DCAM classes
Dynamic Classes for Academic Mastery

8.APPLICATION OF INTEGRALS

Single Correct Answer Type

1. Area enclosed between the curves $|y| = 1 - x^2$ and $x^2 + y^2 = 1$ is
a) $\frac{3\pi - 8}{3}$ sq. units b) $\frac{\pi - 8}{3}$ sq. units c) $\frac{2\pi - 8}{3}$ sq. units d) None of these

2. The area enclosed by the curve $xy^2 = a^2(a - x)$ and $(a - x)y^2 = a^2x$ is
a) $(\pi - 2)a^2$ sq. units b) $(4 - \pi)a^2$ sq. units c) $\pi a^2/3$ sq. units d) None of these

3. The area of the region enclosed between the curves $x = y^2 - 1$ and $x = |y|\sqrt{1 - y^2}$ is
a) 1 sq. units b) 4/3 sq. units c) 2/3 sq. units d) 2 sq. units

4. Area enclosed by the curve $y = (x)$ defined parametrically as $x = \frac{1-t^2}{1+t^2}, y = \frac{2t}{1+t^2}$ is equal to
a) π sq. units b) $\pi/2$ sq. units c) $\frac{3\pi}{4}$ sq. units d) $\frac{3\pi}{2}$ sq. units

5. Area bounded by the curve $xy^2 = a^2(a - x)$ and the y- axis is
a) $\pi a^2/2$ sq. units b) πa^2 sq. units c) $3\pi a^2$ sq. units d) None of these

6. Area bounded by the curves $y = \log_e x$ and $y = (\log_e x)^2$ is
a) $e - 2$ sq. units b) $3 - e$ sq. units c) e sq. units d) $e - 1$ sq. units

7. The area of the loop of the curve, $ay^2 = x^2(a - x)$ is
a) $4a^2$sq. units b) $\frac{8a^2}{15}$ sq. units c) $\frac{16a^2}{9}$ sq. units d) None of these

8. The area bounded by the two branches of curve $(y - x)^2 = x^3$ and the straight line $x = 1$ is
a) 1/5 sq. units b) 3/5 sq. units c) 4/5 sq. units d) 8/4 sq. units

9. The area o the region enclosed by the curves $y = x \log x$ and $y = 2x - 2x^2$is
a) $\frac{7}{12}$sq. units b) $\frac{1}{2}$sq. units c) $\frac{5}{12}$sq. units d) None of these

10. If $f(x) = \sin x, \forall\, x \in \left[0, \frac{\pi}{2}\right], f(x) + f(\pi - x) = 2, \forall x \in \left(\frac{\pi}{2}, \pi\right]$ and $f(x) = f(2\pi - x), \forall x \in (\pi, 2\pi)$, then the area enclosed by $y = f(x)$ and the x-axis is
a) π sq. units b) 2π sq. units c) 2 sq. units d) 4 sq. units

11. The area of the region whose boundaries are defined by the curve $y = 2\cos x, y = 3\tan x$ and the y-axis is
a) $1 + 3\ \text{In}\left(\frac{2}{\sqrt{3}}\right)$ sq. units b) $1 + \frac{3}{2}\text{In}\ 3 - 3\ \text{In}\ 2$ sq. units
c) $1 + \frac{3}{2}\text{In}\ 3 - \text{In}\ 2$ sq. units d) In 3 −In 2 sq. units

12. The area bounded by the curve $y^2 = 1 - x$ and lines $y = \frac{|x|}{x}, x = -1$ and $x = \frac{1}{2}$ is
a) $\frac{3}{\sqrt{2}} - \frac{11}{6}$ sq. units b) $3\sqrt{2} - \frac{11}{4}$ sq. units c) $\frac{6}{\sqrt{2}} - \frac{11}{5}$ sq. units d) None of these

13. Let $f(x)$ =minimum $(x + 1, \sqrt{1 - x})$ for all $x \leq 1$. Then the area bounded by $y = f(x)$ and the x-axis is
a) $\frac{7}{3}$sq. units b) $\frac{1}{6}$sq. units c) $\frac{11}{6}$sq. units d) $\frac{7}{6}$sq. units

14. The area inside the parabola $5x^2 - y = 0$ but outside the parabola $2x^2 - y + 9 = 0$ is
a) $12\sqrt{3}$ sq. units b) $6\sqrt{3}$ sq. units c) $8\sqrt{3}$ sq. units d) $4\sqrt{3}$ sq. units

15. The area of the region of the plane bounded by max $(|x|, |y|) \leq 1$ and $xy \leq \frac{1}{2}$ is
a) 1/2 + In 2 sq. units b) 3+ In 2 sq. units c) 31/4 sq. units d) 1+2 In 2 sq. units

16. The area of the closed figure bounded by $y = \frac{x^2}{2} - 2x + 2$ and the tangents to it at (1, 1/2) and (4, 2) is
a) 9/8 sq. units b) 3/8 sq. units c) 3/2 sq. units d) 9/4 sq. units

17. The value of the parameter a such that the area bounded by $y = a^2x^2 + ax + 1$, coordinate axes and the line $x = 1$ attains its least value, is equal to
a) $-\frac{1}{4}$sq. units b) $-\frac{1}{2}$sq. units c) $-\frac{3}{4}$sq. units d) -1 sq. units

18. The area bounded by the curves $y = \sin^{-1}|\sin x|$ and $y = (\sin^{-1}|\sin x|)^2$, where $0 \leq x \leq 2\pi$, is

a) $\frac{1}{3}+\frac{\pi^2}{4}$ sq. units b) $\frac{1}{6}+\frac{\pi^3}{8}$ sq. units c) 2 sq. units d) None of these

19. The area bounded by the curve $a^2y = x^2(x+a)$ and the x-axis is
a) $a^2/3$ sq. units b) $a^2/4$ sq. units c) $a^2/43$ sq. units d) $a^2/12$ sq. units

20. Area bounded by $y = \frac{1}{x^2-2x+2}$ and x-axis is
a) 2π sq. units b) $\frac{\pi}{2}$ sq. units c) 2 sq. units d) π sq. units

21. The area bounded by the loop of the curve $4y^2 = x^2(4-x^2)$ is
a) 7/3 sq. units b) 8/3 sq. units c) 11/3 sq. units d) 16/3 sq. units

22. The area of the region bounded by $x^2+y^2-2x-3=0$ and $y=|x|+1$ is
a) $\frac{\pi}{2}-1$ sq. units b) 2π sq. units c) 4π sq. units d) $\pi/2$ sq. units

23. The area enclosed between the curves $y=\log_e(x+e), x=\log_e\left(\frac{1}{y}\right)$ and the x –axis is
a) 2 sq. units b) 1 sq. units c) 4 sq. units d) None of these

24. The area of the region bounded by $x=0, y=0, x=2, y=2, y\le e^x$ and $y \ge \text{In}\, x$ is
a) $6-4\,\text{In}\,2$ sq. units b) $4\,\text{In}\,2-2$ sq. units c) $2\,\text{In}\,2-4$ sq. units d) $6-2\,\text{In}\,2$ sq. units

25. Consider two curve $C_1: y^2 = 4[\sqrt{y}]x$ and $C_2: x^2 = 4[\sqrt{x}]y$, where [.] denotes the greatest integer function. Then the area of region enclosed by these two curves within the square formed by the lines $x=1, y=1, x=4, y=4$ is
a) 8/3 sq. units b) 10/3 sq. units c) 11/3 sq. units d) 11/4 sq. units

26. The area of the region in 1st quadrant bounded by the y-axis, $y=\frac{x}{4}, y=1+\sqrt{x}$ and $y=\frac{2}{\sqrt{x}}$ is
a) 2/3 sq. units b) 8/3 sq. units c) 11/3 sq. units d) 13/6 sq. units

27. The area of the region containing the points (x,y) satisfying $4 \le x^2+y^2 \le 2(|x|+|y|)$ is
a) 8 sq. units b) 2 sq. units c) 4π sq. units d) 2π sq. units

28. The area bounded by the curves
$y=(x-1)^2, y=(x+1)^2$ and $y=\frac{1}{4}$ is
a) $\frac{1}{3}$ sq unit b) $\frac{2}{3}$ sq unit c) $\frac{1}{4}$ sq unit d) $\frac{1}{5}$ sq unit

29. The area bounded by $y=\sec^{-1}x, y=\text{cosec}^{-1}x$ and line $x-1=0$ is
a) $\log(3+2\sqrt{2})-\frac{\pi}{2}$ sq. units b) $\frac{\pi}{2}-\log(3+2\sqrt{2})$ sq. units
c) $\pi-\log_e 3$ sq. units d) None of these

30. The area bounded by the curve $f(x)=x+\sin x$ and its inverse function between the ordinates $x=0$ and $x=2\pi$ is
a) 4π sq. units b) 8π sq. units c) 4 sq. units d) 8 sq. units

31. The area enclosed by the curve $y=\sqrt{4-x^2}, y\ge\sqrt{2}\sin\left(\frac{x\pi}{2\sqrt{2}}\right)$ and the x-axis is divided by the y-axis in the ratio
a) $\frac{\pi^2-8}{\pi^2+8}$ b) $\frac{\pi^2-4}{\pi^2+4}$ c) $\frac{\pi^2-4}{\pi-4}$ d) $\frac{2\pi^2}{2\pi^2+\pi^2-8}$

32. The area between the curve $y=2x^4-x^2$, the x-axis and the ordinates of the two minima of the curve is
a) 11/60 sq. units b) 7/120 sq. units c) 1/30 sq. units d) 7/90 sq. units

33. The area bounded by the x-axis, the curve $y=f(x)$ and the lines $x=1, x=b$ is equal to $\sqrt{b^2+1}-\sqrt{2}$ for all $b>1$, then $f(x)$ is
a) $\sqrt{x-1}$ b) $\sqrt{x+1}$ c) $\sqrt{x^2+1}$ d) $\frac{x}{\sqrt{1+x^2}}$

34. The area of the region between the curves
$y=\sqrt{\frac{1+\sin x}{\cos x}}$ and $y=\sqrt{\frac{1-\sin x}{\cos x}}$
Bounded by the line $x=0$ and $x=\frac{\pi}{4}$

a) $\int_0^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}} dt$ b) $\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}} dt$

c) $\int_0^{\sqrt{2}+1} \frac{4t}{(1+t^2)\sqrt{1-t^2}} dt$ d) $\int_0^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}} dt$

35. Let $f(x)$ be a non-negative continuous function such that the area bounded by the curve $y = f(x)$, x-axis and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is $\beta \sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta$. Then $f'\left(\frac{\pi}{2}\right)$ is

a) $\left(\frac{\pi}{2} - \sqrt{2} - 1\right)$ b) $\left(\frac{\pi}{4} + \sqrt{2} - 1\right)$ c) $-\frac{\pi}{2}$ d) $\left(1 - \frac{\pi}{4} - \sqrt{2}\right)$

36. The area of the figure bounded by the parabola $(y-2)^2 = x - 1$, the tangent to it at the point with the ordinate $x = 3$ and the x-axis is

a) 7 sq. units b) 6 sq. units c) 9 sq. units d) None of these

37. The area bounded by $y = 3 - |3 - x|$ and $y = \frac{6}{|x+1|}$ is

a) $\frac{15}{2} - 6$ In 2 sq. units b) $\frac{13}{2} - 3$ In 2 sq. units c) $\frac{13}{2} - 6$ In 2 sq. units d) None of these

38. The area of the closed figure bounded by $x = -1, x = 2$ and $y = \begin{cases} -x^2 + 2, & x \le 1 \\ 2x - 1, & x > 1 \end{cases}$ and the abscissa axis is

a) 16/3 sq. units b) 10/3 sq. units c) 13/3 sq. units d) 7/3 sq. units

39. The area of the closed figure bounded by $x = -1, y = 0,\ y = x^2 + x + 1$ and the tangent to the curve $y = x^2 + x + 1$ at $A(1,3)$ is

a) 4/3 sq. units b) 7/3 sq. units c) 7/6 sq. units d) None of these

40. The area enclosed between the curves $y = ax^2$ and $x = ay^2 (a > 0)$ is 1 sq unit. Then value of a is

a) $\frac{1}{\sqrt{3}}$ b) $\frac{1}{2}$ c) 1 d) $\frac{1}{3}$

41. The area bounded by the curve $y = f(x)$, the x-axis and the ordinates $x = 1$ and $x = b$ is $(b-1)\sin(3b + 4)$. Then $f(x)$ is

a) $(x-1)\cos(3x+4)$ b) $\sin(3x+4)$

c) $\sin(3x+4) + 3(x-1)\cos(3x+4)$ d) None of these

42. The area bounded by the curves $y = xe^x, y = xe^{-x}$ and the line $x = 1$ is

a) $\frac{2}{e}$ sq. units b) $1 - \frac{2}{e}$ sq. units c) $\frac{1}{e}$ sq. units d) $1 - \frac{1}{e}$ sq. units

43. The area bounded by the curves $y = \sqrt{x}, 2y + 3 = x$ and x-axis in the first quadrant is

a) 9 b) 27/4 c) 36 d) 18

44. Let $f(x) = x^3 + 3x + 2$ and $g(x)$ is the inverse of it. Then the area bounded by $g(x)$, the x-axis and the ordinate at $x = -2$ and $x = 6$ is

a) 1/4 sq. units b) 4/3 sq. units c) 5/4 sq. units d) 7/3 sq. units

45. The area enclosed between the curve $y^2(2a - x) = x^3$ and the line $x = 2$ above the x-axis is

a) πa^2 sq. units b) $\frac{3\pi a^2}{2}$ sq. units c) $2\pi a^2$ sq. units d) $3\pi a^2$ sq. units

Multiple Correct Answers Type

46. Area bounded by the curve $y = $ In, $x, y = 0$ and $x = 3$ is

a) (In 9 − 2) sq unit b) (In 27 − 2) sq unit c) In $\left(\frac{27}{e^2}\right)$ sq unit d) (greater than 3) sq unit

47. For which of the following values of m is the area of the regions bounded by the curve $y = x - x^2$ and the line $y = mx$ equal to 9/2?

a) −4 b) −2 c) 2 d) 4

48. Let $A(k)$ be the area bounded by the curves $y = x^2 - 3$ and $y = kx + 2$

a) The range of $A(k)$ is $\left[\frac{10\sqrt{5}}{3}, \infty\right)$

b) The range of $A(k)$ is $\left[\frac{20\sqrt{5}}{3}, \infty\right)$

c) If function $k \to A(k)$ is defined for $k \in [-2, \infty)$, then $A(k)$ is many-one function

d) The value of k for which area is minimum is 1

49. The value (s) of `a' for which the area of the triangle included between the axes and any tangent to the curve $x^a y = \lambda^a$ is constant is/are

a) $-\frac{1}{2}$ b) -1 c) $\frac{1}{2}$ d) 1

50. The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4, y = 4$ and the co-ordinate axes. If S_1, S_2, S_3 are the areas of these parts numbered from top to bottom, respectively, then

a) $S_1 : S_2 \equiv 1:1$ b) $S_2 : S_3 \equiv 1:2$ c) $S_1 : S_3 \equiv 1:1$ d) $S_1 : (S_1 + S_2) = 1:2$

51. The area enclosed by the curves $x = a\sin^3 t$ and $y = a\cos^3 t$ is equal to

a) $12a^2 \int_0^{\pi/2} \cos^4 t \sin^2 t \, dt$ b) $12a \int_0^{\pi/2} \cos^2 t \sin^4 t \, dt$

c) $2 \int_{-a}^{a} \left(a^{2/3} - x^{2/3}\right)^{3/2} dx$ d) $4 \int_0^{a} \left(a^{2/3} - x^{2/3}\right)^{3/2} dx$

52. If the curve $y = ax^{1/2} + bx$ passes through the point (1, 2) and lies above the x-axis for $0 \le x \le 9$ and the area enclosed by the curve, the x-axis and the line $x = 4$ is 8 sq. units. Then

a) $a = 1$ b) $b = 1$ c) $a = 3$ d) $b = -1$

53. Which of the following have the same bounded area

a) $f(x) = \sin x, g(x) = \sin^2 x$, where $0 \le x \le 10\pi$

b) $f(x) = \sin x, g(x) = |\sin x|$, where $0 \le x \le 20\pi$

c) $f(x) = |\sin x|, g(x) = \sin^3 x$, where $0 \le x \le 10\pi$

d) $f(x) = \sin x, g(x) = \sin^4 x$, where $0 \le x \le 10\pi$

54. If A_i is the area bounded by $|x - a_i| + |y| = b_i, i \in N$, where $a_{i+1} = a_i + \frac{3}{2}b_i$ and $b_{i+1} = \frac{b_i}{2}, a_1 = 0, b_1 = 32$, then

a) $A_3 = 128$ b) $A_3 = 256$ c) $\lim_{n\to\infty} \sum_{i=1}^{n} A_i = \frac{8}{3}(32)^2$ d) $\lim_{n\to\infty} \sum_{i=1}^{n} A_i = \frac{4}{3}(16)^2$

Assertion - Reasoning Type

This section contain(s) 0 questions numbered 55 to 54. Each question contains STATEMENT 1(Assertion) and STATEMENT 2(Reason). Each question has the 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

a) Statement 1 is True, Statement 2 is True; Statement 2 **is** correct explanation for Statement 1

b) Statement 1 is True, Statement 2 is True; Statement 2 **is not** correct explanation for Statement 1

c) Statement 1 is True, Statement 2 is False

d) Statement 1 is False, Statement 2 is True

55

Statement 1: Let f be a real values function satisfying $f\left(\frac{x}{y}\right) = f(x) - f(y)$ and $\lim_{x\to 0} \frac{f(1+x)}{x} = 3$. Then, the area bounded by the curve $y = f(x)$, the y-axis and the line $y = 3$ is $3e$ sq unit

Statement 2: The function $f(x)$ is concave down

56

Statement 1: The area bounded by the curves $y = x^2 + 2x - 3$ and the line $y = \lambda x + 1$ is least, if $\lambda = 2$

Statement 2: The area bounded by the curve $y = x^2 + 2x - 3$ and $y = \lambda x + 1$ is $\{(\lambda - 2)^2 + 16)^{3/2}\}$ sq unit

57 $f(x)$ is a polynomial of degree 3 passing through origin having local extrema at $x = \pm 2$

Statement 1: Ratio of areas in which $f(x)$ cuts the circle $x^2 + y^2 = 36$ is 1:1

Statement 2: Both $y = f(x)$ and the circle are symmetric about origin

58

Statement 1: Area enclosed by the curve $y = e^{x^3}$ between the lines $x = a, x = b$ and x-axis is $\int_a^b e^{x^3}\, dx$

Statement 2: e^{x^3} is an increasing function

59 Consider two regions

R_1: Point P is nearer to $(1, 0)$ than to $x = -1$

R_2: Point P is nearer to $(0, 0)$ than to $(8, 0)$

Statement 1: Area of the region common to R_1 and R_2 is $\frac{128}{3}$ sq. units

Statement 2: Area bounded by $x = 4\sqrt{y}$ and $y = 4$ is $\frac{32}{3}$ sq. units

60

Statement 1: If $f(x) = (x - 1)(x - 2)(x - 3)$, then area enclosed by $|f(x)|$ between the lines $x = 2.2, x = 2.8$ and x-axis is equal to $\int_{2.2}^{2.8} (x - 1)(x - 2)(x - 3)\, dx$

Statement 2: $(x - 1)(x - 2)(x - 3) \le 0, \forall\, x \in [2.2, 2.8]$

61

Statement 1: The area enclosed between the parabolas $y^2 - 2y + 4x + 5 = 0$ and $x^2 + 2x - y + 2 = 0$ is same as that of bounded by curves $y^2 = -4x$ and $x^2 = y$

Statement 2: Shifting of origin to point (h, k) does not change the bounded area

62

Statement 1: The area bounded by the curves $y = x^2 - 3$ and $y = kx + 2$ is least if $k = 0$

Statement 2: The area bounded by the curves $y = x^2 - 3$ and $y = -kx + 2$ is $\sqrt{k^2 + 20}$

63

Statement 1: Area enclosed by the curve $|x| + |y| = 2$ is 8 unit

Statement 2: $|x| + |y| = 2$ represents on square of side length $\sqrt{8}$ unit

64

Statement 1: The area of the function $y = \sin^2 x$ from 0 to π will be more than that of curve $y = \sin x$ from 0 to π

Statement 2: $t^2 < t$, if $0 < t < 1$

65

Statement 1: The area bounded by parabola $y = x^2 - 4x + 3$ and $y = 0$ is 4/3 sq. units

Statement 2: The area bounded by curve $y = f(x) \ge 0$ and $y = 0$ between ordinates $x = a$ and $x = b$ (where $b > a$) is $\int_a^b f(x) dx$

66

Statement 1: Area bounded by $y = e^x, y = 0$ and $x = 0$ is 1 sq. units

Statement 2: Area bounded by $y = \log_e x, x = 0$ and $y = 0$ is 1 sq. units

67

Statement 1: Area bounded by $2 \geq \max.\{|x - y|, |x + y|\}$ is 8 sq. units

Statement 2: Area of the square of side length 4 is 16 sq. units

68

Statement 1: The area of the ellipse $2x^2 + 3y^2 = 6$ will be more than the area bounded by $2|x| + 3|y| \leq 6$

Statement 2: The length of major axis of the ellipse $2x^2 + 3y^2 = 6$ is less than the distance between the points of $2|x| + 3|y| \leq 6$ on x-axis

69

Statement 1: The area of the region bounded by the curve $2y = \log_e x, y = e^{2x}$ and the pair of lines $(x + y - 1) \times (x + y - 3) = 0$ is $2k$ sq. units

Statement 2: The area of the region bounded by the curves $y = e^{2x}, y = x$ and the pair of lines $x^2 + y^2 + 2xy - 4x - 4y + 3 = 0$ is k units

Matrix-Match Type

This section contain(s) 0 question(s). Each question contains Statements given in 2 columns which have to be matched. Statements (A, B, C, D) in **columns I** have to be matched with Statements (p, q, r, s) in **columns II.**

70.

Column-I	Column- II
(A) Area enclosed by $[x]^2 = [y]^2$ for $1 \leq x \leq 4$	(p) 8 sq. units
(B) Area enclosed by $[|x|] + [|y|] = 2$	(q) 6 sq. units
(C) Area enclosed by $[|x|][|y|] = 2$	(r) 4 sq. units
(D) Area enclosed by $\frac{[|x|]}{[|y|]} = 2, -5 \leq x \leq 5$	(s) 12 sq. units

CODES :

	A	**B**	**C**	**D**
a)	p	q	s	r
b)	q	s	p	p
c)	s	p	r	q
d)	p	r	q	s

71.

Column-I	Column- II

(A)	Area enclosed by $y = [x]$ and $y = \{x\}$ where $[\cdot]$ and $\{\cdot\}$ represent greatest integer and fractional part functions, respectively	(p)	32/5 sq. units
(B)	The area bounded by the curves $y^2 = x^3$ and $\|y\| = 2x$	(q)	1 sq. units
(C)	The smaller area included between the curves $\sqrt{x} + \sqrt{\|y\|} = 1$ and $\|x\| + \|y\| = 1$	(r)	4 sq. units
(D)	Area bounded by the curves $y = \left[\frac{x^2}{64} + 2\right]$ (where [.] denotes the greatest integer function), $y = x - 1$ and $x = 0$ above the x-axis	(s)	2/3 sq. units

CODES :

	A	B	C	D
a)	r	s	q	p
b)	p	q	r	s
c)	q	p	s	r
d)	s	r	p	q

72.

	Column-I		Column- II
(A)	The area bounded by the curve $y = x\|x\|$, x-axis and the ordinates $x = 1, x = -1$	(p)	10/3 sq. units
(B)	The area of the region lying between the lines $x - y + 2 = 0, x = 0$ and the curve $x = \sqrt{y}$	(q)	64/3 sq. units
(C)	The area enclosed between the curves $y^2 = x$ and $y = \|x\|$	(r)	2/3 sq. units
(D)	The area bounded by parabola $y^2 = x$, straight line $y = 4$ and y-axis	(s)	1/6 sq. units

CODES :

	A	B	C	D
a)	r	p	s	q
b)	p	s	q	r
c)	q	r	p	s
d)	s	q	r	p

Linked Comprehension Type

This section contain(s) 13 paragraph(s) and based upon each paragraph, multiple choice questions have to be answered. Each question has atleast 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 73 to -73

Let $f(x) = x^2 - 3x + 2$ be a function, $\forall x \in R$

On the basis of above information, answer the following questions

73. The area bounded by $f(x)$, the x-axis and y-axis is
a) $\frac{1}{3}$ sq unit b) $\frac{2}{3}$ sq unit c) $\frac{3}{5}$ sq unit d) $\frac{5}{6}$ sq unit

Paragraph for Question Nos. 74 to - 74

Let there are two functions defined by $f(x) = \min(|x|, |x-1|, |x+1|)$ and $g(x) = \min\{e^x, e^{-x}\}$. Now, the roots of he equation $e^{-x} - x = 0$ is $a, \forall\, a \in R$
On the basis of above information, answer the following questions :

74. The area bounded by $f(x)$ in $[-1,1]$ and x-axis is
a) $\frac{1}{5}$ sq unit b) $\frac{1}{4}$ sq unit c) $\frac{1}{3}$ sq unit d) $\frac{1}{2}$ sq unit

Paragraph for Question Nos. 75 to - 75

Let A_r be the area of the region bounded between the curves $y^2 = (e^{-kr})x$ (where $k > 0, r \in N$) and the line $y = mx$ (where $m \neq 0$), k and m are some constants

75. $A_1, A_2, A_3, \ldots$ are in G.P. with common ratio
a) e^{-k} b) e^{-2k} c) e^{-4k} d) None of these

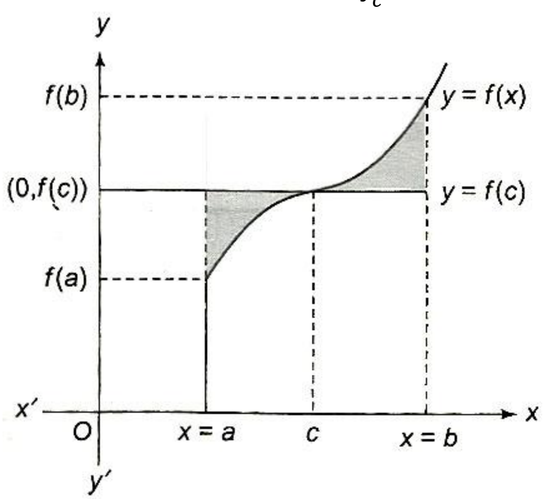
Paragraph for Question Nos. 76 to - 76

If $y = f(x)$ is a monotonic function in (a, b), then the area bounded by the ordinates at $x = a, x = b, y = f(x)$ and $y = f(c)$ (where $c \in (a, b)$) is minimum when $c = \frac{a+b}{2}$

Proof: $A = \int_a^c (f(c) - f(x))\,dx + \int_c^b (f(x) - f(c))dx$

$$= f(c)(c-a) - \int_a^c (f(x))dx + \int_c^b (f(x))dx - f(c)(b-c)$$

$$\Rightarrow A = [2c - (a+b)]f(c) + \int_c^b (f(x))dx - \int_a^c (f(x))dx$$



Differentiating w.r.t. c,

$$\frac{dA}{dc} = [2c - (a+b)]f'(c) + 2f(c) + 0 - f(c) - (f(c) - 0)$$

For maximum and minima $\frac{dA}{dc} = 0$
$\Rightarrow f'(c)[2c - (a+b)] = 0$ (as $f'(c) \neq 0$)
Hence $c = \frac{a+b}{2}$
Also for $c < \frac{a+b}{2}, \frac{dA}{dc} < 0$ and for $c > \frac{a+b}{2}, \frac{dA}{dc} > 0$
Hence A is minimum when $c = \frac{a+b}{2}$

76. If the area bounded by $f(x) = \frac{x^3}{3} - x^2 + a$ and the straight lines $x = 0, x = 2$ and the x-axis is minimum, then the value of a is
a) 1/3 b) 2 c) 1 d) 2/3

Paragraph for Question Nos. 77 to - 77

Consider the areas S_0, S_1, S_2 ... bounded by the x-axis and half-waves of the curve $y = e^{-x} \sin x$, where $x \geq 0$

77. The value of S_0 is
a) $\frac{1}{2}(1 + e^{\pi})$ sq. units b) $\frac{1}{2}(1 + e^{-\pi})$ sq. units c) $\frac{1}{2}(1 - e^{-\pi})$ sq. units d) $\frac{1}{2}(e^{\pi} - 1)$ sq. units

Paragraph for Question Nos. 78 to - 78

Two curves $C_1 \equiv [f(y)]^{2/3} + [f(x)]^{1/3} = 0$ and $C_2 \equiv [f(y)]^{2/3} + [f(x)]^{2/3} = 12$, satisfying the relation $f(x-y)f(x+y) - (x+y)f(x-y) = 4xy(x^2 - y^2)$

78. The area bounded by C_1 and C_2 is
a) $2\pi - \sqrt{3}$ sq. units b) $2\pi + \sqrt{3}$ sq. units c) $\pi + \sqrt{6}$ sq. units d) $2\sqrt{3} - \pi$ sq. units

Paragraph for Question Nos. 79 to - 79

Consider the two curves $C_1: y = 1 + \cos x$ and $C_2: y = 1 + \cos(x - \alpha)$ for $\alpha \equiv \left(0, \frac{\pi}{2}\right)$, where $x \in [0, \pi]$. Also the area of the figure bounded by the curves C_1, C_2 and $x = 0$ is same as that of the figure bounded of $C_2, y = 1$ and $x = \pi$

79. The value of α is
a) $\frac{\pi}{4}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{8}$

Paragraph for Question Nos. 80 to - 80

Consider the function defined implicitly by the equation $y^2 - 2\,ye^{\sin^{-1} x} + x^2 - 1 + [x] + e^{2\sin^{-1} x} = 0$ (where $[x]$ denotes the greatest integer function)

80. The area of the region bounded by the curve and the line $x = -1$ is
a) $\pi + 1$ sq. units b) $\pi - 1$ sq. units c) $\frac{\pi}{2} + 1$ sq. units d) $\frac{\pi}{2} - 1$ sq. units

Paragraph for Question Nos. 81 to - 81

Computing area with parametrically represented boundaries:
If the boundary of a figure is represented by parametric equation, i.e., $x = x(t), y = y(t)$, then the area of the figure is evaluated by one of the three formulas

$$S = -\int_{\alpha}^{\beta} y(t)x'(t)dt,\ S = \int_{\alpha}^{\beta} x(t)y'(t)dt,$$

$$S = \frac{1}{2}\int_{\alpha}^{\beta} (xy' - yx')\, dt,$$

Where α and β are the values of the parameter t corresponding respectively to the beginning and the end of the traversal of the curve corresponding to increasing t

81. The area of the region bounded by an arc of cycloid $x = a(t - \sin t), y = a(1 - \cos t)$ and the x-axis is
a) $6\pi a^2$ sq. units b) $3\pi a^2$ sq. units c) $4\pi a^2$ sq. units d) None of these

Integer Answer Type

82. If 'a' $(a > 0)$ is the value of parameter for each of which the area of the figure bounded by the straight line, $y = \frac{a^2-ax}{1+a^4}$ and the parabola $y = \frac{x^2+2ax+3a^2}{1+a^4}$ is the greatest, then the value of a^4 is
83. If S is the sum of possible values of c for which the area of the figure bounded by the curves $y = \sin 2x$, the straight lines $x = \pi/6, x = c$ and the abscissa axis is equal to $1/2$, then the value of π/S is
84. The area enclosed by the curve $C: y = x\sqrt{9 - x^2}$ $(x \geq 0)$ and the x-axis is
85. If the area enclosed by the curve $y = \sqrt{x}$ and $x = -\sqrt{y}$, the circle $x^2 + y^2 = 2$ above the x-axis, is A then the value of $\frac{16}{\pi}A$ is
86. If the area bounded by the curve $y = x^2 + 1$ and the tangents to it drawn from the origin is A, then the value of $3A$ is
87. Let S be the area bounded by the curve $y = \sin x$ $(0 \leq x \leq \pi)$ and the x-axis and T be the area bounded by the curves $y = \sin x \left(0 \leq x \leq \frac{\pi}{2}\right), y = a\cos x \left(0 \leq x \leq \frac{\pi}{2}\right)$ and the x-axis (where $a \in R^+$)
The value of $(3a)$ such that $S : T = 1 : \frac{1}{3}$ is
88. Area bounded by the relation $[2x] + [y] = 5, x, y > 0$, is (where $[\cdot]$ represents greatest integer function)
89. If the area bounded by the curve $f(x) = x^{1/3}\ (x - 1)$ and x-axis is A, then the value of $28A$ is
90. If S is the sum of cubes of possible value of 'c' for which the area of the figure bounded by the curve $y = 8x^2 - x^5$, then straight lines $x = 1$ and $x = c$ and the abscissa axis is equal to $16/3$, then the value of $[S]$, where $[\cdot]$ denotes the greatest integer function, is
91. Let C be a curve passing through $M(2, 2)$ such that the slope of the tangent at any point to the curve is reciprocal of the ordinate of the point. If the area bounded by curve C and line $x = 2$ is A, then the value of $\frac{3A}{2}$ is
92. Consider two curve $C_1: y = \frac{1}{x}$ and $C_2: y = \text{In } x$ on the xy plane. Let D_1 denotes the region surrounded by C_1, C_2 and the line $x = 1$ and D_2 denotes the region surrounded by C_1, C_2 and the line $x = a$. If $D_1 = D_2$, then the sum of logarithm of possible values of a is
93. If the area of the region $\{(x, y): 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\}$ is A, then the value of $3A - 17$ is
94. The area bounded by the curves $y = x(x - 3)^2$ and $y = x$ is (in sq. units):
95. The area enclosed by $f(x) = 12 + ax + -x^2$ coordinates axes and the ordinates at $x = 3(f(3) > 0)$ is 45 square units. If m and n are the x-axis intercepts of the graph of $y = f(x)$ then the value of $(m + n + a)$ is

96. The value 'a' ($a > 0$) for which the area bounded by the curves $y = \frac{x}{6} + \frac{1}{x^2}, y = 0, x = a$ and $x = 2a$ has the least value is

97. If A is the area bounded by the curves $y = \sqrt{1 - x^2}$ and $y = x^3 - x$, then the value of π/A

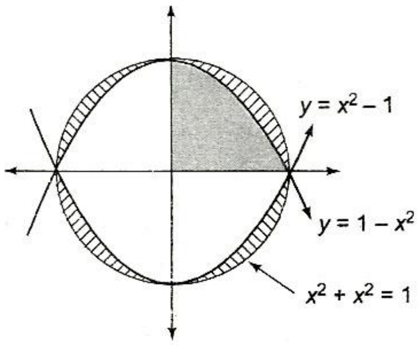
: ANSWER KEY :

1)	a	2)	a	3)	d	4)	a
5)	b	6)	b	7)	b	8)	c
9)	a	10)	b	11)	b	12)	a
13)	d	14)	a	15)	b	16)	a
17)	c	18)	d	19)	d	20)	d
21)	d	22)	a	23)	a	24)	a
25)	c	26)	c	27)	a	28)	a
29)	a	30)	d	31)	d	32)	b
33)	d	34)	b	35)	c	36)	c
37)	c	38)	a	39)	c	40)	a
41)	c	42)	a	43)	a	44)	c
45)	b	1)	b,c,d	2)	b,d	3)	
	b,c	4)	b,d				
5)	a,c,d	6)	a,c,d	7)	c,d	8)	
	a,c,d						

9)	a,c	1)	b	2)	c	3)	a
	4)	b					
5)	d	6)	d	7)	a	8)	c
9)	a	10)	d	11)	b	12)	a
13)	b	14)	d	15)	a	1)	b
	2)	c	3)	a	1)	d	
	2)	d	3)	b	4)	d	
5)	a	6)	b	7)	c	8)	a
9)	b	1)	3	2)	6	3)	9
	4)	8					
5)	2	6)	4	7)	3	8)	9
9)	2	10)	8	11)	1	12)	6
13)	8	14)	8	15)	1	16)	2

: HINTS AND SOLUTIONS :

1 **(a)**



The dotted area is

$$A=\int_0^1(1-x^2)dx=\left(x-\frac{x^3}{3}\right)_0^1=1-\frac{1}{3}=\frac{2}{3}$$

Hence, area bounded by circle $x^2+y^2=1$ and $|y|=1-x^2$

= lined area

= Area of circle – area bounded by $|y|=1-x^2$

$=\pi-4.\left(\frac{2}{3}\right)=\frac{3\pi-8}{3}$ sq. units

2 **(a)**

The two curves are

$xy^2=a^2(a-x)\Rightarrow x=\frac{a^3}{a^2+y^2}$ (1)

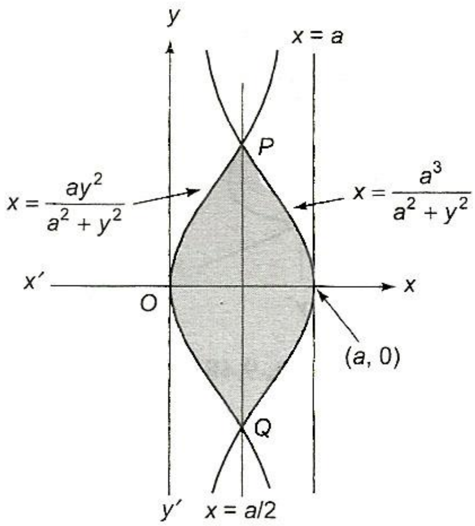
and $(a-x)y^2=a^2x$

$\Rightarrow x=\frac{ay^2}{a^2+y^2}=\frac{ay^2+a^3-a^3}{a^2+y^2}=a-\frac{a^3}{a^2+y^2}$ (2)

Curve (1) is symmetrical about x-axis, and have y-axis as the asymptote

Curve (2) is symmetrical about x-axis, tangent at origin as y-axis and the asymptote $x=a$

The two curves intersect at the point $P(a/2,a)$ and $Q(a/2,-a)$



Required area

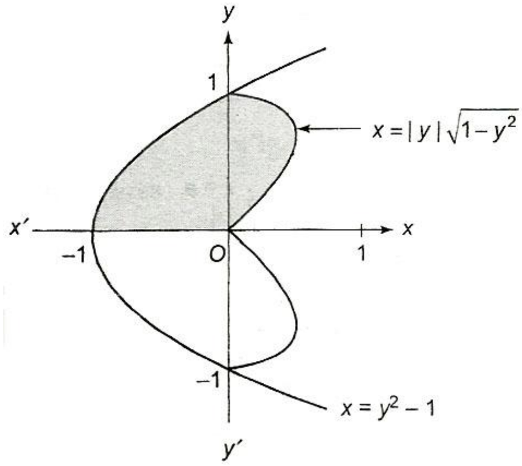
$=2\int_0^a\left[-a+\frac{a^3}{a^2+y^2}+\frac{a^3}{a^2+y^2}\right]dy$ (integrating along y-axis)

$=2\left[-ay+2a^2\tan^{-1}\frac{y}{a}\right]_0^a$

$=2\left[-a^2+2a^2\frac{\pi}{4}\right]$

$=(\pi-2)a^2$ sq. units

3 **(d)**



$$A=2\int_0^1\left[y\sqrt{1-y^2}-(y^2-1)\right]dy$$

= 2 sq. units

4 **(a)**

Clearly t can be any real number

Let $t=\tan\theta\Rightarrow x=\frac{1-\tan^2\theta}{1+\tan^2\theta}$

$\Rightarrow x=\cos 2\theta$, and

$$y=\frac{2\tan\theta}{1+\tan^2\theta}=\sin 2\theta$$

$\Rightarrow x^2+y^2=1$

Thus, required area = π sq. units

5 **(b)**

$xy^2=a^2(a-x)$

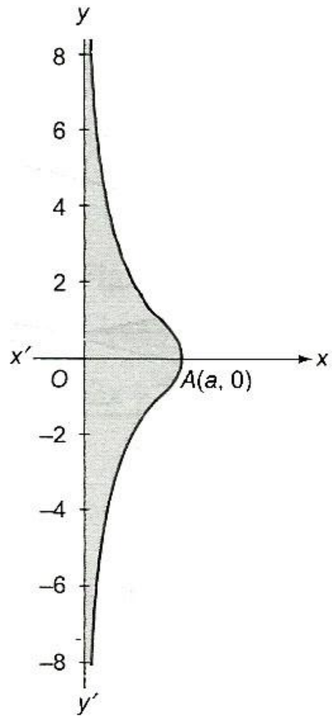
$$\Rightarrow x=\frac{a^3}{y^2+a^2}$$

The given curve is symmetrical about x-axis, and meets it at $(a,0)$

The line $x=0$, i.e., y-axis is an asymptote (tangent at infinitly)

Area $=\int_0^\infty x\,dy=2\int_0^\infty\frac{a^3}{y^2+a^2}dx$

$$=2a^3\frac{1}{a}\left[\tan^{-1}\frac{y}{a}\right]_0^\infty=2a^2\frac{\pi}{2}=\pi a^2\text{ sq. units}$$



6 **(b)**

Given curves are $y = \log_e x$ and $y = (\log_e x)^2$

Solving $\log_e x = (\log_e x)^2 \Rightarrow \log_e x = 0,\ 1 \Rightarrow x = 1$ and $x = e$

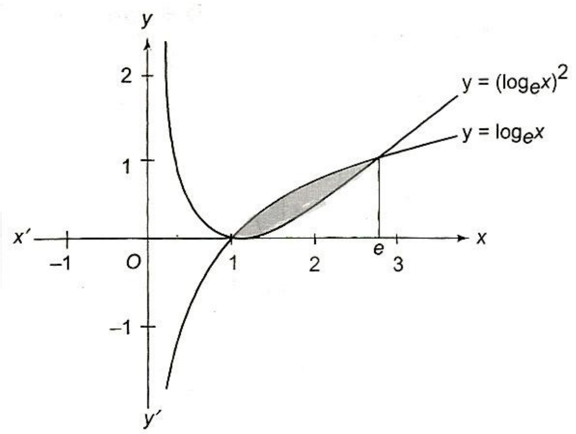
Also, for $1 < x < e, 0 < \log_e x < 1 \Rightarrow \log_e x > (\log_e x)^2$

For $x > e, \log_e x < (\log_e x)^2$

$y = (\log_e x)^2 > 0$ for all $x > 0$

and when $x \to 0, (\log_e x)^2 \to \infty$

From these information, we can plot the graph of the functions



Then the required area $= \int_1^e (\log x - (\log_e x)^2) dx$

$$= \int_1^e \log x dx - \int_1^e (\log_e x)^2 dx$$

$$= [x \log_e x - x]_1^e - [x(\log_e x)^2]_1^e + \int_1^e \frac{2 \log_e x}{x} x dx$$

$= 1 - e + 2[x \log_e x - x]_1^e = 3 - e$ sq. units

7 **(b)**

$$ay^2 = x^2(a - x) \Rightarrow y = \pm x\sqrt{\frac{a - x}{a}}$$

Curve tracing: $y = x\sqrt{\frac{a-x}{a}}$

We must have $x \le a$

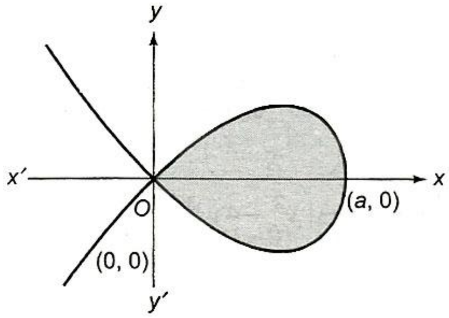
For $0 < x \le a, y > 0$ and for $x < 0, y < 0$

Also $y = 0 \ \Rightarrow x = 0, a$

Curve is symmetrical about x-axis

When $x \to -\infty, y \to -\infty$

Also, it can be verified that y has only one point of maxima for $0 < x < a$



Area $= 2\int_0^a x\sqrt{\frac{a-x}{a}} dx$

$$\sqrt{\frac{a - x}{a}} = t \Rightarrow 1 - \frac{x}{a} = t^2 \Rightarrow x = a(1 - t^2)$$

$$\Rightarrow A = 2\int_1^0 a(1 - t^2)\, t(-2at) dt$$

$$= 4a^2 \int_0^1 (t^2 - t^4) dt$$

$$= 4a^2 \left[\frac{t^3}{3} - \frac{t^5}{5}\right]_0^1$$

$$= 4a^2 \left[\frac{1}{3} - \frac{1}{5}\right] = \frac{8a^2}{15} \text{ sq. units}$$

8 **(c)**

$(y - x)^2 = x^3$, where $x \ge 0 \Rightarrow y - x = \pm x^{3/2}$

$\Rightarrow y = x + x^{3/2}$ (1)

$y = x - x^{3/2}$ (2)

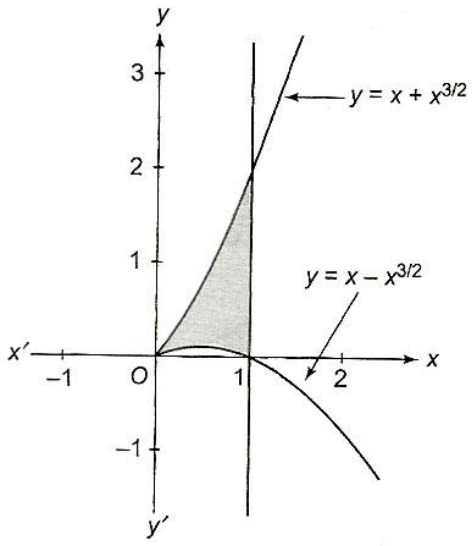
Function (1) is an increasing function

Function (2) meets x-axis, when $x - x^{3/2} = 0$ or $x = 0, 1$

Also, for $0 < x < 1, x - x^{3/2} > 0$ and for $x > 1, x - x^{3/2} < 0$

When $x \to \infty, x - x^{3/2} \to -\infty$

From these information, we can plot the graph as below:



Required area

$= \int_0^1 \left[\left(x + x^{3/2}\right) - \left(x - x^{3/2}\right)\right] dx = 2\int_0^1 x^{3/2} dx$

$= 2\left[\frac{x^{5/2}}{5/2}\right]_0^1 = \frac{4}{5}$ sq. units

9 **(a)**

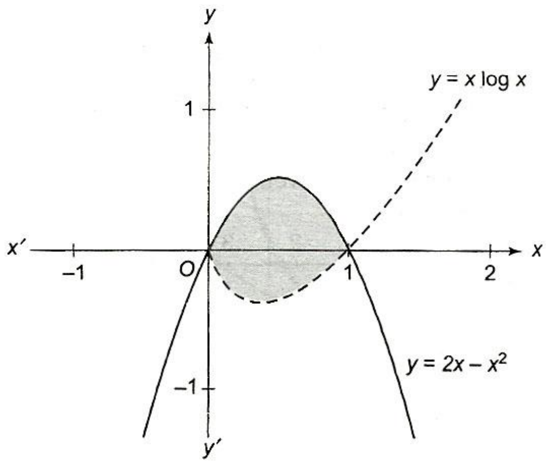
Curve tracing: $y = x \log_e x$

Clearly, $x > 0$

For $0 < x < 1, x \log_e x < 0$, and for $x > 1, x \log_e x > 0$

Also $x \log_e x = 0 \Rightarrow x = 1$

Further, $\frac{dy}{dx} = 0 \Rightarrow 1 + \log_e x = 0 \Rightarrow x = 1/e$, which is a point of minima



Required area

$= \int_0^1 (2x - 2x^2) dx - \int_0^1 x \log x \, dx$

$= \left[x^2 - \frac{2x^3}{3}\right]_0^1 - \left[\frac{x^2}{2}\log x - \frac{x^2}{4}\right]_0^1$

$= \left(1 - \frac{2}{3}\right) - \left[0 - \frac{1}{4} - \frac{1}{2}\lim_{x\to 0} x^2 \log x\right] = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$

10 **(b)**

$f(x) = \sin x$

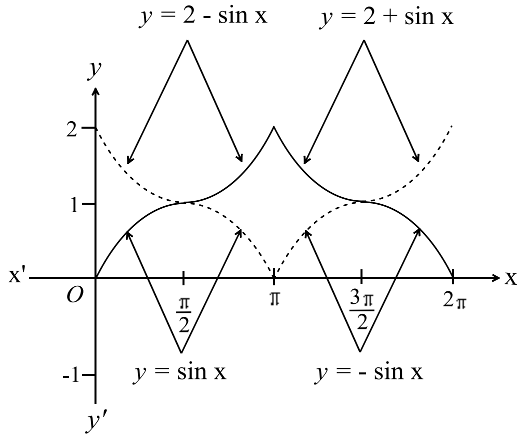
$f(x) + f(\pi - x) = 2$

$f(x) = 2 - f(\pi - x) = 2 - \sin(\pi - x) = 2 - \sin x$, where $x \in \left(\frac{\pi}{2}, \pi\right]$

$f(x) = f(2\pi - x) = 2 - \sin(2\pi - x)$, where $x \in \left(\pi, \frac{3\pi}{2}\right]$

$f(x) = f(2\pi - x) = -\sin x$, where $x \in \left(\frac{3\pi}{2}, 2\pi\right]$

$$f(x) = \begin{cases} \sin x, x \in \left[0, \frac{\pi}{2}\right] \\ 2 - \sin x, x \in \left(\frac{\pi}{2}, \pi\right] \\ 2 + \sin x, x \in \left(\pi, \frac{3\pi}{2}\right] \\ -\sin x, x \in \left(\frac{3\pi}{2}, 2\pi\right] \end{cases}$$



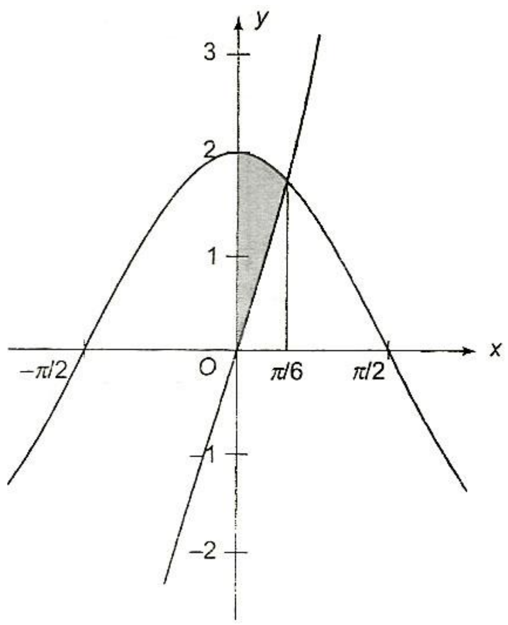
Area $= \int_0^{\pi/2} \sin x \sin dx + \int_{\pi/2}^{\pi} (2 - \sin x) dx + \int_{\pi}^{3\pi/2} (2 + \sin x) dx + \int_{3\pi/2}^{2\pi} (-\sin x)\, dx$

$= 1 + 2 \times \frac{\pi}{2} - 1 + 2.\frac{\pi}{2} - 1 + 1 = 2\pi$ sq. units

11 **(b)**

Solving $2\cos x = 3\tan x$, we get

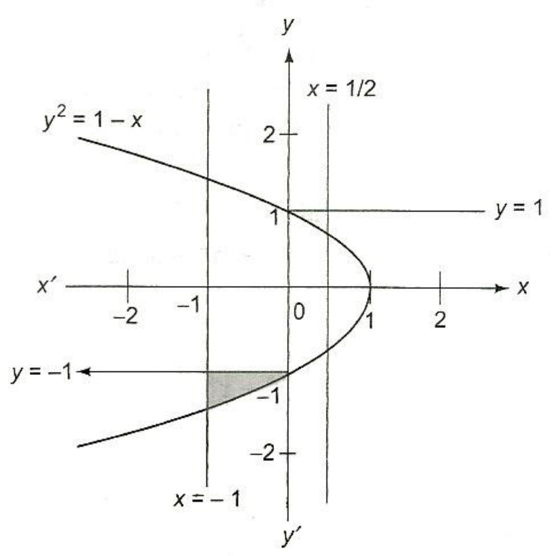
$2 - 2\sin^2 x = 3\sin x \Rightarrow \sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}$



Required area $= \int_0^{\pi/6} (2\cos x - 3\tan x) dx$

$= 2\sin x - 3\log\sec x|_0^{\pi/6} = 1 - 3 \text{ In } 2$
$+ \frac{3}{2} \text{ In } 3 \text{ sq. units}$

12 **(a)**



From figure

$$A = \int_{-1}^{0}\left(-1 - \left(-1\sqrt{1-x}\right)\right)dx + \int_{0}^{1/2}(1 - \sqrt{1-x})dx$$

$$= \left[-x - \frac{(1-x)^{3/2}}{3/2}\right]_{-1}^{0} + \left[x + \frac{(1-x)^{3/2}}{3/2}\right]_{0}^{1/2}$$

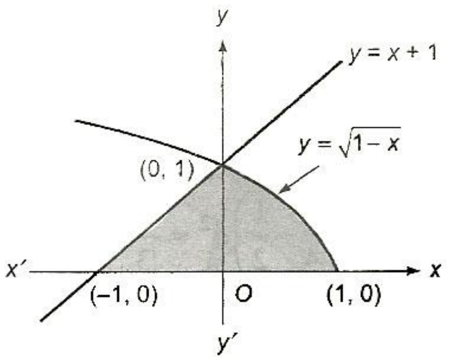
$$= \left[-\frac{2}{3} - \left(1 - \frac{2\times 2^{3/2}}{3}\right)\right] + \left[\frac{1}{2} + \frac{2}{3\times 2^{3/2}} - \frac{2}{3}\right]$$

$$= \frac{2}{3\times 2^{3/2}} + \frac{2\times 2^{3/2}}{3} - \frac{4}{3} - \frac{1}{2}$$

$$= \frac{3}{\sqrt{2}} - \frac{4}{3} - \frac{1}{2}$$

$$= \frac{3}{\sqrt{2}} - \frac{11}{6} \text{ sq. units}$$

13 **(d)**



Required area = shaded region

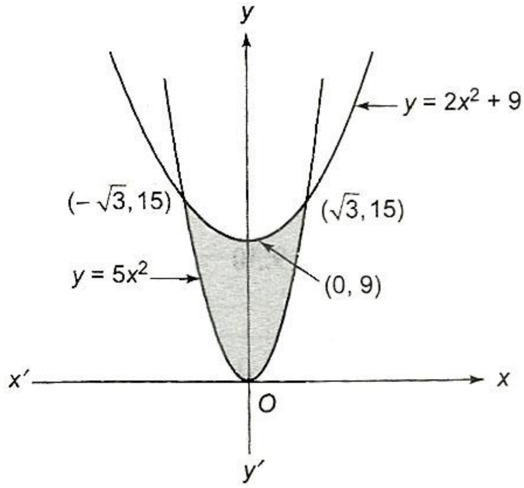
$= \int_0^1 (x_2 - x_1)dy$ (integrating along y-axis)

$$= \int_0^1 [(1-y^2) - (y-1)]dy$$

$$= \frac{7}{6} \text{ sq. units}$$

14 **(a)**

Given $5x^2 - y = 0$, and (1)



$2x^2 - y + 9 = 0$ (2)

Eliminating y, we get

$5x^2 - (2x^2 + 9) = 0$

$\Rightarrow 3x^2 = 9 \Rightarrow x = -\sqrt{3}, \sqrt{3}$

$\therefore$ required area

$$= 2\int_0^{\sqrt{3}}\left((2x^2 + 9) - 5x^2\right)dx$$

$$= 2\int_0^{\sqrt{3}}(9 - 3x^2)\,dx$$

$$= 2[9x - x^3]_0^{\sqrt{3}}$$

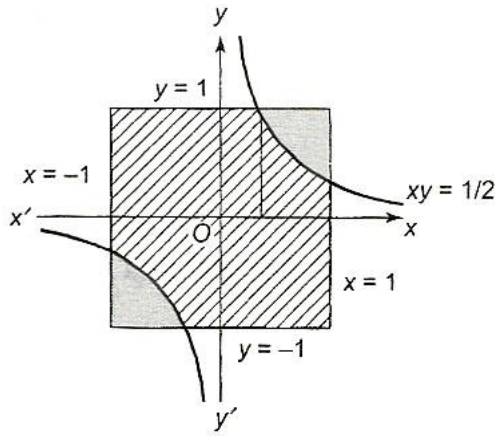
$$= 2\left[9\sqrt{3} - 3\sqrt{3}\right]$$

$$= 12\sqrt{3} \text{ sq. units}$$

15 **(b)**

$\max(|x|, |y|) \le 1 \Rightarrow |x| \le 1$, and $|y| \le 1$

Which represent square bounded by $x = \pm 1$ and $y = \pm 1$



Required area is lined area

Now, shaded area is

$$2\int_{1/2}^{1}\left(1 - \frac{1}{2x}\right)dx = 2\left(x - \frac{1}{2}\text{ In } x\right)_{1/2}^{1}$$

$$= 2\left[(1-0) - \left(\frac{1}{2} - \frac{1}{2}\text{ In}\frac{1}{2}\right)\right]$$

$= 1 - \text{In } 2$ sq. units

$\Rightarrow$ Horizontal lined area $= 4 - (1 - \text{In } 2) = 3 + \text{In } 2$ sq. units

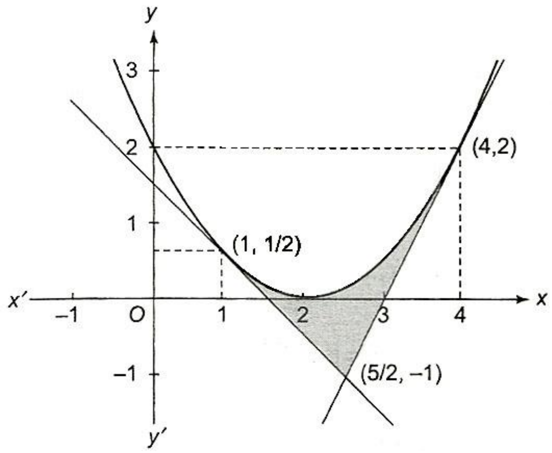
16 **(a)**

$y = \frac{x^2}{2} - 2x + 2 = \frac{(x-2)^2}{2}$,

$\frac{dy}{dx} = x - 2, \left(\frac{dy}{dx}\right)_{x=1} = -1, \left(\frac{dy}{dx}\right)_{x=4} = 2$

$\Rightarrow$ Tangent at $(1, 1/2)$ is $y - 1/2 = -1(x-1)$ or $2x + 2y - 3 = 0$

Tangent at $(4, 2)$ is $y - 2 = 2(x-4)$ or $2x - y - 6 = 0$



Hence, $A = \int_1^{5/2} \left(\frac{x^2}{2} - 2x + 2 - \frac{3-2x}{2}\right) dx + \int_{5/2}^4 \left(\frac{x^2}{2} - 2x + 2 - (2x-6)\right) dx$

$= \int_1^4 \left(\frac{x^2}{2} - 2x + 2\right) dx - \int_1^{5/2} \left(\frac{3-2x}{2}\right) dx - \int_{5/2}^4 (2x-6) dx$

$= \left(\frac{x^3}{6} - x^2 + 2x\right)_1^4 - \frac{1}{2}(3x - x^2)_1^{5/2} - (x^2 - 6x)_{5/2}^4$

$= \left(\frac{63}{6} - 15 + 6\right) - \frac{1}{2}\left(3 \times \frac{3}{2} - \left(\frac{25}{4} - 1\right)\right) - \left(\left(16 - \frac{25}{4}\right) - 6\left(4 - \frac{5}{2}\right)\right)$

$= \frac{3}{2} - \frac{1}{2}\left(\frac{9}{2} - \frac{21}{4}\right) - \left(\frac{39}{4} - 6\left(\frac{3}{2}\right)\right)$

$= \frac{9}{8}$ sq. units

17 **(c)**

$a^2x^2 + ax + 1$ is clearly positive for all real values of x. Area under consideration

$A = \int_0^1 (a^2x^2 + ax + 1)\, dx$

$= \frac{a^2}{3} + \frac{a}{2} + 1$

$= \frac{1}{6}(2a^2 + 3a + 6)$

$= \frac{1}{6}\left(2\left(a^2 + \frac{3}{2}a + \frac{9}{16}\right) + 6 - \frac{18}{16}\right)$

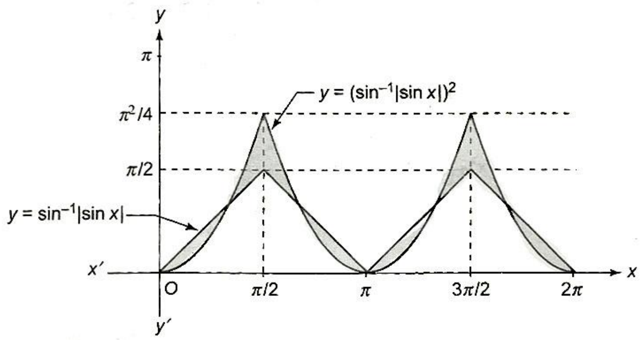
$= \frac{1}{6}\left(2\left(a + \frac{3}{4}\right)^2 + \frac{39}{8}\right)$, which is clearly minimum for $a = -\frac{3}{4}$

18 **(d)**

$$y = \sin^{-1}|\sin x| = \begin{cases} x, & 0 \le x < \frac{\pi}{2} \\ \pi - x, \frac{\pi}{2} \le x < \pi \\ x - \pi, \pi \le x < \frac{3\pi}{2} \\ 2\pi - x, \frac{3\pi}{2} \le x < 2\pi \end{cases}$$

$$y = (\sin^{-1}|\sin x|)^2 = \begin{cases} x^2, & 0 \le x < \frac{\pi}{2} \\ (\pi - x)^2, \frac{\pi}{2} \le x < \pi \\ (x - \pi)^2, \pi \le x < \frac{3\pi}{2} \\ (2\pi - x)^2, \frac{3\pi}{2} \le x < 2\pi \end{cases}$$

The required area A is shown shaded in figure



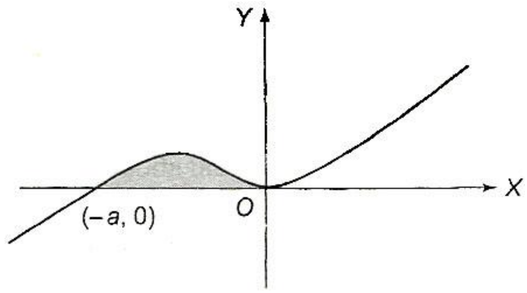
$$\Rightarrow 4\int_0^1 (x - x^2)dx + 4\int_1^{\pi/2} (x^2 - x)\,dx$$

$$= \frac{4}{3} + \pi^2\left[\frac{\pi - 3}{6}\right] \text{ sq. units}$$

19 **(d)**

The curve is $y = \frac{x^2(x+a)}{a^2}$, which is a cubic polynomial

Since $\frac{x^2(x+a)}{a^2} = 0$ has repeated root $x = 0$, it touches x-axis at $(0, 0)$ and intersects at $(-a, 0)$

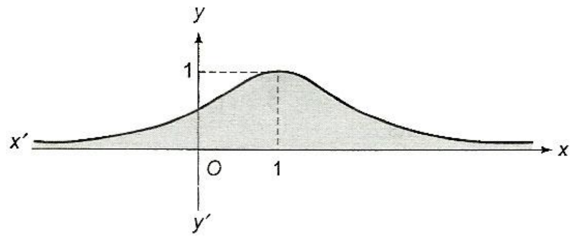


Required area $= \int_{-a}^{0} y dx = \int_{-a}^{0}\left[\frac{x^2(x+a)}{a^2}\right] dx = a^2/12$ sq. units

20 **(d)**

$$y = \frac{1}{(x-1)^2 + 1}$$

y is maximum when $(x-1)^2 = 0$. Also, graph is symmetrical about line $x = 1$



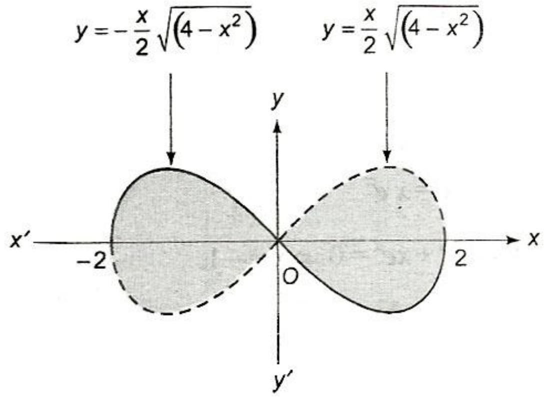
Area $= 2\int_1^{\infty} \frac{1}{(x-1)^2+1} dx = 2[\tan^{-1}(x-1)]_1^{\infty} = \pi$ sq. units

21 **(d)**

$4y^2 = x^2(4 - x^2) \quad (1)$

$$\Rightarrow y = \pm\frac{1}{2}\sqrt{x^2(4 - x^2)}$$

$$\Rightarrow y = \pm\frac{x}{2}\sqrt{(4 - x^2)}$$



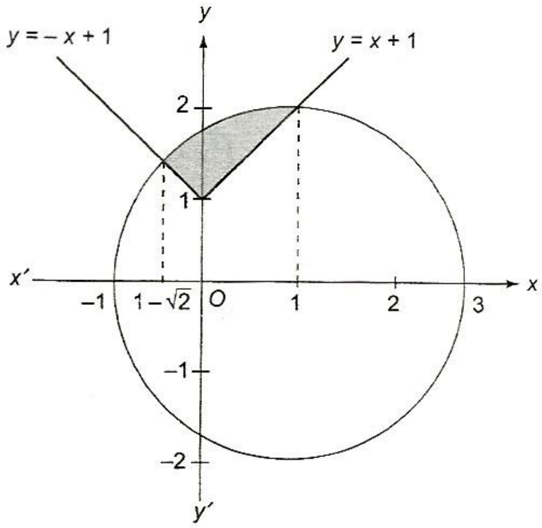
$\therefore$ Area $(A) = 4 \times \int_0^2 \frac{x}{2}\sqrt{(4-x^2)}dx$

Let $4 - x^2 = t \Rightarrow -2x\,dx = dt$

$$\Rightarrow A = \int_0^4 \sqrt{t}\,dt = \left[\frac{t^{3/2}}{3/2}\right]_0^4 = \frac{2}{3} \times [\sqrt{64} - 0]$$

$$\Rightarrow A = \frac{16}{3} \text{ sq. units}$$

22 **(a)**



$x^2 + y^2 - 2x - 3 = 0$

$\Rightarrow (x-1)^2 + y^2 = 4$

$$A = \int_{1-\sqrt{2}}^{0} (\sqrt{4-(x-1)^2} - (-x+1)dx + \int_0^1 \left(\sqrt{4-(x-2)^2} - (x+1)\right)dx$$

$$= \frac{x-1}{2}\sqrt{4-(x-1)^2} + \frac{4}{2}\sin^{-1}\frac{x-1}{2} + \frac{x^2}{2} - x\Big|_{1-\sqrt{2}}^{0}$$

$$+ \frac{x-1}{2}\sqrt{4-(x-1)^2} + \frac{4}{2}\sin^{-2}\frac{x-1}{2} - \frac{x^2}{2} - x\Big|_0^1$$

$$= \left(-\frac{\sqrt{3}}{2} - \frac{\pi}{3}\right) - \left(\frac{-\sqrt{2}}{2}\sqrt{2} - \frac{\pi}{2} + \frac{3-2\sqrt{2}}{2} - 1 + \sqrt{2}\right) + \left(-\frac{1}{2} - 1\right) - \left(-\frac{\sqrt{3}}{2} - \frac{\pi}{3}\right)$$

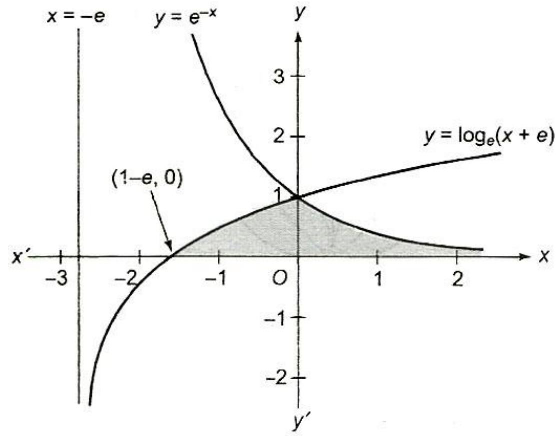
$$= -\left(-1 - \frac{\pi}{2} + \frac{3}{2} - \sqrt{2} - 1 + \sqrt{2}\right) - \frac{3}{2}$$

$$= \frac{\pi}{2} - 1 \text{ sq. units}$$

23 **(a)**

$y = \log_e(x+e),\ x = \log_e\left(\frac{1}{y}\right) \Rightarrow y = e^{-x}$

For $y = \log_e(x+e)$ shift the graph of $y = \log_e x$, e units left hand side



Required area $= \int_{1-e}^{0} \log_e(x+e)dx + \int_0^{\infty} e^{-x}\,dx$

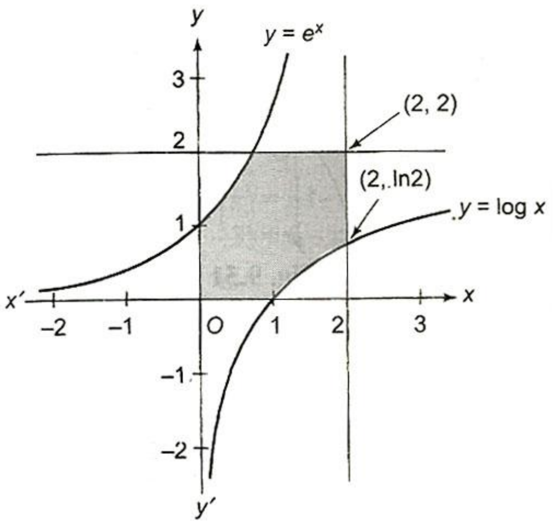
$$= |x\log_e(x+e)|_{1-e}^{0} - \int_{1-e}^{0} \frac{x}{x+e}dx - |e^{-x}|_0^{\infty}$$

$$= \int_0^{1-e} \left(1 - \frac{e}{x+e}\right)dx - e^{-\infty} + e^0$$

$= |x - e\log(x+e)|_0^{1-e} - 0 + 1$

$= 1 - e + e\log e + 1 = 2$ sq. units

24 **(a)**



$$A = \int_1^2 \text{In}\, x\,dx$$

$= [x\log x - x]_1^2$

$= 2\log 2 - 1$

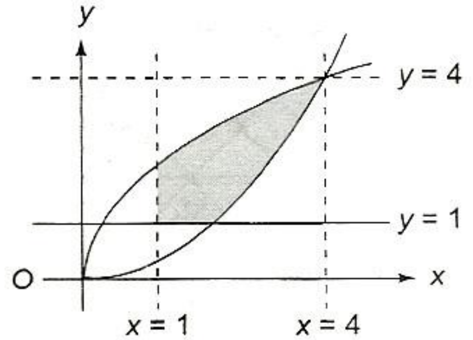
$\Rightarrow$ Required area $= 4 - 2(2\text{ In }2 - 1) = 6 - 4\text{ In }2$ sq. units

25 **(c)**

$y^2 = 4[\sqrt{y}]x$

For $y \in [1,4), [\sqrt{y}] = 1 \Rightarrow y^2 = 4x$

Similarly, for $x \in [1,4)$, $\lfloor\sqrt{x}\rfloor = 1$ and $x^2 = 4\lfloor\sqrt{x}\rfloor y$ would transform into $x^2 = 4y$

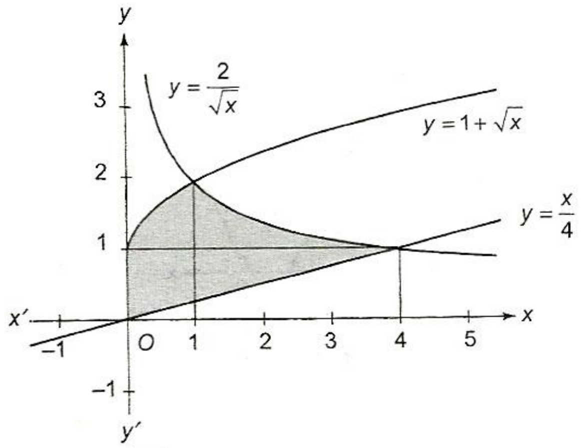


The required area is being shaded

$$A = \int_1^2 (2\sqrt{x} - 1)dx + \int_2^4 \left(2\sqrt{x} - \frac{x^2}{4}\right)dx$$

$$= \left(\frac{4}{3}x^{3/2} - x\right)_1^2 + \left(\frac{4}{3}x^{3/2} - \frac{x^3}{12}\right)_2^4 = \frac{11}{3} \text{ sq. units}$$

26 **(c)**



$$A_1 = \int_0^1 \left(1 + \sqrt{x} - \frac{x}{4}\right)dx$$

$$= \left[x + \frac{2x^{3/2}}{3} - \frac{x^2}{8}\right]_0^1 = 1 + \frac{2}{3} - \frac{1}{8} = \frac{37}{24}$$

$$A_2 = \int_1^4 \left(\frac{2}{\sqrt{x}} - \frac{x}{4}\right)dx$$

$$= \left[4\sqrt{x} - \frac{x^2}{8}\right]_1^4$$

$$= \left[8 - 2 - 4 + \frac{1}{8}\right] = \frac{17}{8}$$

$$\Rightarrow A = A_1 + A_2 = \frac{88}{24} = \frac{11}{3} \text{ sq. units}$$

27 **(a)**

The points in the required region satisfy

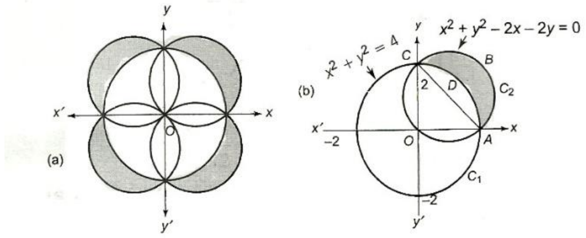
$4 \le x^2 + y^2 \le 2(|x| + |y|)$ (1)

Since the curve (1) is symmetrical about both the axes, the required area is 4 times the area of the region in the first quadrant. Therefore, it is sufficient to sketch the region and to find the area in the first quadrant

In the first quadrant, the curve (1) consist of two curves

$x^2 + y^2 \ge 4$, and (C_1)

$x^2 + y^2 - 2x - 2y \ge 0$ (C_2)



$\therefore$ Required area $= 4$ area $ABCDA$

$= 4$(area of semi-circle $ABCA$) – (area of sector $ADCA$)

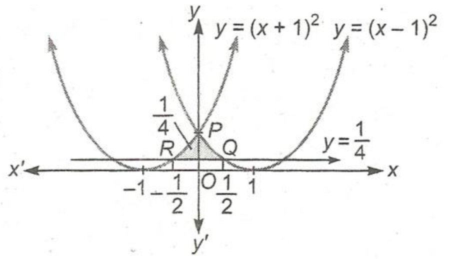
$= 4$(area of semi-circle $ABCA$) – (area of sector $OADCO$ $-$ area of triangle OAC)

$= 4\{\pi - (\pi - 2)\} = 8$ sq. units

28 **(a)**

The points of intersection of given curves and line are

$Q\left(\frac{1}{2}, \frac{1}{4}\right)$ and $R\left(\frac{-1}{2}, \frac{1}{4}\right)$



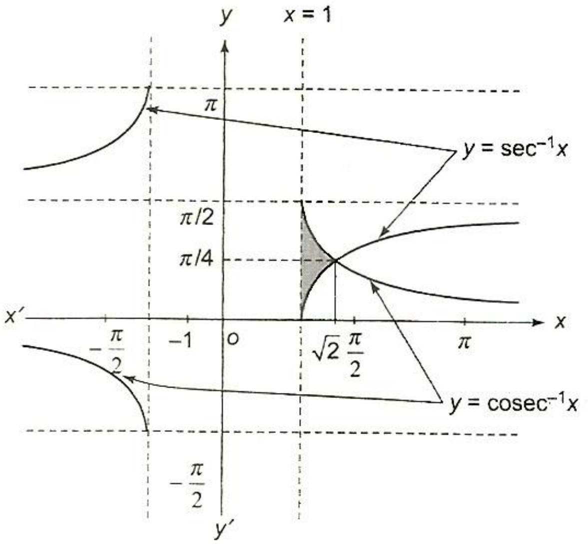
$$\text{Required area} = 2\int_0^{1/2} \left\{(x-1)^2 - \frac{1}{4}\right\}dx$$

$$= 2\left\{\frac{(x-1)^3}{3} - \frac{1}{4}x\right\}_0^{1/2}$$

$$= 2\left\{\frac{(-1/2)^3}{3} - \frac{1}{8} - \left(-\frac{1}{3} - 0\right)\right\}$$

$$= \frac{1}{3} \text{ sq unit}$$

29 **(a)**



Integrating along x-axis, we get

$$A = \int_1^{\sqrt{2}} (\text{cosec}^{-1}x - \sec^{-1} x) dx$$

Integrating along y-axis, we get

$$A = 2\int_0^{\pi/4} (\sec y - 1) dy$$

$$= 2[\log|\sec y + \tan y| - y]_0^{\pi/4}$$

$$= 2\left[\log|\sqrt{2}+1| - \frac{\pi}{4}\right]$$

$$= \log(3 + 2\sqrt{2}) - \frac{\pi}{2} \text{ sq. units}$$

30 **(d)**

Curve tracing : $y = x + \sin x$

$\frac{dy}{dx} = 1 + \cos x \geq 0 \;\; \forall x$

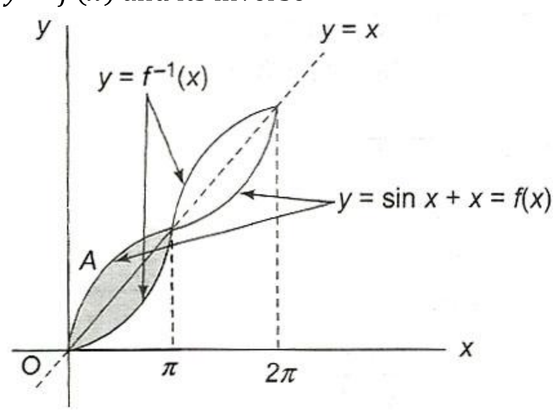
Also $\frac{d^2y}{dx^2} = -\sin x = 0$ when $x = n\pi, n \in Z$

Hence, $x = n\pi$ are points of inflection, where curve changes its concavity

Also for $x \in (0, \pi), \sin x > 0 \Rightarrow x + \sin x > x$,

And for $x \in (\pi, 2\pi), \sin x < 0 \Rightarrow x + \sin x < x$

From these information, we can plot the graph of $y = f(x)$ and its inverse



Required area $= 4A$, where

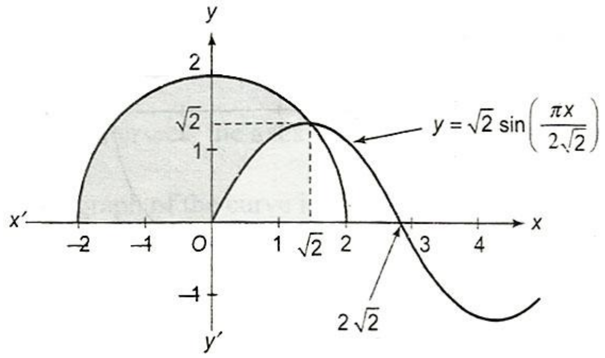
$$A = \int_0^{\pi} (x + \sin x) dx - \int_0^{\pi} x dx$$

$$= \int_0^{\pi} \sin x \, dx = 2 \text{ square units}$$

31 **(d)**

$y = \sqrt{4 - x^2}, y = \sqrt{2}\sin\left(\frac{x\pi}{2\sqrt{2}}\right)$

Intersect at $x = \sqrt{2}$



Area to the left of y-axis is π

Area to the right of y-axis

$$= \int_0^{\sqrt{2}} \left(\sqrt{4 - x^2} - \sqrt{2}\sin\frac{x\pi}{2\sqrt{2}}\right) dx$$

$$= \left(\frac{x\sqrt{4-x^2}}{2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right)_0^{\sqrt{2}} + \left(\frac{4}{\pi}\cos\frac{x\pi}{2\sqrt{2}}\right)_0^{\sqrt{2}}$$

$$= \left(1 + 2 \times \frac{\pi}{4}\right) + \frac{4}{\pi}(0 - 1)$$

$$= 1 + \frac{\pi}{2} - \frac{4}{\pi}$$

$$= \frac{2\pi + \pi^2 - 8}{2\pi} \text{ sq. units}$$

$\therefore$ ratio $= \frac{2\pi^2}{2\pi+\pi^2-8}$

32 **(b)**

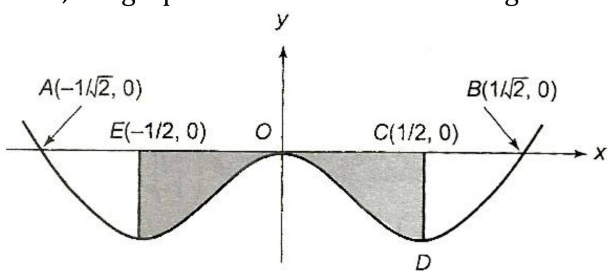
The curve is $y = 2x^4 - x^2 = x^2(2x^2 - 1)$

The curve is symmetrical about the axis of y

Also, it is a polynomial of 4 degree having roots 0, 0, $\pm\frac{1}{\sqrt{2}}$. $x = 0$ is repeated root. Hence, graph touches at (0, 0)

The curve intersects the axes at $O(0, 0), A(-1/\sqrt{2}, 0)$ and $B(1/\sqrt{2}, 0)$

Thus, the graph of the curve is show in figure



Here, $y \leq 0$, as x varies from $x = -1/2$ to $x = 1/2$

$\therefore$ The required area

$= 2$ Area $OCDO$

$$= 2\left|\int_{0}^{1/2} y dx\right|$$
$$= 2\left|\int_0^{1/2}(2x^4 - x^2)dx\right|$$
$= 7/120$ sq. units

33 **(d)**

Area $= \int_1^b f(x)\,dx = \sqrt{b^2+1} - \sqrt{2}$
$$= \sqrt{b^2+1} - \sqrt{1+1}$$
$$= \left|\sqrt{x^2+1}\right|_1^b$$
$$\therefore f(x) = \frac{d}{dx}\left(\sqrt{x^2+1}\right) = \frac{1}{2}\,\frac{2x}{\sqrt{x^2+1}} = \frac{x}{\sqrt{x^2+1}}$$

34 **(b)**

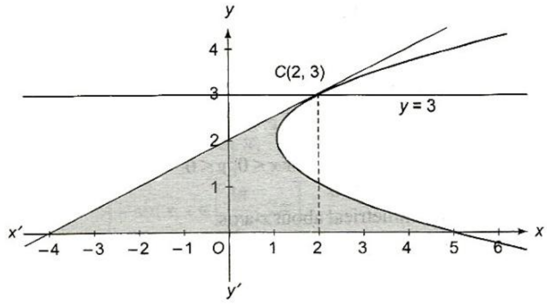
$$\text{Required area} = \int_0^{\pi/4}\left(\sqrt{\frac{1+\sin x}{\cos x}} - \sqrt{\frac{1-\sin x}{\cos x}}\right)dx$$
$$\because \left[\frac{1+\sin x}{\cos x} > \frac{1-\sin x}{\cos x} > 0\right]$$
$$= \int_0^{\pi/4}\left(\sqrt{\frac{1+\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}{\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}}} - \sqrt{\frac{1-\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}{\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}}}\right)dx$$
$$= \int_0^{\pi/4}\frac{1+\tan\frac{x}{2} - 1 + \tan\frac{x}{2}}{\sqrt{1-\tan^2\frac{x}{2}}}dx$$
$$= \int_0^{\pi/4}\frac{2\tan\frac{x}{2}}{\sqrt{1-\tan^2\frac{x}{2}}}dx$$
put $\tan\frac{x}{2} = t \Longrightarrow \frac{1}{2}\sec^2\frac{x}{2}dx = dt$
$$\therefore \text{Required area} = \int_0^{\tan\frac{\pi}{8}}\frac{4t\ \ dt}{(1+t^2)\sqrt{1-t^2}}$$
$$= \int_0^{\sqrt{2}-1}\frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$$
$$\left[\because \tan\frac{\pi}{8} = \sqrt{2}-1\right]$$

35 **(c)**

$$\int_{\pi/4}^{\beta} f(x)dx = \beta\sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\,\beta$$
Differentiating both sides w.r.t. β, we get
$$\therefore f(\beta) = \beta\cos\beta + \sin\beta - \frac{\pi}{4}\sin\beta + \sqrt{2}$$
$$\Rightarrow f'(\beta) = -\beta\sin\beta + \cos\beta + \cos\beta - \frac{\pi}{4}\cos\beta$$
$$\Rightarrow f'\left(\frac{\pi}{2}\right) = -\frac{\pi}{2}$$

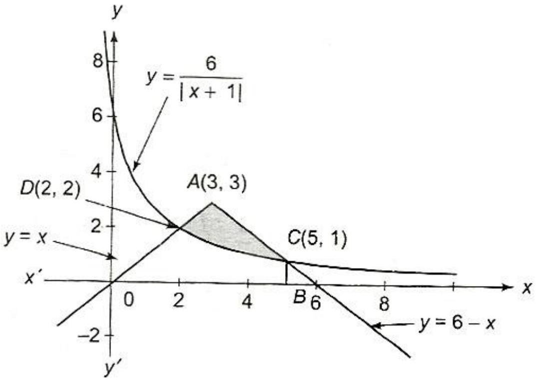
36 **(c)**

Given parabola is $(y-2)^2 = x-1$
$$\Rightarrow \frac{dy}{dx} = \frac{1}{2(y-2)}$$
When $y = 3, x = 2$
$$\therefore \frac{dy}{dx} = \frac{1}{2(3-2)} = \frac{1}{2}$$
Tangent at $(2, 3)$ is $y - 3 = \frac{1}{2}(x-2) \Rightarrow x - 2y + 4 = 0$



$\therefore$ required area
$$= \int_0^3((y-2)^2+1)dy - \int_0^3(2y-4)dy$$
$$= \left|\frac{(y-2)^3}{3} + y\right|_0^3 - |y^2-4y|_0^3$$
$$= \frac{1}{3} + 3 + \frac{8}{3} - (9-12) = 9 \text{ sq. units}$$

37 **(c)**



First consider $y = 3 - |3 - x|$

For $x < 3; y = 3 - (3 - x) = x$

For $x \geq 3; y = 3 - (x - 3) = 6 - x$

Consider $y = \frac{6}{|x+1|}$

For $x < -1; y = \frac{6}{-1-x}$

$\Rightarrow (1 + x)y = -6$

For $x > -1; y = \frac{6}{x+1}$

Required area

$$= \left[\int_2^3 \left(x - \frac{6}{x+1}\right)dx + \int_3^5 \left((6 - x) - \frac{6}{x+1}\right)dx\right]$$

$$= \left[\left(\frac{x^2}{2}\right)_2^3 + \left(6x - \frac{x^2}{2}\right)_3^5 - (6\log(x+1))_2^5\right]$$

$$= \left[\frac{5}{2} + 4 - 6\log 2\right] = \frac{13}{2} - 6 \text{ In } 2 \text{ sq. units}$$

38 **(a)**

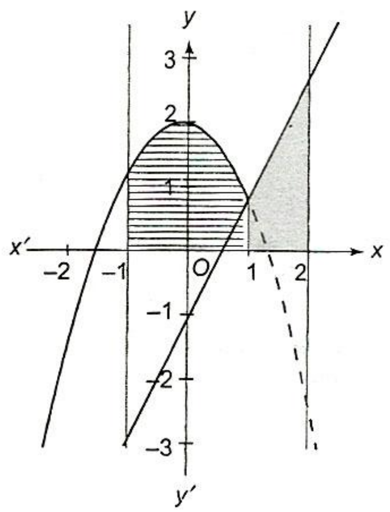


Fig. 9.42

$$A = \int_{-1}^{1}(-x^2 + 2)dx + \int_1^2 (2x - 1)dx$$

$$= \left(-\frac{x^3}{3} + 2x\right)_{-1}^{1} + (x^2 - x)_1^2$$

$$= \frac{16}{3} \text{ sq. units}$$

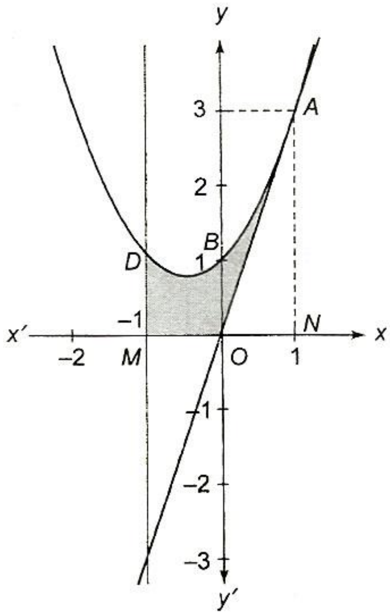
39 **(c)**

Given $y = x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} \Rightarrow y - \frac{3}{4} = \left(x + \frac{1}{2}\right)^2$

This is a parabola with vertex at $\left(-\frac{1}{2}, \frac{3}{4}\right)$ and the curve is concave upwards

$$y = x^2 + x + 1 \Rightarrow \frac{dy}{dx} = 2x + 1 \Rightarrow \left(\frac{dy}{dx}\right)_{(1,3)} = 3$$

Equation of the tangent at $A(1, 3)$ is $y = 3x$



Required (shaded) area = area $ABDMN$ − area ONA

Now, area $ABDMN = \int_{-1}^{1}(x^2 + x + 1)dx$

$$= 2\int_0^1 (x^2 + 1) = \frac{8}{3}$$

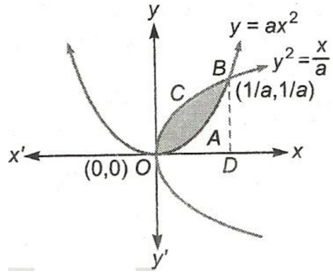
Area of $ONA = \frac{1}{2} \times 1 \times 3 = \frac{3}{2}$

$\therefore$ required area $= \frac{8}{3} - \frac{3}{2} = \frac{16-9}{6} = \frac{7}{6}$ sq. units

40 **(a)**

The points of intersection of given curves are (0,0) and

$\left(\frac{1}{a}, \frac{1}{a}\right)$



$\therefore$ Required area $OABCO$

= area of $OCBDO$

− area of $OABDO$

$$\Rightarrow \int_0^{1/a}\left(\sqrt{\frac{x}{a}} - ax^2\right)dx = 1 \quad \text{[given]}$$

$$\Rightarrow \left(\frac{1}{\sqrt{a}} \cdot \frac{x^{3/2}}{3/2} - \frac{ax^3}{3}\right)_0^{1/a} = 1$$

$$\Rightarrow \frac{2}{3a^2} - \frac{1}{3a^2} = 1$$

$$\Rightarrow a^2 = \frac{1}{3} \Rightarrow a = \frac{1}{\sqrt{3}} \quad [\text{as } a > 0]$$

41 **(c)**

Given $\int_1^b f(x)\,dx = (b-1)\sin(3b+4)$

Differentiating both sides w.r.t. b, we get

$\Rightarrow f(b) = 3(b-1)\cos(3b+4) + \sin(3b+4)$

$\Rightarrow f(x) = \sin(3x+4) + 3(x-1)\cos(3x+4)$

42 **(a)**

Curve tracing: $y = x\,e^x$

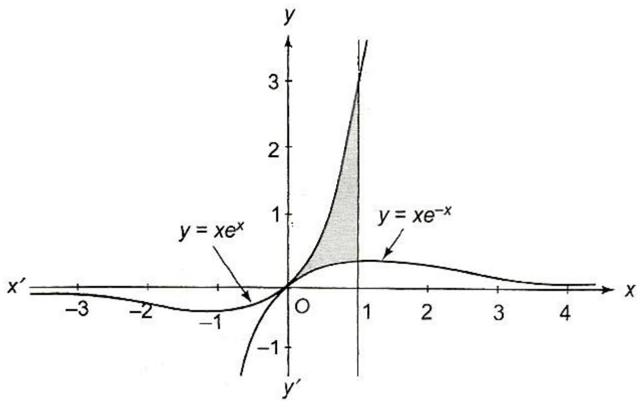
Let $\frac{dy}{dx} = 0 \Rightarrow e^x + xe^x = 0 \Rightarrow x = -1$

Also, at $x = -1, \frac{dy}{dx}$ changes sign from – ve to +ve, hence, $x = -1$ is a point of minima

When $x \to \infty, y \to \infty$

Also $\lim_{x\to\infty} xe^x = \lim_{x\to\infty} \frac{x}{e^{-x}} = \lim_{x\to\infty} \frac{1}{-e^{-x}} = 0$

With similar types of arguments, we can draw the graph of $y = x\,e^{-x}$



Required area

$$= \int_0^1 xe^x dx - \int_0^1 xe^{-x} dx$$

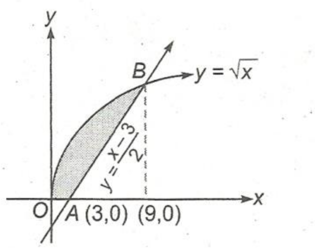
$$= [xe^x]_0^1 - \int_0^1 e^x dx - \left([-xe^{-x}]_0^1 + \int_0^1 e^{-x} dx\right)$$

$$= e - (e-1) - \left(-e^{-1} - (e^{-1} - 1)\right) = \frac{2}{e} \text{ sq. units}$$

43 **(a)**

$$\text{Required area } OABO = \int_0^9 \sqrt{x}\,dx - \int_3^9 \left(\frac{x-3}{2}\right) dx$$

$$= \left(\frac{x^{3/2}}{3/2}\right)_0^9 - \frac{1}{2}\left(\frac{x^2}{2} - 3x\right)_3^9$$



$$= \left(\frac{2}{3} \cdot 27\right) - \frac{1}{2}\left\{\left(\frac{81}{2} - 27\right) - \left(\frac{9}{2} - 9\right)\right\}$$

$= 9$ sq units

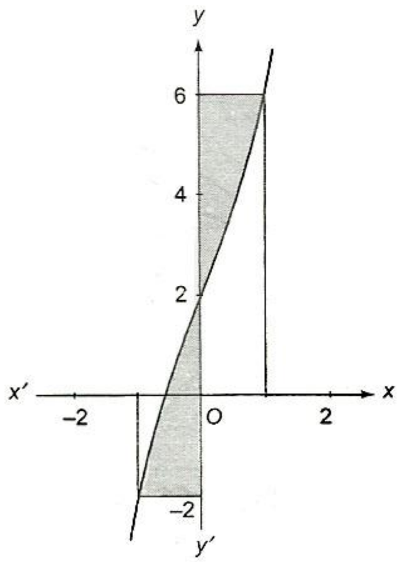
44 **(c)**

The required area will be equal to the area enclosed by $y = f(x)$, y-axis between the abscissa At $y = -2$ and $y = 6$

Hence, $A = \int_0^1 (6 - f(x))dx + \int_{-1}^0 (f(x) -$
$-2dx$

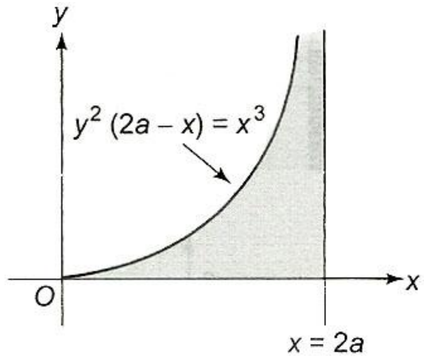
$$= \int_0^1 (4 - x^3 - 3x)dx + \int_{-1}^0 (x^3 + 3x + 4)\,dx$$

$$= \frac{5}{4} \text{ sq. units}$$



45 **(b)**

The required area $A = \int_0^{2a} \sqrt{\frac{x^3}{2a-x}}\, dx$



Put $x = 2a\sin^2\theta$

$\Rightarrow dx = 2a2\sin\theta\cos\theta\, d\theta$

$\Rightarrow A = 8a^2 \int_0^{\pi} \left(\frac{1-\cos 2\theta}{2}\right)^2 d\theta$

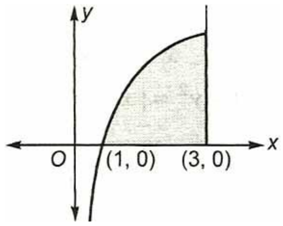
$= 2a^2 \int_0^{\pi} (1 - 2\cos 2\theta + \cos^2 2\theta) d\theta$

$= 2a^2 \int_0^{\pi} \left(1 - 2\cos 2\theta + \frac{1+\cos 4\theta}{2}\right) d\theta$

$= \frac{3\pi a^2}{2}$

46 **(b,c,d)**

Required area $= \int_1^3 \text{In}\, x\, dx$



$= [x\,\text{In}\,x - x]_1^3$

$= (3\,\text{In}\,3 - 2)$

$= \text{In}\,27 - 2$ sq unit (b is correct)

$= \text{In}\,27 - \text{In}\,e^2$

$= \text{In}\,(27/e^2)$ sq unit (c is correct)

Also, In $\left(\frac{27}{e^2}\right) > 3$

$\Rightarrow$ In $\left(\frac{27}{e^2}\right) > 3$ In e

$\Rightarrow$ In $27 >$ In e^5, which is false

47 **(b,d)**

The two curves meet at $mx = x - x^2$ or

$x^2 = x(1-m)$

$\therefore x = 0, 1-m$

$A = \int_0^{1-m} (x - x^2 - mx)\, dx$

$= \left[(1-m)\frac{x^2}{2} - \frac{x^3}{3}\right]_0^{1-m} = \frac{9}{2}$ if $m < 1$

$\Rightarrow (1-m)^3 \left[\frac{1}{2} - \frac{1}{3}\right] = \frac{9}{2}$

$\Rightarrow (1-m)^3 = 27$

$\Rightarrow m = -2$

But if $m > 1$ and $1 - m$ is – ve, then

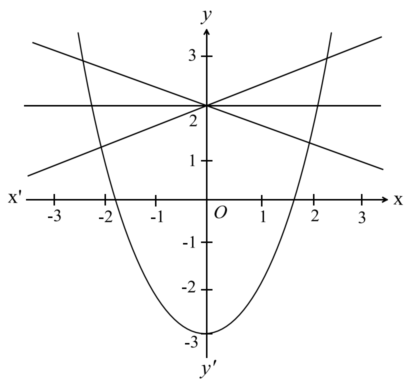
$\left[(1-m)\frac{x^2}{2} - \frac{x^3}{3}\right]_{1-m}^{0} = \frac{9}{2}$

$\Rightarrow -(1-m)^3 \left(\frac{1}{2} - \frac{1}{3}\right) = \frac{9}{2}$

$\Rightarrow -(1-m)^3 = -27$

$\Rightarrow m = 4$

48 **(b,c)**



Line $y = kx + 2$ passes through fixed point (0, 2) for different value of k

Also, it is obvious that minimum $A(k)$ occurs when $k = 0$, as when line is rotated from this position about point (0, 2) the increased part of area is more than the decreased part of area

$\therefore$ Minimum area $= 2\int_0^{\sqrt{5}} (2 - (x^2 - 3))dx$

$= 2\int_0^{\sqrt{5}} (5 - x^2) dx$

$= 2\left[5x - \frac{x^3}{3}\right]_0^{\sqrt{5}}$

$= 2\left[5\sqrt{5} - \frac{5\sqrt{5}}{3}\right]$

$= \dfrac{20\sqrt{5}}{3}$ sq. units

49 **(b,d)**

Given curve $x^a y = \lambda^a$ ...(i)

$(\lambda, 1)$ is a point on the given curve

Now, differentiating Eq. (i) w. r. t. x, we get

$$ax^{a-1}y + x^a \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-ax^{a-1}y}{x^a} = -\frac{ay}{x}$$

At $(\lambda, 1)$ $\frac{dy}{dx} = -\frac{a}{\lambda}$

Equation of tangent at $(\lambda, 1)$ is

$y - 1 = -\frac{a}{\lambda}(x - \lambda)$,

Now, $x = 0$

$\Rightarrow y = 1 + a$

$y = 0$

$$\Rightarrow x = \frac{\lambda}{a} + \lambda = \frac{\lambda(1+a)}{a}$$

Area, $A = \frac{1}{2} \times (1+a)\frac{(1+a)\lambda}{a}$

Now, $\frac{dA}{da} = \frac{1}{2}\lambda\left[\frac{a \cdot 2(1+a) - (1+a)^2}{a^2}\right]$

For maxima or minima, put $\frac{dA}{da} = 0$

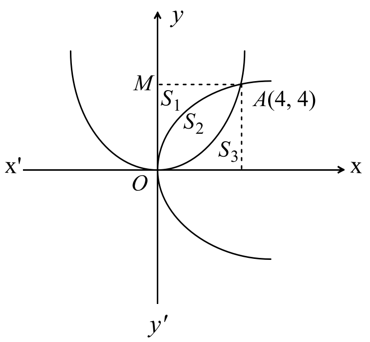
$\Rightarrow (2a - 1 - a)(1 + a) = 0$

$\Rightarrow (a - 1)(a + 1) = 0$

$\Rightarrow a = 1, a = -1$

50 **(a,c,d)**

$y^2 = 4x$ and $x^2 - 4y$ meet at $O(0,0)$ and $A(4,4)$



Now $S_3 = \int_0^4 \frac{x^2}{4}dx = \frac{1}{4}\left[\frac{x^3}{3}\right]_0^4 = \frac{1}{12}[64 - 0] = \frac{16}{3}$

$$S_2 = \int_0^4 2\sqrt{x}dx - S_3 = 2\left[\frac{x^{3/2}}{3/2}\right]_0^4 - \frac{16}{3}$$

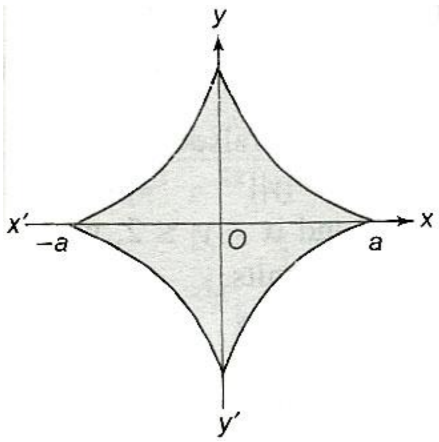
$$= \frac{4}{3}[8 - 0] - \frac{16}{3} = \frac{16}{3}$$

And $S_1 = 4 \times 4 - (S_2 + S_3) = 16 - \left(\frac{16}{3} + \frac{16}{3}\right) = \frac{16}{3}$

Hence, $S_1 : S_2 : S_3 = 1:1:1$

51 **(a,c,d)**

Eliminating t, we have $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}} \Rightarrow y = \left(a^{2/3} - x^{2/3}\right)^{3/2}$



From diagram,

$$\Rightarrow A = 2\int_{-a}^{a}\left(a^{2/3} - x^{2/3}\right)^{3/2}dx$$

$$= 4\int_0^a \left(a^{2/3} - x^{2/3}\right)^{3/2}dx$$

$$A = 4\int_0^a y\, dx$$

$$= 4a^2\int_0^{\pi/2} 3\cos^3 t \sin^2 t \cos t\, dt$$

52 **(c,d)**

Since the curve $y = ax^{1/2} + bx$ passes through the point (1, 2)

$\therefore 2 = a + b$ (1)

By observation the curve also passes through (0, 0)

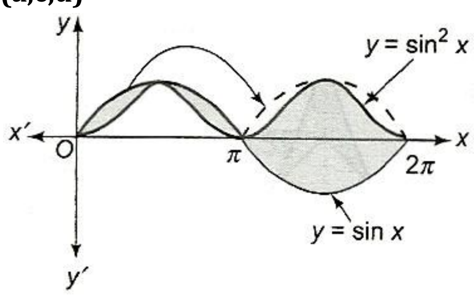
Therefore, the area enclosed by the curve, x-axis and $x = 4$ is given by

$$A = \int_0^4 \left(ax^{1/2} + bx\right)dx = 8 \Rightarrow \frac{2a}{3} \times 8 + \frac{b}{2} \times 16 = 8$$

$\Rightarrow \frac{2a}{3} + b = 1$ (2)

Solving (1) and (2), we get $a = 3, b = -1$

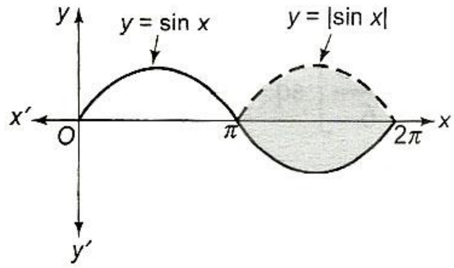
53 **(a,c,d)**



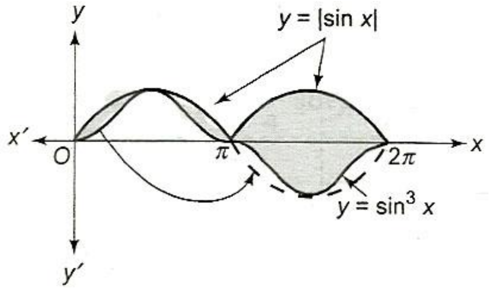
We know that area bounded by $y = \sin x$ and x-axis for $x \in [0, \pi]$ is 2 sq. units

Then area bounded by $y = \sin x$ and $y = \sin^2 x$ is 4 sq. units for $x \in [0, 2\pi]$

Then for $x \in [0, 10\pi]$, the area bounded is 20 sq. units



The area bounded by $y = \sin x$ and $y = |\sin x|$ for $x \in [0, 2\pi]$ is 4 sq. units
Then for $x \in [0, 20\pi]$, the area bounded is 40 sq. units

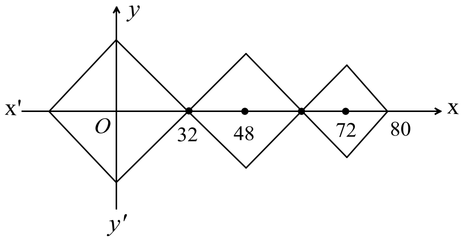


The area bounded by $y = \sin x$ and $y = \sin^3 x$ for $x \in [0, 2\pi]$ is 4 sq. units
Then for $x \in [0, 10\pi]$, the area bounded is 20 sq. units
Similarly, the area bounded by $y = \sin x$ and $y = \sin^4 x$ for $x \in [0, 10\pi]$ is 20 sq. units

54 **(a,c)**

$$a_1 = 0, b_1 = 32, a_2 = a_1 + \frac{3}{2}b_1 = 48, b_2 = \frac{b_1}{2} = 16$$

$$a_3 = 48 + \frac{3}{2} \times 16 = 72, b_3 = \frac{16}{2} = 8$$



So the three loops from $i = 1$ to $i = 3$ are alike
Now area of ith loop (square) $= \frac{1}{2}$ (diagonal)^2

$$A_i = \frac{1}{2}(2b_i)^2 = 2(b_i)^2$$

So, $\frac{A_{i+1}}{A_i} = \frac{2(b_{i+1})^2}{2(b_i)^2} = \frac{1}{4}$
So the areas form a G.P. series
So, the sum of the G.P. upto infinite terms

$$= A_i \frac{1}{1-r} = 2(32)^2 \times \frac{1}{1-\frac{1}{4}}$$

$$= 2 \times (32)^2 \times \frac{4}{3}$$

$$= \frac{8}{3}(32)^2 \text{ square units}$$

55 **(b)**

Given, $f\left(\frac{x}{y}\right) = f(x) - f(y)$ ...(i)

On putting $x = y$, then

$f(1) = 0$

$$\therefore\ f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{f\left(1+\frac{h}{x}\right)}{h} \quad \text{[from Eq. (i)]}$$

$$= \lim_{h\to 0} \frac{f\left(1+\frac{h}{x}\right)}{x \cdot \frac{h}{x}}$$

$$= \frac{3}{x} \quad \left[\because \lim_{x\to 0} \frac{f(1+x)}{x} = 3\right]$$

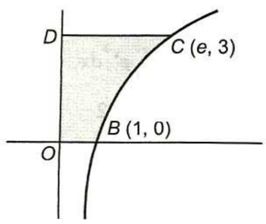
$\therefore\ f(x) = 3 \text{ In } x + c$

Put $x = 1$, then

$f(1) = 0 + c = 0$

$\Rightarrow\ f(x) = 3 \text{ In } x = y$ (say)

$\therefore\ x = e^{y/3}$



$\therefore$ Required area $= \int_{-\infty}^{3} x\, dy$

$$= \int_{-\infty}^{3} e^{y/3} dy = 3\{e^{y/3}\}_{-\infty}^{3}$$

$= 3(e - 0) = 3e$ sq unit

$$\because\ f''(x) = -\frac{3}{x^2} < 0$$

$\Rightarrow f(x)$ is concave down

56 **(c)**

The given curves are

$y = x^2 + 2x - 3$ ...(i)

and $y = \lambda x + 1$ ...(ii)

Solving Eqs. (i) and (ii), we get

$x^2 + (2 - \lambda)x - 4 = 0$

α, β are the roots of the quadratic, then

$\alpha + \beta = \lambda - 2, \alpha\beta = -4$

hence, required area

$$S(\lambda) = \left|\int_\alpha^\beta (\lambda x + 1) - (x^2 + 2x - 3)dx\right|$$

$$= \left|\left\{4x + (\lambda - 2)\frac{x^2}{2} - \frac{x^3}{3}\right\}_\alpha^\beta\right|$$

$$= \left|4(\beta - \alpha) + \frac{(\lambda - 2)}{2}(\beta^2 - \alpha^2) - \frac{1}{3}(\beta^3 - \alpha^3)\right|$$

$$= \sqrt{(\beta - \alpha)^2 - 4\beta\alpha}$$

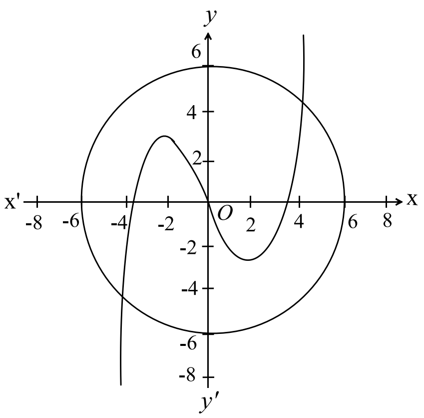
$$\left|\left\{4 + \frac{(\lambda - 2)}{2}(\beta + \alpha) - \frac{1}{3}\{(\alpha + \beta)^2\} - \alpha\right\}\right|$$

$$= \frac{1}{6}\{(\lambda - 2)^2 + 16\}^{3/2}$$

For least value of $S(\lambda), \lambda - 2 = 0$

$\therefore\ \lambda = 2$

57 **(a)**
Statement 2 is correct as $y = f(x)$ is odd and hence statement 1 is correct



58 **(b)**
Since, $y = e^{x^3}$

$$\therefore\ \frac{dy}{dx} = e^{x^3} \cdot 3x^2 > 0$$

$\Rightarrow y$ is an increasing function

And area bounded by the curve $y = e^{x^3}$ between the lines $x = a, x = b$ and x-axis $\int_a^b e^{x^3} dx$

59 **(d)**
R_1: points $P(x, y)$ is nearer to $(1, 0)$ than to $x = -1$

$\Rightarrow \sqrt{(x-1)^2 + y^2} < |x + 1|$

$\Rightarrow y^2 < 4x$

$\Rightarrow$ Point P lies inside parabola $y^2 = 4x$

R_2: Point $P(x, y)$ is nearer to $(0, 0)$ than to $(8, 0)$

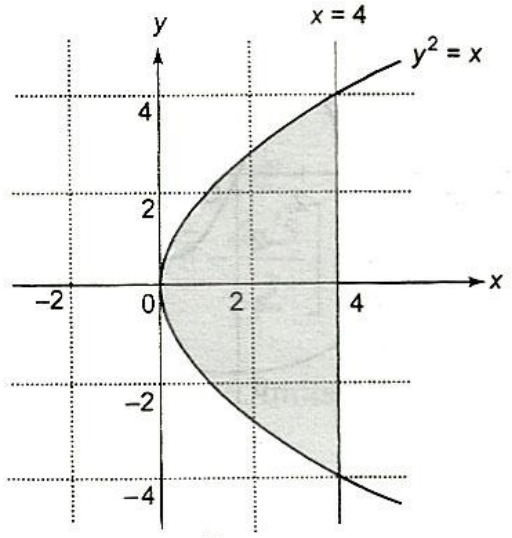
$\Rightarrow |x| < |x - 8|$

$\Rightarrow x^2 < x^2 - 16x + 64$

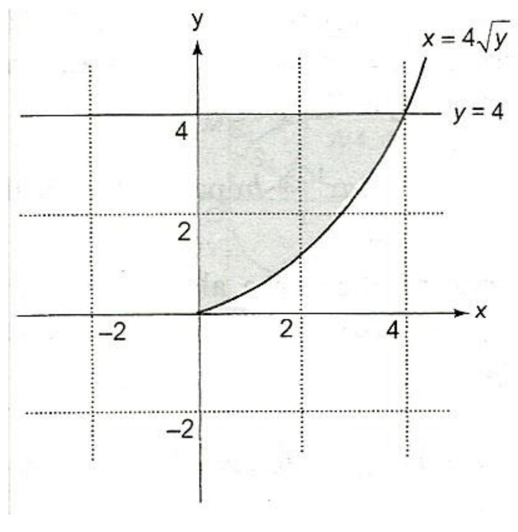
$\Rightarrow x < 4$

$\Rightarrow$ Point P is towards left side of line $x = 4$

The area of common region of R_1 and R_2 is the area bounded by $x = 4$ and $y^2 = 4x$



(a)



This area is twice the area bounded by $x = 4\sqrt{y}$ and $y = 4$

Now, the area bounded by $x = 4\sqrt{y}$ and $y = 4$ is

$$A = \int_0^4 \left(4 - \frac{x^2}{4}\right) dx = \left[4x - \frac{x^3}{12}\right]_0^4 = \left[16 - \frac{64}{12}\right]$$

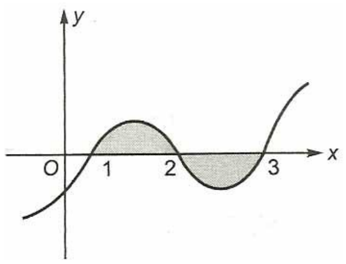
$$= \frac{32}{3} \text{ sq. units}$$

Hence, the area bounded by R_1 and R_2 is $\frac{64}{3}$ sq. units

Thus, statement 1 is false but statement 2 is true

60 **(d)**
It is clear from the figure for $x \in [2.2, 2.8]$

$\Rightarrow\ (x-1)(x-2)(x-3) \le 0$



Required area $= \left|\int_{2.2}^{2.8} f(x)dx\right|$

$$= \left|\int_{2.2}^{2.8} (x-1)(x-2)(x-3)dx\right|$$

61 **(a)**
Given curves are $y^2 - 2y + 4x + 5 = 0$ and $x^2 + 2x - y + 2 = 0$

or $(y-1)^2 = -4(x+1)$ and $(x+1)^2 = y - 1$

Shifting origin to $(-1, 1)$, equation of given curves change to $Y^2 = -4X$ and $X^2 = Y$

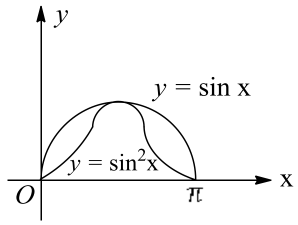
Hence, statement 1 is true and statement 2 is correct explanation of statement 1

64 **(d)**
For $0 < t < 1$

$t^2 < 1$

$\therefore \sin^2 x < \sin x$

$$\Rightarrow \int_0^\pi \sin^2 x\, dx < \int_0^\pi \sin x\, dx$$



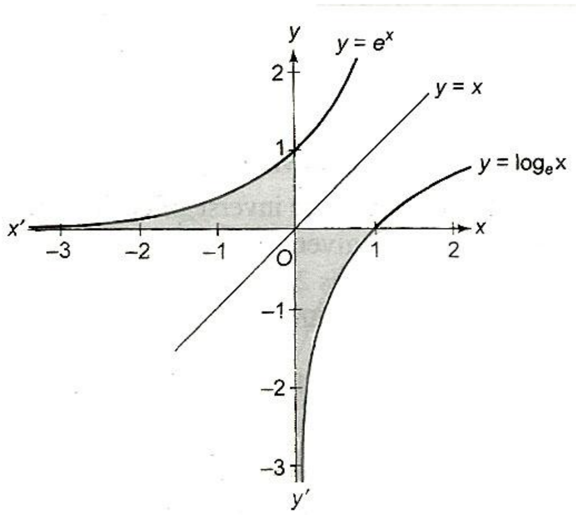
65 **(b)**
Area $= \int_1^3 -(x^2 - 4x + 3)dx = -\left(\frac{x^3}{3} - \frac{4x^2}{2} + \right.$
3x13

$= \frac{4}{3}$ sq. units

$\therefore$ Statement 1 is true

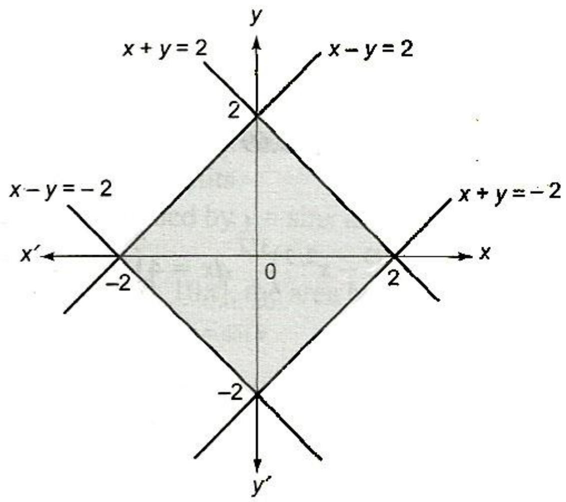
Obviously, statement 2 is true, but does not explain statement 1

66 **(a)**
Since $y = e^x$ and $y = \log_e x$ are inverse to each other



67 **(b)**
$2 \ge \max\{|x-y|, |x+y|\}$

$\Rightarrow |x-y| \le 2$ and $|x+y| \le 2$, which forms a square of diagonal length 4 units



⇒The area of the region is $\frac{1}{2} \times 4 \times 4 = 8$ sq. units

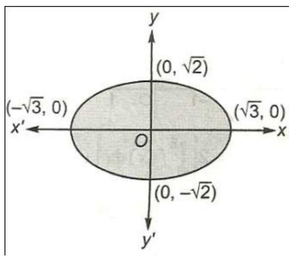
This is equal to the area of the square of side length $2\sqrt{2}$

68 **(d)**

Area of ellipse $\frac{x^2}{3} + \frac{y^2}{2} = 1$ is

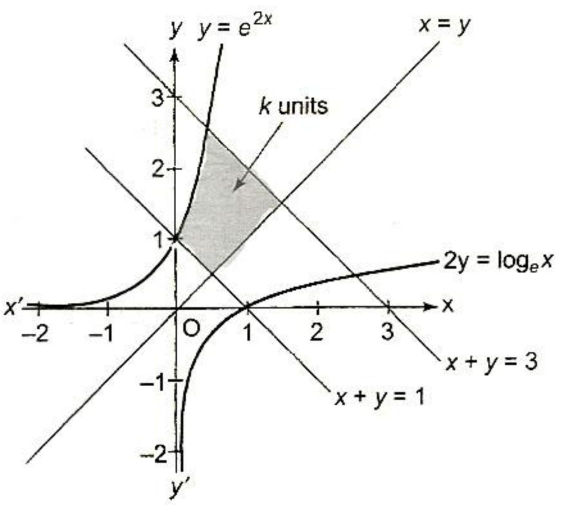
$\pi\sqrt{3}\sqrt{2} = 3.14 \times \sqrt{6} = 7.8$ (approx.) sq unit, the area bounded by $2|x| + 3|y| \leq 6$ is $4 \times \frac{1}{2} \times 3 \times 2$

$= 12$ sq unit



and length of major axis $= 2\sqrt{6} < 3 + 3$

69 **(a)**



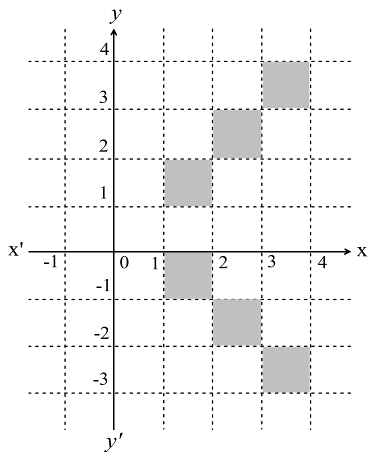
$y = e^{2x}$ and $2y = \log_e x$ are inverse of each other

The shaded area is given as k sq. units

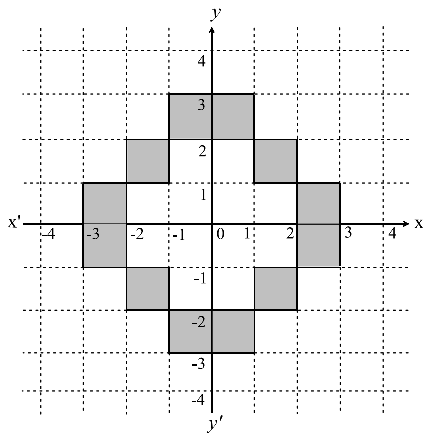
⇒ The required area is $2k$ sq. units

70 **(b)**

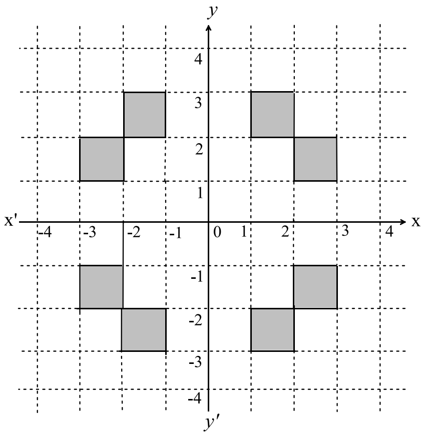
a. $[x]^2 = [y]^2$, where $1 \leq x \leq 4$
⇒ $[x] = \pm[y]$



b. $[|x|] + [|y|] = 2$
The graph is symmetrical about both x-axis and y-axis
For $x, y > 0$; $[x] + [y] = 2$
⇒ $[x] = 0$ and $[y] = 2$, $[x] = 1$ and $[y] = 1$ or $[x] = 2$ and $[y] = 0$



c. $[|x|][|y|] = 2$
The graph is symmetrical about both x-axis and y-axis
For $x, y > 0$; $[x][y] = 2 \Rightarrow [x] = 1$ and $[y] = 2$ or $[x] = 2$ and $[y] = 1$

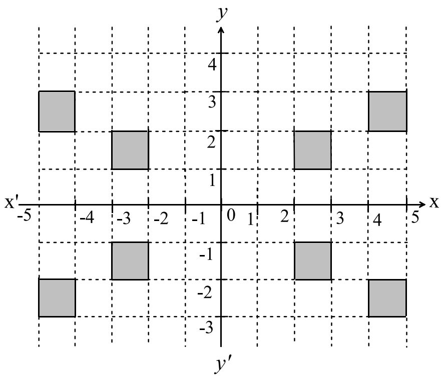


d. $\frac{[|x|]}{[|y|]} = 2$, where $-5 \leq x \leq 5$

The graph is symmetrical about both the axes

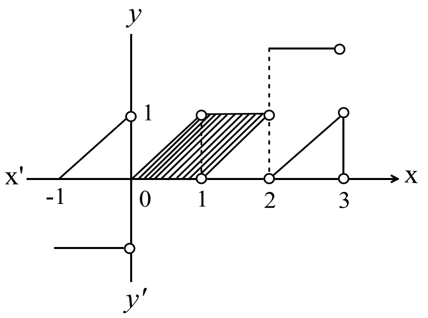
For $x, y > 0, [x] = 2[y], [y] \neq 0$

$\Rightarrow [x] = 2$ and $[y] = 1$ or $[x] = 4$ and $[y] = 2$

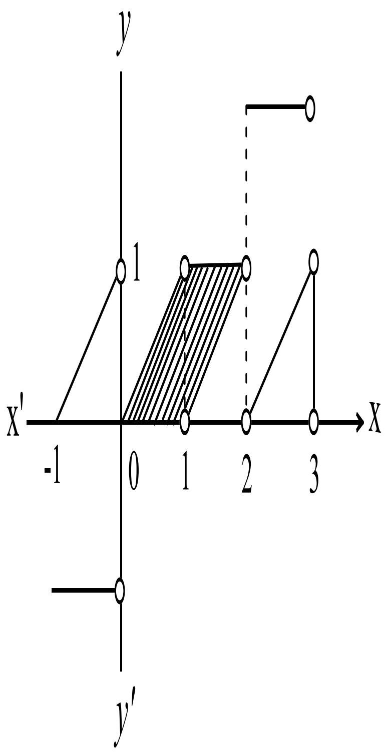


71 **(c)**

a. Area $= 2\left(\frac{1}{2}1.1\right) = 1$ sq. Units



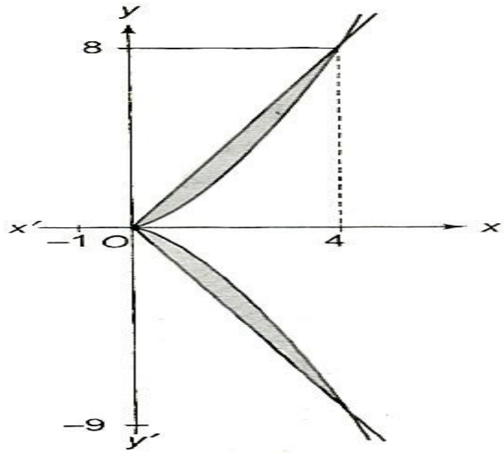
b. $y^2 = x^3$ and $|y| = 2x$, both the curve are symmetric about y-axis



$4x^2 - x^3 \Rightarrow x = 0, 4$

The required area $= 2\int_0^4 (2x - x^{3/2})\, dx = \frac{32}{5}$ sq. units

c. $\sqrt{x} + \sqrt{|y|} = 1$



The curve is symmetrical about x-axis

$\sqrt{|y|} = 1 - \sqrt{x}$ and $\sqrt{x} = 1 - \sqrt{|y|}$

$\Rightarrow$ for $x > 0, y > 0$ $\sqrt{y} = 1 - \sqrt{x}$

$$\frac{1}{2\sqrt{y}}\frac{dy}{dx} = -\frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = -\sqrt{\frac{y}{x}}$$

$\frac{dy}{dx} < 0$, function is decreasing, the required area

$$= 2\int_0^1 ((1 - x) - (1 - 2\sqrt{x} + x)dx$$

$$= 4\int_0^1 (\sqrt{x} - x)dx$$

$= 4\left[\frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^1$

$= 4\left[\frac{2}{3} - \frac{1}{2}\right]$

$= \frac{2}{3}$ sq. units

d. If $-8 < x < 8$, then $y = 2$

If $x \in (-8\sqrt{2}, -8] \cup [8, 8\sqrt{2})$, then $y = 3$, and so on

Intersection of $y = x - 1$ and $y = 2$. We get $x = 3 \in (-8, 8)$

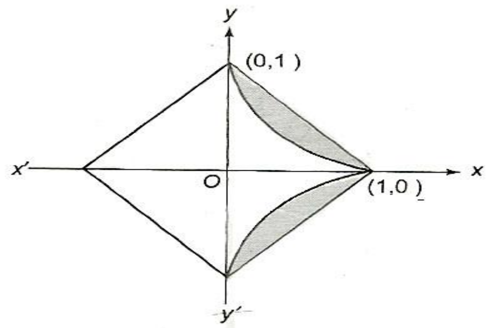
Intersection of $y = x - 1$ and $y = 3$

We get $x = 4 \notin (-8\sqrt{2}, -8] \cup [8, 8\sqrt{2})$

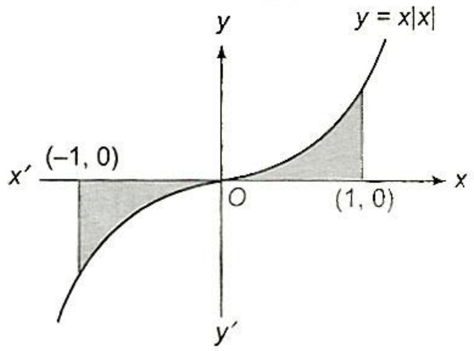
Similarly, $y = x - 1$ will not intersect $y = \left[\frac{x^2}{64} + 2\right]$ at any other integral, except in the interval $x \in (-8, 8)$

The required area (shaded region) $= 2 \times 3 - \frac{1}{2} \times 2 \times 2$

$= 4$ sq. units



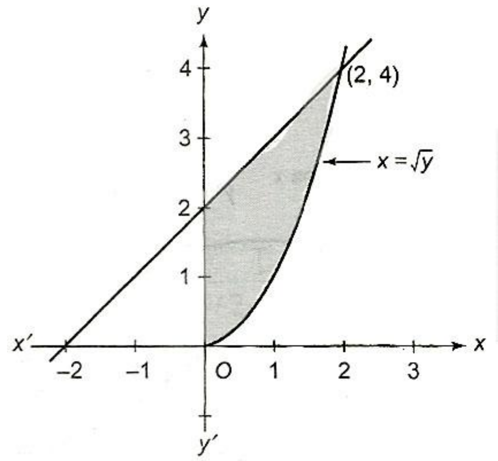
72 **(a)**



a.

Required area $= 2\int_0^1 x|x|\, dx$

$= 2\left(\frac{x^3}{3}\right)_0^1 = \frac{2}{3}$



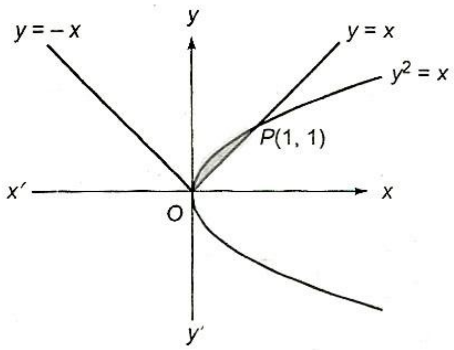
b.

$= \int_0^2 [(x+2) - (x^2)]dx = \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_0^2$

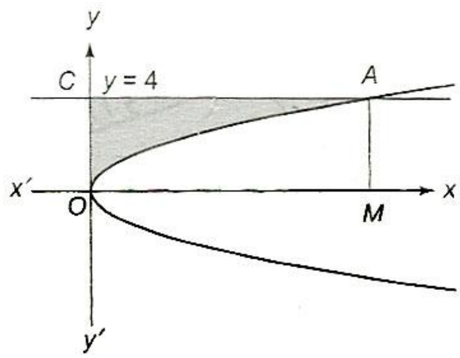
$= 2 + 4 - \frac{8}{3} = \frac{10}{3}$ sq. units

c. Reqd. area $= \int_0^1 (\sqrt{x} - x)dx = \left[\frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^1$

$= \left(\frac{1}{3/2} - \frac{1}{2}\right) = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$ sq. units



d. $y = 4$ meets the parabola $y^2 = x$ at A is (16, 4)



Required area = Area of rectangle $OMAC$ − Area OMA

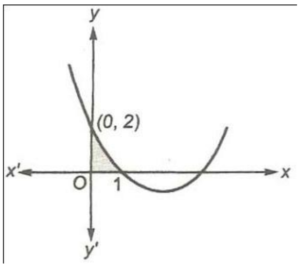
$= 4 \times 16 - \int_0^{16} \sqrt{x}\, dx = 64 - \left|\frac{x^{3/2}}{3/2}\right|_0^{16}$

$= 64 - \frac{2}{3}(4^3) = 64 - \frac{128}{3} = \frac{64}{3}$ sq. units

73 **(d)**

$\because f(x) = x^2 - 3x + 2$

$\therefore$ Required area $= \int_0^1 f(x)dx$

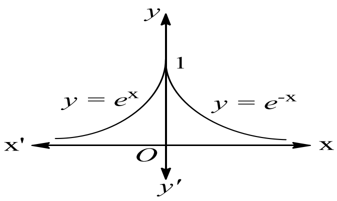


$= \int_0^1 (x^2 - 3x + 2)dx$

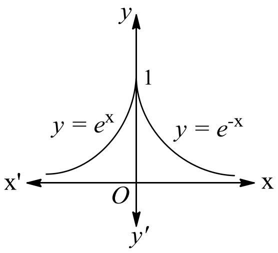
$= \frac{1}{3} - \frac{3}{2} + 2 = \frac{5}{6}$ sq unit

74 **(d)**

$\because$ Graph of $f(x) = \min(|x|, |x-1|, |x+1|)$



and graph of $g(x) = \min(e^x, e^{-x})$



Required area $= 2 \times \frac{1}{2} \times 1 \times \frac{1}{2}$

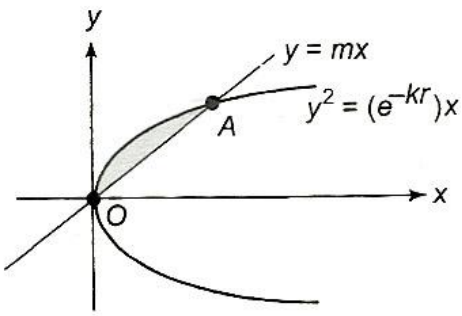
$= \frac{1}{2}$ sq unit

75 **(b)**

Solving the two equations,

$m^2x^2 = (e^{-kr})x$

$x_1 = 0, x_2 = \frac{e^{-kr}}{m^2},$



So, $A_r = \int_0^{x_2} \left(e^{-\frac{kr}{2}}\sqrt{x} - mx\right) dx$

$= \frac{2}{3} e^{-kr/2} x_2^{3/2} - m\frac{x_2^2}{2}$

$= \frac{2}{3} e^{-kr/2} \frac{e^{-3kr/2}}{m^3} - \frac{m}{2}\frac{e^{-2kr}}{m^4} = \frac{e^{-2kr}}{6m^3}$

Now, $\frac{A_{r+1}}{A_r} = \frac{e^{-2k(r+1)}}{e^{-2kr}} = e^{-2k}$ = constant

So, the sequence $A_1, A_2, A_3, \ldots$ is in G.P.

Sum of n terms $= \frac{e^{-2k}}{6m^3} \frac{e^{-2nk}-1}{e^{-2k}-1} = \frac{1}{6m^3} \frac{e^{-2nk}-1}{1-e^{2k}}$

Sum of infinite terms $= A_1 \frac{1}{1-e^{-2k}}$

$= \frac{e^{-2k}}{6m^3} \times \frac{e^{2k}}{e^{2k}-1} = \frac{1}{6m^3(e^{2k}-1)}$

76 **(d)**

$f(x) = \frac{x^3}{3} - x^2 + a$

$f'(x) = x^2 - 2x = x(x-2) < 0$ (note that $f(x)$ is monotonic in $(0, 2)$)

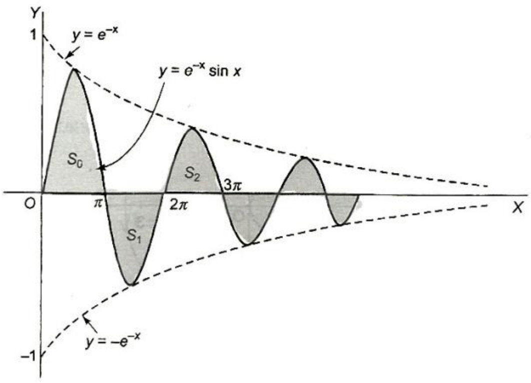
Hence for the minimum and $f(x)$ must cross the x-axis at $\frac{0+2}{2} = 1$

Hence, $f(1) = \frac{1}{3} - 1 + a = 0$

$\Rightarrow a = \frac{2}{3}$

77 **(a)**

Since $-1 \le \sin x \le 1$, the curve $y = e^{-x}\sin x$ is bounded by the curves $y = e^{-x}$ and $y = e^{-x}$



Also, the curve $y = e^{-x}\sin x$ intersects the positive semi-axis OX at the points where $\sin x = 0$, where $x_n = n\pi, n \in Z$

Also $|y_n| = |y$ coordinate in the half-wave $S_n|$

$= (-1)^n e^{-x}\sin x$, and in $S_n, n\pi \le x \le (n+1)\pi$

$\therefore S_n = (-1)^n \int_{n\pi}^{(n+1)\pi} e^{-x}\sin x dx$

$= \frac{(-1)^{n+1}}{2}[e^{-x}(-\sin x + \cos x)]_{n\pi}^{(n+1)\pi}$

$= \frac{(-1)^{n+1}}{2}\left[e^{-(n+1)\pi}(-1)^{n+1} - e^{n\pi}(-1)^n\right]$

$= \frac{e^{-n\pi}}{2}(1 + e^{\pi})$

$\Rightarrow \frac{S_{n+1}}{S_n} = e^{-\pi}$ and $S_0 = \frac{1}{2}(1 + e^{\pi})$

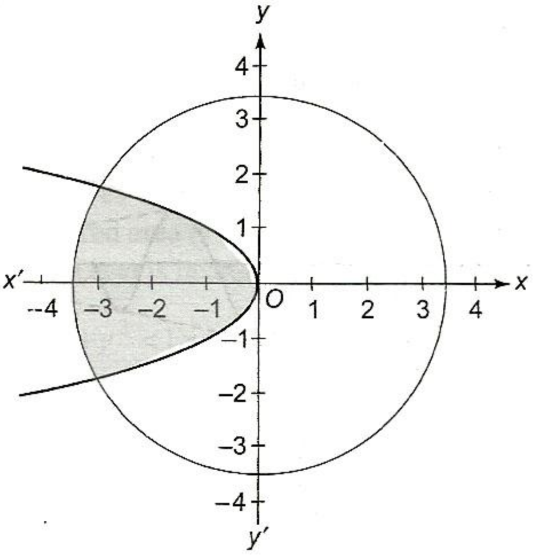
$\therefore$ the sequence $S_0, S_1, S_2, \ldots$ forms an infinite G.P. with common ratio $e^{-\pi}$

$\therefore \sum_{n=0}^{\infty} S_n = \frac{\frac{1}{2}(1 + e^{\pi})}{1 - e^{-\pi}}$

78 **(b)**

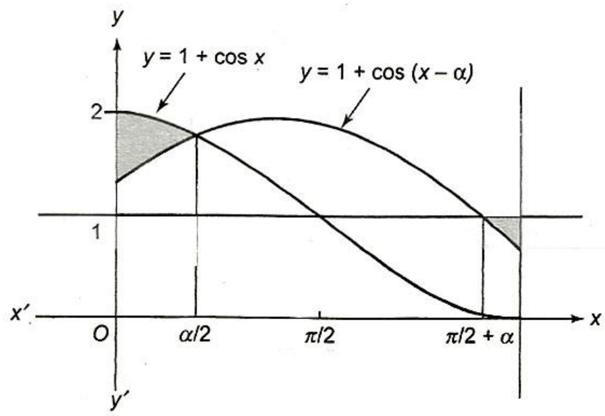
Given

$(x-y)f(x+y)-(x+y)f(x-y)$
$= 4xy(x^2-y^2)$
$= (x^2-y^2)[(x+y)^2-(x-y)^2]$
$= (x-y)(x+y)^3-(x+y)(x-y)^3$
$\Rightarrow f(x+y)=(x+y)^3 \Rightarrow f(x)=x^3, f(y)=y^3$
Now equations of given curves are
$y^2+x=0$ (1)
$x^2+y^2=12$ (2)



Solving equations (1) and (2), we get
$x=-3, y=\pm\sqrt{3}$
The area bounded by curves
$$A=2\left[\left|\int_{-2\sqrt{3}}^{-3}\sqrt{12-x^2}dx\right|+\left|\int_{-3}^{0}\sqrt{-x}dx\right|\right]$$
$$I_1=2\int_{-2\sqrt{3}}^{-3}\sqrt{12-x^2}dx=2\int_{-\pi/2}^{-\pi/3}12\cos^2\theta d\theta$$
$$=12\left[\int_{-\pi/2}^{-\pi/3}(1+\cos 2\theta)d\theta\right]$$
$$=12\left[\theta+\frac{\sin\theta}{2}\right]_{-\pi/2}^{-\pi/3}=12\left[-\frac{\pi}{3}-\frac{\sqrt{3}}{4}+\frac{\pi}{2}\right]$$
$$=12\left[\frac{\pi}{6}-\frac{\sqrt{3}}{4}\right]=2\pi-3\sqrt{3}$$
$$I_2=2\int_{-3}^{0}\sqrt{-x}dx=\frac{2\left[(-x)^{3/2}\right]_{-3}^{0}}{-3/2}$$
$$=-\frac{4}{3}[0-3^{3/2}]$$
$=4\sqrt{3}$
$A=2\pi-3\sqrt{3}+4\sqrt{3}=2\pi+\sqrt{3}$ sq. units

79 **(c)**



$1+\cos x=1+\cos(x-\alpha)$
$x=\alpha-x\Rightarrow x=\frac{\alpha}{2}$
Now $\int_0^{\alpha/2}\big((1+\cos x)-(1+\cos(x-\alpha))\big)dx$
$$=-\int_{\frac{\pi}{2}+\alpha}^{\pi}\big(1-(1+\cos(x-\alpha))\big)dx$$
$$\Rightarrow[\sin x-\sin(x-\alpha)]_0^{\alpha/2}=[\sin(x-\alpha)]_{\pi}^{\frac{\pi}{2}+\alpha}$$
$$\Rightarrow\left[\sin\frac{\alpha}{2}-\sin\left(-\frac{\alpha}{2}\right)\right]-[0-\sin(-\alpha)]$$
$$=\sin\left(\frac{\pi}{2}\right)-\sin(\pi-\alpha)$$
$$\Rightarrow 2\sin\frac{\alpha}{2}-\sin\alpha=1-\sin\alpha$$
Hence, $2\sin\frac{\alpha}{2}=1\Rightarrow\alpha=\frac{\pi}{3}$

80 **(a)**

For $-1\le x<0$
$\left(y-e^{\sin^{-1}x}\right)^2=2-x^2$
$y=e^{\sin^{-1}x}\pm\sqrt{2-x^2}$
$$A=\int_{-1}^{0}\left(e^{\sin^{-1}x}+\sqrt{2-x^2}\right)-\left(e^{\sin^{-1}x}-\sqrt{2-x^2}\right)dx$$
$$=2\int_{-1}^{0}\sqrt{2-x^2}dx$$
$$=2\left(\frac{1}{2}x\sqrt{2-x^2}\Big|_{-1}^{0}+\frac{2}{2}\sin^{-1}\frac{x}{\sqrt{2}}\Big|_{-1}^{0}\right)$$
$$=\left[1+2\left(0-\left(-\frac{\pi}{4}\right)\right)\right]$$
$=\frac{\pi}{2}+1$ sq. units
For $0\le x<1, y=\sin^{-1}x\pm\sqrt{1-x^2}$
$$A=2\int_0^1\sqrt{1-x^2}dx$$
$$=2\left[\frac{x}{2}\sqrt{1-x^2}\Big|_0^1+\frac{1}{2}\sin^{-1}\frac{x}{1}\Big|_0^1\right]$$
$=0+\sin^{-1}(1)=\frac{\pi}{2}$ sq. units
Total area $=\left(\frac{\pi}{2}+1\right)+\frac{\pi}{2}=\pi+1$

81 **(b)**

$$S=\left|-\int_0^{2\pi} a(1-\cos t)a(1-\cos t)dt\right|$$

$$=\left|-a^2\int_0^{2\pi}(1-2\cos t+\cos^2 t)dt\right|$$

$$=\left|-a^2\int_0^{2\pi}\left(1-2\cos t+\left(\frac{1+\cos 2t}{2}\right)\right)dt\right|$$

$$=\left|-\frac{a^2}{2}\int_0^{2\pi}(3-4\cos t+\cos 2t)dt\right|$$

$$=\left|-\frac{a^2}{2}[3t-4\cos t+\cos 2t]_0^{2\pi}\right|$$

$=|-3\pi a^2|=3\pi a^2$ sq. units

82 **(3)**

$y=\frac{a^2-ax}{1+a^4}$ (1)

$y=\frac{x^2+2ax+3a^2}{1+a^4}$ (2)

Point of intersection of (1) and (2)

$$\frac{a^2-ax}{1+a^4}=\frac{x^2+2ax+3a^2}{1+a^4}$$

$(x+a)(x+2a)=0$

$x=-a,-2a$

Req. area $=\int_{-2a}^{-a}\left[\left(\frac{a^2-ax}{1+a^4}\right)-\left(\frac{x^2+2ax+3a^2}{1+a^4}\right)\right]$

$$\therefore f(a)=\frac{a^3}{6(1+a^4)}$$

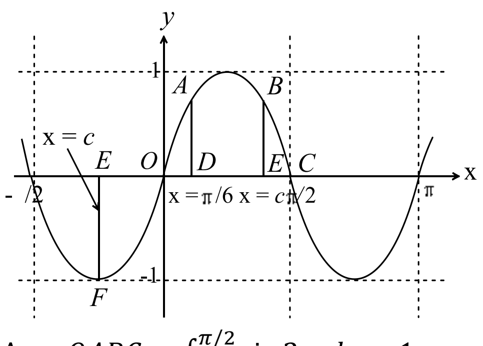
$f(a)$ is max is

Then $f'(a)=0$

$3+3a^4-4a^4=0$

$a^4=3$

83 **(6)**



Area $OABC=\int_0^{\pi/2}\sin 2x\ dx=1$

Area $OAD=\int_0^{\pi/6}\sin 2x\ dx=\frac{1}{4}$

$\because \sin 2x$ is symmetric about origin

So $c=-\frac{\pi}{6}$, because area $OAD=$ area OEF

$$\int_{\frac{\pi}{6}}^{c}\sin 2x\ dx=\frac{1}{2}$$

$\cos 2c=-\frac{1}{2}\cos 2c=\frac{3}{2}$ (not possible)

$c=\frac{\pi}{3}$

So $c=-\frac{\pi}{6},\frac{\pi}{3}$

84 **(9)**

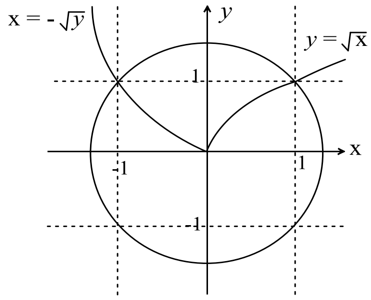
Required area

$A=\int_0^3 x\sqrt{9-x^2}dx$; Put $9-x^2=t^2\Rightarrow -2x\,dx=2t\,dt$

$$\therefore A=\int_0^3 t^2dt=9$$

85 **(8)**

Required area = area of one quadrant of the circle $=\pi/2$



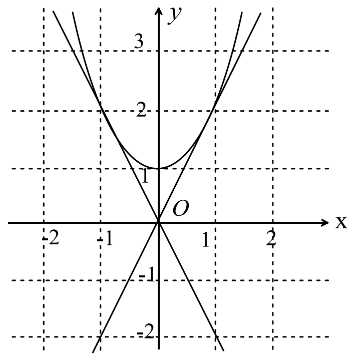
86 **(2)**

Let the point of the curve is (x,x^2+1)

Now, the slope of tangent at this point is $2x$, which is equal to the slope of the line joining (x,x^2+1) and $(0,0)$

Hence $2x=(x^2+1)/x\Rightarrow 2x^2=x^2+1$

$\Rightarrow x^2=1\ \Rightarrow x=\pm 1$



Hence equation of tangent is $y=\pm 2x$

Now area $2\int_0^1(x^2+1-2x)\,dx$

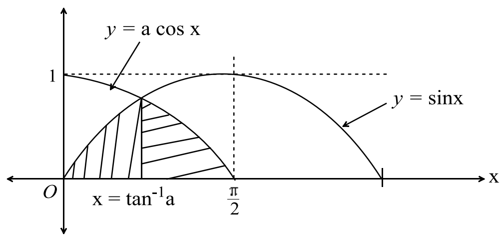
$$=2\int_0^1(x-1)^2\,dx$$

$$=2\left[\frac{(x-1)^3}{3}\right]_0^1=2/3$$

87 **(4)**

We have $S=\int_0^{\pi}\sin x\,dx=2$, so $T=\frac{2}{3}$, where $a>0$

Now $T=\int_0^{\tan^{-1}a}\sin x\,dx+\int_{\tan^{-1}a}^{\pi/2}a\cos x\,dx=\frac{2}{3}$



i. e., $-\cos(\tan^{-1} a) + 1 + a\{[1 - \sin(\tan^{-1} a)]\}$
$= \frac{2}{3}$,

i. e., $-\frac{1}{\sqrt{1+a^2}} + 1 + a - \frac{a^2}{\sqrt{1+a^2}} = \frac{2}{3}$

$\Rightarrow (a+1) - \sqrt{a^2+1} = \frac{2}{3} \Rightarrow a + \frac{1}{3} = \sqrt{a^2+1}$

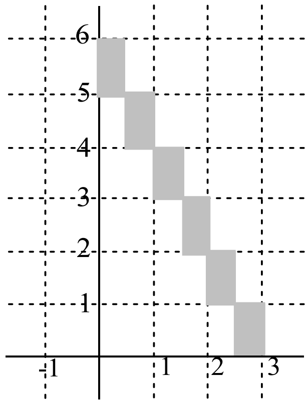
$\Rightarrow a = \frac{4}{3}$

Hence $3a = 4$

88 **(3)**

$[2x] = 0 \Rightarrow 2x \in [0,1) \Rightarrow x \in [0, 1/2) \Rightarrow [y] = 5 \Rightarrow y \in [5,6)$

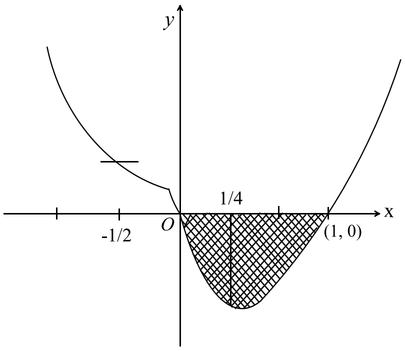
Similarly we can consider $[2x] = 1, 2, 3, 4$ and 5



From the graph, area is 3 sq. units

89 **(9)**

Graph of $f(x)$ is as

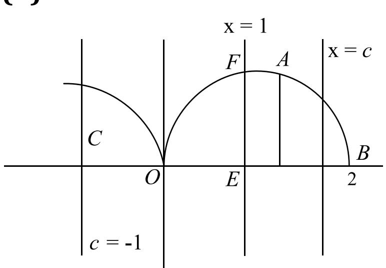


$A = \int_0^1 (x^{4/3} - x^{1/3})\, dx = \frac{3}{7}x^{3/7} - \frac{3}{4}x^{4/3}\Big]_0^1$

$= \left|\frac{3}{7} - \frac{3}{4}\right| = 3\left|\frac{4-7}{28}\right| = \frac{9}{28}$

$\Rightarrow 28A = 9$

90 **(2)**



Given than $\int_1^c y dx = \frac{16}{3}$

$\Rightarrow \int_1^c (8x^2 - x^5) dx = \frac{16}{3}$

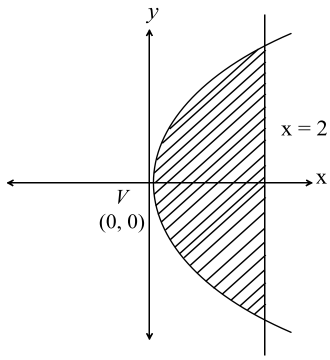
$c = \left(8 - \sqrt{17}\right)^{1/3} \quad (c > 0)$

Area $OFE = \int_0^c (8x^2 - x^5)\, dx = \frac{8}{3} \quad (c > 0)$

So $c = -1$

Hence $c = -1$ and $\left(8 - \sqrt{17}\right)^{1/3}$

91 **(8)**



Let $P(x, y)$ be any point on the curve C

Now, $\frac{dy}{dx} = \frac{1}{y}$

$\Rightarrow ydy = dx \Rightarrow \frac{y^2}{2} = x + k$

Since the curve passes through $M(2, 2)$, so $k = 0$

$\Rightarrow y^2 = 2x$

Hence required area $= 2\int_0^2 \sqrt{2x}\, dx$

$= 2\sqrt{2} \times \frac{2}{3}\left(x^{3/2}\right)_0^2$

$= \frac{4}{3}\sqrt{2} \times 2\sqrt{2}$

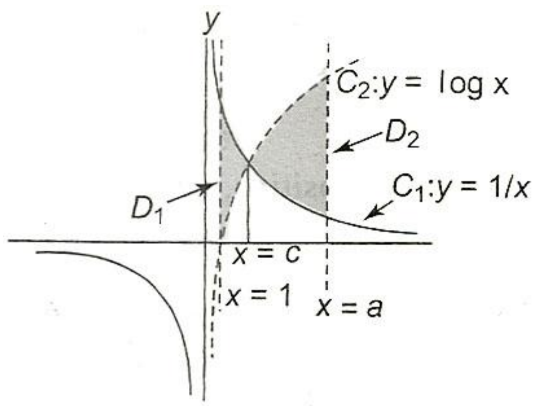
$= \frac{16}{3}$ (square unit)

92 **(1)**

Given that $D_1 = D_2$

$\int_1^c \left(\frac{1}{x} - \log x\right) dx = \int_c^a \left(\log x - \frac{1}{x}\right) dx$

$\left(\frac{-1}{x^2} - x(\log x - 1)\right)_1^c = \left(x(\log x - 1) + \frac{1}{x^2}\right)_c^a$



$\therefore 0 = a(\log a - 1) + \frac{1}{a^2}$

$\therefore a = 1$

93 **(6)**

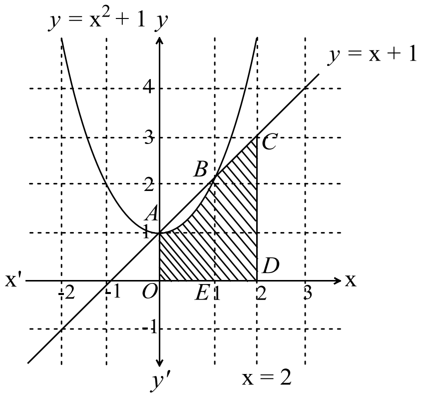
Draw the given region point of intersection of

$y = x^2 + 1$

$y = x + 1$

$x + 1 = x^2 + 1$

$x = 0, 1$

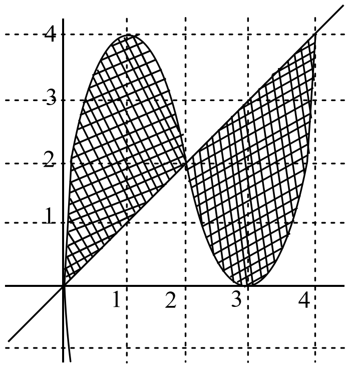


Required area $OABCDE = \int_0^1 (x^2+1)dx + \int_1^2 (x+1)\,dx$

$= \left(\frac{x^3}{3}+x\right)_0^1 + \left(\frac{x^2}{2}+x\right)_1^2 = \frac{23}{6}$ sq. units

94 **(8)**

Required area $= 2\int_0^2 (x(x-3)^2 - x)\,dx = 8$ sq. units



95 **(8)**

$\int_0^3 (-x^2 + ax + 12)\,dx = 45$ gives $a = 4$

Hence $f(x) = 12 + 4x - x^2 = (2+x)(6-x)$

Hence $m = -2$ and $n = 6$

$m + n + a = 6 - 1 + 4 = 8$

96 **(1)**

$f(a) = \int_a^{2a} \left(\frac{x}{6} + \frac{1}{x^2}\right) dx = \left(\frac{x^2}{12} - \frac{1}{x}\right)_a^{2a}$

$= \left(\frac{4a^2}{12} - \frac{1}{2a} - \frac{a^2}{12} + \frac{1}{a}\right) = \frac{a^2}{4} + \frac{1}{2a}$

Let $f'(a) = \frac{2a}{4} - \frac{1}{2a^2} = 0$

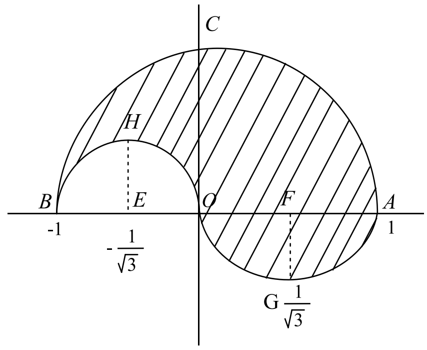
$\Rightarrow a = 1$ which is point of minima

97 **(2)**

$y = \sqrt{1-x^2}$

$y = x^3 - x \quad (1)$

$y = 0$ in (2) $x = 0, 1, -1 \quad (2)$



Required area = area of region $BCAGOHB$

= Area of semi-circle $BCAOB$

$= \frac{\pi}{2}$

($\because$ area of $BHOEB$ = area of $OFAGO$)