Single Correct Answer Type

1. If $D_k = \begin{vmatrix} 1 & n & n \\ 2k & n^2+n+1 & n^2+n \\ 2k-1 & n^2 & n^2+n+1 \end{vmatrix}$ and $\sum_{k=0}^{n} D_k = 56$, then n equals
   a) 4 b) 6 c) 8 d) None of these
2. If $A_1, B_1, C_1, \dots$ are, respectively, the cofactors of the elements $a_1, b_1, c_1, \dots$ of the determinant $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $\Delta \neq 0$, then the value of $\begin{vmatrix} B_2 & C_2 \\ B_3 & C_3 \end{vmatrix}$ is equal to
   a) $a_1^2\Delta$ b) $a_1\Delta$ c) $a_1\Delta^2$ d) $a_1^2\Delta^2$
3. Let $f(x) = \begin{vmatrix} 2\cos^2 x & \sin 2x & -\sin x \\ \sin 2x & 2\sin^2 x & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix}$. Then the value of $\int_0^{\pi/2} [f(x) + f'(x)]dx$ is
   a) π b) $\pi/2$ c) 2π d) $3\pi/2$
4. The number of distinct real root of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\pi/4 \le x \le \pi/4$ is
   a) 0 b) 2 c) 1 d) 3
5. If $\begin{vmatrix} x^n & x^{n+2} & x^{n+3} \\ y^n & y^{n+2} & y^{n+3} \\ z^n & z^{n+2} & z^{n+3} \end{vmatrix} = (x-y)(y-z)(z-x)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$ then n equals
   a) 1 b) -1 c) 2 d) -2
6. Given $a = x/(y-z), b = y/(z-x)$ and $c = z/(x-y)$, where x, y and z are not all zero, then the value of $ab+bc+ca$ is
   a) 0 b) 1 c) -1 d) None of these
7. If $\omega\ (\neq 1)$ is a cube root of unity, then value of the determinant $\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -i+\omega-1 & -1 \end{vmatrix}$ is
   a) 0 b) 1 c) i d) ω
8. If $\begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ c+a & a+b & b+c \end{vmatrix} = k\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$, then the value of k is
   a) 1 b) 2 c) 3 d) 4
9. $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$. The value of $\lim_{x\to 0} \frac{f(x)}{x}$ is equal to
   a) 1 b) -1 c) Zero d) None of these
10. The parameter, on which the value of the determinant $\begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$ does not depend, is
    a) a b) p c) d d) x
11. If $\Delta = \begin{vmatrix} 3 & 4 & 5 & x \\ 4 & 5 & 6 & y \\ 5 & 6 & 7 & z \\ x & y & z & 0 \end{vmatrix} = 0$, then
    a) x, y, z are in A.P. b) x, y, z are in G.P. c) x, y, z are in H.P. d) None of these
12. The determinant $\begin{vmatrix} y^2 & -xy & x^2 \\ a & b & c \\ a' & b' & c' \end{vmatrix}$ is equal to

a) $\begin{vmatrix} bx+ay & cx+by \\ b'x+a'y & c'x+b'y \end{vmatrix}$

b) $\begin{vmatrix} ax+by & bx+cy \\ a'x+b'y & b'x+c'y \end{vmatrix}$

c) $\begin{vmatrix} bx+cy & ax+by \\ b'x+c'y & a'x+b'y \end{vmatrix}$

d) $\begin{vmatrix} ax+by & bx+cy \\ a'x+b'y & b'x+c'y \end{vmatrix}$

13. If $z = \begin{vmatrix} -5 & 3+4i & 5-7i \\ 3-4i & 6 & 8+7i \\ 5+7i & 8-7i & 9 \end{vmatrix}$, then z is

a) Purely real
b) Purely imaginary
c) $a+ib$, where $a \neq 0, b \neq 0$
d) $a+ib$, where $b=4$

14. If a, b and c are non-zero real numbers, then $\Delta = \begin{vmatrix} b^2c^2 & ab & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$ is equal to

a) abc
b) $a^2b^2c^2$
c) $bc+ca+ab$
d) None of these

15. If $\begin{vmatrix} x & 3 & 6 \\ 3 & 6 & x \\ 6 & x & 3 \end{vmatrix} = \begin{vmatrix} 2 & x & 7 \\ x & 7 & 2 \\ 7 & 2 & x \end{vmatrix} = \begin{vmatrix} 4 & 5 & x \\ 5 & x & 4 \\ x & 4 & 5 \end{vmatrix} = 0$, then '$x$' is equal to

a) 0
b) -9
c) 3
d) None of these

16. Let $\vec{a}_r = x_r\hat{i} + y_r\hat{j} + z_r\hat{k}, r = 1, 2, 3$ be three mutually perpendicular unit vectors, then the value of $\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix}$ is equal to

a) Zero
b) ± 1
c) ± 2
d) None of these

17. The value of the determinant $\begin{vmatrix} {}^nC_{r-1} & {}^nC_r & (r+1)\,{}^{n+2}C_{r+1} \\ {}^nC_r & {}^nC_{r+1} & (r+2)\,{}^{n+2}C_{r+2} \\ 2 & {}^nC_{r+2} & (r+3)\,{}^{n+2}C_{r+3} \end{vmatrix}$ is

a) n^2+n-1
b) 0
c) ${}^{n+3}C_{r+3}$
d) ${}^nC_{r-1} + {}^nC_r + {}^nC_{r+1}$

18. If α, β, γ are the roots of $px^3+qx^2+r=0$, then the value of the determinant $\begin{vmatrix} \alpha\beta & \beta\gamma & \gamma\alpha \\ \beta\gamma & \gamma\alpha & \alpha\beta \\ \gamma\alpha & \alpha\beta & \beta\gamma \end{vmatrix}$

a) p
b) q
c) 0
d) r

19. If w is a complex cube root of unity, then value of $\Delta = \begin{vmatrix} a_1+b_1w & a_1w^2+b_1 & c_1+b_1\overline{w} \\ a_2+b_2w & a_2w^2+b_2 & c_2+b_2\overline{w} \\ a_3+b_3w & a_3w^2+b_3 & c_3+b_3\overline{w} \end{vmatrix}$ is

a) 0
b) -1
c) 2
d) None of these

20. If x, y, z are in A.P., then the value of the determinant $\begin{vmatrix} a+2 & a+3 & a+2x \\ a+3 & a+4 & a+2y \\ a+4 & a+5 & a+2z \end{vmatrix}$ is

a) 1
b) 0
c) $2a$
d) a

21. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125$, then the value of $\alpha$is

a) ± 1
b) ± 2
c) ± 3
d) ± 5

22. Value of $\begin{vmatrix} x+y & z & z \\ x & y+z & x \\ y & y & z+x \end{vmatrix}$, where x, y, z are non-zero real numbers, is equal to

a) xyz
b) $2xyz$
c) $3xyz$
d) $4xyz$

23. Roots of the equation $\begin{vmatrix} x & m & n & 1 \\ a & x & n & 1 \\ a & b & x & 1 \\ a & b & c & 1 \end{vmatrix} = 0$ are

a) Independent of m and n
b) Independent of a, b and c
c) Depend on m, n and a, b, c
d) Independent of m, n and a, b, c

24. If $f(x) = a + bx + cx^2$ and α, β, γ are the roots of the equation $x^3 = 1$, then $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is equal to

a) $f(\alpha) + f(\beta) + f(\gamma)$
b) $f(\alpha)f(\beta) + f(\beta)f(\gamma) + f(\gamma)f(\alpha)$
c) $f(\alpha)f(\beta)f(\gamma)$
d) $-f(\alpha)f(\beta)f(\gamma)$

25. If $a > 0$ and discriminant of $ax^2 + 2bx + c$ is negative, then $\Delta \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$ is

a) +ve
b) $(ac - b)^2(ax^2 + 2bx + c)$
c) −ve
d) 0

26. If $a^2 + b^2 + c^2 = -2$ and $f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$, then $f(x)$ is a polynomial of degree

a) 0
b) 1
c) 2
d) 3

27. If a, b, c are non-zero real numbers and if the equations $(a-1)x = y + z, (b-1)y = z + x, (c-1)z = x + y$ have a non-trivial solution, then $ab + bc + ca$ equals

a) $a + b + c$
b) abc
c) 1
d) None of these

28. If $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & a+b & c \end{vmatrix} = 0$, then the line $ax + by + c = 0$ passes through the fixed point which is

a) (1, 2)
b) (1, 1)
c) (−2, 1)
d) (1, 0)

29. The value of determinant $\begin{vmatrix} bc-a^2 & ac-b^2 & ab-c^2 \\ ac-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ac-b^2 \end{vmatrix}$ is

a) Always positive
b) Always negative
c) Always zero
d) Cannot say anything

30. If $f(x) = \begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix} = 0$, then

a) $f'(x) = 0$ and $f''(x) = 0$ has one common root
b) $f(x) = 0$ and $f'(x) = 0$ has one common root
c) Sum of roots of $f(x) = 0$ is $-3a$
d) None of these

31. If $f'(x) = \begin{vmatrix} mx & mx-p & mx+p \\ n & n+p & n-p \\ mx+2n & mx+2n+p & mx+2n-p \end{vmatrix}$, then $y = f(x)$ represents

a) A straight line parallel to x-axis
b) A straight line parallel to y-axis
c) Parabola
d) A straight line with negative slope

32. If $p\lambda^4 + q\lambda^3 + r\lambda^2 + s\lambda + t = \begin{vmatrix} \lambda^2+3\lambda & \lambda-1 & \lambda+3 \\ \lambda^2+1 & 2-\lambda & \lambda-3 \\ \lambda^2-3 & \lambda+4 & 3\lambda \end{vmatrix}$, then p is equal to

a) −5
b) −4
c) −3
d) −2

33. Let $a, b, c \in R$ such that no two of them are equal and satisfy $\begin{vmatrix} 2a & b & c \\ b & c & 2a \\ c & 2a & b \end{vmatrix} = 0$, then equation $24ax^2 + 4bx + c = 0$ has

a) At least one root in $[0, 1]$
b) At least one root in $\left[-\frac{1}{2}, \frac{1}{2}\right]$
c) At least one root in $[-1, 0]$
d) At least two roots in $[0, 2]$

34. Consider the set A of all determinants of order 3 with entries 0 or 1 only. Let B be the subset of A consisting of all determinants with values −1. Then

a) C is empty
b) B has as many elements as C
c) $A = B \cup C$
d) B has twice as many elements as elements as C

35. If $x \neq y \neq z$ and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then the value of xyz is

a) 1 b) 2 c) -1 d) -2

36. If $l_1^2+m_1^2+n_1^2=1$, etc and $l_1l_2+m_1m_2+n_1n_2=0$, etc, and $\Delta=\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}$, then

a) $|\Delta|=3$ b) $|\Delta|=2$ c) $|\Delta|=1$ d) $\Delta=0$

37. Which of the following is not the root of the equation $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix}=0$?

a) 2 b) 0 c) 1 d) -3

38. If $x\neq 0, y\neq 0, z\neq 0$ and $\begin{vmatrix} 1+x & 1 & 1 \\ 1+y & 1+2y & 1 \\ 1+z & 1+z & 1+3z \end{vmatrix}=0$, then $x^{-1}+y^{-1}+z^{-1}$ is equal to

a) -1 b) -2 c) -3 d) None of these

39. If $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}=(a-b)(b-c)(c-a)(a+b+c)$, where a, b, c are all different, then the determinant $\begin{vmatrix} 1 & 1 & 1 \\ (x-a)^2 & (x-b)^2 & (x-c)^2 \\ (x-b)(x-c) & (x-c)(x-a) & (x-a)(c-b) \end{vmatrix}$ vanishes when

a) $a+b+c=0$ b) $x=\frac{1}{3}(a+b+c)$ c) $x=\frac{1}{2}(a+b+c)$ d) $x=a+b+c$

40. If the system of equations $x-ky-z=0, kx-y-z=0, x+y-z=0$ has a non-zero solution then the possible values of k are

a) $-1, 2$ b) $1, 2$ c) $0, 1$ d) $-1, 1$

41. Value of $\begin{vmatrix} 1+x_1 & 1+x_1x & 1+x_1x^2 \\ 1+x_2 & 1+x_2x & 1+x_2x^2 \\ 1+x_3 & 1+x_3x & 1+x_3x^2 \end{vmatrix}$ depends upon

a) x only b) x_1 only c) x_2 only d) None of these

42. The set of equations $\lambda x-y+(\cos\theta)z=0, 3x+y+2z=0, (\cos\theta)x+y+2z=0, 0\leq\theta<2\pi$, has non-trivial solution(s)

a) For non value of λ and θ
b) For all values of λ and θ
c) For all values of λ and only two values of θ
d) For only one value of λ and all values of θ

43. Let $x<1$, then value of $\begin{vmatrix} x^2+2 & 2x+1 & 1 \\ 2x+1 & x+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$ is

a) Non-negative b) Non-positive c) Negative d) Positive

44. Let $\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix}=Ax^4+Bx^3+Cx^2+Dx+E$. Then the value of $5A+4B+3C+2D+E$ is equal to

a) Zero b) -16 c) 16 d) -11

45. The value of the determinant $\Delta=\begin{vmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{vmatrix}$ is equal to

a) 1 b) 0 c) 2 d) 3

46. Let $\{D_1, D_2, D_3, \cdots, D_n\}$ be the set of third-order determinants that can be made with the distinct non-zero real numbers $a_1, a_2, \cdots, a_9$. Then

a) $\sum_{i=1}^{n} D_i=1$ b) $\sum_{i=1}^{n} D_i=0$ c) $D_i=D_j, \forall\, i, j$ d) None of these

47. If α, β, γ are the angles of a triangle and the system of equations
$\cos(\alpha-\beta)x+\cos(\beta-\gamma)y+\cos(\gamma-\alpha)z=0$
$\cos(\alpha+\beta)x+\cos(\beta+\gamma)y+\cos(\gamma+\alpha)z=0$
$\sin(\alpha+\beta)x+\sin(\beta+\gamma)y+\sin(\gamma+\alpha)z=0$
Has non-trivial solutions, then triangle is necessarily

a) Equilateral b) Isosceles c) Right angled d) Acute angled

48. If $c < 1$ and the system of equations $x + y - 1 = 0, 2x - y - c = 0$ and $bx + 3by - c = 0$ is consistent, then the possible real values of b are

a) $b \in \left(-3, \frac{3}{4}\right)$ b) $b \in \left(-\frac{3}{2}, 4\right)$ c) $b \in \left(-\frac{3}{4}, 3\right)$ d) None of these

49. Let a, b, c be the real numbers. Then following system of equation in x, y and z, $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$, $\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, $-\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, has

a) No solution b) Unique solution

c) Many solutions d) Finitely many solutions

50. The value of $\begin{vmatrix} -1 & 2 & 1 \\ 3+2\sqrt{2} & 2+2\sqrt{2} & 1 \\ 3-2\sqrt{2} & 2-2\sqrt{2} & 1 \end{vmatrix}$ is equal to

a) Zero b) $-16\sqrt{2}$ c) $-8\sqrt{2}$ d) None of these

51. The value of $\sum_{r=2}^{n}(-2)^r \begin{vmatrix} {}^{n-2}C_{r-2} & {}^{n-2}C_{r-1} & {}^{n-2}C_r \\ -3 & 1 & 1 \\ 2 & -1 & 0 \end{vmatrix}$ $(n > 2)$ is

a) $2n - 1 + (-1)^n$ b) $2n + 1 + (-1)^{n-1}$ c) $2n - 3 + (-1)^n$ d) None of these

52. The value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ {}^mC_1 & {}^{m+1}C_1 & {}^{m+2}C_1 \\ {}^mC_2 & {}^{m+1}C_2 & {}^{m+2}C_2 \end{vmatrix}$ is equal to

a) 1 b) -1 c) 0 d) None of these

53. The number of positive integral solutions of the equation $\begin{vmatrix} x^3+1 & x^2y & x^2z \\ xy^2 & y^3+1 & y^2z \\ xz^2 & yz^2 & z^3+1 \end{vmatrix} = 11$ is

a) 0 b) 3 c) 6 d) 12

54. In triangle ABC, if

$\begin{vmatrix} 1 & 1 & 1 \\ \cot\frac{A}{2} & \cot\frac{B}{2} & \cot\frac{C}{2} \\ \tan\frac{B}{2}+\tan\frac{C}{2} & \tan\frac{C}{2}+\tan\frac{A}{2} & \tan\frac{A}{2}+\tan\frac{B}{2} \end{vmatrix} = 0$, then the triangle must be

a) Equilateral b) Isosceles c) Obtuse angled d) None of these

55. If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$ then $f(500)$ is equal to

a) 0 b) 1 c) 500 d) -500

56. If $a_1b_1c_1, a_2b_2c_2$ and $a_3b_3c_3$ are 3-digit even natural numbers and

$\Delta = \begin{vmatrix} c_1 & a_1 & b_1 \\ c_2 & a_2 & b_2 \\ c_3 & a_3 & b_3 \end{vmatrix}$, then Δ is

a) Divisible by 2 but not necessarily by 4 b) Divisible by 4 but not necessarily by 8

c) Divisible by 8 d) None of these

57. The system of equations

$ax - y - z = \alpha - 1$

$x - \alpha y - z = \alpha - 1$

$x - y - \alpha z = \alpha - 1$

Has no solution if α is

a) Either -2 or 1 b) -2 c) 1 d) Not -2

58. a, b, c are distinct real numbers, not equal to one. If $ax + y + z = 0, x + by + z = 0$ and $x + y + cz = 0$ have a non-trivial solution, then the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is equal to

a) -1 b) 1 c) Zero d) None of these

59. If $\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = ka^2b^2c^2$, then the value of k is

a) 2 b) 4 c) 0 d) None of these

60. $\Delta_1 = \begin{vmatrix} y^5z^6(z^3-y^3) & x^4z^6(x^3-z^3) & x^4y^5(y^3-x^3) \\ y^2z^3(y^6-z^6) & xz^3(z^6-x^6) & xy^2(x^6-y^6) \\ y^2z^3(z^3-y^3) & xz^3(x^3-z^3) & xy^2(y^3-x^3) \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & y^2 & z^3 \\ x^4 & y^5 & z^6 \\ x^7 & y^8 & z^9 \end{vmatrix}$. Then $\Delta_1\Delta_2$ is equal to

a) Δ_2^3 b) Δ_2^2 c) Δ_2^4 d) None of these

61. If a, b, c are different , then the value of x satisfying $\begin{vmatrix} 0 & x^2-a & x^3-b \\ x^2+a & 0 & x^2+c \\ x^4+b & x-c & 0 \end{vmatrix} = 0$ is

a) c b) c c) b d) 0

62. Let m be a positive integer and $\Delta_r = \begin{vmatrix} 2r-1 & {}^mC_r & 1 \\ m^2-1 & 2^m & m+1 \\ \sin^2(m^2) & \sin^2(m) & \sin^2(m+1) \end{vmatrix} (0 \le r \le m)$

Then the value of $\sum_{r=0}^{m} \Delta_r$ is given by

a) 0 b) m^2-1 c) 2^m d) $2^m \sin^2(2^m)$

63. For the equation $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = 0$

a) There are exactly two distinct roots b) There is one pair of equation real roots
c) There are three pairs of equal roots d) Modulus of each root is 2

64. If a, b, c are in G.P. with common ratio r_1 and α, β, γ are in G.P. with common ratio r_2, and equations $ax + \alpha y + z = 0, bx + \beta y + z = 0, cx + \gamma y + z = 0$ have only zero solution, then which of the following is not true?

a) $r_1 \neq 1$ b) $r_2 \neq 1$ c) $r_1 \neq r_2$ d) None of these

65. The value of the determinant $\begin{vmatrix} (a_1-b_1)^2 & (a_1-b_2)^2 & (a_1-b_3)^2 & (a_1-b_4)^2 \\ (a_2-b_1)^2 & (a_2-b_2)^2 & (a_2-b_3)^2 & (a_2-b_4)^2 \\ (a_3-b_1)^2 & (a_3-b_2)^2 & (a_3-b_3)^2 & (a_3-b_4)^2 \\ (a_4-b_1)^2 & (a_4-b_2)^2 & (a_4-b_3)^2 & (a_4-b_4)^2 \end{vmatrix}$ is

a) Dependant on $a_i, i = 1, 2, 3, 4$ b) Dependant on $b_i, i = 1, 2, 3, 4$
c) Dependant on $a_{ij}, b_i, i = 1, 2, 3, 4$ d) 0

66. If A, B, C are angles of a triangle, then the value of $\begin{vmatrix} e^{2iA} & e^{-iC} & e^{-iB} \\ e^{-iC} & e^{2iB} & e^{-iA} \\ e^{-iB} & e^{-iA} & e^{2iC} \end{vmatrix}$ is

a) 1 b) -1 c) -2 d) -4

67. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then

a) $x = 3, y = 1$ b) $x = 1, y = 3$ c) $x = 0, y = 3$ d) $x = 0, y = 0$

68. If p, q, r are in A.P., then the value of determinant $\begin{vmatrix} a^2+a^{2n+1}+2p & b^2+2^{n+2}+3q & c^2+p \\ 2^n+p & 2^{n+1}+q & 2q \\ a^2+2^n+p & b^2+2^{n+1}+2q & c^2-r \end{vmatrix}$ is

a) 1 b) 0 c) $a^2b^2c^2 - 2^n$ d) $(a^2+b^2+c^2) - 2^nq$

69. If $p + q + r = 0 = a + b + c$, then the value of the determinant

$\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$ is

a) 0 b) $pa + qb + rc$ c) 1 d) None of these

70. If a, b, c, d, e, and f are in G.P., then the value of $\begin{vmatrix} a^2 & d^2 & x \\ b^2 & e^2 & y \\ c^2 & f^2 & z \end{vmatrix}$ depends on

a) x and y b) x and z

c) y and z d) Independent of x, y and z

71. If a, b, c are non-zeros, then the system of equations $(\alpha + a)x + \alpha y + \alpha z = 0, \alpha x + (\alpha + b)y + \alpha z = 0, \alpha x + \alpha y + (\alpha + c)z = 0$ has a non-trivial solution if

a) $\alpha^{-1} = -(a^{-1} + b^{-1} + c^{-1})$ b) $\alpha^{-1} = a + b + c$

c) $\alpha + a + b + c = 1$ d) None of these

72. If $a = \cos\theta + i\sin\theta, b = \cos 2\theta - i\sin 2\theta, c = \cos 3\theta + i\sin 3\theta$ and if $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$, then

a) $\theta = 2k\pi, k \in Z$ b) $\theta = (2k+1)\pi, k \in Z$ c) $\theta = (4k+1)\pi, k \in Z$ d) None of these

73. If $\begin{vmatrix} x^n & x^{n+2} & x^{2n} \\ 1 & x^a & a \\ x^{n+5} & x^{a+6} & x^{2n+5} \end{vmatrix} = 0, \forall x \in R$, where $n \in N$, then value of 'a' is

a) n b) $n - 1$ c) $n + 1$ d) None of these

74. If $a_1, a_2, \dots, a_n, \dots$ from a G.P. and $a_i > 0$, for all $i \geq 1$, then $\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$ is equal to

a) 0 b) 1 c) 2 d) 3

75. The value of $\begin{vmatrix} yz & zx & xy \\ p & 2q & 3r \\ 1 & 1 & 1 \end{vmatrix}$, where x, y, z are, respectively, $p^{\text{th}}, (2q)^{\text{th}}$and $(3r)^{\text{th}}$ terms of an H.P., is

a) -1 b) 0 c) 1 d) None of these

76. If $y = \sin mx$, then the value of the determinant $\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix}$, where $y_n = \frac{d^n y}{dx^n}$, is

a) m^9 b) m^2 c) m^3 d) None of these

77. Suppose $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and$D' = \begin{vmatrix} a_1 + pb_1 & b_1 + qc_1 & c_1 + ra_1 \\ a_2 + pb_2 & b_2 + qc_2 & c_2 + ra_2 \\ a_3 + pb_3 & b_3 + qc_3 & c_3 + ra_3 \end{vmatrix}$. Then

a) $D' = D$ b) $D' = D(1 - pqr)$

c) $D' = D(1 + p + q + r)$ d) $D' = D(1 + pqr)$

78. The value of the determinant $\begin{vmatrix} ka & k^2 + a^2 & 1 \\ kb & k^2 + b^2 & 1 \\ kc & k^2 + c^2 & 1 \end{vmatrix}$ is

a) $k(a+b)(b+c)(c+a)$ b) $k\,abc(a^2 + b^2 + c^2)$

c) $k(a-b)(b-c)(c-a)$ d) $k(a+b-c)(b+c-a)(c+a-b)$

79. If a, b, c are positive and are the $p^{\text{th}}, q^{\text{th}}$ and r^{th} terms, respectively, of a G.P., then $\Delta = \begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$ is

a) 0 b) $\log(abc)$ c) $-(p + q + r)$ d) None of these

80. If the determinant $\begin{vmatrix} b-c & c-a & a-b \\ b'-c' & c'-a' & a'-b' \\ b''-c'' & c''-a'' & a''-b'' \end{vmatrix} = m \begin{vmatrix} a & b & c \\ a' & b' & c' \\ a'' & b'' & c'' \end{vmatrix}$, then the value of m is

a) 0 b) 2 c) -1 d) 1

81. If x, y, z are different from zero and $\Delta = \begin{vmatrix} a & b-y & c-z \\ a-x & b & c-z \\ a-x & b-y & c \end{vmatrix} = 0$, then the value of the expression $\frac{a}{x} + \frac{b}{y} + \frac{c}{z}$ is

a) 0 b) -1 c) 1 d) 2

82. If $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$ are the given determinants, then

a) $\Delta_1 = 3(\Delta_2)^2$ b) $\frac{d}{dx}(\Delta_1) = 3\Delta_2$ c) $\frac{d}{dx}(\Delta_1) = 3(\Delta_2)^2$ d) $\Delta_1 = 3\Delta_2^{3/2}$

83. If a determinant of order 3×3 is formed by using the numbers 1 or -1, then the minimum value of the

determinant is

a) -2 b) -4 c) 0 d) -8

84. If the system of linear equations $x+y+z=6, x+2y+3z=14$ and $2x+5y+\lambda z=\mu(\lambda,\mu\in R)$ has a unique solution, then

a) $\lambda\neq 8$ b) $\lambda=8,\mu\neq 36$ c) $\lambda=8,\mu=36$ d) None of these

85. If $(x_1-x_2)^2+(y_1-y_2)^2=a^2$

$(x_2-x_3)^2+(y_2-y_3)^2=b^2$

$(x_3-x_1)^2+(y_3-y_1)^2=c^2$ and $k\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}=(a+b+c)(b+c-a)(c+a-b)\times(a+b-c)$, then the value of k is

a) 1 b) 2 c) 4 d) None of these

86. If [ ] denotes the greatest integer less than or equal to the real number under consideration, and $-1\leq x<0, 0\leq y<1, 1\leq z<2$, then the value of the determinant

$\begin{vmatrix}[x]+1 & [y] & [z]\\ [x] & [y]+1 & [z]\\ [x] & [y] & [z]+1\end{vmatrix}$ is

a) $[x]$ b) $[y]$ c) $[z]$ d) None of these

87. If $pqr\neq 0$ and the system of equations

$(p+a)x+by+cz=0$

$ax+(q+b)y+cz=0$

$ax+by+(r+c)z=0$

Has a non-trivial solution, then value of $\frac{a}{p}+\frac{b}{q}+\frac{c}{r}$ is

a) -1 b) 0 c) 1 d) 2

88. When the determinant $\begin{vmatrix}\cos 2x & \sin^2 x & \cos 4x\\ \sin^2 x & \cos 2x & \cos^2 x\\ \cos 4x & \cos^2 x & \cos 2x\end{vmatrix}$ is expanded in powers of $\sin x$, then the constant term in that expression is

a) 1 b) 0 c) -1 d) 2

89. If the value of the determinant $\begin{vmatrix}a & 1 & 1\\ 1 & b & 1\\ 1 & 1 & c\end{vmatrix}$ is positive, then $(a,b,c>0)$

a) $abc>1$ b) $abc>-8$ c) $abc<-8$ d) $abc>-2$

90. If $\begin{vmatrix}a^2+\lambda^2 & ab+c\lambda & ca-b\lambda\\ ab-c\lambda & b^2+\lambda^2 & bc+a\lambda\\ ca+b\lambda & bc-a\lambda & c^2+\lambda^2\end{vmatrix}\begin{vmatrix}\lambda & c & -b\\ -c & \lambda & a\\ b & -a & \lambda\end{vmatrix}=(1+a^2+b^2+c^2)^3$, then the value of λ is

a) 8 b) 27 c) 1 d) -1

91. The determinant $\begin{vmatrix}xp+y & x & y\\ yp+z & y & z\\ 0 & xp+y & yp+z\end{vmatrix}=0$ if

a) x,y,z are in A.P. b) x,y,z are in G.P. c) x,y,z are in H.P. d) xy,yz,zx are in A.P.

92. The value of the determinant of n^{th} order, being given by

$\begin{vmatrix}x & 1 & 1 & \cdots\\ 1 & x & 1 & \cdots\\ 1 & 1 & x & \cdots\\ \cdots & \cdots & \cdots & \cdots\end{vmatrix}$ is

a) $(x-1)^{n-1}(x+n-1)$ b) $(x-1)^n(x+n-1)$

c) $(1-x)^{-1}(x+n-1)$ d) None of these

93. If $a+b+c=0$, one root of $\begin{vmatrix}a-x & c & b\\ c & b-x & a\\ b & a & c-x\end{vmatrix}=0$ is

a) $x=1$ b) $x=2$ c) $x=a^2+b^2+c^2$ d) $x=0$

Multiple Correct Answers Type

94. The determinant $\Delta=\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix}$ is equal to zero, if

a) a, b, c are in AP
b) a, b, c are in GP
c) a, b, c are in HP
d) α is the root of $ax^2+2bx+c=0$

95. If $\Delta=\begin{vmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\theta\sin\phi & \sin\theta\cos\phi & 0 \end{vmatrix}$ then

a) Δ is independent of θ
b) Δ is independent of ϕ
c) Δ is a constant
d) $\left.\frac{d\Delta}{d\theta}\right]_{\theta=\pi/2}=0$

96. If $f(x)=\begin{vmatrix} 3 & 3x & 3x^2+2a^2 \\ 3x & 3x^2+2a^2 & 3x^3+6a^2x \\ 3x^2+2a^2 & 3x^3+6a^2x & 3x^4+12a^2x^2+2a^4 \end{vmatrix}$, then

a) $f'(x)=0$
b) $y=f(x)$ is a straight line parallel to x-axis
c) $\int_0^2 f(x)dx=32a^4$
d) None of these

97. If $f(\theta)=\begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cos B & 1 \\ \sin^2 C & \cos C & 1 \end{vmatrix}$, then

a) $\tan A+\tan B+c$
b) $\cot A\cot B\cot C$
c) $\sin^2 A+\sin^2 B+\sin^2 C$
d) 0

98. The determinant $\Delta=\begin{vmatrix} a^2+x & ab & ac \\ ab & b^2+x & bc \\ ac & bc & c^2+x \end{vmatrix}$ is divisible by

a) x
b) x^2
c) x^3
d) None of these

99. Let $f(x)=\begin{vmatrix} n & n+1 & n+2 \\ {}^nP_n & {}^{n+1}P_{n+1} & {}^{n+2}P_{n+2} \\ {}^nC_n & {}^{n+1}C_{n+1} & {}^{n+2}C_{n+2} \end{vmatrix}$, where the symbols have their usual meanings. The $f(x)$ is divisible by

a) n^2+n+1
b) $(n+1)!$
c) $n!$
d) None of the above

100. The determinant $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix}=0$, if

a) a, b, c are in A.P.
b) a, b, c are in G.P.
c) a, b, c are in H.P.
d) α is a root of the equation $ax^2+bx+c=0$

101. The determinant $\Delta=\begin{vmatrix} a^2+x^2 & ab & ac \\ ab & b^2+x^2 & bc \\ ac & bc & c^2+x^2 \end{vmatrix}$ is divisible by

a) x
b) x^2
c) x^3
d) x^4

102. If $g(x)=\begin{vmatrix} a^{-x} & e^{x\log_e a} & x^2 \\ a^{-3x} & e^{3x\log_e a} & x^4 \\ a^{-5x} & e^{5x\log_e a} & 1 \end{vmatrix}$, then

a) Graphs of $g(x)$ is symmetrical about origin
b) Graphs of $g(x)$ is symmetrical about Y-axis
c) $\left.\frac{d^4 g(x)}{dx^4}\right|_{x=0}=0$
d) $f(x)=g(x)\times\log\left(\frac{a-x}{a+x}\right)$ is an odd function

103. If $g(x)=\frac{f(x)}{(x-a)(x-b)(x-c)}$, where $f(x)$ is a polynomial of degree <3, then

a) $\int g(x)dx=\begin{vmatrix} 1 & a & f(a)\log|x-a| \\ 1 & b & f(b)\log|x-b| \\ 1 & c & f(c)\log|x-c| \end{vmatrix}\div\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}+k$
b) $\frac{dg(x)}{dx}=\begin{vmatrix} 1 & a & f(a)(x-a)^{-2} \\ 1 & b & f(b)(x-b)^{-2} \\ 1 & c & f(c)(x-c)^{-2} \end{vmatrix}\div\begin{vmatrix} a^2 & a & 1 \\ b^2 & b & 1 \\ c^2 & c & 1 \end{vmatrix}$

c) $\frac{dg(x)}{dx}=\begin{vmatrix}1 & a & f(a)(x-a)^{-2}\\1 & b & f(b)(x-b)^{-2}\\1 & c & f(c)(x-c)^{-2}\end{vmatrix}\div\begin{vmatrix}1 & a & a^2\\1 & b & b^2\\1 & c & c^2\end{vmatrix}$

d) $\int g(x)dx=\begin{vmatrix}1 & a & f(a)\log|x-a|\\1 & b & f(b)\log|x-b|\\1 & c & f(c)\log|x-c|\end{vmatrix}\div\begin{vmatrix}a^2 & a & 1\\b^2 & b & 1\\c^2 & c & 1\end{vmatrix}+k$

104. Eliminating a,b,c from $x=\frac{a}{b-c},y=\frac{b}{c-a},z=\frac{c}{a-b}$, we get

a) $\begin{vmatrix}1 & x- & x\\1 & -y & y\\1 & -z & z\end{vmatrix}=0$ b) $\begin{vmatrix}1 & -x & x\\1 & 1 & -y\\1 & z & 1\end{vmatrix}=0$ c) $\begin{vmatrix}1 & -x & x\\y & 1 & -y\\-z & z & 1\end{vmatrix}=0$ d) None of these

105. If determinant $\begin{vmatrix}\cos(\theta+\phi) & -\sin(\theta+\phi) & \cos 2\phi\\\sin\theta & \cos\theta & \sin\phi\\-\cos\theta & \sin\theta & \cos\phi\end{vmatrix}$ is

a) Positive b) Independent of θ c) Independent of ϕ d) None of these

106. If $A+B+C=\pi, e^{i\theta}+\cos\theta+\sin\theta$ and $z=\begin{vmatrix}e^{2iA} & e^{-iC} & e^{-iB}\\e^{-iC} & e^{2iB} & e^{-iA}\\e^{-eB} & e^{-iA} & 2^{2iC}\end{vmatrix}$ then

a) $\text{Re}(z)=4$ b) $\text{Im}(z)=0$ c) $\text{Re}(z)=-4$ d) $\text{Im}(z)=-1$

107. If $f(\theta)=\begin{vmatrix}\sin\theta & \cos\theta & \sin\theta\\\cos\theta & \sin\theta & \cos\theta\\\cos\theta & \sin\theta & \sin\theta\end{vmatrix}$, then

a) $f(\theta)=0$ has exactly 2 real solutions in $[0,\pi]$ b) $f(\theta)=0$ has exactly 3 real solutions in $[0,\pi]$

c) Range of function $\frac{f(\theta)}{1-\sin 2\theta}$ is $[-\sqrt{2},\sqrt{2}]$ d) Range of function $\frac{f(\theta)}{\sin 2\theta-1}$ is $[-3,3]$

108. If $\begin{vmatrix}yz-x^2 & zx-y^2 & xy-z^2\\xz-y^2 & xy-z^2 & yz-x^2\\xy-z^2 & yz-x^2 & zx-y^2\end{vmatrix}=\begin{vmatrix}r^2 & u^2 & u^2\\u^2 & r^2 & u^2\\u^2 & u^2 & r^2\end{vmatrix}$, then

a) $r^2=x+y+z$ b) $r^2=x^2+y^2+z^2$ c) $u^2=yz+zx+xy$ d) $u^2=xyz$

109. If a,b,c are non-zero real numbers such that $\begin{vmatrix}bc & ca & ab\\ca & ab & bc\\ab & bc & ca\end{vmatrix}=0$, then

a) $\frac{1}{a}+\frac{1}{b\omega}+\frac{1}{c\omega^2}=0$ b) $\frac{1}{a}+\frac{1}{b\omega^2}+\frac{1}{c\omega}=0$ c) $\frac{1}{a\omega}+\frac{1}{b\omega^2}+\frac{1}{c}=0$ d) None of these

110. If $f(x)=\begin{vmatrix}a & -1 & 0\\ax & a & -1\\ax^2 & ax & a\end{vmatrix}$, then $f(2x)-f(x)$ is divisible by

a) x b) a c) $2a+3x$ d) x^2

111. The values of $k\in R$ for which the system of equations $x+ky+3z=0, kx+2y+2z=0, 2x+3y+4z=0$ admits of non-trivial solution is

a) 2 b) 5/2 c) 3 d) 5/4

112. If $\Delta=\begin{vmatrix}-x & a & b\\b & -x & a\\a & b & -x\end{vmatrix}$, then a factor of Δ is

a) $a+b+x$ b) $x^2-(a-b)x+a^2+b^2+ab$

c) $x^2+(a+b)x+a^2+b^2-ab$ d) $a+b-x$

113. Which of the following has/have value equal to zero?

a) $\begin{vmatrix}8 & 2 & 7\\12 & 3 & 5\\16 & 4 & 3\end{vmatrix}$ b) $\begin{vmatrix}1/a & a^2 & bc\\1/b & b^2 & ac\\1/c & c^2 & ab\end{vmatrix}$

c) $\begin{vmatrix}a+b & 2a+b & 3a+b\\2a+b & 3a+b & 4a+b\\4a+b & 5a+b & 6a+b\end{vmatrix}$ d) $\begin{vmatrix}2 & 43 & 6\\7 & 35 & 4\\3 & 17 & 2\end{vmatrix}$

114. If $\phi(\alpha,\beta)=\begin{vmatrix}\cos\alpha & -\sin\alpha & 1\\\sin\alpha & \cos\alpha & 1\\\cos(\alpha+\beta) & -\sin(\alpha+\beta) & 1\end{vmatrix}$, then

a) $f(300,200)=f(400,200)$ b) $f(200,400)=f(200,600)$
c) $f(100,200)=f(200,200)$ d) None of these

115. The roots of the equation $\begin{vmatrix} {}^{x}C_r & {}^{n-1}C_r & {}^{n-1}C_{r-1} \\ {}^{x+1}C_r & {}^{n}C_r & {}^{n}C_{r-1} \\ {}^{x+2}C_r & {}^{n+1}C_r & {}^{n+1}C_{r-1} \end{vmatrix}=0$ are

a) $x=n$ b) $x=n+1$ c) $x=n-1$ d) $x=n-2$

116. $\Delta=\begin{vmatrix} 1 & 1+ac & 1+bc \\ 1 & 1+ad & 1+bd \\ 1 & 1+ae & 1+be \end{vmatrix}$ is independent of

a) a b) b c) c, d, e d) None of these

117. If $\Delta(x)=\begin{vmatrix} x^2+4x-3 & 2x+4 & 13 \\ 2x^2+5x-9 & 4x+5 & 26 \\ 8x^2-6x+1 & 16x-6 & 104 \end{vmatrix}=ax^3+bx^2+cx+d$, then

a) $a=3$ b) $b=0$ c) $c=0$ d) None of these

118. Let $f(n)=\begin{vmatrix} n & n+1 & n+2 \\ {}^{n}P_n & {}^{n+1}P_{n+1} & {}^{n+2}P_{n+2} \\ {}^{n}C_n & {}^{n+1}C_{n+1} & {}^{n+2}C_{n+2} \end{vmatrix}$ where the symbols have their usual meanings. Then $f(n)$ is divisible by

a) n^2+n+1 b) $(n+1)!$ c) $n!$ d) None of these

Assertion - Reasoning Type

This section contain(s) 0 questions numbered 119 to 118. Each question contains STATEMENT 1(Assertion) and STATEMENT 2(Reason). Each question has the 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

a) Statement 1 is True, Statement 2 is True; Statement 2 **is** correct explanation for Statement 1

b) Statement 1 is True, Statement 2 is True; Statement 2 **is not** correct explanation for Statement 1

c) Statement 1 is True, Statement 2 is False

d) Statement 1 is False, Statement 2 is True

119

Statement 1: If $bc+qr=ca+rp=ab+pq=-1$, then $\begin{vmatrix} ap & a & p \\ bq & b & q \\ cr & c & r \end{vmatrix}=0 \quad (abc, pqr \neq 0)$

Statement 2: If system of equations $a_1x+b_1y+c_1=0, a_2x+b_2y+c_2=0, a_3x+b_3y+c_3=0$ has non-trivial solutions, $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$

120 Consider the system of equation $x+y+z=6, x+2y+3z=10$ and $x+2y+\lambda z=\mu$

Statement 1: If the system has infinite number of solutions, then $\mu=10$

Statement 2: The determinant $\begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 1 & 2 & \mu \end{vmatrix}=0$ for $\mu=10$

121

Statement 1: If A, B and C are the angles of a triangle and

$$\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A+\sin^2 A & \sin B+\sin^2 B & \sin C+\sin^2 C \end{vmatrix}$$

$= 0$, then triangle may not be equilateral

Statement 2: If any two rows of a determinant are the same, then the value of that determinant is zero

122 Let x, y, z are three integers lying between 1 and 9 such that $x51, y41$ and $z31$ are three digit numbers

Statement 1: The value of the determinant $\begin{vmatrix} 5 & 4 & 3 \\ x51 & y41 & z31 \\ x & y & z \end{vmatrix}$ is zero

Statement 2: The value of a determinant is zero, if the entries in any two rows (or columns) of the determinant are correspondingly proportional

123

Statement 1: $\begin{vmatrix} \cos(\theta+\alpha) & \cos(\theta+\beta) & \cos(\theta+\gamma) \\ \sin(\theta+\alpha) & \sin(\theta+\beta) & \sin(\theta+\gamma) \\ \sin(\beta-\gamma) & \sin(\gamma-\alpha) & \sin(\alpha-\beta) \end{vmatrix}$ is independent of θ.

Statement 2: If $f(\theta) = c$, then $f(\theta)$ is independent of θ.

124

Statement 1: If a, b, c are even natural numbers, then $\Delta = \begin{vmatrix} a-1 & a & a+1 \\ b-1 & b & b+1 \\ c-1 & c & c+1 \end{vmatrix}$ is an even natural number.

Statement 2: Sum and product of two even natural numbers is also an even natural number.

125 Consider the determinant $f(x) = \begin{vmatrix} 0 & x^2-a & x^3-b \\ x^2+a & 0 & x^2+c \\ x^4+b & x-c & 0 \end{vmatrix}$

Statement 1: $f(x) = 0$ has one root $x = 0$

Statement 2: The value of skew-symmetric determinant of odd-order is always zero

126

Statement 1: If the system of equations $\lambda x + (b-a)y + (c-a)z = 0$, $(a-b)x + \lambda y + (c-b)z = 0$ and $(a-c)x + (b-c)y + \lambda z = 0$ has a non-trivial solution, then the value of λ is 0

Statement 2: The value of skew-symmetric matrix of order 3 is zero

127

Statement 1: $\Delta = \begin{vmatrix} my+nz & mq+nr & mb+nc \\ kz-mx & kr-mp & kc-ma \\ -nx-ky & -np-kq & -na-kb \end{vmatrix}$ is equal to 0

Statement 2: The value of skew-symmetric matrix of order 3 is zero

128 Consider the system of the equations $kx + y + z = 1$, $x + ky + z = k$ and $x + y + kz = k^2$

Statement 1: System of equations has infinite solutions when $k = 1$

Statement 2: If the determinant $\begin{vmatrix} 1 & 1 & 1 \\ k & k & 1 \\ k^2 & 1 & k \end{vmatrix} = 0$, then $k = -1$

129

Statement 1: If $\Delta(x) = \begin{vmatrix} f_1(x) & f_2(x) \\ g_1(x) & g_2(x) \end{vmatrix}$,

then $\Delta'(x) \neq \begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1'(x) & g_2'(x) \end{vmatrix}$

Statement 2: $\frac{d}{dx}\{f(x)g(x)\} \neq \frac{d}{dx}f(x)\frac{d}{dx}g(x)$

130 Consider the determinant $\Delta = \begin{vmatrix} a_1 + b_1x^2 & a_1x^2 + b_1 & c_1 \\ a_2 + b_2x^2 & a_2x^2 + b_2 & c_2 \\ a_3 + b_3x^2 & a_3x^2 + b_3 & c_3 \end{vmatrix} = 0$, where $a_i, b_i, c_i \in R(i = 1, 2, 3)$ and $x \in R$

Statement 1: The values of x satisfying $\Delta = 0$ are $x = 1, -1$

Statement 2: If $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$, then $\Delta = 0$

131

Statement 1: If $f(x) = \begin{vmatrix} (1+x)^{21} & (1+x)^{22} & (1+x)^{23} \\ (1+x)^{31} & (1+x)^{32} & (1+x)^{33} \\ (1+x)^{41} & (1+x)^{42} & (1+x)^{43} \end{vmatrix}$ then coefficient of x in $f(x)$ is zero.

Statement 2: If $F(x) = A_0 + A_1x + A_2x^2 + \ldots + A_nx^n$, then $A_1 = F'(0)$,where dash denotes the differential coefficient.

Matrix-Match Type

This section contain(s) 0 question(s). Each question contains Statements given in 2 columns which have to be matched. Statements (A, B, C, D) in **columns I** have to be matched with Statements (p, q, r, s) in **columns II.**

132.

Column-I	Column- II
(A) $\begin{vmatrix} 1/c & 1/c & -(a+b)/ \\ -(b+c)/a^2 & 1/a & 1/a \\ -b(b+c)/a^2c & (a+2b+c)/ac & -b(a+b)/ \end{vmatrix}$ is	(p) Independent of a
(B) $\begin{vmatrix} \sin a\cos b & \sin a\sin b & \cos a \\ \cos a\cos b & \cos a\sin b & -\sin a \\ -\sin a\sin b & \sin a\cos b & 0 \end{vmatrix}$ is	(q) Independent of b
(C) $\begin{vmatrix} \frac{1}{\sin a\cos b} & \frac{1}{\sin a\sin b} & \frac{1}{\cos a} \\ \frac{-\cos a}{\sin^2 a\cos b} & \frac{-\cos a}{\sin^2 a\sin b} & \frac{\sin a}{\cos^2 a} \\ \frac{\sin b}{\sin a\cos^2 b} & \frac{-\cos b}{\sin a\sin^2 b} & 0 \end{vmatrix}$ is	(r) Independent of c
(D) If a, b, and c are the sides of a triangle and A, B and C are the angles opposite to a, b, and c, respectively, then $\Delta = \begin{vmatrix} a^2 & b\sin A & c\sin A \\ b\sin A & 1 & \cos A \\ c\sin A & \cos A & 1 \end{vmatrix}$	(s) Dependent on a, b

CODES :

	A	B	C	D
a)	p	r	r	q
b)	s	p	r	s
c)	s	p	q	s

d)	p,q,r	q	s	p, q, r

133.

	Column-I		Column- II
(A)	Coefficient of x in $f(x)=\begin{vmatrix} x & (a+\sin x)^3 & \cos x \\ 1 & \log(1+x) & 2 \\ x^2 & 1+x^2 & 0 \end{vmatrix}$	(p)	10
(B)	Value of $\begin{vmatrix} 1 & 3\cos\theta & 1 \\ \sin\theta & 1 & 3\cos\theta \\ 1 & \sin\theta & 1 \end{vmatrix}$ is	(q)	0
(C)	If a, b, c are in A.P. and $f(x)=\begin{vmatrix} x+a & x^2+1 & 1 \\ x+b & 2x^2-1 & 1 \\ x+c & 3x^2-2 & 1 \end{vmatrix}$	(r)	-12
(D)	If $\begin{vmatrix} x & 2 & x \\ 1 & x & 6 \\ x & x & x+1 \end{vmatrix} = a_4x^4+a_3x^3+a_2x^2+a_1x+a_0$, then a_0 is	(s)	-2

CODES :

	A	B	C	D
a)	r	s	r	r
b)	p	q	p	p
c)	s	p	s	s
d)	q	r	q	q

134.

	Column-I		Column- II
(A)	The value of the determinant $\begin{vmatrix} x+2 & x+3 & x+5 \\ x+4 & x+6 & x+9 \\ x+8 & x+11 & x+15 \end{vmatrix}$ is	(p)	1
(B)	If one of the roots of the equation $\begin{vmatrix} 7 & 6 & x^2-13 \\ 2 & x^2-13 & 2 \\ x^2-13 & 3 & 7 \end{vmatrix}=0$ is $x+2$, then sum of the all other five roots is	(q)	-6
(C)	The value of $\begin{vmatrix} \sqrt{6} & 2i & 3+\sqrt{6} \\ \sqrt{12} & \sqrt{3}+\sqrt{8}i & 3\sqrt{2}+\sqrt{6}i \\ \sqrt{18} & \sqrt{2}+\sqrt{12}\,i & \sqrt{27}+2i \end{vmatrix}$ is	(r)	2
(D)	If $f(\theta)=\begin{vmatrix} \cos^2\theta & \cos\theta\sin\theta & -\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & \cos\theta \\ \sin\theta & -\cos\theta & 0 \end{vmatrix}$ then $f(\pi/3)$	(s)	-2

CODES :

	A	B	C	D
a)	r	s	s	q
b)	s	r	q,r	p
c)	p	s	q	r
d)	q	p	r	s

135. Match the following elements of $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 4 & 2 \\ 3 & -4 & 6 \end{bmatrix}$ with their cofactors and choose the correct answer.

Column-I	Column- II
(A) -1	(1) -2
(B) 1	(2) 32
(C) 3	(3) 4
(D) 6	(4) 6
	(5) -6

CODES :

	A	B	C	D
a)	2	4	1	3
b)	2	4	3	1
c)	4	2	1	3
d)	4	1	2	3

Linked Comprehension Type

This section contain(s) 16 paragraph(s) and based upon each paragraph, multiple choice questions have to be answered. Each question has atleast 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 136 to -136

Let p be an odd prime number and T_p be the following set of

2×2 matrices

$$T_p = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix}; a, b, c \in \{0,1,2,..., p-1\} \right\}$$

136. The number of A inT_p such that A is either symmetric or skew-symmetric or both, and $\det(A)$ is divisible by p is

a) $(p-1)^2$ b) $2(p-1)$ c) $(p-1)^2+1$ d) $2p-1$

Paragraph for Question Nos. 137 to - 137

Let $\Delta \neq 0$ and Δ^c denotes the determinant of cofactors, then $\Delta^c = \Delta^{n-1}$, where $n(>0)$ is the order of Δ.
on the basis of above information, answer the following questions.

137.
If a, b, c are the roots of the equation $x^3 - px^2 + r = 0$, then the value of $\begin{vmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ca-b^2 \end{vmatrix}$ is

a) p^2 b) p^4 c) p^6 d) p^9

Paragraph for Question Nos. 138 to - 138

$f(x) = \begin{vmatrix} x+c_1 & x+a & x+a \\ x+b & x+c_2 & x+a \\ x+b & x+b & x+c_3 \end{vmatrix}$ and $g(x) = (c_1 - x)(c_2 - x)(c_3 - x)$

138. Coefficient of x in $f(x)$ is

a) $\dfrac{g(a) - f(b)}{b-a}$ b) $\dfrac{g(-a) - g(-b)}{b-a}$ c) $\dfrac{g(a) - g(b)}{b-a}$ d) None of these

Paragraph for Question Nos. 139 to - 139

Consider the function $f(x) = \begin{vmatrix} a^2+x & ab & ac \\ ab & b^2+x & bc \\ ac & bc & c^2+x \end{vmatrix}$

139. Which of the following is true?

a) $f(x) = 0$ and $f'(x) = 0$ have one positive common root
b) $f(x) = 0$ and $f'(x) = 0$ have one negative common root
c) $f(x) = 0$ and $f'(x) = 0$ have no common root
d) None of these

Paragraph for Question Nos. 140 to - 140

Given that the system of equations $x = cy + bz, y = az + cx, z = bx + ay$ has non-zero solutions and at least one of the a, b, c is a proper fraction

140. $a^2 + b^2 + c^2$ is

a) >2 b) >3 c) <3 d) <2

Paragraph for Question Nos. 141 to - 141

Consider the system of equations
$x + y + z = 6$
$x + 2y + 3z = 10$
$x + 2y + \lambda z = \mu$

141. The system has unique solution if

a) $\lambda \neq 3$ b) $\lambda = 3, \mu = 10$ c) $\lambda = 3, \mu \neq 10$ d) None of these

Paragraph for Question Nos. 142 to - 142

Let α, β be the roots of the equation $ax^2 + bx + c = 0$. Let $S_n = \alpha^n + \beta^n$

For $n \geq 1$ and $\Delta = \begin{vmatrix} 3 & 1+S_1 & 1+S_2 \\ 1+S_1 & 1+S_2 & 1+S_3 \\ 1+S_2 & 1+S_3 & 1+S_4 \end{vmatrix}$

142. If $\Delta < 0$, then the equation $ax^2 + bx + c = 0$ has

a) Positive real roots
b) Negative real roots
c) Equal roots
d) Imaginary roots

Paragraph for Question Nos. 143 to - 143

Let $\Delta = \begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix}$ and the equation $px^3 + qx^2 + rx + s = 0$ has roots a, b, c where $a, b, c \in R^+$

143. The value of Δ is

a) r^2/p^2
b) r^3/p^3
c) $-s/p$
d) None of these

Paragraph for Question Nos. 144 to - 144

Consider the polynomial function $f(x) = \begin{vmatrix} (1+x)^a & (1+2x)^b & 1 \\ 1 & (1+x)^a & (1+2x)^b \\ (1+2x)^b & 1 & (1+x)^a \end{vmatrix}$, a, b, being positive integers

144. The constant term in $f(x)$ is

a) 2
b) 1
c) -1
d) 0

Paragraph for Question Nos. 145 to - 145

If $x > m, y > n, z > r (x, y, z > 0)$ such that $\begin{vmatrix} x & n & r \\ m & y & r \\ m & n & z \end{vmatrix} = 0$

145. The value of $\frac{x}{x-m} + \frac{y}{y-n} + \frac{z}{z-r}$ is

a) 1
b) -1
c) 2
d) -2

Paragraph for Question Nos. 146 to - 146

Suppose $f(x)$ is a function satisfying the following conditions:

1. $f(0) = 2, f(1) = 1,$
2. f has a minimum value at $x = 5/2$
3. For all $x, f'(x) = \begin{vmatrix} 2ax & 2ax-1 & 2ax+b+1 \\ b & b+1 & -1 \\ 2(ax+b) & 2ax+2b+1 & 2ax+b \end{vmatrix}$

146. The value of $f(2)$ is

a) 1/4 b) 1/2 c) -1 d) 3

Integer Answer Type

147. If $\Delta=\begin{vmatrix} 1 & 3\cos\theta & 1 \\ \sin\theta & 1 & 3\cos\theta \\ 1 & \sin\theta & 1 \end{vmatrix}$, then the value of $(\Delta_{\text{max}})/2$ is

148. If $\begin{vmatrix} x^n & x^{n+2} & x^{n+4} \\ y^n & y^{n+2} & y^{n+4} \\ z^n & z^{n+2} & z^{n+4} \end{vmatrix} = \left(\frac{1}{y^2}-\frac{1}{x^2}\right)\left(\frac{1}{z^2}-\frac{1}{y^2}\right)\left(\frac{1}{x^2}-\frac{1}{z^2}\right)$ then $-n$ is

149. Absolute value of sum of roots of the equation $\begin{vmatrix} x+2 & 2x+3 & 3x+4 \\ 2x+3 & 3x+4 & 4x+5 \\ 3x+5 & 5x+8 & 10x+17 \end{vmatrix} = 0$ is

150. The value of $|\alpha|$ for which the system of equation

$\alpha x + y + z = \alpha - 1$

$x + \alpha y + z = \alpha - 1$

$x + y + \alpha z = \alpha - 1$

Has no solution, is

151. If $a_1, a_2, a_3, 5, 4, a_6, a_7, a_8, a_9$ are in H.P., and $D = \begin{vmatrix} a_1 & a_2 & a_3 \\ 5 & 4 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ then the value of [D]is (where [ ] represents the greatest integer function)

152. Sum of values of p for which, the equations: $x+y+z=1; x+2y+4z=p$ and $x+4y+10z=p^2$ have a solution is

153. Let α, β, γ are the real roots of the equation $x^3+ax^2+bx+c=0 (a, b, c \in R$ and $a \neq 0)$. If the system of equations (in u, v and w) given by

$\alpha u + \beta v + \gamma w = 0$

$\beta u + \gamma v + \alpha w = 0$

$\gamma u + \alpha v + \beta w = 0$

has non-trivial solutions, then the value of a^2/b is

154. Let $D_1 = \begin{vmatrix} a & b & a+b \\ c & d & c+d \\ a & b & a-b \end{vmatrix}$ and $D_2 = \begin{vmatrix} a & c & a+c \\ b & d & b+d \\ a & c & a+b+c \end{vmatrix}$ then the value of $\left|\frac{D_1}{D_2}\right|$ is where $b \neq 0$ and $ad \neq bc$,

155. If $(1+ax+bx^2)^4 = a_0 + a_1 x + a_2 x^2 + \cdots + a_8 x^8$, where $a, b, a_0, a_1, \dots, a_8 \in R$ such that $a_0 + a_1 + a_2 \neq 0$ and $\begin{vmatrix} a_0 & a_1 & a_2 \\ a_1 & a_2 & a_0 \\ a_2 & a_0 & a_1 \end{vmatrix} = 0$ then the value of $5\frac{a}{b}$ is

156. If $a_1, a_2, a_3, \dots, a_{12}$ are in A.P. and $\Delta_1 = \begin{vmatrix} a_1 a_5 & a_1 & a_2 \\ a_2 a_6 & a_2 & a_3 \\ a_3 a_7 & a_3 & a_4 \end{vmatrix}$ $\Delta_3 = \begin{vmatrix} a_2 a_{10} & a_2 & a_3 \\ a_3 a_{11} & a_3 & a_4 \\ a_3 a_{12} & a_4 & a_5 \end{vmatrix}$ then $\Delta_2 : \Delta_2 =$

157. The value of $\begin{vmatrix} 2x_1 y_1 & x_1 y_2 + x_2 y_1 & x_1 y_3 + x_3 y_1 \\ x_1 y_2 + x_2 y_1 & 2x_2 y_2 & x_2 y_3 + x_3 y_2 \\ x_1 y_3 + x_3 y_1 & x_2 y_3 + x_3 y_2 & 2x_3 y_3 \end{vmatrix}$ is

158. If $\begin{vmatrix} (\beta+\gamma-\alpha-\delta)^4 & (\beta+\gamma-\alpha-\delta)^2 & 1 \\ (\gamma+\alpha-\beta-\delta)^4 & (\gamma+\alpha-\beta-\delta)^2 & 1 \\ (\alpha+\beta-\gamma-\delta)^4 & (\alpha+\beta-\gamma-\delta)^2 & 1 \end{vmatrix} = -k(\alpha-\beta)(\alpha-\gamma)(\alpha-\delta)(\beta-\gamma)(\beta-\delta)(\gamma-\delta)$, then the value of $(k)^{1/2}$ is

159. Three distinct points $P(3u^2, 2u^3); Q(3v^2, 2v^2)$ and $R(3w^2, 2w^2)$ are collinear then $uv+vw+wu$ is equal to

160. Given $A = \begin{vmatrix} a & b & 2c \\ d & e & 2f \\ l & m & 2n \end{vmatrix}, B = \begin{vmatrix} f & 2d & e \\ 2n & 4l & 2m \\ c & 2a & b \end{vmatrix}$, then the value of B/A is

161. If $\begin{vmatrix} x & x+y & x+y+z \\ 2x & 3x+2y & 4x+3y+2z \\ 3x & 6x+3y & 10x+6y+3z \end{vmatrix} = 64$, then the real value of x is

: ANSWER KEY :

1)	d	2)	b	3)	a	4)	c	13)	0	14)	2	15)	4
5)	b	6)	c	7)	b	8)	b						
9)	c	10)	b	11)	a	12)	d						
13)	b	14)	d	15)	b	16)	b						
17)	b	18)	c	19)	a	20)	b						
21)	c	22)	d	23)	a	24)	d						
25)	c	26)	c	27)	b	28)	b						
29)	a	30)	b	31)	b	32)	b						
33)	a	34)	b	35)	c	36)	c						
37)	b	38)	c	39)	b	40)	d						
41)	d	42)	a	43)	c	44)	d						
45)	b	46)	b	47)	b	48)	c						
49)	b	50)	b	51)	a	52)	a						
53)	b	54)	b	55)	a	56)	a						
57)	b	58)	b	59)	b	60)	a						
61)	d	62)	a	63)	c	64)	c						
65)	d	66)	d	67)	d	68)	b						
69)	a	70)	d	71)	a	72)	a						
73)	b	74)	a	75)	b	76)	d						
77)	d	78)	c	79)	a	80)	a						
81)	d	82)	b	83)	b	84)	a						
85)	c	86)	c	87)	a	88)	c						
89)	b	90)	c	91)	b	92)	a						
93)	d	1)	b,d	2)	b,d	3)							
	a,b	4)	d										
5)	a,b	6)	a,c	7)	d	8)							
	a,b,c,d												
9)	a,c	10)	a,b	11)	b,c	12)							
	a,b												
13)	b,c	14)	a,c	15)	b,c	16)							
	a,b,c												
17)	a,b,c	18)	a,b	19)	c,d	20)							
	a,b,c												
21)	a,c	22)	a,c	23)	a,b,c	24)							
	b,c												
25)	a,c	1)	a	2)	b	3)	a						
	4)	d											
5)	a	6)	d	7)	a	8)	a						
9)	a	10)	b	11)	d	12)	b						
13)	a	1)	d	2)	c	3)	b						
	4)	c											
1)	d	2)	c	3)	c	4)	d						
5)	c	6)	a	7)	d	8)	a						
9)	d	10)	c	11)	b	1)	5						
	2)	4	3)	4	4)	2							
5)	2	6)	3	7)	3	8)	2						
9)	8	10)	1	11)	0	12)	8						

: HINTS AND SOLUTIONS :

1 **(d)**

$$\sum_{k=1}^{n} D_k = 56$$

$$\Rightarrow \begin{vmatrix} \sum_{k=1}^{n} 1 & n & n \\ \sum_{k=1}^{n} 2k & n^2+n+1 & n^2+n \\ \sum_{k=1}^{n} (2k-1) & n^2 & n^2+n+1 \end{vmatrix} = 56$$

$$\Rightarrow \begin{vmatrix} n & n & n \\ n(n+1) & n^2+n+1 & n^2+n \\ n^2 & n^2 & n^2+n+1 \end{vmatrix} = 56$$

Applying $C_3 \to C_3 - C_1$ and $C_2 \to C_2 - C_1$, we get

$$\begin{vmatrix} n & 0 & 0 \\ n(n+1) & 1 & 0 \\ n^2 & 0 & n+1 \end{vmatrix} = 56 \Rightarrow n(n+1) = 56 \Rightarrow n = 7$$

2 **(b)**

$B_2 = a_1c_3 - a_3c_1, C_2 = -(a_1b_3 - a_3b_1)$

$B_3 = -(a_1c_2 - a_2c_1), C_3 = a_1b_2 - a_2b_1$

$$\therefore \begin{vmatrix} B_2 & C_2 \\ B_3 & C_2 \end{vmatrix} = \begin{vmatrix} a_1c_3 - a_3c_1 & -a_1b_3 + a_3b_1 \\ -a_1c_2 + a_2c_1 & a_1b_2 - a_2b_1 \end{vmatrix}$$

$$= \begin{vmatrix} a_1c_3 & -a_1b_3 \\ -a_1c_2 & a_1b_2 \end{vmatrix} + \begin{vmatrix} a_1c_3 & a_3b_1 \\ -a_1c_2 & -a_2b_1 \end{vmatrix} + \begin{vmatrix} -a_3c_1 & -a_1b_3 \\ a_2c_1 & a_1b_2 \end{vmatrix} + \begin{vmatrix} -a_3c_1 & a_3b_1 \\ a_2c_1 & -a_2b_1 \end{vmatrix}$$

$$= a_1^2 \begin{vmatrix} c_3 & -b_3 \\ -c_2 & b_2 \end{vmatrix} + a_1b_1 \begin{vmatrix} c_3 & a_3 \\ -c_2 & -a_2 \end{vmatrix} + a_1c_1 \begin{vmatrix} -a_3 & -b_3 \\ a_2 & b_2 \end{vmatrix} + b_1c_1 \begin{vmatrix} -a_3 & a_3 \\ a_2 & -a_2 \end{vmatrix}$$

$$= a_1\{a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)\}$$

$$= a_1 \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1\Delta$$

3 **(a)**

Applying $C_1 \to C_1 - 2\sin x C_3$ and $C_2 \to C_2 + 2\cos x C_3$, we get

$$f(x) = \begin{vmatrix} 2 & 0 & -\sin x \\ 0 & 2 & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix}$$

$= 2\cos^2 x + 2\sin^2 x = 2$

$\therefore f'(x) = 0$

$$\therefore \int_0^{\pi/2} [f(x) + f'(x)] = dx = \int_0^{\pi/2} 2dx = \pi$$

4 **(c)**

Using $C_1 \to C_1 + C_2 + C_3$,

$$\Delta = \begin{vmatrix} \sin x + 2\cos x & \cos x & \cos x \\ \sin x + 2\cos x & \sin x & \cos x \\ \sin x + 2\cos x & \cos x & \sin x \end{vmatrix}$$

$$= (\sin x + 2\cos x) \begin{vmatrix} 1 & \cos x & \cos x \\ 1 & \sin x & \cos x \\ 1 & \cos x & \sin x \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\Delta = (\sin x + 2\cos x) \begin{vmatrix} 1 & \cos x & \cos x \\ 0 & \sin x - \cos x & 0 \\ 0 & 0 & \sin x - \cos x \end{vmatrix}$$

$= (\sin x + 2\cos x)(\sin x - \cos x)^2 +$

Thus, $\Delta = 0 \Rightarrow \tan x = -2$ or $\tan x = 1$

As $-\pi/4 \le x \le \pi/4$, we get $-1 \le \tan x \le 1$

$\therefore \tan x = 1 \Rightarrow x = \pi/4$

5 **(b)**

The degree of the determinant is $n + (n+2) + (n+3) = 3n + 5$ and the degree of the expression on R.H.S. is 2

$\therefore 3n + 5 = 2 \Rightarrow n = -1$

6 **(c)**

$a = x/(y-z) \Rightarrow x - ay + az = 0 \quad (1)$

$b = y/(z-x) \Rightarrow bx + y - bz = 0 \quad (2)$

$c = z/(x-y) \Rightarrow -cx + xy + z = 0 \quad (3)$

Since x, y, z are not all zero, the above system has a non-trivial solution. So,

$$\Delta = \begin{vmatrix} 1 & -a & a \\ b & 1 & -b \\ -c & c & 1 \end{vmatrix} = 0$$

$\therefore 1 + ab + bc + ca = 0$

7 **(b)**

$$\begin{vmatrix} 0 & 1+\omega+\omega^2 & 0 \\ 1-i & -1 & \omega^2 - 1 \\ -i & -1+\omega-1 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 0 \\ 1-i & -1 & \omega^2-1 \\ -1 & -i+\omega-1 & -1 \end{vmatrix} \quad [\because 1 + \omega + \omega^2 = 0]$$

(Operating $R_1 \to R_1 - R_2 + R_3$)

8 **(b)**

We have,

$$\begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ c+a & a+b & b+c \end{vmatrix} = k \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 2(a+b+c) & c+a & a+b \\ 2(a+b+c) & b+c & c+a \\ 2(a+b+c) & a+b & b+c \end{vmatrix} = k\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

[Applying $C_1 \to C_1 + (C_2 + C_3)$ on L.H.S.]

$$\Rightarrow 2\begin{vmatrix} a+b+c & -b & -c \\ a+b+c & -a & -b \\ a+b+c & -c & -a \end{vmatrix} = k\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

[Applying $C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$ on L.H.S.]

$$\Rightarrow \begin{vmatrix} a & -b & -c \\ c & -a & -b \\ b & -c & -a \end{vmatrix} = k\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

[Applying $C_1 \to C_1 + C_2 + C_3$ on L.H.S.]

$$\Rightarrow 2\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} = k\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

$\therefore k = 2$

9 **(c)**

$$f'(x) = \begin{vmatrix} -\sin x & 1 & 0 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix} + \begin{vmatrix} \cos x & x & 1 \\ 2\cos x & 2x & 2 \\ \tan x & x & 1 \end{vmatrix} + \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \sec^2 x & 1 & 0 \end{vmatrix}$$

$$\Rightarrow f'(0) = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 1 \\ 2 & 0 & 2 \\ 0 & 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{vmatrix} = 0$$

Now, $\lim_{x\to 0} \frac{f(x)}{x} = \lim_{x\to 0} f'(x)$ [as $f(0) = 0$]

$= f'(0) = 0$

10 **(b)**

$$\text{Let, } \Delta = \begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$$

Expanding along first row, we have

$1[\cos px \sin(p+d)x - \cos(p+d)x \sin px]$

$-a[\cos(p-d)x \sin(p+d)\, x - \cos(p+d)\, x \sin(p-d)\, x]$

$+a^2[\cos(p-d)\, x \sin px - \cos px \sin(p-d)x]$

$= \sin dx - a\sin 2dx + a^2 \sin dx$

Which is independent of p

11 **(a)**

Applying $R_1 \to R_1 + R_3 - 2R_2$, we get

$$\Delta = \begin{vmatrix} 0 & 0 & 0 & x+z-2y \\ 4 & 5 & 6 & y \\ 5 & 6 & 7 & z \\ x & y & z & 0 \end{vmatrix}$$

$= -(x+z-2y)\begin{vmatrix} 4 & 5 & 6 \\ 5 & 6 & 7 \\ x & y & z \end{vmatrix}$ [Expanding along R_1]

$$= -(x+z-2y)\begin{vmatrix} 0 & -1 & 6 \\ 0 & -1 & 7 \\ x-2y+z & y-z & z \end{vmatrix}$$

[Applying $C_1 \to C_1 + C_3 - 2C_2$ and $C_2 \to C_2 - C_3$]

$= -(x+z-2y)^2 \begin{vmatrix} -1 & 6 \\ -1 & 7 \end{vmatrix}$

$= (x-2y+z)^2$

Hence $\Delta = 0 \Rightarrow x, y, z$ are in A.P.

12 **(d)**

$$\text{Let, } \Delta = \begin{vmatrix} y^2 & -xy & x^2 \\ a & b & c \\ a' & b' & c' \end{vmatrix}$$

Then,

$\Delta = \frac{1}{xy}\begin{vmatrix} xy^2 & -xy & x^2y \\ ax & b & cy \\ a'x & b' & c'y \end{vmatrix}$ [Applying $C_1 \to xC_1, C_3 \to yC_3$]

$$= \frac{1}{xy}\begin{vmatrix} 0 & -xy & 0 \\ ax+by & b & bx+cy \\ a'x+b'y & b' & b'x+c'y \end{vmatrix}$$

[Applying $C_1 \to C_1 + yC_2, C_3 \to C_3 + xC_2$]

$= \frac{1}{xy} xy \begin{vmatrix} ax+by & bx+cy \\ a'x+b'y & b'x+c'y \end{vmatrix}$ [Expanding along R_1]

$= \begin{vmatrix} ax+by & bx+cy \\ a'x+b'y & b'x+c'y \end{vmatrix}$

13 **(b)**

$$z = \begin{vmatrix} -5 & 3+4i & 5-7i \\ 3-4i & 6 & 8+7i \\ 5+7i & 8-7i & 9 \end{vmatrix}$$

$$\Rightarrow \bar{z} = \begin{vmatrix} -5 & 3+4i & 5+7i \\ 3+4i & 6 & 8-7i \\ 5-7i & 8+7i & 9 \end{vmatrix}$$

$$= \begin{vmatrix} -5 & 3+4i & 5-7i \\ 3-4i & 6 & 8+7i \\ 5+7i & 8-7i & 9 \end{vmatrix} = z$$

(Taking transpose)

$\Rightarrow z$ is purely real

14 **(d)**

Applying $R_1 \to aR_1, R_2 \to bR_2$ and $R_3 \to cR_3$, we get

$$\Delta = \frac{1}{abc}\begin{vmatrix} ab^2c^2 & abc & ab+ac \\ a^2bc^2 & abc & bc+ab \\ a^2b^2c & abc & ac+bc \end{vmatrix}$$

$$= \frac{a^2b^2c^2}{abc}\begin{vmatrix} bc & 1 & ab+ac \\ ac & 1 & bc+ab \\ ab & 1 & ac+bc \end{vmatrix}$$

Applying $C_3 \to C_3 + C_1$ and taking $(bc+ca+ab)$ common, we get

$$\Delta = abc(bc+ca+ab)\begin{vmatrix} bc & 1 & 1 \\ ac & 1 & 1 \\ ab & 1 & 1 \end{vmatrix} = 0$$

[$\because C_2$ and C_3 are identical]

15 **(b)**

In each determinant applying $R_1 \to R_1 + R_2 + R_3$ and then taking out $(x+9)$ common, we get

$x + 9 = 0 \Rightarrow x = -9$

16 **(b)**

$$\Delta=\begin{vmatrix}x_1 & x_2 & x_3\\ y_1 & y_2 & y_3\\ z_1 & z_2 & z_3\end{vmatrix}=\begin{vmatrix}x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\\ x_3 & y_3 & z_3\end{vmatrix}$$

$$\Delta^2=\begin{vmatrix}x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\\ x_3 & y_3 & z_3\end{vmatrix}\begin{vmatrix}x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\\ x_3 & y_3 & z_3\end{vmatrix}$$

$$=\begin{vmatrix}x_1^2+y_1^2+z_1^2 & x_1x_2+y_1y_2+z_1z_2 & x_1x\\ x_1x_2+y_2y_1+z_2z_1 & x_2^2+y_2^2+z_2^2 & x_2x\\ x_3x_1+y_3y_1+z_3z_1 & x_2x_3+y_2y_3+z_2z_3 & \end{vmatrix}$$

$$=\begin{vmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{vmatrix}=1\Rightarrow\Delta=\pm1$$

17 **(b)**

$$\Delta=\begin{vmatrix}{}^nC_{r-1} & {}^nC_r & (r+1)\,{}^{n+2}C_{r+1}\\ {}^nC_r & {}^nC_{r+1} & (r+2)\,{}^{n+2}C_{r+2}\\ {}^nC_{r+1} & {}^nC_{r+2} & (r+3)\,{}^{n+2}C_{r+3}\end{vmatrix}$$

Applying $C_1\to C_1+C_2$ and using ${}^nC_r=\frac{n}{r}\,{}^{n-1}C_{r-1}$ in C_3, we get

$$\Delta=\begin{vmatrix}{}^{n+1}C_r & {}^nC_r & (n+2)\,{}^{n+1}C_r\\ {}^{n+1}C_{r+1} & {}^nC_{r+1} & (n+2)\,{}^{n+1}C_{r+1}\\ {}^{n+1}C_{r+2} & {}^nC_{r+2} & (n+2)\,{}^{n+1}C_{r+2}\end{vmatrix}$$

$$=(n+2)\begin{vmatrix}{}^{n+1}C_r & {}^nC_r & {}^{n+1}C_r\\ {}^{n+1}C_{r+1} & {}^nC_{r+1} & {}^{n+1}C_{r+1}\\ {}^{n+1}C_{r+2} & {}^nC_{r+2} & {}^{n+1}C_{r+2}\end{vmatrix}$$

$=0$ (as C_1 and C_3 are identical)

18 **(c)**

Operation $C_1\longrightarrow C_1+C_2+C_3$ gives $(\alpha\beta+\beta\gamma+\gamma\alpha$

$$\begin{vmatrix}1 & \beta\gamma & \gamma\alpha\\ 1 & \gamma\alpha & \alpha\beta\\ 1 & \alpha\beta & \beta\gamma\end{vmatrix}$$

From the given equation, $\alpha\beta+\beta\gamma+\gamma\alpha=0$. So the value of determinant is 0

19 **(a)**

$$\Delta=\begin{vmatrix}a_1+b_1w & a_1w^2+b_1 & c_1+b_1\overline{w}\\ a_2+b_2w & a_2w^2+b_2 & c_2+b_2\overline{w}\\ a_3+b_3w & a_3w^2+b_3 & c_3+b_3\overline{w}\end{vmatrix}$$

Operating $C_2\to wC_2$, we have

$$\Delta=\frac{1}{w}\begin{vmatrix}a_1+b_1w & a_1w^3+b_1w & c_1+b_1\overline{w}\\ a_2+b_2w & a_2w^3+b_2w & c_2+b_2\overline{w}\\ a_3+b_3w & a_3w^3+b_3w & c_3+b_3\overline{w}\end{vmatrix}$$

$$=\frac{1}{w}\begin{vmatrix}a_1+b_1w & a_1+b_1w & c_1+b_1\overline{w}\\ a_2+b_2w & a_2+b_2w & c_2+b_2\overline{w}\\ a_3+b_3w & a_3+b_3w & c_3+b_3\overline{w}\end{vmatrix}\quad(\because\omega^3=1)$$

$=0$

20 **(b)**

Since x, y, z are in A.P., therefore, $x+z-2y=0$.

Now,

$$\begin{vmatrix}a+2 & a+3 & a+2x\\ a+3 & a+4 & a+2y\\ a+4 & a+5 & a+2z\end{vmatrix}=\begin{vmatrix}0 & 0 & 2(x+z-2y)\\ a+3 & a+4 & a+2y\\ a+4 & a+5 & a+2z\end{vmatrix}$$

[Applying $R_1\to R_1+R_3-2R_2$]

$$\begin{vmatrix}0 & 0 & 0\\ a+3 & a+4 & a+2y\\ a+4 & a+5 & a+2z\end{vmatrix}\quad[\because x+z-2y=0]$$

$=0$

21 **(c)**

$\because\ |A^3|=|A|^3=125$

$\Rightarrow\begin{bmatrix}\alpha & 2\\ 2 & \alpha\end{bmatrix}=5$

$\Rightarrow\alpha^2-4=5\quad\Rightarrow\quad\alpha=\pm3$

22 **(d)**

Applying $R_1\to R_1-(R_2+R_3)$, we get

$$D=\begin{vmatrix}0 & -2y & -2x\\ x & y+z & x\\ y & y & z+x\end{vmatrix}$$

$$=2\begin{vmatrix}0 & -y & -x\\ x & y+z & x\\ y & y & z+x\end{vmatrix}$$

$$=2\begin{vmatrix}0 & -y & -x\\ x & z & 0\\ y & 0 & z\end{vmatrix}$$ ($R_2\to R_2+R_1$ and $R_3\to R_3+R_1$)

$=4xyz$

23 **(a)**

$$\begin{vmatrix}x & m & n & 1\\ a & x & n & 1\\ a & b & x & 1\\ a & b & c & 1\end{vmatrix}=0\quad[R_1\to R_1-R_2, R_2\to R_2-R_3, R_3\to R_3-R_4]$$

$$\Rightarrow\begin{vmatrix}x-a & m-x & 0 & 0\\ 0 & x-b & n-x & 0\\ 0 & 0 & x-c & a\\ a & b & c & 1\end{vmatrix}=0$$

$$\Rightarrow\begin{vmatrix}x-a & m-x & 0\\ 0 & x-b & n-x\\ 0 & 0 & x-c\end{vmatrix}=0$$

$$\Rightarrow(x-a)\begin{vmatrix}(x-b) & n-x\\ 0 & (x-c)\end{vmatrix}=0$$

$\Rightarrow(x-a)(x-b)(x-c)=0\ \Rightarrow$ roots are independent of m, n

24 **(d)**

$$\begin{vmatrix}a & b & c\\ b & c & a\\ c & a & b\end{vmatrix}=-(a^3+b^3+c^3-3abc)$$

$=-(a+b+c)(a+b\omega^2+c\omega)(a+b\omega+c\omega^2)$

(where ω is cube roots of unity)

$=-f(\alpha)f(\beta)f(\gamma)\quad[\because\alpha=1,\beta=\omega,\gamma=\omega^2]$

25 **(c)**

Here $a>0$ and $4b^2-4ac<0$, i.e., $ac-b^2>0$

$\therefore ax^2+2bx+c>0, \forall x\in R$

Now,

$$\Delta=\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ 0 & 0 & -(ax^2+2bx+c) \end{vmatrix}$$

[Operating $R_3 \to R_3 - xR_1 - R_2$]

$= -(ax^2+2bx+c)(ac-b^2)$

$= -(+\text{ve})(+\text{ve}) = -\text{ve}$

26 **(c)**

Operating $C_1 \to C_1 + C_2 + C_3$, we get

$$f(x)=\begin{vmatrix} 1+2x+(a^2+b^2+c^2)x & (1+b^2)x & (1+c^2)x \\ 1+2x+(a^2+b^2+c^2)x & 1+b^2x & (1+c^2)x \\ 1+2x+(a^2+b^2+c^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$$=\begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix} \quad [\because a^2+b^2+c^2=-2]$$

$$=\begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 0 & 1-x & 0 \\ 0 & 0 & 1+x \end{vmatrix}$$

[Operating $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$]

$= (1)[(1-x)^2 - 0]$

$= (1-x)^2$

Which is a polynomial of degree 2

27 **(b)**

For non-trivial solution

$$\begin{vmatrix} a-1 & -1 & -1 \\ 1 & -(b-1) & 1 \\ 1 & 1 & -(c-1) \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a-1 & -1 & 0 \\ 1 & -(b-1) & b \\ 1 & 1 & -c \end{vmatrix} = 0$$

$\Rightarrow (a-1)(bc-c-b) + 1(-c-b) = 0$

$\Rightarrow abc - ac - ab - bc + b + c - c - b = 0$

$\Rightarrow ab + bc + ac = abc$

28 **(b)**

Applying $C_1 \to aC_1$ and then $C_1 \to C_1 + bC_2 + cC_3$, and taking $(a^2+b^2+c^2)$ common from C_1, we get

$$\Delta= \frac{(a^2+b^2+c^2)}{a}\begin{vmatrix} 1 & b-c & c+b \\ 1 & b & c-a \\ 1 & b+a & c \end{vmatrix}$$

$$= \frac{(a^2+b^2+c^2)}{a}\begin{vmatrix} 1 & b-c & c+b \\ 0 & c & -a-b \\ 0 & a+c & -b \end{vmatrix}$$

$(R_2 \to R_2 - R_1, R_3 \to R_3 - R_1)$

$= \frac{(a^2+b^2+c^2)}{a}(-bc + a^2 + ab + ac + bc)$

(expanding along C_1)

$= (a^2+b^2+c^2)(a+b+c)$

Hence, $\Delta = 0 \Rightarrow a+b+c = 0$

Therefore, line $ax+by+c=0$ passes through the fixed point (1, 1)

29 **(a)**

Determinant formed by the cofactors of

$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is

$$\begin{vmatrix} bc-a^2 & ac-b^2 & ab-c^2 \\ ac-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ac-b^2 \end{vmatrix}$$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$$

30 **(b)**

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\Delta= \begin{vmatrix} x+2a & a & a \\ x+2a & x & a \\ x+2a & a & x \end{vmatrix} = (x+2a)\begin{vmatrix} 1 & a & a \\ 1 & x & a \\ 1 & a & x \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$, we get

$$\Delta= (x+2a)\begin{vmatrix} 0 & a-x & a \\ 0 & x-a & a-x \\ 1 & a & x \end{vmatrix}$$

$= (x-a)^2(x+2a)$

31 **(b)**

$R_3 \to R_3 - 2R_2$, hence two identical rows

$\Rightarrow f(x)$ =constant

32 **(b)**

We divide L.H.S. by λ^4 and $C_1$by λ^2, $C_2$by λ and C_3 by λ on the R.H.S. to obtain

$$p + q\left(\frac{1}{\lambda}\right) + r\left(\frac{1}{\lambda}\right)^2 + s\left(\frac{1}{\lambda}\right)^3 + t\left(\frac{1}{\lambda}\right)^4$$

$$= \begin{vmatrix} 1+3/\lambda & 1-1/\lambda & 1+3/\lambda \\ 1+1/\lambda^2 & 2/\lambda - 1 & 1-3/\lambda \\ 1-3/\lambda^2 & 1+4/\lambda & 3 \end{vmatrix}$$

Taking limit as $\lambda \to \infty$, we get

$$p = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & -2 & 0 \\ 0 & 0 & 2 \end{vmatrix} = -4$$

[Applying $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$]

33 **(a)**

Given determinant,

$2a(bc - 4a^2) + b(2ac - b^2) + c(2ab - c^2) = 0$

$\Rightarrow 6abc - 8a^3 - b^3 - c^3 = 0$

$\Rightarrow (2a+b+c)[(2a-b)^2 + (b-c)^2 + (c-2a)^2] = 0$

$\Rightarrow 2a+b+c = 0 \quad (\because b \neq c)$

Let $f(x) = 8ax^3 + 2bx^2 + cx$

$f(0) = 0$

$$f\left(\frac{1}{2}\right) = a + \frac{b}{2} + \frac{c}{2} = \frac{2a+b+c}{2} = 0$$

So, $f(x)$ satisfies the Roll's theorem and hence, $f'(x) = 0$ has at least one root in $\left[0, \frac{1}{2}\right]$

34 **(b)**

For every 'det. with 1' ($\in B$) we can find a det. with value -1 by changing the sign of one entry of '1'. Hence there are equal number of elements in B and C.

Therefore, (b) is the correct option

35 **(c)**

Since each element of C_1 is the sum of two elements, putting the determinant as sum of two determinants, we get

$$\Delta = \begin{vmatrix} x^3 & x^2 & x \\ y^3 & y^2 & y \\ z^3 & z^2 & z \end{vmatrix} + \begin{vmatrix} 1 & x^2 & x \\ 1 & y^2 & y \\ 1 & z^2 & z \end{vmatrix}$$

$$= xyz \begin{vmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{vmatrix} + \begin{vmatrix} 1 & x^2 & x \\ 1 & y^2 & y \\ 1 & z^2 & z \end{vmatrix}$$

$$= -(xyz+1) \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

$$= -(xyz+1)(x-y)(y-z)(z-x)(x+y+z)$$

Since $\Delta = 0, x, y, z$ all are distinct, we have

$xyz + 1 = 0$ or $xyz = -1$

36 **(c)**

We have,

$$\Delta^2 = \Delta\Delta = \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}$$

$$= \begin{vmatrix} l_1^2 + m_1^2 + n_1^2 & l_1 l_2 + m_1 m_2 + n_1 n_2 & l_1 l \\ l_1 l_2 + m_1 m_2 + n_1 n_2 & l_2^2 + m_2^2 + n_2^2 & l_2 l \\ l_1 l_3 + m_1 m_3 + n_1 n_3 & l_2 l_3 + m_2 m_3 + n_2 n_3 & \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} 1 \Rightarrow \Delta = \pm 1 \Rightarrow |\Delta| = 1$$

37 **(b)**

Operating $R_1 \to R_1 - R_2$, gives

$$\Delta = \begin{vmatrix} x-2 & 3(x-2) & -(x-2) \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix}$$

$$= (x-2) \begin{vmatrix} 1 & 3 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix}$$

$$= (x-2) \begin{vmatrix} 1 & 3 & -1 \\ 0 & -3(x+2) & x-1 \\ 0 & 2x+9 & x-1 \end{vmatrix}$$

$[R_2 \to R_2 - 2R_1, R_3 \to R_3 + 3R_1]$

$$= (x-2)\{-(3x+6)(x-1) - (x-1)(2x+9)\}$$

$$= -(x-2)(x-1)(5x+15)$$

Therefore, $\Delta = 0$ gives $x = 2, 1, -3$

38 **(c)**

$$\begin{vmatrix} 1+x & 1 & 1 \\ 1+y & 1+2y & 1 \\ 1+z & 1+z & 3+3z \end{vmatrix}$$

$$= xyz \begin{vmatrix} 1+\frac{1}{x} & \frac{1}{x} & \frac{1}{x} \\ 1+\frac{1}{y} & 2+\frac{1}{y} & \frac{1}{y} \\ 1+\frac{1}{z} & 1+\frac{1}{z} & 3+\frac{1}{z} \end{vmatrix}$$

$$= xyz\left(3+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right) \begin{vmatrix} 1 & 1 & 1 \\ 1+\frac{1}{y} & 2+\frac{1}{y} & \frac{1}{y} \\ 1+\frac{1}{z} & 1+\frac{1}{z} & 3+\frac{1}{z} \end{vmatrix}$$

$$= xyz\left(3+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right) \begin{vmatrix} 1 & 0 & 0 \\ 1+\frac{1}{y} & 1 & -1 \\ 1+\frac{1}{z} & 0 & 2 \end{vmatrix}$$

$$= 2xyz\left(3+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$$

Hence, the given equation gives $x^{-1} + y^{-1} + z^{-1} = -3$

39 **(b)**

We have,

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)$$

$(c-a)(a+b+c)$ (1)

Also,

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$$

$$= abc \begin{vmatrix} \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \\ 1 & 1 & 1 \\ a^2 & b^2 & c^2 \end{vmatrix}$$ (taking a, b, c common from R_1, R_2, R_3)

$$= \begin{vmatrix} bc & ac & ab \\ 1 & 1 & 1 \\ a^2 & b^2 & c^2 \end{vmatrix}$$ (Multiplying R_1 by abc)

$$= \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ bc & ac & ab \end{vmatrix}$$

Then,

$$D = \begin{vmatrix} 1 & 1 & 1 \\ (x-a)^2 & (x-b)^2 & (x-c)^2 \\ (x-b)(x-c) & (x-c)(x-a) & (x-a)(x-b) \end{vmatrix}$$

$$= (a-b)(b-c)(c-a)(3x-a-b-c)$$

Now given that a, b, c are all different, then $D = 0$

$$\therefore x = \frac{1}{3}(a+b+c)$$

40 **(d)**

For the given homogeneous system of equations to have non-zero solution, determinant of coefficient matrix should be zero, i.e.,

$$\begin{vmatrix} 1 & -k & -1 \\ k & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 1(1+1) + k(-k+1) - (k+1)$$

$$= 0$$

$\Rightarrow 2 - k^2 + k - k - 1 = 0$

$\Rightarrow k^2 = 1$

$\Rightarrow k = \pm 1$

41 **(d)**

$$\begin{vmatrix} 1+x_1 & 1+x_1x & 1+x_1x^2 \\ 1+x_2 & 1+x_2x & 1+x_2x^2 \\ 1+x_3 & 1+x_3x & 1+x_3x^2 \end{vmatrix}$$
$$= \begin{vmatrix} 1 & x_1 & 0 \\ 1 & x_2 & 0 \\ 1 & x_3 & 0 \end{vmatrix} \begin{vmatrix} 1 & 1 & 0 \\ 1 & x & 0 \\ 1 & x^2 & 0 \end{vmatrix}$$
$= 0$

42 **(a)**

$D = \cos\theta - \cos^2 . +6 > 0$. Since $D > 0$ only trivial solution is possible

43 **(c)**

Applying $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$ reduce the determinant to
$$\begin{vmatrix} x^2-2x+1 & x-1 & 0 \\ 2x-2 & x-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$
$= (x-1)^3 - 2(x-1)^2 = (x-1)^2(x-1-2) = (x-1)^2(x-3)$,

Which is clearly negative for $x < 1$

44 **(d)**

Let the given determinant be equal to $\Delta(x)$. Then,

$5A + 4B + 3C + 2D + E = \Delta(1) + \Delta'(1)$

Now, $\Delta(1) = 0$ as R_2 and R_3 are identical
$$\Delta'(x) = \begin{vmatrix} 1 & 0 & 1 \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} + \begin{vmatrix} x & 2 & x \\ 2x & 1 & 0 \\ x & x & 6 \end{vmatrix} + \begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ 1 & 1 & 0 \end{vmatrix}$$
$$\Delta'(1) = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & 6 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 6 \\ 1 & 1 & 0 \end{vmatrix}$$
$= -17 + (12 + 1 - 1 - 6) = -11$

45 **(b)**

$$\Delta = \begin{vmatrix} 1 & 4 & 9 & 16 \\ 4 & 9 & 16 & 25 \\ 9 & 16 & 25 & 36 \\ 16 & 25 & 36 & 49 \end{vmatrix} \quad (R_3 \to R_3 - R_2, R_4 \to R_4 - R_3)$$
$$= \begin{vmatrix} 1 & 4 & 9 & 16 \\ 4 & 9 & 16 & 25 \\ 5 & 7 & 9 & 11 \\ 15 & 21 & 27 & 33 \end{vmatrix}$$
$$= 3\begin{vmatrix} 1 & 4 & 9 & 16 \\ 4 & 9 & 16 & 25 \\ 5 & 7 & 9 & 11 \\ 5 & 7 & 9 & 11 \end{vmatrix} = 0 \quad (R_4 \to R_4 - R_3)$$

46 **(b)**

The total number of third-order determinants is 9! Since the number of determinants is even and in which there are 9!/2 pairs of determinants which are obtained by changing two consecutive rows,

So $\sum_{i=1}^{n} D_i = 0$

47 **(b)**

Let, $$\Delta = \begin{vmatrix} \cos(\alpha-\beta) & \cos(\beta-\gamma) & \cos(\gamma-\alpha) \\ \cos(\alpha+\beta) & \cos(\beta+\gamma) & \cos(\gamma+\alpha) \\ \sin(\alpha+\beta) & \sin(\beta+\gamma) & \sin(\gamma+\alpha) \end{vmatrix}$$

It is clear that either $\alpha = \beta$ or $\beta = \gamma$ or $\gamma = \alpha$ is sufficient to make $\Delta = 0$. It is not necessary that triangle is equilateral. Also, isosceles triangle can be obtuse one

48 **(c)**

The given system is consistent
$$\therefore \ \Delta = \begin{vmatrix} 1 & 1 & -1 \\ 2 & -1 & -c \\ -b & 3b & -c \end{vmatrix} = 0$$
$\Rightarrow c + bc - 6b + b + 2c + 3bc = 0$

$\Rightarrow 3c + 4bc - 5b = 0$

$\Rightarrow c = \dfrac{5b}{4b+3}$

Now,

$c < 1$

$\Rightarrow \dfrac{5b}{4b+3} < 1$

$\Rightarrow \dfrac{5b}{4b+3} - 1 < 0$

$\Rightarrow \dfrac{b-3}{4b+3} < 0$

$\Rightarrow b \in \left(-\dfrac{3}{4}, 3\right)$

49 **(b)**

Let $\frac{x^2}{a^2} = X, \frac{y^2}{b^2} = Y, \frac{z^2}{c^2} = Z$

Then the given system of equations is

$X + Y - Z = 1$

$X - Y + Z = 1$

$-X + Y + Z = 1$

Coefficient determinant is
$$A = \begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{vmatrix}$$
$= 1(-1-1) - 1(1+1) - 1(1-1)$

$= -4 \neq 0$

Hence, the given system of equation has unique solutions

50 **(b)**

Applying $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$, we get
$$\Delta = \begin{vmatrix} -4-2\sqrt{2} & -2\sqrt{2} & 0 \\ 4\sqrt{2} & 4\sqrt{2} & 0 \\ 3-2\sqrt{2} & 2-2\sqrt{2} & 1 \end{vmatrix}$$
$= 1\left(-(4+2\sqrt{2})\right)4\sqrt{2} + 2\sqrt{2} \times 4\sqrt{2}$

$= -16\sqrt{2}$

51 **(a)**

Applying $C_1 \to C_1 + 2C_2 + C_3$, we get
$$S = \sum_{r=2}^{n} (-2)^r \begin{vmatrix} {}^{n}C_r & {}^{n-2}C_{r-1} & {}^{n-2}C_r \\ 0 & 1 & 1 \\ 0 & -1 & 0 \end{vmatrix}$$

$$= \sum_{r=2}^{n} (-2)^{r\,n}C_r$$
$$= \sum_{r=0}^{n} (-2)^r \ {}^nC_r - ({}^nC_0 - 2\,{}^nC_1)$$
$$= (1-2)^n - (1-2n) = 2n - 1 + (-1)^n$$

52 **(a)**

$$\begin{vmatrix} 1 & 1 & 1 \\ {}^mC_1 & {}^{m+1}C_1 & {}^{m+2}C_1 \\ {}^mC_2 & {}^{m+1}C_2 & {}^{m+2}C_2 \end{vmatrix}$$
$$= \begin{vmatrix} 1 & 1 & 1 \\ {}^mC_1 & {}^{m+1}C_1 & {}^{m+1}C_0 + {}^{m+1}C_1 \\ {}^mC_2 & {}^{m+1}C_2 & {}^{m+1}C_1 + {}^{m+1}C_2 \end{vmatrix}$$
$$= \begin{vmatrix} 1 & 1 & 0 \\ {}^mC_1 & {}^{m+1}C_1 & {}^{m+1}C_0 \\ {}^mC_2 & {}^{m+1}C_2 & {}^{m+1}C_1 \end{vmatrix}$$ [Applying $C_3 \to C_3 - C_2$]
$$= \begin{vmatrix} 1 & 1 & 0 \\ {}^mC_1 & {}^mC_0 + {}^mC_1 & {}^{m+1}C_0 \\ {}^mC_2 & {}^mC_1 + {}^mC_2 & {}^{m+1}C_1 \end{vmatrix}$$
$$= \begin{vmatrix} 1 & 1 & 0 \\ {}^mC_1 & {}^mC_0 & {}^{m+1}C_0 \\ {}^mC_2 & {}^mC_1 & {}^{m+1}C_1 \end{vmatrix}$$ [Applying $C_2 \to C_2 - C_1$]
$$= {}^mC_0\,{}^{m+1}C_1 - {}^{m+1}C_0\,{}^mC_1$$
$$= m + 1 - m$$
$$= 1$$

53 **(b)**

$$\begin{vmatrix} x^3+1 & x^2y & x^2z \\ xy^2 & y^3+1 & y^2z \\ xz^2 & yz^2 & z^3+1 \end{vmatrix} = 11$$

Multiplying R_1 by x, R_2 by y and R_3 by z, we get

$$\frac{1}{xyz}\begin{vmatrix} x^4+x & x^3y & x^3z \\ xy^3 & y^4+1 & y^3\,z \\ xz^3 & yz^3 & z^4+1 \end{vmatrix} = 11$$

Taking x, y, z common from C_1, C_2, C_3, respectively, we get

$$\begin{vmatrix} x^3+1 & x^3 & x^3 \\ y^3 & y^3+1 & y^3 \\ z^3 & z^3 & z^3+1 \end{vmatrix} = 11$$

Using $R_1 \to R_1 + R_2 + R_3$, we have

$$(x^3+y^3+z^3+1)\begin{vmatrix} 1 & 1 & 1 \\ y^3 & y^3+1 & y^3 \\ z^3 & z^3 & z^3+1 \end{vmatrix} = 11$$

Using $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, we get

$$(x^3+y^3+z^3+1)\begin{vmatrix} 1 & 0 & 0 \\ y^3 & 1 & 0 \\ z^3 & 0 & 1 \end{vmatrix} = 11$$

Hence, $x^3 + y^3 + z^3 = 10$

Therefore, the ordered triplets are (2, 1, 1,), (1, 2, 1,), (1, 1, 2)

54 **(b)**

Applying $C_1 \to C_1 - C_2, C_2 \to C_2 - C_3$, we get

$$\Delta = \begin{vmatrix} 0 & 0 & 1 \\ \cot\frac{A}{2} - \cot\frac{B}{2} & \cot\frac{B}{2} - \cot\frac{C}{2} & \cot\frac{C}{2} \\ \tan\frac{B}{2} - \tan\frac{A}{2} & \tan\frac{C}{2} - \tan\frac{B}{2} & \tan\frac{A}{2} + \tan\frac{B}{2} \end{vmatrix}$$
$$= \begin{vmatrix} 0 & 0 & 1 \\ \cot\frac{A}{2} - \cot\frac{B}{2} & \cot\frac{B}{2} - \cot\frac{C}{2} & \cot\frac{C}{2} \\ \frac{\cot\frac{A}{2} - \cot\frac{B}{2}}{\cot\frac{A}{2}\cot\frac{B}{2}} & \frac{\cot\frac{B}{2} - \cot\frac{C}{2}}{\cot\frac{B}{2}\cot\frac{C}{2}} & \tan\frac{A}{2} + \tan\frac{B}{2} \end{vmatrix}$$
$$= \left(\cot\frac{A}{2} - \cot\frac{B}{2}\right)\left(\cot\frac{B}{2} - \cot\frac{C}{2}\right)$$
$$\times \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & \cot\frac{C}{2} \\ \tan\frac{A}{2}\tan\frac{B}{2} & \tan\frac{B}{2}\tan\frac{C}{2} & \tan\frac{A}{2}\tan\frac{B}{2} \end{vmatrix}$$
$$= \left(\cot\frac{A}{2} - \cot\frac{B}{2}\right)\left(\cot\frac{B}{2} - \cot\frac{C}{2}\right)\left(\tan\frac{C}{2} - \tan\frac{A}{2}\right)\tan\frac{B}{2}$$

Since $\Delta = 0$, therefore

$\cot\frac{A}{2} = \cot\frac{B}{2}$ or $\cot\frac{B}{2} = \cot\frac{C}{2}$ or $\tan\frac{A}{2} = \tan\frac{C}{2}$

Hence, the triangle is definitely isosceles

55 **(a)**

Taking x common from R_2 and $x(x-1)$ common from R_3, we get

$$f(x) = x^2(x-1)\begin{vmatrix} 1 & x & x+1 \\ 2 & x-1 & x+1 \\ 3 & x-2 & x+1 \end{vmatrix}$$

Applying $C_3 \to C_3 - C_2$, we get

$$f(x) = x^2(x-1)\begin{vmatrix} 1 & x & 1 \\ 2 & x-1 & 2 \\ 3 & x-2 & 3 \end{vmatrix} = 0$$

Thus , $f(500) = 0$

56 **(a)**

As $a_1b_1c_1, a_2b_2c_2$ and $a_3b_3c_3$ are even natural numbers, each of c_1, c_2, c_3 is divisible by 2. Let $c_i = 2k_i$ for $i = 1, 2, 3$. Thus,

$$\Delta = 2\begin{vmatrix} k_1 & a_1 & b_1 \\ k_2 & a_2 & b_2 \\ k_3 & a_3 & b_3 \end{vmatrix} = 2m$$

Where m is some natural number. Thus, Δ is divisible by 2. That Δ may not be divisible by 4 can be seen by taking the three numbers as 112, 122 and 134. Note that

$$\Delta = \begin{vmatrix} 2 & 1 & 1 \\ 2 & 1 & 2 \\ 4 & 1 & 3 \end{vmatrix} = 2$$

Which is divisible by 2 but not by 4

57 **(b)**

For no solution or infinitely many solutions

$$\begin{vmatrix} \alpha & -1 & -1 \\ 1 & -\alpha & -1 \\ 1 & 1 & -\alpha \end{vmatrix} = 0$$

$\Rightarrow\ \alpha(\alpha^2-1)-1(\alpha-1)+1(1-\alpha)=0$

$\Rightarrow\ \alpha(\alpha^2-1)-2\alpha+2=0$

$\Rightarrow\ \alpha(\alpha-1)(\alpha+1)-2(\alpha-1)=0$

$\Rightarrow (\alpha-1)(\alpha^2+\alpha-2)=0$

$\Rightarrow (\alpha-1)(\alpha+2)(\alpha-1)=0$

$\Rightarrow (\alpha-1)^2(\alpha+2)=0$

$\Rightarrow\ \alpha=1,1,-2$

But for $\alpha=1$, there are infinite solutions. When $\alpha=-2$, we have

$-2x-y-z=-3$

$x+2y-z=-3$

$x-y+2z=-3$

Adding, we get $0=-9$, which is not true. Hence there is no solution

58 **(b)**

Since the system has non-trivial solution,

$$\therefore \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

Applying $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$, we get

$$\Delta = \begin{vmatrix} a-1 & 1-b & 0 \\ 0 & b-1 & 1-c \\ 1 & 1 & c \end{vmatrix} = 0$$

$\Rightarrow c(1-a)(1-b)+(1-b)(1-c)$
$\qquad -(1-c)(a-1)=0$

Dividing throughout by $(1-a)(1-b)(1-c)$, we get

$$\frac{c}{1-c}+\frac{1}{1-c}+\frac{1}{1-b}=0$$

$$\Rightarrow\ -1+\frac{1}{1-c}+\frac{1}{1-b}+\frac{1}{1-a}=0$$

$$\Rightarrow \frac{1}{1-c}+\frac{1}{1-a}+\frac{1}{1-b}=1$$

59 **(b)**

$$\Delta = \begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix}$$

Applying $R_1 \to aR_1, R_2 \to bR_2, R_3 \to cR_3$, we get

$$\Delta = \frac{1}{abc} \times \begin{vmatrix} b^2+c^2 & a^2b & a^2c \\ ab^2 & b(c^2+a^2) & cb^2 \\ ac^2 & bc^2 & c(a^2+b^2) \end{vmatrix}$$

Now, applying $C_1 \to \frac{1}{a}C_1, C_2 \to \frac{1}{b}C_2, C_3 \to \frac{1}{c}C_3$, we get

$$\Delta = \frac{abc}{abc} \begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & -2c^2 & -2b^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} \quad [R_1 \to R_1 - R_2 - R_3]$$

$$= 2 \begin{vmatrix} 0 & -c^2 & -b^2 \\ b^2 & a^2 & 0 \\ c^2 & 0 & a^2 \end{vmatrix}$$

(Taking 2 common from R_1 and applying $R_2 \to R_2 + R_1$ and $R_3 \to R_3 + R_1$)

Evaluating along R_1, we get

$\Delta = 2[c^2(a^2b^2) - b^2(-a^2c^2)]$

$= 4a^2b^2c^2$

Hence, $k=4$

60 **(a)**

The given determinant Δ_1 is obtained by corresponding co-factors of determinant Δ_2; hence $\Delta_1 = \Delta_2^2$. Now $\Delta_1\Delta_2 = \ \Delta_2^2\,\Delta_2 = \Delta_2^3$

61 **(d)**

Since for $x=0$, the determinant reduces to the determinant of a skew-symmetric matrix of odd order which is always zero, hence $x=0$ is the solution of the given equation

62 **(a)**

Using the sum property, we get

$$\sum_{r=0}^{m} \Delta_r = \begin{vmatrix} \sum_{r=0}^{m}(2r-1) & \sum_{r=0}^{m} {}^mC_r & \sum_{r=0}^{m} 1 \\ m^2-1 & 2^m & m+1 \\ \sin^2(m^2) & \sin^2(m) & \sin^2(m+1) \end{vmatrix}$$

But $\sum_{r=0}^{m}(2r-1) = \frac{1}{2}(m+1)(2m-1-1) = m^2-1$,

$\sum_{r=0}^{m} {}^mC_r = 2^m$ and $\sum_{r=0}^{m} 1 = m+1$. Therefore,

$$\sum_{r=0}^{m} \Delta_r = \begin{vmatrix} m^2-1 & 2^m & m+1 \\ m^2-1 & 2^m & m+1 \\ \sin^2(m^2) & \sin^2(m) & \sin^2(m+1) \end{vmatrix} = 0$$

63 **(c)**

$$\Delta = (1+x+x^2) \begin{vmatrix} 1 & 1 & 1 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

$$= (1+x+x^2)(x+1)^2$$

Therefore, $\Delta = 0$ has roots $1, 1, \omega, \omega, \omega^2, \omega^2$

64 **(c)**

As a, b, c are in G.P. with common ration r_1 and α, β, γ are in G.P. having common ratio $r_2, a \neq 0, \alpha \neq 0, b = ar_1, c = ar_1^2, \beta = ar_2, \gamma = ar_2^2$

Also the system of equation has only zero (trivial) solution

$$\Delta = \begin{vmatrix} a & \alpha & 1 \\ b & \beta & 1 \\ c & \gamma & 1 \end{vmatrix} \neq 0$$

$$\Rightarrow a\alpha \begin{vmatrix} 1 & 1 & 1 \\ r_1 & r_2 & 1 \\ r_1^2 & r_2^2 & 1 \end{vmatrix} \neq 0$$

$\Rightarrow a\alpha(r_1-1)(r_2-1)(r_1-r_2) \neq 0$

$\Rightarrow r_1 \neq 1, r_2 \neq 1$ and $r_1 \neq r_2$

65 **(d)**

The given determinant, on simplification, gives

$$\Delta_1 = \begin{vmatrix} a_1^2 & -2a_1 & 1 & 0 \\ a_2^2 & -2a_2 & 1 & 0 \\ a_3^2 & -2a_3 & 1 & 0 \\ a_4^2 & --2a_4 & 1 & 0 \end{vmatrix} \times \begin{vmatrix} 1 & b_1 & b_1^2 & 0 \\ 1 & b_2 & b_2^2 & 0 \\ 1 & b_3 & b_3^2 & 0 \\ 1 & b_4 & b_4^2 & 0 \end{vmatrix}$$

$= 0 \times 0 = 0$

66 **(d)**

Since $A + B + C = \pi$ and $e^{i\pi} = \cos\pi + i\sin\pi = -1$,

$e^{i(B+C)} = e^{i(\pi-A)} = -e^{iA}$ and $e^{-i(B+C)} = -e^{iA}$

By taking e^{iA}, e^{iB}, e^{iC} common from R_1, R_2 and R_3, respectively,

We have

$$\Delta = -\begin{vmatrix} e^{iA} & e^{-i(A+C)} & e^{-i(A+B)} \\ e^{-i(B+C)} & e^{iB} & e^{-i(A+B)} \\ e^{-i(B+C)} & e^{-i(A+C)} & e^{iC} \end{vmatrix}$$

$$= -\begin{vmatrix} e^{iA} & -e^{iB} & -e^{iC} \\ -e^{iA} & e^{iB} & -e^{iC} \\ -e^{iA} & -e^{iB} & e^{iC} \end{vmatrix}$$

By taking e^{iA}, e^{iB}, e^{iC} common from C_1, C_2 and C_3, respectively,

We have

$$\Delta = \begin{vmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{vmatrix} = -4$$

67 **(d)**

$$\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy \quad \text{(given)}$$

$$\Rightarrow -3i \begin{vmatrix} 6i & 1 & 1 \\ 4 & -1 & -1 \\ 20 & i & i \end{vmatrix} = x + iy$$

$\Rightarrow x + iy = 0 + i0$

$\Rightarrow x = y = 0$

68 **(b)**

The given determinant is

$$\begin{bmatrix} 2^{n+1} - 2^n + p & 2^{n+2} - 2^{n+1} + q & p + r \\ 2^n + p & 2^{n+1} & p + r \\ a^2 + 2^n + p & b^2 + 2^n + 2q & c^2 - r \end{bmatrix}$$

(Using $R_1 \to R_1 - R_3$ and $2q = p + r$)

$$\begin{bmatrix} 2^n(2-1) + p & 2^{n+1}(2-1) + q & p + r \\ 2^n + p & 2^{n+1} + q & p + r \\ a^2 + 2^n + p & b^2 + 2^{n+1} + 2q & c^2 - r \end{bmatrix}$$

$$= \begin{bmatrix} 2^n + p & 2^{n+1} + q & p + r \\ 2^n + p & 2^{n+1} + q & p + r \\ a^2 + 2^n + p & b^2 + 2^{n+1} + 2q & c^2 - r \end{bmatrix} = 0 \quad (\because R_1 \equiv R_2)$$

69 **(a)**

$$\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$$

$= pqr(a^3 + b^3 + c^3 - 3abc) - abc(p^3 + q^3 + r^3 - 3pqr)$

$= pqr(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
$-abc(p + q + r)(p^2 + q^2 + r^2 - pq - qr - pr)$

$= 0$

70 **(d)**

Since a, b, c, d, e, f are in G.P. and if r is the common ratio of the G.P., then

$b = ar$

$c = ar^2$

$d = ar^3$

$e = ar^4$

$f = ar^5$

Therefore, given determinant is

$$\begin{vmatrix} a^2 & a^2r^6 & x \\ a^2r^2 & a^2r^8 & y \\ a^2r^4 & a^2r^{10} & z \end{vmatrix}$$

$$= a^2a^2r^6 = \begin{vmatrix} 1 & 1 & x \\ r^2 & r^2 & y \\ r^4 & r^4 & z \end{vmatrix}$$

$= a^4r^6(0) = 0 \quad [\because C_1, C_2$, are identical]

71 **(a)**

The given system of equations will have a non-trivial solution if

$$\begin{vmatrix} \alpha + a & \alpha & \alpha \\ \alpha & \alpha + b & \alpha \\ \alpha & \alpha & \alpha + c \end{vmatrix} = 0$$

Operating $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\begin{vmatrix} \alpha + a & \alpha & \alpha \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix} = 0$$

$\Rightarrow \alpha ab + c(\alpha b + ab + a\alpha) = 0$

$\Rightarrow \alpha(bc + ca + ab) + abc = 0$

$\Rightarrow \frac{1}{\alpha} = -\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \quad (\because a, b, c \neq 0)$

72 **(a)**

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2] = 0$

$\Rightarrow a + b + c = 0$ or $a = b = c$

If $a + b + c = 0$, we have

$\cos\theta + \cos 2\theta + \cos 3\theta = 0$ and $\sin\theta - \sin 2\theta + \sin 3\theta = 0$

$\Rightarrow \cos 2\theta(2\cos\theta + 1) = 0$ and $\sin 2\theta(1 - 2\cos\theta) = 0$ (i)

Which is not possible as $\cos 2\theta = 0$ gives $\sin 2\theta \neq 0$, $\cos\theta \neq 1/2$. And $\cos\theta = -1/2$ gives $\sin 2\theta \neq 0$, $\cos\theta \neq 1/2$. Therefore, Eq. (i) does not hold simultaneously

$\therefore a+b+c \neq 0$

$\therefore a=b=c$

or $e^{i\theta}=e^{-2i\theta}=e^{3i\theta}$

Which is satisfied only by $e^{i\theta}=1$ i.e., $\cos\theta=1, \sin\theta=0$ so $\theta=2k\pi, k\in\mathcal{Z}$

73 **(b)**

Taking x^5 common from last row, we get

$$x^5\begin{vmatrix} x^n & x^{n+2} & x^{2n} \\ 1 & x^a & a \\ x^n & x^{a+1} & x^{2n} \end{vmatrix}=0, \forall x\in R$$

$\Rightarrow a+1=n+2 \Rightarrow a=n+1$

(as it will make first and third row is identical)

74 **(a)**

We have,

$a_{n+1}^2=a_n a_{n+2}$

$\Rightarrow 2\log a_{n+1}=\log a_n+\log a_{n+2}$

Similarly,

$2\log a_{n+4}=\log a_{n+3}+\log a_{n+5}$

$2\log a_{n+7}=\log a_{n+6}+\log a_{n+8}$

Substituting these values in second column of determinant, we get

$$\Delta=\frac{1}{2}\begin{vmatrix} \log a_n & \log a_n+\log a_{n+2} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+3}+\log a_{n+5} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+6}+\log a_{n+8} & \log a_{n+8} \end{vmatrix}$$

$=\frac{1}{2}(0)=0$ [Using $C_2\to C_2-C_1-C_3$]

75 **(b)**

Let a be the first term and d be the common difference of corresponding A.P. Then

$$\Delta=xyz\begin{vmatrix} 1/x & 1/y & 1/z \\ p & 2q & 3r \\ 1 & 1 & 1 \end{vmatrix}$$

$$=xyz\begin{vmatrix} a+(p-1)d & a+(2q-1)d & a+(3r-1)d \\ p & 2q & 3r \\ 1 & 1 & 1 \end{vmatrix}$$

Applying $R_1\to R_1-aR_3, R_2\to R_2-R_3$ and then taking d common from R_1, we get

$$\Delta= xyzd\begin{vmatrix} (p-1) & (2q-1) & (3r-1) \\ (p-1) & (2q-1) & (3r-1) \\ 1 & 1 & 1 \end{vmatrix}=0$$

76 **(d)**

We have $y=\sin mx$, therefore

$y_1=m\cos mx,\ y_2=-m^2\sin mx$, etc

$$\therefore\ \Delta=\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix}$$

$$=\begin{vmatrix} \sin mx & m\cos mx & -m^2\sin mx \\ -m^3\cos mx & m^4\sin mx & m^5\cos mx \\ -m^6\sin mx & -m^7\cos mx & m^8\sin mx \end{vmatrix}$$

$$=m^{12}\begin{vmatrix} \sin mx & \cos mx & -\sin mx \\ -\cos mx & \sin mx & \cos mx \\ -\sin mx & -\cos mx & \sin mx \end{vmatrix}=0$$

77 **(d)**

$$D'=\begin{vmatrix} a_1+pb_1 & b_1+qc_1 & c_1+ra_1 \\ a_2+pb_2 & b_2+qc_2 & c_2+ra_2 \\ a_3+pb_3 & b_3+qc_3 & c_3+ra_3 \end{vmatrix}$$

$$=\begin{vmatrix} a_1 & b_1+qc_1 & c_1+ra_1 \\ a_2 & b_2+qc_2 & c_2+ra_2 \\ a_3 & b_3+qc_3 & c_3+ra_3 \end{vmatrix}+\begin{vmatrix} pb_1 & b_1+qc_1 & c_1+ra_1 \\ pb_2 & b_2+qc_2 & c_2+ra_2 \\ pb_3 & b_3+qc_3 & c_3+ra_3 \end{vmatrix}$$

In the first determinant, apply $C_3\to C_3-rC_1$ and then $C_2\to C_2-qC_3$

In second determinant take p common from C_1 and then apply $C_2\to C_2-C_1$. Then take q common from C_2 and then apply $C_3\to C_3-C_2$. Finally taking r common from C_3, we have ultimately $D'=(1+pqr)D$

78 **(c)**

We have,

$$\begin{vmatrix} ka & k^2+a^2 & 1 \\ kb & k^2+b^2 & 1 \\ kc & k^2+c^2 & 1 \end{vmatrix}=\begin{vmatrix} ka & k^2 & 1 \\ kb & k^2 & 1 \\ kc & k^2 & 1 \end{vmatrix}+\begin{vmatrix} ka & a^2 & 1 \\ kb & b^2 & 1 \\ kc & c^2 & 1 \end{vmatrix}$$

$$=0+k\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

$=k(a-b)(b-c)(c-a)$

79 **(a)**

Let first term of G.P. is A and common ration is R. Then,

$a=AR^{p-1}\Rightarrow \log a=\log A+(p-1)\log R$, etc

$$\Rightarrow\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}=\begin{vmatrix} \log A+(p-1)\log R & p & 1 \\ \log A+(q-1)\log R & q & 1 \\ \log A+(r-1)\log R & r & 1 \end{vmatrix}$$

$$=\begin{vmatrix} (p-1)\log R & p & 1 \\ (q-1)\log R & q & 1 \\ (r-1)\log R & r & 1 \end{vmatrix}\quad [C_1\to C_1-(\log A)C_3]$$

$$=\log R\begin{vmatrix} (p-1) & p & 1 \\ (q-1) & q & 1 \\ (r-1) & r & 1 \end{vmatrix}$$

$$=\log R\begin{vmatrix} p & p & 1 \\ q & q & 1 \\ r & r & 1 \end{vmatrix}\quad (C_1\to C_1+C_3)$$

$= 0$

80 **(a)**

Operating $C_1 \to C_1 + C_2 + C_3$ on the L.H.S. we get

$$\Delta = \begin{vmatrix} 0 & c-a & a-b \\ 0 & c'-a' & a'-b' \\ 0 & c''-a'' & a''-b'' \end{vmatrix} = m \begin{vmatrix} a & b & c \\ a' & b' & c' \\ a'' & b'' & c'' \end{vmatrix}$$

$\Rightarrow m = 0$

81 **(d)**

Applying $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$, we get

$$\Delta = \begin{vmatrix} a & b-y & c-z \\ -x & y & 0 \\ -x & 0 & z \end{vmatrix} = 0$$

Expanding along C_3, we get

$$(c-z)\begin{vmatrix} -x & y \\ -x & 0 \end{vmatrix} + z\begin{vmatrix} a & b-y \\ -x & y \end{vmatrix} = 0$$

$\Rightarrow (c-z)(xy) + z(ay + bx - xy) = 0$

$\Rightarrow cxy - xyz + ayz + bxz - xyz = 0$

$\Rightarrow ayz + bzx + cxy = 2xyz$

$\Rightarrow \dfrac{a}{x} + \dfrac{b}{y} + \dfrac{c}{z} = 2$

82 **(b)**

$\Delta_1 = x(x^2 - ab) - b(ax - ab) + b(a^2 - ax)$

$= x^3 - 3abx + ab^2 + a^2b$

$\dfrac{d}{dx}(\Delta_1) = 3x^2 - 3ab = 3(x^2 - ab) = 3\Delta_2$

83 **(b)**

Let $D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

Applying $C_2 \to C_2 - \frac{a_{12}}{a_{11}}C_1, C_3 \to C_3 - \frac{a_{13}}{a_{11}}C_1$, we get

$$D = \begin{vmatrix} a_{11} & 0 & 0 \\ a_{21} & \left(a_{22} - \dfrac{a_{12}}{a_{11}} \times a_{21}\right) & \left(a_{23} - \dfrac{a_{13}}{a_{11}} \times a_{21}\right) \\ a_{31} & \left(a_{32} - \dfrac{a_{12}}{a_{11}} \times a_{31}\right) & \left(a_{32} - \dfrac{a_{13}}{a_{11}} \times a_{31}\right) \end{vmatrix}$$

Which has minimum value of -4

84 **(a)**

The given system of linear equations has a unique solution if

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 5 & \lambda \end{vmatrix} \neq 0$$

i.e., if $\lambda - 8 \neq 0$ or $\lambda = 8$

85 **(c)**

Consider the triangle with vertices $B(x_1, y_1), C(x_2, y_2)$ and $A(x_3, y_3)$, and $AB = c, BC = a$ and $AC = b$. Then area of triangle is

$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \sqrt{s(s-a)(s-b)(s-c)}$ where

$2s = a + b + c$

Squaring and simplifying, we get

$4\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = (a+b+c)(b+c-a)(c+a-$

$ba+b-c$

Hence, $k = 4$

86 **(c)**

$\because -1 \le x < 0 \quad \therefore [x] = -1$

$0 \le y < 1 \quad \therefore [y] = 0$

$1 \le z < 2 \quad \therefore [z] = 1$

Hence, the given determinant is

$$\begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & 0 & 2 \end{vmatrix} = 1 = [z]$$

87 **(a)**

$$\Delta = \begin{vmatrix} p+a & b & c \\ a & q+b & x \\ a & b & r+c \end{vmatrix} = 0$$

Applying $R_1 \to R_2 - R_1, R_3 \to R_3 - R_1$, we get

$$\begin{vmatrix} p+a & b & c \\ -p & q & 0 \\ -q & 0 & r \end{vmatrix} = 0$$

$\Rightarrow pqc + [q(p+a) + bp]r = 0$

Dividing by pqr, we obtain

$\dfrac{a}{p} + \dfrac{b}{q} + \dfrac{c}{r} = -1$

88 **(c)**

$f(x)$

$$= \begin{vmatrix} 1 - 2\sin^2 x & \sin^2 x & 1 - 8\,\text{si} \\ \sin^2 x & 1 - 2\sin^2 x & 1 \\ 1 - 8\sin^2 x(1 - \sin^2 x) & 1 - \sin^2 x & 1 \end{vmatrix}$$

The required constant term is

$$f(0) = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 1(0-1) = -1$$

89 **(b)**

We have,

$$\Delta = \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = abc - (a+b+c) + 2$$

$\therefore \Delta > 0 \Rightarrow abc + 2 > a + b + c$

$\Rightarrow abc + 2 > 3(abc)^{1/3}$ $[\because$ A.M. > G.M. $\Rightarrow$

$a+b+c3>abc13$

$\Rightarrow x^3 + 2 > 3x$, where $x = (abc)^{1/3}$

$\Rightarrow x^3 - 3x + 2 > 0 \Rightarrow (x-1)^2(x+2) > 0$

$\Rightarrow x + 2 > 0 \Rightarrow x > -2 \Rightarrow (abc)^{1/3} > -2 \Rightarrow abc > -8$

90 **(c)**

We observe that the elements in the pre-factor are the cofactor of the corresponding elements of the post-factor. Hence,

$$\begin{vmatrix} \lambda & c & -b \\ -c & \lambda & a \\ b & -a & \lambda \end{vmatrix}^3 = [\lambda(\lambda^2+a^2+b^2+c^2)]^3$$

$$= (1+a^2+b^2+c^2)^3$$

$\Rightarrow \lambda = 1$

Alternative solution:

Writing $a=0, b=0, c=0$ on both sides, we get

$\lambda^6\lambda^3 = 1 \Rightarrow \lambda = 1$

91 **(b)**

Given,

$$\begin{vmatrix} xp+y & x & y \\ yp+z & y & z \\ 0 & xp+y & yp+z \end{vmatrix} = 0$$

Operating $C_1 \to C_1 - pC_2 - C_3$, we get

$$\begin{vmatrix} 0 & x & y \\ 0 & y & z \\ -(xp^2+2py+z) & xp+y & yp+z \end{vmatrix} = 0$$

$\Rightarrow (xz-y^2)(xp^2+2py+z) = 0$

$\Rightarrow xz - y^2 = 0$

$\Rightarrow y^2 = xz$

Hence, x, y, z are in G.P.

92 **(a)**

We have,

$$\begin{vmatrix} x & 1 & 1 & \cdots \\ 1 & x & 1 & \cdots \\ 1 & 1 & x & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{vmatrix}$$

$$= \begin{vmatrix} x & 1 & 1 & \cdots \\ (1-x) & (x-1) & 0 & \cdots \\ (1-x) & 0 & (x-1) & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{vmatrix}$$

[Applying $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1, \dots, R_n \to R_n - R_1$]

$= x(x-1)^{n-1} + |(x-1)^{n-1} + (x-1)^{n-1} + \cdots + (x-1)^{n-1}(n-1) \text{ times}|$

[Expanding along R_1]

$= x(x-1)^{n-1} + (n-1)(x-1)^{n-1}$

$= (x-1)^{n-1}(x+n-1)$

93 **(d)**

Operating $C_1 \to C_1 + C_2 + C_3$, we get

$$(a+b+c-x) \begin{vmatrix} 1 & c & b \\ 1 & b-x & a \\ 1 & a & c-x \end{vmatrix} = 0$$

$\therefore x = a+b+c = 0$

94 **(b,d)**

Since, given that

$$\Delta = \begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix}$$

Applying $R_3 \to R_3 - (\alpha R_1 + R_2)$, we get

$$\Delta = \begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ 0 & 0 & -(a\alpha^2+2b\alpha+c) \end{vmatrix}$$

$\Rightarrow \Delta = (b^2-ac)(a\alpha^2+2b\alpha+c) = 0$

$\Rightarrow b^2 = ac$ or $a\alpha^2 + 2b\alpha + c = 0$

$\Rightarrow a, b, c$ are in GP or α is the root of the equation $ax^2 + 2bx + c = 0$.

95 **(b,d)**

Applying $C_1 \to C_1 - (\cot\phi)C_2$, we get

$$\Delta = \begin{vmatrix} 0 & \sin\theta\sin\phi & \cos\theta \\ 0 & \cos\theta\sin\phi & -\sin\theta \\ -\sin\theta\sin\phi & \sin\theta\cos\phi & 0 \end{vmatrix}$$

$= -\frac{\sin\theta}{\sin\theta}[-\sin\phi\sin^2\theta - \cos^2\theta\sin\phi]$

[expanding along C_1]

$= \sin\theta$

Which is independent of ϕ. Also,

$$\frac{d\Delta}{d\theta} = \cos\theta \Rightarrow \left.\frac{d\Delta}{d\theta}\right]_{\theta=\pi/2} = \cos(\pi/2) = 0$$

96 **(a,b)**

Applying $C_3 \to C_3 - xC_2, C_2 \to C_2 - xC_1$, we obtain

$$\Delta(x) = \begin{vmatrix} 3 & 0 & 2a^2 \\ 3x & 2a^2 & 4a^2x \\ 3x^2+2a^2 & 4a^2x & 6a^2x^2+2a^2 \end{vmatrix}$$

Applying $C_3 \to C_3 - xC_2$, we get

$$\Delta(x) = 4a^4 \begin{vmatrix} 3 & 0 & 1 \\ 3x & 1 & x \\ 3x^2+2a^2 & 2x & x^2+2a^2 \end{vmatrix}$$

Applying $C_1 \to C_1 - 3C_3$, we get

$$\Delta(x) = 4a^4 \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & x \\ -4a^2 & 2x & x^2+2a^2 \end{vmatrix} = 16a^6$$

97 **(d)**

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\Delta = \begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin(B+A)\sin(B-A) & \frac{\sin(A-B)}{\sin A\sin B} & 0 \\ \sin(C+A)\sin(C-A) & \frac{\sin(A-C)}{\sin A\sin C} & 0 \end{vmatrix}$$

$$\left[\because \cot\alpha - \cot\beta = \frac{\sin(\beta-\alpha)}{\sin\alpha\sin\beta}\right]$$

Expanding along C_3, we get

$$\Delta = \frac{\sin(A-B)\sin(A-C)}{\sin A}\left[-\frac{\sin(B+A)}{\sin C} + \frac{\sin(C+A)}{\sin B}\right]$$

$$= \frac{\sin(A-B)\sin(A-C)}{\sin A}\left[-\frac{\sin(\pi-C)}{\sin C} + \frac{\sin(\pi-B)}{\sin B}\right]$$

$$= \frac{\sin(A-B)\sin(A-C)}{\sin A}\left[-\frac{\sin C}{\sin C} + \frac{\sin B}{\sin B}\right] = 0$$

98 **(a,b)**

$$\Delta = \frac{1}{a}\begin{vmatrix} a^3+ax & ab & ac \\ a^2b & b^2+x & bc \\ a^2c & bc & c^2+x \end{vmatrix}$$

Applying $C_1 \to C_1 + bC_2 + cC_3$ and taking $a^2+b^2+c^2+x$ common, we get

$$\Delta = \frac{1}{a}(a^2+b^2+c^2+x)\begin{vmatrix} a & ab & ac \\ b & b^2+x & bc \\ c & bc & c^2+x \end{vmatrix}$$

Applying $C_2 \to C_2 - bC_1$ and $C_3 \to C_3 - cC_1$, we get

$$\Delta = \frac{1}{a}(a^2+b^2+c^2+x)\begin{vmatrix} a & 0 & 0 \\ b & x & 0 \\ c & 0 & x \end{vmatrix}$$

$$= \frac{1}{a}(a^2+b^2+c^2+x)(ax^2)$$

$$= x^2(a^2+b^2+c^2+x)$$

Thus Δ is divisible by x and x^2

99 **(a,c)**

$$\because\ f(x) = \begin{vmatrix} n & n+1 & n+2 \\ {}^nP_n & {}^{n+1}P_{n+1} & {}^{n+2}P_{n+2} \\ {}^nC_n & {}^{n+1}C_{n+1} & {}^{n+2}C_{n+2} \end{vmatrix}$$

$$= \begin{vmatrix} n & n+1 & n+2 \\ n! & (n+1)! & (n+2)! \\ 1 & 1 & 1 \end{vmatrix}$$ $(\because\ {}^nP_n = n!, {}^nC_n = 1)$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

Then, $$f(x) = \begin{vmatrix} n & 1 & 2 \\ n! & n \cdot n! & (n^2+3n+1)n! \\ 1 & 0 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 2 \\ n \cdot n! & (n^2+3n+1)n! \end{vmatrix} = n!\,(n^2+n+1)$$

100 **(d)**

4. Multiplying C_1 by a, C_2 by b and C_3 by c, we obtain

$$\Delta = \frac{1}{abc}\begin{vmatrix} \frac{a}{c} & \frac{b}{c} & -\frac{a+b}{c} \\ -\frac{b+c}{c} & \frac{b}{a} & \frac{c}{a} \\ -\frac{b(b+c)}{ac} & \frac{b(a+2b+c)}{ac} & -\frac{b(a+b)}{ac} \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\Delta = \frac{1}{abc}\begin{vmatrix} 0 & \frac{b}{c} & -\frac{a+b}{c} \\ 0 & \frac{b}{a} & \frac{c}{a} \\ 0 & \frac{b(a+2b+c)}{ac} & -\frac{b(a+b)}{ac} \end{vmatrix}$$

This shows that Δ is independent of a, b and c

5. Applying $C_1 \to C_1 - (\cot b)C_2$, we get

$$\Delta = \begin{vmatrix} 0 & \sin a \sin b & \cos a \\ 0 & \cos a \sin b & -\sin a \\ -\sin a/\sin b & \sin a \cos b & 0 \end{vmatrix}$$

$= -\frac{\sin a}{\sin b}[-\sin b \sin^2 a - \cos^2 a \sin b]$ [Expanding along C_1]

$= \sin a$

6. Taking $1/\sin a \cos b, 1/\sin a \sin b, 1/\cos a$ common from C_1, C_2, C_3, respectively, we get

$$\Delta = \frac{1}{\sin^2 a \cos a \sin b \cos b}\Delta_1$$

Where $$\Delta_2 = \begin{vmatrix} 1 & 1 & 1 \\ -\cot a & -\cot a & \tan a \\ \tan b & -\cot b & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 1 & 1 \\ 0 & -\cot a & \tan a \\ 1/\sin b \cos b & -\cot b & 0 \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2$, we get

$$\Delta = \frac{1}{\sin b \cos b}[\tan a + \cot a]$$

$$= \frac{1}{\sin a \cos a \sin b \cos b}$$

7. $$\begin{vmatrix} a^2 & b \sin A & c \sin A \\ b \sin A & 1 & \cos A \\ c \sin A & \cos A & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a^2 & a \sin B & a \sin C \\ a \sin B & 1 & \cos A \\ a \sin C & \cos A & 1 \end{vmatrix}$$

$$= a^2\begin{vmatrix} 1 & \sin B & \sin C \\ \sin B & 1 & \cos A \\ \sin C & \cos A & 1 \end{vmatrix}$$

$$= a^2\begin{vmatrix} 1 & 0 & 0 \\ \sin B & 1-\sin^2 B & \cos A - \sin B \sin C \\ \sin C & \cos A - \sin B \sin C & 1-\sin^2 C \end{vmatrix}$$

[Applying $C_2 \to C_2 - (\sin B)C_1$ and $C_3 \to C_3 - (\sin C)C_1$]

$$= a^2[\cos^2 B \cos^2 C - (\cos A - \sin B \sin C)^2]$$

$$= a^2[\cos^2 B \cos^2 C - (\cos(B+C) + \sin B \sin C)^2]$$

$$= a^2[\cos^2 B \cos^2 C - \cos^2 B \cos^2 C]$$

$$= 0$$

101 **(a,b,c,d)**

Let $a \neq 0$, then on applying $C_1 \to aC_1$, we get

$$\Delta = \frac{1}{a}\begin{vmatrix} a^3+ax^2 & ab & ac \\ a^2b & b^2+x^2 & bc \\ a^2c & bc & c^2+x^2 \end{vmatrix}$$

Applying $C_1 \to C_1 + bC_2 + cC_3$

$$\Delta=\frac{1}{a}\begin{vmatrix} a(a^2+b^2+c^2+x^2) & ab & ac \\ b(a^2+b^2+c^2+x^2) & b^2+x^2 & bc \\ c(a^2+b^2+c^2+x^2) & bc & c^2+x^2 \end{vmatrix}$$

$$\Delta=\frac{1}{a}(a^2+b^2+c^2+x^2)\begin{vmatrix} a & ab & ac \\ b & b^2+x^2 & bc \\ c & bc & c^2+x^2 \end{vmatrix}$$

Applying $C_2 \to C_2 - bC_1, C_3 \to C_3 - cC_1$

$$\Delta=\frac{1}{a}(a^2+b^2+c^2+x^2)\begin{vmatrix} a & 0 & 0 \\ b & x^2 & 0 \\ c & 0 & x^2 \end{vmatrix}$$

$\therefore\ \Delta=(a^2+b^2+c^2+x^2)x^4 \quad (\because a \neq 0)$

Now, if $a=0$, then $\Delta=0$

Also, it can be easily seen that Δ is divisible by x, x^2, x^3 and x^4.

102 **(a,c)**

$$g(x)=\begin{vmatrix} a^{-x} & e^{\log_e a^x} & x^2 \\ a^{-3x} & e^{\log_e a^{3x}} & x^4 \\ a^{-5x} & e^{\log_e a^{5x}} & 1 \end{vmatrix}$$

$$=\begin{vmatrix} a^{-x} & a^x & x^2 \\ a^{-3x} & a^{3x} & x^4 \\ a^{-5x} & a^{5x} & 1 \end{vmatrix} \left(e^{\log a^x = a^x}\right)$$

$$\Rightarrow g(-x)=\begin{vmatrix} a^{x} & a^{-x} & x^2 \\ a^{3x} & a^{3x} & x^4 \\ a^{5x} & a^{-5x} & 1 \end{vmatrix}$$

$$=-\begin{vmatrix} a^{-x} & a^x & x^2 \\ a^{-3x} & a^{3x} & x^4 \\ a^{-5x} & a^{5x} & 1 \end{vmatrix}$$

[interchanging 1^{st} and 2^{nd} columns]

$=-g(x)$

$\Rightarrow g(x)+g(-x)=0$

$\Rightarrow g(x)$ is an odd function

Hence, the graph is symmetrical about origin.

Also, $g_4(x)$ is an odd function [where $g_4(x)$ is fourth derivative of $g(x)$]. Hence,

$g_4(x)=-g_4(-x)$

$\Rightarrow g_4(0)=-g_4(0)$

$\Rightarrow g_4(0)=0$

103 **(a,b)**

By partial fractions, we have

$$g(x)=\frac{f(a)}{(x-a)(a-b)(a-c)}+\frac{f(b)}{(b-a)(x-b)(b-c)}+\frac{f(c)}{(c-a)(c-b)(x-c)}$$

$$\Rightarrow g(x)=\frac{1}{(a-b)(b-c)(c-a)}\times\left[\frac{f(a)(c-b)}{(x-a)}+\frac{f(b)(a-b)}{(x-b)}+\frac{f(c)(b-a)}{(x-c)}\right]$$

$$\Rightarrow g(x)=\begin{vmatrix} 1 & a & f(a)/(x-a) \\ 1 & b & f(b)/(x-b) \\ 1 & c & f(c)/(x-c) \end{vmatrix} \div \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$\Rightarrow \int g(x)dx=\begin{vmatrix} 1 & a & f(a)\log|x-a| \\ 1 & b & f(b)\log|x-b| \\ 1 & c & f(c)\log|x-c| \end{vmatrix} \div \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}+k$$

and

$$\frac{dg(x)}{dx}=\begin{vmatrix} 1 & a & -f(a)(x-a)^{-2} \\ 1 & b & -f(b)(x-b)^{-2} \\ 1 & c & -f(c)(x-c)^{-2} \end{vmatrix} \div \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$=\begin{vmatrix} 1 & a & f(a)(x-a)^{-2} \\ 1 & b & f(b)(x-b)^{-2} \\ 1 & c & f(c)(x-c)^{-2} \end{vmatrix} \div \begin{vmatrix} a^2 & a & 1 \\ b^2 & b & 1 \\ c^2 & c & 1 \end{vmatrix}$$

104 **(b,c)**

$\because\ x=\frac{a}{b-c}, y=\frac{b}{c-a}, z=\frac{c}{a-b}$

or $-a+bx-cx=0, -ay-b+cy=0, az-bz-c=0$

Now, on eliminating a, b, c, we get

$$\begin{vmatrix} -1 & x & -x \\ -y & -1 & y \\ z & -z & -1 \end{vmatrix}=0$$

$$\Rightarrow (-1)^3\begin{vmatrix} 1 & -x & x \\ y & 1 & -y \\ -z & z & 1 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} 1 & -x & x \\ y & 1 & -y \\ -z & z & 1 \end{vmatrix}=0$$

Also, on applying $C_1 \to C_1+C_2+C_3$, we get

$$\begin{vmatrix} 1 & -x & x \\ 1 & 1 & -y \\ 1 & z & 1 \end{vmatrix}=0$$

105 **(a,b)**

Applying $R_1 \to R_1+\sin\phi(R_2)+\cos\phi(R_3)$,

$$f(x)=\Delta=\begin{vmatrix} 0 & 0 & \cos 2\phi+1 \\ \sin\theta & \cos\theta & \sin\phi \\ -\cos\phi & \sin\theta & \cos\phi \end{vmatrix}$$

$=(\cos 2\phi+1)(\sin^2\theta+\cos^2\theta)$

$=(1+\cos 2\phi)$

Hence, Δ is independent of θ

106 **(b,c)**

$$z = e^{iA}e^{iB}e^{iC}\begin{vmatrix} e^{iA} & e^{-i(C+A)} & e^{-i(B+A)} \\ e^{-i(C+B)} & e^{iB} & e^{-i(A+B)} \\ e^{-e(B+C)} & e^{-i(A+C)} & e^{iC} \end{vmatrix}$$

$$\Rightarrow z = -\begin{vmatrix} e^{iA} & -e^{iB} & -e^{iC} \\ -e^{iA} & e^{iB} & -e^{iC} \\ -e^{iA} & -e^{iB} & e^{iC} \end{vmatrix}$$

$(\because e^{i(A+B+C)} = e^{i\pi} = \cos\pi + i\sin\pi = -1)$

Applying $R_1 \to R_1 + R_3, R_2 \to R_2 + R_3$

$$\Rightarrow z = -\begin{vmatrix} 0 & -2e^{iB} & 0 \\ -2e^{iA} & 0 & 0 \\ -e^{iA} & -e^{iB} & e^{iC} \end{vmatrix}$$

$\Rightarrow z = 2e^{iB}\{2e^{i(A+C)}\}$

$\Rightarrow z = 4e^{i(A+B+C)} = 4e^{i\pi} = -4$

107 **(a,c)**

$f(\theta) = \sin^3\theta + \cos^3\theta - \cos\theta\sin\theta(\sin\theta + \cos\theta)$

$= (\sin\theta + \cos\theta)^3 - 4\sin\theta\cos\theta(\sin\theta + \cos\theta)$

$= (\sin\theta + \cos\theta)[1 - \sin 2\theta]$

Now, $f(\theta) = 0$

$\Rightarrow \tan\theta = -1$ or $\sin 2\theta = 1$

$\Rightarrow f(\theta) = 0$ has 2 real solutions in $[0, \pi]$

Also, $\frac{f(\theta)}{1-\sin 2\theta} = \sin\theta + \cos\theta \in [-\sqrt{2}, \sqrt{2}]$

108 **(b,c)**

In the left-hand determinant, each element is the cofactor of the elements of the determinant

$$\begin{vmatrix} x & y & z \\ y & z & x \\ z & x & y \end{vmatrix} = \Delta^* \text{ (say)}$$

Hence,

$$\Delta^{*2} = \begin{vmatrix} x & y & z \\ y & z & x \\ z & x & y \end{vmatrix}\begin{vmatrix} x & y & z \\ y & z & x \\ z & x & y \end{vmatrix}$$

$$= \begin{vmatrix} x^2+y^2+z^2 & xy+yz+zx & xz+yx+zy \\ \Sigma xy & \Sigma x^2 & \Sigma xy \\ \Sigma xy & \Sigma xy & \Sigma x^2 \end{vmatrix}$$

$$= \begin{vmatrix} r^2 & u^2 & u^2 \\ u^2 & r^2 & u^2 \\ u^2 & u^2 & r^2 \end{vmatrix}$$ [Since $x^2 + y^2 + z^2 = r^2, xy + yz + zx = u^2$]

109 **(a,b,c)**

We have,

$$\begin{vmatrix} bc & ca & ab \\ ca & ab & bc \\ ab & bc & ca \end{vmatrix} = 0$$

$\Rightarrow (ab)^3 + (bc)^3 + (ca)^3 - 3(ab)(bc)(ca) = 0$

$\Rightarrow (ab + bc\omega^2 + ca\omega)(ab\omega + bc\omega^2 + ca)(ab\omega^2 + bc\omega + ca) = 0$

$\Rightarrow ab + bc\omega^2 + ca\omega = 0, ab\omega + bc\omega^2 + ca = 0, ab\omega^2 + bc\omega + ca = 0$

$\Rightarrow \frac{1}{c\omega^2} + \frac{1}{a} + \frac{1}{b\omega} = 0, \frac{1}{c\omega} + \frac{1}{a} + \frac{1}{b\omega^2} = 0, \frac{1}{c} + \frac{1}{a\omega} + \frac{1}{b\omega^2} = 0$

$\Rightarrow \frac{1}{a} + \frac{1}{b\omega} + \frac{1}{c\omega^2} = 0, \frac{1}{a} + \frac{1}{b\omega^2} + \frac{1}{c\omega} = 0, \frac{1}{a\omega} + \frac{1}{b\omega^2} + \frac{1}{c} = 0$

110 **(a,b,c)**

Applying $R_3 \to R_3 - xR_2$ and $R_2 \to R_2 - xR_1$, we get

$$f(x) = \begin{vmatrix} a & -1 & 0 \\ 0 & a+x & -1 \\ 0 & 0 & a+x \end{vmatrix} = a(a+x)^2$$

Hence,

$f(2x) - f(x) = a[(a+2x)^2 - (a+x)^2]$

$= a(a + 2x - a - x)(a + 2x + a + x) = ax(2a + 3x)$

111 **(a,b)**

$$\begin{vmatrix} 1 & k & 3 \\ k & 2 & 2 \\ 2 & 3 & 4 \end{vmatrix} = 0$$

$\Rightarrow 8 + 4k + 9k - 12 - 4k^2 - 6 = 0$

$\Rightarrow 4k^2 - 13k + 10 = 0$

$\Rightarrow 4k^2 - 8k - 5k + 10 = 0$

$\Rightarrow (2k - 5)(k - 2) = 0$

$\Rightarrow k = 5/2, 2$

112 **(c,d)**

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\Delta = \begin{vmatrix} a+b-x & a & b \\ a+b-x & -x & a \\ a+b-x & b & -x \end{vmatrix}$$

$$= (a+b-x)\begin{vmatrix} 1 & a & b \\ 1 & -x & a \\ 1 & b & -x \end{vmatrix}$$

$$= (a+b-x)\begin{vmatrix} 1 & a & b \\ 0 & -x-a & a-b \\ 0 & b-a & -x-b \end{vmatrix}$$

[Applying $R_2 \longrightarrow R_2 - R_1$ and $R_3 \longrightarrow R_3 - R_1$]

$= (a + b - x)[(x + a)(x + b) + (a - b)^2]$

[expanding along C_1]

$= (a + b - x)[x^2 + (a + b)x + a^2 + b^2 - ab]$

113 **(a,b,c)**

$$\begin{vmatrix} 8 & 2 & 7 \\ 12 & 3 & 5 \\ 16 & 4 & 3 \end{vmatrix} = \begin{vmatrix} 8 & 2 & 7 \\ 4 & 1 & -2 \\ 4 & 1 & -2 \end{vmatrix}$$ [$R_3 \to R_3 - R_2$ and $R_2 \to R_2 - R_1$]

$= 0$

$$\begin{vmatrix} 1/a & a^2 & bc \\ 1/b & b^2 & ac \\ 1/c & c^2 & ab \end{vmatrix} = \frac{1}{abc}\begin{vmatrix} 1 & a^3 & abc \\ 1 & b^3 & abc \\ 1 & c^3 & abc \end{vmatrix}$$

[$R_1 \to aR_1, R_2 \to bR_2, R_3 \to bR_3$]

$$= \frac{abc}{abc}\begin{vmatrix} 1 & a^3 & 1 \\ 1 & b^3 & 1 \\ 1 & c^3 & 1 \end{vmatrix}$$ [taking abc common from C_3]

$= 0$

$$\begin{vmatrix} a+b & 2a+b & 3a+b \\ 2a+b & 3a+b & 4a+b \\ 4a+b & 5a+b & 6a+b \end{vmatrix} = \begin{vmatrix} a+b & 2a+b & 3a+b \\ a & a & a \\ 2a & 2a & 2a \end{vmatrix}$$

$[R_3 \to R_3 - R_2, R_2 \to R_2 - R_1]$

$= 0$

$$\begin{vmatrix} 2 & 43 & 6 \\ 7 & 35 & 4 \\ 3 & 17 & 2 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 6 \\ 7 & 7 & 4 \\ 3 & 3 & 2 \end{vmatrix} \quad [C_2 \to C_2 - 7C_3]$$

$$= \begin{vmatrix} 1 & 1 & 6 \\ 0 & 7 & 4 \\ 0 & 3 & 2 \end{vmatrix} \quad [C_1 \to C_1 - C_2]$$

$= 2$

114 **(a,c)**

$$\begin{vmatrix} \cos\alpha & -\sin\alpha & 1 \\ \sin\alpha & \cos\alpha & 1 \\ \cos(\alpha+\beta) & -\sin(\alpha-\beta) & 1 \end{vmatrix} = \begin{vmatrix} \cos\alpha & -\sin\alpha & 1 \\ \sin\alpha & \cos\alpha & 1 \\ 0 & 0 & 1+\sin\beta-\cos\beta \end{vmatrix}$$

[Applying $R_3 \to R_3 - R_1(\cos\beta) + R_2(\sin\beta)$]

$= (1+\sin\beta-\cos\beta)(\cos^2\alpha+\sin^2\alpha) = 1 + \sin\beta - \cos\beta$ which is independent of α

115 **(a,c)**

$$\begin{vmatrix} {}^{x}C_r & {}^{n-1}C_r & {}^{n}C_r \\ {}^{x+1}C_r & {}^{n}C_r & {}^{n+1}C_r \\ {}^{x+2}C_r & {}^{n+1}C_r & {}^{n+2}C_r \end{vmatrix} = 0 \quad \text{(i)}$$

$$\Rightarrow \begin{vmatrix} \frac{x!}{r!\,(x-r)!} & \frac{(n-1)!}{r!\,(n-r-1)!} & \frac{n!}{r!\,(n-r)!} \\ \frac{(x+1)!}{r!\,(x+1-r)!} & \frac{n!}{r!\,(n-r)!} & \frac{(n+1)!}{r!\,(n-r+1)!} \\ \frac{(x+2)!}{r!\,(x+2-r)!} & \frac{(n+1)!}{r!\,(n+1-r)!} & \frac{(n+2)!}{r!\,(n-r+2)!} \end{vmatrix} = 0$$

Taking $\frac{x!}{r!(x+2-r)!}$ common from C_1, we have quadratic equation in x

Now in (i), if we put $x = n-1$, C_1 and C_2 are the same, hence $x = n-1$ is one root of the equation

If we put $x = n$, then C_1 and C_3 are same. Hence, $x = n$ is the other root

116 **(a,b,c)**

Operating $C_1 \to C_2 - C_1, C_3 \to C_3 - C_1$, we get

$$\Delta = \begin{vmatrix} 1 & ac & bc \\ 1 & ad & bd \\ 1 & ae & be \end{vmatrix} = ab \begin{vmatrix} 1 & c & c \\ 1 & d & d \\ 1 & e & e \end{vmatrix} ab(0) = 0$$

117 **(b,c)**

$$\Delta'(x) = \begin{vmatrix} 2x+4 & 2x+4 & 13 \\ 4x+5 & 4x+5 & 26 \\ 16x-6 & 16x-6 & 104 \end{vmatrix} + \begin{vmatrix} x^2+4x-3 & 2 & 13 \\ 2x^2+5x-9 & 4 & 26 \\ 8x^2-6x+1 & 16 & 104 \end{vmatrix}$$

$= 0 + 2 \times 13 \times (0) = 0$

$\Rightarrow \Delta(x) = \text{constant} \Rightarrow a = 0, b = 0, c = 0$

118 **(a,c)**

$$f(n) = \begin{vmatrix} n & n+1 & n+2 \\ n! & (n+1)! & (n+2)! \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} n & 1 & 1 \\ n! & nn! & (n+1)(n+1)! \\ 1 & 0 & 0 \end{vmatrix}$$

[Applying $C_3 \to C_3 - C_2$ and $C_2 \to C_2 - C_1$]

$= (n+1)(n+1)! - nn! = n!\,[(n+1)^2 - n]$

$= n!\,(n^2+n+1)$

Thus, $f(n)$ is divisible by $n!$ and n^2+n+1

119 **(a)**

We are given that

$1 + bc + qr = 0 \quad$ (i)

$1 + ca + pr = 0 \quad$ (ii)

$1 + ab + pq = 0 \quad$ (iii)

The determinant in the question involves a column consisting the elements ap, bq and cr. So multiplying (i), (ii) and (iii) by ap, bq and cr, respectively, we get

$ap + abcp + apqr = 0 \quad$ (iv)

$bq + abcq + bpqr = 0 \quad$ (v)

$cq + abcr + cpqr = 0 \quad$ (vi)

Since abc and pqr occur in all the three equations, putting $abc = x, pqr = y$, we get the system

$ap + px + ay = 0$

$bq + qx + by = 0 \quad$ (vii)

$cr + rx + cy = 0$

System (vii) must have a common solution (i.e., system is consistent). So,

$$\begin{vmatrix} ap & p & a \\ bq & q & b \\ cr & r & c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} ap & a & p \\ bq & b & q \\ cr & c & r \end{vmatrix} = 0$$

120 **(b)**

Let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = 3$. Now,

$$\Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ \mu & 2 & \lambda \end{vmatrix}$$

$$= \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ \mu & 2 & 3 \end{vmatrix} = \mu - 10$$

$$\Delta_2 = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 10 & 3 \\ 1 & \mu & \lambda \end{vmatrix} = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 10 & 3 \\ 1 & \mu & 3 \end{vmatrix} = 20 - 2\mu$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 1 & 2 & \mu \end{vmatrix} = \mu - 10$$

Clearly, for $\mu = 10$, all of $\Delta_1, \Delta_2, \Delta_3$ are zero

121 **(a)**

$\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A + \sin^2 A & \sin B + \sin^2 B & \sin C + \sin^2 C \end{vmatrix} = 0 \quad (1)$

Then $A = B$ or $B = C$ or $C = A$, for which any two rows are same.

For (1) to hold it is not necessary that all the three rows are same or $A = B = C$

122 **(d)**

$$\because \ \Delta = \begin{vmatrix} 5 & 4 & 3 \\ x51 & y41 & z31 \\ x & y & z \end{vmatrix}$$

$$= \begin{vmatrix} 5 & 4 & 3 \\ 100x+51 & 100y+41 & 100z+31 \\ x & y & z \end{vmatrix}$$

$$= \begin{vmatrix} 5 & 4 & 3 \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} \quad (R_2 = R_2 - 100R_3 - 10R_1)$$

Which is zero provided x, y, z are in AP.

123 **(a)**

Let $f(\theta) = \begin{vmatrix} \cos(\theta+\alpha) & \cos(\theta+\beta) & \cos(\theta+\gamma) \\ \sin(\theta+\alpha) & \sin(\theta+\beta) & \sin(\theta+\gamma) \\ \sin(\beta-\gamma) & \sin(\gamma-\alpha) & \sin(\alpha-\beta) \end{vmatrix}$

$\therefore$

$f'(\theta) =$

$\begin{vmatrix} -\sin(\theta+\alpha) & -\sin(\theta+\beta) & -\sin(\theta+\gamma) \\ \sin(\theta+\alpha) & \sin(\theta+\beta) & \sin(\theta+\gamma) \\ \sin(\beta-\gamma) & \sin(\gamma-\alpha) & \sin(\alpha-\beta) \end{vmatrix} +$

$\begin{vmatrix} \cos(\theta+\alpha) & \cos(\theta+\beta) & \cos(\theta+\gamma) \\ \cos(\theta+\alpha) & \cos(\theta+\beta) & \cos(\theta+\gamma) \\ \sin(\beta-\gamma) & \sin(\gamma-\alpha) & \sin(\alpha-\beta) \end{vmatrix} +$

$\begin{vmatrix} \cos(\theta+\alpha) & \cos(\theta+\beta) & \cos(\theta+\gamma) \\ \sin(\theta+\alpha) & \sin(\theta+\beta) & \sin(\theta+\gamma) \\ 0 & 0 & 0 \end{vmatrix}$

$= 0 + 0 + 0 = 0$

$\Rightarrow f'(\theta) = 0 \ \Rightarrow f(\theta) = c$

124 **(d)**

$\Delta = \begin{vmatrix} a-1 & a & a+1 \\ b-1 & b & b+1 \\ c-1 & c & c+1 \end{vmatrix} = \begin{vmatrix} 0 & a & a+1 \\ 0 & b & b+1 \\ 0 & c & c+1 \end{vmatrix}$

$(C_1 \to C_1 + C_3 - 2C_2)$

$\therefore \ \Delta = 0$, which is not a natural number.

125 **(a)**

For $x = 0$, the determinant reduces to the determinant of a skew-symmetric matrix of odd order which is always zero. Hence, $x = 0$ is the solution of the given equation

126 **(a)**

As the given system of equations has non-trivial solutions, hence

$$\begin{vmatrix} \lambda & b-a & c-a \\ a-b & \lambda & c-b \\ a-c & b-c & \lambda \end{vmatrix} = 0$$

When $\lambda = 0$, then the determinant becomes skew-symmetric of odd order, which is equal to zero. Thus, $\lambda = 0$

127 **(a)**

$\Delta = \begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix} \begin{vmatrix} 0 & m & n \\ -m & 0 & k \\ -n & -k & 0 \end{vmatrix}$ where

$\begin{vmatrix} 0 & m & n \\ -m & 0 & k \\ -n & -k & 0 \end{vmatrix}$ is skew symmetric

$\therefore \ \Delta = 0$

128 **(b)**

The system of equations $kx + y + z = 1, x + ky + z = k, x + y + kz = k^2$ is inconsistent if

$\Delta = \begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix} = 0$ and one of $\Delta_1, \Delta_2, \Delta_3$ is non-zero where

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ k & k & 1 \\ k^2 & 1 & k \end{vmatrix}, \Delta_2 = \begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & k^2 & k \end{vmatrix} \Delta_3$$
$$= \begin{vmatrix} k & 1 & 1 \\ 1 & k & k \\ 1 & 1 & k^2 \end{vmatrix}$$

We have, $\Delta = (k+2)(k-1)^2, \Delta_1 = -(k+$
1k−12,

$\Delta_2 = -k(k-1)^2, \Delta_3 = (k+1)^2(k-1)^2$

The determinant give in statement 2 is $\Delta_1 = 0$, for which $k = 1$ or $k = -1$

$k = 1$ makes all the determinants zero. But for $k = -1$, all the determinants are not zero

Hence, both statements are true but statement 2 is not correct explanation of statement 1

129 **(d)**

$$\because \frac{d}{dx} f(x)g(x) = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$$

$$\Rightarrow \frac{d}{dx} f(x)g(x) \neq \frac{d}{dx} f(x) \frac{d}{dx} g(x)$$

Given, $\Delta(x) = \begin{vmatrix} f_1(x) & f_2(x) \\ g_1(x) & g_2(x) \end{vmatrix}$

$= f_1(x)g_2(x) - f_2(x)g_1(x)$

$$\therefore \frac{d}{dx}\{\Delta(x)\} = \{f_1'(x)g_2(x) + g_2'(x)f_1(x)\}$$
$$-\{f_2(x)g_1'(x) + g_1(x)f_2'(x)\}$$

$$\Delta'(x) = \begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1(x) & g_2(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & f_2(x) \\ g_1'(x) & g_2'(x) \end{vmatrix}$$
$$\neq \begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1'(x) & g_2'(x) \end{vmatrix}$$

130 **(b)**

$\Delta = \Delta_1\Delta_2$ where $\Delta_1 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and

$\Delta_2 = \begin{vmatrix} 1 & x^2 & 0 \\ x^2 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

Hence, both the statements are true but statement 2 is not correct explanation of statement 1

131 **(a)**
Let $f(x) = a_0 + a_1x + a_2x^2 + \ldots$

$\therefore f'(x) = 0 + a_1 + 2a_2x + \ldots$

or $f'(0) = a_1$

$$\therefore a_1 = \begin{vmatrix} 21 & 22 & 23 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ 31 & 32 & 33 \\ 1 & 1 & 1 \end{vmatrix}$$
$$+ \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 41 & 42 & 43 \end{vmatrix}$$

$= 0 + 0 + 0 = 0$

132 **(d)**

8. Multiplying C_1 by a, C_2 by b and C_3 by c, we obtain

$$\Delta=\frac{1}{abc}\begin{vmatrix}\frac{a}{c} & \frac{b}{c} & -\frac{a+b}{c}\\ -\frac{b+c}{c} & \frac{b}{a} & \frac{c}{a}\\ -\frac{b(b+c)}{ac} & \frac{b(a+2b+c)}{ac} & -\frac{b(a+b)}{ac}\end{vmatrix}$$

Applying $C_1 \to C_1+C_2+C_3$, we get

$$\Delta=\frac{1}{abc}\begin{vmatrix}0 & \frac{b}{c} & -\frac{a+b}{c}\\ 0 & \frac{b}{a} & \frac{c}{a}\\ 0 & \frac{b(a+2b+c)}{ac} & -\frac{b(a+b)}{ac}\end{vmatrix}$$

This shows that Δ is independent of a, b and c

9. Applying $C_1 \to C_1-(\cot b)C_2$, we get

$$\Delta=\begin{vmatrix}0 & \sin a\sin b & \cos a\\ 0 & \cos a\sin b & -\sin a\\ -\sin a/\sin b & \sin a\cos b & 0\end{vmatrix}$$

$=-\frac{\sin a}{\sin b}[-\sin b\sin^2 a-\cos^2 a\sin b]$ [Expanding along C_1]

$=\sin a$

10. Taking $1/\sin a\cos b, 1/\sin a\sin b, 1/\cos a$ common from C_1, C_2, C_3, respectively, we get

$$\Delta=\frac{1}{\sin^2 a\cos a\sin b\cos b}\Delta_1$$

Where $\Delta_2=\begin{vmatrix}1 & 1 & 1\\ -\cot a & -\cot a & \tan a\\ \tan b & -\cot b & 0\end{vmatrix}$

$$=\begin{vmatrix}0 & 1 & 1\\ 0 & -\cot a & \tan a\\ 1/\sin b\cos b & -\cot b & 0\end{vmatrix}$$

Applying $C_1 \to C_1-C_2$, we get

$$\Delta=\frac{1}{\sin b\cos b}[\tan a+\cot a]$$

$$=\frac{1}{\sin a\cos a\sin b\cos b}$$

11. $\begin{vmatrix}a^2 & b\sin A & c\sin A\\ b\sin A & 1 & \cos A\\ c\sin A & \cos A & 1\end{vmatrix}$

$$=\begin{vmatrix}a^2 & a\sin B & a\sin C\\ a\sin B & 1 & \cos A\\ a\sin C & \cos A & 1\end{vmatrix}$$

$$=a^2\begin{vmatrix}1 & \sin B & \sin C\\ \sin B & 1 & \cos A\\ \sin C & \cos A & 1\end{vmatrix}$$

$$=a^2\begin{vmatrix}1 & 0 & 0\\ \sin B & 1-\sin^2 B & \cos A-\sin B\sin C\\ \sin C & \cos A-\sin B\sin C & 1-\sin^2 C\end{vmatrix}$$

[Applying $C_2 \to C_2-(\sin B)C_1$ and $C_3 \to C_3-(\sin C)C_1$]

$=a^2[\cos^2 B\cos^2 C-(\cos A-\sin B\sin C)^2]$

$=a^2[\cos^2 B\cos^2 C-(\cos(B+C)+\sin B\sin C)^2]$

$=a^2[\cos^2 B\cos^2 C-\cos^2 B\cos^2 C]$

$=0$

133 **(c)**

1. Coefficient of x in $f(x)$ is coefficient of x in $\begin{vmatrix}x & 1 & 1\\ 1 & x & 2\\ x^2 & 1 & 0\end{vmatrix}$

Therefore, coefficient of x is -2

2. Let $D=\begin{vmatrix}1 & 3\cos\theta & 1\\ \sin\theta & 1 & 3\cos\theta\\ 1 & \sin\theta & 1\end{vmatrix}$

$=(3\cos\theta-\sin\theta)^2$

$\Delta_{\max}=10$

3. $f'(x)=0$

$\Rightarrow f'(0)=0$

4. $a_0=\begin{vmatrix}0 & 2 & 0\\ 1 & 0 & 6\\ 0 & 0 & 1\end{vmatrix}=-2(1)=-2$

134 **(b)**

1. The given determinant is

$$\Delta=\begin{vmatrix}x+2 & x+3 & x+5\\ x+4 & x+6 & x+9\\ x+8 & x+11 & x+15\end{vmatrix}$$

Applying $R_2 \to R_2-R_1$ and $R_3 \to R_3-R_2$, we have

$$\Delta=\begin{vmatrix}x+2 & x+3 & x+5\\ 2 & 3 & 4\\ 4 & 5 & 6\end{vmatrix}$$

$= 2\begin{vmatrix} x & x & x+1 \\ 2 & 3 & 4 \\ 1 & 1 & 1 \end{vmatrix}$ [Applying $R_1 \to R_1 - R_2$ and $R_3 \to R_3 - R_2$]

$= 2\begin{vmatrix} x & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix}$ [Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_2$]

$= -2$ [Expanding along R_3]

2. $\begin{vmatrix} 7 & 6 & x^2-13 \\ 2 & x^2-13 & 2 \\ x^2-13 & 3 & 7 \end{vmatrix}$

Let $x^2 - 13 = t$. Then

$t^3 - 67t + 126 = 0$

$\Rightarrow t = -9, 2, 7 \Rightarrow x = \pm 2, \pm\sqrt{20}, \pm\sqrt{15}$

Hence sum of other five roots is 2

3. $\Delta = \begin{vmatrix} \sqrt{6} & 2i & 3+\sqrt{6} \\ \sqrt{12} & \sqrt{3}+\sqrt{8}i & 3\sqrt{2}+\sqrt{6}i \\ \sqrt{18} & \sqrt{2}+\sqrt{12}i & \sqrt{27}+2i \end{vmatrix}$

Taking $\sqrt{6}$ common from C_1, we get

$\Delta = \sqrt{6}\begin{vmatrix} 1 & 2i & 3+\sqrt{6} \\ \sqrt{2} & \sqrt{3}+2\sqrt{2}i & 3\sqrt{2}+\sqrt{6}i \\ \sqrt{3} & \sqrt{2}+2\sqrt{3}i & 3\sqrt{3}+2i \end{vmatrix}$

Applying $R_2 \to R_2 - \sqrt{2}R_1$ and $R_3 \to R_3\sqrt{3}R_1$, we get

$\Delta = \sqrt{6}\begin{vmatrix} 1 & 2i & 3+\sqrt{6} \\ 0 & \sqrt{3} & \sqrt{6}i - 2\sqrt{3} \\ 0 & \sqrt{2} & 2i - 3\sqrt{2} \end{vmatrix}$

$= \sqrt{6}\begin{vmatrix} \sqrt{3} & \sqrt{6i} - 2\sqrt{3} \\ \sqrt{2} & 2i - 3\sqrt{2} \end{vmatrix}$

$= \sqrt{6}\begin{vmatrix} \sqrt{3} & -2\sqrt{3} \\ \sqrt{2} & -3\sqrt{2} \end{vmatrix}$ [Applying $C_2 \to C_2 - \sqrt{2}iC_1$]

$= \sqrt{6}(-3\sqrt{6} + 2\sqrt{6})$

$= -6$, which is an integer

4. $f(\theta) = \begin{vmatrix} \cos^2\theta & \cos\theta\sin\theta & -\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & \cos\theta \\ \sin\theta & -\cos\theta & 0 \end{vmatrix}$

Applying $R_1 \to R_1 + (\sin\theta)R_3$ and $R_2 \to R_2 - (\cos\theta)R_3$, we get

$f(\theta) = \begin{vmatrix} 1 & 0 & -\sin\theta \\ 0 & 1 & \cos\theta \\ \sin\theta & -\cos\theta & 0 \end{vmatrix}$

$= \sin^2\theta + \cos^2\theta = 1$

135 **(c)**

Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 4 & 2 \\ 3 & -4 & 6 \end{bmatrix}$

Cofactor of $(-1) = -\begin{vmatrix} 0 & 2 \\ 3 & 6 \end{vmatrix} = 6$

Cofactor of $(1) = \begin{vmatrix} 4 & 2 \\ -4 & 6 \end{vmatrix} = 24 + 8 = 32$

Cofactor of $(3) = \begin{vmatrix} -1 & 0 \\ 4 & 2 \end{vmatrix} = -2 - 0 = -2$

Cofactor of $(6) = \begin{vmatrix} 1 & -1 \\ 0 & 4 \end{vmatrix} = 4$

136 **(d)**

Given, $A = \begin{bmatrix} a & b \\ c & a \end{bmatrix}$,

a, b, c $\epsilon\{0, 1, 2, \ldots\ldots, p-1\}$

If A is skew-symmetric matrix, then $a = 0, b = -c$

$\therefore \quad |A| = -b^2$.

Thus, p divides $|A|$only when $b = 0$

Again, if A is symmetric matrix, then $\text{b} = c$ and $|A| = a^2 - b^2$

Thus, p divides $|A|$if either p divides $(a-b)$or p divides $(a+b)$.

p divides $(a-b)$,

p divides $(a-b)$, only when $a = b$

$ie, a = b\epsilon\{0, 1, 2, \ldots\ldots, (p-1)\}$

$ie,$ p choices

p divides $(a+b)$.

$\Rightarrow$ p choices, including $a = b = 0$ included in (i)

$\therefore$ Total number of choices are $(p + p - 1) = 2p - 1$

137 **(c)**

$\because a + b + c = p, ab + bc + ca = 0$

$\therefore a^2 + b^2 + c^2 = (a+b+c)^2 - 2(ab+bc+ca)$

$= p^2 - 0 = p^2$

If $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$\therefore \ \Delta^2 = \begin{vmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ca-b^2 \end{vmatrix} = \Delta^{3-1}$

$= \Delta^2 = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$

$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$= \begin{vmatrix} a^2+b^2+c^2 & ab+bc+ca & ab+bc+ca \\ ab+bc+ca & a^2+b^2+c^2 & ab+bc+ca \\ ab+bc+ca & ab+bc+ca & a^2+b^2+c^2 \end{vmatrix}$

$$= \begin{vmatrix} p^2 & 0 & 0 \\ 0 & p^2 & 0 \\ 0 & 0 & p^2 \end{vmatrix} = p^6$$

138 **(c)**

In given determinant applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_2$, we get

$$f(x) = \begin{vmatrix} x + c_1 & a - c_1 & 0 \\ x + b & c_2 - b & a - c_2 \\ x + b & 0 & c_3 - b \end{vmatrix}$$

$$= x \begin{vmatrix} 1 & a - c_1 & 0 \\ 1 & c_2 - b & a - c_2 \\ 1 & 0 & c_3 - b \end{vmatrix} + \begin{vmatrix} c_1 & a - c_1 & 0 \\ b & c_2 - b & a - c_2 \\ b & 0 & c_3 - b \end{vmatrix}$$

So, $f(x)$ is linear. Let $f(x) = Px + Q$. Then

$f(-a) = -aP + Q, f(-b) = -bP + Q$

Then, $f(0) = 0 \times P + Q \Rightarrow Q = \frac{bf(-a) - af(-b)}{(b-a)}$ (1)

Also,

$$f(-a) = \begin{vmatrix} c_1 - a & 0 & 0 \\ b - a & c_2 - a & 0 \\ b - a & b - a & c_3 - a \end{vmatrix}$$

$= (c_1 - a)(c_1 - a)(c_3 - a)$

Similarly,

$f(-b) = (c_1 - b)(c_2 - b)(c_3 - x)$

$g(x) = (c_1 - x)(c_2 - x)(c_3 - x) \Rightarrow g(a) = f(-a)$ and $g(b) = f(-b)$

Now from (1), we get

$$f(0) = \frac{bg(a) - ag(b)}{(b - a)}$$

139 **(d)**

$$\Delta = \frac{1}{a} \begin{vmatrix} a^3 + ax & ab & ac \\ a^2 b & b^2 + x & bc \\ a^2 c & bc & c^2 + x \end{vmatrix}$$

Applying $C_1 \to C_1 + bC_2 + cC_3$ and taking $a^2 + b^2 + c^2 + x$ common, we get

$$\Delta = \frac{1}{a}(a^2 + b^2 + c^2 + x) \begin{vmatrix} a & ab & ac \\ b & b^2 + x & bc \\ c & bc & c^2 + x \end{vmatrix}$$

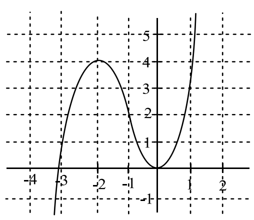
Applying $C_2 \to C_2 + bC_1$ and $C_3 \to C_3 + cC_1$, we get

$$\Delta = \frac{1}{a}(a^2 + b^2 + c^2 + x) \begin{vmatrix} a & 0 & 0 \\ b & x & 0 \\ c & 0 & x \end{vmatrix}$$

$$= \frac{1}{a}(a^2 + b^2 + c^2 + x)(ax^2)$$

$= x^2(a^2 + b^2 + c^2 + x)$

Thus Δ is divisible by x and x^2. Also, graph of $f(x)$ is



140 **(c)**

The system of equations

$-x + cy + bz = 0$ (1)

$cx - y + az = 0$ (2)

$bx + ay - z = 0$ (3)

Has a non-zero solution if

$$\Delta = \begin{vmatrix} -1 & c & b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

$\Rightarrow a^2 + b^2 + c^2 + 2abc - 1 = 0$

$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1$ (4)

Then clearly the system has infinitely many solutions. From (1) and (2), we have

$$\frac{x}{ac + b} = \frac{y}{bc + a} = \frac{z}{1 - c^2}$$

$$\therefore \frac{x^2}{(ac + b)^2} = \frac{y^2}{(bc + a)^2} = \frac{z^2}{(1 - c^2)^2}$$

or $\frac{x^2}{(1-a^2)(1-c^2)} = \frac{y^2}{(1-b^2)(1-c^2)} = \frac{z^2}{(1-c^2)^2}$ [from (4)]

or $\frac{x^2}{1-a^2} = \frac{y^2}{1-b^2} = \frac{z^2}{1-c^2}$ (5)

From (5), we see that $1 - a^2, 1 - b^2, 1 - c^2$ are all positive or all negative. Given that one of a, b, c is proper fraction, so

$1 - a^2 > 0, 1 - b^2 > 0, 1 - c^2 > 0$, which gives

$a^2 + b^2 + c^2 < 3$ (6)

Using (4) and (6), we get

$1 < 3 + 2abc$

or $abc > -1$ (7)

141 **(a)**

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} = 2\lambda + 3 + 2 - 2 - \lambda - 6 = \lambda - 3$$

$$\Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ \mu & 2 & \lambda \end{vmatrix}$$

$= 12\lambda + 3\mu + 20 - 2\mu - 10\lambda - 36$

$= 2\lambda + \mu - 16$

$$\Delta_2 = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 10 & 3 \\ 1 & \mu & \lambda \end{vmatrix} = 10\lambda + 18 + \mu - 10 - 3\mu - 6\lambda$$

$= 4\lambda - 2\mu + 8$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 1 & 2 & \mu \end{vmatrix} = 2\mu + 10 + 12 - 12 - \mu - 20$$

$= \mu - 10$

Thus the system has unique solutions if $\Delta \neq 0$ or $\lambda \neq 3$ and the system has infinite solutions if $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$ or $\lambda = 3$ and $\mu = 10$. System has no solution if $\Delta = 0$ and at least one of $\Delta_1, \Delta_2, \Delta_3$ is non-zero or $\lambda = 3$ and $\mu \neq 10$

142 **(d)**

$$\Delta = \begin{vmatrix} 1 + 1 + 1 & 1 + \alpha + \beta & 1 + \alpha^2 + \beta^2 \\ 1 + \alpha + \beta & 1 + \alpha^2 + \beta^2 & 1 + \alpha^3 + \beta^3 \\ 1 + \alpha^2 + \beta^2 & 1 + \alpha^3 + \beta^3 & 1 + \alpha^4 + \beta^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}$$ [multiplying row by row]

$= D^2$ (say)

Now,

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}$$

$= (1-\alpha)(\alpha-\beta)(\beta-1)$

$= (\beta-\alpha)[\alpha\beta-\alpha-\beta+1]$

$= (\beta-\alpha)\left(\frac{c}{a}+\frac{b}{a}+1\right) = \frac{(\beta-\alpha)}{a}(a+b+c)$

$\therefore \Delta = D^2 = \frac{(\beta-\alpha)^2}{a^2}(a+b+c)^2$

$= \frac{1}{a^2}(a+b+c)^2\left[\frac{b^2}{a^2}-4\frac{c}{a}\right]$

$= \frac{1}{a^4}(a+b+c)^2(b^2-4ac)$

If $\Delta < 0$, i.e., $b^2-4ac<0$, then roots are imaginary

If one root is $1+\sqrt{2}$ and since coefficients are real, the other root is $1-\sqrt{2}$. Hence the equation is $x^2-2x-1=0$. Then the value of Δ is

$(1-2-1)^2\big(4-4(1)(-1)\big) = 32$

If $\Delta > 0$, i.e., $b^2-4ac>0$, then roots are real and distinct but nothing can be said about $f(1)$

143 **(a)**

Multiplying R_1, R_1, R_3 by a, b, c, respectively, and then taking a, b, c common from C_1, C_2 and C_3, we get

$$\Delta = \begin{vmatrix} -bc & ab+ac & ac+ab \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

Now, using $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, and then taking $(ab+bc+ca)$ common from C_2 and C_3, we get

$$\Delta = \begin{vmatrix} -bc & 1 & 1 \\ ab+bc & -1 & 0 \\ ac+bc & 0 & -1 \end{vmatrix} \times (ab+bc+ca)^2$$

Now, applying $R_2 \to R_2 - R_1$, we get

$$\Delta = \begin{vmatrix} -bc & 1 & 1 \\ ab & 0 & 1 \\ ac+bc & 0 & -1 \end{vmatrix} (ab+bc+ca)^2$$

Expanding along C_2, we get

$\Delta = (ab+bc+ca)^2[ac+bc+ab]$

$= (ab+bc+ca)^3$

$= (r/p)^3 = r^3/p^3$

Now given a, b, c are all positive, then

AM. $\geq$ G.M.

$\Rightarrow \frac{ab+bc+ac}{3} \geq (ab\times bc\times ac)^{1/3}$

$\Rightarrow (ab+bc+ac)^3 \geq 27a^2b^2c^2$

$\Rightarrow (ab+bc+ac)^3 \geq 27(s^2/p^2)$

If $\Delta = 27$, then $ab+bc+ca=3$, and given that $a^2+b^2+c^2=3$, from $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$,

we have $a+b+c=\pm 3$

$\Rightarrow a+b+c=3$ (since all roots are positive)

$\Rightarrow 3p+q=0$

144 **(d)**

Let,

$$\begin{vmatrix} (1+x)^a & (1+2x)^b & 1 \\ 1 & (1+x)^a & (1+2x)^b \\ (1+2x)^b & 1 & (1+x)^a \end{vmatrix} = A+Bx+Cx^2+\ldots$$

Putting $x=0$, we get

$$A = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

Now differentiating both sides with respect to x and putting $x=0$, we get

$$B = \begin{vmatrix} a & 2b & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ 0 & a & 2b \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 2b & 0 & a \end{vmatrix} = 0$$

Hence coefficient of x is 0. Since $f(x)=0$ and $f'(0)=0, x=0$ is a repeating root of the equation $f(x)=0$

145 **(c)**

$$\begin{vmatrix} x & n & r \\ m & y & r \\ m & n & z \end{vmatrix} = 0$$

Applying $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$, we get

$$\begin{vmatrix} x-m & n-y & 0 \\ 0 & y-n & r-z \\ m & n & z \end{vmatrix} = 0$$

$$\Rightarrow (x-m)(y-n)z + (n-y)(r-z)m - n(r-z)(x-m) = 0$$

Dividing by $(x-m)(y-n)(z-r)$, we have

$$\frac{z}{z-r} + \frac{m}{x-m} + \frac{n}{y-n} = 0$$

$$\Rightarrow \frac{z}{z-r} + \frac{m}{x-m} + \frac{n}{y-n} = 0$$

$$\Rightarrow \frac{z}{z-r} + \frac{m}{x-m} + 1 + \frac{n}{y-n} + 1 = 2$$

$$\Rightarrow \frac{z}{z-r} + \frac{x}{x-m} + \frac{y}{y-n} = 2$$

$$\Rightarrow \frac{z}{z-r} - 1 + \frac{x}{x-m} - 1 + \frac{y}{y-n} - 1 = -1$$

$$\Rightarrow \frac{m}{x-m} + \frac{n}{y-n} + \frac{r}{z-r} = -1$$

Now, A.M. $\geq$ G.M.

$$\Rightarrow \frac{\frac{z}{z-r} + \frac{x}{x-m} + \frac{y}{y-n}}{3} \geq \left(\frac{z}{(z-r)}\frac{x}{(x-m)}\frac{y}{(y-n)}\right)^{1/3}$$

$$\Rightarrow \frac{z}{z-r}\frac{x}{x-m}\frac{y}{y-n} \leq \frac{8}{27}$$

146 **(b)**

$$f'(x) = \begin{vmatrix} 2ax & 2ax-1 & 2ax+b+1 \\ b & b+1 & -1 \\ 2(ax+b) & 2ax+2b+1 & 2ax+b \end{vmatrix}$$

Applying $C_1 \to C_1 - C_3, C_2 \to C_2 - C_3$

$$f'(x) = \begin{vmatrix} -(b+1) & -(b+2) & 2ax+b+1 \\ (b+1) & (b+2) & -1 \\ b & b+1 & 2ax+b \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2$ and $R_3 \to R_3 - R_2$, we get

$$f'(x) = \begin{vmatrix} 0 & 0 & 2ax+b \\ b+1 & b+2 & -1 \\ -1 & -1 & 2ax+b+1 \end{vmatrix}$$

$= (2ac + b)[-b - 1 + b + 2]$

$\therefore f'(x) = 2ax + b$

$\therefore f(x) = ax^2 + bx + c$

$f(0) = 2 \Rightarrow c = 2$

$f(1) = 1 \Rightarrow a + b + 2 = 1 \Rightarrow a + b = -1$

$f'(5/2) = 0 \Rightarrow 5a + b = 0$

$\Rightarrow a = 1/4, b = -5/4$

Hence, $f(x) = \frac{1}{4}x^2 - \frac{5}{4}x + 2$

Clearly, discriminant (D) of the equation $f(x) = 0$ is less than 0. Hence, $f(x) = 0$ has imaginary roots. Also, $f(2) = 1/2.$ and minimum value of $f(x)$ is

$$\frac{\frac{25}{16} - 4\frac{1}{4}(2)}{4\frac{1}{4}} = \frac{7}{16}$$

Hence, range of the $f(x)$ is $\left[\frac{7}{16}, \infty\right)$

147 **(5)**

$$\Delta = \begin{vmatrix} 1 & 3\cos\theta & 1 \\ \sin\theta & 1 & 3\cos\theta \\ 1 & \sin\theta & 1 \end{vmatrix}$$

Applying $R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 1 & 3\cos\theta & 1 \\ \sin\theta & 1 & 3\cos\theta \\ 0 & \sin\theta - 3\cos\theta & 0 \end{vmatrix}$$

$= -(\sin\theta - 3\cos\theta)(3\cos\theta - \sin\theta)$

$= (3\cos\theta - \sin\theta)^2$

Now, $-\sqrt{9+1} \leq 3\cos\theta - \sin\theta \leq \sqrt{9+1}$

$\Rightarrow (3\cos\theta - \sin\theta\,)^2 \leq 10.1$

$\Rightarrow \Delta_{\max} = 10$

148 **(4)**

$$\Delta = (xyz)^n \begin{vmatrix} 1 & x^2 & x^4 \\ 1 & y^2 & y^4 \\ 1 & z^2 & z^4 \end{vmatrix}$$

$= (xyz)^n (x^2 - y^2)(y^2 - z^2)(z^2 - x^2)$

Clearly when

$$n = -4, \Delta = \left(\frac{1}{y^2} - \frac{1}{x^2}\right)\left(\frac{1}{z^2} - \frac{1}{y^2}\right)\left(\frac{1}{x^2} - \frac{1}{z^2}\right)$$

149 **(4)**

$$\Delta = \begin{vmatrix} x+2 & 2x+3 & 3x+4 \\ 2x+3 & 3x+4 & 4x+5 \\ 3x+5 & 5x+8 & 10x+17 \end{vmatrix} = 0$$

Applying $R_3 \to R_3 - R_2$ and $R_2 \to R_2 - R_1$

$$\Delta = \begin{vmatrix} x+2 & 2x+3 & 3x+4 \\ x+1 & x+1 & x+1 \\ x+2 & 2(x+2) & 6(x+2) \end{vmatrix} = 0$$

$$\therefore \Delta = (x+1)(x+2)\begin{vmatrix} x+2 & 2x+3 & 3x+4 \\ 1 & 1 & 1 \\ 1 & 2 & 6 \end{vmatrix} = 0$$

$$\therefore \Delta = (x+1)(x+2)[(x+2).4 - (2x+3).5 + (3x+4).1] = 0$$

$\Delta = (x+1)(x+2)(-3x-3) = 0$

or $(x+1)^2(x+2) = 0$

$\therefore x = -1, -1, -2$

150 **(2)**

System of equations

$\Rightarrow \alpha x + y + z = \alpha - 1$ (1)

$x + \alpha y + z = \alpha - 1$ (2)

$x + y + \alpha z = \alpha - 1$ (3)

Since system has no solution.

Therefore, (1) $\Delta = 0$ and (2) $\alpha - 1 \neq 0$

$$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0, \alpha \neq 1$$

$R_1 \to R_1 - R_3, R_2 \to R_2 \to R_3$

$$\begin{vmatrix} \alpha-1 & 0 & 1-\alpha \\ 0 & \alpha-1 & 1-\alpha \\ 1 & 1 & \alpha \end{vmatrix} = 0$$

$\Rightarrow (\alpha-1)[\alpha(\alpha-1)-(1-\alpha)]$

$+ (1-\alpha)[-(\alpha-1)] = 0$

$\Rightarrow (\alpha-1)[\alpha(\alpha-1)+(\alpha-1)]+(\alpha-1)^2 = 0$

$\Rightarrow (\alpha-1)^2[(\alpha+1)+1] = 0$

$\Rightarrow \alpha = 1, 1, -2 \Rightarrow \alpha = 1, -2$

Since system has no solution, $\alpha \neq 1$

$\therefore \alpha = -2$

151 **(2)**

We have $D = \begin{vmatrix} a_1 & a_2 & a_3 \\ 5 & 4 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$

Since $a_n = \frac{20}{n}; d = \frac{1}{20}$

Hence, $D = \begin{vmatrix} 20 & \frac{20}{2} & \frac{20}{3} \\ \frac{20}{4} & \frac{20}{5} & \frac{20}{6} \\ \frac{20}{7} & \frac{20}{8} & \frac{20}{9} \end{vmatrix} = \frac{(20)^3}{4\times7} \begin{vmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 1 & \frac{4}{5} & \frac{2}{3} \\ 1 & \frac{7}{8} & \frac{7}{9} \end{vmatrix}$

$R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$= \frac{(20)^3}{4\times7} \begin{vmatrix} 0 & \frac{-3}{10} & \frac{-1}{3} \\ 0 & \frac{-3}{40} & \frac{-1}{9} \\ 1 & \frac{7}{8} & \frac{7}{9} \end{vmatrix} = \frac{50}{21}$$

$\Rightarrow [D] = 2$

152 **(3)**

$x + y + z = 1$ (1)

$x + 2y + 4z = p$ (2)

$x + 4y + 10z = p^2$ (3)

$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{vmatrix}$

$R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$

$= \begin{vmatrix} 0 & -1 & -3 \\ 0 & -2 & -6 \\ 1 & 4 & 10 \end{vmatrix} = 0$

Since $\Delta = 0$, solution is not unique solution.

The system will have infinite solutions if

$\Delta_1 = 0, \Delta_2 = 0, \Delta_3 = 0$

$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ p & 2 & 4 \\ p^2 & 4 & 10 \end{vmatrix} = 0$

$C_3 \to C_3 - C_2$

$\Delta_1 = \begin{vmatrix} 1 & 1 & 0 \\ p & 2 & 2 \\ p^2 & 4 & 6 \end{vmatrix} = 0$

$\Rightarrow 1(12-8) - 1(6p - 2p^2) = 0$

$\Rightarrow 4 - 6p + 2p^2 = 0$

$\Rightarrow 2(p^2 - 3p + 2) = 0$

$\Rightarrow p^2 - 3p + 2 = 0$

$\Rightarrow p = 1$ or 2

Also for these values of $p, \Delta_2, \Delta_3 = 0$

153 **(3)**

Equation $x^3 + ax^2 + bx + c = 0$ has roots α, β, γ

$\therefore \alpha + \beta + \gamma = -a$

$\alpha\beta + \beta\gamma + \gamma\alpha = b$

Since the given system of equations has non-trivial solutions, so

$\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$

$\Rightarrow \alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma = 0$

$\Rightarrow (\alpha+\beta+\gamma)[\alpha^2+\beta^2+\gamma^2-\alpha\beta-\beta\gamma-\gamma\alpha]$

$= 0$

$\Rightarrow (\alpha+\beta+\gamma)[(\alpha+\beta+\gamma)^2 - 3(\alpha\beta+\beta\gamma+\gamma\alpha)]$

$= 0$

$\Rightarrow -a[a^2 - 3b] = 0 \Rightarrow a^2/b = 3$

154 **(2)**

Using $C_3 \to C_3 - (C_1 + C_2)$ in D_1 and D_2, we have

$\therefore \frac{D_1}{D_2} = \frac{-2b(ad-bc)}{b(ad-bc)} = -2$

155 **(8)**

Putting $x = 0, a_0 = 1$

$(1 + ax + bx^2)^4$

$= (1 + ax + bx^2)(1 + ax$

$+ bx^2)(1 + ax + bx^2)(1 + ax$

$+ bx^2)$

Clearly $a_0 = 1, a_1 =$ coefficient of $x = a + a + a + a = 4a$

$a_2 =$ coefficient of $x^2 = 4b + 6a^2$

Now $\Delta = -(a_0^3 + a_1^3 + a_2^3 - 3a_0a_1a_2)$

$\because a_0 + a_1 + a_2 \neq 0$

$\therefore a_0 = a_1 = a_2$

$1 = 4a = 6a^2 + 4b \Rightarrow a = \frac{1}{4}, b = \frac{5}{32}$

156 **(1)**

$\Delta_1 = \begin{vmatrix} a_1^2 + 4a_1d & a_1 & d \\ a_2^2 + 4a_2d & a_2 & d \\ a_3^2 + 4a_3d & a_3 & d \end{vmatrix}, [C_3 \to C_3 - C_2]$

Where d is the common difference of A.P.

$= d\begin{vmatrix} a_1^2 & a_1 & 1 \\ a_2^2 & a_2 & 1 \\ a_3^2 & a_3 & 1 \end{vmatrix} + 4d\begin{vmatrix} a_1 & a_1 & d \\ a_2 & a_2 & d \\ a_3 & a_3 & d \end{vmatrix}$

$= d(a_1 - a_2)(a_2 - a_3)(a_3 - a_1) = -2d^4$

Similarly, $\Delta_2 = -2d^4$

157 **(0)**

$\Delta = \begin{vmatrix} x_1 & y_1 & 0 \\ x_2 & y_2 & 0 \\ x_3 & y_3 & 0 \end{vmatrix}\begin{vmatrix} y_1 & x_1 & 0 \\ y_2 & x_2 & 0 \\ y_3 & x_3 & 0 \end{vmatrix} = 0.0 = 0$

158 **(8)**

$$\text{Let } D = \begin{vmatrix} (\beta+\gamma-\alpha-\delta)^4 & (\beta+\gamma-\alpha-\delta)^2 & 1 \\ (\gamma+\alpha-\beta-\delta)^4 & (\gamma+\alpha-\beta-\delta)^2 & 1 \\ (\alpha+\beta-\gamma-\delta)^4 & (\alpha+\beta-\gamma-\delta)^2 & 1 \end{vmatrix}$$

Applying $R_1 \to R_1 - R_3, R_2 \to R_2 - R_3$

$$= \begin{vmatrix} (\beta+\gamma-\alpha-\delta)^4 - (\alpha+\beta-\gamma-\delta)^4 & (\beta+\gamma-\alpha-\delta)^2 - (\alpha+\beta-\gamma-\delta)^2 & 0 \\ (\gamma+\delta-\beta-\delta)^4 - (\alpha+\beta-\gamma-\delta)^4 & (\gamma+\alpha-\beta-\delta)^2 - (\alpha+\beta-\gamma-\delta)^2 & 0 \\ (\alpha+\beta-\gamma-\delta)^4 & (\alpha+\beta-\gamma-\delta)^2 & 1 \end{vmatrix}$$

$= 4(\beta-\delta)(\gamma-\alpha).4(\alpha-\delta)(\gamma-\beta)$

$$\times \begin{vmatrix} (\beta+\gamma-\alpha-\delta)^2 + (\alpha+\beta-\gamma-\delta)^2 & \\ (\gamma+\alpha-\beta-\delta)^2 + (\alpha+\beta-\gamma-\delta)^2 & \\ (\alpha+\beta-\gamma-\delta)^4 & (\alpha+\beta - \end{vmatrix}$$

Apply $R_1 \to R_1 - R_2$

$= 16(\beta-\delta)(\gamma-\alpha)(\alpha-\delta).4(\gamma-\delta)(\beta-\alpha)$

$$\begin{vmatrix} 1 & 0 \\ (\gamma+\alpha-\beta-\delta)^2 + (\alpha+\beta-\gamma-\delta)^2 & 1 \\ (\alpha+\beta-\gamma-\delta)^4 & (\alpha+\beta-\gamma \end{vmatrix}$$

$= -64(\alpha-\beta)(\alpha-\gamma)(\alpha-\delta)(\beta-\gamma)(\beta-\delta)(\gamma-\delta)$

159 **(0)**

$$\begin{vmatrix} 3u^2 & 2u^3 & 1 \\ 3v^2 & 2v^3 & 1 \\ 3w^2 & 2w^3 & 1 \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$\Rightarrow \begin{vmatrix} u^2-v^2 & u^3-v^3 & 0 \\ v^2-w^2 & v^3-w^3 & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} u+v & u^2+v^2+vu & 0 \\ v+w & v^2+w^2+vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$

$$\Rightarrow \begin{vmatrix} u-w & (u^2-w^2)+v(u-w) & 0 \\ v+w & v^2+w^2+vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & u+w+v & 0 \\ v+w & v^2+w^2+vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0$$

$\Rightarrow (v^2+w^2+vw) - (v+w)[(v+w)+u] = 0$

$\Rightarrow v^2+w^2+vw-(v+w)^2-u(v+w) = 0$

$\Rightarrow uv+vw+wu = 0$

160 **(2)**

$$B = 2.2\begin{vmatrix} f & d & e \\ n & l & m \\ c & a & b \end{vmatrix}$$

[Taking 2 common from R_2 and C_2]

$$= 2\begin{vmatrix} 2f & d & e \\ 2n & l & m \\ 2c & a & b \end{vmatrix}$$

$$= 2\begin{vmatrix} 2c & a & b \\ 2f & d & e \\ 2n & l & m \end{vmatrix}$$

[$R_3 \leftrightarrow R_2$, then $R_2 \leftrightarrow R_1$]

$$= 2\begin{vmatrix} a & b & 2c \\ d & e & 2f \\ l & m & 2n \end{vmatrix} = 2A$$

[$C_1 \leftrightarrow C_2$ and then $C_2 \leftrightarrow C_3$]

161 **(4)**

$$\Delta = x\begin{vmatrix} 1 & x+y & x+y+z \\ 2 & 3x+2y & 4x+3y+2z \\ 3 & 6x+3y & 10x+6y+3z \end{vmatrix}$$

$$= x^2\begin{vmatrix} 1 & 1 & x+y \\ 2 & 3 & 4x+3y \\ 3 & 6 & 10x+6y \end{vmatrix} \begin{vmatrix} C_3 \to C_3 - zC_1 \\ C_2 \to C_2 - yC_1 \end{vmatrix}$$

$$= x^3\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 6 & 10 \end{vmatrix} [C_3 \to C_3 - yC_2]$$

$= x^3(6-8+3) = 64$

$= x^3(6-8+3) = 64$

$\Rightarrow x^3 = 64 \Rightarrow x = 4$

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