9.DIFFERENTIAL EQUATIONS

Single Correct Answer Type

1. The solutions of $(x+y+1)dy = dx$ is
   a) $x+y+2 = Ce^y$ b) $x+y+4 = C\log y$
   c) $\log(x+y+2) = Cy$ d) $\log(x+y+2) = C-y$
2. The solution of the differential equation $y'y''' = 3(y'')^2$ is
   a) $x = A_1y^2 + A_2y + A_3$ b) $x = A_1y + A_2$ c) $x = A_1y^2 + A_2y$ d) None of these
3. An integrating factor of the differential equation $(1+y+x^2y)dx + (x+x^3)dy = 0$ is
   a) $\log x$ b) x c) e^x d) $\frac{1}{x}$
4. The solution of the differential equation
   $x^2\frac{dy}{dx}\cos\frac{1}{x} - y\sin\frac{1}{x} = -1$, where $y \to -1$ as $x \to \infty$ is
   a) $y = \sin\frac{1}{x} - \cos\frac{1}{x}$ b) $y = \frac{x+1}{x\sin\frac{1}{x}}$ c) $y = \cos\frac{1}{x} + \sin\frac{1}{x}$ d) $y = \frac{x+1}{x\cos 1/x}$
5. Tangent to a curve intercepts the y-axis at a point P. A line perpendicular to this tangent through P passes through another point $(1, 0)$. The differential equation of the curve is
   a) $y\frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^2 = 1$
   b) $\frac{xd^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$
   c) $y\frac{dx}{dy} + x = 1$
   d) None of these
6. The solution of the differential equation $y(2x^4+y)\frac{dy}{dx} = (1-4xy^2)x^2$ is given by
   a) $3(x^2y)^2 + y^3 - x^3 = c$ b) $xy^2 + \frac{y^3}{3} - \frac{x^3}{3} + c = 0$
   c) $\frac{2}{5}yx^5 + \frac{y^3}{3} = \frac{x^3}{3} - \frac{4xy^3}{3} + c$ d) None of these
7. The integrating factor of the differential equation $\frac{dy}{dx}(x\log_e x) + y = 2\log_e x$ is given by
   a) x b) e^x c) $\log_e x$ d) $\log_e(\log_e x)$
8. The solution of the differential equation $(x\cot y + \log\cos x)dy + (\log\sin y - y\tan x)dx = 0$
   a) $(\sin x)^y(\cos y)^x = c$ b) $(\sin y)^x(\cos x)^y = c$ c) $(\sin x)^x(\cos y)^y = c$ d) None of these
9. The differential equation whose general solution is given by, $y = \left(c_1\cos(x+c_2) - (c_3e^{(-x+c_4)}\right) + (c_3\sin x)$, where c_1, c_2, c_3, c_4, c_5 are arbitrary constants, is
   a) $\frac{d^4y}{dx^4} - \frac{d^2y}{dx^2} + y = 0$ b) $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$
   c) $\frac{d^5y}{dx^5} + y = 0$ d) $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$
10. A differential equation associated to the primitive $y = a + be^{5x} + ce^{-7x}$ is (where y_n is nth derivative w.r.t. x)
    Where y_n represents nth order derivative
    a) $y_3 + 2y_2 - y_1 = 0$ b) $4y_3 + 5y_2 - 20y_1 = 0$
    c) $y_3 + 2y_2 - 35y_1 = 0$ d) None of these
11. The curve satisfying the equation $\frac{dy}{dx} = \frac{y(x+y^3)}{x(y^3-x)}$ and passing through the point $(4, -2)$ is
    a) $y^2 = -2x$ b) $y = -2x$ c) $y^3 = -2x$ d) None of these
12. The solution of the equation $\log(dy/dx) = ax + by$ is

a) $\frac{e^{by}}{b} = \frac{e^{ax}}{a} + c$ b) $\frac{e^{-by}}{-b} = \frac{e^{ax}}{a} + c$ c) $\frac{e^{-by}}{a} = \frac{e^{ax}}{b} + c$ d) None of these

13. The solution of the differential equation $y''' - 8y'' = 0$ where $y(0) = \frac{1}{8}, y'(0) = 0, y''(0) = 1$ is

a) $y = \frac{1}{8}\left(\frac{e^{8x}}{8} + x - \frac{7}{9}\right)$ b) $y = \frac{1}{8}\left(\frac{e^{8x}}{8} + x + \frac{7}{8}\right)$ c) $y = \frac{1}{8}\left(\frac{e^{8x}}{8} - x + \frac{7}{8}\right)$ d) None of these

14. The general solution of the differential equation $\frac{dy}{dx} + \sin\frac{x+y}{2} = \sin\frac{x-y}{2}$ is

a) $\log\tan\left(\frac{y}{2}\right) = c - 2\sin x$ b) $\log\tan\left(\frac{y}{2}\right) = c - 2\sin\left(\frac{x}{2}\right)$

c) $\log\tan\left(\frac{y}{2} + \frac{\pi}{4}\right) = c - 2\sin x$ d) $\log\tan\left(\frac{y}{4} + \frac{\pi}{4}\right) = c - 2\sin\left(\frac{x}{2}\right)$

15. The curve, with the property that the projection of the ordinate on the normal is constant and has a length equal to a is

a) $a\ln\left(\sqrt{y^2 - a^2} + y\right) = x + c$ b) $x + \sqrt{a^2 - y^2} = c$

c) $(y - a)^2 = cx$ d) $ay = \tan^{-1}(x + c)$

16. The degree of the differential equation satisfying $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$ is

a) 1 b) 2 c) 3 d) None of these

17. If $y = y(x)$ and $\frac{2+\sin x}{y+1}\left(\frac{dy}{dx}\right) = -\cos x, y(0) = 1$, then $y\left(\frac{\pi}{2}\right)$ equals

a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) $-\frac{1}{3}$ d) 1

18. The solution of the differential equation

$2x^2y\frac{dy}{dx} = \tan(x^2y^2) - 2xy^2$, given $y(1) = \sqrt{\frac{\pi}{2}}$, is

a) $\sin x^2y^2 = e^{x-1}$ b) $\sin(x^2y^2) = x$ c) $\cos x^2y^2 + x = 0$ d) $\sin(x^2y^2) = e\,e^x$

19. A normal at any point (x, y) to the curve $y = f(x)$ cuts a triangle of unit area with the axis, the differential equation of the curve is

a) $y^2 - x^2\left(\frac{dy}{dx}\right)^2 = 4\frac{dy}{dx}$ b) $x^2 - y^2\left(\frac{dy}{dx}\right)^2 = \frac{dy}{dx}$ c) $x + y\frac{dy}{dx} = y$ d) None of these

20. Solution of $\frac{dy}{dx} + 2xy = y$ is

a) $y = c\,e^{x-x^2}$ b) $y = c\,e^{x^2-x}$ c) $y = c\,e^x$ d) $y = c\,e^{-x^2}$

21. An object falling from rest in air is subject not only to the gravitational force but also to air resistance. Assume that the air resistance is proportional to the velocity with constant of proportionality as $k > 0$, and acts in a direction opposite to motion ($g = 9.8\ m/s^2$). Then velocity cannot exceed

a) 9.8/k m/s b) 98/k m/s c) $\frac{k}{9.8}$ m/s d) None of these

22. The family of curves represented by $\frac{dy}{dx} = \frac{x^2+x+1}{y^2+y+1}$ and $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$

a) Touch each other b) Are orthogonal c) Are one and the same d) None of these

23. The solution of differential equation

$yy' = x\left(\frac{y^2}{x^2} + \frac{f(y^2/x^2)}{f'(y^2/x^2)}\right)$ is

a) $f(y^2/x^2) = cx^2$ b) $x^2f(y^2/x^2) = c^2y^2$ c) $x^2f(y^2/x^2) = c$ d) $f(y^2x^2) = cy/x$

24. A normal at $P(x, y)$ on a curve meets the x-axis at Q and N is the foot of the ordinate at P. If $NQ = \frac{x(1+y^2)}{1+x^2}$, then the equation of curve given that it passes through the point $(3, 1)$ is

a) $x^2 - y^2 = 8$ b) $x^2 + 2y^2 = 11$ c) $x^2 - 5y^2 = 4$ d) None of these

25. The solution of the differential equation $\frac{d^2y}{dx^2} = \sin 3x + e^x + x^2$ when $y_1(0) = 1$ and $y(0) = 0$ is

a) $\frac{-\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x - 1$ b) $\frac{-\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x$

c) $\frac{-\cos 3x}{3} + e^x + \frac{x^4}{12} + \frac{1}{3}x + 1$ d) None of these

26. The solution of $x^2\frac{dy}{dx} - xy = 1 + \cos\frac{y}{x}$ is

a) $\tan\left(\frac{y}{2x}\right) = c - \frac{1}{2x^2}$ b) $\tan\frac{y}{x} = c + \frac{1}{x}$ c) $\cos\left(\frac{y}{x}\right) = 1 + \frac{c}{x}$ d) $x^2 = (c + x^2)\tan\frac{y}{x}$

27. The solution of $(x^2 + xy)dy = (x^2 + y^2)dx$ is

a) $\log x = \log(x - y) + \frac{y}{x} + c$
b) $\log x = 2\log(x - y) + \frac{y}{x} + c$
c) $\log x = \log(x - y) + \frac{x}{y} + c$
d) None of these

28. If $y + x\frac{dy}{dx} = x\frac{\phi(xy)}{\phi'(xy)}$, then $\phi(xy)$ is equal to

a) $ke^{x^2/2}$ b) $ke^{y^2/2}$ c) $ke^{xy/2}$ d) ke^{xy}

29. Differential equation of the family of circles touching the line $y = 2$ at $(0, 2)$ is

a) $x^2 + (y-2)^2 + \frac{dy}{dx}(y - 2) = 0$
b) $x^2 + (y - 2)\left(2 - 2x\frac{dx}{dy} - y\right) = 0$
c) $x^2 + (y-2)^2 + \left(\frac{dx}{dy} + y - 2\right)(y - 2) = 0$
d) None of these

30. The solution of the differential equation $x(x^2 + 1)(dy/dx) = y(1 - x^2) + x^3\log x$ is

a) $y(x^2 + 1)/x = \frac{1}{4}x^2\log x + \frac{1}{2}x^2 + c$
b) $y^2(x^2 - 1)/x = \frac{1}{2}x^2\log x - \frac{1}{4}x^2 + c$
c) $y(x^2 + 1)/x = \frac{1}{2}x^2\log x - \frac{1}{4}x^2 + c$
d) None of these

31. The x-intercept of the tangent to a curve is equal to the ordinate of the point of contact. The equation of the curve through the point $(1, 1)$ is

a) $ye^{x/y} = e$ b) $xe^{x/y} = e$ c) $xe^{y/x} = e$ d) $ye^{y/x} = e$

32. Integrating factor of differential equation
$\cos x\frac{dy}{dx} + y\sin x = 1$ is

a) $\cos x$ b) $\tan x$ c) $\sec x$ d) $\sin x$

33. The solution of $\frac{dy}{dx} = \frac{x^2+y^2+1}{2xy}$ satisfying $y(1) = 1$ is given by

a) A system of parabolas
b) A system of circles
c) $y^2 = x(1 + x) - 1$
d) $(x - 2)^2 + (y - 3)^2 = 5$

34. Which of the following is not the differential equation of family of curves whose tangent form an angle of $\pi/4$ with the hyperbola $xy = c^2$?

a) $\frac{dy}{dx} = \frac{x - y}{x + y}$ b) $\frac{dy}{dx} = \frac{x}{x - y}$ c) $\frac{dy}{dx} = \frac{x + y}{y - x}$ d) None of these

35. The population of a country increases at a rate proportional to the number of inhabitants. f is the population which doubles in 30 years, then the population will triple in approximately

a) 30 years b) 45 years c) 48 years d) 54 years

36. The general solution of the equation $\frac{dy}{dx} = 1 + xy$ is

a) $y = ce^{-x^2/2}$ b) $y = ce^{x^2/2}$ c) $y = (x + c), e^{-x^2/2}$ d) None of these

37. The form of the differential equation of the central conics $ax^2 + by^2 = 1$ is

a) $x = y\frac{dy}{dx}$
b) $x + y\frac{dy}{dx} = 0$
c) $x\left(\frac{dy}{dx}\right)^2 + xy\frac{d^2y}{dx^2} = y\frac{dy}{dx}$
d) None of these

38. A curve passing through $(2, 3)$ and satisfying the differential equation
$\int_0^x ty(t)\,dt = x^2y(x), (x > 0)$ is

a) $x^2 + y^2 = 13$ b) $y^2 = \frac{9}{2}x$ c) $\frac{x^2}{8} + \frac{y^2}{18} = 1$ d) $xy = 6$

39. If $y(t)$ is a solution of $(1+t)\frac{dy}{dt} - ty = 1$ and $y(0) = -1$, then $y(1)$ is equal to

a) $-\frac{1}{2}$ b) $e + \frac{1}{2}$ c) $e - \frac{1}{2}$ d) $\frac{1}{2}$

40. Solution of the differential equation $\left(y + x\sqrt{xy}(x+y)\right)dx + \left(y\sqrt{xy}(x+y) - x\right)dy = 0$ is

a) $\frac{x^2+y^2}{2} + \tan^{-1}\sqrt{\frac{y}{x}} = c$ b) $\frac{x^2+y^2}{2} + 2\tan^{-1}\sqrt{\frac{x}{y}} = c$

c) $\frac{x^2+y^2}{2} + 2\cot^{-1}\sqrt{\frac{x}{y}} = c$ d) None of these

41. The solution of differential equation $x^2 = 1 + \left(\frac{x}{y}\right)^{-1}\frac{dy}{dx} + \frac{\left(\frac{x}{y}\right)^{-2}\left(\frac{dy}{dx}\right)^2}{2!} + \frac{\left(\frac{x}{y}\right)^{-3}\left(\frac{dy}{dx}\right)^3}{3!} + \cdots$ is

a) $y^2 = x^2(\ln x^2 - 1) + c$ b) $y = x^2(\ln x - 1) + c$

c) $y^2 = x(\ln x - 1) + c$ d) $y = x^2 e^{x^2} + c$

42. The solution of the differential equation

$\frac{x+\frac{x^3}{3!}+\frac{x^5}{5!}+\cdots}{1+\frac{x^2}{2!}+\frac{x^4}{4!}+\cdots} = \frac{dx-dy}{dx+dy}$ is

a) $2y\,e^{2x} = C\,e^{2x} + 1$ b) $2y\,e^{2x} = C\,e^{2x} - 1$ c) $y\,e^{2x} = C\,e^{2x} + 2$ d) None of these

43. A curve is such that the mid point of the portion of the tangent intercepted between the point where the tangent is drawn and the point where the tangent meets the y-axis lies on the line $y = x$. If the curve passes through $(1, 0)$, then the curve is

a) $2y = x^2 - x$ b) $y = x^2 - x$ c) $y = x - x^2$ d) $y = 2(x - x^2)$

44. If $y = \frac{x}{\log|cx|}$ (where c is an arbitrary constant) is the general solution of the differential equation $dy/dx = y/x + \phi(x/y)$ then the function $\phi(x/y)$ is

a) x^2/y^2 b) $-x^2/y^2$ c) y^2/x^2 d) $-y^2/x^2$

45. The solution of $ye^{-x/y}dx - \left(xe^{(-x/y)} + y^3\right) = dy = 0$ is

a) $e^{-x/y} + y^2 = C$ b) $xe^{-x/y} + y = C$ c) $2e^{-x/y} + y^2 = C$ d) $e^{-x/y} + 2y^2 = C$

46. Solution of differential equation $dy - \sin x \sin y dx = 0$ is

a) $e^{\cos x}\tan\frac{y}{2} = c$ b) $e^{\cos x}\tan y = c$ c) $\cos x \tan y = c$ d) $\cos x \sin y = c$

47. The differential equation whose solution is $Ax^2 + By^2 = 1$, where A and B are arbitrary constants, is of

a) Second order and second degree b) First order and second degree

c) First order and first degree d) Second order and first degree

48. The solution of the differential equation

$\left(e^{x^2} + e^{y^2}\right)y\frac{dy}{dx} + e^{x^2}(xy^2 - x) = 0$ is

a) $e^{x^2}(y^2 - 1) + e^{y^2} = C$ b) $e^{y^2}(x^2 - 1) + e^{x^2} = C$

c) $e^{y^2}(y^2 - 1) + e^{x^2} = C$ d) $e^{x^2}(y - 1) + e^{y^2} = C$

49. The solution of the differential equation

$\frac{dy}{dx} = \frac{3x^2y^4+2xy}{x^2-2x^3y^3}$ is

a) $\frac{y^2}{x} - x^3y^2 = c$ b) $\frac{x^2}{y^2} + x^3y^3 = c$ c) $\frac{x^2}{y} + x^3y^2 = c$ d) $\frac{x^2}{3y} - 2x^3y^2 = c$

50. If $= (e^y - x)^{-1}$, where $y(0) = 0$, then y is expressed explicitly as

a) $\frac{1}{2}\ln(1 + x^2)$ b) $\ln(1 + x^2)$ c) $\ln\left(x + \sqrt{1 + x^2}\right)$ d) $\ln\left(x + \sqrt{1 - x^2}\right)$

51. The differential equation of the family of curves $y = e^x(A\cos x + B\sin x)$, where A and B are arbitrary constants, is

a) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$ b) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}-2y=0$ c) $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+y=0$ d) $\frac{d^2y}{dx^2}-7\frac{dy}{dx}+2y=0$

52. The solution of the equation $\frac{dy}{dx}=\frac{x(2\log x+1)}{\sin y+y\cos y}$ is

a) $y\sin y=x^2\log x+\frac{x^2}{2}+c$ b) $y\cos y=x^2(\log x+1)+c$

c) $y\cos y=x^2\log x+\frac{x^2}{1}+c$ d) $y\sin y=x^2\log x+c$

53. Solution of the equation $\cos^2 x\frac{dy}{dx}-(\tan 2x)y=\cos^4 x, |x|<\frac{\pi}{4}$, when $y\left(\frac{\pi}{6}\right)=\frac{3\sqrt{3}}{8}$ is

a) $y=\tan 2x\cos^2 x$ b) $y=\cot 2x\cos^2 x$ c) $y=\frac{1}{2}\tan 2x\cos^2 x$ d) $y=\frac{1}{2}\cot 2x\cos^2 x$

54. The solution of the differential equation

$x=1+xy\frac{dy}{dx}+\frac{x^2y^2}{2!}\left(\frac{dy}{dx}\right)^2+\frac{x^3y^3}{3!}\left(\frac{dy}{dx}\right)^3+\ldots$ is

a) $y=\ln(x)+c$ b) $y^2=(\ln x)^2+c$ c) $y=\log x+xy$ d) $xy=x^y+c$

55. The function $f(\theta)=\frac{d}{d\theta}\int_0^\theta\frac{dx}{1-\cos\theta\cos x}$ satisfies the differential equation

a) $\frac{df(\theta)}{d\theta}+2f(\theta)\cot\theta=0$ b) $\frac{df}{d\theta}-2f(\theta)\cot\theta=0$

c) $\frac{df}{d\theta}+2f(\theta)=0$ d) $\frac{df}{d\theta}-2f(\theta)=0$

56. The solution to the differential equation $y\log y+xy'=0$, where $y(1)=e$, is

a) $x(\log y)=1$ b) $xy(\log y)=1$ c) $(\log y)^2=2$ d) $\log y+\left(\frac{x^2}{2}\right)y=1$

57. The differential equation $\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{y}$ determines a family of circles with

a) Variable radii and a fixed centre t (0,1)

b) Variable radii and a fixed centre at (0,-1)

c) Fixed radius 1 and variable centres along the x-axis

d) Fixed radius 1 and variable centres along the y-axis

58. The solution of the differential equation

$\left\{1+x\sqrt{(x^2+y^2)}\right\}dx+\left\{\sqrt{(x^2+y^2)}-1\right\}y\,dy=0$ is equal to

a) $x^2+\frac{y^2}{2}+\frac{1}{3}(x^2+y^2)^{3/2}=c$

b) $x-\frac{y^3}{3}+\frac{1}{2}(x^2+y^2)^{1/2}=c$

c) $x-\frac{y^2}{2}+\frac{1}{3}(x^2+y^2)^{3/2}=c$

d) None of these

59. If $x\frac{dy}{dx}=y(\log y-\log x+1)$, then the solution of the equation is

a) $\log\frac{x}{y}=cy$ b) $\log\frac{y}{x}=cy$ c) $\log\frac{x}{y}=cx$ d) None of these

60. If integrating factor of

$x(1-x^2)dy+(2x^2y-y-ax^3)dx=0$ is $e^{\int p\,dx}$, then P is equal to

a) $\frac{2x^2-ax^3}{x(1-x^2)}$ b) $2x^3-1$ c) $\frac{2x^2-a}{ax^2}$ d) $\frac{2x^2-1}{x(1-x^2)}$

61. The solution of the equation

$(x^2y+x^2)dx+y^2(x-1)dy=0$ is given by

a) $x^2+y^2+2(x-y)+2\ln\frac{(x-1)(y+1)}{c}=0$

b) $x^2+y^2+2(x-y)+\ln\frac{(x-1)(y+1)}{c}=0$

c) $x^2+y^2+2(x-y)-2\ln\frac{(x-1)(y+1)}{c}=0$

d) None of these

62. A function $y=f(x)$ satisfies $(x+1)f''(x)-2(x^2+x)f(x)=\frac{e^{x^2}}{(x+1)}, \forall\, x>-1$

If $f(0)=5$, then $f(x)$ is

a) $\left(\frac{3x+5}{x+1}\right)e^{x^2}$ b) $\left(\frac{6x+5}{x+1}\right)e^{x^2}$ c) $\left(\frac{6x+5}{(x+1)^2}\right)e^{x^2}$ d) $\left(\frac{5-6x}{x+1}\right)e^{x^2}$

63. The curve for which the normal at any point (x,y) and the line joining the origin to that point form an isosceles triangle with the x-axis as base is

a) An ellipse b) A rectangular hyperbola

c) A circle d) None of these

64. Orthogonal trajectories of family of the curve $x^{2/3}+y^{2/3}=a^{2/3}$, where a is any arbitrary constant, is

a) $x^{2/3}-y^{2/3}=c$ b) $x^{4/3}-y^{4/3}=c$ c) $x^{4/3}-y^{4/3}=c$ d) $x^{1/3}-y^{1/3}=c$

65. The slope of the tangent at (x,y) to a curve passing through a point $(2,1)$ is $\frac{x^2+y^2}{2xy}$, then the equation of the curve is

a) $2(x^2-y^2)=3x$ b) $2(x^2-y^2)=6y$ c) $x(x^2-y^2)=6$ d) $x(x^2+y^2)=10$

66. The normal to a curve at $P(x,y)$ meets the x-axis at G. If the distance of G from the origin is twice the abscissa of P, then the curve is a

a) Parabola b) Circle c) Hyperbola d) Ellipse

67. The solution of $(y+x+5)dy=(y-x+1)dx$ is

a) $\log((y+3)^2+(x+2)^2)+\tan^{-1}\frac{y+3}{y+2}+C$

b) $\log((y+3)^2+(x-2)^2)+\tan^{-1}\frac{y-3}{x-2}=C$

c) $\log((y+3)^2+(x+2)^2)+2\tan^{-1}\frac{y+3}{x+2}+C$

d) $\log((y+3)^2+(x+2)^2)-2\tan^{-1}\frac{y+3}{x+2}+C$

68. The differential equation of the curve for which the initial ordinate of any tangent is equal to the corresponding subnormal

a) Is linear b) Is homogenous of second degree

c) Has separable variables d) Is of second order

69. If $\frac{dy}{dx}=\frac{xy}{x^2+y^2}, y(1)=1$, then one of the values of x_0 satisfying $y(x_0)=e$ is given by

a) $e\sqrt{2}$ b) $e\sqrt{3}$ c) $e\sqrt{5}$ d) $e/\sqrt{2}$

70. The equation of the curves through the point $(1,0)$ and whose slope is $\frac{y-1}{x^2+x}$ is

a) $(y-1)(x+1)+2x=0$ b) $2x(y-1)+x+1=0$

c) $x(y-1)(x+1)+2=0$ d) None of these

71. If $y=y(x)$ and $\frac{2+\sin x}{y+1}\left(\frac{dy}{dx}\right)=-\cos x, y(0)=1$, then $y(\pi/2)$ equals

a) $1/3$ b) $2/3$ c) $-1/3$ d) 1

72. The slope of the tangent at (x,y) to a curve passing through $\left(1,\frac{\pi}{4}\right)$ is given by $\frac{y}{x}=\cos^2\left(\frac{y}{x}\right)$, then the equation of the curve is

a) $y=\tan^{-1}\left(\log\left(\frac{e}{x}\right)\right)$ b) $y=x\tan^{-1}\left(\log\left(\frac{x}{e}\right)\right)$ c) $y=x\tan^{-1}\left(\log\left(\frac{e}{x}\right)\right)$ d) None of these

73. The differential equation of all non-horizontal lines in a plane is

a) $\frac{d^2y}{dx^2}$ b) $\frac{d^2x}{dy^2}=0$ c) $\frac{dy}{dx}=0$ d) $\frac{dx}{dy}=0$

74. The equation of a curve passing through (2, 7/2) and having gradient $1-\frac{1}{x^2}$ at (x, y) is

a) $y=x^2+x+1$ b) $xy=x^2+x+1$ c) $xy=x+1$ d) None of these

75. The differential equation of the curve $\frac{x}{c-1}+\frac{y}{c+1}=1$ is given by

a) $\left(\frac{dy}{dx}-1\right)\left(y+x\frac{dy}{dx}\right)=2\frac{dy}{dx}$ b) $\left(\frac{dy}{dx}+1\right)\left(y-x\frac{dy}{dx}\right)=\frac{dy}{dx}$

c) $\left(\frac{dy}{dx}+1\right)\left(y-x\frac{dy}{dx}\right)=2\frac{dy}{dx}$ d) None of these

76. Solution of the differential equation

$\left\{\frac{1}{x}-\frac{y^2}{(x-y)^2}\right\}dx+\left\{\frac{x^2}{(x-y)^2}-\frac{1}{y}\right\}dy=0$ is

a) $\ln\left|\frac{x}{y}\right|+\frac{xy}{x-y}=c$ b) $\frac{xy}{x-y}=ce^{x/y}$ c) $\ln|xy|=c+\frac{xy}{x-y}$ d) None of these

77. The solution of the equation $dy/dx=\cos(x-y)$ is

a) $y+\cot\left(\frac{x-y}{2}\right)=C$ b) $x+\cot\left(\frac{x-y}{2}\right)=C$ c) $x+\tan\left(\frac{x-y}{2}\right)=C$ d) None of these

78. Differential equation of the family of curves $v=A/r+B$, where A and B are arbitrary constant, is

a) $\frac{d^2v}{dr^2}+\frac{1}{r}\frac{dv}{dr}=0$ b) $\frac{d^2v}{dr^2}-\frac{2}{r}\frac{dv}{dr}=0$ c) $\frac{d^2v}{dr^2}+\frac{2}{r}\frac{dv}{dr}=0$ d) None of these

79. Number of values of $m\in N$ for which $y=e^{mx}$ is a solution of the differential equation $\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}-4\frac{dy}{dx}+12y=0$

a) 0 b) 1 c) 2 d) More than 2

80. The differential equation of all circles which pass through the origin and whose centres lie on the y-axis is

a) $(x^2-y^2)\frac{dy}{dx}-2xy=0$ b) $(x^2-y^2)\frac{dy}{dx}+2xy=0$

c) $(x^2-y^2)\frac{dy}{dx}-xy=0$ d) $(x^2-y^2)\frac{dy}{dx}+xy=0$

81. The differential equation for the family of curve $x^2+y^2-2ay=0$, where a is an arbitrary constant, is

a) $2(x^2-y^2)y'=xy$ b) $2(x^2+y^2)y'=xy$ c) $(x^2-y^2)y''=2xy$ d) $(x^2-y^2)y''=2xy$

82. If $y(t)$ is a solution of $(1+t)\frac{dy}{dt}-ty=1$ and $y(0)=-1$ then $y(1)$ is equal to

a) $-\frac{1}{2}$ b) $e+\frac{1}{2}$ c) $e-\frac{1}{2}$ d) $\frac{1}{2}$

83. The solution of $\frac{dv}{dt}+\frac{k}{m}v=-g$ is

a) $v=ce^{-\frac{k}{m}t}-\frac{mg}{k}$ b) $v=c-\frac{mg}{k}e^{-\frac{k}{m}t}$ c) $ve^{-\frac{k}{m}t}=c-\frac{mg}{k}$ d) $ve^{\frac{k}{m}t}=c-\frac{mg}{k}$

84. The equation of a curve passing through (1, 0) for which the product of the abscissa of a point P and the intercept made by a normal at P on the x-axis equals twice the square of the radius vector of the point P, is

a) $x^2+y^2=x^4$ b) $x^2+y^2=2x^4$ c) $x^2+y^2-4x^4$ d) None of these

85. The solution of the differential equation $(x+2y^3)\frac{dy}{dx}=y$ is

a) $\frac{x}{y^2}=y+c$ b) $\frac{x}{y}=y^2+c$ c) $\frac{x^2}{y}=y^2+c$ d) $\frac{y}{x}=x^2+c$

86. The solution of the differential equation

$\frac{dy}{dx}=\frac{1}{xy[x^2\sin y^2+1]}$ is

a) $x^2\left(\cos y^2-\sin y^2-2\,Ce^{-y^2}\right)=2$

b) $y^2\left(\cos x^2-\sin y^2-2\,Ce^{-y^2}\right)=2$

c) $x^2\left(\cos y^2-\sin y^2-e^{-y^2}\right)=4C$

d) None of these

87. The solution of differential equation $(2y+xy^3)dx+(x+x^2y^2)dy=0$ is

a) $x^2y+\frac{x^3y^3}{3}=c$ b) $xy^2+\frac{x^3y^3}{3}=c$ c) $x^2y+\frac{x^4y^4}{4}=c$ d) None of these

88. Spherical rain drop evaporates at a rate proportional to its surface area. The differential equation corresponding to the rate of change of the radius of the rain drop if the constant of proportionality is $K > 0$ is

a) $\frac{dr}{dt} + K = 0$ b) $\frac{dr}{dt} - K = 0$ c) $\frac{dr}{dt} = Kr$ d) None of these

89. Water is drained from a vertical cylindrical tank by opening a valve at the base of the tank. It is known that the rate at which the water level drops is proportional to the square root of water depth y, where the constant of proportionality $k > 0$ depends on the acceleration due to gravity and the geometry of the hole. If t is measured in minutes and $k = \frac{1}{15}$, then the time to drain the tank if the water is 4 m deep to start with is

a) 30 min b) 45 min c) 60 min d) 80 min

90. The solution of differential equation

$\frac{x+y\frac{dy}{dx}}{y-x\frac{dy}{dx}} = \frac{x\cos^2(x^2+y^2)}{y^3}$ is

a) $\tan(x^2+y^2) = \frac{x^2}{y^2} + c$ b) $\cot(x^2+y^2) = \frac{x^2}{y^2} + c$

c) $\tan(x^2+y^2) = \frac{y^2}{x^2} + c$ d) $\cot(x^2+y^2) = \frac{y^2}{x^2} + c$

91. The differential equation of all parabolas each of which has a latus rectum $4a$ and whose axis are parallel to the x-axis is

a) Of order 1 and degree 2 b) Of order 2 and degree 3

c) Of order 2 and degree 1 d) Of order 2 and degree 2

92. The equation of the curve which is such that the portion of the axis of x cut off between the origin and tangent at any point is proportional to the ordinate of that point is

(b is a constant of proportionality)

a) $x = y(a - b\log x)$ b) $\log x = by^2 + a$ c) $x^2 = y(a - b\log y)$ d) None of these

93. The differential equation of all parabolas whose axis are parallel to the y-axis is

a) $\frac{d^3y}{dx^3} = 0$ b) $\frac{d^2x}{dy^2} = C$ c) $\frac{d^3y}{dx^3} + \frac{d^2x}{dy^2} = 0$ d) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = C$

94. The general solution of the differential equation,

$y' + y\phi'(x) - \phi(x).\phi'(x) = 0$, where $\phi(x)$ is a known function, is

a) $y = ce^{-\phi(x)} + \phi(x) - 1$ b) $y = ce^{+\phi(x)} + \phi(x) - 1$

c) $y = ce^{-\phi(x)} - \phi(x) + 1$ d) $y = ce^{-\phi(x)} + \phi(x) + 1$

Multiple Correct Answers Type

95. Which one of the following function(s) is/are homogeneous?

a) $f(x,y) = \frac{x-y}{x^2+y^2}$

b) $f(x,y) = x^{\frac{1}{3}}y^{-\frac{2}{3}}\tan^{-1}\frac{x}{y}$

c) $f(x,y) = x\left(\ln\sqrt{x^2+y^2} - \ln y\right) + ye^{x/y}$

d) $f(x,y) = x\left[\ln\frac{2x^2+y^2}{x} - \ln(x+y)\right] + y^2\tan\frac{x+2y}{3x-y}$

96. The curve $y = f(x)$ is such that the area of the trapezium formed by the coordinate axes, ordinate of an arbitrary point and the tangent at this point equals half the square of its abscissa. The equation of the curve can be

a) $y = cx^2 \pm x$ b) $y = cx^2 \pm 1$ c) $y = cx \pm x^2$ d) $y = cx^2 \pm x \pm 1$

97. The solution of the differential equation

$\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$ is

a) $y = 2$ b) $y = 2x$ c) $y = 2x - 4$ d) $y = 2x^2 - 4$

98. $y = ae^{-1/x} + b$ is a solution of $\frac{dy}{dx} = \frac{y}{x^2}$, then

a) $a \in R$
b) $b = 0$
c) $b = 1$
d) a takes finite number of values

99. The differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where $c$is a positive parameter, is of

a) Order 1 b) Order 2 c) Degree 3 d) Degree 4

100. The equation of the curve satisfying the differential equation $y_2(x^2 + 1) = 2xy_1$ passing through the point (0, 1) and having slope of tangent at $x = 0$ as 3 (where y_2 and y_1) represents 2nd and 1st order derivative), then

a) $y = f(x)$ is a strictly increasing function
b) $y = f(x)$ is a non-monotonic function
c) $y = f(x)$ has three distinct real roots
d) $y = f(x)$ has only one negative root

101. The equation of the curve satisfying the differential equation $y\left(\frac{dy}{dx}\right)^2 + (x - y)\frac{dy}{dx} - x = 0$ can be a

a) Circle b) Straight line c) Parabola d) Ellipse

102. The tangent at any point P on $y = f(x)$ meets x-axis and y-axis at A and B respectively. If $PA : PB = 2 : 1$, then the equation of the curve, is

a) $| x |y = c$
b) $x^2| y | = c$
c) $| x |y^2 = c$
d) $y^2c| x |$
(where c is arbitrary constant)

103. The curves for which the length of the normal is equal to the length of the radius vector is/are

a) Circles
b) Rectangular hyperbola
c) Ellipse
d) Straight lines

104. The solution of $\frac{dy}{dx} = \frac{ax+h}{by+k}$ represents a parabola if

a) $a = -2, b = 0$ b) $a = -2, b = 2$ c) $a = 0, b = 2$ d) $a = 0, b = 0$

105. A curve $y = f(x)$ passes through the point $P(1, 1)$. The normal to the curve at P is a $(y - 1) + (x - 1) = 0$. If the slope of the tangent at any point on the curve is proportional to the ordinate of the point, then the equation of the curve is

a) $y = e^{K(x-1)}$ b) $y = e^{Ke}$ c) $y = e^{K(x-2)}$ d) None of these

106. The solution of the differential equation
$(x^2y^2 - 1)dy + 2x\, y^3\, dx = 0$ is

a) $1 + x^2y^2 = cx$ b) $1 + x^2y^2 = cy$ c) $y = 0$ d) $y = -\frac{1}{x^2}$

107. For the differential equation whose solution is $(x - h)^2 + (y - k)^2 = a^2$ (a is a constant), is

a) Order is 2 b) Order is 3 c) Degree is 2 d) Degree is 3

108. If $f(x), g(x)$ be twice differential functions on [0, 2] satisfying $f''(x) = g''(x), f'(1) = 2g'(1) = 4$ and $f(2) = 3g(2) = 9$, then

a) $f(4) - g(4) = 10$
b) $|f(x) - g(x)| < 2 \Rightarrow -2 < x < 0$
c) $f(2) = g(2) \Rightarrow x = -1$
d) $f(x) - g(x) = 2x$ has real root

109. Identify the statement(s) which is/are true

a) $f(x, y) = e^{y/x} + \tan\frac{y}{x}$ is a homogeneous of degree zero
b) $x \ln\frac{y}{x} dx + \frac{y^2}{x} \sin^{-1}\frac{y}{x} dy = 0$ is a homogeneous differential equation
c) $f(x, y) = x^2 + \sin x \cos y$ is a not homogeneous
d) $(x^2 + y^2)dx - (xy^2 - y^3)dy = 0$ is a homogeneous differential equation

110. In which of the following differential equation degree is not defined?

a) $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x \log\frac{d^2y}{dx^2}$
b) $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x \sin\left(\frac{d^2y}{dx^2}\right)$

c) $x = \sin\left(\frac{dy}{dx} - 2y\right), |x| < 1$

d) $x - 2y = \log\left(\frac{dy}{dx}\right)$

111. The solution of $\frac{dy}{dx} = \frac{ax+hhhhhh}{by+k}$ represents a parabola when

a) $a = 0, b \neq 0$
b) $a \neq 0, b \neq 0$
c) $b = 0, a \neq 0$
d) $a = 0, b \in R$

112. The solution of $\left(\frac{x\,dx+y\,dy}{x\,dy-y\,dx}\right) = \sqrt{\left(\frac{a^2-x^2-y^2}{x^2+y^2}\right)}$ is

a) $\sqrt{(x^2+y^2)} = a\sin\{\,(\tan^{-1} y/x) + \text{constant}\}$

b) $\sqrt{(x^2+y^2)} = a\cos\{\,(\tan^{-1} y/x) + \text{constant}\,\}$

c) $\sqrt{(x^2+y^2)} = a\,\{\tan(\sin^{-1} y/x + \text{constant}\}$

d) $y = x\tan\{\,\text{constant} + \sin^{-1}\frac{1}{a}\sqrt{\sqrt{(x^2+y^2)}}\}$

113. The solution of $\frac{xdx+ydy}{xdy-ydx} = \sqrt{\frac{1-x^2-y^2}{x^2+y^2}}$ is

a) $\sqrt{x^2+y^2} = \sin\{\tan^{-1}(y/x) + C\}$

b) $\sqrt{x^2+y^2} = \cos\{\tan^{-1}(y/x) + C\}$

c) $\sqrt{x^2+y^2} = (\tan(\sin^{-1} y/x) + C)$

d) $y = x\tan\left(c + \sin^{-1}\sqrt{x^2+y^2}\right)$

114. The graph of the function $y = f(x)$ passing through the point (0, 1) and satisfying the differential equation $\frac{dy}{dx} + y\cos x = \cos x$ is such that

a) It is a constant function
b) It is periodic
c) It is neither an ever nor an odd function
d) It is continuous and differentiable for all x

115. Which of the following equation(s) is/are linear?

a) $\frac{dy}{dx} + \frac{y}{x} = \log x$
b) $y\left(\frac{dy}{dx}\right) + 4x = 0$
c) $(2x + y^3)\left(\frac{dy}{dx}\right) = 3y$
d) None of these

116. For equation of the curve whose subnormal is constant, then

a) Its eccentricity is 1
b) Its eccentricity is $\sqrt{2}$
c) Its axis is the x-axis
d) Its axis is the y-axis

117. The order of the differential equation whose general solution is given by $y = (C_1 + C_2)\cos(x + C_3) - C_4 e^{x+C_5}$, where C_1, C_2, C_3, C_4, C_5, are arbitrary constants, is

a) 5
b) 4
c) 3
d) 2

118. The solution of $\left(\frac{dy}{dx}\right)^2 + 2y\cot x\,\frac{dy}{dx} = y^2$ is

a) $x = 2\sin^{-1}\sqrt{\left(\frac{c}{2y}\right)}$
b) $x = 2\cos^{-1}\sqrt{\left(\frac{c}{2y}\right)}$
c) $y = \frac{c}{1-\cos x}$
d) $y - \frac{c}{1+\cos x} = 0$

Assertion - Reasoning Type

This section contain(s) 0 questions numbered 119 to 118. Each question contains STATEMENT 1(Assertion) and STATEMENT 2(Reason). Each question has the 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

a) Statement 1 is True, Statement 2 is True; Statement 2 **is** correct explanation for Statement 1

b) Statement 1 is True, Statement 2 is True; Statement 2 **is not** correct explanation for Statement 1

c) Statement 1 is True, Statement 2 is False

d) Statement 1 is False, Statement 2 is True

119

Statement 1: The value of $y(2)$, if y satisfies $x^2\frac{dy}{dx} + xy = \sin x, y(1) = 2$ is $\frac{1}{2}\int_1^2 \frac{\sin t}{t}\,dt$.

Statement 2: The solution of linear equation $\frac{dy}{dx} + Py = Q$ can be obtained by multiplying with the factor $e^{\int P\,dx}$

120

Statement 1: Degree of the differential equation $2x - 3y + 2 = \log\left(\frac{dy}{dx}\right)$ is not defined

Statement 2: In the given differential equation, the power of highest order derivative when expressed as the polynomials of derivatives is called degree

121

Statement 1: The differential equation of all circles in a plane must be of order 3

Statement 2: There is only one circle passing through three non –collinear points

122

Statement 1: Order of the differential equation whose solutions is $y = c_1 e^{x+c_2} + c_3 e^{x+c_4}$ is 4.

Statement 2: Order of the differential equation is equal to the number of independent arbitrary constant mentioned in the solution of differential equation.

123

Statement 1: The elimination of four arbitrary constants in $y = (c_1 + c_2 + c_3 e^{c_4})x$ results into a differential equation of the first order $x\frac{dy}{dx} = y$.

Statement 2: Elimination of n arbitrary constants requires in general, a differential equation of the nth order.

124 Let a solution $y = y(x)$ of the differential equation $x\sqrt{x^2-1}dy - y\sqrt{y^2-1}dx = 0$ satisfy $y(2) = \frac{2}{\sqrt{3}}$

Statement 1: $y(x) = \sec(\sec^{-1} x - \frac{\pi}{6})$

Statement 2: $y(x)$ is given by $\frac{1}{y} = \frac{2\sqrt{3}}{x} - \sqrt{1 - \frac{1}{x^2}}$

125

Statement 1: The differential equation of the family of curves represented by $y = Ae^x$ is given by $\frac{dy}{dx} = y$

Statement 2: $\frac{dy}{dx} = y$ is valid for every member of the given family

126

Statement 1: The differential equation of the form $yf(xy)dx + x\phi(xy)dy = 0$ can be converted to homogeneous forms by substitution $xy = v$

Statement 2: All differential equation of first order and first degree become homogeneous, if we put $y = vx$

127

Statement 1: The differential equation of all circles in a plane must be of order 3.

Statement 2: If three point are non-collinear, then only one circle always passing through these points.

128

Statement 1: The equation of curve passing through (3, 9) which satisfies differential equation $\frac{dy}{dx} = x + \frac{1}{x^2}$ is $6xy = 3x^3 + 29x - 6$

Statement 2: The solution of differential equation $\left(\frac{dy}{dx}\right)^2 - \left(\frac{dy}{dx}\right)(e^x + e^{-x}) + 1 = 0$ is $y = c_1 e^x + c_2 e^{-x}$.

129

Statement 1: The degree of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} = \text{In}\left(\frac{d^2y}{dx^2}\right)$ is 2.

Statement 2: The degree of a differential equation which can be written as polynomial in the derivatives is the degree of the derivatives of the highest order occurring in it.

130

Statement 1: The order of the differential equation whose general solution is $y = c_1 \cos 2x +$ *c2*sin*2x+c3*cos*2x+c4e2x+c5e2x+c6* is 3

Statement 2: Total number of arbitrary parameters in the given general solution in the statement (1) is 3

131

Statement 1: Order of a differential equation represents number of arbitrary constants in the general solution

Statement 2: Degree of a differential equation represents number of family of curves

132

Statement 1: $y = a \sin x + b \cos x$ is a general solution of $y'' + y = 0$.

Statement 2: $y = a \sin x + b \cos x$ is a trigonometric function.

Matrix-Match Type

This section contain(s) 0 question(s). Each question contains Statements given in 2 columns which have to be matched. Statements (A, B, C, D) in **columns I** have to be matched with Statements (p, q, r, s) in **columns II**.

133.

Column-I	Column- II
(A) Order 1	(p) Of all parabolas whose axis is the x-axis
(B) Order 2	(q) Of family of curves $y = a(x + a)^2$, where a is an arbitrary constant
(C) Degree 1	(r) $\left(1 + 3\frac{dy}{dx}\right)^{2/3} = \frac{4d^3y}{dx^3}$
(D) Degree 3	(s) Of family of curve $y^2 = 2c\ (x + \sqrt{c})$, where $c > 0$

CODES :

	A	**B**	**C**	**D**
a)	Q,s	p	p	q,r,s
b)	p	q,r,s	p	q,s
c)	q,r,s	p	q,s	p

d)	p	q,s	q,r,s	p

134.

	Column-I		Column- II
(A)	If the function $y = e^{4x} + 2e^{-x}$ is a solution of the differential equation $\frac{\frac{d^3y}{dx^3} - 13\frac{dy}{dx}}{y} = K$, then the value of $K/3$ is	(p)	3
(B)	Number of straight lines which satisfy the differential equation $\frac{dy}{dx} + x\left(\frac{dy}{dx}\right)^2 - y = 0$ is	(q)	4
(C)	If real value of m for which the substitution, $y = u^m$ will transform the differential equation, $2x^4 y \frac{dy}{dx} + y^4 = 4x^6$ into a homogeneous equation, then the value of $2m$ is	(r)	2
(D)	If the solution of differential equation $x^2 \frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 12y$ is $y = Ax^m + Bx^{-n}$, then $\lvert m + n \rvert$ is	(s)	1

CODES :

	A	**B**	**C**	**D**
a)	r	p	s	q
b)	q	r	p	s
c)	p	s	q	r
d)	s	q	r	p

Linked Comprehension Type

This section contain(s) 10 paragraph(s) and based upon each paragraph, multiple choice questions have to be answered. Each question has atleast 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 135 to -135

Newton's law of cooling states that rate at which a substance cools in moving air is proportional to the difference between the temperatures of the substance and that of the air. If the temperature of the air is 290 K. We can write as $\frac{dT}{dt} = -k(T - 290), k > 0$ constants, where T is temperature of substance.

on the basis of above information, anseer the following questions :

135. The substance cools from 370 K to 330 K in 10 min, then

a) $T = 290 + 160e^{-kt}$ b) $T = 290 + 80e^{-kt}$ c) $T = 290 + 40e^{-kt}$ d) $T = 290 + 20e^{-kt}$

Paragraph for Question Nos. 136 to - 136

A right circular cone with radius R and height H contains a liquid which evaporates at a rate proportional to its surface area in contact with air (proportionally constant $= k > 0$). Suppose that $r(t)$ is the radius of liquid cone at time t.

on the basis of above information, anseer the following questions :

136. The time after which the cone is empty, is

a) $\frac{H}{2k}$ b) $\frac{H}{k}$ c) $\frac{H}{3k}$ d) $\frac{2H}{k}$

Paragraph for Question Nos. 137 to - 137

Let $f(x)$ be a non-positive continuous function and $F(x) = \int_0^x f(t)\,dt\ \forall\ x \geq 0$ and $f(x) \geq cF(x)$ where $c > 0$ and let $g: [0, \infty) \to R$ be a function such that $\frac{dg(x)}{dx} < g(x)\ \forall x > 0$ and $g(0) = 0$

137. The total number of root(s) of the equation $f(x) = g(x)$ is/are

a) ∞ b) 1 c) 2 d) 0

Paragraph for Question Nos. 138 to - 138

The differential equation $y = px + f(p)$, (1)
Where $p = \frac{dy}{dx}$, is known as Clairout's Equation. To solve equation (1), differentiate it with respect to x, which gives either
$\frac{dp}{dx} = 0 \Rightarrow p = c$ (2)
Or $x + f''(p) = 0$ (3)
Note:
a. *If p is eliminated between equation* (1)*and* (2), *the solution obtained is ageneral solution of equation* (1),
b. *If p is eliminated between equation* (1)*and* (3), *then solution obtained doesnot contain any arbitrary constant and is not particular solution of equation*(1). *This solution is called singular solution of equation* (1)

138. Which of the following is true about solutions of differential equation $y = xy'' + \sqrt{1 + y'^2}$?

a) The general solution of equation is family of parabolas
b) The general solution of equation is family of circles
c) The singular solution of equation is circle
d) The singular solution of equation is ellipse

Paragraph for Question Nos. 139 to - 139

For certain curves $y = f(x)$ satisfying $\frac{d^2y}{dx^2} = 6x - 4$, $f(x)$ has local minimum value 5 when $x = 1$

139. Number of critical point for $y = f(x)$ for $x \in [0, 2]$

a) 0 b) 1 c) 2 d) 3

Paragraph for Question Nos. 140 to - 140

A certain radioactive material is known to decay at a rate proportional to the amount present. Initially there is 50 kg of the material present and after two hours it is observed that the material has lost 10 percent of its original mass. Based on these data answer the following questions

140. The expression for the mass of the material remaining at any time t

a) $N = 50e^{-(1/2)(\ln 0.9)t}$ b) $50e^{-(1/4)(\ln 9)t}$ c) $N = 50e^{(\ln 0.9)t}$ d) None of these

Paragraph for Question Nos. 141 to - 141

Consider a tank which initially holds V_0ltr. of brine that contains a lbof salt. Another brine solution, containing b lb of salt/ltr., is poured into the tank at the rate of e ltr./min while, simultaneously, the well-stirred solution leaves the tank at the rate of f ltr./min. The problem is to find the amount of salt in the tank at any time t

Let Q denote the amount of salt in the tank at any time. The time rate of change of $Q, dQ/dt$, equals the rate at which salt enters the tank minus the rate at which salt leaves the tank. Salt enters the tank at the rate of be lb/min. To determine the rate at which salt leaves the tank, we first calculate the volume of brine in the tank at any time t, which is the initial volume V_0 plus the volume of brine added et minus the volume of brine removed ft. Thus, the volume of brine at any time is

$V_0 + et - ft$ (a)

The concentration of salt in the tank at any time is $Q/(V_0 + et - ft)$, from which it follows that salt leaves the tank at the rate of $f\left(\frac{Q}{V_0+et-ft}\right)$ lb/min

Thus, $\frac{dQ}{dt} = be - f\left(\frac{Q}{V_0+et-ft}\right)$ (b)

Or $\frac{dQ}{dt} + \frac{f}{V_0+et-ft}Q = be$

141. A tank initially holds 100 ltr. Of a brine solution containing 20 lb of salt. At $t = 0$, fresh water is poured into the tank at the rate of 5 ltr./min, while the well-stirred mixture leaves the tank at the same rate. Then the amount of salt in the tank after 20 min

a) $20/e$ b) $10/e$ c) $40/e^2$ d) $5/e$

Integer Answer Type

142. The perpendicular from the origin to the tangent at any point on a curve is equal to the abscissa of the point contact. Also curve passes through the point (1, 1). Then the length of intercept of the curve on the x-axis is
143. If $x\frac{dy}{dx} = x^2 + y - 2, y(1) = 1$, then $y(2)$ equals
144. The curve passing through the point (1, 1) satisfies the differential equation $\frac{dy}{dx} + \frac{\sqrt{(x^2-1)(y^2-1)}}{xy} = 0$. If the curve passes through the point $(\sqrt{2}, k)$ then the value of $[k]$ is (where $[\cdot]$ represents greatest integer function)
145. If the solution of the differential equation $\frac{dy}{dx} = \frac{1}{x\cos y + \sin 2y}$ is $x = ce^{\sin y} - k(1 + \sin y)$, then the value of k is
146. Tangent is drawn at the point (x_i, y_i) on the curve $y = f(x)$, which intersects the x-axis at $(x_{i+1}, 0)$. Now, again a tangent is drawn at (x_{i+1}, y_{i+1}) on the curve which intersect the x-axis at $(x_{i+2}, 0)$ and the process is repeated n times, i.e., $i = 1,2,3, \dots, n$. If $x_1, x_2, x_3, \dots x_n$ form an arithmetic progression with common

difference equal to $\log_2 e$ and curve passes through $(0, 2)$. Now if curve passes through the point $(-2, k)$, then the value of k is

147. If the solution of the differential equation $\frac{dy}{dx} - y = 1 - e^{-x}$ and $y(0) = y_0$ has a finite value, when $x \to \infty$, then the value of $|2/y_0|$ is

148. If $y = y(x)$ and it follows the relation $4xe^{xy} = y + 5\sin^2 x$, then $y'(0)$ is equal to

149. Let $y = y(t)$ be a solution to the differential equation $y' + 2ty = t^2$, then $16\ \lim_{t\to\infty} \frac{y}{t}$ is

150. If the eccentricity of the curve for which tangent at point P intersects the y-axis at M such that the point of tangency is equidistant from M and the origin is e, then the value of $5e^2$ is

151. If the independent variable x is changed to y, then the differential equation $x\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 - \frac{dy}{dx} = 0$ is changed to $x\frac{d^2x}{dy^2} + \left(\frac{dx}{dy}\right)^2 = k$ where k equals

152. If the dependent variable y is changed to 'z' by the substitution $y = \tan z$ and the differential equation $\frac{d^2y}{dx^2} = 1 + \frac{2(1+y)}{1+y^2}\left(\frac{dy}{dx}\right)^2$ is changed to $\frac{d^2z}{dx^2} = \cos^2 z + k\ \left(\frac{dz}{dx}\right)^2$, then the value of k equals

: ANSWER KEY :

1)	a	2)	a	3)	b	4)	a
5)	a	6)	a	7)	c	8)	b
9)	b	10)	c	11)	c	12)	b
13)	c	14)	b	15)	a	16)	a
17)	a	18)	d	19)	d	20)	a
21)	a	22)	b	23)	a	24)	c
25)	a	26)	a	27)	b	28)	a
29)	d	30)	c	31)	a	32)	c
33)	c	34)	b	35)	c	36)	d
37)	c	38)	d	39)	a	40)	b
41)	a	42)	b	43)	c	44)	d
45)	c	46)	a	47)	d	48)	a
49)	c	50)	c	51)	a	52)	d
53)	c	54)	b	55)	a	56)	a
57)	c	58)	c	59)	d	60)	d
61)	a	62)	b	63)	b	64)	b
65)	a	66)	c	67)	c	68)	a
69)	b	70)	a	71)	a	72)	c
73)	b	74)	b	75)	c	76)	a
77)	b	78)	c	79)	c	80)	a
81)	c	82)	a	83)	a	84)	a
85)	b	86)	a	87)	a	88)	a
89)	c	90)	a	91)	c	92)	a
93)	a	94)	a	1)	a,b,c	2)	
	a,b	3)	c	4)	a,b		
5)	a,c	6)	a,d	7)	a,b	8)	
	c,d						
9)	a,b	10)	a,c	11)	a	12)	b
13)	a,c	14)	a,b,c	15)	a,b,c	16)	
	a,b						
17)	a,c	18)	a,d	19)	a,d	20)	
	a,b,d						
21)	a,c	22)	b	23)	c	24)	
	a,b,c,d						
1)	d	2)	d	3)	a	4)	d
5)	a	6)	c	7)	a	8)	d
9)	a	10)	b	11)	d	12)	a
13)	b	14)	b	1)	a	2)	b
	1)	b	2)	b	3)	b	
	4)	c					
5)	c	6)	a	7)	a	1)	2
	2)	2	3)	3	4)	2	
5)	8	6)	4	7)	4	8)	8
9)	5	10)	1	11)	2		

: HINTS AND SOLUTIONS :

1 **(a)**

Putting $x+y+1=u$, we have $du=dx+dy$ and the given equations reduces to

$u(du-dx)=dx$

$\Rightarrow \dfrac{u\,du}{u+1}=dx$

$\Rightarrow u-\log(u+1)=x+C$

$\Rightarrow \log(x+y+2)=y+C$

$\Rightarrow x+y+2=Ce^{y}$

2 **(a)**

$y'y'''=3(y'')^2$

$\Rightarrow \int \dfrac{y'''}{y''}dx=3\int \dfrac{y''}{y_1'}dx$

$\Rightarrow \ln y''=3\ln y'+\ln c$

$\Rightarrow y''=c(y')^3$

$\Rightarrow \int \dfrac{y''}{(y')^2}dx=\int cy'dx$

$\Rightarrow -\dfrac{1}{y'}=cy+d$

$\Rightarrow -dx=(cy+d)dy$

$\Rightarrow\ -x=\dfrac{cy^2}{2}+dy+e$

3 **(b)**

Given, $\dfrac{dy}{dx}=-\dfrac{1+y+x^2y}{x+x^3}$

$\Rightarrow \dfrac{dy}{dx}+\dfrac{y}{x}=-\dfrac{1}{x(1+x^2)}$

$\therefore\quad IF=e^{\int \frac{1}{x}dx}=x$

4 **(a)**

$x^2\dfrac{dy}{dx}\cos\dfrac{1}{x}-y\sin\dfrac{1}{x}=-1$

$\Rightarrow \dfrac{dy}{dx}-\dfrac{y}{x^2}\tan\dfrac{1}{x}=-\sec\dfrac{1}{x}\dfrac{1}{x^2}$ (linear)

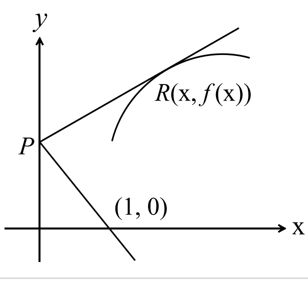
$\text{I. F.}=e^{\int \frac{1}{x^2}\tan\frac{1}{x}dx}=\sec\dfrac{1}{x}$

$\Rightarrow$ solution is $y\sec\frac{1}{x}=-\int \sec^2\left(\frac{1}{x}\right)\frac{1}{x^2}dx=$ tan *1x+c*

Given $y\to -1, x\to\infty \Rightarrow c=-1$

Hence equation of curve is $y=\sin\frac{1}{x}-\cos\frac{1}{x}$

5 **(a)**



The equation of the tangent at the point $R(x,f(x))$ is $Y-f(x)=f'(x)(X-x)$

The coordinates of the point P are $(0,f(x)-xf'(x))$

The slope of the perpendicular line through P is

$\frac{f(x)-xf'(x)}{-1}=-\frac{1}{f'(x)}$

$\Rightarrow f(x)f'(x)-x\big(f'(x)\big)^2=1$

$\Rightarrow \frac{ydy}{dx}-x\left(\frac{dy}{dx}\right)^2=1$ which is the required differential equation to the curve at $y=f(x)$

6 **(a)**

$y(2x^4+y)\dfrac{dy}{dx}=(1-4xy^2)x^2$

$\Rightarrow 2x^4y\,dy+y^2dy+4x^3y^2dx-x^2dx=0$

$\Rightarrow 2x^2y(x^2dy+2xy\,dx)+y^2dy-x^2dx=0$

$2x^2yd(x^2y)+y^2dy-x^2dx=0$

Integrating, we get $(x^2y)^2+\frac{y^3}{3}-\frac{x^3}{3}=c$

Or $3(x^2y)^2+y^3-x^3=c$

7 **(c)**

$\because \dfrac{dy}{dx}+\dfrac{y}{x\log_e x}=\dfrac{2}{x}$

$\therefore \text{I. F.}=e^{\int \frac{1}{x\log_e x}dx}$

$=e^{\log_e \log_e x}$

$=\log_e x$

8 **(b)**

$(x\cot y+\log\cos x)dy+(\log\sin y-y\tan x)dx=0$

$\Rightarrow (x\cot y\,dy+\log\sin y\,dx)+(\log\cos x\,dy-y\tan x\,dx)=0$

$\Rightarrow \int d(x\log\sin y)+\int d(y\log\cos x)=0$

$\Rightarrow x\log\sin y+y\log\cos x=\log c$

$\Rightarrow (\sin y)^x(\cos x)^y=c$

9 **(b)**

$y=c_1\cos(x+c_2)-(c_3e^{-x+c_4})+(c_5\sin x)$

$\Rightarrow y=c_1(\cos x\cos c_2-\sin x\sin c_2)-(c_3e^{c_4}e^{-x})+(c_5\sin x)$

$\Rightarrow y=(c_1\cos c_2)\cos x-(c_1\sin c_2-c_5)\sin x-(c_3e^{c_4})e^{-x}$

$\Rightarrow y=\ l\cos x+m\sin x-n\,e^{-x}$ (1)

Where l,m,n are arbitrary constant

$\Rightarrow \frac{dy}{dx}=-l\sin x+m\cos x+n\,e^{-x}$ (2)

$\Rightarrow \frac{d^2y}{dx^2}=-l\cos x-m\sin x-n\,e^{-x}$ (3)

$\Rightarrow \frac{d^3y}{dx^2}=l\sin x-m\cos x+n\,e^{-x}$ (4)

From equation (1) + (3), $\frac{d^2y}{dx^2} + y = -2ne^{-x}$ (5)

From equation (2) + (4), $\frac{d^3y}{dx^3} + \frac{dy}{dx} = 2n\, e^{-x}$ (6)

From equation (5) + (6), we get $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$

10 **(c)**

Differentiating the given equation successively, we get

$y_1 = 5\, b\, e^{5x} - 7c\, e^{-7x}$ (1)

$y_2 = 25\, b\, e^{5x} + 49\, ce^{-7x}$ (2)

$y_3 = 125\, \text{b}e^{5x} - 343\, ce^{-7x}$ (3)

Multiplying equation (1) by 7 and then adding to equation (2), we get $y_2 + 7y_1 = 60\, \text{b}e^{5x}$ (4)

Multiplying equation (1) by 5 and then subtracting it from equation (2),

We get $y_2 - 5y_1 = 84\, ce^{-7x}$ (5)

Putting the values of b and c, obtained from equation (4) and (5), respectively, in equation (1), we get

$y_3 + 2y_2 - 35y_1 = 0$

11 **(c)**

$(xy^3 - x^2)dy - (xy + y^4)dx = 0$

$\Rightarrow y^3(x\, dy - y\, dx) - x(x\, dy + y\, dx) = 0$

$\Rightarrow x^2y^3 \frac{(x\, dy - y\, dx)}{x^2} - x(x\, dy + y\, dx) = 0$

$\Rightarrow x^2y^3\, d\left(\frac{y}{x}\right) - xd\,(xy) = 0$

$\Rightarrow$ Dividing by x^3y^2, we get

$\Rightarrow \frac{y}{x} d\left(\frac{y}{x}\right) - \frac{d(xy)}{x^2y^2} = 0$

Now integrating $\frac{1}{2}\left(\frac{y}{x}\right)^2 + \frac{1}{xy} = c$

It passes through the point $(4, -2)$

$\Rightarrow \frac{1}{8} - \frac{1}{8} = c \Rightarrow c = 0$

$\therefore y^3 = -2x$

12 **(b)**

$\frac{dy}{dx} = e^{ax+by} = e^{ax}e^{by}$

Or $e^{-by}\, dy = e^{ax}dx$

$\therefore\ -\frac{1}{b}e^{-by} = \frac{1}{a}e^{ax} + c$

13 **(c)**

$\frac{y'''}{y''} = 8 \Rightarrow \log y'' = 8x + c$

When $x = 0, y'' = 1$ and $\log 1 = 0 \therefore c = 0$

$\therefore\ y'' = e^{8x}$. Integrating again

$y' = \frac{e^{8x}}{8} + \lambda$ when $x = 0, y'(0) = 0$

$\therefore \lambda = -1/8$

$\therefore\ y' = \frac{e^{8x}}{8} - \frac{1}{8}$. Integrate again

$y = \frac{e^{8x}}{64} - \frac{x}{8} + k$

Also when $x = 0, y = \frac{1}{8} \therefore k = \frac{7}{64}$

$\therefore y = \frac{1}{8}\left(\frac{e^{8x}}{8} - x + \frac{7}{8}\right)$

14 **(b)**

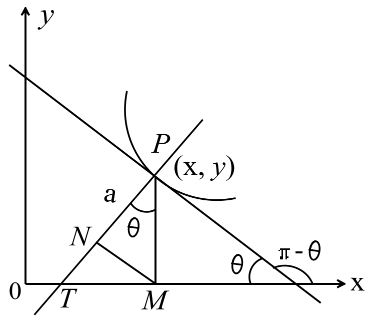
We have $\frac{dy}{dx} = \sin\frac{x-y}{2} - \sin\frac{x+y}{2}$

$= -2\cos\frac{x}{2}\sin\frac{y}{2}$

$\Rightarrow \log\tan\frac{y}{4} = -\frac{\sin\frac{x}{2}}{\frac{1}{2}} + c$

$\Rightarrow \log\tan\left(\frac{y}{4}\right) = c - 2\sin\frac{x}{2}$

15 **(a)**



Ordinate $= PM$. Let $P \equiv (x, y)$

Projection of ordinate on normal $= PN$

$\therefore PN = PM\cos\theta = a$ (given)

$\therefore\ \frac{y}{\sqrt{1 + \tan^2\theta}} = a$

$\Rightarrow y = a\sqrt{1 + (y_1)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{y^2 - a^2}}{a}$

$\Rightarrow \int \frac{a\, dy}{\sqrt{y^2 - a^2}} = \int dx$

$\Rightarrow a\ln\left|y + \sqrt{y^2 - a^2}\right| = x + c$

16 **(a)**

Putting $x = \sin A$ and $y = \sin B$ in the given relation, we get $\cos A + \cos B = a(\sin A - \sin B)$

$\Rightarrow A - B = 2\cot^{-1} a$

$\Rightarrow \sin^{-1} x - \sin^{-1} y = 2\cot^{-1} a$

Differentiatingw. r. t. x, we get

$\frac{1}{\sqrt{1 - x^2}} - \frac{1}{\sqrt{1 - y^2}}\frac{dy}{dx} = 0$

Clearly, it is a differential equation of degree one

17 **(a)**

Given, $\frac{dy}{y+1} = \frac{-\cos x}{2+\sin x}\, dx$

$\Rightarrow \int \frac{dy}{y+1} = -\frac{\int\cos x}{2+\sin x}dx$

$\Rightarrow \log(y + 1) = -\log(2 + \sin x) + \log c$

When $x = 0, y = 1$

$\Rightarrow \qquad c = 4$

$\therefore \qquad y + 1 = \frac{4}{2+\sin x}$

At $x = \frac{\pi}{2}$, $\qquad y + 1 = \frac{4}{2+1}$

$\Rightarrow \qquad y = \frac{1}{3}$

18 **(d)**

$2x^2y\frac{dy}{dx} = \tan(x^2y^2) - 2xy^2$

$\Rightarrow x^2\,2y\frac{dy}{dx} + y^2\,2x = \tan(x^2y^2)$

$\Rightarrow \frac{d}{dx}(x^2y^2) = \tan(x^2y^2)$

$\Rightarrow \int \cot(x^2y^2)d\,(x^2y^2) = \int dx$

$\Rightarrow \log(\sin(x^2y^2)) = x + c$

When $x = 1, y = \sqrt{\frac{\pi}{2}} \Rightarrow c = -1$

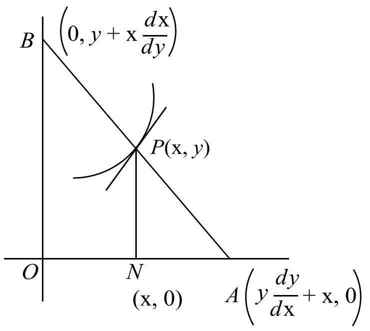
$\Rightarrow$ Equation of curve is $x = \log\sin(x^2y^2) + 1$

$\Rightarrow \log\sin(x^2y^2) = x + 1$

$\Rightarrow \sin(x^2y^2) = e^{x+1}$

19 **(d)**

Equation of normal at point p is $Y - y = \frac{dx}{dy}(X - x)$



Area of ΔOAB *is* 1 $\Rightarrow \frac{1}{2}\left(y\frac{dy}{dx} + x\right)\left(x\frac{dx}{dy} + y\right) = 1$

$\Rightarrow \left(y\frac{dy}{dx} + x\right)\left(y\frac{dy}{dx} + x\right) = 2\frac{dy}{dx}$

$\Rightarrow y^2\left(\frac{dy}{dx}\right)^2 + 2(xy - 1)\frac{dy}{dx} + x^2 = 0$

20 **(a)**

$\frac{dy}{dx} + 2\,xy = y$

$\Rightarrow \frac{dy}{dx} = y(1 - 2x)$

$\Rightarrow \frac{dy}{y} = (1 - 2x)dx$

$\Rightarrow \log y = x - x^2 + c_1$

$\Rightarrow y = e^{x-x^2}e^{c_1} = c\,e^{x-x^2}$ where $c = e^{c_1}$

$\Rightarrow y = ce^{x-x^2}$ is the required solution

21 **(a)**

Let $V(t)$ be the velocity of the object at time t

Given $\frac{dV}{dt} = 9.8 - kV \Rightarrow \frac{dV}{9.8-kV} = dt$

Integrating, we get $\log(9.8 - kV) = -kt + \log C$

$\Rightarrow 9.8 - kV = Ce^{-kt}$

But $V(0) = 0 \Rightarrow C = 9.8$

Thus, $9.8 - kV = 9.8\,e^{-kt}$

$\Rightarrow kV = 9.8(1 - e^{-kt})$

$\Rightarrow V(t) = \frac{9.8}{k}\left(1 - e^{-kt}\right) < \frac{9.8}{k}$

For all t. Hence, $V(t)$ cannot exceed $\frac{9.8}{k}$ m/s

22 **(b)**

For the family of curves represented by the first differential equation the slope of the tangent at any point is given by

$\left(\frac{dy}{dx}\right)_{c_1} = \frac{x^2 + x + 1}{y^2 + y + 1}$

For the family of curves represented by the second differential the slope of the tangent at any point is given by

$\left(\frac{dy}{dx}\right)_{c_2} = \frac{y^2 + y + 1}{x^2 + x + 1}$

Clearly, $\left(\frac{dy}{dx}\right)_{c_1} \times \left(\frac{dy}{dx}\right)_{c_2} = -1$

Hence, the two curves are orthogonal

23 **(a)**

The given equation can be written as

$\frac{y}{x}\frac{dy}{dx} = \left\{\frac{y^2}{x^2} + \frac{f(y^2/x^2)}{f'(y^2/x^2)}\right\}$

Above equation is a homogeneous equation putting $y = vx$, we get

$v\left[v + x\frac{dv}{dx}\right] = v^2 + \frac{f(v^2)}{f'(v^2)}$

$\Rightarrow vx\frac{dv}{dx} = \frac{f(v^2)}{f'(v)}$ variable separable

$\Rightarrow \frac{2vf'(v^2)}{f(v^2)}dv = 2\frac{dv}{x}$

Now integrating both sides, we get

$\log f(v^2) = \log x^2 + \log c \qquad [\log c = \text{constant}]$

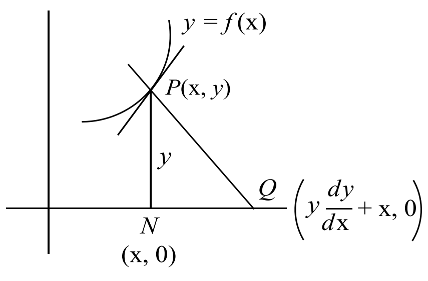
Or $\log f(v^2) = \log cx^2$

Or $f(v^2) = cx^2$

Or $f(y^2/x^2) = cx^2$

24 **(c)**

Equation of normal at point $P(x, y), Y - y = -\frac{dy}{dx}(X - x)$



$NQ = y\dfrac{dy}{dx} = \dfrac{x(1+y^2)}{1+x^2}$

$\Rightarrow \dfrac{xdx}{1+x^2} = \dfrac{ydy}{1+y^2}$

$\Rightarrow \ln(1+x^2) = \ln(1+y^2) + \ln c$

$\Rightarrow 1+y^2 = \dfrac{1+x^2}{c}$

It passes through $(3,1) \Rightarrow 1+1 = \frac{1+(3)^2}{6} \Rightarrow c = 5$

$\Rightarrow$ curve is $5+5y^2 = 1+x^2$ or $x^2 - 5y^2 = 4$

25 **(a)**

Integrating the given differential equation, we have

$\dfrac{dy}{dx} = \dfrac{-\cos 3x}{3} + e^x + \dfrac{x^3}{3} + C_1$

But $y_1(0) = 1$

So $1 = \left(-\frac{1}{3}\right) + 1 + C_1 \Rightarrow C_1 = 1/3$

Again integrating, we get

$y = \dfrac{-\sin 3x}{9} + e^x + \dfrac{x^4}{12} + \dfrac{1}{3}x + C_2$

But $y(0) = 0$ so $0 = 0 + 1 + C_2 \Rightarrow C_2 = -1$

Thus $y = \frac{-\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x - 1$

26 **(a)**

$x^2\dfrac{dy}{dx} - xy = 1 + \cos\dfrac{y}{x}$

$\Rightarrow \dfrac{x(xdy - ydx)}{dx} = 1 + \cos\dfrac{y}{x}$

$\Rightarrow \dfrac{\frac{xdy - ydx}{x^2}}{1+\cos\frac{y}{x}} = \int \dfrac{dx}{x^3}$

$\Rightarrow \int \dfrac{d\left(\frac{y}{x}\right)}{1+\cos\frac{y}{x}} = \int \dfrac{dx}{x^3}$

$\Rightarrow \dfrac{1}{2}\int \dfrac{d\left(\frac{y}{x}\right)}{\cos^2\frac{y}{2x}} = \int \dfrac{dx}{x^3}$

$\Rightarrow \dfrac{1}{2}\int \sec^2\dfrac{y}{2x}.d\left(\dfrac{y}{x}\right) = \int \dfrac{dx}{x^3}$

$\Rightarrow \dfrac{1}{2}.\dfrac{\tan\frac{y}{2x}}{\frac{1}{2}} = \dfrac{x^{-2}}{-2} + c$

$\Rightarrow \tan\dfrac{y}{2x} + \dfrac{1}{2x^2} = c$

27 **(b)**

$(x^2 + xy)dy = (x^2 + y^2)dx \Rightarrow \dfrac{dy}{dx} = \dfrac{x^2+y^2}{x^2+xy}$

Let $\frac{y}{x} = v \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ equation reduces to

$x\dfrac{dv}{dx} = \dfrac{1+v^2}{1+v} - v$

$= \dfrac{1+v^2-v-v^2}{1+v}$

$= \dfrac{1-v}{1+v}$

$\Rightarrow \int \dfrac{1+v}{1-v}dv = \int \dfrac{dx}{x}$

$\Rightarrow -\int \left(1 - \dfrac{2}{1-v}\right)dv = \int \dfrac{dx}{x}$

$\Rightarrow -v - 2\log(1-v) = \log x + \log c$

$\Rightarrow -\dfrac{y}{x} - 2\log\left(\dfrac{x-y}{x}\right) = \log x + \log c$

$\Rightarrow \dfrac{-y}{x} - 2\log(x-y) = 2\log x = \log x + \log c$

$\Rightarrow \log x = 2\log(x-y) + \frac{y}{x} + k$ where $k = \log c$

28 **(a)**

Put $xy = v \ \therefore y + x\frac{dy}{dx} = \frac{dv}{dx}$

$\Rightarrow \dfrac{dv}{dx} = x\ \dfrac{\phi(v)}{\phi'(v)}$

$\therefore \frac{\phi'(v)}{\phi(v)}dv = x\ dx$. Integrating, we get

$\log\phi(v) = \dfrac{x^2}{2} + \log k$

$\Rightarrow \log\dfrac{\phi(v)}{k} = \dfrac{x^2}{2}$

or $\phi(v) = ke^{x^2/2} \Rightarrow \phi(xy) = ke^{x^2/2}$

29 **(d)**

Equation of circle will be $x^2 + (y-2)^2 + \lambda(y-2) = 0$

Differentiating, we get $2x + 2(y-2)\frac{dy}{dx} + \lambda\frac{dy}{dx} = 0$

$\therefore$ the equation is $x^2 + (y-2)^2 - (y-2)\left(2x\frac{dx}{dy} + \right.$

2y−4=0

30 **(c)**

The given equation can be rewritten as

$\frac{dy}{dx} + \frac{x^2-1}{x(x^2+1)}y = \frac{x^2\log x}{(x^2+1)}$ (1)

Which is linear. Also

$P = \frac{x^2-1}{x(x^2+1)}$ and $Q = \frac{x^2\log x}{(x^2+1)}$

$\int P\,dx = \int \left[\dfrac{2x}{x^2+1} - \dfrac{1}{x}\right]dx$

[resolving into partial fractions]

$= \log(x^2+1) - \log x$

$\therefore \text{I.F.} = e^{\log[(x^2+1)/x]} = \dfrac{x^2+1}{x}$

Hence the required solution of equation (1) is

$\dfrac{y(x^2+1)}{x} = \int \dfrac{(x^2+1)}{x}\dfrac{x^2\log x}{(x^2+1)}dx + c$

$= \int x\log x\ dx + c$

$= \dfrac{1}{2}x^2\log x - \int \dfrac{1}{x}\dfrac{x^2}{2}\ dx + c$

$\therefore y(x^2+1)/x = \frac{1}{2}x^2 \log x - \frac{1}{4}x^2 + c$

31 **(a)**

Equations of tangent is $Y - y = \frac{dy}{dx}(X - x)$

For X-intercept $Y = 0 \Rightarrow X = x - y\frac{dx}{dy}$

According to question $x - y\frac{dx}{dy} = y$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x-y}$

Putting $y = vx$, we get

$\Rightarrow v + x\frac{dv}{dx} = \frac{v}{1-v}$

$\Rightarrow x\frac{dv}{dx} = \frac{v}{1-v} - v = \frac{v - v + v^2}{1-v}$

$\Rightarrow \int \frac{1-v}{v^2}dv = \int \frac{dx}{x}$

$\Rightarrow -\frac{1}{v} - \log v = \log x + c$

$\Rightarrow -\frac{x}{y} - \log\frac{y}{x} = \log x + c$

$\Rightarrow -\frac{x}{y} = \log y + c$

Given when $x = 1, y = 1 \Rightarrow c = -1$

Hence equation of curve is $1 - \frac{x}{y} = \log y$

$\Rightarrow y = e\, e^{-x/y} \Rightarrow e^{x/y} = \frac{e}{y}$

$\Rightarrow ye^{x/y} = e$

32 **(c)**

$\cos x\frac{dy}{dx} + y\sin x = 1$

$\Rightarrow \frac{dy}{dx} + y\frac{\sin x}{\cos x} = \sec x$

$\therefore \int P\,dx = \int \frac{\sin x}{\cos x}dx$

$= -\log\cos x$

$= \log\sec x$

$\therefore \text{I. F.} = e^{\int Pdx} = e^{\log\sec x} = \sec x$

33 **(c)**

Rewriting the given equation as

$2xy\frac{dy}{dx} - y^2 = 1 + x^2$

$\Rightarrow 2y\frac{dy}{dx} - \frac{1}{x}y^2 = \frac{1}{x} + x$

Putting $y^2 = u$, we have

$\frac{du}{dx} - \frac{1}{x}u = \frac{1}{x} + x$

$\text{I. F.} = e^{-\int\frac{1}{x}dx} = \frac{1}{x}$

$\therefore$ solution is $u\frac{1}{x} = \int\left(\frac{1}{x^2} + 1\right)dx = -\frac{1}{x} + x + C$

$\Rightarrow y^2 = (x^2 - 1) + Cx$

Since $y(1) = 1$ so $C = 1$

Hence $y^2 = x(1+x) - 1$ which represents a system of hyperbola

34 **(b)**

$xy = C$

$\Rightarrow x\frac{dy}{dx} + y = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-y}{x} = m_1$

By condition,

$\tan\frac{\pi}{4} = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$

$1 = \left|\frac{-\frac{y}{x} - m_2}{1 - \frac{y}{x}m_2}\right|$

$\Rightarrow \frac{y}{x} + m_2 = 1 - \frac{y}{x}m_2$ or $\frac{y}{x}m_2 - 1$

$\Rightarrow m_2 = \frac{1 - \frac{y}{x}}{1 + \frac{y}{x}}$ or $m_2 = \frac{\frac{y}{x} + 1}{\frac{y}{x} - 1}$

$\Rightarrow \frac{dy}{dx} = \frac{x-y}{x+y}$ or $\frac{dy}{dx} = \frac{x+y}{y-x}$

35 **(c)**

Let population $= x$, at time t years. Given $\frac{dx}{dt} \propto x$

$\Rightarrow \frac{dx}{dt} = kx$ where k is a constant of proportionality

Or $\frac{dx}{x} = kdt$. Integrating, we get $\ln x = kt + \ln c$

$\Rightarrow \frac{x}{c} = e^{kt}$ or $x = ce^{kt}$

If initially, i.e., when time $t = 0, x = x_0$

then $x_0 = ce^0 = c$

$\Rightarrow x = x_0e^{kt}$

Given $x = 2x_0$ when $t = 30$ then $2x_0 = x_0e^{30k} \Rightarrow 2 = e^{30k}$

$\therefore \ln 2 = 30k$ (1)

To find t, when t triples, $x = 3x_0$ $\therefore 3x_0 = x_0e^{kt} \Rightarrow 3 = e^{kt}$

$\therefore \ln 3 = kt$ (2)

Dividing equation (2) by (1) then $\frac{t}{30} = \frac{\ln 3}{\ln 2}$ or

$t = 30 \times \frac{\ln 3}{\ln 2} = 30 \times 1.5849 = 48$ years (approx)

36 **(d)**

$\frac{dy}{dx} = 1 + xy$

$\Rightarrow \frac{dy}{dx} - xy = 1$

$\text{I. F.} = e^{\int -xdx} = e^{-x^2/2}$

Hence solution is $y.\,e^{-x^2/2} = \int e^{-x^2/2}dx + c$

$\int e^{-x^2/2}\,dx$ is not further integrable

37 **(c)**

$ax^2 + by^2 = 1$

Differentiating w. r. t. x, we get

$2ax + 2by\,y_1 = 0$

$\Rightarrow ax + byy_1 = 0 \Rightarrow \frac{-a}{b} = \frac{yy_1}{x}$ (1)

Again differentiating w.r.t. x, we get

$\Rightarrow a + by_1^2 + byy_2 = 0 \Rightarrow \frac{-a}{b} = y_1^2 + yy_1$ (2)

From equation (1) and (2), we get

$\frac{yy_1}{x} = y_1^2 + yy_2$

$\Rightarrow yy_1 = xy_1^2 + xyy_2$

38 **(d)**

$\int_0^x t\, y(t)\, dt = x^2 y(x)$

Differentiating w.r.t. x, we get

$xy(x) = x^2 y'(x) + 2xy(x)$

$\Rightarrow xy(x) + x^2 y'(x) = 0$

$\Rightarrow x\frac{dy}{dx} + y = 0$

$\Rightarrow \log y + \log x = \log c$

$\Rightarrow xy = c$

39 **(a)**

Given, $\frac{dy}{dt} - \left(\frac{1}{1+t}\right) y = \frac{1}{(1+t)}$ and $y(0) = -1$

$\therefore \quad IF = e^{\int -\left(\frac{t}{1+t}\right)dt} = e^{-\int\left(1-\frac{1}{1+t}\right)dt}$

$e^{-t+\log(1+t)} = e^{-t}(1+t)$

$\therefore$ Required solution is,

$ye^{-t}(1+t) = \int \frac{1}{1+t}\, e^{-t}\,(1+t)dt + c$

$= \int e^{-t}\, dt + c$

$\Rightarrow ye^{-1}(1+t) = -e^{-1} + c$

Since, $y(0) = -1$

$\Rightarrow \quad c = 0$

$\therefore \quad y = -\frac{1}{(1+t)}$

$\Rightarrow \quad y(1) = -\frac{1}{2}$

40 **(b)**

The given equation is written as $y\,dx - x\,dy + x\sqrt{xy}$

$(x+y)dx + y\sqrt{xy}(x+y)dy = 0$

$\Rightarrow ydx - xdy + (x+y)\sqrt{xy}(xdx + ydy) = 0$

$\Rightarrow \frac{ydx - xdy}{y^2} + \left(\frac{x}{y} + 1\right)\sqrt{\frac{x}{y}}\left(d\left(\frac{x^2+y^2}{2}\right)\right) = 0$

$d\left(\frac{x^2+y^2}{2}\right) + \frac{d\left(\frac{x}{y}\right)}{\left(\frac{x}{y}+1\right)\sqrt{\frac{x}{y}}} = 0$

$\Rightarrow \frac{x^2+y^2}{2} + 2\tan^{-1}\sqrt{\frac{x}{y}} = c$

41 **(a)**

$x^2 = e^{\left(\frac{x}{y}\right)^{-1}\left(\frac{dy}{dx}\right)}$

$\Rightarrow x^2 = e^{\left(\frac{x}{y}\right)\left(\frac{dy}{dx}\right)}$

$\Rightarrow \ln x^2 = \frac{y}{x}\frac{dy}{dx}$

$\Rightarrow \int x\ln x^2 dx = \int y\, dy$

Putting $x^2 = t$, we get $2xdx = dt$

$\Rightarrow \frac{1}{2}\int \ln t\, dt = \frac{y^2}{2}$

$\Rightarrow c + t\ln t - t = y^2$

$\Rightarrow y^2 = x^2\ln x^2 - x^2 + c$

42 **(b)**

Applying componendo and divedendo

We get $\frac{dy}{dx} = \frac{e^{-x}}{e^x} = e^{-2x}$

$\Rightarrow 2y = -e^{-2x} + C$

$\Rightarrow 2y\, e^{2x} = Ce^{2x} - 1$

43 **(c)**

The point on y-axis is $\left(0, y - x\frac{dy}{dx}\right)$

According to given condition,

$\frac{x}{2} = y - \frac{x}{2}\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = 2\frac{y}{x} - 1$

Putting $\frac{y}{x} = v$, we get $x\frac{dv}{dx} = v - 1$

$\Rightarrow \ln\left|\frac{y}{x} - 1\right| = \ln|x| + c$

$\Rightarrow 1 - \frac{y}{x} = x$[as $y(1) = 0$]

44 **(d)**

$\log c + \log|x| = \frac{x}{y}$

Differentiating w.r.t. x, $\frac{1}{x} = \frac{y - x\frac{dy}{dx}}{y^2}$

$\Rightarrow \frac{y^2}{x} = y - x\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \frac{y^2}{x^2}$

$\Rightarrow \phi\left(\frac{x}{y}\right) = -\frac{y^2}{x^2}$

45 **(c)**

$ye^{-x/y}dx - \left(xe^{(-x/y)} + y^3\right)dy = 0$

$\Rightarrow (ydx - xdy)e^{-x/y} - y^3 dy = 0$

$\Rightarrow \frac{ydx - xdy}{y^2}e^{-x/y} - y^3 = ydy$

$\Rightarrow d(x/y)e^{-x/y} = ydy$

$\Rightarrow -e^{-x/y} = \frac{y^2}{2} + C$

$\Rightarrow 2e^{-x/y} + y^2 = C$

46 **(a)**

$dy - \sin x \sin y\, dx = 0$

$\Rightarrow dy = \sin x \sin y\, dx$

$\Rightarrow \int \operatorname{cosec} y dy = \int \sin x dx$

$\Rightarrow \log \tan \frac{y}{2} = -\cos x + \log c$

$\Rightarrow \log \frac{\tan \frac{y}{2}}{c} = -\cos x$

$\Rightarrow \frac{\tan \frac{y}{2}}{c} = e^{-\cos x}$

$\Rightarrow e^{\cos x} \tan \frac{y}{2} = c$

47 **(d)**

$Ax^2 + By^2 = 1 \quad (1)$

Differentiating w.r.t. x, we get

$2Ax + 2By\frac{dy}{dx} = 0 \Rightarrow Ax + By\frac{dy}{dx} = 0 \quad (2)$

Again diff. $A + By\frac{d^2y}{dx^2} + B\left(\frac{dy}{dx}\right)^2 = 0 \quad (3)$

From equations (2) and (3), we get

$x\left[-By\frac{d^2y}{dx^2} - B\left(\frac{dy}{dx}\right)^2\right] + By\frac{dy}{dx} = 0$

$\Rightarrow x\,y\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

$\therefore$ order = 2 and degree = 1

48 **(a)**

$y^2 = t; 2y\frac{dy}{dx} = \frac{dt}{dx}$; hence the differential equation becomes

$\left(e^{x^2} + e^t\right)\frac{dt}{dx} + 2\,e^{x^2}(xt - x) = 0$

$e^{x^2} + e^t + 2e^{x^2}x(t-1)\frac{dx}{dt} = 0$

Put $e^{x^2} = z;\ e^{x^2}\,2x\frac{dx}{dt} = \frac{dz}{dt}$

$\Rightarrow z + e^t + \frac{dz}{dt}(t-1) = 0$

$\Rightarrow \frac{dz}{dt} + \frac{z}{(t-1)} = -\frac{e^t}{(t-1)}$; I. F.

$= \int_e^{\frac{dt}{t-1}} = e^{\ln(t-1)} = t - 1$

$\Rightarrow z(t-1) = -\int (e^t)dt$

$\Rightarrow z(t-1) = -e^t + C$

$\Rightarrow e^{x^2}(y^2 - 1) = -e^{y^2} + C$

$\Rightarrow e^{x^2}(y^2 - 1) + e^{y^2} = C$

49 **(c)**

Re-write the D.E. as

$(2xy\,dx - x^2\,dy) + y^2(3x^2y^2dx + 2x^3y\,dy) = 0$

Dividing by y^2, we get

$\frac{y\,2x\,dx - x^2dy}{y^2} + y^2 3x^2\,dx + x^3 2y\,dy = 0$

Or $d\left(\frac{x^2}{y}\right) + d(x^3y^2) = 0$

Integrating, we get the solution

$\frac{x^2}{y} + x^3y^2 = c$

50 **(c)**

We have $\frac{dy}{dx} = (e^y - x)^{-1} \Rightarrow \frac{dx}{dy} = e^y - x$

$\Rightarrow \frac{dx}{dy} + x = e^y;$

So I.F. $= e^{\int dy} = e^y$

$\therefore$ General solution is given by $xe^y = \frac{1}{2}e^{2y} + C$

$\Rightarrow x = \frac{e^y}{2} + Ce^{-y}$

As $y(0) = 0$, so $C = \frac{-1}{2}$

$\therefore x = \frac{e^y}{2} - \frac{1}{2}e^{-y}$

$\Rightarrow e^y - e^{-y} = 2x$

$\Rightarrow e^2y - 2xe^y - 1 = 0$

$\Rightarrow 2e^y = 2x \pm \sqrt{4x^2 + 4}$

But $e^y = x - \sqrt{x^2 + 1}$ (Rejected)

Hence $y = \ln\left(x + \sqrt{x^2 + 1}\right)$

51 **(a)**

$y = e^x(A\cos x + B\sin x)$

$\frac{dy}{dx} = e^x[-A\sin x + B\cos x] + e^x[A\cos x + B\sin x]$

$\frac{dy}{dx} = e^x[-A\sin x + B\cos x] + y \quad (1)$

Again differentiating w.r.t. x, we get

$\frac{d^2y}{dx^2} = e^x[-A\sin x + B\cos x] + e^x[-A\cos x - B\sin x] + \frac{dy}{dx}$

$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx} - y\right) - y + \frac{dy}{dx}$ [using (1)]

$\frac{d^2y}{dx} - 2\frac{dy}{dx} + 2y = 0$

52 **(d)**

$(y\cos y + \sin y)dy = (2x\log x + x)dx$

$y\sin y - \int \sin y\,dy + \int \sin y\,dx$

$= x^2\log x - \int x^2\frac{1}{x}dx + \int x\,dx + c$

$\therefore y\sin y = x^2\log x + c$

53 **(c)**

The given differential equation can be written as $\frac{dy}{as} - \frac{\tan 2x}{\cos^2 x}y = \cos^2 x$ which is linear differential equation of first order

$\int P\,dx = \int \frac{-\sin 2x}{\cos 2x\cos^2 x}dx$

$= -\int \frac{2\sin 2x\,dx}{\cos 2x\,(1 + \cos 2x)}$

$= \int \frac{dt}{t(1+t)}$

$= \int \left(\frac{1}{t} - \frac{1}{1+t}\right)dt$

$= \log \frac{t}{1+t}$ where $t = \cos 2x$

$= \log \frac{\cos 2x}{1+\cos 2x}\left[\because -\frac{\pi}{2} < 2x < \frac{\pi}{2}\right]$

$\therefore e^{\int P dx} = e^{\log \frac{\cos 2x}{1+\cos 2x}}$

$= \frac{\cos 2x}{1+\cos 2x} = \frac{\cos 2x}{2\cos^2 x}$

$\therefore$ the solution is,

$y\frac{\cos 2x}{1\cos^2 x} = \int \frac{\cos^2 x \cos 2x}{2\cos^2 x}dx + C$

$= \frac{1}{4}\sin 2x + C$

When $x = \frac{\pi}{6}, y = \frac{3\sqrt{3}}{8}$

$\therefore \frac{3\sqrt{3}}{8}\frac{4}{2\times 2\times 3} = \frac{1}{4}\frac{\sqrt{3}}{2} + C \Rightarrow C = 0$

$\therefore y = \frac{1}{2}\tan 2x \cos^2 x$

54 **(b)**

The given equation is reduced to $x = e^{xy(dy/dx)}$

$\Rightarrow \log x = xy\frac{dy}{dx}$

$\Rightarrow \int y dy = \int \frac{1}{x}\log x dx$

$\Rightarrow \frac{y^2}{2} = \frac{(\log x)^2}{2} + C'$

55 **(a)**

We have

$f(\theta) = \frac{d}{d\theta}\int_0^\theta \frac{dx}{1-\cos\theta\cos x} = \frac{1}{1-\cos^2\theta} = \text{cosec}^2\theta$

[using Leibnitz's Rule]

$\Rightarrow \frac{df(\theta)}{d\theta} = -2\,\text{cosec}^2\theta\cot\theta$

$\Rightarrow \frac{df(\theta)}{d\theta} + 2f(\theta)\cot\theta = 0 \quad [\because f(\theta) = \text{cosec}^2\theta]$

56 **(a)**

$x\frac{dy}{dx} + y(\log y) = 0$

$\Rightarrow \int \frac{dx}{x} + \int \frac{dy}{y(\log y)} = c$

$\Rightarrow \log x + \log(\log y) = \log c$

$\Rightarrow x\log y = c$

$y(1) = e \Rightarrow c = 1$

Hence, the equation of the curve is $x\log y = 1$

57 **(c)**

Given, $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$

$\Rightarrow \int \frac{y}{\sqrt{1-y^2}}dy = \int dx$

$\Rightarrow -\sqrt{1-y^2} = x + c$

$\Rightarrow (x+c)^2 + y^2 = 1$

$\therefore$ Centre $(-c,0)$, radius=1

58 **(c)**

$\left\{1 + x\sqrt{(x^2+y^2)}\right\}dx + \left\{\sqrt{(x^2+y^2)} - 1\right\}y\,dy = 0$

$\Rightarrow dx - y\,dy + \sqrt{(x^2+y^2)}(x\,dx + y\,dy) = 0$

$\Rightarrow dx - y\,dy + \frac{1}{2}\sqrt{(x^2+y^2)}\,d(x^2+y^2) = 0$

Integrating, we have

$x - \frac{y^2}{2} + \frac{1}{2}\int \sqrt{t}\,dt = c, \left\{t = \sqrt{(x^2+y^2)}\right\}$

Or $x - \frac{y^2}{2} + \frac{1}{3}(x^2+y^2)^{3/2} = c$

59 **(d)**

$\frac{dy}{dx} = \frac{y}{x}\left[\log\frac{y}{x} + 1\right]$

Put $y = vx$

$v + x\frac{dy}{dx} = v\log v + v$

$\therefore \frac{dv}{v\log v} = \frac{dx}{x}$

$\therefore \log(\log v) = \log x + \log c = \log cx$

$\therefore \log\frac{y}{x} = cx$

60 **(d)**

$x(1-x^2)dy + (2x^2y - ax^3)dx = 0$

$\Rightarrow x(1-x^2)\frac{dy}{dx} + 2x^2y - y - ax^3 = 0$

$\Rightarrow x(1-x^2)\frac{dy}{dx} + y(2x^2-1) = ax^3$

$\Rightarrow \frac{dy}{dx} + \frac{2x^2-1}{x(1-x^2)}y = \frac{ax^3}{x(1-x^2)}$

Which is of the form $\frac{dy}{dx} + Py = Q$

Its integrating factor is $e^{\int P dx}$

Here $P = \frac{2x^2-1}{x(1-x^2)}$

61 **(a)**

$x^2(y+1)dx + y^2(x-1)dy = 0$

$\Rightarrow \frac{x^2dx}{x-1} = -\frac{y^2dy}{y+1}$

$\Rightarrow \int\left[x + 1 + \frac{1}{x-1}\right]ax = -\int\left[y - 1 + \frac{1}{y+1}\right]dy$

$\Rightarrow \frac{x^2}{2} + x + \ln(x-1)$

$= -\left[\frac{y^2}{2} - y + \ln(y+1)\right] + \ln c$

$\Rightarrow \frac{x^2+y^2}{2} + (x-y) + \ln\left(\frac{(x-1)(y+1)}{c}\right) = 0$

62 **(b)**

$f''(x) - \frac{2x(x+1)}{x+1}f(x) = \frac{e^{x^2}}{(x+1)^2}$

$\text{I.F.} = e^{\int -2x dx} = e^{-x^2}$

$\therefore$ solution is $f(x)e^{-x^2} = \int \frac{dx}{(x+1)^2} + C$

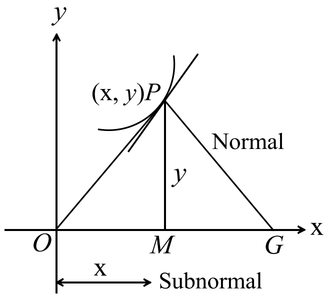
$\Rightarrow f(x)e^{-x^2} = -\dfrac{1}{x+1} + C$

Given $f(0) = 5 \Rightarrow C = 6$

$\therefore f(x) = \left(\dfrac{6x+5}{x+1}\right)e^{x^2}$

63 **(b)**

It is given that the triangle OPG is an isosceles triangle



Therefore, $OM = MG$ =sub-normal

$\Rightarrow x = y\dfrac{dy}{dx} \Rightarrow x\,dx = y\,dx$

On integration, we get $x^2 - y^2 = C$, which is a rectangular hyperbola

64 **(b)**

$x^{2/3} + y^{2/3} = a^{2/3}$

$\Rightarrow \dfrac{2}{3}x^{-1/3} + \dfrac{2}{3}y^{-1/3}\dfrac{dy}{dx} = 0$

$\Rightarrow \dfrac{dy}{dx} = -\dfrac{x^{-1/3}}{y^{-1/3}}$

Replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$

$\Rightarrow \dfrac{dx}{dy} = -\dfrac{x^{-1/3}}{y^{-1/3}}$

$\Rightarrow \int x^{1/3}\,dx = \int y^{1/3}\,dy$

$\Rightarrow x^{4/3} - y^{4/3} = c$

65 **(a)**

$\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$ (1)

Put $y = vx$ $\therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ equation (1) transforms to

$v + x\dfrac{dv}{dx} = \dfrac{x^2 + v^2x^2}{2xvx} = \dfrac{1+v^2}{2v}$

$\Rightarrow x\dfrac{dv}{dx} = \dfrac{1+v^2}{2v} - v = \dfrac{1-v^2}{2v}$

$\Rightarrow \dfrac{2vdv}{1-v^2} = \dfrac{dx}{x}$

$\Rightarrow \log x + \log(1-v^2) = \log C$

$\Rightarrow x(1-v^2) = C$

$\Rightarrow x\left(1 - \dfrac{y^2}{x^2}\right) = C$

$\Rightarrow x^2 - y^2 = Cx$

It passes through (2, 1)

$\therefore 4 - 1 = 2C \Rightarrow C = \dfrac{3}{2}$

$\therefore x^2 - y^2 = \dfrac{3}{2}x \Rightarrow 2(x^2 - y^2) = 3x$

66 **(c)**

Slope of tangent $= \frac{dy}{dx}$

$\therefore$ slope of normal $= -\frac{dx}{dy}$

$\therefore$ the equation of normal is;

$Y - y = -\dfrac{dx}{dy}(X - x)$

This meets x-axis ($y = 0$), where

$-y = -\dfrac{dx}{dy}(X - x) \Rightarrow X = x + y\dfrac{dy}{dx}$

$\therefore G$ is $\left(x + y\dfrac{dy}{dx}, 0\right)$

$\therefore OG = 2x \Rightarrow x + y\dfrac{dy}{dx} = 2x$

$\Rightarrow y\dfrac{dy}{dx} = x \Rightarrow y\,dy = xdx$

Integrating, we get $\frac{y^2}{2} = \frac{x^2}{2} + \frac{C}{2}$

$\Rightarrow y^2 - x^2 = c$, which is a hyperbola

67 **(c)**

The intersection of $y - x + 1 = 0$ and $y + x + 5 = 0$ is $(-2, -3)$. Put $x = X - 2, y = Y - 3$

The given equation reduces to $\frac{dY}{dX} = \frac{Y-X}{Y+X}$

Putting $Y = vX$, we get

$X\dfrac{dv}{dX} = -\dfrac{v^2+1}{v+1}$

$\Rightarrow \left(-\dfrac{v}{v^2+1} - \dfrac{1}{v^2+1}\right)dv = \dfrac{dX}{X}$

$\Rightarrow -\frac{1}{2}\log(v^2+1) - 2\tan^{-1} v = \log|X|$ +constant

$\Rightarrow \log(Y^2 + X^2) + 2\tan^{-1}\frac{Y}{X}$ = constant

$\Rightarrow \log((y+3)^2 + (x+2)^2) + 2\tan^{-1}\dfrac{y+3}{x+2} = C$

68 **(a)**

If $y = f(x)$is the curve,

$Y - y = \frac{dy}{dx}(X - x)$ is the equation of the tangent at (x, y)

Putting $X = 0$, the initial ordinate of the tangent is therefore $y = xf'(x)$

The subnormal at this point is given by $y\frac{dy}{dx}$, so we have $y\frac{dy}{dx} = y - x\frac{dy}{dx} \Rightarrow \frac{y}{x+y} = \frac{dy}{dx}$

This is a homogeneous equation and, by rewriting it as $\frac{dx}{dy} = \frac{x+y}{y} = \frac{x}{y} + 1 \Rightarrow \frac{dx}{dy} - \frac{x}{y} = 1$ we see that it is also a linear equation

69 **(b)**

Given, $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$

Put $y=vx$

$\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore v+x\frac{dv}{dx}=\frac{x^2v}{x^2(1+v^2)}$

$\Rightarrow \int\frac{1+v^2}{v^3}dv=-\int\frac{dx}{x}$

$\Rightarrow -\frac{1}{2v^2}+\log v=-\log x+\log c$

$\Rightarrow -\frac{1}{2}.\frac{x^2}{y^2}+\log|y|=\log c$

$\therefore y(1)=1,\ -\frac{1}{2}=\log c$

$\therefore -\frac{1}{2}.\frac{x^2}{y^2}+\log|y|=-\frac{1}{2}$

$\Rightarrow \log_e|y|+\frac{1}{2}=\frac{x^2}{2y^2}$

Again, when $x=x_0, y=e$

$$1+\frac{1}{2}=\frac{x_0^2}{2e^2}\Rightarrow x_0=\sqrt{3}e$$

70 **(a)**

slope $=\frac{dy}{dx}\Rightarrow\frac{dy}{dx}=\frac{y-1}{x^2+x}$

$\Rightarrow\frac{dy}{y-1}=\frac{dx}{x^2+x}$

$\Rightarrow\int\frac{1}{y-1}dy=\int\left(\frac{1}{x}-\frac{1}{x+1}\right)dx+C$

$\Rightarrow\frac{(y-1)(x+1)}{x}=k$

Putting $x=1, y=0$, we get $k=-2$

The equation is $(y-1)(x+1)+2x=0$

71 **(a)**

$\frac{1}{y+1}dy=-\frac{\cos x}{2+\sin x}dx$

Integrating, we get

$\log(y+1)+\log k+\log(2+\sin x)=0$

$\therefore k(y+1)(2+\sin x)=1$ when $x=0, y=1$

where k is constant

$\therefore 4k=1$ or $k=1/4$

$\therefore (y+1)(2+\sin x)=4$

Now put $x=\pi/2 \therefore (y+1)3=4$

$\therefore y=\frac{1}{3}$

72 **(c)**

We have, $\frac{dy}{dx}=\frac{y}{x}-\cos^2\left(\frac{y}{x}\right)$

Putting $y=vx$, so that $\frac{dy}{dx}=v+x\frac{dv}{dx}$, we get

$v+x\frac{dv}{dx}=v-\cos^2 v$

$\Rightarrow\frac{dv}{\cos^2 v}=-\frac{dx}{x}$

$\Rightarrow \sec^2 u\,du=-\frac{1}{x}dx$

On integration, we get

$\tan u=-\log x+\log C$

$\Rightarrow\tan\left(\frac{y}{x}\right)=-\log x+\log C$

This passes through $(1,\pi/4)$, therefore $1=\log C$

So, $\tan\left(\frac{y}{x}\right)=-\log x+1$

$\Rightarrow\tan\left(\frac{y}{x}\right)=-\log x+\log e$

$\Rightarrow y=x\tan^{-1}\left(\log\left(\frac{e}{x}\right)\right)$

73 **(b)**

The general equation of all non-horizontal lines in xy-plane is $ax+by=1$, where $a\neq 0$

Now, $ax+by=1$

$\Rightarrow a\frac{dx}{dy}+b=0$ [Diff. w.r.t. y]

$\Rightarrow a\frac{d^2x}{dy^2}=0$ [Diff. w.r.t. y]

$\Rightarrow\frac{d^2x}{dy^2}=0$ [$\because a\neq 0$]

Hence, the required differential equation is

$\frac{d^2x}{dy^2}=0$

74 **(b)**

We have, $\frac{dy}{dx}=1-\frac{1}{x^2}\Rightarrow y=x+\frac{1}{x}+C$

This passes through $(2,7/2)$

Therefore, $\frac{7}{2}=2+\frac{1}{2}+C\Rightarrow C=1$

Thus the equation of the curve is

$y=x+\frac{1}{x}+1\Rightarrow xy=x^2+x+1$

75 **(c)**

$\frac{x}{c-1}+\frac{y}{c+1}=1$ (1)

$\Rightarrow\frac{x}{c-1}+\frac{y'}{c+1}=0$ (2)

$\Rightarrow\frac{y'}{1}=\frac{c+1}{1-c}$

$\Rightarrow\frac{y'-1}{y+1}=c$

Put value of c in equation (1)

$\Rightarrow\frac{x}{\frac{y'-1}{y'+1}-1}+\frac{y}{\frac{y'-1}{y'+1}+1}=1$

$\Rightarrow\frac{x(y'+1)}{-2}+\frac{y(y'+1)}{2y'}=1$

$\Rightarrow\frac{(y'+1)}{2}\left(\frac{y}{y'}-x\right)=1$

$\Rightarrow\left(1+\frac{dy}{dx}\right)\left(y-x\frac{dy}{dx}\right)=2\frac{dy}{dx}$

76 **(a)**

$$\left\{\frac{1}{x}-\frac{y^2}{(x-y)^2}\right\}dx+\left\{\frac{x^2}{(x-y)^2}-\frac{1}{y}\right\}dy=0$$

$$\Rightarrow\left(\frac{dx}{x}-\frac{dy}{y}\right)+\left(\frac{x^2dy-y^2dx}{(x-y)^2}\right)=0$$

$$\Rightarrow\left(\frac{dx}{x}-\frac{dy}{y}\right)+\left(\frac{dy/y^2-dx/x^2}{(1/y=1/x)^2}\right)=0$$

Integrating, we get $\ln|x|-\ln|y|-\frac{1}{(1/x-1/y)}=c$

$$\Rightarrow\ln\left|\frac{x}{y}\right|+\frac{xy}{x-y}=c$$

77 **(b)**

Putting $u=x-y$, we get $du/dx\ =1-dy/dx$.

The given equation can be written as

$1-du/dx=\cos u$

$\Rightarrow(1-\cos u)=du/dx$

$$\Rightarrow\int\frac{du}{1-\cos u}=\int dx+C$$

$$\Rightarrow\frac{1}{2}\int\operatorname{cosec}^2(u/2)du=\int dx+C$$

$\Rightarrow x+\cot(u/2)=c$

$$\Rightarrow x+\cot\frac{x-y}{2}=C$$

78 **(c)**

$v=\frac{A}{r}+B\quad(1)$

$\frac{dv}{dr}=\frac{A}{r^2}(2)$

$\frac{d^2v}{dr^2}=\frac{2A}{r^3}(3)$

Eliminating A between equations (2) and (3), we get

$$r\frac{d^2v}{dr^2}=\frac{2A}{r^2}=2\left(-\frac{dv}{dr}\right)$$

$$\therefore\frac{d^2v}{dr^2}+\frac{2}{r}\frac{dv}{dr}=0$$

79 **(c)**

$y=e^{mx}$ satisfies $\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}-4\frac{dy}{dx}+12y=0$

Then $e^{mx}(m^3-3m^2-4m+12)=0$

$\Rightarrow m=\pm2,3$

$m\in N$ hence $m\in\{2,3\}$

80 **(a)**

If $(0,k)$ be the centre on y-axis then its radius will be k as it passes through origin. Hence its equation is

$x^2+(y-k)^2=k^2$

Or $x^2+y^2=2ky\quad(1)$

$$\therefore 2x+2y\frac{dy}{dx}=2k\frac{dy}{dx}$$

$$=\frac{x^2+y^2}{y}\frac{dy}{dx}\quad\text{[by (1)]}$$

$$\therefore 2xy=(x^2+y^2-2y^2)\frac{dy}{dx}$$

Or $(x^2-y^2)\frac{dy}{dx}-2xy=0$

81 **(c)**

The given family of curve is $x^2+y^2-2ay=0$ (1)

Differentiating w.r.t. x, we get $2x+2y\frac{dy}{dx}-2a\frac{dy}{dx}=0$

$\Rightarrow 2x+2y\frac{dy}{dx}-\frac{x^2+y^2}{y}\frac{dy}{dx}=0$ [Using equation (1)]

$\Rightarrow 2xy+(2y^2-x^2-y^2)y'=0$

$\Rightarrow(y^2-x^2)y''+2xy=0$

$\Rightarrow(x^2-y^2)y''=2xy$

82 **(a)**

Given, $\frac{dy}{dt}-\left(\frac{1}{1+t}\right)y=\frac{1}{(1+t)}$ and $y(0)=-1$

$\therefore\quad \text{IF}=e^{\int-\left(\frac{t}{1+t}\right)dt}=e^{-\int\left(1-\frac{1}{1+t}\right)dt}$

$=e^{-t+\log(1+t)}=e^{-t}(1+t)$

$\therefore$ Required solution is

$$ye^{-t}(1+t)=\int\frac{1}{1+t}e^{-t}(1+t)dt+c$$

$$=\int e^{-t}\ dt+c$$

$\Rightarrow\quad ye^{-t}(1+t)=-e^{-t}+c$

Since, $\quad y(0)=-1$

$\Rightarrow\quad c=0$

$\therefore\quad y=-\frac{1}{(1+t)}$

$\Rightarrow\quad y(1)=-\frac{1}{2}$

83 **(a)**

$$\frac{dv}{dt}+\frac{k}{m}v=-\text{g}$$

$$\Rightarrow\frac{dv}{dt}=-\frac{k}{m}\left(v+\frac{m\text{g}}{k}\right)$$

$$\Rightarrow\frac{dv}{v+m\text{g}/k}=-\frac{k}{m}dt$$

$$\Rightarrow\log\left(v+\frac{m\text{g}}{k}\right)=-\frac{k}{m}t+\log c$$

$$\Rightarrow v+\frac{m\text{g}}{k}=ce^{-k/mt}$$

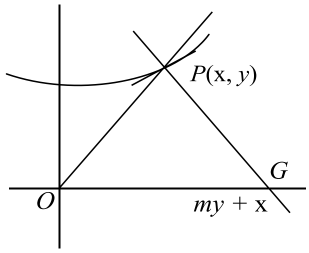
$$\Rightarrow v=ce^{-\frac{k}{m}t}-\frac{m\text{g}}{k}$$

84 **(a)**

Tangent at point P is $Y-y=-\frac{1}{m}(X-x)$

where $m=\frac{dy}{dx}$

Let $Y=0\Rightarrow X=my+x$



According to questions, $x(my+x)=2(x^2+y^2)$

$\Rightarrow \frac{dy}{dx}=\frac{x^2+2y^2}{xy}$ (homogeneous)

Putting $y=vx$, we get

$\Rightarrow v+x\frac{dv}{dx}=\frac{1+2v^2}{v}$

$\Rightarrow x\frac{dv}{dx}=\frac{1+2v^2}{v}-v=\frac{1+v^2}{v}$

$\Rightarrow \int\frac{v\,dv}{1+v^2}=\int\frac{dx}{x}$

$\Rightarrow \frac{1}{2}\log(1+v^2)=\log x+\log c, c>0$

$\Rightarrow x^2+y^2=cx^4$

Also it passes through (1,0) then $c=1$

85 **(b)**

$\frac{dx}{dy}=\frac{x+2y^3}{y}$

$\Rightarrow \frac{dx}{dy}-\frac{1}{y}x=2y^2$ which is linear

$\text{I. F.}=e^{\int-\frac{1}{y}dy}=e^{-\log y}=\frac{1}{y}$

$\therefore$solution is $\frac{1}{y}x=\int\frac{1}{y}2y^2\,dy=y^2+c$

$\Rightarrow \frac{x}{y}=y^2+c$

86 **(a)**

$\frac{dy}{dx}=\frac{1}{xy[x^2\sin y^2+1]}$

$\Rightarrow \frac{dx}{dy}=xy\,[x^2\sin y^2+1]$

$\Rightarrow \frac{1}{x^3}\frac{dx}{dy}-\frac{1}{x^2}y=y\sin y^2$

Putting $-1/x^2=u$, the least equation can be written as $\frac{du}{dy}+2uy=2y\sin y^2$

$\text{I.F.}=e^{y^2}$

$\therefore$ solution is $ue^{y^2}=\int 2y\sin y^2 e^{y^2}dy+C$

$=\int(\sin t)e^t\,dt+C$

$=\frac{1}{2}e^{y^2}(\sin y^2-\cos y^2)+c'$

$\Rightarrow 2u=(\sin y^2-\cos y^2)+2Ce^{-y^2}$

$\Rightarrow 2=x^2[\cos y^2-\sin y^2-2Ce^{-y^2}]$

87 **(a)**

$(2y+xy^3)dx+(x+x^2y^2)dy=0$

$\Rightarrow (2y\,dx+xdy)+(xy^3dx+x^2y^2dy)=0$

Multiplying by x, we get

$(2xy\,dx+x^2\,dy)+(x^2y^3\,dx+x^3y^2dy)=0$

$\Rightarrow d(x^2y)+\frac{1}{3}d(x^3y^3)=0$

Integrating, we get $x^2y+\frac{x^3y^3}{3}=c$

88 **(a)**

$\frac{dV}{dt}=-k4\pi r^2$ (1)

But $V=\frac{4}{3}\pi r^3$

$\Rightarrow \frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$ (2)

Hence, $\frac{dr}{dt}=-K$

89 **(c)**

According to the question

$\frac{dy}{dt}=-k\sqrt{y}$

$\Rightarrow \int_4^0\frac{dy}{\sqrt{y}}=-k\int_0^t dt$

$\Rightarrow 2\sqrt{y}\Big|_4^0=-kt=-\frac{t}{15}$

$\Rightarrow 0-4=-\frac{t}{15}$

$t=60$ min

90 **(a)**

The given equation can be written as

$\frac{x\,dx+y\,dy}{(y\,dx-x\,dy)/y^2}=y^2\frac{x}{y^3}\cos^2(x^2+y^2)$

$\Rightarrow \frac{xdx+ydy}{\cos^2(x^2+y^2)}=\frac{x}{y}\left(\frac{ydx-xdy}{y^2}\right)$

$\Rightarrow \frac{1}{2}\sec^2(x^2+y^2)d\,(x^2+y^2)=\frac{x}{y}d\left(\frac{x}{y}\right)$

On integrating, we get

$\frac{1}{2}\tan(x^2+y^2)=\frac{1}{2}\left(\frac{x}{y}\right)^2+\frac{c}{2}$

Or $\tan(x^2+y^2)=\frac{x^2}{y^2}+c$

91 **(c)**

Equation to the family of parabolas is

$(y-k)^2=4a(x-h)$

$2(y-k)=\frac{dy}{dx}=4a(\text{diff. w. r. t.}\,x)$

$\Rightarrow (y-k)\frac{dy}{dx}=2a$ ...(1)

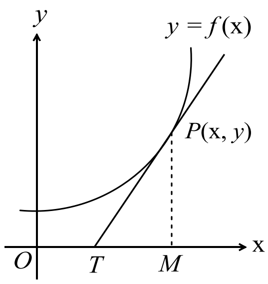
$\Rightarrow (y-k)\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0(\text{diff. w. r. t.}\,x)$

$\Rightarrow 2a\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3=0$ (substituting $y-k$ from equation (1))

Hence the order is 2 and the degree is 1

92 **(a)**

Let the equation of the curve be $y=f(x)$



It is given that $OT \propto y$

$\Rightarrow OT = by$

$\Rightarrow OM - TM = by$

$\Rightarrow x - \frac{y}{dy/dx} = by$ [$\because TM$ = Length of the subtangent]

$\Rightarrow x - y\frac{dx}{dy} = by$

$\Rightarrow \frac{dx}{dy} - \frac{x}{y} = -b$

It is linear differential equation

Its solution is $\frac{x}{y} = -b \log y + a$

$\Rightarrow x = y(a - b \log y)$

93 **(a)**

The equation of a member of the family of parabolas having axis parallel to y-axis is

$y = Ax^2 + Bx + C$ (1)

Where A, B and C are arbitrary constants

Differentiating equation (1) w. r. t., we get

$\frac{dy}{dx} = 2Ax + B$ (2)

Which on again differentiating w. r. t. x gives

$\frac{d^2y}{dx^2} = 2A$ (3)

Differentiating (3) w. r. t. x, we get $\frac{d^3y}{dx^3} = 0$

94 **(a)**

$\frac{dy}{dx} + y\phi''(x) = \phi(x)\phi''(x)$

I. F. $= e^{\int \phi''(x)dx} = e^{\phi(x)}$

Hence, the solution is

$ye^{\phi(x)} = \int e^{\phi(x)}\phi(x)\phi''(x)dx$

$= \int e^t t\, dt$,where $\phi(x) = t$

$= te^t - e^t + c$

$= \phi(x)e^{\phi(x)} - e^{\phi(x)} + c$

$\therefore y = ce^{-\phi(x)} + \phi(x) - 1$

95 **(a,b,c)**

a.$f(\lambda x, \lambda y) = \frac{\lambda(x-y)}{\lambda^2(x^2+y^2)} = \lambda^{-1}f(x, y)$

$\Rightarrow$ homogeneous of degree (-1)

b.$f(\lambda x, \lambda y) = (\lambda x)^{1/3}(\lambda y)^{-2/3}\tan^{-1}\frac{x}{y}$

$\lambda^{-1/3}x^{1/3}y^{-2/3}\tan^{-1}\frac{x}{y}$

$= \lambda^{-\frac{1}{3}}f(x, y)$

$\Rightarrow$ homogeneous

c.$f(\lambda x, \lambda y) = \lambda x\left(\ln\sqrt{\lambda^2(x^2+y^2)} - \ln \lambda y\right) + \lambda y e^{x/y}$

$= \lambda x\left[\ln\left(\frac{\lambda\sqrt{(x^2+y^2)}}{\lambda y}\right)\right] + \lambda y e^{x/y}$

$= \lambda\left[x\left(\ln\sqrt{x^2+y^2} - \ln y\right) + ye^{x/y}\right]$

$= \lambda f\ (x, y)$

$\Rightarrow$ homogeneous

d.$f(\lambda x, \lambda y) = \lambda x\left[\ln\frac{2\lambda^2x^2+\lambda^2y^2}{\lambda x\lambda(x+y)}\right] + \lambda^2x^2\tan\frac{x+2y}{3x-y}$

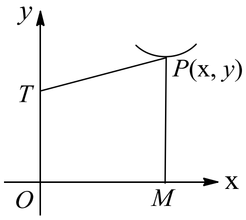
$= \lambda x\left[\ln\frac{2x^2+y^2}{x(x+y)}\right] + \lambda^2x^2\tan\frac{x+2y}{3x-y}$

$\Rightarrow$ non homogeneous

96 **(a,b)**

Let $P(x,y)$ be any point on the curve. Length of intercept on y-axis by any tangent at $P(x,y)$

$= OT = y - x\frac{dy}{dx}$



$\therefore$ Area of trapezium $OMPTO = \frac{1}{2}(PM + OT)OM$

$= \frac{1}{2}\left(y + y - x\frac{dy}{dx}\right) \times x$

$= \frac{1}{2}\left(2y - x\frac{dy}{dx}\right)x$

Given, area of trapezium $OMPTO = \frac{1}{2}x^2$

$\Rightarrow \frac{1}{2}\left(2y - x\frac{dy}{dx}\right)x = \pm\frac{1}{2}x^2$

$\Rightarrow 2y - x\frac{dy}{dx} = \pm x$

or $\frac{dy}{dx} - \frac{2y}{x} = \pm 1$

Which is linear differential equation

An IF$= e^{\int -2/x\,dx} = e^{-2\ln x} = \frac{1}{x^2}$

$\therefore$ The solution is $\frac{y}{x^2} = \int \pm\frac{1}{x^2}dx + c = \pm\frac{1}{x} + c$

$\Rightarrow y = \pm x + cx^2$

$\Rightarrow y = cx^2 \pm x$

97 **(c)**

$\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$

By verification we find that the choice (c), i.e., $y = 2x - 4$ satisfies the given differential equation

Alternate

$\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$

$\Rightarrow \frac{dy}{dx} = \frac{x \pm \sqrt{x^2 - 4y}}{2}$(1)

Let $x^2 - 4y = t^2$

$\Rightarrow 2x - 4\frac{dy}{dx} = 2t\frac{dt}{dx}$

$\Rightarrow x - 2\frac{dy}{dx} = t\frac{dt}{dx}$

Then equation (1) changes to $x - t\frac{dt}{dx} = x \pm t$

$\Rightarrow \frac{dt}{dx} = \pm 1$ or $t = 0$

$\Rightarrow t = \pm x + c$ or $x^2 = 4y$

$\Rightarrow x^2 - 4y = x^2 \pm 2cx + c^2$

$\Rightarrow\ -4y = \pm 2cx + c^2$

For $c = 4$

$4y = \pm 8x - 16$ or $y = 2x - 4$

98 **(a,b)**

$\frac{dy}{dx} = \frac{y}{x^2} \Rightarrow \frac{dy}{y} = \frac{dx}{x^2} \Rightarrow \ln y = -\frac{1}{x} + \ln c \Rightarrow \frac{y}{c} = e^{\frac{1}{x}}$

$\Rightarrow y = ce^{\frac{1}{x}}$

Comparing with $y = ae^{-1/x} + b, a \in R, b = 0$

99 **(a,c)**

$y^2 = 2c(x + \sqrt{c})$

Differentiating w.r.t. x, we get

$2yy' = 2c \Rightarrow c = yy'$

Eliminating c, we get

$y^2 = 2yy_1(x + \sqrt{yy_1})$ or $(y^2 - 2x\,yy_1)^2 = 4y^3y_1^3$

It involves only first order derivative, its order is 1 but its degree is 3 as y_1^3 is there

100 **(a,d)**

The given differential equation is

$y_2(x^2 + 1) = 2xy_1 \Rightarrow \frac{y_2}{y_1} = \frac{2x}{x^2 + 1}$

Integrating both sides, we get

$\log y_1 = \log(x^2 + 1) + \log C$

$\Rightarrow y_1 = C(x^2 + 1)$ (1)

It is given that $y_1 = 3$ at $x = 0$

Putting $x = 0, y_1 = 3$ in equation (1), we get

$3 = C$

Substituting the value of C in (1), we obtain

$y_1 = 3(x^2 + 1)$ (2)

Integrating both sides w.r.t to x, we get

$y = x^3 + 3x + C_2$

This passes through the point (0,1). Therefore,

$1 = C_2$

Hence, the required equation of the curve is

$y = x^3 + 3x + 1$

Obviously it is strictly increasing from equation (2)

Also $f(0) = 1 > 0$, then the only root is negative

101 **(a,b)**

$y\left(\frac{dy}{dx}\right)^2 + (x - y)\frac{dy}{dx} - x = 0$

$\Rightarrow \frac{dy}{dx} = \frac{(y - x) \pm \sqrt{(x - y)^2 + 4xy}}{2y}$

$\Rightarrow \frac{dy}{dx} = 1$ which gives straight line

Or $\frac{dy}{dx} = -\frac{x}{y}$ which gives circle

102 **(c,d)**

The equation of tangent at (x,y) is

$Y - y = \frac{dy}{dx}(X - x)$

Points A and B are respectively $\left(x - y\frac{dx}{dy}, 0\right)$ and $\left(0, y - x\frac{dy}{dx}\right)$.

Now, $\frac{PA}{PB}=\frac{2}{1}$

$\Rightarrow (PA)^2 = 4(PB)^2$

$\Rightarrow \left(y\frac{dx}{dy}\right)^2 + y^2 = 4\left[x^2 + x^2\left(\frac{dy}{dx}\right)^2\right]$

$\Rightarrow 4x^2\left(\frac{dy}{dx}\right)^4 + (4x^2-y^2)\left(\frac{dy}{dx}\right)^2 - y^2 = 0$

$\Rightarrow \left(\frac{dy}{dx}\right)^2$

$= \frac{(y^2-4x^2)\pm\sqrt{(4x^2-y^2)^2+16x^2y^2}}{8x^2}$

$= \frac{(y^2-4x^2)\pm(4x^2+y^2)}{8x^2}$

$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{y^2}{4x^2}$ or $\left(\frac{dy}{dx}\right)^2 = -1$

$\Rightarrow \frac{dy}{dx} = \pm\frac{y}{2x}$

$\Rightarrow 2\,\text{In}|y| = \pm\text{In}\,|x| + c$

$\Rightarrow \text{In}\,\frac{y^2}{|x|} = \text{In}\,c$

or $\text{In}\,y^2|x| = \text{In}\,c$

$\Rightarrow y^2 = c_1|x|$ or $y^2|x| = c_2$

103 **(a,b)**

We have length of the normal=radius vector

$\Rightarrow y\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{x^2+y^2}$

$\Rightarrow y^2\left(1+\left(\frac{dy}{dx}\right)^2\right) = x^2+y^2$

$\Rightarrow y^2\left(\frac{dy}{dx}\right)^2 = x^2$

$\Rightarrow x = \pm y\frac{dy}{dx}$

$\Rightarrow x = y\frac{dy}{dx}$ or $x = -y\frac{dy}{dx}$

$\Rightarrow xdx - ydy = 0$ or $xdx + ydy = 0$

$\Rightarrow x^2 - y^2 = c_1$ or $x^2 + y^2 = c_2$

Clearly, $x^2+y^2=c_2$ represents a rectangular hyperbola and $x^2+y^2=c_2$ represents circles

104 **(a,c)**

We have, $\int(by+k)dy = \int(ax+h)dx$

or $\frac{by^2}{2} + ky = \frac{ax^2}{2} + hx + c$

Clearly, for $a=-2, b=0$

and for $a=0, b=2$

It represents a parabola ($\because y = ax^2+bx+c$ and $x = ay^2+by+c$ represents a parabola.)

105 **(a)**

Slope of the normal at $(1,1) = -\frac{1}{a}$

Slope of tangent at $(1,1) = a$

i.e., $\left(\frac{dy}{dx}\right)_{(1,1)} = a$

Since $\frac{dy}{dx}$ is proportional to y,

$\therefore \frac{dy}{dx} = Ky$

$\Rightarrow \frac{dy}{y} = K\,dx$

$\Rightarrow \log y = Kx + C$

$\Rightarrow y = e^{Kx+C} = Ae^{Kx}$ where $A = e^c$

It passes through (1,1)

$\therefore 1 = Ae^K \;\; \therefore A = e^{-K}$

$\therefore y = e^{-K}e^{Kx} = e^{K(x-1)}$

106 **(b)**

$(x^2y^2-1)dy + 2xy^3\,dx = 0$

$\Rightarrow x^2y^2dy + 2xy^3dx = dy$

$\Rightarrow x^2dy + 2xy\,dx = \frac{dy}{y^2}$

$\Rightarrow \int d(x^2y) = \int\frac{dy}{y^2} + c$

$\Rightarrow x^2y = \frac{y^{-1}}{-1} + c$

$\Rightarrow x^2y^2 = -1 + cy$

i.e., $1 + x^2y^2 = cy$

107 **(a,c)**

We have $(x-h)^2 + (y-k)^2 = a^2$ (1)

Differentiating w.r.t. x, we get

$2(x-h) + 2(y-k)\frac{dy}{dx} = 0$

$\Rightarrow (x-h) + (y-k)\frac{dy}{dx} = 0$ (2)

Differentiating w.r.t. x, we get

$1 + \left(\frac{dy}{dx}\right)^2 + (y-k)\frac{d^2y}{dx^2} = 0$ (3)

From equation (3), $y - k = -\left(\frac{1+p^2}{q}\right)$, where $p = \frac{dy}{dx}$,

$q = \frac{d^2y}{dx^2}$

Putting the value of $y-k$ in equation (2), we get

$x - h = \frac{(1+p^2)p}{q}$

Substituting the values of $x-h$ and $y-k$ in equation (1),

We get

$\left(\frac{1+p^2}{q}\right)^2(1+p^2) = a^2 \Rightarrow \left[1+\left(\frac{dy}{dx}\right)^2\right]^3$

$= a^2\left(\frac{d^2y}{dx^2}\right)^2$

Which is the required differential equation

108 **(a,b,c)**

We have $f''(x) = g''(x)$. On integration, we get

$f'(x) = g'(x) + C$ (1)

Putting $x = 1$, we get

$f'(1) = g'(1) + C \Rightarrow 4 = 2 + C \Rightarrow C = 2$

$\therefore f'(x) = g'(x) + 2$

Integrating w.r.t. x, we get $f(x) = g(x) + 2x + c_1$ (2)

Putting $x = 2$, we get

$f(2) = g(2) + 4 + c_1 \Rightarrow 9 = 3 + 4 + c_1 \Rightarrow c_1 = 2$

$\therefore f(x) = g(x) + 2x + 2$. Putting $x = 4$, we get

$f(4) - g(4) = 10$

$|f(x) - g(x)| < 2 \Rightarrow |2x + 2| < 2 \Rightarrow |x + 1| < 1$

$\Rightarrow -2 < x < 0$

Also $f(2) = g(2) \Rightarrow x = -1$

$f(x) - g(x) = 2x$ has no solution

110 **(a,b)**

$x = \sin\left(\frac{dy}{dx} - 2y\right) \Rightarrow \frac{dy}{dx} - 2y = \sin^{-1} x$

$x - 2y = \log\left(\frac{dy}{dx}\right) \Rightarrow \frac{dy}{dx} = e^{x-2y}$

111 **(a,c)**

$\frac{dy}{dx} = \frac{ax+h}{by+k} \Rightarrow (by + k)dy = (ax + h)dx$

$\Rightarrow b\frac{y^2}{2} + ky = \frac{a}{2}x^2 + hx + C$

For this to represent a parabola, one of the two terms x^2 or y^2 is zero

Therefore, either $a = 0, b \neq 0$ or $a \neq 0, b = 0$

112 **(a,d)**

Taking $x = r\cos\theta$ and $y = r\sin\theta$, so that $x^2 + y^2 = r^2$ and $\frac{y}{x} = \tan\theta$,

We have $x\,dx + y\,dy = r\,dr$

and $x\,dy - y\,dx = x^2\sec^2\theta\ d\theta = r^2\,d\theta$

The given equation can be transformed into

$\frac{r\,dr}{r^2 d\theta}\sqrt{\left(\frac{a^2 - r^2}{r^2}\right)}$

$\Rightarrow \frac{dr}{d\theta} = \sqrt{(a^2 - r^2)}$

$\Rightarrow \frac{dr}{\sqrt{(a^2 - r^2)}} = d\theta$

Integrating both sides, then we get

$\sin^{-1}\left(\frac{r}{a}\right) = \theta + c$

$\Rightarrow \sin^{-1}\left(\frac{\sqrt{(x^2+y^2)}}{a}\right) = \tan^{-1}\left(\frac{y}{x}\right) + c$ .....(i)

$\Rightarrow \sqrt{(x^2 + y^2)} = a\sin\{\tan^{-1}(y/x) + c\}$

$\Rightarrow \sqrt{(x^2 + y^2)} = a\sin\{\tan^{-1}(y/x) + \text{constant}\}$

Also, from Eq. (i),

$\tan^{-1}\left(\frac{y}{x}\right) = \left\{\sin^{-1}\left(\frac{\sqrt{(x^2 + y^2)}}{a}\right) - c\right\}$

$\Rightarrow y = x\tan\left\{\sin^{-1}\left(\frac{\sqrt{(x^2 + y^2)}}{a}\right) - c\right\}$

$= x\tan\left\{\sin^{-1}\left(\frac{\sqrt{(x^2 + y^2)}}{a}\right) + \text{constant}\right\}$

113 **(a,d)**

The D.E. can be re-written as

$\frac{x\,dx + ydy}{\sqrt{1 - (x^2 + y^2)}} = \frac{xdy - ydx}{\sqrt{x^2 + y^2}}$

Since $d\tan^{-1}(y/x) = \frac{xdy - ydx}{x^2 + y^2}$, and $d(x^2 + y^2) = 2(xdx + ydy)$

$\therefore$ we have $\frac{\frac{1}{2}d(x^2+y^2)}{\sqrt{x^2+y^2}\sqrt{1-(x^2+y^2)}} = \frac{xdy - ydx}{x^2+y^2} = d\{\tan^{-1}(y/x)\}$

Put $x^2 + y^2 = t^2$ in the L.H.S and get

$\frac{t\,dt}{t\sqrt{1 - t^2}} = d\{\tan^{-1}(y/x)$

Integrating both sides, we get

$\sin^{-1} t = \tan^{-1}(y/x) + c$

i.e, $\sin^{-1}\sqrt{(x^2 + y^2)} = \tan^{-1}(y/x) + c$

114 **(a,b,d)**

$\frac{dy}{dx} + y\cos x = \cos x$ (linear)

$\text{I.F.} = e^{\int \cos x\,dx} = e^{\sin x}$

$\therefore$ solution is $y\,e^{\sin x} = \int e^{\sin x}\cos x dx = e^{\sin x} + c$

When $x = 0, y = 1$ then $c = 0$

$\Rightarrow y = 1$. Hence options (a), (b),(d) are true

115 **(a,c)**

Obviously (a) is linear D.E. with $P = \frac{1}{x}$ and

$Q = \log x\ y\left(\frac{dy}{dx}\right) + 4x = 0 \Rightarrow \frac{dy}{dx} + \frac{4x}{y} = 0$

Hence not linear

$(2x + y^3)\left(\frac{dy}{dx}\right) = 3y$

$\Rightarrow \frac{dx}{dy} = \frac{2x}{3y} + \frac{y^2}{3}$

$\Rightarrow \frac{dx}{dy} - \frac{2x}{3y} = \frac{y^2}{3}$ which is linear with $P = \frac{2}{3y}$ and $Q = \frac{y^2}{3}$

116 **(b)**

We have $y\frac{dy}{dx} = k$ (constant)

$\Rightarrow ydy = k\,dx \Rightarrow \frac{y^2}{2} = kx + C \Rightarrow y^2 = 2kx + 2C$

$\Rightarrow y^2 = 2ax + b$, where $a = k, b = 2C$

117 **(c)**

$y = (C_1 + C_2)\cos(x + C_3) - C_4 e^{x+C_5}$

$= (C_1 + C_2)\cos(x + C_3) - C_4 e^{C_5} e^x$

$= A\cos(x + C_3) - Be^x$ [Taking $C_1 + C_2 = A, C_4 e^{C_5} = B$]

Thus, there are actually three arbitrary constants and hence this differential equation should be of order 3

118 **(a,b,c,d)**

$\because \dfrac{dy}{dx} = \dfrac{-2y\cot x \pm \sqrt{(4y^2\cot^2 x + 4y^2)}}{2}$

$= y(-\cot x \pm \operatorname{cosec} x)$

$\therefore \dfrac{dy}{dx} = (-\cot x + \operatorname{cosec} x)$

$\Rightarrow \text{In } y = -\text{In}\sin x + \text{In}\tan\dfrac{x}{2} + \text{In } c$

$\Rightarrow y = \frac{c\tan\frac{x}{2}}{\sin x} = \frac{c}{2\cos^2\frac{x}{2}}$ ....(i)

$= \dfrac{c}{1+\cos x}$

On solving, $\frac{dy}{y} = -(\cot x + \operatorname{cosec} x)dx$, we get

$= y = \dfrac{c}{1-\cos x}$

$\Rightarrow x = 2\sin^{-1}\sqrt{\dfrac{c}{2y}}$

Also from Eq. (i),

$x = 2\cos^{-1}\sqrt{\left(\dfrac{c}{2y}\right)}$

119 **(d)**

Given, $\frac{dy}{dx} + \frac{1}{x}y = \frac{\sin x}{x^2}$

$\therefore \text{IF} = e^{\int\frac{1}{x}dx} = x$

$\therefore$ Solution is $xy = \int_0^x \frac{\sin t}{t}\,dt + c$

On putting $x = 1, 2$ respectively, we get

$c = 2 - \displaystyle\int_0^1 \frac{\sin t}{t}dt$

$\Rightarrow y = \dfrac{1}{x}\displaystyle\int_0^x \frac{\sin t}{t}dt + \frac{c}{x}$

$= \dfrac{1}{x}\displaystyle\int_0^x \frac{\sin t}{t}dt + \frac{2}{x} - \frac{1}{x}\int_0^1 \frac{\sin t}{t}dt$

$= \dfrac{2}{x} + \dfrac{1}{x}\displaystyle\int_1^x \frac{\sin t}{t}dt$

$\Rightarrow y(z) = 1 + \dfrac{1}{2}\displaystyle\int_1^2 \frac{\sin t}{t}dt$

120 **(d)**

Statement 2 is obviously true. But statement 1 is false as $2x - 3y + 2 = \log\left(\frac{dy}{dx}\right)$

$\Rightarrow \left(\frac{dy}{dx}\right) = e^{2x-3y+2}$ which has degree 1

121 **(a)**

The equation of circle contains. There independent constants if it passes through three non-collinear points, therefore statement 1 is true and follows from statement 2

122 **(d)**

$\because y = (c_1e^{c_2} + c_3e^{c_4})e^x = ce^x$ (say)

$\dfrac{dy}{dx} = ce^x = y$

$\therefore$ Order is 1.

123 **(a)**

Let $c_1 + c_2 + c_3e^{c_4} = A$ constant

Then, $y = Ax$

$\Rightarrow \dfrac{dy}{dx} = A$

$\Rightarrow y = x\dfrac{dy}{dx}$

$\Rightarrow x\dfrac{dy}{dx} = y$

124 **(c)**

Given, $\frac{dy}{dx} = \frac{y\sqrt{y^2-1}}{x\sqrt{x^2-1}}$

$\Rightarrow \displaystyle\int \frac{dy}{y\sqrt{y^2-1}} = \int \frac{dx}{x\sqrt{x^2-1}}$

$\Rightarrow \sec^{-1} y = \sec^{-1} x + c$

At $x = 2, y = \frac{2}{\sqrt{3}}$

$\Rightarrow \dfrac{\pi}{6} = \dfrac{\pi}{3} + c$

$\Rightarrow c = -\dfrac{\pi}{6}$

Now, $y = \sec\left(\sec^{-1} x - \frac{\pi}{6}\right)$

$= \cos\left(\cos^{-1}\dfrac{1}{x} - \cos^{-1}\dfrac{\sqrt{3}}{2}\right)$

$= \cos\left[\cos^{-1}\left(\dfrac{\sqrt{3}}{2x} + \sqrt{1-\dfrac{1}{x^2}}\cdot\sqrt{1-\dfrac{3}{4}}\right)\right]$

$\Rightarrow \dfrac{1}{y} = \dfrac{\sqrt{3}}{2x} + \dfrac{1}{2}\sqrt{1-\dfrac{1}{x^2}}$

125 **(a)**

$y = Ae^x$

On differentiating, we get $\frac{dy}{dx} = Ae^x$

126 **(d)**

$\because xy = v$

$\therefore x\frac{dy}{dx} + y = \frac{dv}{dx}$

Then, the given equation reduces to

$\frac{v}{x}f(v) + x\phi(v)\left(\frac{1}{x}\left(\frac{dv}{dx} - y\right)\right)$=0

$\Rightarrow \frac{v}{x}f(v) + \phi(v)\frac{dv}{dx} - y\phi(v) = 0$

$\Rightarrow \left\{\frac{v(f(v) - \phi(v))}{x}\right\} + \phi(v)\frac{dv}{dx} = 0$

$\Rightarrow \frac{dx}{x} + \frac{\phi(v)dv}{v(f(v)) - \phi(v)} = 0$

Which is variable seperable form.

127 **(a)**

Let $x^2 + y^2 + 2gx + 2fy + c = 0$

Here, in this equation, there are three constants.

$\therefore$ Order = 3

$\because$ Circle passes through three non-collinear points, then we get three constants g, t, c.

128 **(b)**

From Statement II

Differential equation can be written as

$\left(\frac{dy}{dx} - e^x\right)\left(\frac{dy}{dx} - e^{-x}\right) = 0$

$\Rightarrow \frac{dy}{dx} = e^x$ or $\frac{dy}{dx} = e^{-x}$

$\Rightarrow \int dy = \int e^x dx$ or $\int dy = \int e^{-x} dx$

$\Rightarrow y = e^x + c_1$ or $y = -e^{-x} + c_2$

$\Rightarrow y = c_1e^x + c_2e^{-x}$ will satisfy the above equation.

From Statement I

$\int dy = \int\left(x + \frac{1}{x^2}\right)dx$

$\Rightarrow y = \frac{x^2}{2} - \frac{1}{x} + c$

It is passes through (3, 9).

$\therefore 9 = \frac{9}{2} - \frac{1}{3} + c$

$\Rightarrow c = 9 - \frac{9}{2} + \frac{1}{3} = \frac{29}{6}$

$\therefore y = \frac{x^2}{2} - \frac{1}{x} + \frac{29}{6}$

$\Rightarrow 6xy = 3x^3 + 29x - 6$

129 **(d)**

$\because$ The given equation cannot be written as a polynomial in all the differentials.

$\because$ Degree of the equation is not defined.

130 **(a)**

$$y = c_1\cos 2x + c_2\sin^2 x + c_3\cos^2 x + c_4e^{2x} + c_5e^{2x+c_6}$$

$$= c_1\cos 2x + c_2\left(\frac{1-\cos 2x}{2}\right) + c_3\left(\frac{\cos 2x + 1}{2}\right) + c_4e^{2x} + c_5e^{c_6}e^{2x}$$

$$= \left(c_1 - \frac{c_2}{2} + \frac{c_3}{2}\right)\cos 2x + \left(\frac{c_2}{2} + \frac{c_3}{2}\right) + (c_4 + c_5e^{c_6})e^{2x}$$

$= \lambda_1\cos 2x + \lambda_2e^{2x} + \lambda_3$

$\Rightarrow$ Total number of independent parameters in the given general solution is 3

Hence statement 1 is true, also statement 2 is true which explains statement 1

131 **(b)**

Statement 1 is obviously true

Even statement 2 is also obviously true but it does not explain statement 1

132 **(b)**

$\because y = a\sin x + b\cos x$ ....(i)

$\therefore y' = a\cos x - b\sin x$

$\Rightarrow y'' = -a\sin x - b\cos x = -y$ [from Eq. (i)]

$\Rightarrow y'' + y = 0$

133 **(a)**

Equation of the required parabola is of the form$y^2 = 4a(x-h)$. Differentiating, we have

$2y\frac{dy}{dx} = 4a \Rightarrow y\frac{dy}{dx} = 2a \Rightarrow \left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2} = 0$

The degree of this differential equation is 1 and the order is 2

b. we have $y = a(x+a)^2$ (1)

$\Rightarrow \frac{dy}{dx} = 2a(x+a)$ (2)

Dividing equations (1) by (2), we get $\frac{y}{\frac{dy}{dx}} = \frac{x+a}{2}$

$\Rightarrow x + a = \frac{2y}{y_1}$, where $y_1 = \frac{dy}{dx}$

Substituting $a = \frac{2y}{y_1} - x$ in equation (1)

We get $y = \left(\frac{2y}{y_1} - x\right)\left(\frac{2y}{y_1}\right)^2 \Rightarrow {y_1}^3 y = 4(2y -$

xy1y2

Clearly, it is a differential equation of degree 3

c. The given equation is $\left(1 + 3\frac{dy}{dx}\right)^{2/3} = 4\frac{d^3y}{dx^3}$

Cubing, we get $\left(1 + 3\frac{dy}{dx}\right)^2 = 64\left(\frac{d^3y}{dx^2}\right)^3$

Hence order = degree = 3

d. We have $y^2 = 2c(x + \sqrt{c})$ (1)

Diff. w. r. t. x, we get $2y\frac{dy}{dx} = 2c$

$\Rightarrow c = y\frac{dy}{dx}$

Putting in equation (1), we get $y^2 = 2\left(y\frac{dy}{dx}\right)x + 2\left(y\frac{dy}{dx}\right)^{3/2}$

$\Rightarrow \left(y^2 - 2xy\frac{dy}{dx}\right)^2 = 4y^3\left(\frac{dy}{dx}\right)^3$

Its order is 1 and degree is 3

134 **(b)**

a. $y = e^{4x} + 2e^{-x}; y_1 = 4e^{4x} - 2e^{-x}; y_2 = 16e^{4x} + 2e^{-x}; y_3 = 64e^{4x} - 2e^{-x}$

Now, $y_3 - 13y_1 = (64e^{4x} - 2e^{-x}) - 13(4e^{4x} -$ *2e−x=12e4x+24e−x*

$y_3 - 13y = 12(e^{4x} + 2e^{-x}) = 12y$

$\therefore K = 12$ and $K/3 = 4$

b. Since equation is 2 degree, two linew are possible

c. $y = u^m \Rightarrow \frac{dy}{dx} = m\,u^{m-1}\frac{du}{dx}$

Substituting the value of y and $\frac{dy}{dx}$ in $2x^4y\frac{dy}{dx} + y^4 = 4x^6$

We have $2x^4u^m\,m\,u^{m-1}\frac{du}{dx} = 4x^6$

$\Rightarrow \frac{du}{dx} = \frac{4x^6 - u^{4m}}{2m\,x^4u^{2m-1}}$

For homogeneous $4m = 6 \Rightarrow m = \frac{3}{2}$

and $2m - 1 = 2 \Rightarrow m = \frac{3}{2}$

d. $y = Ax^m + Bx^{-n}$

$\Rightarrow \frac{dy}{dx} = Amx^{m-1} - nBx^{-n-1}$

$\Rightarrow \frac{d^2y}{dx^2} = Am(m-1)x^{m-2} + n(n+1)Bx^{-n-2}$

Putting these values in $x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 12y$

We have $= m(m+1)Ax^m + n(n-1)Bx^{-n} = 12(Ax^m + Bx^{-n})$

$\Rightarrow m(m+1) = 12$ or $n(n-1) = 12$

$\Rightarrow m = 3, -4$ or $n = 4, -3$

135 **(b)**

$\because \frac{dT}{dt} = -k(T - 290)$

$\Rightarrow \frac{dT}{(T-290)} = -k\,dt$

$\Rightarrow \log(T - 290) = -kt + c$ ...(i)

Initially, $T = 370$ K and $t = 0$, then $\log(80) = c$

From Eq. (i),

$\log(T - 290) = -kt + \log 80$

$\log\left(\frac{T-290}{80}\right) = -kt$

$\Rightarrow \frac{T-290}{80} = e^{-kt}$

$\Rightarrow T = 290 + 80e^{-kt}$

137 **(b)**

$f(x) \le 0$ and$F'(x) = f(x)$

$\Rightarrow f(x) \ge cF(x)$

$\Rightarrow F'(x) - cF(x) \ge 0$

$\Rightarrow e^{-cx}F'(x) - ce^{-cx}F(x) \ge 0$

$\Rightarrow \frac{d}{dx}\left(e^{-cx}F(x)\right) \ge 0$

$\Rightarrow e^{-cx}F(x)$ is an increasing function

$\Rightarrow e^{-cx}F(x) \ge e^{-c(0)}F(0)$

$\Rightarrow e^{-cx}F(x) \ge 0$

$\Rightarrow F(x) \geq 0$

$\Rightarrow f(x) \geq 0$ (as $f(x) \geq cF(x)$ and c is positive)

$\Rightarrow f(x) = 0$

Also $\left(\frac{d\,g(x)}{dx}\right) < g(x) \forall\, x > 0$

$\Rightarrow e^{-x}\frac{d(g(x))}{dx} - e^{-x}g(x) < 0$

$\Rightarrow \frac{d}{dx}(e^{-x}g(x)) < 0$

$\Rightarrow e^{-x}g(x)$ is a decreasing function

$\Rightarrow e^{-x}g(x) < e^{-(0)}g(0)$

$\Rightarrow g(x) < 0$ (as $g(0) = 0$)

Thus $f(x) = g(x)$ has one solution $x = 0$

138 **(c)**

Given equation can be rewritten as

$y = xp + \sqrt{(1+p^2)}, p = \frac{dy}{dx}(1)$

Differentiating w.r.t. x, we get

$p = p + x\frac{dp}{dx} + \frac{1}{2\sqrt{1+p^2}}2p\frac{dp}{dx}$

$\Rightarrow \frac{dp}{dx} = 0$ or $\frac{p}{\sqrt{1+p^2}} = -x$

$\Rightarrow p = c$ or $p = \frac{x}{\sqrt{1-x^2}}$

$\Rightarrow y = cx + \sqrt{(1+c^2)}$ gives the general solution and $x^2 + y^2 = 1$

As singular solution

139 **(c)**

Integrating $\frac{d^2y}{dx^2} = 6x - 4$, we get $\frac{dy}{dx} = 3x^2 - 4x + A$

When $x = 1, \frac{dy}{dx} = 0$ so that $A = 1$. Hence

$\frac{dy}{dx} = 3x^2 - 4x + 1 \quad (1)$

Integrating, we get $y = x^3 - 2x^2 + x + B$

When $x = 1, y = 5$, so that $B = 5$

Thus, we have $y = x^2 - 2x^2 + x + 5$

From equation (1), we get the critical points $x = 1/3, x = 1$

At the critical point $x = \frac{1}{3}, \frac{d^2y}{dx^2}$ is –ve

Therefore, at $x = 1/3$, y has a local maximum

At $x = 1, \frac{d^2y}{dx^2}$ is +ve

Therefore, at $x = 1$ y has a local minimum

Also $f(1) = 5, f\left(\frac{1}{3}\right) = \frac{139}{27}, f(0) = 5, f(2) = 7$

Hence the global maximum value = 7

And the global minimum value = 5

140 **(a)**

Let N denote the amount of material present at time t. Then,

$\frac{dN}{dt} - kN = 0$

This differential equation is separable and linear, its solution is $N = ce^{kt} \quad (1)$

At $t = 0$, we are given that $N = 50$. Therefore, from equation (1), $50 = ce^{k(0)}$, or $c = 0$

Thus, $N = 50\,e^{kt} \quad (2)$

At $t = 2$,10 percent of the original mass of 50 mg or 5 mg, has decayed

Hence, at $t = 2, N = 50 - 5 = 45$

Substituting these values into equation (2) and solving for k, we have $45 = 50e^{kt}$ or $k = \frac{1}{2}\log\frac{45}{50}$

Substituting this value into (2), we obtain the amount of mass present at any time t as

$N = 50e^{-(1/20)(\ln 0.9)t} \quad (3)$

Where t is measured in hours

141 **(a)**

Here, $V_0 = 100, a = 20, b = 0$, and $e = f = 5$.

Hence

$\frac{dQ}{dt} + \frac{1}{20}Q = 0$

The solution of this linear equation is $Q = ce^{-t/20}$ (1)

At $t = 0$, we are given that $Q = a = 20$

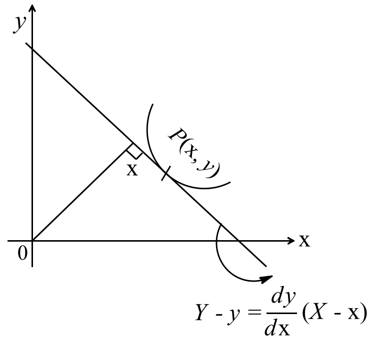
Substituting these values into equation (1), we find that $c = 20$, so that equation (1) can be rewritten as $Q = 20e^{-t/20}$

For $t = 20, Q = 20/e$

142 **(2)**

Equation of tangent is $X\frac{dy}{dx} - y - Y\frac{dy}{dx} + y = 0$

perpendicular distance from origin is



$\therefore \perp$ from (0,0)$= x$

$\left|\frac{0 - 0 - x\frac{dy}{dx} + y}{\sqrt{\left(\frac{dy}{dx}\right)^2 + 1}}\right| = x$

$\therefore \left|\frac{x\frac{dy}{dx} - y}{\sqrt{\left(\frac{dy}{dx}\right)^2 + 1}}\right| = x \Rightarrow \left(x\frac{dy}{dx} - y\right)^2$

$= x^2\left(1 + \left(\frac{dy}{dx}\right)^2\right)$

$\Rightarrow x^2\left(\frac{dy}{dx}\right)^2 + y^2 - 2xy\frac{dy}{dx} = x^2 + x^2\left(\frac{dy}{dx}\right)^2$

$\Rightarrow \frac{y^2-x^2}{2xy} = \frac{dy}{dx}$ (1) (Homogeneous)

Put $y = vx$ in (1)

$$v + x\frac{dv}{dx} = \frac{v^2-1}{2v}$$

$$\int \frac{2v}{v^2+1}dv = -\int \frac{dx}{x}$$

$$\ell n(v^2+1) = -\ell nx + \ell nc$$

$$v^2 + 1 = \frac{c}{x}$$

$$\frac{y^2+x^2}{x^2} = \frac{c}{x} \Rightarrow y^2 + x^2 = cx$$

Passes through (1,1), then $c = 2$

$x^2 + y^2 - 2x = 0$

For intercept of curve on x-axis, put $y = 0$

We have $x^2 - 2x = 0$ or $x = 0,2$

Hence length of intercept is 2

143 **(2)**

Given $\frac{dy}{dx} - \frac{1}{x}y = \left(x - \frac{2}{x}\right)$

$$\text{I.F.} = e^{-\int \frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$$

Now general solution is given by $\frac{y}{x} = \int \left(x - \right.$ $2x1xdx$

$$\Rightarrow \frac{y}{x} = x + \frac{2}{x} + C$$

As $y(1) = 1 \Rightarrow C = -2$

$$\therefore \frac{y}{x} = x + \frac{2}{x} - 2 \Rightarrow y = x^2 - 2x + 2$$

Hence $y(2) = (2)^2 - 2(2) + 2 = 2$

144 **(3)**

$$\frac{dy}{dx} = -\frac{\sqrt{(x^2-1)(y^2-1)}}{xy}$$

$$\int \frac{y}{\sqrt{y^2-1}}dy = -\int \frac{\sqrt{x^2-1}}{x}dx$$

Let $y^2 - 1 = t^2 \Rightarrow 2y\,dy = 2t\,dt$

$$\therefore \int \frac{t}{t}dt = -\int \frac{x^2-1}{x\sqrt{x^2-1}}dx$$

$$\therefore t = -\int \frac{x}{\sqrt{x^2-1}}dx + \int \frac{1}{x\sqrt{x^2-1}}dx$$

$$\therefore \sqrt{y^2-1} = -\sqrt{x^2-1} + \sec^{-1}x + c$$

Curve passes through the point (1,1) then the value of $c = 0$

Hence the curve is $\sqrt{y^2-1} = -\sqrt{x^2-1} + \sec^{-1}\text{x}$

145 **(2)**

$$\frac{dy}{dx} = \frac{1}{x\cos y + 2\sin y\cos y}$$

$$\therefore \frac{dx}{dy} = x\cos y + 2\sin y\cos y$$

$$\therefore \frac{dx}{dy} + (-\cos y)x = 2\sin y\cos y$$

$$\therefore \text{ L.F.} = e^{-\int \cos y\,dx}e = e^{-\sin y}$$

$\therefore$The solution is

$$x \cdot e^{-\sin y} = 2\int e^{-\sin y}\cdot \sin y\cos y\,dy$$

$$= -2\sin y e^{-\sin y} - 2\int \left(-e^{-\sin y}\right)\cos y\,dx$$

$$= -2\sin y\, e^{-\sin y} + 2\int -e^{-\sin y}\cos y\,dy$$

$$= -2\sin y e^{-\sin y} - 2e^{-\sin y} + c$$

i. e. $x = -2\sin y - 2 + ce^{\sin y}$

$= ce^{\sin y} - 2(1 + \sin y)$

$\therefore k = 2$

146 **(8)**

Equation of tangent at $P(x_1, y_1)$of $y = f(x)$

$y - y_1 = \frac{dy}{dx}(x - x_1)$ (1)

This tangent cuts the x-axis so

$$x_2 = x_1 - \frac{y_1}{\left(\frac{dy}{dx}\right)}$$

$\because x_1, x_2, x_3 \dots x_n$ are in AP

$x_2 - x_1 = -\frac{y_1}{\frac{dy}{dx}} = \log_z e$ given

$$-y = \log_z e\frac{dy}{dx}$$

$\frac{dy}{y}\log_z e = -dx$ Integrating both sides

$\log_e y = -x\log_z e + c$

$y = ke^{-x\log_e 2}$

$\because y = f(x)$ passes through (0,2)

$\Rightarrow k = 2$

$\therefore y = 2\cdot e^{-x\log_e 2}$

$\therefore y = 2^{1-x}$

147 **(4)**

$$\frac{dy}{dx} - y = 1 - e^{-x}$$

$P = -1$ $Q = 1 - e^{-x}$

I.F. $= e^{\int Pdx} = e^{\int -1dx} = e^{-x}$

$$\therefore y\cdot e^{-x} = \int e^{-x}(1 - e^{-x})dx + C$$

$$ye^{-x} = -e^{-x} + \frac{1}{2}e^{-2x} + C$$

$$y = -1 + \frac{1}{2}e^{-x} + Ce^x$$

$\because x = 0$ $y = y_0$

So $C = y_0 + \frac{1}{2}$

$$y = -1 + \frac{1}{2}e^{-x} + (y_0 + 1/2)e^x$$

$x \to \infty$ $y \to$ finite value so $y_0 + 1/2 = 0$

$y_0 = -1/2$

148 **(4)**

We have $4xe^{xy} = y + 5\sin^2 x$ (1)

Put $x = 0$, in equation (1), we get $y = 0$

Therefore, (0,0) lies on the curve

Now on differentiating equation (1) w.r.t. x, we get

$$4e^{xy} + 4e^{xy}\left(x\frac{dy}{dx} + y\right) = \frac{dy}{dx} + 10\sin x\cos x$$

$\Rightarrow y'(0) = 4$

149 **(8)**

$$\frac{dy}{dt} + 2t\,y = t^2$$

I.F. $= e^{t^2}$

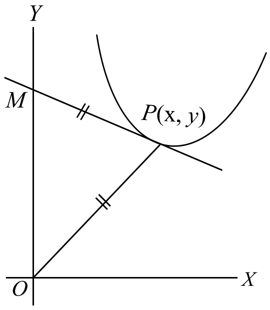
$\therefore$ Solution is $y \cdot e^{t^2} = \int t^2 e^{t^2} dt = \int te^{t^2} dt$

$$\therefore y \cdot e^{t^2} = t \cdot \frac{e^{t^2}}{2} - \frac{1}{2}\int e^{t^2}\,dt + C$$

$$y = \frac{t}{2} - e^{-t^2}\int \frac{e^{t^2}}{2}dt + Ce^{-t^2}$$

$$\lim_{t\to\infty}\frac{y}{t} = \frac{1}{2} - \lim_{t\to\infty}\frac{\int \frac{e^{t^2}}{2}}{te^{t^2}} = \frac{1}{2}$$

150 **(5)**



$\therefore\ OP = OM$

$$y - x\frac{dy}{dx} = \sqrt{x^2 + y^2}$$

$$\frac{dy}{dx} = \frac{y - \sqrt{x^2 + y^2}}{x}$$

$$\therefore \frac{dy}{dx} = \frac{y}{x} - \sqrt{1 + \left(\frac{y}{x}\right)^2}$$

Put $\frac{y}{x} = v \Rightarrow y = vx\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\therefore v + x\frac{dv}{dx} = v - \sqrt{1 + v^2}$$

$$\therefore \log\left(v + \sqrt{1 + v^2}\right) = \log\frac{c}{x}$$

$$\therefore v + \sqrt{1 + v^2} = \frac{c}{x}$$

$$\frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} = \frac{c}{x}$$

$$y + \sqrt{x^2 + y^2} = c$$

Hence curve is parabola, which has eccentricity 1

151 **(1)**

$$\frac{dy}{dx} = \frac{1}{dx/dy};\ \frac{d^2y}{dx^2} = \frac{d}{dy}\left(\frac{1}{dx/dy}\right)\cdot\frac{dy}{dx}$$

$$= -\frac{1}{(dx/dy)^3}\frac{d^2x}{dy^2}$$

Hence $x\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 - \frac{dy}{dx} = 0$

Becomes $-x \cdot \frac{1}{(dx/dy)^3}\frac{d^2x}{dy^2} + \frac{1}{(dx/dy)^3} - \frac{1}{dx/dy} = 0$

Or $x\frac{d^2x}{dy^2} - 1 + \left(\frac{dx}{dy}\right)^2 = 0 \Rightarrow x\frac{d^2x}{dy} + \left(\frac{dx}{dy}\right)^2 = 1;$

$\therefore k = 1$

152 **(2)**

Given $y = \tan z$

$\frac{dy}{dx} = \sec^2 z \cdot \frac{dz}{dx}$ (1)

Now $\frac{d^2y}{dx^2} = \sec^2 z \cdot \frac{d^2x}{dx^2} + \frac{dz}{dx}\cdot\frac{d}{dx}(\sec^2 z)$ [using product rule]

$$= \sec^2 z \cdot \frac{d^2z}{dx^2} + \frac{dz}{dx}\cdot\frac{d}{dz}(\sec^2 z)\frac{dz}{dx}$$

$\frac{d^2y}{dx^2} = \sec^2 z \cdot \frac{d^2z}{dx^2} + \left(\frac{dz}{dx}\right)^2 \cdot 2\sec^2 z \cdot \tan z$ (2)

Now $1 + \frac{2(1+y)}{1+y^2}\left(\frac{dy}{dx}\right)^2$

$$= 1 + \frac{2(1 + \tan z)}{\sec^2 z}\cdot\sec^4 z \cdot \left(\frac{dz}{dx}\right)^2$$

$$= 1 + 2(1 + \tan z)\cdot\sec^2 z \cdot \left(\frac{dz}{dx}\right)^2$$

$= 1 + 2\sec^2 z\left(\frac{dz}{dx}\right)^2 + 2\tan z \cdot \sec^2 z\left(\frac{dz}{dx}\right)^2$ (3)

From (2) and (3), we have RHS of (2)= RHS of (3)

$$\sec^2 z \cdot \frac{d^2z}{dx^2} = 1 + 2\sec^2 z\left(\frac{dz}{dx}\right)^2$$

$$\Rightarrow \frac{d^2z}{dx^2} = \cos^2 z + 2\left(\frac{dz}{dx}\right)^2$$

$\Rightarrow k = 2$