7.INTEGRALS

Single Correct Answer Type

1. $\int_0^{\infty} \frac{x dx}{(1+x)(1+x^2)}$ is equal to
   a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) π d) None of these
2. $\int x \sin x \sec^3 x \, dx$ is equal to
   a) $\frac{1}{2}[\sec^2 x - \tan x] + c$ b) $\frac{1}{2}[x \sec^2 x - \tan x] + c$
   c) $\frac{1}{2}[x \sec^2 x + \tan x] + c$ d) $\frac{1}{2}[\sec^2 x + \tan x] + c$
3. The value of $\int_{-2}^{0}[x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x+1)]\, dx$ is
   a) 0 b) 3 c) 4 d) 1
4. $\int \sqrt{e^x - 1} \, dx$ is equal to
   a) $2\left[\sqrt{e^x-1} - \tan^{-1}\sqrt{e^x-1}\right] + c$ b) $\sqrt{e^x-1} - \tan^{-1}\sqrt{e^x-1} + c$
   c) $\sqrt{e^x-1} + \tan^{-1}\sqrt{e^x-1} + c$ d) $2\left[\sqrt{e^x-1} + \tan^{-1}\sqrt{e^x-1}\right] + c$
5. $\int e^x \frac{(x^2+1)}{(x+1)^2} \, dx$ is equal to
   a) $\left(\frac{x-1}{x+1}\right)e^x + c$ b) $e^x\left(\frac{x+1}{x-1}\right) + c$ c) $e^x(x+1)(x-1) + c$ d) None of these
6. $\int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos x}$ is equal to
   a) 2 b) -2 c) $1/2$ d) $-1/2$
7. The value of the definite integral $\int_2^4 (x(3-x)(4+x)(6-x)(10-x) + \sin x)\, dx$ equals
   a) $\cos 2 + \cos 4$ b) $\cos 2 - \cos 4$ c) $\sin 2 + \sin 4$ d) $\sin 2 - \sin 4$
8. If $I = \int e^{-x} \log(e^x + 1) dx$, then I equals
   a) $x + (e^{-x} + 1)\log(e^x + 1) + C$ b) $x + (e^x + 1)\log(e^x + 1) + C$
   c) $x - (e^{-x} + 1)\log(e^x + 1) + C$ d) None of these
9. If $f(x)$ is monotonic differentiable function on $[a, b]$, then $\int_a^b f(x)dx + \int_{f(a)}^{f(b)} f^{-1}x(dx) =$
   a) $bf(a) - af(b)$ b) $bf(b) - af(a)$ c) $f(a) + f(b)$ d) Cannot be found
10. If $I_{m,n} = \int \cos^m x \sin nx \, dx$, then $7\, I_{4,3} - 4\, I_{3,2}$ is equal to
    a) constant b) $-\cos^2 x + C$ c) $-\cos^4 x \cos 3x + C$ d) $\cos 7x - \cos 4x + C$
11. If $I = \int_{-20\pi}^{20\pi} |\sin x|\, [\sin x] dx$ (where $[.]$ denotes the greatest integer function), then the value of I is
    a) -40 b) 40 c) 20 d) -20
12. If $I = \int\left(\sqrt{\cot x} - \sqrt{\tan x}\right)dx$,then I equals
    a) $\sqrt{2}\log\left(\sqrt{\tan x} - \sqrt{\cot x}\right) + C$ b) $\sqrt{2}\log\left|\sin x + \cos x + \sqrt{\sin 2x}\right| + C$
    c) $\sqrt{2}\log\left|\sin x - \cos x + \sqrt{2}\sin x \cos x\right| + C$ d) $\sqrt{2}\log\left|\sin(x + \pi/4) + \sqrt{2}\sin x \cos x\right| + C$
13. The value of the definite integral $\int_0^{\pi/2} \sqrt{\tan x} \, dx$ is
    a) $\sqrt{2}\pi$ b) $\frac{\pi}{\sqrt{2}}$ c) $2\sqrt{2}\pi$ d) $\frac{\pi}{2\sqrt{2}}$
14. $g(x) = \underbrace{(fofo....of)}_{n \text{ times}}(x).$
    Let $f(x) = \frac{x}{(1+x^n)^{1/n}}$ for $n \geq 2$ and $g(x) = (fofo ... of)(x)$. Then, $\int x^{n-2}\, g(x)dx$ equals
    a) $\frac{1}{n(n-1)}(1 + nx^n)^{1-\frac{1}{n}} + c$
    b) $\frac{1}{n-1}(1 + nx^n)^{1-\frac{1}{n}} + c$

c) $\frac{1}{n(n+1)}(1+nx^n)^{1+\frac{1}{n}}+c$

d) $\frac{1}{n+1}(1+nx^n)^{1+\frac{1}{n}}+c$

15. If $\lambda = \int_0^1 \frac{e^t}{1+t}$, then $\int_0^1 e^t \log_e(1+t)dt$ is equal to

a) 2λ
b) $e \log_e 2 - \lambda$
c) λ
d) $e \log_e 2 + \lambda$

16. $\int \frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}}\, dx, \alpha \neq n\pi, n \in Z$ is equal to

a) $-2 \operatorname{cosec} \alpha(\cos\alpha - \tan x \sin\alpha)^{1/2} + C$
b) $-2\,(\cos\alpha + \cot\, x \sin\alpha)^{1/2} + C$
c) $-2\operatorname{cosec} \alpha(\cos\alpha + \cot x \sin\alpha)^{1/2} + C$
d) $-2\operatorname{cosec} \alpha(\sin\alpha + \cot x \cos\alpha)^{1/2} + C$

17. $\int \frac{\sin 2x}{\sin 5x \sin 3x}\, dx$ is equal to

a) $\log \sin 3x - \log \sin 5x + c$
b) $\frac{1}{3}\log \sin 3x + \frac{1}{5}\log \sin 5x + c$
c) $\frac{1}{3}\log \sin 3x - \frac{1}{5}\log \sin 5x + c$
d) $3 \log \sin 3x - 5 \log \sin 5x + c$

18. $\int \frac{\sin 2x}{\sin^4 x + \cos^4 x}\, dx$ is equal to

a) $\cot^{-1}(\tan^2 x) + c$
b) $\tan^{-1}(\tan^2 x) + c$
c) $\cot^{-1}(\cot^2 x) + c$
d) $\tan^{-1}(\cot^2 x) + c$

19. $\int \frac{3+2\cos x}{(2+3\cos x)^2}\, dx$ is equal to

a) $\left(\frac{\sin x}{3\cos x + 2}\right) + c$
b) $\left(\frac{2\cos x}{3\sin x + 2}\right) + c$
c) $\left(\frac{2\cos x}{3\cos x + 2}\right) + c$
d) $\left(\frac{2\sin x}{3\sin x + 2}\right) + c$

20. Let f be a non-negative function defined on the interval [0,1]. If $\int_0^x \sqrt{1-\left(f'(t)\right)^2}\, dt = \int_0^x f(t)\, dt, 0 \leq x \leq 1$ and $f(0) = 0$, then

a) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$
b) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$
c) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$
d) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

21. If $\int_0^1 \cot^{-1}(1 - x + x^2)\, dx = \lambda \int_0^1 \tan^{-1} x dx$, then $\lambda$is equal to

a) 1
b) 2
c) 3
d) 4

22. If α, β ($\beta > \alpha$) are the roots of $g(x) = ax^2 + bx + c = 0$ and $f(x)$ is an even function, then

$\int_\alpha^\beta \frac{e^{f\left(\frac{g(x)}{x-\alpha}\right)}dx}{e^{f\left(\frac{g(x)}{x-\alpha}\right)} + e^{f\left(\frac{g(x)}{x-\beta}\right)}}$ is equal to

a) $\left|\frac{b}{2a}\right|$
b) $\frac{\sqrt{b^2-4ac}}{|2a|}$
c) $\left|\frac{b}{a}\right|$
d) None of these

23. $\int_{\sin\theta}^{\cos\theta} f(x \tan\theta)dx$ $\left(\text{where } \theta \neq \frac{n\pi}{2}, n \in I\right)$ is equal to

a) $-\cos\theta \int_1^{\tan\theta} f(x \sin\theta)dx$
b) $-\tan\theta \int_{\cos\theta}^{\sin\theta} f(x)dx$
c) $\sin\theta \int_1^{\tan\theta} f(x\cos\theta)dx$
d) $\frac{1}{\tan\theta}\int_{\sin\theta}^{\sin\theta\tan\theta} f(x)dx$

24. The range of the function $f(x) = \int_{-1}^{1} \frac{\sin x\, dt}{(1 - 2t\cos x + t^2)}$is

a) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
b) $[0, \pi]$
c) $\{0, \pi\}$
d) $\left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\}$

25. If $x = \int_c^{\sin t} \sin^{-1} z dz, y = \int_k^{\sqrt{t}} \frac{\sin z^2}{z} dz$, then $\frac{dy}{dx}$ is equal to

a) $\frac{\tan t}{2t}$
b) $\frac{\tan t}{t^2}$
c) $\frac{\tan t}{2t^2}$
d) $\frac{\tan t^2}{2t^2}$

26. $f(x)$ is a continuous function for all real values of x and satisfies $\int_n^{n+1} f(x)dx = \frac{n^2}{2} \forall\, n \in I$, then $\int_{-3}^{5} f(|x|)\, dx$ is equal to

a) 19/2 b) 35/2 c) 17/2 d) None of these

27. If $I = \int \sqrt{\frac{5-x}{2+x}} dx$, then I equals

a) $\sqrt{x+2}\sqrt{5-x} + 3\sin^{-1}\sqrt{\frac{x+2}{3}} + C$
b) $\sqrt{x+2}\sqrt{5-x} + 7\sin^{-1}\sqrt{\frac{x+2}{7}} + C$
c) $\sqrt{x+2}\sqrt{5-x} + 5\sin^{-1}\sqrt{\frac{x+2}{5}} + C$
d) None of these

28. Given $I_m = \int_1^e (\log x)^m\, dx$. If $\frac{I_m}{K} + \frac{I_{m-2}}{L} = e$, then the values of K and L are

a) $\frac{1}{1-m}, \frac{1}{m}$ b) $(1-m), \frac{1}{m}$ c) $\frac{1}{1-m}, \frac{m(m-2)}{m-1}$ d) $\frac{m}{m-1}, m-2$

29. Let $f(x) = \int_2^x \frac{dt}{\sqrt{1+t^4}}$ and g be the inverse of f. Then the value of $g'(0)$ is

a) 1
b) 17
c) $\sqrt{17}$
d) None of these

30. The value of $\int_1^a [x] f'(x)dx$, where $a > 1$,where$[x]$ denotes the greatest integer not exceeding x is

a) $af(a) - \{f(1) + f(2) + \cdots + f([a])\}$
b) $[a]f(a) - \{f(1) + f(2) + \cdots + f([a])\}$
c) $[a]f([a]) - \{f(1) + f(2) + \cdots + fA\}$
d) $af([a]) - \{f(1) + f(2) + \cdots + fA\}$

31. The primitive of the function $x\,|\cos x|$ when$\frac{\pi}{2} < x < \pi$ is given by

a) $\cos x + x \sin x + C$ b) $-\cos x - x\sin x + C$ c) $x \sin\, x - x \cos\, x + C$ d) None of these $+C$

32. Let $g(x) = \int_0^x f(t)dt$, where f is such that $\frac{1}{2} \le f(t) \le 1$, for $t \in [0,1]$ and $0 \le f(t) \le \frac{1}{2}$, for $t \in [1,2]$. Then $g(2)$ satisfies the inequality

a) $-\frac{3}{2} \le g(2) < \frac{1}{2}$ b) $\frac{1}{2} \le g(2) \le \frac{3}{2}$ c) $\frac{3}{2} < g(2) \le \frac{5}{2}$ d) $2 < g(2) < 4$

33. Let $f: R \to R$ be a continuous function and $f(x) = f(2x)$ is true $\forall\, x \in R$. If $f(1) = 3$, then the value of $\int_{-1}^{1} f(f(x))dx$ is equal to

a) 6 b) 0 c) $3f(3)$ d) $2f(0)$

34. The value of the definite integral $\int_0^{\sqrt{\text{In}\left(\frac{\pi}{2}\right)}} \cos\left(e^{x^2}\right) 2xe^{x^2}\, dx$ is

a) 1 b) $1 + (\sin 1)$ c) $1 - (\sin 1)$ d) $(\sin 1) - 1$

35. If $\int \frac{dx}{x^2(x^n+1)^{(n-1)/n}} = -[f(x)]^{1/n} + c$,then $f(x)$ is

a) $(1 + x^n)$ b) $1 + x^{-n}$ c) $x^n + x^{-n}$ d) None of these

36. The value of $\int_a^b (x-a)^3(b-x)^4 dx$ is

a) $\frac{(b-a)^4}{6^4}$ b) $\frac{(b-a)^8}{280}$ c) $\frac{(b-a)^7}{7^3}$ d) None of these

37. If $f(\pi) = 2$ and $\int_0^\pi (f(x) + f''(x)) \sin x\, dx = 5$, then $f(0)$ is equal to (it is given that $f(x)$ is continuous in $[0, \pi]$)

a) 7 b) 3 c) 5 d) 1

38. If $f(x) = \begin{cases} e^{\cos x} \sin x, & \text{for} |x| \le 2 \\ 2, & \text{otherwise} \end{cases}$, then $\int_{-2}^{3} f(x)dx$ is equal to

a) 0 b) 1 c) 2 d) 3

39. If $xf(x) = 3f^2(x) + 2$,then$\int \frac{2x^2 - 12xf(x) + f(x)}{(6f(x)-x)(x^2 - f(x))^2} dx$ equals

a) $\dfrac{1}{x^2 - f(x)} + c$ b) $\dfrac{1}{x^2 + f(x)} + c$ c) $\dfrac{1}{x - f(x)} + c$ d) $\dfrac{1}{x + f(x)} + c$

40. The value of the definite integral $\int_0^{\pi/2} \frac{\sin 5x}{\sin x}\, dx$ is

a) 0 b) $\dfrac{\pi}{2}$ c) π d) 2π

41. If $\int f(x) \sin x \cos x\, dx = \frac{1}{2(b^2-a^2)} \ln f(x) + c$, then $f(x)$ is equal to

a) $\dfrac{1}{a^2\sin^2 x + b^2\cos^2 x}$ b) $\dfrac{1}{a^2\sin^2 x - b^2\cos^2 x}$ c) $\dfrac{1}{a^2\cos^2 x + b^2\sin^2 x}$ d) $\dfrac{1}{a^2\cos^2 x - b^2\sin^2 x}$

42. If $I_1 = \int_0^{\pi/2} \frac{\cos^2 x}{1+\cos^2 x} dx, I_2 = \int_0^{\pi/2} \frac{\sin^2 x}{1+\text{sim}^2 x} dx\, , I_3 = \int_0^{\pi/2} \frac{1+2\cos^2 x \sin^2 x}{4+2\cos^2 x \sin^2 x} dx$, then

a) $I_1 = I_2 > I_3$ b) $I_3 > I_1 = I_2$ c) $I_1 = I_2 = I_3$ d) None of these

43. If $S = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 \frac{1}{3} + \left(\frac{1}{2}\right)^3 \frac{1}{4} + \left(\frac{1}{2}\right)^4 \frac{1}{5} + \ldots$, then

a) $S = \text{In}\, 8 - 2$ b) $S = \text{In}\dfrac{4}{e}$ c) $S = \text{In}\, 4 + 1$ d) None of these

44. If $\int_{-1}^{4} f(x)dx = 4$ and $\int_2^4 (3 - f(x))dx = 7$, then the value of $\int_2^{-1} f(x)\, dx$is

a) 2 b) -3 c) -5 d) None of these

45. The value of $\int_{1/e}^{\tan x} \frac{t\, dt}{1+t^2} + \int_{1/e}^{\cot x} \frac{dt}{t(1+t^2)}$, where $x \in \left(\frac{\pi}{6}, \frac{\pi}{3}\right)$, is equal to

a) 0 b) 2 c) 1 d) None of these

46. The number of possible continuous $f(x)$ defined in $[0, 1]$ for which
$I_1 = \int_0^1 f(x)dx = 1, I_2 = \int_0^1 x\, f(x)dx = a, I_3 = \int_0^1 x^2\, f(x)dx = a^2$, is/are

a) 1 b) ∞ c) 2 d) 0

47. If $\int_1^2 e^{x^2}\, dx = a$, then $\int_e^{e^4} \sqrt{\text{In} x}\, dx$ is equal to

a) $2e^4 - 2e - a$ b) $2e^4 - e - a$ c) $2e^4 - e - 2a$ d) $e^4 - e - a$

48. $\int \frac{\ln(\tan x)}{\sin x \cos x}\, dx$ is equal to

a) $\dfrac{1}{2}\ln(\tan x) + c$ b) $\dfrac{1}{2}\ln(\tan^2 x) + c$ c) $\dfrac{1}{2}(\ln(\tan x))^2 + c$ d) None of these

49. Let f be integrable over $[0, a]$ for any real value of a. If $I_1 = \int_0^{\pi/2} \cos\theta f(\sin\theta + \cos^2\theta)d\theta$ and
$I_2 = \int_0^{\pi/2} \sin 2\theta f(\sin\theta + \cos^2\theta)d\theta$, then

a) $I_1 = -2I_2$ b) $I_1 = I_2$ c) $2I_1 = I_2$ d) $I_1 = -I_2$

50. The value of the integral $\int_0^{\log 5} \frac{e^x\sqrt{e^x - 1}}{e^x + 3}\, dx$ is

a) $3 + 2\pi$ b) $4 - \pi$ c) $2 + \pi$ d) None of these

51. $\int_{2-a}^{2+a} f(x)dx$is equal to (where $f(2 - \alpha) = f(2 + \alpha) \forall \alpha \in R$)

a) $2\displaystyle\int_2^{2+a} f(x)dx$ b) $2\displaystyle\int_0^{a} f(x)dx$ c) $2\displaystyle\int_2^{2} f(x)dx$ d) None of these

52. $\int_{-1}^{2} \left[\frac{[x]}{1+x^2}\right] dx$, where [.] denotes the greatest integer function, is equal to

a) -2 b) -1 c) Zero d) None of these

53. If $f(x) = \cos(\tan^{-1} x)$, then the value of the integral $\int_0^1 xf''(x)dx$ is

a) $\dfrac{3 - \sqrt{2}}{2}$ b) $\dfrac{3 + \sqrt{2}}{2}$ c) 1 d) $1 - \dfrac{3}{2\sqrt{2}}$

54. If $I(m, n) = \int_0^1 x^{m-1}(1 - x)^{n-1}\, dx$, then $(m, n \in 1, m, n \geq 0)$

a) $I(m, n) = \displaystyle\int_0^\infty \frac{x^{m-1}}{(1 + x)^{m-n}}\, dx$ b) $I(m, n) = \displaystyle\int_0^\infty \frac{x^{m}}{(1 + x)^{m+n}}\, dx$

c) $I(m, n) = \displaystyle\int_0^\infty \frac{x^{n-1}}{(1 + x)^{m+n}}\, dx$ d) $I(m, n) = \displaystyle\int_0^\infty \frac{x^{n}}{(1 + x)^{m+n}}\, dx$

55. If $\int_0^t \frac{bx\, \cos 4x - a \sin 4x}{x^2} dx = \frac{a \sin 4t}{t} - 1$, where $0 < t < \frac{\pi}{4}$, then the values of a, b are equal to

a) $\frac{1}{4}, 1$ b) $-1, 4$ c) $2, 2$ d) $2, 4$

56. $\int e^{x^4}(x + x^3 + 2x^5)e^{x^2}\, dx$ is equal to

a) $\frac{1}{2}xe^{x^2}e^{x^4} + c$ b) $\frac{1}{2}x^2e^{x^4} + c$ c) $\frac{1}{2}e^{x^2}e^{x^4} + c$ d) $\frac{1}{2}x^2e^{x^2}e^{x^4} + c$

57. $\int \sqrt{x}\,\left(1 + x^{1/3}\right)^4 dx$ is equal to

a) $2\left\{x^{2/3} + \frac{4}{11}x^{11/6} + \frac{6}{13}x^{13/6} + \frac{4}{15}x^{5/2} + \frac{1}{17}x^{17/6}\right\} + c$

b) $6\left\{x^{2/3} - \frac{4}{11}x^{11/6} + \frac{6}{13}x^{13/6} - \frac{4}{15}x^{5/2} + \frac{1}{17}x^{17/6}\right\} + c$

c) $6\left\{x^{2/3} + \frac{4}{11}x^{11/6} + \frac{6}{13}x^{13/6} + \frac{4}{15}x^{5/2} + \frac{1}{17}x^{17/6}\right\} + c$

d) None of these

58. If $\int \frac{1}{x\sqrt{1-x^3}}\, dx = a \log\left|\frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1}\right| + b$,then a is equal to

a) $1/3$ b) $2/3$ c) $-1/3$ d) $-2/3$

59. If $f'(x) = f(x) + \int_0^1 f(x)dx$, given $f(0) = 1$, then the value of $f(\log_e 2)$ is

a) $\frac{1}{3+e}$ b) $\frac{5-e}{3-e}$ c) $\frac{2+e}{e-2}$ d) None of these

60. The solution for x of the equation $\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{2}$ is

a) π b) $\frac{\sqrt{3}}{2}$ c) $2\sqrt{2}$ d) None of these

61. If $f(x) = 1 + \frac{1}{x}\int_1^x f(t)dt$, then the value of $f(e^{-1})$ is

a) 1 b) 0 c) -1 d) None of these

62. The value of $\int_1^e \frac{1+x^2 \text{ In } x}{x+x^2 \text{In } x}\, dx$ is

a) e b) In $(1 + e)$ c) $e +$ In $(1 + e)$ d) $e -$ In $(1 + e)$

63. $f(x) > 0\ \forall x \in R$and is bounded. If $\lim_{n\to\infty}\left[\int_0^a \frac{f(x)dx}{f(x)+f(a-x)} + a\int_a^{2a} \frac{f(x)dx}{f(x)+f(3a-x)} + a^2\int_{2a}^{3a} \frac{f(x)dx}{f(x)+f(5a-x)} +\right.$ *…+an−1(n−1)anafxdxfx+ f[2n−1 a−x]=7/5*

(where $a < 1$), then a is equal to

a) $\frac{2}{7}$ b) $\frac{1}{7}$ c) $\frac{14}{19}$ d) $\frac{9}{14}$

64. Let f be a positive function. Let $I_1 = \int_{1-k}^{k} xf[x(1-x)]\, dx$, $I_2 = \int_{1-k}^{k} f[x(1-x)]\, dx$, where $2k - 1 > 0$. Then $\frac{I_1}{I_2}$ is

a) 2 b) k c) $\frac{1}{2}$ d) 1

65. $\int \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$ is equal to

a) $x \tan^{-1}x -\ln|\sec(\tan^{-1}x)|+c$ b) $x \tan^{-1}x +\ln|\sec(\tan^{-1}x)|+c$

c) $x \tan^{-1}x -\ln|\cos(\tan^{-1}x)|+c$ d) None of these

66. If $I(m, n) = \int_0^1 t^m(1 + t)^n\, dt$, then the expression for $I(m, n)$ in terms of $I(m + 1, n + 1)$ is

a) $\frac{2^n}{m+1} - \frac{n}{m+1}(m + 1, n - 1)$

b) $\frac{n}{m+1}I(m + 1, n - 1)$

c) $\frac{2^n}{m+1} + \frac{n}{m+1}I(m + 1, n - 1)$

d) $\frac{m}{m+1}I(m + 1, n - 1)$

67. If $\int_0^\infty \frac{x^2\,dx}{(x^2+a^2)\,(x^2+b^2)(x^2+c^2)} = \frac{\pi}{2(a+b)(b+c)(c+a)}$, then the value of $\int_0^\infty \frac{dx}{(x^2+4)\,(x^2+9)}$ is

a) $\frac{\pi}{60}$ b) $\frac{\pi}{20}$ c) $\frac{\pi}{40}$ d) $\frac{\pi}{80}$

68. Given a function $f: [0,4] \to$ R is differentiable, then for some $\alpha, \beta \in (0,2)$, $\int_0^4 f(t)dt$ equals to

a) $f(\alpha^2) + f(\beta^2)$ b) $2\alpha f(\alpha^2) + 2\beta f(\beta^2)$
c) $\alpha f(b^2) + \beta f(\alpha^2)$ d) $f(\alpha)f(\beta)[f(\alpha) + f(\beta)]$

69. $\int \sqrt{\frac{\cos x - \cos^3 x}{1-\cos^3 x}}\, dx$ is equal to

a) $\frac{2}{3}\sin^{-1}(\cos^{3/2} x) + C$ b) $\frac{3}{2}\sin^{-1}(\cos^{3/2} x) + C$ c) $\frac{2}{3}\cos^{-1}(\cos^{3/2} x) + C$ d) None of these

70. The function f and g are positive and continuous. If f is increasing and g is decreasing, then $\int_0^1 f(x)[g(x) - g(1-x)]dx$

a) Is always non-positive b) Is always non-negative
c) Can take positive and negative values d) None of these

71. If $\int_{\sin x}^1 t^2 f(t)dt = 1 - \sin x, \forall\, x \in [0, \pi/2]$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is

a) 3 b) $\sqrt{3}$ c) $\frac{1}{3}$ d) None of these

72. If $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$, then $\int_0^\infty \frac{\sin^3 x}{x} dx$ is equal to

a) $\pi/2$ b) $\pi/4$ c) $\pi/6$ d) $3\pi/2$

73. Let $f: R \to R$ and $g: R \to R$ be continuous functions. Then the value of the integral $\int_{-\pi/2}^{\pi/2}[f(x) + f(-x)][g(x) - g(-x)]\, dx$ is

a) π b) 1 c) -1 d) 0

74. If $S_n = \left[\frac{1}{1+\sqrt{n}} + \frac{1}{2+\sqrt{2n}} + \cdots + \frac{1}{n+\sqrt{n^2}}\right]$, then $\lim_{n\to\infty} S_n$ is equal to

a) log 2 b) log 4 c) log 8 d) None of these

75. $\int_0^\infty \frac{\sin^2 x}{x} dx$ must be same as

a) $\int_0^\infty \frac{\sin x}{x} dx$ b) $\left(\int_0^\infty \frac{\sin x}{x} dx\right)^2$ c) $\int_0^\infty \frac{\cos^2 x}{x^2} dx$ d) None of these

76. If $\int \frac{dx}{\sqrt{\sin^3 x \sin^5 x}} = a\sqrt{\cot x} + b\sqrt{\tan^3 x} + c$, then

a) $a = -1, b = 1/3$ b) $a = -3, b = 2/3$ c) $a = -2, b = 4/3$ d) None of these

77. If $l^r(x)$ means log log log ... x, the log being repeated r times then $\int [xl\,(x) l^2(x) l^3(x) \ldots l^r(x)]^{-1} dx$ is equal to

a) $l^{r+1}(x) + C$ b) $\frac{l^{r+1}(x)}{x+1} + C$ c) $l^r(x) + C$ d) None of these

78. Let a, b, c be non-zero real numbers such that

$$\int_0^1 (1+\cos^8 x)(ax^2 + bx + c)dx = \int_0^2 (1+\cos^8 x)(ax^2 + bx + c)dx$$

Then, the quadratic equation $ax^2 + bx + c = 0$ has

a) No root in (0, 2) b) At least one root in (0, 2)
c) A double root in (0, 2) d) Two imaginary roots

79. If $A = \int_0^1 x^{50}(2-x)^{50}\, dx$; $B = \int_0^1 x^{50}(1-x)^{50}\, dx$, which of the following is true?

a) $A = 2^{50}B$ b) $A = 2^{-50}B$ c) $A = 2^{100}B$ d) $A = 2^{-100}B$

80. $\int_0^\pi \frac{x\tan x}{\sec x + \cos x}\, dx$ is

a) $\frac{\pi^2}{4}$ b) $\frac{\pi^2}{2}$ c) $\frac{3\pi^2}{2}$ d) $\frac{\pi^2}{3}$

81. $\int_0^\infty \frac{dx}{[x+\sqrt{x^2+1}]^3}$ is equal to

a) $\frac{3}{8}$ b) $\frac{1}{8}$ c) $-\frac{3}{8}$ d) None of these

82. If $I_n = \int_0^{\pi} e^x (\sin x)^n dx$, then $\frac{I_3}{I_1}$ is equal to

a) 3/5 b) 1/5 c) 1 d) 2/5

83. $\int \frac{\sqrt{x-1}}{x\sqrt{x+1}}\, dx$ is equal to

a) $\ln\left|x-\sqrt{x^2-1}\right| - \tan^{-1}x + c$ b) $\ln\left|x+\sqrt{x^2-1}\right| - \tan^{-1}x + c$

c) $\ln\left|x-\sqrt{x^2-1}\right| - \sec^{-1}x + c$ d) $\ln\left|x+\sqrt{x^2-1}\right| - \sec^{-1}x + c$

84. If $f(x)$ and $g(x)$ are continuous functions, the $\int_{\ln \lambda}^{\ln 1/\lambda} \frac{f(x^2/4)\,[f(x)-f(-x)]}{g(x^2/4)\,[g(x)+g(-x)]}\, dx$ is

a) Dependent on λ b) A non-zero constant c) Zero d) None of these

85. $\int \frac{\operatorname{cosec}^2 x - 2005}{\cos^{2005} x} dx$ is equal to

a) $\frac{\cot x}{(\cos x)^{2005}} + c$ b) $\frac{\tan x}{(\cos x)^{2005}} + c$ c) $\frac{-\tan x}{(\cos x)^{2005}} + c$ d) None of these

86. If $g(x) = \int_0^x \cos^4 t\, dt$, then $g(x+\pi)$ equals

a) $g(x) + g(\pi)$ b) $g(x) - g(\pi)$ c) $g(x)g(\pi)$ d) $\frac{g(x)}{g(\pi)}$

87. Let $f(x) = \min(\{x\}, \{-x\})\forall x \in R$, where $\{\cdot\}$ denotes the fractional part of x, then $\int_{-100}^{100} f(x)dx$ is equal to

a) 50 b) 100 c) 200 d) None of these

88. If $f(x)$ is differentiable and $\int_0^{t^2} x\, f(x)dx = \frac{2}{5}t^5$, then $f\left(\frac{4}{25}\right)$ equals

a) $\frac{2}{5}$ b) $-\frac{5}{2}$ c) 1 d) $\frac{5}{2}$

89. $\int x \left(\frac{\ln a^{a^{x/2}}}{3a^{5x/2}b^{3x}} + \frac{\ln b^{b^x}}{2a^{2x}b^{4x}}\right) dx$ (where $a, b \in R^+$)is equal to

a) $\frac{1}{6\ln a^2b^3} a^{2x}b^{3x} \ln \frac{a^{2x}b^{3x}}{e} + k$ b) $\frac{1}{6\ln a^2b^3}\frac{1}{a^{2x}b^{3x}} \ln \frac{1}{ea^{2x}b^{3x}} + k$

c) $\frac{1}{6\ln a^2b^3}\frac{1}{a^{2x}b^{3x}} \ln(a^{2x}b^{3x}) + k$ d) $-\frac{1}{6\ln a^2b^3}\frac{1}{a^{2x}b^{3x}} \ln(a^{2x}b^{3x}) + k$

90. $\int_1^4 \{x - 0,4\}\, dx$ equals (where $\{x\}$ is a fractional part of x)

a) 13 b) 6.3 c) 1.5 d) 7.5

91. $\int_{-\pi/3}^{0} \left[\cot^{-1}\left(\frac{2}{2\cos x - 1}\right) + \cot^{-1}\left(\cos x - \frac{1}{2}\right)\right] dx$ is equal to

a) $\frac{\pi^2}{6}$ b) $\frac{\pi^2}{3}$ c) $\frac{\pi^2}{8}$ d) $\frac{3\pi^2}{8}$

92. If $I_1 = \int_{-100}^{101} \frac{dx}{(5+2x-2x^2)(1+e^{2-4x})}$ and $I_2 = \int_{-100}^{101} \frac{dx}{5+2x-2x^2}$, then $\frac{I_1}{I_2}$ is

a) 2 b) $\frac{1}{2}$ c) 1 d) $-\frac{1}{2}$

93. The value of $\int_0^{\pi} \frac{\sin\left(n+\frac{1}{2}\right)x}{\sin\left(\frac{x}{2}\right)} dx$ is, $n \in I, n \geq 0$

a) $\frac{\pi}{2}$ b) 0 c) π d) 2π

94. If $I = \int \frac{dx}{(a^2-b^2x^2)^{3/2}}$,then I equals

a) $\frac{x}{\sqrt{a^2-b^2x^2}} + C$ b) $\frac{x}{a^2\sqrt{a^2-b^2x^2}} + C$ c) $\frac{ax}{\sqrt{a^2-b^2x^2}} + C$ d) None of these

95. $\int_3^{10} [\log (x)] dx$ is equal to (where [.] represents the greatest integer function)

a) 9 b) $16 - e$ c) 10 d) $10 + e$

96. The value of the integral $\int_0^{1/\sqrt{3}} \frac{dx}{(1+x^2)\sqrt{1-x^2}}$ must be

a) $\frac{\pi}{2\sqrt{2}}$ b) $\frac{\pi}{4\sqrt{2}}$ c) $\frac{\pi}{8\sqrt{2}}$ d) None of these

97. The value of the integral $\int \frac{\cos^3 x+\cos^5 x}{\sin^2 x+\sin^4 x} dx$is

a) $\sin x - 6\tan^{-1}(\sin x) + C$ b) $\sin x - 2(\sin x)^{-1} + C$

c) $\sin x - 2(\sin x)^{-1} - 6\tan^{-1}(\sin x) + C$ d) $\sin x - 2(\sin x)^{-1} + 5\tan^{-1}(\sin x) + C$

98. If $f(x)$ satisfies the condition of Rolle's theorem in $[1, 2]$, then $\int_1^2 f'(x)dx$ is equal to

a) 1 b) 3 c) 0 d) None of these

99. If $y^r = \frac{n!\,^{n+r-1}C_{r-1}}{r^n}$, where $n = kr$ (k is constant), then $\lim_{r\to\infty} y$ is equal to

a) $(k-1)\log_e(1+k) - k$ b) $(k+1)\log_e(k-1) + k$

c) $(k+1)\log_e(k-1) - k$ d) $(k-1)\log_e(k-1) + k$

100. The value of $\int_0^{2\pi}[2\sin x]dx$, where $[.]$ represents the greatest integral function, is

a) $\frac{-5\pi}{3}$ b) $-\pi$ c) $\frac{5\pi}{3}$ d) -2π

101. $\int_{-3}^{3} x^8\{x^{11}\}dx$ is equal to (where $\{.\}$ is the fractional part of x)

a) 3^8 b) 3^7 c) 3^9 d) None of these

102. $\int_{-\pi/2}^{\pi/2} \frac{e^{|\sin x|}\cos x}{(1+e^{\tan x})} dx$ is equal to

a) $e+1$ b) $1-e$ c) $e-1$ d) None of these

103. If$\int \frac{dx}{x^2(x^n+1)^{(n-1)/n}} = [f(x)]^{1/n} + C$, then $f(x)$is

a) $(1+x^n)$ b) $1+x^{-n}$ c) $x^n + x^{-n}$ d) None of these

104. $\int_0^x \frac{2^t}{2^{[t]}} dt$, where [.] denotes the greatest integer function, and $x \in R^+$, is equal to

a) $\frac{1}{\text{In } 2}([x] + 2^{\{x\}} - 1)$ b) $\frac{1}{\text{In } 2}([x] + 2^{\{x\}})$ c) $\frac{1}{\text{In } 2}([x] - 2^{\{x\}})$ d) $\frac{1}{\text{In } 2}([x] + 2^{\{x\}} + 1)$

105. If $f(x) = \int_0^{\pi} \frac{t\sin t\,dt}{\sqrt{1+\tan^2 x\sin^2 t}}$for $0 < x < \frac{\pi}{2}$, then

a) $f(0^+) = -\pi$

b) $f\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8}$

c) f is continuous and differentiable in $\left(0, \frac{\pi}{2}\right)$

d) f is continuous but not differentiable in $\left(0, \frac{\pi}{2}\right)$

106. $\int_0^{\infty}\left(\frac{\pi}{1+\pi^2x^2} - \frac{1}{1+x^2}\right)\log x\,dx$ is equal to

a) $-\frac{\pi}{2}\text{In }\pi$ b) 0 c) $\frac{\pi}{2}$ In 2 d) None of these

107. $\int \frac{\sec x\,dx}{\sqrt{\sin(2x+A)+\sin A}}$ is equal to

a) $\frac{\sec A}{\sqrt{2}}\sqrt{\tan x\cos A - \sin A} + c$ b) $\sqrt{2}\sec A\sqrt{\tan x\cos A - \sin A} + c$

c) $\sqrt{2}\sec A\sqrt{\tan x\cos A + \sin A} + c$ d) None of these

108. The value of the integral $\int_0^1 \frac{dx}{x^2+2x\cos\alpha+1}$ is equal to

a) $\sin\alpha$ b) $\alpha\sin\alpha$ c) $\frac{\alpha}{2\sin\alpha}$ d) $\frac{\alpha}{2}\sin\alpha$

109. The value of $\int_0^{\pi/2}\frac{dx}{1+\tan^3 x}$is

a) 0 b) 1 c) $\pi/2$ d) $\pi/4$

110. $f(x)$ is a continuous function for all real values of x and satisfies $\int_0^x f(t)dt = \int_x^1 t^2 f(t)dt + \frac{x^{16}}{8} + \frac{x^6}{3} + a$, then the value of a is equal to

a) $-\frac{1}{24}$ b) $\frac{17}{168}$ c) $\frac{1}{7}$ d) $-\frac{167}{840}$

111. $\int \frac{dx}{x(x^n+1)}$ is equal to

a) $\frac{1}{n}\log\left(\frac{x^n}{x^n+1}\right)+c$ b) $\frac{1}{n}\log\left(\frac{x^n+1}{x^n}\right)+c$ c) $\log\left(\frac{x^n}{x^n+1}\right)+c$ d) None of these

112. $\int \frac{3e^x-5e^{-x}}{4e^x+5e^{-x}}dx = ax + b\ln(4e^x+5e^{-x})+C$,then

a) $a=-\frac{1}{8}, b=\frac{7}{8}$ b) $a=\frac{1}{8}, b=\frac{7}{8}$ c) $a=-\frac{1}{8}, b=-\frac{7}{8}$ d) $a=\frac{1}{8}, b=-\frac{7}{8}$

113. If $A=\int_0^\pi \frac{\cos x}{(x+2)^2}\,dx$, then $\int_0^{/2} \frac{\sin 2x}{x+1}\,dx$ is equal to

a) $\frac{1}{2}+\frac{1}{\pi+2}-A$ b) $\frac{1}{\pi+2}-A$ c) $1+\frac{1}{\pi+2}-A$ d) $A-\frac{1}{2}-\frac{1}{\pi+2}$

114. If $\int_0^x f(t)dt = x+\int_x^1 tf(t)dt$, then the value of $f(1)$ is

a) 1/2 b) 0 c) 1 d) −1/2

115. $\int e^x\left(\frac{2\tan x}{1+\tan x}+\cot^2\left(x+\frac{\pi}{4}\right)\right)dx$ is equal to

a) $e^x\tan\left(\frac{\pi}{4}-x\right)+c$ b) $e^x\tan\left(x-\frac{\pi}{4}\right)+c$ c) $e^x\tan\left(\frac{3\pi}{4}-x\right)+c$ d) None of these

116. The value of $\int_0^x[\cos t]dt, x\in\left[(4n+1)\frac{\pi}{2},(4n+3)\frac{\pi}{2}\right]$ and $n\in N$ is equal to (where $[\cdot]$ represents greatest integer function

a) $\frac{\pi}{2}(2n-1)-2x$ b) $\frac{\pi}{2}(2n-1)+x$ c) $\frac{\pi}{2}(2n+1)-x$ d) $\frac{\pi}{2}(2n+1)+x$

117. Which of the following is incorrect?

a) $\int_{a+c}^{b+c} f(x)dx = \int_a^b f(x+c)dx$

b) $\int_{ac}^{bc} f(x)dx = c\int_a^b f(cx)dx$

c) $\int_{-a}^{a} f(x)dx = \frac{1}{2}\int_{-a}^{a}(f(x)+f(-x)dx$

d) None of these

118. For any integer n, the integral $\int_0^\pi e^{\cos^2 x}\cos^3(2n+1)x\,dx$ has the value

a) π b) 1 c) 0 d) None of these

119. If $f(y)=e^y, g(y)=y, y>0$ and $F(t)=\int_0^t f(t-y)g(y)dt$, then

a) $F(t)=e^t-(1+t)$ b) $F(t)=te^t$ c) $F(t)=te^{-t}$ d) $F(t)=1-e^t(1+t)$

120. If for a real number y, $[y]$ is the greatest integral function less than or equal to y, then the value of the integral $\int_{\pi/2}^{3\pi/2}[2\sin x]dx$ is

a) $-\pi$ b) 0 c) $-\pi/2$ d) $\pi/2$

121. $I_1=\int_0^{\frac{\pi}{2}}\text{In}(\sin x)dx, I_2=\int_{-\pi/4}^{\pi/4}\text{In}(\sin x+\cos x)dx$, then

a) $I_1=2I_2$ b) $I_2=2I_1$ c) $I_1=4I_2$ d) $I_2=4I_1$

122. The equation of the curve is $y=f(x)$. The tangents at $[1,f(1)]$, $[2,f(2)]$ and $[3,f(3)]$ make angle $\frac{\pi}{6},\frac{\pi}{3}$ and $\frac{\pi}{4}$, respectively, with the positive direction of x-axis, then the value of $\int_2^3 f'(x)f''(x)dx+\int_1^3 f''(x)dx$ is equal to

a) $-1/\sqrt{3}$ b) $1/\sqrt{3}$ c) 0 d) None of these

123. f is an odd function. It is also known that $f(x)$ is continuous for all values of x and is periodic with period 2 If $g(x)=\int_0^x f(t)dt$, then

a) $g(x)$ is odd b) $g(n)=0, n\in N$ c) $g(2n)=0, n\in N$ d) $g(x)$ is non-periodic

124. For $x\in R$ and a continuous function f, let $I_1=\int_{\sin^2 t}^{1+\cos^2 t} xf\{x(2-x)\}\,dx$ and $I_2=\int_{\sin^2 t}^{1+\cos^2 t} f\{x(2-x)\}\,dx$. Then $\frac{I_1}{I_2}$ is

a) -1 b) 1 c) 2 d) 3

125. Let $\int e^x \{f(x) - f'(x)\}dx = \phi(x)$. Then $\int e^x f(x)\, dx$ is

a) $\phi(x) = e^x f(x)$ b) $\phi(x) - e^x f(x)$ c) $\frac{1}{2}\{\phi(x) + e^x f(x)\}$ d) $\frac{1}{2}\{\phi(x) + e^x f'(x)\}$

126. $I_1 = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x}\, dx, I_2 = \int_0^{2\pi} \cos^6 x\, dx,$

$I_3 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 x\, dx, I_4 = \int_0^1 \left(\frac{1}{x} - 1\right) dx$, then

a) $I_2 = I_3 = I_4 = 0, I_1 \neq 0$ b) $I_1 = I_2 = I_3 = 0, I_4 \neq 0$

c) $I_1 = I_3 = I_4 = 0, I_2 \neq 0$ d) $I_1 = I_2 = I_3 = 0, I_4 \neq 0$

127. $\int \frac{2 \sin x}{(3+\sin 2x)}\, dx$ is equal to

a) $\frac{1}{2}\ln\left|\frac{2 + \sin x - \cos x}{2 - \sin x + \cos x}\right| - \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right) + c$

b) $\frac{1}{2}\ln\left|\frac{2 + \sin x - \cos x}{2 - \sin x + \cos x}\right| - \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right) + c$

c) $\frac{1}{4}\ln\left|\frac{2 + \sin x - \cos x}{2 - \sin x + \cos x}\right| - \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right) + c$

d) None of these

128. If $f'(x) = \frac{1}{-x+\sqrt{x^2+1}}$ and $f(0) = -\frac{1+\sqrt{2}}{2}$, then $f(1)$, is equal to

a) $-\log(\sqrt{2} + 1)$ b) 1 c) $1 + \sqrt{2}$ d) None of these

129. The value of the integral $\int_0^1 e^{x^2}\, dx$ lies in the interval

a) $(0, 1)$ b) $(-1, 0)$ c) $(1, e)$ d) None of these

130. If $f(x)$ is continuous for all real values of x, then $\sum_{r=1}^{n} \int_0^1 f(r - 1 + x)dx$is equal to

a) $\int_0^n f(x)dx$ b) $\int_0^1 f(x)dx$ c) $n\int_0^1 f(x)dx$ d) $(n - 1)\int_0^1 f(x)dx$

131. $\int \sqrt{1 + \sin x}\; dx$ is equal to

a) $-2\sqrt{1 - \sin x} + C$ b) $\sin(x/2) + \cos(x/2) + C$

c) $\cos(x/2) - \sin(x/2) + C$ d) $2\sqrt{1 - \sin x} + C$

132. $\int \frac{x^3 dx}{\sqrt{1+x^2}}$is equal to

a) $\frac{1}{3}\sqrt{1 + x^2}(2 + x^2) + C$ b) $\frac{1}{3}\sqrt{1 + x^2}(x^2 - 1) + C$

c) $\frac{1}{3}(1 + x^2)^{3/2} + C$ d) $\frac{1}{3}\sqrt{1 + x^2}(x^2 - 2) + C$

133. If $I = \int \frac{dx}{x^3\sqrt{x^2-1}}$, then Iequals

a) $\frac{1}{2}\left(\frac{\sqrt{x^2 - 1}}{x^3} + \tan^{-1}\sqrt{x^2 - 1}\right) + C$ b) $\frac{1}{2}\left(\frac{\sqrt{x^2 - 1}}{x^2} + x\tan^{-1}\sqrt{x^2 - 1}\right) + C$

c) $\frac{1}{2}\left(\frac{\sqrt{x^2 - 1}}{x} + \tan^{-1}\sqrt{x^2 - 1}\right) + C$ d) $\frac{1}{2}\left(\frac{\sqrt{x^2 - 1}}{x^2} + \tan^{-1}\sqrt{x^2 - 1}\right) + C$

134. $\int_0^1 \frac{\tan^{-1} x}{x}\, dx$ is equal to

a) $\int_0^{\frac{\pi}{2}} \frac{\sin x}{x} dx$ b) $\int_0^{\frac{\pi}{2}} \frac{x}{\sin x} dx$ c) $\frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{\sin x}{x} dx$ d) $\frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{x}{\sin x} dx$

135. Let $I = \int \frac{e^x}{e^{4x}+e^{2x}+1} dx, J = \int \frac{e^{-x}}{e^{-4x}+e^{-2x}+1} dx$. Then, for an arbitrary constant c, the value of $J - I$ equals

a) $\frac{1}{2}\log\left|\frac{e^{4x} - e^{2x} + 1}{e^{4x} + e^{2x} + 1}\right| + c$

b) $\frac{1}{2}\log\left|\frac{e^{2x}+e^{x}+1}{e^{2x}-e^{x}+1}\right|+c$

c) $\frac{1}{2}\log\left|\frac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}\right|+c$

d) $\frac{1}{2}\log\left|\frac{e^{4x}+e^{2x}+1}{e^{4x}-e^{2x}+1}\right|+c$

136. The value of $\int \frac{(ax^2-b)dx}{x\sqrt{c^2x^2-(ax^2+b)^2}}$ is equal to

a) $\frac{1}{c}\sin^{-1}\left(ax+\frac{b}{x}\right)+k$ b) $c\sin^{-1}\left(a+\frac{b}{x}\right)+c$ c) $\sin^{-1}\left(\frac{ax+\frac{b}{x}}{c}\right)+k$ d) None of these

137. $\int 4\sin x\cos\frac{x}{2}\cos\frac{3x}{2}\ dx$ is equal to

a) $\cos x+\frac{1}{2}\cos 2x-\frac{1}{3}\cos 3x+C$ b) $\cos x-\frac{1}{2}\cos 2x-\frac{1}{3}\cos 3x+C$

c) $\cos x+\frac{1}{2}\cos 2x+\frac{1}{3}\cos 3x+C$ d) $\cos x-\frac{1}{2}\cos 2x+\frac{1}{3}\cos 3x+C$

138. The value of $\int_0^1 \left(\prod_{r=1}^{n}(x+r)\right)\left(\sum_{k=1}^{n}\frac{1}{x+k}\right)dx$ equals

a) n b) $n!$ c) $(n+1)!$ d) $n\cdot n!$

139. Suppose that $F(x)$ is an anti-derivative of $f(x)=\frac{\sin x}{x}$, where $x>0$, then $\int_1^3 \frac{\sin 2x}{x}\ dx$ can be expressed as

a) $F(6)-F(2)$ b) $\frac{1}{2}(F(6)-F(2))$ c) $\frac{1}{2}(F(3)-F(1))$ d) $2(F(6)-F(2))$

140. If $y=\int \frac{dx}{(1+x^2)^{\frac{3}{2}}}$ and $y=0$ when $x=0$, find the value of y when $x=1$ is

a) $\frac{1}{\sqrt{2}}$ b) $\sqrt{2}$ c) $2\sqrt{2}$ d) None of these

141. If the function $f:[0,8]\to R$ is differentiable, then for $0<a,b<2$, $\int_0^8 f(t)dt$ is equal to

a) $3[\alpha^3 f(\alpha^2)+\beta^2 f(\beta^2)]$ b) $3[\alpha^3 f(\alpha)+\beta^3 f(\beta)]$

c) $3[\alpha^2 f(\alpha^3)+\beta^2 f(\beta^3)]$ d) $3[\alpha^2 f(\alpha^2)+\beta^2 f(\beta^2)]$

142. If $I_k=\int_1^e (\text{In } x)^k\ dx$ (where $k\in I^+$), then I_4 equals

a) $9e-24$ b) $12-2e$ c) $24-9e$ d) $6e-12$

143. $\int\left(\frac{x+2}{x+4}\right)^2 e^x\ dx$ is equal to

a) $e^x\left(\frac{x}{x+4}\right)+c$ b) $e^x\left(\frac{x+2}{x+4}\right)+c$ c) $e^x\left(\frac{x-2}{x+4}\right)+c$ d) $\left(\frac{2xe^2}{x+4}\right)+c$

144. $\int \frac{\ln\left(\frac{x-1}{x+1}\right)}{x^2-1}\ dx$ is equal to

a) $\frac{1}{2}\left(\ln\left(\frac{x-1}{x+1}\right)\right)^2+C$ b) $\frac{1}{2}\left(\ln\left(\frac{x+1}{x-1}\right)\right)^2+C$ c) $\frac{1}{4}\left(\ln\left(\frac{x-1}{x+1}\right)\right)^2+C$ d) $\frac{1}{4}\left(\ln\left(\frac{x+1}{x-1}\right)\right)$

145. If $f(x)=\int_{-1}^{x}|t|dt$, then for any $x\geq 0, f(x)$ equals

a) $\frac{1}{2}(1-x^2)$ b) $\frac{1}{2}x^2$ c) $\frac{1}{2}(1+x^2)$ d) None of these

146. If $\int\frac{dx}{(x+2)(x^2+1)}=a\ln(1+x^2)+b\tan^{-1}x\ \frac{1}{5}\text{In}|x+2|+C$, then

a) $a=-\frac{1}{10}, b=-\frac{2}{5}$ b) $a=\frac{1}{10}, b=-\frac{2}{5}$ c) $a=-\frac{1}{10}, b=\frac{2}{5}$ d) $a=\frac{1}{10}, b=\frac{2}{5}$

147. If $f(x)=\frac{e^x}{1+e^x}, I_1=\int_{f(-a)}^{f(a)} x g(x(1-x))\ dx$ and $I_2=\int_{f(-a)}^{f(a)} g(x(1-x))\,dx$, then the value of $\frac{I_2}{I_1}$ is

a) -1 b) -2 c) 2 d) 1

148. The value of the integral $\int_{e^{-1}}^{e^2}\left|\frac{\log_e x}{x}\right|dx$ is

a) 3/2 b) 5/2 c) 3 d) 5

149. The value of the integral $\int_{-\pi}^{\pi} \sin mx \sin nx\, dx$ for $m \neq n$ $(m, n \in 1)$ is

a) 0 b) π c) $\pi/2$ d) 2π

150. Given $\int_0^{\pi/2} \frac{dx}{1+\sin x+\cos x} = \log 2$, then the value of the definite integral $\int_{\pi}^{\pi/2} \frac{\sin x}{1+\sin x+\cos x}\, dx$ is equal to

a) $\frac{1}{2}\log 2$ b) $\frac{\pi}{2} - \log 2$ c) $\frac{\pi}{4} - \frac{1}{2}\log 2$ d) $\frac{\pi}{2} + \log 2$

151. $\int_0^{\pi} x \sin^4 x\ dx$ is equal to

a) $\frac{3\pi}{16}$ b) $\frac{3\pi^2}{16}$ c) $\frac{16\pi}{3}$ d) $\frac{16\pi^2}{3}$

152. The value of $\int_{-2}^{1} \left[x\left[1+\cos\left(\frac{\pi x}{2}\right)\right]+1\right] dx$, where [.] denotes the greatest integer function, is

a) 1 b) 1/2 c) 2 d) None of these

153. Let $f(x) = \int \frac{x^2\, dx}{(1+x^2)(1+\sqrt{1+x^2})}$ and $f(0) = 0$, then the value of $f(1)$ be

a) $\log(1+\sqrt{2})$ b) $\log(1+\sqrt{2}) - \frac{\pi}{4}$ c) $\log(1+\sqrt{2}) + \frac{\pi}{2}$ d) None of these

154. The value of $\int_1^{\frac{1+\sqrt{5}}{2}} \frac{x^2+1}{x^4-x^2+1} \log\left(1+x-\frac{1}{x}\right) dx$ is

a) $\frac{\pi}{8}\log_e 2$ b) $\frac{\pi}{2}\log_e 2$ c) $-\frac{\pi}{2}\log_e 2$ d) None of these

155. $4\int \frac{\sqrt{a^6+x^8}}{x}\, dx$ is equal to

a) $\sqrt{a^6+x^8} + \frac{a^3}{2}\ln\left|\frac{\sqrt{a^6+x^8}+a^3}{\sqrt{a^6+x^8}-a^3}\right| + c$ b) $a^6 \ln\left|\frac{\sqrt{a^6+x^8}-a^3}{\sqrt{a^6+x^8}+a^3}\right| + c$

c) $\sqrt{a^6+x^8} + \frac{a^3}{2}\ln\left|\frac{\sqrt{a^6+x^8}-a^3}{\sqrt{a^6+x^8}+a^3}\right| + c$ d) $a^6 \ln\left|\frac{\sqrt{a^6+x^8}+a^3}{\sqrt{a^6+x^8}-a^3}\right| + c$

156. The value of integral $\int e^x \left(\frac{1}{\sqrt{1+x^2}} + \frac{1-2x^2}{\sqrt{(1+x^2)^5}}\right) dx$ is equal to

a) $e^x\left(\frac{1}{\sqrt{1+x^2}} + \frac{x}{\sqrt{(1+x^2)^3}}\right) + c$ b) $e^x\left(\frac{1}{\sqrt{1+x^2}} - \frac{x}{\sqrt{(1+x^2)^3}}\right) + c$

c) $e^x\left(\frac{1}{\sqrt{1+x^2}} + \frac{x}{\sqrt{(1+x^2)^5}}\right) + c$ d) None of these

157. If $I = \int \frac{\sin 2x}{(3+4\cos x)^3}\, dx$, then I equals

a) $\frac{3\cos x+8}{(3+4\cos x)^2} + C$ b) $\frac{3+8\cos x}{16(3+4\cos x)^2} + C$ c) $\frac{3+\cos x}{(3+4\cos x)^2} + C$ d) $\frac{3-8\cos x}{16(3+4\cos x)^2} + C$

158. If $\int x^5 (1+x^3)^{2/3} dx = A(1+x^3)^{8/3} + B(1+x^3)^{5/3} + c$, then

a) $A = \frac{1}{4}, B = \frac{1}{5}$ b) $A = \frac{1}{8}, B = -\frac{1}{5}$ c) $A = -\frac{1}{8}, B = \frac{1}{5}$ d) None of these

159. If $\int \frac{1-x^7}{x(1+x^7)}\, dx = a\ln|x| + b\ln|x^7+1| + c$, then

a) $a = 1, b = \frac{2}{7}$ b) $a = -1, b = \frac{2}{7}$ c) $a = 1, b = -\frac{2}{7}$ d) $a = -1, b = -\frac{2}{7}$

160. If $\int xe^x \cos x dx = ae^x (b(1-x)\sin x + cx\cos x) + d$, then

a) $a = 1, b = 1, c = -1$ b) $a = \frac{1}{2}, b = -1, c = 1$ c) $a = 1, b = -1, c = 1$ d) $a = \frac{1}{2}, b = 1, c = -1$

161. The value of the integral $\int_{-3\pi/4}^{5\pi/4} \frac{(\sin x+\cos x)}{e^{x-\pi/4}+1}\, dx$ is

a) 0 b) 1 c) 2 d) None of these

162. Let $x = f''(t)\cos\ t + f'(t)\sin\ t$ and $y = -f''(t)\sin\ t + f'(t)\cos\ t$. Then $\int \left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right]^{1/2} dt$ equals

a) $f'(t) + f''(t) + c$ b) $f''(t) + f'''(t) + c$ c) $f(t) + f''(t) + c$ d) $f'(t) - f''(t) + c$

163. $\int \frac{px^{p+2q-1} - qx^{q-1}}{x^{2p+2q} + 2x^{p+q} + 1} dx$ is equal to

a) $-\dfrac{x^p}{x^{p+q}+1}+C$ b) $\dfrac{x^q}{x^{p+q}+1}+C$ c) $-\dfrac{x^q}{x^{p+q}+1}+C$ d) $\dfrac{x^p}{x^{p+q}+1}+C$

164. If $f(x)=\int_0^1 \frac{dt}{1+|x-t|}$, then $f'\left(\frac{1}{2}\right)$ is equal to

a) 0 b) $\dfrac{1}{2}$ c) 1 d) None of these

165. If $I=\int \frac{dx}{\sec x+\operatorname{cosec} x}$, then I equals

a) $\dfrac{1}{2}\left(\cos x+\sin x-\dfrac{1}{\sqrt{2}}\log(\operatorname{cosec} x-\cos x)\right)+C$

b) $\dfrac{1}{2}\left(\sin x-\cos x-\dfrac{1}{\sqrt{2}}\log|\operatorname{cosec} x+\cot x|\right)+C$

c) $\dfrac{1}{\sqrt{2}}\left(\sin x+\cos x+\dfrac{1}{2}\log(\operatorname{cosec} x-\cos x)\right)+C$

d) $\dfrac{1}{2}[\sin x-\cos x]-\dfrac{1}{\sqrt{2}}\log|\operatorname{cosec}(x+\pi/4)-\cot(x+\pi/4)|+C$

166. If $g(x)=\int_0^x(|\sin t|+|\cos t|)dt$, then $g\left(x+\frac{\pi n}{2}\right)$ is equal to, where $n\in N$

a) $g(x)+g(\pi)$ b) $g(x)+g\left(\dfrac{n\pi}{2}\right)$ c) $g(x)+g\left(\dfrac{\pi}{2}\right)$ d) None of these

167. $f(x)=\int_1^x \frac{e^t}{t}\,dt$,where $x\in R^+$. Then the complete set of values of x for which $f(x)\leq \text{In}\, x$ is

a) $(0,1]$ b) $[1,\infty)$ c) $(0,\infty)$ d) None of these

168. The value of $\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}dx$, where $a>0$, is

a) π b) $a\pi$ c) $\pi/2$ d) 2π

169. $\int_{-\pi}^{\pi}\frac{2x\,(1+\sin x)}{1+\cos^2 x}\,dx$ is equal to

a) π b) π^2 c) 0 d) None of these

170. $\int_0^x|\sin t|dt$, where $x\in(2n\pi,(2n+1)\pi)$, where $n\in N$, is equal to

a) $4n-\cos x$ b) $4n-\sin x$ c) $4n+1-\cos x$ d) $4n-1-\cos x$

171. If $\int_0^{f(x)} t^2dt=x\cos\pi x$, then $f'(9)$ is

a) $-\dfrac{1}{9}$ b) $-\dfrac{1}{3}$ c) $\dfrac{1}{3}$ d) Non-existent

172. $\int_0^{\pi/2}|\sin x-\cos x|\,dx$ is equal to

a) 0 b) $2(\sqrt{2}-1)$ c) $\sqrt{2}-1$ d) $2(\sqrt{2}+1)$

173. Let $I_1=\int_{-2}^{2}\frac{x^6+3x^5+7x^4}{x^4+2}dx$ and $I_2=\int_{-3}^{1}\frac{2(x+1)^2+11(x+1)+14}{(x+1)^4+2}dx$, then the value of I_1+I_2 is

a) 8 b) 200/3 c) 100/3 d) None of these

174. $\int\frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}\,dx$ is equal to

a) $\dfrac{1}{2}\sin 2x+C$ b) $-\dfrac{1}{2}\sin 2x+C$ c) $-\dfrac{1}{2}\sin x+C$ d) $-\sin^2 x+C$

175. $\int\frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}}dx$ is equal to

a) $\dfrac{\sqrt{2x^4-2x^2+1}}{x^3}+C$ b) $\dfrac{\sqrt{2x^4-2x^2+1}}{x}+C$ c) $\dfrac{\sqrt{2x^4-2x^2+1}}{x^2}+C$ d) $\dfrac{\sqrt{2x^4-2x^2+1}}{2x^2}+C$

176. If $I_n=\int(\ln\, x)^n\ dx$, thenI_n+I_{n-1}

a) $\dfrac{(\text{In}\, x)^n}{x}+C$ b) $x\,(\text{In}\, x)^{n-1}+C$ c) $x\,(\text{In}\, x)^n+C$ d) None of these

177. $\int\frac{\cos 4x-1}{\cot x-\tan x}\,dx$ is equal to

a) $\dfrac{1}{2}\ln|\sec 2x|-\dfrac{1}{4}\cos^2 2x+c$ b) $\dfrac{1}{2}\ln|\sec 2x|+\dfrac{1}{4}\cos^2 x+c$

c) $\dfrac{1}{2}\ln|\cos 2x|-\dfrac{1}{4}\cos^2 2x+c$ d) $\dfrac{1}{2}\ln|\cos 2x|+\dfrac{1}{4}\cos^2 x+c$

178. If $\int \frac{\cos 4x+1}{\cot x-\tan x}\,dx = A\cos 4x + B,$ then

a) $A=-1/2$ b) $A=-1/8$ c) $A=-1/4$ d) None of these

179. $\int_{-1}^{1/2} \frac{e^x(2-x^2)dx}{(1-x)\sqrt{1-x^2}}$ is equal to

a) $\frac{\sqrt{e}}{2}(\sqrt{3}+1)$ b) $\frac{\sqrt{3e}}{2}$ c) $\sqrt{3e}$ d) $\sqrt{\frac{e}{3}}$

180. $\int_0^a \frac{dx}{x+\sqrt{a^2-x^2}}$ is

a) $\frac{a^2}{4}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) π

181. $\int e^{\tan x}(\sec x-\sin x)dx$, is equal to

a) $e^{\tan x}\cos x + C$ b) $e^{\tan x}\sin x + C$ c) $-e^{\tan x}\cos x + C$ d) $e^{\tan x}\sec x + C$

182. The value of the expression $\frac{\int_0^a x^4\sqrt{a^2-x^2}\,dx}{\int_0^a x^2\sqrt{a^2-x^2}\,dx}$ is equal to

a) $\frac{a^2}{6}$ b) $\frac{3a^2}{2}$ c) $\frac{3a^2}{4}$ d) $\frac{a^2}{2}$

183. Given that f satisfies $|f(u)-f|v|\le|u-v|$ for u and v in $[a,b]$, then $|\int_a^b f(x)dx-(b-a)f(a)|\le$

a) $\frac{(b-a)}{2}$ b) $\frac{(b-a)^2}{2}$ c) $(b-a)^2$ d) None of these

184. The value of $\int_1^e \left(\frac{\tan^{-1}x}{x}+\frac{\log x}{1+x^2}\right)dx$ is

a) $\tan e$ b) $\tan^{-1} e$ c) $\tan^{-1}(1/e)$ d) None of these

185. The value of $\lim_{n\to\infty}\sum_{r=1}^{4n}\frac{\sqrt{n}}{\sqrt{r}(3\sqrt{r}+4\sqrt{n})^2}$ is equal to

a) $\frac{1}{35}$ b) $\frac{1}{14}$ c) $\frac{1}{10}$ d) $\frac{1}{5}$

186. The value of $\lim_{n\to\infty}\left[\tan\frac{\pi}{2n}\tan\frac{2\pi}{2n}\cdots\tan\frac{n\pi}{2n}\right]^{1/n}$ is

a) e b) e^2 c) 1 d) e^3

187. If $I=\int\frac{dx}{(2ax+x^2)^{3/2}}$, then I is equal to

a) $-\frac{x+a}{\sqrt{2ax+x^2}}+c$ b) $-\frac{1}{a}\frac{x+a}{\sqrt{2ax+x^2}}+c$ c) $-\frac{1}{a^2}\frac{x+a}{\sqrt{2ax+x^2}}+c$ d) $-\frac{1}{a^3}\frac{x+a}{\sqrt{2ax+x^3}}+c$

188. $\int\frac{x^9\,dx}{(4x^2+1)^6}$ is equal to

a) $\frac{1}{5x}\left(4+\frac{1}{x^2}\right)^{-5}+c$ b) $\frac{1}{5}\left(4+\frac{1}{x^2}\right)^{-5}+c$ c) $\frac{1}{10}(1+4x^2)^{-5}+c$ d) $\frac{1}{10}\left(4+\frac{1}{x^2}\right)^{-5}+c$

189. If $\int x\log(1+1/x)dx = f(x)\log(x+1)+g(x)x^2+Ax+C$, then

a) $f(x)=\frac{1}{2}x^2$ b) $g(x)=\log x$ c) $A=1$ d) None of these

190. If $P(x)$ is a polynomial of the least degree that has a maximum equal to 6 at $x=1$, and a minimum equal to 2 at $x=3$, then $\int_0^1 P(x)dx$ equals

a) $\frac{17}{4}$ b) $\frac{13}{4}$ c) $\frac{19}{4}$ d) $\frac{5}{4}$

191. If $\int\frac{3\sin x+2\cos x}{3\cos x+2\sin x}\,dx = ax+b\ln|2\sin x+3\cos x|+C$, then

a) $a=-\frac{12}{13}, b=\frac{15}{39}$ b) $a=-\frac{7}{13}, b=\frac{6}{13}$ c) $a=\frac{12}{13}, b=-\frac{15}{39}$ d) $a=-\frac{7}{13}, b=-\frac{6}{13}$

192. If $\int_0^1 e^{x^2}(x-\alpha)dx=0$, then

a) $1<\alpha<2$ b) $\alpha<0$ c) $0<\alpha<1$ d) $\alpha=0$

193. The value of the integral $\int_{-1}^{3}\left(\tan^{-1}\frac{x}{x^2+1}+\tan^{-1}\frac{x^2+1}{x}\right)dx$ is equal to

a) π b) 2π c) 4π d) None of these

194. The value of the integral $\int \frac{(1-\cos\theta)^{2/7}}{(1+\cos\theta)^{9/7}} d\theta$ is

a) $\frac{7}{11}\left(\tan\frac{\theta}{2}\right)^{\frac{11}{7}} + C$ b) $\frac{7}{11}\left(\cos\frac{\theta}{2}\right)^{\frac{11}{7}} + C$ c) $\frac{7}{11}\left(\sin\frac{\theta}{2}\right)^{\frac{11}{7}} + C$ d) None of these

195. If $f(x) = \cos x - \int_0^x (x-t)f(t)dt$, then $f''(x) + f(x)$ is equal to

a) $-\cos x$ b) $-\sin x$ c) $\int_0^x (x-t)f(t)dt$ d) 0

196. If $\int_{\cos x}^{1} t^2 f(t)dt = 1 - \cos x \ \forall x \in \left(0, \frac{\pi}{2}\right)$, then the value of $\left[f\left(\frac{\sqrt{3}}{4}\right)\right]$ is ([.] denotes the greatest integer function)

a) 4 b) 5 c) 6 d) -7

197. $\int \frac{dx}{(1+\sqrt{x})\sqrt{(x-x^2)}}$ is equal to

a) $\frac{1+\sqrt{x}}{(1-x)^2} + c$ b) $\frac{1+\sqrt{x}}{(1+x)^2} + c$ c) $\frac{1-\sqrt{x}}{(1-x)^2} + c$ d) $\frac{2(\sqrt{x}-1)}{\sqrt{(1-x)}} + c$

198. The value of $\int \frac{(x^2-1)dx}{x^3\sqrt{2x^4-2x^2+1}}$ is

a) $2\sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}} + c$

b) $2\sqrt{2 + \frac{2}{x^2} + \frac{1}{x^4}} + c$

c) $\frac{1}{2}\sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}} + c$

d) None of the above

199. If $\int \sqrt{1+\sin x}\, f(x)dx = \frac{2}{3}(1+\sin x)^{3/2} + c$, then $f(x)$ equals

a) $\cos x$ b) $\sin x$ c) $\tan x$ d) 1

200. $\int e^{\tan^{-1}x}(1+x+x^2)\, d(\cot^{-1}x)$ is equal to

a) $-e^{\tan^{-1}x} + c$ b) $e^{\tan^{-1}x} + c$ c) $-x\, e^{\tan^{-1}x} + c$ d) $x\, e^{\tan^{-1}x} + c$

201. If $\int_{-\pi/4}^{3\pi/4} \frac{e^{\pi/4}\, dx}{(e^x + e^{\pi/4})(\sin x + \cos x)} = k\int_{-\pi/2}^{\pi/2} \sec x dx$, then the value of k is

a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{2}}$ c) $\frac{1}{2\sqrt{2}}$ d) $-\frac{1}{\sqrt{2}}$

202. $\int \frac{x+2}{(x^2+3x+3)\sqrt{x+1}} dx$ is equal to

a) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3(x+1)}}\right)$ b) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3(x+1)}}\right)$

c) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{x+1}}\right)$ d) None of these

203. The value of the definite integral $\int_0^1 (1+e^{-x^2})dx$ is

a) -1 b) 2 c) $1+e^{-1}$ d) None of these

204. Let f be a real-valued function defined on the interval $(-1,1)$ such that $e^{-x}f(x) = 2 + \int_0^x \sqrt{t^4+1}\, dt$, for all $x \in (-1,1)$ and let f^{-1} be the inverse function of f. Then, $(f^{-1})'(2)$ is equal to

a) 1 b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{1}{e}$

205. If $\int x\frac{\ln(x+\sqrt{1+x^2})}{\sqrt{1+x^2}} dx = a\sqrt{1+x^2}\ln\left(x+\sqrt{1+x^2}\right) + bx + c$,then

a) $a=1, b=-1$ b) $a=1, b=1$ c) $a=-1, b=1$ d) $a=-1, b=-1$

206. Let $I_1 = \int_0^1 \frac{e^x dx}{1+x}$ and $I_2 = \int_0^1 \frac{x^2 dx}{e^{x^3}(2-x^3)}$, then $\frac{I_1}{I_2}$ is equal to

a) $3/e$ b) $e/3$ c) $3e$ d) $1/3e$

207. If $\int_0^1 \frac{\sin t}{1+t} dt = \alpha$, then the value of the integral $\int_{4\pi-2}^{4\pi} \frac{\sin\frac{t}{2}}{4\pi+2-1} dt$ is

a) 2α b) -2α c) α d) $-\alpha$

208. If $a > 0$ and $A = \int_0^a \cos^{-1} x dx$, then $\int_{-a}^{a}(\cos^{-1} x - \sin^{-1}\sqrt{1-x^2})dx = \pi a - \lambda A$, then λ is

a) 0 b) 2 c) 3 d) None of these

209. $\int_0^4 \frac{(y^2-4y+5)\sin(y-2)dy}{[2y^2-8y+11]}$ is equal to

a) 0 b) 2 c) -2 d) None of these

210. The value of the integral $\int_0^\infty \frac{x\log x}{(1+x^2)^2} dx$ is

a) 0 b) log 7 c) 5 log 13 d) None of these

211. $\int_0^x [\sin t]dt$, where $x \in (2n\pi, (2n+1)\pi), n \in N$ and $[.]$ denotes the greatest integer function, is equal to

a) $-n\pi$ b) $-(n+1)\pi$ c) $-2n\pi$ d) $-(2n+1)\pi$

212. A function f is continuous for all x (and not every where zero) such that $f^2(x) = \int_0^x f(t)\frac{\cos t}{2+\sin t} dt$, then $f(x)$ is

a) $\frac{1}{2}\ \text{In}\left(\frac{x+\cos x}{2}\right); x \neq 0$

b) $\frac{1}{2}\text{In}\left(\frac{3}{x+\cos x}\right); x \neq 0$

c) $\frac{1}{2}\text{In}\left(\frac{2+\sin x}{2}\right); x \neq n\pi, n \in I$

d) $\frac{\cos x + \sin x}{2+\sin x}; x \neq n\pi + \frac{3\pi}{4}, n \in I$

213. The value of the integral $\int(x^2+x)(x^{-8}+2x^{-9})^{1/10} dx$ is

a) $\frac{5}{11}(x^2+2x)^{11/10} + c$ b) $\frac{5}{6}(x+1)^{11/10} + c$ c) $\frac{6}{7}(x+1)^{11/10} + c$ d) None of these

214. If $\int \frac{dx}{\cos^3 x\sqrt{\sin 2x}} = a(\tan^2 x + b)\sqrt{\tan x} + c$, then

a) $a = \frac{\sqrt{2}}{5}, b = \frac{1}{\sqrt{5}}$ b) $a = \frac{\sqrt{2}}{5}, b = 5$ c) $a = \frac{\sqrt{2}}{5}, b = -\frac{1}{\sqrt{5}}$ d) $a = \frac{\sqrt{2}}{5}, b = \sqrt{5}$

215. $\lim_{x\to 0}\frac{1}{x}\left[\int_y^a e^{\sin^2 t} dt - \int_{x+y}^a e^{\sin^2 t} dt\right]$is equal to

a) $e^{\sin^2 y}$ b) $\sin 2y\, e^{\sin^2 y}$ c) 0 d) None of these

216. If $f(x) = A\sin\left(\frac{\pi x}{2}\right) + B, f'\left(\frac{1}{2}\right) = \sqrt{2}$ and$\int_0^1 f(x)\,dx = \frac{2A}{\pi}$, then constants A and $B$are

a) $\frac{\pi}{2}$and $\frac{\pi}{2}$ b) $\frac{2}{\pi}$ and $\frac{3}{\pi}$ c) 0 and $\frac{-4}{\pi}$ d) $\frac{4}{\pi}$ and 0

217. The value of the integral $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}} dx$is

a) $\pi/4$ b) $\pi/2$ c) π d) None of these

Multiple Correct Answers Type

218. If $f(2-x) = f(2+x)$ and $f(4-x) = f(4+x)$ for all x and $f(x)$ is a function for which $\int_0^2 f(x)dx = 5$, then $\int_0^{50} f(x)dx$ is equal to

a) 125 b) $\int_{-4}^{46} f(x)dx$ c) $\int_1^{51} f(x)dx$ d) $\int_2^{52} f(x)dx$

219. If $\int(\cos^{-1} x + \cos^{-1}\sqrt{(1-x^2)})dx = Ax + f(x)\sin^{-1} x - 2\sqrt{(1-x^2)} + c, \forall x \in [-1,0)$, then

a) $f(x) = x$ b) $f(x) = -2x$ c) $A = \frac{\pi}{4}$ d) $A = \frac{\pi}{2}$

220. If $I_n = \int_0^1 \frac{dx}{(1+x^2)^n}$, where $n \in N$, which of the following statements hold good?

a) $2n\, I_{n+1} = 2^{-n} + (2n-1)I_n$

b) $I_2 = \frac{\pi}{8} + \frac{1}{4}$

c) $I_2 = \frac{\pi}{8} - \frac{1}{4}$

d) $I_3 = \frac{3\pi}{32} + \frac{1}{4}$

221. The value of $\int_0^1 \frac{2x^2+3x+3}{(x+1)(x^2+2x+2)}dx$ is

a) $\frac{\pi}{4}+2\log 2-\tan^{-1}2$ b) $\frac{\pi}{4}+2\log 2-\tan^{-1}\frac{1}{3}$ c) $2\log 2-\cot^{-1}3$ d) $-\frac{\pi}{4}+\log 4-\cot^{-1}2$

222. If $\int \sin x d(\sec x)=f(x)-g(x)+c$, then

a) $f(x)=\sec x$ b) $f(x)=\tan x$ c) $g(x)=2x$ d) $g(x)=x$

223. If $\int \frac{x^2-x+1}{(x^2+1)^{\frac{3}{2}}}e^x\,dx=e^x f(x)+c$, then

a) $f(x)$ is an even function

b) $f(x)$ is a bounded function

c) The range of $f(x)$ is $(0,1]$

d) $f(x)$ has two points of extrema

224. $\int \frac{x^2+\cos^2 x}{x^2+1}\operatorname{cosec}^2 x dx$ is equal to

a) $\cot x-\cot^{-1}x+c$ b) $c-\cot x+\cot^{-1}x$

c) $-\tan^{-1}x-\frac{\operatorname{cosec} x}{\sec x}+c$ d) $-e^{\log\tan^{-1}x}-\cot x+c$

225. $\int\sqrt{1+\operatorname{cosec} x}\,dx$ equals

a) $2\sin^{-1}\sqrt{\sin x}+c$ b) $\sqrt{2}\cos^{-1}\sqrt{\cos x}+c$

c) $c-2\sin^{-1}(1-2\sin x)$ d) $\cos^{-1}(1-2\sin x)+c$

226. The value of $\int_0^1 e^{x^2-x}\,dx$is

a) <1 b) >1 c) $>e^{-\frac{1}{4}}$ d) $<e^{-\frac{1}{4}}$

227. If $g(x)=\int_0^x 2|t|\,dt$,then

a) $g(x)=x|x|$ b) $g(x)$is monotonic

c) $g(x)$ is differentiable at $x=0$ d) $g'(x)$ is differentiable at $x=0$

228. If $\int\frac{x^4+1}{x^6+1}dx=\tan^{-1}f(x)-\frac{2}{3}\tan^{-1}g(x)+C$, then

a) Both $f(x)$ and $g(x)$ are odd functions b) $f(x)$ is monotonic function

c) $f(x)=g(x)$has no real roots d) $\int\frac{f(x)}{g(x)}dx=-\frac{1}{x}+\frac{3}{x^3}+c$

229. If $f(x)$ is integrable over $[1,2]$, then $\int_1^2 f(x)dx$ is equal to

a) $\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}f\left(\frac{r}{n}\right)$ b) $\lim_{n\to\infty}\frac{1}{n}\sum_{r=n+1}^{2n}f\left(\frac{r}{n}\right)$ c) $\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}f\left(\frac{r+n}{n}\right)$ d) $\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n}f\left(\frac{r}{n}\right)$

230. The value of $\int_0^\infty\frac{dx}{1+x^4}$ is

a) Same as that of $\int_0^\infty\frac{x^2+1dx}{1+x^4}$ b) $\frac{\pi}{2\sqrt{2}}$

c) Same as that of $\int_0^\infty\frac{x^2dx}{1+x^4}$ d) $\frac{\pi}{\sqrt{2}}$

231. If$\int\frac{e^{x-1}}{(x^2-5x+4)}2x\,dx=AF(x-1)+BF(x-4)+C$ and $F(x)=\int\frac{e^x}{x}dx$, then

a) $A=-2/3$ b) $B=(4/3)e^3$ c) $A=2/3$ d) $B=(8/3)e^3$

232. Let $f(x)=-[x]$, for every real number x, where $[x]$ is the integral part of x. Then $\int_{-1}^1 f(x)\,dx$ is

a) 1 b) 2 c) 0 d) 1/2

233. If $\int\sqrt{\operatorname{cosec} x+1}\,dx=k\,\mathrm{fog}\,(x)+c$, where k is a real constant, then

a) $k=-2, f(x)=\cot^{-1}x, g(x)=\sqrt{\operatorname{cosec} x-1}$

b) $k=-2, f(x)=\tan^{-1}x, g(x)=\sqrt{\operatorname{cosec} x-1}$

c) $k=2, f(x)=\tan^{-1}x, g(x)=\frac{\cot x}{\sqrt{\operatorname{cosec} x-1}}$

d) $k=2, f(x)=\cot^{-1}x, g(x)=\frac{\cot x}{\sqrt{\operatorname{cosec} x+1}}$

234. If $\int_0^x f(t) = x + \int_x^1 t\, f(t)dt$, then the value of $f(1)$is

a) 1/2 b) 0 c) 1 d) $-1/2$

235. If $\int \sin^{-1} x \cos^{-1} x\, dx = f^{-1}(x)\left[Ax - x\, f^{-1}(x) - 2\sqrt{1-x^2}\right] + 2x + C$, then

a) $f(x) = \sin x$ b) $f(x) = \cos x$ c) $A = \frac{\pi}{4}$ d) $A = \frac{\pi}{2}$

236. If$I = \int \frac{\sin x + \sin^3 x}{\cos 2x} dx = P \cos x + Q \log|f(x)| + R$, then

a) $P = 1/2, Q = -\frac{3}{4\sqrt{2}}$ b) $P = 1/4, Q = -\frac{1}{\sqrt{2}}$ c) $f(x) = \frac{\sqrt{2}\cos x + 1}{\sqrt{2}\cos x - 1}$ d) $f(x) = \frac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}$

237. Let $f(a) > 0$ and let $f(x)$ be a non-decreasing continuous function in $[a, b]$, then $\frac{1}{b-a}\int_a^b f(x)\, dx$ has the

a) Maximum value $f(b)$

b) Minimum value $f(a)$

c) Maximum value $bf(b)$

d) Minimum value $\frac{f(a)}{b-a}$

238. If $I_n = \int_0^{\pi/4} \tan^n x\, dx$ ($n > 1$ and is an integer), then

a) $I_n + I_{n-2} = \frac{1}{n+1}$

b) $I_n + I_{n-2} = \frac{1}{n-1}$

c) $I_2 + I_4, I_4 + I_6, \ldots$, are in H.P.

d) $\frac{1}{2(n+1)} < I_n < \frac{1}{2(n-1)}$

239. Let $f: [1, \infty] \to R$ and $f(x) = x\int_1^x \frac{e^t}{t} dt - e^x$, then

a) $f(x)$ is an increasing function

b) $\lim_{x\to\infty} f(x) \to \infty$

c) $f'(x)$ has a maxima at $x = e$

d) $f(x)$ is a decreasing function

240. The values of a for which the integral $\int_0^2 |x - a| dx \geq 1$ is satisfied are

a) $[2, \infty)$ b) $(-\infty, 0]$ c) $(0, 2)$ d) None of these

241. If $A_n = \int_0^{\pi/2} \frac{\sin(2n-1)x}{\sin x} dx; B_n = \int_0^{\pi/2} \left(\frac{\sin nx}{\sin x}\right)^2 dx$, for $n \in N$, then

a) $A_{n+1} = A_n$ b) $B_{n+1} = B_n$ c) $A_{n+1} - A_n = B_{n+1}$ d) $B_{n+1} - B_n = A_{n+1}$

242. A function $f(x)$ which satisfies the relation$f(x) = e^x + \int_0^1 e^x\, f(t)dt$, then

a) $f(0) < 0$

b) $f(x)$ is a decreasing function

c) $f(x)$ is an increasing function

d) $\int_0^1 f(x)dx > 0$

243. If $\int \frac{\cos 4x + 1}{\cot x - \tan x} dx = Af(x) + B$, then

a) $A = -\frac{1}{8}$

b) $B = \frac{1}{2}$

c) $f(x)$has fundamental period $\frac{\pi}{2}$

d) $f(x)$ is an odd function

244. If the primitive of $\sin(\log x)$ is $f(x)\{\sin g(x) - \cos h(x)\} + c$ (c being the constant of integration), then

a) $\lim_{x\to 2} f(x) = 1$ b) $\lim_{x\to 1} \frac{g(x)}{h(x)} = 1$ c) $g(e^3) = 3$ d) $h(e^5) = 5$

245. If $\int_a^b |\sin x| dx = 8$ and $\int_0^{a+b} |\cos x| dx = 9$, then

a) $a + b = \frac{9\pi}{2}$ b) $|a - b| = 4\pi$ c) $\frac{a}{b} = 15$ d) $\int_a^b \sec^2 x\, dx = 0$

246. Let $I = \int_1^3 \sqrt{3 + x^3}\, dx$, then the values of I will lie in the interval

a) $[4, 6]$ b) $[1, 3]$ c) $[4, 2\sqrt{30}]$ d) $[\sqrt{15}, \sqrt{30}]$

247. $\int \frac{dx}{x^2 + ax + 1} = f(g(x)) + c$, then

a) $f(x)$ is inverse trigonomeric function for$|a| > 2$

b) $f(x)$is logarithmic function for$|a| < 2$

c) $g(x)$ quadratic function for$|a| > 2$

d) $g(x)$ is rational function for $|a| < 2$

248. If $\int x^2 e^{-2x} dx = e^{-2x}(ax^2 + bx + c) + d$, then

a) $a = 1$ b) $b = 2$ c) $c = \frac{1}{2}$ d) $d \in R$

249. If $l = \int \sec^2 c \operatorname{cosec}^4 x dx = A\cot^3 x + B\tan x + C\cot x + D$, then

a) $A = -\frac{1}{3}$ b) $B = 2$ c) $C = -2$ d) None of these

250. Let $f(x) = \int_1^x \frac{3^t}{1+t^2}\, dt$, where $x > 0$, then

a) For $0 < \alpha < \beta, f(\alpha) < f(\beta)$
b) For $0 < \alpha < \beta, f(\alpha) > f(\beta)$
c) $f(x) + \pi/4 < \tan^{-1} x, \forall\, x \geq 1$
d) $f(x) + \pi/4 > \tan^{-1} x, \forall x \geq 1$

251. If $f(x) = \int \frac{x^8+4}{x^4-2x^2+2} dx$ and $f(0) = 0$, then

a) $f(x)$ is an odd function
b) $f(x)$ has range R
c) $f(x)$has at least one real root
d) $f(x)$is a monotonic function

252. A curve $g(x) = \int x^{27}(1 + x + x^2)^6(6x^2 + 5x + 4)dx$ is passing through origin, then

a) $g(1) = \frac{3^7}{7}$ b) $g(1) = \frac{2^7}{7}$ c) $g(-1) = \frac{1}{7}$ d) $g(-1) = \frac{3^7}{14}$

253. If $f(x) = \int_0^x |t-1| dt$, where $0 \leq x \leq 2$, then

a) Range of $f(x)$ is $[0, 1]$
b) $f(x)$ is differentiable at $x = 1$
c) $f(x) = \cos^{-1} x$has two real roots
d) $f'(1/2) = 1/2$

254. Which of the following statement(s) is/are true?

a) If function $y = f(x)$ is continuous at $x = c$ such that $f(c) \neq 0$, then $f(x)f(c) > 0\ \forall\, x \in (c-h, c+h)$ where h is sufficiently small positive quantity

b) $\lim_{n\to\infty} \frac{1}{n} \text{In} \left(\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\cdots\left(1+\frac{n}{n}\right)\right) = 1 + 2\text{ In } 2$

c) Let f be a continuous and non-negative function defined on $[a, b]$.If $\int_a^b f(x)dx = 0$, then $f(x) = 0\ \forall\, x \in [a, b]$

d) Let f be a continuous function defined on $[a, b]$ such that $\int_a^b f(x)dx = 0$, then there exists at least one $c \in (a, b)$ for which $f(c) = 0$

255. A primitive of $\sin 6x$ is

a) $\frac{1}{3}(\sin^6 x - \sin^3 x) + c$
b) $-\frac{1}{3}\cos^2 3x + c$
c) $\frac{1}{3}\sin^2 3x + c$
d) $\frac{1}{3}\sin\left(3x + \frac{\pi}{7}\right)\sin\left(3x - \frac{\pi}{7}\right) + c$

256. The point of extremum of $\int_0^{x^2}\left(\frac{t^2-5t+4}{2+e^t}\right) dt$ are

a) $x = -2$ b) $x = 1$ c) $x = 0$ d) $x = -1$

257. If $f(x) = \int_a^x [f(x)]^{-1} dx$ and $\int_a^1 [f(x)]^{-1} dx = \sqrt{2}$, then

a) $f(2) = 2$ b) $f'(2) = 1/2$ c) $f^{-1}(2) = 2$ d) $\int_0^1 f(x)dx = \sqrt{2}$

258. If $f(x) = \int_0^x (\cos(\sin t) + \cos(\cos t))dt$, then $f(x+\pi)$ is

a) $f(x) + f(\pi)$ b) $f(x) + 2f(\pi)$ c) $f(x) + f\left(\frac{\pi}{2}\right)$ d) $f(x) + 2f\left(\frac{\pi}{2}\right)$

259. If $\int \frac{3x+4}{x^3-2x-4} dx = \log|x-2| + k\log f(x) + c$, then

a) $f(x) = |x^2 + 2x + 2|$ b) $f(x) = x^2 + 2x + 2$ c) $k = \frac{1}{4}$ d) $k = -\frac{1}{2}$

260. The value of α, which satisfy $\int_{\pi/2}^{\alpha} \sin x\, dx = \sin 2\alpha\ (\alpha \in [0, 2\pi])$ are equal to

a) $\pi/2$ b) $3\pi/2$ c) $7\pi/6$ d) $11\pi/6$

261. If $f(x) = \lim_{n\to\infty} e^{x\tan(1/n)\log(1/n)}$ and $\int \frac{f(x)}{\sqrt[3]{(\sin^{11} x\cos x)}} dx = g(x) + c$, then

a) $g\left(\frac{\pi}{4}\right) = \frac{3}{2}$
b) $g(x)$ is continuous for all x

c) $g\left(\frac{\pi}{4}\right) = -\frac{15}{8}$ d) $g(x)$ is non-differentiable at infinitely many points

262. $\int_0^x \{\int_0^u f(t)dt\}\, du$ is equal to

a) $\int_0^x (x-u)f(u)du$ b) $\int_0^x uf(x-u)du$ c) $x\int_0^x f(u)du$ d) $x\int_0^x uf(u-x)du$

263. If $\int_a^b \frac{f(x)}{f(a)+f(a+b-x)}\, dx = 10$, then

a) $b = 22, a = 2$ b) $b = 15, a = -5$ c) $b = 10, a = -10$ d) $b = 10, a = -2$

Assertion - Reasoning Type

This section contain(s) 0 questions numbered 264 to 263. Each question contains STATEMENT 1(Assertion) and STATEMENT 2(Reason). Each question has the 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

a) Statement 1 is True, Statement 2 is True; Statement 2 **is** correct explanation for Statement 1

b) Statement 1 is True, Statement 2 is True; Statement 2 **is not** correct explanation for Statement 1

c) Statement 1 is True, Statement 2 is False

d) Statement 1 is False, Statement 2 is True

264

Statement 1: $\int_0^{\pi/2}(\sin^6 x + \cos^6 x)\, dx$ lie in the interval $\left(\frac{\pi}{8}, \frac{\pi}{2}\right)$

Statement 2: $\sin^6 x + \cos^6 x$ is periodic with period $\pi/2$

265

Statement 1: $\int_{\pi/2}^{3\pi/2}[2\sin x]\, dx = 0$, where $[\cdot]$ denotes the greatest integer function

Statement 2: $2\sin x$ is a decreasing function in $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

266

Statement 1: If $f(x)$ is continuous on $[a, b]$, then there exists a point $c \in (a, b)$such that $\int_a^b f(x)dx = fcb{-}a$

Statement 2: For $a < b$, if m and M are, respectively, the smallest and greatest values of $f(x)$ on $[a, b]$, then $m(b-a) \le \int_a^b f(x)dx \le (b-a)M$

267

Statement 1: $f(x)$ is symmetrical about $x = 2$, then $\int_{2-a}^{2+a} f(x)dx$ is equal to $2\int_2^{2+a} f(x)dx$

Statement 2: If $f(x)$ is symmetrical about $x = b$, then $f(b-\alpha) = f(b+\alpha)\forall(\alpha \in R)$

268 Let f be a polynomial function of degree n

Statement 1: There exist a number $x \in [a, b]$ such that $\int_a^x f(t)dt = \int_x^b f(t)dt$

Statement 2: $f(x)$ is a continuous function

269

Statement 1: $\int_0^\pi x\sin x\cos^2 x\, dx = \frac{\pi}{2}\int_0^\pi \sin x\cos^2 x\, dx$

Statement 2: $\int_a^b x\,f(x)\,dx = \frac{a+b}{2}\int_a^b f(x)dx$

270

Statement 1: $\int e^{x^2} dx = e^{x^2} + c$

Statement 2: $\int e^{x^2} dx = e^{x} + c$

271

Statement 1: A polynomial of least degree that has a maximum equal to 6 at $x = 1$ minimum equal to 2 at $x = 3$ is $x^3 - 6x^2 + 9x + 2$

Statement 2: The polynomial is everywhere differentiable and the points of extremum can only be roots of derivative

272

Statement 1: The value of $\int_{-4}^{-5} \sin(x^2 - 3)dx + \int_{-2}^{-1} \sin(x^2 + 12x + 33)$ is zero

Statement 2: $\int_{-a}^{a} f(x)dx = 0$ if $f(x)$ is an odd function

273

Statement 1: $\int_0^{2\pi} \sin^3 x\,dx = 0$

Statement 2: $\sin^3 x$ is an odd function

274

Statement 1: $\int \frac{dx}{e^x + e^{-x} + 2} = -\frac{1}{e^x + 1} + c$

Statement 2: $\int \frac{d(f(x))}{\left(f(x)\right)^2} = -\frac{1}{f(x)} + c$

275 Consider $I_1 = \int_0^{\pi/4} e^{x^2}\,dx$, $I_2 = \int_0^{\pi/4} e^{x}\,dx$, $I_3 = \int_0^{\pi/4} e^{x^2}\cos x\;dx$, $I_4 = \int_0^{\pi/4} e^{x^2}\sin x\;dx$,

Statement 1: $I_2 > I_1 > I_3 > I_4$

Statement 2: For $x \in (0, 1)$, $x > x^2$ and $\sin x > \cos x$

276

Statement 1: If $I_n = \int \cot^n x\,dx$, then $5(I_6 + I_4) = -\cot^5 x$

Statement 2: If $I_n = \int \cot^n x\,dx$, then $I_n = \frac{\cot^{n-1}}{n} - I_{n-2}$, where $n \geq 2$

277

Statement 1: $\int \frac{(2-2x)}{\sqrt{(4+2x-x^2)}}dx = 2\sqrt{(4 + 2x - x^2)} + \sin^{-1}\left(\frac{x-1}{\sqrt{5}}\right) + c$

Statement 2: $\int \frac{dx}{\sqrt{a^2 - x^2}} = \frac{x}{2}\sqrt{(a^2 - x^2)} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{2}\right)$

278

Statement 1: $\int_a^x f(t)dt$ is an even function if $f(x)$ is an odd function

Statement 2: $\int_a^x f(t)dt$ is an odd function if $f(x)$ is an even function

279

Statement 1: The value of $\int_0^{2\pi} \cos^{99} x dx$is 0

Statement 2: $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$, if $f(2a-x) = f(x)$

280

Statement 1: The function $F(x) = \int \sin^2 x\, dx$ satisfies $F(x+\pi) = F(x), \forall\, x \in R$

Statement 2: $\sin^2(x+\pi) = \sin^2 x$

281

Statement 1: $\int \frac{\{f(x)\phi'(x) - f'(x)\phi(x)\}}{f(x)\phi(x)} \{\log\phi(x) - \log f(x)\}dx = \frac{1}{2}\left\{\log\frac{\phi(x)}{f(x)}\right\}^2 + c$

Statement 2: $\int \left(h(x)\right)^n h'(x)dx = \frac{(h(x))^{n+1}}{n+1} + c$

282

Statement 1: If $\int_0^1 e^{\sin x}dx = \lambda$, then $\int_0^{200} e^{\sin x}dx = 200\lambda$

Statement 2: $\int_0^{na} f(x)dx = n\int_0^a f(x)dx, n \in I$ and $f(a+x) = f(x)$

283

Statement 1: $\int_0^{\pi} \sqrt{1-\sin^2 x}\, dx = 0$

Statement 2: $\int_0^{\pi} \cos x dx = 0$

284

Statement 1: $\int \frac{dx}{x^3\sqrt{1+x^4}} = \frac{1}{2}\sqrt{1+\frac{1}{x^4}} + C$

Statement 2: For integrations by parts we have to follow ILATE rule

285

Statement 1: For $-1 < a < 4, \int \frac{dx}{x^2+2(a-1)x+a+5} = \lambda \log|g(x)| + c$, where λ and c are constants

Statement 2: For $-1 < a < 4, \frac{1}{x^2+2(a-1)x+a+5}$is a continuous function

286

Statement 1: $\int_0^x |\sin t| dt$, for $x \in [0, 2\pi]$ is a non-differentiable function

Statement 2: $|\sin t|$ is non-differentiable at $x = \pi$

287

Statement 1: The value of $\int_0^{\pi/4} \log(1+\tan\theta)d\theta = \frac{\pi}{8}\log 2$

Statement 2: The value of $\int_0^{\pi/2} \log\sin\theta d\theta = -\pi\log 2$

288

Statement 1: The value of $\int_0^1 \tan^{-1} \frac{2x-1}{(1+x-x^2)}\ dx = 0$

Statement 2: $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

289 Let $f(x)$ is continuous and positive for $x \in [a,b]$, $g(x)$ is continuous for $x \in [a,b]$ and $\int_a^b |g(x)|dx >$ /*abgxdx*/, then

Statement 1: The value of $\int_a^b f(x)g(x)dx$ can be zero

Statement 2: Equation $g(x) = 0$ has at least one root for $x \in (a,b)$

290

Statement 1: $\int e^x \sin x dx = \frac{e^x}{2}(\sin x - \cos x) + c$

Statement 2: $\int e^x(f(x) + f'(x))dx = e^x f(x) + c$

291

Statement 1: Let m be any integer. Then the value of $I_m = \int_0^\pi \frac{\sin 2mx}{\sin x} dx$ is zero

Statement 2: $I_1 = I_2 = I_3 = \cdots = I_m$

292 Let $F(x)$ be an indefinite integral of $\sin^2 x$.

Statement 1: The function $F(x)$ satisfies $F(x+\pi) = F(x)$ for all real x

Statement 2: $\sin^2(x+\pi) = \sin^2 x$ for all real x

293

Statement 1: On the interval $\left[\frac{5\pi}{4}, \frac{4\pi}{3}\right]$, the least value of the function $f(x) = \int_{5\pi/4}^{x}(3\sin t + 4\cos t)dt$ is 0

Statement 2: If $f(x)$ is a decreasing function on the interval $[a,b]$, then the least value of $f(x)$ is $f(b)$

294

Statement 1: If $\int \frac{1}{f(x)} dx = 2\log|f(x)| + c$, then $f(x) = \frac{x}{2}$

Statement 2: When $f(x) = \frac{x}{2}$, then $\int \frac{1}{f(x)} dx = \int \frac{2}{x} dx = 2\log|x| + c$

295

Statement 1: $\int_0^6 \{x+5\}^2 dx = 41$, where $\{\cdot\}$ denotes the fractional part function

Statement 2: $\{x+5\}$ is a periodic function

296

Statement 1: $\int \tan 5x \tan 3x \tan 2x\ dx = \frac{\log|\sec 5x|}{5} - \frac{\log|\sec 3x|}{3} - \frac{\log|\sec 2x|}{2} + c$

Statement 2: $\tan 5x - \tan 3x - \tan 2x = \tan 5x \tan 3x \tan 2x$

297

Statement 1: $\int \frac{\sin x\, dx}{x} (x > 0)$cannot be evaluated

Statement 2: Only differentiable functions can be integrated

298 If $n > 1$, then

Statement 1: $\int_0^{\infty} \frac{dx}{1 + x^n} = \int_0^1 \frac{dx}{(1 - x^n)^{1/n}}$

Statement 2: $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$

299

Statement 1: The value of $\int_0^{\pi} \sin^{100} x \cos^{99} x\, dx$ is zero

Statement 2: $\int_a^b f(x)dx = \int_{a+c}^{b+c} f(x - c)dx$ and for odd function $\int_{-a}^{a} f(x)dx = 0$

300

Statement 1: $\int \frac{xe^x}{(x + 1)^2} dx = \frac{e^x}{x + 1} + c$

Statement 2: $\int e^x(f(x) + f'(x)dx = e^x f(x) + c$

301 Consider the function $f(x)$ satisfying the relation $f(x + 1) + f(x + 7) = 0, \forall x \in R$,

Statement 1: The possible least value of t for which $\int_a^{a+t} f(x)dx$ is independent of a is 12

Statement 2: $f(x)$ is a periodic function

302

Statement 1: The value of the integral $\int_0^{\pi} \frac{\sin\left(n+\frac{1}{2}\right)x}{\sin\frac{x}{2}} dx$ $(n \in N)$ is π

Statement 2: $\int_0^{\pi} \sin mx\, dx = 0\ (m \in N)$

303

Statement 1: $\int_0^2 f(x)dx = \frac{4(\sqrt{2} - 1)}{3}$,

Where $f(x) = \begin{cases} x^2, \text{for } 0 \le x < 1 \\ \sqrt{x}, \text{for } 1 \le x \le 2 \end{cases}$

Statement 2: $f(x)$ is continuous in $[0, 2]$

304

Statement 1: If the primitive of $f(x) = \pi \sin \pi x + 2x - 4$ has the value 3 for $x = 1$, then there are exactly two values of x for which primitive of $f(x)$ vanishes

Statement 2: $\cos \pi x$ has period 2

305

Statement 1: $\int_{-\pi/2}^{\pi/2} |\sin x|\, dx = 2$

Statement 2: $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, where $c \in (a, b)$

306

Statement 1: If $y(x-y)^2 = x$, then $\int \frac{dx}{(x-3y)} = \frac{1}{2}\log\{(x-y)^2 - 1\}$

Statement 2: $\int \frac{dx}{(x-3y)} = \log(x-3y) + c$

307 Observe the following statements

Then, which of the following is true?

Statement 1: $\int \left(\frac{x^2-1}{x^2}\right) e^{\frac{x^2+1}{x}}, dx = e^{\frac{x^2+1}{x}} + c$

Statement 2: $\int f'(x) e^{f(x)} dx = f(x) + c$

308

Statement 1: $\int_0^1 e^{-x^2} \cos^2 x \, dx < \int_0^1 e^{-x^2} \cos^2 x \, dx$

Statement 2: $\int_a^b f(x) dx < \int_a^b g(x) \, dx, \forall f(x) \geq g(x)$

Matrix-Match Type

This section contain(s) 0 question(s). Each question contains Statements given in 2 columns which have to be matched. Statements (A, B, C, D) in **columns I** have to be matched with Statements (p, q, r, s) in **columns II**.

309.

	Column-I		Column- II
(A)	If $f(x)$ is an integrable function for $x \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ and $I_1 = \int_{\pi/6}^{\pi/3} \sec^2 \theta f\,(2\sin 2\theta) d\theta$ and $I_2 = \int_{\pi/6}^{\pi/3} \text{cosec}^2\,\theta f\,(2\sin 2\theta) d\theta$,then I_1/I_2	(p)	3
(B)	If $f(x+1) = f(3+x)$ for $\forall\, x$, and the value of $\int_a^{a+b} f(x) dx$is independent of a then the value of b can be	(q)	1
(C)	The value of $\int_1^4 \frac{\tan^{-1}[x^2]}{\tan^{-1}[x^2]+\tan^{-1}[25+x^2-10x]}$ (where [.] denotes the greatest integer function) is	(r)	2
(D)	If $I = \int_0^2 \sqrt{x + \sqrt{x + \sqrt{x + \cdots \infty}}}\, dx$ (where $x > 0$), then $[I]$ is equal to (where [.] denotes the greatest integer function)	(s)	4

CODES :

	A	**B**	**C**	**D**
a)	Q	r,s	p	p
b)	r	r	p	q
c)	s	p	q	s

d)	p	q	s	r

310.

	Column-I		Column- II
(A)	$\lim_{n\to\infty}\left[\int_0^2 \frac{\left(1+\frac{t}{n+1}\right)^n}{n+1}dt\right]$ is equal to	(p)	$e-\frac{1}{2}e^2-\frac{3}{2}$
(B)	Let $f(x)$ be a function satisfying $f'(x)=f(x)$ with $f(0)=1$ and g be the function satisfying $f(x)+g(x)=x^2$, then the value of the integral $\int_0^1 f(x)g(x)dx$ is	(q)	e^2
(C)	$\int_0^1 e^{ex}(1+xe^x)dx$ is equal to	(r)	e^2-1
(D)	$\lim_{k\to 0}\frac{1}{k}\int_0^k(1+\sin 2x)^{\frac{1}{x}}dx$ is equal to	(s)	e^e

CODES :

	A	B	C	D
a)	P	r	q	s
b)	r	p	s	q
c)	s	q	r	p
d)	q	s	p	r

311. If [·] denotes the greatest integer function, then match the following columns:

	Column-I		Column- II
(A)	$\int_{-1}^{1}\left[x+\left[x+[x]\right]\right]dx$	(p)	3
(B)	$\int_2^5([x]+[-x])\,dx$	(q)	5
(C)	$\int_{-1}^{3}\sin(x-[x])dx$	(r)	4
(D)	$25\int_0^{\pi/4}(\tan^6(x-[x])+\tan^4(x-[x]))\,dx$	(s)	−3

CODES :

	A	B	C	D
a)	S	s	r	q
b)	q	p	s	r
c)	p	q	r	s
d)	r	s	q	p

312.

Column-I		Column- II	
(A)	$\int \frac{x^2 - x + 1}{x^3 - 4x^2 + 4x} dx$	(p)	$\log\lvert x\rvert$
(B)	$\int \frac{x^2 - 1}{x(x-2)^3} dx$	(q)	$\log\lvert x-2\rvert$
(C)	$\int \frac{x^3 + 1}{x(x-2)^2} dx$	(r)	$\frac{1}{(x-2)}$
(D)	$\int \frac{x^5 + 1}{x(x-2)^3} dx$	(s)	x

CODES :

	A	B	C	D
a)	P,q,r,s	p,q,r	r,s	p,q
b)	r,s	p,q,r,s	p,q,r	r,s
c)	p,q,r	p,q,r	pq,r,s	p,q,r,s
d)	p,q	r,s	p,q,r	p,q,r,s

313.

Column-I		Column- II	
(A)	If $I = \int_{-2}^{2} (\alpha x^3 + \beta x + \gamma) dx$,then I is	(p)	Independent of α
(B)	Let α, β be the distinct positive roots of the equation $\tan x = 2x$, then $\gamma \int_0^1 (\sin \alpha x \sin \beta x) dx$ (where $\gamma \neq 0$) is	(q)	Independent of β
(C)	If $(x + \alpha) + f(x) = 0$, where $\alpha > 0$, then $\int_{\beta}^{\beta + 2\gamma\alpha} f(x) dx$, where $\gamma \in N$, is	(r)	Independent of γ
(D)	$\gamma \int_0^{\alpha} [\sin x] dx$ is, where $\gamma \neq 0$, $\alpha \in [(2\beta + 1)\pi, (2\beta + 2)\pi] n \in N$, and where $[.]$ denotes the greatest integer function	(s)	Depends on α

CODES :

	A	B	C	D
a)	p, q	p, q, r	q, s	s
b)	s	p, q	p, q, r	q, s
c)	p, q, r	s	p, q	q, s
d)	q, s	p, q	s	p, q, r

314.

Column-I		Column- II	
(A)	If $\int \frac{2^x}{\sqrt{1-4^x}} dx = k \sin^{-1}(f(x)) + C$, then k is greater than	(p)	0

(B) If $\int \frac{(\sqrt{x})^5}{(\sqrt{x})^7+x^6} dx = a \ln \frac{x^k}{x^k+1} + c$, then ak is less than (q) 1

(C) $\int \frac{x^4+1}{x(x^2+1)^2} dx = k \ln|x| \frac{m}{1+x^2} + n$, where n is the constant of integration, then mk is grater than (r) 3

(D) $\int \frac{dx}{5+4\cos x} = k \tan^{-1}\left(m \tan\frac{x}{2}\right) + C$, then k/m is greater than (s) 4

CODES :

	A	B	C	D
a)	P,q	r,s	p	p,q
b)	r,s	p	p,q	s
c)	p	p,q	r,s	q
d)	q	p	q	r,s

315.

Column-I	Column- II
(A) $\int \frac{e^{2x}-1}{e^{2x}+1} dx$ is equal to	(p) $x - \log\left[1 + \sqrt{1-e^{2x}}\right] + c$
(B) $\int \frac{1}{(e^x+e^{-x})} dx$ is equal to	(q) $\log(e^x + 1) - x - e^{-x} + c$
(C) $\int \frac{e^{-x}}{1+e^x} dx$ is equal to	(r) $\log(e^{2x} + 1) - x + c$
(D) $\int \frac{1}{\sqrt{1-e^{2x}}} dx$ is equal to	(s) $-\frac{1}{2(e^{2x}+1)} + c$

CODES :

	A	B	C	D
a)	s	r	p	q
b)	r	s	q	p
c)	p	q	r	s
d)	q	p	s	r

Linked Comprehension Type

This section contain(s) 22 paragraph(s) and based upon each paragraph, multiple choice questions have to be answered. Each question has atleast 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 316 to -316

Let $f(x)$ be a continuous function defined on the closed interval $[a, b]$, then $\lim_{n\to\infty} \sum_{r=0}^{n-1} \frac{1}{n} f\left(\frac{r}{n}\right) = \int_0^1 f(x)dx$

316. The value of $\lim_{n\to\infty} \frac{1}{n}\left\{\frac{1}{n+1} + \frac{2}{n+2} + \ldots + \frac{3n}{4n}\right\}$ is

a) $5 - 2$ In 2 b) $4 - 2$ In 2 c) $3 - 2$ In 2 d) $2 - 2$ In 2

Paragraph for Question Nos. 317 to - 317

If m and M are the smallest and greatest values of a function $f(x)$ defined on an interval $[a, b]$, then answer the following questions

317. If $a \leq \int_0^1 e^{x^2} dx \leq b$ then

a) $a = 0, b = 1$ b) $a = e, b = 1$ c) $a = 2, b = 1$ d) $a = 1, b = e$

Paragraph for Question Nos. 318 to - 318

If $f(x)$ and $g(x)$ be two functions, such that $f(a) = g(a) = 0$ and f and g are both differentiable at everywhere in some neighbourhood of point a except possibly $'a'$.

Then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$ provided $f'(a)$and $g'(a)$ are not both zero

318. The value of $\lim_{x \to 0} \frac{\int_0^{x^2} \sin\sqrt{t}\, dt}{x^3}$ is

a) 0 b) 2/9 c) 1/3 d) 2/3

Paragraph for Question Nos. 319 to - 319

Repeated application of integration by parts gives us, the reduction formula if the integrand is dependent of $n, n \in N$.

On the basis of above information, answer the following question :

319. If $I_n = \int \tan^n x\, dx$ and $I_n = -\frac{\tan^{n-1} x}{(n-1)} + \lambda I_{n-2}$, then λ is equal to

a) $\frac{1}{(n-1)}$ b) $\frac{1}{(n-2)}$ c) $\frac{1}{n}$ d) None of these

Paragraph for Question Nos. 320 to - 320

If the integrand is a rational function of x and fractional powers of a linear fractional function of the form $\frac{ax+b}{cx+d}$, then rationalization of the integral is affected by the substitution $\frac{ax+b}{cx+d} = t^m$, where m is LCM of fractional powers of $\frac{ax+b}{cx+d}$.

on the basis of above information, answer the following questions :

320. If $I = \int \frac{dx}{\sqrt[4]{(x-1)^3(x+2)^5}} = A\sqrt[4]{\frac{x-1}{x+2}} + c$, then A is equal to

a) 1/3 b) 2/3 c) 3/4 d) 4/3

Paragraph for Question Nos. 321 to - 321

$y = f(x)$ is a polynomial function passing through point (0, 1) and which increases in the intervals (1, 2) and

$(3,\infty)$ and decreases in the interval $(-\infty, 1)$ and $(2, 3)$

321. If $f(1) = -8$, then the value of $f(2)$ is

a) $1 - 3$ b) -6 c) -20 d) -7

Paragraph for Question Nos. 322 to - 322

If A is square matrix and e^A if defined as $e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots = \frac{1}{2}\begin{bmatrix} fx & g(x) \\ g(x) & f(x) \end{bmatrix}$, where $A = \begin{bmatrix} x & x \\ x & x \end{bmatrix}$ and $0 < x < 1, I$ is an identify matrix

322. $\int \frac{g(x)}{f(x)} dx$ is equal to

a) $\log(e^x + e^{-x}) + c$ b) $\log|e^x - e^{-x}| + c$ c) $\log|e^x - 1| + c$ d) None of these

Paragraph for Question Nos. 323 to - 323

Euler's substitution

Integrals of the form $\int R(x, \sqrt{ax^2 + bx + c})\, dx$ are calculated with the aid of one of the three Euler substitutions

1. $\sqrt{ax^2 + bx + c} = t \pm x\sqrt{a}$ if $a > 0$;
2. $\sqrt{ax^2 + bx + c} = tx \pm \sqrt{c}$ if $c > 0$;
3. $\sqrt{ax^2 + bx + c} = (x - a)t$ if $ax^2 + bx + c = a(x - a)(x - b)$ i.e., if α is a real root of $ax^2 + bx + c = 0$

323. Which of the following functions does not appear in the primitive of $\frac{1}{1+\sqrt{x^2+2x+2}}$ if t is a function of x?

a) $\log_e|t + 1|$ b) $\log_e|t + 2|$ c) $\frac{1}{t + 2}$ d) None of these

Paragraph for Question Nos. 324 to - 324

$y = f(x)$ satisfies the relation $\int_2^x f(t)dt = \frac{x^2}{2} + \int_x^2 t^2 f(t)dt$

324. The range of $y = f(x)$ is

a) $[0, \infty)$ b) R c) $[-\infty, 0)$ d) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

Paragraph for Question Nos. 325 to - 325

Let $f: R \to R$ be a differentiable function such that

$$f(x) = x^2 + \int_0^x e^{-1} f(x - t)dt$$

325. $f(x)$ increases for

a) $x > 1$ b) $x < -2$ c) $x > 2$ d) None of these

Paragraph for Question Nos. 326 to - 326

$f(x)$ satisfies the relation $f(x) - \lambda \int_0^{\pi/2} \sin x \cos t\, f(t) dt = \sin x$

326. If $\lambda > 2$, then $f(x)$ decreases in which of the following interval?

a) $(0, \pi)$ b) $(\pi/2, 3\pi/2)$ c) $(-\pi/2, \pi/2)$ d) None of these

Paragraph for Question Nos. 327 to - 327

Let $f(x)$ and $\phi(x)$ are two continuous functions on R satisfying $\phi(x) = \int_a^x f(t)dt, a \neq 0$and another continuous function g(x) satisfying g$(x + \alpha)$ + g$(x) = 0\ \forall\ x \in R, \alpha > 0$and $\int_h^{2k}$ g$(t)dt$ is independent of b

327. If $f(x)$ is an odd function, then

a) $\phi(x)$ is also an odd function

b) $\phi(x)$is an even function

c) $\phi(x)$ is neither as even nor an odd function

d) For $\phi(x)$ to be an even function, it must satisfy $\int_0^a f(x)dx = 0$

Paragraph for Question Nos. 328 to - 328

Evaluating integrals Dependent on a Parameter

Differentiate I with respect to the parameter within the sign of integrals taking variable of the integrand as constant. Now, evaluate the integral so obtained as a function of the parameter and then integrate the result to get I. Constant of integration can be computed by giving some arbitrary values to the parameter and the corresponding value of I

328. The value of $\int_0^1 \frac{x^a - 1}{\log x} dx$ is

a) $\log(a-1)$ b) $\log(a+1)$ c) $a \log(a+1)$ d) None of these

Paragraph for Question Nos. 329 to - 329

$$f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + t \cos x) f(t) dt$$

329. The range of $f(x)$ is

a) $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right]$ b) $\left[-\frac{\sqrt{5}}{3}, \frac{\sqrt{5}}{3}\right]$ c) $\left[-\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2}\right]$ d) None of these

Integer Answer Type

330. Let $f(x) = \int_0^x \frac{dt}{\sqrt{1+t^3}}$and g$(x)$ be the inverse of $f(x)$, then the value of $4\frac{g''(x)}{(g(x))^2}$ is

331. If $I_n = \int_0^1 (1 - x^5)^n dx$, then $\frac{55}{7} \frac{I_{10}}{I_{11}}$ is equal to

332. The value of $\int_0^{\frac{3\pi}{2}} \frac{|\tan^{-1}\tan x|-|\sin^{-1}\sin x|}{|\tan^{-1}\tan x|+|\sin^{-1}\sin x|}\,dx$ is equal to

333. If $\int \frac{2\cos x-\sin x+\lambda}{\cos x+\sin x-2}dx = A\ln|\cos x+\sin x-2|+Bx+C$. Then the value of $A+B+|\lambda|$ is

334. If the value of the definite integral $\int_0^1 \frac{\sin^{-1}\sqrt{x}}{x^2-x+1}dx = \frac{\pi^2}{\sqrt{n}}$ (where $n \in N$), then the value of $n/27$ is

335. If $\int x^2 \cdot e^{-2x}\,dx = e^{-2x}(ax^2+bx+c)+d$, then the value of $|a/bc|$ is

336. If $\int_0^{\infty} x^{2n+1} \cdot e^{-x^2 dx} = 360$, then the value of n is

337. The value of the definite integral $\int_{\sqrt{2}-1}^{\sqrt{2}+1} \frac{x^4+x^2+2}{(x^2+1)^2}dx$ equals

338. Let $f:[0,\infty] \to R$ be a continuous strictly increasing function, such that $f^3(x) = \int_0^x t \cdot f^2(t)dt$ for every $x \geq 0$, then value of $f(6)$ is

339. If f is continuous function and
$F(x) = \int_0^x \left((2t+3)\cdot\int_t^2 f(u)du\right)dt$, then $|F''(2)/f(2)|$ is equal to

340. If $F(x) = \frac{1}{x^2}\int_4^x[4t^2-2F'(t)]\,dt$, then $(9F'(4))/4$ is

341. $\lim_{n\to\infty}\frac{n}{2^n}\int_0^2 x^n\,dx$ equals

342. A continuous real function f satisfies $f(2x) = 3f(x)\forall x \in R$. If $\int_0^1 f(x)dx = 1$, then the value of definite integral $\int_1^2 f(x)dx$is

343. The value of $2^{2010}\,\frac{\int_0^1 x^{1004}(1-x)^{1004}\,dx}{\int_0^1 x^{1004}(1-x^{2010})^{1004}\,dx}$ is

344. Let $f(x) = \int x^{\sin x}(1+x\cos x\cdot\ln x+\sin x)dx$ and $f\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4}$, then the value of $\left|\cos\left(f(\pi)\right)\right|$ is

345. If the value of $\text{Lim}_{n\to\infty}\left(n^{-3/2}\right)\cdot\sum_{j=1}^{6n}\sqrt{j}$ is equal to $\sqrt{N}$, then the value of $N/12$ is

346. If the value of the definite integral $\int_0^1 {}^{207}C_7 x^{200}\cdot(1-x)^7dx$ is equal to $\frac{1}{k}$ where $k \in N$, then the value of $k/26$ is

347. Let $g(x) = \int \frac{1+2\cos x}{(\cos x+2)^2}dx$ and $g(0) = 0$, then the value of $8g(\pi/2)$ is

348. Let $J = \int_{-5}^{-4}(3-x^2)\tan(3-x^2)dx$ and $K = \int_{-2}^{-1}(6-6x+x^2)\tan(6x-x^2-6)dx$, then $(J+K)$ equals

349. If $f(x) = \int \frac{3x^2+1}{(x^2-1)^3}dx$ and $f(0) = 0$, then the value of $|2/f(2)|$ is

350. Let $f(x) = x^3 - \frac{3x^2}{2} + x + \frac{1}{4}$.Then the value of $\left(\int_{1/4}^{3/4} f(f(x))dx\right)^{-1}$ is

351. Let $f(x)$ is a derivable function satisfying $f(x) = \int_0^x e^t\sin(x-t)dt$and $g(x) = f''(x)-f(x)$, then the possible integers in the range of $g(x)$ is

352. Let $g(x)$ be differentiable on $R$and $\int_{\sin t}^1 x^2g(x)dx = (1-\sin t)$, where $t \in \left(0,\frac{\pi}{2}\right)$. Then the value of $g\left(\frac{1}{\sqrt{2}}\right)$ is

353. If $\int_0^{100} f(x)dx = 7$, then $\sum_{r=1}^{100}\left(\int_0^1 f(r-1+x)dx\right) =$

354. Consider the polynomial $f(x) = ax^2+bx+c$. If $f(0) = 0, f(2) = 2$, then the minimum value of $\int_0^2 |f'(x)|\,dx$ is

355. If $f(x) = \sqrt{x}, g(x) = e^x-1$, and $\int fog\,(x)\,dx = A\,fog(x)+B\tan^{-1}(fog(x))+C$, then $A+B$ is equal to

356. Let $k(x) = \int \frac{(x^2+1)dx}{\sqrt[3]{x^3+3x+6}}$ and $k(-1) = \frac{1}{\sqrt[3]{2}}$, then the value of $k(-2)$ is

357. If $f(x) = x + \int_0^1 t(x+t)f(t)dt$, then the value of $\frac{23}{2}f(0)$ is equal to

358. If $\int\left[\left(\frac{x}{e}\right)^x + \left(\frac{e}{x}\right)^x\right]\ln x\,dx = A\left(\frac{e}{x}\right)^x + B\left(\frac{e}{x}\right)^x + C$, then the value of $A+B$ is

359. If $I = \int_0^{3\pi/5}\left((1+x)\sin x+(1-x)\cos x\right)dx$, then the value of $(\sqrt{2}-1)I$ is

360. Consider a real valued continuous function f such that $f(x) = \sin x + \int_{-\pi/2}^{\pi/2}(\sin x + t\,f(t))dt$. If M and m are maximum and minimum value of the function f, then the value of M/m is

361. If $U_n = \int_0^1 x^n (2-x)^n dx$and $V_n = \int_0^1 x^n (1-x)^n dx$ $n \in N$, and if $\frac{V_n}{U_n} = 1024$, then the value of n is

: ANSWER KEY :

1)	a	2)	b	3)	c	4)	a	189)	d	190)	c	191)	c	192)	c
5)	a	6)	a	7)	b	8)	c	193)	a	194)	a	195)	a	196)	b
9)	b	10)	c	11)	a	12)	b	197)	d	198)	c	199)	a	200)	c
13)	b	14)	a	15)	b	16)	c	201)	c	202)	b	203)	d	204)	b
17)	c	18)	b	19)	a	20)	c	205)	a	206)	c	207)	d	208)	b
21)	b	22)	b	23)	a	24)	d	209)	a	210)	a	211)	a	212)	c
25)	c	26)	b	27)	b	28)	b	213)	a	214)	b	215)	a	216)	d
29)	c	30)	b	31)	b	32)	b	217)	a	1)	a,b,d	2)	b,d	3)	
33)	a	34)	c	35)	b	36)	b		a,b,d	4)	a,c,d				
37)	b	38)	c	39)	a	40)	b	5)	b,d	6)	a,b,c	7)	b,c,d	8)	
41)	a	42)	c	43)	b	44)	c		a,d						
45)	c	46)	d	47)	b	48)	c	9)	a,c	10)	a,b,c	11)	a,c,d	12)	
49)	b	50)	b	51)	a	52)	b		b,c						
53)	d	54)	c	55)	a	56)	d	13)	b,c	14)	a,d	15)	a	16)	
57)	c	58)	a	59)	b	60)	d		b,d						
61)	b	62)	d	63)	c	64)	c	17)	a	18)	a,d	19)	a,c	20)	
65)	d	66)	a	67)	a	68)	b		a,b						
69)	c	70)	a	71)	a	72)	b	21)	b,c,d	22)	a,b	23)	a,b,c	24)	
73)	d	74)	b	75)	a	76)	d		a,d						
77)	a	78)	b	79)	c	80)	a	25)	a,b	26)	a,c	27)	a,b,c,d	28)	
81)	a	82)	a	83)	d	84)	c		a,b,d						
85)	d	86)	a	87)	a	88)	a	29)	c	30)	a,b,d	31)	a,c,d	32)	
89)	b	90)	c	91)	a	92)	b		a,c						
93)	c	94)	b	95)	a	96)	b	33)	a,d	34)	a,b,c,d	35)	a,c	36)	
97)	c	98)	c	99)	a	100)	b		a,b,d						
101)	b	102)	c	103)	b	104)	a	37)	a,c,d	38)	b,c,d	39)	a,b,c,d	40)	
105)	c	106)	a	107)	c	108)	c		a,b,c						
109)	d	110)	d	111)	a	112)	a	41)	a,d	42)	a,b,d	43)	a,b,c,d	44)	
113)	a	114)	a	115)	b	116)	c		c,d						
117)	d	118)	c	119)	a	120)	c	45)	a,b	46)	a,b,c	1)	b	2)	d
121)	a	122)	a	123)	c	124)	b		3)	a	4)	a			
125)	c	126)	c	127)	c	128)	d	5)	a	6)	c	7)	d	8)	a
129)	c	130)	a	131)	a	132)	d	9)	b	10)	b	11)	a	12)	c
133)	d	134)	d	135)	c	136)	c	13)	c	14)	c	15)	c	16)	b
137)	b	138)	d	139)	a	140)	a	17)	a	18)	a	19)	d	20)	d
141)	c	142)	a	143)	a	144)	c	21)	b	22)	d	23)	d	24)	c
145)	c	146)	c	147)	c	148)	b	25)	a	26)	a	27)	a	28)	a
149)	a	150)	c	151)	b	152)	c	29)	d	30)	d	31)	a	32)	d
153)	b	154)	a	155)	c	156)	a	33)	a	34)	b	35)	b	36)	a
157)	b	158)	b	159)	c	160)	b	37)	a	38)	a	39)	c	40)	d
161)	a	162)	c	163)	c	164)	a	41)	b	42)	b	43)	c	44)	c
165)	d	166)	b	167)	a	168)	c	45)	b	1)	a	2)	b	3)	a
169)	b	170)	c	171)	a	172)	b		4)	c					
173)	c	174)	b	175)	d	176)	c	5)	a	6)	a	7)	b	1)	c
177)	c	178)	b	179)	c	180)	c		2)	d	3)	d	4)	d	
181)	c	182)	d	183)	b	184)	b	5)	d	6)	d	7)	a	8)	d
185)	c	186)	c	187)	c	188)	d	9)	d	10)	b	11)	c	12)	b

13)	**b**	**14)**	**b**	**1)**	**6**	**2)**	**8**
	3)	**0**	**4)**	**3**			
5)	**4**	**6)**	**4**	**7)**	**6**	**8)**	**2**
9)	**6**	**10)**	**7**	**11)**	**8**	**12)**	**2**
13)	**5**	**14)**	**4**	**15)**	**1**	**16)**	**8**
17)	**8**	**18)**	**4**	**19)**	**0**	**20)**	**9**
21)	**4**	**22)**	**3**	**23)**	**2**	**24)**	**7**
25)	**2**	**26)**	**0**	**27)**	**2**	**28)**	**9**
29)	**0**	**30)**	**2**	**31)**	**3**	**32)**	**5**

: HINTS AND SOLUTIONS :

1 **(a)**

Put $x = \tan\theta \ \therefore dx = \sec^2\theta d\theta$

When $x = \infty, \tan\theta = \infty, \therefore \theta = \pi/2$

$\therefore \ I = \int_0^{\pi/2} \frac{\tan\theta \sec^2\theta}{(1+\tan\theta)(\sec^2\theta)} d\theta \quad (1)$

Now changing equation (1) into $\sin\theta$ and $\cos\theta$

$\therefore I = \int_0^{\pi/2} \frac{\sin\theta d\theta}{\cos\theta + \sin\theta} = \frac{\pi}{4}$

2 **(b)**

$\int x \sin x \sec^3 x \, dx$

$= \int x \sin x \frac{1}{\cos^3 x} dx$

$= \int x \tan x \ \sec^2 x \, dx$

$= x \int \sec x (\sec x \tan x) dx$

$- \int [\sec x (\sec x \tan x) dx] dx + C$

$= x \frac{\sec^2 x}{2} - \int \frac{\sec^2 x}{2} dx + C$

$= x \frac{\sec^2 x}{2} - \frac{\tan x}{2} + C$

3 **(c)**

$I = \int_{-2}^{0} [x^3 + 3x^2 + 3x + 3$

$+ (x+1)\cos(x+1)] dx$

$= \int_{-2}^{0} [(x+1)^3 + 2 + (x+1)\cos(x+1)] dx$

Put, $x + 1 = t \Rightarrow dx = dt$

$\therefore \ I = \int_{-1}^{1} t^3 dt + 2\int_{-1}^{1} dt + \int_{-1}^{1} t \cos t \, dt$

$= 0 + 2[1 - (-1)] + 0$

$\Rightarrow I = 4 \left[\begin{array}{c} \because t^3 \text{and } t\cos t \text{ are odd functions.} \\ \therefore \int_{-1}^{1} t^3 dt = \int_{1}^{1} t \cos t \, dt = 0 \end{array} \right]$

4 **(a)**

$I = \int \sqrt{e^x - 1} \, dx$

Let $e^x - 1 = t^2 \Rightarrow e^x \, dx = 2t \, dt \ \Rightarrow dx = \frac{2t}{t^2+1} dt$

$\Rightarrow I = \int t \frac{2t}{t^2+1} dt = \int \frac{2t^2}{t^2+1} dt$

$= \int \frac{2(t^2+1) - 2}{t^2+1} dt = \int 2dt - \int \frac{2dt}{t^2+1}$

$= 2t - 2\tan^{-1} t + C$

$= 2\sqrt{e^x - 1} - 2\tan^{-1}\sqrt{e^x - 1} + C$

5 **(a)**

$\int \frac{e^x(x^2+1)}{(x+1)^2} dx$

$= \int \frac{e^x(x^2 - 1 + 2)}{(x+1)^2} dx$

$= \int e^x \left[\frac{x-1}{x+1} + \frac{2}{(x+1)^2}\right] dx$

$= \int e^x [f(x) + f'(x)] dx$, where $f(x) = \frac{x-1}{x+1}$ and

$f'(x) = \frac{2}{(x+1)^2}$

$= e^x \left(\frac{x-1}{x+1}\right) + C$

6 **(a)**

$I = \int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos x} \ (1)$

$= \int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos(\pi - x)}$

$\left[\text{Using the property } \int_a^b f(x) dx = \int_a^b (f(a+b-x)) dx\right]$

$= \int_{\pi/4}^{3\pi/4} \frac{dx}{1-\cos x} \ (2)$

Adding (1) and (2), we get

$2I = \int_{\pi/4}^{3\pi/4} \left(\frac{1}{1+\cos x} + \frac{1}{1-\cos x}\right) dx$

$= \int_{\pi/4}^{3\pi/4} 2\,\text{cosec}^2 x \, dx$

$= 2(-\cot x)_{\pi/4}^{3\pi/4}$

$= -2[\cot 3\pi/4 - \cot \pi/4]$

$= -2(-1-1) = 4$

$\Rightarrow \ I = 2$

7 **(b)**

$I = \int_2^4 (x(3-x)(4+x)(6-x)(10-x) +$
$\sin x) dx) \ (1)$

$= \int_2^4 \Big((6-x)\big(3-(6-x)\big)\big(4+(6-x)\big)\big(6-(6$

$-x)\big)\big(10-(6-x)\big)$

$+ \sin(6-x)\Big) dx$

$= \int_2^4 \big((6-x)(x-3)(10-x)x(4+x) +$
$\sin 6-x dx \ (2)$

Adding equations (1) and (2), we get

$$2I = \int_2^4 (\sin x + \sin(6-x))dx$$

$= (-\cos x + \cos(6-x))_2^4$

$= -\cos 4 + \cos 2 + \cos 2 - \cos 4$

$= 2(\cos 2 - \cos 4)$

$\Rightarrow \ I = \cos 2 - \cos 4$

8 **(c)**

$$I = -e^{-x}\log(e^x+1) + \int \frac{e^{-x}e^x}{e^x+1}dx$$

$$= -e^{-x}\log(e^x+1) + \int \frac{e^{-x}}{e^{-x}+1}dx$$

$= -e^{-x}\log(e^x+1) - \log(e^{-x}+1) + C$

$= -e^{-x}\log(e^x+1) - \log(1+e^x) + x + C$

$= -(e^{-x}+1)\log(e^x+1) + x + C$

9 **(b)**

$\int_a^b f(x)dx = [xf(x)]_a^b - \int_a^b xf'(x)dx (1)$

Now, put $f(x) = t \ \therefore \ x = f^{-1}(t)$

and $f'(x)dx = dt$and adjust the limits

Therefore, $\int_a^b f(x)dx = [bf(b) - af(a)] -$

f(a)f(b)f−1tdt by (1)

$$\therefore \int_a^b f(x) + \int_{f(a)}^{f(b)} f^{-1}(x)dx = bf\ (b) - af(a)$$

10 **(c)**

$$I_{4,3} = \int \cos^4 x \sin 3x dx$$

Integrating by parts, we have

$$I_{4,3} = -\frac{\cos 3x \cos^4 x}{3} - \frac{4}{3}\int \cos^3 x \sin x \cos 3x dx$$

But $\sin x \cos 3x = -\sin 2x + \sin 3x \cos x$, so

$$I_{4,3} = -\frac{\cos x \cos^4 x}{3} + \frac{4}{3}\int \cos^3 x \sin 2x dx - \frac{4}{3}\int \cos^4 x \sin 3x dx + C$$

$$= -\frac{\cos 3x \cos^4 x}{3} + \frac{4}{3}I_{3,2} - \frac{4}{3}I_{4,3} + C$$

Therefore, $\frac{7}{3}I_{4,3} - \frac{4}{3}I_{3,2} = -\frac{\cos 3x \cos^3 x}{3} + C$

Or $7I_{4,3} - 4I_{3,2} = -\cos 3x \cos^4 x + C$

11 **(a)**

$$\int_{-20\pi}^{20\pi} |\sin x|[\sin x]dx$$

$$= \int_0^{20\pi} |\sin x|([\sin x] + [-\sin x])\ dx$$

$$= -20\int_0^{\pi} (\sin x)dx = -20\,(-\cos x)_0^{\pi} = 20\,(-2)$$

$= -40$

12 **(b)**

$$I = \int \frac{\cos x - \sin x}{\sqrt{\cos x \sin x}}dx$$

Putsin $x + \cos x = t$, so that $2\sin x \cos x = \ t^2 - 1$

$$\therefore \ I = \sqrt{2}\int \frac{dt}{\sqrt{t^2-1}}$$

$$= \sqrt{2}\log\left|t + \sqrt{t^2-1}\right| + c$$

$= \sqrt{2}\log\left|\sin x + \cos x + \sqrt{\sin 2x}\right| + C$

13 **(b)**

$I = \int_0^{\pi/2} \sqrt{\tan x}\ dx \quad (1)$

$\Rightarrow \ I = \int_0^{\pi/2} \sqrt{\cot x}\, dx \quad (2)$

Adding equations (1) and (2), we get

$$2I = \int_0^{\pi/2} \left(\sqrt{\tan x} + \sqrt{\cot x}\right)dx$$

$$= \sqrt{2}\int_0^{\pi/2} \frac{\sin x + \cos x}{\sqrt{\sin 2x}}dx$$

$$= \sqrt{2}\int_0^{\pi/2} \frac{\sin x + \cos x}{\sqrt{1-(\sin x - \cos x)^2}}dx$$

$= \sqrt{2}\int_{-1}^{1} \frac{dt}{\sqrt{1-t^2}}$(where $\sin x - \cos x = t$)

$$= 2\sqrt{2}\int_0^1 \frac{dt}{\sqrt{1-t^2}} = \sqrt{2}\pi$$

$$\Rightarrow I = \frac{\pi}{\sqrt{2}}$$

14 **(a)**

Here, $ff(x) = \frac{f(x)}{[1+f(x)^n]^{1/n}} = \frac{x}{(1+2x^n)^{1/n}}$

and $fff(x) = \frac{x}{(1+3x^n)^{1/n}}$

$$g(x) = \underbrace{(f o f o \ldots . o f)}_{n \text{ times}}(x).$$

$$\therefore \quad = \frac{x}{(1+nx^n)^{1/n}}$$

Let $I = \int x^{n-2}\, g(x)dx = \int \frac{x^{n-1}dx}{(1+nx^n)^{1/n}}$

$$= \frac{1}{n^2}\int \frac{n^2 x^{n-1}dx}{(1+nx^n)^{1/n}}$$

$$= \frac{1}{n^2}\int \frac{\frac{d}{dx}(1+nx^n)}{(1+nx^n)^{1/n}}\ dx$$

$$\therefore \ I = \frac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}} + c$$

15 **(b)**

Given $\lambda = \int_0^1 \frac{e^t}{1+t}\ dt$

$$\int_0^1 e^t \log_e(1+t)dt$$
$$= [\log_e(1+t)e^t]_0^1 - \int_0^1 \frac{e^t}{1+t} = e\log_e 2 - \lambda$$

16 **(c)**

$\sin^3 x \sin(x+\alpha)$
$= \sin^3 x(\sin x\cos\alpha + \cos x\sin\alpha)$
$= \sin^4 x(\cos\alpha + \cot x\sin\alpha)$

$$I = \int \frac{1}{\sqrt{\sin^3 x\sin(x+\alpha)}}dx$$
$$= \int \frac{1}{\sin^2 x\sqrt{\cos\alpha + \cot x\sin\alpha}}dx$$
$$= \int \frac{\mathrm{cosec}^2 x}{\sqrt{\cos\alpha + \cot x\sin\alpha}}dx$$

Putting

$\cos\alpha + \cot x\sin\alpha = t$ and$-\mathrm{cosec}^2 x\sin\alpha dx = dt$, we have

$$I = \int -\frac{1}{\sin\alpha\sqrt{t}}\,dt = -\frac{1}{\sin\alpha}\int t^{-1/2}\,dt$$
$$= \frac{1}{\sin\alpha}\left(\frac{t^{1/2}}{1/2}\right) + C$$
$$\Rightarrow I = -2\,\mathrm{cosec}\,\alpha\sqrt{t} + C$$
$$= -2\,\mathrm{cosec}\,\alpha(\cos\alpha + \cot x\sin\alpha)^{1/2} + C$$

17 **(c)**

$$\int \frac{\sin 2x}{\sin 5x\sin 3x}\,dx$$
$$= \int \frac{\sin(5x-3x)}{\sin 5x\sin 3x}$$
$$= \int \frac{\sin 5x\cos 3x - \cos 5x\sin 3x}{\sin 5x\sin 3x}\,dx$$
$$= \frac{1}{3}\log\sin 3x - \frac{1}{5}\log\sin 5x + C$$

18 **(b)**

$$I = \int \frac{\sin 2x}{\sin^4 x + \cos^4 x}dx$$
$$= \int \frac{2\sin x\cos x}{\sin^4 x + \cos^4 x}dx$$
$$= \int \frac{2\tan x\sec^2 x}{1+\tan^4 x}dx$$

Let $\tan^2 x = t \Rightarrow 2\tan x\ \sec^2 x\,dx = dt$

$$\Rightarrow I = \int \frac{dt}{1+t^2} = \tan^{-1} + C = \tan^{-1}(\tan^2 x) + C$$

19 **(a)**

Let $I = \int \frac{3+2\cos x}{(2+3\cos x)^2}dx$, Multiplying N^r and D^r by $\mathrm{cosec}^2 x$, we get

$$\Rightarrow I = \int \frac{(3\,\mathrm{cosec}^2 x + 2\cot x\,\mathrm{cosec}\,x)}{(2\,\mathrm{cosec}\,x + \cot x)^2}dx$$
$$= -\int \frac{-3\,\mathrm{cosec}^2 x - 2\cot x\,\mathrm{cosec}\,x}{(2\,\mathrm{cosec}\,x + 3\cot x)^2}dx$$
$$= \frac{1}{2\,\mathrm{cosec}\,x + 3\cot x} + C = \left(\frac{\sin x}{2+3\cos x}\right) + C$$

20 **(c)**

Given, $\int_0^x \sqrt{1-(f'(t))^2}\ dt = \int_0^x f(t)\ dt, 0 \le x \le 1$

Applying Leibnitz theorem, we get

$$\sqrt{1-(f'(x))^2} = f(x)$$
$$\Rightarrow 1-(f'(x))^2 = f^2(x)$$
$$\Rightarrow (f'(x))^2 = 1 - f^2(x)$$
$$\Rightarrow f'(x) = \pm\sqrt{1-f^2(x)}$$
$$\Rightarrow \frac{dy}{dx} = \pm\sqrt{1-y^2}, \text{where } y = f(x)$$
$$\Rightarrow \frac{dy}{\sqrt{1-y^2}} = \pm dx$$

On integrating both sides, we get

$\sin^{-1}(y) = \pm x + C$

$\because f(0) = 0 \Rightarrow C = 0$

$\therefore y = \pm\sin x$

$y = \sin x = f(x)$ given $f(x) \ge 0$ for $x \in [0,1]$

It is known that $\sin x < x, \forall\ x \in R^+$

$\therefore \sin\left(\frac{1}{2}\right) < \frac{1}{2} \Rightarrow f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $\sin\left(\frac{1}{3}\right) < \frac{1}{3}$

$\Rightarrow f\left(\frac{1}{3}\right) < \frac{1}{3}$

21 **(b)**

$$\int_0^1 \cot^{-1}(1-x+x^2)dx$$
$$= \int_0^1 \tan^{-1}\left(\frac{1}{1-x+x^2}\right)dx$$
$$= \int_0^1 \tan^{-1}\left(\frac{x+(1-x)}{1-x(1-x)}\right)dx$$
$$= \int_0^1 \tan^{-1} x dx + \int_0^1 \tan^{-1}(1-x)dx$$
$$= \int_0^1 \tan^{-1} x dx + \int_0^1 \tan^{-1}[1-(1-x)]\,dx$$
$$= 2\int_0^1 \tan^{-1} x dx\ \Rightarrow \lambda = 2$$

22 **(b)**

$$2I=\int_{\alpha}^{\beta}\frac{e^{f\left(\frac{g(x)}{x-\alpha}\right)}dx}{e^{f\left(\frac{g(x)}{x-\alpha}\right)}+e^{f\left(\frac{g(x)}{x-\beta}\right)}}+\int_{\alpha}^{\beta}\frac{e^{f\left(\frac{g(\alpha+\beta-x)}{\beta-x}\right)}dx}{e^{f\left(\frac{g(\alpha+\beta-x)}{\beta-x}\right)}+e^{f\left(\frac{g(\alpha+\beta-x)}{\alpha-x}\right)}}$$

$$\Rightarrow I=\frac{1}{2}(\beta-\alpha)=\frac{\sqrt{b^2-4ac}}{2a}$$

($\because f(x)$ is even function $\Rightarrow \alpha+\beta=0$)

23 **(a)**

Putting $x\tan\theta=z\sin\theta \Rightarrow dx=\cos\theta dz$

$$\Rightarrow I=\cos\theta\int_{\tan\theta}^{1}f(z\sin\theta)dz$$

$$=-\cos\theta\int_{1}^{\tan\theta}f(x\sin\theta)dx$$

24 **(d)**

We have $f(x)=\int_{-1}^{1}\frac{\sin x\,dt}{\sin^2 x+(t-\cos x)^2}$

$$=\frac{\sin x}{\sin x}\tan^{-1}\left(\frac{t-\cos x}{\sin x}\right)\Big|_{-1}^{1}$$

$$=\tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)-\tan^{-1}\left(\frac{-1-\cos x}{\sin x}\right)$$

$$=\tan^{-1}(\tan x/2)+\tan^{-1}(\cot x/2)$$

Now, we know that $\tan^{-1}x+\tan^{-1}\frac{1}{x}=$

π2, x>0−π2, x<0

$$\Rightarrow \tan^{-1}\left(\tan\frac{x}{2}\right)+\tan^{-1}\left(\frac{1}{\tan\frac{x}{2}}\right)=\begin{cases}\frac{\pi}{2},\tan\frac{x}{2}>0\\-\frac{\pi}{2},\tan\frac{x}{2}<0\end{cases}$$

Hence, range of $f(x)$ is $\left\{-\frac{\pi}{2},\frac{\pi}{2}\right\}$

25 **(c)**

$$\frac{dx}{dt}=\sin^{-1}(\sin t)\cos t=t\cos t$$

and $\frac{dy}{dt}=\frac{\sin t}{\sqrt{t}}\cdot\frac{1}{2\sqrt{t}}=\frac{\sin t}{2t}\Rightarrow\frac{dy}{dx}=\frac{\sin t}{2t.t\cos t}=\frac{\tan t}{2t^2}$

26 **(b)**

$$\int_{-3}^{5}f(|x|)dx=\int_{-3}^{3}f(|x|)dx+\int_{3}^{5}f(|x|)dx$$

$$=2\int_{0}^{3}f(x)dx+\int_{3}^{5}f(x)dx$$

$$=2\left(\int_{0}^{1}f(x)dx+\int_{1}^{2}f(x)dx+\int_{2}^{3}f(x)dx\right)+\left(\int_{3}^{4}f(x)dx+\int_{4}^{5}f(x)dx\right)$$

$$=2\left(0+\frac{1}{2}+\frac{2^2}{2}\right)+\left(\frac{9}{2}+\frac{16}{2}\right)=\frac{35}{2}$$

27 **(b)**

Put $2+x=t^2$, so that $dx=2t\,dt$ and

$$I=\int\frac{\sqrt{7-t^2}}{t}(2t)\,dt\;2\int\sqrt{7-t^2}\,dt$$

$$=t\sqrt{7-t^2}+7\sin^{-1}\left(\frac{t}{\sqrt{7}}\right)+C$$

$$=\sqrt{x+2}\sqrt{5-x}+7\sin^{-1}\left(\frac{\sqrt{x+2}}{\sqrt{7}}\right)+C$$

28 **(b)**

$$I_m=\int_{1}^{e}(\log x)^m dx$$

$I_m=[x(\log x)^m]_1^e-\int_1^e x\frac{m(\log x)^{m-1}}{x}dx$

(integrating by parts)

$\Rightarrow I_m=e-m\int_1^e(\log x)^{m-1}\,dx=e-mI_{m-1}(1)$

Replacing m by $m-1$

$I_{m-1}=e-(m-1)I_{m-2}(2)$

From equations (1) and (2), we have

$I_m=e-m[e-(m-1)I_{m-2}]$

$\Rightarrow I_m-\ m(m-1)I_{m-1}=e(1-m)$

$\Rightarrow\frac{I_m}{1-m}+\ mI_{m-2}=e$

$\Rightarrow\ K=1-m$ and $L=\frac{1}{m}$

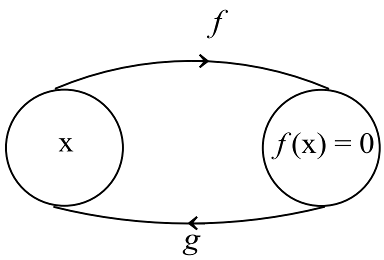
29 **(c)**

$$f(x)=\int_{2}^{x}\frac{dt}{\sqrt{1+t^4}}$$

$$\Rightarrow f'(x)=\frac{1}{\sqrt{1+x^4}}=\frac{dy}{dx}$$

Now $g'(x)=\frac{dx}{dy}=\ \sqrt{1+x^4}$

When $y=0$, i.e., $\int_2^x\frac{dt}{\sqrt{1+t^4}}=0$ then $x=2$



Hence, $g'(0)=\sqrt{1+16}=\sqrt{17}$

30 **(b)**

Let $I=\int_1^a[x]f'(x)dx, a>1$

Let $a = k + h$, where $[a] = k$, and $0 \le h < 1$

$$\therefore \int_1^a [x]f'(x)dx = \int_1^2 1f'(x)dx + \int_2^3 2f'(x)dx$$

$$+ \cdots + \int_{k-1}^{k} (k-1)f'(x)dx + \int_k^{k+h} kf'(x)dx$$

$$= [f(2) - f(1)] + 2[f(3) - f(2)] + \cdots + (k-1)[f(k) - f(k-1)] + k[f(k+h) - f(k)]$$

$$= -f(1) - f(2) - f(3) \cdots - f(k) + kf(k+h)$$

$$= [a]f(a) - [f(1) + f(2) + \cdots + f([a])]$$

31 **(b)**

$f(x) = x|\cos x|, \frac{\pi}{2} < x < \pi = -x\cos x$, because $\cos x$ is negative in $\left(\frac{\pi}{2}, \pi\right)$

$\therefore$ the required primitive function $= \int -x\cos x dx$

Now, use integration by parts

32 **(b)**

$$g(x) = \int_0^x f(t)dt,$$

$$\Rightarrow f(2) = \int_0^2 f(t) = \int_0^1 f(t)dt + \int_1^2 f(t)dt$$

Now, $\frac{1}{2} \le f(t) \le 1$ for $t \in [0, 1]$

$$\Rightarrow \int_0^1 \frac{1}{2}dt \le \int_0^1 f(t)dt \le \int_0^1 1\, dt$$

$$\Rightarrow \frac{1}{2} \le \int_0^1 f(t)dt \le 1 \quad (1)$$

Again, $0 \le f(t) \le \frac{1}{2}$ for $t \in [1, 2]$

$$\Rightarrow \int_1^2 0\, dt \le \int_1^2 f(t) \le \int_1^2 \frac{1}{2}dt$$

$$\Rightarrow 0 \le \int_1^2 f(t)\, dt \le \frac{1}{2} \qquad (2)$$

From equations (1) and (2), we get

$$\frac{1}{2} \le \int_0^1 f(t) + \int_1^2 f(t)dt \le \frac{3}{2}$$

$$\Rightarrow \frac{1}{2} \le g(2) \le \frac{3}{2}$$

33 **(a)**

$$f(2x) = f(x) = f\left(\frac{x}{2}\right) = f\left(\frac{x}{2^2}\right) = \cdots = f\left(\frac{x}{2^n}\right)$$

So, when $n \to \infty \Rightarrow f(2x) = f(0)$ ($f(x)$ is continuous)

i.e., $f(x)$ is a constant function

$$\Rightarrow f(x) = f(1) = 3, \int_{-1}^{1} f(f(x))dx = \int_{-1}^{1} 3dx = 6$$

34 **(c)**

$$I = \int_0^{\sqrt{\ln\left(\frac{\pi}{2}\right)}} \cos\left(e^{x^2}\right) 2xe^{x^2}\, dx$$

Put $e^{x^2} = t \Rightarrow e^{x^2}\, 2x\, dx = dt$

$$\Rightarrow I = \int_1^{\pi/2} \cos t\, dt = [\sin t]_1^{\pi/2} = 1 - (\sin 1)$$

35 **(b)**

We have $\int \frac{dx}{x^2(x^n+1)^{(n-1)/n}}$

$$= \int \frac{dx}{x^2\, x^{n-1}\left(1 + \frac{1}{x^n}\right)^{(n-1)/n}}$$

$$= \int \frac{dx}{x^{n+1}(1 + x^{-n})^{(n-1)/n}}$$

Put $1 + x^{-n} = t$

$$\therefore -nx^{-n-1}dx = dt \Rightarrow \frac{dx}{x^{n+1}} = -\frac{dt}{n}$$

$$\Rightarrow \int \frac{dx}{x^2(x^n+1)^{(n-1)/n}} = -\frac{1}{n}\int \frac{dt}{t^{(n-1)/n}}$$

$$= -\frac{1}{n}\int t^{1/n-1}dt = -\frac{1}{n}\frac{t^{1/n-1+1}}{1/n - 1 + 1} + C$$

$$= -t^{1/n} + C = -(1 + x^{-n})^{1/n} + C$$

36 **(b)**

Put $x = a\cos^2\theta + b\sin^2\theta, \Rightarrow dx = 2\,(b - a\sin\theta\cos\theta\; d\theta$, then

$$\int_a^b (x-a)^3(b-x)^4 dx$$

$$= 2(b-a)\int_0^{\pi/2} (a\cos^2\theta + b\sin^2\theta - a)^3(b - a\cos^2\theta - b\sin^2\theta)^4 \sin\theta\cos\theta\, d\theta$$

$$= 2(b-a)^8 \int_0^{\pi/2} \sin^7\theta\cos^9\theta\, d\theta$$

$$= 2(b-a)^8 \int_0^{\pi/2} \sin^7\theta\,(1 - \sin^2\theta)^4\cos\theta\, d\theta$$

$$= 2(b-a)^8 \int_0^1 x^7(1-x^2)^4\, dx$$

$$= 2(b-a)^8 \int_0^1 x^7(1-x^2)^4 dx$$

$$= 2(b-a)^8 \int_0^1 x^7(1 - 4x^2 + 6x^4 - 4x^6 + x^8)dx$$

$$= 2(b-a)^8\left[\frac{1}{8} - \frac{4}{10} + \frac{6}{12} - \frac{4}{14} + \frac{1}{16}\right] = \frac{(b-a)^8}{280}$$

37 **(b)**

$$\int_0^{\pi}[f(x)+f''(x)]\sin x dx$$

$$=\int_0^{\pi} f(x)\sin x\, dx+\int_0^{\pi} f''(x)\sin x dx$$

$$=\left(f(x)(-\cos x)\right)_0^{\pi}+\int_0^{\pi} f'(x)\cos x\, dx$$

$$+\sin x f'(x)|_0^{\pi}-\int_0^{\pi}\cos x\, f'(x)dx$$

$=f(\pi)+f(0)=5$(given)

$\Rightarrow f(0)=5-f(\pi)=5-2=3$

38 **(c)**

If $f(x)=\begin{cases}e^{\cos x}\sin x \text{ for } |x|\le 2\\ 2 \quad\quad \text{otherwise}\end{cases}$

$$\Rightarrow\int_{-2}^{3} f(x)dx=\int_{-2}^{2} f(x)dx+\int_{2}^{3} f(x)dx$$

$$=\int_{-2}^{2} e^{\cos x}\sin x\, dx+\int_{2}^{3} 2\, dx=0+2\,[x]_2^3=2$$

[$\because e^{\cos x}\sin x$ is an odd function]

39 **(a)**

$$f'(x)=\frac{f(x)}{6f(x)-x}$$

Now $I=\int\frac{2x(x-6f(x))+f(x)}{(6f(x)-x)(x^2-f(x))^2}dx$

$$\Rightarrow I=-\int\frac{2x-f'(x)}{(x^2-f(x))^2}dx=\frac{1}{x^2-f(x)}+C$$

40 **(b)**

$\sin nx-\sin(n-2)x=2\cos(n-1)x\sin x$

$$\Rightarrow\int\frac{\sin nx}{\sin x}dx=\int 2\cos(n-1)dx+\int\frac{\sin(n-2)x}{\sin x}dx$$

$$\therefore\int_0^{\pi/2}\frac{\sin 5x}{\sin x}dx=\int_0^{\pi/2} 2\cos 4x\, dx+\int_0^{\pi/2}\frac{\sin 3x}{\sin x}dx$$

$$=0+\int_0^{\pi/2}\frac{\sin 3x}{\sin x}dx=\int_0^{\pi/2} dx=\frac{\pi}{2}$$

41 **(a)**

Differentiating, we get

$$\frac{f'(x)}{f(x)^2}=2(b^2-a^2)\sin x\cos x$$

Integrating both sides w.r.t. x

$$\Rightarrow-\frac{1}{f(x)}=-b^2\cos^2 x-a^2\sin^2 x$$

$$\Rightarrow f(x)=\frac{1}{a^2\sin^2 x+b^2\cos^2 x}$$

42 **(c)**

$$I_1=\int_0^{\pi/2}\frac{\cos^2 x}{1+\cos^2 x}\,dx$$

$$=\int_0^{\pi/2}\frac{\cos^2(\pi/2-x)}{1+\cos^2(\pi/2-x)}\,dx$$

$$=\int_0^{\pi/2}\frac{\sin^2 x}{1+\sin^2 x}dx=I_2$$

Also $I_1+I_2=\int_0^{\pi/2}\left(\frac{\sin^2 x}{1+\sin^2 x}+\frac{\cos^2 x}{1+\cos^2 x}\right)dx$

$$=\int_0^{\pi/2}\frac{\sin^2 x+\sin^2 x\cos^2 x+\cos^2 x+\sin^2 x\cos^2 x}{1+\sin^2 x+\cos^2 x+\sin^2 x\cos^2 x}dx$$

$$=\int\frac{1+2\sin^2\cos^2 x}{2+\sin^2 x\cos^2 x}\,dx=2I_3$$

$2I_1=2I_3\Rightarrow I_1=I_3\Rightarrow I_1=I_2=I_3$

43 **(b)**

Let $S'=1+x+x^2+x^3+\cdots=\frac{1}{1-x}$

Integrating w.r.t. x, we get $\left|\left(x+\frac{x^2}{2}+\frac{x^3}{3}+\cdots\right)\right|_0^{1/2}$

$=-|\text{In}(1-x)|_0^{1/2}$

$\Rightarrow\frac{1}{2}+\frac{1}{2}(S)=\text{In }2\Rightarrow S=\text{In}\frac{4}{e}$

44 **(c)**

We have $\int_2^4(3-f(x))\,dx=7$

$$\Rightarrow\ 6-\int_2^4 f(x)dx=7\Rightarrow\int_2^4 f(x)dx=-1$$

Now,

$$\int_2^{-1} f(x)dx=-\int_{-1}^{2} f(x)dx$$

$$=-\left[\int_{-1}^{4} f(x)dx+\int_4^2 f(x)dx\right]$$

$$=-\left[\int_{-1}^{4} f(x)dx-\int_2^4 f(x)dx\right]=-[4+1]=-5$$

45 **(c)**

$$f(x)=\int_{\frac{1}{e}}^{\tan x}\frac{tdt}{(1+t^2)}+\int_{\frac{1}{e}}^{\cot x}\frac{dt}{t(1+t^2)}$$

$$\Rightarrow f'(x)=\frac{\tan x}{1+\tan^2 x}\sec^2 x+\frac{1}{\cot x\,(1+\cot^2 x)}(-\text{cosec}^2 x)$$

$=\tan x-\tan x=0$

$\Rightarrow\ f(x)$ is a constant function

$$f\left(\frac{\pi}{4}\right)=\int_{\frac{1}{e}}^{1}\frac{tdt}{(1+t^2)}+\int_{\frac{1}{e}}^{1}\frac{dt}{t(1+t^2)}$$

$$=\int_{\frac{1}{e}}^{1}\frac{1}{t}\,dt=\text{In}t|_{1/e}^{1}=1$$

46 **(d)**

Since $a^2I_1 - 2aI_2 + I_3 = 0$

$\Rightarrow \int_0^1 (a-x)^2 f(x)dx = 0$

Hence, no such positive function $f(x)$

47 **(b)**

$I_1 = \int_e^{e^4} \sqrt{\ln x}\, dx$, putting $t = \sqrt{\ln x}$, i.e.,

$dt = \frac{dx}{2x\sqrt{\ln x}}$

$\Rightarrow dx = 2t\, e^{t^2} dt$

$\Rightarrow \int_e^{e^4} \sqrt{\ln x} dx$

$= \int_1^2 2t^2 e^{t^2}\, dt$

$= t\, e^{t^2}\Big|_1^2 - \int_1^2 e^{t^2}\, dt = 2e^4 - e - a$

48 **(c)**

$I = \int \frac{\ln(\tan x)}{\sin x \cos x} dx$, let $t = \ln(\tan x)$

$\Rightarrow \frac{dt}{dx} = \frac{\sec^2 x}{\tan x}$

$\Rightarrow dt = \frac{dx}{\sin x \cos x}$

$\Rightarrow I = \int t dt = \frac{1}{2}t^2 + C = \frac{1}{2}(\ln(\tan x))^2 + C$

49 **(b)**

$I_1 - I_2 = \int_0^{\pi/2} (\cos\theta - \sin 2\theta) f(\sin\theta + \cos^2\theta) d\theta$

Put $t = \sin\theta + \cos^2\theta \Rightarrow dt = (\cos\theta - \sin 2\theta) d\theta$

$\Rightarrow I_1 - I_2 = \int_1^1 f(t)dt = 0$

50 **(b)**

Putting $e^x - 1 = t^2$ in the given integral, we have

$\int_0^{\log 5} \frac{e^x\sqrt{e^x - 1}}{e^x + 3} dx = 2\int_0^2 \frac{t^2}{t^2 + 4} dt$

$= 2\left(\int_0^2 1dt - 4\int_0^2 \frac{dt}{t^2 + 4}\right)$

$= 2\left[\left(t - 2\tan^{-1}\left(\frac{t}{2}\right)\right)_0^2\right]$

$= 2[(2 - 2\times\pi/4)] = 4 - \pi$

51 **(a)**

$f(2-\alpha) = f(2+\alpha)$

$\Rightarrow$ function is symmetric about the line $x = 2$

$\int_{2-a}^{2+a} f(x)dx = 2\int_2^{2+a} f(x)dx$

52 **(b)**

$[x] = 0, \forall x \in [0,1)$

For $x \in [1,2), [x] = 1$

$\Rightarrow \frac{[x]}{1+x^2} = \frac{1}{1+x^2} < 1, \forall x \in [1,2) \Rightarrow \left[\frac{[x]}{1+x^2}\right] = 0$

For $x \in [-1,0), [x] = -1 \Rightarrow \frac{[x]}{1+x^2} = -\frac{1}{1+x^2}$

Clearly, $2 \geq 1 + x^2 > 1, \forall x \in [-1,0)$

$\Rightarrow \frac{1}{2} \leq \frac{1}{1+x^2} < 1 \Rightarrow -\frac{1}{2} \geq -\frac{1}{1+x^2} > -1$

$\Rightarrow \left[\frac{[x]}{1+x^2}\right] = -1\ \forall x \in [-1,0)$

Thus, the given integral $= -\int_{-1}^0 dx = -1$

53 **(d)**

$f(x) = \cos(\tan^{-1} x)$

$\Rightarrow f'(x) = -\frac{\sin(\tan^{-1} x)}{1+x^2}$

$\Rightarrow I = \int_0^1 x\, f''(x)dx$

$= [x\, f'(x)]_0^1 - \int_0^1 f'(x)dx$(Integrating by parts)

$= [f'(1)] - [f(x)]_0^1$

$= f'(1) - f(1) + f(0)$

Now $f(0) = 1; f'(1) = -\frac{1}{2\sqrt{2}}; f(1) = \frac{1}{\sqrt{2}}$

$\Rightarrow I = 1 - \frac{3}{2\sqrt{2}}$

54 **(c)**

Putting $x = \frac{1}{1+y}, dx = -\frac{1}{(1+y)^2}dy,$

We get $I_{(m,n)} = \int_0^1 x^{m-1}(1-x)^{n-1}\, dx$

$= \int_\infty^0 \frac{1}{(1+y)^{m-1}}\left(1 - \frac{1}{1+y}\right)^{n-1} \frac{(-1)}{(1+y)^2} dy$

$= \int_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}} dy = \int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}} dx$

Since, $I(m,n) = I(n,m)$

Therefore, $I(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx = \int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}} dx$

55 **(a)**

$I = b\int_0^t \frac{1}{x}\cos 4x\, dx - a\int_0^t \frac{1}{x^2}\sin 4x\, dx$

$= bI_1 - aI_2$

$I_2 = \int_0^t \frac{1}{x^2}\sin 4x\, dx$

$= \left\{\left[-\frac{1}{x}\sin 4x\right]_0^t + 4\int_0^t \frac{\cos 4x}{x}\, dx\right\}$

$= \left[-\frac{\sin 4t}{t} + 4 + 4I_1\right], \left\{\lim_{x\to 0}\frac{\sin 4x}{x} = 4\right\}$

$\therefore\ I = bI_1 - \ a\left\{-\frac{\sin 4t}{t} + 4 + 4I_1\right\}$

$= (b - 4a)\int_0^t \frac{1}{x}\cos 4x\, dx + \frac{a\sin 4t}{t} - 4a$

$= \frac{a\sin 4t}{t} - 1$

Therefore, $(b - 4a)\int_0^t \frac{1}{x}\cos 4x\, dx = 4a - 1$

L.H.S. is a function of t, whereas R.H.S. is a constant. Hence, we must have $b - 4a = 0$ and $4a - 1 = 0$

$\therefore a = \frac{1}{4}, b = 1$

56 **(d)**

Putting $x^2 = t$,

$I = \frac{1}{2}\int e^{t^2}(1 + t + 2t^2)\, e^t dt$

$= \frac{1}{2}\int e^t[te^{t^2} + (e^{t^2} + 2t^2e^{t^2})]dt$

$= \frac{1}{2}\int e^t[f(t) + f'(t)]\, dt = \frac{1}{2}e^t(te^{t^2}) + C$ where $t = x^2$

57 **(c)**

Let $x = t^6 \Rightarrow dx = 6t^5dt$

$\Rightarrow I = \int t^3\, (1 + t^2)^4\, 6t^5dt$

$\Rightarrow I = 6\int t^8(1 + 4t^2 + 6t^4 + 4t^6 + t^8)\, dt$

$= 6\int (t^8 + 4t^{10} + 6t^{12} + 4t^{14} + t^{16})\, dt$

$= 6\left\{\frac{t^9}{9} + \frac{4t^{11}}{11} + \frac{6t^{13}}{13} + \frac{4t^{15}}{15} + \frac{t^{17}}{17}\right\} + C$

$= 6\left\{x^{2/3} + \frac{4}{11}x^{11/6} + \frac{6}{13}x^{13/6} + \frac{4}{15}x^{5/2} + \frac{1}{17}x^{17/6}\right\} + C$

58 **(a)**

Putting $1 - x^3 = y^2, -3x^2dx = 2y\, dy$, we get

$\int \frac{1}{x\sqrt{1 - x^3}}dx$

$= -\frac{2}{3}\int \frac{1}{1 - y^2}dy$

$= \frac{1}{3}\log\left|\frac{y - 1}{y + 1}\right| + C$

$= \frac{1}{3}\log\left|\frac{\sqrt{1 - x^3} - 1}{\sqrt{1 - x^3} + 1}\right| + C \Rightarrow a = \frac{1}{3}$

59 **(b)**

Differentiating, we get $f''(x) = f'(x)$

$\Rightarrow \int \frac{df'(x)}{f'(x)} = \int dx\ \Rightarrow \text{In} f'(x) = x + c\ \Rightarrow f'(x) = Aex$ (1)

$\Rightarrow \int f'(x)dx = \int Ae^x\, dx\ \Rightarrow f(x) = Ae^x + B$ (2)

Now, $f(0) = 1 \Rightarrow A + B = 1$

$\therefore\ f'(x) = f(x) + \int_0^1 (Ae^x +\ 1 - A)dx$

$Ae^x = (Ae^x + 1 - A) + |Ae^x +\ (1 - A)x|_0^1$

$\Rightarrow 1 - A + (Ae + 1 - A - A) = 0$

$\Rightarrow A(e - 3) = -2$

$\Rightarrow A = \frac{2}{3 - e}$ and $B = 1 - \frac{2}{3 - e} = \frac{1 - e}{3 - e}$

$\Rightarrow f(\log_e 2) = \frac{4}{3 - e} + \frac{1 - e}{3 - e} = \frac{5 - e}{3 - e}$

60 **(d)**

$\int_{\sqrt{2}}^{x}\frac{dt}{t\sqrt{t^2 - 1}} = \frac{\pi}{2}$

$\Rightarrow [\sec^{-1} t]_{\sqrt{2}}^{x} = \frac{\pi}{2}$

$\Rightarrow \sec^{-1} x - \sec^{-1}\sqrt{2} = \frac{\pi}{2}$

$\Rightarrow \sec^{-1} x - \frac{\pi}{4} = \frac{\pi}{2}$

$\Rightarrow \sec^{-1} x = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}, \Rightarrow\ x = -\sqrt{2}$

61 **(b)**

Given $xf(x) = x + \int_1^x f(t)dt$

$f(x) + xf'(x) = 1 + f(x)$

$\Rightarrow\ f(x) = \log|x| + c$

$f(1) = 1\ \Rightarrow\ f(x) = \log|x| + 1$

$\Rightarrow f(e^{-1}) = 0$

62 **(d)**

$I = \int_1^e \left(\frac{1}{x} + 1\right)dx - \int_1^e \frac{1 + \text{In}\, x}{1 + x\,\text{In}\, x}\, dx$

$= [\text{In}\, x + x]_1^e - [\text{In}(1 + x\,\text{In}\, x)]_1^e$

$= e - \text{In}\,(1 + e)$

63 **(c)**

$\int_0^a \frac{f(x)}{f(x) + f(a - x)} = \frac{a}{2}$

$\Rightarrow \lim_{n\to\infty}\left[\frac{a}{2} + \frac{a^2}{2} + \frac{a^3}{2} + \cdots + \frac{a^n}{2}\right] = \frac{7}{5}$

$\Rightarrow \frac{a}{1 - a} = \frac{14}{5}$

$\Rightarrow 5a = 14 - 14a$

$\Rightarrow a = \frac{14}{19}$

64 **(c)**

Given f is a positive function, and

$I_1 = \int_{1-k}^{k} xf\big(x(1 - x)\big)dx$

$I_2 = \int_{1-k}^{k} f[x(1 - x)]dx$

Now, $I_1 = \int_{1-k}^{k} f[x(1-x)]dx$ (1)

$= \int_{1-k}^{k} (1-x) f[(1-x)x]dx$ (2)

$$\left[\text{Using the property} \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$$

Adding equations (1) and (2), we get

$$2I_1 = \int_{1-k}^{k} f[x(1-x)]dx = I_2 \Rightarrow \frac{I_1}{I_2} = \frac{1}{2}$$

65 **(d)**

$I = \int \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$, let $x = \tan\theta$

$\Rightarrow dx = \sec^2\theta \, d\theta$

$$\Rightarrow I = \int \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) \sec^2\theta d\theta$$

$$= 2\int \theta \sec^2\theta \, d\,\theta$$

$= 2(\theta \tan\theta - \ln|\sec\theta|) + C$

$= 2(x \tan^{-1} x - \ln|\sec(\tan^{-1} x)|) + C$

66 **(a)**

Here, $I(m,n) = \int_0^1 t^m (1+t)^n dt$

$$\Rightarrow I(m,n) = \left\{(1+t)^n . \frac{t^{m+1}}{m+1}\right\}_0^1 - \int_0^1 n(1+t)^{n-1} . \frac{t^{m+1}}{m+1} dt$$

$$= \frac{2^n}{m+1} - \frac{n}{m+1}\int_0^1 (1+t)^{n-1} . t^{m+1} dt$$

$$\therefore I(m,n) = \frac{2^n}{m+1} - \frac{n}{m+1} . I(m+1, n-1)$$

67 **(a)**

Putting $a = 2, b = 3, c = 0$, we get

$$\int_0^\infty \frac{dx}{(x^2+4)(x^2+9)} = \frac{\pi}{2(2+3)(3+0)(0+2)} = \frac{\pi}{60}$$

68 **(b)**

$I = \int_0^4 f(t)dt$, put $t = x^2$

$\Rightarrow dt = 2xdx$, then

$$I = 2\int_0^2 xf(x^2)dx$$

From Lagrange's Mean Value Theorem

$\frac{\int_0^2 2xf(x^2)dx - \int_0^0 2xf(x^2)dx}{2-0} = 2yf(y^2)$ for some $y \in (0,2)$

$$\Rightarrow \int_0^2 2xf(x^2)dx = 2 \times 2yf(y^2)$$

$$= \left\{\frac{2\alpha f(\alpha^2) + 2\beta f(\beta^2)}{2}\right\}$$

(where $0 < \beta < y < \alpha < 2$, and using intermediate Mean Value Theorem)

69 **(c)**

$$\int \sqrt{\frac{\cos x - \cos^3 x}{1-\cos^3 x}} dx = \int \sqrt{\frac{\cos x}{1-\cos^3 x}} \sin x dx$$

$= \int \sqrt{\frac{t}{1-t^3}} dt = -\int \frac{\sqrt{t}}{\sqrt{1-(t^{3/2})^2}} dt$, where $t = \cos x$

$$= -\frac{2}{3}\int \frac{\frac{3}{2}\sqrt{t}}{\sqrt{1-(t^{3/2})^2}} dt = \frac{2}{3}\cos^{-1}(t^{3/2}) + C$$

70 **(a)**

$$I = \int_0^1 f(x)[g(x) - g(1-x)]dx$$

$$= -\int_0^1 f(1-x)[g(x) - g(1-x)]dx$$

$$\Rightarrow 2I = \int_0^1 [f(x) - f(1-x)][g(x) - g(1-x)]dx \leq 0$$

71 **(a)**

Given, $\int_{\sin x}^1 t^2 \, f(t)dt = 1 - \sin x$

Now, $\frac{d}{dx}\int_{\sin x}^1 t^2 \, f(t)dt = \frac{d}{dx}(1 - \sin x)$

$\Rightarrow [1^2 f(1)].(0) - (\sin^2 x).f(\sin x).\cos x = -\cos x$

[by Leibnitz formula]

$\Rightarrow$ Put $\sin x = 1/\sqrt{3}$

$$\therefore f\left(\frac{1}{\sqrt{3}}\right) = (\sqrt{3})^2 = 3$$

72 **(b)**

$$\sin^3 x = \frac{3}{4}\sin x - \frac{1}{4}\sin 3x$$

$$\Rightarrow \int_0^\infty \frac{\sin^3 x}{x} dx$$

$$= \frac{3}{4}\int_0^\infty \frac{\sin x}{x} dx - \frac{1}{4}\int_0^\infty \frac{\sin 3x}{x} dx$$

$$= \frac{3}{4}\int_0^\infty \frac{\sin x}{x} dx - \frac{1}{4}\int_0^\infty \frac{\sin u}{x} du \ (u = 3x)$$

$$= \frac{3}{4}\frac{\pi}{2} - \frac{1}{4}\frac{\pi}{2} = \frac{\pi}{4}$$

73 **(d)**

Since $h(x) = (f(x) + f(-x))(g(x) - g(-x))$

$\Rightarrow\ h(-x) = (f(-x) + f(x))(g(-x) - g(x))$
$\Rightarrow h(-x) = -h(x)$
$\therefore\ h(x)$ is odd function,

$$\Rightarrow \int_{-\pi/2}^{\pi/2} (f(x) + f(-x))(g(x) - g(-x))dx = 0$$

74 **(b)**

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left[\frac{1}{1+\sqrt{n}} + \frac{1}{2+\sqrt{2n}} + \cdots + \frac{1}{n+\sqrt{n^2}}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[\frac{1}{\frac{1}{n}+\frac{1}{\sqrt{n}}} + \frac{1}{\frac{2}{n}+\sqrt{\frac{2}{n}}} + \cdots + \frac{1}{\frac{n}{n}+\sqrt{\frac{n}{n}}}\right]$$

$$= \int_0^1 \frac{dx}{\sqrt{x}(\sqrt{x}+1)}$$

Put $\sqrt{x} = z, \therefore \frac{1}{2\sqrt{x}}\ dx = dz$

$$\Rightarrow \lim_{n\to\infty} S_n = \int_0^1 \frac{2dz}{z+1} = 2|\log(z+1)|_0^1$$

$= 2(\log 2 - \log 1)$
$= 2\log 2 = \log 4$

75 **(a)**

On integrating by parts taking $\sin^2 x$ as first function and $\frac{1}{x^2}$ as second function, we get

$$\int_0^\infty \frac{\sin^2 x}{x^2}dx = \left|\sin^2 x\left(-\frac{1}{x}\right)\right|_0^\infty - \int_0^\infty 2\sin x\cos x\left(-\frac{1}{x}\right)dx$$

Now, $\lim_{x\to\infty}\sin^2 x\left(-\frac{1}{x}\right) = 0$, and

$$\lim_{x\to\infty}\frac{\sin^2 x}{x} = \lim_{x\to\infty}(\sin x)\frac{\sin x}{x} = 0$$

Thus, $\int_0^\infty \frac{\sin^2 x}{x^2}dx = 0 + \int_0^\infty \frac{\sin 2x}{x}\ dx$

Now, put $2x = t$, then $dx = dt/2$

$$\int_0^\infty \frac{\sin 2x}{x}\ dx = \int_0^\infty \frac{\sin t}{t/2}\frac{dt}{2} = \int_0^\infty \frac{\sin t}{t}\ dt = \int_0^\infty \frac{\sin x}{x}\ dx$$

76 **(d)**

$$I = \int \frac{dx}{\sqrt{\sin^3 x\cos^5 x}}$$

$$= \int \frac{dx}{\sqrt{\frac{\sin^3 x}{\cos^3 x}\cos^8 x}}$$

$$= \int \frac{\sec^4 x}{\sqrt{\tan^3 x}}dx$$

$$= \int \frac{(1+\tan^2 x)\sec^2 x}{\sqrt{\tan^3 x}}dx$$

Let $t = \tan x \Rightarrow dt = \sec^2 x\ dx$

$$\Rightarrow I = \int \frac{1+t^2}{t^{3/2}}dt$$

$$= \int (t^{-3/2} + t^{1/2})\,dt$$

$$= -2t^{-\frac{1}{2}} + \frac{2}{3}t^{3/2} + C$$

$$= -2\sqrt{\cot x} + \frac{2}{3}\sqrt{\tan^3 x} + C$$

$$\Rightarrow a = -2, b = \frac{2}{3}$$

77 **(a)**

Putting,

$l^{r+1}(x) = t$ and $\frac{1}{xl(x)l^2(x)\dots l^r(x)}\,dx = dt$, we get

$$\int \frac{1}{xl^2(x)l^3(x)\dots l^r(x)} = \int 1dt = t + C = l^{r+1}(x) + C$$

78 **(b)**

Let $f(x) = \int(1+\cos^8 x)(ax^2+bx+c)dx$
$\therefore\ f'(x) = (1+\cos^8 x)\,(ax^2+\ bx+c)$ (1)
From the given conditions
$f(1) - f(0) = 0 \Rightarrow\ f(0) = f(1)$ (2)
and $f(2) - f(0) = 0\ \Rightarrow\ f(0) = f(2)$ (3)
From equations (2) and (3), we get $f(0) = f(1) = f(2)$
By Rolle's theorem for $f(x)$ in [0, 1] :$f'(\alpha) = 0$, at least one α such that $0 < \alpha < 1$
By Rolle's theorem for $f(x)$ in [1, 2] :$f'(\beta) = 0$, at least one β such that $1 < \beta < 2$
Now, from equation (1), $f'(\alpha) = 0$
$\Rightarrow (1+\cos^8\alpha)(a\alpha^2 + b\alpha + c) = 0$ $(\because 1+\cos^8\alpha \neq 0)$
$\Rightarrow\ a\alpha^2 + b\alpha + c = 0$
i.e., α is a root of the equation $ax^2+bx+c=0$
Similarly, β is a root of the equation $ax^2+\ bx+c=0$
But equation $ax^2+\ bx+c=0$ being a quadratic equation cannot have more than two roots
Hence, equation $ax^2+bx+c=0$ has one root α between 0 and 1, and other root β between 1 and 2

79 **(c)**

Given $A = \int_0^1 x^{50}(2-x)^{50}\ dx; B = \int_0^1 x^{50}(1-$ $x50\ dx$
In A, put $x = 2t\ \Rightarrow dx = 2dt$
$\Rightarrow A = 2\int_0^{1/2} 2^{50}.t^{50}2^{50}(1-t)^{50}dt$ (1)
Now, $B = 2\int_0^{1/2} x^{50}(1-x)^{50}\ dx$ (2)

$$\left[\text{using} \int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx \text{ if } f(2a-x)=f(x)\right]$$

From equations (1) and (2), we get

$A = 2^{100} B$

80 **(a)**

Let $I = \int_0^{\pi} \frac{x \tan x}{\sec x + \cos x} dx$ (1)

$$= \int_0^{\pi} \frac{(\pi - x)\tan(\pi - x)}{\sec(\pi - x) + \cos(\pi - x)} dx$$

$= \int_0^{\pi} \frac{(\pi - x)\tan x}{\sec x + \cos x} dx$ (2)

Adding equations (1) and (2) gives

$$2I = \pi \int_0^{\pi} \frac{\tan x}{\sec x + \cos x} dx$$

$$= \pi \int_0^{\pi} \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x} + \cos x} dx = \pi \int_0^{\pi} \frac{\sin x}{1 + \cos^2 x} dx$$

Put $\cos x = z$, therefore $-\sin x\, dx = dz$

When $x = 0, z = 1, x = \pi, z = -1$

$$\therefore 2I = \pi \int_1^{-1} \frac{-dz}{1+z^2} = \pi \int_{-1}^{1} \frac{dz}{1+z^2}$$

$= \pi\, |\tan^{-1} z|^1 - 1$

$= \pi\, [\tan^{-1} 1 - \tan^{-1}(-1)]$

$$= \pi \left(\frac{\pi}{4} + \frac{\pi}{4}\right) = \frac{2\pi^2}{4}$$

$$\Rightarrow I = \frac{\pi^2}{4}$$

81 **(a)**

Putting $x = \tan\theta$, we get

$$\int_0^{\pi/2} \frac{dx}{\left[x + \sqrt{x^2+1}\right]^3} = \int_0^{\infty} \frac{\sec^2\theta\, d\theta}{(\tan\theta + \sec\theta)^3}$$

$$= \int_0^{\pi/2} \frac{\cos\theta}{(1+\sin\theta)^3}\, d\theta$$

$$= \left[-\frac{1}{2\,(1+\sin\theta)^2}\right]_0^{\pi/2} = -\frac{1}{8} + \frac{1}{2} = \frac{3}{8}$$

82 **(a)**

$$I_3 = \int_0^{\pi} e^x (\sin x)^3\, dx$$

$$= e^x (\sin x)^3|_0^{\pi} - 3\int_0^{\pi} (\sin x)^2 \cos x\, e^x dx$$

$$= 0 - 3\,(\sin x)^2 \cos x\, e^x|_0^{\pi}$$

$$+ 3\int_0^{\pi} (2\sin x \cos x \cos x - \sin x \sin^2 x) e^x dx$$

$$= 0 + 6\int_0^{\pi} \sin x \cos^2 x e^x dx - 3\int_0^{\pi} \sin^3 x\, e^x dx$$

$$= 6\int_0^{\pi} \sin x\,(1 - \sin^2 x) e^x dx - 3\int_0^{\pi} \sin^3 x e^x dx$$

$$= 6\int_0^{\pi} \sin x e^x dx - 9\int_0^{\pi} \sin x^3 e^x dx$$

$= 6I_1 - 9I_3$

$\Rightarrow 10I_3 = 6I_1$

$$\Rightarrow \frac{I_3}{I_1} = \frac{3}{5}$$

83 **(d)**

$$I = \int \frac{\sqrt{x-1}}{x\sqrt{x+1}} dx$$

$$= \int \frac{x-1}{x\sqrt{x^2-1}} dx$$

$$= \int \frac{dx}{\sqrt{x^2-1}} - \int \frac{dx}{x\sqrt{x^2-1}}$$

$$= \ln\left|x + \sqrt{x^2+1}\right| - \sec^{-1} x + c$$

84 **(c)**

$$I = \int_{\log\lambda}^{\log\frac{1}{\lambda}} \frac{f\,(x^2/4)\,[f(x) - f(-x)]}{g(x^2/4)[g(x) + g(-x)]}\, dx$$

$$= \int_{\log\lambda}^{-\log\lambda} \frac{f(x^2/4)\,[f(x) - f(-x)]}{g(x^2/4)[g(x) + g(-x)]} = 0$$

(as function inside the integration is odd)

85 **(d)**

$$\int \frac{\text{cosec}^2 x - 2005}{\cos^{2005} x} dx$$

$$= \int (\cos x)^{-2005} \text{cosec}^2\, x dx - 2005\int \frac{dx}{\cos^{2005} x}$$

$= (\cos x)^{-2005}\,(-\cot x)$

$$-\int (-2005)(\cos x)^{-2006}\,(-\sin x)(-\cot x) dx$$

$$- 2005\int \frac{dx}{\cos^{2005} x}$$

$$= -\frac{\cot x}{(\cos x)^{2005}} + C$$

86 **(a)**

$$g(x) = \int_0^{x} \cos^4 t dt$$

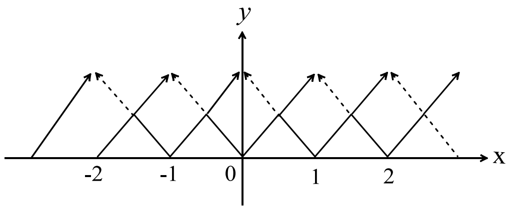
$$\Rightarrow g(x+\pi) = \int_0^{x+\pi} \cos^4 t dt$$

$$= \int_0^{x} \cos^4\, t dt + \int_x^{x+\pi} \cos^4 t dt$$

$$= g(x) + \int_0^{\pi} \cos^4 t dt \quad [\because \text{ period of} \cos^4 t \text{ is } \pi]$$

$$= g(x) + g(\pi)$$

87 **(a)**



The graph with solid line is the graph of $f(x) = \{x\}$ and the graph with dotted lines is the graph of $f(x) = \{-x\}$. Now the graph of min $(\{x\}, \{-x\})$ is the graph with dark solid lines $\int_{-100}^{100} f(x)dx =$ area of 200 triangles shown as solid dark lines in the diagram $= 200\frac{1}{2}(1)\left(\frac{1}{2}\right) = 50$

88 **(a)**

Here, $\int_0^{t^2} \{x\ f(x)\}dx = \frac{2}{5}t^5$

(Using Newton Leibnitz formula): differentiating both sides, we get

$$t^2\{f(t^2)\}.\left\{\frac{d}{dt}(t^2)\right\} - 0.f(0)\left\{\frac{d}{dt}(0)\right\} = 2t^4$$

$$\Rightarrow\ t^2 f(t^2).2t = 2t^4$$

$$\Rightarrow\ f(t^2) = t$$

$$\therefore\ f\left(\frac{4}{25}\right) = \pm\frac{2}{5} \qquad \left[\text{putting } t = \pm\frac{2}{5}\right]$$

$$\Rightarrow\ f\left(\frac{4}{25}\right) = \frac{2}{5} \quad [\text{neglecting negative}]$$

89 **(b)**

$$I = \int \lambda\left(\frac{\ln a^{a^{x/2}}}{3a^{5x/2}b^{3x}} + \frac{\ln b^{b^x}}{2a^{2x}b^{4x}}\right)dx$$

$$= \int \frac{\ln a^{2x}b^{3x}}{6a^{2x}b^{3x}}dx$$

Let $a^{2x}b^{3x} = t$,then $t \ln a^2b^3 dx = dt$

$$\Rightarrow I = \int \frac{1}{6\ln a^2b^3}\frac{\ln t}{t^2}dt$$

$$= \frac{1}{6\ln a^2b^3}\left(\frac{-\ln t}{t} - \int \frac{-1}{t^2}dt\right)$$

$$= -\frac{1}{6\ln a^2b^3}\left(\frac{\ln et}{t}\right) + k$$

$$= -\frac{1}{6\ln a^2b^3}\left(\frac{\ln a^{2x}b^{3x}e}{a^{2x}b^{3x}}\right) + k$$

90 **(c)**

Put $x - 0.4 = t \Rightarrow \int_{0.6}^{3.6}\{t\}dt = \int_{0.6}^{0.6+3}\{t\}dt$

$$= 3\int_0^1 (t - [t])\, dt = 3\left(\frac{t^2}{2}\right)_0^1 = \frac{3}{2} = 1.5$$

91 **(a)**

For $x \in \left(-\frac{\pi}{3}, 0\right), 2\cos x - 1 > 0$

$$\Rightarrow I = \int_{-\pi/3}^{0} \frac{\pi}{2}\, dx = \frac{\pi^2}{6}$$

92 **(b)**

$$I_1 = \int_{-100}^{101} \frac{dx}{(5 + 2x - 2x^2)(1 + e^{2-4x})}$$

$$= \int_{-100}^{101} \frac{dx}{(5 + 2(1 - x) - 2(1 - x)^2)(1 + e^{2-4(1-x)})}$$

$$= 2I_1 = \int_{-100}^{101} \frac{dx}{5 + 2x - 2x^2} = I_2$$

$$\Rightarrow \frac{I_1}{I_2} = \frac{1}{2}$$

93 **(c)**

We have $I_{n+1} - I_n = 2\int_0^{\pi} \cos(n+1)x dx = 0$

$\therefore I_{n+1} = I_n \Rightarrow I_{n+1} = I_n = \cdots = I_0 \Rightarrow I_n = \pi$for all $n \geq 0$

94 **(b)**

Write $I = \int \frac{dx}{x^3(a^2/x^2 - b^2)^{3/2}}$

and put $a^2/x^2 = t + b^2$, so that $(-2a^2/x^3)dx = dt$

$$\therefore I = \int \frac{(-1/2a^2)dt}{t^{3/2}}$$

$$= -\frac{1}{2a^2}\int t^{-3/2}dt = \frac{1}{a^2\sqrt{t}} + C$$

$$= \frac{1}{a^2(a^2/x^2 - b^2)^{1/2}} + C$$

$$= \frac{x}{a^2(a^2 - b^2x^2)^{1/2}} + C$$

95 **(a)**

When $e \leq [x] \leq e^2 \quad 1 < \log[x] < 2$

When $e^2 \leq [x] \leq e^3 \quad 2 < \log[x] < 3$

$$\therefore \int_3^8 1\, dx + \int_8^{10} 2\, dx = 9$$

96 **(b)**

On putting $x = \sin\theta$, we get $dx = \cos\theta\, d\theta$

Integral (without limits) $= \int \frac{\cos \quad \theta d\theta}{(1+\sin^2\theta)(\cos\theta)}$

$$= \int \frac{d\theta}{1 + \sin^2\theta} = \int \frac{\text{cosec}^2\theta\, d\theta}{2 + \cot^2\theta}$$

$= \int \frac{-dt}{2+t^2}$where $t = \cot\theta$

$$= -\frac{1}{\sqrt{2}}\tan^{-1}\frac{t}{\sqrt{2}} = -\frac{1}{\sqrt{2}}\tan^{-1}\frac{\cot\theta}{\sqrt{2}}$$

$$= -\frac{1}{\sqrt{2}}\tan^{-1}\frac{1}{\sqrt{2}}\left(\frac{\sqrt{1 - x^2}}{x}\right)$$

$\Rightarrow$Definite integral $= -\frac{1}{\sqrt{2}}\tan^{-1} 1 + \frac{1}{\sqrt{2}}\tan^{-1}\infty$

$$= -\frac{\pi}{4\sqrt{2}} + \frac{\pi}{2\sqrt{2}} = \frac{\pi}{4\sqrt{2}}$$

97 **(c)**

Let

$$I = \int \frac{\cos^3 + \cos^5 x}{\sin^2 x + \sin^4 x} dx$$

$$= \int \frac{(\cos^2 x + \cos^4 x)\cos x}{\sin^2 x\,(1 + \sin^2 x)} dx$$

$$= \int \frac{[1 - \sin^2 x + (1 - \sin^2 x)^2]\cos x}{\sin^2 x\,(1 + \sin^2 x)} dx$$

$$= \int \frac{(2 - 3\sin^2 x + \sin^4 x)\cos x}{\sin^2 x\,(1 + \sin^2 x)} dx$$

Put $\sin x = t \Rightarrow \cos x\, dx = dt$

$$\Rightarrow I = \int \frac{2 - 3t^2 + t^4}{t^4 + t^2} dt$$

$$= \int \left(1 + \frac{2}{t^2} - \frac{6}{t^2 + 1}\right) dt$$

$$= t - \frac{2}{t} - 6\tan^{-1}(t) + C$$

$$= \sin x - 2(\sin x)^{-1} - 6\tan^{-1}(\sin x) + C$$

98 **(c)**

As $f(x)$satisfies the conditions of Rolle's theorem in [1, 2], $f(x)$ is continuous in the interval and $f(1) = f(2)$

Therefore, $\int_1^2 f'(x)dx = [f(x)]_1^2 = f(2) - f(1) = 0$

99 **(a)**

$$y^r = \left(1 + \frac{1}{r}\right)\left(1 + \frac{2}{r}\right)\left(1 + \frac{3}{r}\right)\ldots\left(1 + \frac{n-1}{r}\right)$$

$$\Rightarrow \log y = \frac{1}{r}\sum_{p=1}^{n-1} \log\left(1 + \frac{p}{r}\right)$$

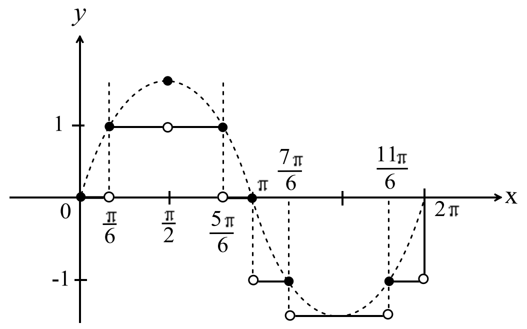
$$\Rightarrow \lim_{n\to\infty} y = \lim_{r\to\infty} y$$

$$= \int_0^k \log(1 + x)\, dx$$

$$= (k - 1)\log_e(1 + k) - k$$

100 **(b)**

$$I = \int_\pi^{2\pi} [2\sin x] dx$$



From the graph in figure

$$\therefore I = \int_{\pi/6}^{5\pi/6} 1dx + \int_{\pi}^{7\pi/6} -1\,dx + \int_{7\pi/6}^{11\pi/6} -2dx$$

$$+ \int_{11\pi/6}^{2\pi} -1\,dx$$

$$= \left(\frac{5\pi}{6} - \frac{\pi}{6}\right) + \left(-\frac{7\pi}{6} + \pi\right) + 2\left(-\frac{11\pi}{6} + \frac{7\pi}{6}\right) + \left(-2\pi + \frac{11\pi}{6}\right)$$

$$= \frac{2\pi}{3} - \frac{\pi}{6} - \frac{8\pi}{6} - \frac{\pi}{6} = -\pi$$

101 **(b)**

$I = \int_{-3}^{3} x^8\{x^{11}\}dx (1)$

Replacing x by $-x$, we have $I = \int_{-3}^{3} x^8\{-x^{11}\}\, dx$ (2)

Adding equations (1) and (2), we get

$$2I = \int_{-3}^{3} x^8(\{x^{11}\} + \{-x^{11}\})dx$$

$$= 2\int_0^3 x^8 dx = 2\left(\frac{x^9}{9}\right)_0^3 = 2.3^7$$

$\Rightarrow I = 3^7$ [as$\{x\} + \{-x\} = 1$ for x is not an integer]

102 **(c)**

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{e^{|\sin x|}\cos x}{(1 + e^{\tan x})} dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{e^{|\sin x|}\cos x}{1 + e^{\tan x}} + \frac{e^{|\sin x|}\cos x}{1 + e^{-\tan x}}\right) dx$$

$$= \int_0^{\frac{\pi}{2}} e^{|\sin x|}\cos x\, dx$$

$$= \int_0^{\frac{\pi}{2}} e^{\sin x}\cos x\, dx$$

$$= e^{\sin x}\Big|_0^{\frac{\pi}{2}} = e - 1$$

103 **(b)**

We have $\int \frac{dx}{x^2(x^n+1)^{(n-1)/n}}$

$$= \int \frac{dx}{x^2 x^{n-1}\left(1 + \frac{1}{x^n}\right)^{(n-1)/n}}$$

$$= \int \frac{dx}{x^{n+1}(1 + x^{-n})^{(n-1)/n}}$$

Put $1 + x^{-n} = t \therefore -nx^{-n-1}dx = dt \Rightarrow \frac{dx}{x^{n+1}} = -\frac{dt}{n}$

$$\Rightarrow \int \frac{dx}{x^2(x^n+1)^{(n-1)/n}} = -\frac{1}{n}\int \frac{dt}{t^{(n-1)/n}}$$
$$= -\frac{1}{n}\int t^{-1+\frac{1}{n}}\, dt = \frac{-1}{n}\cdot\frac{t^{1/n}}{1/n} + C$$
$$= -t^{1/n} + C$$

104 **(a)**

Let $n \le x < n+1$ where $n \in I$

$$I = \int_0^x \frac{2^t}{2^{[t]}}dt = \int_0^n 2^{\{t\}}dt + \int_0^x 2^{\{t\}}\, dt$$
$$= n\int_0^1 2^{\{t\}}\, dt + \int_n^x 2^{\{t\}}dt$$
$$= n\int_0^1 2^t\, dt + \int_n^x 2^{t-n}\, dt$$
$$= n\left.\frac{2^t}{\text{In } 2}\right|_0^1 + \frac{1}{2^n}\left.\frac{2^t}{\text{In } 2}\right|_n^x$$
$$= \frac{n}{\text{In } 2}(2-1) + \frac{1}{2^n \text{In } 2}(2^x - 2^n)$$
$$= \frac{n}{\text{In } 2} + \frac{1}{\text{In } 2}\,(2^{x-n} - 1)$$
$$= \frac{[x] + 2^{\{x\}} - 1}{\text{In } 2}$$

105 **(c)**

$f(x) = \int_0^\pi \frac{t\sin t}{\sqrt{1+\tan^2 x \sin^2 t}}dt (1)$

Replacing t by $\pi - t$ and then adding $f(x)$ with equation (1)

$$f(x) = \frac{\pi}{2}\int_0^\pi \frac{\sin t}{\sqrt{1+\tan^2 x \sin^2 t}}\, dt$$
$$= \pi\int_0^{\pi/2} \frac{\sin t}{\sqrt{1+\tan^2 x(1-\cos^2 t)}}\, dt$$
$$= \pi\int_0^{\pi/2} \frac{\sin t}{\sqrt{\sec^2 x - \tan^2 x\, \cos^2 t}}\, dt$$

Let $y = \cos t$

$\therefore\ dy = -\sin t\, dt$

$$\Rightarrow\ f(x) = \pi\int_0^1 \frac{dy}{\sqrt{\sec^2 x - (\tan^2 x)y^2}}$$
$$= \frac{\pi}{\tan x}\int_0^1 \frac{dy}{\sqrt{\text{cosec}^2 x - y^2}}$$
$$= \frac{\pi}{\tan x}\left\{\sin^{-1}\frac{y}{\text{cosec } x}\right\}_0^1$$
$$= \frac{\pi}{\tan x}\sin^{-1}(\sin x) = \frac{\pi x}{\tan x}$$

106 **(a)**

$$\int_0^\infty \left(\frac{\pi}{1+\pi^2x^2} - \frac{1}{1+x^2}\right)\log x\, dx$$
$$= \int_0^\infty \frac{\log\left(\frac{y}{\pi}\right)dy}{1+y^2} - \int_0^\infty \frac{\log x}{1+x^2}\, dx$$
$$= -\int_0^\infty \frac{\log \pi}{1+y^2}dy = -\frac{\pi}{2}\text{In } \pi$$

107 **(c)**

$$I = \int \frac{\sec x\, dx}{\sqrt{2\sin(x+A)\cos x}}$$
$$= \int \frac{\sec^2 x\, dx}{\sqrt{\frac{2\sin(x+A)}{\cos x}}}$$
$$= \frac{1}{\sqrt{2}}\int \frac{\sec^2 x\, dx}{\sqrt{\tan x\cos A + \sin A}}$$
$$= \frac{\sec A}{\sqrt{2}}\int \frac{2pdp}{p}$$

($\tan x\cos A + \sin A = p^2$, then $\cos A\ \sec^2 x\, dx =$ *2pdp)*

$$I = \sqrt{2}\sec A\int dp$$
$$= \sqrt{2}\sec A\sqrt{\tan x\cos A + \sin A} + C$$

108 **(c)**

Given integral

$$= \int_0^1 \frac{dx}{(x+\cos\alpha)^2 + (1-\cos^2\alpha)}$$
$$= \int_0^1 \frac{dx}{(x+\cos\alpha)^2 + \sin^2\alpha}$$
$$= \frac{1}{\sin\alpha}\left|\tan^{-1}\frac{x+\cos\alpha}{\sin\alpha}\right|_0^1$$
$$= \frac{1}{\sin\alpha}\left[\tan^{-1}\frac{1+\cos\alpha}{\sin\alpha} - \tan^{-1}\frac{\cos\alpha}{\sin\alpha}\right]$$
$$= \frac{1}{\sin\alpha}\left[\tan^{-1}\cot\frac{\alpha}{2} - \tan^{-1}(\cot\alpha)\right]$$
$$= \frac{1}{\sin\alpha}\left[\tan^{-1}\tan\left(\frac{\pi}{2}-\frac{\alpha}{2}\right) - \tan^{-1}\tan\left(\frac{\pi}{2}-\alpha\right)\right]$$
$$= \frac{1}{\sin\alpha}\left[\left(\frac{\pi}{2}-\frac{\alpha}{2}\right) - \left(\frac{\pi}{2}-\alpha\right)\right] = \frac{\alpha}{2\sin\alpha}$$

109 **(d)**

Let $I = \int_0^{\pi/2}\frac{dx}{1+\tan^3 x}$

$= \int_0^{\pi/2}\frac{\cos^3 x}{\sin^3 x+\cos^3 x}dx$ (1)

$$= \int_0^{\pi/2} \frac{\cos^3\left(\frac{\pi}{2}-x\right)}{\sin^3\left(\frac{\pi}{2}-x\right)+\cos^3\left(\frac{\pi}{2}-x\right)}\, dx$$

$= \int_0^{\pi/2}\frac{\sin^3 x}{\cos^3 x+\sin^3 x}dx$ (2)

Adding equation (1) and (2), we get

$$2I = \int_0^{\pi/2} 1dx$$

$$\Rightarrow\ I = \frac{\pi}{4}$$

110 **(d)**

$\int_0^x f(t)dt = \int_x^1 t^2 f(t)dt + \frac{x^{16}}{8} + \frac{x^6}{3} + \ a\ \ (1)$

For $x = 1, \int_0^1 f(t)dt = 0 + \frac{1}{8} + \frac{1}{3} + a = \frac{11}{24} + \ a$

Differentiating both sides of equation (1) w.r.t. x we get,

$f(x) = 0 - x^2\, f(x) + 2x^{15} + 2x^5$

$$\Rightarrow\ f(x) = \frac{2(x^{15} +\ x^5)}{1 + x^2}$$

$$\Rightarrow\ 2\int_0^1 \frac{x^{15} +\ x^5}{1 + x^2} dx = \frac{11}{24} + a$$

$$\Rightarrow 2\int_0^1 (x^{13} - x^{11} + x^9 - x^7 +\ x^5)dx = \frac{11}{24} + a$$

$$\Rightarrow 2\left(\frac{1}{14} - \frac{1}{12} + \frac{1}{10} - \frac{1}{8} + \frac{1}{6}\right) = \frac{11}{24} + a$$

$$\Rightarrow\ a = -\frac{167}{840}$$

111 **(a)**

$$I = \int \frac{dx}{x(x^n + 1)} = \int \frac{x^{n-1}}{x^n(x^n + 1)} dx$$

Putting $x^n = t$ so that $n\, x^{n-1}\, dx = dt$

$$\Rightarrow x^{n-1} dx = \frac{1}{n}\ dt$$

$$\therefore\ I = \int \frac{\frac{1}{n} dt}{t(t + 1)} = \frac{1}{n}\int \left(\frac{1}{t} - \frac{1}{t + 1}\right) dt$$

$$= \frac{1}{n}(\log t - \log(t + 1)) + C$$

$$= \frac{1}{n}\log\left(\frac{x^n}{x^n + 1}\right) + C$$

112 **(a)**

$$\int \frac{3e^x - 5e^{-x}}{4e^x + 5e^{-x}} = ax + b\ln(4e^x + 5e^{-x}) + C$$

Differentiating both sides, we get

$$\frac{3e^x - 5e^{-x}}{4e^x + 5e^{-x}} = a + b\frac{(4e^x - 5e^{-x})}{4e^x + 5e^{-x}}$$

$\Rightarrow 3e^x - 5e^{-x} = a(4e^x - 5e^{-x}) + b(4e^x - 5e^{-x})$

Comparing the coefficient of like terms on both sides, we get

$$3 = 4(a + b), -5 = 5a - 5b \Rightarrow a = -\frac{1}{8}, b = \frac{7}{8}$$

113 **(a)**

$I = \int_0^{\pi/2} \frac{\sin 2x}{x+1} dx$. Put $x = y/2$

$$\Rightarrow\ I = \int_0^{\pi} \frac{\sin y}{y + 2} dy$$

$= \left(\frac{-\cos y}{y+2}\right)_0^{\pi} - \int_0^{\pi} \frac{\cos y}{(y+2)^2} dy$ (integrating by parts)

$$\Rightarrow\ I = \frac{1}{\pi + 2} + \frac{1}{2} - A$$

114 **(a)**

$$\int_0^x f(t)dt = x + \int_x^1 tf(t)dt$$

$$\Rightarrow \frac{d}{dx}\left(\int_0^x f(t)dt\right) = \frac{d}{dx}\left(x + \int_x^1 tf(t)dt\right)$$

$\Rightarrow f(x) = 1 + 0 - xf(x)$ [using Leibnitz's Rule]

$\Rightarrow f(x) = 1 - xf(x)$

$$\Rightarrow f(x) = \frac{1}{x + 1} \Rightarrow f(1) = \frac{1}{2}$$

115 **(b)**

$$\int e^x \left(\frac{2\tan x}{1 + \tan x} + \tan^2\left(x - \frac{\pi}{4}\right)\right) dx$$

$$= \int e^x \left(\tan\left(x - \frac{\pi}{4}\right) + \sec^2\left(x - \frac{\pi}{4}\right)\right) dx$$

$$= e^x \tan\left(x - \frac{\pi}{4}\right) + C$$

116 **(c)**

$$I = \int_0^x [\cos t]dt = \int_0^{2n\pi} [\cos t]dt + \int_{2n\pi}^x [\cos t]dt$$

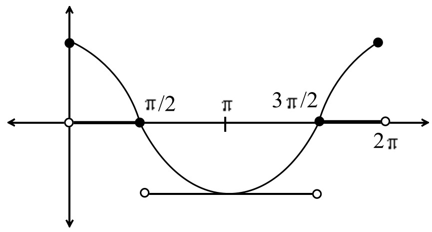
$$= n\int_0^{2\pi} [\cos t]dt + \int_{2n\pi}^{2n\pi+\pi/2} [\cos t]dt$$

$$+ \int_{2n\pi+\frac{\pi}{2}}^x [\cos t]dt$$

$= -n\pi + 0 + (x - (2n\pi + \pi/2))\,(-1)$

$= -n\pi + 2n\pi + \pi/2 - x$

$= (2n + 1)\pi/2 - x$



117 **(d)**

$I = \int_{a+c}^{b+c} f(x)dx$, putting $x = t + c$

$\Rightarrow dx = dt$, we get $I = \int_a^b f(t + c)dt =$ *abfx+cdx*

$$I = \int_{ac}^{bc} f(x)dx$$

Putting $x = tc\ \Rightarrow dx = c\ dt$,

We get $I = c\int_a^b f(ct)dt = c\int_a^b f(cx)dx$

$$f(x)=\frac{1}{2}\big(f(x)+f(-x)+f(x)-f(-x)\big)$$
$$\Rightarrow \int_{-a}^{a} f(x)dx$$
$$=\frac{1}{2}\int_{-a}^{a}(f(x)+f(-x)+f(x)-f(-x))\,dx$$
$$=\frac{1}{2}\int_{-a}^{a}(f(x)+f(-x))dx$$
$$+\frac{1}{2}\int_{-a}^{a}\big(f(x)-f(-x)\big)dx$$
$$=\frac{1}{2}\int_{-a}^{a}\big(f(x)+f(-x)\big)dx$$
As $f(x)+f(-x)$ is even and $f(x)-f(-x)$ is odd

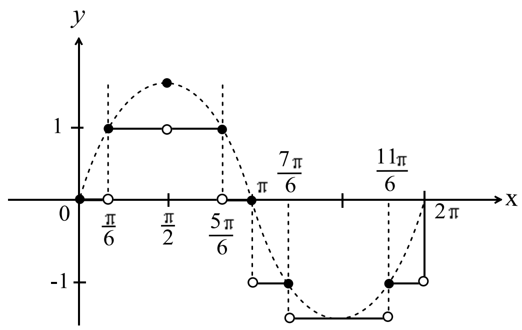
118 **(c)**

$I=\int_0^{\pi} e^{\cos^2 x}\cos^3(2n+1)x\,dx, n\in Z$ (1)
$$=\int_0^{\pi} e^{\cos^2(\pi-x)}\cos^3[(2n+1)(\pi-x)]\,dx$$
$$\left[\text{Using}\int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$$
$$=\int_0^{\pi} e^{\cos^2 x}\cos^3[(2n+1)\pi-(2n+1)x]\,dx$$
$$=-\int_0^{\pi} -e^{\cos^2 x}\cos^3(2n+1)x\,dx$$
$$=-I$$
$$\Rightarrow I=0$$

119 **(a)**

We have $f(y)=e^y, g(y)= y: y>0$
$$F(t)=\int_0^1 f(t-y)g(y)dy$$
$$=\int_0^t e^{t-y}\,y\,dy$$
$$=e^t\int_0^t e^{-y}\,y\,dy$$
$$=e^t\left([-ye^{-y}]_0^t+\int_0^t e^{-y}\,dy\right)$$
$$=e^t(-te^{-t}-[e^{-y}]_0^t)$$
$$=e^t(-te^{-t}-e^{-t}+1)$$
$$=e^t-(1+t)$$

120 **(c)**



we have
$$\int_{\pi/2}^{3\pi/4}[2\sin x]dx$$
$$=\int_{\pi/2}^{5\pi/6}1dx+\int_{\pi}^{7\pi/6}-1dx+\int_{7\pi/6}^{3\pi/2}-2\,dx$$
$$=\left[\frac{5\pi}{6}-\frac{\pi}{2}\right]-\left[\frac{7\pi}{6}-\pi\right]-2\left[\frac{3\pi}{2}-\frac{7\pi}{6}\right]$$
$$=\frac{-\pi}{2}$$

121 **(a)**
$$I_2=\int_{-\pi/4}^{\pi/4}\text{In}\,(\sin x+\cos x)dx$$
$$=\int_0^{\pi/4}\text{In}\,(\sin x+\cos x)+\text{In}(\sin(-x)+\cos(-x)))dx$$
$$=\int_0^{\pi/4}\text{In}\,(\sin x+\cos x)+\text{In}(\cos x+\sin x))dx$$
$$=\int_0^{\pi/4}\text{In}\,(\cos^2 x-\sin^2 x)dx$$
$$=\int_0^{\pi/4}\text{In}(\cos 2x)dx$$
Putting $2x=t$, i.e.,$\frac{dt}{2}=dx$, we get
$$I_2=\frac{1}{2}\int_0^{\pi/2}\text{In}\,(\cos t)\,dt$$
$$=\frac{1}{2}\int_0^{\pi/2}\text{In}\left(\cos\left(\frac{\pi}{2}-t\right)\right)dt$$
$$=\frac{1}{2}\int_0^{\pi/2}\text{In}(\sin t)dt=\frac{1}{2}I_1\Rightarrow I_1=2I_2$$

122 **(a)**

Given $f'(1)=\tan\pi/6, f'(2)=\tan\pi/3, f'(3)=\tan\pi/4$

Now, $\int_2^3 f'(x)f''(x)dx+\int_1^3 f''(x)dx$
$$=\left[\frac{(f'(x))^2}{2}\right]_2^3+[f'(x)]_1^3$$
$$=\frac{(f'(3))^2-(f'(2))^2}{2}+f'(3)-f'(1)$$

$$= \frac{(1)^2 - (\sqrt{3})^2}{2} + \left(1 - \frac{1}{\sqrt{3}}\right)$$
$$= \frac{1-3}{2} + 1 - \frac{1}{\sqrt{3}} = \frac{-1}{\sqrt{3}}$$

123 **(c)**

$$g(x) = \int_0^x f(t)dt$$
$$g(-x) = \int_0^{-x} f(t)dt = -\int_0^x f(-t)dt$$
$$= \int_0^x f(t)dt \text{ as } f(-t) = -f(t)$$
$\Rightarrow g(-x) = g(x)$, thus $g(x)$is even

Also, $g(x+2) = \int_0^{x+2} f(t)dt$
$$= \int_0^2 f(t)dt + \int_2^{2+x} f(t)dt$$
$$= g(2) + \int_0^x f(t+2)dt$$
$$= g(2) + \int_0^x f(t)dt$$
$$= g(2) + g(x)$$
Now, $g(2) = \int_0^2 f(t)dt = \int_0^1 f(t)dt + \int_1^2 f(t)dt$
$$= \int_0^1 f(t)dt + \int_{-1}^0 f(t+2)dt$$
$$= \int_0^1 f(t)dt + \int_{-1}^0 f(t)dt$$
$$= \int_{-1}^1 f(t)dt = 0 \text{ as } f(t) \text{ is odd}$$
$\Rightarrow g(2) = 0 \Rightarrow g(x+2) = g(x) \Rightarrow g(x)$ is periodic with period 2

$\Rightarrow g(4) = 0 \Rightarrow f(6) = 0, g(2n) = 0, n \in N$

124 **(b)**

$$I_1 = \int_{\sin^2 t}^{1+\cos^2 t} xf\,(x(2-x))dx$$
$$= \int_{\sin^2 t}^{1+\cos^2 t} (2-x)f\big(x(2-x)\big)dx = 2\,I_2 - I_1$$
$$\Rightarrow 2I_1 = 2I_2 \Rightarrow \frac{I_1}{I_2} = 1$$

125 **(c)**

Here, $\int e^x\{f(x) - f'(x)\}dx = \phi(x)$

and $\int e^x\{f(x) + f'(x)\}dx = e^x f(x)$

On adding, we get $2\int e^x f(x)dx = \phi(x) + e^x f(x)$

126 **(c)**

$$I_1 = \int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x}\,dx$$
$$= \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2} - x\right) - \cos\left(\frac{\pi}{2} - x\right)}{1 + \sin\left(\frac{\pi}{2} - x\right)\cos\left(\frac{\pi}{2} - x\right)}\,dx$$
$$= \int_0^{\pi/2} \frac{\cos x - \sin x}{1 + \sin x \cos x}\,dx = -I_1$$
$\Rightarrow I_1 = 0$

$I_3 = 0$ as $\sin^3 x$is odd
$$I_4 = \int_0^1 \text{In}\left(\frac{1-x}{x}\right)dx$$
$$= \int_0^1 \text{In}\left(\frac{1-(1-x)}{1-x}\right)dx$$
$$= \int_0^1 \text{In}\frac{x}{1-x}dx = -I_4$$
$\Rightarrow I_4 = 0$
$$I_2 = \int_0^{2\pi} \cos^6 x dx = 2\int_0^{\pi} \cos^6 x\,dx \neq 0$$

127 **(c)**

$$I = \frac{2\sin x}{(3 + \sin 2x)}dx$$
$$= \int \frac{\sin x + \cos x + \sin x - \cos x}{(3 + \sin 2x)}$$
$$= \int \frac{\sin x + \cos x}{3 + \sin 2x}dx - \int \frac{-\sin x + \cos x}{(3 + \sin 2x)}dx$$
Putting $t_1 = \sin x - \cos x$ in I_1 and $t_2 = \sin x + \cos x$ in I_2, we get
$$I = \int \frac{dt_1}{[3 + (1 - t_1^2)]} - \int \frac{dt_2}{[3 + (t_2^2 - 1)]}$$
$$= \int \frac{dt_1}{4 - t_1^2} - \int \frac{dt_2}{2 + t_2^2}$$
$$= \frac{1}{4}\ln\left|\frac{2 + t_1}{2 - t_1}\right| - \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t_2}{\sqrt{2}}\right) + C$$
$$= \frac{1}{4}\ln\left|\frac{2 + \sin x - \cos x}{2 - \sin x + \cos x}\right| - \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right) + C$$

128 **(d)**

By rationalizing the integrand, the given integral can be written as

$f(x)=\int\left(x+\sqrt{x^2+1}\right)dx$

$=\frac{x^2}{2}+\frac{x}{2}\sqrt{x^2+1}+\frac{1}{2}\log\left|x+\sqrt{x^2+1}\right|+C$

Putting $x=0$,we have $f(0)=C$ so $C=-1/2-1/\sqrt{2}$

and $f(1)=\frac{1}{2}+\frac{1}{2}\sqrt{2}+\frac{1}{2}\log\left|1+\sqrt{2}\right|+\left(-\frac{1}{2}-\frac{1}{\sqrt{2}}\right)$

$=\frac{1}{2}\log\left(1+\sqrt{2}\right)=-\log\left(\sqrt{2}-1\right)$

129 **(c)**

Since e^{x^2} is an increasing function on $(0,1)$, therefore $m=e^0=1, M=e^1=e$(m and M are minimum and maximum values of $f(x)=e^{x^2}$ in the interval $(0,1)$)

$\Rightarrow 1<e^{x^2}<e$, for all $x\in(0,1)$

$\Rightarrow 1(1-0)<\int_0^1 e^{x^2}\,dx<e(1-0)$

$\Rightarrow 1<\int_0^1 e^{x^2}\,dx<e$

130 **(a)**

$\sum_{r=1}^{n}\int_0^1 f(r-1+x)dx$

$=\int_0^1 f(x)dx+\int_0^1 f(1+x)dx$

$+\int_0^1 f(2+x)dx+\cdots$

$+\int_0^1 f(n-1+x)dx$

$=\int_0^1 f(x)dx+\int_1^2 f(x)dx$

$+\int_2^3 f(x)dx+\int_{r-1}^{2} f(x)dx+\cdots$

$+\int_{n-1}^{1} f(x)dx=\int_0^n f(x)dx$

131 **(a)**

$I=\int\frac{\sqrt{1+\sin x}\sqrt{1-\sin x}}{\sqrt{1-\sin x}}\,dx$

$=\int\frac{\cos x}{\sqrt{1-\sin x}}\,dx=-2\sqrt{1-\sin x}+C$

132 **(d)**

$I=\int\frac{x^3dx}{\sqrt{1+x^2}}=\int\frac{x\times x^2\,dx}{\sqrt{1+x^2}}$, let $t=\sqrt{1+x^2}$

$\Rightarrow\frac{dt}{dx}=\frac{x}{\sqrt{1+x^2}}$

$\Rightarrow I=\int(t^2-1)dt$

$=\frac{t^3}{3}-t+C=\frac{t}{3}(t^2-3)+C$

$=\frac{1}{3}\sqrt{1+x^2}(x^2-2)+C$

133 **(d)**

$I=\int\frac{xdx}{x^4\sqrt{x^2-1}}$

Let $x^2-1=t^2\Rightarrow 2x\,dx=2tdt$

$\Rightarrow I=\int\frac{t}{(t^2+1)^2t}dt=\int\frac{dt}{(t^2+1)^2}$

But $\tan^{-1}t=\int\frac{dt}{t^2+1}=\int 1\cdot\frac{1}{t^2+1}dt$

$=\frac{t}{t^2+1}+\int t\frac{2t}{(t^2+1)^2}dt$

$=\frac{t}{t^2+1}+2\int\frac{t^2+1-1}{(t^2+1)^2}dt$

$=\frac{t}{t^2+1}+2\tan^{-1}t-2I$

$\therefore I=\frac{1}{2}\frac{t}{t^2+1}+\frac{1}{2}\tan^{-1}t+C$

$=\frac{1}{2}\left(\frac{\sqrt{x^2-1}}{x^2}+\tan^{-1}\sqrt{x^2-1}\right)+C$

134 **(d)**

$I=\int_0^1\frac{\tan^{-1}x}{x}\,dx$

Putting $x=\tan\theta\ \Rightarrow\ dx=\sec^2\theta\,d\theta$

$\Rightarrow\ I=\int_0^{\pi/4}\frac{\theta}{\tan\theta}\sec^2\theta\,d\theta$

$=\int_0^{\pi/4}\frac{2\theta}{\sin 2\theta}d\theta$

Putting $2\theta=t$, i.e., $2d\theta=dt$,

We get $I=\frac{1}{2}\int_0^{\pi/2}\frac{t}{\sin t}\,dt$

$=\frac{1}{2}\int_0^{\pi/2}\frac{x}{\sin x}dx$

135 **(c)**

Since, $J=\int\frac{e^{3x}}{1+e^{2x}+e^{4x}}dx$

$\therefore\ J-I=\int\frac{(e^{3x}-e^x)}{1+e^{2x}+e^{4x}}\,dx$

$=\int\frac{(u^2-1)}{1+u^2+u^4}du\quad[u=e^x]$

$=\int\frac{\left(1-\frac{1}{u^2}\right)}{1+\frac{1}{u^2}+u^2}\,du=\int\frac{\left(1-\frac{1}{u^2}\right)}{\left(u+\frac{1}{u}\right)^2-1}\,du$

$=\int\frac{dt}{t^2-1}\quad\left[\text{put } u+\frac{1}{u}=t\Rightarrow\left(1-\frac{1}{u^2}\right)du=dt\right]$

$$=\frac{1}{2}\log\left|\frac{t-1}{t+1}\right|+c=\frac{1}{2}\log\left|\frac{u^2-u+1}{u^2+u+1}\right|+c$$
$$=\frac{1}{2}\log\left|\frac{e^{2x}-e^x+1}{e^{2x}+e^x+1}\right|+c$$

136 **(c)**

Let$I=\int\frac{(ax^2-b)dx}{x\sqrt{c^2x^2-(ax^2+b)^2}}$

$$=\int\frac{\left(a-\frac{b}{x^2}\right)dx}{\sqrt{c^2-\left(ax+\frac{b}{x}\right)^2}},\begin{cases}\text{put } ax+\frac{b}{x}=t\\ \therefore\left(a-\frac{b}{x^2}\right)dx=dt\end{cases}$$
$$=\int\frac{dt}{\sqrt{c^2-t^2}}=\sin^{-1}\left(\frac{t}{c}\right)+k$$
$$=\sin^{-1}\left(\frac{ax+\frac{b}{x}}{c}\right)+C$$

137 **(b)**

$$I=\int 4\sin x\cos\frac{x}{2}\cos\frac{3x}{2}dx$$
$$=\int 2\sin x(\cos 2x+\cos x)dx$$
$$=\int(\sin 3x-\sin x+\sin 2x)\,dx$$
$$=\cos x-\frac{1}{3}\cos 3x-\frac{1}{2}\cos 2x+C$$

138 **(d)**

The given integrand is a perfect differential coeff. of

$$\prod_{r=1}^{n}(x+r)$$
$$\Rightarrow I=\left[\prod_{r=1}^{n}(x+r)\right]_0^1=(n+1)!-n!=n\cdot n!$$

139 **(a)**

Let $I=\int_1^3\frac{\sin 2x}{x}dx$

Put $2x=t,\Rightarrow\ dx=\frac{dt}{2}$

$$\Rightarrow\ I=\frac{2}{2}\int_2^6\frac{\sin t}{t}\,dt=\int_2^6\frac{\sin t}{t}dt$$

But given $\int\frac{\sin x}{x}dx=F(x)$

$$\Rightarrow\int_2^6\frac{\sin t}{t}\,dt=F(6)-F(2)$$

140 **(a)**

Let $x=\tan\theta$,then $dx=\sec^2\theta d\theta$

Now $y=\int\frac{dx}{(1+x^2)^{\frac{3}{2}}}=\int\frac{\sec^2\theta}{(1+\tan^2\theta)^{\frac{3}{2}}}d\theta$

$$=\int\frac{\sec^2\theta}{(\sec^2\theta)^{\frac{3}{2}}}d\theta$$
$$=\int\frac{\sec^2\theta}{\sec^3\theta}\,d\theta=\int\frac{d\theta}{\sec\theta}=\int\cos\theta d\theta$$

Hence,$y=\sin\theta+c=\frac{x}{\sqrt{1+x^2}}+c$ (1)

$$\left[\because\tan\theta=x=\frac{x}{1}\ \therefore\sin\theta=\frac{x}{\sqrt{1^2+x^2}}\right]$$

Given when $x=0,y=0\Rightarrow$ from equation (1),$0=0+c$

$\Rightarrow c=0$

$\Rightarrow$from equation (1),$y=\frac{x}{\sqrt{1+x^2}}$

$\Rightarrow$when $x=1,y=\frac{1}{\sqrt{2}}$

141 **(c)**

Let $g(x)=\int_0^{x^3}f(t)dt$

Now $\int_0^8 f(t)dt=g(2)=\frac{g(2)-g(1)}{2-1}+\frac{g(1)-g(0)}{1-0}$

$=g'(\alpha)+g'(\beta)$

$=3[\alpha^2 f(\alpha^3)+\beta^2 f(\beta^3)]$

142 **(a)**

$$I_k=\int_1^e(\text{In }x)^k\,dx=\left|x\,(\text{In }x)^k\right|_1^e-k\int_1^e(\text{In }x)^{k-1}dx$$

$\Rightarrow I_k=e-kI_{k-1}$

$\Rightarrow I_4=e-4I_3$

$=e-4\,[e-3(e-2I_1)]$

$=9e-24\ \ (\because I_1=1)$

143 **(a)**

$$I=\int\left(\frac{x+2}{x+4}\right)^2e^x\,dx=\int e^x\left[\frac{x^2+4x+4}{(x+4)^2}\right]dx$$
$$\Rightarrow I=\int e^x\left[\frac{x(x+4)}{(x+4)^2}+\frac{4}{(x+4)^2}\right]dx$$
$$=\int e^x\left[\frac{x}{x+4}+\frac{4}{(x+4)^2}\right]dx$$
$$=e^x\left(\frac{x}{x+4}\right)+C$$

144 **(c)**

$$I=\int\frac{\ln\left(\frac{x-1}{x+1}\right)}{x^2-1}dx,\text{let } t=\ln\left(\frac{x-1}{x+1}\right)$$

$$\Rightarrow \frac{dt}{dx}=\frac{x+1}{x-1}\left\{\frac{x+1-(x-1)}{(x+1)^2}\right\}=\frac{2}{(x^2-1)}$$

$$\Rightarrow \frac{dx}{x^2-1}=\frac{dt}{2}$$

$$\Rightarrow I=\frac{1}{2}\int tdt=\frac{1}{4}t^2+C=\frac{1}{4}\left(\ln\left(\frac{x-1}{x+1}\right)\right)^2+C$$

145 **(c)**

$$f(x)=\begin{cases}\int_{-1}^{x}-tdt & -1\le x\le 0\\ \int_{-1}^{0}-tdt+\int_{0}^{x}tdt & x\ge 0\end{cases}$$

$$=\begin{cases}\frac{1}{2}(1-x^2), & -1\le x\le 0\\ \frac{1}{2}(1+x^2), & x\ge 0\end{cases}$$

146 **(c)**

$$\int\frac{dx}{(x+2)(x^2+1)}=a\ln(1+x^2)$$
$$+b\tan^{-1}x+\frac{1}{5}\ln|x+2|+C$$

Differentiating both sides, we get

$$\frac{1}{(x+2)(x^2+1)}=\frac{2ax}{(1+x^2)}+\frac{b}{(1+x^2)}+\frac{1}{5(x+2)}$$

$$\Rightarrow \frac{1}{(x+2)(x^2+1)}$$
$$=\frac{(x+2)(5b+10ax)+1+x^2}{5(1+x^2)(x+2)}$$

$\Rightarrow 5=(1+x^2)+5(b+2ax)(x+2)$

Comparing the like powers of x on both sides, we get

$1+10a=0, b+4a=0, 10b+1=5$

$$\Rightarrow a=-\frac{1}{10}, b=\frac{2}{5}$$

147 **(c)**

$f(x)=\frac{e^x}{1+e^x}$ $\therefore$ $f(a)=\frac{e^a}{1+e^a}$ and $f(-a)=\frac{e^{-a}}{1+e^{-a}}$

$$=\frac{e^{-a}}{1+\frac{1}{e^a}}=\frac{1}{1+e^a}$$

$$\Rightarrow f(a)+f(-a)=\frac{e^a+1}{1+e^a}=1$$

Let $f(-a)=\alpha$ $\therefore$ $f(a)=1-a$

Now, $I_1=\int_\alpha^{1-\alpha}xg\big(x(1-x)\big)dx$

$$=\int_\alpha^{1-\alpha}(1-x)g((1-x)\big(1-(1-x)\big)dx$$

$$=\int_\alpha^{1-\alpha}(1-x)g\big(x(1-x)\big)dx$$

$$\therefore 2I_1=\int_\alpha^{1-\alpha}g\big(x(1-x)\big)dx=I_2 \ \therefore \frac{I_2}{I_1}=2$$

148 **(b)**

Let $I=\int_{e^{-1}}^{e^2}\left|\frac{\log_e x}{x}\right|dx$

For $\frac{1}{e}<x<1, \log_e x<0$, hence $\frac{\log_e x}{x}<0$

For $1<x<e^2, \log x>0$, hence $\frac{\log_e x}{x}>0$

$$\therefore I=\int_{1/e}^{1}-\frac{\log_e x}{x}dx+\int_{1}^{2}\frac{\log_e x}{x}dx$$

$$=-\frac{1}{2}[(\log_e x)^2]_{1/e}^{1}+\frac{1}{2}[(\log_e x)^2]_1^{e^2}$$

$$=-\frac{1}{2}[0-(-1)^2]+\frac{1}{2}[(2)^2-0]$$

$$=\frac{1}{2}+2=\frac{5}{2}$$

149 **(a)**

$$\int_{-\pi}^{\pi}\sin n\,x\sin mx\,dx$$

$$=\int_0^\pi 2\sin mx\sin nx\,dx$$

$$=\int_0^\pi(\cos(m-n)x-\cos(m+n)x]\,dx$$

$$=\left|\frac{\sin(m-n)x}{m-n}-\frac{\sin(m+n)x}{m+n}\right|_0^\pi=0$$

150 **(c)**

$$I=\int_0^{\pi/2}\frac{\sin xdx}{1+\sin x+\cos x}$$

$$=\int_0^{\pi/2}\frac{\cos xdx}{1+\sin x+\cos x}$$

$$\Rightarrow 2I=\int_0^{\pi/2}\frac{\sin x+\cos x+1-1}{\sin x+\cos x+1}dx$$

$$\Rightarrow 2I=\frac{\pi}{2}-\log 2$$

$$\Rightarrow I=\frac{\pi}{4}-\frac{1}{2}\log 2$$

151 **(b)**

Let $I=\int_0^\pi x\sin^4 x\ dx$ ...(i)

$I=\int_0^\pi(\pi-x)\sin^4 x\ dx$ ...(ii)

On adding Eqs. (i) and (ii), we get

$$2I=\pi\int_0^\pi\sin^4 x\ dx$$

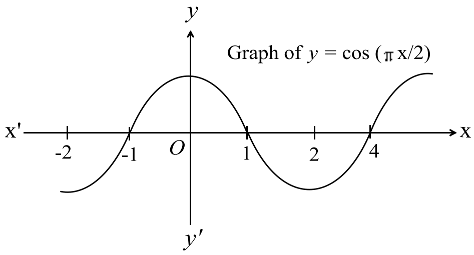
$$=2\pi\int_0^{\pi/2}\sin^4 x\ dx$$

$$=2\pi.\frac{4-1}{4}.\frac{4-3}{4-2}.\frac{\pi}{2}$$

$$=2\pi.\frac{3}{4}.\frac{1}{2}.\frac{\pi}{2}=\frac{3\pi^2}{8}$$

$\Rightarrow\ I=\dfrac{3\pi^2}{16}$

152 **(c)**



From graph, $\int_{-2}^{1}\left[x\left[1+\cos\frac{\pi x}{2}\right]+1\right]dx$

$$=\int_{-2}^{-1}[x[1+(-1)]+1]dx+\int_{-1}^{1}[x[1+0]+1]dx$$

$$=(x)_{-2}^{-1}+\int_{-1}^{1}[x+1]dx$$

$$=\left(-1-(-2)\right)+\int_{-1}^{0}0dx$$

$$+\int_{0}^{1}1dx=2$$

153 **(b)**

$$f(x)=\int\frac{x^2dx}{(1+x^2)(1+\sqrt{1+x^2})}$$

Let $x=\tan\theta\Rightarrow dx=\sec^2\theta d\theta=(1+x^2)d\theta$

$$\Rightarrow f(x)=\int\frac{x^2dx}{(1+x^2)(1+\sqrt{1+x^2})}$$

$$=\int\frac{\tan^2\theta\sec^2\theta d\theta}{\sec^2\theta\,(1+\sec\theta)}$$

$$=\int\frac{\tan^2\theta\, d\theta}{1+\sec\theta}$$

$$=\int\frac{\sin^2\theta d\theta}{\cos\theta\,(1+\cos\theta)}$$

$$=\int\frac{1-\cos^2\theta d\theta}{\cos\theta\,(1+\cos\theta)}$$

$$=\int\frac{(1-\cos\theta)d\theta}{\cos\theta}$$

$$=\int\sec\theta d\theta-\int d\theta$$

$$=\log\left(x+\sqrt{1+x^2}\right)-\tan^{-1}x+C$$

Given $f(0)=0$

$\Rightarrow\ 0=\log 1-0+C$

$\Rightarrow\ C=0$

$\Rightarrow\ f(1)=\log\left(1+\sqrt{1+1}\right)-\tan^{-1}(1)$

$=\log\left(1+\sqrt{2}\right)-\dfrac{\pi}{4}$

154 **(a)**

$$\int_1^{\frac{1+\sqrt{5}}{2}}\frac{1+\frac{1}{x^2}}{x^2-1+\frac{1}{x^2}}\log\left(1+x-\frac{1}{x}\right)dx$$

$$=\int_1^{\frac{1+\sqrt{5}}{2}}\frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+1}\log\left(1+x-\frac{1}{x}\right)dx$$

Put $x-\frac{1}{x}=t\ \therefore\left(1+\frac{1}{x^2}\right)dx=dt$

If $x=1, t=0$, and $x=\frac{\sqrt{5}+1}{2}, t=1$

$\Rightarrow I=\int_0^1\frac{\text{In}(1+t)dt}{1+t^2}$ Put $t=\tan\theta\ \therefore dt=\sec^2\theta d\theta$

$$I=\int_0^{\pi/4}\text{In}\,(1+\tan\theta)d\theta\ =\frac{\pi}{8}\log_e 2$$

155 **(c)**

Putting $a^6+x^8=t^2$, we get

$$\Rightarrow I=\int\frac{t^2}{t^2-a^6}dt=t+\frac{a^3}{2}\ln\left|\frac{t-a^3}{t+a^3}\right|+C$$

156 **(a)**

$$\int e^x\left(\frac{1}{\sqrt{1+x^2}}-\frac{x}{\sqrt{(1+x^2)^3}}+\frac{x}{\sqrt{(1+x^2)^3}}+\frac{1-2x^2}{\sqrt{(1+x^2)^5}}\right)$$

$$=e^x\frac{1}{\sqrt{1+x^2}}+e^x\frac{x}{\sqrt{(1+x^2)^3}}$$

$$=e^x\left(\frac{1}{\sqrt{1+x^2}}+\frac{x}{\sqrt{(1+x^2)^3}}\right)+C$$

Using $\int e^x\left(f(x)+f'(x)\right)dx$, we get

$=e^xf(x)+c$

157 **(b)**

$$I=\int\frac{\sin 2x}{(3+4\cos x)^3}dx$$

and put $3+4\cos x=t$, so that $-4\sin x\,dx=dt$

$$I=\frac{-1}{8}\int\frac{(t-3)}{t^3}dt=\frac{1}{8}\left(\frac{1}{t}-\frac{3}{2}\frac{1}{t^2}\right)+C$$

$$=\frac{2t-3}{16t^2}=\frac{8\cos x+3}{16(3+4\cos x)^2}+C$$

158 **(b)**

Here, $\int x^5(1+x^3)^{2/3}dx$

Let $1+x^3=t^2$ and $3x^2dx=2t\,dt$

$$\therefore\int x^5\,(1+x^3)^{2/3}dx$$

$$=\int x^3\,(1+x^3)^{2/3}x^2dx$$

$$=\int(t^2-1)(t^2)^{2/3}x^2dx$$

$$=\frac{2}{3}\int(t^2-1)\,t^{7/3}dt$$

$$=\frac{2}{3}\int\left(t^{13/3}-\ t^{7/3}\right)dt$$

$$=\frac{2}{3}\left\{\frac{3}{16}t^{16/3}-\frac{3}{10}t^{10/3}\right\}+C$$

$$= \frac{1}{8}(1+x^3)^{8/3} - \frac{1}{5}(1+x^3)^{5/3} + C$$

159 **(c)**

$$I = \int \frac{1-x^7}{x(1+x^7)}dx = a\ln|x| + b\ln|1+x^7| + C$$

Diff. both sides , we get $\frac{1-x^7}{x(1+x^7)} = \frac{a}{x} + b\frac{7x^6}{1+x^7}$

$\Rightarrow 1 - x^7 = a(1+x^7) + 7bx^7$

$\Rightarrow a = 1, a + 7b = -1$

$\Rightarrow b = -2/7$

160 **(b)**

$$I = \int xe^x \cos x dx$$

$$= xe^x \sin x - xe^x(-\cos x) - \int (xe^x + e^x)\cos x\, dx - \int e^x \sin x dx$$

$$= xe^x \sin x + x\, e^x \cos x - \int xe^x \cos x dx - \int e^x(\cos x + \sin x)dx$$

$\Rightarrow 2I = xe^x(\sin x + \cos x) - e^x \sin x + d$

$\Rightarrow 2I = e^x((x-1)\sin x + x\cos x) + d$

$\Rightarrow I = \frac{1}{2}e^x\big((x-1)\sin x + x\cos x\big) + d$

$\Rightarrow a = \frac{1}{2}, b = -1, c = 1$

161 **(a)**

Let $I = \int_{-3\pi/4}^{5\pi/4} \frac{(\sin x + \cos x)}{e^{x-\pi/4}+1}dx$

$$\Rightarrow\ I = \int_{-3\pi/4}^{5\pi/4} \frac{\sqrt{2}\cos\left(x - \frac{\pi}{4}\right)}{e^{x-\pi/4}+1}\,dx$$

Putting $x - \frac{\pi}{4} = t\ \Rightarrow\ dx = dt$

$\Rightarrow\ I = \int_{-\pi}^{\pi} \frac{\sqrt{2}\cos t}{e^t+1}dt(1)$

Replacing t by $\pi + (-\pi) - t$ or $-t$, we get

$I = \int_{-\pi}^{\pi} \frac{\sqrt{2}\cos(-t)}{e^{-t}+1}dt = \int_{-\pi}^{\pi} \frac{e^t\sqrt{2}\cos t}{e^t+1}dt(2)$

Adding equation (1) and (2), we get

$$2I = \sqrt{2}\int_{-\pi}^{\pi} \cos t\, dt\ \Rightarrow I = 0$$

162 **(c)**

$$\frac{dx}{dt} = f'''(t)\cos t - f''(t)\sin t + f''(t)\sin t + f'(t)\cos t$$

$$= [f'''(t) + f'(t)]\cos t$$

$$\frac{dy}{dt} = -f'''(t)\sin t - f''(t)\cos t + f''(t)\cos t - f'(t)\sin t$$

$$= -[f'''(t) + f'(t)]\sin t$$

$$\Rightarrow \left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right]^{1/2}$$

$$= [(f'''(t) + f'(t))^2(\cos^2 t + \sin^2 t)]^{1/2}$$

$$= f'''(t) + f'(t)$$

$$\Rightarrow \int \left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right]^{1/2} dt = f''(t) + f(t) + C$$

163 **(c)**

$$\int \frac{px^{p+2q-1} - qx^{q-1}}{(x^{p+q}+1)^2}dx$$

$$= \int \frac{px^{p-1} - qx^{-q-1}}{(x^p + x^{-q})^2}dx$$

(Dividing $N^r$and D^rbyx^{2q})

$$= \int \frac{dt}{t^2} = -\frac{1}{t} + C = -\frac{1}{x^p + x^{-q}} + C$$

$$= -\frac{x^q}{x^{p+q}+1} + C$$

164 **(a)**

$$f(x) = \int_0^1 \frac{dt}{1+|x-t|} = \int_0^x \frac{dt}{1+x-1} + \int_x^1 \frac{dt}{1-x+t}$$

$$\Rightarrow f'(x) = \frac{1}{1+x-x} - \frac{1}{1-x+x} = 0$$

165 **(d)**

$$I = \int \frac{\sin x \cos x}{\sin x + \cos x}dx$$

$$= \frac{1}{2}\int \frac{(\sin x + \cos x)^2 - 1}{\sin x + \cos x}dx$$

$$= \frac{1}{2}\int \left[\sin x + \cos x - \frac{1}{\sqrt{2}\sin(x + \pi/4)}\right]dx$$

$$= \frac{1}{2}[\sin x + \cos x] - \frac{1}{2\sqrt{2}}\log|\mathrm{cosec}(x + \pi/4) - \cot(x + \pi/4)| + C$$

166 **(b)**

$$\mathrm{g}\left(x + \frac{\pi n}{2}\right) = \int_0^{x+\frac{n\pi}{2}} (|\sin t| + |\cos t|)dt$$

$$= \int_0^x (|\sin t| + |\cos t|)\, dt$$

$$+ \int_x^{x+\frac{n\pi}{2}} (|\sin t| + |\cos t|)\, dt$$

$= g(x) + \int_0^{\frac{n\pi}{2}} (|\sin t| + |\cos t|)\, dt$(as $|\sin t| +$ $/\cos t/$ has a period $\pi/2$)

$= g(x) + g\left(\frac{n\pi}{2}\right)$

167 **(a)**

$f(x) = \int_1^x \frac{e^t}{t} dt \Rightarrow f(1) = 0$ and $f'(x) = \frac{e^x}{x}$

Let $g(x) = f(x) - \text{In}(x), x \in R^+$

$\Rightarrow g'(x) = f'(x) - \frac{1}{x} = \frac{e^x - 1}{x} > 0 \ \forall\ x \in R^+$

$\Rightarrow$ g(x) is increasing for $x \in R^+$,

$g(1) = f(1) - \text{In } 1 = 0 - 0 = 0$

$\Rightarrow g(x) > 0\ \forall x > 1$ and $g(x) \le 0\ \forall\ x \in (0, 1]$

$\Rightarrow \text{In}x \ge f(x) \forall x \in (0,1]$

168 **(c)**

$$I = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1 + a^x}\, dx \quad (1)$$

$$= \int_{-\pi}^{\pi} \frac{\cos^2(0 - x)}{1 + a^{(0-x)}}\, dx$$

$$\left[\text{Using the property} \int_a^b f(x)dx = \int_a^b \big(f(a + b - x)\big)dx\right]$$

$$\Rightarrow I = \int_{-\pi}^{\pi} \frac{a^x \cos^2 x}{1 + a^x}\, dx \quad (2)$$

Adding equations (1) and (2), we get

$$2I = \int_{-\pi}^{\pi} \cos^2 x\, dx \quad (3)$$

$$= 2\int_0^{\pi} \cos^2 x\, dx$$

$$= 4\int_0^{\pi/2} \cos^2 x dx$$

$$\left[\because \int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx \text{ if } f(2a - x) = f(x)\right]$$

$$= 4\int_0^{\pi/2} \sin^2 x\, dx \quad (4)$$

Adding equations (3) and (4), we get

$$4I = 4\int_0^{\pi/2} 1\, dx$$

$\Rightarrow I = \pi/2$

169 **(b)**

$$I = 0 + 2\int_0^{\pi} \frac{2x \sin x}{1 + \cos^2 x}$$

$$= 4\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx = 4\frac{\pi^2}{4} = \pi^2$$

170 **(c)**

$$\int_0^x |\sin t| dt = \int_0^{2n\pi} |\sin t| dt + \int_{2n\pi}^{x} |\sin t|\, dt$$

$= 2n \int_0^{\pi} |\sin t| dt + \int_{2n\pi}^{x} \sin t dt$ (as x lies in either 1st or 2nd quadrant)

$= 2n\, (-\cos t)_0^{\pi} + (-\cos t)_{2n\pi}^{x} = 4n - \cos x + 1$

171 **(a)**

$\int_0^{f(x)} t^2 dt = x \cos \pi x \quad (1)$

$$\Rightarrow \left.\frac{t^3}{3}\right|_0^{f(x)} = x \cos \pi x$$

$\Rightarrow [f(x)]^3 = 3x \cos \pi x \quad (2)$

$\Rightarrow [f(9)]^3 = -27$

$\Rightarrow f(9) = -3$

Also, differentiating equation (1) w.r.t. x, we get

$[f(x)]^2 f'(x) = \cos \pi x - x\, \pi \sin \pi x$

$\Rightarrow [f(9)]^2 f'(9) = -1$

$$\Rightarrow f'(9) = -\frac{1}{\big(f(9)\big)^2} = -\frac{1}{9}$$

172 **(b)**

$$\int_0^{\pi/2} |\sin x - \cos x| dx$$

$$= \int_0^{\pi/4} -(\sin x - \cos x) dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x) dx$$

$= |\cos x + \sin x|_0^{\pi/4} + |-\cos x - \sin x|_{\pi/4}^{\pi/2}$

$$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 - 0\right) + \left(-0 - 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$

$$= \frac{4}{\sqrt{2}} - 2 = 2\sqrt{2} - 2 = 2(\sqrt{2} - 1)$$

173 **(c)**

In I_2, Put $x + 1 = t$, then

$$I_2 = \int_{-2}^{2} \frac{2t^2 + 11t + 14}{t^4 + 2}\, dt$$

$$= \int_{-2}^{2} \frac{2x^2 + 11x + 14}{x^4 + 2}\, dx$$

$\therefore\ I_1 + I_2$

$$= \int_{-2}^{2} \frac{x^6 + 3x^5 + 7x^4 + 2x^2 + 11x + 14}{x^4 + 2} dx$$

$$= \int_{-2}^{2} \frac{(x^2 + 3x + 7)(x^4 + 2) + 5x}{x^4 + 2} dx$$

$$= \int_{-2}^{2} (x^2 + 3x + 7)\, dx + 5 \int_{-2}^{2} \frac{x}{x^4 + 2} dx$$

$$= 2\int_{0}^{2} (x^2 + 7) dx = \frac{100}{3}$$

(The other integrals are zero, being integrals of odd functions)

174 **(b)**

$$\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} dx$$

$$= \int \frac{(\sin^2 x - \cos^2 x)(\sin^4 x + \cos^4 x)}{1 - 2\sin^2 x \cos^2 x}$$

$$= \int -\cos 2x dx = -\frac{1}{2}\sin 2x + C$$

175 **(d)**

$$I = \int \frac{x^2 - 1}{x^3\sqrt{2x^4 - 2x^2 + 1}} dx$$

$$= \frac{1}{4}\int \frac{\frac{4}{x^3} - \frac{4}{x^5}}{\sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}}} dx$$

$\Rightarrow$ Put $2 - \frac{2}{x^2} + \frac{1}{x^4} = t \Rightarrow \left(\frac{4}{x^3} - \frac{4}{x^3}\right) dx = dt$

$$\Rightarrow I = \frac{1}{4}\int \frac{dt}{\sqrt{t}} = \frac{2\sqrt{t}}{4} + C$$

$$= \frac{\sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}}}{2} + C$$

$$= \frac{\sqrt{2x^4 - 2x^2 + 1}}{2x^2} + C$$

176 **(c)**

$$I_n = x(\ln x)^n - \int \frac{x(n)(\ln x)^{n-1}}{x}\, dx$$

$= x(\ln x)^n - n\, I_{(n-1)}$

$\Rightarrow I_n + nI_{n-1} = x(\ln x)^n$

177 **(c)**

$$I = \int \frac{\cos 4x - 1}{\cot x - \tan x} dx$$

$$= \int \frac{-2\sin^2 2x\,(\sin x \cos x)}{(\cos^2 x - \sin^2 x)} dx$$

$$= -\int \frac{\sin^2 2x \sin 2x}{\cos 2x} x$$

$$= \int \frac{(\cos^2 2x - 1)\sin 2x}{\cos 2x} dx$$

Let $t = \cos 2x \Rightarrow dt = -2\sin 2x\, dx$

$$\Rightarrow I = \frac{1}{2}\int \frac{(1 - t^2)}{t} dt = \frac{1}{2}\ln|t| - \frac{t^2}{4} + C$$

$$= \frac{1}{2}\ln|\cos 2x| - \frac{1}{4}\cos^2 2x + c$$

178 **(b)**

$$\int \frac{\cos 4x + 1}{\cot x - \tan x} dx$$

$$= \int \frac{2\cos^2 2x}{\cos^2 x - \sin^2 x} \sin x \cos x\, dx$$

$$= \int \cos 2x \sin 2x dx$$

$$= \frac{1}{4}\int \sin 4x dx = -\frac{1}{8}\cos 4x + C$$

179 **(c)**

$$\int_{-1}^{1/2} \frac{e^x(2 - x^2)dx}{(1 - x)\sqrt{1 - x^2}}$$

$$= \int_{-1}^{1/2} \frac{e^x(1 - x^2 + 1)}{(1 - x)\sqrt{1 - x^2}}$$

$$= \int_{-1}^{1/2} e^x \left[\sqrt{\frac{1 + x}{1 - x}} + \frac{1}{(1 - x)\sqrt{1 - x^2}}\right] dx$$

$$= e^x \sqrt{\frac{1 + x}{1 - x}}\Bigg|_{-1}^{1/2}$$

$= \sqrt{3e}$

180 **(c)**

Put $x = a\sin\theta\ \therefore\ dx = a\cos\theta\, d\theta$

When $x = 0, \theta = 0; x = a, \theta = \frac{\pi}{2}$

$\therefore$ given integral $I = \int_0^{\pi/2} \frac{a\cos\theta\, d\theta}{a\sin\theta + a\cos\theta}$

$$= \int_0^{\pi/2} \frac{\cos\theta\, d\theta}{\sin\theta + \cos\theta}$$

Also, $I = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2} - \theta\right) d\theta}{\sin\left(\frac{\pi}{2} - \theta\right) + \cos\left(\frac{\pi}{2} - \theta\right)}$

$$= \int_0^{\pi/2} \frac{\sin\theta\, d\theta}{\cos\theta + \sin\theta}$$

$$\therefore\ 2I = \int_0^{\pi/2} \frac{\cos\theta + \sin\theta}{\cos\theta + \sin\theta} d\theta = \int_0^{\pi/2} d\theta = \frac{\pi}{2}$$

$$\therefore\ I = \frac{\pi}{4}$$

181 **(c)**

$$I = \int e^{\tan x}(\sin x - \sec x)dx$$

$$= \int \sin x\, e^{\tan x} dx - \int \sec x\, e^{\tan x}\, dx$$
$$= -e^{\tan x} \cos x + \int \cos x\ e^{\tan x} \sec^2 x dx - \int \sec x\ e^{\tan x} dx$$
$$= -\cos x\ e^{\tan x} + C$$

182 **(d)**

$$\int_0^a x^4\sqrt{a^2-x^2}\,dx$$
$$= \left[\frac{-x^3(a^2-x^2)^{3/2}}{3}\right]_0^a + a^2 \cdot \frac{3}{6}\int_0^a x^2\sqrt{a^2-x^2}\ dx$$

(Integrating by parts with x^3 as first function and $x\sqrt{a^2-x^2}$as second function)

$$= \frac{a^2}{2}\int_0^a x^2\sqrt{a^2-x^2}\,dx$$
$$\Rightarrow \frac{\int_0^a x^4\sqrt{a^2-x^2}\,dx}{\int_0^a x^2\sqrt{a^2-x^2}\,dx} = \frac{a^2}{2}$$

183 **(b)**

$$\left|\int_a^b f(x)dx - (b-a)f(a)\right|$$
$$= \left|\int_a^b f(x)dx - \int_a^b f(a)dx\right|$$
$$= \left|\int_a^b (f(x)-f(a))dx\right|$$
$$\le \int_a^b |f(x)-f(a)|dx$$
$$\le \int_a^b |x-a|dx = \int_a^b (x-a)dx = \frac{(b-a)^2}{2}$$

184 **(b)**

$$\int_1^e \left(\frac{\tan^{-1}x}{x} + \frac{\log x}{1+x^2}\right)dx$$
$$= \int_1^e \frac{\tan^{-1}x}{x}dx + \int_1^e \frac{\log x}{1+x^2}\ dx$$
$$= \int_1^e \frac{\tan^{-1}x}{x}dx + (\log x \tan^{-1}x)_1^e - \int_1^e \frac{\tan^{-1}x}{x}\ dx$$
$$= \tan^{-1} e$$

185 **(c)**

$$\lim_{n\to\infty}\sum_{r=1}^{4n}\frac{\sqrt{n}}{\sqrt{r}(3\sqrt{r}+4\sqrt{n})^2}$$
$$T_r = \frac{1}{\sqrt{\frac{r}{n}}\,n\left(3\sqrt{\frac{r}{n}}+4\right)^2}$$
$$\Rightarrow S = \lim_{n\to\infty}\frac{1}{n}\sum_1^{4n}\frac{1}{\left(3\sqrt{\frac{r}{n}}+4\right)^2\sqrt{\frac{r}{n}}}$$
$$= \int_0^4 \frac{dx}{\sqrt{x}(3\sqrt{x}+4)^2}$$

Put $3\sqrt{x}+4 = t \Rightarrow \frac{3}{2}\frac{1}{\sqrt{x}}\ dx = dt$

$$= \frac{2}{3}\int_4^{10}\frac{dt}{t^2} = \frac{2}{3}\left[\frac{1}{t}\right]_{10}^4 = \frac{1}{10}$$

186 **(c)**

Let $A = \lim_{n\to\infty}\left[\tan\frac{\pi}{2n}\tan\frac{2\pi}{2n}\cdots\tan\frac{n\pi}{2n}\right]^{1/n}$

$$\therefore \log A = \lim_{n\to\infty}\frac{1}{n}\left[\log\tan\frac{\pi}{2n} + \log\tan\frac{2\pi}{2n} + \cdots + \log\tan\frac{n\pi}{2n}\right]$$
$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\log\tan\frac{\pi r}{2n} = \int_0^1 \log\tan\left(\frac{\pi}{2}x\right)dx$$

$= \frac{2}{\pi}\int_0^{\pi/2}\log\tan y\ dy$ (1)

$\left[\text{Putting} \frac{1}{2}\ \pi x = y\ \therefore dx = (2/\pi)dy\right]$

Now let $I = \int_0^{\pi/2}\log\tan y\ dy$

$I = \int_0^{\pi/2}\log\tan\left(\frac{1}{2}\pi - y\right)dy$(by property IV)

$$= \int_0^{\pi/2}\log\cot y\ dy$$
$$= -\int_0^{\pi/2}\log\tan y\ dy = -I$$

or $I + I = 0$ or $2I = 0$ or $I = 0$

$\therefore$ from equation (1), $\log A = 0\ \therefore\ A = e^0 = 1$

187 **(c)**

Write $2ax + x^2 = (x+a)^2 - a^2$,and put $x + a = a\sec\theta$,

So that $dx = a\sec\theta\ \tan\theta\ d\theta$

$$\therefore I = \int \frac{a\sec\theta\tan\theta}{a^3\tan^3\theta}d\theta$$
$$= \frac{1}{a^2}\int\frac{\cos\theta}{\sin^2\theta}d\theta$$
$$= -\frac{1}{a^2\sin\theta} + C$$
$$= -\frac{1}{a^2}\frac{\sec\theta}{\tan\theta} + C = -\frac{1}{a^2}\frac{x+a}{\sqrt{2ax+x^2}} + C$$

188 **(d)**

$$I = \int \frac{x^9 dx}{(4x^2+4)^6}$$

$$\int \frac{dx}{x^3\left(4+\frac{1}{x^2}\right)^6}$$
$$= -\frac{1}{2}\int \frac{d\left(4+\frac{1}{x^2}\right)}{\left(4+\frac{1}{x^2}\right)^6}$$
$$= -\frac{1}{2}\frac{\left(4+\frac{1}{x^2}\right)^{-5}}{-5} + C = \frac{1}{10}\left(4+\frac{1}{x^2}\right)^{-5} + C$$

189 **(d)**

$$\int x\log\left(1+\frac{1}{x}\right)dx$$
$$= \int x\log(x+1)dx - \int x\log x dx$$
$$= \frac{x^2}{2}\log(x+1) - \frac{1}{2}\int\frac{x^2}{x+1}dx - \frac{x^2}{2}\log x + \frac{1}{2}\int\frac{x^2}{x}dx$$
$$= \frac{x^2}{2}\log(x+1) - \frac{1}{2}\int\left(x-1+\frac{1}{x+1}\right)dx - \frac{x^2}{2}\log x + \frac{1}{4}x^2$$
$$= \frac{x^2}{2}\log(x+1) - \frac{x^2}{2}\log x - \frac{1}{2}\left(\frac{x^2}{2}-x\right) - \frac{1}{2}\log(x+1) + \frac{1}{4}x^2 + C$$
$$= \frac{x^2}{2}\log(x+1) - \frac{x^2}{2}\log x - \frac{1}{2}\log(x+1) + \frac{1}{2}x + C$$

Hence, $f(x) = \frac{x^2}{2} - \frac{1}{2}, \mathrm{g}(x) = -\frac{1}{2}\log x$ and $A = \frac{1}{2}$

190 **(c)**

The polynomial function is differentiable everywhere. Therefore, the points of extremum can only be the roots of the derivative. Further, the derivative of a polynomial is a polynomial. The polynomial of the least degree with roots $x = 1$ and $x = 3$ has the form $a\,(x-1)(x-3)$

Hence, $P'(x) = a(x-1)(x-3)$

Since at $x = 1$, we must have $P(1) = 6$, we have

$$P(x) = \int_1^x P'(x)dx + 6$$
$$= a\int_1^x (x^2 - 4x + 3)dx + 6$$
$$= a\left(\frac{x^3}{3} - 2x^2 + 3x - \frac{4}{3}\right) + 6$$

Also, $P(3) = 2$so $a = 3$. Hence, $P(x) = x^3 - 6x^2 + 9x + 2$

Thus, $\int_0^1 P(x)dx = \frac{1}{4} - 2 + \frac{9}{2} + 2 = \frac{19}{4}$

191 **(c)**

Differentiating both sides, we get

$$\frac{3\sin x + 2\cos x}{3\cos x + 2\sin x} = a + \frac{b(2\cos x - 3\sin x)}{(2\cos x + 3\cos x)}$$
$$= \frac{\sin x(2a-3b) + \cos x(3a+2b)}{(3\cos x + 2\sin x)}$$

Comparing like terms on both sides, we get

$$3 = 2a - 3b, 2 = 3a + 2b \Rightarrow a = \frac{12}{13}, b = -\frac{15}{39}$$

192 **(c)**

We have $\int_0^1 e^{x^2}(x-\alpha)dx = 0$

$$\Rightarrow \int_0^1 e^{x^2}x\,dx = \int_0^1 e^{x^2}\,\alpha\,dx$$

$\Rightarrow \frac{1}{2}\int_0^1 e^x dt = \alpha\int_0^1 e^{x^2}\,dx$, where $t = x^2$

$\Rightarrow \frac{1}{2}(e-1) = \alpha\int_0^1 e^{x^2}\,dx(1)$

Since, e^{x^2} is an increasing function for $0 \le x \le 1$, therefore,

$1 \le e^{x^2} \le e$ when $0 \le x \le 1$

$$\Rightarrow 1(1-0) \le \int_0^1 e^{x^2}dx \le e(1-0)$$

$\Rightarrow 1 \le \int_0^1 e^{x^2}\,dx \le \mathrm{e}(2)$

From equations (1) and (2), we find that L.H.S. of equation (1) is positive and $\int_0^1 e^{x^2}\;dx$ lies between 1 and e. Therefore, $\alpha$is a positive real number.

Now, from equation (1), $\alpha = \frac{\frac{1}{2}(e-1)}{\int_0^1 e^{x^2}\,dx}$ (3)

The denominator of equation (3) is greater than unity and the numerator lies between 0 and 1. Therefore, $0 < \alpha < 1$

193 **(a)**

$$\int_{-1}^3\left(\tan^{-1}\frac{x}{x^2+1} + \tan^{-1}\frac{x^2+1}{x}\right)dx$$
$$= \int_{-1}^0\left(\tan^{-1}\frac{x}{x^2+1} + \tan^{-1}\frac{x^2+1}{x}\right)dx + \int_0^3\left(\tan^{-1}\frac{x}{x^2+1} + \tan^{-1}\frac{x^2+1}{x}\right)dx$$
$$= \int_{-1}^0 -\frac{\pi}{2}dx + \int_0^3\frac{\pi}{2}\,dx$$
$$= \left[-\frac{\pi}{2}x\right]_{-1}^0 + \left[\frac{\pi}{2}x\right]_0^3$$
$$= \pi$$

194 **(a)**

Let $I = \int\frac{(1-\cos\theta)^{2/7}}{(1+\cos\theta)^{9/7}}d\theta$

$$I=\int \frac{(2\sin^2\theta/2)^{2/7}}{(2\cos^2\theta/2)^{9/2}}d\theta=\frac{1}{2}\int\frac{(\sin\theta/2)^{4/7}}{(\cos\theta/2)^{18/7}}d\theta$$

Put $\frac{\theta}{2}=t\ \therefore \frac{d\theta}{2}=dt$

$\Rightarrow I=\int\frac{(\sin t)^{4/7}}{(\cos t)^{18/7}}\,dt$ (Here $m+n=-2$)

$$=\int(\tan t)^{4/7}\sec^2 t dt$$

Put $\tan t=u\ \therefore \sec^2 t\,dt=du$

$$\Rightarrow I=\int u^{4/7}du=\frac{u^{11/7}}{11/7}+c=\frac{7}{11}(\tan t)^{11/7}+C$$

$$=\frac{7}{11}\left(\tan\frac{\theta}{2}\right)^{11/7}+C$$

195 **(a)**

$$f(x)=\cos x-\int_0^x(x-t)f(t)dt$$

$$\Rightarrow f(x)=\cos x-x\int_0^x f(t)dt+\int_0^x tf(t)dt$$

$$\Rightarrow f'(x)=-\sin x-xf(x)-\int_0^x f(t)dx+xf(x)$$

$$\Rightarrow f'(x)=-\sin x-\int_0^x f(t)dt$$

$\Rightarrow f''(x)=-\cos x-f(x)$

$\Rightarrow f''(x)+f(x)=-\cos x$

196 **(b)**

$$\int_{\cos x}^{1}t^2f(t)dt=1-\cos x$$

Differentiating both sides w.r.t. x

$$\frac{d}{dx}\int_{\cos x}^{1}t^2f(t)dt=\frac{d}{dx}(1-\cos x)$$

$\Rightarrow -\cos^2 xf(\cos x)(-\sin x)=\sin x$

$\Rightarrow \cos^2 xf\ (\cos x)\sin x=\sin x$

$$\Rightarrow f(\cos x)=\frac{1}{\cos^2 x}$$

Now $f\left(\frac{\sqrt{3}}{4}\right)$ is attained when $\cos x=\frac{\sqrt{3}}{4}$

$$f\left(\frac{\sqrt{3}}{4}\right)=\frac{16}{3}=5.33$$

$$\left[f\left(\frac{\sqrt{3}}{4}\right)\right]=5$$

197 **(d)**

Let $I=\int\frac{dx}{(1+\sqrt{x})\sqrt{(x-x^2)}}$

If $\sqrt{x}=\sin p$, then $\frac{1}{2\sqrt{x}}dx=\cos p\,dp$

$$I=\int\frac{2\sin p\cos p\,dp}{(1+\sin p)\sin p\cos p}$$

$$=2\int\frac{dp}{(1+\sin p)}$$

$$=2\int\frac{(1-\sin p)dp}{\cos^2 p}$$

$$=2\left\{\int\sec^2 p\,dp-\int(\tan p\sec p)\,dp\right\}$$

$=2(\tan p-\sec p)+C$

$$=2\left(\sqrt{\frac{x}{(1-x)}}-\frac{1}{\sqrt{(1-x)}}\right)+C$$

$$=\frac{2(\sqrt{x}-1)}{\sqrt{(1-x)}}+C$$

198 **(c)**

Let $I=\int\frac{(x^2-1)dx}{x^3\sqrt{2x^4-2x^2+1}}$

On dividing Nr and Dr by x^5, we get

$$I=\int\frac{\left(\frac{1}{x^3}-\frac{1}{x^5}\right)dx}{\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}}$$

Put $2-\frac{2}{x^2}+\frac{1}{x^4}=t\quad\Rightarrow\quad\left(\frac{4}{x^3}-\frac{4}{x^5}\right)dx=dt$

$$\therefore I=\frac{1}{4}\int\frac{dt}{\sqrt{t}}=\frac{1}{2}\sqrt{t}+c=\frac{1}{2}\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}+c$$

199 **(a)**

Differentiating both sides, we get

$$\sqrt{1+\sin x}\,f(x)=\frac{2}{3}\frac{3}{2}(1+\sin x)^{1/2}\cos x$$

$\Rightarrow f(x)=\cos x$

200 **(c)**

$$I=\int e^{\tan^{-1}x}(1+x+x^2)\left(-\left(\frac{1}{1+x^2}\right)dx\right)$$

$$=-\int e^{\tan^{-1}x}\left(1+\frac{x}{1+x^2}\right)dx$$

$$=-\int e^{\tan^{-1}x}dx-\int x\frac{e^{\tan^{-1}x}}{1+x^2}dx$$

$$=-\int e^{\tan^{-1}x}dx-x\,e^{\tan^{-1}x}+\int e^{\tan^{-1}x}dx+C$$

$=-x\,e^{\tan^{-1}x}+C$

201 **(c)**

$$I=\int_{-\pi/4}^{3\pi/4}\frac{dx}{\sqrt{2}(e^{x-\pi/4}+1)\cos\left(x-\frac{\pi}{4}\right)}$$

Putting $x-\frac{\pi}{4}=t$, we get

$$\Rightarrow I=\frac{1}{\sqrt{2}}\int_{-\pi/2}^{\pi/2}\frac{dt}{(e^t+1)\cos t}$$

$$=\frac{1}{\sqrt{2}}\int_{-\pi/2}^{\pi/2}\frac{e^t dt}{(e^t+1)\cos t}$$

Adding, we get $2I=\frac{1}{\sqrt{2}}\int_{-\pi/2}^{\pi/2}\sec t\,dt$

$$\therefore\ I=\frac{1}{2\sqrt{2}}\int_{-\pi/2}^{\pi/2}\sec x dx\ \therefore k=\frac{1}{2\sqrt{2}}$$

202 **(b)**

Let $I=\int\frac{x+2}{(x^2+3x+3)\sqrt{x+1}}dx$

Putting $x+1=t^2, dx=2t\,dt$, we get

$$I=2\int\frac{t^2+1}{t^4+t^2+1}dt$$

$$=2\int\frac{1+(1/t)^2}{\left(t-\frac{1}{t}\right)^2+3}$$

$$=\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{3}}\right)+C$$

$$=\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3(x+1)}}\right)+C$$

203 **(d)**

$$\int_0^1(1+e^{-x^2})dx$$

$$=\int_0^1\left(1+1-\frac{x^2}{1!}+\frac{x^4}{2!}-\frac{x^6}{3!}+\cdots\infty\right)dx$$

$$=\left[2x-\frac{x^3}{3.1!}+\frac{x^5}{5.2!}-\frac{x^7}{7.3!}+\cdots\infty\right]_0^1$$

$$=\left[2-\frac{1}{3.1!}+\frac{1}{5.2!}-\frac{1}{7.3!}+\cdots\infty\right]$$

Clearly 'd' is the correct alternative

204 **(b)**

We have,

$$e^{-x}f(x)=2+\int_0^x\sqrt{t^4+1}\ dt, x\epsilon(-1,1)$$

On differentiating w.r.t x, we get

$$e^{-x}\big(f'(x)-f(x)\big)=\sqrt{x^4+1}$$

$$\Rightarrow f'(x)=f(x)+\sqrt{x^4+1}\,e^x$$

$\because f^{-1}$ is the inverse of f

$$\therefore f^{-1}\big(f(x)\big)=x$$

$$\Rightarrow f^{-1'}\big(f(x)\big)f'(x)=1$$

$$\Rightarrow\ f^{-1'}\big(f(x)\big)=\frac{1}{f'(x)}$$

$$\Rightarrow\ f^{-1'}\big(f(x)\big)=\frac{1}{f(x)+\sqrt{x^4+1}\,e^x}$$

As $x=0,\ f(x)=2$

and $f^{-1}(2)=\frac{1}{2+1}=\frac{1}{3}$

205 **(a)**

$I=\int x\frac{\ln\left(x+\sqrt{x^2+1}\right)}{\sqrt{x^2+1}}dx$,let $t=\sqrt{x^2+1}$

$$\Rightarrow\frac{dt}{dx}=\frac{x}{\sqrt{x^2+1}}$$

$$\Rightarrow I=\int\ln\left(t+\sqrt{t^2-1}\right)dt$$

$$=\ln\left(t+\sqrt{t^2-1}\right)t-\int\frac{1+\frac{t}{\sqrt{t^2-1}}}{t+\sqrt{t^2-1}}tdt$$

$$=t\ln\left(t+\sqrt{t^2-1}\right)-\frac{1}{2}\int\frac{2t}{\sqrt{t^2-1}}dt$$

$$=t\ln\left(t+\sqrt{t^2-1}\right)-\sqrt{t^2-1}+C$$

$$=\sqrt{1+x^2}\ln\left(x+\sqrt{1+x^2}\right)-x+C$$

$$\Rightarrow a=1, b=-1$$

206 **(c)**

$$I_1=\int_0^1\frac{e^xdx}{1+x}, I_2=\int_0^1\frac{x^2\,dx}{e^{x^3}(2-x^3)}$$

In I_2, put $1-x^3=t$

$$\Rightarrow I_2=\frac{1}{3}\int_1^0\frac{-dt}{e^{1-t}\,(1+t)}$$

$$=\frac{1}{3e}\int_0^1\frac{e^t\,dt}{1+t}=\frac{1}{3e}I_1$$

$$\Rightarrow\frac{I_1}{I_2}=3e$$

207 **(d)**

$$I=\int_{4\pi-2}^{4\pi}\frac{\sin\frac{t}{2}}{4\pi+2-t}\ dt=\frac{1}{2}\int_{4\pi-2}^{4\pi}\frac{\sin\frac{t}{2}}{1+\left(2\pi-\frac{t}{2}\right)}dt$$

Put $2\pi-\frac{t}{2}=z$

$$\therefore -\frac{1}{2}\ dt=dz,\text{i. e.}, dt=-2\ dz$$

When $t=4\pi-2, z=2\pi-2\pi\ +1=1$

When $t=4\pi, z=2\pi-2\pi=0$

$$\Rightarrow\ I=\frac{1}{2}\int_1^0\frac{\sin(2\pi-z)(-2dz)}{1+z}$$

$$=\int_0^1\frac{-\sin z\,dz}{z+1}=-\int\frac{\sin t}{1+t}dt=-\alpha$$

208 **(b)**

$$I=\int_{-a}^{a}\left(\cos^{-1}x-\sin^{-1}\sqrt{1-x^2}\right)dx$$

$$=\int_{-a}^{0}\cos^{-1}x\,dx+A-2\int_0^a\sin^{-1}\sqrt{1-x^2}\,dx$$

$$=\int_0^a(\pi-\cos^{-1}x)dx+A-2A$$

$$=a\pi-2A\Rightarrow\lambda=2$$

209 **(a)**

$I=\int_0^4\frac{(y^2-4y+5)\sin(y-2)}{(2y^2-8y+1)}dy$, put $y-2=z$

$$\Rightarrow I = \int_{-2}^{2} \frac{z^2+1}{2z^2-7}\sin(z)dz = 0$$

210 **(a)**

$$I = \int_0^{\infty} \frac{x\log x dx}{(1+x^2)^2}$$

Let $x = \frac{1}{t}$

$$\Rightarrow I = \int_{\infty}^{0} \frac{\left(\frac{1}{t}\right)\log\left(\frac{1}{t}\right)\left(-\frac{1}{t^2}\right) dt}{\left(1+\frac{1}{t^2}\right)^2}$$

$$= -\int_0^{\infty} \frac{t\log t}{(1+t^2)^2}dt = -I$$

$\Rightarrow I = 0$

211 **(a)**

$$I = \int_0^x [\sin t]\,dt = \int_0^{2n\pi} [\sin t]dt + \int_{2n\pi}^{x} [\sin t]dt$$

$= n\int_0^{2\pi}[\sin t]\,dt + \int_{2n\pi}^{x} [\sin t]dt$ (as $[\sin x]$ is periodic with period 2π)

$= -n\pi + 0 = -n\pi$

212 **(c)**

$$f^2(x) = \int_0^x f(t)\frac{\cos t}{2+\sin t}dt$$

$\Rightarrow 2f(x)f'(x) = f(x)\frac{\cos x}{2+\sin x}$ (differentiating w.r.t. x using Leibnitz rule)

$\Rightarrow 2f'(x) = \frac{\cos x}{2+\sin x}$[as $f(x)$ is not zero everywhere]

$$\Rightarrow 2\int f'(x)dx = \int \frac{\cos x}{2+\sin x}dx$$

$\Rightarrow 2f(x) = \log_e(2+\sin x) + \log C$

Put $x = 0$ we have $2f(0) = \log 2 + \log C$, or $\log C = -\log 2$

$$\Rightarrow f(x) = \frac{1}{2}\text{In}\left(\frac{2+\sin x}{2}\right);\ x \neq n\pi, n \in I$$

213 **(a)**

Given that $I = \int (x^2+x)\,(x^{-8}+2x^{-9})^{1/10}dx$

or $I = \int (x+1)(x^2+2x)^{1/10}\,dx$

Now put $x^2+2x = t \Rightarrow (x+1)dx = \frac{dt}{2}$

$$\Rightarrow I = \int t^{1/10}\frac{dt}{2} = \frac{1}{2}\times\frac{10}{11}t^{11/10} = \frac{5}{11}t^{11/10} + C$$

$$= \frac{5}{11}(x^2+2x)^{11/10} + C$$

214 **(b)**

$$I = \int \frac{dx}{\cos^3 x\sqrt{\sin 2x}}$$

$$= \int \frac{dx}{\cos^3 x\sqrt{\frac{2\sin x\cos x}{\cos^2 x}\cos^2 x}}$$

$$= \int \frac{\sec^4 dx}{\sqrt{2\tan x}} = \frac{1}{\sqrt{2}}\int \frac{\sec^2 x(1+\tan^2 x)}{\sqrt{\tan x}}dx$$

Let $t = \sqrt{\tan x}$

$$\Rightarrow dt = \frac{\sec^2 x dx}{2\sqrt{\tan x}}$$

$$\Rightarrow I = \frac{2}{\sqrt{2}}\int (1+t^4)\,dt$$

$$= \sqrt{2}\left(t + \frac{t^5}{5}\right) + C$$

$$= \frac{\sqrt{2}}{5}t(t^4+5) + C = \frac{\sqrt{2}}{5}\sqrt{\tan x}\,(\tan^2 x + 5) + C$$

$$\Rightarrow a = \frac{\sqrt{2}}{5}, b = 5$$

215 **(a)**

$$\lim_{x\to 0}\frac{1}{x}\left[\int_y^a e^{\sin^2 t}\,dt + \int_a^{x+y} e^{\sin^2 t}dt\right]$$

$$= \lim_{x\to 0}\frac{1}{x}\int_y^{x+y} e^{\sin^2 t}\,dt\left(\frac{0}{0}\text{form}\right)$$

Apply L'Hospital Rule

$$= \lim_{x\to 0}\frac{e^{\sin^2(x+y)}\left(1+\frac{dy}{dx}\right) - e^{\sin^2 y}\frac{dy}{dx}}{1}$$

$$= e^{\sin^2 y}\left[1 + \frac{dy}{dx} - \frac{dy}{dx}\right] = e^{\sin^2 y}$$

216 **(d)**

$f(x) = A\sin(\pi x/2) + B$

$$\Rightarrow f'(x) = \frac{A\pi}{2}\cos\left(\frac{\pi x}{2}\right)$$

$\Rightarrow f'\left(\frac{1}{2}\right) = \frac{A\pi}{2}\cos\frac{\pi}{4} = \sqrt{2}$ (given)

$\Rightarrow A = 4/\pi$

Also, given $\int_0^1 f(x)dx = \frac{2A}{\pi}$

$$\Rightarrow \int_0^1 \left[A\sin\left(\frac{\pi x}{2}\right) + B\right]dx = \frac{2A}{\pi}$$

$$\Rightarrow \left|-\frac{2A}{\pi}\cos\left(\frac{\pi x}{2}\right) + Bx\right|_0^1 = \frac{2A}{\pi}$$

$$\Rightarrow B + \frac{2A}{\pi} = \frac{2A}{\pi} \Rightarrow B = 0$$

217 **(a)**

$I = \int_0^{\pi/2}\frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}}dx$ (1)

$\Rightarrow I = \int_0^{\pi/2}\frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}\,dx$ (2)

$$\left[\text{Using}\int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

Adding equation (1) and (2), we get $2I = \int_0^{\pi/2} 1dx$

$\Rightarrow I = \pi/4$

218 **(a,b,d)**

$f(2-x)=f(2+x), f(4-x)=f(4+x)$

$\Rightarrow f(4+x)=f(4-x)=f(2+2-x)$

$=f(2-(2-x))=f(x)$

$\Rightarrow$ 4 is a period of $f(x)$

$$\int_0^{50} f(x)dx=\int_0^{48} f(x)dx+\int_{48}^{50} f(x)dx$$

$$=12\int_0^4 f(x)dx+\int_0^2 f(x)dx$$

(in second integral replacing x by $x+48$ and then using $f(x)=f(x+48)$)

$$=12\left(\int_0^2 f(x)dx+\int_0^2 f(4-x)dx\right)+5$$

$$=12\left(\int_0^2 f(x)dx+\int_0^2 f(4+x)dx\right)+5$$

$$=24\int_0^2 f(x)dx+5=125$$

$$\int_{-4}^{46} f(x)dx=\int_{-4}^{-2} f(x)dx+\int_{-2}^{-2+48} f(x)dx$$

$$=\int_0^2 f(x+4)dx+12\int_0^4 f(x)dx$$

$$=\int_0^2 f(x)dx+24\int_0^2 f(x)dx$$

$=125$

Also $\int_2^{52} f(x)dx=\int_2^4 f(x)dx+\int_4^{4+48} f(x)dx$

$$=\int_0^2 f(4-x)dx+12\int_0^4 f(x)dx$$

$$=\int_0^2 f(4+x)dx+24\int_0^2 f(x)dx$$

$$=\int_0^2 f(x)dx+24\int_0^2 f(x)dx$$

$=125$

$$\int_1^{51} f(x)dx=\int_1^3 f(x)dx+\int_3^{3+48} f(x)dx$$

$$=\int_1^3 f(x)dx+12\int_0^4 f(x)dx$$

$$=\int_0^2 f(x+1)dx+24\int_0^2 f(x)dx$$

$\neq 125$

219 **(b,d)**

$\because x\in[-1,0)$ or $-1\le x<0$

For $-1\le x<0$

$\cos^{-1}\sqrt{(1-x^2)}=-\sin^{-1}x$

$$\therefore \int\{\cos^{-1}x+\cos^{-1}\sqrt{1-x^2}\}dx$$

$$=\int(\cos^{-1}x-\sin^{-1}x)\,dx$$

$$=\int\left(\frac{\pi}{2}-2\sin^{-1}x\right)dx$$

$$=\frac{\pi}{2}x-2\left\{\sin^{-1}x\cdot x-\int\frac{x}{\sqrt{1-x^2}}dx\right\}$$

$$=\frac{\pi}{2}x-2x\sin^{-1}x+2\{-\sqrt{(1-x^2)}\}+c$$

On comparing, we get

$A=\frac{\pi}{2}, f(x)=-2x$

220 **(a,b,d)**

$$I_n=\int_0^1\frac{dx}{(1+x^2)^n}=\int_0^1(1+x^2)^{-n}\,dx$$

$$=\frac{x}{(1+x^2)^n}\Big|_0^1-\int_0^1(-n)(1+x^2)^{-n-1}\,2x\times xdx$$

$$=\frac{1}{2^n}+2n\int_0^1\frac{x^2\,dx}{(1+x^2)^{n+1}}$$

$$=\frac{1}{2^n}+2n\int_0^1\frac{1+x^2-1}{(1+x^2)^{n+1}}dx$$

$$=\frac{1}{2^n}+2n\,I_n-2nI_{n+1}$$

$\Rightarrow 2nI_{n+1}=2^{-n}+(2n-1)I_n$

$\Rightarrow 2I_2=\frac{1}{2}+I_1=\frac{1}{2}+\tan^{-1}x|_0^1$

$\Rightarrow I_2=\frac{1}{4}+\frac{\pi}{8}$

Also $4I_3=2^{-2}+3I_2$

$=\frac{1}{4}+3\left(\frac{1}{4}+\frac{\pi}{8}\right)=\frac{1}{4}+\frac{3\pi}{32}$

221 **(a,c,d)**

$$I=\int_0^1\frac{2x^2+3x+3}{(x+1)(x^2+2x+2)}$$

$$=\int_0^1\frac{2(x^2+2x+2)-(x+1)}{(x+1)(x^2+2x+2)}dx$$

$$=\int_0^1\left(\frac{2}{x+1}-\frac{1}{x^2+2x+2}\right)dx$$

$=[2\log(x+1)-\tan^{-1}(x+1)]_0^1$

$=2\log 2-\tan^{-1}2+\tan^{-1}1 \qquad (1)$

$= 2\log 2 - \tan^{-1} 2 + \frac{\pi}{4}$

$= \log 4 - \left(\frac{\pi}{2} - \cot^{-1} 2\right) + \frac{\pi}{4}$

$= -\frac{\pi}{4} + \log 4 + \cot^{-1} 2$

From equation (1), $I = 2\log 2 - \tan^{-1}\left(\frac{2-1}{1+2\times 1}\right)$

$= 2\log 2 - \tan^{-1}\frac{1}{3}$

$= 2\log 2 - \cot^{-1} 3$

222 **(b,d)**

$\int \sin x d(\sec x)$

$= \int \sin x \frac{d(\sec x)}{dx} dx = \int \sin x \sec x \tan x dx$

$= \int \tan^2 x dx = \int (\sec^2 x - 1)\, dx = \tan x - x + C$

$\Rightarrow f(x) = \tan x, g(x) = x$

223 **(a,b,c)**

$I = \int \frac{x^2 - x + 1}{(x^2+1)^{3/2}} e^x dx$

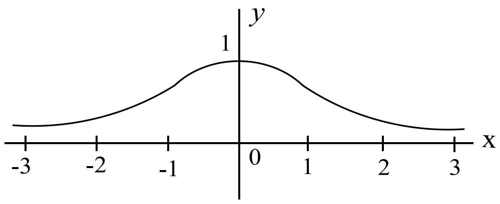
$= \int e^x \left[\frac{x^2+1}{(x^2+1)^{3/2}} - \frac{x}{(x^2+1)^{3/2}}\right] dx$

$= \int e^x \left[\frac{1}{\sqrt{x^2+1}} + \left\{\frac{-x}{(x^2+1)^{3/2}}\right\}\right] dx$

$= \int e^x [f(x) + f'(x)]dx$, where $f(x) = \frac{1}{\sqrt{x^2+1}}$

$= e^x f(x) + c = \frac{e^x}{\sqrt{x^2+1}} + c$

The graph of $f(x)$ is given in Fig 7.1



From the graph, $f(x)$ is even, bounded function and has the range (0,1]

224 **(b,c,d)**

$I = \int \frac{x^2 + \cos^2 x}{x^2 + 1} \text{cosec}^2 x dx$

$= \int \frac{x^2 + 1 + \cos^2 x - 1}{x^2+1} \text{cosec}^2 x dx$

$= \int \left(1 - \frac{\sin^2 x}{x^2+1}\right) \text{cosec}^2 x dx$

$= \int \left(\text{cosec}^2 x - \frac{1}{x^2+1}\right) dx$

$= -\cot x - \tan^{-1} x + C$

$= -\cot x + \cot^{-1} x - \frac{\pi}{2} + C$

$= -\cot x + \cot^{-1} x + C$

225 **(a,d)**

$I = \int \frac{\sqrt{(1+\sin x)(1-\sin x)}}{\sqrt{\sin x(1-\sin x)}} dx$

$= \int \frac{\cos x}{\sqrt{\sin x(1-\sin x)}} dx$

$= \int \frac{\cos x}{\sqrt{\frac{1}{4} - \left(\frac{1}{2} - \sin x\right)^2}} dx$

$= \int \frac{-dt}{\sqrt{\left(\frac{1}{2}\right)^2 - t}} \left(\text{Putting} \frac{1}{2} - \sin x = t\right)$

$= -\sin^{-1}\left(\frac{t}{1/2}\right) + C = -\sin^{-1}(1 - 2\sin x) + C$

$= \cos^{-1}(1 - 2\sin x) + C - \frac{\pi}{2}$

$= \cos^{-1}(1 - 2\sin x) + C$

$= \cos^{-1}\left(1 - 2\left(\sqrt{\sin x}\right)^2\right) + C$

$= \cos^{-1}(1 - 2\sin^2 t) + C$ (Putting $\sqrt{\sin x} = \sin t$)

$= \cos^{-1}(\cos 2t) + C$

$= 2t + C \quad \left(\because \sqrt{\sin x} > 0 \Rightarrow \sin t > 0 \Rightarrow t \in \left(0 \frac{\pi}{2}\right)\right)$

$= 2\sin^{-1}\left(\sqrt{\sin x}\right) + C$

226 **(a,c)**

$\int_0^1 e^{x^2 - x}\, dx$

For $x \in (0,1), x^2 - x \in (-1/4, 0)$

$\Rightarrow e^{-1/4} < e^{x^2-x} < e^0$

$\Rightarrow e^{-\frac{1}{4}} < \int_0^1 e^{x^2-x}\, dx < 1$

227 **(a,b,c)**

$g(x) = \int_0^x 2|t|\, dt$

$= \begin{cases} \int_0^x -2t dt, x < 0 \\ \int_0^x 2t dt, x \geq 0 \end{cases}$

$= \begin{cases} [-t^2]_0^x, x < 0 \\ [t^2]_0^x, \quad x \geq 0 \end{cases}$

$= \begin{cases} -x^2, x < 0 \\ x^2, x \geq 0 \end{cases}$

$= x|x|$

Clearly, continuous and differentiable at $x = 0$

Also, $g'(x) = \begin{cases} -2x, x < 0 \\ 2x, x > 0 \end{cases}$ which is non-differentiable at $x = 0$

228 **(a,c,d)**

$$I = \int \frac{(x^4+1)}{(x^6+1)} dt$$

$$= \int \frac{(x^2+1)^2 - 2x^2}{(x^2+1)(x^4-x^2+1)} dx$$

$$= \int \frac{(x^2+1)dx}{(x^4-x^2+1)} - 2\int \frac{x^2 dx}{(x^6+1)}$$

$$= \int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x^2-1+\frac{1}{x^2}\right)} - 2\frac{x^2 dx}{(x^3)^2+1}$$

In the first integral, put $x - \frac{1}{x} = t$

$$\therefore \left(1+\frac{1}{x^2}\right)dx = dt$$

and in the second integral put$x^3 = u$

$$\therefore x^2 dx = \frac{du}{3}$$

then$I = \int \frac{dt}{1+t^2} - \frac{2}{3}\int \frac{du}{1+u^2}$

$$= \tan^{-1} t - \frac{2}{3}\tan^{-1} u + C$$

$$= \tan^{-1}\left(x - \frac{1}{x}\right) - \frac{2}{3}\tan^{-1}(x^3) + C$$

Here , $f(x) = x - \frac{1}{2}$ and $g(x) = x^3$

Both the function are one-one

Also $f'(x) = 1 + \frac{1}{x^2} \neq 0$. Hence, $f(x)$is monotonic

Also $\int \frac{f(x)}{g(x)} dx = \int \frac{x-\frac{1}{x}}{x^3} dx = \int \left(\frac{1}{x^2} - \frac{1}{x^4}\right) dx$

$$= -\frac{1}{x} + \frac{3}{x^3} + C$$

229 **(b,c)**

$$\lim_{n\to\infty} \frac{1}{n} \sum_{r=n+1}^{2n} f\left(\frac{r}{n}\right) = \int_1^2 f(x)dx$$

$$\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{2n} f\left(\frac{r+n}{n}\right) = \int_0^1 f(1+x)dx$$

$$= \int_1^2 f(t)dt = \int_1^2 f(x)dx$$

$$\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} f\left(\frac{r}{n}\right) = \int_0^1 f(x)dx$$

$$\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{2n} f\left(\frac{r}{n}\right) = \int_0^2 f(x)dx$$

230 **(b,c)**

$$I = \int_0^\infty \frac{dx}{1+x^4} \quad (1)$$

$$= \int_0^\infty \frac{x^2 + 1 - x^2}{1+x^4} dx$$

$$= \int_0^\infty \frac{x^2}{1+x^4} dx + \int_0^\infty \frac{1-x^2}{1+x^4} dx = I_1 + I_2$$

$$I_2 = \int_0^\infty \frac{\frac{1}{x^2} - 1}{\frac{1}{x^2} + x^2} dx$$

Put $x + \frac{1}{x} = y$

$$\Rightarrow I_2 = \int_\infty^\infty \frac{-1}{y^2-2} dy = 0$$

$$\Rightarrow I = \int_0^\infty \frac{dx}{1+x^4} = \int_0^\infty \frac{x^2 dx}{1+x^4} \quad (2)$$

Adding equations (1) and (2), we get

$\Rightarrow 2I = \int_0^\infty \frac{1+x^2 dx}{1+x^4} = \int_0^\infty \frac{\frac{1}{x^2}+1}{\frac{1}{x^2}+x^2} dx$, put$x - \frac{1}{x} = y$

$$\Rightarrow 2I = \int_{-\infty}^\infty \frac{dy}{y^2+2} = \left[\frac{1}{\sqrt{2}} \tan^{-1} \frac{y}{\sqrt{2}}\right]_{-\infty}^\infty = \frac{\pi}{\sqrt{2}}$$

$$\Rightarrow I = \frac{\pi}{2\sqrt{2}}$$

231 **(a,d)**

$$\frac{2x}{(x-1)(x-4)} = \frac{C}{x-1} + \frac{D}{x-4}$$

$$2x = C(x-4) + D(x-1)$$

$$\therefore C = -2/3, D = 8/3$$

$$\therefore \int \frac{e^{x-1}}{(x-1)(x-4)} 2x dx$$

$$= \int e^{x-1}\left(\frac{-2/3}{x-1} + \frac{8/3}{x-4}\right) dx$$

$$= -\frac{2}{3}F(x-1) + \frac{8}{3}e^3 F(x-4) + C$$

$$\therefore A = -2/3, B = 8/3\, e^3$$

232 **(a)**

$$\int_{-1}^1 f(x)\,dx = \int_{-1}^1 (x - [x])\,dx$$

$$= \int_{-1}^1 x\,dx - \int_{-1}^1 [x]\,dx$$

$$= 0 - \int_{-1}^1 [x]\,dx \quad (1)$$

[$\because x$ is an odd function]

$$= -\int_{-1}^0 (-1)\,dx - \int_0^1 0\,dx$$

$$= 1$$

233 **(b,d)**

$$I = \int \sqrt{\operatorname{cosec} x + 1}\,dx = \int \frac{\cot x}{\sqrt{\operatorname{cosec} x - 1}} dx$$

Put $\operatorname{cosec} x - 1 = t^2 \Rightarrow -\operatorname{cosec} x \cot x\, dx = 2tdt$

$$\Rightarrow I = -\int \frac{-\cot x \operatorname{cosec} x}{\operatorname{cosec} x\sqrt{\operatorname{cosec} x - 1}}dx = -\int \frac{2dt}{1+t^2}$$

$$= -2\tan^{-1} t + c = -2\tan^{-1}\sqrt{\operatorname{cosec} x - 1} + C$$

$$= -2\left[\frac{\pi}{2} - \cot^{-1}\sqrt{\operatorname{cosec} x - 1}\right] + C$$

$$= 2\cot^{-1}\sqrt{\operatorname{cosec} x - 1} + C$$

$$= 2\cot^{-1}\frac{\cot x}{\sqrt{\operatorname{cosec} x + 1}} + C$$

234 **(a)**

$$\int_0^x f(t)\,dt = x + \int_x^1 tf(t)\,dt$$

Differentiating both sides w.r.t. x, we get

$f(x) = 1 + 0 - xf(x)$

$\Rightarrow (x+1)f(x) = 1$

$$\Rightarrow f(x) = \frac{1}{x+1}$$

$$\Rightarrow f(1) = \frac{1}{2}$$

235 **(a,d)**

$$\int \sin^{-1} x \cos^{-1} x dx$$

$$= \int \left[\frac{\pi}{2}\sin^{-1} x - (\sin^{-1} x)^2\right] dx$$

$$= \frac{\pi}{2}\left(x\sin^{-1} x + \sqrt{1-x^2}\right) - \left(x(\sin^{-1} x)^2 + \sin^{-1} x\sqrt{1-x^2} - x\right) + C$$

(intergrating by parts)

$$= \sin^{-1} x\left[\frac{\pi}{2}x - x\sin^{-1} x - 2\sqrt{1-x^2}\right] + \frac{\pi}{2}\sqrt{1-x^2} + 2x + C$$

$\therefore f^{-1}(x) = \sin^{-1} x, f(x) = \sin x$

236 **(a,c)**

Let $\cos x = t, \Rightarrow \cos x = t \Rightarrow \cos 2x = 2t^2 - 1$ and $dt = -\sin x\, dx$. Thus

$$I = \int \frac{t^2-2}{2t^2-1}dt = \frac{1}{2}\int \frac{2t^2-4}{2t^2-1}dt$$

$$= \frac{1}{2}\int dt - \frac{3}{2}\int \frac{dt}{2t^2-1}$$

$$= \frac{1}{2}t - \frac{3}{2\sqrt{2}} \times \frac{1}{2}\log\left|\frac{\sqrt{2}t-1}{\sqrt{2}t+1}\right| + C$$

$$= \frac{1}{2}\cos x - \frac{3}{4\sqrt{2}} \times \log\left|\frac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}\right| + C$$

So, $P = 1/2, Q = -\frac{3}{4\sqrt{2}}, f(x) = \frac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}$

or $P = 1/2, Q = \frac{3}{4\sqrt{2}}, f(x) = \frac{\sqrt{2}\cos x + 1}{\sqrt{2}\cos x - 1}$

237 **(a,b)**

Here, $f'(x) \geq 0$ in $[a, b]$. So, $f(x)$ is monotonically increasing.

Hence, $f(a) \leq f(x) \leq f(b)$

$$\therefore \int_a^b f(a)dx \leq \int_a^b f(x)dx \leq \int_a^b f(b)dx$$

$$\Rightarrow f(a)\cdot(b-a) \leq \int_a^b f(x)dx \leq \int_a^b f(b)(b-a)$$

$$\therefore f(a) \leq \frac{1}{(b-a)}\int_a^b f(x)dx \leq f(b)$$

238 **(b,c,d)**

$$I_n = \int_0^{\pi/4} \tan^n x\, dx$$

$$= \int_0^{\pi/4} \tan^{n-2} x \tan^2 x\, dx$$

$$= \int_0^{\pi/4} \sec^2 x \tan^{n-2} x\, dx - \int_0^{\pi/4} \tan^{n-2} x\, dx$$

$= \int_0^1 t^{n-2}\, dt - I_{n-2}$ where $t = \tan x$

$$I_n + I_{n-2} = \left(\frac{t^{n-1}}{n-1}\right)_0^1$$

$$\Rightarrow I_n + I_{n-2} = \frac{1}{n-1}$$

$\Rightarrow I_2 + I_4, I_4 + I_6, \ldots$are in H.P.

For $0 < x < \pi/4$, we have $0 < \tan^n x < \tan^{n-2} x$

So that $0 < I_n < I_{n-2} \Rightarrow I_n + I_{n+2} < 2I_n < I_n + I_{n-2}$

$$\Rightarrow \frac{1}{n+1} < 2I_n < \frac{1}{n-1} \Rightarrow \frac{1}{2\,(n+1)} < I_n < \frac{1}{2(n-1)}$$

239 **(a,b)**

$$f(x) = x\int_1^x \frac{e^t}{t}\,dt - e^x$$

$$\Rightarrow f'(x) = x\frac{e^x}{x} + \int_1^x \frac{e^t}{t}\,dt - e^x$$

$$\Rightarrow f'^{(x)} = \int_1^x \frac{e^t}{t}\,dt > 0\ [\because x \in (1, \infty)]$$

$\Rightarrow f(x)$ is an increasing function

240 **(a,b,c)**

For $a \leq 0$,

Given equation becomes

$$\int_0^2 (x-a)dx \geq 1 \Rightarrow a \leq \frac{1}{2} \Rightarrow a \leq 0$$

For $0 < a < 2$,

$$\int_0^2 |x-a|dx \geq 1$$

$$\Rightarrow \int_0^a (a-x)dx + \int_a^2 (x-a)dx \geq 1$$

$$\Rightarrow \frac{a^2}{2} + 2 - 2a + \frac{a^2}{2} \geq 1 \Rightarrow a^2 - 2a + 1 \geq 0$$

$$\Rightarrow (a-1)^2 \geq 0$$

For $a \geq 2$,

$$\int_0^2 |x-a|dx \geq 1$$

$$\Rightarrow \int_0^2 (a-x)dx \geq 1 \Rightarrow 2a - 2 \geq 1$$

$$\Rightarrow a \geq \frac{3}{2}$$

$\Rightarrow a \geq 2$

241 **(a,d)**

$A_{n+1} - A_n$

$$= \int_0^{\pi/2} \frac{\sin(2n+1)x - \sin(2n-1)x}{\sin x}$$

$$= \int_0^{\pi/2} 2\cos 2nx\, dx = 0$$

$\Rightarrow A_{n+1} = A_n$

$B_{n+1} - B_n$

$$= \int_0^{\pi/2} \frac{\sin^2(n+1)x - \sin^2 nx}{\sin^2 x} dx$$

$$= \int_0^{\pi/2} \frac{\sin(2n+1)x}{\sin x} dx$$

$= A_{n+1}$

242 **(a,b)**

$f(x) = e^x + \int_0^1 e^x f(t)dt = e^x + ke^x$ where $k = \int_0^1 f(t)dt$

$$\therefore k = \int_0^1 (e^t + ke^t)\, dt = e + ke - 1 - k$$

$\therefore k = \frac{e-1}{2-e}$, thus $f(x) = e^x\left(1 + \frac{e-1}{2-e}\right) = \frac{e^x}{2-e}$

Obviously, $f(0) = \frac{1}{2-e} < 0$

Also, $f'(x) = \frac{e^x}{2-e} < 0$ for $\forall x \in R$

Hence, $f(x)$ is a decreasing function

Also, $\int_0^1 f(x)dx$

$$= \int_0^1 \frac{e^x}{2-e}\, dx$$

$$= \left[\frac{e^x}{2-e}\right]_0^1$$

$$= \frac{e-1}{2-e} < 0$$

243 **(a,c)**

$$\int \frac{\cos^2 2x \sin 2x\, dx}{\cos 2x}$$

$$= \frac{1}{2}\int \sin 4x\, dx = -\frac{1}{8}\cos 4x + B$$

244 **(a,b,c,d)**

Let $I = \int \sin(\log x)dx$

Put $\log x = t$

$\therefore x = e^t$

$\Rightarrow dx = e^t\, dt$

Then, $I = \int e^t \sin t\, dt$

$= \frac{e^t}{2}(\sin t - \cos t) + c$

$= \frac{x}{2}\{\sin(\log x) - \cos(\log x)\} + c$

On comparing, we get

$f(x) = \frac{x}{2}, g(x) = \log x, h(x) = \log x$

Option (a) $\lim_{x\to 2} f(x) = \lim_{x\to 2}\frac{x}{2} = \frac{2}{2} = 1$

Option (b) $\lim_{x\to 1}\frac{g(x)}{h(x)} = \lim_{x\to 1}\frac{\log x}{\log x} = 1$

Option (c) $g(e^3) = \log e^3 = 3\log e = 3$

Option (d) $h(e^5) = \log e^5 = 5\log e = 5$

245 **(a,b,d)**

We know $\int_a^b |\sin x|\, dx$ represents the area under the curve from $x = a$ to $x = b$. We also know that area from $x = a$ to $x = a + \pi$is 2

$\therefore \int_a^b |\sin x| dx = 8 \Rightarrow b - a = \frac{8\pi}{2}$ (1)

Similarly, $\int_0^{a+b} |\cos x| dx = 9 \Rightarrow a + b - 0 =$ $9\pi 2$(2)

From (1) and (2), $a = \frac{\pi}{4}$ and $b = \frac{17\pi}{4}$

$\Rightarrow |a+b| = \frac{9\pi}{2}, |a-b| = 4\pi, \frac{a}{b} = 17$ and

$$\int_a^b \sec^2 x dx = [\tan x]_{\pi/4}^{17\pi/4} = 0$$

246 **(c)**

Let $f(x) = \sqrt{3 + x^3}$

Clearly, $f(x)$ is increasing in $[1, 3]$

$\Rightarrow$ The least value of the function, $m = f(1) = \sqrt{3 + 1^3} = 2$

and the greatest value of the function,

$M = f(3) = \sqrt{3 + 3^3}$

$= \sqrt{30}$

Therefore, $(3-1)2 \leq \int_1^3 \sqrt{3 + x^3}\, dx \leq$

$(3-1)\sqrt{30}$

Here, $4 \le \int_1^3 \sqrt{3+x^3}\,dx \le 2\sqrt{30}$

247 **(a,b,d)**

$$\int \frac{dx}{x^2+ax+1} = \int \frac{dx}{\left(x+\frac{a}{2}\right)^2 + \left(1-\frac{a^2}{4}\right)}$$

248 **(a,c,d)**

$$\int x^2 e^{-2x} dx = e^{-2x}(ax^2+bx+c)+d$$

Differentiating both sides, we get

$x^2 e^{-2x} = e^{-2x}(2ax+b)$
$+ (ax^2+bx+c)(-2e^{-2x})$

$= e^{-2x}(-2ax^2+2(a-b)x+b-2c)$

$\Rightarrow a = 1, 2(a-b)=0, b-2c=0$

$\Rightarrow b = 1, c = \frac{1}{2}$

249 **(a,c)**

$I = \int \sec^2 x \operatorname{cosec}^4 x dx$

$= \int \frac{(\sin^2 x + \cos^2 x)^2}{\cos^2 x \sin^4 x} dx$

$= \int \frac{\sin^4 x + \cos^4 x + 2\sin^2 x \cos^2 x}{\cos^2 x \sin^4 x}$

$= \int \left(\sec^2 x + 2\operatorname{cosec}^2 x + \frac{\cos^2 x}{\sin^4 x}\right) dx$

$= \tan x - 2\cot x + \int \cot^2 x \operatorname{cosec}^2 x dx$

$= \tan x - 2\cot x - \frac{\cot^3 x}{3} + D$

250 **(a,d)**

$f'(x) = \frac{3^x}{1+x^2} > 0\ \forall\, x > 0 \Rightarrow f'(x) = \frac{3^x}{1+x^2}$
$> \frac{1}{1+x^2}, \forall\, x \ge 1$

$\Rightarrow \int_1^x f'(x)dx > \int_1^x \frac{1}{1+x^2}\,dx$

$\Rightarrow f(x) > \tan^{-1} x$
$- \tan^{-1} 1 \Rightarrow f(x) + \pi/4$
$> \tan^{-1} x$

251 **(a,b,c,d)**

$\int \frac{(x^8+4+4x^4)-4x^4}{x^4-2x^2+2} dx$

$= \int \frac{(x^4+2)^2-(2x^2)^2}{(x^4-2x^2+2)} dx$

$= \int \frac{(x^4+2-2x^2)(x^4+2+2x^2)}{(x^4-2x^2+2)} dx$

$= \frac{x^5}{5} + \frac{2x^3}{3} + 2x + C$

252 **(a,c)**

$g(x) = \int x^{27}(1+x+x^2)^6(6x^2+5x+4)dx$

$= \int (x^4+x^5+x^6)^6\,(6x^5+5x^4+4x^3)dx$

let $x^6+x^5+x^4 = t \Rightarrow (6x^5+5x^4+4x^3)dx = dt$

$\therefore g(x) = \int t^6 dt = \frac{t^7}{7} + C$

$= \frac{1}{7}(x^4+x^5+x^6)^7 + C$

$g(0) = 0 \Rightarrow x = 0 \Rightarrow g(1) = \frac{3^7}{7}$ also $g(-1) = \frac{1}{7}$

253 **(a,b,d)**

Given that $f(x) = \int_0^x |t-1|dt$

$\Rightarrow f(x) = \int_0^x (1-t)dt, 0 \le x \le 1$

$= x - \frac{x^2}{2}$

Also $f(x) = \int_0^1 (1-t)dt + \int_1^x (t-1)dt$, where $1 \le x \le 2$

$= \frac{1}{2} + \frac{x^2}{2} - x + \frac{1}{2} = \frac{x^2}{2} - x + 1$

Thus, $f(x) = \begin{cases} x - \frac{x^2}{2}, & 0 \le x \le 1 \\ \frac{x^2}{2} - x + 1, & 1 < x \le 2 \end{cases}$

$\Rightarrow f'(x) = \begin{cases} 1-x, 0 \le x < 1 \\ x-1, 1 < x < 2 \end{cases}$

Thus, $f(x)$ is continuous as well as differentiable at $x = 1$. Also, $f(x) = \cos^{-1} x$ has one real root, draw the graph and verify

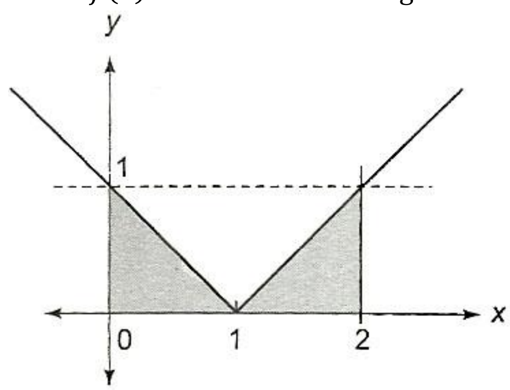
For range of $f(x)$:

$f(x) = \int_0^x |t-1|dt$ is the value of area bounded by the curve $y = |t-1|$ and x-axis between the limits $t = 0$ and $t = x$

Obviously, minimum area is obtained when $t = 0$ and $t = x$ coincide or $x = 0$

Maximum value of area occurs when $t = 2$,

Hence $f(2) =$ area of shaded region $= 1$



254 **(a,c,d)**

The expression $f(x)f(c)\forall\, x \in (c-h, c+h)$ where $h \to 0^+$ is equivalent to $\text{Lim}_{x\to 0} f(x)f(c)$ which equals to $(f(c))^2$ because $f(x)$ is continuous

Therefore, $f(x)f(c) > 0 \ \forall\ x \in (c-h, c+h)$ where $h \to 0^+$

a. We have $I = \lim_{n\to\infty} \frac{1}{n} \text{In} \left[\left(1+\frac{1}{n}\right)\left(1+\right.\right.$
2n⋯1+nn

$$= \lim_{n\to\infty} \frac{1}{n} \text{In} \prod_{k=1}^{n} \left(1+\frac{k}{n}\right)$$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \text{In}\left(1+\frac{k}{n}\right)$$

$$= \int_1^2 \text{In}\, x\, dx = [x\,(\text{In}\, x - 1)]_1^2 = -1 + 2\ \text{In}\ 2$$

c. Given $f(x) \ge 0 \Rightarrow \int_a^b f(x)dx \ge 0$

But given $\int_a^b f(x)dx = 0$, so this can be true only when $f(x) = 0$

d.$\int_a^b f(x)dx = 0 \Rightarrow\ y = f(x)$ cuts x axis at least once

So, there exists at least one $c \in (a, b)$ for which $f(c) = 0$

255 **(b,c,d)**

$$\int \sin 6x\, dx = -\frac{1}{6}\cos 6x + c$$

$$= -\frac{1}{6}(1 - 2\sin^2 3x) + c$$

$$= -\frac{1}{6} + \frac{1}{3}\sin^2 3x + c = \frac{1}{3}\sin^2 3x + d$$

$$= -\frac{1}{6}\cos 6x + c$$

$$= -\frac{1}{6}(2\cos^2 3x - 1) + c$$

$$= -\frac{1}{3}\cos^2 3x + c$$

Also, derivative of $\frac{1}{3}\sin\left(3x+\frac{\pi}{7}\right)\sin\left(3x-\frac{\pi}{7}\right)$ is $\sin 6x$.

256 **(a,b,c,d)**

Let $f(x) = \int_0^{x^2}\left(\frac{t^2-5t+4}{2+e^t}\right)dt$

$$\therefore f'(x) = \left(\frac{x^4-5x^2+4}{2+e^{x^2}}\right) \times 2x$$

For extremum $f'(x) = 0$

$\therefore\ x = 0, \pm 1, \pm 2$

257 **(a,b,c)**

$$f(x) = \int_a^x \frac{1}{f(x)}dx \ \Rightarrow f'(x) = \frac{1}{f(x)} \cdot 1 - 0$$

$$\Rightarrow f(x)f'(x) = 1$$

$$\Rightarrow \int f(x)f'(x)dx = \int 1dx$$

$$\Rightarrow \frac{1}{2}[f(x)]^2 = x + c \qquad (1)$$

Now given that $\int_a^1 [f(x)]^{-1}\, dx = \sqrt{2} \Rightarrow f(1) = \sqrt{2}$

$\Rightarrow$ From (1), $\frac{1}{2}[f(1)]^2 = 1 + c \ \Rightarrow\ c = 0$

$\Rightarrow f(x) = \pm\sqrt{2x}$

But $f(1) = \sqrt{2} \Rightarrow f(x) = \sqrt{2x} \ \Rightarrow f(2) = 2$

Also, $f'(x) = \frac{1}{\sqrt{2x}} \ \Rightarrow f'(2) = 1/2$

$$\int_0^1 f(x)dx = \int_0^1 \sqrt{2xdx} = \left[\frac{(2x)^{3/2}}{3}\right]_0^1 = \frac{(2)^{3/2}}{3}$$

Also, $f^{-1}(x) = \frac{x^2}{2} \ \Rightarrow f^{-1}(2) = 2$

258 **(a,d)**

$$f(x+\pi) = \int_0^{x+\pi} (\cos(\sin t) + \cos(\cos t))dt$$

$$= \int_0^{\pi} (\cos(\sin t) + \cos(\cos t))dt$$

$$+ \int_0^{x+\pi} (\cos(\sin t) + \cos(\cos t))dt$$

$$= f(\pi) + \int_0^{x} (\cos(\sin t) + \cos(\cos t))dt$$

($\because$ for g$(x) = \cos(\sin x) + \cos(\cos x), f(x+\pi) =$ *f(x))*

$= f(\pi) + f(x)$

$= f(\pi) + 2f\left(\frac{\pi}{2}\right)$ ($\because$ g(x) has period $\pi/2$)

259 **(a,b,d)**

$$\frac{3x+4}{x^3-2x-4} = \frac{3x+4}{(x-2)(x^2+2x+2)}$$

$$= \frac{A}{x-2} + \frac{Bx+C}{x^2+2x+2}$$

$\Rightarrow 3x + 4 = A(x^2+2x+2) + (Bx+C)(x-2)$

$\therefore A + B = 0$

$2A - 2B + C = 3$

$2A - 2C = 4$

$\Rightarrow A = 1, B = C = -1$

$$\therefore \int \frac{3x+4}{x^3-2x-4}dx$$

$$= \int \frac{dx}{x-2} - \frac{1}{2}\int \frac{2x+2}{x^2+2x+2}dx$$

$$= \log_e|x-2| - \frac{1}{2}\log|x^2+2x+2| + c$$

$$\Rightarrow k = -\frac{1}{2} \text{ and } f(x) = |x^2+2x+2|$$

260 **(a,b,c,d)**

$$\because \int_{\pi/2}^{\alpha} \sin x\, dx = \sin 2\alpha$$

$\Rightarrow\ -[\cos x]_{\pi/2}^{\alpha} = \sin 2\alpha$

$\Rightarrow\ -(\cos\alpha - 0) = \sin 2\alpha$

$\Rightarrow \cos\alpha\,(2\sin\alpha + 1) = 0$

$\therefore \cos\alpha = 0$ and $\sin\alpha = -\frac{1}{2}$

$\therefore\ \alpha = \frac{\pi}{2}, \frac{3\pi}{2}$ and $\alpha = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$\therefore\ \alpha = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

261 **(c,d)**

$\lim_{n\to\infty} \tan(1/n)\log(1/n)$

$= \lim_{n\to\infty} \frac{\tan(1/n)}{(1/n)} \cdot \frac{\log(1/n)}{n}$

$= -\lim_{n\to\infty} \frac{\tan(1/n)}{(1/n)} \cdot \frac{\log(n)}{n}$

$= -1 \lim_{n\to\infty} \frac{1/n}{1}$

$= 0$

Then, $f(x) = e^{o} = 1$

$\therefore\ \int \frac{f(x)}{\sqrt[3]{(\sin^{11} x \cos x)}} dx = \int \frac{1}{\sin^{11/3} x \cos^{1/3} x} dx$

$= \int \sin^{-11/3} x \cdot \cos^{-1/3} x\, dx$

$= \int (\tan x)^{-11/3} \cos^{-4} x\, dx$

$= \int (\tan x)^{-11/3} \cdot \sec^4 x\, dx$

$= \int (\tan x)^{-11/3} \cdot (1 + \tan^2 x) \cdot \sec^2 x\, dx$

$= \frac{(\tan x)^{\frac{-11}{3}+1}}{\left(\frac{-11}{3} + 1\right)} + \frac{(\tan x)^{-2/3}}{(-2/3)} + c$

$= -\frac{3}{8}(\tan x)^{-8/3} - \frac{3}{2}(\tan x)^{-2/3} + c$

$\therefore g(x) = -\frac{3}{8}(\tan x)^{-8/3} - \frac{3}{2}(\tan x)^{-2/3}$

$\therefore g(\pi/4) = -\frac{3}{8} - \frac{3}{2} = -\frac{15}{8}$

and $g(x)$ is non-differentiable at $\tan x = 0$

Or $x = n\pi, n \in I$

262 **(a,b)**

$\text{L.H.S.} = \int_0^x \left\{\int_0^u f(t)dt\right\} du$

Integrating by parts choose '1' as the second function

$= \left\{u \int_0^u f(t)dt\right\}_0^x - \int_0^x f(u)u\, du$

$= x\int_0^x f(t)dt - \int_0^x f(u)u\, du$

$= x\int_0^x f(u)\, du - \int_0^x f(u)u\, du$

$= -\int_0^x f(u)(x-u)du$

= R.H.S.

263 **(a,b,c)**

Let $I = \int_a^b \frac{f(x)}{f(x) + f(a+b-x)} dx (1)$

$= \int_a^b \frac{f(a+b-x)}{f(a+b-x) + f(x)} dx\ (2)$

Adding equations (1) and (2), we get

$\Rightarrow 2I = \int_a^b 1\, dx = b - a$

$\Rightarrow I = \left(\frac{b-a}{2}\right) = 10$ (given)

$\therefore\ b - a = 20$

264 **(b)**

$\because \sin^6 x + \cos^6 x = (\sin^2 x)^3 + (\cos^2 x)^3$

$= (\sin^2 x + \cos^2 x)^3 - 3\sin^2 x \cos^2 x(\sin^2 x + \cos^2 x)$

$= 1 - 3\sin^2 x \cos^2 x$

$= 1 - \frac{3}{4}\sin^2 2x \quad \left(\because \text{period} \frac{\pi}{2}\right)$

$\therefore$ Least and greatest value of $\sin^6 x +$ cos*6x* are *14* and *1*

Hence, $\left(\frac{\pi}{2} - 0\right) \times \frac{1}{4} < \int_0^{\pi/2} (\sin^6 x + \cos^6 x)dx <$ *π2−0×1*

$\Rightarrow \frac{\pi}{8} < \int_0^{\pi/2} (\sin^6 x + \cos^6 x)dx < \frac{\pi}{2}$

265 **(d)**

$\because\ \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} [2\sin x]dx = \int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} [2\sin x]dx + \int_{\frac{5\pi}{6}}^{\pi} [2\sin x]dx$

$+ \int_{\pi}^{7\pi/6} [2\sin x]dx + \int_{\pi/2}^{3\pi/2} [2\sin x]dx$

$= \int_{\pi/2}^{5\pi/6} 1 \cdot dx + 0 - \int_{\pi}^{7\pi/6} 1 \cdot dx - 2\int_{7\pi/6}^{3\pi/2} 1 \cdot dx$

$= \frac{\pi}{3} - \frac{\pi}{6} - \frac{2\pi}{3}$

$= -\frac{\pi}{2} \quad \left[\because 2\sin x \text{ is decreasing function in } \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)\right]$

266 **(a)**

For $a < b$. If m and M are the smallest and greatest values of $f(x)$ on $[a, b]$

Then $m\,(b-a) \le \int_a^b f(x)dx \le (b-a)M$

or $m \le \frac{1}{(b-a)}\int_a^b f(x)dx \le M$

Since $f(x)$ is continuous on $[a, b]$, it takes on all intermediate values between m and M

Therefore, some values $f(c)(a \le f(c) \le b)$, we will have $\frac{1}{(b-a)}\int_a^b f(x)dx = f(c)$ or $\int_a^b f(x)dx =$ *fc(b−a)*

Hence, both the statements are true and statement 2 is a correct explanation of statement 1

267 **(a)**

Statement 2 is a fundamental concept, also we have $f(2-a) = f(2+a)$

$$\int_{2-a}^{2+a} f(x)dx = 2\int_{2}^{2+a} f(x)dx$$

268 **(a)**

Let $g(x) = \int_a^x f(t)dt - \int_x^b f(t)dt$, where $x \in [a, b]$

We have $g(a) = -\int_a^b f(t)dt$ and $g(b) = \int_a^b f(t)dt$

$$\Rightarrow\; g(a)g(b) = -\left(\int_a^b f(t)dt\right)^2 \le\; 0$$

Clearly, $g(x)$ is continuous in $[a, b]$ and $g(a)g(b) \le 0$

It implies that $g(x)$will becomes zero at least once in $[a, b]$. Hence, $\int_a^x f(t)dt = \int_x^b f(t)dt$ for at least one value of $x \in [a, b]$

Hence, both the statements are true and statement 2 is a correct explanation of statement 1

269 **(c)**

$$\int_a^b xf(x)dx = \int_a^b (a+b-x)f(a+b-x)dx$$

$$= (a+b)\int_a^b f(a+b-x)dx$$

$$-\int_a^b x\,f(a+b-x)dx$$

Therefore, statement 2 is true only when $f(a+b-x) = f(x)$ which holds in statement 1

Therefore, statement 2 is false and statement 1 is true

270 **(d)**

$\int e^{x^2}dx$ cannot be expressed in terms of elementary function, then integral is known as inexpressible or that is " cannot be found ".

271 **(a)**

Let $p'(x) = a(x-1)(x-3)$

$$\Rightarrow\; p(x) = \int_1^x a(x^2 - 4x + 3)dx + c$$

$$\Rightarrow\; p(x) = a\left[\frac{x^3}{3} - 2x^2 + 3x\right]_1^x + 60\quad [\because p(1) = 6]$$

$$\Rightarrow\; p(x) = a\left(\frac{x^3}{3} - 2x^2 + 3x - \frac{4}{3}\right) + 6$$

Since, $p(3) = 2$, then $a = 3$

$\therefore\; p(x) = x^3 - 6x^2 + 9x + 2$

Statement II is also true and it is a correct explanation for Statement I

272 **(b)**

$$I = \int_{-4}^{-5} \sin(x^2 - 3)dx$$

$$+\int_{-2}^{-1} \sin(x^2 + 12x + 33)dx$$

$$= I_1 + I_2$$

$$I_2 = \int_{-2}^{-1} \sin(x^2 + 12x + 33)dx$$

$$= \int_{-2}^{-1} \sin((x+6)^2 - 3)\,dx,$$

Put $x + 6 = -y$

$$\Rightarrow I_2 = -\int_{-4}^{-5} \sin(y^2-3)dy = -I_1$$

$$\Rightarrow I_1 + I_2 = 0 \Rightarrow I = 0$$

273 **(b)**

$$\because I = \int_0^{2\pi} \sin^3 x\, dx = \int_0^{2\pi} (1-\cos^2 x)\sin x\, dx$$

Put $\cos x = t \Rightarrow \sin x\, dx = -dt$

Then, $1 = \int_1^1 (1-t^2)(-dt) = 0$

274 **(a)**

Statement II is true.

Now, $\int \frac{dx}{e^x+e^{-x}+2} = \int \frac{e^x dx}{(e^x+1)^2}$

$$= \int \frac{d(e^x+1)}{(e^x+1)^2}$$

$$= -\frac{1}{e^x+1} + c \quad \text{(By using statement II)}$$

275 **(c)**

$x > x^2, \forall x \in \left(0, \frac{\pi}{4}\right) \Rightarrow e^x > e^{x^2}\ \forall x \in \left(0, \frac{\pi}{4}\right)$

$\cos x > \sin x\ \forall \in \left(0, \frac{\pi}{4}\right)$

$\Rightarrow e^{x^2}\cos x > e^{x^2}\sin x$

$\Rightarrow e^x > e^{x^2} > e^{x^2}\cos x > e^{x^2}\sin x\ \forall x \in \left(0, \frac{\pi}{4}\right)$

$\Rightarrow I_2 > I_1 > I_3 > I_4$

276 **(c)**

Given, $I_n = \int \cot^n x\, dx = \int \cot^{n-2} x(\text{cosec}^2 x - 1dx$

$$= \int \cot^{n-2} x\, \text{cosec}^2 x\, dx - I_{n-2}$$

$$= -\frac{\cot^{n-1} x}{n-1} - I_{n-2}$$

Put $n = 6, 5(I_6 + I_4) = -\cot^5 x$

277 **(c)**

Let $I = \int \frac{(2-2x)}{\sqrt{(4+2x-x^2)}} dx + \int \frac{dx}{\sqrt{(4+2x-x^2)}}$

$$= 2\sqrt{4+2x-x^2} + \int \frac{dx}{\sqrt{5-(x-1)^2}}$$

$$= 2\sqrt{4+2x-x^2} + \sin^{-1}\left(\frac{x-1}{\sqrt{5}}\right) + c$$

278 **(c)**

Statement 1 is true as it is a fundamental property.

Let $g(x) = \int_a^x f(t)dt$

If $f(x)$ is an even function

Then $g(-x) = \int_a^{-x} f(t)dt$

$$= -\int_{-a}^{x} f(-y)dy$$

$$= -\int_{-a}^{x} f(y)dy$$

$$= -\int_{-a}^{a} f(y)dy - \int_{a}^{x} f(y)dy$$

$\neq -g(x)$

Hence, statement 2 is false

279 **(b)**

Let $I = \int_0^{2\pi} \cos^{99} x dx$

Then,

$$I = 2\int_0^{\pi} \cos^{99} x dx\ [\because \cos^{99}(2\pi - x) = \cos^{99} x]$$

Now, $\int_0^{\pi} \cos^{99} x dx = 0[\because \cos^{99}(\pi - x) = -\cos 99x]$

$\Rightarrow I = 2 \times 0 = 0$

280 **(a)**

$$F(x+\pi) = \int \sin^2(x+\pi)\, dx$$

$$= \int \sin^2 x\, dx \quad [\because \sin^2(\pi + x) = \sin^2 x]$$

$= F(x)$

281 **(a)**

$$I = \int \frac{\{f(x)\phi'(x) - f'(x)\phi(x)\}}{f(x)\phi(x)} \{\log\phi(x) - \log f(x)\}dx$$

$$= \int \log\frac{\phi x}{f(x)} d\left\{\log\frac{\phi(x)}{f(x)}\right\} = \frac{1}{2}\left\{\log\frac{\phi(x)}{f(x)}\right\}^2 + c$$

282 **(d)**

$\because$ Period of $e^{\sin x}$ is 2π

$$\therefore \int_0^{200} e^{\sin x} dx \neq 200\lambda$$

283 **(d)**

$$\int_0^{\pi} \sqrt{1-\sin^2 x}\, dx$$

$$= \int_0^{\pi} |\cos x| dx$$

$$= \int_0^{\pi/2} \cos x dx + \int_{\pi/2}^{\pi} -\cos x dx$$

$$= 1 + 1 = 2$$

Hence, statement 1 is false. However, statement 2 is true

284 **(b)**

$$I = \int \frac{dx}{x^3\sqrt{1+x^4}} = \int \frac{dx}{x^5\sqrt{\frac{1}{x^4}+1}}$$

Let $\frac{1}{x^4} + 1 = t \Rightarrow dt = \frac{-4}{x^5} dx$

$$\Rightarrow I = -\frac{1}{4}\int \frac{dt}{\sqrt{t}} = -\frac{1}{2}\sqrt{t} = -\frac{1}{2}\sqrt{1+\frac{1}{x^4}} + C$$

Thus, both the statements are true but statement 2 is not a correct explanation of statement 1

285 **(d)**

For $x^2 + 2(a-1)x + a + 5 = 0$

If $D < 0 \Rightarrow 4(a-1)^2 - 4(a+5) < 0$

$\Rightarrow a^2 - 3a - 4 < 0$ or $(a-4)(a+1) < 0$ or $-1 < a < 4$

Thus for these value of $a, x^2 + 2(a-1)x + a + 5$ cannot be factorized, hence

$$\int \frac{dx}{x^2 + 2(a-1)x + a + 5} = \lambda \tan^{-1}|g(x)| + c$$

Hence, statement 1 is false and statement 2 is true

286 **(d)**

Obviously, $|\sin t|$ is non-differentiable at $x = \pi$

But

$\int_0^x |\sin t|\, dt =$

*0x*sin*t, 0≤x<π0π*sin*tdt+πx*−sin*tdt, π≤x≤*2π

$$= \begin{cases} -\cos x + 1, 0 \le x < \pi \\ 3 + \cos x \;\; \pi \le x \le 2\pi \end{cases}$$

Which is continuous as well as differentiable at $x = \pi$

Hence, statement 1 is false

287 **(c)**

Both the statements are true independently, but statement 2 is not a correct explanation of statement 1

288 **(a)**

$$I = \int_0^1 \tan^{-1}\frac{2(1-x)-1}{1+(1-x)-(1-x)^2}\, dx$$

$$= \int_0^1 \tan^{-1}\frac{1-2x}{1+x-x^2} dx$$

$$= -I$$

$$\Rightarrow I = 0$$

289 **(a)**

Given that $\int_a^b |g(x)| dx > \left|\int_a^b g(x)\, dx\right| \Rightarrow y = g(x)$ cuts the graph at least once, then $y = f(x)g(x)$ changes sign at least once in (a, b), hence $\int_a^b f(x)g(x)dx$ can be zero

290 **(a)**

$$\int e^x \sin x dx$$

$$= \frac{1}{2}\int e^x (\sin x + \cos x + \sin x - \cos x) dx$$

$$= \frac{1}{2}\left(\int e^x(\sin x + \cos x)\, dx - \int e^x (\cos x - \sin x) dx\right)$$

$$=\frac{1}{2}(e^x \sin x - e^x \cos x) + c$$

$$=\frac{1}{2}e^x(\sin x - \cos x) + c$$

291 **(a)**

Let $I_m = \int_0^{\pi} \frac{\sin 2\,mx}{\sin x}dx$, Then,

$$I_m - I_{m-1} = \int_0^{\pi} \frac{\sin 2mx - \sin 2\,(m-1)x}{\sin x}dx$$

$$= \int_0^{\pi} 2\cos(2m-1)\,xdx$$

$$= \frac{2}{2m-1}[\sin(2m-1)x]_0^{\pi} = 0$$

$I_m = I_{m-1}$ for all $m \in N$

$\Rightarrow\; I_m = I_{m-1} = I_{m-2} = \ldots = I_1$

But, $I_1 = \int_0^{\pi} \frac{\sin 2x}{\sin x}dx = 2\int_0^{\pi} \cos x dx = 0$

$\therefore\; I_m = 0$ for all $m \in\; N$

292 **(d)**

$$F(x) = \int \sin^2 x\;\, dx = \int \frac{1-\cos 2x}{2}\; dx$$

$\Rightarrow\; F(x) = \frac{1}{4}(2x - \sin 2x) + c$

Since, $\quad F(x+\pi) \neq F(x)$

Hence, statement I is false.

But statement II is true as $\sin^2 x$ is possible with period π.

293 **(d)**

$$f(x) = \int_{5\pi/4}^{x} (3\sin t + 4\cos t)\,dt$$

$$\Rightarrow\; f'(x) = 3\sin x + 4\cos x, x \in \left[\frac{5\pi}{4}, \frac{4\pi}{3}\right]$$

These values of $x$are in third quadrant where both $\sin x$ and $\cos x$are negative

Then, $f'(x) < 0$ for$x \in \left[\frac{5\pi}{4}, \frac{4\pi}{3}\right]$

Hence, $f(x)$ is decreasing for these values of x

Then, the least value of function occurs at $x = \frac{4\pi}{3}$

$$\Rightarrow f_{\min} = \int_{5\pi/4}^{4\pi/3} (3\sin t + 4\cos t)dt$$

$$= \frac{3}{2} + \frac{1}{\sqrt{2}} - 2\sqrt{3}$$

294 **(a)**

$$\because\; \int \frac{1}{f(x)}dx = 2\log|f(x)| + c$$

On differentiating both sides w.r.t. x, then

$$\frac{1}{f(x)} = \frac{2}{f(x)}f'(x)$$

or $f'(x) = \frac{1}{2}$

$$\therefore f(x) = \frac{x}{2} + c$$

If $f(0) = 0$, then $f(x) = \frac{x}{2}$

295 **(d)**

$$\because\; \int_0^6 \{x+5\}^2 dx = \int_0^5 \{x+6\}^2 dx$$

$= \int_0^5 \{x\}^2 dx = 5\int_0^1 \{x\}^2 dx$ ($\because$ $\{\cdot\}$ is periodic with period 1)

$$= 5\int_0^1 x^2 dx = \frac{5}{3}$$

296 **(a)**

$\because 5x = 3x + 2x$

$$\Rightarrow \tan 5x = \frac{\tan 3x + \tan 2x}{1 - \tan 3x \tan 2x}$$

$\therefore \tan 5x - \tan 3x - \tan 2x = \tan 5x \tan 3x \tan 2x$

297 **(b)**

$\int \frac{\sin x dx}{x}$cannot be evaluated as there does not exist any method for evaluating this (integration by parts also does not works);however,$\frac{\sin x}{x}(x > 0)$ is a differentiable function. Hence, both the statements are true but statement 2 is not a correct explanation of statement 1

298 **(b)**

In LHS, put $x^n = \tan^2 \theta$

$\Rightarrow nx^{n-1}dx = 2\tan\theta \sec^2\theta\; d\theta$

$\therefore \int_0^\infty \frac{dx}{1+x^n} = \frac{2}{n}\int_0^{\pi/2} \tan^{1-2+2/n}\theta\, d\theta$

$= \frac{2}{n}\int_0^{\pi/2} \tan^{(2/n)-1}\theta\, d\theta$

In RHS, put $x^n = \sin^2\theta$

$\Rightarrow nx^{n-1}dx = 2\sin\theta\cos\theta\, d\theta$

$\therefore \int_0^1 \frac{dx}{(1-x^n)^{1/n}}$

$= \frac{2}{n}\int_0^{\frac{\pi}{2}} \frac{1}{\cos^{2/n}\theta}\sin^{\frac{2}{n}-1}\theta\cos\theta\, d\theta$

$= \frac{2}{n}\int_0^{\pi/2} \tan^{(2/n)-1\,\theta}\, d\theta$

Hence, option (b) is correct

299 **(a)**

To prove $\int_a^b f(x)dx = \int_{a+c}^{b+c} f(x-c)dx$

Put $z = x - c$, then $dz = dx$

When $x = a + c, z = a$ and when $x = b + c, z = b$

$\therefore \int_{a+c}^{b+c} f(x-c)dx = \int_a^b f(z)dz = \int_a^b f(x)dx$

Thus, statement 2 is true

$\int_a^b f(x)dx = \int_{a+c}^{b+c} f(x-c)dx$

Putting $f(x) = \sin^{100} x \cos^{99} x, a = 0, b = \pi$ and $c = -\frac{\pi}{2}$, we get

$\int_0^\pi \sin^{100} x \cos^{99} x\, dx$

$= \int_{-\pi/2}^{\pi/2} \sin^{100}\left(x + \frac{\pi}{2}\right)\cos^{99}\left(x + \frac{\pi}{2}\right) dx$

$= -\int_{-\pi/2}^{\pi/2} \cos^{100} x \sin^{99} x\, dx$

$=0$ [$\because \cos^{100} x \sin^{99} x$ is an odd function]

300 **(a)**

Statement II is true.

Now, $\int \frac{xe^x}{(x+1)^2}dx = \int \frac{(x+1-1)e^x}{(x+1)^2}dx$

$\int e^x\left\{\frac{1}{x+1} - \frac{1}{(x+1)^2}\right\}dx = \frac{e^x}{x+1} + c$

(By using statement II)

301 **(a)**

Given $f(x+1) + f(x+7) = 0, \forall\, x \in R$

Replace x by $x - 1$, we have $f(x) + f(x+6) = 0$ (1)

Now, replace x by $x + 6$, we have $f(x+6) + f(x+12) = 0$ (2)

From equations (1) and (2), we have
$f(x) = f(x+12)$ (3)

Hence, $f(x)$ is periodic with period 12

$\Rightarrow \int_a^{a+1} f(x)\, dx$ is independent of a if t is positive integral multiple of 12 then possible value of t is 12

302 **(c)**

$\because \sin\frac{x}{2}[1 + 2(\cos x + \cos 2x + \cos 3x + \ldots + \cos nx)]$

$= \sin\left(n + \frac{1}{2}\right)x$

$\therefore \int_0^\pi \frac{\sin\left(n+\frac{1}{2}\right)x}{\sin\frac{x}{2}}dx$

$= \int_0^\pi dx + 2\left[\int_0^\pi \cos dx + \int_0^\pi \cos 2x\, dx + \ldots + \int_0^\pi \cos nx\, dx\right]$

$= \pi + 2(0 + 0 + \ldots + 0)$

$= \pi$

$\Rightarrow$ Statement I is true.

$\because \int_0^\pi \sin mx\, dx = -\frac{1}{m}[\cos mx]_0^\pi$

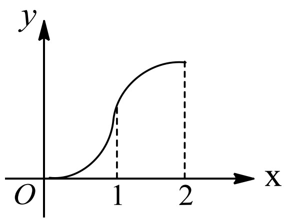
$= -\frac{1}{m}(\cos m\pi - 1)$

$= -\frac{1}{m}[(-1)^m - 1]$

$\neq 0$ when m is odd

303 **(d)**

$\because f(x)$ is continuous in $[0, 2]$



$\therefore \int_0^2 f(x)dx = \int_0^1 f(x)dx + \int_1^2 f(x)dx$

$= \int_0^1 x^2 dx + \int_1^2 \sqrt{x}\, dx$

$= \frac{1}{3} + \frac{2}{3}(2^{3/2} - 1)$

$= \frac{1}{3} + \frac{4\sqrt{2}}{3} - \frac{2}{3}$

$= \left(\frac{4\sqrt{2} - 1}{3}\right)$

304 **(b)**

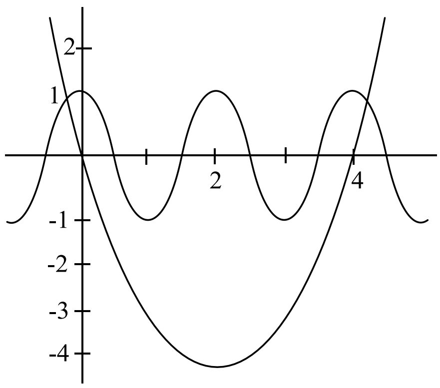
$f(x) = \pi \sin \pi x + 2x - 4$

$\Rightarrow g(x) = \int (\pi \sin \pi x + 2x - 4)dx$

$= -\cos \pi x + x^2 - 4x + c$

Also $f(1) = 3 \Rightarrow 1 + 1 - 4 + c = 3 \Rightarrow c = 0$

$\Rightarrow g(x) = -\cos \pi x + x^2 - 4x$



Hence, both the statements are true but statement 2 is not a correct explanation of statement 1

305 **(b)**

$\because |\sin x|$ is an even function.

$\therefore \int_{-\pi/2}^{\pi/2} |\sin x|\, dx = 2\int_0^{\pi/2} |\sin x| dx$

$= 2\int_0^{\pi/2} \sin x\, dx$

$= -2(\cos x)_0^{\pi/2} = -2(0 - 1) = 2$

306 **(c)**

Let $P = \int \frac{dx}{(x-3y)} = \frac{1}{2}\log\{(x-y)^2 - 1\}$

$\because P = \int \frac{dx}{(x-3y)}$

$\Rightarrow \frac{dP}{dx} = \frac{1}{(x-3y)}$ ....(i)

Also, $P = \frac{1}{2}\log\{ (x-y)^2 - 1\}$

$\therefore \frac{dP}{dx} = \frac{2(x-y)\left(1-\frac{dy}{dx}\right)}{2\{(x-y)^2-1\}} = \frac{(x-y)\left(1-\frac{dy}{dx}\right)}{(x-y)^2-1}$ .....(ii)

Given, $y(x-y)^2 = x$

$\Rightarrow \log y + 2\log(x-y) = \log x$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} + \frac{2}{(x-y)}\left(1 - \frac{dy}{dx}\right) = \frac{1}{x}$

$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y} - \frac{2}{x-y}\right) = \frac{1}{x} - \frac{2}{x-y} = \frac{x-y-2y}{x(x-y)}$

$\Rightarrow \frac{dy}{dx}\left(\frac{x-3y}{y(x-y)}\right) = -\frac{(x+y)}{x(x-y)}$

$\Rightarrow \frac{dy}{dx} = -\frac{y(x+y)}{x(x-3y)}$

Now, from Eq. (ii),

$\frac{dP}{dx} = \frac{(x-y)\left\{1 + \frac{y(x+y)}{x(x-3y)}\right\}}{(x-y)^2 - 1}$

$= \frac{(x-y)\left\{\frac{x^2-2xy+y^2}{x(x-3y)}\right\}}{\left(\frac{x}{y} - 1\right)}$

$= \frac{y(x-y)^2}{x(x-3y)} = \frac{1}{x-3y}$ ....(iii)

$\therefore$ It is true from Eq. (i).

$\therefore \int \frac{dx}{x-3y} = \frac{1}{2}\log\{ \{x-y)^2 - 1\}$

$\because y$ is variable.

$\therefore \int \frac{dx}{x-2y} \neq \log(x-3y)$

307 **(c)**

1. Let $I = \int \left(\frac{x^2-1}{x^2}\right) e^{\left(\frac{x^2+1}{x}\right)} dx =$

$\int \left(1 - \frac{1}{x^2}\right) e^{\left(x+\frac{1}{x}\right)} dx$

put $x + \frac{1}{x} = t \Rightarrow \left(1 - \frac{1}{x^2}\right) dx = dt$

$\therefore I = \int e^t \, dt = e^t + c = e^{\frac{x^2+1}{x}} + c$

(R) Let $I = \int f'(x)\, e^{f(x)} dx$

Put $f(x) = t \Rightarrow f'(x)dx = dt$

$\therefore I = \int e^t dt = e^{f(x)} + c$

Thus, A is true but R is false

308 **(b)**

For $0 < x < 1$, then

$x > x^2$

$\Rightarrow -x < -x^2$

$\Rightarrow e^{-x} < e^{-x^2}$

$\Rightarrow \int_0^1 e^{-x}\cos^2 x\, dx < \int_0^1 e^{-x^2}\cos^2 x\, dx$

If $f(x) \geq \mathrm{g}(x)$, then

$\int_a^b f(x)dx \geq \int_a^b \mathrm{g}(x)dx$

309 **(a)**

a. $I_1 = \int_{\pi/6}^{\pi/3} \sec^2\theta\, f(2\sin 2\theta)d\theta$

Applying property $\int_a^b f(a+b-x)dx =$ $abfxdx$

$I_1 = \int_{\pi/6}^{\pi/3} \sec^2\left(\frac{\pi}{2} - \theta\right) f\left(2\sin 2\left(\frac{\pi}{2} - \theta\right)\right) d\theta$

$I_1 = \int_{\pi/6}^{\pi/3} \operatorname{cosec}^2\theta\, f\,(2\sin 2\theta)d\theta = I_2$

b. $f(x+1) = f(x+3) \Rightarrow f(x) = f(x+2)$

$\Rightarrow f(x)$ is periodic with period 2

Then $\int_a^{a+b} f(x)dx$ is independent of a, for which b is multiple of 2

$\Rightarrow b = 2, 4, 6 \dots$

c. Let $I = \int_1^4 \frac{\tan^{-1}[x^2]}{\tan^{-1}[x^2]+\tan^{-1}[25+x^2-10x]}$ (1)

Applying $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$, we get

$I = \int_1^4 \frac{\tan[(5-x)^2]}{\tan^{-1}[(5-x)^2]+\tan^{-1}[x^2]} dx (2)$

Adding equations (1) and (2), we get

$$2I = \int_1^4 dx \Rightarrow 2I = 3 \Rightarrow I = 3/2$$

d. Let $y = \sqrt{x + \sqrt{x + \sqrt{x + \cdots}}} = \sqrt{x+y}$

$\Rightarrow y^2 - y - x = 0$

$\Rightarrow y = \frac{1 \pm \sqrt{1+4x}}{2.1}$

$\Rightarrow y = \frac{1 + \sqrt{1+4x}}{2} \quad (\because y > 1)$

$\Rightarrow I = \int_0^2 \frac{1+\sqrt{1+4x}}{2} dx = \left[\frac{x}{2} + \frac{(1+4x)^{3/2}}{\frac{3}{2}.2.4}\right]_0^2$

$= \left[\left(1 + \frac{27}{12}\right) - \left(0 + \frac{1}{12}\right)\right] = 1 + \frac{26}{12} = \frac{19}{6}$

$\Rightarrow [I] = 3$

310 **(b)**

a. $\lim_{n\to\infty}\left[\frac{\int_0^2\left(1+\frac{t}{n+1}\right)^n dt}{n+1}\right]$

$= \lim_{n\to\infty}\left[\left(1 + \frac{t}{n+1}\right)^{n+1}\right]_0^2$

$= \lim_{n\to\infty}\left(1 + \frac{2}{n+1}\right)^{n+1} - 1$

$= e^2 - 1$

b. $f'(x) = f(x) \Rightarrow f(x) = Ce^x$ and since $f(0) = 1$

$\therefore 1 = f(0) = C$

$\therefore f(x) = e^x$ and hence $g(x) = x^2 - e^x$

Thus, $\int_0^1 f(x)g(x)dx$

$= \int_0^1 (x^2e^x - e^{2x})dx = x^2e^x\Big|_0^1$

$- 2\int_0^1 xe^x dx - \frac{e^{2x}}{2}\Big|_0^1$

$= (e-0) - 2xe^x|_0^1 + +2e^x|_0^1 - \frac{1}{2}(e^2-1)$

$= (e-0) - 2e + 2e - 2 - \frac{1}{2}(e^2-1)$

$= e - \frac{1}{2}e^2 - \frac{3}{2}$

c. $I = \int_0^1 e^{e^x}(1 + xe^x)dx$

Let $e^x = t$

$\Rightarrow \int_1^e e^t(1 + t\log t)\frac{dt}{t}$

$= \int_1^e e^t\left(\frac{1}{t} + \log t\right)dt$

$= [e^t \log t]_1^e$

$= e^e$

d. $L = \lim_{k\to0} \frac{\int_0^k (1+\sin 2x)^{\frac{1}{x}} dx}{k} \left(\text{form } \frac{0}{0}\right)$

$\Rightarrow L = \lim_{k\to0}(1 + \sin 2k)^{\frac{1}{k}}$

$= e^{\lim_{k\to0}\frac{1}{k}(\sin 2k)} = e^2$

311 **(a)**

a. $\int_{-1}^1 \left[x + [x + [x]]\right] dx$ (use property $[x+n] = [x] + n$ if n is integer)

$= \int_{-1}^1 3[x]dx = 3\int_{-1}^1 [x]dx = 3\int_0^1 ([x] + [-x])\, dx$

$= -3$ (as $[x] + [-x] = -1$)

b. $\int_2^5 ([x] + [-x])\, dx = \int_2^5 -1 dx = -3$

c. $\text{sgn}\,(x - [x]) = \begin{cases} 1, \text{if } x \text{ is not an integer} \\ 0, \text{ if } x \text{ is an integer} \end{cases}$

Hence, $\int_{-1}^3 \text{sgn}\,(x - [x])\, dx = 4\,(1-0) = 4$

d. Let $I = 25\int_0^{\pi/4} (\tan^6(x - [x]) + \tan^4(x - x))dx$

$\left\{\because 0 < x \le \frac{\pi}{4} \Rightarrow [x] = 0\right\}$

$\therefore I = 25\int_0^{\pi/4} (\tan^6 x + \tan^4 x)dx$

$= 25\int_0^{\pi/4} \tan^4 x\,(\tan^2 x + 1)dx$

$= 25\int_0^{\pi/4} \tan^4 x\, \sec^2 x\, dx$

$= 25\left(\frac{\tan^5 x}{5}\right)_0^{\pi/4}$

$= 25 \times \frac{1}{5} = 5$

312 **(c)**

a. $\int \frac{x^2-x+1}{x^3-4x^2+4x} dx = \int \left[\frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}\right] dx$

b. $\int \frac{x^2-1}{x(x-2)^3} dx = \int \left[\frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2} + \frac{D}{(x-2)^3}\right] dx$

c. $\int \frac{x^3+1}{x(x-2)^2} dx = \int \left[\left(\frac{x^3-1}{x(x-2)^2} - 1\right) + 1\right] dx$

$= \int \left[\left(\frac{x^3 + 1 - x(x-2)^2}{x(x-2)^2}\right) + 1\right] dx$

$= \int \left[\left(\frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}\right) + 1\right] dx$

d. $\int \frac{x^5+1}{x(x-2)^3}dx = \int \left[x + k + \frac{g(x)}{x(x-2)^3}\right]dx,$

Where k is constant $a \neq 0$ and $g(x)$ is a polynomial of degree less than 4

313 **(a)**

a. $I = \int_{-2}^{2}(\alpha x^3 + \beta x + \gamma)\,dx$

$\alpha x^3 + \beta x$ is an odd function

$$I = 0 + 2\int_0^2 \gamma dx = 2.2\gamma = 4\gamma$$

b. $I = \frac{1}{2}\int_0^1 2\sin\alpha x \sin\beta x\,dx$

$$= \frac{1}{2}\int_0^1 (\cos(\alpha-\beta)x - \cos(\alpha+\beta)x)dx$$

$$= \frac{1}{2}\left[\frac{\sin(\alpha-\beta)x}{\alpha-\beta} - \frac{\sin(\alpha+\beta)x}{\alpha+\beta}\right]_0^1$$

$= \frac{1}{2}\left[\frac{\sin(\alpha-\beta)}{\alpha-\beta} - \frac{\sin(\alpha+\beta)}{\alpha+\beta}\right]$ (1)

Also, $2\alpha = \tan\alpha$ and $2\beta = \tan\beta$

$\Rightarrow 2(\alpha-\beta) = \tan\alpha - \tan\beta$ and $2(\alpha+\beta) = \tan\alpha + \tan\beta$

$2(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta}$ and $2(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$

Substituting these values, we get,

$I = (\cos\alpha\cos\beta) - (\cos\alpha\cos\beta) = 0$

c. $f(x+\alpha) + f(x) = 0$

$\Rightarrow f(x+2\alpha) + f(x+\alpha) = 0$

$\Rightarrow f(x+2\alpha) = f(x)$

$\Rightarrow f(x)$ is periodic with period 2α

$$\Rightarrow \int_\beta^{\beta+2\gamma\alpha}(\alpha x^3 + \beta x + \gamma)dx = \gamma\int_0^{2\alpha} f(x)dx$$

d. Let $I = \int_0^\alpha [\sin x]dx, \alpha \in [(2\beta+1)\pi, (2\beta + 2\pi, \beta \in N,$

[where $[\cdot]$ denotes the greatest integer function]

$$I = \int_0^{2\beta\pi}[\sin x]dx + \int_{2\beta\pi}^{(2\beta+1)\pi}[\sin x]dx + \int_{(2\beta+1)\pi}^{\alpha}[\sin x]dx$$

$$= \beta\int_0^{2\pi}[\sin x]\,dx + 0 + \int_{(2\beta+1)\pi}^{\alpha}(-1)dx$$

$= -\beta\pi + (2\beta+1)\pi - \alpha$

$= (\beta+1)\pi - \alpha$

$\Rightarrow \gamma\int_0^\alpha [\sin x]dx$ depends on α, β and γ

314 **(a)**

a. Let $I = \int \frac{2^x}{\sqrt{1-4^x}}dx = \frac{1}{\log 2}\int \frac{1}{\sqrt{1-t^2}}dt$

Putting $2^x = t, 2^x \log 2\,dx = dt$

$$I = \frac{1}{\log 2}\sin^{-1}\left(\frac{t}{1}\right) + C = \frac{1}{\log 2}\sin^{-1}(2^x) + C$$

$$\therefore K = \frac{1}{\log 2}$$

b. $\int \frac{dx}{(\sqrt{x})^2 + (\sqrt{x})^7} = \int \frac{dx}{(\sqrt{x})^7\left(1+\frac{1}{(\sqrt{x})^5}\right)}$

Put $\frac{1}{(\sqrt{x})^5} = y, \frac{dy}{dx} = -\frac{5}{2(\sqrt{x})^7}$

$$\therefore I = \int \frac{-2dy}{5(1+y)} = -\frac{2}{5}\ln|1+y| + C$$

$$= \frac{2}{5}\ln\left(\frac{1}{1+\frac{1}{(\sqrt{x})^5}}\right)$$

$$\Rightarrow a = \frac{2}{5}, k = \frac{5}{2}$$

c. Add and subtract $2x^2$ in the numerator, then $k = 1$ and $m = 1$

d. $I = \int \frac{dx}{5+4\cos x}$

$$= \int \frac{dx}{5\left(\sin^2\frac{x}{2} + \cos^2\frac{x}{2}\right) + 4\left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right)}$$

$$= \int \frac{dx}{9\cos^2\frac{x}{2} + \sin^2\frac{x}{2}} = \int \frac{\sec^2\frac{x}{2}}{9 + \tan^2\frac{x}{2}}dx$$

Let $t = \tan\frac{x}{2} \Rightarrow 2dt = \sec^2\frac{x}{2}dx$

$$\Rightarrow I = \int \frac{2dt}{9+t^2} = \frac{2}{3}\tan^{-1}\left(\frac{t}{3}\right) + C$$

$$= \frac{2}{3}\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{3}\right) + C$$

$$\Rightarrow k = \frac{2}{3}, m = \frac{1}{3}$$

315 **(b)**

a. $\int \frac{e^{2x}-1}{e^{2x}+1}dx$

$$= \int \frac{e^x - e^{-x}}{e^x + e^{-x}}dx$$

$$= \int \frac{(e^x + e^{-x})'}{e^x + e^{-x}}dx$$

$= \log(e^x + e^{-x})$

$= \log(e^{2x} + 1) - x + C$

b. $I = \int \frac{1}{(e^x+e^{-x})^2} dx = \int \frac{e^{2x}}{(e^{2x}+1)^2} dx$

Put$e^{2x} + 1 = t \Rightarrow 2e^{2x}dx = dt$, we get

$$\Rightarrow I = \frac{1}{2}\int \frac{1}{t^2} dt = -\frac{1}{2}\frac{1}{t} + C = -\frac{1}{2(e^{2x}+1)} + C$$

c. $I = \int \frac{e^{-x}}{1+e^x} dx = \int \frac{e^{-x}e^{-x}}{e^{-x}+1} dx$

Put $e^{-x} + 1 = t \Rightarrow -e^{-x}dx = dt$

$$\Rightarrow\ I = -\int \frac{(t-1)}{t} dt = \int \left(\frac{1}{t} - 1\right) dt$$

$= \log\ t - t + C$

$= \log(e^{-x} + 1) - (e^{-x} + 1) + C$

$= \log(e^x + 1) - x - e^{-x} - 1 + C$

$= \log(e^x + 1) - x - e^{-x} + C$

d.$I = \int \frac{1}{\sqrt{1-e^{2x}}} dx = \int \frac{e^{-x}}{\sqrt{e^{-2x}-1}} dx$

Put $e^{-x} = t \Rightarrow -e^{-x}dx = dt$,

$$\Rightarrow I = -\int \frac{1}{\sqrt{t^2-1}} dt$$

$$= -\log\left[t + \sqrt{t^2-1}\right] + C$$

$$= -\log\left[e^{-x} + \sqrt{e^{-2x}-1}\right] + C$$

$$= -\log\left[\frac{1}{e^x} + \frac{\sqrt{1-e^{2x}}}{e^x}\right] + C$$

$$= -\log\left[1 + \sqrt{1-e^{2x}}\right] + \log e^x + C$$

$$= x - \log\left[1 + \sqrt{1-e^{2x}}\right] + C$$

316 **(c)**

$$\lim_{n\to\infty} \frac{1}{n}\left\{\frac{1}{n+1} + \frac{2}{n+2} + \ldots + \frac{3n}{n+3n}\right\}$$

$$= \lim_{n\to\infty} \sum_{r=1}^{3n} \frac{1}{n}\left(\frac{r}{n+r}\right) = \int_0^3 \frac{x}{1+x} dx$$

$$= \int_0^3 \left(1 - \frac{1}{1+x}\right) dx$$

$= [x - \text{In}\ (1+x)]_0^3 = 3 - \text{In}\ 4$

$= 3 - 2\ \text{In}\ 2$

317 **(d)**

For $0 \le x \le 1$, we have

$0 \le x^2 \le 1$

$\Rightarrow e^0 \le e^{x^2} \le e^1$

$\Rightarrow 1 \le e^{x^2} \le e$

$\therefore\ m = 1, M = e$

$$\Rightarrow\ 1 \cdot (1-0) \le \int_0^1 e^{x^2} dx \le e \cdot (1-0)$$

$$\Rightarrow\ 1 \le \int_0^1 e^{x^2} dx \le e$$

318 **(d)**

$$\lim_{x\to 0} \frac{\int_0^{x^2} \sin\sqrt{t}dt}{x^3} \qquad \left(\frac{0}{0}\text{form}\right)$$

$$= \lim_{x\to 0} \frac{\sin x \cdot 2x}{3x^2}$$

$$= \frac{2}{3}\lim_{x\to 0} \frac{\sin x}{x} = \frac{2}{3} \cdot 1 = \frac{2}{3}$$

319 **(d)**

$$\because I_n = \int \tan^{n-2} x \cdot (\sec^2 x - 1)dx$$

$$= \frac{\tan^{n-1} x}{(n-1)} - I_{n-2}$$

$\because\ \lambda = -1$

320 **(d)**

Given, $I = \int \frac{dx}{(x-1)^2 \sqrt[4]{\left(\frac{x+2}{x-1}\right)^5}}$

Put $\frac{x+2}{x-1} = t \Rightarrow\ \frac{-3}{(x-1)^2} dx = dt$

$$\therefore I = -\frac{1}{3}\int \frac{dt}{x^{5/4}} = \frac{4}{3}\left[\frac{1}{t^{-1/4}}\right] + c = \frac{4}{3}\left[\sqrt[4]{\frac{x-1}{x+2}}\right] + c$$

$$\therefore A = \frac{4}{3}$$

321 **(d)**

From the given data, we can conclude that $\frac{dy}{dx} = 0$, at $x = 1, 2, 3$

Hence, $f'(x) = a(x-1)(x-2)(x-3), a > 0$

$$\Rightarrow f(x) = \int a(x^3 - 6x^2 + 11x - 6)dx$$

$$= a\int (x^3 - 6x^2 + 11x - 6)\ dx$$

$$= a\left(\frac{x^4}{4} - 2x^3 + \frac{11x^2}{2} + 6x\right) + C$$

Also $f(0) = 1 \Rightarrow c = 1$

$\Rightarrow f(x) = a\left(\frac{x^4}{4} - 2x^3 + \frac{11x^2}{2} - 6x\right) + 1 \quad (1)$

$f(1) = a\left(-\frac{9}{4}\right) + 1, f(2) = -2a + 1,$

$f(3) = a\left(-\frac{9}{4}\right) + 1 \qquad (2)$

$\Rightarrow$ The graph is symmetrical about line $x = 2$ and the range is $[f(1), \infty)$ or $[f(3), \infty)]$

$f(1) = -8 \Rightarrow a = 4$ (from(2))

$\Rightarrow f(2) = -7$

322 **(a)**

$A = \begin{bmatrix} x & x \\ x & x \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 2x^2 & 2x^2 \\ 2x^2 & 2x^2 \end{bmatrix}, A^3$

$= \begin{bmatrix} 2^2x^3 & 2^2x^3 \\ 2^2x^3 & 2^2x^3 \end{bmatrix}$

and so on

Then $e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots +$

$$= \begin{bmatrix} 1 + x + \frac{2x^2}{2!} + \frac{2^2x^3}{3!} + \cdots & x + \frac{2x^2}{2!} + \frac{2^2x^3}{3!} + \cdots \\ x + \frac{2x^2}{2!} + \frac{2^2x^3}{3!} + \cdots & 1 + x + \frac{2x^2}{2!} + \frac{2^2x^3}{3!} + \cdots \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}\left(1 + 2x + \frac{2^2x^2}{2!} + \frac{2^3x^3}{3!} + \cdots\right) + \frac{1}{2} & \frac{1}{2}\left(1 + 2x + \frac{2^2x^2}{2!} + \cdots\right) - \frac{1}{2} \\ \frac{1}{2}\left(1 + 2x + \frac{2^2x^2}{2!} + \frac{2^3x^3}{3!} + \cdots\right) - \frac{1}{2} & \frac{1}{2}\left(1 + 2x + \frac{2^2x^2}{2!} + \cdots\right) + \frac{1}{2} \end{bmatrix}$$

$= \frac{1}{2}\begin{bmatrix} e^{2x} + 1 & e^{2x} - 1 \\ e^{2x} - 1 & e^{2x} + 1 \end{bmatrix}$

$\Rightarrow f(x) = e^{2x} + 1$ and $g(x) = e^{2x} - 1$

$\int \frac{e^{2x} - 1}{e^{2x} + 1} dx = \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$

$= \log|e^x - e^{-x}| + C$

323 **(d)**

Here $a = 1 > 0$; therefore we make the substitution $\sqrt{x^2 + 2x + 2} = t - x$. Squaring both sides of this equality and reducing the similar terms, we get

$2x + 2tx = t^2 - 2 \Rightarrow x$

$= \frac{t^2 - 2}{2(1 + t)} \Rightarrow dx = \frac{t^2 + 2t + 2}{2(1 + t)^2} dt;$

$1 + \sqrt{x^2 + 2x + 2}$

$= 1 + t - \frac{t^2 - 2}{2(1 + t)} = \frac{t^2 + 4t + 4}{2(1 + t)}$

Substituting into the integral, we get

$I = \int \frac{2(1 + t)(t^2 + 2t + 2)}{(t^2 + 4t + 4)2(1 + t)^2} dt$

$= \int \frac{(t^2 + 2t + 2)dt}{(1 + t)(t + 2)^2}$

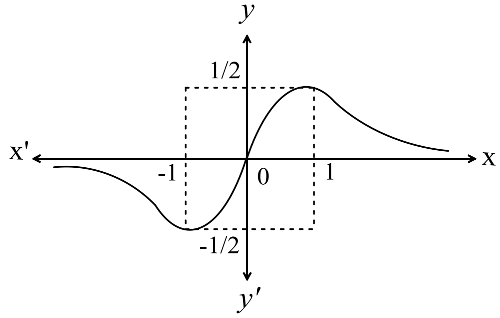
Now let us expand the obtained proper rational fraction into partial fractions:

$\frac{t^2 + 2t + 2}{(t + 1)(t + 2)^3} = \frac{A}{t + 1} + \frac{B}{t + 2} + \frac{D}{(t + 2)^2}$

324 **(d)**

$\int_2^x f(t)dt = \frac{x^2}{2} + \int_x^2 t^2 f(t)dt$

Differentiating w.r.t. x, we get



$f(x) = x + \left(-x^2 f(x)\right)$

$\Rightarrow f(x)[1 + x^2] = x$

$\Rightarrow y = f(x) = \frac{x}{1 + x^2}$

$\Rightarrow yx^2 - x + y = 0$

Since x is real, $D \geq 0$

$\Rightarrow 1 - 4y^2 \geq 0$

$\Rightarrow y \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Also, $f(x)$ is an odd function, hence $\int_{-2}^{2} f(x)dx = 0$

$f'(x) = \frac{1 + x^2 - 2x^2}{1 + x^2} = \frac{1 - x^2}{1 + x^2} \geq 0$

$\Rightarrow x^2 - 1 \leq 0$

$\Rightarrow x \in [-1, 1]$

325 **(b)**

$f(x) = x^2 + \int_0^x e^{-t} f(x - t)dt$ (1)

$= x^2 + \int_0^x e^{-(x-t)} f\left(x - (x - t)\right)dt$

$\left[\text{Using} \int_a^b f(x)dx = \int_a^b f(a + b - x)\, dx\right]$

$x^2 + e^{-x} \int_0^x e^t f(t)dt$ (2)

Differentiating w.r.t. x, we get

$\Rightarrow f'(x) = 2x - e^{-x} \int_0^x e^t f(t)dt + e^{-x} e^x f(x)$

$= 2x - e^{-x} \int_0^x e^t f(t)dt + f(x)$

$\Rightarrow f'(x) = 2x + x^2$ [using equation (2)]

$\Rightarrow f(x) = \frac{x^3}{3} + x^2 + c$

Also $f(0) = 0$[from equation (1)]

$\Rightarrow f(x) = \frac{x^3}{3} + x^2$

$\Rightarrow f'(x) = x^2 + 2x$

$\Rightarrow f'(x) = 0$has real roots, hence $f(x)$ is non-monotonic. Hence $f(x)$ is many-one, but range is R, hence surjective

$$\int_0^1 f(x)dx = \int_0^1 \left(\frac{x^3}{3} + x^2\right) dx$$

$$= \left[\frac{x^4}{12} + \frac{x^3}{3}\right]_0^1$$

$$= \frac{1}{12} + \frac{1}{3} = \frac{5}{12}$$

326 **(c)**

$$f(x) - \lambda \int_0^{\pi/2} \sin x \cot t\, f(t)dt = \sin x$$

$$\Rightarrow f(x) - \lambda \sin x \int_0^{\pi/2} \cos t\, f(t)dt = \sin x$$

$\Rightarrow f(x) - A \sin x = \sin x$ or

$f(x) = (A + 1) \sin x$, where

$A = \lambda \int_0^{\pi/2} \cos t\, f(t)dt$

$$\Rightarrow A = \lambda \int_0^{\pi/2} \cos t\ (A + 1) \sin t dt$$

$$= \frac{\lambda(A + 1)}{2} \int_0^{\pi/2} \sin 2t\, dt$$

$$= \frac{\lambda(A + 1)}{2} \left[\frac{-\cos 2t}{2}\right]_0^{\pi/2}$$

$$= \frac{\lambda\,(A + 1)}{2}$$

$$\Rightarrow A = \frac{\lambda}{2 - \lambda}$$

$$\Rightarrow f(x) = \left(\frac{\lambda}{2 - \lambda} + 1\right) \sin x$$

$$\Rightarrow f(x) = \left(\frac{2}{2 - \lambda}\right) \sin x$$

$$\left(\frac{2}{2 - \lambda}\right) \sin x = 2$$

$\Rightarrow \sin x = (2 - \lambda)$

$\Rightarrow |2 - \lambda| \leq 1$

$\Rightarrow -1 \leq \lambda - 2 \leq 1$

$\Rightarrow 1 \leq \lambda \leq 3$

$$\int_0^{\pi/2} f(x)dx = 3$$

$$\Rightarrow \int_0^{\pi/2} \frac{2}{2 - \lambda} \sin x dx = 3$$

$$\Rightarrow -\left[\frac{2}{2 - \lambda} \cos x\right]_0^{\pi/2} = 3$$

$$\Rightarrow \frac{2}{2 - \lambda} = 3$$

$\Rightarrow \lambda = 4/3$

327 **(b)**

$f(x)$is an odd function $\Rightarrow f(x) = -f(-x)$

$\phi(-x) = \int_a^{-x} f(t)dt$, put $t = -y$

$$\Rightarrow \phi(-x) = \int_{-a}^{x} f(-t)(-dt)$$

$$= \int_{-a}^{x} f(t)dt = \int_{-a}^{a} f(t)dt$$

$$+ \int_a^x f(t)dt = 0 + \int_a^x f(t)dt = \phi(x)$$

328 **(b)**

Let $I(a) = \int_0^1 \frac{x^a - 1}{\log x} dx$ (1)

Differentiating w.r.t. a keeping x as constant

$$\therefore \frac{dI(a)}{da} = \int_0^1 \frac{d}{da}\left(\frac{x^a - 1}{\log x}\right) dx$$

$$= \int_0^1 \frac{x^a \log x}{\log x}\ dx$$

$$= \int_0^1 x^a dx$$

$$= \left.\frac{x^{a+1}}{a + 1}\right|_0^1$$

$$= \frac{1}{(a + 1)}$$

Integrating both sides w.r.t. a, we get

$I(a) = \log(a + 1) + c$

For $a = 0, I(0) = \log 1 + c$[from equation (1)]

$0 = 0 + c$

$\therefore I = \log(a + 1)$

329 **(b)**

$$f(x)=\sin x+\sin x\int_{-\pi/2}^{\pi/2} f(t)dt + \cos x\int_{-\pi/2}^{\pi/2} tf(t)dt$$

$$=\sin x\left(1+\int_{-\pi/2}^{\pi/2} f(t)dt\right)+\cos x\int_{-\pi/2}^{\pi/2} tf\,(t)dt$$

$=A\sin x+B\cos x$

Thus, $A=1+\int_{-\pi/2}^{\pi/2} f(t)dt$

$$=1+\int_{-\pi/2}^{\pi/2}(A\sin t+B\cos t)dt$$

$$=1+2B\int_{0}^{\pi/2}\cos t dt$$

$\Rightarrow\ A=1+2B\ \ (1)$

$$B=\int_{-\pi/2}^{\pi/2} tf\,(t)dt$$

$$=\int_{-\pi/2}^{\pi/2} t(A\sin t+B\cot t)dt$$

$$=2A\int_{0}^{\pi/2} t\sin t dt$$

$=2A\,[-t\cos t+\sin t]_0^{\pi/2}$

$\Rightarrow\ B=2A(2)$

From equations (1) and (2), we get

$A=-1/3, B\ =-2/3$

$\Rightarrow f(x)=-\frac{1}{3}(\sin x+2\cos x)$

Thus, the range of $f(x)$ is $\left[-\frac{\sqrt{5}}{3},\frac{\sqrt{5}}{3}\right]$

$f(x)=-\frac{1}{3}(\sin x+2\cos x)$

$=-\frac{\sqrt{5}}{3}\sin(x+\tan^{-1}2)$

$=-\frac{\sqrt{5}}{3}\cos\left(x-\tan^{-1}\frac{1}{2}\right)$

$f(x)$ is invertible if $-\frac{\pi}{2}\le x+\tan^{-1}2\ \le\frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{2}-\tan^{-1}2\le x\le\frac{\pi}{2}-\tan^{-1}2$

or $0\le\ x-\tan^{-1}\frac{1}{2}\le\ \pi$

$\Rightarrow \tan^{-1}\frac{1}{2}\le\ x\le\pi+\tan^{-1}\frac{1}{2}$

or $\pi\le x-\tan^{-1}\frac{1}{2}\le 2\pi$

$\Rightarrow\ x\in[\pi\ +\cot^{-1}2, 2\pi+\cot^{-1}2]$

$$\int_{0}^{\pi/2} f(x)dx=-\frac{1}{3}\int_{0}^{\pi/2}(\sin x+2\cos x)dx$$

$=-\frac{1}{3}[-\cos x+2\sin x\,]_0^{\pi/2}$

$=-1$

330 **(6)**

$y=f(x)\Rightarrow\ x=f^{-1}(y)\Rightarrow\ x=g(y)$

Given $y=f(x)=\int_0^x\frac{dt}{\sqrt{1+t^3}}$

$\frac{dy}{dx}=\frac{1}{\sqrt{1+x^3}}\Rightarrow\frac{dx}{dy}=\sqrt{1+x^3}$

$g'(y)=\sqrt{1+g^3(y)}$

$g''(y)=\frac{3g^2(y)g'(y)}{2\sqrt{1+g^3(y)}}$

$\Rightarrow 2g''(y)=3g^2(y)\frac{g'(y)}{\sqrt{1+g^3(y)}}$

$=3g^2(y)\frac{\sqrt{1+g^3(y)}}{\sqrt{1+g^3(y)}}=3g^2(y)$

$\Rightarrow 2g''(y)=3g^2(y)$

331 **(8)**

$I_{11}=\int_0^1\underbrace{(1-x^5)^{11}}_{\text{I}}\cdot\underset{\text{II}}{1}\,dx$

$$=(1-x^5)^{11}\cdot x]_0^1+11\int_0^1(1-x^5)^{10}\,5x^4\cdot x\,dx$$

$$=0-55\int_0^1(1-x^5)^{10}(1-x^5-1)dx$$

$$=-55\int_0^1(1-x^5)^{11}dx+55I_{10}$$

$\Rightarrow\ 56I_{11}=55I_{10}$

$\Rightarrow\frac{I_{10}}{I_{11}}=\frac{56}{55}$

332 **(0)**

$\because$ Integrand is discontinuous at $\frac{\pi}{2}$, then

$\int_0^{\pi/2}0\cdot dx+\ \int_{\pi/2}^{3\pi/2}0\cdot dx=0$

$\because 0<x<\frac{\pi}{2}, |\tan^{-1}\tan x|=|\sin^{-1}\sin x|$ and

$\frac{\pi}{2}<x<\frac{3\pi}{2}, |\tan^{-1}\tan x|=|\sin^{-1}\sin x|$

333 **(3)**

$\frac{d}{dx}(A\ln|\cos x+\sin x-2|+Bx+C)$

$=A\ \frac{\cos x-\sin x}{\cos x+\sin x-2}+B$

$=\frac{A\cos x-A\sin x+B\cos x+B\sin x-2B}{\cos x+\sin x-2}$

$\therefore 2=A+B, -1=-A+B, \lambda=-2B$

$\therefore A=3/2, B=1/2, \lambda=-1$

$\Rightarrow A+B+|\lambda|=3$

334 **(4)**

$I = \int_0^1 \frac{\sin^{-1}\sqrt{x}}{x^2-x+1}\,dx(1)$

$I = \int_0^1 \frac{\sin^{-1}\sqrt{1-x}}{x^2-x+1}\,dx = \int_0^1 \frac{\cos^{-1}\sqrt{x}}{x^2-x+1}\,dx(2)$

On adding equations (1) and (2), we get

$$2I = \int_0^1 \frac{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}{x^2 - x + 1}dx$$

$$= \frac{\pi}{2}\int_0^1 \frac{dx}{x^2 - x + 1}\,dx$$

$$= \frac{\pi}{2}\int_0^1 \frac{dx}{\left(x - \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\,dx$$

$$2I = \frac{\pi}{2}\frac{1}{\left(\frac{\sqrt{3}}{2}\right)}\left[\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right]_0^1 = \frac{\pi^2}{3\sqrt{3}}$$

Hence, $I = \frac{\pi^2}{6\sqrt{3}} = \frac{\pi^2}{\sqrt{108}} \equiv \frac{\pi^2}{\sqrt{n}}$

335 **(4)**

$$\int x^2 \cdot e^{-2x}\,dx = e^{-2x}(ax^2 + bx + c) + d$$

Differentiating both sides, we get

$$x^2 \cdot e^{-2x} = e^{-2x}(2ax + b) + (ax^2 + bx + c)(-2e^{-2x})$$

$$= e^{-2x}(-2ax^2 + 2(a-b)x + b - 2c)$$

$$\Rightarrow a = -\frac{1}{2}, 2(a-b) = 0, b - 2c = 0$$

$$\Rightarrow a = -\frac{1}{2}, b = -\frac{1}{2}, c = -\frac{1}{4}$$

336 **(6)**

$$I = \int_0^\infty (x^2)^n \cdot x\,e^{-x^2}\,dx$$

Put $x^2 = t \Rightarrow\ x\,dx = dt/2$

$$\Rightarrow\ I = \frac{1}{2}\int_0^\infty t^n e^{-t}dt$$

$$= \frac{1}{2}\left[-t^n e^{-t}]_0^\infty + n\int_0^\infty t^{n-1}e^{-t}\,dt\right]$$

$$= \frac{1}{2}\left[0 + n\int_0^\infty t^{n-1}e^{-t}\,dt\right]$$

$$\Rightarrow\ I = \frac{n!}{2} = 360$$

$$\Rightarrow\ n = 6$$

337 **(2)**

$$I = \int_{\sqrt{2}-1}^{\sqrt{2}+1} \frac{(x^2+1)^2 - (x^2-1)}{(x^2+1)^2}dx$$

$$= \int_{\sqrt{2}-1}^{\sqrt{2}+1}\left(1 - \frac{(x^2-1)}{(x^2+1)^2}\right)dx$$

$$= 2 - \underbrace{\int_{\sqrt{2}-1}^{\sqrt{2}+1}\frac{(x^2-1)}{(x^2+1)^2}dx}_{I_1}$$

$I_1 = \int_{1/a}^{a}\frac{(x^2-1)}{(x^2+1)^2}dx$ where $\left(a = \sqrt{2}+1\right)$;

Put $x = \frac{1}{t} \Rightarrow\ dx = -\frac{1}{t^2}\,dt$

$$= \int_a^{1/a}\frac{\frac{1}{t^2}-1}{\left(\frac{1}{t^2}+1\right)^2}\cdot\left(-\frac{1}{t^2}\right)dt = -\int_a^{1/a}\frac{(1-t^2)t^4}{t^4(1+t^2)^2}dt$$

$$= -\int_a^{1/a}\frac{(1-t^2)}{(1+t^2)^2}dt = \int_a^{1/a}\frac{t^2-1}{(t^2+1)^2}\,dt$$

$$= -\int_{1/a}^{a}\frac{t^2-1}{(t^2+1)^2}dt = -I_1$$

$$\Rightarrow\ 2I_1 = 0$$

$$\Rightarrow I_1 = 0$$

$$\Rightarrow\ I = 2$$

338 **(6)**

Given $f^3(x) = \int_0^x t\cdot f^2(t)dt$

Differentiating, $3f^2(x)f'(x) = xf^2(x)$

$f(x) \neq 0\ \therefore\ f'(x) = \frac{x}{3};\ \therefore\ f(x) = \frac{x^2}{6} + C$

But $f(0) = 0\ \Rightarrow C = 0$

$f(6) = 6$

339 **(7)**

$$F'(x) = (2x+3)\int_x^2 f(u)du$$

$$\therefore\ F''(x) = -(2x+3)f(x) + \left(\int_x^2 f(u)du\right)\cdot 2$$

$$F''(2) = -7f(2) + 0$$

340 **(8)**

$$\frac{d}{dx}\int_4^x [4t^2 - 2F'(t)]dt = [4x^2 - 2F'(x)]\cdot 1 - 0$$

$$\Rightarrow F'(x) = \frac{1}{x^2}[4x^2 - 2F'(x)] + \frac{-2}{x^3}\int_4^x [4t^2 - 2F'(t)]\,dt$$

$$\Rightarrow F'(4)=\frac{1}{16}[64-2F'(4)]-\frac{1}{32}\int_4^4 g(x)dx$$

$$\Rightarrow\left(1+\frac{1}{8}\right)F'(4)=4$$

$$\Rightarrow F'(4)=\frac{32}{9}$$

341 **(2)**

$$\lim_{n\to\infty}\frac{n}{2^n}\cdot\frac{x^{n+1}}{n+1}\Big]_0^2$$

$$=\lim_{n\to\infty}\frac{n}{2^n}\cdot\frac{2^{n+1}}{n+1}$$

$$=\lim_{n\to 0}\frac{2}{1+(1/n)}=2$$

342 **(5)**

We have $f(2x)=3f(x)$ (1)

and $\int_0^1 f(x)dx=1$ (2)

From equations (1) and (2), $\frac{1}{3}\int_0^1 f(2x)dx=1$

Put $2x=t, \frac{1}{6}\int_0^2 f(t)dt=1$

$$\Rightarrow\int_0^2 f(t)dt=6$$

$$\Rightarrow\int_0^1 f(t)dt+\int_1^2 f(t)dt=6$$

Hence, $\int_1^2 f(t)dt=6-\int_0^1 f(t)dt=6-1=5$

343 **(4)**

$$I_1=\int_0^1 x^{1004}(1-x)^{1004}dx$$

$=2\int_0^{1/2} x^{1004}(1-x)^{1004}\,dx(1)$

And $I_2=\int_0^1 x^{1004}(1-x^{2010})^{1004}\,dx$

Put $x^{1005}=t\Rightarrow 1005\,x^{1004}\,dx=dt$

$$\Rightarrow I_2=\frac{1}{1005}\int_0^1(1-t^2)^{1004}\,dt$$

$$=\frac{1}{1005}\int_0^1\big(t(2-t)\big)^{1004}\,dt$$

$$=\frac{1}{1005}\int_0^1 t^{1004}(2-t)^{2004}\,dt$$

Now put $t=2y\Rightarrow dt=2dy$

$$\Rightarrow I_2=\frac{1}{1005}\int_0^{1/2}(2y)^{1004}(2-2y)^{1004}\,dt$$

$$=\frac{1}{1005}\,2\cdot 2^{1004}\cdot 2^{1004}\int_0^{1/2} y^{1004}(1-y)^{1004}\,dy$$

$$=\frac{1}{1005}2^{2009}\int_0^{1/2} y^{1004}(1-y)^{1004}\,dy$$

$$=\frac{1}{1005}2^{2008}I_1$$

$$\Rightarrow\frac{I_1}{I_2}=\frac{1005}{2^{2008}}$$

$$\Rightarrow\frac{2^{2010}}{1005}\frac{I_1}{I_2}=4$$

344 **(1)**

$$f(x)=\int x^{\sin x}(1+x\cos\cdot\ln x+\sin x)dx$$

If $F(x)=x^{\sin x}=e^{\sin x\ln x}$

$$\therefore f(x)=\int\big(F(x)+xF'(x)\big)=xF(x)+C$$

$$f(x)=x\cdot x^{\sin x}+C$$

$$f\left(\frac{\pi}{2}\right)=\frac{\pi}{2}\cdot\frac{\pi}{2}+C\Rightarrow C=0$$

$$\therefore f(x)=x(x)^{\sin x};\ f(\pi)=\pi(\pi)^0=\pi$$

345 **(8)**

$$I=\lim_{n\to\infty}\frac{\sqrt{1}+\sqrt{2}+\sqrt{3}+\cdots+\sqrt{6n}}{n\sqrt{n}}$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{6n}\sqrt{\frac{r}{n}}=\int_0^6\sqrt{x}\ dx=\left[\frac{2}{3}x^{3/2}\right]_0^6=\frac{2}{3}\cdot 6\sqrt{6}$$

$$=\sqrt{96}$$

346 **(8)**

Let $I=\int_0^1 {}^{207}C_7\cdot\underbrace{x^{200}}_{II}\cdot\underbrace{(1-x)^7}_{I}\ dx$

$$I={}^{207}C_7\left[\underbrace{(1-x)^7\cdot\frac{x^{201}}{201}\Big|_0^1}_{zero}+\frac{2}{201}\int_0^1(1-x)^6\cdot x^{201}\,dx\right]$$

$$={}^{207}C_7\cdot\frac{7}{201}\int_0^1(1-x)^6\cdot x^{201}\,dx$$

Integrating by parts again 6 more times

$$={}^{207}C_7\cdot\frac{7!}{201.202.203.204.205.206.207}\int_0^1 x^{207}\,dx$$

$$=\frac{(207)!}{7!\,(200)!}\cdot\frac{7!}{201.202\cdots 207}\cdot\frac{1}{208}$$

$$=\frac{(207)!}{(207)!\,7!}\cdot\frac{7!}{208}=\frac{1}{208}=\frac{1}{k}\Rightarrow k=208$$

347 **(4)**

$$g(x)=\int\frac{\cos x\,(\cos x+2)+\sin^2 x}{(\cos x+2)^2}dx$$

$$= \int \underbrace{\cos x}_{\text{II}} \cdot \underbrace{\frac{1}{(\cos x + 2)}}_{\text{I}} dx + \int \frac{\sin^2 x}{\cos x + 2} dx$$

$$= \frac{1}{\cos x + 2} \cdot \sin x - \int \frac{\sin^2 x}{(\cos x + 2)^2} dx + \int \frac{\sin^2 x}{(\cos x + 2)^2} dx$$

$$\therefore g(x) = \frac{\sin x}{\cos x + 2} + C$$

$g(0) = 0 \Rightarrow C = 0$

$$\therefore g(x) = \frac{\sin x}{\cos x + 2} \Rightarrow g\left(\frac{\pi}{2}\right) = \frac{1}{2}$$

348 **(0)**

We have $J = \int_{-5}^{-4} (3 - x^2) \tan(3 - x^2) dx$

Put $(x + 5) = t$, we get

$$J = \int_0^1 (3 - (t - 5)^2) \tan(3 - (t - 5)^2) dt$$

$$= \int_0^1 (-22 + 10t - t^2) \tan(-22 + 10t - t^2) dt$$

Now, $K = \int_{-2}^{-1} (6 - 6x + x^2) \tan(6x - x^2 - 6)\, dx$

Put $(x + 2) = z$, we get

$$K = \int_0^1 (6 - 6(z - 2) + (z - 2)^2) \tan(6(z - 2) - (z - 2)^2 - 6) dz$$

$$= \int_0^1 (22 - 10z + z^2) \tan(-22 + 10z - z^2) dz$$

Hence, $(J + K) = 0$

349 **(9)**

$$f(x) = \int \frac{3x^2 + 1}{(x^2 - 1)^3} dx$$

$$= \int \frac{-(x^2 - 1)}{(x^2 - 1)^3} dx + \int \frac{4x^2}{(x^2 - 1)^3} dx$$

$$= \int \left[\frac{-1}{(x^2 - 1)^2} + x \cdot \frac{4x}{(x^2 - 1)^3}\right] dx$$

$$= -\int \frac{dx}{(x^2 - 1)^2} + x \int \frac{4x\, dx}{(x^2 - 1)^3} - \int \left((x)' \int \frac{4x}{(x^2 - 1)^3} dx\right) dx$$

$$= x\left(\frac{-1}{(x^2 - 1)^2}\right) + C$$

$$= -\frac{x}{(x^2 - 1)^2} + C$$

$f(0) = 0 \Rightarrow C = 0$

$$\Rightarrow f(x) = -\frac{x}{(x^2 - 1)^2}$$

Now $f(2) = -\frac{2}{9}$

350 **(4)**

Given $f(x) = x^3 - \frac{3x^2}{2} + x + \frac{1}{4} = \frac{1}{4}(4x^3 - 6x^2 + 4x + 1)$

$$= \frac{1}{4}(4x^3 - 6x^2 + 4x - 1 + 2)$$

$$f(x) = \frac{1}{4}[x^4 - (1 - x)^4] + \frac{2}{4}$$

$$\therefore f(1 - x) = \frac{1}{4}[(1 - x)^4 - x^4] + \frac{2}{4}$$

$\therefore f(x) + f(1 - x) = \frac{2}{4} + \frac{2}{4} = 1 (1)$

Replacing x by $f(x)$ we have

$f[f(x)] + f[1 - f(x)] = 1$ (2)

Now $I = \int_{1/4}^{3/4} f(f(x)) dx (3)$

Also$I = \int_{1/4}^{3/4} f(f(1 - x)) dx = \int_{1/4}^{3/4} f(1 - fxdx(4)$

{using (1)}

Adding (3) and (4),

$$2I = \int_{1/4}^{3/4} [f(f(x)) + f(1 - f(x))]\, dx = \int_{1/4}^{3/4} dx$$

$$\Rightarrow 2I = \frac{1}{2} \Rightarrow I = \frac{1}{4}$$

$$\therefore I = \frac{1}{4}$$

$\therefore I^{-1} = 4$

351 **(3)**

$$f(x) \int_0^x e^t \sin(x - t) dt$$

$$= \int_0^x e^{x-t} \sin(x - (x - t)) dt$$

$$= e^x \int_0^x e^{-t} \sin t dt$$

$$\Rightarrow f'(x) = e^x e^{-x} \sin x + e^x \int_0^x e^{-t} \sin t dt$$

$$= \sin x + e^x \int_0^x e^{-t} \sin t dt$$

$$\Rightarrow f''(x) = \cos x + e^x e^{-x} \sin x + e^x \int_0^x e^{-t} \sin t dt$$

$= \cos x + \sin x + f(x)$

$\Rightarrow f''(x) - f(x) = \cos x + \sin x$

Range of $g(x) = f''(x) - f(x)$ is $[-\sqrt{2}, \sqrt{2}]$

Number of integers in the range is 3

352 **(2)**

We have $\int_{\sin t}^{1} x^2 g(x) dx = (1 - \sin t)$ (1)

Differentiating both the sides of (1) with respect to 't', we get

$0-(\sin^2 t)\, g(\sin t)\,(\cos t) = -\cos t$

$\Rightarrow\ g(\sin t) = \frac{1}{\sin^2 t}$ (2)

Putting $t = \frac{\pi}{4}$in (2),

We get $g\left(\frac{1}{\sqrt{2}}\right) = 2$

353 **(7)**

$$\sum_{r=1}^{100}\left(\int_0^1 f(r-1+x)dx\right)$$

$$= \int_0^1 f(x)dx + \int_0^1 f(1+x)dx + \int_0^1 f(2+x)dx + \cdots + \int_0^1 f(99+x)dx$$

$$= \int_0^1 f(x)dx + \int_1^2 f(x)dx + \int_2^3 f(x)dx + \cdots + \int_{99}^{100} f(x)dx$$

$$= \int_0^{100} f(x)dx = 7$$

354 **(2)**

$$\int_0^2 |f'(x)|dx \ge \left|\int_0^2 f'(x)dx\right|$$

$$\Rightarrow \int_0^2 |f'(x)|dx \ge f(2)| = 2$$

355 **(0)**

$fog(x) = \sqrt{e^x-1}$

$\therefore I = \int \sqrt{e^x-1}\, dx$

$= \int \frac{2t^2}{t^2+1} dt\ \{\text{where} \sqrt{e^x-1} = t\}$

$= 2t - 2\tan^{-1} t + C$

$= 2\sqrt{e^x-1} - 2\tan^{-1}\left(\sqrt{e^x-1}\right) + C$

$= 2\, fog(x) - 2\tan^{-1}(fog(x)) + C$

$\therefore A + B = 2 + (-2) = 0$

356 **(2)**

$$k(x) = \int \frac{(x^2+1)dx}{(x^3+3x+6)^{1/3}}$$

Put $x^3+3x+6 = t^3 \ \Rightarrow 3(x^2+1)dx = 3t^2 dt$

$$k(x) = \int \frac{t^2 dt}{t} = \frac{t^2}{2} + C$$

$$k(x) = \frac{1}{2}(x^3+3x+6)^{2/3} + C$$

$$k(-1) = \frac{1}{2}(2)^{2/3} + C \Rightarrow C = 0$$

$$\therefore k(x) = \frac{1}{2}(x^3+3x+6)^{2/3}; f(-2) = \frac{1}{2}(-8)^{2/3}$$

$$= \frac{1}{2}[(-2)^3]^{2/3} = 2$$

357 **(9)**

$$f(x) = x + x\int_0^1 t\, f(t)dt + \int_0^1 t^2 f(t)dt$$

$\therefore\ f(x) = x(1+A) + B$; where $A = \int_0^1 t\, f(t)dt$ and $B = \int_0^1 t^2 f(t)dt$

Now, $A = \int_0^1 t[t(1+A)+B]dt = \frac{t^3}{3}(1+A)\Big|_0^1 +$ *B2t201*

$$\Rightarrow\ A = \frac{1+A}{3} + \frac{B}{2}$$

$\Rightarrow\ 4A - 3B = 2$ (1)

Again $B = \int_0^1 t^2[t(1+A)+B]dt = \frac{t^4\ (1+A)}{4} +$ *Bt3301*

$$= \frac{1+A}{4} + \frac{B}{3}$$

$\Rightarrow\ 8B - 3A = 3$ (2)

Solving equations (1) and (2) we have

$B = \frac{18}{23} = f(0)$

358 **(0)**

$$\int\left[\left(\frac{x}{e}\right)^x+\left(\frac{e}{x}\right)^x\right]\ln x\,dx$$

Put $\left(\frac{x}{e}\right)^x=t$

Or $x\ln\left(\frac{x}{e}\right)=\ln t$

$\therefore \left(x\cdot\frac{1}{x/e}\cdot\frac{1}{e}+\ln\left(\frac{x}{e}\right)\right)dx=\frac{1}{t}dt$

$\therefore (1+\ln x-\ln e)dx=\frac{1}{t}dt$

$\therefore (\ln e+\ln x-\ln e)dx=\frac{1}{t}dt$

$\therefore (\ln x)dx=\frac{1}{t}dt$

Or $I=\int\left(1+\frac{1}{t}\right)\frac{1}{t}dt=\int 1\cdot dt+\int\frac{1}{t^2}dt$

$=t-\frac{1}{t}+C$

Or $I=\left(\frac{x}{e}\right)^x-\left(\frac{e}{x}\right)^x+C$

359 **(2)**

$$I=\int_0^{3\pi/4}(\sin x+\cos x)dx$$

$$+\int_0^{3\pi/4}\underbrace{x}_{\text{I}}\underbrace{(\sin x-\cos x)}_{\text{II}}\,dx$$

$$=\int_0^{3\pi/4}(\sin x+\cos x)dx+\underbrace{x\,(-\cos x-\sin x)|_0^{3\pi/4}}_{\text{zero}}$$

$$+\int_0^{3\pi/4}(\sin x+\cos x)dx$$

$$=2\int_0^{3\pi/4}(\sin x+\cos x)dx=2\,(\sqrt{2}+\ 1)$$

360 **(3)**

We have $f(x)=\sin x+\int_{-\pi/2}^{\pi/2}(\sin x+t\,f(t))dt=$ $\sin x+\pi\sin x+-\pi/2\pi/2t\,ftdt$

$\therefore f(x)=(\pi+1)\sin x+A\quad(1)$

Now, $A=\int_{-\pi/2}^{\pi/2}t((\pi+1)\sin t+A)dt=$ $2\pi+10\pi/2t\sin tdtAIIBy part

$\Rightarrow A=2(\pi+1)$

Hence, $f(x)=(\pi+1)\sin x+2(\pi+1)$

Therefore, $f_{\max}=3(\pi+1)=M$

and $f_{\min}=(\pi+1)=m$

$\Rightarrow\frac{M}{m}=3$

361 **(5)**

Given $U_n=\int_0^1 x^n\cdot(2-x)^n\,dx; V_n=\int_0^1 x^n\cdot(1-$ $xndx$

In U_n put $x=2t\ \Rightarrow\ dx=2dt$

$\therefore U_n=2\int_0^{1/2}2^n\cdot t^n2^n(1-t)^n dt\qquad(1)$

Now $V_n=2\int_0^{1/2}x^n(1-x)^n dx(2)$

From equations (1) and (2) we get $U_n=2^{2n}\cdot V_n$

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