Single Correct Answer Type

1. The value of the expression $x^4 - 8x^3 + 18x^2 - 8x + 2$ when $x = 2 + \sqrt{3}$
   a) 2 b) 1 c) 0 d) 3
2. If $z = x + iy(x, y \in R, x \neq -1/2)$, the number of value of z safisfying $|z|^n = z^2|z|^{n-2} + z|z|^{n-2} + 1 \cdot (n \in N,\ n > 1)$ is
   a) 0 b) 1 c) 2 d) 3
3. If α, β, γ are the roots of $x^3 - x^2 - 1 = 0$ then the value of $(1+\alpha)/(1-\alpha) + (1+\beta)/(1-\beta) + (1+\gamma)/(1-\gamma)$ is equal to
   a) -5 b) -6 c) -7 d) -2
4. If the equation $|x^2 + bx + c| = k$ has four roots, then
   a) $b^2 - 4c > 0$ and $0 < k < \frac{4c-b^2}{4}$
   b) $b^2 - 4c < 0$ and $0 < k < \frac{4c-b^2}{4}$
   c) $b^2 - 4c > 0$ and $k > \frac{4c-b^2}{4}$
   d) None of these
5. The value of z satisfying the equation $\log z + \log z^2 + \cdots + \log z^n = 0$ is
   a) $\cos\frac{4m\pi}{n(n+1)} + i\sin\frac{4m\pi}{n(n+1)}, m = 1, 2, \ldots$
   b) $\cos\frac{4m\pi}{n(n+1)} - i\sin\frac{4m\pi}{n(n+1)}, m = 1, 2, \ldots$
   c) $\sin\frac{4m\pi}{n(n+1)} + i\cos\frac{4m\pi}{n(n+1)}, m = 1, 2, \ldots$
   d) 0
6. If $a(p+q)^2 + 2bpq + c = 0$ and $a(p+r)^2 + 2bpr + c = 0$ $(a \neq 0)$, then
   a) $qr = p^2$ b) $qr = p^2 + \frac{c}{a}$ c) $qr = -p^2$ d) None of these
7. The value of m for which one of the roots of $x^2 - 3x + 2m = 0$ is double of one of the roots of $x^2 - x + m = 0$ is
   a) -2 b) 1 c) 2 d) None of these
8. Roots of the equations are $(z+1)^5 = (z-1)^5$ are
   a) $\pm i\tan\left(\frac{\pi}{5}\right), \pm i\tan\left(\frac{2\pi}{5}\right)$
   b) $\pm i\cot\left(\frac{\pi}{5}\right), \pm i\cot\left(\frac{2\pi}{5}\right)$
   c) $\pm i\cot\left(\frac{\pi}{5}\right), \pm i\tan\left(\frac{2\pi}{5}\right)$
   d) None of these
9. Total number of integral values of 'a' so that $x^2 - (a+1)x + a - 1 = 0$ has integral roots is equal to
   a) 1 b) 2 c) 4 d) None of these
10. z_1, z_2, z_3, z_4 are distinct complex numbers representing the vertices of a quadrilateral $ABCD$ taken in order. If $z_1 - z_4 = z_2 - z_3$ and $\arg[(z_4 - z_1)/(z_2 - z_1)] = \pi/2$, then the quadrilateral is
    a) Rectangle b) Rhombus c) Square d) Trapezium
11. If the roots of the equation $ax^2 + bx + c = 0$ are of the form $(k+1)/k$ and $(k+2)/(k+1)$, then $(a+b+c)^2$ is equal to
    a) $2b^2 - ac$ b) $\sum a^2$ c) $b^2 - 4ac$ d) $b^2 - 2ac$
12. Let r, s and t be the roots of the equation, $8x^3 + 1001x + 2008 = 0$. The value of $(r+s)^3 + (s+t)^3 + (t+r)^3$ is
    a) 251 b) 751 c) 735 d) 753
13. If $b > a$, then the equation $(x-a)(x-b) - 1 = 0$ has
    a) Both roots in (a, b)
    b) Both roots in $(-\infty, a)$
    c) Both roots in $(b, +\infty)$
    d) One root in $(-\infty, a)$ and the other in $(b, +\infty)$
14. If l, m, n are real $l \neq m$, then the roots of the equation $(l-m)x^2 - 5(l+m)x - 2(l-,m) = 0$ are

a) Real and equal b) Complex c) Real and unequal d) None of these

15. If the expression $x^2 + 2(a+b+c)x + 3(bc+ca+ab)$ is a perfect square, then
a) $a = b = c$ b) $a = \pm b = \pm c$ c) $a = b \neq c$ d) None of these

16. If $|z| < \sqrt{2} - 1$, then $|z^2 + 2z\cos\alpha|$ is
a) Less than 1 b) $\sqrt{2} + 1$ c) $\sqrt{2} - 1$ d) None of these

17. If ω be a complex n^{th} root of unity, then $\sum_{i=1}^{n}(ar+b)\,\omega^{r-1}$ is equal to
a) $\frac{n(n+1)a}{2}$ b) $\frac{nb}{1-n}$ c) $\frac{na}{\omega-1}$ d) None of these

18. If $a, b \in R, a \neq 0$ and the quadratic equation $ax^2 - bx + 1 = 0$ has imaginary roots then $(a+b+1)$ is
a) Positive b) Negative
c) Zero d) Dependent on the sign of b

19. Sum of the non-real roots of $(x^2 + x - 2)(x^2 + x - 3) = 12$ is
a) -1 b) 1 c) -6 d) 6

20. Let $z = \cos\theta + i\sin\theta$. Then, the value of $\sum_{m=1}^{15}\mathrm{Im}(z^{2m-1})$ at $\theta = 2°$ is
a) $\frac{1}{\sin 2°}$ b) $\frac{1}{3\sin 2°}$ c) $\frac{1}{2\sin 2°}$ d) $\frac{1}{4\sin 2°}$

21. If α, β be the roots of the equation $u^2 - 2u + 2 = 0$ and if $\cot\theta = x + 1$, then $[(x+\alpha)^n - (x+\beta)^n]/[\alpha - \beta]$ is equal to
a) $\frac{\sin n\theta}{\sin^n\theta}$ b) $\frac{\cos n\theta}{\cos^n\theta}$ c) $\frac{\sin n\theta}{\cos^n\theta}$ d) $\frac{\cos n\theta}{\sin^n\theta}$

22. If the cube roots of unity are $1, \omega, \omega^2$, then the roots of the equation $(x-1)^3 + 8 = 0$ are
a) $-1, 1+2\omega, 1+2\omega^2$ b) $-1, 1-2\omega, 1-2\omega^2$ c) $-1, -1, -1$ d) None of these

23. Suppose A is a complex number and $n \in N$, such that $A^n = (A+1)^n = 1$, then the least value of n is
a) 3 b) 6 c) 9 d) 12

24. If a, b, c, d are four consecutive terms of an increasing A.P. then the roots of the equation $(x-a)(x-c) + 2(x-b)(x-d) = 0$ are
a) Non-real complex b) Real and equal c) Integers d) Real and distinct

25. If the equations $ax^2 + bx + c = 0$ and $x^3 + 3x^2 + 3x + 2 = 0$ have two common roots, then
a) $a = b = c$ b) $a = b \neq c$ c) $a = -b = c$ d) None of these

26. If α and β, α and γ, α and δ are the roots of the equations $ax^2 + 2bx + c = 0$, $2bx^2 + cx + a = 0$ and $cx^2 + ax + 2b = 0$, respectively, where a, b and c are positive real numbers, then $\alpha + \alpha^2 =$
a) abc b) $a + 2b + c$ c) -1 d) 0

27. If α, β are the roots of $x^2 + px + q = 0$ and $x^{2n} + p^n x^n + q^n = 0$ and if $(\alpha/\beta), (\beta/\alpha)$ are the roots of $x^n + 1 + (x+1)^n = 0$, then $n (\in N)$
a) Must be an odd integer b) May be any integer
c) Must be an even integer d) Cannot say anything

28. If $z^2 + z|z| + |z^2| = 0$, then the locus of z is
a) A circle b) A straight line
c) A pair of straight lines d) None of these

29. If $|z| = 1$ and $w = \frac{z-1}{z+1}$ (where $z \neq -1$), then $\mathrm{Re}(w)$ is
a) 0 b) $\frac{1}{|z+1|^2}$ c) $\left|\frac{1}{z+1}\right| \cdot \frac{1}{|z+1|^2}$ d) $\frac{\sqrt{2}}{|z+1|^2}$

30. If the equation $x^2 + ax + b = 0$ has distinct real roots and $x^2 + a|x| + b = 0$ has only one real root, then which of the following is true
a) $b = 0, a > 0$ b) $b = 0, a < 0$ c) $b > 0, a < 0$ d) $b < 0, a > 0$

31. Let $f(x) = ax^2 - bx + c^2, b \neq 0$ and $f(x) \neq 0$ for all $x \in R$. Then
a) $a + c^2 < b$ b) $4a + c^2 > 2b$ c) $9a - 3b + c^2 < 0$ d) None of these

32. The number of real roots of the equation $x^2 - 3|x| + 2 = 0$ is
a) 2 b) 1 c) 4 d) 3

33. If z_1 and z_2 are the complex roots of the equation $(x-3)^3 + 1 = 0$, then $z_1 + z_2$ equals to

a) 1 b) 3 c) 5 d) 7

34. If x^2+px+1 is factor of the expression ax^3+bx+c, then
a) $a^2-c^2=ab$ b) $a^2+c^2=-ab$ c) $a^2-c^2=-ab$ d) None of these

35. If $z=(i)^{(i)^{(i)}}$ where $i=\sqrt{-1}$, then $|z|$ is equal to
a) 1 b) $e^{-\pi/2}$ c) $e^{-\pi}$ d) None of these

36. The number of roots of the equation $\sqrt{x-2}(x^2-4x+3)=0$ is
a) Three b) Four c) One d) Two

37. Total number of values of a so that $x^2-x-a=0$ has integral roots, where $a \in N$ and $6 \le a \le 100$, is equal to
a) 2 b) 4 c) 6 d) 8

38. If $a, b, c$are the sides of the triangle ABC such that $a \neq b \neq c$ and $x^2-2(a+b+c)x+3\lambda(ab+bc+ca=0$ has real roots, then
a) $\lambda<\frac{4}{3}$ b) $\lambda>\frac{5}{3}$ c) $\lambda \in \left(\frac{4}{3}, \frac{5}{3}\right)$ d) $\lambda \in \left(\frac{1}{3}, \frac{5}{3}\right)$

39. Suppose A, B, C are defined as $A=a^2b+ab^2-a^2c-ac^2, B=b^2c+bc^2-a^2b-ab^2$ and $C=a^2c+ac^2-b^2c-bc^2$, where $a>b>c>0$ and the equation $Ax^2+Bx+C=0$ has equal roots, then a, b, c are in
a) A.P. b) G.P. c) H.P. d) A.G.P.

40. Consider the equation $x^2+2x-n=0$, where $n \in N$ and $n \in [15, 100]$. Total number of different values of 'n' so that the given equation has integral roots is
a) 8 b) 3 c) 6 d) 4

41. If $x^2+x+1=0$, then the value of $(x+1/x)^2+(x^2+1/x^2)^2+\cdots+(x^{27}+1/x^{27})^2$ is
a) 27 b) 72 c) 45 d) 54

42. If (x^2+px+1) is a factor of (ax^3+bx+c), then
a) $a^2+c^2=-ab$ b) $a^2-c^2=-ab$ c) $a^2-c^2=ab$ d) None of these

43. Let z, w be complex numbers such that $\bar{z}+i\bar{w}=0$ and $\arg zw=\pi$. Then $\arg z$ equals
a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\frac{3\pi}{4}$ d) $\frac{5\pi}{4}$

44. If $a>0, b>0$ and $c>0$ then the roots of the equation $ax^2+bx+c=0$
a) Are real and negative b) Have positive real parts
c) Have negative real parts d) None of these

45. Which of the following is equal to $\sqrt[3]{-1}$?
a) $\frac{\sqrt{3}+\sqrt{-1}}{2}$ b) $\frac{-\sqrt{3}+\sqrt{-1}}{\sqrt{-4}}$ c) $\frac{\sqrt{3}-\sqrt{-1}}{\sqrt{-4}}$ d) $-\sqrt{-1}$

46. The interval of a for which the equation $\tan^2 x-(a-4)\tan x+4-2a=0$ has at least one solution $\forall\, x \in [0, \pi/4]$
a) $a \in (2, 3)$ b) $a \in [2, 3]$ c) $a \in (1, 4)$ d) $a \in [1, 4]$

47. Which of the following represents a point in an Argand plane, equidistant from the roots of the equation $(z+1)^4=16z^4$?
a) $(0, 0)$ b) $\left(-\frac{1}{3}, 0\right)$ c) $\left(\frac{1}{3}, 0\right)$ d) $\left(0, \frac{2}{\sqrt{5}}\right)$

48. If α, β, γ are such that $\alpha+\beta+\gamma=2, \alpha^2+\beta^2+\gamma^2=6, \alpha^3+\beta^3+\gamma^3=8$, then $\alpha^4+\beta^4+\gamma^4$ is
a) 18 b) 10 c) 15 d) 36

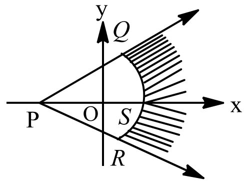
49. The minimum value of $|a+b\omega+c\omega^2|$, where a, b and c are all not equal integers and $\omega(\neq 1)$ is a cube root of unity, is
a) $\sqrt{3}$ b) 1/2 c) 1 d) 0

50. If $|z_1|=|z_2|=|z_3|=1$ and $z_1+z_2+z_3=0$, then area of the triangle whose vertices are z_1, z_2, z_3 is
a) $3\sqrt{3}/4$ b) $\sqrt{3}/4$ c) 1 d) 2

51. Number of positive integers n for which n^2+96 is a perfect square is
a) 8 b) 12 c) 4 d) Infinite

52. The greatest positive argument of complex number satisfying $|z-4|=\text{Re}(z)$ is
a) $\frac{\pi}{3}$ b) $\frac{2\pi}{3}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$
53. If x and y are complex numbers, then the system of equations $(1+i)x+(1-i)y=1, 2ix+2y=1+i$ has
a) Unique solution b) No solution
c) Infinite number of solutions d) None of these
54. For the equation $3x^2+px+3=0, p>0$, if one of the root is square of the other, then p is equal to
a) 1/3 b) 1 c) 3 d) 2/3
55. If α, β are the roots of the equation $x^2-2x+3=0$. Then the equation whose roots are $P=\alpha^3-3\alpha^2+5\alpha-2$ and $Q=\beta^3-\beta^2+\beta+5$ is
a) $x^2+3x+2=0$ b) $x^2-3x-2=0$ c) $x^2-3x+2=0$ d) None of these
56. If centre of a regular hexagon is at origin and one of the vertices on Argand diagram is $1+2i$, then its perimeter is
a) $2\sqrt{5}$ b) $6\sqrt{2}$ c) $4\sqrt{5}$ d) $6\sqrt{5}$
57. If $z_1 z_2 \in C, z_1^2+z_2^2 \in R, z_1(z_1^2-3z_2^2)=2$ and $z_2(3z_1^2-z_2^2)=11$, then the value of $z_1^2+z_2^2$ is
a) 10 b) 12 c) 5 d) 8
58. $P(x)$ is a polynomial with integral coefficients such that for four distinct integers $a, b, c, d; P(a)=P(b)=P(c)=P(d)=3$. If $P(e)=5$ (e is an integer), then
a) $e=1$ b) $e=3$ c) $e=4$ d) No real value of e
59. If α, β are the roots of $ax^2+bx+c=0$ and $a+b, \beta+h$ are the roots of $px^2+qx+r=0$, then $h=$
a) $-\frac{1}{2}\left(\frac{a}{b}-\frac{p}{q}\right)$ b) $\left(\frac{b}{a}-\frac{q}{p}\right)$ c) $\frac{1}{2}\left(\frac{b}{a}-\frac{q}{p}\right)$ d) None of these
60. If t and c are two complex numbers such that $|t| \neq |c|, |t|=1$ and $z=(at+b)/(t-c), z=x+iy$. Locus of z is (where a, b are complex numbers)
a) Line segment b) Straight line c) Circle d) None of these
61. The complex numbers $z=x+iy$ which satisfy the equation $|(z-5i)/(z+5i)|=1$ lie on
a) The x-axis b) The straight line $y=5$
c) A circle passing through the origin d) None of these
62. If α and β $(\alpha<\beta)$ are the roots of the equation $x^2+bx+c=0$, where $c<0<b$, then
a) $0<\alpha<\beta$ b) $\alpha<0<\beta<|\alpha|$ c) $\alpha<\beta<0$ d) $\alpha<0<|\alpha|<\beta$
63. All the values of m for which both the roots of the equation $x^2-2mx+m^2-1=0$ are greater than -2 but less than 4, lie in the interval
a) $-2<m<0$ b) $m>3$ c) $-1<m<3$ d) $1<m<4$
64. Two towns A and B are 60 km apart. A school is to be built to serve 150 students in town A and 50 students in town B. If the total distance to be travelled by all 200 students is to be as small as possible, then the school be built at
a) Town B b) 45 km from town A c) Town A d) 45 km from town B
65. If $z=\left[(\sqrt{3}/2)+i/2\right]^5+\left[(\sqrt{3}/2)-i/2\right]^5$, then
a) $\text{Re}(z)=0$ b) $\text{Im}(z)=0$ c) $\text{Re}(z)>0, \text{Im}(z)>0$ d) $\text{Re}(z)>0, \text{Im}(z)<0$
66. Let p and q be real numbers such that $p\neq 0, p^3\neq q$ and $p^3\neq -q$. If α and β are non-zero complex numbers satisfying $\alpha+\beta=-p$ and $\alpha^3+\beta^3=q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is
a) $(p^3+q)x^2-(p^3+2q)x+(p^3+q)=0$ b) $(p^3+q)x^2-(p^3-2q)x+(p^3+q)=0$
c) $(p^3-q)x^2-(5p^3-2q)x+(p^3-q)=0$ d) $(p^3-q)x^2-(5p^3+2q)x+(p^3-q)=0$
67. Let p and q be roots of the equation $x^2-2x+A=0$ and let r and s be the roots of the equation $x^2-18x+B=0$. If $p<q<r<s$ are in arithmetic progression, then the values of A and B are
a) $3, -77$ b) $3, 77$ c) $-3, -77$ d) $-3, 77$
68. If α and β are the roots of the equation $x^2+px+q=0$, and α^4 and β^4 are the roots of $x^2-rx+q=0$, then the roots of $x^2-4qx+2q^2-r=0$ are always
a) Both non-real b) Both positive c) Both negative d) Opposite in sign

69. The shaded region, where
$P \equiv (-1,0), Q \equiv (-1+\sqrt{2}, \sqrt{2})$
$R \equiv \left(-1+\sqrt{2}, -\sqrt{2}\right), S \equiv (1,0)$ is represented by



a) $|z+1| > 2, |\arg(z+1) < \frac{\pi}{4}|$
b) $|z+1| < 2, \arg(z+1) < \frac{\pi}{2}$
c) $|z-1| > 2, \arg(z+1) > \frac{\pi}{4}$
d) $|z-1| < 2, |\arg(z+1) > \frac{\pi}{4}$

70. Number of values of a for which equations $x^3 + ax + 1 = 0$ and $x^4 + ax^2 + 1 = 0$ have a common root
a) 0 b) 1 c) 2 d) Infinite

71. If $|z - 2 - i| = |z| \left|\sin\left(\frac{\pi}{4} - \arg z\right)\right|$, then locus of z is
a) A pair of straight lines
b) Circle
c) Parabola
d) Ellipse

72. If $z = i \log\left(2 - \sqrt{-3}\right)$, then $\cos z =$
a) -1 b) $-1/2$ c) 1 d) $1/2$

73. If $x, y \in R$ satisfy theEquation $x^2 + y^2 - 4x - 2y + 5 = 0$, then the value of the expression $\left[\left(\sqrt{x} - \sqrt{y}\right)^2 + 4xy)/(x+xy)\right.$ is
a) $\sqrt{2} + 1$ b) $\frac{\sqrt{2}+1}{2}$ c) $\frac{\sqrt{2}-1}{2}$ d) $\frac{\sqrt{2}+1}{\sqrt{2}}$

74. The least value of the expression $x^2 + 4y^2 + 3z^2 - 2x - 12y - 6z + 14$ is
a) 1 b) No least value c) 0 d) None of these

75. If $A(z_1), B(z_2), C(z_3)$ are the vertices of the triangle ABC such that $(z_1 - z_2)/(z_3 - z_2) = (1/\sqrt{2}) + (i/\sqrt{2})$, the triangle ABC is
a) Equilateral b) Right angled c) Isosceles d) Obtuse angled

76. If the roots of the equation, $x^2 + 2ax + b = 0$, are real and distinct and they differ by at most $2m$, then b lies in the interval
a) $(a^2, a^2 + m^2)$ b) $(a^2 - m^2, a^2)$ c) $[a^2 - m^2, a^2)$ d) None of these

77. If x is real, then the maximum value of $(3x^2 + 9x + 17)/(3x^2 + 9x + 7)$ is
a) ¼ b) 41 c) 1 d) 17/7

78. If $a < 0, b > 0$ then $\sqrt{a}\sqrt{b}$ is equal to
a) $-\sqrt{|a|\,b}$ b) $\sqrt{|a|\,b}\,i$ c) $\sqrt{|a|\,b}$ d) None of these

79. The inequality $|z - 4| < |z - 2|$ represents the region given by
a) $\mathrm{Re}(z) \geq 0$ b) $\mathrm{Re}(z) < 0$ c) $\mathrm{Re}(z) > 0$ d) None of these

80. If $x = 9^{1/3}9^{1/9}9^{1/27} \cdots \infty, y = 4^{1/3}4^{-1/9}4^{1/27} \cdots \infty$, and $z = \sum_{r=1}^{\infty}(1+i)^{-r}$, then $\arg(x + yz)$ is equal to
a) 0 b) $\pi - \tan^{-1}\left(\frac{\sqrt{2}}{3}\right)$ c) $-\tan^{-1}\left(\frac{\sqrt{2}}{3}\right)$ d) $-\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

81. The set of values of a for which $(a-1)x^2 - (a+1)x + a - 1 \geq 0$ is true for all $x \geq 2$
a) $(-\infty, 1)$ b) $\left(1, \frac{7}{3}\right)$ c) $\left(\frac{7}{3}, \infty\right)$ d) None of these

82. If $w = \alpha + i\beta$, where $\beta \neq 0$ and $z \neq 1$, satisfies the condition that $\left(\frac{w - \bar{w}z}{1-z}\right)$ is purely real, then the set of values of z is
a) $|z| = 1, z \neq 2$ b) $|z| = 1$ and $z \neq 1$ c) $z = \bar{z}$ d) None of these

83. The number of points of intersection of two curves $y = 2\sin x$ and $y = 5x^2 + 2x + 3$ is
a) 0 b) 1 c) 2 d) ∞

84. If roots of an equation $x^n - 1 = 0$ are $1, a_1, a_2, \dots a_{n-1}$, then the value of $(1 - a_1)(1 - a_2)(1 - a_3) \dots (1 - a_{n-1})$ will be

a) n b) n^2 c) n^n d) 0

85. If one root of the equation $ax^2 + bx + c = 0$ is square of the other, then $a(c - b)^3 = cX$, where X is

a) $a^3 - b^3$ b) $a^3 + b^3$ c) $(a - b)^3$ d) None of these

86. Let x, y, z, t be real numbers $x^2 + y^2 = 9, z^2 + t^2 = 4$ and $xt - yz = 6$. Then the greatest value of $P = xz$ is

a) 2 b) 3 c) 4 d) 6

87. Let $\lambda \in R$, the origin and the non-real roots of $2z^2 + 2z + \lambda = 0$ form the three vertices of an equilateral triangle in the Argand plane then λ is

a) 1 b) $\frac{2}{3}$ c) 2 d) -1

88. The number of values of k for which $[x^2 - (k - 2)x + k^2] \times [x^2 + kx + (2k - 1)]$ is a perfect square is

a) 2 b) 1 c) 0 d) None of these

89. Let $p(x) = 0$ be a polynomial equation of the least possible degree, with rational coefficients, having $\sqrt[3]{7} + \sqrt[3]{49}$ as one of its roots. Then the product of all the roots of $p(x) = 0$ is

a) 56 b) 63 c) 7 d) 49

90. The number of real solutions of the equation $|x|^2 - 3|x| + 2 = 0$ is

a) 4 b) 1 c) 2 d) 0

91. The number of integral values of a for which the quadratic equation $(x + a)(x + 1991) + 1 = 0$ has integral roots are

a) 3 b) 0 c) 1 d) 2

92. Let z and ω be two complex numbers such that $|z| \le 1, |\omega| \le 1$ and $|z - i\omega| = |z - i\overline{\omega}| = 2$ then z equals

a) 1 or i b) i or $-i$ c) 1 or -1 d) i or -1

93. z_1 and z_2 lie on a circle with centre at the origin. The point of intersection z_3 of the tangents at z_1 and z_2 is given by

a) $\frac{1}{2}(\overline{z}_1 + \overline{z}_2)$ b) $\frac{2z_1z_2}{z_1 + z_2}$ c) $\frac{1}{2}\left(\frac{1}{z_1} + \frac{1}{z_2}\right)$ d) $\frac{z_1 + z_2}{\overline{z}_1\overline{z}_2}$

94. If α and β be the roots of the equation $x^2 + px - 1/(2p^2) = 0$ where $p \in R$. Then the minimum value of $\alpha^4 + \beta^4$ is

a) $2\sqrt{2}$ b) $2 - \sqrt{2}$ c) 2 d) $2 + \sqrt{2}$

95. If α, β are the roots of $ax^2 + c = bx$, then the equation $(a + cy)^2 = b^2y$ in y has the roots

a) $\alpha\beta^{-1}, \alpha^{-1}\beta$ b) α^{-2}, β^{-2} c) α^{-1}, β^{-1} d) α^2, β^2

96. If $(\cos\theta + i\sin\theta)(\cos 2\theta + i\sin 2\theta) \cdots (\cos n\theta + i\sin n\theta) = 1$, then the value of θ is, $m \in N$

a) $4m\pi$ b) $\frac{2m\pi}{n(n+1)}$ c) $\frac{4m\pi}{n(n+1)}$ d) $\frac{m\pi}{n(n+1)}$

97. The roots of the cubic equation $(z + ab)^3 = a^3$, such that $a \ne 0$, represent the vertices of a triangle of sides of length

a) $\frac{1}{\sqrt{3}}|ab|$ b) $\sqrt{3}\,|a|$ c) $\sqrt{3}\,|b|$ d) $|a|$

98. A quadratic equation whose product of roots x_1 and x_2 is equal to 4 and satisfying the relation $x_1/(x_1 - 1) + x_2/(x_2 - 1) = 2$ is

a) $x^2 - 2x + 4 = 0$ b) $x^2 + 2x + 4 = 0$ c) $x^2 + 4x + 4 = 0$ d) $x^2 - 4x + 4 = 0$

99. If the equation $\cot^4 x - 2\ \mathrm{cosec}^2 x + a^2 = 0$ has at least one solution then, sum of all possible integral values of a is equal to

a) 4 b) 3 c) 2 d) 0

100. The number of irrational roots of the equation $4x/(x^2 + x + 3) + 5x/(x^2 - 5x + 3) = -3/2$ is

a) 4 b) 0 c) 1 d) 2

101. If z_1, z_2 and z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = |(1/z_1) + (1/z_2) + (1/z_2)| = 1$, then $|z_1 + z_2 + z_3|$ is

a) Equal to 1 b) Less than 1 c) Greater than 3 d) Equal to 3

102. If $z_1$and z_2 be complex numbers such that $z_1 \ne z_2$ and $|z_1|$= $|z_2|$. If $z_1$has positive real part and z_2 has

negative imaginary part, then$[(z_1+z_2)/(z_1-z_2)]$ may be
a) Purely imaginary b) Real and positive c) Real and negative d) None of these

103. If the expression $[mx-1+(1/x)]$ is non-negative for all positive real x, then the minimum value of m must be
a) $-1/2$ b) 0 c) ¼ d) ½

104. If for complex numbers $z_1$and z_2, $\arg(z_1)-\arg(z_2)=0$, then $|z_1-z_2|$ is equal to
a) $|z_1|+|z_2|$ b) $|z_1|-|z_2|$ c) $[|z_1|-|z_2|]$ d) 0

105. Locus of z if $\arg[z-(1+i)]=\begin{cases}\frac{3\pi}{4} & \text{when } |z|\le|z-2| \\ \frac{-\pi}{4} & \text{when } |z|>|z-4|\end{cases}$ is
a) Straight lines passing through (2, 0) b) Straight lines passing through (2, 0), (1, 1)
c) A line segment d) A set of two rays

106. For positive integers n_1, n_2 the value of the expression $(1+i)^{n_1}+(1+i^3)^{n_1}+(1+i^5)^{n_2}+(1+i^7)^{n_2}$, where $i=\sqrt{-1}$ is a real number if and only if
a) $n_1=n_2+1$ b) $n_1=n_2-1$ c) $n_1=n_2$ d) $n_1>0, n_2>0$

107. If $|z_2+iz_1|=|z_1|+|z_2|$ and $|z_1|=3$ and $|z_2|=4$, then area of ΔABC, if affixes of A, B and C are z_1, z_2 and $[(z_2-iz_1)/(1-i)]$ respectively, is
a) $\frac{5}{2}$ b) 0 c) $\frac{25}{2}$ d) $\frac{25}{4}$

108. $x^2-xy+y^2-4x-4y+16=0$ represents
a) A point b) A circle
c) A pair of straight lines d) None of these

109. Sum of common roots of the equations $z^3+2z^2+2z+1=0$ and $z^{1985}+z^{100}+1=0$ is
a) -1 b) 1 c) 0 d) 1

110. If the roots of the equation $ax^2-bx+c=0$ are α, β then the roots of the equation $b^2cx^2-ab^2x+a^3=0$ are
a) $\frac{1}{\alpha^3+\alpha\beta}, \frac{1}{\beta^3+\alpha\beta}$ b) $\frac{1}{\alpha^2+\alpha\beta}, \frac{1}{\beta^2+\alpha\beta}$ c) $\frac{1}{\alpha^4+\alpha\beta}, \frac{1}{\beta^4+\alpha\beta}$ d) None of these

111. If $|z^2-1|=|z|^2+1$, then z lies on
a) A circle b) A parabola c) An ellipse d) None of these

112. The locus of point z satisfying $\text{Re}\left(\frac{1}{z}\right)=k$, where k is a non-zero real number, is
a) A straight line b) A circle c) An ellipse d) A hyperbola

113. The number of integral values of x satifying $\sqrt{-x^2+10x-16}<x-2$ is
a) 0 b) 1 c) 2 d) 3

114. Let α, β be the roots of the equation $x^2-px+r=0$ and $\frac{\alpha}{2}, 2\beta$ be the roots of the equation $x^2-qx+r=0$. Then the value of r is
a) $\frac{2}{9}(p-q)(2q-p)$ b) $\frac{2}{9}(q-p)(2p-q)$ c) $\frac{2}{9}(q-2p)(2q-p)$ d) $\frac{2}{9}(2p-q)(2q-p)$

115. For $x^2-(a+3)\,|x|+4=0$ to have real solutions, the range of a is
a) $(-\infty,-7]\cup[1,\infty)$ b) $(-3,\infty)$ c) $(-\infty,-7]$ d) $[1,\infty)$

116. If $z=3/(2+\cos\theta+i\sin\theta)$, then locus of z is
a) A straight line b) A circle having centre on y-axis
c) A parabola d) A circle having centre on x-axis

117. The number of real solutions of the equation $(9/10)^x=-3+x-x^2$ is
a) 2 b) 0 c) 1 d) None of these

118. If p, q, r are +ve and are in A.P., in the roots of quadratic equation $px^2+qx+r=0$ are all real for
a) $\left|\frac{r}{p}-7\right|\ge 4\sqrt{3}$ b) $\left|\frac{p}{r}-7\right|\ge 4\sqrt{3}$ c) All p and r d) No p and r

119. If $\arg(z)<0$, then $\arg(-z)-\arg(z)=$

a) π b) $-\pi$ c) $-\frac{\pi}{2}$ d) $\frac{\pi}{2}$

120. If $\alpha, \beta, \gamma, \sigma$ are the roots of the equation $x^4 + 4x^3 - 6x^2 + 7x - 9 = 0$, then the value of $(1 + \alpha^2)(1 +$ $\beta 21+\gamma 2(1+\sigma 2)$ is

a) 9 b) 11 c) 13 d) 5

121. If $\left|\frac{z_1}{z_2}\right| = 1$ and $\arg(z_1 z_2) = 0$, then

a) $z_1 = z_2$ b) $|z_2|^2 = z_1 z_2$ c) $z_1 z_2 = 1$ d) None of these

122. The complex numbers $\sin x + i \cos 2x$ and $\cos x - i \sin 2x$ are conjugate to each other for

a) $x = n\pi$ b) $x = 0$ c) $x = (n + 1/2)\pi$ d) No value of x

123. $P(z)$ be a variable point in the Argand plane such that $|z|$ =minimum $\{|z - 1|, |z + 1|\}$ then $z + \bar{z}$ will be equal to

a) -1 or 1 b) 1 but not equal to -1 c) -1 but not equal to 1 d) None of these

124. If $z = (\lambda + 3) - i\sqrt{5 - \lambda^2}$, then the locus of z is

a) Ellipse b) Semicircle c) Parabola d) Straight line

125. For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$, the minimum value of $|z_1 - z_2|$ is

a) 0 b) 2 c) 7 d) 17

126. If α, β be the roots of the equation $(x - a)(x - b) + c = 0 (c \neq 0)$, then the roots of the equation $(x - c - \alpha)(x - c - \beta) = c$ are

a) $a + c$ and $b + c$ b) $a - c$ and $b - c$ c) a and $b + c$ d) $a + c$ and b

127. If $\tan\theta_1, \tan\theta_2, \tan\theta_3$ are the real roots of the $x^3 - (a + 1)x^2 + (b - a)x - b = 0$, where $\theta_1 + \theta_2 + \theta_3 \in (0, \pi)$, then $\theta_1 + \theta_2 + \theta_3$ is equal to

a) $\pi/2$ b) $\pi/4$ c) $3\pi/4$ d) π

128. If $|z| = 1$ then the point representing the complex number $-1 + 3z$ will lie on

a) A circle b) A straight line c) A parabola d) A hyperbola

129. The set of all possible real values of a such that the inequality $\left(x - (a - 1)\right)\left(x - (a^2 + 2)\right) < 0$ holds for all $x \in (-1, 3)$ is

a) $(0, 1)$ b) $(\infty, -1]$ c) $(-\infty, -1)$ d) $(1, \infty)$

130. If the root of the equation $(a - 1)(x^2 + x + 1)^2 = (a + 1)(x^4 + x^2 + 1)$ are real and distinct then the value of $a \in$

a) $(-\infty, 3]$ b) $(-\infty, -2) \cup (2, \infty)$ c) $[-2, 2]$ d) $[-3, \infty)$

131. The equation $x - 2/(x - 1) = 1 - 2/(x - 1)$ has

a) No root b) One root c) Two equals roots d) Infinitely many roots

132. Let α, β be the roots of the equation $(x - a)(x - b) = c, c \neq 0$. Then the roots of the equation $(x - \alpha)(x -$ $\beta + c = 0$ are

a) a, c b) b, c c) a, b d) $a + c, b + c$

133. Number of complex numbers z such that $|z| = 1$ and $|z/\bar{z} + \bar{z}/z| = 1$ is $(\arg(z) \in [0, 2\pi))$

a) 4 b) 6 c) 8 d) More than 8

134. If $x = 1 + i$ is a root of the equation $x^3 - ix + 1 - i = 0$, then the other real root is

a) 0 b) 1 c) -1 d) None of these

135. If $(m_r, 1/m_r), r = 1, 2, 3, 4$ be four pairs of values of x and y that satisfy the equation $x^2 + y^2 + 2gx +$ $2fy + c = 0$, then value of $m_1 m_2 m_3 m_4$ is

a) 0 b) 1 c) -1 d) None of these

136. If $k + |k + z^2| = |z|^2 (k \in R^-)$, then possible argument of z is

a) 0 b) π c) $\pi/2$ d) None of these

137. Let $a \neq 0$ and $p(x)$ be a polynomial of degree greater than 2. If $p(x)$ leaves remainders a and $-a$ when divided respectively by $x + a$ and $x - a$, then remainder when $p(x)$ is divided by $x^2 - a^2$ is

a) $2x$ b) $-2x$ c) x d) $-x$

138. If z is a complex number lying in the fourth quadrant of Argand plane and $|[kz/(k + 1)] + 2i| > \sqrt{2}$ for all real value of $k (k \neq -1)$, then range of $\arg(z)$ is

a) $\left(-\frac{\pi}{8}, 0\right)$ b) $\left(-\frac{\pi}{6}, 0\right)$ c) $\left(-\frac{\pi}{4}, 0\right)$ d) None of these

139. The largest interval for which $x^{12} - x^9 + x^4 - x + 1 > 0$ is

a) $-4 < x \le 0$ b) $0 < x < 1$ c) $-100 < x < 100$ d) $-\infty < x < \infty$

140. If $|2z - 1| = |z - 2|$ and z_1, z_2, z_3 are complex numbers such that $|z_1 - \alpha| < \alpha, |z_2 - \beta| < \beta$, then $\left|\frac{z_1+z_2}{\alpha+\beta}\right|$

a) $< |z|$ b) $< 2|z|$ c) $> |z|$ d) $> 2|z|$

141. If $a, b, c, d \in R$, then the equation $(x^2 + ax - 3b)(x^2 - cx + b)(x^2 - dx + 2b) = 0$ has

a) 6 real roots b) At least 2 real roots c) 4 real roots d) 3 real roots

142. If a, b, c be distinct positive numbers, then the nature of roots of the equation $1/(x - a) + 1/(x - b) + 1/(x - c) = 1/x$ is

a) All real and distinct b) All real and at least two are distinct
c) At least two real d) All non-real

143. If the roots of the quadratic equation $(4p - p^2 - 5)x^2 - (2p - 1)x + 3p = 0$ lie on either side of unity, then the number of integral values of p is

a) 1 b) 2 c) 3 d) 4

144. If $k > 0, |z| = |w| = k$ and $\alpha = \frac{z-\bar{w}}{k^2+z\bar{w}}$, then $\text{Re}(\alpha)$ equals

a) 0 b) $k/2$ c) k d) none of these

145. Let $f(x) = (1 + b^2)x^2 + 2bx + 1$ and let $m(b)$ be the minimum value of $f(x)$. As b varies, the range of $m(b)$ is

a) $[0, 1]$ b) $\left(0, \frac{1}{2}\right]$ c) $\left[\frac{1}{2}, 1\right]$ d) $(0, 1]$

146. The coefficient of x in the equation $x^2 + px + q = 0$ was wrongly written as 17 in place of 13 and the roots thus found was -2 and -15. Then the roots of the correct equation are

a) $-3, 10$ b) $-3, -10$ c) $3, -10$ d) None of these

147. If $8iz^3 + 12z^2 - 18z + 27i = 0$, then

a) $|z| = \frac{3}{2}$ b) $|z| = \frac{2}{3}$ c) $|z| = 1$ d) $|z| = \frac{3}{4}$

148. If 'z' lies on the circle $|z - 2i| = 2\sqrt{2}$ then the value of $\arg[(z - 2)/(z + 2)]$ is equal to

a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{2}$

149. If 'p' and 'q' are distinct prime numbers, then the number of distinct imaginary numbers which are p^{th} as well as q^{th} roots of unity are

a) $\min(p, q)$ b) $\max(p, q)$ c) 1 d) Zero

150. Let a be a complex number such that $|a| < 1$ and $z_1, z_2, z_3, \dots$ be the vertices of a polygon such that $z_k = 1 + a + a^2 + \dots + a^{k-1}$ for all $k = 1, 2, 3, \dots$ then $z_1, z_2, \dots$. lie within the circle

a) $\left|z - \frac{1}{1-a}\right| = \frac{1}{|a-1|}$ b) $\left|z + \frac{1}{a+1}\right| = \frac{1}{|a+1|}$ c) $\left|z - \frac{1}{1-a}\right| = |a - 1|$ d) $\left|z + \frac{1}{a+1}\right| = |a + 1|$

151. The sum of values of x satisfying the equation

$\left(31 + 8\sqrt{15}\right)^{x^2-3} + 1 = \left(32 + 8\sqrt{15}\right)^{x^2-3}$ is

a) 3 b) 0 c) 2 d) None of these

152. Let a, b, c be real numbers, $a \ne 0$. If α is a root of $a^2x^2 + bx + c = 0$. β is the root of $a^2x^2 - bx - c = 0$ and $0 < \alpha < \beta$, then the equation $a^2x^2 + 2bx + 2c = 0$ has a root γ that always satisfies

a) $\gamma = \frac{\alpha+\beta}{2}$ b) $\gamma = \alpha + \frac{\beta}{2}$ c) $\gamma = \alpha$ d) $\alpha < \gamma < \beta$

153. If α, β be the non-zero roots of $ax^2 + bx + c = 0$ and α^2, β^2 be the roots of $a^2x^2 + b^2x + c^2 = 0$, then a, b, c are in

a) G.P. b) H.P. c) A.P. d) None of these

154. The equation $2^{2x} + (a - 1)2^{x+1} + a = 0$ has roots of opposite signs then exhaustive set of values of a is

a) $a \in (-1, 0)$ b) $a < 0$ c) $a \in (-\infty, 1/3)$ d) $a \in (0, 1/3)$

155. Let $z = x + iy$ be a complex number where x and y are integers. Then the area of the rectangle whose

vertices are the roots of the equation $z\bar{z}^3 + \bar{z}z^3 = 350$ is

a) 48 b) 32 c) 40 d) 80

156. If the quadratic equation $4x^2 - 2(a + c - 1)x + ac - b = 0$ $(a > b > c)$

a) Both roots are greater than a
b) Both roots are less than c
c) Both roots lie between $c/2$ and $a/2$
d) Exactly one of the roots lies between $c/2$ and $a/2$

157. Let $|z_r - r| \le r, \forall r = 1, 2, 3, \dots n$. Then $|\sum_{r=1}^{n} z_r|$ is less than

a) n b) $2n$ c) $n(n+1)$ d) $\frac{n(n+1)}{2}$

158. If $(ax^2 + c)y + (a'x^2 + c') = 0$ and x is a rational function of y and ac is negative, then

a) $ac' + a'c = 0$ b) $a/a' = c/c'$ c) $a^2 + c^2 = a'^2 + c'^2$ d) $aa' + cc' = 1$

159. Let z and ω be two non-zero complex numbers such that $|z| = |\omega|$ and $\arg z = \pi - \arg \omega$, then z equals

a) ω b) $-\omega$ c) $\overline{\omega}$ d) $-\overline{\omega}$

160. Suppose that $f(x)$ is a quadratic expression positive for all real x. If $g(x) = f(x) + f'(x) + f''(x)$, then for any real x (where $f'(x)$ and $f''(x)$ represent 1st and 2nd derivative respectively)

a) $g(x) < 0$ b) $g(x) > 0$ c) $g(x) = 0$ d) $g(x) \ge 0$

161. If z_1 is a root of the equation $a_0 z^n + a_1 z^{n-1} + \dots + a_{n-1} z + a_n = 3$, where $|a_i| < 2$ for $i = 0, 1, \dots, n$. Then

a) $|z_1| > \frac{1}{3}$ b) $|z_1| < \frac{1}{4}$ c) $|z_1| > \frac{1}{4}$ d) $|z| < \frac{1}{3}$

162. The complex numbers z_1, z_2 and z_3 satisfying $[(z_1 - z_3)/(z_2 - z_3)] = [(1 - i\sqrt{3})/2]$ are the vertices of a triangle which is

a) Of area zero
b) Right-angled isosceles
c) Equilateral
d) Obtuse-angled isosceles

163. The number of solutions of the equation $z^2 + \bar{z} = 0$ is

a) 1 b) 2 c) 3 d) 4

164. If α is the n^{th} root of unity, then $1 + 2\alpha + 3\alpha^2 + \cdots$ to n terms equal to

a) $\frac{-n}{(1-\alpha)^2}$ b) $\frac{-n}{1-\alpha}$ c) $\frac{-2n}{1-\alpha}$ d) $\frac{-2n}{(1-\alpha)^2}$

165. If α, β be the roots of the equation $2x^2 - 35x + 2 = 0$, then the value of $(2\alpha - 35)^3(2\beta - 35)^3$ is equal to

a) 8 b) 1 c) 64 d) None of these

166. If z is a complex number such that $-\pi/2 \le \arg z \le \pi/2$, then which of the following inequality is true?

a) $|z - \overline{z}| \le |z|(\arg z - \arg \overline{z})$
b) $|z - \overline{z}| \ge |z|(\arg z - \arg \overline{z})$
c) $|z - \overline{z}| < (\arg z - \arg \overline{z})$
d) None of these

167. If $x^2 + ax - 3x - (a + 2) = 0$ has real and distinct roots, then minimum value of $(a^2 + 1)/(a^2 + 2)$ is

a) 1 b) 0 c) $\frac{1}{2}$ d) $\frac{1}{4}$

168. If α and β are the roots of the equation $x^2 - ax + b = 0$ and $A_n = \alpha^n + \beta^n$, then which of the following is true?

a) $A_{n+1} = aA_n + bA_{n-1}$ b) $A_{n+1} = bA_n + aA_{n-1}$ c) $A_{n+1} = aA_n - bA_{n-1}$ d) $A_{n+1} = bA_n - aA_{n-1}$

169. The polynomial $x^6 + 4x^5 + 3x^4 + 2x^3 + x + 1$ is divisible by where w is cube root of units
Where ω is one of the imaginary cube roots of unity

a) $x + \omega$ b) $x + \omega^2$ c) $(x + \omega)(x + \omega^2)$ d) $(x - \omega)(x - \omega^2)$

170. Let $f(x) = ax^2 + bx + c, a \ne 0$ and $\Delta = b^2 - 4ac$. If $\alpha + \beta, \alpha^2 + \beta^2$ and $\alpha^3 + \beta^3$ are in GP, then

a) $\Delta \ne 0$ b) $b\Delta = 0$ c) $c\Delta = 0$ d) $bc \ne 0$

171. z_1 and z_2 are two distinct points in an Argand plane. If $a|z_1| = b|z_2|$ (where $a, b \in R$), then the point $(az_1/bz_2) + (bz_2/az_1)$ is a point on the

a) Line segment $[-2,2]$ of the real axis
b) Line segment $[-2, 2]$ of the imaginary axis
c) Unit circle $|z| = 1$
d) The line with $\arg z = \tan^{-1} 2$

172. The number of complex numbers z satifying $|z - 3 - i| = |z - 9 - i|$ and $|z - 3 + 3i| = 3$ are

a) One b) Two c) Four d) None of these

173. If 'z' is complex number then the locus of 'z' satisfying the condition $|2z - 1| = |z - 1|$ is

a) Perpendicular bisector of line segment joining $1/2$ and 1

b) Circle
c) Parabola
d) None of the above curves

174. If $xy = 2(x + y), x \leq y$ and $x, y \in N$, then the number of solutions of the equation are
a) Two
b) Three
c) No solution
d) Infinitely many solutions

175. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3, then
a) $a < 2$
b) $2 \leq a \leq 3$
c) $3 < a \leq 4$
d) $a > 4$

176. Let $f(x) = ax^2 + bx + c, a, b, c \in R$. If $f(x)$ takes real values for real values of x and non-real values of x, then
a) $a = 0$
b) $b = 0$
c) $c = 0$
d) Nothing can be said about a, b, c

177. If $z(1 + a) = b + ic$ and $a^2 + b^2 + c^2 = 1$, then$[(1+iz)/(1 - iz)=$
a) $\dfrac{a + ib}{1 + c}$
b) $\dfrac{b - ic}{1 + a}$
c) $\dfrac{a + ic}{1 + b}$
d) None of these

178. The number of positive integral solutions of $x^4 - y^4 = 3789108$ is
a) 0
b) 1
c) 2
d) 4

179. Let a, b and c be real numbers such that $4a + 2b + c = 0$ and $ab > 0$. Then the equation $ax^2 + bx + c = 0$ has
a) Complex roots
b) Exactly one root
c) Real roots
d) None of these

180. If $\omega(\neq 1)$ is a cube root of unity and $(1 + \omega)^7 = A + B\omega$ then A and B are respectively
a) 0, 1
b) 1, 1
c) 1, 0
d) $-1, 1$

181. Consider the equation $10z^2 - 3iz - k = 0$, where z is a complex variable and $i^2 = -1$. Which of the following statements is true?
a) For real positive numbers k, both roots are purely imaginary
b) For all complex numbers k, neither roots is real
c) For real purely imaginary numbers k, both roots are real and irrational
d) For real negative numbers k, both roots are purely imaginary

182. The expression $\left[\dfrac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right]^8 =$
a) 1
b) -1
c) i
d) $-i$

183. If roots of $x^2 - (a - 3)x + a = 0$ are such that at least one of them is greater than 2, then
a) $a \in [7, 9]$
b) $a \in [7, \infty]$
c) $a \in [9, \infty]$
d) $a \in [7, 9)$

184. If a, b, c are three distinct positive real numbers, then the number of real roots of $ax^2 + 2b|x| - c = 0$ is
a) 0
b) 4
c) 2
d) None of these

185. If $i = \sqrt{-1}$, then $4 + 5\left[(-1/2) + i\sqrt{3}/2\right]^{334} + 3\left[(-1/2) + (i\sqrt{3}/2)\right]^{365}$ is equal to
a) $1 - i\sqrt{3}$
b) $-1 + i\sqrt{3}$
c) $i\sqrt{3}$
d) $-i\sqrt{3}$

186. The smallest positive integer n for which $[(1 + i)/(1 - i)]^n = 1$ is
a) $n = 8$
b) $n = 16$
c) $n = 12$
d) None of these

187. If x, y and z are real and different and $u = x^2 + 4y^2 + 9z^2 - 6yz - 3zx - 2xy$, then u is always
a) Non-negative
b) Zero
c) Non-positive
d) None of these

188. If the ratio of the roots of $ax^2 + 2bx + c = 0$ is same as the ratio of the $px^2 + 2qx + r = 0$, then
a) $\dfrac{2b}{ac} = \dfrac{q^2}{pr}$
b) $\dfrac{b}{ac} = \dfrac{q}{pr}$
c) $\dfrac{b^2}{ac} = \dfrac{q^2}{pr}$
d) None of these

189. If one root of $x^2 - x - k = 0$ is square of the other, then $k =$
a) $2 \pm \sqrt{5}$
b) $2 \pm \sqrt{3}$
c) $3 \pm \sqrt{2}$
d) $5 \pm \sqrt{2}$

190. If $n \in N > 1$ then sum of real part of roots of $z^n = (z + 1)^n$ is equal to
a) $\dfrac{n}{2}$
b) $\dfrac{(n - 1)}{2}$
c) $-\dfrac{n}{2}$
d) $\dfrac{(1 - n)}{2}$

191. If z is a complex number having least absolute value and $|z - 2 + 2i| = 1$, then $z =$

a) $(2-1/\sqrt{2})(1-i)$ b) $(2-1/\sqrt{2})(1+i)$ c) $(2+1/\sqrt{2})(1-i)$ d) $(2+1/\sqrt{2})(1+i)$

192. If $x=2+2^{2/3}+2^{1/3}$, then the value of x^3-6x^2+6x is
a) 3 b) 2 c) 1 d) -2

193. The principal argument of the complex number $\left[(1+i)^5\left(1+\sqrt{3i}\right)^2\right]/\left[-2i\left(-\sqrt{3}+i\right)\right]$ is
a) $\frac{19\pi}{12}$ b) $-\frac{7\pi}{12}$ c) $-\frac{5\pi}{12}$ d) $\frac{5\pi}{12}$

194. The range of a for which the equation $x^2+ax-4=0$ has its smaller root in the interval $(-1,2)$ is
a) $(-\infty,-3)$ b) $(0,3)$ c) $(0,\infty)$ d) $(-\infty,-3)\cup(0,\infty)$

195. If α and β are the roots of $x^2+px+q=0$ and α^4,β^4 are the roots of $x^2-rx+s=0$, then the equation $x^2-4qx+2q^2-r=0$ has always
a) One positive and one negative root b) Two positive roots
c) Two negative roots d) Cannot say anything

196. The number of solutions of the equation $\sin(e^x)=5^x+5^{-x}$ is
a) 0 b) 1 c) 2 d) Infinitely many

197. If $p(q-r)x^2+q(r-p)x+r(p-q)=0$ has equal roots, then $2/q=$
a) $\frac{1}{p}+\frac{1}{r}$ b) $p+r$ c) p^2+r^2 d) $\frac{1}{p^2}+\frac{1}{r^2}$

198. The roots of the equation $t^3+3at^2+3bt+c=0$ are z_1,z_2,z_3 which represent the vertices of an equilateral triangle, then
a) $a^2=3b$ b) $b^2=a$ c) $a^2=b$ d) $b^2=3a$

199. The integral values of m for which the roots of the equation $mx^2+(2m-1)x+(m-2)=0$ are rational are given by the expression [where n is integer]
a) n^2 b) $n(n+2)$ c) $n(n+1)$ d) None of these

200. Let $z=1-t+i\sqrt{t^2+t+2}$, where t is a real parameter. The locus of z in the Argand plane is
a) A hyperbola b) An ellipse c) A straight line d) None of these

201. $1,z_1,z_2,z_3,\dots,z_{n-1}$ are the n^{th} roots of unity, then the value of $1/(3-z_1)+1/(3-z_2)+\cdots+1/(3-z_{n-1})$ is equal to
a) $\frac{n3^{n-1}}{3^n-1}+\frac{1}{2}$ b) $\frac{n3^{n-1}}{3^n-1}-1$ c) $\frac{n3^{n-1}}{3^n-1}+1$ d) None of these

202. Given $z=(1+i\sqrt{3})^{100}$, then $[\mathrm{RE}(z)/\mathrm{IM}(z)]$ equals
a) 2^{100} b) 2^{50} c) $\frac{1}{\sqrt{3}}$ d) $\sqrt{3}$

203. The quadratic $x^2+ax+b+1=0$ has roots which are positive integers, then (a^2+b^2) can be equal to
a) 50 b) 37 c) 61 d) 19

204. x_1 and x_2 are the roots of $ax^2+bx+c=0$ and $x_1x_2<0$. Roots of $x_1(x-x_2)^2+x_2(x-x_1)^2=0$ are
a) Real and opposite sign b) Negative
c) Positive d) Non-real

205. Both the roots of the equation $(x-b)(x-c)+(x-a)(x-c)+(x-a)(x-b)=0$ are always
a) Positive b) Real c) Negative d) None of these

206. If $a^2+b^2+c^2=1$, then $ab+bc+ca$ lies in the interval
a) $\left[\frac{1}{2},2\right]$ b) $[-1,2]$ c) $\left[-\frac{1}{2},1\right]$ d) $\left[-1,\frac{1}{2}\right]$

207. If $|z_1|=|z_2|$ and arg arg$(z_1/z_2)=\pi$, then $z_1+z_2$is equal to
a) 0 b) Purely imaginary c) Purely real d) None of these

208. If $b_1b_2=2(c_1+c_2)$, then at least one of the equations $x^2+b_1x+c_1=0$ and $x^2+b_2x+c_2=0$ has
a) Imaginary roots b) Real roots
c) Purely imaginary roots d) None of these

209. If $\cos\alpha+2\cos\beta+3\cos\gamma=\sin\alpha+2\sin\beta+3\sin\gamma=0$, then the value of $\sin 3\alpha+8\sin 3\beta+27\sin 3\gamma$ is
a) $\sin(a+b+\gamma)$ b) $3\sin(\alpha+\beta+\gamma)$ c) $18\sin(\alpha+\beta+\gamma)$ d) $\sin(\alpha+2\beta+3)$

210. If $z=x+iy$ and $\omega=(1-iz)/(z-i)$, then $|\omega|=1$ implies that, in the complex plane

a) z lies on the imaginary axis
b) z lies on the real axis
c) z lies on the unit circle
d) None of these

211. The maximum area of the triangle formed by the complex coordinate z, z_1, z_2 which satisfy the relations $|z - z_1| = |z - z_2|$ and $|z - (z_1 + z_2)/2| \leq r$, where $r > |z_1 - z_2|$ is
a) $\frac{1}{2}|z_1 - z_2|^2$
b) $\frac{1}{2}|z_1 - z_2|\, r$
c) $\frac{1}{2}|z_1 - z_2|^2\, r^2$
d) $\frac{1}{2}|z_1 - z_2| r^2$

212. The points z_1, z_2, z_3, z_4 in the complex plane are the vertices of a parallelogram taken in order if and only if
a) $z_1 + z_4 = z_2 + z_3$
b) $z_1 + z_3 = z_2 + z_4$
c) $z_1 + z_2 = z_3 + z_4$
d) None of these

213. The equation $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$ has
a) No solution
b) One solution
c) Two solutions
d) More than two solutions

214. If $(b^2 - 4ac)^2(1 + 4a^2) < 64a^2, a < 0$, then maximum value of quadratic expression $ax^2 + bx + c$ is always less than
a) 0
b) 2
c) -1
d) -2

215. The curve $y = (\lambda + 1)x^2 + 2$ intersects the curve $y = \lambda x + 3$ in exactly one point, if λ equals
a) $\{-2, 2\}$
b) $\{1\}$
c) $\{-2\}$
d) $\{2\}$

216. If $a^2 + b^2 = 1$,then $(1 + b + ia)/(1 + b - ia) =$
a) 1
b) 2
c) $b + ia$
d) $a + ib$

217. If z_1, z_2, z_3 are the vertices of an equilateral triangle ABC such that $|z_1 - i| = |z_2 - i| = |z_3 - i|$, then $|z_1 + z_2 + z_3|$ equals to
a) $3\sqrt{3}$
b) $\sqrt{3}$
c) 3
d) $\frac{1}{3\sqrt{3}}$

218. Let C_1 and C_2 are concentric circles of radius 1 and 8/3, respectively, having centre at (3, 0) on the Aragnd plane. If the complex number z satisfies the inequality
$\log_{1/3}\left(\frac{|z-3|^2+2}{11|z-3|-2}\right) > 1$ then
a) z lies outside C_1 but inside C_2
b) z lies inside of both C_1 and C_2
c) z lies outside both of C_1 and C_2
d) None of these

219. If a, b and c are real numbers such that $a^2 + b^2 + c^2 = 1$, then $ab + bc + ca$ lies in the interval
a) $[1/2, 2]$
b) $[-1, 2]$
c) $[-1/2, 1]$
d) $[-1, 1/2]$

220. If $z = x + iy$ and $x^2 + y^2 = 16$, then the range of $||x| - |y||$ is
a) [0,4]
b) [0,2]
c) [2,4]
d) None of these

221. If $|z - 1| \leq 2$ and $|\omega z - 1 - \omega^2| = a$ (where ω is a cube root of unity) then complete set of values of a is
a) $0 \leq a \leq 2$
b) $\frac{1}{2} \leq a \leq \frac{\sqrt{3}}{2}$
c) $\frac{\sqrt{3}}{2} - \frac{1}{2} \leq a \leq \frac{1}{2} + \frac{\sqrt{3}}{2}$
d) $0 \leq a \leq 4$

222. If α, β are real and α^2, β^2 are the roots of the equation $a^2x^2 + x + 1 - a^2 = 0 (a > 1)$, then $\beta^2 =$
a) a^2
b) $1 - \frac{1}{a^2}$
c) $1 - a^2$
d) $1 + a^2$

223. If the complex number z satisfies the condition $|z| \geq 3$, then the least value of $[z + (1/z)|$ is equal to
a) 5/3
b) 8/3
c) 11/3
d) None of these

224. The points $z_1 = 3 + \sqrt{3}i$ and $z_2 = 2\sqrt{3} + 6i$ are given on a complex plane. The complex number lying on the bisector of the angle formed by the vectors z_1 and z_2 is
a) $z = \frac{(3 + 2\sqrt{3})}{2} + \frac{\sqrt{3} + 2}{2}i$
b) $z = 5 + 5i$
c) $z = -1 - i$
d) None of these

225. If $|z^2 - 3| = 3|z|$ then the maximum value of $|z|$ is
a) 1
b) $\frac{3 + \sqrt{21}}{2}$
c) $\frac{\sqrt{21} - 3}{2}$
d) None of these

226. If a complex number z satisfies $|2z + 10 + 10i| \leq 5\sqrt{3} - 5$, then the least principle argument of z is
a) $-\frac{5\pi}{6}$
b) $-\frac{11\pi}{12}$
c) $-\frac{3\pi}{4}$
d) $-\frac{2\pi}{3}$

227. If x be real, then $x/(x^2-5x+9)$ lies between

a) -1 and $-1/11$ b) 1 and $-1/11$ c) 1 and $1/11$ d) None of these

228. The complex number associated with the vertices A, B, C of ΔABC are $e^{\text{th}}, \omega, \overline{\omega}$, respectively [where $\omega, \overline{\omega}$ are the complex cube roots of unity and $\cos\theta > Re(\omega)$], then the complex number of the point where angle bisector of A meets the circumcircle of the triangle, is

a) $e^{i\theta}$ b) $e^{-i\theta}$ c) $\omega\overline{\omega}$ d) $\omega+\overline{\omega}$

229. If $\arg\left(\frac{z_1-\frac{z}{|z|}}{\frac{z}{|z|}}\right)=\frac{\pi}{2}$ and $\left|\frac{z}{|z|}-z_1\right|=3$ then $|z_1|$ equals to

a) $\sqrt{26}$ b) $\sqrt{10}$ c) $\sqrt{3}$ d) $2\sqrt{2}$

230. Dividing $f(z)$ by $z-i$, we obtain the remainder i and dividing it by $z+i$, we get the remainder $1+i$, then remainder upon the division of $f(z)$ by z^2+1 is

a) $\frac{1}{2}(z+1)+i$ b) $\frac{1}{2}(iz+1)+i$ c) $\frac{1}{2}(iz-1)+i$ d) $\frac{1}{2}(z+i)+1$

231. If both roots of the equation $ax^2+x+c-a=0$ are imaginary and $c>-1$, then

a) $3a>2+4c$ b) $3a<2+4c$ c) $c<a$ d) None of these

232. Let α, β be the roots of $x^2-x+p=0$ and γ, δ be roots of $x^2-4x+q=0$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then the integral values of p and q, respectively are

a) $-2, -32$ b) $-2, 3$ c) $-6, 3$ d) $-6, -32$

233. If α, β be the roots of the equation $ax^2+bx+c=0$, then value of $(a\alpha^2+c)/(a\alpha+b)+(a\beta^2+c)/(a\beta+b)$ is

a) $\frac{b(b^2-2ac)}{4a}$ b) $\frac{b^2-4ac}{2a}$ c) $\frac{b(b^2-2ac)}{a^2c}$ d) None of these

234. Number of solutions of the equation $z^3+[3(\bar{z})^2]/|z|=0$ where z is a complex number is

a) 2 b) 3 c) 6 d) 5

235. Given z is a complex number with modulus 1. Then the equation $[(1+ia)/(1-ia)]^4=z$ has

a) All roots real and distinct
b) Two real and two imaginary
c) Three roots real and one imaginary
d) One root real and three imaginary

236. If the equation $z^4+a_1z^3+a_2z^2+a_3z+a_4=0$, where a_1, a_2, a_3, a_4 are real coefficients different from zero, has a purely imaginary root, then the expression $a_3/(a_1a_2)+(a_1a_4)/(a_2a_3)$ has the value equal to

a) 0 b) 1 c) -2 d) 2

237. If z_1, z_2, z_3 are three complex numbers and $A=\begin{bmatrix}\arg z_1 & \arg z_2 & \arg z_3\\ \arg z_2 & \arg z_3 & \arg z_1\\ \arg z_3 & \arg z_1 & \arg z_2\end{bmatrix}$ then A is divided by

a) $\arg(z_1+z_2+z_3)$ b) $\arg(z_1z_2z_3)$ c) All numbers d) cannot say

238. Let z_1 and z_2 be n^{th} roots of unity which subtend a right angle at the origin. Then n must be of the form

a) $4k+1$ b) $4k+2$ c) $4k+3$ d) $4k$

Multiple Correct Answers Type

239. If a, b, c are distinct numbers in arithmetic progressions then both the roots of the quadratic equation $(a+2b-3c)x^2+(b+2c-3a)x+(c+2a-3b)=0$ are

a) Real b) Positive c) Negative d) Rational

240. If the equation $ax^2+bx+c=0, a, b\ c\in R$ have non-real roots, then

a) $c(a-b+c)>0$ b) $c(a+b+c)>0$ c) $c(4a-2b+c)>0$ d) None of these

241. Let z be a complex number satisfying equation $z^p=\overline{z}^q$, where $p, q\in N$, then

a) If $p=q$, then number of solutions of equation will be infinite
b) If $p=q$, then number of solutions of equation will be finite
c) If $p\neq q$, then number of solutions of equation will be $p+q+1$
d) If $p\neq q$, then number of solutions of equation will be $p+q$

242. Let $a, b, c\in Q'$ satisfying $a>b>c$. Which of the following statement(s) hold true for the quadratic

polynomial $f(x) = (a+b-2c)x^2 + (b+c-2a)x + (c+a-2b)$?
a) The mouth of the parabola $y = f(x)$ opens upwards
b) Both roots of the equation $f(x) = 0$ are rational
c) x-coordinate of vertex of the graph is positive
d) Product of the roots is always negative

243. Let $P(x)$ and $Q(x)$ be two polynomials. Suppose that $f(x) = P(x^3) + xQ(x^3)$ is divisible by $x^2 + x + 1$, then
a) $P(x)$ is divisible by $(x-1)$ but $Q(x)$ is not divisible by $(x-1)$
b) $Q(x)$ is divisible by $(x-1)$ but $P(x)$ is not divisible by $(x-1)$
c) Both $P(x)$ and $Q(x)$ are divisible by $(x-1)$
d) $f(x)$ is divisible by $(x-1)$

244. z_1 and z_2 are the roots of the equation $z^2 - az + b = 0$, where $|z_1| = |z_2| = 1$ and a, b are non-zero complex numbers, then
a) $|a| \le 1$
b) $|a| \le 2$
c) $\arg(a^2) = \arg(b)$
d) $\arg a = \arg(b^2)$

245. Let z_1 and z_2 be complex numbers such that $z_1 \ne z_2$ and $|z_1| = |z_2|$. If z_1 has positive real part and z_2 has negative imaginary part, then $(z_1 + z_2)/(z_1 - z_2)$ may be
a) Zero
b) Real and positive
c) Real and negative
d) Purely imaginary

246. Given that the complex numbers which satisfy the equation $z\bar{z}^3 + \bar{z}z^3 = 350$ form a rectangle in the Argand plane with the length of its diagonal having an integral number of units, then
a) Area of rectangle is 48 sq. units
b) If z_1, z_2, z_3, z_4 are vertices of rectangle then $z_1 + z_2 + z_3 + z_4 = 0$
c) Rectangle is symmetrical about real axis
d) $\arg(z_1 - z_3) = \frac{\pi}{4}$ or $\frac{3\pi}{4}$

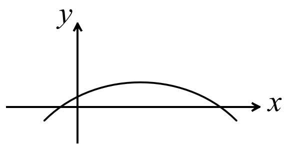
247. If $z_1 = 5 + 12i$ and $|z_2| = 4$ then
a) Maximum $(|z_1 + iz_2|) = 17$
b) Minimum $(|z_1 + (1+i)z_2|) = 13 - 4\sqrt{2}$
c) Minimum $\left|\frac{z_1}{z_2 + \frac{4}{z_2}}\right| = \frac{13}{4}$
d) Maximum $\left|\frac{z_1}{z_2 + \frac{4}{z_2}}\right| = \frac{13}{3}$

248. For the quadratic equation $x^2 + 2(a+1)x + 9a - 5 = 0$, which of the following is/are true?
a) If $2 < a < 5$, then roots are of opposite sign
b) If $a < 0$, then roots are of opposite sign
c) If $a > 7$, then both roots are negative
d) If $2 \le a \le 5$, then roots are unreal

249. Given $z = f(x) + i\,g(x)$ where $f, g: (0,1) \to (0,1)$ are real valued functions. Then, which of the following does not hold good?
a) $z = \frac{1}{1-ix} + i\left(\frac{1}{1+ix}\right)$
b) $z = \frac{1}{1+ix} + i\left(\frac{1}{1-ix}\right)$
c) $z = \frac{1}{1+ix} + i\left(\frac{1}{1+ix}\right)$
d) $z = \frac{1}{1-ix} + i\left(\frac{1}{1-ix}\right)$

250. If S is the set of all real x such that $(2x-1)/(2x^3 + 3x^2 + x)$ is positive, then S contains
a) $\left(-\infty, -\frac{3}{2}\right)$
b) $\left(-\frac{3}{2}, -\frac{1}{4}\right)$
c) $\left(-\frac{1}{4}, \frac{1}{2}\right)$
d) $\left(\frac{1}{2}, 3\right)$

251. If the following figure shows the graph of $f(x) = ax^2 + bx + c$, then



a) $ac < 0$
b) $bc > 0$
c) $ab > 0$
d) $abc < 0$

252. If the equation $ax^2 + bx + c = 0 (a > 0)$ has two roots α and β such that $\alpha < -2$ and $\beta > 2$, then
a) $b^2 - 4ac > 0$
b) $c < 0$
c) $a + |b| + c < 0$
d) $4a + 2|b| + c < 0$

253. If $|z-1|=1$, then

a) $\arg((z-1-i)/z)$ can be equal to $-\pi/4$
b) $(z-2)/z$ is purely imaginary number
c) $(z-2)/z$ is purely real number
d) If $\arg(z)=\theta$, where $z\neq 0$ and θ is acute, then $1-2/z=i\tan\theta$

254. If the points $A(z), B(-z)$ and $C(1-z)$ are the vertices of an equilateral triangle ABC, then

a) Sum of possible z is $1/2$
b) Sum of possible z is1
c) Product of possible z is $1/4$
d) Product of possible z is $1/2$

255. If every pair from among the equations $x^2+ax+bc=0, x^2+bx+ca=0$ and $x^2+cx+ab=0$ has a common root, then

a) The sum of the three common roots is $-1/2(a+b+c)$
b) The sum of the three common roots is $2(a+b+c)$
c) The product of the three common roots is abc
d) The product of the three common roots is $a^2b^2c^2$

256. $|z^2|^3$ is equal to

a) $|z^3|^2$
b) $|\bar{z}^3|^2$
c) $|z|^6$
d) $|z^6|$

257. If $ax^2+(b-c)x+a-b-c=0$ has unequal real roots for all $c\in R$, then

a) $b<0<a$
b) $a<0<b$
c) $b<a<0$
d) $b>a>0$

258. If $\cos x-y^2-\sqrt{y-x^2-1}\geq 0$, then

a) $y\geq 1$
b) $x\in R$
c) $y=1$
d) $x=0$

259. Equation of tangent drawn to circle $|z|=r$ at the point $A(z_0)$ is

a) $\mathrm{Re}\left(\frac{z}{z_0}\right)=1$
b) $z\bar{z}_0+z_0\bar{z}=2r^2$
c) $\mathrm{Im}\left(\frac{z}{z_0}\right)=1$
d) $\mathrm{Im}\left(\frac{z_0}{z}\right)=1$

260. If $a,b,c\in R$ and $abc<0$, then the equation $bcx^2+2(b+c-a)x+a=0$, has

a) Both positive roots
b) Both negative roots
c) Real roots
d) One positive and one negative root

261. If $\sqrt{5-12i}+\sqrt{-5-12i}=z$, then principle value of $\arg z$ can be

a) $-\frac{\pi}{4}$
b) $\frac{\pi}{4}$
c) $\frac{3\pi}{4}$
d) $-\frac{3\pi}{4}$

262. If α,β are the roots of the quadratic equation $ax^2+bx+c=0$ then which of the following expression will be the symmetric function of roots

a) $\left|\log\frac{\alpha}{\beta}\right|$
b) $\alpha^2\beta^5+\beta^2\alpha^5$
c) $\tan(\alpha-\beta)$
d) $\left(\log\frac{1}{\alpha}\right)^2+(\log\beta^2)$

263. Let z_1,z_2,z_3 be the three non-zero complex numbers such that $z_2\neq 1, a=|z_1|, b=|z_2|$ and $c=|z_3|$. Let,

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}=0$$

Then

a) $\arg\left(\frac{z_3}{z_2}\right)=\arg\left(\frac{z_3-z_1}{z_2-z_1}\right)^2$
b) Orthocenter of triangle formed by z_1,z_2,z_3 is $z_1+z_2+z_3$
c) If triangle formed by z_1,z_2,z_3 is equilateral, then its area is $\frac{3\sqrt{3}}{2}|z_1|^2$
d) If triangle formed by z_1,z_2,z_3 is equialateral then $z_1+z_2+z_3=0$

264. If the quadratic equation $ax^2+bx+c=0$ $(a>0)$ has $\sec^2\theta$ and $\mathrm{cosec}^2\,\theta$ as its roots, then which of the following must hold good?

a) $b+c=0$
b) $b^2-4ac\geq 0$
c) $c\geq 4a$
d) $4a+b\geq 0$

265. If $|z_1|=15$ and $|z_2-3-4i|=5$, then

a) $|z_1-z_2|_{\min}=5$
b) $|z_1-z_2|_{\min}=10$
c) $|z_1-z_2|_{\max}=20$
d) $|z_1-z_2|_{\max}=25$

266. z_1,z_2,z_3 and z_1',z_2',z_3' are non-zero complex numbers such that $z_3=(1-\lambda)z_1+\lambda z_2$ and $z_3'=(1-\mu)z_1'+\mu z_2'$ then which of the following statements is/are true?

a) If $\lambda, \mu \in R-\{0\}$, then z_1, z_2 and z_3 are collinear and z_1', z_2', z_3' are collinear separately

b) If λ, μ are complex numbers, where $\lambda = \mu$ then triangles formed by points z_1, z_2, z_3 and z_1', z_2', z_3' are similar

c) If λ, μ are distinct complex numbers, then points z_1, z_2, z_3 and z_1', z_2', z_3' are not connected by any well defined geometry

d) If $0 < \lambda < 1$, then z_3 divides the line joining $z_1$and z_2 internally and if $\mu > 1$ then z_3' divides the line joining of z_1', z_2' externally

267. If from a point P representing the complex number z_1 on the curve $|z| = 2$, two tangents are drawn from P to the curve $|z| = 1$, meeting at points $Q(z_2)$ and $R(z_3)$, then

a) Complex number $(z_1 + z_2 + z_3)/3$ will be on the curve $|z| = 1$

b) $\left(\frac{4}{\bar{z}_1} + \frac{1}{\bar{z}_2} + \frac{1}{\bar{z}_3}\right)\left(\frac{4}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right) = 9$

c) $\arg\left(\frac{z_2}{z_3}\right) = \frac{2\pi}{3}$

d) Orthocentre and circumcentre of ΔPQR will coincide

268. If every pair from among the equations $x^2 + px + qr = 0, x^2 + qx + rp = 0$ and $x^2 + rx + pq = 0$, where p, q, r are unequal non-zero numbers, have a common root, then the value of $\left(\frac{\text{sum of common roots}}{\text{product of common roots}}\right)$ is

a) $\frac{\sum p}{pqr}$ b) $\left(\sum p\right)^2$ c) $\sum \frac{1}{p}$ d) 0

269. If $z_1 = a + ib$ and $z_2 = c + id$ are complex numbers such that $|z_1| = |z_2| = 1$ and $\operatorname{Re}(z_1\bar{z}_2) = 0$, then the pair of complex number $w_1 = a + ic$ and $w_2 = b + id$ satisfies

a) $|w_1| = 1$ b) $|w_2| = 1$ c) $\operatorname{Re}|w_1\bar{w}_2| = 0$ d) None of these

270. If $\arg(z + a) = \pi/6$ and $\arg(z - a) = 2\pi/3$ $(a \in R^+)$, then

a) $|z| = a$ b) $|z| = 2a$ c) $\arg(z) = \frac{\pi}{2}$ d) $\arg(z) = \frac{\pi}{3}$

271. A rectangle of maximum area is inscribed in the circle $|z - 3 - 4i| = 1$. If one vertex of the rectangle is $4 + 4i$, then another adjacent vertex of this rectangle can be

a) $2 + 4i$ b) $3 + 5i$ c) $3 + 3i$ d) $3 - 3i$

272. If amp $(z_1z_2) = 0$ and $|z_1| = |z_2| = 1$, then

a) $z_1 + z_2 = 0$ b) $z_1z_2 = 1$ c) $z_1 = \bar{z}_2$ d) None of these

273. If the equations $4x^2 - x - 1 = 0$ and $3x^2 + (\lambda + \mu)x + \lambda - \mu = 0$ have a root common then the rational values of λ and μ are

a) $\lambda = \frac{-3}{4}$ b) $\lambda = 0$ c) $\mu = \frac{3}{4}$ d) $\mu = 0$

274. If $|ax^2 + bx + c| \le 1$ for all x in $[0, 1]$, then

a) $|a| \le 8|$ b) $|b| > 8$ c) $|c| \le 1$ d) $|a| + |b| + |c| \le 17$

275. If $x, y \in R$ and $2x^2 + 6xy + 5y^2 = 1$, then

a) $|x| \le \sqrt{5}$ b) $|x| \ge \sqrt{5}$ c) $y^2 \le 2$ d) $y^2 \le 4$

276. Let z_1 and z_2 be two distinct complex numbers and let $z = (1-t)z_1 + tz_2$ for some real number t with $0 < t < 1$. If $\arg(w)$ denotes the principle argument of a non-zero complex number w, then

a) $|z - z_1| + |z - z_2| = |z_1 - z_2|$ b) $\arg(z - z_1) = \arg(z - z_2)$

c) $\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$ d) $\arg(z - z_1) = \arg(z_2 - z_1)$

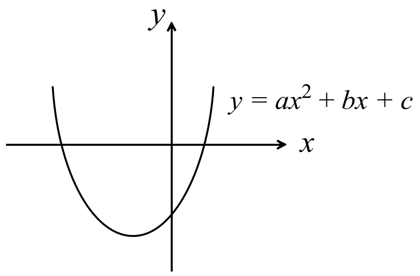
277. Value $(s)(-i)^{1/3}$ is/are

a) $\frac{\sqrt{3} - i}{2}$ b) $\frac{\sqrt{3} + i}{2}$ c) $\frac{-\sqrt{3} - i}{2}$ d) $\frac{-\sqrt{3} + i}{2}$

278. z_0 is a root of the equation $z^n \cos\theta_0 + z^{n-1}\cos\theta_1 + \cdots + z\cos\theta_{n-1} + \cos\theta_n = 2$, where $\theta_i \in R$, then

a) $|z_0| > 1$ b) $|z_0| > \frac{1}{2}$ c) $|z_0| > \frac{1}{4}$ d) $|z_0| > \frac{3}{2}$

279. Let $f(x) = ax^2 + bx + c$. Consider the following diagram. Then



a) $c < 0$ b) $b > 0$ c) $a + b - c > 0$ d) $abc < 0$

280. If α is a complex constant such that $\alpha z^2 + z + \overline{\alpha} = 0$ has a real root, then
a) $\alpha + \overline{\alpha} = 1$
b) $\alpha + \overline{\alpha} = 0$
c) $\alpha + \overline{\alpha} = -1$
d) The absolute value of the real roots is 1

281. If $p = a + b\omega + c\omega^2, q = b + c\omega + a\omega^2$ and $r = c + a\omega + b\omega^2$ where $a, b, c \neq 0$ and ω is the complex cube root of unity, then
a) If p, q, r lie on the circle $|z| = 2$, the triangle formed by these points is equilateral
b) $p^2 + q^2 + r^2 = a^2 + b^2 + c^2$
c) $p^2 + q^2 + r^2 = 2(pq + qr + rp)$
d) None of these

282. $P(z_1), Q(z_2), R(z_3)$ and $S(z_4)$ are four complex numbers representing the vertices of a rhombus taken in order on the complex plane, then which one of the following is/are correct?
a) $\frac{z_1-z_4}{z_2-z_3}$ is purely real
b) amp $\frac{z_1 - z_4}{z_2 - z_4} = \text{amp}\frac{z_2 - z_4}{z_3 - z_4}$
c) $\frac{z_1-z_3}{z_2-z_4}$ is purely imaginary
d) It is not necessary that $|z_1 - z_3| \neq |z_2 - z_4|$

283. If z_1, z_2 be two complex numbers $(z_1 \neq z_2)$ satisfying $\left|z_1^2 - z_2^2\right| = |\overline{z}_1^2 + \overline{z}_2^2 - 2\overline{z}_1\overline{z}_2|$, then
a) $\frac{z_1}{z_2}$ is purely imaginary b) $\frac{z_1}{z_2}$ is purely real c) $|\arg z_1 - \arg z_2| = \pi$ d) $|\arg z_1 - \arg z_2| = \frac{\pi}{2}$

284. If $|z - (1/z)| = 1$ then
a) $|z|_{\max} = \frac{1 + \sqrt{5}}{2}$ b) $|z|_{\min} = \frac{\sqrt{5} - 1}{2}$ c) $|z|_{\max} = \frac{\sqrt{5} - 2}{2}$ d) $|z|_{\min} = \frac{\sqrt{5} - 1}{\sqrt{2}}$

285. A quadratic equation whose difference of roots is 3 and the sum of the squares of the roots is 29, is given by
a) $x^2 + 9x + 14 = 0$ b) $x^2 + 7x + 10 = 0$ c) $x^2 - 7x - 10 = 0$ d) $x^2 - 7x + 10 = 0$

286. If α and β are the roots of $ax^2 + bx + c = 0$ and $\alpha + h, \beta + h$ are the roots of $px^2 + qx + r = 0$, then
a) $h = \frac{1}{2}\left(\frac{b}{a}, \frac{q}{p}\right)$
b) $\frac{b^2 - 4ac}{a^2} = \frac{q^2 - 4pr}{p^2}$
c) $\frac{a}{p} = \frac{b}{q} = \frac{c}{r}$
d) None of these

287. If α is one root of the equation $4x^2 + 2x - 1 = 0$, then its other root is given by
a) $4\alpha^3 - 3\alpha$ b) $4\alpha^3 + 3\alpha$ c) $\alpha - \frac{1}{2}$ d) $-\alpha - \frac{1}{2}$

288. Locus of complex number satisfying $\arg[(z - 5 + 4i)/(z + 3 - 2i)] = -\pi/4$ is the arc of a circle
a) Whose radius is $5\sqrt{2}$
b) Whose radius is 5
c) Whose length (of arc) is $\frac{15\pi}{\sqrt{2}}$
d) Whose centre is $-2 - 5i$

289. If the roots of the equation, $x^3 + px^2 + qx - 1 = 0$ form an increasing G.P., where p and q are real, then
a) $p + q = 0$
b) $p \in (-3, \infty)$
c) One of the root is unity
d) One root is smaller than 1 and one root is greater than 1

290. If $|(z - z_1)/(z - z_2)| = 3$, where z_1 and z_2 are fixed complex numbers and z is a variable complex number, then 'z' lies on a

a) Circle with 'z_1' as its interior point
b) Circle with 'z_2' as its interior point
c) Circle with 'z_1' as its exterior point
d) Circle with 'z_2' as its exterior point

291. The value of x satisfying the equation $2^{2x} - 8 \times 2^x = -12$ is
a) $1 + \frac{\log 3}{\log 2}$
b) $\frac{1}{2}\log 6$
c) $1 + \log\frac{3}{2}$
d) 1

292. The equation $x^2 + a^2x + b^2 = 0$ has two roots each of which exceeds a number c, then
a) $a^4 > 4b^2$
b) $c^2 + a^2c + b^2 > 0$
c) $-\frac{a^2}{2} > c$
d) None of these

293. If the equations $x^2 + bx - a = 0$ and $x^2 - ax + b = 0$ have a common root, then
a) $a + b = 0$
b) $a = b$
c) $a - b = 1$
d) $a + b = 1$

294. If $\cos^4\theta + a, \sin^4\theta + \alpha$ are the roots of the equation $x^2 + 2bx + b = 0$ and $\cos^2\theta + \beta, \sin^2\theta + \beta, \sin^2\theta + \beta$ are the roots of the equation $x2+4x+2=0$, then values of b are
a) 2
b) -1
c) -2
d) 1

295. If a, b, c are in G.P. then the roots of the equation $ax^2 + bx + c = 0$ are in the ratio
a) $\frac{1}{2}(-1 + i\sqrt{3})$
b) $\frac{1}{2}(1 - i\sqrt{3})$
c) $\frac{1}{2}(-1 - i\sqrt{3})$
d) $\frac{1}{2}(1 + i\sqrt{3})$

296. If $x^3 + 3x^2 - 9x + c$ is of the form $(x - \alpha)^2(x - \beta)$, then c is equal to
a) 27
b) -27
c) 5
d) -5

297. If $p, q, r \in R$ and the quadratic equation $px^2 + qx + r = 0$ has no real root, then
a) $p(p + q + r) > 0$
b) $r(p + q + r) > 0$
c) $q(p + q + r) > 0$
d) $(p + q + r) > 0$

298. If the equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root, then it must be equal to
a) $\frac{pq' - p'q}{q - q'}$
b) $\frac{q - q'}{p' - p}$
c) $\frac{p' - p}{q - q'}$
d) $\frac{pq' - p'q}{p - p'}$

299. A complex number z is rotated in anticlockwise direction by an angle α and we get z' and if the same complex number z is rotated by an angle α in clockwise direction and we get z'' then
a) z', z, z'' are in G.P.
b) z', z, z'' are in H.P.
c) $z' + z'' = 2z\cos\alpha$
d) $z'^2 + z''^2 = 2z^2\cos 2\alpha$

300. Given that the two curves $\arg(z) = \pi/6$ and $|z - 2\sqrt{3}i| = r$ intersect in two distinct points, then
a) $[r] \neq 2$
b) $0 < r < 3$
c) $r = 6$
d) $3 < r < 2\sqrt{3}$

301. If $c \neq 0$ and the equation $\frac{p}{2x} = \frac{a}{x+c} + \frac{b}{x-c}$ has two equal roots, then p can be
a) $(\sqrt{a} - \sqrt{b})^2$
b) $(\sqrt{a} + \sqrt{b})^2$
c) $a + b$
d) $a - b$

302. If $c \neq 0$ and the equation $p/(2x) = a/(x + c) + b/(x - c)$ has two equal roots, then p can be
a) $(\sqrt{a} - \sqrt{b})^2$
b) $(\sqrt{a} + \sqrt{b})^2$
c) $a + b$
d) $a - b$

303. The equation $x^{\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}} = \sqrt{2}$ has
a) At least one real solution
b) Exactly three solutions
c) Exactly one irrational solution
d) Complex roots

304. If P and Q are represented by the complex number z_1 and z_2, such that $|1/z_2 + 1/z_1| = |1/z_2 - 1/z_1|$, then
a) ΔOPQ (where O is the origin) is equilateral
b) ΔOPQ is right angled
c) The circumcentre of ΔOPQ is $\frac{1}{2}(z_1 + z_2)$
d) The circumcentre of ΔOPQ is $\frac{1}{3}(z_1 + z_2)$

305. If $(x^2 + ax + 3)/(x^2 + x + a)$ takes all real values for possible real values of x, then
a) $4a^3 + 39 < 0$
b) $4a^3 + 39 \geq 0$
c) $a \geq \frac{1}{4}$
d) $a < \frac{1}{4}$

306. Given than α, γ are roots of the equation $Ax^2 - 4x + 1 = 0$, and β, δ the roots of the equation of $Bx^2 - 6x + 1 = 0$, such that α, β, γ and δ are in H.P., then
a) $A = 3$
b) $A = 4$
c) $B = 2$
d) $B = 8$

307. Let $P(x) = x^2 + bx + c$, where b and c are integer. If $P(x)$ is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$, then
a) $P(x) = 0$ has imaginary roots
b) $P(x) = 0$ has roots of opposite sign
c) $P(1) = 4$
d) $P(1) = 6$

308. If n is natural number ≥ 2, such that $z^n = (z+1)^n$, then
a) Roots of equation lie on a straight line parallel to y-axis
b) Roots of equation lie on a straight line parallel to x-axis
c) Sum of the real parts of the roots is $-[(n-1)/2]$
d) None of these
309. If $z^3 + (3+2i)z + (-1+ia) = 0$ has one real root, then the value of 'a' lies in interval $(a \in R)$
a) $(-2, 1)$ b) $(-1, 0)$ c) $(0, 1)$ d) $(-2, 3)$
310. For $a > 0$, the roots of the equation $\log_{ax} a + \log_x a^2 + \log_{a^2x} a^3 = 0$ are given by
a) $a^{-4/3}$ b) $a^{-3/4}$ c) $a^{-1/2}$ d) a^{-1}
311. If $z_1 = a + ib$ and $z_2 = c + id$ are complex numbers such that $|z_1| = |z_2| = 1$ and $\text{Re}(z_1\bar{z}_2) = 0$, then the pair of complex numbers $\omega_1 = a + ic$ and $\omega_2 = b + id$ satisfies
a) $|\omega_1| = 1$ b) $|\omega_2| = 1$ c) $\text{Re}(\omega_1\bar{\omega}_2) = 0$ d) $\omega_1\bar{\omega}_2 = 0$
312. If $z = x + iy$, then the equation $|(2z - i)/(z+1)| = m$ represnts a circle then m can be
a) $1/2$ b) 1 c) 2 d) $3 < r < 2\sqrt{2}$
313. If $1, z_1, z_2, z_3, \dots, z_{n-1}$ be the n^{th} roots unity and ω be a non-real complex cube root of unity, then the product $\prod_{r=1}^{n-1}(\omega - z_r)$ can be equal to
a) 0 b) 1 c) -1 d) $1 + \omega$
314. If $z = \omega, \omega^2$, where ω is a non-real complex cube root of unity, are two vertices of an equilateral triangle in the Argand plane then the third vertex may be represented by
a) $z = 1$ b) $z = 0$ c) $z = -2$ d) $z = -1$
315. If $(\sin\alpha)x^2 - 2x + b \geq 2$, for all real values of $x \leq 1$ and $\alpha \in (0, \pi/2) \cup (\pi/2, \pi)$, then possible real values of b is/are
a) 2 b) 3 c) 4 d) 5
316. If the roots of the equation $x^2 + ax + b = 0$ are c and d, then roots of the equation $x^2 + (2c + a)x + c^2 + ac + b = 0$ are
a) c b) $d - c$ c) $2c$ d) 0
317. Let $P(x)$ and $Q(x)$ be two polynomials. Suppose that $f(x) = P(x^3) + xQ(x^3)$ is divisible by $x^2 + x + 1$, then
a) $P(x)$ is divisible by $(x-1)$ but $Q(x)$ is not divisible by $x - 1$
b) $Q(x)$ is divisible by $(x-1)$ but $P(x)$ is not divisible by $x - 1$
c) Both $P(x)$ and $Q(x)$ are divisible by $x - 1$
d) $f(x)$ is divisible by $x - 1$
318. If the equation $ax^2 + bx + c = 0$ $(a > 0)$ has two real roots α and β such that $\alpha < -2$ and $\beta > 2$, then which of the following statements is/are true?
a) $a - |b| + c < 0$ b) $c < 0, b^2 - 4ac > 0$ c) $4a - 2|b| + c < 0$ d) $9a - 3|b| + c < 0$
319. For real x, then function $(x-a)(x-b)/(x-c)$ will assume all real values provided
a) $a > b > c$ b) $a < b < c$ c) $a > c > b$ d) $a < c < b$
320. The real value of θ for which the expression $\frac{1+i\cos\theta}{1-2i\cos\theta}$ is a real number is
a) $2n\pi + \frac{\pi}{2}, n \in I$ b) $2n\pi - \frac{\pi}{2}, n \in I$ c) $2n\pi \pm \frac{\pi}{2}, n \in I$ d) $2n\pi \pm \frac{\pi}{4}, n \in I$
321. If the equation whose roots are the squares of the roots of the cubic $x^3 - ax^2 + bx - 1 = 0$ is identical with the given cubic equation, then
a) $a = 0, b = 3$ b) $a = b = 0$
c) $a = b = 3$ d) a, b are roots of $x^2 + x + 2 = 0$
322. If $|z - 3| = \min\{|z - 1|, |z - 5|\}$, then $\text{Re}(z)$ equals to
a) 2 b) $\frac{5}{2}$ c) $\frac{7}{2}$ d) 4
323. The graph of the quadratic trinomial $y = ax^2 + bx + c$ has its vertex at $(4, -5)$ and two x-intercepts one positive and one negative. Which of the following holds good?
a) $a > 0$ b) $b < 0$ c) $c < 0$ d) $8a = b$

324. If $|z_1| = |z_2| = 1$ and amp z_1 + amp $z_2 = 0$, then

a) $z_1 z_2 = 1$ b) $z_1 + z_2 = 0$ c) $z_1 = \bar{z}_2$ d) None of these

325. If the equation, $z^3 + (3+i)z^2 - 3z - (m+i) = 0$, where $m \in R$, has at least one real root, then m can have the value equal to

a) 1 b) 2 c) 3 d) 5

Assertion - Reasoning Type

This section contain(s) 0 questions numbered 326 to 325. Each question contains STATEMENT 1(Assertion) and STATEMENT 2(Reason). Each question has the 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

a) Statement 1 is True, Statement 2 is True; Statement 2 **is** correct explanation for Statement 1

b) Statement 1 is True, Statement 2 is True; Statement 2 **is not** correct explanation for Statement 1

c) Statement 1 is True, Statement 2 is False

d) Statement 1 is False, Statement 2 is True

326

Statement 1: The greatest integral value of λ for which $(2\lambda - 1)x^2 - 4x + (2\lambda - 1) = 0$ has real roots, is 2.

Statement 2: For real roots of $ax^2 + bx + c = 0, D \geq 0$.

327

Statement 1: If roots of the equation $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4c = 1$

Statement 2: If a, b, c are odd integer, then the roots of the equation $4\,abc\,x^2 + (b^2 - 4ac)x - b = 0$ are real and distinct

328 Let $ax^2 + bx + c = 0, a \neq 0\ (a, b, c \in R)$ has no real roots and $a + b + 2c = 2$

Statement 1: $ax^2 + bx + c > 0, \forall\, x \in R$

Statement 2: $a + b$ is positive

329

Statement 1: If $\cos(1-i) = a + ib$, where $a, b \in R$ and $i = \sqrt{-1}$, then $a = \frac{1}{2}\left(e + \frac{1}{e}\right)\cos 1, b =$ *12e−1e*sin *1*

Statement 2: $e^{i\theta} = \cos\theta + i\sin\theta$

330 Let fourth roots of unity z_1, z_2, z_3 and z_4 respectively

Statement 1: $z_1^2 + z_2^2 + z_3^2 + z_4^2 = 0$

Statement 2: $z_1 + z_2 + z_3 + z_4 = 0$

331

Statement 1: The equation $(x-p)(x-r) + \lambda(x-q)(x-s) = 0$, where $p < q < r < s$, has non-real roots

Statement 2: The equation $px^2 + qx + r = 0\ (p, q, r \in R)$ has non-real roots if $q^2 - 4pr < 0$

332

Statement 1: If both roots of the equation $2x^2 - x + a = 0 (a \in R)$ lies in $(1, 2)$, then$-1 < a \le 1/8$.

Statement 2: If $F(x) = 2x^2 - x + a$, then $D \ge 0, f(1) > 0, f(2) > 0$ yield $-1 < a \le 1/8$.

333

Statement 1: The number of values of a for which $(a^2 - 3a + 2)x^2 + (a^2 - 5a + 6)x + a^2 - 4 = 0$ is an identity in x is 2

Statement 2: If $a = b = c = 0$, then equation $ax^2 + bx + c = 0$ is an identity in x

334

Statement 1: If equations $ax^2 + bx + c = 0, (a, b, c \in R)$ and $2x^2 + 3x + 4 = 0$ have a common root, then $a : b : c = 2 : 3 : 4$.

Statement 2: Roots of $2x^2 + 3x + 4 = 0$ are imaginary

335

Statement 1: If z_1 and z_2 are two complex numbers such that $|z_1| = |z_2| + |z_1 - z_2|$, then $\text{Im}\left(\frac{z_1}{z_2}\right) = 0$

Statement 2: $\arg(z) = 0 \Rightarrow z$ is purely real

336

Statement 1: If $a, b, c \in Z$ and $ax^2 + bx + c = 0$ has an irrational root, then $|f(\lambda)| \ge 1/q^2$, where $\lambda \in (\lambda = \frac{p}{q}; p, q \in Z)$ and $f(x) = ax^2 + bx + c$

Statement 2: If $a, b, c \in Q$ and $b^2 - 4ac$ is positive but not a perfect square, then roots of equation $ax^2 + bx + c = 0$ are irrational and always occur in conjugate pair like $2 + \sqrt{3}$ and $2 - \sqrt{3}$

337

Statement 1: If equations $ax^2 + bx + c = 0$ and $x^2 - 3x + 4 = 0$ have exactly one root common, then at least one of a, b, c is imaginary

Statement 2: If a, b, c are not all real, then equation $ax^2 + bx + c = 0$ can have one root real and one root imaginary

338

Statement 1: Locus of z, satisfying the equation $|z - 1| + |z - 8| = 5$ is an ellipse

Statement 2: Sum of focal distances of any point on ellipse is constant

339

Statement 1: If $\cos^2 \pi/8$ is a root of the equation $x^2 + ax + b = 0$ where $a, b \in Q$, then ordered pair (a, b) is $[-1, (1/8)]$

Statement 2: If $a + mb = 0$ and m is irrational, then $a, b = 0$

340 Consider the function $f(x) = \log_e(ax^3 + (a + b)x^2 + (b + c)x + c)$

Statement 1: Domain of the functions is $(-1, \infty) \sim \{-(b/2a)\}$, where $a > 0, b^2 - 4ac = 0$

Statement 2: $ax^2 + bx + c = 0$ has equal roots when $b^2 - 4ac = 0$

341 If $z_1 \ne -z_2$ and $|z_1 + z_2| = |(1/z_1) + (1/z_2)|$ then

Statement 1: $z_1 z_2$ is unimodular

Statement 2: z_1 and z_2 both are unimodular

342

Statement 1: If $z_1 + z_2 = a$ and $z_1 z_2 = b$, where $a = \overline{a}$ and $b = \overline{b}$, then $\arg(z_1 z_2) = 0$

Statement 2: The sum and product of two complex numbers are real if and only if they are conjugate of each other

343

Statement 1: If all real values of x obtained from the equation $4^x - (a-3)2^x + (a-4) = 0$ are non-positive, then $a \in (4, 5]$

Statement 2: If $ax^2 + bx + c$ is non-positive for all real values of x, then $b^2 - 4ac$ must be negative or zero and 'a' must be negative

344

Statement 1: If $px^2 + qx + r = 0$ is a quadratic equation $(p, q, r \in R)$ such that its roots are α, β and $p + q + r < 0, p - q + r < 0$ and $r > 0$, then $[\alpha] + [\beta] = -1$, where $[\cdot]$ denotes greatest integer function

Statement 2: If for any two real numbers a and b, function $f(x)$ is such that $f(a)f(b) < 0 \Rightarrow f(x)$ has at least one real root lying in (a, b)

345

Statement 1: If $a > 0$ and $b^2 - ac < 0$, then domain of the function $f(x) = \sqrt{ax^2 + 2bx + c}$ is R

Statement 2: If $b^2 - ac < 0$, then $ax^2 + 2bx + c = 0$ has imaginary roots

346

Statement 1: If $|z_1| = 1, |z_2| = 2, \ |z_3| = 3$ and $|z_1 + 2z_2 + 3z_3| = 6$, then the value of $|z_2 z_3 + 8z_3 z_1 + 27z_1 z_2|$ is 36

Statement 2: $|\ z_1 + z_2 + z_3| \leq |z_1| + |z_2| + |z_3|$

347

Statement 1: If $f(x)$ is a quadratic polynomial satisfying $f(2) + f(4) = 0$. If unity is a root of $f(x) = 0$, then the other root is 3.5

Statement 2: If $\mathrm{g}(x) = px^2 + qx + r = 0$ has roots α, β, then $\alpha + \beta = -q/p$ and $\alpha\beta = (r/p)$

348

Statement 1: If both roots of the equation $4x^2 - 2x + a = 0, a \in R$ lie in the interval $(-1,1)$, then$-2 < a \leq \frac{1}{4}$.

Statement 2: If $f(x) = 4x^2 - 2x + a$, then $D \geq 0, f(-1) > 0$ and $f(1) > 0 \Rightarrow -2 < a \leq \frac{1}{4}$.

349

Statement 1: If $a^2 + b^2 + c^2 < 0$, then if roots of the equation $ax^2 + bx + c = 0$ are imaginary, then they are not complex conjugates

Statement 2: equation $ax^2 + bx + c = 0$ has complex conjugate roots when a, b, c are real

350

Statement 1: The equation $x^2 + (2m+1)x + (2n+1) = 0$, where m and n are integer cannot have any rational roots

Statement 2: The quantity $(2m+1)^2 - 4(2n+1)$, where $m, n \in I$ can never be a perfect square

351

Statement 1: If n is an odd integer greater than 3 but not a multiple of 3, then $(x+1)^n - x^n - 1$ is divisible by $x^3 + x^2 + x$

Statement 2: If n is an odd integer greater than 3 but not a multiple of 3, we have $1 + \omega^n + \omega^{2n} = 3$

352

Statement 1: If $x + (1/x) = 1$ and $p = x^{4000} + (1/x^{4000})$ and q be the digit at unit place in the number $2^{2^n} + 1, n \in N$ and $n > 1$, then the value of $p + q = 8$

Statement 2: If ω, ω^2 are the roots of $x + 1/x = -1$, then $x^2 + 1/x^2 = -1, x^3 + (1/x^3) = 2$

353

Statement 1: If $0 < \alpha < (\pi/4)$, then the equation $(x - \sin\alpha) \times (x - \cos\alpha) - 2 = 0$ has both roots in $(\sin\alpha, \cos\alpha)$

Statement 2: If $f(a)$ and $f(b)$ possess opposite signs, then there exist at least one solution of the equation $f(x) = 0$ in open interval (a, b)

354

Statement 1: Let z_1 and z_2 are two complex numbers such that $|z_1 - z_2| = |z_1 + z_2|$ then the orthocentre of ΔAOB is $[(z_1 + z_2)/2]$ (where O is origin)

Statement 2: In case of right angled triangle, orthocentre is that point at which the triangle is right angled

355

Statement 1: If $\left|\frac{z\,z_1 - z_2}{z\,z_1 + z_2}\right| = k, (z_1, z_2 \neq 0)$, then the locus of z is circle

Statement 2: As $\left|\frac{z - z_1}{z - z_2}\right| = \lambda$ represents a circle, if $\lambda \notin \{0, 1\}$

356

Statement 1: If z_1, z_2 are the roots of the quadratic equation $az^2 + bz + c = 0$ such that $\text{Im}\,(z_1 z_2) \neq 0$, then at least one of a, b, c is imaginary

Statement 2: If quadratic equation having real coefficients has complex roots, then roots are always conjugate to each other

357

Statement 1: If the equation $ax^2 + bx + c = 0, 0 < a < b < c$, has non-real complex roots z_1 and z_2, then $|z_1| > 1, |z_2| > 1$

Statement 2: Complex roots always occur in conjugate pairs

358

Statement 1: equation $ix^2 + (i-1)x - (1/2) - i = 0$ has imaginary roots

Statement 2: If $a = i, b = i - 1$ and $c = -(1/2) - i$, then $b^2 - 4ac < 0$

359

Statement 1: The question $-x^2 + x - 1 = \sin^4 x$ has only one solution.

Statement 2: If the curve $y = f(x)$and $y = g(x)$ cut at one point, the number of solution is 1.

360

Statement 1: If a, b, c, a_1, b_1, c_1 are rational and equations $ax^2 + 2bx + c = 0$ and $a_1x^2 + 2b_1x + c_1 = 0$ have one and only one root in common, then both $b^2 - ac$ and $b_1^2 - a_1c_1$ must be perfect squares

Statement 2: If two quadratic equations with rational coefficient have a common irrational root $p + \sqrt{q}$, then both roots will be common

361

Statement 1: If $a + b + c = 0$ and a, b, c are rational, then the roots of the equation $(b + c - a)x^2 + (c + a - b)x + (a + b - c) = 0$ are rational.

Statement 2: Discriminant of equation $(b + c - a)x^2 + (c + a - b)x + (a + b - c) = 0$ is a perfect square.

362 Let a, b, c, p, q be real numbers. Suppose α, β are the roots of the equation $x^2 + 2px + q = 0$ and $\alpha, \frac{1}{\beta}$ are the roots of the equation $x^2 + 2bx + c = 0$, where $\beta^2 \notin (-1, 0, 1)$

Statement 1: $(p^2 - q)(b^2 - ac) \geq 0$

Statement 2: $b \neq pa$ or $c \neq qa$

363

Statement 1: If $(a^2 - 4)x^2 + (a^2 - 3a + 2)x + (a^2 - 7a + 10) = 0$ is an identity, then the value of a is 2

Statement 2: If $a - b = 0$, then $ax^2 + bx + c = 0$ is an identity

364

Statement 1: If $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$, then $\frac{z_1}{z_2}$ is purely imaginary

Statement 2: If z is purely imaginary, then $z + \bar{z} = 0$

365 Let $f(x) = -x^2 + (a + 1)x + 5$

Statement 1: $f(x)$ is positive for some $\alpha < x < \beta$ and for all $a \in R$

Statement 2: $f(x)$ is positive for all $x \in R$ and for some real a

366

Statement 1: $|z_1 - a| < a, |z_2 - b| < b, |z_3 - c| < c$, where a, b, c are positive real numbers, then $|z_1 + z_2 + z_3|$ is greater than $2|a + b + c|$

Statement 2: $|z_1 \pm z_2| \leq |z_1| + |z_2|$

367

Statement 1: If $|z_1| = |z_2| = |z_3|, z_1 + z_2 + z_3 = 0$ and $(z_1), B(z_2), C(z_3)$ are the vertices of ΔABC, then one of the values of $\arg(z_2 + z_3 - 2z_1)/(z_3 - z_2)$ is $\pi/2$

Statement 2: In equilateral triangle orthocentre coincides with centroid

368 Let a, b, c be real such that $ax^2 + bx + c = 0$ and $x^2 + x + 1 = 0$ have a common root

Statement 1: $a = b = c$

Statement 2: Two quadratic equations with real coefficients cannot have only one imaginary root common

369

Statement 1: If the roots of $x^5 - 40x^4 + Px^3 + Qx^2 + Rx + S = 0$ are in G.P. and sum of their reciprocal is 10, then $|S| = 64$

Statement 2: $x_1\, x_2\, x_3\, x_4\, x_5 = -S$, where x_1, x_2, x_3, x_4, x_5 are the roots of given equation

370

Statement 1: The product of all values of $(\cos\alpha + i\sin\alpha)^{3/5}$ is $\cos 3\alpha + i\sin 3\alpha$

Statement 2: The product of fifth roots of unity is 1

371

Statement 1: If $\arg(z_1 z_2) = 2\pi$, then both z_1 and z_2 are purely real (z_1 and z_2 have principle arguments)

Statement 2: Principle argument of complex number lies in $(-\pi, \pi)$

372

Statement 1: Let $f(x)$ be quadratic expression such that $f(0) + f(1) = 0$. If -2 is one of the root of $f(x) = 0$, then other root is $3/5$.

Statement 2: If α, β are the zero's of $f(x) = ax^2 + bx + c$, then sum of zero's $= -b/a$, product of zero's $= c/a$.

373

Statement 1: Let z be a complex number, then the equation $z^4 + z + 2 = 0$ cannot have a root, such that $|z| < 1$

Statement 2: $|z_1 + z_2| \le |z_1| + |z_2|$

374 Consider a general expression of degree 2 in two variables as $f(x, y) = 5x^2 + 2y^2 - 2xy - 6x - 6y + 9$

Statement 1: $f(x, y)$ can be resolved into two linear factors over real coefficients

Statement 2: If we compare $f(x, y)$ with $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$, we have $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

Matrix-Match Type

This section contain(s) 0 question(s). Each question contains Statements given in 2 columns which have to be matched. Statements (A, B, C, D) in **columns I** have to be matched with Statements (p, q, r, s) in **columns II**.

375.

	Column-I		Column- II
(A)	If a, b, c and d are four zero real number such that $(d + a - b)^2 + (d + b - c)^2 = 0$ and the roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are real and equal then	(p)	$a + b + c = 0$
(B)	If the roots of the equation $(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2 = 0)$ are real and equal	(q)	a, b, c are in A.P

then

(C)	If the equation $ax^2+bx+c=0$ and $x^3-3x^2+3x-1=0$ have a common real root, then	(r)	a,b,c are in G.P.
(D)	Let a,b,c be positive real numbers such that the expression $bx^2+\left(\sqrt{(a+c)^2+4b^2}\right)x+(a+c)$ is non-negative $\forall\ x \in R$, then	(s)	a,b,c are in H.P.

CODES :

	A	**B**	**C**	**D**
a)	r	p	qr	s
b)	p	q	q,r	s
c)	q,r,s	r	p	q
d)	q,r	s	r	p

376.

	Column-I		**Column- II**
(A)	$\|z-1\|=\|z-i\|$	(p)	Pair of straight line
(B)	$\|z+\bar{z}\|+\|z-\bar{z}\|=2$	(q)	A line through the origin
(C)	$\|z+\bar{z}\|=\|z-\bar{z}\|$	(r)	Circle
(D)	If $\|z\|=1$, then $2/z$ lies on	(s)	Square

CODES :

	A	**B**	**C**	**D**
a)	p	r	q	s
b)	r	q	s	p
c)	q	s	p	r
d)	s	p	r	q

377. If $a=\frac{1-i\sqrt{3}}{2}$, then the correct matching of list I from list II is

	Column-I		**Column- II**
(A)	$a\bar{a}$	(p)	$-\frac{\pi}{3}$
(B)	$\arg\left(\frac{1}{a}\right)$	(q)	$-i\sqrt{3}$
(C)	$a-\bar{a}$	(r)	$2i\sqrt{3}$
(D)	$\mathrm{Im}\left(\frac{4}{3a}\right)$	(s)	1
		(t)	$\frac{\pi}{3}$

(u) $\frac{2}{\sqrt{3}}$

CODES :

	A	**B**	**C**	**D**
a)	d	e	c	b
b)	d	a	b	f
c)	f	e	b	c
d)	d	a	b	c

378.

Column-I	**Column- II**
(A) If $x^2 + ax + b = 0$ has roots α, β and $x^2 + px + q = 0$ has roots $-\alpha, \gamma$, then	(p) $(1 - bq)^2 = (a - pb)(p - aq)$
(B) If $x^2 + ax + b = 0$ has roots α, β and $x^2 + px + q = 0$ has roots $1/\alpha, \gamma$, then	(q) $(4 - bq)^2 = (4a + 2pb)(-2p - aq)$
(C) If $x^2 + ax + b = 0$ has roots α, β and $x^2 + px + q = 0$ has roots $-2/\alpha, \gamma$, then	(r) $(1 - 4bq)^2 = (a + 2bp)(-2p - 4aq)$
(D) If $x^2 + ax + b = 0$ has roots α, β and $x^2 + px + q = 0$ has roots $-1/(2\alpha), \gamma$, then	(s) $(q - b)^2 = (aq + bp)(p - a)$

CODES :

	A	**B**	**C**	**D**
a)	s	p	q	r
b)	q	p	s	r
c)	r	s	p	q
d)	p	r	q	s

379.

Column-I	**Column- II**
(A) If $\|z - 2i\| + \|z - 7i\| = k$, then locus of z is an ellipse if $k =$	(p) 7
(B) If $\|(2z - 3)/(3z - 2)\| = k$, then locus of z is a circle if $2/3$ is a point inside circle and $3/2$ is outside the circle if $k =$	(q) 8
(C) If $\|z - 3\| - \|z - 4i\| = k$, then locus of z is a hyperbola if k is	(r) 2
(D) If $\|z - (3 + 4i) = (k/50)\|a\bar{z} + \bar{a}z + b\|$, where $a = 3 + 4i$, then locus of z is a hyperbola with $k =$	(s) 4
	(t) 5

CODES :

	A	B	C	D
a)	P,q	p,q,r,s,t	r,s	p,q
b)	p,q,r,s,t	r,s	p,q	p,q
c)	r,s	p,q	p,q	p,q,r,s,t
d)	p,q	p,q	p,q,r,s,t	r,s

380.

	Column-I		Column- II
(A)	$y = \frac{x^2-2x+4}{x^2+2x+4}, x \in R$, then y can be	(p)	1
(B)	$y = \frac{x^2-3x-2}{2x-3}, x \in R$, then y can be	(q)	4
(C)	$y = \frac{2x^2-2x+4}{x^2-4x+3}, x \in R$, then y can be	(r)	-3
(D)	$x^2 - (a-3)x + 2 < 0, \forall\, x \in (-2, 3)$, then a can be	(s)	-10

CODES :

	A	B	C	D
a)	p	p,q,r,s	p,q,s	r,s
b)	r,s	p,q	q,s	p
c)	p,q	r,s	q,s	p
d)	q,s	r,s	p,q	r

381.

	Column-I		Column- II
(A)	One root is positive and the other is negative for the equation $(m-2)x^2 - (8-2m)x - (8-3m) = 0$	(p)	0
(B)	Exactly one root of equation $x^2 - m(2x-8) - 15 = 0$ lies in interval $(0, 1)$	(q)	Infinite
(C)	The equation $x^2 + 2(m+1)x + 9m - 5 = 0$ has both roots negative	(r)	1
(D)	The equation $x^2 + 2(m-1)x + m + 5 = 0$ has both roots lying on either sides of 1	(s)	2

CODES :

	A	B	C	D
a)	p	q	s	r
b)	q	s	p	r
c)	s	p	r	q

d)	r	r	q	p

382.

	Column-I		Column- II
(A)	The value of $\sum_{n=1}^{5}(x^n + 1/x^n)^2$ when $x^2 - x + 1 = 0$ is	(p)	2
(B)	If $\left[\frac{1+\cos\theta+i\sin\theta}{\sin\theta+i\,(1+\cos\theta)}\right]^4 = \cos n\theta + i\sin n\theta$, then $n =$	(q)	4
(C)	The adjacent vertices of a regular polygon of n sides having centre at origin are the points z and $\overline{z}$. If $\text{Im}(z)/\text{Re}(z) = \sqrt{2} - 1$, then the value of $n/4$ is	(r)	9
(D)	$(1/50)\{\sum_{r=1}^{10}(r-\omega)(r-\omega^2)\} =$ (where ω is cube root of unity)	(s)	8

CODES :

	A	B	C	D
a)	q	r	p	s
b)	s	q	p	r
c)	r	p	s	q
d)	p	e	q	r

383. Match the statements of Column I with these in Column II.
(***Note*** *: Here z takes values in the complex plane and* Im *(z) and* Re *(z) denote respectively, the imaginary part and the real part of z*)

	Column-I		Column- II
(A)	The set of points z satisfying $\|z - i\|z\|\|=$ *z+iz/* is contained in or equal to	(p)	An ellipse with eccentricity 4/5
(B)	The set of points z satisfying $\|z + 4\| + \|z - 4\| = 0$ is contained in or equal to	(q)	The set of points z satisfying Im $(z) = 0$
(C)	If $\|w\| = 2$, then the set of points $z = w - \frac{1}{w}$ is contained in or equal to	(r)	The set of points z satisfying $\|\text{Im}\, z\| \leq 1$
(D)	If $\|w\| = 1$, then the set of points $z = w + \frac{1}{w}$ is contained in or equal to	(s)	The set of points satisfying $\|\text{Re}\, z\| \leq 2$
		(t)	The set of points z satisfying $\|z\| \leq 3$

CODES :

	A	B	C	D
a)	s	q	q	p
b)	p	s	t, s	q,
c)	q,	p	p, s, t	q, r, s, t

d)	t, s	q	p	s

384. Match the following for the equation $x^2 + a|x| + 1 = 0$, where a is a parameter

Column-I	**Column- II**
(A) No real roots	(p) $a < -2$
(B) Two real roots	(q) ϕ
(C) Three real roots	(r) $a = -2$
(D) Four distinct real roots	(s) $a \geq 0$

CODES :

	A	**B**	**C**	**D**
a)	r	p	q	s
b)	s	r	q	p
c)	p	q	r	s
d)	q	s	p	r

385.

Column-I	**Column- II**
(A) $z^4 - 1 = 0$	(p) $z = \cos\frac{\pi}{8} + i\sin\frac{\pi}{8}$
(B) $z^4 + 1 = 0$	(q) $z = \cos\frac{\pi}{8} - i\sin\frac{\pi}{8}$
(C) $iz^4 + 1 = 0$	(r) $z = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$
(D) $iz^4 - 1 = 0$	(s) $z = \cos 0 + i\sin 0$

CODES :

	A	**B**	**C**	**D**
a)	s	r	p	q
b)	r	p	q	s
c)	p	q	r	s
d)	q	r	s	p

386. Let α, β, γ are the roots of the equation $x^3 - 10x^2 + 7x + 8 = 0$. Match the following and choose the correct answer

Column-I	**Column- II**
(A) $\alpha + \beta + \gamma$	(1) $-\frac{43}{4}$
(B) $\alpha^2 + \beta^2 + \gamma^2$	(2) $-\frac{7}{8}$

(C)	$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$	(3)	86
(D)	$\frac{\alpha}{\beta\gamma}+\frac{\beta}{\gamma\alpha}+\frac{\gamma}{\alpha\beta}$	(4)	0
		(5)	10

CODES :

	A	B	C	D
a)	5	3	1	2
b)	4	3	1	2
c)	5	3	2	1
d)	5	2	3	1

387. Which of the condition/conditions in column II are satisfied by the quadrilateral formed by z_1, z_2, z_3, z_4 in order given in column I?

	Column-I		Column- II
(A)	Parallelogram	(p)	$z_1 - z_4 = z_2 - z_3$
(B)	Rectangle	(q)	$\lvert z_1 - z_3\rvert = \lvert z_2 - z_4\rvert$
(C)	Rhombus	(r)	$\frac{z_1-z_2}{z_3-z_4}$ is purely real
(D)	Square	(s)	$\frac{z_1-z_3}{z_2-z_4}$ is purely imaginary
		(t)	$\frac{z_1-z_2}{z_3-z_2}$ is purely imaginary

CODES :

	A	B	C	D
a)	P,q,r,t	p,r,s	p,q,r,s,t	p,r
b)	p,r,s	p,q,r,s,t	p,r	p,q,r,t
c)	p,q,r,s,t	p,r	p,q,r,t	p,r,s
d)	p,r	p,q,r,t	p,r,s	p,q,r,s,t

388. Let α and β be the roots of the quadratic equation $ax^2 + bx + c = 0$. Observe the lists given below

	Column-I		Column- II
(A)	(i) $\alpha = \beta$	(p)	(A) $(ac^2)^{1/3} + (a^2c)^{1/3} + b = 0$
(B)	(ii) $\alpha = 2\beta$	(q)	(C) $b^2 = 6ac$
(C)	(iii) $\alpha = 3\beta$	(r)	(D) $3b^2 = 16ac$
(D)	(iv) $\alpha = \beta^2$	(s)	(E) $b^2 = 4ac$

(t) (F) $(ac^2)^{1/3} + (a^2c)^{1/3} = b$

CODES :

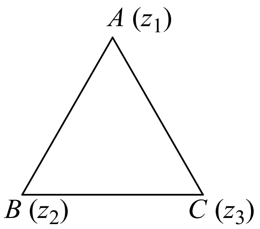
	A	B	C	D
a)	e	b	d	f
b)	e	b	a	d
c)	e	d	b	f
d)	e	b	d	a

Linked Comprehension Type

This section contain(s) 41 paragraph(s) and based upon each paragraph, multiple choice questions have to be answered. Each question has atleast 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 389 to -389

Suppose z_1, z_2 and z_3 represent the vertices A, B and C of an equilateral triangle ABC on the Argand plane. Then, $AB = BC = CA$



$\Rightarrow |z_2 - z_1| = |z_3 - z_2| = |z_1 - z_3|$

Also, $\angle CAB = \frac{\pi}{3} \Rightarrow \arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right) = \pm\frac{\pi}{3}$

$\therefore \ \frac{z_3 - z_1}{z_2 - z_1} = \left|\frac{z_3 - z_1}{z_2 - z_1}\right| \left\{\cos\left(\pm\frac{\pi}{3}\right)\right\} = \frac{1}{2} \pm \frac{\sqrt{3}}{2} i$

$\Rightarrow \ \frac{z_3 - z_1}{z_2 - z_1} - \frac{1}{2} = \pm\frac{\sqrt{3}}{2} i$

$\Rightarrow \ \frac{2z_3 - z_1 - z_2}{2(z_2 - z_1)} = \pm\frac{\sqrt{3}}{2} i$

On squaring, we get

$(2z_3 - z_1 - z_2)^2 = -3(z_2 - z_1)^2$

On the basis of above information, answer the following questions

389. If the complex z_1, z_2, z_3 represent the vertices of an equilateral triangle such that $|z_1| = |z_2| = |z_3|$, then $z_1 + z_2 + z_3$ is equal to

a) 0 b) 3 c) ω d) ω^2

Paragraph for Question Nos. 390 to - 390

Let $z = a + ib = (a, b)$ be any complex number, $\forall a, b \in R$ and $i = \sqrt{-1}$. If $(a, b) \neq (0, 0)$, then $\arg(z) =$ tan−1ba, where arg*z*≤π andarg*z*+arg−*z*=π, ifarg*z*<*0*−π, ifarg*z*>*0*

On the basis of above information, answer the following questions

390. If $\arg(z) > 0$, then $\arg(-z) - \arg(z) = \lambda_1$ and if $\arg(z) < 0$, then $\arg(z) - \arg(-z) = \lambda_2$, then
a) $\lambda_1 + \lambda_2 = 0$ b) $\lambda_1 - \lambda_2 = 0$ c) $3\lambda_1 - 2\lambda_2 = 0$ d) $2\lambda_1 - 3\lambda_2 = 0$

Paragraph for Question Nos. 391 to - 391

The equation $z^n - 1 = 0$ has n roots which are called the nth roots of unity. The nth roots of unity are $1, \alpha, \alpha^2, \dots, \alpha^{n-1}$ which are in GP, where $\alpha = \cos\left(\frac{2\pi}{n}\right) + i\sin\left(\frac{2\pi}{n}\right)$; $i = \sqrt{-1}$, then we have following results

1. $\sum_{r=0}^{n-1} \alpha^r = 0$ or $\sum_{r=0}^{n-1} \cos\left(\frac{2\pi r}{n}\right) = 0$
and $\sum_{r=0}^{n-1} \sin\left(\frac{2\pi r}{n}\right) = 0$
2. $z^n - 1 = \sum_{r=0}^{n-1}(z - \alpha^r)$

On the basis of above information, answer the following questions

391. The value of $\sum_{r=1}^{n-1} \frac{1}{(2-\alpha^r)}$ is equal to
a) $(n-2)\,2^n$ b) $\dfrac{(n-2)2^{n-1}+1}{2^n-1}$ c) $\dfrac{(n-2)2^{n-1}}{2^n-1}$ d) $\dfrac{(n-1)2^{n-1}}{2^n-1}$

Paragraph for Question Nos. 392 to - 392

Directions (Q. No. 34 to 36) Consider the quadratic equation
$(1+m)x^2 - 2(1+3m)x + (1+8m) = 0$,
where $m \in R - \{-1\}$.
On the basis of above information, answer the following questions.

392. The number of integral values of m such that given quadratic equation has imaginary roots, are
a) 0 b) 1 c) 2 d) 3

Paragraph for Question Nos. 393 to - 393

Let the roots of $f(x) = x$ be α and β, where $f(x)$ is a quadratic polynomial $ax^2 + bx + c$, α and β are also the roots of $f\big(f(x)\big) = x$. Let the other two roots of $f(f(x)) = x$ be λ and δ.
On the basis of above information, answer the following questions.

393. The correct statement (s) is/ are
I. if α and β are real and unequal ,then λ and δ are also real.
II. if α and β are imaginary, then λ and δ are also imaginary.
a) I only b) II only c) Both I and II d) Neither I nor II

Paragraph for Question Nos. 394 to - 394

Directions (Q. No. 40 and 41) If $x = 2 + i\sqrt{3}$ is a root of $x^2 + px + q = 0$, where p, q are real, then
On the basis of above information, answer the following questions.

394. The value of p is

a) -3 b) -4 c) 4 d) 3

Paragraph for Question Nos. 395 to - 395

Directions (Q. No.42 and 43) Let $f(x) = x^2 + b_1x + c_1$, $g(x) = x^2 + b_2x + c_2$, real roots of $f(x) = 0$ be α, β and real roots of $g(x) = 0$ be $\alpha + \delta, \beta + \delta$. Also, assume that the least value of $f(x) be -\frac{1}{4}$ and the least value of $g(x)$ occurs at $x = \frac{7}{2}$.
On the basis of above information, answer the following questions.

395. The least value of $g(x)$ is

a) -1 b) $-\dfrac{1}{2}$ c) $-\dfrac{1}{4}$ d) $-\dfrac{1}{3}$

Paragraph for Question Nos. 396 to - 396

Consider the complex numbers z_1 and z_2 satisfying the relation $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$

396. Complex number $z_1\bar{z}_2$ is

a) Purely real b) Purely imaginary c) Zero d) None of these

Paragraph for Question Nos. 397 to - 397

Consider the complex numbers $z = (1 - i\sin\theta)/(1 + i\cos\theta)$

397. The value of θ for which z is purely real are

a) $n\pi - \dfrac{\pi}{4}, n \in I$ b) $n\pi + \dfrac{\pi}{4}, n \in I$ c) $n\pi, n \in I$ d) None of these

Paragraph for Question Nos. 398 to - 398

Consider a quadratic equation $az^2 + bz + c = 0$ where a, b, c are complex numbers

398. The condition that the equation has one purely imaginary root is

a) $(c\bar{a} - a\bar{c})^2 = -(b\bar{c} + c\bar{b})(a\bar{b} + \bar{a}b)$ b) $(c\bar{a} + a\bar{c})^2 = (b\bar{c} + c\bar{b})(a\bar{b} + \bar{a}b)$

c) $(c\bar{a} - a\bar{c})^2 = (b\bar{c} - c\bar{b})(a\bar{b} - \bar{a}b)$ d) None of these

Paragraph for Question Nos. 399 to - 399

Consider the equation $az + b\bar{z} + c = 0$, where $a, b, c \in Z$

399. If $|a| \neq |b|$, then z represents

a) Circle b) Straight line c) One point d) Ellipse

Paragraph for Question Nos. 400 to - 400

Let z be a complex number satisfying $z^2 + 2z\lambda + 1 = 0$, where λ is a parameter which can take any real value

400. The roots of this equation lie on a certain circle if

a) $-1 < \lambda < 1$ b) $\lambda > 1$ c) $\lambda < 1$ d) None of these

Paragraph for Question Nos. 401 to - 401

Consider the equation $az^2 + z + 1 = 0$ having purely imaginary root where $a = \cos\theta + i\sin\theta$, $i = \sqrt{-1}$ and function $f(x) = x^3 - 3x^2 + 3(1 + \cos\theta)x + 5$, then answer the following questions

401. Which of the following is true about $f(x)$?

a) $f(x)$ decreases for $x \in [2n\pi, (2n+1)\pi], n \in Z$

b) $f(x)$ decreases for $x \in \left[(2n-1)\frac{\pi}{2}, (2n+1)\frac{\pi}{2}\right], n \in Z$

c) $f(x)$ is non-monotonic function

d) $f(x)$ increases for $x \in R$

Paragraph for Question Nos. 402 to - 402

Complex numbers z satisfy the equation $|z - (4/z)| = 2$

402. The difference between the least and the greatest moduli of complex numbers is

a) 2 b) 4 c) 1 d) 3

Paragraph for Question Nos. 403 to - 403

Consider ΔABC in Argand plane. Let $A(0)$, $B(1)$ and $C(1+i)$ be its vertices and M be the mid-point of CA. Let z be a variable complex number on the line BM. Let u be another variable complex number defined as $u = z^2 + 1$

403. Locus of u is

a) Parabola b) Ellipse c) Hyperbola d) None of these

Paragraph for Question Nos. 404 to - 404

In an Argand plane z_1, z_2 and z_3 are respectively, the vertices of an isosceles triangle ABC with $Ac = BC$ and $\angle CAB = \theta$. If z_4 is the centre of triangle, then

404. The value of $AB \times AC/(IA)^2$ is

a) $\frac{(z_3 - z_1)(z_3 - z_1)}{(z_4 - z_1)^2}$ b) $\frac{(z_2 - z_1)(z_1 - z_3)}{(z_4 - z_1)^2}$ c) $\frac{(z_4 - z_1)}{(z_2 - z_1)(z_3 - z_1)}$ d) None of these

Paragraph for Question Nos. 405 to - 405

$A(z_1), B(z_2), C(z_3)$ are the vertices of a triangle ABC inscribed in the circle $|z| = 2$. Internal angle bisector of the angle A meets the circumcircle again at $D(z_4)$

405. Complex number representing point D is

a) $z_4 = \frac{1}{z_2} + \frac{1}{z_3}$ b) $\sqrt{\frac{z_2 + z_3}{z_1}}$ c) $\sqrt{\frac{z_2 z_3}{z_1}}$ d) $z_4 = \sqrt{z_2 z_3}$

Paragraph for Question Nos. 406 to - 406

Consider an unknown polynomial which when divided by $(x - 3)$ and by $(x - 4)$ leaves remainders as 2 and 1, respectively. Let $R(x)$ be the remainder when this polynomial is divided by $(x - 3)(x - 4)$

406. If equation $R(x) = x^2 + ax + 1$ has two distinct real root then exhaustive values of a are

a) $(-2, 2)$ b) $(-\infty, -2) \cup (2, \infty)$ c) $(-2, \infty)$ d) All real numbers

Paragraph for Question Nos. 407 to - 407

Consider the quadratic equation $ax^2 - bx + c = 0, a, b, c \in N$, which has two distinct real root belonging to the interval $(1, 2)$

407. The least value of a is

a) 4 b) 6 c) 7 d) 5

Paragraph for Question Nos. 408 to - 408

Consider the equation $x^4 + 2ax^3 + x^2 + 2ax + 1 = 0$, where $a \in R$. Also range of function $f(x) = x + 1/x$ is $(-\infty, -2] \cup [2, \infty)$

408. If equation has at least two distinct positive real roots then all possible values of a are

a) $(-\infty, -1/4)$ b) $(5/4, \infty)$ c) $(-\infty, -3/4)$ d) None of these

Paragraph for Question Nos. 409 to - 409

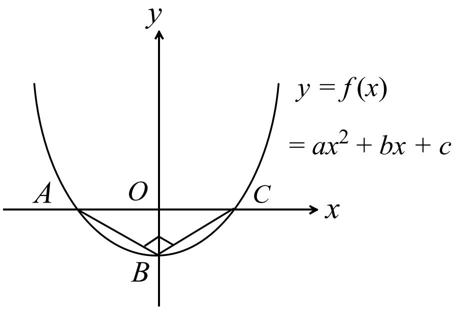
Let $f(x) = x^2 + b_1 x + c_1, \mathrm{g}(x) = x^2 + b_2 x + c_2$. Let the real roots of $f(x) = 0$ be α, β and real roots of $\mathrm{g}(x) = 0$ be $\alpha + h, \beta + h$. The least value of $f(x)$ is $-1/4$. The least value of $\mathrm{g}(x)$ occurs at $x = 7/2$

409. The least value of $\mathrm{g}(x)$ is

a) $-\frac{1}{4}$ b) -1 c) $-\frac{1}{3}$ d) $-\frac{1}{2}$

Paragraph for Question Nos. 410 to - 410

In the given figure, vertices of ΔABC lie on $y = f(x) = ax^2 + bx + c$. The ΔABC is right angled isosceles triangle whose hypotenuse $AC = 4\sqrt{2}$ units



410. $y = f(x)$ is given by

a) $y = x^2 - 2\sqrt{2}$ b) $y = x^2 - 12$ c) $y = \frac{x^2}{2} - 2$ d) $y = \frac{x^2}{2\sqrt{2}} - 2\sqrt{2}$

Paragraph for Question Nos. 411 to - 411

Consider the inequality $9^x - a3^x - a + 3 \leq 0$, where '$a$' is a real parameter

411. The given inequality has at least one negative solution for $a \in$

a) $(-\infty, 2)$ b) $(3, \infty)$ c) $(-2, \infty)$ d) $(2, 3)$

Paragraph for Question Nos. 412 to - 412

Consider the in equation $x^2 + x + a - 9 < 0$

412. The value of the real parameter 'a' so that the given in equation has at least one positive solution:

a) $(-\infty, 37/4)$ b) $(-\infty, \infty)$ c) $(3, \infty)$ d) $(-\infty, 9)$

Paragraph for Question Nos. 413 to - 413

'$af(\mu) < 0$' is the necessary and sufficient condition for a particular real number μ to lie between the roots of a quadratic equation $f(x) = 0$, where $f(x) = ax^2 + bx + c$. Again if $f(\mu_1)f(\mu_2) < 0$, then exactly one of the roots will lie between μ_1 and μ_2

413. If $|b| > |a + c|$, then

a) One root of $f(x) = 0$ is positive, the other is negative

b) Exactly one of the roots of $f(x) = 0$ lies in $(-1, 1)$

c) 1 lies between the roots of $f(x) = 0$

d) Both the roots of $f(x) = 0$ are less than 1

Paragraph for Question Nos. 414 to - 414

The real numbers x_1, x_2, x_3 satisfying the equation $x^3 - x^2 + \beta x + \gamma = 0$ are in A.P.

414. All possible values of β are

a) $\left(-\infty, \frac{1}{3}\right)$ b) $\left(-\infty, -\frac{1}{3}\right)$ c) $\left(\frac{1}{3}, \infty\right)$ d) $\left(-\frac{1}{3}, \infty\right)$

Integer Answer Type

415. Given α and β are the roots of the quadratic equation $x^2-4x+k=0(k\neq 0)$. If $\alpha\beta, \alpha\beta^2+\alpha^2\beta, \alpha^3+\beta^3$ are in geometric progression, then the value of $7k/2$ equals
416. Let $P(x)=x^4+ax^3+bx^2+cx+d$ be a polynomial such that $P(1)=1, P(2)=8, P(3)=27, P(4)=64$, then the value of $P(5)$ is divisible by prime number
417. Let $|z|=2$ and $w=\frac{z+1}{z-1}$ where $z, w\in C$ (where C is the set of complex numbers) Then product of least and greatest value of modulus of w is
418. If the equation $2x^2+4xy+7y^2-12x-2y+t=0$ where 't' is a parameter has exactly one real solution of the form (x, y). Then the sum of $(x+y)$ is equal to
419. If set of values of 'a' for which $f(x)=ax^2-(3+2a)x+6, a\neq 0$ is positive for exactly three distinct negative integral values of x is $(c, d]$, then the value of $(c^2+4|d|)$ is equal to
420. Let $P(x)=\frac{5}{3}-6x-9x^2$ and $Q(y)=-4y^2+4y+\frac{13}{2}$. If there exist unique pair of real numbers (x, y) such that $P(x)Q(y)=20$, then the value of $(6x+10y)$ is
421. Let 'a' is a real number satisfying $a^3+\frac{1}{a^3}=18$. Then the value of $a^4+\frac{1}{a^4}-39$ is
422. If ω is the imaginary cube root of unity, then find the number of pairs of integers (a, b) such that $|a\omega+b|=1$
423. If $\left[\frac{1+\cos\theta+i\sin\theta}{\sin\theta+i(1+\cos\theta)}\right]^4=\cos n\theta+i\sin n\theta$, then n is
424. If the expression $(1+ir)^3$ is of the form of $s(1+i)$ for some real 's' where 'r' is also real and, then the sum of all possible values of r is
425. Given that $x^2-3x+1=0$, then the value of the expression $y=x^9+x^7+x^{-9}+x^{-7}$ is divisible by prime number
426. The minimum value of the expression $E=|z|^2+|z-3|^2+|z-6i|^2$ is m then the value of $m/5$ is
427. a, b, c are reals such that $a+b+c=3$ and $\frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}=\frac{10}{3}$. The value of $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}$ is
428. If a and b are positive numbers and each of the equations $x^2+ax+2b=0$ and $x^2+2bx+a=0$ has real roots, then the smallest possible value of $(a+b)$ is
429. Suppose a, b, c are the roots of the cubic $x^3-x^2-2=0$. Then the value of $a^3+b^3+c^3$ is
430. Let $P(x)=x^3-8x^2+cx-d$ be a polynomial with real coefficients and with all its roots being distinct positive integers. Then number of possible value of 'c' is
431. If complex number $z(z\neq 2)$ satisfies the equation $z^2=4z+|z|^2+\frac{16}{|z|^3}$ then the value of $|z|^4$ is
432. Let $x^2+y^2+xy+1\geq a(x+y)\forall x, y\in R$, then the number of possible integer(s) in the range of a is
433. The quadratic polynomial $p(x)$ has the following properties: $p(x)\geq 0$ for all real numbers, $p(1)=0$ and $p(2)=2$. Find the value of $p(3)$ is
434. Suppose $a, b, c, \in$ I such that greatest common divisor of x^2+ax+b and x^2+bx+c is $(x+1)$ and the least common multiple of x^2+ax+b and x^2+bx+c is (x^3-4x^2+x+6). Then the value of $|a+b+c|$ is equal to
435. If $a, b\in R$ such that $a+b=1$ and $(1-2ab)(a^3+b^3)=12$. The value of (a^2+b^2) is equal to
436. If $x+y+z=12$ and $x^2+y^2+z^2=96$ and $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=36$. Then the value $x^3+y^3+z^3$ is divisible by prime number
437. Suppose that z is a complex number that satisfies $|z-2-2i|\leq 1$. The maximum value of $|2iz+4|$ is equal to
438. If $|z+2-i|=5$ and maximum value of $|3z+9-7i|$ is M then the value of $M/4$ is
439. Let a, b and c be real numbers which satisfy the equations $a+\frac{1}{bc}=\frac{1}{5}, b+\frac{1}{ac}=\frac{-1}{15}$ and $c+\frac{1}{ab}=\frac{1}{3}$. The value of $\frac{c-b}{c-a}$ is equal to
440. If the complex numbers x and y satisfy $x^3-y^3=98i$ and $x-y=7i$ then $xy=a+ib$ where $a, b\in R$. The value of $(a+b)/3$ equals

441. The quadratic equation $x^2 + mx + n = 0$ has roots which are twice those of $x^2 + px + m = 0$ and m, n and $p \neq 0$. Then the value n/p is
442. The complex number z satisfies $z + |z| = 2 + 8i$. The value of $(|z| - 8)$ is
443. Let α_1, β_1 are the roots of $x^2 - 6x + p = 0$ and α_2, β_2 are the roots of $x^2 - 54x + q = 0$. If $\alpha_1, \beta_1, \alpha_2, \beta_2$, from an increasing G.P., then sum of the digits of the value of $(q - p)$ is
444. Number of positive integers x for which $f(x) = x^3 - 8x^2 + 20x - 13$ is a prime number is
445. If $\sqrt{\sqrt{\sqrt{x}}} = \sqrt[4]{\sqrt[4]{\sqrt[4]{3x^4 + 4}}}$, then the value of x^4 is
446. All the values of k for which the quadratic polynomial $f(x) = -2x^2 + kx + k^2 + 5$ has two distinct zeroes and only one of them satisfying $0 < x < 2$, lie in the interval (a, b). The value of $(a + 10b)$ is
447. If z be a complex number satisfying $z^4 + z^3 + 2z^2 + z + 1 = 0$ then $|z|$ is equal to
448. Let $Z_1 = (8 + i)\sin\theta + (7 + 4i)\cos\theta$ and $Z_2 = (1 + 8i)\sin\theta + (4 + 7i)\cos\theta$ are two complex numbers. If $Z_1.Z_2 = a + ib$ where $a, b \in R$. If M is the greatest value of $(a + b)\forall\theta \in R$, then the value of $M^{1/3}$ is
449. If the roots of the cubic, $x^3 + ax^2 + bx + c = 0$ are three consecutive positive integers. Then the value of $\frac{a^2}{b+1}$ is equal to
450. Let α and β be the solutions of the quadratic equation $x^2 - 1154x + 1 = 0$, then the value of $\sqrt[4]{\alpha} + \sqrt[4]{\beta}$ is equal to
451. Let $1, w, w^2$ be the cube root of unity. The least possible degree of a polynomial with real coefficients having roots $2w, (2 + 3w), (2 + 3w^2), (2 - w - w^2)$, is
452. If $x = \omega - \omega^2 - 2$, then the value of $x^4 + 3x^3 + 2x^2 - 11x - 6$ is (where ω is cube root of unity)
453. If $x = a + bi$ is a complex number such that $x^2 = 3 + 4i$ and $x^3 = 2 + 11i$ where $i = \sqrt{-1}$, then $(a + b)$ equal to
454. $f: R \to R, f(x) = \frac{3x^2+mx+n}{x^2+1}$. If the range of this function is $[-4, 3)$, then find the value of $|m + n|$ is
455. The function $f(x) = ax^3 + bx^2 + cx + d$ has three positive roots. If the sum of the roots of $f(x)$ is 4, the largest possible integral values of c/a is
456. If equation $x^4 - (3m + 2)x^2 + m^2 = 0 (m > 0)$ has four real solutions which are in A.P., then the value of 'm' is
457. If the cubic $2x^3 - 9x^2 + 12x + k = 0$ has two equal roots, then maximum value of $|k|$ is
458. Modulus of non zero complex number z, satistying $\overline{z} + z = 0$ and $|z|^2 - 4zi = z^2$ is
459. a, b, and c are all different and non-zero real numbers in arithmetic progression. If the roots of quadratic equation $ax^2 + bx + c = 0$ are α and β such that $\frac{1}{\alpha} + \frac{1}{\beta}, \alpha + \beta$ and $\alpha^2 + \beta^2$ are in geometric progression, then the value of a/c will be
460. Let $A = \{a \in R |$ the equation $(1 + 2i)x^3 - 2(3 + i)x^2 + (5 - 4i)x + 2a^2 = 0\}$ has the at least one real root. Then the value of $\frac{\sum a^2}{2}$ is
461. If $a^2 - 4a + 1 = 4$, then the value of $\frac{a^3-a^2+a-1}{a^2-1} (a^2 \neq 1)$ is equal to
462. Polynomial $P(x)$ contains only terms of odd degree. When $P(x)$ is divided by $(x - 3)$, the remainder is 6. If $P(x)$ is divided by $(x^2 - 9)$, then the remainder is $g(x)$. Then the value of $g(2)$ is
463. Let a, b and c be distinct non zero real numbers such that $\frac{1-a^3}{a} = \frac{1-b^3}{b} = \frac{1-c^3}{c}$. The value of $(a^3 + b^3 + c^3)$, is
464. If the equation $x^2 + 2(\lambda + 1)x + \lambda^2 + \lambda + 7 = 0$ has only negative roots, then the least value of λ equals
465. If a, b, c are non-zero real numbers, then the minimum value of the expression $\left(\frac{(a^4+3a^2+1)(b^4+5b^2+1)(c^4+7c^2+1)}{a^2b^2c^2}\right)$ is not divisible by prime number
466. Let $z = 9 + bi$ where b is non zero real and $i^2 = -1$. If the imaginary part of z^2 and z^3 are equal, then $b/3$ is

: ANSWER KEY :

1)	b	2)	b	3)	a	4)	a	189)	a	190)	d	191)	a	192)	b
5)	a	6)	b	7)	a	8)	b	193)	c	194)	a	195)	a	196)	a
9)	a	10)	a	11)	c	12)	d	197)	a	198)	c	199)	c	200)	a
13)	d	14)	c	15)	a	16)	a	201)	d	202)	c	203)	a	204)	a
17)	c	18)	a	19)	a	20)	d	205)	b	206)	c	207)	a	208)	b
21)	a	22)	b	23)	b	24)	d	209)	c	210)	b	211)	b	212)	b
25)	a	26)	c	27)	c	28)	c	213)	a	214)	b	215)	c	216)	c
29)	a	30)	a	31)	b	32)	c	217)	c	218)	a	219)	c	220)	a
33)	d	34)	a	35)	a	36)	d	221)	d	222)	b	223)	b	224)	b
37)	d	38)	a	39)	c	40)	a	225)	b	226)	a	227)	b	228)	d
41)	d	42)	c	43)	c	44)	c	229)	b	230)	b	231)	b	232)	a
45)	b	46)	b	47)	c	48)	a	233)	c	234)	d	235)	a	236)	b
49)	c	50)	a	51)	c	52)	d	237)	b	238)	d	1)	a,d	2)	
53)	c	54)	c	55)	c	56)	d		a,b,c	3)	a,c	4)	a,b,c		
57)	c	58)	d	59)	c	60)	c	5)	c,d	6)	b,c	7)	a,d	8)	
61)	a	62)	b	63)	c	64)	c		a,b,c						
65)	b	66)	b	67)	d	68)	d	9)	a,b,d	10)	b,c,d	11)	a,c,d	12)	
69)	a	70)	b	71)	c	72)	d		a,d						
73)	d	74)	a	75)	c	76)	c	13)	a,b,d	14)	c,d	15)	a,b,d	16)	
77)	b	78)	b	79)	d	80)	c		a,c						
81)	c	82)	b	83)	a	84)	a	17)	a,c	18)	a,b,c,d	19)	c,d	20)	
85)	c	86)	b	87)	b	88)	b		c,d						
89)	a	90)	a	91)	d	92)	c	21)	a,b	22)	c,d	23)	a,b,c,d	24)	
93)	b	94)	d	95)	b	96)	c		a,b,d						
97)	b	98)	a	99)	d	100)	d	25)	a,b,d	26)	a,b,c	27)	a,d	28)	
101)	a	102)	a	103)	c	104)	c		a,b,c,d						
105)	d	106)	d	107)	d	108)	a	29)	a,b,c,d	30)	a,d	31)	a,b,c	32)	
109)	a	110)	b	111)	d	112)	b		a,d						
113)	d	114)	d	115)	d	116)	d	33)	b,c	34)	b,c	35)	a,d	36)	
117)	b	118)	b	119)	a	120)	c		a,d						
121)	b	122)	d	123)	a	124)	b	37)	a,c	38)	a,c,d	39)	a,c	40)	
125)	b	126)	a	127)	b	128)	a		a,b,c						
129)	b	130)	b	131)	a	132)	c	41)	a,b,c,d	42)	a,c,d	43)	a,c	44)	
133)	c	134)	c	135)	b	136)	c		a,b,c,d						
137)	d	138)	c	139)	d	140)	b	45)	a,d	46)	a,b	47)	b,d	48)	
141)	b	142)	a	143)	b	144)	a		a,b						
145)	d	146)	b	147)	a	148)	b	49)	a,d	50)	a,c,d	51)	a,c,d	52)	
149)	d	150)	a	151)	b	152)	d		b,c						
153)	a	154)	c	155)	a	156)	d	53)	a,d	54)	a,b,c	55)	a,c	56)	
157)	c	158)	b	159)	d	160)	b		a,b						
161)	a	162)	c	163)	d	164)	b	57)	a,c	58)	b,c	59)	a,b	60)	
165)	c	166)	a	167)	c	168)	c		a,b						
169)	d	170)	c	171)	a	172)	a	61)	a,c,d	62)	a,d	63)	a,b	64)	
173)	b	174)	a	175)	a	176)	a		a,b						
177)	a	178)	a	179)	c	180)	b	65)	a,b,c	66)	b,c	67)	a,d	68)	
181)	d	182)	b	183)	c	184)	c		a,d						
185)	c	186)	d	187)	a	188)	c	69)	a,c	70)	a,c	71)	a,b,d	72)	

	a,c						
73)	**a,b,c**	**74)**	**a,b,d**	**75)**	**a,b,d**	**76)**	
	a,c						
77)	**a,b**	**78)**	**b,d**	**79)**	**c,d**	**80)**	
	a,b,c						
81)	**c,d**	**82)**	**a,b,c**	**83)**	**b,c,d**	**84)**	
	a,d						
85)	**a,b,c**	**86)**	**a,c**	**87)**	**a,d**	**1)**	**d**
	2)	**b**	**3)**	**c**	**4)**	**a**	
5)	**b**	**6)**	**d**	**7)**	**a**	**8)**	**d**
9)	**b**	**10)**	**a**	**11)**	**a**	**12)**	**a**
13)	**d**	**14)**	**a**	**15)**	**b**	**16)**	**c**
17)	**a**	**18)**	**b**	**19)**	**a**	**20)**	**b**
21)	**b**	**22)**	**a**	**23)**	**a**	**24)**	**a**
25)	**a**	**26)**	**c**	**27)**	**d**	**28)**	**d**
29)	**d**	**30)**	**d**	**31)**	**a**	**32)**	**a**
33)	**b**	**34)**	**d**	**35)**	**a**	**36)**	**a**
37)	**b**	**38)**	**c**	**39)**	**c**	**40)**	**c**
41)	**d**	**42)**	**b**	**43)**	**a**	**44)**	**d**
45)	**b**	**46)**	**a**	**47)**	**a**	**48)**	**a**
49)	**d**	**1)**	**c**	**2)**	**c**	**3)**	**b**
	4)	**a**					
5)	**a**	**6)**	**a**	**7)**	**d**	**8)**	**b**
9)	**c**	**10)**	**b**	**11)**	**a**	**12)**	**c**
13)	**d**	**14)**	**d**	**1)**	**a**	**2)**	**b**
	3)	**b**	**4)**	**c**			
5)	**c**	**6)**	**b**	**7)**	**c**	**8)**	**b**
9)	**a**	**10)**	**a**	**11)**	**c**	**12)**	**a**
13)	**d**	**14)**	**a**	**15)**	**a**	**16)**	**a**
17)	**d**	**18)**	**d**	**19)**	**d**	**20)**	**c**
21)	**a**	**22)**	**d**	**23)**	**d**	**24)**	**d**
25)	**b**	**26)**	**a**	**1)**	**8**	**2)**	**3**
	3)	**1**	**4)**	**3**			
5)	**4**	**6)**	**3**	**7)**	**8**	**8)**	**6**
9)	**4**	**10)**	**3**	**11)**	**3**	**12)**	**6**
13)	**7**	**14)**	**6**	**15)**	**7**	**16)**	**2**
17)	**4**	**18)**	**3**	**19)**	**8**	**20)**	**6**
21)	**3**	**22)**	**2**	**23)**	**3**	**24)**	**5**
25)	**3**	**26)**	**7**	**27)**	**8**	**28)**	**9**
29)	**9**	**30)**	**3**	**31)**	**4**	**32)**	**7**
33)	**1**	**34)**	**5**	**35)**	**3**	**36)**	**6**
37)	**5**	**38)**	**1**	**39)**	**3**	**40)**	**4**
41)	**5**	**42)**	**6**	**43)**	**5**	**44)**	**2**
45)	**3**	**46)**	**9**	**47)**	**4**	**48)**	**4**
49)	**3**	**50)**	**6**	**51)**	**2**	**52)**	**5**

: HINTS AND SOLUTIONS :

1 **(b)**

$x = 2 + \sqrt{3}$

$\Rightarrow (x-2)^2 = 3$

$\Rightarrow x^2 - 4x + 1 = 0 \quad (1)$

$\Rightarrow (x-2)^4 = 9$

$\Rightarrow x^4 - 8x^3 + 24x^2 - 32x + 16 = 9$

$\Rightarrow x^4 - 8x^3 + 18x^2 - 8x + 2 + 6(x^2 - 4x + 1) - 1 = 0$ Using (1), we get

$x^4 - 8x^3 + 18x^2 - 8x + 2 = 1$

2 **(b)**

The given equation is

$|z|^n = (z^2 + z)|z|^{n-2} + 1$

$\Rightarrow z^2 + z$ is real

$\Rightarrow z^2 + z = \overline{z}^2 + \overline{z}$

$\Rightarrow (z - \overline{z})(z + \overline{z} + 1) = 0$

$\Rightarrow z = \overline{z} = x$ as $z + \overline{z} + 1 \neq 0 (x \neq -1/2)$

Hence, the given equation reduces to

$x^n = x^n + x|x|^{n-2} + 1$

$\Rightarrow x|x|^{n-2} = -1$

$\Rightarrow x = -1$

So number of solutions is 1

3 **(a)**

$\sum \alpha = 1, \sum \alpha\beta = 0, \alpha\beta\gamma = 1$

$$\sum \frac{1+\alpha}{1-\alpha} = -\sum \frac{-\alpha + 1 - 2}{1-\alpha} = \sum \left(\frac{2}{1-\alpha} - 1\right)$$

$$= 2\sum \frac{1}{1-\alpha} - 3$$

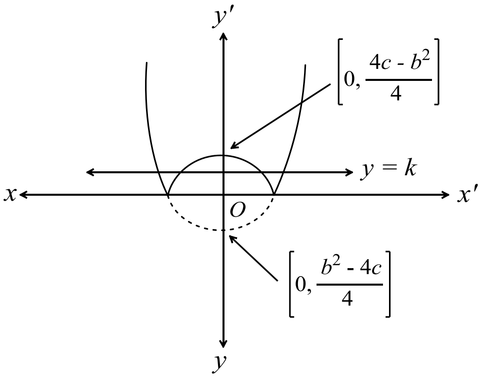
Now,

$$\frac{1}{(x-\alpha)} + \frac{1}{(x-\beta)} + \frac{1}{(1-\gamma)} = \frac{3x^2 - 2x}{x^3 - x^2 - 1}$$

$$\Rightarrow \frac{1}{1-\alpha} + \frac{1}{1-\beta} + \frac{1}{1-\gamma} = \frac{3-2}{1-1-1} = -1$$

$$\Rightarrow \frac{1+\alpha}{1-\alpha} = -5$$

4 **(a)**



For the equation to have four real roots, the line $y = k$ must intersect $y = |x^2 + bx + c|$ at four points

$\therefore D > 0$ and $k \in \left(0, -\frac{D}{4}\right)$

5 **(a)**

We have,

$\log z + \log z^2 + \log z^3 + \cdots + \log z^n = 0$

$\Rightarrow \log(zz^2z^3 \cdots z^n) = 0$

$\Rightarrow \log\left(z^{\frac{n(n+1)}{2}}\right) = 0$

$\Rightarrow z^{\frac{n(n+1)}{2}} = 1$

$\Rightarrow z = 1^{\frac{2}{n(n+1)}}$

$= (\cos 0° + i \sin 0°)^{\frac{2}{n(n+1)}}$

$= (\cos 2m\pi + i \sin 2m\pi)^{\frac{2}{n(n+1)}}, m = 0, 1, 2, 3, \ldots$

$$= \cos \frac{4m\pi}{n(n+1)} + i \sin \frac{4m\pi}{n(n+1)}, m = 0, 1, 2, \ldots$$

6 **(b)**

Given,

$a(p+q)^2 + 2bpq + c = 0$ and $a(p+r)^2 + 2bpr + c = 0$

$\Rightarrow q$ and r satisfy the equation $a(p+x)^2 + 2bpx + c = 0$

$\Rightarrow q$ and r are the roots of

$ax^2 + 2(ap + bp)x + c + ap^2 = 0$

$\Rightarrow qr =$ product of roots $= \frac{c + ap^2}{a} = p^2 + \frac{c}{a}$

7 **(a)**

Let α be the root of $x^2 - x + m = 0$ and 2α be the root of $x^2 - 3x + 2m = 0$. Then,

$\alpha^2 - \alpha + m = 0$ and $4\alpha^2 - 6\alpha + 2m = 0$

Eliminating $\alpha, m^2 = -2m \Rightarrow m = 0, m = -2$

8 **(b)**

For $z \neq 1$, the given equation can be written as

$$\left(\frac{z+1}{z-1}\right)^5 = 1$$

$$\Rightarrow \frac{z+1}{z-1} = e^{2k\pi i/5}$$

Where $k = -2, -1, 1, 2$

If we denote this value of z by z_k, then

$$z_k = \frac{e^{2k\pi i/5} + 1}{e^{2k\pi i/5} - 1}$$

$$= \frac{e^{k\pi i/5} + e^{-k\pi i/5}}{e^{k\pi i/5} - e^{-k\pi i/5}}$$

$$= -i \cot\left(\frac{k\pi}{5}\right), k = -2, -1, 1, 2$$

Therefore, roots of the equation are $\pm i \cot(\pi/5), \pm i \cot(2\pi/5)$

9 **(a)**

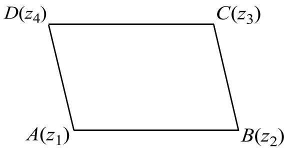
$x^2-(a+1)x+a-1=0$

$\Rightarrow (x-a)(x-1)=1$

Now, $a \in I$ and we want x to be an integer. Hence, $x-a=1, x-1=1$ or $x-\mathrm{a}=-1, x-1=-1$

$\Rightarrow a=1$ in both cases

10 **(a)**



The first condition implies that $(z_1+z_3)/2=(z_2+z_4)/2$, i.e., diagonals AC and BD bisect each other. Hence, quadrilateral is a parallelogram. The second condition implies that the angle between AD and AB is $90°$. Hence the parallelogram is a rectangle

11 **(c)**

We have,

$\frac{k+1}{k}+\frac{k+2}{k+1}=-\frac{b}{a}$ (1)

and $\frac{k+1}{k}+\frac{k+2}{k+1}=\frac{c}{a}$

$\Rightarrow \frac{k+2}{k}=\frac{c}{a}$ or $\frac{2}{k}=\frac{c}{a}-1=\frac{c-a}{a}$ or $k=\frac{2a}{c-a}$ (2)

Now, eliminate k. Putting the value of k in Eq. (1), we get

$\frac{c+a}{2a}+\frac{2c}{c+a}=-\frac{b}{a}$

$\Rightarrow (c+a)^2+4ac=-2b(a+c)$

$\Rightarrow (a+c)^2+2b(a+c)=-4ac$

Adding b^2 to both sides, we have

$(a+b+c)^2=b^2-4ac$

12 **(d)**

Equation $8x^3+1001x+2008=0$ has roots r, s and t

$r+s+r=0, rst=-\frac{2008}{8}=-251$

Now, let $r+s=A, s+1=B, t+r=C$,

$\therefore A+B+C=2(r+s+t)=0$

Hence,

$A^3+B^3+C^3=3ABC$

$\therefore (r+s)^3+(s+t)^3+(t+r)^3$

$=3(r+s)(s+t)(t+r)$

$=3(r+s+t-t)(s+t+r-r)(t+r+s-s)$

$=-3rst(\text{as } r+s+t=0)$

$=3(251)=753$

13 **(d)**

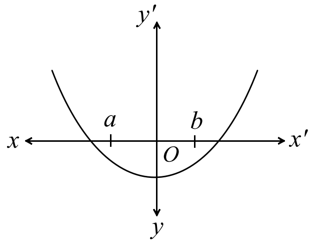
Given equation is

$(x-a)(x-b)-1=0$

Let $f(x)=(x-a)(x-b)-1$. Then,

$f(a)=-1$ and $f(b)=-1$

Also, graph of $f(x)$ is concave upward; hence, a and b lie between the roots. Also, if $b>a$, then one root lies in $(-\infty, a)$ and the other root lies in $(b,+\infty)$



14 **(c)**

l, m, n are real and $l \neq m$. Given equation is

$(l-m)x^2-5(l+m)x-2(l-m)=0$

$D=25(l+m)^2+8(l-m)^2>0, l, m \in R$

Therefore, the roots are real and unequal

15 **(a)**

Given quadratic expression is $x^2+2(a+b+$ *cx+3(bc+ca+ab)*, this quadratic expression will be a perfect square if the discriminant of its corresponding equation is zero. Hence,

$4(a+b+c)^2-4\times 3(bc+ca+ab)=0$

$\Rightarrow (a+b+c)^2-3(bc+ca+ab)=0$

$\Rightarrow a^2+b^2+c^2+2ab+2bc+2ca$
$\quad -3(bc+ca+ab)=0$

$\Rightarrow a^2+b^2+c^2-ab-bc-ca=0$

$\Rightarrow \frac{1}{2}[2a^2+2b^2+2c^2-2ab-2bc-2ca]=0$

$\Rightarrow \frac{1}{2}[(a^2+b^2-2ab)+(b^2+c^2-2bc)$
$\quad +(c^2+a^2-2ca)]=0$

$\Rightarrow \frac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2]=0$

Which is possible only when $(a-b)^2=0, (b-c)^2=0$ and $(c-a)^2=0$, i.e., $a=b=c$

16 **(a)**

$|z^2+2z\cos\alpha| \leq |z^2|+|2z\cos\alpha|$

$=|z|^2+2|z||\cos\alpha|$

$\leq |z|^2+2|z|$

$<(\sqrt{2}-1)^2+2(\sqrt{2}-1)=1$

17 **(c)**

$E=\sum_{i=1}^{n}(ar+b)\ \omega^{r-1}$

$=(a+b)+(2a+b)\omega+(3a+b)\omega^2+\cdots$
$\quad +(na+b)\omega^{n-1}$

$=b\ \underbrace{(1+\omega+\omega^2+\cdots+\omega^{n-1})}_{\text{zero}}+a(1+2\omega+3\omega^2$
$\quad +\cdots+n\omega^{n-1}$

Now,

$S=1+2\omega+3\omega^2+\cdots+n\omega^{n-1}$

$S\omega=\omega+2\omega^2+\cdots+(n-1)\omega^{n-1}+n\omega^n$

$\therefore S(1-0) = \underbrace{1+\omega+\omega^2+\ldots+\omega^{n-1}}_{\text{zero}} - n\omega^n$

$= -n \ (\because \omega^n = 1)$

$\Rightarrow S = \dfrac{n}{\omega - 1}$

$\Rightarrow E = \dfrac{na}{\omega - 1}$

18 **(a)**

$D = b^2 - 4a < 0 \Rightarrow a > 0$

Therefore the graph is concave upwards

$f(x) > 0, \forall x \in R$

$\Rightarrow f(-1) > 0$

$\Rightarrow a + b + 1 > 0$

19 **(a)**

Put $x^2 + x = y$, so that Eq.(1) becomes

$(y-2)(y-3) = 12$

$\Rightarrow y^2 - 5y - 6 = 0$

$\Rightarrow (y-6)(y+1) = 0 \ \Rightarrow y = 6, -1$

When $y = 6$, we get

$x^2 + x - 6 = 0$

$\Rightarrow (x+3)(x-2) = 0$ or $x = -3, 2$

When $y = -1$, we get

Which has non-real roots and sum of roots is -1

20 **(d)**

Given, $z = \cos\theta + i\sin\theta = e^{i\theta}$

$\therefore \displaystyle\sum_{m=1}^{15} \text{Im}\,(z^{2m-1}) = \sum_{m=1}^{15} \text{Im}\left(e^{i\theta}\right)^{2m-1}$

$= \displaystyle\sum_{m=1}^{15} \text{Im}\, e^{i(2m-1)\theta}$

$= \sin\theta + \sin 3\theta + \sin 5\theta + \ldots + \sin 29\,\theta$

$= \dfrac{\sin\left(\frac{\theta+29\theta}{2}\right)\sin\left(\frac{15\times 2\theta}{2}\right)}{\sin\left(\frac{2\theta}{2}\right)}$

$= \dfrac{\sin(15\,\theta)\sin(15\,\theta)}{\sin\theta} = \dfrac{1}{4\sin 2°}$

21 **(a)**

$u^2 - 2u + 2 = 0 \ \Rightarrow u = 1 \pm i$

$\Rightarrow \dfrac{(x+\alpha)^n - (x+\beta)^n}{\alpha - \beta}$

$= \dfrac{[(\cot\theta - 1) + (1+i)]^n - [(\cot\theta - 1) + (1-i)]^n}{2i} \quad (\because \cot\theta - 1 = x)$

$= \dfrac{(\cos\theta + i\sin\theta)^n - (\cos\theta - i\sin\theta)^n}{\sin^n\theta\ 2i}$

$= \dfrac{2i\sin n\theta}{\sin^n\theta\, 2i}$

$= \dfrac{\sin n\theta}{\sin^n\theta}$

22 **(b)**

$\left(\dfrac{x-1}{-2}\right)^3 = 1$

$\Rightarrow \dfrac{x-1}{-2} = 1, \omega, \omega^2$

$\Rightarrow x = -1, 1-2\omega, 1, -2\omega^2$

23 **(b)**

Let $A = x + iy$. Given,

$|A| = 1 \Rightarrow x^2 + y^2 = 1$

and

$|A+1| = 1 \ \Rightarrow (x+1)^2 + y^2 = 1$

$\Rightarrow x = -\frac{1}{2}$ and $y = \pm\frac{\sqrt{3}}{2}$

$\Rightarrow A = \omega$ or ω^2

$\Rightarrow (\omega)^n = (1+\omega)^n = (-\omega^2)^n$

Therefore, n must be even and divisible by 3

24 **(d)**

Given that $a < b < c < d$. Let

$f(x) = (x-a)(x-c) + 2(x-b)(x-d)$

$\Rightarrow f(b) = (b-a)(b-c) < 0$

and $f(d) = (d-a)(d-c) > 0$

Hence, $f(x) = 0$ has one root in (b, d). Also,

$f(a)f(c) < 0$. So the other root lies in (a, c).

Hence, roots of the equation are real and distinct

25 **(a)**

By observation $x = -2$ satisfies equation

$x^3 + 3x^2 + 3x + 2 = 0$ then we have $(x+2)(x^2 + x+1=0$

$x^2 + x + 1 = 0$ has non-real roots

Since non-real roots occur in conjugate pair,

$x^2 + x + 1 = 0$ and $ax^2 + bx + c = 0$ are identical

$\Rightarrow a = b = c$

26 **(c)**

Since α is root of all equations

$a\alpha^2 + 2b\alpha + c = 0$

$2b\alpha^2 + c\alpha + a = 0$

$c\alpha^2 + a\alpha + 2b = 0$

Adding we get $(a + 2b + c)(\alpha^2 + \alpha + 1) = 0$

$a + 2b + c \neq 0$ as $a, b, c > 0$

$\Rightarrow \alpha^2 + \alpha + 1 = 0$ or $\alpha^2 + \alpha = -1$

27 **(c)**

We have,

$\alpha + \beta = -p$ and $\alpha\beta = q \quad (1)$

Also, since α, β are the root of $x^{2n} + p^n x^n + q^n = 0$,

we have

$\alpha^{2n} + p^n\alpha^n + q^n = 0$ and $\beta^{2n} + p^n\beta^n + q^n = 0$

Subtracting the above relations, we get

$(\alpha^{2n} - \beta^{2n}) + p^n(\alpha^n - \beta^n) = 0$

$\therefore \alpha^n + \beta^n = -p^n \quad (2)$

Given, α/β or β/α is a root of $x^n + 1 + (x+1)^n = 0$. So,

$(\alpha/\beta)^n + 1 + [(\alpha/\beta) + 1]^n = 0$

$\Rightarrow (\alpha^n + \beta^n) + (\alpha + \beta)^n = 0$

$\Rightarrow -p^n + (-p)^n = 0$ [Using (1) and (2)]

It is possible only when n is even

28 **(c)**

$$z^2 + z|z| + |z|^2 = 0 \Rightarrow \left(\frac{z}{|z|}\right)^2 + \frac{z}{|z|} + 1 = 0$$

$$\Rightarrow \frac{z}{|z|} = \omega, \omega^2 \Rightarrow z = \omega|z| \text{ or } z = \omega^2|z|$$

$$\Rightarrow x + iy = |z|\left(\frac{-1}{2} + \frac{i\sqrt{3}}{2}\right) \text{ or } x + iy = |z|\left(\frac{-1}{2} - \frac{i\sqrt{3}}{2}\right)$$

$$\Rightarrow x = -\frac{1}{2}|z|, y = |z|\frac{\sqrt{3}}{2} \text{ or } x = -\frac{|z|}{2}, y = -\frac{|z|\sqrt{3}}{2}$$

$$\Rightarrow y + \sqrt{3}x = 0 \text{ or } y - \sqrt{3}x = 0 \Rightarrow y^2 - 3x^2 = 0$$

29 **(a)**

Since, $|z| = 1$ and $w = \frac{z-1}{z+1} \Rightarrow z = \frac{1+w}{1-w}$

$\Rightarrow \quad |z| = \dfrac{|1+w|}{|1-w|} \quad \Rightarrow \quad |1 - w| = |1 + w| \quad [\because |z| = 1]$

$\Rightarrow \quad 1 + |w|^2 - 2\,\text{Re}\,(w) = 1 + |w|^2 + 2\,\text{Re}\,(w)$

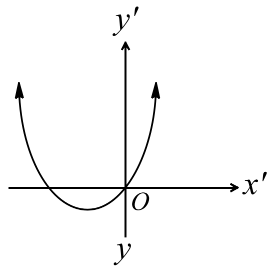
$\Rightarrow \quad \text{Re}\,(w) = 0$

30 **(a)**

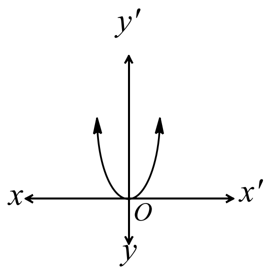
Since the equation $x^2 + ax + b = 0$ has distinct real roots and $x^2 + a|x| + b = 0$ has only one real root, so one root of the equation $x^2 + ax + b = 0$ will be zero and other root will be negative.

Hence, $b = 0$ and $a > 0$

Graph of $y = ax^2 + bx + c$ according to conditions given in question



Graph of $y = ax^2 + b|x| + c$



31 **(b)**

Here, $ax^2 - bx + c^2 = 0$ does not have real roots. So,

$D < 0 \Rightarrow b^2 - 4ac^2 < 0 \Rightarrow a > 0$

Therefore, $f(x)$ is always positive. So,

$f(2) > 0 \Rightarrow 4a - 2b + c^2 > 0$

32 **(c)**

We have,

$|x|^2 - 3|x| + 2 = 0$

$\Rightarrow (|x| - 1)(|x| - 2) = 0$

$\Rightarrow |x| = 1, 2 \Rightarrow x = \pm 1, \pm 2$

33 **(d)**

$(x - 3)^3 + 1 = 0$

$$\Rightarrow \left(\frac{x-3}{-1}\right)^3 = 1$$

$$\Rightarrow \frac{x-3}{-1} = 1, \omega, \omega^2$$

$\Rightarrow x = 2, 3 - \omega, 3 - \omega^2$

Hence, the run of complex root is $6 - (\omega + \omega)^2 = 6 + 1 = 7$

34 **(a)**

Given that $x^2 + px + 1$ is a factor of $ax^3 + bx + c$. Then let $ax^3 + bx + c = (x^2 + px + 1)(ax + \lambda)$, where λ is a constant. Then equation the coefficients of like powers of x on both sides, we get

$0 = ap + \lambda, b = p\lambda + a, c = \lambda$

$$\Rightarrow p = -\frac{\lambda}{a} = -\frac{c}{a}$$

Hence,
$b = \left(-\frac{c}{a}\right)c + a$
or $ab = a^2 - c^2$

35 **(a)**
$i = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = e^{\frac{i\pi}{2}}$
$\Rightarrow i^i = \left(e^{\frac{i\pi}{2}}\right)^i = e^{-\frac{\pi}{2}}$
$\Rightarrow z = (i)^{(i)^i} = i^{e^{-\frac{\pi}{2}}}$
$\Rightarrow |z| = 1$

36 **(d)**
Given equation is satisfied by $x = 1, 2, 3$. But for $x = 1, \sqrt{x-2}$ is not defined. Hence, number of roots is 2 and the roots are $x = 2$ and 3

37 **(d)**
$x^2 - x - a = 0, D = 1 + 4a = \text{odd}$
D must be perfect square of some odd integer. Let $D = (2\lambda + 1)^2$
$\Rightarrow 1 + 4a = 1 + 4\lambda^2 + 4\lambda$
$\Rightarrow a = \lambda(\lambda + 1)$
Now,
$a \in [6, 100]$
$\Rightarrow a = 6, 12, 20, 30, 42, 56, 72, 90$
Thus a can attain eight different values

38 **(a)**
Since, a, b and c are the sides of a $\Delta\, ABC$, then
$|a-b| < |c| \Rightarrow a^2 + b^2 - 2ab < c^2$
Similarly, $b^2 + c^2 - 2bc < a^2$, $c^2 + a^2 - 2ca < b^2$
On adding, we get
$(a^2 + b^2 + c^2) < 2(ab + bc + ca)$
$\Rightarrow \dfrac{a^2 + b^2 + c^2}{ab + bc + ca} < 2 \quad \ldots\ldots(i)$
Also, $D \geq 0, (a + b + c)^2 - 3\lambda(ab + bc + ca) \geq 0$
$\Rightarrow \dfrac{a^2 + b^2 + c^2}{ab + bc + ca} > 3\lambda - 2 \quad \ldots\ldots\ldots(ii)$
From Eqs. (i) and (ii),
$3\lambda - 2 < 2 \Rightarrow \lambda < \frac{4}{3}$

39 **(c)**
$A = a(b - c)(a + b + c)$
$B = b(c - a)(a + b + c)$
$C = c(a - b)(a + b + c)$
Now,
$Ax^2 + Bx + C = 0$
$$\Rightarrow (a + b + c)\{a(b - c)x^2 + b(c - a)x + c(a - b)\} = 0$$
Given that roots are equal. Hence,
$D = 0$
$\Rightarrow b^2(c - a^2) - 4ac(b - c)(a - b) = 0$
$$\Rightarrow b^2c^2 - 2ab^2c + b^2a^2 - 4a^2bc + 4acb^2 + 4a^2c^2 - 4abc^2 = 0$$
$$\Rightarrow b^2c^2 + b^2a^2 + 4a^2c^2 + 2ab^2c - 4a^2bc - 4abc^2 = 0$$
$\Rightarrow (bc + ab - 2ac)^2 = 0$
$\Rightarrow bc + ab = 2ac$
$\Rightarrow \frac{1}{a} + \frac{1}{c} = \frac{2}{b}$
$\Rightarrow a, b, c$ are in H.P.

40 **(a)**
$x^2 + 2x - n = 0 \Rightarrow (x + 1)^2 = n + 1$
$\Rightarrow x = -1 \pm \sqrt{n+1}$
Thus, $n + 1$ should be a perfect square. Now,
$n \in [5, 100] \Rightarrow n + 1 \in [6, 101]$
Perfect square values of $n + 1$ are
$9, 16, 25, 36, 49, 64, 81, 100$. Hence, number of values is 8

41 **(d)**
$x^2 + x + 1 = 0 \Rightarrow x = \omega$ or ω^2
Let $x = \omega$. Then,
$x + \frac{1}{x} = \omega + \frac{1}{\omega} = \omega + \omega^2 = -1$
$x^2 + \frac{1}{x^2} = \omega^2 + \frac{1}{\omega^2} = \omega^2 + \omega = -1$
$x^3 + \frac{1}{x^3} = \omega^3 + \frac{1}{\omega^3} = 2$
$x^4 + \frac{1}{x^4} = \omega^4 + \frac{1}{\omega^4} = \omega + \omega^2 = -1$, etc
$$\therefore \left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \left(x^3 + \frac{1}{x^3}\right)^2 + \cdots + \left(x^{27} + \frac{1}{x^{27}}\right)^2$$
$$= \left[\left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \left(x^4 + \frac{1}{x^4}\right)^2 + \cdots + \left(x^{26} + \frac{1}{x^{26}}\right)^2\right] + \left[\left(x^3 + \frac{1}{x^3}\right)^2 + \left(x^6 + \frac{1}{x^6}\right)^2 + \left(x^9 + \frac{1}{x^9}\right) + \cdots + \left(x^{27} + \frac{1}{x^{27}}\right)^2\right]$$
$= 18 + 9(2^2) = 54$

42 **(c)**
$$x^2+px+1\overline{\smash{)}\,ax^3 + bx + c}\,\big(ax - sp$$
$ax^3 + apx^2 + ax$
$\overline{}$
$-apx^2 + (b - a)x + c$
$\underline{-apx^2 + ap^2x - ap}$
$(b - a + ap^2)x + c + ap$
Now, remainder must be zero. Hence,

$b - a + ap^2 = 0$ and $c + ap = 0$

$\Rightarrow p = -\frac{c}{a}$ and $p^2 = \frac{a-b}{a}$

$\Rightarrow \left(\frac{-c}{a}\right)^2 = \frac{a-b}{a}$

$\Rightarrow c^2 = a^2 - ab$

$\Rightarrow a^2 - c^2 = ab$

43 **(c)**

$\overline{z} + i\overline{w} = 0$

$\Rightarrow z - iw = 0 \quad (1)$

$\Rightarrow z = iw$

$\arg zw = \pi$

$\Rightarrow \arg z + \arg w = \pi$

$\Rightarrow \arg z + \arg \frac{z}{i} = \pi \quad$ [Using (1)]

$\Rightarrow \arg z + \arg z - \arg i = \pi$

$\Rightarrow 2 \arg z - \frac{\pi}{2} = \pi$

$\Rightarrow 2 \arg z = \frac{3\pi}{2}$

$\Rightarrow \arg z = \frac{3\pi}{4}$

44 **(c)**

As $a, b, c > 0$, so a, b, c should be real (note that other relation is not defined in the set of complex numbers). Therefore, the roots of equation are either real or complex conjugate

Let α, β be the roots of $ax^2 + bx + c = 0$. Then,

$\alpha + \beta = -\frac{b}{a} = -$ve and $\alpha\beta = \frac{c}{a} = +$ve

Hence, either both α, β are – ve (if roots are real) or both α, β have – ve real part (if roots are complex conjugate)

45 **(b)**

$x = \sqrt[3]{-1}$

$\Rightarrow x^3 = -1$

$\Rightarrow (-x)^3 = 1$

$\Rightarrow -x = 1, \omega, \omega^2$

$\Rightarrow x = -1, -\omega, -\omega^2$

$= -1, \frac{1 + \sqrt{3}i}{2}, \frac{1 - \sqrt{3}i}{2}$

$= -1, \frac{-\sqrt{3} + i}{2i}, \frac{\sqrt{3} + i}{2i}$

$= -1, \frac{-\sqrt{3} + \sqrt{-1}}{\sqrt{-4}}, \frac{\sqrt{3} + \sqrt{-1}}{\sqrt{-4}}$

46 **(b)**

$\tan x = \frac{a - 4 - \sqrt{(a-4)^2 - 4(4 - 2a)}}{2}$

$= \frac{a - 4 - a}{2} = a - 2, -2$

$\therefore \tan x = a - 2 \ \ (\because \tan x \neq -2)$

$\because x \in \left[0, \frac{\pi}{4}\right]$

$\therefore 0 \leq a - 2 \leq 1$

$\Rightarrow 2 \leq a \leq 3$

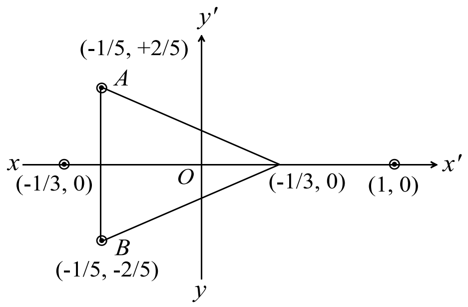
47 **(c)**

$\left(\frac{z+1}{z}\right)^4 = 16$

$\Rightarrow \frac{z+1}{z} = \pm 2, \pm 2i$

The roots are $1, -1/3, (-1/5 - (2/5)i)$ and $(-1/5 + (2/5)i)$

Note that $(-1/3, 0)$ and $(1, 0)$ are equidistant from $(1/3, 0)$ and since it lies on the perpendicular bisector of AB, it will be equidistant from A and B also



48 **(a)**

We have,

$(\alpha + \beta + \gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2(\beta\gamma + \gamma\alpha + \alpha\beta)$

$\Rightarrow 4 = 6 + 2\ (\beta\gamma + \gamma\alpha + \alpha\beta)$

$\Rightarrow \beta\gamma + \gamma\alpha + \alpha\beta = -1$

Also, $\alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma = (\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - \beta\gamma - \gamma\alpha - \alpha\beta)$

$\Rightarrow 8 - 3\alpha\beta\gamma = 2(6 + 1)$

$\Rightarrow 3\alpha\beta\gamma = 8 - 14 = -6$ or $\alpha\beta\gamma = -2$

Now,

$(\alpha^2 + \beta^2 + \gamma^2)^2 = \sum \alpha^4 + 2 \sum \alpha^2\beta^2$

$= \sum \alpha^4 + 2\left[\left(\sum \alpha\beta\right)^2 - 2\alpha\beta\gamma\left(\sum \alpha\right)\right]$

$\Rightarrow \sum \alpha^4 = 36 - 2[(-1)^2 - 2(-2)(2)] = 18$

49 **(c)**

Let $x = |a + b\omega + c\omega^2|$

$\Rightarrow \quad x^2 = (a^2 + b^2 + c^2 - ab - bc - ca)$

$\Rightarrow \quad x^2 = \frac{1}{2}\{(a-b)^2 + (b-c)^2 + (c-a)^2\}$

...(i)

Since a, b, c are all integers but not all simultaneously equal

$\Rightarrow \quad$ If $a = b$, then $a \neq c$ and $b \neq c$

As, difference of integers=integer

$\Rightarrow \quad (b - c)^2 \geq 1$

[as minimum difference of two consecutive integers is (± 1)]

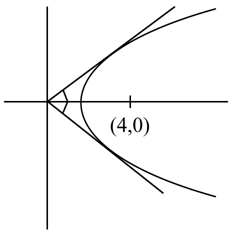
Also, $\ (c - a)^2 \geq 1$

$\therefore$ From Eq. (i),

$x^2 = \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2]$
$\geq \frac{1}{2}[0+1+1]$
$\Rightarrow \quad x^2 \geq 1$
Hence, minimum value of x is 1

50 **(a)**
$|z_1| = |z_2| = |z_3| = 1$
Hence, the circumcentre of triangle is origin. Also, centroid $(z_1 + z_2 + z_3)/3 = 0$, which coincides with the circumcentre. So the triangle is equilateral. Since radius is 1, length of side is $a = \sqrt{3}$. Therefore, the area of the triangle is $(\sqrt{3}/4)a^2 = (3\sqrt{3}/4)$

51 **(c)**
Let m be a positive integer for which
$n^2 + 96 = m^2$
$\Rightarrow m^2 - n^2 = 96 \Rightarrow (m+n)(m-n) = 96$
$\Rightarrow (m+n)\{(m+n) - 2n\} = 96$
$\Rightarrow m+n$ and $m-n$ must be both even
As $96 = 2 \times 48$ or 4×24 or 6×16 or 8×12, hence, number of solutions is 4

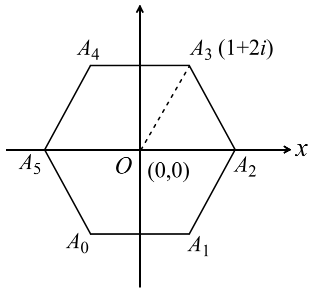
52 **(d)**


$|z-4| = \text{Re}(z)$
$\Rightarrow \sqrt{(x-4)^2 + y^2} = x$
$\Rightarrow x^2 - 8x + 16 + y^2 = x^2$
$\Rightarrow y^2 = 8(x-2)$
The given relation represents the part of the parabola with focus (4, 0) lying above x-axis and the imaginary axis as the directrix. The two tangents from directrix are at right angle. Hence greatest positive argument of z is $\pi/4$

53 **(c)**
Observing carefully the system of equations, we find
$\frac{1+i}{2i} = \frac{1-i}{2} = \frac{1}{1+i}$
Hence, there are infinite number of solutions

54 **(c)**
Let α, α^2 be the roots of $3x^2 + px + 3 = 0$. Now,
$S = \alpha + \alpha^2 = -p/3, p = \alpha^3 = 1$
$\Rightarrow \alpha = 1, \omega, \omega^2 \quad \left(\text{where } \omega = \frac{-1+\sqrt{3}i}{2}\right)$
$\alpha + \alpha^2 = -p/3 \Rightarrow \omega + \omega^2 = -p/3$
$\Rightarrow -1 = -p/3 \Rightarrow p = 3$

55 **(c)**
Given, α, β are roots of equation
$x^2 - 2x + 3 = 0$
$\Rightarrow \alpha^2 - 2\alpha + 3 = 0 \quad (1)$
And $\beta^2 - 2\beta + 3 = 0 \quad (2)$
$\Rightarrow \alpha^2 = 2\alpha - 3 \Rightarrow \alpha^3 = 2\alpha^2 - 3\alpha$
$\Rightarrow P = (2\alpha^2 - 3\alpha) - 3\alpha^2 + 5\alpha - 2$
$\Rightarrow -\alpha^2 + 2\alpha - 2 = 3 - 2 = 1, \quad$ [Using (1)]
Similarly, we have $Q = 2$
Now, sum of root is 3 and product of roots is 2.
Hence, the required equation is $x^2 - 3x + 2 = 0$

56 **(d)**


Let the vertices be $z_0, z_1, \ldots, z_5$ w.r.t. centre O at origin and $|z_0| = \sqrt{5}$
Now ΔOA_2A_3 is equilateral $\Rightarrow OA_2 = OA_3 = A_2A_3 = \sqrt{5}$
$= |z_0||\cos\theta + i\sin\theta - 1|$
Perimeter $= 6\sqrt{5}$

57 **(c)**
We have,
$z_1(z_1^2 - 3z_2^2) = 2 \quad (1)$
$z_2(3z_1^2 - z_2^2) = 11 \quad (2)$
Multiplying (2) by i and adding it to (1), we get
$z_1^3 - 3z_2^2 z_1 + i(3z_1^2 z_2 - z_2^3) = 2 + 11i$
$\Rightarrow (z_1 + iz_2)^3 = 2 + 11i \quad (3)$
Multiplying (2) by i and subtracting it from (1), we get
$z_1^3 - 3z_2^2 z_1 - i(3z_1^2 z_2 - z_2^3) = 2 - 11i$
$\Rightarrow (z_1 - iz_2)^3 = 2 - 11i \quad (4)$
Multiplying (3) and (4), we get
$(z_1^2 + z_2^2)^3 = 2^2 - 121i^2 = 4 + 121 = 125$
$\Rightarrow z_1^2 + z_2^2 = 5$

58 **(d)**
$P(a) = P(b) = P(c) = P(d) = 3$
$\Rightarrow P(x) = 3$ has a, b, c, d as its roots
$\Rightarrow P(x) - 3 = (x-a)(x-b)(x-c)(x-d)Q(x)$
[$\because Q(x)$ has integral coefficient]
Given $P(e) = 5$, then
$(e-a)(e-b)(e-c)(e-d)\,Q(e) = 5$
This is possible only when at least three of the five integers $(e-a), (e-b)(e-c)(e-d)Q(e)$ are equal to 1 or -1. Hence, two of them will be equal,

which is not possible. Since a, b, c, d are distinct integers, therefore $P(e) = 5$ is not possible

59 **(c)**

We have

$\alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$

$a + h + \beta + h = -\frac{q}{p}, (\alpha + h)(\beta + h) = \frac{r}{p}$

$\Rightarrow\ \alpha + \beta + 2h = -\frac{q}{p}$

$\Rightarrow -\frac{b}{a} + 2h = \frac{-q}{p}\ \ [\because a + b = -\frac{b}{a}]$

$\Rightarrow h = \frac{1}{2}\left(\frac{b}{a} - \frac{q}{p}\right)$

60 **(c)**

$z = \frac{at + b}{t - c}\ \Rightarrow t = \frac{b + cz}{z - a}$

Now, $|t| = 1$

$\Rightarrow \left|\frac{b + cz}{z - a}\right| = 1$

$\Rightarrow \left|\frac{z + \frac{b}{c}}{z - a}\right| = \frac{1}{|c|} = (\neq 1 \text{ as } |c| \neq |t|)$

$\Rightarrow$ locus of z is a circle

61 **(a)**

We know that $|z - z_1| = |z - z_2|$. Then locus of z is the line, which is a perpendicular bisector of line segment joining z_1 and z_2

Hence,

$z = x + iy$

$\Rightarrow |z - 5i| = |z + 5i|$

Therefore, z remains equidistant from $z_1 = 5i$ and $z_2 = 5i$. Hence, z lies on perpendicular bisector of line segment joining z_1 and z_2, which is clearly the real axis or $y = 0$

Alternative solution:

$\left|\frac{z - 5i}{z + 5i}\right| = 1$

$\Rightarrow |x + iy - 5i| = |x + iy + 5i|$

$\Rightarrow |x + (y - 5)i| = |x + (y + 5)i|$

$\Rightarrow x^2 + (y - 5)^2 = x^2 + (y + 5)^2$

$\Rightarrow x^2 + y^2 - 10y + 25 = x^2 + y^2 + 10y + 25$

$\Rightarrow 20y = 0$

$\Rightarrow y = 0$

62 **(b)**

Here $D = b^2 - 4c > 0$ because $c < 0 < b$. So, roots are real and unequal. Now,

$\alpha + \beta = -b < 0$ and $\alpha\beta = c < 0$

Therefore, one root is positive and the other root is negative, the negative root being numerically bigger. As $\alpha < \beta$, so α is the negative root while β is the positive root. So, $|\alpha| > \beta$ and $\alpha < 0 < \beta < |\alpha|$

63 **(c)**

The given equation is

$x^2 - 2mx + m^2 - 1 = 0$

$\Rightarrow (x - m)^2 - 1 = 0$

$\Rightarrow (x - m + 1)(x - m - 1) = 0$

$\Rightarrow x = m - 1, m + 1$

From given condition,

$m - 1 > -2$ and $m + 1 < 4$

$\Rightarrow m > -1$ and $m < 3$

Hence, $-1 < m < 3$

64 **(c)**

Let the distance of the school from A be x. Therefore, the distance of the school from B is $60 - x$. The total distance covered by 200 students is

$[150x + 50(60 - x)] = [100x + 3000]$

This is minimum when $x = 0$. Hence, the school should be at town A

65 **(b)**

$z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$

$= \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^5 + \left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)^5$

$= \left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right) + \left(\cos\frac{5\pi}{6} - i\sin\frac{5\pi}{6}\right)$

$= 2\cos\frac{5\pi}{6}$

$= -\sqrt{3}$

$\Rightarrow \text{Re}(z) < 0$ and $\text{Im}(z) = 0$

Alternative solution:

$z = \bar{z}_1 + \bar{z}_2$

Where

$\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5$

$\Rightarrow z$ is real

$\Rightarrow \text{Im}(z) = 0$

66 **(b)**

Sum of roots$= \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$ and product $=1$

Given, $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$

$\Rightarrow (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2) = q$

$\therefore\ \alpha^2 + \beta^2 - \alpha\beta = \frac{-q}{p}$ ...(i)

And $(\alpha + \beta)^2 = p^2$

$\Rightarrow\ \alpha^2 + \beta^2 + 2\alpha\beta = p^2$

From Eqs. (i) and (ii), we get

$\alpha^2 + \beta^2 = \frac{p^3 - 2q}{3p}$

And $\alpha\beta = \frac{p^3 + q}{3p}$

$\therefore$ Required equation is

$x^2 - \frac{(p^3 - 2q)x}{(p^3 + q)} + 1 = 0$

$\Rightarrow (p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

67 **(d)**

Let the four numbers in A.P. be $p = a - 3d, q = a - d, r = a + d, s = a + 3d$. Therefore,

$p + q = 2, r + s = 18$

Given that $pq = A, rs = B$

$\therefore p + q + r + s = 4a = 20$

$\Rightarrow a = 5$

Now, $p + q = 2 \Rightarrow 10 - 4d = 2$

$r + s = 18 \Rightarrow 10 + 4d = 18$

$\therefore d = 2$

Hence, the numbers are $-1, 3, 7, 11$

$pq = A = -3, rs = B = 77$

68 **(d)**

α, β are roots of $x^2 + px + q = 0$. Hence,

$\alpha + \beta = -p$

$\alpha\beta = q$

Now,

α^4, β^4 are roots $x^2 - px + q = 0$. Hence,

$\alpha^4 + \beta^4 = r, \alpha^4\beta^4 = q$

Now, for equation $x^2 - 4qx + 2q^2 - r = 0$, product of roots is

$2q^2 - r = 2(\alpha\beta)^2 - (\alpha^4 + \beta^4)$

$= -(\alpha^2 - \beta^2)^2$

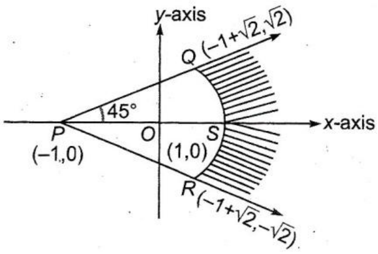
< 0

As product of roots is negative, so the roots must be real

69 **(a)**

Here, $|PQ| = |PS| = |PR| = 2$

$\therefore$ Shaded part represents the external part of circle having centre (-1, 0) and radius 2



As we know equation of circle having centre z_0 and radius r, is

$|z - z_0| = r$

$\therefore \quad |z - (-1 + 0i)| > 2$

$\Rightarrow \quad |z + 1| > 2 \qquad$ ...(i)

Also, argument of $z + 1$ with represent to positive direction of x-axis is $\pi/4$

$\therefore \ \arg(z + 1) \leq \dfrac{\pi}{4}$

And argument of $z + 1$ in anti-clockwise direction is $-\pi/4$.

$\therefore \quad -\dfrac{\pi}{4} \leq \arg(z + 1)$

$\Rightarrow \quad |\arg(z + 1)| \leq \dfrac{\pi}{4}$

70 **(b)**

Given equations are

$x^3 + ax + 1 = 0$

or $x^4 + ax^2 + x = 0$ (1)

and $x^4 + ax^2 + 1 = 0$ (2)

From (1) – (2), we get $x = 1$. Thus, $x = 1$ is the common roots.

Hence,

$1 + a + 1 = 0 \Rightarrow a = -2$

71 **(c)**

We have,

$$|(x-2) + i(y-1)| = |z| \left| \frac{1}{\sqrt{2}} \cos\theta - \frac{1}{\sqrt{2}} \sin\theta \right|$$

Where $\theta = \arg z$

$$\sqrt{(x-2)^2 + (y-1)^2} = \frac{1}{\sqrt{2}} |x - y|$$

Which is a parabola

72 **(d)**

$z = i \log(2 - \sqrt{3})$

$\Rightarrow e^{iz} = e^{i^2 \log(2-\sqrt{3})} = e^{-\log(2-\sqrt{3})}$

$\Rightarrow e^{iz} = e^{\log(2-\sqrt{3})^{-1}} = e^{\log(2-\sqrt{3})} = (2 + \sqrt{3})$

$$\Rightarrow \cos z = \frac{e^{iz} + e^{-iz}}{2} = \frac{(2+\sqrt{3}) + (2-\sqrt{3})}{2} = 2$$

73 **(d)**

$f(x, y) = (x-2)^2 + (y-1)^2 = 0$

$\Rightarrow x = 2$ and $y = 1$

$$\therefore E = \frac{(\sqrt{2}-1)^2 + 4\sqrt{2}}{2 + \sqrt{2}} = \frac{(\sqrt{2}+1)^2}{\sqrt{2}(\sqrt{2}+1)} = \frac{\sqrt{2}+1}{\sqrt{2}}$$

74 **(a)**

Let, $f(x, y, z) = x^2 + 4y^2 + 3z^2 - 2x - 12y - 6z + 14$

$= (x-1)^2 + (2y-3)^2 + 3(z-1)^2 + 1$

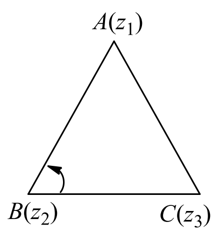
For the least value of $f(x, y, z)$,

$x - 1 = 0, 2y - 3 = 0$ and $z - 1 = 0$

$\therefore \ x = 1, y = 3/2, z = 1$

Hence the least value of $f(x, y, z)$ is $f(1, 3/2, 1) = 1$

75 **(c)**



$$\frac{z_1 - z_2}{z_3 - z_2} = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} = e^{i\pi/4}$$

$\angle CBA = \dfrac{\pi}{4}$

Also,

$|z_1 - z_2| = |z_3 - z_2|$

Hence, ΔABC is isosceles

76 **(c)**

Let the roots be α, β

$\therefore \alpha+\beta=-2a$ and $\alpha\beta=b$

Given,

$|\alpha-\beta| \le 2m$

$\Rightarrow |\alpha-\beta|^2 \le (2m)^2$

$\Rightarrow (\alpha+\beta)^2-4ab \le 4m^2$

$\Rightarrow 4a^2-4b \le 4m^2$

$\Rightarrow a^2-m^2 \le b$ and discriminant $D>0$ or $4a^2-4b>0$

$\Rightarrow a^2-m^2 \le b$ and $b<a^2$

Hence, $b \in [a^2-m^2, a^2]$

77 **(b)**

Let,

$$y=\frac{3x^2+9x+17}{3x^2+9x+7}$$

$\Rightarrow 3x^2(y-1)+9x(y-1)+7y-17=0$

Since x is real, so,

$D \ge 0$

$\Rightarrow 81(y-1)^2-4\times 3(y-1)(7y-17) \ge 0$

$\Rightarrow (y-1)(y-41) \le 0 \Rightarrow 1 \le y \le 41$

Therefore, the maximum value of y is 41

78 **(b)**

Verify by selecting particular values of a and b

Let $a=-9$ and $b=4$. Then,

$\sqrt{a}\sqrt{b}=\sqrt{-9}\sqrt{4}=(3i)(2)=6i$

From option (a), we have

$-\sqrt{|a|b}=-\sqrt{|-9|\times 4}=-\sqrt{36}=-6$

From option (b), we have

$\sqrt{|a|b}i=\sqrt{|-9|\times 4}\, i=6i$

79 **(d)**

$|z-4|<|z-2|$

$\Rightarrow |(x-4)+iy|<|(x-2)+iy|$

$\Rightarrow (x-4)^2+y^2<(x-2)^2+y^2$

$\Rightarrow -8x+16<-4x+4$

$\Rightarrow 4x-12>0$

$\Rightarrow x>3$

$\Rightarrow \mathrm{Re}(z)>3$

80 **(c)**

$$x=9^{\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots}=9^{\frac{\frac{1}{3}}{1-\frac{1}{3}}}=9^{\frac{1}{2}}=3$$

$$y=4^{\frac{1}{3}-\frac{1}{9}+\frac{1}{27}-\cdots}=4^{\frac{\frac{1}{3}}{1+\frac{1}{3}}}=4^{\frac{1}{4}}=\sqrt{2}$$

$$z=\sum_{r=1}^{\infty}(1+i)^{-r}$$

$$=\frac{1}{1+i}+\frac{1}{(1+i)^2}+\frac{1}{(1+i)^3}+\cdots$$

$$=\frac{\frac{1}{1+i}}{1-\frac{1}{1+i}}=\frac{1}{i}=-i$$

Let $\alpha=x+yz=3-\sqrt{2}i$ (fourth quadrant). Then,

$$\arg\alpha=-\tan^{-1}\left(\frac{\sqrt{2}}{3}\right)$$

81 **(c)**

Given,

$(a-1)x^2-(a+1)x+a-1 \ge 0$

$\Rightarrow a(x^2-x+1)-(x^2+x+1) \ge 0$

$$\Rightarrow a \ge \frac{x^2+x+1}{x^2-x+1}$$

$$=1+\frac{2x}{x^2-x+1}$$

$$=1+\frac{2}{x+\frac{1}{x}-1} \quad (1)$$

Let $y=x+1/x$. Now, y is increasing in $[2,\infty)$. Hence,

$$1+\frac{2}{x+\frac{1}{x}-1} \in \left(1,\frac{7}{3}\right]$$

For all $x \ge 2$, Eq. (1) should be true. Hence, $a>7/3$

82 **(b)**

Let $z_1=\frac{w-\overline{w}z}{1-z}$ be purely real

$\Rightarrow \quad z_1=\overline{z_1}$

$$\therefore \quad \frac{w-\overline{w}z}{1-z}=\frac{\overline{w}-w\overline{z}}{1-\overline{z}}$$

$\Rightarrow \quad w-w\overline{z}-\overline{w}z+\overline{w}z\,\overline{z}$

$= \quad \overline{w}-z\overline{w}-w\overline{z}+wz\,\overline{z}$

$\Rightarrow \quad (w-\overline{w})+(\overline{w}-w)|z|^2=0$

$\Rightarrow \quad (w-\overline{w})+(1-|z|^2)=0$

$\Rightarrow \quad |z|^2=1$

[as, $w-\overline{w}\neq 0$, since $\beta \neq 0$]

$\Rightarrow \quad |z|=1$ and $z\neq 1$

83 **(a)**

Minimum value of $5x^2+2x+3$ is

$$-\frac{D}{4a}=-\frac{(2)^2-4(5)(3)}{4(5)}>2$$

Where maximum value of $2\sin x$ is 2. Therefore, the two curves do not meet at all

84 **(a)**

Clearly,

$x^n-1=(x-1)(x-a_1)(x-a_2)\ldots(x-a_{n-1})$

$$\Rightarrow \frac{x^n-1}{x-1}=(x-a_1)(x-a_2)\ldots(x-a_{n-1})$$

$\Rightarrow 1+x+x^2+\cdots+x^{n-1}$

$=(x-a_1)(x-a_2)\ldots(x-a_{n-1})$

$\Rightarrow n=(1-a_1)(1-a_2)\ldots(1-a_{n-1})$ [putting $x=1$]

85 **(c)**

If one root is square of the other root of the equation $ax^2 + bx + c = 0$, then

$\beta = \alpha^2 \Rightarrow \alpha^2 + \alpha = -b/a$ and $\alpha^2 a = c/a$

By eliminating α, we get

$b^3 + ac^2 + a^2c = 3abc$

Which can be written in the form $a(c-b)^3 = c(a-b)^3$

Alternative solution:

Let the roots be 2 and 4. Then the equation is

$x^2 - 6x + 8 = 0$

Here obviously,

$$X = \frac{a(c-b)^3}{c} = \frac{1(14)^3}{8} = \frac{14}{2} \times \frac{14}{2} \times \frac{14}{2} = 7^3$$

Which is given by $(a-b)^3 = 7^3$

86 **(b)**

$x = 3\cos\theta\,; y = 3\sin\theta$

$z = 2\cos\phi\,; t = 2\sin\phi$

$\therefore 6\cos\theta\sin\phi - 6\sin\theta\cos\phi = 6$

$\Rightarrow \sin(\phi - \theta) = 1$

$\Rightarrow \phi = 90° + \theta$

$\Rightarrow P = xz = -6\sin\theta\cos\theta = -3\sin 2\theta$

$\Rightarrow P_{\max} = 3$

87 **(b)**

$2z^2 + 2z + \lambda = 0$

Let the roots be z_1, z_2. Then,

$z_1 + z_2 = -1$ and $z_1 z_2 = \frac{\lambda}{2}$

$0, z_1, z_2$ form, an equilateral triangle

$\therefore z_1^2 + z_2^2 = z_1 z_2$

$\Rightarrow (z_1 + z_2)^2 = 3z_1 z_2$

$\Rightarrow 1 = 3\frac{\lambda}{2}$

$\Rightarrow \lambda = \frac{2}{3}$

88 **(b)**

For given situation, $x^2 - (k-2)x + k^2 = 0$ and $x^2 + kx + 2k - 1 = 0$ should have both roots common or each should have equal roots. If both roots are common, then

$$\frac{1}{1} = \frac{-(k-2)}{k} = \frac{k^2}{2k-1}$$

$\Rightarrow k = -k + 2$ and $2k - 1 = k^2 \Rightarrow k = 1$

If both the equations have equal roots, then

$(k-2)^2 - 4k^2 = 0$ and $k^2 - 4(2k-1) = 0$

$\Rightarrow (3k-2)(-k-2) = 0$ and $k^2 - 8k + 4 = 0$ (no common value)

Therefore, $k = 1$ is the only possible value

89 **(a)**

$x = \sqrt[3]{7} + \sqrt[3]{49}$

$\Rightarrow x^3 = 7 + 49 + 3\sqrt[3]{7}\ \sqrt[3]{49}\left(\sqrt[3]{7} + \sqrt[3]{49}\right)$

$= 56 + 21x$

$\Rightarrow x^3 - 21x - 56 = 0$

Therefore, the product of roots is 56

90 **(a)**

$|x|^2 - 3|x| + 2 = 0$

$\Rightarrow (|x| - 2)(|x| - 1) = 0$

$\Rightarrow |x| = 1$ or 2

$\Rightarrow x = \pm 1, \pm 2$

Hence, there are four real solutions

91 **(d)**

$(x + a)(x + 1991) + 1 = 0$

$\Rightarrow (x + a)(x + 1991) = -1$

$\Rightarrow (x + a) = 1$ and $x + 1991 = -1$

$\Rightarrow a = 1993$

or $x + a = -1$ and $x + 1991 = 1 \Rightarrow a = 1989$

92 **(c)**

We have,

$2 = |z - i\omega| \le |z| + |\omega|$ $(\because |z_1 + z_2| \le |z_1| + |z_2|)$

$\therefore |z| + |\omega| \ge 2$ (i)

But given that $|z| \le 1$ and $|\omega| \le 1$. Hence

$\Rightarrow |z| + |\omega| \le 2$ (ii)

From (i) and (ii),

$|z| = |\omega| = 1$

Also, $|z + i\omega| = |z - i\overline{\omega}|$

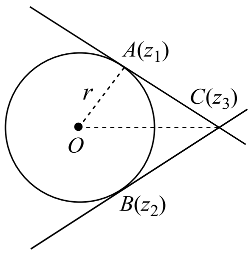
$\Rightarrow |z - (-i\omega)| = |z - i\overline{\omega}|$

Hence, z lies on perpendicular bisector of the line segment joining $(-i\omega)$ and $(i\overline{\omega})$, which is a real axis, as $(-i\omega)$ and $(i\overline{\omega})$ are conjugate to each other. For z, $\mathrm{Im}(z) = 0$. If $z = x$, then

$|z| \le 1 \Rightarrow x^2 \le 1$

$\Rightarrow -1 \le x \le 1$

93 **(b)**



As ΔOAC is a right-angled triangle with right angle at A, so

$|z_1|^2 + |z_3 - z_1|^2 = |z_3|^2$

$\Rightarrow 2|z_1|^2 - \overline{z}_3 z_1 - \overline{z}_1 z_3 = 0$

$\Rightarrow 2\overline{z}_1 - \overline{z}_3 - \frac{\overline{z}_1}{z_1} z_3 = 0$ (1)

Similarly,

$2\overline{z}_2 - \overline{z}_3 - \frac{\overline{z}_2}{z_2} z_3 = 0$ (2)

Subtracting (2) from (1), we get

$$2(\overline{z}_2 - z_1) = z_3\left(\frac{\overline{z}_1}{z_1} - \frac{\overline{z}_2}{z_2}\right)$$

$$\Rightarrow \frac{2r^2(z_1-z_2)}{z_1z_2} = z_3r^2\left(\frac{z_2^2-z_1^2}{z_1^2z_2^2}\right) \quad [\because |z_1|^2 = |z_2|^2 = r^2]$$

$$\Rightarrow z_3 = \frac{2z_1z_2}{z_2+z_1}$$

94 **(d)**

Here,

$$\alpha^4+\beta^4 = (\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2$$

$$= \{(\alpha+\beta)^2 - 2\alpha\beta\}^2 - 2(\alpha\beta)^2$$

$$= \left(p^2+\frac{1}{p^2}\right)^2 - \frac{1}{2p^4}$$

$$= p^4 + \frac{1}{2p^4} + 2$$

$$= \left(p^2 - \frac{1}{\sqrt{2}p^2}\right)^2 + 2 + \sqrt{2} \geq 2+\sqrt{2}$$

Thus, the minimum value of $\alpha^4+\beta^4$ is $2+\sqrt{2}$

95 **(b)**

$ax^2 - bx + c = 0$

$\alpha+\beta = \frac{b}{a}, \alpha\beta = \frac{c}{a}$

Also, $(a+cy)^2 = b^2y$

$\Rightarrow c^2y^2 - (b^2-2ac)y + a^2 = 0$

$$\Rightarrow \left(\frac{c}{a}\right)^2 y^2 - \left(\left(\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right)\right)y + 1 = 0$$

$\Rightarrow (\alpha\beta)^2y^2 - (\alpha^2+\beta^2)y + 1 = 0$

$\Rightarrow y^2 - (\alpha^{-2}+\beta^{-2})y + \alpha^{-2}\beta^{-2} = 0$

$\Rightarrow (y-\alpha^{-2})(y-\beta^{-2}) = 0$

Hence the roots are α^{-2}, β^{-2}

96 **(c)**

We have,

$$(\cos\theta + i\sin\theta)(\cos 2\theta + i\sin 2\theta)\ldots \times (\cos n\theta + i\sin n\theta) = 1$$

$$\Rightarrow \cos(\theta+2\theta+3\theta+\cdots+n\theta) + i\sin(\theta+2\theta+\cdots+n\theta) = 1$$

$$\Rightarrow \cos\left(\frac{n(n+1)}{2}\theta\right) + i\sin\left(\frac{n(n+1)}{2}\theta\right) = 1$$

$$\Rightarrow \cos\left(\frac{n(n+1)}{2}\theta\right) = 1 \text{ and } \sin\left(\frac{n(n+1)}{2}\theta\right) = 0$$

$\Rightarrow \frac{n(n+1)}{2}\theta = 2m\pi \;\Rightarrow\; \theta = \frac{4m\pi}{n(n+1)}$, where $m \in Z$

97 **(b)**

Taking cube roots of both sides, we get

$z + ab = a(1)^{1/3} = a, a\omega, a\omega^2$

Where

$\therefore z_1 = a - ab, z_2 = a\omega - ab, z_3 = a\omega^2 - ab$

$|z_1 - z_2| = |a(1-\omega)|$

$$= |a|\left|1 - \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)\right|$$

$$= |a|\left|\frac{3}{2} - i\frac{\sqrt{3}}{2}\right|$$

$$= |a|\left(\frac{9}{4}+\frac{3}{4}\right)^{1/2} = \sqrt{3}|a|$$

Similarly,

$|z_2 - z_3| = |z_3 - z_1| = \sqrt{3}\,|a|$

98 **(a)**

We have,

$x_1x_2 = 4$

$\Rightarrow x_2 = \frac{4}{x_1}$

$$\therefore \frac{x_1}{x_1-1} + \frac{x_2}{x_2-1} = 2$$

$$\Rightarrow \frac{x_1}{x_1-1} + \frac{\frac{4}{x_1}}{\frac{4}{x_1}-1} = 2$$

$$\Rightarrow \frac{x_1}{x_1-1} + \frac{4}{4-x_1} = 2$$

$\Rightarrow 4x_1 - x_1^2 + 4x_1 - 4 = 2(x_1-1)(4-x_1)$

$\Rightarrow x_1^2 - 2x_1 + 4 = 0$

$\Rightarrow x^2 - 2x + 4 = 0$

99 **(d)**

$\cot^4 x - 2(1+\cot^2 x) + a^2 = 0$

$\Rightarrow \cot^4 x - 2\cot^2 x + a^2 - 2 = 0$

$\Rightarrow (\cot^2 x - 1)^2 = 3 - a^2$

Now, for at least one solution

$3 - a^2 \geq 0$

$\Rightarrow a^2 - 3 \leq 0$

$\therefore a \in [-\sqrt{3}, \sqrt{3}]$

Integral values are $-1, 0, 1$

$\therefore$ sum$= \;0$

100 **(d)**

Here, $x = 0$ is not a root. Divide both the numerator and denominator by x and put $x + 3/x = y$ to obtain

$$\frac{4}{y+1} + \frac{5}{y-5} = -\frac{3}{2} \Rightarrow y = -5, 3$$

$x + 3/x = -5$ has two irrational roots and $x + 3/x = 3$ has imaginary roots

101 **(a)**

$|z_1| = |z_2| = |z_3|$ (given)

Now,

$|z_1| = 1 \;\Rightarrow |z_1|^2 = 1 \;\Rightarrow z_1\bar{z}_1 = 1$

Similarly,

$z_2\bar{z}_2 = 1, z_3\bar{z}_3 = 1$

Now,

$$\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right| = 1$$

$\Rightarrow |\bar{z}_1 + \bar{z}_2 + \bar{z}_3| = 1$

$\Rightarrow |\overline{z_1+z_2+z_3}| = 1$

$\Rightarrow |z_1+z_2+z_3| = 1$

102 **(a)**

Let $z_1 = a + ib$ and $z_2 = c - id$, where $a > 0$ and $d > 0$. Then,

$|z_1| = |z_2| \Rightarrow a^2 + b^2 = c^2 + d^2 \quad (1)$

Now,

$$\frac{z_1 + z_2}{z_1 - z_2} = \frac{(a + ib) + (c - id)}{(a + ib) - (c - id)}$$

$$= \frac{[(a + c) + i(b - d)][(a - c) - i(b + d)]}{[(a - c) + i(b + d)][(a - c) - i(b + d)]}$$

$$= \frac{(a^2 + b^2) - (c^2 + d^2) - 2(ad + bc)i}{a^2 + c^2 - 2ac + b^2 + d^2 + 2bd}$$

$$= \frac{-(ad+bc)i}{a^2+b^2-ac+bd} \quad \text{[Using (1)]}$$

Hence, $(z_1 + z_2)/(z_1 - z_2)$ is purely imaginary. However, if $ad + bc = 0$, then $(z_1 + z_2)/(z_1 - z_2)$ will be equal to zero. According to the conditions of the equation, we can have $ad + bc = 0$

103 **(c)**

We know that $ax^2 + bx + c \geq 0, \forall x \in R$,

If $a > 0$ and $b^2 - 4ac \leq 0$. So,

$$mx - 1 + \frac{1}{x} \geq 0 \Rightarrow \frac{mx^2 - x + 1}{x} \geq 0$$

$\Rightarrow mx^2 - x + 1 \geq 0$ as $x > 0$

Now, $mx^2 - x + 1 \geq 0$ if $m > 0$ and $1 - 4m \leq 0$

$\Rightarrow m > 0$ and $m \geq 1/4$

Thus, the minimum value of m is ¼

104 **(c)**

We have,

$|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2|z_1||z_2| \cos(\theta_1 - \theta_2)$

Where $\theta_1 = \arg(z_1)$ and $\theta_2 = \arg(z_2)$. Given, $\arg(z_1 - z_2) = 0$

$\Rightarrow |z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2|z_1||z_2|$

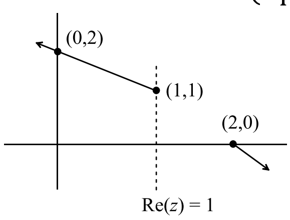
$= (|z_1| - |z_2|)^2$

$\Rightarrow |z_1 - z_2| = ||z_1| - |z_2||$

105 **(d)**

The given equation is written as

$$\arg(z - (1 + i)) = \begin{cases} \frac{3\pi}{4}, & \text{when } x \leq 2 \\ \frac{-\pi}{4}, & \text{when } x > 2 \end{cases}$$



Therefore, the locus is a set of two rays

106 **(d)**

$(1 + i)^{n_1} + (1 + i^3)^{n_1} + (1 + i^5)^{n_2} + (1 + i^7)^{n_2}$

$= [(1 + i)^{n_1} + (1 - i)^{n_1}] + [(1 + i)^{n_2} + (1 - i)^{n_2}]$

$= \left[(1 + i)^{n_1} + \overline{(1 + i)^{n_1}}\right] + [(1 + i)^{n_2} + \overline{(1 + i)^{n_2}}]$

=[purely real number]+[purely real number]

Hence, n_1 and n_2 are any integers

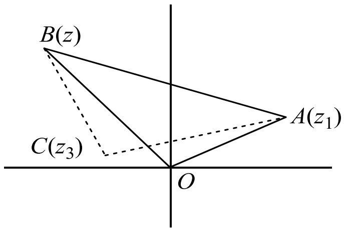
107 **(d)**

$|z_2 + iz_1| = |z_1| + |z_2|$

$\Rightarrow iz_1, 0 + i0$ and z_2 are collinear

$\Rightarrow \arg(iz_1) = \arg(z_2)$

$$\Rightarrow \arg(z_2) - \arg(z_1) = \frac{\pi}{2}$$



Let,

$$z_3 = \frac{z_2 - iz_1}{1 - i}$$

$\Rightarrow (1 - i)z_3 = z_2 - iz_1$

$\Rightarrow z_2 - z_3 = i(z_1 - z_3)$

$$\therefore \angle ACB = \frac{\pi}{2}$$

and

$|z_1 - z_3| = |z_2 - z_3|$

$\Rightarrow AC = BC$

$\because AB^2 = AC^2 + BC^2$

$$\Rightarrow AC = \frac{5}{\sqrt{2}} \quad (\because AB = 5)$$

Therefore area of ΔABC is $(1/2)AC \times BC = AC^2/2 = 25/4$ sq. units

108 **(a)**

Given equation is

$x^2 - (y + 4)x + y^2 - 4y + 16 = 0$

Since x is real, so,

$D \geq 0$

$\Rightarrow (y + 4)^2 - 4(y^2 - 4y + 16) \geq 0$

$\Rightarrow -3y^2 + 24y - 48 \geq 0$

$\Rightarrow y^2 - 8y + 16 \leq 0$

$\Rightarrow (y - 4)^2 \leq 0$

$\Rightarrow y - 4 = 0$

$\Rightarrow y = 4$

Since the equation is symmetric in x and y, therefore $x = 4$ only

109 **(a)**

We have,

$z^3 + 2z^2 + 2z + 1 = 0$

$\Rightarrow (z^3 + 1) + 2z(z + 1) = 0$

$\Rightarrow (z + 1)(z^2 + z + 1) = 0$

$\Rightarrow z = -1, \omega, \omega^2$

Since $z = -1$ does not satisfy $z^{1985} + z^{100} + 1 = 0$ while $z = \omega, \omega^2$ satisfy it, hence sum is $\omega + \omega^2 = -1$

110 **(b)**

Multiplying the given equation by c/a^3, we get

$$\frac{b^2c^2}{a^3}x^2 - \frac{b^2c}{a^2}x + c = 0$$

$$\Rightarrow a\left(\frac{bc}{a^2}x\right)^2 - b\left(\frac{bc}{a^2}\right)x + c = 0$$

$$\Rightarrow \frac{bc}{a^2}x = \alpha, \beta$$

$$\Rightarrow (\alpha+\beta)\alpha\beta x = \alpha, \beta$$

$$\Rightarrow x = \frac{1}{(\alpha+\beta)\alpha}, \frac{1}{(\alpha+\beta)\beta}$$

111 **(d)**

Let $z = x + iy$. Then,

$|z^2 - 1| = |z|^2 + 1$

$\Rightarrow |(x^2 - y^2 - 1) + 2ixy| = x^2 + y^2 + 1$

$\Rightarrow (x^2 - y^2 - 1)^2 + 4x^2y^2 = (x^2 + y^2 + 1)^2$

$\Rightarrow x = 0$

Hence, z lies on imaginary axis

112 **(b)**

Let $z = x + iy$. Then,

$$\text{Re}\left(\frac{1}{z}\right) = k$$

$$\Rightarrow \text{Re}\left(\frac{1}{x+iy}\right) = k$$

$$\Rightarrow \text{Re}\left(\frac{x}{x^2+y^2} - \frac{iy}{x^2+y^2}\right) = k$$

$$\Rightarrow \frac{x}{x^2+y^2} = k$$

$$\Rightarrow x^2 + y^2 - \frac{1}{k}x = 0$$

Which is circle

113 **(d)**

$\sqrt{-x^2 + 10x - 16} < x - 2$

We must have

$-x^2 + 10x - 16 \geq 0$

$\Rightarrow x^2 - 10x + 16 \leq 0$

$\Rightarrow 2 \leq x \leq 8 \quad (1)$

Also, $-x^2 + 10x - 16 < x^2 - 4x + 4$

$\Rightarrow 2x^2 - 14x + 20 > 0$

$\Rightarrow x^2 - 7x + 10 > 0$

$\Rightarrow x > 5$ or $x < 2 \quad (2)$

From (1) and (2)

$5 < x \leq 8 \Rightarrow x = 6, 7, 8$

114 **(d)**

Since, the equation $x^2 - px + r = 0$ has roots (α, β) and the equation $x^2 - qx + r = 0$ has roots $\left(\frac{\alpha}{2}, 2\beta\right)$

$\therefore\ \alpha + \beta = p$ and $r = \alpha\beta$ and $\frac{\alpha}{2} + 2\beta = q$

$\Rightarrow\ \beta = \frac{2q-p}{3}$ and $\alpha = \frac{2(2p-q)}{3}$

$$\therefore\ \alpha\beta = r = \frac{2}{9}(2q-p)(2p-q)$$

115 **(d)**

$$a = \frac{x^2+4}{|x|} - 3$$

$$= |x| + \frac{4}{|x|} - 3 = \left(\sqrt{|x|} - \frac{2}{\sqrt{|x|}}\right) + 1$$

$\Rightarrow a \geq 1$

116 **(d)**

Given,

$$z = \frac{3}{2 + \cos\theta + i\sin\theta}$$

$$\cos\theta + i\sin\theta = \frac{3}{z} - 2 = \frac{3-2z}{z}$$

$$\Rightarrow 1 = \frac{|3-2z|}{|z|} \quad \text{[taking modulus]}$$

$$\Rightarrow \frac{\left|z - \frac{3}{2}\right|}{|z|} = \frac{1}{2}$$

Hence, locus of z is a circle

117 **(b)**

Let $f(x) = -3 + x - x^2$. Then $f(x) < 0$ for all x, because coefficient of x^2 is less than 0 and $D < 0$. Thus, L.H.S. of the given equation is always positive whereas the R.H.S. is always less than zero. Hence, there is no solution

118 **(b)**

For real roots,

$q^2 - 4pr \geq 0$

$\Rightarrow \left(\frac{p+r}{2}\right)^2 - 4pr \geq 0 \quad (\because p, q, r$ are in A.P.)

$\Rightarrow p^2 + r^2 - 14pr \geq 0$

$$\Rightarrow \frac{p^2}{r^2} - 14\frac{p}{r} + 1 \geq 0$$

$$\Rightarrow \left(\frac{p}{r} - 7\right)^2 - 48 \geq 0$$

$$\Rightarrow \left|\frac{p}{r} - 7\right| \geq 4\sqrt{3}$$

119 **(a)**

$$\arg(-z) - \arg(z) = \arg\left(\frac{-z}{z}\right) = \arg(-1) = \pi$$

120 **(c)**

Since $\alpha, \beta, \gamma, \sigma$ are the roots of the given equation, therefore

$x^4 + 4x^3 - 6x^2 + 7x - 9$
$\qquad = (x-\alpha)(x-\beta)(x-\gamma)(x-\sigma)$

Putting $x = i$ and then $x = -i$, we get

$1 - 4i + 6 + 7i - 9 = (i-\alpha)(i-\beta)(i-\gamma)(i-\sigma)$

and $1 + 4i + 6 - 7i - 9 = (-i-\alpha)(-i-\beta)(-i-\gamma(-i-\sigma)$

Multiplying these two equations, we get

$(-2+3i)(-2-3i)$
$\qquad = (1+\alpha^2)(1+\beta^2)(1+\gamma^2)(1+\sigma^2)$

$\Rightarrow 13 = (1+\sigma^2)(1+\beta^2)(1+\gamma^2)(1+\sigma^2)$

121 **(b)**

Let $z_1 = |z_1|(\cos\theta_1 + i\sin\theta_1)$. Now,

$\left|\frac{z_1}{z_2}\right| = 1 \Rightarrow |z_1| = |z_2|$

Also,

$\arg(z_1 z_2) = 0 \ \Rightarrow \arg(z_1) + \arg(z_2) = 0$

$\Rightarrow \arg(z_2) = -\theta_1$

$\Rightarrow z_2 = |z_2|(\cos(-\theta_1) + i\sin(\theta_1))$

$= |z_1|(\cos\theta_1 - i\sin\theta_1) = \overline{z}_1$

$\Rightarrow \overline{z}_2 = \overline{(\overline{z}_1)} = z_1$

$\Rightarrow |z_2|^2 = z_1 z_2$

122 **(d)**

Let $z_1 = \sin x + i\cos 2x$; $z_2 = \cos x - i\sin 2x$. Then

$\overline{z}_1 = z_2$

$\Rightarrow \sin x - i\cos 2x = \cos x - i\sin 2x$

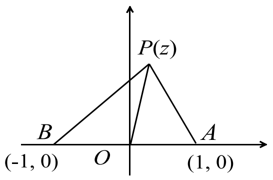
$\Rightarrow \sin x = \cos x$ and $\cos 2x = \sin 2x$

$\Rightarrow \tan x = 1$ and $\tan 2x = 1$

$\Rightarrow x = \frac{\pi}{4}$ and $x = \frac{\pi}{8}$

Which is not possible. Hence, there is no value of x

123 **(a)**



When $|z-1| < |z+1|$ (or $x > 0$)

$|z| = |z-1|$

$\Rightarrow x^2 + y^2 = (x-1)^2 + y^2$

$\Rightarrow x = 1/2$

$\Rightarrow z + \overline{z} = 1$

When $|z-1| > |z+1|$(or $x < 0$)

$|z| = |z+1|$

$\Rightarrow x^2 + y^2 = (x+1)^2 + y^2$

$\Rightarrow x = -1/2$

$\Rightarrow z + \overline{z} = -1$

124 **(b)**

Let $z = x + iy$. Then,

$x = \lambda + 3$ and $y = -\sqrt{5-\lambda^2}$

$\Rightarrow (x-3)^2 = \lambda^2 \quad (1)$

and

$y^2 = 5 - \lambda^2 \qquad (2)$

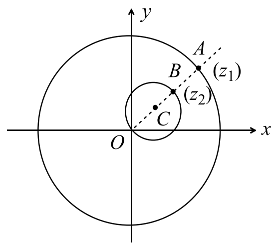
From (1) and (2),

$(x-3)^2 = 5 - y^2 \Rightarrow (x-3)^2 + y^2 = 5$

Obviously it is a semicircle as $y < 0$. Hence part of the circle lies below the x-axis

125 **(b)**

$|z_1| = 12$. Therefore, z_1 lies on a circle with centre (0, 0) and radius 12 units. As $|z_2 - 3 - 4i| = 5$, so z_2 lies on a circle with centre (3, 4) and radius 5 units



From the above figure it is clear that $|z_1 - z_2|$, i.e., distance between z_1 and z_2 will be minimum when they lie at A and B, respectively. i.e., on diagram as shown. Then $|z_1 - z_2| = AB = OA - OB = 12 - 2(5) = 2$. As it is the minimum value, we must have $|z_1 - z_2| \geq 2$

126 **(a)**

Given, α, β are roots of the equation $(x-a)(x-b+c=0$

Then, by factor theorem,

$(x-a)(x-b) + c = (x-\alpha)(x-\beta)$

Replacing x by $x - c$,

$(x-c-a)(x-c-b) + c$

$\qquad = (x-c-\alpha)(x-c-\beta)$

$\Rightarrow (x-c-\alpha)(x-c-\beta) - c$

$\qquad = [x-(c+a)][x-(c+b)]$

Then, again by factor theorem roots of the equation $(x-c-\alpha)(x-c-\beta) - c = 0$ are $a + c$ and $b + c$

127 **(b)**

$\tan\theta_1 + \tan\theta_2 + \tan\theta_3 = (a+1)$

$\sum \tan\theta_1 \tan\theta_2 = (b-a)$

$\tan\theta_1 \tan\theta_2 \tan\theta_3 = b$

$\therefore \tan(\theta_1 + \theta_2 + \theta_3) = \frac{\sum\tan\theta_1 - \prod\tan\theta_1}{1 - \sum\tan\theta_1\tan\theta_2}$

$= \frac{a+1-b}{1-(b-a)} = 1$

$\Rightarrow \theta_1 + \theta_2 + \theta_3 = \frac{\pi}{4}$

128 **(a)**

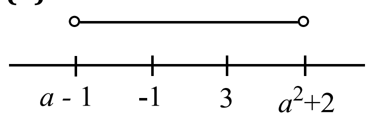
$|z| = 1$, let $\alpha = -1 + 3z$

$\Rightarrow \alpha + 1 = 3z$

$\Rightarrow |\alpha + 1| = 3|z| = 3$

Hence, 'α' lies on a circle centred at -1 and radius equal to 3

129 **(b)**



We have,

$a - 1 \leq -1$ and $a^2 + 2 \geq 3$

$a \leq 0$ and $a^2 \geq 1$

Hence, $a \leq -1$

130 **(b)**

$x^4 + x^2 + 1 = (x^2+1)^2 - x^2$

$= (x^2 + x + 1)(x^2 - x + 1)$
$x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} \neq 0\ \forall\ x$
Therefore, we can cancel this factor and we get
$(a - 1)(x^2 - x + 1) = (a + 1)(x^2 - x + 1)$
or $x^2 - ax + 1 = 0$
It has real and distinct roots if $D = a^2 - 4 > 0$

131 **(a)**
Given equation is
$x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$
Clearly, $x \neq 1$ for the given equation to be defined if $x - 1 \neq 0$. We can cancel the common term $-2/(x-1)$ on both sides to get $x = 1$, but it is not possible. So, given equation has no roots

132 **(c)**
α, β are roots of the equation $(x - a)(x - b) = c, c \neq 0$
$\therefore\ (x - a)(x - b) - c = (x - \alpha)(x - \beta)$
$\Rightarrow (x - \alpha)(x - \beta) + c = (x - a)(x - b)$
Hence, the roots of $(x - \alpha)(x - \beta) + c = 0$ are a and b

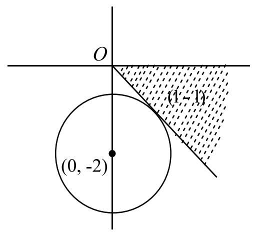
133 **(c)**
Let $z = \cos x + i \sin x, x \in [0, 2\pi)$. Then,
$1 = \left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right|$
$= \frac{|z^2 + \bar{z}^2|}{|z|^2}$
$= |\cos 2x + i \sin 2x + \cos 2x - i \sin 2x|$
$= 2\,|\cos 2x|$
Now,
$\cos 2x = 1/2$
$\Rightarrow x_1 = \frac{\pi}{6}, x_2 = \frac{5\pi}{6}, x_3 = \frac{7\pi}{6}, x_4 = \frac{11\pi}{6}$
$\cos 2x = -\frac{1}{2}$
$\Rightarrow x_5 = \frac{\pi}{3}, x_6 = \frac{2\pi}{3}, x_7 = \frac{4\pi}{3}, x_8 = \frac{5\pi}{3}$

134 **(c)**
Clearly, $x = -1$ satisfies the equation

135 **(b)**
If $[m_r, (1/m_r)]$ satisfy the given equation $x^2 + y^2 + 2gx + 2fy + c = 0$, then
$m_r^2 + \frac{1}{m_r^2} + 2gm_r + \frac{2f}{m_r} + c = 0$
$\Rightarrow m_r^4 + 2gm_r^3 + cm_r^2 + 2fm_r + 1 = 0$
Now, roots of given equation are m_1, m_2, m_3, m_4.
The product of roots
$m_1m_2m_3m_4 = \frac{\text{contant term}}{\text{coefficient of } m_r^4} = \frac{1}{1} = 1$

136 **(c)**
$|k + z^2| = |z^2| - k = |z^2| + |k|$
$\Rightarrow k, z^2$ and $0 + i0$ are collinear
$\Rightarrow \arg(z^2) = \arg(k)$
$\Rightarrow 2\arg(z) = \pi$
$\Rightarrow \arg(z) = \frac{\pi}{2}$

137 **(d)**
We are given that $p(-a) = a$ and $p(a) = -a$ [since when a polynomial $f(x)$ is divided by $x - a$, remainder is $f(a)$]. Let the remainder, when $p(x)$ is divided by $x^2 - a^2$, be $Ax + B$. Then
$p(x) = Q(x)(x^2 - a^2) + Ax + B$ (1)
Where $Q(x)$ is the quotient. Putting $x = a$ and $-a$ in (1), we get
$p(a) = 0 + Aa + B \Rightarrow -a = Aa + B$ (2)
and $p(-a) = 0 - aA + B \Rightarrow a = -aA + B$ (3)
Solving (2) and (3), we get
$B = 0$ and $A = -1$
Hence, the required remainder is $-x$

138 **(c)**


$kz/(k + 1)$ represents any point lying on the line joining origin and z
Given,
$\left|\frac{kz}{k+1} + 2i\right| > \sqrt{2}$
Hence, $kz/(k + 1)$ should lie outside the circle $|z + 2i| > \sqrt{2}$. So, z should lie in the shaded region
$\therefore\ -\frac{\pi}{4} < \arg(z) < 0$

139 **(d)**
Given expression is
$x^{12} - x^9 + x^4 - x + 1 = f(x)$
For $x < 0$, put $x = -y$, where $y > 0$. Thus, we get
$f(x) = y^{12} + y^9 + y^4 + y + 1 > 0$ for $y > 0$
For $0 < x < 1$,
$x^9 < x^4 \Rightarrow -x^9 + x^4 > 0$
Also, $1 - x > 0$ and $x^{12} > 0$
$\Rightarrow x^{12} - x^9 + x^4 + 1 - x > 0 \Rightarrow f(x) > 0$
For $x > 1$,
$f(x) = x(x^3 - 1)(x^8 + 1) + 1 > 0$
So $f(x) > 0$ for $-\infty < x < \infty$

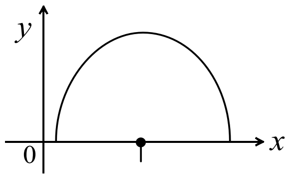
140 **(b)**
$|2z - 1| = |z - 2|$
$\Rightarrow |2z - 1|^2 = |z - 2|^2$
$\Rightarrow (2z - 1)(2\bar{z} - 1) = (z - 2)(\bar{z} - 2)$

$\Rightarrow 4z\bar{z}-2\bar{z}-2z+1=z\bar{z}-2\bar{z}-2z+4$
$\Rightarrow 3|z|^2=3$
$\Rightarrow |z|=1$
Again,
$|z_1+z_2|=|z_1-\alpha+z_2-\beta+\alpha+\beta|$
$\leq |z_1-\alpha|+|z_2-\beta|+|\alpha+\beta|$
$<\alpha+\beta+|\alpha+\beta|$
$=2|\alpha+\beta|\quad [\because \alpha,\beta>0]$
$\therefore \left|\dfrac{z_1+z_2}{\alpha+\beta}\right|<2$
$\Rightarrow \left|\dfrac{z_1+z_2}{\alpha+\beta}\right|<2|z|$

141 **(b)**
The discriminant of the given equations are
$D_1=a^2+12b, D_2=c^2-4b$ and $D_3=d^2-8b$
$\therefore D_1+D_2+D_3=a^2+c^2+d^2\geq 0$
Hence, at least one of D_1, D_2, D_3 is non-negative.
Therefore, the equation has at least two real roots

142 **(a)**
The equation on simplifying gives
$x(x-b)(x-c)+x(x+c)(x-a)$
$+x(x-a)(x-b)$
$-(x-a)(x-b)(x-c)=0\quad (1)$
Let,
$f(x)=x(x-b)(x-c)+x(x-c)(x-a)$
$+x(x-a)(x-b)$
$-(x-a)(x-b)(x-c)$
We can assume without loss of generality that $a<b<c$. Now,
$f(a)=a(a-b)(a-c)>0$
$f(b)=b(b-c)(b-a)<0$
$f(c)=c(c-a)(c-b)>0$
So, one root of (1) lies in (a,b) and one root in (b,c). Obviously the third root must also be real

143 **(b)**
Note that coefficient of x^2 is $(4p-p^2-5)<0$. Therefore the graph is concave downward.
According to the question, 1 must lie between the roots. Hence,
$f(1)>0$
$\Rightarrow 4p-p^2-5-2p+1+3p>0$
$\Rightarrow -p^2+5p-4>0$
$\Rightarrow p^2-5p+4<0$



$\Rightarrow (p-4)(p-1)<0$
$\Rightarrow 1<p<4$
$\Rightarrow p\in[2,3]$

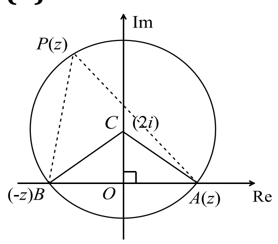
144 **(a)**
$\alpha=\dfrac{z-\bar{w}}{k^2+z\bar{w}} \Rightarrow \bar{\alpha}=\dfrac{\bar{z}-w}{k^2+\bar{z}w}$
But $z\bar{z}=w\bar{w}=k^2$. Hence
$\Rightarrow \bar{\alpha}=\dfrac{\frac{k^2}{z}-\frac{k^2}{\bar{w}}}{k^2+\frac{k^2}{z}\frac{k^2}{\bar{w}}}=\dfrac{\bar{w}-z}{z\bar{w}+k^2}=-\alpha$
$\Rightarrow \alpha+\bar{\alpha}=0$
$\Rightarrow \mathrm{Re}(\alpha)=0$

145 **(d)**
Minimum value of $f(x)=(1+b^2)x^2+2bx+1$ is
$m(b)=-\dfrac{(2b)^2-4(1+b^2)}{4(1+b^2)}=\dfrac{1}{1+b^2}$
Clearly, $m(b)$ has range $(0,1]$

146 **(b)**
Correct equation is
$x^2+13x+q=0\quad (1)$
Incorrect equation is
$x^2+17x+q=0\quad (2)$
Given that roots of Eq. (1) are -2 and -15.
Therefore, product of the roots of incorrect equation is $q=(-2)(-15)=30$. From (1), the correct equation is
$x^2+13x+30=0$
$\therefore x=-3,-10$

147 **(a)**
$8iz^3+12z^2-18z+27i=0$
$\Rightarrow 4iz^2(2z-3i)-9(2z-3i)=0$
$\Rightarrow (2z-3i)(4iz^2-9)=0$
$\Rightarrow z=\dfrac{3i}{2}$ and $z^2=\dfrac{9}{4i}$
$\Rightarrow |z|=\dfrac{3}{2}$ and $|z^2|=\dfrac{9}{4}$
$\Rightarrow |z|=\dfrac{3}{2}$

148 **(b)**



$CA=CB=2\sqrt{2}, OC=2$
$\Rightarrow OA=OB=2$
$\Rightarrow A\equiv 2+0i, B=-2+0i$
Clearly,
$\angle BCA=\pi/2$
$\Rightarrow \angle BPA=\pi/4$
$\Rightarrow \arg\left(\dfrac{z-2}{z+2}\right)=\dfrac{\pi}{4}$

149 **(d)**

We have,

$e^{\frac{2\pi ri}{p}} = e^{\frac{2\pi m}{q}}$

$r = 0, 1, \dots, p-1$

$m = 0, 1, \dots, q-1$

This is possible iff $r = m = 0$ but for $r = m = 0$ we get I which is not an imaginary number

150 **(a)**

Given,

$$z_k = 1 + a + a^2 + \cdots + a^{k-1} = \frac{1-a^k}{1-a}$$

$$\Rightarrow z_k - \frac{1}{1-a} = -\frac{a^k}{1-a}$$

$$\Rightarrow \left|z_k - \frac{1}{1-a}\right| = \frac{|a|^k}{|1-a|} < \frac{1}{|1-a|} \quad [\because |a| < 1]$$

Hence, z_k lies within the circle

$$\therefore \left|z - \frac{1}{1-a}\right| = \frac{1}{|1-a|}$$

151 **(b)**

$$\left(31 + 8\sqrt{15}\right)^{x^2-3} + 1 = \left(32 + 8\sqrt{15}\right)^{x^2-3}$$

$$\Rightarrow \left(31 + 8\sqrt{15}\right)^{x^2-3} + 1^{x^2-3} = \left(32 + 8\sqrt{15}\right)^{x^2-3}$$

$\Rightarrow x^2 - 3 = 1$ or $x = \pm 2$ $[\because a^n + b^n = (a+b)^n]$

152 **(d)**

We know that if $f(\alpha)$ and $f(\beta)$ are of opposite signs then there must be a value γ between α and β such that $f(\gamma) = 0$. Hence, a, b, c are real numbers and $a \neq 0$. As α is a root of $a^2x^2 + bx + c = 0$, so

$a^2\alpha^2 + b\alpha + c = 0 \quad (1)$

Also, β is a root of $a^2x^2 - bx - c = 0$, so

$a^2\beta^2 - b\beta - c = 0 \quad (2)$

Now, let $f(x) = a^2x^2 + 2bx + 2c$. Then,

$f(\alpha) = a^2\alpha^2 + 2b\,\alpha + 2c$

$= a^2\alpha^2 + 2(b\alpha + c)$

$= a^2\alpha^2 + 2(-a^2\alpha^2) \quad$ [Using (1)]

$= -a^2\alpha^2 < 0$

and $f(\beta) = a^2\beta^2 + 2b\,\beta + 2c$

$= a^2\beta^2 + 2(b\,\beta + c)$

$= a^2\beta^2 + 2(a^2\beta^2) \quad$ [Using (2)]

$= 3a^2\beta^2 > 0$

Since $f(\alpha)$ and $f(\beta)$ are of opposite signs and γ is a root of equation $f(x) = 0$, therefore, γ must lie between α and β. Thus, $\alpha < \gamma < \beta$

153 **(a)**

α, β are roots of $ax^2 + bx + c = 0$. Hence,

$\alpha + \beta = -\frac{b}{a} \quad (1)$

$\alpha\beta = \frac{c}{a} \quad (2)$

α^2, β^2 are roots of $a^2x^2 + b^2x + c^2 = 0$. Hence,

$\alpha^2 + \beta^2 = -\frac{b^2}{a^2} \quad (3)$

$\alpha^2\beta^2 = \frac{c^2}{a^2} \quad (4)$

Now, from (3),

$$(\alpha + \beta)^2 - 2\alpha\beta = -\frac{b^2}{a^2}$$

$$\Rightarrow \left(\frac{-b}{a}\right)^2 - 2\frac{c}{a} = \frac{-b^2}{a^2}$$

$$\Rightarrow 2\frac{b^2}{a^2} = \frac{2c}{a}$$

$\Rightarrow b^2 = ac \Rightarrow a, b, c$ are in G.P.

154 **(c)**

The given equation is

$2^{2x} + (a-1)2^{x+1} + a = 0$

or

$t^2 + 2(a-1)t + a = 0$, where $2^x = t$

Now, $t = 1$ should lie between the roots of this equation

$$\therefore\ 1 + 2(a-1) + a < 0 \Rightarrow a < \frac{1}{3}$$

155 **(a)**

Since, $\quad z\bar{z}(z^2 + \bar{z}^2) = 350$

$\Rightarrow \quad 2(x^2 + y^2)(x^2 - y^2) = 350$

$\Rightarrow \quad (x^2 + y^2)(x^2 - y^2) = 175$

Since, $x, y \in I$, the only possible case which gives integral solution, is

$x^2 + y^2 = 25 \qquad$ ...(i)

$x^2 - y^2 = 7 \qquad$ ...(ii)

From Eqs. (i) and (ii), we get

$x^2 = 16, \quad y^2 = 9$

$\Rightarrow \qquad x = \pm 4, \quad y = \pm 3$

$\therefore$ Area of rectangle$= 8 \times 6 = 48$

156 **(d)**

Here, $f(x) = (2x-a)(2x-c) + (2x-b)$. So,

$$f\left(\frac{a}{2}\right) = a - b, f\left(\frac{c}{2}\right) = c - b$$

Now,

$$f\left(\frac{a}{2}\right)f\left(\frac{c}{2}\right) = (a-b)(c-b) < 0 (a > b > c)$$

Hence, exactly one of the roots lies between $c/2$ and $a/2$

157 **(c)**

$$\left|\sum_{r=1}^{n} z_r\right| \leq \sum_{r=1}^{n} |z_r| \leq \sum_{r=1}^{n} |z_r - r| + \sum_{r=1}^{n} r \leq 2\sum_{r=1}^{n} r$$

158 **(b)**

$(ax^2 + c)y + (a'x^2 + c') = 0$

or $x^2(ay + a') + (cy + c') = 0$

It x is rational, then the discriminant of the above equation must be a perfect square. Hence,

$0 - 4(ay + a')(cy + c')$ must be a perfect square

$\Rightarrow -acy^2 - (ac' + a'c)y - a'c'$ must be a perfect square

$\Rightarrow (ac' + a'c)^2 - 4ac\, a'c' = 0 \quad [\because D = 0]$
$\Rightarrow (ac' - a'c)^2 = 0$
$\Rightarrow ac' = a'c$
$\Rightarrow \frac{a}{a'} = \frac{c}{c'}$

159 **(d)**
We have,
$|z| = |\omega|$ and $\arg z = \pi - \arg \omega$
Let $\omega = re^{i\theta}$. Then
$z = re^{i(\pi-\theta)}$
$\Rightarrow z = re^{i\pi}e^{-i\theta}(re^{-i\theta})(\cos \pi + i \sin \pi)$
$= \overline{\omega}(-1) = -\overline{\omega}$

160 **(b)**
Let $f(x) = ax^2 + bx + c$ be a quadratic expression such that $f(x) > 0$ for all $x \in R$. Then, $a > 0$ and $b^2 - 4ac < 0$. Now, $g(x) = f(x) + f'(x) + f''(x)$
$\Rightarrow g(x) = ax^2 + x(b + 2a) + (b + 2a + c)$
Discriminant of $g(x)$ is
$D = (b + 2a)^2 - 4a(b + 2a + c)$
$= b^2 - 4a^2 - 4ac$
$= (b^2 - 4ac) - 4a^2$
< 0 ($\because b^2 - 4ac < 0$)
Therefore, $g(x) > 0$ for all $x \in R$

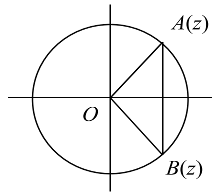
161 **(a)**
$a_0z^n + a_1z^{n-1} + a_2z^{n-2} + \cdots + a_{n-1}z + a_n = 3$
$\Rightarrow |3| = |a_0z^n + a_1z^{n-1} + \cdots + a_{n-1}z + a_n|$
$\Rightarrow 3 \leq |a_0||z|^n + |a_1||z|^{n-1} + \cdots + |a_{n-1}||z| + |a_n|$
$\Rightarrow 3 < 2(|z|^n + |z|^{n-1} + \cdots + |z| + 1)$
$\Rightarrow 1 + |z| + |z|^2 + \cdots + |z|^n > \frac{3}{2}$
If $|z| \geq 1$, the inequality is clearly satisfied. For $|z| < 1$, we must have,
$\frac{1 - |z|^{n+1}}{1 - |z|} > \frac{3}{2}$
$\Rightarrow 2 - 2|z|^{n+1} > 3 - 3|z|$
$\Rightarrow 2|z|^{n+1} < 3|z| - 1$
$\Rightarrow 3|z| - 1 > 0$
$\Rightarrow |z| > \frac{1}{3}$

162 **(c)**
$\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$
$\Rightarrow \arg\left(\frac{z_1 - z_3}{z_2 - z_3}\right) = \arg\left(\frac{1 - i\sqrt{3}}{2}\right)$
Hence, the angle between $z_1 - z_3$ and $z_2 - z_3$ is $60°$. Also,
$\left|\frac{z_1 - z_3}{z_2 - z_3}\right| = \left|\frac{1 - i\sqrt{3}}{2}\right|$
$\Rightarrow \left|\frac{z_1 - z_3}{z_2 - z_3}\right| = 1$
$\Rightarrow |z_1 - z_2| = |z_2 - z_3|$
Hence, the triangle with vertices z_1, z_2 and z_3 is isosceles with vertices angle $60°$. Hence rest of the two angles should also be $60°$each. Therefore, the required triangle is an equilateral triangle

163 **(d)**
Let $z = x + iy$, so that $\overline{z} = x - iy$
$\therefore z^2 + \overline{z} = 0$
$\Rightarrow (x^2 - y^2 + x) + i(2xy - y) = 0$
Equating real and imaginary parts, we get
$x^2 - y^2 + x = 0$
and $2xy - y = 0 \Rightarrow y = 0$ or $x = \frac{1}{2}$
If $y = 0$, then (1) gives $x^2 + x = 0 \Rightarrow x = 0$ or $x = -1$
If $x = 1/2$, then from (1),
$y^2 = \frac{1}{4} + \frac{1}{2} = \frac{3}{4} \Rightarrow y = \pm\frac{\sqrt{3}}{2}$
Hence, there are four solutions in all

164 **(b)**
Let,
$S = 1 + 2a + 3a^2 + \cdots + n\alpha^{n-1}$
$\Rightarrow \alpha S = \alpha + 2\alpha^2 + 3\alpha^3 + \cdots + (n-1)\alpha^{n-1} + n\alpha^n$
On subtracting, we get
$S(1 - \alpha) = 1 + [\alpha + \alpha^2 + \alpha^{n-1}] - n\alpha^n$
$= 1 + \frac{\alpha(1 - \alpha^{n-1})}{1 - \alpha} - n\alpha^n$
$\Rightarrow S = \frac{1}{1 - \alpha} + \frac{\alpha - \alpha^n}{(1 - \alpha)^2} - \frac{n\alpha^n}{1 - \alpha} \quad [\because \alpha^n = 1]$
$= \frac{1}{1 - \alpha} + \frac{\alpha - 1}{(1 - \alpha)^2} - \frac{n}{1 - \alpha} = -\frac{n}{1 - \alpha}$

165 **(c)**
Since α, β are the roots of the equation $2x^2 - 35x + 2 = 0$, therefore,
$2\alpha^2 - 35\alpha = -2$ or $2\alpha - 35 = \frac{-2}{\alpha}$
and $2\beta^2 - 35\beta = -2$ or $2\beta - 35 = \frac{-2}{\beta}$
Now,
$(2\alpha - 35)^3(2\beta - 35)^3 = \left(\frac{-2}{\alpha}\right)^3\left(\frac{-2}{\beta}\right)^3$
$= \frac{8 \times 8}{\alpha^3\beta^3} = \frac{64}{1} = 64 \quad (\because \alpha\beta = 1)$

166 **(a)**


$|z - \overline{z}|$ is the length AB while $|z|(\arg z - \arg \overline{z})$ is arc length AB
$\therefore |z - \overline{z}| \leq |z|(\arg z - \arg \overline{z})$

167 **(c)**

$D > 0 \Rightarrow (a-3)^2 + 4(a+2) > 0$

$\Rightarrow a^2 - 6a + 9 + 4a + 8 > 0$

$\Rightarrow a^2 - 2a + 17 > 0$

$\Rightarrow a \in R$

$\therefore \dfrac{a^2+1}{a^2+2} = 1 - \dfrac{1}{a^2+2} \geq \dfrac{1}{2}$

168 **(c)**

$A_{n+1} = \alpha^{n+1} + \beta^{n+1}$

$= \alpha^{n+1} + \alpha^n\beta + \beta^{n+1} + \alpha\beta^n - \alpha^n\beta - \alpha\beta^n$

$= \alpha^n(\alpha+\beta) + \beta^n(\beta+\alpha) - \alpha\beta(\alpha^{n-1} + \beta^{n-1})$

$= \alpha^n(\alpha+\beta) + \beta^n(\beta+\alpha) - \alpha\beta(\alpha^{n-1} + \beta^{n-1})$

$= (\alpha+\beta)(\alpha^n + \beta^n) - \alpha\beta(\alpha^{n-1} + \beta^{n-1})$

$= aA_n - bA_{n-1}$

169 **(d)**

Let $f(x) = x^6 + 4x^5 + 3x^4 + 2x^3 + x + 1$. Hence,

$f(\omega) = \omega^6 + 4\omega^5 + 3\omega^4 + 2\omega^3 + \omega + 1$

$= 1 + 4\omega^2 + 3\omega + 2 + \omega + 1$

$= 4(\omega^2 + \omega + 1)$

$= 0$

Hence, $f(x)$ is divisible by $x - \omega$. Then $f(x)$ is also divisible by $x - \omega^2$ (as complex roots occur in conjugate pairs)

$f(-\omega) = (-\omega)^6 + 4(-\omega)^5 + 3(-\omega)^4 + 2(-\omega)^3 + (-\omega) + 1$

$= \omega^6 - 4\omega^5 + 3\omega^4 - 2\omega^3 - \omega + 1$

$= 1 - 4\omega^2 + 3\omega - 2 - \omega + 1$

$\neq 0$

170 **(c)**

Since, $(\alpha+\beta), (\alpha^2+\beta^2)$ and $(\alpha^3+\beta^3)$ are in GP.

$(\alpha^2+\beta^2)^2 = (\alpha+\beta)(\alpha^3+\beta^3)$

$\Rightarrow \alpha^4 + \beta^4 + 2\alpha^2\beta^2 = \alpha^4 + \beta^4 + \alpha\beta^3 + \beta\alpha^3$

$\Rightarrow \alpha\beta(\alpha^2 + \beta^2 - 2\alpha\beta) = 0$

$\Rightarrow \alpha\beta(\alpha-\beta)^2 = 0$

$\Rightarrow \alpha\beta = 0$ or $\alpha = \beta$

$ie, \frac{c}{a} = 0$ or $\Delta = 0$

$\Rightarrow c\Delta = 0$

171 **(a)**

Assuming $\arg z_1 = \theta$ and $\arg z_2 = \theta + \alpha$,

$\dfrac{az_1}{bz_2} + \dfrac{bz_2}{az_1} = \dfrac{a|z_1|e^{i\theta}}{b|z_2|e^{i(\theta+\alpha)}} + \dfrac{b|z_2|e^{i(\theta+\alpha)}}{a|z_1|e^{i\theta}}$

$= e^{-i\alpha} + e^{i\alpha} = 2\cos\alpha$

Hence, the point lies on the line segment $[-2, 2]$ of the real axis

172 **(a)**

Let $z = x + iy$. Then,

$|z - 3 - i| = |z - 9 - i|$

$\Rightarrow \sqrt{(x-3)^2 + (y-1)^2} = \sqrt{(x-9)^2 + (y-1)^2}$

$\Rightarrow x = 6$

$|z - 3 + 3i| = 3$

$\Rightarrow \sqrt{(x-3)^2 + (y+3)^2} = 3$

For $x = 6, y = -3$

$\therefore\ z = 6 - 3i$

173 **(b)**

$2\left|z - \frac{1}{2}\right| = |z - 1|$

$\therefore \dfrac{|z-1|}{\left|z - \frac{1}{2}\right|} = 2$

So, locus of z is a circle

174 **(a)**

$xy = 2(x+y) \Rightarrow y(x-2) = 2x$

$\therefore y = \dfrac{2x}{x-2} = 2 + \dfrac{4}{x-2} \Rightarrow x$

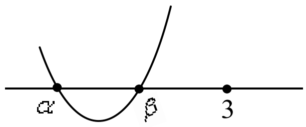
$= 3, 4\ (x \neq 6 \text{ as } x < y)$

By trial, $x = 3, 4, 6$. Then $y = 6, 4, 3$. But $x \leq y$.

Therefore, $x = 3, 4$ and $y = 6, 4$ are two solutions

175 **(a)**

If both the roots of a quadratic equation $ax^2 + bx + c = 0$ are less than k, then $af(k) > 0, -b/2a < k$ and $D \geq 0$. Now,



$f(x) = x^2 - 2ax + a^2 + a - 3$

$\Rightarrow f(3) > 0, a < 3, -4a + 12 \geq 0$

$\Rightarrow a^2 - 5a + 6 > 0, a < 3, -4a + 12 \geq 0$

$\Rightarrow a < 2$ or $a > 3, a < 3, a \leq 3$

$\Rightarrow a < 2$

176 **(a)**

Suppose $a \neq 0$. We rewrite $f(x)$ as follows:

$f(x) = a\left\{x^2 + \frac{b}{a}x + \frac{c}{a}\right\}$

$= a\left\{\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a^2}\right\}$

$f\left(-\frac{b}{2a} + i\right) = a\left\{\left(-\frac{b}{2a} + i + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a^2}\right\}$

$= a\left\{-1 + \frac{4ac-b^2}{4a^2}\right\}$, which is real number. This is against the hypothesis. Therefore, $a = 0$

177 **(a)**

$\dfrac{1+iz}{1-iz} = \dfrac{1 + i(b+ic)/(1+a)}{1 - i(b+ic)/(1+a)}$

$= \dfrac{1+a-c+ib}{1+a+c-ib}$
$= \dfrac{(1+a-c+ib)(1+a+c+ib)}{(1+a+c)^2+b^2}$
$= \dfrac{1+2a+a^2-b^2-c^2+2ib+2iab}{1+a^2+c^2+b^2+2ac+2(a+c)}$
$= \dfrac{2a+2a^2+2ib+2iab}{2+2ac+2(a+c)} \quad (\because a^2+b^2+c^2=1)$
$= \dfrac{a+a^2+ib+iab}{1+ac+(a+c)}$
$= \dfrac{a(a+1)+ib(a+1)}{(a+1)(c+1)}$
$= \dfrac{a+ib}{c+1}$

178 **(a)**
Since R.H.S. is an even integer. So L.H.S. is also an even integer. So, either both x and y are even integers, or both of them are odd integers. Now,
$x^4-y^4=(x-y)(x+y)(x^2+y^2)$
$\Rightarrow x-y, x+y, x^2+y^2$ must be an even integer
Therefore, $(x-y)(x+y)(x^2+y^2)$ must be divisible by 8. But R.H.S. is not divisible by 8.
Hence, the given equation has no solution

179 **(c)**
Clearly, $x=2$ is a root of the equation and imaginary roots always occur in pairs. Therefore, the other root is also real

180 **(b)**
$(1+\omega)^7=A+B\omega$
$\Rightarrow (-\omega^2)^7=A+B\omega \quad (\because 1+\omega+\omega^2=0)$
$\Rightarrow -\omega^{14}=A+B\omega$
$\Rightarrow -\omega^2=A+B\omega \quad (\because \omega^3=1)$
$\Rightarrow 1+\omega=A+B\omega$
$\Rightarrow A=1, B=1$

181 **(d)**
$10z^2-3iz-k=0$
$\Rightarrow z=\dfrac{3i\pm\sqrt{-9+40k}}{20}$
Now, $D=-9+40k$. If $k=1$, then $D=31$. So (a) is false
If k is a negative real number, then D is a negative real number. So (d) is true
If $k=i$, then $D=-9+40i=16+40i-25=(4+5i)^2$, and the roots are $(1/5)+(2/5)i$. So (c) is false
If $k=0$ (which is a complex number), then the roots are 0 and $(3/10)i$. So (b) is false

182 **(b)**
Let,
$\sin\dfrac{\pi}{8}+i\cos\dfrac{\pi}{8}=z$
$\Rightarrow \left[\dfrac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right]^8$
$= \left(\dfrac{1+z}{1+\frac{1}{z}}\right)^8$
$= z^8$
$= \left(\sin\dfrac{\pi}{8}+i\cos\dfrac{\pi}{8}\right)^8$
$= \left(\cos\left(\dfrac{\pi}{2}-\dfrac{\pi}{8}\right)+i\sin\left(\dfrac{\pi}{2}-\dfrac{\pi}{8}\right)\right)^8$
$= \left(\cos\dfrac{3\pi}{8}+i\sin\dfrac{3\pi}{8}\right)^8$
$= \cos 3\pi = -1$

183 **(c)**
$x^2-(a-3)x+a=0$
$\Rightarrow D=(a-3)^2-4a$
$= a^2-10a+9$
$= (a-1)(a-9)$
Case I:
Both the roots are greater than 2
$D\ge 0, f(2)>0, -\dfrac{B}{2A}>2$
$\Rightarrow (a-1)(a-9)\ge 0; 4-(a-3)2+a > 0; \dfrac{a-3}{2}>2$
$\Rightarrow a\in(-\infty,1]\cup[9,\infty); a<10; a>7$
$\Rightarrow a\in[9,10) \qquad (1)$
Case II:
One root is greater than 2 and the other root is less than or equal to 2. Hence,
$f(2)\le 0$
$\Rightarrow 4-(a-3)2+a\le 0$
$\Rightarrow a\ge 10 \qquad (2)$
From (1) and (2),
$a\in[9,10)\cup[10,\infty)\Rightarrow a\in[9,\infty)$

184 **(c)**
Here a, b, c are positive. So,
$|x|=-b+\sqrt{b^2+ac}$
Hence, x has two real values, neglecting
$|x|=-b-\sqrt{b^2+ac}$, as $|x|\ge 0$

185 **(c)**
$E=4+5(\omega)^{334}+3(\omega)^{365}$
$= 4+5\omega+3\omega^2$
$= 1+2\omega+3(1+\omega+\omega^2)$
$= 1+(-1+i\sqrt{3})$
$= i\sqrt{3}$

186 **(d)**
$\dfrac{1+i}{1-i}=\dfrac{(1+i)^2}{(1-i)(1+i)}=\dfrac{1-1+2i}{2}=\mathrm{i}$
Now $i^n=1$. Hence, the smallest positive integral value of n should be 4

187 **(a)**

$u = x^2 + 4y^2 + 9z^2 - 6yz - 2zx - 2xy$

$= \frac{1}{2}[2x^2 + 8y^2 + 18z^2 - 12yz - 6zx - 4xy]$

$= \frac{1}{2}[(x^2 - 4xy + 4y^2) + (4y^2 + 9z^2 - 12yz)$
$+ (x^2 + 9z^2 - 6zx)]$

$= \frac{1}{2}[(x - 2y)^2 + (2y - 3z)^2 + (3z - x)^2] \geq 0$

Hence, u is always non-negative

188 **(c)**

Let roots of the equation $ax^2 + 2bx + c = 0$ be α and β and roots of the equation $px^2 + 2qx + r = 0$ be γ and δ. Given,

$\frac{\alpha}{\beta} = \frac{\gamma}{\delta} \Rightarrow \frac{\alpha}{\gamma} = \frac{\beta}{\delta}$

$\Rightarrow \frac{\alpha + \beta}{\gamma + \delta} = \sqrt{\frac{\alpha\beta}{\gamma\delta}}$

$\Rightarrow \frac{-\frac{2b}{a}}{-\frac{2q}{p}} = \sqrt{\frac{\frac{c}{a}}{\frac{r}{p}}}$

$\Rightarrow \frac{b^2}{ac} = \frac{q^2}{pr}$

189 **(a)**

Let α and α^2 be the roots of $x^2 - x - k = 0$. Then, $\alpha + \alpha^2 = 1$ and $\alpha^3 = -k$

$\Rightarrow (-k)^{1/3} + (-k)^{2/3} = 1$

$\Rightarrow -k^{1/3} + k^{2/3} = 1$

$\Rightarrow \left(k^{2/3} - k^{1/3}\right)^3 = 1$

$\Rightarrow k^2 - k - 3k(k^{2/3} - k^{1/3}) = 1$

$\Rightarrow k^2 - k - 3k(1) = 1$

$\Rightarrow k^2 - 4k - 1 = 0$

$\Rightarrow k = 2 \pm \sqrt{5}$

190 **(d)**

The equation $z^n = (z + 1)^n$ will have exactly $n - 1$ roots. We have,

$\left(\frac{z + 1}{z}\right)^n = 1$

$\Rightarrow \left|\frac{z + 1}{z}\right| = 1$

$\Rightarrow |z + 1| = |z|$

Therefore, 'z' lies on the right bisector of the segment connecting the points $(0, 0)$ and $(-1, 0)$. Thus $\text{Re}(z) = -1/2$. Hence, roots are collinear and will have their real parts equal to $-1/2$.

Hence, roots are collinear and will have their real parts equal to $-1/2$

Hence sum of the real parts of roots is $(-1/2)(n - 1)$

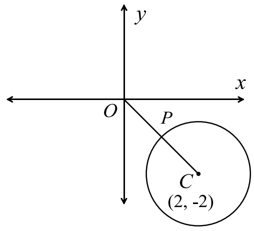
191 **(a)**

We have,

$|z - 2 + 2i| = 1$

$\Rightarrow |z - (2 - 2i)| = 1$

Hence, z lies on a circle having centre at $(2, -2)$ and radius 1. It is evident from the figure that the required complex number z is given by the point P. We find that OP makes an angle $\pi/4$ with OX and



$OP = OC - CP = \sqrt{2^2 + 2^2} - 1 = 2\sqrt{2} - 1$

So, coordinates of P are $\left[2\sqrt{2} - 1\right) \cos(\pi/4)]$
$-\left(2\sqrt{2} - 1\right) \sin(\pi/4)$, i.e., $((\sqrt{2} - 1/\sqrt{2}), -(2 - 1/\sqrt{2}))$. Hence,

$z = \left(2 - \frac{1}{\sqrt{2}}\right) + \left\{-\left(2 - \frac{1}{\sqrt{2}}\right)\right\} i = \left(2 - \frac{1}{\sqrt{2}}\right)(1 - i)$

192 **(b)**

Given $x - 2 = 2^{2/3} + 2^{1/3}$

Cubing both sides, we get

$(x - 2)^3 = 2^2 + 2 + 3 \times 2^{2/3} \times 2^{1/3}(x - 2)$
$= 6 + 6(x - 2)$

or $x^3 - 6x^2 + 12x - 8 = -6 + 6x$

$\therefore\ x^3 - 6x^2 + 6x = 2$

193 **(c)**

$\frac{(1 + i)^5\left(1 + \sqrt{3}i\right)^2}{-2i\left(-\sqrt{3} + i\right)}$

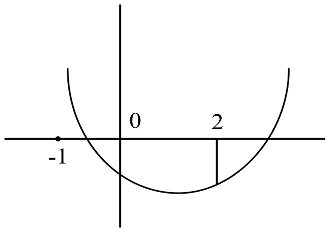
$= \frac{\left(\sqrt{2}\right)^5 \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^5 2^2 \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^2}{2i2\left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)}$

$\Rightarrow$ Argument $= \frac{5\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{2} + \frac{\pi}{6} = \frac{19\pi}{12}$

Therefore, the principle argument is $-5\pi/12$

194 **(a)**

Clearly, $f(-1) > 0, f(2) < 0$. Now,



$f(0) = -4 < 0$

$\Rightarrow f(-1) = 1 - a - 4 > 0$ and $f(2) = 4 + 2a - 4 < 0$

$\Rightarrow a < -3$ and $a < 0$

$\Rightarrow a \in (-\infty, -3)$

195 **(a)**

α, β are roots of $x^2 + px + q = 0$. Hence,

$\alpha + \beta = -p$ and $\alpha\beta = q$

α^4, β^4 are roots of $x^2 - rx + s = 0$. Hence,

$\alpha^4 + \beta^4 = r, \alpha^4\beta^4 = q$

Now for equation $x^2 - 4qx + 2q^2 - r = 0$, the product of roots is

$2q^2 - r = 2(\alpha\beta)^2 - (\alpha^4 + \beta^4)$

$= -(\alpha^2 - \beta^2)^2$

< 0

Therefore, the product of roots is negative. So, the roots must be real and of opposite signs

196 **(a)**

The given equation is $\sin(e^x) = 5^x + 5^{-x}$. We know that 5^x and 5^{-x} both are +ve real numbers

Now, $5^x + 5^{-x} = \left(\sqrt{5^x} - \sqrt{5^{-x}}\right)^2 + 2 \geq 2$

But L.H.S. $= \sin(e^x) \leq 1$

Hence, no solution

197 **(a)**

Since $p(q - r) + q(r - q) + r(p - q) = 0$, so one root is 1 and the other root is $r(p - q)/[p(q - r)]$. Since both the roots are equal, we have

$$\frac{rp - rq}{pq - pr} = 1$$

$\Rightarrow rp - rq = pq - pr$

$\Rightarrow 2\,rp = q\,(p + r)$

$$\Rightarrow \frac{2}{q} = \frac{p + r}{pr} = \frac{1}{p} + \frac{1}{r}$$

198 **(c)**

$S_1 = \sum z_1 = -3a, S_2 = \sum z_1z_2 = 3b$

Since the triangle is equilateral, we have

$\sum z_1^2 = \sum z_1z_2$

$\Rightarrow \left(\sum z_1\right)^2 - 2\sum z_1z_2 = \sum z_1z_2$

$\Rightarrow \left(\sum z_1\right)^2 = 3\sum z_1z_2$

$\Rightarrow (-3a)^2 = 3(3b)$

$\Rightarrow 9a^2 = 9b$

$\Rightarrow a^2 = b$

199 **(c)**

Discriminant $D = (2m - 1)^2 - 4(m - 2)m = 4m + 1$ must be perfect square. Hence,

$4m + 1 = k^2$, say for some $k \in I$

$$\Rightarrow m = \frac{(k - 1)(k + 1)}{4}$$

Clearly, k must be odd. Let $k = 2n + 1$

$$\therefore\ m = \frac{2n(2n + 2)}{4} = n(n + 1), n \in I$$

200 **(a)**

$x + iy = 1 - t + i\sqrt{t^2 + t + 2}$

$\Rightarrow x = 1 - t, y = \sqrt{t^2 + t + 2}$

Eliminating t,

$$y^2 = t^2 + t + 2 = (1 - x)^2 + 1 - x + 2$$
$$= \left(x - \frac{3}{2}\right)^2 + \frac{7}{4}$$

$\Rightarrow y^2 - \left(x - \frac{3}{2}\right)^2 = \frac{7}{4}$, which is a hyperbola

201 **(d)**

$(z^n - 1) = (z - 1)(z - z_1)(z - z_2)\ldots(z - z_{n-1})$ (1)

Differentiating w.r.t. x, and then dividing by (1), we have

$$\frac{nz^{n-1}}{z^n - 1} = \frac{1}{z - 1} + \frac{1}{z - z_1} + \frac{1}{z - z_2} + \cdots + \frac{1}{z - z_{n-1}}$$

Putting $z = 3$, we get

$$\frac{n3^{n-1}}{3^n - 1} = \frac{1}{2} + \frac{1}{3 - z_1} + \frac{1}{3 - z_2} + \cdots + \frac{1}{3 - z_{n-1}}$$

$$\Rightarrow \frac{1}{3 - z_1} + \frac{1}{3 - z_2} + \cdots + \frac{1}{3 - z_{n-1}} = \frac{n3^{n-1}}{3^n - 1} - \frac{1}{2}$$

202 **(c)**

$$z = \left(1 + i\sqrt{3}\right)^{100} = 2^{100}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{100}$$

$$= 2^{100}\left(\cos\frac{100\pi}{3} + i\sin\frac{100\pi}{3}\right)$$

$$= 2^{100}\left(-\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)$$

$$= 2^{100}\left(-\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)$$

$\Rightarrow \frac{\text{Re}(z)}{\text{Im}(z)} = \frac{-1/2}{-\sqrt{3}/2} = \frac{1}{\sqrt{3}}$

203 **(a)**

$x^2 + ax + b + 1 = 0$ has positive integral roots α and β. Hence,

$(\alpha + \beta) = -a$ and $\alpha\beta = b + 1$

$\Rightarrow (\alpha + \beta)^2 + (\alpha\beta - 1)^2 = a^2 + b^2$

$\Rightarrow a^2 + b^2 = (\alpha^2 + 1)(\beta^2 + 1)$

$\Rightarrow a^2 + b^2$ can be equal to 50 (since other options have prime numbers)

204 **(a)**

$x_1(x - x_2)^2 + x_2(x - x_1)^2 = 0$

$\Rightarrow x^2(x_1 + x_2) - 4x\, x_1x_2(x_1 + x_2)^2 > 0 \quad (\because x_1x_2 < 0)$

The product of roots is $x_1x_2 < 0$. Thus, the roots are real and of opposite signs

205 **(b)**

The given equation is

$(x - b)(x - c) + (x - a)(x - c) + (x - a)(x - b) = 0$

$\Rightarrow 3x^2 - 2(a + b + c)x + (ab + bc + ca) = 0$

$D = 4(a + b + c)^2 - 12(ab + bc + ca)$

$= 4[a^2 + b^2 + c^2 - ab - bc - ca]$

$= 2[(a - b)^2 + (b - c)^2 + (c - a)^2] \geq 0, \forall\, a, b, c$

Therefore, the roots of the given equation are always real

206 **(c)**

Given that

$a^2 + b^2 + c^2 = 1 \quad (1)$

We know that

$(a + b + c)^2 \geq 0$

$\Rightarrow a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \geq 0$

$\Rightarrow 2(ab + bc + ca) \geq -1$ [Using (1)]

$\Rightarrow ab + bc + ca \geq -1/2 \quad (2)$

Also, we know that

$\frac{1}{2}[(a - b)^2 + (b - c)^2 + (c - a)^2] \geq 0$

$\Rightarrow a^2 + b^2 + c^2 - ab\, bc - ca \geq 0$

$\Rightarrow ab + bc + ca \leq 1$ [Using (1)] (3)

Combining (2) and (3), we get

$-1/2 \leq ab + bc + ca \leq 1$

$\Rightarrow ab + bc + ca \in [-1/2, 1]$

207 **(a)**

We have,

$\arg\left(\frac{z_1}{z_2}\right) = \pi$

$\Rightarrow \arg(z_1) - \arg(z_2) = \pi$

$\Rightarrow \arg(z_1) = \arg(z_2) + \pi$

Let $\arg(z_2) = \theta$. Then $\arg(z_1) = \pi + \theta$

$\therefore\ z_1 = |z_1|\cos(\pi + \theta) + i\sin(\pi + \theta)$

$= |z_1|(-\cos\theta - i\sin\theta)$

and $z_2 = |z_2|(\cos\theta + i\sin\theta)$

$= |z_1|(\cos\theta + i\sin\theta) \quad (\because |z_1| = |z_2|)$

$= -z_1$

$\Rightarrow z_1 + z_2 = 0$

208 **(b)**

Let D_1 and D_2 be discriminants of $x^2 + b_1x + c_1 = 0$ and $x^2 + b_2x + c_2 = 0$, respectively. Then,

$D_1 + D_2 = b_1^2 - 4c_1 + b_2^2 - 4c_2$

$= (b_1^2 + b_2^2) - 4(c_1 - c_2)$

$= b_1^2 + b_2^2 - 2b_1b_2 \quad [\because b_1b_2 = 2(c_1 + c_2)]$

$= (b_1 - b_2)^2 \geq 0$

$\Rightarrow D_1 \geq 0$ or $D_2 \geq 0$ and D_1 and D_2 both are positive

Hence, at least one of the equations has real roots

209 **(c)**

Let

$a = \cos\alpha \sin\alpha$

$b = \cos\beta + i\sin\beta$

$c = \cos\gamma + i\sin\gamma$

Then,

$a + 2b + 3c = (\cos\alpha + 2\cos\beta + 3\cos\gamma) + i(\sin\alpha + 2\sin\beta + 3\sin\gamma) = 0$

$\Rightarrow a^3 + 8b^3 + 27c^3 = 18\,abc$

$\Rightarrow \cos 3\alpha + 8\cos 3\beta + 27\cos 3\gamma = 18\sin(\alpha + \beta + \gamma)$

and $\sin 3\alpha + 8\sin 3\beta + 27\sin 3\gamma = 18\sin(\alpha + \beta + \gamma)$

210 **(b)**

$|\omega| = 1$

$\Rightarrow \left|\frac{1 - iz}{z - i}\right| = 1$

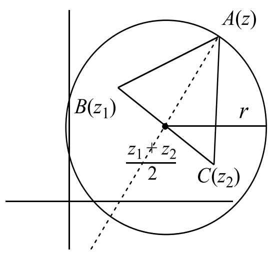
$\Rightarrow |1 - iz| = |z - i|$

$\Rightarrow |-i||z + i| = |z - i|$

$\Rightarrow |z + i| = |z - i|$

Hence, z is equidistant from $(0, -1)$ and $(0, 1)$. So, z lies on perpendicular bisector of $(0, -1)$ and $(0, 1)$ i.e., x-axis, and $y = 0$. Therefore, z lies on real axis

211 **(b)**



By the given conditions, the area of the triangle ABC is given by $(1/2)|z_1 - z_2|r$

212 **(b)**

If vertices of a parallelogram are z_1, z_2, z_3, z_4, then as diagonals bisect each other as given,

$\frac{z_1 + z_3}{2} = \frac{z_2 + z_4}{2}$

$\Rightarrow z_1 + z_3 = z_2 + z_4$

213 **(a)**

The given equation is

$\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$

Squaring both sides, we get

$x + 1 + x - 1 - 2\sqrt{x^2-1} = 4x - 1$

$\Rightarrow -2\sqrt{x^2-1} = 2x - 1$

Again squaring both sides, we get

$\Rightarrow 4(x^2-1) = 4x^2 - 4x + 1$

$\Rightarrow -4x = -5$

$\Rightarrow x = 5/4$

Substituting this value of x in given equation, we get

$$\sqrt{\frac{5}{4}+1} - \sqrt{\frac{5}{4}-1} = \sqrt{4 \times \frac{5}{4} - 1}$$

$\Rightarrow \frac{3}{2} - \frac{1}{2} = 2$ (not satisfied)

Therefore, $5/4$ is not a solution of given equation. Hence, the given equation has no solution

214 **(b)**

$\frac{(b^2-4ac)^2}{16a^2} < \frac{4}{1+4a^2}$ (1)

Now,

$$\max(ax^2 + bx + c) = -\frac{b^2-4ac}{4a}$$

Also, $\frac{-2}{\sqrt{1+4a^2}} < -\frac{b^2-4ac}{4a} < \frac{2}{\sqrt{1+4a^2}}$ [From (1)]

So, maximum value is always less than 2 (when $a \to 0$)

215 **(c)**

As $(\lambda - 1)x^2 + 2 = \lambda x + 3$ has only one solution, so $D = 0$

$\Rightarrow \lambda^2 - 4(\lambda+1)(-1) = 0$

$\Rightarrow \lambda^2 + 4\lambda + 4 = 0$

$\Rightarrow (\lambda + 2)^2 = 0$

$\therefore \lambda = -2$

216 **(c)**

Given that $a^2 + b^2 = 1$. Therefore,

$$\frac{1+b+ia}{1+b-ia} = \frac{(1+b+ia)(1+b+ia)}{(1+b-ia)(1+b+ia)}$$

$$= \frac{(1+b)^2 - a^2 + 2ia(1+b)}{1+b^2+2b+a^2}$$

$$= \frac{(1-a^2) + 2b + b^2 + 2ia(1+b)}{2(1+b)}$$

$$= \frac{2b^2 + 2b + 2ia(1+b)}{2(1+b)}$$

$= b + ia$

217 **(c)**

Given that

$|z_1 - i| = |z_2 - i| = |z_3 - i|$

Hence, z_1, z_2, z_3 lie on the circle whose centre is i.

Also given that the triangle is equilateral. Hence centroid and circumcentre coincides

$$\therefore \frac{z_1 + z_2 + z_3}{3} = i$$

$\Rightarrow |z_1 + z_2 + z_3| = 3$

218 **(a)**

$$\log_{1/3}\left(\frac{|z-3|^2+2}{11|z-3|-2}\right) > 1$$

$$\Rightarrow \frac{|z-3|^2+2}{11|z-3|-2} < \frac{1}{3}$$

$\Rightarrow (3t-8)(t-1) < 0$ (where $|z-3| = t$)

$\Rightarrow 1 < |z-3| < 8/3$

Hence z lies between the two concentric circles

219 **(c)**

Put $ab + bc + ca = t$. Now,

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2t$

$\Rightarrow (a+b+c)^2 = 1 + 2t$

$\Rightarrow 1 + 2t \geq 0$

$$\Rightarrow -\frac{1}{2} \leq t$$

Again, $(a-b)^2 + (b-c)^2 + (c-a)^2 = 2 - 2t$

$\Rightarrow 2 - 2t \geq 0$

$\Rightarrow t \leq 1$

$$\Rightarrow -\frac{1}{2} \leq t \leq t \leq 1$$

220 **(a)**

Here $x = 4\cos\theta, y = 4\sin\theta$

$\therefore ||x| - |y||$

$= |4|\cos\theta|-4|\sin\theta||$

$= 4||\cos\theta| - \sin\theta||$

$= 4\sqrt{1 - 2|\cos\theta||\sin\theta|}$

$= 4\sqrt{1 - |\sin 2\theta|}$

Hence, the range is $[0, 4]$

221 **(d)**

$|\omega z - 1 - \omega^2| = a$

$\Rightarrow |z+1| = a \Rightarrow |z - 1 + 2| = a$

$\Rightarrow |z-1| + 2 \geq a \Rightarrow 0 \leq a \leq 4$

222 **(b)**

We have,

$$x = \frac{-1 \pm \sqrt{1 - 4a^2(1-a^2)}}{2a^2}$$

$$= \frac{-1 \pm (2a^2-1)}{2a^2}$$

$= 1 - \frac{1}{a^2}$ or $-a^2$

$$\Rightarrow \beta^2 = 1 - \frac{1}{a^2}$$

223 **(b)**

$$\left|z + \frac{1}{z}\right| \geq \left||z| - \frac{1}{|z|}\right|$$

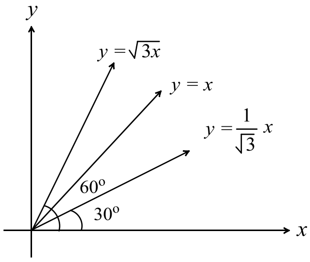
Hence the least value occurs when $|z| = 3$

$\therefore \left|z + \frac{1}{z}\right|_{\text{least}} = 3 - \frac{1}{3} = \frac{8}{3}$

224 **(b)**

Note that $z_1 = 3 + \sqrt{3}i$ lies on the line $y = (1/\sqrt{3})x$ and $z_2 = 2\sqrt{3} + 6i$ lies on the line $y = \sqrt{3}x$

Hence $z = 5 + 5i$ will only lie on the bisector of z_1 and z_2, i.e. $y = x$



225 **(b)**

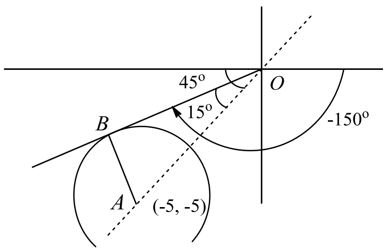
$|z^2 - 3| \geq |z|^2 - 3$

$\Rightarrow 3|z| \geq |z|^2 - 3$

$\Rightarrow |z|^2 - 3|z| - 3 \leq 0$

$\Rightarrow 0 < |z| \leq \frac{3 + \sqrt{21}}{2}$

226 **(a)**



$|2z + 10 + 10i| \leq 5\sqrt{3} - 5$

$\Rightarrow |z + 5 + 5i| \leq \frac{5(\sqrt{3} - 1)}{2}$

Point B has least principle argument. Now,

$AB = \frac{5(\sqrt{3} - 1)}{2}$

$OA = 5\sqrt{2}$

$\angle AOB = \frac{\pi}{12}$

$\therefore \arg(z) = -\frac{5\pi}{6}$

227 **(b)**

Let,

$\frac{x}{x^2 - 5x + 9} = y$

$\Rightarrow yx^2 - 5yx + 9y = x$

$\Rightarrow yx^2 - (5y + 1)x + 9y = 0$

Now, x is real, so

$D \geq 0$

$\Rightarrow \left(-(5y + 1)\right)^2 - 4 \cdot y \cdot (9y) \geq 0$

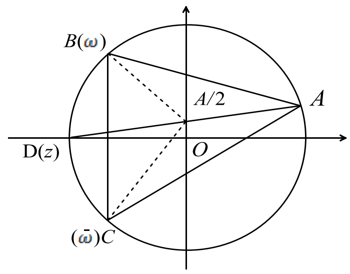
$\Rightarrow -11y^2 + 10y + 1 \geq 0$

$\Rightarrow 11y^2 - 10y - 1 \leq 0$

$\Rightarrow (11y + 1)(y - 1) \leq 0$

$\Rightarrow -\frac{1}{11} \leq y \leq 1$

228 **(d)**



Clearly,

$\angle DOB = \angle COD = A$

$\Rightarrow z = \omega e^{iA}$ and $\overline{\omega} = ze^{iA}$ (Applying rotation about O)

$\Rightarrow z^2 = \omega\overline{\omega} = 1$

$\Rightarrow z = -1$ (As A and D are non opposite sides of BC)

229 **(b)**

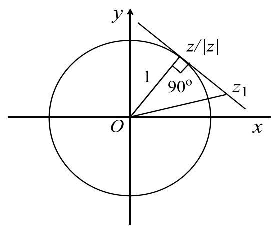
Given that

$\arg\left(\frac{z_1 - \frac{z}{|z|}}{\frac{z}{|z|}}\right) = \frac{\pi}{2}$

and

$\left|\frac{z}{|z|} - z_1\right| = 3$

From which we can establish the following geometry



From the diagram,

$\left|\frac{z}{|z|} - z_1\right| = 3, |z_1| = \sqrt{9 + 1} = \sqrt{10}$

230 **(b)**

$f(z) = g(z)(z - i)(z + i) + az + b; a, b \in C$

Given,

$f(i) = i \Rightarrow ai + b = i$ (1)

and $f(-i) = 1 + i$

$\Rightarrow a(-i) + b = 1 + i$ (2)

From (1) and (2), we have

$a = \frac{i}{2}, b = \frac{1}{2} + i$

Hence, the required remainder is $az + b = (1/2)iz + (1/2) + i$

231 **(b)**

Let,

$f(x) = ax^2 + x + c - a$

$f(1) = c + 1 > 0 \quad (\because c > -1)$

Therefore, given expression is positive $\forall\, x \in R$.

So,

$f\left(\frac{1}{2}\right) > 0$

$\Rightarrow \frac{a}{4} + \frac{1}{2} + c - a > 0$

$\Rightarrow 4c - 3a + 2 > 0$

$\Rightarrow 4c + 2 > 3a$

232 **(a)**

Clearly, $\alpha + \beta = 1, \alpha\beta = p, \gamma + \delta = 4, \gamma\delta = q\ (p, q \in I)$

Since $\alpha, \beta, \gamma, \delta$ are in G.P. (with common ratio r), so

$\alpha + \alpha r = 1, \alpha(r^2 + r^3) = 4$

$\Rightarrow \alpha(1 + r) = 1, \alpha r^2(1 + r) = 4$

$\Rightarrow r^2 \times 1 = 4 \Rightarrow r^2 = 4 \Rightarrow r = 2, -2$

If $r = 2$,

$\alpha + 2\alpha = 1 \Rightarrow \alpha = \frac{1}{3}$

If $r = -2$,

$\alpha - 2\alpha = 1 \Rightarrow \alpha = -1$

But $p = \alpha\beta \in I$

$\therefore r = -2$ and $\alpha = -1$

$\Rightarrow p = -2$,

$q = \alpha^2 r^5 = 1\,(-2)^5 = -32$

233 **(c)**

$a\alpha^2 + c = -b\alpha, a\alpha + b = -\frac{c}{\alpha}$

Hence, the given expression is

$\frac{b}{c}(\alpha^2 + \beta^2) = \frac{b(b^2 - 2ac)}{a^2 c}$

234 **(d)**

Given,

$z^3 + \frac{3(\bar{z})^2}{|z|} = 0$

Let,

$z = re^{i}\theta$

$\Rightarrow r^3 e^{i3\theta} + 3re^{-i2\theta} = 0$

Since 'r' cannot be zero, so

$re^{i5\theta} = -3$

Which will hold for $r = 3$ and five distinct value of 'θ'. Thus there are five solutions

235 **(a)**

$\left(\frac{1 + ia}{1 - ia}\right)^4 = z$

$\Rightarrow \left|\frac{1 + ia}{1 - ia}\right|^4 = |z|$

$\Rightarrow \left|\frac{a - i}{a + i}\right|^4 = 1$

$\Rightarrow |a - i| = |a + i|$

Therefore, a lies on the perpendicular bisector of i and $-i$, which is real axis. Hence all the roots are real

236 **(b)**

Let xi be the root where $x \neq 0$ and $x \in R$

$x^4 - a_1 x^3 i - a_2 x^2 + a_3 xi + a_4 = 0$

$\Rightarrow x^4 - a_2 x^2 + a_4 = 0 \quad (1)$

and

$a_1 x^3 - a_3 x = 0 \qquad (2)$

From Eq. (2),

$a_1 x^2 - a_3 = 0$

$\Rightarrow x^2 = a_3/a_1 \quad (\text{as } x \neq 0)$

Putting the value of x^2 in Eq. (1), we get

$\frac{a_3^2}{a_1^2} - \frac{a_2\, a_3}{a_1} + a_4 = 0$

$\Rightarrow a_3^2 + a_4 a_1^2 = a_1 a_2 a_3$

$\Rightarrow \frac{a_3}{a_1\, a_2} + \frac{a_1 a_4}{a_2 a_3} = 1$ (dividing by $a_1 a_2 a_3$)

237 **(b)**

If z_1, z_2, z_3 are three complex numbers, then

$A = \begin{vmatrix} \arg z_1 & \arg z_2 & \arg z_3 \\ \arg z_2 & \arg z_3 & \arg z_1 \\ \arg z_3 & \arg z_1 & \arg z_2 \end{vmatrix}$

$\Rightarrow A = (\arg z_1 + \arg z_1 + \arg z_3) \begin{vmatrix} 1 & \arg z_2 & \arg z_3 \\ 1 & \arg z_3 & \arg z_1 \\ 1 & \arg z_1 & \arg z_2 \end{vmatrix}$

(Using $C_1 \to C_1 + C_2 + C_3$)

$\Rightarrow A = \arg(z_1 z_2 z_3) \begin{vmatrix} 1 & \arg z_2 & \arg z_3 \\ 1 & \arg z_3 & \arg z_1 \\ 1 & \arg z_1 & \arg z_2 \end{vmatrix}$

Hence, A is divisible by $\arg(z_1 z_2 z_3)$

238 **(d)**

Let,

$z = (1)^{1/n} = (\cos 2k\pi + i \sin 2k\pi)^{1/n}$

$= \cos\frac{2k\pi}{n} + i \sin\frac{2k\pi}{n}, k = 0, 1, 2, \dots, n - 1$

Let

$z_1 = \cos\left(\frac{2k_1\pi}{n}\right) + i \sin\left(\frac{2k_1\pi}{n}\right)$

and

$z_2 = \cos\left(\frac{2k_2\pi}{n}\right) + i \sin\left(\frac{2k_2\pi}{n}\right)$

Be the two values of z such that they subtend angle of 90° at origin. Then,

$\Rightarrow \frac{2k_1\pi}{n} - \frac{2k_2\pi}{n} = \pm\frac{\pi}{2} \Rightarrow 4(k_1 - k_2) = \pm n$

As k_1 and k_2 are integers and $k_1 \neq k_2$, therefore $n = 4m, m \in Z$

239 **(a,d)**

Let $A = a + 2b - 3c, B = b + 2c - 3a, C = c +$

$2a-3b$

$\therefore A+B+C=0$

Hence, roots are 1 and $\frac{C}{A}$.

Thus, roots are real and rational.

240 **(a,b,c)**

Since the roots of $ax^2+bx+c=0$ are non-real, so, $f(x)=ax^2+bx+c$ will have same sign for every value of x. Hence,

$f(0)=c, f(1)=a+b+c, f(-1)=a-b+c$

$f(-2)=4a-2b+c$

$\Rightarrow c(a+b+c)>0, c(a-b+c)>0, c(4a-2b+c)>0$

241 **(a,c)**

If $p=q$, then equation become $z^p=\bar{z}^q$ and it has infinite number of solutions because any $z\in R$ will satisfy it. If $p\neq q$, let $p>q$, then $z^p=\bar{z}^q$

$\therefore |z|^p=|z|^q$

$\Rightarrow |z|^p(|z|^{p-q}-1)=0$

$\Rightarrow |z|=0$ or $|z|=1$

$|z|=0 \Rightarrow z=0+i0$

$|z|=1 \Rightarrow z=e^{i\theta}$

$\Rightarrow e^{(p+q)\theta i}=1$

$\Rightarrow z=1^{1/(p+q)}$

Therefore, the number of solutions is $p+q+1$

242 **(a,b,c)**

$f(x)=Ax^2+Bx+C$

$A=a+b-2c=(a-c)+(b-c)>0$

$\Rightarrow A>0$

Hence, the graph is concave upwards. Also, $x=1$ is obvious solution; therefore, both roots are rational

$b+c-2a=\underbrace{(b-a)}_{-\text{ve}}+\underbrace{(c-a)}_{-\text{ve}}<0$

$\Rightarrow B<0$

$\therefore$ vertex $=-\dfrac{B}{2A}>0$

Hence, abscissa of the vertex is positive. Option (d) need not be correct as with $a=5, b=4, c=2, P<0$ and with $a=6, b=3, c=2, P>0$

243 **(c,d)**

We have, $x^2+x+1=(x-\omega)(x-\omega^2)$

Since, $f(x)$ is divisible by $x^2+x+1, f(\omega)=0, f(\omega^2)=0$

$\therefore\ P(\omega^3)+\omega Q(\omega^3)=0$

$\Rightarrow\ P(1)+\omega Q(1)=0$ ...(i)

and $P(\omega^6)+\omega^2 Q(\omega^6)=0$

$\Rightarrow\ P(1)+\omega^2 Q(1)=0$ ...(ii)

On solving Eqs. (i) and (ii), we get

$P(1)=0$ and $Q(1)=0$

$\therefore$ Both $P(x)$ and $Q(x^3)$ are divisible by $(x-1)$

$\Rightarrow\ P(x^3)$ and $Q(x^3)$ are divisible by x^3-1 and hence by $(x-1)$

Since, $f(x)=P(x^3)+xQ(x^3)$, we get $f(x)$ is divisible by $x-1$

244 **(b,c)**

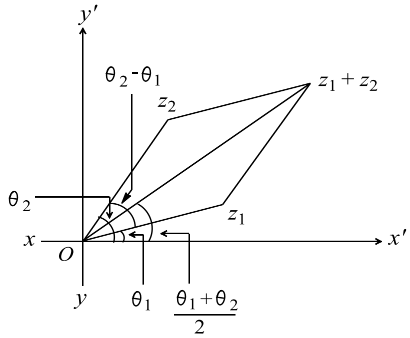
z_1 and z_2 are the roots of the equation

$z^2-az+b=0$. Hence, $z_1+z_2=a, z_1z_2=b$

Now,

$|z_1+z_2|\leq|z_1|+|z_2|$

$\Rightarrow |z_1+z_2|=|a|\leq 1+1=2$ $(\because |z_1|=|z_2|=1)$



$\Rightarrow \arg(a)=\dfrac{1}{2}[\arg(z_2)+\arg(z_1)]$

Also, $\arg(b)=\arg(z_1z_2)=\arg(z_1)+\arg(z_2)$

$\Rightarrow 2\arg(a)=\arg(b)$

245 **(a,d)**

Let $z_1=a+ib, a>0$ and $b\in R; z_2=c+id, d<0, c\in R$

Given,

$|z_1|=|z_2|$

$\Rightarrow a^2+b^2=c^2+d^2$

$\Rightarrow a^2-c^2=d^2-b^2$ (1)

Now,

$$\frac{z_1+z_2}{z_1-z_2}=\frac{(a+c)+i(b+d)}{(a-c)+i(b-d)}$$

$$=\frac{[(a^2-c^2)+(b^2-d^2)]+i[(a-c)(b+d)-(a+c)(b-d)]}{(a-c)^2+(b-d)^2}$$

Which is a purely imaginary number or zero in case $a+c=b+d=0$

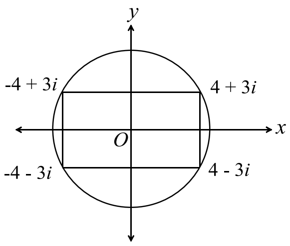
246 **(a,b,c)**

Let $z=x+iy$ where x,y satisfy the given equation. Hence,

$(x^2+y^2)(x^2-y^2)=175$

$\Rightarrow x^2+y^2=25$ and $x^2-y^2=7$ (as all other possibilities will give non-integral solutions)

Hence, possible values of z will be $4+3i, 4-3i, -4+3i$ and $-4-3i$. Clearly, it will form a rectangle having length of the diagonal 10



From the diagram, options (a), (b), (c) are correct

247 **(a,b,d)**

$z_1 = 5 + 12i, |z|_2 = 4$

$|z_1 + iz_2| \leq |z_1| + |z_2| = 13 + 4 = 17$

$\therefore |z_1 + (1+i)z_2| \geq \big||z_1| - |1+i||z_2|\big|$

$= 13 - 4\sqrt{2}$

$\therefore \min(|z_1 + (1+i)z_2|) = 13 - 4\sqrt{2}$

$$\left|z_2 + \frac{4}{z_2}\right| \leq |z_2| + \frac{4}{|z_2|} = 4 + 1 = 5$$

$$\left|z_2 + \frac{4}{z_2}\right| \geq |z_2| - \frac{4}{|z_2|} = 4 - 1 = 3$$

$$\therefore \max\left|\frac{z_1}{z_2 + \frac{4}{z_2}}\right| = \frac{13}{3} \text{ and } \min\left|\frac{z_1}{z_2 + \frac{4}{z_2}}\right| = \frac{13}{5}$$

248 **(b,c,d)**

Given equation is

$x^2 + 2(a+1)x + 9a - 5 = 0$

$D = 4(a+1)^2 - 4(9a-5) = 4(a-1)(a-6)$

$\therefore D \geq 0 \Rightarrow a \leq 1$ or $a \geq 6 \Rightarrow$ roots are real

If $a < 0$, then $9a - 5 < 0$. Hence, the products of roots is less than 0. So, the roots are of opposite sign. If $a > 7$, then sum of roots is $-2(a+1) < 0$. Product of roots is greater than 0

249 **(a,c,d)**

Choice (a) on simplification gives

$$z = \frac{1+x}{1+x^2} + i\frac{1+x}{1+x^2}$$

For $x = 0.5, f(0.5) > 1$ which is out of range. Hence, (a) is not correct. From choice (b),

$$z = \frac{1-x}{1+x^2} + i\frac{1-x}{1+x^2}$$

$f(x)$ and $g(x) \in (0,1)$ if $x \in (0,1)$. Hence, (b) is correct. From choice (c),

$$z = \frac{1+x}{1+x^2} + \frac{1-x}{1+x^2}\, i$$

Hence, (c) is not correct. From choice (d),

$$z = \frac{1-x}{1+x^2} + \frac{1+x}{1+x^2} i$$

Hence, (d) is not correct

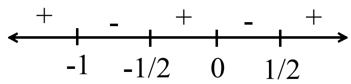
250 **(a,d)**

We have,

$$f(x) = \frac{2x-1}{2x^3 + 3x^2 + x} = \frac{2x-1}{x(2x+1)(x+1)}$$

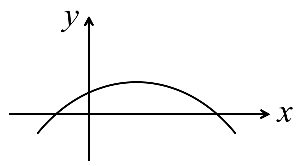
Critical points are $x = 1/2, 0, -1/2, -1$

On number line by sign scheme method, we have



For $f(x) > 0, x \in (-\infty, -1) \cup (-1/2, 0) \cup (1/2, \infty)$. Clearly, S contains $(-\infty, -3/2)$ and $(1/2, 3)$

251 **(a,b,d)**



From the graph,

$f(0) = c > 0$ (1)

Also, the graph is concave downward. Hence,

$a < 0$ (2)

Further, abscissa of the vertex,

$\frac{b}{2a}$ (3)

From (1), (2), (3),

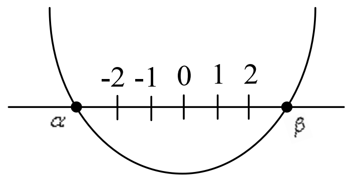
$ac < 0, ab < 0$ and $bc > 0$

252 **(c,d)**

Since, the equation has two distinct roots α and β, the discriminant, $b^2 - 4ac > 0$,

we must have

$f(x) = ax^2 + bx + c < 0$ for $\alpha < x < \beta$



Since, $\alpha < 0 < \beta$, we must have $f(0) = c < 0$

Also, as $\alpha < -1, 1 < \beta$, we get

$f(-1) = a - b + c < 0$

and $f(1) = a + b + c < 0$

$\Rightarrow a + |b| + c < 0$

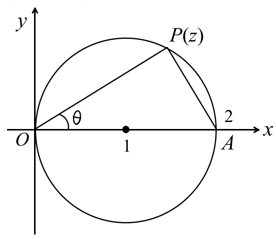
Since, $\alpha < -2, 2 < \beta$.

$f(-2) = 4a - 2b + c < 0$

and $f(2) = 4a + 2b + c < 0$

$\Rightarrow 4a + 2|b| + c < 0$

253 **(a,b,d)**



Since $\arg((z-1-i)/z)$ is the angle subtended by the chord joining the points O and $1+i$ at the circumference of the circle $|z-1| = 1$, so it is equal to $-\pi/4$. The line joining the points $z = 0$ and $z = 2 + 0i$ is the diameter

$\arg\dfrac{z-2}{z}=\pm\dfrac{\pi}{2}$

$\Rightarrow \frac{z-2}{z-0}$ is purely imaginary

We have,

$\angle OPA=\dfrac{\pi}{2}$

$\Rightarrow \arg\left(\dfrac{2-z}{0-z}\right)=\dfrac{\pi}{2} \Rightarrow \dfrac{z-2}{z}=\dfrac{AP}{OP}i$

Now in ΔOAP,

$\tan\theta=\dfrac{AP}{OP}$

Thus,

$\dfrac{z-2}{z}=i\tan\theta$

254 **(a,c)**

Triangle ABC is equilateral. Hence,

$z^2+(-z)^2+(1-z)^2$

$=z(-z)+z(1-z)+(-z)(1-z)$

$\Rightarrow 3z^2-2z+1=-z^2$

$\Rightarrow 4z^2-2z+1=0$

Sum of roots $=2$

and product of roots is $=\frac{1}{4}$

255 **(a,c)**

Since each pair has common root, let the roots be α,β for Eq. (1); β,γ for Eq. (2) and γ,α for Eq. (3). Therefore,

$\alpha+\beta=-\alpha,\alpha\beta=bc$

$\beta+\gamma=-b,\beta\gamma=ca$

$\gamma+\alpha=-c,\gamma\alpha=ab$

Adding, we get

$2(\alpha+\beta+\gamma)=-(a+b+c)$

$\Rightarrow\ \alpha+\beta+\gamma=-\dfrac{1}{2}(a+b+c)$

Also by multiplying product of roots, we have

$\alpha^2\beta^2\gamma^2=a^2b^2c^2\Rightarrow\alpha\beta\gamma=abc$

257 **(c,d)**

We have,

$D=(b-c)^2-4a(a-b-c)>0$

$\Rightarrow b^2+c^2-2bc-4a^2+4ab+4ac>0$

$\Rightarrow c^2+(4a-2b)c-4a^2+4ab+b^2>0$ for all $c\in R$

Discriminant of the above expression in c must be negative. Hence,

$(4a-2b)^2-4(-4a^2+4ab+b^2)<0$

$\Rightarrow 4a^2-4ab+b^2+4a^2-4ab-b^2<0$

$\Rightarrow a(a-b)<0$

$\Rightarrow a<0$ and $a-b>0$ or $a>0$ and $a-b>0$

$\Rightarrow b<a<0$ or $b>a>0$

258 **(c,d)**

$\cos x-y^2-\sqrt{y-x^2-1}\geq 0$ (1)

Now, $\sqrt{y-x^2-1}$ is defined when $y-x^2-1\geq 0$ or $y\geq x^2+1$. So minimum value of y is 1. From (1),

$\cos x-y^2\geq\sqrt{y-x^2-1}$

Where $\cos x-y^2\leq 0$ [as when $\cos x$ is maximum $(=1)$ and y^2 is minimum $(=1)$, so $\cos x-y^2$ is maximum]. Also,

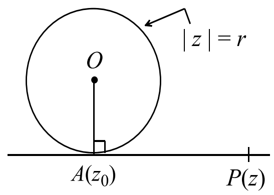
$\sqrt{y-x^2-1}\geq 0$

Hence, $\cos x-y^2=\sqrt{y-x^2-1}=0$

$\Rightarrow y=1$ and $\cos x=1, y=x^2+1$

$\Rightarrow x=0,y=1$

259 **(a,b)**



$OAP=\dfrac{\pi}{2}$

$\Rightarrow\frac{z-z_0}{z_0}$ is purely imaginary

$\Rightarrow\dfrac{z-z_0}{z_0}+\dfrac{\bar{z}-\bar{z}_0}{\bar{z}_0}=0$

$\Rightarrow\dfrac{z}{z_0}+\dfrac{\bar{z}}{\bar{z}_0}=2\qquad(1)$

$\Rightarrow \operatorname{Re}\left(\dfrac{z}{z_0}\right)=1$

From (1),

$z\bar{z}_0+z_0\bar{z}=2|z_0|^2=2r^2$

260 **(c,d)**

Product of roots is

$\dfrac{a}{bc}<0\ [\because abc<0]$

Hence, roots are real and of opposite sign

261 **(a,b,c,d)**

$\sqrt{5-12i}=\sqrt{(3-2i)^2}=\pm(3-2i)$

$\sqrt{-5-12i}=\sqrt{(2-3i)^2}=\pm(2-3i)$

$\Rightarrow z=\sqrt{5-12i}+\sqrt{-5-12i}$

$=-1,-i,-5+5i,5-5i,1+i$

Therefore, principle values of $\arg z$ are

$-3\pi/4,3\pi/4,-\pi/4,\pi/4$

262 **(a,b,d)**

Symmetric functions are those which do not change by interchanging α and β

263 **(a,b,d)**

Given,

$\begin{vmatrix}a&b&c\\b&c&a\\c&a&b\end{vmatrix}=0$

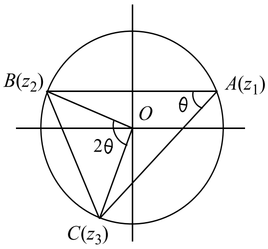
$\Rightarrow\ a^3+b^3+c-3abc=0$

$\Rightarrow(a+b+c)(a^2+b^2+c^2-ab-bc-ca)=0$

$$\Rightarrow \frac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2] = 0$$

$\Rightarrow (a-b)^2+(b-c)^2+(c-a)^2=0$

$\Rightarrow a=b=c \quad [\because a+b+c\neq 0, \because z_1 \neq 0, \therefore |z_1| = a \neq 0 \text{ etc}]$



Hence, $OA = OB = OC$, where O is the origin and A, B, C are the points representing z_1, z_2 and z_3, respectively. Therefore, O is circumcentre of $\angle ABC$. Now,

$\arg\left(\frac{z_3}{z_2}\right) = \angle BOC \quad$ (i)

$= 2\angle BAC = 2\arg\left(\frac{z_3-z_1}{z_2-z_1}\right) \quad$ (ii)

$= \arg\left(\frac{z_3-z_1}{z_2-z_1}\right)^2 \quad [\because \angle BOC = 2\angle BAC]$

Hence,

$\arg\left(\frac{z_3}{z_2}\right) = \arg\left(\frac{z_3-z_1}{z_2-z_1}\right)^2$

Also, centroid is $(z_1+z_2+z_3)/3$. Since $HG:GO \equiv 2:1$ (where H is orthocenter and G is centroid), then orthocenter is $z_1+z_2+z_3$ (by section formula). When triangle is equilateral centroid coincides with circumcentre; hence $z_1+z_2+z_3=0$

Also, the area for equilateral triangle is $(\sqrt{3}/4)L^2$, where L is length of side. Since radius is $|z_1|$, $L=\sqrt{3}|z_1|$, hence area is $(3\sqrt{3}/4)|z_1|^2$

264 **(a,b,c)**

$\sec^2\theta + \text{cosec}^2\theta = \sec^2\theta\ \text{cosec}^2\theta$

Sum of the roots is equal to their product and the roots are real. Hence,

$-\frac{b}{a} = \frac{c}{a}$

$\Rightarrow b+c=0$

Also $b^2-4ac \geq 0$

$\Rightarrow c^2-4ac \geq 0$

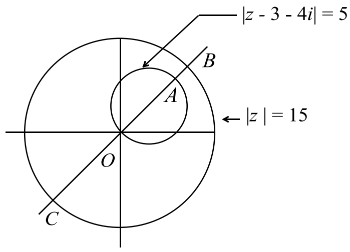
$\Rightarrow c(c-4a) \geq 0$

$\Rightarrow c-4a \geq 0 \ (\because c>0)$

Further $b^2+4ab \geq 0$

$\Rightarrow b+4a \leq 0 \ (\because b<0)$

265 **(a,d)**

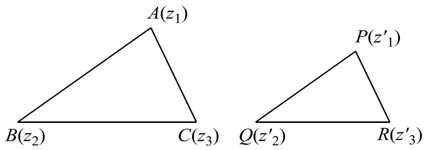


We have,

$|z_1| = 15, |z_2-3-4i| = 5$

Minimum value of $|z_1-z_2|$ is $AB = OB - OA = 15-10 = 5$. Maximum value of $|z_1-z_2|$ is $CA = OA + OC = 10+15 = 25$

266 **(a,b,c,d)**



$$z_3 = (1-\lambda)z_1 + z_2 = \frac{(1-\lambda)z_1+\lambda z_2}{1-\lambda+\lambda}$$

Hence, z_3 divides the line joining $A(z_1)$ and $B(z_2)$ in the ratio $\lambda:(1-\lambda)$. That means the given points are collinear. Also, the ratio $\lambda/(1-\lambda) > 0$ (or $0<\lambda<1$) if z_3 divides the line joining z_1 and z_2 internally and $\mu/(1-\mu) < 0$ (or $\mu < 0$ or $\mu > 1$) if z_3' divides the line joining z_1', z_2' externally

When λ, μ are complex numbers, where $\lambda = \mu$, we have $z_3 = (1-\lambda)z_1 + \lambda z_2$ and $z_3' = (1-\lambda)z_1' + \lambda z_2'$. Comparing the value of λ, we have

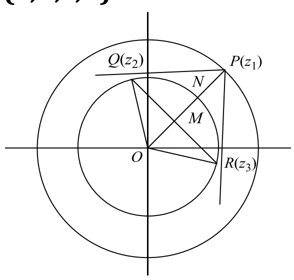
$$\frac{z_3-z_1}{z_2-z_1} = \frac{z_3'-z_1'}{z_2'-z_1'}$$

$\Rightarrow \left|\frac{z_3-z_1}{z_2-z_1}\right| = \left|\frac{z_3'-z_1'}{z_2'-z_1'}\right|$ and $\arg\left(\frac{z_3-z_1}{z_2-z_1}\right) = \arg\left(\frac{z_3'-z_1'}{z_2'-z_1'}\right)$

$\Rightarrow \frac{AC}{AB} = \frac{PR}{PQ}$ and $\angle BAC = \angle QPR$

Hence, triangles ABC and PQR are similar

267 **(a,b,c,d)**



Since $OQ = 1$ and $OP = 2$, so $\sin(\angle OPQ) = 1/2$ and hence $\angle QPR = \pi/3$. Then $\angle PQR$ is equilateral. Also, $OM \perp QR$. Then from $\angle OMQ$, $OM = 1/2$. Hence $MN = 1/2$. Then centroid of $\angle PQR$ lies on $|z| = 1$

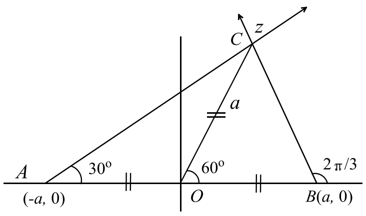
As PQR is an equilateral triangle, so orthocenter, circumcentre and centroid will coincide. Now,

$\Rightarrow \left|\frac{z_1+z_2+z_3}{3}\right| = 1$
$\Rightarrow |z_1+z_2+z_3|^2 = 9$
$\Rightarrow (z_1+z_2+z_3)(\bar{z}_1+\bar{z}_1+\bar{z}_3) = 9$
$\Rightarrow \left(\frac{4}{\bar{z}_1}+\frac{1}{\bar{z}_2}+\frac{1}{\bar{z}_3}\right)\left(\frac{4}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right) = 9$
and
$\angle QOR = 120°$

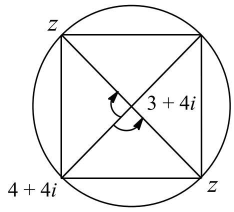
268 **(a,d)**
Let the roots of three given equations be (α,β); (β,γ) and (γ,α), then on substituting β in first two equations.
We get, $\beta^2+p\beta+qr=0$ and $\beta^2+q\beta+rp=0$.
On subtracting, we get $(p-q)\beta+r(q-p)=0$
$\Rightarrow \beta = r$. If $p \neq q$
$\therefore \frac{\alpha+\beta+\gamma}{\alpha\beta\gamma} = \frac{p+q+r}{pqr} = \frac{\sum p}{pqr}$
and $p+q+r=0$

269 **(a,b,c)**
We have, $z_1 = a+ib$ and $z_2 = c+id$
$\therefore |z_1|^2 = a^2+b^2 = 1$ and $|z_2|^2 = c^2+d^2 = 1$ …(i)
Also, $\mathrm{Re}(z_1\,\bar{z}_2) = 0 \Rightarrow ac+bd = 0$
$\Rightarrow \frac{a}{b} = -\frac{d}{c} = \lambda$ …(ii)
From Eqs. (i) and (ii),
$b^2\lambda^2+b^2 = c^2+\lambda^2c^2$
$\Rightarrow b^2 = c^2$ and $a^2 = d^2$
Now, $|w_1| = \sqrt{a^2+c^2} = \sqrt{a^2+b^2} = 1$
and $|w_2| = \sqrt{b^2+d^2} = \sqrt{a^2+b^2} = 1$
Re $|w_1\,\overline{w_2} = (ab+cd)$
$= (b\lambda)b + c(-\lambda c)$
$= \lambda(b^2-c^2) = 0$
Hence, (a), (b) and (c) are correct answers

270 **(a,d)**
Refer the figure, z lies on the point of intersection of the rays from A and B. $\angle ACB$ is a right angle and OBC is an equilateral triangle. Hence,
$OC = a \Rightarrow z = a\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$



271 **(b,c)**



Clearly, the inscribed rectangle is square. Let the adjacent vertex be z. Then,
$\frac{3+4i-(z)}{(3+4i)-(4+4i)} = e^{\pm i\pi/2}$ (by rotation about centre)
$\Rightarrow 3+4i-z = \pm i(-1)$
$\Rightarrow z = 3+4i \pm i = 3+5i$ or $3+3i$

272 **(b,c)**
amp $(z_1z_2) = 0 \Rightarrow$ amp z_1 + amp $z_2 = 0$
$\therefore$ amp $z_1 = -$amp $z_2 =$ amp $\bar{z}_2$
Since $|z_1| = |z_2|$, we get $|z_1| = |\bar{z}_2|$. So, $z_1 = \bar{z}_2$.
Also, $z_1z_2 = \bar{z}_2z_2 = |z_2|^2$
$=1$ because $|z_2| = 1$

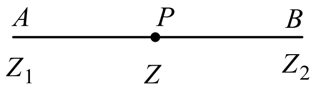
273 **(a,d)**
Roots of $4x^2-x-1=0$ are irrational. So, one root common implies both roots are common. Therefore,
$\frac{4}{3} = \frac{-1}{\lambda+\mu} = \frac{-1}{\lambda-\mu}$
$\Rightarrow \lambda = \frac{-3}{4}, \mu = 0$

274 **(a,d)**
On putting $x = 0, 1$ and $1/2$, we get
$-1 \le c \le 1$ (1)
$-1 \le a+b+c \le 1$ (2)
$-4 \le a+2b+4c \le 4$ (3)
From (1), (2), (3), we get
$|b| \le 8$ and $|a| \le 8$
$\Rightarrow |a|+|b|+|c| \le 17$

275 **(a,c)**
$2x^2+6xy+5y^2 = 1$ (1)
Equation (1) can be rewriter as
$2x^2+(6y)x+5y^2-1 = 0$
Since x is real,
$36y^2-8(5y^2-1) \ge 0$
$\Rightarrow y^2 \le 2$
$\Rightarrow -\sqrt{2} \le y \le \sqrt{2}$
Equation (1) can also be rewriter as
$5y^2+(6x)y+2x^2-1 = 0$
Since y is real,
$36x^2-20(2x^2-1) \ge 0$
$\Rightarrow 36x^2-40x^2+20 \ge 0$
$\Rightarrow -4x^2 \ge -20$
$\Rightarrow x^2 \le 5$
$\Rightarrow -\sqrt{5} \le x \le \sqrt{5}$

276 **(a,c,d)**

Given, $z = \frac{(1-t)z_1 + tz_2}{(1-t)+t}$



t: (1- t)

Clearly, z divides z_1 and z_2 in the ratio of $t:(1-t), 0 < t < 1$

$\Rightarrow \quad AP + BP = AB$

$ie, \quad |z - z_1| + |z - z_2| = |z_1 - z_2|$

$\Rightarrow$ Option (ac) is true

And $\arg(z - z_1) = \arg(z_2 - z) = \arg(z_2 - z_1)$

$\Rightarrow$ (b) is false and (d) is true

Also, $\arg(z - z_1) = \arg(z_2 - z_1)$

$\Rightarrow \quad \arg\left(\frac{z - z_1}{z_2 - z_1}\right) = 0$

$\therefore \quad \frac{z - z_1}{z_2 - z_1}$ is purely real

$\Rightarrow \quad \frac{z - z_1}{z_2 - z_1} = \frac{\bar{z} - \overline{z_1}}{\overline{z_2} - \overline{z_1}}$ or $\begin{vmatrix} z - z_1 & \bar{z} - \overline{z_1} \\ z_2 - z_1 & \overline{z_2} - \overline{z_1} \end{vmatrix} = 0$

$\therefore$ Options (c) is correct

277 **(a,c)**

$(-i)^{1/3} = (i^3)^{1/3} = i, i\omega, i\omega^2$

Where

$\omega = \frac{-1 + \omega\sqrt{3}}{2}$

Hence roots are, $i, (-\sqrt{3} - i)/2, (\sqrt{3} - i)/2$

278 **(a,b,c)**

$z^n \cos\theta_0 + z^{n-1}\cos\theta_1 + \cdots + z\cos\theta_{n-1} + \cos\theta_n = 2$

$\Rightarrow 2 = |z_0^n \cos\theta_0 + z_0^{n-1}\cos\theta_1 + \cdots + z_0\cos\theta_{n-1} + \cos\theta_n|$

$\Rightarrow 2 \le |z^n||\cos\theta_0| + |z|^{n-1}|\cos\theta_1| + \cdots + |z_0||\cos\theta_{n-1}| + |\cos\theta_n|$

$\Rightarrow 2 \le |z_0|^n + |z_0|^{n-1} + |z_0|^{n-2} + \cdots + |z_0| + 1$

Which is clearly satisfied for $|z_0| \ge 1$. If $|z_0| < 1$, then

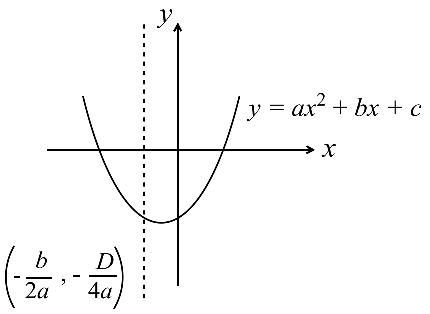
$2 < 1 + |z_0| + |z_0|^2 + \cdots + |z|^n + \cdots \infty$

$\Rightarrow 2 < \frac{1}{1 - |z_0|}$

$\Rightarrow |z_0| > \frac{1}{2}$

279 **(a,b,c,d)**

Let $f(x) = ax^2 + bx + c$



From the diagram, we can see that $a > 0, c < 0$ and $-[b(2a)] < 0$. Hence, $b > 0$

$\therefore \; a + b - c > 0$

280 **(a,c,d)**

Let $z = c$ be a real root. Then,

$\alpha c^2 + c + \overline{\alpha} = 0 \quad (1)$

Putting $\alpha = p + iq$, we have

$(p + iq)c^2 + c + p - iq = 0$

$\Rightarrow pc^2 + c + p = 0$ and $qc^2 - q = 0 \Rightarrow c = \pm 1 \quad (\because q \ne 0)$

$\therefore (1) \Rightarrow \alpha \pm 1 + \overline{\alpha} = 0$

Also,

$|c| = 1$

281 **(a,c)**

$p + q + r = a + b\omega + c\omega^2$

$+b + c\omega + a\omega^2$

$+c + a\omega + b\omega^2$

$\therefore p + q + r = (a + b + c)(1 + \omega + \omega^2) = 0 \quad (1)$

p, q, r lie on the circle $|z| = 2$, whose circumcentre is origin. Also, $(p + q + r)/3 = 0$. Hence the centroid coincides with circumcentre. So, the triangle is equilateral. Now,

$(p + q + r)^2 = 0$

$\Rightarrow p^2 + q^2 + r^2 = -2pqr\left[\frac{1}{p} + \frac{1}{q} + \frac{1}{r}\right]$

$= -2pqr\left[\frac{1}{a + b\omega + c\omega^2} + \frac{1}{b + c\omega + a\omega^2} + \frac{1}{c + a\omega + b\omega^2}\right]$

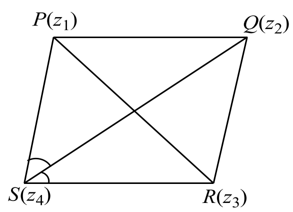
$= -2pqr\left[\frac{1}{\omega^2(a\omega + b\omega^2 + c)} + \frac{1}{\omega(b\omega^2 + c + a\omega)} + \frac{1}{c + a\omega + b\omega^2}\right]$

$= \frac{-2pqr}{a\omega + b\omega^2 + c}\left[\frac{1}{\omega^2} + \frac{1}{\omega} + \frac{1}{1}\right] = 0$

Hence,

$p^2 + q^2 + r^2 = 2(pq + qr + rp)$

282 **(a,b,c,d)**



1. $PS||QR \Rightarrow \arg\left(\frac{z_1 - z_4}{z_2 - z_3}\right) = 0 \Rightarrow \frac{z_1 - z_4}{z_2 - z_3}$ is purely real
2. Since diagonal bisect the angle

   $\Rightarrow \text{amp}\left(\frac{z_1 - z_4}{z_2 - z_4}\right) = \text{amp}\left(\frac{z_2 - z_4}{z_3 - z_4}\right)$
3. Diagonals of rhombus are perpendicular.

Hence, $(z_1 - z_3)/(z_2 - z_4)$ is purely imaginary

4. Diagonals of rhombus are not equal.
Hence, $|z_1 - z_3| \neq |z_2 - z_4|$

283 **(a,d)**

$|z_1^2 - z_2^2| = |\bar{z}_1^2 + \bar{z}_1^2 - 2\bar{z}_1\bar{z}_2|$

$\Rightarrow |z_1 - z_2||z_1 + z_2| = |\bar{z}_1 - \bar{z}_2|^2$

$\Rightarrow |z_1 + z_2| = |\bar{z}_1 - \bar{z}_2|$

$|z_1 + z_2| = |z_1 - z_2|$

$\Rightarrow \left|\frac{z_1}{z_2} + 1\right| = \left|\frac{z_1}{z_2} - 1\right|$

$\Rightarrow \frac{z_1}{z_2}$ lies on $\perp$ bisector of 1 and -1

$\Rightarrow \frac{z_1}{z_2}$ lies on imaginary axis

$\Rightarrow \frac{z_1}{z_2}$ is purely imaginary

$\Rightarrow \arg\left(\frac{z_1}{z_2}\right) = \pm\frac{\pi}{2}$

$|\arg(z_1) - \arg(z_2)| = \frac{\pi}{2}$

284 **(a,b)**

$\left|z - \frac{1}{z}\right| = 1$

$\Rightarrow 1 \geq \left||z| - \frac{1}{|z|}\right|$

$\Rightarrow -1 \leq |z| - \frac{1}{|z|} \leq 1$

$\Rightarrow -|z| \leq |z|^2 - 1 \leq |z|$

From $|z|^2 - 1 \geq |z|$, we get

$|z|^2 + |z| - 1 \geq 0$

$\Rightarrow |z| \geq \frac{-1+\sqrt{5}}{2}$ (1)

From $|z|^2 - 1 \leq |z|$, we get

$|z|^2 - |z| - 1 \leq 0$

$\Rightarrow \frac{1-\sqrt{5}}{2} \leq |z| \leq \frac{1+\sqrt{5}}{2}$ (2)

From (1) and (2), we get

$\Rightarrow \frac{-1+\sqrt{5}}{2} \leq |z| \leq \frac{1+\sqrt{5}}{2}$

$\Rightarrow |z|_{\min} = \frac{\sqrt{5}-1}{2}, |z|_{\max} = \frac{1+\sqrt{5}}{2}$

285 **(b,d)**

Let α and β be the roots.

$\therefore |\alpha - \beta| = 3$ and $\alpha^2 + \beta^2 = 29$

$\Rightarrow |\alpha - \beta|^2 = 9$

$\Rightarrow \alpha^2 + \beta^2 - 2\alpha\beta = 9$

$\Rightarrow \alpha\beta = 10$

Now, $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$= 29 + 20 = 49$

$\therefore \alpha + \beta = \pm 7$

Hence, required equation is

$x^2 \pm 7x + 10 = 0$

Hence, options (b) and (d) are correct.

286 **(a,b)**

$\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

$\alpha + h + \beta + h = \frac{-q}{p}$

and $(\alpha + h)(\beta + h) = \frac{r}{p}$

$\therefore -\frac{q}{p} = \alpha + \beta + 2h = -\frac{b}{a} + 2h$

$\Rightarrow h = \frac{1}{2}\left(\frac{b}{a} - \frac{q}{p}\right)$

Now, $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$= \left(-\frac{b}{a}\right)^2 - \frac{4c}{a} = \frac{b^2 - 4ac}{a^2}$

Also, $(\alpha - \beta)^2 = [(\alpha + h) - (\beta + h)]^2$

$= \frac{q^2 - 4pr}{p^2}$

$\Rightarrow \frac{b^2 - 4ac}{a^2} = \frac{q^2 - 4pr}{p^2}$

287 **(a,d)**

Let α and β be the roots of the equation

$4x^2 + 2x - 1 = 0$

$\therefore \alpha + \beta = -\frac{2}{4} = -\frac{1}{2}$ and $4\alpha^2 + 2\alpha - 1 = 0$

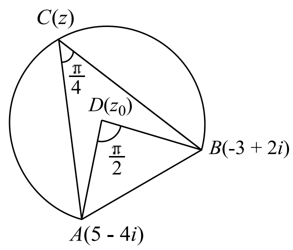
$\Rightarrow \beta = -\frac{1}{2} - \alpha$ and $4\alpha^2 = 1 - 2\alpha$

$\Rightarrow 4\alpha^3 - \alpha(1 - 2\alpha) = \alpha - 2\alpha^2$

$= \alpha - \frac{1}{2} + \alpha = 2\alpha - \frac{1}{2}$

$\Rightarrow 4\alpha^3 - 3\alpha = -\alpha - \frac{1}{2} = \beta$

288 **(a,c,d)**



$\frac{z_0 - (-3 + 2i)}{z_0 - (5 - 4i)} = \frac{BD}{AD} = e^{i\pi/2} = i$

$\Rightarrow z_0 + 3 - 2i = iz_0 - 5i - 4$

$\Rightarrow z_0 = -2 - 5i$

$\Rightarrow$ Radius $AD = |5 - 4i - (-2 - 5i)|$

$= |17 + i|$

$= \sqrt{50} = 5\sqrt{2}$

Length of arc $= \frac{3}{4}$ (perimeter of circle)

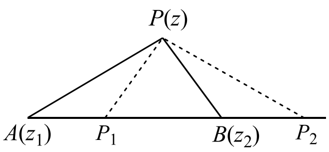
$= \frac{3}{4}(2\pi \times 5\sqrt{2})$

$= \frac{15\pi}{\sqrt{2}}$

289 **(a,c,d)**

Let the roots be $a/r, a, ar$, where $a > 0, r > 1$.
Now,
$a/r + a + ar = -p$ (1)
$a(a/r) + a(ar) + (ar)(a/r) = q$ (2)
$(a/r)(a)(ar) = 1$ (3)
$\Rightarrow a^3 = 1$
$\Rightarrow a = 1$
Hence, (c) is correct. From (1), putting $a = 1$, we get
$-p - 3 > 0$ $\left(\because r + \frac{1}{r} > 2\right)$
$\Rightarrow p < -3$
Hence, (b) is not correct. Also,
$1/r + 1 + r = -p$ (4)
From (2), putting $a = 1$, we get
$1/r + r + 1 = q$ (5)
From (4) and (5), we have
$-p = q \Rightarrow p + q = 0$
Hence, (a) is correct. Now, as $r > 1$
$a/r = 1/r < 1$
and $ar = r > 1$
Hence, (d) is correct

290 **(b,c)**



Let internal and external bisectors of $\angle APB$ meet the line joining A and B at P_1 and P_2, respectively. Hence,
$AP_1 : P_1B \equiv PA : PB \equiv 3:1$ (internal division)
$AP_2 : P_2B \equiv PA : PB \equiv 3:1$ (external division)
Thus, P_1 and P_2 are fixed points. Also,
$\angle P_1PP_2 = \frac{\pi}{2}$
Thus 'P' lies on a circle having P_1P_2 as its diameter. Clearly, $B(z_2)$ lies inside this circle

291 **(a,d)**
$2^x = t$
$t^2 - 8t + 12 = 0$
$(t-6)(t-2) = 0$
$2^x = 6 \Rightarrow x = \log_2 6 = 1 + \frac{\log 3}{\log 2}$
$2^x = 2 \Rightarrow x = 1$

292 **(a,b,c)**
Since, the roots of equation are real.
$\therefore\ B^2 - 4AC > 0$
$\Rightarrow\ a^4 > 4b^2$
Hence, option (a) is correct.
If $f(x) = x^2 + a^2x + b^2$ ($\because c$ lies outside the roots)
$\therefore f(c) > 0$, then $c^2 + a^2c + b^2 > 0$
Hence, option (b) is correct.
Also, (x-coordinate of vertex)$> c$
$\Rightarrow\ -\frac{a^2}{2} > c$
Hence, option (c) is correct.

293 **(a,c)**
If α be the common root, then
$\alpha^2 + b\alpha - a = 0$ and $\alpha^2 - a\alpha + b = 0$
Subtracting,
$\alpha(b + a) - (a + b) = 0$
$\Rightarrow (a - b)(\alpha - 1) = 0$
$\Rightarrow a + b = 0$ or $\alpha = 1$
When $\alpha = 1$, then from any equation we have
$a - b = 1$

294 **(a,b)**
Here,
$\cos^2\theta - \sin^2\theta = \cos 2\theta$
$\Rightarrow \cos^4\theta - \sin^4\theta = \cos 2\theta$
$\Rightarrow (-2b)^2 - 4b = (-4)^2 - 4 \times 2$
(since L.H.S. is difference of roots of first equation and R.H.S. is difference of roots of second equation)
$\Rightarrow 4b^2 - 4b = 16 - 8 = 8$
$\Rightarrow 4b^2 - 4b - 8 = 0$
$\Rightarrow b^2 - b - 2 = 0$
$\Rightarrow (b + 1)(b - 2) = 0$
$\Rightarrow b = 2, -1$

295 **(a,c)**
Given, $b^2 = ac$
$\Rightarrow \left(\frac{b}{a}\right)^2 = \frac{c}{a}$
$\Rightarrow \left(-\frac{b}{a}\right)^2 = \frac{c}{a}$
$\Rightarrow (\alpha + \beta)^2 = \alpha\beta$
$\Rightarrow \alpha^2 + \beta^2 + \alpha\beta = 0$
$\Rightarrow \left(\frac{\alpha}{\beta}\right)^2 + \left(\frac{\alpha}{\beta}\right) + 1 = 0$
$\Rightarrow \frac{\alpha}{\beta} = \frac{-1 \pm \sqrt{3}i}{2}$ (where $i = \sqrt{-1}$)

296 **(b,c)**
$f(x) = x^3 + 3x^2 - 9x + c$ is of the form $(x - \alpha)^2(x - \beta)$, showing that α is a double root so that $f'(x) = 0$ has also one root α, i.e., $3x^2 + 6x - 9 = 0$ has one root α. Hence, $x^2 + 2x - 3 = 0$ or $(x + 3)(x - 1) = 0$ has the root α which can be either -3 or 1. If $\alpha = 1$, then $f(x) = 0$ gives $c - 5 = 0$ or $c = 5$. If $\alpha = -3$, then $f(x) = 0$ gives
$-27 + 27 + 27 + c = 0$

$\therefore c = -27$

297 **(a,b)**

The equation $px^2 + qx + r = 0$ has no real root, therefore $D < 0$.

$\therefore pf(x) > 0, \forall x \in R$, where $f(x) = px^2 + qx + r$

In general, $pf(1) > 0 \Rightarrow p(p+q+r) > 0$ ...(i)

and $pf(0) > 0 \Rightarrow pr > 0$ ....(ii)

From relations (i) and (ii), we get

$r(p+q+r) > 0$

298 **(a,b)**

Equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root. Therefore,

$(q - q')^2 = (pq' - p'q)(p' - p)$ (1)

Subtracting two equations, we have

$x = \dfrac{q - q'}{p' - p}$

Also using (1),

$x = \dfrac{q - q'}{p' - p} = \dfrac{pq' - p'q}{q - q'}$

299 **(a,c,d)**

$z' = ze^{i\alpha}$ (1)

$z'' = ze^{-i\alpha}$ (2)

$\therefore z'z'' = z^2$

$\Rightarrow z', z, z''$ are in G.P.

Also,

$\left(\dfrac{z'}{z}\right)^2 + \left(\dfrac{z''}{z}\right)^2 = 2\cos 2\alpha$

$\Rightarrow z'^2 + z''^2 = 2z^2 \cos 2\alpha$

$\Rightarrow z'^2 + z''^2 = 2z^2(2\cos^2\alpha - 1)$

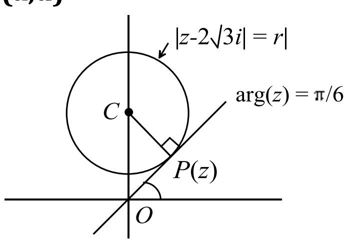
$\Rightarrow z'^2 + z''^2 + 2z^2 = 4z^2\cos^2\alpha$

$\Rightarrow z'^2 + z''^2 + 2z'z'' = 4z^2\cos^2\alpha$

$\Rightarrow (z' + z'')^2 = 4z^2\cos^2\alpha$

$\Rightarrow z' + z'' = 2z\cos\alpha$

300 **(a,d)**



$CP = r, OC = 2\sqrt{3}, \angle COP = \pi/3$

$\Rightarrow CP = OC \sin\dfrac{\pi}{3} = 2\sqrt{3}\dfrac{\sqrt{3}}{2} = 3$

Thus, when $r = 3$, the circle touches the line.

Hence, for two distinct points of intersection

$3 < r < 2\sqrt{3}$

301 **(a,b)**

The given equation can be written as

$\dfrac{p}{2x} = \dfrac{(a+b)x + c(b-a)}{x^2 - c^2}$

or $p(x^2 - c^2) = 2(a+b)x^2 - 2c(a-b)x$

or $(2a + 2b - p)x^2 - 2c(a-b)x + pc^2 = 0$

Now, $c^2(a-b)^2 - pc^2(2a + 2b - p) = 0$ ($\because$ equal roots)

$\Rightarrow (a-b)^2 - 2p(a+b) + p^2 = 0$ ($\because c^2 \neq 0$)

$\Rightarrow [p - (a+b)]^2 = (a+b)^2 - (a-b)^2$

$\Rightarrow p = a + b \pm 2\sqrt{ab} = \left(\sqrt{a} \pm \sqrt{b}\right)^2$

302 **(a,b)**

We can write the given equation as

$\dfrac{p}{2x} = \dfrac{(a+b)x + c(b-a)}{x^2 - c^2}$

$\Rightarrow p(x^2 - c^2) = 2(a+b)x^2 - 2c(a-b)x$

$\Rightarrow (2a + 2b - p)x^2 - 2c(a-b)x + pc^2 = 0$

For this equation to have equal roots,

$c^2(a-b)^2 - pc^2(2a + 2b - p) = 0$

$\Rightarrow (a-b)^2 - 2p(a+b) + p^2 = 0$ $[\because c^2 \neq 0]$

$\Rightarrow [p - (a+b)]^2 = (a+b)^2 - (a-b)^2 = 4ab$

$\Rightarrow p - (a+b) = \pm 2\sqrt{ab}$

$\Rightarrow p = a + b \pm 2\sqrt{ab} = \left(\sqrt{a} \pm \sqrt{b}\right)^2$

303 **(a,b,c)**

$x^{\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}} = \sqrt{2}$

$\Rightarrow \left(\dfrac{3}{4}(\log_2 x)^2 + \log_2 x - \dfrac{5}{4}\right)\log_2 x = \log_2 \sqrt{2}$

(taking logarithm both sides on base 2)

$\Rightarrow \left(\frac{3}{4}t^2 + t - \frac{5}{4}\right)t = \frac{1}{2}$ (putting $\log_2 x = t$)

$\Rightarrow 3t^3 + 4t^2 - 5t - 2 = 0$

$\Rightarrow 3t^3 - 3t^2 + 7t^2 - 7t + 2t - 2 = 0$

$\Rightarrow (3t^2 + 7t + 2)(t - 1) = 0$

$\Rightarrow (3t + 1)(t + 2)(t - 1) = 0$

$\Rightarrow t = \log_2 x = 1, -2, -\dfrac{1}{3}$

$\Rightarrow x = 2, 2^{-2}, 2^{-\frac{1}{3}}$

304 **(b,c)**

We have,

$\left|\dfrac{1}{z_2} + \dfrac{1}{z_1}\right| = \left|\dfrac{1}{z_2} - \dfrac{1}{z_1}\right| \Rightarrow |z_1 + z_2| = |z_1 - z_2|$

Squaring both sides, we have

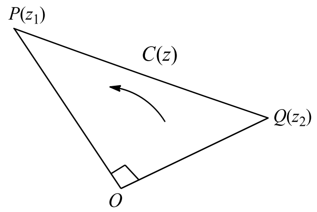
$|z_1|^2 + |z_2|^2 + 2(z_1\bar{z}_2 + \bar{z}_1 z_2)$

$= |z_1|^2 + |z_2|^2 - 2(z_1\bar{z}_2 + \bar{z}_1 z_2)$

$\Rightarrow 4(z_1\bar{z}_2 + \bar{z}_1 z_2) = 0$

$\Rightarrow \dfrac{z_1}{z_2} + \dfrac{\bar{z}_1}{\bar{z}_2} = 0$

$\Rightarrow \arg\left(\dfrac{z_1}{z_2}\right) = \dfrac{\pi}{2} = \arg\left(\dfrac{z_1 - 0}{z_2 - 0}\right)$



That is angle between z_2, O and z_1 is a right angle, taken in order, as shown in the above diagram. Now, the circumcentre of the above diagram will lie on the line PQ as diameter and is represented by C which is the centre of PQ, such that $z = (z_1 + z_2)/2$, where z is the affix of circumcentre

305 **(a,d)**

Let,

$$\frac{x^2 + ax + 3}{x^2 + x + a} = y$$

$\Rightarrow x^2(1-y) - x(y-a) + 3 - ay = 0$

$\because x \in R$

$(y-a)^2 - 4(1-y)(3-ay) \geq 0$

$\Rightarrow (1-4a)y^2 + (2a+12)y + a^2 - 12 \geq 0$ (1)

Now, (1) is true for all $y \in R$, if $1 - 4a > 0$ and $D \leq 0$. Hence,

$a < \frac{1}{4}$ and $4(a+6)^2 - 4(a^2-12)(1-4a) \leq 0$

$\Rightarrow a < \frac{1}{4}$ and $4a^3 - 36a + 48 \leq 0$

$\Rightarrow a < \frac{1}{4}$ and $4a^3 \leq 36a - 48$

$\Rightarrow 4a^3 < 36\left(\frac{1}{4}\right) - 48$

$\Rightarrow 4a^3 + 39 < 0 \quad \left[\because a < \frac{1}{4}\right]$

306 **(a,d)**

Since $\alpha, \beta, \gamma, \delta$ are in H.P., hence $1/\alpha, 1/\beta, 1/\gamma, 1/\delta$ are in A.P. and they may be taken as $a-3d, a-d, a+d, a+3d$. Replacing x by $1/x$, we get the equation whose roots are $1/\alpha, 1/\beta, 1/\gamma, 1/\delta$. Therefore, equation $x^2 - 4x + A = 0$ has roots $a-3d, a+d$ and equation $x^2 - 6x + B = 0$ has roots $a-d, a+3d$. Sum of the roots is

$2(a-d) = 4, 2(a+d) = 6$

$\therefore a = 5/2, d = 1/2$

Product of the roots is

$(a-3d)(a+d) = A = 3$

$(a-d)(a+3d) = B = 8$

307 **(a,c)**

Since $P(x)$ divides both of them, hence $P(x)$ also divides

$(3x^4 + 4x^2 + 28x + 5) - 3(x^4 + 6x^2 + 25)$

$= -14x^2 + 28x - 70$

$= -14(x^2 - 2x + 5)$

Which is a quadratic. Hence,

$P(x) = x^2 - 2x + 5 \Rightarrow P(1) = 4$

308 **(a,c)**

Given,

$z^n = (z+1)^n \Rightarrow |z^n| = |(z+1)^n|$

$\therefore |z|^n = |z+1|^n \Rightarrow |z| = |z+1|$

$\Rightarrow |z|^2 = |z+1|^2$

$\Rightarrow x^2 + y^2 = (x+1)^2 + y^2$, where $z = x + iy$

$\Rightarrow x = -\frac{1}{2}$

Hence, z lies on the line $x = -1/2$. Hence sum of real parts of the roots is $-(n-1)/2$ (since equation has $n-1$ roots)

309 **(a,b,d)**

Let $z = \alpha$ be a real root. Then,

$\alpha^3 + (3+2i)\alpha + (-1+ia) = 0$

$\Rightarrow (\alpha^3 + 3\alpha - 1) + i(a + 2\alpha) = 0$

$\Rightarrow \alpha^3 + 3\alpha - 1 = 0$ and $\alpha = -a/2$

$\Rightarrow -\frac{a^3}{8} - \frac{3a}{2} - 1 = 0$

$\Rightarrow a^3 + 12a + 8 = 0$

Let $f(a) = a^3 + 12a + 8$

$\therefore f(-1) < 0, f(0) > 0, f(-2) < 0, f(1) > 0$ and $f(3) > 0$

Hence, $a \in (-1, 0)$ or $a \in (-2, 1)$ or $a \in (-2, 3)$

310 **(a,c)**

The given equation can be rewritten as

$\frac{1}{1+\log_a x} + \frac{2}{\log_a x} + \frac{3}{2+\log_a x}$ ...(i)

Let $\log_a x = t$

Then, $\frac{1}{1+t} + \frac{2}{t} + \frac{3}{2+t} = 0$

$\Rightarrow 6t^2 + 11t + 4 = 0$

$\Rightarrow (2t+1)(3t+4) = 0$

$\Rightarrow t = -\frac{1}{2}$ and $t = -\frac{4}{3}$

$\Rightarrow \log_a x = -\frac{1}{2}$ and $\log_a x = -\frac{4}{3}$

$\Rightarrow x = a^{-1/2}$ and $x = a^{-4/3}$

311 **(a,b,c)**

$|z_1| = |z_2| = 1 \Rightarrow a^2 + b^2 = c^2 + d^2 = 1$ (1)

and

$\text{Re}(z_1\overline{z}_2) = 0 \Rightarrow Re\{(a+ib)(c-id)\} = 0 \Rightarrow ac + bd = 0$ (2)

Now from (1) and (2),

$a^2 + b^2 = 1 \Rightarrow a^2 + \frac{a^2c^2}{d^2} = 1 \Rightarrow a^2 = d^2$ (3)

Also,

$c^2 + d^2 = 1 \Rightarrow c^2 + \frac{a^2c^2}{b^2} = 1 \Rightarrow b^2 = c^2$ (4)

$|\omega_1| = \sqrt{a^2 + c^2} = \sqrt{a^2 + b^2} = 1$ [From (1) and (4)]

and

$|\omega_2| = \sqrt{b^2 + d^2} = \sqrt{a^2 + b^2} = 1$ [From (1) and (4)]

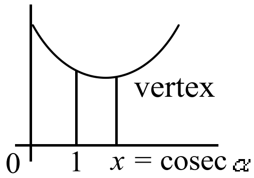
Further,

$\text{Re}(\omega_1\overline{\omega}_2) = \text{Re}\{(a+ic)(b-id)\}$
$= ab + cd$
$= ab + \left(-\frac{ac^2}{b}\right)$ [From (2)]
$= \frac{ab^2 - ac^2}{b} = 0$ [From (4)]
Also,
$$\text{Im}(\omega_1\overline{\omega}_2)bc - ad = bc - a\left(-\frac{ac}{b}\right) = \frac{(a^2+b^2)c}{b}$$
$$= \frac{c}{b} = \pm 1 \neq 0$$
$\therefore |\omega_1| = 1, |\omega_2| = 1$ and $\text{Re}(\omega_1\overline{\omega}_2) = 0$

312 **(a,b,d)**
$$\left|\frac{2z-i}{z+1}\right| = m \Rightarrow \left|z - \frac{i}{2}\right| = \frac{m}{2}|z+1|$$
This shows that the given equation will represent a circle, if $m/2 \neq 1$, i.e., $m \neq 2$

313 **(a,b,d)**
$$x^n - 1 = (x-1)(x-z_1)(x-z_2)\cdots(x-z_{n-1})$$
$$\Rightarrow \frac{x^n-1}{x-1} = (x-z_1)(x-z_2)\cdots(x-z_{n-1})$$
Putting $x = \omega$, we have
$$\prod_{r=1}^{n-1}(\omega - z_r) = \frac{\omega^n - 1}{\omega - 1}$$
$$= \begin{cases} 0, & \text{if } n = 3k, k \in Z \\ 1, & \text{if } n = 3k+1, k \in Z \\ 1+\omega, & \text{if } n = 3k+2, k \in Z \end{cases}$$

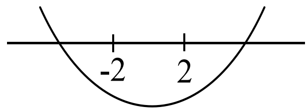
314 **(a,c)**
Clearly, we have to find it for real z. Let $z = x$. Then,
$$|z - w| = |x - w^2| = |w - w^2|$$
$$\Rightarrow \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} = \left|\frac{-1+\sqrt{3}i}{2} - \frac{-1-\sqrt{3}i}{2}\right|^2 = 3$$
$$\Rightarrow x + \frac{1}{2} = \pm\frac{3}{2}$$
$\Rightarrow x = 1, -2$

315 **(a,b)**
Given, $(\sin\alpha)x^2 - 2x + b \geq 2$. Let $f(x) = (\sin\alpha)x^2 - 2x + b - 2$. Abscissa of the vertex is given by
$$x = \frac{1}{\sin\alpha} > 1$$


The graph of $f(x) = (\sin\alpha)x^2 - 2x + b - 2, \forall\, x \leq 1$, is shown in the figure. Therefore, minimum of $f(x) = (\sin\alpha)x^2 - 2x + b - 2$ must be greater than zero but minimum is at $x = 1$. That is,
$\sin\alpha - 2 + b - 2 \geq 0, b \geq 4 - \sin\alpha, \alpha \in (0,\pi)$

316 **(b,d)**
Let $f(x) = x^2 + ax + b$. Then,
$x^2 + (2c+a)x + c^2 + ac + b = f(x+c)$
Thus, the roots of $f(x+c) = 0$ will be $0, d-c$

317 **(c,d)**
We have,
$x^2 + x + 1 = (x-\omega)(x-\omega^2)$
Since $f(x)$ is divisible by $x^2+x+1, f(\omega) = 0, f(\omega^2) = 0$, so
$P(\omega^3) + \omega Q(\omega^3) = 0 \Rightarrow P(1) + \omega Q(1) = 0$ (1)
$P(\omega^6) + \omega^2 Q(\omega^6) = 0 \Rightarrow P(1) + \omega^2 Q(1) = 0$ (2)
Solving (1) and (2), we obtain
$P(1) = 0$ and $Q(1) = 0$
Therefore, both $P(x)$ and $Q(x)$ are divisible by $x-1$. Hence, $P(x^3)$ and $Q(x^3)$ are divisible by x^3-1 and so by $x-1$. Since $f(x) = P(x^3) + xQ(x^3)$, we get $f(x)$ is divisible by $x-1$

318 **(a,b,c)**


$f(x) = ax^2 + bx + c$
$f(0) = c < 0, D > 0 \Rightarrow b^2 - 4ac > 0$
$f(1) < 0$ and $f(-1) < 0$
$\Rightarrow a - |b| + c < 0$
$f(2) < 0$ and $f(-2) < 0$
$\Rightarrow 4a - 2|b| + c < 0$
Nothing can be said about $f(3)$ or $f(-3)$, whether it is positive or negative

319 **(c,d)**
Let,
$$y = \frac{(x-a)(x-b)}{(x-c)}$$
$\Rightarrow (x-c)y = x^2 - (a+b)x + ab$
$\Rightarrow x^2 - (a+b+y)x + ab + cy = 0$
Since x is real, so
$D \geq 0$
$\Rightarrow (a+b+y)^2 - 4(ab+cy) \geq 0, \forall x \in R$
$\Rightarrow y^2 + 2y(a+b-2c) + (a-b)^2 \geq 0, \forall x \in R$
$\Rightarrow 4(a+b-2c)^2 - 4(a-b)^2 < 0$
$\Rightarrow (a+b-2c+a-b)(a+b-2c-a+b) < 0$
$\Rightarrow 4(a-c)(b-c) < 0$
$\Rightarrow a-c < 0$ and $b-c > 0$ or $a-c > 0$ and $b-c < 0$
$\Rightarrow a < c < b$ or $a > c > b$

320 **(a,b,c)**
We have, $\frac{1+i\cos\theta}{1-2i\cos\theta} = \frac{(1+i\cos\theta)(1+2i\cos\theta)}{(1-2i\cos\theta)(1+2i\cos\theta)}$
$$= \frac{(1-2\cos^2\theta) + i\,3\cos\theta}{1+4\cos^2\theta}$$

Thus, $\frac{(1+i\cos\theta)}{(1-2i\cos\theta)}$ is a real number if $\cos\theta = 0$

$\Rightarrow\ \theta = 2n\pi \pm \frac{\pi}{2}$

Where n is an integer

321 **(b,c,d)**

Given equation is $x^3 - ax^2 + bx - 1 = 0$. If roots of the equation be α, β, γ, then

$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$

$= a^2 - 2b$

$\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$

$= (\alpha\beta + \beta\gamma + \gamma\alpha)^2 - 2\alpha\beta\gamma(\alpha + \beta + \gamma)$

$= b^2 - 2a$

$\alpha^2\beta^2\gamma^2 = 1$

So, the equation whose roots are $\alpha^2, \beta^2, \gamma^2$ is given by

$x^3 - (a^2 - 2b)x^2 + (b^2 - 2a)x - 1 = 0$

It is identical to

$x^3 - ax^2 + bx - 1 = 0$

$\Rightarrow a^2 - 2b = a$ and $b^2 - 2a = b$

Eliminating b, we get

$\frac{(a^2 - a)^2}{4} - 2a = \frac{a^2 - a}{2}$

$\Rightarrow a\{a(a-1)^2 - 8 - 2(a-1)\} = 0$

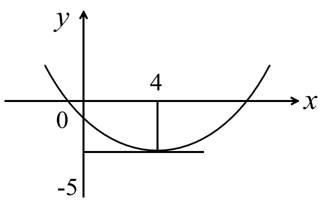
$\Rightarrow a(a^3 - 2a^2 - a - 6) = 0$

$\Rightarrow a(a-3)(a^2 + a + 2) = 0$

$\Rightarrow a = 0$ or $a = 3$ or $a^2 + a + 2 = 0$

Which gives $b = 0$ or $b = 3$ or $b^2 + b + 2 = 0$. So, $a = b = 0$ or $a = b = 3$ or a, b are roots of $x^2 + x + 2 = 0$

323 **(a,b,c)**



From figure,

$a > 0$

$-\frac{b}{2a} = 4 \Rightarrow -\frac{b}{2a} > 0$

$\therefore\ b < 0$

$f(0) = c < 0$

Also, $-\frac{b}{2a} = 4\ \Rightarrow 8a + b = 0$

324 **(a,c)**

Let $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

$\Rightarrow\ |z_2| = r_2$

Also, $\arg(z_1) + \arg(z_2) = 0$

$\Rightarrow\ \arg(z_1) = -\arg(z_2) = -\theta_2$

$\therefore\ z_1 = r_2[\cos(-\theta_2) + i\sin(-\theta_2)]$

$= r_2[\cos\theta_2 - i\sin\theta_2]$

$\Rightarrow\ z_2 = \bar{z}_2$

$\Rightarrow\ z_1 = \frac{1}{z_2}$

$\Rightarrow\ z_1 z_2 = 1$

325 **(a,d)**

If α is a real root, then

$\alpha^3 + (3+i)\alpha^2 - 3\alpha - (m+i) = 0$

$\therefore \alpha^3 + 3\alpha^2 - 3\alpha - m = 0$ and $\alpha^2 - 1 = 0$

$\Rightarrow \alpha = 1$ or -1

$\alpha = 1\ \Rightarrow m = 1$

$\alpha = -1\ \Rightarrow m = 5$

326 **(d)**

For real roots, $D \geq 0$

$\Rightarrow (-4)^2 - 4(2\lambda - 1)(2\lambda - 1) \geq 0$

$\Rightarrow\ (2\lambda - 1)^2 \leq 4$

$\Rightarrow\ -2 \leq 2\lambda - 1 \leq 2$

$\Rightarrow\ -\frac{1}{2} \leq \lambda \leq \frac{3}{2}$

$\therefore$ Integral values of λ are 0 and 1.

Hence, greatest integral value of λ=1

327 **(b)**

According to statement 1, given equation is

$x^2 - bx + c = 0$

Let α, β be two roots such that

$|\alpha - \beta| = 1$

$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 1$

$\Rightarrow b^2 - 4c = 1$

According to statement 2, given equation is $4abc\, x^2 + (b^2 - 4ac)x - b = 0$. Hence,

$D = (b^2 - 4ac)^2 + 16ab^2c$

$= (b^2 + 4ac)^2 > 0$

Hence, roots are real and unequal

328 **(c)**

$f(x) = ax^2 + bx + c$

Given, $f(0) + f(1) = 2$

$\Rightarrow f(x) > 0 \ \forall \ x \in R$

Hence, statement 1 is true. Let,

$f(x) = x^2 - x + 1$

$a + b = 0$

Hence, statement 2 is false

329 **(a)**

$\because \ e^{i\theta} = \cos\theta + i\sin\theta$

$\Rightarrow e^{-i\theta} = \cos\theta - i\sin\theta$

$\therefore \ \cos\theta = \dfrac{e^{i\theta} + \mathrm{e}^{i\theta}}{2}$

Now, $\cos(1 - i) = \frac{e^{i(1-i)} + e^{-i(1-i)}}{2} = \frac{e^{(i+1)} + e^{-(1+i)}}{2}$

$= \dfrac{e(e^i) + e^{-1}(e^{-i})}{2}$

$= \dfrac{e(\cos 1 + i\sin 1 + e^{-1}(\cos 1 - i\sin 1)}{2}$

$= \dfrac{1}{2}\left(e + \dfrac{1}{e}\right)\cos 1 + \dfrac{i}{2}\left(e - \dfrac{1}{e}\right)\sin 1$

$\therefore \ a = \dfrac{1}{2}\left(e + \dfrac{1}{e}\right)\cos 1, b = \dfrac{1}{2}\left(e - \dfrac{1}{e}\right)\sin 1$

330 **(b)**

Fourth roots of unity are $-1, 1, -i$ and i

$\therefore z_1^2 + z_2^2 + z_3^2 + z_4^2 = 0$ and $z_1 + z_2 + z_3 + z_4 = 0$

331 **(d)**

Statement 2 is obviously true. Let,

$f(x) = (x - p)(x - r) + \lambda(x - q)(x - s) = 0$

Then, $f(p) = \lambda(p - q)(p - s)$

$f(r) = \lambda(r - q)(r - s)$

$\Rightarrow f(p)f(r) < 0$

Hence, there is a root between p and r. Thus, statement 1 is false

332 **(a)**

Here, coefficient of $x^2 = 2 > 0$ then conditions are $D \geq 0, f(1) > 0$ and $f(2) > 0$

$\Rightarrow 1 - 8a \geq 0, 2 - 1 + a > 0$ and $8 - 2 + a > 0$

$\Rightarrow a \leq \dfrac{1}{8}, a > -1$ and $a > -6$

$\Rightarrow \ -1 < a \leq \dfrac{1}{8}$

Hence, option (a) is correct.

333 **(d)**

$a^2 - 3a + 2 = 0 \Rightarrow a = 1,2$

$a^2 - 5a + 6 = 0 \Rightarrow a = 2, 3$

$a^2 - 4 = 0 \ \Rightarrow a = \pm 2$

Therefore, $a = 2$ is the only solution

Hence, statement 1 is false. Statement 2 is true by definition

334 **(b)**

$D = (3)^2 - 4 \cdot 2 \cdot 4 = -23 < 0$

$\therefore$ Roots of $2x^2 + 3x + 4 = 0$ are imaginary

Now, $\because$ $2, 3, 4 \in R$

$\therefore$ Roots are conjugate to each other.

$\because$ One root is common in $ax^2 + bx + c = 0$ and $2x^2 + 3x + 4 = 0$ (given) then other roots is also common.

$\because$ Roots are conjugate $(a, b, c \ \in R)$

Hence, both equation are identical

$\therefore \ a : b : c = 2 : 3 : 4$

335 **(a)**

We have, $\arg(z) = 0 \Rightarrow z$ is purely real.

$\therefore$ Statement (II) is true.

Also, $|z_1| = |z_2 + |z_1 - z_2|$

$\Rightarrow |z_1 - z_2|^2 = (|z_1| - |z_2|)^2$

$\Rightarrow |z_1|^2 + |z_2|^2 - 2|z_1||z_2|\cos(\theta_1 - \theta_2)$

$= |z_1|^2 + |z_2|^2 - 2|z_1||z_2|$

$\Rightarrow \cos(\theta_1 - \theta_2) = 1$

$\Rightarrow \theta_1 - \theta_2 = 0$

$\Rightarrow \arg(z_1) - \arg(z_2) = 0$

$\Rightarrow \arg\left(\frac{z_1}{z_2}\right) = 0 \Rightarrow \frac{z_1}{z_2}$ is purely real

$\Rightarrow \text{Im}\left(\frac{z_1}{z_2}\right) = 0$

Statement (I) and Statement (II) are true and Statement (II) is a correct explanation of Statement (I)

336 **(a)**

Let $f(x) = ax^2 + bx + c$. Since coefficient are integers and one root is irrational, so both the roots are irrational. Hence, for any $\lambda \in Q$,

$f(\lambda) \neq 0 \Rightarrow |f(\lambda)| > 0$

$\Rightarrow \left|\frac{ap^2}{q^2} + \frac{bp}{q} + c\right| > 0$ where $\lambda = \frac{p}{q}, p, q \in Z$

$\Rightarrow \frac{1}{q^2}|ap^2 + bpq + cq^2| > 0$

Now, $a, b, c, p, q \in I$. Hence,

$|ap^2 + bpq + cq^2| \geq 1$

$\Rightarrow |f(\lambda)| \geq \frac{1}{q^2}$

337 **(a)**

$ax^2 + bx + c = 0$ has two complex conjugate roots only if all the coefficients are real. If all the coefficients are not real then it is not necessary that both the roots are imaginary. Hence, statement 2 is true

Now, equation $x^2 - 3x + 4 = 0$ has two complex conjugate roots. If $ax^2 + bx + c = 0$ has all coefficients real, then there will be two common roots. But if there is only one root common, then at least one of a, b, c must be non-real

Thus, both the statements are true and statement 2 is correct explanation of statement 1

338 **(d)**

Statement 2 is true as it is the definition of an ellipse. Statement 1 is false as distance between 1 and 8 is 7 but $|z-1| + |z-8| = 5 < 7$. Hence not such z exists

339 **(a)**

$\cos\frac{\pi}{4} = 2\cos^2\frac{\pi}{8} - 1$

$\Rightarrow \cos^2\frac{\pi}{8} = \left(\frac{1}{\sqrt{2}} + 1\right)\frac{1}{2}$

$\Rightarrow \cos^4\frac{\pi}{8} = \frac{1}{4}\left(\frac{1}{2} + 1 + \frac{2}{\sqrt{2}}\right) = \left(\frac{3}{2} + \sqrt{2}\right)\frac{1}{4}$

$\Rightarrow \frac{1}{4}\left(\frac{3}{2} + \sqrt{2}\right) + \frac{a}{2}\left(\frac{1}{\sqrt{2}} + 1\right) + b = 0$

($\because \cos^2\pi/8$ a root of equation)

$\Rightarrow \left(\frac{3}{8} + \frac{a}{2} + b\right) + \sqrt{2}\left(\frac{1}{4} + \frac{a}{4}\right) = 0$

Since a and b are rational, so

$\frac{1}{4} + \frac{a}{4} = 0, \frac{3}{8} + \frac{a}{2} + b = 0$

$\Rightarrow a = -1, b = \frac{1}{8}$

Thus, both the statements are correct and statement 2 is correct explanation of statement 1

340 **(b)**

We must have

$ax^3 + (a+b)x^2 + (b+c)x + c > 0$

$\Rightarrow ax^2(x+1) + bx(x+1) + c(x+1) > 0$

$\Rightarrow (x+1)(ax^2 + bx + c) > 0$

$\Rightarrow a(x+1)\left(x + \frac{b}{2a}\right)^2 > 0$ as $b^2 = 4ac$

$\Rightarrow x > -1$ and $x \neq -\frac{b}{2a}$

341 **(c)**

$|z_1 + z_2| = \left|\frac{z_1 + z_2}{z_1 z_2}\right|$

$\Rightarrow |z_1 + z_2| \left(1 - \frac{1}{|z_1 z_2|}\right) = 0$

$\Rightarrow |z_1 z_2| = 1$

Hence, statement 1 is true. However, it is not necessary that $|z_1| = |z_2| = 1$. Hence, statement 2 is false

342 **(a)**

First, let the two complex numbers be conjugate of each other. Let complex numbers be $z_1 = x + iy$ and $z_2 = x - iy$. Then, $z_1 + z_2 = (x + iy) + (x - ix) = 2$, which is real and $z_1 z_2 = (x + iy)(x - iy) = x^2 - i^2 y^2 = x^2 + y^2$, which is real

Conversely, let z_1 and z_2 be two complex numbers such that their sum $z_1 + z_2$ and product $z_1 z_2$ both are real. Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$. Then $z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$ and $z_1 z_2 = (x_1 x s_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1)$

Now, $z_1 + z_2$ and $z_1 z_2$ are real. Hence, $b_1 + b_2 = 0$ and $a_1 b_2 + a_2 b_1 = 0$ $[\because z$ is real $\Rightarrow \text{Im}(z) = 0]$

$\Rightarrow b_2 = -b_1$ and $a_1 b_2 + a_2 b_1 = 0$

$= -b_1$ and $-a_1 b_1 + a_2 b_1 = 0$

$= -b_1$ and $(a_2 - a_1) b_1 = 0$

$= -b_1$ and $a_2 - a_1 = 0$

$= -b_1$ and $a_2 = a_1$

$\Rightarrow z_2 = a_2 + ib_2 = a_1 - ib_1$

$= \bar{z}_1$

Hence, z_1 and z_2 are conjugate of each other.

Hence, statement 2 is true

Also in statement 1, $a = \bar{a}$ and $b = \bar{b}$, then a and b are real. Thus, $z_1 + z_2$ and $z_1 z_2$ are real. So, $z_2 = \bar{z}_2$

$\Rightarrow \arg(z_1 z_2) = \arg(z_1 \bar{z}_1) = \arg(|z_1|^2) = 0$

Hence, statement 1 is correct and statement 2 is correct explanation of statement 1

343 **(b)**

The equation can be writer as

$(2^x)^2 - (a - 3)2^x + (a - 4) = 0$

$\Rightarrow 2^x = 1$ and $2^x = a - 4$

We have,

$x \leq 0$ and $2^x = a - 4$ $[\because x$ is non-positive$]$

$\therefore\ 0 < a - 4 \leq 1 \Rightarrow 4 < a \leq 5$

$\therefore a \in (4, 5)$

344 **(a)**

Given equation is

$px^2 + qx + r = 0$

Let, $f(x) = px^2 + qx + r$

$f(0) = r > 0$

$f(1) = p + q + r < 0$

$f(-1) = p - q + r < 0$

Hence, one root lies in $(-1, 0)$ and the other in $(0, 1)$

$\therefore [\alpha] = -1$ and $[\beta] = 0$

$\Rightarrow [\alpha] + [\beta] = -1$

Therefore, statement 2 is true and is correct explanation of statement 1

345 **(b)**

If $a > 0$, then graph of $y = ax^2 + 2bx + c$ is concave upward. Also if $b^2 - ac < 0$, then the graph always lies above x-axis; hence $ax^2 + 2bx + c > 0$ for all real values of x. Thus, domain of function $f(x) = \sqrt{ax^2 + 2bx + c}$ is R

If $b^2 - ac < 0$, then $ax^2 + 2bx + c = 0$ has imaginary roots. Then the graph of $y = ax^2 + 2bx + c$ never cuts x-axis, or y is either always positive or always negative. Hence, both the statements are correct but statement 2 is not correct explanation of statement 1

346 **(b)**

$|z_2 z_3 + 8 z_3 z_1 + 27 z_1 z_2|$

$= \left|z_1 z_2 z_3 \left(\frac{1}{z_1} + \frac{8}{z_2} + \frac{27}{z_3}\right)\right|$

$= |z_1||z_2||z_3| \left|\frac{1}{z_1} + \frac{8}{z_2} + \frac{27}{z_3}\right|$

$= |z_1||z_2||z_3| \left|\frac{\bar{z}_1}{|z_1|^2} + \frac{8\bar{z}_2}{|z_2|^2} + \frac{27\bar{z}_3}{|z_3|^2}\right|$

$= |z_1||z_2||z_3| \left|\frac{\bar{z}_1}{1} + \frac{8\bar{z}_2}{4} + \frac{27\bar{z}_3}{9}\right|$

$= |z_1||z_2||z_3| |\overline{z_1 + 2z_2 + 3z_3}|$

$= |z_1||z_2||z_3| |z_1 + 2z_2 + 3z_3|$

$= 1 \cdot 2 \cdot 3 \cdot 6 = 36$

347 **(a)**
$f(x) = (x-1)(ax+b)$

$f(2) = 2a + b$

$f(4) = 3(4a+b) = 12a + 3b$

$f(2) + f(4) = 14a + 4b = 0$

$\Rightarrow \frac{-b}{a} = 3.5$

Now, sum of roots is $(a-b)/a = 1 - (b/a) = 1 + 3.5 = 4.5$. Hence, the other root is 3.5

348 **(a)**
Let $f(x) = 4x^2 - 2x + a$

Also, both roots of $f(x) = 0$ lie in the interval $(-1, 1)$.

$\therefore D \geq 0, f(-1) > 0$ and $f(1) > 0$

Now, $D \geq 0$

$\Rightarrow (-2)^2 - 4 \cdot 4 \cdot a \geq 0$

$\Rightarrow a \leq \frac{1}{4}$ ....(i)

and $f(-1) > 0$

$\Rightarrow 4(-1)^2 - 2(-1) + a > 0$

$\Rightarrow a > -6$ ...(ii)

Also, $f(1) > 0 \Rightarrow 4(1)^2 - 2(1) + a > 0$

$\Rightarrow a > -2$ ...(iii)

From Eqs. (i), (ii) and (iii), we get

$-2 < a \leq \frac{1}{4}$

349 **(a)**
If $a^2 + b^2 + c^2 < 0$, then all a, b, c are not real or at least one of a, b, c is imaginary number. Hence roots of equation $ax^2 + bx + c = 0$ has no complex conjugate roots, even through the roots are complex. Hence statement 1 is true. Statement 2 is obviously true. Also, statement 2 is correct explanation of statement 1

350 **(a)**
$$D = \underbrace{\underbrace{(2m+1)^2}_{\text{odd}} - \underbrace{4(2n+1)}_{\text{even}}}_{\text{odd}}$$

For rational root, D must be a perfect square. As D is odd, let D be perfect square of $2l + 1$, where $l \in Z$

$(2m+1)^2 - 4(2n+1) = (2l+1)^2$

$\Rightarrow (2m+1)^2 - (2l+1)^2 = 4(2n+1)$

$\Rightarrow [(2m+1) + (2l+1)][2(m-l)] = 4(2n+1)$

$\Rightarrow (m+l+1)(m-l) = (2n+1) \quad (1)$

R.H.S. of (1) is always odd but L.H.S. is always even. Hence, D cannot be a perfect square. So, the roots cannot be rational

Hence, statement 1 is true, statement 2 is true and statement 2 is correct explanation for statement 1

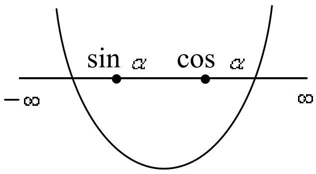
351 **(c)**
$x^3 + x^2 + x = x(x^2 + x + 1) = x(x-\omega)(x-\omega^2)$
Now $f(x) = (x+1)^n - x^n - 1$ is divisible by $x^3 + x^2 + x$. Then $f(0) = 0, f(\omega) = 0, f(\omega^2) = 0$.
Now,
$f(0) = (0+1)^n - 0^n - 1 = 0$
$f(\omega) = (\omega+1)^n - \omega^n - 1 = (-\omega^2)^n - \omega^n - 1$
$= -(\omega^{2n} + \omega^n + 1)$
$= 0$ (as n is not a multiple of 3)
Similarly, we have $f(\omega^2) = 0$
Hence statement 1 is correct but statement 2 is false

352 **(d)**
$x + \frac{1}{x} = 1$
$\Rightarrow x^2 - x + 1 = 0$
$\therefore x = -\omega, -\omega^2$
Now for $x = -\omega$,
$p = \omega^{4000} + \frac{1}{\omega^{4000}} = \omega + \frac{1}{\omega} = -1$
Similarly for $x = -\omega^2, P = -1$. For $n > 1$,
$2^n = 4k$
$\therefore 2^{2^n} = 2^{4k} = (16)^k =$ a number with last digit 6
$\Rightarrow q = 6 + 1 = 7$
Hence, $p + q = -1 + 7 = 6$

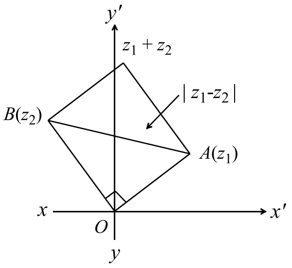
353 **(d)**
Let $f(x) = (x - \sin\alpha)(x - \cos\alpha) - 2$. Then,

$f(\sin\alpha) = -2 < 0, f(\cos\alpha) = -2 < 0$

Also, as $0 < \alpha < \pi/4$, hence, $\sin\alpha < \cos\alpha$. Therefore, equation $f(x) = 0$ has one root in $(-\infty, \sin\alpha)$ and other in $(\cos\alpha, \infty)$



354 **(d)**



From the diagram when $|z_1 - z_2| = |z_1 + z_2|, OAB$ is right-angled triangle. Hence orthocentre is 0

355 **(d)**

$$\left|\frac{z_1 z - z_2}{z_1 z + z_2}\right| = k$$

$$\Rightarrow \left|\frac{z - \frac{z_2}{z_1}}{z + \frac{z_2}{z_1}}\right| = k$$

Clearly, if $k \neq 0, 1$, then z would lie on a circle

If $k = 1, z$ would lie on a perpendicular bisector of line segment joining $\frac{z_2}{z_1}$ and $-\frac{z_2}{z_1}$ and represents a points if $k = 0$

∴ Statement (I) is false and Statement (II) is true

356 **(a)**

We have,

$az^2 + bz + c = 0 \quad (1)$

and z_1, z_2 [roots of (1)] are such that $\text{Im}(z_1 z_2) \neq 0$. So, z_1 and z_2 are not conjugate of each other. That is complex roots of (1) are not conjugate of each other, which implies that coefficients a, b, c cannot all be real. Hence, at least one of a, b, c is imaginary

357 **(a)**

If roots of $ax^2 + bx + c = 0, 0 < a < b < c$, are non-real, then they will be the conjugate of each other. Hence,

$z_2 = \bar{z}_1 \Rightarrow |z_1| = |z_2|$

Now,

$z_1 z_2 = \dfrac{c}{a} > 1 \Rightarrow |z_1|^2 > 1$

$\Rightarrow |z_1| > 1$

$\Rightarrow |z_2| > 1$

358 **(b)**

$$ix^2 + (i-1)x - \frac{1}{2} - i = 0$$

$$\Rightarrow x = \frac{-(i-1) \pm \sqrt{(i-1)^2 - 4(i)\left(-\frac{i}{2} - i\right)}}{2i}$$

$$= \frac{-(i-1) \pm \sqrt{-4}}{2i}$$

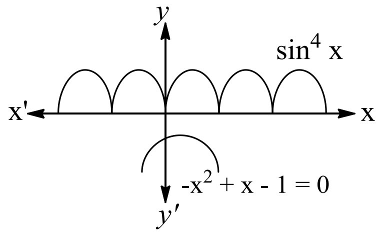
Thus, roots are imaginary. Also, we have $b^2 - 4ac = -4 < 0$, but this is not the correct reason for which roots are imaginary as coefficients of the equation are imaginary

Hence, both the statements are correct but statement 2 is not correct explanation of statement 1

359 **(d)**

Let $f(x) = -x^2 + x - 1$

Here, $a < 0$ and $D = (1)^2 - 4(1) < 0$, then $f(x) < 0$



But $\sin^4 x \geq 0$

$\therefore -x^2 + x - 1 \neq \sin^4 x$

Hence, number of solutions is 0.

Hence, option (d) is correct.

360 **(a)**

Given equations are

$ax^2 + 2bx + c = 0 \qquad (1)$

$a_1 x^2 + 2b_1 x + c_1 = 0 \quad (2)$

Since Eqs. (1) and (2) have only one common root and a, b, c, a_1, b_1, c_1 are rational, therefore, common root cannot be imaginary or irrational (as irrational roots occur in conjugate pair when coefficients are rational, and complex roots always occur in conjugate pair)

Hence, the common root must be rational. Therefore, both the roots of Eqs. (1) and (2) will be rational. Therefore, $4(b^2 - ac)$ and

$4(b_1^2 - a_1c_1)$ must be perfect squares (squares of rational numbers). Hence, $b^2 - ac$ and $b_1^2 - a_1c_1$ must be perfect squares

361 **(a)**
Let $A = b + c - a, B = c + a - b, C = a + b - c$

$\therefore A + B + C = 0$

And given equation becomes

$Ax^2 + Bx + C = 0$

$\therefore D = B^2 - 4AC$

$= (A + C)^2 - 4AC = (A - C)^2$

$= 4[c - a]^2 =$ perfect square

Hence, roots of $(b + c - a)x^2 + (c + a - b)x + (a + b - c) = 0$ are rationals.

Hence option (a) is correct.

362 **(b)**
Given $x^2 + 2px + q = 0$

$\therefore\ \alpha + \beta = -2p$ ...(i)

$\alpha\beta = q$ ...(ii)

And $ax^2 + 2bx + c = 0$

$\therefore\ \alpha + \frac{1}{\beta} = -\frac{2b}{a}$ .....(iii)

and $\frac{\alpha}{\beta} = \frac{c}{a}$ ...(iv)

Now, $(p^2 - q)(b^2 - ac)$

$$= \left[\left(\frac{\alpha + \beta}{-2}\right)^2 - \alpha\beta\right]\left[\left(\frac{\alpha + \frac{1}{\beta}}{2}\right)^2 - \frac{\alpha}{\beta}\right]a^2$$

$$= \frac{(\alpha - \beta)^2}{16}\left(\alpha - \frac{1}{\beta}\right)^2 \cdot a^2 \geq 0$$

$\therefore$ Statement I is true.

Again, now $pa = -\left(\frac{\alpha+\beta}{2}\right)a = -\frac{a}{2}(\alpha + \beta)$

and $b = -\frac{a}{2}\left(\alpha + \frac{1}{\beta}\right)$

Since, $pa \neq b\ \Rightarrow \alpha + \frac{\beta}{2} \neq \alpha + \beta$

$\Rightarrow\ \beta^2 \neq 1, \beta \neq \{-1, 0, 1\}$, which is correct.

Similarly, if $c \neq qa\ \Rightarrow a\frac{\alpha}{\beta} \neq a\alpha\beta$

$\Rightarrow\ \alpha\left(\beta - \frac{1}{\beta}\right) \neq 0$

$\Rightarrow\ \alpha \neq 0$ and $\beta - \frac{1}{\beta} \neq 0$

$\Rightarrow\ \beta \neq \{-1, 0, 1\}$

Statement II is true.

Both Statement I and Statement II are true. But statement II does not explain Statement I.

363 **(c)**
Clearly, Statement 1 is true but Statement 2 is false, since, $ax^2 + bx + c = 0$ is an identity when $a = b = c = 0$

364 **(c)**
We have, $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$

$$\Rightarrow |z_1|^2 + |z_2|^2 + 2|z_1||z_2|\cos(\theta_1 - \theta_2) = |z_1|^2 + |z_2|^2$$

where, $\theta_1 = \arg(z_1),\ \theta_2 = \arg(z_2)$

$\Rightarrow\ \cos(\theta_1 - \theta_2) = 0\ \Rightarrow\ \theta_1 - \theta_2 = \frac{\pi}{2}$

$\Rightarrow\ \arg\left(\frac{z_1}{z_2}\right) = \frac{\pi}{2}$

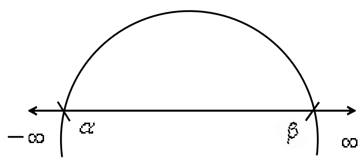
$\Rightarrow\ \operatorname{Re}\left(\frac{z_1}{z_2}\right) = \frac{|z_1|}{|z_2|}\cos\frac{\pi}{2} = 0$

$\therefore\ \frac{z_1}{z_2}$ is purely imaginary

$\therefore$ Statement I is true

If z is purely imaginary then $z - \bar{z} = 0$

365 **(c)**
Here, $f(x)$ is a downward parabola



$D = (a + 1)^2 + 20 > 0$

From the graph, clearly, statement 1 is true but statement 2 is false

366 **(d)**

$|z_1 + z_2 + z_3| = |z_1 - a + z_2 - b + z_3 - c + (a + b + c)|$

$\leq |z_1 - a| + |z_2 - b| + |z_3 - c| + |a + b + c|$

$\leq 2|a + b + c|$

Hence, $|z_1 + z_2 + z_3|$ is less than $2|a + b + c|$

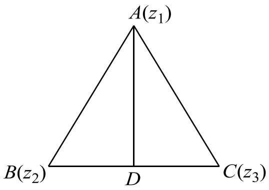
367 **(b)**

Since $|z_1| = |z_2| = |z_3|$, circumcentre of Δ is origin

Also $\frac{z_1+z_2+z_3}{3} = 0$

Centroid concide with circumcentre

$\Rightarrow$ Δ is equilateral



$\frac{z_2 + z_3}{2}$

$$\arg\left(\frac{z_2 + z_3 - 2z_1}{z_3 - z_2}\right) = \arg\left(2\left(\frac{\frac{z_2+z_3}{2} - z_1}{z_3 - z_2}\right)\right)$$

$$= \arg\left(\frac{\frac{z_2+z_3}{2} - z_1}{z_3 - z_2}\right)$$

$(z_2 + z_3)/2$ is the mid-point of side BC. Clearly, line joining A and mid-point of BC will be perpendicular to side BC. Thus,

$$\arg\left(\frac{\frac{z_2+z_3}{2} - z_1}{z_3 - z_2}\right) = \frac{\pi}{2}$$

Hence, statement 2 is also true. However, it does not explain statement 1

368 **(a)**

$x^2 + x + 1 = 0$

$D = -3 < 0$

Therefore, $x^2 + x + 1 = 0$ and $ax^2 + bx + c = 0$ have both the roots common. Hence, $a = b = c$

369 **(d)**

Roots of the equation $x^5 - 40x^4 + Px^3 + Qx^2 + Rx + S = 0$ are in G.P. Let the roots be a, ar, ar^2, ar^3, ar^4. Therefore,

$a + ar + ar^2 + ar^3 + ar^4 = 40$ (1)

and

$\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} + \frac{1}{ar^4} = 10$ (2)

From (1) and (2),

$ar^2 = \pm 2$ (3)

Now, the product of roots is $a^5 r^{10} = (ar^2)^5 = \pm 32$

$\therefore$ $|S| = 32$

370 **(b)**

We have,

Let $x = (\cos\theta + i\sin\theta)^{3/5}$

$\Rightarrow x^5 = (\cos\theta + i\sin\theta)^3$

$\Rightarrow x^5 - (\cos 3\theta + i\sin 3\theta) = 0$

$\Rightarrow$ Product of roots $= \cos 3\theta + i\sin 3\theta$

Also product of roots of the equation $x^5 - 1 = 0$ is 1. Hence statement 2 is true. But it is not correct explanation of statement 1

371 **(a)**

$\arg(z_1 z_2) = 2\pi \Rightarrow \arg(z_1) + \arg(z_2) = 2\pi \Rightarrow$ $\arg(z_1) = \arg(z_2) = \pi$, as principal arguments are from $-\pi$ to π

Hence both the complex numbers are purely real.

Hence both the statements are true and statement 2 is correct explanation of statement 1

372 **(a)**

Since, $x = -2$ is a root of $f(x)$.

$\therefore f(x) = (x + 2)(ax + b)$

But $f(0) + f(1) = 0$

$\therefore$ $2b + 3a + 3b = 0$

$\Rightarrow$ $-\frac{b}{a} = \frac{3}{5}$

Hence, option (a) is correct.

373 **(a)**

Suppose there exists a complex number z which satisfies the given equation and is such that $|z| < 1$. Then,

$z^4 + z + 2 = 0 \Rightarrow -2 = z^4 + z \Rightarrow |-2| = |z^4 + z|$

$\Rightarrow 2 \leq |z^4| + |z| \Rightarrow 2 < 2$, because $|z| < 1$

But $2 < 2$ is not possible. Hence given equation cannot have a root z such that $|z| < 1$

374 **(d)**

$f(x, y) = (2x - y)^2 + (x + y - 3)^2$

Therefore, statement 1 is false as it represents a point (1, 2)

375 **(c)**

a. $d + a - b = 0$ and $d + b - c = 0$
$d = b - a$ and $d = c - b$
$\therefore b - a = c - b \Rightarrow 2b = a + b \Rightarrow a, b, c$ are in A.P.
Also $x = 1$ satisfies the second equation.
Therefore, the other root is also 1. Product of roots is 1
$\therefore\ c(a - b) = a(b - c) \Rightarrow b = \frac{2ac}{a+c} \Rightarrow a, b, c$ are in H.P.
Therefore, a, b, c are in A.P. and a, b, c in H.P
Hence, a, b, c are in G.P.
b. $(a^2 + b^2)x^2 - 2b(a + c)(b^2 + c^2) = 0$
The roots are real and equal. Hence,
$4b^2(a + c)^2 - 4(a^2 + b^2)(b^2 + c^2) = 0$
$\Rightarrow b^2(a^2 + c^2 + 2ac) - (a^2b^2 + a^2c^2 + b^4 + b^2c^2) = 0$
$\Rightarrow b^2a^2 + b^2c^2 + 2ab^2c - a^2b^2 - a^2c^2 - b^4 - b^2c^2 = 0$
$\Rightarrow 2ab^2c - a^2c^2 - b^4 = 0 \Rightarrow (b^2 - ac)^2 = 0$
Hence, $b^2 = ac$. Thus a, b, c are in G.P.
c. $(x - 1)^3 = 0 \Rightarrow x = 1$ is the common root.
Hence, $a + b + c = 0$
d. $(a + c)^2 + 4b^2 - 4b(a + c) \le 0, \forall\, x \in R$
$\Rightarrow \left((a + c) - 2b\right)^2 \le 0$
$\Rightarrow a + c = 2b$
$\Rightarrow a, b, c$ in A.P.

376 **(c)**
a. $|z - 1| = |z - i|$
Hence it lies on the perpendicular bisector of the line joining (1, 0) and (0, 1) which is a straight line passing through the origin
b. $|z + \bar{z}| + |z - \bar{z}| = 2$
$\Rightarrow |x| + |y| = 1$
Hence, z lies on a square,
c. Let $z = x + iy$. Then,
$|z + \bar{z}| = |z - \bar{z}|$
$\Rightarrow |2x| = |2iy|$
$\Rightarrow |x| = |y|$
$\Rightarrow x = \pm y$
Hence, the locus of z is a pair of straight lines
d. Let $Z = 2/z$. Then,
$$|Z| = \left|\frac{2}{z}\right| = \frac{2}{|z|} = \frac{2}{1} = 2$$
This shows that Z lies on a circle with centre at the origin and radius 2 units

377 **(b)**
Given, $a = \frac{1-i\sqrt{3}}{2} = \frac{1}{2} - \frac{i\sqrt{3}}{2}$
$\therefore\quad \bar{a} = \frac{1}{2} + \frac{i\sqrt{3}}{2}$ and $\frac{1}{\bar{a}} = \frac{1-i\sqrt{3}}{2}$

5. $\quad a\bar{a} = \left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)$
$$= \left(\frac{1}{2}\right)^2 - i^2\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1$$
6. $\quad \arg\left(\frac{1}{\bar{a}}\right) = \tan^{-1}\left(\frac{\sqrt{3}}{2} \times \frac{2}{1}\right) = -\frac{\pi}{3}$
7. $\quad a - \bar{a} = \left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right) - \left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right) = -i\sqrt{3}$
8. $\quad \mathrm{Im}\left(\frac{4}{3a}\right) = \mathrm{Im}\left(\frac{4}{3\left(\frac{1-i\sqrt{3}}{2}\right)}\right)$
$$= \mathrm{Im}\left(\frac{8(1 + i\sqrt{3})}{3(1 - i\sqrt{3})(1 + i\sqrt{3})}\right)$$
$$= \mathrm{Im}\left(\frac{2 + 2i\sqrt{3}}{3}\right) = \frac{2}{\sqrt{3}}$$

378 **(a)**
a. $x^2 + ax + b = 0$ has root α. Hence,
$\alpha^2 + a\alpha + b = 0$ (1)
$x^2 + px + q = 0$ has roots $-\alpha, \gamma$. Hence,
$\alpha^2 - p\alpha + q = 0$ (2)
Eliminating α from (1) and (2), we get
$(q - b)^2 = (aq + bp)(-p - a)$
$\Rightarrow (q - b)^2 = -(aq + bp)(p + a)$
b. $x^2 + ax + b = 0$ has root is α, β. Hence,
$\alpha + a\alpha + b = 0$ (1)
$x^2 + px + q = 0$ has root $1/\alpha$. Hence,
$q\alpha^2 + p\alpha + 1 = 0$ (2)
Eliminating α from (1) and (2), we get
$(1 - bq)^2 = (a - pb)(p - aq)$
c. $x^2 + ax + b = 0$ has roots α, β. Hence,
$\alpha^2 + a\alpha + b = 0$ (1)
$x^2 + px + q = 0$ has roots $-2/\alpha, \gamma$. Hence,
$q\alpha^2 - 2p\alpha + 4 = 0$ (2)
Eliminating α from (1) and (2), we get
$(4 - bq)^2 = (4a + 2pb)(-2p - aq)$
d. $x^2 + ax + b = 0$ has roots α, β. Hence,
$\alpha^2 + a\alpha + b = 0$ (1)
$x^2 + px + q = 0$ has roots $-1/2\alpha, \gamma$. Hence,
$4q\alpha^2 - 2p\alpha + 1 = 0$ (2)
Eliminating α from (1) and (2), we get
$(1 - 4bq)^2 = (a + 2bp)(-2p - 4aq)$

379 **(a)**
1. $\quad |z - 2i| + |z - 7i| = k$ is ellipse if $k > |7i - 2i|$ or $k > 5$
2. $\quad \left|\frac{2z-3}{3z-2}\right| = k \Rightarrow \left|\frac{z-\frac{3}{2}}{z-\frac{2}{3}}\right| = \frac{3k}{2} \Rightarrow 3k/2 > 1 \Rightarrow$

$k > 2/3$

3. $|z-3|-|z-4i| = k$ is hyperbola, if $k > |3-4i| \Rightarrow 0 < k < 5$

4. $|z-(3+4i)| = \frac{k}{50}|a\bar{z}+\bar{a}z+b|$

$\Rightarrow |z-(3+4i)| = \frac{k}{5}\frac{|a\bar{z}+\bar{a}z+b|}{2|3+4i|}$

This is hyperbola if $k/5 > 1 \Rightarrow k > 5$

380 **(a)**

a. $y = \frac{x^2-2x+4}{x^2+2x+4}$

$\Rightarrow x^2y+2xy+4y = x^2-2x+4$

$\Rightarrow (y-1)x^2+2(y+1)x+4(y-1) = 0$

$D \geq 0$

$\Rightarrow 4(y+1)^2-16(y-1)^2 \geq 0$

$\Rightarrow (y+1)^2-(2y-2)^2 \geq 0$

$\Rightarrow (3y-1)(3-y) \geq 0$

$\Rightarrow (3y-1)(y-3) \leq 0 \Rightarrow y \in \left[\frac{1}{3}, 3\right]$

$\Rightarrow \{1\} \Rightarrow P$

b. $y = \frac{x^2-3x-2}{2x-3}$

$\Rightarrow x^2-3x-2 = 2xy-3y$

$\Rightarrow x^2-(3+2y)x+(3y-2) = 0$

$D \geq 0$

$\Rightarrow (3+2y)^2-4(3y-2) \geq 0$

$\Rightarrow 9+4y^2+12y-12y+8 \geq 0$

$\Rightarrow 4y^2+17 \geq 0$

Which is always true. Hence,

$y \in R \Rightarrow \{1,4,-3,-10\} \Rightarrow p,q,r,s$

c. $y = \frac{2x^2-2x+4}{x^2-4x+3}$

$\Rightarrow x^2y-4xy+3y = 2x^2-2x+4$

$(y-2)x^2+2(1-2y)x+3y-4 = 0$

$D \geq 0$

$4(1-2y)^2-4(y-2)(3y-4) \geq 0$

$\Rightarrow 1+4y^2-4y-(3y^2-10y+8) \geq 0$

$\Rightarrow y^2+6y-7 \geq 0$

$\Rightarrow (y+7)(y-1) \geq 0$

$\Rightarrow y \geq 1$ or $y \leq -7$

$\Rightarrow \{1,4,-10\} \Rightarrow p,q,s$

d. $f(x) = x^2-(a-3)x+2 < 0, \forall x \in [-2,-1]$

$\Rightarrow f(-2) < 0$ and $f(-1) < 0$

$\Rightarrow 4+2(a-3)+2 < 0$ and $1+(a-2)+2 < 0$

$\Rightarrow a < 0$ and $a < -1$

$\Rightarrow a < -1$

$\Rightarrow a \in \{-10,-3\}$

381 **(d)**

a. $(m-2)x^2-(8-2m)x-(8-3m) = 0$ has root of opposite signs. The product of roots is

$-\frac{8-3m}{m-2} < 0$

$\Rightarrow \frac{3m-8}{m-2} < 0$

$\Rightarrow 2 < m < 8/3$

b. Exactly one root of equation $x^2-m(2x-8)-15 = 0$ lies in interval $(0, 1)$

$f(0)f(1) < 0$

$\Rightarrow (0-m(-8)-15)(1-m(-6)-15) < 0$

$\Rightarrow (8m-15)(6m-15) < 0$

$\Rightarrow 15/8 < m < 15/6$

c. $x^2+2(m+1)x+9m-5 = 0$ has both roots negative. Hence, sum of roots is

$-2(m+1) < 0$ or $m > -1$

Product of roots is

$9m-5 > 0$ or $m > 5/9$

Discriminant,

$D \geq 0 \Rightarrow 4(m+1)^2-4(9m-5) \geq 0$

$\Rightarrow m^2-7m+6 \geq 0$

$\Rightarrow m \leq 1$ or $m \geq 6$

Hence, for (1), (2) and (3), we get

$m \in \left(\frac{5}{9}, 1\right] \cup [6, \infty)$

d. $f(x) = x^2+2(m-1)x+m+5 = 0$ has one root less than 1 and the other root greater than 1. Hence,

$f(1) < 0$

$\Rightarrow 1+2(m-1)+m+5 < 0$

$\Rightarrow m < -4/3$

382 **(b)**

a. $x^2-x+1 = 0$

$\Rightarrow x = \frac{1 \pm i\sqrt{3}}{2}$

$= -\omega, -\omega^2$

$\Rightarrow \left(x^n+\frac{1}{x^n}\right)^2 = (-1)^{2n}\left(\omega^n+\frac{1}{\omega^n}\right)^2$

$= (\omega^n+\omega^{2n})^2 \quad (1)$

$\because \frac{1}{\omega^n} = \frac{\omega^{2n}}{\omega^{3n}} = \omega^{2n}$

Now,

$1+\omega^n+\omega^{2n} = \frac{1-\omega^{3n}}{1-\omega^n} = 0$ for $n \neq 3p$

$\therefore\ \omega^n+\omega^{2n} = -1$ for $n \neq 3p$

$= 2$ for $n = 3p$

$\therefore \sum_{n=1}^{5}\left(x^n+\frac{1}{x^n}\right)^2 = 8$

b. In the expression,

$\left[\frac{1+\cos\theta+i\sin\theta}{\sin\theta+i(1+\cos\theta)}\right]$

Numerator is

$1+\cos\theta+i\sin\theta = 2\cos\frac{\theta}{2}\left[\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right]$

$= 2\cos\frac{\theta}{2}\ e^{i\theta/2}$

and denominator is

$-i^2 \sin\theta + i(1+\cos\theta) = i$[conjugate of numerator]

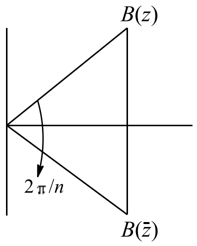
$= i\ 2\cos\frac{\theta}{2}\ e^{-\theta/2}$

$\therefore E = \left(\frac{N^r}{D^r}\right) = \left[\frac{1}{i}\frac{e^{-i\theta/2}}{e^{-i\theta/2}}\right]^4 = \frac{1}{i^4}e^{4i\theta}$

$= \cos 4\theta + i\sin 4\theta$

$\therefore n = 4$

c. We know that if $z = re^{i\theta}$, then $\bar{z} = re^{-i\theta}$



$\therefore \frac{\text{Im}(z)}{\text{Re}(z)} = \frac{r\sin\theta}{r\cos\theta} = \tan\theta = \tan\frac{\pi}{n} = \sqrt{2} - 1$

$\Rightarrow \tan\frac{\pi}{n} = \tan\frac{\pi}{8} \Rightarrow n = 8$

d. $\sum_{r=1}^{10}(r-\omega)(r-\omega^2) = \sum_{r=1}^{10}(r^2 + r + 1)$

$= \sum r^2 + \sum r + 10$

$= \frac{10\times 11\times 21}{6} + \frac{10\times 11}{2} + 10$

$= 450$

$\Rightarrow \frac{1}{50}\left\{\sum_{r=1}^{10}(r - \omega(r-\omega^2)\right\} = 9$

383 **(c)**

5. z is equidistant from the points $i\,|z|$ and $-\,i\,|z|$, whose perpendicular bisector is Im $(z) = 0$
6. Sum of distance of z from (4, 0) and (-4, 0) is a constant 10, hence locus of z is ellipse with semi-major axis 5 and focus at $(\pm 4, 0)$, $ae = 4$
7. $|z| \le |w| + \left|\frac{1}{w}\right| = \frac{5}{2} < 3$
8. $|z| \le |w| + \left|\frac{1}{w}\right| = 2$

$\therefore \quad \text{Re}(z) \le |z| \le 2$

384 **(b)**

Obviously when $a \ge 0$, we have no roots as all the terms are followed by +ve sign. Also for $a = -2$, we have

$x^2 - 2|x| + 1 = 0$

or $|x| - 1 = 0 \Rightarrow x = \pm 1$

Hence, the equation has two roots

Also when $a < -2$, for given equation

$|x| = \frac{-a \pm \sqrt{a^2-4}}{2} > 0$

Hence, the equation has four roots as $|-a| > \sqrt{a^2-4}$. Obviously, the equation has no three real roots for any value of a

385 **(a)**

1. $z^4 - 1 = 0 \Rightarrow z^4 = 1 = \cos 0 + i\sin 0 \Rightarrow z = (\cos 0 + i\sin 0)^{1/4}$

$= \cos 0 + i\sin 0$

2. $z^4 + 1 = 0 \Rightarrow z^4 = -1 = \cos\pi + i\sin\pi \Rightarrow z = (\cos\pi + i\sin\pi)^{1/4}$

$= \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$

3. $iz^4 + 1 = 0 \Rightarrow z^4 = i = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} \Rightarrow z = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^{1/4}$

$= \cos\frac{\pi}{8} + i\sin\frac{\pi}{8}$

4. $iz^4 - 1 = 0 \Rightarrow z^4 = -i = \cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$

$\Rightarrow z = \left(\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}\right)^{1/4} = \cos\frac{\pi}{8} - i\sin\frac{\pi}{8}$

386 **(c)**

(A) Here, $\alpha + \beta + \gamma = 10$ ...(i)

$\alpha\beta + \beta\gamma + \gamma\alpha = 7$ ...(ii)

And $\alpha\beta\gamma = -8$ ...(iii)

(B) On squaring Eq. (i) both sides, we get

$\alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \gamma\alpha) = 100$

$\Rightarrow \alpha^2 + \beta^2 + \gamma^2 = 100 - 2(7)$ [from Eq. (ii)]

$\Rightarrow \alpha^2 + \beta^2 + \gamma^2 = 86$ ...(iv)

(C) Now, $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\beta + \gamma\alpha}{\alpha\beta\gamma} = \frac{7}{-8}$

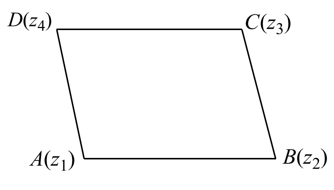
[from Eqs. (ii) and (iii)]

(D) Again now, $\frac{\alpha}{\beta\gamma} + \frac{\beta}{\alpha\gamma} + \frac{\gamma}{\alpha\beta}$

$= \frac{\alpha^2 + \beta^2 + \gamma^2}{\alpha\beta\gamma}$

$= \frac{86}{-8} = -\frac{43}{4}$ [from Eqs. (iii) and (iv)]

387 **(d)**



In parallelogram, the mid-points of diagonals coincide

$$\frac{z_1+z_3}{2}=\frac{z_2+z_4}{2}$$
$\Rightarrow z_1-z_4=z_2-z_3$

Also in parallelogram, $AB||CD$. Hence,

$$\arg\left(\frac{z_1-z_2}{z_3-z_4}\right)=0$$
$\Rightarrow \frac{z_1-z_2}{z_3-z_4}$ is purely real

In rectangle, adjacent sides are perpendicular. Hence,

$$\arg\left(\frac{z_1-z_2}{z_3-z_2}\right)=\frac{\pi}{2}$$
$\Rightarrow \frac{z_1-z_2}{z_3-z_2}$ is purely imaginary

Also in rectangle,

$AC=BD \Rightarrow |z_1-z_3|=|z_2-z_4|$

In rhombus,

$AC\perp BD \Rightarrow \frac{z_1-z_3}{z_2-z_4}$ is purely imaginary

388 **(d)**

Using the condition that the roots of $ax^2+bx+c=0$ may be in the ratio $m:n$ is $mnb^2=ac(m+n)^2$

(i) If the roots are $\alpha=\beta$, then

$\alpha.\alpha b^2=ac(\alpha+\alpha)^2$

$\Rightarrow b^2=4ac$

(ii) If the roots are $\alpha=2\beta$, then

$\beta.2\beta b^2=ac(\beta+2\beta)^2$

$\Rightarrow 2b^2=9ac$

(iii) if the roots are $\alpha=3\beta$, then

$\beta.3\beta b^2=ac(\beta+3\beta)^2$

$\Rightarrow 3b^2=16ac$

(iv) If the roots are $\alpha=\beta^2$, then

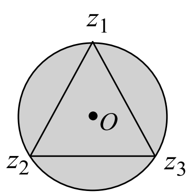
$(a^2c)^{\frac{1}{2+1}}+(ac^2)^{\frac{1}{2+1}}=-b$

$\Rightarrow (a^2c)^{\frac{1}{3}}+(ac^2)^{\frac{1}{3}}=-b$

389 **(a)**

$\because |z_1|=|z_2|=|z_3|$

$\therefore$ They lie on a circle having centre at origin shown as in the figure



Since, triangle is equilateral

$\therefore\ z_1+z_2+z_3=0$

390 **(b)**

Given, $\arg(z)>0$ and $\arg(-z)-\arg(z)=\lambda_1$,

then $-\arg z-\arg(-z)]=\lambda 1$

$\Rightarrow\ -\pi=\lambda_1\ \Rightarrow\ \lambda_1=-\pi$

Again, given $\arg(z)<0, \arg(z)-\arg(-z)=\lambda_2$

$\Rightarrow\ \lambda_2=-\pi$

$\therefore\ \lambda_1=\lambda_2$

391 **(b)**

Since, $z^n-1=\sum_{r=0}^{n-1}(z-\alpha^r)$

$$\Rightarrow \log(z^n-1)=\sum_{r=0}^{n-1}\log(z-\alpha^r)$$
$$\Rightarrow \frac{nz^{n-1}}{z^n-1}=\sum_{r=0}^{n-1}\frac{1}{(z-\alpha^r)} \qquad \text{(diferentiating)}$$
$$\Rightarrow \frac{n(2)^{n-1}}{2^n-1}=\sum_{r=0}^{n-1}\frac{1}{(2-\alpha^r)} \qquad (\text{put } z=2)$$
$$=1+\sum_{r=1}^{n-1}\frac{1}{(2-\alpha^r)}$$
$$\therefore \sum_{r=1}^{n-1}\frac{1}{(2-\alpha^r)}=\frac{n(2)^{n-1}-2^n+1}{(2^n-1)}$$
$$=\frac{(n-2)-2^{n-1}+1}{2^n-1}$$

392 **(c)**

If α,β are the roots and D be the discriminant of the given quadratic equation, then

$\alpha+\beta=\frac{2(1+3m)}{1+m}, \alpha\beta=\frac{1+8m}{1+m}$ ...(i)

and $D=4(1+3m)^2-4(1+m)(1+8m)$

$=4m(m-3)$

If roots are real, then $D\geq 0$.

$\therefore m\in(-\infty,0]\cup[3,\infty)$ ...(ii)

$D<0$

$\Rightarrow 4m(m-3)<0$

$\Rightarrow 0<m<3$

$\Rightarrow m=1,2$

393 **(c)**

Since, the other roots of $f(f(x))=x$ are λ and δ, we have

$f(f(\lambda))=\lambda$

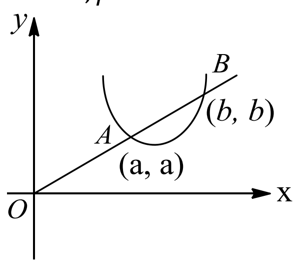
Let $f(\lambda)=\gamma$

$\Rightarrow f(\gamma)=\lambda \Rightarrow$ other roots γ and δ lie on the line $y=-x+c$

$\Rightarrow$ There must be two points C and D on the parabola

$y=ax^2+bx+c$ which are images of each other in the line $y=x$

$\Rightarrow$ If α,β are real so are λ and δ.



If $\alpha+\beta=\lambda+\delta \Rightarrow$ middle points of AB and CD become same.

This is not possible $\Rightarrow$ α, β and γ, δ can't be real.
Also α and β are equal, then λ, δ can't be real.

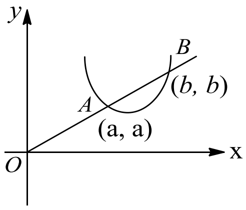
394 **(b)**

If $x = 2 + i\sqrt{3}$ is one root, then the other root is $2 - i\sqrt{3}$.

Sum of roots $= 4 \Rightarrow p = -4$

Product of roots $= 7 \Rightarrow q = 7$ is given below.

Graph of $y = x^2 - 4x + 7$



395 **(c)**

Now, $(\beta - \alpha) = [(\beta + \delta) - (\alpha + \delta)]$

$(\beta + \alpha)^2 - 4\alpha\beta$

$$= [(\beta + \delta) + (\alpha + \delta)]^2 - 4(\beta + \delta)(\alpha + \delta)$$

$\Rightarrow (-b_1)^2 - 4c_1 = (-b_2)^2 - 4c_2$

$\Rightarrow D_1 = D_2$ ...(i)

Since, least value of $f(x)$ is

$-\frac{D_1}{4} = \frac{-1}{4}$

$\Rightarrow D_1 = D_2 = 1$

Hence, least value of $g(x)$ is $-D_2 = -\frac{1}{4}$

396 **(b)**

Given that

$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$

$\Rightarrow |z_1|^2 + |z_2|^2 + z_1\bar{z}_2 + \bar{z}_1 z_2 = |z_1|^2 + |z_2|^2$

$\Rightarrow z_1\bar{z}_2 + \bar{z}_1 z_2 = 0$ (1)

$\Rightarrow \frac{z_1}{z_2} + \frac{\bar{z}_1}{z_2} = 0$ (dividing by $z_2\bar{z}_2$)

$\Rightarrow \frac{z_1}{z_2} + \overline{\left(\frac{z_1}{z_2}\right)} = 0$ (2)

From (1), $z_2, \bar{z}_2$ is purely imaginary. From (2), z_1/z_2 is purely imaginary. Hence,

$\arg\left(\frac{z_1}{z_2}\right) = \pm\frac{\pi}{2} \Rightarrow \arg(z_1) - \arg(z_2) = \pm\frac{\pi}{2}$

Also, $i(z_1/z_2)$ is purely real. Hence its possible arguments are 0 and π

397 **(a)**

$$z = \frac{1 - i\sin\theta}{1 + i\cos\theta} = \frac{(1 - i\sin\theta)(1 - i\cos\theta)}{(1 + i\cos\theta)(1 - i\cos\theta)}$$

$$= \frac{(1 - \sin\theta\cos\theta) - i(\cos\theta + \sin\theta)}{(1 + \cos^2\theta)}$$

If z is purely real, then

$\cos\theta + \sin\theta = 0$

or $\tan\theta = -1$

$\Rightarrow \theta = n\pi - \frac{\pi}{4}, n \in I$

If z is purely imaginary, $1 - \sin\theta\cos\theta = 0$ or $\sin\theta\cos\theta = 1$, which is not possible

$$|z| = \left|\frac{1 - i\sin\theta}{1 + i\cos\theta}\right| = \frac{\sqrt{1 + \sin^2\theta}}{\sqrt{1 + \cos^2\theta}}$$

If $|z| = 1$, then

$\cos^2\theta = \sin^2\theta \Rightarrow \tan^2\theta = 1 \Rightarrow \theta = n\pi \pm \frac{\pi}{4}, n \in I$

We have,

$$\arg(z) = \tan^{-1}\left(\frac{-(\cos\theta + \sin\theta)}{1 - \sin\theta\cos\theta}\right)$$

Now,

$\arg(z) = \pi/4$

$\Rightarrow \frac{-(\cos\theta + \sin\theta)}{(1 - \sin\theta\cos\theta)} = 1$

$\Rightarrow \cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta$

$= 1 + \sin^2\theta\cos^2\theta - 2\sin\theta\cos\theta$

$\Rightarrow 1 + 4\sin\theta\cos\theta = 1 + \sin^2\theta\cos^2\theta$

$\Rightarrow \sin^2\theta\cos^2\theta - 4\sin\theta\cos\theta = 0$

$\Rightarrow \sin\theta\cos\theta(\sin\theta\cos\theta - 4) = 0$

$\Rightarrow \sin\theta\cos\theta = 0$ ($\because \sin\theta\cos\theta = 4$ is not possible)

$\Rightarrow \theta = (2n + 1)\pi$ or $\theta = (4n - 1)\pi/2, n \in I$

($\because -\cos\theta - \sin\theta > 0$)

398 **(a)**

Let z_1(purely imaginary) be a root of the given equation. Then,

$z_1 = -\bar{z}_1$

and

$az_1^2 + bz_1 + c = 0$ (1)

$\overline{}$

$\Rightarrow az_1^2 + bz_1 + c = 0$

$\Rightarrow \bar{a}\,\bar{z}_1^2 + \bar{b}\bar{z}_1 + \bar{c} = 0$

$\Rightarrow \bar{a}z_1^2 - \bar{b}z_1 + \bar{c} = 0$ (as $\bar{z}_1 = -z_1$) (2)

Now Eqs. (1) and (2) must have one common root

$\therefore (c\bar{a} - a\bar{c})^2 = (b\bar{c} - + + c\bar{b})(-a\bar{b} - \bar{a}b)$

Let z_1 and z_2 be two purely imaginary roots. Then,

$\bar{z}_1 = -z_1, \bar{z}_2 = -z_2$

Now,

$az^2 + bz + c = 0$ (1)

$\overline{}$

$\Rightarrow az^2 + bz + c = \bar{0}$

$\Rightarrow \bar{a}\,\bar{z}^2 + \bar{b}\bar{z} + \bar{c} = 0$

$\Rightarrow \bar{a}z^2 - \bar{b}z + \bar{c} = 0$ (2)

Equations (1) and (2) must be identical as their roots are same,

$\therefore \frac{a}{\bar{a}} = -\frac{b}{\bar{b}} = \frac{c}{\bar{c}}$

$\Rightarrow a\bar{c} = \bar{a}c, a\bar{b} + \bar{a}b = 0$ and $b\bar{c} + \bar{b}c = 0$

Hence, $a\bar{c}$ is purely real and $a\bar{b}$ and $\bar{b}c$ are purely imaginary. Let z_1 (purely real) be a root of the given equation. Then,

$z_1 = \bar{z}_1$

and

$az_1^2 + bz_1 + c = 0$

$\Rightarrow \overline{az_1^2 + bz_1 + c} = \overline{0}$

$\Rightarrow \bar{a}\,\bar{z}_1^2 + \bar{b}\bar{z}_1 + \bar{c} = 0$

$\Rightarrow \bar{a}z_1^2 + \bar{b}z_1 + \bar{c} = 0 \quad (2)$

Now (1) and (2) must have one common root. Hence,

$(c\bar{a} - a\bar{c})^2 = (b\bar{c} - c\bar{b})(a\bar{b} - \bar{a}b)$

399 **(c)**

$az + b\bar{z} + c = 0 \quad (1)$

$\Rightarrow \overline{az} + \bar{b}z + \bar{c} = 0 \quad (2)$

Eliminating z from (1) and (2), we get

$z = \dfrac{c\bar{a} - b\bar{c}}{|b|^2 - |a|^2}$

If $|a| \neq |b|$, then z represents one point on the Argand plane. If $|a| = |b|$ and $\bar{a}c \neq b\bar{c}$, then no such z exists. Adding (1) and (2)

$(\bar{a} + b)\bar{z} + (a + \bar{b})z + (c + \bar{c}) = 0$

This is of the form $A\bar{z} + \bar{A}z + B = 0$, where $B = c + \bar{c}$ is real. Hence locus of z is a straight line

400 **(a)**

$z = -\lambda \pm \sqrt{\lambda^2 - 1}$

Case I:

When $-1 < \lambda < 1$, we have

$\lambda^2 < 1 \Rightarrow \lambda^2 - 1 < 0$

$z = -\lambda \pm i\sqrt{1 - \lambda^2}$ or $x = -\lambda, y = \pm\sqrt{1 - \lambda^2}$

$\Rightarrow y^2 = 1 - x^2$ or $x^2 + y^2 = 1$

Case II:

$\lambda > 1 \Rightarrow \lambda^2 - 1 > 0$

$z = -\lambda \pm \sqrt{\lambda^2 - 1}$ or $x = -\lambda \pm \sqrt{\lambda^2 - 1}, y = 0$

Roots are $(-\lambda, +\sqrt{\lambda^2 - 1}, 0), (-\lambda, -\sqrt{\lambda^2 - 1}, 0)$.

One root lies inside the unit circle and the other root lies outside the unit circle

Case III:

When λ is very larger, then

$z = -\lambda - \sqrt{\lambda^2 - 1} \approx -2\lambda$

$z = -\lambda + \sqrt{\lambda^2 - 1}$

$= \dfrac{(-\lambda + \sqrt{\lambda^2 - 1})(-\lambda - \sqrt{\lambda^2 - 1})}{(-\lambda - \sqrt{\lambda^2 - 1})}$

$= \dfrac{1}{-\lambda - \sqrt{\lambda^2 - 1}} = -\dfrac{1}{2\lambda}$

401 **(d)**

We have,

$az^2 + z + 1 = 0 \quad (1)$

$\Rightarrow \overline{az^2 + z + 1} = 0$ (taking conjugate of both sides)

$\Rightarrow \bar{a}z^2 - z + 1 = 0 \quad (2)$

[since z is purely imaginary $\bar{z} = -z$]

Eliminating z from (1) and (2) by cross-multiplication rule,

$(\bar{a} - a)^2 + 2(a + \bar{a}) = 0 \Rightarrow \left(\dfrac{\bar{a} - a}{2}\right)^2 + \dfrac{a + \bar{a}}{2} = 0$

$\Rightarrow -\left(\dfrac{a - \bar{a}}{2i}\right)^2 + \left(\dfrac{a + \bar{a}}{2}\right) = 0 \Rightarrow -\sin^2\theta + \cos\theta = 0$

$\Rightarrow \cos\theta = \sin^2\theta \quad (3)$

Now, $f(x) = x^3 - 3x^2 + 3(1 + \cos\theta)x + 5$

$f'(x) = 3x^2 - 6x + 3(1 + \cos\theta)$

Its discriminant is

$36 - 36(1 + \cos\theta) = -36\cos\theta = -36\sin^2\theta < 0$

$\Rightarrow f'(x) > 0\ \forall x \in R$

Hence, $f(x)$ is increasing $\forall\ x \in R$. Also, $f(0) = 5$, then $f(x) = 0$ has one negative root. Now,

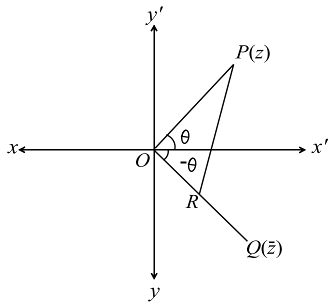
$\cos 2\theta = \cos\theta \Rightarrow 1 - 2\sin^2\theta = \cos\theta$

$\Rightarrow 1 - 2\cos\theta = \cos\theta$

$\Rightarrow \cos\theta = 1/3$

Which has four roots for $\theta \in [0, 4\pi]$

402 **(a)**



We have,

$\left||z| - \left|\dfrac{4}{z}\right|\right| \leq \left|z - \dfrac{4}{z}\right| = 2$

$\Rightarrow -2 \leq |z| - \dfrac{4}{|z|} \leq 2$

$\Rightarrow |z|^2 + 2|z| - 4 \geq 0$ and $|z|^2 - 2|z| - 4 \leq 0$

$\Rightarrow (|z| + 1)^2 - 5 \geq 0$ and $(|z| - 1)^2 \leq 5$

$\Rightarrow (|z| + 1 + \sqrt{5})(|z| + 1 - \sqrt{5}) \geq 0$

and $(|z| - 1 + \sqrt{5}) \times (|z| - 1 - \sqrt{5}) \leq 0$

$\Rightarrow |z| \leq -\sqrt{5} - 1$ or $|z| \geq \sqrt{5} - 1$ and $\sqrt{5} - 1 \leq |z| \leq \sqrt{5} + 1$

$\Rightarrow \sqrt{5} - 1 \leq |z| \leq \sqrt{5} + 1$

Hence, the least modulus is $\sqrt{5} - 1$ and the greatest modulus is $\sqrt{5} + 1$. Also,

$|z| = \sqrt{5} + 1 \Rightarrow \frac{4}{|z|} = \sqrt{5} - 1$

Now,

$\frac{4}{z} = \frac{4\bar{z}}{|z|^2}$

Hence, $4/z$ lies in the direction of $\bar{z}$

$\left|z - \frac{4}{z}\right| = PR = 2$ (given)

We have,

$OP = \sqrt{5} + 1$ and $OR = \sqrt{5} - 1$

$\Rightarrow \cos 2\theta = \frac{OP^2 + OR^2 - PR^2}{2OP \cdot OR}$

$= \frac{(\sqrt{5}+1)^2 + (\sqrt{5}-1)^2 - 4}{2(5-1)} = 1$

$\Rightarrow 2\theta = 0, 2\pi$

$\Rightarrow \theta = 0, \pi$

$\Rightarrow z$ is purely real

$\Rightarrow z = \pm(\sqrt{5} + 1)$

Similarly for $|z| = \sqrt{5} - 1$, we have $z = \pm(\sqrt{5} - 1)$

403 **(a)**

$BM \equiv y - 0 = -1(x - 1)$

$x + y = 1$

$\therefore \sqrt{u - 1} = t + i(1 - t)$

$u = 2t + 2it(1 - t)$

$x = 2t$ and $y = 2t(1 - t)$

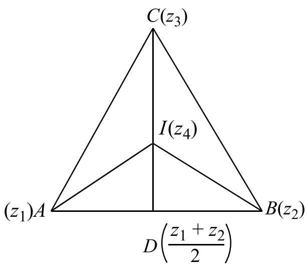
$y = x(1 - x/2)$

$2y = 2x - x^2$

$\Rightarrow (x - 1)^2 = -2\left(y - \frac{1}{2}\right)$

Which is a parabola. Its axis is $x = 1$, i.e, $z + \bar{z} = 2$ and directrix is $y = 1$, i.e, $z - \bar{z} = 2i$

404 **(a)**



$\frac{AB \times AC}{(IA)^2} = \frac{AB}{IA} \times \frac{AC}{IA}$

$\angle IAB = \frac{\theta}{2}, \angle IAC = \frac{\theta}{2}$

$\frac{z_2 - z_1}{z_4 - z_1} = \frac{|z_2 - z_1|}{|z_4 - z_1|} e^{-\frac{i\theta}{2}}$

and

$\frac{z_3 - z_1}{z_4 - z_1} = \frac{|z_3 - z_1|}{|z_4 - z_1|} e^{\frac{i\theta}{2}}$

Multiplying,

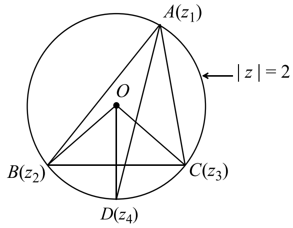
$\frac{z_2 - z_1}{z_4 - z_1} \frac{z_3 - z_1}{z_4 - z_1} = \frac{|z_2 - z_1|}{|z_4 - z_1|} \frac{|z_3 - z_1|}{|z_4 - z_1|}$

$\Rightarrow \frac{(z_2 - z_1)(z_3 - z_1)}{(z_4 - z_1)^2} = \frac{AB \times AC}{IA^2}$ (1)

405 **(d)**

$\angle BOD = 2\angle BAD = A$

$\angle COD = 2\angle CAD = A$



$\frac{z_4}{z_2} = e^{iA}, \frac{z_3}{z_4} = e^{iA}$ (From rotation about the point 'O')

$\Rightarrow z_4^2 = z_2 z_3$

406 **(d)**

Let unknown polynomial be $P(x)$. Let $Q(x)$ and $R(x)$ be the quotient and remainder, respectively, when it is divided by $(x - 3)(x - 4)$. Then,

$P(x) = (x - 3)(x - 4)Q(x) + R(x)$

Then, we have

$R(x) = ax + b$

$\Rightarrow P(x) = (x - 3)(x - 4)Q(x) + ax + b$

Given that $P(3) = 2$ and $P(4) = 1$. Hence,

$3a + b = 2$ and $4a + b = 1$

$\Rightarrow a = -1$ and $b = 5$

$\Rightarrow R(x) = 5 - x$

$5 - x = x^2 + ax + 1 \Rightarrow x^2 + (a + 1)x - 4 = 0$

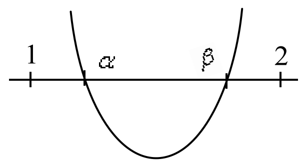
Given that roots are real and distinct

$\therefore D > 0 \Rightarrow (a + 1)^2 + 16 > 0$

Which is true or all real x

407 **(d)**

$ax^2 - bx + c = 0$



Let $f(x) = ax^2 - bx + c$ be the corresponding quadratic expression and α, β be the roots of $f(x) = 0$. Then,

$f(x) = a(x - \alpha)(x - \beta)$

Now,

$af(1) > 0, af(2) > 0, 1 < \frac{b}{2a} < 2, b^2 - 4ac > 0$

$\Rightarrow a(1 - \alpha)(1 - \beta) > 0, a(2 - \alpha)(2 - \beta) > 0, 2a < b < 4a, b^2 - 4ac > 0$

$\Rightarrow a^2(1 - \alpha)(1 - \beta)(2 - \alpha)(2 - \beta) > 0$

$\Rightarrow a^2(\alpha - 1)(2 - \alpha)(\beta - 1)(2 - \beta) > 0$

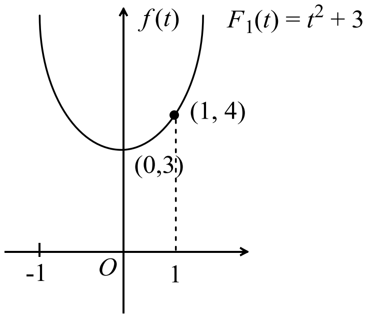
As $f(1)$ and $f(2)$ both are integers and $f(1) > 0$, and $f(2) > 0$, so $f(1)f(2) > 0$
$\Rightarrow f(1)\,f(2) \geq 1$
$\Rightarrow 1 \leq a^2(\alpha-1)(2-\alpha)(\beta-1)(2-\beta)$
Now,
$$\frac{(\alpha-1)+(2-\alpha)}{2} \geq \left((\alpha-1)(2-\alpha)\right)^{1/2}$$
$$\Rightarrow (\alpha-1)(2-\alpha) \leq \frac{1}{4}$$
Similarly, $(\beta-1)(2-\beta) \leq \frac{1}{4}$
$$\Rightarrow (\alpha-1)(2-\alpha)(\beta-1)(2-\beta) < \frac{1}{16}$$
As $\alpha \neq \beta$, so
$a^2 > 16 \Rightarrow a \geq 5$
$\Rightarrow b^2 > 20c$ and $b > 10 \Rightarrow b \geq 11$
Also, $b^2 > 100 \Rightarrow c > 5 \Rightarrow c \geq 6$

408 **(c)**
Given equation is
$x^4 + 2ax^3 + x^2 + 2ax = 0$ (1)
or $\left(x^2 + \frac{1}{x^2}\right) + 2a\left(x + \frac{1}{x}\right) + 1 = 0$
$$\Rightarrow \left(x + \frac{1}{x}\right)^2 + 2a\left(x + \frac{1}{x}\right) - 1 = 0$$
$\Rightarrow t^2 + 2at - 1 = 0$ (2)
Where $t = x + (1/x)$. Now,
$$\left(x + \frac{1}{x}\right) \geq 2$$
or $\left(x + \frac{1}{x}\right) \leq -2$
$\therefore t \geq 2$ or $t \leq -2$
Now, eq. (1) will have at least two positive roots, when at least one root of Eq. (2) will be greater than 2. From eq. (2),
$D = 4a^2 - 4(-1) = 4(1 + a^2) > 0, \forall\, a \in R$ (3)
Let the roots of eq. (2) be α, β. If $\alpha, \beta \leq 2$, then
$\Rightarrow f(2) \geq 0$ and $\frac{-B}{2A} < 2$
$\Rightarrow 4 + 4a - 1 \geq 0$ and $-\frac{2a}{2} < 2$
$\Rightarrow a \geq -\frac{3}{4}$ and $a > -2$
$$\Rightarrow a \geq -\frac{3}{4}$$
Therefore, at least one root will be greater than 2. Then,
$a < -\frac{3}{4}$ (4)
Combining (3) and (4), we get
$$a < -\frac{3}{4}$$
Hence, at least one root will be positive if
$a \in [-\infty, -(3/4)]$

409 **(a)**
$(\beta - \alpha) = \left((\beta + h) - (\alpha + h)\right)$
$(\beta + \alpha)^2 - 4\alpha\beta$
$= [(\beta + h) + (\alpha + h)]^2 - 4(\beta + h)(\alpha + h)$
$(-b_1)^2 - 4c_1 = (-b_2)^2 - 4c_2$
$D_1 = D_2$ (i)
The least value of $f(x)$ is
$-\frac{D_1}{4} = -\frac{1}{4} \Rightarrow D_1 = 1$ and $D_2 = 1$
Therefore, the least value of
$g(x)$ is $-\frac{D_2}{4} = -\frac{1}{4}$
The least value of $g(x)$ occurs at
$$-\frac{b_2}{2} = \frac{7}{2} \Rightarrow b_2 = -7$$
$\Rightarrow b_2^2 - 4c_2 = D_2$
$$\Rightarrow 49 - 4c_2 = 1 \Rightarrow \frac{48}{4} = c_2 \Rightarrow c_2 = 12$$
$\Rightarrow x^2 - 7x + 12 = 0 \Rightarrow x = 3, 4$

410 **(d)**
$\because AC = 4\sqrt{2}$
$\therefore AB = BC = \frac{4\sqrt{2}}{\sqrt{2}} = 4$ units
$$OB = \sqrt{4^2 - \left(2\sqrt{2}\right)^2} = 2\sqrt{2}$$
$\therefore A\left(-2\sqrt{2}, 0\right), B\left(2\sqrt{2}, 0\right), C\left(0, -2\sqrt{2}\right)$
Since $y = ax^2 + bx + c$ passes through A, B and C, we get $y = \frac{x^2}{2\sqrt{2}} - 2\sqrt{2}$

411 **(d)**
Given that $9^x - a3^x - a + 3 \leq 0$
Let $t = 3^x$. Then,
$t^2 - at - a + 3 \leq 0$
or $t^2 + 3 \leq a(t + 1)$ (1)
Where $t \in R^+, \forall x \in R$

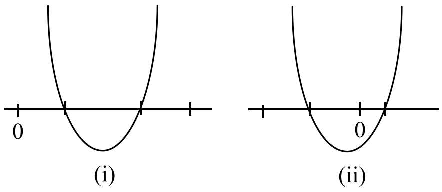


Let $f_1(t) = t^2 + 3$ and $f_2(t) = a(t + 1)$
For $x < 0, t \in (0, 1)$. That means (1) should have at least one solution in $t \in (0, 1)$. From (1), it is obvious that $a \in R^+$. Now $f_2(t) = a(t + 1)$ represents a straight line. It should meet the curve $f_1(t) = t^2 + 3$, at least once in $t \in (0, 1)$
$f_1(0) = 3, f_1(1) = 4, f_2(0) = a, f_2(1) = 2a$
If $f_1(0) = f_2(0)$, Then $a = 3$; if $f_1(1) = f_2(1)$, then $a = 2$. Hence, the required range is $a \in (2, 3)$

412 **(d)**

Let $f(x) = x^2 + x + a - 9$

$x^2 + x + a - 9 < 0$ has at least one positive solution, then either both the roots of equation $x^2 + x + a - 9 = 0$ are non-negative or 0 lies between the roots



Now sum of roots $= -\frac{1}{2}$, hence case I is not possible. For case II,

$f(0) < 0 \Rightarrow a - 9 < 0 \Rightarrow a < 9$

413 **(b)**

$b^2 > (a+c)^2$

$\Rightarrow (a + c - b)(a + c + b) < 0$

$\Rightarrow f(-1)f(1) < 0$

So, there is exactly one root in $(-1, 1)$

414 **(a)**

From the question, the real roots of $x^3 - x^2 + \beta x + \gamma = 0$ are x_1, x_2, x_3 and they are in A.P. As x_1, x_2, x_3 are in A.P., let $x_1 = a - d, x_2 = a, x_3 = a + d$. Now,

$x_1 + x_2 + x_3 = -\dfrac{-1}{1} = 1$

$\Rightarrow a - d + a + a + d = 1$

$\Rightarrow a = \frac{1}{3}$ (1)

$x_1x_2 + x_2x_3 + x_3x_1 = \dfrac{\beta}{1} = \beta$

$\Rightarrow (a - d)a + a(a + d) + (a + d)(a - d) = \beta$ (2)

$x_1x_2x_3 = -\dfrac{\gamma}{1} = -\gamma$

$\Rightarrow (a - d)a(a + d) = -\gamma$ (3)

From (1) and (2), we get

$3a^2 - d^2 = \beta$

$\Rightarrow 3\frac{1}{9} - d^2 = \beta$, so $\beta = \frac{1}{3} - d^2 < \frac{1}{3}$

From (1) and (3), we get

$\dfrac{1}{3}\left(\dfrac{1}{9} - d^2\right) = -\gamma$

$\Rightarrow \gamma = \dfrac{1}{3}\left(d^2 - \dfrac{1}{9}\right) > \dfrac{1}{3}\left(-\dfrac{1}{9}\right) = -\dfrac{1}{27}$

$\gamma \in \left(-\dfrac{1}{27}, +\infty\right)$

415 **(8)**

Given $\alpha\beta;\ \alpha\beta(\alpha+\beta);\ \alpha^3 + \beta^3$ are in G.P.

$\alpha + \beta = 4;\ \alpha\beta = k;\ \alpha\beta^2 + \alpha^2\beta = \alpha\beta(\alpha+\beta)$
$= 4k$

$\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$= 64 - 3k(4) = 4(16 - 3k)$

$\therefore k; 4k; 4(16 - 3k)$ are in G.P.

$16k^2 = 4k(16 - 3k)$

$4k(4k - 16 + 3k) = 0$

$k = 0; k = \dfrac{16}{7}$

416 **(3)**

Clearly, $P(x) - x^3 = 0$ has roots 1, 2, 3, 4

$\therefore P(x) - x^3 = (x-1)(x-2)(x-3)(x-4)$

$\Rightarrow P(x) = (x-1)(x-2)(x-3)(x-4) + x^3$

Hence, $P(5) = 1 \times 2 \times 3 \times 4 + 125 = 129$

417 **(1)**

Let $z = a + ib$

Given $|z| = 2 \Rightarrow a^2 + b^2 = 4 \Rightarrow a, b \in [-2, 2]$

Now $w = \frac{(a+1)+ib}{(a-1)+ib}$;

$\Rightarrow |w| = \sqrt{\dfrac{(a+1)^2 + b^2}{(a-1)^2 + b^2}}$

$= \sqrt{\dfrac{a^2 + b^2 + 2a + 1}{a^2 + b^2 - 2a + 1}} = \sqrt{\dfrac{5 + 2a}{5 - 2a}}$

$|w|_{\max} = \sqrt{\dfrac{5+4}{1}} = 3$ (when $a = 2$)

$|w|_{\min} = \sqrt{\dfrac{5-4}{9}} = \dfrac{1}{3}$ (when $a = -2$)

Hence, required product is 1

418 **(3)**

$2x^2 + 4x(y - 3) + 7y^2 - 2y + t = 0$

$D = 0$ (for one solution)

$\Rightarrow 16(y-3)^2 - 8(7y^2 - 2y + t) = 0$

$\Rightarrow 2(y-3)^2 - (7y^2 - 2y + t) = 0$

$\Rightarrow 2(y^2 - 6y + 9) - (7y^2 - 2y + t) = 0$

$\Rightarrow -5y^2 - 10y + 18 - t = 0$

$\Rightarrow 5y^2 + 10y + t - 18 = 0$

Again $D = 0$ (for one solution)

$\Rightarrow 100 - 20(t - 18) = 0$

$\Rightarrow 5 - t + 18 = 0$

$\Rightarrow t = 23$

For $t = 23; 5y^2 + 10y + 5 = 0$

$(y+1)^2 = 0 \Rightarrow y = -1$

For $y = -1; 2x^2 - 16x + 32 = 0$

$x^2 - 8x + 16 = 0$

$x = 4 \Rightarrow x + y = 3$

419 **(4)**

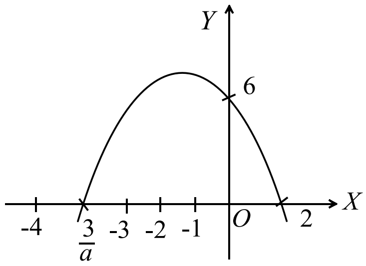
$f(x) = ax^2 - (3 + 2a)x + 6$

$= (ax - 3)(x - 2)$

Here, roots of the equation $f(x) = 0$ are 2 and $3/a$, and $f(0) = 6$. $f(x)$ should be positive for exactly three negative integral values of x

Therefore, graph of $f(x)$ must be a downward parabola passing through $x = 2$ and $x = 3/a$ and $-4 \leq \frac{3}{a} < -3$

$\therefore\ a \in \left(-1 - \frac{3}{4}\right]$



$\therefore\ c = -1, d = -\frac{3}{4}$

$\Rightarrow c^2 + 4|d| = 1 + 3 = 4$

420 **(3)**

We have $P(x) = \frac{5}{2} - 6x - 9x^2 = -(3x+1)^2 + \frac{8}{3}$

$\Rightarrow P_{\max} = \frac{8}{3}$

Similarly, $Q(y) = -4y^2 + 4y + \frac{13}{2} = -(2y -$

12+152

$\Rightarrow Q_{\max} = \frac{15}{2}$

Now, $P_{\max} \times Q_{\max} = \frac{8}{3} \times \frac{15}{2} = 20$

So $(x, y) \equiv \left(-\frac{1}{3}, \frac{1}{2}\right)$

Hence, $6x + 10y = 6\left(\frac{-1}{3}\right) + 10\left(\frac{1}{2}\right) = -2 + 5 = 3$

421 **(8)**

Let $\left(a + \frac{1}{a}\right) = t$

$\Rightarrow a^3 + \frac{1}{a^3} = 18$

$t^3 - 3t - 18 = 0 \quad (1)$

$t = 3$ satisfies (1)

Hence factorizing (1)

$(t - 3)(t^2 + 3t + 6) = 0$

$t = 3$ only is the solution

$\therefore\ a + \frac{1}{a} = 3\ \Rightarrow a^2 + \frac{1}{a^2} = 7\ \Rightarrow a^4 + \frac{1}{a^4} = 47$

422 **(6)**

We have $|a\omega + b| = 1$

$\Rightarrow |a\omega + b|^2 = 1$

$\Rightarrow (a\overline{\omega} + b) = 1$

$\Rightarrow a^2 + ab(\omega + \overline{\omega}) + b^2 = 1$

$\Rightarrow a^2 - ab + b^2 = 1$

$\Rightarrow (a - b)^2 + ab = 1 \quad (1)$

When $(a - b)^2 = 0$ and $ab = 1$ then

$(1, 1); (-1, -1)$

When $(a - b)^2 = 1$ and $ab = 0$ then $(0, 1); (1, 0); (0, -1); (-1, 0)$

Hence there are 6 ordered pairs

423 **(4)**

$= \left[\frac{1 + \cos\theta + i\sin\theta}{\sin\theta + i\,(1 + \cos\theta)}\right]^4$

$= i^4\left[\frac{1 + \cos\theta + i\sin\theta}{i\sin\theta + i^2(1 + \cos\theta)}\right]^4$

$= \left[\frac{2\cos^2\frac{\theta}{2} + i2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2} - i2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right]$

$= \left[\frac{\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}}{\cos\frac{\theta}{2} - i\sin\frac{\theta}{2}}\right]^4$

$= \left[\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)^2\right]^4$

$= \cos 8\frac{\theta}{2} + i\sin 8\frac{\theta}{2} = \cos 4\theta + i\sin 4\theta\ \Rightarrow\ n = 4$

424 **(3)**

$(1 + ri)^3 = s(1 + i)$

$\Rightarrow 1 + 3ri + 3r^2i^2 + r^3i^3 = s(1 + i)$

$\Rightarrow 1 - 3r^2 + i(3r - r^3) = s + si$

$\Rightarrow 1 - 3r^2 = s = 3r - r^3$

Hence, $1 - 3r^2 = 3r - r^3$

$\Rightarrow r^3 - 3r^2 - 3r + 1 = 0$

$\Rightarrow$ sum of three roots is 3

425 **(3)**

The given equation $x + \frac{1}{x} = 3$

$\therefore\ x^2 + \frac{1}{x^2} = 7\ \Rightarrow x^4 + \frac{1}{x^4} = 47\ \Rightarrow x^8 + \frac{1}{x^8}$

$= (47)^2 - 2$

$\therefore\ x^8 + x^{-8} = 2207 \quad (1)$

Now $E = x^9 + x^7 + x^{-9} + x^{-7}$

$= x^8\left(x + \frac{1}{x}\right) + x^{-9} + x^{-7}$

$= x^8\left(x + \frac{1}{x}\right) + x^{-8}\left(x + \frac{1}{x}\right)$

$E = \left(x + \frac{1}{x}\right)\ (x^8 + x^{-8}) \quad (2)$

Substitute the value of $x^8 + x^{-8} = 2207$ from (1) and $x + \frac{1}{x} = 3$

$E = (3)(2207) = 6621$

426 **(6)**

Let $z = x + iy$

$\therefore E = z\,\overline{z} + (z - 3)(\overline{z} - 3) + (z - 6i)(\overline{z} + 6i)$

$= 3z\overline{z} - 3(z + \overline{z}) + 9 + 6(z - \overline{z})i + 36$

$= 3(x^2 + y^2) - 6x - 12y + 45$

$= 3[x^2 + y^2 - 2x - 4y + 15]$

$= 3[(x - 1)^2 + (y - 2)^2 + 10]$

$\therefore E_{\min} = 30$ when $x = 1$ and $y = 2$

427 **(7)**

Let $E = \frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$

Now $3+E=\frac{a}{b+c}+1+\frac{b}{c+a}+1+\frac{c}{a+b}+1$

$\Rightarrow 3+E=(a+b+c)\left[\frac{1}{b+c}+\frac{1}{c+a}+\frac{1}{a+b}\right]$

$\Rightarrow 3+E=3\times\frac{10}{3}=10 \Rightarrow E=7$

428 **(6)**

$a^2\geq 8b$ and $4b^2\geq 4a$

Now $b^2\geq a\Rightarrow b^4\geq a^2\geq 8b\ (a>0,b>0)$

$\therefore \Rightarrow\ b^3\geq 8\Rightarrow b\geq 2\ \ (1)$

Again $a^2\geq 8b$ and $b\geq 2$

$\Rightarrow a^2\geq 16$

$\Rightarrow a\geq 4\quad (2)$

From (1) and (2), $(a+b)_{\text{least}}=6$

429 **(7)**

Given $a+b+c=1\ \ (1)$

$ab+bc+ca=0\quad (2)$

$abc=2\quad (3)$

Now $(a+b+c)^2=1$

$a^2+b^2+c^2+2\sum ab=1$

$\therefore a^2+b^2+c^2=1$

Now, $a^3+b^3+c^3-3abc=(a+b+c)[\sum a^2-ab$

$=1(1-0)=1$

$a^3+b^3+c^3=1+3abc=1+3\times 2=7$

430 **(2)**

We have $x_2+x_2+x_3=8$

$x_1x_2x_3=d$

$x_1x_2+x_2x_3+x_3x_1=c$

Possible roots 1, 2, 5 or 1, 3, 4

$\therefore d=10$ or $d=12$

$\Rightarrow c=2+10+5=17$ or $3+12+4=19$

Hence, $d=10$ and $c=17$ or $d=12$ and $c=19$

431 **(4)**

We have $|z|^2+\frac{16}{|z|^3}=z^2-4z=\overline{z}^2-4\overline{z}$

$\Rightarrow (z-\overline{z})(z+\overline{z}-4)=0$

$\Rightarrow z=\overline{z}=x(x\neq 2)$

So, $x^2=4x+x^2+\frac{16}{|x|^3}$

$\Rightarrow x=\frac{-4}{|x|^3}\ \ \Rightarrow x=-\sqrt{2}$

$\therefore z=-\sqrt{2}$

$\therefore |z|^4=4$

432 **(3)**

$x^2+x(y-a)+y^2-ay+1\geq 0x\ \in R$

$\Rightarrow (y-a)^2-4(y^2-ay+1)\leq 0$

$\Rightarrow -3y^2+2ay+a^2-4\leq 0$

$\therefore 3y^2-2ay+4-a^2\geq 0\ y\in R$

$D\leq 0$

$\Rightarrow 4a^2-4.3(4-a^2)\leq 0\Rightarrow a^2-3(4-a^2)\leq 0$

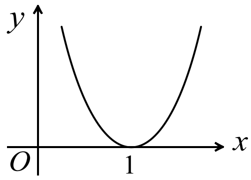
$\Rightarrow 4a^2-12\leq 0$

$\therefore$ range of $a\in(-\sqrt{3},\sqrt{3})\Rightarrow$ number of integer $\{-1,0,1\}$

433 **(8)**

As $P(1)=0$

and $p(x)\geq 0$ hence let $p(x)=k(x-1)^2,k>0$



$p(2)=k=2\ \Rightarrow k=2$

$\therefore\ p(x)=2(x-1)^2\Rightarrow p(3)=8$

434 **(6)**

$x^2+ax+b\equiv(x+1)(x+b)\Rightarrow b+1=a\ \ (1)$

Also $x^2+bx+c\equiv(x+1)(x+c)\Rightarrow c+1=b$

or $b+1=c+2\quad (2)$

hence $b+1=a=c+2$

also $(x+1)(x+b)(x+c)\equiv x^3-4x^2+x+6$

$\Rightarrow x^3+(1+b+c)x^2+(b+bc+c)x+bc$

$\equiv x^3-4x^2+x+6$

$\Rightarrow 1+b+c=-4$

$\Rightarrow 2c+2=-4\ \Rightarrow c=-3; b=-2$ and $a=-1$

$\Rightarrow a+b+c=-6$

435 **(3)**

Let $a^2+b^2=x$

$1-2ab=(a+b)^2-2ab=a^2+b^2$

$=x(a+b=1);$

Also $ab=\frac{1-x}{2}$

and $a^3+b^3=(a+b)(a^2+b^2-ab)$

$a^3+b^3=x-ab\qquad (1)$

But $ab=\frac{1-x}{2}$

Hence $a^3+b^3=x-\frac{1-x}{2}=\frac{3x-1}{2}$

Hence the equation

$(1-2ab)(a^3+b^3)=12$, becomes

$x\left(\frac{3x-1}{2}\right)=12$

$\Rightarrow 3x^2-x-24=0$

$\Rightarrow (x-3)(3x+8)=0$

$\Rightarrow x=3$ or $x=-\frac{8}{3}$ (not possible) as $x=a^2+b^2\not<0$

$\therefore x=3\ \Rightarrow a^2+b^2=3$

436 **(2)**

$(x+y+z)^2=144$ (given)

$\Rightarrow \sum x^2+2\sum xy=144$

$\Rightarrow 96+2\sum xy=144\Rightarrow \sum xy=24$

Again $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=36\Rightarrow xyz=\frac{24}{36}=\frac{2}{3}$

Now $x^3+y^3+z^3-3xyz=(x+y+$

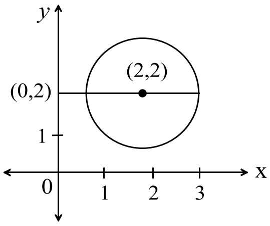
$z)(\sum x^2 - \sum xy)$

$\Rightarrow \sum x^3 - 2 = (12)(96 - 24) = (12)(72) = 864$

$\Rightarrow \sum x^3 = 866$

437 **(3)**

$|z - 2 - 2i| \leq 1$



Denotes the region inside a circle with centre (2, 2) and radius is 1

$|2iz + 4| = |2i\,(2 - 2i)|$

$= |2i||z - 2i|$

$= 2|z - 2i|$

$|z - 2i| =$ distance of z from $P(0, 2)$

Hence, maximum value is 3

438 **(5)**

$|3z + 9 - 7i| = |(3z + 6 - 3i) + (3 - 4i)|$

$\leq |3z + 6 - 3i| + |3 - 4i|$

$= 3|z + 2 - i| + 5$

$= 20$

439 **(3)**

We have $abc + 1 = \frac{bc}{5} = \frac{-ac}{15} = \frac{ab}{3}$

$\Rightarrow \frac{a}{b} = -3$ and $\frac{c}{b} = -5$

Now $\frac{c-b}{c-a} = \frac{\frac{c}{b}-1}{\frac{c}{b}-\frac{a}{b}} = \frac{-5-1}{-5-(-3)} = 3$

440 **(7)**

We have

$x^3 - y^3 = 98i$

$\Rightarrow (x - y)^3 + 3xy(x - y) = 98i$

$\Rightarrow\ -343i + 3(a + ib)(7i) = 98i$

$\Rightarrow\ -343 + 3(a + bi)7 = 98$

$\Rightarrow a + ib = 21$

$\Rightarrow a = 21$ and $b = 0$

$\Rightarrow a + b = 21$

441 **(8)**

$x^2 + mx + n = 0 \left\langle \begin{matrix} 2\alpha \\ 2\beta \end{matrix} \right.$ and $x^2 + px + m = 0 \left\langle \begin{matrix} \alpha \\ \beta \end{matrix} \right.$

$2(\alpha + \beta) = -m \quad (1)$

$4\alpha\beta = n \quad (2)$

and $\alpha + \beta = -p \quad (3)$

$\alpha\beta = m \quad (4)$

$\therefore$ (1) and (3) $\Rightarrow 2p = m$

and (2) and (4) $\Rightarrow 4m = n$

$\Rightarrow \frac{n}{p} = \frac{4m}{m/2} = 8$

442 **(9)**

Let $z = a + bi$

$\Rightarrow |z|^2 = a^2 + b^2$

Now $z + |z| = 2 + 8i$

$\Rightarrow a + bi + \sqrt{a^2 + b^2} = 2 + 8i$

$\Rightarrow a + \sqrt{a^2 + b^2} = 2, b = 8$

$\Rightarrow a + \sqrt{a^2 + 64} = 2$

$\Rightarrow a^2 + 64 = (2 - a)^2 = a^2 - 4a + 4,$

$\Rightarrow 4a = -60, a = -15$

Thus, $a^2 + b^2 = 225 + 64 = 289$

$\therefore |z| = \sqrt{a^2 + b^2} = \sqrt{289} = 17$

443 **(9)**

Let $\alpha_1 = A, \beta_1 = \text{AR}, \alpha_2 = \text{AR}^2, \beta_2 = \text{AR}^3$

We have $\alpha_1 + \beta_1 = 6 \Rightarrow A(1 + R) = 6 \;\; (1)$

$\alpha_1\beta_1 = p \Rightarrow A^2\,R = p \quad (2)$

Also $\alpha_2 + \beta_2 = 54 \Rightarrow \text{AR}^2(1 + R) = 54 \;\; (3)$

$\alpha_2\beta_2 = q \Rightarrow A^2R^5 = q \;\; (4)$

Now, on dividing Eq. (3) by Eq. (1), we get

$\frac{AR^2(1 + R)}{A(1 + R)} = \frac{54}{6} = 9 \ \Rightarrow R^2 = 9$

$\therefore\ R = 3$ (As it is an increasing G.P.)

$\therefore$ On putting $R = 3$ in eq. (1), we get

$A = \frac{6}{4} = \frac{3}{2}$

$\therefore p = A^2R = \frac{9}{4} \times 3 = \frac{27}{4}$ and $q = A^2R^5 = \frac{9}{4} \times 243 = \frac{2187}{4}$

Hence, $q - p = \frac{2187-27}{4} = \frac{2160}{4} = 540$

444 **(3)**

$f(x) = (x - 1)(x^2 - 7x + 13)$

For $f(x)$ to be prime at least one of the factors must be prime

Hence, $x - 1 = 1 \Rightarrow x = 2$ or

$x^2 - 7x + 13 = 1 \Rightarrow x^2 - 7x + 12 = 0 \ \Rightarrow x = 3$ or 4

$\Rightarrow x = 2,3,4$

445 **(4)**

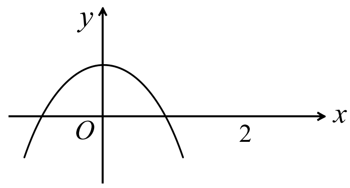
$x^{1/8} = (3x^4 + 4)^{1/64} \Rightarrow x^8 = 3x^4 + 4 \Rightarrow x^4 = 4$

446 **(7)**

For two distinct roots, $D > 0$ i.e., $k^2 + 8(k^2 + 5) > 0$ which is always true

Also let $f(x) = -2x^2 + kx + k^2 + 5 = 0$

But $f(0) > 0$ and $f(2) < 0$



$-8 + 2k + k^2 + 5 < 0 \Rightarrow k^2 + 2k - 3 < 0$

$\Rightarrow (k + 3)(k - 1) < 0$

$k \in (-3, 1) \Rightarrow a = -3; b = 1 \ \Rightarrow a + 10b$

$= -3 + 10 = 7$

447 **(1)**

$z^4 + z^3 + z^2 + z^2 + z + 1 = 0$

$\Rightarrow (z^2 (z^2 + z + 1) + (z^2 + z + 1) = 0$

$\Rightarrow (z^2 + z + 1)(z^2 + 1) = 0$

$\therefore z = \mathrm{i}, -\mathrm{i}, \omega, \omega^2$. For each, $|z| = 1$

448 **(5)**

$Z_1 = (8\sin\theta + 7\cos\theta) + i\,(\sin\theta + 4\cos\theta)$

$Z_2 = (\sin\theta + 4\cos\theta) + i(8\sin\theta + 4\cos\theta)$

Hence, $Z_1 = x + iy$ and $Z_2 = y + ix$

Where $x = (8\sin\theta + 7\cos\theta)$ and $y = (\sin\theta + 4\cos\theta)$

$Z_1 \cdot Z_2 = (xy - xy) + i(x^2 + y^2) = i(x^2 + y^2)$
$= a + ib$

$\Rightarrow a = 0; b = x^2 + y^2$

Now, $x^2 + y^2 = (8\sin\theta + 7\cos\theta)^2 + (\sin\theta + 4\cos\theta)^2$

$= 65\sin^2\theta + 65\cos^2\theta + 120\sin\theta.\cos\theta$

$= 65 + 60\sin 2\theta$

$\Rightarrow Z_1 \cdot Z_2|_{\max} = 125$

449 **(3)**

$n, n+1, n+2$

sum $= 3(n+1) = -a$

$\therefore\ a^2 = 9(n+1)^2$

Sum of the roots taken 2 at a time $= \pm b$

$\therefore\ n(n+1) + (n+1)(n+2) + (n+2)n + 1$
$= b + 1$

$n^2 + n + n^2 + 3n + 2 + n^2 + 2n + 1 = b + 1$

$\therefore b + 1 = 3n^2 + 6n + 3$

$b + 1 = 3(n+1)^2 = \dfrac{a^2}{3};\ \therefore \dfrac{a^2}{b+1} = 3$

450 **(6)**

$\alpha + \beta = 1154$ and $\alpha\beta = 1$

$\left(\sqrt{\alpha} + \sqrt{\beta}\right)^2 = \alpha + \beta + 2\sqrt{\alpha\beta} = 1154 + 2$
$= 1156 = (34)^2$

$\Rightarrow \sqrt{\alpha} + \sqrt{\beta} = 34$

Again $\left(\alpha^{1/4} + \beta^{1/4}\right)^2 = \sqrt{\alpha} + \sqrt{\beta} = +2(\alpha\beta)^{1/4} = 34 + 2 = 36$

$\alpha^{1/4} + \beta^{1/4} = 6$

451 **(5)**

Roots are $2\omega, (2 + 3\omega), (2 + 3\omega^2), (2 - \omega - \omega^2)$

$2 + 3\omega$ and $2 + 3\omega^2$ are conjugate to each other

2ω is complex root, then other root must be $2\omega^2$ (as complex roots occur in conjugate pair)

$2 - \omega - \omega^2 = 2 - (-1) = 3$ which is real

Hence least degree of the polynomial is 5

452 **(1)**

We have $x = \omega - \omega^2 - 2$ or $x + 2 = \omega - \omega^2$

Squaring, $x^2 + 4x + 4 = \omega^2 + \omega^4 - 2\omega^3 = \omega^2 + \omega - 2 = -3$

$\Rightarrow x^2 + 4x + 7 = 0$

Dividing, $x^4 + 3x^3 + 2x^2 - 11x - 6$ by $x^2 + 4x + 7$, we get

$x^4 + 3x^3 + 2x^2 - 11x - 6$
$= (x^2 + 4x + 7)(x^2 - x - 1) + 1$

$= (0)(x^2 - x - 1) + 1 = 0 + 1 = 1$

453 **(3)**

$$x = \frac{x^3}{x^2} = \frac{2 + 11i}{3 + 4i} \times \frac{3 - 4i}{3 - 4i} = \frac{50 + 25i}{25} = 2 + i$$

454 **(4)**

$$y = \frac{3x^2 + mx + n}{x^2 + 1}$$

$x^2(y - 3) - mx + y - n = 0$

$x \in R$

$D \geq 0$

$\Rightarrow m^2 - 4(y - 3)(y - n) \geq 0$

$\Rightarrow m^2 - 4(y^2 - ny - 3y + 3n) \geq 0$

$4y^2 - 4y(n + 3) + 12n - m^2 \leq 0$ (1)

Also given $(y + 4)(y - 3) \leq 0$

$y^2 + y - 12 \leq 0$ (2)

Compare (1) and (2), we get $\frac{4}{1} = -\frac{4(n+3)}{1} = \frac{12n - m^2}{-12}$

$\Rightarrow m = 0$ and $n = -4$

455 **(5)**

Let $ax^3 + bx^2 + cx + d = 0$ has roots p, q, r

$pq + qr + rp = \frac{c}{a}$ (1)

But $pq + qr + rp \leq p^2 + q^2 + r^2$

$= (p + q + r)^2 - 2\sum pq$

$\therefore 3(pq + qr + rp) \leq (p + q + r)^2 = 16$

$\therefore 3\frac{c}{a} \leq 16 \Rightarrow \frac{c}{a} \leq \frac{16}{3} \Rightarrow$ largest possible integral value of $\frac{c}{a}$ is 5

456 **(6)**

Let the roots be $a - 3d, a - d, a + d, a + 3d$

Sum of roots $= 4a = 0 \Rightarrow a = 0$

Hence, roots are $-3d, -d, d, 3d$

Product of roots $= 9d^4 = m^2 \Rightarrow d^2 = \frac{m}{3}$ (1)

Again $\sum x_1x_2 = 3d^2 - 3d^2 - 9d^2 - d^2 - 3d^2 + 3d^2 = -10d^2$

$= -(3m + 2)$

$\Rightarrow \dfrac{10m}{3} = 3m + 2 \Rightarrow 10m = 9m + 6$

$\Rightarrow m = 6$

457 **(5)**

We have $2x^3 - 9x^2 + 12x + k = 0$

Let the roots are α, α, β

$2\alpha + \beta = \frac{9}{2}$ (1)

$\alpha^2 + 2\alpha\beta = \frac{12}{2} = 6$ (2)

are $\alpha^2\beta = -\frac{k}{2}$ (3)

putting $\beta = \left(\frac{9}{2} - 2\alpha\right)$ from (1) in (2)

$\alpha^2 + 2\alpha\left(\frac{9}{2} - 2\alpha\right) = 6 \Rightarrow \alpha^2 + 9\alpha - 4\alpha^2 = 6$
$\Rightarrow 3\alpha^2 - 9\alpha + 6 = 0$
$\Rightarrow \alpha^2 - 3\alpha + 2 = 0$
$\Rightarrow (\alpha - 2)(\alpha - 1) = 0 \ \Rightarrow \alpha = 2$ or 1
If $\alpha = 2$ then $\beta = \frac{1}{2}$; if $\alpha = 1$ then $\beta = \frac{5}{2}$
$\therefore k = -2(4)\frac{1}{2} = -4$ or $k = -2(1^2)\left(\frac{5}{2}\right) = -5$

458 **(2)**
$\overline{z} + z = 0$
$\Rightarrow \overline{z} = -z \quad (1)$
Now $|z|^2 - 4zi = z^2$
$\Rightarrow \ -z^2 - 4zi = z^2$ (from (1))
$\Rightarrow 2z = -4i$
$\Rightarrow z = -2i$
$\Rightarrow |z| = 2$

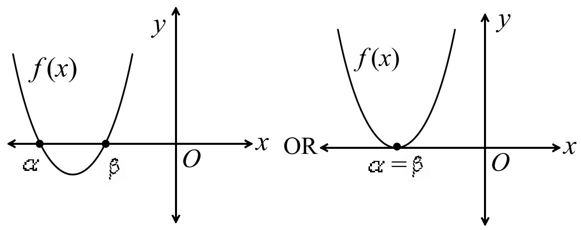
459 **(3)**
We have $(\alpha + \beta)^2 = \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)(\alpha^2 + \beta^2)$
$\Rightarrow (\alpha + \beta)^2 = \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)[(\alpha + \beta)^2 - 2\alpha\beta]$
Substituting $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$ we have
$\frac{b^2}{a^2} = \frac{-b}{c}\left(\frac{b^2}{a^2} - \frac{2c}{a}\right)$
$\Rightarrow cb^2 + b(b^2 - 2ac) = 0$
$b \neq 0, \therefore bc + b^2 - 2ac = 0$
a, b, c are in AP, $\therefore b = \frac{a+c}{2}$
$\therefore$ we have $\frac{(a+c)c}{2} + \left(\frac{a+c}{2}\right)^2 - 2ac = 0$
$\Rightarrow a^2 - 4ac + 3c^2 = 0 \ \Rightarrow (a - c)(a - 3c) = 0$
a $\neq c \ \therefore a = 3c \ \therefore \frac{a}{c} = 3$

460 **(9)**
$A \equiv (1 + 2i)x^3 - 2(3 + i)x^2 + (5 - 4i)x + 2a^2 = 0$
Let the real root of equation be α
Then $(1 + 2i)\alpha^3 - 2(3 + i)\alpha^2 + (5 - 4i)\alpha + 2a^2 = 0$
Equating imaginary part zero, we get
$2\alpha^3 - 2\alpha^2 - 4\alpha = 0$
$\Rightarrow$ or $\alpha(\alpha^2 - \alpha - 2) = 0$
$\Rightarrow \ \alpha = 0$ or $\alpha = -1, 2$
Now equating real part zero
$\alpha^3 - 6\alpha^2 + 5\alpha + 2a^2 = 0$
$\alpha = 0 \ \Rightarrow a = 0$
$\alpha = -1 \ \Rightarrow a = \pm\sqrt{6}$
$\alpha = 2 \ \Rightarrow a = \pm\sqrt{3}$
$$\Rightarrow \sum a^2 = (0)^2 + \left(+\sqrt{6}\right)^2 + \left(-\sqrt{6}\right)^2 + \left(+\sqrt{3}\right)^2 + \left(-\sqrt{3}\right)^2 = 18$$

461 **(4)**
Given $a^2 - 4a + 1 = 4 \Rightarrow a^2 + 1 = 4(1 + a)$
$$y = \frac{(a - 1)(1 + a^2)}{a^2 - 1} = \frac{a^2 + 1}{a + 1} = \frac{4(a + 1)}{a + 1} = 4$$

462 **(4)**
As $P(x)$ is an odd function
Hence, $P(-x) = -P(x) \Rightarrow P(-3) = -P(3) = -6$
Let $P(x) = Q(x^2 - 9) + ax + b$
(where Q is quotient and $(ax + b) = \text{g}(x) =$ remainder)
Now $P(3) = 3a + b = 6 \quad (1)$
$P(-3) = -3a + b = -6 \quad (2)$
Hence, $b = 0$ and $a = 2$
Hence, $\text{g}(x) = 2x \Rightarrow \text{g}(2) = 4$

463 **(3)**
Let $\frac{1-a^3}{a} = \frac{1-b^3}{b} = \frac{1-c^3}{c} = k \Rightarrow \frac{1-x^3}{x} = k,$
Where x take 3 values a, b and c
$\Rightarrow x^3 + kx - 1 = 0$ has roots a, b, c
Now $a + b + c = 0 \qquad (1)$
$abc = 1 \qquad (2)$
Hence $a^3 + b^3 + c^3 = 3abc = 3$

464 **(6)**


Let $f(x) = x^2 + 2(\lambda + 1)x + \lambda^2 + \lambda + 7$
If both roots of $f(x) = 0$ are negative, then
$D = b^2 - 4ac = 4(\lambda + 1)^2 - 4(\lambda^2 + \lambda + 7) \geq 0$
$\Rightarrow \lambda - 6 \geq 0$
$\Rightarrow \lambda \in [6, \infty) \quad (1)$
Sum of roots $= -2(\lambda + 1) < 0$
$\Rightarrow \lambda \in (-1, \infty) \quad (2)$
And product of roots $= \lambda^2 + \lambda + 7 > 0 \ \forall \lambda \in R$ (3)
$\therefore$ From (1), (2), (3), we get $\lambda \in [6, \infty)$
(As (1), (2), (3) must be satisfied simultaneously)
Hence, the least value of $\lambda = 6$

465 **(2)**
We have $\left(\frac{a^4+3a^2+1}{a^2}\right)\left(\frac{b^4+5b^2+1}{b^2}\right)\left(\frac{c^4+7c^2+1}{c^2}\right)$
$$= \left(a^2 + \frac{1}{a^2} + 3\right)\left(b^2 + \frac{1}{b^2} + 5\right)\left(c^2 + \frac{1}{c^2} + 7\right)$$
$$= \left(\left(a - \frac{1}{a}\right)^2 + 5\right)\left(\left(b - \frac{1}{b}\right)^2 + 7\right)\left(\left(c - \frac{1}{c}\right)^2 + 9\right)$$

466 **(5)**
$z^2 = 81 - b^2 + 18bi$
$z^3 = 729 + 243bi - 27b^2 - b^3i$
$z^2 = z^3 \Rightarrow 243b - b^3 = 18b$ and $243 - b^2 = 18 \Rightarrow b = 15$