8.BINOMIAL THEOREM

Single Correct Answer Type

1. $\binom{30}{0}\binom{30}{10}-\binom{30}{1}\binom{30}{11}+\ldots\binom{30}{20}\binom{30}{30}$ is equal to
   a) ${}^{30}C_{11}$ b) ${}^{60}C_{10}$ c) ${}^{30}C_{10}$ d) ${}^{65}C_{55}$
2. The coefficient of $a^8b^4c^9d^9$ in $(abc+abd+acd+bcd)^{10}$ is
   a) 10! b) $\frac{10!}{8!\,4!\,9!\,9!}$ c) 2520 d) None of these
3. The value of $\sum_{r=1}^{15}\frac{r2^r}{(r+2)!}$ is equal to
   a) $\frac{(17)!-12^{16}}{(17)!}$ b) $\frac{(18)!-2^{17}}{(18)!}$ c) $\frac{(16)!-2^{15}}{(16)!}$ d) $\frac{(15)!-2^{14}}{(15)!}$
4. If $(1-x^2)^n\ \sum_{r=0}^{n}a_rx^r(1-x)^{2n-r}$, then a_r is equal to
   a) ${}^nC_r$ b) ${}^nC_r\,3^r$ c) ${}^{2n}C_r$ d) ${}^nC_r2^r$
5. The term independent of a in the expansion of $\left(1+\sqrt{a}+\frac{1}{\sqrt{a}-1}\right)^{-30}$ is
   a) ${}^{30}C_{20}$ b) 0 c) ${}^{30}C_{10}$ d) None of these
6. If $(1+2x+x^2)^n=\sum_{r=0}^{2n}a_rx^r$ then $a_r=$
   a) $({}^nC_r)^2$ b) ${}^nC_r\cdot{}^nC_{r+1}$ c) ${}^{2n}C_r$ d) ${}^{2n}C_{r+1}$
7. The value of $\sum_{r=0}^{50}(-1)^r\frac{{}^{50}C_r}{r+2}$ is equal to
   a) $\frac{1}{50\times 51}$ b) $\frac{1}{52\times 50}$ c) $\frac{1}{52\times 51}$ d) None of these
8. Maximum sum of coefficient in the expansion of $(1-x\sin\theta+x^2)^n$ is
   a) 1 b) 2^n c) 3^n d) 0
9. In the expansion of $[(1+x)/(1-x)]^2$, the coefficient of x^n will be
   a) $4n$ b) $4n-3$ c) $4n+1$ d) None of these
10. In the expansion of $\left(3^{-x/4}+3^{5x/4}\right)^n$ the sum of binomial coefficient is 64 and term with the greatest binomial coefficient exceeds the third by $(n-1)$, the value of x must be
    a) 0 b) 1 c) 2 d) 3
11. For $r=0,\ldots,10$ let A_r, B_r and C_r denotes, respectively, the coefficient of x^r in the $(1+x)^{10}, (1+x)^{20}$, and $(1+x)^{30}$. Then
    $$\sum_{r=1}^{10}A_r(B_{10}B_r-C_{10}A_r)$$
    is equal to
    a) $B_{10}-C_{10}$ b) $A_{10}(B_{10}^2-C_{10}A_{10})$
    c) 0 d) $C_{10}-B_{10}$
12. The value of ${}^{15}C_0^2-{}^{15}C_1^2+{}^{15}C_2^2-\cdots-{}^{15}C_{15}^2$ is
    a) 15 b) -15 c) 0 d) 51
13. If $a_n=\sum_{r=0}^{n}\frac{1}{{}^nC_r}$, then $\sum_{r=0}^{n}\frac{r}{{}^nC_r}$ equals
    a) $(n-1)a_n$ b) na_n c) $(1/2)na_n$ d) None of the above
14. If the coefficient of x^n in $(1+x)^{101}(1-x+x^2)^{100}$ is non-zero, then n cannot be of the form
    a) $3r+1$ b) $3r$ c) $3r+2$ d) None of these
15. The number of integral terms in the expansion of $\left(\sqrt{3}+\sqrt[8]{5}\right)^{256}$ is
    a) 33 b) 34 c) 35 d) None of these
16. $\sum_{r=0}^{300}a_rx^r=(1+x+x^2+x^3)^{100}$. If $a=\sum_{r=0}^{300}a_r$, then $\sum_{r=0}^{300}r\,a_r$ is equal to
    a) $300a$ b) $100a$ c) $150a$ d) $75a$
17. The value of $\sum_{r=1}^{n}(-1)^{r+1}\frac{{}^nC_r}{r+1}$ is equal of

a) $-\frac{1}{n+1}$ b) $-\frac{1}{n}$ c) $\frac{1}{n+1}$ d) $\frac{n}{n+1}$

18. If x is positive, the first negative term in the expansion of $(1+x)^{27/5}$ is $(|x|<1)$

a) 5^{th} term b) 8^{th} term c) 6^{th} term d) 7^{th} term

19. The value of $\sum_{r=0}^{10} r\ {}^{10}C_r 3^r(-2)^{10-r}$ is

a) 20 b) 10 c) 300 d) 30

20. The value of ${}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + {}^{20}C_3 + {}^{20}C_4 + {}^{20}C_{12} + {}^{20}C_{13} + {}^{20}C_{14} + {}^{20}C_{15}$ is

a) $2^{19} - \frac{({}^{20}C_{10} + {}^{20}C_9)}{2}$ b) $2^{19} - \frac{({}^{20}C_{10} + 2 \times {}^{20}C_9)}{2}$

c) $2^{19} - \frac{{}^{20}C_{10}}{2}$ d) None of these

21. If $|x|<1$, then $1 + n\left(\frac{2x}{1+x}\right) + \frac{n(n+1)}{2!}\left(\frac{2x}{1+x}\right)^2 + \cdots$ is equal to

a) $\left(\frac{2x}{1+x}\right)^n$ b) $\left(\frac{1+x2x}{2x}\right)^n$ c) $\left(\frac{1-x}{1+x}\right)^n$ d) $\left(\frac{1+x}{1-x}\right)^n$

22. The coefficient of x^{28} in the expansion of $(1+x^3-x^6)^{30}$ is

a) 1 b) 0 c) ${}^{30}C_6$ d) ${}^{30}C_3$

23. The number of distinct terms in the expansion of $\left(x + \frac{1}{x} + x^2 \frac{1}{x^2}\right)^{15}$ is/are (with respect to different power of x)

a) 255 b) 61 c) 127 d) None of these

24. The value of $\sum_{r=0}^{40} r\,{}^{40}C_r\,{}^{30}C_r$ is

a) $40\ {}^{69}C_{29}$ b) $40\ {}^{70}C_{30}$ c) ${}^{69}C_{29}$ d) ${}^{70}C_{30}$

25. $1 + \frac{1}{3}x + \frac{1\times4}{3\times6}x^2 + \frac{1\times4\times7}{3\times6\times9}x^3 + \ldots$ is equal to

a) x b) $(1+x)^{1/3}$ c) $(1-x)^{1/3}$ d) $(1-x)^{-1/3}$

26. The coefficient of the term independent of x in the expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}}\right)^{10}$ is

a) 210 b) 105 c) 70 d) 112

27. The total number of terms which are dependent on the value of x, in the expansion of $\left(x^2 - 2 + \frac{1}{x^2}\right)^2$ is equal to

a) $2n+1$ b) $2n$ c) n d) $n+1$

28. The value $\sum_{r=0}^{20} r(20-r)\left({}^{20}C_r\right)^2$ is equal to

a) $400\ {}^{39}C_{20}$ b) $400\ {}^{40}C_{19}$ c) $400\ {}^{39}C_{19}$ d) $400\ {}^{38}C_{20}$

29. If $C_0, C_1, C_2, \ldots, C_n$ are the binomial coefficients, then $2 \times C_1 + 2^3 \times C^3 + 2^5 \times C_5 + \cdots$ equals

a) $\frac{3^n + (-1)^n}{2}$ b) $\frac{3^n - (-1)^n}{2}$ c) $\frac{3^n + 1}{2}$ d) $\frac{3^n - 1}{2}$

30. If the coefficients of three consecutive terms in the expansion of $(1+x)^n$ are in the ration 1:7:42, then the value of n is

a) 60 b) 70 c) 55 d) None of these

31. The number of real negative terms in the binomial expansion of $(1+ix)^{4n-2}, n \in N, x > 0$ is

a) n b) $n+1$ c) $n-1$ d) $2n$

32. The sum of $1 + n\left(1 - \frac{1}{x}\right) + \frac{n(n+1)}{2!}\left(1 - \frac{1}{x}\right)^2 + \cdots\infty$ will be

a) x^n b) x^{-n} c) $\left(1 - \frac{1}{x}\right)^n$ d) None of these

33. For $2 \le r \le n$, $\binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2} =$

a) $\binom{n+1}{r-1}$ b) $2\binom{n+1}{r+1}$ c) $2\binom{n+1}{r}$ d) $\binom{n+2}{r}$

34. The sum of series ${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \cdots + {}^{20}C_{10}$ is

a) $\frac{1}{2}\ {}^{20}C_{10}$ b) 0 c) ${}^{20}C_{10}$ d) $-{}^{20}C_{10}$

35. The coefficient of $x^r [0 \le r \le (n-1)]$ in the expansion of $(x+3)^{n-1} + (x+3)^{n-2}(x+2) + (x+$ $3n-3x+22+...+x+2n-1$ are

a) ${}^nC_r(3^r - 2^n)$ b) ${}^nC_r(3^{n-r} - 2^{n-r})$ c) ${}^nC_r(3^r + 2^{n-r})$ d) None of these

36. If $(3 + x^{2008} + x^{2009})^{2010} = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$, then the value of $a0 - \frac{1}{2}a_1 - \frac{1}{2}a_2 + a_3 - \frac{1}{2}a_4 - \frac{1}{2}a_5 + a_6 - \cdots$ is

a) 3^{2010} b) 1 c) 2^{2010} d) None of these

37. $(n+2)\,{}^nC_0 2^{n+1} - (n+1)\,{}^nC_1 2^n + n\,{}^nC_2 2^{n-1} - \cdots$ is equal to

a) 4 b) $4n$ c) $4(n+1)$ d) $2(n+2)$

38. The coefficient of x^{53} in the expansion $\sum_{m=0}^{100} 100\, C_m (x-3)^{100-m} 2^m$ is

a) ${}^{100}C_{47}$ b) ${}^{100}C_{53}$ c) $-{}^{100}C_{53}$ d) $-{}^{100}C_{100}$

39. The fractional part of $2^{4n}/15$ is $(n \in N)$

a) $\frac{1}{15}$ b) $\frac{2}{15}$ c) $\frac{4}{15}$ d) None of these

40. If $(1 + 2x + 3x^2)^{10} = a_0 + a_1x + a_2x^2 + \ldots + a_{20}x^{20}$, then a_1 equals

a) 10 b) 20 c) 210 d) None of these

41. The approximate value of $(1.0002)^{3000}$ is

a) 1.6 b) 1.4 c) 1.8 d) 1.2

42. $[({}^nC_0 + {}^nC_3 + \ldots) - 1/2({}^nC_1 + {}^nC_2 + {}^nC_4 + {}^nC_5 + \ldots)]^2 + 3/4({}^nC_1 - {}^nC_2 + {}^nC_4 - {}^nC_5 + \ldots)^2 =$

a) 3 b) 4 c) 2 d) 1

43. The last two digits of the number 3^{400} are

a) 81 b) 43 c) 29 d) 01

44. $\sum_{k=1}^{\infty} k\left(1 - \frac{1}{n}\right)^{k-1} =$

a) $n(n-1)$ b) $n(n+1)$ c) n^2 d) $(n+1)^2$

45. If the coefficients of r^{th}, $(r+1)^{\text{th}}$ and $(r+2)^{\text{th}}$ terms in the binomial expansion of $(1+y)^m$ are in AP., then m and r satisfy the equation

a) $m^2 - m(4r+1) + 4r^2 + 2 = 0$ b) $m^2 - m(4r-1) + 4r^2 - 2 = 0$

c) $m^2 - m(4r-1) + 4r^2 + 2 = 0$ d) $m^2 - m(4r+1) + 4r^2 - 2 = 0$

46. If $|x| < 1$, then the coefficient of x^n in expansion of $(1 + x + x^2 + x^3 + \ldots)^2$ is

a) n b) $n-1$ c) $n+2$ d) $n+1$

47. If $(1-x)^{-n} = a_0 + a_1x + a_2x^2 + \ldots + a_rx^r + \ldots$, then $a_0 + a_1 + a_2 + \ldots + a_r$ is equal to

a) $\frac{n(n+1)(n+2)\cdots(n+r)}{r!}$ b) $\frac{(n+1)(n+2)\cdots(n+r)}{r!}$

c) $\frac{n(n+1)(n+2)\cdots(n+r-1)}{r!}$ d) None of these

48. The value of x for which the sixth term in the expansion of $\left[2^{\log_2 \sqrt{9^{x-1}+7}} + \frac{1}{2^{\frac{1}{5}\log_2(3^{x-1}+1)}}\right]^7$ is 84 is

a) 4 b) 1 or 2 c) 0 or 1 d) 3

49. In the expansion of $\left(5^{1/2} + 7^{1/8}\right)^{1024}$, the number of integral terms is

a) 128 b) 129 c) 130 d) 131

50. If $\frac{x^2+x+1}{1-x} = a_0 + a_1x + a_2x^2 + \ldots$, then $\sum_{r=1}^{50} a_r$ is equal to

a) 148 b) 146 c) 149 d) None of these

51. If $p = \left(8 + 3\sqrt{7}\right)^n$ and $f = p - [p]$, where $[\cdot]$ denotes the greatest integer function, then the value of $p(1-f)$ is equal to

a) 1 b) 2 c) 2^n d) 2^{2n}

52. The value of $\sum_{r=1}^{n+1}\left(\sum_{k=1}^{n} {}^kC_{r-1}\right)$ (where $r, k, n \in N$) is equal to

a) $2^{n+1} - 2$ b) $2^{n+1} - 1$ c) 2^{n+1} d) None of these

53. Value of $\sum_{k=1}^{\infty}\sum_{r=0}^{k} \frac{1}{3^k}\left({}^kC_r\right)$ is

a) $\frac{2}{3}$ b) $\frac{4}{3}$ c) 2 d) 1

54. The coefficient of x^4 in $(x/2-3/x^2)^{10}$ is
a) $\frac{405}{256}$ b) $\frac{504}{259}$ c) $\frac{450}{263}$ d) None of these

55. The coefficient of the middle term in the binomial expansion in power of x of $(1-\alpha x)^4$ and of $(1-\alpha x)^6$ is the same, if α equals
a) $-\frac{5}{3}$ b) $\frac{10}{3}$ c) $-\frac{3}{10}$ d) $\frac{3}{5}$

56. If x^m occurs in the expansion of $(x+1/x^2)^{2n}$, then the coefficient of x^m is
a) $\frac{(2n)!}{(m)!\,(2n-m)!}$ b) $\frac{(2n)!\,3!\,3!}{(2n-m)!}$ c) $\frac{(2n)!}{\left(\frac{2n-m}{3}\right)!\left(\frac{4n+m}{3}\right)!}$ d) None of these

57. The last two digits of the number $(23)^{14}$ are
a) 01 b) 03 c) 09 d) None of these

58. If $(1+x-2x^2)^6=1+a_1x+a_2x^2+a_3x^3+\ldots$, then the value of $a_2+a_4+a_6+\ldots+a_{12}$ will be
a) 32 b) 31 c) 64 d) 1024

59. If $n-{}^1C_r=(k^2-3)\,{}^nC_{r+1}$, then $k\in$
a) $(-\infty,-2)$ b) $[2,\infty)$ c) $[-\sqrt{3},\sqrt{3}]$ d) $(\sqrt{3},2]$

60. The coefficient of x^{10} in the expansion of $(1+x^2-x^3)^8$ is
a) 476 b) 496 c) 506 d) 528

61. If the 6th term in the expansion of $\left(\frac{1}{x^{8/3}}+x^2\log_{10}x\right)^8$ is 5600, then x equals
a) 1 b) $\log_e 10$ c) 10 d) x does not exist

62. The expression $\left(\sqrt{2x^2+1}+\sqrt{2x^2-1}\right)^6+\left(\frac{2}{\sqrt{2x^2+1}+\sqrt{2x^2-1}}\right)^6$ is a polynomial of degree
a) 6 b) 8 c) 10 d) 12

63. If the last term in the binomial expansion of $\left(2^{1/3}-\frac{1}{\sqrt{2}}\right)^n$ is $\left(\frac{1}{3^{5/3}}\right)^{\log_3 8}$, then the 5th term from the beginning is
a) 210 b) 420 c) 105 d) None of these

64. $1+\frac{1}{4}+\frac{1\times 3}{4\times 8}+\frac{1\times 3\times 5}{4\times 8\times 12}+\cdots=$
a) $\sqrt{2}$ b) $\frac{1}{\sqrt{2}}$ c) $\sqrt{3}$ d) $\frac{1}{\sqrt{3}}$

65. If the coefficients of 5th, 6th and 7th terms in the expansion of $(1+x)^n$ be in A.P., then $n=$
a) 7 only b) 14 only c) 7 or 14 d) None of these

66. The coefficient of x^n in the expansion of $(1-x)(1-x)^n$ is
a) $n-1$ b) $(-1)^n(1+n)$ c) $(-1)^{n-1}(n-1)^2$ d) $(-1)^{n-1}n$

67. The expression $\left(x+(x^3-1)^{\frac{1}{2}}\right)^5+\left(x+(x^3+1)^{\frac{1}{2}}\right)^5$ is a polynomial of degree
a) 5 b) 6 c) 7 d) 8

68. The sum of rational term in $\left(\sqrt{2}+\sqrt[3]{3}+\sqrt[6]{5}\right)^{10}$ is equal to
a) 12632 b) 1260 c) 126 d) None of these

69. If C_r stands for ${}^nC_r$, then the sum for the series $\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}[C_0^2-2C_1^2+3C_2^2-\cdots+(-1)^n(n+1)C_n^2]$, where n is an even positive integer is equal to
a) 0 b) $(-1)^{n/2}(n+1)$ c) $(-1)^n(n+2)$ d) $(-1)^n n$

70. The coefficient of x^5 in the expansion of $(1+x)^{21}+(1+x)^{22}+\cdots+(1+x)^{30}$ is
a) ${}^{51}C_5$ b) ${}^9C_5$ c) ${}^{31}C_6-{}^{21}C_6$ d) ${}^{30}C_5+{}^{20}C_5$

71. If $f(x)=1-x+x^2-x^3+\cdots-x^{15}+x^{16}-x^{17}$, then the coefficient of x^2 in $f(x-1)$ is
a) 826 b) 816 c) 822 d) None of these

72. The coefficient of $1/x$ in the expansion of $(1+x)^m(1+1/x)^n$ is

a) $\frac{n!}{(n-1)!\,(n+1)!}$ b) $\frac{(2n)!}{(n-1)!\,(n+1)!}$ c) $\frac{(2n)!}{(2n-1)!\,(2n+1)!}$ d) None of these

73. In the expansion of $(1+3x+2x^2)^6$, the coefficient of x^{11} is

a) 144 b) 288 c) 216 d) 576

74. The coefficient of x^5 in the expansion of $(x^2-x-2)^5$ is

a) -83 b) -82 c) -86 d) -81

75. If in the expansion of $(1+x)^n, a, b, c$ are three consecutive coefficients, then $n=$

a) $\frac{ac+ab+bc}{b^2+ac}$ b) $\frac{2ac+ab+bc}{b^2-ac}$ c) $\frac{ab+bc}{b^2-ac}$ d) None of these

76. Let $f(x)=a_0+a_1x+a_2x^2+\cdots+a_nx^n+\cdots$ and $\frac{f(x)}{1-x}=b_0+b_1x+b_2x^2+\cdots+b_nx^n+\cdots$, then

a) $b_n+b_{n-1}=a_n$ b) $b_n-b_{n-1}=a_n$ c) $b_n/b_{n-1}=a_n$ d) None of these

77. If in the expansion of $(a-2b)^n$, the sum of 5^{th} and 6^{th} terms in 0, then the values of $a/b=$

a) $\frac{n-4}{5}$ b) $\frac{2(n-4)}{5}$ c) $\frac{5}{n-4}$ d) $\frac{5}{2(n-4)}$

78. If x is so small that x^3 and higher powers of x may be neglected, then $\frac{(1+x)^{3/2}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}}$ may be approximated as

a) $3x+\frac{3}{8}x^2$ b) $1-\frac{3}{8}x^2$ c) $\frac{x}{2}-\frac{3}{x}x^2$ d) $-\frac{3}{8}x^2$

79. The sum of the coefficients of even power of x in the expansion $(1+x+x^2+x^3)^5$ is

a) 256 b) 128 c) 512 d) 64

80. ${}^{404}C_4-{}^4C_1\,{}^{303}C_4+{}^4C_2\,{}^{202}C_4-{}^4C_3\,{}^{101}C_4$ is equal to

a) $(401)^4$ b) $(101)^4$ c) 0 d) $(201)^4$

81. If n is an integer between 0 and 21, then the minimum value of $n!\,(21-n)!$ is attained for $n=$

a) 1 b) 10 c) 12 d) 20

82. The value of $\frac{{}^nC_0}{n}+\frac{{}^nC_1}{n+1}+\frac{{}^nC_2}{n+2}+\cdots+\frac{{}^nC_n}{2n}$ is equal to

a) $\int_0^1 x^{n-1}(1-x)^n dx$ b) $\int_1^2 x^n(x-1)^{n-1}dx$ c) $\int_1^2 x^{n-1}(1+x)^n dx$ d) $\int_0^1 (1-x)^n x^{n-1}dx$

83. If the term independent of x in the $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$ is 405, then k equals

a) $2,-2$ b) $3,-3$ c) $4,-4$ d) $1,-1$

84. $(1+x)^n=C_0+C_1x+C_2x^2+\cdots+C_nx^n$ then $C_0C_2+C_1C_3+C_2C_4+\cdots+C_{n-2}C_n=$

a) $\frac{(2n)!}{(n!)^2}$ b) $\frac{(2n)!}{(n-1)!\,(n+1)!}$ c) $\frac{(2n)!}{(n-2)!\,(n+2)!}$ d) None of these

85. If the sum of the coefficients in the expansion of $(1-3x+10x^2)^n$ is a and if the sum of the coefficients in the expansion of $(1+x^2)^n$ is b, then

a) $a=3b$ b) $a=b^3$ c) $b=a^3$ d) None of these

86. Given positive integers $r>1, n>2$ and that the coefficient of $(3r)^{\text{th}}$ and $(r+2)^{\text{th}}$ terms in the binomial expansion of $(1+x)^{2n}$ are equal. Then

a) $n=2r$ b) $n=2r+1$ c) $n=3r$ d) None of these

87. If the coefficient of x^7 in $\left[ax^2+\left(\frac{1}{bx}\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax^2-\left(\frac{1}{bx^2}\right)\right]^{11}$, then a and b satisfy the relation

a) $a+b=1$ b) $a-b=1$ c) $ab=1$ d) $\frac{a}{b}=1$

88. The value of $\binom{30}{0}\binom{30}{10}-\binom{30}{1}\binom{30}{11}+\binom{30}{2}\binom{30}{12}+\cdots+\binom{30}{20}\binom{30}{30}=$

a) ${}^{60}C_{20}$ b) ${}^{30}C_{10}$ c) ${}^{60}C_{30}$ d) ${}^{40}C_{30}$

89. If the expansion in powers of x of the function $1/[(1-ax)(1-bx)]$ is $a_0+a_1x+a_2x^2+a_2x^3+\cdots$, then a_n is

a) $\frac{b^n - a^n}{b-a}$ b) $\frac{a^n - b^n}{b-a}$ c) $\frac{a^{n+1} - b^{n+1}}{b-a}$ d) $\frac{b^{n+1} - a^{n+1}}{b-a}$

90. The coefficient of x^5 in $(1+2x+3x^2+\ldots)^{-3/2}$ is $(|x|<1)$
a) 21 b) 25 c) 26 d) None of these

91. The value of $\sum_{r=0}^{10}(r)\ {}^{20}C_r$ is equal to
a) $20(2^{18}+\ {}^{19}C_{10})$ b) $10(2^{18}+\ {}^{19}C_{10})$ c) $20(2^{18}+\ {}^{19}C_{11})$ d) $10(2^{18}+\ {}^{19}C_{11})$

92. The coefficient of x^2y^3 in the expansion of $(1-x+y)^{20}$ is
a) $\frac{20!}{2!\,3!}$ b) $-\frac{20!}{2!\,3!}$ c) $\frac{20!}{5!\,2!\,3!}$ d) None of these

93. 'p' is a prime number and $n<p<2n$. If $N=\ {}^{2n}C_n$, then
a) p divides N b) p^2Divides N c) p cannot divide N d) None of these

94. If ${}^{n+1}C_{r+1}:{}^{n}C_r:{}^{n-1}C_{r-1}=11:6:3$, then $nr=$
a) 20 b) 30 c) 40 d) 50

95. In the binomial expansion of $(a-b)^n n\geq 5$, the sum of the 5th and 6th terms is zero. Then a/b equals
a) $(n-5)/6$ b) $(n-4)/5$ c) $n/(n-4)$ d) $6/(n-5)$

96. If the coefficients of r^{th} and $(r+1)^{\text{th}}$ terms in the expansion of $(3+7x)^{29}$ are equal, then r equals
a) 15 b) 21 c) 14 d) None of these

97. If $f(x)=x^n$, then the value of $f(1)+\frac{f^1(1)}{1}+\frac{f^2(1)}{2!}+\cdots+\frac{f^n(1)}{n!}$, where $f^r(x)$ denotes the r^{th} order derivative of $f(x)$ with respect to x is
a) n b) 2^n c) 2^{n-1} d) None of these

98. In the expansion of $(1+x+x^3+x^4)^{10}$, the coefficient of x^4 is
a) ${}^{40}C_4$ b) ${}^{10}C_4$ c) 210 d) 310

99. The coefficient of x^4 in the expansion of $\left\{\sqrt{1+x^2}-x\right\}^{-1}$ in ascending powers of x, when $|x|<1$, is
a) 0 b) $\frac{1}{2}$ c) $-\frac{1}{2}$ d) $-\frac{1}{8}$

100. If the sum of the coefficients in the expansion of $(a+b)^n$ is 4096, then the greatest coefficient in the expansion is
a) 924 b) 792 c) 1594 d) None of these

Multiple Correct Answers Type

101. If $(1+x)^n=C_0+C_1x+C_2x^2+\cdots+C_nx^n$, then $C_0-(C_0+C_1)+(C_0+C_1+C_2)-(C_0+C_1+C_2+C_3)+\cdots+(-1)^{n-1}(C_0+C_1+\cdots+C_{n-1})$, where n is even integer is
a) A positive value b) A negative value c) Divisible by 2^{n-1} d) Divisible by 2^n

102. The last digit of $3^{3^{4n}}+1, n\in N$, is
a) ${}^4C_3$ b) ${}^8C_7$ c) 8 d) 4

103. In the expansion of $(2-2x+x^2)^9$
a) Number of distinct terms is 10
b) Coefficient of x^4 is 97
c) Sum of coefficients is 1
d) Number of distinct terms is 55

104. Which of the following is/are correct?
a) $101^{50}-99^{50}>100^{50}$ b) $101^{50}-100^{50}>99^{50}$
c) $(1000)^{1000}>(1001)^{999}$ d) $(1001)^{999}>(1000)^{1000}$

105. For which of the following values of x, 5^{th} term is the numerically greatest term in the expansion of $(1+x/3)^{10}$
a) -2 b) 1.8 c) 2 d) -1.9

106. The middle term in the expansion of $(x/2+2)^8$ is 1120; then $x\in R$ is equal to
a) -2 b) 3 c) -3 d) 2

107. The sum of the coefficient in the expansion of $(1+ax-2x^2)^n$ is

a) Positive, when $a<1$ and $n=2k, k\in N$
b) Negative, when $a<1$ and $n=2k+1, k\in N$
c) Positive, when $a>1$ and $n\in N$
d) Zero, when $a=1$

108. For natural numbers m, n if $(1-y)^m(1+y)^n = 1+a_1y+a_2y^2+\ldots$, and $a_1=a_2=10$, then

a) $m<n$
b) $m>n$
c) $m+n=80$
d) $m-n=20$

109. If the coefficients of r^{th}, $(r+1)^{\text{th}}$ and $(r+2)^{\text{th}}$ terms in the expansion of $(1+x)^{14}$ are in AP., then r is/are

a) 5
b) 12
c) 10
d) 9

110. In the expansion of $\left(7^{1/3}+11^{1/9}\right)^{6561}$,

a) There are exactly 730 rational terms
b) There are exactly 5831 irrational terms
c) The term which involves greatest binomial coefficients is irrational
d) The term which involves greatest binomial coefficients is rational

111. The number of values of r satisfying the equation ${}^{69}C_{3r-1} - {}^{69}C_{r^2} = {}^{69}C_{r^2-1} - {}^{69}C_{3r}$ is

a) 1
b) 2
c) 3
d) 7

112. In the expansion of $\left(x^2+1+\frac{1}{x^2}\right)^n, n\in N$,

a) Number of terms is $2n+1$
b) Coefficient of constant term is 2^{n-1}
c) Coefficient of x^{2n-2} is n
d) Coefficient of x^2 in n

113. For the expansion $(x\sin p + x^{-1}\cos p)^{10}, (p\in R)$,

a) The greatest value of the term independent of x is $10!\,2^5(5!)^2$
b) The least value of sum of coefficient is zero
c) The greatest value of sum coefficient is 32
d) The least value of the term independent of x occurs when $p=(2n+1)\frac{\pi}{4}, n\in \mathcal{Z}$

114. If $\left(4+\sqrt{15}\right)^n = I+f$, where n is an odd natural number, I is an integer and $0<f<1$, then

a) I is an odd integer
b) I is an even integer
c) $(I+f)(1-f)=1$
d) $1-f=\left(4-\sqrt{5}\right)^n$

115. If the 4th term in the expansion of $(ax+1/x)^n$ is 5/2, then

a) $a=\frac{1}{2}$
b) $n=8$
c) $a=\frac{2}{3}$
d) $n=6$

116. If $f(m)=\sum_{i=0}^{m}\binom{30}{30-i}\binom{20}{m-i}$

Where $\binom{p}{q} = {}^{p}C_q$, then

a) Maximum value of $f(m)$ is ${}^{50}C_{25}$
b) $f(0)+f(1)+\ldots+f(50)=2^{50}$
c) $f(m)$ is always divisible by 50 $(1\le m\le 49)$
d) The value of $\sum_{m=0}^{50}\left(f(m)\right)^2 = {}^{100}C_{50}$

117. If $(1+x)^n = C_0+C_1x+C_2x^2+\cdots+C_nx^n, n\in N$, then $C_0-C_1+C_2-\cdots+(-1)^{m-1}C_{m-1}$ is equal to $(m<n)$

a) $\frac{(n-1)(n-2)\cdots(n-m+1)}{(m-1)!}(-1)^{m-1}$
b) ${}^{n-1}C_{m-1}(-1)^{m-1}$
c) $\frac{(n-1)(n-2)\cdots(n-m)}{(m-1)!}(-1)^{m-1}$
d) ${}^{n-1}C_{n-m}(-1)^{m-1}$

118. Let $(1+x^2)^2(1+x)^n = \sum_{k=0}^{n+4} a_kx^k$. If a_1, a_2 and a_3 are in arithmetic progression, then the possible value/values of n is/are

a) 5
b) 4
c) 3
d) 2

119. If for z as real or complex, $(1+z^2+z^4)^8 = C_0+C_1z^2+C_2z^4+\ldots+C_{16}z^{32}$, then

a) $C_0-C_1+C_2-C_3+\ldots+C_{16}=1$
b) $C_0+C_3+C_6+C_9+C_{12}+C_{15}=3^7$
c) $C_2+C_5+C_8+C_{11}+C_{14}=3^6$
d) $C_1+C_4+C_7+C_{10}+C_{13}+C_{16}=3^7$

120. 10th term of $\left(3-\sqrt{\frac{17}{4}+3\sqrt{2}}\right)^{20}$

a) An irrational number
b) A rational number
c) A positive integer
d) A negative integer

121. In the expansion of $(x+a)^n$ if the sum of odd terms be P and sum of even terms be Q, then

a) $P^2 - Q^2 = (x^2 - a^2)^n$
b) $4PQ - (x+a)^{2n} - (x-a)^{2n}$
c) $2(P^2 + Q^2) = (x+a)^{2n} + (x-a)^{2n}$
d) None of these

122. If n is a positive integer and if $(1 + x + x^2)^n = \sum_{r=0}^{2n} a_r x^r$, then
a) $a_1 = a_{2n-\mathrm{r}}$, for $0 \le r \le 2n$
b) $a_0 + a_1 + \ldots + a_{n-1} = \frac{1}{2}(3^n - a_n)$
c) $a_0^2 - a_1^2 + a_2^2 - a_3^2 + \ldots + a_{2n}^2 = a_n$
d) $a_0 + a_1 + \ldots + a_{2n} = \frac{1}{2}(3^n + 1)$

123. The value of ${}^{n}C_1 + {}^{n+1}C_2 + {}^{n-2}C_3 + \cdots + {}^{n+m-1}C_m$ is equal to
a) ${}^{m+\mathrm{n}}C_{n-1}$
b) ${}^{m+n}C_{n-1}$
c) ${}^{m}C_1 + {}^{m+1}C_2 + {}^{m+2}C_3 + \cdots + {}^{m+n-1}C_n$
d) ${}^{m+n}C_{m-1}$

124. The value/values of x in the expression $\left(x + x^{\log_{10} x}\right)^5$ if the third term in the expansion is 10,00,000 is /are
a) 10
b) 100
c) $10^{-5/2}$
d) $10^{-3/2}$

125. The number $101^{100} - 1$ is divisible by
a) 100
b) 1000
c) 10000
d) 100000

Assertion - Reasoning Type

This section contain(s) 0 questions numbered 126 to 125. Each question contains STATEMENT 1(Assertion) and STATEMENT 2(Reason). Each question has the 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

a) Statement 1 is True, Statement 2 is True; Statement 2 **is** correct explanation for Statement 1

b) Statement 1 is True, Statement 2 is True; Statement 2 **is not** correct explanation for Statement 1

c) Statement 1 is True, Statement 2 is False

d) Statement 1 is False, Statement 2 is True

126 Let $S_1 = \sum_{j=1}^{10} j(j-1)^{10}C_j, S_2 = \sum_{j=1}^{10} j\ 10_{C_j}$ and $S_3 = \sum_{j=1}^{10} j^2\ {}^{10}C_j$

Statement 1: $S_3 = 55 \times 2^9$

Statement 2: $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$

127

Statement 1: If n is an odd prime, then the integral part of $\left(\sqrt{5} + 2\right)^n - 2^{n+1}$ is divisible by $2n$)

Statement 2: If n is prime, then ${}^{n}C_1, {}^{n}C_2, \ldots, {}^{n}C_{n-1}$ must be divisible by n

128

Statement 1: The coefficient of x^n in the binomial expansion of $(1-x)^{-2}$ is $(n+1)$

Statement 2: The coefficient of x^r in $(1-x)^{-n}$ when $n \in N$ is ${}^{n+r-1}C_r$

129

Statement 1: The coefficient of $x^{3\lambda+2}$ in the expansion of $(a+x)^{\lambda}(b+x)^{\lambda+1}(c+x)^{\lambda+2}\ \forall \lambda \in N$ is $\lambda(a+b+c)$

Statement 2: The coefficient of x^m in the expansion $(a+x)^n$ is ${}^nC_m a^{n-m}$

130

Statement 1: The sum of coefficients in the expansion of $\left(3^{-x/4}+3^{5x/4}\right)^n$ is 2^n

Statement 2: The sum of coefficients in the expansion of $(x+y)^n$ is 2^n when we put $x=y=1$

131

Statement 1: The term independent of x in the expansion of $\left(x+\frac{1}{x}+2\right)^m$ is $\frac{(4m)!}{(2m!)^2}$

Statement 2: The coefficient of x^6 in the expansion $(1+x)^n$ is ${}^nC_6$

132

Statement 1: The number of terms in the expansion $\left(x+\frac{1}{x}+1\right)^n$ is $2n+1$

Statement 2: The number of terms in the expansion $(a_1+a_2+a_3+\ldots+a_m)^n$ is ${}^{n+m-1}C_{m-1}$

133

Statement 1: $\sum_{0\le i<}\sum_{j\le n}\left(\frac{i}{{}^nC_i}+\frac{j}{{}^nC_j}\right)$ is equal to $\frac{n^2}{2}a$, where $a=\sum_{r=0}^{n}\frac{1}{{}^nC_r}$

Statement 2: $$\sum_{r=0}^{n}\frac{r}{{}^nC_r}=\sum_{r=0}^{n}\frac{n-r}{{}^nC_r}$$

134

Statement 1: Three consecutive binomial coefficients are always in A.P.

Statement 2: Three consecutive binomial coefficients are not in H.P. or G.P.

135

Statement 1: The total number of dissimilar terms in the expansion of $(x_1+x_2+\cdots+x_n)^3$ is $\frac{n(n+1)(n+2)}{6}$

Statement 2: The total number of dissimilar terms in the expansion of $(x_1+x_2+x_3)^n$ is $\frac{n(n+1)(n+2)}{6}$

136

Statement 1: If p is a prime number $(p\ne 2)$, then $\left[\left(2+\sqrt{5}\right)^p\right]-2^{p+1}$ is always divisible by p (where [.] denotes the greatest integer function)

Statement 2: If n is prime, then ${}^nC_1$, ${}^nC_2$, ${}^nC_3,\cdots,$ ${}^nC_{n-1}$ must be divisible by n

137

Statement 1: $3^{2n+2}-8n-9$ is divisible by $64, \forall\, n\in N$

Statement 2: $(1+x)^n-nx-1$ is divisible by $x^2, \forall\, n\in N$

138

Statement 1: The coefficient of x^n in $\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots+\frac{x^n}{n!}\right)$ is $\frac{3^n}{n!}$

Statement 2: The coefficient of x^n in e^{3x} is $\frac{3^n}{n!}$

139

Statement 1: The value of $({}^{10}C_0) + ({}^{10}C_0 + {}^{10}C_1) + ({}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2) + \cdots + ({}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + \cdots + {}^{10}C_9)$ is $10 \cdots 2^9$

Statement 2: ${}^{n}C_1 + 2\,{}^{n}C_2 + 3\,{}^{n}C_3 + \cdots + n\,{}^{n}C_n = n2^{n-1}$

140

Statement 1: For every natural number $n \geq 2$.
$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} > \sqrt{n}$

Statement 2: For every natural number $n \geq 2$
$\sqrt{n(n+1)} < n + 1$

141

Statement 1: In the expansion of $(1+x)^{41}(1-x+x^2)^{40}$, the coefficient of x^{85} is zero

Statement 2: In the expansion of $(1+x)^{41}$ and $(1-x+x^2)^{40}$, x^{85} term does not occur

142

Statement 1: ${}^{m}C_r + {}^{m}C_{r-1}\,{}^{n}C_1 + {}^{m}C_{r-2}\,{}^{n}C_2 + \cdots + {}^{n}C_r = 0$, if $m + n < r$

Statement 2: ${}^{n}C_r = 0$ if $n < r$

143

Statement 1: The number of distinct terms in $(1 + x + x^2 + x^3 + x^4)^{1000}$ is 4001

Statement 2: The number of distinct terms in the expansion $(a_1 + a_2 + \cdots + a_m)^n$ is ${}^{n+m-1}C_{m-1}$

144 Let n be a positive integer and k be a whole number, $k \leq 2n$

Statement 1: The maximum value of ${}^{2n}C_k$ is ${}^{2n}C_n$

Statement 2: $\frac{{}^{2n}C_{k+1}}{{}^{2n}C_k} < 1$, for $k = 0,1,2,\ldots,n-1$ and $\frac{{}^{2n}C_k}{{}^{2n}C_{k-1}} > 1$ for $k = n+1, n+2, \ldots, 2n$

145

Statement 1: If $\sum_{r=1}^{n} r^3 \left(\frac{{}^{n}C_r}{{}^{n}C_{r-1}}\right) = 196$, then the sum of the coefficients of power x in the expansion of the polynomial $(x - 3x^2 + x^3)^n$ is -1

Statement 2: $\frac{{}^{n}C_r}{{}^{n}C_{r-1}} = \left(\frac{n-r+1}{r}\right) \forall n \in N$ and $r \in W$

146

Statement 1: Remainder when 3456^{2222} is divided by 7 is 4

Statement 2: Remainder when 5^{2222} is divided 7 is 4

147 In the expansion of $(1 + x + x^2 + x^3)^6$, then coefficient of x^{14} is

Statement 1: 130

Statement 2: 120

148

Statement 1: Greatest term in the expansion of $(1+x)^{12}$, when $x = 11/10$ is 7^{th}

Statement 2: 7^{th} term in the expansion of $(1+x)^{12}$ has the factor ${}^{12}C_6$ which is greatest value of ${}^{12}C_r$

149

Statement 1: $\sum_{r=0}^{n} (r+1)\cdot {}^{n}C_r = (n+2)2^{n-1}$

Statement 2: $\sum_{r=0}^{n} (r+1)\, {}^{n}C_r \cdot x^r = (1+x)^n + nx(1+x)^{n-1}$

150 The height of a communication satellite. (G=6.67 $\times 10^{-11}$ N m^2/kg^2) ,(M=5.98$\times 10^{24}$ $\times$kg , R=6.4$\times 10^{6}$ m,)

Statement 1: 35850 km

Statement 2: 3585 km

151

Statement 1: If $n \in N$ and 'n' is not a multiple of 3 and $(1+x+x^2)^n = \sum_{r=0}^{2n} a_r x^r$, then the value of $\sum_{r=0}^{n}(-1)^r a_r\, {}^{n}C_r$ is zero

Statement 2: The coefficient of x^n in the expansion of $(1-x^3)$ is zero, if $n = 3k+1$ or $n = 3k+2$

152

Statement 1: If n is even, then ${}^{2n}C_1 + {}^{2n}C_3 + \ldots + {}^{2n}C_{n-1} = 2^{2n-1}$

Statement 2: ${}^{2n}C_1 + {}^{2n}C_3 + \ldots + {}^{2n}C_{2n-1} = 2^{2n-1}$

Matrix-Match Type

This section contain(s) 0 question(s). Each question contains Statements given in 2 columns which have to be matched. Statements (A, B, C, D) in **columns I** have to be matched with Statements (p, q, r, s) in **columns II**.

153.

Column-I	Column- II
(A) $\sum\sum_{i\neq j} {}^{10}C_i\, {}^{10}C_j$	(p) $\frac{2^{20} - {}^{20}C_{10}}{2}$
(B) $\sum_{0\le i}\sum_{\le j \le n} {}^{10}C_i\, {}^{10}C_j$	(q) $2^{20} - {}^{20}C_{10}$
(C) $\sum_{0\le i}\sum_{< j \le n} {}^{10}C_i\, {}^{10}C_j$	(r) 2^{20}
(D) $\sum_{i=0}^{10}\sum_{j=0}^{10} {}^{10}C_i\, {}^{10}C_j$	(s) $\frac{2^{20} + {}^{20}C_{10}}{2}$

CODES :

A **B** **C** **D**

a)	p	q	r	s
b)	q	s	p	r
c)	s	r	q	p
d)	r	p	s	q

154.

	Column-I		Column- II
(A)	The sum of binomial coefficients of terms containing power of x more than x^{20} in $(1+x)^{41}$ is divisible by	(p)	2^{39}
(B)	The sum of binomial coefficients of rational terms in the expansion of $\left(1+\sqrt{2}\right)^{42}$ is divisible by	(q)	2^{40}
(C)	Id $\left(x+\frac{1}{x}+x^2+\frac{1}{x^2}\right)^{21}=a_0x^{-42}+a_1x^{-41}+a_2x^{-40}+\cdots a_{84}x^{42}$, then $a_0+a_2+\cdots+a_{84}$ is divisible by	(r)	2^{41}
(D)	The sum of binomial coefficients of positive real terms in the expansion of $(1+ix)^{42}(x>$ 0 is divisible by	(s)	2^{38}

CODES :

	A	B	C	D
a)	Q,s,p	r,s,p,q	r,s,p,q	q,s,p
b)	s,p,q	s,p,q,r	s,p,q,r	s,p,q
c)	p,s,q	q,r,s,p	q,r,s,p	p,s,q
d)	p,q,s	p,q,r,s	p,q,r,s	p,q,s

155.

	Column-I		Column- II
(A)	If ${}^{(n+1)}C_4+{}^{(n+1)}C_3+{}^{(n+2)}C_3>{}^{(n+3)}C_3$, then possible value/values of n is/are	(p)	4
(B)	The remainder when $(3053)^{456}-(2417)^{333}$ is divided by 9 is less than	(q)	5
(C)	The digit in the unit place of the number $183!+3^{183}$ is greater than	(r)	6
(D)	If sum of the coefficients of the first, second and third terms of the expansion of $\big(x^2+$ $1xm$is 46, then the index of the term that does not contain x is greater than	(s)	7

CODES :

	A	B	C	D

a)	R,s,q	q,r,s,p	q,r,p	q,p
b)	s,q,r	r,s,p,q	r,p,q	q,p
c)	r,s	s,r,p,q	q,r	q,p
d)	q,r,s	p,q,r,s	p,q,r	p,q

156. The correct matching of List I from List II

	Column-I		**Column- II**
(A)	$(1-x)^{-n}$	(1)	$\frac{x}{x+1}$
(B)	$(1+n)^{-n}$	(2)	$1-nx+\frac{n(n+1)}{2!}x^2-\cdots$ If $\|x\|<1$
(C)	If $x>1$, Then $1+\frac{1}{x}+\frac{3}{x^2}+\cdots$ is	(3)	$1+nx+\frac{n(n+1)}{2!}x^2+\cdots$ If $\|x\|<1$
(D)	If $\|x\|>1$,then $1-\frac{2}{x^2}+\frac{3}{x^4}+\frac{4}{x^6}+\cdots$ is	(4)	$\frac{x}{x-1}$
		(5)	$\frac{x^4}{(x^2+1)^2}$
		(6)	$\frac{x^4}{(x^2-1)^2}$

CODES :

	A	**B**	**C**	**D**
a)	1	3	4	5
b)	2	3	4	5
c)	3	2	4	5
d)	2	3	1	5

157.

	Column-I		**Column- II**
(A)	The coefficient of tow consecutive terms in the expansion of $(1+x)^n$ will be equal, then n can be	(p)	9
(B)	If 15^n+23^n is divided by 19, then n can be	(q)	10
(C)	${}^{10}C_0\,{}^{20}C_{10}-{}^{10}C_1\,{}^{18}C_{10}+{}^{10}C_2\,{}^{16}C_{10}-\cdots$ is divisible by 2^n, then n can be	(r)	11
(D)	If the coefficients of T_r, T_{r+1}, T_{r+2} term of $(1+x)^{14}$ are in AP., then r is less than	(s)	12

CODES :

	A	**B**	**C**	**D**

a)	P,r	p,r	p,q	q,r,s
b)	r,p	r,p	q,p	r,s,q
c)	r,p	r,p	q,p	s,q,r
d)	r,p	r,p	q,p	r,q,s

158.

	Column-I		Column- II
(A)	${}^{32}C_0^2 - {}^{32}C_1^2 + {}^{32}C_2^2 - \cdots + {}^{32}C_{32}^2 =$	(p)	${}^{63}C_{16}$
(B)	${}^{32}C_0^2 + {}^{32}C_1^2 + {}^{32}C_2^2 - \cdots + {}^{32}C_{32}^2 =$	(q)	${}^{32}C_{16}$
(C)	$\frac{1}{32}(1 \times {}^{32}C_1^2 + 2 \times {}^{32}C_2^2 + \cdots + 32 \times {}^{32}C_{32}) =$	(r)	0
(D)	${}^{32}C_0^2 - {}^{31}C_1^2 + {}^{32}C_2^2 - \cdots - {}^{31}C_{31}^2 =$	(s)	${}^{64}C_{32}$

CODES :

	A	**B**	**C**	**D**
a)	P	q	r	s
b)	s	r	q	p
c)	q	s	p	r
d)	r	p	s	q

Linked Comprehension Type

This section contain(s) 16 paragraph(s) and based upon each paragraph, multiple choice questions have to be answered. Each question has atleast 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 159 to -159

If $a, b \in$ prime numbers and $n \in N$. then free from radical terms or rational terms in the expansion of $(a^{1/p} + b^{1/q})^n$ are the terms in which indices of a and b are integers.
On the basis of above information, answer the following questions

159. In the expansion of $(7^{1/3} + 11^{1/9})^{6561}$, the number of terms free from radicals is

a) 715 b) 725 c) 730 d) 745

Paragraph for Question Nos. 160 to - 160

If $C = {}^{n}C_{\mathrm{r}}$, then evaluate the expression $P = \sum_{0 \le \mathrm{r} < s \le n} \sum \ (C_{\mathrm{r}} C_s)$ we make use of $C_0^2 + C_1^2 + \ldots + C_n^2 = {}^{2n}C_n$ and expansion of $(C_0 + C_1 + \ldots + C_n)^2$.
On the basis of above information, answer the following questions

160. The value of $P = \underset{0 \le \mathrm{r} < s \le n}{\sum C_{\mathrm{r}} C_s}$ is

a) $2^{2n}-\frac{1}{2}({}^{2n}C_n)$
b) $2^{2n-1}-\frac{1}{2}({}^{2n}C_n)$
c) $2^{2n}-{}^{2n}C_n$
d) None of these

Paragraph for Question Nos. 161 to - 161

The sixth term in the expansion of $\left[\sqrt{\{2^{\log(10-3^x)}\}}+\sqrt[5]{\{2^{(x-2)\log 3}\}}\right]^m$ is equal to 21, if it is known that the binomial coefficient of the 2^{nd}, 3^{rd} and 4^{th} terms in the expansion represents, respectively, the first, third and fifth terms of an AP. (the symbol log stands for logarithm to the base 10)

161. The value of m is
a) 6
b) 7
c) 8
d) 9

Paragraph for Question Nos. 162 to - 162

The 2^{nd} , 3^{rd} and 4^{th} terms in the expansion of $(x+a)^n$ are 240, 720 and 1080, respectively

162. The value of $(x+a)^n$ can be
a) 64
b) -1
c) -32
d) None of these

Paragraph for Question Nos. 163 to - 163

If $(1+x+x^2)^{20}=a_0+a_1x+a_2x^2\cdots+a_{40}x^{40}$, then answer the following questions

163. The value of $a_0+a_1+a_2+\ldots+a_{19}$ is
a) $\frac{1}{2}(9^{10}+a_{20})$
b) $\frac{1}{2}(9^{10}-a_{20})$
c) $\frac{9^{10}}{2}$
d) None of these

Paragraph for Question Nos. 164 to - 164

An equation $a_0+a_1x+a_2x^2+\cdots+a_{99}x^{99}+x^{100}=0$ has roots ${}^{99}C_0$, ${}^{99}C_1$, ${}^{99}C_2,\cdots$, ${}^{99}C_{99}$

164. The value of a_{99} is equal to
a) 2^{98}
b) 2^{99}
c) -2^{99}
d) None of these

Paragraph for Question Nos. 165 to - 165

Any complex number in polar form can be an expression in Euler's form as $\cos\theta+i\sin\theta=e^{i\theta}$. This form of the complex number is useful in finding the sum of series $\sum_{r=0}^{n}{}^nC_r(\cos\theta+i\sin\theta)^r$

$$\sum_{r=0}^{n}{}^nC_r(\cos r\theta+i\sin r\theta)=\sum_{r=0}^{n}{}^nC_re^{ir\theta}$$

$$=\sum_{r=0}^{n}{}^nC_r\left(r^{i\theta}\right)^r$$

$$=\left(1+e^{i\theta}\right)^n$$

Also, we know that the sum of binomial series does not change if r is replaced by $n-r$.
Using these facts, answer the following questions

165. The value of $\sum_{r=0}^{100} {}^{100}C_r\,(\sin rx)$ is equal to

a) $2^{100}\cos^{100}\frac{x}{2}\sin 50x$
b) $2^{100}\sin(50x)\cos\frac{x}{2}$
c) $2^{101}\cos^{100}(50x)\sin\frac{x}{2}$
d) $2^{101}\sin^{100}(50x)\cos(50x)$

Paragraph for Question Nos. 166 to - 166

Let $P=\sum_{r=1}^{50}\frac{{}^{50+r}C_{r}(2r-1)}{{}^{50}C_r(50+r)}$, $Q=\sum_{r=0}^{50}({}^{50}C_r)^2$, $R=\sum_{r=0}^{100}(-1)^r({}^{100}C_r)^2$

166. The value of $P-Q$ is equal to

a) 1
b) -1
c) 2^{50}
d) 2^{100}

Paragraph for Question Nos. 167 to - 167

P is a set containing n elements. A subset A of P is chosen and the set P is reconstructed by replacing the elements of A. A subset B of P is chosen again

167. The number of ways of choosing A and B such that A and B have no common elements is

a) 3^n
b) 2^n
c) 4^n
d) None of these

Integer Answer Type

168. The largest real value for x such that $\sum_{k=0}^{4}\left(\frac{3^{4-k}}{(4-k)!}\right)\left(\frac{x^k}{k!}\right)=\frac{32}{3}$ is
169. Sum of last three digits of the number $N=7^{100}-3^{100}$ is
170. Number of values in set of values of 'r' for which ${}^{23}C_r+2.{}^{23}C_{r+1}+{}^{23}C_{r+2}\geq{}^{25}C_{15}$ is
171. Let $a=3^{\frac{1}{223}}+1$ and for all $n\geq 3$, let $f(n)={}^nC_0.a^{n-1}-{}^nC_1.a^{n-2}+{}^nC_2.a^{n-3}-\cdots+(-1)^{n-1}.{}^nC_{n-1}.a^0$. If the value of $f(2007)+f(2008)=3^k$ where $k\in N$, then the value of k is
172. If the three consecutive coefficient in the expansion of $(1+x)^n$ are 28, 56 and 70, then the value of n is
173. If R is remainder when $6^{83}+8^{83}$ is divided by 49, then the value of $R/5$ is
174. Let a and b be the coefficient of x^3 in $(1+x+2x^2+3x^3)^4$ and $(1+x+2x^2+3x^3+4x^4)^4$ respectively. Then the value of $4a/b$ is
175. If the constant term in the binomial expansion of $\left(x^2-\frac{1}{x}\right)^n$, $n\in N$ is 15, then the value of n is equal to
176. The value of $\lim_{n\to\infty}\sum_{r=1}^{n}\left(\sum_{t=0}^{r-1}\frac{1}{5^n}.{}^nC_r.{}^rC_t.3^t\right)$ is equal to
177. If the middle term in the expansion of $\left(\frac{x}{2}+2\right)^8$ is 1120; then the sum of possible real values of x is
178. Least positive integer just greater then $(1+0.00002)^{50000}$ is
179. Let $1+\sum_{r=1}^{10}(3^r.{}^{10}C_r+r.{}^{10}C_r)=2^{10}(\alpha\cdot 4^5+\beta)$ where $\alpha,\beta\in N$ and $(x)=x^2-2x-k^2+1$. If α,β lies between the roots of $f(x)=0$, then find the smallest positive integral value of k
180. Degree of the polynomial $\left[\sqrt{x^2+1}+\sqrt{x^2-1}\right]^8+\left[\frac{1}{\sqrt{\sqrt{x^2+1}+\sqrt{x^2-1}}}\right]^8$ is
181. If the coefficients of the $(2r+4)^{\text{th}}$, $(r+2)^{\text{th}}$ terms in the expansion of $(1+x)^{18}$ are equal, then the value of r is

182. If the second term of the expansion $\left[a^{1/13} + \frac{a}{\sqrt{a^{-1}}}\right]^n$ is $14a^{5/2}$, then the value of $\frac{{}^nC_3}{{}^nC_2}$ is

183. Given $(1 - 2x + 5x^5 - 10x^3)(1 + x)^n = 1 + a_1x + a_2x^2 + \cdots$ and that $a_1^2 = 2a_2$ then the value of n is

184. If the coefficients of the r^{th}, $(r + 1)^{\text{th}}$, $(r - 2)^{\text{th}}$ terms in the expansion of $(1 + x)^{14}$ are in AP, then the largest value of r is

185. If the coefficients x^7 in $\left(ax^2 + \frac{1}{bx}\right)^{11}$ and coefficient of x^{-7} in $\left(ax - \frac{1}{bx^2}\right)^{11}$ are equal then the value of ab is

186. The remainder, if $1 + 2 + 2^2 + 2^3 + \cdots + 2^{1999}$ is divided by 5 is

187. The largest value of x for which the fourth term in the expansion, $\left(5^{\frac{2}{5}\log_5\sqrt{4^x+44}} + \frac{1}{5^{\log_5\sqrt[3]{\sqrt{2^{x-1}+7}}}}\right)^8$ is 336 is

: ANSWER KEY :

1)	c	2)	c	3)	a	4)	d		a,c						
5)	b	6)	c	7)	c	8)	c	9)	a,d	10)	a,b,c	11)	c,d	12)	
9)	a	10)	a	11)	d	12)	c		a,c,						
13)	c	14)	c	15)	a	16)	c	13)	a,b,c	14)	a,c,d	15)	a,d	16)	
17)	d	18)	b	19)	d	20)	b		a,b,d						
21)	d	22)	b	23)	b	24)	a	17)	a,b,d	18)	b,c,d	19)	a,b,d	20)	a
25)	d	26)	a	27)	c	28)	d	21)	a,b,c	22)	a,b,c,d	23)	a,c,d	24)	
29)	b	30)	c	31)	a	32)	a		a,c						
33)	d	34)	a	35)	b	36)	c	25)	a,b,c	1)	c	2)	a	3)	a
37)	c	38)	c	39)	a	40)	b		4)	d					
41)	a	42)	d	43)	d	44)	c	5)	b	6)	d	7)	b	8)	a
45)	d	46)	d	47)	b	48)	b	9)	d	10)	b	11)	a	12)	a
49)	b	50)	c	51)	a	52)	a	13)	a	14)	a	15)	a	16)	b
53)	c	54)	a	55)	c	56)	c	17)	a	18)	b	19)	a	20)	d
57)	c	58)	b	59)	d	60)	a	21)	a	22)		23)	b	24)	a
61)	c	62)	a	63)	a	64)	a	25)		26)	a	27)	d	1)	b
65)	c	66)	b	67)	c	68)	d		2)	d	3)	d	4)	c	
69)	c	70)	c	71)	b	72)	b	5)	a	6)	c	1)	c	2)	b
73)	d	74)	d	75)	b	76)	b		3)	b	4)	b			
77)	b	78)	d	79)	c	80)	b	5)	b	6)	c	7)	a	8)	b
81)	b	82)	b	83)	b	84)	c	9)	a	1)	1	2)	0	3)	5
85)	b	86)	a	87)	c	88)	b		4)	9					
89)	d	90)	d	91)	a	92)	d	5)	8	6)	7	7)	4	8)	6
93)	a	94)	d	95)	b	96)	b	9)	1	10)	0	11)	3	12)	5
97)	b	98)	d	99)	d	100)	a	13)	8	14)	6	15)	4	16)	6
1)	b,c	2)	a,d	3)	c,d	4)		17)	9	18)	1	19)	0	20)	4
	a,b,c														
5)	a,b,c,d	6)	a,d	7)	a,b,c,d	8)									

: HINTS AND SOLUTIONS :

1 **(c)**

Given, $A = {}^{30}C_0 \cdot {}^{30}C_{10} - {}^{30}C_1 \cdot {}^{30}C_{11} + {}^{30}C_2 \cdot {}^{30}C_{12} + \cdots + {}^{30}C_{20} \cdot {}^{30}C_{30}$

$=$ coefficient of x^{20} in $(1+x)^{30}(1-x)^{30}$

$=$ coefficient of x^{20} in $(1+x^2)^{30}$

$=$ coefficient of x^{20} in $\sum_{r=0}^{30} (-1)^r \ {}^{30}C_r (x^2)^r$

$= (-1)^{10} \cdot {}^{30}C_{10}$ {for coefficient of x^{20}, let $r = 10$}

$= {}^{30}C_{10}$

2 **(c)**

$$a^{10}b^{10}c^{10}d^{10}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)^{10}$$

Therefore the required coefficient is equal to the coefficient

of $a^{-2}b^{-6}c^{-1}d^{-1}$ in $\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)^{10}$, which is given by

$$\frac{10!}{2!\,6!\,1!\,1!} = \frac{10 \times 9 \times 8 \times 7}{2} = 2520$$

3 **(a)**

$$\frac{r \times 2^r}{(r+2)!} = \frac{(r+2-2)2^r}{(r+2)!}$$

$$= \frac{2^r}{(r+1)!} - \frac{2^{r+1}}{(r+2)!}$$

$$= -\left(\frac{2^{r+1}}{(r+2)!} - \frac{2^r}{(r+1)!}\right)$$

$$= -\big(V(r) - V(r-1)\big)$$

$$\Rightarrow \sum_{r=1}^{15} \frac{r \times 2^r}{(r+2)!} = -\big(V(15) - V(0)\big)$$

$$= -\left(\frac{2^{16}}{17!} - \frac{2}{2!}\right)$$

$$= 1 - \frac{2^{16}}{(17)!}$$

4 **(d)**

$$(1-x)^n(1+x)^n = \sum_{r=0}^{n} a_r x^r (1-x)^n (1-x)^{n-r}$$

$$\Rightarrow (1-x+2x)^n = \sum_{r=0}^{n} a_r x^r (1-x)^{n-r}$$

$$\Rightarrow \sum_{r=0}^{n} {}^nC_r (1-x)^{n-r}(2x)^r = \sum_{r=0}^{n} a_r x^r (1-x)^{n-r}$$

Comparing general term, we get $a_r = {}^n C_r 2^r$

5 **(b)**

$$\left(1+\sqrt{a}+\frac{1}{\sqrt{a}-1}\right)^{-30}$$

$$= \left(\frac{a}{\sqrt{a}-1}\right)^{-30}$$

$$= \left(\frac{\sqrt{a}-1}{a}\right)^{30}$$

$$= \frac{1}{a^{30}}\left(1-\sqrt{a}\right)^{30}$$

$$= \frac{1}{a^{30}}\left\{{}^{30}C_0 - {}^{30}C_1\sqrt{a} + \cdots + {}^{30}C_{30}\left(\sqrt{a}\right)^{30}\right\}$$

There is no term independent of a

6 **(c)**

$$(1+2x+x^2)^n = \sum_{r=0}^{2n} a_r x^r \Rightarrow [(1+x)^2]^n$$

$$= \sum_{r=0}^{2n} a_r x^r$$

$$\Rightarrow (1+x)^{2n} = \sum_{r=0}^{2n} a_r x^r$$

$$\Rightarrow \sum_{r=0}^{2n} {}^{2n}C_r x^r = \sum_{r=0}^{2n} a_r x^r$$

$$\Rightarrow a_r = {}^{2n} C_r$$

7 **(c)**

Here,

$$T_r = (-1)^r \frac{{}^{50}C_r}{r+2}$$

$$= (-1)^r (r+1) \frac{{}^{50}C_r}{(r+1)(r+2)}$$

$$= (-1)^r (r+1) \frac{{}^{52}C_{r+2}}{51 \times 52}$$

$$= (-1)^r \frac{[(r+2)-1]\,{}^{52}C_{r+2}}{51 \times 52}$$

$$= (-1)^r \frac{[52\ {}^{51}C_{r+1} - {}^{52}C_{r+2}]}{51 \times 52}$$

$$= \frac{[-52\ {}^{51}C_{r+1}(-1)^{r+1} - {}^{52}C_{r+2}(-1)^{r+2}]}{51 \times 52}$$

$$\sum_{r=0}^{50} (-1)^r \frac{{}^{50}C_r}{r+2}$$

$$= \sum_{r=0}^{50} \frac{[-52\ {}^{51}C_{r+1}(-1)^{r+1} - {}^{52}C_{r+2}(-1)^{r+2}]}{51 \times 52}$$

$$= -52 \frac{(1-1)^{51} - {}^{51}C_0}{51 \times 52}$$

$$- \frac{(1-1)^{52} - {}^{52}C_0 + {}^{52}C_1}{51 \times 52}$$

$= \frac{1}{51} - \frac{1}{52}$
$= \frac{1}{51 \times 52}$
Alternative solution:
$(1-x)^n = \sum_{r=0}^{n} {}^nC_r(-1)^r x^r$
$\Rightarrow x(1-x)^n = \sum_{r=0}^{n} (-1)^r\, {}^nC_r x^{r+1}$
Integration both sides within the limits 0 to 1, we get
$\int_0^1 x(1-x)^n dx = \sum_{r=0}^{n} (-1)^r \frac{{}^nC_r}{r+2}$
$\Rightarrow \sum_{r=0}^{n} (-1)^r \frac{{}^nC_r}{r+2} = \int_0^1 x(1-x)^n dx$
$\int_0^1 (1-x)x^n dx$ (replace x by $1-x$)
$= \frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\Big|_0^1$
$= \frac{1}{n+1} - \frac{1}{n+2}$
$= \frac{1}{(n+1)(n+2)}$
Now put $n = 50$

8 **(c)**
Sum of coefficient in $(1 - x\sin\theta + x^2)^n$ is $(1 - \sin\theta + 1)^n$
(putting $x = 1$)
This sum is greatest when $\sin\theta = -1$, then maximum sum is 3^n

9 **(a)**
Given term can be written as
$(1+x)^2(1-x)^{-2}$
$= (1 + 2x + x^2)[1 + 2x + 3x^2 + \cdots + (n-1) \times x^{n-2} + nx^{n-1} + (n+1)x^n + \cdots]$
Coefficient of x^n is $(n + 1 + 2n + n - 1) = 4n$

10 **(a)**
To get sum of coefficients put $x = 0$. Given that sum of coefficients is
$2^n = 64$
$\Rightarrow n = 6$
The greatest binomial coefficient is ${}^6C_3$
Now given that
$T_4 - T_3 = 6 - 1 = 5$
$\Rightarrow$
${}^6C_3\left(3^{-x/4}\right)^3\left(3^{5x/4}\right)^3 - {}^6C_2\left(3^{-x/4}\right)^2\left(3^{5x/4}\right)^4 = 5$
Which is satisfied by $x = 0$

11 **(d)**
A_r = Coefficient of x^r in $(1+x)^{10} = {}^{10}C_r$
B_r = Coefficient of x^r in $(1+x)^{20} = {}^{20}C_r$
C_r = Coefficient of x^r in $(1+x)^{30} = {}^{30}C_r$
$\therefore \sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$
$= \sum_{r=1}^{10} A_r B_{10} B_r, - \sum_{r=1}^{10} A_r C_{10} A_r$
$= \sum_{r=1}^{10} {}^{10}C_r\, {}^{20}C_{10}\, {}^{20}C_r \sum_{r=1}^{10} {}^{10}C_r\, {}^{30}C_{10}\, {}^{10}C_r\text{l}$
$\sum_{r=1}^{10} {}^{10}C_{10-rl}\, {}^{20}C_{10}\, {}^{20}C_r - \sum_{r=1}^{10} {}^{10}C_{10-r}\, {}^{30}C_{10}\, {}^{10}C_r l$
$= {}^{20}C_{10} \sum_{r=1}^{10} {}^{10}C_{10-r}\, {}^{20}C_r$
$- {}^{30}C_{10} \sum_{r=1}^{10} {}^{10}C_{10-r}\, {}^{10}C_r$
$= {}^{20}C_{10}({}^{30}C_{10} - 1) - {}^{30}C_{10}({}^{20}C_{10} - 1)$
$= {}^{20}C_{10}({}^{30}C_{10} - 1) - {}^{30}C_{10}({}^{20}C_{10} - 1)$
$= {}^{30}C_{10} - {}^{20}C_{10} = C_{10} - B_{10}$

12 **(c)**
As we know that ${}^nC_0 - {}^nC_1^2 + {}^nC_2^2 - {}^nC_3^2 + \cdots + (-1)^n\, {}^nC_n^2 = 0$
(if n is odd) and in the question $n = 15$ (odd).
Hence, sum of given series is 0

13 **(c)**
Let,
$b = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$ (1)
$= \sum_{r=0}^{n} \frac{n-r}{{}^nC_{n-r}}$ (we can replace r by $n-r$)
$= \sum_{r=0}^{n} \frac{n-r}{{}^nC_r}$ (2)
Adding (1) and (2), we have
$2b = \sum_{r=0}^{n} \frac{r}{{}^nC_r} + \sum_{r=0}^{n} \frac{n-r}{{}^nC_r}$
$= n\sum_{r=0}^{n} \frac{1}{{}^nC_r}$
$= na_n$
$\Rightarrow b = \frac{n}{2}a_n$

14 **(c)**
We have,

$(1+x)^{101}(1-x+x^2)^{100}$

$= (1+x)\big((1+x)(1+x+x^2)\big)^{100}$

$= (1+x)(1+x^3)^{100}$

$= (1+x)\{C_0 + C_1x^3 + C_2x^6 + \cdots + C_{100}\, x^{300}\}$

$= (1+x)\sum_{r=0}^{n} {}^nC_r x^3$

$= \sum_{r=0}^{n} {}^nC_r x^{3r} + \sum_{r=0}^{n} {}^nC_r\, x^{3r+1}$

Hence, there will be no term containing $3r+2$

15 **(a)**

General term,

$T_{r+1} = {}^{256}C_1\left(\sqrt{3}\right)^{256-r}\left(\sqrt[8]{5}\right)^r$

$={}^{256}C_r 3^{\frac{256-r}{2}} 5^{\frac{r}{8}}$

The terms are integral if $\frac{256-r}{2}$ and $\frac{r}{8}$ are both positive integers

$\therefore\ r = 0, 8, 16, 24, \dots, 256$

Hence, there are 33 integral terms

16 **(c)**

$\sum_{r=0}^{300} a_r \times x^r = (1+x+x^2+x^3)^{100}$

Clearly, 'a_r' is the coefficient of x^r in the expansion of $(1+x+x^2+x^3)^{100}$

Replacing x by $1/x$ in the given equation, we get

$\sum_{r=0}^{300} a_r\left(\frac{1}{x}\right)^r = \frac{1}{x^{300}}(x^3+x^2+x+1)^{100}$

$\Rightarrow \sum_{r=0}^{300} a_r x^{300-r} = (1+x+x^2+x^3)^{100}$

Here, a_r represents the coefficient of x^{300-r} in $(1+x+x^2+x^3)^{100}$

Thus, $a_r = a_{300-r}$

Let $I = \sum_{r=0}^{300} r \times a_r$

$= \sum_{r=0}^{300}(300-r)a_{300-r}$

$= \sum_{r=0}^{300}(300-r)a_r$

$= 300\sum_{r=0}^{300} a_r - \sum_{r=0}^{300} ra_r$

$\Rightarrow 2I = 300a$

$\Rightarrow I = 150a$

17 **(d)**

$\sum_{r=1}^{n}(-1)^{r+1}\frac{{}^nC_r}{(r+1)} = \frac{1}{n+1}\sum_{r=1}^{n}(-1)^{r+1n+1}C_{r+1}$

$= \frac{1}{n+1}\big(0-1+(n+1)\big) = \frac{n}{n+1}$

18 **(b)**

T_{r+1} in $(1+x)^n$ is

$\frac{n(n-1)(n-2)\cdots(n-r+1)}{r!}x^r$

For first negative term,

$n-r+1<0$

$\Rightarrow \frac{27}{5} - r + 1 < 0$

$\Rightarrow r > \frac{32}{5}$

Thus, first negative term occurs when $r=7$

19 **(d)**

$\sum_{r=0}^{10} r^{10}C_r 3^r(-2)^{10-r}$

$= 10\sum_{r=0}^{10} {}^9C_{r-1}3^r(-2)^{10-r}$

$= 10\times 3\sum_{r=0}^{10} {}^9C_{r-1}3^{r-1}(-2)^{10-r}$

$= 30(3-2)^{10}$

$= 30$

20 **(b)**

Given series is ${}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \cdots + {}^{20}C_8$

$= \frac{1}{2}(2\cdot {}^{20}C_0 + 2\,{}^{20}C_1 + \cdots 2\cdot {}^{20}C_8)$

$= \frac{1}{2}[({}^{20}C_0 + {}^{20}C_1 + \cdots + {}^{20}C_8 + {}^{20}C_9 + {}^{20}C_{10} + {}^{20}C_{11} + \cdots + {}^{20}C_{20}) - ({}^{20}C_9 + {}^{20}C_{10} + {}^{20}C_{11})]$

$= \frac{1}{2}[2^{20} - 2\cdot{}^{20}C_9 - {}^{20}C_{10}]$

$= 2^{19} - \frac{(2\cdot {}^{20}C_9 + {}^{10}C_{10})}{2}$

$= \frac{(2^{20} - {}^{20}C_{10})}{2} - {}^{20}C_9$

$= 2^{19} - \frac{({}^{20}C_{10} + 2\times {}^{20}C_9)}{2}$

21 **(d)**

Required value is

$\left(1-\frac{2x}{1+x}\right)^{-n} = \left(\frac{1+x-2x}{1+x}\right)^{-n} = \left(\frac{1-x}{1+x}\right)^{-n}$

$= \left(\frac{1+x}{1-x}\right)^{-n}$

22 **(b)**

$(1+x^3-x^6)^{30}$

$= \{1+x^3(1-x^3)\}^{30}$

$= {}^{30}C_0 + {}^{30}C_1x^3(1-x^3) + {}^{30}C_2x^6(1-x^3)^2 + \dots$

Obviously, each term will contain $x^{3m}, m\in N$. But 28 is not divisible by 3. Therefore, there will be no term containing x^{28}

23 **(b)**

$$\left(x+\frac{1}{x}+x^2+\frac{1}{x^2}\right)^{15}$$
$$=\left(\frac{x^3+x+x^4+1}{x^2}\right)^{15}$$
$$=\frac{a_0+a_1x+a_2x^2+\cdots+a_{60}x^{60}}{x^{30}}$$

Hence, the total number of terms is 61

24 **(a)**

$$\sum_{r=0}^{40} r\,{}^{40}C_r\ {}^{30}C_r$$
$$=40\sum_{r=0}^{40}{}^{39}C_{r-1}\,{}^{30}C_r$$
$$=40\sum_{r=0}^{40}{}^{39}C_{r-1}\ {}^{30}C_{30-r}$$
$$=40^{39+30}C_{r-1+30-r}$$
$$=40\ {}^{69}C_{29}$$

25 **(d)**

Let,

$$(1+y)^n=1+\frac{1}{3}x+\frac{1\times4}{3\times6}x^2+\frac{1\times4\times7}{3\times6\times9}x^3+\cdots$$
$$=1+ny+\frac{n(n-1)}{2!}y^2+\cdots$$

Comparing the terms, we get

$$ny=\frac{1}{3}x,\frac{n(n-1)}{2!}y^2=\frac{1\times4}{3\times6}x^2$$

Solving, $n=-1/3, y=-x$. Hence, the given series is $(1-x)^{-1/3}$

26 **(a)**

We have,

$$\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}$$
$$=\frac{\left(x^{1/3}\right)^3+1^3}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x^{1/2}(x^{1/2}-1)}$$
$$=\frac{\left(x^{1/3}+1\right)(x^{2/3}-x^{1/3}+1)}{x^{2/3}-x^{1/3}+1}-\frac{x^{1/2}+1}{x^{1/2}}$$
$$=x^{1/3}+1-1-x^{-1/2}=x^{1/3}-x^{-1/2}$$
$$\therefore\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}=\left(x^{1/3}-x^{-1/2}\right)^{10}$$

Let T_{r+1} be the general term in $\left(x^{1/3}-x^{-1/2}\right)^{10}$.

Then,

$$T_{r+1}={}^{10}C_r\left(x^{1/3}\right)^{10-r}(-1)^r\left(x^{-1/2}\right)^r$$

For this term to be independent of x, we must have

$$\frac{10-r}{3}-\frac{r}{2}=0\Rightarrow20-2r-3r=0\Rightarrow r=4$$

So, the required coefficient is ${}^{10}C_4(-1)^4=210$

27 **(c)**

$$\left(x^2-2+\frac{1}{x^2}\right)^n=\frac{1}{x^{2n}}(x^4-2x^2+1)^n$$
$$=\frac{(x^2-1)^{2n}}{x^{2n}}$$

Total number of terms that are dependent on x is equal to number of terms in the expansion of $(x^2-1)^{2n}$ that have degree of x different from $2n$, which is given by $(2n+1)-1=2n$

28 **(d)**

$$\sum_{r=0}^{20} r(20-r)\times({}^{20}C_r)^2$$
$$=\sum_{r=0}^{20} r\times{}^{20}C_r\,(20-r)\,{}^{20}C_{20-r}$$
$$\Rightarrow\sum_{r=0}^{20}20\,{}^{19}C_{r-1}\times20\times{}^{19}C_{19-r}$$
$$=400\times\sum_{r=0}^{20}{}^{19}C_{r-1}\times{}^{19}C_{19-r}$$
$$=400\times\text{coefficient of }x^{18}\text{ in }(1+x)^{19}(1+x)^{19}$$
$$=400\times{}^{38}C_{18}$$
$$=400\times{}^{38}C_{20}$$

29 **(b)**

$$(1+x)^n=C_0+C_1x+C_2x+C_2x^2+C_3x^3+\cdots+C_nx^n$$
$$(1-x)^n=C_0-C_1x+C_2x^2-C_3x^3+\cdots+(-1)^nC_nx^n$$
$$\Rightarrow[(1+x)^n-(1-x)^n]=2[C_1x+C_3x^3+C_5x^5+\cdots]$$
$$\Rightarrow\frac{1}{2}[(1+x)^n-(1-x)^n]=C_1x+C_3x^3+C_5x^5+\cdots]$$

Putting $x=2$, we have

$$2C_1+2^3C_3+2^5C_5+\cdots=\frac{3^n-(-1)^n}{2}$$

30 **(c)**

Let $(r+1)^{\text{th}}, (r+2)^{\text{th}}$ and $(r+3)^{\text{th}}$ be three consecutive terms

Then,

$${}^nC_r:{}^nC_{r+1}:{}^nC_{r+2}=1:7:42$$

Now,

$$\frac{{}^nC_r}{{}^nC_{r+1}}=\frac{1}{7}\Rightarrow\frac{r+1}{n-r}=\frac{1}{7}\Rightarrow n-8r=7\quad\text{(i)}$$
$$\frac{{}^nC_{r+1}}{{}^nC_{r+2}}=\frac{7}{42}\Rightarrow\frac{r+2}{n-r-1}=\frac{1}{6}\Rightarrow n-7r=13\quad\text{(ii)}$$

Solving (i) and (ii), we get $n=55$

31 **(a)**

$$T_{r+1}={}^{4n-2}C_r(ix)^r$$

T_{r+1} is negative, if i^r is negative and real

$i^r = -1 \Rightarrow r = 2, 6, 10, \ldots$, which form an A.P.
$0 \le r \le 4n-2$
$4n-2 = 2 + (r-1)4 \Rightarrow r = n$
The required number of terms is n

32 **(a)**

$$1 + n\left(1-\frac{1}{x}\right) + \frac{n(n+1)}{2!}\left(1-\frac{1}{x}\right)^2 + \cdots \infty$$
$$= 1 - n\left[-\left(1-\frac{1}{x}\right)\right] + \frac{-n(-n-1)}{2!}\left[-\left(1-\frac{1}{x}\right)\right]^2 + \cdots \infty$$
$$= \left[1-\left(1-\frac{1}{x}\right)\right]^{-n}$$
$$= x^n$$

33 **(d)**

$$\binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2}$$
$$= \left[\binom{n}{r} + \binom{n}{r-1}\right] + \left[\binom{n}{r-1} + \binom{n}{r-2}\right]$$
$$= \binom{n+1}{r} + \binom{n+1}{r-1} = \binom{n+2}{r} \quad [\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r]$$

34 **(a)**

We know that
$$(1-1)^{20} = {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \cdots + {}^{20}C_{10} - {}^{20}C_{11} + {}^{20}C_{12} - \cdots + {}^{20}C_{20} = 0$$
$$2({}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \cdots - {}^{20}C_9) + {}^{20}C_{10} = 0$$
$[\because {}^{20}C_{20} = {}^{20}C_0, {}^{20}C_{19} = {}^{20}C_1$, etc$]$
$$\Rightarrow {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \cdots - {}^{20}C_9 + {}^{20}C_{10}$$
$$= -\frac{1}{2}\,{}^{20}C_{10} + {}^{20}C_{10} = \frac{1}{2}\,{}^{20}C_{10}$$

35 **(b)**

We have
$$(x+3)^{n-1} + (x+3)^{n-2}(x+2) + (x+3)^{n-3}(x+2)^2 + \cdots + (x+2)^{n-1}$$
$$= \frac{(x+3)^n - (x+2)^n}{(x+3)-(x+2)} = (x+3)^n - (x+2)^n$$
$$\left(\because \frac{x^n - a^n}{x-a} = x^{n-1} + x^{n-2}a^1 + x^{n-3}a^2 + \cdots + a^{n-1}\right)$$
Therefore, coefficient of x^r in the given expression is equal to
Coefficient of x^r in $[(x+3)^n - (x+2)^n]$, which is given by
$${}^nC_r 3^{n-r} - {}^nC_r 2^{n-r} = {}^nC_r(3^{n-r} - 2^{n-r})$$

36 **(c)**

Put $x = \omega, \omega^2$
$$(3 + \omega + \omega^2)^{2010} = a_0 + a_1\omega + a_2\omega^2 + \cdots$$
$$\Rightarrow 2^{2010} = a_0 + a_1\omega^2 + a_2\omega + a_3 + a_4\omega + \cdots$$
(1)
and $2^{2010} = a_0 + a_1\omega^2 + a_2\omega + a_3 + a_4\omega + \cdots$
(2)
Adding (1) and (2), we have
$$2 \times 2^{2010} = 2a_0 - a_1 + a_2 + 2a_3 - a_4 - a_5 + 2a_6 - \cdots$$
$$\Rightarrow 2^{2010} = a_0 - \frac{1}{2}a_1 - \frac{1}{2}a_2 + a_3 - \frac{1}{2}a_4 - \frac{1}{2}a_5 + a_6 \cdots$$

37 **(c)**

$$t_{r+1} = (-1)^r(n-r+2)\,{}^nC_r 2^{n-r+1}$$
$$= (n+2)2^{n+1}(-1)^r\,{}^nC_r\left(\frac{1}{2}\right)^r - 2^{n+1}(-1)^r\, r\,{}^nC_r\left(\frac{1}{2}\right)^r$$
$$= (n+2)2^{n+1}\,{}^nC_r\left(-\frac{1}{2}\right)^r + 2^n n\,{}^{n-1}C_{r-1}\left(-\frac{1}{2}\right)^{r-1}$$
$\therefore$ Sum $= (n+2)2^{n+1}\left\{{}^nC_0 - {}^nC_1 \times \frac{1}{2} + {}^nC_2 \times\right.$
122−...+n2n n−1C0− n−1C1×12+
n−1C2×122+...

$$= (n+2)2^{n+1}\left(1-\frac{1}{2}\right)^n + n2^n\left(1-\frac{1}{2}\right)^{n-1}$$
$$= 2(n+2) + 2n$$
$$= 4n + 4$$

38 **(c)**

The given sigma is the expansion of $[(x-3)+$
2100=x−1100=1−x100
Therefore, x^{53} will occur in T_{54}
$T_{54} = {}^{100}C_{53}(-x)^{53}$
Therefore, the coefficient is $-{}^{100}C_{53}$

39 **(a)**

$$\frac{2^{4n}}{15} = \frac{(15+1)^n}{15}$$
$$= \frac{({}^nC_0 15^n + {}^nC_1 15^{n-1} + \ldots + {}^nC_{n-1}15 + {}^nC_n)}{15}$$
$= \text{Integer} + \frac{1}{5}$
Hence, the fractional part of $\frac{2^{4n}}{15}$ is $\frac{1}{15}$

40 **(b)**

a_1 = coefficient of x in $(1+2x+3x^2)^{10}$
= coefficient of x in $\left((1+2x)+3x^2\right)^{10}$
= coefficient of x in
$({}^{10}C_0(1+2x)^{10} + {}^{10}C_1(1+2x)^9(3x^2) + \cdots)$
= coefficient of x in ${}^{10}C_0(1+2x)^{10}$
$= {}^{10}C_0 2.\,{}^{10}C_1 = 20$

41 **(a)**

$$(1.0002)^{3000} = (1+0.0002)^{3000}$$

$= 1 + (3000)(0.0002)$
$+ \frac{(3000)(2999)}{1.2}(0.0002)^2 + ..$
$= 1 + (3000)(0.0002) = 1.6$

42 **(d)**
$(1+\omega)^n = {}^nC_0 + {}^nC_1\omega + \cdots$
$= ({}^nC_0 + {}^nC_3 + \cdots)$
$+ ({}^nC_1 + {}^nC_4 + \cdots)\left(\frac{-1+\sqrt{3}i}{2}\right)$
$+({}^nC_2 + {}^nC_5 + \cdots)\left(\frac{-1-\sqrt{3}i}{2}\right)$
$= ({}^nC_0 + {}^nC_3 + \cdots)$
$-\frac{1}{2}({}^nC_1 + {}^nC_2 + {}^nC_4 + {}^nC_5 \ldots)$
$+\frac{i\sqrt{3}}{2}({}^nC_1 - {}^nC_2 + {}^nC_4 - {}^nC_5 + \cdots)$
Equating the modulus, we get $|(-\omega^2)^n| = 1$

43 **(d)**
$3^{400} = 81100 = (1+80)^{100}$
$= {}^{100}C_0 + {}^{100}C_1 80 + \cdots + {}^{100}C_{100} 80^{100}$
$\Rightarrow$ Last two digits are 01

44 **(c)**
$\sum_{k=1}^{n} k\left(1-\frac{1}{n}\right)^{k-1}$
$= 1 + 2\left(1-\frac{1}{n}\right)^1 + 3\left(1-\frac{1}{n}\right)^2 + \cdots$
$= 1 + 2t + 3t^2 + \cdots$
$= (1-t)^{-2}$
$\left[1-\left(1-\frac{1}{n}\right)\right]^{-2} = \left(\frac{1}{n}\right)^{-2} = n^2$

45 **(d)**
Here, the coefficients of T_r, T_{r+1} and T_{r+2} in $(1+y)^m$ are in A.P.
$\Rightarrow {}^mC_{r-1}\ {}^mC_r$ and ${}^mC_{r+1}$ are in A.P.
$\Rightarrow 2\,{}^mC_r = {}^mC_{r-1} + {}^mC_{r+1}$
$\Rightarrow 2\frac{m!}{r!\,(m-r)!} = \frac{m!}{(r-1)!\,(m-r+1)!} + \frac{m!}{(r+1)!\,(m-r-1)!}$
$\Rightarrow \frac{2}{r(m-r)} = \frac{1}{(m-r+1)(m-r)} + \frac{1}{(r+1)r}$
$\Rightarrow m^2 - m(4r+1) + 4r^2 - 2 = 0$

46 **(d)**
$(1+x+x^2+\cdots)^2 = ((1-x)^{-1})^2 = (1-x)^{-2}$
$= 1 + 2x + 3x^2 + \cdots$
Therefore, coefficient of x^n is $n+1$

47 **(b)**
We have,
$(1-x)^{-n} = a_0 + a_1x + a_2x^2 + \cdots + a_rx^r + \cdots$
And
$(1-x)^{-1} = 1 + x + x^2 + x^3 + .. + x^r + \cdots$
Hence,
$a_0 + a_1 + a_2 + \cdots + a_r$
= Coefficient of x^r in the product of the two series
= Coefficient of x^r in $(1-x)^{-n}(1-x)^{-1}$
= Coefficient of x^r in $(1-x)^{-(n+1)}$
$= \frac{(n+1)(n+2)\cdots(n+r)}{r!}$
$= {}^{r+n+1-1}C_{n+1-1} = {}^{n+r}C_n$

48 **(b)**
By the given condition,
$84 = T_6 = T_{5+1}$
$= {}^7C_5\left(2^{\log_2\sqrt{9^{x-1}+7}}\right)^2\left(\frac{1}{2^{\frac{1}{5}\log_2(3^{x-1}+1)}}\right)^5$
$= 21\ 2^{\log_2(9^{x-1}+7)}2^{-\log_2(3^{x-1}+1)}$
$\Rightarrow 4 = 2^{\log_2\frac{9^{x-1}+7}{3^{x-1}+1}} = \frac{9^{x-1}+7}{3^{x-1}+1}$
$\Rightarrow (3^{x-1})^2 - 4\times 3^{x-1} + 3 = 0$
$\Rightarrow (3^{x-1}-1)(3^{x-1}-3) = 0$
$\Rightarrow 3^{x-1} = 1$ or 3
$\Rightarrow 3^{x-1} = 3^0$ or 3^1
$\Rightarrow x - 1 = 0$ or 1
$\Rightarrow x = 1, 2$

49 **(b)**
$T_{r+1} = {}^{1024}C_r\left(5^{1/2}\right)^{1024-r}\left(7^{1/8}\right)^r$
Now this term is an integer if $1024 - r$ is an even integer, for which $r = 0, 2, 4, 6, \ldots, 1022, 1024$ of which $r = 0, 8, 16, 2424, \ldots, 1024$ are divisible by 8 which makes $r/8$ an integer
For A.P., $r = 0, 8, 16, 24, \ldots, 1024$,
$1024 = 0 + (n-1)8 \Rightarrow n = 129$

50 **(c)**
$\frac{(x^2+x+1)(1-x)}{(1-x)^2} = (1-x^3)(1-x)^{-2}$
$= (1-x^3)(1+2x+3x^2+\cdots)$
Now, $a_r = (r+1) - (r-2) = 3$
But $a_1 = 2$
So, $\sum_{r=1}^{50} a_r = 2 + 49\times 3 = 149$

51 **(a)**

$p=\left(8+3\sqrt{7}\right)^n = {}^nC_0 8^n + {}^nC_1 8^{n-1}\left(3\sqrt{7}\right)+\cdots$

Let,

$p_1=\left(8-3\sqrt{7}\right)^n = {}^nC_0 8^n - {}^7C_1 8^{n-1}\left(3\sqrt{7}\right)+\cdots$

$p_1+p_2 = 2\left({}^nC_0 8^n + {}^nC_2 8^{n-2}\left(3\sqrt{7}\right)^2+\cdots\right)=$ even integer p_1 clearly belongs to (0,1)

$\Rightarrow [p]+f+p_1 =$ even integer

$\Rightarrow f+p_1 =$ integer

$f\in(0,1), p_1\in(0,1)$

$\Rightarrow f+p\in(0,2)$

$\Rightarrow f+p_1=1$

$\Rightarrow p_1 = 1-f$

Now, $p(1-f)=pp_1-\left[\left(8+3\sqrt{7}\right)^n(8-\right.$

37n=1

52 **(a)**

$$\sum_{r=1}^{n+1}\left(\sum_{k=1}^{n}{}^kC_{r-1}\right)$$

$$=\sum_{r=1}^{n+1}\left(\sum_{k=1}^{n}\left({}^{k+1}C_r-{}^kC_r\right)\right)$$

$$=\sum_{r=1}^{n+1}\left({}^{n+1}C_r-{}^1C_r\right)$$

$$=2^{n+1}-2$$

53 **(c)**

$$\sum_{k=1}^{\infty}\sum_{r=0}^{k}\frac{1}{3^k}\left({}^kC_r\right)$$

$$=\sum_{k=1}^{\infty}\left(\frac{1}{3^k}\left(\sum_{r=0}^{k}{}^kC_r\right)\right)$$

$$=\sum_{k=0}^{\infty}\left(\frac{2^k}{3^k}\right)$$

$$=\frac{2}{3}+\left(\frac{2}{3}\right)^2+\cdots\infty$$

$$=\frac{2/3}{1-\frac{2}{3}}=2$$

54 **(a)**

$$\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$$

General term in this expansion is

$$T_{r+1}={}^{10}C_r\left(\frac{x}{2}\right)^{10-r}\left(\frac{-3}{x^2}\right)^r$$

$$={}^{10}C_r x^{10-3r}\frac{(-1)^r3^r}{2^{10-r}}$$

For coefficient of x^4, we should have $r=2$

Therefore, coefficient of x^4 is ${}^{10}C_2\frac{(-1)^2 3^2}{2^8}=\frac{405}{256}$

55 **(c)**

Middle term of $(1+\alpha x)^4$ is T_3

Its coefficient is ${}^4C_2(\alpha)^2=6\alpha^2$

Middle term of $(1-\alpha x)^6$ is T_4

Its coefficient is ${}^6C_3\left(-\alpha^3\right)=-20\alpha^3$

According to question,

$6\alpha^2=-20\alpha^3$

$\Rightarrow 3\alpha^2+10\alpha^3=0$

$\Rightarrow \alpha^2(3+10\alpha)=0$

$\Rightarrow \alpha=-\frac{3}{10}$

56 **(c)**

$$T_{r+1}={}^{2n}C_r x^{2n-r}\left(\frac{1}{x^2}\right)^r={}^{2n}C_r x^{2n-3r}$$

This contains x^m. If $2n-3r=m$, then

$$r=\frac{2n-m}{3}$$

$\Rightarrow$ Coefficient of $x^m={}^{2n}C_r, r=\frac{2n-m}{3}$

$$=\frac{2n!}{(2n-r)!\,r!}=\frac{2n!}{\left(2n-\frac{2n-m}{3}\right)!\left(\frac{2n-m}{3}\right)!}$$

$$=\frac{(2n)!}{\left(\frac{4n+m}{3}\right)!\left(\frac{2n-m}{3}\right)!}$$

57 **(c)**

$(23)^{14}=(529)^7=(530-1)^7$

$={}^7C_0(530)^7-{}^7C_1(530)^6+\cdots-{}^7C_5(530)^2+{}^7C_6 530-1$

$={}^7C_0(530)^7-{}^7C_1(530)^6+\cdots+3710-1$

$=100m+3709$

Therefore, last two digits are 09

58 **(b)**

$(1+x-2x^2)^6=1+a_1x+a_2x^2+\cdots$

Putting $x=1$, we get

$0=1+a_1+a_2+a_3+\cdots+a_{12}$ (1)

Putting $x=-1$, we get

$64=1-a_1+a_2+a_3+\cdots+a_{12}$ (2)

(1)+(2) gives

$64=2[1+a_2+a_4+\cdots+a_{12}]$

$\Rightarrow 1+a_2+a_4+\cdots+a_{12}=32$

$\Rightarrow a_2+a_4+\cdots+a_{32}=31$

59 **(d)**

Here ${}^{n-1}C_r=(k^2-3)\,{}^nC_{r+1}$

$$\Longrightarrow {}^{n-1}C_r=(k^2-3)\frac{n}{r+1}\,{}^{n-1}C_r$$

$$\Longrightarrow k^2-3=\frac{r+1}{n}$$

$$\left[\text{since}, n-1\ge r\Longrightarrow\frac{r+1}{n}\le 1\text{and } n,r\ge 0\right]$$

$\Longrightarrow 0<k^2-3\le 1\Longrightarrow 3<k^2\le 4$

$\Longrightarrow k\in\left[-2,-\sqrt{3}\right)\cup\left(\sqrt{3},2\right]$

60 **(a)**

We rewrite the given expression as [1 +

$x^2(1-x)]^8$ and expand by using the binomial theorem. We have,

$[1+x^2(1-x)]^8$

$={}^8C_0+{}^8C_1x^2(1-x)+{}^8C_2x^4(1-x)^2+{}^8C_3x^6(1-x)^3+{}^8C_4x^8(1-x)^4+{}^8C_5x^{10}(1-x)^5+\cdots$

The two terms which contain $x^{10}$are ${}^8C_4x^8(1-$ *x8*and *8C5x101−x5.*

Thus, the coefficient of x^{10} in the given expression is given by ${}^8C_4$ [coefficient of x^2 in the expansion of $(1-x)^4$] $+{}^8C_5$

$={}^8C_4(6)+{}^8C_5=\frac{8!}{4!\,4!}(6)+\frac{8!}{3!\,5!}$

$=(70)(6)+56=476$

61 **(c)**

It is given that 6^{th} term in the expansion of $\left(\frac{1}{x^{8/3}}+x^2\log_{40}x\right)^8$ is 5600, therefore

${}^8C_5(x^2\log_{10}x)^5\left(\frac{1}{x^{8/3}}\right)^3=5600$

$\Rightarrow 56\,x^{10}(\log_{10}x)^5\frac{1}{x^8}=5600$

$\Rightarrow x^2(\log_{10}x)^5=100$

$\Rightarrow x^2(\log_{10}x)^5=10^2(\log_{10}10)^5$

$\Rightarrow x=10$

62 **(a)**

We have,

$\frac{2}{\sqrt{2x^2+1}+\sqrt{2x^2-1}}$

$=\frac{2\left(\sqrt{2x^2+1}-\sqrt{2x^2-1}\right)}{(2x^2+1)-(2x^2-1)}$

$=\sqrt{2x^2+1}-\sqrt{2x^2-1}$

Thus, the given expression can be written as

$\left(\sqrt{2x^2+1}+\sqrt{2x^2-1}\right)^6+\left(\sqrt{2x^2+1}-\sqrt{2x^2-1}\right)^6$

But

$(a+b)^6+(a-b)^6=2[a^6+{}^6C_2a^4b^2+{}^6C_4a^2b^4+b^6]$

Therefore, $\left(\sqrt{2x^2+1}+\sqrt{2x^2-1}\right)^6+\left(\sqrt{2x^2+1}-\sqrt{2x^2-1}\right)^6$

$=2[(2x^2+1)^3+15(2x^2+1)^2(2x^2-1)+15(2x^2+1)\times(2x^2-1)^2+(2x^2-1)^3]$

Which is a polynomial of degree 6

63 **(a)**

Last term of $\left(2^{1/3}-\frac{1}{\sqrt{2}}\right)^n$ is

$T_{n+1}={}^nC_n\left(2^{1/3}\right)^{n-n}\left(-\frac{1}{\sqrt{2}}\right)^n={}^nC_n(-1)^n\frac{1}{2^{n/2}}$

$=\frac{(-1)^n}{2^{n/2}}$

Also, we have

$\left(\frac{1}{3^{5/3}}\right)^{\log_3 8}=\frac{1}{(3^{5/3})^{3\log_3 2}}=3^{-(5/3)\log_3 2^3}=2^{-5}$

Thus,

$\frac{(-1)^n}{2^{n/2}}=2^{-5}$

$\Rightarrow\frac{(-1)^n}{2^{n/2}}=\frac{(-1)^{10}}{2^5}$

$\Rightarrow n/2=5$

$\Rightarrow n=10$

Now,

$T_5=T_{4+1}={}^{10}C_4\left(2^{1/3}\right)^{10-4}\left(-\frac{1}{\sqrt{2}}\right)^4$

$=\frac{10!}{4!\,6!}\left(2^{1/3}\right)^6(-1)^4\left(2^{-1/2}\right)^4$

$=210(2^2)(1)(2^{-2})=210$

64 **(a)**

Let the given series be identical with

$(1+x)^n=1+nx+\frac{n(n-1)}{1\times 2}x^2+\cdots\infty$

$\Rightarrow nx=\frac{1}{4}\Rightarrow n^2x^2=\frac{1}{16}$

Also,

$\frac{n(n-1)}{2}x^2=\frac{3}{32}\Rightarrow\frac{2n}{n-1}=\frac{\frac{1}{16}}{\frac{3}{32}}=\frac{2}{3}$

$\Rightarrow 3n=n-1$

$\Rightarrow 2n=-1$

$\Rightarrow n=-\frac{1}{2}$

$\Rightarrow\ x=-\frac{1}{2}$

$\Rightarrow$ Required sum $=\left(1-\frac{1}{2}\right)^{-\frac{1}{2}}=\left(\frac{1}{2}\right)^{-\frac{1}{2}}$

$=(2)^{\frac{1}{2}}=\sqrt{2}$

65 **(c)**

Coefficient of T_5 is ${}^nC_4$ that of T_6 is ${}^nC_5$ and that of T_7 is ${}^nC_6$

According to the condition, $2{}^nC_5={}^nC_4+{}^nC_6$.

Hence,

$2\left[\frac{n!}{(n-5)!\,5!}\right]=\left[\frac{n!}{(n-4)!\,4!}+\frac{n!}{(n-6)!\,6!}\right]$

$\Rightarrow 2\left[\frac{1}{(n-5)5}\right]=\left[\frac{1}{(n-4)(n-5)}+\frac{1}{6\times 5}\right]$

After solving, we get $n=7$ or 14

66 **(b)**

$(1-x)(1-x)^n$

$= (1-x)[1+n(-x)$
$+\cdots+{}^nC_{n-1}(-x)^{n-1}+{}^nC_n(-x)^n]$
Therefore, coefficient of x^n is
${}^nC_n(-1)^n-{}^nC_{n-1}(-1)^{n-1} = (-1)^n + (-1)^n n$
$= (-1)^n(1+n)$

67 **(c)**
The given expression is $\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3+1}\right)^5$
We know that
$(x+a)^n + (x-a)^n$
$= 2[\,{}^nC_0x^n + {}^nC_2x^{n-2}a^2 + {}^nC_4x^{n-4}a^4+\cdots]$
Therefore the given expression is equal to
$2[\,{}^5C_0x^5 + {}^5C_2x^3(x^3-1) + {}^5C_4x(x^3-1)^2]$
Maximum power of x involved here is 7, also only +ve integral powers of x are involved, therefore the given expression is a polynomial of degree 7

68 **(d)**
General term in the expansion of
$\left(\sqrt{2+\sqrt[3]{3}+\sqrt[6]{5}}\right)^{10}$ is
$\frac{10!}{a!b!c!}(\sqrt{2})^a(\sqrt[3]{3})^b(\sqrt[6]{5})^c$ where $a+b+c=10$
For rational term, we have the following:

Value of a, b, c	Value of term
$a=4, b=0, c=6$	$\frac{10!}{4!\,0!\,6!}(\sqrt{2})^4(\sqrt[3]{3})^0(\sqrt[6]{5})^6 = 4200$
$a=10, b=0, c=0$	$\frac{10!}{10!\,0!\,0!}(\sqrt{2})^{10}(\sqrt[3]{3})^0(\sqrt[6]{5})^0 = 32$
$a=4, b=6, c=0$	$\frac{10!}{4!\,6!\,0!}(\sqrt{2})^4(\sqrt[3]{3})^6(\sqrt[6]{5})^0 = 7560$

69 **(c)**
Since n is even, let $n=2m$. Then,
$$\text{L.H.S.} = S = \frac{2m!\,m!}{(2m)!}[C_0^2 - 2C_1^2 + 3C_2^2 + \cdots + (-1)^{2m}\times(2m+1)C_{2m}^2] \quad (1)$$
$$\Rightarrow S = \frac{2m!\,m!}{(2m)!}[(2m+1)C_0^2 - 2mC_1^2 + (2m-1)\times C_2^2 + \cdots + C_0^2] \quad (2)\ (\text{using } C_r = C_{n-r})$$
Adding (1) and (2), we get
$$2S = 2\frac{m!\,m!}{(2m)!}(2m+2)[C_0^2 - C_1^2 + C_2^2 + \cdots + C_{2m}^2]$$
Now keeping in mind that $C_0^2 - C_1^2 + C_2^2 - \cdots + C_n^2 = (-1)^{n/2}\,{}^nC_{n/2}$
If n is even, we get
$$S = 2\frac{2m!\,m!}{(2m)!}(m+1)[(-1)^m\,{}^{2m}C_m]$$
$$= 2\left(\frac{n}{2}+1\right)(-1)^{n/2}$$
$$= (-1)^{n/2}(n+2)$$

70 **(c)**
$(1+x)^{21} + (1+x)^{22} + \cdots + (1+x)^{30}$
$$= (1+x)^{21}\left[\frac{(1+x)^{10}-1}{(1+x)-1}\right]$$
$$= \frac{1}{x}[(1+x)^{31} - (1+x)^{21}]$$
$\Rightarrow$ Coefficient of x^5 in the given expression
$=$ Coefficient of x^5 in $\left\{\frac{1}{x}[(1+x)^{31} - (1+x)^{21}]\right\}$
$=$ Coefficient of x^6 in $[(1+x)^{31} - (1+x^{21})]$
$= {}^{31}C_6 - {}^{21}C_6$

71 **(b)**
$f(x) = 1 - x + x^2 - x^3 + \cdots - x^{15} + x^{16} - x^{17}$
$$= \frac{1-x^{18}}{1+x}$$
$$\Rightarrow f(x-1) = \frac{1-(x-1)^{18}}{x}$$
Therefore, required coefficient of x^2 is equal to coefficient of x^3 in $1-(x-1)^{18}$, which is given by ${}^{18}C_3 = 816$

72 **(b)**
c.e. of x^{-1} in $(1+x)^n\left(1+\frac{1}{x}\right)^n$
=c.e. of x^{-1} in $\frac{(1+x)^{2n}}{x^n}$
=c.e. of x^{n-1} in $(1+x)^{2n}$
$= {}^{2n}C_{n-1}$
$$= \frac{(2n)!}{(n-1)!\,(n+1)!}$$

73 **(d)**
$(1+3x+2x^2)^6 = [1+x(3+2x)]^6$
$= 1+{}^6C_1x(3+2x)+{}^6C_2\,x^2(3+2x)^2+{}^6C_3x^3(3+2x)^3+{}^6C_4x^4(3+2x)^4 + {}^6C_5x^5(3+2x)^5+{}^6C_6x^6(3+2x)^6$
We get x^{11} only from ${}^6C_6x^6\,(3+2x)^6$. Hence, coefficient of x^{11} is ${}^6C_5\times3\times2^5 = 576$

74 **(d)**
$(x-2)^5(x+1)^5$
$= [\,{}^5C_0x^5 - {}^5C_1\,x^4\times2+\cdots][\,{}^5C_0+{}^5C_1x+\cdots]$
$\Rightarrow$ Coefficient of x^5
$= {}^5C_0\,{}^5C_5 - {}^5C_1\times2\times{}^5C_4 + {}^5C_2\times2^2\times{}^5C_3 - {}^5C_3\times2^3\times{}^5C_2 + {}^5C_4\times2^4\times{}^5C_1 - {}^5C_5\times2^5\times{}^5C_0$
$= 1 - 5\times5\times2 + 10\times10\times4 - 10\times10\times8 + 5\times5\times16 - 32$
$= -81$

75 **(b)**

Here $a = {}^nC_r, b = {}^nC_{r+1}$ and $c = {}^nC_{r+2}$

Put $n = 2, r = 0$, then option (b) holds the condition, i.e.,

$$n = \frac{2ac + ab + bc}{b^2 - ac}$$

76 **(b)**

$$\frac{f(x)}{1-x} = b_0 + b_1x + b_2x^2 + \cdots + b_nx^n + \cdots$$

$\Rightarrow a_0 + a_1x + a_2x^2 + \cdots + a_nx^n + \cdots$

$= (1-x)(b_0 + b_1x + b_2x^2 + \cdots + b_nx^n + \cdots)$

Comparing the coefficient of x^n on the both sides,

$a_n = b_n - b_{n-1}$

77 **(b)**

$T_5 = {}^nC_4a^{n-4}(-2b)^4$

and $T_6 = {}^nC_5\, a^{n-5}(-2b)^5$

As $T_5 + T_6 = 0$, we get

${}^nC_4 2^4\, a^{n-4}b^4 = {}^nC_5 2^5 a^{n-5}b^5$

$$\Rightarrow \frac{a^{n-4}b^4}{a^{n-5}b^5} = \frac{n!\,2^5}{5!\,(n-5)!}\cdot\frac{4!\,(n-4)!}{n!\,2^4}$$

$$\Rightarrow \frac{a}{b} = \frac{2(n-4)}{5}$$

78 **(d)**

$$\frac{(1+x)^{3/2} - \left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}}$$

$$= \frac{\left(1+\frac{3}{2}x+\frac{3}{8}x^2\right) - \left(1+\frac{3}{2}x+3\frac{x^2}{4}\right)}{(1-x)^{1/2}}$$

$$= \frac{-3}{8}x^2(1-x)^{-1/2}$$

$$= -\frac{3}{8}x^2\left(1+\frac{x}{2}\right)$$

$$= -\frac{3}{8}x^2$$

79 **(c)**

$(1 + x + x^2 + x^3)^5$

$= a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + a_{15}x^{15}$

Putting $x = 1$ and $x = -1$ alternatively, we have

$a_0 + a_1 + a_2 + a_3 + \cdots + a_{15} = 4^5$ (1)

$a_0 - a_1 + a_2 - a_3 + \cdots - a_{15} = 0$ (2)

Adding (1) and (2), we have

$2(a_0 + a_2 + a_4 + \cdots + a_{14}) = 4^5$

$\Rightarrow a_0 + a_2 + a_4 + \cdots + a_{14} = 2^9 = 512$

80 **(b)**

The given expression is the coefficient of x^4 in

${}^4C_0(1+x)^{404} - {}^4C_1(1+x)^{303} + {}^4C_2(1+x)^{202} - {}^4C_3(1+x)^{101} + {}^4C_4$

=Coefficient of x^4 in $[(1+x)^{101} - 1]^4$

=Coefficient of x^4 in $({}^{101}C_1x + {}^{101}C_2x^2 + \cdots)^4$

$= (101)^4$

81 **(b)**

$n!\,(21-n)! = 21!\frac{n!(21-n)!}{21!} = \frac{21!}{{}^{21}C_n}$ which is minimum

When ${}^{21}C_n$ is maximum which occurs when $n = 10$

82 **(b)**

Let,

$$S = \frac{{}^nC_0}{n} + \frac{{}^nC_1}{n+1} + \frac{{}^nC_2}{n+2} + \cdots + \frac{{}^nC_n}{2n}$$

$$= {}^nC_0\int_0^1 x^{n-1}dx + {}^nC_1\int_0^1 x^n dx + \cdots + {}^nC_n\int_0^1 x^{2n-1}dx$$

$$= \int_0^1 [{}^nC_0x^{n-1} + {}^nC_1\,x^n + \cdots + {}^nC_nx^{2n-1}]dx$$

$$= \int_0^1 x^{n-1}(1+x)^n dx$$

$$= \int_1^2 x^n(x-1)^{n-1}dx$$

83 **(b)**

$$t_{r+1} = {}^{10}C_r\left(\sqrt{x}\right)^{10-r}\left(\frac{-k}{x^2}\right)^r = {}^{10}C_rx^{5-5r/2}(-k)^r$$

For this to be independent of x, r must be 2, so that

${}^{10}C_2\, k^2 = 405 \Rightarrow k = \pm 3$

84 **(c)**

$(1+x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + \cdots + C_{n-1}x^{n-1} + C_nx^n$ (1)

$(x+1)^n = C_0x^n + C_1x^{n-1} + C_2x^{n-2} + \cdots + C_{n-1}x + C_n$ (2)

Multiplying Eqs. (1) and (2) and equating the coefficient of x^{n-2}, we get

$C_0C_2 + C_1C_3 + C_2C_4 + \cdots + C_{n-2}C_n$

= Coefficient of x^{n-2} in $(1+x)^{2n}$

$= {}^{2n}C_{n-2}$

$$= \frac{(2n)!}{(n+2)!\,(n+2)!}$$

85 **(b)**

We have, a = sum of the coefficients in the expansion of

$(1 - 3x + 10x^2)^n = (1 - 3 + 10)^n = (8)^n = (2)^{3n}$ (putting $x = 1$)

Now, b = sum of the coefficients in the expansion of $(1+x^2)^n$

$= (1+1)^n = 2^n$. Clearly, $a = b^3$

86 **(a)**

Given that r and n are +ve integers such that $r > 1, n > 2$. Also, in the expansion of $(1+x)^{2n}$,
Coefficient of $3r^{\text{th}}$ term = coefficient of $(r+2)^{\text{th}}$ term
$\Rightarrow\ {}^{2n}C_{3r-1} = {}^{2n}C_{r+1}$
$\Rightarrow 3r-1 = r+1$ or $3r-1+r+1$
$= 2n$ [using ${}^{n}C_x \Rightarrow {}^{n}C_y \Rightarrow x = y$ or $x+y=n$]
$\Rightarrow r = 1$ or $2r = n$
But $r > 1$
$\therefore n = 2r$

87 **(c)**
For $\left(ax^2+\left(\frac{1}{bx}\right)\right)^{11}, T_{r+1} = {}^{11}C_r(ax^2)^{11-r}\left(\frac{1}{bx}\right)^r$
$= {}^{11}C_r a^{11-r}\frac{1}{b^r}x^{22-3r}$
For x^7,
$22-3r=7$
$\Rightarrow 3r = 15$
$\Rightarrow r = 5$
$\Rightarrow T_6 = {}^{11}C_5 a^6\frac{1}{b^5}x^7$
$\Rightarrow$ Coefficient of x^7 is ${}^{11}C_5\frac{a^6}{b^5}$
Similarly, coefficient of x^{-7} in $\left(ax-\left(\frac{1}{bx^2}\right)\right)^{11}$ is ${}^{11}C_6\frac{a^5}{b^6}$
Given that
${}^{11}C_5\frac{a^6}{b^5} = {}^{11}C_6\frac{a^5}{b^6}$
$\Rightarrow\ a = \frac{1}{b}$
$\Rightarrow ab = 1$

88 **(b)**
$(1-x)^{30} = {}^{30}C_0x^0 - {}^{30}C_1x^1 + {}^{30}C_2x^2 + \cdots + (-1)^{30}\,{}^{30}C_{30}x^{30}$ (1)
$(x+1)^{30} = {}^{30}C_0x^{30} + {}^{30}C_1x^{29} + {}^{30}C_2x^{28} + \cdots + {}^{30}C_{10}x^{20} + \cdots + {}^{30}C_{30}x^0$ (2)
Multiplying (1) and (2) and equating the coefficient of x^{20} on both sides, we get required sum is equal to coefficient of x^{20} in $(1-x^2)^{30}$, which is given by ${}^{30}C_{10}$

89 **(d)**
$\frac{1}{(1-ax)(1-bx)}$
$= a_0 + a_1x + a_2x^2 + \cdots + a_nx^n + \cdots$
But $(1-ax)^{-1}(1-bx)^{-1} = (1+ax+a^2x^2+$ *...1+bx+b2x2+...*
$\Rightarrow$ Coefficient of x^2 is $b^n + ab^{n-1} + a^2b^{n-2} + \cdots + a^{n-1}b + a^n$
$= \frac{b^{n+1}-a^{n+1}}{b-a}$
$\Rightarrow a_n = \frac{b^{n+1}-a^{n+1}}{b-a}$

90 **(d)**
$(1+2x+3x^2+\cdots)^{-3/2} = [(1-x)^2]^{-3/2}$
$= (1-x)^3 = 1-3x+3x^2-x^3$
Therefore, coefficient of x^5 is 0

91 **(a)**
$\sum_{r=0}^{10}(r)\ {}^{20}C_r = \sum_{r=1}^{10} 20\times{}^{19}C_{r-1}$
$= 20({}^{19}C_0 + {}^{19}C_1 + \cdots + {}^{19}C_{10})$
$= 20({}^{19}C_0 + {}^{19}C_1 + \cdots + {}^{19}C_{10})$
$= 20\left(\frac{1}{2}\times 2^{19} + {}^{19}C_{10}\right)$
$= 20(2^{18} + {}^{19}C_{10})$

92 **(d)**
The general term in the expansion of $(1-x+$ *y20* is
$\frac{20!}{r!s!t!}1^r(-x)^5(y)^r$, where $r+s+t=20$
For x^2y^3, we have the term
$\frac{20!}{15!\,2!\,3!}1^{15}(-x)^2(y)^3$
Hence, the coefficient of x^2y^3 is
$\frac{20!}{15!\,2!\,3!}$

93 **(a)**
$N = {}^{2n}C_n = \frac{(2n)!}{(n!)^2} = \frac{(n+1)(n+2)\cdots(n+n)}{(n!)}$
$\Rightarrow (n!)N = (n+1)(n+2)\cdots(n+n)$
Since $n < p < 2n$, so p divides $(n+1)(n+$ *2⋯n+n*

94 **(d)**
$\frac{{}^{n+1}C_{r+1}}{{}^{n}C_r} = \frac{11}{6} \Rightarrow \frac{\frac{n+1}{r+1}\times{}^{n}C_r}{{}^{n}C_r} = \frac{11}{6}$
$\Rightarrow 6n+6 = 11r+11 \Rightarrow 6n-11r = 5$ (1)
Also,
$\frac{{}^{n}C_r}{{}^{n-1}C_{r-1}} = \frac{6}{3} \Rightarrow \frac{\frac{n}{r}\times{}^{n-1}C_{r-1}}{{}^{n-1}C_{r-1}} = \frac{6}{3} \Rightarrow n \Rightarrow 2r$ (2)
From (1) and (2), $r = 5$ and $n = 10$,
$\therefore\ nr = 50$

95 **(b)**
$(a-1)^n, n \geq 5$
In the binomial expansion,
$T_5 + T_6 = 0$
$\Rightarrow\ {}^{n}C_4a^{n-4}b^4 - {}^{n}C_5a^{n-5}b^5 = 0$

$\Rightarrow \frac{^nC_4}{^nC_5}\frac{a}{b} = 1 \Rightarrow \frac{4+1}{n-4}\frac{a}{b}$

$= 1 \left[\text{using } \frac{^nC_r}{^nC_{r+1}} = \frac{r+1}{n-r}\right]$

$\Rightarrow \frac{a}{b} = \frac{n-4}{5}$

96 **(b)**

We have $T_{r+1} = {}^{29}C_r 3^{29-r}(7x)^r = ({}^{29}C_r \times$ *329−r×7rxr*

Coefficient of $(r+1)^{\text{th}}$ term is ${}^{29}C_r \times 3^{29-r} \times 7^r$

And coefficient of r^{th} term is ${}^{29}C_{r-1} \times 3^{30-r} \times 7^{r-1}$

From given condition,

${}^{29}C_r \times 3^{29-r} \times 7^r = {}^{29}C_{r-1} \times 3^{30-r} \times 7^{r-1}$

$\Rightarrow \frac{^{29}C_r}{^{29}C_{r-1}} = \frac{3}{7} \Rightarrow \frac{30-r}{r} = \frac{3}{7} \Rightarrow r = 21$

97 **(b)**

We have, $f(x) = x^n$. So,

$f^1(x) = nx^{n-1} \Rightarrow f(1) = n$

$f^2(x) = n(n-1)x^{n-2} \Rightarrow f^2(1) = n(n-1)$

$f^3(x) = n\big(n-1(n-2)\big)x^{n-3} \Rightarrow f^3(1)$

$= n(n-1)(n-2)$

⋮

$f^n(x) = n(n-1)(n-2)\dots 1 \Rightarrow f^n(1)$

$= n(n-1)(n-2)\dots 1$

$\Rightarrow f(1) + \frac{f^1(1)}{1} + \frac{f^2(1)}{2!} + \cdots \frac{f^n(1)}{n!}$

$= 1 + \frac{n}{1} + \frac{n(n-1)}{2!} + \frac{n(n-1)(n-2)}{3!} \cdots$

$+ \frac{n(n-1)(n-2)\cdots 1}{n!}$

$= {}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n$

$= 2^n$

98 **(d)**

$(1+x+x^3+x^4)^{10} = (1+x)^{10}(1+x^3)^{10}$

$= (1 + {}^{10}C_1x + {}^{10}C_2x^2 + {}^{10}C_3x^3 + {}^{10}C_4x^4 \cdots)$

$(1 + {}^{10}C_1x^3 + {}^{10}C_2x^6 + \cdots)$

Therefore, coefficient of x^4 is ${}^{10}C_1\,{}^{10}C_1 + {}^{10}C_4 = 310$

99 **(d)**

$\left[\sqrt{1+x^2} - x\right]^{-1} = \frac{1}{\sqrt{1+x^2}-x} \times \frac{(\sqrt{1+x^2}+x)}{(\sqrt{1+x^2}+x)}$

$= \frac{\sqrt{1+x^2}+x}{1+x^2-x^2} = x + \sqrt{1+x^2} = x + (1+x^2)^{1/2}$

$= x + 1 + \frac{1}{2}x^2 + \frac{1}{2}\left(-\frac{1}{2}\right)\frac{x^4}{2!} + \cdots$

Obviously, the coefficient of x^4 is $-1/8$

100 **(a)**

We know that the sum of the coefficients in a binomial expansion is obtained by replacing each variable by unit in the given expression.

Therefore, sum of the coefficients in $(a+b)^n$ is given by $(1+1)^n$

$\therefore 4096 = 2^n \Rightarrow 2^n = 2^{12} \Rightarrow n = 12$

Hence, n is even. So, the greatest coefficient is ${}^nC_{n/2}$, i.e.,

${}^{12}C_6 = 924$

101 **(b,c)**

For $n = 2m$, the given expression is

$C_0 - (C_0 + C_1) + (C_0 + C_1 + C_2)$

$- (C_0 + C_1 + C_2 + C_3)$

$+ \cdots (-1)^{n-1}(C_0 + C_1 + \cdots + C_{n-1})$

$= C_0 - (C_0 + C_1) + (C_0 + C_1 + C_2)$

$- (C_0 + C_1 + C_2 + C_3) + \cdots - (C_0$

$+ C_1 + \cdots + C_{2m-1})$

$= -(C_1 + C_3 + C_5 + \cdots + C_{2m-1})$

$= -(C_1 + C_3 + C_5 + \cdots + C_{n-1}) = -2^{n-1}$

102 **(a,d)**

$\because 3^{4n} = 81^n = (1+80)^n = 1 + 80\lambda, \lambda \in N$

$\therefore 3^{3^{4n}} = 3^{1+80\lambda} = 3 \cdot 3^{80\lambda} = 3 \cdot (9)^{40\lambda}$

$= 3(10-1)^{40\lambda}$

$= 3(1 + 10\mu) = 3 + 30\mu$

$\therefore$ Last digit of $3^{3^{4n}} + 1$ is 4

103 **(c,d)**

$\because$ Number of distinct terms $= {}^{9+3-1}C_{3-1} = {}^{11}C_2 = 55$

Sum of coefficients $= (2-2+1)^9 = 1^9 = 1$

and $(2 - 2x + x^2)^9 = \sum \frac{9!}{\alpha!\,\beta!\,\gamma!}(2)^\alpha(-2x)^\beta(x^2)^\gamma$

Here, $\beta + 2\gamma = 4, \alpha + \beta + \gamma = 9$

	α	β	γ
	5	4	0
∴	6	2	1
	7	0	2

$\therefore$ Coefficient of x^4

$= \frac{9!}{5!\,4!\,0!} \cdot 2^5 \cdot (-2)^4 + \frac{9!}{6!\,2!\,1!}(2)^6(-2)^2$

$+ \frac{9!}{7!\,0!\,2!}(2)^7(-2)^0$

$= 2^9(126 + 126 + 9) = 133632$

104 **(a,b,c)**

$\because (101)^{50} - (99)^{50} = (100+1)^{50} - (100-1)^{50}$

$= 2\{ {}^{50}C_1(100)^{49} + {}^{50}C_3(100)^{47} + {}^{50}C_5(100)^{45} + \ldots\}$

$= (100)^{50} + 2\{ {}^{50}C_3(100)^{47} + {}^{50}C_5(100)^{45} + \ldots\}$

$> (100)^{50}$

$\Rightarrow (101)^{50} - (99)^{50} > (100)^{50}$

$\Rightarrow (101)^{50} - (100)^{50} > (99)^{50}$

Also, $\left(\frac{1001}{1000}\right)^{999} = \left(1 + \frac{1}{1000}\right)^{999}$

$= 1 + {}^{999}C_1\left(\frac{1}{1000}\right) + {}^{999}C_2\left(\frac{1}{1000}\right)^2 + \ldots$

$< 1+1+1+1+\ldots+1$

$= 1000$

$\therefore \left(\frac{1001}{1000}\right)^{999} < 1000$

$\Rightarrow (1001)^{999} < (1000)^{1000}$

105 **(a,b,c,d)**

Let T_5 be numerically the greatest term in the expansion of $(1+x/3)^{10}$

Then,

$\left[\frac{T_5}{T_4}\right] \geq 1$ and $\left[\frac{T_6}{T_5}\right] \leq 1$

Now,

$\frac{T_{r+1}}{T_r} = \frac{10-r+1}{r}\frac{x}{3}$

$\Rightarrow \left|\frac{7}{4} \times \frac{x}{3}\right| \geq 1$ and $\left|\frac{6}{5} \times \frac{x}{3}\right| \leq 1$

$\Rightarrow |x| \geq \frac{12}{7}$ and $|x| \leq \frac{5}{2}$ (1)

$\Rightarrow \frac{12}{7} \leq |x| \leq \frac{5}{2}$

$\Rightarrow x \in \left[-\frac{5}{2}, -\frac{12}{7}\right] \cup \left[\frac{12}{7}, \frac{5}{2}\right]$

106 **(a,d)**

Middle term is $\left(\frac{n}{2}+1\right)^{\text{th}}$ or $(4+1)^{\text{th}}$ or T_5

$\Rightarrow T_5 = {}^8C_4\left(\frac{x}{2}\right)^4 \times 2^4 = 1120$

$\Rightarrow \frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4}x^4 = 1120$

$\Rightarrow x^4 = \frac{1120}{70} = 16$

$\Rightarrow (x^2+4)(x^2-4) = 0$

$\Rightarrow x = \pm 2 \ (\because x \in R)$

107 **(a,b,c,d)**

We know that to get the sum of coefficients, we put $x = 1$

Then, sum of coefficients is $(1+ax-2x^2)^n$ is $(a-1)^n$

Obviously, when $a > 1$, sum is positive for any n

108 **(a,c)**

$(1-y)^m(1+y)^n$

$= (1 - {}^mC_1 y + {}^mC_2 - \ldots)(1 + {}^nC_1 y + {}^nC_2 y^2 + \ldots)$

$= 1 + (n-m)y + \left\{\frac{m(m-1)}{2} + \frac{n(n-1)}{2} - mn\right\}y^2 + \ldots$

Given,

$a_1 = 10$

$\Rightarrow a_1 = n - m = 10$ (1)

$a_2 = \frac{m^2 + n62 - m - n - 2mn}{2} = 10$

$(m-n)^2 - (m+n) = 20$

$\Rightarrow m + n = 80$ (2)

Solving (1) and (2), we get $m = 35, n = 45$

109 **(a,d)**

Coefficients of r^{th}, $(r+1)^{\text{th}}$ and $(r+2)^{\text{th}}$ term are ${}^{14}C_{r-1}$, ${}^{14}C_r$ and ${}^{14}C_{r+1}$

If these coefficients are in A.P., then

$2({}^{14}C_r) = {}^{14}C_{r-1} + {}^{14}C_{r+1}$

$\Rightarrow \frac{2(14)!}{r!\,(14-r)} = \frac{(14)!}{(r-1)!\,(15-r)!} + \frac{(14)!}{(r+1)!\,(13-r)}$

$\Rightarrow \frac{2(14)!}{r!\,(14-r)!}$

$= \frac{(14)!\,[(r+1)r + (15-r)(14-r)]}{(r+1)!\,(15-r)!}$

$\Rightarrow 2(15-r)(r+1) = 2r^2 - 28r + 210$

$\Rightarrow r^2 - 14r + 45 = 0$ or $(r-5)(r-9) = 0$

$\Rightarrow r = 5$ or 9

110 **(a,b,c)**

General term is ${}^{6561}C_r 7^{\frac{6561-r}{3}} 11^{\frac{r}{9}}$

To make the term free of radical sign, r should be a multiple of 9

$\therefore r = 0, 9, 18, 27, \ldots 6561$

Hence, there are 730 terms. The greatest binomial coefficients are

${}^{6561}C_{\frac{6561-1}{2}}$ and ${}^{6561}C_{\frac{6561-3}{2}}$ or ${}^{6561}C_{3280}$ and ${}^{6561}C_{3279}$

Now, 3280 are 3279 are not a multiple of 3; hence, both terms involving greatest binomial coefficients are irrational

111 **(c,d)**

${}^{69}C_{3r-1} + {}^{69}C_{3r} = {}^{69}C_{r^2-1} + {}^{69}C_{r^2}$

$\Rightarrow {}^{70}C_{3r} = {}^{70}C_{r^2}$

Thus, $r^2 = 3r$ or $70 - 3r = r^2$ so that $r = 0,3$ or $7,10$

Hence, $r = 3$ and 7(as the given equation is nor defined for $r = 0$ and -10)

112 **(a,c,)**

$\left(x^2+1+\frac{1}{x^2}\right)$

$$= {}^nC_0 + {}^nC_1\left(x^2+\frac{1}{x^2}\right) + {}^nC_2\left(x^2+\frac{1}{x^2}\right)^2 + \cdots$$
$$+ {}^nC_n + \left(x^2+\frac{1}{x^2}\right)^n$$

This contains each of the term

$x^0, x^2, x^4, \ldots x^2n, x^{-2}, x^{-4}, \ldots, x^{-2n}$

Coefficient of constant term $= nC_0 + (nC_2)(2) + (nC_4)(4C_2) + (nC_6)(6C_3) + \cdots \neq 2^{n-1}$ coefficient of x^{2n-2} in $nC_{n-1} = n$ coefficient of x^2 is

${}^nC_1 + ({}^nC_3)({}^3C_1) + (nC_5)({}^5C_2) + \cdots > n$

113 **(a,b,c)**

$(x\sin p + x^{-1}\cos p)^{10}$

The general term in the expansion is

$T_{r+1} = {}^{10}C_r(x\sin p)^{10-r}(x^{-1}\cos p)^r$

For the term independent of x, we have

$10-2r=0$ or $r=5$

Hence, the independent term is

$${}^{10}C_5 \sin^5 p\cos^5 p = {}^{10}C_5\frac{\sin^5 2p}{32}$$

Which is the greatest when $\sin 2p = 1$

The least value of ${}^{10}C_5\frac{\sin^5 2p}{32}$ is $-\frac{10!}{2^5(5!)^2}$ when $\sin 2p$

$= -1$ or $p = (4n-1)\frac{\pi}{4}, n \in Z$

Sum of coefficient is $(\sin p + \cos p)^{10}$, when $x=1$ or $(1+\sin 2p)^5$, which is least when $\sin 2p = -1$

Hence, least sum of coefficients is zero. Greatest sum of coefficient occurs when $\sin 2p = 1$. Hence, greatest sum is $2^5 = 32$

114 **(a,c,d)**

$I + f = (4+\sqrt{15})^n$

Let $f' = (4-\sqrt{15})^n$. then $0 < f' < 1$

$$I+f = {}^nC_0 4^n + {}^nC_1 4^{n-1}\sqrt{15} + {}^nC_2 4^{n-2}15 + {}^nC_3 4^{n-3}(\sqrt{15})^3 + \cdots$$
$$f' = {}^nC_0 4^n - {}^nC_1 4^{n-1}\sqrt{15} + {}^nC_2 4^{n-2}\cdot 15 - {}^nC_3 4^{n-3}(\sqrt{15})^3 + \cdots$$

$\therefore I + f + f' = 2({}^nC_0 4^n + {}^nC_2 4^{n-2}\times 15 + \cdots) =$ even integer

$\because 0 < f + f' < 2 \Rightarrow f + f' = 1 \Rightarrow 1 - f = f'$

Thus, I is an odd integer. Now,

$1 - f = f' = (4-\sqrt{15})^n$

$(I+f)(1-f) = (I+f)f' = 1$

115 **(a,d)**

It is given that the fourth term in the expansion of $\left(ax+\frac{1}{x}\right)^n$ is $\frac{5}{2}$, therefore

${}^nC_3(ax)^{n-3}\left(\frac{1}{x}\right)^3 = \frac{5}{2} \Rightarrow {}^nC_3 a^{n-3}x^{n-6} = \frac{5}{2}$ (i)

[$\because$ R.H.S is independent of x]

Putting $n=6$ in (i), we get ${}^6C_3 a^3 = \frac{5}{2} \Rightarrow a^3 = \frac{1}{8} \Rightarrow a = \frac{1}{2}$

116 **(a,b,d)**

$$f(m) = \sum_{i=0}^{m}\binom{30}{30-i}\binom{20}{m-i}$$
$$= \sum_{i=0}^{m}\binom{30}{i}\binom{20}{m-i} = {}^{50}C_m$$

$f(m)$ is greatest when $m = 25$. Also,

$f(0) + f(1) + \cdots + f(50)$

$= {}^{50}C_0 + {}^{50}C_1 + {}^{50}C_2 + \cdots + {}^{50}C_{50} = 2^{50}$

Also, ${}^{50}C_m$ is not divisible by 50 for any m as 50 is not a prime number

$$\sum_{m=0}^{50}(f(m))^2 = ({}^{50}C_0)^2 + ({}^{50}C_1)^2 + ({}^{50}C_2)^2 + \cdots + ({}^{50}C_{50})^2 = {}^{100}C_{50}$$

117 **(a,b,d)**

$$\frac{(n-1)(n-2)\cdots(n-m+1)}{(m-1)!}$$
$$= \frac{(n-1)(n-2)\cdots(n-m+1)(n-m)\cdots 2.1}{(n-m)!\,(m-1)!}$$

$= {}^{n-1}C_{m-1}$

= Coefficient of x^{m-1} in $(1+x)^{n-1}$

= Coefficient of x^{m-1} in $(1+x)^n(1+x)^{-1}$

Now,

$(1+x)^n = C_0 + C_1x + C_2x^2 + \cdots + C_{m-1}x^{m-1} + \cdots + C_nx^n$ (1)

$(1+x)^{-1} =$

$1 - x + x^2 - x^3 + \cdots + (-1)^{m-1}x^{m-1} + \cdots$

(2)

Collecting the coefficient of x^{m-1} in the product of (1) and (2), we get

$(-1)^{m-1}C_0 + (-1)^{m-2}C_1 + \cdots + C_{m-1}$

= Coefficient of x^{m-1} in $(1+x)^{n-1}$

$= {}^{n-1}C_{m-1}$

$\therefore C_0 - C_1 + C_2 - \cdots + (-1)^{m-1}C_{m-1}$

$= {}^{n-1}C_{m-1}(-1)^{m-1}$

$$= \frac{(n-1)(n-2)\cdots(n-m+1)}{(m-1)!}(-1)^{m-1}$$

118 **(b,c,d)**

L.H.S $= (1+2x^2+x^4)(1+C_1x+C_2x^2+C_3x^3+\cdots$

R.H.S$= a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$

Comparing the coefficient of $x, x^2, x^3, \cdots$

$a_1 = C_1, a_2 = C_2 + 2, a_3 = C_3 + 2C_1$ (1)

Now, $2a_2 = a_1 + a_3$(A.P.)

$\Rightarrow 2({}^nC_2 + 2) = {}^nC_1 + ({}^nC_3 + 2{}^nC_1)$ [Using (1)]

$\Rightarrow 2\frac{n(n-1)}{2} + 4 = 3n + \frac{n(n-1)(n-2)}{6}$

$\Rightarrow n^3 - 9n^2 + 26n - 24 = 0$

$\Rightarrow (n-2)(n^2 - 7n + 12) = 0$

$\Rightarrow (n-2)(n-3)(n-4) = 0$

$\Rightarrow n = 2, 3, 4$

119 **(a,b,d)**

$(1 + z^2 + z^4)^8 = C_0 + C_1 z^2 + C_2 z^4 + \cdots + C_{16} z^{32}$ (1)

Putting $z = i$, where $i = \sqrt{-1}$,

$(1 - 1 + 1)^8 = C_0 - C_1 + C_2 - C_3 + \cdots + C_{16}$

$\Rightarrow C_0 - C_1 + C_2 - C_3 + \cdots + C_{16} = 1$

Also, putting $z = \omega$,

$(1 + \omega^2 + \omega^4)^8 = C_0 + C_1\omega^2 + C_2\omega^4 + \cdots + C_{16}\omega^{32}$

$\Rightarrow C_0 + C_1\omega^2 + C_2\omega + C_3 + \cdots + C_{16}\omega^{32} = 0$ (2)

Putting $x = \omega^2$,

$(1 + \omega^4 + \omega^8)^8 = C_0 + C_1\omega^4 + C_2\omega^8 + \cdots + C_{16}\omega^{64}$

$\Rightarrow C_0 + C_1\omega + C_2\omega^2 + \cdots + C_{16}\omega = 0$ (3)

Putting $x = 1$,

$3^8 = C_0 + C_1 + C_2 + \cdots + C_{16}$ (4)

Adding (2), (3) and (4), we have

$3(C_0 + C_3 + \cdots + C_{15}) = 3^8$

$\Rightarrow C_0 + C_3 + \cdots + C_{15} = 3^7$

Similarly, first multiplying (1) by z and then putting $1, \omega, \omega^2$ and adding we get

$C_1 + C_4 + C_7 + C_{10} + C_{13} + C_{16} = 3^7$

Multiplying (1) by z^2 and then putting $1, \omega, \omega^2$ and adding, we get

$C_2 + C_5 + C_8 + C_{11} + C_{14} = 3^7$

120 **(a)**

We have,

$\frac{17}{4} + 3\sqrt{2} = \frac{1}{4}(9 + 8 - 12\sqrt{2})$

$= \frac{1}{4}(3 - 2\sqrt{2})^2$

$\therefore\ 3 - \sqrt{\frac{17}{4} + 3\sqrt{2}} = 3 - \frac{1}{2}(3 + 2\sqrt{2})$

$= \frac{3}{2} - \sqrt{2}$

Hence, the 10th term of $\left(3 - \sqrt{\frac{17}{4} + 3\sqrt{2}}\right)^{20} = \left(\frac{3}{2} - \sqrt{2}\right)^{20}$ is

${}^{20}C_9\left(\frac{3}{2}\right)^{20-9}(-\sqrt{2})^9$

Which is an irrational number

121 **(a,b,c)**

We have,

$(x + a)^n = {}^nC_0\, x^n + {}^nC_1 x^{n-1}a + {}^nC_2 x^{n-2}a^2 + \cdots + {}^nC_n a^n$

$= [{}^nC_0 x^n + {}^nC_2 x^{n-2}a^n + \cdots] + [{}^nC_1 x^{n-1}a + {}^nC_3 x^{n-3}a^3 + \cdots]$

or

$(x + a)^n = P + Q$ (1)

Similarly,

$(x - a)^n = P - Q$ (2)

1. $(1) \times (2) \Rightarrow P^2 - Q^2 = (x^2 - a^2)^n$
2. Squaring (1) and (2) and subtracting (2) from (1), we get $4PQ = (x + a)^{2n} - (x - a)^{2n}$
3. Squaring (1) and (2) and adding, $2(P^2 + Q^2) = (x + a)^{2n} + (x - a)^{2n}$

122 **(a,b,c,d)**

On putting $x = \frac{1}{x}$ in given equation, we get

$\sum_{r=0}^{2n} a_r\left(\frac{1}{x}\right)^r = \left(1 + \frac{1}{x} + \frac{1}{x^2}\right)^n = \frac{1}{x^{2n}}(x^2 + x + 1)^n$

$\Rightarrow \sum_{r=0}^{2n} a_r x^{2n-r} = (x^2 + x + 1)^n = \sum_{r=0}^{2n} a_r x^r$

$= \sum_{r=0}^{2n} a_{2n-r} x^{2n-r}$ ...(i)

On equating the coefficients of x^{2n-r} on both sides, we get $a_r = a_{2n-r}$ for $0 \le r \le 2n$

Now, on putting $x = 1$ in Eq. (i), we get

$a_0 + a_1 + a_2 + \ldots + a_{2n} = (1 + 1 + 1)^n = 3^n$ ...(ii)

But $a_r = a_{2n-r}$, for $0 \le r \le n - 1$

$2(a_0 + a_1 + \ldots + a_{n-1}) + a_n = 3^n$

$a_0 + a_1 + \ldots + a_{n-1} = \frac{1}{2}(3^n - a_n)$

Again

$(1 + x + x^2)^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots + a_{2n} x^{2n}$ ...(iii)

On replacing x by $-\frac{1}{x}$, we get

$\left(1 - \frac{1}{x} + \frac{1}{x^2}\right)^n = a_0 - \frac{a_1}{x} + \frac{a_2}{x^2} - \frac{a_3}{x^3} + \ldots + \frac{a_{2n}}{x^{2n}}$ ...(iv)

On multiplying Eqs. (iii) and (iv) and comparing constant terms, then

RHS$= a_0^2 - a_1^2 + a_2^2 - a_3^2 + \ldots + a_{2n}^2$

$=$ Constant term in $(1 + x + x^2)^n\left(1 - \frac{1}{x} + \frac{1}{x^2}\right)^n$

$\therefore$ Coefficient of x^{2n} in $(1 + x^2 + x^4)^n$ is a_n

Again putting $x = -1$ in Eq.(i), we get

$a_0 - a_1 + a_2 - a_3 + \ldots + a_{2n} = 1$ ...(v)

On adding Eqs. (ii) and (v) and dividing by 2, we get

$a_0 + a_2 + a_4 + \ldots + a_{2n} = \frac{1}{2}(3^n + 1)$

123 **(a,c,d)**

${}^nC_1 + {}^{n+1}C_2 + {}^{n+2}C_3 + \cdots + {}^{n+m-1}C_m$

$= {}^n C_{n-1} + {}^{n+1}C_{n-1} + {}^{n+2}C_{n-1} + \cdots + {}^{n+m-1}C_{n-1}$

$=$ Coefficient of x^{n-1} in $(1+x)^n + (1+x)^{n+1} + (1+x)^{n+2} + \cdots + (1+x)^{n+m-1}$

$=$ Coefficient of x^{n-1} in $(x+1)^n \left[\frac{(1+x)^m - 1}{(1+x) - 1}\right]$

$=$ Coefficient of x^{n-1} in $\frac{(1+x)^{m+n} - (1+x)^n}{x}$

$=$ Coefficient of x^n in $[(1+x)^{m+n} - (1+x)^n]$

$= {}^{m+n} C_n - 1$

Similarly, we can prove

${}^mC_1 + {}^{m+1}C_2 + {}^{m+2}C_3 + \cdots + {}^{m+n-1}C_n$

$= {}^{m+n}C_m - 1$

124 **(a,c)**

Inclusion of $\log x$ implies $x > 0$

Now, 3rd term in the expansion is

$T_{2+1} = {}^5C_2 x^{5-2}\left(x^{\log_{10} x}\right)^2 = 1000000$ (given)

or

$x^{3 + 2\log_{10} x} = 10^5$

Taking logarithm of both sides, we get

$(3 + 2\log_{10} x)\log_{10} x = 5$

or

$2y^2 + 3y - 5 = 0,$

where $\log_{10} x = y$

or

$(y-1)(2y+5) = 0$ or $y = 1$ or $-5/2$

or

$\log_{10} x = 1$ or $-5/2$

$\therefore x = 10^1 = 10$ or $10^{-5/2}$

125 **(a,b,c)**

$(101)^{100} - 1 = (1 + 100)^{100} - 1$

$= 1 + {}^{100}C_1(100)$

$+ {}^{100}C_2(100)^2 + \ldots\ldots + {}^{100}C_{100}(100)^{100} - 1$

$= 10^4 \lambda \forall \lambda \in N$

126 **(c)**

$$S_1 = \sum_{j=1}^{10} j(j-1)\frac{10!}{j(j-1)(j-2)!\,(10-j)!}$$

$$= 90 = \sum_{j=2}^{10} \frac{8!}{(j-2)\big(8-(j-2)\big)!}$$

$= 90.\,2^8$

$$\text{and } S_2 = \sum_{j=1}^{10} j(j-1)\frac{10!}{j(j-1)!\,\big(9-(j-1)\big)!}$$

$$= 10 = \sum_{j=1}^{10} \frac{9!}{j(j-1)!\,\big(9-(j-1)\big)!} = 10.\,2^9$$

$$\text{and } S_3 = \sum_{j=1}^{10} [j(j-1)+j]\,{}^{10}C_j$$

$$= \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j + \sum_{j=1}^{10} j\,{}^{10}C_j$$

$= 90.\,2^8 + 10.\,2^9$

$= 90.2^8 + 20.2^8 = 110.\,2^8 = 55.\,2^9$

127 **(a)**

Let $\left(\sqrt{5}+2\right)^n = N + f$, where N is an integer and $0 < f < 1$

Let $\left(\sqrt{5}-2\right)^n = f'$, then $0 < f' < 1$

Let $\left(\sqrt{5}+2\right)^n - \left(\sqrt{5}-2\right)^n =$ integer ($\because n$ is odd)

$\therefore N = \left(\sqrt{5}+2\right)^n - \left(\sqrt{5}-2\right)^n$

$= 2[{}^nC_1 \cdot 2 \cdot 5^{(n-1)/2} + {}^nC_3 \cdot 2^3 \cdot 5^{(n-3)/2} + \ldots]$

$\Rightarrow N$ is divisible by $2n$ on using statement II

(If n is prime and $r < n$, then there is no factor which will cut $n \Rightarrow {}^nC_r$ will be divisible by n)

128 **(a)**

Since, coefficient of x^r in $(1-x)^{-n} = {}^{n+r-1}C_r$

$\therefore$ Coefficient of x^n in $(1-x)^{-2} = {}^{2+n-1}C_n$

$= {}^{n+1}C_n = (n+1)$

Hence, option (a) is correct

129 **(d)**

$\because (a+x)^\lambda (b+x)^{\lambda+1} (c+x)^{\lambda+2}$

$= \{(x+a)(x+a) \ldots \lambda \text{ times}\}$

$\{(x+b)(x+b) \ldots (\lambda+1)\text{times}\}$

$\{(x+c)(x+c) \ldots (\lambda+2)\text{times}\}$

$= x^{3\lambda+3} + \{a\lambda + b(\lambda+1) + c(\lambda+2)\}x^{3\lambda+2} + \ldots$

$\therefore$ Coefficient of $x^{3\lambda+2}$ is $\lambda(a+b+c) + b + 2c$

130 **(b)**

Obviously, statement 2 is true. But to get the sum of coefficient in the expansion of $\big(3^{-x/4} +$ *35x4n,* we must put *x=0*

131 **(d)**

$$\left(x+\frac{1}{x}+2\right)^m=\left(\frac{x^2+2x+1}{x}\right)^m=\frac{(1+x)^{2m}}{x^m}$$

Term independent of x is coefficient of x^m in the expansion of $(1+x)^{2m} = {}^{2m}C_m = \frac{(2m)!}{(m!)^2}$

Hence, statement I is false and statement II is true

132 **(b)**

Given expression $= \left\{1+\left(x+\frac{1}{x}\right)\right\}^n$

$$= 1 + {}^nC_1\left(x+\frac{1}{x}\right) + {}^nC_2\left(x+\frac{1}{x}\right)^2 + {}^nC_3\left(x+\frac{1}{x}\right)^3 + \ldots + \left(x+\frac{1}{x}\right)^n$$

This will be of the form

$$= a_0 + a_1x + a_2x^2 + \ldots + a_nx^n + \frac{b_1}{x} + \frac{b_2}{x^2} + \frac{b_3}{x^3} + \ldots + \frac{b_n}{x^n}$$

$\therefore$ Number of terms $= 1 + n + n = 2n + 1$

133 **(a)**

$$S = \sum_{0\le i<}\sum_{j\le n}\left(\frac{i}{{}^nC_i}+\frac{j}{{}^nC_j}\right)$$

$$= \sum_{0\le i<}\sum_{j\le n}\left(\frac{n-i}{{}^nC_{n-i}}+\frac{n-j}{{}^nC_{n-j}}\right)$$

$$= n\sum_{0\le i<}\sum_{j\le n}\left(\frac{1}{{}^nC_i}+\frac{1}{{}^nC_j}\right) - S$$

$$\Rightarrow S = \frac{n}{2}\sum_{0\le i<}\sum_{j\le n}\left(\frac{i}{{}^nC_i}+\frac{j}{{}^nC_j}\right)$$

$$= \frac{n}{2}\left(\sum_{r=0}^{n-1}\frac{n-r}{{}^nC_r}+\sum_{r=1}^{n}\frac{r}{{}^nC_r}\right)$$

$$= \frac{n}{2}\left(\sum_{r=0}^{n}\frac{n}{{}^nC_r}\right)$$

$$= \frac{n^2}{2}a$$

134 **(d)**

Statement 2 is true as it is the property of binomial coefficients. But statement 1 is false as three consecutive binomial coefficients may be in A.P. but not always

135 **(b)**

We know that the total number of terms in $(x_1 + x_2 + \cdots + x_r)^n$ is C_{r-1}. So, the total number of terms in $(x_1 + x_2 + \cdots + x_n)^3$ is

$$ {}^{3+n-1}C_{n-1} = {}^{n+2}C_{n-1} = {}^{n+2}C_3 = \frac{(n+2)(n+1)n}{6}$$

and the total number of terms in $(x_1 + x_2 + x_3)^n$ is

$$ {}^{n+3-1}C_{n-1} = {}^{n+2}C_3 = \frac{(n+2)(n+1)n}{6}$$

136 **(a)**

We have,

$(2+\sqrt{5})^p + (2-\sqrt{5})^p =$
$2[2^p + {}^pC_2 2^{p-5}5 + {}^pC_4\, 2^{p-4}\, 5^2 + \cdots + {}^pC_{p-1}2 \times$
5p–1/2 (1)

From, (1), $(2+\sqrt{5})^p + (2-\sqrt{5})^p$ is an integer and

$$-1 < (2-\sqrt{5})^p < 0 \quad (\because p \text{ is odd})$$

So, $\left[(2+\sqrt{5})^p\right] = (2+\sqrt{5})^p + (2-\sqrt{5})^p$

$$= 2^{p+1} + {}^pC_2 2^{p-1}5 + \cdots + {}^pC_{p-1}2^2 5^{(p-1)/2}$$

$$\therefore \left[(2+\sqrt{5})^p\right] - 2^{p+1} = 2\left[{}^pC_2 2^{p-2}5 + {}^pC_4 2^{p-4}5^2 + \cdots + {}^pC_{p-1}2 \times 5^{(p-1)/2}\right]$$

Now, all the binomial coefficients

$$ {}^pC_2 = \frac{p(p-1)}{1\times 2},$$

$$ {}^pC_2 = \frac{p(p-1)(p-2)(p-3)}{1\times 2\times 3\times 4}, \ldots, {}^pC_{p-1} = p$$

are divisible by the prime p. Thus, R.H.S. is divisible by p

137 **(a)**

$(1+x)^n - nx - 1 =$
$(1+{}^nC_1x + {}^nC_2x^2 + \ldots + {}^nC_nx^n) - nx - 1 \quad (1)$

$= {}^nC_2x^2 + \cdots + {}^nC_nx^n$

$= x^2({}^nC_2 + {}^nC_3x + \cdots + {}^nC_nx^{n-2})$

Hence, $(1+x)^n - nx - 1$ is divisible by x^2

Now in (1), replace x by $8n+1$. Then, we have

$$(1+8)^{n+1} - (n+1)8 - 1 = 8^2({}^nC_2 + {}^nC_3 8 + \cdots + {}^nC_n 8^{n-2})$$

$$\Rightarrow 9^{2n+2} - 8n - 9 = 8^2({}^nC_2 + {}^nC_3 8 + \cdots + {}^nC_n 8^{n-2})$$

Which is divisible by 64

Hence, both the statements are correct and statement 2 is a correct explanation of statement 1

138 **(a)**

Coefficient of x^n in $\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \frac{x^n}{n!}\right)^3$

$= $ Coefficient of x^n in $\left(1 + x + \frac{x^2}{2!} + \cdots \frac{x^n}{n!} + \cdots\right)^3$

(as higher powers of x are not counted while calculating the coefficient of x^n)

$= $ Coefficient of x^n in $e^{3x} = \frac{3^n}{n!}$

139 **(a)**

$$({}^{10}C_0) + ({}^{10}C_0 + {}^{10}C_1) + ({}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2) + \cdots + ({}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + \cdots + {}^{10}C_9)$$

$= 10\,{}^{10}C_0 + 9\,{}^{10}C_1 + 8\,{}^{10}C_2 + \cdots + {}^{10}C_9$

$= {}^{10}C_1 + 2\,{}^{10}C_2 + 3\,{}^{10}C_3 + \cdots + 10\,{}^{10}C_{10}$

$$= \sum_{r=1}^{10} r\,{}^{10}C_r = 10\sum_{r=1}^{10} {}^9C_{r-1} = 10 \times 2^9$$

140 **(a)**

$n(n+1) = n^2 + n < n^2 + n + n + 1 = (n+1)^2$
$\Longrightarrow \sqrt{n(n+1)} < n+1, \forall\, n \geq 2$
$\Longrightarrow \sqrt{n} < \sqrt{n+1}$
$\Longrightarrow \frac{1}{\sqrt{n}} > \frac{1}{\sqrt{n+1}}; n \geq 2$
Statement II is true.

Also, $\frac{1}{\sqrt{1}} > \frac{1}{\sqrt{n}}, \frac{1}{\sqrt{2}} > \frac{1}{\sqrt{n}}, \frac{1}{\sqrt{3}} > \frac{1}{\sqrt{n}}, \ldots\ldots, \forall\, n \geq 2$
On adding all of them, we get
$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} > \frac{n}{\sqrt{n}} = \sqrt{n}, \forall\, n \geq 2$
Clearly, Statements I and II are true and Statement II is a correct explanation of Statement I.

141 **(b)**

$(1+x)^{41}(1-x+x^2)^{40}$

$= (1+x)(1+x)^{40}(1-x+x^2)^{40}$

$= (1+x)(1+x^3)^{40}$

$= (1+x^3)^{40} + x(1+x^3)^{40}$

$$= (1 + {}^{40}C_1x^3 + {}^{40}C_2x^6 + \cdots + {}^{40}C_{40}x^{120}) + ({}^{40}C_0 + {}^{40}C_1x^4 + {}^{40}C_2x^7 + \cdots + {}^{40}C_{40}x^{121})$$

Hence, the coefficient of x^{85} is zero as there is no term in the above expansion which has x^{85}

Also, statement 2 is correct but it is not a correct explanation of statement 1

142 **(a)**

We know that

${}^mC_r + {}^mC_{r-1}\,{}^nC_1 + {}^mC_{r-2}\,{}^nC_2 + \cdots + {}^nC_r$

$= $ Coefficient of x^r in $(1+x)^m(1+x)^n$

$= $ Coefficient of x^r in $(1+x)^{m+n}$

$= {}^{m+n}C_r$

$= 0$ as $m + n < r$

143 **(b)**

$$(1+x+x^2+x^3+x^4)^{1000} = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + a_{4000}x^{4000}$$

Clearly, there are 4001 terms. Also, number of term in the expansion

$(a_1 + a_2 + \cdots + a_m)^n$ is ${}^{n+m-1}C_{m-1}$

Clearly, statement 2 has nothing to do with statement 1

144 **(a)**

Statement 2 is true(can be checked easily) and that is why

$$^{2n}C_0 < {}^{2n}C_1 < {}^{2n}C_2 < \cdots < {}^{2n}C_{n-1} < {}^{2n}C_n > {}^{2n}C_{n+1} \cdots > {}^{2n}C_{2n}$$

145 **(d)**

$$\sum_{r=1}^{n} r^3 \left(\frac{n-r+1}{r}\right)^2 = \sum_{r=1}^{n} r(n-r+1)^2$$

$$= \sum_{r=1}^{n} r\{(n+1)^2 - 2(n+1)r + r^2\}$$

$$= (n+1)^2 \sum r - 2(n+1)\sum r^2 + \sum r^3$$

$$= (n+1)^2 \times \frac{n(n+1)}{2} - \frac{2(n+1) \times n(n+1)(2n+1)}{6} + \left[\frac{n(n+1)}{2}\right]^2$$

$$= \frac{(n+1)^2 \cdot n \cdot (n+2)}{12} = 14^2 \text{ (given)}$$

$$= 7^2 \times 2^2 = \frac{7^2 \cdot 6 \cdot 8}{12}$$

$\therefore\ n = 6$

Sum of coefficients$= (1-3+1)^6 = (-1)^6 = 1$

146 **(a)**

$$3456^{2222} = (7 \times 493 + 5)^{2222}$$

$$= (7k+5)^{2222}$$

$$= 7m + 5^{2222}$$

Now,

$$5^{2222} = 5^2(5^3)^{740}$$

$$= 25(125)^{740}$$

$$= 25(126-1)^{740}$$

$$= 25[7n+1]$$

$$= 175n + 25$$

Remainder when $175n + 25$ is divided by 7 is 4

Hence, both the statements are correct and statement 2 is a correct explanation of statement 1

147 **()**

115

148 **(b)**

$$\frac{T_{r+1}}{T_r} = \frac{12-r+1}{r}\frac{11}{10}$$

Let,

$T_{r+1} \geq T_r \Rightarrow 13 - r \geq 1.1x$

$\Rightarrow 13 \geq 2.1r$

$\Rightarrow r \leq 6.19$

Hence, the greatest term occurs for $r = 6$. Hence, 7th term is the greates term. Also, the binomial coefficient of 7th term is $^{12}C_6$ which is the greatest binomial coefficient.

But this is not the reason for which T_7 is the greatest. Here, it is coincident that the greatest term has the greatest binomial coefficient

Hence, statement 1 is true, statement 2 is true; but statement 2 is not correct explanation of statement 1

149 **(a)**

Since, $\sum_{r=0}^{n} {}^nC_r x^r = (1+x)^n$

On multiplying by x on both sides, we get

$$\sum_{r=0}^{n} {}^nC_r \cdot x^{r+1} = x(1+x)^n$$

On differentiating w.r.t. x, we get

$$\sum_{r=0}^{n} (r+1) \cdot {}^nC_r \cdot x^r = (1+x)^n + nx(1+x)^{n-1}$$

Statement II is true

If $x = 1$, then

$$\sum_{r=0}^{n} (r+1) \cdot {}^nC_r = 2^n + n(2)^{n-1} = (n+2)2^{n-1}$$

$\therefore$ Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I.

151 **(a)**

$$(1+x+x^2)^n = \sum_{r=0}^{2n} a_r x^r \qquad (1)$$

We have that

$$(1-r)^n = \sum_{r=0}^{n} (-1)^{n-r}\, {}^nC_r x^r$$

$$= \sum_{r=0}^{n} (-1)^{n-r}\ {}^nC_r x^{n-r} \qquad (2)$$

Multiplying (1) and (2), we get

$$\sum_{r=0}^{n} (-1)^{n-r}\ {}^nC_r a_r$$

$$= \text{coefficient of } x^n \text{ in } (1-x^3)^n$$

Since $n \neq 3k$, therefore

$$\sum_{r=0}^{n} (-1)^{n-r} a_r\ {}^nC_r = 0$$

$$\Rightarrow \sum_{r=0}^{n} (-1)^{r} a_r\ {}^nC_r = 0$$

Hence, both the statements are correct and statement 2 is a correct explanation of statement 1

152 **(d)**

Since, n is even, put $n = 2$

LHS= ${}^4C_1 = 4$ and RHS= $2^3 = 8$

Hence, Statement I is false, but Statement II is true

153 **(b)**

In the sum of series $\sum_{i=1}^{n} \sum_{j=1}^{n} f(i) \times f(j) =$ *i=1nf(i)j=1nf(j)*

i and j are independent. In this summation, three types of terms occur, for which $i < j, i > j$ and $i = j$. Also, the sum of terms when $i < j$ is equal to the sum of the terms when $i > j$ if $f(i)$ and $f(j)$ are symmetrical. So, in that case

$$\sum_{i=0}^{n}\sum_{j=0}^{n} f(i) \times f(j) = \sum_{0\le i<}\sum_{j\le n} f(i) \times f(j) +$$

$$\sum_{0\le j<}\sum_{i\le n} f(i)f(j) + \sum_{i=}\sum_{j} f(i)f(j)$$

$$= 2\sum_{0\le i<}\sum_{j\le n} f(i)f(j) + \sum_{i=}\sum_{j} f(i)f(j)$$

$$\Rightarrow \sum_{0\le i<}\sum_{j\le n} f(i)f(j)$$

$$= \frac{\sum_{i=0}^{n}\sum_{j=0}^{n} f(i) \times f(j) - \sum_{i=j}\sum f(i)f(j)}{2}$$

1. $\sum\sum_{i\neq j}\ {}^{10}C_i\, {}^{10}C_j = \sum_{i=0}^{10}\sum_{j=0}^{10}\ {}^{10}C_i\, {}^{10}C_j -$ $\sum_{i=0}^{10}\ {}^{10}C_i^2 = 2^{20} -\ {}^{20}C_{10}$

2. $\sum_{0\le i\le}\sum_{j\le 10}\ {}^{10}C_i\, {}^{10}C_j =$ *i=010j=010 10Ci 10Cj+i=010 10Ci22=220+ 20C102*

3. $\sum_{0\le i<}\sum_{j\le 10}\ {}^{10}C_i\, {}^{10}C_j =$ *i=010j=010 10Ci 10Cj−i=010 10Ci22*

$$= \frac{2^{20} - {}^{20}C_{10}}{2}$$

4. $\sum_{i=0}^{10}\sum_{j=0}^{10}\ {}^{10}C_i\, {}^{10}C_j =$ *i=010 10Ci j=010 10Cj=220*

154 **(d)**

1. In $(1+x)^{41} = {}^{41}C_0 + {}^{41}C_1 x + {}^{41}C_2 x^2 + \dots + {}^{41}C_{20}x^{20} + {}^{41}C_{21}x^{21} + \dots + {}^{41}C_{41}x^{41}$

$$\Rightarrow {}^{41}C_{21} + {}^{41}C_{22} + \dots + {}^{41}C_{41} = 2^{40}$$

2. $\left(1+\sqrt{2}\right)^{42} = {}^{42}C_0 + {}^{42}\left(C_1\sqrt{2}\right) + {}^{42}C_2\left(\sqrt{2}\right)^2 + {}^{42}C_1\left(\sqrt{2}\right)^3 + \dots + {}^{42}C_{42}\left(\sqrt{2}\right)^{42}$

Sum of binomial coefficients of rational terms is

$${}^{42}C_0 + {}^{42}C_2 + {}^{42}C_4 + \dots + {}^{42}C_{42} = 2^{41}$$

3. $\left(x + \frac{1}{x} + x^2 + \frac{1}{x^2}\right)^{21} = \left(\frac{x^3+x+x^4+1}{x^2}\right)^{21}$

$$= \frac{a_0 + a_1x + a_2x^2 + \dots + a_{82}x^{82}}{x^{42}} \qquad (1)$$

Now, putting $x = 1$, we get

$$4^{21} = a_0 + a_1 + a_2 + \dots + a_{82}$$

Putting $x = -1$, we get

$$0 = a_0 - a_1 + a_2 - a_3 + \dots + a_{82}$$

Adding, we get

$$4^{21} = 2(a_0 + a_2 + \dots + a_{82})$$

$$\Rightarrow a_0 + a_2 + \dots + a_{82} = 2^{41}$$

4. We know that

$${}^nC_0 - {}^nC_2 + {}^nC_4 - {}^nC_6 + \dots = 2^{n/2}\cos\frac{n\pi}{4} \qquad (1)$$

and

$$^{n}C_0 + {}^{n}C_2 + {}^{n}C_4 + {}^{n}C_6 + \cdots = 2^{n-1} \quad (2)$$

$$\Rightarrow {}^{n}C_0 + {}^{n}C_4 + {}^{n}C_8 + \cdots = \frac{1}{2}\left(2^{n/2} \times \cos\frac{n\pi}{4} + 2^{n-1}\right)$$

For $n = 42$,

$$^{42}C_0 + {}^{42}C_4 + {}^{42}C_8 + \cdots = \frac{1}{2}\left(2^{21} \times \cos\frac{21\pi}{2} + 2^{41}\right) = 2^{40}$$

155 **(d)**

1. $^{(n+1)}C_4 + {}^{(n+1)}C_3 + {}^{(n+2)}C_3 = {}^{(n+3)}C_4$

$$\Rightarrow {}^{(n+3)}C_4 > {}^{(n+3)}C_3 \Rightarrow \frac{^{n+3}C_4}{^{n+3}C_3} > 1$$

$\Rightarrow n > 4$ or $n \geq 5$

2. $(3053)^{456} - (2417)^{333}$

$= (339 \times 9 + 2)^{456} - (269 \times 9 - 4)^{333}$

Remainder of given number is same as remainder of $2^{456} + 4^{333}$

and

$2^{456} + 4^{333} = (64)^{76} + (64)^{111}$

$= (1 + 63)^{76} + (1 + 63)^{111}$

$= (1 + 9 \times 7)^{76} + (1 + 9 \times 7)^{111}$

Hence, the remainder is 2

3. We know that $n!$ terminates in 0 for $n \geq 5$ and 3^{4n} terminates in $1 (\because 3^4 = 81)$

Therefore, $3^{180} = (3^4)^{45}$ terminates in 1

Also, $3^3 = 27$ terminates in 7

Hence, $183! + 3^{183}$ terminates in 7

That is, the digit in the unit place is 7

4. We are given

$^{m}C_0 + {}^{m}C_1 + {}^{m}C_2 = 46$

$\Rightarrow 2m + m(m-1) = 90$

$\Rightarrow m^2 + m - 90 = 0$

$\Rightarrow m = 9$ as $m > 0$

Now, $(r+1)^{\text{th}}$ term of $\left(x^2 + \frac{1}{x}\right)^m$ is

$$^{m}C_r (x^2)^{m-r} \left(\frac{1}{x}\right)^r = {}^{m}C_r x^{2m-3r}$$

For this to be independent of x, $2m - 3r = 0 \Rightarrow r = 6$

156 **(c)**

(A) $(1-x)^{-n} = 1 + nx + \frac{n(n+1)}{2!}x^2 + \cdots$ if $|x| < 1$

(B) $(1-x)^{-n} = 1 + nx + \frac{n(n+1)}{2!}x^2 - \ldots$ if $|x| < 1$

(C) $1 + \frac{1}{x} + \frac{1}{x^2} + \cdots = \frac{1}{1 - \frac{1}{x}} = \frac{x}{x-1} \quad [\because x > 1]$

(D) $1 - \frac{2}{x^2} + \frac{3}{x^4} - \frac{4}{x^6} + \ldots = \left(1 + \frac{1}{x^2}\right)^{-2} = \frac{x^4}{(x^2+1)^2}$

157 **(a)**

Let consecutive coefficients be $^{n}C_r$ and $^{n}C_{r+1}$. Then,

$$\frac{n!}{(n-r)!\,r!} = \frac{n!}{(n-r-1)!\,(r+1)!}$$

$$\Rightarrow \frac{1}{(n-r)(n-r-1)!\,r!} = \frac{1}{(n-r-1)!\,(r+1)r!}$$

$\Rightarrow r + 1 = n - r$

$\Rightarrow n = 2r + 1$

Hence, n is odd

$E = (19-4)^n + (19+4)^n$

$2[{}^{n}C_0 19^n + {}^{n}C_2 19^{n-2} 4^2 + \cdots + {}^{n}C_n 4^n]$ when n is even

or

$2[{}^{n}C_0 19^n + {}^{n}C_2 \cdot 19^{n-2} \cdot 4^2 + \cdots + {}^{n}C_{n-1} 19 \cdot 4^{n-1}]$ then n is odd

$\Rightarrow E$ is divisible by 19 when n is odd

$^{10}C_0\,{}^{20}C_{10} - {}^{10}C_1\,{}^{18}C_{10} + {}^{10}C_2\,{}^{16}C_{10} - \cdots$

=Coefficient of x^{10} in $[{}^{10}C_0(1+x)^{20} -$ *10C1×1+x18+ 10C21+x16−…*

=Coefficient of x^{10} in $[{}^{10}C_0((1+x)^2)^{10} -$

$^{10}C_1 \times ((1+x)^2)^9 + {}^{10}C_2((1+x)^2)^8 - \cdots]$

$= \text{Coefficient of } x^{10} \text{ in } [(1+x)^2 - 1]^{10}$

$= \text{Coefficient of } x^{10} \text{in } [2x + x^2]^{10}$

$= 2^{10}$

$T_r = {}^{14}C_{r-1}x^{r-1}; T_{r+1} = {}^{14}C_r x^r T_{r+2} = {}^{14}C_{r+1}x^{r+1}$

By the given condition,

$2\ {}^{14}C_r = {}^{14}C_{r-1} + {}^{14}C_{r+1} \quad (1)$

$\Rightarrow 2 = \dfrac{{}^{14}C_{r-1}}{{}^{14}C_r} + \dfrac{{}^{14}C_{r+1}}{{}^{14}C_r}$

$\Rightarrow 2 = \dfrac{r}{14-r+1} + \dfrac{14-(r+1)+1}{r+1}$

$\Rightarrow 2 = \dfrac{r}{15-r} + \dfrac{14-r}{r+1}$

$\Rightarrow r = 9$

158 **(c)**
We know that
$^nC_0^2 + {}^nC_1^2 + \cdots + {}^nC_n^2 = {}^{2n}C_n$
And
$^nC_0^2 - {}^nC_1^2 + \cdots + (-1)^n\, {}^nC_n^2$
$= \begin{cases} 0, & \text{if } n \text{ is odd} \\ {}^nC_{n/2}(-1)^n, & \text{if } n \text{ is even} \end{cases}$
From this, $^{31}C_0^2 - {}^{31}C_1^2 + {}^{31}C_2^2 - \cdots - {}^{31}C_{31}^2 = 0$
$^{32}C_0^2 - {}^{32}C_1^2 + {}^{32}C_2^2 - \cdots + {}^{32}C_{32}^2 = {}^{32}C_{16}$
$^{32}C_0^2 - {}^{32}C_1^2 + {}^{32}C_2^2 - \cdots + {}^{32}C_{32}^2 = {}^{64}C_{32}$
Also, $(1/32)(1 \times {}^{32}C_1^2 + 2 \times {}^{32}C_2^2 - \cdots + 32 \times {}^{32}C_{32}^2)$
$= \dfrac{1}{32}\sum_{r=1}^{32} r({}^{32}C_r)^2$
$= \dfrac{1}{32}\sum_{r=1}^{32} r\,{}^{32}C_r\, {}^{32}C_{32-r}$
$= \dfrac{1}{32}\sum_{r=1}^{32} 32\, {}^{31}C_{r-1}\, {}^{32}C_{32-r}$
$= {}^{63}C_{31} = {}^{63}C_{32}$

159 **(c)**
General term, $T_{r+1} = {}^{6561}C_r(7^{1/3})^{6561-r} \cdot (11^{1/9})^r$

$= {}^{6561}C_r \cdot 7^{2187-\frac{r}{3}} \cdot 11^{\frac{r}{9}}$

If T_{r+1} is rational

then $\frac{r}{9}$ and $\frac{r}{3}$ are integers

$\therefore r$ is a multiple of 9

$\because 0 \le r \le 6561$

$\Rightarrow\ 0 \le \dfrac{r}{9} \le 729$

$\therefore\ \dfrac{r}{9} = 0, 1, 2, 3, \dots, 729$

$\therefore$ Total terms=730

160 **(b)**
Now, $(C_0 + C_1 + \ldots + C_n)^2 = \sum_{r=0}^{n} C_r^2 + 2P$
$\Rightarrow\ 2P = (2^n)^2 - \sum_{r=0}^{n} C_r^2$
$\Rightarrow\ P = 2^{2n-1} - \dfrac{1}{2}({}^{2n}C_n)$

161 **(b)**
The coefficient of the 2nd, 3rd and 4th terms in the expansion are $^mC_1$, $^mC_2$ and $^mC_3$, which are given in A.P. Hence,
$2\,{}^mC_2 = {}^mC_1 + {}^mC_3$
$\Rightarrow \dfrac{2m(m-1)}{2!} = m + \dfrac{m(m-1)(m-2)}{3!}$
$\Rightarrow m(m^2 - 9m + 14) = 0$
$\Rightarrow m(m-2)(m-7) = 0$
$\Rightarrow m = 7 (\because m \neq 0$ or 2 as 6th term is given equal to 21)
Now, 6th term in the expansion, when $m = 7$, is
$^7C_5\left[\sqrt{\{2^{\log(10-3^x)}\}}\right]^{7-5} \times \left[\sqrt[5]{\{2^{(x-3)\log 3}\}}\right]^5 = 21$
$\Rightarrow \dfrac{7 \times 6}{2!} 2^{\log(10-3^x)} \times 2^{(x-2)\log 3} = 21$
$\Rightarrow 2^{\log(10-3^x)+(x-2)\log 3} = 1 = 2^0$
$\Rightarrow \log(10-3^x) + (x-2)\log 3 = 0$
$\Rightarrow \log(10-3^x)(3)^{(x-2)} = 0$
$\Rightarrow (10-3^x) \times 3^x \times 3^{-2} = 1$
$\Rightarrow 10 \times 3^x - (3^x)^2 = 9$
$\Rightarrow (3^x)^2 - 10 \times 3^x + 9 = 0$
$\Rightarrow (3^x - 1)(3^x - 9) = 0$
$\Rightarrow 3^x - 1 = 0 \Rightarrow 3^x = 1 = 3^0 \Rightarrow x = 0$
$\Rightarrow 3^x - 9 = 0 \Rightarrow 3^x = 3^2 \Rightarrow x = 2$
Hence, $x = 0$ or 2. When $x = 2$
$\left[\sqrt{\{2^{\log(10-3^x)}\}} + \sqrt[5]{\{2^{(x-2)\log 3}\}}\right]^m$
$= [1+1]^7 = 128$
When $x = 0$,
$\left[\sqrt{\{2^{\log(10-3^x)}\}} + \sqrt[5]{\{2^{(x-2)\log 3}\}}\right]^m$

$= \left[\sqrt{\{2^{\log 9}\}} + {}^5\sqrt{\{2^{-2\log 3}\}}\right]^7$

$= \left[2^{\frac{\log 9}{2}} + \frac{1}{2^{\frac{\log 9}{5}}}\right]^7 > 2^7$

Hence, the minimum value is 128

162 **(b)**

2nd term is ${}^nC_1 x^{n-1} a = 240$ (1)

3rd term is ${}^nC_2 x^{n-2} a^2 = 720$ (2)

4th term is ${}^nC_3 x^{n-3} a^3 = 1080$ (3)

Multiplying (1) and (3) and dividing by the square of (2), we get

$\frac{{}^nC_1 \times {}^nC_3}{({}^nC_2)^2} = \frac{240 \times 1080}{(720)^2}$

$\Rightarrow \frac{n \times n(n-1)(n-2)(2!)^2}{n^2(n-1)^2 \times 3!} = \frac{1}{2}$

$\Rightarrow 4(n-2) = 3(n-1)$ $(\because n \neq 1)$

$\Rightarrow n = 5$

Putting $n = 5$, from (1) and (2), we get

$5x^4 a = 240$ and $10x^3 a^2 = 720$

$\Rightarrow \frac{(5x^4 a)^2}{10x^3 a^2} = \frac{(240)^2}{720}$

or

$x^5 = 32$

$\therefore x = 2$

$\therefore a = \frac{240}{5x^4} = \frac{48}{2^4} = 3$

Hence, $x = 2, a = 3$ and $n = 5$

$(x-a)^n = (2-3)^5 = -1$

Also,

$(2+3)^5 = 2^5 + {}^5C_1 2^4 \times 3 + {}^5C_2 2^3 \times 3^2 + {}^5C_3 2^2 \times 3^3 + {}^5C_4 2 \times 3^4 + {}^5C_5 3^5$

$= 32 + 240 + 720 + 1080 + 810 + 243$

Hence, least value of the term is 32

Sum of odd-numbered terms is $32 + 720 + 810 = 1562$

163 **(b)**

Let,

$(1 + x + x^2)^{20} = \sum_{r=0}^{40} a_r\ x^r$ (1)

Replacing x by $1/x$, we get

$$\left(1 + \frac{1}{x} + \frac{1}{x^2}\right)^{20} = \sum_{r=0}^{40} a_r \left(\frac{1}{x}\right)^r$$

$\Rightarrow (1 + x + x^2)^{20} = \sum_{r=0}^{40} a_r\, x^{40-r}$ (2)

Since (1) and (2) are same series, coefficient of x^r in (1)= coefficient of x^r in (2)

$\Rightarrow a_r = a_{40-r}$

In (1), putting $x = 1$, we get

$3^{20} = a_0 + a_1 + a_2 + \cdots + a_{40}$

$= (a_0 + a_1 + a_2 + \cdots + a_{19}) + a_{20} + (a_{21} + a_{n+2} + \cdots + a_{40})$

$= 2(a_0 + a_1 + a_2 + \cdots + a_{19}) + a_{20}$ $(\because a_r = a_{40-r})$

$\Rightarrow a_0 + a_1 + a_2 + \cdots + a_{19} = \frac{1}{2}(3^{20} - a_{20})$

$= \frac{1}{2}(9^{10} - a_{20})$

Also,

$a_0 + 3a_1 + 5a_2 + \cdots + 81a_{40}$

$= (a_0 + 81a_{40}) + (3a_1 + 79a_{39}) + \cdots + (39a_{19} + 43a_{21}) + 41a_{20}$

$= 82(a_0 + a_1 + a_2 + \cdots + a_{19}) + 41a_{20}$

$= 41(9^{10} - a_{20}) + 41a_{20}$

$= 41 \times 3^{20}$

$a_0^2 - a_1^2 + a_2^2 - a_3^2 + \cdots$ suggests that we have to multiply the two expansions.

Replacing x by $-1/x$ in (1), we get

$$\left(1 - \frac{1}{x} + \frac{1}{x^2}\right)^{20} = a_0 - \frac{a_1}{x} + \frac{a_2}{x^2} - \cdots + \frac{a_{40}}{x^{40}}$$

$\Rightarrow (1 - x + x^2)^{20}$

$= a_0 x^{40} - a_1 x^{39} + a_2 x^{38} - \cdots + a_{40}$ (3)

Clearly,

$a_0^2 - a_1^2 + a_2^2 + \cdots + a_{40}^2$ is the coefficient of x^{40} in $(1 + x + +x^2)^{20}(1 - x + x^2)^{20}$

$=$ Coefficient of x^{40} in $(1 + x^2 + x^4)^{20}$

In $(1 + x^2 + x^4)^{20}$, replace x^2 by y, then the coefficient of y^{20} in $(1 + y + y^2)^{20}$ is a_{20}. Hence,

$a_0^2 - a_1^2 + a_2^2 - \cdots + a_{40}^2 = a_{20}$

$\Rightarrow (a_0^2 - a_1^2 + a_2^2 - \cdots - a_{19}^2) + a_{20}^2 + (-a_{21}^2 + \cdots + a_{40}^2) = a_{20}$

$\Rightarrow (a_0^2 - a_1^2 + a_2^2 - \cdots - a_{19}^2) + a_{20}^2 = a_{20}$

$\Rightarrow a_0^2 - a_1^2 + a_2^2 - \cdots - a_{19}^2 = \frac{a_{20}}{2}[1 - a_{20}]$

164 **(c)**

$a_0 + a_1 x + a_2 x^2 + \cdots + a_{99} x^{99} + x^{100} = 0$ has roots ${}^{99}C_0, {}^{99}C_1, {}^{99}C_2, \ldots, {}^{99}C_{99}$

$\Rightarrow a_0 + a_1 x + a_1 x^2 + \cdots + a_{99} x^{99} + x^{100}$

$= (x - {}^{99}C_0)(x - {}^{99}C_1)(x - {}^{99}C_2) \ldots (x - {}^{99}C_{99})$

Now, sum of roots is

${}^{99}C_0 + {}^{99}C_1 + {}^{99}C_2 + \cdots + {}^{99}C_{99}$

$= -\frac{a_{99}}{\text{coefficient of } x^{100}}$

$\Rightarrow a_{99} = -2^{99}$

Also, sum of product of roots taken two at a time is

$\frac{a_{99}}{\text{coefficient of } x^{100}}$

$\therefore \sum_{a\le i<}\sum_{j\le 99} {}^{99}C_i\, {}^{99}C_j$

$= \frac{\left(\sum_{i=0}^{99}\sum_{j=0}^{99} {}^{99}C_i\, {}^{99}C_j\right) - \sum_{i=0}^{99}({}^{99}C_i)^2}{2}$

$= \frac{\left(\sum_{i=0}^{99} {}^{99}C_i 2^{99}\right) - \sum_{i=0}^{99}({}^{99}C_i)^2}{2}$

$= \frac{2^{99}2^{99} - \sum_{i=0}^{99}({}^{99}C_i)^2}{2}$

$= \frac{2^{198} - {}^{198}C_{99}}{2}$

$({}^{99}C_0)^2 + ({}^{99}C_1)^2 + \cdots + ({}^{99}C_{99})^2$

$= ({}^{99}C_0 + {}^{99}C_1 + {}^{99}C_2 + \cdots + {}^{99}C_{99})^2$

$- 2\sum_{0\le i<}\sum_{j\le 99} {}^{99}C_i\, {}^{99}C_j$

$= (-a_{99})^2 - 2a_{98}$

$= a_{99}^2 - 2a_{98}$

165 **(a)**

$\sum_{r=0}^{100} {}^{100}C_r \sin rx = \mathrm{Im}\left(\sum_{r=0}^{100} {}^{100}C_r\, e^{irx}\right)$

(Im=imaginary part)

$= \mathrm{Im}\left(\sum_{r=0}^{100} {}^{100}C_r \left(e^{ix}\right)^r\right)$

$= \mathrm{Im}\left(1 + e^{ix}\right)^{100})$

$= \mathrm{Im}(1 + \cos x + i\sin x)^{100}$

$= \mathrm{Im}\left(2\cos^2\frac{x}{2} + 2i\sin\frac{x}{2}\times\cos\frac{x}{2}\right)^{100}$

$= \mathrm{Im}\left(2\cos\frac{x}{2}\left(\cos\frac{x}{2} + i\sin\frac{x}{2}\right)\right)^{100}$

$= 2^{100}\cos^{100}\frac{x}{2}\sin(50x)$

$\sum_{r=0}^{50} {}^{50}C_r\, a^r \times b^{50-r} \times \cos(rB - (50-r)A)$

$= \mathrm{Re}\left(\sum_{r=0}^{50} {}^{50}C_r a^r \times b^{50-r} \times e^{i(rB-(50-r)A)}\right)$

$= \mathrm{Re}\left(\sum_{r=0}^{50} {}^{50}C_r\left(a \times e^{iB}\right)^r \times \left(b \times e^{-iA}\right)^{50-r}\right)$

$= \mathrm{Re}\left(ae^{iB} + be^{-iA}\right)^{50}$

$= \mathrm{Re}(a\cos B + ia\sin B + b\cos A - ib\sin A)^{50}$

$= \mathrm{Re}(a\cos B + b\cos A)^{50} = c^{50}(\because a\sin B = b\sin A)$

$\frac{\sum_{r=0}^{50} {}^{50}C_r \sin 2rx}{\sum_{r=0}^{50} {}^{50}C_r \cos 2rx} = \frac{\sum_{r=0}^{50} {}^{50}C_{50-r} \sin 2(50-r)x}{\sum_{r=0}^{50} {}^{50}C_{50-r} \cos 2(50-r)x}$

$= \frac{\sum_{r=0}^{50} {}^{50}C_r\,[\sin 2rx + \sin 2(50-r)x]}{\sum_{r=0}^{50} {}^{50}C_r\,[\cos 2rx + \cos 2(50-r)x]} \left(\because \frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}\right)$

$= \frac{\sum_{r=0}^{50} {}^{50}C_r\, 2\sin(50x)\cos(2r-50)x}{\sum_{r=0}^{50} {}^{50}C_r\, 2\cos(50x)\cos(2r-50)x}$

$= \tan(50x)$

$\Rightarrow f(\pi/8) = \tan(25\pi/4) = \tan(6\pi + \pi/4) = 1$

166 **(b)**

General term of the series is

$T(r) = \sum_{r=0}^{50} \frac{{}^{50+r}C_r(2r-1)}{{}^{50}C_r(50+r)}$

$= \frac{{}^{50+r}C_r}{{}^{50}C_r}\left(1 - \frac{50-r+1}{50+r}\right)$

$= \frac{{}^{50+r}C_r}{{}^{50}C_r} - \frac{{}^{50+r}C_r}{{}^{50}C_r}\left(\frac{50-r+1}{50+r}\right)$

Now,

$\frac{{}^{50+r}C_r}{{}^{50}C_r}\left(\frac{50-r+1}{50+r}\right)$

$= \frac{(50-r+1)(50+r)!\,r!\,(50-r)!}{r!\,50!\,(50+r)50!}$

$= \frac{(50-r+1)!\,(50+r-1)!}{50!\,50!}$

$= \frac{{}^{50+r-1}C_{r-1}}{{}^{50}C_{r-1}}$

$\Rightarrow T(r) = \frac{{}^{50+r}C_r}{{}^{50}C_r} - \frac{{}^{50+r-1}C_{r-1}}{{}^{50}C_{r-1}}$

$= V(r) - V(r-1)$

Where $V(r) = \frac{{}^{50+r}C_r}{{}^{50}C_r}$

Now, sum of the given series

$P = \sum_{r=1}^{50} T(r) = V(50) - V(0)$

$= \frac{{}^{100}C_{50}}{{}^{50}C_{50}} - \frac{{}^{50}C_0}{{}^{50}C_0} = {}^{100}C_{50} - 1$

Also,

$Q = \sum_{r=0}^{50}({}^{50}C_r)^2$

$= {}^{50}C_0^2 + {}^{50}C_1^2 + {}^{50}C_2^2 + \cdots + {}^{50}C_{50}^2 = {}^{100}C_{50}$

$\Rightarrow P - Q = -1$

We know that

$C_0^2 - C_1^2 + C_2^2 + \cdots + (-1)^n C_n^2$

$= \begin{cases} 0, & \text{if } n \text{ is odd} \\ (-1)^n\, {}^nC_{n/2}, & \text{if } n \text{ is even} \end{cases}$

$\Rightarrow \sum_{r=0}^{100}(-1)^r({}^{100}C_r)^2 = (-1)^{100}\,{}^{100}C_{50} = {}^{100}C_{50}$

$\Rightarrow P - R = -1$

$Q + R = 2\,{}^{100}C_{50} = 2P + 2$

167 **(a)**

Suppose A contains $r(0 \le r \le n)$ elements

Then, B is constructed by selecting some elements from the remaining $n-r$ elements Here, A can be chosen in ${}^{n}C_r$ ways and B in ${}^{n-r}C_0 + {}^{n-r}C_1 + \cdots + {}^{n-r}C_{n-r} = 2^{n-r}$ ways.

So, the total number of ways of choosing A and B is ${}^{n}C_r \times 2^{n-r}$

But r can vary from 0 to n. So, total number of ways is

$$\sum_{r=0}^{n} {}^{n}C_r \times 2^{n-r} = (1+2)^n = 3^n$$

If A contains r elements, then B contains $(r+1)$ elements

Then, the number of ways of choosing A and B is ${}^{n}C_r \times {}^{n}C_{r+1} = C_r \; C_{r+1}$

But r can vary from 0 to $(n-10$.

So, total number of ways is

$$\sum_{r=0}^{n-1} C_r C_{r+1} = C_0C_1 + C_1C_2 + \cdots . + C_{n-1}C_n$$
$$= {}^{2n}C_{n-1}$$

Let A contains $r (0 \le r \le n)$ elements.

Then, A can be chosen in ${}^{n}C_r$ ways. The subset B of A can have at most r elements, and the number of ways of choosing B is 2^r

Therefore, the number of ways of choosing A and B is ${}^{n}C_r \times 2^r$

But r can vary from 0 to n

So, the total number of ways is

$$\sum_{r=0}^{n} {}^{n}C_r \times 2^r = (1+2)^n = 3^n$$

168 **(1)**

$$= \sum_{k=0}^{4} \left(\frac{3^{4-k}}{(4-k)!}\right)\left(\frac{x^k}{k!}\right)$$
$$= \sum_{k=0}^{4} \left(\frac{3^{4-k}}{(4-k)!}\right)\left(\frac{x^k}{k!}\right)\frac{4!}{4!}$$
$$= \sum_{k=0}^{4} \frac{{}^{4}C_k \cdot 3^{4-k} \cdot x^k}{4!} = \frac{(3+x)^4}{4!}$$

According to the question,

$$\frac{(3+x)^4}{4!} = \frac{32}{3}$$
$$\Rightarrow (3+x)^4 = 256$$
$$\Rightarrow x+3 = 4 \Rightarrow x = 1$$

169 **(0)**

Consider $(5+2)^{100} - (5-2)^{100}$

$$= 2[{}^{100}C_1 5^{99} \cdot 2 + {}^{100}C_3 5^{97} \cdot 2^3 + \cdots + {}^{100}C_{99} 5 \cdot 2^{99}]$$
$$= 2[1000 \cdot 5^{98} + 1000. {}^{100}C_3 \cdot 5^{94} + \cdots + 1000 \cdot 2^{98}]$$

$\Rightarrow$ minimum 000 as last three digits

170 **(5)**

$${}^{23}C_r + 2. \; {}^{23}C_{r+1} + {}^{23}C_{r+2} = {}^{24}C_{r+1} + {}^{24}C_{r+2}$$
$$= {}^{25}C_{r+2} \ge {}^{25}C_{15}$$

$\therefore (r+2)$ can be 10, 11, 12, 13 and 15 so 5 elements

171 **(9)**

$$f(n) = {}^{n}C_0 a^{n-1} - {}^{n}C_1 a^{n-2} + {}^{n}C_2 a^{n-3} + \cdots + (-1)^{n-1n}C_{n-1}\, a^0$$
$$= \frac{1}{a}({}^{n}C_0 a^n - {}^{n}C_1 a^{n-1} + {}^{n}C_2 a^{n-2} + \cdots + (-1)^{n-1}C_{n-1}a')$$
$$= \frac{1}{a}((a-1)^n - (-1)^n \; {}^{n}C_n)$$
$$= \frac{1}{a}\left(\left(\frac{1}{3^{223}} - (-1)^n\right)\right)$$
$$f(x) = \frac{3^{\frac{n}{223}} - (-1)^n}{\left(3^{\frac{1}{223}} + 1\right)}$$
$$\Rightarrow f(2007) = \frac{3^{\frac{2007}{223}} + 1}{3^{\frac{1}{223}} + 1}$$
$$\Rightarrow f(2008) = \frac{3^{\frac{2008}{223}} - 1}{3^{\frac{1}{223}} + 1}$$
$$\Rightarrow f(2007) + f(2008) = \frac{3^{\frac{2007}{223}} + 3^{\frac{2008}{223}}}{3^{\frac{1}{223}+1}}$$
$$= \frac{3^9 + 3^{9+\frac{1}{223}}}{3^{\frac{1}{223}} + 1}$$
$$= 3^9 \frac{\left(1 + 3^{\frac{1}{223}}\right)}{1 + 3^{\frac{1}{223}}} = 3^9$$
$$\Rightarrow 3^9 = 3^k \text{ then } k = 9$$

172 **(8)**

Let the three consecutive coefficients be ${}^{n}C_{r-1} = 28$, ${}^{n}C_r = 56$ and ${}^{n}C_{r+1} = 70$,

So that $\frac{{}^{n}C_r}{{}^{n}C_{r-1}} = \frac{n-r+1}{r} = \frac{56}{28} = 2$ and $\frac{{}^{n}C_{r+1}}{{}^{n}C_r} = \frac{n-r}{r+1} = \frac{70}{56} = \frac{5}{4}$

This gives $n+1 = 3r$ and $4n-5 = 9r$

$$\therefore \frac{4n-5}{n+1} = 3 \Rightarrow n = 8$$

173 **(7)**

$$(1+7)^{83} + (7-1)^{83} = (1+7)^{83} - (1-7)^{83}$$
$$= 2[{}^{83}C_1 \cdot 7 + {}^{83}C_3 \cdot 7^3 + \cdots + {}^{83}C_{83} \cdot 7^{83}] =$$

$(2 \cdot 7 \cdot 83) + 49I$ where I is an integer

Now $14 \times 83 = 1162$

$$\therefore \frac{1162}{49} = 23\frac{35}{49}$$

$\therefore$ Reminder is 35

174 **(4)**

We have b = coefficient of x^3 in $\big((1 + x + 2x^2 +$

$3x^3) + 4x^4)^4$

=coefficient of x^3 in $[{}^4C_0(1 + x + 2x^2 +$

3x344x40+ 4C11+x+2x2+3x334x41+…]

=coefficient of x^3in $(1 + x + 2x^2 + 3x^3)^4 = a$

Hence, $4a/b = 4$

175 **(6)**

$T_{r+1} = {}^nC_r(x^2)^{n-r}(-1)^r x^{-r}$

$= {}^nC_r x^{2n-3r}(-1)^r$

Constant term = ${}^nC_r(-1)^r$ if $2n = 3r$

i.e., coefficient of $x = 0$

hence, ${}^nC_{2n/3}(-1)^{2n/3} = 15 = {}^6C_4\ n = 6$

176 **(1)**

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{5^n}\cdot {}^nC_r\left(\sum_{t=0}^{r-1} {}^rC_t.3^t\right)$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{5^n}.\ {}^nC_r\,(4^r - 3^r)$$

$$= \lim_{n\to\infty}\frac{1}{5^n}\left(\sum_{r=1}^{n} {}^nC_r 4^r - \sum_{r=0}^{n} {}^nC_r\,3^r\right)$$

$$= \lim_{n\to\infty}\frac{1}{5^n}(5^n - 4^n) = 1$$

177 **(0)**

Middle term is $\left(\frac{n}{2}+1\right)^{\text{th}}$, i. e., $(4+1)^{\text{th}}$, i. e., T_5

$$\therefore\ T_5 = {}^8C_4\left(\frac{x}{2}\right)^4.2^4 = 1120 \Rightarrow x^4\frac{8.7.6.5}{1.2.3.4}x^4 = 1120$$

$$\Rightarrow x^4 = \frac{1120}{70} = 16$$

$\Rightarrow (x^2 + 4)(x^2 - 4) = 0$

$\therefore x = \pm 2$ only as $x \in R$

178 **(3)**

$$(1 + 0.00002)^{50000} = \left(1 + \frac{1}{50000}\right)^{50000}$$

Now we know that $2 \le \left(1 + \frac{1}{n}\right)^n < 3\ \forall\, n \ge 1 \Rightarrow$

Least integer is 3

179 **(5)**

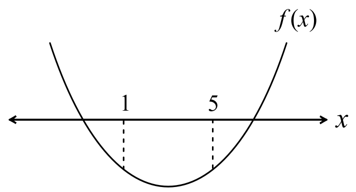
We have $1 + \sum_{r=1}^{10}(3^r \cdot {}^{10}C_r + r \cdot {}^{10}C_r)$

$$= 1 + \sum_{r=1}^{10} 3^r \cdot {}^{10}C_r + 10\sum_{r=1}^{10} {}^9C_{r-1}$$

$= 1 + 4^{10} - 1 + 10 \cdot 2^9$

$= 4^{10} + 5.2^{10} = 2^{10}(4^5 + 5)$

$= 2^{10}(\alpha \cdot 4^5 + \beta)$, so $\alpha = 1$ and $\beta = 5$



Now $f(1) < 0$ and $f(5) < 0$

$f(1) < 0 \Rightarrow -k^2 < 0\ \Rightarrow k \ne 0$ and $f(5) < 0$

$\Rightarrow 16 - k^2 < 0$

$\Rightarrow k^2 - 16 > 0$

$\Rightarrow k \in (-\infty, 4) \cup (4, \infty)$

Hence, the smallest positive integral value of $k = 5$

180 **(8)**

$$= \left[\sqrt{x^2+1} + \sqrt{x^2-1}\right]^8 + \left[\sqrt{x^2+1} - \sqrt{x^2-1}\right]^8$$

$$= 2\begin{bmatrix} {}^8C_0\left(\sqrt{x^2+1}\right)^8 + {}^8C_2\left(\sqrt{x^2+1}\right)^6 \\ \left(\sqrt{x^2-1}\right)^2 + \\ {}^8C_4\left(\sqrt{x^2+1}\right)^4\left(\sqrt{x^2-1}\right)^4 \\ {}^8C_6\left(\sqrt{x^2+1}\right)^2\left(\sqrt{x^2-1}\right)^6 \\ + {}^8C_8\left(\sqrt{x^2-1}\right)^8 \end{bmatrix}$$

Which has degree 8

181 **(6)**

Coefficients of $(2r + 4)^{\text{th}}$ and $(r - 2)^{\text{th}}$ terms are equal

$\Rightarrow\ {}^{18}C_{2r+3} = {}^{18}C_{r-3}$ (when ${}^nC_x = {}^nC_y$, then $x = y$ or $x + y = n$)

$\Rightarrow 2r + 3 + r - 3 = 18\ \Rightarrow r = 6$

182 **(4)**

$T_2 = {}^nC_1\left(a^{1/13}\right)^{n-1}.\,a\sqrt{a} = 14a^{5/2}$

$\Rightarrow n \cdot a^{\frac{n-1}{13}} = 14a$

$\Rightarrow\ n \cdot a^{\frac{n-14}{13}} = 14$

$$\Rightarrow \frac{n-14}{13} = 0$$

$\Rightarrow n = 14$

$$\Rightarrow \frac{{}^{14}C_3}{{}^{14}C_2} = \frac{14!}{3!\cdot 11!}\frac{2!\cdot 12!}{14!} = \frac{12}{3} = 4$$

183 **(6)**

$(1 - 2x + 5x^2 - 10x^3)[\,{}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \cdots] = 1 + a_1x + a_2x^2 + \cdots$

$\Rightarrow a_1 = n - 2$ and $a_2 = \frac{n(n-1)}{2} - 2n + 5$

Given that $a_1^2 = 2a_2$

$\Rightarrow (n + 2)^2 = n(n - 1) - 4n + 10$

$\Rightarrow n^2 - 4n + 4 = n^2 - 5n + 10$

$\Rightarrow n = 6$

184 **(9)**

According to the question,

${}^{14}C_{r-1}$, ${}^{14}C_r$, ${}^{14}C_{r+1}$ are in A.P., so $\left\{b = \frac{a+c}{2}\right\}$

$\Rightarrow 2.\ {}^{14}C_r = {}^{14}C_{r-1} + {}^{14}C_{r+1}$

$$\Rightarrow \frac{2.14!}{(14-r)!\,r!} = \frac{14!}{(14-r+1)!\,(r-1)} + \frac{14!}{(14-r-1)!\,(r+1)!}$$

$$\Rightarrow \frac{2}{(14-r)(13-r)!\,r(r-1)!} = \frac{1}{(15-r)(14-r)(13-r)!\,(r-1)!} + \frac{1}{(13-r)!\,(r+1)r(r-1)!}$$

$$\Rightarrow \frac{2}{(14-r)r} = \frac{1}{(15-r)(14-r)} + \frac{1}{r(r+1)}$$

$$\Rightarrow \frac{2}{(14-r)r} - \frac{1}{r(r+1)} = \frac{1}{(15-r)(14-r)}$$

$$\Rightarrow \frac{3r-12}{r(r+1)} = \frac{1}{(15-r)}$$

$\Rightarrow r = 5$ or 9

185 **(1)**

Let x^7 occurs in T_{r+1} term, then

$$T_{r+1} = {}^{n}C_r (ax^2)^{n-r} \left(\frac{1}{bx}\right)^r$$

$$= {}^{11}C_r \frac{a^{11-r}}{b^r} . x^{22-2r-r}$$

For $x^7 \Rightarrow 22 - 3r = 7 \Rightarrow r = 5$

Hence, coefficients of x^7 is ${}^{11}C_5 \frac{a^6}{b^5}$

Let x^{-7} occur in T_{r+1} term, then

$$T_{r+1} = {}^{11}C_r (ax)^{11-r} \left(-\frac{1}{bx^2}\right)^r$$

$$= {}^{11}C_r \frac{a^{11-r}}{(-b)^r} x^{11-3r}$$

For $x^7 \Rightarrow 11 - 3r = -7 \Rightarrow r = 6$

Hence, coefficient of x^{-7} is ${}^{11}C_6 \frac{a^5}{b^6}$

Now ${}^{11}C_5 \frac{a^5}{b^6} = {}^{11}C_6 \frac{a^6}{b^5}$

$$\Rightarrow {}^{11}C_5 a = {}^{11}C_6 \frac{a^5}{b^6}$$

$$\Rightarrow {}^{11}C_5 a = {}^{11}C_{11-6} \frac{1}{b}$$

$$\Rightarrow {}^{11}C_5 a = {}^{11}C_5 \frac{1}{b}$$

$\Rightarrow ab = 1$

186 **(0)**

$$1 + 2 + 2^2 + 2^3 + \cdots + 2^{1999}$$

$$= \frac{1(2^{2000} - 1\,)}{1}$$

$$= 2^{2000} - 1$$

$$= (1-5)^{1000} - 1$$

$$= 1 - {}^{1000}C_1 \cdot 5 + {}^{1000}C_2 \cdot 5^2 + \cdots + {}^{1000}C_{1000} \cdot 5^{1000} - 1$$

Which is divisible by 5

187 **(4)**

$$\left(5^{\frac{2}{5}\log_5 \sqrt{4^x+44}} + \frac{1}{5^{\log_5 \sqrt[3]{2^{x-1}+7}}}\right)^8$$

$$= \left(\left(\sqrt{4^x + 44}\right)^{2/5} + \left(\frac{1}{\sqrt[3]{2^{x-1} + 7}}\right)\right)^8$$

$$= \left((4^x + 44)^{1/5} + \frac{1}{(2^{x-1} + 7)^{1/3}}\right)^8$$

Now

$$T_4 = T_{3+1} = {}^{8}C_3 \left((4^x + 44)^{1/5}\right)^{8-3} \frac{1}{\left((2^{x-1}+7)^{1/3}\right)^3}$$

Given $336 = {}^{8}C_3 \left(\frac{4^x+44}{2^{x-1}+7}\right)$

Let $2^x = y$

$$\Rightarrow 336 = {}^{8}C_3 \left(\frac{y^2 + 44}{(y/2) + 7}\right)$$

$$\Rightarrow 336 = \frac{8 \times 7 \times 6}{3 \times 2 \times 1}\left(\frac{2(y^2 + 44)}{y + 14}\right)$$

$\Rightarrow y^2 - 3y + 2 = 0 \;\Rightarrow y = 0, 2$