Single Correct Answer Type

1. If the normal to the given hyperbola at the point $\left(ct, \frac{c}{t}\right)$ meets the curve again at $\left(ct', \frac{c}{t'}\right)$, then
   a) $t^3t' = 1$ b) $t^3t' = -1$ c) $tt' = 1$ d) $tt' = -1$
2. The curve represented by the equation $\sqrt{px} + \sqrt{qy} = 1$, where $p, q \in R, p, q > 0$ is
   a) A circle b) A parabola c) An ellipse d) A hyperbola
3. If two distinct tangents can be drawn from the point $(\alpha, 2)$ on different branches of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$, then
   a) $|\alpha| < \frac{3}{2}$ b) $|\alpha| > \frac{2}{3}$ c) $|\alpha| > 3$ d) none of these
4. If OA and OB are equal perpendicular chord of the circles $x^2 + y^2 - 2x + 4y = 0$, then equations of OA and OB are where O is origin
   a) $3x + y = 0$ and $3x - y = 0$ b) $3x + y = 0$ or $3y - x = 0$
   c) $x + 3y = 0$ and $y - 3x = 0$ d) $x + y = 0$ or $x - y = 0$
5. A variable chord of circle $x^2 + y^2 = 4$ is drawn from the point $P(3, 5)$ meeting the circle at the points A and B. A point Q is taken on this chord such that $2PQ = PA + PB$. Locus of $'Q'$ is
   a) $x^2 + y^2 + 3x + 4y = 0$ b) $x^2 + y^2 = 36$
   c) $x^2 + y^2 = 16$ d) $x^2 + y^2 - 3x - 5y = 0$
6. If S_1 and S_2 are the foci of the hyperbola whose transverse axis length is 4 and conjugate axis length is 6, S_3 and S_4 are the foci of the conjugate hyperbola, then the area of the quadrilateral $S_1S_3S_2S_4$ is
   a) 24 b) 26 c) 22 d) none of these
7. The equation of the tangent to the circle $x^2 + y^2 = a^2$, which makes a triangle of area a^2 with the coordinates axes, is
   a) $x \pm y = a\sqrt{2}$ b) $x \pm y = \pm a\sqrt{2}$ c) $x \pm y = 2a$ d) $x + y = \pm 2a$
8. Minimum area of circle which touches the parabolas $y = x^2 + 1$ and $y^2 = x - 1$ is
   a) $\frac{9\pi}{16}$ sq.unit b) $\frac{9\pi}{32}$ sq.unit c) $\frac{9\pi}{8}$ sq.unit d) $\frac{9\pi}{4}$ sq.unit
9. The circles having radii r_1 and r_2 intersect orthogonally. Length of their common chord is
   a) $\frac{2r_1r_2}{\sqrt{r_1^2 + r_2^2}}$ b) $\frac{\sqrt{r_1^2 + r_2^2}}{2r_1r_2}$ c) $\frac{r_1r_2}{\sqrt{r_1^2 + r_2^2}}$ d) $\frac{\sqrt{r_1^2 + r_2^2}}{r_1r_2}$
10. Two parabolas have the same focus. If their directrices are the x–axis and the y-axis, respectively, then the slope of their common chord is
    a) ± 1 b) $\frac{4}{3}$ c) $\frac{3}{4}$ d) None of these
11. If $ax + by = 1$ is tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $a^2 - b^2$ equals to
    a) $\frac{1}{a^2e^2}$ b) a^2e^2 c) b^2e^2 d) None of these
12. The focal chord to $y^2 = 16x$ is tangent to $(x - 6)^2 + y^2 = 2$, then the possible values of the slope of this chord, are
    a) $\{-1, 1\}$ b) $\{-2, 2\}$ c) $\{-2, 1/2\}$ d) $\{2, -1/2\}$
13. Let C be a circle with two diameters intersecting at an angle of 30°. A circle S is tangent to both the diameters and to C, and has radius unity. The largest radius of C is
    a) $1 + \sqrt{6} + \sqrt{2}$ b) $1 + \sqrt{6} - \sqrt{2}$ c) $\sqrt{6} + \sqrt{2} - 11$ d) None of these
14. The length of the chord of the parabola $y^2 = x$ which is bisected at the point (2,1) is
    a) $2\sqrt{3}$ b) $4\sqrt{3}$ c) $3\sqrt{2}$ d) $2\sqrt{5}$
15. Two circles with radii a and b touch each other externally such that θ is the angle between the direct common tangents $(a > b \geq 2)$, then

a) $\theta = 2\cos^{-1}\left(\frac{a-b}{a+b}\right)$ b) $\theta = 2\tan^{-1}\left(\frac{a+b}{a-b}\right)$ c) $\theta = 2\sin^{-1}\left(\frac{a+b}{a-b}\right)$ d) $\theta = 2\sin^{-1}\left(\frac{a-b}{a+b}\right)$

16. In triangle ABC, equation of side BC is $x - y = 0$. Circumcentre and orthocenter of the triangle are $(2, 3)$ and $(5, 8)$, respectively. Equation of circumcircle of the triangle is
a) $x^2 + y^2 - 4x + 6y - 27 = 0$ b) $x^2 + y^2 - 4x - 6y - 27 = 0$
c) $x^2 + y^2 + 4x + 6y - 27 = 0$ d) $x^2 + y^2 + 4x + 6y - 27 = 0$

17. If a circle of radius r is touching the lines $x^2 - 4xy + y^2 = 0$ in the first quadrant at points A and B, then area of triangle OAB (O being the origin) is
a) $\frac{3\sqrt{3}\,r^2}{4}$ b) $\frac{\sqrt{3}\,r^2}{4}$ c) $\frac{3r^2}{4}$ d) r^2

18. A square is inscribed in the circle $x^2 + y^2 - 2x + 4y + 3 = 0$. Its sides are parallel to the coordinates axes. The one vertex of the square is
a) $(1 + \sqrt{2}, -2)$ b) $(1 - \sqrt{2}, -2)$ c) $(1, -2 + \sqrt{2})$ d) None of these

19. On the line segment joining $(1, 0)$ and $(3, 0)$, an equilateral triangle is drawn having its vertex in the fourth quadrant, then radical centre of the circles described on its sides as diameter is
a) $\left(3, -\frac{1}{\sqrt{3}}\right)$ b) $(3, -\sqrt{3})$ c) $\left(2, -\frac{1}{\sqrt{3}}\right)$ d) $(2, -\sqrt{3})$

20. If the angle of intersection of the circles $x^2 + y^2 + x + y = 0$ and $x^2 + y^2 + x - y = 0$ is θ, then equation of the line passing through $(1, 2)$ and making an angle θ with the y-axis is
a) $x = 1$ b) $y = 2$ c) $x + y = 3$ d) $x - y = 3$

21. The equations of four circles are $(x \pm a)^2 + (y \pm a)^2 = a^2$. The radius of a circle touching all the four circles is
a) $(\sqrt{2} + 2)a$ b) $2\sqrt{2}a$ c) $(\sqrt{2} + 1)a$ d) $(2 + \sqrt{2})a$

22. An ellipse is sliding along the co-ordinate axes. If the foci of the ellipse are $(1,1)$ and $(3,3)$, then area of the director circle of the ellipse (in sq.units) is
a) 2π b) 4π c) 6π d) 8π

23. If the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is inscribed in a rectangle whose length to breadth ratio is 2:1 then the area of the rectangle is
a) $4\frac{a^2 + b^2}{7}$ b) $4\frac{a^2 + b^2}{3}$ c) $12\frac{a^2 + b^2}{5}$ d) $8\frac{a^2 + b^2}{5}$

24. Tangents are drawn to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b)$ and the circle $x^2 + y^2 = a^2$ at the points where a common ordinate cuts them (on the same side of the x-axis). Then the greatest acute angle between these tangents is given by
a) $\tan^{-1}\left(\frac{a-b}{2\sqrt{ab}}\right)$ b) $\tan^{-1}\left(\frac{a+b}{2\sqrt{ab}}\right)$ c) $\tan^{-1}\left(\frac{2ab}{\sqrt{a-b}}\right)$ d) $\tan^{-1}\left(\frac{2ab}{\sqrt{a+b}}\right)$

25. If $(\sqrt{3})bx + ay = 2ab$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then the eccentric angle of the point of contact is
a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

26. The circle $x^2 + y^2 = 4$ cuts the line joining the points $A(1, 0)$ and $B(3, 4)$ in two points P and Q. Let $\frac{BP}{PA} = \alpha$ and $\frac{BQ}{QA} = \beta$. Then α and β are roots of the quadratic equation
a) $3x^2 + 2x - 21 = 0$ b) $3x^2 + 2x + 21 = 0$ c) $2x^2 + 3x - 21 = 0$ d) None of these

27. Equation of a rectangular hyperbola whose asymptotes are $x = 3$ and $y = 5$ and passing through $(7, 8)$ is
a) $xy - 3y + 5x + 3 = 0$ b) $xy + 3y + 4x + 3 = 0$
c) $xy - 3y + 5x - 3 = 0$ d) $xy - 3y - 5x + 3 = 0$

28. For the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ with vertices A and A', tangent drawn at the point P in the first quadrant meets the y-axis and Q and the chord $A'P$ meets the y-axisat M. If O is the origin, then $OQ^2 - MQ^2$ equales to
a) 9 b) 13 c) 4 d) 5

29. The ratio in which the line segment joining the points $(4, -6)$ and $(3,1)$ is divided by the parabola $y^2 = 4x$

is

a) $\frac{-20 \pm \sqrt{155}}{11} : 1$ b) $\frac{-2 \pm 2\sqrt{155}}{11} : 2$ c) $-20 \pm 2\sqrt{155}:\ 11$ d) $-20\ \pm\ \sqrt{155}:\ 11$

30. If $y = 2x - 3$ is a tangent to the parabola $y^2 = 4a\left(x - \frac{1}{3}\right)$, then '$a$' is equal to

a) $\frac{22}{3}$ b) -1 c) $\frac{14}{3}$ d) $\frac{-14}{3}$

31. From the point $(15, 12)$ three normals are drawn to the parabola $y^2 = 4ax$ then centroid of triangle formed by three co-normals points is

a) $\left(\frac{16}{3}, 0\right)$ b) $(4, 0)$ c) $\left(\frac{26}{3}, 0\right)$ d) $(6, 0)$

32. PQ is a normal chord of the parabola $y^2 = 4ax$ at P, A being the vertex of the parabola. Through P a line is drawn parallel to AQ meeting the x–axis in R. Then line length of AR is

a) Equal to the length of the latus rectum
b) Equal to the focal distance of the point P
c) Equal to twice the focal distance of the point P
d) Equal to the distance of the point P from the directrix

33. If d is the distance between parallel tangents with positive slope to $y^2 = 4x$ and $x^2 + y^2 - 2x + 4y - 11 = 0$, then

a) $10 < d < 2$ b) $4 < d < 6$ c) $d < 4$ d) None of these

34. If the pair of straight line $x\,y\sqrt{3} - x^2 = 0$ is tangent to the circle at P and Q from origin O such that area of smaller sector formed by CP and CQ is 3π sq. unit, where C is the centre of circle, then OP equals to

a) $(3\sqrt{3})/2$ b) $3\sqrt{3}$ c) 3 d) $\sqrt{3}$

35. If the radius of the circle $(x-1)^2 + (y-2)^2 = 1$ and $(x-7)^2 + (y-10)^2 = 4$ are increasing uniformly w.r.t. times as 0.3 and 0.4 unit/sec, then they will touch each other at t equal to

a) 45 sec b) 90 sec c) 11 sec d) 135 sec

36. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle of area 154 sq. units. Then the equation of this circle is

a) $x^2 + y^2 + 2x - 2y = 62$ b) $x^2 + y^2 + 2x - 2y = 47$
c) $x^2 + y^2 - 2x + 2y = 47$ d) $x^2 + y^2 - 2x + 2y = 62$

37. Tangent and normal drawn to parabola at $A(at^2, 2at)$, $t \neq 0$ meet the x–axis at point B and D respectively. If the rectangle $ABCD$ is completed, then locus of 'C' is

a) $y = 2a$ b) $y + 2a = c$ c) $x = 2a$ d) $x + 2a = 0$

38. If the chord $y = mx + 1$ of the circles $x^2 + y^2 = 1$ subtends an angle 45° at the major segment of the circle, then value of m is

a) 2 b) -2 c) -1 d) None of these

39. The chords of contact of a point 'P' w.r.t. a hyperbola and its auxiliary circle are at right angle, then the point P lies on

a) conjugate hyperbola b) one of the directrix
c) one of the asymptotes d) none of these

40. If A_1B_1 and A_2B_2 are two focal chords of the parabola $y^2 = 4ax$, then the chord A_1A_2 and B_1B_2 intersect on

a) Directrix b) Axis c) Tangent at vertex d) None of these

41. The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9}$, and having its centre$(0, 3)$ is

a) 4 b) 3 c) $\sqrt{12}$ d) $\frac{7}{2}$

42. For a hyperbola whose centre is at $(1, 2)$ and asymptotes are parallel to lines $2x + 3y = 0$ and $x + 2y = 1$, then equation of hyperbola passing through $(2, 4)$ is

a) $(2x + 3y - 5)(x + 2y - 8) = 40$ b) $(2x + 3y - 8)(x + 2y - 5) = 40$
c) $(2x + 3y - 8)(x + 2y - 5) = 30$ d) None of these

43. From any point P lying in first quadrant on the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1, PN$ is drawn perpendicular to the major axis and produced at Q so that NQequals to PS, where S is a focus. Then the locus of Q is

a) $5y-3x-25=0$ b) $3x+5y+25=0$ c) $3x-5y-25=0$ d) None of these

44. The end points of two normal chords of a parabola are concyclic, then the tangents at the feet of the normals will intersect at

a) Tangent at vertex of the parabola b) Axis of the parabola
c) Directrix of the parabola d) None of these

45. The eccentricity of locus of point $(3h+2, k)$ where (h, k) lies on the circle $x^2+y^2=1$ is

a) $\frac{1}{3}$ b) $\frac{\sqrt{2}}{3}$ c) $\frac{2\sqrt{2}}{3}$ d) $\frac{1}{\sqrt{3}}$

46. A tangent at a point on the circle $x^2+y^2=a^2$ intersects a concentric circle C at two points P and Q. The tangents to the circle X at P and Q meet at a point on the circle $x^2+y^2=b^2$, then the equation of circle is

a) $x^2+y^2=ab$ b) $x^2+y^2=(a-b)^2$
c) $x^2+y^2=(a+b)^2$ d) $x^2+y^2=a^2+b^2$

47. There are exactly two points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose distance from its centre is the same and is equal to $\frac{\sqrt{a^2+2b^2}}{2}$. Then the eccentricity of the ellipse is

a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{2}}$ c) $\frac{1}{3}$ d) $\frac{1}{3\sqrt{2}}$

48. If the line $2x+\sqrt{6}y=2$ touches the hyperbola $x^2-2y^2=4$, then the point of contact is

a) $(-2, \sqrt{6})$ b) $(-5, 2\sqrt{6})$ c) $\left(\frac{1}{2}, \frac{1}{\sqrt{6}}\right)$ d) $(4, -\sqrt{6})$

49. If a variable line has its intercepts on the coordinate axes e, e', where $\frac{e}{2}, \frac{e'}{2}$ are the eccentricities of a hyperbola and its conjugate hyperbola, then the line always touches the circle $x^2+y^2=r^2$, where $r=$

a) 1 b) 2 c) 3 d) cannot be decided

50. If two chords drawn from the point $A(4, 4)$ to the parabola $x^2=4y$ are bisected by line $y=mx$, the interval in which lies is

a) $(-2\sqrt{2}, 2\sqrt{2})$ b) $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$
c) $(-\infty. -2\sqrt{2}-2) \cup (2\sqrt{2}-2, \infty)$ d) None of these

51. If the distance between the foci and the distance between the two directries of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ are in the ratio 3:2, then $b:a$ is

a) $1:\sqrt{2}$ b) $\sqrt{3}:\sqrt{2}$ c) 1:2 d) 2:1

52. If P be a point on the parabola $y^2=3(2x-3)$ and M is the foot perpendicular drawn from P on the directrix of the parabola, then length of each side of an equilateral triangle SMP, where S is focus of the parabola is

a) 2 b) 4 c) 6 d) 8

53. Let P_i and P_i' be the feet of the perpendiculars drawn from foci S, S' on a tangent T_i to an ellipse whose length of semi-major axis is 20, if $\sum_{i=1}^{10}(SP_i)(S'P_i')=2560$, then the value of eccentricity is

a) $\frac{1}{5}$ b) $\frac{2}{5}$ c) $\frac{3}{5}$ d) $\frac{4}{5}$

54. A tangent to the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ at any point P meets the line $x=0$ at a point Q. Let R be the image of Q in the line $y=x$, then the circle whose extremities of a diameter are Q and R passes through a fixed point. The fixed point is

a) (3, 0) b) (5, 0) c) (0, 0) d) (4, 0)

55. A tangent is drawn to the parabola $y^2=4ax$ at the point 'P' whose abscissa lies in the interval (1, 4). The maximum possible area of the triangle formed by the tangent at 'P' ordinates of the point 'P' and the $x-$axis is equal to

a) 8 b) 16 c) 24 d) 32

56. If the angle between the tangents from the point $(\lambda, 1)$ to the parabola $y^2 = 16x$ be $\frac{\pi}{2}$ then λ is

a) 4 b) -4 c) -1 d) 2

57. The locus of a point on the variable parabola $y^2 = 4ax$, whose distance from focus is always equal to k, is equal to: (a is parameter)

a) $4x^2 + y^2 - 4kx = 0$ b) $x^2 + y^2 - 4kx = 0$ c) $2x^2 + 4y^2 - 8kx = 0$ d) $4x^2 - y^2 + 4kx = 0$

58. The graph of the curve $x^2 + y^2 - 2xy - 8x - 8y + 32 = 0$ falls wholly in the

a) First quadrant b) Second quadrant c) Third quadrant d) None of these

59. The equation $16x^2 - 3y^2 - 32x + 12y - 44 = 0$ represents a hyperbola

a) The length of whose transverse axis is $4\sqrt{3}$ b) The length of whose conjugate axis is 4

c) Whose centre is $(-1,2)$ d) Whose eccentricity is $\sqrt{\frac{19}{3}}$

60. B and C are fixed points having co-ordinates $(3, 0)$ and $(-3, 0)$, respectively. If the vertical angle BAC is $90°$, then the locus of the centroid of the ΔABC has the equation

a) $x^2 + y^2 = 1$ b) $x^2 + y^2 = 2$ c) $9(x^2 + y^2) = 1$ d) $9(x^2 + y^2) = 4$

61. The family of the curves which intersects the family of rectangular hyperbola $xy = c^2$ orthogonally is

a) family of parabola b) family of ellipse

c) family of circle d) family of rectangular hyperbola

62. If two points P and Q on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, whose centre C be such that CP is perpendicular to $CQ, a < b,$ then the value of $\frac{1}{CP^2} + \frac{1}{CQ^2}$ is

a) $\frac{b^2 - a^2}{2ab}$ b) $\frac{1}{a^2} + \frac{1}{b^2}$ c) $\frac{2ab}{b^2 - a^2}$ d) $\frac{1}{a^2} - \frac{1}{b^2}$

63. The area of the triangle formed by joining the origin to the points of intersection of the line $x\sqrt{5} + 2y = 3\sqrt{5}$ and circle $x^2 + y^2 = 10$ is

a) 3 b) 4 c) 5 d) 6

64. The ends of a quadrant of a circle have the coordinates $(1, 3)$ and $(3, 1)$. Then the centre of such a circle is

a) $(2, 2)$ b) $(1, 1)$ c) $(4, 4)$ d) $(2, 6)$

65. C_1 is a circle of radius 1 touching the x-axis and the y-axis. C_2 is another circle of radius $>$ 1 and touching the axes as well as the circle C_1. Then, the radius of C_2 is

a) $3 - 2\sqrt{2}$ b) $3 + 2\sqrt{2}$ c) $3 + 2\sqrt{3}$ d) None of these

66. A parabola $y = ax^2 + bx + c$ crosses the x–axis at $(\alpha, 0)$ $(\beta, 0)$ both to the right of the origin. A circle also passes through these two points. The length of a tangent from the origin to the circle is

a) $\sqrt{\frac{bc}{a}}$ b) ac^2 c) $\frac{b}{a}$ d) $\sqrt{\frac{c}{a}}$

67. If the parabola $y = ax^2 - 6x + b$ passes through $(0, 2)$ and has its tangent at $x = \frac{3}{2}$ parallel to the x–axis then

a) $a = 2, b = -2$ b) $a = 2, b = 2$ c) $a = -2, b = 2$ d) $a = -2, b = -2$

68. P and Q are the foci of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and B is an end of the minor axis. If PBQ is an equilateral triangle, then eccentricity of the ellipse is

a) $\frac{1}{\sqrt{2}}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{\sqrt{3}}{2}$

69. Each of the four inequalities given below defines a region in the xy plane. One of these four regions does not have the following property. For any two points (x_1, y_1) and (x_2, y_2) in the region, the point $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ is also in the region. The inequality defining this region is

a) $x^2 + 2y^2 \leq 1$ b) $\max\{|x|, |y|\} \leq 1$ c) $x^2 - y^2 \leq 1$ d) $y^2 - x \leq 0$

70. If e_1 is the eccentricity of the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$ and e_2 is the eccentricity of the hyperbola passing through the foci of the ellipse and $e_1 e_2 = 1$, then equation of the hyperbola is

a) $\frac{x^2}{9}-\frac{y^2}{16}=1$ b) $\frac{x^2}{16}-\frac{y^2}{9}=-1$ c) $\frac{x^2}{9}-\frac{y^2}{25}=1$ d) None of these

71. Vertex of the parabola whose parametric equation is $x=t^2-t+1, y=t^2+t+1; t\epsilon\ R$, is
a) $(1, 1)$ b) $(2, 2)$ c) $\left(\frac{1}{2},\frac{1}{2}\right)$ d) $(3, 3)$

72. The tangent and normal at the point $P(at^2, 2at)$ to the parabola $y^2=4ax$ meet the x-axis in T and G, respectively, then the angle at which the tangent at P to the parabola is inclined to the tangent at P to the circle through P, T, G is
a) $\tan^{-1}(t^2)$ b) $\cot^{-1}(t^2)$ c) $\tan^{-1}(t)$ d) $\cot^{-1}(t)$

73. From points (3, 4), chords are drawn to the circle $x^2+y^2-4x=0$. The locus of the midpoints of the chords is
a) $x^2+y^2-5x-4y+6=0$ b) $x^2+y^2+5x-4y+6=0$
c) $x^2+y^2-5x+4y+6=0$ d) $x^2+y^2-5x-4y-6=0$

74. The centres of a set of circle, each of radius 3, lies on the circle $x^2+y^2=25$. The locus of any point in the set is
a) $4\le x^2+y^2\le 64$ b) $x^2+y^2\le 25$ c) $x^2+y^2\ge 25$ d) $3\le x^2+y^2\le 9$

75. Tangents and normal drawn to parabola $y^2=4ax$ at point $P(at^2, 2at), t\ne 0$, meet the x–axis at points T and N, respectively. If 'S' is the focus of the parabola, then
a) $SP=ST\ne SN$ b) $SP\ne ST=SN$ c) $SP=ST=SN$ d) $SP\ne ST\ne SN$

76. The straight line joining any point P on the parabola $y^2=4ax$ to the vertex and perpendicular from the focus to the tangent at P, intersect at R, then the equation of the locus of R is
a) $x^2+2y^2-ax=0$ b) $2x^2+y^2-2ax=0$ c) $2x^2+2y^2-ay=0$ d) $2x^2+y^2-2ay=0$

77. The sides AC and AB of a ΔABC touch the conjugate hyperbola of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If the vertex A lies on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then the side BC must touch
a) Parabola b) Circle c) Hyperbola d) Ellipse

78. A line L passing through the focus of the parabola $y^2=4(x-1)$ intersects the parabola in two distinct points. If 'm' be the slope of the line L then
a) $-1<m<1$ b) $m<-1$ or $m>1$ c) $m\in\ R$ d) None of these

79. If two circles $(x-1)^2+(y-3)^2=r^2$ and $x^2+y^2-8x+2y+8=0$ intersect in two distinct points, then
a) $2<r<8$ b) $r<2$ c) $r=2$ d) $r>2$

80. The equation of the chord joining two points (x_1, y_1) and (x_2, y_2) on the rectangular hyperbola $xy=c^2$ is
a) $\frac{x}{x_1+x_2}+\frac{y}{y_1+y_2}=1$ b) $\frac{x}{x_1-x_2}+\frac{y}{y_1-y_2}=1$
c) $\frac{x}{y_1+y_2}+\frac{y}{x_1+x_2}=1$ d) $\frac{x}{y_1-y_2}+\frac{y}{x_1-x_2}=1$

81. The tangent at a point P on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ passes through the point $(0, -b)$ and the normal at P passes through the point $(2a\sqrt{2}, 0)$ then eccentricity of the hyperbola is
a) 2 b) $\sqrt{2}$ c) 3 d) $\sqrt{3}$

82. The centre of a rectangular hyperbola lies on the line $y=2x$. If one of the asymptotes is $x+y+c=0$, then the other asymptotes is
Hence, equation of other asymptotes is $x+y-3c=0$
a) $x-y-3c=0$ b) $2x-y+c=0$ c) $x-y-c=0$ d) None of these

83. The ellipse $4x^2+9y^2=36$ and the hyperbola $a^2x^2-y^2=4$ intersects at right angles then the equation of the circle through the points of intersection of two conic is
a) $x^2+y^2=5$ b) $\sqrt{5}(x^2+y^2)-3x-4y=0$
c) $\sqrt{5}(x^2+y^2)+3x+4y=0$ d) $x^2+y^2=25$

84. The slopes of the common tangents of the ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ and the circle $x^2+y^2=3$ are

a) ± 1 b) $\pm\sqrt{2}$ c) $\pm\sqrt{3}$ d) None of these

85. The length of the sides of square which can be made by four perpendicular tangents to the ellipse $\frac{x^2}{7}+\frac{2y^2}{11}=1$ is
a) 10 units b) 8 units c) 6 units d) 5 units

86. Equation of conjugate axis of hyperbola $xy-3y-4x+7=0$ is
a) $y+x=3$ b) $y+x=7$ c) $y-x=3$ d) none of these

87. The equation of a circle which has normals $(x-1)<(y-2)=0$ and a tangent $3x+4y=6$ is
a) $x^2+y^2-2x-4y+4=0$ b) $x^2+y^2-2x-4y+5=0$
c) $x^2+y^2=5$ d) $(x-3)^2+(y-4)^2=5$

88. Suppose the circle having equation $x^2+y^2=3$ intersects the rectangular hyperbola $xy=1$ at the points A, B, C and D. The equation $x^2+y^2-3+\lambda(xy-1)=0, \lambda\in R$, represents
a) A pair of lines through origin for $\lambda=-3$ b) An ellipse through A, B, C and D for $\lambda=-3$
c) a parabola through A, B, C and D for $\lambda=-3$ d) A circle for any $\lambda\in R$

89. Suppose $ax+by+c=0$, where a, b, c are in A.P. be normal to a family of circles. The equation of the circle of the family intersects the circle $x^2+y^2-4x-4y-1=0$ orthogonally is
a) $x^2+y^2-2x+4y-3=0$ b) $x^2+y^2+2x-4y-3=0$
c) $x^2+y^2-2x+4y-5=0$ d) $x^2+y^2-2x-4y+3=0$

90. If the sum of the slopes of the normal from a point P to the hyperbola $xy=c^2$ is equal to $\lambda(\lambda\in R^+)$, then locus of point P is
a) $x^2=\lambda c^2$ b) $y^2=\lambda c^2$ c) $xy=\lambda c^2$ d) None of these

91. The chord PQ of the rectangular hyperbola $xy=a^2$ meets the axis of x at A; C is the midpoint of PQ and 'O' is the origin. Then the ΔACO is
a) Equilateral b) Isosceles c) Right angled d) Right isosceles

92. If the ellipse $\frac{x^2}{a^2-7}+\frac{y^2}{13-5a}=1$ is inscribed in a square of side length $\sqrt{2}a$, then a is equal to
a) $\frac{6}{5}$ b) $(-\infty.-\sqrt{7})\cup(\sqrt{7}, 13/5)$
c) $(-\infty,-\sqrt{7})\cup(13/5,\sqrt{7})$ d) No such a exists

93. Tangents PA and PB drawn to $x^2+y^2=9$ from any arbitrary point $'P'$ on the line $x+y=25$. Locus of midpoint of chord AB is
a) $25(x^2+y^2)=9(x+y)$ b) $25(x^2+y^2)=3(x+y)$
c) $5(x^2+y^2)=3(x+y)$ d) None of these

94. Let P be any point on any directrix of an ellipse. Then chords of contact of point P with respect to the ellipse and its auxiliary circle intersect at
a) Some point on the major axis depending upon the position of point P
b) Midpoint of the line segment joining the centre to the corresponding focus
c) Corresponding focus
d) None of these

95. Length of the shortest normal chord of the parabola $y^2=4ax$ is
a) $2a\sqrt{27}$ b) $9a$ c) $a\sqrt{54}$ d) None of these

96. The angle between a pair of tangents drawn from a point P to the circle $x^2+y^2+4x-6y+9\sin^2\alpha+13\cos^2\alpha=0$ is 2α. Then equation of the locus of the point P is
a) $x^2+y^2+4x-6y+4=0$ b) $x^2+y^2+4x-6y-9=0$
c) $x^2+y^2+4x-6y-4=0$ d) $x^2+y^2+4x-6y+9=0$

97. If a tangent of slope 2 of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is normal to the circle $x^2+y^2+4x+1=0$, then the maximum value of ab is
a) 4 b) 2 c) 1 d) None of these

98. The co-ordinates of two points P and Q are (x_1, y_1) and (x_2, y_2) and O is the origin. If circles be described on OP and OQ as diameters then length of their common chord is

a) $\frac{|x_1 y_2 + x_2 y_1|}{PQ}$ b) $\frac{|x_1 y_2 - x_2 y_1|}{PQ}$ c) $\frac{|x_1 x_2 + y_1 y_2|}{PQ}$ d) $\frac{|x_1 x_2 + y_2 y_2|}{PQ}$

99. If $P = (x, y), F_1 = (3, 0), F_2 = (-3, 0)$ and $16x^2 + 25y^2 = 400$, then $PF_1 + PF_2$ equals
a) 8 b) 6 c) 10 d) 12

100. The area bounded by the circles $x^2 + y^2 = 1, x^2 + y^2 = 4$ and the pair of lines $\sqrt{3}(x^2 + y^2) = 4xy$, is equal to
a) $\frac{\pi}{2}$ b) $\frac{5\pi}{2}$ c) 3π d) $\frac{\pi}{4}$

101. An ellipse having foci at $(3, 3)$ and $(-4, 4)$ and passing through the origin has eccentricity equal to
a) $\frac{3}{7}$ b) $\frac{2}{7}$ c) $\frac{5}{7}$ d) $\frac{3}{5}$

102. The auxiliary circle of a family of ellipse passes through origin and makes intercept of 8 and 6 units on the x-axis and the y-axis, respectively. If eccentricity of all such family of ellipse is $\frac{1}{2}$, then locus of the focus will be
a) $\frac{x^2}{16} + \frac{y^2}{9} = 25$ b) $4x^2 + 4y^2 - 32x - 24y + 75 = 0$
c) $\frac{x^2}{16} + \frac{y^2}{9} = 25$ d) None of these

103. Consider a circle with its centre lying on the focus of the parabola $y^2 = 2px$ such that it touches the directrix of the parabola. Then a point of intersection of the circle and parabola is
a) $\left(\frac{p}{2}, p\right)$ or $\left(\frac{p}{2}, -p\right)$ b) $\left(\frac{p}{2}, -\frac{p}{2}\right)$ c) $\left(-\frac{p}{2}, p\right)$ d) $\left(-\frac{p}{2}, -\frac{p}{2}\right)$

104. If the tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ make angles α and β with the major axis such that $\tan\alpha + \tan\beta = \lambda$, then the locus of their point of intersection is
a) $x^2 + y^2 = a^2$ b) $x^2 + y^2 = b^2$ c) $x^2 - a^2 = 2\lambda\, xy$ d) $\lambda(x^2 - a^2) = 2xy$

105. Consider a square $ABCD$ of side length 1. Let P be the set of all segments of length 1 with end points on adjacent sides of square $ABCD$. The midpoints of segments in P enclose a region with area A, the value of A is
a) $\frac{\pi}{4}$ b) $1 - \frac{\pi}{4}$ c) $4 - \frac{\pi}{4}$ d) None of these

106. A line is drawn form $A(-2, 0)$ to intersect the curve $y^2 = 4x$ in P and Q in the first quadrant such that $\frac{1}{AP} + \frac{1}{AQ} < \frac{1}{4}$, then slope of the line is always
a) $> \sqrt{3}$ b) $< \frac{1}{\sqrt{3}}$ c) $> \sqrt{2}$ d) $> \frac{1}{\sqrt{3}}$

107. The circle $x^2 + y^2 + 2\lambda x = 0, \lambda \in R$, touches the parabola $y^2 = 4x$ externally. Then
a) $\lambda > 0$ b) $\lambda < 0$ c) $\lambda > 1$ d) None of these

108. The normal at a variable point P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, of eccentricity e meets the axes of the ellipse at Q and R, then the locus of midpoint of QR is a conic with an eccentricity e' such that
a) e' is independent of e b) $e' = 1$ c) $e' = e$ d) $e' = 1/e$

109. The locus of the point which is such that the chord of contact of tangents drawn from it to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ forms a triangle of constant area with the coordinate axes is
a) A straight line b) A hyperbola c) An ellipse d) A circle

110. The line $x = t^2$ meets the ellipse $x^2 + \frac{y^2}{9} = 1$ in the real and distinct points if and only if
a) $|t| < 2$ b) $|t| < 1$ c) $|t| < 1$ d) None of these

111. The equation of the circle passing through (1,1) and the points of intersection of $x^2 + y^2 + 13x - 3y = 0$ and $2x^2 + 2y^2 + 4x - 7y - 25 = 0$ is
a) $4x^2 + 4y^2 - 30x - 10y - 25 = 0$ b) $4x^2 + 4y^2 + 30x - 13y - 25 = 0$
c) $4x^2 + 4y^2 - 17x - 10y + 25 = 0$ d) None of these

112. The circle $x^2 + y^2 = 5$ meets the parabola $y^2 = 4x$ at P and Q. Then the length PQ is equal to

a) 2 b) $2\sqrt{2}$ c) 4 d) None of these

113. The equation $\frac{x^2}{1-r} - \frac{y^2}{1+r} = 1, r > 1$ represents

a) A ellipse b) A hyperbola c) A circle d) None of these

114. Locus of point whose chord of contact with respect to the circle $x^2 + y^2 = 4$ is a tangent to the hyperbola $xy = 1$ is a/an

a) Ellipse b) Circle c) Hyperbola d) Parabola

115. S_1, S_2 foci of an ellipse of major axis of length 10 units and P is any point on the ellipse such that perimeter of triangle PS_1S_2 is 15. Then eccentricity of the ellipse is

a) 0.5 b) 0.25 c) 0.28 d) 0.75

116. The vertex of a parabola is the point (a, b) and latus rectum in of length l. If the axis of the parabola is along the positive direction of y-axis, then its equation is

a) $(x + a)^2 = \frac{l}{2}(2y - 2b)$ b) $(x - a)^2 = \frac{l}{2}(2y - 2b)$

c) $(x + a)^2 = \frac{l}{4}(2y - 2b)$ d) $(x - a)^2 = \frac{l}{8}(2y - 2b)$

117. The largest value of a for which the circle $x^2 + y^2 = a^2$ falls totally in the interior of the parabola $y^2 = 4(x + 4)$ is

a) $4\sqrt{3}$ b) 4 c) $4\frac{\sqrt{6}}{7}$ d) $2\sqrt{3}$

118. Radius of the tangent circle that can be dawn to pass through the point (0, 7), (0, 6) and touching the x-axis is

a) $\frac{5}{2}$ b) $\frac{3}{2}$ c) $\frac{7}{2}$ d) $\frac{9}{2}$

119. The locus of the midpoints of a chord of the circle $x^2 + y^2 = 4$, which subtends a right angle at the origin, is

a) $x + y = 2$ b) $x^2 + y^2 = 1$ c) $x^2 + y^2 = 2$ d) $x + y = 1$

120. Any circle through the point of intersection of the lines $x + \sqrt{3}y = 1$ and $\sqrt{3}\,x - y = 2$ if intersects these lines at points P and Q, then the angle subtended by the are PQ at its centre is

a) 180° b) 90°

c) 120° d) Depends on centre and radius

121. $f(x, y) = x^2 + y^2 + 2ax + 2by + c = 0$ represents a circle. If $f(x, 0) = 0$ has equal roots, each being 2 and $f(0, y) = 0$ has 2 and 3 as it's roots, then centre of circle is

a) $(2, 5/2)$ b) Data are not sufficient

c) $(-2, -5/2)$ d) Data are inconsistent

122. Minimum radius of circle which is orthogonal with both the circles $x^2 + y^2 - 12x + 35 = 0$ and $x^2 + y^2 + 4x + 3 = 0$ is

a) 4 b) 3 c) $\sqrt{15}$ d) 1

123. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord to the circle with centre (2, 1), then the radius of the circle is

a) $\sqrt{3}$ b) $\sqrt{2}$ c) 3 d) 2

124. Let any double ordinate PNP' of the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$ be produced on both sides to meet the asymptotes in Q and Q', then $PQ \cdot P'Q$ is equal to

a) 25

b) 16

c) 41

d) None of these

125. If (0,6)and (0,3) are respectively the vertex and focus of a parabola, then its equation is

a) $x^2 + 12y = 72$ b) $x^2 - 12y = 72$ c) $y^2 - 12x = 72$ d) $y^2 + 12x = 72$

126. P is a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, N is the foot of the perpendicular from P on the transverse axis.

The tangent to the hyperbola at P meets the transverse axis at T. If O is the centre of the hyperbola, the OT. ON is equal to

a) e^2 b) a^2 c) b^2 d) $\frac{b^2}{a^2}$

127. If the locus of middle of point of contact of tangent drawn to the parabola $y^2 = 8x$ and foot of perpendicular drawn from its focus to the tangents is a conic then length of latus rectum of this conic is

a) $\frac{9}{4}$ b) 9 c) 18 d) $\frac{9}{2}$

128. Consider the parabola $y^2 = 4x$. $A \equiv (4, -4)$ and $B \equiv (9, 6)$ be two fixed points on the parabola. Let 'C' be a moving point on the parabola between A and B such that the area of the triangle ABC is maximum, then coordiante of 'C'

a) $\left(\frac{1}{4}, 1\right)$ b) $(4, 4)$ c) $(3, 2\sqrt{3})$ d) $(3, -2\sqrt{3})$

129. If O is the origin and OP, OQ are the tangents from the origin to the circle $x^2 + y^2 - 6x + 4y + 8 = 0$, then circumcenter of the triangle OPQ is

a) $(3, -2)$ b) $\left(\frac{3}{2}, -1\right)$ c) $\left(\frac{3}{4}, -\frac{1}{2}\right)$ d) $\left(-\frac{3}{2}, 1\right)$

130. A straight line moves such that the algebraic sum of the perpendicular drawn to it from two fixed points is equal to $2k$. Then , the straight line always touches a fixed circle of radius

a) $2k$ b) $k/2$ c) k d) None of these

131. The exhaustive set of values of α^2 such that there exists a tangent to the ellipse $x^2 + \alpha^2 y^2 = \alpha^2$ such that the portion of the tangent intercepted by the hyperbola $\alpha^2 x^2 - y^2 = 1$ subtends a right angle at the centre of the curves is

a) $\left[\frac{\sqrt{5}+1}{2}, 2\right]$ b) $(1, 2]$

c) $\left[\frac{\sqrt{5}-1}{2}, 1\right)$ d) $\left[\frac{\sqrt{5}-1}{2}, 1\right) \cup \left(1, \frac{\sqrt{5}+1}{2}\right]$

132. If $P(x_1, y_1), Q(x_2, y_2), R(x_3, y_3)$ and $S(x_4, y_4)$ are four con-cyclic points on the rectangular hyperbola $xy = c^2$, then coordinates of the orthocentre of the triangle PQR is

a) $(x_4, -y_4)$ b) (x_4, y_4) c) $(-x_4, -y_4)$ d) $(-x_4, y_4)$

133. Angle subtended by common tangents of two ellipses $4(x-4)^2 + 25y^2 = 100$ and $4(x+1)^2 = 4$ at origin is

a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{2}$

134. From a point $(\sin\theta, \cos\theta)$ if three normals can be drawn to the parabola $y^2 = 4ax$ then the value of 'a' is

a) $\left(\frac{1}{2}, 1\right)$ b) $\left[\frac{1}{2} 0\right)$ c) $\left[\frac{1}{2}, 1\right]$ d) $\left(-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right)$

135. The tangent at a point P on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meets one of the directrix in F. If PF subtends an angle θ at the corresponding focus, then θ equals

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\frac{3\pi}{4}$ d) π

136. Let $y = f(x)$ be a parabola, having its axis parallel to y–axis, which is touched by the line $y = x$ at $x = 1$, then

a) $2f(0) = 1 - f'(0)$ b) $f(0) + f'(0) + f''(0) = 1$

c) $f'(1) = 1$ d) $f'(0) = f'(1)$

137. The curve described parametrically by $x = t^2 + t + 1, y = t^2 - t + 1$ represents

a) A pair of straight lines b) An ellipse

c) A parabola d) A hyperbola

138. Locus of the feet of the perpendiculars drawn from either focus on a variable tangent to the hyperbola $16y^2 - 9x^2 = 1$ is

a) $x^2 + y^2 = 9$ b) $x^2 + y^2 = \frac{1}{9}$ c) $x^2 + y^2 = \frac{7}{144}$ d) $x^2 + y^2 = \frac{1}{16}$

139. A line of fixed length $a + b$ moves so that its ends are alwayes on two fixed perpendicular straight lines, then the locus of a point, which divides this line into portions of lenghts a and b is a/an

a) Ellipse b) Parabola c) Straight line d) None of these

140. A variable chord of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $(b > a)$ subtends a right angle at the centre of the hyperbola, if this chord touches

a) A fixed circle concentric with the hyperbola

b) A fixed ellipse concentric with the hyperbola

c) A fixed hyperbola concentric with the hyperbola

d) A fixed parabola having vertex at (0, 0)

141. The number of common chords of the parabolas $x = y^2 - 6y + 11$ and $y = x^2 - 6x + 1$ are

a) 1 b) 2 c) 4 d) 6

142. Equation of the chord of contact of pair of tangents drawn to the ellipse $4x^2 + 9y^2 = 36$ from the point (m, n) where $m.n = m + n, m, n$being non-zero positive integers is

a) $2x + 9y = 18$ b) $2x + 2y = 1$ c) $4x + 9y = 18$ d) None of these

143. $ABCD$ is a square of unit area. A circle is tangent to two sides of $ABCD$ and passes through exactly one of its vertices. The radius of the circle is

a) $2 - \sqrt{2}$ b) $\sqrt{2} - 1$ c) $\frac{1}{2}$ d) $\frac{1}{\sqrt{2}}$

144. Let LL' be the latus rectum through the focus of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and A' be the farther vertex. If $\Delta A'LL'$ is equilateral, then the eccentricity of the hyperbola is (axes are coordinate axes)

a) $\sqrt{3}$ b) $\sqrt{3} + 1$ c) $\frac{\sqrt{3} + 1}{\sqrt{2}}$ d) $\frac{\sqrt{3} + 1}{\sqrt{3}}$

145. The locus of the point of intersection of the lines $\sqrt{3}x - y - 4\sqrt{3}t = 0$ and $\sqrt{3}tx + ty - 4\sqrt{3} = 0$ (where t is a parameter) is a hyperbola whose eccentricity is

a) $\sqrt{3}$ b) 2 c) $\frac{2}{\sqrt{3}}$ d) $\frac{4}{3}$

146. From any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ tangents are drawn to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2$. Then area cut-off by the chord of contact on the asymptotes is equal to

a) $\frac{a}{2}$ b) ab c) $2ab$ d) $4ab$

147. If the angle between tangents drawn to $x^2 + y^2 + 2gx + 2fy + c = 0$ form (0,0) is $\pi/2$, then

a) $g^2 + f^2 = 3c$ b) $g^2 + f^2 = 2c$ c) $g^2 + f^2 = 5c$ d) $g^2 + f^2 = 4c$

148. The angle at which the circles $(x - 1)^2 + y^2 = 10$ and $x^2 + (y - 2)^2 = 5$ intersect is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

149. Angle between the tangents to the curve $y = x^2 - 5x + 6$ at the point (2, 0) and (3,0) is

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{4}$

150. The equation of the transverse and conjugate axes of a hyperbola are respectively $x + 2y - 3 = 0, 2x - y + 4 = 0$, and their respective lengths are $\sqrt{2}$ and $\frac{2}{\sqrt{3}}$. The equation of the hyperbola is

a) $\frac{2}{5}(x + 2y - 3)^2 - \frac{3}{5}(2x - y + 4)^2 = 1$ b) $\frac{2}{5}(2x - y + 4)^2 - \frac{3}{5}(x + 2y - 3)^2 = 1$

c) $2(2x - y + 4)^2 - 3(x + 2y - 3)^2 = 1$ d) $2(x + 2y - 3)^2 - 3(2x - y + 4)^2 = 1$

151. Which one of the following equations represented parametrically equation to a parabolic curve?

a) $x = 3\cos t; y = 4\sin t$ b) $x^2 - 2 = 2\cos t; y = 4\cos^2\frac{t}{2}$

c) $\sqrt{x} = \tan t;\ \sqrt{y} = \sec t$ d) $x = \sqrt{1 - \sin t}\,; y = \sin\frac{t}{2} + \cos\frac{t}{2}$

152. The latus rectum of the hyperbola $9x^2 - 16y^2 - 18x - 32y - 151 = 0$ is

a) $\frac{9}{4}$ b) 9 c) $\frac{3}{2}$ d) $\frac{9}{2}$

153. Parabolas $y^2 = 4a(x - c_1)$ and $x^2 = 4a(y - c_2)$, where $c_1$and c_2 are variable, are such that they touch each other. Locus of their point of contact is

a) $xy = 2a^2$ b) $xy = 4a^2$ c) $xy = a^2$ d) None of these

154. Let d_1 and d_2 be the lengths of the perpendicular drawn from foci S and S' of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ to the tangent at any point Pon the ellipse. Then, $SP: S'P =$

a) $d_1: d_2$ b) $d_2: d_1$ c) $d_1^2: d_2^2$ d) $\sqrt{d_1}/\sqrt{d_2}$

155. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$ meets its auxiliary circle at the point M. Then, the area of the triangle with vertices at A, M and the origin O is

a) $\frac{31}{10}$ b) $\frac{29}{10}$ c) $\frac{21}{10}$ d) $\frac{27}{10}$

156. If the tangent at the point $P(2,4)$ to the parabola $y^2 = 8x$ meets the parabola $y^2 = 8x + 5$ at Q and R then the midpoint of QR is

a) $(4, 2)$ b) $(2, 4)$ c) $(7, 9)$ d) None of these

157. A circle of constant radius $'a'$ passes through origin $'O'$ and cuts the axes of co-ordinates in points P and Q, then the equation of the locus of the foot of perpendicular from O to PQ is

a) $(x^2 + y^2)\left(\frac{1}{x^2} + \frac{1}{y^2}\right) = 4a^2$ b) $(x^2 + y^2)^2\left(\frac{1}{x^2} + \frac{1}{y^2}\right) = a^2$

c) $(x^2 + y^2)^2\left(\frac{1}{x^2} + \frac{1}{y^2}\right) = 4a^2$ d) $(x^2 + y^2)\left(\frac{1}{x^2} + \frac{1}{y^2}\right) = a^2$

158. The condition that the chord $x \cos a + y \sin \alpha - p = 0$ of $x^2 + y^2 - a^2 = 0$ may subtend a right angle at the centre of the circle is

a) $a^2 = 2p^2$ b) $p^2 = 2a^2$ c) $a = 2p$ d) $p = 2a$

159. Six points $(x_i, y_i), i = 1, 2, \dots, 6$ are taken on the circle $x^2 + y^2 = 4$ such that $\sum_{i=1}^{6} x_i = 8$ and $\sum_{i=1}^{6} y_i = 4$. The line segment joining orthocenter of a triangle made by any three points and the centroid of the triangle made by other three points passes through a fixed points (h, k), then $h + k$ is

a) 1 b) 2 c) 3 d) 4

160. An equation for the line that passes through $(10, -1)$ and is perpendicular to $y = \frac{x^2}{4} - 2$ is

a) $4x + y = 39$ b) $2x + y = 19$ c) $x + y = 9$ d) $x + 2y = 8$

161. Two mutually perpendicular tangent of the parabola $y^2 = 4ax$ meet the axis in P_1 and P_2. If S is the focus of the parabola, then $\frac{1}{(SP_1)} + \frac{1}{(SP_2)}$ is equal to

a) $\frac{4}{a}$ b) $\frac{2}{a}$ c) $\frac{1}{a}$ d) $\frac{1}{4a}$

162. If the vertex of a hyperbola bisects the distance between its centre and the corresponding focus, then ratio of square of its conjugate axis to the square of its transverse axis is

a) 2 b) 4 c) 6 d) 3

163. If the line $ax + by = 2$ is a normal to the circle $x^2 + y^2 - 4x - 4y = 0$ and a tangent to the circle $x^2 + y^2 = 1$, then

a) $a = \frac{1}{2}, b = \frac{1}{2}$ b) $a = \frac{1 + \sqrt{7}}{2}, b = \frac{1 - \sqrt{7}}{2}$

c) $a = \frac{1}{4}, b = \frac{3}{4}$ d) $a = 1, b = \sqrt{3}$

164. From point $P(8,27)$ tangent PQ and PR are drawn to the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$. Then the angle subtended by QR at origin is

a) $\tan^{-1}\frac{\sqrt{6}}{65}$ b) $\tan^{-1}\frac{4\sqrt{6}}{65}$ c) $\tan^{-1}\frac{8\sqrt{2}}{65}$ d) $\tan^{-1}\frac{48\sqrt{6}}{455}$

165. Two straight lines are perpendicular to each other. One of them touches the parabola $y^2 = 4a(x + a)$ and the other touches $y^2 = 4b(x + b)$. Their point of intersection lies on the line

a) $x - a + b = 0$ b) $x + a - b = 0$ c) $x + a + b = 0$ d) $x - a - b = 0$

166. P, Q, R are the feet of the normals drawn to a parabola $(y - 3)^2 = 8(x - 2)$. A circle cuts the above parabola in points P, Q, R and S. Then this circle always passes through the point

a) $(2, 3)$ b) $(3, 2)$ c) $(0, 3)$ d) $(2, 0)$

167. Let 'C' be a curve which is locus of the point of the intersection of lines $x = 2 + m$ and $my = 4 - m$. A circle $s \equiv (x - 2)^2 + (y + 1)^2 = 25$ intersects the curve C at four points P, Q, R and S. If O is centre of the curve 'C' then $OP^2 + OQ^2 + OR^2 + OS^2$ is

a) 50 b) 100 c) 25 d) $\frac{25}{2}$

168. The locus of the centre of the circles such that the point $(2, 3)$ is the midpoint of the chord $5x + 2y = 16$ is

a) $2x - 5y + 11 = 0$ b) $2x + 5y - 11 = 0$ c) $2x + 5y + 11 = 0$ d) None

169. If the segment intercepted by the parabola $y = 4ax$ with the line $lx + my + n = 0$ subtends a right angle at the vertex, then

a) $4al + n = 0$ b) $4al + 4am + n = 0$

c) $4am + n = 0$ d) $al + n = 0$

170. If S and S' are the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{3} = 1$, and P is any point on it then range of values of $SP.S'P$ is

a) $9 \le f(\theta) \le 16$ b) $9 \le f(\theta) \le 25$ c) $16 \le f(\theta) \le 25$ d) $1 \le f(\theta) \le 16$

171. An equilateral triangle SAB is inscribed in the parabola $y^2 = 4ax$ having its focus at 'S'. If chord AB lies towards the left of S, then side length of this triangel is

a) $2a(2 - \sqrt{3})$ b) $4a(2 - \sqrt{3})$ c) $a(2 - \sqrt{3})$ d) $8a(2 - \sqrt{3})$

172. If $\alpha - \beta =$ constant, then the locus of the point of intersection of tangents at $P(a \cos \alpha, b \sin \alpha)$ and $Q(a \cos \beta, b \sin \beta)$ to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

a) A circle b) A straight line c) An ellipse d) A parabola

173. A straight line I_1 with equation $x - 2y + 10 = 0$ meets the circle with equation $x^2 + y^2 = 100$ at B in the first quadrant. A line through B, perpendicular to l_1 cuts y-axis at $P(0, t)$. The value of 't' is

a) 12 b) 15 c) 20 d) 25

174. If the equation of any two diagonals of a regular pentagon belongs to family of lines $(1 + 2\lambda)y - (2 + \lambda)x + 1 - \lambda = 0$ and their lengths are $\sin 36°$, then locus of centre of circle circumscribing the given pentagon (the triangles formed by these diagonals with sides of pentagon have no side common) is

a) $x^2 + y^2 - 2x - 2y + 1 + \sin^2 72° = 0$ b) $x^2 + y^2 - 2x - 2y + \cos^2 72° = 0$

c) $x^2 + y^2 - 2x - 2y + 1 + \cos^2 72° = 0$ d) $x^2 + y^2 - 2x - 2y + \sin^2 72° = 0$

175. If the line $x - 1 = 0$ is the directrix of the parabola $y^2 - kx + 8 = 0$, then one of the values of k is

a) $\frac{1}{8}$ b) 8 c) 4 d) $\frac{1}{4}$

176. The length of the major axis of the ellipse $(5x - 10)^2 + (5y + 15)^2 = \frac{(3x-4y+7)^2}{4}$ is

a) 10 b) $\frac{20}{3}$ c) $\frac{20}{7}$ d) 4

177. If the circumference of the circle $x^2 + y^2 + 8x + 8y - b = 0$ is bisected by the circle $x^2 + y^2 - 2x + 4y + a = 0$, then $a + b$ equals to

a) 50 b) 56 c) -56 d) -34

178. The line $x - y = 1$ intersects the parabola $y^2 = 4x$ at A and B. Normals at A and B intersect at C. If D is the point at which line CD is normal to the parabola, then coordinates of D are

a) $(4, -4)$ b) $(4, 4)$ c) $(-4, -4)$ d) None of these

179. The locus of a point which moves such that the sum of the squares of its distance from three vertices of a triangle is constant is a/an

a) Circle b) Straight line c) Ellipse d) None of these

180. A circle with centre (a, b) passes through the origin. The equation of the tangent to the circle at the origin

is

a) $ax - by = 0$ b) $ax + by = 0$ c) $bx - ay = 0$ d) $bx + ay = 0$

181. If r_1 and r_2 are the radii of smallest and largest circles which passes through (5, 6) and touches the circle $(x-2)^2 + y^2 = 4$, then $r_1 r_2$ is

a) $\frac{4}{41}$ b) $\frac{41}{4}$ c) $\frac{5}{41}$ d) $\frac{41}{6}$

182. The common chord of the circle $x^2 + y^2 + 6x + 8y - 7 = 0$ and a circle passing through the origin, and touching the line $y = x$, always passes through the point

a) $(-1/2, 1/2)$ b) $(1, 1)$ c) $(1/2, 1/2)$ d) None of these

183. The distance from the centre of the circle $x^2 + y^2 - 2x$ to the common chord of the circles $x^2 + y^2 + 5x - 8y + 1 = 0$ and $x^2 + y^2 - 3x + 7y - 25 = 0$ is

a) 2 b) 4 c) $\frac{34}{13}$ d) $\frac{26}{17}$

184. If two normals to a parabola $y^2 = 4ax$ intersect at right angles, then the chord joining their feet passes through a fixed point whose co-ordinates are

a) $(-2a, 0)$ b) $(a, 0)$ c) $(2a, 0)$ d) None

185. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is (axes are coordinate axes)

a) $\frac{4}{3}$ b) $\frac{4}{\sqrt{3}}$ c) $\frac{2}{\sqrt{3}}$ d) none of these

186. The point of intersection of the tangents at the point P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and its corresponding point Q on the auxiliary circle meet on the line

a) $x = a/e$ b) $x = 0$ c) $y = 0$ d) None of these

187. If a line $y = 3x + 1$ cuts the parabola $x^2 - 4x - 4y + 20 = 0$ at A and B, then the tangent of the angle subtended by line segment AB at origin is

a) $\frac{8\sqrt{3}}{205}$ b) $\frac{8\sqrt{3}}{209}$ c) $\frac{8\sqrt{3}}{215}$ d) None of these

188. If $C_1: x^2 + y^2 = (3 + 2\sqrt{2})^2$ be a circle and PA and PB are pair of tangents on C_1, where P is any point on the director circle of C_1, then the radius of smallest circle which touch C_1 externally and also the two tangents PA and PB is

a) $2\sqrt{3} - 3$ b) $2\sqrt{2} - 1$ c) $2\sqrt{2} - 1$ d) 1

189. The equation of a line inclined at an angle $\pi/4$ to the X-axis, such that the two circles $x^2 + y^2 = 4$, $x^2 + y^2 - 10x - 14y + 65 = 0$ intercept equal lengths on it, is

a) $2x - 2y - 3 = 0$ b) $2x - 2y + 3 = 0$ c) $x - y + 6 = 0$ d) $x - y - 6 = 0$

190. The number of possible tangents which can be drawn to the curve $4x^2 - 9y^2 = 36$, which are perpendicular to the straight line $5x + 2y - 10 = 0$ is

a) Zero b) 1 c) 2 d) 4

191. A circle passes through the points $A(1, 0)$, $B(5, 0)$ and touches the y-axis at $C(0, h)$. If $\angle ACB$ is maximum then

a) $h = 3\sqrt{5}$ b) $h = 2\sqrt{5}$ c) $h = \sqrt{5}$ d) $h = 2\sqrt{10}$

192. The equation of the circle passing through the point of intersection of the circles $x^2 + y^2 - 4x - 2y = 8$ and $x^2 + y^2 - 2x - 4y = 8$ and the point $(-1, 4)$ is

a) $x^2 + y^2 + 4x + 4y - 8 = 0$ b) $x^2 + y^2 - 3x + 4y + 8 = 0$

c) $x^2 + y^2 + x + y - 8 = 0$ d) $x^2 + y^2 - 3x - 3y - 8 = 0$

193. The length of the latus rectum of the parabola whose focus is $\left(\frac{u^2}{2g}\sin 2\alpha, -\frac{u^2}{2g}\cos 2\alpha\right)$ and directrix is $y = \frac{u^2}{2g}$ is

a) $\frac{u^2}{g}\cos^2\alpha$ b) $\frac{u^2}{g}\cos 2\alpha$ c) $\frac{2u^2}{g}\cos 2\alpha$ d) $\frac{2u^2}{g}\cos^2\alpha$

194. If the line $y - \sqrt{3}x + 3 = 0$ cuts the parabola $y^2 = x + 2$ at P and Q, then $AP \cdot AQ$ is equal to [where

$A \equiv(\sqrt{3}, 0)]$

a) $\frac{2(\sqrt{3}+2)}{3}$ b) $\frac{4\sqrt{3}}{2}$ c) $\frac{4(2-\sqrt{2})}{3}$ d) $\frac{4(\sqrt{3}+2)}{3}$

195. If the tangents are drawn to the ellipse $x^2 + 2y^2 = 2$, then the locus of the mid point of the intercept made by the tangents between the coordinate axes is

a) $\frac{1}{2x^2}+\frac{1}{4y^2}=1$ b) $\frac{1}{4x^2}+\frac{1}{2y^2}=1$ c) $\frac{x^2}{2}+\frac{y^2}{4}=1$ d) $\frac{x^2}{4}+\frac{y^2}{2}=1$

196. A circle with radius $|a|$ and centre on y-axis slides along it and a variable lines through $(a, 0)$ cuts the circle at points P and Q. The region in which the point of intersection of tangents to the circle at point P and Q lies is represented by

a) $y^2 \geq 4(ax-a^2)$ b) $y^2 \leq 4(ax-a^2)$ c) $y \geq 4(ax-a^2)$ d) $y \leq 4(ax-a^2)$

197. The range of values of $\lambda(\lambda > 0)$ such that the angle θ between the pair of tangents drawn from $(\lambda, 0)$ to the circle $x^2+y^2=4$ lies in $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ is

a) $\left(\frac{4}{\sqrt{3}}, 2\sqrt{2}\right)$ b) $(0, \sqrt{2})$ c) $(1, 2)$ d) None of these

198. The eccentric angle of a point on the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$ at a distance of $\frac{5}{4}$ units from the focus on the positive x –axis, is

a) $\cos^{-1}\left(\frac{3}{4}\right)$ b) $\pi-\cos^{-1}\left(\frac{3}{4}\right)$ c) $\pi+\cos^{-1}\left(\frac{3}{4}\right)$ d) None of these

199. The number of common tangent (s) to the circles $x^2+y^2+2x+8y-23=0$ and $x^2+y^2-4x-10y-19=0$ is

a) 1 b) 2 c) 3 d) 4

200. If tangents PQ and PR are drawn from a point on the circle $x^2+y^2=25$ to the ellipse $\frac{x^2}{4}+\frac{y^2}{b^2}=1, (b<4)$, so that the fourth vertex S of parallelogram $PQSR$ lies on the circumcircle of triangle PQR, then eccentricity of the ellipse is

a) $\frac{\sqrt{5}}{4}$ b) $\frac{\sqrt{7}}{3}$ c) $\frac{\sqrt{7}}{4}$ d) $\frac{\sqrt{5}}{3}$

201. If the ellipse $\frac{x^2}{4}+y^2=1$ meets the ellipse $x^2+\frac{y^2}{a^2}=1$ in four distinct points and $a=b^2-5b+7$, then b does not lie in

a) $[4, 5]$ b) $(-\infty, 2) \cup (3, \infty)$ c) $(-\infty, 0)$ d) $[2, 3]$

202. A circle is inscribed into a rhombous $ABCD$ with one angle 60°. The distance from the centre of the circle to the nearest vertex is equal to 1. If P is any point of the circle, then $|PA|^2+|PB|^2+|PC|^2+|PD|^2$ is equal to

a) 12 b) 11 c) 9 d) None of these

203. A circle of radius unity is centred at origin. Two particles start moving at the same time from the point (1, 0) and move around the circle in opposite direction. One of the particle moves counterclockwise with constant speed v and the other moves clockwise with constant speed $3v$. After leaving (1, 0), the two particles meet first at a point P, and continue until they meet next at point Q. The coordinates of the point Q is

a) $(1, 0)$ b) $(0, 1)$ c) $(0, -1)$ d) $(-1, 0)$

204. The mirror image of the parabola $y^2=4x$ in the tangent ot the parabola at the point (1, 2) is

a) $(x-1)^2=4(y+1)$ b) $(x+1)^2=4(y+1)$ c) $(x+1)^2=4(y-1)$ d) $(x-1)^2=4(y-1)$

205. Two circles of radii 'a' and 'b' touching each other externally, are inscribed in the area bounded by $y=\sqrt{1-x^2}$ and the x-axis. If $b=\frac{1}{2}$, then a is equal to

a) $\frac{1}{4}$ b) $\frac{1}{8}$ c) $\frac{1}{2}$ d) $\frac{1}{\sqrt{2}}$

206. The locus of the centre of the circle touching the line $2x-y=1$ at (1, 1) is

a) $x+3y=2$ b) $x+2y=0$ c) $x+y=2$ d) None of these

207. Normals at two point (x_1, y_1) and (x_2, y_2) of parabola $y^2 = 4x$ meet again on the parabola where $x_1 + x_2 = 4$, then $|y_1 + y_2|$ equals to

a) $\sqrt{2}$ b) $2\sqrt{2}$ c) $4\sqrt{2}$ d) None of these

208. There are two circles whose equations are $x^2 + y^2 = 9$ and $x^2 + y^2 - 8x - 6y + n^2 = 0, n \in Z$. If the two circles have exactly two common tangents then the number of possible values of n is

a) 2 b) 8 c) 9 d) None of these

209. If maximum distance of any point on the ellipse $x^2 + 2y^2 + 2xy = 1$ from its centre be r, then r is equal to

a) $3 + \sqrt{3}$ b) $2 + \sqrt{2}$ c) $\dfrac{\sqrt{2}}{\sqrt{3 - \sqrt{5}}}$ d) $\sqrt{2 - \sqrt{2}}$

210. Let $P(a \sec\theta, b \tan\theta)$ and $Q(a \sec\phi, b \tan\phi)$, where $\theta + \phi = \frac{\pi}{2}$, be two points on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If (h, k) is the point of intersection of the normals at P and Q, then k is equal to

a) $\dfrac{a^2 + b^2}{a}$ b) $-\left(\dfrac{a^2 + b^2}{a}\right)$ c) $\dfrac{a^2 + b^2}{b}$ d) $-\left(\dfrac{a^2 + b^2}{b}\right)$

211. If $x + y = k$ is normal to $y^2 = 12x$, then k is

a) 3 b) 9 c) -9 d) -3

212. From a arbitrary point 'P' on the circle $x^2 + y^2 = 9$, tangents are drawn to the circle $x^2 + y^2 = 1$, which meet $x^2 + y^2 = 9$ at A and B. Locus of the point of intersection of tangents at A and B to the circle $x^2 + y^2 = 9$ is

a) $x^2 + y^2 = \left(\frac{27}{7}\right)^2$ b) $x^2 - y^2 = \left(\frac{27}{7}\right)^2$ c) $y^2 - x^2 = \left(\frac{27}{7}\right)^2$ d) None of these

213. Portion of asymptote of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (between centre and the tangent at vertex) in the first quadrants is cut by the line $y + \lambda(x - a) = 0$(λ is a parameter) then

a) $\lambda \in R$ b) $\lambda \in (0, \infty)$ c) $\lambda \in (-\infty, 0)$ d) None of these

214. A region in the $x - y$ plane is bounded by the curve $y = \sqrt{25 - x^2}$ and the line $y = 0$. If the point $(a, a + 1)$ lies in the interior of the region, then

a) $a \in (-4, 3)$ b) $a \in (-\infty, -1) \in (3, \infty)$
c) $a \in (-1, 3)$ d) None of these

215. A circle touches the x-axis and also touches the circle with centre $(0, 3)$ and radius 2. The locus of the centre of the circle is

a) A circle b) An ellipse c) A parabola d) A hyperbola

216. Tangents drawn from the point $P(1, 8)$ to the circle $x^2 + y^2 - 6x - 4y - 11 = 0$ touch the circle at the points $A$and$B$. The equation of the circumcircle of the triangle PAB is

a) $x^2 + y^2 + 4x - 6y + 19 = 0$ b) $x^2 + y^2 - 4x - 10y + 19 = 0$
c) $x^2 + y^2 - 2x + 6y - 29 = 0$ d) $x^2 + y^2 - 6x - 4y + 19 = 0$

217. A parabola is drawn with focus is at one of the foci of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$(where $a > b$) and directrix passing through the other focus and perpendicular to the major axes of the ellipse. If latus rectum of the ellipse and the parabola are same, then the eccentricity of the ellipse is

a) $1 - \dfrac{1}{\sqrt{2}}$ b) $2\sqrt{2} - 2$ c) $\sqrt{2} - 1$ d) None of these

218. Let P be the point $(1, 0)$ and Q a point on the locus $y^2 = 8x$. The locus of the midpoint of PQ is

a) $y^2 + 4x + 2 = 0$ b) $y^2 - 4x + 2 = 0$ c) $x^2 - 4y + 2 = 0$ d) $x^2 + 4y + 2 = 0$

219. If (α, β) is a point on the circle whose centre is on the x-axis and which touches the line $x + y = 0$ at $(2, -2)$, then the greatest values of α is

a) $4 - \sqrt{2}$ b) 6 c) $4 + 2\sqrt{2}$ d) $4 + \sqrt{2}$

220. 't_1' and 't_2' are two points on the parabola $y^2 = 4ax$. If the focal chord joining them coincides with the normal chord, then

a) $t_1(t_1 + t_2) + 2 = 0$ b) $t_1 + t_2 = 0$ c) $t_1 t_2 = -1$ d) None of these

221. AB is a chord of the parabola $y^2 = 4ax$ with vertex A. BC is drawn is drawn perpendicular AB to meeting

the axis at C. The projection of BC on the axis of the parabola is

a) a b) $2a$ c) $4a$ d) $8a$

222. $(-6,0),(0,6)$ and $(-7,7)$ are the vertices of a ΔABC. The incircle of the triangle has the equation

a) $x^2+y^2-9x-9y+36=0$ b) $x^2+y^2+9x-9y+36=0$

c) $x^2+y^2+9x+9y-36=0$ d) $x^2+y^2+18x-18y+36=0$

223. The eccentricity of the conic represented by $x^2-y^2-4x+4y+16=0$ is

a) 1 b) $\sqrt{2}$ c) 2 d) $\frac{1}{2}$

224. Normals AO, AA_1, AA_2 are drawn to parabola $y^2=8x$ from the point $A(h,0)$. If triangle OA_1A_2 is equilateral, then possible values of 'h' is

a) 26 b) 24 c) 28 d) none of these

225. The number of integral values of y for which the chord of the circle $x^2+y^2=125$ passing through the point $P(8,y)$ gets bisected at the point $P(8,y)$ and has integral slope is

a) 8 b) 6 c) 4 d) 2

226. An ellipse has the points $(1,-1)$ and $(2,-1)$ as its foci and $x+y-5=0$ as one of its tangents. Then the point where this line touches the ellipse from origin is

a) $\left(\frac{32}{9},\frac{22}{9}\right)$ b) $\left(\frac{23}{9},\frac{2}{9}\right)$ c) $\left(\frac{34}{9},\frac{11}{9}\right)$ d) None of these

227. If $x=9$ is the chord of contact of the hyperbola $x^2-y^2=9$, then the equation of the corresponding pair of tangent is

a) $9x^2-8y^2+18x-9=0$ b) $9x^2-8y^2-18x+9=0$

c) $9x^2-8y^2-18x-9=0$ d) $9x^2-8y^2+18x+9=0$

228. The triangle PQR is inscribed in the circle $x^2+y^2=25$. If Q and R have co-ordinates $(3,4)$ and $(-4,3)$ respectively, then $\angle QPR$ is equal to

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$

229. If PQR is an equilateral triangle inscribed in the auxiliary circle of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 (a>b)$ and $P'Q'R'$ is corresponding triangle inscribed within the ellipse then centroid of the triangle $P'Q'R$ lies at

a) Centre of ellipse b) Focus of ellipse

c) Between focus and centre on major axis d) None of these

230. A rhombus is inscribed in the region common to the two circles $x^2+y^2-4x-12=0$ and $x^2+y^2+4x-12=0$ with two of its vertices on the line joining the centres of the circles. The area of the rhombus is

a) $8\sqrt{3}$ sq. units b) $4\sqrt{3}$ sq. units c) $6\sqrt{3}$ sq. units d) None

231. If the circle $x^2+y^2+2gx+2fy+c=0$ is touched by $y=x$ at P such that $OP=6\sqrt{2}$, then the value of c is

a) 36 b) 144 c) 72 d) None of these

232. The equation $2x^2+3y^2-8x-18y+35=k$ represents

a) No locus if $k>0$ b) An ellipse if $k>0$

c) A point if $k=0$ d) A hyperbola if $k>0$

233. A light ray gets reflected from the $x=-2$. If the reflected ray touches the circle $x^2+y^2=4$ and point of incident is $(-2,-4)$, then equation of incident ray is

a) $4y+3x+22=0$ b) $3y+4x+20=0$ c) $4y+2x+20=0$ d) $y+x+6=0$

234. P is a point (a,b) in the first quadrant. If the two circles which pass through P and touch both the co-ordinates axes cut at right angles, then

a) $a^2-6ab+b^2=0$ b) $a^2+2ab-b^2=0$

c) $a^2-4ab+b^2=0$ d) $a^2-8ab+b^2=0$

235. If parabola $y^2=\lambda x$ and $25\left[(x-3)^2+(y+2)^2\right]=(3x-4y-2)^2$ are equal, then value of λ is

a) 9 b) 3 c) 7 d) 6

236. The range of values of r for which the point $\left(-5+\frac{r}{\sqrt{2}},-3+\frac{r}{\sqrt{2}}\right)$ is an interior point of the major segment of

the circle $x^2 + y^2 = 16$, cut-off by the line $x + y = 2$, is

a) $(-\infty, 5\sqrt{2})$
b) $(4\sqrt{2} - \sqrt{14}, 5\sqrt{2})$
c) $(4\sqrt{2} - \sqrt{14}, 4\sqrt{2} + \sqrt{14})$
d) None of these

237. The locus of the point which is such that the chord of contact of tangents drawn from it to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ forms a triangle of constant area with the coordinate axes is

a) A straight line
b) A hyperbola
c) An ellipse
d) A circle

238. $(x - 1)(y - 2) = 5$ and $(x - 1)^2 + (y + 2)^2 = r^2$ intersects at four points A, B, C, D and if centroid of ΔABC lies on line $y = 3x - 4$, then locus of D is

a) $y = 3x$
b) $x^2 + y^2 + 3x + 1 = 0$
c) $3y = x + 1$
d) $y = 3x + 1$

239. The straight line $x \cos\theta + y \sin\theta = 2$ will touch the circle $x^2 + y^2 - 2x = 0$, if

a) $\theta = n\pi, n \in I$
b) $A = (2n + 1)\pi, n \in I$
c) $\theta = 2n\pi, n \in I$
d) None of these

240. The set of points on the axis of the parabola $y^2 = 4ax + 8$ from which the three normals to the parabola are all real and different is

a) $\{(k, 0) | k \leq -2\}$
b) $\{(k, 0) | k > -2\}$
c) $\{(0, k) | k > -2\}$
d) None of these

241. Any ordinate MP of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ meets the auxiliary circle at Q, then locus of the point of intersection of normals at $P$and Q to the respective curves is

a) $x^2 + y^2 = 8$
b) $x^2 + y^2 = 34$
c) $x^2 + y^2 = 64$
d) $x^2 + y^2 = 15$

242. The line $2x - y + 1 = 0$ is tangent to the circle at the point (2, 5) and the centre of the circles lies on $x - 2y = 4$. The radius of the circle is

a) $3\sqrt{5}$
b) $5\sqrt{3}$
c) $2\sqrt{5}$
d) $5\sqrt{2}$

243. If the line $x + y = 1$ touches the parabola $y^2 - y + x = 0$, then the coordinates of the point of contact are

a) (1, 1)
b) $\left(\frac{1}{2}, \frac{1}{2}\right)$
c) (0, 1)
d) (1, 0)

244. A pair of tangents are drawn to a unit circle with centre at the origin and these tangents intersect at A enclosing an angle of 60°. The area enclosed by these tangents and the arc of the circle is

a) $\frac{2}{\sqrt{3}} - \frac{\pi}{6}$
b) $\sqrt{3} - \frac{\pi}{3}$
c) $\frac{\pi}{3} - \frac{\sqrt{3}}{6}$
d) $\sqrt{3}\left(1 - \frac{\pi}{6}\right)$

245. Through the vertex O of the parabola $y^2 = 4ax$, two chords OP and OQ are drawn and the circles on OP and OQ as diameters intersect in R. If θ_1, θ_2 and $\emptyset$ are the angles made with axis by the tangents at P and Q on the parabola and by OR, then the value of, $\cot\theta_1 + \cot\theta_2$

a) $-2\tan\emptyset$
b) $-2\tan(\pi - \emptyset)$
c) 0
d) $2\cot\emptyset$

246. Number of points on the ellipse $\frac{x^2}{50} + \frac{y^2}{20} = 1$ from which pair of perpendicular tangents are drawn to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ is

a) 0
b) 2
c) 1
d) 4

247. The equation of the directrix of the parabola $y^2 + 4y + 4x + 2 = 0$ is

a) $x = -1$
b) $x = 1$
c) $x = -\frac{3}{2}$
d) $x = \frac{3}{2}$

248. If normals are drawn from a point $P(h, k)$ to the parabola $y^2 = 4ax$, then the sum of the intercepts which the normals cutoff from the axis of the parabola is

a) $(h + a)$
b) $3(h + a)$
c) $2(h + a)$
d) None of these

249. If $C_1: x^2 + y^2 - 20x + 64 = 0$ and $C_2: x^2 + y^2 + 30x + 144 = 0$. Then the length of the shortest line segment PQ which touches C_1 at P and to C_2 at Q is

a) 20
b) 15
c) 22
d) 27

250. If values of m for which the line $y = mx + 2\sqrt{5}$ touches the hyperbola $16x^2 - 9y^2 = 144$ are roots of the equation $x^2 - (a + b)x - 4 = 0$, then value of $(a + b)$ is equal to

a) 2 b) 4 c) Zero d) None of these

251. Which of the following is independent of α in the hyperbola $\left(0<\alpha<\frac{\pi}{2}\right)\frac{x^2}{\cos^2\alpha}-\frac{y^2}{\sin^2\alpha}=1$?

a) Eccentricity b) Abscissa of foci c) Directrix d) Vertex

252. If the normal to a parabola $y^2=4ax$ at P meets the curve again in Q and if PQ and the normal at Q makes angle α and β, respectively, with the x–axis then tan α(tanα +tanβ) has the value equal to

a) 0 b) -2 c) $-\frac{1}{2}$ d) -1

253. In a triangle ABC, right angled at A, on the leg AC as diameter, a semicircle is described. If a chord joins A with the point of intersection D of the hypotenuse and the semicircle, then the length of AC equals to

a) $\frac{AB.AD}{\sqrt{AB^2+AD^2}}$ b) $\frac{AB.AD}{AB+AD}$ c) $\sqrt{AB.AD}$ d) $\frac{AB.AD}{\sqrt{AB^2-AD^2}}$

254. If a circle of constant radius $3k$ passes through the origin 'O' and meets co-ordinate axes at A and B, then the locus of the centroid of the triangle OAB is

a) $x^2+y^2=(2k)^2$ b) $x^2+y^2=(3k)^2$ c) $x^2+y^2=(4k)^2$ d) $x^2+y^2=(6k)^2$

255. If a and c are the lengths of segments of any focal chord of the parabola $y^2=2bx(b>0)$, then the roots of the equation $ax^2+bx+c=0$ are

a) Real and distinct b) Real and equal c) Imaginary d) None of these

256. The equation of the line passing through the centre and bisecting the chord $7x+y-1=0$ of the ellipse $\frac{x^2}{1}+\frac{y^2}{7}=1$ is

a) $x=y$ b) $2x=y$ c) $x=2y$ d) $x+y=0$

257. From a point $P(1,2)$ two tangents are drawn to a hyperbola 'H' in which one tangent is drawn to each arm of the hyperbola. If the equations of the asymptotes of hyperbola H are $\sqrt{3}x-y+5=0$ and $\sqrt{3}x+y-1=0$, then eccentricity of 'H' is

a) 2 b) $\frac{2}{\sqrt{3}}$ c) $\sqrt{2}$ d) $\sqrt{3}$

258. The locus of the centre of a circle, which touches externally the circle $x^2+y^2-6x-6y+14=0$ and also touches the y-axis, is given by equation

a) $x^2-6x-10y+14=0$ b) $x^2-10x-6y+14=0$

c) $y^2-6x-10y+14=0$ d) $y^2-10x-6y+14=0$

259. The asymptotes of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ form with any tangent to the hyperbola a triangle whose area is $a^2\tan\lambda$ in magnitude then its eccentricity is

a) $\sec\lambda$ b) $\operatorname{cosec}\lambda$ c) $\sec^2\lambda$ d) $\operatorname{cosec}^2\lambda$

260. Let E be the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and C be the circle $x^2+y^2=9$. Let P and Q be the point $(1,2)$ and $(2,1)$ respectively. Then

a) Q lies inside C but outside E b) Q lies outside both C and E

c) P lies inside both C and E d) P lies inside C but outside E

261. A ray of light travels along a line $y=4$ and strikes the surface of a curves $y^2=4(x+y)$, then equations of the line along which reflected ray travel is

a) $x=0$ b) $x=2$ c) $x+y=4$ d) $2x+y=4$

262. Let P be point on the circle $x^2+y^2=9$, Q a point on the line $7x+y+3=0$, and the perpendicular bisector of PQ be the line $x-y+1=0$. Then the coordinate of P are

a) $(0,-3)$ b) $(0,3)$ c) $\left(\frac{72}{25},-\frac{21}{25}\right)$ d) $\left(-\frac{72}{25},\frac{21}{25}\right)$

263. A hyperbola passes through $(2,3)$ and has asymptotes $3x-4y+5=0$ and $12x+5y-40=0$, then the equation of its transverse axis is

a) $77x-21y-265=0$ b) $21x-77y+265=0$

c) $21x-77y-265=0$ d) $21x+77y-265=0$

264. The locus of a point from which the lengths of the tangents to the circles $x^2+y^2=4$ and $2(x^2+y^2)-10x+3y-2=0$ are equal to

a) A straight line inclined at $\pi/4$ with the line joining the centres of the circles
b) A circle
c) An ellipse
d) A straight line perpendicular to the line joining the centres of the circles

265. The locus of the midpoint of a chord of the circle $x^2 + y^2 = 4$ which subtends a right angle at the origin is
a) $x + y = 2$ b) $x^2 + y^2 = 1$ c) $x^2 + y^2 = 2$ d) $x + y = 1$

266. The radius of the circumcircle of the triangle TPQ, where PQ is chord of contact corresponding to point T with respect to circle $x^2 + y^2 - 2x + 4y - 11 = 0$, is 12 units, then minimum distance of T from the director circle of the given circle is
a) 6 b) 12 c) $6\sqrt{2}$ d) $12 - 4\sqrt{2}$

267. A straight lines with slope 2 and y-intercept 5 touches the circle, $x^2 + y^2 + 16x + 12y + c = 0$ at a point Q. Then the co-ordinates of Q are
a) $(-6, 11)$ b) $(-9, -13)$ c) $(-10, -15)$ d) $(-6, -7)$

268. The family of the curves for which the length of the normal at any point is equal to the radius vector of that point is
a) Hyperbola b) straight line c) Parabola d) Ellipse

269. AB is a double ordinate of the parabola $y^2 = 4ax$. Tangents drawn to parabola at A and B meet y–axis at A_1 and B_1, respectively. If the area of trapezium AA_1B_1B is equal to $12a^2$, then angle subtended by A_1B_1 at the focus of the parabola is equal to
a) $2\tan^{-1}(3)$ b) $\tan^{-1}(3)$ c) $2\tan^{-1}(2)$ d) $\tan^{-1}(2)$

270. Double ordinate AB of the parabola $y^2 = 4ax$ subtends an angle $\pi/2$ at the focus of the parabola, then tangents drawn to parabola at A and B will intersect at
a) $(-4a, 0)$ b) $(-2a, 0)$ c) $(-3a, 0)$ d) None of these

271. A line meets the co-ordinate axes in A and B. A circle is circumscribed about the triangle OAB. If d_1 and d_2 are the distance of the tangent to the circle at the origin O from the points A and B, respectively, then the diameter of the circle is
a) $\dfrac{2d_1 + d_2}{2}$ b) $\dfrac{d_1 + 2d_2}{2}$ c) $d_1 + d_2$ d) $\dfrac{d_1 d_2}{d_1 + d_2}$

272. The co-ordinates of a point on the hyperbola, $\frac{x^2}{24} - \frac{y^2}{18} = 1$, which is nearest to the lines $3x + 2y + 1 = 0$ are
a) $(6, 3)$ b) $(-6, -3)$ c) $(6, -3)$ d) $(-6, 3)$

273. If two distinct chords, drawn from the point (p, q) on the circle $x^2 + y^2 - px - qy = 0$ (where $pq \neq 0$) are bisected by the x-axis, then
a) $p^2 = q^2$ b) $p^2 = 8q^2$ c) $p^2 < 8q^2$ d) $p^2 > 8q^2$

274. Number of distinct normal lines that can be drawn to the ellipse $\frac{x^2}{169} + \frac{y^2}{25} = 1$ from the point P (0,6) is
a) One b) Two c) Three d) Four

275. Let S and S' be two foci of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. If a circle described on SS' as diameter interests the ellipse in real and distinct points, then the eccentricity e of the ellipse satisfies
a) $e = 1\sqrt{2}$ b) $e \in \left(1/\sqrt{2}, 1\right)$ c) $e \in \left(0, 1\sqrt{2}\right)$ d) None of these

276. The sum of the square of the perpendiculars on any tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ from two points on the minor axis each at a distance ae from the centre is
a) $2a^2$ b) $2b^2$ c) $a^2 + b^2$ d) $a^2 - b^2$

277. The difference between the radii of the largest and the smallest circles which have their centre on the circumference of the circle $x^2 + y^2 + 2x + 4y - 4 = 0$ and pass through the point (a, b) lying outside the given circle, is
a) 6
b) $\sqrt{(a+1)^2 + (b+2)^2}$
c) 3
d) $\sqrt{(a+1)^2 + (b+2)^2} - 3$

278. The locus of the centers of the circles which cut the circles $x^2 + y^2 + 4x - 6y + 9 = 0$ and $x^2 + y^2 - 5x +$

$4y-2=0$ orthogonally is

a) $9x+10y-7=0$ b) $x-y+2=0$ c) $9x-10y+11=0$ d) $9x+10y+7=0$

279. If the normals at $P(\theta)$ and $Q(\pi\,/\,+\theta)$ to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meet the major axis at G and g, respectively, then $PG^2+Q\mathrm{g}^2=$

a) $b^2(1+e^2)(2-e^2)$ b) $a^2(e^4-e^2+2)$ c) $a^2(1+e^2)(2+e^2)$ d) $b^2(1+e^2)(2+e^2)$

280. The curve $xy=c\ (c>0)$ and the circle $x^2+y^2=1$ touch at two points, then distance between the points of contacts is

a) 1 b) 2 c) $2\sqrt{2}$ d) None of these

281. Two parabola have the focus $(3,-2)$. Their directrices are the x–axis, and the y–axis, respectively. Then the slope of their common chord is

a) -1 b) $-\frac{1}{2}$ c) $-\frac{\sqrt{3}}{2}$ d) None of these

282. The equation of the parabola whose vertex and focus lie on the axis of x of distances a and a_1 from the origin respectively is

a) $y^2=4(a_1-a)x$ b) $y^2=4(a_1-a)(x-a)$

c) $y^2=4(a_1-a)(x-a_1)$ d) None of these

283. The equation of the common tangent touching the circle $(x-3)^2+y^2=9$ and the parabola $y^2=4x$ above the x-axis is

a) $\sqrt{3}y=3x+1$ b) $\sqrt{3}y=-(x+3)$ c) $\sqrt{2}y=x+3$ d) $\sqrt{3}y=-(3x+1)$

284. The circles which can be drawn to pass through (1, 0) and (3, 0) and to touch the y-axis, intersect at an angle θ, then $\cos\theta$ is equal to

a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) $\frac{1}{4}$ d) $-\frac{1}{4}$

285. Let P be any moving point on the circle $x^2+y^2-2x=1$. AB be the chord of contact of this point w.r.t. the circle $x^2+y^2-2x=0$. The locus of the circumcentre of the triangle CAB (C being centre of the circles) is

a) $2x^2+2y^2-4x+1=0$ b) $x^2+y^2-4x+2=0$

c) $x^2+y^2-4x+1=0$ d) $2x^2+2y^2-4x+3=0$

286. Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and PQ intersect at a point X on the circumference of the circle, then $2r$ equals

a) $\sqrt{PQ.RS}$ b) $\frac{(PQ+RS)}{2}$ c) $\frac{2\,PQ\times RS}{PQ+RS}$ d) $\frac{\sqrt{(PQ^2+RS^2)}}{2}$

287. If PSQ is the focal chord of the parabola $y^2=8x$ such that $SP=\ 6$. Then the length of SQ is

a) 6 b) 4 c) 3 d) None of these

288. The locus of a point, from where tangents to the rectangular hyperbola $x^2-y^2=a^2$ contain an angle of 45°, is

a) $(x^2+y^2)^2+a^2(x^2-y^2)=4a^2$ b) $2(x^2+y^2)^2+4a^2(x^2-y^2)=4a^2$

c) $(x^2+y^2)^2+4a^2(x^2-y^2)=4a^2$ d) $(x^2+y^2)^2+a^2(x^2-y^2)=a^4$

289. The radius of circle touching parabola $y^2=x$ at (1,1) and having directrix of $y^2=x$ as its normal is

a) $\frac{5\sqrt{5}}{8}$ b) $\frac{10\sqrt{5}}{3}$ c) $\frac{5\sqrt{5}}{4}$ d) None of these

290. The equation of circumcircle of an equilateral triangle is $x^2+y^2+2\mathrm{g}x+2fy+c=0$ and one vertex of the triangle is (1, 1). The equation of incircle of the triangle is

a) $4(x^2+y^2)=\mathrm{g}^2+f^2$

b) $4(x^2+y^2)+8\mathrm{g}x+8fy=(1-\mathrm{g})(1+3\mathrm{g})+(1-f)(1+3f)$

c) $4(x^2+y^2)+8\mathrm{g}x+8fy=\mathrm{g}^2+f^2$

d) None of these

291. Normals drawn to $y^2=4ax$ at the points where it is intersected by the line $y=mx+c$ intersect at P. Foot of the another normal drawn to the parabola from the point 'P' is

a) $\left(\frac{a}{m^2},-\frac{2a}{m}\right)$ b) $\left(\frac{9a}{m},\frac{-6a}{m}\right)$ c) $(am^2,-2am)$ d) $\left(\frac{4a}{m^2},-\frac{4a}{m}\right)$

292. If $y = m_1x + c$ and $y = m_2x + c$ are two tangents to the parabola $y^2 + 4a(x + a) = 0$, then

a) $m_1 + m_2 = 0$ b) $1 + m_1 + m_2 = 0$ c) $m_1m_2 - 1 = 0$ d) $1 + m_1m_2 = 0$

293. If tangents OQ and OR are dawn to variable circles having radius r and the centre lying on the rectangular hyperbola $xy = 1$, then locus of circumcentre of triangle OQR is (O being the origin)

a) $xy = 4$ b) $xy = \frac{1}{4}$ c) $xy = 1$ d) none of these

294. Locus of the point which divides double ordinates of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the ratio $1:2$ internally is

a) $\frac{x^2}{a^2} + \frac{9y^2}{b^2} = 1$ b) $\frac{x^2}{a^2} + \frac{9y^2}{b^2} = \frac{1}{9}$ c) $\frac{9x^2}{a^2} + \frac{9y^2}{b^2} = 1$ d) None of these

295. C is the centre of the circle with centre (0,1) and radius unity. P is the parabola $y = ax^2$. The set of values of 'a' for which they meet at a point other than the origin, is

a) $a > 0$ b) $a \in \left(0, \frac{1}{2}\right)$ c) $\left(\frac{1}{4}, \frac{1}{2}\right)$ d) $\left(\frac{1}{2}, \infty\right)$

296. If the normals at points 't_1' and 't_2' meet on the parabola, then

a) $t_1\, t_2 = -1$ b) $t_2 = -t_1 - \frac{2}{t_1}$ c) $t_1t_2 = 2$ d) None of these

297. If foci of hyperbola lie on $y = x$ and one of the asymptote is $y = 2x$, then equation of the hyperbola, given that it passes through (3, 4) is

a) $x^2 - y^2 - \frac{5}{2}xy + 5 = 0$ b) $2x^2 - 2y^2 + 5xy + 5 = 0$

c) $2x^2 + 2y^2 - 5xy + 10 = 0$ d) None of these

298. The value of 'c' for which the set $\{(x, y)|x^2 + y^2 + 2x \le 1\} \cap \{(x, y)|x - y + c \ge 0\}$ contains only one point on common is

a) $(-\infty, -1] \cup [3, \infty)$ b) $\{-1, 3\}$ c) $\{-3\}$ d) $\{-1\}$

299. If two different tangents of $y^2 = 4x$ are the normals to $x^2 = 4by$ then

a) $|b| > \frac{1}{2\sqrt{2}}$ b) $|b| < \frac{1}{2\sqrt{2}}$ c) $|b >| \frac{1}{\sqrt{2}}$ d) $|b| < \frac{1}{\sqrt{2}}$

300. The normal at the point $P(ap^2, 2ap)$ meets the parabola $y^2 = 4ax$ again at $Q(aq^2, 2aq)$ such that the lines joining the origin to P and Q are at right angle. Then

a) $p^2 = 2$ b) $q^2 = 2$ c) $p = 2q$ d) $q = 2p$

301. The number of values of c such that the straight line $y = 4x + c$ touches the curve $\frac{x^2}{4} + \frac{y^2}{1} = 1$ is

a) 0 b) 1 c) 2 d) Infinite

302. Let S be the focus of $y^2 = 4x$ and a point P is moving on the curve such that its abscissa is increasing at the rate of 4 units/sec, then the rate of increase of projection of SP on $x + y = 1$ when P is at (4,4) is

a) $\sqrt{2}$ b) -1 c) $-\sqrt{2}$ d) $-\frac{3}{\sqrt{2}}$

303. The locus of the centre of a circle which cuts orthogonally the parabola $y^2 = 4x$ at (1, 2) will pass though points

a) (3, 4) b) (4, 3) c) (5, 3) d) (2, 4)

304. The area of the triangle formed by the tangent and the normal to the parabola $y^2 = 4ax$, both drawn at the same end of the latus rectum, and the axis of the parabola is

a) $2\sqrt{2}a^2$ b) $2a^2$ c) $4a^2$ d) None of these

305. The centre of the circle passing through the points (0, 0), (1, 0) and touching the circle $x^2 + y^2 = 9$ is

a) $\left(\frac{3}{2}, \frac{1}{2}\right)$ b) $\left(\frac{1}{2}, \frac{3}{2}\right)$F c) $\left(\frac{1}{2}, \frac{1}{2}\right)$ d) $\left(\frac{1}{2}, -2^{\frac{1}{2}}\right)$

306. A normal to the hyperbola $\frac{x^2}{4} - \frac{y^2}{1} = 1$ has equal intercepts on positive x- and y-axes. If this normal touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then $a^2 + b^2$ is equal to

a) 5 b) 25 c) 16 d) none of these

307. If the tangents are drawn from any point on the line $x + y = 3$ to the circle $x^2 + y^2 = 9$, then the chord of

contact passes through the point

a) (3, 5) b) (3, 3) c) (5, 3) d) None of these

308. A straight line has its extremities on two fixed straight lines and cuts off from them a triangle of constant area c^2. Then the locus of the middle point of the line is

a) $2xy = c^2$ b) $xy + c^2 = 0$ c) $4x^2y^2 = c$ d) None of these

309. If the normals to the parabola $y^2 = 4ax$ at three points P, Q and R meet at A and S be the focus, then $SP.SQ.SR$ is equal to

a) a^2SA b) SA^3 c) aSA^2 d) None of these

310. If the line $x\cos\theta + y\sin\theta = 2$ is the equation of a transverse common tangent to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6\sqrt{3}\,x - 6y + 20 = 0$, then the value of θ is

a) $5\pi/6$ b) $2\pi/3$ c) $\pi/3$ d) $\pi/6$

311. The point of intersection of the tangents of the parabola $y^2 = 4x$, drawn at end points of the chord $x + y = 2$ lies on

a) $x - 2y = 0$ b) $x + 2y = 0$ c) $y - x = 0$ d) $x + y = 0$

312. If the eccentricity of the hyperbola $x^2 - y^2\sec^2\alpha = 5$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2\sec^2\alpha + y^2 = 25$, then a value of α is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

313. Tangents are drawn to the circle $x^2 + y^2 = 1$ at the points where it is met by the circles, $x^2 + y^2 = (\lambda + 6)x + (8 - 2\lambda)y - 3 = 0, \lambda$ being the variable. The locus of the point of intersection of these tangents is

a) $2x - y + 10 = 0$ b) $x + 2y - 10 = 0$ c) $x - 2y + 10 = 0$ d) $2x + y - 10 = 0$

314. At what point on the parabola $y^2 = 4x$ the normal makes equal angle with axes?

a) (4, 4) b) (9, 6) c) (4,−4) d) (1,±2)

315. The angle between the tangents to the parabola $y^2 = 4ax$ at the points where it intersects with the line $x - y - a = 0$ is

a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{2}$

316. Let $ABCD$ be a quadrilateral with are 18, with side AB parallel to the side CD and $AB = 2CD$. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral $ABCD$ touching all the side, then its radius is

a) 3 b) 2 c) $\frac{3}{2}$ d) 1

317. $\min\left[(x_1 - x_2)^2 + \left(5 + \sqrt{1 - x_1^2} - \sqrt{4x_2}\right)^2\right] \forall\, x_1, x_2 \in R$ is

a) $4\sqrt{5} + 1$ b) $4\sqrt{5} - 1$ c) $\sqrt{5} + 1$ d) $\sqrt{5} - 1$

318. The locus of the vertex of the family of parabolas $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$ is

a) $xy = \frac{105}{64}$ b) $xy = \frac{3}{4}$ c) $xy = \frac{35}{16}$ d) $xy = \frac{64}{105}$

319. The two circles which passes through $(0, a)$ and $(0, -a)$ and touch the line $y = mx + c$ will intersect each other at right angle, if

a) $a^2 = c^2(2m + 1)$ b) $a^2 = c^2(2 + m^2)$ c) $c^2 = a^2(2 + m^2)$ d) $c^2 = a^2(2m + 1)$

320. If the normals to the parabola $y^2 = 4ax$ at the ends of the latus rectum meet the parabola at Q, Q', then QQ' is

a) $10a$ b) $4a$ c) $20a$ d) $12a$

321. The equation the tangent parallel to $y - x + 5 = 0$, drawn to $\frac{x^2}{3} - \frac{y^2}{2} = 1$ is

a) $x - y - 1 = 0$ b) $x - y + 2 = 0$ c) $x + y - 1 = 0$ d) $x + y + 2 = 0$

322. The number of rational point(s) (a point (a, b) is called rational, if a and b both are rational numbers) on the circumference of a circle having centre (π, e) is

a) At most one b) At least two c) Exactly two d) Infinite

323. A tangent drawn to hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at $P\left(\frac{\pi}{6}\right)$ forms a triangle of area $3a^2$ square units, with coordinate axes, then the square of its eccentricity is

a) 15 b) 24 c) 17 d) 14

324. With a given point and line as focus and directrix, a series of ellipses are described, the locus of the extremities of their minor axis is

a) Ellipse
b) Parabola
c) Hyperbola
d) None of these

325. The combined equation of the asymptotes of the hyperbola $2x^2+5xy+2y^2+4x+5y=0$ is

a) $2x^2+5xy+2y^2+4x+5y+2=0$ b) $2x^2+5xy+2y^2+4x+5y-2=0$
c) $2x^2+5xy+2y^2=0$ d) none of these

326. A tangent and normal is drawn at the point $P\equiv(16,16)$ of the parabola $y^2=16x$ which cut axis of the parabola at the points A and B, respectively. If the centre of the circle through P, A and B is C, then the angle between PC and the axis of is

a) $\tan^{-1}\frac{1}{2}$ b) $\tan^{-1}2$ c) $\tan^{-1}\frac{3}{4}$ d) $\tan^{-1}\frac{4}{3}$

327. Tangent at a point of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is drawn which cuts the coordinate axes at A and B. The minimum area of the ΔOAB is (O being the origin)

a) ab b) $\frac{a^3+ab+b^3}{3}$ c) a^2+b^2 d) $\frac{(a^2+b^2)}{4}$

328. Two circles of radii 4 cm and 1 cm touch each other externally and θ is the angle contained by their direct common tangents. Then $\sin\theta$ is equal to

a) $\frac{24}{25}$ b) $\frac{12}{25}$ c) $\frac{3}{4}$ d) None of these

329. Four points are such that the line joining any two points is perpendicular to the line joining other two points. If three points out of these lie on a rectangular hyperbola then the fourth point will lie on

a) The same hyperbola b) Conjugate hyperbola
c) One of the directrix d) One of the asymptotes

330. A point $P(x,y)$ moves in xy plane such that $x=a\cos^2\theta$ and $y=2a\sin\theta$, where θ is a parameter. The locus of the point P is

a) Circle b) Ellipse
c) Unbounded parabola d) Part of the parabola

331. Locus of the midpoint of any normal chords of $y^2=4ax$ is

a) $x=a\left(\frac{4a^2}{y^2}-2+\frac{y^2}{2a^2}\right)$ b) $x=a\left(\frac{4a^2}{y^2}+2+\frac{y^2}{2a^2}\right)$
c) $x=a\left(\frac{4a^2}{y^2}-2-\frac{y^2}{2a^2}\right)$ d) $x=a\left(\frac{4a^2}{y^2}+2-\frac{y^2}{2a^2}\right)$

332. If the chord of contact of tangents from a point P to a given circle passes through Q, then the circle on PQ as diameter

a) Cuts the given circle orthogonally b) Touches the given circle externally
c) Touches the given circle internally d) None of these

333. The locus of the foot of the perpendicular form the centre of the hyperbola $xy=1$ on a variable tangent is

a) $(x^2-y^2)^2=4xy$ b) $(x^2+y^2)^2=2xy$ c) $(x^2+y^2)=4xy$ d) $(x^2+y^2)^2=4xy$

334. The locus of the midpoints of the chords of the circle $x^2+y^2-ax-by=0$ which subtend a right angle at $\left(\frac{a}{2},\frac{b}{2}\right)$ is

a) $ax+by=0$ b) $ax+by=a^2+b^2$

c) $x^2+y^2-ax-by+\dfrac{a^2+b^2}{8}=0$ d) $x^2+y^2-ax-by-\dfrac{a^2+b^2}{8}=0$

335. The range of values of α for which the line $2y=\mathrm{g}x+\alpha$ is a normal to the circle $x^2+y^2+2\mathrm{g}x+2\mathrm{g}y-2=0$ for all values of g is

a) $[1,\infty)$ b) $[-1,\infty)$ c) $(0,1)$ d) $(-\infty,1]$

336. Consider a circle $x^2+y^2+ax+by+c=0$ lying completely in first quadrant. If m_1 and m_2 are the maximum and minimum values of y/x for all ordered pairs (x,y) on the circumference of the circle, then the value of (m_1+m_2) is

a) $\dfrac{a^2-4c}{b^2-4c}$ b) $\dfrac{2ab}{b^2-4c}$ c) $\dfrac{2ab}{4c-b^2}$ d) $\dfrac{2ab}{b^2-4ac}$

337. The chord of contact of tangents from a point P to a circle passes through Q. If I_1 and I_2 are the lengths of the tangents from P and Q to the circle, then PQ is equal to

a) $\dfrac{I_1+I_2}{2}$ b) $\dfrac{I_1-I_2}{2}$ c) $\sqrt{I_1^2+I_2^2}$ d) $2\sqrt{I_1^2+I_2^2}$

338. If $S=0$ be the equation of the hyperbola $x^2+4xy+3y^2-4x+2y+1=0$, then the value of k for which $S+K=0$ represents its asymptotes is

a) 20 b) -16 c) -22 d) 18

339. If angle between asymptotes of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $120°$ and product of perpendiculars drawn from foci upon its any tangent is 9, then locus of point of intersection of perpendicular tangents of the hyperbola can be

a) $x^2+y^2=6$ b) $x^2+y^2=9$ c) $x^2+y^2=3$ d) $x^2+y^2=18$

340. If bisector of the angle APB, where PA and PB are the tangents to the parabola $y^2=4ax$, is equally inclined to the coordinate axes, then the point P lies on

a) Tangent at vertex of the parabola b) Directrix of the parabola

c) Circle with centre at the origin and radius a d) The line of latus rectum

341. If point A and B are $(1,0)$ and $B(0,1)$. If point C is on the circle $x^2+y^2=1$, then locus of the orthocenter of the triangle ABC is

a) $x^2+y^2=4$ b) $x^2+y^2-x-y=0$

c) $x^2+y^2-2x-2y+1=0$ d) $x^2+y^2+2x-2y+1=0$

342. A man running round a race course notes that the sum of the distances of two flagposts from him is always 10 m and the distance between the flag posts is 8 m. Then the area of the path he encloses in square meters is

a) 15π b) 20π c) 27π d) 30π

343. Let P be any point on a dirextrix of an ellipse of eccentricity e. S be the corresponding focus and C the centre of the ellipse. The line PC meets the ellipse at A. The angel between PS and tangent at A is α, the α is equal to

a) $\tan^{-1}e$ b) $\dfrac{\pi}{2}$ c) $\tan^{-1}(1-e^2)$ d) None of these

344. Equation of incircle of equilateral triangle ABC where $B\equiv(2,0)C\equiv(4,0)$ and A lies in fourth quadrant is

a) $x^2+y^2-6x+\dfrac{2y}{\sqrt{3}}+9=0$ b) $x^2+y^2-6x-\dfrac{2y}{\sqrt{3}}+9=0$

c) $x^2+y^2+6x+\dfrac{2y}{\sqrt{3}}+9=0$ d) None of these

345. Axis of a parabola is $y=x$ and vertex and focus are at a distance $\sqrt{2}$ and $2\sqrt{2}$ Respectively from the origin. Then, equation of the parabola is

a) $(x-y)^2=8(x+y-2)$ b) $(x+y)^2=2(x+y-2)$

c) $(x-y)^2=4(x+y-2)$ d) $(x+y)^2=2(x-y+2)$

346. The equation of the parabola whose focus is the point (0,0) and the tangent at the vertex is $x-y+1=0$ is

a) $x^2+y^2-2xy-4x-4y-4=0$ b) $x^2+y^2-2xy+4x-4y-4=0$

c) $x^2+y^2+2xy-4x+4y-4=0$ d) $x^2+y^2+\ 2xy-4x-4y+4=0$

347. If tangents PQ and PR are drawn from variable point P to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1(a>b)$ so that the fourth vertex S of parallelogram $PQSR$ lies on circumcircle of triangle PQR, then locus of P is
a) $x^2+y^2=b^2$ b) $x^2+y^2=a^2$ c) $x^2+y^2=(a^2-b^2)$ d) None of these

348. If $a\neq 0$ and the line $2bx+3cy+4d=0$, passes through the points of intersection of the parabolas$y^2=4ax\ x^2=4ay$, then
a) $d^2+(2b+3c)^2=0$ b) $d^2+(3b+2c)^2=0$ c) $d^2+(2b-3c)^2=0$ d) $d^2+(3b-2c)^2=0$

349. Asymptotes of the hyperbola $\frac{x^2}{a_1^2}-\frac{y^2}{b_1^2}=1$ and $\frac{x^2}{a_2^2}-\frac{y^2}{b_2^2}=1$ are perpendicular to each other, then
a) $\frac{a_1}{a_2}=\frac{b_1}{b_2}$ b) $a_1a_2=b_1b_2$ c) $a_1a_2+b_1b_2=0$ d) $a_1-a_2=b_1-b_2$

350. The set of values of m for which it is possible to draw the chord $y=\sqrt{m}\,x+1$ to the curve $x^2+2xy+(2+\sin^2\alpha)y^2=1$, which subtends a right angle at the origin for some value of α is
a) $[2,3]$ b) $[0,1]$ c) $[1,3]$ d) None of these

351. The coordinates of the middle point of the chord cut-off by $2x-5y+18=0$ by the circle $x^2+y^2-6x+2y-54=0$ are
a) $(1,4)$ b) $(2,4)$ c) $(4,1)$ d) $(1,1)$

352. Set of values of α for which the point $(\alpha,1)$ lies inside the curves $c_1\colon x^2+y^2-4=0$ and $c_2\colon y^2=4x$ is
a) $|\alpha|<\sqrt{3}$ b) $|\alpha|<2$ c) $\frac{1}{4}<\alpha<\sqrt{3}$ d) None of these

353. The locus of the point of intersection of tangents to an ellipse at two points, sum of whose eccentric angles is constant, is a/an
a) Parabola b) Circle c) Ellipse d) Straight line

354. A line of slope $\lambda(0<\lambda<1)$ touches parabola $y+3x^2=0$ at P. If S is the focus and M is the foot of the perpendicular of directrix from P, then $\tan\angle MPS$ equals
a) 2λ b) $\frac{2\lambda}{-1+\lambda^2}$ c) $\frac{1-\lambda^2}{1+\lambda^2}$ d) None of these

355. A circle has the same centre as an ellipse and passes through the foci F_1 and F_2 of the ellipse, such that the two curves intersect in 4 points. Let $'P'$ be any one of their point of intersection. If the major axis of the ellipse is 17 and the area of the triangle PF_1F_2 is 30, then the distance between the focii is
a) 13 b) 10 c) 11 d) None of these

356. Number of points on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=3$, from which mutually perpendicular tangents can be drawn to the circle $x^2+y^2=a^2$, is/are
a) 0 b) 2 c) 3 d) 4

357. The equation of the circle passing through the point of intersection of the circle $x^2+y^2=4$ and the line $2x+y=1$ and having minimum possible radius is
a) $5x^2+5y^2+18x+6y-5=0$ b) $5x^2+5y^2+9x+8y-15=0$
c) $5x^2+5y^2+4x+9y-5=0$ d) $5x^2+5y^2-4x-2y-18=0$

358. The length of the transverse axis of the rectangular hyperbola $xy=18$ is
a) 6 b) 12 c) 18 d) 9

359. Consider a family of circle which are passing through the point $(-1,1)$ and are tangent to x-axis. If (h,k) are the coordinates of the centre of the circles, then the set of values of k given by the interval
a) $k\geq\frac{1}{2}$ b) $-\frac{1}{2}\leq k\leq\frac{1}{2}$ c) $k\leq\frac{1}{2}$ d) $0<k<\frac{1}{2}$

360. If $a\neq 0$ and the line $2bx+3cy+4d=0$ passes through the points of intersection of the parabola $y^2=4ax$ and $x^2=4ay$, then
a) $d^2+(2b+3c)^2=0$ b) $d^2+(3b+2c)^2=0$ c) $d^2+(2b-3c)^2=0$ d) None of these

361. If the distance between two parallel tangents drawn to the hyperbola $\frac{x^2}{9}-\frac{y^2}{49}=1$ is 2, then their slope is equal to

a) $\pm\frac{5}{2}$ b) $\pm\frac{4}{5}$ c) $\pm\frac{7}{2}$ d) None of these

362. If y_1, y_2 and y_3 are the ordinates of the vertices of a triangle inscribed in the parabola $y^2 = 4ax$, then its area is

a) $\frac{1}{2a}(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)$ b) $\frac{1}{4a}(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)$

c) $\frac{1}{8a}(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)$ d) None of these

363. The chord of contact of tangents from three points A, B, C to the circle $x^2 + y^2 = a^2$ are concurrent, then A, B, C will

a) Be concylic b) Be collinear

c) Form the vertices of a triangle d) None of these

364. If the eccentricity of the ellipse $\frac{x^2}{a^2+1} + \frac{y^2}{a^2+2} = 1$ is $\frac{1}{\sqrt{6}}$, then latus rectum of ellipse is

a) $\frac{5}{\sqrt{6}}$ b) $\frac{10}{\sqrt{6}}$ c) $\frac{8}{\sqrt{6}}$ d) None of these

365. Let a and b represent the length of a right triangle's legs. If d is the diameter of a circle inscribed into the triangle, and D is the diameter of a circle circumscribed on the triangle, then $d + D$ equals

a) $a + b$ b) $2(a + b)$ c) $\frac{1}{2}(a + b)$ d) $\sqrt{a^2 + b^2}$

366. A water jet from a fountain reaches its maximum height of 4 m at a distance 0.5 m from the vertical passing through the point O of water outlet. The height of the jet above the horizontal OX at a distance of 0.75 m from the point O is

a) 5 m b) 6 m c) 3 m d) 7 m

367. The equation of the transverse axis of the hyperbola $(x - 3)^2 + (y + 1)^2 = (4x + 3y)^2$ is

a) $x + 3y = 0$ b) $4x + 3y = 9$ c) $3x - 4y = 13$ d) $4x + 3y = 0$

368. The eccentricity of the conjugate hyperbola of the hyperbola $x^2 - 3y^2 = 1$ is

a) 2 b) $\frac{2}{\sqrt{3}}$ c) 4 d) $\frac{4}{5}$

369. Two congruent circles with centres at (2, 3) and (5, 6), which intersect at right angles, have radius equal to

a) $2\sqrt{2}$ b) 3 c) 4 d) None

370. If $y = mx + c$ touches the parabola $y^2 = 4a(x + a)$, then

a) $c = \frac{a}{m}$ b) $c = am + \frac{a}{m}$ c) $c = a + \frac{a}{m}$ d) None of these

371. Locus of the point of intersection of the tangent at the end points of the focal chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (b < a)$ is a/an

a) Circle b) Ellipse

c) Hyperbola d) Pair of straight lines

372. The line $y = mx - \frac{(a^2-b^2)m}{\sqrt{a^2+b^2m^2}}$ is normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ for all values of m belongs to

a) $(0, 1)$ b) $(0, \infty)$ c) R d) None of these

373. Length of the normal chord of the parabola $y^2 = 4x$ which makes an angle of $\frac{\pi}{4}$ with the axis of x is

a) 8 b) $8\sqrt{2}$ c) 4 d) $4\sqrt{2}$

374. The equation of the ellipse whose axes are coincident with the co-ordinates axes and which touches the straight lines $3x - 2y - 20 = 0$ and $x + 6y - 20 = 0$ is

a) $\frac{x^2}{40} + \frac{y^2}{10} = 1$ b) $\frac{x^2}{5} + \frac{y^2}{8} = 1$ c) $\frac{x^2}{10} + \frac{y^2}{40} = 1$ d) $\frac{x^2}{40} + \frac{y^2}{30} = 1$

375. If the curves $\frac{x^2}{4} + y^2 = 1$ and $\frac{x^2}{a^2} + y^2 = 1$ for suitable value of a cut on four concyclic points, the equation of the circle passing through these four points is

a) $x^2 + y^2 = 2$ b) $x^2 + y^2 = 1$ c) $x^2 + y^2 = 4$ d) None of these

376. If a ray of light incident along the line $3x + (5 - 4\sqrt{2})y = 15$ gets reflected from the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$, then its reflected ray goes along the line

a) $x\sqrt{2} - y + 5 = 0$ b) $\sqrt{2}y - x + 5 = 0$ c) $\sqrt{2}y - x - 5 = 0$ d) None of these

377. Maximum number of common normals of $y^2 = 4ax$ and $x^2 = 4by$ is equal to

a) 3 b) 4 c) 6 d) 5

378. The area of the triangle formed by the positive x-axis and the normal and tangent to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$ is

a) $2\sqrt{3}$ sq. units b) $3\sqrt{2}$ sq. units c) $\sqrt{6}$ sq. units d) None of these

379. The shortest distance between the parabola $2y^2 = 2x - 1, 2x^2 = 2y - 1$ is

a) $2\sqrt{2}$ b) $\frac{1}{2\sqrt{2}}$ c) 4 d) $\sqrt{\frac{36}{5}}$

380. If $y + 3 = m_1(x + 2)$ and $y + 3 = m_2(x + 2)$ are two tangents to the parabola $y^2 = 8x$ then

a) $m_1 + m_2 = 0$ b) $m_1 m_2 = -1$ c) $m_1 m_2 = 1$ d) None

381. $y = x + 2$ is any tangent to the parabola $y^2 = 8x$. The point P on this tangent is such that the other tangent from it which is perpendicular to it is

a) (2,4) b) (−2,0) c) (−1,1) d) (2,0)

382. A tangent having slope of $-\frac{4}{3}$ to the ellipse $\frac{x^2}{18} + \frac{y^2}{32} = 1$ intersects the major and minor axes at points A and B respectively. If C is the centre of the ellipse, then the area of the triangle ABC is

a) 12 sq.units b) 24 sq.units c) 36 sq.units d) 48 sq.units

383. The angle between lines joining the origin to the points of intersection of the line $\sqrt{3}x + y = 2$ and the curve $y^2 - x^2 = 4$ is

a) $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ b) $\frac{\pi}{6}$ c) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ d) $\frac{\pi}{2}$

384. Let these base AB of a triangle ABC be fixed and the vertex C lie on a fixed circle of radius r. Lines through A and B are drawn to intersect CB and CA, respectively, at E and F such that $CE: EB = 1: 2$ and $CF: FA = 1: 2$. If the point of intersection P of these lines lies on the median through AB for all positions of AB then the locus of P is

a) A circle of radius $\frac{r}{2}$ b) A circle of radius $2r$
c) A parabola of latus rectum $4r$ d) A rectangular hyperbola

385. The set of points on the axis of the parabola $(x - 1)^2 = 8(y + 2)$, from where three distinct normals can be drawn to the parabola is the set (h, k) of points satisfying

a) $h > 2$ b) $h > 1$ c) $k > 2$ d) None of these

386. If the circles $x^2 + y^2 + 2x + 2ky + 6 = 0, x^2 + y^2 + 2ky + k = 0$ intersect orthogonally, then k is

a) 2 or $-\frac{3}{2}$ b) -2 or $-\frac{3}{2}$ c) 2 or $\frac{3}{2}$ d) -2 or $\frac{3}{2}$

387. An isosceles triangles ABC is inscribed in a circle $x^2 + y^2 = a^2$ with the vertex A at $(a, 0)$ and the base angle B and C each equal to 75°, then coordinates of an end point of the base are

a) $\left(\frac{-\sqrt{3}a}{2}, \frac{a}{2}\right)$ b) $\left(-\frac{\sqrt{3}a}{2}, a\right)$ c) $\left(\frac{a}{2}, \frac{\sqrt{3}a}{2}\right)$ d) $\left(\frac{\sqrt{3}a}{2}, -\frac{a}{2}\right)$

388. A set of parallel chords of the parabola $y^2 = 4ax$ have their midpoints on

a) Any straight line through the vertex b) Any straight line through the focus
c) A straight line parallel to the axis d) Another parabola

389. $P(x, y)$ is a variable point on the parabola $y^2 = 4ax$ and $Q(x + c, y + c)$ is another variable point, where 'c' is a constant. The locus of the midpoint of PQ is

a) Parabola b) Ellipse c) Hyperbola d) Circle

390. Locus of midpoints of the chords of contact of $x^2 + y^2 = 2$ from the points on the line $3x + 4y = 10$ is a circle with centre P. If O be the origin then OP is equal to

a) 2 b) 3 c) $\frac{1}{2}$ d) $\frac{1}{3}$

391. From a point $A(t)$ on the parabola $y^2 = 4ax$, a focal chord and a tangent is drawn. Two circles are drawn in which one circle is drawn taking focal chord AB as diameter and other is drawn by taking intercept of tangent between point A and point P on the directrix, as diameter. Then the common chord of the circles is

a) Line joining focus and P b) Line joining focus and A
c) Tangent to the parabola at point A d) None of these

392. The locus of the midpoint of a line segment that is drawn from a given external point P to a given circle with centre O (where O is origin) and radius r, is

a) A straight line perpendicular to PO
b) A circle with centre P and radius r
c) A circle with centre P and radius $2r$
d) A circle with centre at the midpoint PO and radius $r/2$

393. An ellipse with major and minor axes length as $2a$ and $2b$ touches coordinate axes in first quadrant and having foci (x_1, y_1) and (x_2, y_2) then the value x_1x_2 and y_1y_2 is

a) a^2 b) b^2 c) a^2b^2 d) $a^2 + b^2$

394. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = k^2$ orthogonally, then the equation of the locus of its centre is

a) $2ax + 2by - (a^2 + b^2 + k^2) = 0$ b) $2ax + 2by - (a^2 - b^2 + k^2) = 0$
c) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - k^2) = 0$ d) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - k^2) = 0$

395. Radius of the circle that passes through origin and touches the parabola $y^2 = 4ax$ at the point $(a, 2a)$ is

a) $\frac{5}{\sqrt{2}}a$ b) $2\sqrt{2}a$ c) $\sqrt{\frac{5}{2}}a$ d) $\frac{3}{\sqrt{2}}a$

396. If the conics whose equations are $S \equiv \sin^2\theta\, x^2 + 2hxy + \cos^2\theta\ y^2 + 32x + 16y + 19 = 0, S' \equiv \cos^2\theta\, x^2 + 2h'xy + \sin^2\theta\, y^2 + 16x + 32y + 19 = 0$ intersects in four concyclic points then, (where $\theta \in R$)

a) $h + h'' = 0$ b) $h = h'$ c) $h + h' = 1$ d) None of these

397. The asymptotes of the hyperbola $xy = hx + ky$ are

a) $x - k = 0$ and $y - h = 0$ b) $x + h = 0$ and $y + k = 0$
c) $x - k = 0$ and $y + h = 0$ d) $x + k = 0$ and $y - h = 0$

398. The tangent at any point P on the parabola $y^2 = 4ax$ intersects the y–axis at$Q$. The tangent to the circum circle of triangle PQS (S is the focus) at Q is

a) A line parallel to x–axis b) y-axis
c) a line parallel to y–axis d) None of these

399. Let AB be a chord of the circle $x^2 + y^2 = r^2$ subtending a right angle at the centre. Then, the locus of the centroid of the triangle PAB as P moves on the circle is

a) A parabola b) A circle
c) An ellipse d) A pair of straight lines

400. If (a, b) is the midpoint of a chord passing through the vertex of the parabola $y^2 = 4x$, then

a) $a = 2b$ b) $2a = b$ c) $a^2 = 2b$ d) $2a = b^2$

401. Two circles $x^2 + y^2 = 6$ and $x^2 + y^2 - 6x + 8 = 0$ are given. Then, the equation of the circle through their points of intersection and the point $(1, 1)$ is

a) $x^2 + y^2 - 6x + 4 = 0$ b) $x^2 + y^2 - 3x + 1 = 0$
c) $x^2 + y^2 - 4y + 2 = 0$ d) None of these

402. Locus of the point $\sqrt{3h},\ \sqrt{3k+2}$ if it lies on the line $x - y - 1 = 0$ is a

a) Straight line b) Circle c) Parabola d) None of these

403. The equation $2x^2 + 3y^2 - 8x - 18y + 35 = k$ represents

a) No locus if $k > 0$ b) An ellipse if $k < 0$
c) A point if $k = 0$ d) A hyperbola if $k > 0$

404. From a point $R(5, 8)$ two tangents RP and RQ are drawn to a given circle $S = 0$ whose radius is 5. If circumcentre of the triangle PQR is $(2, 3)$, then the equation of circle $S = 0$ is

a) $x^2 + y^2 + 2x + 4y - 20 = 0$ b) $x^2 + y^2 + x + 2y - 10 = 0$

c) $x^2 + y^2 - x - 2y - 20 = 0$ d) $x^2 + y^2 - 4x - 6y - 12 = 0$

405. A square is inscribed in the circle $x^2 + y^2 - 2x + 4y - 93 = 0$ with its sides parallel to the coordinate axis. The co-ordinates of its vertices are

a) $(-6, -9), (-6, 5), (8, -9), (8, 5)$ b) $(-6, 9), (-6, -5), (8, -9), (8, 5)$

c) $(-6, -9), (-6, 5), (8, 9), (8, 5)$ d) $(-6, -9), (-6, 5), (8, -9), (8, -5)$

406. Equation of chord of the circle $x^2 + y^2 - 3x - 4y - 4 = 0$, which passes through the origin such that the origin divides it in the ratio 4: 1, is

a) $x = 0$ b) $24x + 7y = 0$ c) $7x + 24y = 0$ d) $7x - 24y = 0$

407. The triangle PQR of area 'A' is inscribed in the parabola $y^2 = 4ax$ such that the vertex P lies at the vertex of the parabola and the base QR is a focal chord. The modules of the difference of the ordinates of the points Q and R is

a) $\frac{A}{2a}$ b) $\frac{A}{a}$ c) $\frac{2A}{a}$ d) $\frac{4A}{a}$

408. If $2x + y + \lambda = 0$ is a normal to the parabola $y^2 = -8x$, then λ is

a) 12 b) -12 c) 24 d) -24

409. With one focus of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$ as the centre, a circle is drawn which is tangent to the hyperbola with no part of the circle being outside the hyperbola. The radius of the circle is

a) less than 2 b) 2 c) 1/3 d) none of these

Multiple Correct Answers Type

410. The equation of the circle which touches the axes of coordinates and the line $\frac{x}{3} + \frac{y}{4} = 1$ and whose centre lies in the first quadrant is $x^2 + y^2 - 2cx - 2cy + c^2 = 0$, where c is

a) 1 b) 2 c) 3 d) 6

411. A normal drawn to parabola $y^2 = 4ax$ meet the curve again at Q such that angel subtended by PQ at vertex is 90°, then coordinated of P can be

a) $(8a, 4\sqrt{2}a)$ b) $(8a, 4a)$ c) $(2a, -2\sqrt{2}a)$ d) $(2a, 2\sqrt{2}a)$

412. Which of the following is true about the parabola $y^2 = 4ax$ $(a > 0)$?

a) If t_1, t_2 are end points of a focal chord, then $t_1 t_2 = -1$

b) Tangent at the end of a focal chord cuts at right angle at directrix

c) Distance of any point on the parabola from directrix is equal to the sum of a abscissa of the point

d) End points of latusrectum are $(a, 2a)$ and $(-a, 2a)$

413. Co-ordinates of the centre of a circle, whose radius is 2 unit and which touches the line pair $x^2 - y^2 - 2x + 1 = 0$, are

a) $(4, 0)$ b) $(1 + 2\sqrt{2}, 0)$ c) $(4, 1)$ d) $(1, 2, \sqrt{2})$

414. Which of the following line can be normal to parabola $y^2 = 12x$?

a) $x + y - 9 = 0$ b) $2x - y - 32 = 0$ c) $2x + y - 36 = 0$ d) $3x - y - 72 = 0$

415. If the equation of the ellipse is $3x^2 + 2y^2 + 6x - 8y + 5 = 0$, then which of the following is/are true?

a) $e = \frac{1}{\sqrt{3}}$ b) Centre is $(-1, 2)$

c) Foci are $(-1, 1)$ and $(-1, 3)$ d) Directrices are $y = 2 \pm \sqrt{3}$

416. Equation of a circle with centre $(4, 3)$ touching the circle $x^2 + y^2 = 1$ is

a) $x^2 + y^2 - 8x - 6y - 9 = 0$ b) $x^2 + y^2 - 8x - 6y + 11 = 0$

c) $x^2 + y^2 - 8x - 6y - 11 = 0$ d) $x^2 + y^2 - 8x - 6y + 9 = 0$

417. If the tangent drawn at point $(t^2, 2t)$ on the parabola $y^2 = 4x$ is same as the normal drawn at point $(\sqrt{5}\cos\theta, 2\sin\theta)$ on the ellipse $4x^2 + 5y^2 = 20$. Then

a) $\theta = \cos^{-1}\left(-\frac{1}{\sqrt{5}}\right)$ b) $\theta = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ c) $t = -\frac{2}{\sqrt{5}}$ d) $t = -\frac{1}{\sqrt{5}}$

418. Consider the circle $x^2 + y^2 - 10x - 6y + 30 = 0$. Let O be the centre of the circle and tangent at $A(7, 3)$ and $B(5, 1)$ meet at C. Let $S = 0$ represents family of circles passing through A and B, then
a) Area of quadrilateral $OACB = 4$
b) The radical axis for the family of circles $S = 0$ is $x + y = 10$
c) The smallest possible circle of the family $S = 0$ is $x^2 + y^2 - 12x - 4y + 38 = 0$
d) The coordinates of point C are $(7, 1)$

419. The equation of the lines parallel to $x - 2y = 1$ which touches (touch) the circle $x^2 + y^2 - 4x - 2y - 15 = 0$ is (are)
a) $x - 2y + 2 = 0$ b) $x - 2y - 10 = 0$ c) $x - 2y - 5 = 0$ d) $x - 2y + 10 = 0$

420. If the focus of the parabola $x^2 - ky + 3 = 0$ is (0,2), then a value of k is/are
a) 4 b) 6 c) 3 d) 2

421. The equation $(x - \alpha)^2 + (y - \beta)^2 = k(lx + my + n)^2$ represents
a) A parabola fox $k < (l^2 + m^2)^{-1}$ b) An ellipse for $0 < k < (l^2 + m^2)^{-1}$
c) A hyperbola for $k > (1^2 + m^2)^{-1}$ d) A point circle for $k = 0$

422. Parabola $y^2 = 4x$ and the circle having it's centre at $(6, 5)$ intersect at right angle. Possible point of intersection of these curves can be
a) $(9, 6)$ b) $(2, \sqrt{8})$ c) $(4, 4)$ d) $(3, 2\sqrt{3})$

423. Let L_1 be a straight line passing through the origin and L_2 be the straight line $x + y = 1$. If the intercepts made by the circles $x^2 + y^2 - x + 3y = 0$ on L_1 and L_2 are equal, then which of the following equations can represents L_1
a) $x + y = 0$ b) $x - y = 0$ c) $x + 7y = 0$ d) $x - 7y = 0$

424. A point $P(\sqrt{3}, 1)$ moves on the circle $x^2 + y^2 = 4$ and after covering a quarter of the circle leaves it tangentially. The equation of a line along which the point moves after leaving the circle is
a) $y = \sqrt{3}\,x + 4$ b) $\sqrt{3}\,y = x + 4$ c) $y = \sqrt{3}\,x - 4$ d) $\sqrt{3}y = x - 4$

425. Equation $x^2 - 2x - 2y + 5 = 0$ represents
a) A circle with centre $(1, 1)$ b) A parabola with vertex $(1, 2)$
c) A parabola with directrix $y = 5/2$ d) A parabola with directrix $y = -1/3$

426. The equation of circle passing through $(3, -6)$ and touching both the axes is
a) $x^2 + y^2 - 6x + 6y + 9 = 0$
b) $x^2 + y^2 + 6x - 6y + 9 = 0$
c) $x^2 + y^2 + 30x - 30y + 225 = 0$
d) $x^2 + y^2 - 30x + 30y + 225 = 0$

427. The points on the line $x = 2$ from which the tangents drawn to the circle $x^2 + y^2 = 16$ are at right angles is (are)
a) $(2, 2\sqrt{7})$ b) $(2, 2\sqrt{5})$ c) $(2, -2\sqrt{7})$ d) $(2, -2\sqrt{5})$

428. $\frac{x^2}{r^2-r-6} + \frac{y^2}{r^2-6r+5} = 1$ will represents the ellipse, if r lies in the interval
a) $(-\infty, -2)$ b) $(3, \infty)$ c) $(5, \infty)$ d) $(1, \infty)$

429. The equation of a circle C_1 is$x^2 + y^2 = 4$. The locus of the intersection of orthogonal tangents to the circle is the curve C_2 and the locus of the intersection of perpendicular tangents to the curve C_2 is the curveC_3. Then,
a) C_3 is a circle b) The area enclosed by the curve C_3 is 8π
c) C_2 and C_3 are circles with the same centre d) None of the above

430. The circle $x^2 + y^2 + 2a_1x + c = 0$ lies completely inside the circle $x^2 + y^2 + 2a_2x + c = 0$, then
a) $a_1a_2 > 0$ b) $a_1a_2 < 0$ c) $c > 0$ d) $c < 0$

431. Which of the following lines have the intercepts of equal lengths on the circle, $x^2 + y^2 + 4y = 0$?
a) $3x - y = 0$ b) $x + 3y = 0$ c) $x + 3y + 10 = 0$ d) $3x - y - 10 = 0$

432. Which of the following is/are true about the ellipse $x^2 + 4y^2 - 2x - 16y + 13 = 0$?

a) The latus rectum of the ellipse is 1

b) Distance between foci of the ellipse is $4\sqrt{3}$

c) Sum of the focal distances of a point $P(x, y)$ on the ellipse is 4

d) $y = 3$ meets the tangents drawn at the vertices of the ellipse at points P and Q then PQ subtends a right angle at any of its foci

433. On the ellipse $4x^2 + 9y^2 = 1$, the points at which the tangents are parallel to the line $8x = 9y$ are

a) $\left(\frac{2}{5}, \frac{1}{5}\right)$ b) $\left(-\frac{2}{5}, \frac{1}{5}\right)$ c) $\left(-\frac{2}{5}, -\frac{1}{5}\right)$ d) $-\left(\frac{2}{5}, -\frac{1}{5}\right)$

434. The co-ordinates $(2, 3)$ and $(1, 5)$ are the foci of an ellipse which passes through the origin, then the equation of

a) Tangent at the origin is $(3\sqrt{2} - 5)x + (1 - 2\sqrt{2})y = 0$

b) Tangent at the origin is $(3\sqrt{2} + 5)x + (1 + 2\sqrt{2}y) = 0$

c) Normal at the origin is $(3\sqrt{2} + 5)x - (2\sqrt{2} + 1)y = 0$

d) Normal at the origin is $x(3\sqrt{2} - 5) - y(1 - 2\sqrt{2}) = 0$

435. The normals to the parabola $y^2 = 4ax$ from the point $(5a, 2a)$ are

a) $y = x - 3a$ b) $y = -2x + 12a$ c) $y = -3x + 33a$ d) $y = x + 3a$

436. The equation of a tangent to the circle $x^2 + y^2 = 25$ passing through $(-2, 11)$ is

a) $4x + 3y = 25$ b) $3x + 4y = 38$ c) $24x - 7y + 125 = 0$ d) $7x + 24y = 230$

437. The circles $x^2 + y^2 - 2x - 4y + 1 = 0$ and $x^2 + y^2 + 4x + 4y - 1 = 0$

a) Touch internally

b) Touch externally

c) Have $3x + 4y - 1 = 0$ as the common tangent at the point of contact

d) Have $3x + 4y + 1 = 0$ as the common tangent at the point of contact

438. The centre(s) of the circle(s) passing through the points $(0, 0)$, $(1, 0)$ and touching the circle $x^2 + y^2 = 9$ is/are

a) $\left(\frac{3}{2}, \frac{1}{2}\right)$ b) $\left(\frac{1}{2}, \frac{3}{2}\right)$ c) $\left(\frac{1}{2}, 2^{1/2}\right)$ d) $\left(\frac{1}{2}, -2^{1/2}\right)$

439. A straight line touches the rectangular hyperbola $9x^2 - 9y^2 = 8$ and the parabola $y^2 = 32x$. The equation of the line is

a) $9x + 3y - 8 = 0$ b) $9x - 3y + 8 = 0$ c) $9x + 3y + 8 = 0$ d) $9x - 3y - 8 = 0$

440. Three sides of a triangle have the equations $L_i \equiv y - m_i\, x = 0; i = 1, 2, 3$. Then $L_1L_2 + \lambda L_2L_3 + \mu L_3L_1 = 0$, where $\lambda \neq 0, \mu \neq 0$, is the equation of the circumcircle of the triangle if

a) $1 + \lambda + \mu = m_1m_2 + \lambda m_2m_3 + \lambda m_3m_1$

b) $m_1(1 + \mu) + m_2(1 + \lambda) + m_3(\mu + \lambda) = 0$

c) $\frac{1}{m_3} + \frac{1}{m_1} + \frac{1}{m_2} = 1 + \lambda + \mu$

d) None of these

441. Consider the ellipse $\frac{x^2}{f(k^2+2k+5)} + \frac{y^2}{f(k+11)} = 1$ and $f(x)$ is a positive decreasing function, then

a) The set of values of k, for which the major axis is x-axis is$(-3, 2)$

b) The set of values of k, for which the major axis is y-axis is $(-\infty, 2)$

c) The set of values of k, for which the major axis is y-axis is $(-\infty, -3) \cup (2, \infty)$

d) The set of values of k, for which the major axis is y-axis is $(-3, \infty)$

442. Let x, y be real variable satisfying the $x^2 + y^2 + 8x - 10y - 40 = 0$. Let $a = \max\left\{\sqrt{(x+2)^2 + (y-3)^2}\right\}$ and $b = \min\left\{\sqrt{(x+2)^2 + (y-3)^2}\right\}$, then

a) $a + b = 18$ b) $a + b = \sqrt{2}$ c) $a - b = 4\sqrt{2}$ d) $a \cdot b = 73$

443. If the circle $x^2 + y^2 = a^2$ intersect the hyperbola $xy = c^2$ in four points $P(x_1, y_1), Q(x_2, y_2), R(x_3, y_3), S(x_4, y_4)$, then

a) $x_1 + x_2 + x_3 + x_4 = 0$

b) $y_1 + y_2 + y_3 + y_4 = 0$

c) $x_1x_2x_3x_4 = c^4$

d) $y_1y_2y_3y_4 = c^4$

444. If a pair of variable straight lines $x^2 + 4y^2 + \alpha xy = 0$ (where α is a real parameter) cut the ellipse

$x^2 + 4y^2 = 4$ at two points A and B, then the locus of the point of intersection of tangents at A and B is

a) $x - 2y = 0$ b) $2x - y = 0$ c) $x + 2y = 0$ d) $2x + y = 0$

445. Point M moved on the circle $(x-4)^2 + (y-8)^2 = 20$. Then it broke away from it and moving along a tangent to the circle, cuts the x-axis at the point $(-2, 0)$. The co-ordinates of a point on the circle at which the moving point broke away is

a) $\left(\frac{42}{5}, \frac{36}{5}\right)$ b) $\left(-\frac{2}{5}, \frac{44}{5}\right)$ c) $(6, 4)$ d) $(2, 4)$

446. The range of values of 'a' such that the angle θ between the pair of tangent drawn from $(a, 0)$ to the circle $x^2 + y^2 = 1$ satisfies $\frac{\pi}{2} < \theta < \pi$, lies in

a) $(1, 2)$ b) $(1, \sqrt{2})$ c) $(-\sqrt{2}, -1)$ d) $(-2, -1)$

447. If the circles $x^2 + y^2 - 9 = 0$ and $x^2 + y^2 + 2ax + 2y + 1 = 0$ touch each other, then $a =$

a) $-4/3$ b) 0 c) 1 d) $4/3$

448. If two concentric ellipses are such that the foci of one are on the other and their major axes are equal. Let e and e' be their eccentricities, then

a) The quadrilateral formed by joining the foci of the two ellipses is a parallelogram

b) The angle θ between their axes is given by $\theta = \cos^{-1}\sqrt{\frac{1}{e^2} + \frac{1}{e'^2} - \frac{1}{e^2 e'^2}}$

c) If $e^2 + e'^2 = 1$, then the angel between the axes of the two ellipses is 90^0

d) None of these

449. If $P(x_1, y_1), Q(x_2, y_2), R(x_3, y_3)$ *and* $S(x_4, y_4)$ are four concyclic points on the rectangular hyperbola $xy = c^2$, then coordinates of the orthocenter of the ΔPQR are

a) $(x_4, -y_4)$ b) (x_4, y_4) c) $(-x_4, -y_4)$ d) $(-x_4, y_4)$

450. If $y = 2$ be the directrix and (0,1) be the vertex of the parabola $x^2 + \lambda y + \mu = 0$ then

a) $\lambda = 4$ b) $\mu = 8$ c) $\lambda = -8$ d) $\mu = 4$

451. The tangents from which of the following points to the ellipse $5x^2 + 4y^2 = 20$ are perpendicular?

a) $(1, 2\sqrt{2})$ b) $(2\sqrt{2}, 1)$ c) $(2, \sqrt{5})$ d) $(\sqrt{5}, 2)$

452. Let P be a point whose coordinated differ by unity and the point does not lie on any of the axes of reference. If the parabola $y^2 = 4x + 1$ passes through P, then the ordinate of P may by

a) 3 b) -1 c) 5 d) 1

453. The equation of the tangents drawn from the origin to the circle $x^2 + y^2 - 2rx - 2hy + h^2 = 0$, are

a) $x = 0$ b) $y = 0$

c) $(h^2 - r^2)x - 2rhy = 0$ d) $(h^2 - r^2)x + 2rhy = 0$

454. The locus of the midpoint of the focal distance of a variable point moving on the parabola, $y^2 = 4ax$ is a parabola whose

a) Latus rectum is half the latus rectum of the original parabola

b) Vertex is $\left(\frac{a}{2}, 0\right)$

c) Directrix is y-axis

d) Focus has the co-ordinates $(a, 0)$

455. The circles $x^2 + y^2 + 2x + 4y - 20 = 0$ and $x^2 + y^2 + 6x - 8y + 10 = 0$

a) Are such that the number of common tangents on them is 2

b) Are orthogonal

c) Are such that the length of their common tangent is $5(12/5)^{1/4}$

d) Are such that the length of their common chord is $5\sqrt{\frac{3}{2}}$

456. The extremities of latus rectum of a parabola are (1,1) and (1,–1), then the equation of the parabola can be

a) $y^2 = 2x - 1$ b) $y^2 = 1 - 2x$ c) $y^2 = 2x - 3$ d) $y^2 = 2x - 4$

457. Which of the following is/are true?

a) There are infinite positive integral values of a for which $(13x - 1)^2 + (13y - 2)^2 = \left(\frac{5x + 12y - 1}{a}\right)^2$

represents an ellipse

b) The minimum distance of a point $(1, 2)$ from the ellipse $4x^2 + 9y^2 + 8x - 36y + 4 = 0$ is 1

c) If from a point $P(0, \alpha)$ two normlas other than axes are drawn to the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, then $|\alpha| < \frac{9}{4}$

d) If the length of latus rectum of an ellipse is one-third of its major axis, then its eccentricity is equal to $\frac{1}{\sqrt{3}}$

458. The equation of tangent parallel to $y = x$ drawn to $\frac{x^2}{3} - \frac{y^2}{2} = 1$ is

a) $x - y + 1 = 0$ b) $x - y - 2 = 0$ c) $x + y - 1 = 0$ d) $x - y - 1 = 0$

459. A $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ is a point on the circle $x^2 + y^2 = 1$ and B is another point on the circle such that arc length $AB = \frac{\pi}{2}$ units. Then, co-ordinates of B can be

a) $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ b) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ c) $\left(-\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ d) None of these

460. The equation of the directrix of the parabola with vertex at the origin and having the axis along the x-axis and a common tangent of slope 2 with the circle $x^2 + y^2 = 5$ is/are

a) $x = 10$ b) $x = 20$ c) $x = -10$ d) $x = -20$

461. Circles are drawn on chords of the rectangular hyperbola $xy = 4$ parallel to the line $y = x$ as diameters. All such circles pass thorough two fixed points whose coordinates are

a) $(2, 2)$ b) $(2, -2)$ c) $(-2, 2)$ d) $(-2, -2)$

462. If the chord through the points whose eccentric angles are θ and ϕ on the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ passes through a focus, then the calue of $\tan(\theta/2)\tan(\phi/2)$ is

a) $\frac{1}{9}$ b) -9 c) $-\frac{1}{9}$ d) 9

463. From point $(2, 2)$ tangents are drawn to the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ then point of contact lies in

a) I quadrant b) II quadrant c) quadrant d) IV quadrant

464. The locus of the point of intersection of two perpendicular tangents to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

a) Director circle b) $x^2 + y^2 = a^2$ c) $x^2 + y^2 = a^2 - b$ d) $x^2 + y^2 = a^2 + b^2$

465. For the hyperbola $9x^2 - 16y^2 - 18x + 32y - 151 = 0$

a) One of the directrix is $x = \frac{21}{5}$ b) Length of latus rectum $= \frac{9}{2}$

c) Focii are $(6, 1)$ and $(-4, 1)$ d) Eccentricity is $\frac{5}{4}$

466. If a circle passes through the point of intersection of the lines $x + y + 1 = 0$ and $x + \lambda y - 3 = 0$ with the co-ordinates axes, then

a) $\lambda = -1$ b) $\lambda = 1$

c) $\lambda = 2$ d) λ can have any real value

467. The points, where the normals to the ellipse $x^2 + 3y^2 = 37$ be parallel to the line $6x - 5y + 7 = 7$ is/are

a) $(5, 2)$ b) $(2, 5)$ c) $(1, 3)$ d) $(-5, -2)$

468. A point on the ellipse $x^2 + 3y^2 = 37$ where the normal is parallel to the line $6x - 5y = 2$ is

a) $(5, -2)$ b) $(5, 2)$ c) $(-5, 2)$ d) $(-5, -2)$

469. The equations of tangents to the circles $x^2 + y^2 - 6x - 6y + 9 = 0$ drawn from the origin are

a) $x = 0$ b) $x = y$ c) $y = 0$ d) $x + y = 0$

470. Tangent is drawn at any point (x_1, y_1) other than vertex on the parabola $y^2 = 4ax$. If tangents are drawn from any point on this tangent to the circle $x^2 + y^2 = a^2$ such that all the chords of contact pass through a fixed point (x_2, y_2), then

a) x_1, a, x_2 are in G.P. b) $\frac{y_1}{2}, a, y_2$ are in G.P.

c) $-4, \frac{y_1}{y_2}, \frac{x_1}{x_2}$ are in G.P. d) $x_1x_2 + y_1y_2 = a^2$

471. If $(5, 12)$ and $(24, 7)$ are the foci of a hyperbola passing through the origin, then

a) $e = \frac{\sqrt{386}}{12}$ b) $e = \frac{\sqrt{386}}{13}$ c) $LR = \frac{121}{6}$ d) $LR = \frac{121}{3}$

472. For hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, let n be the number of points on the plane through which perpendicular tangents are drawn

a) If $n=1$, then $e=\sqrt{2}$
b) If $n>1$, then $0<e<\sqrt{2}$
c) If $n=0$, then $e>\sqrt{2}$
d) None of these

473. Let A and B be two distinct points on the parabola $y^2=4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be

a) $-\frac{1}{r}$
b) $\frac{1}{r}$
c) $\frac{2}{r}$
d) $-\frac{2}{r}$

474. If the two intersecting lines intersect the hyperbola and neither of them is a tangent to it, then number of intersecting points are

a) 1
b) 2
c) 3
d) 4

475. If $(5,12)$ and $(24,7)$ are the foci of a conic passing through the origin, then the eccentricity of conic is

a) $\frac{\sqrt{386}}{38}$
b) $\frac{\sqrt{386}}{12}$
c) $\frac{\sqrt{386}}{13}$
d) $\frac{\sqrt{386}}{25}$

476. The equation $\left|\sqrt{x^2+(y-1)^2}-\sqrt{x^2+(y+1)^2}\right|=K$ will represent a hyperbola for

a) $K\in(0,2)$
b) $K\in(-2,1)$
c) $K\in(1,\infty)$
d) $K\in(0,\infty)$

477. If $x,y\in R$ then the equation $3x^4-2(19y+8)x^2+(361y^2+2(100+y^4)+64)=2(190y+2y^2)$ represents in rectangular Cartesian system

a) Parabola
b) Hyperbola
c) Circle
d) Ellipse

478. The equation $3x^2+4y^2-18x+16y+43=k$

a) Represents empty set, if $k<0$
b) Represents an ellipse, if $k>0$
c) A point, if $k=0$
d) Cannot represent a real pair of straight lines for any value of k

479. For which of the hyperbolas, we can have more than one pair of perpendicular tangents?

a) $\frac{x^2}{4}-\frac{y^2}{9}=1$
b) $\frac{x^2}{4}-\frac{y^2}{9}=-1$
c) $x^2-y^2=4$
d) $xy=44$

480. If the normal at P to the rectangular hyperbola $x^2-y^2=4$ meets the axes in G and g and C is the centre of the hyperbola, then

a) $PG=PC$
b) $Pg=PC$
c) $PG=Pg$
d) $Gg=2PC$

481. The distance of a point on the ellipse $\frac{x^2}{6}+\frac{y^2}{2}=1$ from the centre is 2. Then the eccentric angel of the point is

a) $\frac{\pi}{4}$
b) $\frac{3\pi}{4}$
c) $\frac{5\pi}{6}$
d) $\pi/6$

482. If $(5,12)$ and $(24,7)$ are the foci of a conic passing through the origin the eccentricity of conic is

a) $\frac{\sqrt{386}}{12}$
b) $\frac{\sqrt{386}}{13}$
c) $\frac{\sqrt{386}}{25}$
d) $\frac{\sqrt{386}}{38}$

483. If the area of the quadrilateral formed by the tangents from the origin to the circle $x^2+y^2+6x-10y+c=0$ and the radii corresponding to the points of contact is 15, then a value of c is

a) 9
b) 4
c) 5
d) 25

484. A square has one vertex at the vertex of the parabola $y^2=4ax$ and the diagonal through the vertex lies along the axis of the parabola. If the ends of the other diagonal lie on the parabola, the coordinates of the vertices of the square are

a) $(4a,4a)$
b) $(4a,-4a)$
c) $(0,0)$
d) $(8a,0)$

485. If the circles $x^2+y^2-9=0$ and $x^2+y^2+2ax+2y+1=0$ touch each other, then α is

a) $-\frac{4}{3}$
b) 0
c) 1
d) $\frac{4}{3}$

486. The equation of a circle of radius 1 touching the circles $x^2+y^2-2|x|=0$ is

a) $x^2+y^2+2\sqrt{2}\,x+1=0$
b) $x^2+y^2-2\sqrt{3}\,y+2=0$

c) $x^2+y^2+2\sqrt{3}\,y+2=0$ d) $x^2+y^2-2\sqrt{2}\ +1=0$

487. The equations of the tangents drawn from the origin to the circle $x^2+y^2-2rx-2hy+h^2=0$, are

a) $x=0$ b) $y=0$

c) $(h^2-r^2)x-2rhy=0$ d) $(h^2-r^2)x+2rhy=0$

488. If the tangent at the point $P(\theta)$ to the ellipse $16x^2+11y^2=256$ is also a tangent to the circle $x^2+y^2-2x=15$, then $\theta=$

a) $\frac{2\pi}{3}$ b) $\frac{4\pi}{3}$ c) $\frac{5\pi}{3}$ d) $\frac{\pi}{3}$

489. The equation of a tangent to the circle $x^2+y^2=25$ passing through $(-2,11)$ is

a) $4x+3y=15$ b) $3x+4y=38$ c) $24x-7y+125=0$ d) $7x+24y=250$

490. The equation of tangent to the ellipse $x^2+3y^2=3$ which is perpendicular to the line $4y=x-5$ is

a) $4x+y+7=0$ b) $4x+y-7=0$ c) $4x+y+3=0$ d) $4x+y-3=0$

491. Which of the following line can be tangent to parabola $y^2=8x$?

a) $x-y+2=0$ b) $9x-3y+2=0$ c) $x+2y+8=0$ d) $x+3y+12=0$

492. The lines parallel to normal to the curvy $xy=1$ is/are

a) $3x+4y+5=0$ b) $3x-4y+5=0$ c) $4x+3y+5=0$ d) $3y-4x+5=0$

493. If the circle $x^2+y^2+2x+2ky+6=0$ and $x^2+y^2+2ky+k=0$ intersect orthogonally, then k is

a) 2 b) -2 c) $-\frac{3}{2}$ d) $\frac{3}{2}$

494. A quadrilateral is inscribed in parabola, then

a) Quadrilateral may be cyclic

b) Diagonal of the quadrilateral may be equal

c) All possible pairs of adjacent sides may be perpendicular

d) None of these

495. The locus of the image of the focus of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1(a>b)$ with respect to any of the tangents to the ellipse is

a) $(x+4)^2+y^2=100$ b) $(x+2)^2+y^2=50$

c) $(x-4)^2+y^2=100$ d) $(x-2)^2+y^2=50$

496. If foci of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ coincide with the foci of $\frac{x^2}{25}+\frac{y^2}{9}=1$ and eccentricity of the hyperbola is 2, then

a) $a^2+b^2=16$ b) There is no director circle to the hyperbola

c) Centre of the director circle is (0, 0) d) Length of latus rectum of the hyperbola = 12

497. If P is a point on a hyperbola, then

a) locus of excentre of the circle described opposite to $\angle P$ for $\Delta PSS'(S,S'$are foci) is tangents at vertex

b) locus of excentre of the circle described opposite to $\angle S'$ is hyperbola

c) locus of excentre of the circle described opposite to $\angle P$ for $\Delta PSS'(S,S'$are foci), is hyperbola

d) locus of excentre of the circle described opposite to $\angle S'$, is tangent at vertex

498. The equation of the tangent to the parabola $y^2=9x$ which goes through the point $(4,10)$ is

a) $x+4y+1=0$ b) $9x+4y+4=0$ c) $x-4y+36=0$ d) $9x-4y+4=0$

499. If equation $x^2+y^2+2hxy+2gx+2fy+c=0$ represents a circle, then the condition for that circle to pass through three quadrants only but not passing through the origin is

a) $f^2>c$ b) $g^2>c$ c) $c>0$ d) $h=0$

500. The differential equation $\frac{dy}{dx}=\frac{3y}{2x}$ represents a family of hyperbolas (except when it represents a pair of lines) with eccentricity

a) $\sqrt{\frac{3}{5}}$ b) $\sqrt{\frac{5}{3}}$ c) $\sqrt{\frac{2}{5}}$ d) $\sqrt{\frac{5}{2}}$

501. If the circle $x^2+y^2=a^2$ intersects of the hyperbola $xy=c^2$ in four points $P(x_1,y_1),Q(x_2,y_2),R(x_3,y_3),S(x_4,y_4)$ then

a) $x_1+x_2+x_3+x_4=0$ b) $y_1+y_2+y_3+y_4=0$

c) $x_1x_2x_3x_4 = c^4$ d) $y_1y_2y_3y_4 = c^4$

502. If equation of directrix of the parabola $x^2 + 4y - 6x + k = 0$ is $y + 1 = 0$, then

a) $k = 17$ b) $k = -17$ c) Focus is $(3, -3)$ d) Vertex is $(3, -3)$

503. On the $x - y$ plane, the eccentricity of an ellipse is fixed (in size and position) by

a) Both foci b) Both directrices

c) One focus and corresponding directrix d) Length of major axis

504. Let F_1, F_2 be two foci of the ellipse and PT and PN be the tangent and the normal respectively to the ellipse at point P. Then,

a) PN bisects $\angle F_1PF_2$ b) PT bisects $\angle F_1PF_2$

c) PT bisects angle $(180° - \angle F_1PF_2)$ d) None of the above

505. Let E_1 and E_2 be two ellipses $\frac{x^2}{a^2} + y^2 = 1$ and $x^2 + \frac{y^2}{a^2} = 1$ (where a is a parameter). Then the locus of the points of intersection of the ellipses E_1 and E_2 is a set of curves comprising

a) Two straight lines b) One straight line c) One circle d) One parabola

506. The line $y = x + 5$ touches

a) The parabola $y^2 = 20x$ b) The ellipse $9x^2 + 16y^2 = 144$

c) The hyperbola $\frac{x^2}{29} - \frac{y^2}{4} = 1$ d) The circle $x^2 + y^2 = 25$

Assertion - Reasoning Type

This section contain(s) 0 questions numbered 507 to 506. Each question contains STATEMENT 1(Assertion) and STATEMENT 2(Reason). Each question has the 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

a) Statement 1 is True, Statement 2 is True; Statement 2 **is** correct explanation for Statement 1

b) Statement 1 is True, Statement 2 is True; Statement 2 **is not** correct explanation for Statement 1

c) Statement 1 is True, Statement 2 is False

d) Statement 1 is False, Statement 2 is True

507

Statement 1: Ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and $12x^2 - 4y^2 = 27$ intersect each other at right angle

Statement 2: Given ellipse and hyperbola have same foci

508 Consider two circles $x^2 + y^2 - 4x - 6y - 8 = 0$ and $x^2 + y^2 - 2x - 3 = 0$

Statement 1: Both circles intersect each other at two distinct points

Statement 2: Sum of radii of two circles in greater than distance between the centres of two circles

509

Statement 1: The normals at the point (4, 4) and $\left(\frac{1}{4}, -1\right)$ of the parabola $y^2 = 4x$ are perpendicular

Statement 2: The tangents to the parabola at the end of a focal chord are perpendicular

510 Observe the following statements

Statement 1: The circle $x^2 + y^2 - 6x - 4y - 7 = 0$ touches y-axis

Statement 2: The circle $x^2 + y^2 + 6x + 4y - 7 = 0$ touches x-axis

511

Statement 1: Number of circles touching lines $x + y = 1, 2x - y = 5$ and $3x + 5y - 1 = 0$ is four

Statement 2: In any triangle, four circles can be drawn touching all the three sides of triangle

512

Statement 1: Locus of the centre of a variable circle touching two circles $(x + 1)^2 + (y - 2)^2 = 25$ and $(x - 2)^2 + (y - 1)^2 = 16$ is an ellipse

Statement 2: If a circle $S_2 = 0$ lies completely inside the circle $S_1 = 0$, then locus of the centre of a variable circle $S = 0$ that touches both the circles is an ellipse

513

Statement 1: If normal at the ends of double ordinate $x = 4$ of parabola $y^2 = 4x$ meet the curve again at P and P' respectively, then $PP' = 12$ unit

Statement 2: If normal at t_1 and $y^2 = 4ax$ meet the parabola again at $t_2, t_2 = t_1 - \frac{2}{t_1}$

514

Statement 1: For the ellipse $\frac{x^2}{5} + \frac{y^2}{3} = 1$, the product of the perpendiculars drawn from foci on any tangent is 3

Statement 2: For ellipse $\frac{x^2}{5} + \frac{y^2}{3} = 1$, the foot of the perpedicualrs drawn from foci on any tangent lies on the cirlce $x^2 + y^2 = 5$ which is an auxiliary circle of the ellipse

515

Statement 1: If straight line $x = 8$ meets the parabola $y^2 = 8x$ at P and Q, then PQ substends a right angle at the origin

Statement 2: Double ordinate equal to twice of latus rectum of a parabola subtends a right angle at the vertex

516

Statement 1: In parabola $y^2 = 4ax$, the circle drawn taking focal radii as diameter touches y-axis

Statement 2: The portion of the tangent intercepted between point of contact and directix subtends $90°$ angle at focus

517

Statement 1: There are no common tangents between circle $x^2 + y^2 - 4x + 3 = 0$ and parabola $y^2 = 2x$

Statement 2: Given circle and parabola do not intersect

518

Statement 1: The values of α for which the point (α, α^2) lies inside the triangle formed by the lines $x = 0, x + y = 2$ and $3y = x$ is $(0,1)$

Statement 2: Parabola $y = x^2$ meets the line $x + y = 2$ at $(1, 1)$

519

Statement 1: A hyperbola whose asymptotes include a right angle is said to be equilateral hyperbola.

Statement 2: Eccentricity of an equilateral hyperbola is $\sqrt{2}$.

520

Statement 1: Equation of circle through the origin and belonging to the co-axial system of which the limiting points are (1, 1) and (3, 3) is $2x^2 + 2y^2 - 3x - 3y = 0$

Statement 2: Equation of a circle passing through the point (1, 1) and (3, 3) is $x^2 + y^2 - 2x - 6y + 6 = 0$

521

Statement 1: The locus of a moving point (x, y) satisfying $\sqrt{(x-2)^2 + y^2} + \sqrt{(x-2)^2 + y^2} = 4$ is ellipse

Statement 2: Distance between $(-2, 0)$ and $(2, 0)$ is 4

522

Statement 1: From the point $(\lambda, 3)$ tangents are drawn to $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and are perpendicular to each other, then $\lambda = \pm 2$

Statement 2: The locus of point of intersection of perpendicular tangents to $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is $x^2 + 3y = 13$

523

Statement 1: In a ΔABC, if base BC is fixed and perimeter of triangle is constant, then vertex A move on an ellipse

Statement 2: If sum of distance of a point P from two fixed points is constant, then locus of P is a real ellipse

524 Let C_1 be the circle with centre O_1 (0, 0) and radius 1 and C_2 be the circle with centre $O_2(t, t^2 + 1)(t \in R)$ and radius 2

Statement 1: Circles C_1 and C_2 always have at least one common tangent for any value of t

Statement 2: For the two circles, $O_1O_2 \geq |r_1 - r_2|$, where r_1 and r_2 are their radii for any value of t

525

Statement 1: Line $x - y - 5 = 0$ cannot be normal to parabola $(5x - 15)^2 + (5y + 10)^2 = (3x - 4y + 2)^2$

Statement 2: Normal to parabola never passes through its focus

526

Statement 1: Through $(\lambda, \lambda + 1)$ there can't be more than one normal to the parabola $y^2 = 4x$, if $\lambda < 2$

Statement 2: The point $(\lambda, \lambda + 1)$ lies outside the parabola for all $\lambda \neq 1$

527

Statement 1: Asymptotes of hyperbola $3x + 4y = 2$ and $4x - 3y = 5$ are bisectors of transverse and conjugate axes of hyperbola

Statement 2: Transverse and conjugate axes of hyperbola are bisectors of the asymptotes

528

Statement 1: The line $bx - ay = 0$ will not meet the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $(a > b > 0)$.

Statement 2: The line $y = mx + c$ does not meet the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, if $c^2 = a^2m^2 - b^2$.

529

Statement 1: If a, b are real numbers and $c > 0$, then the locus represented by the equation $|ay - bx| = c\sqrt{(x-a)^2 + (y-b)^2}$ is an ellipse

Statement 2: An ellipse is the locus of a point which moves in a plane such that ratio of its distances from a fixed point (i.e., focus) to the fixed line (i.e., directrix) is constant and less than 1

530

Statement 1: Two orthogonal circles intersect to generate a common chord which subtends complimentary angles at their circumferences

Statement 2: Two orthogonal circles intersect to generate a common chord which subtends supplementary angles at their centres

531

Statement 1: If circle with centre $P(t, 4-2t), t \in R$ cuts the circles $x^2 + y^2 = 16$ and $x^2 + y^2 - 2x - y - 12 = 0$; then both the intersections are orthogonal

Statement 2: Length of tangent from P for $t \in R$ is same for both the given circles

532

Statement 1: Tangents are drawn to ellipse $\frac{x^2}{4} + \frac{y^2}{2} = 1$ at the points, where it is intersected by the line $2x + 3y = 1$. Point of intersection of these tangents is $(8, 6)$

Statement 2: Equation of the chord of contact to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ from an external point is given by $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1 = 0$

533

Statement 1: The equations of the straight lines joining origin to the points of intersection of $x^2 + y^2 - 4x - 2y = 4$ and $x^2 + y^2 - 2x - 4y - 4 = 0$ is $x - y = 0$

Statement 2: $y + x = 0$ is common chord of $x^2 + y^2 - 4x - 2y = 4$ and $x^2 + y^2 - 2x - 4y - 4 = 0$

534 Tangents are drawn from the point $(17, 7)$ to the circle $x^2 + y^2 = 169$

Statement 1: The tangents are mutually perpendicular

Statement 2: The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$

535

Statement 1: The sum of focal distances of a point on the ellipse $9x^2 + 4y^2 - 18x - 24y + 9 = 0$ is 4.

Statement 2: The equation $9x^2 + 4y^2 - 18x - 24y + 9 = 0$ can be expressed as $9(x-1)^2 + 4(y-3)^2 = 36$.

536

Statement 1: The circle $x^2 + y^2 + 2px + r = 0, x^2 + y^2 + 2qy + r = 0$ touch, if $\frac{1}{p^2} + \frac{1}{q^2} = \frac{1}{r}$

Statement 2: Two circles with centre C_1, C_2 and radii r_1, r_2 touch each other if $|r_1 \pm r_2| = c_1c_2$

537

Statement 1: Length of focal chord of a parabola $y^2 = 8x$ making on angle of $60°$ with x-axis is 32

Statement 2: Length of focal chord of a parabola $y^2 = 4ax$ making an angle a with x-axis is $4a \operatorname{cosec}^2\alpha$

538

Statement 1: The equation $x^2 + y^2 - 2x - 2ay - 8 = 0$ represents, for different values of 'a', a system of circles passing through two fixed points lying on the x-axis

Statement 2: $S = 0$ is a circle and $L = 0$ is a straight line, then $S + \lambda L = 0$ represents the family of circles passing through the points of intersection of circle and straight line (where λ is arbitrary parameter)

539

Statement 1: The chord of contact of tangent from three points A, B, C to the circle $x^2 + y^2 = a^2$ are concurrent, then A, B, C will be collinear

Statement 2: Lines $(a_1x + b_1y + c_1) + k(a_2x + b_2y + c_2) = 0$ always pass through a fixed point for $k \in R$

540

Statement 1: The equation of the director circle to the hyperbola $4x^2 - 3y^2 = 12$ is $x^2 + y^2 = 1$.

Statement 2: Director circle is the locus of the point of intersection of perpendicular tangents to a hyperbola.

541

Statement 1: Circle $x^2 + y^2 = 9$, and the circle $\left(x - \sqrt{5}\right)\left(\sqrt{2}x - 3\right) + y\left(\sqrt{2}y - 2\right) = 0$ touches each other internally

Statement 2: Circle described on the focal distance as diameter of ellipse $4x^2 + 9y^2 = 36$ touches the auxiliary circle $x^2 + y^2 = 9$ internally

542

Statement 1: Number of circles passing through (1, 2), (4, 8) and (0, 0) is one

Statement 2: Every triangle has one circumcircle

543

Statement 1: Diagonals of any parallelogram inscribed in an ellipse always intersect at the centre of the ellipse

Statement 2: Centre of the ellipse is the point at which chord passing through the centre of the ellipse gets bisected at the centre

544 For the parabola $y^2 + 6y - 2x + 5 = 0$

Statement 1: The vertex is $(-2, -3)$

Statement 2: The directrix is $y + 3 = 0$

545

Statement 1: The smaller possible radius of circle which pass through (1, 0) and(0, 1) is $\frac{1}{\sqrt{2}}$

Statement 2: Circle passes through origin

546

Statement 1: Feet of perpendiculars drawn from foci of an ellipse $4x^2 + y^2 = 16$ on the line $2\sqrt{3}x + y = 8$ lie on the circle $x^2 + y^2 = 16$

Statement 2: If perpendiculars are drawn from foci of an ellipse to its tangent, then feet of there perpendicular lies on director circle of the ellipse

547

Statement 1: A ray of light incident at the point $(-3,-1)$ gets reflected from the tangent at $(0,-1)$ to the circle$x^2 + y^2 = 1$. If the reflected ray touches the circle, then equation of the reflected ray is $4y - 3x = 5$

Statement 2: The angle of incidence =angle of reflection $ie, \angle i = \angle r$

548

Statement 1: If a circle $\mathrm{S} = 0$ intersects a hyperbola $xy = 4$ at four points. Three of them are (2, 2), (4, 1) and (6, 2/3) then coordinates of the fourth point are (1/4, 16)

Statement 2: If a circles $\mathrm{S} = 0$ intersects a hyperbola $xy = c^2$ at t_1, t_2, t_3, t_4, then $t_1 - t_2 - t_3 - t_4 = 1$

549

Statement 1: If end points of two normal chords AB and CD (normal at A and C) of a parabola $y^2 = 4ax$ are concyclic, then the tangents at A and C will intersect on the axis of the parabola

Statement 2: If four point on the parabola $y^2 = 4ax$ are concyclic, then sum of their ordinates is zero

550

Statement 1: Circle $x^2 + y^2 - 6x - 4y + 9 = 0$ bisects the circumference of the circle $x^2 + y^2 - 8x - 6y + 23 = 0$

Statement 2: Centre of first circle lie on the second circle

551

Statement 1: The major and minor axes of the ellipse $5x^2 + 9y^2 - 54y + 36 = 0$ are 6 and 10 respectively.

Statement 2: The equation $5x^2 + 9y^2 - 54y + 36 = 0$ can be expressed as $5x^2 + 9(y - 3)^2 = 45$.

552

Statement 1: The point $(a, -a)$ lies inside the circle $x^2 + y^2 - 4x + 2y - 8 = 0$ whenever $a \in (-1, 4)$

Statement 2: Point (x_1, y_1) lies inside the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, if $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c < 0$

553

Statement 1: Chord of contact of the circle $x^2 + y^2 = 1$ w.r.t. points (2, 3), (3, 5) and (1, 1) are concurrent

Statement 2: Points (1, 1), (2, 3) and (3, 5) are collinear

554

Statement 1: Every line which cuts the hyperbola in two distinct points has slope lies in $(-2,2)$

Statement 2: Slope of tangents of hyperbola lies in $(-\infty, -2) \cup (2, \infty)$

555

Statement 1: If line $x + y = 3$ is a tangent to an ellipse with foci $(4, 3)$ and $(6, y)$ at the point $(1, 2)$ then $y = 17$

Statement 2: Tangent and normal to the ellipse at any point bisects the angle subtended by foci at that point

556

Statement 1: A triangle ABC right angled at A moves so that its perpendicular sides touch the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ all the time. Then loci of the points A, B and C are circle

Statement 2: Locus of point of intersection of two perpendicular tangents to the curve is a director circle

557

Statement 1: The point $(5, -4)$ is inside the hyperbola $y^2-9x^2+1=0$.

Statement 2: The point (x_1, y_1) is inside the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, then $\frac{x_1^2}{a^2}-\frac{y_1^2}{b^2}-1<0$.

558

Statement 1: The lines from the vertex to the two extremities of a focal chord of the parabola $y^2=4ax$ are at an angle of $\frac{\pi}{2}$

Statement 2: If extremities of focal chord of parabola are $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$, then $t_1t_2=-1$

559

Statement 1: Centre of the circle having $x+y=3$ and $x-y=1$ as its normal is $(1, 2)$

Statement 2: Normals to the circle always passes through its centre

560

Statement 1: There are infinite points from which two mutually perpendicular tangents can be drawn to the hyperbola $\frac{x^2}{9}-\frac{y^2}{16}=1$

Statement 2: The locus of point of intersection of perpendicular tangents lies on the circle

561

Statement 1: The equation of the image of the circle $x^2+y^2+16x-24y+183=0$ by the line mirror $4x+7y+13=0$ is $x^2+y^2+32x+4y+235=0$

Statement 2: Equation of perpendicular bisector of line segment joining the points $(-8, 12)$ and $(-16, -2)$ is $4x+7y+13=0$

562

Statement 1: The point of intersection of the tangents at three distinct points A, B, C on the parabola $y^2=4x$ can be collinear

Statement 2: If a line L does not intersect the parabola $y^2=4x$, then from every point of the line two tangents can be drawn to the parabola

563

Statement 1: Normal chord drawn at the point $(8, 8)$ of the parabola $y^2=8x$ subtends a right angle at the vertex of the parabola

Statement 2: Every chord of the parabola $y^2=4ax$ passing through the point $(4a, 0)$ subtends a right angle at the vertex of the parabola

564 Consider $L_1: 2x+3y+p-3=0$ $L_2: 2x+3y+p+3$, where p is a real number and $C: x^2+y^2+6x-10y+30=0$

Statement 1: If line L_1 is a chord of circle C, then L_2 is not always a diameter of circle C

Statement 2: If line L_1 is a diameter of circle C, then L_2 is not a chord of circle C

565

Statement 1: There can be maximum two points on the line $px + qy + r = 0$, from which perpendicular tangents can be drawn to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Statement 2: Circle $x^2 + y^2 = a^2 + b^2$ and the given line can intersect in maximum two distinct points

566

Statement 1: The equation of the tangents drawn at the ends of the major axis of the ellipse $9x^2 + 5y^2 - 30y = 0$ is $y = 0, y = 6$

Statement 2: The equation of the tangent drawn at the ends of major axis of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is always parallel to y- axis

567

Statement 1: In an ellipse the sum of the distances between foci is always less than the sum of focal distances of any point on it

Statement 2: The eccentricity of any ellipse is less than 1

568

Statement 1: If a triangle ABC, if base BC is fixed and perimeter of the triangle is constant, then vertex A moves on an ellipse

Statement 2: If the sum of distances of a point P from two fixed points is constant, then locus of P is a real ellipse

569

Statement 1: Director circle of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} + 1 = 0$ is defined only when $b > a$.

Statement 2: Director circle of hyperbola $\frac{x^2}{25} - \frac{y^2}{9} = 1$ is $x^2 + y^2 = 16$.

570

Statement 1: If there exists points on the circle $x^2 + y^2 = a^2$ from which two perpendicular tangents can be drawn to parabola $y^2 = 2x$, then $a \geq 1/2$

Statement 2: Perpendicular tangents to parabola meet on the directrix

571

Statement 1: Equations of tangents to the hyperbola $2x^2 - 3y^2 = 6$ which is parallel to the line $y = 3x + 4$ is $y = 3x - 5$ and $y = 3x + 5$

Statement 2: For given slope two parallel tangents can be drawn to the hyperbola

572

Statement 1: Circles $x^2 + y^2 = 144$ and $x^2 + y^2 - 6x - 8y = 0$ do not have any common tangent

Statement 2: If tow circles are concentric, then they do not have common tangents

573

Statement 1: Points $A(1, 0)$, $B(2, 3)$, $C(5, 3)$ and $D(6, 0)$ are concyclic

Statement 2: Points A, B, C, D forms isosceles trapezium or AB and CD meet in E then $EA \cdot EB = EC \cdot ED$

574

Statement 1: The line $y = x + 2a$ touches the parabola $y^2 = 4a(x + a)$

Statement 2: The line $y = mx + am + a/m$ touches $y^2 = 4a(x + a)$ for all real values of m

575

Statement 1: Circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 8x + 7 = 0$ intersect each other at two distinct points

Statement 2: Circles with centres C_1 and C_2 and radii r_1 and r_2 intersect at two distinct points, if $|C_1C_2| < r_1 + r_2$

576

Statement 1: The condition on a and b for which two distinct chord of the ellipse $\frac{x^2}{2a^2} + \frac{y^2}{2b^2} = 1$ passing through $(a, -b)$ are bisected by the line $x + y = b$ is $a^2 + 6ab - 7b^2 \geq 0$

Statement 2: Equation of chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose mid point (x_1, y_1), is $T = S_1$

577

Statement 1: Tangents drawn from any point on the circle $x^2 + y^2 = 13$ to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ are at right angles

Statement 2: Equation of the auxiliary circle of the ellipse $\frac{x^2}{13} + \frac{y^2}{4} = 1$ is $x^2 + y^2 = 13$

578

Statement 1: If there is exactly one point on the line $3x + 4y + 5\sqrt{5} = 0$, from which perpendicular tangents can be drawn to the ellipse $\frac{x^2}{a^2} + y^2 = 1$ $(a > 1)$, then the eccentricity of the ellipse is $\frac{1}{3}$

Statement 2: For the condition given in statement 1, given line must touch the circle $x^2 + y^2 = a^2 + 1$

579

Statement 1: The line $ax + by + c = 0$ is a normal to the parabola $y^2 = 4ax$, then the equation of tangent at the foot of this normal is $y = (b/a)x + (a^2/b)$

Statement 2: Equation of normal at any point $P(at^2, 2at)$ to the parabola $y^2 = 4ax$ is $y = -tx + 2at + at^3$

580

Statement 1: Let $(2, \sqrt{2})$ be any point on hyperbola$x^2 - y^2 = 2$, then the product of distance of foci from P is equal to 6.

Statement 2: If S and S' be the foci, C the centre and P be any point on a hyperbola $x^2 - y^2 = a^2$, then $SP \cdot S'P = CP^2$.

581

Statement 1: Any chord of the conic $x^2 + y^2 + xy = 1$ through $(0, 0)$ is bisected at (0,0).
2 the centre of a conic is point through which every chord is bisected at $(0, 0)$

Statement 2: The centre of a conic is a point through which every chord is bisected

582

Statement 1: The product of the focal distances of a point on an ellipse is equal to the square of the semi-diameter which is conjugate d the diameter through the point

Statement 2: If $y = mx$ and $y = m_1x$ be two conjugate diameters of an ellipse, then $mm_1 = \frac{b^2}{a^2}$

583

Statement 1: If parabola $y^2 = 4ax$ and circle $x^2 + y^2 + 2bx = 0$ touch each other externally, then roots of the equation, $f(x) = x^2 - (b + a + 1)x + a = 0$ has real roots

Statement 2: For parabola and circle externally touching a and b must have the same sign

584

Statement 1: Let C_1 and C_2 be two circles with C_2 lying insideC_1. A circle C lying inside C_1 touches C_1 internally and C_2 externally. Then, the locus of the centre of C is an ellipse

Statement 2: If A and B are foci and P be any point on the ellipse, then $AP + BP$ =Constnat

585

Statement 1: If tangent at point P(in first quadrant) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$, meets corresponding directrix $x = a/e$ at point Q, then circle with minimum radius having PQ as chord passes through the corresponding focus

Statement 2: PQ subtends right angle at corresponding focus

586 Tangents are drawn from the origin to the circle $x^2 + y^2 - 2hx - 2hy + h^2 = 0(h \geq 0)$

Statement 1: Angle between the tangents is $\pi/2$

Statement 2: The given circle is touching the co-ordinate axes

587

Statement 1: A bullet is fired and it hits a target. An observer in the same plane heard two sounds, the crack of the rifle and the thud of the bullet striking the target at the same instant, then locus of the observer is hyperbola where velocity of sound is smaller than velocity of bullet

Statement 2: If difference of distances of a point 'P' from the two fixed points is constant and less than the distance between the fixed points then locus of 'P' is a hyperbola

588

Statement 1: The length of latusrectum of the parabola $(x - y + 2)^2 = 8\sqrt{2}(x + y - 6)$ is $8\sqrt{2}$

Statement 2: The length of latusrectum of the parabola $(y - a)^2 = 8\sqrt{2}(x - b)$ is $8\sqrt{2}$

589

Statement 1: The line joining points $(8, -8)$ and $\left(\frac{1}{2}, 2\right)$ which are on parabola $y^2 = 8x$, passes through focus of parabola

Statement 2: Tangents drawn at $(8,-8)$ and $\left(\frac{1}{2}, 2\right)$ on the parabola $y^2 = 4ax$ are perpendicular

590

Statement 1: If two circles $x^2 + y^2 + 2gx + 2fy = 0$ and $x^2 + y^2 + 2g'x + 2f'y = 0$ touch each other, then $f'g = fg'$

Statement 2: Two circles touch other, if line joining their centres is perpendicular to all possible common tangents

591

Statement 1: Slope of tangents drawn from(4,10) to parabola $y^2 = 9x$ are $\frac{1}{4}, \frac{9}{4}$

Statement 2: Two tangents can be drawn to parabola from any point lying outside parabola

592

Statement 1: The equation of chord through the point $(-2, 4)$ which is farthest from the centre of the circle $x^2 + y^2 - 6x + 10y - 9 = 0$ is $x + y - 2 = 0$

Statement 2: In notations, the equation of such chord of the circle $S = 0$ bisected at (x_1, y_1) must be $T = S_1$

593

Statement 1: The latusrectum of a parabola is 4 unit, axis is the line $3x + 4y - 4 = 0$ and the tangent at the vertex is the line $4x - 3y + 7 = 0$, then the equation of directrix of the parabola is $4x - 3y + 8 = 0$

Statement 2: If P be any point on the parabola and let PM and PN are perpendiculars from P on the axis and tangent at the vertex respectively, then $(PM)^2 = (\text{latusrectum})(PN)$

594

Statement 1: A hyperbola and its conjugate hyperbola have the same asymptotes.

Statement 2: The difference between the second degree curve and pair of asymptotes is constant.

595

Statement 1: AA' and BB' are double ordinates of the parabola. Then points A, A', B, B' are concyclic

Statement 2: Circle can cut parabola in maximum four points

596

Statement 1: If two circles $x^2 + y^2 + 2gx + 2fy = 0$ and $x^2 + y^2 + 2g'x + 2f'y = 0$ touch each other, then $f'g = fg'$

Statement 2: Two circles touch each other, if line joining their centres is perpendicular to all possible common tangents

597

Statement 1: Number of circles passing through $(-2, 1), (-1, 0), (-4, 3)$ is 1

Statement 2: Through three non-collinear points in a plane only one circle can be drawn

598

Statement 1: The circle having equation $x^2 + y^2 - 2x + 6y + 5 = 0$ intersects both the coordinate axes

Statement 2: The lengths of x and y intercepts made by the circle having equation $x^2 + y^2 + 2gx + 2fy + c = 0$ are $2\sqrt{g^2 - c}$ and $2\sqrt{f^2 - c}$, respectively

599

Statement 1: Given the base BC of the triangle and the ratio radius of the ex-circles opposite to the angles B and C. Then locus of the vertex A is hyperbola

Statement 2: $|S'P - SP| = 2a$, where S and S' are the two foci, $2a$ = length of the transverse axis and P be any point on the hyperbola

600

Statement 1: The least and greatest distance of the point $P(10, 7)$ from the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ are 5 and 15 units, respectively

Statement 2: A point (x_1, y_1) lies outside a circle $S = x^2 + y^2 + 2gx + 2fy + c = 0$, if $S_1 > 0$, where $S_1 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$

601 From the point $P(\sqrt{2}, \sqrt{6})$, tangents PA and PB are drawn to the circle $x^2 + y^2 = 4$

Statement 1: Area of the quadrilateral $OAPB$ (being origin) is 4

Statement 2: Area of square is a^2 where a is length of side

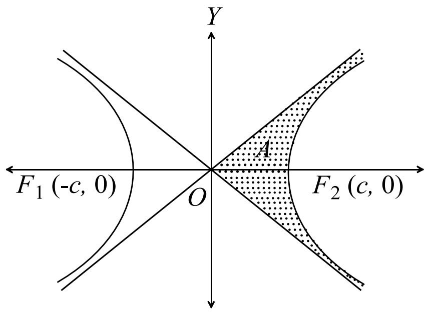
602

Statement 1: Equation $(5x - 5)^2 + (5y + 10)^2 = (3x + 4y + 5)^2$ is parabola

Statement 2: If distance of the point from the given line and from the given point (not lying on the given line) is equal, then locus of variable point is parabola

603

Statement 1: If a point (x_1, y_1) lies in the shaded region $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, shown in the figure, then $\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} < 0$



Statement 2: If $P(x_1, y_1)$ lies outside the a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} < 1$

604

Statement 1: If (3, 4) is a point on a hyperbola having focus (3, 0) and $(\lambda, 0)$ and length of the transverse axis being 1 unit then λ can take the value 0 or 3

Statement 2: $|S'P - SP| = 2a$, where S and S' are the two foci, $2a =$ length of the transverse axis and P be any point on the hyperbola

605

Statement 1: The number of circles that pass through the points $(1, -7)$ and $(-5, 1)$ and of radius 4, is two

Statement 2: The centre of any circle that pass through the points A and B lies on the perpendicular bisector of AB

606

Statement 1: The area of the ellipse $2x^2 + 3y^2 = 6$ is more than the area of the circle $x^2 + y^2 - 2x + 4y + 4 = 0$

Statement 2: The length of semi-major axes of an ellipse is more than the radius of the circle

607

Statement 1: If the parabola $y = (a - b)x^2 + (b - c) + (c - a)$ touches the x-axis in the interval (0, 1), then the line $ax + by + c = 0$ always passes through a fixed point

Statement 2: The equation $L_1 + \lambda L_2 = 0$ or $\mu L_1 + \nu L_2 = 0$ represent a line passing through the intersection of the lines $L_1 = 0$ and $L_2 = 0$
Which is a fixed point, when λ, μ, ν are constants

608

Statement 1: If from any point $P(x_1, y_1)$ on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$, tangents are draws to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then corresponding chord of contact lies on another branch of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

Statement 2: From any point outside the hyperbola two tangents can be drawn to the hyperbola

609

Statement 1: Let $S_1: x^2 + y^2 - 10x - 12y - 39 = 0$
$S_2: x^2 + y^2 - 2x - 4y + 1 = 0$
and $S_3: 2x^2 + 2y^2 - 20x - 24y + 78 = 0$
The radical centre of these circles taken pairwise as $(-2, -3)$

Statement 2: Point of intersection of three radical axis of three circles taken in pairs is known as radical centre

610

Statement 1: The curve $y = -\frac{x^2}{2} + x + 1$ is symmetric with respect to the line x=1.

Statement 2: A parabola is symmetric about it's axis

611

Statement 1: $\frac{5}{3}$ and $\frac{5}{4}$ are the eccentricities of two conjugate hyperbolas.

Statement 2: If e and e_1 are the eccentricities of two conjugate hyperbolas, then $ee_1 > 1$.

612

Statement 1: The equation $7y^2 - 9x^2 + 54x - 28y - 116 = 0$ represents a hyperbola.

Statement 2: The square of the coefficient of xy is greater than the product of coefficient of x^2 and y^2 and $\Delta \neq 0$.

613

Statement 1: Any chord of ellipse $x^2 + y^2 + xy = 1$ through $(0, 0)$ is bisected at $(0, 0)$

Statement 2: The centre of an ellipse is a point through which every chord is bisected

614

Statement 1: The chord of contact of tangent from three points A, B, C to the circle $x^2 + y^2 = a^2$ are concurrent, then A, B, C will be collinear

Statement 2: A, B, C always lies on the normal to the circle $x^2 + y^2 = a^2$

615

Statement 1: Circumcircle of a triangle formed by the line $x = 0, x + y + 1 = 0$ and $x - y + 1 = 0$ also passes through the point$(1, 0)$

Statement 2: Circumcircle of a triangle formed by three tangents of a parabola passes through its focus

Matrix-Match Type

This section contain(s) 0 question(s). Each question contains Statements given in 2 columns which have to be matched. Statements (A, B, C, D) in **columns I** have to be matched with Statements (p, q, r, s) in **columns II**.

616.

	Column-I		Column- II
(A)	If two circles $x^2+y^2+2a_1x+b=0$ and $x^2+y^2+2a_2x+b=0$ touch each other then triplet (a_1,a_2,b) can be	(p)	(2, 2, 2)
(B)	If two circles $x^2+y^2+2a_1x+b=0$ and $x^2+y^2+2a_2y+b=0$ touch each other then triplet (a_1,a_2,b) can be	(q)	$\left(1,1,\frac{1}{2}\right)$
(C)	If the straight line $a_1x-by+b^2=0$ touches the circle $x^2+y^2=a_2x+by$, then triplet (a_1,a_2,b) can be	(r)	(2, 1, 0)
(D)	If the line $3x+4y-4=0$ touches the circle $(x-a_1)^2+(y-a_2)^2=b^2$, then triplet (a_1,a_2,b) can be	(s)	$\left(1,1,\frac{3}{5}\right)$

CODES :

	A	**B**	**C**	**D**
a)	r	p,q	q,r	p,s
b)	p,q	r	p,s	q
c)	s	q	q,r	p
d)	p	q,r	r	s

617.

	Column-I		Column- II
(A)	If z is a complex number such that Im $(z^2)=3$, then eccentricity of the locus is	(p)	$\sqrt{3}$
(B)	If the latus rectum of a hyperbola through one focus subtends 60° angle at the other focus, then its eccentricity is	(q)	2
(C)	If $A(3,0)$ and $B(-3,0)$ and $PA-PB=4$, then eccentricity of conjugate hyperbola is	(r)	$\sqrt{2}$
(D)	If the angle between the asymptotes of hyperbola is $\pi/3$, then the eccentricity of its conjugate hyperbola is	(s)	$\frac{3}{\sqrt{5}}$

CODES :

	A	**B**	**C**	**D**
a)	p	q	r	s
b)	s	r	q	p
c)	q	s	p	r

d)	r	p	s	q

618. Let $x^2 + y^2 + 2gx + 2fy + c = 0$ be an equation of circle

	Column-I		**Column- II**
(A)	If circle lie in first quadrant, then	(p)	g<0
(B)	If circle lie above x-axis, then	(q)	g>0
(C)	If circle lie on the left of y-axis, then	(r)	$g^2 - c < 0$
(D)	If circle touches positive x-axis and does not intersect y-axis then	(s)	$c > 0$

CODES :

	A	**B**	**C**	**D**
a)	P,r,s	r,s	q,s	p,s
b)	r	p,s	p,r	q,s
c)	p,r	q,s	s	r
d)	s	r	q	p

619.

	Column-I		**Column- II**
(A)	If $ax + by - 5 = 0$ is the equation of the chord of the circle $(x-3) + (y-4)^2 = 4$, which passes through (2, 3) and at the greatest distance from the centre of the circle, then $\|a + b\|$ is equal to	(p)	6
(B)	Let O be the origin and P be a variable point on the circle $x^2 + y^2 + 2x + 2y = 0$. If the locus of midpoint of OP is $x^2 + y^2 + 2gx + 2fy + c = 0$, then $(g + f)$ is equal to	(q)	3
(C)	The x-coodinates of the centre of the smallest circle which cuts the circles $x^2 + y^2 - 2x - 4y - 4 = 0$ and $x^2 + y^2 - 10x + 12y + 52 = 0$ orthogonally is	(r)	2
(D)	If θ be the angle between two tangents which are drawn to the circles $x^2 + y^2 - 6\sqrt{3}x - 6y + 27 = 0$ from the origin, then $2\sqrt{3}\tan\theta$ equals to	(s)	1

CODES :

	A	**B**	**C**	**D**
a)	s	q	r	p
b)	q	s	p	r

c)	r	s	q	p
d)	p	r	s	q

620.

	Column-I		Column- II
(A)	If the tangent to the ellipse $x^2 + 4y^2 = 16$ at the point $P(\phi)$ is a normal to the circle $x^2 + y^2 - 8x - 4y = 0$, then $\frac{\phi}{2}$ may be	(p)	0
(B)	The eccentric angle(s) of a point on the ellipse $x^2 + 3y^2 = 6$ at a distance 2 units from the centre of the ellipse is/are	(q)	$\cos^{-1}\left(-\frac{2}{3}\right)$
(C)	The eccentric angle of intersection of the ellipse $x^2 + 4y^2 = 4$ and the parabola $x^2 + 1 = y$ is	(r)	$\frac{\pi}{4}$
(D)	If the normal at the point $P(\theta)$ to the ellipse $\frac{x^2}{14} + \frac{y^2}{5} = 1$ intersects it again at the point $Q(2\theta)$, then θ is	(s)	$\frac{5\pi}{4}$

CODES :

	A	B	C	D
a)	P, r	r, s	p	q
b)	r	s	p	q
c)	q	p	s	r
d)	r	q	p	s

621. The tangents drawn from a point P to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ make angle α and β with the major axis

	Column-I		Column- II
(A)	If $\alpha + \beta = \frac{c\pi}{2} (c \in N)$	(p)	Circle
(B)	If $\tan\alpha \tan\beta = c$ {where $c \in R$}, then locus of P can be	(q)	Ellipse
(C)	If $\tan\alpha + \tan\beta = c$ {where $c \in R$}, then locus of P can be	(r)	Hyperbola
(D)	If $\cot\alpha + \cot\beta = c${where $c \in R$}, then locus of P can be	(s)	Pair of straight lines

CODES :

	A	B	C	D
a)	R, s	p,q,r,s	r,s	r,s
b)	s	r	p	q
c)	q	p	r	s
d)	s	r	q	p

622.

	Column-I		Column- II
(A)	Tangents are drawn from point (2,3) to the parabola $y^2 = 4x$, then points of contact are	(p)	$(9, -6)$
(B)	From a point P on the circle $x^2 + y^2 = 5$, the equation of chord of contact to the parabola $y^2 = 4x$ is $y = 2(x - 2)$, then the coordinate of point P will be	(q)	$(1, 2)$
(C)	$P(4, -4), Q$ are points on parabola $y^2 = 4x$ such that area of ΔPOQ is 6 sq. units where O is the vertex, then coordinates of Q may be	(r)	$(-2, 1)$
(D)	The common chord of circle $x^2 + y^2 = 5$ and parabola $6y = 5x^2 + 7x$ will pass through point(s)	(s)	$(4, 4)$

CODES :

	A	B	C	D
a)	P,r	q,	q,s	r,q
b)	q,s	r,	p,q	q,r
c)	r,p	p	s,p	s,q
d)	s,q	s	r,r	p,s

623. Consider the parabola $y^2 = 12x$

	Column-I		Column- II
(A)	Equation of tangent can be	(p)	$2x + y - 6 = 0$
(B)	Equation of normal can be	(q)	$3x - y + 1 = 0$
(C)	Equation of chord of contact w.r.t. any point on the directrix can be	(r)	$x - 2y - 12 = 0$
(D)	Equation of chord which subtends right angle at the vertex can be	(s)	$2x - y - 36 = 0$

CODES :

	A	B	C	D
a)	P	q	r	s
b)	r	p	s	q
c)	q	s	p	r
d)	s	r	q	p

624.

Column-I	Column- II

(A)	If vertices of a rectangle of maximum area inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are extremities of latus rectum. Then eccentricity of ellipse is	(p)	$\frac{2}{\sqrt{5}}$
(B)	If extremities of diameter of the circle $x^2+y^2=16$ are foci of a ellipse, then eccentricity of the ellipse, if its size is just sufficient to contain the circle, is	(q)	$\frac{1}{\sqrt{2}}$
(C)	If normal at point $(6,2)$ to the ellipse passes through its nearest focus $(5,2)$, having centre at $(4,2)$ then its eccentricity is	(r)	$\frac{1}{3}$
(D)	If extremities of latus rectum of the parabola $y^2=24x$ are foci of ellipse and if ellipse passes through the vertex of the parabola, then its eccentricity is	(s)	$\frac{1}{2}$

CODES :

	A	**B**	**C**	**D**
a)	q	q	s	p
b)	s	p	q	s
c)	s	p	r	q
d)	p	r	s	q

625. Let C_1 and C_2 be two circles whose equations are $x^2+y^2-2x=0$ and $x^2+y^2+2x=0$. $P(\lambda_1,\lambda)$ is a variable point. Then match the following

	Column-I		**Column- II**
(A)	P lies inside C_1 but outside C_2	(p)	$\lambda \in (-\infty,-1) \cup (0,\infty)$
(B)	P lies inside C_2 but outside C_1	(q)	$\lambda \in (-\infty,-1) \cup (1,\infty)$
(C)	P lies outside C_1 but outside C_2	(r)	$\lambda \in (-1,0)$
(D)	P does not lie inside C_2	(s)	$\lambda \in (0,1)$

CODES :

	A	**B**	**C**	**D**
a)	r	q	s	p
b)	q	s	p	r
c)	p	q	r	s
d)	s	r	q	p

626.

Column-I **Column- II**

(A)	The length of the common chord of two circles of radii 3 and 4 units which intersect orthogonally is $\frac{k}{5}$, then k equals to	(p)	1
(B)	The circumference of the circle $x^2+y^2+4x+12y+p=0$ is bisected by the circle $x^2+y^2-2x+8y-q=0$, then $p+q$ is equal to	(q)	24
(C)	Number of distance chord of the circle $2x(x-\sqrt{2})+y(2y-1)=0$ chords are passing through the point $\left(\sqrt{2},\frac{1}{2}\right)$ and are bisected on x-axis is	(r)	32
(D)	One of the diameter of the circle circumscribing the rectangle $ABCD$ is $4y=x+7$. If A and B are the points $(-3,4)$ and (5,4), respectively, then the area of the rectangle is	(s)	36

CODES :

	A	B	C	D
a)	s	q	r	p
b)	q	s	p	r
c)	r	q	s	p
d)	p	q	r	s

627.

	Column-I		Column- II
(A)	Points from which perpendicular tangents can be drawn to parabola $y^2=4x$	(p)	$(-1,2)$
(B)	Points from which only one normal can be drawn to parabola $y^2=4x$	(q)	(3, 2)
(C)	Point at which chord $x-y+1=0$ of parabola $y^2=4x$ is bisected	(r)	$(-1,-5)$
(D)	Points from which tangents cannot be drawn to parabola $y^2=4x$	(s)	(5,–2)

CODES :

	A	B	C	D
a)	P,r	p,r	q	q,s
b)	q,s	r,p	p	s,r
c)	r,p	s,q	s	p,q
d)	s,q	q,s	r	r,p

628.

Column-I		Column- II	
(A)	The points common to the hyperbola $x^2 - y^2 = 9$ and circle $x^2 + y^2 = 41$ are	(p)	$(-5, -4)$
(B)	Tangents are drawn from point $\left(0, -\frac{9}{4}\right)$ to the hyperbola $x^2 - y^2 = 9$, then the point of tangency may have coordinate(s)	(q)	$(5, 4)$
(C)	The point which is diametrically opposite of point $(5, 4)$ with respect to the hyperbola $x^2 - y^2 = 9$ is	(r)	$(-5, 4)$
(D)	If P and Q lie on the hyperbola $x^2 - y^2 = 9$ such that area of the isosceles triangle PQR where $PR = QR$ is 10 sq. units, where $R \equiv (0, -6)$, then P can have the co-ordinate(s)	(s)	$(5, -4)$

CODES :

	A	B	C	D
a)	Q,r	p	p,s	p,q,r,s
b)	p,q,r,s	q,r	p	p,s
c)	p,s	p	p,q,r,s	q,r
d)	p	p,q,r,s	q,r	p,s

629. Let the foci of the hyperbola $\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$ be the vertices of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the foci of the ellipse be the vertices of the hyperbola. Let the eccentricities of the ellipse and hyperbola be e_E and e_H, respectively, then match the following

Column-I		Column- II	
(A)	$\frac{b}{B}$ is equal to	(p)	1
(B)	$e_H + e_E$ is always greater than	(q)	2
(C)	If angle between the asymptotes of hyperbola is $\frac{2\pi}{3}$, then $4e_E$ is equal to	(r)	3
(D)	If $e_E^2 = \frac{1}{2}$ and (x, y) is point of intersection of ellipse and the hyperbola then $\frac{9x^2}{2y^2}$ is	(s)	4

CODES :

	A	B	C	D
a)	p	p	q	s
b)	q	s	r	p
c)	s	r	p	q
d)	q	p	s	p

630. If e_1 and e_2 are the roots of the equation $x^2 - ax + 2 = 0$ then match the following

Column-I		Column- II	
(A)	If e_1 and e_2 are the eccentricities of the ellipse and hyperbola, respectively then the values of a are	(p)	6
(B)	If both e_1 and e_2 are the eccentricities of the hyperbola, then values of a are	(q)	$\frac{5}{2}$
(C)	If e_1 and e_2 are eccentricities of hyperbola and conjugate hyperbola then values of a are	(r)	$2\sqrt{2}$
(D)	If e_1 is the eccentricity of the hyperbola for which there exists infinite points from which perpendicular tangents can be drawn and e_2 is the eccentricity of the hyperbola in which no such points exist then the values of a are	(s)	5

CODES :

	A	B	C	D
a)	P,	q,r	r	p,s
b)	q,r	r	p,s	p
c)	r	p,s	p	q,r
d)	p,s	p	q,r	r

631. $A(-2,0)$ and $B(2,0)$ are the two fixed points and P is a point such that $PA - PB = 2$. Let S be the circle $x^2 + y^2 = r^2$, then match the following

Column-I		Column- II	
(A)	If $r = 2$, then the number of points P satifying $PA - PB = 2$ and lying on $x^2 + y^2 = r^2$ is	(p)	2
(B)	If $r = 1$, then he number of points satisfying $PA - PB = 2$ and lying on $x^2 + y^2 = r^2$ is	(q)	4
(C)	For $r = 2$ the number of common tangents is	(r)	0
(D)	For $r = 1/2$, the number of common tangents is	(s)	1

CODES :

	A	B	C	D
a)	s	p	q	r
b)	q	r	p	s
c)	p	s	r	p
d)	r	q	s	p

632.

Column-I		Column- II	
(A)	A stick of length 10 m slides on coordinate axes, then locus of a point dividing this stick	(p)	$\sqrt{6}$

	from x-axis in the ration 6: 4 is a curve whose eccentricity is e, then $3e$ is equal to		
(B)	AA' is a major axis of an ellipse $3x^2 + 2y^2 + 6x - 4y - 1 = 0$ and P is a variable point on it, then greatest area of triangle APA' is	(q)	$2\sqrt{7}$
(C)	Distance between foci of the curve represented by the equation $x = 1 + 4\cos\theta, y = 2 + 3\sin\theta$ is	(r)	$\frac{128}{3}$
(D)	Tangents are drawn to the ellipse $\frac{x^2}{16} + \frac{y^2}{7} = 1$ at end points of the latus rectum. The area of equadrilateral so formed is	(s)	$\sqrt{5}$

CODES :

	A	**B**	**C**	**D**
a)	s	p	q	r
b)	p	s	r	q
c)	q	r	s	p
d)	r	s	p	q

633.

	Column-I		**Column- II**
(A)	Number of circles touching given three non-concurrent lines	(p)	1
(B)	Number of circles touching $y = x$ at $(2, 2)$ and also touching line $x + 2y - 4 = 0$	(q)	2
(C)	Number of circles touching lines $x \pm y = 2$ and passing through the point $(4, 3)$	(r)	4
(D)	Number of circle intersecting given three circles orthogonally	(s)	Infinite

CODES :

	A	**B**	**C**	**D**
a)	r	s	p	q
b)	r	q	q	p
c)	q	q	p	r
d)	p	r	q	s

634.

	Column-I		**Column- II**
(A)	An ellipse passing through the origin has its foci $(3, 4)$ and $(6, 8)$, then length of its minor axis is	(p)	8
(B)	If PQ is focal chord of ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ which	(q)	$10\sqrt{2}$

passes through $S \equiv (3, 0)$ and $PS = 2$ then length of chord PQ is

(C) If the line $y = x + K$ touches the ellipse $9x^2 + 16y^2 = 144$, then the difference of values of K is — (r) 10

(D) Sum of distances of a point on the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$ from the foci — (s) 12

CODES :

	A	B	C	D
a)	P	q	r	s
b)	r	s	q	p
c)	q	r	r	p
d)	s	p	q	r

635.

	Column-I		Column- II
(A)	Distance between the points on the curve $4x^2 + 9y^2 = 1$, where tangent is parallel to the line $8x = 9y$, is less than	(p)	1
(B)	Sum of distance between the foci of the curve $25(x + 1)^2 + 9(y + 2)^2 = 225$ from $(-1, 0)$ is more than	(q)	4
(C)	Sum of distances from the x-axis of the points on the ellipse $\frac{x^2}{9} + \frac{y^2}{4}$, where the normal is parallel to the line $2x + y = 1$, is less than	(r)	7
(D)	Tangents are drawn from points on the line $x - y + 2 = 0$ to the ellipse $x^2 + 2y^2 = 2$, then all the chords of contact pass through the point whose distance from $(2, 1/2)$ is more than	(s)	5

CODES :

	A	B	C	D
a)	s	q	r	p
b)	p, q, r, s	p,q,r,s	q, r, s	p
c)	s	r	p	r
d)	p	s	q	r

636. Consider the parabola $(x - 1)^2 + (y - 2)^2 = \frac{(12x - 5y + 3)^2}{169}$

	Column-I		Column- II
(A)	Locus of point of intersection of perpendicular tangent	(p)	$12 - 5y - 2 = 0$

(B)	Locus of foot of perpendicular from focus upon any tangent	(q)	$5x + 12y - 29 = 0$
(C)	Line along which minimum length of focal chord occurs	(r)	$12x - 5y + 3 = 0$
(D)	Line about which parabola is symmetrical	(s)	$24x - 10y + 1 = 0$

CODES :

	A	**B**	**C**	**D**
a)	P	q	r	s
b)	q	r	s	p
c)	s	p	q	r
d)	r	s	p	q

Linked Comprehension Type

This section contain(s) 70 paragraph(s) and based upon each paragraph, multiple choice questions have to be answered. Each question has atleast 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 637 to -637

Tangents are drawn from the point $P(3,4)$ to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ touching the ellipse at points A and B

637. Equation of the circle with AB as its diameter is

a) $x^2 + y^2 - 12x + 24 = 0$
b) $x^2 + y^2 + 12x + 24 = 0$
c) $x^2 + y^2 + 24x - 12 = 0$
d) $x^2 + y^2 - 24x - 12 = 0$

Paragraph for Question Nos. 638 to - 638

Tangents are drawn from the point $P(3,4)$ to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ touching the ellipse at points A and B

638. The coordinates of A and B are

a) (3, 0) and (0, 2)
b) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and $\left(-\frac{9}{5}, \frac{8}{5}\right)$
c) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and (0,2)
d) (3,0) and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

Paragraph for Question Nos. 639 to - 639

If we rotate the axes of the rectangular hyperbola $x^2 - y^2 = a^2$ through an angle $\pi/4$ in the clockwise direction, then the equation $x^2 - y^2 = a^2$ reduces to $xy = \frac{a^2}{2} = \left(\frac{a}{\sqrt{2}}\right)^2 = c^2$(say). Since, $x = ct, y = \frac{c}{t}$ satisfies $xy = c^2$. Therefore, $(x, y) = \left(ct, \frac{c}{t}\right)$ $(t \neq 0)$ is called a $'t'$ point on the rectangular hyperbola.
On the basis of above information, answer the following question:

639. If t_1 and t_2 are the roots of the equation $x^2 - 4x + 2 = 0$, then the point of intersection of tangents at $''t_1''$ and $''t_2''$ on $xy = c^2$ is

a) $\left(\frac{c}{2}, 2c\right)$ b) $\left(2c, \frac{c}{2}\right)$ c) $\left(\frac{c}{2}, c\right)$ d) $\left(c, \frac{c}{2}\right)$

Paragraph for Question Nos. 640 to - 640

Let the curves be
$x^2 - y^2 = 9, P: y^2 = 4(x-5), L: x = 9$
On the basis of above information, answer the following questions:

640. If L is the chord of contact of the hyperbola H, then the equation of the corresponding pair of tangents is
a) $9x^2 - 8y^2 + 18x - 9 = 0$ b) $9x^2 - 8y^2 - 18x + 9 = 0$
c) $9x^2 - 8y^2 - 18x - 9 = 0$ d) $9x^2 - 8y^2 + 18x + 9 = 0$

Paragraph for Question Nos. 641 to - 641

If the normals at $(x_i, y_i), i = 1,2,3,4$ on the rectangular hyperbola $xy = c^2$, meet at the point (α, β).
On the basis of above information, answer the following questions:

641. The value of $\sum x_i$ is
a) $c\,\beta$ b) $c\,\alpha$ c) α d) β

Paragraph for Question Nos. 642 to - 642

Three normals AA_1, BB_1 and CC_1 are drawn from a point $P(h, k)$ to the parabola $y^2 = 4ax$, at A, B and C points. The following conditions are satisfied by the three normals
(I) Any two of three normals are coincide
(II) $S(a, 0)$ be the focus of the parabola
(III) Three normals be real, then $h > 2a$
(IV) Slopes of the normals are m_1, m_2 and m_3. If $m_1 m_2 = \lambda$, then the locus of P is a parabola
(V) P lies on the line $y = \mu$, then the side of the triangle ABC touch the parabola $S' = 0$
On the basis of above information, answer the following questions

642. Locus of point P is, if (I) is satisfied
a) $ay^2 = 4(x - 2a)^3$ b) $27ay^2 = 4(x - 2a)^3$
c) $ax^2 = 4(y - 2a)^3$ d) $27ax^2 = 4(y - 2a)^3$

Paragraph for Question Nos. 643 to - 643

Consider the standard equation of an ellipse whose focus and corresponding foot of directrix are $(\sqrt{7}, 0)$ and $\left(\frac{16}{\sqrt{7}}, 0\right)$ and a circle with equation $x^2 + y^2 = r^2$. If in the first quadrant, the common tangent to a circle of this family and the above ellipse meets the coordinate axes at A and B
on the basis of above information, answer the following questions:

643. The equation of the ellipse is
a) $16x^2 + 9y^2 = 144$ b) $9x^2 + 16y^2 = 144$ c) $16x^2 + y^2 = 144$ d) $x^2 + 9y^2 = 144$

Paragraph for Question Nos. 644 to - 644

An ellipse E has its centre $C(1,3)$, focus $(6,3)$ and is passing through the point $P(4,7)$, then on the basis of above information, answer the following questions:

644. The product of the lengths of the perpendicular segments from the foci on tangent at point P is

a) 20
b) 45
c) 40
d) Cannot be determined

Paragraph for Question Nos. 645 to - 645

$C: x^2+y^2=9, E: \dfrac{x^2}{9}+\dfrac{y^2}{4}=1, L: y=2x$

on the basis of above information, answer the following questions:

645. P is a point on the circle C, the perpendicular PQ to the major-axis of the ellipse E meets the ellipse at M, then $\frac{MQ}{PQ}$ is equal to

a) 1/3
b) 2/3
c) 1/2
d) None of these

Paragraph for Question Nos. 646 to - 646

P Is a variable point on the line$L=0$. Tangents are drawn to the circle $x^2+y^2=4$ from P to touch it at Q and$R$. The parallelogram $PQRS$ is completed
On the basic of above information, answer the following questions

646. If $L=2x+y=6$, then the locus of circumcentre of $\Delta\ PQR$ is

a) $2x-y=4$
b) $2x+y=3$
c) $x-2y=4$
d) $x+2y=3$

Paragraph for Question Nos. 647 to - 647

Let the points $A(3,7)$ and $B(6,5)$ and equation of circle, $C: x^2+y^2-4x-6y-3=0$
On the basis above information, answer the following questions :

647. The chord in which the circle C cuts the members of the family S of the circles through A and B are concurrent at

a) $(2,3)$
b) $\left(2,\dfrac{23}{3}\right)$
c) $\left(3,\dfrac{23}{2}\right)$
d) $(3,2)$

Paragraph for Question Nos. 648 to - 648

A circle C of radius 1 is inscribed in an equilateral$\Delta\ PQR$. The point of contact of C with the sides PQ, QR, RP are D, E, F respectively. The line PQ is given by the equation $\sqrt{3}x+y-6=0$ and the point D is$\left(\frac{3\sqrt{3}}{2},\frac{3}{2}\right)$. Further, it is given that the origin and centre of C are on the same side of the line PQ
On the basis of above information, answer the following questions

648. The equation of circle C is

a) $\left(x-2\sqrt{3}\right)^2+(y-1)^2=1$
b) $\left(x-2\sqrt{3}\right)^2+\left(y+\frac{1}{2}\right)^2=1$
c) $\left(x-\sqrt{3}\right)^2+(y+1)^2=1$
d) $\left(x-\sqrt{3}\right)^2+(y-1)^2=1$

Paragraph for Question Nos. 649 to - 649

If $7l^2-9m^2+8l+1=0$ and we have to find equation of circle having $lx+my+1=0$ is a tangent and we can adjust given condition as
$16l^2+8l+1=9(l^2+m^2)$
or $(4l+1)^2=9(l^2+m^2)$
$\Rightarrow \frac{|4l+1|}{\sqrt{(l^2+m^2)}}=3$
Centre of circle $=(4, 0)$ and radius $=3$ when any two non-parallel lines touching a circle, then centre of circle lies on angular bisector of lines
On the basis of above information, answer the following questions

649. If $16m^2-8l-1=0$then the equation of the circle having $lx+my+1=0$ as a tangent is
a) $x^2+y^2+8x=0$
b) $x^2+y^2-8x=0$
c) $x^2+y^2+8y=0$
d) $x^2+y^2-8y=0$

Paragraph for Question Nos. 650 to - 650

Each side of a square has length 4 units and its centre is at (3, 4). If one of the diagonals is parallel to the line $y=x$, then answer the following questions

650. Which of the following is not the vertex of the square?
a) (1, 6)
b) (5, 2)
c) (1, 2)
d) (4, 6)

Paragraph for Question Nos. 651 to - 651

Tangents PA and PB are drawn to the circle $(x-4)^2+(y-5)^2=4$ from the point P on the curve $y=\sin x$, where A and B lie on the circle. Consider the function $y=f(x)$ represented by the locus of the center of the circumcircle of triangle PAB, then answer the following questions

651. Range of $y=f(x)$ is
a) $[-2, 1]$
b) $[-1, 4]$
c) $[0, 2]$
d) $[2, 3]$

Paragraph for Question Nos. 652 to - 652

Consider a family of circles passing through the points (3, 7) and (6, 5). Answer the following questions

652. Number of circles which belong to the family and also touching x-axis are
a) 0
b) 1
c) 2
d) Infinite

Paragraph for Question Nos. 653 to - 653

Consider the relation $4l^2 - 5m^2 + 6I + 1 = 0$, where $l, m \in R$, then the line $lx + my + 1 = 0$ touches a fixed circle whose

653. Centre and radius of circle one

a) $(2, 0), 3$ b) $(-3, 0), \sqrt{3}$ c) $(3, 0), \sqrt{5}$ d) None of these

Paragraph for Question Nos. 654 to - 654

A circle C whose radius is 1 unit, touches x-axis at point A. The centre Q of C lies in first quadrant. The tangent from origin O to the circle touches it at T and a point P lies on it such that ΔOAP is a right-angled triangle at A and its perimeter is 8 units

654. The length of PQ is

a) $\frac{1}{2}$ b) $\frac{4}{3}$ c) $\frac{5}{3}$ d) None of these

Paragraph for Question Nos. 655 to - 655

P is a variable point on the line $L = 0$. Tangents are drawn to the circles $x^2 + y^2 = 4$ from P to touch it at Q and R. The parallelogram $PQSR$ is completed

655. If $L \equiv 2x + y - 6 = 0$, then the locus of circumcentre of ΔPQR is

a) $2x - y = 4$ b) $2x + y = 3$ c) $x - 2y = 4$ d) $x + 2y = 3$

Paragraph for Question Nos. 656 to - 656

To the circle $x^2 + y^2 = 4$, two tangents are drawn from $P(-4, 0)$, which touches the circle at T_1 and T_2, a rhombus $PT_1P'T_2$ is completed

656. Circumcentre of the triangle PT_1T_2 is at

a) $(-2, 0)$ b) $(2, 0)$ c) $\left(\frac{\sqrt{3}}{2}, 0\right)$ d) None of these

Paragraph for Question Nos. 657 to - 657

Let α chord of a circle be that chord of the circle which subtends an angle α at the centre

657. If $x + y = 1$ is a chord of $x^2 + y^2 = 1$ is 1, then α is equal to

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{6}$ d) $x + y = 1$ is not a chord

Paragraph for Question Nos. 658 to - 658

Two variable chords AB and BC of a circles $x^2 + y^2 = a^2$ are such that $AB = BC = a$, and M and N are the midpoints of AB and BC, respectively, such that line joining MN intersect the circles at P and Q, where P is closer to AB and O is the centre of the circle

658. $\angle OAB$ is
a) $30°$ b) $60°$ c) $45°$ d) $15°$

Paragraph for Question Nos. 659 to - 659

Given two circles intersecting orthogonally having length of common chord 24/5 units. Radius of one of the circles is 3 units

659. Radius of other circle is
a) 6 units b) 5 units c) 2 units d) 4 units

Paragraph for Question Nos. 660 to - 660

A tangents is drawn at any point $P(t)$ on the parabola $y^2 = 8x$ and on it is taken a point $Q(\alpha, \beta)$ from which pair of tangents QA and QB are drawn to the circle $x^2 + y^2 = 4$. Using this information answer the following questions

660. The locus of the point of concurrency of the chord of contact AB of the circle $x^2 + y^2 = 4$ is
a) $y^2 - 2x = 0$ b) $y^2 - x^2 = 4$ c) $y^2 + 4x = 0$ d) $y^2 - 2x^2 = 4$

Paragraph for Question Nos. 661 to - 661

Tangent to the parabola $y = x^2 + ax + 1$, at the point of intersection of y-axis also touches the circle $x^2 + y^2 = r^2$. Also no point of the parabola is below x-axis

661. The radius of circle when a attains its maximum value
a) $\frac{1}{\sqrt{10}}$ b) $\frac{1}{\sqrt{5}}$ c) 1 d) $\sqrt{5}$

Paragraph for Question Nos. 662 to - 662

If the locus of the circumcentre of a variable triangle having sides y-axis, $y = 2$ and $lx + my = 1$, where (l, m) lies on the parabola $y^2 = 4x$ is a curve C, then

662. Coordinates of the vertex of this curve C is
a) $\left(-2, \frac{3}{2}\right)$ b) $\left(-2, -\frac{3}{2}\right)$ c) $\left(2, \frac{3}{2}\right)$ d) $\left(-2, -\frac{3}{2}\right)$

Paragraph for Question Nos. 663 to - 663

$y = x$ is tangent to the parabola $y = ax^2 + c$

663. If $a = 2$, then the value of c is

a) 1 b) $-\frac{1}{2}$ c) $\frac{1}{2}$ d) $\frac{1}{8}$

Paragraph for Question Nos. 664 to - 664

If l, m are variable real numbers such that $5l^2 + 6m^2 - 4lm + 3l = 0$, then variable line $lx + my = 1$ always touches a fixed parabola, whose axes is parallel to x –axis

664. Vertex of the parabola is

a) $\left(-\frac{5}{3}, \frac{4}{3}\right)$ b) $\left(-\frac{7}{4}, \frac{3}{4}\right)$ c) $\left(\frac{5}{6}, -\frac{7}{6}\right)$ d) $\left(\frac{1}{2}, -\frac{3}{4}\right)$

Paragraph for Question Nos. 665 to - 665

Consider the parabola whose focus is at (0,0) and tangent at vertex is $x - y + 1 = 0$

665. The length of latus rectum is

a) $4\sqrt{2}$ b) $2\sqrt{2}$ c) $8\sqrt{2}$ d) $3\sqrt{2}$

Paragraph for Question Nos. 666 to - 666

Two tangents on a parabola are $x - y = 0$ and $x + y = 0$. If (2, 3) is focus of the parabola, then

666. The equation of tangent at vertex is

a) $4x - 6y + 5 = 0$ b) $4x - 6y + 3 = 0$ c) $4x - 6y + 1 = 0$ d) $4x - 6y + 3/2 = 0$

Paragraph for Question Nos. 667 to - 667

$y^2 = 4x$ and $y^2 = -8(x - a)$ intersect at points A and C. Points $O(0,0), A, B(a, 0), C$ are concyclic

667. The length of common chord of parabolas is

a) $2\sqrt{6}$ b) $4\sqrt{3}$ c) $6\sqrt{5}$ d) $8\sqrt{2}$

Paragraph for Question Nos. 668 to - 668

PQ is double ordinate of the parabola $y^2 = 4x$ which passes through the focus S. ΔPQA is isosceles right angle triangle, where A is on the axis of the parabola. Line PA meets the parabola at C and QA meets the parabola at B

668. Area of the trapezium $PBCQ$ is

a) 96 sq. units b) 64 sq. units c) 72 sq. units d) 48 sq. units

Paragraph for Question Nos. 669 to - 669

Consider the inequality, $9^x - a.3^x - a + 3 \leq 0$, where '$a$' is a real parameter

669. The given inequality has at least one negative solution for $a \in$

a) $(-\infty, 2)$ b) $(3, \infty)$ c) $(-2, \infty)$ d) $(2, 3)$

Paragraph for Question Nos. 670 to - 670

An ellipse $(E)\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, centred at point O have AB and CD as its major and minor axes, respectively, let the S_1be one of the foci of the ellipse, radius of incircle of triangle OCS_1 be 1 unit and $OS_1 = 6$ units. Then

670. Perimeter of ΔOCS_1 is

a) 20 units b) 10 units c) 15 units d) 25 units

Paragraph for Question Nos. 671 to - 671

Consider the ellipse whose major and minor axes are x-axis and y-axis, respectively. If ϕ is the angle between the $CP$and the normal at point P on the ellipse, and the greatest value $\tan\phi$ is $\frac{3}{2}$ (where C is the centre of the ellipse). Also semi-major axis is 10 units

671. The eccentricity of the ellipse is

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{\sqrt{3}}{2}$ d) none of these

Paragraph for Question Nos. 672 to - 672

A curve is represented by $C = 21x^2 - 6xy + 29y^2 + 6x - 58y - 151 = 0$

672. Eccentricity of curve is

a) $1/3$ b) $1/\sqrt{3}$ c) $2/3$ d) $2/\sqrt{5}$

Paragraph for Question Nos. 673 to - 673

For all real p, the line $2px + y\sqrt{1-p^2} = 1$ touches a fixed ellipse whose axes are coordinate axes

673. The eccentricity of the ellipse is

a) $\frac{2}{3}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{1}{\sqrt{3}}$ d) $\frac{1}{2}$

Paragraph for Question Nos. 674 to - 674

Let S, S' be the foci of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose eccentricity is e. P is a variable point on the ellipse. Consider the locus of the incentre of the $\Delta PSS'$

674. The locus of incentre is

a) Ellipse b) Hyperbola c) Parabola d) Circle

Paragraph for Question Nos. 675 to - 675

$C_1: x^2+y^2=r^2$and $C_2: \frac{x^2}{16}+\frac{y^2}{9}=1$ intersect at four distinct points A, B, C, and D. Their common tangents form a parallelogram $A'B'C'D'$

675. If $ABCD$ is a square then r is equal to

a) $\frac{12}{5}\sqrt{2}$ b) $\frac{12}{5}$ c) $\frac{12}{5\sqrt{5}}$ d) None of these

Paragraph for Question Nos. 676 to - 676

A coplanar beam of light emerging from a point source has the equation $\lambda x-y+2(1+\lambda)=0, \lambda \in R$, the rays of the beam strike an elliptical surface and get reflected. The reflected rays form another convergent beam having equation $\mu x-y+2(1-\mu)=0, \mu \in R$. Further it is found that the foot of the perpendicular from the point $(2, 2)$ upon any tangent to the ellipse lies on the circle $x^2+y^2-4y-5=0$

676. The eccentricity of the ellipse is equal to

a) $\frac{1}{3}$ b) $\frac{1}{\sqrt{3}}$ c) $\frac{2}{3}$ d) $\frac{1}{2}$

Paragraph for Question Nos. 677 to - 677

The tangent at any point P of the circle $x^2+y^2=16$ meets the tangent at a fixed point A at T, and T is joined to B, the other end of the diameter through A

677. The locus of the intersection of AP and BT is conic whose eccentricity is

a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{2}}$ c) $\frac{1}{3}$ d) $\frac{1}{\sqrt{3}}$

Paragraph for Question Nos. 678 to - 678

The ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is such that it has the least area but contains the circle $(x-1)^2+y^2=1$

678. The eccentricity of the ellipse is

a) $\sqrt{\frac{2}{3}}$ b) $\frac{1}{\sqrt{3}}$ c) $\frac{1}{2}$ d) None of these

Paragraph for Question Nos. 679 to - 679

A conic passes through the point (2, 4) and is such that the segment of any of its tangents at any contained between the coordinate axes is bisected at the point of tangency

679. The eccentricity of the conic is

a) 2 b) $\sqrt{2}$ c) $\sqrt{3}$ d) $\sqrt{\frac{3}{2}}$

Paragraph for Question Nos. 680 to - 680

The locus of foot of perpendicular from any focus of a hyperbola upon any tangent to the hyperbola is the auxillary circle of the hyperbola. Consider the foci of a hyperbola as $(-3, -2)$ and $(5, 6)$ and the foot of perpendicular from the focus $(5, 6)$ upon a tangent to the hyperbola as $(2, 5)$

680. The conjugate axis of the hyperbola is

a) $4\sqrt{11}$ b) $2\sqrt{11}$ c) $4\sqrt{22}$ d) $2\sqrt{22}$

Paragraph for Question Nos. 681 to - 681

Let $P(x, y)$ is a variable point such that

$\left|\sqrt{(x-1)^2+(y-2)^2}-\sqrt{(x-5)+(y-5)^2}\right| = 3$ which represents hyperbola

681. The eccentricity e' of the corresponding conjugate hyperbola is

a) $\frac{5}{3}$ b) $\frac{4}{3}$ c) $\frac{5}{4}$ d) $\frac{3}{\sqrt{7}}$

Paragraph for Question Nos. 682 to - 682

In hyperbola portion of tangent intercepted between asymptotes is bisected at the point of contact
Consider a hyperbola whose centre is at origin. A line $x + y = 2$ touches this hyperbola at $P(1, 1)$ and interests the asymptotes at A and B such that $AB = 6\sqrt{2}$ units

682. Equation of asymptotes area

a) $5xy + 2x^2 + 2y^2 = 0$ b) $3x^2 + 4y^2 + 6xy = 0$
c) $2x^2 + 2y^2 - 5xy = 0$ d) None of these

Paragraph for Question Nos. 683 to - 683

A point P moves such that sum of the slopes of the normals drawn from it to the hyperbola $xy = 16$ is equal to the sum of ordinates of feet of normals. The locus of P is a curve C

683. The equation of the curve C is

a) $x^2 = 4y$ b) $x^2 = 16y$ c) $x^2 = 12y$ d) $y^2 = 8x$

Paragraph for Question Nos. 684 to - 684

The vertices of ΔABC lie on a rectangular hyperbola such that the orthocentre of the triangle is (3, 2) and the asymptotes of the rectangular hyperbola are parallel to the coordinate axes. The two perpendicular tangents of the hyperbola intersect at the point (1, 1)

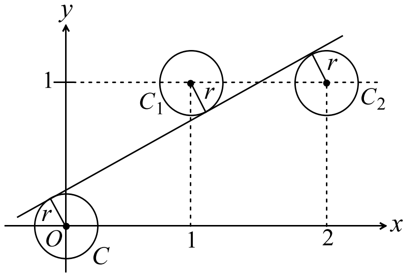
684. The equation of the asymptotes is
a) $xy - 1 = x - y$ b) $xy + 1 = x + y$ c) $2xy = x + y$ d) None of thee

Integer Answer Type

685. If hyperbola $x^2 - y^2 = 4$ is rotated by 45° in anticlockwise direction about its center keeping the axis intact then equation of hyperbola is $xy = a^2$, where a^2 is equal to
686. If a tangent of slope 2 of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is normal to the circle $x^2 + y^2 + 4x + 1 = 0$, then the maximum value of ab is
687. If the circle $x^2 + y^2 + (3 + \sin\beta)x + (2\cos\alpha)y = 0$ and $x^2 + y^2 + (2\cos\alpha)x + 2cy = 0$ touches each other then the maximum value of 'c' is
688. If the line $x + y = 6$ is a normal to the parabola $y^2 = 8x$ at point (a, b) then the value of $a + b$ is
689. If the point $P(4,-2)$ is the one end of the focal chord PQ of the $y^2 = x$, then the slope of the tangent at Q is
690. Two circles C_1 and C_2 both passes through the points $A(1, 2)$ and $E(2, 1)$ and touch the line $4x - 2y = 9$ at B and D respectively. The possible coordinates of a points C such that the quadrilateral $ABCD$ is a parallelogram is (a, b) then the value of $|ab|$ is
691. Lines segments AC and BD are diameter of circle of radius one. If $\angle BDC = 60°$, the length of line segment AB is
692. Consider locus of center of circle which touches circle $x^2 + y^2 = 4$ and line $x = 4$. The distance of the vertex of the locus from origin is
693. If a variable line has its intercepts on the co-ordinates axes, e, e', where $\frac{e}{2}, \frac{e'}{2}$ are the eccentricities of a hyperbola and its conjugate hyperbola, then the line always touches the circle $x^2 + y^2 = r^2$, where $r =$
694. The locus of the mid-points of the portion of the normal to the parabola $y^2 = 16x$ intercepted between the curve and the axis is another parabola whose latus rectum is
695. If the vertex of a hyperbola bisects the distance between its centre and the corresponding focus, then ratio of square of its conjugate axis to the square of its transverse axis is
696. Let $A(-4, 0)$ and $B(4, 0)$. If the number of points on the circle $x^2 + y^2 = 16$ such that the area of the triangle whose vertices are A, B and C is a positive integer, is N then the value of $[N/7]$ is, where N represents greatest integer function
697. If locus of a point, whose chord of contact with respect to the circle $x^2 + y^2 = 4$ is a tangent to the hyperbola $xy = 1$ is $xy = c^2$, then value of c^2 is
698. The length of a common internal tangent to two circles is 7 and a common external tangent is 11. If the product of the radii of the two circles is p, then the value of $p/2$ is
699. Let the distance between a focus and corresponding directrix of an ellipse be 8 and the eccentricity be $\frac{1}{2}$. If the length of the minor axis is k, then $\sqrt{3}\,k/2$ is
700. Consider an ellipse $(E)\ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ centered at point 'O' and having AB and CD as its major and minor axes respectively if S_1 be one of the foci of the ellipse, radius of incircle of triangle OCS_1 be 1 unit and $OS_1 = 6$ units, then the value of $(a - b)/2$ is

701. The value of a for the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b)$, if the extremities of the latus rectum of the ellipse having positive ordinate lies on the parabola $x^2=-2(y-2)$, is

702. As shown in fig, three circles which have the same radius r, have centres at $(0,0)$, $(1,1)$ and $(2,1)$. If they have a common tangent line, as shown, then the value of $10\sqrt{5}r$ is



703. If $x, y \in R$, satisfying the equation $\frac{(x-4)^2}{4}+\frac{y^2}{9}=1$, then the difference between the largest and smallest value of the expression $\frac{x^2}{4}+\frac{y^2}{9}$ is

704. If the variable line $y=kx+2h$ is tangent to an ellipse $2x^2+3y^2=6$, then locus of $P(h,k)$ is a conic C whose eccentricity is e then the value of $3e^2$ is

705. Let the lines $(y-2)=m_1(x-5)$ and $(y+4)=m_2(x-3)$ intersect at right angles at P (where m_1 and m_2 are parameters). If locus of P is $x^2+y^2+gx+fy+7=0$, then the value of $|f+g|$ is

706. If the mid point of a chord of the ellipse $\frac{x^2}{16}+\frac{y^2}{25}=1$ is $(0,3)$, and length of the chord is $\frac{4k}{5}$, then k is

707. Tangents drawn from the point $P\ (2,3)$ to the circle $x^2+y^2-8x+6y+1=0$ touch the circle at the points A and B. The circumcircle of the ΔPAB cuts the director circle of ellipse $\frac{(x+5)^2}{9}+\frac{(y-3)^2}{b^2}=1$ orthogonally. Then the value of $b^2/6$ is

708. If L is the length of latus rectum of hyperbola for which $x=3$ and $y=2$ are the equations of asymptotes and which passes through the point $(4,6)$, then the value of $L/\sqrt{2}$ is

709. If two perpendicular tangents can be drawn from the origin to the circle $x^2-6x+y^2-2py+17=0$, then the value of $|p|$ is

710. If the length of focal chord to the parabola $y^2=12x$ drawn from the point $(3,6)$ on it is L then the value of $L/3$ is

711. If real numbers x and y satisfy $(x+5)^2+(y-12)^2=(14)^2$, then the minimum value of $\sqrt{x^2+y^2}$ is

712. Rectangle $ABCD$ has area 200. An ellipse with area 200π passes through A and C and has foci at B and D. If the perimeter of the rectangle is P, then the value of $P/20$ is

713. If distance between two parallel tangents having slope m drawn to the hyperbola $\frac{x^2}{9}-\frac{y^2}{49}=1$ is 2, then the value of $2|m|$ is

714. Line $y=2x-b$ cuts the parabola $y=x^2-4x$ at points A and B. Then the value of b for which the $\angle AOB$ is a right angle is

715. Tangents are drawn from the point (α,β) to the hyperbola $3x^2-2y^2=6$ and are inclined at angle θ and ϕ to the x-axis. If $\tan\theta\,.\tan\phi=2$, then the value of $2\alpha^2-\beta^2$ is

716. The acute angle between the line $3x-4y=5$ and the circle $x^2+y^2-4x+2y-4=0$ is θ, then $9\cos\theta$

717. An ellipse passing through the origin has its foci $(3,4)$ and $(6,8)$ and length of its semi-minor axis is b, then the value of $b/\sqrt{2}$ is

718. A line through the origin intersects the parabola $5y=2x^2-9x+10$ at two points whose x-coordinates add up to 17. Then the slope of the line is

719. The sum of the slopes of the lines tangent to both circles $x^2+y^2=1$ and $(x-6)^2+y^2=4$ is

720. Two tangents are drawn from the point $(-2,-1)$ to the parabola $y^2=4x$. If θ is the angle between these tangents then $\tan\theta=$

721. $y=x+2$ is any tangent to the parabola $y^2=8x$. The ordinate of the point P on this tangent such that the other tangent from it which is perpendicular to it is

722. The equation of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is $ax + by + c = 0$ then the value of $a + b + c$ is
723. If circle and $(x-6)^2 + y^2 = r^2$ and parabola $y^2 = 4x$ have maximum number of common chord then least integral value of r is
724. Difference in values of radius of a circle whose centre is at the origin and which touches the circles $x^2 + y^2 - 6x - 8y + 21 = 0$ is
725. Two circles are externally tangent. Lines PAB and $PA'B'$ are common tangents with A and A' on the smaller circle and B and B'on the larger circle. If $PA = AB = 4$, then the square of radius of circle is
726. If tangents drawn from the point $(a, 2)$ to the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ are perpendicular, then the value of a^2 is
727. Eccentricity of the hyperbola $\left|\sqrt{(x-3)^2 + (y-2)^2} - \sqrt{(x+1)^2 + (y+1)^2}\right| = 1$ is
728. If the eccentricity of the hyperbola $x^2 - y^2 \sec^2 \theta = 5$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2 \sec^2 \theta + y^2 = 25$, then smallest positive value of θ is $\frac{\pi}{P}$, value of 'p' is
729. If $y = mx + c$ is tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, having eccentricity 5, then the least positive integral value of m is
730. If the length of the latus rectum of the parabola $169\{(x-1)^2 + (y-3)^2\} = (5x - 12y + 17)^2$ is L then the value of $\frac{13L}{4}$ is
731. A circle $x^2 + y^2 + 4x - 2\sqrt{2}\, y + c = 0$ is the director circle of circle S_1 and S_1 is the director circle of circle S_2 and so on. If the sum of radii of all these circles is 2, then the value of $c = k\sqrt{2}$, where value of k is
732. The area of triangle formed by the tangents from point (3, 2) to hyperbola $x^2 - 9y^2 = 9$ and the chord of contact w.r.t. point (3, 2)
733. Consider the graphs of $y = Ax^2$ and $y^2 + 3 = x^2 + 4y$, where A is a positive constant and $x, y \in R$. Number of points in which the two graphs intersect is
734. If from a point $P(0, \alpha)$ two normals other than axes are drawn to ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, such that $|\alpha| < k$, then the value of $4k$ is
735. PQ is any focal chord of the parabola $y^2 = 8x$. Then the length of PQ can never be less than
736. A tangent drawn to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $P\left(\frac{\pi}{6}\right)$ forms a triangle of area $3a^2$ square units, with coordinate axes. If the eccentricity of hyperbola is e, then the value of $e^2 - 9$ is
737. Suppose x and y are real numbers and that $x^2 + 9y^2 - 4x + 6y + 4 = 0$, then the maximum value of $(4x - 9y)/2$ is
738. If on a given base BC ($B(0,0)$ and $C(2,0)$) a triangle be described such that the sum of the tangents of the base angles is 4, then equation of locus of opposite vertex A is parabola whose directrix is $y = k$, then the value of $8k - 9$ is
739. The focal chord of $y^2 = 16x$ is tangent to $(x-6)^2 + y^2 = 2$, then the possible value of the square of slope of this chord is
740. The line $3x + 6y = k$ intersect the curve $2x^2 + 2xy + 3y^2 = 1$ at points A and B. The circle on AB as diameter passes through the origin. Then the value of k^2 is
741. If the chord $x \cos \alpha + y \sin \alpha = p$ of the hyperbola $\frac{x^2}{16} - \frac{y^2}{18} = 1$ subtends a right angle at the centre, and the diameter of the circle, concentric with the hyperbola, to which the given chord is a tangent is d then the value of $d/4$ is
742. The number of points $P(x, y)$ lying inside or on the circle $x^2 + y^2 = 9$ and satisfying the equation $\tan^4 x + \cot^4 x + 2 = 4 \sin^2 y$, is
743. Consider the family of circles $x^2 + y^2 - 2x - 2\lambda y - 8 = 0$ passing through two fixed points A and B. Then the distance between the points A and B is
744. From the point $(-1, 2)$ tangent lines are drawn to the parabola $y^2 = 4x$. If the area of the triangle formed

by the chord of contact & the tangents is A the value of $\frac{A}{\sqrt{2}}$ is

: ANSWER KEY :

1)	b	2)	b	3)	a	4)	c	189)	a	190)	a	191)	c	192)	d
5)	d	6)	b	7)	b	8)	b	193)	d	194)	d	195)	a	196)	a
9)	a	10)	a	11)	a	12)	a	197)	a	198)	a	199)	c	200)	c
13)	a	14)	d	15)	d	16)	b	201)	d	202)	b	203)	d	204)	c
17)	d	18)	d	19)	c	20)	b	205)	a	206)	b	207)	c	208)	c
21)	c	22)	d	23)	d	24)	a	209)	c	210)	d	211)	b	212)	a
25)	a	26)	a	27)	d	28)	c	213)	b	214)	c	215)	c	216)	b
29)	c	30)	c	31)	c	32)	c	217)	c	218)	b	219)	c	220)	d
33)	c	34)	b	35)	b	36)	c	221)	c	222)	b	223)	b	224)	c
37)	b	38)	c	39)	c	40)	a	225)	b	226)	c	227)	b	228)	c
41)	a	42)	b	43)	b	44)	b	229)	a	230)	a	231)	c	232)	c
45)	c	46)	a	47)	b	48)	d	233)	a	234)	c	235)	d	236)	b
49)	b	50)	c	51)	a	52)	c	237)	b	238)	a	239)	c	240)	d
53)	c	54)	c	55)	b	56)	b	241)	c	242)	a	243)	c	244)	b
57)	a	58)	a	59)	d	60)	a	245)	a	246)	d	247)	d	248)	c
61)	d	62)	d	63)	c	64)	b	249)	a	250)	c	251)	b	252)	b
65)	b	66)	d	67)	b	68)	c	253)	d	254)	a	255)	c	256)	a
69)	c	70)	b	71)	a	72)	c	257)	b	258)	d	259)	a	260)	d
73)	a	74)	a	75)	c	76)	b	261)	a	262)	d	263)	d	264)	d
77)	d	78)	d	79)	a	80)	a	265)	c	266)	d	267)	d	268)	a
81)	b	82)	a	83)	a	84)	b	269)	c	270)	a	271)	c	272)	c
85)	d	86)	b	87)	a	88)	a	273)	d	274)	c	275)	b	276)	a
89)	a	90)	a	91)	b	92)	d	277)	a	278)	c	279)	b	280)	b
93)	a	94)	c	95)	a	96)	d	281)	a	282)	b	283)	c	284)	a
97)	a	98)	b	99)	c	100)	d	285)	a	286)	a	287)	c	288)	c
101)	c	102)	b	103)	a	104)	d	289)	c	290)	b	291)	d	292)	d
105)	b	106)	a	107)	a	108)	c	293)	b	294)	a	295)	d	296)	c
109)	b	110)	b	111)	b	112)	c	297)	c	298)	d	299)	b	300)	a
113)	d	114)	c	115)	a	116)	d	301)	c	302)	c	303)	a	304)	c
117)	d	118)	c	119)	c	120)	a	305)	d	306)	d	307)	b	308)	a
121)	d	122)	c	123)	c	124)	b	309)	c	310)	d	311)	c	312)	b
125)	a	126)	b	127)	b	128)	a	313)	a	314)	d	315)	d	316)	b
129)	b	130)	c	131)	a	132)	c	317)	b	318)	a	319)	c	320)	d
133)	d	134)	d	135)	b	136)	a	321)	a	322)	a	323)	c	324)	b
137)	c	138)	d	139)	a	140)	a	325)	a	326)	d	327)	a	328)	a
141)	d	142)	c	143)	a	144)	d	329)	a	330)	d	331)	b	332)	a
145)	b	146)	d	147)	b	148)	b	333)	d	334)	c	335)	b	336)	c
149)	a	150)	b	151)	b	152)	d	337)	c	338)	c	339)	d	340)	d
153)	b	154)	a	155)	d	156)	b	341)	c	342)	a	343)	b	344)	a
157)	c	158)	a	159)	c	160)	d	345)	a	346)	c	347)	c	348)	a
161)	c	162)	c	163)	b	164)	d	349)	c	350)	a	351)	a	352)	c
165)	c	166)	a	167)	b	168)	a	353)	d	354)	b	355)	a	356)	a
169)	a	170)	c	171)	b	172)	c	357)	d	358)	b	359)	a	360)	d
173)	c	174)	a	175)	c	176)	b	361)	a	362)	c	363)	b	364)	b
177)	c	178)	b	179)	a	180)	b	365)	a	366)	c	367)	c	368)	a
181)	b	182)	c	183)	a	184)	b	369)	b	370)	d	371)	d	372)	c
185)	c	186)	c	187)	b	188)	d	373)	b	374)	a	375)	b	376)	d

377) d 378) a 379) b 380) b
381) b 382) b 383) c 384) a
385) c 386) c 387) a 388) c
389) a 390) c 391) c 392) d
393) b 394) a 395) a 396) a
397) a 398) b 399) b 400) d
401) b 402) c 403) c 404) a
405) a 406) b 407) c 408) c
409) b 1) a,d 2) c,d 3)
a,b,c 4) b,d
5) a,c,d 6) a, b, c 7) c,d 8)
a, d
9) a,c,d 10) b,d 11) b,d 12)
b,c,d
13) a,c 14) b,c 15) b,d 16)
b,c
17) a,d 18) a,c 19) a,c 20)
a,c
21) a,c 22) a,b,c,d 23) a,c,d 24)
b,d
25) a, c 26) a,b 27) a,c 28)
b,c
29) c,d 30) a,b,c,d 31) a,b 32)
a, c
33) a,c,d 34) a,b,c,d 35) a, c 36)
b,c
37) b,c 38) a,d 39) a, b, c 40)
b,c
41) a,d 42) a,b,c,d 43) a,c 44)
a,c
45) a,b,c,d 46) a,b,c,d 47) a,c 48)
a, b, c
49) a,d 50) a,b 51) a,c 52)
a,d
53) c, d 54) c,d 55) a,c 56)
a,b,c,d
57) b 58) a,d 59) b, d 60)
a,c
61) b,c,d 62) a,c 63) a,b,c 64)
c,d
65) b,c,d 66) a,b 67) a 68)
a,b,c
69) a, b, c, d 70) b 71) a,b,c,d
72) a,b
73) a,d 74) a,d 75) a,b,c,d 76)
a,d
77) b,c 78) a,c 79) c,d 80)
a,c
81) a,b 82) a,b,c 83) b,d 84)
a,c
85) a,b 86) a, c 87) a,b,d 88)
a,b
89) c,d 90) a,b,c,d 91) b,d 92)
a,b,c,d
93) a,c 94) a,c 95) a,c 96)
a, c
97) a,b,c 1) a 2) b 3) a
4) b
5) d 6) d 7) c 8) b
9) a 10) b 11) b 12) a
13) b 14) b 15) d 16) c
17) c 18) a 19) c 20) b
21) b 22) c 23) d 24) a
25) a 26) a 27) c 28) a
29) d 30) a 31) d 32) a
33) a 34) d 35) a 36) d
37) a 38) b 39) a 40) c
41) b 42) a 43) a 44) b
45) d 46) a 47) a 48) a
49) d 50) d 51) c 52) d
53) d 54) b 55) b 56) d
57) d 58) c 59) a 60) c
61) a 62) c 63) b 64) a
65) d 66) b 67) a 68) c
69) c 70) a 71) b 72) d
73) a 74) a 75) a 76) b
77) a 78) a 79) a 80) a
81) a 82) c 83) a 84) c
85) b 86) d 87) d 88) a
89) b 90) c 91) d 92) d
93) d 94) b 95) a 96) d
97) d 98) b 99) d 100) b
101) a 102) d 103) d 104) a
105) b 106) a 107) a 108) c
109) a 1) a 2) d 3) a
4) c
5) a 6) a 7) b 8) c
9) a 10) d 11) b 12) a
13) b 14) a 15) a 16) c
17) a 18) b 19) c 20) b
21) d 1) a 2) d 3) d
4) b
5) c 6) b 7) b 8) a
9) b 10) b 11) b 12) d
13) b 14) d 15) d 16) c
17) c 18) c 19) b 20) a
21) b 22) b 23) b 24) c
25) b 26) a 27) d 28) a
29) b 30) a 31) d 32) b
33) d 34) c 35) c 36) b
37) b 38) a 39) a 40) c
41) b 42) a 43) b 44) d

45)	c	46)	a	47)	b	48)	b
1)	3	2)	4	3)	1	4)	6
5)	4	6)	4	7)	1	8)	3
9)	2	10)	4	11)	3	12)	8
13)	4	14)	9	15)	8	16)	4
17)	2	18)	5	19)	8	20)	7
21)	6	22)	8	23)	9	24)	4
25)	5	26)	4	27)	1	28)	4
29)	5	30)	7	31)	7	32)	3

33)	5	34)	5	35)	0	36)	3
37)	0	38)	3	39)	5	40)	4
41)	2	42)	3	43)	5	44)	4
45)	5	46)	7	47)	4	48)	8
49)	4	50)	9	51)	8	52)	8
53)	8	54)	8	55)	1	56)	9
57)	6	58)	8	59)	6	60)	8

: HINTS AND SOLUTIONS :

1 **(b)**

Equation of normals is given by $ty = t^3x - ct^4 + c = 0$. It passes though $(ct', c/t')$. Hence,

$$\frac{c}{t'} = t^3ct' - ct^4 + c = 0$$

$$t = t^3t^2 - tt^4 + t'$$

$$t^3t' = -1$$

2 **(b)**

We have, $\sqrt{px} + \sqrt{qy} = 1$

$\Rightarrow \left(\sqrt{px} + \sqrt{qy}\right)^2 = 1$

$\Rightarrow\ px + qy + 2\sqrt{(pq)(xy)} = 1$

$\Rightarrow (px + qy - 1)^2 = 4(pq)(xy)$

$\Rightarrow p^2x^2 - 2(pq)(xy) + q^2y^2 - 2px - 2qy + 1 = 0$

On comparing this equation with the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, we get

$a = p^2, b = q^2, c = 1, g = -p,$

$f = -q$ and $h = -pq$

$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

$= p^2q^2 - 2p^2q^2 - p^2q^2 - p^2q^2\ - p^2q^2$

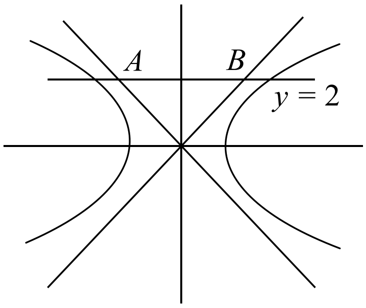
$= -4p^2q^2 \neq 0$

and $h^2 - ab = p^2q^2 - p^2q^2 = 0$

Thus, we have $\Delta \neq 0$ and $h^2 = ab$

Hence, the given curve is parabola

3 **(a)**



For two distinct tangents on different branches the point should lie on the line $y = 2$ and between A and B (where A and B are the points on the asymptotes)

Equation of asymptotes are $4x = \pm 3y$

Solving with $y = 2$, we have

$$x = \pm\frac{3}{2}$$

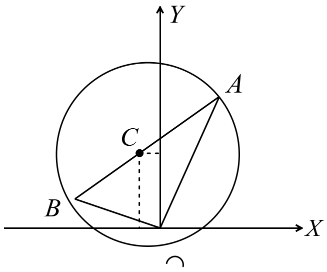
$$\therefore -\frac{3}{2} < \alpha < \frac{3}{2}$$

4 **(c)**

Let the equation of the chord OA of the circle

$x^2 + y^2 - 2x + 4y = 0 \quad$ ...(i)

Be $y = mx\ \ $ ...(ii)



Solving (i) and (ii), we get

$\Rightarrow x^2 + m^2x^2 - 2x + 4mx = 0$

$\Rightarrow (1 + m^2)x^2 - (2 - 4m)x = 0$

$\Rightarrow x = 0$ and $x = \frac{2-4m}{1+m^2}$

Hence, the points of intersection are

$(0, 0)$ and $A\left(\frac{2-4m}{1+m^2}, \frac{m(2-4m)}{1+m^2}\right)$

$$\Rightarrow OA^2 = \left(\frac{2 - 4m}{1 + m^2}\right)^2 (1 + m^2)$$

$$= \frac{(2 - 4m)^2}{1 + m^2}$$

Since OAB is an isosceles right-angled triangle

$OA^2 = \frac{1}{2}AB^2$

Where AB is a diameter of the given circle

$OA^2 = 10$

$\Rightarrow \dfrac{(2-4m)^2}{1+m^2} = 10$

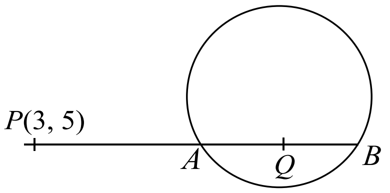
$\Rightarrow 4 - 16m + 16m^2 = 10(1+m)^2$

$\Rightarrow 3m^2 - 8m - 3 = 0$

$\Rightarrow m = 3$ or $-\frac{1}{3}$

Hence, the required equations are $y = 3x$ or $x + 3y = 0$

5 **(d)**



$2PQ = PA + PB$

$\Rightarrow PQ - PA = PB - PQ$

$\Rightarrow AQ = QB$

$\Rightarrow$ Q is midpoint of AB

Let Q has coordinates (h, k)

Then equation of chord AB is given by $T = S_1$ or $hx + ky - 4 = h^2 + k^2 - 4$

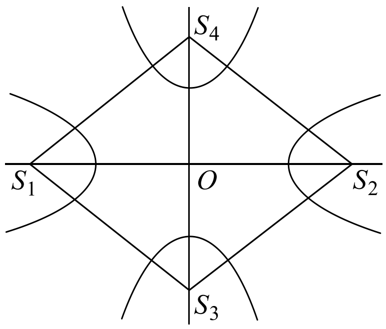
This variable chord passes through the point $P(3, 5)$

$\Rightarrow 3h + 5k = h^2 + k^2$

$\Rightarrow x^2 + y^2 - 3x - 5y = 0$

Which is required locus

6 **(b)**



Required area $= 4$ area $\Delta S_2OS_4 = 4 \times \frac{1}{2} ae \times 8\, be_1 - 4 \times \frac{1}{2} \times 2 \times 3 \times ee_1$ (i)

$b^2 = a^2(e^2 - 1) \Rightarrow e^2 = \dfrac{9}{4} + 1 = \dfrac{13}{4}$

Also $\frac{1}{e_1^2} = 1 - \frac{1}{e^2} = 1 - \frac{4}{13} = \frac{9}{13}$

$e_1^2 = \dfrac{13}{9}$

Required area $= 12 \times \frac{\sqrt{13}}{2} \times \frac{\sqrt{13}}{3}$

$= 2 \times 13 = 26$

7 **(b)**

Let the tangent be of form $\frac{x}{x_1} + \frac{y}{y_1} = 1$ and area of Δ formed by it with coordinate axes is

$\frac{1}{2} x_1 y_1 = a^2$ ...(i)

Again, $y_1 x + x_1 y - x_1 y_1 = 0$

Applying conditions of tangency

$\frac{|-x_1y_1|}{\sqrt{x_1^2 + y_1^2}} = a$ or $(x_1^2 + y_1^2) = \frac{x_1^2 y_1^2}{a^2}$ ...(ii)

From Eqs. (i) and (ii), we get x_1, y_1, which gives equation of tangent as $x \pm y = \pm a\sqrt{2}$

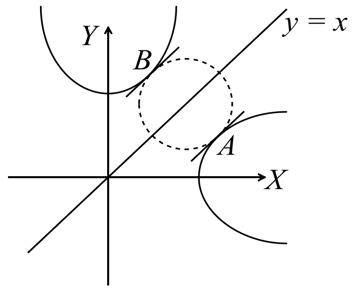
8 **(b)**

Parabola $y = x^2 + 1$ and $x = y^2 + 1$ are symmetrical about $y = x$

Therefore, tangent at point A is parallel to $y = x$

$\Rightarrow \dfrac{dy}{dx} = 2x \Rightarrow 2x = 1$

$\Rightarrow x = \frac{1}{2}$ and $y = \frac{5}{4}$



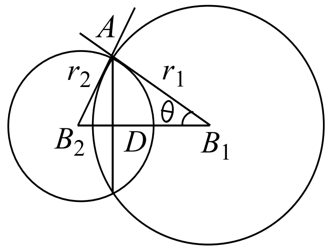
$A\left(\frac{1}{2}, \frac{5}{4}\right)$ and $B\left(\frac{5}{4}, \frac{1}{2}\right)$

Therefore, radius $= \frac{1}{2}\sqrt{\left(\frac{1}{2} - \frac{5}{4}\right)^2 + \left(\frac{5}{4} - \frac{1}{2}\right)^2} = \frac{1}{2}\sqrt{\frac{9}{16} + \frac{9}{16}}$

$= \dfrac{3}{8}\sqrt{2}$

Therefore, area $= \frac{9\pi}{32}$

9 **(a)**



Let $\angle AB_1B_2 = \theta$

$\Rightarrow AD = r_1 \sin\theta$

and $AD = r_2 \cos\theta$

$\Rightarrow AD^2\left(\dfrac{1}{r_1^2} + \dfrac{1}{r_2^2}\right) = 1$

$\Rightarrow AD = \dfrac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$

Thus, length of common chord $= \frac{2r_1r_2}{\sqrt{r_1^2+r_2^2}}$

10 **(a)**

Let focus be (a, b)

Equations are

$S_1: (x-a)^2 + (y-b)^2 = x^2$

and $S_2: (x-a)^2 + (y-b)^2 = y^2$

Common chord $S_1 - S_2 = 0$ gives $x^2 - y^2 = 0$

$\Rightarrow y = \pm x$

11 **(a)**

Any tangent to hyperbola is

$\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$ (i)

Given tangent is

$ax + by = 1$ (ii)

Comparing Eqs. (i) and (ii), we have

$\sec\theta = a^2$ and $\tan\theta = -b^2$

Eliminating θ, we have

$a^4 - b^4 = 1$

$\Rightarrow (a^2 - b^2)(a^2 + b^2) = 1$

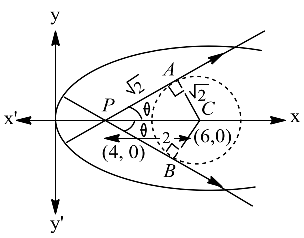
Also $a^2 + b^2 = a^2e^2$

$\Rightarrow a^2 - b^2 = \frac{1}{a^2e^2}$

12 **(a)**

Here, the focal chord to $y^2 = 16x$ is tangent to circle $(x-6)^2 + y^2 = 2$

$\Rightarrow$ focus of parabola is (4, 0)



Now, tangent are drawn from $(4, 0)$ to $(x-6)^2 + y^2 = 2$

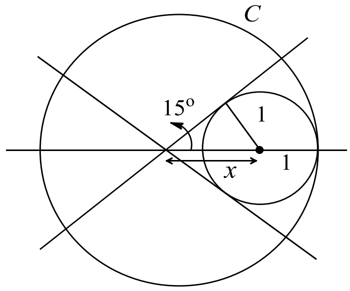
Since, PA is tangent to circle

$\tan\theta =$ slope of tangent $= \frac{AC}{AP} = \frac{\sqrt{2}}{\sqrt{2}} = 1$

or $\tan\theta = \frac{BC}{BP} = -1$

$\therefore$ Slope of focal chord as tangent to circle$= \pm 1$

13 **(a)**



$\text{cosec}\ 15° = \frac{x}{1}$

$\Rightarrow x = \text{cosec}\ 15°$

$\Rightarrow R = x + 1 = 1 + \text{cosec}\ 15°$

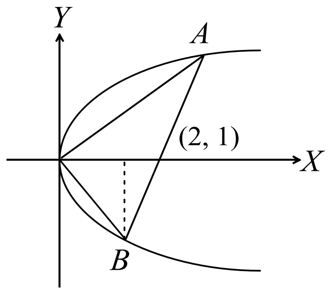
$= 1 + \frac{2\sqrt{2}}{\sqrt{3}-1}$

$= 1 + \frac{4}{\sqrt{6}-\sqrt{2}}$

$= 1 + \sqrt{6} + \sqrt{2}$

14 **(d)**

Chord through (2, 1) is $\frac{x-2}{\cos\theta} = \frac{y-1}{\sin\theta} = r$ (i)



Solving Eq.(i)with parabola $y^2 = x$, we have

$(1 + r\sin\theta)^2 = 2 + r\cos\theta$

$\Rightarrow \sin^2\theta\, r^2 + (2\sin\theta - \cos\theta)r - 1 = 0$

This equation has two roots $r_1 = AC$ and $r_2 = -BC$

Then, sum of roots $r_1 + r_2 = 0$

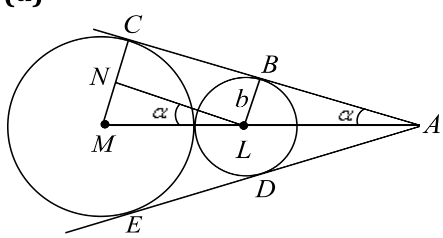
$\Rightarrow 2\sin\theta - \cos\theta = 0 \Rightarrow \tan\theta = \frac{1}{2}$

$AB = |r_1 - r_2|$

$= \sqrt{(r_1 + r_2)^2 - 4r_1r_2}$

$= \sqrt{4\frac{1}{\sin^2\theta}} = 2\sqrt{5}$

15 **(d)**



From $\Delta\, MLN$

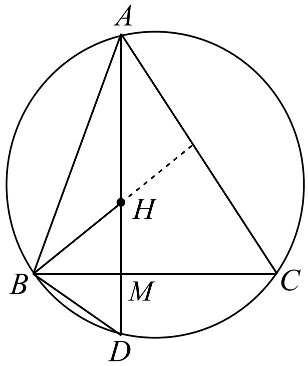
$\sin\alpha = \dfrac{a-b}{a+b}$

$\therefore\ a = \sin^{-1}\left(\dfrac{a-b}{a+b}\right)$

$\therefore$ Angle between AB and AD

$= 2\alpha = \sin^{-1}\left(\dfrac{a-b}{a+b}\right)$

16 **(b)**



Let orthocenter be $H(5, 8)$

Now, $\angle HBM = \pi/2 - C$

Also, $\angle DBC = \angle DAC = \pi/2 - C$

Hence, ΔBMH and ΔBMD are congruent

$\Rightarrow HM = MD$

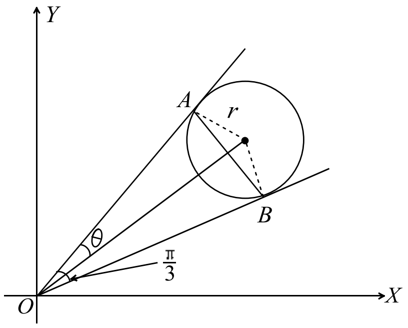
$\Rightarrow D$ is image of H in the line $x - y = 0$ which is $D(8, 5)$

Thus, equation of circumcircle is

$(x-2)^2 + (y-3)^2 = (8-2)^2 + (5-3)^2$

i.e. $x^2 + y^2 - 4x - 6y - 27 = 0$

17 **(d)**



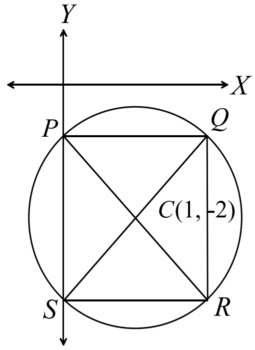
Here $\tan 2\theta = \frac{2\sqrt{4-1}}{2} = \sqrt{3}$

$\Rightarrow\ \theta = \pi/6$

Area of $\Delta OAB = \frac{1}{2}\,(r\cot\theta)^2(\sin 2\theta)$

$= \dfrac{1}{2}\left(r\sqrt{3}\right)^2 \dfrac{\sqrt{3}}{2}$

18 **(d)**



Radius of the circle $CQ = \sqrt{2}$

Since $\angle QSR = 45°$

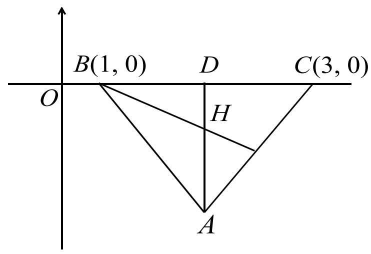
Coordinates of Q and S are given by $\big(1 \pm 2\cos 45°, -2 \pm 2\sin 45°$

or $Q(2, -1)$ and $S(0, -3)$

Coordinates of P and R are given by $\big(1 \pm 2\cos 135°, -2 \pm 2\sin 135°$

or $P(0, -1)$ and $S(2, -3)$

19 **(c)**



Radical centre of the circles described on the sides of a triangle as diameters is the orthocenter of the triangle

$\therefore D = (2, 0)$

$DH = -BD\tan\dfrac{\pi}{6} = -\dfrac{1}{\sqrt{3}}$

$\therefore$ Coordinates of H are $\left(2, -\frac{1}{\sqrt{3}}\right)$

20 **(b)**

Let A and B be the centres and r_1 and r_2 the radii of the two circles, then

$A = \left(-\dfrac{1}{2}, -\dfrac{1}{2}\right), B \equiv \left(-\dfrac{1}{2}, \dfrac{1}{2}\right),$

$r_1 = \dfrac{1}{\sqrt{2}}, r_2 = \dfrac{1}{\sqrt{2}}$

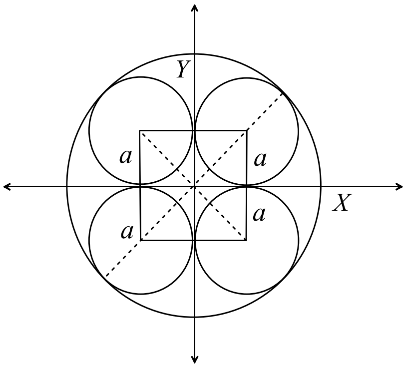
$\cos\theta = \dfrac{r_1^2 + r_2^2 - AB^2}{2r_1r_2}$

$= \dfrac{\frac{1}{2} + \frac{1}{2} - 1}{2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}} = 0$

$\therefore\ \theta = \dfrac{\pi}{2}$

$\therefore$ Required line is parallel to x-axis and since it passes through $(1, 2)$, therefore its equation will be $y = 2$

21 **(c)**



The four circles are as shown in the fig

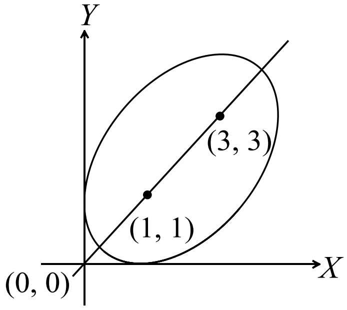
The smallest circle touching all of them has the radius $= \sqrt{2}a - a$ and the greatest circle touching all of them has the radius $= \sqrt{2}a + a$

22 **(d)**

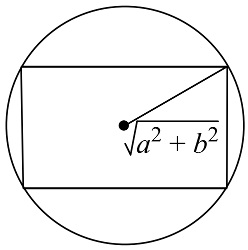
Since x –axis and y-axis are perpendicular tangents to the ellipse. $(0, 0)$ lies on the director circle and midpoint of foci $(2, 2)$ is centre of the circle.

Hence, radius $= 2\sqrt{2}$

$\Rightarrow$ The area is 8π units



23 **(d)**



Since mutually perpendicular tangents can be drawn from vertices of rectangle. So all the vertices of rectangle should lie on director circle $x^2 + y^2 = a^2 + b^2$.

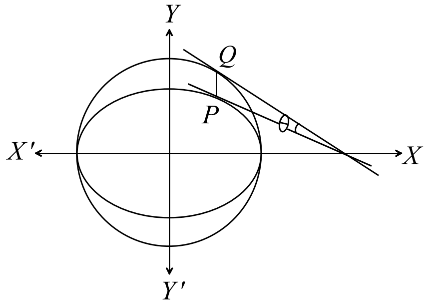
Let breadth $= 2l$ and length $= 4l$, then

$l^2 + (2l)^2 = a^2 + b^2$

$\Rightarrow l^2 = \dfrac{a^2 + b^2}{5}$

$\Rightarrow$ Area $= 4l \times 2l = 8\,\frac{a^2+b^2}{5}$

24 **(a)**



Tangent to the ellipse at $P(a\cos\alpha, b\sin\alpha)$ is

$\frac{x}{a}\cos\alpha + \frac{y}{b}\sin\alpha = 1$ (i)

Tangent to the circle at $Q(a\cos\alpha, a\sin\alpha)$ is

$\cos\alpha x + \sin\alpha y = a$ (ii)

Now angel between tangents is θ,

Then $\tan\theta = \left|\frac{-\frac{b}{a}\cot\alpha - (-\cot\alpha)}{1 + \left(-\frac{b}{a}\cot\alpha\right)(-\cot\alpha)}\right|$

$= \left|\dfrac{\cot\alpha\left(1 - \frac{b}{a}\right)}{1 + \frac{b}{a}\cot^2\alpha}\right|$

$= \left|\dfrac{a - b}{a\tan\alpha + b\cot\alpha}\right|$

$\left|\dfrac{a - b}{\left(\sqrt{a\tan\alpha} - \sqrt{b\cot\alpha}\right)^2 + 2\sqrt{ab}}\right|$

Now the greatest value of the above expression is $\left|\frac{a-b}{2\sqrt{ab}}\right|$

When $\sqrt{a\tan\alpha} = \sqrt{b\cot\alpha}$

$\Rightarrow\ \theta_{\text{maximum}} = \tan^{-1}\left(\dfrac{a-b}{2\sqrt{ab}}\right)$

25 **(a)**

Equation of tangent $\frac{x}{a}\frac{\sqrt{3}}{2} + \frac{y}{b}\frac{1}{2} = 1$ (i)

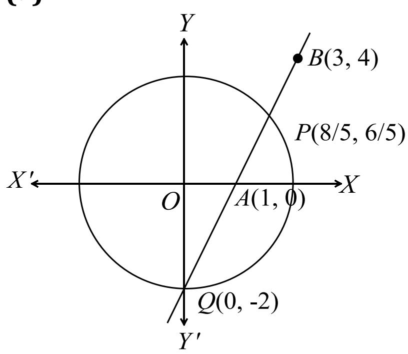
and equation of tangent at the point $(a\cos\phi, b\sin\phi)$ is

$\frac{x}{a}\cos\phi + \frac{y}{b}\sin\phi = 1$ ...(ii)

Comparing (i) and (ii), we have $\cos\phi = \frac{\sqrt{3}}{2}$ and $\sin\phi = \frac{1}{2}$

Hence, $\phi = \frac{\pi}{6}$

26 **(a)**



The equation of the line joining $A(1, 0)$ and

$B(3,4)$ is $y = 2x - 2$
This cuts the circle $x^2 + y^2 = 4$ at $Q(0,-2)$ and $P\left(\frac{8}{5}, \frac{6}{5}\right)$
We have $BQ = 3\sqrt{5}, QA = \sqrt{5}, BP = \frac{7}{\sqrt{5}}$ and $PA = \frac{3}{\sqrt{5}}$

$$\therefore\ \alpha = \frac{BP}{PA} = \frac{7/\sqrt{5}}{3/\sqrt{5}} = \frac{7}{3}$$

and $\beta = \frac{BQ}{QA} = \frac{3\sqrt{5}}{-\sqrt{5}} = -3$
$\therefore\ \alpha, \beta$ are roots of the equation $x^2 - x(\alpha+\beta) + \alpha\beta = 0$
i.e. $x^2 - x\left(\frac{7}{3} - 3\right) + \frac{7}{3}(-3) = 0$
or $3x^2 + 2x - 21 = 0$

27 **(d)**
The equation of rectangular hyperbola is

$(x-3)(y-5) + \lambda = 0$

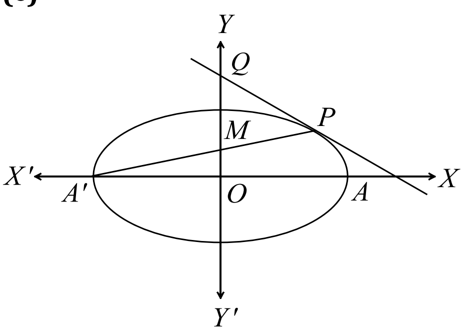
Which passes through $(7, 8)$. Hence,

$4 \times 3 + \lambda \Rightarrow \lambda = -12$

$\therefore\ xy - 5x - 3y + 15 - 12 = 0$

$\Rightarrow\ xy - 3y - 5x + 3 = 0$

28 **(c)**



Let point P be $(a\cos\theta, b\sin\theta)$
Equation of the tangent at point P is

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$$

Then point Q is $(b \operatorname{cosec}\theta, 0)$
Equation of chord $A'P$ is

$$y - 0 = \frac{b\sin\theta}{a\cos\theta + a}(x+a)$$

Putting $x = 0$, we have $y = \frac{b\sin\theta}{\cos\theta+1}$
Then

$$OQ^2 - MQ^2 = b^2\operatorname{cosec}^2\theta - \left(b\operatorname{cosec}\theta - \frac{b\sin\theta}{\cos\theta+1}\right)^2$$

$$= \frac{2b^2}{\cos\theta+1} - \frac{b^2\sin^2\theta}{(\cos\theta+1)^2}$$

$$= \frac{b^2}{\cos\theta+1}\left(\frac{2\cos\theta + 2 - \sin^2\theta}{(\cos\theta+1)}\right)$$

$$= \frac{b^2}{\cos\theta+1}\left(\frac{2\cos\theta + 1 + \cos^2\theta}{(\cos\theta+1)}\right)$$

$$= b^2 = 4$$

29 **(c)**
Let the point $P(h,k)$ on the parabola divides the line joining $A(4,-6)$ and $B(3,1)$ in ratio λ
Then, we have $(h,k) \equiv \left(\frac{3\lambda+4}{\lambda+1}, \frac{\lambda-6}{\lambda+1}\right)$
This point lies on the parabola,

$$\therefore\ \left(\frac{\lambda-6}{\lambda+1}\right)^2 = 4\left(\frac{3\lambda+4}{\lambda+1}\right)$$

$\Rightarrow (\lambda-6)^2 = 4(3\lambda+4)(\lambda+1)$
$\Rightarrow 11\lambda^2 + 40\lambda - 20 = 0$
$\Rightarrow \lambda = \frac{-20\pm2\sqrt{155}}{11} : 1$

30 **(c)**
Solving $y = 2x - 3$ and $y^2 = 4a\left(x - \frac{1}{3}\right)$, we have

$$(2x-3)^2 = 4a\left(x - \frac{1}{3}\right)$$

$$\Rightarrow 4x^2 + 9 - 12x = 4ax - \frac{4a}{3}$$

$$\Rightarrow 4x^2 - 4(3+a)x + 9 + \frac{4a}{3} = 0$$

This equation must have equal roots $\Rightarrow D = 0$

$$\Rightarrow 16(3+a)^2 - 16\left(9 + \frac{4a}{3}\right) = 0$$

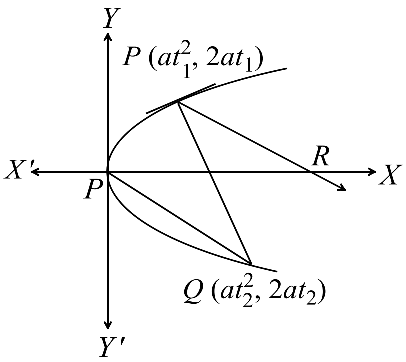
$\Rightarrow\ 9 + a^2 + 6a = 9 + \frac{4a}{3}$
$\Rightarrow\ a^2 + \frac{14a}{3} = 0$
$\Rightarrow\ a = 0$ or $a = \frac{14}{3}$

31 **(c)**
The equation of any normal be $y = -tx + 2t + t^3$
Since it passes through the points $(15, 12)$
$\therefore\ 12 = -15t + 2t + t^3$
$\Rightarrow t^3 - 13t - 12 = 0$
One root is -1, then
$(t+1)(t^2 + t - 12) = 0$
$\Rightarrow t = 1, 3, 4$
Therefore, the co-normal points are
$(1,-2), (9,-6), (16, 8)$
Therefore, centroid is $\left(\frac{26}{3}, 0\right)$

32 **(c)**

$$t_2 = -t_1 - \frac{2}{t_1} \Rightarrow t_1t_2 = -t_1^2 - 2$$

Equation of the line through P parallel to AQ



$y - 2at_1 = \dfrac{2}{t_2}(x - at_1^2)$

Put $y = 0 \Rightarrow x = at_1^2 - at_1t_2$

$= at_1^2 - a(-2 - t_1^2)$

$= 2a + 2at_1^2$

$= 2(a + at_1^2)$

$=$ twice the focal distance of P

33 **(c)**

Tangent to parabola $y^2 = 4x$ having slope m is

$y = mx + \frac{1}{m}$

Tangent to circle $(x-1)^2 + (y+2)^2 = 16$ having slope m is

$(y+2) = m(x-1) + 4\sqrt{1+m^2}$

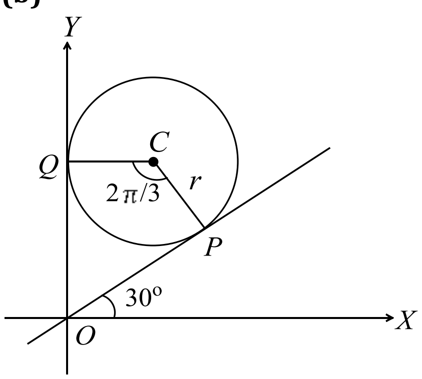
Distance between tangents

$= \left|\dfrac{4\sqrt{1+m^2} - m - 2 - 1/m}{\sqrt{1+m^2}}\right|$

$= \left|4 - \dfrac{2}{\sqrt{1+m^2}} - \dfrac{\sqrt{m^2+1}}{m}\right|$

As $m > 0 \Rightarrow d < 4$

34 **(b)**



$\tan 30° = \dfrac{r}{OP}$

$\Rightarrow OP = r\sqrt{3}$

Also $r^2 \frac{1}{2}\frac{2\pi}{3} = 3\pi$

$\therefore r = 3$

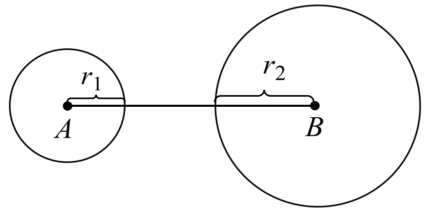
$\therefore OP = 3\sqrt{3}$

35 **(b)**

Given circles are

$(x-1)^2 + (y-2)^2 = 1$ ...(i)

and $(x-7)^2 + (y-10)^2 = 4$ ...(ii)



Let $A \equiv (1,2), B \equiv (7,10), r_1 = 1, r_2 = 2$

$AB \equiv 10, r_1 + r_2 = 3$

$AB > r_1 + r_2$, hence the two circles are separated

Radius of the two circles at time t are $(1 + 0.3t)$ and $(2 + 0.4t)$

For the two circle to touch each other

$AB^2 = [(r_1 + 0.3t) \pm (r_2 + 0.4t)]^2$

or $100 = [(1 + 0.3t) \pm (2 + 0.4t)]^2$

or $100 = (3 + 0.7t)^2 \pm [(0.1)t + 1]^2$

or $3 + 0.7t = \pm 10, 0.1t + 1 = \pm 10$

$\therefore t = 10, t = 90$ $[\because t > 0]$

The two circles will touch each other externally in 10 seconds and internally in 90 seconds

36 **(c)**

The given diameter are $2x - 3y = 5$ ...(i)

and $3x - 4y = 7$ ...(ii)

Solving Eqs. (i) and (ii), $x = 1, y = -1$

Thus $(1, -1)$ is the centre

Now area of the circle, $\pi r^2 = 154 \Rightarrow r^2 = \frac{154}{22} \times 7 = 49$

Hence, the equation of the circle is: $(x-1)^2 + (y+1)^2 = 49$

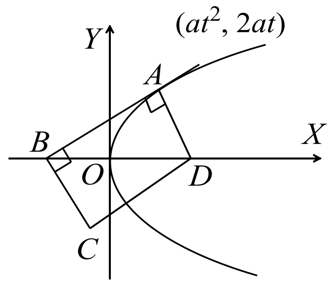
$\Rightarrow x^2 + y^2 - 2x + 2y = 47$

37 **(b)**

Equations of tangent and normal at A are

$yt = x + at^2$ and $y = -tx + 2at + at^3$

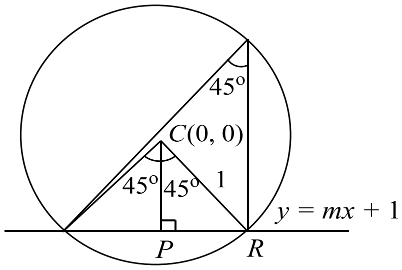
$\Rightarrow B \equiv (-at^2, 0), D \equiv (2a + at^2, 0)$. If $ABCD$ is a rectangle, then midpoints of BD and AC will be coincident



$\Rightarrow h + at^2 = 2a + at^2 - at^2, k + 2at = 0$

$\Rightarrow h = 2a, t = -\dfrac{k}{2a}$

38 **(c)**



Given circle is $x^2 + y^2 = 1$

$C(0,0)$ and radius $= 1$ and chord is $y = mx + 1$

$\cos 45° = \dfrac{CP}{CR}$

$CP =$ perpendicular distance from $(0,0)$ to chord $y = mx + 1$

$CP = \dfrac{1}{\sqrt{m^2+1}}$ $\quad (CR = \text{radius} = 1)$

$\Rightarrow \cos 45° = \dfrac{1/\sqrt{m^2+1}}{1}$

$\Rightarrow \dfrac{1}{\sqrt{2}} = \dfrac{1}{\sqrt{m^2+1}}$

$\Rightarrow m^2 + 1 = 2$

$\Rightarrow m = \neq 1$

39 **(c)**

Let P be (h,k) be any point. The chord of contact of P w.r.t. the hyperbola is

$\frac{hx}{a^2} - \frac{ky}{b^2} = 1 \qquad$ (i)

The chord of contact of P w.r.t. the auxiliary circle is

$hx + ky = a^2 - b^2 \qquad$ (ii)

Now, $\frac{h}{a^2} \times \frac{b^2}{k} \times \left(-\frac{h}{k}\right) = -1$

$\Rightarrow \dfrac{h^2}{a^2} - \dfrac{k^2}{b^2} = 0$

Therefore, P lies on one of the asymptotes

40 **(a)**

$A_1B_1$is a focal chord, then

$A_1(at_1^2, 2at_1)$ and $B_1\left(\frac{a}{t_1^2}, \frac{-2a}{t_1}\right)$

$A_2\,B_2$is a focal chord, then $A_2(at_2^2, 2at_2)$ and $B_2\left(\frac{a}{t_2^2}, \frac{-2a}{t_2}\right)$

Now equation of chord A_1A_2 is

$y(t_1 + t_2) - 2x - 2at_1t_2 = 0 \quad$ (i)

Chord B_1B_2 is

$y\left(-\dfrac{1}{t_1} - \dfrac{1}{t_2}\right) - 2x - 2a\left(-\dfrac{1}{t_1}\right)\left(-\dfrac{1}{t_2}\right) = 0$

Or $y(t_1 + t_2) + 2xt_1t_2 + 2a = 0 \quad$ (ii)

For their intersection, we subtract them and get

$2x(t_1t_2 + 1) + 2a(t_1t_2 + 1) = 0$

Or $(x + a)(1 + t_1t_2) = 0$

$\Rightarrow x + a = 0$

Hence, they interest on directrix

41 **(a)**

The given ellipse is $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Here $a^2 = 16$ and $b^2 = 9$

$\therefore b^2 = a^2(1 - e^2) \Rightarrow 9 = 16(1 - e^2)$

$\Rightarrow e = \dfrac{\sqrt{7}}{4}$

Hence, the foci are $(\pm\sqrt{7}, 0)$

Radius of the circle=distance between $(\pm\sqrt{7}, 0)$ and $(0,3) = \sqrt{7+9} = 4$

42 **(b)**

Let the asymptotes be $2x + 3y + \lambda_1 = 0$ and $x + 2y + \lambda_2 = 0$

It will pass through centre (1, 2). Hence,

$\Rightarrow \lambda_1 = -8, \lambda_2 = -5$

The equation of the hyperbola is

$(2x + 3y - 8)(x + 2y - 5) + \lambda = 0$

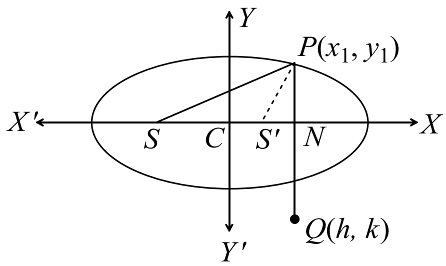
It passes through (2, 4), therefore

$(4 + 12 - 8)(2 + 8 - 5) + \lambda = 0 \Rightarrow \lambda = -40$

Hence , equation of hyperbola is

$(2x + 3y - 8)(x + 2y - 5) = 40$

43 **(b)**



$a^2 = 25$

and $b^2 = 16$

$\Rightarrow e = \sqrt{1 - \dfrac{16}{25}}$

$= \dfrac{3}{5}$

Let point Q be (h,k), where $k < 0$

Given that $k = SP = a + ex_1$, where $P(x_1, y_1)$ lies on the ellipse

$\Rightarrow |\,k| = a + eh$ (as $x_1 = h$)

$\Rightarrow -y = a + ex$

$\Rightarrow 3x+5y+25=0$

44 **(b)**

Let the concyclic point be t_1, t_2, t_3 and t_4

$\Rightarrow t_1+t_2+t_3+t_4=0$

Here, t_1 and t_3 are feet of the normals

$\Rightarrow t_2=-t_1-\frac{2}{t_1}$ and $t_4=-t_3-\frac{2}{t_3}$

$\Rightarrow t_1+t_2=-\frac{2}{t_1}$ and $t_4+t_3=-\frac{2}{t_3}$

Adding,

$-2\left(\frac{1}{t_1}+\frac{1}{t_3}\right)=0$

$\Rightarrow t_1+t_3=0$

$\Rightarrow$ Point of intersection of tangents at t_1 and $t_3\left(at_1t_3, a(t_1+t_3)\right)\equiv(at_1t_3, 0)$

$\Rightarrow$ This point lies on the axis of the parabola

45 **(c)**

Let $p=3h+2$ and $q=k$

$\Rightarrow h=\frac{p-2}{3}$ and $k=q$

Since (h,k) lies on $x^2+y^2=1$

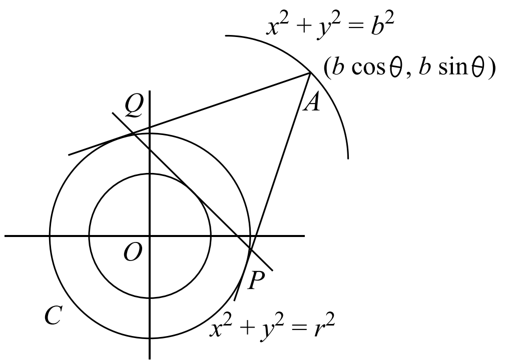
$\Rightarrow h^2+k^2=1$

$\Rightarrow \left(\frac{p-2}{3}\right)^2+q^2=1$

Locus is $\left(\frac{x-2}{3}\right)^2+y^2=1$

Which has eccentricity $e=\sqrt{1-\frac{1}{9}}=\frac{2\sqrt{2}}{3}$

46 **(a)**



Chord of contact of the points A w.r.t. $x^2+y^2=r^2$ is

$xb\cos\theta+yb\sin\theta=r^2$ (i)

This must be a tangent to the circle $x^2+y^2=a^2$

$\Rightarrow \left[\frac{r^2}{\sqrt{b^2\cos^2\theta+b^2\sin^2\theta}}\right]=a\Rightarrow r^2=ab$

Hence, equation of circle is $x^2+y^2=ab$

47 **(b)**

Since there are exactly two points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, whose distance from centre is same, the points would be either end points of the major axis or of the minor axis

But $\sqrt{\frac{a^2+2b^2}{2}}>b$, so the points are the vertices of major axis

Hence, $a=\sqrt{\frac{a^2+2b^2}{2}}$

$\Rightarrow a^2=2b^2$

$\Rightarrow e=\sqrt{1-\frac{b^2}{a^2}}=\frac{1}{\sqrt{2}}$

48 **(d)**

As we know equation of tangent to the given hyperbola at (x_1, y_1) is $xx_1-2yy_1=4$ which is same as $2x+\sqrt{6}y=2$

$\Rightarrow x_1=4$ and $y_1=\sqrt{6}$

Thus, the point of contact is $(4, -\sqrt{6})$

49 **(b)**

Since $\frac{e}{2}$ and $\frac{e'}{2}$ are eccentricities of a hyperbola and its conjugate hyperbola, therefore

$\frac{4}{e^2}+\frac{4}{e'^2}=1$

$\Rightarrow 4=\frac{e^2e'^2}{e'^2+e'^2}$

The line passing through the points $(e,0)$ and $(0,e')$ is

$e'x+ey-ee'=0$

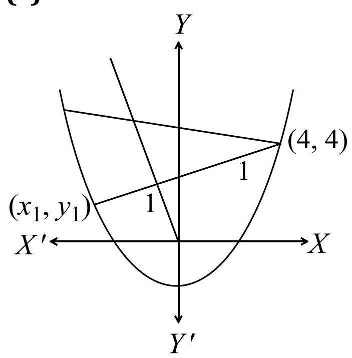
It is tangents to the circle $x^2+y^2=r^2$

Hence, $\frac{ee'}{\sqrt{e^2+e'^2}}=r$

$\Rightarrow r^2=\frac{e^2e'^2}{e^2+e'^2}=4$

$\Rightarrow r=2$

50 **(c)**



Point (4, 4) lies on the parabola

Let the point of intersection of the line $y=mx$

with the chords be $(\alpha, m\alpha)$, then

$\alpha = \dfrac{4+x_1}{2}$

$\Rightarrow x_1 = 2\alpha - 4$

and $m\alpha = \frac{4+y_1}{2}$

$\Rightarrow\ y_1 = 2m\alpha - 4$

as (x_1, y_1) lies on the curve

$\therefore\ (2\alpha-4)^2 = 4(2m\alpha - 4)$

$\Rightarrow 4\alpha^2 + 16 - 16\alpha = 8(m\alpha - 2)$

$\Rightarrow 4\alpha^2 - 8\alpha(2+m) + 32 = 0$

For two distinct chords

$\therefore\ D > 0$

$\left(8(2+m)\right)^2 - 4(4)(32) > 0$

$\Rightarrow (2+m)^2 - 8 > 0$

$2 + m > 2\sqrt{2}$

Or $2 + m < -2\sqrt{2}$

$\Rightarrow m > 2\sqrt{2} - 2$

Or $\ m < -2\sqrt{2} - 2$

51 **(a)**

Given that

$$\frac{\text{Distance between foci}}{\text{Distance between two directrix}} = \frac{3}{2}$$

$\Rightarrow\ \dfrac{2ae}{2\frac{a}{e}} = \dfrac{3}{2}$

$\Rightarrow e^2 = \dfrac{3}{2}$

$\Rightarrow 1 + \dfrac{b^2}{a^2} = \dfrac{3}{2}$

$\Rightarrow\ \dfrac{b}{a} = \dfrac{1}{\sqrt{2}}$

52 **(c)**

$y^2 = 6\left(x - \dfrac{3}{2}\right)$

Equation of directrix is

$x - \frac{3}{2} = -\frac{3}{2},$ i.e, $x = 0$

Let coordinates of P be $\left(\frac{3}{2} + \frac{3}{2}t^2, 3t\right)$

Therefore, coordinate of M are $(0,3t)$

$\Rightarrow MS = \sqrt{9+9t^2}$

$MP = \dfrac{3}{2} + \dfrac{3}{2}t^2$

$\therefore\ 9 + 9t^2 = \left(\dfrac{3}{2} + \dfrac{3}{2}t^2\right)^2 = \dfrac{9}{4}(1+t^2)^2$

$\therefore\ 4 = 1 + t^2$

$\therefore$ length of side $=6$

53 **(c)**

$\displaystyle\sum_{i=1}^{10}(SP_i)(S'P_i') = 2560$

$\Rightarrow 10b^2 = 2560$

$\Rightarrow b^2 = 256$

$\Rightarrow b = 16$

$\Rightarrow 256 = 400(1-e^2)$

$\Rightarrow 1 - e^2 = \dfrac{16}{25}$

$\Rightarrow e = \dfrac{3}{5}$

54 **(c)**

Equation of the tangent to the ellipse at $P(5\cos\theta, 4\sin\theta)$ is

$\dfrac{x\cos\theta}{5} + \dfrac{y\sin\theta}{4} = 1$

It meets the line $x = 0$ at $Q(0, 4\operatorname{cosec}\theta)$

Image of Q in the line $y = x$ is $R(4\operatorname{cosec}\theta, 0)$

$\therefore$ Equation of the circle is

$x(x - 4\operatorname{cosec}\theta) + y(y - \operatorname{cosec}\theta) = 0$

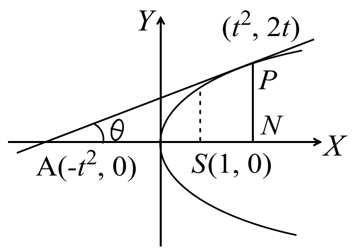
i.e., $x^2 + y^2 - 4(x+y)\operatorname{cosec}\theta = 0$

$\therefore$ Each member of the family passes through the intersection of $x^2 + y^2 = 0$ and $x + y = 0$, i.e., the point $(0, 0)$

55 **(b)**

Tangent at point P is $ty = x + t^2$, when slope of tangent is $\tan\theta = \frac{1}{t}$

Now required area is $A = \frac{1}{2}(AN)(PN) = \frac{1}{2}(2t^2)(2t)$



$A = 2t^3 = 2(t^2)^{3/2}$

Now $t^2 \in [1, 4]$, then $A_{\max}$ occurs when $t^2 = 4$

$\Rightarrow\ A_{\max} = 16$

56 **(b)**

Since tangents are perpendicular, they intersect on the directrix

$\Rightarrow (\lambda, 1)$ lies on the line $x = -4$

$\Rightarrow\ \lambda = -4$

57 **(a)**

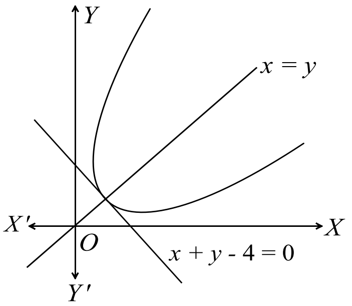
Let the point be $P(at^2, 2at)$

Then according to question, $SP = at^2 + a = k$

Let (α, β) is the moving point, then $\alpha = at^2, \beta = 2at$

$\Rightarrow \dfrac{\alpha}{\beta}=\dfrac{t}{2}$
and $a=\frac{\beta^2}{4\alpha}$
($\because$ point (α,β) lies on $y^2=4ax$)
On substituting these values in Eq. (i),
$\dfrac{\beta^2}{4\alpha}\left(1+\dfrac{4\alpha^2}{\beta^2}\right)=k$
$\Rightarrow \beta^2+4\alpha^2=4k\alpha$
$\Rightarrow 4x^2+y^2-4kx=0$ is the required locus

58 **(a)**
$x^2+y^2-2xy-8x-8y+32=0$
$\Rightarrow (x-y)^2=8(x+y-4)$
Is a parabola whose axis is $x-y=0$ and the tangent at the vertex is $x+y-4=0$



Also, when $y=0$, we have
$x^2-8x+32=0$
Which gives no real values of x
When $x=0$, we have $y^2-8y+32=0$ which gives no real values of y
So, the parabola does not intersect the axes.
Hence, the graph falls in the quadrant

59 **(d)**
We have $16(x^2-2x)-3(y^2-4y)=44$

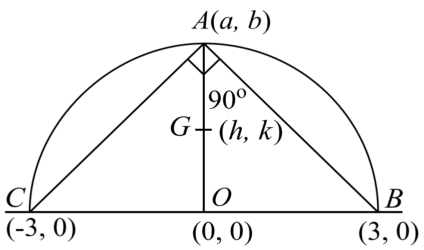
$\Rightarrow 16(x-1)^2-3(y-2)^2=48$

$\Rightarrow \dfrac{(x-1)^2}{3}-\dfrac{(y-2)^2}{16}=1$

This equation represents a hyperbola with eccentricity

$e=\sqrt{1+\frac{16}{3}}=\sqrt{\frac{19}{3}}$

60 **(a)**



Let $A(a,b)$ and $G(h,k)$

Now A,G,O are collinear with $AG:GO\equiv 2:1$
$\Rightarrow h=\dfrac{2.0+a}{3}$
$\Rightarrow a=3h$ and similarly $b=3k$
Now (a,b) lies on the circle $x^2+y^2=9$
Therefore, locus of (h,k) is $x^2+y^2=1$

61 **(d)**
$xy=c^2$

$\Rightarrow x\dfrac{dy}{dx}+y=0$

Replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$, we have

$-x\dfrac{dx}{dy}+y=0$

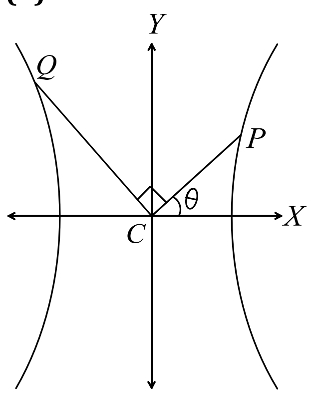
$\Rightarrow ydy-xdx=0$

Integrating, we have

$x^2-y^2=k^2$

Where k is the parameter which represents family of hyperbolas

62 **(d)**



Let $CP=r_1$ be inclined to transverse axis at an angle θ so that P is $(r_1\cos\theta, r_1\sin\theta)$ and P lies on the hyperbola. It gives

$r_1^2\left(\dfrac{\cos^2\theta}{a^2}-\dfrac{\sin^2\theta}{b^2}\right)=1$

Replacing θ by $90°+\theta$, we have

$r_2^2\left(\dfrac{\sin^2\theta}{a^2}-\dfrac{\cos^2\theta}{b^2}\right)=1$

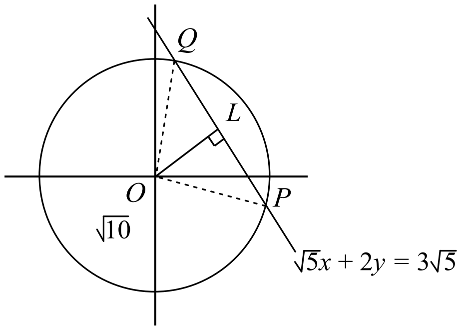
$\Rightarrow \dfrac{1}{r_1^2}+\dfrac{1}{r_2^2}=\dfrac{\cos^2\theta}{a^2}-\dfrac{\sin^2\theta}{b^2}+\dfrac{\sin^2\theta}{a^2}-\dfrac{\cos^2\theta}{b^2}$

$$\Rightarrow \frac{1}{r_1^2}+\frac{1}{r_2^2}=\cos^2\theta\left(\frac{1}{a^2}-\frac{1}{b^2}\right)$$
$$+\sin^2\theta\times\left(\frac{1}{a^2}-\frac{1}{b^2}\right)$$

$$\Rightarrow \frac{1}{r_1^2}+\frac{1}{r_2^2}=\frac{1}{a^2}-\frac{1}{b^2}$$

$$\Rightarrow \frac{1}{CP^2}+\frac{1}{CQ^2}=\frac{1}{a^2}-\frac{1}{b^2}$$

63 **(c)**



Length of perpendicular from origin to the line $x\sqrt{5}+2y=3\sqrt{5}$ is

$$OL=\frac{3\sqrt{5}}{\sqrt{\left(\sqrt{5}\right)^2+2^2}}=\frac{3\sqrt{5}}{\sqrt{9}}=\sqrt{5}$$

Radius of the given circle $=\sqrt{10}=OQ=OP$

$PQ=2QL=2\sqrt{OQ^2-OL^2}$

$=2\sqrt{10-5}=2\sqrt{5}$

Thus, area of $\Delta OPQ=\frac{1}{2}\times PQ\times OL$

$=\frac{1}{2}\times 2\sqrt{5}\times\sqrt{5}=5$

64 **(b)**

If (α,β) is the centre

Then $(\alpha-1)^2+(\beta-3)^2=(\alpha-3)^2+(\beta-1)^2$ ...(i)

and $\frac{\beta-3}{\alpha-1}\cdot\frac{\beta-1}{\alpha-3}=-1$

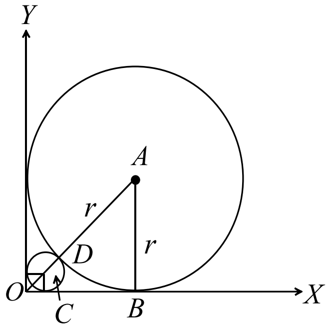
or $(\alpha-1)(\alpha-3)+(\beta-1)(\beta-3)=0$ ...(ii)

(i) $\Rightarrow 4\alpha-4\beta=0 \quad \therefore \alpha=\beta$

(ii) $\Rightarrow 2(\alpha-1)(\alpha-3)=0 \quad \therefore \alpha=1,3$

$\therefore (\alpha,\beta)=(1,1),(3,3)$

65 **(b)**



or $25-n^2>4$

From the diagram $\sin 45°=\frac{AB}{OA}$

$=\frac{r}{OC+CD+DA}$

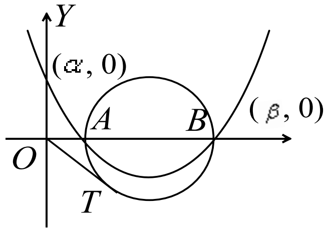
$=\frac{r}{\sqrt{2}+1+r}$

$\Rightarrow \sqrt{2}+1+r=\sqrt{2}\,r$

$\Rightarrow r=\frac{\sqrt{2}+1}{\sqrt{2}-1}=3+2\sqrt{2}$

66 **(d)**

$OT^2=OA\cdot OB=\alpha\beta=\frac{c}{a}\Rightarrow OT=\sqrt{\frac{c}{a}}$



67 **(b)**

$y=ax^2-6x+b$ passes through (0,2)

Here, $2=a(0^2)-6(0)+b$

$\therefore\ b=2$

Also, $\frac{dy}{dx}=2ax-6$

$\therefore \left(\frac{dy}{dx}\right)_{x=\frac{3}{2}}=2a\left(\frac{3}{2}\right)-6$

$=3a-6=0$

$\therefore\ a=2$

68 **(c)**

We have $PQ=BP$

$\Rightarrow 2ae=\sqrt{a^2e^2+b^2}=\sqrt{a^2}=a$

$\Rightarrow e=\frac{1}{2}$

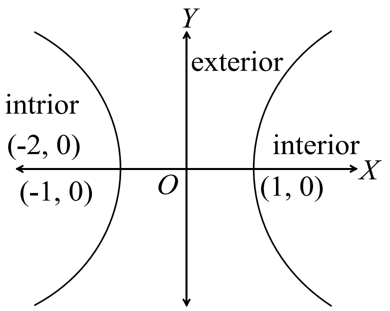
69 **(c)**

$x^2+2y^2\le 1$ represents interior region of circle, where on taking any two points the midpoint of that segment will also lie inside that circle

Max$\{|x|,|y|\}\le 1 \Rightarrow |x|\le 1,|y|\le 1\Rightarrow -1\le x\le 1$ and $-1\le y\le 1$

Which represents the interior region of a square with its sides $x=\pm1$ and $y=\pm1$ in which for any two points, their midpoint also lies inside the region

$x^2-y^2\le 1$ represents the exterior region of hyperbola in which we take two points (4, 3) and (4, −3). Then their midpoint (4, 0) does not lie in the same region (as shown in the figure)



$y^2 \leq x$ represents interior region of parabola in which for any two points, their midpoint also lies inside the region

70 **(b)**

The eccentricity of $\frac{x^2}{16} + \frac{y^2}{25} = 1$ is $e_1 = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

$\therefore\ e_2 = \frac{5}{3}\ (\because e_1 e_2 = 1)$

and foci of given ellipse $(0, \pm 3)$

$\therefore 2b = 3 + 3 = 6\ \Rightarrow\ b = 3\ \Rightarrow\ b^2 = 9$

$\Rightarrow\ a^2 = 16$

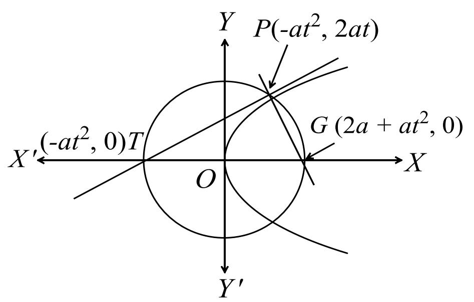
$\Rightarrow$equation of hyperbola is $\frac{x^2}{16} - \frac{y^2}{9} = -1$

Hence, (b) is the correct answer

71 **(a)**

$x = t^2 - t + 1, y = t^2 + t + 1$
$\Rightarrow x + y = 2(t^2 + 1)$ and $y - x = 2t$
$\Rightarrow\ \frac{x+y}{2} = 1 + \left(\frac{y-x}{2}\right)^2$
$\Rightarrow (y - x)^2 = 2(x + y) - 4$
$\Rightarrow (y - x)^2 = 2(x + y - 2)$
Vertex will be the point where lines $y - x = 0$ and $x + y - 2 = 0$ meet, i.e., the point (1,1)

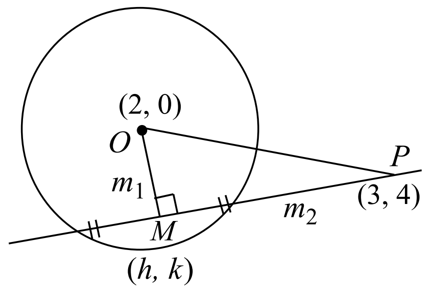
72 **(c)**



Tangent and normal at $P(at^2, 2at)$ to the parabola $y^2 = 4ax$ is
$ty = x + at^2$ (i)
and $y = -tx + 2at + at^3$ (ii)

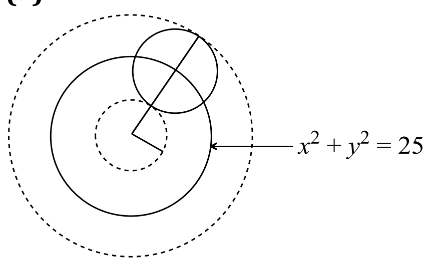
Equations (i) and (ii) meet the x-axis where $y = 0$
From Eq. (i), $x = -at^2$
$\Rightarrow T$ is $(-at^2, 0)$
From Eq. (ii), $tx = 2at + at^3$
$\Rightarrow G$ is $(2a + at^2, 0)$
Midpoint of $TG = \left(\frac{2a+at^2-at^2}{2}, 0\right)$
$= O(a, 0)$
Since $\angle TPG = 90°$, therefore centre of the circle of the circle through PTG is $(a, 0)$
If θ is the angle between tangents at P to the parabola and circle through P, T, G, then $(90° - \theta)$ is the angle between PT and OP
Slope of $PT = \frac{2at}{2at^2} = \frac{1}{t}$
Slope of $OP = \frac{2at}{a(t^2-1)} = \frac{2t}{t^2-1}$
$\therefore\ \tan(90° - \theta) = \left|\frac{\frac{1}{t} - \frac{2t}{t^2-1}}{1 + \frac{1}{t}\left(\frac{2t}{t^2-1}\right)}\right| = \frac{1}{t}$
$\therefore \cot\theta = \frac{1}{t}\ \Rightarrow \tan\theta = t$
$\Rightarrow\ \theta = \tan^{-1}(t)$

73 **(a)**



$\Rightarrow\ m_1 m_2 = -1$
$\Rightarrow\ \left(\frac{4-k}{3-h}\right)\left(\frac{k-0}{h-2}\right) = -1$
Hence, locus is $x^2 + y^2 - 5x - 4y + 6 = 0$

74 **(a)**



Let (h, k) be any point in the set, then equation of circle is
$(x - h)^2 + (y - k)^2 = 9$
But (h, k) lies on $x^2 + y^2 = 25$
Then $h^2 + k^2 = 25$
$\therefore 2 \leq$ Distance between the centres of two circles ≤ 8
$4 \leq h^2 + k^2 \leq 64$
Therefore, locus of (h, k) is $4 \leq x^2 + y^2 \leq 64$

75 **(c)**

Equations of tangent and normal at $P(at^2, 2at)$ are $ty = x + at^2$ and $y = -tx + 2at + at^3$, respectively

Thus, $T \equiv (-at^2, 0)$,

$N \equiv (2a + at^2, 0)$

Also, $S \equiv (a, 0)$

Hence, $SP = a + at^2, ST = a + at^2$

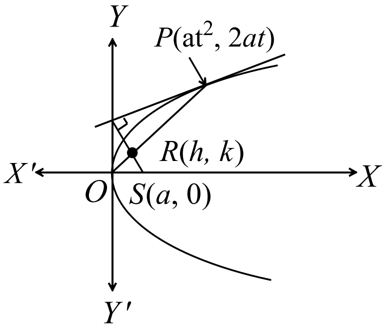
and $SN = a + at^2$

Thus, $SP = ST = SN$

76 **(b)**

Tangent at point P is $ty = x + at^2$

Line perpendicular to Eq. (i) passes through $(a,0)$



$\therefore y - 0 = -t(x - a)$ or $tx + y = ta$ or

$y = t(a - x)$ (ii)

Equation of OP

$y - \frac{2}{t}x = 0$ or $y = \frac{2}{t}x$ (iii)

From Eqs. (ii) and (iii), eliminating t, we get

$y^2 = 2x(a - x)$

Or $2x^2 + y^2 - 2ax = 0$

77 **(d)**

Let the vertex A be $(a\cos\theta, b\sin\theta)$

Since AC and AB touch the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

BC is the chord of contact. Its equation is

$$\frac{x\cos\theta}{a} - \frac{y\sin\theta}{b} = -1$$

or $-\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$

or $\frac{x\cos(\pi-\theta)}{a} + \frac{y\sin(\pi-\theta)}{b} = 1$

which is the equation of the tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

at the point $(a\cos(\pi-\theta), b\sin(\pi-\theta))$

Hence, BC touches the given ellipse

78 **(d)**

Any line passing through focus other than axis always meets parabola in two distinct points

Hence, $m \in R - \{0\}$

79 **(a)**

If d is the distance between the centres of two circles of radii r_1 and r_2, then intersect in two distinct point if $|r_1 - r_2| < d < r_1 + r_2$

Here, radii of two circles are r and 3 and distance between the centres is 5

Thus, $|r - 3| < 5 < r + 3$

$\Rightarrow 2 < r < 8$ and $r < 2$

$\Rightarrow 2 < r < 8$

80 **(a)**

The midpoint of the chord is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

The equation of the chord in terms of its midpoint is

$$\Rightarrow x\left(\frac{y_1+y_2}{2}\right) + y\left(\frac{x_1+x_2}{2}\right) - c^2 = 2\left(\frac{x_1+x_2}{2}\right)\left(\frac{y_1+y_2}{2}\right) - c^2$$

$$\Rightarrow x(y_1+y_2) + y(x_1+x_2) = (x_1+x_2)(y_1+y_2)$$

$$\Rightarrow \frac{x}{x_1+x_2} + \frac{y}{y_1+y_2} = 1$$

81 **(b)**

Equation of tangent at (x_1, y_1) is

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

It passing through $(0, -b)$. So,

$$0 + \frac{y_1}{b} = 1 \Rightarrow y_1 = b$$

Equation of normal is

$$\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2e^2$$

Which passes through $(2a\sqrt{2}, 0)$. Hence,

$$\frac{a^2 2a\sqrt{2}}{x_1} = a^2e^2$$

$$\Rightarrow x_1 = \frac{2a\sqrt{2}}{e^2}$$

Now (x_1, y_1) lies on the hyperbola

$$\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} = 1$$

$$\Rightarrow \frac{8}{e^4} - 1 = 1$$

$$\Rightarrow e^2 = 2$$

82 **(a)**
The asymptotes of a rectangular hyperbola are perpendicular to each other

Given one asymptote,

$x + y + c = 0$

Let the other asymptote be

$x - y + \lambda = 0$

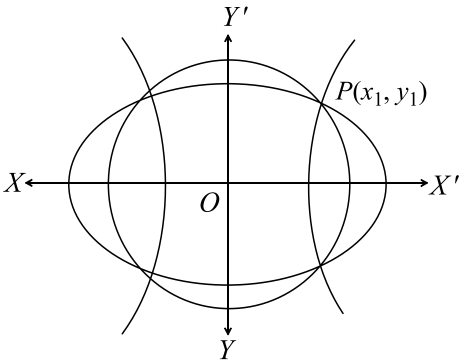
We also know that the asymptotes pass through centre of the hyperbola. Therefore, the line $2x - y = 0$ and the asymptotes must be concurrent

Thus, we have

$$\begin{vmatrix} 2 & -1 & 0 \\ 1 & 1 & c \\ 1 & -1 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda = -3c$$

83 **(a)**



Since ellipse and hyperbola intersect orthogonally, they are confocal

Hence, $a = 2$ (equating foci)

Let point of intersection in the first quadrant be $P(x_1, y_1)$. P lies on both the curves. Therefore,

$4x_1^2 + 9y_1^2 = 36$ and $4x_1^2 - y_1^2 = 4$

Adding these two results, we get

$$8(x_1^2 + y_1^2) = 40$$

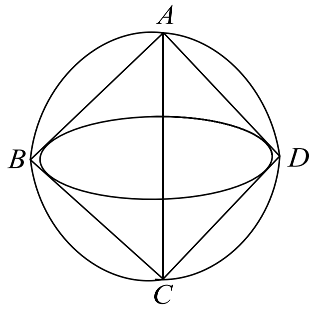
$$\Rightarrow x_1^2 + y_1^2 = 5 \ \Rightarrow r = \sqrt{5}$$

Hence, equation of the circle is

$$x^2 + y^2 = 5$$

84 **(b)**
Let m be the slope of the common tangent, then
$\pm\sqrt{3}\sqrt{1 + m^2} = \pm\sqrt{4m^2 + 1}$
$\Rightarrow 3 + 3m^2 = 4m^2 + 1$
$\Rightarrow m^2 = 2$
$\Rightarrow m = \pm\sqrt{2}$

85 **(d)**

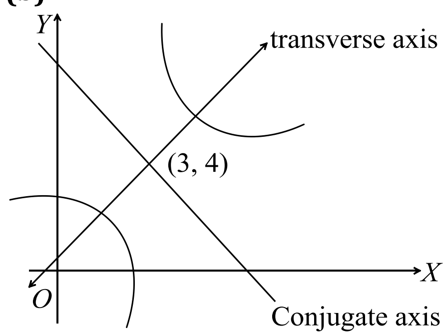


Clearly, vertices of the square lie on the director circle of the ellipse $\frac{x^2}{7} + \frac{2y^2}{11} = 1$
Which is $x^2 + y^2 = 7 + \frac{11}{2}$ or $x^2 + y^2 = \frac{25}{2}$
Clearly, $AC = 2\sqrt{\frac{25}{2}}$
Now $AB = BC = CD = AD$
and in ΔACD, $AC^2 = CD^2 + AD^2$

$$\Rightarrow 2AD^2 = \left(2\sqrt{\frac{25}{2}}\right)^2$$

$\Rightarrow AD^2 = 25$
$\Rightarrow AD = 5$ units

86 **(b)**



$(x - 3)(y - 4) = 5$

The axes of the hyperbola are $x = 3$ and $y = 4$

Since the hyperbola is rectangular, axes are bisectors of these axes

Hence, their slope is ± 1, out of which conjugate axis has slope $m = -1$ and passes through $(3, 4)$

Hence, its equation is

$y - 4 = -1(x - 3)$

87 **(a)**
The two normals are $x = 1$ and $y = 2$
Their point of intersection $(1, 2)$ is the centre of the required circle
Radius $\frac{|3+8-6|}{5} = 1$
$\therefore$ Required circle is
$(x-1)^2 + (y-2)^2 = 1$
i.e. $x^2 + y^2 - 2x - 4y + 4 = 0$

88 **(a)**
For $\lambda = -3$, the equation becomes

$x^2 + y^2 - 3xy = 0$

Which represents a pair of lines through origin

89 **(a)**
a, b, c are in A.P., so $ax + by + c = 0$ represents a family of lines passing through the point $(1, -2)$. So, the family of circles (concentric) will be given by $x^2 + y^2 - 2x + 4y + c = 0$. It intersects given circle orthogonally
$\Rightarrow 2(-1 \times -2) + (2 \times -2) = -1 + c \Rightarrow c = -3$

90 **(a)**
Equation of normal at any point $\left(ct, \frac{c}{t}\right)$ is

$ct^4 - xt^3 + ty - c = 0$

$\Rightarrow$ Slope of normal $= t^2$

Let P be (h, k)

$\Rightarrow ct^4 - ht^3 + tk - c = 0$

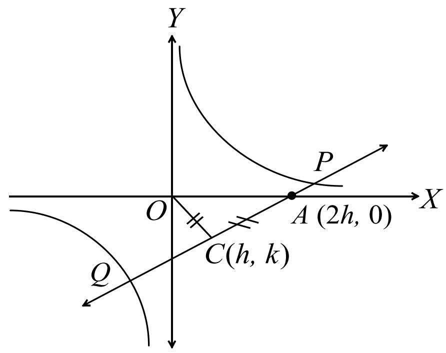
$\Rightarrow \Sigma\, t_i = \frac{h}{c}$ and $\Sigma t_i t_j = 0$

$\Rightarrow \Sigma\, t_i^2 = (\Sigma t_i)^2$

$\Rightarrow h^2 = c^2 \lambda$

Therefore, the required locus is $x^2 = \lambda c^2$

91 **(b)**



Hyperbola is $xy = a^2$

or $2xy - 2a^2 = 0$

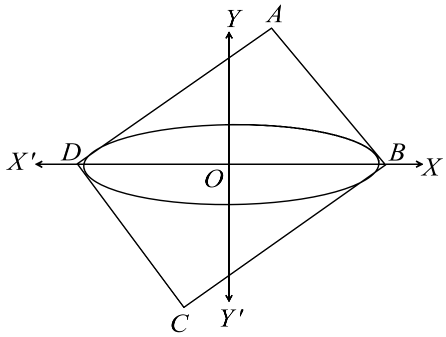
Chord with a given middle point is given by

$hy + kx - a^2 - 2hk - a^2$

$\Rightarrow \frac{x}{h} + \frac{y}{k} = 2$

From the diagram ΔOCA is isosceles with $OC = CA$

92 **(d)**



Since sides of the square are tangent and perpendicular to each other, so the vertices lie on director circle
$\Rightarrow x^2 + y^2(a^2 - 7) + (13 - 5a)$
$= a^2\left(\sqrt{2}\, a \text{ is side of the square}\right)$
$\Rightarrow (a^2 - 7)(13 - 5a) = a^2$
$\Rightarrow a = \frac{6}{5}$
But for an ellipse to exist $a^2 - 7 > 0$ and $13 - 5a > 0$
$\Rightarrow a \in \left(-\infty, -\sqrt{7}\right)$
Hence, $a \neq \frac{6}{5}$
Hence, no such a exists

93 **(a)**
Any point on link $x + y = 25$ is $P \equiv (a, 25 - a, a \in R$
Equation of chord AB is $T = 0$
i.e., $xa + y(25 - a) = 9$ ...(i)
If midpoint of chord AB is $C(h, k)$, then equation

of chord AB is

$T = S_1$, i.e. $xh + yk = h^2 + k^2$ ...(ii)

Comparing the ratio of coefficients of Eqs. (i) and (ii), we get

$$\frac{a}{h} = \frac{25-a}{k} = \frac{9}{h^2+k^2}$$

$$\Rightarrow \frac{a+25-a}{h+k} = \frac{9}{h^2+k^2}$$

Thus, locus of 'C' is $25(x^2+y^2) = 9(x+y)$

94 **(c)**

Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Any point on the directrix is $P\left(\frac{a}{e}, k\right)$

Chord of contact of P with respect to the ellipse is

$\frac{a}{e}\frac{x}{a^2} + \frac{ky}{b^2} = 1$ (i)

Chord of contact of P with respect to the auxiliary circle is

$\frac{a}{e}x + ky = a^2$ (ii)

Equation (i) and (ii) intersect at $(ae, 0)$

95 **(a)**

Let AB be a normal chord where $A \equiv (at^2, 2at)$, $B \equiv (at_1^2, 2at_1)$

We have $t_1 = -t - \frac{2}{t}$

Now, $AB^2 = [a^2(t^2 - t_1^2)]^2 + 4a^2(t-t_1)^2$

$= a^2(t-t_1)^2[(t+t_1)^2 + 4]$

$$= a^2\left(t + t + \frac{2}{t}\right)^2\left(\frac{4}{t^2} + 4\right)$$

$$= \frac{16a^2(1+t^2)^3}{t^4}$$

$$\Rightarrow \frac{d(AB)^2}{dt}$$

$$= 16a^2\left(\frac{t^4[3(1+t^2)^2.2t] + (1+t^2)^3.4t^3}{t^8}\right)$$

$$= 32a^2(1+t^2)^2\left(\frac{3t^2-2-2t^2}{t^5}\right)$$

$$= \frac{a^2 \times 32(1+t^2)^2}{t^5}(t^2-2)$$

For $\frac{d(AB^2)}{dt}$ =0 $\Rightarrow t - \sqrt{2}$ for which AB^2 is minimum

Thus, $AB_{\min} = \frac{4a}{2}(1+2)^{3/2} = 2a\sqrt{27}$ units

96 **(d)**

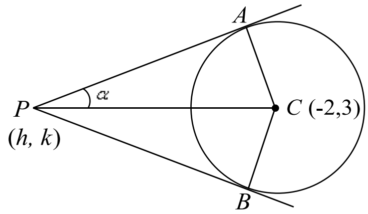
Centre of the circle

$x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$

is $C(-2, 3)$ and its radius is

$\sqrt{2^2 + (-3)^2 - 9\sin^2\alpha - 13\cos^2\alpha}$

$\sqrt{4 + 9 - 9\sin^2\alpha - 13\cos^2\alpha} = |2\sin\alpha|$



Let $P(h, k)$ be any point on the locus. Then $\angle APC = \alpha$

From the diagram $\sin\alpha = \frac{AC}{PC} = \frac{2\sin\alpha}{\sqrt{(h+2)^2+(k-3)^2}}$

$\Rightarrow (h+2)^2 + (k-3)^2 = 4$

or $h^2 + k^2 + 4h - 6k + 9 = 0$

Thus, required equation of the locus is

$x^2 + y^2 + 4x - 6y + 9 = 0$

97 **(a)**

A tangent of slope 2 to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

$y = 2x \pm \sqrt{4a^2 + b^2}$ (i)

This is normal to the circle $x^2 + y^2 + 4x + 1 = 0$

$\Rightarrow$ Eq. (i), passes through $(-2,0)$

$\Rightarrow 0 = -4 \pm \sqrt{4a^2 + b^2}$

$\Rightarrow 4a^2 + b^2 = 16$

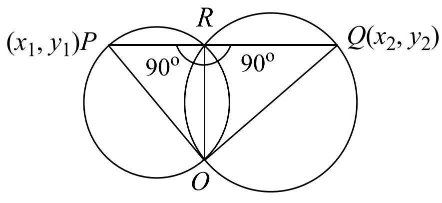
Using A. M. $\geq$ G. M., we get

$$\frac{4a^2+b^2}{2} \geq \sqrt{4a^2b^2}$$

$\Rightarrow ab \leq 4$

98 **(b)**

$$OR = \frac{2 \text{ area of } \Delta OPQ}{PQ}$$



$$= \frac{2 \cdot \left|\frac{1}{2}(x_1y_2 - x_2y_1)\right|}{PQ}$$

$$= \frac{|x_1y_2 - x_2y_1|}{PQ}$$

99 **(c)**

The ellipse can be written as

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

Here $a^2 + 25, b^2 = 16$

Now, $b^2 = a^2(1-e^2)$

$$\Rightarrow \frac{16}{25} = 1 - e^2$$

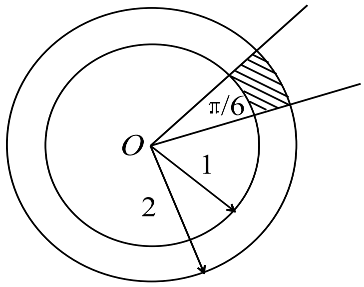
$$\Rightarrow e^2 = 1 - \frac{16}{25} = \frac{9}{25}$$

$$\Rightarrow e = \frac{3}{5}$$

Foci of the ellipse are $(\pm ae, 0) \equiv (\pm 3, 0)$, i.e. F_1 and F_2 are foci of the ellipse.

Therefore, we have $PF_1 + PF_2 = 2a = 10$ for every point P on the ellipse

100 **(d)**



The angle θ between the lines represented by $\sqrt{3}\,x^2 - 4xy + \sqrt{3}y^2 = 0$ is given by

$\theta = \tan^{-1}\dfrac{\sqrt{h^2 - ab}}{|a+b|}$

$= \tan^{-1}\dfrac{2\sqrt{2^2 - 3}}{\sqrt{3}+\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

Gives $\theta = \frac{\pi}{6}$

Hence, the shaded area $= \frac{\pi/6}{2\pi} \times \pi(2^2 - 1^2) = \frac{\pi}{4}$

101 **(c)**

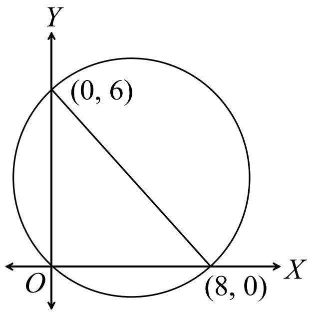
Ellipse passing through $O(0,0)$ and having foci $P(3,3)$ and $Q(-4,4)$,

Then $e = \frac{PQ}{OP+OQ}$

$= \dfrac{\sqrt{50}}{3\sqrt{2} + 4\sqrt{2}}$

$= \dfrac{5}{7}$

102 **(b)**

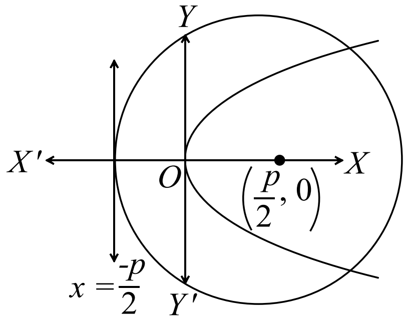


Centre of family of ellipse is $(4, 3)$ and distance of focus from centre $= a = \frac{5}{2}$

Hence, locus $(x-4)^2 + (y-3)^2 = \frac{25}{4}$

103 **(a)**

The focus of parabola $y^2 = 2px$ is $\left(\frac{p}{2}, 0\right)$ and directrix $x = -\frac{p}{2}$



$\therefore$ Centre of circle is $\left(\frac{p}{2}, 0\right)$ and radius $= \frac{p}{2} + \frac{p}{2} = p$

$\therefore$ Equation of circle is $\left(x - \frac{p}{2}\right)^2 + y^2 = p^2$

Or $4x^2 + 4y^2 - 4px - 3p^2 = 0$

Solving this circle with the given parabola, we have (eliminating y)

$4x^2 + 8px - 4px - 3p^2 = 0$

$\Rightarrow 4x^2 + 4px - 3p^2 = 0$

$\Rightarrow (2x + 3p)(2x - p) = 0$

$\Rightarrow x = \dfrac{-3p}{2}, \dfrac{p}{2}$

$\Rightarrow y^2 = -3p^2$(not possible),

$\Rightarrow y^2 = 2p.\dfrac{p}{2} \Rightarrow = \pm p$

Therefore, required points are $\left(\frac{p}{2}, p\right), \left(\frac{p}{2}, -p\right)$

104 **(d)**

Tangent to the ellipse having slope m is

$y = mx + \sqrt{a^2m^2 + b^2}$

If it passes through the point $P(h,k)$, then

$k = mh + \sqrt{a^2m^2 + b^2}$

or $(a^2 - h^2)m^2 + 2hkm + b^2 - k^2 = 0$

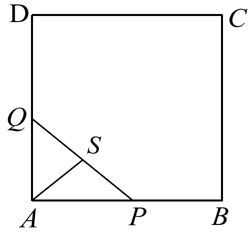
Now given $\tan\alpha + \tan\beta = \lambda$

$\Rightarrow\ m_1 + m_2 = \lambda$

$\Rightarrow\ \dfrac{-2hk}{a^2 - h^2} = \lambda$

$\Rightarrow$ locus is $\lambda(x^2 - a^2) = 2xy$

105 **(b)**



Let S be the midpoint of PQ

Since $\angle PAQ = \frac{\pi}{2}$, we get $AS = SP = SQ = \frac{1}{2}$

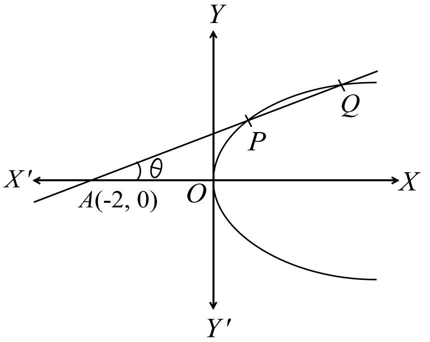
$\Rightarrow S$ lies on the quarter circle of radius $\frac{1}{2}$ with centre at A

Similarly S can also lie on quarter circle of radius $\frac{1}{2}$ with centre at B, C or D

$\Rightarrow$ area $A = 1 - \frac{\pi}{4}$

106 **(a)**

Let $P(-2 + r\cos\theta, r\sin\theta)$ and Q lies on parabola



$\Rightarrow r^2\sin^2\theta - 4(-2 + r\cos\theta) = 0$

$\Rightarrow r_1 + r_2 = \dfrac{4\cos\theta}{\sin^2\theta}, r_1r_2 = \dfrac{8}{\sin^2\theta}$

Now $\frac{1}{AP} + \frac{1}{AQ} = \frac{r_1+r_2}{r_1r_2} = \frac{\cos\theta}{2}$

Given that $\frac{1}{AP} + \frac{1}{AQ} < \frac{1}{4}$

$\Rightarrow \cos\theta < \dfrac{1}{2}$

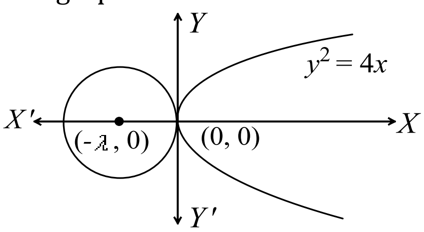
$\Rightarrow \tan\theta > \sqrt{3}$

($\because \cos\theta$ is decreasing and $\tan\theta$ is increasing in $(0, \pi/2)$)

$\Rightarrow m > \sqrt{3}$

107 **(a)**

The graph shows $\lambda > 0$



108 **(c)**

Normal at point $P(a\cos\theta, b\sin\theta)$ is

$\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2$ (i)

It meets axes at $Q\left(\frac{(a^2-b^2)\cos\theta}{a}, 0\right)$

and $R\left(0, -\frac{(a^2-b^2)\sin\theta}{b}\right)$

Let $T(h, k)$ is a midpoint of QR

Then $2h = \frac{(a^2-b^2)\cos\theta}{a}$

and $2k = -\frac{(a^2-b^2)\sin\theta}{b}$

$\Rightarrow \cos^2\theta + \sin^2\theta = \dfrac{4h^2a^2}{(a^2-b^2)^2} + \dfrac{4k^2b^2}{(a^2-b^2)^2} = 1$

$\Rightarrow$Locus is $\frac{x^2}{\frac{(a^2-b^2)^2}{4a^2}} + \frac{y^2}{\frac{(a^2-b^2)^2}{4b^2}} = 1$ (ii)

Which is an ellipse, having eccentricity e', given by

$e'^2 = 1 - \dfrac{\frac{(a^2-b^2)^2}{4a^2}}{\frac{(a^2-b^2)^2}{4b^2}} = 1 - \dfrac{b^2}{a^2} = e^2$

$\Rightarrow e' = e$

Note:

***In Eq.* (ii),** $\dfrac{(a^2-b^2)}{4a^2} < \dfrac{(a^2-b^2)}{4b^2}$. ***Hence, x* − *axis is minor axis*.**

109 **(b)**

The chord of contact of tangents from (x_1, y_1) is

$\dfrac{xx_1}{a^2} + \dfrac{yy_1}{b^2} = 1$

It meets the axes at the points $\left(\frac{a^2}{x_1}, 0\right)$ and $\left(0, \frac{b^2}{y_1}\right)$

Area of the triangle is $\frac{1}{2}\frac{a^2}{x_1}\frac{b^2}{y_1} = k$ (constant)

$\Rightarrow x_1y_1 = \frac{a^2b^2}{2k} = c^2$ (c is constant)

$\Rightarrow xy = c^2$ is the required locus

110 **(b)**

Solving the given line and the ellipse, we get

$t^2 + \dfrac{y^2}{9} = 1$

$\Rightarrow y^2 = 9(1 - t^2)$,

Which gives real and distinct values of y, if

$1 - t^2 > 0$

$\Rightarrow t \in (-1, 1)$

112 **(c)**

Solving circle $x^2 + y^2 = 5$ and parabola $y^2 = 4x$, we have

$x^2 + 4x - 5 = 0$

$\Rightarrow x = 1$

Or $x = -5$(not possible)

$\Rightarrow$ Point of intersection are $P(1,2)$; $Q(1, -2)$

Hence, $PQ = 4$

113 **(d)**

Given that $\frac{x^2}{1-r} - \frac{y^2}{1+r} = 1$

As $r > 1$, so $1 - r < 0$ and $1 + r > 0$

Let $1 - r = -a^2, 1 + r = b^2$

Then we get

$\dfrac{x^2}{-a^2} - \dfrac{y^2}{b^2} = 1 \Rightarrow \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = -1$

114 **(c)**
Let the point be (h, k)

Then equation of the chord of contact is
$hx + ky = 4$

Since $hx + ky = 4$ is tangent to $xy = 1$

$\therefore x\left(\frac{4-hx}{k}\right) = 1$ has two equal roots

$\Rightarrow hx^2 - 4x + k = 0$

$\Rightarrow hk = 4$

$\Rightarrow$ locus of (h, k) is $xy = 4$

$\Rightarrow c^2 = 4$

115 **(a)**
$2a(1 + e) = 15$
$1 + e = \frac{3}{2}$
$e = 0.5$

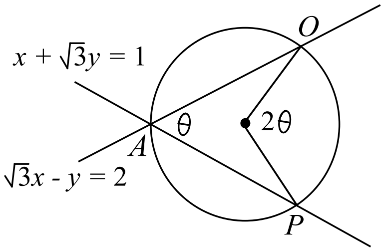
116 **(d)**
Parabola having axis parallel to y-axis is
$(x - a)^2 = 4A(y - b)$
According to question, length of latus rectum
$4A = l$
Hence, equation of parabola is
$(x - a)^2 = \frac{l}{4}(y - b)$
or $(x - a)^2 = \frac{l}{8}(2y - 2b)$

117 **(d)**
Solving the equations,
$x^2 + 4(x + 4) = a^2$
If circle and parabola touch each other, then
$D = 0$
$\Rightarrow 16 - 4(16 - a^2) = 0$
$\Rightarrow a = 2\sqrt{3}$

118 **(c)**
Equation of any circles through (0, 1) and (0, 6) is
$x^2 + (y - 1)(y - 6) + \lambda x = 0$
$\Rightarrow x^2 + y^2 + \lambda x - 7y + 6 = 0$
If it touches x-axis, then $x^2 + \lambda x + 6 = 0$ should have equal roots
$\Rightarrow \lambda^2 = 24 \Rightarrow \lambda = \pm\sqrt{24}$
Radius of these circles $= \sqrt{6 + \frac{49}{4} - 6} = \frac{7}{2}$ units
That means we can draw two circles but radius of both circles is $\frac{7}{2}$

119 **(c)**
Let the midpoint of the chord be $P(h, k)$
Then $CP = \sqrt{h^2 + k^2}$, where C is centre of the circle
Since chord subtends right angle at the centre
Radius $= \sqrt{2}\sqrt{h^2 + k^2}$
$\Rightarrow 2 = \sqrt{2}\sqrt{h^2 + k^2}$
$\Rightarrow$ locus of P is $x^2 + y^2 = 2$

120 **(a)**

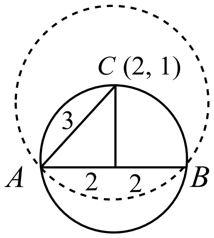


Let the point of intersection of two lines is A
$\therefore$ The angle subtended by PQ on centre C
$= 2 \times$ the angle subtended by PQ on point A
For $x + \sqrt{3}y = 1, m_1 = \frac{-1}{\sqrt{3}}$ and for $\sqrt{3}x - y = 2, m_2 = \sqrt{3}$
$\because\ m_1 \times m_2 = \frac{-1}{\sqrt{3}} \times \sqrt{3} = -1,$
$\therefore\ \angle A = 90°$
$\therefore$ The angle subtended by are PQ at its centre
$= 2 \times 90° = 180°$

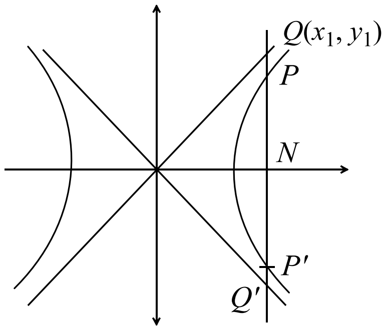
121 **(d)**
$x^2 + 2ax + c = (x - 2)^2$
$\Rightarrow\ -2a = 4, c = 4$
$\Rightarrow a = -2, c = 4$
$y^2 + 2by + c = (y - 2)(y - 3)$
$\Rightarrow -2b = 5, c = 6$
$\Rightarrow b = -\frac{5}{2}, c = 6$ clearly the data are not consistent

122 **(c)**
$x^2 + y^2 - 12x + 35 = 0$ ...(i)
$x^2 + y^2 + 4x + 3 = 0$ ...(ii)
Equation of radical axis of circles (i) and (ii) is
$-16x + 32 = 0 \Rightarrow x = 2$
It intersects the line joining the centres, i.e. $y = 0$ at the point (2, 0)
$\therefore$ Required radius $= \sqrt{4 - 24 + 35} = \sqrt{15}$ (length of tangent from (2, 0)

123 **(c)**
The centre of given circle is (1, 3) and radius is 2.
So, AB is a diameter of the given circle has its mid point as (1, 3). The radius of the required circle is 3



124 **(b)**



$$NP = \frac{4}{5}\sqrt{x_1^2 - 25}$$

Q is on $y = \frac{4}{5}x$

$$NQ = \frac{4}{5}x_1$$

$$PQ = NQ - NP = \frac{4}{5}(x_1 - \sqrt{x_1^2 - 25})$$

$$P'Q = \frac{4}{5}(x_1 + \sqrt{x_1^2 - 25})$$

$$PQ \cdot P'Q = 16$$

125 **(a)**

Since, the focus and vertex of the parabola are on y-axis, therefore its axis of the parabola is $y -$ axis

Let the equation of the directrix be $y = k$ the directrix meets the axis of the parabola at $(0, k)$. But vertex is the mid point of the line segment joining the focus to the point where directrix meets axis of the parabola

$$k + \frac{3}{2} = 6 \Rightarrow k = 9$$

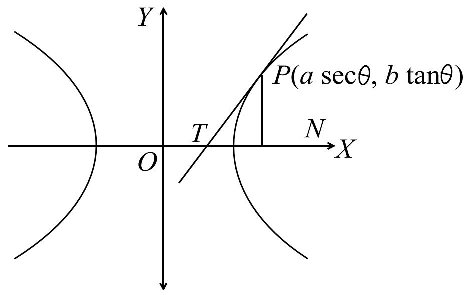
Thus, the equation of directrix is $y = 9$

Equation of parabola is

$$(x - 0)^2 + (y - 3)^2 = (y - 9)^2$$

$$\Rightarrow x^2 + 12y - 72 = 0$$

126 **(b)**



Tangent at point P is

$$\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$$

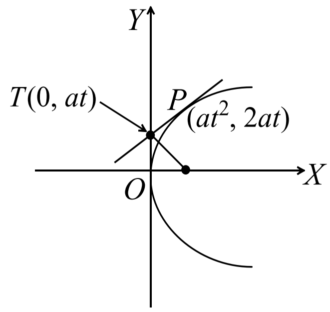
It meets the x-axis at point $T(a\cos\theta, 0)$ and foot of perpendicular from P to x-axis is $N(a\sec\theta, 0)$

From the diagram,

$OT = a\cos\theta$ and $ON = a\sec\theta$

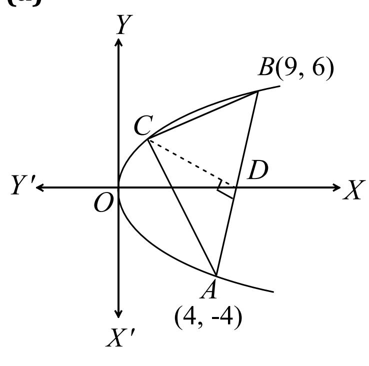
$\Rightarrow OT \cdot ON = a^2$

127 **(b)**



Let middle point of P and T be (h, k)
$\therefore\ 2h = at^2$
and $2k = 3at$
$\therefore 2h = a \cdot \frac{4k^2}{9a^2}$
Locus of (h, k) is $2y^2 = 9ax$
As $a = 2\ \therefore y^2 = 9x$

128 **(a)**



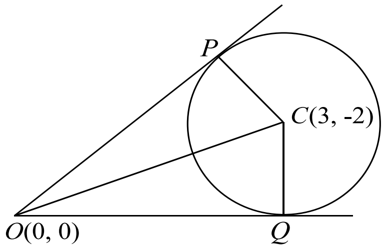
Area of triangle ABC is maximum if CD is maximum, because AB is fixed
That means tangent drawn to parabola at 'C' should be parallel to AB
Slope of $AB = \frac{6+4}{9-4} = 2$

For $y^2 = 4x, \frac{dy}{dx} = \frac{2}{y} = 2$

$\Rightarrow y = 1$

$\Rightarrow x = \frac{1}{4}$

129 **(b)**



Clearly, $OPCQ$ is cyclic quadrilateral, then circumcircle of ΔOPQ passes through the point C

For this circle, OC is diameter, then centre is midpoint of OC which is $\left(\frac{3}{2}, -1\right)$

130 **(c)**

Let the coordinates $A(a, 0)$ and $B(-a, 0)$ and let the straight line be $y = mx + c$. Then,

$$\frac{mx + c}{\sqrt{1+m^2}} + \frac{-mx + c}{\sqrt{1+m^2}} = 2k$$

$\Rightarrow c = k\sqrt{1+m^2}$

So, the straight line is $y = mx + k\sqrt{1+m^2}$

Clearly, it touches the circle $x^2 + y^2 = k^2$ of radius k

131 **(a)**

Equation of tangent at point P $(\alpha \cos\theta, \sin\theta)$ is

$\frac{x}{\alpha}\cos\theta + \frac{y}{1}\sin\theta = 1$ (i)

Let it cut the hyperbola at points P and Q

Homogenizing the hyperbola $\alpha^2 x^2 - y^2 = 1$ with the help of the above the equation, we get

$$\alpha^2 x^2 - y^2 = \left(\frac{x}{\alpha}\cos\theta + y\sin\theta\right)^2$$

This is a pair of straight lines OP and OQ

Given $\angle POQ = \frac{\pi}{2}$

$\Rightarrow$ Coefficient of x^2 + coefficient of $y^2 = 0$

$$\Rightarrow \alpha^2 - \frac{\cos^2\theta}{\alpha^2} - 1 - \sin^2\theta = 0$$

$$\Rightarrow \alpha^2 - \frac{\cos^2\theta}{\alpha^2} - 1 - 1 + \cos^2\theta = 0$$

$$\Rightarrow \cos^2\theta = \frac{\alpha^2(2-\alpha^2)}{\alpha^2 - 1}$$

Now, $0 \le \cos^2\theta \le 1$

$$\Rightarrow 0 \le \frac{\alpha^2(2-\alpha^2)}{\alpha^2 - 1} \le 1$$

Solving, we get

$$\alpha^2 \in \left[\frac{\sqrt{5}+1}{2}, 2\right]$$

132 **(c)**

A rectangular hyperbola circumscribing a triangle also passes through its orthocentre.

If $\left(ct_i, \frac{c}{t_i}\right)$, where $i = 1, 2, 3$, are the vertices of the triangle then the orthocentre is $\left(\frac{-c}{t_1 t_2 t_3}, -ct_1 t_2 t_3\right)$, where $t_1\, t_2\, t_3\, t_4 = 1$

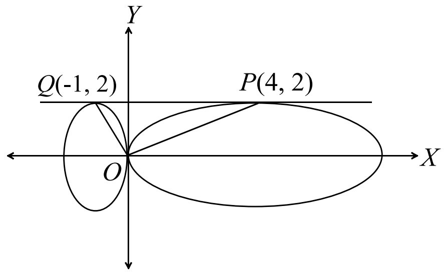
Hence, orthocentre is $\left(-ct_4, \frac{-c}{t_4}\right) = (-x_4, -y_4)$

133 **(d)**

$\frac{(x-4)^2}{25} + \frac{y^2}{4} = 1$ and $(x-1)^2 + \frac{y^2}{4} = 1$

Clearly $m_{OP}.m_{OQ} = -1$

$\Rightarrow OP$and OQ are perpendicular to each other



134 **(d)**

Point $(\sin\theta, cos\theta)$ lies on the circle $x^2 + y^2 = 1$ for $\forall\ \theta \in R$

Now three normals can be drawn to the parabola $y^2 = 4ax$ if $x = |2a|$ meets this circle

Hence, we must have $\cos\theta > |2a|$

$\Rightarrow 0 < |2a| < 1$

$\Rightarrow 0 < |a| < \frac{1}{2}$

$\Rightarrow a \in \left(-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right)$

135 **(b)**

Let directrix be $x = \frac{a}{e}$ and focus be $S(ae, 0)$. Let $P(a\sec\theta, b\tan\theta)$ be any point on the curve

Equation of tangent at P is

$$\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$$

Let F be the intersection point of tangent of directrix

Then $F=\left(\frac{a}{e},\frac{b(\sec\theta-e)}{e\tan\theta}\right)$

$$\Rightarrow m_{SF}=\frac{b(\sec\theta-e)}{-e\tan\theta(a^2-1)}$$

$$m_{PS}=\frac{b\tan\theta}{a(\sec\theta-e)}$$

$$\Rightarrow m_{SF}\cdot m_{PS}=-1$$

136 **(a)**

The general equation of a parabola having its axis parallel to y-axis is

$y=ax^2+bx+c$ (i)

This is touched by the line $y=x$ at $x=1$

Therefore, slope of the tangent at (1, 1) is 1 and,

$x=ax^2+bx+c$ must have equal roots

$\Rightarrow\left(\frac{dy}{dx}\right)_{(1,1)}=1$ and $(b-1)^2=4ac$

$\Rightarrow 2a+b=1$ and $(b-1)^2=4ac$

Also, (1,1) lies on Eq.(i)

$\Rightarrow a+b+c=1$

From $2a+b=1$ and $a+b+c$

$=1, a-c=0$

$\Rightarrow a=c$

Then from $a+b+c=1, 2c+b=1$

$\Rightarrow 2f(0)+f'(0)=1$ [$\because f(0)=c$ and $f'(0)=b$]

137 **(c)**

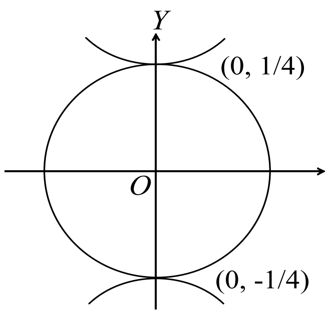
$\frac{x+y}{2}=t^2+1, \frac{x-y}{2}=t$

Eliminating t, $2(x+y)=(x-y)^2+4$

Since 2nd degree terms form a perfect square, it represents a parabola (also$\Delta\neq 0$)

138 **(d)**

$$\frac{y^2}{\frac{1}{16}}-\frac{x^2}{\frac{1}{9}}=1$$



Locus will be the auxiliary circle

$$x^2+y^2=\frac{1}{16}$$

139 **(a)**

Let AB be the line

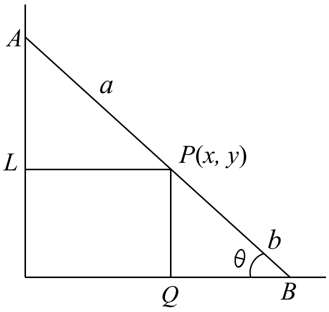
Let $AP=a, PB=b$,

So that $AB=a+b$

If AB makes an angle θ with x-axis and coordinates of P are (x,y),

Then in $\Delta APL, x=a\cos\theta$

In $\Delta PBQ, y=b\sin\theta$



$\therefore$ Locus of $P(x,y)$ is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Which is an ellipse

140 **(a)**

Let the variable chord be

$$x\cos\alpha+y\sin\alpha=p$$

Let this chord intersect the hyperbola at A and B. Then the combined equation of OA and OB is given by

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=\left(\frac{x\cos\alpha+y\sin\alpha}{p}\right)^2$$

$$x^2\left(\frac{1}{a^2}-\frac{\cos^2\alpha}{p^2}\right)-y^2\left(\frac{1}{b^2}+\frac{\sin^2\alpha}{p^2}\right)-\frac{2\sin\alpha\cos\alpha}{p}xy=0$$

This chord subtends a right angle at centre.

Therefore, Coefficient of x^2 + coefficient of $y^2=0$

$$\Rightarrow\frac{1}{a^2}-\frac{\cos^2\alpha}{p^2}-\frac{1}{b^2}-\frac{\sin^2\alpha}{p^2}=0$$

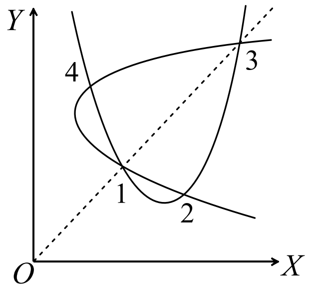
$$\Rightarrow\frac{1}{a^2}-\frac{1}{b^2}=\frac{1}{p^2}$$

$$\Rightarrow p^2=\frac{a^2b^2}{b^2-a^2}$$

Hence, p is constant, i.e., it touches the fixed circle

141 **(d)**

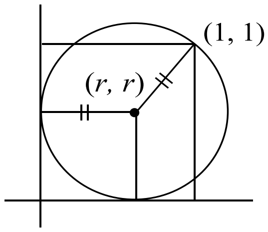
The given parabolas are symmetrical about the line $y=x$ as shown in the figure



They intersect to each other at four distinct points
Hence, the number of common chords
$= 4C_2 = \frac{4\times3}{2} = 6$

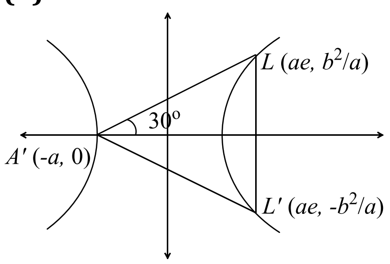
142 **(c)**
Given $m(n-1) = n$
n is divisible by $n-1$
$\Rightarrow n = 2 \Rightarrow m = 2$
Hence, chord of contact of tangents drawn from $(2, 2)$ to
$\frac{x^2}{9} + \frac{y^2}{4} = 1$ is $\frac{2x}{9} + \frac{2y}{4} = 1$
$\Rightarrow 4x + 9y = 18$

143 **(a)**



From Fig
$2(1-r)^2 = r^2$
$\Rightarrow \sqrt{2}(1-r) = r$
$\Rightarrow r(\sqrt{2}+1) = \sqrt{2}$
$$\Rightarrow r = \frac{\sqrt{2}}{\sqrt{2}+1} = \sqrt{2}(\sqrt{2}-1) = 2-\sqrt{2}$$

144 **(d)**



$$\tan 30° = \frac{\frac{b^2}{a}}{a+ae}$$

$$\Rightarrow \frac{1+e}{\sqrt{3}} = e^2 - 1$$

$$\Rightarrow e - 1 = \frac{1}{\sqrt{3}}$$

$$\Rightarrow e = \frac{\sqrt{3}+1}{\sqrt{3}}$$

145 **(b)**
Eliminating t from the given two equation, we have

$$\frac{x^2}{16} - \frac{y^2}{48} = 1$$

whose eccentricity is

$$e = \sqrt{1 + \frac{48}{16}} = 2$$

146 **(d)**
Let $P(x_1, y_1)$ be a point on the hyperbola
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

The chord of contact of tangents from P to the hyperbola is given by

$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$ (i)

The equations of the asymptotes are

$$\frac{x}{a} - \frac{y}{b} = 0$$

and $\frac{x}{a} + \frac{y}{b} = 0$

The points of intersection of (i) with the two asymptotes are given by

$$x_1 = \frac{2a}{\frac{x_1}{a} - \frac{y_1}{b}}, y_1 = \frac{2b}{\frac{x_1}{a} - \frac{y_1}{b}}$$

$$x_2 = \frac{2a}{\frac{x_1}{a} + \frac{y_1}{b}}, y_2 = \frac{2b}{\frac{x_1}{a} + \frac{y_1}{b}}$$

Area of the said triangle $= \frac{1}{2}(x_1y_2 - x_2y_1)$

$$= \frac{1}{2}\left|\left(-\frac{4ab \times 2}{\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2}}\right)\right| = 4ab$$

147 **(b)**
Clearly $(0, 0)$ lies on director circle of the given circle
Now, equation of director circle is
$(x+\mathrm{g})^2 + (y+f)^2 = 2(\mathrm{g}^2 + f^2 - c)$
If $(0, 0)$ lies on it, then
$\mathrm{g}^2 + f^2 = 2(\mathrm{g}^2 + f^2 - c)$

$\Rightarrow g^2 + f^2 = 2c$

148 **(b)**

For given $r_1 = \sqrt{10}, C_1(1, 0)$
and $r_2 = \sqrt{5}, C_2(0, 2)$
$d = C_1C_2 = \sqrt{5}$
If θ is the angle between the circle, then

$$\cos\theta = \frac{|d^2 - r_1^2 - r_2^2|}{2r_1r_2}$$

$$= \frac{|5 - 10 - 5|}{2\sqrt{10}\sqrt{5}}$$

$$= \frac{1}{\sqrt{2}}$$

Hence, $\theta = \frac{\pi}{4}$

149 **(a)**

$$\frac{dy}{dx} = 2x - 5$$

$$m_1 = \left(\frac{dy}{dx}\right)_{(2,0)} = 4 - 5 = -1 \text{ and}$$

$$m_2 = \left(\frac{dy}{dx}\right)_{(3,0)} = 6 - 5 = 1$$

$\Rightarrow m_1m_2 = 1 \Rightarrow$ angle between tangents $= \frac{\pi}{2}$

150 **(b)**

The equation of the hyperbola is

$$\frac{\left(\frac{2x-y+4}{\sqrt{5}}\right)^2}{\frac{1}{2}} - \frac{\left(\frac{x+2y-3}{\sqrt{5}}\right)^2}{\frac{1}{3}} = 1$$

or $\frac{2}{5}(2x - y + 4)^2 - \frac{3}{5}(x + 2y - 3)^2 = 1$

151 **(b)**

$x = 3\cos t, y = 4\sin t$
Eliminating t, we have
$\frac{x^2}{9} + \frac{y^2}{16} = 1$, which is ellipse
$\therefore \quad x^2 - 2 = 2\cos t; y = 4\cos^2\frac{t}{2}$
$\Rightarrow y = 2(1 - 2\cos t)$
$y = 2\left(1 + \frac{x^2-2}{2}\right)$
Which is parabola
$\sqrt{x} = \tan t;\ \sqrt{y} = \sec t$
Eliminating t, we have
$y - x = 1$, which is straight line
$x = \sqrt{1 - \sin t}; y$
$y = \sin\frac{t}{2} + \cos\frac{t}{2}$
Eliminating t, we have
$x^2 + y^2 = 1 - \sin t + 1 + \sin t = 2$, which is circle

152 **(d)**

Hyperbola $9x^2 - 16y^2 - 18x - 32y - 151 = 0$ can be written as

$$9(x^2 - 2x) - 16(y^2 + 2y) = 151$$

$$\Rightarrow 9(x-1)^2 - 16(y+1)^2 = 151 + 9 - 16 = 144$$

$$\Rightarrow \frac{(x-1)^2}{16} - \frac{(y+1)^2}{9} = 1$$

or $\frac{X^2}{16} - \frac{Y^2}{9} = 1$ [where $X = x - 1, Y = y + 1$]

Here $a^2 = 16, b^2 = 9$

Latus rectum $= 2\frac{b^2}{a} = \frac{2(9)}{4} = \frac{9}{2}$

153 **(b)**

Let $P(x, y)$ be the point of contact. At 'P' both of them must have same slope. We have,
$2y\frac{dy}{dx} = 4a, 2x = 4a\frac{dy}{dx}$
Eliminating $\frac{dy}{dx}$, we get $xy = 4a^2$

154 **(a)**

Tangent at P$(a\cos\alpha, b\sin\alpha)$ is
$\frac{x}{a}\cos\alpha + \frac{y}{b}\sin\alpha = 1$ (i)
Distance of focus $S(ae, o)$ from this tangent is

$$d_1 = \frac{|e\cos\alpha - 1|}{\sqrt{\frac{\cos^2\alpha}{a^2} + \frac{\sin^2\alpha}{b^2}}}$$

$$= \frac{1 - e\cos\alpha}{\sqrt{\frac{\cos^2\alpha}{a^2} + \frac{\sin^2\alpha}{b^2}}}$$

Distance of focus $S(-ae, 0)$ from this line

$$d_2 = \frac{1 + e\cos\alpha}{\sqrt{\frac{\cos^2\alpha}{a^2} + \frac{\sin^2\alpha}{b^2}}}$$

$$\Rightarrow \frac{d_1}{d_2} = \frac{1 - e\cos\alpha}{1 + e\cos\alpha}$$

Now $SP = a - ae\cos\alpha$ and $S'P = a + ae\cos\alpha$

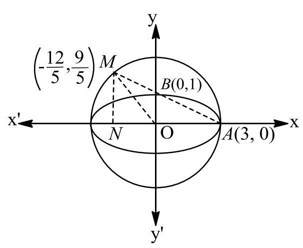
$$\Rightarrow \frac{SP}{S'P} = \frac{1 - e\cos\alpha}{1 + e\cos\alpha}$$

$$\Rightarrow \frac{SP}{S'P} = \frac{d_1}{d_2}$$

155 **(d)**

Equation of auxiliary circle is

$x^2 + y^2 = 9$ ... (i)



Equation of AM is $\frac{x}{3}+\frac{y}{1}=1$ ... (ii)

On solving Eqs. (i) and (ii) , we get $M\left(-\frac{12}{5},\frac{9}{5}\right)$

Now, area of $\Delta AOM=\frac{1}{2}.OA\times MN$

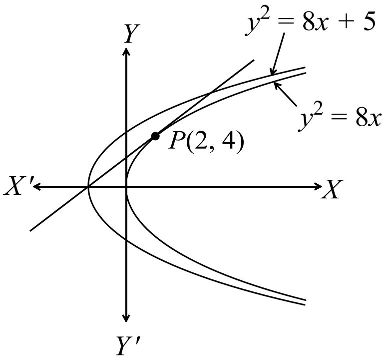
$=\frac{27}{10}$ sq unit

156 **(b)**

Equation of tangent to the parabola $y^2=8x$ at $P(2,4)$ is

$4y=4(x+2)$

Or $x-y+2=0$ (i)



Equation of chord of parabola $y^2=8x+5$ whose middle point is (h,k) is $T=S_1$

i.e., $ky-4(x+h)-5=k^2-8h-5$

or $4x-ky+k^2-4h=0$ (ii)

Equations (i) and (ii) must be identical

$\therefore \frac{4}{1}=\frac{k}{1}=\frac{k^2-4h}{2}$

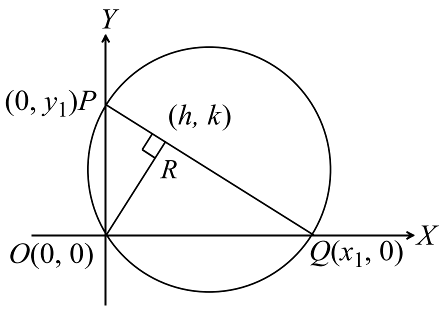
By comparing Eqs.(i), (ii)

$k=4$

and $8=k^2-4h$

Hence, the required point is (2, 4)

157 **(c)**



Equation of line PQ is $y-k=-\frac{h}{k}(x-h)$

or $hx+ky=h^2+k^2$

$\Rightarrow$ Points $Q\left(\frac{h^2+k^2}{h},0\right)$ and $P\left(0,\frac{h^2+k^2}{k}\right)$

Also $2a=\sqrt{x_1^2+y_1^2}$

$\Rightarrow x_1^2+y_1^2=4a^2$

Eliminating x_1 and y_1 we have

$(x^2+y^2)^2\left(\frac{1}{x^2}+\frac{1}{y^2}\right)=4a^2$

158 **(a)**

Distance of given line from the centre of the circle is $|p|$

Now line subtends right angle at the centre

Hence, radius $=\sqrt{2}|p|$

$\Rightarrow a=\sqrt{2}\,|p|$

$\Rightarrow a^2=2p^2$

159 **(c)**

Let $\sum_{i=1}^{6} x_i=\alpha$ ad $\sum_{i=1}^{6} y_i=\beta$

Let O be the orthocenter of the triangle made by (x_1,y_1), (x_2,y_2) and (x_3,y_3)

$\Rightarrow O$ is $(x_1+x_2+x_3,y_1+y_2+y_3)\equiv(\alpha_1,\beta_1)$

Similarly let G be the centroid of the triangle made by other three points

$\Rightarrow G$ is $\left(\frac{x_4+x_5+x_6}{3},\frac{y_4+y_5+y_6}{3}\right)$

$\Rightarrow G$ is $\left(\frac{\alpha-\alpha_1}{3},\frac{\beta-\beta_1}{3}\right)$

The point dividing OG is the ratio 3: 1 is

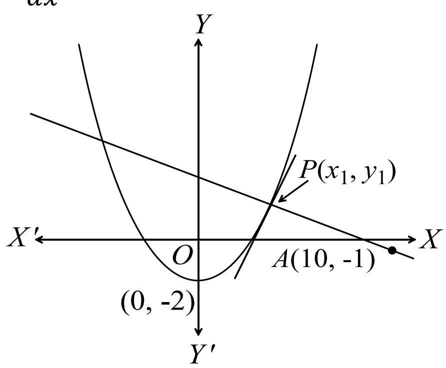
$\left(\frac{\alpha}{4},\frac{\beta}{4}\right)\equiv(2,1)$

$\Rightarrow h+k=3$

160 **(d)**

$4y=x^2-8$

$4\frac{dy}{dx}=2x$



Therefore, slope of normal $=-\frac{2}{x_1}$; but slope of normal

$=\frac{y_1+1}{x_1-10}$

$\therefore \frac{y_1+1}{x_1-10}=-\frac{2}{x_1}$

$\Rightarrow x_1y_1+x_1=-2x_1+20$

$\Rightarrow x_1y_1+3x_1=20$

Substituting $y_1=\frac{x_1^2-8}{4}$

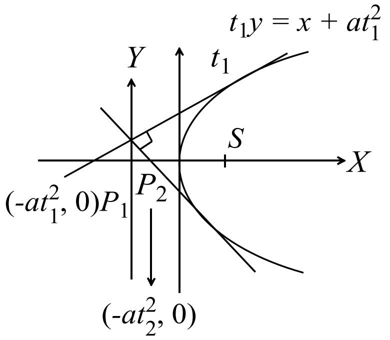
(from the given equation)

$x_1\left(\frac{x_1^2-8}{4}+3\right)=20$

$\Rightarrow x_1(x_1^2+4)=80$

$\Rightarrow x_1^3 + 4x_1 - 80 = 0,$
Which has one root $x_1 = 4$
Hence, $x_1 = 4; y_1 = 2$
$\therefore P = (4, 2)$
Therefore, equation of PA is
$y + 1 = -\frac{1}{2}(x - 10)$
$\Rightarrow 2y + 2 = -x + 10$
$\Rightarrow x + 2y - 8 = 0$

161 **(c)**
$SP_1 = a(1 + t_1^2); SP_2 = a(1 + t_2^2)$
$\Rightarrow t_1 t_2 = -1$



$\frac{1}{SP_1} = \frac{1}{a(1 + t_1^2)}$
$\frac{1}{SP_2} = \frac{1}{a(1 + t_2^2)} = \frac{t_1^2}{a(t_1^2 + 1)}$
$\therefore \frac{1}{SP_1} + \frac{1}{SP_2} = \frac{1}{a}$

162 **(c)**
Let the hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

then $2a = ae$, i.e., $e = 2$

$\therefore \frac{b^2}{a^2} = e^2 - 1 = 3$

$\therefore \frac{(2b)^2}{(2a)^2} = 3$

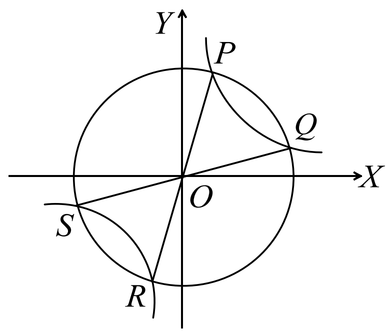
163 **(b)**
The centre of $x^2 + y^2 - 4x - 4y = 0$ is $(2, 2)$
It is $ax + by = 2$
$\therefore 2a + 2b = 2$ or $a + b = 1$
$ax + by = 2$ touches $x^2 + y^2 = 1$
So, $1 = \left|\frac{-2}{\sqrt{a^2+b^2}}\right|$
$\therefore a^2 + b^2 = 4$ or $a^2 + (1 - a)^2 = 4$
or $2a^2 - 2a - 3 = 0$
$\therefore a = \frac{2 \pm \sqrt{4 + 24}}{4} = \frac{1 \pm \sqrt{7}}{2}$
$\therefore b = 1 - a = 1 - \frac{1 \pm \sqrt{7}}{2} = \frac{1 \mp \sqrt{7}}{2}$

164 **(d)**
Equation of QR is $T = 0$ (chord of contact)
$\frac{8x}{4} + \frac{27y}{9} = 1$
$\Rightarrow 2x + 3y = 1$ (i)
Now, equation of the pair of lines passing through origin and points Q, R is given by
$\left(\frac{x^2}{4} + \frac{y^2}{9}\right) = (2x + 3y)^2$ (Making equation of ellipse homogeneous using Eq.(i))
$\Rightarrow 9x^2 + 4y^2 = 36\,(4x^2 + 12xy + 9y^2)$
$\Rightarrow 135x^2 + 432xy + 320y^2 = 0$
$\therefore$ Required angle is $\tan^{-1}\frac{2\sqrt{216^2 - 135 \times 320}}{455}$
$= \tan^{-1}\frac{8\sqrt{2916 - 2700}}{455}$
$= \tan^{-1}\frac{8\sqrt{216}}{455}$
$= \tan^{-1}\frac{48\sqrt{6}}{455}$

165 **(c)**
Any tangent to $y^2 = 4a(x + a)$ is
$y = m(x + a) + \frac{a}{m}$ (i)
Any tangent to $y^2 = 4b(x + b)$ which is perpendicular to Eq. (i) is
$y = -\frac{1}{m}x + (x + b) - bm$ (ii)
Subtracting, we get
$\left(m + \frac{1}{m}\right)x + (a + b)\left(m + \frac{1}{m}\right) = 0$
Or $x + a + b = 0$ which is a locus of their point of intersection

166 **(a)**
A circle through three co-normal points of a parabola always passes through the vertex of the parabola. Hence, the circle through P, Q, R, S out of which P, Q, R are co- normals points will always pass through vertex (2, 3) of parabola

167 **(b)**
$x - 2 = m$



$y + 1 = \frac{4}{m}$

$\therefore (x-2)(y+1)=4$

$\Rightarrow XY=4$, where $X=x-2, Y=y+1$

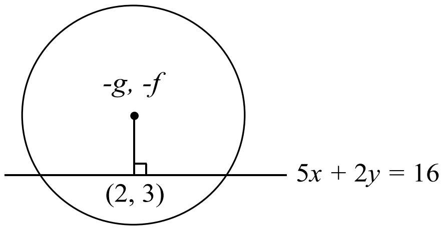
$S \equiv (x-2)^2+(y+1)^2=25$

$\Rightarrow X^2+Y^2=25$

Curve 'C' and circle S both are concentric

$\therefore OP^2+OQ^2+OR^2+OS^2=4r^2=4\times 25$
$=100$

168 **(a)**



Slope of the given line $=-\frac{5}{2}$

$\Rightarrow \left(\frac{5}{2}\right)\left(\frac{3+f}{2+g}\right)=-1$

$\Rightarrow 15+5f=4+2g$

$\Rightarrow$ Locus is $2x-5y+11=0$

169 **(a)**

Make homogeneous and put $A+B=0$

$y^2=4ax\left(\frac{lx+my}{-n}\right)$

$\therefore 4al+n=0$

170 **(c)**

Let $P(5\cos\theta, 4\sin\theta)$ be any point on the ellipse

Then $SP=5+5e\cos\theta$

$S'P=5-5e\cos\theta$

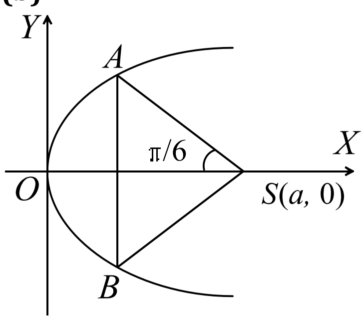
$SP.S'P=25-25e^2\cos^2\theta$

$=25\sin^2\theta+16\cos^2\theta$

$=16+9\sin^2\theta=f(\theta)$ (say)

$\Rightarrow 16\le f(\theta)\le 25$

171 **(b)**



Let $A=(at_1^2, 2at_1), B\equiv(at_2^2, -2at_1)$

We have

$m_{AS}=\tan\left(\frac{5\pi}{6}\right)$

$\Rightarrow \frac{2at_1}{at_1^2-a}=-\frac{1}{\sqrt{3}}$

$\Rightarrow t_1^2+2\sqrt{3}t_1-1=0$

$\Rightarrow t_1=-\sqrt{3}\pm 2$

Clearly $t_1=-\sqrt{3}-2$ is rejected

Thus, $t_1=(2-\sqrt{3})$

Hence, $AB=4at_1=4a(2-\sqrt{3})$

172 **(c)**

Tangent to the ellipse at P and Q are

$\frac{x}{a}\cos\alpha+\frac{y}{b}\sin\alpha=1$ (i)

and $\frac{x}{a}\cos\beta+\frac{y}{b}\sin\beta=1$ (ii)

Solving Eqs. (i) and (ii), we get

$$\frac{x}{\begin{vmatrix}\frac{\sin\alpha}{b} & 1\\ \frac{\sin\beta}{b} & 1\end{vmatrix}}=\frac{-y}{\begin{vmatrix}\frac{\cos\alpha}{a} & 1\\ \frac{\cos\beta}{a} & 1\end{vmatrix}}=\frac{1}{\begin{vmatrix}\frac{\cos\alpha}{a} & \frac{\sin\alpha}{b}\\ \frac{\cos\beta}{a} & \frac{\sin\beta}{b}\end{vmatrix}}$$

$\Rightarrow x=\frac{a(\sin\alpha-\sin\beta)}{\sin(\beta-\alpha)}$,

$y=\frac{-b(\cos\alpha-\cos\beta)}{\sin(\beta-\alpha)}$

$\Rightarrow \frac{x\sin(\beta-\alpha)}{b}=\sin\alpha-\sin\beta$,

$\frac{y\sin(\beta-\alpha)}{b}=-(\cos\alpha-\cos\beta)$

Squaring and adding, we get

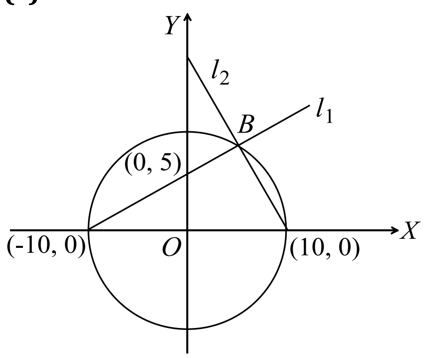
$\sin^2(\beta-\alpha)\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)=2$

$\Rightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{2}{\sin^2 c}$

(where $\beta-\alpha=c$ (constant) given)

Which is an ellipse

173 **(c)**



Slope of $l_1=\frac{1}{2}$

Slope of $l_2=-2$

Equation of $l_2, y=-2(x-10)$

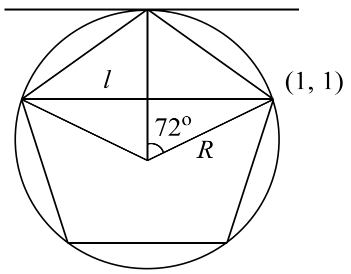
$\Rightarrow y+2x=20$

Hence, $t=20$

174 **(a)**

Point of intersection of diagonals lie on circumcircle

i.e. (1, 1), since $(y-2x+1)+\lambda(2y-x-1)=0$



$l = 2R\sin 72°$

$R = \dfrac{\sin 36°}{2\sin 72°} = \cos 72°$

$\Rightarrow$ Locus is $(x-1)^2 + (y-1)^2 = \cos^2 72°$

$\Rightarrow x^2 + y^2 - 2x - 2y + 1 + \sin^2 72° = 0$

175 **(c)**

$y^2 = kx - 8$

$\Rightarrow y^2 = k\left(x - \dfrac{8}{k}\right)$

Directrix of parabola is $x = \frac{8}{k} - \frac{k}{4}$

$\Rightarrow \frac{8}{k} - \frac{k}{4} = 1$, we get $k = 4$

176 **(b)**

$(5x-10)^2 + (5y+15)^2$

$= \dfrac{(3x-4y+7)^2}{4}$

$\Rightarrow (x-2)^2 + (y+3)^2 = \left(\dfrac{1}{2}\dfrac{3x-4y-7}{5}\right)^2$

$\Rightarrow \sqrt{(x-2)^2 + (y+3)^2} = \frac{1}{2}\frac{|3x-4y-7|}{5}$ is in ellipse,

Whose focus is $(2,-3)$, directrix $3x - 4y + 7 = 0$ and eccentricity is $\frac{1}{2}$

Length of $\perp$ from focus to directrix is

$\dfrac{|3\times 2 - 4(-3) + 7|}{5} = 5$

$\Rightarrow \dfrac{a}{e} - ae = 5$

$\Rightarrow 2a - \dfrac{a}{2} = 5$

$\Rightarrow a = \dfrac{10}{3}$

So length of major axis is $\frac{20}{3}$

177 **(c)**

Equation of radical axis (i.e. common chord) of the two circles is

$10x + 4y - a - b = 0$ ...(i)

Centre of first circle is $H(-4,-4)$

Since second circle bisects the circumference of the first circle, therefore, centre $H(-4,-4)$ of the first circle must lie on the common chord Eq. (i)

$\therefore\ -40 - 16 - a - b = 0$

$\Rightarrow a + b = -56$

178 **(b)**

Solving the line $y = x - 1$ and parabola $y^2 = 4x$, we have

$(x-1)^2 = x$

$\Rightarrow x^2 - 6x + 1 = 0$

$\Rightarrow x = 3 \pm \sqrt{8}$

$\therefore y = 2 \pm \sqrt{8}$

Suppose point D is (x_3, y_3), then

$y_1 + y_2 + y_3 = 0$

$\Rightarrow 2 + \sqrt{8} + 2 - \sqrt{8} + y_3 = 0$

$\Rightarrow y_3 - 4$, then $x_3 = 4$

Therefore, the point is (4, 4)

179 **(a)**

Let $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of the triangle ABC, and let $P(h, k)$ be any point on the locus

Then, $PA^2 + PB^2 + PC^2 = c$ (constant)

$\Rightarrow \displaystyle\sum_{i=1}^{3}(h - x_i)^2 + (k - y_i)^2 = c$

$\Rightarrow h^2 + k^2 - \dfrac{2h}{3}(x_1 + x_2 + x_3)$

$\qquad - \dfrac{2k}{3}(y_1 + y_2 + y_3)$

$\qquad + \displaystyle\sum_{i=1}^{3}(x_i^2 + y_i^2) - c = 0$

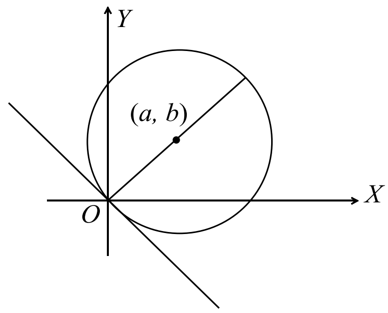
So, locus of (h, k) is

$x^2 + y^2 - \dfrac{2x}{3}(x_1 + x_2 + x_3) - \dfrac{2y}{3}(y_1 + y_2 + y_3)$

$\qquad + \lambda = 0,$

Where $\lambda = \sum_{i=1}^{3}(x_i^2 + y_1^2) - c = 0$ constant

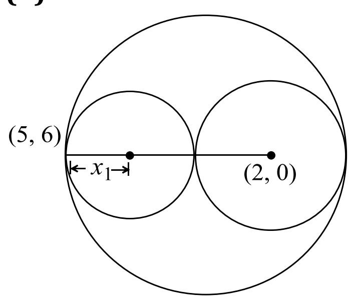
180 **(b)**



Obviously, the slope of the tangent will be $-\left(\frac{1}{b/a}\right)$, i.e., $-\frac{a}{b}$

Hence, the equation of the tangent is $y = -\frac{a}{b}x$, i.e., $by + ax = 0$

181 **(b)**



Given circle is $(x-2)^2+y^2=4$
Centre is $(2, 0)$ and radius $= 2$
Therefore, distance between $(2, 0)$ and $(5, 6)$ is
$\sqrt{9+36}=3\sqrt{5}$
$\Rightarrow r_1=\dfrac{3\sqrt{5}-2}{2}$
and $r_2=\frac{3\sqrt{5}+2}{2}$
$=r_1 r_2=\dfrac{41}{4}$

182 **(c)**
Let the second circle be $x^2+y^2+2gx+2fy=0$
But $y=x$ touches the circle
Hence, $x^2+x^2+2gx+2fx=0$ has equal roots, i.e. $f+g=0$
Therefore, the equation of the common chord is
$2(g-3)x+2(-g-4)y+7=0$
or $(-6x-8y+7)+g(2x-2y)=0$ which passes through the point of intersection of $-6x-8y+7=0$ and $2x-2y=0$ which is (1/2, 1/2)

183 **(a)**
Centre of the circle $x^2+y^2=2x$ is $(1, 0)$
Common chord of the other two circles is
$8x-15y+26=0$
Distance from $(1, 0)$ to $8x-15y+26=0$
$=\dfrac{|8+26|}{\sqrt{15^2+8^2}}=2$

184 **(b)**
Slope of normal at point $P(t_1)$ and $Q(t_2)$ is $-t_1$ and $-t_2$, respectively
Equation of chord joining $P(t_1)$ and $Q(t_2)$ is
$y-2at_1=\dfrac{2}{t_1+t_2}(x-at_1^2)$
Or $2x-(t_1+t_2)y+2at_1t_2=0$
But $t_1t_2=-1$
Chord PQ is $2x-(t_1+t_2)y-2a=0$
Or $(2x-2a)-(t_1+t_2)y=0$
Which passes through the fixed point $(a, 0)$

185 **(c)**
We have

$\dfrac{2b^2}{a}=8$

and $2b=\frac{1}{2}(2ae)$

$\therefore \dfrac{2}{a}\left(\dfrac{ae}{2}\right)^2=8$

$\Rightarrow ae^2=16$ (i)

Also, $2\frac{b^2}{a}=8$

$\Rightarrow b^2=4a$

$\Rightarrow a^2(e^2-1)=4a$

$\Rightarrow ae^2-a=4$ (ii)

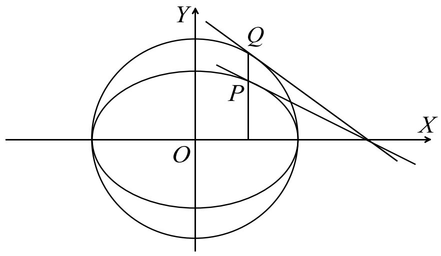
From (i) and (ii), we have

$16-\dfrac{16}{e^2}=4$

$\Rightarrow \dfrac{16}{e^2}=12$

$\Rightarrow e=\dfrac{2}{\sqrt{3}}$

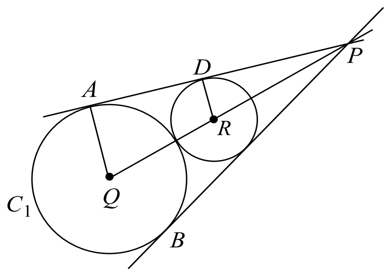
186 **(c)**



Tangent to the ellipse at point $P(a\cos\theta, b\sin\theta)$ is
$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$ (i)
Tangent to the circle at point $Q(a\cos\theta, a\sin\theta)$ is
$x\cos\theta+y\sin\theta=a$ (ii)
Equation (i) and (ii) intersect at $\left(\frac{a}{\cos\theta}, 0\right)$ which lies on $y=0$

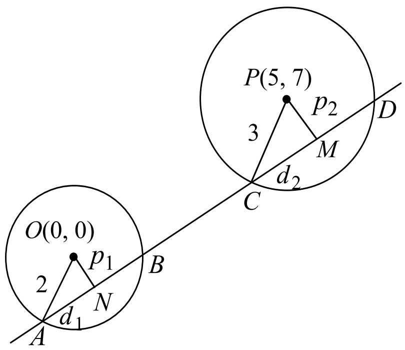
187 **(b)**
Joint equation of OA and OB is
$x^2-4x(y-3x)-4y(y-3x)+20(y-3x)^2=0$
Making equation of parabola homogeneous using straight line
$\Rightarrow x^2(1+12+180)-y^2(4-20)-xy(4-12+120=0$
$\Rightarrow 193x^2+16y^2-112xy=0$
$\tan\theta=\dfrac{2\sqrt{h^2-ab}}{a+b}$
$=\dfrac{2\sqrt{56^2-193\times 16}}{193+16}=\dfrac{8\sqrt{3}}{209}$

188 **(d)**



$AQ = 3 + 2\sqrt{2}$
$PQ = 3\sqrt{2} + 4$
Let r be required radius
$\therefore 3\sqrt{2} + 4 = 3 + 2\sqrt{2} + r + r\sqrt{2}$ $\left(\because \angle RPD = \frac{\pi}{4}\right)$
$\sqrt{2} + 1 = r(1 + \sqrt{2}) \Rightarrow r = 1$

189 **(a)**



Let equation of line be $y = x + c$ or $y - x = c$ ...(i)
Perpendicular from (0, 0) on line (i) is $\left|\frac{-c}{\sqrt{2}}\right| = \frac{c}{\sqrt{2}}$
In ΔAON, $\sqrt{2^2 - \left(\frac{c}{\sqrt{2}}\right)^2} = AN$
and in ΔCPM, $\sqrt{3^2 - \left(\frac{2-c}{\sqrt{2}}\right)^2} = CM$
Given $AN = CM \Rightarrow 4 - \frac{c^2}{2} = 9 - \frac{(2-c)^2}{2}$
$\Rightarrow c = -\frac{3}{2}$
Therefore, Equation of line $y = x - \frac{3}{2}$ or
$2x - 2y - 3 = 0$

190 **(a)**
Tangent to $\frac{x^2}{9} - \frac{y^2}{4} = 1$ at $P(3\sec\theta, 2\tan\theta)$ is

$\frac{x}{3}\sec\theta - \frac{y}{2}\tan\theta = 1$

This is perpendicular to

$5x + 2y - 10 = 0$

$\Rightarrow \frac{2\sec\theta}{3\tan\theta} = \frac{2}{5}$

$\Rightarrow \sin\theta = \frac{5}{3}$ which is not possible

Hence, there is no such tangent

191 **(c)**
Equation of any circles passing through (1, 0) and (5, 0) is
$y^2 + (x - 1)(x - 5) + \lambda y = 0$
i.e. $x^2 + y^2 + \lambda y - 6x + 5 = 0$
If $\angle ACB$ is maximum, then this circle must touch the y-axis at $(0, h)$. Putting $x = 0$ in the equation of circle, we get $y^2 + \lambda y + 5 = 0$. It should have $y = h$ as it is a repeated root
$\Rightarrow h^2 = 5$ and $\lambda = -2h$
$\Rightarrow |h| = \sqrt{5}$

192 **(d)**
Equation of any circle through the points of intersection of given circles is
$x^2 + y^2 - 4x - 2y - 8 + k(x^2 + y^2 - 2x - 4y - 8 = 0$ ...(i)
Since circle Eq. (i) passes through $(-1, 4)$
$\therefore k = 1$
$\therefore$ Required circle is
$x^2 + y^2 - 3x - 3y - 8 = 0$

193 **(d)**
Length of latus rectum
=2×distance of focus form directrix
$= 2 \times \left|\frac{-\frac{u^2}{2g}\cos 2\alpha - \frac{u^2}{2g}}{\sqrt{1}}\right|$
$= \frac{2u^2}{g}\cos^2\alpha$

194 **(d)**
$y - \sqrt{3}\,x + 3 = 0$ can be rewritten as
$\frac{y-0}{\sqrt{3}/2} = \frac{x-\sqrt{3}}{1/2} = r$ (i)
On solving Eq. (i) with the parabola $y^2 = x + 2$
$\frac{3r^2}{4} = \frac{r}{2} + \sqrt{3} + 2$
$\Rightarrow 3r^2 - 2r - (4\sqrt{3} + 8) = 0$
$\Rightarrow AP \cdot AQ = |r_1 r_2|$
$= \frac{4(\sqrt{3} + 2)}{3}$
(product of roots)

195 **(a)**
Let the point be $P(\sqrt{2}\cos\theta, \sin\theta)$ on $\frac{x^2}{2} + \frac{y^2}{1} = 1$

$\Rightarrow$ Equation of tangent is

$\frac{x\sqrt{2}}{2}\cos\theta + y\sin\theta = 1$

Whose intercept on coordinate axes are
$A(\sqrt{2}\sec\theta, 0)$ and $B(0, \operatorname{cosec}\theta)$

$\therefore$ Mid point of its intercept between axes is

$\left(\frac{\sqrt{2}}{2}\sec\theta, \frac{1}{2}\operatorname{cosec}\theta\right) = (h, k)$

$$\cos\theta = \frac{1}{\sqrt{2h}} \text{ and } \sin\theta = \frac{1}{2k}$$

Thus, locus of mid point M is

$$\cos^2\theta + \sin^2\theta = \frac{1}{2h^2} + \frac{1}{4k^2}$$

$$\Rightarrow \frac{1}{2x^2} + \frac{1}{4y^2} = 1$$

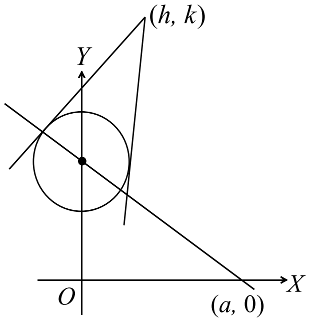
196 **(a)**

Let the centre be $(0, \alpha)$ equation of circle

$x^2 + (y-\alpha)^2 = |a|^2$

$\therefore$ Equation of chord of contact for $P(h,k)$ is

$xh + yk - \alpha(y+k) + \alpha^2 - a^2 = 0$



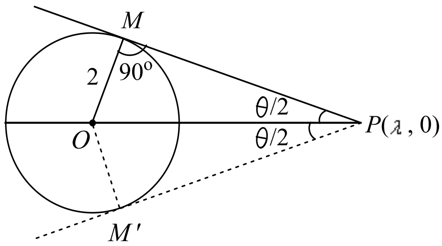
It passes through $(a, 0)$

$\Rightarrow \alpha^2 - \alpha k + ah - a^2 = 0$

As α is real

$\Rightarrow k^2 - 4(ah - a^2) \geq 0$

197 **(a)**



We have $\frac{\pi}{2} < \theta < \frac{2\pi}{3}$, i.e., $\frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{3}$

$$\Rightarrow \frac{1}{\sqrt{2}} < \sin\frac{\theta}{2} < \frac{\sqrt{3}}{2}$$

But, $\sin\frac{\theta}{2} = \frac{2}{\lambda} \Rightarrow \frac{1}{\sqrt{2}} < \frac{2}{\lambda} < \frac{\sqrt{3}}{2}$

$$\Rightarrow \frac{4}{\sqrt{3}} < \lambda < 2\sqrt{2}$$

198 **(a)**

Any point on the ellipse is $(2\cos\theta, \sqrt{3}\sin\theta)$. The focus on the positive x-axis is $(1, 0)$

Given that

$$(2\cos\theta - 1)^2 + 3\sin^2\theta = \frac{25}{16}$$

$$\Rightarrow \cos\theta = \frac{3}{4}$$

199 **(c)**

$C_1 = (-1, -4); C_2 = (2, 5);$

$r_1 = \sqrt{1 + 16 + 23} = 2\sqrt{10};$

$r_2 = \sqrt{4 + 25 + 19} = \sqrt{10};$

$C_1C_2 = \sqrt{9 + 18} = 3\sqrt{10}$

$\Rightarrow C_1C_2 = r_1 + r_2$

Hence, circles touch externally

200 **(c)**

A cyclic parallelogram will be a rectangle or square

So, $\angle QPR = 90^0$

$\Rightarrow P$ lies on director circle of the ellipse

$\frac{x^2}{16} + \frac{y^2}{b^2} = 1$

$\Rightarrow x^2 + y^2 = 25$ is director circle of $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$

$\Rightarrow 16 + b^2 = 25$

$\Rightarrow b^2 = 9$

$\Rightarrow a^2(1 - e^2) = 9$

$$\Rightarrow 1 - e^2 = \frac{9}{16}$$

$$\Rightarrow e^2 = \frac{7}{16}$$

$$\Rightarrow e = \frac{\sqrt{7}}{4}$$

201 **(d)**

For the two ellipses to intersect at four distinct points, $a > 1$

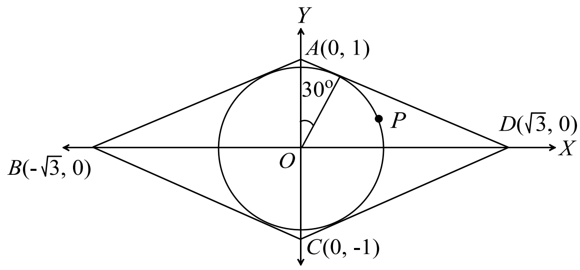
$\Rightarrow b^2 - 5b + 7 > 1$

$\Rightarrow b^2 - 5b + 6 > 0$

$\Rightarrow b \in (-\infty, 2) \cup (3, \infty)$

$\Rightarrow b$ does not lie in $[2, 3]$

202 **(b)**



$OA = 1$

$$r = OA\cos 30° = \frac{\sqrt{3}}{2}$$

Equation of circle is $x^2 + y^2 = 3/4$

$PA^2 + PB^2 + PC^2 + PD^2$

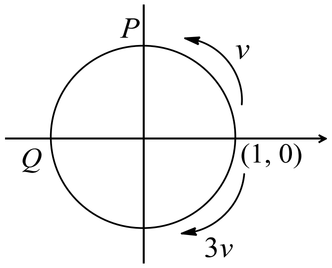
$= x_1^2 + (y_1 - 1)^2 + \left(x_1 + \sqrt{3}\right)^2 + y_1^2 + x_1^2 + (y_1 + 1)^2 + \left(x_1 - \sqrt{3}\right)^2 + y_1^2$

$= 4x_1^2 + 4y_1^2 + 8 = 4(x_1^2 + y_1^2) + 8$

$$= 4 \times \frac{3}{4} + 8$$

$= 11$

203 **(d)**



The particle which moves clockwise is moving three times as fast as the particle moving anticlockwise. This means the clockwise particle travels (3/4)th of the way around the circle, the anticlockwise particle will travel (1/4)th of the way around the circle and so the second particle will meet at $P(0,1)$

Using the same logic they will meet at $Q(-1,0)$ when they meet the second time

204 **(c)**

Any point on the given parabola is $(t^2,\ 2t)$

The equation of the tangent at (1, 2) is

$x-y+1=0$

The image (h,k) of the point $(t^2,2t)$ in

$x-y+1=0$ is given by

$$\frac{h-t^2}{1}=\frac{k-2t}{-1}=-\frac{2(t^2-2t+1)}{1+1}$$

$\therefore h=t^2-t^2+2t-1=2t-1$

And $k=2t+t^2-2t+1=t^2+1$

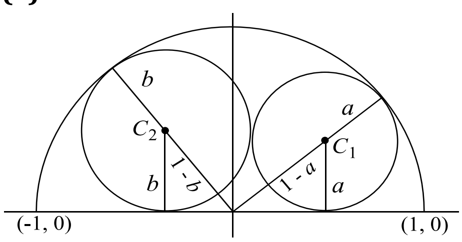
Eliminating t from $h=2t-1$ and $k=t^2+1$, we get

$(h+1)^2=4(k-1)$

The required equation of reflection is

$(x+1)^2=4(y-1)$

205 **(a)**



Let centre of the circles be C_1 and C_2

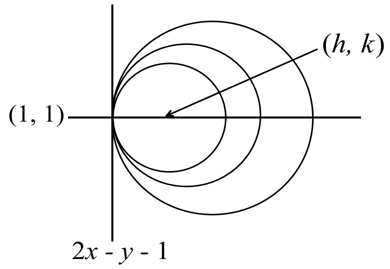
$\Rightarrow C_1$ is $(\sqrt{1-2a},a)$ and C_2 is $(\sqrt{1-2b},b)$

Now $C_1C_2=a+b=a+\frac{1}{2}$

$$\Rightarrow 1-2a+\left(a-\frac{1}{2}\right)^2=\left(a+\frac{1}{2}\right)^2$$

$$\Rightarrow a=\frac{1}{4}$$

206 **(b)**



Obviously locus of centre is line perpendicular to the given line

Hence, locus is $\frac{k-1}{h-1}=-\frac{1}{2}$ or $x+2y=0$

207 **(c)**

Normal at point $P(x_1,y_1)\equiv(at_1^2,2at_1)$meets the parabola at $R(at^2,2at)$

$\Rightarrow t=-t_1-\frac{2}{t_1}$ (i)

Normal at point $Q(x_2,y_2)\equiv(at_2^2,2at_2)$ meets the parabola at $R(at^2,2at)$

$\Rightarrow\ t=-t_2-\frac{2}{t_2}$ (ii)

From Eqs. (i) and (ii)

$$-t_1-\frac{2}{t_1}=-t_2-\frac{2}{t_2}$$

$\Rightarrow\ t_1t_2=2$

Now given that $x_1+x_2=4$

$\Rightarrow t_1^2+t_2^2=4$

$\Rightarrow (t_1+t_2)^2=4+4=8$

$\Rightarrow |t_1+t_2|=2\sqrt{2}$

$\Rightarrow |y_1+y_2|=4\sqrt{2}$

208 **(c)**

For $x^2+y^2=9$, the centre $=(0,0)$ and the radius $=3$

For $x^2+y^2-8x-6y+n^2=0$,

The centre $=\ (4,3)$ and the radius $=$ $\sqrt{4^2+3^2-n^2}$

$\therefore 4^2+3^2-n^2>0$ or $n^2<5^2$ or $-5<n<5$

Circles should cut to have exactly two common tangents

So, $r_1+r_2>d$ (distance between centres)

$\therefore 3+\sqrt{25-n^2}>\sqrt{4^2+3^2}$

or $\sqrt{25-n^2}>2$

or $25-n^2>4$

$\therefore n^2<21$ or $-\sqrt{21}<n<\sqrt{21}$

Therefore, common values of n should satisfy

$-\sqrt{21}<n<\sqrt{21}$

But $n\in Z$. So, $n=-4,-3,\dots,4$

209 **(c)**

Here centre of the ellipse is (0, 0)

Let $P(r\cos\theta,r\sin\theta)$ be any point on the given ellipse then

$r^2\cos^2\theta+2r^2\sin^2\theta+2r^2\sin\cos\theta=1$

$$\Rightarrow r^2 = \frac{1}{\cos^2\theta + 2\sin^2\theta + \sin 2\theta}$$
$$= \frac{1}{\sin^2\theta + 1 + \sin 2\theta}$$
$$= \frac{2}{1 - \cos 2\theta + 2 + 2\sin 2\theta}$$
$$= \frac{2}{3 - \cos 2\theta + 2\sin 2\theta}$$
$$\Rightarrow r_{\max} = \frac{\sqrt{2}}{\sqrt{3 - \sqrt{5}}}$$

210 **(d)**

Normals at $p(\theta), Q(\phi)$ are

$$ax\cos\theta + by\cot\theta = a^2 + b^2$$

$$ax\cos\phi + by\cot\phi = a^2 + b^2$$

Where $\varphi = \frac{\pi}{2} - \theta$ and these pass through (h, k). Therefore,

$$ah\cos\theta + bk\cot\theta = a^2 + b^2$$

and $ah\sin\theta + bk\tan\theta = a^2 + b^2$

Eliminating h, we have

$$bk\,(\cot\theta\sin\theta - \tan\theta\cos\theta) = (a^2 + b^2)(\sin\theta - \cos\theta)$$

$$\Rightarrow k = -\left(\frac{a^2 + b^2}{b}\right)$$

211 **(b)**

$y = mx + c$ is normal to the parabola

$y^2 = 4ax$ if $c = -2am - am^3$

Here $m = -1$ and $c = k$ and $a = 3$

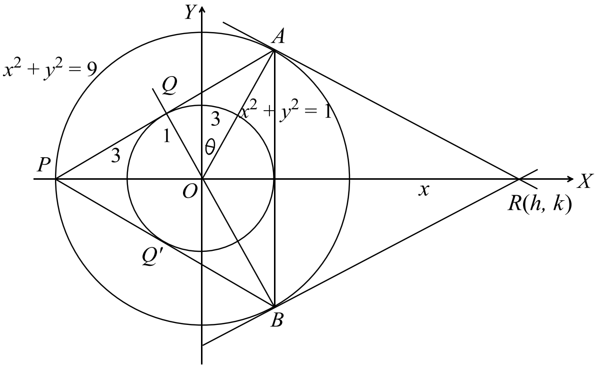
$\therefore\ c = k = -2(3)(-1) - 3(-1)^3$

$= 9$

212 **(a)**

Since ΔPOQ and ΔAOQ are congruent

Hence, $\angle POQ = \angle QOA = \theta$



$\cos\theta = \frac{1}{3}$, since $\angle PQR = 180°$

$\Rightarrow \angle AOR = \pi - 2\theta$

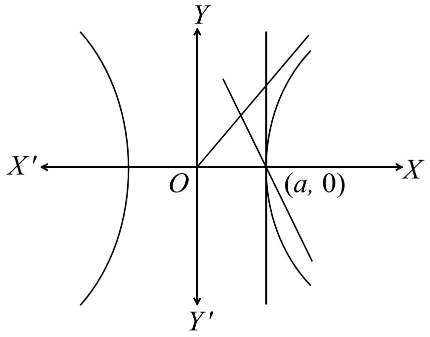
Now in triangle AOR, $\angle AOR = \pi - 2\theta$ and $AO = 3$ unit

$$\Rightarrow \cos(\pi - 2\theta) = \frac{OA}{OR} = \frac{3}{\sqrt{h^2 + k^2}}$$

$$\Rightarrow \sqrt{h^2 + k^2} = \frac{27}{7}$$

$$\Rightarrow x^2 + y^2 = \left(\frac{27}{7}\right)^2$$

213 **(b)**



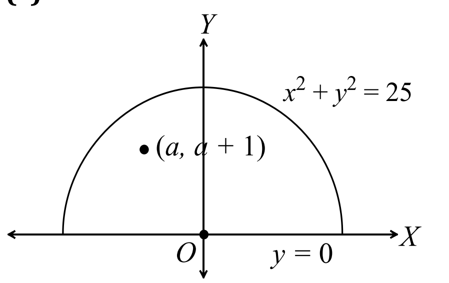
The line $y + \lambda(x - a) = 0$ will intersect the portion of the asymptote in the first quadrant only if its slope is negative. Hence,

$$-\lambda < 0$$

$$\Rightarrow\ \lambda > 0$$

$$\therefore \lambda \in (0, \infty)$$

214 **(c)**



$y = \sqrt{25 - x^2}, y = 0$ bound the semicircle above the x-axis

$\therefore a + 1 > 0$ ...(i)

and $a^2 + (a+1)^2 - 25 < 0 \Rightarrow 2a^2 + 2a - 24 < 0$

$\Rightarrow a^2 + a - 12 < 0$

$\Rightarrow\ -4 < a < 3$ ...(ii)

From Eqs. (i) and (ii)

$-1 < a < 3$

215 **(c)**

Let $C_1(h, k)$ be the centre of the circle

Circle touches the x-axis then its radius is $r_1 = k$

Also circle touches the circle with centre $C_2(0,3)$ and radius $r_2 = 2$

$\therefore |C_1C_2| = r_1 + r_2$

$\Rightarrow \sqrt{(h-0)^2+(k-3)^2} = |k+2|$

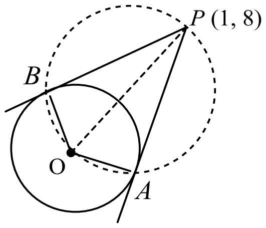
Squaring

$h^2 - 10k + 5 = 0$

$\Rightarrow$ Locus is $x^2 - 10y + 5 = 0$, which is parabola

216 **(b)**

The centre of the given circle is $O(3,2)$



Since, OA and OB are perpendicular to $PA$and PB.

Also, OP is the diameter of the circumcircle of $\triangle PAB$

Its equation is

$(x-3)(x-1)+(y-2)(y-8)=0$

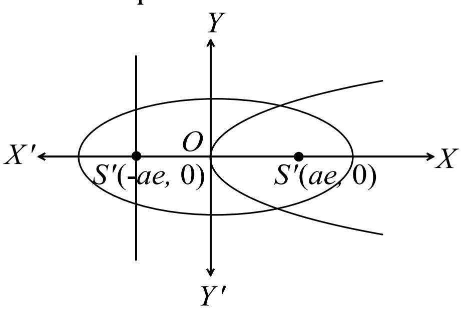
$\Rightarrow \quad x^2+y^2-4x-10y+19=0$

217 **(c)**

Equation of the ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Equation of the parabola with focus $S(ae,0)$ and directrix $x+ae=0$ is $y^2=4aex$

Now length of latus rectum of the ellipse is $\frac{2b^2}{a}$ and that of the parabola is $4ae$.



For the two latus recta to be equal, we get

$$\frac{2b^2}{a}=4ae$$

$$\Rightarrow \frac{2a^2(1-e^2)}{a}=4ae$$

$\Rightarrow 1-e^2=2e$

$\Rightarrow e^2+2e-1=0$

Therefore, $e=-\frac{2\pm\sqrt{8}}{2}=-1\pm\sqrt{2}$

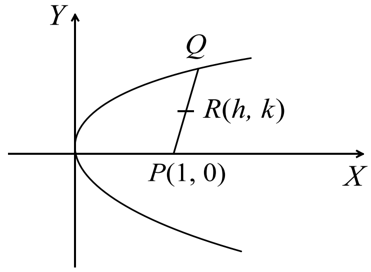
Hence, $e=\sqrt{2}-1$

218 **(b)**

Let $R(h,k)$ be the midpoint of PQ. Therefore, Q is $(2h-1,2k)$

Since Q lies on $y^2=8x$

$\therefore (2k)^2=8(2h-1)$



Hence, locus of $Q(h,k)$ is

$y^2=2(2x-1)$

or $y^2=4x-2$

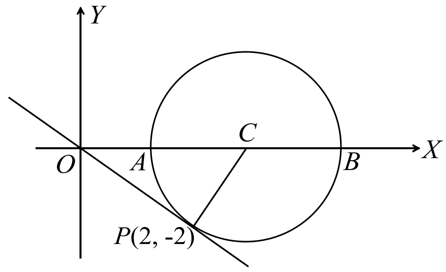
$\Rightarrow y^2-4x+2=0$

219 **(c)**

If $(a,0)$ is the centre C and P is $(2,-2)$, then $\angle COP=45°$

Since the equation of OP is $x+y=0$

$\therefore OP=2\sqrt{2}=CP$. Hence, $OC=4$



The point on the circle with the greatest x coordinates is B

$\alpha=OB=OC+CB=4+2\sqrt{2}$

220 **(d)**

For a focal chord $t_1t_2=-1$ and for the normal $t_1(t_1+t_2)+2=0$

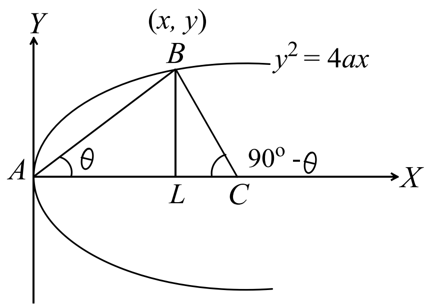
$\therefore t_1^2+t_1t_2+2=0 \Rightarrow t_1^2=-1$

Therefore, t_1 is imaginary

221 **(c)**

$$\tan\theta=\frac{y}{x}$$

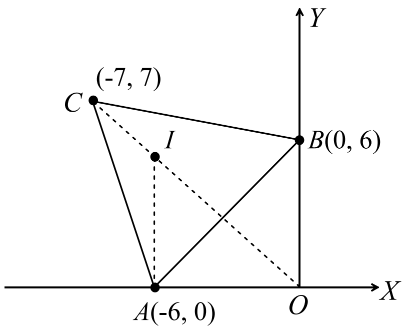
Projection of BC on the x-axis



$$LC=\frac{y}{\tan(90°-\theta)}=y\tan\theta$$

$$\frac{y^2}{x}=4a$$

222 **(b)**



The triangle is evidently isosceles and therefore the median through C is the angle bisector of $\angle C$

The equation of the angle bisector is $y = -x$ and in centre $I = (-a, a)$ where a is positive

Equation of AC is $y - 0 = -7(x + 6)$ or $7x + y + 42 = 0$ and equation of AB is $x - y + 6 = 0$

The length of the perpendicular from I to AB and AC are equal

$\therefore \left|\dfrac{-7a + a + 42}{\sqrt{50}}\right| = \left|\dfrac{-a - a + 6}{\sqrt{2}}\right|$

$\therefore a = \dfrac{9}{2}$ $(\because a > 0)$

$\therefore$ Centre is $\left(-\frac{9}{2}, \frac{9}{2}\right)$ and radius $= \frac{3}{\sqrt{2}}$

$\therefore$ The equation of the circle is $\left(x + \frac{9}{2}\right)^2 + \left(y - \frac{9}{2}\right)^2 = \frac{9}{2}$

$\therefore$ $x^2 + y^2 + 9x - 9y + 36 = 0$

223 **(b)**

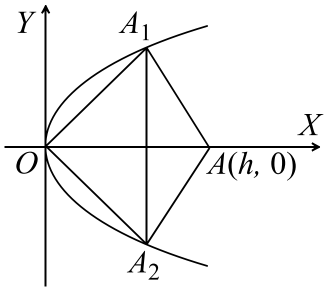
We have $x^2 - y^2 - 4x + 4y + 16 = 0$

$\Rightarrow (x - 2)^2 - (y - 2)^2 = -16$

$\Rightarrow \dfrac{(x - 2)^2}{4^2} - \dfrac{(y - 2)^2}{4^2} = -1$

This is a rectangular hyperbola, whose eccentricity is always $\sqrt{2}$

224 **(c)**



Let $A_1 \equiv (2at_1^2, 4t_1), A_2 \equiv (2t_1^2, -4t_1)$

Clearly, $\angle A_1OA = \frac{\pi}{6}$

$\Rightarrow \dfrac{2}{t_1} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow t_1 = 2\sqrt{3}$

Equation of normal at A_1 is $y = -t_1x + 4t_1 + 2t_1^3$

$\Rightarrow h = 4 + 2t_1^2 = 4 + 2(12) = 28$

225 **(b)**

The slope of the chord is $m = -\frac{8}{y}$

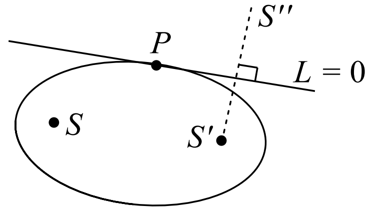
$\Rightarrow y = \pm 1, \pm 2, \pm 4, \pm 8$

But $(8, y)$ must also lie inside the circle $x^2 + y^2 = 125$

$\Rightarrow y$ can be equal to $\pm 1, \pm 2, \pm 4 \Rightarrow 6$ values

226 **(c)**

Equation has three roots, hence three normal can be drawn



Let image of S' be with respect to $x + y - 5 = 0$

$\Rightarrow \dfrac{h - 2}{1} = \dfrac{k + 1}{1} = \dfrac{-2(-4)}{2}$

$\Rightarrow S' = (6, 3)$

Let P be the point of contact

Because the line $L = 0$ is tangent to the ellipse, there exists a point P uniquely on the line such that $PS + PS' = 2a$

Since $PS = PS''$

There exists one and only one point P on $L = 0$ such that $PS + PS' = 2a$

Hence, P should be the collinear with SS''

Hence, P is a point of intersection of SS'' $(4x - 5y=9,$ and $x+y-5=0$ i.e. $P=349,119$

227 **(b)**

Let a pair of tangents be drawn from point (x_1, y_1) to hyperbola

$x^2 - y^2 = 9$

Then chord of contact will be

$xx_1 - yy_1 = 9$ (i)

But the given chord contact is

$x = 9$ (ii)

As Eqs. (i) and (ii) represents the same line, theses equation should be identical and hence

$\dfrac{x_1}{1} = -\dfrac{y_1}{0} = \dfrac{9}{9} \Rightarrow x_1 = 1, y_1 = 0$

Therefore, the equation of pair of tangents drawn from (1, 0) to $x^2 - y^2 = 9$ is

$(x^2 - y^2 - 9)(1^2 - 0^2 - 9) = (x.1 - y.0 - 9)^2$ (using $SS_1 = T^2$)

$\Rightarrow (x^2 - y^2 - 9)(-8) = (x - 9)^2$

$\Rightarrow\ -8x^2 + 8y^2 + 72 = x^2 - 18x + 81$

$\Rightarrow 9x^2 - 8y^2 - 18x + 9 = 0$

229 **(a)**

Let $P(\theta), Q\left(\theta + \frac{2\pi}{3}\right), R\left(\theta + \frac{4\pi}{3}\right)$

Then $P' \equiv\ (a\cos\theta, b\sin\theta)$,

$$Q' \equiv \left(a\cos\left(\theta + \frac{2\pi}{3}\right), b\sin\left(\theta + \frac{2\pi}{3}\right)\right)$$

$$R' \equiv \left(a\cos\left(\theta + \frac{4\pi}{3}\right), b\sin\left(\theta + \frac{4\pi}{3}\right)\right)$$

Let centroid of $\Delta P'Q'R' \equiv\ (x', y')$

$$x' = a\left[\frac{\cos\theta + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{4\pi}{3}\right)}{3}\right]$$

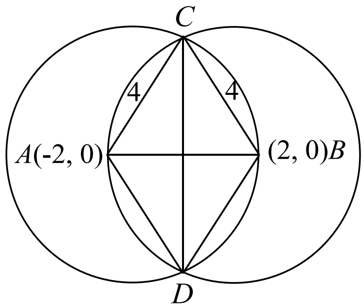
$$= \frac{a}{3}\left[\cos\theta + 2\cos\ (\theta + \pi)\cos\frac{\pi}{3}\right] = 0$$

$$y' \equiv \frac{a}{3}\left[\sin\theta + \sin\left(\theta + \frac{2\pi}{3}\right) + \sin\left(\theta + \frac{4\pi}{3}\right)\right]$$

$$= \frac{a}{3}\left[\sin\theta + 2\sin(\theta + \pi)\sin\frac{\pi}{3}\right] = 0$$

$= 0$

230 **(a)**



Circles with centre (2, 0) and $(-2, 0)$ each with radius 4

$\Rightarrow$ y-axis is their common chord

ΔABC is equilateral. Hence, area of $ADBC$ is

$\frac{2.\sqrt{3}}{4}\ (4)^2 = 8\sqrt{3}$

231 **(c)**

The equation of the line $y = x$ in distance form is

$\frac{x}{\cos\theta} = \frac{y}{\sin\theta} = r$, whose $\theta = \frac{\pi}{4}$

For point P, $r = 6\sqrt{2}$. Therefore, coordinates of P are given by $\frac{x}{\cos\frac{\pi}{4}} = \frac{y}{\sin\frac{\pi}{4}} = 6\sqrt{2}\ \Rightarrow x = 6, y = 6$

Since $P(6, 6)$ lies on $x^2 + y^2 + 2gx + 2fy + c = 0$, therefore

$72 + 12(g + f) + c = 0$ ...(i)

Since $y = x$ touches the circle, therefore the equation $2x^2 + 2x(g + f) + c = 0$ has equal roots

$\Rightarrow 4(g + f)^2 = 8c$

$(g + f)^2 = 2c$ ...(ii)

From (i), we get

$[12(g + f)]^2 = [-(c + 72)]^2$

$\Rightarrow 144(g + f)^2 = (c + 72)^2$

$\Rightarrow 144(2c) = (c + 72)^2$

$\Rightarrow (c - 72)^2 = 0\ \Rightarrow c = 72$

232 **(c)**

We have

$2x^2 + 3y^2 - 8x - 18y + 35 = k$

$\Rightarrow 2(2x^2 - 4x) + 3(y^2 - 6y) + 35 = k$

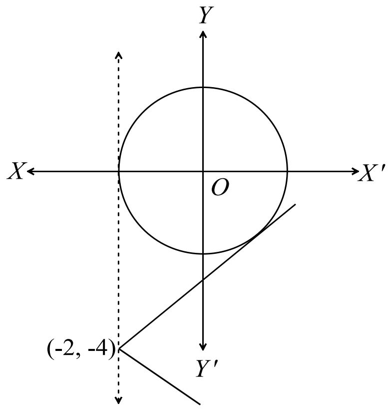
$\Rightarrow 2(x - 2)^2 + 3(y - 3)^2 = k$

For $k = 0$, we get

$2(x - 2)^2 + 3(y - 3)^2 = 0$

Which represents the point (2, 3)

233 **(a)**



Any tangent of $x^2 + y^2 = 4$ is $y = mx \pm 2\sqrt{1 + m^2}$ if it passes through $(-2, -4)$ then

$(2m - 4)^2 = 4(1 + m^2)$

$\Rightarrow 4m^2 + 16 - 16m = 4 + 4m^2$

$\Rightarrow m = \infty, m = \frac{3}{4}$

Hence, slope of reflected ray is $\frac{3}{4}$

Thus, equation of incident ray is $(y + 4) = -\frac{3}{4}(x + 2)$, i.e. $4y + 3x + 22 = 0$

234 **(c)**
Equation of the two circles be $(x-r)^2+(y-r)^2=r^2$
i.e. $x^2+y^2-2rx-2ry+r^2=0$, where $r=r_1$ and r_2. Condition of orthogonality gives
$2r_1r_2+2r_1r_2=r_1^2+r_2^2 \Rightarrow 4r_1r_2=r_1^2+r_2^2$
Circle passes through (a,b)
$\Rightarrow a^2+b^2-2ra-2rb+r^2=0$
i.e. $r^2-2r(a+b)+a^2+b^2=0$
$r_1+r_2=2(a+b)$ and $r_1r_2=a^2+b^2$
$\therefore\ 4(a^2+b^2)=4(a+b)^2-2(a^2+b^2)$
i.e. $a^2-4ab+b^2=0$

235 **(d)**
Two parabolas are equal if the length of their latus rectum are equal
Length of the latus rectum of $y^2=\lambda x$ is λ
The equation of the second parabola is
$25\{(x-3)^2+(y+2)^2\}=(3x-4y-2)^2$
$$\Rightarrow \sqrt{(x-3)^2+(y+2)^2}=\frac{|3x-4y-2|}{\sqrt{3^2+4^2}}$$
Here focus is (3,–2), and equation of the directrix is $3x-4y-2=0$
Therefore, length of the latus rectum = 2×distance between focus and directrix
$=2\left|\frac{3\times3-4\times(-2)-2}{\sqrt{3^2+(-4)^2}}\right|=6$
Thus, the two parabolas are equal, if $\lambda=6$

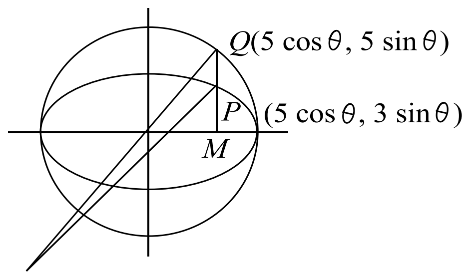
236 **(b)**
The give point is a interior point
$$\Rightarrow \left(-5+\frac{r}{\sqrt{2}}\right)^2+\left(-3+\frac{r}{\sqrt{2}}\right)^2-16<0$$
$\Rightarrow r^2-8\sqrt{2}\,r+18<0$
$\Rightarrow 4\sqrt{2}-\sqrt{14}<r<4\sqrt{2}+\sqrt{14}$ ...(i)
The point is on the major segment
⇒ The centre and the point are on the same side of the line $x+y=2$
$$\Rightarrow -5+\frac{r}{\sqrt{2}}-3+\frac{r}{\sqrt{2}}-2<0$$
$\Rightarrow r<5\sqrt{2}$ ...(ii)
From Eqs. (i) and (ii). $4\sqrt{2}-\sqrt{14}<r<5\sqrt{2}$

237 **(b)**
The chord of contact of tangents from (h,k) is
$\frac{xh}{a^2}+\frac{yk}{b^2}=1$
It meets the axes points $\left(\frac{a^2}{h},0\right)$ and $\left(0,\frac{b^2}{k}\right)$
Area of the triangle $=\frac{1}{2}\times\frac{a^2}{h}\times\frac{b^2}{k}=c$ (constant)
$\Rightarrow hk=\frac{a^2b^2}{2c}$ (c is constant)
$xy=c^2$ is the required locus

238 **(a)**
If (x_i,y_i) is the point of intersection of given curves, then
$\frac{\sum_{j=1}^4 x_i}{4}=\frac{1+1}{2}$ and $\frac{\sum_{j=1}^4 y_i}{4}=0$
Now $\frac{\sum_{i=1}^3 x_i}{3}=\frac{4-x_4}{3}$ and $\frac{\sum_{i=1}^3 y_i}{3}=-\frac{y_4}{4}$
Centroid $\left(\frac{\sum_{j=1}^3 x_i}{3},\frac{\sum_{i=1}^3 y_i}{3}\right)$ lies on the line $y=3x-4$
Hence ,
$$\frac{-y_4}{3}=\frac{3(4-x_4)}{3}-4$$
$\Rightarrow y_4=3x_4$
Therefore, the locus of D is $y=3x$

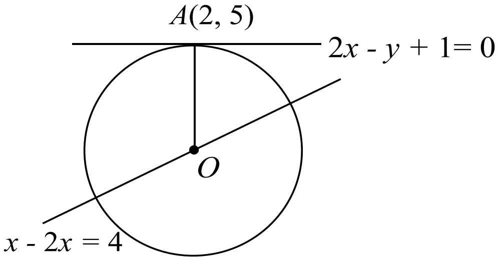
239 **(c)**
Centre of circle is (1, 0) and radius is 1. Line will touch the circle if $|\cos\theta-2|=1 \Rightarrow \cos\theta=1,3$
Thus, $\cos\theta=1 \Rightarrow \theta=2n\pi, n\in I$

240 **(d)**
Given parabola is $y^2=4x+8$ or $y^2=4(x+2)$
Equation of normal to parabola at any point $P(t)$ is $y=-t(x+2)+2t+t^3$
It passes through $(k,0)$ if $tk=t^3 \Rightarrow t(t^2-k)=0$
Hence, it has three real values of t if $k>0$

241 **(c)**
Equation of normal to the ellipse at P is
$5x\sec\theta-3y\ cosec\ \theta=16$ (i)
Equation of normal to the circle $x^2+y^2=25$ at point Q is


$y=x\tan\theta$ (ii)
Eliminating θ from (i) and (ii), we get
$x^2+y^2=64$

242 **(a)**



$2x - y + 1 = 0$ is tangent

Slope of lie $OA = -\frac{1}{2}$

Equation of OA, $(y-5) = -\frac{1}{2}(x-2)$

or $x + 2y = 12$

$\therefore$ Intersection with $x - 2y = 4$ will given coordinates of centre which are $(8, 2)$

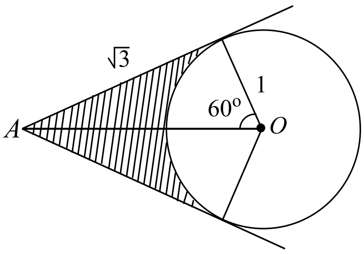
$\therefore\ r = OA = \sqrt{(8-2)^2 + (2-5)^2} = 3\sqrt{5}$

243 **(c)**

The required point is obtained by solving $x + y = 1$ and $y^2 - y + x = 0$

244 **(b)**

Area of quadrilateral $= \sqrt{3}$

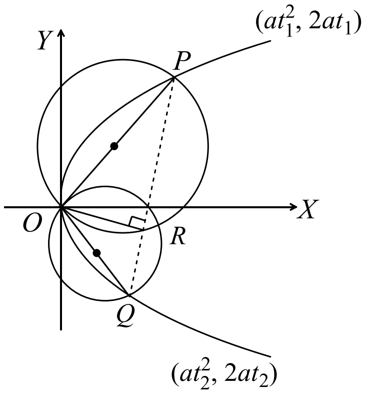


Sector $= \frac{1}{2}.1.\frac{2\pi}{3} = \frac{\pi}{3}$

Shaded region $= \sqrt{3} - \frac{\pi}{3}$

245 **(a)**

Slope of tangent at P is $\frac{1}{t_1}$ and at Q is $\frac{1}{t_2}$



$\Rightarrow \cot\theta_1 = t_1$ and $\cot\theta_2 = t_2$

Slope of $PQ = \frac{2}{t_1 + t_2}$

$\Rightarrow$ Slope of OR is $-\frac{t_1+t_2}{2} = \tan\emptyset$

(Note angle in a semicircle is 90°)

$\Rightarrow \tan\emptyset = -\frac{1}{2}(\cot\theta_1 + \cot\theta_2)$

$\Rightarrow \cot\theta_1 + \cot\theta_2 = -2\tan\emptyset$

246 **(d)**

For the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, equation of director circle is $x^2 + y^2 = 25$. The director circle will cut the ellipse $\frac{x^2}{50} + \frac{y^2}{20} = 1$ at 4 points

Hence, number of points = 4

247 **(d)**

$y^2 + 4y + 4x + 2 = 0$

$y^2 + 4y + 4 = -4x + 2$

$(y+2)^2 = -4(x - 1/2)$

It is of the form $Y^2 = 4AX$ whose directrix is given by $X = A$

$\therefore$ Required equation is $x - \frac{1}{2} = 1$

$\Rightarrow x = \frac{3}{2}$

248 **(c)**

Equation of normal $y = mx - 2am - am^3$

Put $y = 0$, we get

$x_1 = 2a + am_1^2$

$x_2 = 2a + am_2^2$

$x_3 = 2a + am_3^2$

Where x_1, x_2, x_3 are the intercepts on the axis of the parabola

The normal passes through (h, k)

$\Rightarrow\ am^3 + (2a - h)m + k = 0,$

Which has roots m_1, m_2, m_3 which are slopes of the normals

$\Rightarrow m_1 + m_2 + m_3 = 0$

and $m_1m_2 + m_2m_3 + m_3m_1 = \frac{2a-h}{a}$

$\Rightarrow m_1^2 + m_2^2 + m_3^2$

$= (m_1 + m_2 + m_3)^2 - 2(m_1m_2 + m_2m_3 + m_3m_1)$

$= -\frac{2(2a-h)}{a}$

$\Rightarrow\ x_1 + x_2 + x_3 = 6a - 2(2a - h) = 2(h + a)$

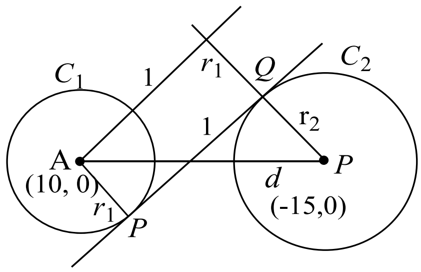
249 **(a)**

Centres are $(10,0)$ and $(-15, 0)$

and radii are $r_1 = 6$; $r_2 = 9$

Also $d = 25$

$r_1 + r_2 < d$



$\Rightarrow$ circles are neither intersecting nor touching

$PQ = \sqrt{d^2 - (r_1 + r_2)^2}$

$= \sqrt{625 - 225}$

$= 20$

250 **(c)**

Equation of hyperbola is

$$\frac{x^2}{9}-\frac{y^2}{16}=1$$

Equation of tangent is

$$y=mx+\sqrt{9m^2-16}$$

$$\Rightarrow \sqrt{9m^2-16}=2\sqrt{5}$$

$$\Rightarrow m=\pm 2$$

$$\Rightarrow a+b=\text{sum of roots}=0$$

251 **(b)**

$$e^2=1+\frac{b^2}{a^2}=1+\frac{\sin^2\alpha}{\cos^2\alpha}=\frac{1}{\cos^2\alpha}$$

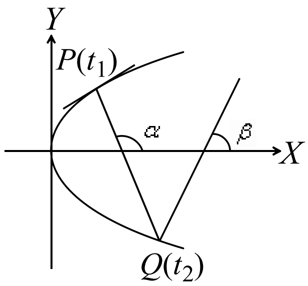
$$a^2=\cos^2\alpha$$

$$\therefore a^2\,e^2=1$$

Hence, the foci are $(\pm ae,0)=(\pm 1,0)$, which are independent of α

252 **(b)**

$\tan\alpha=-t_1$ and $\tan\beta=-t_2$

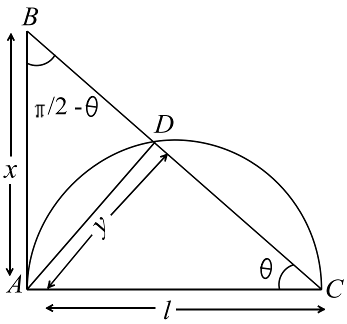


Also $t_2=-t_1-\frac{2}{t_1}$

$t_1t_2+t_1^2=-2$

$\tan\alpha\tan\beta+\tan^2\alpha=-2$

253 **(d)**



ΔABC and ΔDBA are similar

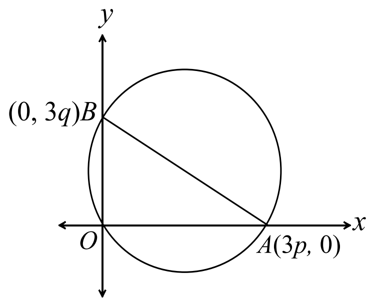
$\Rightarrow l\cdot x=y\sqrt{l^2+x^2}$

$\Rightarrow l^2x^2=y^2(l^2+x^2)$

$\Rightarrow l^2(x^2-y^2)=x^2y^2$

$$\Rightarrow l=\frac{xy}{\sqrt{x^2-y^2}}=\frac{AB.AD}{\sqrt{AB^2-AD^2}}$$

254 **(a)**



Let the centroid of triangle OAB is (p,q)

Hence, points A and B are $(3p,3q)$

But diameter of triangle $AB=6k$

Hence, $\sqrt{9p^2+9q^2}=6k$

Therefore, locus of (p,q) is $x^2+y^2=4k^2$

255 **(c)**

Latus rectum of $y^2=2bx$ is $2b$

Semi=latus rectum is b

We know that semi-latus rectum is H.M. of segments of focal chord

Then $\frac{1}{a}+\frac{1}{c}=\frac{2}{b}$

$$\Rightarrow\; b=\frac{2ac}{a+c}$$

Now for $ax^2+bx+c=0$,

$D=b^2-4ac$

$$=\left(\frac{2ac}{a+c}\right)^2-4ac$$

$$=-4ac\left(\frac{a^2+c^2-ac}{(a+c)^2}\right)<0$$

Hence, roots are imaginary

256 **(a)**

Let (h,k)be the midpoint of the chord

$7x+y-1=0$

$\Longrightarrow \frac{hx}{1}+\frac{ky}{7}=\frac{h^2}{1}+\frac{k^2}{7}$(i)

And $7x+y=1$(ii)

Represents same straight line

$$\Longrightarrow \frac{h}{7}=\frac{k}{7}\Longrightarrow h=k$$

$\Rightarrow$Equation of the line joining $(0,0)$ and (h,k) is $y-x=0$

257 **(b)**

Since $c_1c_2(a_1a_2+b_1b_2),<0$, therefore origin lies in acute angle. $P(1,2)$ lies in obtuse angle

Acute angle between the asymptotes is $\frac{\pi}{3}$. Hence,

$$e=\sec\frac{\theta}{2}=\sec\frac{\pi}{6}=\frac{2}{\sqrt{3}}$$

258 **(d)**
Let the centre of the circle be (h, k)
Since the circle touches the axis of y, therefore radius $= h$
The radius of the circle $x^2 + y^2 - 6x - 6y + 14 = 0$ is 2 and it has its centre at (3,3)
Since the two circles touch each other externally, therefore
Distance between the centres = sum of the radii
$\Rightarrow \sqrt{(h-3)^2 + (k-3)^2} = |h + 2|$
$\Rightarrow k^2 - 10h - 6k + 14 = 0$
Hence, the locus of (h, k) is $y^2 - 10x - 6y + 14 = 0$

259 **(a)**
Any tangent to hyperbola forms a triangle with the asymptotes which has constant area ab

Given

$ab = a^2 \tan\lambda$

$\Rightarrow \frac{b}{a} = \tan\lambda$

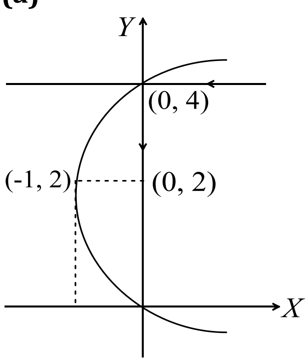
$\Rightarrow e^2 - 1 = \tan^2\lambda$

$\Rightarrow e^2 = 1 + \tan^2\lambda = \sec^2\lambda$

$\Rightarrow e = \sec\lambda$

260 **(d)**
Since $1^2 + 2^2 = 5 < 9$ and $2^2 + 1^1 = 5 < 9$ both P and Q lie inside C. Also $\frac{1^2}{9} + \frac{2^2}{4} = \frac{1}{9} + 1 >$ 1 and $\frac{2^2}{9} + \frac{1}{4} = \frac{25}{36} < 1$. Hence, P lies outside E and Q lies inside$E$. Thus P lies inside C but outside E

261 **(a)**



Given curve is $(y-2)^2 = 4(x+1)$
Focus is (0, 2)
Point of intersection of the curve and $y = 4$ is (0, 4)
From the reflection property of parabola, reflected ray passes through the focus
$\therefore\ x = 0$ is required line

262 **(d)**
Any point on the line $7x + y + 3 = 0$ is $Q(t, -3, -7t), t \in R$
Now $P(h, k)$ is image of point Q in the line $x - y + 1 = 0$
Then, $\frac{h-t}{1} = \frac{k-(-3-7t)}{-1}$
$= -\frac{2(t-(-3-7t)+1)}{1+1}$
$= -8t - 4$
$\Rightarrow\ (h, k) \equiv (-7t - 4, t + 1)$
This point lies on the circle $x^2 + y^2 = 9$
$\Rightarrow (7t-4)^2 + (t+1)^2 = 9$
$\Rightarrow 50t^2 + 58t + 8 = 0$
$\Rightarrow 25t^2 + 29t + 4 = 0$
$\Rightarrow\ (25t + 4)(t + 1) = 0$
$\Rightarrow\ t = -4/25, t = -1$
$\Rightarrow (h, k) \equiv \left(-\frac{72}{25}, \frac{21}{25}\right)$ or $(3, 0)$

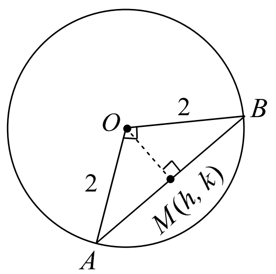
263 **(d)**
Transverse axis is the equation of the angle bisector passing containing point (2, 3), which is given by

$$\frac{3x - 4y + 5}{5} = \frac{12x + 5y - 40}{13}$$

$\Rightarrow 21x + 77y = 265$

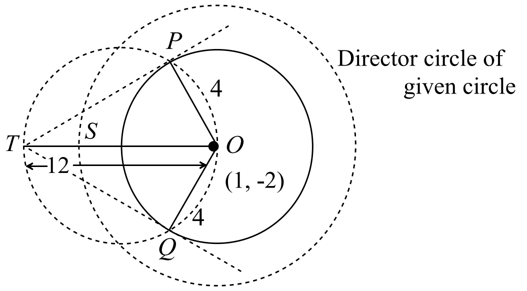
264 **(d)**
The locus is the radical axis which is perpendicular to the line joining the centres of the circles

265 **(c)**
Let AB be the chord with its midpoint $M(h, k)$



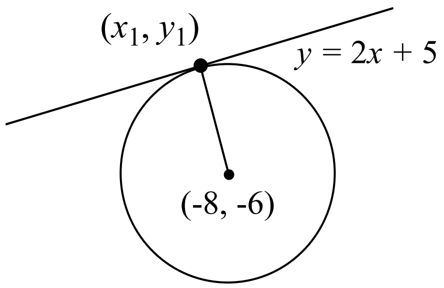
As $\angle AOB = 90°$
$\therefore AB = \sqrt{2^2 + 2^2} = 2\sqrt{2}$
$\therefore AM = \sqrt{2}$
By property of right angled $\Delta, AM = MB = OM$
$\therefore OM = \sqrt{2}\ \Rightarrow h^2 + k^2 = 2$
$\therefore$ Locus of (h, k) is $x^2 + y^2 = 2$

266 **(d)**
Given circle $(x-1)^2 + (y+2)^2 = 16$
Its director circle is $(x-1)^2 + (y+2)^2 = 32$
$\Rightarrow\ OS = 4\sqrt{2}$



Therefore, required distance, $TS = OT - SO = 12 - 4\sqrt{2}$

267 **(d)**



From the figure, we have

$y_1 = 2x_1 + 5$ and $\frac{y_1+6}{x_1+8} \times 2 = -1$

$\Rightarrow x_1 = -6$ and $y_1 = -7$

268 **(a)**

Given that $y\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{x^2 + y^2}$

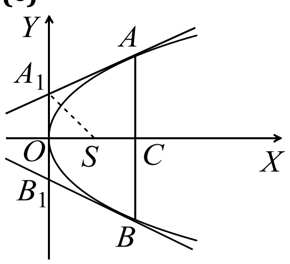
$\Rightarrow y^2\left(\frac{dy}{dx}\right) = x^2$

$\Rightarrow ydy = \pm xdx$

$\Rightarrow y^2 \pm x^2 = k^2$

$\Rightarrow$ family of curves may be a circle or rectangular hyperbola

269 **(c)**



Let $A \equiv (at_1^2, 2at_1)$,

$B \equiv (at_1^2, -2at_1)$

Equation of tangents at A and B are

$t_1y = x + at_1^2$

and $-t_1y = x + at_1^2$, respectively

These tangents meet y-axis at

$A_1 \equiv (0, at_1)$

and $B_1 \equiv (0, -at_1)$

Area of trapezium $AA_1B_1B = \frac{1}{2}(AB + A_1B_1) \times OC$

$\Rightarrow 24a^2 = \frac{1}{2}(4at_1 + 2at_1)(at_1^2)$

$\Rightarrow t_1^3 = 8 \Rightarrow t_1 = 2$

$\Rightarrow A_1 \equiv (0, 2a)$

If $\angle OSA_1 = \theta \Rightarrow \tan\theta = \frac{2a}{a} = 2$

$\Rightarrow \theta = \tan^{-1}(2)$

Thus, required angle is $2\tan^{-1}(2)$

270 **(a)**

Let $A \equiv (at^2, 2at), B \equiv (at^2, -2at)$

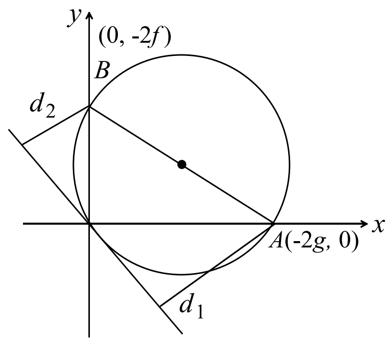
$m_{OA} = \frac{2}{t}, m_{OB} = \frac{-2}{t}$

Thus, $\left(\frac{2}{t}\right)\left(\frac{-2}{t}\right) = -1$

$\Rightarrow t^2 = 4$

Thus, tangents will intersect at $(-4a, 0)$

271 **(c)**



Let the circle be $x^2 + y^2 + 2gx + 2fy = 0$

Tangent at the origin is

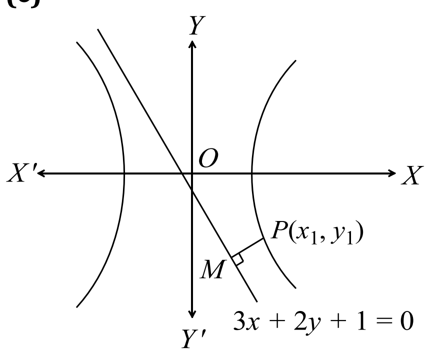
$gx + fy = 0$

$d_1 = \frac{2g^2}{\sqrt{g^2+f^2}}$ and $d^2 = \frac{2f^2}{\sqrt{g^2+f^2}}$

$\Rightarrow d_1 + d_2 = 2\sqrt{g^2 + f^2}$

= diameter of the circle

272 **(c)**



Point P is nearest to the given line if tangent at P is parallel to the given line

Now slope of tangent at $P(x_1, y_1)$ is

$\left(\frac{dy}{dx}\right)_{(x_1,y_1)} = \frac{18y_1}{24x_1} = \frac{3}{4}\frac{y_1}{x_1}$ which must be equal to $-\frac{3}{2}$

$\Rightarrow \frac{3}{4}\frac{y_1}{x_1} = -\frac{3}{2}$

$\Rightarrow y_1 = -2x_1$ (i)

Also(x_1, y_1) lies on the curve. Hence,

$\frac{x_1^2}{24} - \frac{y_1^2}{18} = 1$ (ii)

Solving (i) and (ii) we get two points $(6, -3)$ and $(-6,3)$ of which $(6, -3)$ is nearest

273 **(d)**

Let $B(h, 0)$ is the midpoint of the chord drawn from point $A(p, q)$

Also centre is $C\left(\frac{p}{2}, \frac{q}{2}\right)$

Then, we have $BC \perp AB$

$\Rightarrow \left(\frac{\frac{q}{2} - 0}{\frac{p}{2} - h}\right)\left(\frac{q-0}{p-h}\right) = -1$

$\Rightarrow \left(\frac{q}{p-2h}\right)\left(\frac{q-0}{p-h}\right) = -1$

$\Rightarrow 2h^2 - 3ph + p^2 + q^2 = 0$

Since two such chords exist, the above equation must have two distinct real roots,

$\Rightarrow$ Discriminant > 0

$\Rightarrow 9p^2 - 8(p^2 + q^2) > 0$

$\Rightarrow p^2 > 8q^2$

274 **(c)**

$\frac{x^2}{169} + \frac{y^2}{25} = 1$

Equation of normal at the point $(13\cos\theta, 5\sin\theta)$ is

$\frac{13x}{\cos\theta} - \frac{5y}{\sin\theta} = 144$, it passes through $(0, 6)$

$\Rightarrow (15 + 72\sin\theta) = 0$

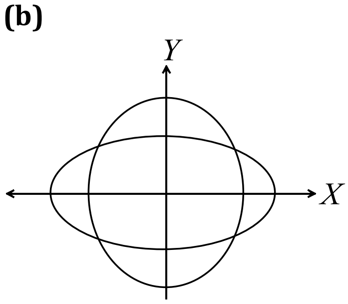
$\Rightarrow \sin\theta = -\frac{5}{24}$

$\Rightarrow \theta = 2\pi - \sin^{-1}\left(\frac{5}{24}\right)$,

and $\pi + \sin^{-1}\frac{5}{24}$

Also y-axis is one of the normals

275 **(b)**



Radius of the circle having SS' as diameter is $r = ae$ If it cuts an ellipse, then $r > b$

$\Rightarrow ae > b$

$\Rightarrow e^2 > \frac{b^2}{a^2}$

$\Rightarrow e^2 > 1 - e^2$

$\Rightarrow e^2 > \frac{1}{2}$

$\Rightarrow e > \frac{1}{\sqrt{2}}$

$\Rightarrow e \in \left(\frac{1}{\sqrt{2}}, 1\right)$

276 **(a)**

Any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ having slope m is

$y = mx + \sqrt{a^2m^2 + b^2}$

Points on the minor axis are $(0, ae)(0, -ae)$

$\therefore$ Sum of squares of the perpendicular on the tangent from $(0, ae)$ and

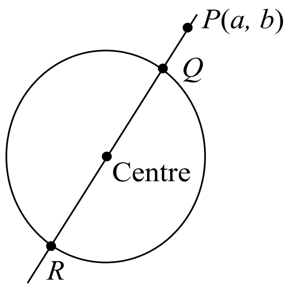
$(0, -ae) = \left[\frac{\sqrt{a^2m^2+b^2} - ae}{\sqrt{m^2+1}}\right]^2 + \left[\frac{\sqrt{a^2m^2+b^2} - ae}{\sqrt{m^2+1}}\right]^2$

$= \frac{2(a^2m^2 + b^2 + a^2e^2)}{m^2 + 1}$

$= \frac{2(a^2m^2 + a^2 - a^2e^2 + a^2e^2)}{m^2 + 1}$

$= \frac{2a^2(m^2 + 1)}{m^2 + 1} = 2a^2$

277 **(a)**



The give circle is $(x + 1)^2 = (y + 2)^2 = 9$, which has radius $= 3$

The points on the circle which are nearest and farthest to the point $P(a, b)$ are Q and R, respectively

Thus, the circle centred at Q having radius PQ will be the smallest circle while the circle centred at R having radius PR will be the largest required circle

Hence, difference between their radii

$= PR - PQ = QR = 6$

278 **(c)**

Locus of the centre of the circle cutting $S_1 = 0$ and $S_2 = 0$ orthogonally is the radical axis between $S_1 = 0$ and $S_2 = 0$, i.e., $S_1 - S_2 = 0$ or

$9x - 10y + 11 = 0$

279 **(b)**

Normal $P(\theta)$ is $\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$ (i)

Normal at $P\left(\frac{\pi}{2}+0\right)$ is $\frac{ax}{\cos\left(\frac{\pi}{2}+\theta\right)}-\frac{by}{\sin\left(\frac{\pi}{2}+\theta\right)}$

$=a^2-b^2$

or $-\frac{ax}{\sin\theta}-\frac{by}{\cos\theta}=a^2-b^2$ (ii)

Equations (i) and (ii) meet major axis at

$G\left(\frac{(a^2-b^2)\cos\theta}{a},0\right)$

and g $\left(-\frac{(a^2-b^2)\sin\theta}{a},0\right)$

Now PG^2+Qg^2

$$=\left(\frac{(a^2-b^2)\cos\theta}{a}-a\cos\theta\right)^2+(0-b\sin\theta)^2$$

$$+\left(-\frac{(a^2-b^2)\sin\theta}{a}-a\sin\theta\right)^2+(0-b\cos\theta)^2$$

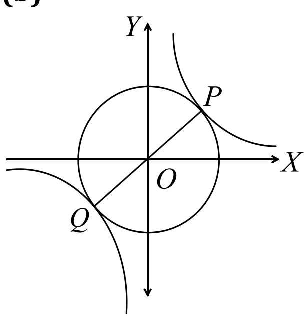
$$=\frac{(a^2-b^2)^2}{a^2}+b^2+a^2$$

$$=a^2\left(\frac{(a^2-b^2)^2}{a^4}+\frac{b^2}{a^2}+1\right)$$

$$=a^2\left(\left(1-\frac{b^2}{a^2}\right)^2+\frac{b^2}{a^2}+1\right)$$

$$=a^2(e^4+2-e^2)$$

280 **(b)**



From the diagram $PQ=$ diameter of the circle $=2$

281 **(a)**

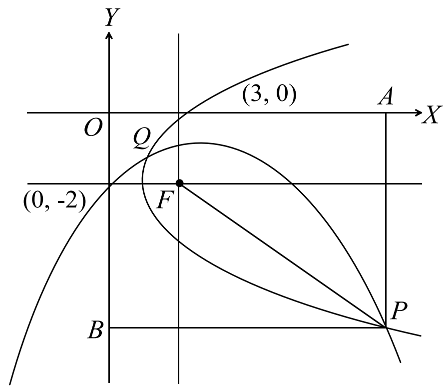
Let the focus be F. The parabolas are open down and open right, respectively. Let the parabolas intersect at points P and Q. From P perpendiculars are drawn on the x-axis and y-axis at A and B, respectively, then

$PA=PF=PB$

$\Rightarrow P$ lies on the line$y=-x$

Similarly, Q lies on the line $y=-x$

$\Rightarrow$ slope of $PQ=-1$



282 **(b)**

The coordinates of the focus and vertex of the required parabola are $S(a_1,0)$ and $A(a,0)$, respectively. Therefore, the distance between the vertex and focus is $AS=a_1-a$ and so the length of the latus rectum $=4(a_1,-a)$.Thus, the equation of the parabola is

$y^2=4(a_1-a)(x-a)$

283 **(c)**

Equation of tangent to the given parabola having slope m is

$y=mx+\frac{1}{m}$ (i)

Equation of tangent to the given circle having slope m is

$y=m(x-3)\pm3\sqrt{1+m^2}$ (ii)

Equations (i) and (ii) are identical,

$\Rightarrow \dfrac{1}{m}=-3m\pm3\sqrt{1+m^2}$

$\Rightarrow 1+3m^2=\pm3m\sqrt{1+m^2}$

$\Rightarrow 1+6m^2+9m^4=9(m^2+m^4)$

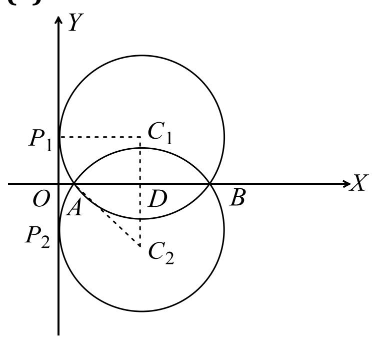
$\Rightarrow 3m^2=1$

$\Rightarrow m=\pm\dfrac{1}{\sqrt{3}}$

Hence, equation of common tangent is

$\sqrt{3}y=x+3$ (as tangent is lying above x-axis)

284 **(a)**



Let $A\equiv(1,0)$, $B\equiv(3,0)$ and C_1,C_2 be the centre of circles passing through A,B and touching the y-axis at P_1 and P_2. If r be the radius of circle (here radius of both circles will be same), $C_1A=C_2A=r=OD=2$ and $C_1\equiv(2,h)$

Where $h^2=AC_1^2-AD^2=4-1=3$

$\Rightarrow C_1 \equiv (2, \sqrt{3}), C_2 \equiv (2, -\sqrt{3})$

If $\angle C_1AC_2 = \theta$

$$\Rightarrow \cos\theta = \frac{AC_1^2 + AC_2^2 - C_1C_2^2}{2AC_1.AC_2} = \frac{1}{2}$$

285 **(a)**

Let P be $(1+\sqrt{2}\cos\theta, \sqrt{2}\sin\theta)$ and C is $(1, 0)$.

Circumcentre of triangle ABC is midpoint of PC

$\Rightarrow 2h = 1 + \sqrt{2}\cos\theta + 1$

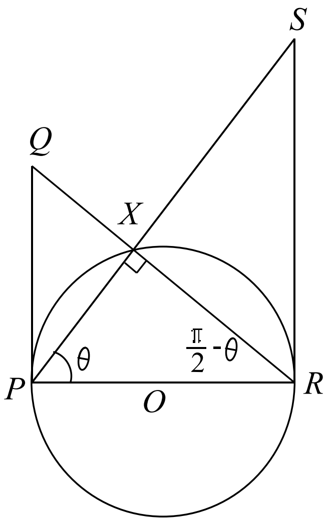
and $2k = \sqrt{2}\sin\theta$

$\Rightarrow [2(h-1)^2] + (2k)^2 = 2$

$\Rightarrow 2(h-1)^2 + k^2 - 1 = 0$

$\Rightarrow 2x^2 + 2y^2 - 4x + 1 = 0$

286 **(a)**



From the above figure, we have $\frac{PQ}{PR} = \tan(\pi/2 - \theta) = \cot\theta$

and $\frac{RS}{PR} = \tan\theta$

$$\Rightarrow \frac{PQ}{PR} . \frac{RS}{PR} = 1$$

$\Rightarrow (PR)^2 = PQ.RS$

$\Rightarrow (2r)^2 = PQ.RS$

$\Rightarrow 2r = \sqrt{PQ.RS}$

287 **(c)**

Since the semi-latus rectum of parabola is the harmonic mean between the segments of any focal chord of a parabola, therefore $SP, 4, SQ$ are in HP

$$\Rightarrow 4 = \frac{2SP.SQ}{SP+SQ}$$

$$\Rightarrow 4 = \frac{2(6)(SQ)}{6+SQ}$$

$\Rightarrow 24 + 4(SQ) = 12(SQ)$

$\Rightarrow SQ = 3$

288 **(c)**

Let $y = mx \pm \sqrt{m^2a^2 - a^2}$ be two tangents and passes through (h, k). Then,

$(k - mh)^2 = m^2a^2 - a^2$

$\Rightarrow m^2(h^2 - a^2) - 2khm + k^2 + a^2 = 0$

$$\Rightarrow m_1 + m_2 = \frac{2kh}{h^2 - a^2}$$

and $m_1m_2 = \frac{k^2+a^2}{h^2-a^2}$

Now, $\tan 45° = \frac{m_1 - m_2}{1 + m_1m_2}$

$$\Rightarrow 1 = \frac{(m_1+m_2)^2 - 4m_1m_2}{(1+m_1m_2)^2}$$

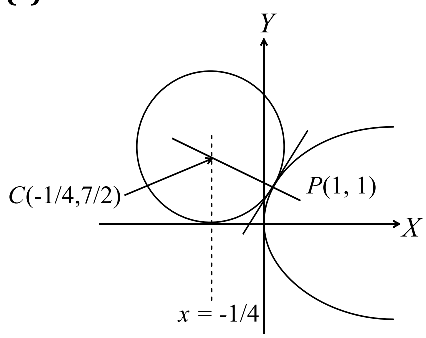
$$\Rightarrow \left(1 + \frac{k^2+a^2}{h^2-a^2}\right)^2 = \left(\frac{2kh}{h^2-a^2}\right)^2 - 4\left(\frac{k^2+a^2}{h^2-a^2}\right)$$

$\Rightarrow (h^2+k^2)^2 = 4h^2k^2 - 4(k^2+a^2)(h^2-a^2)$

$\Rightarrow (x^2+y^2)^2 = 4(a^2y^2 - a^2x^2 + a^4)$

$\Rightarrow (x^2+y^2)^2 + 4a^2(x^2 - y^2) = 4a^2$

289 **(c)**



Equation of normal at $P(1,1)$ is

$y - 1 = -2(x - 1)$

Or $y + 2x = 3$ (i)

Directrix of the parabola $y^2 = x$ is

$x = -\frac{1}{4}$ (ii)

Centre of the circle is intersection of two normals to the circle, i.e., Eqs. (i) and (ii) which is $\left(-\frac{1}{4}, \frac{7}{2}\right)$

Hence, radius of the circle is

$$\sqrt{\left(1+\frac{1}{4}\right)^2 + \left(1 - \frac{7}{2}\right)^2} = \sqrt{\frac{25}{16} + \frac{25}{4}} = \frac{5\sqrt{5}}{4}$$

290 **(b)**

In an equilateral triangle, circumcentre and in centre are coincident

$\therefore$ Incentre $= (-g, -f)$

$\Rightarrow 1^2 + 1^2 + 2g + 2f + c = 0$

$\Rightarrow c = -2(g + f + 1)$

Also, in an equilateral triangle,

Circumradius $= 2 \times$ inradius

Therefore, inradius $= \frac{1}{2} \times \sqrt{g^2 + f^2 - c}$ it is continuous single word

$\therefore$ The equation of the incircle is

$(x + g)^2 + (y + f)^2 = \frac{1}{4}(g^2 + f^2 - c)$

$= \frac{1}{4}\left(g^2 + f^2 + 2(g + f + 1)\right)$

291 **(d)**

Let $y = mx + c$, intersect $y^2 = 4ax$ at $A(at_1^2, 2at_1)$ and $B(at_2^2, 2at_2)$

Then, $\frac{2}{t_1+t_2} = m$

$\Rightarrow\ t_1 + t_2 = \frac{2}{m}$

Let the foot of another normal be $C(at_3^2, 2at_3)$

Then,

$t_1 + t_2 + t_3 = 0$

$\Rightarrow\ t_3 = -(t_1 + t_2) = -\frac{2}{m}$

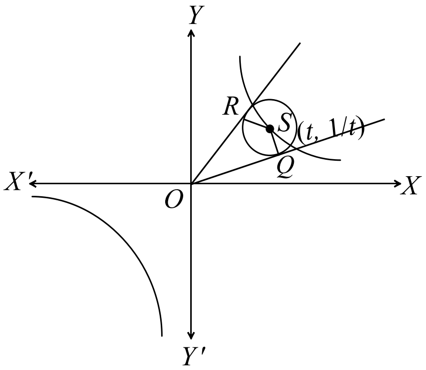
Thus, other foot is $\left(\frac{4a}{m^2}, \frac{-4a}{m}\right)$

292 **(d)**

Tangents $y = m_1x + c$ and $y = m_2x + c$ intersect at $(0, c)$ which lies on the directrix of the given parabola

Hence, tangents are perpendicular for which, $m_1m_2 = -1$

293 **(b)**



Let S be a point on the rectangular hyperbola $\left[\text{say}\left(t, \frac{1}{t}\right)\right]$

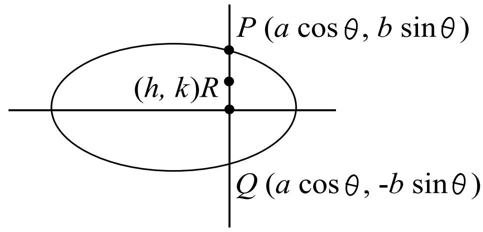
Now, circumcircle of ΔOQR also passes through S

Therefore, circumcentre is the midpoint of OS. Hence,

$x = \frac{t}{2}, y = \frac{1}{2t}$

So, the locus of the circum centre is $xy = \frac{1}{4}$

294 **(a)**



Let $P(a\cos\theta, b\sin\theta), Q(a\cos\theta, -b\sin\theta)$

$PR: RQ = 1: 2$

Therefore , $h = a\cos\theta$

$\Rightarrow \cos\theta = \frac{h}{a}$ (i)

and $k = \frac{b}{3} = \sin\theta$

$\Rightarrow \sin\theta = \frac{3k}{b}$ (ii)

On squaring and adding Eqs. (i) and (ii), we get

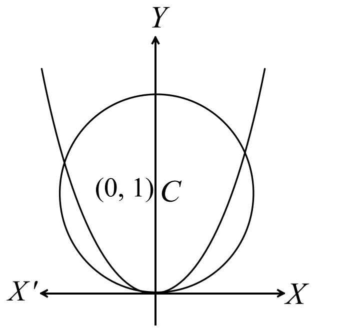
$\frac{x^2}{a^2} + \frac{9y^2}{b^2} = 1$

295 **(d)**

Put $x^2 = \frac{y}{a}$ in circle, $x^2 + (y - 1)^2 = 1$, we get

(Note that for $a < 0$ they cannot intersect other than origin) $\frac{y}{a} + y^2 - 2y = 0$. Hence, we get $y = 0$ or $y = 2 - \frac{1}{a}$

Substituting $y = 2 - \frac{1}{a}$ in $y = ax^2$, we get



$ax^2 = 2 - \frac{1}{a}$

$\Rightarrow x^2 = \frac{2a - 1}{a^2} > 0$

$\Rightarrow a > \frac{1}{2}$

296 **(c)**

Normal at point $P(t_1)$ meets the parabola again at point $R(t_3)$, then

$t_3 = -t_1 - \frac{2}{t_1}$

Also normal at point $Q(t_2)$ meets the parabola at the same point $R(t_3)$, then

$t_3 = -t_2 - \frac{2}{t_2}$

Comparing these values of t_3, we have

$-t_1 - \frac{2}{t_1} = -t_2 - \frac{2}{t_2}$ or $t_1t_2 = 2$

297 **(c)**

Foci of hyperbola lie on $y = x$. So, the major axis is $y = x$

Major axis of hyperbola bisects the asymptote

$\Rightarrow$ equation of other asymptote is $x = 2y$

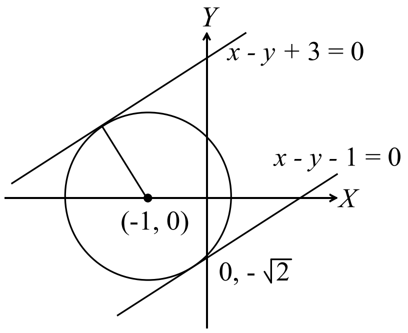
$\Rightarrow$ equation of hyperbola is $(y - 2x)(x - 2y) + k = 0$

Given that it passes through $(3, 4) \Rightarrow k = 10$

Hence required equation is

$2x^2 + 2y^2 - 5xy + 10 = 0$

298 **(d)**



$x^2 + y^2 + 2x - 1 = 0$

Centre $(-1, 0)$ and radius $= \sqrt{2}$

Line $x - y + c = 0$ must be tangent to the circle

$\Rightarrow \left|\dfrac{-1 + c}{\sqrt{2}}\right| = \sqrt{2}$

$\Rightarrow |c - 1| = 2$

$\Rightarrow c - 1 = \pm 2$

$\Rightarrow c = 3$ or -1

$\Rightarrow c = 1$ ($\because$ for $c = 3$ there will be infinite points common lying inside circle)

299 **(b)**

Tangent to $y^2 = 4x$ in terms of 'm' is

$y = mx + \frac{1}{m}$

Normal to $x^2 = 4by$ in terms of 'm' is

$y = mx + 2b + \dfrac{b}{m^2}$

If these are same lines, then

$\dfrac{1}{m} = 2b + \dfrac{b}{m^2}$

$\Rightarrow 2bm^2 - m + b = 0$

For two different tangents

$1 - 8b^2 > 0$

$\Rightarrow |b| < \dfrac{1}{\sqrt{8}}$

300 **(a)**

Since the normal at $(ap^2, 2ap)$ to $y^2 = 4ax$ meets the parabola at $(aq^2, 2aq)$,

$\therefore q = -p - \frac{2}{p}$ (i)

Since $OP \perp OQ$,

$\therefore \dfrac{2ap - 0}{ap^2 - 0} \times \dfrac{2aq - 0}{aq^2 - 0} = -1 \Rightarrow pq = -4$

$\Rightarrow p\left(-p - \frac{2}{p}\right) = -4$ [Using(i)]

$\Rightarrow p^2 = 2$

301 **(c)**

For given slope there exists two parallel tangents to ellipse. Hence, there are two values of c

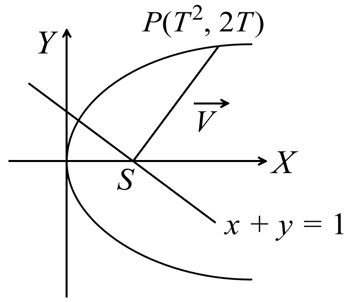
302 **(c)**

$\vec{V} = (T^2 - 1)\hat{\imath} = 2T\hat{\jmath}$

$\vec{n} = \hat{\jmath} - \hat{\imath}$

Projection of $\vec{V}$ on $\vec{n}$

$y = \dfrac{\vec{V}.\vec{n}}{|\vec{n}|} = \dfrac{(1 - T^2) + 2T}{\sqrt{2}}$



Given $\frac{dx}{dt} = 4$; but $x = T^2$

$\Rightarrow \dfrac{dx}{dt} = 2T\dfrac{dT}{dt}$

When $P(4, 4)$ then $T = 2$

$\Rightarrow 4 = 2(2)\dfrac{dT}{dt}$

$\Rightarrow \dfrac{dT}{dt} = 1$

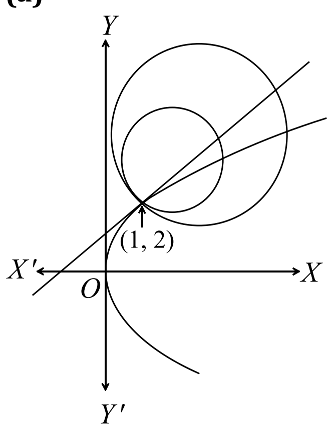
Now $\frac{dy}{dt} = \left(\frac{-2T+2}{\sqrt{2}}\right)\frac{dT}{dt}$

Therefore, at $T = 2$,

$\sqrt{2}\dfrac{dy}{dt} = -4 + 2 = -2$

$\Rightarrow \dfrac{dy}{dt} = -\sqrt{2}$

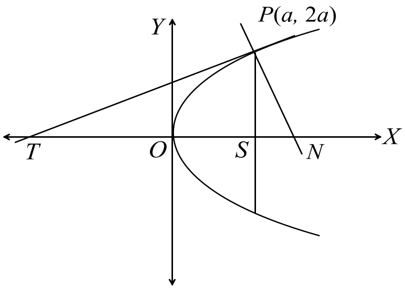
303 **(a)**



Tangent to parabola $y^2 = 4x$ at(1,2) will be the locus

i.e., $2y = 2(x+1)$
$\Rightarrow y = x + 1$

304 **(c)**



One end of the latus rectum, $P(a, 2a)$
The equation of the tangent PT at $P(a, 2a)$ is
$2ya = 2a(x+a)$, i.e.,$y = x + a$
The equation of normal PN at $P(a, 2a)$ is
$y + x = 2a + a$, i.e.,$y + x = 3a$
Solving $y = 0$ and $y = x + a$, we get
$x = -a, y = 0$
Solving $y = 0, y + x = 3a$, we get
$x = 3a, y = 0$
The area of the triangle with vertices
$P(a, 2a), T(-a, 0), N(3a, 0)$is $4a^2$

306 **(d)**
The equation of the normal to the hyperbola
$\frac{x^2}{4} - \frac{y^2}{1} = 1$ at $(2\sec\theta, \tan\theta)$ is $2x\cos\theta + y\cot\theta = 5$

Slope of the normal is $-2\ \sin\theta = -1$

$$\Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$$

Y-intercept of the normal $= \frac{5}{\cot\theta} = \frac{5}{\sqrt{3}}$

As it touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

We have $a^2 + b^2 = \frac{25}{9}$

307 **(b)**
Let $(\alpha, 3-\alpha)$ be any point on $x + y = 3$
$\therefore$ Equation of chord of contact is $\alpha x + (3-\alpha)y = 9$
i.e. $\alpha(x-y) + 3y - 9 = 0$
$\therefore$ The chord passes through the point $(3, 3)$ for all values of α

308 **(a)**
Let the given straight line be axis of coordinates and let the equation of the variable line be

$$\frac{x}{a} + \frac{y}{b} = 1$$

This line cuts the coordinate axis at the point $A(a, 0)$ and $B(0, b)$

Therefore, the area of ΔAOB is

$$\frac{1}{2}ab = c^2$$

$\Rightarrow ab = 2c^2$ (i)

If (h, k) be the coordinates of the middle point of AB, then

$h = \frac{a}{2}$ and $k = \frac{b}{2}$ (ii)

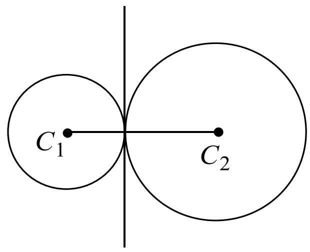
On eliminating a and b from Eqns. (i) and (ii), we get

$2hk = c^2$

Hence, the locus of (h, k) is $2xy = c^2$

309 **(c)**
Let $A = (\alpha, \beta)$
The normal at $(at^2, 2at)$ is $y = -tx + 2at + at^3$
$\therefore at^3 + (2a-\alpha)t - \beta = 0$ (i)
Let t_1, t_2, t_3 be roots of Eq.(i), then
$at^3 + (2a-\alpha)t - \beta = a(t-t_1)(t-t_2)(t-t3$ (ii)
Let $P = (at_1^2, 2at_1), Q = (at_2^2, 2at_2)$, and
$R = (at_3^2, 2at_3)$
Since the focus S is $(a, 0)$
$\therefore SP = a(t_1^2 + 1)$
Similarly, $SQ = a(t_2^2 + 1)$,
and $SR = a(t_3^2 + 1)$
Put $t = i$
$= \sqrt{-1}$ in Eq. (ii), we have
$-ai + (2a-\alpha)i - \beta = a(i-t_1)(i-t_2)(i-t_3)$
$\Rightarrow |(a-\alpha)i - \beta| = a|(i-t_1)(i-t_2)(i-t_3)|$
$\Rightarrow \sqrt{(a-\alpha)^2 + \beta^2} = a\sqrt{1+t_1^2}\sqrt{1+t_2^2}\sqrt{1+t_3^2}$
$\Rightarrow a\sqrt{(a-\alpha)^2 + \beta^2}$
$= \sqrt{a + at_1^2}\sqrt{a + at_2^2}\sqrt{a + at_3^2}$
$\Rightarrow aSA^2 = SP \cdot SQ \cdot SR$

310 **(d)**



$C_1C_2 = r_1 + r_2$
$C_1 = (0,0); C_2 = (3\sqrt{3}, 3)$
and $r_1 = 2, r_2 = 4$
$\Rightarrow$ Circles touch each other externally
Equation of common tangent is $\sqrt{3}x + y - 4 = 0$
...(i)
Comparing it with $x\cos\theta + y\sin\theta = 2$, we get
$\theta = \frac{\pi}{6}$

311 **(c)**
Let point of intersection be (α, β)
Therefore, chord of contact w.r.t. this point is
$\beta y = 2x + 2\alpha$
Which is same as $x + y = 2$
$\Rightarrow \alpha = \beta = -2$
Which satisfy $y - x = 0$

312 **(b)**
For hyperbola

$$\frac{x^2}{5} - \frac{y^2}{5\cos^2\alpha} = 1$$

We have $e_1^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{5\cos^2\alpha}{5}$

$= 1 + \cos^2\alpha$

For ellipse

$$\frac{x^2}{25\cos^2\alpha} + \frac{y^2}{25} = 1$$

We have

$$e_2^2 = 1 - \frac{25\cos^2\alpha}{25} = \sin^2\alpha$$

Given that

$e_1 = \sqrt{3}e_2$

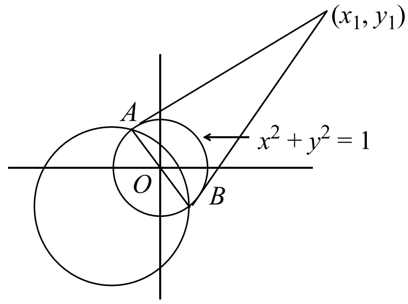
$\Rightarrow e_1^2 = 3e_2^2$

$\Rightarrow 1 + \cos^2\alpha = 3\sin^2\alpha$

$\Rightarrow 2 = 4\sin^2\alpha$

$\Rightarrow \sin\alpha = \frac{1}{\sqrt{2}}$

313 **(a)**
Locus of point of intersection of tangents chord of contact of (x_1, y_1) w.r.t.
$x^2 + y^2 = 1$ is $xx_1 + yy_1 = 1$ (AB) ...(i)
AB is also common chord between two circles
$\therefore -1 + (\lambda + 6)x - (8 - 2\lambda)y + 3 = 0$



$\Rightarrow (\lambda + 6)x - (8 - 2\lambda)y + 2 = 0$ ...(ii)
Comparing Eqs. (i) and (ii), we get

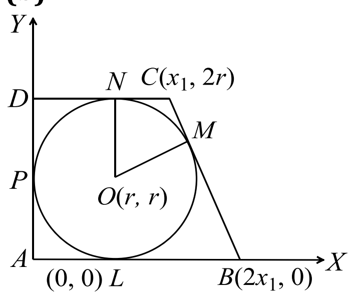
$$\frac{x_1}{\lambda + 6} = \frac{y_1}{2\lambda - 8} = \frac{-1}{2}$$

Eliminate $\lambda \Rightarrow 2x - y + 10 = 0$ which is required locus

314 **(d)**
We have $a = 1$
Normal at $(m^2, -2m)$ is $y = mx - 2m - m^3$
Given that normal makes equal angle with axes, then its slope $m = \pm 1$
Therefore, point P is $(m^2, -2m) = (1, \pm 2)$

315 **(d)**
The coordinates of the focus of the parabola $y^2 - 4ax$ are$(a, 0)$. The line $y - x - a = 0$ pass through this point. Therefore, it is a focal chord of the parabola. Hence, the tangent intersects at right angle

316 **(b)**



Here $\frac{1}{2} \times 3x_1 \times 2r = 18$
$\Rightarrow x_1 \times r = 6$ ...(i)
Equation of BC is: $y = -\frac{2r}{x_1}(x - 2x_1)$
BC is tangent to the circle $(x - r)^2 + (y - r)^2 = r^2$
$\therefore$ Perpendicular distance of BC from centre = radius

$$\Rightarrow \frac{\left|r + \frac{2r}{x_1}(r - 2x_1)\right|}{\sqrt{1 + \frac{4r^2}{x_1^2}}} = r$$

$$\Rightarrow \frac{2r^2}{x_1} - 3r = r\sqrt{1 + 4\frac{r^2}{x_1^2}}$$

$\Rightarrow (2r - 3x_1)^2 = x_1^2 + 4r^2$
$\Rightarrow r.x_1 = \frac{2}{3}x_1^2 \Rightarrow 3r = 2x_1$ ...(ii)

From Eqs. (i) and (ii), $r = 2$ units

317 **(b)**

Let $y_1 = 5 + \sqrt{1 - x_1^2}$ and $y_2 = \sqrt{4x_2}$ or
$x_1^2 + (y_1 - 5)^2 = 1$ and $y_2^2 = 4x_2$
Thus (x_1, y_1) lies on the circle $x^2 + (y - 5)^2 = 1$ and (x_2, y_2) lies on the parabola $y^2 = 4x$
Thus, given expression is the shortest distance between the curves $x^2 + (y - 5)^2 = 1$ and $y^2 = 4x$
Now the shortest distance always occur along common normal to the curves and normal to circle passes through the centre of the circle
Normal to the parabola $y^2 = 4x$ is $y = mx - 2m - m^3$ passes through (0,5) gives
$m^3 + 2m + 5 = 0$, which has only one root $m = -2$
Hence, corresponding point on the parabola is (4, 4)
Thus, required minimum distance$= \sqrt{4^2 + 8^2} - 1 = 4\sqrt{5} - 1$

318 **(a)**

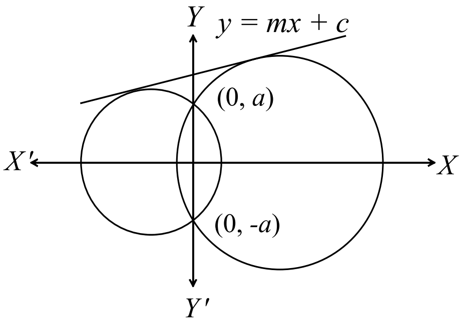
The family of parabolas is $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$and the vertex is $A\left(\frac{-B}{2A}, \frac{-D}{4A}\right) = (h, k)$

$$\Rightarrow\ h = -\frac{\frac{a^2}{2}}{2\frac{a^3}{3}} = -\frac{3}{4a}$$

and $k = \dfrac{\left(\frac{a^2}{2}\right)^2 - \frac{4a^3(-2a)}{3}}{4\frac{a^3}{3}}$

$\Rightarrow h = -\frac{3}{4a}$ and $k = -\frac{35a}{16}$
Eliminating a, we have hk =105/64
Hence, the required locus is $xy = 105/64$

319 **(c)**



Equation of family circles through $(0, a)$ and $(0, -a)$ is
$[x^2 + (y - a)(y + a)] + \lambda x = 0, \lambda \in R$
$\Rightarrow x^2 + y^2 + \lambda x - a^2 = 0$
and $\sqrt{\left(\frac{\lambda}{2}\right)^2 + a^2} = \frac{-\frac{m\lambda}{2} + c}{\sqrt{1+m^2}}$

$$\Rightarrow (1 + m^2)\left[\frac{\lambda^2}{4} + a^2\right] = \left(\frac{m\lambda}{2} - c\right)^2$$

$$\Rightarrow (1 + m^2)\left[\frac{\lambda^2}{4} + a^2\right] = \frac{m^2\lambda^2}{4} - mc\lambda + c^2$$

$\Rightarrow \lambda^2 + 4mc\lambda + 4a^2(1 + m^2) - 4c^2 = 0$
$\therefore\ \lambda_1\lambda_2 = 4[a^2(1 + m^2) - c^2]$
$\Rightarrow g_1g_2 = [a^2(1 + m^2) - c^2]$
and $g_1g_2 + f_1f_2 = \frac{c_1 + c_2}{2}$
$\Rightarrow a^2(1 + m^2) - c^2 = -a^2$
Hence, $c^2 = a^2(2 + m^2)$

320 **(d)**

Ends of latus rectum are $P(a, 2a)$ and $P'(a, 2a)$
Point P has parameter $t_1 = 1$ and point P' has parameter $t_2 = -1$
Normal at point P meets the curve again at point Q whose parameter $t_1' = -t_1 - \frac{2}{t_1} = -3$
Normal at point P' meets the curve again at point Q' whose parameter $t_2' = -t_2 - \frac{2}{t_2} = 3$
Hence, point Q and Q' have coordinates $(9a, -6a)$ and $(9a, 6a)$, respectively
Hence, $QQ' = 12a$

321 **(a)**

Equation of AB is

322 **(a)**

If there are more than one rational points on the circumference of the circle $x^2 + y^2 - 2\pi x - 2ey + c = 0$ (as (π, e) is the centre), then e will be a rational multiple of π, which is not possible. Thus, the number of rational points on the circumference of the circle is at most one

323 **(c)**

$$P\left(a \sec\frac{\pi}{2}, b\tan\frac{\pi}{6}\right) \equiv P\left(\frac{2a}{\sqrt{3}}, \frac{b}{\sqrt{3}}\right)$$

Therefore, equation of tangent at P is

$$\frac{x}{\frac{\sqrt{3}a}{2}} - \frac{y}{\sqrt{3}\,b} = 1$$

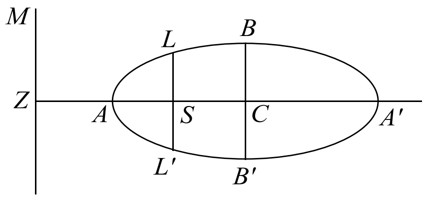
$\therefore$ Area of triangle $= \frac{1}{2} \times \frac{\sqrt{3}a}{2} \times \sqrt{3}b = 3a^2$

$$\therefore \frac{b}{a} = 4$$

$$\therefore e^2 = 1 + \frac{b^2}{a^2} = 17$$

324 **(b)**

Let S be the given focus and ZM be the given line



Then $SZ = \frac{a}{e} - ae$

$= \frac{a}{e}(1-e^2)$

$= \frac{b^2}{ae} = k(\text{say})$

as $b^2 = a^2(1-e^2)$

Now take SC as x-axis and LSL' as y-axis. Let (x, y) be the coordinates of B with respect to these axes, then $x = SC = ae, y = CB = b$

Hence, $\frac{y^2}{x} = \frac{b^2}{ae} = SZ$, which is constant

$\therefore\ y^2 = kx$ is the required locus which is a parabola

325 **(a)**

Let the equation of asymptotes by

$2x^2 + 5xy + 2y^2 + 4x + 5y + \lambda = 0 \quad \text{(i)}$

This equation represents a pair of straight lines

Therefore, $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

We have

$$4\lambda + 25 - \frac{25}{2} - 8 - \lambda\frac{25}{4} = 0$$

$$\Rightarrow\ -\frac{9\lambda}{4} + \frac{9}{2} = 0$$

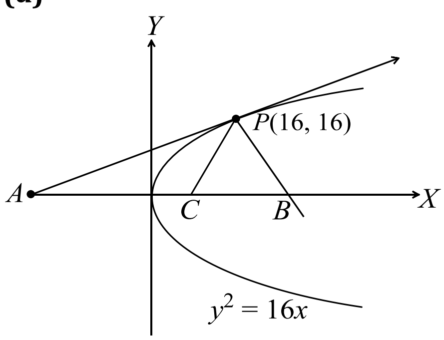
$$\Rightarrow \lambda = 2$$

Putting the value of λ in(i), we get

$2x^2 + 5xy + 2y^2 + 4x + 5y + 2 = 0$

This is the equation of the asymptotes

326 **(d)**



By property centre of circle coincides with focus of parabola

$\Rightarrow C \equiv (4,0)$

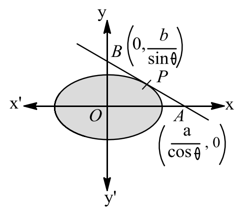
$\tan\alpha = \text{slope of } PC = \frac{16}{12}$

$\Rightarrow \alpha = \tan^{-1}\left(\frac{4}{3}\right)$

327 **(a)**

Equation of tangent at $P(a\cos\theta, b\sin\theta)$ is

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$$



Whose point of intersection of axes are

$$A\left(\frac{a}{\cos\theta}, 0\right) \text{ and } B\left(0, \frac{b}{\sin\theta}\right)$$

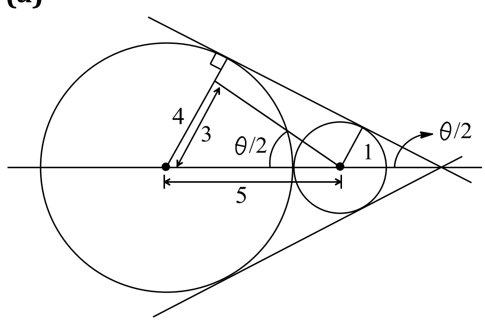
$$\therefore \text{Area of } \Delta AOB = \frac{1}{2}\left|\frac{a}{\cos\theta} \cdot \frac{b}{\sin\theta}\right|$$

$$\Delta = \frac{ab}{|\sin 2\theta|}$$

Now area is minimum when $|\sin 2\theta|$ is maximum *ie*, $|\sin 2\theta| = 1$

$\therefore\ \Delta_{\text{minimum}} = ab$

328 **(a)**



$$\sin\frac{\theta}{2} = \frac{3}{5}$$

and $\cos\frac{\theta}{2} = \frac{4}{5}$

$$\therefore \sin\theta = 2.\frac{3}{5}.\frac{4}{5} = \frac{24}{25}$$

329 **(a)**

The points are such that one of the points is the orthocentre of the triangle formed by other three points. When the vertices of a triangle lie on a rectangular hyperbola the orthocentre also lies on the same hyperbola

330 **(d)**

Eliminating θ from the given equations, we get

$y^2 = -4a(x-a)$;
Which is a parabola but $0 \le \cos^2\theta \le 1$
$\Rightarrow 0 \le x \le a$
and $-1 \le \sin\theta \le 1$
$\Rightarrow -2a \le y \le 2a$
Hence, the locus of the point P is not exactly the parabola, rather it is a part of the parabola

331 **(b)**
Let AB be a normal chord where $\equiv (at_1^2, 2at_1)$, $B \equiv (at_2^2, 2at_2)$. If it's midpoint is $P(h,k)$, then
$2h = a(t_1^2 + t_2^2)$
$= a[(t_1+t_2)^2 - 2t_1t_2]$
and $2k = 2a(t_1+t_2)$
We also have
$$t_2 = -t_1 - \frac{2}{t_1}$$
$\Rightarrow t_1 + t_2 = \frac{-2}{t_1}$ and $t_1t_2 = -t_1^2 - 2$
$\Rightarrow t_1 = -\frac{2a}{k}$ and $h = a\left(t_1^2 + 2 + \frac{2}{t_1^2}\right)$
Thus, required locus is $x = a\left(\frac{4a^2}{y^2} + 2 + \frac{y^2}{2a^2}\right)$

332 **(a)**
Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be the given points and $x^2 + y^2 = a^2$ be the circle
The chord of contact of tangents drawn from $P(x_1, y_1)$ to $x^2 + y^2 = a^2$ is $xx_1 + yy_1 = a^2$
If it passes through $Q(x_2, y_2)$, then
$x_1x_2 + y_1y_2 = a^2$ ...(i)
The equation of the circle on PQ as diameter is
$(x - x_1(x - x_2) + (y - y_1)(y - y_2) = 0$
$$\Rightarrow x^2 + y^2 - x(x_1 + x_2) - y(y_1 + y_2) + x_1x_2 + y_1y_2 = 0$$
This circle will cut the given circle orthogonally, if
$0(x_1 + x_2) + 0(y_1 + y_2) = -a^2 + x_1x_2 + y_1y_2$
$\Rightarrow x_1x_2 + y_1y_2 - a^2 = 0$, which is true by Eq. (i)

333 **(d)**
Let the foot of perpendicular from $O(0,0)$ to tangent to hyperbola is $P(h,k)$. Slope of $OP = \frac{k}{h}$

Then equation of tangent to hyperbola is
$$y - k = -\frac{h}{k}(x - h)$$
or $hx + ky = h^2 + k^2$

Solving it with $xy = 1$, we have
$$hx + \frac{k}{x} = h^2 + k^2$$
or $hx^2 - (h^2 + k^2)x + k = 0$

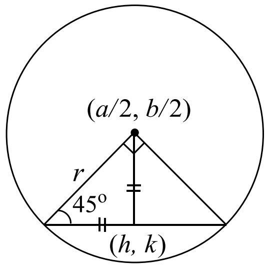
This equation must have real and equal roots. Hence,

$D = 0$

$\Rightarrow (h^2 + k^2)^2 - 4hk = 0$

$\Rightarrow (x^2 + y^2)^2 = 4xy$

334 **(c)**



$$r = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = \frac{\sqrt{a^2 + b^2}}{2}$$
$$\sin 45° = \frac{\sqrt{\left(h - \frac{a}{2}\right)^2 + \left(h - \frac{b}{2}\right)^2}}{\frac{\sqrt{a^2+b^2}}{2}}$$
$$\Rightarrow \frac{1}{2} = 4\left[\frac{\frac{(2h-a)^2}{4} + \frac{(2k-b)^2}{4}}{a^2 + b^2}\right]$$
Simplify to get locus $x^2 + y^2 - ax - by - \frac{a^2+b^2}{8} = 0$

335 **(b)**
The line $2y = \mathrm{g}x + \alpha$ should pass through $(-\mathrm{g}, -\mathrm{g})$, so $-2\mathrm{g} = -\mathrm{g}^2 + \alpha \Rightarrow \alpha = \mathrm{g}^2 - 2\mathrm{g} = (\mathrm{g} - 1)^2 - 1 \ge -1$

336 **(c)**
Substituting $y = mx$ in the equation of circle we get $x^2 + m^2x^2 = ax + bmx + c = 0$ (y/x denotes the slope of the tangent from the origin on the circle)

Since line is touching the circle, we must have discriminant
$\Rightarrow (a + bm)^2 - 4c(1 + m^2) = 0$
$\Rightarrow a^2 + b^2m^2 + 2abm - 4c - 4cm^2 = 0$
$\Rightarrow m^2(b^2 - 4c) + 2abm + a^2 - 4c = 0$
This equation has two roots m_1 and m_2
$$\Rightarrow m_1 + m_2 = -\frac{2ab}{b^2 - 4c} = \frac{2ab}{4c - b^2}$$

337 **(c)**
Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points and $x^2 + y^2 = a^2$ be the given circle. Then, the chord of contact of tangents drawn from P to the given circle is $xx_1 + yy_1 = a^2$

It will pass through $Q(x_2, y_2)$, if

$x_1x_2 + y_1y_2 = a^2$ ...(i)

Now, $l_1 = \sqrt{x_1^2 + y_1^2 - a^2}$,

$l_2 = \sqrt{x_2^2 + y_2^2 - a^2}$

and $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 + y_1)^2}$

$= \sqrt{(x_2^2 + y_2^2) + (x_1^2 + y_1^2) - 2(x_1x_2 + y_1y_2)}$

$\therefore\ PQ = \sqrt{[(x_2^2 + y_2^2) + (x_1^2 + y_1^2) - 2a^2]}$

[Using Eq. (i)]

$\Rightarrow\ PQ = \sqrt{(x_1^2 + y_1^2 - a^2) + (x_2^2 + y_2^2 - a^2)}$

$\Rightarrow\ PQ = \sqrt{l_1^2 + l_2^2}$

338 **(c)**

For equation $S + K = 0$ to represent a pair of lines,

$$\begin{vmatrix} 1 & 2 & -2 \\ 2 & 3 & 1 \\ -2 & 1 & 1+k \end{vmatrix} = 0$$

$\Rightarrow 3(1 + k) - 1 - 2(2 + 2k + 2) - 2(2 + 6) = 0$

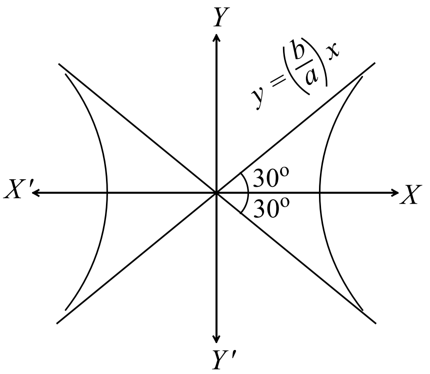
$\Rightarrow k = -22$

339 **(d)**

Product of perpendiculars drawn from foci upon any of its tangents = 9

$\Rightarrow b^2 = 9$

Also $\frac{b}{a} = \tan 30° = \frac{1}{\sqrt{3}}$



$\therefore a^2 = 3b^2 = 27$

Therefore, required locus is the director circle of the hyperbola which is given by $x^2 + y^2 = 27 - 9$

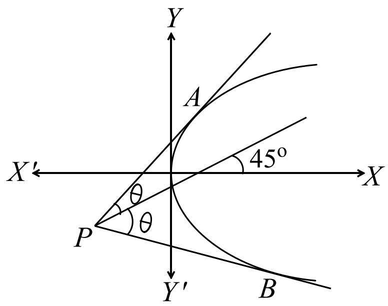
or $x^2 + y^2 = 18$

If $\frac{b}{a} = \tan 60°$, then

$$a^2 = \frac{b^2}{3} = \frac{9}{3} = 3$$

Hence, the required locus is $x^2 + y^2 = 3 - 9 = -6$ which is not possible

340 **(d)**



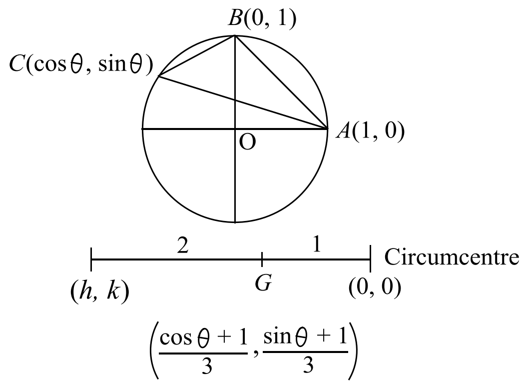
Here $\frac{1}{t_1} = \tan\left(\frac{\pi}{4} + \theta\right)$

and $\frac{1}{t_2} = \tan\left(\frac{\pi}{4} - \theta\right)$

So, $t_1t_2 = 1$

$\Rightarrow$the x-coordinate of $P = at_1t_2 = a$

341 **(c)**



Let C $(\cos\theta, \sin\theta)$; $H(h, k)$ is the orthocenter of the ΔABC

Since circumcentre of the triangle is (0, 0), for orthocenter $h = 1 + \cos\theta$ and $k = 1 + \sin\theta$

Eliminating θ, $(x - 1)^2 + (y - 1)^2 = 1$

$\therefore\ x^2 + y^2 - 2x - 2y + 1 = 0$

342 **(a)**

Let $P(x, y)$ be the position of the man at any time.

Let $S(4, 0)$ and $S'(-4, 0)$ be the fixed flag, post, with Cas the origin

Since $SP + S'P = 10$ m i.e., a constant, the locus of P is an ellipse with S and S' as foci

$\Rightarrow ae = 4$, and $2a = 10$

$\Rightarrow e = \frac{4}{5}$

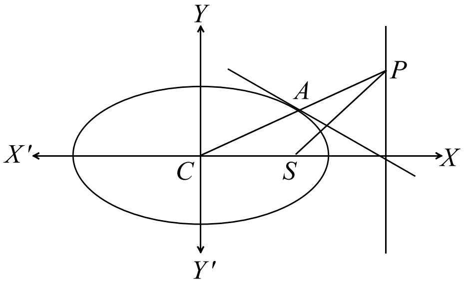
Now $b^2 = a^2(1 - e^2)$

$\Rightarrow b^2 = 25\left(1 - \frac{16}{25}\right) = 9$

$\Rightarrow b = 3$

Hence, the area of the ellipse $= \pi ab = \pi \times 5 \times 3 = 15\pi$

343 **(b)**



Let the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

and let $A\equiv(a\cos\theta, b\sin\theta)$

Equation of AC will be $y=\frac{b}{a}\tan\theta x$

Solving with $x=\frac{a}{e}$, we get

$P\equiv\left(\frac{a}{e},\frac{b}{e}\tan\theta\right)$

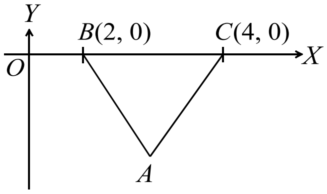
Slope of tangent at A is $-\frac{b}{a\tan\theta}$

Slope of PS

$$=\frac{\frac{b}{e}\tan\theta}{\frac{a}{e}-ae}=\frac{b\tan\theta}{a(1-e^2)}=\frac{a}{b}\tan\theta$$

So $\alpha=\frac{\pi}{2}$

344 **(a)**



Clearly, $A\equiv(3,-\sqrt{3})$

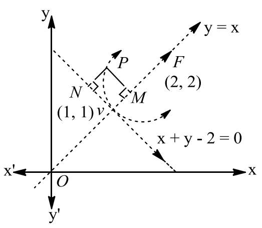
Centroid of triangle ABC is $\left(3,-\frac{1}{\sqrt{3}}\right)$, thus equation of incircle is

$$(x-3)^2+\left(y+\frac{1}{\sqrt{3}}\right)^2=\frac{1}{3}$$

$$\Rightarrow x^2+y^2-6x+\frac{2y}{\sqrt{3}}+9=0$$

345 **(a)**

As distance of vertex from origin is $\sqrt{2}$ and focus is $2\sqrt{2}$



$\therefore V(1,1)$ and $F(2,2)$ (ie, lying on $y=x$)

Length of latusrectum $=4a=4\sqrt{2}$ [where $a=\sqrt{2}$]

$\therefore$ By definition of parabola

$$PM^2=(4a)(PN)$$

where, PN is length of perpendicular upon $x+y-2=0$ (ie, tangent at vertex)

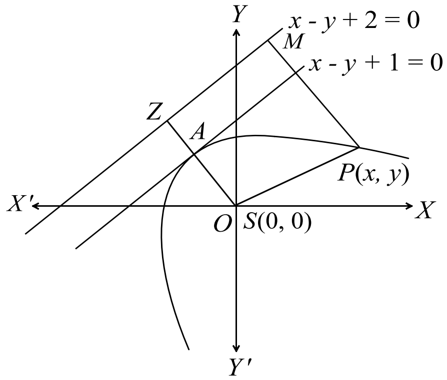
$$\Rightarrow \frac{(x-y)^2}{2}=4\sqrt{2}\left(\frac{x+y-2}{\sqrt{2}}\right)$$

$$\Rightarrow (x-y)^2=8(x+y-2)$$

346 **(c)**

Tangent at the vertex is

$x-y+1=0$ (i)



Therefore, equation of axis the parabola is

$x+y=0$ (ii)

Now solving Eqs. (i) and (ii), we get $A\left(-\frac{1}{2},\frac{1}{2}\right)$

$\therefore$ Z is $(-1,1)$

Now directrix is

$x-y+k=0$

But this passes through $Z(-1,1)$

$\Rightarrow k=2$

$\Rightarrow$ Directrix is $x-y+2=0$

Therefore, by definition equation of parabola is given by

$OP=PM$

$\Rightarrow OP^2=PM^2$

$$\left(\frac{x-y+2}{\sqrt{2}}\right)^2=x^2+y^2$$

$\Rightarrow (x-y+2)^2=2x^2+2y^2$

$\Rightarrow x^2+y^2+4-2xy+4x-4y=2x^2+2y^2$

$\Rightarrow x^2+y^2+2xy-4x+4y-4=0$

347 **(c)**

Fourth vertex of parallelogram lies on circumcircle

$\Rightarrow$ Parallelogram is cyclic

$\Rightarrow$ Parallelogram is a rectangle

$\Rightarrow$ Tangents are perpendicular

$\Rightarrow$ Locus of P is the director circle

349 **(c)**

Slopes of asymptotes are

$m_1 = \frac{b_1}{a_1}, m_2 = \frac{b_2}{a_2}$

According to the question,

$m_1 m_2 = -1$

$\Rightarrow a_1 a_2 + b_1 b_2 = 0$

350 **(a)**

Combined equation of pair of lines through the origin joining the points of intersection of line $y = \sqrt{m}x + 1$ with the given curve is $x^2 + 2xy + (2 + \sin\alpha)y^2 - \left(y - \sqrt{m}x\right)^2 = 0$

For the chord to subtend a right angle at the origin $(1 - m) + (2 + \sin^2\alpha - 1) = 0$ (as sum of the coefficients of $x^2 + y^2 = 10$)

$\Rightarrow \sin^2\alpha = m - 2$

$\Rightarrow 0 \le m - 2 \le 1$

$\Rightarrow 2 \le m \le 3$

351 **(a)**

The midpoint is the intersection of the chord and perpendicular line to it from the centre $(3, -1)$

The equation of perpendicular line is

$5x + 2y - 13 = 0$. Solving this with the given line, we get the point $(1, 4)$

352 **(c)**

$\alpha^2 + 1 - 4 < 0$

$\Rightarrow \ \alpha^2 < 3, |\alpha| < \sqrt{3}$

$\Rightarrow \ 1 - 4\alpha < 0$

$\Rightarrow \alpha > \frac{1}{4}$

353 **(d)**

As in above question point of intersection is

$(h, k) \equiv \left(\frac{a\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}, \frac{b\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)} \right)$

It is given that $\alpha + \beta = c =$ constant

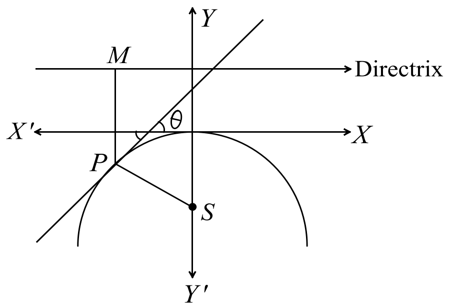
$\Rightarrow h = \frac{a\cos\frac{c}{2}}{\cos\left(\frac{\alpha-\beta}{2}\right)}$ and $k = \frac{b\sin\frac{c}{2}}{\cos\left(\frac{\alpha-\beta}{2}\right)}$

$\Rightarrow \frac{h}{k} = \frac{a}{b}\cot\left(\frac{c}{2}\right)$

$\Rightarrow k = \frac{b}{a}\tan\left(\frac{c}{2}\right) h$

$\Rightarrow (h, k)$ lies on the straight line

354 **(b)**



Slope of line $\lambda = \tan\theta$

$\Rightarrow \tan(\angle MPS) = \tan 2\left(\frac{\pi}{2} - \theta\right) = \tan(\pi - 2\theta)$

$= -\tan 2\theta = \frac{2\lambda}{\lambda^2 - 1}$

355 **(a)**

Let ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and circle $x^2 + y^2 = a^2e^2$

Radius of circle$= ae$

Point of intersection of circle and ellipse is$\left[\frac{a}{e}\sqrt{2e^2 - 1}, \frac{a}{e}(1 - e^2)\right]$

Now area of ΔPF_1F_2

$= \frac{1}{2}\begin{vmatrix} \frac{a}{e}\sqrt{2e^2 - 1} & \frac{a}{e}(1 - e^2) & 1 \\ ae & 0 & 1 \\ -ae & 0 & 1 \end{vmatrix} = \frac{1}{2}\left|\frac{a}{e}(1 - \right.$

e22ae=30

$\Rightarrow a^2(1 - e^2) = 30$ (given)

$\Rightarrow a^2e^2 = a^2 - 30 = \left(\frac{17}{2}\right)^2 - 30 = \frac{169}{4}$

$\Rightarrow 2ae = 13$

356 **(a)**

Director circle of circle $x^2 + y^2 = a^2$ is $x^2 + y^2 = 2a^2$

The semi-transverse axis is $\sqrt{3}a$

Radius of the circle is $\sqrt{2}a$

Hence, director circle and hyperbola do not intersect

357 **(d)**

Let the equation of circle be

$x^2 + y^2 - 4 + k(2x + y - 1) = 0$

Where k is a real number

Radius $= \sqrt{\frac{5k^2}{4} + 4 + k}$

Radius is minimum when $k = -\frac{2}{5}$

$\therefore$ The required equation will be

$5x^2 + 5y^2 - 4x - 2y - 18 = 0$

358 **(b)**

Transverse axis is along the line $y = x$

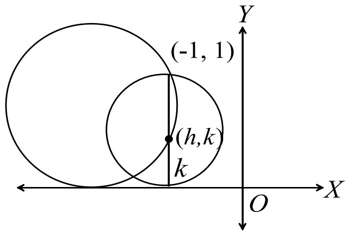
Solving $y = x$ and $xy = 18$, we have $x^2 = 18$ or

$x = \pm 3\sqrt{2}$

Then two vertices of the hyperbola are $(\pm 3\sqrt{2}, \pm 3\sqrt{2})$

Distance between them $= \sqrt{72 + 72} = 12$

359 **(a)**



From the figure, $k \geq \frac{1}{2}$

360 **(d)**

The given parabolas are

$y^2 = 4ax$ (i)

and $x^2 = 4ay$ (ii)

From Eq. (ii),

$y = \dfrac{x^2}{4a}$

Putting in Eq.(i),

$\frac{x^4}{16a^2} = 4ax$

$\Rightarrow x = 0$ or $x = 4a$

When $x = 0, y = 0,$

and when $x = 4a, y = \frac{16a^2}{4a} = 4a$

Thus, Eqs. (i)and meet at (0,0) and $(4a, 4a)$

Now $2bx + 3cy + 4d = 0$

Passes through $(4a, 4a)$ and (0,0)

$\Rightarrow d = 0$

and $2b(4a) + 3c(4a) = 0$

$\Rightarrow 2b + 3c = 0$

$\Rightarrow d^2 + (2b + 3c)^2 = a^2$

361 **(a)**

According to the question $\frac{2\sqrt{9m^2-49}}{\sqrt{1+m^2}} = 2$

$\Rightarrow 9m^2 - 49 = 1 + m^2$

$\Rightarrow 8m^2 = 50$

$\Rightarrow m = \pm \dfrac{5}{2}$

362 **(c)**

Let x_1, x_2 and x_3 be the abscissae of the points on the parabola whose ordinates are $y_1, y_2$and y_3, respectively. Then $y_1^2 = 4ax_1, y_2^2 = 4ax_2$ and $y_3^2 = 4ax_3$. Therefore, the area of the triangle whose vertices are $(x_1, y_1)(x_2, y_2)$ and is (x_3, y_3) is

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} \frac{y_1^2}{4a} & y_1 & 1 \\ \frac{y_2^2}{4a} & y_2 & 1 \\ \frac{y_3^2}{4a} & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{8a}\begin{vmatrix} y_1^2 & y_1 & 1 \\ y_2^2 & y_2 & 1 \\ y_3^2 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{8a}(y_1 - y_2)(y_2 - y_3)$$

$(y_3 - y_1)$

363 **(b)**

Let the coordinates of A, B and C be $(x_1, y_2), (x_2, y_2)$ and (x_3, y_3), respectively. Then, the chords of contact of tangents drawn from A, B and C are

$xx_1 + yy_1 = a^2$, $xx_2 + yy_2 = a^2$ and $xx_3 + yy_3 = a^2$, respectively. These three lines will be concurrent, if

$$\begin{vmatrix} x_1 & y_1 & -a^2 \\ x_2 & y_2 & -a^2 \\ x_3 & y_3 & -a^2 \end{vmatrix} = 0$$

$$\Rightarrow -a^2 \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

$\Rightarrow$ Points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) are collinear

364 **(b)**

Here $a^2 + 2 > a^2 + 1$

$\Rightarrow a^2 + 1 = (a^2 + 2)(1 - e^2)$

$\Rightarrow a^2 + 1 = (a^2 + 2)\dfrac{5}{6}$

$\Rightarrow 6a^2 + 6 = 5a^2 + 10$

$\Rightarrow a^2 = 10 - 6 = 4$

$\Rightarrow a = \pm 2$

Latus rectum $= \frac{2(a^2+1)}{\sqrt{a^2+2}} = \frac{2\times 5}{\sqrt{6}} = \frac{10}{\sqrt{6}}$

365 **(a)**

$AB = \sqrt{a^2 + b^2}$

Hence, $D = \sqrt{b^2 + a^2}$ ...(i)

Now, $\frac{d}{2} = \frac{\Delta}{s} = \frac{ab}{2s}$ (where s is semiperimeter)

$\therefore \dfrac{d}{2} = \dfrac{ab}{a + b + \sqrt{a^2 + b^2}}$

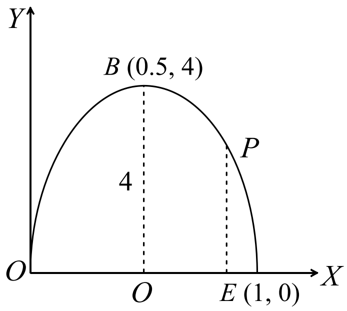
or $d = \frac{2ab}{a+b+\sqrt{a^2+b^2}}$ ...(ii)

From Eqs. (i) and (ii)

$$d+D=\frac{\sqrt{a^2+b^2}\left[(a+b)+\sqrt{a^2+b^2}\right]+2ab}{a+b+\sqrt{a^2+b^2}}$$

$$=\frac{(a+b)^2+(a+b)\sqrt{a^2+b^2}}{a+b+\sqrt{a^2+b^2}}=a+b$$

366 **(c)**



The path of the water jet is a parabola

Let its equation be

$y=ax^2+bx+c$

It should pass through (0,0),(0.5,4),(1,0)

$\Rightarrow c=0, a=-16, b=16$

$\Rightarrow y=-16x^2+16x$

If $x=0.75$,we get $y=3$

367 **(c)**

$(x-3)^2+(y+1)^2=(4x+3y)^2$

$\Rightarrow (x-3)^2+(y+1)^2=25\left(\frac{4x+3y}{5}\right)^2$

$\Rightarrow PS=5PM$

$\Rightarrow$ directrix is $4x+3y=0$ and focus $(3,-1)$

So equation of transverse axis is $y+1=\frac{3}{4}(x-3)$

$\Rightarrow 3x-4y=13$

368 **(a)**

Given hyperbola is

$$\frac{x^2}{1}-\frac{y^2}{\frac{1}{3}}=1$$

Its eccentricity 'e' is given by

$$\frac{1}{3}=1(e^2-1)$$

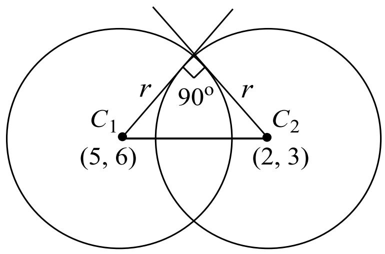
Hence, eccentricity e' of the conjugate hyperbola is given by

$$1=\frac{1}{3}(e'^2-1)$$

$\Rightarrow e'^2=4$

$\Rightarrow e'=2$

369 **(b)**



$2r^2=3^2+3^2=18 \Rightarrow r^2=9$

$\Rightarrow r=3$

370 **(d)**

Equation of tangent to given parabola having slope m is

$$y=m(x+a)+\frac{a}{m}$$

Or $y=mx+am+\frac{a}{m}$

Comparing Eq. (i) with $y=mx+c$, we have

$$c=am+\frac{a}{m}$$

371 **(d)**

Since locus of the point of intersection of the tangent at the end points of a focal chord is directrix

$\therefore$ Required locus is $x=\pm\frac{a}{e}$, which is pair of straight lines

372 **(c)**

The equation of the normal to the given ellipse at the point $P(a\cos\theta, b\sin\theta)$ is $ax\sec\theta -$ by cosec θ=a2−b2

$\Rightarrow y=\left(\frac{a}{b}\tan\theta\right)x-\frac{(a^2-b^2)}{b}\sin\theta$ (i)

Let $\frac{a}{b}\tan\theta=m$, so that

$$\sin\theta=\frac{bm}{\sqrt{a^2+b^2m^2}}$$

Hence, the equation of the normal Eq. (i) becomes

$$y=mx-\frac{(a^2-b^2)m}{\sqrt{a^2+b^2m^2}}$$

$\therefore m\in R$, as $m=\frac{a}{b}\tan\theta\in R$

373 **(b)**

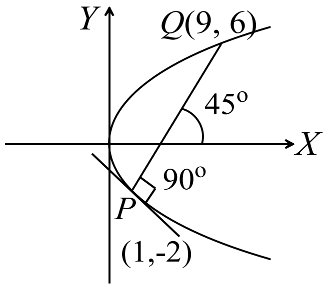
Normal at point $(t^2, 2t)$ is $y=-tx+2t+t^3$

Slope of the tangent is 1

Hence, $-t=1\Rightarrow t=-1$

$\Rightarrow$ Coordinates of P are $(1,-2)$

Hence, parameter at Q is $t_2=-t_1-2/t_1=1+2=3$



Therefore, coordinates at Q are (9,6)

$\therefore l(PQ)=\sqrt{64+64}=8\sqrt{2}$

374 **(a)**

Let the equation of the ellipse be

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

We know that the general equation of the tangent to the ellipse is

$y=mx\pm\sqrt{a^2m^2+b^2}$ (i)

Since $3x-2y-20=0$ or $y=\frac{3}{2}x-10$ is tangent to the ellipse

Comparing with Eq. (i), $m=\frac{3}{2}$ and $a^2m^2+b^2=100$

$\Rightarrow a^2\times\frac{9}{4}+b^2=100$

$\Rightarrow 9a^2+4b^2=400$ (ii)

Similarly, since $x+6y-20=0$, i.e., $y=-\frac{1}{6}x+\frac{10}{3}$

is tangent to the ellipse, therefore comparing with Eq. (i),

$m=\frac{1}{6}$ and $a^2m^2+b^2=\frac{100}{9}$

$\Rightarrow \frac{a^2}{36}+b^2=\frac{100}{9}$

$\Rightarrow a^2+36b^2=400$ (iii)

Solving Eqs. (ii) and (iii), we get $a^2=40$ and $b^2=40$ and $b^2=10$

Therefore, the required equation of the ellipse is

$\frac{x^2}{40}+\frac{y^2}{10}=1$

375 **(b)**

Equation of conic through point of intersection of given two ellipse is

$\left(\frac{x^2}{4}+y^2-1\right)+\lambda\left(\frac{x^2}{a^2}+y^2-1\right)=0$

$\Rightarrow x^2\left(\frac{1}{4}+\frac{\lambda}{a^2}\right)+y^2(1+\lambda)=1+\lambda$

$\Rightarrow x^2\left(\frac{a^2+4\lambda}{4a^2(1+\lambda)}\right)+y^2=1$

This equation is a circle if $\frac{a^2+4\lambda}{4a^2(1+\lambda)}=1$

$\Rightarrow$ Circle is $x^2+y^2=1$

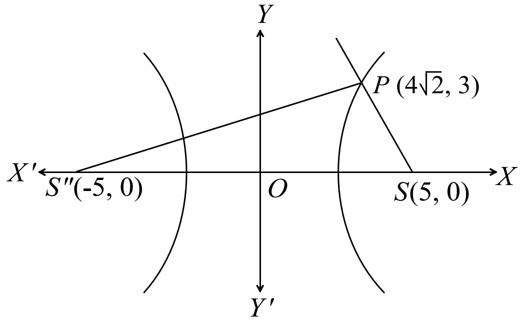
376 **(d)**

The given hyperbola is

$\frac{x^2}{16}-\frac{y^2}{9}=1$

$\Rightarrow e=\frac{5}{4}$

Its foci are $(\pm 5, 0)$



The ray is incident at $P(4\sqrt{2}, 3)$

The incident ray passes through (5, 0); so the reflected ray must pass through $(-5,0)$

Its equation is $\frac{y-0}{x+5}=\frac{3}{4\sqrt{2}+5}$

or $3x-y(4\sqrt{2}+5)+15=0$

377 **(d)**

Normals to $y^2=4ax$ and $x^2=4by$ in terms of 'm' are

$y=mx-2am-am^3$

and $y=mx+2b+\frac{b}{m^2}$

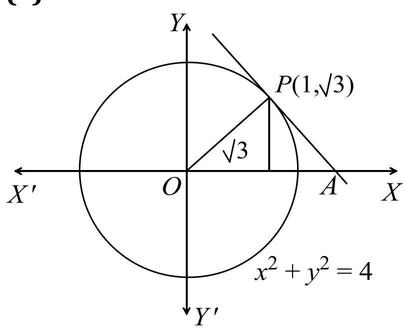
For a common normal,

$2b+b/m^2+2am+am^3=0$

$\Rightarrow am^5+2am^3+2bm^2+b=0$

This means there can be most '5' common normals

378 **(a)**

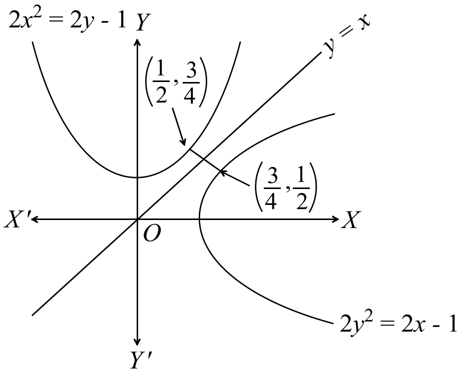


The equation of the tangent and normal to $x^2+y^2=4$ at $(1,\sqrt{3})$ are $x+\sqrt{3}y=4$ and $y=\sqrt{3}\,x$

The tangent meets x-axis at (4, 0)

Therefore, area of $\Delta OAP = \frac{1}{2}(4)\sqrt{3} = 2\sqrt{3}$ sq. units

379 **(b)**



Given parabolas $2y^2 = 2x - 1, 2x^2 = 2y - 1$ are symmetrical about the line $y = x$

Also shortest distance occurs along the common normal which perpendicular to the line $y = x$

Differentiating $2y^2 = 2x - 1$ w.r.t. x,

$2y\dfrac{dy}{dx} = 1$

We have $\frac{dy}{dx} = \frac{1}{2y} = 1 \Rightarrow y = \frac{1}{2}$

Hence, points are as shown in the figure

Then, the shortest distance, $d = \sqrt{\frac{1}{16} + \frac{1}{16}} = \frac{1}{2\sqrt{2}}$

380 **(b)**

Here a =2 for parabola and the two tangents pass through the points $(-2, -3)$, which lie on the directrix, then tangents are perpendicular or $m_1 m_2 = -1$

381 **(b)**

Clearly P is the point of intersction of two perpendicular tangents ot the parabola $y^2 = 8x$

Hence, P must lie on the directrix $x + a = 0$ or $x + 2 = 0$

$\therefore\ x = -2$

Hence, the point is $(-2, 0)$

382 **(b)**

One of the tangents of slope m to the given ellipse is

$y = mx + \sqrt{18m^2 + 32}$

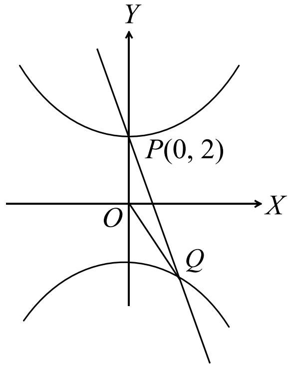
For $m = -\frac{4}{3}$,

We have $y = -\frac{4}{3}x + 8$

Then points on the axis where tangents meet are $A(6, 0)$ and $B(0, 8)$

Then area of triangle ABC is $\frac{1}{2}(6)(8) = 24$ units

383 **(c)**



Homogenizing the hyperbola using the straight line, we get pair of straight lines OP and OQ, which are given by

$$y^2 - x^2 = 4\left(\frac{\sqrt{3}x + y}{2}\right)^2$$

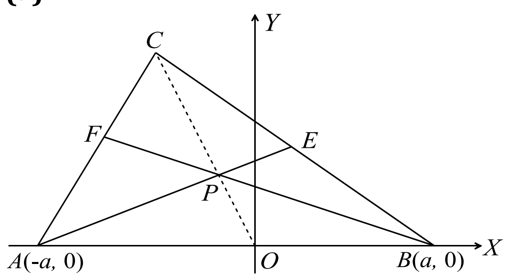
$$\Rightarrow y^2 - x^2 = 3x^2 + y^2 + 2\sqrt{3}\,xy$$

$$\Rightarrow 4x^2 + 2\sqrt{3}\,xy = 0$$

$$\Rightarrow x = 0 \text{ and } 2x + \sqrt{3}y = 0$$

Angle between the line is $\frac{\pi}{2} - \tan^{-1}\left(\frac{2}{\sqrt{3}}\right) = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

384 **(a)**



Let A and B be $(-a, 0)$ and $(a, 0)$. Also let P be (h, k)

Then by geometry, we know $\frac{CP}{PO} = \frac{CF}{FA} + \frac{CE}{EB}$

$\therefore\ \dfrac{CP}{PO} = 1$

If $C(\alpha, \beta)$ lies on $x^2 + y^2 + 2gx + 2fy + c = 0$, then $\alpha = 2h$ and $\beta = 2k$

$\Rightarrow 4(h^2 + k^2 + gh + fk) + c = 0$

$\therefore$ Locus of $P(h, k)$ is $x^2 + y^2 + gx + fy + \frac{c}{4} = 0$

which is a circle of radius $= \sqrt{\left(\frac{g}{2}\right)^2 + \left(\frac{f}{2}\right)^2 - \frac{c}{4}}$

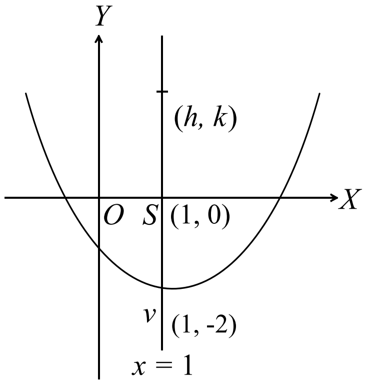
$= \dfrac{1}{2}\sqrt{g^2 + f^2 - c}$

$= \dfrac{r}{2}$

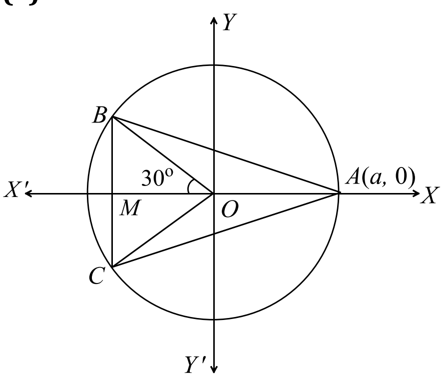
385 **(c)**

Axis of the parabola is $x = 1$. Any point on it is

$(1, k)$. Now distance of $(1, k)$ from $(1, -2)$ should be more than the semi-latus rectum and $(1, k)$ should be inside the parabola, hence $k > 2$



387 **(a)**



Since $\angle B = \angle C = 75°$

$\Rightarrow \angle BAC = 30°$

$\Rightarrow \angle BOC = 60°$

$\Rightarrow B$ has coordinates $(-a \cos 30°, a \sin 30°)$

or $\left(\frac{-\sqrt{3}a}{2}, \frac{a}{2}\right)$ and those of C are $\left(-\frac{\sqrt{3}a}{2}, -\frac{a}{2}\right)$

388 **(c)**

Let points $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ lied on the parabola $y^2 = 4ax$

Here point P and Q are variable but slope of the chord PQ,

$$m_{PQ} = \frac{2}{t_1 + t_2}$$

Now let midpoint PQ be $R(h, k)$,

$$k = \frac{2at_1 + 2at_2}{2}$$

Or $k = a(t_1 + t_2) = \frac{2}{m}$

$$\Rightarrow y = \frac{2}{m},$$

Which is a line parallel to the axis of parabola

389 **(a)**

Let the midpoint of PQ be (α, β)

$\Rightarrow \alpha = x + \frac{c}{2}$ and $\beta = y + \frac{c}{2}$

$$\Rightarrow \left(\beta - \frac{c}{2}\right)^2 = 4a\left(\alpha - \frac{c}{2}\right)$$

$$\Rightarrow \left(y - \frac{c}{2}\right)^2 - 4a\left(x - \frac{c}{2}\right)$$

Which is required locus

390 **(c)**

Chord with midpoint (h, k) is

$hx + ky = h^2 + k^2$ ...(i)

Chord of contact of (x_1, y_1) is

$xx_1 + yy_1 = 2$ ...(i)

Comparing, we get

$x_1 = \frac{2h}{h^2+k^2}$ and $y_1 = \frac{2k}{h^2+k^2}$

(x_1, y_1) lies on $3x + 4y = 10 \Rightarrow 6h + 8k = 10(h^2 + k^2)$

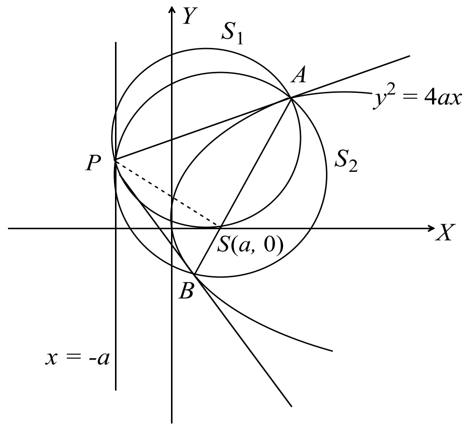
$\therefore$ Locus of (h, k) is $x^2 + y^2 - \frac{3}{5}x - \frac{4}{5}y = 0$

Which is circle with centre $P\left(\frac{3}{10}, \frac{4}{10}\right)$

$$\therefore OP = \frac{1}{2}$$

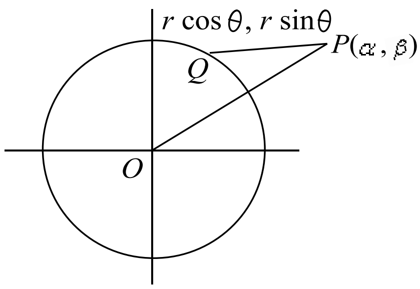
391 **(c)**

Circle S_2, taking focal chord AB as diameter will touch directrix at point P and circle S_1, taking AP as diameter will pass through focus S (since AP subtends 90° at focus of parabola)



Hence, common chord of given circles is line AP (which is intercept of tangent at point 'A' between point A and directrix)

392 **(d)**



$2h = \alpha + r\cos\theta$

$2k = \beta + r\sin\theta$

$\Rightarrow (2h - \alpha)^2 + (2k - \beta)^2 = r^2$

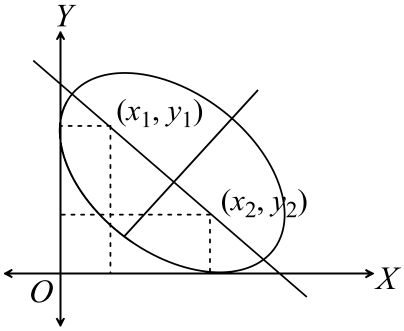
$$\left(h - \frac{\alpha}{2}\right)^2 + \left(k - \frac{\beta}{2}\right)^2 = \left(\frac{r}{2}\right)^2$$

Locus is $\left(x - \frac{\alpha}{2}\right)^2 + \left(y - \frac{\beta}{2}\right)^2 = \left(\frac{r}{2}\right)^2$

Which is a circle with centre as midpoint of OP and radius $r/2$

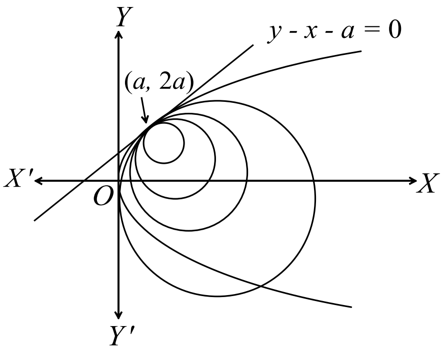
393 **(b)**

We know that product of length of perpendiculars from foci upon any tangents to ellipse is b^2
Hence, from the diagram, x_1 and x_2 are length of perpendiculars from foci upon tangent y-axis of the given ellipse, hence $x_1x_2 = b^2$
Similarly $y_1y_2 = b^2$



394 **(a)**
Let the equation of the circles through (a, b) be
$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)
So, $a^2 + b^2 + 2ag + 2bf + c = 0$ ...(ii)
Since circle (i) cuts $x^2 + y^2 = k^2$ orthogonally,
$\therefore 2g(0) + 2f(0) = c - k^2 \Rightarrow c = k^2$
Putting $c = k^2$ in Eq. (ii), we get
$2ag + 2bf + (a^2 + b^2 + k^2) = 0$
So, the locus of the centre $(-g, -f)$ is
$-2ax - 2by + (a^2 + b^2 + k^2) = 0$
or $2ax + 2by - (a^2 + b^2 + k^2) = 0$

395 **(a)**



Equation of tangent of parabola at $(a, 2a)$ is
$2ya = 2a(x + a)$, i.e., $y - x - a = 0$
Equation of circle touching the parabola at $(a, 2a)$ is $(x - a)^2 + (y - 2a)^2 + \lambda(y - x - a) = 0$
It passes through (0,0)
$\Rightarrow a^2 + 4a^2 + \lambda(-a) = 0 \Rightarrow \lambda = 5a$
Thus, required circle is $x^2 + y^2 - 7ax - ay = 0$
It's radius is $\sqrt{\frac{49}{4}a^2 + \frac{a^2}{4}} = \frac{5}{\sqrt{2}} = a$

396 **(a)**
Curve passing through point of intersection of S and S' is
$\Rightarrow S + \lambda S'' = 0$
$\Rightarrow x^2(\sin^2\theta + \lambda\cos^2\theta) + y^2(\cos^2\theta + \lambda\sin^2\theta) + 2\,xy$
$(h + \lambda h'') + x(32 + 16\lambda) + y(16 + 32\lambda) + 19(1 + \lambda) = 0$ for this equation to be a circle
$\sin^2\theta + \lambda\cos^2\theta = \cos^2\theta + \lambda\sin^2\theta \Rightarrow \lambda = 1$
and $h + \lambda h'' = 0 \Rightarrow h + h' = 0$

397 **(a)**
The given hyperbola is

$xy - hx - ky = 0$ (i)

The equation of asymptotes is given by

$xy - hx - ky + c = 0$ (ii)

Equation (ii) gives a pair of straight lines, So,

$$\begin{vmatrix} A & H & G \\ H & B & F \\ G & F & C \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 0 & \frac{1}{2} & -\frac{h}{2} \\ \frac{1}{2} & 0 & -\frac{k}{2} \\ -\frac{h}{2} & -\frac{k}{2} & c \end{vmatrix} = 0$$

$$\Rightarrow \frac{hk}{8} + \frac{hk}{8} - \frac{c}{4} = 0$$

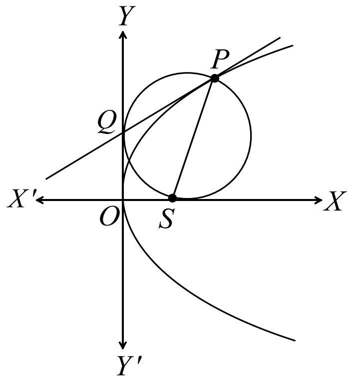
$\Rightarrow c = hk$

Hence, asymptotes are

$xy - hx - ky + hk = 0$

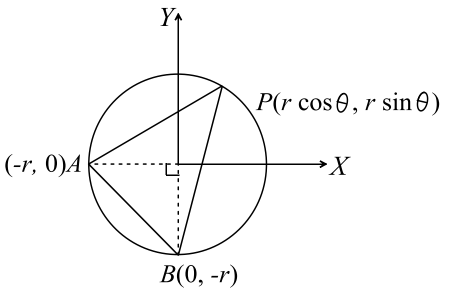
or $(x - k)(y - h) = 0$

398 **(b)**



Tangent at P intersects y-axis at $Q \equiv (0, at)$
Also circle with PS as diameter touches the y-axis at$(0, at)$
$\Rightarrow y$-axis is the tangent to circumcircle of ΔPQS at Q

399 **(b)**
$x^2 + y^2 = r^2$ is a circle with centre at (0, 0) and radius r units



Any arbitrary point P on it is $(r\cos\theta, r\sin\theta)$
choosing A and B as $(-r, 0)$ and $(0, -r)$
For locus of centroid of ΔABP
$\left(\frac{r\cos\theta - r}{3}, \frac{r\sin\theta - r}{3}\right) = (x, y)$
$\Rightarrow r\cos\theta - r = 3x$ and $r\sin\theta - r = 3y$
$\Rightarrow r\cos\theta = 3x + r$ and $r\sin\theta = 3y + r$
Squaring and adding $(3x + r)^2 + (3y + r)^2 = r^2$
which is a circle

400 **(d)**
Let $P(x, y)$be the coordinates of the other end of the chord. OP
Then $\frac{x+0}{2} = a, \frac{y+0}{2} = b$
But (x, y)lies on the parabola
$\therefore y^2 = 4x$
$\Rightarrow (2b)^2 = 4(2a)$
$\Rightarrow b^2 = 2a$

401 **(b)**
The circle through points of intersection of the two circle $x^2 + y^2 - 6 = 0$ and $x^2 + y^2 - 6x + 8 = 0$ is
$(x^2 + y^2 - 6) + \lambda(x^2 + y^2 - 6x + 8) = 0$
As it passes through (1, 1)
$(1 + 1 - 6) + \lambda(1 + 1 - 6 + 8) = 0$
$\Rightarrow \lambda = 1$
$\therefore$ The required circle is
$2x^2 + 2y^2 - 6x + 2 = 0$
or $x^2 + y^2 - 3x + 1 = 0$

402 **(c)**
$\left(\sqrt{3h}, \sqrt{3k+2}\right)$ lies on the line $x - y - 1 = 0$
$\Rightarrow \left(\sqrt{3h}\right)^2 = \left(\sqrt{3k+2} + 1\right)^2$
$\Rightarrow 3h = 3k + 2 + 1 + 2\sqrt{3k+2}$
$\Rightarrow 3^2(h - k - 1)^2 = 2^2\left(\sqrt{3k+2}\right)^2$
$\Rightarrow 9(h^2 + k^2 + 1 - 2hk - 2h + 2k) = 4(3k + 2)$
$\Rightarrow 9(x^2 + y^2) - 18xy - 18x + 6y + 1 = 0$
Now $h^2 = ab$ and $\Delta \neq 0$
Therefore, locus is a parabola

403 **(c)**
We have

$2x^2 + 3y^2 - 18y + 35 = k$

$\Rightarrow 2(2x^2 - 4x) + 3(y^2 - 6y) + 35 = k$

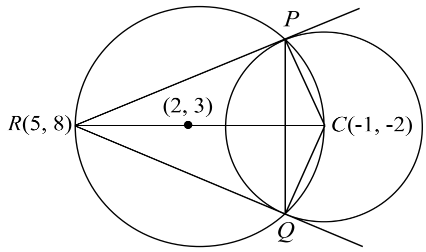
$\Rightarrow 2(x - 2)^2 + 3(y - 3)^2 = k$

For $k = 0$, we get

$2(x - 2)^2 + 3(y - 3)^2 = 0$

Which represents the point (2, 3)

404 **(a)**

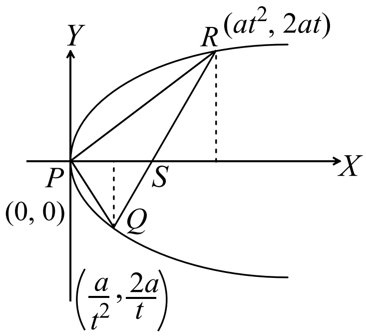


Let C be the centre of the given circle
Then, circumcircle of the ΔRPQ passes through C
$\therefore$ (2, 3) is the midpoint of RC
$\therefore$ Coordinates of C are $(-1, -2)$
$\therefore$ Equation of the circle is $x^2 + y^2 + 2x + 4y - 20 = 0$

405 **(a)**
Let $x = a, x = b, y = c, y = d$ be the sides of the square. The length of each diagonal of the square is equal to the diameter of the circle, i.e. $2\sqrt{98}$
Let l be the length of each side of the square.
Then, $2l^2 = (\text{Diagonal})^2$
$\Rightarrow l = 14$
Therefore, each side of the square is at a distance 7 from the centre $(1, -2)$ of the given circle. This implies that $a = -6, b = 8, c = -9, d = 5$
Hence, the vertices of the square are
$(-6, -9), (-6, 5), (8, -9), (8, 5)$

406 **(b)**
Then point of intersection are given by
$x^2(1 + m^2) - x(3 + 4m) - 4 = 0$
$\therefore x_1 + x_2 = \frac{3+4m}{1+m^2}$ and $x_1x_2 = \frac{-4}{1+m^2}$
Since (0, 0) divides chord in the ratio 1 : 4
$\therefore x_2 = -4x_1$
$\therefore -3x_1 = \frac{3+4m}{1+m^2}$ and $4x_1^2 = -\frac{-4}{1+m^2}$
$\therefore 9 + 9m^2 = 9 + 16m^2 + 24m$
i.e. $m = 0, -\frac{24}{7}$
Therefore, the lines are $y = 0$ and $y + 24x = 0$

407 **(c)**
Difference of the ordinate
$$d = \left|2at + \frac{2a}{t}\right| = 2a\left|t + \frac{1}{t}\right|$$



Now area $A = \frac{1}{2}\begin{vmatrix} at^2 & 2at & 1 \\ \frac{a}{t^2} & -\frac{2a}{t} & 1 \\ 0 & 0 & 1 \end{vmatrix} = a^2\left(t + \frac{1}{t}\right)$

$\Rightarrow 2a\left(t + \frac{1}{t}\right) = \frac{2A}{a}$

408 **(c)**

$y = mx + c$ is a normal to $y^2 = 4ax$ if

$c = -2am - am^3, y = -2x - \lambda$

$\Rightarrow m = -2, a = -2$

$\Rightarrow -\lambda = -2am - am^3$

$= -2(-2)(-2) - (-2)(-2)^3$

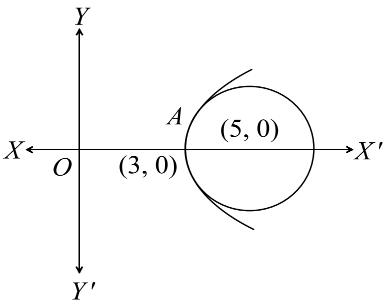
$= -24$

$\Rightarrow \lambda = 24$

409 **(b)**

Given hyperbola is

$\frac{x^2}{9} - \frac{y^2}{16} = 1 \qquad$ (i)



$\Rightarrow e^2 = 1 + \frac{16}{9} = \frac{25}{9}$

$\Rightarrow \ e = \frac{5}{3}$

Hence, its foci are $(\pm 5, 0)$

The equation of the circle with (5, 0) as centre is

$(x - 5)^2 + y^2 = r^2 \qquad$ (ii)

Solving (i) and (ii), we have

$16x^2 - 9[r^2 - (x - 5)^2] = 144$

or $\ 25x^2 - 90x - 9r^2 + 81 = 0$

Since the circle touches the hyperbola, above equation must have equal roots. Hence,

$90^2 - 4(25)(81 - 9r^2) = 0$

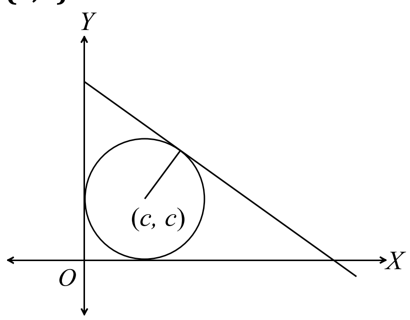
$\Rightarrow 9 - (9 - r^2) = 0$

$\Rightarrow r = 0$ which is not possible

Hence, the circle cannot touch at two points

It can only be tangent at the vertex. Hence, $r = 5 - 3 = 2$

410 **(a,d)**



We must have $\left|\frac{\frac{c}{3} + \frac{c}{4} - 1}{\sqrt{\frac{1}{3^2} + \frac{1}{4^2}}}\right| = c$

$\Rightarrow c = 6, 1$

411 **(c,d)**

$t_2 = -t_1 - \frac{2}{t_1}$

Also $\frac{2at_1}{at_1^2} \times \frac{2at_2}{at_2^2} = -1$

$\Rightarrow t_1 t_2 = -4$

$\therefore \ \frac{-4}{t_1} = -t_1 - \frac{2}{t_1}$

$\Rightarrow t_1^2 + 2 = 4$ and $t_1 = \pm\sqrt{2}$

So point can be $(2a, \pm 2\sqrt{2a})$

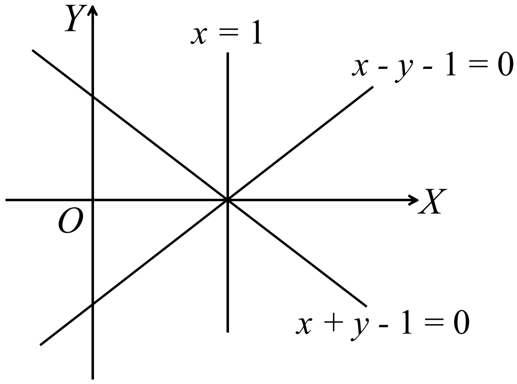
412 **(a,b,c)**

By properties of parabola options (a), (b) and (c) are always correct

413 **(b,d)**

Line pair is $(x - 1)^2 - y^2 = 0$, i.e. $x + y - 1 = 0, x - y - 1 = 0$

Let the centre be $(\alpha, 0)$, then its distance from $x + y - 1 = 0$ is



$\left|\frac{\alpha - 1}{\sqrt{2}}\right| = 2$ (radius)

i.e. $\alpha = 1 \pm 2\sqrt{2}$

$\therefore$ Centre may be $\left(1 + 2\sqrt{2}, 0\right), (1 - 2\sqrt{2}, 0)$

Now let the centre be $(1, \beta)$, then

$\left|\frac{1 + \beta - 1}{\sqrt{2}}\right| = 2$

$\Rightarrow \ \beta = \pm 2\sqrt{2}$

$\therefore$ Centre may be $\left(1, 2\sqrt{2}\right), (1, -2\sqrt{2})$

414 **(a,c,d)**

Equation of normal to parabola $y^2 = 12x$ having slope m is $y = mx - 6m - 3m^3$. Options $(a), (c), (d)$ are normal for $m - 1, -2$ and 3 respectively

415 **(a, b, c)**

$3x^2 + 2y^2 + 6x - 8y + 5 = 0$

$\Rightarrow \frac{(x+1)^2}{2} + \frac{(y-2)^2}{3} = 1$

Therefore, centre is $(-1, 2)$ and ellipse is vertical $(\because b > a)$

$a^2 = 2, b^2 = 3$

Now $2 = 3(1 - e^2)$

$\Rightarrow e = \frac{1}{\sqrt{3}}$

Foci are $(-1, 2 \pm be)$ and $(-1, 2 \pm 1) \equiv (-1, 3)$ and $(-1, 1)$

and directrix are $y = 2 \pm \frac{b}{e} \Rightarrow y = 5$ and $y = -1$

416 **(c,d)**

Let radius of required circle is r

Since, it touches the given circle

$\therefore$ Distance between centres

=radius of I circle$\pm$ radius of II circle

$\Rightarrow 5 = r \pm 1$

$\Rightarrow r = 5 \pm 1 = 6, 4$

$\therefore$ equation of circles are

$(x - 4)^2 + (y - 3)^2 = 6^2$

and $(x - 4)^2 + (y - 3)^2 = 4^2$

or $x^2 + y^2 - 8x - 6y - 11 = 0$

and $x^2 + y^2 - 8x - 6y + 9 = 0$

417 **(a, d)**

The equation of the tangent at $(t^2, 2t)$ to the parabola

$y^2 = 4x$ is

$2ty = 2(x + t^2)$

$\Longrightarrow ty = x + t^2$

$\Longrightarrow x - ty + t^2 = 0$ (i)

The equation of the normal at point $\left(\sqrt{5}\cos\theta, 2\sin\theta\right)$ on the ellipse $5x^2 + 5y^2 = 20$ is

$\Longrightarrow \left(\sqrt{5}\sec\theta\right)x - (2\operatorname{cosec}\theta)y = 5 - 4$

$\Longrightarrow \left(\sqrt{5}\sec\theta\right)x - (2\operatorname{cosec}\theta)y = 1$ (ii)

Given that Eqs. (i) and (ii) represent the same line

$\Longrightarrow \frac{\sqrt{5}\sec\theta}{1} = \frac{-2\operatorname{cosec}\theta}{-t} = \frac{-1}{t^2}$

$\Longrightarrow t = \frac{2}{\sqrt{5}}\cot\theta$ and $t = -\frac{1}{2}\sin\theta$

$\Longrightarrow \frac{2}{\sqrt{5}}\cot\theta = -\frac{1}{2}\sin\theta$

$\Longrightarrow 4\cos\theta = -\sqrt{5}\sin^2\theta$

$\Longrightarrow 4\cos\theta = -\sqrt{5}(1 - \cos^2\theta)$

$\Longrightarrow \sqrt{5}\cos^2\theta - 4\cos\theta - \sqrt{5} = 0$

$\Longrightarrow \left(\cos\theta - \sqrt{5}\right)\left(\sqrt{5}\cos\theta + 1\right) = 0$

$\Longrightarrow \cos\theta = -\frac{1}{\sqrt{5}}$ $\left[\because \cos\theta \neq -\sqrt{5}\right]$

$\Longrightarrow \theta = \cos^{-1}\left(-\frac{1}{\sqrt{5}}\right)$

Putting $\cos\theta = -\frac{1}{\sqrt{5}}$ in $t = -\frac{1}{2}\sin\theta$, we get

$t = -\frac{1}{2}\sqrt{1 - \frac{1}{5}} = -\frac{1}{\sqrt{5}}$

Hence, $\theta = \cos^{-1}\left(-\frac{1}{\sqrt{5}}\right)$ and $t = -\frac{1}{\sqrt{5}}$

418 **(a,c,d)**

Coordinates of O are $(5, 3)$ and radius $= 2$

Equation of tangent at $A(7, 3)$ is $7x + 3y - 5(x + 7) - 3(y + 3) + 30 = 0$

i.e. $2x - 14 = 0$, i.e. $x = 7$

equation of tangent as $B(5, 1)$ is $5x + y - 5(x + 5) - 3(y + 1) + 30 = 0$, i.e., $-2y + 2 = 0$, i.e. $y = 1$

$\therefore$ Coordinate of C are $(7, 1)$

$\therefore$ Area of $OACB = 4$

Equation of AB is $x - y = 4$ (radical axis)

Equation of the smallest circles is

$(x - 7)(x - 5) + (y - 3)(y - 1) = 0$

i.e., $x^2 + y^2 - 12x - 4y + 38 = 0$

419 **(b,d)**

Let the equation of the tangent be

$x - 2y = k$ ...(i)

$\because$ Line Eq.(i) touches the circle

$\therefore$ Distance from centre to line Eq. (i)
= radius of the circle

$$\therefore \frac{|2-2-k|}{\sqrt{5}} = \sqrt{4+1+15}$$

$|k| = 10 \Rightarrow k = \pm 10$

$\therefore$ The tangents can be $x - 2y \pm 10 = 0$

420 **(b,d)**

Given parabola is

$x^2 - ky + 3 = 0$

or $x^2 = k\left(y - \frac{3}{k}\right)$

Le $x = Y, y - \frac{3}{k} = X$

Then the parabola is

$Y^2 = kX$

Whose focus is $\left(0, \frac{k}{4}\right)$

Therefore, the focus of $x^2 = k\left(y - \frac{3}{k}\right)$ is

$$\left(0, \frac{3}{k} + \frac{k}{4}\right) \equiv (0,2)$$

$$\therefore \frac{3}{k} + \frac{k}{4} = 2$$

$\Rightarrow 12 + k^2 = 8k$

$\Rightarrow k^2 - 8k + 12 = 0$

$\Rightarrow (k-6)(k-2) = 0$

$\Rightarrow k = 2, 6$

421 **(b,c,d)**

$(x-\alpha)^2 + (y-\beta)^2 = k(lx + my + n)^2$

$$\Rightarrow \sqrt{(x-\alpha)^2 + (y-\beta)^2} = \sqrt{k}\sqrt{l^2+m^2}\frac{(lx+my+n)}{\sqrt{l^2+m^2}}$$

$$\Rightarrow \frac{PS}{PM} = \sqrt{k}\sqrt{l^2+m^2}$$

Where point $P(x, y)$ is any point on the curve

Fixed point $S(\alpha, \beta)$ is focus and fixed line $lx + my + n = 0$ is directrix

If $k(l^2+m^2) = 1, P$ lies on parabola

If $k(l^2+m^2) < 1, P$ lies on ellipse

If $k(l^2+m^2) > 1, P$ lies on hyperbola

$K = 0, P$ lies on a point circle

422 **(a,c)**

Let the possible point be $(t^2, 2t)$. Equation of tangent at this point is

$yt = x + t^2$

It must pass through (6, 5). (Since normal to circle always passes through its centre)

$\Rightarrow t^2 - 5t + 6 = 0$

$\Rightarrow t = 2, 3$

$\Rightarrow$ Possible points are (4, 4),(9, 6)

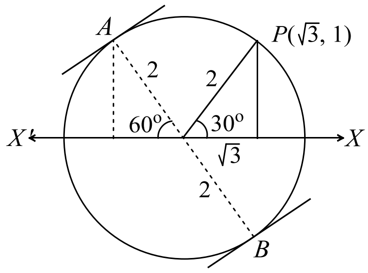
423 **(b,c)**

Distance of line $x + y - 1 = 0$ from the centre $\left(\frac{1}{2}, -\frac{3}{2}\right)$ is $\frac{\left|\frac{1}{2}-\frac{3}{2}-1\right|}{\sqrt{2}} = \sqrt{2}$

Now distance of line in options (b) and (c) from the centre is also $\sqrt{2}$

Hence, given lies are $x - y = 0$ and $x + 7y = 0$

424 **(b,d)**



Clearly, $A = (-2\cos 60°, 2\sin 60°)$ and $B = (2\cos 60°, -2\sin 60°)$

The tangent at A is $x(-2\cos 60°) + y(2\sin 60°) = 4$ and the tangent at B is $x(2\cos 60°) + y(-2\sin 60°) = 4$

425 **(b,c)**

Given equation is $x^2 - 2x = 2y - 5$

Or $(x-1)^2 = 2(y-2)$ ...(i)

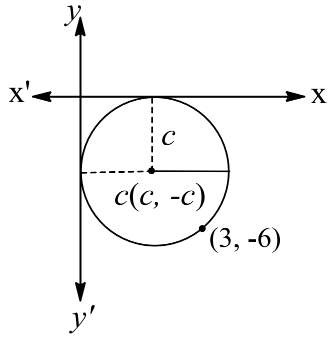
Eq. (i) is a parabola whose vertex is $(1, 2)$. Its directrix is

$y - 2 = a = \frac{1}{2}$ or $y = \frac{5}{2}$

426 **(a,d)**

Circle possible in IV quadrant. Equation of circle is

$(x-c)^2 + (y+c)^2 = c^2$ ...(i)



Its passes through $(3, -6)$

$\Rightarrow (3-c)^2 + (c-6)^2 = c^2$

$\Rightarrow c^2 - 18c + 45 = 0$

$\Rightarrow (c-15)(c-3) = 0$

$\therefore c = 3, 15$

From Eq. (i), equation of circles are

$x^2 + y^2 - 6x + 6y + 9 = 0$

and $x^2 + y^2 - 30x + 30y + 225 = 0$

427 **(a,c)**

The point from which the tangents drawn are at right angle lie on the director circle

Equation of director circle is $x^2 + y^2 = 2 \times 16 = 32$

Putting $x = 2$, we get

$y^2 = 28$

$\Rightarrow y = \pm 2\sqrt{7}$

$\therefore$ The points can be $(2, 2\sqrt{7})$ or $(2, -2\sqrt{7})$

428 **(a,c)**

$r^2 - r - 6 > 0$ and $r^2 - 6r + 5 > 0$

$\Rightarrow (r-3)(r+2) > 0$ and $(r-1)(r-5) > 0$

$\Rightarrow (r < -2$ or $r > 3)$ and $(r < 1$ or $r > 5)$

$\Rightarrow r < -2$ or $r > 5$

Also $r^2 - r - 6 \neq r^2 - 6r + 5$

$\Rightarrow r \neq \frac{11}{5}$

429 **(a,c)**

$\because C_2$ is the director circle of C_1

$\therefore$ Equation of C_2 is $x^2 + y^2 = 2(2)^2 = 8$

Again, C_3 is the director circle of C_2. Hence, the equation of C_3 is

$x^2 + y^2 = 2(8) = 16$

430 **(a,c)**

Equation of radical axis of the give circle is $x = 0$. If one circle lies completely inside the other, centre of both circles should lie on the same side of radical axis and radical axis should not intersect the circles

$\Rightarrow (-a_1)(-a_2) > 0$

$\Rightarrow a_1 a_2 > 0$ and $y^2 + c = 0$ should have imaginary roots

$\Rightarrow c > 0$

431 **(a,b,c,d)**

Chords equidistance from the centre are equal

432 **(a,c,d)**

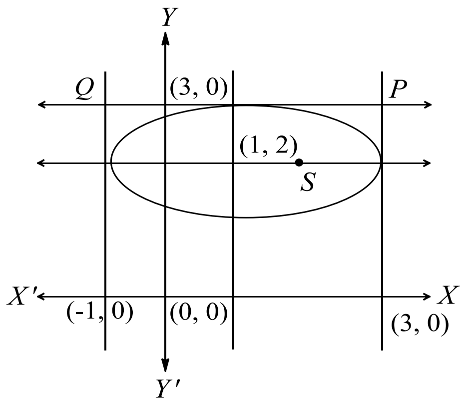
$x^2 + 4y^2 - 2x - 16y + 13 = 0$

$\Rightarrow (x^2 - 2x + 1) + 4(y^2 - 4y + 4) = 4$

$\Rightarrow \frac{(x-1)^2}{4} + \frac{(y-2)^2}{1} = 1$

$\therefore$ Length of latus rectum $= \frac{2\times1}{2} = 1$

Also $e = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$



$\Rightarrow 2ae = 2 \times 2 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$

Sum of the focal distance $= 2a = 4$

Tangents at the vertices are $x - 1 = \pm 2$

or $x = 3, -1$

Therefore, the line $y = 3$ intersect these at points $P(3,3)$ and $Q(-1,3)$

Coordinate of focus are $S(\sqrt{3}+1, 2)$

Slope of PS is $\frac{1}{2-\sqrt{3}}$, slope of QS is $\frac{1}{-2-\sqrt{3}}$

$\Rightarrow$Product of slopes $= \frac{1}{2-\sqrt{3}} \times \frac{1}{-2-\sqrt{3}} = -1$

433 **(b,d)**

Let (x_1, y_1) be the point at which tangents to ellipse $4x^2 + 9y^2 = 1$ are parallel to $8x = 9y$

Then slope of the tangent $= \frac{8}{9}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = \frac{8}{9}$ (i)

Differentiating equation of ellipse w.r.t. x, we get

$8x + 18y\frac{dy}{dx} = 0$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = \frac{-8x_1}{18y_1} = \frac{-4x_1}{9y_1}$

Substituting in equation (i), we get

$\frac{-4x_1}{9y_1} = \frac{8}{9} \Rightarrow -x_1 = 2y_1$ (ii)

Also (x_1, y_1) is the point of contact which must be on curve. Hence,

$4x_1^2 + 9y_1^2 = 1$

$\Rightarrow 4x \times 4x_1^2 + 9y_1^2 = 1$ [using (2)]

$\Rightarrow y_1^2 = \frac{1}{25}$

$\Rightarrow y_1 = \pm\frac{1}{5}$

$\Rightarrow x_1 = \mp\frac{2}{5}$

Thus the required points are $\left(\frac{-2}{5}, \frac{1}{5}\right)$ and $\left(\frac{2}{5}, -\frac{1}{5}\right)$

Alternative Method:

Let $= \frac{8}{9}x + c$ be the tangent to $\frac{x^2}{1/4} + \frac{y^2}{1/9} = 1$

Where $c = \pm\sqrt{a^2m^2b^2} = \pm\sqrt{\frac{1}{4} \times \frac{64}{81} + \frac{1}{9}} = \pm\frac{5}{9}$

So, points of contact are $\left(\frac{-a^2m}{c}, \frac{b^2}{c}\right) =$
$\left(\frac{2}{5}, \frac{-1}{5}\right)$ or $\left(\frac{-2}{5}, \frac{1}{5}\right)$

434 **(a, c)**

Tangent and normal are bisectors of $\angle SPS'$

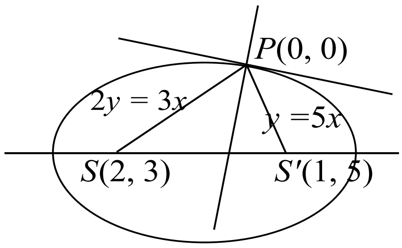
Now equation of SP is SP is $y = 3x/2$ and that of $S'P$ is $y = 5x$

Then their equations are $\frac{3x-2y}{\sqrt{13}} = \pm\frac{5x-y}{\sqrt{26}}$

or $3x - 2y = \pm\frac{5x-y}{\sqrt{2}}$

$\Rightarrow$ lines are $(3\sqrt{2} - 5)x + (1 - 2\sqrt{2})y = 0$ and $(3\sqrt{2} + 5)x - (2\sqrt{2} + 1)y = 0$

Now $(2, 3)$ and $(1, 5)$ lie on the same side of $(3\sqrt{2} - 5)x + (1 - 2\sqrt{2})y = 0$, which is equation of tangent



Points $(2, 3)$ and $(1, 5)$ lie on the different sides of $(3\sqrt{2} + 5)x - (2\sqrt{2} + 1)y = 0$, which is equation of normal

435 **(a,b)**

Any normal to the parabola $y^2 = 4ax$ may be taken as

$y - 2at = -t(x - at^2)$

Or $y + tx - 2at - at^3 = 0$ ...(i)

If Eq. (i) passes through $(5a, 2a)$, then

$2a + 5at - 2at - at^3 = 0$

Or $t^3 - 3t - 2 = 0$

Or $(t + 1)(t^2 - t - 2) = 0$

$\Rightarrow t = -1, 2, -1$

$\therefore$ From Eq. (i) normals are

$y - x + 3a = 0$ and $y + 2x - 12a = 0$

436 **(a,c)**

The equation of tangent in terms of slope of $x^2 + y^2 = 25$ is

$y = mx \pm 5\sqrt{(1 + m^2)}$ ...(i)

If Eq. (i), passes through $(-2, 11)$, then

$11 = -2m \pm 5\sqrt{(1 + m^2)}$

On squaring both sides, we get

$21m^2 - 44m - 96 = 0$

$\Rightarrow (7m - 24)(3m + 4) = 0$

$\therefore\ m = -\frac{4}{3}, \frac{24}{7}$

Thus, from Eq. (i), the required tangents are

$24x - 7y \pm 125 = 0$

and $4x + 3y = \pm 25$

437 **(b,c)**

For given circle $S_1: x^2 + y^2 - 2x - 4y + 1 = 0$

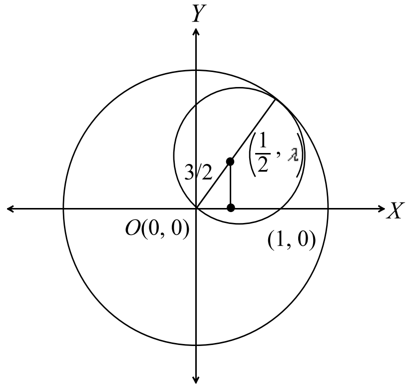
and $S_2: x^2 + y^2 + 4x + 4y - 1 = 0$

$C_1(1, 2), r_1 = 2$ and $C_2(-2, -2), r_2 = 3$

Now $r_1 + r_2 = 5$ and $C_1C_2 = 5$

Hence, circles touch externally. Also common tangent at point of contact is $S_1 - S_2 = 0$ or $3x + 4y - 1 = 0$

438 **(c,d)**



From the diagram

$$\sqrt{\left(\frac{1}{2}\right)^2 + \lambda^2} = \frac{3}{2} \Rightarrow 1 = \pm\sqrt{2}$$

Hence, centres of the circle are $\left(\frac{1}{2}, \pm\sqrt{2}\right)$

439 **(a,b,c,d)**

The equation of tangent in terms of slope of $y^2 = 32x$ is

$y = mx + \dfrac{8}{m}$

Which is also tangent of the hyperbola

$9x^2 - 9y^2 = 8$

$ie, x^2 - y^2 = \dfrac{8}{9}$

Then , $\left(\frac{8}{m}\right)^2 = \frac{8}{9}m^2 - \frac{8}{9}$

$\Rightarrow \dfrac{8}{m^2} = \dfrac{m^2}{9} - \dfrac{1}{9}$

$\Rightarrow 72 = m^4 - m^2$

$\Rightarrow m^4 - m^2 - 72 = 0$

$\Rightarrow\ (m^2 - 9)(m^2 + 8) = 0$

$\Rightarrow m^2 = 9, \quad (\because m^2 + 8 \neq 0)$

$\Rightarrow m = \pm 3$

From Eq.(i), we get

$y = \pm 3x \pm \dfrac{8}{3}$

$\Rightarrow 3y = \pm 9x \pm 8$

$\Rightarrow\ \pm 9x - 3y \pm 8 = 0$

$\Rightarrow 9x - 3y + 8 = 0, 9x - 3y - 8 = 0$

$-9x - 3y + 8 = 0, -9x - 3y - 8 = 0$

or $9x - 3y + 8 = 0, 9x - 3y - 8 = 0$

$\Rightarrow 9x+3y-8=0$
and $9x+3y+8=0$

440 **(a,b)**

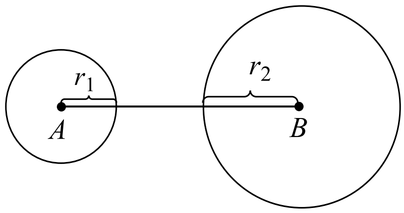
$L_1L_2+\lambda L_2L_3+\mu L_3L_1=0$
$\Rightarrow (y-m_1x)(y-m_2x)+\lambda(y-m_2x)(y-$
$m3x+\mu y-m3xy-m1x=0$ ...(i)
Clearly Eq.(i) represents a curve passing through points of intersection of lines L_1, L_2 and L_3
Equation (i) will represent a circle if coefficient of x^2 = coefficient of y^2 and coefficient of $xy=0$
$\therefore 1+\lambda+\mu=m_1m_2+\lambda m_2m_3+\mu m_1m_3$
and $m_1(1+\mu)+m_2(1+\lambda)+m_3(\mu+\lambda)=0$

441 **(a, c)**

$f(x)$ is a decreasing function and for major axis to be x-axis
$f(k^2+2k+5)>f(k+11)$
$\Rightarrow k^2+2k+5<k+11$
$\Rightarrow k\in(-3,2)$
Then for the remaining values of k, i.e.,
$k\in(-\infty,-3)\cup(2,\infty)$, major axis is y-axis

442 **(a,c,d)**

$x^2+y^2+8x-10y-40=0$
Centre of the circle is $(-4,5)$
Its radius $=9$
Distance of the centre $(-4,5)$ from the point $(-2,3)$ is $\sqrt{4+4}=2\sqrt{2}$



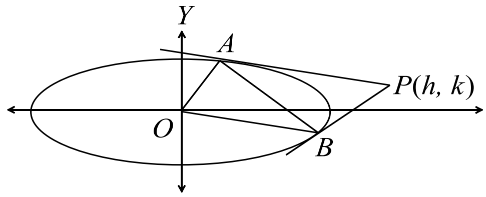
$\therefore\ a=2\sqrt{2}+9$ and $b=-2\sqrt{2}+9$
$\therefore\ a+b=18$
$a-b=4\sqrt{2}$
$a\cdot b=81-8=73$

443 **(a,b,c,d)**

Putting $y=c^2/x$ in $x^2+y^2=a^2$, we obtain
$x^2+c^4/x^2=a^2$
$\Rightarrow x^4-a^2x^2+c^4=0$ ...(i)
As x_1,x_2,x_3 and x_4 are roots of Eq. (i),
$\Rightarrow x_1+x_2+x_3+x_4=0$ and $x_1x_2x_3x_4=c^4$
Similarly, forming equation in y, we get
$y_1+y_2+y_3+y_4=0$ and $y_1y_2y_3y_4=c^4$

444 **(a, c)**

Let the point of equation of intersection of tangents A and B be $P(h,k)$, then equation of AB is
$\frac{xh}{4}+\frac{yk}{1}=1$ (i)



Homogenizing the equation of ellipse using Eq. (i), we get
$$\frac{x^2}{4}+\frac{y^2}{1}=\left(\frac{xh}{4}+\frac{yk}{1}\right)^2$$
$\Rightarrow x^2\left(\frac{h^2-4}{16}\right)+y^2(k^2-1)+\frac{2hk}{4}xy=0$ (ii)
Given equation of OA and OB is
$x^2+4y^2+\alpha xy=0$ (iii)
$\because$ Equation (ii) and (iii) represent same line,
Hence, $\frac{h^2-4}{16}=\frac{k^2-1}{4}=\frac{hk}{2a}$
$\Longrightarrow h^2-4=4(k^2-1)$
$\Rightarrow h^2-4k^2=0$
$\Rightarrow$ Locus $(x-2y)(x+2y)=0$

445 **(b,c)**

$x^2+y^2-8x-16y+60=0$ ...(i)
Equation of chord of contact from $(-2,0)$ is
$-2x-4(x-2)-8y+60=0$
$3x+4y-34=0$ ...(ii)
Solving Eqs. (i) and (ii)
$$x^2+\left(\frac{34-3x}{4}\right)^2-8x-16\left(\frac{34-3x}{4}\right)+60=0$$
$$\Rightarrow 16x^2+1156-204x+9x^2-128x-2176$$
$$+192x+960=0$$
$\Rightarrow 5x^2-28x-12=0$
$\Rightarrow (x-6)(5x+2)=0$
$\Rightarrow x=6,-\frac{2}{5}$
$\Rightarrow$ Points are $(6,4),\left(-\frac{2}{5},\frac{44}{5}\right)$

446 **(b,c)**

Equation of pair of tangent by $SS'=T^2$ is
$(ax+0-1)^2=(x^2+y^2-1)(a^2+0-1)$
or $(a^2-1)y^2-x^2+2ax-a^2=0$
If θ be the angle between the tangents, then
$$\tan\theta=\frac{2\sqrt{H^2+AB}}{A+B}$$
$$=\frac{2\sqrt{-(a^2-1)(-1)}}{a^2-2}$$
$$=\frac{2\sqrt{a^2-1}}{a^2-2}$$
If θ lies in II quadrant, then $\tan\theta<0$
$$\therefore\ \frac{2\sqrt{a^2-1}}{a^2-2}<0$$
$\Rightarrow a^2-1>0$
and $a^2-2<0$
$\Rightarrow |a|>1$ and $|a|<2$

$\Rightarrow a \in \left(-\sqrt{2}, -1\right) \cup (1, \sqrt{2})$

447 **(a,d)**

When two circles touch each other externally, then

$r_1 + r_2 = \sqrt{\{0-(-a)\}^2 + \{0-(-1)\}^2}$

$\Rightarrow 3 + a = \sqrt{a^2+1}$

$\Rightarrow a = -\dfrac{4}{3}$

When two circles touch each other internally, then

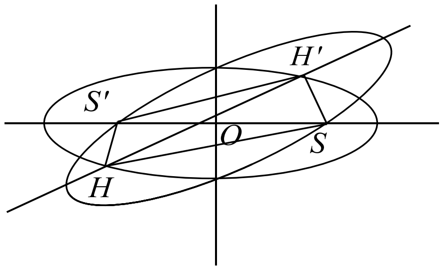
$\Rightarrow \ |r_1 - r_2| = \sqrt{\{0-(-a)\}^2 + \{0-(-1)\}^2}$

$\Rightarrow \ |3-a| = \sqrt{a^2+1}$

$\Rightarrow a = \dfrac{4}{3}$

448 **(a, b, c)**

Clearly O is the midpoint of SS' and HH'



$\Rightarrow$ Diagonals of quadrilateral $HSH'S'$ bisect each other, so it is a parallelogram

Let $H'OH = 2r \Rightarrow OH = r = ae'$

H lies on $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (suppose)

$\therefore \dfrac{r^2\cos^2\theta}{a^2} + \dfrac{r^2\sin^2\theta}{b^2} = 1$

$e'^2\cos^2\theta + \dfrac{e'^2\sin^2\theta}{1-e^2}$

$= 1 \quad [\because b^2 = a^2(1-e'^2)]$

$\Rightarrow e'^2\cos^2\theta - \dfrac{e'^2\cos^2\theta}{1-e^2} = 1 - \dfrac{e'^2}{1-e^2}$

$\Rightarrow \cos^2\theta = \dfrac{1}{e^2} + \dfrac{1}{e'^2} - \dfrac{1}{e^2e'^2}$

$\theta = \cos^{-1}\sqrt{\dfrac{1}{e^2} + \dfrac{1}{e'^2} - \dfrac{1}{e^2e'^2}}$

For $\theta = 90^0, \frac{e^2+e'^2}{e^2e'^2} = \frac{1}{e^2e'^2}$

$\Longrightarrow e^2 + e'^2 = 1$

449 **(b,c)**

Let P, Q, R, S lie on the circle

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

And also lies on

$xy = c^2$ ...(ii)

On solving Eqs.(i) and (ii), we get

$x^2 + \left(\dfrac{c^2}{x}\right)^2 + 2gx + \dfrac{2fc^2}{x} + c = 0$

$\Rightarrow x^4 + 2gx^3 + cx^2 + 2fc^2x + c^4 = 0$

$\Rightarrow x_1x_2x_3x_4 = c^4$ ...(iii)

And $P \equiv (x_1, y_1) \equiv \left(x_1, \frac{c^2}{x_3}\right), Q \equiv \left(x_2, \frac{c^2}{x^2}\right)$

And $R \equiv \left(x_3, \frac{c^2}{x_3}\right)$

Let orthocenter is $O(h,k)$.

Then, slope of QR $\times$slope of $OP = -1$

$= \left(\dfrac{\frac{c^2}{x_3} - \frac{c^2}{x_2}}{x_3 - x_2}\right) \times \left(\dfrac{k - \frac{c^2}{x_1}}{h - x_1}\right) = -1$

$\Rightarrow \ -\dfrac{c^2}{x_2x_3} \times \left(\dfrac{k - \frac{c^2}{x_1}}{h - x_1}\right) = -1$

$\Rightarrow k - \frac{c^2}{x_1} = \frac{hx_2x_3}{c^2} - \frac{x_1x_2x_3}{c^2}$ ...(iv)

Also, slope of PQ = slope of $OR = -1$

$\Rightarrow k - \frac{c^2}{x_3} = \frac{hx_1x_2}{c^2} - \frac{x_1x_2x_3}{c^2}$...(v)

From Eqs.(iii) and (iv), we get

$h = -\frac{c^4}{x_1x_2x_3}$ and $k = -\frac{x_1x_2x_3}{c^2}$

From Eqs.(iii), we get

$h = -x_4$ and $k = -\frac{c^2}{x_4}$

$\therefore$ Orthocentre lies on $xy = c^2$ $ie, (x_4, y_4)$ and $(-x_4, -y_4)$

450 **(a,d)**

Here $x^2 = -\lambda\left(y + \frac{\mu}{\lambda}\right)$

Therefore, vertex= $\left(0, -\frac{\mu}{\lambda}\right)$

And the directrix is

$\left(y + \dfrac{\mu}{\lambda}\right) + \dfrac{-\lambda}{4} = 0$

Comparing with the given data, $-\frac{\mu}{\lambda} = 1$ and

$\frac{\mu}{\lambda} - \frac{\lambda}{4} = -2$

$\therefore -1 - \dfrac{\lambda}{4} = -2$

Or $\lambda = 4 \Rightarrow \mu = 4$

451 **(a,b,c,d)**

$\because 5x^2 + 4y^2 = 20$

$\Rightarrow \frac{x^2}{4} + \frac{y^2}{5} = 1$ ...(i)

$\therefore$ The locus of point of intersection of perpendicular tangents of Eq. (i) is

$x^2 + y^2 = 4 + 5$ [Director circle of Eq. (i)]

$\Rightarrow x^2 + y^2 = 9$ ...(ii)

Hence, $(1, 2\sqrt{2}), (2\sqrt{2}, 1), (2, \sqrt{5})$ and $(\sqrt{5}, 2)$ are lie on Eq. (ii)

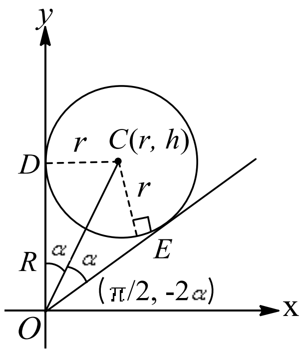
452 **(a,c)**

$P = (\alpha, \alpha+1)$ where $\alpha \neq 0, -1$

Or $P = (\alpha, \alpha - 1)$ where $\alpha \neq 0, 1$

$(\alpha, \alpha+1)$ is on $y^2 = 4x+1$
$\Rightarrow (\alpha+1)^2 = 4\alpha+1$
$\Rightarrow \alpha^2 - 2\alpha = 0$
$\Rightarrow \alpha = 2 \quad (\because \alpha \neq 0)$
Therefore, ordinate of P is 3
$(\alpha, \alpha-1)$ is on $y^2 = 4x+1$
$\Rightarrow (\alpha-1)^2 = 4\alpha+1$
$\Rightarrow \alpha^2 - 6\alpha = 0$
$\Rightarrow \alpha = 6 \quad (\because \alpha \neq 0)$
Therefore, ordinate of P is 5

453 **(a,c)**
The given equation is
$(x-r)^2 + (y-h)^2 = r^2$
Tangents are $x = 0$
And $y = x\tan\left(\frac{\pi}{2} - 2\alpha\right)$
$= x\cot 2\alpha$
$= \dfrac{x(1-\tan^2\alpha)}{2\tan\alpha}$



$\Rightarrow y = \dfrac{x\left(1-\frac{r^2}{h^2}\right)}{2\left(\frac{r}{h}\right)} \quad \left(\because \text{ in } \Delta ODC, \tan\alpha = \dfrac{r}{h}\right)$
or $(h^2 - r^2)x - 2rhy = 0$

454 **(a,b,c,d)**
Any point on the parabola is $P(at^2, 2at)$
Therefore, midpoint of $S(a,0)$ and $P(at^2, 2at)$ is
$R\left(\dfrac{a+at^2}{2}, at\right) \equiv (h,k)$
$\therefore h = \dfrac{a+at^2}{2}, k = at$
Eliminate 't'
i.e., $2x = a\left(1+\frac{y^2}{a^2}\right) = a + \frac{y^2}{a}$
i.e., $2ax = a^2 + y^2$
i.e., $y^2 = 2a\left(x - \frac{a}{2}\right)$
It's a parabola with vertex at $\left(\frac{a}{2}, 0\right)$, latus rectum $=2a$
Directrix is
$x - \dfrac{a}{2} = -\dfrac{a}{2}$
i.e., $x = 0$
Focus is
$x - \dfrac{a}{2} = \dfrac{a}{2}$,
i.e., $x = a$
i.e., $(a, 0)$

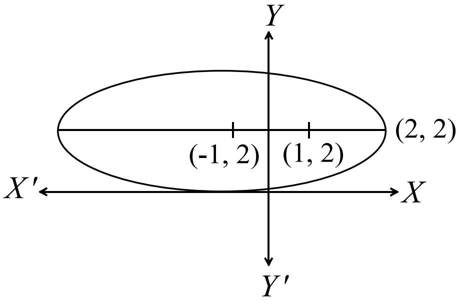
455 **(a,b,c,d)**
$r_1 = 5; r_2 = \sqrt{15}; C_1C_2 = \sqrt{40}$
$\Rightarrow r_1 + r_2 > C_1C_2 > r_1 - r_2$
Hence, circles intersect in two distinct points
There are two common tangents
Also $2g_1g_2 + 2f_1f_2 = 2(1)(3) + 2(2)(-4) = -10$
and $c_1 + c_2 = -20 + 10 = -10$
Thus, two circle are orthogonal
Length of common chord is $\frac{2r_1r_2}{\sqrt{r_1^2+r_2^2}} = 5\sqrt{\frac{3}{2}}$
Length of common tangent is $\sqrt{C_1C_2^2 - (r_1-r_2)^2}$
$= 5\left(\dfrac{12}{5}\right)^{\frac{1}{4}}$

456 **(a,c)**
Given that the extremities of the latus rectum are $(1,1)$ and $(1,-1)$
$\Rightarrow 4a = 2 \Rightarrow a = \dfrac{1}{2}$
$\Rightarrow$ The focus of the parabola is $(1,0)$
$\Rightarrow$ The vertex can be $\left(\frac{1}{2}, 0\right)$ and $\left(\frac{3}{2}, 0\right)$
$\Rightarrow$ The equations of the parabola can be
$y^2 = 2\left(x - \dfrac{1}{2}\right)$
Or $y^2 = 2\left(x - \frac{3}{2}\right)$
$\Rightarrow y^2 = 2x - 1$
Or $y^2 = 2x - 3$

457 **(a, b, c)**
The given equation is $\left(x - \frac{1}{13}\right)^2 + \left(y - \frac{2}{13}\right)^2$
$= \dfrac{1}{a^2}\left(\dfrac{5x+12y-1}{13}\right)^2$
It represents ellipse if $\frac{1}{a^2} < 1 \Longrightarrow a^2 > 1 \Longrightarrow a > 1$

$4x^2 + 8x + 9y^2 - 36y = -4$

$\Longrightarrow 4(x^2+2x+1) + 9(y^2-4y+4) = 36$



$$\Rightarrow \frac{(x+1)^2}{9}+\frac{(y-2)^2}{4}=1$$

Hence, $(-1, 2)$ is focus and $(1, 2)$ lies on the major axis

Then required minimum distance is 1

Equation of normal at $P(\theta)$ is $5\sec\theta\, x - 4\operatorname{cosec}\theta\, y = 25-16$, and it passes through $P(0,\alpha)$

$$\therefore\ \alpha = \frac{-9}{4\operatorname{cosec}\theta}$$

$$\Longrightarrow \alpha = \frac{-9}{4}\sin\theta$$

$$\Rightarrow\ |\alpha| < \frac{9}{4}$$

$$\frac{2b^2}{a} = \frac{2a}{3} \Rightarrow 3b^2 = a^2$$

$\Rightarrow$ From $b^2 = a^2(1-e^2), 1 = 3(1-e^2) \Rightarrow e = \sqrt{2/3}$

458 **(a,d)**

Let the equation of tangent is $y = x + c$.

$\therefore c^2 = a^2m^2 - b^2$

$\Rightarrow c^2 = (3)(1)^2 - 2 = 1$

$\Rightarrow c = \pm 1$

Hence, equation of tangent are $y = x \pm 1$.

459 **(a,b)**

Let $O \equiv (0,0)$ be the centre of the circle

$\because$ Arc length $AB = \frac{\pi}{2} = \frac{1}{4}$ (circumference of the circle)

$\therefore\ \angle AOB = \dfrac{\pi}{2}$

$\therefore$ Slope of $OB = -\frac{1}{\text{slope of } OA}$

$\Rightarrow$ slope of $OB = -\frac{1}{1} = -1$ ...(i)

Let $B \equiv (\alpha \pm \sqrt{1-\alpha^2})$

$\therefore\ \pm\frac{\sqrt{1-\alpha^2}}{\alpha} = -1$ [from Eq. (i)]

$\therefore B$ can be $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}},\frac{-1}{\sqrt{2}}\right), \left(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$ but possible points are $\left(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$

460 **(a,c)**

The line $y = 2x + c$ is a tangent to $x^2 + y^2 = 5$

If $c^2 = 25$

$\Rightarrow\ c = \pm 5$

Let the equation of parabola be $y^2 = 4ax$. Then

$\dfrac{a}{2} = \pm 5$

$\Rightarrow a = \pm 10$

$\Rightarrow$ Equation of the parabola is $y^2 = \pm 40x$

$\Rightarrow$Equation of the directrix are $x = \pm 10$

461 **(a,d)**

Circle with points $P(2t_1, 2/t_1)$ and $Q(2t_2, 2/t_2)$ as diameter is given by

$$(x-2t_1)(x-2t_2) + \left(y - \frac{2}{t_1}\right)\left(y - \frac{2}{t_2}\right) = 1$$

Also, slope of PQ is given by

$$-\frac{1}{t_1t_2} = 1 \Rightarrow t_1t_2 = -1$$

Hence, from (1), circle is

$$(x^2+y^2-8) - (t_1+t_2)(x-y) = 0$$

Which is of the form $S + \lambda L = 0$

Hence, circles pass through the points of intersection of the circle $x^2 + y^2 - 8 = 0$ and the line $x = y$

The points of intersection are $(2, 2)$ and $(-2, -2)$

462 **(c, d)**

The equation of the line joining θ and ϕ is

$$\frac{x}{5}\cos\left(\frac{\theta+\phi}{2}\right) + \frac{y}{3}\sin\left(\frac{\theta+\phi}{2}\right) = \cos\left(\frac{\theta-\phi}{2}\right)$$

If it passes through the point $(4, 0)$, then

$$\frac{4}{5}\cos\left(\frac{\theta+\phi}{2}\right) = \cos\left(\frac{\theta-\phi}{2}\right)$$

$$\Rightarrow \frac{4}{5} = \frac{\cos\left(\frac{\theta-\phi}{2}\right)}{\cos\left(\frac{\theta+\phi}{2}\right)}$$

$$\Rightarrow \frac{4+5}{4-5} = \frac{\cos\left(\frac{\theta-\phi}{2}\right)+\cos\left(\frac{\theta-\phi}{2}\right)}{\cos\left(\frac{\theta-\phi}{2}\right)-\cos\left(\frac{\theta+\phi}{2}\right)}$$

$$= \frac{2\cos\frac{\theta}{2}\cos\frac{\phi}{2}}{2\sin\frac{\phi}{2}\ \sin\frac{\theta}{2}}$$

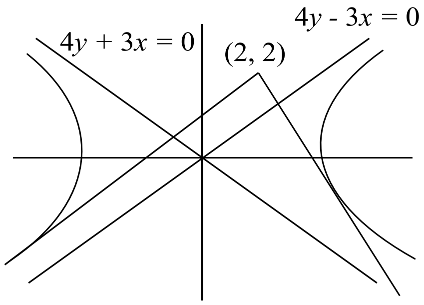
$$\Rightarrow \tan\frac{\theta}{2}\tan\frac{\phi}{2} = -\frac{1}{9}$$

If it passes through the point $(-5, 0)$, then

$\tan\frac{\phi}{2}\tan\frac{\theta}{2} = 9$

463 **(c,d)**

Equations of asymptotes are $4y - 3x = 0$ and $4y + 3x = 0$



As point (2, 2) lies above the asymptote

Hence, points of contact of the tangents are in III and IV quadrants

464 **(a,c)**

Equation of any tangent in terms of slope m is

$y = mx + \sqrt{(a^2m^2 - b^2)}$.

It passes through (h, k), then

$(k - mh)^2 = a^2m^2 - b^2$

$\Rightarrow\ m^2(h^2 - a^2) - 2mhk + k^2 + b^2 = 0$

Which is quadratic in m.

Let slopes of tangents are m_1 and m_2, then

$m_1m_2 = -1$

$\Rightarrow\ \dfrac{k^2 + b^2}{h^2 - a^2} = -1$

$\Rightarrow\ h^2 + k^2 = a^2 - b^2$

Hence, required locus is $x^2 + y^2 = a^2 - b^2$.

Which is director circle of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

465 **(a,b,c,d)**

Given hyperbola can be written as

$$\frac{(x-1)^2}{16} - \frac{(y-1)^2}{9} = 1$$

$$\Rightarrow\ \frac{X^2}{16} - \frac{Y^2}{9} = 1$$

(where $X = x - 1, Y = y - 1$)

$$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{9}{16}} = \frac{5}{4}$$

Directrices are

$$X = \pm\frac{a}{e}$$

$\Rightarrow x - 1 = \pm\frac{16}{5} \Rightarrow x = \frac{21}{5}$ and $x = -\frac{11}{5}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{9}{2}$

The foci are given by

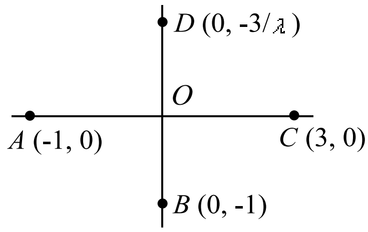
$X = \pm ae, Y = 0$

$\Rightarrow\ (6, 1), (-4, 1)$

466 **(b)**

Since A, B, C, D are concyclic

$OA \cdot OC = OB \cdot OD$



$\Rightarrow 1 \times 3 = 1 \times \left|\dfrac{3}{\lambda}\right|$

$\Rightarrow\ \lambda = \pm 1$

But when $\lambda = -1$, B and D will not lie on the circle simultaneously

$\therefore\ \lambda = 1$

467 **(a,d)**

Let (x_1, y_1) be a point, then $x_1^2 + 3y_1^2 = 37$ ...(i)

Equation of tangent at (x_1, y_1) is

$xx_1 + 3yy_1 = 37$

Slope of tangent $= -\frac{x_1}{3y_1}$

Then, slope of normal $= \frac{3y_1}{x_1} = \frac{6}{5}$ (given)

$\Rightarrow x_1 = \frac{5y_1}{2}$ ...(ii)

From Eqs. (i) and (ii), we get

$\dfrac{25y_1^2}{4} + 3y_1^2 = 37$

$\Rightarrow y_1^2 = 4$

$\therefore y_1 = \pm 2$

From Eq. (ii), $x_1 = \pm 5$

$\therefore$ Required points are $(5, 2)$ and $(-5, -2)$

468 **(b, d)**

Differentiating the equation of ellipse

$x^2 + 3y^2 = 37$ w.r.t. x,

$\dfrac{dy}{dx} = -\dfrac{x}{3y}$

Slope of the given line is $\frac{6}{5}$, which is normal to the ellipse

Hence, $\frac{3y}{5} = \frac{6}{5}$ or $2x = 5y$

Points in the option (b) and (d) are satisfying the above relation

469 **(a,c)**
Since the given circle is $(x-3)^2+(y-3)^2=9$ is touching both the axis, tangents from the origin are x-axis and y-axis or $y=0$ and $x=0$

470 **(b,c,d)**
Let $(x_1, y_1) \equiv (at^2, 2at)$
Tangent at this point is $ty = x + at^2$
Any point on this tangent is $\left(h, \left(\frac{h+at^2}{t}\right)\right)$
Chord of contact of this point with respect to the circle $x^2+y^2=a^2$ is
$hx + \left(\frac{h+at^2}{t}\right)y = a^2$
or $(aty - a^2) + h\left(x + \frac{y}{t}\right) = 0$
which is a family of straight lines passing through point of intersection of
$ty - a = 0$ and $x + \frac{y}{t} = 0$
So, the fixed point is $\left(-\frac{a}{t^2}, \frac{a}{t}\right)$
$\therefore x_2 = -\frac{a}{t^2}, y_2 = \frac{a}{t}$
Clearly, $x_1x_2 = -a^2, y_1y_2 = 2a^2$
Also, $\frac{x_1}{x_2} = -t^4$
$\frac{y_1}{y_2} = 2t^2$
$\Rightarrow 4\frac{x_1}{x_2} + \left(\frac{y_1}{y_2}\right)^2 = 0$

471 **(a,c)**
$|PS_1 - PS_2| = 2a$

$2a = K$

$\Rightarrow 2a = \sqrt{(24-0)^2+(7-0)^2} - \sqrt{12^2+5^2} = 12$

$\therefore a = 6$

$2ae = \sqrt{(24-5)^2+(12-7)^2}$

$= \sqrt{386}$

$\therefore e = \frac{\sqrt{386}}{12}$

$LR = \frac{2b^2}{a} = \frac{2a^2(e^2-1)}{a}$

$= 2 \times 6\left(\frac{386}{144} - 1\right) = \frac{121}{6}$

472 **(a,b,c)**
Locus of point of intersection of perpendicular tangents is director circle $x^2+y^2=a^2-b^2$

$e^2 = 1 + \frac{b^2}{a^2}$

If $a^2 > b^2$, then there are infinite (or more than 1) points on the circle $\Rightarrow e^2 < 2 \Rightarrow e < \sqrt{2}$

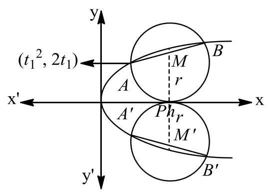
If $a^2 < b^2$, there does not exist any point on the plane $\Rightarrow e^2 > 2 \Rightarrow e > \sqrt{2}$

If $a^2 = b^2$, there is exactly one point (centre of the hyperbola) $\Rightarrow e = \sqrt{2}$

473 **(c,d)**
Let $A(t_1^2, 2t_1)$and $B(t_2^2, 2t_2)$

Then, coordinate of $M = \left(\frac{t_1^2+t_2^2}{2}, t_1+t_2\right)$ ie, mid point of chord AB

Let axis of parabola and circle touch each other at point P, then



$MP = t_1 + t_2 = r$ ... (i)

Also, $m_{AB} = \frac{2t_2 - 2t_1}{t_2^2 - t_1^2}$

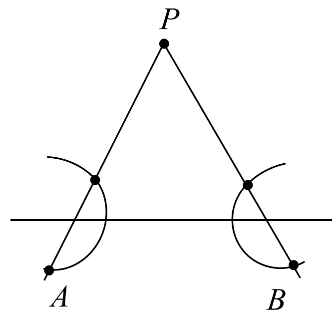
$= \frac{2}{t_2+t_1}$ (when AB is chord)

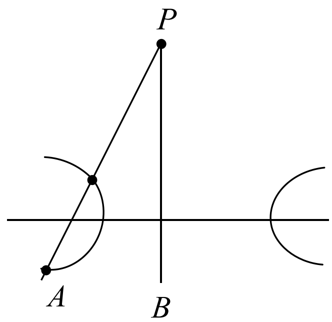
$\Rightarrow m_{AB} = \frac{2}{r}$ [from Eq. (i)]

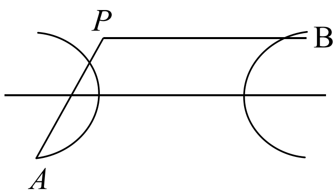
Also, $m_{A''B''} = -\frac{2}{r}$ (when $A''B''$is chord)

Hence, (c, d) are the correct options

474 **(b,c,d)**







Different cases have been shown in the above diagrams. Therefore, number of points can be 2, 3 or 4

475 **(a,b)**

Let $S \equiv (5, 12)$ and $S' \equiv (24, 7)$

And let $P \equiv (0, 0)$

Now, $SS' = \sqrt{(24-5)^2 + (7-12)^2}$

$= \sqrt{(361+25)} = \sqrt{386}$

For ellipse $SP + S'P = 2a$

$\Rightarrow 13 + 25 = 2a$

$\therefore 2a = 38$

$\because SS' = 2ae$

$\Rightarrow e = \dfrac{\sqrt{386}}{38}$

And for hyperbola $S'P - SP = 2a$

$\Rightarrow 25 - 13 = 2a$

$\therefore 2a = 12$

$SS' = 2ae$

$\Rightarrow e = \dfrac{SS'}{2a} = \dfrac{\sqrt{386}}{12}$

476 **(a)**

We have

$$\left|\sqrt{x^2 + (y-1)^2} - \sqrt{x^2 + (y+1)^2}\right| = K$$

which is equivalent to $|S_1P - S_2P| =$ constant, where $S_1 \equiv (0,1), S_2 \equiv (0,-1)$ and $P \equiv (x, y)$

The above equation represents a hyperbola. So, we have

$2a = K$

and $2ae = S_1S_2 = 2$

Where $2a$ is the transverse axis and e is the eccentricity. Dividing, we have

$$e = \frac{2}{k}$$

Since, $e > 1$ for a hyperbola, therefore $K < 2$

Also, K must be a positive quantity

So, we have $K \in (0, 2)$

477 **(a,b,c)**

$$3x^4 - 2(19y + 8)x^2 + [(19y)^2 + (10)^2 + (10)^2 + y^4 + y^4 + 8^2]$$

$$= 2(19 \times 10y + 10y^2 - 8y^2)$$

$$\Rightarrow 3x^4 - 2(19y + 8)x^2 + (19y - 10)^2 + (10 - y^2)^2 + (y^2 + 8)^2 = 0$$

$$\Rightarrow 3x^4 - 2(19y - 10 + 10 - y^2 + y^2 + 8)x^2 + (19y - 10)^2 + (10 - y^2)^2 + (y^2 + 8)^2 = 0$$

$$\Rightarrow [x^2 - (19y - 10)]^2 + [x^2 - (10 - y)^2]^2 + [x^2 - (y^2 + 8)]^2 = 0$$

$\Rightarrow x^2 - 19y + 10 = 0, x^2 - 10 + y^2 = 0$ and $x^2 - y^2 - 8 = 0$

The three curves represented by the give equation are $x^2 = 19y - 10$ (parabola), $x^2 + y^2 = 10$ (circle) and $x^2 - y^2 = 8$ (hyperbola)

478 **(a, b, c, d)**

$\left(\sqrt{3}x - 3\sqrt{3}\right)^2 + (2y + 4)^2 = k$

So no locus for $k < 0$

Ellipse for $k > 0$ and point for $k = 0$

479 **(b)**

Locus of point of intersection of perpendicular tangents is director circle for $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Equation of director circle is $x^2 + y^2 = a^2 - b^2$

which is real if $a > b$

480 **(a,b,c,d)**

Normal at point $P(2\sec\theta, 2\tan\theta)$ is

$$\frac{2x}{\sec\theta}+\frac{2y}{\tan\theta}=8$$

It meets the axes at points $G(4\sec\theta, 0)$ and g$(0, 4\tan\theta)$

Then

$$PG=\sqrt{4\sec^2\theta+4\tan^2\theta}$$

$$Pg=\sqrt{4\sec^2\theta+4\tan^2\theta}$$

$$PC=\sqrt{4\sec^2\theta+4\tan^2\theta}$$

$$Gg=\sqrt{16\sec^2\theta+16\tan^2\theta}$$

$$=2\sqrt{4\sec^2\theta+4\tan^2\theta}=2\,PC$$

481 **(a,b)**

Any point on this ellipse is $(\sqrt{6}\cos\phi, \sqrt{2}\sin\phi)$

Here centre is $(0, 0)$, so $6\cos^2\phi+2\sin^2\phi=4$

$\Rightarrow 2\cos^2\phi=1$

$\Rightarrow \cos^2\phi=\left(\frac{1}{\sqrt{2}}\right)^2=\cos^2\frac{\pi}{4}$

$\Rightarrow \phi=\frac{\pi}{4}$ or $\frac{3\pi}{4}$

482 **(a,d)**

Let $A(5, 12)$ and $B(24, 7)$ be two fixed points

So, $|OA-OB|=12$ and $|OA+OB|=38$

If the conic is an ellipse, then

$e=\frac{\sqrt{386}}{38}(\because 2ea=\sqrt{386}$ and $a=19)$

If the conic is a hyperbola, then

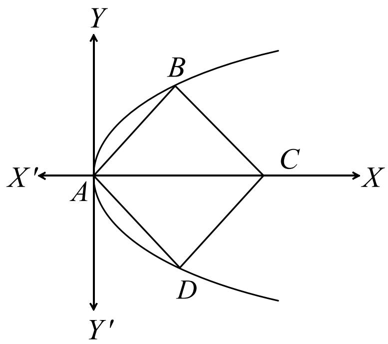
$e=\frac{\sqrt{386}}{12}$ $(\because 2ae=\sqrt{386}$ and $a=6)$

483 **(a,d)**

Area of the quadrilateral $=\sqrt{c}\times\sqrt{9+25-c}=15$

$\therefore\ c=9, 25$

484 **(a,b,c,d)**



AC is one diagonal along x-axis, then the other diagonal is BD where both B and D lie on parabola. Also slope of AB is $\tan\frac{\pi}{4}=1$. If B is $(at^2, 2at)$ then the slope of AB

$=\frac{2at}{at^2}=\frac{2}{t}=1$

$\therefore\ t=2$

Therefore, B is $(4a, 4a)$ and hence D is $(4a, -4a)$

Clearly, C is $(8a, 0)$

485 **(a,d)**

Equation of the radical axis is

$2\alpha x+2y+10=0$

i.e. $\alpha x+y+5=0$ ...(i)

Putting the value of y from Eq. (i) in the circle $x^2+y^2=9$,

we get

$(1+\alpha^2)x^2+10\alpha x+16=0$

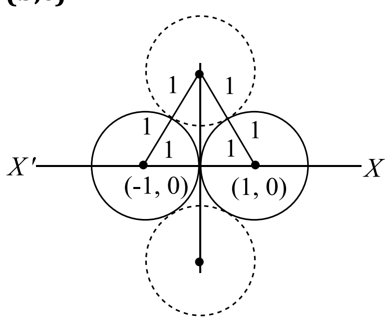
$\because$ Radical axis is tangent

$\therefore D=0$

$\Rightarrow 36\alpha^2-64=0$

$\Rightarrow a=\pm\frac{4}{3}$

486 **(b,c)**



The given circles are $x^2+y^2-2x=0, x>0$, and $x^2+y^2+2x=0, x<0$

From the above figure, the centres of the required circles will be $(0, \sqrt{3})$ and $(0, -\sqrt{3})$

$\therefore$ The equation of the circles are $(x-0)^2+(y\mp\sqrt{3})^2=1^2$

487 **(a,c)**

Equation of any line through the origin (0, 0) is

$y=mx$ ...(i)

If line (i) is tangent to the circle x^2+y^2-2rx-

$2hy + h^2 = 0,$

Then the length of $\perp$ from centre (r, h) on (i) = radius of circle

$\Rightarrow \dfrac{|mr - h|}{\sqrt{m^2+1}} = \sqrt{r^2 + h^2 - h^2}$

$\Rightarrow (mr - h)^2 = (m^2 + 1)r^2$

$\Rightarrow 0. m^2 + (2hr)m + (r^2 - h^2) = 0$

$\Rightarrow m = \infty, \dfrac{h^2 - r^2}{2hr}$

Substituting these values in Eq. (i), we get tangents

as $x = 0$ and $(h^2 - r^2)x - 2rhy = 0$

488 **(c,d)**

Ellipse is $16x^2 + 11y^2 = 256$

Equation of tangent at $\left(4\cos\theta, \frac{16}{\sqrt{11}}\sin\theta\right)$ is $16x$

$(4\cos\theta) + 11y\left(\dfrac{16}{\sqrt{11}}\sin\theta\right) = 256$

It touches $(x-1)^2 + y^2 = 4^2$

If $\left|\frac{4\cos\theta - 16}{\sqrt{16\cos^2\theta + 11\sin^2\theta}}\right| = 4$

$\Rightarrow (\cos\theta - 4)^2 = 16\cos^2\theta + 11\sin^2\theta$

$\Rightarrow 4\cos^2\theta + 8\cos\theta - 5 = 0$

$\Rightarrow \cos\theta = \dfrac{1}{2}$

$\therefore \theta = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$

489 **(a,c)**

Equation of any tangent to the circle $x^2 + y^2 = 25$ is of the form

$y = mx + 5\sqrt{1+m^2}$ (where m is the slope)

$\because$ It passes through $(-2, 11)$

$\therefore 11 = -2m + 5\sqrt{1+m^2}$

$\Rightarrow (11 + 2m)^2 = 25(1 + m^2)$

$\Rightarrow m = \dfrac{24}{7}, -\dfrac{4}{3}$

Therefore, equation of the tangents are

$24x - 7y + 125 = 0$

or $4x + 3y = 25$

490 **(a,b)**

$x^2 + 3y^2 = 3$

$\Rightarrow \dfrac{x^2}{3} + \dfrac{y^2}{1} = 1$

$\Rightarrow \dfrac{x^2}{\left(\sqrt{3}\right)^2} + \dfrac{y^2}{(1)^2} = 1$

Equation of tangent in terms of slope is

$y = mx \pm \sqrt{(3m^2 + 1)}$ ...(i)

Here, $m = -4$

($\because$ Tangent is perpendicular to $4y = x - 5$)

From Eq. (i), we get

$y = -4x \pm \sqrt{49}$

$\Rightarrow y = -4x \pm 7$

$\Rightarrow 4x + y \pm 7 = 0$

491 **(a,b,c)**

Equation of tangent to parabola $y^2 = 8x$ having slope m is $y = mx + \frac{2}{m}$

Options $(a), (b), (c)$ are tangents for $m = 1, 3, -\frac{1}{2}$ respectively

492 **(b,d)**

Differentiating $xy = 1$ w.r.t. x, we have

$\dfrac{dy}{dx} = -\dfrac{1}{x^2} < 0$

Hence, the slope of normal at any point $P(x_1, y_1)$ is

$x_1^2 > 0$

There, slope of the normal must always be positive.

Hence, possible lines are (b) and (d)

493 **(a,c)**

$2gg' + 2ff' = c + c'$

$\Rightarrow 2 \times 1 \times 0 + 2 \cdot k \cdot k = 6 + k$

$\Rightarrow 2k^2 - k - 6 = 0$

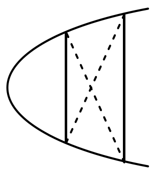
$\Rightarrow (2k + 3)(k - 2) = 0$

$\therefore k = 2, -\dfrac{3}{2}$

494 **(a,b)**

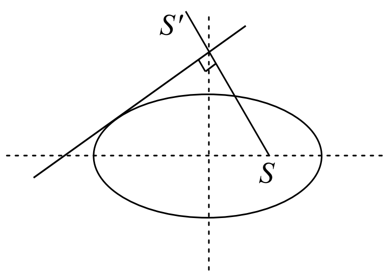
As a circle can intersect a parabola in four points, so quadrilateral may be cyclic.

The diagonals of the quadrilateral may be equal as the quadrilateral may be an isosceles trapezium



A rectangle cannot be inscribed in a parabola. So (C) is wrong

495 **(a, c)**



Let $S'(h, k)$ be the image

SS' cuts a tangent at a point which lies on the auxiliary circle of the ellipse

$\Rightarrow \left(\frac{h \pm 4}{2}\right)^2 + \frac{k^2}{4} = 25$

$\Rightarrow$ locus is $(x \pm 4)^2 + y^2 = 100$

496 **(a,b,d)**

For the ellipse,

$a = 5$ and $e = \sqrt{\frac{25-9}{25}} = \frac{4}{5}$

$\therefore ae = 4$

Hence, the foci are $(-4, 0)$ and $(4, 0)$

For the hyperbola,

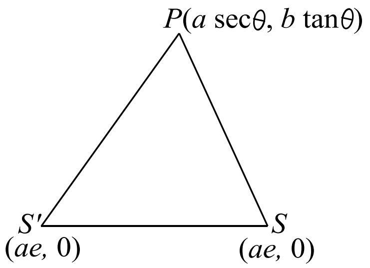
$ae = 4, e = 2$

$\therefore a = 2$

$b^2 = 4(4-1) = 12$

$\Rightarrow b = \sqrt{12}$

497 **(a,b)**



Let(h, k) be excentre, then

h
$= \frac{a(ae \sec\theta + a) - ae(ae\sec\theta - a) - 2ae(a\sec}{2ae\ (\sec\theta - 1)}$

$h = -a \Rightarrow x = -a$ (for $a\ \sec\theta > 0$)

Similarly, $x = a$ for $a\sec\theta < 0$

$\Rightarrow$ locus is $x^2 = a^2$

Again let (h, k) be excentre opposite $\angle S'$,

$\therefore h$
$= \frac{2a^2e\ \sec\theta + a^2e^2\sec\theta + a^2e + a^2e^2\sec\theta - a^2}{2a + 2ae}$

$\Rightarrow h = ae\ \sec\theta\ , k = \frac{2aeb\tan\theta}{2a + 2ae}$

$\Rightarrow$ locus is hyperbola

498 **(c,d)**

Equation of any tangent to $y^2 = 9x$ is of the form

$yt = x + \frac{9}{4}t^2$

$\because$ It passed through $(4, 10)$

$\therefore 10t = 4 + \frac{9}{4}t^2$

$\Rightarrow 9t^2 - 40t + 16 = 0$

$\therefore t = 4, \frac{4}{9}$

$\therefore$ Equation of tangents can be

$x - 4y + 36 = 0$

Or $9x - 4y + 4 = 0$

499 **(a,b,c,d)**

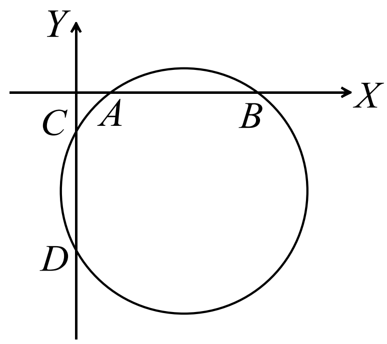
Given circle is

$x^2 + y^2 + 2hxy + 2gx + 2fy + c = 0$ ...(i)

For Eq. (i) to represent a circle, $h = 0$

$\therefore$ Given circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(ii)



For circle Eq. (ii) to pass through three quadrants only

i. $AB > 0\ \therefore g^2 - c > 0$

ii. $CD > 0\ \therefore g^2 - c > 0$

iii. Origin should lie outside circle Eq. (ii)

$\therefore\ c > 0$

Therefore, required conditions are $g^2 > c, f^2 > c, c > 0, h = 0$

500 **(b,d)**

$\frac{dx}{dy} = \frac{3y}{2x}$

$\Rightarrow \int 2xdx = \int 3ydy$

$\Rightarrow x^2 = \frac{3y^2}{2} + c$

Or $\frac{x^2}{3} - \frac{y^2}{2} = \frac{c}{3}$

If c is positive, then

$e = \sqrt{1 + \frac{2}{3}} = \sqrt{\frac{5}{3}}$

If c is negative, then

$$e = \sqrt{1+\frac{3}{2}} = \sqrt{\frac{5}{2}}$$

501 **(a,b,c,d)**

Solving $xy = c^2$ and $x^2 + y^2 = a^2$, we have

$$x^2 + \frac{c^4}{x^2} = a^2$$

$$\Rightarrow\ x^4 - a^2x^2 + c^4 = 0$$

$$\Rightarrow \Sigma x_i = 0 \text{ and } x_1\, x_2\, x_3\, x_4 = c^4$$

Similarly, if we eliminate y, then $\Sigma y_i = 0$ and $y_1\, y_2\, y_3\, y_4 = c^4$

502 **(a,c)**

$\because (x-3)^2 = -4y - k + 9 = -4\left(y + \frac{k-9}{4}\right)$

Equation of directrix is

$y + \frac{k-9}{4} = 1$

Or $y = 1 - \left(\frac{k-9}{4}\right) = \frac{13-k}{4}$

According to question, $\frac{13-k}{4} = -1$

$\Rightarrow k = 17$

Vertex is $\left(3, \frac{-k+9}{4}\right) = (3, -2)$

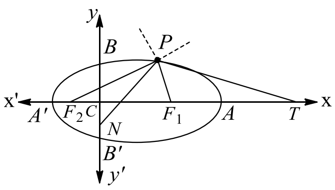
And focus is $(3, -3)$ $\quad (\because |a| = 1)$

503 **(a,c)**

If both foci are fixed, then the ellipse is fixed, that is, both the directrices can be decided (eccentricity is given). Similar is the case for option (c). Thus (a) and (c) are the correct choices. In the remaining cases, size of the ellipse is fixed, but its position is not fixed

504 **(a,c)**

$\because \frac{PF_1}{PF_2} = \frac{NF_1}{NF_2}$



$\therefore PN$ bisects the $\angle F_1PF_2$

$\because$ Bisectors are perpendicular to each other.

$\therefore PT$ bisects the angle $(180° - \angle F_1PF_2)$

505 **(a, c)**

Let $P(h, k)$ be the point of intersection of E_1 and E_2

$\Rightarrow \frac{h^2}{a^2} + k^2 = 1$

$\Rightarrow h^2 = a^2(1 - k^2)$ (i)

and $\frac{h^2}{1} + \frac{k^2}{a^2} = 1$

$\Rightarrow k^2 = a^2(1 - h)^2$ (ii)

Eliminating a from Eqs. (i) and (ii), we get

$\frac{h^2}{1-k^2} = \frac{k^2}{1-h^2}$

$\Rightarrow h^2(1-h^2) = k^2(1-k^2)$

$\Rightarrow (h-k)(h+k)(h^2+k^2-1) = 0$

Hence, the locus is a set of curves consisting of the straight lines

$y = x, y = -x$ and circle $x^2 + y^2 = 1$

506 **(a,b,c)**

Given, $y = x + 5$

On comparing with $y = mx + c$, we get

$m = 1, c = 5$

Option (a) Condition of tangency is

$c = \frac{a}{m} \Rightarrow 5 = \frac{5}{1}$

Which is true

Option (b) The equation of the ellipse is

$9x^2 + 16y^2 = 144$

$\Rightarrow \frac{x^2}{16} + \frac{y^2}{9} = 1$

$\because$ Condition of tengency is

$c^2 = a^2m^2 + b^2$

$\Rightarrow 25 = 16 \times 1 + 9 = 25$

Which is true.

Option (c) The equation of the hyperbola is

$\frac{x^2}{29} - \frac{y^2}{4} = 1$

$\because$ Condition of tangency for the hyperbola is

$c^2 = a^2m^2 - b^2$

$\Rightarrow 25 = 29 \times 1 - 4 = 25$

Which is true.

Option (d) Now, length f perpendicular from centre $(0, 0)$ to the line $y = x + 5$ is $\frac{|5|}{\sqrt{2}}$.

ie, length of perpendicular from the centre is not equal to the radius.

507 **(a)**

For ellipse $\frac{x^2}{27/12} - \frac{y^2}{27/4} = 1$

$$e = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$$

$a = 5$

The foci are $(\pm 3, 0)$

For hyperbola $\frac{x^2}{27/12} - \frac{y^2}{27/4} = 1$

$$e = \sqrt{1 + \frac{12}{4}} = 2$$

$$a = \frac{3}{2}$$

The foci are $(\pm 3, 0)$

Therefore, the two conics are confocal. Hence, curves are orthogonal

508 **(b)**
Circles $S_1: x^2 + y^2 - 4x - 6y - 8 = 0$ and
$S_2: x^2 + y^2 - 2x - 3 = 0$

$C_1(2,3), r_1 = \sqrt{21}, C_2(1,0), r_2 = 2$

$C_1C_2 = \sqrt{10}, r_1 + r_2 = 2 + \sqrt{21}, r_2 - r_1 = \sqrt{21} - 2$

Here $r_2 - r_1 < C_1C_2 < r_1 + r_2$. Hence, two circle intersect at two distinct points. Statement 2 is true, but does not explain statement 1

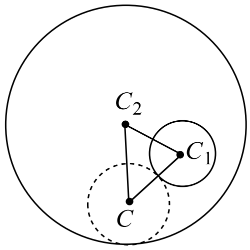
509 **(a)**
For parabola $y^2 = 4x$, $(4, 4)$ and $\left(\frac{1}{4}, -1\right)$ are extremities of the focal chord. Hence, tangents are perpendicular
Then obviously normals at these points are also perpendicular

510 **(b)**
1. Since the equation of circle is not in the from of $x^2 + y^2 - 2hx - 2ky + k^2 = 0$, then the circle does not touch y-axis
2. Since the equation of the circle is not in the from of $x^2 + y^2 - 2hx - 2ky + h^2 = 0$, then the circle does not touch x-axis

$\therefore$ Neither statement I nor statement II is true.

511 **(d)**
Statement 2 is true as in any triangle in-circle and three ex-circle touches the three sides of the triangle. But Statement 1 is false as given lines are concurrent, hence triangle is not formed

512 **(d)**
Let C_1, C_2 the centers and r_1, r_2 be the radii of the two circles. Let $S_1 = 0$ lies completely inside the circle. $S_2 = 0$ Let C and r be centre and radius of the variable circle, respectively



Then $CC_2 = r_2 - r$ and $C_1C = r_1 + r$
$\Rightarrow C_1C + C_2C = r_1 + r_2$ (constant)
$\Rightarrow$ Locus of C is an ellipse
$\Rightarrow S_2$ is true
Statement 1 is false (two circles are intersecting)

513 **(c)**
End points of double ordinate $x = 4$ of parabola $y^2 = 4x$ are $(4, \pm 4)$

$\Rightarrow t_1 = \pm 2$

$\Rightarrow t_2 = -t_1 - \frac{2}{t_1} = \pm 3$

$\Rightarrow P(9, 6)$ and $P'(9, -6)$

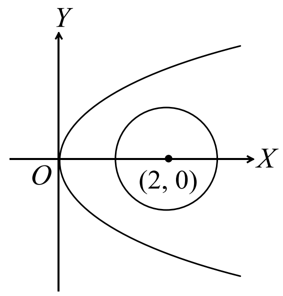
$\therefore PP' = 12$ unit

514 **(b)**
By formula $p_1p_2 = b^2 = 3$
Also foot of perpendicular lies on auxiliary circle of the ellipse
Thus both the statements are true. But statement 2 is not correct explanation of statement 1

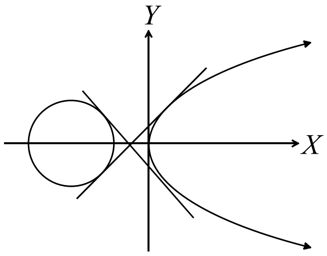
515 **(a)**
Let $y^2 = 4ax$ be a parabola. Consider a line $x = 4a$ (this is a double ordinate which is twice of latus rectum), which cuts the parabola at $A(4a, -4a)$ and $B(4a, -4a)$
Slope of $OA = 1$,
Slope of $OB = -1$, where O is given
Therefore, AB subtends 90° at the origin
$\Rightarrow$ Statement 2 is correct and it clearly explains statement 1

516 **(b)**
Both the statements are true, but statement 2 is not correct explanation of statement 1

517 **(b)**
Statement 2 is true as circle is lying inside parabola without intersecting it
But this cannot be considered the explanation of the statement 1, as even if they are not intersecting we can have common tangents as shown in the figure

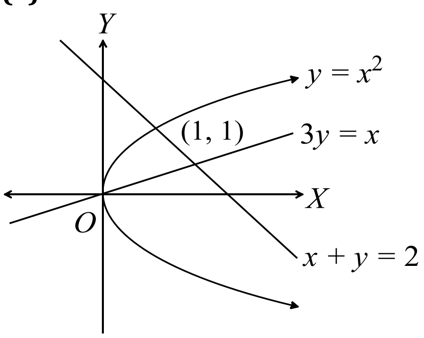


a.



b.

518 **(a)**



Point (α, α^2) lies on the parabola $y^2 = x$
As shown in the figure, we have to find the value of α for which the part of the parabola lies inside the triangle formed by three lines
Now line $x + y = 2$ meets the parabola at point $(0, 0)$ and $(1, 1)$.
Hence, $\alpha \in (0,1)$

519 **(b)**

$\because 2\tan^{-1}\left(\frac{b}{a}\right) = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}\left(\frac{b}{a}\right) = \frac{\pi}{4}$

$\Rightarrow \frac{b}{a} = 1$ or $a = b$

Then hyperbola convert in $x^2 - y^2 = a^2$

and $e = \sqrt{\left(1 + \frac{a^2}{a^2}\right)} = \sqrt{2}$

521 **(d)**
Statement 1 is false as locus of (x, y) is a line segment joining points $(2, 0)$ and $(-2, 0)$

522 **(c)**
$\because$ Locus of point of intersection of perpendicular tangents to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is director circle
$x^2 + y^2 = a^2 + b^2$

$\therefore (\lambda, 3)$ lie on $x^2 + y^2 = 4 + 9$

$\Rightarrow \lambda^2 + 9 = 4 + 9$

$\therefore \lambda = \pm 2$

524 **(a)**
Here $(O_1O_2)^2 = t^2 + (t^2 + 1)^2 = t^4 + 3t^2 + 1 \geq 0$

$\Rightarrow O_1O_2 \geq 1$ and $|r_1 - r_2| = 1$

$\Rightarrow O_1O_2 \geq |r_1 - r_2|$ hence the two circles have at least one common tangent

525 **(c)**
Statement 2 is false, as axis of parabola is normal to parabola which passes through the focus.
However, normal other axis never passes through focus.
Statement 1 is correct as $x - y - 5 = 0$ passes through focus $(3, -2)$, hence it cannot be normal

526 **(b)**
Any normal to $y^2 = 4x$ is
$y = -tx + 2t + t^3$
If only one normal can be drawn to parabola from $(\lambda, \lambda + 1)$, then $\lambda < 2$
Hence, statement 1 is true
Statement 2 is also true as $(\lambda + 1)^2 > 4\lambda$ is true $\forall \lambda \in R - \{1\}$, but does not explain statement 1, as it is not necessary that from every outside points only one normal can be drawn

527 **(b)**
Statement 1 is correct.

Also statement 2 is true as asymptotes are perpendicular, they are bisectors of transverse and conjugate axes of hyperbola

But statement 2 does not explain statements 1, as in hyperbolas other than rectangular hyperbolas asymptotes are not bisectors of transverse of transverse and conjugate axes

528 **(c)**
The intersection of line $bx - ay = 0$ and the hyperbola

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{x^2}{a^2}-\left(\frac{b}{a}\right)^2\cdot\frac{x^2}{b^2}=1$

$\Rightarrow 0=1$

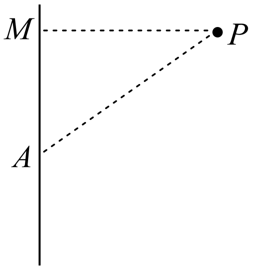
Hence, no solution exist

But Statement II is false

529 **(d)**

$|ay-bx|=c\sqrt{(x-a)^2+(y-b)^2}$

$\Rightarrow \frac{|ay-bx|}{\sqrt{a^2+b^2}}=\frac{v}{\sqrt{a^2+b^2}}\sqrt{(x-a)^2+(y-b)^2}$



or $PM=kPA$, where m is the length of perpendicular from P on the line $ay-bx=0$ and PA is the length of line segment joining P to the point $A(a,b)$ and A lies through A inclined at an angle $\sin^{-1}\frac{c}{\sqrt{a^2+b^2}}$ to the given line (provided $c<\sqrt{a^2+b^2}$)

530 **(a)**

Common chord of two orthogonal circles subtend supplementary angles at the centre and so complementary angles on the circumferences of the two circle

∴ Both the statements are correct and statement 2 is the correct explanation of statement 1

531 **(a)**

We know that the radical axis of the circle is the locus of point from which length of tangents to given two circles is same, also it the locus of the centre of the circle which intersect the given two circles orthogonally

Now radical axis of the given two circles is $2x+y-4=0$. Any point on this line is $(t,4-2t), t\in R$

Hence, both the statements are true and statement 2 is correct explanation of statement

532 **(a)**

Chord of contact of the ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ with respect to point $(8,6)$ is

$\frac{8x}{4}+\frac{6x}{2}=1$

or $2x+3y=1$

Hence, statement 1 is correct, also statement 2 is correct and explains the statement 1

533 **(c)**

Statement 1 is true because common chord itself passes through origin

Statement 2 is false (common chord is $x-y=0$)

534 **(a)**

Since, the tangents are perpendicular

So, locus of perpendicular tangents to circle $x^2+y^2=169$ is a director circle having equation

$x^2+y^2=2\times 169=338$

535 **(d)**

$9x^2+4y^2-18x-24y+9=0$

$\Rightarrow 9(x-1)^2+4(y-3)^2=36$

$\Rightarrow \frac{(x-1)^2}{2^2}+\frac{(y-3)^2}{3^2}=1$

Here, $b>a$

∴ Sum of focal distances of a point is $2b=6$

536 **(a)**

Two circles touch each other $C_1C_2=|r_1\pm r_2|$

$\Rightarrow \sqrt{p^2+q^2}=\sqrt{p^2-r}=\sqrt{q^2-r}$

$\Rightarrow p^2+q^2=p^2-r+q^2-r+2\sqrt{(p^2-r)(q^2-r)}$

$\Rightarrow \frac{1}{r}=\frac{1}{p^2}+\frac{1}{q^2}$

537 **(d)**

Statement 2 is correct .

Then length of the focal chord according to the statement 1 is

$4(2)\left(\frac{4}{3}\right)=\frac{32}{3}$

538 **(a)**

$x^2+y^2-2x-2ay-8=0$

$\Rightarrow (x^2+y^2-2x-8)-2a(y)=0$

$S+\lambda L=0$

Solving circle $x^2 + y^2 - 2x - 8 = 0$ and line $y = 0$

$\therefore x^2 - 4x + 2x - 8 = 0$

$\therefore x = 4, x = -2$

So, $(4, 0)$, $(-2, 0)$ are the points of intersection which lie on x-axis

539 **(a)**
We know that chords of contact of given circle generated by any point on given line passes through the fixed point, as they form family of straight lines, hence both the statements are true and statement 2 is the correct explanation of statement 1

540 **(d)**
Now, $4x^2 - 3y^2 = 12$

$\Rightarrow \dfrac{x^2}{3} - \dfrac{y^2}{4} = 1$

Then, director circle is $x^2 + y^2 = 3 - 4 = -1$ which does not exist for existence $a > b$

541 **(a)**
Statement 2 is true as it is one of the properties of ellipse

Ellipse is $\frac{x^2}{9} + \frac{y^2}{4} = 1$

Focus $\equiv (\sqrt{5}, 0)$, $e = \frac{\sqrt{5}}{3}$

One of the points on the ellipse $\equiv \left(\frac{3}{\sqrt{2}}, \frac{2}{\sqrt{2}}\right)$

Equation of the circle as the diameter joining the points $(3/\sqrt{2}, 2/\sqrt{2})$ and focus $(\sqrt{5}, 0)$ is

$(x - \sqrt{5})(\sqrt{2}x - 3) + y(\sqrt{2}y - 2) = 0$

Hence, statement 1 is true and statement 2 is correct explanation of statement 1

542 **(d)**
Given points are collinear, hence circle is not possible. Hence, statement 1 is false, however statement 2 is true

543 **(a)**
Statement 2 is correct as ellipse is a central conic and it also explains statement 1

544 **(b)**
Given parabola can be rewritten as

$\Rightarrow (y + 3)^2 = 2(x + 2)$

Vertex of parabola is $(-2. -3) = (h, k)$

Equation of directrix is $x - h + a = 0$

$\Rightarrow x + 2 + \dfrac{1}{2} = 0$

$\Rightarrow 2x + 5 = 0$

$\therefore$ I is true and II is false

545 **(a)**
Let the equation of circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$

Since, it passes through $(1, 0)$ and $(0, 1)$, then

$1 + 0 + 2g + 0 + c = 0$

$\Rightarrow g = -\dfrac{(1 + c)}{2}$

and $0 + 1 + 0 + 2f + c = 0$

$\Rightarrow f = -\dfrac{(1 + c)}{2}$

$\therefore$ Radius $= \sqrt{(g^2 + f^2 - c^2)}$

$= \sqrt{\left(\dfrac{(1 + c)^2}{4} + \dfrac{(1 + c)^2}{4} - c\right)} = \sqrt{\left(\dfrac{1 + c^2}{2}\right)}$

For minimum radius, c must be equal to zero

$\therefore$ Radius $= \sqrt{\dfrac{1}{2}} = \dfrac{1}{\sqrt{2}}$

Then $g = -\frac{1}{2}, f = -\frac{1}{2}, c = 0$

$\therefore$ Circle is $x^2 + y^2 - x - y = 0$

Which pass through origin

547 **(b)**
Equation of tangent of the given circle is

$y = mx \pm \sqrt{(1 + m^2)}$

If it passes through $(-3, -1)$, then, we have

$-1 = -3m \pm \sqrt{(1 + m^2)}$

$\Rightarrow 1 + m^2 = (3m - 1)^2$

$\Rightarrow 8m^2 - 6m = 0$

$\therefore m = 0, m = \frac{3}{4}$

Thus, reflected ray is $y + 1 = \frac{3}{4}(x + 3)$

$\Rightarrow 4y - 3x = 5$

549 **(a)**

Let normals at points $A(at_1^2, 2at_1)$ and $C(at_3^2, 2at_3)$ meets the parabola again at points $B(at_2^2, 2at_2)$ and $D(at_4^2, 2at_4)$, then $t_2 = -t_1 - \frac{2}{t_1}$ and $t_4 = -t_3 - \frac{2}{t_3}$

Adding $t_2 + t_4 = -t_1 - t_3 - \frac{2}{t_1} - \frac{2}{t_3}$

$\Rightarrow t_1 + t_2 + t_3 + t_4 = -\frac{2}{t_1} - \frac{2}{t_3}$

$\Rightarrow \frac{1}{t_1} + \frac{1}{t_3} = 0$

$\Rightarrow t_1 + t_3 = 0$

Now, point of intersection of tangent at A and C will be $\left(at_1t_3, a(t_1 + t_3)\right)$

Since $t_1 + t_3 = 0$, so this point will lie on x-axis, which is axis of parabola

550 **(b)**

Equation of common chord of these circles is

$$(x^2 + y^2 - 6x - 4y + 9) - (x^2 + y^2 - 8x - 6y + 23) = 0$$

$\Rightarrow 2x + 2y - 14 = 0$

Or $x + y - 7 = 0$

Since, centre of the second circle *ie*, $(4, 3)$ lie on it. Hence, $x + y - 7 = 0$ is a diameter of second circle and hence, first circle bisects the circumference of the second circle

551 **(d)**

$\because 5x^2 + 9y^2 - 54y + 36 = 0$

$\Rightarrow 5x^2 + 9(y - 3)^2 = 45$

$\Rightarrow \frac{x^2}{3^2} + \frac{(y - 3)^2}{(\sqrt{5})^2} = 1$

$\therefore$ Length of major axis $= 2 \times 3 = 6$

And length of minor axis $= 2 \times \sqrt{5} = 2\sqrt{5}$

552 **(a)**

Since point lies inside the circle

$\Rightarrow a^2 + a^2 - 4a - 2a - 8 < 0$

$\Rightarrow a^2 - 3a - 4 < 0$

$\Rightarrow -1 < a < 4$

553 **(a)**

Given points are $A(1, 1)$, $B(2, 3)$ and $C(3, 5)$ which are collinear as slope AB =slope $BC = 2$. Hence, statement 2 is true

Chord of contact are concurrent then

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Hence, point (x_1, y_1), (x_1, y_1) and (x_3, y_3) are collinear

554 **(a)**

Tangent to hyperbola having slope m is

$y = mx \pm \sqrt{4m^2 - 16}$

which is real line if

$4m^2 - 16 > 0 \Rightarrow m^2 > 4 \Rightarrow m \in (-\infty, -2) \cup (2, \infty)$

Hence, statement 2 is correct

Also statement 1 is correct and statement 2 is correct explanation of statement 1

555 **(d)**

Statement 2 is true,

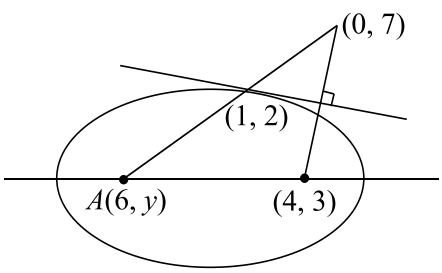


Image of $(4, 3)$in the line $x + y - 3 = 0$ is

$\frac{x - 4}{1} = \frac{y - 3}{1} = -2\frac{(4 + 3 - 3)}{2} = -4$

$\Rightarrow x = 0$ and $y = 7$

Now points $(0, 7)$, $(1, 2)$ and $(6, y)$ are collinear

$\Rightarrow \frac{7 - 2}{0 - 1} = \frac{y - 2}{6 - 1}$

$\Rightarrow y = 27$

556 **(d)**

Statement 1: Locus of point of intersection of only perpendicular lines is a circle, and other vertices B and C do not form a circle
Statement 2 is obvious (standard definition)

557 **(c)**
$\because y^2 - 9x^2 + 1 = 0$

$\Rightarrow 9x^2 - y^2 - 1 = 0$

Coordinates of the point are (5,–4).

Then, $9(5)^2 - (-4)^2 - 1 = 225 - 17 = 208 > 0$

$\therefore$ Point $(5, -4)$ is inside the hyperbola.

558 **(d)**
If $P \equiv (at_1^2, 2at_1)$ and $Q \equiv (at_2^2, 2at_2)$, vertex $A \equiv (0, 0)$

$\therefore$ Slope of $AP \times$ Slope of AQ

$$= \left(\frac{2at_1 - 0}{at_1^2 - 0}\right) \times \left(\frac{2at_2 - 0}{at_2^2 - 0}\right)$$

$$= \frac{2}{t_1} \times \frac{2}{t_2} = \frac{4}{-1} = -4 \neq -1 \quad (\because t_1 t_2 = -1)$$

$$\therefore \angle PAQ \neq \frac{\pi}{2}$$

559 **(d)**
Point of intersection of $x + 7 = 3$ and $x - y = 1$ is $(2, 1)$

560 **(b)**
Given hyperbola is

$$\frac{x^2}{3} - \frac{y^2}{2} = 1$$

Now line having slope $m = 3$ is tangent to the hyperbola. So, its equation is

$$y = 3x \pm \sqrt{3(3)^2 - 2}$$

or $y = 3x \pm 5$

Hence, statement 1 is correct

Also statement 2 is correct, but information is not enough to get the equation of tangents

561 **(b)**
Statement II is true. Centre of required circle will be image of centre of given first circle about the line mirror and radius of the required circle will be same as that of radius of first circle.

$\Rightarrow$ Given line mirror will be perpendicular bisector of the line segment joining their centres. Also, radius of first circle

$= \sqrt{64 + 144 - 183} = 5$

and that of second circle $= \sqrt{256 + 4 - 235} = 5$

562 **(d)**
Area of the triangle formed by the intersection points of tangents at point $A(t_1), B(t_2)$ and $C(t_3)$ is

$$\frac{1}{2}|t_1 - t_2||t_2 - t_3||t_3 - t_1| \neq 0$$

Hence, statement 1 is wrong. However, statement 2 is correct

563 **(d)**
Statement 1 is false
Since here $t^2 = 4$
Therefore, the normal chord subtends a right angle at the focus (not at the vertex)
However, statement 2 true (a standard result)

564 **(c)**
Distance between two parallel lines L_1 and L_2 is

$$d = \left|\frac{p - 3 - p - 3}{\sqrt{4 + 9}}\right| = \frac{6}{\sqrt{13}}$$

And radius of given circle= q clearly, $d < 2$

$\Rightarrow$ Statement II is false and statement I is true

565 **(a)**
Locus of point of intersection of perpendicular tangents is director circle, which is $x^2 + y^2 = a^2 + b^2$
Now line $px + qy + r = 0$ may intersect this circle maximum at two points.
Thus there can be maximum two points on the line from which perpendicular tangents can be drawn to the ellipse

566 **(c)**

$$\frac{x^2}{5} + \frac{(y - 3)^2}{9} = 1$$

Ends of the major axis are $(0, 6)$ and $(0, 0)$
Equation of tangent at $(0, 6)$ and $(0, 0)$ is $y = 6$ and $y = 0$
Hence, statement 1 is true.
But statement 2 is false, as tangents at the ends of major axis may be lines parallel to y-axis when $a < b$

567 **(a)**

It is fundamental property of an ellipse

568 **(c)**

Statement 2 is false (locus of P may be a line segment also). Statement 1 is true

569 **(b)**

$\because$ Director circle is the locus of point of intersection of perpendicular tangents.

Any tangent in terms of m of $\frac{x^2}{a^2} - \frac{y^2}{b^2} + 1 = 0$ is

$y = mx \pm \sqrt{(b^2 - a^{2m})}$

$\Rightarrow (y - mx)^2 = b^2 - a^2m^2$

$\Rightarrow m^2(x^2 + a^2) - 2\,mxy + y^2 - b^2 = 0$

$\because m_1m_2 = -1$

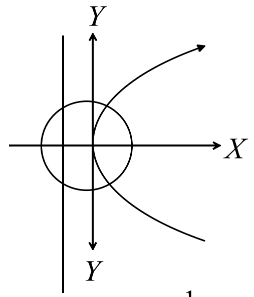
$\Rightarrow \dfrac{y^2 - b^2}{x^2 + a^2} = -1$

$\Rightarrow x^2 + y^2 = b^2 - a^2, b < a$

Also, director circle of $\frac{x^2}{25} - \frac{y^2}{9} = 1$ is

$x^2 + y^2 = 25 - 9 = 16.$

570 **(a)**



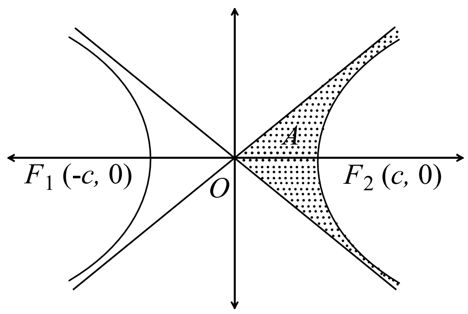
Directrix, $x = \frac{1}{2}$

Statement 2 is true as it is the property of the parabola.
Now such points exists on the circle $x^2 + y^2 = a^2$ if it meets the directrix at least on point, for which radius of the circle $a \geq 1/2$

571 **(d)**

Statement 1 is false as points in region A lie below the asymptote

$y = \dfrac{b}{a}x \Rightarrow \dfrac{x_1^2}{a^2} - \dfrac{y_1^2}{b^2} > 0$



Statement 2 is true (standard result). Indeed for points in region A

$0 < \dfrac{x_1^2}{a^2} - \dfrac{y_1^2}{b^2} < 1$

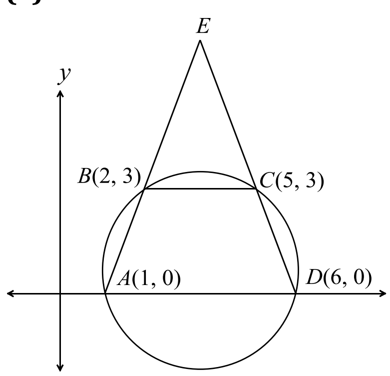
572 **(b)**

For circle $x^2 + y^2 = 144$, centre $C_1(0,0)$ and radius $r_1 = 12$

For circle $x^2 + y^2 - 6x - 8y = 0$, centre $C_2 = (3,4)$ and radius $r_2 = 5$

Now $C_1C_2 = 5$ and $r_1 - r_2 = 7$, thus $C_1C_2 < r_1 - r_2$, hence one circle is completely lying inside other without touching it, hence there is no common tangent. Therefore, statement 1 is true. Therefore, both the statements are true but statement 2 is not correct explanation of statement 1

573 **(a)**



From the figure, it is clear that $ABCD$ is isosceles trapezium as $AB = CD$. Also ΔEAD is isosceles $\Rightarrow EA \times EB = EC \times ED$

Hence, both the statements are correct and statement 1 is correct explanation of statement 1

574 **(c)**

Any tangent having slope m is

$y = m(x + a) + \dfrac{a}{m}$

Or $y = mx + am + \frac{a}{m}$

Is tangent to the given parabola for all
$m \in R - \{0\}$
Hence, statement 2 is false
However, statement 1 is true as when $m = 1$, tangent is
$y = x + 2a$

575 **(c)**
The two circles having centres at C_1 and C_2 and radii r_1 and r_2 respectively intersect at two distinct points, if

$|r_1 - r_2| < C_1C_2 < r_1 + r_2$

$\Rightarrow$ Statement II is not true

Here, $C_1 = (0, 0), C_2 = (4, 0)$

and $r_1 = 2, r_2 = 3$

$\therefore C_1C_2 = 4, |r_1 - r_2| = 1, r_1 + r_2 = 5$

That satisfies above condition

576 **(a)**
Let $(t, b - t)$ be a point on the line $x + y = b$, then equation of chord whose mid point $(t, b - t)$ is

$\frac{tx}{2a^2} + \frac{(b-t)y}{2b^2} - 1 = \frac{t^2}{2a^2} + \frac{(b-t)^2}{2b^2} - 1$ ...(i)

$(a, -b)$ lies on Eq.(i), then

$$\frac{ta}{2a^2} + \frac{b(b-t)}{2b^2} = \frac{t^2}{2a^2} + \frac{(b-t)^2}{2b^2}$$

$\Rightarrow t^2(a^2 + b^2) - ab(3a + b)t + 2a^2b^2 = 0$

$\because t$ is real

$\therefore B^2 - 4AC \geq 0$

$\Rightarrow a^2b^2(3a + b)^2 - 4(a^2 + b^2)2a^2b^2 \geq 0$

$\Rightarrow 9a^2 + 6ab + b^2 - 8a^2 - 8b^2 \geq 0$

$\therefore a^2 + 6ab - 7b^2 \geq 0$

577 **(b)**
The equation of the director circle of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is $x_1 + x_2 = 13$

Statement II is also true but it is not a correct explanation for Statement I

578 **(d)**
Locus of point of intersection of perpendicular tangents is director circle.
If there exists exactly one such point on the line $3x + 4y + 5\sqrt{5 = 0}$, then it must touch the director circle
$x^2 + y^2 = a^2 + 1$
$\Rightarrow 5 = a^2 + 1$
$\Rightarrow a^2 = 4$
$\Rightarrow a = 2$
Hence, eccentricity $= \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$

579 **(a)**
Let the foot of normal be $P(at^2, 2at)$ then
$ax + by + c = 0$
and $y = -tx + 2at + at^3$
are identical line,
$\Rightarrow \frac{1}{b} = \frac{t}{a} = \frac{2at + at^3}{-c}$
$\Rightarrow t = \frac{a}{b}$
Thus, 'P' is $\left(\frac{a^3}{b^2}, \frac{2a^2}{b}\right)$
Hence, equation of required tangent is
$ty = x + at^2$
Or $\frac{a}{b}y = x + a\left(\frac{a}{b}\right)^2$
Or $y = \frac{b}{a}x + \frac{a^2}{b}$

580 **(a)**
First of all will verify statement II.

Let $P(a \sec \theta, a \tan \theta)$ be any point on $x^2 - y^2 = a^2$, then

$SP \cdot S'P = (ea \sec \theta - a)(\text{ea} \sec \theta + a)$

$= e^2a^2 \sec^2 \theta - a^2$

$= 2a^2 \sec^2 \theta - a^2 \quad (\because e = \sqrt{2})$

and $CP^2 = a^2 \sec^2 \theta + a^2 \tan^2 \theta$

$= a^2 \sec^2 \theta + a^2 \sec^2 \theta - a^2$

$= 2a^2 \sec^2 \theta - a^2$

$\Rightarrow SP \cdot S'P = CP^2$

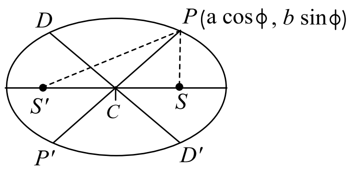
$\therefore$ Statement II is true.

If we put $a = \sqrt{2}, \theta = \frac{\pi}{4}$, then statement I is verified.

581 **(a)**
Let $y = mx$ be any chord through $(0, 0)$. This will meet conic at points whose x-coordinates are given by $x^2 + m^2x^2 + mx^2 = 1$

$\Rightarrow (1+m+m^2)x^2-1=0$
$\Rightarrow x_1+x_2=0$
$\Rightarrow \dfrac{x_1+x_2}{2}=0$
Also $y_1=mx_1, y_2=mx_2$
$\Rightarrow y_1+y_2=m(x_1+x_2)=0$
$\Rightarrow \dfrac{y_1+y_2}{2}=0$
$\Rightarrow$ Midpoint of chord is $(0,0)\forall m$
Hence, statement 1 is true as $(0,0)$ is also centre of the ellipse
Statement 2 is fundamental property of the ellipse, hence statement 2 is correct explanation of statement 1

582 **(b)**
Let PCP'' and DCD'' be the conjugate diameter of an ellipse and let the eccentric angle of P is ϕ, then coordinate of P is $(a\cos\phi, b\sin\phi)$



$\therefore$ coordinate of Q is $(-a\sin\phi, b\cos\phi)$

Let S and S'' be two foci of the ellipse

Then, $SP\cdot S''P=(a-ae\cos\phi)\cdot(a+ae\cos\phi)$

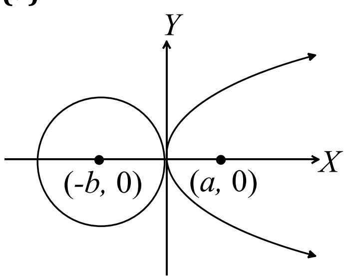
$=a^2-a^2e^2\cos^2\phi \quad [\because b^2=a^2(1-e^2)]$

$=a^2-(a^2-b^2)\cos^2\phi \quad (\because a^2-b^2=a^2e^2)$

$=a^2\sin^2\phi+b^2\cos^2\phi$

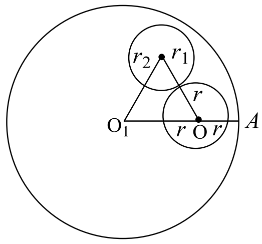
$=CD^2$

583 **(a)**



As shown in the figure, circle and parabola touch when a and b have same sign.
Now for
$f(x)=x^2-(b+a+1)x+a,$
$\Rightarrow f(0)=a$
and $f(1)=1-(b+a+1)+a=-b$
$\Rightarrow f(0)\cdot f(1)=-ab<0$
Hence, one root lies in (0, 1)
$\Rightarrow$ Both the statements are true and statement 2 is correct explanation of statement 1

584 **(a)**
Let the given circles C_1 and C_2 have centres O_1 and O_2 with radii r_1 and r_2 respectively. Let centre of circle C is at O radius is r



$\because OO_2=r+r_2$

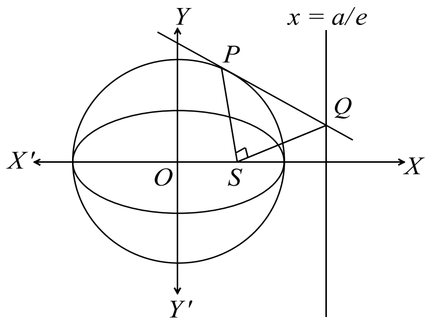
$OO_1=r_1-r$

$\Rightarrow OO_1+OO_2=r_1+r_2$

Which is greater than O_1O_2 as $O_1O_2<r_1+r_2$

$\therefore$ Locus of O is an ellipse with foci O_1 and O_2

585 **(a)**
Statement 2 is true as it is one of the properties of the ellipse.

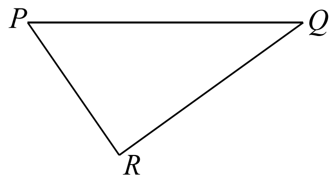


Circle with minimum radius having PQ as chord when PQ is diameter of the circle, hence as shown in the figure it passes through the focus

586 **(a)**
The centre of circle is (h,h) and radius $=h$

$\Rightarrow$ The circle is touching the co-ordinates axes

587 **(a)**
Let P be the position of the gun and Q be the position of the target. Let u be the velocity of sound, v be the velocity of bullet and R be the position of the man. Then, we have



$\frac{PR}{u} = \frac{QR}{u} + \frac{PQ}{v}$

$\Rightarrow \frac{PR}{u} - \frac{QR}{u} = \frac{PQ}{v}$

$\Rightarrow PR - QR = \frac{u}{v} PQ = \text{constant}$

and $\frac{u}{v} PQ < PQ$

Therefore, locus of R is a hyperbola

588 **(c)**

$\because x - y + 2 = 0$ and $x + y - 6 = 0$

Are perpendicular to each other

$\therefore 2\left(\frac{x-y+2}{\sqrt{2}}\right)^2 = 8\sqrt{2}\left(\frac{x+y-6}{\sqrt{2}}\right)\sqrt{2}$

$\therefore \left(\frac{x-y+2}{\sqrt{2}}\right)^2 = 8\left(\frac{x+y-6}{\sqrt{2}}\right)$

Let $\frac{x-y+2}{\sqrt{2}} = Y, \frac{x+y-6}{\sqrt{2}} = X$

$\therefore Y^2 = 8X$

$\therefore$ Length of latusrectum $= 8$

589 **(a)**

Differentiating $y^2 = 8a$ w.r.t. x., we have

$2y\frac{dy}{dx} = 8$

$\Rightarrow \frac{dy}{dx} = \frac{4}{y}$

Now slopes of tangents at $(8, -8)$ and $\left(\frac{1}{2}, 2\right)$ are $-\frac{1}{2}$ and 2. Hence, tangents are perpendicular.

Also tangents at the extremities of the focal chord are perpendicular and meet on the directrix. Hence, both the statements are true and statement 2 is correct explanation of statement 1

590 **(c)**

Statement 2 is false because line joining centres may not be parallel to common tangents

Statement 1 can be proved easily by using distance between centres = sum of radii

591 **(b)**

Any tangent having slope m is

$y = mx + \frac{a}{m}$

Or $y = mx + \frac{9/4}{m}$

It passes through the point (4, 10), then

$10 = 4m + \frac{9/4}{m}$

$\Rightarrow 16m^2 - 40m + 9 = 0$

$\Rightarrow m_1 = \frac{1}{4}, m_2 = \frac{9}{4}$

$\Rightarrow$ Statement 1 is correct

Also statement 2 is correct but it does not say anything about slope of the tangents, hence it is not correct explanation of statement 1

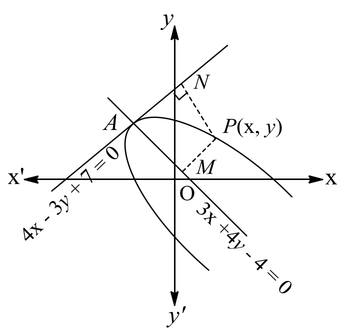
592 **(d)**

The statement 2 is well-known result, but if applied to the data give in statement 1 will yield $5x - 9y + 46 = 0$

$\Rightarrow$ Statement 1 is false, statement 2 is true

593 **(d)**

Let $P(x, y)$ be any point on the parabola and let PM and PN are perpendiculars from P on the axis and tangent at the vertex respectively, then



$(PM)^2 = (\text{latusrectum})(PN)$

$\Rightarrow \left(\frac{3x+4y-4}{\sqrt{\{3^2+4^2\}}}\right)^2 = 4\left(\frac{4x+3y+7}{\sqrt{\{4^2+(-3)^2\}}}\right)$

$\Rightarrow Y^2 = 4\rho X$

$\rho = 1, Y = \frac{3x+4y-4}{5}, X = \frac{4x-3y+7}{5}$

$\therefore$ Directrix is $X + \rho = 0$

$\Rightarrow \frac{4x-3y+7}{5} + 1 = 0$

Or $4x - 3y + 12 = 0$

594 **(a)**

Let hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, then pair of asymptotes is $\frac{x^2}{a^2}-\frac{y^2}{b^2}+\lambda=0$, then $\Delta=0$,

$\therefore\ \lambda=0$

$\therefore$ Pair of asymptotes is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$

And equation of conjugate hyperbola is $-\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Then, pair of asymptotes is $-\frac{x^2}{a^2}+\frac{y^2}{b^2}+\mu=0$.

Then, $\Delta=0$. $\therefore\ \mu=0$

$\therefore$ Pair of asymptotes is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$.

595 **(b)**
Obviously, statement 2 is true, but it is not the correct explanation of statement 1 as A, A', B, B' form an isosceles trapezium, hence points are concyclic

596 **(c)**
Let $C_1\equiv(-g,-f), r_1=\sqrt{(g^2+f^2)}$

and $C_2\equiv(-g',-f'), r_2=\sqrt{(g'^2+f'^2)}$

$\therefore |C_1C_2|=r_1\pm r_2$

$$\Rightarrow \sqrt{(g-g')^2+(f-f')^2} = \sqrt{(g^2+f^2)}\pm\sqrt{(g'^2+f'^2)}$$

$$\Rightarrow (g-g')^2+(f-f')^2 = g^2+f^2+g'^2+f'^2 \mp 2\sqrt{(g^2+f^2)(g'^2+f'^2)}$$

or $(-gg'-ff')^2=(g^2+f^2)(g'^2+f'^2)$

$\Rightarrow\ g^2f'^2+g'^2f'^2-2gg'ff'=0$

or $(gf'-g'f)^2=0$

$\Rightarrow gf'=g'f$

And line joining centres may not be parallel to common tangents

597 **(d)**
Statement II is true

Now, $\begin{vmatrix}-2 & 1 & 1\\ -1 & 0 & 1\\ -4 & 3 & 1\end{vmatrix}=-2(0-3)-1(-1+4)+1(-3-0)$

$=6-3-3=0$

$\therefore$ Given points are collinear

$\Rightarrow$ No circle will pass through the given points

598 **(d)**
Statement 2 is correct (a known fact)

Using statement 2, x intercept made by $x^2+y^2-2x+6y+5=0$ is $2\sqrt{(-1)^2-5}$ an imaginary number. Thus, $x^2+y^2-2x+6y+5=0$ is away from x-axis. Hence, statement 1 is false

599 **(d)**
We have

$\sqrt{(\lambda-3)^2+16}-4=1\ \Rightarrow \lambda=0$ or 6

600 **(b)**
Centre of the circle $C(2,1)$ and radius $r=5$

Distance of $P(10,7)$ from $C(2,1)$ is 10 units, hence required distances are 5, 15, respectively. Therefore, statement 1 is true. Statement 2 is true but not the correct explanation of statement 1, as the information is not sufficient to get distance said in Statement 1

601 **(a)**
Clearly $(\sqrt{2},\sqrt{6})$ lies on $x^2+y^2=8$, which is the director circle of $x^2+y^2=4$

$\Rightarrow$ Tangents PA and PB are perpendicular to each other

$\therefore$ $(OAPB)$ is a square

$\therefore$ Area of $OAPB=4$

602 **(d)**
Statement 2 is true as it is the definition of parabola.
From statement 1, we have
$\sqrt{(x-1)^2+(y+2)^2}=\frac{|3x+4y+5|}{5}$,
Which is not parabola as point $(1,-2)$ lies on the line $3x+4y+5=0$. Hence, statement 1 is false

603 **(d)**

Statement 2 is true.

For the points (2, 2),(4, 1) and (6, 2/3), $t_1 = 1, t_2 = 2$ and $t_3 = 3$, respectively

For the point (1/4, 16), $t_4 = \frac{1}{8}$

Now $t_1\, t_2\, t_3\, t_4 = \frac{3}{4} \neq 1$

Hence, statement 1 is false

604 **(b)**

Chord of contact of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ w.r.t. point $P(x_1, y_1)$ is $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$ (i)

Equation (i) can be written as

$$\frac{x(-x_1)}{a^2} - \frac{y(-y_1)}{b^2} = -1$$

Which is tangent to the hyperbola

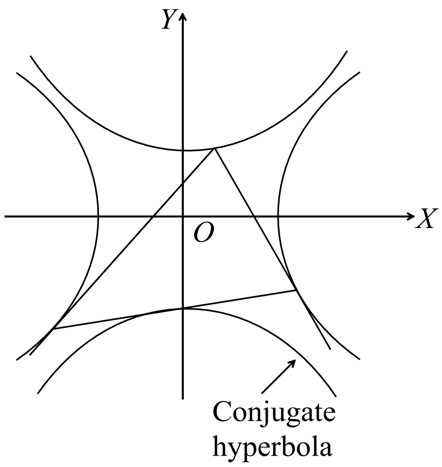
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

At point $(-x_1, -y_1)$

Obviously, points (x_1, y_1) and $(-x_1, -y_1)$ lie on the different branches of hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

Hence, statement 1 is correct



Statement 2 is also correct but does not explain statement I

605 **(d)**

Statement 2 is true as the centre is equidistant from A and B, hence lies on the perpendicular bisector of AB

Statement 1 is false as the distance between the given points is 10 and hence any circle through A and B has radius more than or equal to 5, and hence there is no circle of radius 4 through A and B is possible

606 **(b)**

Given ellipse is $\frac{x^2}{3} + \frac{y^2}{2} = 1$, whose area is $= \pi\sqrt{3}\sqrt{2} = \pi\sqrt{6}$

Circle is $x^2 + y^2 - 2x + 4y + 4 = 0$ or $(x-1)^2 + (y-2)^2 = 1$

Its area is π. Hence, statement 1 is true.

Also statement 2 is true but it is not the correct explanation of statement 1.

Consider the ellipse $\frac{x^2}{25} + \frac{y^2}{1} = 1$, whose area is 5π and circle $x^2 + y^2 = 16$ whose area is 16π.

Also here semi-major axis of ellipse (= 5) is more than the radius of the circle (= 4)

607 **(a)**

$\because (a-b) + (b-c) + (c-a) = 0$

$\therefore x = 1$ is a root

Solving equation of parabola with x-axis ($ie, y = 0$)

We get, $(a-b)x^2 + (b-c)x + (c-a) = 0$ which should have two equal values of x, as x-axis touches the parabola

$$\therefore 1 \times 1 = \frac{c-a}{a-b}$$

$\Rightarrow -2a + b + c = 0$ ...(i)

And given $ax + by + c = 0$ ...(ii)

From Eqs. (i) and (ii), we get

$a(x+2) + b(y-1) = 0$

Which is a family of lines

$\therefore x + 2 = 0, y - 1 = 0$

$\Rightarrow (-2, 1)$

$\Rightarrow ax + by + c = 0$ always passes through $(-2, 1)$

608 **(d)**

The locus of point of intersection of two mutually perpendicular tangents drawn on to hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is its director circle whose equation is $x^2+y^2=a^2-b^2$

For $\frac{x^2}{9}-\frac{y^2}{16}=1, x^2+y^2=9-16$

So director circle does not exist

609 **(d)**
Since $S_1=0$ and $S_3=0$ has no radical axis

$\therefore$ Radical centre does not exist

610 **(a)**
Given, $y=-\frac{x^2}{2}+x+1 \ \Rightarrow \ y-\frac{3}{2}=-\frac{1}{2}(x-1)^2$

$\Rightarrow$ It is symmetric about $x=1$

611 **(b)**
The equation of two conjugate hyperbolas are

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

and $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$.

$$\therefore e^2=1+\frac{b^2}{a^2}=\frac{a^2+b^2}{a^2}$$

and $e_1^2=1+\frac{a^2}{b^2}=\frac{a^2+b^2}{b^2}$

$$\therefore \frac{1}{e^2}+\frac{1}{e_1^2}=\frac{a^2}{a^2+b^2}+\frac{b^2}{a^2+b^2}=1$$

$$\Rightarrow \frac{1}{(5/3)^2}+\frac{1}{(5/4)^2}=\frac{9}{25}+\frac{16}{25}=1$$

and $ee_1=\frac{5}{3}\times\frac{5}{4}=\frac{25}{12}>1$

612 **(a)**
Here, $a=-9, b=7, c=-116, g=27, f=-14, h=0$

$$\therefore \ \Delta=abc+2fgh-af^2-bg^2-ch^2$$

$$=-9\times7\times(-116)+0+9\times(14)^2-7(27)^2-0$$

$$=7308+1764-5103$$

$$=3969\neq0$$

and $h^2>ab \ \Rightarrow 0>-63$

613 **(a)**
Let $y=mx$ be any chord through $(0,0)$, then

solving $y=mx$ and $x^2+y^2xy=1$

$$\Rightarrow x^2+m^2x^2+x(mx)=1$$

$$\Rightarrow x^2(1+m+m^2)-1=0$$

$$\therefore x_1+x_2=0$$

$$\Rightarrow \frac{x_1+x_2}{2}=0$$

Also, $\frac{y_1+y_2}{2}=\frac{mx_1+mx_2}{2}=m\left(\frac{x_1+x_2}{2}\right)=0$

$\Rightarrow$ Mid point of chord is $(0,0)$ for all m

614 **(c)**
Equation of chord of contact from $A(x_1,y_1)$ is

$$xx_1+yy_1-a^2=0$$

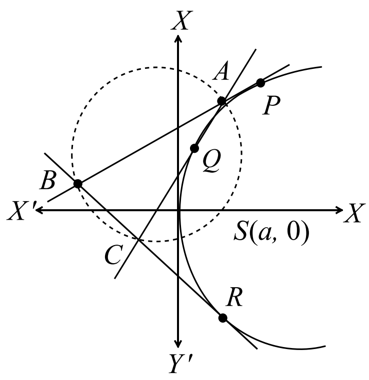
$$xx_2+yy_2-a^2=0$$

$$xx_3+yy_3-a^2=0$$

i.e. $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}=0$

$\Rightarrow A, B, C$ are collinear

615 **(a)**
Let parabola be $y^2=4x$
Clearly $x=0$ is tangent to the parabola at $(0,0)$
And lines $y=-x-1$ and $y=x+1$ are tangents to the parabola at $(1,2)$ and $(1,-2)$ which are extremities of the latus rectum. These tangents meet on the directrix at right angle at $(-a,0)$.
Hence, circle passing through the point A, B, C also passes through its focus, as shown in the figure



Now consider a parabola $y^2=4ax$
Let $P(t_1), Q(t_2)$ and $R(t_3)$ be three points on it
Tangents are drawn at these points which intersect at
$A\equiv\left(at_1t_2, a(t_1+t_2)\right)$

$B \equiv (at_1t_3, a(t_1+t_3))$
$C \equiv (at_2t_3, a(t_2+t_3))$
Let $\angle SAC = \alpha$ and $\angle SBC = \beta$

$\Rightarrow \tan\alpha = \left|\frac{\frac{1}{t_2} - \frac{t_1+t_2}{t_1t_2-1}}{1+\frac{1}{t_2}\left(\frac{t_1+t_2}{t_1t_2-1}\right)}\right| = \left|\frac{1}{t_1}\right|$

Similarly $\tan\beta = \left|\frac{1}{t_1}\right|$
$\Rightarrow \alpha = \beta$ or $\alpha + \beta = \pi$
$\Rightarrow$ A, B, C and S are concyclic

616 **(a)**

a. Radical axis of $x^2+y^2+2a_1x+b=0$ and $x^2+y^2+2a_2x+b=0$ is $(a_1-a_2)x=0$ or $x=0$, it must touch both of the circle. Solving it with one of the circles we get $y^2+b=0 \Rightarrow b \le 0$

b. Radical axis of $x^2+y^2+2a_1x+b=0$ and $x^2+y^2+2a_2y+b=0$ is $a_1x-a_2y=0$
Solving it with one the circles, we have
$x^2+(a_1/a_2)^2x^2+2a_1x+b=0$
This equation must have equal roots
Hence, $4a_1^2-4b[1+(a_1^2/a_2^2)]=0$
$\Rightarrow a_1^2-b[1+(a_1^2/a_2^2)]=0$
Options p and q satisfy this condition

c. If the straight line $a_1x-by+b^2=0$ touches the circle $x^2+y^2=a_2x+by$

$\Rightarrow \frac{\left|a_1\frac{a_2}{2}-b\frac{b}{2}+b^2\right|}{\sqrt{a_1^2+b^2}} = \sqrt{\frac{a_2^2}{4}+\frac{b^2}{4}}$

$\Rightarrow \frac{|a_1a_2+b^2|}{\sqrt{a_1^2+b^2}} = \sqrt{a_2^2+b^2}$

$\Rightarrow a_1^2a_2^2+2b^2a_1a_2+b^4$
$= a_1^2a_2^2+a_1^2b^2+a_2^2b^2+b^4$
$\Rightarrow b^2=0$ or $2a_1a_2=a_1^2+a_2^2$
$\Rightarrow$ Option (q) and (r)

d. Line $3x+4y-4=0$ touches the circle $(x-a_1)^2+(y-a_2)^2=b^2$,

$\Rightarrow \frac{|3a_1+4a_2-4|}{5} = b$

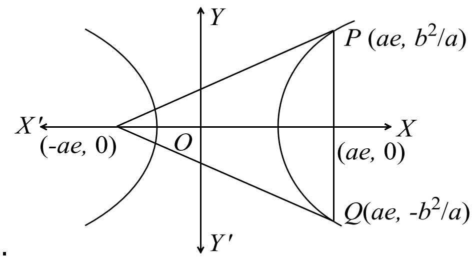
617 **(d)**

a. $\text{Im}(z^2)=3$

$\Rightarrow \text{Im}((x+iy)^2)=3$

$\Rightarrow 2xy=3$, which is a rectangular hyperbola having eccentricity $\sqrt{2}$

b.



$\tan 30° = \frac{b^2/a}{2ae}$

$\Rightarrow \frac{2}{\sqrt{3}}e = e^2-1$

$\Rightarrow \sqrt{3}e^2-2e-\sqrt{3}=0$

$\Rightarrow e = \frac{2\pm\sqrt{4+12}}{2\sqrt{3}} = \frac{2\pm 4}{2\sqrt{3}}$

$\Rightarrow e = \frac{3}{\sqrt{3}} = \sqrt{3}$

c. Eccentricity of the hyperbola $= \frac{AB}{PA-PB} = \frac{6}{4} = \frac{3}{2}$

If eccentricity of conjugate hyperbola is e', then

$\frac{1}{\left(\frac{3}{2}\right)^2} + \frac{1}{e'^2} = 1$

$\Rightarrow e' = \frac{3}{\sqrt{5}}$

d. Angle between the asymptotes in
$\tan^{-1}\left|\frac{2ab}{a^2-b^2}\right| = \frac{\pi}{3}$

$\Rightarrow \left|\frac{2\frac{a}{b}}{\frac{a^2}{b^2}-1}\right| = \sqrt{3}$

$\Rightarrow \frac{2\sqrt{e'^2-1}}{|e'^2-2|} = \sqrt{3}$ (where e' is eccentricity of conjugate hyperbola)

$\Rightarrow e'=2$

618 **(a)**

a. $(-g, -f)$ lies in first quadrant, then $g<0$ and $f<0$ also x-axis and y-axis must not cut the circle
Solving circle and x-axis, we have $x^2+2gx+c=0$, which must have imaginary roots; then $g^2-c<0$, then c must be positive. Also $f^2-c<0$

b. If circle lies above x-axis then $x^2+2gx+c=0$ must have imaginary roots, then $g^2-c<0$ and $c>0$

c. $(-g, -f)$ lies in third or fourth quadrant, then $g > 0$. Also y-axis must not cut the circle
Solving circle and y-axis, we have $y^2 + 2gy + c = 0$, which must have imaginary roots, then
$f^2 - c < 0$, then c must be positive
d. $x^2 + 2gx + c = 0$ must have equal roots, then $g^2 = c$, hence $c > 0$
Also $-g > 0 \Rightarrow g < 0$

619 **(c)**

a. Since $(2, 3)$ lies inside circle, such chord is bisected at $(2, 3)$, which has equation
$y - 3 = -(x - 2)$

or $x + y - 5 = 0 \Rightarrow a = b = 1$

b. Let P be the point (α, β), then $\alpha^2 + \beta^2 + 2\alpha + 2\beta = 0$

Midpoint of OP is $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

$\therefore$ Locus of $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$ is

$4x^2 + 4y^2 + 4x + 4y = 0$

i.e. $x^2 + y^2 + x + y = 0$

$\therefore 2g = 1, 2f = 1$

$\therefore g + f = 1$

c. Centres of the circle are $(1, 2), (5, -6)$

Equation of C_1C_2 is $y - 2 = -\frac{8}{4}(x - 1)$, i.e.
$2x + y - 4 = 0$

Equation of radical axis is $8x - 16y - 56 = 0$

i.e., $x - 2y - 7 = 0$

Points of intersection is $(3, -2)$

d. If θ is angle between tangents, then

$$\sin\frac{\theta}{2} = \frac{\text{radius}}{\text{distance between } (-3\sqrt{3}\tan\theta) \text{and} (0, 0)}$$

$$= \frac{1}{2}$$

$$\Rightarrow \frac{\theta}{2} = \frac{\pi}{6} \Rightarrow \theta = \frac{\pi}{3} = 2\sqrt{3}\tan\theta = 6$$

620 **(a)**

Tangent to ellipse at $P(\phi)$ is $\frac{x}{4}\cos\phi + \frac{y}{2}\sin\phi = 1$

It must pass through the centre of the circle.
Hence,
$$\frac{4}{4}\cos\phi + \frac{2}{2}\sin\phi = 1$$
$\Rightarrow \cos\phi + \sin\phi = 1$
$\Rightarrow 1 + \sin 2\phi = 1$
or $\sin 2\phi = 0$
$\Rightarrow 2\phi = 0$ or π
$\Rightarrow \frac{\phi}{2} = 0$ or $\frac{\pi}{4}$

Consider any point $P(\sqrt{6}\cos\theta, \sqrt{2}\sin\theta)$ on ellipse $\frac{x^2}{6} + \frac{y^2}{2} = 1$

Given that $OP = 2$

$\Rightarrow 6\cos^2\theta + 2\sin^2\theta = 4$

$\Rightarrow 4\cos^2\theta = 2$

$$\Rightarrow \cos\theta = \pm\frac{1}{\sqrt{2}}$$

$\Rightarrow \theta = \frac{\pi}{4}$ or $\frac{5\pi}{4}$

Solving the equation of ellipse and parabola (eliminating x^2), we have

$y - 1 + 4y^2 = 4$

$\Rightarrow 4y^2 + y - 5 = 0$

$\Rightarrow (4y + 5)(y - 1) = 0$

$\Rightarrow y = 1, x = 0$

The curves touch at $(0, 1)$. So the angle of intersection is 0

The normal at $P(a\cos\theta, b\sin\theta)$ is

$$\frac{ax}{\cos\theta} - \frac{bx}{\sin\theta} = a^2 - b^2$$

Where $a^2 = 14, b^2 = 5$

It meets the curve again at $Q(2\theta)$, i.e.
$(a\cos 2\theta, b\sin 2\theta)$. Hence,

$$\frac{a}{\cos\theta}a\cos 2\theta - \frac{b}{\sin\theta}(b\sin 2\theta) = a^2 - b^2$$

$$\Rightarrow \frac{14}{\cos\theta}\cos 2\theta - \frac{5}{\sin\theta}(\sin 2\theta) = 14 - 5$$

$\Rightarrow 28\cos^2\theta - 14 - 10\cos^2\theta = 9\cos\theta$

$\Rightarrow 18\cos^2\theta - 9\cos\theta - 14 = 0$

$\Rightarrow (6\cos\theta - 7)(3\cos\theta - 2) = 0$

$\Rightarrow \cos\theta = -\dfrac{2}{3}$

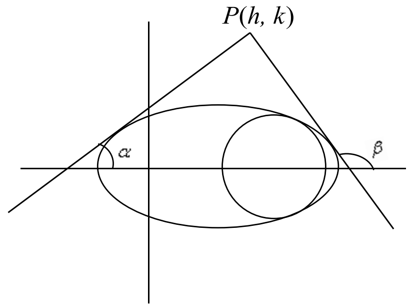
621 **(a)**

Equation of any tangent to

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (i)

is given by

$y = mx + \sqrt{a^2m^2 + b^2}$



Since it passes through $P(h, k)$

$k = mh + \sqrt{a^2m^2 + b^2}$

$\Rightarrow m^2(h^2 - a^2) - 2kmh + (k^2 - b^2) = 0$ (ii)

As (ii) is quadratic in m, having two roots m_1 and m_2 (say),

Therefore,

$m_1 + m_2 = \frac{2hk}{h^2-a^2}, m_1m_2 = \frac{k^2-b^2}{h^2-a^2}$ (iii)

$\Rightarrow \tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= \dfrac{m_1+m_2}{1 - m_1m_2}$

$= \dfrac{\frac{2hk}{h^2-a^2}}{1 - \frac{k^2-b^2}{h^2-a^2}}$

$= \dfrac{2hk}{h^2 - k^2}$

1. $\alpha + \beta = \frac{c\pi}{2}$

When c is even,

$m_1 + m_2 = 0$

$\dfrac{2kh}{h^2 - a^2} = 0 \Rightarrow 2kh = 0$

$\Rightarrow xy = 0$,which is the equation of a pair of straight lines

When c is odd,

$1 - m_1m_2 = 0$

$\Rightarrow \dfrac{k^2 - b^2}{h^2 - a^2} = 1$

Therefore, the locus of (h, k) is

$y^2 - b^2 = x^2 - a^2$

Which is a hyperbola

2. $m_1 m_2 = c$

$\Rightarrow \dfrac{k^2 - b^2}{h^2 - a^2} = c$

When $c = 0, k = \pm b$, the locus is pair of straight lines

When $c = 1, h^2 - k^2 = a^2 - b^2$ the locus is hyperbola

When $c = -1, h^2 + k^2 = a^2 + b^2$, the locus is circle

When $c = -2, 2h^2 + k^2 = 2a^2 + b^2$, the locus is ellipse

3. $\tan\alpha + \tan\beta = c$

$\Rightarrow m_1 + m_2 = c$

$\Rightarrow \dfrac{2hk}{h^2 - a^2} = c$

When $c = 0, kh = 0$, the locus is pair of straight lines

When $c \neq 0$

$c(h^2 - a^2) - 2kh = 0$

Locus of (h, k) is

$cx^2 - 2xy - ca^2 = 0$

$\Delta = -ca^2 \neq 0$

Also, $h^2 - ab = 1 > 0$

Therefore, the locus is a hyperbola for $c \neq 0$

4. $\cot\alpha + \cot\beta = c$

$\Rightarrow \dfrac{1}{m_1} + \dfrac{1}{m_2} = c$

$\Rightarrow \dfrac{m_1 + m_2}{m_1 m_2} = c$

$\Rightarrow \dfrac{2kh}{k^2 - b^2} = c$

$\Rightarrow c(k^2 - b^2) - 2kh = 0$

When $c = 0$, locus is a pair of straight lines

When $c \neq 0$, locus is a hyperbola (as in previous case c)

622 **(b)**

Tangent to parabola having slope m is $ty = x + t^2$, it passes through point $(2, 3)$ then $3t = 2 + t^2 \Rightarrow t = 1$ or $2 \Rightarrow$ point of contact $(t^2 + 2t) \equiv (1, 2)$or $(4,4)$

Let point on the circle be $P(x_1, y_1)$, then chord of contact of parabola w.r.t. P is $yy_1 = 2(x + x_1)$. Comparing with $y = 2(x - 2)$, we have $y_1 = 1$ and $x_1 = -2$, which also satisfy the circle

Point Q on the parabola $Q(t^2, 2t)$

Now area of triangle OPQ is $\left\| \frac{1}{2} \begin{vmatrix} 0 & 0 \\ 4 & -4 \\ t^2 & 2t \\ 0 & 0 \end{vmatrix} \right\| = 6 \Rightarrow 8t$

For $t^2 + 2t - 3 = 0, (t - 1)(t + 3) = 0$, then $t = 1$ or $t = -3$

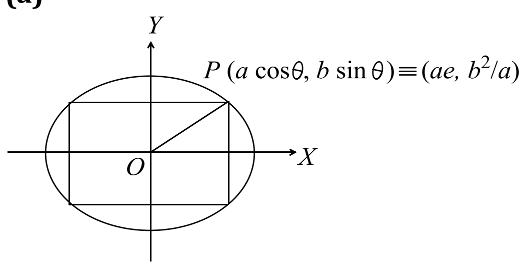
Then point Q are $(1,2)$or $(9, -6)$

Points $(1, 2)$and $(-2, 1)$satisfy both the curves

623 **(c)**

Equation of tangent having slope m is
$y = mx - 6m - 3m^3$
line $3x - y + 1 = 0$ is tangent for $m = 3$
Equation of normal having slope m is
$y = mx - 6m - 3m^3$
Line $2x - y - 36 = 0$ is normal for $m = 2$
Chord of contact w.r.t. any point on the directrix is the focal chord which passes through the focus (3, 0)
Line $2x - y - 36 = 0$ passes through the focus
Chord which subtends right angle at the vertex are concurrent at point $(4 \times 3, 0)$ or $(12, 0)$
Line $x - 2y - 12 = 0$ passes through the point $(12, 0)$

624 **(a)**



a. Let one of the vertices of the rectangle be

$P(a\cos\theta, b\sin\theta)$

Then its area $A = (2a\cos\theta)(2b\sin\theta) = 2ab\sin 2\theta$

Hence, $A\_\max = 2ab$

Now area of rectangle formed by extremities of $LR = (2ae)(2b^2 / a) = 4eb^2$.

Given that $2ab = 4eb^2 \Rightarrow \frac{2b}{a}e = 1$

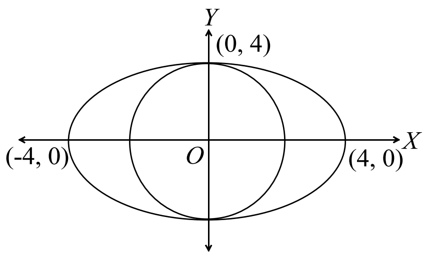
$\Rightarrow \dfrac{4b^2}{a^2}e^2 = 1$

$\Rightarrow 4(1-e^2)e^2 = 1$

$\Rightarrow 4e^4 - 4e^2 + 1 = 0$

$\Rightarrow (2e^2 - 1)^2 = 0$

$\Rightarrow e = \dfrac{1}{\sqrt{2}}$



b.

For ellipse, distance c between the foci, $2ae = 8$ and length of semi-minor axis, is $b = 4$

Now,

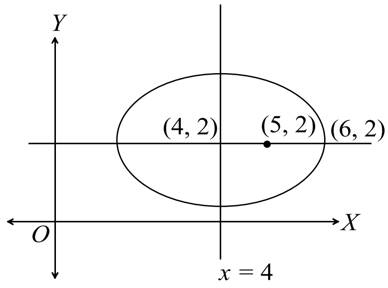
$b^2 = a^2 - a^2e^2$

$\Rightarrow 16 = a^2 - 16$

$\Rightarrow a^2 = 32$

$\Rightarrow e = \sqrt{1 - \dfrac{16}{32}} = \dfrac{1}{\sqrt{2}}$

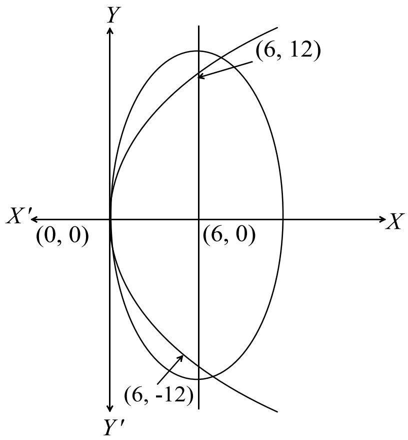
c. Normal at point $P(6,2)$ to the ellipse passes through its focus Q $(5,2)$.Then P must be extremely of the major axis. Now $ae = QR = 1$ (where R is centre) and $a - ae = 1$



$\therefore\ a = 2$

$b^2 = a^2 - a^2e^2 = 4 - 1 = 3$

$\Rightarrow e = \sqrt{1 - \dfrac{3}{4}} = \dfrac{1}{2}$



d.

Extremities of LR of parabola $y^2 = 24x$ are $(6, \pm 12)$

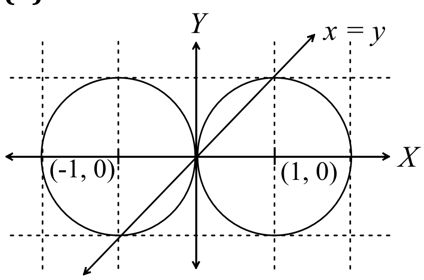
For ellipse, $2be = 24$ and extremity of minor axis is $(0,0)$. Hence, $a = 6$

Now, $a^2 = b^2 - b^2e^2$

$\Rightarrow b^2 = 36 + 144 = 180$

$\Rightarrow e^2 = \sqrt{1 - \dfrac{36}{180}} = \sqrt{1 - \dfrac{1}{5}} = \dfrac{2}{\sqrt{5}}$

625 **(d)**



(λ, λ) lies on the line $y = x$
From the diagram $a \to s, b \to r, c \to q, d \to p$

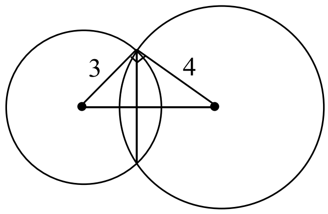
626 **(b)**

a. Let length of common chord be $2a$, then

$\sqrt{9-a^2}+\sqrt{16-a^2}=5$

$\sqrt{16-a^2}=5-\sqrt{9-a^2}$

$16-a^2=25+9-a^2-10\sqrt{9-a^2}$



$10\sqrt{9-a^2}=18$

$\Rightarrow 100(9-a^2)=324$, i.e., $100a^2=576$

$\therefore a=\sqrt{\frac{576}{100}}=\frac{24}{10}$

$\therefore 2a=\frac{24}{5}=\frac{k}{5}\Rightarrow k=24$

b. Equation of common chord is $6x+4y+p+q=0$

Common chord pass through centre $(-2,-6)$ of circle $x^2+y^2+4x+12y+p=0$

$\therefore p+q=36$

c. Equation of the circle is $2x^2+2y^2-2\sqrt{2}\,x-y=0$

Let $(\alpha,0)$ be midpoint of a chord. Then, equation of the chord is

$2\alpha x-\sqrt{2}(x+\alpha)-\frac{1}{2}(y+0)=2\alpha^2-2\sqrt{2}\alpha$

Since it passes through the point $\left(\sqrt{2},\frac{1}{2}\right)$

$\therefore 2\sqrt{2}\alpha-\sqrt{2}(\sqrt{2}+\alpha)-\frac{1}{4}=2\alpha^2-2\sqrt{2}\alpha$

i.e. $8\alpha^2-12\sqrt{2}\alpha+9=0$,

i.e. $\left(2\sqrt{2}\alpha-3\right)^2=0$

i.e., $\alpha=\frac{3}{2\sqrt{2}},\frac{3}{2\sqrt{2}}$

$\therefore$ Number of chords is 1

d. Midpoint of $AB=(1,4)$

$\therefore$ Equation perpendicular bisector of AB is $x=1$

A diameter is $4y=x+7$

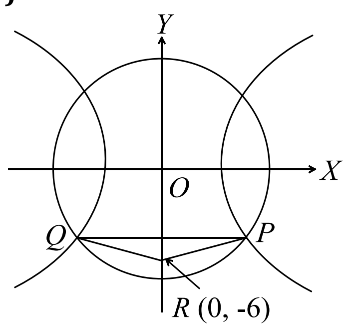
$\therefore$ Centre of the circle is (1,2)

$\therefore$ sides of the rectangular are 8 and 4

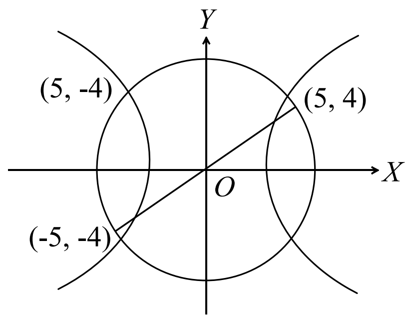
$\therefore$ Area =32

627 **(a)**
Points through which perpendicular tangent can be drawn to the parabola $y^2=4x$ lie on the directrix. Points $(-1,2)$ and $(-1,-5)$ lie on the directirx. Also from these points only one normal can be drawn

628 **(b)**



(a)



(b)

a. Obviously all the points in column II are common to the hyperbola and circle

b. Chord of contact of hyperbola w.r.t. $\left(0,-\frac{9}{4}\right)$ is $\theta(x)-\left(-\frac{9}{4}\right)y=9$ or $y=4$

Solving this with hyperbola we have

$x^2-16=9\Rightarrow x^2=25\ \Rightarrow x=\pm 5$

Hence, points of contact are $(\pm 5,4)$

c. Obviously the required point is $(-5,-4)$

d. Let the points on the hyperbola be $P(h,k)$ and

$Q(-h, k)$

Then area of triangle is $\frac{1}{2}|2h||-6-k| = 10$

$\Rightarrow |h||6+k| = 10 \quad$ (i)

Also points P and Q lie on the hyperbola. Hence,

$h^2 - k^2 = 9 \quad$ (ii)

Obviously points $(\pm 5, -4)$ satisfy both Eqs. (i) and (2)

629 **(a)**
We have

$A = ae_E$ and $a = Ae_H$

$\Rightarrow e_E\, e_H = 1 \Rightarrow e_E + e_H > 2$

$B^2 = A^2(e_H^2 - 1) = a^2\,(1 - e_E^2)$

$= b^2$

$\Rightarrow \frac{b}{B} = 1$

Also the angle between the asymptotes is

$2\tan^{-1}\frac{B}{A} = \frac{2\pi}{3}$

Also, $\frac{B}{A} = \sqrt{3} \Rightarrow \frac{b}{ae_E} = \sqrt{3} \Rightarrow e_E^2 = \frac{1}{4}$

Solving $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2e_E^2} - \frac{y^2}{b^2} = 1$

We get $x^2 = \frac{2a^2e_E^2}{b^2(1-e_E^2)} = 4$

630 **(a)**
a. We must have

$e_1 < 1 < e_2 \Rightarrow f(1) < 0 \Rightarrow 1 - a + 2 < 0 \Rightarrow a > 3$

b. We must have both the roots greater than 1

i. $D > 0$ or $a^2 - 4 > 0$ or $a \in (-\infty, -2) \cup (2, \infty)$

ii. $1 \cdot f(1) > 0$ or $1 - a + 2 > 0$ or $a < 3$

iii. $\frac{a}{2} >= 1 \Rightarrow a > 2$

from Eq. (i), (iii) and (iii) we have $a \in (2, 3)$

c. We must have

$\frac{I}{e_1^2} + \frac{1}{e_2^2} = 1$

$\Rightarrow \frac{(e_1+e_2)^2 - 2e_1e_2}{e_1^2e_2^2} = 1$

$\Rightarrow \frac{a^2-4}{4} = 1$

$\Rightarrow a = \pm 2\sqrt{2}$

d. We must have

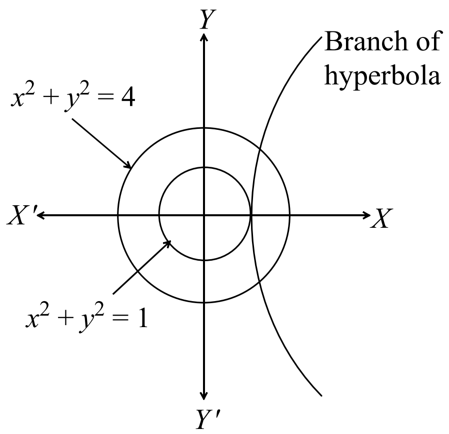
$e_2 < \sqrt{2} < e_1$

$\Rightarrow f(\sqrt{2}) < 0$

$\Rightarrow 2 - a\sqrt{2} + 2 < 0$

$\Rightarrow a > 2\sqrt{2}$

631 **(c)**



Locus of point P satisfying $PA - PB = 2$ is a branch of the hyperbola $x^2 - y^2/3 = 1$

For $r = 2$ the circle and the branch of the hyperbola intersect at two points. For $r = 1$ there is no point of intersection. If m be the slope of the common tangent, then

$m^2 - 3 = r^2\,(1 + m^2)$

$\Rightarrow m^2 = \frac{r^2+3}{1-r^2}$

Hence, there are no common tangents for $r > 1$ and two common tangents for $r < 1$

632 **(a)**

1. The locus is

$$\Rightarrow \frac{x^2}{16}+\frac{y^2}{36}=1$$

$$\Rightarrow e=\sqrt{1-\frac{16}{36}}=\sqrt{\frac{20}{36}}=\frac{\sqrt{5}}{3}$$

$$\Rightarrow\ 3e=\sqrt{5}$$

2. $3(x^2+2x+1)+2(y^2-2y+1)=3+2+1$

$$\Rightarrow \frac{(x+1)^2}{2}+\frac{(y-1)^2}{3}=1$$

$$\Rightarrow e=\sqrt{1-\frac{2}{3}}=\frac{1}{\sqrt{3}}$$

$$\therefore a=\sqrt{3}, b=\sqrt{2}$$

$$\therefore \text{Area}=\frac{1}{2}\times 2\sqrt{3}\times\sqrt{2}=\sqrt{6}$$

3. Eliminating θ from $x=1+4\cos\theta, y=2+3\sin\theta$, we have

$$\frac{(x-1)^2}{16}+\frac{(y-2)^2}{9}=1$$

Hence, $a=4$ and $e=\frac{\sqrt{7}}{4}$

$$\Rightarrow ae=\sqrt{7}$$

$\Rightarrow$ Distance between the foci $=2\sqrt{7}$

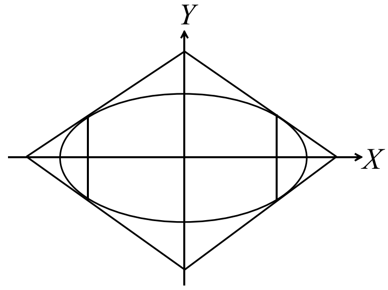
4. $\frac{x^2}{16}+\frac{y^2}{7}=1$

$$e=\sqrt{1-\frac{7}{16}}=\frac{3}{4}$$

One end of latus rectum is

$$\left(ae,\frac{b^2}{a}\right)=\left(3,\frac{7}{4}\right)$$

Therefore, equation of tangent is



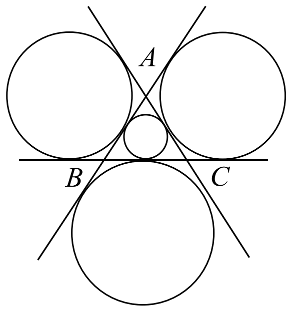
$$\frac{3x}{16}+\frac{7}{4}\frac{y}{7}=1$$

Or $\frac{3x}{16}+\frac{y}{4}=1$

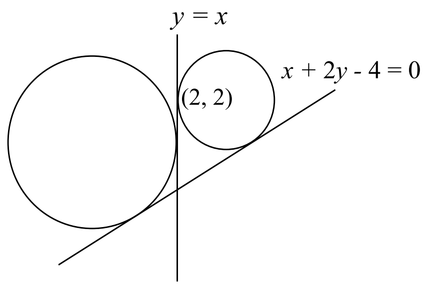
It meets x-axis at $\left(\frac{16}{3},0\right)$ and y-axis at $(0,4)$

Hence, the area of quadrilateral $=2\times\frac{16}{3}\times 4=\frac{128}{3}$

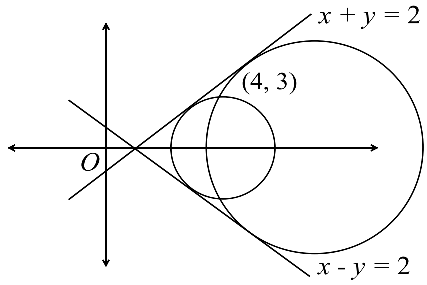
633 **(b)**



a.



b.



c.

d. Obviously one

634 **(c)**

Points are $O(0,0), P(3,4)$ and $Q(6,8)$

$2a = OP + OQ$

$= 5 + 10 = 15$

$\Rightarrow a = \frac{15}{2}$

Also distance between foci,

$2ae = \sqrt{(6-3)^2 + (8-4)^2} = 5$

$\Rightarrow e = \frac{1}{3}$

$\Rightarrow b^2 = \frac{225}{4}\left(1 - \frac{1}{9}\right) = 50$

$\Rightarrow b = 5\sqrt{2}$

$\Rightarrow 2b = 10\sqrt{2}$

We know that $\frac{1}{SP} + \frac{1}{SQ} = \frac{2a}{b^2}$

$\Rightarrow \frac{1}{2} + \frac{1}{SQ} = \frac{10}{16}$

$\Rightarrow SQ = 8$

$\Rightarrow PQ = 10$

If the line $y = x + k$ touches the ellipse $9x^2 + 16y^2 = 144$, then

$k^2 = 16(1)^2 + 9$

$\Rightarrow\ k = \pm 5$

Sum of the distances of a point on the ellipse from the foci

$= 2a = 8$

635 **(b)**

Equation of tangent at $\left(\frac{\cos\theta}{2}, \frac{\sin\theta}{3}\right)$ is $2x\cos\theta + 3y\sin\theta = 1$

Which is parallel to the given line $8x = 9y$

$\therefore \cos\theta = \pm\frac{4}{5}, \sin\theta = \mp\frac{3}{5}$

Hence, points are $\left(\frac{2}{5}, -\frac{1}{5}\right)$ and $\left(-\frac{2}{5}, \frac{1}{5}\right)$

Distance between the points is

$\sqrt{\frac{16}{25} + \frac{4}{25}} = \frac{2}{\sqrt{5}}$

Which is less than 1

The given equation is

$\frac{(x+1)^2}{9} + \frac{(y+2)^2}{25} = 1$

$\Longrightarrow e^2 = 1 - \frac{9}{25} = \frac{16}{25} \Rightarrow e = \frac{4}{5}$

Hence, the foci are $S, S' \equiv (-1, -2 \pm 4) \equiv S(-1,2)$ and $S'(-1,-6)$

The required sum of distances $= 2 + 6 = 8$

Equation of normal at $(3\cos\theta, 2\sin\theta)$ is $3x\sec\theta - 2y\operatorname{cosec}\theta = 5$

Which is parallel to the given line $2x + y = 1$. Therefore,

$\cos\theta = \mp\frac{3}{5}, \sin\theta = \pm\frac{4}{5}$

Hence, points are $\left(\frac{-9}{5}, \frac{8}{5}\right)$ and $\left(\frac{9}{5}, -\frac{8}{5}\right)$

The required sum of distances $= \frac{16}{5}$

Consider any point $(t, t+2), t \in R$, on the line $x - y + 2 = 0$

The chord of contact of ellipse w.r.t. this point is

$xt + 2y(t+2) = 2$

$\Longrightarrow (4y - 2) + (x + 2y) = 0$

This line p[asses through point of intersection of lines

$4y - 2 = 0$ and $x + 2y = 0$

$\therefore\ x = -1$

Hence, the point is $(-1, 1/2)$, whose distance from $(2, 1/2)$ is 3

636 **(d)**

Locus of point of intersection of perpendicular tangent is directrix which is $12x - 5y + 3 = 0$ Parabola is symmetrical about its axis, which is a line passing through the focus (1,2) and perpendicular to the directrix, which has equation

$5x + 12y - 29 = 0$

Minimum length of focal chord along the latus rectum line, which is a line passing through the focus and parallel to directrix, i.e., $12x - 5y - 2 = 0$

Locus of foot of perpendicular from focus upon any tangent is tangent at the vertex, which is parallel to directrix and equidistant from directrix and latus rectum line, i.e., $12x - 5y + \lambda = 0$

Where $\frac{|\lambda-3|}{\sqrt{12^2+5^2}} = \frac{|\lambda+2|}{\sqrt{12^2+5^2}} \Rightarrow \lambda = \frac{1}{2}$

Hence, equation of tangent at vertex is

$24x - 10y + 1 = 0$

637 **(a)**

The equation of the hyperbola is $\frac{x^2}{9} - \frac{y^2}{4} = 1$ and that of circle is

$x^2 + y^2 - 8x - 0$

For their points of intersection

$$\frac{x^2}{9} + \frac{x^2 - 8x}{4} = 1$$

$\Rightarrow\ 4x^2 + 9x^2 - 72x = 36$

$\Rightarrow\ 13\,x^2 - 72\,x - 36 = 0$

$\Rightarrow\ 13x^2 - 78x + 6x - 36 = 0$

$\Rightarrow\ 13x(x - 6) + 6(x - 6) = 0$

$\Rightarrow\ x = 6, x = -\frac{13}{6}$

$x = -\frac{13}{6}$ not acceptable

Now, for $x = 6, y = \pm 2\sqrt{3}$

Required equation is

$(x - 6)^2 + (y + 2\sqrt{3})(y - 2\sqrt{3}) = 0$

$\Rightarrow\ x^2 - 12x + y^2 + 24 = 0$

$\Rightarrow\ x^2 + y^2 - 12x + 24 = 0$

639 **(d)**

$\because\ t_1 + t_2 = 4, t_1t_2 = 2$

Then, the equation of the tangent at t_1 is

$\frac{x}{t_1} + yt_1 = 2c$ ...(i)

And the equation of the tangent at $''t_2''$ is

$\frac{x}{t_2} + yt_2 = 2c$ ...(ii)

On solving Eqs. (i) and (ii), we get

$\left(\frac{2ct_1t_2}{t_1 + t_2}, \frac{2c}{t_1 + t_2}\right)$ ie $\left(\frac{4c}{4}, \frac{2c}{4}\right)$ or $\left(c, \frac{c}{2}\right)$

640 **(b)**

Let $R(h, k)$ be the point of intersection of the tangents to the extremities of the chord $L: x = 9$ to the hyperbola, then equation of L is

$hx - ky = 9\ \Rightarrow h = 1, k = 0.$

$\therefore$ Coordinates of R are $(1, 0)$.

Equation of the pair of tangents from R to the hyperbola is

$(x^2 - y^2 - 9)(1 - 9) = (x - 9)^2\ \ (\because S_1 = T^2)$

$\Rightarrow 9x^2 - 8y^2 - 18x + 9 = 0$

641 **(c)**

Let $(x_i, y_i) = \left(ct, \frac{t}{t}\right), i = 1, 2, 3, 4$ are the points on the rectangular hyperbola $xy = c^2$.

Equation of normal to the hyperbola $xy = c^2$ at $\left(ct, \frac{c}{t}\right)$ is

$ct^4 - t^3x + ty - c = 0$

It passes through (α, β), then

$ct^4 - t^3\alpha + t\beta - c = 0$

It's a biquadratic equation in$t$. Let the roots of this equation are t_1, t_2, t_3, t_4, then

$\sum t_1 = \frac{\alpha}{c}$ ...(i)

$\sum t_1t_2 = 0$ ...(ii)

$\sum t_1t_2t_3 = -\beta/c$ and $t_1t_2t_3t_4 = -1$ ...(iii)

$\sum x_i = c\sum t_1 = c\left(\frac{\alpha}{c}\right) = \alpha$ [from Eq.(i)]

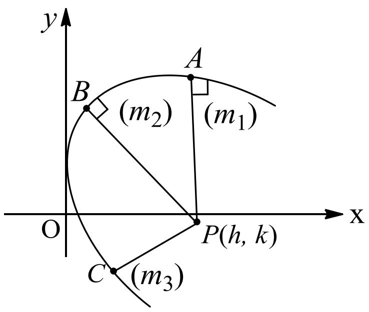
642 **(b)**

The equation of the normal to $y^2 = 4ax$ is

$y = mx - 2am - am^3$ ...(i)

$\because$ It passes through (h, k), then

$am^3 + m(2a - h) + k = 0$ ...(ii)



$\because$ Roots of Eq. (ii) be m_1, m_2, m_3

Then, $m_1 + m_2 + m_3 = 0$ ....(iii)

$m_1m_2 + m_2m_3 + m_3m_1 = \frac{(2a-h)}{a}$ ...(iv)

And $m_1m_2m_3 = -\frac{k}{a}$ ...(v)

Here two of the three normals are given to be coincident $ie, m_1 = m_2$

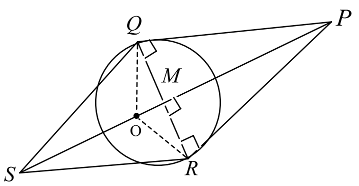
On putting $m_1 = m_2$ in Eq. (iii) and (v), we get

$2m_1 + m_3 = 0$ ...(vi)

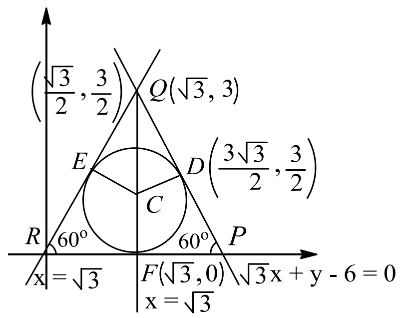
And $m_1^2 m_3 = -\frac{k}{a}$ ...(vii)
From Eqs. (vi) and (vii), we get
$$m_1^3 = \frac{k}{2a}$$
Since m_1 is a root of Eq. (i)
$\therefore am_1^3 + m_1(2a-h) + k = 0$
$$\Rightarrow \left(\frac{k}{2} + k\right)^3 = -m_1^3(2a-h)^3$$
$$\Rightarrow 27\frac{k^3}{8} = -\frac{k}{2a}(2a-h)^3$$
$\Rightarrow 27ak^2 = 4(h-2a)^3$
$\therefore$ Locus of P is
$27ay^2 = 4(x-2a)^3$

643 **(b)**
Given, $ae = \sqrt{7}, \frac{a}{e} = \frac{16}{\sqrt{17}}$
$\therefore a^2 = 16 \Rightarrow a = 4$
Then, $e = \frac{\sqrt{7}}{4}$
And $b^2 = a^2(1-e^2) = 16\left(1 - \frac{7}{16}\right)$
$= 16 - 7 = 9$
$\therefore$ Equation of ellipse is $\frac{x^2}{16} + \frac{y^2}{9} = 1$
$\Rightarrow 9x^2 + 16y^2 = 144$

645 **(b)**
Let the coordinates of P be $(3\cos\theta, 3\sin\theta)$, then the eccentric angle of M, the point where the ordinate PQ through P meets the ellipse is θ and the coordinates of M are $(3\cos\theta, 2\sin\theta)$, $PQ = 3\sin\theta$
$MQ = 2\sin\theta$,
So, $\frac{MQ}{PQ} = \frac{2}{3}$

646 **(b)**
$\because\ L \equiv 2x + y = 6$
Let $P \equiv (\lambda, 6 - 2\lambda)$
$$\because\ \angle PQO = \angle PRO = \frac{\pi}{2}$$


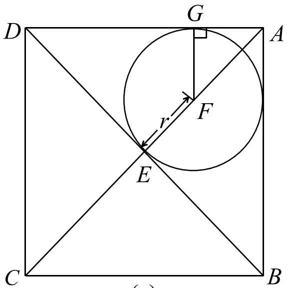
$\therefore OP$ is diameter of circumcircle PQR, then centre is $\left(\frac{\lambda}{2}, 3 - \lambda\right)$
$$\therefore x = \frac{\lambda}{2} \Rightarrow \lambda = 2x$$
and $y = 3 - \lambda$
Then $2x + y = 3$
Which is the required locus

648 **(d)**


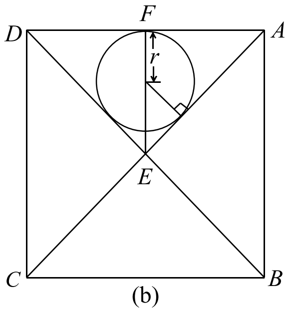
Equation of CD is $\frac{x - \frac{3\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}} = \frac{y - \frac{3}{2}}{\frac{1}{2}} = -1$
$\Rightarrow C = (\sqrt{3}, 1)$
Equation of the circle is $\left(x - \sqrt{3}\right)^2 + (y-1)^2 = 1$

649 **(b)**
Equation $16m^2 = 8l + 1$
$\Rightarrow 16(l^2 + m^2) = 16l^2 + 8l + 1$
$= (4l + 1)^2$
$\Rightarrow 4\sqrt{(l^2 + m^2)} = |4l + 1|$
$$\Rightarrow \frac{|4l + 1|}{\sqrt{(l^2 + m^2)}} = 4$$
$\therefore$ Centre $\equiv (4, 0)$ and radius=4
Equation of circle is $(x-4)^2 + (y-0)^2 = 4^2$
$\Rightarrow\ x^2 + y^2 - 8x = 0$

650 **(d)**
It is given that one of the diagonals of the square is parallel to the line $y = x$
Also the length of the diagonal of the square is $4\sqrt{2}$
Hence, the equation of the one of diagonals is
$$\frac{x-3}{\frac{1}{\sqrt{2}}} = \frac{y-4}{\frac{1}{\sqrt{2}}} = r = \pm 2\sqrt{2}$$
Hence, $x - 3 = y - 4 = \pm 2$
$\Rightarrow x = 5, 1$ and $y = 6, 2$
Hence, two of the vertices are $(1, 2)$ and $(5, 6)$
The other diagonal is parallel to the line $y = -x$, so that its equation is
$$\frac{x-3}{\frac{1}{\sqrt{2}}} = \frac{y-4}{\frac{1}{\sqrt{2}}} = r = \pm 2\sqrt{2}$$
Hence, the two vertices on this diagonal are $(1, 6)$ and $(5, 2)$



(a)



(b)

$AB = 4, AC = 4\sqrt{2}$

$\Rightarrow AE = 2\sqrt{2}$

In Fig (a), $EF + FA = AE$

$\Rightarrow r + \sqrt{2}r = 2\sqrt{2}$

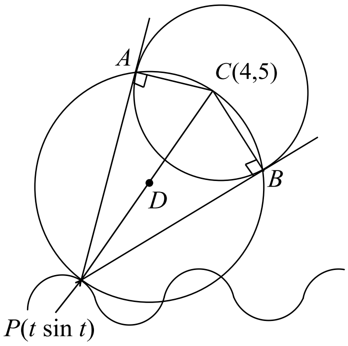
$$\Rightarrow r = \frac{2\sqrt{2}}{\sqrt{2}+1} = 2\sqrt{2}(\sqrt{2}-1)$$

In Fig. (b), $EG + GF = EF$

$\Rightarrow \sqrt{2}\,r + r = 2$

$$\Rightarrow r = \frac{2}{\sqrt{2}+1} = 2(\sqrt{2}-1)$$

651 **(d)**



Centre of the given circle is $C(4,5)$. Points P, A, C, B are concyclic such that PC is diameter of the circle. Hence, centre D of the circumcircle of ΔABC is midpoint of PC, then we have

$h = \frac{t+4}{2}$ and $k = \frac{\sin t - 5}{2}$

Eliminating t, we have $k = \frac{\sin(2h-4)+5}{2}$

or $y = \frac{\sin(2x-4)+5}{2}$

$$\Rightarrow f^{-1}(x) = \frac{\sin^{-1}(2x-5)+4}{2}$$

Thus range of $y = \frac{\sin(2x-4)+5}{2}$ is $[2, 3]$ and period is π

Also $f(x) = 4 \Rightarrow \sin(2x-4) = 3$ which has no real solutions

For $f(x) = 1 \Rightarrow \sin(2x-4) = -3$ which has no real solutions

But range of $y = \frac{\sin^{-1}(2x-5)+4}{2}$ is $\left[-\frac{\pi}{4}+2, \frac{\pi}{4}+2\right]$

652 **(c)**

Equation of line passing through the points $A(3,7)$ and $B(6,5)$ is

$$y - 7 = -\frac{2}{3}(x-3)$$

or $2x + 3y - 27 = 0$

Also equation of circle with A and B as diameter end points is

$(x-3)(x-6) + (y-7)(y-5) = 0$

Now family of circle through A and B is

$(x-3)(x-6) + (x-7)(y-5) + \lambda(2x + 3y - 27=0$ ...(i)

If circle belonging to this family touches the x-axis, then equation $(x-3)(x-6) + (0-7)(0-5+\lambda 2x+30-27=0$ has two equal roots, for which Discriminant $D = 0$, which gives two values of λ

Equation of common chord of (i) and $x^2 + y^2 - 4x - 6y - 3 = 0$ is radical axis, which is

$$[(x-3)(x-6) + (y-7)(y-5) + \lambda(2x+3y-27)] - [x^2+y^2-4x-6y-3] = 0$$

or $(2\lambda - 5)x + (3\lambda - 6)y + (-27\lambda + 56) = 0$

or $(-5x - 6y + 56) + \lambda(2x + 3y - 27) = 0$

This is family of lines which passes through the point of intersection of $-5x - 6y + 56 = 0$ and $2x + 3y - 27 = 0$ which is $(2, 23/3)$

If circle (i) cuts $x^2 + y^2 = 29$ orthogonally, then $0 + 0 = -29 + 56 - 27\lambda = 0 \Rightarrow \lambda = 1$

$\Rightarrow$ Required circle is cuts $x^2 + y^2 - 7x - 9y + 26 = 0$, centre is $(7/2, 9/2)$

653 **(c)**

Let the equation of the circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ (i)

The line $lx + my + 1 = 0$, will touch circle (i), if the length of $\perp$ from the centre $(-g, -f)$ of the circle on the line is equal to radius

i.e. $\frac{|-gl - mf + 1|}{\sqrt{(l^2+m^2)}} = \sqrt{(g^2 + f^2 - c)}$

$(gl + mf - 1)^2 = (l^2 + m^2)(g^2 + f^2 - c)$

$\Rightarrow (c - f^2)l^2 + (c - g^2)m^2 - 2gl - 2fm + 2gf\ lm + 1 = 0$ ...(ii)

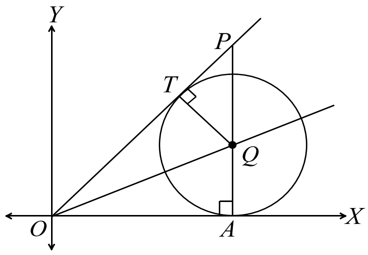
But the given condition of tangency is

$4l^2 - 5m^2 + 6l + 1 = 0$ ...(iii)

$\therefore$ Comparing Eqs. (ii) and (iii), we get

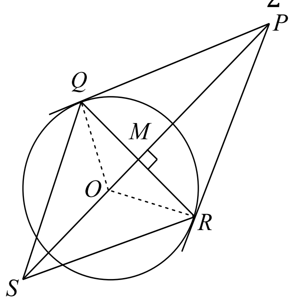
$c - f^2 = 4, c - g^2$
$= -5, -2g = 6, -2f = 0, 2gf = 0$
Solving, we get $f = 0, g = -3, c = 4$
Substituting these values in Eq. (i), the equation of the circle is $x^2 + y^2 - 6x + 4 = 0$. Any point on the line $x + y - 1 = 0$ is $(t, 1-t), t \in R$
Chord of contact generated by this point for the circle is $tx + y(1-t) - 3(t+x) + 4 = 0$ or $t(x - y - 3) + (-3x + y + 4) = 0$, which are concurrent at point of intersection of the lines $x - y - 3 = 0$ and $-3x + y + 4 = 0$ for all values of t. Hence, lines are concurrent at $\left(\frac{1}{2}, -\frac{5}{2}\right)$
Also point $(2, -3)$ lies outside the circle from which two tangents can be drawn

654 **(c)**
Given $QT = QA = 1$



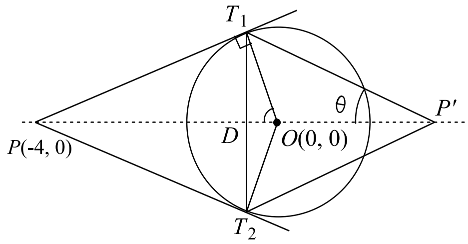
Let $PQ = x$, then $PT = \sqrt{x^2 - 1}$
ΔTQP and ΔAPO are similar triangles
Then, $OT = OA = \frac{x+1}{\sqrt{x^2-1}}$
$$\Rightarrow 1 + x + \frac{2(x+1)}{\sqrt{x^2-1}} + \sqrt{x^2-1} = 8$$
$$\Rightarrow x = \frac{5}{3}$$

655 **(b)**
$\because PQ = PR$, i.e., parallelogram $PQRS$ is a rhombus
$\therefore$ Midpoint of QR = midpoint of PS and $QR \perp PS$
$\therefore$ S is the mirror image of P w.r.t. QR
$\because L \equiv 2x + y = 6$
Let $P \equiv (k, 6 - 2k)$
$\because \angle PQO = \angle PRO = \frac{\pi}{2}$



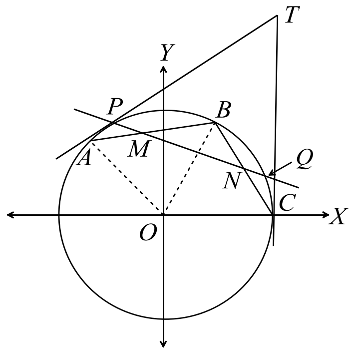
$\therefore$ OP is diameter of circumcircle PQR,then centre is $\left(\frac{k}{2}, 3 - k\right)$
$\therefore x = \frac{k}{2} \Rightarrow k = 2x$ and $y = 3 - k$
$\therefore$ Required locus is $2x + y = 3$

656 **(a)**



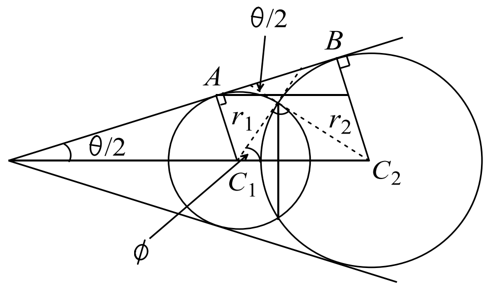
$PT_2 = PT_1 = \sqrt{(-4)^2 + 0^2 - 4} = 2\sqrt{3}$
Circumcentre of triangle PT_1T_2 is midpoint of PO as
$\angle PT_1O = \angle PT_2O = 90°$
So, $\left(\frac{-4+0}{2}, \frac{0+0}{2}\right) = (-2, 0)$

658 **(b)**



From the figure,
Since ΔOAB is equilateral triangle
$\therefore\ \angle OAB = 60°$

659 **(b)**



We have $\sin\phi = \frac{d}{r_1}, \cos\phi = \frac{d}{r_2}$, (where $2d$ = length of common chord)
$$\Rightarrow 1 = \frac{d^2}{r_1^2} + \frac{d^2}{r_2^2}$$
$$\Rightarrow d = \frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$$
$\Rightarrow 2d = \frac{2r_1r_2}{\sqrt{r_1^2+r_2^2}} = \frac{24}{5}$, where $r_1 = 3$
$$\Rightarrow d = \frac{6r_2}{\sqrt{9 + r_2^2}} = \frac{24}{5}$$
$\Rightarrow r_2 = 4$
From the figure, $\sin\frac{\theta}{2} = \frac{r_2 - r_1}{C_1C_2}$
Where $C_1^2C_2^2 = r_1^2 + r_1^2$
$\Rightarrow C_1C_2 = 5$

$\therefore \sin\frac{\theta}{2}=\frac{1}{5}$

$\Rightarrow \cos\frac{\theta}{2}=\frac{\sqrt{24}}{5}$

$\Rightarrow \sin\theta = 2\frac{1}{5}\cdot\frac{\sqrt{24}}{5}$

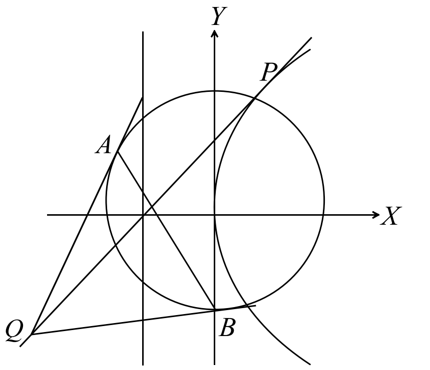
$=\frac{4\sqrt{6}}{25}$

$\Rightarrow\ \theta=\sin^{-1}\frac{4\sqrt{6}}{25}$

$\Rightarrow$ Also $AB=\sqrt{C_1C_2^2-(r_1-r_2)^2}$

$=\sqrt{25-1}=\sqrt{24}$

660 **(c)**



Equation of the tangent at point P of the parabola $y^2=8x$ is

$yt=x+2t^2$ (i)

Equation of the chord of contact of the circle $x^2+y^2=8$ w.r.t. $Q(\alpha,\beta)$ is

$x\alpha+y\beta=8$ (ii)

$Q(\alpha,\beta)$ lies in Eq. (i)

Hence, $\beta t=\alpha+2t^2$ (iii)

$x\alpha+y\left(\frac{\alpha}{t}+2t\right)-8=0$ [from Eqs. (ii) and (iii)]

$2(ty-4)+\alpha\left(x+\frac{y}{t}\right)=0$

For point of concurrency

$x=-\frac{y}{t}$ and $y=\frac{4}{t}$

Therefore, locus is $y^2+4x=0$

661 **(b)**

Since no point of the parabola is below x-axis

$D=a^2-4\le 0$

Therefore, maximum value of a is 2

Equation of the parabola, when $a=2$, is

$y=x^2+2x+1$

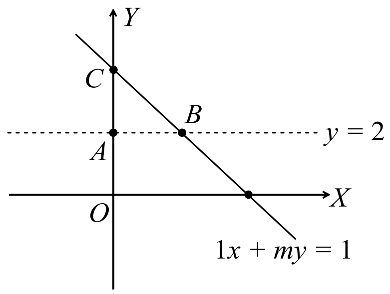
It intersects y-axis at (0, 1)

$y=2x+1$

Since $y=2x+1$ touches the circle $x^2+y^2=r^2$

$\therefore r=\frac{1}{\sqrt{5}}$

662 **(a)**



$C\equiv\left(0,\frac{1}{m}\right), B\equiv\left(\frac{1-2m}{l},2\right), A\equiv(0,2)$

Let (h,k) be the circumcentre of ΔABC which is mid-point of BC

$\Rightarrow h=\frac{1-2m}{2l}; k=\frac{1+2m}{2m},$

$\Rightarrow m=\frac{1}{2k-2}; l=\frac{k-2}{2h(k-1)}$

Given that (l,m) lies on $y^2=4x$

$\therefore m^2=4l$

$\Rightarrow\left(\frac{1}{2k-2}\right)^2=4\left\{\frac{k-2}{2h(k-1)}\right\}$

$\Rightarrow h=8(k^2-3k+2)$

Therefore, locus of (h,k) is

$x=8(y^2-3y+2)$

Or $\left(y-\frac{3}{2}\right)^2=\frac{1}{8}(x+2)$

Therefore, vertex is $\left(-2,\frac{3}{2}\right)$

Length of smallest focal chord = length of latus rectum $=\frac{1}{8}$

From the equation of curve C, it is clear that it is symmetric about line $y=\frac{3}{2}$

663 **(d)**

$y=ax^2+c$

$\therefore\ \frac{dy}{dx}=2ax=1$

Therefore, point of contact of the tangent is $\left(\frac{1}{2a},\frac{1}{4a}+c\right)$

Since it lies on $y=x$

$\therefore c=\frac{1}{4a}$, thus $c=\frac{1}{8}$ for $a=2$

664 **(a)**

Any parabola whose axes is parallel to x-axis will be of the form

$(y-a)^2=4b(x-c)$ (i)

Now, $lx+my=1$, can be rewritten as

$y-a=-\frac{l}{m}(x-c)+\frac{1-am-lc}{m}$

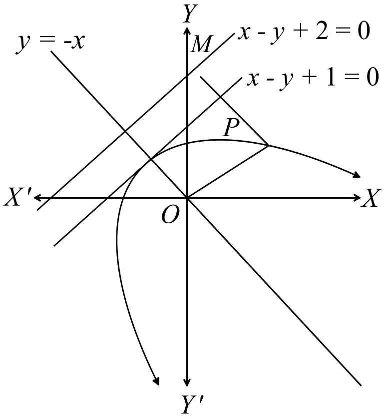
Equation (ii) will touch Eq. (i) if

$\frac{1-am-lc}{m}=\frac{b}{-l/m}$

$\Rightarrow -\frac{l}{m}=\frac{bm}{1-am-lc}$

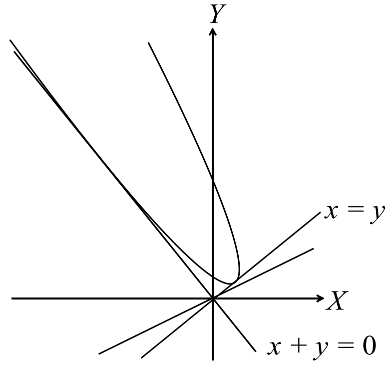
$\Rightarrow cl^2 - bm^2 + alm - l = 0$ (iii)
But given that $5l^2 + 6m^2 - 4lm - 3l = 0$ (iv)
Comparing Eqs. (iii) and (iv), we get
$$\frac{c}{5} = \frac{-b}{6} = \frac{a}{-4} = \frac{-1}{3}$$
$\Rightarrow c = \frac{-5}{3}, b = 2$ and $a = \frac{4}{3}$
So parabola is $\left(y - \frac{4}{3}\right)^2 = 8\left(x + \frac{5}{3}\right)$ whose focus is $\left(\frac{1}{3}, \frac{4}{3}\right)$ and directrix is $3x + 11 = 0$

665 **(b)**



The distance between the focus and the tangent at the vertex$= \frac{|0-0+1|}{\sqrt{1^2+1^2}} = \frac{1}{\sqrt{2}}$
The directrix is the line parallel to the tangent at vertex and at a distance $2 \times \frac{1}{\sqrt{2}}$ from the focus
Let equation of directrix is
$x - y + \lambda = 0$,
Where $\frac{\lambda}{\sqrt{1^2+1^2}} = \frac{2}{\sqrt{2}}$
$\Rightarrow \lambda = 2$
Let $P(x, y)$ be any moving point on the parabola, then
$OP = PM$
$$\Rightarrow x^2 + y^2 = \left(\frac{x - y + 2}{\sqrt{1^2 + 1^2}}\right)^2$$
$\Rightarrow 2x^2 + 2y^2 = (x - y + 2)^2$
$\Rightarrow x^2 + y^2 + 2xy - 4x + 4y - 4 = 0$
Latus rectum length $=2\times$(distance of focus from directrix)
$$= 2\left|\frac{0 - 0 + 2}{\sqrt{1^2 + 1^2}}\right|$$
Solving parabola with x-axis,
$x^2 - 4x - 4 = 0$
$$\Rightarrow x = \frac{4 \pm \sqrt{32}}{2} = 2 \pm 2\sqrt{2}$$
$\Rightarrow$Length of chord on x-axis is $4\sqrt{2}$
Since the chord $3x + 2y = 0$ passes through the focus, it is focal chord
Hence, tangents at the extremities of chord are perpendicular

666 **(a)**

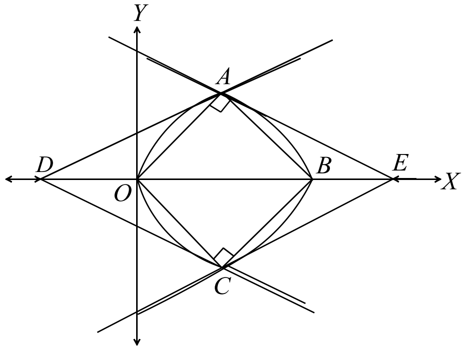


We know that foot of perpendicular from focus upon tangent lies on the tangent at vertex of the parabola
Now, if foot of perpendicular of (2, 3) on the line $x - y = 0$ is (x_1, y_1), then
$$\frac{x_1 - 2}{1} = \frac{y_1 - 3}{-1} = \frac{2 - 3}{2}$$
$\Rightarrow x_1 = \frac{5}{2}$ and $y_1 = \frac{5}{2}$
If foot of perpendicular of (2, 3) on the line $x + y = 0$ is (x_2, y_2), then
$$\frac{x_2 - 2}{1} = \frac{y_2 - 3}{1} = \frac{2 + 3}{2}$$
$\Rightarrow x_2 = -\frac{1}{2}$ and $y_2 = \frac{1}{2}$
Now tangent at vertex passes through the points $\left(\frac{5}{2}, \frac{5}{2}\right)$ and $\left(-\frac{1}{2}, \frac{1}{2}\right)$. Then, its equation is
$$y - \frac{1}{2} = \frac{2}{3}\left(x + \frac{1}{2}\right)$$
Or $4x - 6y + 5 = 0$
Latus rectum of the parabola
$= 4 \times$(distance of focus from tangent at vertex)
$$= 4 \times \left|\frac{8 - 18 + 5}{\sqrt{52}}\right| = \frac{10}{\sqrt{13}}$$
Also, distance between the focus and tangent at vertex$= \frac{5}{\sqrt{13}}$
Since tangents $x + y = 0$ and $x - y = 0$ are perpendicular, they meet at (0, 0) which lies on the directrix
Also, it is parallel to the tangent at vertex, hence its equation is $4x - 6y = 0$
We know that $\frac{1}{SP} + \frac{1}{SQ} = \frac{1}{a}$, where a is $\left(\frac{1}{4}\right)$th of latus rectum
$$\Rightarrow \frac{1}{SP} + \frac{1}{SQ} = \frac{2\sqrt{13}}{5}$$

667 **(d)**
Solving given parabolas, we have
$-8(x - a) = 4x$
$$\Rightarrow x = \frac{2a}{3}$$

$\Rightarrow$ Points of intersection are $\left(\frac{2a}{3}, \pm\sqrt{\frac{8a}{3}}\right)$

Now $OABC$ is concyclic



Hence, $\angle OAB$ must be right angle

$\Rightarrow$ Slope of $OA \times$ Slope of $AB = -1$

$\Rightarrow \frac{\sqrt{\frac{8a}{3}}}{\frac{2a}{3}} \times \frac{\sqrt{\frac{8a}{3}}}{a - \frac{2a}{3}} = -1$

$\Rightarrow a = 12$

$\Rightarrow$ Coordinates of A and B are $(8, 4\sqrt{2})$ and $(8, -4\sqrt{2})$ respectively

$\Rightarrow$ Length of common chord $= 8\sqrt{2}$

Area of quadrilateral $= \frac{1}{2} OB \times AC$

$= \frac{1}{2} \times 12 \times 8\sqrt{2}$

$= 48\sqrt{2}$

Tangent to parabola $y^2 = 4x$ at point $\left(8, 4\sqrt{2}\right)$ is $4\sqrt{2}y = 2(x + 8)$ or $x - 2\sqrt{2}y + 8 = 0$ which meets the x-axis at $D(-8,0)$

Tangent to parabola $y^2 = -8(x - 12)$ at point $(8, 4\sqrt{2})$ is $4\sqrt{2}y = -4(x + 8) + 96$ or $x + \sqrt{2}y - 16 = 0$, which meets the x-axis at $E(16, 0)$

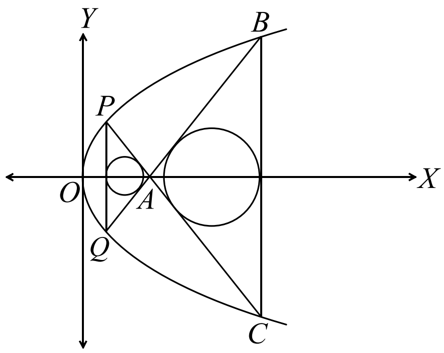
Hence, area of quadrilateral $DAEC = \frac{1}{2} DE \times AC$

$= \frac{1}{2} \times 24 \times 8\sqrt{2}$

$= 96\sqrt{2}$

668 **(b)**

For $y^2 = 4x$, coordinates of end of latus rectum are $P(1,2)$ and $Q(1, -2)$



ΔPAQ is isosceles right angled. Therefore, slope of PA is -1 and its equation is $y - 2 = -(x - 1)$ or $x + y - 3 = 0$

Similarly, equation of line QB is $x - y - 3 = 0$

Solving $x + y - 3 = 0$ with the parabola $y^2 = 4x$, we have

$(3 - x)^2 = 4x$ or $x^2 - 10x + 9 = 0$

$\therefore\ x = 1, 9$

Therefore, coordinates of B and C are $(9, -6)$ and $(9, 6)$ respectively

Area of trapezium $PBCQ = \frac{1}{2} \times (12 + 4) \times 8$

$= 64$ sq. units

Let the circumcentre of trapezium $PBCQ$ is $T(h, 0)$

Then $PT = PB$

$\Rightarrow \sqrt{(h - 1)^2 + 4} = \sqrt{(h - 9)^2 + 36}$

$\Rightarrow -2h + 5 = -18h + 81 + 36$

$\Rightarrow 16h = 112$

$\Rightarrow\ h = 7$

Hence, radius is $\sqrt{40} = 2\sqrt{10}$

Let inradius of ΔAPQ be r_1, then $r_1 = \frac{\Delta_1}{s_1}$

$= \frac{\frac{1}{2} \times 4 \times 2}{4 + 2\sqrt{4 + 4}}$

$= \frac{1}{1 + \sqrt{2}} = \sqrt{2} - 1$

Let inradius of ΔABC be r_2, then

$r_2 = \frac{\Delta_2}{s_2}$

$= \frac{\frac{1}{2} \times 12 \times 6}{12 + 2\sqrt{36 + 36}}$

$= \frac{3}{1 + \sqrt{2}} = 3\left(\sqrt{2} - 1\right)$

$\Rightarrow \frac{r_2}{r_1} = 3$

669 **(d)**

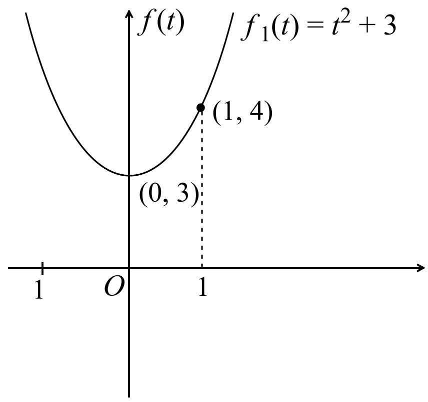
$9x - a.3^x - a + 3 \leq 0$

Let $t = 3^x$

$\Rightarrow t^2 - at - a + 3 \leq 0$

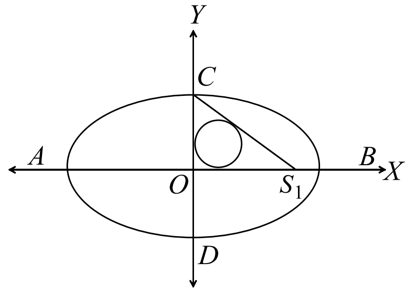
Or $t^2 + 3 \leq a(t + 1)$

Where $t \in R^+$ for $\forall x \in R$



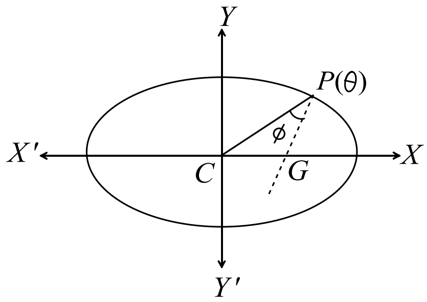
Let $f_1(t)$ be t^2+3 and $f_2(t)$ be $a(t+1)$
From $x<0$, $t\in(0,1)$. That means (1) should have at least one solution in $t\in(0,1)$
From (1), it is obvious that $a\in R^+$
Now $f_2(t)=a(t+1)$ represents a straight line. It should meet the curve.
$f_1(t)=t^2+3$, at least once in $t\in(0,1)$
$f_1(0)=3, f_1(1)=4, f_2(0)=a, f_2(1)=2a$
If $f_1(0)=f_2(0)\Rightarrow a=3$; if $f_1(1)=f_2(1)=a=2$
Hence, required a $\in$ (2, 3)

670 **(c)**



$\because OS_1=ae=6, OC=b$
Also $CS_1=a$
$\Longrightarrow$ Area of $\Delta OCS_1=\frac{1}{2}(OS_1)\times(OC)=3b$
$\Longrightarrow$ Semi-perimeter of $\Delta OCS_1=\frac{1}{2}(OS_1+OC+$
CS1
$=\frac{1}{2}(6+a+b)$ (i)
$\Longrightarrow$ In radius of $\Delta OCS_1=1$
$$\Longrightarrow \frac{3b}{\frac{1}{2}(6+a+b)}=1$$
$\Longrightarrow 5b=6+a$ (ii)
also $b^2=a^2-a^2e^2$
$=a^2-36$ (iii)
$\Longrightarrow$From (ii), we get
$25(a^2-36)=36+a^2+12a$
$\Longrightarrow 2a^2-a-78=0$
$$\Longrightarrow a=\frac{13}{2},-6$$
$$\Longrightarrow a=\frac{13}{2}$$
and $b=\frac{5}{2}$
Area of ellipse $=\pi ab=\frac{65\pi}{4}$ sq.unit
Perimeter of $\Delta OCS_1=6+a+b=6+\frac{13}{2}+\frac{5}{2}=$ 15 units
Equation director circle is $x^2+y^2=a^2+b^2$
or $x^2+y^2=\frac{97}{2}=r^2$

671 **(c)**



Any point P on the ellipse at $(a\,cos\theta, b\sin\theta)$
$\therefore$ Equation of CP is $y=\left(\frac{b}{a}\tan\theta\right)x$
The normal to the ellipse at P is $\frac{ax}{\cos\theta}-\frac{bx}{\sin\theta}=a^2-b^2$
Slope of the lines CP and the normal GP are $\frac{b}{a}\tan\theta$ and $\frac{a}{b}\tan\theta$, respectively
$$\therefore \tan\phi=\frac{\frac{a}{b}\tan\theta-\frac{b}{a}\tan\theta}{1+\frac{a}{b}\tan\theta\frac{b}{a}\tan\theta}$$
$$=\frac{a^2-b^2}{ab}\frac{\tan\theta}{\sec^2\theta}$$
$$=\frac{a^2-b^2}{ab}\sin\theta\cos\theta=\frac{a^2-b^2}{2ab}\sin 2\theta$$
Therefore, the greatest value of $\tan\phi=\frac{a^2-b^2}{2ab}\times$
1=a2−b22ab

Given that $\frac{a^2-b^2}{ab}=\frac{3}{2}$ Let $\frac{a}{b}=t$
$$\Rightarrow\ t-\frac{1}{t}=\frac{3}{2}$$
$\Rightarrow\ 2t^2-3t-2=0$
$\Longrightarrow 2t^2-4t+t-2=0$
$$\Longrightarrow (2t+1)(t-2)\Rightarrow\frac{a}{b}=2$$
$$\Longrightarrow e^2=1-\frac{1}{4}$$
$$\Longrightarrow e=\frac{\sqrt{3}}{2}$$
Rectangle inscribed in the ellipse whose one vertex is $(a\cos\theta, b\sin\theta)$ is $(2a\cos\theta)(2b\sin\theta)=2a\,b\sin(2\theta)$ which has maximum value $2ab$.
Given that $a=10$, than $b=5\Rightarrow$ maximum area is 100
Locus of intersection point of perpendicular tangents is $x^2+y^2=10^2+5^2$ or $x^2+y^2=125$ (director circle)

672 **(b)**

$21x^2-6xy+29y^2+6x-58y-151=0$
$3(x-3y+3)^2+2(3x+y-1)^2=180$
$$\Longrightarrow \frac{(x-3y+3)^2}{60}+\frac{(3x+y-1)^2}{90}=1$$
$$\Rightarrow\left(\frac{x-3y+3}{\sqrt{1+3^2}\sqrt{6}}\right)^2\left(\frac{3x+y-1}{\sqrt{1+3^2}3}\right)^2=1$$

Thus C is an ellipse whose length of axes are $6, 2\sqrt{6}$

The minor and the major axes are $x - 3y + 3 = 0$ and $3x + y - 1 = 0$, respectively

Their point of intersection gives the centre of the conic

$\therefore$ Centre $\equiv (0, 1)$

673 **(b)**

Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

The line $y = mx \pm \sqrt{a^2m^2 + b^2}$ touches the ellipse for all m

Hence, it is identical with

$$y = -\frac{2px}{\sqrt{1-p^2}} + \frac{1}{\sqrt{1-p^2}}$$

Hence, $m = -\frac{2p}{\sqrt{1-p^2}}$

and $a^2m^2 + b^2 = \frac{1}{1-p^2}$

$$\Rightarrow \; a^2\frac{4p^2}{1-p^2} + b^2 = \frac{1}{1-p^2}$$

$\Rightarrow \; p^2(4a^2 - b^2) + b^2 - 1 = 0$

This equation is true for all real p if $b^2 = 1$ and $4a^2 = b^2$

$\Rightarrow b^2 = 1$ and $a^2 = \frac{1}{4}$

Therefore, the equation of the ellipse is

$$\frac{x^2}{1/4} + \frac{y^2}{1} = 1$$

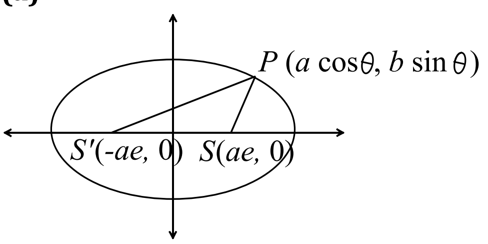
If e is its eccentricity, then

$$\frac{1}{4} = 1 - e^2 \Rightarrow e^2 \Rightarrow e = \frac{\sqrt{3}}{2}$$

be $= \frac{\sqrt{3}}{2}$, hence foci are $\left(0, \pm\frac{\sqrt{3}}{2}\right)$

Equation of director circle is $x^2 + y^2 = \frac{5}{4}$

674 **(a)**



Let the coordinates of P be $(a\cos\theta, b\sin\theta)$

Here $SP =$ focal distance of the point

$P = a - ae\cos\theta$

$S'P = a + ae\cos\theta$

$SS' = 2ae$

If (h, k) be the coordinates of the incentre of $\Delta PSS'$, then

$$h = \frac{2ae(b\cos\theta) + a(1 - e\cos\theta)(-ae) + a(1 + e\cos\theta)ae}{2ae + a(1 - e\cos\theta) + a(1 + e\cos\theta)}$$

$\Rightarrow \; h = ae\cos\theta \quad$ (i)

and

$$k = \frac{2ae(b\sin\theta) + a(1 - e\cos\theta) \times 0 + a(1 + e\cos\theta) \times 0}{2ae + a(1 - e\cos\theta) + a(1 + e\cos\theta)}$$

$\Longrightarrow \quad k = \frac{eb\sin\theta}{(e+1)} \quad$ (ii)

Eliminating θ from (i) and (ii), we get

$$\frac{x^2}{a^2e^2} + \frac{y^2}{\left(\frac{be}{e+1}\right)^2} = 1$$

Which clearly represents an ellipse. Let e_1 be its eccentricity. Then

$$\frac{b^2e^2}{(e+1)^2} = a^2e^2(1 - e_1^2)$$

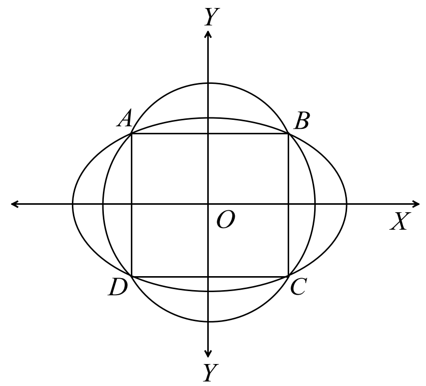
$$\Longrightarrow \; e_1^2 = 1 - \frac{b^2}{a^2(e+1)^2}$$

$$\Rightarrow \; e_1^2 = 1 - \frac{1 - e^2}{(e+1)^2} = 1 - \frac{1 - e}{1 + e}$$

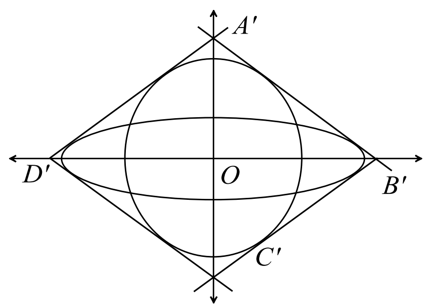
$$\Rightarrow \; e_1^2 = \frac{2e}{e+1} \Rightarrow e_1 = \sqrt{\frac{2e}{e+1}}$$

Maximum area of rectangle is $2(ae)\left(\frac{be}{e+1}\right) = \frac{2abe^2}{e+1}$

675 **(a)**



(a)



Solving the curves given (eliminating x^2, we have)

$$\frac{r^2 - y^2}{16} + \frac{y^2}{9} = 1$$

$$\Rightarrow y^2 = \frac{144 - 9r^2}{7}$$

Solving the curves given (eliminating y^2), we have

$$\frac{x^2}{16} = \frac{r^2 - x^2}{9} = 1$$

$\Rightarrow x^2 = \dfrac{16r^2 - 144}{7}$

If $ABCD$ is a square, then

$x^2 = y^2$

or $\frac{144-9r^2}{7} = \frac{16r^2-144}{7}$

or $25r^2 = 288$

or $r = \frac{12}{5}\sqrt{2}$

676 **(c)**

$\lambda x - y + 2(1+\lambda) = 0$

$\Longrightarrow \lambda(x+2) - (y-2) = 0$

This line passes through $(-2, 2)$

$\mu x - y + 2(1-\mu) = 0$

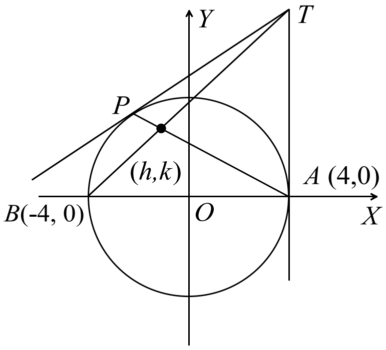
$\Longrightarrow \mu(x-2) - (y-2) = 0$

This line passes through $(2, 2)$

Clearly these represent the foci of the ellipse. So $2ae = 4$

The circle $x^2 + y^2 - 4y - 5 = 0 \Rightarrow x^2 + (y-2)^2 = 9$ represents auxiliary circle. Thus $a^2 = 9 \Rightarrow e = \frac{2}{3}$ and $b^2 = 5$

677 **(b)**



Tangents at $P(4\cos\theta, 4\sin\theta)$ to $x^2 + y^2 = 16$ is

$x\cos\theta + y\sin\theta = 4$ (i)

Equation of AP is

$y = \frac{\sin\theta}{\cos\theta - 1}(x-4)$ (ii)

From (i), coordinates of the point T are given by

$\left(4, \dfrac{4(1-\cos\theta)}{\sin\theta}\right)$

Equation of BT is

$y = \frac{1-\cos\theta}{2\sin\theta}(x+4)$ (iii)

Let (h, k) be the point of intersection of the lines (ii) and (iii). Then we have

$k^2 = -\dfrac{1}{2}(h^2 - 16)$

$\Rightarrow \dfrac{h^2}{16} + \dfrac{y^2}{8} = 1$

Therefore, locus of (h, k) is

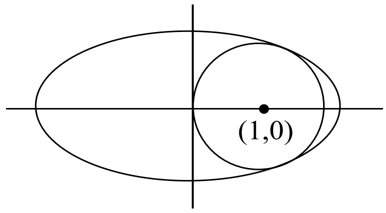
$\dfrac{x^2}{16} + \dfrac{y^2}{8} = 1$

Which is an ellipse with eccentrically $e = \frac{1}{\sqrt{2}}$

Sum of focal distance of any point $= 2a = 8$

Considering circle $x^2 + y^2 = a^2$, we find that the eccentricity of the ellipse is $\frac{1}{\sqrt{2}}$ which is constant and does not change by changing the radius of the circle

678 **(a)**



Solving both equations, we have

$\dfrac{x^2}{a^2} + \dfrac{1-(x-1)^2}{b^2} = 1$

$\Rightarrow\ b^2x^2 + a^2[1-(x-1)^2] = a^2b^2$

$\Rightarrow\ (b^2 - a^2)x^2 + 2a^2x - a^2b^2 = 0$ (i)

For least area circle must touch the ellipse

$\Rightarrow$ Discriminant of (1) is zero

$\Rightarrow\ 4a^4 + 4a^2b^2(b^2 - a^2) = 0$

$\Rightarrow a^2 + b^2(b^2 - a^2) = 0$

$\Rightarrow\ a^2 + b^2(-a^2e^2) = 0$

$\Rightarrow\ 1 - b^2e^2 = 0 \Rightarrow b = \dfrac{1}{e}$

Also $a^2 = \frac{b^2}{1-e^2} = \frac{1}{e^2(1-e^2)}$

$\Rightarrow\ a = \dfrac{1}{e\sqrt{1-e^2}}$

Let S be the area of the ellipse

$\Rightarrow\ S = \pi ab = \dfrac{\pi}{e^2\sqrt{1-e^2}}$

$= \dfrac{\pi}{\sqrt{e^4 - e^6}}$

Area is minimum if $f(e) = e^4 - e^6$ is maximum

When $f''^{(e)} = 4e^3 - 6e^5 = 0$

or $e = \sqrt{\frac{2}{3}}$ (which is point of maxima for $f(e)$)

$\Rightarrow S$ is least when $e = \sqrt{\frac{2}{3}}$

$\Rightarrow$ Ellipse is $2x^2 + 6y^2 = 9$

Equation of auxiliary circle of ellipse is

$x^2 + y^2 = 4.5$

Length of latus rectum of ellipse is $\frac{2b^2}{a} = \frac{2\frac{9}{4}}{\frac{9}{2}} = 1$

679 **(b)**

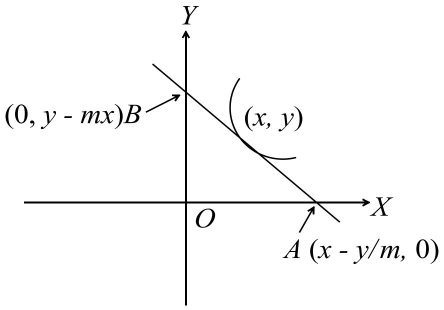
Let the curve be $y = f(x)$

Now tangent at point P to the curve is

$Y - y = m(X - x)$

It meets y-axis when

$X = 0 \Rightarrow Y = y - mx$



and x-axis when

$$Y = 0 \Rightarrow X = x - \frac{y}{m}$$

Given that P is midpoint of AB. Hence,

$$x - \frac{y}{m} = 2x$$

$$\Rightarrow \frac{y}{m} = -x$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$

$$\Rightarrow \frac{dy}{y} + \frac{dx}{x} = 0$$

$$\Rightarrow \log_e xy = c$$

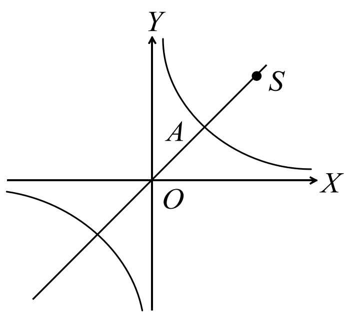
$$\Rightarrow xy = c$$

As the curve passes through (2, 4), so

$$xy = 8$$

Solving with $y = x$, we get

$$x = 2\sqrt{2}$$



$$\therefore OA = \sqrt{8+8} = 4$$

$$\Rightarrow OS = 4\sqrt{2}$$

Hence, coordinates of S are (4, 4) or (−4, −4)

Directrix is at distance $4/\sqrt{2}$ from origin

Hence, its equation $x + y = \pm 4$

680 **(d)**

Centre $\equiv (1, 2)$

Radius of auxiliary circle

$$= a = \sqrt{(2-1)^2 + (5-2)^2}$$

$$= \sqrt{10}$$

$$2ae = \sqrt{8^2 + 8^2} = 8\sqrt{2} \Rightarrow e = \frac{4}{\sqrt{5}}$$

$$b^2 = a^2e^2 - a^2 = 32 - 10 = 22$$

$$\Rightarrow 2b = 2\sqrt{22}$$

681 **(c)**

$2a = 3$

Distance between the foci (1, 2) and (5, 5) is 5

$$\therefore 2ae = 5$$

$$\therefore e = \frac{5}{3}$$

Now if e' is eccentricity of the corresponding conjugate hyperbola, then

$$\frac{1}{e^2} + \frac{1}{e'^2} = 1$$

$$\Rightarrow e' = \frac{5}{4}$$

682 **(a)**

Equation of tangent in parametric form is given by

$$\frac{x-1}{-1/\sqrt{2}} = \frac{y-1}{1/\sqrt{2}} = \pm 3\sqrt{2}$$

$$\Rightarrow A \equiv (4, -2), B \equiv (-2, 4)$$

Equations of asymptotes (OA and OB) are given by

$$y + 2 = \frac{-2}{4}(x - 4) \Rightarrow 2y + x = 0$$

and $y - 4 = \frac{4}{-2}(x + 2) \Rightarrow 2x + y = 0$

Hence, the combined equation of asymptotes is

$$(2x + y)(x + 2y) = 0$$

$\Rightarrow 2x^2 + 2y^2 + 5xy = 0$

683 **(b)**

Any point on the hyperbola $xy = 16$ is $\left(4t, \frac{4}{t}\right)$

Normal at this point is $y - 4/t = t^2(x = 4t)$

If the normal passes through $P(h, k)$, then
$k - 4/t = t^2(h - 4t)$

$\Rightarrow 4t^4 - t^3h + tk - 4 = 0$

This equation has roots t_1, t_2, t_3, t_4 which are parameters of the four feet of normals on the hyperbola. Therefore,

$$\sum t_1 = \frac{h}{4}$$

$$\sum t_1t_2 = 0$$

$$\sum t_1t_2t_3 = -\frac{k}{4}$$

$$t_1t_2t_3t_4 = -1$$

$$\therefore \frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + \frac{1}{t_4} = \frac{k}{4}$$

$\Rightarrow y_1 + y_2 + y_3 + y_4 = k$

According to the question,

$$t_1^2 + t_2^2 + t_3^2 + t_4^2 = \frac{h^2}{16} = k$$

Hence, the locus of (h, k) is

$x^2 = 16y$

684 **(b)**

Perpendicular tangents intersect at the centre of rectangular hyperbola. Hence, centre of hyperbola is (1, 1) and equation of asymptotes are $x - 1 = 0$ and $y - 1 = 0$

685 **(3)**

$(a, 2)$ lies on director circle $x^2 + y^2 = 7$

$\therefore a^2 = 3$

686 **(4)**

Equation of tangent is $y = 2x \pm \sqrt{4a^2 + b^2}$
$\Rightarrow$ this is normal to the circle $x^2 + y^2 + 4x + 1 = 0$
$\Rightarrow$ this tangent passes through $(-2, 0)$
$\Rightarrow 0 = -4 \pm \sqrt{4a^2 . b^2} \Rightarrow 4a^2 + b^2 = 16$
$\Rightarrow$ Using A.M $\geq$ G.M, we get
$\frac{4a^2 + b^2}{2} \geq \sqrt{4a^2 + b^2} \Rightarrow ab \leq 4$

687 **(1)**

$x^2 + y^2 + (3 + \sin\beta)x + (2\cos\alpha)y = 0$ (1)
$x^2 + y^2 + (2\cos\alpha)x + 2cy = 0$ (2)
Since both the circles are passing through the origin (0, 0), equation of tangent at (0, 0) will be same tangent at (0, 0) to circle (1),
$(3 + \sin\beta)x + (2\cos\alpha)y = 0$ (3)
Tangent at (0, 0) to circle (2),
$(2\cos\alpha)x + 2cy = 0$
$\therefore$ (1) and (2) must be identical
comparing (1) and (2)
$$\frac{3 + \sin\beta}{2\cos\alpha} = \frac{2\cos\alpha}{2c}$$
$$\Rightarrow c = \frac{2\cos^2\alpha}{3 + \sin\beta}$$
$\Rightarrow c_{\max} = 1$ when $\sin\beta = -1$ and $\alpha = 0$

688 **(6)**

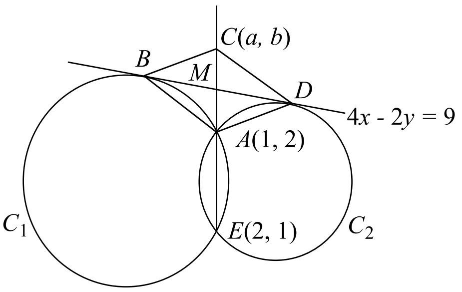
Slope of the line -1
From the curve, $\frac{dy}{dx} = \frac{4}{y}$
Hence slope of normal $= -\frac{y}{4} = -1$ or $y = 4$
Putting $y = 4$ in equation of curve we have $x = 2$
Hence point is (4, 2)

689 **(4)**

$y^2 = x \quad \therefore 4a = 1,$
$P(at_1^2, 2at_1) = (4, -2)$
$\therefore t_1 = -4$
Also $t_1t_2 = -1$ as PQ is a focal chord
Slope of tangent at t_2 is $\frac{1}{t_2} = -t_1 = 4$

690 **(4)**

The radical axis bisects the common tangent BD
Hence M is the mid point of BD
Let $C(a, b)$



Now $C(a, b)$ lies on common chord AE which is
$y - 2 = -1(x - 1)$ or $x + y = 3$
$\therefore a + b = 3$ (1)
Also M $\left(\frac{a+1}{2}, \frac{b+2}{2}\right)$ lies on $4x - 2y = 9$

$\Rightarrow 4\left(\frac{a+1}{2}\right) - 2\left(\frac{b+2}{2}\right) = 9$

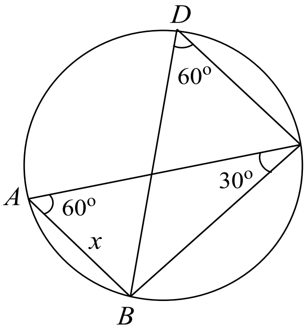
$\Rightarrow 2a + 2 - b - 2 = 9$

$\Rightarrow 2a - b = 9 \quad ...(2)$

Solving (1) and (2) $a = 4$ and $b = -1$

$\Rightarrow a + b = 3$

691 **(1)**



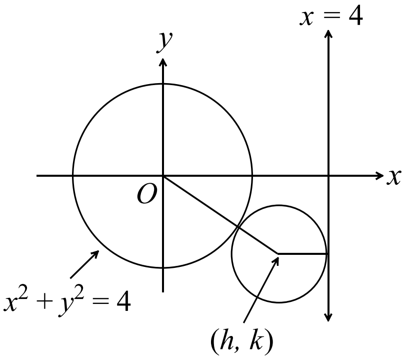
$\angle A = 60° = \angle D$

$AC = 2$ (given)

$\angle ABC = 90°$

$\Rightarrow x = 1$

692 **(3)**



Radius of variable circle is $4 - h$

If touches $x^2 + y^2 = 4$

$\therefore 2 + 4 - h = \sqrt{h^2 + k^2}$

Or $x^2 + y^2 = x^2 - 12x + 36$

$\Rightarrow y^2 = -12(x + 3)$

The vertex (3,0)

693 **(2)**

Since $\frac{e}{2}$ and $\frac{e'}{2}$ are eccentricities of a hyperbola and its conjugate

$$\therefore \frac{4}{e^2} + \frac{4}{e'^2} = 1$$

$$\therefore 4 = \frac{e^2 e'^2}{e'^2 + e'^2}$$

Line passing through the points $(e, 0)$ and $(0, e')$

$e'x + ey - ee' = 0$

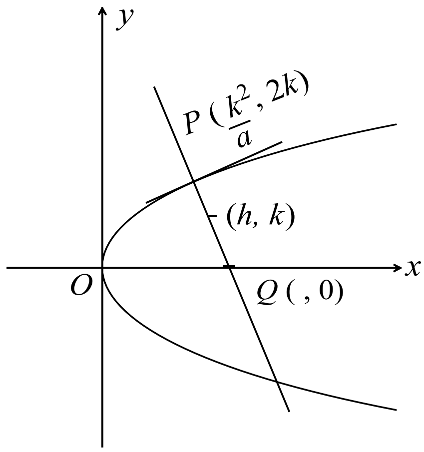
It is tangent to the circle $x^2 + y^2 = r^2$

$$\therefore \frac{ee'}{\sqrt{e^2 + e'^2}} = r$$

$$\therefore r^2 = \frac{e^2 e'^2}{e^2 + e'^2} = 4$$

$\therefore r = 2$

694 **(4)**



Consider the parabola $y^2 = 4ax$

We have to find the locus of $R(h, k)$, since Q has ordinate 'O', ordinate of P is $2k$

Also P is on the curve, then abscissa of P is k^2/a

Now PQ is normal to curve

Slope of tangent to curve at any point $\frac{dy}{dx} = \frac{2a}{y}$

Hence slope of normal at point P is $-\frac{k}{a}$

Also slope of normal joining P and R (h, k) is $\frac{2k-k}{\frac{k^2}{a}-h}$

Hence comparing slopes $\frac{2k-k}{\frac{k^2}{a}-h} = -\frac{k}{a}$

or $y^2 = a(x - a)$

For $y^2 = 16x, a = 4$, hence locus us $y^2 = 4(x - 4)$

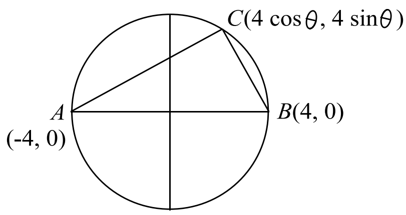
695 **(3)**

Let the hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ then $2a = ae$, i.e., $e = 2$

$$\therefore \frac{b^2}{a^2} = e^2 - 1 = 3$$

$$\therefore \frac{(2b)^2}{(2a)^2} = 3$$

696 **(8)**



Required area

$A = \frac{1}{2} \cdot 8 \cdot 4 \sin\theta = |16 \sin\theta|$

Now area is integer than the possible values of $\sin\theta$ are $\frac{1}{16}, \frac{2}{16}, \ldots\ldots \frac{15}{16}$

i.e. 15 points in each quadrant

$\Rightarrow 60 + 2$ more with $\sin\theta = 1$

$\Rightarrow N = 62$

697 **(4)**

Let the point be (h, k). Then equation of the chord of contact is $hx + ky = 4$

Since $hx + ky = 4$ is tangent to $xy = 1$

$\therefore x\left(\frac{4-hx}{k}\right) = 1$ has two equal roots

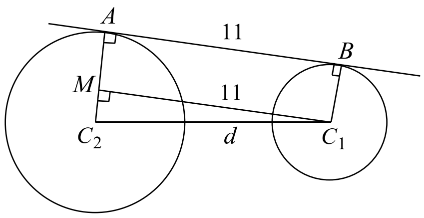
i.e., $hx^2 - 4x + k = 0$ has equal roots

$\therefore hk = 4$

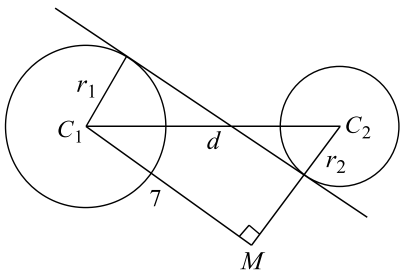
$\therefore$ locus of (h, k) is $xy = 4$

i.e., $c^2 = 4$

698 **(9)**



From the figure $d^2 = 11^2 + (r_1 - r_2)^2$ (1)



From the figure

$d^2 = 7^2 + (r_1 + r_2)^2$ (2)

From (1) and (2), $4r_1r_2 = 72 \Rightarrow r_1r_2 = 18$

699 **(8)**

$\frac{a}{e} - ae = 8 \Rightarrow 2a - \frac{a}{2} = 8$, i.e. $a = \frac{16}{3}$

$b^2 = a^2(1-e)^2 = \frac{256}{9}\left(1 - \frac{1}{4}\right) = \frac{64}{3}$

$\therefore$ length of minor axis $= 2b = \frac{16}{\sqrt{3}}$

$\therefore k = 8$

700 **(4)**

$\because OS_1 = ae = 6, OC = b$ (let)

Also $CS_1 = a$

$\because$ Area of $\Delta OCS_1 = \frac{1}{2}(OS_1) \times (OC) = 3b$

$\because$ semi-perimeter of $\Delta OCS_1 = 1/2\ (OS_1 + OC + CS_1)$

$= 1/2(6 + a + b)$ (1)

$\because$ Inradius of $\Delta OCS_1 = 1$

$\Rightarrow \frac{3b}{\frac{1}{2}(6+a+b)} = 1 \Rightarrow 5b = 6 + a$ (2)

Also $b^2 = a^2 - a^2e^2 = a^2 - 36$ (3)

$\Rightarrow$ from (2)

$25b^2 = 36 + 12a + a^2$

$\therefore 25(a^2 - 36) = 36 + a^2 + 12a$

$2a^2 - a - 78 = 0$

$\therefore a = \frac{13}{2}, -6$

$a = \frac{13}{2} \therefore b = \frac{5}{2}$

701 **(2)**

$\left(\pm ae, \frac{b^2}{a}\right)$ are extremities of the latus-rectum having positive ordinates

$\Rightarrow a^2e^2 = -2\left(\frac{b^2}{a} - 2\right)$ (1)

But $b^2 = a^2(1 - e^2)$ (2)

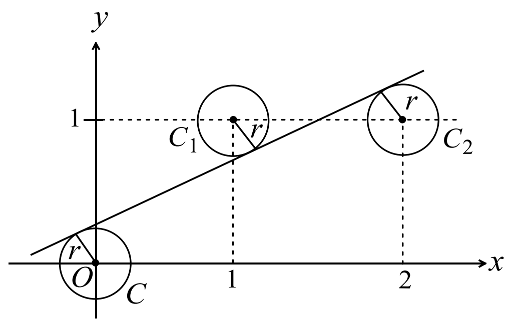
$\therefore$ From (1) and (2), we get $a^2e^2 - 2ae^2 + 2a - 4 = 0$

$\Rightarrow ae^2(a-2) + 2(a-2) = 0$

$\therefore (ae^2 + 2)(a - 2) = 0$

Hence $a = 2$

702 **(5)**



Equation of line joining origin and centre of circle $C_2 \equiv (2, 1)$ is, $y = \frac{x}{2}$

$\Rightarrow x - 2y = 0$

Let equation of common tangent is $x - 2y + c = 0$ (1)

$\therefore$ perpendicular distance from (0, 0) on this line = perpendicular distance from (1, 1)

$\Rightarrow \left|\frac{c}{\sqrt{5}}\right| = \left|\frac{c-1}{\sqrt{5}}\right|$

$\Rightarrow c = 1 - c \Rightarrow c = \frac{1}{2}$

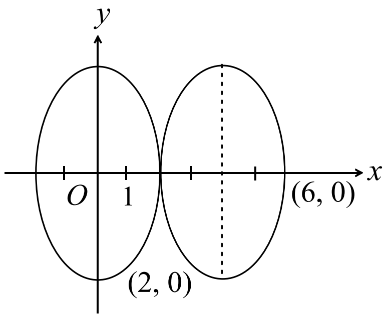
Equation of common tangent is

$x - 2y + \frac{1}{2} = 0$ or $2x - 4y + 1 = 0$ (2)

Perpendicular from (2, 1) on the line (2)

$r = \left|\frac{4 - 4 + 1}{\sqrt{20}}\right| = \frac{1}{2\sqrt{5}} = \frac{\sqrt{5}}{10}$

703 **(8)**



Let $x - 4 = 2\cos\theta \Longrightarrow x = 2\cos\theta + 4$ and $y = 3\sin\theta$

Now $E = \frac{x^2}{4} + \frac{y^2}{9}$

$= \dfrac{(2\cos\theta + 4)^2}{4} + \sin^2\theta$

$= \dfrac{4\cos^2\theta + 16 + 16\cos\theta + 4\sin^2\theta}{4}$

$= \dfrac{20 + 16\cos\theta}{4}$

$= 5 + 4\cos\theta$

Hence $E_{\text{max}} - E_{\text{min}} = (9 - 1) = 8$

704 **(7)**

By using condition of tangency, we get

$4h^2 = 3k^2 + 2$

$\therefore$ Locus of $P(h, k)$ is $4x^2 - 3y^2 = 2$ (which is hyperbola)

Hence $e^2 = 1 + \frac{4}{3} \Rightarrow e = \sqrt{\frac{7}{3}}$

705 **(6)**

Clearly locus of point of intersection of lines is

$(x - 5)(x - 3) + (y - 2)(y + 4) = 0$

$\Rightarrow x^2 + y^2 - 8x + 2y + 7 = 0$

Hence $|f + g| = |2 + (-8)| = 6$

706 **(8)**

Equation of the chord whose mid point is $(0, 3)$ is

$\frac{3y}{25} - 1 = \frac{9}{25} - 1$ i.e. $y = 3$

Intersects the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$

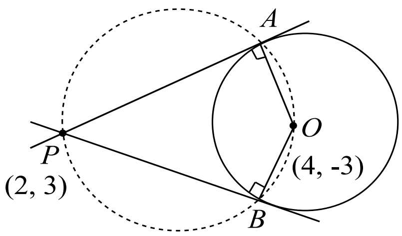
At $\frac{x^2}{16} = 1 - \frac{9}{25} = \frac{16}{25} \Rightarrow x = \pm\frac{16}{5}$

$\therefore$ length of the chord $= \frac{32}{5}$

Thus $\frac{4k}{5} = \frac{32}{5}$ $\therefore k = 8$

707 **(9)**

Center of the given circle is $O(4, -3)$



The circumcircle of ΔPAB will circumscribe the quadrilateral $PBOA$ also, hence one of the diameters must be OP

$\therefore$ Equation of circumcircle of ΔPAB will be

$(x - 2)(x - 4) + (y - 3)(y + 3) = 0$

$\Rightarrow x^2 + y^2 - 6x - 1 = 0$ (1)

Director circle of given ellipse will be

$(x + 5)^2 + (y - 3)^2 = 9 + b^2$

$\Rightarrow x^2 + y^2 + 10x - 6y + 25 - b^2 = 0$ (2)

$\therefore$ From (1) and (2), by applying condition of orthogonality, we get

$2[-3(5) + 0(-3)] = -1 + 25 - b^2 \Rightarrow -30$
$= 24 - b^2$

Hence $b^2 = 54$

708 **(4)**

Equation of hyperbola $(x - 3)(y - 2) = c^2$

Or $xy - 2x - 3y + 6 = c^2$

It passes through $(4, 6)$, then

$4 \times 6 - 2 \times 4 - 3 \times 6 + 6 = c^2$

$\Rightarrow c = 2$

$\therefore$ Latus rectum $= 2\sqrt{2}c = 2\sqrt{2} \times 2 = 4\sqrt{2}$

709 **(5)**

The equation of given circle is $x^2 + y^2 - 6x - 2py + 17 = 0$

or $(x - 3)^2 + (y - p)^2 = (p^2 - 8)$ (1)

Also $(0, 0)$ line outside the circle

Equation of director circle of $S = 0$ will be

$(x - 3)^2 + (y - p)^2 = 2(p^2 - 8)$ (2)

Tangents drawn from $(0, 0)$ to circle (i) are perpendicular to each other

$\therefore$ $(0, 0)$ must lie on director circle

$\therefore (0 - 3)^2 + (0 - p)^2 = 2(p^2 - 8)$

$\Rightarrow p^2 = 25$

$\Rightarrow p = \pm 5$

710 **(4)**

$a = 3$, comparing point $(3, 6)$ with $(3t^2, 6t)$, we have $t = 1$,

Then length of chord$= a\left(t + \frac{1}{t}\right)^2 = 3(1 + 1)^2 = 12$

711 **(1)**

Let $x + 5 = 14\cos\theta$ and $y - 12 = 14\sin\theta$

$\therefore x^2 + y^2 = (14\cos\theta - 5)^2 + (14\sin\theta + 12)^2$

$= 196 + 25 + 144 + 28\,(12\sin\theta - 5\cos\theta)$

$= 365 + 28\,(12\sin\theta - 5\cos\theta)$

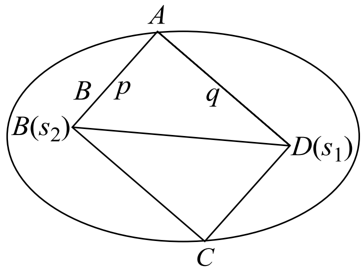
$\therefore \sqrt{x^2 + y^2}\Big|_{\text{min}} = \sqrt{365 - 28 \times 13}$

$= \sqrt{365 - 364} = 1$

712 **(4)**

Let sides of rectangle be p and q

Area of rectangle= $pq = 200$ (1)



Area of ellipse $= \pi ab = 200\pi$

$\therefore ab = 200$ (2)

We have to find the perimeter of rectangle

$= 2(p+q)$

From triangle ABD

Distance $BD = \sqrt{p^2+q^2}$ = distance between foci

or $p^2+q^2 = 4a^2e^2$

or $(p+q)^2 - 2pq = 4(a^2-b^2)$ (3)

Also from the definition of ellipse sum of focal length is $2a$,

Then $AB + AD = p + a = 2a$ (4)

Putting value of $(p + q)$ in equation (3) and (4)

We have $(2a)^2 - 2pq = 4a^2 - 4b^2$ (using equation (1))

$\Rightarrow 4a^2 - 2\times 200 = 4(a^2-b^2)$

$\Rightarrow a^2 - 100 = a^2 - b^2$

$\Rightarrow b = 10$

From equation (2), $ab = 200 \Rightarrow a = 20$

Since $p + q = 2a$ (from equation (4))

Therefore perimeter $= 2(p+q) = 4a = 4\times 20 = 80$

713 **(5)**

Equation of tangents to hyperbola having slope m are $y = mx \pm \sqrt{9m^2-49}$

Distance between tangents is 2

$$\Rightarrow \frac{2\sqrt{9m^2-49}}{\sqrt{1+m^2}} = 2$$

$$\Rightarrow 9m^2 - 49 = 1 + m^2$$

$$\Rightarrow 8m^2 = 50 \Rightarrow m = \pm\frac{5}{2}$$

714 **(7)**

Line $y = 2x - b$

$$\Rightarrow 1 = \frac{2x-y}{b}$$

Homogenizing parabola with line

$$x^2 - 4x\left(\frac{2x-y}{b}\right) - y\left(\frac{2x-y}{b}\right) = 0$$

Since $\angle AOB = 90°$

$\therefore$coefficient of x^2 =coefficient of $y^2 = 0$

$$\Rightarrow 1 - \frac{8}{b} + \frac{1}{b} = 0$$

$\Rightarrow b = 7$

715 **(7)**

Given hyperbola is

$$3x^2 - 2y^2 = 5 \text{ or } \frac{x^2}{2} - \frac{y^2}{3} = 1$$

Tangents from the point (α, β)

$$y = mx \pm \sqrt{a^2m^2 - b^2}$$

Or $(y - mx)^2 = a^2m^2 - b^2$

Or $(\beta - m\alpha)^2 = 2m^2 - 3$ ($\because a^2 = 2$ and $b^2 = 3$)

Or $m^2\alpha^2 + \beta^2 - 2m\alpha\beta - 2m^2 + 3 = 0$

$$m^2(\alpha^2-2) - 2\alpha\beta m + \beta^2 + 3 = 0$$

$$m_1.m_2 = \frac{\beta^2+3}{\alpha^2-3} = 2 = \tan\theta\cdot\tan\phi$$

$\therefore\ \beta^2 + 3 = 2(\alpha^2 - 2)$

Or $2\alpha^2 - \beta^2 = 7$

716 **(3)**

θ is the angle between the tangent and the line

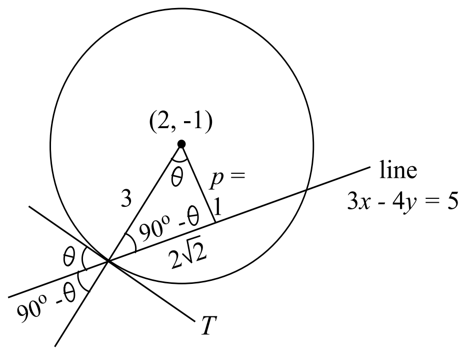
Circle with centre $(2,-1)$ and $r = 3$

perpendicular from centre on $3x - 4y = 5$ is

$$p = \left|\frac{6+4-5}{5}\right| = 1$$

$$\Rightarrow \sin(90-\theta) = \frac{1}{3}$$

$$\Rightarrow \cos\theta = \frac{1}{3}$$



717 **(5)**

Points are $A\ (3,4), B\ (6,8)$ and $O\ (0,0)$.

$OA + OB = 2a$ (where a is semi-major axis)

$2a = 5 + 10 = 15$

$$\therefore\ a = \frac{15}{2}$$

Now $2ae = \sqrt{(6-3)^2(8-4)^2} = 5$

$e = \dfrac{1}{3}$

$\therefore\ b^2 = \dfrac{225}{4}\left(1 - \dfrac{1}{9}\right) = 50$

718 **(5)**

Let the line be $y = mx$

Solving it with

$5y = 2x^2 - 9x + 10$, we get

$5mx = 2x^2 - 9x + 10$

$2x^2 - (9 + 5m)x + 10 = 0$

Sum of the roots $= \frac{9+5m}{2} = 17$

$\Rightarrow 9 + 5m = 34$

$\Rightarrow 5m = 25$

$\Rightarrow m = 5$

719 **(0)**

Since both the circles are symmetric about the x-axis, sum of slopes of tangents is 0

720 **(3)**

Any tangent to parabola $y^2 = 4x, (a = 1)$is

$y = mx + \frac{1}{m}$. It passes through $(-2, -1)$

$\therefore -1 = -2m + \frac{1}{m}$ or $2m^2 - m - 1 = 0$

Or $(2m + 1)(m - 1) = 0$

Or $m = 1/2$ and $m = 1$

Then angle between lines is

$\tan\theta = \left|\dfrac{m_1 + m_2}{1 - m_1 m_2}\right| = 3$

721 **(0)**

Clearly P is the point of intersection of two perpendicular tangents to the parabola

$y^2 = 8x, 4a = 8$ or $a = 2$

Hence, P must lie on the directrix $x + a = 0$ or $x + 2 = 0$

$\therefore x = -2$. hence the point is $(-2,0)$

722 **(3)**

Equation of tangent in terms of slope of parabola $y^2 = 4x$ is

$y = mx + \dfrac{1}{m}$

$\because$Eq.(i) is also tangent of $x^2 = -32y$

Then $x^2 = -32\left(mx + \frac{1}{m}\right)$

$\Rightarrow x^2 + 32mx + \dfrac{32}{m} = 0$

Above equation must have equal roots,

Hence its discriminant must be zero

$\Rightarrow (32m)^2 = 4.1.\dfrac{32}{m}$

$\Rightarrow m^3 = \frac{1}{8}$ or $m = \frac{1}{2}$

Form Eq. (i), $y = \frac{x}{2} + 2$

$\Rightarrow x - 2y + 4 = 0$

723 **(5)**

For maximum number of common chord, circle and parabola must intersect in 4 distinct points

Let us first find the value of r when circle and parabola touch each other

For that solving the given curves we have

$(x-6)^2 + 4x = r^2$ or $x^2 - 8x + 36 - r^2 = 0$

Curves touch if discriminant $D = 64 - 4(36 - r^2) = 0$ or $r^2 = 20$

Hence least integral value of r for which the curves intersect is 5

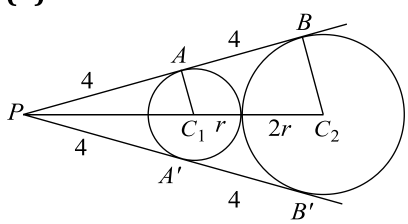
724 **(4)**

Let r be the radius of required circle

Now, if two circles touches each other then distance between their centres $= |r \pm 2| = 5$ (given)

$\therefore r = 3, 7$

725 **(2)**



$\dfrac{AC_1}{PA} = \dfrac{BC_2}{PB} \Rightarrow BC_2 = 2\,AC_1$

$PC_1 = \sqrt{16 + r^2}$

and $PC_2 = \sqrt{64 + 4r^2} = 2\sqrt{16 + r^2}$

$PC_2 - PC_1 = 3r$

$\Rightarrow 2\sqrt{16 + r^2} - \sqrt{16 + r^2} = 3r$

$\Rightarrow \sqrt{16 + r^2} = 3r$

$\Rightarrow 16 + r^2 = 9r^2$

$\Rightarrow r^2 = 2$

726 **(3)**

Since tangent drawn from the point $A(a, 2)$ are perpendicular then A must lie on the director circle $x^2 + y^2 = 7$. Putting $y = 2$ we get the value of $x^2 = a^2 = 3$

727 **(5)**

For given equation of hyperbola foci are $S(3, 2)$ and $S'(-1, -1)$,

Using definition of hyperbola $|SP - S'P| = 2a$,

We have $SS' = 5$ and $2a = 1$

Hence eccentricity is $\frac{SS'}{2a} = 5$

728 **(4)**

Eccentricity of the hyperbola $x^2 - y^2 \sec^2\theta = 5$ is

$$e_1 = \sqrt{\frac{1+\sec^2\theta}{\sec^2\theta}} = \sqrt{1+\cos^2\theta}$$

Eccentricity of the ellipse $x^2 \sec^2\theta + y^2 = 25$ is

$$e_2 = \sqrt{\frac{\sec^2\theta - 1}{\sec^2\theta}} = |\sin\theta|$$

Given $e_1 = \sqrt{3}\, e_2$

$\Rightarrow 1 + \cos^2\theta = 3\sin^2\theta$

$\Rightarrow \cos\theta = \pm\frac{1}{\sqrt{2}}$

$\therefore$ least positive value of θ is $\frac{\pi}{4}$

$\therefore p = 4$

729 **(5)**

$e^2 = \frac{b^2}{a^2} + 1 \;\Rightarrow \frac{b^2}{a^2} = e^2 - 1 = 24$

Now $y = mx + c$ is tangent to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then we must have $a^2m^2 - b^2 \geq 0$

Or $m^2 \geq b^2/a^2$ or $m^2 \geq 24$ then least positive integral value of m is 5

730 **(7)**

Here, $(x-1)^2 + (y-3)^2 = \left\{\frac{5x-12y+17}{\sqrt{5^2+(-12)^2}}\right\}^2$

$\therefore$ the focus $= (1,3)$ and the directrix is $5x - 12y + 17 = 0$

The distance of the focus from the directrix

$= \left|\frac{5\times 1 - 12\times 3 + 17}{\sqrt{5^2+(-12)^2}}\right| = \frac{14}{13}$

$\therefore$ latus rectum $= 2 \times \frac{14}{13} = \frac{28}{13}$

731 **(4)**

Radius of given circle $x^2 + y^2 + 4x - 2\sqrt{2}\,y + c = 0$ is $\sqrt{4+2-c} = \sqrt{6-c} = a$ (let)

Now radius of circle $S_1 = \frac{a}{\sqrt{2}}$,

Radius of circle $S_2 = \frac{a}{2}$ and so on

Now $a + \frac{a}{\sqrt{2}} + \frac{a}{\sqrt{2}} + \ldots \infty = 2$ (given)

$\Rightarrow a\left(\frac{1}{1-\frac{1}{\sqrt{2}}}\right) = 2$

$\Rightarrow \frac{a\sqrt{2}}{\sqrt{2}-1} = 2$

$\Rightarrow a = 2 - \sqrt{2} = \sqrt{6-c}$

$\Rightarrow 4 + 2 - 4\sqrt{2} = 6 - c$

$\Rightarrow c = 4\sqrt{2}$

732 **(8)**

Hyperbola is $x^2 - 9y^2 = 9$ or $\frac{x^2}{9} - \frac{y^2}{1} = 1$

Equation of tangent is $y = mx \pm \sqrt{a^2m^2 - b^2}$ (1)

It passes through (3, 2)

$\Rightarrow 2 = 3m \pm \sqrt{9m^2 - 1}$

Or $4 + 9m^2 - 12m = 9m^2 - 1$

Solving we get values of m as $m_1 = \frac{5}{12}$ and $m_2 = \infty$

Equation of tangent (1) for $m_1 = \frac{5}{12}$

$$y = \frac{5}{12}x \pm \sqrt{9\left(\frac{5}{12}\right)^2 - 1}$$

Or $y = \frac{5}{12}x \pm \frac{3}{4}$

On taking $(-)$ve sign point $P(3,2)$ does not satisfy the equation of tangent therefore rejecting $(-)$ve sign. Hence equation of tangent is $y = \frac{5x}{12} + \frac{3}{4}$ (2)

Now equation of tangent (1) for $m_2 = \infty$ is $x \pm 3 = 0$ rejecting $(+)$ sign (since taking $(+)$ sign point $P(3,2)$ does not satisfy this equation)

Hence, equation of tangent is $x - 3 = 0$ (3)

Now equation of chord of contact w.r.t. point $P(3,2)$ is $T = 0$

Or $3x - 18y = 9$

Or $x - 6y = 3$ (4)

Solving (2) and (4); $x = -5, y = -\frac{4}{3}$

Solving (3) and (4); $x = 3, y = 0$

Now vertices of triangle are $(3,2), (3,0), (-5,-4/3)$

$\therefore$ Area $= \frac{1}{2}\begin{vmatrix} 3 & 2 & 1 \\ 3 & 0 & 1 \\ -5 & -\frac{4}{3} & 1 \end{vmatrix}$

$= \frac{1}{2} \times \left|3\left(\frac{4}{3}\right) - 2(3+5) + 1(-4)\right|$

$= \frac{1}{2}|4 - 16 - 4|$

$= 8$ sq. units

733 **(4)**

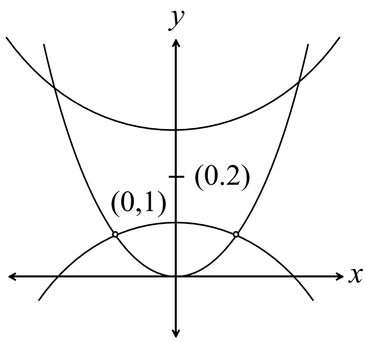
We have $y = Ax^2, y^2 + 3 = x^2 + 4y; A > 0$

Now $y^2 - 4y = x^2 - 3$

$\Rightarrow (y-2)^2 = x^2 + 1$

$\Rightarrow (y-2)^2 - x^2 = 1$

If $x = 0, y - 2 = 1$ or $-1 \Rightarrow y = 3$ or 1



Hence the two graphs of $y = Ax^2$ $(A > 0)$ and the hyperbola $(y-2)^2 - x^2 = 1$ are as shown which intersects in 4 points

734 **(9)**

Equation of normal at $P(\theta)$ is $5\sec\theta x - 4\operatorname{cosec}\theta y = 25 - 16$ and it passes through $P0, \alpha$

$\alpha = \frac{-9}{4\operatorname{cosec}\theta}$ i.e. $\alpha = \frac{-9}{4}\sin\theta \Rightarrow |\alpha| < \frac{9}{4}$

735 **(8)**

Length of focal chord having one extremity $(at^2, 2at)$is $a\left(t + \frac{1}{t}\right)^2$

$\left|r + \frac{1}{t}\right| \geq 2 \Rightarrow a\left(1 + \frac{1}{t}\right)^2 \geq 4a = 8 \Rightarrow$ length of coal chord $\nless 8$

736 **(8)**

The point $P\left(\frac{\pi}{6}\right)$ is $\left(a\sec\frac{\pi}{6}, b\tan\frac{\pi}{6}\right)$ or $P\left(\frac{2a}{\sqrt{3}}, \frac{b}{\sqrt{3}}\right)$

$\therefore$ Equation of tangent at P is $\frac{x}{\frac{\sqrt{3}a}{2}} - \frac{y}{\sqrt{3}b} = 1$

$\therefore$ Area of the triangle $= \frac{1}{2} \times \frac{\sqrt{3}a}{2} \times \sqrt{3}b = 3a^2$

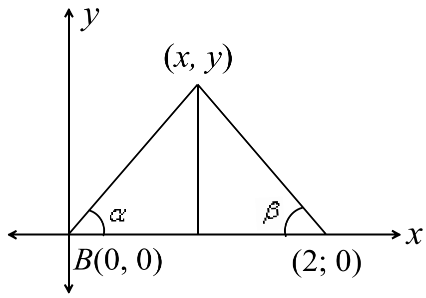
$\therefore \frac{b}{a} = 4$

$\therefore e^2 = 1 + \frac{b^2}{a^2} = 17$

737 **(8)**

$x^2 + 9y^2 - 4x + 6y + 4 = 0$
$\Rightarrow x^2 - 4x + 9y^2 + 6y + 4 = 0$
$\Rightarrow (x-2)^2 + (3y+1)^2 = 1$
$\Rightarrow (x-2)^2 + \frac{\left(y + \frac{1}{3}\right)^1}{\frac{1}{9}} = 1$
Which is an equation of ellipse having centre at $\left(2, -\frac{1}{3}\right)$
General point on ellipse is
$P(x,y) = (2 + a\cos\theta, -1/3 + b\sin\theta)$
$= (2 + \cos\theta, -1/3 + 1/3\sin\theta)$
$x = 2 + \cos\theta$ and $y = -1/3 + 1/3\sin\theta$
$\therefore 4x - 9y = 4(2 + \cos\theta) - 9\left(-\frac{1}{3} + \frac{1}{3}\sin\theta\right)$
$\Rightarrow f(\theta) = 8 + 4\cos\theta + 3 - 3\sin\theta$
$= 11 + 4\cos\theta - 3\sin\theta$
$\therefore f(\theta)_{\max} = 11 + 5 = 16$

738 **(8)**



Given $\tan\alpha \cdot \tan\beta = 4$
$\Rightarrow y \cdot \frac{y}{x} + \frac{y}{2-x} = 4 \Rightarrow y = 2x(2-x)$
$\Rightarrow -\frac{y}{2} = x^2 - 2x = (x-1)^2 - 1$
$\Rightarrow (x-1)^2 = -\frac{1}{2}(y-2)$
$\Rightarrow$ Directrix $y - 2 = \frac{1}{8} \Rightarrow y = \frac{17}{8}$

739 **(1)**

Focus of $y^2 = 16x$ is $(4,0)$
Any focal chord is $y - 0 = m(x-4)$
Or $mx - y - 4m = 0$
This focal chord touches the circle $(x-6)^2 + y^2 = 2$
Then distance from the center of circle to this chord is equal to radius of the circle

Or $\frac{|6m-4m|}{\sqrt{m^2+1}} = \sqrt{2}$ or $2m = \sqrt{2}.\sqrt{m^2+1}$

Or $2m^2 = m^2 + 1 \Rightarrow m^2 = 1$

$\therefore\ m = \pm 1$

740 **(9)**

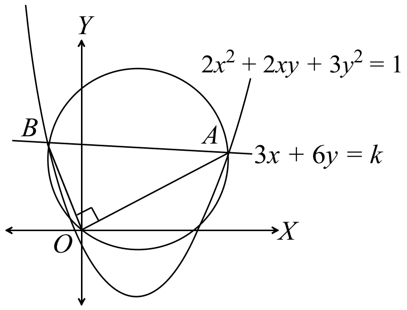
$3x + 6y = k$

$\Rightarrow \frac{3x+6y}{k} = 1 \quad (1)$

Also $2x^2 + 2xy + 3y^2 - 1 = 0 \quad (2)$

Now homogenising (2) with the help of (1), we gets

$\Rightarrow 2x^2 + 2xy + 3y^2 - \left(\frac{3x + 6y}{k}\right)^2 = 0$



$\Rightarrow k^2(2x^2 + 2xy + 3y^2) - (3x + 6y)^2 = 0 \quad (3)$

This is the equation of pair of straight lines OA and OB

Now AB is diameter, then OA and OB are perpendicular

$\Rightarrow$ coefficient of x^2 + coeffiecient of $y^2 = 0$

$\Rightarrow (2k^2 - 9) + (3k^2 - 36) = 0$

$\Rightarrow 5k^2 = 45$

$\Rightarrow k^2 = 9$

741 **(6)**

Equation of hyperbola is $\frac{x^2}{16} - \frac{y^2}{18} = 1$

Or $9x^2 - 8y^2 - 144 = 0$

Homogenization of this equation using

$\frac{x\cos\alpha + y\sin\alpha}{p} = 1$

We have $9x^2 - 8y^2 - 144\ \left(\frac{x\cos\alpha + y\sin\alpha}{p}\right)^2 = 0$

Since these lines are perpendicular to each other

$\therefore 9p^2 - 8p^2 - 144\ (\cos^2\alpha + \sin^2\alpha) = 0$

$p^2 = 144$ or $p = \pm 12$

$\therefore$ radius of the circle $= 12$

$\therefore$ diameter of the circle $= 24$

742 **(8)**

We have $\tan^4 x + \cot^4 x + 2 = 4\sin^2 y$

$\Rightarrow (\tan^2 x - \cot^4 x)^2 + 4 = 4\sin^2 y$

Now L.H.S ≥ 4 and R.H.S ≤ 4

$\therefore \tan^2 x = 1$ and $\sin^2 y = 1\ \Rightarrow \tan x = \pm 1$ and $\sin y = \pm 1$

But $-3 \leq x \leq 3$ and $-3 \leq y \leq 3$

$\therefore$ Acceptable values of x are $\pm\frac{\pi}{4}$ and $\pm\frac{3\pi}{4}$ and acceptable values of y are $\pm\frac{\pi}{2}$

Hence the number of points $P(x, y)$ are 8

743 **(6)**

$x^2 + y^2 - 2x - 2\lambda y - 8 = 0$

$\Rightarrow (x^2 + y^2 - 2x - 8) - 2\lambda y = 0$, which is of the form of $S + \lambda L = 0$

All the circles passing through the point of intersection of circle $x^2 + y^2 - 2x - 8 = 0$ and $y = 0$

Solving we get $x^2 - 2x + 8 = 0$ or $(x - 4)(x + 2=0$

$\Rightarrow A \equiv (4, 0)$ and $B \equiv (-2, 0)$

$\Rightarrow$ Distance between A and B is 6

744 **(8)**

Chord of the contact w.r.t. point $O(-1, 2)$ is

$y = (x - 1)$ (using $yy_1 = 2a(x + x_1)$)

Solving $y = x + 1$, with parabola, we get points of intersection as

$P(3 + 2\sqrt{2}, 2 + 2\sqrt{2})$ and $Q(3 - 2\sqrt{2}, 2 - 2\sqrt{2})$

$\therefore PQ^2 = 32 + 32 = 64$

$\therefore PQ = 8$

Also length of perpendicular from $O(-1,2)$ on $PQ = \frac{4}{\sqrt{2}}$

Then required area of triangle is

$A = \frac{1}{2}.8.\left(\frac{4}{\sqrt{2}}\right) = 8\sqrt{2}$ sq. units