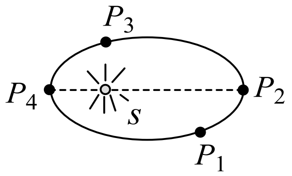
8.GRAVITATION

Single Correct Answer Type

1. Figure shows a planet in an elliptical orbit around the Sun S. Where is the kinetic energy of the planet maximum?

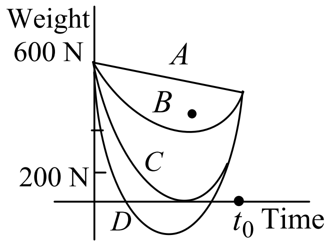


a) P_1 b) P_2 c) P_3 d) P_4

2. The distance between the centre of the Moon and the earth is D. The mass of the earth is 81 times the mass of the Moon. At what distance from the centre of the earth, the gravitational force will be zero?

a) $\frac{D}{2}$ b) $\frac{2D}{3}$ c) $\frac{4D}{3}$ d) $\frac{9D}{10}$

3. Suppose, the acceleration due to gravity at the earth''s surface is 10 $\mathrm{ms^{-2}}$ and at the surface of Mars is 4.0 $\mathrm{m\ s^{-2}}$. A 60 kg passenger goes from the earth of Mars in a spaceship moving with a constant velocity. Neglect all other objects in the sky. Which part of the figure best represents the weight (net gravitational force) of the passenger as a function of time?



a) A b) B c) C d) D

4. The charge in the value of g at a height 'h' above the surface of earth is the same as at a depth 'd' below the earth. When both d and h are much smaller than the radius of earth, then which one of the following is correct?

a) $d=\frac{h}{2}$ b) $d=\frac{3h}{2}$ c) $d=2h$ d) $d=h$

5. The gravitational force between two objects is proportional to $1/R$ (and not as $1/R^2$) where R is separation between them, then a particle in circular orbit under such a force would have its orbital speed v proportional to

a) $\frac{1}{R^2}$ b) R^0 c) R^1 d) $\frac{1}{R}$

6. A skylab of mass m kg is first launched from the surface of the earth in a circular orbit of radius $2R$ (from the centre of the earth) and then it is shifted from this circular orbit to another circular orbit of radius $3R$. The minimum energy required to place the lab in the first orbit and to shift the lab from first orbit to the second orbit are

a) $\frac{3}{4}mgR, \frac{mgR}{6}$ b) $\frac{3}{4}mgR, \frac{3gR}{12}$ c) mgR, mgR d) $2mgR, mgR$

7. A tunnel is dug along a diameter of the earth. If M_e and R_e are the mass and radius, respectively, of the earth, then the force on a particle of mass m placed in the tunnel at a distance r from the centre is

a) $\frac{GM_em}{R_e^3}r$ b) $\frac{GM_em}{R_e^3r}$ c) $\frac{GM_emR_e^3}{r}$ d) $\frac{GM_em}{R_e^2}r$

8. A comet is in highly elliptical orbit around the Sun. The period of the comet's orbit is 90 days. Some statements are given are given regarding the collision between the comet and the earth. Mark the correct statement. [Mass of the Sun= 2×10^{30} kg, mean distance between the earth and the Sun= 1.5×10^{11} m]

a) Collision is there b) Collision is not possible

c) Collision may or may not be there d) Enough information is not given

9. Two planets have radii r_1 and r_2 and their densities are ρ_1 and ρ_2 respectively. The ratio of acceleration due to gravity on them will be
a) $r_1\rho_1 : r_2\rho_2$ b) $r_1\rho_1^2 : r_2\rho_2^2$ c) $r_1^2\rho_1 : r_2^2\rho_2$ d) $r_1\rho_2 : r_2\rho_1$

10. A ball of mass m is fired vertically upwards from the surface of the earth with velocity nv_e, where v_e is the escape velocity and $n > 1$. To what height will the ball rise? Neglecting air resistance, take radius of the earth as R
a) $\frac{R}{n^2}$ b) $\frac{R}{(1-n^2)}$ c) $\frac{Rn^2}{(1-n^2)}$ d) Rn^2

11. Two bodies of masses M_1 and M_2 are placed at a distance R apart. Then at the position where the gravitational field due to them is zero, the gravitational potential is
a) $-G\frac{\sqrt{M_1}}{R}$ b) $-G\frac{\sqrt{M_2}}{R}$ c) $-\left(\sqrt{M_1}+\sqrt{M_2}\right)^2\frac{G}{R}$ d) $-\left(\sqrt{M_1}+\sqrt{M_2}\right)^2\frac{G}{R}$

12. Two satellite of masses m_1 and m_2 $(m_1 > m_2)$ are revolving around the earth in a circular orbit of radii r_1 and r_2 $(r_1 > r_2)$, respectively. Which of the following statements is true regarding their speeds v_1 and v_2?
a) $v_1 = v_2$ b) $v_1 > v_2$ c) $v_1 < v_2$ d) $\frac{v_1}{r_1} = \frac{v_2}{v_2}$

13. A satellite is seen after each 8 h over the equator at a place on the earth when its sense of rotation is opposite to the earth. The time interval after which it can be seen at the same place when the sense of rotation of the earth and the satellite is the same will be
a) 8 h b) 12 h c) 24 h d) 6 h

14. A body starts from rest from a point distant r_0 from the centre of the earth. It reaches the surface of the earth whose radius is R. The velocity acquired by the body is
a) $2GM\sqrt{\frac{1}{R}-\frac{1}{r_0}}$ b) $\sqrt{2GM\left(\frac{1}{R}-\frac{1}{r_0}\right)}$ c) $GM\sqrt{\frac{1}{R}-\frac{1}{r_0}}$ d) $\sqrt{GM\left(\frac{1}{R}-\frac{1}{R_0}\right)}$

15. A rocket is launched vertically from the surface of earth with an initial velocity v. How far above the surface of earth it will go? Neglect the air resistance
a) $R\left(\frac{2gR}{v^2}-1\right)^{-1/2}$ b) $R\left(\frac{2gR}{v^2}-1\right)$ c) $R\left(\frac{2gR}{v^2}-1\right)^{-1}$ d) $R\left(\frac{2gR}{v^2}-1\right)^{2}$

16. In problem 86, what is the gravitational field strength at the location of m?
a) $\frac{GM}{l^2}$ b) $\frac{4GM}{l^2}$ c) $\frac{4GM}{3l^2}$ d) $\frac{GM}{3l^2}$

17. A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work done against the gravitational force between them to take the particle far away from the sphere
a) 13.34×10^{-10}J b) 3.33×10^{-10}J c) 6.67×10^{-9}J d) 6.67×10^{-10}J

18. A body is fired with a velocity of magnitude $\sqrt{gR} < V < \sqrt{2gR}$ at an angle of 30° with the radius vector of the earth. If at the highest point, the speed of the body is $V/4$, the maximum height attained by the body is equal to
a) $\frac{V^2}{8g}$ b) R c) $\sqrt{2}R$ d) None of these

19. Two particles of equal mass go around a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is
a) $v = \frac{1}{2R}\sqrt{\left(\frac{1}{GM}\right)}$ b) $v = \sqrt{\left(\frac{GM}{2R}\right)}$ c) $v = \frac{1}{2}\sqrt{\left(\frac{GM}{R}\right)}$ d) $v = \sqrt{\left(\frac{4GM}{R}\right)}$

20. Imagine that you are in a spacecraft orbiting around the earth in a circle of radius 7000 km (from the centre of the earth). If you decrease the magnitude of mechanical energy of the spacecraft -earth system by 10% by firing the rockets, then what is the greatest height you can take your spacecraft above the surface of the earth? [$R = 6400$ km]
a) 6400 km b) 540 km c) 2140 km d) 3000 km

21. A tunnel is dug along the diameter of the earth (radius R and mass M). There is a particle of mass 'm' at the centre of the tunnel. The minimum velocity given to the particle so that it just reaches to the surface of the earth is

a) $\sqrt{\frac{GM}{R}}$ b) $\sqrt{\frac{GM}{2R}}$

c) $\sqrt{\frac{2GM}{R}}$ d) It will reach with the help of a negligible velocity

22. Three uniform spheres, each having mass m and radius r, are kept in such a way that each touches the other two. The magnitude of the gravitational force on any sphere due to the other two is

a) $\frac{Gm^2}{r^2}$ b) $\frac{Gm^2}{4r^2}$ c) $\sqrt{2}\frac{Gm^2}{4r^2}$ d) $\sqrt{3}\frac{Gm^2}{4r^2}$

23. Two equal masses each m are hung from a balance whose scale pans differ in vertical height by 'h'. The error in weighing in terms of density of the earth ρ is

a) $\pi G\rho mh$ b) $\frac{1}{3}\pi G\rho mh$ c) $\frac{8}{3}\pi G\rho mh$ d) $\frac{4}{3}\pi G\rho mh$

24. What should be the angular velocity of rotation of the earth about its own axis so that the weight of a body at the equator reduces to 3/5 or its present value? (Take R as the radius of the earth)

a) $\sqrt{\frac{g}{3R}}$ b) $\sqrt{\frac{2g}{3R}}$ c) $\sqrt{\frac{2g}{5R}}$ d) $\sqrt{\frac{2g}{7R}}$

25. Consider two solid uniform spherical objects of the same density ρ. One has radius R and te other has radius $2R$. They are in outer space where the gravitational fields from other objects are negligible. If they are arranged with their surface touching, what is the contact force between the objects due to their traditional attraction?

a) $G\pi^2R^4$ b) $\frac{128}{81}G\pi^2R^4\rho^2$ c) $\frac{128}{81}G\pi^2$ d) $\frac{128}{87}\pi^2R^2G$

26. A satellite of mass m is in an elliptical orbit around the earth. The speed of the satellite at its nearest position is $(6GM_e)/(5r)$ where r is the perigee (nearest point) distance from the centre of the earth. It is desired to transfer the satellite to the circular orbit of radius equal to its apogee (farthest point) distance from the centre of the earth. The change in orbital speed required for this purpose is

a) $0.35\sqrt{\frac{GM_e}{r}}$ b) $0.085\sqrt{\frac{GM_e}{r}}$ c) $\sqrt{\frac{2GM_e}{r}}$ d) Zero

27. Two satellites of the same mass are launched in the same orbit around the earth so as to rotate opposite to each other. If they collide inelastically and stick together as wreckage, the total energy of the system just after collision is

a) $-\frac{2GMm}{r}$ b) $-\frac{GMm}{r}$ c) $\frac{GMm}{2r}$ d) $\frac{GMm}{4r}$

28. Te radius of the earth is about 6400 km and that of Mars is about 3200 km. The mass of the earth is about 10 times the mass of Mars. An object weights 200 N on the surface of the earth. Its weight on the surface of mars would be

a) 6 N b) 20 N c) 40 N d) 80 N

29. Two astronauts have deserted their spaceship in a region of space far from the gravitational attraction of any other body. Each has a mass of 100 kg and they are 100 m apart. They are initially at rest relative to one another. How long will it be before the gravitational attraction brings them 1 cm closer together?

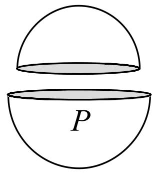
a) 2.52 days b) 1.41 days c) 0.70 days d) 1.41 s

30. The distances from the centre of the earth, where the weight of a body is zero and one-fourth that of the weight of the body on the surface of the earth are (assume R is the radius of the earth)

a) $0, \frac{R}{4}$ b) $0, \frac{3R}{4}$ c) $\frac{R}{4}, 0$ d) $\frac{3R}{4}, 0$

31. The mass of the earth is 81 times the mass of the Moon and the distance between the earth and the Moon is 60 times the radius of the earth. If R is the radius of the earth, then the distance between the Moon and the point on the line joining the Moon and the earth where the gravitational force becomes zero is
a) $30R$ b) $15R$ c) $6R$ d) $5R$

32. A spherical shell is cut into two pieces along a chord as shown in the figure. P is a point on the plane of the chord. The gravitational field at P due to the upper part is I_1 and due to the lower part is I_2. What is the relation between them?



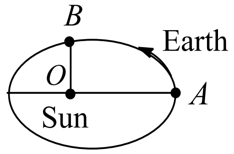
a) $I_1 > I_2$ b) $I_1 < I_2$ c) $I_1 = I_2$ d) No definite relation

33. Two bodies with masses M_1, and M_2 are initially at rest and a distance R apart. Then they move directly towards one another under the influence of their mutual gravitational attraction. What is the ratio of the distances travelled by $M_1$to the distance travelled by M_2?
a) $\frac{M_1}{M_2}$ b) $\frac{M_2}{M_1}$ c) 1 d) $\frac{1}{2}$

34. A body of mass m rises to a height $h = R/5$ from the earth's surface where R is earth's radius. If g is acceleration due to gravity at the earth's surface, the increase in potential energy is
a) mgh b) $\frac{4}{5}mgh$ c) $\frac{5}{6}mgh$ d) $\frac{6}{7}mgh$

35. In problem 86, what is the gravitational potential energy of the mass m?
a) $-\frac{2}{\sqrt{3}}\frac{GMm}{l}(1-2\sqrt{3})$ b) $-\frac{2}{\sqrt{3}}\frac{GMm}{l}(1+2\sqrt{3})$ c) $-\frac{\sqrt{3}}{2}\frac{GMm}{l}(1-2\sqrt{3})$ d) $-\frac{\sqrt{3}}{2}\frac{GMm}{l}(1+2\sqrt{3})$

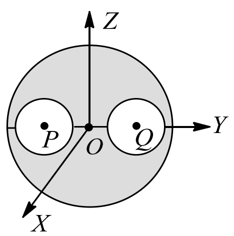
36. The earth moves around the Sun in an elliptical orbit as shown in figure. The ratio $OA/OB = x$. The ratio of the speed of the earth at B to that at A is nearly



a) $\sqrt{x}$ b) x c) $x\sqrt{x}$ d) x^2

37. A space station is set up in space at a distance equal to the earth's radius from the surface of the earth. Suppose a satellite can be launched from the space station. Let v_1 and v_2 be the escape velocities of the satellite on the earth's surface and space station, respectively. Then
a) $v_2 = v_1$ b) $v_2 < v_1$
c) $v_2 > v_1$ d) (a), (b) and (c) are valid depending on the mass of satellite

38. A solid sphere of uniform density and mass M has radius 4 m. Its centre is at the origin of the coordinate system. Two spheres of radii 1 m are taken out so that their centres are at $P(0, -2, 0)$ and $Q(0, 2, 0)$ respectively. This leaves two spherical cavities. What is the gravitational field at the origin of the coordinate axes?



a) $\frac{31GM}{1024}$ b) $\frac{Gm}{1024}$ c) $31\ GM$ d) Zero

39. The value of 'g' at a certain height h above the free surface of the earth is $x/4$ where x is the value of 'g' at

the surface of the earth. The height h is

a) R b) $2R$ c) $3R$ d) $4R$

40. The radius of a planet is R. A satellite revolves around it in a circle of radius r with angular velocity ω_0. The acceleration due to the gravity on planet's surface is

a) $\frac{r^2\omega_0}{R}$ b) $\frac{r^3\omega_0^3}{R^2}$ c) $\frac{r^3\omega_0^2}{R}$ d) $\frac{r^3\omega_0^2}{R^2}$

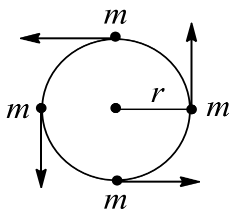
41. The gravitational potential due to earth at infinite distance from it is zero. Let the gravitational potential at a point P be $-5\ \mathrm{J\ kg^{-1}}$. Suppose, we arbitarily assume the gravitational potential at infinity to be $+10\ \mathrm{J\ kg^{-1}}$, then the gravitational potential at P will be

a) $-5\ \mathrm{J\ kg^{-1}}$ b) $+5\ \mathrm{J\ kg^{-1}}$ c) $-15\ \mathrm{J\ kg^{-1}}$ d) $+15\ \mathrm{J\ kg^{-1}}$

42. Two satellite A and B of masses m_1 and $m_2 (m_1 = 2\,m_2)$ are moving in circular orbits of radii r_1 and $r_2 (r_1 = 4r_2)$, respectively, around the earth. If their periods are T_A and T_B, then the ratio T_A/T_B is

a) 4 b) 16 c) 2 d) 8

43. Four similar particles of mass m are orbiting in a circle of radius r in the same angular direction because of their mutual gravitational attractive force. Velocity of a particle is given by

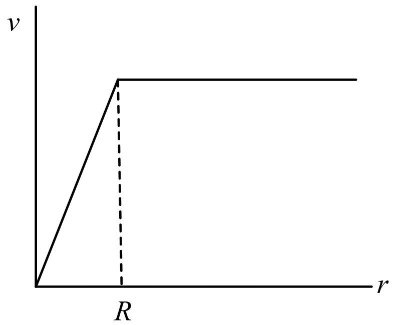


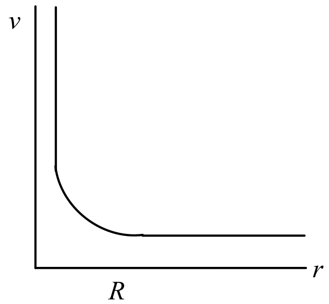
a) $\left[\frac{Gm}{r}\left(\frac{1+2\sqrt{2}}{4}\right)\right]^{\frac{1}{2}}$ b) $\sqrt{\frac{Gm}{r}}$ c) $\sqrt{\frac{Gm}{r}(1+2\sqrt{2})}$ d) $\left[\frac{1}{2}\frac{Gm}{r}\left(\frac{1+2\sqrt{2}}{2}\right)\right]^{\frac{1}{2}}$

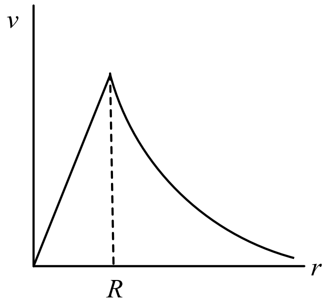
44. A spherically symmetric gravitational system of particles has a mass density

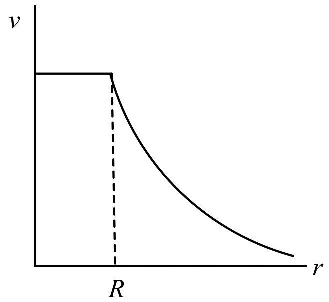
$\rho = \begin{cases} \rho_0 \text{ for } r \le R \\ 0 \text{ for } r > R \end{cases}$

where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed v as a function of distance $r (0 < r < \infty)$ from the centre of the system is represented by

a) 

b) 

c) 

d) 

45. If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the earth is

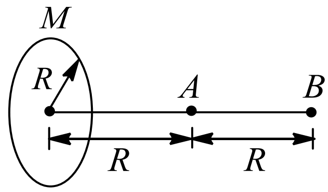
a) $\frac{1}{2}mgR$ b) $2\,mgR$ c) mgR d) $\frac{1}{4}mgR$

46. A satellite of mass m is orbitiing around the earth at a height h above the surface of the earth. Mass of the

earth is M and its radius is R. The angular momentum of the satellite is independent of

a) m b) M c) h d) None of these

47. A ring having non-uniform distribution of mass M and radius R is being considered. A point mass m_0 is taken slowly towards the ring. In doing so, work done by the external force against the gravitational force exerted by ring is



a) $\frac{GMm_0}{\sqrt{2}R}$

b) $\frac{GMm_0}{R}\left[\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{5}}\right]$

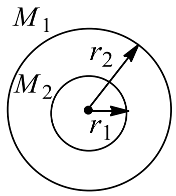
c) $\frac{GMm_0}{R}\left[\frac{1}{\sqrt{5}}-\frac{1}{\sqrt{2}}\right]$

d) It is not possible to find the required work as the nature of distribution of mass is not known

48. Four particles, each of mass M, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is

a) $\frac{GM}{R}$ b) $\sqrt{2\sqrt{2}\frac{GM}{R}}$ c) $\sqrt{\frac{GM}{R}(2\sqrt{2}+1)}$ d) $\sqrt{\frac{GM}{R}\left(\frac{2\sqrt{2}+1}{4}\right)}$

49. Two concentric shells of masses M_1 and M_2 are having radii r_1 and r_2. Which of the following is the correct expression for the gravitational filed on a mass m?



a) $F=\frac{G(M_1+M_2)}{r^2}$, for $r<r_1$ b) $F=\frac{G(M_1+M_2)}{r^2}$, for $r<r_2$

c) $F=\frac{GM_2}{r^2}$, for $r_1<r<r_2$ d) $F=\frac{GM_1}{r^2}$, for $r_1<r<r_2$

50. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}$ g, where g is the gravitational acceleration on the surface of earth. The average mass density of the planet is $\frac{2}{3}$ times that of the earth. If the escape speed on the surface of the earth is taken on be 11 kms^{-1}, the escape speed on the surface of the planet in kms^{-1} will be

a) 5 b) 7 c) 3 d) 11

51. If a man at the equator would weigh (3/5)th of his weight, the angular speed of the earth is

a) $\sqrt{\frac{2}{5}\frac{g}{R}}$ b) $\sqrt{\frac{g}{R}}$ c) $\sqrt{\frac{R}{g}}$ d) $\sqrt{\frac{2}{5}\frac{R}{g}}$

52. A projectile is fired vertically upwards from the surface of the earth with a velocity Kv_e, where v_e is the escape velocity and $k<1$. If R is the radius of the earth, the maximum height to which it will rise, measured from the centre of the earth, will be (neglect air resistance)

a) $\frac{1-k^2}{R}$ b) $\frac{R}{1-k^2}$ c) $R(1-k)^2$ d) $\frac{R}{1+k^2}$

53. A satellite is orbiting around the earth in a circular orbit of radius r. A particle of mass m is projected from the satellite in a forward direction with a velocity $v=2/3$ times the orbital velocity (this velocity is given w.r.t. earth). During subsequent motion of the particle, its minimum distance from the centre of earth is

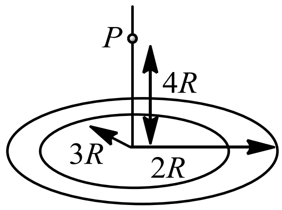
a) $\frac{r}{2}$ b) r c) $\frac{2r}{3}$ d) $\frac{4r}{5}$

54. The escape velocity for a body projected vertically upwards from the surface of the earth is 11.2 km s^{-1}. If the body is projected in a direction making an angle $45°$ with the vertical, the escape velocity will be
a) $\frac{11.2}{\sqrt{2}} \text{ km s}^{-1}$ b) $11.2 \times \sqrt{2} \text{ km s}^{-1}$ c) $11.2 \times 2 \text{ km s}^{-1}$ d) 11.2 km s^{-1}

55. If the distance between the earth and the Sun were half its present value, the number of days in a year would have been
a) 64.5 b) 129 c) 182.5 d) 730

56. If an artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of the escape velocity from the earth, the height of the satellite above the surface of the earth is
a) $2R$ b) $\frac{R}{2}$ c) R d) $\frac{R}{4}$

57. A thin uniform annular disc (see figure) of mass M has outer radius $4R$ and inner radius $3R$. The work



required to take a unit mass from point P on its axis to infinity is
a) $\frac{2GM}{7R}(4\sqrt{2}-5)$ b) $-\frac{2GM}{7R}(4\sqrt{2}-5)$ c) $\frac{GM}{4R}$ d) $\frac{2GM}{5R}(\sqrt{2}-1)$

58. The gravitational potential energy of an isolated system of three particles, each of mass m, at the three corners of an equilateral triangle of side l is
a) $-\frac{Gm^2}{l}$ b) $-\frac{Gm^2}{2l}$ c) $-\frac{2Gm^2}{l}$ d) $-\frac{3Gm^2}{l}$

59. If the radius of the earth decreases by 10%, the mass remaining unchanged, what will happen to the acceleration due to gravity?
a) Decreases by 19% b) Increases by 19%
c) Decreases by more than 19% d) Increases by more than 19%

60. A body is thrown from the surface of the earth with velocity $(gR_e)/2$, where R_e is the radius of the earth at some angle from the vertical. If the maximum height reached by the body is $R_e/4$, then the angle of projection with the vertical is
a) $\sin^{-1}\left(\frac{\sqrt{5}}{4}\right)$ b) $\cos^{-1}\left(\frac{\sqrt{5}}{4}\right)$ c) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ d) None of these

61. If g is same at a height h and at a depth d, then
a) $R = 2d$ b) $d = 2h$ c) $h = d$ d) None

62. Suppose the gravitational force varies inversely as the n^{th} power of the distance. Then, the time period of a planet in a circular orbit of radius R around the Sun will be proportional to
a) R^n b) $R^{\frac{(n+1)}{2}}$ c) $R^{\frac{(n-1)}{2}}$ d) R^{-n}

63. Two satellites A and B of the same mass are revolving around the earth in the concentric circular orbits such that the distance of satellite B from the centre of the earth is thrice as compared to the distance of the satellite A from the centre of the earth. The ratio of the centripetal force acting on B as compared to that on A is
a) $\frac{1}{3}$ b) 3 c) $\frac{1}{9}$ d) $\frac{1}{\sqrt{3}}$

64. A planet is revolving in an elliptical orbit around the Sun. Its closest distance from the Sun is $r_{\min}$ and the farthest distance is $r_{\max}$. If the velocity of the planet at the distance of the closest approach is v_1 and that at the farthest distance from the Sun is v_2, then $\frac{v_1}{v_2}$
a) $\frac{r_{\max}}{r_{\min}}$ b) $\frac{r_{\min}}{r_{\max}}$ c) $\frac{r_{\min}+r_{\max}}{r_{\max}-r_{\min}}$ d) None

65. A simple pendulum has a time period T_1 when on the earth's surface and T_2 when taken to a height R above the earth's surface, where R is the radius of the earth. The value of T_2/T_1 is

a) 1 b) $\sqrt{2}$ c) 4 d) 2

66. A diameter tunnel is dug across the earth. A ball is dropped into the tunnel from one side. The velocity of the ball when it reaches the centre of the earth is
[Given: gravitational potential at the centre of earth $= -3/(2\,GM/R)$]

a) $\sqrt{R}$ b) $\sqrt{gR}$ c) $\sqrt{2.5\text{ g}R}$ d) $\sqrt{7.1\text{ g}R}$

67. In order to shift a body of mass m from a circular orbit of radius $3R$ to a higher orbit of rdaius $5R$ around the earth, the work done is

a) $\frac{3GMm}{5R}$ b) $\frac{GMm}{2R}$ c) $\frac{2}{15}\frac{GMm}{R}$ d) $\frac{GMm}{5R}$

68. In problem 83, what is the gravitational potential at any point on the circle $x^2 + z^2 = 6$?

a) $-\frac{GM}{6}$ b) $-\frac{GM}{64\sqrt{10}}$ c) $-\frac{GM}{2}\left[\frac{1}{3}-\frac{1}{32\sqrt{10}}\right]$ d) $-\frac{GM}{2}\left[\frac{1}{3}+\frac{1}{32\sqrt{10}}\right]$

69. A cavity of radius $R/2$ is made inside a solid sphere of radius R. The centre of the cavity is located at a distance $R/2$ from the centre of the sphere. The gravitational force on a particle of mass 'm' at a distance $R/2$ from the centre of the sphere on the line joining both the centres of the sphere and the cavity is (opposite to the centre of the cavity)
[Here $\text{g} = (GM)/R^2$, where M is the mass of the sphere]

a) $\frac{m\text{g}}{2}$ b) $\frac{3m\text{g}}{8}$ c) $\frac{m\text{g}}{16}$ d) None of these

70. In problem 80, if x is the distance from the common centre, then what is the gravitational potential at a point for which $r < x < R$?

a) $-G\left[\frac{M}{x}+\frac{m}{r}\right]$ b) $-G\left[\frac{M}{x}-\frac{m}{r}\right]$ c) $-G\left[\frac{M}{R}+\frac{m}{x}\right]$ d) $-G\left[\frac{M}{R}-\frac{m}{x}\right]$

71. In the solar system, the Sun is in the focus of the system for Sun-earth binding system. Then the binding energy for the system will be [given that radius of the earth's orbit round the Sun is 1.5×10^{11}m and mass of the earth $= 6 \times 10^{24}$ kg]

a) 2.7×10^{33} J b) 5.4×10^{33} J c) 2.7×10^{30} J d) 5.4×10^{30} J

72. Two particles of equal mass go around a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is

a) $v = \frac{1}{2R}\sqrt{\left(\frac{1}{Gm}\right)}$ b) $v = \sqrt{\left(\frac{Gm}{2R}\right)}$ c) $v = \frac{1}{2}\sqrt{\left(\frac{Gm}{R}\right)}$ d) $v = \sqrt{\left(\frac{4Gm}{R}\right)}$

73. What is the mass of the planet that has a satellite whose time period is T and orbital radius is r?

a) $\frac{4\pi^2 r^3}{GT^2}$ b) $\frac{4\pi^2 r^3}{GT^2}$ c) $\frac{4\pi^2 r^3}{GT^3}$ d) $\frac{4\pi^2 T}{GT^2}$

74. The escape velocity corresponding to a planet of mass M and radius R is 50 km s^{-1}. If the planet's mass and radius were $4M$ and R, respectively, then the corresponding escape velocity would be

a) 100 km s^{-1} b) 50 km s^{-1} c) 200 km s^{-1} d) 25 km s^{-1}

75. A point P is on the axis of a fixed ring of mass M and radius R, at a distance $2R$ from the centre O. A small particle starts from P and reaches O under the gravitational attraction only. Its speed at O will be

a) Zero b) $\sqrt{\frac{2GM}{R}}$ c) $\sqrt{\frac{2GM}{R}(\sqrt{5}-1)}$ d) $\sqrt{\frac{2GM}{R}\left(1-\frac{1}{\sqrt{5}}\right)}$

76. Masses of 1 kg each are placed 1 m, 2 m, 4 m, 8 m,... from a point P. The gravitational field intensity at P due to these masses is

a) G b) $-G$ c) $4G$ d) $4G/3$

77. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth

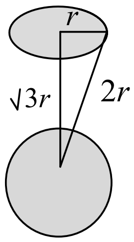
a) The acceleration of S is always directed towards the centre of the earth

b) The angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant

c) The total mechanical energy of S varies periodically with time

d) The linear momentum of S remains constant in magnitude

78. A uniform ring of mass m and radius r is placed directly above a uniform sphere of mass M and of equal radius. The centre of the ring is directly above the centre of the sphere at a distance $r\sqrt{3}$ as shown in the figure. The gravitational force exerted by the sphere on the ring will be



a) $\frac{GMm}{8r^2}$ b) $\frac{GMm}{4r^2}$ c) $\sqrt{3}\frac{GMm}{8r^2}$ d) $\frac{GMm}{8r^3\sqrt{3}}$

79. The minimum energy required to launch a m kg satellite from the earth's surface in a circular orbit at an altitude $2R$, where R is the radius of earth is

a) $\frac{5}{3}mgR$ b) $\frac{4}{3}mgR$ c) $\frac{5}{6}mgR$ d) $\frac{5}{4}mgR$

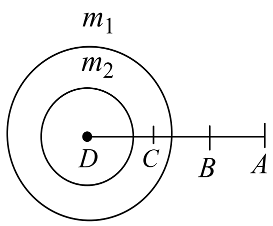
80. The masses and radii of the earth and the Moon are M_1, R_1 and M_2, R_2, respectively. Their centres are at distance d apart. The minimum speed with which a particle of mass m should be projected from a point midway the two centres so as to escape to infinity is

a) $\sqrt{\frac{2G(M_1+M_2)}{d}}$ b) $\sqrt{\frac{4G(M_1+M_2)}{d}}$ c) $\sqrt{\frac{4GM_1M_2}{d}}$ d) $\sqrt{\frac{G(M_1+M_2)}{d}}$

81. Two concentric shells have masses M and m and their radii are R and r, respectively, where $R > r$. What is the gravitational potential at their common centre?

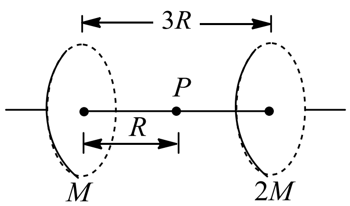
a) $-\frac{GM}{R}$ b) $-\frac{GM}{r}$ c) $-G\left[\frac{M}{R}-\frac{m}{r}\right]$ d) $-G\left[\frac{M}{R}+\frac{m}{r}\right]$

82. The following figure shows two shells of masses m_1 and m_2. The shells are concentric. At which point, a particle of mass m shall experience zero force?



a) A b) B c) C d) D

83. Two rings having masses M and $2M$, respectively, having the same radius are placed coaxially as shown in the figure



If the mass distribution on both the rings is non-uniform, then the gravitational potential at point P is

a) $-\frac{GM}{R}\left[\frac{1}{\sqrt{2}}+\frac{2}{\sqrt{5}}\right]$ b) $-\frac{GM}{R}\left[1+\frac{2}{2}\right]$

c) Zero d) Cannot be determined from the given information

84. The gravitational force exerted by the Sun on the Moon is about twice as great as the gravitational force

exerted by the earth on the Moon, but still Moon is not escaping from the gravitational influence of the earth. Mark the option which correctly explains the above system

a) Escape speed is independent of the direction in which it is projected
b) The rotational effect of the earth plays a role in computation of escape speed, however small it may be
c) A body thrown in the eastward direction has less escape speed
d) None of the above

85. A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is

a) gx b) $\frac{gR}{R-x}$ c) $\frac{gR^2}{R+x}$ d) $\left(\frac{gR^2}{R+x}\right)^{\frac{1}{2}}$

86. In problem 83, what is the gravitational field at the centre of the cavities?

a) $\frac{31GM}{1024}$ b) $\frac{Gm}{1024}$ c) $31GM$ d) GM

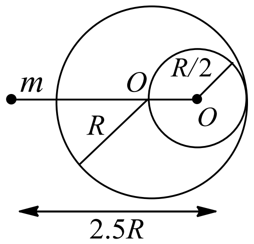
87. If the mass of a planet is 10% less than that of the earth and the radius is 20% greater than that of the earth, the acceleration due to gravity on the planet will be

a) 5/8 times that on the surface of the earth
b) 3/4 times that on the surface of the earth
c) 1/2 times that on the surface of the earth
d) 9/10 times that on the surface of the earth

88. A satellite is moving with a constant speed v in a circular orbit about the earth. An object of mass m is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is

a) $\frac{1}{2}mv^2$ b) mv^2 c) $\frac{3}{2}mv^2$ d) $2mv^2$

89. A solid sphere of radius $R/2$ is cut out of a solid sphere of radius R such that the spherical cavity so formed touches the surface on one side and the centre of the sphere on the other side, as shown. The initial mass of the soild sphere was M. If a particle of mass m is placed at a distnace 2.5 R from the centre of the cavity, then what is the gravitational attraction on the mass m?



a) $\frac{GMm}{R^2}$ b) $\frac{GMm}{2R^2}$ c) $\frac{GMm}{8R^2}$ d) $\frac{23}{100}\frac{GMm}{R^2}$

90. The gravitational potential of two homogenous spherical shells A and B of same surface density at their respective centres are in the ratio 3:4. If the two shells coalesce into a single one such that surface charge density remains the same, then the ratio of potential at an internal point of the new shell to shell A is equal to

a) 3:2 b) 4:3 c) 5:3 d) 5:6

91. A satellite moves around the earth in a circular orbit with speed v. If m is the mass of the satellite, its total energy is

a) $-\frac{1}{2}mv^2$ b) $\frac{1}{2}mv^2$ c) $\frac{3}{2}mv^2$ d) $\frac{1}{4}mv^2$

92. If R is the radius of the earth and g the acceleration due to gravity on the earth's surface, the mean density of the earth is

a) $\frac{4\pi G}{3gR}$ b) $\frac{3\pi R}{4gG}$ c) $\frac{3g}{4\pi RG}$ d) $\frac{\pi R}{12G}$

93. The value of g (acceleration due to gravity) at earth's surface is 10 ms^{-2}. Its value in m s^{-2} at the centre of the earth which is assumed to be a sphere of radius R meter and uniform mass density is

a) 5 b) $\frac{10}{R}$ c) $\frac{10}{2R}$ d) Zero

94. A space vehicle approaching a planet has a speed v, when it is very far from the planet. At that moment tangent of its tranjectory would miss the centre of the planet by distance R. If the planet has mass M and radius r, what is the smallest value of R in order that the resulting orbit of the space vehicle will just miss the surface of the planet?

a) $\frac{r}{v}\left[v^2+\frac{2GM}{r}\right]^{\frac{1}{2}}$ b) $vr\left[1+\frac{2GM}{r}\right]$ c) $\frac{r}{v}\left[v^2+\frac{2GM}{r}\right]$ d) $\frac{2GMv}{r}$

95. Imagine a light planet revolving around a very massive star in a circular orbit of radius r with a period of revolution T. If the gravitational force of attraction between the planet and the star is proportional to $r^{5/2}$, then the square of the time period will be proportional to

a) r^3 b) r^2 c) $r^{2.5}$ d) $r^{3.5}$

96. A space ship is launched into a circular orbit close to the surface of the earth. The additional velocity now imparted to the space-ship in the orbit to overcome the gravitational pull is

a) 11.2 km s^{-1} b) 8 km s^{-1} c) 3.2 km s^{-1} d) 1.414×8 km s^{-1}

97. A man weighs 80 kg on the surface of earth of radius R. At what height above the surface of earth his weight will be 40 kg?

a) $\frac{R}{2}$ b) $\sqrt{2}\,R$ c) $(\sqrt{2}-1)R$ d) $(\sqrt{2}+1)R$

98. A planet is revolving around the Sun in an elliptical orbit. Its closest distance from the Sun is r and farthest distance is R. If the orbital velocity of the planet closest to the Sun is v, then what is the velocity at the farthest point?

a) $\frac{vr}{R}$ b) $\frac{vR}{r}$ c) $v\sqrt{\frac{r}{R}}$ d) $v\sqrt{\frac{R}{r}}$

99. A geostationary satellite orbits around the earth in a circular orbit of radius 36000 km. Then, the time period of a spy satellite orbiting a few 100 km above the earth's surface ($R_{\text{earth}} = 6400$ km) will approximately be

a) $\frac{1}{2}h$ b) $1h$ c) $2\,h$ d) $4h$

100. A projectile is fired vertically upwards from the surface of the earth with a velocity kv_e where v_e is the escape velocity and $k<1$. If R is the radius of the earth, the maximum height to which it will rise measured from the centre of earth will be (neglect air resistance)

a) $\frac{1-k^2}{R}$ b) $\frac{R}{1-k^2}$ c) $R(1-k^2)$ d) $\frac{R}{1+k^2}$

101. If the radius of the earth were to shrink by one per cent, its mass remaining the same, acceleration due to gravity on the earth's surface would

a) Decrease b) Remain unchanged c) Increase d) Be zero

102. The orbital velocity of an artificial satellite in a circular orbit just above the earth's surface is v. For a satellite orbiting at an altitude of half of the earth's radius, the orbital velocity is

a) $\left(\frac{3}{2}\right)v$ b) $\sqrt{\left(\frac{3}{2}\right)}v$ c) $\sqrt{\left(\frac{2}{3}\right)}v$ d) $\left(\frac{2}{3}\right)v$

103. A body is released from a point of distance R' from the centre of earth. Its velocity at the time of striking the earth will be $(R' > R_e)$

a) $\sqrt{2gR_e}$ b) $\sqrt{R_e g}$ c) $\sqrt{2g(R'-R_e)}$ d) $\sqrt{2gR_e\left(1-\frac{R_e}{R'}\right)}$

104. If three particles, each of mass M, are placed at the three corners of an equilateral triangle of side a, the forces exerted by this system on another particles of mass M placed (i) at the midpoint of a side and (ii) at the centre of the triangle are, respectively,

a) $0, \frac{4GM^2}{3a^2}$ b) $\frac{4GM^2}{3a^2}, 0$ c) $\frac{3GM^2}{a^2}, \frac{GM^2}{a^2}$ d) $0, 0$

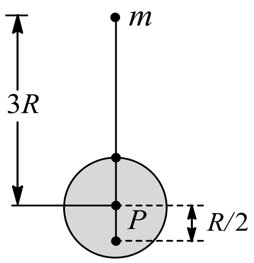
105. An artificial satellite of the earth is launched in circular orbit in the equatorial plane of the earth and the satellite is moving from west to east. With respect to a person on the equator, the satellite is completing one round trip in 24 h. Mass of the earth is $M = 6 \times 10^{24}$ kg. For this situation, the orbital radius of the satellite is

a) 2.66×10^4 km b) 6400 km c) 36,000 km d) 29, 600 km

106. How many hours would make a day if the earth were rotating at such a high speed that the weight of a body on the equator were zero

a) 6.2 h b) 1.4 h c) 28 h d) 5.6 h

107. A point mass m is released from rest at a distance of $3R$ from the centre of a thin-walled hollow sphere of rdaius R and mass M as show. The hollow sphere is fixed in position and the only force on the point mass is the gravitational attraction of the hollow sphere. There is a very small hole in the hollow sphere through which the point mass falls as shown. The velocity of a point mass when it passes through point P at a distance $R/2$ from the centre of the sphere is

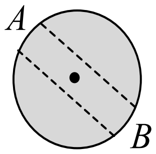


a) $\sqrt{\dfrac{2GM}{3R}}$ b) $\sqrt{\dfrac{5GM}{3R}}$ c) $\sqrt{\dfrac{25GM}{24R}}$ d) None of these

108. A system of binary stars of masses m_A and m_B are moving in circular orbits of radii r_A and r_B, respectively. If T_A and T_B are the time periods of masses m_A and m_B respectively, then

a) $\dfrac{T_A}{T_B} = \left(\dfrac{r_A}{r_B}\right)^{\frac{3}{2}}$ b) $T_A > T_B (\text{if } r_A > r_B)$ c) $T_A > T_B (\text{if } m_A > m_B)$ d) $T_A = T_B$

109. A tunnel has been dug into a solid sphere of non-uniform mass density as shown in the figure
As one moves from A to B, the magnitude of gravitational field intensity



a) Will continuously decrease
b) Will decrease up to the centre of the sphere and then increase
c) May increase or decrease
d) Will continuously increase

110. The distance of two planets from the Sun are 10^{13} and 10^{12}m, respectively. The ratio of time periods of these two planets is

a) $\dfrac{1}{\sqrt{10}}$ b) 100 c) $\dfrac{10}{\sqrt{10}}$ d) $\sqrt{10}$

111. In problem 81, what is the gravitational intensity at a point for which $x < r$?

a) $\dfrac{Gm}{r^2}$ b) $\dfrac{Gm}{x^2}$ c) $\dfrac{Gm}{R^2}$ d) Zero

112. The percentage change in the acceleration of the earth towards the Sun from a total eclipse of the Sun to the point where the Moon is on a side of earth directly opposite to the Sun is

a) $\dfrac{M_s}{M_m}\dfrac{r_2}{r_1} \times 100$ b) $\dfrac{M_s}{M_m}\left(\dfrac{r_2}{r_1}\right)^2 \times 100$ c) $2\left(\dfrac{r_1}{r_2}\right)^2 \dfrac{M_m}{M_s} \times 100$ d) $\left(\dfrac{r_1}{r_2}\right)^2 \dfrac{M_m}{M_s} \times 100$

113. The two planets with radii R_1, R_2 have densities ρ_1, ρ_2, and atmospheric pressure p_1 and p_2, respectively. Therefore, the ratio of masses of their atmospheres, neglecting variation of g and ρ within the limits of atmosphere, is

a) $\frac{p_1 R_2 \rho_1}{p_2 R_1 \rho_2}$ b) $\frac{p_1 R_2 \rho_2}{p_2 R_1 \rho_1}$ c) $\frac{p_1 R_1 \rho_1}{p_2 R_2 \rho_2}$ d) $\frac{p_1 R_1 \rho_2}{p_2 R_2 \rho_1}$

114. If g is the acceleration due to gravity on the earth's surface, the change in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the earth is

a) $\frac{mgR}{2}$ b) $2mgR$ c) mgR d) $-mgR$

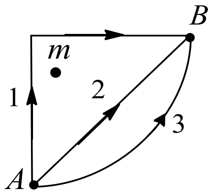
115. The value of g at a particular point is 10 m s^{-2}. Suppose the earth shrinks uniformly to half of its present size without losing any mass. The value of g at the same point (assuming that the distance of the point from the centre of the earth does not change) will now be

a) 5 ms^{-2} b) 10 ms^{-2} c) 3 m s^{-2} d) 20 m s^{-2}

116. If g is acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of earth to a height equal to the radius R of the earth is

a) $\frac{1}{2} mgR$ b) $2mgR$ c) mgR d) $\frac{1}{4} mgR$

117. If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3, respectively, (as shown in the figure) in the gravitational field of a point mass m, find the correct relation between W_1, W_2 and W_3



a) $W_1 > W_2 > W_3$ b) $W_1 = W_2 = W_3$ c) $W_1 < W_2 < W_3$ d) $W_2 > W_1 > W_3$

118. The maximum vertical distance through which a fully dressed astronaut can jump on the earth is 0.5 m. If mean density of the Moon is two-third that of the earth and radius is one quarter that of the earth, the maximum vertical distance through which he can jump on the Moon and the ratio of the time of duration of the jump on the Moon to hold on the earth are

a) 3 m, 6:1 b) 6 m, 3:1 c) 3m, 1:6 d) 6 m, 1:6

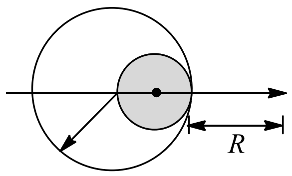
119. A satellite of mass m is revolving around the earth at height R (radius of the earth) from the earth's surface. Its potential energy will be

a) mgR b) 0.67 mgR c) $-\frac{mgR}{2}$ d) 0.33 mgR

120. Three particles, each of mass M, are placed at the three corners of an equilateral triangle of side l. What is the force due to this system of particles on another particle of mass m placed at the midpoint of any side?

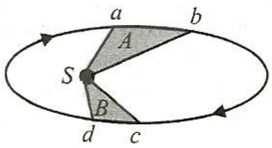
a) $\frac{3GMm}{4l^2}$ b) $\frac{4GMm}{3l^2}$ c) $\frac{GMm}{4l^2}$ d) $\frac{4GMm}{l^2}$

121. A solid sphere of uniform density and radius R applies a gravitational force of attraction equal to F_1 on a particle placed at a distance $2R$ from the centre of the sphere. A spherical cavity of radius $R/2$ is now made in the sphere as shown in the figure. The sphere with the cavity now applies a gravitational force F_2 on the same particle. The ratio F_1/F_2 is



a) $\frac{1}{2}$ b) $\frac{3}{4}$ c) $\frac{7}{8}$ d) $\frac{9}{7}$

122. The given figure shows the motion of a planet around the Sun S in an elliptical orbit with the Sun at the focus. The shaded areas A and B are also shown in the figure which can be assumed to be equal. If t_1 and t_2 represent the time taken for the planet to move from a to b and c to d, respectively, then



a) $t_1 < t_2$
b) $t_1 > t_2$
c) $t_1 = t_2$
d) From the given information the relation between t_1 and t_2 cannot be determined

123. Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a speed of revolution T. If the gravitational force of attraction between the planet and the star is proportional to $R^{-5/2}$, then
a) T^2 is proportional to R^2
b) T^2 is proportional to $R^{7/2}$
c) T^2 is proportional to $R^{3/2}$
d) T^2 is proportional to $R^{3.75}$

124. The atmosphere is held to the earth by
a) Winds
b) Gravity
c) Clouds
d) None of the above

125. Two spherical bodies of masses m and $5M$ and radii R and $2R$, respectively, are released in free space with initial separation between their centres equal to $12R$. If they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is
a) 2.5 R
b) 4.5 R
c) 7.5 R
d) 1.5 R

126. If three uniform spheres, each having mass M and radius R, are kept in such a way that each touches the other two, the magnitude of the gravitational force on any sphere due to the other two is
a) $\frac{GM^2}{4r^2}$
b) $\frac{2GM^2}{r^2}$
c) $\frac{2GM^2}{4r^2}$
d) $\frac{\sqrt{3}GM^2}{4r^2}$

127. Suppose that the acceleration of a free fall at the surface of a distant planet was found to be equal to that at the surface of the earth. If the diameter of the planet were twice the diameter of the earth, then the ratio of mean density of the planet to that of the earth would be
a) 4:1
b) 2:1
c) 1:1
d) 1:2

Multiple Correct Answers Type

128. Choose the incorrect statements from the following
a) It is possible to shield a body from the gravitational field of another body by using a thick shielding material between them
b) The escape velocity of a body is independent of the mass of the body and the angle of projection
c) The acceleration due to gravity increases due to the rotation of the earth
d) The gravitational force exerted by the earth on a body is greater than that exerted by the body on the earth

129. If both the mass and radius of the earth decreases by 1%, the value of
a) Acceleration due to gravity would decrease by nearly 1%
b) Acceleration due to gravity would increase by 1%
c) Escape velocity from the earth's surface would decrease by 1%
d) The gravitational potential energy of a body on earth's surface will remain unchanged

130. Suppose a smooth tunnel is dug along a straight line joining two points on the surface of the earth and a particle is dropped from rest at its one end. Assume that mass of the earth is uniformly distributed over its volume. Then, which of the following statements are not correct?
a) The particle will emerge from the other end with velocity $(GM_e)/(2R_e)$, where M_e and R_e are earth's mass and radius, respectively
b) The particle will come to rest at the centre of the tunnel because at this position, the particle is closest to the earth's centre
c) Potential energy of the particle will be equal to zero at centre of the tunnel if it is along a diameter

d) Acceleration of the particle will be proportional to its distance from the mid-point of the tunnel

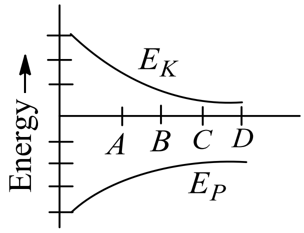
131. The radius and mass of earth are increased by 0.5%. Which of the following statements are true at the surface of the earth

a) g will increase
b) g will decrease
c) Escape velocity will remain unchanged
d) Potential energy will remain unchanged

132. For two satellites at distances R and $7R$ above the earth's surface, the ratio of their

a) Total energies is 4 and potential and kinetic energies is 2
b) Potential energies is 4
c) Kinetic energies is 4
d) Total energies is 4

133. Figure shows the kinetic energy (E_k) and potential energy (E_p) curves for a two-particle system. Name the point at which the system is bound system



a) A
b) B
c) C
d) D

134. An orbiting satellite will escape if

a) Its speed is increased by 41%
b) Its speed in the orbit is made $\sqrt{(1.5)}$ times of its initial value
c) Its KE is doubled
d) It stops moving in the orbit

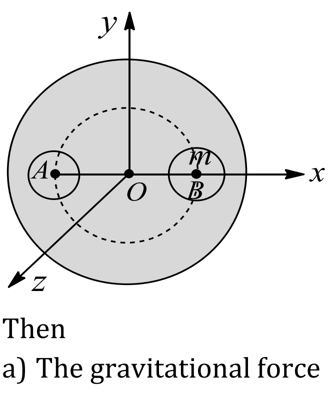
135. Which of the following are correct?

a) An astronaut going from the earth to the Moon will experience weightlessness once
b) When a thin uniform spherical shell gradually shrinks maintaining its shape, the gravitational potential at its centre decreases
c) In the case of a spherical shell, the plot of V versus r is continuous
d) In the case of a spherical shell, the plot of gravitational field intensity I versus r is continuous

136. If two satellites of different masses are revolving in the same orbit, they have the same

a) Angular momentum
b) Energy
c) Time period
d) Speed

137. A solid sphere of uniform density and radius 4 units is located with its centre at the origin O of coordinates. Two spheres of equal radii 1 unit, with their centres at $A(-2, 0, 0)$ and $B(2, 0, 0)$, respectively, are taken out of the solid leaving behind spherical cavities as shown in the figure



Then

a) The gravitational force due to this object at the origin zero
b) The gravitational force at point $B(2, 0, 0)$ is zero
c) The gravitational potential is the same at all points of circle $y^2 + z^2 = 36$
d) The gravitational potential is the same at all points on the circle $y^2 + z^2 = 4$

138. An astronaut, inside an earth satellite, experience weightlessness because

a) No external force is acting on him
b) He is falling freely

c) No reaction is exerted by the floor of the satellite

d) He is far away from the earth's surface

139. A body is imparted a velocity v from surface of the earth. If v_0 is orbital velocity and v_e be the escape velocity then for

a) $v = v_0$, the body follows a circular track around the earth.

b) $v > v_0$, but $< v_e$, the body follows elliptical path and returns to surface of earth.

c) $v < v_0$, the body follows elliptical path and returns to surface of earth

d) $v > v_e$, the body follows hyperbolic path and escapes the gravitational pull of the earth

140. Suppose an earth satellite, revolving in a circular orbit experiences a resistance due to cosmic dust. Then

a) Its kinetic energy will increase

b) Its potential energy will decrease

c) It will spiral towards the earth and in the process its angular momentum will remain conserved

d) It will burn off ultimately

141. The magnitudes of the gravitational field at distance r_1 and r_2 from the centre of a uniform sphere of radius R and mass M are r_1 and r_2 respectively. Then,

a) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ for $r_1 < R$ and $r_1 = R$

b) $\frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$ for $r_1 > R$ and $r_1 > R$

c) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ for $r_1 > R$ and $r_2 > R$

d) $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2}$ for $r_1 < R$ and $r_2 < R$

142. Which of the following are correct?

a) Out of electrostatic, electromagnetic, nuclear and gravitational interactions, the gravitational interaction is the weakest

b) If the earth were to rotate faster than its present speed, the weight of an object would decrease at the equator but remain unchanged at the poles

c) The mass of the earth in terms of g, R and G is (gR^2/G)

d) If the earth stops rotating in its orbit around the Sun there will be no variation in the weight of a body on the surface of earth

143. Which of the following are correct?

a) If R is the radius of a planet, g is the accleration due to gravity, the mean density of the planet is $3g/4\pi GR$

b) Acceleration due to gravity is a universal constant

c) The escape velocity of a body from earth is 11.2 km s^{-1}. The escape velocity from a planet which has double the mass of earth and half its radius is 22.4 km s^{-1}

d) The radio of gravitational mass and inertial mass of a body at the surface of earth is 1

144. An object is taken from a point P to another point Q in a gravitational field

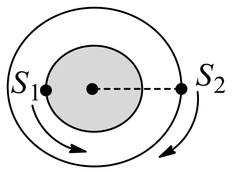
a) Assuming the earth to be spherical, if both P and Q lie on the earth's surface, the work done is zero

b) If P is on the earth's surface and Q above it, the work done is minimum when it is taken along the straight line PQ

c) The work done depends only on the position of P and Q and is independent of the path along which the particle is taken

d) There is no work done if the object is taken from P to Q and then brought back to P along any path

145. Two satellites S_1 and S_2 are revolving around the earth in coplanar concentric orbits in the opposite sense. At $t = 0$, the positions of satellites are shown in the diagram. The periods of S_1 and S_2 are 4 h and 24 h, respectively. The radius of orbit of S_1 is 1.28×10^4 km. For this situation, mark the correct statement(s)



a) The angular velocity of S_2 as observed by S_1 at $t = 12\ h$ is 0.468π rad s^{-1}

b) The two satellite are closest to each other for the first time at $t = 12$ h and then after every 24 h they are closest to each other

c) The orbital velocity of S_1 is $0.64\pi \times 10^4$ km
d) The velocity of S_1 relative to S_2 is continuously changing in magnetic and direction both

146. A particle of mass m lies at a distance r from the centre of earth. The force of attraction between the particle and earth is (r)
a) $F(r) \propto \frac{1}{r^2}$ for $r < R$
b) $F(r) \propto \frac{1}{r^2}$ for $r \geq R$
c) $F(r) \propto r$ for $r < R$
d) $F(r) \propto \frac{1}{2}$ for $r < R$

147. Which of the following statements are true? For a particle on the surface of the earth
a) The linear speed is minimum at the equator
b) The angular speed is maximum at the equator
c) The linear speed is minimum at the poles
d) The angular speed is 7.3×10^{-5} rad s^{-1} at the equator

148. A particle of mass m is moved from the surface of the earth to a height h. The work done by an external agency to do this is
a) mgh for $h \ll R$
b) mgh for all R
c) $\frac{1}{2}mgR$ for $h \gg R$
d) $\frac{1}{2}mgR$ for $h = R$

149. A small mass m is moved slowly from the surface of the earth to a height h above the surface. The work done (by an external agent) in doing this is
a) mgh, for all values of h
b) mgh, for $h \ll R$
c) $1/2\ mgR$, for $h = R$
d) $-1/2\ mgR$, for $h = R$

150. A planet is revolving round the sun. Its distance from the sun at Apogee is r_A and that at Perigee is r_P. The mass of planet and sun is m and M respectively, v_A and v_P is the velocity of planet at Apogee and Perigee respectively and T is the time period of revolution of planet round the sun.
a) $T^2 = \frac{\pi^2}{2Gm}(r_A + r_P)^3$
b) $T^2 = \frac{\pi^2}{2Gm}(r_A + r_P)^3$
c) $v_A r_A = v_P r_P$
d) $v_A < v_P; r_A > r_P$

151. Two spherical planets P and Q have the same uniform density ρ, masses M_P and M_Q, and surface areas A and $4A$, respectively. A spherical planet R also has uniform density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R, are V_P, V_Q and V_R respectively. Then
a) $V_Q > V_R > V_P$
b) $V_R > V_Q > V_P$
c) $V_R/V_P = 3$
d) $V_P/V_Q = \frac{1}{2}$

152. The magnitudes of the gravitational field at distance r_1 and r_2 from the centre of a uniform sphere of radius R and mass m are F_1 and F_2, respectively. Then
a) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$
b) $\frac{r_2^2}{r_2}$ if $r_1 > R$ and $r_2 > R$
c) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 > R$ and $r_2 > R$
d) $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2}$ if $r_1 < R$ and $r_2 < R$

153. Choose the correct statements from the following
a) The magnitude of the gravitational force between two bodies of mass 1 kg each and separated by a distance of 1 m is 9.8 N
b) The higher the value of the escape velocity for a planet, the higher is the abundance of the lighter gases in its atmosphere
c) The gravitational force of attraction between two bodies of ordinary mass is not noticeable because the value of the gravitational constant is extremely small
d) Force of friction arises due to gravitational attraction

154. The gravitation potential on the surface of a planet of radius R mass M is
a) g
b) $\frac{gM}{R}$
c) $\frac{-GM}{R}$
d) $-gR$

155. Suppose universal gravitational constant starts to decrease, then
a) Length of the day on the earth will increase
b) Length of the year will increase
c) The earth will follow a spiral path of decreasing radius
d) Kinetic energy of the earth will decrease

156. Mark the correct statements

a) Gravitational potential at the centre of curvature of a thin hemispherical shell of radius R and mass M is equal to GM/R

b) Gravitational field strength at a point lying on the axis of a thin, uniform circular ring of radius R and mass M is equal to $GMx/[(R^2+x^2)^{3/2}]$, where x is distance of that point from the centre of the ring

c) Newton's law of gravitation for gravitational force between two bodies is applicable only when bodies have spherical symmetric distribution of mass

d) None of these

157. A planet of mars m is revolving round the sun (of mass m_s) in an elliptical orbit. If $\vec{\mathbf{v}}$ is the velocity of the planet when its position vector from sun r then if the planet rotates in counter clockwise direction then areal velocity has direction

a) Given by "Right Hand Thumb Rule"

b) Given by "Left Hand Thumb Rule"

c) Normal to the plane of orbit upwards

d) Normal to the plane of orbit downwards

158. A double star consists of two stars having masses M and $2M$. The distance between their centres is equal to r. They revolve under their mutual gravitational interaction. Then, which of the following statements are not correct?

a) Heavier star revolves in orbit of radius $2r/3$

b) Both the stars revolve with the same speed, period of which is equal to $(2\pi/r^3)(2GM^2/3)$

c) Kinetic energy of the heavier star is twice that of other star

d) None of the above

159. Consider an attractive central fore of the form $F(r) = -\frac{k}{r^n}$, k is constant. For a stable circular orbit to exist

a) $n = 2$ b) $n < 3$ c) $n > 3$ d) $n = -1$

160. Choose the correct statements from the following:

a) The gravitational forces between two particles are an action and reaction pair

b) Gravitational constant (G) is scalar but acceleration due to gravity (g) is a vector

c) The value of G and g are to be determined experimentally

d) G and g are constant everywhere

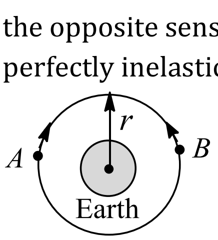
161. Let V and E denote the gravitational potential and gravitational field at a point, respectively. It is possible to have

a) $V =$and $E = 0$ b) $V = 0$ and $E \neq 0$ c) $V \neq$ and $E = 0$ d) $V \neq 0$ and $E \neq 0$

162. A double star is a system of two stars of masses m and $2m$, rotating about their centre of mass only under their mutual gravitational attraction. If r is the separation between these two starts then their period of rotation about their centre of mass will be proportional to

a) $r^{\frac{3}{2}}$ b) r c) $m^{\frac{1}{2}}$ d) $m^{-\frac{1}{2}}$

163. Consider two satellite A and B of equal mass m, moving in the same circular orbit about the earth, but in the opposite sense as shown in figure. The orbital radius is r. The satellites undergo a collision which is perfectly inelastic. For this situations, mark out the correct statement(s). [Take mass of earth as M]



a) The total energy of the two satellite plus earth system just before collision is $-(GMm)/r$

b) The total energy of the two satellite plus earth system just after collision is $-(2GMm)/r$

c) The total energy of two satellites plus earth system just after collision is $-(GMm)/2r$

d) The combined mass (two satellites) will fall towards the earth just after collision

164. If the radius of the earth suddenly decreases to 80% of its present value, the mass of the earth remaining the same, the value of the acceleration due to gravity will

a) Remain unchanged b) Become $9.8/0.64\ \mathrm{m\ s^{-2}}$

c) Increase by 36%
d) Increase by about 56%

165. Which of the following statements are true about acceleration due to gravity?

a) g decreases in moving away from the centre if $r > R$
b) g decreases in moving away from the centre if $r < R$
c) g is zero at the centre of earth
d) g decreases if earth stops rotating on its axis

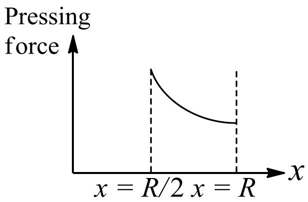
166. Which of the following are not correct?

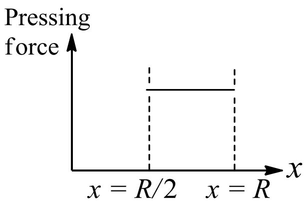
a) The escape velocity for the Moon is 6 km s^{-1}
b) The escape velocity from the surface of Moon is v. The orbital velocity for a satellite to orbit very close to the surface of Moon is $v/2$
c) When an earth satellite is moved from one stable orbit to a further stable orbit, the gravitational potential energy increases
d) The orbital velocity of a satellite revolving in a circular path close to the planet is independent of the density of the planet

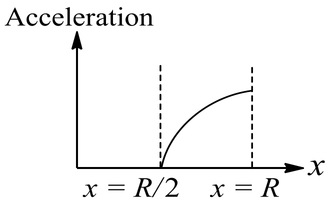
167. A satellite is orbiting the earth, if its distance from the earth is increased, its

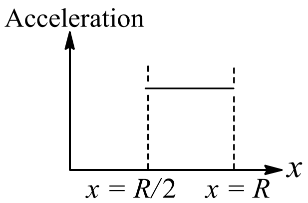
a) Angular velocity would increase
b) Linear velocity would increase
c) Angular velocity would decrease
d) Time period would increase

168. A tunnel is dug along a chord of the earth at a perpendicular distance $R/2$ from the earth's centre. The wall of the tunnel may be assumed to be frictionless. A particle is released from one end of the tunnel. The pressing force by the particle on the wall, and the acceleration of the particle vary with x (distance of the particle from the centre) according to

a) 

b) 

c) 

d) 

169. If a body is projected with a speed lesser than escape velocity, then

a) The body can reach a certain height and may fall down following a straight line path
b) The body can reach a certain height and may fall down following a parabolic path
c) The body may orbit the earth in a circular orbit
d) The body may orbit the earth in an elliptical orbit

170. Consider a planet moving in an elliptical orbit around the Sun. The work done on the planet by the gravitational force of the Sun

a) Is zero in any small part of the orbit
b) Is zero in some parts of the orbit
c) Is zero in complete revolution
d) Is zero in no part of the motion

171. In case of an orbiting satellite, if the radius of orbit is decreased
a) Its KE decreases b) Its PE decreases c) Its ME decreases d) Its speed decreases

172. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth
a) The acceleration of S is always directed towards the centre of the earth
b) The angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant
c) The total mechanical energy of S varies periodically with time
d) The linear momentum of S remains constant in magnitude

Assertion - Reasoning Type

This section contain(s) 0 questions numbered 173 to 172. Each question contains STATEMENT 1(Assertion) and STATEMENT 2(Reason). Each question has the 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

a) Statement 1 is True, Statement 2 is True; Statement 2 **is** correct explanation for Statement 1

b) Statement 1 is True, Statement 2 is True; Statement 2 **is not** correct explanation for Statement 1

c) Statement 1 is True, Statement 2 is False

d) Statement 1 is False, Statement 2 is True

173

Statement 1: The speed of satellite always remains constant in an orbit

Statement 2: The speed of a satellite depends on its path

174

Statement 1: If a pendulum is suspended in a lift and lift is falling freely, then its time period becomes infinite

Statement 2: Free falling body has acceleration equal to acceleration due to gravity

175

Statement 1: The magnitude of the gravitational potential at the surface of solid sphere is less than that of the centre of sphere

Statement 2: Due to the solid sphere, the gravitational potential is the same within the sphere

176

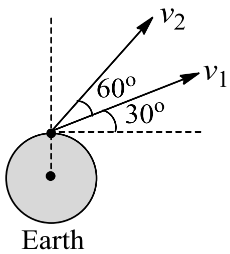
Statement 1: Consider a satellite moving in an elliptical orbit around the earth. As the satellite moves, the work done by the gravitational force of the earth on the satellite for any small part of the orbit is zero

Statement 2: KE of the satellite in the above described case is not constant as it moves around the earth

177

Statement 1: The value of escape velocity from the surface of earth at 30° and 60° is $v_1 = 2v_e, v_2 = 2/3\ v_e$

Statement 2: The value of escape velocity is independent of angle of projection



178

Statement 1: Kepler's second law can be understood by conservation of angular momentum principle

Statement 2: Kepler's second law is related with areal velocity which can further be proved to be based on conservation of angular momentum as $(dA/dt) = (r^2\omega)/2$

179

Statement 1: For a mass M kept at the centre of a cube of side 'a', the flux of gravitational field passing through its sides is $4\,\pi$ GM

Statement 2: If the direction of a field due to a point source is radial and its dependence on the distance 'r' from the source is given as $\frac{1}{r^2}$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface

180

Statement 1: For a satellite revolving very near to the earth's surface the time period of revolution is given by 1 h 24 min

Statement 2: The period of revolution of a satellite depends only upon its height above the earth's surface

181

Statement 1: Two satellites are following one another in the same circular orbit. If one satellite tries to catch another (leading one) satellite, then it can be done by increasing its speed without changing the orbit

Statement 2: The energy of earth satellites system in circular orbit is given by $E = -\frac{GMm}{2r}$, where r is the radius of the circular orbit

182

Statement 1: If earth suddenly stops rotating about its axis then the value of acceleration due to gravity will becomes same at all the places

Statement 2: The value of acceleration due to gravity is independent of rotation of earth

183

Statement 1: Two satellites are following one another in the same circular orbit. If one satellite tires to catch another (leading one) satellite, then it can be done by increasing its speed without changing the orbit

Statement 2: The energy of earth-satellite system in circular orbit is given by $E = -(-Gms)/(2a)$, where r is the radius of the circular orbit

184

Statement 1: The binding energy of a satellite does not depend upon the mass of the satellite

Statement 2: Binding energy is the negative value of total energy of satellite

185

Statement 1: Two different planets have same escape velocity

Statement 2: Value of escape velocity is a universal constant

186

Statement 1: The earth does not retain hydrogen molecules and helium atoms in its atmosphere, but does retain much heavier molecules, such as oxygen and nitrogen

Statement 2: Lighter molecules in the atmosphere have translational speed that is greater or closer to escape speed of earth

187

Statement 1: The speed of revolution of an artificial satellite revolving very near the earth is 8 kms^{-1}

Statement 2: Orbital velocity of a satellite, become independent of height of satellite

188

Statement 1: The time period of geostationary satellite is 24 hours

Statement 2: Geostationary satellite must have the same time period as the time taken by the earth to complete one revolution about its axis

189

Statement 1: There is no effect of rotation of earth on acceleration due to gravity at poles

Statement 2: Rotation of earth is about polar axis

190

Statement 1: The smaller the orbit of a planet around the Sun, the shorter is the time it takes to complete

Statement 2: According to Kepler's third law of planetary motion, square of time period is proportional to cube of mean distance from Sun

191

Statement 1: A force act upon the earth revolving in a circular orbit about the sun. Hence work should be done on the earth

Statement 2: The necessary centripetal force for circular motion of earth comes from the gravitational force between earth and sun

192

Statement 1: If time period of a satellite revolving in circular orbit in equatorial plane is 24 h, then it must be a geostationary satellite

Statement 2: Time period of a geostationary satellite is 24 h

193

Statement 1: The difference in the value of acceleration due to gravity at pole and equator is proportional to square of angular velocity of earth

Statement 2: The value of acceleration due to gravity is minimum at the equator and maximum at the pole

194

Statement 1: A body becomes weightless at the centre of earth

Statement 2: As the distance from centre of earth decreases, acceleration due to gravity increases

195

Statement 1: If the earth suddenly stops rotating about its axis, then the acceleration due to gravity will become the same at all the plates

Statement 2: The value of acceleration due to gravity is independent of rotation of the earth

196

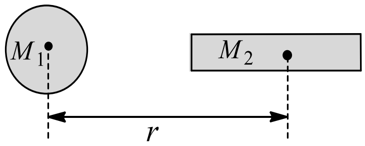
Statement 1: An astronaut in an orbiting space station above the earth experience weightlessness

Statement 2: An object moving around the earth under the influence of earth's gravitational force is in a state of free fall

197

Statement 1: The force of gravitation between a sphere and a rod of mass M_2 is $= (GM_1M_2)/r$

Statement 2: Newton''s law of gravitation holds correct for point masses



198

Statement 1: The principle of superposition is not valid for gravitational force

Statement 2: Gravitational force is a conservative force

199

Statement 1: Even when orbit of a satellite is elliptical, its plane of rotation passes through the centre of earth

Statement 2: According to law of conservation of angular momentum plane of rotation of satellite always remain same

200

Statement 1: We can not move even a finger without disturbing all the stars

Statement 2: Every body in this universe attracts every other body with a force which is inversely proportional to the square of distance between them

201

Statement 1: Gravitational potential of earth at every place on it is negative

Statement 2: Every body on earth is bound by the attraction of earth

202

Statement 1: When distance between two bodies is doubled and also mass of each body is also doubled. Gravitational force between them remains the same

Statement 2: According to Newton's law of gravitation, force is directly proportional to mass of bodies and inversely proportional to distance between them

203

Statement 1: Earth has an atmosphere but the moon does not

Statement 2: Moon is very small in comparison to earth

204

Statement 1: For the planets orbiting around the Sun, angular speed, linear speed and KE change with time, but angular momentum remains constant

Statement 2: No torque is acting on the rotating planet. So its angular momentum is constant

205

Statement 1: The time period of revolution of a satellite close to surface of earth is smaller than that revolving away from surface of earth

Statement 2: The square of time period of revolution of a satellite is directly proportional to cube of its orbital radius

206

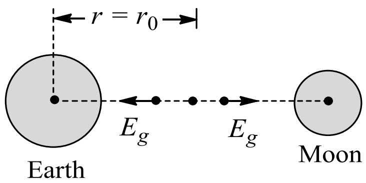
Statement 1: If a particle projected horizontally just above the surface of the earth with a speed greater than escape speed, then it will escape from gravitational influence of the earth. Assume that particle has a clear path

Statement 2: Escape velocity is independent of its direction

207

Statement 1: It takes more fuel for a spacecraft to travel from the earth to the Moon than for the return trip

Statement 2: The point of zero gravitational field intensity due to the earth and the Moon is lying nearer to the Moon, i.e., in the diagram shown, for $r < r_0, E_g$ is towards the earth''s centre and for $r > r_0, E_g$ is towards the Moon''s centre, and at $r = r_0, E_g$ is zero



208

Statement 1: Orbital velocity of a satellite is greater than its escape velocity

Statement 2: Orbit of a satellite is within the gravitational field of earth whereas escaping is beyouned the gravitational field of earth

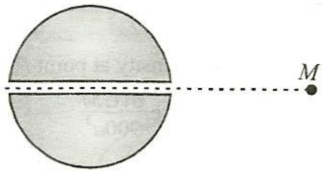
209

Statement 1: Gravitational force between two particles is negligible small compared to the electrical force

Statement 2: The electrical force is experienced by charged particles only

210

Statement 1: In free space a uniform spherical planet of mass M has a smooth narrow tunnel along its diameter. This planet and another superdense small particle of mass M start approaching towards each other from rest under action of their gravitational forces. When the particle passes through the centre of the planet, sum of kinetic energies of both the bodies is maximum



Statement 2: When the resultant of all forces acting on a particle or a particle like object (initially at rest) is constant in direction, the kinetic energy of the particle keeps on decreasing

211

Statement 1: Space rockets are usually launched in the equatorial line from west to east

Statement 2: The acceleration due to gravity is minimum at the equator

212

Statement 1: Generally the path of a projectile from the earth is parabolic but it is elliptical for projectiles going to a very great height

Statement 2: Upto ordinary height the projectile moves under a uniform gravitational force, but for great heights, projectile moves under a variable force

213

Statement 1: The value of acceleration due to gravity does not depend upon the mass of the body

Statement 2: Acceleration due to gravity is a constant quantity

214

Statement 1: An astronaut in an orbiting space station above the Earth experiences weightlessness

Statement 2: An object moving around the Earth under the influence of Earth's gravitational force is in a state of 'free-fall'

215

Statement 1: A planet moves faster, when it is closer to the sun in its orbit and vice versa

Statement 2: Orbital velocity in the orbit of planet is constant

Matrix-Match Type

This section contain(s) 0 question(s). Each question contains Statements given in 2 columns which have to be matched. Statements (A, B, C, D) in **columns I** have to be matched with Statements (p, q, r, s) in **columns II**.

216. For a planet orbiting about the Sun in a elliptical orbit, some incomplete statements regarding physical quantities are given in Column I, which can be completed by using the entries of Column II. Match the entries of Column I with the entries of Column II

Column-I	Column- II
(A) Maximum PE of the Sun planet system	(p) Is at perihelion
(B) Maximum speed of the planet	(q) Is at aphelion
(C) Minimum PE of Sun-planet system	(r) Is independent of mass of planet
(D) Minimum kinetic energy of planet	(s) Is independent of semi-major axis of orbit

CODES :

	A	B	C	D
a)	A,c,d	a,d	b,c	b,d
b)	b,d	a,c,d	a,d	b,c
c)	a,d	b,c	b,d	a,c,d
d)	b,c	b,d	a,c,d	a,d

217. A satellite is revolving around the earth in a circular orbit of radius 'a' with velocity v_0. A particle is projected from the satellite in forward direction with relative velocity $V = [(\sqrt{5}/4) - 1]v_0$, During the subsequent motion of the particle, match the following

Column-I		Column- II	
(A)	Total energy of particle	(p)	$(3GM_e m)/a$
(B)	Minimum distance of particle from the earth	(q)	$(5GM_e m)/a$
(C)	Maximum distance of particle from the earth	(r)	$5a/3$
		(s)	$2a$
		(t)	$2a/3$
		(u)	a

CODES :

	A	B	C	D
a)	a	b	c	
b)	c	a	b	
c)	c	b	a	
d)	a	c	b	

218. Consider the earth to be a homogeneous sphere but keeping in mind its spin, match the following

Column-I		Column- II	
(A)	Acceleration due to gravity	(p)	May change from point to point
(B)	Orbital angular momentum of the earth as seen from a distant star	(q)	Does not depend on direction of projection
(C)	Escape velocity from the earth	(r)	Remains constant
(D)	Gravitational potential due to earth at a particular point	(s)	Depend on direction of projection

CODES :

	A	B	C	D
a)	A,b	c	a	c

b)	a,c	b	c	d
c)	a	c,d	d	a
d)	c	b	d	a,b

219.

	Column-I		Column- II
(A)	Gravitational potential	(p)	On the surface of planets with density ratio 1:2
(B)	Escape velocity	(q)	Conservation of angular momentum
(C)	Ratio of the acceleration due to gravity 1:3	(r)	Varies with the reference point
(D)	Orbiting satellite	(s)	Does not depend on the angle
		(t)	Similar to an atom

CODES :

	A	B	C	D
a)	a	b,e	c,d	d
b)	b,e	c,d	d	a
c)	c,d	d	a	b,e
d)	d	a	b,e	c,d

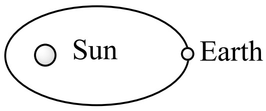
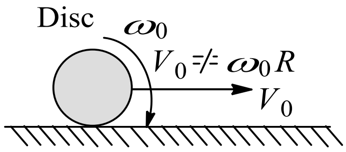
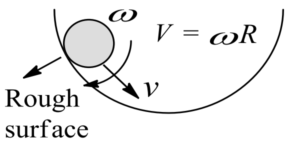
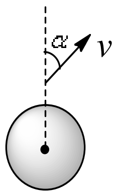
220. An artificial satellite is in circular orbit around the earth. One of the rockets of the satellite is momentarily fired, the direction of firing of rocket is mentioned in Column I and corresponding change (s) are given in Column II. Match the entries of Column I with the entries of Column II

	Column-I		Column- II
(A)	Towards the earth's centre	(p)	Orbit changes and becomes elliptical
(B)	Away from the earth's centre	(q)	Orbit plane changes
(C)	At right angle to the plane or orbit	(r)	Semi-major axis of orbit increases
(D)	In forward direction	(s)	Energy of earth-satellite system increases

CODES :

	A	B	C	D
a)	a	a	a,c	a,b,c,d
b)	a,b,c,d	a	a	a,c
c)	a	a,c,d	a,b,c,d	a
d)	a,c	a,b,c,d	a	a

221.

Column-I	Column- II
(A) The earth moving in an elliptical orbit (only earth in system) 	(p) Conservation of linear momentum along any direction
(B) A disc having translation and rotation motion both (both slipping) on a rough surface (only disc in system) 	(q) Conservation of linear momentum along specific direction
(C) A sphere rolling without slipping on a curved surface (only the sphere in system) 	(r) Conservation of angular momentum about any point in the space
(D) Projection of a particle from the surface of the earth (only particle in system) 	(s) Conservation of angular momentum about a specific point in the space

CODES :

	A	**B**	**C**	**D**
a)	a	c	d	a
b)	a	b	d	c
c)	b	c	a	d
d)	c	d	no match	d

222.

Column-I	Column- II
(A) Geostationary satellite	(p) Circular orbit
(B) Total energy of earth-satellite system is constant in	(q) Elliptical orbit
(C) Angular momentum of satellite about centre of earth is constant in	(r) Equatorial plane orbit
(D) Orbital speed of satellite is/may be constant	(s) Non-equatorial plane orbit

CODES :

	A	B	C	D
a)	A,c	b,d	a,c	b,c
b)	a,c	a,b,c,d	a,b,c	a,c,d
c)	a,d	a,c	c,d	b,d
d)	a,b	a,d	a,d,c	d,c

223. Let V and E denote the gravitational potential and gravitational field, respectively, at a point due to certain uniform mass distribution described in four different situations of Column I. Assume the gravitational potential at infinity to be zero. The values of E and V are given in Column II. Match the statement in column I with the results in Column II

	Column-I		Column- II
(A)	At the centre of thin spherical shell	(p)	$E = 0$
(B)	At the centre of solid sphere	(q)	$E \neq 0$
(C)	A solid sphere has a non-concentric spherical cavity. At the centre of the spherical cavity	(r)	$V \neq 0$
(D)	At the centre of the joining two point masses of equal magnitude	(s)	$V = 0$

CODES :

	A	B	C	D
a)	A,c	a,c	a,c	a,c
b)	a,c	b,d	c,a	d,c
c)	b,d	c,a	d,c	a,c
d)	c,a	d,c	a,c	b,d

Linked Comprehension Type

This section contain(s) 19 paragraph(s) and based upon each paragraph, multiple choice questions have to be answered. Each question has atleast 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

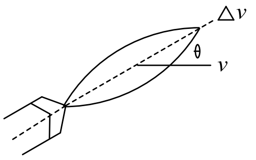
Paragraph for Question Nos. 224 to -224

A rocket is fired vertically upwards with a speed of $v(= 5\ \text{kms}^{-1})$ from the surface of earth. It goes up to a height h before returning to earth. At height h a body is thrown from the rocket with speed v_0 in such a way so that the body becomes a satellite of earth. Let the mass of the earth, $M = 6 \times 10^{24}$ kg; mean radius of the earth, $R = 6.4 \times 10^6\text{m}$;
$G = 6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2}$;
$\text{g} = 9.8\ \text{ms}^{-2}$

224. The value of h is
a) $1.5 \times 10^5\text{m}$ b) $3.2 \times 10^5\text{m}$ c) $3.2 \times 10^6\text{m}$ d) $1.6 \times 10^6\text{m}$

Paragraph for Question Nos. 225 to - 225

A spaceship is in a circular orbit of radius r_0 around a star of mass M. The spaceship's rocket engine can alter its velocity (instantaneously) by an $\Delta\vec{\mathbf{v}}$. Amounts direction of firing is measured by angle θ between the ship's velocity $\vec{\mathbf{v}}$ and the vector from the tail to the nose of the ship. To conserve fuel in a sequence of N firings, it is desirable to minimise $\Delta V = \sum_{i=1}^{N}|\Delta\vec{\mathbf{v}}_i|$. Δv is known as the specific impulse. We want to use the ship's engine to cause it to crash into the star (assume the radius of the star to be negligible)



225. What is the minimum specific impulse required to escape from the star if the engine is fired in a single rapid burst?

a) $\sqrt{\frac{GM}{r_0}}$ b) $\sqrt{\frac{GM^2}{r_0}}(\sqrt{2}-1)$ c) $\sqrt{\frac{GM}{r_0}}\sqrt{2}$ d) $\sqrt{\frac{GM}{r_0}}(\sqrt{2}-1)$

Paragraph for Question Nos. 226 to - 226

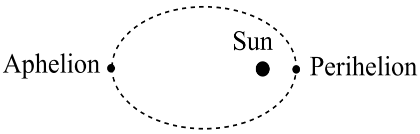
The minimum and maximum distances of a satellite from the centre of the earth are $2R$ and $4R$, respectively, where R is the radius of the earth and M is the mass of the earth

226. The minimum and maximum speeds are

a) $\sqrt{\frac{GM}{9R}}, \sqrt{\frac{2GM}{R}}$ b) $\sqrt{\frac{GM}{5R}}, \sqrt{\frac{3GM}{2R}}$ c) $\sqrt{\frac{GM}{6R}}, \sqrt{\frac{2GM}{3R}}$ d) $\sqrt{\frac{GM}{3R}}, \sqrt{\frac{5GM}{2R}}$

Paragraph for Question Nos. 227 to - 227

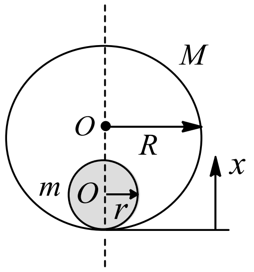
The orbit of Pluto is much more eccentric than the orbits of the other planets. That is, instead of being nearly circular, the orbit is noticeably elliptical. The point in the orbit nearest to the Sun is called the perihelion and the point farthest from the Sun is called the aphelion



227. At perihelion, the gravitational potential energy of Pluto in its orbit has
   a) Its maximum value
   b) Its minimum value
   c) The same value as at every other point in the orbit
   d) The value which depends on the sense of rotation

Paragraph for Question Nos. 228 to - 228

A solid sphere of mass m and radius r is placed inside a hollow thin spherical shell of mass M and radius R as shown in the figure. A particle of masses m is placed on the line joining the two centres at a distance x from the point of contact of the sphere and the shell. Find the magnitude of the resultant gravitational force on this particle due to the sphere and the shell if



228. $r < x < 2r$

a) $\frac{Gmm'(2r-x)}{2r^3}$ b) $\frac{Gmm'(x-r)}{2r^3}$ c) $\frac{Gmm'(x-r)}{r^3}$ d) $\frac{Gmm'(2x-r)}{r^3}$

Paragraph for Question Nos. 229 to - 229

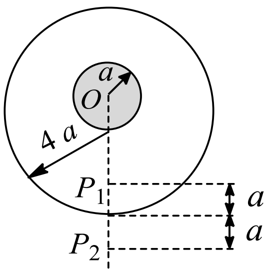
The gravitational field in a region is given by $\vec{E} = (5\text{N kg}^{-1})\vec{\imath} + (12\text{ N kg}^{-1})\vec{\jmath}$

229. Find the magnitude of the gravitational force acting on a particle of mass 2 kg placed at the origin

a) 26 N b) 30 N c) 20 N d) 35 N

Paragraph for Question Nos. 230 to - 230

A uniform metal sphere of radius a and mass M is surrounded by a thin uniform spherical shell of equal mass and radius $4a$. The centre of the shell falls on the surface of the inner sphere

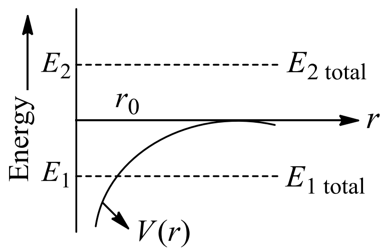


230. Find the gravitational field intensity at point P_1

a) $\frac{GM}{16a^2}$ b) $\frac{GM}{8a^2}$ c) $\frac{GM}{2a^2}$ d) $\frac{GM}{4a^2}$

Paragraph for Question Nos. 231 to - 231

In the graph shown, the PE of earth-satellite system is shown by a solid line as a function of distance r(the separation between earth's centre and satellite). The total energy of the two objects which may or may not be bounded to the earth are shown in the figure by dotted lines

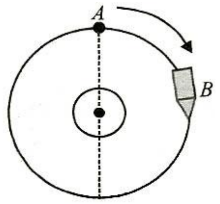


Based on the above information, answer the following questions

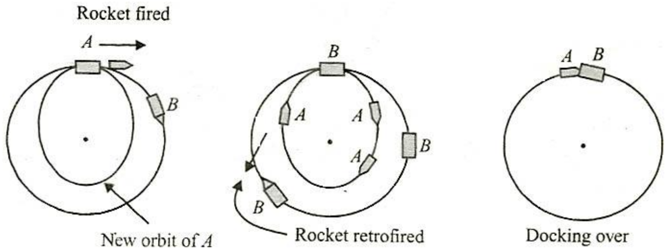
231. Mark the correct statements(s)
a) The object having the total energy E_1 is bounded one
b) The object having the total energy E_2 is bounded one
c) Both the objects are bounded
d) Both the objects are unbounded

Paragraph for Question Nos. 232 to - 232

An unmanned satellite A and a spacecraft B are orbiting around the earth in the same circular orbit as shown



The spacecraft is ahead of the satellite by some time. Let us consider that some technical problem has arisen in the satellite and the astronaut from B has made it correct. For this to be done docking of two (A and B) is required (in layman terms connecting A and B). To achieve this, the rockets of A have been fired in forward direction and docking takes place as shown in the figure below :



Take mass of the earth= 5.98×10^{24} kg
Radius of the earth= 6400 km
Orbital radius= 9600 km
Mass of satellite A = 320 kg
Mass of spacecraft= 3200 kg
Assume that initially spacecraft B leads satellite A by 100 s, i.e., A arrive at any particular position after 100 s of B's arrival. Based on the above information answer the following questions

232. To dock A and B in the above-described situation, one can use the rocket system of either one, i.e., either of A or of B. To accomplish docking in the minimum possible time which is the best way?
a) To use rocket system of A
b) To use rocket system of B
c) Either (a) or (b)
d) Information insufficient

Paragraph for Question Nos. 233 to - 233

The satellite when launched from the earth are not given the orbital velocity initially, a multistage rocket propeller carrier the spacecraft up to its orbit and during each stage rocket has been fired to increase the velocity to acquire the desired velocity for a particular orbit. The last stage of the rocket brings the satellite in circular/elliptical (desired) orbit
Consider a satellite of mass 150 kg in a low circular orbit. In this orbit, we cannot neglect the effect of air drag. This air opposes the motion of satellite and hence the total mechanical energy of earth satellite system decreases. That means the total energy becomes more negative and hence the orbital radius decreases which cause the increase in KE When the satellite comes in the low enough orbit, excessive thermal energy generation

due to air friction may cause the satellite to burn up. Based on the above information, answer the following questions

233. It has been mentioned in the passage that as r decreases, E decreases but K increases. The increase in K is [E = total mechanical energy, r =orbital radius, K =kinetic energy]
   a) Due to increase in gravitational PE
   b) Due to decrease in gravitational PE
   c) Due to work done by air friction force
   d) Both (b) and (c)

Integer Answer Type

234. Imagine a new planet having the same density as that of the earth but it is 3 times bigger than the earth is size. If the acceleration due to gravity on the surface of the earth is g and that on the new planet is g′, then what is the value of g'/g?
235. The earth (mass= 10^{24} kg) revolves round the Sun with an angular velocity 2×10^{7} rad s^{-1} in a circular orbit of radius 1.5×10^{8} km. Find the force exerted by the Sun on the earth (in $\times 10^{21}$ N)
236. A particle of mass m is subjected to an attraction central force of magnitude k/r^2, k being a constant. If at the instant when the particle is at an extreme position in its closed orbit, at a distance a from the centre of force, its speed is $(k/2ma)$, if the distance of other extreme position is 'b' . Find a/b
237. The density of a newly discovered planet is twice that of the earth. The acceleration due to gravity at the surface of the planet is equal to that at the surface of the earth. If the radius of the earth is R and the radius of the planet is 'R'. Then what is the value of R/R'?

: ANSWER KEY :

1)	d	2)	d	3)	c	4)	c
5)	b	6)	b	7)	a	8)	b
9)	a	10)	c	11)	c	12)	c
13)	c	14)	b	15)	c	16)	c
17)	d	18)	b	19)	c	20)	c
21)	a	22)	d	23)	c	24)	c
25)	b	26)	b	27)	a	28)	d
29)	b	30)	a	31)	c	32)	c
33)	b	34)	c	35)	b	36)	b
37)	b	38)	d	39)	a	40)	d
41)	b	42)	d	43)	a	44)	c
45)	a	46)	d	47)	b	48)	d
49)	c	50)	c	51)	a	52)	b
53)	a	54)	d	55)	b	56)	c
57)	a	58)	d	59)	d	60)	a
61)	b	62)	b	63)	c	64)	a
65)	d	66)	b	67)	c	68)	c
69)	b	70)	c	71)	a	72)	c
73)	a	74)	a	75)	d	76)	d
77)	a	78)	c	79)	c	80)	b
81)	d	82)	d	83)	a	84)	c
85)	d	86)	a	87)	a	88)	b
89)	d	90)	c	91)	a	92)	c
93)	d	94)	a	95)	d	96)	c
97)	c	98)	a	99)	c	100)	b
101)	c	102)	c	103)	d	104)	b
105)	a	106)	b	107)	d	108)	d
109)	c	110)	c	111)	d	112)	c
113)	d	114)	a	115)	b	116)	a
117)	b	118)	a	119)	c	120)	b
121)	d	122)	c	123)	b	124)	b
125)	c	126)	d	127)	d	1)	
	a,c,d	2)	b,d	3)	a,b,c	4)	
	b,c,d						
5)	b,c,d	6)	a,b,c,d	7)	a,c	8)	
	a,b,c						
9)	c,d	10)	a,c,d	11)	b,c	12)	

	a,b,c,d						
13)	a,b,d	14)	a,b	15)	a,b,c,d	16)	
	a,c,d						
17)	a,c,d	18)	a,b,c,d	19)	b,c	20)	
	c,d						
21)	a,d	22)	b,c	23)	b,c,d	24)	
	b,d						
25)	a,b	26)	b,c	27)	c,d	28)	
	b,d						
29)	b,c	30)	a, c	31)	a,c	32)	
	a,b,d						
33)	a,b,c	34)	a,c,d	35)	a,d	36)	
	a,b,d						
37)	b,d	38)	a,c	39)	a,d	40)	
	c,d						
41)	b,c	42)	a,b,c,d	43)	b,c	44)	
	b,c						
45)	a,c	1)	e	2)	a	3)	c
	4)	d					
5)	d	6)	a	7)	a	8)	a
9)	d	10)	c	11)	d	12)	d
13)	d	14)	a	15)	a	16)	b
17)	a	18)	a	19)	e	20)	d
21)	b	22)	c	23)	c	24)	a
25)	d	26)	e	27)	a	28)	a
29)	a	30)	a	31)	b	32)	a
33)	a	34)	a	35)	a	36)	d
37)	b	38)	a	39)	b	40)	c
41)	c	42)	a	43)	c	1)	b
	2)	d	3)	a	4)	c	
5)	c	6)	d	7)	b	8)	a
1)	d	2)	d	3)	c	4)	b
5)	c	6)	a	7)	a	8)	a
9)	c	10)	b	1)	3	2)	6
	3)	3	4)	2			

: HINTS AND SOLUTIONS :

1 **(d)**

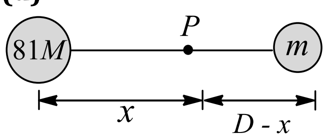
Here, angular momentum is conserved. According to it,

$I_1\omega_1 = I_2\omega_2$

or $MR_1^2\omega_1 = MR_1^2\omega_2$ or $R_1v_1 = R_2v_2$

At point P_4, the value of R is minimum and hence the velocity is maximum or KE is maximum

2 **(d)**



$$\frac{81GM}{x^2} = \frac{GM}{(D-x)^2}$$

$$\frac{9}{x} = \frac{1}{(D-x)}$$

$$D = \frac{9}{10}x$$

$$x = \frac{9}{10}D$$

3 **(c)**

Note that there must be some point where the gravitational field of the earth is balanced by the gravitational field of Mars

4 **(c)**

At height h above the surface of the earth, therefore

$$g' = g\left(1 - \frac{2g}{R}\right) \Rightarrow \Delta g_1 = \frac{2h}{R}g$$

At depth d below the surface of the earth

$$g' = g\left(1 - \frac{d}{R}\right) \Rightarrow \Delta g_2 = \frac{d}{R}g$$

$\Delta g_1 = \Delta g_2 \Rightarrow d = 2h$

5 **(b)**

$F_G = \frac{GMm}{R}$ (as $F_G \propto \frac{1}{R}$ given)

So $\frac{mv^2}{R} = \frac{GMm}{R} \Rightarrow v^2 \propto R^0$

6 **(b)**

Energy of the skylab in the first orbit is

$$-\frac{GMm}{2(2R)} = -\frac{GMm}{4R}$$

Total energy required to place the skylab into the orbit of radius $2R$ from the surface of earth is

$$-\frac{GMm}{4R} - \left(-\frac{GMm}{R}\right) = \frac{3GMm}{4R}$$

$$= \frac{3gR^2m}{4R} = \frac{3}{4}mgR$$

Energy of the skylab in the second orbit=

$-(GMm)/6R$

Energy needed to shift the skylab from the first orbit to the second orbit is

$$-\frac{GMm}{4R} - \frac{GMm}{6R} = \frac{GMm}{R} \times \frac{2}{24} = \frac{mgR}{12}$$

7 **(a)**

Let us first calculate the mass of the inner solid sphere of radius r

Mass of the inner solid sphere is

$$M' = \frac{M_e}{\frac{4}{3}\pi R_e^3} \times \frac{4}{3} = \pi r^2 = \frac{M_e}{R_e^3 r^3}$$

Now, $g = \frac{GM_e r^3}{R_e^3} \times \frac{1}{r^2}$ or $g = \frac{GM_e r}{R_e^3}$

Force on the particle of mass $m = mg$

$$= \frac{GM_e mr}{R_e^3}$$

Note: ***Those layers of the sphere which have the gravitational force***

8 **(b)**

Using Kepler's third law, the semi-major axis of the comet's orbit is given by

$$T^4 = \frac{4\pi^2}{GM_s} \times a^3$$

$$a = \left[\frac{T^2 \times GM_s}{4\pi^2}\right]^{\frac{1}{3}}$$

$$= \left[\frac{(90 \times 24 \times 3600) \times 6.67 \times 10^{-11} \times 2 \times 10^{30}}{4\pi^2}\right]^{\frac{1}{3}}$$

$= 5.89 \times 10^{10}$ m

So, the major axis of the comet's orbit would be

$2a = 1.178 \times 10^{11}$ m

As $2a <$ mean distance between the earth and the Sun, so the collision is not possible

9 **(a)**

$$g = \frac{GM}{r^2} = G\frac{4}{3}\frac{\pi r^3 \rho}{r^2} = G\frac{4}{3}\pi r\rho$$

$$\frac{g_1}{g_2} = \frac{r_1\rho_1}{r_2\rho_2}$$

10 **(c)**

$$\frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{R+h}$$

$$= \frac{GMmh}{R(R+h)}$$

$$= \frac{mghR}{R+h}$$

Since, $v = nv_e$ and $v_e = \sqrt{2}gR$, hence

$= (Rn^2)/(1+n^2)$

11 **(c)**

$M_1 \longleftarrow (x) \longrightarrow (R - x) \quad M_2$

$$\frac{GM_1}{x^2}=\frac{GM_2}{(R-x)^2}$$

$$\frac{M_2}{M_1}x^2=R^2+x^2-2Rx$$

Let $\frac{M_2}{M}=k$

$$x^2(k-1)+2Rx-R^2=0$$

$$x=-\frac{2R+\sqrt{4R^2+4(k-1)R^2}}{2(k-1)}$$

$$=\frac{R\sqrt{M_1}}{\sqrt{M_1}+\sqrt{M_2}}$$

$$R-x=\frac{R\sqrt{M_2}}{\sqrt{M_1}+\sqrt{M_2}}$$

Gravitational potential at point P is

$$-\left(\frac{GM_1}{x}+\frac{GM_2}{R-x}\right)$$

$$=-\left[\frac{GM_1(\sqrt{M_1}+\sqrt{M_2})}{R\sqrt{M_1}}+\frac{GM_2(\sqrt{M_1}+\sqrt{M_2})}{R\sqrt{M_2}}\right]$$

$$=-\left[\frac{G\left(\sqrt{M_2}+\sqrt{M_1}\right)}{R}\left(\sqrt{M_1}+\sqrt{M_2}\right)\right]$$

$$=-\frac{G\left(\sqrt{M_1}+\sqrt{M_2}\right)^2}{R}$$

12 **(c)**

$$\frac{m_1v_1^2}{r_1}=m_1\frac{GM}{r_1^2},\frac{m_2v_2^2}{r_2}=m_2\frac{GM}{r_2^2}$$

$$\frac{v_1^2}{v_2^2}=\frac{r_2}{r_1};r_2<r_1\Rightarrow v_1^2<v_2^2$$

Hence, $v_1<v_2$

13 **(c)**

Given $8=\frac{2\pi}{\omega_1+\omega_2}=\frac{2\pi}{\frac{2\pi}{T_1}+\frac{2\pi}{T_2}}$

$\Rightarrow T_1=24$ h for the earth

$\Rightarrow T_1=12$ h (T_2being the time period of the satellite, it will remain the same as the distance from the centre of the earth remains constant)

$$\Rightarrow \quad T=\frac{2\pi}{\omega_2+\omega_1}=\frac{2\pi}{\frac{2\pi}{T_1}-\frac{2\pi}{T_2}}=24\text{ h}$$

14 **(b)**

$$\frac{GMm}{r_0}=\frac{1}{2}mv^2-\frac{GMm}{R}$$

or $\frac{1}{2}mv^2=\frac{GMm}{R}-\frac{GMm}{r_0}$

or $\frac{v^2}{v}=\frac{GM}{R}-\frac{GM}{r_0}$

or $v^2=2GM\left[\frac{1}{R}-\frac{1}{r_0}\right]$

or $v=\sqrt{2GM\left(\frac{1}{R}-\frac{1}{r_0}\right)}$

15 **(c)**

On the surface of the earth,

Total energy=Kinetic energy + Potential energy

$$=\frac{1}{2}mv^2\frac{GmM}{R}$$

At the highest point, $v=0$,

Potential energy $=-(GmM)/(R+h)$

Where h is the maximum height

$$\frac{1}{2}mv^2-\frac{GmM}{R}=-\frac{GmM}{R+h}$$

$\frac{1}{2}v^2=\frac{gRh}{R}+h$ or $\frac{R+h}{h}=\frac{2gR}{v^2}$

$$h=R\left(\frac{2gR}{v^2}-1\right)^{-1}$$

16 **(c)**

$$l=\frac{F}{m}=\frac{4GMm/(3l^2)}{m}=\frac{4GM}{3l^2}$$

17 **(d)**

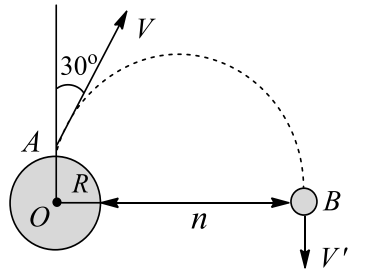
Work done = change in GPE $=U_\infty-U_R$

$$W=0-\left(\frac{GMm}{R}\right)=\frac{GMm}{R}$$

$$=\frac{6.67\times10^{-11}\times100\times10^{-2}}{10^{-1}}=6.67\times10^{-10}\text{ J}$$

18 **(b)**

Conservation of angular momentum of the body about O yield the following :



$(mv\sin30°)\ R=nV'(R+h)$

$$\frac{V}{2}R=\frac{V}{4}(R+h)\quad\left[\therefore V'=\frac{V}{4}\right]$$

Therefore, $h=R$

19 **(c)**

$$F_G=\frac{GM^2}{4R^2}\Rightarrow\frac{mv^2}{R}=\frac{GM^2}{4R^2}$$

$$v=\frac{1}{2}\sqrt{\frac{GM}{R}}$$

20 **(c)**

As we decrease the magnitude of mechanical energy of the spacecraft-earth system, it means we are increasing the energy of the spacecraft-earth system as the total energy of the bounded system is negative

As we change the energy, the circular orbit of the spacecraft will become elliptical. Let a be the semi-major axis of this new elliptical orbit

$E_{\text{final}} = -\frac{GM_m}{2a}$
$E_{\text{initial}} = \frac{GM_m}{2r}$, where $r = 7000$ km
$= \frac{GM_m}{2r} = -\frac{GM_m}{2a}$
$a = \frac{r}{0.9} = 1.11\, r$
$r_{\max} = 2a$ [where $r_{\max}$ is the distance corresponding to aphelion] $= 2.22r - r = 1.22\, r$
Required greatest height, $h = r_{\max} - R_e = 2140$ km

21 **(a)**
Let the minimum speed imparted to the particle of mass m so that it just reaches the surface of the earth is v. Applying the principle of conservation of energy,
$\frac{1}{2}mv^2 + \left(-\frac{3}{2}\frac{GM}{R}m\right) = \frac{GM}{R}m + 0$
Solving, we get $v = \sqrt{\frac{GM}{R}}$

22 **(d)**
The given system may be regarded as a system of three particles locates at the three vertices of an equilateral triangle of side $2r$
Now, $F_A = F_B$
$= \frac{Gm^2}{(2r)^2} = \frac{GM^2}{4r^2}$
F_A and F_B are inclined to each other at an angle of 60°. If F is the resultant of F_A and F_B, then
$F = \sqrt{3} \times \frac{GM^2}{4r^2}$

23 **(c)**
$g_1 = \frac{g}{\left[1 + \frac{h}{r}\right]^2} = g\left[1 - \frac{2h}{R}\right]$
$W_2 - W_1 =$error in weighing
$= 2mg\left[\frac{h_1}{R} - \frac{h_2}{R}\right] = 2m\frac{GM}{R^2}\frac{h}{R}$
$\left[\because g = \frac{GM}{R^2} \text{ and } h_1 - h_2 = h\right]$
Therefore, $W_2 - W_1 =$error in weighing
$= 2mG\frac{4}{3}\pi R^3\rho\frac{h}{R^3} = \frac{8\pi}{3}Gm\rho h$

24 **(c)**
$mg' = mg - mR\omega^2 \cos^2\phi$
Now, $\frac{3}{5}mg = mg - mR\omega^2$
or $mR\omega^2 = mg - \frac{3}{5}mg$
$= \frac{3}{5}mg$ or $\omega = \sqrt{\frac{2g}{5R}}$

25 **(b)**
$m_1 = 4/3\pi R^2\rho, m_2 = 4/2\pi(2R)^3\rho$, distance between the centres of the two spherical objects, $r = 3R$
$F = \frac{GM_1m_2}{r_2} = G\left(\frac{4}{3}\pi R^3\rho\right)\left(8 \times \frac{4}{3}\pi R^2\rho\right) \times \frac{1}{(3R)^2}$
$= \frac{128}{81}G\pi^2R^4\rho^2$

26 **(b)**
$E = \frac{m}{2} \times \frac{6GM_e}{5r} - \frac{GM_em}{2r} = -\frac{2}{3}\frac{GM_em}{r}$
Which is the total energy of the earth-satellite system
So, semi-major axis of the elliptical orbit is $a = \frac{5r}{4}$
Speed of the satellite at the apogee position is
$v_A = \frac{v_P \times r}{2a - r}$
$= \frac{2}{3}\sqrt{\frac{6GM_e}{5r}}$
For orbit to change to a circle of radius $3r/2 = (2a - r)$, the rocket has to be fired when the satellite is at the apogee position
New orbital speed is $v_0 = \sqrt{\frac{GM_e}{3r/2}} = \sqrt{\frac{2GM_e}{3r}}$
Required change in the orbital speed is
$\Delta v = v_A - v_0 = 0.085\sqrt{\frac{GM_e}{R}}$

27 **(a)**
Energy of each satellite in the orbit $= \frac{-GMm}{2r}$
Total energy of the system before collision,
$E_i = E_1 = E_2 = 2E = -2 \times \frac{GMm}{2r} = -\frac{GMm}{r}$
As the satellites of equal mass are moving in the opposite directions and collide inelastically, the velocity of the wreckage just after the collision is $mv - mv = 2mV$,i.e., $V = 0$
The energy of the wreckage just after the collision will be totally potential and will be
$E_f = \frac{GM(2m)}{r} = -\frac{2GMm}{r}$
As after collision the wreckage comes to standstill in the orbit, it will move along the radius towards the earth under gravity

28 **(d)**
$g_E = (GM)/R^2$
$g_M = \frac{GM^1}{R^{12}} = \frac{GM/10}{R^2/4}$
$= \frac{2}{5}\frac{GM}{R^2}$
$= \frac{2}{5}g_E$

$W_M + W_E \dfrac{2}{5}$

$= 200 \times \frac{2}{5} = 80$ N

29 **(b)**

$a_1 =$ acceleration of first

$= \dfrac{Gm_1m_2}{r_2} \times \dfrac{1}{m_1} = \dfrac{Gm_2}{r^2} = \dfrac{6.67 \times 10^{-11} \times 100}{(100)^2}$

$= 6.67 \times 10^{-13}$ ms^{-1}

$a_2 =$acceleration of second

$= \dfrac{Gm_1}{r^2} = \dfrac{6.67 \times 10^{-11} \times 100}{(100)^2}$

$= 6.67 \times 10^{-13}$ms^{-1}

Net acceleration of approach

$a = a_1 + a_2 = 2 \times 6.67 \times 10^{-13}$ ms^{-1}

Now, $s = \frac{1}{2}at^2$

$1 \times 10^{-2} = \dfrac{1}{2} \times 6.67 \times 10^{-13} \times 2 \times t^2$

Solving, we get $t = 1.41$ days

30 **(a)**

(i)The weight of the body at the centre of the earth is equal to zero because

$g_{\text{centre}} = g\left(1 - \dfrac{d}{R}\right) = g\left(1 - \dfrac{R}{R}\right) = 0$

$\dfrac{g_1}{g} = \left(1 - \dfrac{d}{R}\right) = \dfrac{1}{4} \Rightarrow d = \dfrac{3R}{4}$

So from the centre, $d' = \frac{R}{4}$

31 **(c)**

The required distance is

$x \times \dfrac{d}{\sqrt{\frac{m_2}{m_2} + 1}} = \dfrac{60R}{\sqrt{\frac{81}{1}} + 1} = 6R$

32 **(c)**

At point P, we have $l_1 - l_2 = 0$ (because the gravitational field inside a shell is zero). Hence, $l_1 = l_2$

33 **(b)**

Because the gravitational force is the mutual force, hence the position of centre of mass remains unaffected

$\therefore M_1R_1 = M_2R_2$ or $\frac{R_1}{R_2} = \frac{M_2}{M_1}$

34 **(c)**

Increase in gravitational potential energy is

$\left[\dfrac{GMm}{R + \frac{R}{5}}\right]\left[-\dfrac{GMm}{R}\right] = \dfrac{GMm}{R} - \dfrac{GMm \times 5}{6R}$

$= \dfrac{GMm}{R}\left[1 - \dfrac{5}{6}\right] = \dfrac{GMm}{R} \times \dfrac{1}{6}$

$= \dfrac{mgR^2}{6R} = \dfrac{1}{6}mgR = \dfrac{5}{6}mgh\ [\because R = 5h]$

35 **(b)**

$U_P = -\sum \dfrac{GMm}{r}$

$= -GMm\left[\dfrac{1}{l \sin 60°} + \dfrac{1}{l/2} + \dfrac{1}{l2}\right]$

$= -\dfrac{2}{\sqrt{3}}\dfrac{GMm}{l}[1 + 2\sqrt{3}]$

36 **(b)**

Applying conservation of angular momentum at position A and B

$mv_A \times OA = mv_B \times OB$

Hence, $\frac{v_B}{v_A} = \frac{OA}{OB} = x$

37 **(b)**

From the surface of the earth, the escape velocity is $\sqrt{(2GM)/R}$. From the satellite, the escape velocity is calculated as fallows. By conservation of energy,

$\dfrac{1}{2}mv_2^2 - \dfrac{GMm}{2R} = 0$

$\Rightarrow v_2^2 = \sqrt{\dfrac{GM}{R}} \Rightarrow v_2 < v_1$

38 **(d)**

Let ρ =density of sphere, R =radius of sphere, r =radius of the spherical cavities

Mass of the complete sphere $= \frac{4}{3}\pi R^3\rho = M$

Mass of the removed sphere $= \frac{4}{3}\pi r^3\rho = m$

Here $m = \frac{Mr^3}{R^3} = \frac{M(1)^3}{(4)^3} = \frac{M}{64}$

Now $\vec{I}_R = \vec{I} + \vec{I}_P + \vec{I}_Q$

Here $I = 0$, also $\vec{I}_P = -\vec{I}_Q \Rightarrow I_R = 0$

39 **(a)**

$x = \dfrac{GM}{R^2}$

Again, $\frac{x}{4} = \frac{GM}{(R+h)^2}$ or $x = GM\left(\frac{2}{R+h}\right)^2$

$\therefore \dfrac{1}{R^2} = \left(\dfrac{2}{R+h}\right)^2$

$= \frac{2}{R+h}$ or $R + h = 2R$ or $h = R$

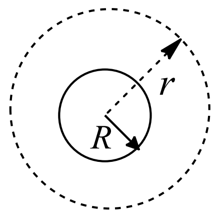
40 **(d)**

$\dfrac{GMm}{r^2} = m\omega_0^2 r$

$\Rightarrow GM = \omega_0^2 r^3$

$\Rightarrow gR^2 = \omega_0^2 r^3$

$\Rightarrow g = \dfrac{\omega_0^2 r^3}{R^2}$



41 **(b)**
According to the problem, as the potential at ∞ incresees by $+10\ \text{J kg}^{-1}$, hence potential will increase by the same amount everywhere (potential gardient will remain constant). Hence, potential at point $P = 10 - 5 = +5\ \text{J kg}^{-1}$

42 **(d)**

$$m_1 = 2m_2 \Rightarrow \frac{m_1}{m_2} = 2$$

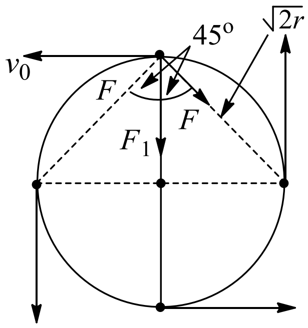
$$r_1 = 4r_2 \Rightarrow \frac{r_1}{r_2} = 4$$

$$T_A^2 \propto r_1^2 \quad \text{and} \quad T_B^2 \propto r_2^3$$

$$\therefore \frac{T_A}{T_B} = \left(\frac{r_1}{r_2}\right)^{3/2} = (4^3)^{1/2}$$

$$\Rightarrow \frac{T_A}{T_B} = 8$$

43 **(a)**



Centripetal force= net gravitational force

$$\Rightarrow mv_0^2 = 2F\cos 45° + F_1 = \frac{2Gm^2}{(\sqrt{2})^2}\frac{1}{\sqrt{2}} + \frac{Gm^2}{4r^2}$$

$$\frac{mv_0^2}{r} = \frac{GM^2}{4r^2}[2\sqrt{2}+1]$$

$$\Rightarrow v_0 = \left[\frac{Gm(2\sqrt{2}+1)}{4r}\right]^{\frac{1}{2}}$$

44 **(c)**

For $r \leq R$:

$$\frac{mv^2}{r} = \frac{Gmm'}{r^2}$$

Here, $\quad m' = \left(\frac{4}{3}\pi r^3\right)\rho_0$

Substituting in Eq. (i) we get

$$v \propto r$$

ie, $v-r$ graph is a straight line passing through orgine.

For $r > R$:

$$\frac{mv^2}{r} = \frac{G\,m\left(\frac{4}{3}\pi R^3\right)\rho_0}{r^2}$$

or $\quad v \propto \frac{1}{\sqrt{r}}$

The corresponding $v-r$ graph will be as shown in option (c).

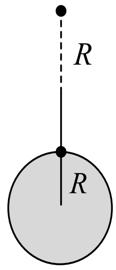
45 **(a)**

$U_t = -GMm/R$ = Initial potential energy of the system

$U_f = -GMm/2R$ = Final PE of the system

$$\therefore \Delta U = U_f - U_t = -GMm\left[\frac{1}{2R} - \frac{1}{R}\right] = \frac{GMm}{2R} \quad \text{(i)}$$

But $g = GM/R^2$



$$dK = -dU + W_{\text{air friction}}$$

$$\therefore GM = gR^2 \quad \text{(ii)}$$

From Eqs. (i) and (ii), $\Delta U = \frac{gR^2m}{2R} = \frac{gRm}{2}$

46 **(d)**

$$L = mv_0(R+h)$$

$$= m\sqrt{\frac{GM}{(R+h)}}(R+h) = m\sqrt{GM(R+h)}$$

i.e., L depends on m, M as well as h

47 **(b)**

Even through the distribution of the mass is unknown, we can find the potential due to the ring on any axial point because from any axial point the entire mass is at the same distance (whatever would be the nature of distribution)

Potential at A due to the ring is $V_A = \frac{GM}{\sqrt{2}R}$

Potential at B due to the ring is $V_B = \frac{GM}{\sqrt{5}R}$

$$d_U = U_f - U_f - U_i = U_B - U_A = m_0(V_B - V_A)$$

$$= \frac{GMm_0}{R}\left[-\frac{1}{\sqrt{5}} + \frac{1}{\sqrt{2}}\right]$$

$$W_{\text{gr}} = -W_{\text{ext}}$$

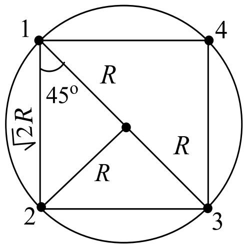
$$W_{gr} = dU = -W_{\text{ext}}$$

$$W_{\text{ext}} = \text{dU} = \frac{GMm_0}{R}\left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{5}}\right]$$

48 **(d)**

$$F_{12} = \frac{GM^2}{2R^2}$$

$$F_{14} = \frac{GM^2}{2R^2}$$



The resultant of these two forces is $(\sqrt{2}GM^2)/$

$2R^2$. Now, $F_{12} = (GM^2)/(4R^2)$
The combined resultant of all the forces is
$\frac{\sqrt{2}GM^2}{2R^2} + \frac{GM^2}{4R^2}$ or $\frac{GM^2}{R^2}\left[\frac{\sqrt{2}}{2} + \frac{1}{4}\right]$
Equating this with centripetal force, we get
$$\frac{Mv^2}{R} = \frac{GM^2}{R^2}\left[\frac{2\sqrt{2}+1}{4}\right]$$
or $v^2 = \frac{GM}{R}\left[\frac{2\sqrt{2}+1}{4}\right]$ or $v = \sqrt{\frac{GM}{R}\left(\frac{2\sqrt{2}+1}{4}\right)}$

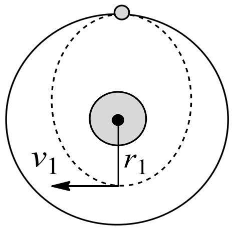
49 **(c)**
Gravitational field inside a shell is zero, but for points outside it, the shell behaves as if whole of its mass is concentrated at its centre. Hence, for a point lying in between the shells there will be a field only due to the inner shell $(= GM_2/r^2)$ because the point will be an external point for the inner shell but internal for the outer shell

50 **(c)**
$$g = \frac{GM}{R^2} = \frac{G\left(\frac{4}{3}\pi R^3\right)\rho}{R^2}$$
or $\quad g \propto \rho R$
or $\quad R \propto \frac{g}{\rho}$
Now escape velocity, $v_e = \sqrt{2gR}$
or $\quad v_e \propto \sqrt{gR}$
or $\quad v_e \propto \sqrt{g \times \frac{g}{\rho}} \propto \sqrt{\frac{g^2}{\rho}}$
$$\therefore (v_e)_{\text{panet}} = (11\text{ ms}^{-1})\sqrt{\frac{6}{121} \times \frac{3}{2}}$$
$$= 3\text{ km s}^{-1}$$

51 **(a)**
$3/5\, mg = mg - mR\omega^2$
$$\omega^2 = g - \frac{3}{5}g \Rightarrow \omega\sqrt{\frac{2}{5}\frac{g}{R}}$$

52 **(b)**
If a body is projected from the surface of the earth with a velocity v and reaches a height h, then according to law of conservation of energy,
$$\frac{1}{2}mv^2 = \frac{mgh}{1 + h/R}$$
Here $v = kv_e = k\sqrt{2gR}$
Therefore, $\frac{1}{2}mk^2 3gR = \frac{mg(r-R)}{1+\frac{(r-R)}{R}}$
$k^2R\left[1 + \frac{r-R}{R}\right] = r - R$ or $k^2r = r - R$
or $r = \frac{R}{1-k^2}$

53 **(a)**
As the velocity of the particle is less than the orbital velocity of the satellite, the particle goes in the elliptical orbit of the semi-major axis less than r



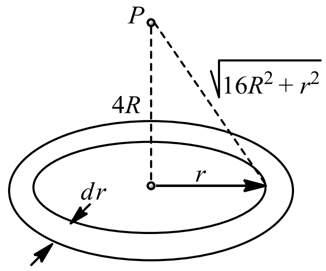
Let r_1 be the minimum distance and v_1 be the velocity of the particle at this position, then
$m_0 \times \sqrt{\frac{2}{3}} \times v_0 r = m_0 v_1 r_1$, where m_0 is the mass of the particle and v_0 is the orbital speed equal to $\sqrt{GM/r}$
$$v_1 r_1 = \sqrt{\frac{2}{3}}v_0 r$$
From energy conservation,
$$\frac{m_0 \times \frac{2}{3}v_0^2}{2} - \frac{Gm_0}{r} = \frac{m_0 v_1^2}{2} - \frac{GMm_0}{r_1}$$
Solving the equations, we get $r_1 = r/2$

54 **(d)**
Change in energy $= 1/(2(GMm)/R) = 1/2Mv_e^2$
So it is independent of angle as gravitational field is conservative in nature

55 **(b)**
According to Kepler's law,
$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}, T_1 = 365\text{ days}, T_2 = ?,\quad R_1 = R, R_2 = \frac{R}{2}$$
$$\Rightarrow T_2 = T_1\left(\frac{R_2}{R_1}\right)^{3/2} = 365\left[\frac{R/2}{R}\right]^{3/2} = 129\text{ days}$$

56 **(c)**
$$\sqrt{\frac{GM}{R+h}} = \frac{1}{2}\sqrt{\frac{2GM}{R}} \Rightarrow h = R$$

57 **(a)**
$$W = \Delta U = U_f - U_i = U_\infty - U_P$$
$$= -U_P = -mV_P$$
$$= -V_P(\text{as } m = 1)$$
Potential at point P will be obtained by in integration as given below. Let dM be the mass of small rings as shown



$$dM = \frac{M}{\pi(4R)^2 - \pi(3R)^2}(2\pi r)dr$$
$$= \frac{2Mr\,dr}{7R^2}$$
$$dV_P = -\frac{G.dM}{\sqrt{16R^2 + r^2}}$$
$$= -\frac{2GM}{7R^2}\int_{3R}^{4R} \frac{r}{\sqrt{16R^2 + r^2}} \cdot dr$$
$$= -\frac{2GM}{7R}\left(4\sqrt{2} - 5\right)$$
$$\therefore \quad W = +\frac{2GM}{7R}(4\sqrt{2} - 5)$$

58 **(d)**

Gravitational potential energy of a pair of particles$= -(Gm^2)/l$

Since we have three pairs, the total gravitational potential energy is $-3Gm^2/l$

59 **(d)**

$g = \frac{GM}{R^2}$ (i)

$g' = \frac{Gm}{\left(\frac{90}{100}R\right)^2} = \frac{100}{81}\frac{GM}{R^2}$ (ii)

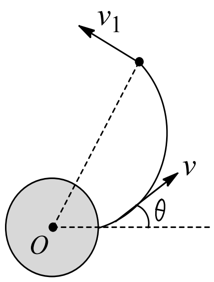
From Eqs. (i) and (ii),

$$g' = \frac{100}{91}g \Rightarrow \frac{g'}{g} = \frac{100}{81}$$
$$\frac{g'}{g} - 1 = \frac{100}{81} - 1$$
$$\therefore \Delta g = \frac{19}{81}g = 23\ \%\text{ of g}$$

So increase is more than 19% of g

60 **(a)**

Let the speed at the maximum height be v_1, then from energy be conservation



$$\frac{mv^2}{2} - \frac{GM_m}{R_e} = \frac{mv_1^2}{2} - \frac{GMm}{5R_e/4}$$

From angular momentum conservation,

$$mv_{R_e}\sin\theta = mv_1 \times \frac{5R_e}{4}$$

Using $v = \sqrt{\frac{gR_e}{2}}$ and solving the equation, we get

$\theta = \sin^{-1}\left(\frac{\sqrt{5}}{4}\right)$

61 **(b)**

$$g' = g\left[1 - \frac{2h}{R}\right] = g\left[1 - \frac{d}{R}\right]$$
$$\frac{2h}{R} = \frac{d}{R} \Rightarrow d = 2h$$

62 **(b)**

$$T = \frac{2\pi R}{v}$$
$$E = \frac{1}{2}mv^2 = \frac{GMm}{R^{n-1}}$$

or $v = \left[\frac{2GM}{R^{n-1}}\right]^{1/2}$

$$\therefore \quad T = \left[\frac{2\pi R}{\sqrt{2GM/R^{n-1}}}\right] = \frac{2\pi}{\sqrt{2GM}} \times R^{(n+1)/2}$$
$$\Rightarrow T \propto R^{(n+1)/2}$$

63 **(c)**

$V_A = \sqrt{\frac{GM}{r_A}}$ and $V_B = \sqrt{\frac{GM}{r_B}}$

Given $r_B = 3r_A$

Now $F_A = \frac{mv_A^2}{r_A} = \frac{M}{r_A}\frac{GM}{r_A} = \frac{GMm}{r_A^2}$

$$F_B = \frac{GMm}{r_B^2}$$
$$\frac{F_B}{F_A} = \frac{R_A^2}{r_B^2} = \frac{1}{9}$$

64 **(a)**

Conserving angular momentum,

$mv_1r_{\min} = mv_2r_{\max}$ or $\frac{v_1}{v_2} = \frac{r_{\max}}{r_{\min}}$

65 **(d)**

We have that $T_1 = 2\pi\sqrt{l/g}$ and $T_2 = 2\pi\sqrt{l/g'}$

$\therefore \frac{T_2}{T_1} = \sqrt{\frac{g}{g'}}$ (i)

Also $g = \frac{GM}{R^2}$

$$\therefore \quad g' = \frac{GM}{(2R)^2} = \frac{GM}{4RL}$$
$$\therefore \quad \frac{g}{g'} = 4 \Rightarrow \frac{T_2}{T_1} = 2$$

66 **(b)**

Gravitational potential at a point on the surface of earth is $-\frac{GM}{R}$

If the earth is assumed to be a solid sphere, then the gravitational potential at the centre of the earth is $-(3/(2GM)/R)$

Decreases in gravitational potential is

$$\frac{R}{2} \times \frac{GM}{R^2} = \frac{Rg}{2}$$

Loss in potential energy is

$$\frac{R}{2} \times \frac{GM}{R^2} \times m$$

Now, gain in kinetic energy = loss in potential energy

Therefore,

$$\frac{1}{2}mv^2 = \frac{1}{2}mgR \text{ or } v = \sqrt{gR}$$

67 **(c)**

$$W = \left[-\frac{GMm}{5R}\right] - \left[\frac{GMm}{3R}\right]$$

$$= \frac{GMm}{3R} - \frac{GMm}{5R}$$

$$= \frac{GMm}{R}\left[\frac{1}{3} - \frac{1}{5}\right] = \frac{2}{15}\frac{GMm}{R}$$

68 **(c)**

The circle lies outside the bigger sphere

Hence, $V_R = V + V_P + V_Q$

Or $V = V_R + V_P + V_Q$

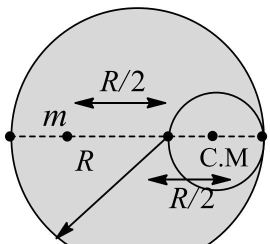
$$= -\frac{Gm}{6} - \left(-\frac{Gm}{r}\right) \times 2$$

Where $r = \sqrt{6^2 + 2^2} = \sqrt{40}$

$$\therefore\ V = -\frac{GM}{2}\left[\frac{1}{3} - \frac{1}{32\sqrt{10}}\right]$$

69 **(b)**

Mass of sphere $M \propto R^3$. If mass of complete sphere is M, then, the mass of cavity will be $M/8$



Using the figure, F = force due to whole sphere−force due to cavity

$$F = \frac{GMm}{R^3}\left(\frac{R}{2}\right) - \frac{G(M/8)m}{(R)^2}$$

$$\frac{GMm}{R^2}\frac{R^2}{2} - \frac{G(M/8)m}{R^2}$$

$$\frac{GMm}{R^2}\left[\frac{4}{8} - \frac{1}{8}\right] = \frac{3}{8}\frac{GMm}{R^2} = \frac{3}{8}mg$$

70 **(c)**

$$V = -\frac{GM}{R} + \left(-\frac{Gm}{x}\right) = -G\left[\frac{M}{R} + \frac{m}{x}\right]$$

71 **(a)**

Binding energy= GMm/R

M = mass of the Sun= 10^{30} kg

m =mass of the earth = 6×10^{24} kg

$R = 1.5 \times 10^{11}$ m

Binding energy of the system is

$$\frac{6.67\times10^{-11}\times10^{30}\times6\times10^{24}}{1.5\times10^{11}} = 2.7 \times 10^{33}\ \text{J}$$

72 **(c)**

$$F_G = \frac{Gm^2}{4R^2} \Rightarrow \frac{Mv^2}{R} = \frac{Gm^2}{4R^2}$$

$$\therefore\ v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$$

73 **(a)**

Suppose that a satellite of mass m describe a circular orbit around a planet of mass M

$$F = \frac{GmM}{r^2}$$

This force must be mass times the centripetal acceleration

$$\therefore F = \frac{mv^2}{r} = m\omega^2 r = m\frac{4\pi^2}{T^2}r$$

$$\therefore\ M = \frac{4\pi^2 r^3}{GT^2}$$

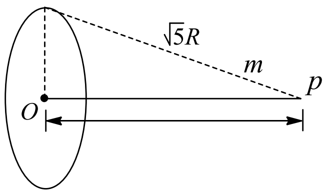
74 **(a)**

In the first case, $v_e = \sqrt{\frac{2GM}{R}}$ or $50 = \sqrt{\frac{2GM}{R}}$

In the second case, $v_e = \left[\frac{2G(4M)^2}{R}\right]^{\frac{1}{2}} = 2\sqrt{\frac{2GM}{R}}$

$= 2 \times 50 = 100\ \text{km s}^{-1}$

75 **(d)**



Gravitational potential at 'P',

$$v_P = \frac{-GM}{\sqrt{5R}}$$

Gravitational potential at 'O',

$$v_o = -\frac{GM}{R}$$

Using work energy theorem,

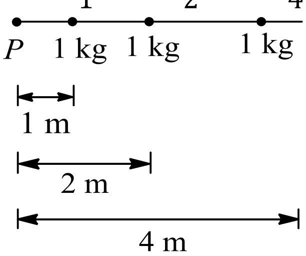
$$W = \Delta K \ \Rightarrow\ m[v_P - v_O] = \frac{1}{2}mv^2$$

$$m = \left[\frac{GM}{R} - \frac{GM}{\sqrt{5R}}\right] = \frac{1}{2}mv^2$$

$$\sqrt{\frac{2GM}{R}\left[1 - \frac{1}{\sqrt{5}}\right]} = V$$

76 **(d)**

$$E = \frac{G \times 1}{1^2} + \frac{G \times 1}{2^2} + \frac{G \times 1}{4^2} + \ldots +$$



$$E = G\left[1 + \frac{1}{4} + \frac{1}{16} + \ldots\right]$$

$$= G\frac{1}{1 - \frac{1}{4}} = \frac{4G}{3}$$

77 **(a)**

Force on the satellite is always towards the earth, therefore, acceleration of satellite S is always directed towards the centre of the earth. Net torque of this gravitational force F about the centre of the earth is zero. Therefore, angular momentum (both in magnitude and direction) of S about the centre of the earth is constant throughout. Since force F is conservative in nature, therefore mechanical energy of the satellite remains constant. Speed of S is maximum when it is nearest to the earth and minimum when it is farthest

78 **(c)**

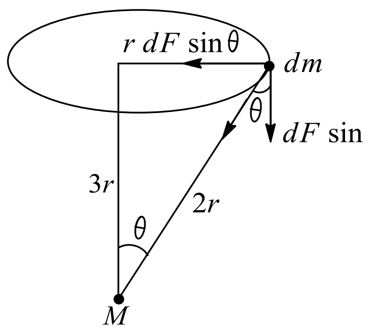
$dF = G\frac{Mdm}{4r^2}$

$F = \sum dF \cos\theta$

$= \sum \frac{GMdm}{4r^2}\cos\theta$

$= \frac{GM}{4r^2} \times \frac{\sqrt{3}r}{2r}\sum dm$

$= \frac{\sqrt{3}GMm}{8r^2}$



Alternative solution :

The gravitational field due to the ring at a distance $\sqrt{3}r$ is given by

$E = \frac{Gm(\sqrt{3}r)}{\left[r^2 + (\sqrt{3}r)^2\right]^{3/2}}$ or $E = \frac{\sqrt{3}Gm}{8r^2}$

The required force is EM, i.e., $(\sqrt{3}Gm)M/8r^2$

Note: ***Follow the central argument in this so point on the ring due to a sphere is equal to mass M located at the centre of the sphere***

79 **(c)**

Total energy, $E = \frac{1}{2}mv^2 - \frac{GmM}{r}$

$= \frac{GmM}{2r} - \frac{GMm}{r} = -G\frac{mM}{2r}$

$r = 2R = R = 3R$

$E = \frac{GmM}{6R}$

Potential energy$= -\frac{GMm}{R}$

Minimum energy required$= \frac{1}{6}\frac{GMm}{R} - \left(\frac{-GMm}{R}\right) =$

$\frac{5}{6}\frac{GMm}{R}$

$= \frac{5}{6}\,mgR$

80 **(b)**

Potential energy of mass m when it is midway between masses M_1 and M_2 is

$U = -\frac{GM_1m}{d/2} - \frac{GM_2m}{d/2}$

$= -\frac{2Gm}{d}(M_1 + M_2)$

According to law of conservation of energy,

$\frac{1}{2}mv_e^2 = \frac{2Gm}{d}(M_1 + M_2)$

Therefore, escape velocity,

$v_e = \sqrt{\frac{4G(M_1 + M_2)}{d}}$

81 **(d)**

$V = \frac{GM}{R} + \left(-\frac{Gm}{r}\right) = -G\left[\frac{M}{R} + \frac{m}{r}\right]$

82 **(d)**

The gravitational field intensity at a point inside the spherical shell is zero

Note: There is NO gravitational field inside a s

83 **(a)**

As all the points on the periphery of either ring are at the same distance from point P, the potential at point P due to the whole ring can be calculated as $V = -(GM)/(\sqrt{R^2 + x^2})$ where x is the axial distance from the centre of the ring. This expression is independent of the fact whether the distribution of mass is uniform or non-uniform

So, at $P, V = -\frac{GM}{\sqrt{2}R} - \frac{G\times 2M}{\sqrt{5}R} = -\frac{GM}{R}\left[\frac{1}{\sqrt{2}} + \frac{2}{\sqrt{5}}\right]$

84 **(c)**

A minimum amount of energy equal to total energy of the moon-earth system has to be given to break (unbound) the system. The Sun is exerting force on the Moon but not providing any energy

85 **(d)**

For the satellite, the gravitational force provides the necessary centripetal force, i.e.,

$\frac{GM_em}{(R+x)^2} = \frac{mv_0^2}{(R+x)}$ or $\frac{GM_e}{R^2} = \text{g}$

$v_0 = \sqrt{\frac{\text{g}R^2}{R + x}}$

86 **(a)**

At the centre of the cavities, if the smaller spheres are not removed, then

$I_R = \frac{GM}{R^3} \times \frac{GM}{64} \times 2 = \frac{GM}{32}$

Also, $\vec{I}_R = \vec{I} + \vec{I}_P + \vec{I}_Q$

$\vec{I} = \vec{I}_R + \vec{I}_P + \vec{I}_Q$

At P, we have $I_P = 0$ and

$I_Q = \frac{Gm}{2} = G\left(\frac{M}{64}\right) \times \frac{1}{4^2}$

Hence, $I = \frac{31GM}{1024}$

87 **(a)**

$g_p = \frac{G\left[M - \frac{10}{100}M\right]}{\left[R + \frac{20}{100}R\right]^2} = \frac{G \times 9M}{10} \times \frac{25}{36R^2} = \frac{5}{8}g$

88 **(b)**

In circular orbit of a satellite of potential energy

$= -2 \times (\text{kinetic energy})$

$= -2 \times \frac{1}{2}m^v = -mv^2$

Just to escape from the gravitational pull, its total mechanical energy should be zero. Therefore , its kinetic energy should be $+mv^2$

89 **(d)**

Let mass of the cavity= M'

Density of the sphere = $M/(4/3\pi R^3)$

Mass of the cavity cut out= M'

$= \frac{4}{3}\pi \frac{R^3}{8} \times \frac{M}{\frac{4}{3}\pi R^3}$

$\therefore \; M' = \frac{M}{8} \Rightarrow F_{\text{net}} = F_{Mm} - F_{M'm}$

$= \frac{GMm}{4R^2} \frac{GM'm}{\left(\frac{5}{2}R\right)^2} = \frac{GMm}{4R^2} - \frac{GMm}{50R^2}$

$F_{\text{net}} = \frac{23}{100} \frac{GMm}{R^2}$

90 **(c)**

$M_A = \sigma 4\pi R_A^3, M_B = \sigma 4\pi R_B^2$

Where σ is surface density

$\Rightarrow \; V_A = \frac{-GM_A}{R_A}, V_B = \frac{-GM_B}{R_B}$

$\frac{V_A}{V_B} = \frac{M_A}{M_B}\frac{R_B}{R_A} = \frac{\sigma 4\pi R_A^2}{\sigma 4\pi R_B^2} \; \frac{R_B}{R_A} = \frac{R_A}{R_B}$

Given $\frac{V_A}{V_B} = \frac{M_A}{M_B} = \frac{3}{4}$

Then $R_B = \frac{4}{3}R_A$

For new shell of mass M and radius R,

$M = M_A + M_B = \sigma 4\pi R_A^2 + \sigma 4\pi R_B^2$

$\sigma 4\pi R^2 = 4\pi(R_A^2 + R_B^2)$

Then $\frac{V}{V_A} = \frac{M}{R}\frac{R_A}{M_A} = \frac{\sigma 4\pi[R_A^2+R_B^2]}{(R_A^2+R_B^2)^{\frac{1}{2}}} \frac{R_A}{\sigma 4\pi R_A^2} = \frac{\sqrt{R_A^2+R_B^2}}{R_A} = \frac{5}{3}$

91 **(a)**

Total mechanical energy is given by

$E = K + u = -\frac{GMm}{2a} - \frac{GMm}{a}$

$= \frac{-GMm}{2a}$

$\frac{GM}{a} = v^2 \; \Rightarrow E = -\frac{1}{2}mv^2$

92 **(c)**

$g = \frac{Gm}{R^2}$

$M = \frac{4}{3}\; \omega R^2 \rho$

So $\rho = \frac{3g}{4\pi RG}$

93 **(d)**

Inside the earth $g' = g\left[1 - \frac{h}{R}\right]$

At centre, $g' = g\left[1 - \frac{R}{R}\right] = 0$

94 **(a)**

From the principle of conserving angular momentum, we have

$mvR = mv'r \quad$ (i)

[v' =speed when spaceship is just touching the planet]

From conserving of energy, we have

$\frac{1}{2}mv^2 = \frac{1}{2}mv'^2 - \frac{GMm}{r} \quad$ (ii)

Solving Eqs. (i) and (ii), we get

$R = \frac{r}{v}\left[v^2 + \frac{2GM}{r}\right]^{1/2}$

95 **(d)**

$\frac{mv^2}{r} = \frac{GMm}{r^{5/2}}\left[\because F \propto \frac{1}{r^{5/2}}\right]$

or $\; r\omega^2 = \frac{GM}{r^{5/2}}$

or $\; r\frac{4\pi^2}{T^2} = \frac{GM}{r^{5/2}}$

or $\; T^2 \propto r^{5/2}$ or $T^2 \propto r^{3.5}$

96 **(c)**

$V_e = 11.2$ km/s

$\frac{mv'^2}{R} = \frac{GMm}{R^2}$

$v'^2 = \frac{GM}{R^2}R$

$v' = \sqrt{10 \times 6400000} = 8$ km/s

Therefore, the additional velocity = $(11.2 - 8) =$ 3.2 km/s

97 **(c)**

$\frac{40}{80} = \frac{R^2}{(R+h)^2} = \sqrt{2}R = R + h$

$h = (\sqrt{2} - 1)R$

98 **(a)**

As angular momentum is conserved, hence

$I_1\omega_1 = I_2\omega_2$

Or $\; MR_1^2\omega_1 = MR_1^2\omega_2$ or $\; MR_1v_1 = MR_2v_2$

$\left(\because \omega = \frac{v}{R}\right)$

Or $R_1 v_1 = R_2 v_2$ or $VR = vr$

$\therefore V = \frac{vr}{R}$

99 **(c)**

$T^2 \propto R^3$

$T^2 = KR^3 \Rightarrow (24)^2 = K(36000)^3$

$K = \frac{1}{9 \times 10^4 \sqrt{10}}$

$T' = \frac{1}{9 \times 10^4 \sqrt{10}} (6400)^{3/2} = 2h$

100 **(b)**

Using conservation of energy:

$\frac{1}{2} m (K v_e)^2 - \frac{GMm}{R} = -\frac{GMm}{R+h}$

Use $v_e = \sqrt{2gR}$ and $GM = gR^2$

and to solve we get $h + R = R/(1 - K^2)$

101 **(c)**

$g = \frac{GM}{R^2}, g' = \frac{GM}{(0.99R)^2}$

$\therefore \frac{g'}{g} = \left(\frac{R^2}{0.99R}\right)^2 \Rightarrow g' > R$

102 **(c)**

$\frac{mv^2}{R} = \frac{GMm}{R^2}$ (i)

and $\frac{mv'^2}{R'} = \frac{GMm}{R'^2}$ (ii)

Dividing Eq. (i) by Eq. (ii), $\frac{v^2}{v'^2} = \frac{R'}{R}$

$\Rightarrow \frac{v'}{v} = \sqrt{\frac{R}{R'}} = \sqrt{\frac{2}{3}} \quad \left[\because R' = \frac{3R}{2}\right]$

$\Rightarrow v' = \sqrt{\frac{2}{3}} v$

103 **(d)**

Let the mass of the particle be m

PE at a distance of $R' = (GMm)/R'$

PE at a distance of $R_e = -(GMm)/R_e$

Decrease in PE = Increase in KE

$\Rightarrow -\frac{GMm}{R'} + \frac{GMm}{R_e} = \frac{1}{2} mv^2$

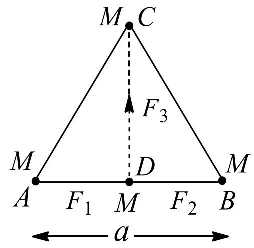
$v^2 = 2GM\left[\frac{1}{R_e} - \frac{1}{R'}\right] \Rightarrow v^2 = \frac{2GM}{R_e}\left[1 - \frac{R_e}{R'}\right]$

$\Rightarrow v^2 = \frac{2GM}{R_e}\left(1 - \frac{R_e}{R'}\right) \Rightarrow v = \sqrt{\frac{2GMR_e}{R_e^2}\left(1 - \frac{R_e}{R'}\right)}$

$\therefore v = \sqrt{2gR_e\left(1 - \frac{R_e}{R'}\right)}$

104 **(b)**

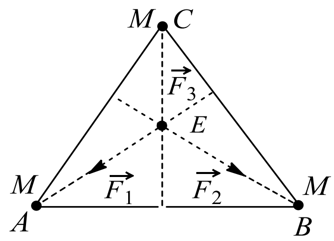
1. Gravitational force on the particle placed at the midpoint D of side AB of length a is $= \vec{F}_1 + \vec{F}_2 + \vec{F}_3$. But $\vec{F}_1$ and $\vec{F}_3$ are equal in magnitude and opposite in direction



$F = F_3 = \frac{4GMm}{3a^2}$

$= \frac{4GM^2}{3a^2} \quad \left[\because CD^2 = \frac{3a^2}{4}\right]$

2. Gravitational force on the particle placed at the point of intersection of three medians $= \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$, since the resultant of $\vec{F}_1$ and $\vec{F}_2$ is equal and opposite to $\vec{F}_3$



105 **(a)**

Here time period of the satellite w.r.t. an observer on the equator is 24 h and the satellite is moving from west to east, so angular velocity of the satellite w.r.t. earth's axis of rotation (considered as fixed) is $\omega = \frac{2\pi}{T_s} + \frac{2\pi}{T_e}$, where T_s and T_e are time periods of satellite and earth, respectively

$\omega = \frac{\pi}{6} (h^{-1}) = 1.45 \times 10^{-4}$ rad s^{-1}

From, $v = \sqrt{\frac{GM}{r}}$

$r\omega = \sqrt{\frac{GM}{r}}$

$r^{\frac{3}{2}} = \sqrt{\frac{GM}{r}}$

$r = 2.66 \times 10^7 \text{m} = 2.66 \times 10^4 \text{km}$

106 **(b)**

$mg = mR\omega^2$

$R =$ radius of earth

$\omega = \sqrt{\frac{g}{R}}$

$T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{64000}$

$= 2\pi \times 800 \text{ s} = \frac{2\pi \times 800}{3600} \text{h} = 1.36$

$= 0.14$ h

107 **(d)**

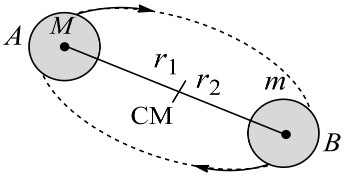
Inside the spherical shell, V is constant, so from energy conservation,

$\frac{-GMm}{3R} = \frac{mv^2}{2} - \frac{GMm}{R}$

$\frac{v^2}{2} = \frac{GM}{R}\left[1 - \frac{1}{3}\right] = \frac{GM}{R} \times \frac{2}{3}$ or $v = \sqrt{\frac{4GM}{3R}}$

108 **(d)**

If the binary stars are rotating about their common centre of mass, then they have to be on the same line all the time, otherwise the centre of mass will be changing. Their angular velocities have to be the same although in the same time, the smaller mass will describe a bigger circle



But as $\omega = \frac{2\pi}{T}$ so, $T_A = T_B$

109 **(c)**

As the sphere is having non-uniform mass density, so nothing can be predicted about the variation of gravitational field intensity

110 **(c)**

$T^2 \propto a^3$

$T_1^2 \propto (10^{13})^3$

$(T^2)^2 \propto (10^{12})^3$

$\therefore \frac{T^1}{T^2} = \left(\frac{10^{13}}{10^{12}}\right)^{\frac{3}{2}} \Rightarrow \frac{T^1}{T^2} = 10/\sqrt{10}$

111 **(d)**

The point lies inside both the shells, hence gravitational field due to both is zero

112 **(c)**

During total eclipse, total attraction due to the Sun and the Moon,

$F_1 = \frac{GM_sM_e}{r_1^2} + \frac{GM_mM_e}{R_2^2}$

When the Moon goes on the opposite side of the earth, the effective force of attraction is

$F_2 = \frac{GM_sM_e}{r_1^2} - \frac{GM_mM_e}{r_2^2}$

Change in force, $\Delta F = F_1 - F_2 = \frac{2GM_mM_e}{r_2^2}$

Change in acceleration of the earth,

$\Delta a = \frac{\Delta F}{M_e} = \frac{2GMm}{r_2^2}$

Average force on the earth,

$F_{\text{av}} = \frac{F_1 + F_2}{2} = \frac{GM_sM_e}{r_1^2}$

Average acceleration of the earth,

$a_{\text{av}} = \frac{F_{\text{av}}}{M_e} = \frac{GM_s}{r_1^2}$

Percentage change in acceleration is

$\frac{\Delta a}{a_{\text{av}}} \times 100 = \frac{2GM_m}{r_2^2} \times \frac{r_1^2}{GM_s} \times 100$

$= 2\left(\frac{r_1}{r_2}\right)^2 \frac{M_m}{M_s} \times 100$

113 **(d)**

$p = \rho_{\text{atm}} gh = \frac{m}{\frac{4}{3}\pi[(R+h)^3 - R^3]} \frac{GM}{R^2} h$

$\Rightarrow p = \frac{m}{\frac{4}{3}\pi R^3\left[\left(1 + \frac{h}{R}\right)^3 - 1\right]} \times \frac{G\frac{4}{3}\pi R^3 \ell}{R^2} \times h$

$= \frac{m}{\frac{4}{3}\pi R^3\left[1 + \frac{3h}{R} - 1\right]}$

$\times G\frac{4}{3}\pi R \ell h \left(\text{Here } M = \frac{4}{3}\pi R^2 \ell\right)$

Hence, $p = \frac{mG\rho}{3R}$

$\therefore \ m \propto \frac{pR}{\rho}$

114 **(a)**

$U_g = -\frac{GM_1M_2}{R}$

$\Rightarrow U_f - U_i = -\frac{GMm}{2R} - \frac{GMm}{R}$

$\Delta u = \frac{GMm}{2R} = \frac{mgR}{2} \quad \left(\because g = \frac{GM}{R^2}\right)$

115 **(b)**

$F = (GMm)/R^2 = mg$

As the mass remains the same and radius, i.e., distance from the centre is also same, therefore g will remain the same, i.e., 10 ms^{-2}

116 **(a)**

At a distance x from the centre of earth, the gravitational force

$F = \frac{GMm}{x^2}$

$dW = Fdx = \frac{GMm}{x^2} dx$

We get $W = \int_R^{R+h} \frac{GMm}{x^2} dx = GMm\left[\frac{1}{R} - \frac{1}{R+h}\right]$

$= gR^2 m\left[\frac{1}{R} - \frac{1}{R+h}\right]$

Therefore, the gain in potential energy is

$$gR^2m\left[\frac{1}{R}-\frac{1}{2R}\right]=\frac{mgR}{2}$$

117 **(b)**

As gravitational force is a conservative force, work done is independent of path

$\therefore\ W_1=W_2=W_3$

118 **(a)**

i. g' =accleration due to gravity on the Moon

$$=G\frac{4}{3}\pi R'\rho'=G\frac{4}{3}\pi\times\frac{R}{4}\times\frac{2}{3}\rho=\frac{g}{6}$$

Now, $h_m=\frac{u^2}{2g}$ or $h_m\propto\frac{1}{8}$

Hence $h'_m g'=h_m g$

or $h'_m=6h_m=6\times0.5=3$ m

ii. $t=\frac{2u}{g}$ or $t\propto\frac{1}{g}$

$\therefore t'g'=tg$ or $\frac{t'}{t}=\frac{g}{g'}=\frac{6}{1}$

119 **(c)**

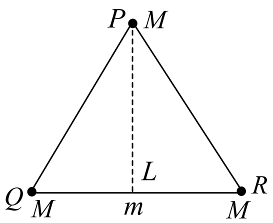
$PE=\frac{-GM}{2R}$ (at height R)

$$=-\frac{GMR_m}{2R^2}=-\frac{1}{2}mgR$$

120 **(b)**

Force on m due to masses at Q and R is zero. So, the net force is due to the mass at P

Hence, $F=(GMm)/(PL)^2$

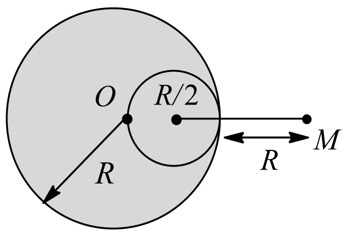


Now, $PL=l\sin60°=l\frac{\sqrt{3}}{2}$

$$F=\frac{4GMm}{3l^2}$$

121 **(d)**

Mass of the cavity= $M/8$ if mass of the sphere $=M$ as volume of the cavity is 1/8th of the sphere



$$F_2=\frac{GMm}{4R^2}=\frac{GMm}{8\left(\frac{3}{2}R\right)^2}=\frac{GMm}{R^2}\left[\frac{1}{4}-\frac{1}{18}\right]$$

$$=\frac{GMm}{R^2}\left[\frac{9-2}{36}\right]=\frac{GMm}{R^2}\frac{7}{36}$$

$$\therefore\frac{F_1}{F_2}=\frac{36}{4\times7}=\frac{9}{7}$$

122 **(c)**

Equal time is taken to cover equal area

123 **(b)**

$\frac{mv^2}{R}\propto R^{-5/2}$, $\therefore v\propto R^{-3/4}$

Now, $T=\frac{2\pi R}{v}$ or $T^2\propto\left(\frac{R}{v}\right)^2$

$T^2\propto\left(\frac{R}{R^{-3/4}}\right)^2$ or $T^2\propto R^{7/2}$

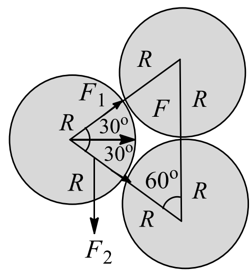
125 **(c)**

If x_1 and x_2 are the distances covered by the two bodies, then $x_1+x_2=9R$

Also, $Mx_1=5Mx_2\ \Rightarrow x_2=\frac{x_1}{5}$

$x_1+\frac{x_1}{5}=9R\ \Rightarrow\ x_1=7.5\,R$

126 **(d)**



$\vec{F}=\vec{F}_1+\vec{F}_2$

As $|\vec{F}_1|=|\vec{F}_2|$

$\therefore\ |\vec{F}|=2F_1\cos30°$

$$=2\frac{GM^2}{(2r)^2}\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{4}\frac{GM^2}{r^2}$$

127 **(d)**

$g_p=g_e;\frac{GM_p}{R_p^2}=\frac{GM_e}{R_e^2};\frac{4}{3}\pi R_p d_p=\frac{4}{3}\pi R_e d_e$

or $2R_e d_p=R_e d_e\Rightarrow\frac{d_p}{d_e}=\frac{1}{2}$

128 **(a,c,d)**

Statement (a) is incorrect; there is no method by which a body can be shielded from the gravitational field of another body. Statement (b) is correct. Statement (c) is incorrect. As the earth rotates about its axis, a body on the surface of the earth also rotates with it. Since the body is in a rotating (non-inertial) frame, it experiences an outward centrifugal force against the inward force of gravity. As a result, the acceleration due to gravity decreases due to rotation. Statement (d) is incorrect, the forces are equal in magnitude but opposite in direction. Hence, choices (a), (c) and (d) are wrong

129 **(b,d)**

The acceleration due to gravity is $g = \frac{GM}{R^2}$

The new value of g would be

$$g' = \frac{G(0.99M)}{(0.99R)^2} = 1.01\frac{GM}{R^2}$$

Thus, g would increase by about 1%. The new escape velocity would be

$$v_e = \sqrt{\frac{2 \times 0.99M \times G}{0.99R}} = \sqrt{\frac{2MG}{R}} = v_e$$

Thus, the escape velocity will remain unchanged. The potential energy of a body of mass m on the earth's surface would be

$$-\frac{Gm(0.99\,M)}{(0.99\,R)} = -\frac{GmM}{R}$$

Thus, the potential energy will also remain unchanged. Hence, the correct choices are (b) and (d)

130 **(a,b,c)**

When the particle is dropped in the tunnel at its one end, it starts to execute simple harmonic motion with mean position as the centre of the tunnel. Hence, acceleration of the particle is directly proportional to its distance from the centre of the tunnel. It means, option (d) is correct

The particle has zero velocity at one end because it is dropped from rest at that end, therefore, ends of the tunnel are its extreme positions. Hence, at the other end, the velocity will become equal to zero. It means option (a) is incorrect

Velocity of a particle executing S.H.M. is maximum at its mean position. Hence, at the centre of the tunnel, its velocity will be maximum possible. Hence, option (b) is incorrect

When the particle moves from extreme position to centre of the tunnel, its velocity increases. It means KE increases or PE decreases. But initial potential energy of the particle is negative. Hence, potential energy becomes more negative and it is least at the mid-point of the tunnel because KE is maximum there. It means the gravitational PE can never be equal to zero. Hence, option (c) is also incorrect

131 **(b,c,d)**

$g = \frac{GM}{R^2}, v_e = \sqrt{\frac{2GM}{R}}$ and $U = \frac{-GMm}{R}$

$\therefore g \propto \frac{M}{R^2}, v_e \propto \sqrt{\frac{M}{R}}$ and $U \propto \frac{M}{R}$

If both mass and radius are increased by 0.5% then v_e and U remains unchanged where as g decrease by 0.5%

132 **(b,c,d)**

Distances of the two satellites from the centre of the earth are $r_1 = 2R$ and $r_2 = -8R$, respectively (R =earth's) radius. Their potential energies are

$V_1 = -\frac{GmM}{r_1}$ and $V_2 = -\frac{GmM}{r_2}$

Their ratio is $\frac{V_1}{V_2} = \frac{r_2}{r_1} = \frac{8R}{2R} = 4$

The kinetic energy of a satellite can be obtained from the relation

$$\frac{mv^2}{r} = \frac{GmM}{r^2}$$

or $K = \frac{1}{2}mv^2 = \frac{GmM}{2r}$

$K_1 = \frac{GmM}{2r_1}$ and $K_2 = \frac{GmM}{2r_2}$

The ratio of their kinetic energies is

$$\frac{K_1}{K_2} = \frac{r_2}{r_1} = \frac{8R}{2R} = 4$$

Their total energies are

$$E_1 = -\frac{GmM}{r_1} + \frac{GmM}{2r_1} = -\frac{GmM}{2r_1}$$

and $E_2 = -\frac{GmM}{r_2} + \frac{GmM}{2r_2} = \frac{GmM}{2r_2}$

Their ratio is $\frac{E_1}{E_2} = \frac{r_2}{r_1} = \frac{8R}{2R} = 4$

Hence, the correct choices are (b), (c) and (d)

133 **(a,b,c,d)**

For all the points, $E_P > E_K$ (numerically). So, the total energy is negative. Thus, the system is a bound system is a bound system corresponding to all the points

134 **(a,c)**

If the speed of the orbiting satellite is made $\sqrt{2}$ times, then the satellite will escape because its velocity becomes equal to escape velocity

This implies that the speed is increased by 41% and the KE is doubled

135 **(a,b,c)**

1. For a proper explanation, the earth-Moon system will be regarded as an isolated system. At a particular point in space, the gravitational force of attraction of the earth on astronaut will be balanced by the gravitational force of attraction of the Moon

2. $V = -GM/R$. As R decreases, GM/R increases or – GM/R decraeses

3. In case of the spherical shell, plot of I

versus r is discontinuous

136 **(c,d)**

If two satellites of different masses are revolving in the same orbit, then they have the same time period and speed because

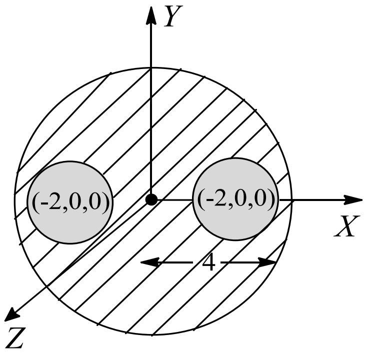
$$v_0 = \sqrt{\frac{GM}{r}}$$

and $T = 2\pi\sqrt{\frac{r^3}{GM}}$

Angular momentum ($= mvr$) and energy ($= -(GMm)/2r$) both depend on the mass of the satellite. So, options (c) and (d) are correct

137 **(a,c,d)**

The gravitational field intensity at point O is zero (as the cavities are symmetrical with respect to O)



Now the force acting on a test mass m_0 placed at O is given by

$F = m_0 E = m_0 \times 0 = 0$

Now, $y^2 + z^2 = 36$ represents the equation of a circle with centre (0, 0, 0) and radius 6 units. The plane of the circle is perpendicular to the x-axis. Since the spherical mass distribution behaves as if the whole mass is at its centre (for a point outside the sphere) and since all the points on the circle are equidistant from the centre of the sphere, the circle is a gravitational equipotential

138 **(b,c)**

For an earth satellite moving in a circular orbit, the centripetal force required for its circular motion is provided by the gravitational force exerted by the earth on it

It means the resultant force on the astronaut is equal to the gravitational force exerted by the earth on him. Hence, no reaction is exerted by the floor of the satellite on him

In other words, his acceleration towards the earth's centre (centripetal acceleration) is exactly equal to acceleration caused by the gravitational force

Hence, options (b) and (c) are correct

139 **(a,b,c,d)**

Velocity of nature of path of satellite

(i)$v = v_0$ Circular path around the earth

(ii)$v < v_0$ Elliptical path and body returns to earth

(iii)$v > v_0$ but v_eElliptical path around the earth and will not escape

(iv)$v = v_e$Parabolic path and it escape from the earth

(v)$v > v_e$Hyperbolic path and escape from earth

140 **(a,b,d)**

If a satellite is revolving in a circular orbit then the magnitude of its KE is equal to half of the magnitude of gravitational potential energy. But the potential energy is negative. That is why the total energy of a revolving satellite is negative. If the satellite experience resistance due to cosmic dust, then it follows a spiral path of decreasing radius. During the process of motion, its potential energy decreases. It means PE becomes more negative or the magnitude of PE increase. Hence, magnitude of KE also increases. Therefore, options (a) and (b) are correct

Since cosmic dust exerts a tangential force on the satellite, therefore it experiences a retarding moment, hence, its angular momentum does not remain conserved. Hence, option (c) is incorrect. Since KE of the satellite increases, therefore speed of the satellite increases. Power acting against the resisting force is equal to force × speed, therefore it increases. Hence, thermal power generated increases with time. Hence, ultimately it will turn off. Therefore, option (d) is correct

141 **(a,b)**

$g = \frac{4}{3} m\rho G r \quad \therefore g \propto r$ if $r < R$

$g = \frac{GM}{r^2} \quad \therefore g \propto \frac{1}{r^2}$ if $r < R$

if $r_1 < R$ and $r_2 < R$

then $\frac{F_1}{F_2} = \frac{g_1}{g_2} = \frac{r_1}{r_2}$

if $r_1 > R$ and $r_2 > R$

then $\frac{F_1}{F_2} = \frac{g_1}{g_2} = \left(\frac{r_2}{r_1}\right)^2$

142 **(a,b,c,d)**

1. Factual statement

2. $g = \frac{GM}{R^2}$ or $M = \frac{gR^2}{G}$

3. The weight of a body on the surface of earth is not determined by the orbital motion of earth around Sun

143 **(a,c,d)**

1. $g = \frac{GM}{R^2} = \frac{G}{R^2} \times \frac{4}{3}\pi R^3 \rho$

or $\rho = \frac{3g}{4\pi GR}$

2. Acceleration due to gravity varies from plane to place, hence, not correct

3. $v_e' = \sqrt{\frac{2G(2M)}{\frac{R}{2}}} = 2\sqrt{\frac{2GM}{R}} = 2 \times$ $11.2 \text{ km s}^{-2} = 22.4 \text{ km s}^{-1}$

4. Factual statement

144 **(a,c,d)**

Work done is independent of the path chosen and depends only on the initial and final positions of the object. Also the work done on any closed path in a gravitational field will be zero. Since, every point on the surface of the earth is at the same potential, no work is done for points on the surface of the earth. Hence, the correct choices are (a), (c) and (d)

145 **(a,b,c,d)**

From Kepler's law, $T^2 \propto R^3$

So, $\frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{\frac{3}{2}}$

$R_2 = \left(\frac{T_2}{T_1}\right)^{\frac{2}{3}} \times R_1 = \left(\frac{24}{6}\right)^{\frac{2}{3}} \times 1.28 \times 10^4 \text{ km}$

$= 3.22 \times 10^4$ km

Orbit velocity of S_1 is

$v_1 = \frac{2\pi R_1}{T_1} = \frac{2\pi \times 1.28 \times 10^4}{4}$

$= 0.64\pi \times 10^4$ km

Orbital velocity of S_2 is

$v_2 = \frac{2\pi R_2}{T_2} = \frac{2\pi \times 1.28 \times 10^4}{4}$

$= 0.64\pi \times 10^4$ km

At $t = 12$ h the two satellites are closest to each other and after every 24 h they come at the same position relative to each other. It is clear that direction of v_2 w.r.t. v_1 is changing continuously in both magnitude and direction

Angular velocity of S_2 w.r.t. S_1 at $t = 12$ h is

$\omega = v_1 + v_2/R_2 - R_1 = 0.468\pi \text{ rad s}^{-1}$

146 **(b,c)**

$$F = \begin{cases} \frac{GMm}{r^2} & r \geq R \\ \frac{4\pi G\rho rm}{3} & r < R \end{cases}$$

where, ρ is density of earth

147 **(c,d)**

Linear speed is $v = r\omega$. At the equator, the radius of the earth r is maximum. Therefore, $\omega = \frac{v}{r}$ is minimum. Also,

$\omega = \frac{2\pi}{T} = \frac{2 \times 3.14}{24 \times 60 \times 60}$

$= 7.3 \times 10^{-5} \text{ rad s}^{-1}$

Hence, the correct choices are (c) and (d)

148 **(a,d)**

$U_i = -\frac{GMm}{R}$

$U_f = -\frac{GMm}{R+h}$

Work done = Change in gravitational potential energy

$\Rightarrow W = U_f - U_i = -GMm\left(\frac{1}{R+h} - \frac{1}{R}\right)$

$\Rightarrow W = -\frac{GMm}{R}\left[\left(1 + \frac{h}{R}\right)^{-1} - 1\right]$

$\Rightarrow W = -\frac{GMm}{R}\left[1 - \frac{h}{R} - 1\right]$

$\Rightarrow W = \frac{GMmh}{R^2}$

$\Rightarrow W = mgh$ for $h \ll R$

Also, $U_i = -\frac{GMm}{R}$, and

$U_f(h = R) = -\frac{GMm}{R+R} = -\frac{GMm}{2R}$

$\Rightarrow W = U_f - U_i = \frac{GMm}{2R}$

$\Rightarrow W = \frac{1}{2} mgR \left\{\because g = \frac{GM}{R^2}\right\}$

149 **(b,c)**

$W = U_2 - U_1 = \frac{GMm}{R+h} - \left(-\frac{GMm}{R}\right)$

$= -GMm\left[\frac{1}{R} - \frac{1}{R+h}\right]$

$= gR^2m\left[\frac{h}{R(R+h)}\right] = \frac{mgRh}{R+h}$

For $h << R, W = mgh$

For $h = R, W = \frac{mgR}{2}$

150 **(b,c,d)**

$T^2 = \frac{4\pi^2}{GM}\left(\frac{r_A + r_P}{2}\right)^3 \left\{\because r = \frac{r_A + r_P}{2}\right\}$

$\Rightarrow T^2 = \frac{\pi^2}{2GM}(r_A + r_P)^3$

By law of conservation of angular momentum

$mv_Ar_A = mv_Pr_P$

$\Rightarrow\ v_A r_A = v_P r_P$

151 **(b,d)**

Escape velocity $V_{es} = \sqrt{\frac{2GM}{R}}$

Surface area of $P = A = 4\pi R_P^2$

Surface area of $Q = 4A = 4\pi R_Q^2$

$\Rightarrow R_Q = 2R_P$

If $M_P = M = \rho\frac{4}{3}\pi R^3$ (Let R be the radius of P)

So $M_Q = \rho\frac{4}{3}\pi(2R)^3 = 8M$

So, $M_R = M_P + M_Q = 9M$

For planet $R, 9M = \rho\frac{4}{3}\pi R_r^3$

So, $R_r = (9)^{1/3}.R$

Escape velocity from $P, V_P = \sqrt{\frac{2GM}{R}}$

$$V_Q = \sqrt{\frac{2G(8M)}{2R}} = 2\sqrt{\frac{2GM}{R}}$$

$$V_R = \sqrt{2G\frac{(9M)}{(9)^{1/3}R}}$$

$$= 9^{1/3}\sqrt{\frac{2GM}{R}}$$

So $V_R > V_Q > V_P$ and $\frac{V_P}{V_Q} = \frac{1}{2}$

152 **(a,b)**

For $r > R$, the gravitational field is $F = GM/r^2$

$\therefore\ F_1 = \frac{GM}{r_1^2}$ and $F_2 = \frac{GM}{r_2^2}\ \Rightarrow\ \frac{F_1}{F_2} = \frac{r_2^2}{R_1^2}$

For $r > R$

The gravitational field is $F = \frac{GM}{R^3}\times r$

$\therefore\ F_1 = \frac{GM}{R^3}\times r_1$ and $F_2 = \frac{GM}{R^3}\times r_2$

$$\Rightarrow\ \frac{F_1}{F_2} = \frac{r_1}{r_2}$$

153 **(b,c)**

Statement (a) is incorrect; the magnitude of the force is 6.67×10^{-11} N. Statement (b) is correct. If the escape velocity for a planet is high, gases are not able to escape. Statement (c) is also correct. Statement (d) is incorrect; force of friction arises due to electrical forces. Hence, correct choices are (b) and (c)

154 **(c,d)**

$$V = \frac{GM}{R}$$

$$\Rightarrow\ V = \frac{GM}{R^2}R$$

$$\Rightarrow\ V = gR$$

155 **(b,d)**

If universal gravitational constant G starts to decrease, the gravitational force between the Sun and the earth will also start to decrease. Therefore, the earth will try to follow a path of larger radius. Hence, its period of revolution round the Sun will increase. Therefore, duration of the year will increase. But rotational motion of the earth about its own axis will remain unchanged; hence, period of rotational will remain unchanged or length of day will remain unchanged. Hence, option (a) is incorrect, (b) is correct and (c) is incorrect

Since the radius of the circular path of the earth will increase or the earth will follow a spiral path of increasing radius, therefore its PE will increase but PE is always negative, so the magnitude of PE will decrease. Hence, KE will also decrease. Therefore, option (d) is correct

156 **(b,c)**

Since every element of the hemispherical shell is at a distance R from the centre of curvature, therefore gravitational potential at its centre$= -GM/R$. It is not equal to GM/R. Hence, option (a) is incorrect

Gravitational field strength at a point, lying on the axis of a thin uniform circular ring of radius R, can be calculated by considering two equal elemental lengths of the ring such that these are lengths lying on the ends of a diameter and thus the expression given in option (b) is obtained. Hence, option (b) is correct

Newton's law of gravitation is applicable to only those bodies which have spherically symmetric distribution of mass. For example, if we consider two solid hemispheres of equal radii; one made of wood and the other made of iron. These two hemispheres are joined together to form a shape of complete sphere, we cannot apply Newton's gravitational law to this sphere. So, option (c) is correct

157 **(a, c)**

According to right Hand Thumb Rule "curl the fingers of right hand in the direction of rotation then thumb gives the direction of the areal velocity/angular momentum".

158 **(a,c)**

The centre of mass of the double star system remains stationary and both the stars revolve around in circular orbits which are concentric with the centre of mass

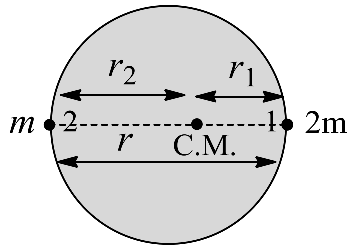
The distance of centre of mass from the heavier

star is equal to $\left(\frac{Mr+2\,M.0}{M+2M}\right)=\frac{r}{3}$
Hence the heavier star revolves in a circle of radius $r/3$ while the lighter star in a circle of radius $2r/3$. Hence, option (a) is incorrect
To calculate period of revolution of a double star system, concept of reduced mass can be used
The reduced mass of the system is equal to
$$\frac{(M)(2M)}{(M+2M)}=\frac{2M}{3}$$
Hence, the period of revolution will be equal to $\frac{2\pi}{\sqrt{2\frac{GM}{3}}}r^{\frac{3}{2}}$ where r is distance between two stars.
Hence, option (b) is correct
Kinetic energy of a star will be equal to $1/2\ mv^2$ where v is speed of the star which is equal to (radius of its circular orbit)$\times\ \omega$
Hence, KE of heavier star is
$$E_1=\frac{1}{2}(2M)\left(\frac{r}{3}\omega\right)^2$$
And that of lighter star, $E_2=\frac{1}{2}M\left(\frac{2r}{3}\omega\right)^2$
It means, KE of the lighter star is twice of the heavier star. Hence, option (c) is incorrect

159 **(a,b,d)**
$$F(r')=-\frac{k}{r^n}$$
$$\Rightarrow\ U(r)=-\int F(r)dr=-\frac{k}{(n-1)}\frac{1}{r^{n-1}}$$
If L is the angular momentum of the particle of mass m in an orbit of radius r then
Kinetic energy $=\frac{L^2}{2I}=\frac{L^2}{2mr^2}=k(r)$
Since, total energy $=E(r)=U(r)+K(r)$
$$\Rightarrow\ \ E(r)=-\frac{k}{(n-1)}\frac{1}{r^{n-1}}+\frac{L^2}{2mr^2}$$
The criterion that a circular orbit of radius r_0 be stable is that $E(r)$ is minimum
For $E(r)$ to be minimum two conditions must be fulfilled.
$$\Rightarrow\ \left.\frac{\partial E}{\partial r}\right|_{r=r_0}=0\text{ and }\left.\frac{\partial^2 E}{\partial r^2}\right|_{r=r_0}>0$$
Using both these conditions, we get
$$(3-n)\frac{L^2}{m}>0$$
This is possible only when $n<3$
We also note that inverse square law belongs to this category
$n=-1$ also gives stable circular orbits (Law of direct distance)
But $n=3$ gives circular orbits which are unstable (Inverse cube law)

160 **(a,b,c)**
Choices (a), (b) and (c) are correct. When the particle exerts a force on another particle; the second particle exerts an equal and opposite force on the first particle. Choice (d) is incorrect because although G is constant everywhere, the value of g varies with height and depth and also from place to place

161 **(a,c,d)**
From relation $|E|=\frac{dV}{dr}$
When $V=0, E=0$; when $V=$ constant, $E=0$; when $V\neq 0, E\neq 0$
So, options (a), (c) and (d) are correct

162 **(a,d)**


$$r_2=\frac{2mr}{m+2m}=\frac{2r}{3}$$
$$T_2^2=\frac{4\pi^2 r_2^2}{Gm}\ \Rightarrow\ T_2^2=\frac{32\pi^2 r^3}{27Gm}$$
$T_2\propto r^{\frac{3}{2}}$ and $T_2\propto m^{\frac{1}{2}}$

163 **(a,b,d)**
Just before collision, the total energy of two satellite is
$$E=-\frac{GMm}{2r}-\frac{GMm}{2r}=-\frac{GMm}{r}$$
Let the orbital velocity be v, then from momentum conservation,
$mv-mv=2m\times v_1$
$v_1=0$
As velocity of combined mass just after collision is zero, the combined mass will fall towards the earth. At this instant, the total energy of the system only consists of the gravitational potential energy given by
$$U=\frac{GM\times 2m}{2r}$$

164 **(b,d)**
Now, $g=GM/R^2$
If R reduces to $R'=0.8R$, the value of g becomes
$$g'=\frac{GM}{R^2}=\frac{GM}{0.64\,R^2}=\frac{g}{0.64}=\frac{9.8\text{ m s}^{-2}}{0.64}$$
Which is choice (b)
Increase in the value of g, $=\frac{g}{0.64}=\frac{0.36g}{0.64}$
Therefore, the percentage increase $=\frac{0.36\text{ g}}{0.64\text{ g}}\times$

$100 = 56.25\%$
Hence, choices (b) and (d) are correct

165 **(a,c)**

If $r > R, g' = g\frac{R^2}{(R+h)^2} = g\frac{R^2}{r^2}$

If $r < R, g' = g\left[\frac{R-d}{R}\right]$

Where d is the depth below the surface of earth

If $d = R, g = 0$

Further, due to rotation: $g' = g - R\omega^2\cos^2\theta$

If $\omega = 0$, then g increases

166 **(a,d)**

1. For Moon, escape velocity is 2.4 km s^{-1}
2. $v_0 = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{M} \times \frac{4}{3}\pi R^3 \rho}$

or, $v_0 \propto \sqrt{\rho}$

167 **(c,d)**

Linear velocity or orbital velocity is $v = \sqrt{GM/r}$ where r is the distance of the satellite from the centre of the earth. Therefore, v decareses as r is increased

Also $v = r\omega$, where ω is the angular velocity

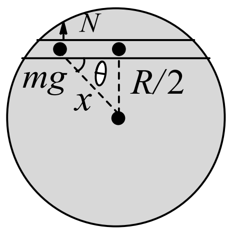
Therefore, $\omega = \frac{v}{r} = \frac{1}{r}\sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{r^3}}$

Thus, ω decreases with increase in r. The time period of the satellite is given by

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

Thus, T incraeses as r is increased. Hence, the correct choices are (c) and (d)

168 **(b,c)**



Net force towards the centre of the earth

$= mg' = \frac{mGMx}{R^3} = \frac{mgR^2x}{R^3} = (mgx)/R$

Normal force, $N = mg'\sin\theta$

Thus, pressing force, $N = \frac{mgx}{R}\frac{R}{2x}$

$N = \frac{mg}{2}$ is constant and independent of x

Hence, option (b) is correct

Tangential force, $F = ma = mg'\cos\theta$

$$a = g'\cos\theta = \frac{gx}{R}\frac{\sqrt{\frac{R^2}{4} - x^2}}{x} \Rightarrow a = \frac{gx}{R}\sqrt{R^2 - 4x^2}$$

Curve is parabolic and at $x = \frac{R}{2}, a = 0$

Hence, option (c) is correct

169 **(a,b,c,d)**

1. If $v < v_e$ and the body is projected vertically upwards, the body will rise up to that height where its velocity becomes zero. After that it will fall freely due to gravity following a straight line path
2. If $v < v_e$ but the body is projected at some angle w.r.t. vertical direction, the body will reach up to a certain height (where vertical component of the velocity becomes zero) and then fall done following a parabolic path (this will be a case of projectile motion)
3. If $v = v_0 = \sqrt{GM/R}$, then the orbit will be circular

But if $v_0 < v < v_e$ (escape velocity), then the orbit will be an ellipse

So, all the options are correct

170 **(b,c)**

When a planet is on the major axis of the orbit, gravitational force on the planet is normal to its motion. So, no work is done. As that energy remains the same, no work is done in complete revolution

171 **(b,c)**

If radius of an orbiting satellite is decreased, then its PE gets decreased. As ME = KE + PE, so ME also decreases

$\left[\text{PE} = -\frac{GMm}{r}, \text{KE} = +\frac{GMm}{2r} \text{ and } \text{ME} = -\frac{GMm}{2r}\right]$

172 **(a,c)**

Force on the satellite is always the earth; therefore, acceleration of satellite S is always directed towards the centre of the earth. Net torque of this gravitational force F about the centre of the earth is zero. Therefore, angular momentum (both in magnitude and direction) of S about the centre of the earth is constant throughout. Since force F is conservative in nature, therefore mechanical energy of the satellite remains constant. Speed of S is maximum when it is nearest to the earth and minimum when it is farthest

173 **(e)**

If the orbital path of a satellite is circular, then its speed is constant and if the orbital path of a satellite is elliptical, then its speed in its orbit is not constant. In that case its areal velocity is constant

174 **(a)**
If a pendulum is suspended in a lift and lift is moving downward with some acceleration a, then time period of pendulum is given by, $T = 2\pi\sqrt{\frac{l}{g-a}}$

In the case of free fall, $a = g$ then $T = \infty$

$i.e.$, the time period of pendulum becomes infinite

175 **(c)**
$$V_{\text{in}} = \frac{GM}{2R^3}[3R^2 - r^2]$$

At surface, $V_s = \frac{GM}{R}$ [at $r = R$]

$$V_{\text{in}} = \frac{3}{2}V_s$$

$$V_{\text{in}} > V_s$$

V is not the same everywhere as indicated by V_{in}

176 **(d)**
For Statement 1: Gravitational force is not perpendicular to velocity of the satellite. So for any small part of the orbit work done is not zero, although when satellite is at perihelion or aphelion position then work done by gravitational force for small part would be zero

177 **(d)**
The value of escape velocity is derived from the method of conservation of total mechanical energy and energy is independent of direction

178 **(a)**
$$\frac{dA}{dt} = \frac{1}{2}r^2 \Rightarrow \frac{d\theta}{dt} = \frac{1}{2}r^2\omega$$

$$\frac{mr^2\omega}{2m} = \frac{2}{m} = \text{constant}$$

$\therefore\ L =$constant

179 **(a)**
Gravitational Flux $(\phi_g) = \int \vec{E}.d\vec{s}$

For any closed surface $\phi_g = 4\pi\ GM$

and gravitational field $E \propto \frac{1}{r^2}$

180 **(a)**
The time period of the satellite which is very near to the earth is given by $T = 2\pi\sqrt{\frac{R}{g}} \approx 84$ min $=$ 1 h 24 min

182 **(c)**
The value of g at any place is given by the relation,

$g' = g - \omega^2 R_e \cos^2\lambda$. When λ is angle of latitude and ω is the angular velocity of earth

If $\omega = 0$, $\therefore g' = g$. If there is no rotation

Assertion is true but reason is false

183 **(d)**
Here, Statement 1 is incorrect as speed of one satellite increases, its kinetic energy and hence total energy increases, i.e., total energy becomes less negative and hence r increases, i.e., orbit changes

184 **(d)**
Binding energy is the minimum energy required to free a satellite from the gravitational attraction. It is the negative value of total energy of satellite. Let a satellite of mass m be revolving around earth of mass M_e and radius R_e total energy of satellite $=$ PE $+$ KE $= \frac{-GM_e m}{R_e} + \frac{1}{2}mv^2$

$$= \frac{-GM_e m}{R_e} + \frac{GM_e m}{2R_e}$$

$$= -\frac{GMm}{2R_e}$$

$\therefore$ Binding energy of satellite $= -$(total energy of satellite)

which depend on mass of the satellite

Assertion is false but reason is true

185 **(d)**
As, escape velocity $= \sqrt{\frac{2GM}{R}}$, so its value depends on mass of planet and radius of the planet. The two different planets have same escape velocity, when these quantities (mass and radius) are equal

186 **(a)**

From kinetic theory of gas, $v = \sqrt{\frac{3RT}{M}}$

So, for lighter gas molecules, v is greater which is enough to take these molecules away from the earth's atmosphere

187 **(a)**

$v_0 = R_e\sqrt{\frac{g}{R_e+h}}$ for a satellite revolving very near the earth surface $R_e + h = R_e$

$v_0 = \sqrt{R_e g}$

$= \sqrt{64 \times 10^5 \times 10}$

$= 8 \times 10^3 \text{ms}^{-1} = 8\text{kms}^{-1}$

Which is independent of height of satellite

Both Assertion and Reason are true and reason is the correct explanation of assertion

188 **(b)**

As the geostationary satellite is established in an orbit in the plane of the equator at a particular place, so it move in the same sense as the earth and hence its time period of revolution is equal to 24 hours, which is equal to time period of revolution of earth about its axis

189 **(a)**

As a rotation of earth takes place about polar axis therefore, body places at poles will not feel any centrifugal force and its weight or acceleration due to gravity remains unaffected

190 **(a)**

According to Kepler's third law of motion, the square of the time period of a planet about the Sun is proportional to the cube of the semi-major axis of the ellipse or mean distance of the planet from the Sun, i.e., $T^2 \propto a^3$. When a is smaller, the time period is shorter

191 **(e)**

Earth revolves around the sun in circular path and required centripetal force is provided by gravitational force between earth and sun but the work done by this centripetal force is zero

192 **(d)**

For a satellite to be geostationary, the necessary requirements are as follows:

1. Its orbit must be in equatorial plane and circular;
2. Its time period must be 24 h;
3. Its sense of rotation must be the same as that of the earth about its own axis

193 **(b)**

Acceleration due to gravity,

$g' = g - R\omega^2 \cos^2 \lambda$

At equator, $\lambda = 0°$ $i.e.\cos 0° = 1 \therefore g_e = g - R\omega^2$

At poles, $\lambda = 90°$ $i.e.\cos 90° = 0 \ \therefore g_p = g$

Thus, $g_p = g_e = g - g + R\omega^2 = R\omega^2$

Also, the value of g is maximum at poles and minimum at equators

194 **(c)**

Variation of g with depth from surface of earth is given by

$\text{g}' = \text{g}R\left(1 - \frac{d}{R}\right)$

At the centre of earth, $d = R$

$\therefore \ \ \text{g}' = \text{g}\left(1 - \frac{d}{R}\right) = 0$

$\therefore$ Apparent weight of body $= m\text{g}' = 0$

Assertion is true but reason is false

195 **(c)**

The value of g at any place is given by

$\text{g}' = \text{g} - \omega^2 R_e \cos^2 \lambda$

If $\omega = 0$, then $\text{g}' = \text{g}$. The value of g will be same at all places

196 **(a)**

Force acting on astronant is utilised in providing necessary centripetal force, thus he feels weighlessness, as he in a state of free fall.

197 **(d)**

We can take the sphere as a point mass lying at its centre, but the rod will not be taken as point mass lying at its centre of mass

198 **(e)**

The total gravitational force on one particle due to number of particles is the resultant force of attraction (or gravitational force) exerted on the given particle due to individual particles, $i.e., \vec{F} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} + \cdots$ It means the principle of superposition is valid

199 **(a)**

As no torque is acting on the planet, its angular momentum must remain constant in magnitude as well as direction. Therefore, plane of rotation must pass through the centre of earth

200 **(a)**

According to Newton's law of gravitation, every body in this universe attracts every other body with a force which is inversely proportional to the square of the distance between them. When we move our finger, the distance of the objects with respect to finger changes, hence the force of attraction changes, disturbing the entire universe, including stars

201 **(a)**

Because gravitational force is always attractive in nature and every body is bound by this gravitational force of attraction of earth

202 **(a)**

According to Newton's law of gravitation, $F = \frac{Gm_1m_2}{r^2}$

When m_1, m_2 and r all are doubled,

$$F = \frac{v^2}{2} = \frac{GM}{R} \qquad \frac{GM}{(R+h)} = \frac{gR}{R}$$

ie, remains the same.

Both assertion reason are true and reason is correct explanation of assertion

203 **(b)**

If root mean square velocity of the gas molecules is less than escape velocity from that planet (or satellite) then atmosphere will remain attached with that planet and if $v_{rms} > u_{escape}$ then there will be no atmosphere on the planet. This is the reason for no atmosphere at moon

204 **(a)**

The torque on a body is given by $\vec{\tau} = \overrightarrow{dL}/dt$. In case of a planet orbiting around the Sun, no torque is acting on it

$$\frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L} = \text{constant}$$

205 **(a)**

According to kepler's law $T^2 \propto r^3 \propto (R+h)^3$

$i.e.$ if distance of satellite is more then its time period will be more

206 **(a)**

Both the statements are true and Statement 2 correctly explains Statement 1

207 **(a)**

Here, both the statements are correct and Statement 2 correctly explains Statement 1

This can be easily proved by writing the basic equations

208 **(d)**

The orbital velocity, if a satellite close to earth is $v_0 = \sqrt{gR_e}$, while the escape velocity for a body thrown from the earths surface $v_e = \sqrt{2gR_e}$

Thus $\frac{v_0}{v_e} = \frac{\sqrt{gR_e}}{\sqrt{2gR_e}} = \frac{1}{\sqrt{2}}$ or $v_e = \sqrt{2}v$

Assertion is false but reason is true

209 **(b)**

If r is the distance between two electrons then according to Newtons law, the gravitational force between them is

$$F_G = G\frac{m^2}{r^2} = 6.67 \times 10^{-11} \times \frac{(9.1 \times 10^{-31})^2}{r^2}$$
$$\cong \frac{5 \times 10^{-71}}{r^2}$$

and according to Coulomb's law, the electrical force between electron is

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{q \times q}{r^2} = 9 \times 10^{-9} \times \frac{(1.6 \times 10^{-19})^2}{r^2}$$
$$\cong \frac{2 \times 10^{-28}}{r^2}$$

$$\therefore \frac{F_G}{F_e} \cong \frac{10^{-71}}{10^{-28}} \cong 10^{-43} \qquad ie, F_G = 10^{-43}F_e$$

ie, gravitational force between two particles is negligible compared to the electrical force.

Both assertion and reason are true but reason is not the correct explanation of assertion

210 **(a)**
Till the particle reaches the centre of the planet, force on both the bodies are in direction of their respective velocities, hence kinetic energies of both keep on increasing. After the particle crosses the centre of the planet, forces on both are retarding in nature. Hence, as the particle passes through the centre of the planet, sum of kinetic energies of both the bodies is maximum.
Therefore, Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1

211 **(b)**
We know that earth revolves from west to east about its polar axis. Therefore, all the particles on the earth have velocity from west to east

This velocity is maximum in the equatorial line, as $v = R\omega$, where R is the radius of earth and ω is the angular velocity of revolution of earth about its polar axis

When a rocket is launched from west to east in equatorial plane, the maximum linear velocity is added to the launching velocity of the rocket, due to which launching becomes easier

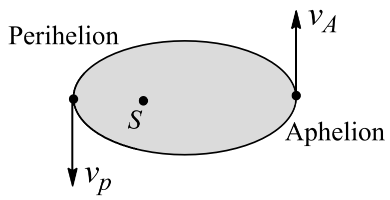
212 **(c)**
Upto ordinary heights the change in the distance of a projectile from the centre of the earth is negligible compared to the radius of the earth. Hence, projectile moves under a nearly uniform gravitation force and its path is parabolic. But for projectile going to great height, the gravitational force decreases in inverse proportion to the square of the distance of the projectile from the centre of the earth. Under such a variable force the path of projectile is elliptical.

Both Assertion and Reason are true and reason is the correct explanation of Assertion.

213 **(c)**
Acceleration due to gravity is given by g $= GM/l^2$. Thus, it does not depend on the mass of the body on which it is acting. Also, it is not a constant quality. It changes with changes in value of both M and R (distance between two bodies)

215 **(c)**
According to Kepler's law of planetary motion, a planet revolves around the sun in such a way that its areal velocity is constant, $i.e.$, it move faster, when it is closer the sun and vice-versa

216 **(b)**
Speed of the planet is maximum at perihelion and minimum at aphelion. So, KE is maximum at perihelion and minimum at aphelion
Where KE is maximum, PE is minimum



PE and KE of the planet are dependent on both mass of planet and on semi-major axis

217 **(d)**
Angular momentum of the particle $= m(v_0 + v)a$

$= \sqrt{\frac{5}{4}} m v_0 a \left[v_0 = \sqrt{\frac{GM_e}{a}} \right]$

Total energy of the particle $= \frac{1}{2}m(v_0+v)^2 - \frac{GM_e m}{a}$

$= \frac{1}{2} \times \frac{5}{4} m v_0^2 - \frac{GM_e m}{a}$

$= \frac{5}{8}\frac{GM_e m}{a} - \frac{GM_e m}{a} = -\frac{3GM_e m}{a}$

At any distance 'r', T.E. $= \frac{1}{2}mu^2 - \frac{GM_e m}{a}$

But angular momentum conservation gives

$mur = \sqrt{\frac{5Gm_e}{4a}}$

$\Rightarrow u = \sqrt{\frac{5}{4}\frac{GM_e a}{r^2}} a$

T.E. at any distance 'r' $= \frac{1}{2}m\frac{5}{4}\frac{Gmea}{r^2} - \frac{GM_e m}{r}$

But through conservation of total energy, we have

$\frac{1}{2}m\frac{5}{4}\frac{GM_e a}{r^2} - \frac{GM_e m}{r} = -\frac{3GM_e m}{a}$

On solving, we get

$3r^2 - 8ar + 5a^2 = 0$

$(r-a)(3r-5a) = 0$

$R = a, r = \frac{5a}{3}$

Minimum distance $= a$

Maximum distance $= \frac{5a}{3}$

218 **(a)**

1. Acceleration due gravity is different at different points and it does not depend upon direction of projection
2. Angular momentum of the earth remains constant about the Sun
3. Escape velocity depends upon g and independent of direction of projection
4. Gravitational potential will remain constant

219 **(c)**

Gravitational potential is defined as the work done in carrying a unit mass from a point outside the field (zero potential) to a point inside the field. So when reference point is altered the potential will vary and the potential difference will remain the same. Also the energy is not dependent on the angle by any means and depends only on the two points (initial and final) in any conservation field like gravitational field
So i→c, d
Escape velocity is given by $\sqrt{GM/R}$ and is dependent on the mass of the planet and does not depend on the angle of projection
So ii → d
Acceleration due to gravity is given by
g $= GM/R^2 = 4/3\pi R_\rho G$ and so its ratio in two planets depends on their densities when their mass is not the same
So iii → a
As the Kepler's law, the angular momentum will remain conserved for keeping the area swept in equal to remain the same. The same concept was proposed by Bohr's model of atom
So iv → b, c

220 **(c)**

As rocket has been fired the speed of the satellite increases in all these three cases which leads to the increase in kinetic energy and hence total energy becomes less negative or we can say the semi-major axis increases
Due to thrust exerted by firing of the rocket in a direction perpendicular to the plane of the orbit, the plane of the orbit changes
iv. Here speed decreases and hence the kinetic energy, the total energy and the semi-major axis decrease

221 **(d)**

1. For earth moving in an elliptical orbit. Centripetal force passes through centre of mass. So its torque will be zero at all points of motion. Here, angular momentum can be conserved at all points of motion. But as external force is acting, so linear momentum will be unconserved
2. Angular momentum can be conserved about the point of contact as work done about this point by force of friction is zero. Linear momentum will not be conserved as force of friction is an external force here
3. External forces acting on the system are normal reaction, $mg \sin \theta$ and friction. Torque due to normal reaction and $mg \sin \theta$ will be zero about centre of mass but not due to friction. So, angular momentum will not be conserved. Also, in the presence of external forces, the linear momentum will not be conserved
4. For the projected particle, force of gravity which is external in this case will render linear momentum unconserved. But torque of this force will be zero as it will pass through centre of mass. So angular momentum is conserved instantaneously

222 **(b)**

1. A geostationary satellite has circular orbit over equator
2. Total energy will remain constant in any orbit
3. Angular momentum will remain constant in any orbit
4. In circular orbit, speed remains the same, whereas in elliptical orbit speed changes

223 **(a)**

1. At centre of thin spherical shell, $V \neq 0, E = 0$
2. At centre of solid sphere,$V \neq 0, E = 0$
3. At centre of spherical cavity inside solid sphere, $V \neq 0, E \neq 0$

4. At centre of two point masses,
$V \neq 0, E = 0$

224 **(d)**

According to law of conservation of total mechanical energy.

Total energy of rocket at the surface of earth = total energy of rocket at the highest point

or $\frac{1}{2}mv^2 + \left(\frac{-GMm}{R}\right) = 0 + \left(\frac{-GMm}{R+h}\right)$

or $\frac{v^2}{2} = \frac{GM}{R} - \frac{GM}{(R+h)} = \frac{gR^2}{R} - \frac{gR^2}{(R+h)}$

$= gR\left(1 - \frac{R}{R+h}\right) = gR\left(\frac{h}{R+h}\right)$

or $v^2(R+h) = 2gRh$

or $Rv^2 = 2gRh - v^2h = (2gR - v^2)h$

or $h = \frac{Rv^2}{(2gR - v^2)}$

$= \frac{6.4 \times 10^6 (5 \times 10^3)^2}{(2 \times 9.8 \times 6.4 \times 10^6) - (5 \times 10^3)^2}$

$= 1.6 \times 10^6 \mathrm{m}$

225 **(d)**

Let v_0 be the speed of the spaceship n the circular orbit of radius r_0, and v_e be the escape velocity for the orbit. Then,

$$\frac{mv_0^2}{r_0} = \frac{GM_m}{r_0^2}, \frac{mv_e^2}{2} = \frac{GM_m}{r_0} \text{ or } v_0 = \sqrt{\frac{GM}{r_0}}, v_e = \sqrt{\frac{2GM}{r_0}}$$

As $v_e = v_0 + |\Delta\vec{\mathbf{v}}| \cos\theta = v_0 + \Delta v \cos\theta$

The specific impulse required for escape is the least for $\theta = 0$ *ie*, the initial velocity of the spaceship and the impulse are in the same direction, and is given by

$$\Delta v = v_e - v_0 = \sqrt{\frac{GM}{r_0}}(\sqrt{2} - 1)$$

226 **(c)**

Applying the principle of conservation of angular momentum,

$mv_1(2R) = mv_2(4R)$

$v_1 = 2v_2$ (i)

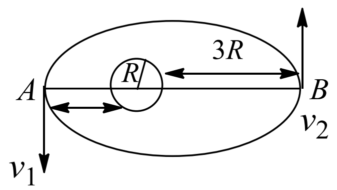
From conservation of energy,

$\frac{1}{2}mv_1^2 - \frac{GMm}{2R} = \frac{1}{2}mv_2^2 - \frac{GMm}{4R}$ (ii)

Solving Eqs. (i) and (ii), we get

$$v_2 = \sqrt{\frac{GM}{6R}}, v_1 = \sqrt{\frac{2GM}{3R}}$$

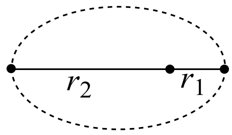
If r is the radius of curvature at point A



$$\frac{mv_1^2}{r} = \frac{GMm}{(2R)^2}$$

$r = \frac{4v_1^2}{GM} = \frac{8R}{3}$ (putting value of v_1)

227 **(b)**

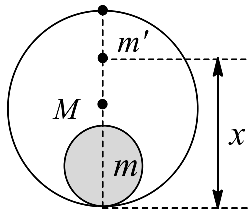


Gravitational PE at perihelion $= -GMm/r_1$ as r_1 is minimum

Therefore, PE is minimum

228 **(c)**

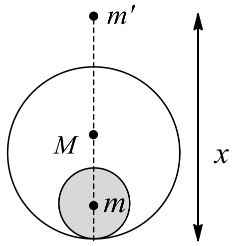
When $2r < x < 2R$



Then the force will be due to sphere only

$$F = \frac{Gmm'}{(x-r)^2}$$

When $x > 2R$



The sphere and the shell both will contribute to the

$$F_{\text{sphere}} = \frac{GMm'}{(x-r)^2}$$

$$F_{\text{shell}} = \frac{GMm'}{(x-R)^2}$$

$$F = \frac{GMm'}{(x-R)^2} + \frac{GMm'}{(x-r)^2}$$

229 **(a)**

$\vec{F} = \vec{E}m = (10\hat{\imath} + 24\hat{\jmath})\mathrm{N}$

$= \sqrt{(100) + 576} = 26 \mathrm{~N}$

230 **(a)**

$E_P = \frac{GM}{(4a)^2} = \frac{GM}{16a^2}$ and $E_P = \frac{GM}{(6a)^2} + \frac{GM}{5a^2} = \frac{61GM}{900a^2}$

231 **(a)**

For the system to be bounded one, total energy of the system must be negative. So, object having total energy E_1 is bounded and other is unbounded

232 **(c)**

As it is not known that by how much the velocity has changed, which will cause the change in orbital radius and hence time speed, so we can use either rocket system to carry out the docking in the minimum possible time, though the extent of firing rockets may differ

233 **(b)**

From work – energy theorem, $dK = -dU + W_{\text{air friction}}$

$W_{\text{air friction}}$ is negative, so

$dK = -dU +$(a negative quantity)

As K increases, it means U decreases by an amount greater than magnitude of $W_{\text{air friction}}$

234 **(3)**

Given $R' = 3R, M = \frac{4}{3}\pi R^3 \rho$

$g' = GM'R'^2$

Putting $M' = \frac{4}{3}\pi R'^3 \rho,$

$$g' = \frac{G}{R'^2}\frac{4}{3}\pi R'^3 \rho = \frac{27}{9}\frac{G}{R^2}\frac{4}{3}\pi R^3 \rho$$

$$= 3\frac{GM}{R^2} = 3g$$

235 **(6)**

$$F = mr\omega^2 = 10^{24} \times 1.5 \times 10^8 \times 10^3 \times (2 \times 10^{-7})^2$$

$$= 6 \times 10^{21}\ N$$

236 **(3)**

$F = -K/r^2$ (negative sign is for attractive force)

Potential energy $U = -\int F\,dr = \int \frac{K}{r_2} dr = -\frac{K}{r}$

Conservation of energy gives (let at other extreme position $r = b$)

$K_1 + U_1 = K_2 + U_2$

$\frac{1}{2}mv_1^2 - \frac{K}{a} = \frac{1}{2}mv_2^2 - \frac{K}{b}$ (i)

Where $v_1 = \sqrt{\frac{K}{2ma}}$

Conservation of angular momentum gives

$mv_1 a = mv_2 b$

$$v_2 = \frac{a}{b}v_1 = \frac{a}{b}\sqrt{\frac{K}{2ma}}$$

Therefore, from Eq. (i) $\Rightarrow \frac{1}{2}m\frac{K}{2ma} - \frac{K}{a} = \frac{1}{2}m\left(\frac{a}{b}\right)^2\frac{K}{2ma} - \frac{K}{b}$

$$-\frac{3K}{4a} = \frac{aK}{4b^2} - \frac{K}{b} \Rightarrow b^2 - \frac{4a}{3}b + \frac{a^3}{3} = 0$$

Hence, $b = a/3 \Rightarrow a/b = 3$

237 **(2)**

$$g = \frac{GM}{R^2} = G\frac{4}{3}\pi R^3 \rho$$

$$g = \frac{4}{3}G\pi R\rho \Rightarrow g \propto R\rho$$

$g' \propto R'\rho' \Rightarrow \rho' = 2\rho$

Given, $\frac{g}{g'} = 1$

$$\frac{R}{R'} = 2$$

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