Single Correct Answer Type

1. Which of the following functions is non-differentiable?
a) $f(x) = (e^x - 1)|e^{2x} - 1|$ in R
b) $f(x) = \dfrac{x-1}{x^2+1}$ in R
c) $f(x) = \begin{cases} \left||x-3|-1\right|, x < 3 \\ \frac{x}{3}[x] - 2, \ x \geq 3 \end{cases}$ at $x = 3$
Where [.] represents the greatest integer function
d) $f(x) = 3(x-2)^{1/3} + 3$ in R

2. Given that $\prod_{n=1}^{n} \cos\frac{x}{2^n} = \frac{\sin x}{2^n \sin\left(\frac{x}{2^n}\right)}$ and
$$f(x) = \begin{cases} \lim_{n\to\infty} \sum_{n=1}^{n} \frac{1}{2^n} \tan\left(\frac{x}{2^n}\right), x \in (0, \pi) - \left\{\frac{\pi}{2}\right\} \\ \frac{2}{\pi}, \quad x = \frac{\pi}{2} \end{cases}$$
Then which one of the following is true?
a) $f(x)$ has non-removable discontinuity of finite type at $x = \frac{\pi}{2}$
b) $f(x)$ has removable discontinuity at $x = \frac{\pi}{2}$
c) $f(x)$ is continuous at $x = \frac{\pi}{2}$
d) $f(x)$ has non-removable discontinuity of infinite type at $x = \frac{\pi}{2}$

3. If $f(x) = \{x^2\} - (\{x\})^2$, where $\{x\}$ denotes the fractional part of x, then
a) $f(x)$ is continuous at $x = -2$ but not at $x = 2$
b) $f(x)$ is continuous at $x = 2$ but not at $x = -2$
c) $f(x)$ is continuous at $x = 2$ and at $x = -2$
d) $f(x)$ is discontinuous at $x = -2$ and at $x = 2$

4. The function $f(x) = |12 \text{ sgn } 2x| + 2$ has
a) Jump discontinuity
b) Removal discontinuity
c) Infinite discontinuity
d) No discontinuity at $x = 0$

5. $f(x) = \lim_{n\to\infty} \sin^{2n}(\pi x) + \left[x + \frac{1}{2}\right]$, where [.] denotes the greatest integer function is
a) Continuous at $x = 1$ but discontinuous at $x = 3/2$
b) Continuous at $x = 1$ and $x = 3/2$
c) Discontinuous at $x = 1$ and $x = 3/2$
d) Discontinuous at $x = 1$ but continuous at $x = 3/2$

6. If $f(x) = a|\sin x| + be^{|x|} + c|x|^3$ is differentiable at $x = 0$, then
a) $a = b = c = 0$
b) $a = 0, b = 0, c \in R$
c) $b = c = 0, a \in R$
d) $c = 0, a = 0, b \in R$

7. The function $f(x)$ defined by
$$f(x) = \begin{cases} \log_{(4x-3)}(x^2 - 2x + 5), \frac{3}{4} < x < 1 \text{ and } x > 1 \\ 4, \qquad x = 1 \end{cases}$$
a) Is continuous at $x = 1$
b) Is discontinuous at $x = 1$ since $f(1^+)$ does not exists though $f(1^-)$ exists
c) Is discontinuous at $x = 1$ since $f(1^-)$ does not exist though $f(1^+)$ exists
d) Is discontinuous at $x = 1$ since neither $f(1^+)$ nor $f(1^-)$ exists

8. If $f(x) = \begin{cases} x + 2, x < 0 \\ -x^2 - 2, 0 \leq x < 1 \\ x, x \geq 1 \end{cases}$, then the number of points of discontinuity of $|f(x)|$ is

a) 1 b) 2 c) 3 d) None of these

9. The set of all points, where $f(x)=\sqrt[3]{x^2|x|}-|x|-1$ is not differentiable, is

a) $\{0\}$ b) $\{-1,0,1\}$ c) $\{0,1\}$ d) None of these

10. For a real number y, let $[y]$ denotes the greatest integer less than or equal to y. Then the function $f(x)=\frac{\tan(\pi[x-\pi])}{1+[x]^2}$ is

a) Discontinuous at some x

b) Continuous at all x, but the derivative $f'(x)$ does not exist for some x

c) $f'(x)$ exists for all x, but the derivative $f'(x_0)$ does not exist second for some x

d) $f'(x)$ exists for all x

11. $f(x)=[x^2]-\{x\}^2$, where [.] and {.} denote the greatest integer function and the fractional part, respectively, is

a) Continuous at $x=1,-1$ b) Continuous at $x=-1$ but not at $x=1$

c) Continuous at $x=-1$ but not at $x=-1$ d) Discontinuous at $x=1$ and $x=-1$

12. If $f(x)=\cos\pi(|x|+[x])$, (where [.] denotes the greatest integral function), then which is not true?

a) Continuous at $x=1/2$ b) Continuous at $x=0$

c) Differentiable in $(-1,0)$ d) Differentiable in $(0,1)$

13. If $f(x)=\begin{cases}\sin x, & x<0\\ \cos x-|x-1|, & x\geq 0\end{cases}$ then $g(x)=f(|x|)$ is non-differentiable for

a) No value of x b) Exactly one value of x

c) Exactly three values of x d) None of these

14. If $f(x)=\begin{cases}e^{-1/x^2}, & x>0\\ 0, & x\leq 0\end{cases}$, then $f(x)$ is

a) Differentiable at $x=0$ b) Continuous but not differentiable at $x=0$

c) Discontinuous at $x=0$ d) None of these

15. The function $f(x)=[x]^2-[x^2]$ (where $[y]$ is the greatest integer less than or equal to y), is discontinuous at

a) All integers b) All integers except 0 and 1

c) All integers except 0 d) All integers except 1

16. If $f(x)=\text{sgn}(\sin^2 x-\sin x-1)$ has exactly four points of discontinuity for $x\in(0,n\pi), n\in N$, then

a) Minimum value of n is 5 b) Maximum value of n is 6

c) There are exactly two possible values of n d) None of these

17. If $f(2+x)=f(-x)$ for all $x\in R$, then differentiability at $x=4$ implies differentiability at

a) $x=1$ b) $x=-1$ c) $x=-2$ d) Cannot say anything

18. The left-hand derivatives of $f(x)=[x]\sin(\pi x)$ at $x=k$, k an integer, is

a) $(-1)^k(k-1)\pi$ b) $(-1)^{k-1}(k-1)\pi$ c) $(-1)^k k\pi$ d) $(-1)^{k-1}k\pi$

19. Which of the following is true about

$$f(x)=\begin{cases}\frac{(x-2)}{|x-2|}\left(\frac{x^2-1}{x^2+1}\right), & x\neq 2\\ \frac{3}{5}; & x=2\end{cases}$$

a) $f(x)$ is continuous at $x=2$

b) $f(x)$ has removable discontinuity at $x=2$

c) $f(x)$ has non-removable discontinuity at $x=2$

d) Discontinuity at $x=2$ can be removed by redefining function at $x=2$

20. $f(x)=[\sin x]+[\cos x], x\in[0,2\pi]$, where [.] denotes the greatest integer function. The total number of points, where $f(x)$ is non-differentiable, is equal to

a) 2 b) 3 c) 5 d) 4

21. Let $f(x)=||x|-1|$, then points where $f(x)$ is not differentiable, is/(are)

a) $0,\pm 1$ b) ± 1 c) 0 d) 1

22. The number of values of $x \in [0, 2]$ at which $f(x) = \left|x - \frac{1}{2}\right| + |x - 1| + \tan x$ is not differentiable at

a) 0 b) 1 c) 3 d) None of these

23. If $f(x) = \begin{cases} x - 1, & x < 0 \\ x^2 - 2x, & x \geq 0 \end{cases}$, then

a) $f(|x|)$ is discontinuous at $x = 0$ b) $f(|x|)$ is differentiable at $x = 0$
c) $|f(x)|$ is non-differentiable at $x = 0, 2$ d) $|f(x)|$ is continuous at $x = 0$

24. The function defined by $f(x) = (-1)^{[x^3]}$ ([.] denotes the greatest integer function) satisfies

a) Discontinuous for $x = n^{1/3}$, where n is any integer b) $f(3/2) = 1$
c) $f'(x) = 1$ for $-1 < x < 1$ d) None of these

25. Given that $f(x) = xg(x)/|x|, g(0) = g'(0) = 0$ and $f(x)$ is continuous at $x = 0$. Then the value of $f'(0)$

a) Does not exist b) Is -1 c) Is 1 d) Is 0

26. The number of points, where the function $f(x) = \max(|\tan x|, \cos |x|)$ is non-differentiable in the interval $(-\pi, \pi)$, is

a) 4 b) 6 c) 3 d) 2

27. If $f(x) = \begin{cases} 2x - [x] + x\sin(x - [x]); & x \neq 0 \\ 0; & x = 0 \end{cases}$, where [.] denotes the greatest integer function, then n cannot be

a) 4 b) 2 c) 5 d) 6

28. Let $f: R \to R$ be given by $f(x) = 5x$, if $x \in Q$ and $f(x) = x^2 + 6$ if $x \in R \sim Q$, then

a) f is continuous at $x = 2$ and $x = 3$ b) f is continuous at $x = 2$ and $x = 3$
c) f is continuous at $x = 2$ but not at $x = 3$ d) f is continuous at $x = 3$ but not at $x = 2$

29. If $f(x) = \frac{x - e^x + \cos 2x}{x^2}, x \neq 0$, is continuous at $x = 0$, then

Where $[x]$ and $\{x\}$ denote the greatest integer and fractional part function, respectively

a) $f(0) = 5/2$ b) $[f(0)] = -2$ c) $\{f(0)\} = -0.5$ d) $[f(0)]\{f(0)\} = -1.5$

30. Let $f(x)$ be a function for all $x \in R$ and $f'(0) = 1$. Then $g(x) = f(|x|) - \sqrt{\frac{1 - \cos 2x}{2}}$, at $x = 0$

a) Is differentiable at $x = 0$ and its value is 1
b) Is differentiable at $x = 0$ and its value is 0
c) Is non-differentiable at $x = 0$ as its graph has sharp turn at $x = 0$
d) Is non-differentiable at $x = 0$ as its graph has vertical tangent at $x = 0$

31. If $f(x) = \begin{cases} x^a \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous but non-differentiable at $x = 0$, then

a) $a \in (-1, 0)$ b) $a \in (0, 2]$ c) $a \in (0, 1]$ d) $a \in [1, 2)$

32. Let $f(x) = \begin{cases} \sin 2x, 0, x \leq \pi/6 \\ ax + b, \pi/6 < x < 1 \end{cases}$. If $f(x)$ and $f'(x)$ are continuous, then

a) $a = 1, b = \frac{1}{\sqrt{2}} + \frac{\pi}{6}$ b) $a = \frac{1}{\sqrt{2}}, b = \frac{1}{\sqrt{2}}$ c) $a = 1, b = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$ d) None of these

33. $f(x) = \begin{cases} 3 - \left[\cot^{-1} \frac{2x^3 - 3}{x^2}\right] & \text{if } x > 0 \\ \{x^2\}\cos(e^{1/x}), & \text{if } x < 0 \end{cases}$ is continuous at $x = 0$, then the value of $f(0)$, (where $[x]$ and $\{x\}$ denotes the greatest integer and fractional part functions, respectively)

a) 0 b) 1 c) -1 d) None of these

34. The number of points of non-differentiability for $f(x) = \max\{||x| - 1|, 1/2|\}$ is

a) 4 b) 3 c) 2 d) 5

35. If $f(x) = \begin{cases} x^2 - ax + 3, & x \text{ is rational} \\ 2 - x, & x \text{ is irrational} \end{cases}$ is continuous at exactly two points, then the possible values of a are

a) $(2, \infty)$ b) $(-\infty, 3)$ c) $(-\infty, -1) \cup (3, \infty)$ d) None of these

36. Let $f: R \to R$ be a function defined by $f(x) = \max\{x, x^3\}$. The set of all point where $f(x)$ is NOT differentiable is

a) $\{-1, 1\}$ b) $\{-1, 0\}$ c) $\{0, 1\}$ d) $\{-1, 0, 1\}$

37. A point where function $f(x)$ is not continuous where $f(x)=[\sin[x]]$ in $(0,2\pi)$; [.] denotes the greatest integer $\leq x$ is

a) $(3,0)$ b) $(2,0)$ c) $(1,0)$ d) None of these

38. Let $[\cdot]$ denotes the greatest integer function and $f(x)=[\tan^2 x]$, then

a) $\lim_{x\to 0} f(x)$ does not exist
b) $f(x)$ is continuous at $x=0$
c) $f(x)$ is not differentiable at $x=0$
d) $f''(0)=1$

39. A function $f(x)$ is defined as

$f(x)=\begin{cases} x^m \sin\frac{1}{x}, x\neq 0, m\in N \\ 0, \quad \text{if} \quad x=0 \end{cases}$. The least value of m for which $f'(x)$ is continuous at $x=0$ is

a) 1 b) 2 c) 3 d) None

40. The function $f(x)=\frac{(3^x-1)^2}{\sin x.\text{In}(1+x)}, x\neq 0$, is continuous at $x=0$. Then the value of $f(0)$ is

a) $2\log_e 3$ b) $(\log_e 3)^2$ c) $\log_e 6$ d) None of these

41. The function $f(x)=(x^2-1)|x^2-3x+2|+\cos(|x|)$ is NOT differentiable at

a) -1 b) 0 c) 1 d) 2

42. A function $f(x)$ is defined as

$f(x)=\begin{cases} \sin x, x \text{ is rational} \\ \cos x, x \text{ is irrational} \end{cases}$ is continuous at

a) $x=n\pi+\pi/4, n\in I$ b) $x=n\pi+\pi/8, n\in I$ c) $x=n\pi+\pi/6, n\in I$ d) $x=n\pi+\pi/3, n\in I$

43. $f(x)=\begin{cases} \frac{x}{2x^2+|x|}, x\neq 0 \\ 1, \quad x=0 \end{cases}$ then $f(x)$ is

a) Continuous but non-differentiable at $x=0$
b) Differentiable at $x=0$
c) Discontinuous at $x=0$
d) None of these

44. If both $f(x)$ and $g(x)$ are differentiable functions at $x=x_0$, then the function defined as $h(x)$= maximum $\{f(x), g(x)\}$:

a) Is always differentiable at $x=x_0$
b) Is never differentiable at $x=x_0$
c) Is differentiable at $x=x_0$ provided $f(x_0)\neq g(x_0)$
d) Cannot be differentiable at $x=x_0$ if $f(x_0)=g(x_0)$

45. Let $f(x)=\begin{cases} 1-\sqrt{1-x^2}, \text{if} -1\leq x\leq 1 \\ 1+\log\frac{1}{x}, \quad \text{if } x>1 \end{cases}$ is

a) Continuous and differentiable at $x=1$
b) Continuous but not differentiable at $x=1$
c) Neither continuous nor differentiable at $x=1$
d) None of these

46. The function $f(x)=\sin^{-1}(\cos x)$ is

a) Not differentiable at $x=\frac{\pi}{2}$
b) Differentiable at $\frac{3\pi}{2}$
c) Differentiable at $x=0$
d) Differentiable at $x=2\pi$

47. Which of the following statement is always true? ([.] represents the greatest integer function)

a) If $f(x)$ is discontinuous, then $|f(x)|$ is discontinuous
b) If $f(x)$ is discontinuous, then $f(|x|)$ is discontinuous
c) $f(x)=[g(x)]$ is discontinuous when $g(x)$ is an integer
d) None of these

48. If $f(x)=\begin{cases} ax^2+1, x\leq 1 \\ x^2+ax+b, x>1 \end{cases}$ is differentiable at $x=1$, then

a) $a=1, b=1$ b) $a=1, b=0$ c) $a=2, b=0$ d) $a=2, b=1$

49. If $f(x)=x^3 \text{ sgn } x$, then

a) f is derivable at $x=0$
b) f is continuous but not derivable at $x=0$
c) L.H.D. at $x=0$ is 1
d) R.H.D. at $x=0$ is 1

50. Let $f(x)=\begin{cases} g(x)\cos\frac{1}{x}, x\neq 0 \\ 0, \quad x=0 \end{cases}$, where $g(x)$ is an even function differentiable at $x=0$, passing through the origin. The $f'(0)$

a) Is equal to 1 b) Is equal to 0 c) Is equal to 2 d) Does not exist

51. The function $f(x) = \frac{4-x^2}{4x-x^3}$ is

a) Discontinuous at only one point
b) Discontinuous exactly at two points
c) Discontinuous exactly at three points
d) None of these

52. If $f(x) = \frac{\tan\left(\frac{\pi}{4}-x\right)}{\cot 2x}$, $(x \neq \pi/4)$, is continuous at $x = \pi/4$, then the value of $f\left(\frac{\pi}{4}\right)$ is

a) 1 b) 1/2 c) 1/3 d) −1

53. The function $f(x) = [x]\cos\left(\frac{2x-1}{2}\right)\pi$, where [.] denotes the greatest integer function, is discontinuous at

a) All x
b) All integer points
c) No x
d) x which is not an integer

54. Let $f(x) = \lim_{n\to\infty}(\sin x)^{2n}$, then which of the following is not true?

a) Discontinuous at infinite number of points
b) Discontinuous at $x = \frac{\pi}{2}$
c) Discontinuous at $x = -\frac{\pi}{2}$
d) None of these

55. Let a function $f(x)$ be defined by $f(x) = \frac{x-|x-1|}{x}$, then which of the following is not true

a) Discontinuous at $x = 0$
b) Discontinuous at $x = 1$
c) Not differentiable at $x = 0$
d) Not differentiable at $x = 1$

56. The function $f(x) = \sin(\log_e|x|),\ x \neq 0$, and 1 if $x = 0$

a) Is continuous at $x = 0$
b) Has removable discontinuity at $x = 0$
c) Has jump of discontinuity at $x = 0$
d) Has oscillating discontinuity at $x = 0$

57. Number of points where the function

$$f(x) = \begin{cases} 1+\left[\cos\frac{\pi x}{2}\right], 1 < x \leq 2 \\ 1-\{x\},\quad 0 \leq x < 1 \\ |\sin \pi x|, -1 \leq x < 0 \end{cases}$$

and $f(1) = 0$ is continuous but non-differentiable is/are (where [·] and {·} represent greatest integer and fractional part function, respectively)

a) 0 b) 1 c) 2 d) None of these

58. If $f(x) = |1-x|$, then the points where $\sin^{-1}(f|x|)$ is non-differentiable are

a) {0, 1} b) {0, −1} c) {0, 1, −1} d) None of these

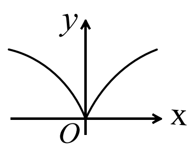
59. $$f(x) = \begin{cases} x^2\left(\frac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{-1/x}}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$ Then

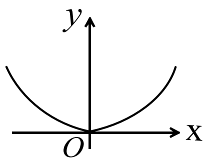
a) $f(x)$ is discontinuous at $x = 0$
b) $f(x)$ is continuous but non-differentiable at $x = 0$
c) $f(x)$ is differentiable at $x = 0$
d) $f'(0) = 2$

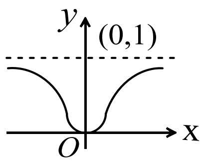
60. $f(x) = \begin{cases} xe^{-\left(\frac{1}{x}+\frac{1}{|x|}\right)}, & x \neq 0 \\ a, & x = 0 \end{cases}$. The value of a, such that $f(x)$ is differentiable at $x = 0$, is equal to

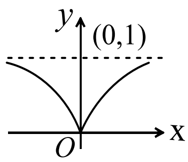
a) 1 b) −1 c) 0 d) None of these

61. Let $y = f(x) = \begin{cases} e^{-\frac{1}{x^2}} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$. Then which of the following can best represent the graph of $y = f(x)$?

a) 

b) 

c) 

d) 

62. If $f(x) = \begin{cases} |1-4x^2|, 0 \leq x < 1 \\ [x^2-2x],\ 1 \leq x < 2 \end{cases}$, where [.] denotes the greatest integer function, then $f(x)$ is

Discuss the continuity and differentiability of $f(x)$ in [0, 2)

a) Differentiable for all x b) Continuous at $x = 1$
c) $f(x)$ is non-differentiable at $x = 1$ d) None of these

63. If $f(x) = (x^2 - 4)|x^3 - 6x^2 + 11x - 6| + \frac{x}{1+|x|}$, then the set of point at which the function $f(x)$ is not differentiable is
a) $\{-2, 2, 1, 3\}$ b) $\{-2, 0, 3\}$ c) $\{-2, 2, 0\}$ d) $\{1, 3\}$

64. $f(x) = \{x\}^2 - \{x^2\}$($\{.\}$ denotes the fractional part function)
a) $f(x)$ is discontinuous at infinite number of integers but not all integers
b) $f(x)$ is discontinuous at finite number of integers
c) $f(x)$ is discontinuous at all integers
d) $f(x)$ is continuous at all integers

65. If $f(x) = \begin{cases} e^{x^2+x}, x > 0 \\ ax + b, x \le 0 \end{cases}$ is differentiable at $x = 0$, then
a) $a = 1, b = -1$ b) $a = -1, b = 1$ c) $a = 1, b = 1$ d) $a = -1, b = -1$

66. Let g(x) be a polynomial of degree one and $f(x)$ be defined by $f(x) = \begin{cases} g(x), x \le 0 \\ |x|^{\sin x}, x > 0 \end{cases}$. If $f(x)$ is continuous satisfying $f'(1) = f(-1)$, then g(x) is
a) $(1 + \sin 1)x + 1$ b) $(1 - \sin 1)x + 1$ c) $(1 - \sin 1)x - 1$ d) $(1 + \sin 1)x - 1$

67. If $f(x) = \frac{x^2 - bx + 25}{x^2 - 7x + 10}$ for $x \ne 5$ is continuous at $x = 5$, then the value of $f(5)$ is
a) 0 b) 5 c) 10 d) 25

68. Let $f(x) = [x]$ and g$(x) = \begin{cases} 0, & x \in Z \\ x^2, & x \in R - Z \end{cases}$. Then which of the following is not true ([.] represents greatest integer function)
a) $\lim_{x \to 1}$ g(x) exists but g(x) is not continuous at $x = 1$
b) $\lim\limits_{x \to 1} f(x)$ does not exist and $f(x)$ is not continuous at $x = 1$
c) gof is a discontinuous function
d) fog is a discontinuous function

69. Let$g(x) = \frac{(x-1)^n}{\log \cos^m (x-1)}$; $0 < x < 2, m$ and n are integers, $m \ne 0, n > 0$, and let p be the left hand derivative of $|x - 1|$ at $x = 1$. If $\lim_{x \to 1^+} g(x) = p$, then
a) $n = 1, m = 1$ b) $n = 1, m = -1$ c) $n = 2, m = 2$ d) $n > 2, m = n$

70. Let $f(x) = \begin{cases} \min(\{x, x^2\})\ x \ge 0 \\ \max\{2x, x^2 - 1\} x < 0 \end{cases}$. Then which of the following is not true
a) $f(x)$ is continuous at $x = 0$ b) $f(x)$ is not differentiable at $x = 1$
c) $f(x)$ is not differentiable at exactly three point d) None of these

71. Let f be a continuous function on R such that $f(1/4n) = (\sin e^n)e^{-n^2} + \frac{n^2}{n^2+1}$. Then the value of $f(0)$ is
a) 1 b) 1/2 c) 0 d) None of these

72. $f(x) = \max\{x/n, |\sin \pi x|\}, n \in N$ has maximum points of non-differentiability for $x \in (0, 4)$, then n cannot be
a) 4 b) 2 c) 5 d) 6

73. If $f(x) = \begin{cases} \frac{8^x - 4^x - 2^x + 1}{x^2}, x > 0 \\ e^x \sin x + \pi x + \lambda \text{ In } 4,\ x \le 0 \end{cases}$
Is continuous at $x = 0$. Then the value of λ is
a) $4 \log_e 2$ b) $2 \log_e 2$ c) $\log_e 2$ d) None of these

74. $f(x) = \lim_{n \to \infty} \frac{(x-1)^{2n} - 1}{(x-1)^{2n} + 1}$ is discontinuous at
a) $x = 0$ only b) $x = 2$ only c) $x = 0$ and 2 d) None of these

75. Let $f(x) = \lim_{n \to \infty} \frac{(x^2 + 2x + 3 + \sin \pi x)^n - 1}{(x^2 + 2x + 3 + \sin \pi x)^n + 1}$, then
a) $f(x)$ is continuous and differentiable for all $x \in R$
b) $f(x)$ is continuous but not differentiable for all $x \in R$

c) $f(x)$ is discontinuous at infinite number of points

d) $f(x)$ is discontinuous at finite number of points

76. Which of the following function is not differentiable at $x = 1$?

a) $f(x) = (x^2 - 1)|(x-1)(x-2)|$ b) $f(x) = \sin(|x-1|) - |x-1|$

c) $f(x) = \tan(|x-1|) + |x-1|$ d) None of these

77. If $x + 4|y| = 6y$, then y as a function of x is

a) Continuous at $x = 0$ b) Derivable at $x = 0$ c) $\frac{dy}{dx} = \frac{1}{2}$ for all x d) None of these

78. Which of the following functions have finite number of points if discontinuity in R([·] represents greatest integer function)?

a) $\tan x$ b) $x[x]$ c) $\frac{|x|}{x}$ d) $\sin[\pi x]$

79. The function $f(x) = \{x\}\sin(\pi[x])$, where [.] denotes the greatest integer function and {.} is the fractional part function, is discontinuous at

a) All x b) All integer points

c) No x d) x which is not an integer

80. The value of $f(0)$, so that the function $f(x) = \frac{2x - \sin^{-1} x}{2x + \tan^{-1} x}$ is continuous at each point in its domain, is equal to

a) 2 b) 1/3 c) 2/3 d) $-1/3$

81. Which of the following functions is differentiable at $x = 0$?

a) $\cos(|x|) + |x|$ b) $\cos(|x|) - |x|$ c) $\sin(|x|) + |x|$ d) $\sin(|x|) - |x|$

82. The number of points $f(x) = \begin{cases} [\cos \pi x], & 0 \le x \le 1 \\ |2x-3|[x-2], & 1 < x \le 2 \end{cases}$ is discontinuous at ([.] denotes the greatest integer function)

a) Two points b) Three points c) Four points d) No points

83. If $f(x) = \sqrt{1 - \sqrt{1 - x^2}}$, then $f(x)$ is

a) Continuous on $[-1, 1]$ and differentiable on $(-1, 1)$

b) Continuous $[-1, 1]$ and differentiable on $(-1, 0) \cup (0, 1)$

c) Continuous and differentiable on $[-1, 1]$

d) None of these

84. If $f(x) = \begin{cases} \frac{1-|x|}{1+x}; & x \ne -1 \\ 1; & x = -1 \end{cases}$, then $f([2x])$ where [·] represents the greatest integer function is

a) Discontinuous at $x = -1$ b) Continuous at $x = 0$

c) Continuous at $x = 1/2$ d) Continuous at $x = 1$

85. Let $f(x)$ be defined in the interval $[0, 4]$ such that

$$f(x) = \begin{cases} 1 - x, 0 \le x \le 1 \\ x + 2, 1 < x < 2 \\ 4 - x, 2 \le x \le 4 \end{cases}$$

Then number of points where $f(f(x))$ is discontinuous is

a) 1 b) 2 c) 3 d) None of these

86. If $f(x) = \frac{a\cos x - \cos bx}{x^2}, x \ne 0$ and $f(0) = 4$ is continuous at $x = 0$, then the ordered pair (a, b) is

a) $(\pm 1, 3)$ b) $(1, \pm 3)$ c) $(-1, -3)$ d) $(1, 3)$

87. Let $f(x) = \lim_{n\to\infty} \frac{\log(2+x) - x^{2n}\sin x}{1 + x^{2n}}$. Then

a) f is continuous at $x = 1$ b) $\lim_{x\to 1^+} f(x) = \log 3$

c) $\lim_{x\to 1^+} f(x) = -\sin 1$ d) $\lim_{x\to 1^-} f(x)$ does not exist

88. The set of points where $x^2|x|$ is thrice differentiable is

a) R b) $R - \{0, \pm 1\}$ c) $R - \{0\}$ d) None of these

89. Let $f(x)=\begin{cases}\frac{x-4}{|x-4|}+a, x<4\\ a+b,\ x=4\\ \frac{x-4}{|x-4|}+b, x>4\end{cases}$. Then $f(x)$ is continuous at $x=4$ when,

a) $a=0, b=0$ b) $a=1, b=1$ c) $a=-1, b=1$ d) $a=1, b=-1$

90. If $f(x)=\begin{cases}x^3, x^2<1\\ x, x^2\geq 1\end{cases}$, then $f(x)$ is differentiable at

a) $(-\infty,\infty)-\{1\}$

b) $(-\infty,\infty)\sim\{1-1\}$

c) $(-\infty,\infty)\sim\{1-1,0\}$

d) $(-\infty,\infty)\sim\{-1\}$

91. If $f(x)=[\log_e x]+\sqrt{\{\log_e x\}}, x>1$, where [.] and {.} denote the greatest integer function and the fractional part function, respectively, then

a) $f(x)$ is continuous but non-differentiable at $x=e$

b) $f(x)$ is differentiable at $x=e$

c) $f(x)$ is discontinuous at $x=e$

d) None of these

Multiple Correct Answers Type

92. If $f(x)=\begin{cases}(\sin^{-1}x)^2\cos(1/x), x\neq 0\\ 0, \quad x=0\end{cases}$ then

a) $f(x)$ is continuous everywhere in $x\in(-1,1)$

b) $f(x)$ is discontinuous in $x\in[-1,1]$

c) $f(x)$ is differentiable everywhere in $x\in(-1,1)$

d) $f(x)$ is non-differentiable nowhere in $x\in[-1,1]$

93. Which of the following function is thrice differentiable at $x=0$?

a) $f(x)=|x^3|$ b) $f(x)=x^3|x|$ c) $f(x)=|x|\sin^3 x$ d) $f(x)=x|\tan^3 x|$

94. A function $f(x)$ satisfies the relation $f(x+y)=f(x)+f(y)+xy(x+y)\ \forall x,y\in R$. If $f'(0)=-1$, then

a) $f(x)$ is a polynomial function

b) $f(x)$ is an exponential function

c) $f(x)$ is twice differentiable for all $x\in R$

d) $f'(3)=8$

95. Let $f(x)=[x]$ and $g(x)=\begin{cases}0, x\in Z\\ x^2, x\in R-Z\end{cases}$ ([.] represents greatest integer function). Then

a) $\lim\limits_{x\to 1} g(x)$ exists but $g(x)$ is not continuous at $x=1$ b) $f(x)$ is not continuous at $x=1$

c) gof is continuous for all x d) fog is continuous for all x

96. Let $h(x)=\min\{x,x^2\}$, for every real number of x, then

a) h is continuous for all x

b) h is differentiable for all x

c) $h'(x)=1$, for all $x>1$

d) h is not differentiable at two values of x

97. Let $f: R\to R$ be any function and $g(x)=\frac{1}{f(x)}$. Then which of following is/are not true

a) g is onto if f is onto b) g is one-one if f is one-to-one

c) g is continuous if f is continuous d) g is differentiable if f is differentiable

98. The function $f(x)=\max\{(1-x),(1+x),2\}, x\in(-\infty,\infty)$ is

a) Continuous at all points

b) Differentiable at all points

c) Differentiable at all points except at $x=1$ and $x=-1$

d) Continuous at all points except at $x=1$ and $x=-1$, where it is discontinuous

99. The function $f(x)=\begin{cases}1, |x|\geq 1\\ \frac{1}{n^2}, \frac{1}{n}<|x|<\frac{1}{n-1}, n=2,3,\ldots\\ 0,\ x=0\end{cases}$

a) Is discontinuous at infinite points

b) Is continuous everywhere

c) Is discontinuous only at $x=\frac{1}{n}, n\in Z-\{0\}$

d) None of these

100. Which of the statement(s) is/are incorrect?

a) If $f+g$ is continuous at $x=a$, then f and g are continuous at $x=a$

b) If $\lim_{x\to a}(fg)$ exists, then both $\lim_{x\to a} f$ and $\lim_{x\to a} g$ exist

c) Discontinuity at $x=a \Rightarrow$ non-existence of limit

d) All functions defined on a closed interval attain a maximum or a minimum value in that interval

101. Let $g(x)=xf(x)$, where $f(x)=\begin{cases}x\sin\frac{1}{x}, x\neq 0\\ 0,\ x=0\end{cases}$. At $x=0$

a) g is differentiable but g' is not continuous

b) g is differentiable while f is not

c) Both f and g are differentiable

d) g is differentiable and g' is continuous

102. Let $f(x)=[\sin^4 x]$, then (where [.] represents the greatest integer function)

a) $f(x)$ is continuous at $x=0$

b) $f(x)$ is differentiable at $x=0$

c) $f(x)$ is non-differentiable at $x=0$

d) $f'(0)=1$

103. Which of the following function f has/have a removable discontinuity at the indicated point?

a) $f(x)=\dfrac{x^2-2x-8}{x+2}$ at $x=-2$

b) $f(x)=\dfrac{x-7}{|x-7|}$ at $x=7$

c) $f(x)=\dfrac{x^3+64}{x+4}$ at $x=-4$

d) $f(x)=\dfrac{3-\sqrt{x}}{9-x}$ at $x=9$

104. $f(x)$ is differentiable function and $\big(f(x).g(x)\big)$ is differentiable at $x=a$, then

a) $g(x)$ must be differentiable at $x=a$

b) If $g(x)$ is discontinuous, then $f(a)=0$

c) $f(a)\neq 0$, then $g(x)$ must be differentiable

d) None of these

105. $f(x)=\frac{[x]+1}{\{x\}+1}$ for $f:\left[0,\frac{5}{2}\right)\to\left(\frac{1}{2},3\right]$, where [.] represents the greatest integer function and {.} represents the fractional part of x, then which of the following is true

a) $f(x)$ is injective discontinuous function

b) $f(x)$ surjective non-differentiable function

c) $\min\left(\lim_{x\to 1^-} f(x), \lim_{x\to 1^+} f(x)\right)=f(1)$

d) max (x values of point of discontinuity)$=f(1)$

106. Let $f(x)=\begin{cases}0, x<0\\ x^2, x\geq 0\end{cases}$ then for all x

a) f' is differentiable

b) f is differentiable

c) f' is continuous

d) f is continuous

107. Which of the following is/are true for $f(x)=\text{sgn}(x)\times\sin x$

a) Discontinuous no where

b) An even function

c) $f(x)$ is periodic

d) $f(x)$ is differentiable for all x

108. $f(x)=\begin{cases}x+a, x\geq a\\ 2-x, x<0\end{cases}$ and $g(x)=\begin{cases}\{x\}, x<0\\ \sin x+b, x\geq 0\end{cases}$ and if $f\big(g(x)\big)$ is continuous at $x=0$ then which of the following is/are true (where $\{x\}$ represents the fractional part function)

a) If $b=1$, then a can take any real value

b) If $b<-1$, then $a+b=1$

c) No values of a and b are possible

d) There exist finite ordered pairs (a,b)

109. Let $f(x)=\text{sgn}(\cos 2x-2\sin x+3)$, where sgn $(\cdot)$ is the signum function, then $f(x)$

a) Is continuous over its domain

b) Has a missing point discontinuity

c) Has isolated point discontinuity

d) Irremovable discontinuity

110. The function $f(x) = \begin{cases} |x-3|, x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, x < 1 \end{cases}$ is

a) Continuous at $x = 1$

b) Differentiable at $x = 1$

c) Continuous at $x = 3$

d) Differentiable at $x = 3$

111. If $x + |y| = 2y$, then y as a function of x is

a) Defined for all real x

b) Continuous at $x = 0$

c) Differentiable for all x

d) Such that $\frac{dy}{dx} = \frac{1}{3}$ for $x < 0$

112. If $f(x) = \begin{cases} |x| - 3, x < 1 \\ |x-2| + a, x \geq 1 \end{cases}$ and $g(x) = \begin{cases} 2 - |x|, x < 2 \\ \text{sgn}(x) - b, x \geq 2 \end{cases}$ and $h(x) = f(x) + g(x)$ is discontinuous at exactly one point then which of the following values of a and b are possible

a) $a = -3, b = 0$

b) $a = 2, b = 1$

c) $a = 2, b = 0$

d) $a = -3, b = 1$

113. If $f(x) = \begin{cases} x^2(\text{sgn}[x]) + \{x\}, 0 \leq x < 2 \\ \sin x + |x-3|, \quad 2 \leq x < 4 \end{cases}$, where [ ] and { } represents the greatest integer and the fractional part function, respectively

a) $f(x)$ is differentiable at $x = 1$

b) $f(x)$ is continuous but non-differentiable at $x = 1$

c) $f(x)$ is non-differentiable at $x = 2$

d) $f(x)$ is discontinuous at $x = 2$

114. Let $f(x) = \begin{cases} \frac{e^x - 1 + ax}{x^2}, x > 0 \\ b, \quad x = 0 \\ \sin\frac{\frac{x}{2}}{x}, \quad x < 0 \end{cases}$, then

a) $f(x)$ is continuous at $x = 0$ if $a = -1, b = \frac{1}{2}$

b) $f(x)$ is discontinuous at $x = 0$ if $b \neq \frac{1}{2}$

c) $f(x)$ has irremovable discontinuity at $x = 0$ if $a \neq -1$

d) $f(x)$ has removable discontinuity at $x = 0$ if $a = -1, b \neq \frac{1}{2}$

115. If $f(x) = \text{sgn}(x^2 - ax + 1)$ has maximum number of points of discontinuity, then

a) $a \in (2, \infty)$

b) $a \in (-\infty, -2)$

c) $a \in (-2, 2)$

d) None of these

116. If $f(x) = [|x|]$, where [.] denotes the greatest integer function, then which of the following is not true?

a) $f(x)$ is continuous $\forall x \in R$

b) $f(x)$ is continuous from right and discontinuous from left $\forall x \in N$

c) $f(x)$ is continuous from left and discontinuous from right $\forall x \in I$

d) $f(x)$ is continuous at $x = 0$

117. The function $f(x) = \begin{cases} 5x - 4 \text{ for } 0 < x \leq 1 \\ 4x^2 - 3x \text{ for } 1 < x < 2 \\ 3x + 4 \text{ for } x \geq 2 \end{cases}$ is

a) Continuous at $x = 1$ and $x = 2$

b) Continuous at $x = 1$ but not derivable at $x = 2$

c) Continuous at $x = 2$ but not derivable at $x = 1$

d) Continuous at $x = 1$ and 2 but not derivable at $x = 1$ and $x = 2$

118. Which of the following function(s) has/have removable discontinuity at $x = 1$?

a) $f(x) = \frac{1}{\text{In } |x|}$

b) $f(x) = \frac{x^2 - 1}{x^3 - 1}$

c) $f(x) = 2^{-2^{\frac{1}{1-x}}}$

d) $f(x) = \frac{\sqrt{x+1} - \sqrt{2x}}{x^2 - x}$

119. $f(x) = \begin{cases} \left(\frac{3}{2}\right)^{(\cot 3x)/(\cot 2x)} & ; 0 < x < \frac{\pi}{2} \\ b+3; & x = \frac{\pi}{2} \\ (1+|\cot x|)^{(a|\tan x|)/b}; & \frac{\pi}{2} < x < \pi \end{cases}$ is continuous at $x = \pi/2$, then

a) $a = 0$ b) $a = 2$ c) $b = -2$ d) $b = 2$

120. Let $g(x) = x\,f(x)$, where

$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$

At $x = 0$

a) g is differentiable but g' is not continuous
b) g is differentiable while f is not differentiable
c) Both f and g are differentiable
d) g is differentiable but g' is continuous

121. The function $f(x) = 1 + |\sin x|$ is

a) Continuous nowhere
b) Continuous everywhere
c) Not differentiable at $x = 0$
d) Not differentiable at infinite number of points

122. Let $[x]$ denotes the greatest integer less than or equal to x. If $f(x) = [x \sin \pi x]$, then $f(x)$ is

a) Continuous at $x = 0$ b) Continuous in $(-1, 0)$
c) Differentiable at $x = 1$ d) Differentiable in $(-1, 1)$

123. The set of all points, where the function $f(x) = \frac{x}{1+|x|}$, is differentiable is

a) $(-\infty, \infty)$ b) $[0, \infty)$ c) $(-\infty, 0) \cup (0, \infty)$ d) $(0, \infty)$

124. If $f(x) = \frac{x-2}{2}$, then in $[0, \pi]$

a) Both $\tan(f(x))$ and $\frac{1}{f(x)}$ are continuous
b) $\tan(f(x))$ is continuous but $f^{-1}(x)$ is not continuous
c) $\tan(f^{-1}(x))$ and $f^{-1}(x)$ are discontinuous
d) None of these

125. The following functions are continuous on $(0, \pi)$

a) $\tan x$ b) $\int_0^x t \sin\frac{1}{t}\,dt$

c) $\begin{cases} 1, & 0 < x \leq \frac{3\pi}{4} \\ 2\sin\frac{2}{9}x, & \frac{3\pi}{4} < x < \pi \end{cases}$ d) $\begin{cases} x \sin x, & 0 < x \leq \frac{\pi}{2} \\ \frac{\pi}{2}\sin(\pi + x), & \frac{\pi}{2} < x < \pi \end{cases}$

126. If $f(x) = x + |x| + \cos([\pi^2]x)$ and $g(x) = \sin x$, where [.] denotes the greatest integer function, then

a) $f(x) + g(x)$ is continuous everywhere
b) $f(x) + g(x)$ is differentiable everywhere
c) $f(x) \times g(x)$ is differentiable everywhere
d) $f(x) \times g(x)$ is continuous but not differentiable at $x = 0$

127. If $f(x) = \begin{cases} \frac{x \log \cos x}{\log(1+x^2)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ then

a) $f(x)$ is not continuous at $x = 0$
b) $f(x)$ is continuous at $x = 0$
c) $f(x)$ is continuous at $x = 0$ but not differentiable at $x = 0$
d) $f(x)$ is differentiable at $x = 0$

128. The function defined as

$$f(x)=\lim_{n\to\infty}\begin{bmatrix}\cos^{2n}x & \text{if } x<0\\ \sqrt[n]{\sqrt{1+x^n}} & \text{if } 0\le x\le 1\\ \dfrac{1}{1+x^n} & \text{if } x>1\end{bmatrix},$$

Which of the following does not hold good?

a) Continuous at $x=0$ but discontinuous at $x=1$

b) Continuous at $x=1$ but discontinuous at $x=0$

c) Continuous both at $x=1$ and $x=0$

d) Discontinuous both at $x=1$ and $x=0$

129. If $f(x)=\sin ln\left(\frac{\sqrt{9-x^2}}{2-x}\right)$, then

a) Domain of $f(x)$ is $x\in(-3,2)$

b) Range of $f(x)$ is $y\in(-1,1)$

c) $f(x)$ is continuous at $x=0$

d) The right hand limit of $y=(x-3)f(x)$ at $x=-3$ is zero

130. A function f is defined on an interval $[a,b]$. Which of the following statement(s) is/are incorrect?

a) If $f(a)$ and $f(b)$ have opposite signs, then there must be a point $c\in(a,b)$ such that $f(c)=0$

b) If f is continuous on $[a,b]$, $f(a)<0$ and $f(b)>0$, then there must be a point $c\in(a,b)$ such that $f(c)=0$

c) If f is continuous on $[a,b]$, then there is a point c in (a,b) such that $f(c)=0$, then $f(a)$ and $f(b)$ have opposite signs

d) If f has no zeros on $[a,b]$, then $f(a)$ and $f(b)$ have the same sign

131. If $f(x)=\lim_{t\to\infty}\frac{|a+\sin\pi x|^t-1}{|a+\sin\pi x|^t+1}$, $x\in(0,6)$, then

a) If $a=1$, then $f(x)$ has 5 points of discontinuity

b) If $a=3$, then $f(x)$ has no point of discontinuity

c) If $a=0.5$, then $f(x)$ has 6 points of discontinuity

d) If $a=0$, then $f(x)$ has 6 points of discontinuity

Assertion - Reasoning Type

This section contain(s) 0 questions numbered 132 to 131. Each question contains STATEMENT 1(Assertion) and STATEMENT 2(Reason). Each question has the 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

a) Statement 1 is True, Statement 2 is True; Statement 2 **is** correct explanation for Statement 1

b) Statement 1 is True, Statement 2 is True; Statement 2 **is not** correct explanation for Statement 1

c) Statement 1 is True, Statement 2 is False

d) Statement 1 is False, Statement 2 is True

132

Statement 1: $f(x)=(2x-5)^{3/5}$ is non-differentiable at $x=5/2$

Statement 2: If the graph of $y=f(x)$ has sharp turn at $x=a$, then it is non-differentiable

133

Statement 1: The function $f(x)=a_1e^{|x|}+a_2|x|^5$, where a_1,a_2 are constants, is differentiable at $x=0$ if $a_1=0$

Statement 2: $e^{|x|}$ is a many-one function

134

Statement 1: Let $f(x) = \lim_{m\to\infty}\{\lim_{n\to\infty} \cos^{2m}(n!\,\pi x)\}$, and $g(x) = \begin{cases} 0, \text{if } x \text{ is rational} \\ 1, \text{if } x \text{ is irrational} \end{cases}$. Then $h(x) = f(x) + g(x)$ is continuous for all x

Statement 2: $f(x)$ and $g(x)$ are discontinuous for all $x \in R$

135

Statement 1: If $f(x)$ is a continuous function such that $f(0) = 1$ and $f(x) \neq x, \forall x \in R$, then $f\left(f(x)\right) > x$

Statement 2: If $f: R \to R, f(x)$ is a onto function, then $f(x) = 0$ has at least one solution

136

Statement 1: $f(x) = |x| \sin x$ is non-differentiable at $x = 0$

Statement 2: If $f(x)$ is not differentiable and $g(x)$ is differentiable at $x = a$, then $f(x)g(x)$ can still be differentiable at $x = a$

137

Statement 1: $f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ is non-differentiable at $x = \pm 1$

Statement 2: Principal value of $\tan^{-1} x$ are $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

138

Statement 1: If $|f(x)| \leq |x|$ for all $x \in R$, then $|f(x)|$ is continuous at 0

Statement 2: If $f(x)$ is continuous, then $|f(x)|$ is also continuous

139

Statement 1: If $f(x)$ and $g(x)$ are two differentiable functions $\forall x \in R$, then $y = \max\{f(x), g(x)\}$ is always continuous but not differentiable at the point of intersection of graphs of $f(x)$ and $g(x)$

Statement 2: $y = \max\{f(x), g(x)\}$ is always differentiable in between the two consecutive roots of $f(x) - g(x) = 0$ if both the functions $f(x)$ and $g(x)$ are differentiable $\forall x \in R$

140

Statement 1: $y = \sin x$ and $y = \sin^{-1} x$, both are differentiable functions

Statement 2: Differentiable of $f(x) \Rightarrow$ differentiability of $y = f^{-1}(x)$

141

Statement 1: Both the functions $|\text{In } x|$ and In x are both continuous for all x

Statement 2: Continuity of $|f(x)| \Rightarrow$ continuity of $f(x)$

142

Statement 1: $f(x) = (\sin \pi x)(x-1)^{1/5}$ is differentiable at $x = 1$

Statement 2: Product of two differentiable function is always differentiable

143

Statement 1: The function $f(x) = [\sqrt{x}]$ is discontinuous for all integral values of x in its domain (where $[x]$ is the greatest integer $\leq x$)

Statement 2: $[g(x)]$ will be discontinuous for all x given by $g(x) = k$, where k is any integer

144

Statement 1: $f(x) = \text{sgn}(x^2 - 2x + 3)$ is continuous for all x

Statement 2: $ax^2 + bx + c = 0$ has no real roots if $b^2 - 4ac < 0$

145

Statement 1: $f(x) = ||x^2| - 3|x| + 2|$ is not differentiable at 5 points

Statement 2: If the graph of $f(x)$ crosses the x-axis at m distinct points, then $g(x) = |f(x)|$ is always non-differentiable at least at m distinct points

146

Statement 1: The function $f(x) = \begin{cases} \frac{e^{1/x}-1}{e^{1/x}+1}, & x \neq 0 \\ \cos x & x = 0 \end{cases}$ is discontinuous at $x = 0$

Statement 2: $f(0) = 1$

147 Consider the functions $f(x) = x^2 - 2x$ and $g(x) = -|x|$

Statement 1: The composite function $F(x) = f(g(x))$ is not derivable at $x = 0$

Statement 2: $F'(0^+) = 2$ and $F'(0^-) = -2$

148

Statement 1: $f(x) = \text{sgn}\ x$ is discontinuous at $x = 0 \Rightarrow f(x) = |\text{sgn}\ x|$ is discontinuous at $x = 0$

Statement 2: Discontinuity of $f(x) \Rightarrow$ discontinuity of $|f(x)|$

149 Consider $[\cdot]$ and $\{\cdot\}$ denote the greatest integer function and the fractional part function, respectively
Let $f(x) = \{x\} + \sqrt{\{x\}}$

Statement 1: f is not differentiable at integral values of x

Statement 2: f is not continuous at integral points

150

Statement 1: $f(x) = [\sin x] - [\cos x]$ is discontinuous at $x = \pi/2$, where [.] represent the greatest integer function

Statement 2: If $f(x)$ and $g(x)$ are discontinuous at $x = a$, then $f(x) + g(x)$ is discontinuous at $x = a$

151 Let $f(x) = x|x|$ and $g(x) = \sin x$

Statement 1: gof is differentiable at $x = 0$ and its derivative is continuous at that point

Statement 2: gof is twice differentiable at $x = 0$

152 Consider the function $f(x) = \text{sgn}\ (x - 1)$ and $g(x) = \cot^{-1}[x - 1]$, where $[\cdot]$ denotes the greatest integer function

Statement 1: The function $F(x) = f(x), g(x)$ is discontinuous at $x = 1$

Statement 2: If $f(x)$ is discontinuous at $x = a$ and $g(x)$ is also discontinuous at $x = a$, then the product function $f(x)g(x)$ is discontinuous at $x = a$

153

Statement 1: $f(x) = \lim_{x\to\infty} \frac{x^{2n}-1}{x^{2n}+1}$ is discontinuous at $x = 1$

Statement 2: If limit of function exists at $x = a$ but not equal to $f(a)$, then $f(x)$ is discontinuous at $x = a$

154

Statement 1: If $f(x)$ is discontinuous at $x = e$ and $\lim_{x\to a} g(x) = e$, then $\lim_{x\to a} f(g(x))$ cannot be equal to $f\left(\lim_{x\to a} g(x)\right)$

Statement 2: If $f(x)$ is continuous at $x = e$ and $\lim_{x\to a} g(x) = e$, then $\lim_{x\to a} f(g(x)) = f\left(\lim_{x\to a} g(x)\right)$

155 Consider the function

$f(x) = \cot^{-1}\left(\text{sgn}\left(\frac{[x]}{2x-[x]}\right)\right)$, where $[\cdot]$ denotes the greatest integer function

Statement 1: $f(x)$ is discontinuous at $x = 1$

Statement 2: $f(x)$ is non-differentiable at $x = 1$

156

Statement 1: If $f'(x)$ exists then $f'(x)$ is continuous

Statement 2: Every differentiable function is continuous

157

Statement 1: $f(x) = \sin x + [x]$ is discontinuous at $x = 0$, where [.] denotes the greatest integer function

Statement 2: If $g(x)$ is continuous and $h(x)$ is discontinuous at $x = a$, then $g(x) + h(x)$ will necessary be discontinuous at $x = a$

158

Statement 1: $f(x) = \min\{\sin x, \cos x\}$ is non-differentiable at $x = \pi/2$

Statement 2: Non-differentiability of $\max\{g(x), h(x)\} \Rightarrow$ non-differentiability of $\min\{g(x)\ h(x)\}$

Matrix-Match Type

This section contain(s) 0 question(s). Each question contains Statements given in 2 columns which have to be matched. Statements (A, B, C, D) in **columns I** have to be matched with Statements (p, q, r, s) in **columns II**.

159. Consider the function $f(x) = x^2 + bx + c$, where $D = b^2 - 4c > 0$

Column-I	Column- II
(A) $b < 0, c > 0$	(p) 1
(B) $c = 0, b < 0$	(q) 2

(C) $c = 0, b > 0$	(r)	3
(D) $b = 0, c < 0$	(s)	5

CODES :

	A	**B**	**C**	**D**
a)	p	q	s	r
b)	q	p	r	s
c)	r	s	q	p
d)	s	r	p	q

160. Let $f(x) = \begin{cases} \frac{5e^{1/x}+2}{3-e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Column-I	**Column- II**
(A) $y = f(x)$ is	(p) Continuous at $x = 0$
(B) $y = xf(x)$ is	(q) Discontinuous at $x = 0$
(C) $y = x^2 f(x)$ is	(r) Differentiable at $x = 0$
(D) $y = x^{-1} f(x)$ is	(s) Non-differentiable at $x = 0$

CODES :

	A	**B**	**C**	**D**
a)	P,s	q,s	p,q	p,r
b)	p,r	q,s	p,s	q
c)	q,s	p,s	p,r	q,s
d)	p,s	p,q	q,s	p,r

161.

Column-I	**Column- II**
(A) $f(x) = \lim_{n\to\infty} \cos^{2n}(2\pi x) + \left\{x + \frac{1}{2}\right\}$, where {.} denotes the fractional part function at $x = \frac{1}{2}$	(p) Continuous
(B) $f(x) = (\log_e x)(x-1)^{1/5}$ at $x = 1$	(q) Discontinuous
(C) $f(x) = [\cos 2\pi x] + \sqrt{\left\{\sin \pi \frac{x}{2}\right\}}$, where [.] and {.} denote the greatest integer and the fractional part function, respectively at $x = 1$	(r) Differentiable
(D) $f(x) = \begin{cases} \cos 2x, & x \in Q \\ \sin x, & x \notin Q \end{cases}$ at $x = \frac{\pi}{6}$	(s) Non-differentiable

CODES :

	A	B	C	D
a)	Q,s	p,r	p,r	p,s
b)	p,r	q,s	p,s	p,r
c)	q,s	p,s	p,r	p,q
d)	p,q	q,s	p,s	q,r

162.

	Column-I		Column- II
(A)	$f(x) = \begin{cases} \frac{1}{\lvert x\rvert} \text{ for } \lvert x\rvert \geq 1 \\ ax^2 + b \text{ for } \lvert x\rvert < 1 \end{cases}$ is differentiable everywhere and $\lvert k\rvert = a + b$, then the value of k is	(p)	2
(B)	If $f(x) = \text{sgn}(x^2 - ax + 1)$ has exactly one point of discontinuity, then the value of a can be	(q)	-2
(C)	$f(x) = [2 + 3\lvert n\rvert \sin x], n \in N; x \in (0, \pi)$ has exactly 11 points of discontinuity, then the value of n is	(r)	1
(D)	$f(x) = \left\lvert \left\lvert \lvert x\rvert - 2\right\rvert + a \right\rvert$ has exactly three points of non-differentiability, then the value of a is	(s)	-1

CODES :

	A	B	C	D
a)	P,q	p,r	p,s	q,s
b)	r,s	p,q	p,q	p,r
c)	p,r	q,s	p,q	r,s
d)	q,s	p,r	r,s	p,q

163.

	Column-I		Column- II
(A)	$f(x) = \lvert x^3\rvert$ is	(p)	Continuous in $(-1, 1)$
(B)	$f(x) = \sqrt{\lvert x\rvert}$ is	(q)	Differentiable in $(-1, 1)$
(C)	$f(x) = \lvert \sin^{-1} x \rvert$ is	(r)	Differentiable in $(0, 1)$
(D)	$f(x) = \cos^{-1} \lvert x\rvert$ is	(s)	Not differentiable at least at one point in $(-1, 1)$

CODES :

	A	B	C	D
a)	P,q,r	p,r,s	p,r,s	p,r,s

b)	p,q	p,r,s	q,r	p,r
c)	q,r	p,s	p,r	p,r,s
d)	p,q	s,q	p,r	q,s

Linked Comprehension Type

This section contain(s) 11 paragraph(s) and based upon each paragraph, multiple choice questions have to be answered. Each question has atleast 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 164 to -164

Let $f(x = \begin{cases} \frac{a(1-x\sin x)+b\cos x+5}{x^2}, & x<0 \\ 3, & x=0 \\ \left\{1+\left(\frac{P(x)}{x^2}\right)\right\}^{1/x}, & x>0 \end{cases}$, where $P(x)$ is a cubic function and f is continuous at $x=0$

164. The range of function $g(x) = 3a\sin x - b\cos x$ is
a) $[-10, 10]$ b) $[-5, 5]$ c) $[-12, 12]$ d) None of these

Paragraph for Question Nos. 165 to - 165

Let $f(x) = \begin{cases} x+2, 0 \le x < 2 \\ 6-x, \quad x \ge 2 \end{cases}$,

$g(x) = \begin{cases} 1+\tan x, 0 \le x < \frac{\pi}{4} \\ 3-\cot x, \frac{\pi}{4} \le x < \pi \end{cases}$

165. $f(g(x))$ is
a) Discontinuous at $x = \pi/4$
b) Differentiable at $x = \pi/4$
c) Continuous but non-differentiable at $x = \pi/4$
d) Differentiable at $x = \pi/4$, but derivative is not continuous

Paragraph for Question Nos. 166 to - 166

Consider $f(x) = x^2 + ax + 3$ and $g(x) = x + b$ and $F(x) = \lim_{n\to\infty} \frac{f(x)+x^{2n}g(x)}{1+x^{2n}}$

166. If $F(x)$ is continuous at $x = 1$, then
a) $b = a + 3$ b) $b = a - 1$ c) $a = b - 2$ d) None of these

Paragraph for Question Nos. 167 to - 167

Let $f(x) = \begin{cases} [x], -2 \le x \le -\frac{1}{2} \\ 2x^2-1, -\frac{1}{2} < x \le 2 \end{cases}$ and $g(x) = f(|x|) + |f(x)|$, where $[\cdot]$ represencts greatest integer function

167. The number of points where $|f(x)|$ is non-differentiable is

a) 3 b) 4 c) 2 d) 5

Paragraph for Question Nos. 168 to - 168

Given the continuous function

$$y = f(x) = \begin{cases} x^2 + 10x + 8, x \le -2 \\ ax^2 + bx + c, -2 < x < 0, a \ne 0 \\ x^2 + 2x, x \ge 0 \end{cases}$$

If a line L touches the graph of $y = f(x)$ at three points, then

168. The slope of the line 'L' is equal to

a) 1 b) 2 c) 4 d) 6

Integer Answer Type

169. $f(x) = \frac{x}{1+(\ln x)(\ln x)\cdots\infty}$ $\forall\, x \in [1,3]$ is non-differentiable at $x = k$. Then the value of $[k^2]$ is (where $[\cdot]$ represents greatest integer function)

170. Number of points of discontinuity for $f(x) = \text{sgn}(\sin x), x \in [0, 4\pi]$ is

171. Let $f(x)$ and $g(x)$ be two continuous functions and $h(x) = \lim_{n\to\infty} \frac{x^{2n} \cdot f(x) + x^{2m} \cdot g(x)}{(x^{2n}+1)}$. If limit of $h(x)$ exists at $x = 1$, then one root of $f(x) - g(x) = 0$ is

172. Let $f(x) = \begin{cases} \frac{x}{2} - 1, 0 \le x < 1 \\ \frac{1}{2}, 1 \le x \le 2 \end{cases}$ and $g(x) = (2x+1)(x-k) + 3, 0 \le x\, \infty$. Then $g(f(x))$ is continuous at $x = 1$ if $12k$ is equal to

173. A differentiable function f satisfying a relation $f(x+y) = f(x) + f(y) + 2xy(x+y) - \frac{1}{3}$ $\forall x, y \in R$ and $\lim_{h\to 0} \frac{3f(h)-1}{6h} = \frac{2}{3}$. Then the value of $[f(2)]$ is (where $[x]$ represents greatest integer function)

174. If the function $f(x) = \frac{\tan(\tan x) - \sin(\sin x)}{\tan x - \sin x}$ $(x \ne 0)$ is continuous at $x = 0$, then the value of $f(0)$ is

175. Let $f(x) = \lim_{n\to\infty} \frac{x^{2n-1} + ax^2 + bx}{x^{2n}+1}$. If $f(x)$ is continuous for all $x \in R$, then the value of $a + 8b$ is

176. If $f(x)$ is a continuous function $\forall x \in R$ and the $f(x) \in (1, \sqrt{30})$, and $g(x) = \left[\frac{f(x)}{a}\right]$, where $[\cdot]$ denotes the greatest integer function, is continuous $\forall x \in R$, then the least positive integral value of a is

177. Number of points where $f(x) = \text{sgn}(x^2 - 3x + 2) + [x - 3], x \in [0, 4]$ is discontinuous is (where $[\cdot]$ denotes the greatest integer function)

178. Let $g(x) = \begin{bmatrix} a\sqrt{x+1} \text{ if } 0 < x < 3 \\ bx + 2 \text{ if } 3 \le x < 5 \end{bmatrix}$, if $g(x)$ is differentiable on $(0, 5)$ then $(a + b)$ equals

179. Number of points of non-differentiability of function $f(x) = \max\{\sin^{-1}|\sin x|, \cos^{-1}|\sin x|\},\ 0 < x < 2\pi$ is

180. Given $\frac{\int_{f(y)}^{f(x)} e^t dt}{\int_y^x (1/t) dt} = 1, \forall\, x, y \in \left(\frac{1}{e^2}, \infty\right)$ where $f(x)$ is continuous and differentiable function and $f\left(\frac{1}{e}\right) = 0$. If $g(x) = \begin{cases} e^x, x \ge k \\ e^{x^2}, 0 < x < k \end{cases}$; then the value of '$k$' for which $f(g(x))$ is continuous $\forall\, x \in R^+$ is

181. Number of points where $f(x) = [x] + [x + 1/3] + [x + 2/3]$, then ($[\cdot]$ denotes the greatest integer function) is discontinuous for $x \in (0, 3)$

182. The least integer value of p for which $f''(x)$ is everywhere continuous where

$$f(x) = \begin{cases} x^p \sin\left(\frac{1}{x}\right) + x|x|, x \ne 0 \\ 0, \quad x = 0 \end{cases}$$

: ANSWER KEY :

1)	d	2)	c	3)	b	4)	a
5)	a	6)	b	7)	d	8)	a
9)	d	10)	d	11)	d	12)	b
13)	c	14)	a	15)	d	16)	c
17)	c	18)	a	19)	c	20)	c
21)	a	22)	c	23)	c	24)	a
25)	d	26)	a	27)	d	28)	a
29)	d	30)	b	31)	c	32)	c
33)	a	34)	d	35)	c	36)	d
37)	d	38)	b	39)	c	40)	b
41)	d	42)	a	43)	c	44)	c
45)	b	46)	b	47)	d	48)	c
49)	a	50)	b	51)	c	52)	b
53)	c	54)	d	55)	b	56)	d
57)	b	58)	c	59)	c	60)	d
61)	c	62)	c	63)	d	64)	a
65)	c	66)	b	67)	a	68)	c
69)	c	70)	d	71)	a	72)	b
73)	c	74)	c	75)	a	76)	c
77)	a	78)	c	79)	c	80)	b
81)	d	82)	b	83)	b	84)	b
85)	b	86)	b	87)	c	88)	c
89)	d	90)	b	91)	a		
1)	a,c	2)	b,c,d	3)	a,c,d	4)	a,b,c
5)	a,c,d	6)	a,c,d	7)	a,c	8)	a,c
9)	a,b,c,d	10)	a,b	11)	a,b	12)	a,c,d
13)	b,c	14)	a,b,d	15)	b,c,d	16)	a,b
17)	a,b	18)	b,d	19)	a,b,c	20)	a,b,d
21)	a,b	22)	a,c,d	23)	a,b,c,d	24)	a,b
25)	b,d	26)	a,b	27)	b,d	28)	a,c
29)	a,b	30)	b,d,e	31)	a,b,d	32)	a
33)	d	34)	b,c	35)	a,c	36)	b,d
37)	b,c	38)	a, c	39)	a,c,d	40)	a,b,c,d
1)	b	2)	b	3)	b	4)	b
5)	d	6)	b	7)	b	8)	d
9)	c	10)	c	11)	b	12)	c
13)	a	14)	c	15)	b	16)	a
17)	c	18)	a	19)	c	20)	c
21)	c	22)	b	23)	d	24)	b
25)	d	26)	a	27)	c		
1)	d	2)	c	3)	a	4)	b
5)	a						
1)	b	2)	c	3)	a	4)	a
5)	c						
1)	7	2)	5	3)	1	4)	6
5)	8	6)	2	7)	8	8)	6
9)	4	10)	2	11)	7	12)	1
13)	8	14)	5				

: HINTS AND SOLUTIONS :

1 **(d)**

$f(x)=(e^x-1)|e^{2x}-1|$

$=(e^x-1)|e^x-1||e^x+1|$

$=(e^x+1)(e^x-1)|e^x-1|$

Now, both e^x+1 and $(e^x-1)|e^x-1|$ are differentiable

[as $g(x)|g(x)|$ is differentiable when $g(x)=0$]

Hence, $f(x)$ is differentiable

$f(x)=\frac{x-1}{x^2+1}$ is rational function is which denominator never becomes zero

Hence, $f(x)$ is differentiable

$$f(x)=\begin{cases}\big||x-3|-1\big|, x<3\\ \dfrac{x}{3}[x]-2, x\geq 3\end{cases}$$

$$=\begin{cases}|3-x-1|, x<3\\ \dfrac{x}{3}3-2, 3\leq x<4\end{cases}$$

$$=\begin{cases}|x-2|, x<3\\ x-2, 3\leq x<4\end{cases}$$

$=x-2, x\in[2,4)$

Hence, $f(x)$ is differentiable at $x=3$

$$f(x)=3(x-2)^{3/4}+3\Rightarrow f'(x)=\frac{9}{4}(x-2)^{-1/4}$$

Which is non-differentiable at $x=2$

Here $f(x)$ is continuous and the graph has vertical tangent at $x=2$; however, graph is smooth in neighbourhood of $x=2$

2 **(c)**

Given that $\cos\frac{x}{2}\cos\frac{x}{2^2}\cos\frac{x}{2^3}\ldots\cos\frac{x}{2^n}=\frac{\sin x}{2^n\sin\left(\frac{x}{2^n}\right)}$

(1)

Taking logarithm to the base 'e' on both sides of equation (1) and then differentiating w.r.t. x, we get

$$\sum_{n=1}^{n}\frac{1}{2^n}\tan\frac{x}{2^n}=\left(\frac{1}{2^n}\cot\frac{x}{2^n}-\cot x\right)$$

$$\therefore \lim_{n\to\infty}\sum_{n=1}^{n}\frac{1}{2^n}\tan\frac{x}{2^n}=\lim_{n\to\infty}\left(\frac{1}{x}\times\frac{\frac{x}{2^n}}{\tan\frac{x}{2^n}}-\cot x\right)$$

$$=\left(\frac{1}{x}-\cot x\right)$$

$$\therefore \text{We have } f(x)=\begin{cases}\frac{1}{x}-\cot x, x\in(0,\pi)-\left\{\frac{\pi}{2}\right\}\\ \frac{2}{\pi},\ x=\frac{\pi}{2}\end{cases}$$

Clearly $\lim_{x\to\frac{\pi}{2}}f(x)=\lim_{x\to\frac{\pi}{2}}\left(\frac{1}{x}-\cot x\right)=\frac{2}{\pi}=f\left(\frac{\pi}{2}\right)$

Hence $f(x)$ is continuous at $x=\frac{\pi}{2}$

3 **(b)**

$f(2)=0$,

$f(2^+)=\{4^+\}-\{2^+\}^2=0-0=0$

$f(2^-)=\{4^-\}-\{2^-\}^2=1-1=0$

Hence $f(x)$ is continuous at $x=2$

$f(-2)=0$,

$f(-2^+)=\{4^-\}-\{-2^+\}^2=1-0=1$

Hence $f(x)$ is discontinuous at $x=-2$

4 **(a)**

$$f(x)=2|\mathrm{sgn}(2x)|+2=\begin{cases}4, x>0\\ 2, x=0\\ 0, x<0\end{cases}$$

Thus, $f(x)$ has non-removable discontinuity at $x=0$

5 **(a)**

$$f(x)=\lim_{n\to\infty}(\sin^2[\pi x])^n+\left[x+\frac{1}{2}\right]$$

Now $g(x)=\lim_{n\to\infty}(\sin^2(\pi x))^n$ is discontinuous when $\sin^2(\pi x)=1$ or $\pi x=(2n+1)\frac{\pi}{2}$ or $x=\frac{(2n+1)}{2}, n\in z$

Thus, $g(x)$ is discontinuous at $x=3/2$

Also $h(x)=\left[x+\frac{1}{2}\right]$ is discontinuous at $x=3/2$

But $f(3/2)=\lim_{n\to\infty}(\sin^2(3\pi/2))^n+\left[\frac{3}{2}+\frac{1}{2}\right]=1+2=3$

$$f(3/2^+)=\lim_{n\to\infty}(\sin^2((3\pi/2)^+))^n+\left[\left(\frac{3}{2}\right)^++\frac{1}{2}\right]$$

$$=0+2=2$$

Hence, $f(x)$ is discontinuous at $x=3/2$

Both $g(x)$ and $h(x)$ are continuous at $x=1$, hence, $f(x)$ is continuous at $x=1$

6 **(b)**

$|\sin x|$ and $e^{|x|}$ are not differentiable at $x=0$ and $|x^3|$ is differentiable at $x=0$

Therefore, for $f(x)$ to be differentiable at $x=0$, We must have $a=0, b=0$ and c can be any real number

7 **(d)**

We have $\lim_{x\to1^-}f(x)=\lim_{h\to0}f(1-h)$

$$=\lim_{h\to0}\frac{\log(4+h^2)}{\log(1-4h)}=-\infty$$

And, $\lim_{x\to1^+}f(x)=\lim_{h\to0}f(1+h)=\lim_{h\to0}\frac{\log(4+h^2)}{\log(1+4h)}=\infty$

So, $f(1^-)$ and $f(1^+)$ do not exist

8 **(a)**

$$f(x)=\begin{cases} x+2, & x<0 \\ -x^2-2, 0\le x<1 \\ x, & x\ge 1\end{cases}$$

$$\therefore |f(x)|=\begin{cases} -x-2, x<-2 \\ x+2, -2\le x<0 \\ x^2+2, 0\le x<1 \\ x, & x\ge 1\end{cases}$$

Discontinuous at $x=1$ $\therefore$ number of points of discount 1

9 **(d)**

$f(x)=\sqrt[3]{|x|^3}-|x|-1$

$\Rightarrow |x|-|x|-1=-1$

Hence, differentiable for all x

10 **(d)**

$$f(x)=\frac{\tan(\pi[x-\pi])}{1+[x]^2}$$

By definition, $[x-\pi]$ is an integer whatever be the value of x and so $\pi[x-\pi]$ is an integral multiple of π

Consequently, $\tan(\pi[x-\pi])=0, \forall x$

And since $1+[x]^2\neq 0$ for any x, we conclude that $f(x)=0$

Thus $f(x)$ is constant function and so it is continuous and differentiable

11 **(d)**

$f(x)=[x^2]-\{x\}^2$

$f(-1)=1, f(-1^-)=1-1=0$

$f(1)=1, f(1^+)=1-0=1$

$f(1^-)=0-1=-1$

Thus, $f(x)$ is discontinuous at $x=1,-1$

12 **(b)**

$f(x)=\cos\pi\,(|x|+[x])$

$$=\begin{cases}\cos\pi\left(-x+(-1)\right), -1\le x<0 \\ \cos\pi\,(x+0), \quad 0\le x<1\end{cases}$$

$$=\begin{cases}-\cos\pi x, -1\le x<0 \\ \cos\pi x, 0\le x<1\end{cases}$$

Obviously, $f(x)$ is discontinuous at $x=0$, otherwise $f(x)$ is continuous and differentiable in $(-1,0)$ and $(0,1)$

13 **(c)**

$$f(|x|)=\begin{cases}\sin|x|, & |x|<0 \\ \cos(x)-\big||x|-1\big|, & |x|\ge 0\end{cases}$$

$\Rightarrow f(|x|)=\cos(x)-\big||x|-1\big|, x\in R$

[as $|x|<0$ is not possible and $|x|\ge 0$ is true $\forall\, x\in R$]

Which is non-differentiable at $x=0$ and when $|x|-1=0$ or $x=\pm 1$

Hence, $f(|x|)$ has exactly three points of non-differentiability

14 **(a)**

Clearly $f(x)$ is continuous at $x=0$

Now $f'(0^+)=\lim\limits_{h\to 0}\frac{e^{-1/h^2}-0}{h}=\lim\limits_{h\to 0}\frac{1/h}{e^{1/h^2}}$

$=\lim\limits_{h\to 0}\frac{-1/h^2}{-2/h^3\,e^{1/h^2}}$ (applying L' Hopital's rule)

$=\frac{1}{2}\lim\limits_{h\to 0}\frac{h}{e^{1/h^2}}=0$

Also $f(0^-)=0$

Thus, $f(x)$ is differentiable at $x=0$

15 **(d)**

Let k is integer

$f(k)=0, f(k-0)=(k-1)^2-(k^2-1)$

$=2-2k$

$f(k+0)=k^2-(k^2)=0$

If $f(x)$ is continuous at $x=k$, then $2-2k=0$

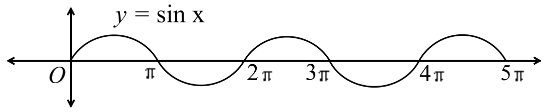
$\Rightarrow k=1$

16 **(c)**

$f(x)=\text{sgn}(\sin^2 x-\sin x-1)$ is discontinuous when $\sin^2 x-\sin x-1=0$

or $\sin x=\frac{1\pm\sqrt{5}}{2}$ or $\sin x=\frac{1-\sqrt{5}}{2}$

For exactly four point of discontinuity, n can take value 4 or 5 as shown in the diagram



17 **(c)**

$f(2+x)=f(-x)$

Replace x by $x-1$, we have $f(2+x-1)=f(-x+1)$ or $f(1+x)=f(1-x)$

Hence $f(x)$ is symmetrical about line $x=1$

Now put $x=2$ in (1), we get $f(4)=f(-2)$, hence differentiability at $x=4$ implies differentiability at $x\to 2$

18 **(a)**

L.H.D. at $x=k$

$=\lim_{h\to 0}\frac{f(k)-f(k-h)}{h}$ $(k=\text{integer})$

$=\lim\limits_{h\to 0}\frac{[k]\sin k\pi-[k-h]\sin(k-h)\pi}{h}$

$=\lim\limits_{h\to 0}\frac{-(k-1)\sin(k\pi-h\pi)}{h}$ $[\because \sin k\pi=0]$

$=\lim\limits_{h\to 0}\frac{-(k-1)(-1)^{k-1}\sin h\pi}{h\pi}\times\pi$

$=\pi(k-1)(-1)^k$

19 **(c)**

$$\lim_{x\to 2^+}\frac{(x-2)}{|x-2|}\left(\frac{x^2-1}{x^2+1}\right)=\lim_{x\to 2^+}\frac{(x-2)}{(x-2)}\left(\frac{x^2-1}{x^2+1}\right)$$

$$=\lim_{x\to 2^+}\left(\frac{x^2-1}{x^2+1}\right)=\frac{3}{5}$$

$$=\lim_{x\to 2^-}\frac{(x-2)}{|x-2|}\left(\frac{x^2-1}{x^2+1}\right)$$

$$= \lim_{x \to 2^-} \frac{(x-2)}{(2-x)} \left(\frac{x^2-1}{x^2+1}\right) = -\frac{3}{5}$$

Thus, L.H.L. ≠ R.H.L.

Hence, the function has non-removable discontinuity at $x = 2$

20 **(c)**

$[\sin x]$ is non-differentiable at $x = \frac{\pi}{2}, \pi, 2\pi$ and

$[\cos x]$ is non-differentiable at $x = 0, \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi$

Thus, $f(x)$ is definitely non-differentiable at

$x = \pi, \frac{3\pi}{2}, 0$

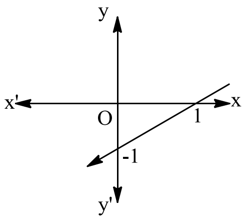
Also, $f\left(\frac{\pi}{2}\right) = 1, f\left(\frac{\pi}{2} - 0\right) = 0$

$f(2\pi) = 1, f(2\pi - 0) = -1$

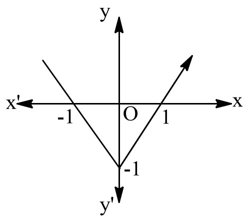
Thus, $f(x)$ is also non-differentiable at $x = \frac{\pi}{2}$ and 2π

21 **(a)**

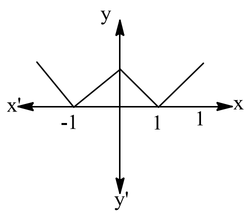
Using graphical transformation



(i) y = x - 1



(ii) y = |x| - 1



(iii) y = ||x| - 1|

As, we know the function is not differentiable at6 sharp edges and in figure (iii) $y = \left||x| - 1\right|$ we have 3 sharp edges at $x = -1, 0, 1$

$\therefore f(x)$ is not differentiable at $\{0, \pm 1\}$

22 **(c)**

$\left|x - \frac{1}{2}\right|$ is continuous everywhere but not differentiable at $x = \frac{1}{2}$, $|x - 1|$ is continuous everywhere but not differentiable at $x = 1$, and $\tan x$ is continuous in $[0, 2]$ except at $x = \frac{\pi}{2}$

Hence $f(x)$ is not differentiable at $x = \frac{1}{2}, 1, \frac{\pi}{2}$

23 **(c)**

$$f(x) = \begin{cases} |x| - 1, & |x| < 0 \\ |x|^2 - 2|x|, & |x| \geq 0 \end{cases}$$

Where $|x| < 0$ is not possible thus, neglecting we get,

$f(|x|) = |x|^2 - 2|x|, |x| \geq 0$

$$f(|x|) = \begin{cases} x^2 + 2x, x < 0 \\ x^2 - 2x, x \geq 0 \end{cases} \quad (1)$$

$$\Rightarrow f'(|x|) = \begin{cases} 2x + 2, & x < 0 \\ 2x - 2, & x > 0 \end{cases}$$

Clearly $f(|x|)$ is continuous at $x = 0$, but non-differentiable at $x = 0$

$$f(|x|) = \begin{cases} |x| - 1, & |x| < 0 \\ |x|^2 - 2|x|, |x| \geq 0 \end{cases}$$

$$g(x) = |f(x)| = \begin{cases} 1 - x, \ x < 0 \\ -x^2 + 2x, \ 0 \leq x < 2 \\ x^2 - 2x, \ x \geq 2 \end{cases} \quad (2)$$

Clearly $|f(x)|$ is discontinuous at $x = 0$, but continuous at $x = 2$

$$\text{Also, } g'(x) = \begin{cases} -1, \ x < 0 \\ -2x + 2, \ 0 < x < 2 \\ 2x - 2, \ x > 2 \end{cases}$$

$|f(x)|$ is non-differentiable at $x = 0$ and $x = 2$

24 **(a)**

$f(x) = (-1)^{[x^3]}$ is discontinuous

When $x^3 = n, n \in Z \Rightarrow x = n^{1/3}$

$f\left(\frac{3}{2}\right) = (-1)^3 = -1$

For $x \in (-1, 0), f(x) = (-1)^{-1} = -1$

$\Rightarrow f'(x) = 0$

For $x \in [0, 1), f(x) = (-1)^{\circ} = 1$

$\Rightarrow f'(x) = 0$

25 **(d)**

$f(x)$ is continuous at $x = 0 \Rightarrow \lim_{x \to 0} f(x) = f(0)$

$$\Rightarrow f(0) = \lim_{x \to 0} f(0 + h) = \lim_{h \to 0} \frac{hg(h)}{|h|} = \lim_{h \to 0} g(h)$$

$$= g(0) = 0$$

Now $f'(0^+) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{\frac{hg(h)}{|h|}}{h}$

$$= \lim_{h \to 0} \frac{g(h)}{h} = \lim_{h \to 0} \frac{g(h) - g(0)}{h}$$

$= g'(0)$ (as $g(0) = 0$) $= 0$

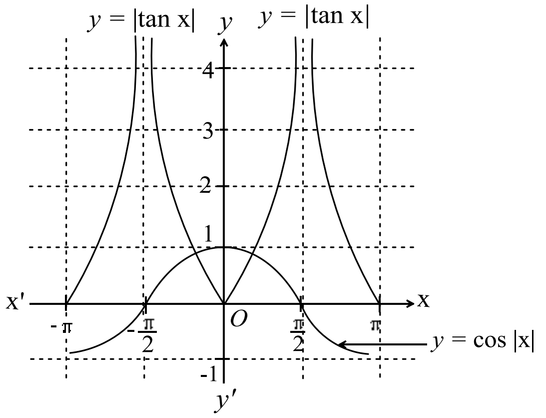
$$f'(0^-) = \lim_{h \to 0} \frac{f(0 - h) - f(0)}{-h}$$

$$= \lim_{h \to 0} \frac{\frac{-hg(-h)}{|-h|}}{-h} = \lim_{h \to 0} \frac{g(-h)}{h}$$

$$= -\lim_{h \to 0} \frac{g(-h) - g(0)}{-h} = -g'(0) = 0$$

Hence, $f'(0)$ exists and $f'(0) = 0$

26 **(a)**



The functions is not differentiable and continuous at two points between $x = -\pi/2$ and $x = \pi/2$. Also the function is not continuous at $x = \frac{\pi}{2}$ and $x = -\frac{\pi}{2}$. Hence, at four points, the function is not differentiable

27 **(d)**

$f(2^+) = 2 + 2\sin(0) = 2$

$f(2^-) = 3 + 2\sin(1)$

Hence, $f(x)$ is discontinuous at $x = 2$

Also $f(0^+) = 2(0) - 0 - 0\sin(0 - 0) = 0$

and $f(0^-) = 2(0) - (-1) - 0\sin(0 - (-1)) = 1$

Hence, $f(x)$ is discontinuous at $x = 0$

28 **(a)**

$f(x)$ is continuous when $5x = x^2 + 6 \Rightarrow x = 2, 3$

29 **(d)**

$$\lim_{x\to 0} \frac{x - e^x + 1 - (1 - \cos 2x)}{x^2}$$

$$= \lim_{x\to 0}\left[\frac{x - e^x + 1}{x^2} - \frac{(1 - \cos 2x)}{x^2}\right]$$

$$= \lim_{x\to 0}\left[\frac{x + 1 - \left(1 + x + \frac{x^2}{2}\right)}{x^2} - \frac{2\sin^2 x}{x^2}\right]$$ (Using expansion of e^x)

$$= -\frac{1}{2} - 2$$

$= -\frac{5}{2}$; hence for continuous $f(0) = -\frac{5}{2}$

Now $[f(0)] = -3; \{f(0)\} = \left\{-\frac{5}{2}\right\} = \frac{1}{2}$

Hence, $[f(0)]\{f(0)\} = -\frac{3}{2} = -1.5$

30 **(b)**

$$g'(0^+) = \lim_{h\to 0} \frac{f(|h|) - |\sin h| - f(0)}{h}$$

$$= \lim_{h\to 0} \frac{f(h) - f(0)}{h} - \lim_{h\to 0} \frac{\sin h}{h}$$

$= 1 - 1 = 0$

$$= g'(0^-) = \lim_{h\to 0} \frac{f(|-h|) - |\sin(-h)| - f(0)}{-h}$$

$$= \lim_{h\to 0} \frac{f(-h) - f(0)}{-h} + \lim_{h\to 0} \frac{\sin h}{h}$$

$= -1 + 1 = 0$

Thus, $g(x)$ is differentiable and $g'(0) = 0$

31 **(c)**

$$f'(0) = \lim_{h\to 0} \frac{f(h) - f(0)}{h}$$

$$= \lim_{h\to 0} \frac{h^a \sin\left(\frac{1}{h}\right)}{h} = \lim_{h\to 0} h^{a-1} \sin\left(\frac{1}{h}\right)$$

This limit will not exist if $a - 1 \le 0 \Rightarrow a \le 1$

Now $\lim_{x\to 0} f(x) = \lim_{x\to 0} x^a \sin\left(\frac{1}{x}\right) = 0$ if $a > 0$

Thus, $a \in (0, 1]$

32 **(c)**

Clearly, $f(x)$ is continuous for all x except possibly at $x = \pi/6$

For $f(x)$ to be continuous at $x = \pi/6$, we must have

$$\lim_{x\to \pi/6^-} f(x) = \lim_{x\to \pi/6^+} f(x)$$

$$\Rightarrow \lim_{x\to \pi/6} \sin 2x = \lim_{x\to \pi/6} ax + b$$

$\Rightarrow \sin(\pi/3) = (\pi/6)a + b$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\pi}{6}a + b \qquad (1)$$

For $f(x)$ to be differentiable at $x = \pi/6$, we must have L.H.D. at $x = \pi/6$ = R.H.D. at $x = \pi/6$

$$\Rightarrow \lim_{x\to \pi/6} 2\cos 2x = \lim_{x\to \pi/6} a$$

$\Rightarrow 2\cos \pi/3 = a \Rightarrow a = 1$

Putting $a = 1$ in equation (1), we get $b = (\sqrt{3}/2) - \pi/6$

33 **(a)**

$$\lim_{x\to 0+}\left(3 - \left[\cot^{-1}\frac{2x^3 - 3}{x^2}\right]\right) = (3 - [\cot^{-1}(-\infty)])$$

$$= (3 - [\pi])$$

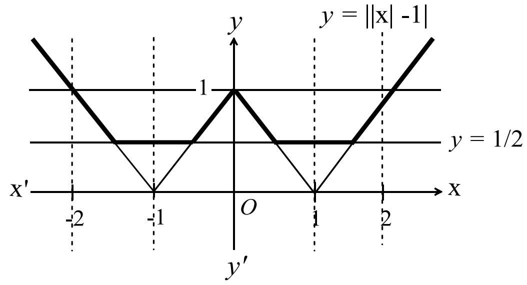
$$= \lim_{x\to 0-} \{x^2\} \cos\left(e^{1/x}\right)$$

$$= \left(\lim_{x\to 0-} \{x^2\}\right)\left(\lim_{x\to 0-} \cos\left(e^{1/x}\right)\right)$$

$= (0)(\cos(e^{-\infty})) = 0$

Thus $f(x)$ has irremovable discontinuity at $x = 0$, hence $f(0)$ does not exist

34 **(d)**



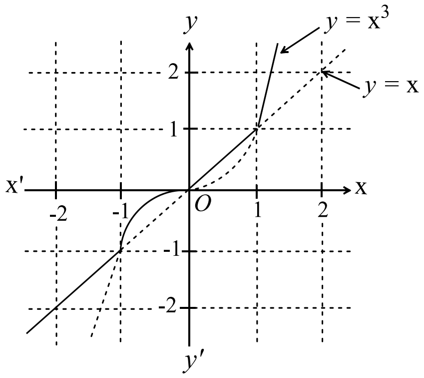
Clearly from the graph, $f(x)$ is non-differentiable at five points, $x = -2, -1, 0, 1, 2$

35 **(c)**

$$f(x) = \begin{cases} x^2 - ax + 3, & x \text{ is rational} \\ 2 - x, & x \text{ is rational} \end{cases}$$

Is continuous when $x^2 - ax + 3 = 2 - x$ or
$x^2 - (a-1)x + 1 = 0$
Which must have two distinct roots for
$(a-1)^2 - 4 > 0$
$\Rightarrow (a-1-2)(a-1+2) > 0$
$\Rightarrow a \in (-\infty, -1) \cup (3, \infty)$

36 **(d)**



From the graph $f(x) = \max\{x, x^3\} =$
$$\begin{cases} x, & x < -1 \\ x^3, & -1 \le x \le 0 \\ x, & 0 < x < 1 \\ x^3, & x \ge 1 \end{cases}$$
Clearly, f is not differentiable at $-1, 0$ and 1

37 **(d)**

For $0 \le x < 1, f(x) = [\sin 0] = 0, \quad 1 \le x < 2, f(x) = [\sin 1] = 0$
$2 \le x < 3, f(x) = [\sin 2] = 0, \ \ 3 \le x < 4, f(x) = [\sin 3] = 0$
$4 \le x < 5, f(x) = [\sin 4] = -1$
Hence, there is discontinuity at point $(4, -1)$

38 **(b)**

$0 \le \tan^2 x < 1$ when $-\frac{\pi}{4} < x < \frac{\pi}{4}$
$\Rightarrow f(x) = 0 - \frac{\pi}{4} < x < \frac{\pi}{4}$
Hence, $f(x)$ is continuous and differentiable at $x = 0$, also $f'(0) = 0$

39 **(c)**

$f'(0^+) = \lim_{h\to 0} \frac{h^m \sin\frac{1}{h}}{h}$ must exist $\Rightarrow m > 1$
For
$m > 1, h'(x) =$
$$\begin{cases} m\, x^{m-1} \sin\frac{1}{x} - x^{m-2}\cos\frac{1}{x}, \text{if } x \neq 0 \\ \quad 0, \ \text{if } x = 0 \end{cases}$$
Now $\lim_{h\to 0} h(x) = \lim_{h\to 0}\left(mh^{m-1}\sin\frac{1}{h} - h^{m-2}\cos\frac{1}{h}\right)$
Limit exists if $m > 2$
$\therefore m \in N \Rightarrow m = 3$

40 **(b)**

Given $f(x)$ is continuous at $x = 0$
$\Rightarrow \lim_{x\to 0} f(x) = f(0)$
$$\Rightarrow \lim_{x\to 0} \frac{(3^x - 1)^2}{\sin x \ \ \text{In}(1+x)} = f(0)$$
$$\Rightarrow f(0) = \lim_{x\to 0} \frac{\left(\frac{3^x-1}{x}\right)^2}{\left(\frac{\sin x}{x}\right)\left(\frac{\text{In}(1+x)}{x}\right)} = (\text{In } 3)^2$$

41 **(d)**

$f(x) = (x^2 - 1)|x^2 - 3x + 2| + \cos(|x|)$
$= [(x-1)|x-1|]\,|x-2| + \cos x$
$(x-1)|x-1|$ and $\cos x$ are differentiable for all x
But $|x-2|$ is non-differentiable at $x = 2$
Hence, $f(x)$ is non-differentiable at $x = 2$

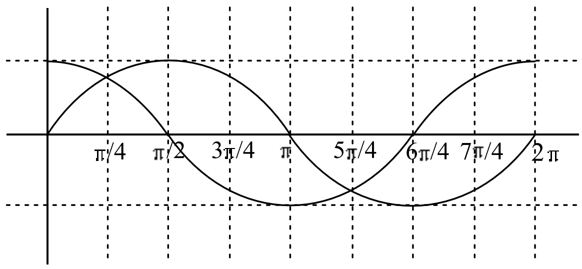
42 **(a)**

$f(x)$ is continuous at some x where $\sin x = \cos x$
or $\tan x = 1$ or $x = n\pi + \pi/4, n \in I$

43 **(c)**

$f(0+0) = \lim_{h\to 0} f(h)$
$$= \lim_{h\to 0}\frac{h}{2h^2 + h} = \lim_{h\to 0}\frac{1}{2h+1} = 1$$
and $f(0-0) = \lim_{h\to 0} f(-h) = \lim_{h\to 0}\frac{-h}{2h^2 + |-h|}$
$$\lim_{h\to 0}\frac{-h}{2h^2+h} = \lim_{h\to 0}\frac{-1}{2h+1} = -1$$

44 **(c)**



Consider the graph of $f(x) = \max(\sin x, \cos x)$, which is non-differentiable at $x = \pi/4$, hence statement (a) is false
From the graph $y = f(x)$ is differentiable at $x = \pi/2$, hence statement (b) is false
Statement (c) is always true
Statement (d) is false as consider $g(x) = \max(x, x^2)$ at $x = 0$, for which $x = x^2$ at $x = 0$, but $f(x)$ is differentiable at $x = 0$

45 **(b)**

$f(1)=1-\sqrt{1-1^2}=1$

$f(1^-)=\lim_{x\to 1^+}(1-\sqrt{1-x^2})=1$

$f(1^+)=\lim_{x\to 1^-}\left(1+\log\frac{1}{x}\right)=1+\log\frac{1}{1}=1$

Hence, $f(x)$ is continuous at $x=1$

$f'(1^+)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$

$=\lim_{h\to 0}\frac{1+\log\frac{1}{1+h}-1}{h}$

$=-\lim_{h\to 0}\frac{\log(1+h)}{h}=-1$

$f'(1^-)=\lim_{h\to 0}\frac{f(1-h)-f(1)}{-h}$

$=\lim_{h\to 0}\frac{1-\sqrt{1-(1-h)^2}-1}{-h}=\lim_{h\to 0}\frac{\sqrt{2-h}}{\sqrt{h}}=\infty$

Hence, $f(x)$ is non-differentiable at $x=1$

46 **(b)**

Since both $\cos x$ and $\sin^{-1}x$ are continuous function. $f(x)=\sin^{-1}(\cos x)$ is also a continuous function. Now

$f'(x)=\frac{-\sin x}{\sqrt{1-\cos^2 x}}=\frac{-\sin x}{|\sin x|}$

Hence, $f(x)$ is non-differentiable at $x=n\pi, n\in Z$

48 **(c)**

$f(x)=\begin{cases}ax^2+1, x\le 1\\ x^2+ax+b, x>1\end{cases}$ is differentiable at $x=1$

Then $f(x)$ is continuous at $x=1$

$\Rightarrow f(1^-)=f(1^+)\Rightarrow a+1=1+a+b\Rightarrow b=0$

Also $f'(x)=\begin{cases}2ax, x<1\\ 2x+a, x>1\end{cases}$

We must have $f'(1^-)=f'(1^+)\Rightarrow 2a=2+a\Rightarrow a=2$

49 **(a)**

We have $f(x)=\begin{cases}x^3, x>0\\ 0, x=0\\ -x^3, x<0\end{cases}$

Clearly, $f(x)$ is continuous at $x=0$

(L.H.D. at $x=0$) $=\left[\frac{d}{dx}(-x^3)\right]_{x=0}=[-3x^2]_{x=0}=0$

Similar (R.H.D. at $x=0$)=0

So, $f(x)$ is differentiable at $x=0$

50 **(b)**

$g(x)$ is an even function, then $g(x)=g(-x)$

$\Rightarrow g'(x)=-g'(-x)\Rightarrow g'(0)=-g'(0)\Rightarrow g'(0)=0$

Now $f'(0)=\lim_{h\to 0}\frac{g(0+h)\cos(1/h)-0}{h}$

$=\lim_{h\to 0}\frac{g(h)\cos(1/h)}{h}=\lim_{h\to 0}g'(0)\cos(1/h)=0$

51 **(c)**

We have $f(x)=\frac{4-x^2}{x(4-x^2)}$

Clearly, there are three points of discontinuity, viz., $0, 2, -2$

52 **(b)**

$f(x)=\frac{\tan\left(\frac{\pi}{4}-x\right)}{\cot 2x}, (x\ne\pi/4)$ is continuous at $x=\pi/4$

$\Rightarrow f\left(\frac{\pi}{4}\right)=\lim_{x\to\frac{\pi}{4}}f(x)$

$=\lim_{x\to\frac{\pi}{4}}\frac{\tan\left(\frac{\pi}{4}-x\right)}{\cot 2x}$

Now by applying L' Hopital's rule,

$=\lim_{x\to\frac{\pi}{4}}\frac{-\sec^2\left(\frac{\pi}{4}-x\right)}{-2\,\mathrm{cosec}^2(2x)}=\frac{1}{2}$

53 **(c)**

When x is not an integer, both the functions $[x]$ and $\cos\left(\frac{2x-1}{2}\right)\pi$ are continuous

$\therefore f(x)$ is continuous on all non-integral points

For $x=n\in I$

$\lim_{x\to n-}f(x)=\lim_{x\to n-}[x]\cos\left(\frac{2x-1}{2}\right)\pi$

$=(n-1)\cos\left(\frac{2n-1}{2}\right)\pi=0$

$\lim_{x\to n+}f(x)=\lim_{x\to n+}[x]\cos\left(\frac{2x-1}{2}\right)\pi$

$=n\cos\left(\frac{2n-1}{2}\right)\pi=0$

Also $f(n)=n\cos\frac{(2n-1)\pi}{2}=0$

$\therefore f$ is continuous at all integral points as well.

Thus, f is continuous everywhere

54 **(d)**

Since $\lim_{n\to\infty}x^{2n}=\begin{cases}0, \text{if}|x|<1\\ 1, \text{if }|x|=1\end{cases}$

$\therefore f(x)=\lim_{x\to\infty}(\sin x)^{2n}=\begin{cases}0, \text{if }|\sin x|<1\\ 1, \text{if }|\sin x|=1\end{cases}$

Thus, $f(x)$ is continuous at all x, except for those values of x for which $|\sin x|=1$, i.e., $x=(2k+1)\frac{\pi}{2}, k\in Z$

55 **(b)**

We have

$$f(x)=\frac{x-|x-1|}{x}=\begin{cases}\frac{x+x-1}{x}, x<1, x\neq 0\\ \frac{x-(x-1)}{x}, x\geq 1\end{cases}$$
$$=\begin{cases}\frac{2x-1}{x}, x<1, x\neq 0\\ \frac{1}{x}, \quad x\geq 1\end{cases}$$
Clearly, $f(x)$ is discontinuous at $x=0$ as it is not defined at $x=0$. Since $f(x)$ is not defined at $x=0$, therefore $f(x)$ cannot be differentiable at $x=0$. Clearly $f(x)$ is continuous at $x=1$, but it is not differentiable at $x=1$, because $Lf'(1)=1$ and $Rf'(1)=-1$

56 **(d)**

We have $\lim_{x\to 0^-} f(x)=\lim_{h\to 0}\sin(\log_e|-h|)=\lim_{h\to 0}\sin(\log_e h)$ which does not exist and oscillates between -1 and 1. Similarly, $\lim_{x\to 0^+} f(x)$ lies between -1 and 1

57 **(b)**

$$f(x)=\begin{cases}1+\left[\cos\frac{\pi x}{2}\right], 1<x\leq 2\\ 1-\{x\},\ 0\leq x<1\\ |\sin\pi x|, -1\leq x<0\end{cases}=$$
$$\begin{cases}1-1,\ 1<x\leq 2\\ 1-x,\ 0\leq x<1\\ -\sin\pi x, -1\leq x<0\end{cases}$$
$f(x)$ is continuous at $x=1$ but not differentiable

58 **(c)**

Given that $f(x)=|1-x|$
$$\Rightarrow f(|x|)=\begin{cases}x-1, & x>1\\ 1-x, & 0<x\leq 1\\ 1+x, & -1\leq x\leq 0\\ -x-1, & x<-1\end{cases}$$
Clearly, the domain of $\sin^{-1}(f|x|)$ is $[-2,2]$

$\Rightarrow$ It is non-differentiable at the points $\{-1,0,1\}$

59 **(c)**

At $x=0$,

L.H.L. $=\lim_{x\to 0-} f(x)=\lim_{h\to 0} f(0-h)$
$$=\lim_{h\to 0} h^2\left(\frac{e^{-1/h}-e^{1/h}}{e^{-1/h}+e^{1/h}}\right)$$
$$=\lim_{h\to 0} h^2\left(\frac{e^{-2/h}-1}{e^{-2/h}+1}\right)$$
$$=0\left(\frac{0-1}{0+1}\right)=0$$
R.H.L. $=\lim_{x\to 0+} f(x)=\lim_{h\to 0} f(0+h)$
$$=\lim_{h\to 0} h^2\left(\frac{e^{1/h}-e^{-1/h}}{e^{1/h}+e^{-1/h}}\right)$$
$$=\lim_{h\to 0} h^2\left(\frac{1-e^{-2/h}}{1+e^{-2/h}}\right)$$
$$=0\left(\frac{1-0}{1+0}\right)=0$$
and $f(0)=0$

$\Rightarrow$ L.H.L $=$ R.H.L. $=f(0)$

Hence, $f(x)$ is continuous at $x=0$

Also L.H.D. $=\lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}$
$$=\lim_{h\to 0}\frac{h^2\frac{e^{-1/h}-e^{1/h}}{e^{-1/h}+e^{1/h}}-0}{-h}$$
$$=-\lim_{h\to 0} h\frac{e^{-2/h}-1}{e^{-2/h}+1}=0$$
and R.H.D. $=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$
$$=\lim_{h\to 0}\frac{h^2\frac{e^{1/h}-e^{-1/h}}{e^{1/h}+e^{-1/h}}-0}{-h}$$
$$=-\lim_{h\to 0} h\frac{1-e^{-2/h}}{1+e^{-2/h}}=0$$
Hence, $f(x)$ is differentiable at $x=0$ and $f'(0)=0$

60 **(d)**

Clearly, $f(x)$ is continuous at $x=0$ if $a=0$

Now, $f'(0+0)=\lim_{h\to 0}\frac{he^{-\left(\frac{1}{h}+\frac{1}{h}\right)}-0}{h}$
$$=\lim_{h\to 0}\frac{he^{-2/h}-0}{h}=0$$
$$f'(0-0)=\lim_{h\to 0}\frac{-he^{-\left(\frac{1}{h}+\frac{1}{h}\right)}-0}{-h}=1$$
Thus, no values of a exists

61 **(c)**

Obviously $\lim_{x\to 0+} e^{-1/x^2}=\lim_{x\to 0-} e^{-1/x^2}=0$,

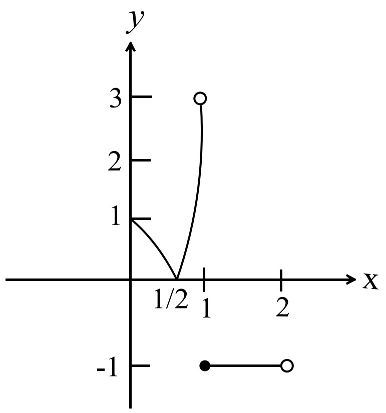
Hence $f(x)$ is continuous at $x=0$
$$f'(0)=\lim_{h\to 0}\frac{e^{-1/h^2}}{h}=\lim_{h\to 0}\frac{1/h}{e^{1/h^2}}$$
$$=\lim_{h\to 0}\frac{-1/h^2}{-e^{1/h^2}\cdot\frac{2}{h^3}}=\lim_{h\to 0}\frac{2h^3}{h^2 e^{1/h^2}}=0$$
Hence f is differentiable at $x=0$. Also
$$\lim_{x\to\pm\infty} e^{-\frac{1}{x^2}}\to 1$$

62 **(c)**

Since $1\leq x<2\Rightarrow 0\leq x-1<1$

$\Rightarrow [x^2-2x]=[(x-1)^2-1]=[(x-1)^2]-1$

$=0-1=-1$
$$\therefore f(x)=\begin{cases}1-4x^2, & 0\leq x<\frac{1}{2}\\ 4x^2-1, & \frac{1}{2}\leq x<1\\ -1, & 1\leq x<2\end{cases}$$
$\therefore$ graph of $f(x)$:



It is clear from graph that $f(x)$ is discontinuous at $x=1$ and differentiable at $x=\frac{1}{2}$ and $x=1$

63 **(d)**

$\frac{x}{1+|x|}$ is always differentiable (also at $x=0$)

Also $(x-2)(x+2)|(x-1)(x-2)(x-3)|$ is not differentiable at $x=1,3$

So, $f(x)$ is not differentiable at $x=1,3$

64 **(a)**

Hence check continuity at $x=k, k\in Z$

For positive integers

$f(k)=\{k\}^2-\{k^2\}=0$

$f(k^+)=\{k^+\}^2-\{(k^+)^2\}=0-0$

$f(k^-)=\{k^-\}^2-\{(k^-)^2\}=1-1=0$

For negative integers,

$f(k)=\{k^2\}-\{k^2\}=0$

$f(k^+)=\{k^+\}^2-\{(k^+)^2\}=0-1=-1$

$f(k^-)=\{k^-\}^2-\{(k^-)^2\}=1-0=1$

Hence, $f(x)$ is continuous at positive integers and discontinuous at negative intergers

65 **(c)**

For $f(x)$ to be continuous at $x=0$, we have

$f(0^-)=f(0^+)\Rightarrow a(0)+b=1\Rightarrow b=1$

$$f'(0^+)=\lim_{h\to 0}\frac{f(h)-f(0)}{h}=\lim_{h\to 0}\frac{e^{h^2+h}-b}{h}$$

$$=\lim_{h\to 0}\frac{e^{h^2+h}-1}{h}=\lim_{h\to 0}\frac{e^{h^2+h}-1}{h(h+1)}(h+1)=1$$

$\therefore f'(0^-)=a$

Hence, $a=1$

66 **(b)**

$f(0^+)=\lim_{x\to 0+}|x|^{\sin x}=e^{\lim_{x\to 0}\sin x\log|x|}$

$=e^{\lim_{x\to 0}\frac{\log x}{\operatorname{cosec} x}}=e^0=1$ (Using L' Hopital rule)

$f(0^-)=g(0)=1$

Let $g(x)=ax+b$

$\Rightarrow b=1 \Rightarrow g(x)=ax+1$

For $x>0, f'(x)=e^{\sin x\,\text{In}(|x|)}\Big[\cos x\,\text{In}(|x|)+$

$\sin xx$

$f'(1)=1[0+\sin 1]=\sin 1$

$f(-1)=-a+1\Rightarrow a=1-\sin 1$

$\Rightarrow g(x)=(1-\sin 1)x+1$

67 **(a)**

$$f(x)=\frac{x^2-bx+25}{x^2-7x+10}, x\neq 5$$

$f(x)$ is continuous at $x=5$, only if

$\lim_{x\to 5}\frac{x^2-bx+25}{x^2-7x+10}$ is finite

Now $x^2-7x+10\to 0$ when $x\to 5$

Then we must have $x^2-bx+25\to 0$ for which $b=10$

Hence, $\lim_{x\to 5}\frac{x^2-10x+25}{x^2-7x+10}=\lim_{x\to 5}\frac{x-5}{x-2}=0$

68 **(c)**

Since, $\lim_{x\to 1^-}g(x)=\lim_{x\to 1^+}g(x)=1$ and $g(1)=0$

So, $g(x)$ is not continuous at $x=1$ but $\lim_{x\to 1}g(x)$ exists

We have $\lim_{x\to 1^-}f(x)=\lim_{h\to 0}f(1-h)=\lim_{h\to 0}[1-h]=0$

and, $\lim_{x\to 1^+}f(x)=\lim_{h\to 0}f(1+h)=\lim_{h\to 0}[1+h]=1$

So, $\lim_{x\to 1}f(x)$ does not exist and so $f(x)$ is not continuous at $x=1$

We have $gof(x)=g(f(x))=g([x])=0, \forall x\in R$

We have $fog(x)=f(g(x))$

$$=\begin{cases} f(0), & x\in Z \\ f(x^2), & x\in R-Z\end{cases}=\begin{cases} 0, & x\in z \\ [x^2], & x\in R-Z\end{cases}$$

Which is clearly not continuous

69 **(c)**

Given, $g(x)=\frac{(x-1)^n}{\log\cos^m(x-1)}$; $0<x<2$, $m\neq 0$, n are integers and $|x-1|=\begin{cases} x-1; & x\geq 1 \\ 1-x; & x<1\end{cases}$

The left hand derivative of $|x-1|$ at $x=1$ is $p=-1$

Also, $\lim_{x\to 1^+}g(x)=p=-1$

$$\Rightarrow \lim_{h\to 0}\frac{(1+h-1)^n}{\log\cos^m(1+h-1)}=-1$$

$$\Rightarrow \lim_{h\to 0}\frac{h^n}{m\log\cos h}=-1$$

$$\Rightarrow \lim_{h\to 0}\frac{n.h^{n-1}}{m\frac{1}{\cos h}(-\sin h)}=-1$$

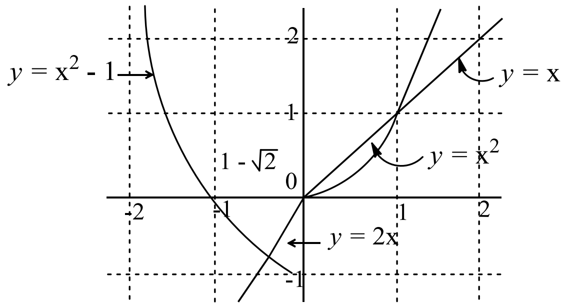
[using L 'Hospital's rule]

$$\Rightarrow \left(\frac{n}{m}\right)\lim_{h\to 0}\frac{h^{n-2}}{\left(\frac{\tan h}{h}\right)}=1$$

$\Rightarrow n=2$ and $\frac{n}{m}=1$

$\Rightarrow m=n=2$

70 **(d)**



From the graph it is clear that $f(x)$ is everywhere continuous but not differentiable at $x = 1 - \sqrt{2}, 0, 1$

71 **(a)**

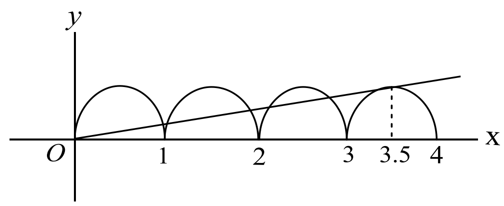
As f is continuous so $f(0) = \lim_{x\to 0} f(x)$

$\Rightarrow f(0) = \lim_{n\to\infty} f(1/4n)$

$$= \lim_{n\to\infty}\left((\sin e^{n})e^{-n^2} + \frac{1}{1+1/n^2}\right) = 0 + 1 = 1$$

72 **(b)**

$f(x) = \max\left\{\frac{x}{n}, |\sin \pi x|\right\}$



Thus, for the maximum points of non-differentiability, graphs of $y = \frac{x}{n}$ and $y = |\sin \pi x|$ must intersect at maximum number of points which occurs when $n > 3.5$

Hence, the least value of n is 4

73 **(c)**

$f(0) = 0 + 0 + \lambda \text{In } 4 = \lambda \text{ In } 4 \quad (1)$

R.H.L. $= \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h)$

$$= \lim_{h\to 0}\frac{8^h - 4^h - 2^h + 1^h}{h^2}$$

$$= \lim_{h\to 0}\frac{(4^h - 1)(2^h - 1)}{h.h}$$

$$= \lim_{h\to 0}\left(\frac{4^h - 1}{h}\right)\lim_{h\to 0}\left(\frac{2^h - 1}{h}\right)$$

$= \text{In } 4 \text{ In } 2 \quad (2)$

$\therefore f(0) =$ R.H.L.

$\Rightarrow \lambda = \text{ In } 2$

74 **(c)**

$$f(x) = \lim_{n\to\infty}\frac{[(x-1)^2]^n - 1}{[(x-1)^2]^n + 1}$$

$$= \lim_{n\to\infty}\frac{1 - \frac{1}{[(x-1)^2]^n}}{1 + \frac{1}{[(x-2)^2]^n}}$$

$$= \begin{cases} -1, 0 \le (x-1)^2 < 1 \\ 0, (x-1)^2 = 1 \\ 1, (x-1)^2 > 1 \end{cases}$$

$$= \begin{cases} 1, & x < 0 \\ 0, & x = 0 \\ -1, 0 < x < 2 \\ 0, & x = 2 \\ 1, & x > 2 \end{cases}$$

Thus, $f(x)$ is discontinuous at $x = 0, 2$

75 **(a)**

$x^2 + 2x + 3 + \sin \pi x = (x+1)^2 + 2 + \sin \pi x > 1$

$\therefore f(x) = 1 \ \forall\ x \in R$

76 **(c)**

$f(x) = (x^2 - 1)|(x-1)(x-2)|$

$f(x) = (x^2 - 1)|(x-1)(x-2)|$

$= (x+1)[(x-1|x-1|]|x-2|$

Which is differentiable at $x = 1$

For $f(x) = \sin(|x-1|) - |x-1|$

$$f'(1^+) = \lim_{h\to 0}\frac{\sin h - h - 0}{-h} = 0$$

$$f'(1^-) = \lim_{h\to 0}\frac{\sin|-h| - |-h|}{-h} = \lim_{h\to 0}\frac{\sin h - h}{-h}$$

$= 0$

Hence, $f(x)$ is differentiable at $x = 1$

For $f(x) = \tan(|x-1|) + |x-1|$

$$f'(1^+) = \lim_{h\to 0}\frac{\tan h + h - 0}{h} = 2$$

$$f'(1^-) = \lim_{h\to 0}\frac{\tan|-h| + |-h|}{-h} = \lim_{h\to 0}\frac{\tan h + h}{-h}$$

$= -2$

Hence, $f(x)$ is non-differentiable at $x = 1$

77 **(a)**

We have $x + 4|y| = 6y$

$$\Rightarrow \begin{cases} x - 4y = 6y, & \text{if } y < 0 \\ x + 4y = 6y, & \text{if } y \ge 0 \end{cases}$$

$$\Rightarrow y = \begin{cases} \frac{1}{2}x, & \text{if } x \ge 0 \\ \frac{1}{10}x, & \text{if } x < 0 \end{cases} \Rightarrow f'(x) = \begin{cases} \frac{1}{2}, & x > 0 \\ \frac{1}{10}, & x < 0 \end{cases}$$

Clearly, $f(x)$ is continuous at $x = 0$ but non-differentiable at $x = 0$

78 **(c)**

$f(x) = \tan x$ is discontinuous when $x = (2n+1)\pi/2, n \in Z$

$f(x) = x[x]$ is discontinuous when $x = k, k \in Z$

$f(x) = \sin[n\pi x]$ is discontinuous when $n\pi x = k, k \in Z$

Thus, all the above functions have infinite number of points of discontinuity

But $f(x) = \frac{[x]}{x}$ is discontinuous when $x = 0$ only

79 **(c)**

$f(x) = \{x\}\sin(\pi[x])$

$= \{x\}$ sin (integral multiple of π)

$= 0$

Hence, $f(x)$ is continuous for all x

80 **(b)**

The function f is clearly continuous at each point in its domain except possibly at $x = 0$. Given that $f(x)$ is continuous at $x = 0$

Therefore, $f(0) = \lim_{x\to 0} f(x)$

$= \lim_{x\to 0} \frac{2x - \sin^{-1} x}{2x + \tan^{-1} x}$

$= \lim_{x\to 0} \frac{2 - (\sin^{-1} x)/x}{2 + (\tan^{-1} x)x} = \frac{1}{3}$

81 **(d)**

$f(x) = \cos(|x|) + |x| = \cos x + |x|$ is non-differentiable at $x = 0$ as $|x|$ is non-differentiable at $x = 0$. Similarly $f(x) = \cos(|x|) - |x|$ is non-differentiable at $x = 0$

$f(x) = \sin|x| + |x| = \begin{cases} -\sin x - x, & x < 0 \\ +\sin x + x, & x \ge 0 \end{cases}$

$\Rightarrow f'(x) = \begin{cases} -\cos x - 1, & x < 0 \\ +\cos x + 1\ , & x \ge 0 \end{cases}$

Which is not differentiable at $x = 0$

$f(x) = \sin|x| - |x| = \begin{cases} -\sin x + x, & x < 0 \\ \sin x - x, & x \ge 0 \end{cases}$

$\Rightarrow f'(x) = \begin{cases} -\cos x + 1, & x < 0 \\ +\cos x - 1\ , & x \ge 0 \end{cases}$

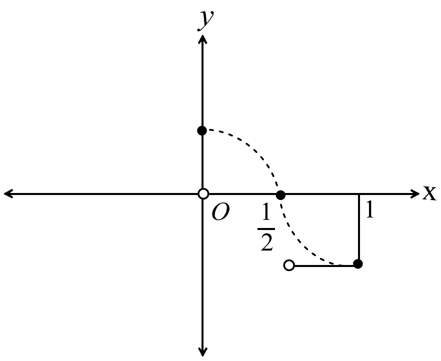
$\therefore f$ is differentiable at $x = 0$

82 **(b)**

Consider $x \in [0, 1]$

From the graph given in figure, it is clear that $[\cos \pi x]$ is discontinuous at

$x = 0, 1/2$ (1)



Now consider $x \in (1, 2]$

$f(x) = [x - 2]|2x - 3|$

For $x \in (1, 2)$; $[x - 2] = -1$ and for $x = 2$; $[x - 2] = 0$

Also $|2x - 3| = 0 \Rightarrow x = 3/2$

$\Rightarrow x = 3/2$ and 2 may be the points at which $f(x)$ is discontinuous (2)

$$f(x) = \begin{cases} 1, & x = 0 \\ 0, & 0 < x \le \frac{1}{2} \\ -1, & \frac{1}{2} < x \le 1 \\ -(3 - 2x), & 1 < x \le 3/2 \\ -(2x - 3), & 3/2 < x \le 2 \\ 0, & x = 2 \end{cases}$$

Thus, $f(x)$ is continuous when $x \in [0, 2] - \{0, 1/2, 2\}$

83 **(b)**

We have $f(x) = \sqrt{1 - \sqrt{1 - x^2}}$

The domain of definition of $f(x)$ is $[-1, 1]$

For $x \ne 0, x \ne \pm 1$, we have

$f'(x) = \frac{1}{\sqrt{1 - \sqrt{1 - x^2}}} \times \frac{x}{\sqrt{1 - x^2}}$

Since $f(x)$ is not defined on the right side of $x = 1$ and on the left side of $x = -1$

Also, $f'(x) \to \infty$ when $x \to -1^+$ or $x \to 1^-$

So, we check the differentiability at $x = 0$

Now, L.H.D. at $x = 0$

$= \lim_{x\to 0^-} \frac{f(x) - f(0)}{x - 0}$

$= \lim_{h\to 0} \frac{f(0 - h) - f(0)}{-h}$

$= \lim_{h\to 0} \frac{\sqrt{1 - \sqrt{1 - h^2}} - 0}{-h}$

$= -\lim_{h\to 0} \frac{\sqrt{1 - (1 - (1/2)h^2 + (3/8)h^4 + \cdots)}}{h}$

$= -\lim_{h\to 0} \sqrt{\frac{1}{2} - \frac{3}{8}h^2 + \cdots} = -\frac{1}{\sqrt{2}}$

Similarly, R.H.D. at $x = 0$ is $\frac{1}{\sqrt{2}}$

Hence, $f(x)$ is not differentiable at $x = 0$

84 **(b)**

We have $f(x) = \begin{cases} \frac{1-|x|}{1+x}, & x \ne -1 \\ 1, & x = -1 \end{cases}$

$= \begin{cases} 1, & x < 0, \\ \frac{1-x}{1+x}, & x \ge 0 \end{cases}$ ($\because f(-1) = 1$ is given)

$\Rightarrow f([2x]) = \begin{cases} 1, & [2x] < 0 \\ \frac{1 - [2x]}{1 + [2x]}, & [2x] \ge 0 \end{cases}$

$= \begin{cases} 1, & x < 0 \\ 1, & 0 \le x < 1/2 \\ 0, & 1/2 \le x < 1 \\ -1/3, & 1 \le x < \frac{3}{2} \end{cases}$

Clearly, $f(x)$ is continuous for all $x < \frac{1}{2}$ and discontinuous at $x = \frac{1}{2}, 1$

85 **(b)**

$f(x)$ is discontinuous at $x = 1$ and $x = 2$

$\Rightarrow f(f(x))$ may be discontinuous when $f(x) = 1$ or 2

Now $1 - x = 1 \Rightarrow x = 0$, where $f(x)$ is continuous

$x + 2 = 1 \Rightarrow x = -1 \notin (1, 2)$

$4 - x = 1 \Rightarrow x = 3 \in [2, 4]$

Now $1 - x = 2 \Rightarrow x = -1 \notin [0, 1]$

$x + 2 = 2 \Rightarrow x = 0 \notin (0, 2]$

$4 - x = 2 \Rightarrow x = 2 \in [2, 4]$

Hence $f(f(x))$ is discontinuous at $x = 2, 3$

86 **(b)**

We must have $\lim_{x\to 0} \frac{a\cos x - \cos bx}{x^2} = 4$

$$\Rightarrow \lim_{x\to 0} \frac{a\left(1 - \frac{x^2}{2!}\right) - \left(1 - \frac{b^2x^2}{2!}\right)}{x^2} = 4$$

$$\Rightarrow \lim_{x\to 0}\left[\frac{(a-1)}{x^2} - \left(\frac{a}{2} - \frac{b^2}{2}\right)\right] = 4$$

$\Rightarrow a = 1$ and $\frac{a}{2} - \frac{b^2}{2} = -4$

$\Rightarrow a = 1$ and $b^2 = 9$

$\Rightarrow a = 1$ and $b = \pm 3$

87 **(c)**

For $|x| < 1, x^{2n} \to 0$ as $n \to \infty$ and for $|x| > 1, 1/x^{2n} \to 0$ as $n \to \infty$. So

$f(x)$

$$= \begin{cases} \log(2+x), & |x < 1| \\ \lim_{n\to\infty} \frac{x^{-2n}\log(2+x) - \sin x}{x^{-2n} + 1} = -\sin x, & \text{if } |x| > \\ \frac{1}{2}[\log(2+x) - \sin x], & |x| = 1 \end{cases}$$

Thus, $\lim_{x\to 1+} f(x) = \lim_{x\to 1}(-\sin x) = -\sin 1$ and $\lim_{x\to 1-} f(x) = \lim_{x\to 1}\log(2+x) = \log 3$

88 **(c)**

Let $f(x) = x^2|x|$ which could be expressed as

$$f(x) = \begin{cases} -x^3, & x < 0 \\ 0, & x = 0 \\ x^3, & x > 0 \end{cases} \Rightarrow f'(x) = \begin{cases} -3x^2, x < 0 \\ 0, x = 0 \\ 3x^2, x > 0 \end{cases}$$

So, $f'(x)$ exists for all real x

$$f''(x) = \begin{cases} -6x, & x < 0 \\ 0, & x = 0 \\ 6x, & x > 0 \end{cases}$$

So, $f''(x)$ exists for all real x

$$f'''(x) = \begin{cases} -6, & x < 0 \\ 0, & x = 0 \\ 6, & x > 0 \end{cases}$$

However, $f'''(0)$ does not exist since $f'''(0^-) = -6$ and $f'''(0^+) = 6$ which are not equal. Thus, the set of points where $f(x)$ is thrice differentiable is $R - \{0\}$

89 **(d)**

We have,

L.H.L. $= \lim_{x\to 4^-} f(x)$

$$= \lim_{h\to 0} f(4 - h)$$

$$= \lim_{h\to 0} \frac{4 - h - 4}{|4 - h - 4|} + a$$

$$= \lim_{h\to 0}\left(-\frac{h}{h} + a\right) = a - 1$$

R.H.L. $= \lim_{x\to 4^+} f(x)$

$$= \lim_{h\to 0} f(4 + h)$$

$$= \lim_{h\to 0} \frac{4 + h - 4}{|4 + h - 4|} + b = b + 1$$

$\Rightarrow f(4) = a + b$

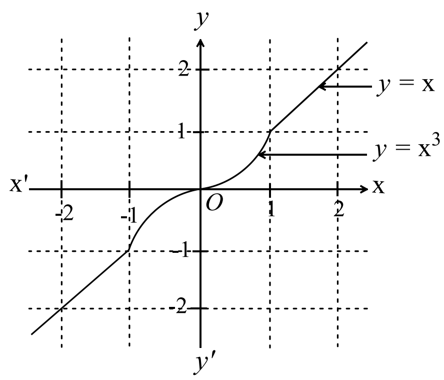
Since $f(x)$ is continuous at $x = 4$, therefore

$\lim_{x\to 4^-} f(x) = f(4) = \lim_{x\to 4^+} f(x)$

$\Rightarrow a - 1 = a + b = b + 1 \Rightarrow b = -1$ and $a = 1$

90 **(b)**

$f(x)$ is clearly continuous for $x \in R$



$$f'(x) = \begin{cases} 3x^2, x^2 < 1 \\ 1, x^2 > 1 \end{cases}$$

Thus, $f(x)$ is non-differentiable at $x = 1, -1$

91 **(a)**

$$f(e) = [\log_e e] + \sqrt{\{\lg_e e\}} = [1] + \sqrt{\{1\}} = 1 + 0 = 1$$

$$f(e^+) = [\log_e e^+] + \sqrt{\{\log_e e^+\}}$$

$$= \lim_{h\to 0}[1 + h] + \sqrt{\{1 + h\}} = 1 + 0 = 1$$

$$f(e^-) = [\log_e e^-] + \sqrt{\{\log_e e^-\}}$$

$$= \lim_{h\to 0}[1 - h] + \sqrt{\{1 - h\}} = 0 + 1 = 1$$

Hence, $f(x)$ is continuous at $x = e$

Now $f'(e^+) = \lim_{h\to 0} \frac{f(e+h) - f(e)}{h}$

$$= \lim_{h\to 0} \frac{[1 + h] + \sqrt{\{1 + h\}} - 1}{h}$$

$$= \lim_{h\to 0} \frac{1 + \sqrt{h} - 1}{h} = \lim_{h\to 0} \frac{1}{\sqrt{h}} \to \infty$$

Hence, $f(x)$ is non-differentiable at $x = 0$

92 **(a,c)**

$$f(x)=\begin{cases}(\sin^{-1}x)^2\cos\left(\frac{1}{x}\right), x\neq 0\\ 0, \quad x=0\end{cases}$$

$$\lim_{x\to 0} f(x)=\lim_{x\to 0}(\sin^{-1}x)^2\cos\left(\frac{1}{x}\right)$$

$= 0 \times$ (any value between -1 to 1) $= 0$

Hence $f(x)$ is continuous at $x=0$

$$f'(0^+)=\lim_{h\to 0}\frac{(\sin^{-1}h)^2\cos\left(\frac{1}{h}\right)-0}{h}$$

$$=\left(\lim_{h\to 0}\frac{\sin^{-1}h}{h}\right)\left(\lim_{h\to 0}\sin^{-1}h\right)\left(\lim_{h\to 0}\cos\left(\frac{1}{h}\right)\right)$$

$= 1 \times (0) \times$ (any value between -1 to 1) $= 0$

Similarly, $f'(0^-)=0$

Hence, $f(x)$ is continuous and differentiable in $[-1,1]$ and $(-1,1)$, respectively

93 **(b,c,d)**

$$f(x)=|x^3|=\begin{cases}-x^3, x<0\\ x^3, x\geq 0\end{cases}\Rightarrow f'''(x)\begin{cases}-6, x<0\\ 6, x>0\end{cases}$$

Hence $f'''(0)$ does not exist

$$f(x)=x^3|x|=\begin{cases}-x^4, x<0\\ x^4, x\geq 0\end{cases}\Rightarrow f'''(x)$$

$$=\begin{cases}-24x, x<0\\ 24x, x>0\end{cases}$$

Hence $f'''(0)=0$ and exists

Similarly for $f(x)=|x|\sin^3 x$ and $f(x)=x|\tan^3 x|$, also $f'''(0)=0$ and exists

94 **(a,c,d)**

Differentiating w.r.t. x, keeping y as constant, we get $f'(x+y)=f'(x)+2xy+y^2$

Now put $x=0$

$f'(y)=f'(0)+y^2=y^2-1$

$\therefore f'(x)=x^2-1$

$$\therefore f(x)=\frac{x^3}{3}-x+c$$

Also $f(0+0)=f(0)+f(0)+0 \therefore f(0)=0$

$\therefore f(x)=\frac{x^3}{3}-x, f(x)$ is twice differentiable for all $x\in R$ and $f'(3)=3^2-1=8$

95 **(a,b,c)**

Since, $\lim_{x\to 1^-} g(x)=\lim_{x\to 1^+} g(x)=1$ and $g(1)=0$

So, $g(x)$ is not continuous at $x=1$ but $\lim_{x\to 1} g(x)$ exists

We have $\lim_{x\to 1^-} f(x)=\lim_{h\to 0} f(1-h)=\lim_{h\to 0}[1-h]=0$

and $\lim_{x\to 1^+} f(x)=\lim_{h\to 0} f(1+h)=\lim_{h\to 0}[1+h]=1$

So, $\lim_{x\to 1} f(x)$ does not exist and hence $f(x)$ is not continuous at $x=1$

We have $gof(x)=g(f(x))=g([x])=0, \forall x, \in R$

So, gof is continuous for all x

We have $fog(x)=f(g(x))=\begin{cases} f(0), x\in Z\\ f(x^2), x\in R-Z\end{cases}$

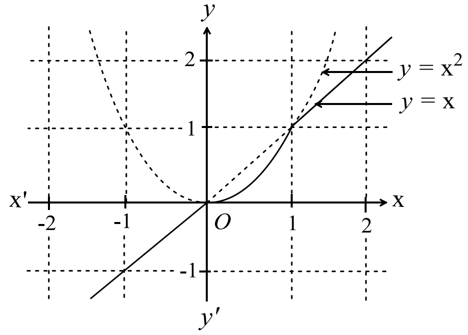
$$=\begin{cases}0, \quad x\in Z\\ [x^2], \quad x\in R-Z\end{cases}$$

Which is clearly not continuous

96 **(a,c,d)**

From the figure, it is clear that $h(x)=$

$$\begin{cases}x, \quad \text{if } x\leq 0\\ x^2, \text{ if } 0<x<1\\ x, \quad \text{if } x\geq 1\end{cases}$$



From the graph, it is clear that $h(x)$ is continuous for all $x\in R, h'(x)=1$ for all $x>1$, and h is not differentiable at $x=0$ and 1

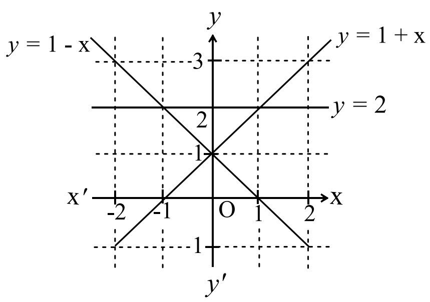
97 **(a,c,d)**

a is not correct as $f(x)=x$ from R to R is onto but its reciprocal function $g(x)=\frac{1}{x}$ is not onto on R

b is obviously true

Also $g(x)$ is not continuous, hence not differentiable though $f(x)$ is continuous and differentiable in the above case

98 **(a,c)**

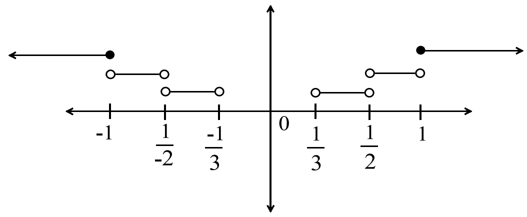


From the graph, it is clear that $f(x)$ is continuous everywhere and also differentiable everywhere except at $x=1$ and -1

99 **(a,c)**

$$f(x)=\begin{cases}1, \quad |x|\geq 1\\ \frac{1}{n^2}, \frac{1}{n}<|x|<\frac{1}{n-1}, n=2,3,\ldots\\ 0, \quad x=0\end{cases}$$

$$= \begin{cases} 1, & x \le \text{ or } x \ge 1 \\ \frac{1}{4}, & x \in \left(-1, -\frac{1}{2}\right) \cup \left(\frac{1}{2}, 1\right) \\ \frac{1}{9}, & x \in \left(\frac{-1}{2}, \frac{-1}{3}\right) \cup \left(\frac{1}{3}, \frac{1}{2}\right) \\ \vdots \end{cases}$$



The function f is clearly continuous for $|x| > 1$

We observe that

$\lim_{x \to -1^+} f(x) = 1, \lim_{x \to -1^-} f(x) = \frac{1}{4}$

Also, $\lim_{x \to \frac{1+}{n}} f(x) = \frac{1}{n^2}$ and $\lim_{x \to \frac{1-}{n}} f(x) = \frac{1}{(n+1)^2}$

Thus f is discontinuous for $x = \pm\frac{1}{n}, n = 1, 2, 3, \dots$

Hence **a** and **c** are the correct answers

100 **(a,b,c,d)**

a, **b**, and **c** are false. Refer to definitions for **d**, f must be continuous ⇒ False

101 **(a,b)**

We have $g(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \ne 0 \\ 0, & x = 0 \end{cases}$

If $x \ne 0, g'(x) = x^2 \cos\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right) + 2x \sin\left(\frac{1}{x}\right)$

$= -\cos\left(\frac{1}{x}\right) + 2x \sin\left(\frac{1}{x}\right)$

Which exists for $\forall x \ne 0$

If $x = 0$,

Then

$g'(0) = \lim_{x \to 0} \frac{g(x) - g(0)}{x - 0} = \lim_{x \to 0} \frac{x^2 \sin(1/x) - 0}{x - 0}$

$= \lim_{x \to 0} x \sin\left(\frac{1}{x}\right) = 0$

$\Rightarrow g'(x) = \begin{cases} -\cos\left(\frac{1}{x}\right) + 2x \sin\frac{1}{x}, & x \ne 0 \\ 0, & x = 0 \end{cases}$

At $x = 0, \cos\left(\frac{1}{x}\right)$ is not continuous, therefore $g'(x)$ is not continuous at $x = 0$. At $x = 0$

$Lf' = \lim_{x \to 0} \frac{0 - (-x) \sin \sin\left(-\frac{1}{x}\right)}{x} = \sin\left(\frac{1}{x}\right)$

Which does not exist

102 **(a,b)**

$\sin^4 x \in (0, 1)$ for $x \in (-\pi/2, \pi/2)$,

$\Rightarrow f(x) = 0$ for $x \in (-\pi/2, \pi/2)$

Hence $f(x)$ is continuous and differentiable at $x = 0$

103 **(a,c,d)**

$f(x) = \frac{x^2 - 2x - 8}{x + 2} = \frac{(x + 2)(x - 4)}{x + 2} = x - 4, x \ne -2$

Hence $f(x)$ has removable discontinuity at $x = -2$

Similarly $f(x)$ in options (c) and (d) has also removable discontinuity

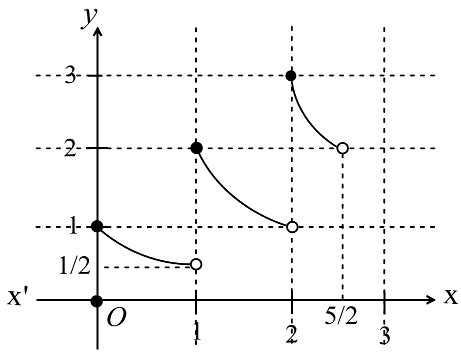
$f(x) = \frac{x - 7}{|x - 7|} = \begin{cases} -1, x < 7 \\ 1, x > 7 \end{cases}$

Hence $f(x)$ has non-removable discontinuity at $x = 7$

104 **(b,c)**

Option (a) is wrong as $f(x) = \sin x$ and $g(x) = |x|$, $g(x)$ is non-differentiable at $x = 0$, but $f(x)\, g(x)$ is differentiable at $x = 0$

105 **(a,b,d)**



$f(x) \begin{cases} \frac{1}{x + 1}, 0 \le x < 1 \\ \frac{2}{x}, 1 \le x < 2 \\ \frac{3}{x - 1}, 2 \le x < \frac{5}{2} \end{cases}$

Clearly, $f(x)$ is discontinuous and bijective function

$\lim_{x \to 1^-} f(x) = \frac{1}{2}, \lim_{x \to 1^+} f(x) = 2$

$\min\left(\lim_{x \to 1^-} f(x), \lim_{x \to 1^+} f(x)\right) = \frac{1}{2} \ne f(1)$

$\max(1, 2) = 2 = f(1)$

106 **(b,c,d)**

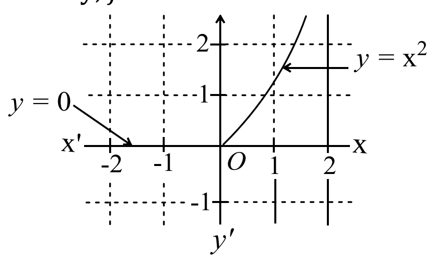
$f(x) = \begin{cases} 0, & x < 0 \\ x^2, & x \ge 0 \end{cases} \Rightarrow f'(x) = \begin{cases} 0, & x < 0 \\ 2x, & x > 0 \end{cases}$

which exists $\forall x$ except possibly at $x = 0$

At $x = 0, Lf' = 0 = Rf'$

$\Rightarrow f$ is differentiable

Clearly, f' is non-differentiable



107 **(a,b)**

$f(x) = \text{sgn}(x)\sin x$

$f(0^+) = \text{sgn}(0^+)\sin(0^+) = 1 \times (0) = 0$

$f(0^-) = \text{sgn}(0^-)\sin(0^-) = (-1) \times (0) = 0$

Also $f(0) = 0$

Hence, $f(x)$ is continuous everywhere

Both $\text{sgn}(x)$ and $\sin(x)$ are odd functions

Hence, $f(x)$ is an even function

Obviously, $f(x)$ is non-periodic

Now $f'(0^+) = \lim_{h\to 0}\frac{f(h)-f(0)}{h}$

$= \lim_{h\to 0}\frac{\text{sgn}(h)\sin h - 0}{h} = \lim_{h\to 0}\frac{\sin h}{h} = 1$

and $f'(0^+) = \lim_{h\to 0}\frac{\text{sgn}(-h)\sin(-h)-0}{-h}$

$= \lim_{h\to 0}\frac{-1\times(-\sin h)}{-h} = -1$

Hence, $f(x)$ is non-differentiable at $x = 0$

108 **(a,b)**

For $b = 1$, we have $f(\text{g}(0)) = f(\sin(0)+1) = f(1) = 1 + a$

Also $f(\text{g}(0^+)) = \lim_{x\to 0^+} f(\sin x + 1) = f(1) = 1 + a$

and $f(\text{g}(0^-)) = \lim_{x\to 0^-} f(\{x\}) = f(1^-) = 1 + a$

Hence, $f(\text{g}(x))$ is continuous for $b = 1$

For $b < 0$,

$f(\text{g}(0)) = f(\sin(0) + b) = f(b) = 2 - b$

$f(\text{g}(0^+)) = \lim_{x\to 0^+} f(\sin x + b) = f(b) = 2 - b$

and $f(\text{g}(0^-)) = \lim_{x\to 0^-} f(\{x\}) = f(1) = 1 + a$

For continuity at $x = 0$, we must have

$2 - b = 1 + a$ or $a + b = 1$

109 **(b,d)**

$f(x) = \text{sgn}\,(\cos 2x - 2\sin x + 3)$

$= \text{sgn}(1 - 2\sin^2 x - 2\sin x + 3)$

$= \text{sgn}(-2\sin^2 x - 2\sin x + 4)$

$f(x)$ is discontinuous when $-2\sin^2 x - 2\sin x + 4=0$ or $\sin 2x+\sin x-2=0$

or $(\sin x - 1)(\sin x + 2) = 0$ or $\sin x = 1$

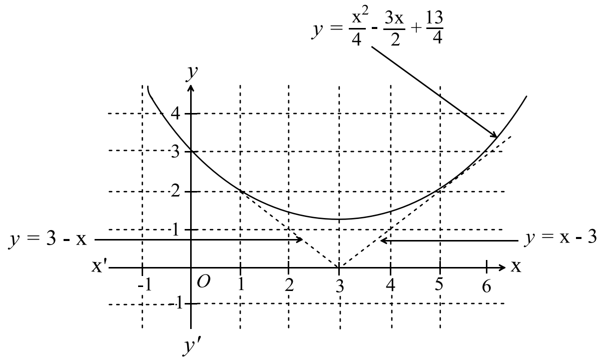
Hence $f(x)$ is discontinuous

110 **(a,b,c)**

$$f(x) = \begin{cases} |x-3|, & x \ge 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$$

$$= \begin{cases} \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \\ 3 - x, & 1 \le x < 3 \\ x - 3, & x \ge 3 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} \frac{x}{2} - \frac{3}{2}, & x < 1 \\ -1, & 1 < x < 3 \\ 1, & x > 3 \end{cases}$$



Clearly, $f(x)$ is non-differentiable at $x = 3$

For $x = 1$, where function changes its definition

$f(1^-) = \lim_{x\to 1}\left[\frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}\right] = \frac{1}{4} - \frac{3}{2} + \frac{13}{4} = 2$

$f(1^+) = \lim_{x\to 1}|x - 3| = 2$

$Lf'(1^-) = -1, Rf'(1^+) = -1$

Hence, $f(x)$ is differentiable at $x = 1$

Hence, $f(x)$ is continuous for all x but non-differentiable at $x = 3$

111 **(a,b,d)**

Given that $x + |y| = 2y$

If $y < 0$, then $x - y = 2y \Rightarrow y = x/3 \Rightarrow x < 0$

If $y = 0$, then $x = 0$

If $y > 0$, then $x + y = 2y \Rightarrow y = x \Rightarrow x > 0$

Thus, we can define $f(x) = y = \begin{cases} x/3, x < 0 \\ x, \ x \ge 0 \end{cases}$

$\Rightarrow \frac{dy}{dx} = \begin{cases} 1/3, x < 0 \\ 1, \quad x > 0 \end{cases}$

Clearly, y is continuous but non-differentiable at $x = 0$

112 **(a,b)**

$f(x)$ is continuous for all x if it is continuous at $x = 1$ for which $|1| - 3 = |1 - 2| + a$ or $a = -3$

$\text{g}(x)$ is continuous for all x if it is continuous at $x = 2$ for which $2 - |2| = \text{sgn}(2) - b = 1 - b$ or $b = 1$

thus, $f(x) + \text{g}(x)$ is continuous for all x if $a = -3, b = 1$

hence, $f(x)$ is discontinuous at exactly one point for options **a** and **b**

113 **(a,c,d)**

For continuity at $x = 1$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(x^2\text{sgn}[x] + \{x\}) = 1 + 0 = 1$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(x^2\,\text{sgn}[x] + \{x\})$

$= 1\,\text{sgn}\,(0) + 1 = 1$

Also, $f(1) = 1$

$\therefore$ L.H.L. = R.H.L. $= f(1)$. Hence, $f(x)$ is continuous at $x = 1$

Now for differentiability,

$f'(1^+) = \lim_{h\to 0}\frac{f(1+h) - f(1)}{h}$

$= \lim_{h\to 0} \frac{(1+h)^2 \text{sgn}\,[1+h] + \{1+h\} - 1}{h}$

$= \lim_{h\to 0} \frac{(1+h)^2 + h - 1}{h} = \lim_{h\to 0} \frac{h^2+3h}{h} = 3$

and $f'(1^-) = \lim_{h\to 0} \frac{f(1-h)-f(1)}{-h}$

$= \lim_{h\to 0} \frac{(1-h)^2\,\text{sgn}[1-h] + \{1-h\} - 1}{-h}$

$= \lim_{h\to 0} \frac{(1-h)^2 + 1 - h - 1}{-h}$

$= \lim_{h\to 0} \frac{h^2-3h}{-h} = 3$

$f'(1^+) = f'(1^-)$

Hence, $f(x)$ is differentiable at $x = 1$

Now at $x = 2$,

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (x^2 \text{sgn}[x] + \{x\}) = 4\times 0 + 1$

$\lim_{h\to 2^+} f(x) = \lim_{h\to 2^+} (\sin x + |x-3|) = 1 + \sin 2$

Hence, L.H.L $\neq$ R.H.L.

Hence, $f(x)$ is discontinuous at $x = 2$ and then $f(x)$ is also non-differentiable at $x = 2$

114 **(a,b,c,d)**

a. $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{e^x+a}{2x} = \frac{1}{2} \Rightarrow a = -1$

If $a = -1$, then $\lim_{x\to 0^+} f(x) = \frac{1}{2}$, $\lim_{x\to 0^-} f(x) = \frac{1}{2}$

$\therefore$ $f(x)$ is continuous at $x = 0$ if $b = \frac{1}{2}$

c. If $a \neq -1$, then $\lim_{x\to 0} \frac{e^x+a}{2x}$ does not exist

$\therefore x = 0$ is a point of irremovable type of discontinuity

d. if $a = -1$, then $\lim_{x\to 0} f(x) = \frac{1}{2}$

$\therefore b \neq \frac{1}{2} \Rightarrow$ removable type of discontinuity at $x = 0$

115 **(a,b)**

For maximum points of discontinuity of $f(x) = \text{sgn}(x^2 - ax + 1)$,

$x^2 - ax + 1 = 0$ must have two distinct roots, for which $D = a^2 - 4 > 0$

$\Rightarrow a \in (-\infty, -2) \cup (2, \infty)$

117 **(a,b)**

$f(1^-) = 1; f(1^+) = 1; f(1) = 1$

$f'(1^-) = 5; f'(1^+) = 5$

$f(2^+) = 10, f(2^-) = 10$

$f'(2^+) = 3; f'(2^-) = 13$

118 **(b,d)**

a. $\lim_{x\to 1^+} \frac{1}{\text{In}|x|} = \infty$ and $\lim_{x\to 1^-} \frac{1}{\text{In}\,|x|} = -\infty$,

hence $f(x)$ has non-removable discontinuity

b. $\lim_{x\to 1} \frac{x^2-1}{x^3-1} = \frac{2}{3}$

$\therefore f(x)$ has removable discontinuity at $x = 1$

c. $\lim_{x\to 1^+} \left(2^{-2^{\frac{1}{1-x}}}\right) = 1$ and $\lim_{x\to 1^-} \left(2^{-2^{\frac{1}{1-x}}}\right) = 0$

Hence, the limit does not exist

d. $\lim_{x\to 1} \frac{\sqrt{x+1}-\sqrt{2x}}{x^2-x} = \frac{-1}{2\sqrt{2}}$ (Rationalizing)

$\therefore f(x)$ has removable discontinuity at $x = 1$

119 **(a,c)**

$f\left(\frac{\pi^-}{2}\right) = \lim_{h\to 0} \left(\frac{3}{2}\right)^{\cot\left(3\left(\frac{\pi}{2}-h\right)\right)/\cot\left(2\left(\frac{\pi}{2}-h\right)\right)}$

$= \lim_{h\to 0} \left(\frac{3}{2}\right)^{\frac{\tan 3h}{-\cot 2h}}$

$= \lim_{h\to 0} \left(\frac{3}{2}\right)^{-(\tan 3h)(\tan 2h)} = 1$

$f\left(\frac{\pi^+}{2}\right) = \lim_{h\to 0} \left[1 + \left|\cot\left(\frac{\pi}{2}+h\right)\right|\right]^{\left[a\left|\tan\left(\frac{\pi}{2}+h\right)\right|\right]/b}$

$= \lim_{h\to 0} (1+\tan h)^{\frac{a\cot h}{b}}$

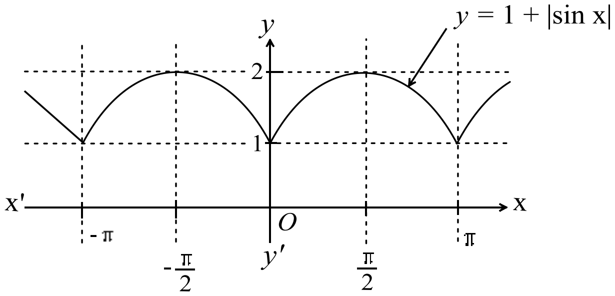
$= e^{\lim_{h\to 0}(1+\tan h-1)\frac{a\cot h}{b}} = e^{a/b}$

Also $f\left(\frac{\pi}{2}\right) = b + 3$

$f(x)$ is continuous at $x = \pi/2$

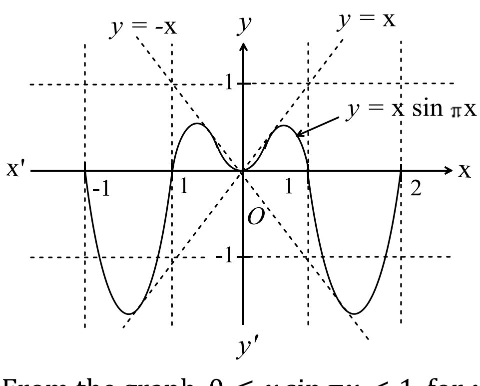
$\Rightarrow 1 = b + 3 = e^{a/b} \Rightarrow b = -2$ and $a = 0$

121 **(b,d,e)**



$|\sin x|$ is continuous for all but not differentiable when $\sin x = 0$ (where $\sin x$ crosses x-axis) or $x = n\pi, n \in Z$

122 **(a,b,d)**



From the graph, $0 \leq x \sin \pi x < 1$, for $x \in [-1, 1]$

Hence, $f(x) = 0, x \in [-1, 1]$

123 **(a)**

$f(x) = \frac{x}{1+|x|}$ is differentiable everywhere except probably at $x = 0$

For $x = 0$

$$Lf'(0)=\lim_{h\to0}\frac{f(0-h)-f(0)}{-h}=\lim_{h\to0}\frac{\frac{-h}{1+h}-0}{-h}=1$$
$$Rf'(0)=\lim_{h\to0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to0}\frac{\frac{h}{1+h}-0}{h}=1$$
$Lf'(0)=Rf'(0)$

$\Rightarrow f$ is differentiable at $x=0$

Hence, f is differentiable in $(-\infty,\infty)$

124 **(d)**

$x\in[0,\pi]\Rightarrow\frac{x-2}{2}\in\left[-1,\frac{\pi}{2}-1\right]$

$\frac{1}{f(x)}=\frac{2}{x-2}$, which is continuous in $(-\infty,\infty)\sim\{2\}$

$\tan(f(x))$ is continuous in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

$f^{-1}(x)=2(x+1)$ which is clearly continuous but $\tan\left(f^{-1}(x)\right)$ is not continuous

125 **(b,c)**

On $(0,\pi)$

a. $\tan x=f(x)$

we know $\tan x$ is discontinuous at $x=\pi/2$

b. $f(x)=\int_0^x t\sin\left(\frac{1}{t}\right)dt$

$\Rightarrow f'(x)=x\sin\left(\frac{1}{x}\right)$ which is well-defined on $(0,\pi)$

$\therefore f(x)$ being differentiable is continuous on $(0,\pi)$

c. $f(x)=\begin{cases}1, & 0<x\le 3\pi/4\\ 2\sin\frac{2x}{9}, & 3\pi/4<x<\pi\end{cases}$

Clearly, $f(x)$ is continuous on $(0,\pi)$ except possibly at $x=3\pi/4$, where

$$\text{L.H.L.}=\lim_{h\to0}f\left(\frac{3\pi}{4}-h\right)=\lim_{x\to0}1=1$$
$$\text{R.H.L.}=\lim_{h\to0}f\left(\frac{3\pi}{4}+h\right)=\lim_{x\to0}2\sin\frac{2}{9}\left(\frac{3\pi}{4}+h\right)$$
$$=\lim_{h\to0}2\sin\left(\frac{\pi}{6}+\frac{2h}{9}\right)=2\sin\frac{\pi}{6}=2\times\frac{1}{2}=1$$
Also $f\left(\frac{3\pi}{4}\right)=1$

As L.H.L.$=$ R.H.L.$=f\left(\frac{3\pi}{4}\right)$ $\therefore f(x)$ is continuous on $(0,\pi)$

d. $f(x)=\begin{cases}x\sin x, & 0<x\le\pi/2\\ \frac{\pi}{2}\sin(\pi+x), & \frac{\pi}{2}<x<\pi\end{cases}$

Here $f(x)$ will be continuous on $(0,\pi)$ if it is continuous at $x=\pi/2$. At $x=\pi/2$

$$\text{L.H.L.}=\lim_{h\to0}f\left(\frac{\pi}{2}-h\right)$$
$$=\lim_{h\to0}\left(\frac{\pi}{2}-h\right)\sin\left(\frac{\pi}{2}-h\right)=\frac{\pi}{2}\sin\frac{\pi}{2}=\frac{\pi}{2}$$
$$\text{R.H.L.}=\lim_{h\to0}f\left(\frac{\pi}{2}+h\right)=\lim_{h\to0}\frac{\pi}{2}\sin\left(\pi+\frac{\pi}{2}+h\right)$$
$$=\frac{\pi}{2}\sin\left(\pi+\frac{\pi}{2}\right)=\frac{-\pi}{2}\sin\frac{\pi}{2}=-\frac{\pi}{2}$$
As L.H.L. $\neq$ R.H.L. $\therefore f(x)$ is not continuous

126 **(a,c)**

$f(x)=x+|x|+\cos 9x$, $g(x)=\sin x$

Since both $f(x)$ and $g(x)$ are continuous everywhere, $f(x)+g(x)$ is also continuous everywhere

$f(x)$ is non-differentiable and $x=0$

Hence $f(x)+g(x)$ is non-differentiable at $x=0$

Now $h(x)=f(x)\times g(x)$

$$=\begin{cases}(\cos 9x)(\sin x), & x<0\\ (2x+\cos 9x)(\sin x), & x\ge0\end{cases}$$
Clearly, $h(x)$ is continuous at $x=0$

Also

$h'(x)$

$$=\begin{cases}\cos x\cos 9x-9\sin x\sin 9x, & x\\ (2-9\sin 9x)\sin x+\cos x\,(2x+\cos 9x), & x\end{cases}$$
$h'(0^-)=1, h'(0^+)=1$

$\Rightarrow f(x)\times g(x)$ is differentiable everywhere

127 **(b,d)**

We have $\lim_{x\to0}\frac{f(x)-f(0)}{x-0}=\lim_{x\to0}\frac{\log\cos x}{\log(1+x^2)}$

$$=\lim_{x\to0}\frac{\log(1-1+\cos x)}{\log(1+x^2)}\frac{1-\cos x}{1-\cos x}$$
$$=\lim_{x\to0}\frac{\log\{1-(1-\cos x)\}}{1-\cos x}\frac{1-\cos x}{\log(1+x^2)}$$
$$=-\lim_{x\to0}\frac{\log[1-(1-\cos x)]}{-(1-\cos x)}\frac{2\sin^2\frac{x}{2}}{4\left(\frac{x}{2}\right)^2}\frac{x^2}{\log(1+x^2)}$$
$$=-\frac{1}{2}$$
Hence, $f(x)$ is differentiable at $x=0$

Hence, **b** and **d** are the correct answers

128 **(b,c)**

$f(0^-)=\lim_{n\to\infty}\left[\lim_{x\to0^-}(\cos^2 x)^n\right]$

$=(\text{a value lesser than }1)^\infty=0$

$f(0^+)=\lim_{n\to\infty}\left[\lim_{x\to0^+}(1+x^n)^{1/n}\right]=1$

Also $f(0)=1$ $\Rightarrow$ discontinuous at $x=0$

Further, $f(1^-)=1; f(1^+)=0; f(1)=1$

$\Rightarrow$ discontinuous at $x=1$

129 **(a, c)**

Clearly, $f(x)$ is defined for all x satisfying $9-x^2>0$ and $2-x>0\Rightarrow x\in(-3,2)$

So, domain of $f(x)=(-3,2)$

Clearly, range of $f(x)=[-1,1]$

Also, $\lim_{x\to0^-}f(x)=\lim_{x\to0^+}f(x)=f(0)$

So, $f(x)$ is continuous at $x=0$

Now,

$\lim_{x\to-3^+}(x-3)f(x)$

$$=\lim_{h\to0}(h-6)\sin\left\{\log\left(\frac{9-(-3+h)^2}{2-(-3+h)}\right)\right\}$$

$\Rightarrow \lim_{x\to -3^+}(x-3)f(x)$

$$= \lim_{h\to 0}(h-6)\sin\left\{\log\left(\frac{h(6-h)}{5-h}\right)\right\}$$

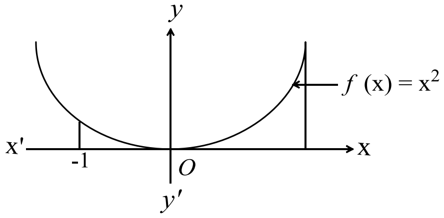
$\Rightarrow \lim_{x\to -3^+}(x-3)f(x) = (h-6)\times$ (An oscillating number)

$\therefore \lim_{x\to -3^+}(x-3)f(x)$ does not exist

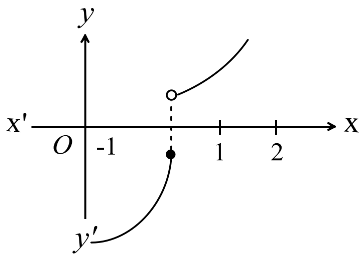
130 **(a,c,d)**

a is wrong as continuity is a must for $f(x)$

b is the correct form of intermediate value theorem



c as per the graph (in figure), is incorrect



d is wrong if f is discontinuous

131 **(a,b,c,d)**

Given function is discontinuous when

$a + \sin \pi x = 1$

Now if $a = 1 \Rightarrow \sin \pi x = 0 \Rightarrow x = 1, 2, 3, 4, 5$

If $a = 3 \Rightarrow \sin \pi x = -2$ not possible

If $a = 0.5 \Rightarrow \sin \pi x = 0.5$

$\Rightarrow x$ has 6 values, 2 each for one cycle of period 2

If $a = 0 \Rightarrow \sin \pi x = +1 \Rightarrow x = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \frac{11}{2}$

Hence, all the options are correct

132 **(b)**

$$f(x) = (2x-5)^{3/5} \Rightarrow f'(x) = \frac{3}{5(2x-5)^{2/5}}$$

Statement 2 as it is fundamental concept for non-differentiability

But given function is non-differentiable at $x = 5/2$, as it has vertical tangent at $x = 5/2$, but not due to sharp turn

The graph of the function is smooth in the neighbourhood of $x = 5/2$

133 **(b)**

Statement 1 is correct as $e^{|x|}$ is non-differentiable at $x = 0$

134 **(b)**

We know that $0 \le \cos^2(n!\ \pi x) \le 1$

Hence, $\lim_{m\to\infty} \cos^{2m}(n!\pi x) = 0$ or 1, as

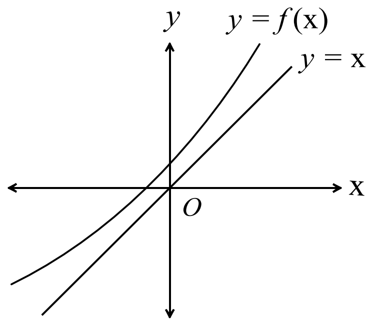
$0 \le \cos^2(n!\pi x) < 1$ or $\cos^2(n!\ \pi x) = 1$

Also, since $n \to \infty$, then $n!\,x =$ integer if $x \in Q$ and $n!\,x \ne$ integer if $x \in Q$ and $n!\,x \ne$ integer, if $x \in$ irrational

Hence, $f(x) = \begin{cases} 1, \text{if } x \text{ is rational} \\ 0, \text{if } x \text{ is irrational} \end{cases}$

$\Rightarrow h(x) = 1$ when $\forall\ x \in R$ which is continuous for all x; however, statement 2 does not correctly explain statement 1 as the addition of discontinuous functions may be continuous

135 **(b)**



Since $f(x)$ is a continuous function such that $f(0) = 1$ and $f(x) \ne x, \forall x \in R$

The graph of $y = f(x)$ always lies above the graph of $y = x$

Hence $f(x) > x$

Hence, $f(f(x)) > x$ (as $f(x)$ is onto function, $f(x)$ takes all real values which acts as x)

Statement 2 is a fundamental property of continuous function, but does not explain statement 1

136 **(d)**

$f(x) = |x| \sin x$

$$\text{L. H. D} = \lim_{h\to 0}\frac{|0-h|\sin(0-h) - 0}{-h} = \lim_{h\to 0}\frac{-h\sin h}{-h} = 0$$

$$\text{R. H. D.} = \lim_{h\to 0}\frac{|0+h|\sin(0+h) - 0}{h} = \lim_{h\to 0}\frac{h\sin h}{h} = 0$$

$\Rightarrow$ $f(x)$ is differentiable at $x = 0$

137 **(b)**
Statement 2 is obviously true

But $f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ is non-differentiable at $x = \pm 1$ as $\frac{2x}{1-x^2}$ is not defined at $x = \pm 1$. Hence statement 1 is true but statement 2 is not the correct explanation of statement 1

138 **(b)**
$|f(x)| \le |x|$

$\Rightarrow 0 \le |f(x)| \le |x|$

$\Rightarrow$ Graph of $y = |f(x)|$ lies between the graph of $y = 0$ and $y = |x|$

Also $|f(0)| \le 0 \Rightarrow f(0) = 0$

Also from Sandwich theorem, $\lim_{x\to 0} 0 \le$ limx→0fx≤limx→0/x/

$\Rightarrow \lim_{x\to 0}|f(x)| = 0$

$\Rightarrow y = f(x)$ is continuous at $x = 0$

Also statement 2 is correct but it has no link with statement 1

139 **(d)**
Statement 1 is false, as consider the function $f(x) = \max\{0, x^3\}$ which is equivalent to

$$f(x) = \begin{cases} 0, x < 0 \\ x^3, x \ge 0 \end{cases}$$

Here $f(x)$ is continuous and differentiable at $x = 0$

However, statement 2 is obviously true

140 **(c)**
Statement 1 is obviously true

But statement 2 is false as $f(x) = x^3$ is differentiable, but $f^{-1}(x) = x^{1/3}$ is non-differentiable at $x = 0$

$f^{-1}(x) = x^{1/3}$ has vertical tangent at $x = 0$

141 **(c)**
Consider $f(x) = \begin{cases} 1, & \text{if } \ x \ge 0 \\ -1, & \text{if } \ x < 0 \end{cases}$

Hence $|f(x)| = 1$ for all x is continuous at $x = 0$

but $f(x)$ is discontinuous at $x = 0$

142 **(b)**
$f(x) = (\sin \pi x)(x-1)^{1/5}$ is continuous function as both $(\sin \pi x)$ and $(x-1)^{1/5}$ are continuous

But $(x-1)^{1/5}$ is not differentiable at $x = 1$

However, $f'(1^-) = \lim_{h\to 0} \frac{f(1-h)-f(1)}{-h}$

$$= \lim_{h\to 0} \frac{\sin[\pi(1-h)](1-h-1)^{1/5} - 0}{-h}$$

$$= \lim_{h\to 0} \frac{\sin(\pi h) - (-h)^{1/5}}{h} = 0$$

and $f'(1^+) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$

$$= \lim_{h\to 0} \frac{\sin[\pi(1+h)]\,(1+h-1)^{1/5} - 0}{h}$$

$$= \lim_{h\to 0} \frac{-\sin(\pi h)(h)^{1/5}}{h} = 0$$

Hence, $f(x)$ is differentiable at $x = 1$, through $(x-1)^{1/5}$ is not differentiable at $x = 1$

However, statement 2 is correct but it is not a correct explanation of statement 1

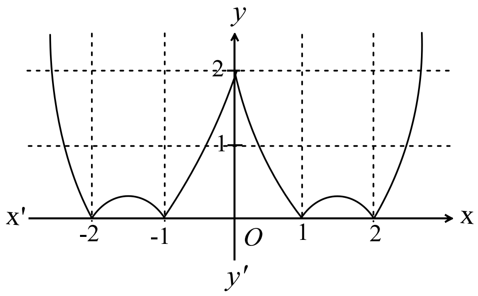
143 **(c)**
Statement 1 is true as $\sqrt{x}$ is monotonic function. But statement 2 is false as $f(x) = [\sin\ x]$ is continuous at $x = 3\pi/2$, though $\sin(3\pi/2) = -1$ (integer)

144 **(a)**
Statement 2 is true as it is a fundamental concept

Also, $f(x) = \text{sgn}\big(\text{g}(x)\big)$ is discontinuous when $\text{g}(x) = 0$

Now the given function $f(x) = \text{sgn}(x^2 - 2x + 3)$ may be discontinuous when $x^2 + 2x + 3 = 0$, which is not possible: it has imaginary roots as its discriminant is < 0

145 **(c)**
See the graph of $f(x) = \big||x^2| - 3|x| + 2\big|$,



Which is non-differentiable at 5 points, $x = 0, \pm 1, \pm 2$

However, statement 2 is false,

As $f(x) = x^3$ crosses x-axis at $x = 0$,

But $|f(x)| = |x^3|$ is differentiable at $x = 0$

146 **(b)**
Statement 2 is true as $\cos 0 = 1$

Now $\lim_{x \to 0+} \frac{e^{1/x}-1}{e^{1/x}+1} = \lim_{h \to 0} \frac{e^{1/h}-1}{e^{1/h}+1} = \lim_{h \to 0} \frac{1-e^{-1/h}}{1+e^{-1/h}} = 1$

and $\lim_{x \to 0-} \frac{e^{1/x}-1}{e^{1/x}+1} = \lim_{h \to 0} \frac{e^{-1/h}-1}{e^{-1/h}+1} = -1$

Thus L.H.L. ≠ R.H.L.

Hence, the function has no-removable discontinuity at $x = 0$

Hence, statement 2 is not a correct explanation of statement 1

147 **(a)**
$F(x) = f(\mathrm{g}(x))$,

$\Rightarrow F(x) = x^2 + 2|x|$

$\Rightarrow F'(x) = \begin{cases} 2x - 2, x < 0 \\ 2x + 2, x > 0 \end{cases}$

Hence, $F'(0^+) = 2$ and $F'(0^-) = -2$

Hence, both statement are correct and statement 2 is a correct explanation of statement 1

148 **(c)**
We know that $\mathrm{sgn}(x)$ is discontinuous at $x = 0$

Also $f(x) = |\mathrm{sgn} x| = \begin{cases} 1, x \neq 0 \\ 0, x = 0 \end{cases}$ which is discontinuous at $x = 0$

Consider $\mathrm{g}(x) = \begin{cases} -1, x < 0 \\ 1, \ x \geq 0 \end{cases}$. Here $\mathrm{g}(x)$ is discontinuous at $x = 0$ but $|\mathrm{g}(x)| = 1$ for all x is continuous at $x = 0$

Hence, answer is **c**

149 **(a)**
Let $x = k, k \in Z \Rightarrow f(k) = \{k\} + \sqrt{\{k\}} = 0$

$f(k^+) = 0 + 0 = 0, f(k^-) = 1 + 1 = 2$

Hence, $f(x)$ is not continuous at integral points

Hence, correct answer is **a**

150 **(c)**
We know that both $[\sin x]$ and $[\cos x]$ are discontinuous at $x = \pi/2$

Also $f(x) = [\sin x] - [\cos x]$ is discontinuous at $x = \pi/2$

As $f(\pi/2) = 1 - 0 = 1$ and $f(\pi/2^+) = 0 - (-1) = 1$

$f(\pi/2^-) = 0 - 0 = 0$

But the difference of two discontinuous function is not necessarily discontinuous

151 **(c)**
$f(x) = x|x|$ and $g(x) = \sin x$

$gof\ (x) = \sin(x|x|) = \begin{cases} -\sin x^2, & x < 0 \\ \sin x^2, & x \geq 0 \end{cases}$

$\therefore \quad (gof)'(x) = \begin{cases} -2x \cos x^2, & x < 0 \\ 2x \cos x^2, & x \geq 0 \end{cases}$

Clearly, $L(gof)'(0) = 0 = R(gof)'(0)$

$\therefore gof$ is differentiable at $x = 0$ and also its derivative is continuous at $x = 0$

Now,
$(gof)''(x) = \begin{cases} -2x \cos x^2 + 4x^2 \sin x^2, & x < 0 \\ 2 \cos x^2 - 4x^2 \sin x^2, & x > 0 \end{cases}$

$\therefore L(gof)''(0) = -2$ and $R(gof)''(0) = 2$

$\therefore L(gof)''(0) \neq R(gof)''(0)$

$\therefore gof(x)$ is not twice differentiable at $x = 0$

152 **(c)**
$F(1) = 0, F(1^+) = \frac{\pi}{2}$ and $F(1^-) = -\frac{3\pi}{4}$

$\Rightarrow F$ is discontinuous

But for $f(x) = \begin{bmatrix} 1, \text{if } x \geq 0 \\ -1, \text{if } x < 0 \end{bmatrix}$ and $g(x) = \begin{bmatrix} -1, \text{if } x \geq 0 \\ 1, \quad \text{if } x < 0 \end{bmatrix}$ then $f(x)g(x)$ is continuous at $x = 0$

153 **(b)**

$f(x) = \lim_{n\to\infty} \frac{x^{2n}-1}{x^{2n}+1}$ is discontinuous at $x = 1$

$$= \begin{cases} -1, & x^2 < 1 \\ 1, & x^2 > 1 \\ 0, & x^2 = 1 \end{cases}$$

$\Rightarrow f(1^+) = 1$ and $f(1^{-1}) = -1$

Hence, $f(x)$ is discontinuous at $x = 1$ as the limit of the function does not exist

154 **(d)**

Statement 1 is incorrect because if $\lim\limits_{x\to a} g(x)$ and $\lim\limits_{x\to a} f(g(x))$ approach e from the same side of e (say right side), and $\lim\limits_{x\to e} f(x) = f(e) \neq \lim\limits_{x\to e} f(x)$, then $\lim\limits_{x\to a} f(g(x)) = f(e^+) = f(e)$

Statement 2 is correct

155 **(b)**

$f(x) = \begin{bmatrix} \pi/4, x > 1 \\ \pi/4, x = 1 \\ \pi/2, x < 1 \end{bmatrix}$ [in the interval $(1-8, 1+8)$]

Hence, f is discontinuous and non-derivable, but non-derivability does not imply discontinuity

156 **(d)**

Consider $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, x \neq 0 \\ 0, \quad x = 0 \end{cases}$ which is differentiable at $x = 0$, but derivative is not continuous at $x = 0$

However, statement 2 is correct

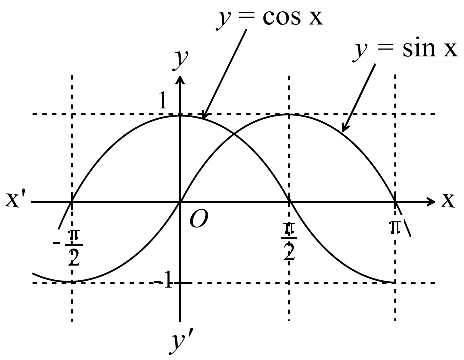
157 **(a)**

$\lim\limits_{x\to 0^+} (\sin x + [x]) = 0$, $\lim\limits_{x\to 0^-} (\sin x + [x]) = -1$

Thus, limit does not exist, hence $f(x)$ is discontinuous at $x = 0$

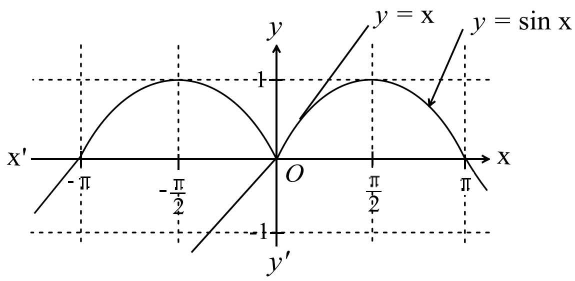
Statement 2 is a fundamental property and is a correct explanation of statement 1

158 **(c)**

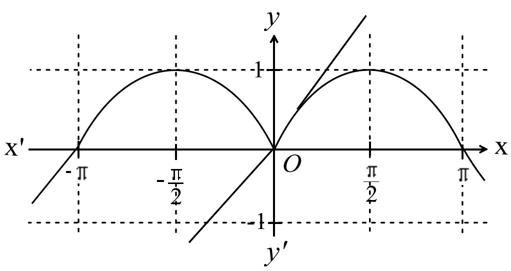


From the graph, statement 1 is true

Consider $f(x) = \min\{x, \sin|x|\}$ is differentiable at $x = 0$, through $g(x) = \max\{x, \sin|x|\}$ is non-differentiable at $x = 0$



Graph of $y = \min\{x, \sin|x|\}$



Graph of $y = \max\{x, \sin|x|\}$

160 **(c)**

a. $f(x) = \begin{cases} \frac{5e^{1/x}+2}{3-e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

$$f(0^+) = \lim_{h\to 0} \frac{5e^{1/h}+2}{3-e^{1/h}} = \lim_{h\to 0} \frac{5+2e^{-1/h}}{3e^{-1/h}-1} = -5$$

Hence, $f(x)$ is discontinuous and non-differentiable at $x = 0$

b. $g(x) = xf(x) = \begin{cases} x\frac{5e^{1/x}+2}{3-e^{1/x}}, x \neq 0 \\ 0, \quad x = 0 \end{cases}$

$$f(0^+) = \lim_{h\to 0} h\frac{5e^{1/h}+2}{3-e^{1/h}} = \lim_{h\to 0} h\frac{5+2e^{-1/h}}{3e^{-1/h}-1}$$
$$= 0 \times (5) = 0$$

$$f(0^-) = \lim_{h\to 0} h\frac{5e^{-1/h}+2}{3-e^{-1/h}} = 0 \times (2/3) = 0$$

Hence, $f(x)$ is continuous at $x = 0$

$$Lg'(0) = \lim_{h\to 0} \frac{g(0-h)-g(0)}{-h}$$

$$= \lim_{h\to 0} \frac{-hf(-h)-0}{-h}$$
$$= \lim_{h\to 0} f(-h)$$
$$= \lim_{h\to 0} \frac{5e^{-1/h}+2}{3-e^{-1/h}} = \frac{0+2}{3-0} = \frac{2}{3}$$
$$Rg'(0) = \lim_{h\to 0} \frac{g(0+h)-g(0)}{h}$$
$$= \lim_{h\to 0} \frac{g(h)-0}{h}$$
$$= \lim_{h\to 0} f(h) = \lim_{h\to 0} \frac{5e^{1/h}+2}{3-e^{1/h}}$$
$$= \lim_{h\to 0} \frac{5+2e^{-1/h}}{3e^{-1/h}-1}$$
$$= \frac{5+0}{0-1} = -5$$
$\because LF'(0) \neq RF'(0)$

Hence, $F(x)$ is not differentiable, but continuous at $x = 0$

c. For $x^2 f(x)$,

Let $F(x) = x^2 f(x)$

$$\therefore LF'(0) = \lim_{h\to 0} \frac{F(0-h)-F(0)}{-h}$$
$$= \lim_{h\to 0} \frac{h^2 f(-h)-0}{-h} = 0$$
$$RF'(0) = \lim_{h\to 0} \frac{F(0+h)-F(0)}{h}$$
$$= \lim_{h\to 0} \frac{h^2 f(h)-0}{h} = 0$$
$\therefore LF'(0) = RF'(0)$

Hence, $F(x)$ is differentiable at $x = 0$, then it is always continuous at $x = 0$

d. Clearly from the above discussion $y = x^{-1} f(x)$ is discontinuous and hence non-differentiable at $x = 0$

161 **(a)**

a. $f(x) = \lim_{n\to\infty} [\cos^2(2\pi x)]^n + \left\{x + \frac{1}{2}\right\}$

obviously, $\lim_{x\to \frac{1}{2}^+} f(x) = 0 + 0 = 0$

and $\lim_{x\to \frac{1}{2}^-} f(x) = 0 + 1$

$\therefore f(x)$ is discontinuous at $x = \frac{1}{2}$

b. $f(x) = (\log x)(x-1)^{1/5}$

Obviously, $f(x)$ is continuous at $x = 1$

$$f'(1^+) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$$
$$= \lim_{h\to 0} \frac{\log(1+h) h^{1/5}}{h} = 0$$
$$f'(1^-) = \lim_{h\to 0} \frac{f(1-h)-f(1)}{h}$$
$$= \lim_{h\to 0} \frac{\log(1-h)(-h)^{1/5}}{-h} = 0$$

Hence, $f(x)$ is differentiable at $x = 1$

c. $f(x) = [\cos 2\pi x] + \sqrt{\left\{\sin\left(\frac{\pi x}{2}\right)\right\}}$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} [\cos 2\pi x] + \lim_{x\to 1^-} \sqrt{\left\{\sin\left(\frac{\pi x}{2}\right)\right\}}$$
$= 0 + 1 = 1$
$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} [\cos(2\pi x)] + \lim_{x\to 1^+} \sqrt{\left\{\sin\left(\frac{\pi x}{2}\right)\right\}}$$
$= 0 + 1 = 1$

Also $f(1) = 1 + 0 = 1$

$f(x)$ is continuous at $x = 1$

$f'(1^+)$

$$= \lim_{h\to 0} \frac{[\cos 2\pi(1+h)] + \sqrt{\left\{\sin\left(\frac{\pi(1+h)}{2}\right)\right\}} - 1}{h}$$
$$= \lim_{h\to 0} \frac{[\cos 2\pi h] + \sqrt{\left\{\cos\left(\frac{\pi h}{2}\right)\right\}} - 1}{h}$$
$$= \lim_{h\to 0} \frac{\sqrt{\cos\left(\frac{\pi h}{2}\right)} - 1}{h} = \lim_{h\to 0} \frac{-\frac{\pi}{2}\sin\left(\frac{\pi h}{2}\right)}{2\sqrt{\cos\left(\frac{\pi h}{2}\right)}} = 0$$

Similarly, $f'(1^-) = 0$

d. $f(x) = \begin{cases} \cos 2x, & x \in Q \\ \sin x, & x \notin Q \end{cases}$ at $\frac{\pi}{6}$

$f(x)$ is continuous when $\cos 2x = \sin x$ which has $x = \frac{\pi}{6}$ as one of the solutions. Hence, it is continuous

Also in the neighbourhood of $x = \frac{\pi}{6}$,

$$f'(x) = \begin{cases} -2\sin 2x, & \frac{\pi}{6} - \delta < x < \frac{\pi}{6} \\ \cos x, & \frac{\pi}{6} < x < \frac{\pi}{6} + \delta \end{cases}$$

Here, $f'\left(\frac{\pi^-}{6}\right) \neq f'\left(\frac{\pi^+}{6}\right)$

$\Rightarrow f(x)$ is not differentiable at $x = \frac{\pi}{6}$

162 **(b)**

a. The given function is clearly continuous at all points except possibly at $x = \pm 1$

As $f(x)$ is an even function, so we need to check its continuity only at $x = 1$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$

$\Rightarrow \lim_{x\to 1^-} (ax^2 + b) = \lim_{x\to 1^+} \frac{1}{|x|} \Rightarrow a + b = 1 \quad (1)$

Clearly, $f(x)$ is differentiable for all x, except possible at $x = \pm 1$. As $f(x)$ is an even function, so we need to check its differentiability at $x = 1$ only

$$\lim_{x\to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^+} \frac{f(x) - f(1)}{x - 1}$$

$$\Rightarrow \lim_{x\to 1} \frac{ax^2 + b - 1}{x - 1} = \lim_{x\to 1^+} \frac{\frac{1}{|x|} - 1}{x - 1}$$

$$\Rightarrow \lim_{x\to 1} \frac{ax^2 - a}{x - 1} = \lim_{x\to 1} \frac{-1}{x} \Rightarrow 2a = -1 \Rightarrow a = -\frac{1}{2}$$

Putting $a = -1/2$ in (1) we get $b = 3/2 \Rightarrow |k| = 1 \Rightarrow k = \pm 1$

b. If $f(x) = \text{sgn}(x^2 - ax + 1)$ is discontinuous then $x^2 - ax + 1 = 0$ must have only one real root. Hence $a = \pm 2$

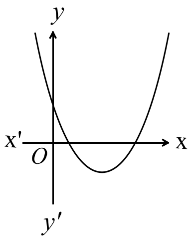
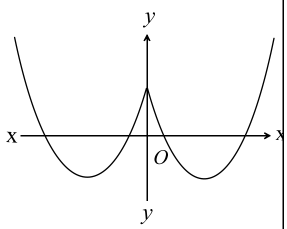
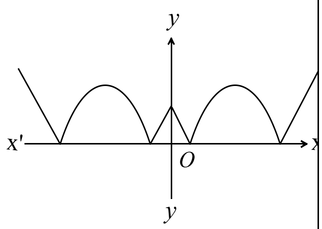
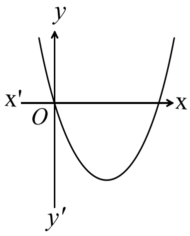
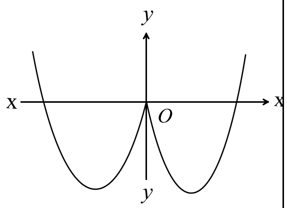
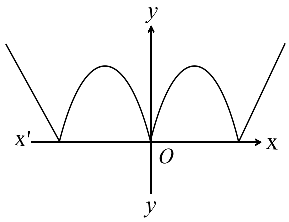
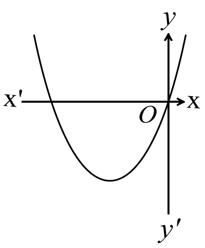
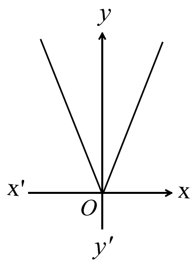
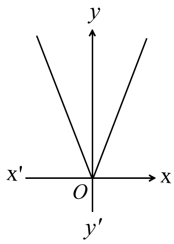
c. $f(x) = [2 + 3|n| \sin x], n \in N$ has exactly 11 points of discontinuity in $x \in (0, \pi)$

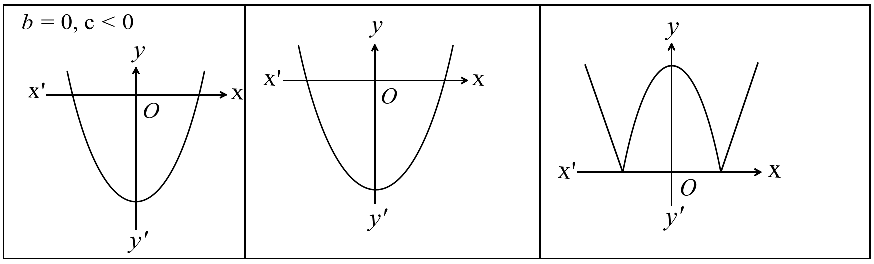
The required number of points are $1 + 2\,(3|n| - 1) = 6|n| - 1 = 11 \Rightarrow n = \pm 2$

d. $f(x) = \left|\left||x| - 2\right| + a\right|$ has exactly three points of non-differentiability

$f(x)$ is non-differentiable at $x = 0, |x| - 2 = 0$ or $x = 0, \pm 2$

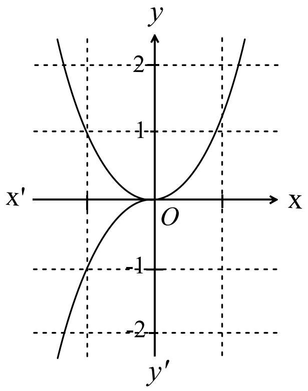
Hence, the value of a must be positive, as negative value of a allows $\left||x| - 2\right| + a = 0$ to have real roots, which given more points of non-differentiability

$g(x) = x^2 + bx + c$	$g(\lvert x\rvert) = x^2 + b\lvert x\rvert + c$	$f(x) = \lvert g(\lvert x\rvert)\rvert = \lvert x^2 + b\lvert x\rvert + c\rvert$
$b < 0, c > 0$ 		
$c = 0, b < 0$ 		
$c = 0, b > 0$ 		

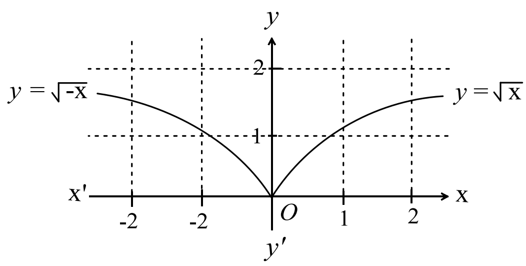


163 **(a)**

a. $f(x) = |x^3| = x(x|x|)$ is continuous and differentiable

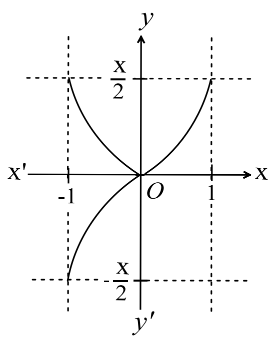


b. $f(x) = \sqrt{|x|}$ is continuous



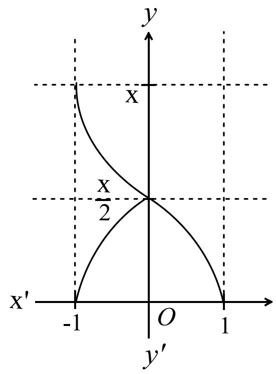
Clearly from the graph, $f(x)$ is non-differentiable at $x = 0$

c. $f(x) = |\sin^{-1} x|$ is continuous



Clearly from the graph, $f(x)$ is non-differentiable at $x = 0$

d. $f(x) = \cos^{-1}|x|$ is continuous



Clearly from the graph, $f(x)$ is no-differentiable at $x = 0$

164 **(b)**

$$f(x) = \begin{cases} \dfrac{a(1 - x\sin x) + b\cos x + 5}{x^2}, & x < 0 \\ 3, & x = 0 \\ \left\{1 + \left(\dfrac{P(x)}{x}\right)\right\}^{1/x}, & x > 0 \end{cases}$$

Where $P(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

$f(0) = 3$

$\text{R. H. L.} = \lim_{x\to 0+} f(x)$

$= \lim_{h\to 0} f(0 + h) = \lim_{h\to 0} f(h) = \lim_{h\to 0}\left\{1 + \left(\dfrac{P(h)}{h}\right)\right\}^{1/h}$

$\because f$ is continuous at $x = 0$

$\therefore$ R.H.L. exists

For the existence of R.H.L., $a_0, a_1 = 0$

$\Rightarrow \text{R. H. L.} = \lim_{h\to 0}(1 + a_2 h + a_3 h^2)^{1/h}$ (1^∞ form)

$= e^{\lim_{h\to 0}(1 + a_2 h + a_3 h^2 - 1)(1/h)} = e^{a_2}$

L.H.L. $= \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0 - h)$

$= \lim_{h\to 0} \dfrac{a(1 - (-h)\sin(-h)) + b\cos(-h) + 5}{(-h)^2}$

$= \lim_{h\to 0} \dfrac{a(1 - h(h)) + b\left(1 - \frac{h^2}{2!}\right) + 5}{h^2}$

For finite value of L.H.L., $a + b + 5 = 0$ and $-a - \frac{b}{2} = 3$

Solving, we get $a = -1, b = -4$

Now $g(x) = 3a\sin x - b\cos x = -3\sin x + 4\cos x$ which has the range $[-5, 5]$

Also $P(x) = a_3 x^3 + (\log_e 3)x^2$

$P''(x) = 6a_3 x + 2\log_e 3$

$\Rightarrow P''(0) = 2\log_e 3$

Further, $P(x) = b \Rightarrow a_3x^3 + (\log_e 3)x^2 = -4$ has only one real root, as the graph of $P(x) = a_3x^3 + (\log_e 3)x^2$ meets $y = -4$ only once for negative value of x

165 **(c)**

For $0 \le x < \frac{\pi}{4}$, $g(x) = 1 + \tan x$

$x \in \left[0, \frac{\pi}{4}\right) \Rightarrow 1 + \tan x \in [1, 2)$

So $f(g(x)) = f(1 + \tan x) = 1 + \tan x + 2$

and for $x \in \left[\frac{\pi}{4}, \pi\right)$, $g(x) = 3 - \cot x$

$x \in \left[\frac{\pi}{4}, \pi\right) \Rightarrow 3 - \cot x \in [2, \infty)$

So $f(g(x)) = f(3 - \cot x) = 6 - (3 - \cot x)$

Let $h(x) = f(g(x)) = \begin{cases} 3 + \tan x, 0 \le x < \frac{\pi}{4} \\ 3 + \cot x, \frac{\pi}{4} \le x < \pi \end{cases}$

Clearly, $f(g(x))$ is continuous in $[0, \pi)$

Now $h'\left(\frac{\pi^+}{4}\right) = \lim_{x\to\frac{\pi^+}{4}} (-\mathrm{cosec}^2 x) = -2$

$h'\left(\frac{\pi^-}{4}\right) = \lim_{x\to\frac{\pi^-}{4}} (\sec^2 x) = 2$

So $f(g(x))$ is differentiable everywhere in $[0, \pi)$ other than at $x = \frac{\pi}{4}$

$$f(g(x)) = \begin{cases} |3 + \tan x|, 0 \le x < \frac{\pi}{4} \\ |3 + \cot x|, \frac{\pi}{4} \le x < \pi \end{cases}$$

Which is non-differentiable at $x = \pi/4$ and where $3 + \cot x = 0$ or $x = \cot^{-1}(-3)$

For $x \in \left[0, \frac{\pi}{4}\right)$, $3 + \tan x \in [3, 4)$

For $x \in \left[\frac{\pi}{4}, \pi\right)$, $3 + \cot x \in (-\infty, 4]$

Hence, the range is $[-\infty, 4)$

166 **(a)**

$$F(x) = \lim_{n\to\infty} \frac{f(x) + x^{2n}g(x)}{1 + x^{2n}}$$

$$= \begin{cases} f(x), & 0 \le x^2 < 1 \\ \frac{f(x) + g(x)}{2}, & x^2 = 1 \\ g(x), & x^2 > 1 \end{cases}$$

$$= \begin{cases} g(x), & x < -1 \\ \frac{f(-1) + g(-1)}{2}, & x = -1 \\ f(x), & -1 < x < 1 \\ \frac{f(1) + g(1)}{2}, & x = 1 \\ g(x), & x > 1 \end{cases}$$

If $F(x)$ is continuous $\forall x \in R$, $F(x)$ must be made continuous out at $x = \pm 1$

For continuity at $x = -1$, $f(-1) = g(-1) \Rightarrow 1 - a + 3 = b - 1 \Rightarrow$

$a + b = 5$ (1)

For continuity at $x = 1$, $f(1) = g(1) \Rightarrow 1 + a + 3 = 1 + b$

$\Rightarrow a - b = -3$ (2)

Solving equations (1) and (2), we get $a = 1$ and $b = 4$

$f(x) = g(x) \Rightarrow x^2 + x + 3 = x + 4 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

167 **(a)**

$$f(x) = \begin{cases} [x], -2 \le x \le -\frac{1}{2} \\ 2x^2 - 1, -\frac{1}{2} < x \le 2 \end{cases}$$

$$= \begin{cases} -2, -2 \le x < -1 \\ -1, -1 \le x \le -\frac{1}{2} \\ 2x^2 - 1, -\frac{1}{2} < x \le 2 \end{cases}$$

$$|f(x)| = \begin{cases} 2, & -2 \le x < -1 \\ 1, & -1 \le x \le -\frac{1}{2} \\ |2x^2 - 1|, & -\frac{1}{2} < x \le 2 \end{cases}$$

$$= \begin{cases} 2, & -2 \le x < -1 \\ 1, & -1 \le x \le -\frac{1}{2} \\ 1 - 2x^2, & -\frac{1}{2} < x \le \frac{1}{\sqrt{2}} \\ 2x^2 - 1, & \frac{1}{\sqrt{2}} < x \le 2 \end{cases}$$

$$f(|x|) = \begin{cases} -2, & -2 \le |x| < -1 \\ -1, & -1 \le |x| \le -\frac{1}{2} \\ 2|x|^2 - 1, & -\frac{1}{2} < |x| \le 2 \end{cases} = 2x^2 - 1, -2 \le x \le 2$$

$\Rightarrow g(x) = f(|x|) + |f(x)|$

$$= \begin{cases} 2x^2 + 1, & -2 \le x < -1 \\ 2x^2, & -1 \le x \le -\frac{1}{2} \\ 0, & -\frac{1}{2} < x < \frac{1}{\sqrt{2}} \\ 4x^2 - 2, & \frac{1}{\sqrt{2}} \le x \le 2 \end{cases}$$

$g(-1^-) = \lim_{x\to-1} (2x^2 + 1) = 3$, $g(-1^+) = \lim_{x\to-1} 2x^2 = 2$

$g\left(-\frac{1^-}{2}\right) = \lim_{x\to-\frac{1}{2}} 2x^2 = \frac{1}{2}$, $g\left(-\frac{1^+}{2}\right) = \lim_{x\to-\frac{1}{2}} 0 = 0$

$g\left(\frac{1^-}{\sqrt{2}}\right) = \lim_{x\to-\frac{1}{\sqrt{2}}} 0 = 0$, $g\left(\frac{1^+}{\sqrt{2}}\right) = \lim_{x\to-\frac{1}{\sqrt{2}}} (4x^2 - 2) = 0$

Hence, $g(x)$ is discontinous at $x = -1, -\frac{1}{2}$

$g(x)$ is continuous at $x = \frac{1}{\sqrt{2}}$

Now, $g'\left(\frac{1^-}{\sqrt{2}}\right) = 0, g'\left(\frac{1^+}{\sqrt{2}}\right) = 8\left(\frac{1}{\sqrt{2}}\right) = \frac{8}{\sqrt{2}}$

Hence, $g(x)$ is non-differentiable at $x = \frac{1}{\sqrt{2}}$

168 **(c)**

$$f(x) = \begin{cases} x^2 + 10x + 8, & x \le -2 \\ ax^2 + bx + c, & -2 < x < 0, a \ne 0 \\ x^2 + 2x, & x \le 0 \end{cases}$$

For continuous at $x = 0 \Rightarrow c = 0$

Continuous at $x = -2 \Rightarrow 4 - 20 + 8 = 4a - 2b$

$\Rightarrow 2a - b = -4$ (1)

Now let the line $y = mx + p$ is tangent to all the three curves

Solving $y = mx + p$ and $y = x^2 + 2x$

$x^2 + 2x = mx + p$

$x^2 + (2 - m)x - p = 0$

$D = 0$

$(2 - m)^2 + 4p = 0$ (2)

Again solving $y = mx + p$ and $y = x^2 + 10x + 8$

$x^2 + 10x + 8 = mx + p$

$\Rightarrow x^2 + (10 - m)x + 8 - p = 0$

$D = 0 \Rightarrow (10 - m)^2 - 4(8 - p) = 0$

$\Rightarrow (10 - m)^2 - (2 - m)^2 = 42$

$\Rightarrow (100 - 20m) - (4 - 4m) = 32$

$\Rightarrow m = 4$ and $p = -1$

Hence equation of the tangent to first and last curves is

$y = 4x - 1$ (3)

Now solving this with $y = ax^2 + bx$(as $c = 0$)

$ax^2 + bx = 4x - 1 \Rightarrow ax^2 + (b - 4)x + 1 = 0$

$D = 0$

$\Rightarrow (b - 4)^2 = 4a$

Also $b = 2a + 4$ (from (1))

$\therefore 4a^2 = 4a \Rightarrow a = 1$ and $b = 6$ (as $a \ne 0$)

$f'(0^-) = \lim_{x\to 0}(2ax + b) = b$

$f'(0^+) = \lim_{x\to 0}(2ax + 2) = 2 \Rightarrow b = 2$

169 **(7)**

Let $g(x) = (\ln x)(\ln x) \cdots \infty$

$$g(x) = \begin{cases} 0, & 1 < x < e \\ 1, & x = e \\ \infty, & x > e \end{cases}$$

Therefore $f(x) = \begin{cases} x, & 1 < x < e \\ x/2, & x = e \\ 0, & e < x < 3 \end{cases}$

Hence $f(x)$ is non-differentiable at $x = e$

170 **(5)**

$f(x) = \text{sgn}(\sin x)$ is discontinuous when $\sin x = 0$

$\Rightarrow x = 0, \pi, 2\pi, 3\pi, 4\pi$

171 **(1)**

$$\lim_{x\to 1^-} h(x) = \lim_{x\to 1^-}\lim_{n\to\infty} \frac{x^{2n}\cdot f(x) + x^{2m}\cdot g(x)}{(1 + x^{2n})} = g(1)$$

$$\lim_{x\to 1^-} h(x) = \lim_{x\to 1^+}\lim_{n\to\infty} \frac{x^{2n}\cdot f(x) + x^{2m}\cdot g(x)}{(1 + x^{2n})} = f(1)$$

$\because \lim_{x\to 1} h(x)$ exists $\Rightarrow f(1) = g(1)$

$\Rightarrow f(x) - g(x) = 0$ has a root at $x = 1$

172 **(6)**

$$g(f(x)) = \begin{cases} g\left(\frac{x}{2} - 1\right), & 0 \le x < 1 \\ g\left(\frac{1}{2}\right), & 1 \le x \le 2 \end{cases}$$

$$= \begin{cases} \frac{(x-1)(x-2-2k)}{2} + 3, & 0 \le x < 1 \\ 4 - 2k, & 1 \le x < 2 \end{cases}$$

$\lim_{x\to 1^-} g(f(x)) = 3, g(f(1)) = 4 - 2k$ and $\lim_{x\to 1^+} g(f(x)) = 4 - 2k$ for $g(f(x))$ to be continuous at $x = 1, 4 - 2k = 3 \Rightarrow k = \frac{1}{2}$

173 **(8)**

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0}\frac{f(x) + f(h) + 2xh(x+h) - \frac{1}{3} - \left(f(x) + f(0) - \frac{1}{3}\right)}{h}$$

$$= \lim_{h\to 0}\frac{f(h) - f(0)}{h} + 2x^2 = f'(0) + 2x^2$$

$$\lim_{h\to 0}\frac{3f(h) - 1}{6h} = \lim_{h\to 0}\frac{f(h) - \frac{1}{3}}{2h} = \lim_{h\to 0}\frac{f(h) - f(0)}{2h}$$

$$= \frac{f'(0)}{2} = \frac{2}{3} \Rightarrow f'(0) = \frac{4}{3}$$

$$\therefore f'(x) = \frac{4}{3} + 2x^2$$

$$f(x) = \lambda + \frac{4}{3}x + \frac{2x^3}{3} \Rightarrow f(0) = \lambda = \frac{1}{3}$$

$$\therefore f(x) = \frac{2x^3}{3} + \frac{4}{3}x + \frac{1}{3} \Rightarrow f(2) = \frac{25}{3}$$

174 **(2)**

$$f(0) = \lim_{x\to 0}\frac{\tan(\tan x) - \sin(\sin x)}{\tan x - \sin x}$$

$$= \lim_{x\to 0}\frac{\tan(\tan x) - \sin(\sin x)}{\frac{\tan x}{x}\left(\frac{1-\cos x}{x^2}\right)x^3}$$

$$= 2\lim_{x\to 0}\frac{\tan(\tan x) - \sin(\sin x)}{x^3}$$

$$= 2\lim_{x\to 0}\frac{\left(\tan x + \frac{\tan^3 x}{3} + \frac{2}{15}\tan^5 x + \cdots\right) - \left(\sin x - \frac{\sin^3 x}{3!} + \frac{\sin^5 x}{5!}\cdots\right)}{x^3}$$

$$= 2\lim_{x\to 0}\left(\left(\frac{\tan x - \sin x}{x^3}\right) + \frac{\left(\frac{\tan^3 x}{3} + \frac{\sin^3 x}{3!}\right)}{x^3} + \cdots\right)$$

$$= 2\lim_{x\to 0}\left(\left(\frac{\tan x}{x}\right)\left(\frac{1-\cos x}{x^2}\right) + \frac{1}{3} + \frac{1}{6}\right) =$$

212+12=2

175 **(8)**

$$f(x)\begin{cases} ax^2 + bx & \text{for } -1 < x < 1 \\ \frac{a-b-1}{2} & x = -1 \\ \frac{a+b+1}{2} & x = 1 \\ \frac{1}{x} & \text{for } x > 1 \text{ or } x < -1 \end{cases}$$

For continuity at $x = 1$ we have $a + b = \frac{a+b+1}{2}$

Hence, $a + b = 1$ (1)

For continuity at $x = -1$

$a - b = -1$ $a - b = -1$ (2)

Hence $a = 0$ and $b = 1$

176 **(6)**

$g(x) = \left[\frac{f(x)}{a}\right]$ is continuous if $\left[\frac{f(x)}{a}\right] = 0$ for

$\forall f(x) \in (1, \sqrt{30})$, for which we must have

$a > \sqrt{30}$

Hence the least value of a is 6

177 **(4)**

$\text{sgn}(x^2 - 3x + 2)$ is discontinuous when

$x^2 - 3x + 2 = 0$ or $x = 1, 2$

$[x - 3] = [x] - 3$ is discontinuous at $x = 1, 2, 3, 4$

Thus $f(x)$ is discontinuous at $x = 3, 4$

Now both $\text{sgn}(x^2 - 3x + 2)$ and $[x - 3]$ are discontinuous at $x = 1$ and 2

Then $f(x)$ may be continuous at $x = 1$ and 2

But $f(1) = -2$ and $f(1^+) = -1 + 0 - 3 = -4$

Thus $f(x)$ is discontinuous at $x = 1$

Also $f(2) = -1$ and $f(2^+) = 1 - 1 = 0$

Hence $f(x)$ is discontinuous at $x = 2$ also

178 **(2)**

$g'(3^-) = \lim_{h\to 0}\frac{g(3-h)-g(3)}{-h} = \lim_{h\to 0}\frac{a\sqrt{4-h}-(3b+2)}{-h}$

(1)

For existence of limit $\lim_{h\to 0} N^r = 0$

$\therefore 2a - 3b = 2$ (2)

Now $g'(3^+) = \lim_{h\to 0}\frac{b(3+h)+2-(3b+2)}{h} = b$ (3)

Substituting $3b + 2 = 2a$ in equation (1)

$$g'(3^-) = \lim_{h\to 0}\frac{a\sqrt{4-h} - 2a}{-h}$$

$$= \lim_{h\to 0}\left(\frac{(4-h)-4}{(-h)(\sqrt{4-h}+2)}\right) = \frac{a}{4}$$

Hence $g'(3^-) = g'(3^+)$

$\frac{a}{4} = b \Rightarrow a = 4b$ (4)

From equation (2) and (4)

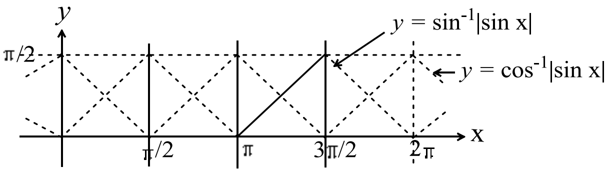
$8b - 3b = 2$

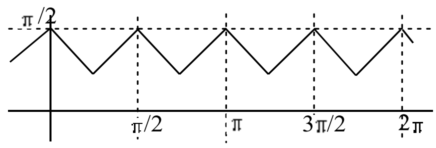
$\Rightarrow b = \frac{2}{5}$ and $a = \frac{8}{5}$

$\Rightarrow a + b = 2$

179 **(7)**

$\sin^{-1}|\sin x|$ is periodic with period π





180 **(1)**

Given $\frac{\int_{f(y)}^{f(x)} e^t dt}{\int_y^x (1/t)dt} = 1$

$\Rightarrow e^{f(x)} - e^{f(y)} = \text{In } x - \text{In } y$

$\Rightarrow e^{f(x)} - \text{In } x = c \Rightarrow f(x) = \text{In }(\text{In } x + c)$

Since $f\left(\frac{1}{e}\right) = 0 \Rightarrow c = 2$

Now $f(g(x)) = \begin{cases} \text{In }(x+2); & x \geq k \\ \text{In }(2 + x^2); & 0 < x < k \end{cases}$

For continuity at $x = k$

$\text{In }(k + c) = \text{In }(k^2 + c) \Rightarrow$ either $k = 0$ or $k = 1$

$\because k > 0 \Rightarrow k = 1$

181 **(8)**

We have $f(x) = [x] + [x + 1/3] + [x + 2/3] = [3x]$

Which is discontinuous when $3x = k$ or $x = k/3, k \in I$

Hence points of discontinuity are 1/3, 2/3, 3/3, 4/3, 5/3, 6/3, 7/3, 8/3

182 **(5)**

$$\because f''(x) = \begin{cases} x^p \sin\left(\frac{1}{x}\right) + x^2, & x > 0 \\ x^p \sin\left(\frac{1}{x}\right) - x^2, & x < 0 \\ 0, & x = 0 \end{cases}$$

$$f'''(x) = \begin{cases} -x^{p-4}\sin\left(\frac{1}{x}\right) - (p-2)x^{p-3}\cos\left(\frac{1}{x}\right) \\ -px^{p-3}\cos\left(\frac{1}{x}\right) \\ \quad +p(p-1)x^{p-2}\sin\left(\frac{1}{x}\right) + 2, & x > 0 \\ -x^{p-4}\ \sin\left(\frac{1}{x}\right) - (p-2)x^{p-3}\cos\left(\frac{1}{x}\right) \\ px^{p-3}\cos\left(\frac{1}{x}\right) \\ \quad +p(p-1)x^{p-2}\sin\left(\frac{1}{x}\right) - 2, & x < 0 \\ 0, & x = 0 \end{cases}$$

RHL $=$ LHL $= f(0) = 0$

$\because$ $\sin\infty$ and $\cos\infty$ lie between -1 to 1. For $p \geq 5$,

RHL $= 2$ LHL $= -2$

$f(0) = 0$

For $p \in [5, \infty)$, $f''(x)$ is not continuous

DCAM classes
Dynamic Classes for Academic Mastery