$ is zero

Statement 2: Range of $\cot^{-1} x$ and $\cos^{-1} x$ is $(0, \pi)$ and $[0, \pi]$, respectively

148

Statement 1: $\sin^{-1} 2x + \sin^{-1} 3x = \frac{\pi}{3}$

$\Rightarrow x = \sqrt{\left(\frac{3}{76}\right)}$ only

Statement 2: Sum of two negative angles cannot be positive

149

Statement 1: $\operatorname{cosec}^{-1}\left(\frac{3}{2}\right) + \cos^{-1}\left(\frac{2}{3}\right) - 2\cot^{-1}\left(\frac{1}{7}\right) - \cot^{-1}(7)$ is equal to $\cot^{-1} 7$.

Statement 2: $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$,

$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$,

$\operatorname{cosec}^{-1} x = \sin^{-1}\left(\frac{1}{x}\right)$

$\cot^{-1}(x) = \tan^{-1}\left(\frac{1}{x}\right)$

150

Statement 1: $\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \frac{\pi}{4}$

Statement 2: For $x > 0, y > 0, \tan^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y-x}{y+x}\right) = \frac{\pi}{4}$

151

Statement 1: Principal value of $\cos^{-1}(\cos 30)$is $30 - 9\pi$

Statement 2: $30 - 9\pi \in [0, \pi]$

152

Statement 1: The equation $\sin^{-1} x - \cos^{-1} x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ has unique solution

Statement 2: $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}; \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

153

Statement 1: If $p > q > 0$ and $pr < -1 < qr$, then

$\tan^{-1}\left(\frac{p-q}{1+pq}\right) + \tan^{-1}\left(\frac{q-r}{1+qr}\right) + \tan^{-1}\left(\frac{r-p}{1+rp}\right) = \pi$

Statement 2: $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$ for all x, y

154

Statement 1: If $p > q > 0$ and $pr < -1 < qr$, then

$\tan^{-1}\left(\frac{p-q}{1+pq}\right) + \tan^{-1}\left(\frac{q-r}{1+qr}\right) + \tan^{-1}\left(\frac{r-p}{1+rp}\right) = \pi$

Statement 2: $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$ for all x, y

155 Let $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Statement 1: $f'(2) = -\frac{2}{5}$

Statement 2: $\sin^{-1}\left(\frac{2x}{1+x_7^2}\right) = \pi - 2\tan^{-1}x, \forall\, x > 1$

156

Statement 1: $\tan\left[\cos^{-1}\left(\frac{1}{\sqrt{82}}\right) - \sin^{-1}\left(\frac{5}{\sqrt{26}}\right)\right]$ is equal to $\frac{29}{3}$

Statement 2: $\{x\cos(\cot^{-1}x) + \sin(\cot^{-1}x)\}^2 = \frac{51}{50,}$

if $x = \frac{1}{5\sqrt{2}}$

157

Statement 1: The value of $\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right)$ is $\frac{\pi}{4}$

Statement 2: If $x > 0, y > 0$, then

$\tan^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y-x}{y+x}\right) = \frac{\pi}{4}$

158

Statement 1: Domain of $\tan^{-1}x$ and $\cot^{-1}x$ is R

Statement 2: $f(x) = \tan x$ and $g(x) = \cot x$ are unbounded functions

159

Statement 1: $\sin^{-1}2x + \sin^{-1}3x = \frac{\pi}{3}$

$\Rightarrow x = \sqrt{\left(\frac{3}{76}\right)}$ only

Statement 2: Sum of two negative angles cannot be positive

Matrix-Match Type

This section contain(s) 0 question(s). Each question contains Statements given in 2 columns which have to be matched. Statements (A, B, C, D) in **columns I** have to be matched with Statements (p, q, r, s) in **columns II**.

160.

	Column-I		Column- II
(A)	$(\sin^{-1}x)^2 + (\sin^{-1}y)^2 = \frac{\pi^2}{2}$ $\Rightarrow x^3 + y^3 =$	(p)	1
(B)	$(\cos^{-1}x)^2 + (\cos^{-1}y)^2 = 2\pi^2$ $\Rightarrow x^5 + y^5$	(q)	-2
(C)	$(\sin^{-1}x)^2(\cos^{-1}y)^2 = \frac{\pi^4}{4} \Rightarrow \lvert x - y \rvert$	(r)	0
(D)	$\lvert \sin^{-1}x - \sin^{-1}y \rvert = \pi \Rightarrow x^y$	(s)	2

CODES :

	A	**B**	**C**	**D**
a)	q	r,s	p	q

b)	q,r,s	q	r,s	p
c)	p	q,r,s	q	r,s
d)	r,s	p	q,r,s	q

161.

	Column-I		**Column- II**
(A)	Range of $f(x)=\sin^{-1}x+\cos^{-1}x+\cot^{-1}x$ is	(p)	$\left[0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right]$
(B)	Range of $f(x)=\cot^{-1}x+\tan^{-1}x+\operatorname{cosec}^{-1}x$ is	(q)	$\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$
(C)	Range of $f(x)=\cot^{-1}x+\tan^{-1}x+\cos^{-1}x$ is	(r)	$\{0,\pi\}$
(D)	Range of $f(x)=\sec^{-1}x+\operatorname{cosec}^{-1}x+\sin^{-1}x$ is	(s)	$\left[\frac{3\pi}{4},\frac{5\pi}{4}\right]$

CODES :

	A	**B**	**C**	**D**
a)	p	q	r	s
b)	r	s	p	q
c)	q	r	s	p
d)	s	p	q	r

162.

	Column-I		**Column- II**
(A)	$\sin^{-1}\frac{4}{5}+2\tan^{-1}\frac{1}{3}=$	(p)	$\pi/6$
(B)	$\sin^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}=$	(q)	$\pi/2$
(C)	If $A=\tan^{-1}\frac{x\sqrt{3}}{2\lambda-x}$ and $B=\tan^{-1}\left(\frac{2x-\lambda}{\lambda\sqrt{3}}\right)$, then the value of $A-B$ is	(r)	$\pi/4$
(D)	$\tan^{-1}\frac{1}{7}+2\tan^{-1}\frac{1}{3}=$	(s)	π

CODES :

	A	**B**	**C**	**D**
a)	q	s	p	r
b)	s	r	q	p
c)	p	q	r	s
d)	r	p	s	q

163.

Column-I	**Column- II**

(A)	$\sin^{-1} x + x > 0$, for	(p)	$x < 0$
(B)	$\cos^{-1} x - x \geq 0$, for	(q)	$x \in (0, 1]$
(C)	$\tan^{-1} x + x < 0$, for	(r)	$x \in [-1, 0)$
(D)	$\cot^{-1} x + x > 0$, for	(s)	$x > 0$

CODES :

	A	**B**	**C**	**D**
a)	P,r	q,r,s	q	r
b)	q	r	p,r	q,r,s
c)	r	q	q,r,s	p,r
d)	q,r,s	p,r	r	q

164.

	Column-I		**Column- II**
(A)	$x \in [\pi, 2\pi] \Rightarrow |\tan^{-1}(\tan x)|$ can be	(p)	$|x - 2\pi|$
(B)	$x \in [\pi, 2\pi] \Rightarrow |\cot^{-1}(\cot x)|$ can be	(q)	$|x - \pi|$
(C)	$x \in [-\pi, \pi] \Rightarrow |\sin^{-1}(\sin x)|$ can be	(r)	$|x|$
(D)	$x \in [-\pi, \pi] \Rightarrow |\cos^{-1}(\cos x)|$ can be	(s)	$|x + \pi|$

CODES :

	A	**B**	**C**	**D**
a)	Q,r,s	p,r	p,q	q
b)	q	p,q	p,r	q,r,s
c)	p,q	q	q,r,s	p,r
d)	p,r	q,r,s	q	p,q

165.

	Column-I		**Column- II**
(A)	$\cos^{-1}(4x^3 - 3x) = 3\cos^{-1} x$, then x can take values	(p)	$[1/2, 1]$
(B)	$\sin^{-1}(3x - 4x^3) = 3\sin^{-1} x$, then x can take values	(q)	$[-1/2, 0]$
(C)	$\cos^{-1}(4x^3 - 3x) = 3\sin^{-1} x$, then x can take values	(r)	$[0, \sqrt{3}/2]$
(D)	$\sin^{-1}(3x - 4x^3) = 3\cos^{-1} x$, then x can take values	(s)	$[0, 1/2]$

CODES :

	A	B	C	D
a)	p	q,s	r,s	r,s
b)	r,s	p	q,s	r,s
c)	q,s	r,s	p	q
d)	q	q,s	q,s	r,s

Linked Comprehension Type

This section contain(s) 15 paragraph(s) and based upon each paragraph, multiple choice questions have to be answered. Each question has atleast 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 166 to -166

$$\sum_{r=1}^{n} \tan^{-1}\left(\frac{x_1 - r_{r-1}}{1 + x_{r-1}x_r}\right) = \sum_{r=1}^{n} (\tan^{-1} x_r - \tan^{-1} x_{r-1})$$

$= \tan^{-1} x_n - \tan^{-1} x_0, \forall n \in N$

On the basis of above information, answer the following questions:

166. The sum to infinite terms of the series

$$\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{2}{9}\right) + \ldots + \tan^{-1}\left(\frac{2^{n-1}}{1 + 2^{2n-1}}\right) + \ldots \text{ is}$$

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) π d) none of these

Paragraph for Question Nos. 167 to - 167

$f(x) = \sin\{\cot^{-1}(x + 1)\} - \cos(\tan^{-1} x)$

And $a = \cos \tan^{-1} \sin \cot^{-1} x$

On the basis of above information, answer the following question:

167. The value of x for which $f(x) = 0$ is

a) $-\frac{1}{2}$ b) 0 c) $\frac{1}{2}$ d) 1

Paragraph for Question Nos. 168 to - 168

$$\sum_{r=1}^{n} \tan^{-1}\left(\frac{x_1 - r_{r-1}}{1 + x_{r-1}x_r}\right) = \sum_{r=1}^{n} (\tan^{-1} x_r - \tan^{-1} x_{r-1})$$

$= \tan^{-1} x_n - \tan^{-1} x_0, \forall n \in N$

On the basis of above information, answer the following questions:

168. The sum to infinite terms of the series

$$\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{2}{9}\right) + \ldots + \tan^{-1}\left(\frac{2^{n-1}}{1 + 2^{2n-1}}\right) + \ldots \text{ is}$$

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) π d) none of these

Paragraph for Question Nos. 169 to - 169

$f(x) = \sin\{\cot^{-1}(x+1)\} - \cos(\tan^{-1} x)$
And $a = \cos\tan^{-1}\sin\cot^{-1} x$
On the basis of above information, answer the following question:

169. The value of x for which $f(x) = 0$ is
a) $-\frac{1}{2}$
b) 0
c) $\frac{1}{2}$
d) 1

Paragraph for Question Nos. 170 to - 170

For $x, y, z, t \in R, \sin^{-1} x + \cos^{-1} y + \sec^{-1} z \geq t^2 - \sqrt{2\pi}\, t + 3\pi$

170. The value of $x + y + z$ is equal to
a) 1
b) 0
c) 2
d) -1

Paragraph for Question Nos. 171 to - 171

$ax + b\,(\sec(\tan^{-1} x)) = c$ and $ay + b\,(\sec(\tan^{-1} y)) = c$

171. The value of xy is
a) $\frac{2ab}{a^2 - b^2}$
b) $\frac{c^2 - b^2}{a^2 - b^2}$
c) $\frac{c^2 - b^2}{a^2 + b^2}$
d) None of these

Paragraph for Question Nos. 172 to - 172

Consider the system of equations $\cos^{-1} x + (\sin^{-1} y)^2 = \frac{p\pi^2}{4}$ and $(\cos^{-1} x)(\sin^{-1} y)^2 = \frac{\pi^4}{16}, p \in Z$

172. The value of p for which system has a solution is
a) 1
b) 2
c) 0
d) -1

Paragraph for Question Nos. 173 to - 173

Let $\cos^{-1}(4x^3 - 3x) = a + b\cos^{-1} x$

173. If $x \in \left[-\frac{1}{2}, -1\right)$, then the value of $a + b\pi$ is
a) 2π
b) 3π
c) π
d) -2π

Integer Answer Type

174. If $x = \sin^{-1}(a^6 + 1) + \cos^{-1}(a^4 + 1) - \tan^{-1}(a^2 + 1), a \in R$, then the value of $\sec^2 x$ is ________
175. Number of values of x for which $\sin^{-1}\left(x^2 - \frac{x^4}{3} + \frac{x^6}{9} \cdots\right) + \cos^{-1}\left(x^4 - \frac{x^8}{3} + \frac{x^{12}}{9} \cdots\right) = \frac{\pi}{2}$, where $0 \leq |x| <$

$\sqrt{3}$, is ________

176. If $\tan^{-1}\left(x+\frac{3}{x}\right)-\tan^{-1}\left(x-\frac{3}{x}\right)=\tan^{-1}\frac{6}{x}$, then the value of x^4 is ________

177. Let $\cos^{-1}(x)+\cos^{-1}(2x)+\cos^{-1}(3x)$ be π. If x satisfies the equation $ax^3+bx^2+cx-1=0$, then the value of $(b-a-c)$ is ________

178. If the area enclosed by the curves $f(x)=\cos^{-1}(\cos x)$ and $g(x)=\sin^{-1}(\cos x)$ in $x\in[9\pi/4, 15\pi/4]$ is $a\pi^2/b$ (where a and b are coprime), then the value of $(a-b)$ is ________

179. If $0<\cos^{-1}x<1$ and $1+\sin(\cos^{-1}x)+\sin^2(\cos^{-1}x)+\sin^3(\cos^{-1}x)+\cdots\infty=2$, then the value of $12x^2$ is ________

180. Number of integral values of x satisfying the equation $\tan^{-1}(3x)+\tan^{-1}(5x)=\tan^{-1}(7x)+\tan^{-1}(2x)$ is ________

181. The solution set of inequality $(\cot^{-1}x)(\tan^{-1}x)+\left(2-\frac{\pi}{2}\right)\cot^{-1}x-3\tan^{-1}x-3\left(2-\frac{\pi}{2}\right)>0$ is (a,b), then the value of $\cot^{-1}a+\cot^{-1}b$ is ________

182. If the roots of the equation $x^3-10x+11=0$ are u, v and w. Then the value of $3\text{cosec}^2\ (\tan^{-1}u+$ tan $-1v$+tan $-1w$ is ________

183. If range of the function $f(x)=\sin^{-1}x+2\tan^{-1}x+x^2+4x+1$ is $[p,q]$, then the value of $(p+q)$ is ________

184. If the domain of the function $f(x)=\sqrt{3\cos^{-1}(4x)-\pi}$ is $[a,b]$, then the value of $4a+64b$ is ________

185. Absolute value of sum of all integers in the domain of $f(x)=\cot^{-1}\sqrt{(x+3)x}+\cos^{-1}\sqrt{x^2+3x+1}$ is ________

186. If n is the number of terms of the series $\cot^{-1}3, \cot^{-1}7, \cot^{-1}13, \cot^{-1}21, \ldots$, whose sum is $\frac{1}{2}\cos^{-1}\left(\frac{24}{145}\right)$, then the value of $n-5$ is ________

187. The least value of $(1+\sec^{-1}x)(1+\cos^{-1}x)$ is ________

: ANSWER KEY :

1)	c	2)	d	3)	a	4)	b	9)	a,c,d	10)	b,c,d	11)	a,d	12)	
5)	d	6)	a	7)	b	8)	b		a,d						
9)	c	10)	d	11)	a	12)	d	13)	b	14)	c,d	15)	a,c	16)	
13)	b	14)	c	15)	b	16)	b		a,b,c						
17)	d	18)	b	19)	b	20)	d	17)	a,b,c	18)	a,b,d	19)	a,c	20)	
21)	d	22)	a	23)	d	24)	b		a,b,c						
25)	c	26)	b	27)	c	28)	a	21)	b,c,d	22)	a,b,c	23)	a,b	24)	
29)	d	30)	a	31)	b	32)	c		a,c						
33)	a	34)	d	35)	b	36)	b	25)	b	26)	a,b,c,d	27)	a,d	28)	
37)	c	38)	a	39)	c	40)	c		b,c						
41)	d	42)	c	43)	c	44)	d	29)	a,b,d	30)	a,b,d	31)	a,b,c	32)	
45)	c	46)	c	47)	c	48)	d		a,c,d						
49)	a	50)	c	51)	d	52)	b	33)	a,b,c	1)	d	2)	a	3)	d
53)	d	54)	c	55)	b	56)	a		4)	a					
57)	d	58)	a	59)	c	60)	b	5)	a	6)	a	7)	a	8)	d
61)	c	62)	c	63)	d	64)	a	9)	a	10)	c	11)	d	12)	d
65)	a	66)	a	67)	b	68)	c	13)	a	14)	a	15)	d	16)	a
69)	b	70)	c	71)	d	72)	a	17)	d	18)	a	19)	d	20)	d
73)	d	74)	c	75)	b	76)	c	21)	a	22)	d	23)	a	24)	b
77)	c	78)	c	79)	b	80)	d	25)	a	1)	b	2)	d	3)	a
81)	c	82)	e	83)	b	84)	a		4)	b					
85)	b	86)	d	87)	d	88)	d	5)	c	6)	a	1)	a	2)	a
89)	c	90)	d	91)	d	92)	c		3)	a	4)	a			
93)	c	94)	c	95)	c	96)	a	5)	d	6)	b	7)	b	8)	c
97)	b	98)	a	99)	c	100)	b	1)	2	2)	3	3)	9	4)	3
101)	c	1)	a,c,d	2)	a,b,c	3)		5)	1	6)	9	7)	1	8)	5
	a,b	4)	a,d					9)	6	10)	4	11)	7	12)	3
5)	a, b	6)	a,c	7)	a,d	8)		13)	6	14)	1				
	a,b,d														

: HINTS AND SOLUTIONS :

1 **(c)**

Let $\tan^{-1} 2 = \alpha \Rightarrow \tan\alpha = 2$

and $\cot^{-1} 3 = \beta \Rightarrow \cot\beta = 3$

$\sec^2(\tan^{-1} 2) + \mathrm{cosec}^2(\cot^{-1} 3)$
$= \sec^2\alpha + \mathrm{cosec}^2\beta$
$= 1 + \tan^2\alpha + 1 + \cot^2\beta$

$= 2 + (2)^2 + (3)^2 = 15$

2 **(d)**

$\lim_{|x|\to\infty} \cos(\tan^{-1}(\sin(\tan^{-1} x)))$
$= \cos(\tan^{-1}(\sin(\tan^{-1}\infty)))$

$= \cos(\tan^{-1}(\sin(\pi/2)))$

$= \cos(\tan^{-1}(1)) = \cos(\pi/4) = \frac{1}{\sqrt{2}}$

3 **(a)**

$\sin^{-1}(\sin 12) + \cos^{-1}(\cos 12)$
$= \sin^{-1}(\sin(12 - 4\pi))$
$+ \cos^{-1}(\cos(4\pi - 12))$

$= 12 - 4\pi + 4\pi - 12 = 0$

4 **(b)**

Since $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$

$\therefore \sin^{-1} x + \sin^{-1} y = \pi - \sin^{-1} z$

$\Rightarrow \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right) = \pi - \sin^{-1}(z)$

$\Rightarrow x\sqrt{1-y^2} + y\sqrt{1-x^2} = \sin(\pi - \sin^{-1}(z))$
$= \sin(\sin^{-1} z) = z$

$\Rightarrow x^2(1-y^2) = z^2 + y^2(1-x^2) - 2zy\sqrt{1-x^2}$
$\Rightarrow (x^2 - y^2 - z^2)^2$
$= 4y^2z^2(1-x^2)$

$\Rightarrow x^4 + y^4 + z^4 - 2x^2y^2 - 2x^2z^2 + 2y^2z^2$
$= 4y^2z^2 - 4x^2y^2z^2$

$\Rightarrow x^4 + y^4 + z^4 + 4x^2y^2z^2$
$= 2(x^2y^2 + y^2z^2 + z^2x^2) \Rightarrow K$
$= 2$

5 **(d)**

$\cot^{-1}\left[\frac{\sqrt{1-\sin x} + \sqrt{1+\sin x}}{\sqrt{1-\sin x} - \sqrt{1+\sin x}}\right]$

$= \cot^{-1}\left[\frac{\left(\sqrt{1-\sin x} + \sqrt{1+\sin x}\right)}{\left(\sqrt{1-\sin x} - \sqrt{1+\sin x}\right)} \frac{\left(\sqrt{1-\sin x} + \sqrt{1+\sin x}\right)}{\left(\sqrt{1-\sin x} + \sqrt{1+\sin x}\right)}\right]$

$= \cot^{-1}\left[\frac{(1-\sin x) + (1+\sin x) + 2\sqrt{1-\sin^2 x}}{(1-\sin x) - (1+\sin x)}\right]$
$= \cot^{-1}\left[\frac{2(1+\cos x)}{-2\sin x}\right]$

$= \cot^{-1}\left[-\frac{2\cos^2(x/2)}{2\sin(x/2)\cos(x/2)}\right] = \cot^{-1}\left(-\cot\frac{x}{2}\right)$
$= \cot^{-1}\left[\cot\left(\pi - \frac{x}{2}\right)\right] = \pi - \frac{x}{2}$

6 **(a)**

$\frac{\pi}{2} - \cos^{-1}\cos\left(\frac{2(x^2+5|x|+3)-2}{x^2+5|x|+3}\right)$
$= \cot\cot^{-1}\left(\frac{2}{9|x|} - 2\right) + \frac{\pi}{2}$

$\frac{\pi}{2} - 2 + \frac{2}{x^2+5|x|+3} = \frac{2}{9|x|} - 2 + \frac{\pi}{2}$

$\Rightarrow |x|^2 - 4|x| + 3 = 0$

$|x| = 1, 3 \Rightarrow x = \pm 1, \pm 3$

7 **(b)**

Let $x = \sin\theta$ and $\sqrt{x} = \sin\phi$, where $x \in [0,1] \rightarrow \theta, \phi \in [0, \pi/2]$

$\Rightarrow \theta - \phi \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

Now, $\sin^{-1}\left(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right) =$
$\sin^{-1}\left(\sin\theta\ \sqrt{1-\sin^2\phi} - \sin\phi\sqrt{1-\sin^2\theta}\right)$

$= \sin^{-1}(\sin\theta\cos\phi - \sin\phi\cos\theta)$

$= \sin^{-1}\sin(\theta - \phi) = \theta - \phi$
$= \sin^{-1}(x) - \sin^{-1}(\sqrt{x})$

8 **(b)**

We have $\frac{xy}{zr}\frac{yz}{xr} = \frac{y^2}{r^2} = \frac{y^2}{x^2+y^2+z^2} < 1$

$$\Rightarrow \tan^{-1}\left(\frac{xy}{zr}\right)+\tan^{-1}\left(\frac{yz}{xr}\right)+\tan^{-1}\left(\frac{xz}{yr}\right)$$
$$=\tan^{-1}\left(\frac{\frac{xy}{zr}+\frac{yz}{xr}}{1-\frac{xy}{zr}\frac{yz}{xr}}\right)+\tan^{-1}\left(\frac{xz}{yr}\right)$$

$$=\tan^{-1}\left(\frac{\frac{y(x^2+z^2)}{xzr}}{\frac{r^2-y^2}{r^2}}\right)+\tan^{-1}\left(\frac{xz}{yr}\right)$$

$$=\tan^{-1}\left(\frac{\frac{yr(x^2+z^2)}{xz}}{(x^2+z^2)}\right)+\tan^{-1}\left(\frac{xz}{yr}\right)$$

$$=\tan^{-1}\left(\frac{yr}{xz}\right)+\tan^{-1}\left(\frac{xz}{yr}\right)=\frac{\pi}{2}$$

9 **(c)**

$$\tan^{-1}x+2\cot^{-1}x=\frac{2\pi}{3}$$

$$\Rightarrow \tan^{-1}x=2\left(\frac{\pi}{3}-\cot^{-1}x\right)$$
$$=2\left(\frac{\pi}{3}-\left(\frac{\pi}{2}-\tan^{-1}x\right)\right)$$
$$=2\left(-\frac{\pi}{6}+\tan^{-1}x\right)$$

$$\Rightarrow \tan^{-1}x=\frac{\pi}{3}\Rightarrow x=\tan\frac{\pi}{3}=\sqrt{3}$$

10 **(d)**

$$\sin^{-1}(x^2-4x+6)+\cos^{-1}(x^2-4x+6)$$
$$=\sin^{-1}((x-2)^2+2)$$
$$+\cos^{-1}((x-2)^2+2)$$

$(x-2)^2+2\geq 2$, for which $\sin^{-1}x$ and $\cos^{-1}x$ are not defined

11 **(a)**

Expanding, we have

$$(\tan^{-1}x)^3+(\tan^{-1}2x)^3+(\tan^{-1}3x)^3$$
$$=3\tan^{-1}x\tan^{-1}2x\tan^{-1}3x$$

$$\Rightarrow x=0$$

12 **(d)**

$$\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)-\cos^{-1}\left(\frac{\sqrt{6}+1}{2\sqrt{3}}\right)$$

$$=\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)-\tan^{-1}\left(\frac{\sqrt{3}-\sqrt{2}}{1+\sqrt{6}}\right)$$
$$=\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$$
$$-\left[\tan^{-1}\sqrt{3}-\tan^{-1}\sqrt{2}\right]$$

$$=\left(\tan^{-1}\frac{1}{\sqrt{2}}+\tan^{-1}\sqrt{2}\right)-\tan^{-1}\sqrt{3}$$

$$=\frac{\pi}{2}-\frac{\pi}{3}=\frac{\pi}{6}$$

13 **(b)**

$$\sin^{-1}\left(x-\frac{x^2}{2}+\frac{x^3}{3}+\cdots\right)+\cos^{-1}\left(x^2-\frac{x^4}{2}+\frac{x^6}{4}\right)$$
$$=\frac{\pi}{2}$$

$$\Rightarrow \cos^{-1}\left(x^2-\frac{x^4}{2}+\frac{x^6}{4}\ldots\right)$$
$$=\frac{\pi}{2}-\sin^{-1}\left(x-\frac{x^2}{2}+\frac{x^3}{4}\right)$$
$$=\cos^{-1}\left(x-\frac{x^2}{2}+\frac{x^3}{4}\right)$$

14 **(c)**

$$\sin^{-1}x=2\sin^{-1}a$$

Now $-\frac{\pi}{2}\leq\sin^{-1}x\leq\frac{\pi}{2}$

$$\Rightarrow -\frac{\pi}{2}\leq 2\sin^{-1}a\leq\frac{\pi}{2}$$

$$\Rightarrow -\frac{\pi}{4}\leq\sin^{-1}a\leq\frac{\pi}{4}$$

$$\Rightarrow \frac{1}{\sqrt{2}}\leq a\leq\frac{1}{\sqrt{2}}\ \Rightarrow |a|\leq\frac{1}{\sqrt{2}}$$

15 **(b)**

The given equation is $ax^2+\sin^{-1}((x-1)^2+$ *1+*cos*−1x−12+1=0*

Now, $-1\leq(x-1)^2+1\leq 1\Rightarrow x=1$

So, we have $a+\frac{\pi}{2}=0\Rightarrow a=-\frac{\pi}{2}$

16 **(b)**

$$\tan\left[\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right]+\tan\left[\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right]$$

Let $\frac{1}{2}\cos^{-1}\frac{a}{b}=\theta\Rightarrow\cos 2\theta=\frac{a}{b}$

Thus, $\tan\left[\frac{\pi}{4}+\theta\right]+\tan\left[\frac{\pi}{4}-\theta\right]=\frac{1+\tan\theta}{1-\tan\theta}+\frac{1-\tan\theta}{1+\tan\theta}$

$$= \frac{1 + \tan^2\theta + 2\tan\theta + 1 + \tan^2\theta - 2\tan\theta}{(1 - \tan^2\theta)}$$

$$= \frac{2(1 + \tan^2\theta)}{1 - \tan^2\theta} = \frac{2}{\cos 2\theta} = \frac{2}{(a/b)} = \frac{2b}{a}$$

17 **(d)**

$\frac{3\pi}{2} < 5 < \frac{5\pi}{2}$

$\Rightarrow \sin^{-1}(\sin 5) = 5 - 2\pi$

Given $\sin^{-1}(\sin 5) > x^2 - 4x$

$\Rightarrow x^2 - 4x + 4 < 9 - 2\pi$

$\Rightarrow (x-2)^2 < 9 - 2\pi$

$\Rightarrow -\sqrt{9 - 2\pi} < x - 2 < \sqrt{9 - 2\pi}$

$\Rightarrow 2 - \sqrt{9 - 2\pi} < x < 2 + \sqrt{9 - 2\pi}$

18 **(b)**

$0 \le x^2 + x + 1 \le 1$ and $0 \le x^2 + x \le 1$

$\therefore x = -1, 0$

For $x = -1$

$\text{L.H.S.} = 2\sin^{-1}1 + \cos^{-1}0 = \frac{3\pi}{2}$

$\therefore x = -1$ is a solution

For $x = 0$, $\text{L.H.S.} = 2\sin^{-1}1 + \cos^{-1}0 = \frac{3\pi}{2}$

Therefore, $x = 0$ is a solution and sum of the solutions $= -1$

19 **(b)**

$$\tan^{-1}\left(\frac{x\cos\theta}{1 - x\sin\theta}\right) - \cot^{-1}\left(\frac{\cos\theta}{x - \sin\theta}\right) = \tan^{-1}\left(\frac{x\cos\theta}{1 - x\sin\theta}\right) - \tan^{-1}\left(\frac{x - \sin\theta}{\cos\theta}\right)$$

$$= \tan^{-1}\left(\frac{\frac{x\cos\theta}{1 - x\sin\theta} - \frac{x - \sin\theta}{\cos\theta}}{1 + \left(\frac{x\cos\theta}{1 - x\sin\theta}\right)\left(\frac{x - \sin\theta}{\cos\theta}\right)}\right)$$

$$= \tan^{-1}\left(\frac{x\cos^2\theta - x + \sin\theta + x^2\sin\theta - x\sin^2\theta}{\cos\theta - x\cos\theta\sin\theta + x^2\cos\theta - x\cos\theta\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{-x\sin^2\theta + \sin\theta + x^2\sin\theta - x\sin^2\theta}{\cos\theta - 2x\cos\theta\sin\theta + x^2\cos\theta}\right)$$

$$= \tan^{-1}\left(\frac{-2x\sin^2\theta + \sin\theta + x^2\sin\theta}{\cos\theta - 2x\cos\theta\sin\theta + x^2\cos\theta}\right)$$

$$= \tan^{-1}\left(\frac{\sin\theta\,(-2x\sin\theta + 1 + x^2)}{\cos\theta\,(1 - 2x\sin\theta + x^2)}\right) = \tan^{-1}(\tan\theta) = \theta$$

20 **(d)**

$a\sin^{-1}x - b\cos^{-1}x = c$

We have $b\sin^{-1}x + b\cos^{-1}x = \frac{b\pi}{2} \Rightarrow$ $(a + b)\sin^{-1}x = \frac{b\pi}{2} + c$

$$\Rightarrow \sin^{-1}x = \frac{\frac{(b\pi)}{2} + c}{a + b}$$

$$\Rightarrow \cos^{-1}x = \frac{\pi ab + c(a - b)}{a + b}$$

21 **(d)**

$$\tan^{-1}\left(\frac{\tan x}{4}\right) + \tan^{-1}\left(\frac{3\sin 2x}{5 + 3\cos 2x}\right) = \tan^{-1}\left(\frac{\tan x}{4}\right) + \tan^{-1}\left(\frac{\frac{6\tan x}{1+\tan^2 x}}{5 + \frac{3(1-\tan^2 x)}{1+\tan^2 x}}\right)$$

$$= \tan^{-1}\left(\frac{\tan x}{4}\right) + \tan^{-1}\left(\frac{6\tan x}{8 + 2\tan^2 x}\right)$$

$$= \tan^{-1}\left(\frac{\tan x}{4}\right) + \tan^{-1}\left(\frac{3\tan x}{4 + \tan^2 x}\right)$$

$$= \tan^{-1}\left(\frac{\frac{\tan x}{4} + \frac{3\tan x}{4+\tan^2 x}}{1 - \frac{3\tan^2 x}{4(4+\tan^2 x)}}\right)\left[\text{as } \left|\frac{\tan x}{4}\frac{3\tan x}{4 + \tan^2 x}\right| < 1\right]$$

$$= \tan^{-1}\left(\frac{16\tan x + \tan^3 x}{16 + \tan^2 x}\right)$$

$$= \tan^{-1}(\tan x) = x$$

22 **(a)**

We have $\sin^{-1}\left(\cot\left(\sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}} + \cos^{-1}\frac{\sqrt{12}}{4} + \sec^{-1}\sqrt{2}\right)\right)$

$$= \sin^{-1}\left(\cot\left(\sin^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)+\cos^{-1}\frac{\sqrt{3}}{2}+\cos^{-1}\frac{1}{\sqrt{2}}\right)\right)$$

$$= \sin^{-1}[\cot(15^\circ + 30^\circ + 45^\circ)]$$

$$= \sin^{-1}(\cot(90^\circ)) = \sin^{-1}(0) = 0$$

23 **(d)**

$$2\tan^{-1} 2x = \sin^{-1}\frac{4x}{1+4x^2}$$

$$\Rightarrow -\frac{\pi}{2} \le 2\tan^{-1} 2x \le \frac{\pi}{2}$$

$$\Rightarrow -\frac{\pi}{4} \le \tan^{-1} 2x \le \frac{\pi}{4}$$

$$\Rightarrow -1 \le 2x \le 1$$

$$\Rightarrow -\frac{1}{2} \le x \le \frac{1}{2}$$

24 **(b)**

We have

$$\left|\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right| < \frac{\pi}{3} \Rightarrow -\frac{\pi}{3} < \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) < \frac{\pi}{3}$$

$$\Rightarrow 0 \le \cos^{-1}\frac{1-x^2}{1+x^2} < \frac{\pi}{3} \Rightarrow \frac{1}{2} < \frac{1-x^2}{1+x^2} \le 1$$

$$\Rightarrow 1 + x^2 < 2(1-x^2) \le 2(1+x^2) \Rightarrow 0 \le x^2 < \frac{1}{3}$$

$$\Rightarrow -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$$

25 **(c)**

$$\sin^{-1}\left(\frac{\sqrt{r}-\sqrt{r-1}}{\sqrt{r(r+1)}}\right) = \tan^{-1}\left(\frac{\sqrt{r}-\sqrt{r-1}}{1+\sqrt{r(r-1)}}\right)$$

$$\Rightarrow \sum_{r=1}^{n} \sin^{-1}\left(\frac{\sqrt{r}-\sqrt{r-1}}{\sqrt{r(r+1)}}\right) = \sum_{r=1}^{n}\left(\tan^{-1}\sqrt{r} - \tan^{-1}\sqrt{r-1}\right) = \tan^{-1}\sqrt{n}$$

26 **(b)**

$$\cos^{-1}\left(\cos\frac{5\pi}{4}\right) = \cos^{-1}\left(\cos\left(2\pi - \frac{5\pi}{4}\right)\right) = \cos^{-1}\left(\cos\frac{3\pi}{4}\right) = \frac{3\pi}{4}$$

27 **(c)**

Given equation is $\tan^{-1}\frac{a+x}{a} + \tan^{-1}\frac{a-x}{a} = \frac{\pi}{6}$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{a+x}{a}+\frac{a-x}{a}}{1-\frac{a+x}{a}\frac{a-x}{a}}\right) = \frac{\pi}{6}$$

$$\Rightarrow \frac{2a^2}{x^2} = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}} \Rightarrow x^2 = 2\sqrt{3}a^2$$

28 **(a)**

$$2\tan^{-1}\left[\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\right]$$

$$= \cos^{-1}\left[\frac{1-\left(\frac{a-b}{a+b}\right)\tan^2\frac{\theta}{2}}{1+\left(\frac{a-b}{a+b}\right)\tan^2\frac{\theta}{2}}\right] \quad \left[\because 2\tan^{-1} x = \cos^{-1}\frac{1-x^2}{1+x^2}\right]$$

$$= \cos^{-1}\left[\frac{(a+b)-(a-b)\tan^2\frac{\theta}{2}}{(a+b)+(a-b)\tan^2\frac{\theta}{2}}\right]$$

$$= \cos^{-1}\left[\frac{a\left(1-\tan^2\frac{\theta}{2}\right)+b\left(1+\tan^2\frac{\theta}{2}\right)}{a\left(1+\tan^2\frac{\theta}{2}\right)+b\left(1-\tan^2\frac{\theta}{2}\right)}\right]$$

$$= \cos^{-1}\left[\frac{\frac{a\left(1-\tan^2\frac{\theta}{2}\right)}{1+\tan^2\frac{\theta}{2}}+b}{a+b\left(\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right)}\right]$$

$$= \cos^{-1}\left[\frac{a\cos\theta + b}{a + b\cos\theta}\right]$$

29 **(d)**

Given that $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$

$$\Rightarrow \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-xz}\right] = \frac{\pi}{2}$$

Hence, $xy + yz + zx - 1 = 0$

30 **(a)**

Let $\tan^{-1} x = \theta$, where $-\frac{\pi}{2} < \theta < \frac{\pi}{2} \Rightarrow -\pi < 2\theta < \pi$

Let $\frac{\pi}{2} < 2\theta < \pi \Rightarrow \frac{\pi}{4} < \theta < \frac{\pi}{2} \Rightarrow \frac{\pi}{4} < \tan^{-1} x < \frac{\pi}{2} \Rightarrow x > 1$

$$\Rightarrow \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) = \tan^{-1}(\tan 2\theta)$$
$$= \tan^{-1}(\tan(2\theta - \pi)) = 2\theta - \pi$$
$$= 2\tan^{-1} x - \pi$$

31 **(b)**

If $x < 0$, then $\sin^{-1} x < 0$ but $\cos^{-1}\sqrt{1-x^2}$ is always positive

So, $\sin^{-1} x = -\cos^{-1}\sqrt{1-x^2}$

32 **(c)**

$$\sin^{-1}\sqrt{1-x^2} + \cos^{-1} x$$
$$= \cot^{-1}\frac{\sqrt{1-x^2}}{x} - \sin^{-1} x$$

or $\frac{\pi}{2} + \sin^{-1}\sqrt{1-x^2} = \cot^{-1}\frac{\sqrt{1-x^2}}{x}$

$$\tan^{-1}\frac{\sqrt{1-x^2}}{x} + \sin^{-1}\sqrt{1-x^2} = 0$$

$\Rightarrow x \in [-1, 0) \cup \{1\}$

33 **(a)**

Let $\sin^{-1} a = A, \sin^{-1} b = B$ and $\sin^{-1} c = C$

$\Rightarrow \sin A = a, \sin B = b, \sin C = c$

and $A + B + C = \pi \Rightarrow \sin 2A + \sin 2B + \sin 2C = 4\sin A \sin B \sin C$ (i)

$$\Rightarrow \sin A\cos A + \sin B\cos B + \sin C\cos C$$
$$= 2\sin A\sin B\sin C$$

$\Rightarrow \sin A\sqrt{(1-\sin^2 A)} + \sin B\sqrt{(1-\sin^2 B)} + \sin C\sqrt{1-\sin^2 C} = 2\sin A\sin B\sin C$ (ii)

$$\Rightarrow a\sqrt{(1-a^2)} + b\sqrt{(1-b^2)} + c\sqrt{(1-c^2)}$$
$$= 2abc$$

Trick: Let $a = \frac{1}{\sqrt{2}}, b = \frac{1}{\sqrt{2}}, c = 1$

Then $a\sqrt{1-a^2} + b\sqrt{1-b^2} + c\sqrt{1-c^2} = \frac{1}{\sqrt{2}}\sqrt{1-\frac{1}{2}} + \frac{1}{\sqrt{2}}\sqrt{1-\frac{1}{2}} + 1\sqrt{1-1} = 1$

34 **(d)**

$$2^{2\pi/\sin^{-1}x} - 2(a+2)2^{\pi/\sin^{-1}x} + 8a < 0$$

$$\left(2^{\pi/\sin^{-1}x} - 4\right)\left(2^{\pi/\sin^{-1}x} - 2a\right) < 0$$

Now $2^{\pi/\sin^{-1}x} \in \left(0, \frac{1}{4}\right] \cup [4, \infty)$

Now for $2^{\pi/\sin^{-1}x} \in \left(0, \frac{1}{4}\right]$, we have $\left(2^{\pi/\sin^{-1}x} - 4<0\right.$

$\Rightarrow 2^{\pi/\sin^{-1}x} - 2a > 0$

$\Rightarrow 2a < 2^{\pi/\sin^{-1}x} \Rightarrow 2a < \frac{1}{4}$

$\Rightarrow 0 \le a < \frac{1}{8}$

Similarly, for $2^{\pi/\sin^{-1}x} \in [4, \infty), a > 2$, we get

$a \in \left[0, \frac{1}{8}\right) \cup (2, \infty)$

35 **(b)**

Given that $\sin^{-1} x + \tan^{1} x = 2k + 1$

The range of the function $\sin^{-1} x + \tan^{-1} x$ is $\left[\frac{-3\pi}{4}, \frac{3\pi}{4}\right]$ [as both functions are increasing]

Therefore, the integral values of k are -1 and 0

36 **(b)**

$\tan(\sin^{-1}(\cos(\sin^{-1} x)))\tan(\cos^{-1}(\sin(\cos^{-1} x)))$

$= \tan\left(\sin^{-1}\left(\cos\left(\cos^{-1}\sqrt{1-x^2}\right)\right)\right)$

$\tan\left\{\cos^{-1}\left(\sin\left(\sin^{-1}\sqrt{1-x^2}\right)\right)\right\}$

$= \tan\left(\sin^{-1}\sqrt{1-x^2}\right)\tan\left(\cos^{-1}\sqrt{1-x^2}\right)$

$= \tan(\cos^{-1} x)\tan(\sin^{-1} x)$

$$= \tan(\cos^{-1} x)\tan(\pi/2 - \cos^{-1} x)$$
$$= \tan(\cos^{-1} x)\cot(\cos^{-1} x) = 1$$

37 **(c)**

$\cot^{-1}\frac{n}{\pi} > \frac{\pi}{6}$

$\Rightarrow \frac{n}{\pi} < \cot\frac{\pi}{6}$ [as $\cot^{-1} x$ is a decreasing function]

$\Rightarrow \frac{n}{\pi} < \sqrt{3} \Rightarrow n < \sqrt{3}\pi \Rightarrow n < 5.46 \Rightarrow$ maximum value of n is 5

38 **(a)**
We have $\sum_{m=1}^{n}\tan^{-1}\left(\frac{2m}{m^4+m^2+2}\right)=\sum_{m=1}^{n}\tan^{-1}\left(\frac{2m}{1+(m^2+m+1)(m^2-m+1)}\right)$

$$=\sum_{m=1}^{n}\tan^{-1}\left(\frac{(m^2+m+1)-(m^2-m+1)}{1+(m^2+m+1)(m^2-m+1)}\right)$$

$$=\sum_{m=1}^{n}[\tan^{-1}(m^2+m+1)-\tan^{-1}(m^2-m+1)]$$

$$=(\tan^{-1}3-\tan^{-1}1)+(\tan^{-1}7-\tan^{-1}3)+(\tan^{-1}13-\tan^{-1}7)+\cdots+[\tan^{-1}(n^2+n+1)-\tan^{-1}(n^2-n+1)]$$

$$=\tan^{-1}(n^2+n+1)-\tan^{-1}1=\tan^{-1}\left(\frac{n^2+n}{2+n^2+n}\right)$$

For $n\to\infty$, sum $=\tan^{-1}(1)=\frac{\pi}{4}$

39 **(c)**
$$\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x\right)=\frac{1+\tan\left(\frac{1}{2}\cos^{-1}x\right)}{1-\tan\left(\frac{1}{2}\cos^{-1}x\right)}+\frac{1-\tan\left(\frac{1}{2}\cos^{-1}x\right)}{1+\tan\left(\frac{1}{2}\cos^{-1}x\right)}$$

$$=\frac{\left(1+\tan\left(\frac{1}{2}\cos^{-1}x\right)\right)^2+\left(1-\tan\left(\frac{1}{2}\cos^{-1}x\right)\right)^2}{1-\tan^2\left(\frac{1}{2}\cos^{-1}x\right)}$$

$$=2\frac{1+\tan^2\left(\frac{1}{2}\cos^{-1}x\right)}{1-\tan^2\left(\frac{1}{2}\cos^{-1}x\right)}$$

$$=\frac{2}{\cos(\cos^{-1}x)}=\frac{2}{x}$$

40 **(c)**
We have $\sin^{-1}x+\sin^{-1}(1-x)=\cos^{-1}x$

$\Rightarrow \sin(\sin^{-1}x+\sin^{-1}(1-x))=\sin(\cos^{-1}x)$

$\Rightarrow x\sqrt{1-(1-x)^2}+\sqrt{1-x^2}\,(1-x)=\sqrt{1-x^2}$

$\Rightarrow x\sqrt{1-(1-x)^2}=x\sqrt{1-x^2}$

$\Rightarrow x=0$ or $2x-x^2=1-x^2 \Rightarrow x=0$ or $x=\frac{1}{2}$

41 **(d)**
Given that $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$

$\Rightarrow \cos^{-1}(x)+\cos^{-1}(y)=\pi-\cos^{-1}(z)$

$\Rightarrow \cos^{-1}\left(xy-\sqrt{1-x^2}\sqrt{1-y^2}\right)=\cos^{-1}(-z)$

$\Rightarrow xy-\sqrt{(1-x^2)(1-y^2)}=-z$

$\Rightarrow (xy+z)=\sqrt{(1-x^2)(1-y^2)}$

Squaring both sides, we get $x^2+y^2+z^2+2xyz=1$

Trick: Put $x=y=z=\frac{1}{2}$ so that $\cos^{-1}\frac{1}{2}+\cos^{-1}\frac{1}{2}+\cos^{-1}\frac{1}{2}=\pi$

42 **(c)**
$[\cot^{-1}x]+[\cos^{-1}x]=0$

As $\cos^{-1}x, \cot^{-1}x\geq 0$, $[\cot^{-1}x]=[\cos^{-1}x]=0$

$[\cot^{-1}x]=0 \Rightarrow x\in(\cot 1,\infty)$ (i)

$[\cos^{-1}x]=0 \Rightarrow x\in(\cos 1, 1]$ (ii)

Hence, from Eqs. (i) and (ii), $x\in(\cot 1, 1]$

43 **(c)**
$\sin^{-1}(x-1)\Rightarrow -1\leq x-1\leq 1 \Rightarrow 0\leq x\leq 2$

$\cos^{-1}(x-3) \Rightarrow -1\leq x-3\leq 1 \Rightarrow 2\leq x\leq 4$

$\therefore x=2$

So, $\sin^{-1}(2-1)+\cos^{-1}(2-3)+\tan^{-1}\frac{2}{2-4}=\cos^{-1}k+\pi$

$\Rightarrow \sin^{-1}1+\cos^{-1}(-1)+\tan^{-1}(-1)=\cos^{-1}k+\pi$

$\Rightarrow \frac{\pi}{2}+\pi-\frac{\pi}{4}=\cos^{-1}k+\pi$

$\Rightarrow \cos^{-1}k=\frac{\pi}{4}\Rightarrow k=\frac{1}{\sqrt{2}}$

44 **(d)**

$\sin^{-1}(-(x-1)(x-2)) + \cos^{-1}((x-3)(x-1)) = \frac{\pi}{4}$

For $x \in [1,2] \Rightarrow \sin^{-1}(-(x-1)(x-2)) \in [0, \pi/2)$

and $\cos^{-1}((x-3)(x-1)) \in [\pi/2, \pi]$ $\Rightarrow$ no solution in the given range

Also, $-1 \le 3x - 2 - x^2 \le 1$ and $-1 \le x^2 - 4x + 3 \le 1$ $\Rightarrow 2 - \sqrt{2} \le x \le \frac{3+\sqrt{5}}{2}$

45 **(c)**

$\cos^{-1}(\cos(2\cot^{-1}(\sqrt{2}-1))) = \cos^{-1}(\cos(2(67.5°)))$

$= \cos^{-1}(\cos(135°)) = 135° = \frac{3\pi}{4}$

46 **(c)**

$2\sin^{-1} x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Range of the right-hand angle is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Rightarrow -\frac{\pi}{2} \le 2\sin^{-1} x \le \frac{\pi}{2}$

$\Rightarrow \frac{-\pi}{2} \le \sin^{-1} x \le \frac{\pi}{4}$

$\Rightarrow x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

47 **(c)**

$f(x) = \sin^{-1} x + \tan^{-1} x + \sec^{-1} x$, clearly domain of $f(x)$ is $x = \pm 1$

Thus, the range is $\{f(1), f(-1)\}$, i.e., $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$

48 **(d)**

Given, $\sin[\cot^{-1}(1+x)] = \cos(\tan^{-1} x)$

$\therefore \sin\left(\sin^{-1}\frac{1}{\sqrt{1+(1+x^2)}}\right) = \cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right)$

$\Rightarrow \frac{1}{\sqrt{1+(1+x^2)}} = \frac{1}{\sqrt{1+x^2}}$

$\Rightarrow 1 + x^2 + 2x + 1 = x^2 + 1$

$\Rightarrow \quad x = -\frac{1}{2}$

49 **(a)**

$1 + x^2 \ge 2|x| \Rightarrow \frac{2|x|}{1+x^2} \le 1$

$\Rightarrow -1 \le \frac{2x}{1+x^2} \le 1 \Rightarrow \tan^{-1}\left(\frac{2x}{1+x^2}\right) \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

50 **(c)**

Put $\sin^{-1}\frac{5}{x} = A \Rightarrow \frac{5}{x} = \sin A$

$\sin^{-1}\frac{12}{x} = B \Rightarrow \frac{12}{x} = \sin B \Rightarrow A + B = \frac{\pi}{2}$

$\Rightarrow \sin A = \sin\left(\frac{\pi}{2} - B\right) = \cos B = \sqrt{1 - \sin^2 B}$

$\Rightarrow \frac{5}{x} = \sqrt{1 - \frac{144}{x^2}} \Rightarrow \frac{169}{x^2} = 1$

$\Rightarrow x^2 = 169 \Rightarrow x = 13$ [$\because x = -13$ doses not satisfy the given equation]

51 **(d)**

We have $\cos^{-1} x - \cos^{-1}\frac{y}{2} = \alpha$

$\Rightarrow x = \cos\left(\cos^{-1}\frac{y}{2} + \alpha\right)$

$= \cos\left(\cos^{-1}\frac{y}{2}\right)\cos\alpha - \sin\left(\cos^{-1}\frac{y}{2}\right)\sin\alpha$

$= \frac{y}{2}\cos\alpha - \sqrt{1 - \frac{y^2}{4}}\sin\alpha$

$\Rightarrow 2x = y\cos\alpha - \sin\alpha\sqrt{4 - y^2}$

$\Rightarrow 2x - y\cos\alpha = -\sin\alpha\sqrt{4 - y^2}$

Squaring, we get

$4x^2 + y^2\cos^2\alpha - 4xy\cos\alpha = 4\sin^2\alpha - y^2\sin^2\alpha$

$\Rightarrow 4x^2 - 4xy\cos\alpha + y^2 = 4\sin^2\alpha$

52 **(b)**

$\cos^{-1}\left(\frac{1+x^2}{2x}\right) = \frac{\pi}{2} + (\sin^{-1} x + \cos^{-1} x)$

$\Rightarrow \cos^{-1}\left(\frac{1+x^2}{2x}\right) = \pi \Rightarrow \left(\frac{1+x^2}{2x}\right) = \cos\pi = -1 \Rightarrow x^2 + 1 + 2x = 0 \Rightarrow x = -1$

53 **(d)**

$\sin^{-1}(\cos(\cos^{-1}(\cos x) + \sin^{-1}(\sin x)))$
$= \sin^{-1}(\cos(x + \pi - x))$ [as $x \in (\pi/2, \pi)$]

$= \sin^{-1}(\cos\pi) = \sin^{-1}(-1) = -\frac{\pi}{2}$

54 **(c)**

$x_1 = 2\tan^{-1}\left(\frac{1+x}{1-x}\right)$ and $x_2 = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$
$= \tan^{-1}\left(\frac{1-x^2}{2x}\right)$

Now $\frac{1+x}{1-x} > 1 \Rightarrow x_1 = \pi + \tan^{-1}\left(\frac{2\left(\frac{1+x}{1-x}\right)}{1-\left(\frac{1+x}{1-x}\right)^2}\right) = \pi + \tan^{-1}\left(\frac{1-x^2}{-2x}\right) = \pi - \tan^{-1}\left(\frac{1-x^2}{2x}\right)$

$\Rightarrow x_1 + x_2 = \pi$

55 **(b)**

$\cot^{-1}x + \cot^{-1}y + \cot^{-1}z = \frac{\pi}{2}$

$\Rightarrow \frac{\pi}{2} - \tan^{-1}x + \frac{\pi}{2} - \tan^{-1}y + \frac{\pi}{2} - \tan^{-1}z = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$

$\Rightarrow \tan^{-1}x + \tan^{-1}y = \pi - \tan^{-1}z$

$\Rightarrow \tan(\tan^{-1}x + \tan^{-1}y) = \tan(\pi - \tan^{-1}z)$

$\Rightarrow \frac{x+y}{1-xy} = -z$

$\Rightarrow x + y + z = xyz$

56 **(a)**

$\tan^{-1}\left[\frac{\cos x}{1+\sin x}\right] = \tan^{-1}\left[\frac{\sin[(\pi/2)-x]}{1+\cos[(\pi/2)-x]}\right]$

$= \tan^{-1}\left[\frac{2\sin\left[\left(\frac{\pi}{4}\right)-\left(\frac{x}{2}\right)\right]\cos[(\pi/4)-(x/2)]}{2\cos^2[(\pi/4)-(x/2)]}\right]$

$= \tan^{-1}\tan\left(\frac{\pi}{4}-\frac{x}{2}\right) = \frac{\pi}{4}-\frac{x}{2}$

$\Rightarrow -\frac{\pi}{2} < \frac{\pi}{4} - \frac{x}{2} < \frac{\pi}{2}$

$\Rightarrow -\frac{3\pi}{4} < -\frac{x}{2} < \frac{\pi}{4}$

$\Rightarrow -\frac{\pi}{4} < \frac{x}{2} < \frac{3\pi}{4}$

$\Rightarrow -\frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{2}$

57 **(d)**

Since $\sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}x$ for $x \in (-1, 1)$

$\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \sin^{-1}\left(\frac{2b}{1+b^2}\right) = 2\tan^{-1}x$

$\Rightarrow 2\tan^{-1}a + 2\tan^{-1}b = 2\tan^{-1}x$

$\Rightarrow \tan^{-1}a + \tan^{-1}b = \tan^{-1}x$

$\Rightarrow \tan^{-1}\left(\frac{a+b}{1-ab}\right) = \tan^{-1}x$

$\Rightarrow x = \frac{a+b}{1-ab}$

58 **(a)**

$\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$

$\Rightarrow \tan^{-1}\left(\frac{1}{\sqrt{\cos\alpha}}\right) - \tan^{-1}(\sqrt{\cos\alpha}) = x$

$\Rightarrow \tan^{-1}\frac{\frac{1}{\sqrt{\cos\alpha}} - \sqrt{\cos\alpha}}{1 + \frac{1}{\sqrt{\cos\alpha}}\sqrt{\cos\alpha}} = x$

$\Rightarrow \tan^{-1}\frac{1-\cos\alpha}{2\sqrt{\cos\alpha}} = x$

$\Rightarrow \tan x = \frac{1-\cos\alpha}{2\sqrt{\cos\alpha}}$

$\Rightarrow \cot x = \frac{2\sqrt{\cos\alpha}}{1-\cos\alpha}$

$\Rightarrow \operatorname{cosec} x = \sqrt{1 + \frac{4\cos\alpha}{(1-\cos\alpha)^2}} = \frac{1+\cos\alpha}{1-\cos\alpha}$

$\Rightarrow \sin x = \frac{1-\cos\alpha}{1+\cos\alpha} = \frac{2\sin^2(\alpha/2)}{2\cos^2(\alpha/2)} = \tan^2\alpha/2$

59 **(c)**

$\tan^{-1}\frac{x}{y} - \tan^{-1}\left(\frac{x-y}{x+y}\right)$
$= \tan^{-1}\frac{x}{y} - \tan^{-1}\left(\frac{1-(y/x)}{1+(y/x)}\right)$

$= \tan^{-1}\frac{x}{y} - \left(\tan^{-1}1 - \tan^{-1}\frac{y}{x}\right)$

$$= \tan^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x} - \frac{\pi}{4}$$

$$= \tan^{-1}\frac{x}{y} + \cot^{-1}\frac{x}{y} - \frac{\pi}{4} = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

60 **(b)**

Obviously, $x = \sin(\theta + \beta)$ and $y = \sin(\theta - \beta)$

$$\Rightarrow 1 + xy = 1 + \sin(\theta + \beta)\sin(\theta - \beta)$$
$$= 1 + \sin^2\theta - \sin^2\beta$$
$$= \sin^2\theta + \cos^2\beta$$

61 **(c)**

$$2\tan^{-1}(\operatorname{cosec}\tan^{-1}x - \tan\cot^{-1}x)$$
$$= 2\tan^{-1}[\operatorname{cosec}\tan^{-1}x - \tan\cot^{-1}x]$$

$$= 2\tan^{-1}\left[\operatorname{cosec}\left\{\operatorname{cosec}^{-1}\frac{\sqrt{1+x^2}}{x}\right\} - \tan^{-1}\left\{\tan^{-1}\left(\frac{1}{x}\right)\right\}\right]$$

$$= 2\tan^{-1}\left[\sqrt{\frac{1+x^2}{x}} - \frac{1}{x}\right] = 2\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$$

$$= 2\tan^{-1}\left[\frac{\sec\theta - 1}{\tan\theta}\right] \quad [\text{putting } x = \tan\theta]$$

$$= 2\tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right] = 2\tan^{-1}\left[\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right]$$

$$= 2\tan^{-1}\tan\frac{\theta}{2} = 2 \times \frac{\theta}{2} = \theta = \tan^{-1}x$$

62 **(c)**

Let $\cos^{-1}\left(\frac{\sqrt{5}}{3}\right) = \alpha$. Then $\cos\alpha = \frac{\sqrt{5}}{3}$, where $0 < \alpha < \frac{\pi}{2}$

Now, $\tan\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \sqrt{\frac{1-\sqrt{5}/3}{1+\sqrt{5}/3}} = \sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}} = \sqrt{\frac{(3-\sqrt{5})^2}{9-5}} = \frac{1}{2}(3-\sqrt{5})$

63 **(d)**

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

$$= \frac{\frac{\sqrt{3}\,x}{2K-x} - \frac{2x-K}{\sqrt{3}\,K}}{1 + \frac{\sqrt{3}\,x}{2K-x}\frac{2x-K}{\sqrt{3}\,K}}$$

$$= \frac{3Kx - (2x-K)(2K-x)}{(2K-x)\sqrt{3}\,K + \sqrt{3}\,x(2x-K)}$$

$$= \frac{3Kx - (4Kx - 2x^2 - 2K^2 + Kx)}{2\sqrt{3}\,K^2 - \sqrt{3}\,Kx + 2\sqrt{3}\,x^2 - \sqrt{3}\,Kx}$$

$$= \frac{2x^2 - 2Kx + 2K^2}{2\sqrt{3}\,x^2 - 2\sqrt{3}\,Kx + 2\sqrt{3}\,K^2} = \frac{1}{\sqrt{3}} = \tan 30°$$

$\therefore A - B = 30°$

64 **(a)**

$$\sin^{-1}\sin\left(\frac{22\pi}{7}\right) = \sin^{-1}\sin\left(3\pi + \frac{\pi}{7}\right) = -\frac{\pi}{7}$$

$$\cos^{-1}\cos\left(\frac{5\pi}{3}\right) = \cos^{-1}\cos\left(2\pi - \frac{\pi}{3}\right) = \frac{\pi}{3}$$

$$\tan^{-1}\tan\left(\frac{5\pi}{7}\right) = \tan^{-1}\tan\left(\pi - \frac{2\pi}{7}\right) = -\frac{2\pi}{7}$$

$$\sin^{-1}\cos(2) = \frac{\pi}{2} - \cos^{-1}\cos 2 = \frac{\pi}{2} - 2$$

Therefore, the required value $= -\frac{\pi}{7} + \frac{\pi}{3} - \frac{2\pi}{7} + \frac{\pi}{2} - 2$

$$= \frac{(-18+35)\pi}{42} - 2 = \frac{17\pi}{42} - 2$$

65 **(a)**

$$\tan^{-1}\left(\frac{1}{1+r+r^2}\right) = \tan^{-1}\left(\frac{r+1-r}{1+r(r+1)}\right)$$

$$= \tan^{-1}(r+1) - \tan^{-1}(r)$$

$$\Rightarrow \sum_{r=0}^{n}[\tan^{-1}(r+1) - \tan^{-1}(r)]$$
$$= \tan^{-1}(n+1) - \tan^{-1}(0)$$

$$= \tan^{-1}(n+1)$$

$$\Rightarrow \sum_{r=0}^{\infty}\tan^{-1}\left(\frac{1}{1+r+r^2}\right) = \tan^{-1}(\infty) = \frac{\pi}{2}$$

66 **(a)**

Let $\cos^{-1}\left(\frac{1}{8}\right) = \theta$, where $0 < \theta < \pi$, then $\frac{1}{2}\cos^{-1}\frac{1}{8} = \frac{1}{2}\theta$

$$\Rightarrow \cos\left(\frac{1}{2}\cos^{-1}\frac{1}{8}\right) = \cos\frac{\theta}{2}$$

Now, $\cos^{-1}\frac{1}{8} = \theta \Rightarrow \cos\theta = \frac{1}{8} \Rightarrow \cos^2\frac{\theta}{2} = \frac{9}{16} \Rightarrow$

$\cos\frac{\theta}{2} = \frac{3}{4}$

67 **(b)**

$\cos^{-1}(2x^2 - 1) = 2\pi - 2\cos^{-1}x$ (as $x < 0$)

$\Rightarrow \cos^{-1}(2x^2 - 1) - 2\sin^{-1}x$
$= 2\pi - 2\cos^{-1}x - 2\sin^{-1}x$

$= 2\pi - 2(\cos^{-1}x + \sin^{-1}x)$

$= 2\pi - 2\frac{\pi}{2} = \pi$

68 **(c)**

Let $y = \tan^{-1}\left(\frac{1-x^2}{2x}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Put $x = \tan\theta$. As $x \in (0,1), \theta \in \left(0, \frac{\pi}{4}\right)$ and $\frac{\pi}{2} - 2\theta \in (0, \pi/2)$

$\therefore y = \tan^{-1}(\cot 2\theta) + \cos^{-1}(\cos 2\theta)$
$= \tan^{-1}\left(\tan\left(\frac{\pi}{2} - 2\theta\right)\right)$
$+ \cos^{-1}(\cos 2\theta)$

$= \frac{\pi}{2} - 2\theta + 2\theta = \frac{\pi}{2}$

69 **(b)**

Let $x = \sin\theta$ where $-\frac{1}{2} \le x \le 1 \Rightarrow -\frac{\pi}{6} \le \theta \le \frac{\pi}{2}$

Then $f(x) = \sin^{-1}\left(\frac{\sqrt{3}}{2}x - \frac{1}{2}\sqrt{1-x^2}\right)$

$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta\right)$

$= \sin^{-1}\left(\sin\left(\theta - \frac{\pi}{6}\right)\right)$

$= \theta - \frac{\pi}{6} = \sin^{-1}x - \frac{\pi}{6}$ $\left[\because \theta - \frac{\pi}{6} \in \left(\frac{-\pi}{3}, \frac{\pi}{3}\right)\right]$

70 **(c)**

$\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}(1+x) = \frac{\pi}{2} - \tan^{-1}(1-x)$

$= \cot^{-1}(1-x)$

$= \tan^{-1}\left(\frac{1}{1-x}\right)$

$\Rightarrow 1 + x = \frac{1}{1-x} \Rightarrow 1 - x^2 = 1 \Rightarrow x = 0$

71 **(d)**

$\tan\left(\cos^{-1}\frac{1}{5\sqrt{2}} - \sin^{-1}\frac{4}{\sqrt{17}}\right)$
$= \tan[\tan^{-1}7 - \tan^{-1}4]$
$= \tan\left(\tan^{-1}\left(\frac{3}{29}\right)\right) = \frac{3}{29}$

72 **(a)**

Put $x = \tan\theta$

$\therefore 3\sin^{-1}\frac{2\tan\theta}{1+\tan^2\theta} - 4\cos^{-1}\frac{1-\tan^2\theta}{1+\tan^2\theta}$
$+ 2\tan^{-1}\frac{2\tan\theta}{1-\tan^2\theta} = \frac{\pi}{3}$

$\Rightarrow 3\sin^{-1}(\sin 2\theta) - 4\cos^{-1}(\cos 2\theta)$
$+ 2\tan^{-1}(\tan 2\theta) = \frac{\pi}{3}$

$\Rightarrow 3(2\theta) - 4(2\theta) + 2(2\theta) = \frac{\pi}{3} \Rightarrow 2\theta = \frac{\pi}{3} \Rightarrow \theta$
$= \frac{\pi}{6} \Rightarrow x = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

73 **(d)**

$[\cot^{-1}x] + 2[\tan^{-1}x] = 0 \Rightarrow [\cot^{-1}x]$
$= 0, [\tan^{-1}x] = 0$

or $[\cot^{-1}x] = 2, [\tan^{-1}x] = -1$

Now $[\cot^{-1}x] = 0 \Rightarrow x \in (\cot 1, \infty)$

$[\tan^{-1}x] = 0 \Rightarrow x \in (0, \tan 1)$

Therefore, for $[\cot^{-1}x] = [\tan^{-1}x] = 0, x \in (\cot 1, \tan 1)$

$[\cot^{-1}x] = 2 \Rightarrow x \in (\cot 3, \cot 2]$

$[\tan^{-1}x] = -1 \Rightarrow x \in [-\tan 1, 0)$ $\Rightarrow$No such x exists

Thus, the solution set is $(\cot 1, \tan 1)$

74 **(c)**

Let $\tan^{-1}(x) = \theta \Rightarrow x = \tan\theta \Rightarrow \cos\theta = x \Rightarrow \frac{1}{\sqrt{1+x^2}} = x$

$\Rightarrow x^2(1+x^2) = 1 \Rightarrow x^2 = \frac{-1 \pm \sqrt{5}}{2} \Rightarrow x^2$
$= \frac{\sqrt{5}-1}{2} \Rightarrow \frac{x^2}{2} = \frac{\sqrt{5}-1}{4}$

Now $\cos^{-1}\left(\frac{\sqrt{5}-1}{4}\right) = \cos^{-1}\left(\sin\frac{\pi}{10}\right) = \cos^{-1}\left(\cos\frac{2\pi}{5}\right) = \frac{2\pi}{5} = \frac{2\pi}{5}$

75 **(b)**

$$\sum_{r=1}^{n}\tan^{-1}\left(\frac{2^{r-1}}{1+2^{2r-1}}\right)=\sum_{r=1}^{n}\tan^{-1}\left(\frac{2^{r-1}}{1+2^{r}\,2^{r-1}}\right)$$

$$=\sum_{r=1}^{n}\tan^{-1}\left(\frac{2^{r}-2^{r-1}}{1+2^{r}\,2^{r-1}}\right)$$

$$=\sum_{r=1}^{n}[\tan^{-1}(2^{r})-\tan^{-1}(2^{r-1})]$$

$$=\tan^{-1}(2^{n})-\tan^{-1}(1)$$
$$=\tan^{-1}(2^{n})-\frac{\pi}{4}$$

76 **(c)**

Since

$\sqrt{x^2-3x+2}\ge 0\Rightarrow 0\le\tan^{-1}\sqrt{x^2-3x+2}<\frac{\pi}{2}$

and

$\sqrt{4x-x^2-3}\ge 0\Rightarrow 0<\cos^{-1}\sqrt{4x-x^2-3}\le\frac{\pi}{2}$

Adding, we have $0<\text{L. H. S.}<\pi$

Therefore, the given equation has no solution

77 **(c)**

$$\tan^{-1}\sqrt{[x(x+1)]}=(\pi/2)-\sin^{-1}\sqrt{(x^2+x+1)}$$
$$=\cos^{-1}\sqrt{x^2+x+1}$$
$$=\tan^{-1}\frac{\sqrt{-x^2-x}}{\sqrt{x^2+x+1}}$$

$\Rightarrow\sqrt{x(x+1)}=\frac{\sqrt{-x^2-x}}{\sqrt{x^2+x+1}}\Rightarrow x=0,-1$ are the only real solutions

78 **(c)**

From the given equation $\sin^2\theta-2\sin\theta+3=5^{\sec^2 y}+1$, we get

$(\sin\theta-1)^2+2=5^{\sec^2 y}+1$

$\text{L. H. S.}\le 6, \text{R. H. S.}\ge 6$

Possible solution is $\sin\theta=-1$ when $\text{L. H. S.}=\text{R. H. S}\Rightarrow\cos^2\theta=0\Rightarrow\cos^2\theta-\sin\theta=1$

79 **(b)**

$$\sin^{-1}x+\sin^{-1}y=\frac{\pi}{2}\Rightarrow\sin^{-1}x=\frac{\pi}{2}-\sin^{-1}y$$
$$\Rightarrow\sin^{-1}x=\sin^{-1}\sqrt{1-y^2}$$
$$\Rightarrow x^2+y^2=1$$

$$\Rightarrow\frac{1+x^4+y^4}{x^2-x^2y^2+y^2}=\frac{1+(x^2+y^2)^2-2x^2y^2}{1-x^2y^2}$$
$$=\frac{1+1-2x^2y^2}{1-x^2y^2}=2$$

80 **(d)**

$$\tan\left[\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{2}{3}\right)\right]$$
$$=\tan\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\right)$$
$$=\tan\left(\tan^{-1}\left(\frac{(3/4)+(2/3)}{1-(3/4)\times(2/3)}\right)\right)$$
$$=\frac{17}{12}\times\frac{12}{6}=\frac{17}{6}$$

81 **(c)**

We have $\cos^{-1}x+\cos^{-1}(2x)=-\pi$, which is not possible as $\cos^{-1}x$ and $\cos^{-1}2x$ never take negative values

82 **(e)**

The principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)=$ principal value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)=\pi/3$

83 **(b)**

$\sin\cos^{-1}(\cos(\tan^{-1}x))=p$

For $x\in R\ \tan^{-1}x\in(-\pi/2,\pi/2)$

$\cos(\tan^{-1}x)\in(0,1]$

$\cos^{-1}\cos(\tan^{-1}x)\in[0,\pi/2)$

$\sin(\cos^{-1}(\cos(\tan^{-1}x)))\in[0,1)$

84 **(a)**

Let $\sqrt{\tan\alpha}=\tan x$, then $u=\cot^{-1}(\tan x)-\tan^{-1}(\tan x)=\frac{\pi}{2}-x-x=\frac{\pi}{2}-2x$

$$\Rightarrow 2x=\frac{\pi}{2}-u\quad\Rightarrow\frac{\pi}{4}-\frac{u}{2}$$

$$\Rightarrow\tan x=\tan\left(\frac{\pi}{4}-\frac{u}{2}\right)$$

$$\Rightarrow\sqrt{\tan\alpha}=\tan\left(\frac{\pi}{4}-\frac{u}{2}\right)$$

85 **(b)**

The given equation can be written as

$$3\tan^{-1}(2-\sqrt{3})=\tan^{-1}\left(\frac{1}{x}\right)+\tan^{-1}\left(\frac{1}{3}\right)$$

$$\Rightarrow 3(15°) = \tan^{-1}\frac{\frac{1}{x}+\frac{1}{3}}{1-\frac{1}{x}\frac{1}{3}} \Rightarrow 1 = \frac{3+x}{3x-1} \Rightarrow x = 2$$

86 **(d)**

$$f(x) = \tan^{-1}\left(\frac{(\sqrt{12}-2)x^2}{x^4+2x^2+3}\right)$$

$$= \tan^{-1}\left(\frac{2(\sqrt{3}-1)}{x^2+\frac{3}{x^2}+2}\right)$$

As $x^2+\frac{3}{x^2} \geq 2\sqrt{3}$ [using A.M. ≥ G.M.]

$$\Rightarrow x^2+\frac{3}{x^2}+2 \geq 2+2\sqrt{3}$$

$$\therefore (f(x))_{\max} = \tan^{-1}\left(\frac{2(\sqrt{3}-1)}{2(\sqrt{3}+1)}\right) = \frac{\pi}{12}$$

87 **(d)**

$\sin(2\sin^{-1}(0.8))$

$$= \sin\left(\sin^{-1}\left(2 \times 0.8\sqrt{1-(0.8)^2}\right)\right)$$

$$= \sin(\sin^{-1} 0.96) = 0.96$$

88 **(d)**

$$\frac{x}{\sqrt{1+x^2}} = \frac{x+1}{\sqrt{(x+1)^2+1}}$$

$$\Rightarrow x^2[(x+1)^2+1] = (x+1)^2[(x^2+1)]$$

$$\Rightarrow x^2(x+1)^2+x^2 = x^2(x+1)^2+(x+1)^2$$

$\Rightarrow x^2 = (x+1)^2 \Rightarrow x+1 = x$ not possible as $x \to \infty$

$\Rightarrow x+1 = -x \Rightarrow x = -1/2$ which is also not possible as for this L. H. S. < 0 but R. H. S. > 0

89 **(c)**

We have $\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1} x$

$$\Rightarrow \tan^{-1}\left[\frac{1-\tan\theta}{1+\tan\theta}\right] = \frac{1}{2}\theta \text{ (putting } x = \tan\theta)$$

$$\Rightarrow \tan^{-1}\left[\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}\right] = \frac{\theta}{2}$$

$$\Rightarrow \tan^{-1}\tan\left(\frac{\pi}{4}-\theta\right) = \frac{\theta}{2}$$

$$\Rightarrow \frac{\pi}{4}-\theta = \frac{\theta}{2}$$

$$\Rightarrow \theta = \frac{\pi}{6} = \tan^{-1} x$$

$$\Rightarrow x = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

90 **(d)**

$f(x)+f(-x) = 2$

Now $(\sin^{-1}(\sin 8)) = 3\pi-8 = y$

and $(\tan^{-1}(\tan 8)) = (8-3\pi)$

Hence, $f(y)+f(-y) = 2$

Given, $f(y) = \alpha$ we have $f(-y) = 2-\alpha$

91 **(d)**

Let $\alpha = \cos^{-1}\sqrt{p}, \beta = \cos^{-1}\sqrt{1-p}$ and $\gamma = \cos^{-1}\sqrt{1-q}$

$\Rightarrow \cos\alpha = \sqrt{p}, \cos\beta = \sqrt{1-p}$ and $\cos\gamma = \sqrt{1-q}$

Therefore, $\sin\alpha = \sqrt{1-p}, \sin\beta = \sqrt{p}$ and $\sin\gamma = \sqrt{q}$

The given equation may be written as

$$\alpha+\beta+\gamma = \frac{3\pi}{4}$$

$$\Rightarrow \alpha+\beta = \frac{3\pi}{4}-\gamma$$

$$\Rightarrow \cos(\alpha+\beta) = \cos\left(\frac{3\pi}{4}-\gamma\right)$$

$$\Rightarrow \cos\alpha\cos\beta-\sin\alpha\sin\beta = \cos\left(\pi-\left(\frac{\pi}{4}+\gamma\right)\right)$$

$$= -\cos\left(\frac{\pi}{4}+\gamma\right)$$

$$\Rightarrow \sqrt{p}\sqrt{1-p}-\sqrt{1-p}\sqrt{p} = -\left(\frac{1}{\sqrt{2}}\sqrt{1-q}-\frac{1}{\sqrt{2}}\sqrt{q}\right)$$

$$\Rightarrow 0 = \sqrt{1-q}-\sqrt{q} \Rightarrow 1-q = q \Rightarrow q = \frac{1}{2}$$

92 **(c)**

$\sin^{-1}(\sin 10) = \sin^{-1}[\sin(3\pi-10)] = 3\pi-10$

93 **(c)**

$$\frac{\alpha^3}{2}\operatorname{cosec}^2\left(\frac{1}{2}\tan^{-1}\frac{\alpha}{\beta}\right)+\frac{\beta^3}{2}\sec^2\left(\frac{1}{2}\tan^{-1}\frac{\beta}{\alpha}\right)$$

$$=\alpha^3\frac{1}{1-\cos\left(\tan^{-1}\left(\frac{\alpha}{\beta}\right)\right)}+\beta^3\frac{1}{1+\cos\left(\tan^{-1}\frac{\beta}{\alpha}\right)}$$

$$=\alpha^3\frac{1}{1-\cos\left(\cos^{-1}\left(\frac{\beta}{\sqrt{\alpha^2+\beta^2}}\right)\right)}$$

$$+\beta^3\frac{1}{1+\cos\left(\cos^{-1}\frac{\alpha}{\sqrt{\alpha^2+\beta^2}}\right)}$$

$$=\alpha^3\frac{1}{1-\frac{\beta}{\sqrt{\alpha^2+\beta^2}}}+\beta^3\frac{1}{1+\frac{\alpha}{\sqrt{\alpha^2+\beta^2}}}$$

$$=\sqrt{\alpha^2+\beta^2}\left(\frac{\alpha^3}{\sqrt{\alpha^2+\beta^2}-\beta}+\frac{\beta^3}{\sqrt{\alpha^2+\beta^2}+\alpha}\right)$$

$$=\sqrt{\alpha^2+\beta^2}\left(\alpha^3\frac{\left(\sqrt{\alpha^2+\beta^2}+\beta\right)}{\alpha^2}\right.$$

$$\left.+\beta^3\frac{(\sqrt{\alpha^2+\beta^2}-\alpha)}{\beta^2}\right)$$

$$=\sqrt{\alpha^2+\beta^2}\left[\alpha\left(\sqrt{\alpha^2+\beta^2}+\beta\right)\right.$$

$$\left.+\beta\left(\sqrt{\alpha^2+\beta^2}-\alpha\right)\right]$$

$$=\sqrt{\alpha^2+\beta^2}(\alpha+\beta)\sqrt{\alpha^2+\beta^2}$$

$$=(\alpha+\beta)(\alpha^2+\beta^2)$$

94 **(c)**

We have $(\sin^{-1}x)^3+(\cos^{-1}x)^3=(\sin^{-1}x+$
cos−1x3−3sin−1cos−1xsin−1x+cos−1x

$$=\frac{\pi^3}{8}-3(\sin^{-1}x\cos^{-1}x)\frac{\pi}{2}$$

$$=\frac{\pi^3}{8}-\frac{3\pi}{2}\sin^{-1}x\left(\frac{\pi}{2}-\sin^{-1}x\right)$$

$$=\frac{\pi^3}{8}-\frac{3\pi^2}{4}\sin^{-1}x+\frac{3\pi}{2}\ (\sin^{-1}x)^2$$

$$=\frac{\pi^3}{8}+\frac{3\pi}{2}\left[(\sin^{-1}x)^2-\frac{\pi}{2}\sin^{-1}x\right]$$

$$=\frac{\pi^3}{8}+\frac{3\pi}{2}\left[\left(\sin^{-1}x-\frac{\pi}{4}\right)^2\right]-\frac{3\pi^3}{32}$$

$$=\frac{\pi^3}{32}+\frac{3\pi}{2}\left(\sin^{-1}x-\frac{\pi}{4}\right)^2$$

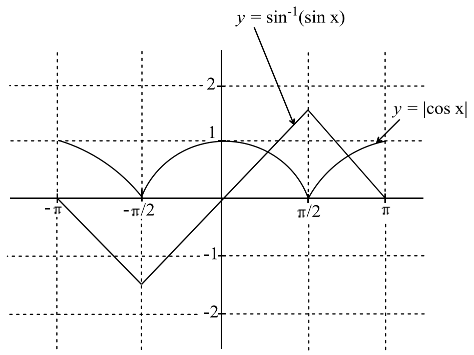
So, the least value is $\frac{\pi^3}{32}$ when $\left(\sin^{-1}x-\frac{\pi}{4}\right)=0$

And the greatest value occurs when $\left(\sin^{-1}x-\right.$
π42=−π2−π42=9π216

Therefore, the greatest value is $\frac{\pi^3}{32}+\frac{9\pi^2}{16}\times\frac{3\pi}{2}=$
$\frac{7\pi^3}{8}$

95 **(c)**

Here $|\cos x|=\sin^{-1}(\sin x)$



From the graph, number of solutions is 2

96 **(a)**

$$2\cos^{-1}x=\cot^{-1}\left(\frac{2x^2-1}{2x\sqrt{1-x^2}}\right)$$

Put $x=\cos\theta$: LHS $=2\theta;0\le\theta\le\pi$ and
$-1\le x\le 1$ (i)

R. H. S. $= \cot^{-1}\left(\frac{\cos 2\theta}{2\cos\theta\,|\sin\theta|}\right) = \cot^{-1}(\cot 2\theta) = 2\theta$ if $0 < 2\theta < \pi$ (ii)

From Eqs. (i) and (ii), we get $0 < \theta < \pi/2$

$\therefore x \in (0, 1)$

97 **(b)**

$$\tan^{-1}\frac{b+a}{b-a} - \tan^{-1}\frac{a}{b} = \tan^{-1}\frac{\frac{b+a}{b-a} - \frac{a}{b}}{1 + \frac{b+a}{b-a}\frac{a}{b}}$$

$$= \tan^{-1}\frac{b^2 + ab - ab + a^2}{b^2 - ab + ab + a^2} = \tan^{-1}\frac{a^2+b^2}{a^2+b^2}$$
$$= \tan^{-1} 1 = \frac{\pi}{4}$$

Therefore, the required value $= \sec\left(\frac{\pi}{4}\right) = \sqrt{2}$

98 **(a)**

$\sin^{-1} x - \cos^{-1} x = \sin^{1}(3x - 2)$

$\Rightarrow \frac{\pi}{2} - \cos^{-1} x - \cos^{-1} x = \frac{\pi}{2} - \cos^{-1}(3x-2)$

$\Rightarrow 2\cos^{-1} x = \cos^{-1}(3x-2)$. Also $x \in [-1, 1]$

$\Rightarrow \cos^{-1}(2x^2 - 1) = \cos^{-1}(3x-2)$ and $(3x - 2\in -1, 1$, i.e., $-1 \le 3x-2 \le 1$

$\Rightarrow 2x^2 - 1 = 3x - 2$; hence, $x \in \left[\frac{1}{3}, 1\right]$

$\Rightarrow 2x^2 - 3x + 1 = 0 \Rightarrow x = 1$ or $\frac{1}{2}$

99 **(c)**

$\sqrt{1+x^2}[\{x\cos(\cot^{-1}x) + \sin(\cot^{-1}x)\}^2 - 1]^{1/2}$

$$= \sqrt{1+x^2}\left[\left\{x\cos\left(\cos^{-1}\frac{x}{\sqrt{1+x^2}}\right) + \sin\left(\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right)\right\}^2 - 1\right]^{1/2}$$

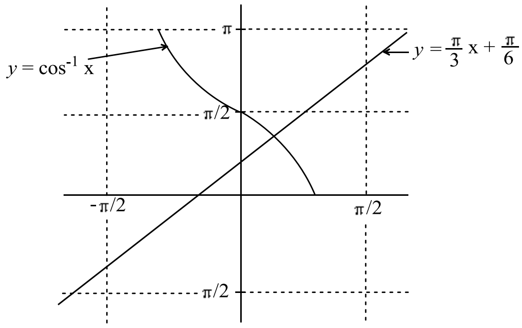
$$= \sqrt{1+x^2}\left[\left\{x.\frac{x}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+x^2}}\right\}^2 - 1\right]^{1/2}$$

$= \sqrt{1+x^2}[1 + x^2 - 1]^{1/2}$

$= x\sqrt{1+x^2}$

100 **(b)**

$3\cos^{-1} x - \pi x - \frac{\pi}{2} = 0 \Rightarrow \cos^{-1} x = \frac{\pi x}{3} + \frac{\pi}{6}$



101 **(c)**

$x = \tan^{-1} 3 \Rightarrow \tan x = 3$

$\tan(x + y) = 33$

$\Rightarrow \frac{\tan x + \tan y}{1 - \tan x \tan y} = 33$

$\Rightarrow \frac{3 + \tan y}{1 - 3\tan y} = 33$

$\Rightarrow 3 + \tan y = 33 - 99\tan y$

$\Rightarrow 100\tan y = 30$

$\Rightarrow \tan y = 0.3 \ \Rightarrow y = \tan^{-1}(0.3)$

102 **(a,c,d)**

The given equation holds, if

$x^2 + x + 1 = \lambda x + 1$

And $-1 \le x^2 + x + 1 \le 1$

$\Rightarrow x(x + 1 - \lambda) = 0$ and $-1 \le x \le 0$

$\Rightarrow x = 0$ or $\lambda - 1$ and $-1 \le x \le 0$

$\therefore x = 0$ is one solution and for another different solution

$-1 \le \lambda - 1 < 0$.

$\Rightarrow 0 \le \lambda < 1$, so only integral value λ can have is 0.

103 **(a,b,c)**

The solution of $y = \sqrt{y}$ is $y = 0$ or $y = 1$

if $\sin^{-1}|\sin x| = 1$

$\Rightarrow x = 1$ or $\pi - 1$ [in the interval $(0, \pi)$]

But $y = \sin^{-1}|\sin x|$ is periodic with period π, so

$x = n\pi + 1$ or $n\pi - 1$

Again, if $\sin^{-1}|\sin x| = 0$

$\Rightarrow x = n\pi$

104 **(a,b)**

We know that

if $|x| \le 1$, then $2\tan^{-1} x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

if $x > 1$, $2\tan^{-1} x = \pi - \sin^{-1}\frac{2x}{1+x^2}$

if $x < -1, 2\tan^{-1} x = -\pi - \sin^{-1} \frac{2x}{1+x^2}$

Hence, the required values are $x < -1$ or $x > 1$

105 **(a,d)**
We have,
$\tan^{-1} x + \cot^{-1} x + \sin^1 x = \frac{\pi}{2} + \sin^{-1} x$ ... (i)
Since, $-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$
$\Rightarrow 0 \leq \frac{\pi}{2} + \sin^{-1} x \leq \pi$
$\Rightarrow 0 \leq \tan^{-1} x + \cot^{-1} x + \sin^{-1} x \leq \pi$ [from Eq.(i)]
$\therefore a = 0$ and $b = \pi$

106 **(a, b)**
a. $\cos(\tan^{-1}(\tan(4-\pi))) = \cos(4-\pi) = \cos(\pi - 4) = -\cos 4 > 0$

b. $\sin(\cot^{-1}(\cot(4-\pi))) = \sin(4-\pi) = -\sin 4 > 0$ (as $\sin 4 < 0$)

c. $\tan(\cos^{-1}(\cos(2\pi - 5))) = \tan(2\pi - 5) = -\tan 5 > 0$ (as $\tan 5 < 0$)

d. $\cot(\sin^{-1}(\sin(\pi - 4))) = \cot(\pi - 4) = -\cot 4 < 0$

107 **(a,c)**
Given equation is $x^2 + 2x\sin(\cos^{-1} y) + 1 = 0$.
Since x is real, $D \geq 0$

$\therefore 4(\sin(\cos^{-1} y))^2 - 4 \geq 0$

$\Rightarrow (\sin(\cos^{-1} y))^2 \geq 1$

$\Rightarrow \sin(\cos^{-1} y) = \pm 1$

$\Rightarrow \cos^{-1} y = \pm\frac{\pi}{2} \Rightarrow y = 0$

Putting value of y in the original equation, we have $x^2 + 2x + 1 = 0 \Rightarrow x = -1$

Hence, the equation has only one solution

108 **(a,d)**
For the given equation $0 \leq x, y \leq 1$

Also, $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$

$\Rightarrow \sin^{-1} x = \cos^{-1} y = \sin^{-1}\sqrt{1-y^2}$

$\Rightarrow x = \sqrt{1-y^2} \Rightarrow x^2 + y^2 = 1$ (i)

Again, $\sin 2x = \cos 2y$

$\Rightarrow \cos\left(\frac{\pi}{2} - 2x\right) = \cos 2y$

$\Rightarrow \frac{\pi}{2} - 2x = 2n\pi \pm 2y$, where $n \in I$

$\Rightarrow x \pm y = \frac{\pi}{4} - n\pi$ (ii)

From Eqs. (i) and (ii), we get

$x = \frac{\pi}{8} + \sqrt{\frac{1}{2} - \frac{\pi^2}{64}}$ and $y = \sqrt{\frac{1}{2} - \frac{\pi^2}{64}} - \frac{\pi}{8}$

109 **(a,b,d)**
$2\cot^{-1} 7 = 2\tan^{-1}\left(\frac{1}{7}\right)$
$= \cos^{-1}\left(\frac{1-\frac{1}{49}}{1+\frac{1}{49}}\right) = \cos^{-1}\frac{24}{25}$
Now ,$2\cot^{-1} 7 + \cos^{-1}\frac{3}{5}$
$= \cos^{-1}\frac{24}{25} + \cos^{-1}\frac{3}{5}$
$= \cos^{-1}\left(\frac{24}{25}\cdot\frac{3}{5} - \frac{7}{25}\cdot\frac{4}{5}\right) = \cos^{-1}\frac{44}{125}$
Since, $\frac{44}{125} > 0$
$\therefore 0 < \cos^{-1}\frac{44}{125} < \frac{\pi}{2}$
Let $\cos^{-1}\frac{44}{125} = \theta$, then $\cos\theta = \frac{44}{125}$
$\therefore \operatorname{cosec}\theta = \frac{125}{117}$ or $\theta = \operatorname{cosec}^{-1}\frac{125}{117}$
Also, $\cot\theta = \frac{44}{117}$
$\therefore \theta = \cot^{-1}\frac{44}{117}$

110 **(a,c,d)**
The given equation holds, if
$x^2 + x + 1 = \lambda x + 1$
And $-1 \leq x^2 + x + 1 \leq 1$
$\Rightarrow x(x + 1 - \lambda) = 0$ and $-1 \leq x \leq 0$
$\Rightarrow x = 0$ or $\lambda - 1$ and $-1 \leq x \leq 0$
$\therefore x = 0$ is one solution and for another different solution
$-1 \leq \lambda - 1 < 0$.
$\Rightarrow 0 \leq \lambda < 1$, so only integral value λ can have is 0.

111 **(b,c,d)**
$6x^2 + 11x + 3 = 0$

$\Rightarrow (2x+3)(3x+1) = 0$

$\Rightarrow x = -3/2, -1/3$

For $x = -3/2$, $\cos^{-1} x$ is not defined as domain of $\cos^{-1} x$ is $[-1, 1]$ and for $x = -1/3$, $\operatorname{cosec}^{-1} x$ is not defined as domain of $\operatorname{cosec}^{-1}x$ is $R-(-1,1)$. However, $\cot^{-1}x$ is defined for both of these values as domain of $\cot^{-1}x$ is R

112 **(a,d)**

We have,

$\tan^{-1}x+\cot^{-1}x+\sin^{1}x=\frac{\pi}{2}+\sin^{-1}x$ ... (i)

Since, $-\frac{\pi}{2}\le\sin^{-1}x\le\frac{\pi}{2}$

$\Rightarrow 0\le\frac{\pi}{2}+\sin^{-1}x\le\pi$

$\Rightarrow 0\le\tan^{-1}x+\cot^{-1}x+\sin^{-1}x\le\pi$ [from Eq.(i)]

$\therefore a=0$ and $b=\pi$

113 **(a,d)**

Let $f(x)=(\sin^{-1}x)^2+(\cos^{-1}x)^2$

$=(\sin^{-1}x+\cos^{-1}x)^2-2\sin^{-1}x\cos^{-1}x$

$=\frac{\pi^2}{4}-2\sin^{-1}x\left[\frac{\pi}{2}-\sin^{-1}x\right]$

$=\frac{\pi^2}{4}-\pi\sin^{-1}x+2(\sin^{-1}x)^2$

$=2\left[(\sin^{-1}x)^2-\frac{\pi}{2}\sin^{-1}x+\frac{\pi^2}{8}\right]$

$=2\left(\sin^{-1}x-\frac{\pi}{4}\right)^2+2\left[\frac{\pi^2}{16}\right]$

Now, $-\frac{\pi}{2}\le\sin^{-1}x\le\frac{\pi}{2}$

$\Rightarrow -\frac{3\pi}{4}\le\sin^{-1}x-\frac{\pi}{4}\le\frac{\pi}{4}$

$\Rightarrow 0\le\left(\sin^{-1}x-\frac{\pi}{4}\right)^2\le\frac{9\pi^2}{16}$

$\Rightarrow 0\le 2\left(\sin^{-1}x-\frac{\pi}{4}\right)^2\le\frac{9\pi^2}{8}$

$\Rightarrow\frac{\pi^2}{8}\le\left(\sin^{-1}x-\frac{\pi}{4}\right)^2+\frac{\pi^2}{8}\le\frac{5\pi^2}{4}$

114 **(b)**

$f(x)=\sin^{-1}|\sin kx|+\cos^{-1}(\cos kx)$

Let $g(x)=\sin^{-1}|\sin x|+\cos^{-1}(\cos x)$

$$g(x)\begin{cases}2x, & 0\le x\le\frac{\pi}{2}\\ \pi, & \frac{\pi}{2}<x<\frac{3\pi}{2}\\ 4\pi-2x, & \frac{3\pi}{2}<x\le 2\pi\end{cases}$$

$g(x)$ is periodic with period 2π and is constant in the continuous interval

$\left[2n\pi+\frac{\pi}{2},2n\pi+\frac{3\pi}{2}\right]$

(where $n\in I$) and $f(x)=g(kx)$

So, $f(x)$ is constant in the interval $\left[\frac{2n\pi}{k}+\frac{\pi}{2k},\frac{2n\pi}{k}+\right.$ $3\pi 2k$

$\Rightarrow\frac{\pi}{4}=\frac{3\pi}{2k}-\frac{\pi}{2k}\Rightarrow\frac{\pi}{k}=\frac{\pi}{4}\Rightarrow k=4$

115 **(c,d)**

$xy<0\Rightarrow x+\frac{1}{x}\ge 2, y+\frac{1}{y}\le -2$

or $x+\frac{1}{x}\le -2, y+\frac{1}{y}\ge 2$

$x+\frac{1}{x}\ge 2\Rightarrow\sec^{-1}\left(x+\frac{1}{x}\right)\in\left[\frac{\pi}{3},\frac{\pi}{2}\right)$

$y+\frac{1}{y}\le -2\Rightarrow\sec^{-1}\left(y+\frac{1}{y}\right)\in\left(\frac{\pi}{2},\frac{2\pi}{3}\right]\Rightarrow z\in\left(\frac{5\pi}{6},\frac{7\pi}{6}\right)$

116 **(a,c)**

The given relation is possible when $a-\frac{a^2}{3}+\frac{a^3}{9}+\cdots=1+b+b^2+\cdots$

Also, $-1\le a-\frac{a^2}{3}+\frac{a^3}{9}+\cdots\le 1$ and $-1\le 1+b+b^2+\cdots\le 1$

$\Rightarrow |b|<1\Rightarrow |a|<3$ and $\frac{a}{1+\frac{a}{3}}=\frac{1}{1-b}$

$\Rightarrow\frac{3a}{a+3}=\frac{1}{1-b}$, there are infinitely many solutions

$\Rightarrow 3a-3ab=a+3\Rightarrow 2a-3ab=3$

$\Rightarrow b=\frac{2a-3}{3a}$ and $a=\frac{3}{2-3b}$

117 **(a,b,c)**

(a) $\sin\left(\tan^{-1}3+\tan^{-1}\frac{1}{3}\right)=\sin\frac{\pi}{2}=1$

(b) $\cos\left(\frac{\pi}{2}-\sin^{-1}\frac{3}{4}\right)=\cos\left(\cos^{-1}\frac{3}{4}\right)=\frac{3}{4}$

(c) $\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$

Let $\sin^{-1}\frac{\sqrt{63}}{8} = \theta$

$$\Rightarrow \sin\theta = \frac{\sqrt{63}}{8} \Rightarrow \cos\theta = \frac{1}{8}$$

We have $\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}} = \frac{3}{4}$

$$\Rightarrow \sin\frac{\theta}{4} = \sqrt{\frac{1-\cos\frac{\theta}{2}}{2}} = \frac{1}{2\sqrt{2}}$$

Now, $\log_2 \sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right) = \log_2\frac{1}{2\sqrt{2}} = -\frac{3}{2}$

(d) $\cos^{-1}\frac{\sqrt{5}}{3} = \theta \Rightarrow \cos\theta = \frac{\sqrt{5}}{3}$

$\therefore \tan\frac{\theta}{2} = \frac{3-\sqrt{5}}{2}$ which is irrational

118 **(a,b,c)**

a. $\cos(\tan^{-1}(\tan(4-\pi))) = \cos(4-\pi) = \cos(\pi-4) = -\cos 4 > 0$

b. $\sin(\cot^{-1}(\cot(4-\pi))) = \sin(4-\pi) = -\sin 4 > 0$ (as $\sin 4 < 0$)

c. $\tan(\cos^{-1}(\cos(2\pi-5))) = \tan(2\pi-5) = -\tan 5 > 0$ (as $\tan 5 < 0$)

d. $\cot(\sin^{-1}(\sin(\pi-4))) = \cot(\pi-4) = -\cot 4 < 0$

119 **(a,b,d)**

$$\tan^{-1}(x-1) + \tan^{-1}(x) + \tan^{-1}(x+1) = \tan^{-1}3x$$

$$\Rightarrow \tan^{-1}(x-1) + \tan^{-1}(x) = \tan^{-1}3x - \tan^{-1}(x+1)$$

$$\Rightarrow \tan^{-1}\left[\frac{(x-1)+x}{1-(x-1)(x)}\right] = \tan^{-1}\left[\frac{3x-(x+1)}{1+3x(x+1)}\right]$$

$$\Rightarrow \frac{2x-1}{1-x^2+x} = \frac{2x-1}{1+3x^2+3x}$$

$$\Rightarrow (1-x^2+x)(2x-1) = (1+3x^2+3x)(2x-1)$$

$$\Rightarrow x = 0, \pm\frac{1}{2}$$

120 **(a,c)**

We have

$$\cot^{-1}\left(\frac{n^2-10n+21.6}{\pi}\right) > \frac{\pi}{6}$$

$$\Rightarrow \frac{n^2-10n+21.6}{\pi} < \cot\frac{\pi}{6} \quad \text{(as } \cot x \text{ is decreasing for } 0 < x < \pi)$$

$$\Rightarrow n^2-10n+21.6 < \pi\sqrt{3}$$

$$\Rightarrow n^2-10n+25+21.6-25 < \pi\sqrt{3}$$

$$\Rightarrow (n-5)^2 < \pi\sqrt{3}+3.4$$

$$\Rightarrow -\sqrt{\sqrt{3}\pi+3.4} < n-5 < \sqrt{\sqrt{3}\pi+3.4}$$

$$\Rightarrow 5-\sqrt{\sqrt{3}\pi+3.4} < n < 5+\sqrt{\sqrt{3}\pi+3.4} \qquad \text{(i)}$$

Since $\sqrt{3}\pi = 5.5$ nearly, $\sqrt{\sqrt{3}\pi+3.4} \sim \sqrt{8.9} \sim 2.9$

$$\Rightarrow 2.1 < n < 7.9$$

$\therefore n = 3, 4, 5, 6, 7$ {as $n \in N$}

121 **(a,b,c)**

If we put $x = \tan\theta$, then given equality becomes $\tan^{-1}y = 4\theta$

$$\Rightarrow y = \tan 4\theta = \frac{2\tan 2\theta}{1-\tan^2 2\theta}$$

$$= \frac{2\left[\frac{2\tan\theta}{1-\tan^2\theta}\right]}{1-\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)^2}$$

$$= \frac{2\times 2x(1-x^2)}{(1-x^2)^2-4x^2} = \frac{4x(1-x^2)}{1-6x^2+x^4}$$

So that y is infinite, if $x^4-6x^2+1=0$

$$\Rightarrow x^2 = \frac{6\pm\sqrt{36-4}}{2} = 3\pm 2\sqrt{2}$$

122 **(b,c,d)**

$$\cos\left(-\frac{14\pi}{5}\right) = \cos\frac{14\pi}{5} = \cos\frac{4\pi}{5}$$

Hence , $\cos\frac{1}{2}\cos^{-1}\left(\cos\frac{4\pi}{5}\right) = \cos\frac{2\pi}{5}$

123 **(a,b,c)**

The solution of $y = \sqrt{y}$ is $y = 0$ or $y = 1$

if $\sin^{-1}|\sin x| = 1$

$\Rightarrow x = 1$ or $\pi - 1$ [in the interval $(0,\pi)$]

But $y = \sin^{-1}|\sin x|$ is periodic with period π, so $x = n\pi + 1$ or $n\pi - 1$

Again, if $\sin^{-1}|\sin x| = 0$

$\Rightarrow x = n\pi$

124 **(a,b)**

$\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$

$$\Rightarrow \sin^{-1} + \sin^{-1} y + \sin^{-1} z = \pi/2$$

$\Rightarrow \cos^{-1} x + \cos^{-1} y = \cos^{-1}(-z)$

$$\Rightarrow xy - \sqrt{1-x^2}\sqrt{1-y^2} = -z$$
$$\Rightarrow x^2 + y^2 + z^2 + 2xyz = 1$$

125 **(a,c)**

Domain of $f(x) = \log_e \cos^{-1} x$ is $x \in [-1, 1)$

$\therefore [\alpha] = -1$ or 0

126 **(b)**

We know that $\sin^{-1} x$ is defined for $x \in [-1, 1]$ and $\sec^{-1} x$ is defined for $x \in (-\infty, -1] \cup [1, \infty)$

Hence, the given function is defined for $x \in \{-1, 1\}$

Therefore, $f(1) = \pi/2, f(-1) = \pi/2$

127 **(a,b,c,d)**

Since $|\tan^{-1} x| = \begin{cases} \tan^{-1} x, & \text{if } x \geq 0 \\ -\tan^{-1} x, & \text{if } x < 0 \end{cases}$

$\Rightarrow |\tan^{-1} x| = \tan^{-1}|x| \ \forall x \in R$

$\Rightarrow \tan|\tan^{-1} x| = \tan \tan^{-1}|x| = |x|$

Also $|\cot^{-1} x| = \cot^{-1} x\,; \forall\, x \in R$

$\Rightarrow \cot|\cot^{-1} x| = x, \forall\, x \in R$

$\tan^{-1}|\tan x| = \begin{cases} x, & \text{if} \tan x > 0 \\ -x, & \text{if} \tan x < 0 \end{cases}$

$\sin|\sin^{-1} x| = \begin{cases} x, & x \in [0, 1] \\ -x, & x \in [-1, 0) \end{cases}$

128 **(a,d)**

Case 1: If $0 \leq x \leq \frac{1}{2}$, then

$$\cos^{-1}\left(\frac{x}{2} + \frac{1}{2}\sqrt{3-3x^2}\right)$$
$$= \cos^{-1}\left(x\frac{1}{2} + \sqrt{1-x^2}\frac{\sqrt{3}}{2}\right)$$
$$= \cos^{-1} x - \cos^{-1}\frac{1}{2}$$

Case 2: If $\frac{1}{2} \leq x \leq 1$, then

$$\cos^{-1}\left(\frac{x}{2} + \frac{1}{2}\sqrt{3-3x^2}\right) = \cos^{-1}\frac{1}{2} - \cos^{-1} x$$

129 **(b,c)**

$1 \leq \dfrac{\pi}{\cos^{-1} x} < \infty \Rightarrow 2 \leq 2^{\frac{\pi}{\cos^{-1} x}} < \infty$

Hence, 2 should lie between or on the roots of $t^2 - \left(a + \frac{1}{2}\right)t - a^2 = 0$ where $t = 2^{\pi/\cos^{-1} y}$

$\Rightarrow f(2) \leq 0 \Rightarrow a^2 + 2a - 3 \geq 0 \Rightarrow a \in (-\infty, -3] \cup [1, \infty)$

130 **(a,b,d)**

$2\cot^{-1} 7 = 2\tan^{-1}\left(\frac{1}{7}\right)$

$= \cos^{-1}\left(\dfrac{1-\frac{1}{49}}{1+\frac{1}{49}}\right) = \cos^{-1}\dfrac{24}{25}$

Now ,$2\cot^{-1} 7 + \cos^{-1}\frac{3}{5}$

$= \cos^{-1}\dfrac{24}{25} + \cos^{-1}\dfrac{3}{5}$

$= \cos^{-1}\left(\dfrac{24}{25}\cdot\dfrac{3}{5} - \dfrac{7}{25}\cdot\dfrac{4}{5}\right) = \cos^{-1}\dfrac{44}{125}$

Since, $\frac{44}{125} > 0$

$\therefore 0 < \cos^{-1}\dfrac{44}{125} < \dfrac{\pi}{2}$

Let $\cos^{-1}\frac{44}{125} = \theta$, then $\cos\theta = \frac{44}{125}$

$\therefore \operatorname{cosec}\theta = \dfrac{125}{117}$ or $\theta = \operatorname{cosec}^{-1}\dfrac{125}{117}$

Also, $\cot\theta = \frac{44}{117}$

$\therefore \theta = \cot^{-1}\dfrac{44}{117}$

131 **(a,b,d)**

Let t_r denote the rth term of the series $3, 7, 13, 21, \ldots$ and

$$S = 3 + 7 + 13 + 21 + \cdots + t_n$$

$$\begin{array}{l} -S = \quad 3 + 7 + 13 + \cdots + t_{n-1} + t_n \\ \hline 0 = 3 + 4 + 6 + 8 + \cdots + 2n - t_n \end{array}$$

$$\Rightarrow t_n = 3 + 4 + 6 + \cdots + 2n = 1 + 2 \times \frac{1}{2}n(n+1)$$
$$= n^2 + n + 1$$

Let $T_r = \cot^{-1}(r^2 + r + 1) = \tan^{-1}\left(\frac{1}{r^2+r+1}\right) = \tan^{-1}\left(\frac{r+1-r}{1+r(r+1)}\right) = \tan^{-1}(r+1) - \tan^{-1} r$

Thus, the sum of the first n terms of the given series is

$$\sum_{r=1}^{n} [\tan^{-1}(r+1) - \tan^{-1} r]$$
$$= \tan^{-1}(n+1) - \tan^{-1}(1)$$
$$= \tan^{-1}\left[\frac{n+1-n}{1+1(n+1)}\right] = \tan^{-1}\left(\frac{n}{n+2}\right)$$
$$= \tan^{-1}\left(\frac{1}{1+\frac{2}{n}}\right)$$

$$\Rightarrow S_\infty = \lim_{n\to\infty} \tan^{-1}\left(\frac{1}{1+\frac{2}{n}}\right) = \frac{\pi}{4}, S_{10} = \tan^{-1}\frac{10}{12}$$
$$= \tan^{-1}\frac{5}{6}$$

$$S_6 = \tan^{-1}\frac{3}{4} = \sin^{-1}\frac{3}{5}$$

$$S_{20} = \tan^{-1}\frac{10}{11} = \cot^{-1} 1.1$$

132 **(a,b,c)**

If we put $x = \tan\theta$, then given equality becomes $\tan^{-1} y = 4\theta$

$$\Rightarrow y = \tan 4\theta = \frac{2\tan 2\theta}{1 - \tan^2 2\theta}$$
$$= \frac{2\left[\frac{2\tan\theta}{1-\tan^2\theta}\right]}{1 - \left(\frac{2\tan\theta}{1-\tan^2\theta}\right)^2}$$
$$= \frac{2 \times 2x(1-x^2)}{(1-x^2)^2 - 4x^2} = \frac{4x(1-x^2)}{1 - 6x^2 + x^4}$$

So that y is infinite, if $x^4 - 6x^2 + 1 = 0$

$$\Rightarrow x^2 = \frac{6 \pm \sqrt{36-4}}{2} = 3 \pm 2\sqrt{2}$$

133 **(a,c,d)**

$$(\sin^{-1} x + \sin^{-1} w)(\sin^{-1} y + \sin^{-1} z) = \pi^2$$

$$\Rightarrow \sin^{-1} x + \sin^{-1} w = \sin^{-1} y + \sin^{-1} z = \pi$$

or $\sin^{-1} x + \sin^{-1} w = \sin^{-1} y + \sin^{-1} z = -\pi$

$$\Rightarrow x = y = z = w = 1 \text{ or } x = y = z = w = -1$$

Hence, the maximum value of $\begin{vmatrix} x^{N_1} & y^{N_2} \\ z^{N_3} & w^{N_4} \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2$ and minimum value $\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -2$

Also, there are 16 different determinants as each place value is either 1 or -1

134 **(a,b,c)**

Let $\tan^{-1}(-2) = \theta \;\; \Rightarrow \tan\theta = -2 \;\Rightarrow \theta = (-\pi/2, 0)$

$$\Rightarrow 2\theta = (-\pi, 0)$$

$$\cos(-2\theta) = \cos 2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta} = -\frac{3}{5}$$

$$\Rightarrow -2\theta = \cos^{-1}\left(\frac{-3}{5}\right) = \pi - \cos^{-1}\frac{3}{5}$$

$$\Rightarrow 2\theta = -\pi + \cos^{-1}\frac{3}{5} = -\pi + \tan^{-1}\frac{4}{3}$$
$$= -\pi + \cot^{-1}\frac{3}{4}$$
$$= -\pi + \frac{\pi}{2} - \tan^{-1}\frac{3}{4} = -\frac{\pi}{2} - \tan^{-1}\frac{3}{4}$$

$$= -\frac{\pi}{2} + \tan^{-1}\left(-\frac{3}{4}\right)$$

135 **(d)**

$$\tan\left[\cos^{-1}\left(\frac{1}{\sqrt{82}}\right)-\sin^{-1}\left(\frac{5}{\sqrt{26}}\right)\right]$$

$$=\tan(\tan^{-1}9-\tan^{-1}5)$$

$$=\tan\tan^{-1}\left(\frac{9-5}{1+9\times 5}\right)=\frac{2}{23}$$

In statement II, put $\cot^{-1}x=y\Rightarrow x=\cot y$

LHS=$(\cot y\cos y+\sin y)^2$

$$=\frac{(\cos^2 y+\sin^2 y)^2}{\sin^2 y}$$

$$=1+\cot^2 y=1+x^2$$

$$=1+\frac{1}{50}=\frac{51}{50}\left(\because x=\frac{1}{5\sqrt{2}}\right)$$

136 **(a)**

Statement II is true.

Given, $\sin^{-1}x-\cos^{-1}x=\frac{\pi}{6}$ ... (i)

And from statement II

$\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$ ... (ii)

Adding Eqs. (i) and (ii), we get

$$2\sin^{-1}x=\frac{2\pi}{3}$$

$$\Rightarrow \sin^{-1}x=\frac{\pi}{3}$$

$$\Rightarrow x=\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$$

∴ Given equation has unique solution.

⇒ Statement I is true.

137 **(d)**

$$\sin^{-1}\left(\frac{2}{3}\right)+\cos^{-1}\left(\frac{2}{3}\right)-\tan^{-1}7-\cot^{-1}7-\cot^{-1}\left(\frac{1}{7}\right)$$

$$=\frac{\pi}{2}-\frac{\pi}{2}-\cot^{-1}\left(\frac{1}{7}\right)=-\tan^{-1}7$$

138 **(a)**

$\because$ AM ≥ GM

$$\therefore \frac{\cos^{-1}x+(\sin^{-1}y)^2}{2}\geq\sqrt{(\cos^{-1}x)(\sin^{-1}y)^2}$$

$$\Rightarrow \frac{p\pi^2}{8}\geq\frac{\pi^2}{4}$$

$$\Rightarrow P\geq 2$$

Thus , we conclude that the only value of p that satisfies all conditions is $p=2$.

Then ,$\cos^{-1}x=(\sin^{-1}y)^2$

$$\Rightarrow (\cos^{-1}x)^2=\frac{\pi^4}{16}$$

$$\Rightarrow \cos^{-1}x=\pm\frac{\pi^2}{4}$$

$$\Rightarrow x=\cos\left(\pm\frac{\pi^2}{4}\right)$$

$$\therefore x=\cos\left(\frac{\pi^2}{4}\right)$$

Also , $(\sin^{-1}y)^4=\frac{\pi^4}{16}$

$$\Rightarrow \sin^{-1}y=\pm\frac{\pi}{2}$$

$$\therefore y=\sin\left(\pm\frac{\pi}{2}\right)=\pm 1$$

139 **(a)**

$\because$ AM ≥ GM

$$\therefore \frac{\cos^{-1}x+(\sin^{-1}y)^2}{2}\geq\sqrt{(\cos^{-1}x)(\sin^{-1}y)^2}$$

$$\Rightarrow \frac{p\pi^2}{8}\geq\frac{\pi^2}{4}$$

$$\Rightarrow P\geq 2$$

Thus , we conclude that the only value of p that satisfies all conditions is $p=2$.

Then ,$\cos^{-1}x=(\sin^{-1}y)^2$

$$\Rightarrow (\cos^{-1}x)^2=\frac{\pi^4}{16}$$

$$\Rightarrow \cos^{-1}x=\pm\frac{\pi^2}{4}$$

$$\Rightarrow x = \cos\left(\pm\frac{\pi^2}{4}\right)$$

$$\therefore x = \cos\left(\frac{\pi^2}{4}\right)$$

Also , $(\sin^{-1} y)^4 = \frac{\pi^4}{16}$

$$\Rightarrow \sin^{-1} y = \pm\frac{\pi}{2}$$

$$\therefore y = \sin\left(\pm\frac{\pi}{2}\right) = \pm 1$$

141 **(a)**

$$\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\cdot\frac{1}{7}}\right) = \frac{\pi}{4}$$

$$= \tan^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y-x}{x+y}\right) = \frac{\pi}{4}$$

Both statement I and statement II are true and statement II is correct explanation of statement I.

142 **(d)**

Obviously, statement 2 is correct, but when $x \in [-1, 1]$ we have $\tan^{-1} x \in [-\pi/4, \pi/4]$.

It implies that $\frac{\pi}{2} + \tan^{-1} x \in [\pi/4, 3\pi/4]$

Hence, statement 2 is true but statement 1 is false

143 **(a)**

$$\sin^{-1} x = \tan^{-1}\frac{x}{\sqrt{1-x^2}} > \tan^{-1} x$$

$$> \tan^{-1} y \quad \left[\because x > y, \frac{x}{\sqrt{1-x^2}} > x\right]$$

Therefore, statement 2 is true

Now, $e < \pi \quad \Rightarrow \frac{1}{\sqrt{e}} > \frac{1}{\sqrt{\pi}}$

By statement 2, we have

$$\sin^{-1}\left(\frac{1}{\sqrt{e}}\right) > \tan^{-1}\left(\frac{1}{\sqrt{e}}\right) > \tan^{-1}\left(\frac{1}{\sqrt{\pi}}\right)$$

Therefore, statement 1 is true

144 **(c)**

$\text{cosec}^{-1} x > \sec^{-1} x$

$$\Rightarrow \text{cosec}^{-1} x > \frac{\pi}{2} - \text{cosec}^{-1} x$$

$$\Rightarrow \text{cosec}^{-1} x > \frac{\pi}{4}$$

$$\Rightarrow 1 \le x < \sqrt{2} \text{ and } \left(\frac{1}{2} + \frac{1}{\sqrt{2}}\right) \in [1, \sqrt{2})$$

But statement 2 is false

145 **(d)**

If $x < 0, \tan^{-1}\left(\frac{1}{x}\right) = -\pi + \cot^{-1} x$

$$\tan^{-1} x + \tan^{-1}\frac{1}{x} = \tan^{-1} x - \pi + \cot^{-1} x$$

$$= -\pi + \frac{\pi}{2} = -\frac{\pi}{2}$$

Statement I is false but statement II is true

146 **(d)**

If $x < 0, \tan^{-1}\left(\frac{1}{x}\right) = -\pi + \cot^{-1} x$

$$\tan^{-1} x + \tan^{-1}\frac{1}{x} = \tan^{-1} x - \pi + \cot^{-1} x$$

$$= -\pi + \frac{\pi}{2} = -\frac{\pi}{2}$$

Statement I is false but statement II is true

147 **(a)**

Statement 2 is correct, from which we can say $\cot^{-1} x + \cos^{-1} 2x = -\pi$ is not possible. Hence, both the statements are correct and statement 2 is the correct explanation of statement 1

148 **(a)**

$$\because \sin^{-1} 2x + \sin^{-1} 3x = \frac{\pi}{3}$$

$$\Rightarrow \frac{\pi}{2} - \cos^{-1} 2x + \frac{\pi}{2} - \cos^{-1} 3x = \frac{\pi}{3}$$

$$\Rightarrow \cos^{-1} 2x + \cos^{-1} 3x = \frac{2\pi}{3}$$

$$\Rightarrow \cos^{-1}\{6x^2 - \sqrt{1-(2x)^2}\sqrt{1-(3x)^2}\} = \frac{2\pi}{3}$$

$$\Rightarrow 6x^2 - \sqrt{(1 - 13x^3 + 36x^4)} = -\frac{1}{2}$$

$$\Rightarrow \left(6x^2 + \frac{1}{2}\right)^2 = 1 - 13x^2 + 36x^4$$

$$\Rightarrow 19x^2 = \frac{3}{4}$$

$\Rightarrow x = \pm\sqrt{\frac{3}{76}}$

But sum of two negative number cannot be $\frac{\pi}{3}$.

$\therefore x = \sqrt{\frac{3}{76}}$ is the only solution

149 **(d)**

$$\sin^{-1}\left(\frac{2}{3}\right) + \cos^{-1}\left(\frac{2}{3}\right) - \tan^{-1} 7 - \cot^{-1} 7 - \cot^{-1}\left(\frac{1}{7}\right)$$

$$= \frac{\pi}{2} - \frac{\pi}{2} - \cot^{-1}\left(\frac{1}{7}\right) = -\tan^{-1} 7$$

150 **(a)**

For $x > 0, y > 0$,

$\tan^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y-x}{y+x}\right)$ (i)

$$= \tan^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{1-\frac{x}{y}}{1+\frac{x}{y}}\right)$$
$$= \tan^{-1}\left(\frac{x}{y}\right) + \tan^{-1} 1 - \tan^{-1}\frac{x}{y}$$
$$= \frac{\pi}{4}$$

Now, in Eq. (i), putting $\frac{x}{y} = \frac{3}{4}$, we get

$$\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \frac{\pi}{4}$$

Hence, both the statements are correct and statement 2 is the correct explanation of statement 1

151 **(d)**

$30 - 9\pi \in [0, \pi]$ is true but it is not principal value of $\cos^{-1}(\cos 30)$ as $\cos^{-1}(\cos 30) = \cos^{-1}\big(\cos\big(9\pi + (30 - 9\pi)\big)\big) = \cos^{-1}(-\cos(30 -$ *$9\pi=\pi-30-9\pi=10\pi-30$.*

Hence, statement 2 is true but statement 1 is false

152 **(a)**

Statement II is true.

Given, $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$ ... (i)

And from statement II

$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ ... (ii)

Adding Eqs. (i) and (ii), we get

$$2\sin^{-1} x = \frac{2\pi}{3}$$

$$\Rightarrow \sin^{-1} x = \frac{\pi}{3}$$

$$\Rightarrow x = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$\therefore$ Given equation has unique solution.

$\Rightarrow$ Statement I is true.

153 **(d)**

Since ,$p, q > 0$ therefore $pq > 0$

And $\tan^{-1}\left(\frac{p-q}{1+pq}\right) = \tan^{-1} p - \tan^{-1} q$ ... (i)

Since, $qr > -1$

$\therefore \tan^{-1}\left(\frac{q-r}{1+qr}\right) = \tan^{-1} q - \tan^{-1} r$ ... (ii)

And since $pr < -1$ and $r<0$

$\therefore \tan^{-1}\left(\frac{r-p}{1+rp}\right) = \pi + \tan^{-1} r - \tan^{-1} p$ ... (iii)

On adding Eqs. (i), (ii) and (iii), we get

$$\tan^{-1}\left(\frac{p-q}{1+pq}\right) + \tan^{-1}\left(\frac{q-r}{1+qr}\right) + \tan^{-1}\left(\frac{r-p}{1+rp}\right) = \pi$$

154 **(d)**

Since ,$p, q > 0$ therefore $pq > 0$

And $\tan^{-1}\left(\frac{p-q}{1+pq}\right) = \tan^{-1} p - \tan^{-1} q$ ... (i)

Since, $qr > -1$

$\therefore \tan^{-1}\left(\frac{q-r}{1+qr}\right) = \tan^{-1} q - \tan^{-1} r$ ... (ii)

And since $pr < -1$ and $r<0$

$\therefore \tan^{-1}\left(\frac{r-p}{1+rp}\right) = \pi + \tan^{-1} r - \tan^{-1} p$ ... (iii)

On adding Eqs. (i), (ii) and (iii), we get

$$\tan^{-1}\left(\frac{p-q}{1+pq}\right)+\tan^{-1}\left(\frac{q-r}{1+qr}\right)+\tan^{-1}\left(\frac{r-p}{1+rp}\right)=\pi$$

155 **(a)**

$$f(x)=\sin^{-1}\left(\frac{2x}{1+x^2}\right)=\pi-2\tan^{-1}x\,, x\geq 1$$

$$\Rightarrow f'(x)=-\frac{2}{1+x^2}\Rightarrow f'(2)=-\frac{2}{5}$$

Thus statement 1 is true, statement 2 is true and statement 2 is the correct explanation of statement 1

156 **(d)**

$$\tan\left[\cos^{-1}\left(\frac{1}{\sqrt{82}}\right)-\sin^{-1}\left(\frac{5}{\sqrt{26}}\right)\right]$$

$$=\tan(\tan^{-1}9-\tan^{-1}5)$$

$$=\tan\tan^{-1}\left(\frac{9-5}{1+9\times 5}\right)=\frac{2}{23}$$

In statement II, put $\cot^{-1}x=y\Rightarrow x=\cot y$

LHS=$(\cot y\cos y+\sin y)^2$

$$=\frac{(\cos^2 y+\sin^2 y)^2}{\sin^2 y}$$

$$=1+\cot^2 y=1+x^2$$

$$=1+\frac{1}{50}=\frac{51}{50}\ \left(\because x=\frac{1}{5\sqrt{2}}\right)$$

157 **(a)**

$$\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{1}{7}\right)=\tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\cdot\frac{1}{7}}\right)=\frac{\pi}{4}$$

$$=\tan^{-1}\left(\frac{x}{y}\right)+\tan^{-1}\left(\frac{y-x}{x+y}\right)=\frac{\pi}{4}$$

Both statement I and statement II are true and statement II is correct explanation of statement I.

158 **(b)**

We know that $\tan^{-1}x$ and $\cot^{-1}x$ have domain R, also $\tan x$ and $\cot x$ are unbounded functions. On the other hand, $\sec x$ is an unbounded function, but its range is $R-(-1,1)$, and not R

159 **(a)**

$$\because \sin^{-1}2x+\sin^{-1}3x=\frac{\pi}{3}$$

$$\Rightarrow \frac{\pi}{2}-\cos^{-1}2x+\frac{\pi}{2}-\cos^{-1}3x=\frac{\pi}{3}$$

$$\Rightarrow \cos^{-1}2x+\cos^{-1}3x=\frac{2\pi}{3}$$

$$\Rightarrow \cos^{-1}\{6x^2-\sqrt{1-(2x)^2}\sqrt{1-(3x)^2}\}=\frac{2\pi}{3}$$

$$\Rightarrow 6x^2-\sqrt{(1-13x^3+36x^4)}=-\frac{1}{2}$$

$$\Rightarrow \left(6x^2+\frac{1}{2}\right)^2=1-13x^2+36x^4$$

$$\Rightarrow 19x^2=\frac{3}{4}$$

$$\Rightarrow x=\pm\sqrt{\frac{3}{76}}$$

But sum of two negative number cannot be $\frac{\pi}{3}$.

$\therefore x=\sqrt{\frac{3}{76}}$ is the only solution

160 **(b)**

a. $(\sin^{-1}x)^2+(\sin^{-1}y)^2=\frac{\pi^2}{2}$

$$\Rightarrow (\sin^{-1}x)^2=(\sin^{-1}y)^2=\frac{\pi^2}{4}$$

$$\Rightarrow \sin^{-1}x=\pm\frac{\pi}{2},\sin^{-1}y=\pm\frac{\pi}{2}$$

$\Rightarrow x=\pm 1$ and $y=\pm 1$

$$\Rightarrow x^3+y^3=-2,0,2$$

b. $(\cos^{-1}x)^2+(\cos^{-1}y)^2=2\pi^2$

$$\Rightarrow (\cos^{-1}x)^2=(\cos^{-1}y)^2=\pi$$

$$\Rightarrow x=y=-1$$

$$\Rightarrow x^5+y^5=-2$$

c. $(\sin^{-1}x)^2(\cos^{-1}y)^2=\frac{\pi^4}{4}$

$\Rightarrow (\sin^{-1}x)^2=\frac{\pi^2}{4}$ and $(\cos^{-1}y)^2=\pi^2$

$\Rightarrow (\sin^{-1}x)=\pm\frac{\pi}{2}$ and $(\cos^{-1}y)=\pi$

$\Rightarrow x=\pm 1$ and $y=-1$

$$\Rightarrow -|x-y|=0,2$$

d. $|\sin^{-1}x - \sin^{-1}y| = \pi$

$\Rightarrow \sin^{-1}x = -\frac{\pi}{2}$ and $\sin^{-1}y = \frac{\pi}{2}$

or $\sin^{-1}x = \frac{\pi}{2}$ and $\sin^{-1}y = -\frac{\pi}{2}$

$\Rightarrow x^y = 1^{-(1)}$ or $(-1)^1 = 1$ or -1

161 **(d)**

a. $f(x) = \sin^{-1}x + \cos^{-1}x + \cot^{-1}x$

$= \dfrac{\pi}{2} + \cot^{-1}x, x \in [-1, 1]$

For $x \in [-1,1], \cot^{-1}x \in \left[\frac{\pi}{4}, \frac{3\pi}{4}\right] \Rightarrow \frac{\pi}{2} + \cot^{-1}x \in \left[\frac{3\pi}{4}, \frac{5\pi}{4}\right]$

b. $f(x) = \cot^{-1}x + \tan^{-1}x + \operatorname{cosec}^{-1}x$

$= \frac{\pi}{2} + \operatorname{cosec}^{-1}x$, where $x \in (-\infty, -1] \cup [1, \infty)$

Now $\operatorname{cosec}^{-1}x \in \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right] \Rightarrow \frac{\pi}{2} + \operatorname{cosec}^{-1}x \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

c. $f(x) = \cot^{-1}x + \tan^{-1}x + \cos^{-1}x$

$= \frac{\pi}{2} + \cos^{-1}x$, where $x \in [-1, 1] \Rightarrow \frac{\pi}{2} + \cos^{-1}x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

d. $\sec^{-1}x + \operatorname{cosec}^{-1}x + \sin^{-1}x$, where $x \in \{-1, 1\}$

$= \frac{\pi}{2} + \sin^{-1}x$, where $x \in \{-1, 1\}$

Hence, $f(x) \in \{0, \pi\}$

162 **(a)**

a. $\sin^{-1}\frac{4}{5} = \tan^{-1}\frac{4}{3}$

$2\tan^{-1}\dfrac{1}{3} = \tan^{-1}\dfrac{3}{4} = \cot^{-1}\dfrac{4}{3}$

and $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

b. $\tan^{-1}\frac{12}{5} + \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{63}{16} = \pi + \tan^{-1}\frac{48+15}{20-36} + \tan^{-1}\frac{63}{16}$

$= \pi - \tan^{-1}\dfrac{63}{16} + \tan^{-1}\dfrac{63}{16} = \pi$

c. $A = \tan^{-1}\frac{x\sqrt{3}}{2\lambda - x}$ and $B = \tan^{-1}\left(\frac{2x-\lambda}{\lambda\sqrt{3}}\right)$

Now, $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$$= \frac{\frac{x\sqrt{3}}{2\lambda - x} - \frac{2x-\lambda}{\lambda\sqrt{3}}}{1 + \frac{x\sqrt{3}}{2\lambda - x}\frac{2x-\lambda}{\lambda\sqrt{3}}}$$

$$= \frac{3x\lambda + (2x - \lambda)(x - 2\lambda)}{\sqrt{3}[\lambda(2\lambda - x) + x(2x - \lambda)]}$$

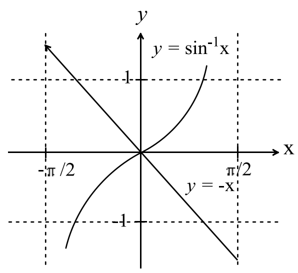
$$= \frac{1}{\sqrt{3}}\left[\frac{2x^2 - 2\lambda x + 2\lambda^2}{2x^2 - 2\lambda x + 2\lambda^2}\right] = \frac{1}{\sqrt{3}} = \tan 30°$$

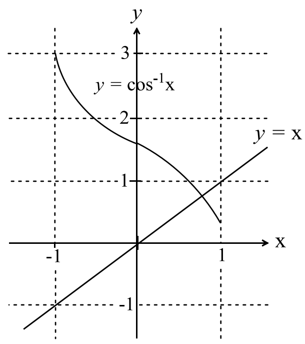
$\therefore A - B = 30°$

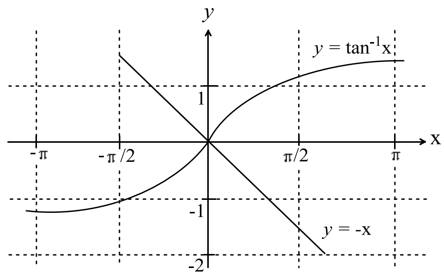
d. $\tan^{-1}\frac{1}{7} + 2\tan^{-1}\frac{1}{3} = \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{2\frac{1}{3}}{1 - \frac{1}{9}} = \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{3}{4}$

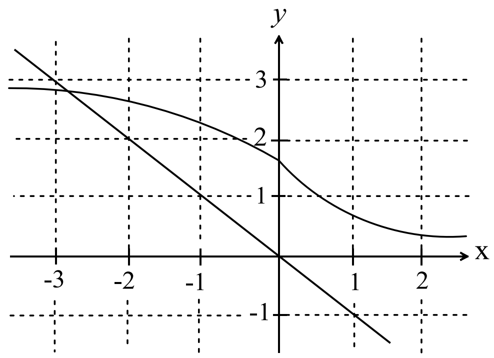
$$= \tan^{-1}\frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{1}{7}\frac{3}{4}} = \tan^{-1}1 = \pi/4$$

163 **(b)**









Refer graph for solution

164 **(c)**

$y = \sin^{-1}(\sin x),\ y = \cos^{-1}(\cos x), y = \tan^{-1}(\tan x)$ and $y = \cot^{-1}(\cot x)$

165 **(a)**

a. $\cos^{-1}(4x^3 - 3x) = 3\cos^{-1}x$

$0 \le \cos^{-1}(4x^3 - 3x) \le \pi$

$\Rightarrow 0 \le 3\cos^{-1}x \le \pi \Rightarrow 0 \le \cos^{-1}x \le (\pi/3)$

$\Rightarrow (1/2) \le x \le 1$

b. $\sin^{-1}(3x - 4x^3) = 3\sin^{-1}x$

$(-\pi/2) \le \sin^{-1}(3x - 4x^3) \le (\pi/2)$

$\Rightarrow (-\pi/2) \le 3\sin^{-1}x \le (\pi/2)$

$\Rightarrow (-\pi/6) \le \sin^{-1}x \le (\pi/6)$

$\Rightarrow (-1/2) \le x \le (1/2)$

c. $\cos^{-1}(4x^3-3x)=3\sin^{-1}x$
$0\le\cos^{-1}(3x-4x^3)\le\pi$
$\Rightarrow 0\le 3\sin^{-1}x\le\pi$
$\Rightarrow 0\le\sin^{-1}x\le\pi/3$
$\Rightarrow 0\le x\le(\sqrt{3}/2)$
d. $\sin^{-1}(3x-4x^3)=3\cos^{-1}x$
$(-\pi/2)\le\sin^{-1}(3x-4x^3)\le(\pi/2)$
$\Rightarrow(-\pi/2)\le 3\cos^{-1}x\le(\pi/2)$
$\Rightarrow(-\pi/6)\le\cos^{-1}x\le(\pi/6)$
$\Rightarrow 0\le\cos^{-1}x\le(\pi/6)$
$\Rightarrow 0\le x\le\left(\sqrt{3}/2\right)$

166 **(a)**

$$\lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{2^{r-1}}{1+2^{2r-1}}\right)$$
$$=\lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{2^r-2^{r-1}}{1+2^r\cdot 2^{r-1}}\right)$$
$$=\lim_{n\to\infty}\sum_{r=1}^{n}\{\tan^{-1}(2^r)-\tan^{-1}(2^{r-1})\}$$
$$=\lim_{n\to\infty}(\tan^{-1}2^n-\tan^{-1}2^0)$$
$$=\tan^{-1}2^\infty-\tan^{-1}1$$
$$=\tan^{-1}\infty-\tan^{-1}1=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}$$

167 **(a)**

Since, $f(x)=0\Rightarrow\sin\{\cot^{-1}(x+1)\}=$ cos (tan−1x)
$$\Rightarrow\sin\sin^{-1}\frac{1}{\sqrt{1+(x+1)^2}}=\cos\cos^{-1}\frac{1}{\sqrt{1+x^2}}$$
$$\Rightarrow\frac{1}{\sqrt{1+(x+1)^2}}=\frac{1}{\sqrt{1+x^2}}$$
$$\Rightarrow 1+x^2=2+x^2+2x$$
$$\Rightarrow x=-\frac{1}{2}$$

168 **(a)**

$$\lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{2^{r-1}}{1+2^{2r-1}}\right)$$
$$=\lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{2^r-2^{r-1}}{1+2^r\cdot 2^{r-1}}\right)$$
$$=\lim_{n\to\infty}\sum_{r=1}^{n}\{\tan^{-1}(2^r)-\tan^{-1}(2^{r-1})\}$$
$$=\lim_{n\to\infty}(\tan^{-1}2^n-\tan^{-1}2^0)$$
$$=\tan^{-1}2^\infty-\tan^{-1}1$$
$$=\tan^{-1}\infty-\tan^{-1}1=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}$$

169 **(a)**

Since, $f(x)=0\Rightarrow\sin\{\cot^{-1}(x+1)\}=$ cos (tan−1x)
$$\Rightarrow\sin\sin^{-1}\frac{1}{\sqrt{1+(x+1)^2}}=\cos\cos^{-1}\frac{1}{\sqrt{1+x^2}}$$
$$\Rightarrow\frac{1}{\sqrt{1+(x+1)^2}}=\frac{1}{\sqrt{1+x^2}}$$
$$\Rightarrow 1+x^2=2+x^2+2x$$
$$\Rightarrow x=-\frac{1}{2}$$

170 **(d)**

$\sin^{-1}x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$\cos^{-1}y\in[0,\pi]$

$\sec^{-1}z\in\left[0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right]$
$$\Rightarrow\sin^{-1}x+\cos^{-1}y+\sec^{-1}z\le\frac{\pi}{2}+\pi+\pi=\frac{5\pi}{2}$$

Also, $t^2-\sqrt{2\pi}\,t+3\pi$

$$=t^2-2\sqrt{\frac{\pi}{2}}\,t+\frac{\pi}{2}-\frac{\pi}{2}+3\pi=\left(t-\sqrt{\frac{\pi}{2}}\right)^2+\frac{5\pi}{2}\ge\frac{5\pi}{2}$$

The given inequation exists if equality holds, i.e.,

$$\text{L.H.S.}=\text{R.H.S.}=\frac{5\pi}{2}$$

$\Rightarrow x=1,y=-1,z=-1$ and $t=\sqrt{\frac{\pi}{2}}\Rightarrow$
$\cos^{-1}(\cos 5t^2)=\cos^{-1}\left(\cos\left(\frac{5\pi}{2}\right)\right)=\frac{\pi}{2}$

$\cos^{-1}(\min\{x,y,z\})=\cos^{-1}(-1)=\pi$

171 **(b)**

Given $ax + b(\sec(\tan^{-1} x)) = c$ and $ay + b(\sec(\tan^{-1} y)) = c$

Let $\tan^{-1} x = \alpha$ and $\tan^{-1} y = \beta$, then the given relations are

$a \tan\alpha + b \sec\alpha = c$ and $a \tan\beta + b \sec\beta = c$

From these two relations, we can conclude that equation $a \tan\theta + b \sec\theta = c$ has roots α and β

$a \tan\theta + b \sec\theta = c$

$\Rightarrow b \sec\theta = c - a \tan\theta$

$\Rightarrow b^2 \sec^2\theta = c^2 - 2\,ac \tan\theta + a^2 \tan^2\theta$

$\Rightarrow b^2 + b^2 \tan^2\theta = c^2 - 2\,ac \tan\theta + a^2 \tan^2\theta$

$\Rightarrow (a^2 - b^2)\tan^2\theta - 2\,ac \tan\theta + c^2 - b^2 = 0$

Therefore, sum of the roots, $\tan\alpha + \tan\beta = x + y = \frac{2\,ac}{a^2 - b^2}$

and the product of roots, $\tan\alpha \tan\beta = xy = \frac{c^2 - b^2}{a^2 - b^2}$

and $\frac{x+y}{1-xy} = \frac{\frac{2ac}{a^2-b^2}}{1-\frac{c^2-b^2}{a^2-b^2}} = \frac{2ac}{a^2-c^2}$

172 **(b)**

Let $\cos^{-1} x = a \Rightarrow a \in [0, \pi]$

and $\sin^{-1} y = b \Rightarrow b \in [-\pi/2, \pi/2]$

We have $a + b^2 = \frac{p\pi^2}{4}$ (i)

and $ab^2 = \frac{\pi^4}{16}$ (ii)

Since $b^2 \in [0, \pi^2/4]$, we get $a + b^2 \in [0, \pi + \pi^2/4]$

So, from Eq.(i) we get $0 \le \frac{p\pi^2}{4} \le \pi + \frac{\pi^2}{4}$

i.e., $0 \le p \le \frac{4}{\pi} + 1$

Since $p \in Z$, so $p = 0, 1$ or 2

Substituting the value of b^2 from Eq. (i) in Eq. (ii), we get

$$a\left(\frac{p\pi^2}{4} - a\right) = \frac{\pi^2}{16} \Rightarrow 16a^2 - 4p\,\pi^2 a + \pi^4 = 0 \quad \text{(iii)}$$

Since $a \in R \Rightarrow D \ge 0$

i.e., $16p^2\pi^4 - 64\pi^4 \ge 0 \Rightarrow p^2 \ge 4 \Rightarrow p \ge 2 \Rightarrow p = 2$

Substituting $p = 2$ in Eq. (iii), we get

$16a^2 - 8\pi^2\,a + \pi^4 = 0$

$$\Rightarrow (4a - \pi^2)^2 = 0 \Rightarrow a = \frac{\pi^2}{4} = \cos^{-1} x \Rightarrow x = \cos\frac{\pi^2}{4}$$

From Eq.(ii), we get $\frac{\pi^2}{4}\,b^2 = \frac{\pi^4}{16} \Rightarrow b = \pm\frac{\pi}{2} = \sin^{-1} y \Rightarrow y = \pm 1$

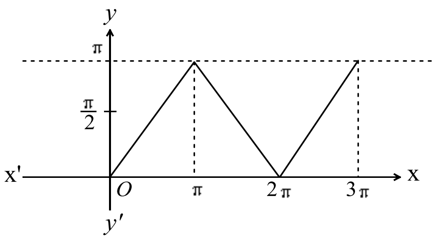
173 **(c)**

Let $\cos^{-1} x = \theta \Rightarrow x = \cos\theta$, where $\theta \in [0, \pi]$

$$\cos^{-1}(4x^3 - 3x) = \cos^{-1}(4\cos^3\theta - 3\cos\theta) = \cos^{-1}(\cos 3\theta) = \cos^{-1}(\cos\alpha)$$

where $\alpha = 3\theta \in [0, 3\pi]$

Refer the graph of $y = \cos^{-1}(\cos\alpha)$, $\alpha \in [0, 3\pi]$



From the graph,

$\cos^{-1}(4x^3 - 3x) = \cos^{-1}(\cos\alpha)$

$$= \begin{cases} \alpha, & 0 \le \alpha < \pi \\ 2\pi - \alpha, & \pi \le \alpha \le 2\pi \\ \alpha - 2\pi, & 2\pi < \alpha \le 3\pi \end{cases}$$

$$= \begin{cases} 3\cos^{-1}x, & 0 \le 3\cos^{-1}x < \pi \\ 2\pi - 3\cos^{-1}x, & \pi \le 3\cos^{-1}x \le 2\pi \\ 3\cos^{-1}x - 2\pi, & 2\pi < 3\cos^{-1}x \le 3\pi \end{cases}$$

$$= \begin{cases} 3\cos^{-1}x, & 0 \le \cos^{-1}x < (\pi/3) \\ 2\pi - 3\cos^{-1}x, & (\pi/3) \le \cos^{-1}x \le (2\pi/3) \\ 3\cos^{-1}x - 2\pi, & (2\pi/3) < 3\cos^{-1}x \le \pi \end{cases}$$

$$= \begin{cases} 3\cos^{-1}x, & (1/2) < x \le 1 \\ 2\pi - 3\cos^{-1}x, & (-1/2) \le x \le (1/2) \\ 3\cos^{-1}x - 2\pi, & -1 \le x < -(1/2) \end{cases}$$

$$= \begin{cases} 3\cos^{-1}x - 2\pi, & -1 \le x < (-1/2) \\ 2\pi - 3\cos^{-1}x, & (-1/2) \le x \le (1/2) \\ 3\cos^{-1}x, & (1/2) < x \le 1 \end{cases}$$

For $x \in \left[-1, -\frac{1}{2}\right)$, $\cos^{-1}(4x^3 - 3x) = 3\cos^{-1}x - 2\pi$

$\Rightarrow a = -2\pi$ and $b = 3 \Rightarrow a + b\pi = \pi$

For $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$, $\cos^{-1}(4x^3 - 3x) = 2\pi - 3\cos^{-1}x$

$\Rightarrow a = 2\pi$ and $b = -3 \Rightarrow \sin^{-1}\left(\sin\frac{a}{b}\right) = \sin^{-1}\left(\sin\frac{2\pi}{-3}\right)$

$= \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

For $x \in \left(\frac{1}{2}, 1\right]$, $\cos^{-1}(4x^3 - 3x) = 3\cos^{-1}x \Rightarrow$ $a = 0$ and $b = 3$

$\therefore \lim_{y\to a} b\cos(y) = \lim_{y\to 0} 3\cos(y) = 3$

174 **(2)**

Since $\sin^{-1}$ is defined for $[-1, 1]$

$\therefore a = 0$

$\therefore x = \sin^{-1}1 + \cos^{-1}1 - \tan^{-1}1 = \frac{\pi}{4}$

$\Rightarrow \sec^2 x = 2$

175 **(3)**

$$\sin^{-1}\left(x^2 - \frac{x^4}{3} + \frac{x^6}{9} - \cdots\right) + \cos^{-1}\left(x^4 - \frac{x^8}{3} + \frac{x^{12}}{9}\cdots\right) = \frac{\pi}{2}$$

$$\Rightarrow \left(x^2 - \frac{x^4}{3} + \frac{x^6}{9}\dots\dots\right) = \left(x^4 - \frac{x^8}{3} + \frac{x^{12}}{9}\dots\dots\right)$$

$$\Rightarrow \frac{x^2}{1 + \frac{x^2}{3}} = \frac{x^4}{1 + \frac{x^4}{3}}$$

$$\Rightarrow \frac{3}{3 + x^2} = \frac{3x^2}{3 + x^4} \text{ or } x = 0$$

$\Rightarrow 9 + 3x^4 = 9x^2 + 3x^4$ or $x = 0$

$\Rightarrow x^2 = 1 \Rightarrow x = 0, 1$ or -1

Therefore, the number of values is equal to 3

176 **(9)**

$$\tan^{-1}\left(x + \frac{3}{x}\right) - \tan^{-1}\left(x - \frac{3}{x}\right) = \tan^{-1}\frac{6}{x}$$

$$\Rightarrow \tan^{-1}\left(\frac{\left(x + \frac{3}{x}\right) - \left(x - \frac{3}{x}\right)}{1 + \left(x + \frac{3}{x}\right)\left(x - \frac{3}{x}\right)}\right) = \tan^{-1}\frac{6}{x}$$

$$\Rightarrow x^2 - \frac{9}{x^2} = 0 \Rightarrow x^4 = 9$$

177 **(3)**

$\cos^{-1}(x) + \cos^{-1}(2x) + \cos^{-1}(3x) = \pi$

$\Rightarrow \cos^{-1}(2x) + \cos^{-1}(3x) = \pi - \cos^{-1}(x) = \cos^{-1}(-x)$

$\Rightarrow \cos^{-1}\left[(2x)(3x) - \sqrt{1 - 4x^2}\sqrt{1 - 9x^2}\right] = \cos^{-1}(-x)$

$\Rightarrow 6x^2 - \sqrt{1 - 4x^2}\sqrt{1 - 9x^2} = -x$

$\Rightarrow (6x^2 + x)^2 = (1 - 4x^2)(1 - 9x^2)$

$\Rightarrow x^2 + 12x^3 = 1 - 13x^2$

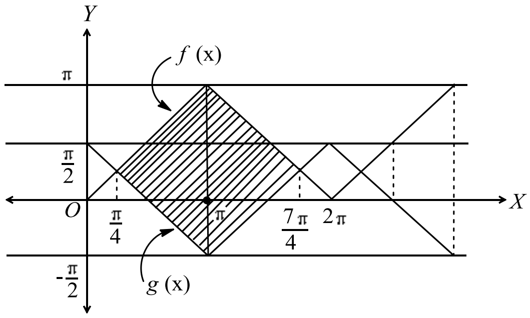
$\Rightarrow 12x^3 + 14x^2 - 1 = 0$

$\Rightarrow a = 12; b = 14; c = 0$

$\Rightarrow b - a - c = 14 - 12 + 1 = 3$

178 **(1)**

We have $g(x) = \sin^{-1}(\cos x) = \frac{\pi}{2} - \cos^{-1}(\cos x)$



Both the curves bound the regions of same area in $\left[\frac{\pi}{4}, \frac{7\pi}{4}\right], \left[\frac{9\pi}{4}, \frac{15\pi}{4}\right]$ and so on

Therefore, the required are = area of shaded square $= \frac{9\pi^2}{8} = \frac{a\pi^2}{b}$

$\therefore a = 9$ and $b = 8 \Rightarrow a - b = 1$

179 **(9)**

$1 + \sin(\cos^{-1} x) + \sin^2(\cos^{-1} x) + \cdots \infty = 2$

$$\Rightarrow \frac{1}{1 - \sin(\cos^{-1} x)} = 2$$

$$\Rightarrow \frac{1}{2} = 1 - \sin(\cos^{-1} x)$$

$$\Rightarrow \sin(\cos^{-1} x) = \frac{1}{2} \Rightarrow \cos^{-1} x = \frac{\pi}{6}$$

$$\Rightarrow x = \frac{\sqrt{3}}{2} \Rightarrow 12x^2 = 9$$

180 **(1)**

$\tan^{-1}(3x) + \tan^{-1}(5x) = \tan^{-1}(7x) + \tan^{-1}(2x)$

$$\Rightarrow \tan^{-1}(3x) - \tan^{-1}(2x) = \tan^{-1}(7x) - \tan^{-1}(5x)$$

$$\Rightarrow \tan^{-1}\left(\frac{3x - 2x}{1 + 6x^2}\right) = \tan^{-1}\left(\frac{7x - 5x}{1 + 35x^2}\right)$$

$$\Rightarrow \frac{x}{1 + 6x^2} = \frac{2x}{1 + 35x^2}$$

$\Rightarrow x = 0$ or $1 + 35x^2 = 2 + 12x^2$

$$\Rightarrow x = 0 \text{ or } x = \frac{1}{\sqrt{23}} \text{ or } -\frac{1}{\sqrt{23}}$$

181 **(5)**

$$(\cot^{-1} x)(\tan^{-1} x) + \left(2 - \frac{\pi}{2}\right)\cot^{-1} x - 3\tan^{-1} x - 3\left(2 - \frac{\pi}{2}\right) > 0$$

$\Rightarrow \cot^{-1} x > 0$

$\Rightarrow (\cot^{-1} x - 3)(2 - \cot^{-1} x) > 0$

$\Rightarrow (\cot^{-1} x - 3)(\cot^{-1} x - 2) < 0$

$\Rightarrow 2 < \cot^{-1} x < 3$

$\Rightarrow \cot 3 < x < \cot 2$ [as $\cot^{-1} x$ is a decreasing function]

$\Rightarrow$ Hence, $x \in (\cot 3, \cot 2)$

$$\Rightarrow \cot^{-1} a + \cot^{-1} b = \cot^{-1}(\cot 3) + \cot^{-1}(\cot 2) = 5$$

182 **(6)**

Let $\tan^{-1} u = \alpha \Rightarrow \tan \alpha = u$

$\tan^{-1} v = \beta \Rightarrow \tan \beta = v$

$\tan^{-1} w = \gamma \Rightarrow \tan \gamma = w$

$$\tan(\alpha + \beta + \gamma) = \frac{s_1 - s_3}{1 - s_2} = \frac{0 - (-11)}{1 - (-10)} = \frac{11}{11} = 1$$

$$\therefore \alpha + \beta + \gamma = \tan^{-1}(1) = \frac{\pi}{4}$$

$\Rightarrow 3\,\text{cosec}^2(\tan^{-1} u + \tan^{-1} v + \tan^{-1} w) = 6$

183 **(4)**

$f(x) = \sin^{-1} x + 2\tan^{-1} x + (x + 2)^2 - 3$

Domain of $f(x)$ is $[-1, 1]$

Also $f(x)$ is an increasing function in the domain

$$\therefore p = f_{\text{min.}}(x) = f(-1) = -\frac{\pi}{2} + 2\left(\frac{-\pi}{4}\right) + 1 - 3 = -\pi - 2$$

and $q = f_{\text{max.}}(x) = f(1) = \frac{\pi}{2} + 2\left(\frac{\pi}{4}\right) + 9 - 6 = \pi + 6$

Therefore, the range of $f(x)$ is $[-\pi - 2, \pi + 6]$

Hence, $(p + q) = 4$

184 **(7)**

$f(x) = \sqrt{3\cos^{-1}(4x) - \pi}$ is defined

If $\cos^{-1} 4x \geq \frac{\pi}{3} \Rightarrow 4x \leq \frac{1}{2} \Rightarrow x \leq \frac{1}{8}$ (i)

Also, $-1 \leq 4x \leq 1 \Rightarrow \frac{-1}{4} \leq x \leq \frac{1}{4}$ (ii)

Therefore, from Eqs. (i) and (ii), we have domain: $x \in \left[\frac{-1}{4}, \frac{1}{8}\right]$

$\Rightarrow 4a + 64b = 7$

185 **(3)**

We must have $x(x+3) \geq 0$

$\Rightarrow x \geq 0$ or $x \leq -3$ (i)

Also, $-1 \leq x^2 + 3x + 1 \leq 1$

$\Rightarrow x(x+3) \leq 0 \Rightarrow -3 \leq x \leq 0$ (ii)

From Eqs. (i) and (ii), we get $x = \{0, -3\}$

Hence, required sum is 3

186 **(6)**

$$T_n = \tan^{-1}\left(\frac{n+1-1}{1+(n+1)1}\right)$$

$$= \tan^{-1}(n+1) - \tan^{-1}(n)$$

Hence, $S_n = \tan^{-1}(n+1) - \tan^{-1} 1$

$$= \tan^{-1}\left(\frac{n+1-1}{1+(n+1)\cdot 1}\right) = \left(\tan^{-1}\frac{n}{n+2}\right) = \frac{1}{2}\cos^{-1}\left(\frac{24}{145}\right)$$

$$\Rightarrow 2\left(\tan^{-1}\frac{n}{n+2}\right) = \cos^{-1}\left(\frac{24}{145}\right)$$

$$\Rightarrow \cos^{-1}\left(\frac{2(n+1)}{n^2+2n+2}\right) = \cos^{-1}\left(\frac{24}{145}\right)$$

$$\Rightarrow \left(\frac{2(n+1)}{n^2+2n+2}\right) = \left(\frac{24}{145}\right)$$

$$\Rightarrow 12(n+1)^2 - 145(n+1) + 12 = 0$$

$$\Rightarrow \big((n+1) - 12\big)(12(n+1) - 1) = 0$$

$$\Rightarrow n + 1 = 12 \Rightarrow n = 11$$

187 **(1)**

Given expression is defined only for $x = 1$ and -1

$\therefore f(1) = 1$ and $f(-1) = (1+\pi)(1+\pi) = (1+\pi)^2$

Hence, the least value is 1