Single Correct Answer Type

1. Let $\vec{a} = \hat{\imath} + \hat{\jmath}$ and $\vec{b} = 2\hat{\imath} - \hat{k}$, then the point of intersection of the lines $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ and $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ is
   a) $(3, -1, 1)$ b) $(3,1,-1)$ c) $(-3, 1, 1)$ d) $(-3, -1, -1)$
2. The distance between the line: $\vec{r} = 2\hat{\imath} - 2\hat{\jmath} + 3\hat{k} + \lambda(\hat{\imath} - \hat{\jmath} + 4\hat{k})$ and the plane $\vec{r} \cdot (\hat{\imath} + 5\hat{\jmath} + \hat{k}) = 5$ is
   a) $\frac{10}{3\sqrt{3}}$ b) $\frac{10}{9}$ c) $\frac{10}{3}$ d) $\frac{3}{10}$
3. The projection of point $P(\vec{p})$ on the plane $\vec{r} \cdot \vec{n} = q$ is $(\vec{s})$, then
   a) $\vec{s} = \frac{(q - \vec{p} \cdot \vec{n})\vec{n}}{|\vec{n}|^2}$ b) $\vec{s} = \vec{p} + \frac{(q - \vec{p} \cdot \vec{n})\vec{n}}{|\vec{n}|^2}$ c) $\vec{s} = \vec{p} - \frac{(\vec{p} \cdot \vec{n} \cdot)\vec{n}}{|\vec{n}|^2}$ d) $\vec{s} = \vec{p} - \frac{(q - \vec{p} \cdot \vec{n})\vec{n}}{|\vec{n}|^2}$
4. The intercepts made on the axes by the plane which bisects the line joining the points (1,2, 3) and $(-3, 4, 5)$ at right angles are
   a) $\left(-\frac{9}{2}, 9, 9\right)$ b) $\left(\frac{9}{2}, 9, 9\right)$ c) $\left(9, -\frac{9}{2}, 9\right)$ d) $\left(9, \frac{9}{2}, 9\right)$
5. The plane $\vec{r} \cdot \vec{n} = q$ will contain the line $\vec{r} = \vec{a} + \lambda\vec{b}$, if
   a) $\vec{b} \cdot \vec{n} \neq 0, \vec{a} \cdot \vec{n} \neq q$ b) $\vec{b} \cdot \vec{n} = 0, \vec{a} \cdot \vec{n} \neq q$ c) $\vec{b} \cdot \vec{n} = 0, \vec{a} \cdot \vec{n} = q$ d) $\vec{b} \cdot \vec{n} \neq 0, \vec{a} \cdot \vec{n} = q$
6. The vector equation of the plane passing through the origin and the line of intersection of the planes $\vec{r} \cdot \vec{a} = \lambda$ and $\vec{r} \cdot \vec{b} = \mu$ is
   a) $\vec{r} \cdot (\lambda\vec{a} - \mu\vec{b}) = 0$ b) $\vec{r} \cdot (\lambda\vec{b} - \mu\vec{a}) = 0$ c) $\vec{r} \cdot (\lambda\vec{a} + \mu\vec{b}) = 0$ d) $\vec{r} \cdot (\lambda\vec{b} + \mu\vec{a}) = 0$
7. The line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1}$ intersects the curve $xy = c^2, z = 0$ if c is equal to
   a) ± 1 b) $\pm 1/3$ c) $\pm\sqrt{5}$ d) None of these
8. L_1 and L_2 are two lines whose vector equations are
   $L_1: \vec{r} = \lambda\left((\cos\theta + \sqrt{3})\hat{\imath} + (\sqrt{2}\sin\theta)\hat{\jmath} + (\cos\theta - \sqrt{3})\hat{k}\right)$
   $L_2: \vec{r} = \mu(a\hat{\imath} + b\hat{\jmath} + c\hat{k})$, where λ and μ are scalars and α is the acute angle between L_1 and L_2. If the angle 'α' is independent of θ, then the value of 'α' is
   a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$
9. Distance of the point $P(\vec{p})$ from the line $\vec{r} = \vec{a} + \lambda\vec{b}$ is
   a) $\left|(\vec{a} - \vec{p}) + \frac{((\vec{p} - \vec{a}) \cdot \vec{b})\vec{b}}{|\vec{b}|^2}\right|$ b) $\left|(\vec{b} - \vec{p}) + \frac{((\vec{p} - \vec{a}) \cdot \vec{b})\vec{b}}{|\vec{b}|^2}\right|$
   c) $\left|(\vec{a} - \vec{p}) + \frac{((\vec{p} - \vec{b}) \cdot \vec{b})\vec{b}}{|\vec{b}|^2}\right|$ d) None of these
10. The coordinates of the foot of the perpendicular drawn from the origin to the line joining the points $(-9, 4, 5)$ and $(10, 0, -1)$ will be
    a) $(-3, 2, 1)$ b) $(1, 2, 2)$ c) $(4, 5, 3)$ d) None of these
11. For the line $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$, which one of the following is incorrect?
    a) It lies in the plane $x - 2y + z = 0$
    b) It is same as line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$
    c) It passes through $(2, 3, 5)$
    d) It is parallel to the plane $x - 2y + z - 6 = 0$
12. The equation of a plane which passes through the point of intersection of lines $\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-3}{2}$, and $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and at greatest distance from point $(0, 0, 0)$ is
    a) $4x + 3y + 5z = 25$ b) $4x + 3y = 5z = 50$ c) $3x + 4y + 5z = 49$ d) $x + 7y - 5z = 2$

13. Let L_1 be the line $\vec{r}_1 = 2\hat{\imath} + \hat{\jmath} - \hat{k} + \lambda(\hat{\imath} + 2\hat{k})$ and let L_2 be the line $\vec{r}_2 = 3\hat{\imath} + \hat{\jmath} + \mu(\hat{\imath} + \hat{\jmath} - \hat{k})$. Let π be the plane which contains the line L_1 and is parallel to L_2. The distance of the plane π from the origin is
a) $\sqrt{2/7}$ b) $1/7$ c) $\sqrt{6}$ d) None

14. The lines $\vec{r} = \vec{a} + \lambda(\vec{b} \times \vec{c})$ and $\vec{r} = \vec{b} + \mu(\vec{c} \times \vec{a})$ will intersect if
a) $\vec{a} \times \vec{c} = \vec{b} \times \vec{c}$ b) $\vec{a} \cdot \vec{c} = \vec{b} \cdot \vec{c}$ c) $\vec{b} \times \vec{a} = \vec{c} \times \vec{a}$ d) None of these

15. The projection of the line $\frac{x+1}{-1} = \frac{y}{2} = \frac{z-1}{3}$ on the plane $x - 2y + z = 6$ is the line of intersection of this plane with the plane
a) $2x + y + 2 = 0$ b) $3x + y - z = 2$ c) $2x - 3y + 8z = 3$ d) None of these

16. A plane makes intercepts OA, OB and OC whose measurements are b and c on the OX, OY and OZ axes. The area of triangle ABC is
a) $\frac{1}{2}(ab + bc + ca)$ b) $\frac{1}{2}abc(a + b + c)$
c) $\frac{1}{2}(a^2b^2 + b^2c^2 + c^2a^2)^{1/2}$ d) $\frac{1}{2}(a + b + c)^2$

17. Given $\vec{\alpha} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\vec{\beta} = \hat{\imath} - 2\hat{\jmath} - 4\hat{k}$ are the position vectors of the points A and B. Then the distance of the point $-\hat{\imath} + \hat{\jmath} + \hat{k}$ from the plane passing through B and perpendicular to AB is
a) 5 b) 10 c) 15 d) 20

18. If the distance of the point $P(1, -2, 1)$ from the plane $x + 2y - 2z = \alpha$, where $\alpha > 0$, is 5, then the foot of the perpendicular from P to the plane is
a) $\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$ b) $\left(\frac{4}{3}, -\frac{4}{3}, \frac{1}{3}\right)$ c) $\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$ d) $\left(\frac{2}{3}, -\frac{1}{3}, \frac{5}{2}\right)$

19. Shortest distance between the lines $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-1}{1}$ and $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{1}$ is equal to
a) $\sqrt{14}$ b) $\sqrt{7}$ c) $\sqrt{2}$ d) None of these

20. A line with positive direction cosines passes through the point $P(2, -1, 2)$ and makes equal angles with the coordinate axes. The line meets the plane $2x + y + z = 9$ at point Q. The length of the line segment PQ equals
a) 1 b) $\sqrt{2}$ c) $\sqrt{3}$ d) 2

21. A tetrahedron has vertices $O(0, 0, 0), A(1, 2, 1), B(2, 1, 3)$ and $C(-1, 1, 2)$, then angle between faces OAB and ABC will be:
a) $\cos^{-1}\left(\frac{17}{31}\right)$ b) $30°$ c) $90°$ d) $\cos^{-1}\left(\frac{19}{35}\right)$

22. Let the equations of a line and a plane be $\frac{x+3}{2} = \frac{y-4}{3} = \frac{z+5}{2}$ and $4x - 2y - z = 1$, respectively, the
a) The line is parallel to the plane
b) The line is perpendicular to the plane
c) The line lies in the plane
d) None of these

23. The length of the perpendicular from the origin to the plane passing through the point $\vec{a}$ and containing the line $\vec{r} = \vec{b} + \lambda\vec{c}$ is
a) $\frac{[\vec{a}\vec{b}\vec{c}]}{|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}$ b) $\frac{[\vec{a}\vec{b}\vec{c}]}{|\vec{a} \times \vec{b} + \vec{b} \times \vec{c}|}$
c) $\frac{[\vec{a}\vec{b}\vec{c}]}{|\vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}$ d) $\frac{[\vec{a}\vec{b}\vec{c}]}{|\vec{c} \times \vec{a} + \vec{a} \times \vec{b}|}$

24. The number of planes that are equidistant from four non-coplanar points is
a) 3 b) 4 c) 7 d) 9

25. The intercept made by the plane $\vec{r} \cdot \vec{n} = q$ on the x-axis is
a) $\frac{q}{\hat{\imath} \cdot \vec{n}}$ b) $\frac{\hat{\imath} \cdot \vec{n}}{q}$ c) $\frac{\hat{\imath} \cdot \vec{n}}{q}$ d) $\frac{q}{|\vec{n}|}$

26. What is the nature of the intersection of the set of planes $x + ay + (b + c)z + d = 0, x + by + (c + a)z +$

$d = 0$ and $x + cy + (a + b)z + d = 0$?

a) They meet at a point

b) They form a triangular prism

c) They pass through a line

d) They are at equal distance from the origin

27. Which of the following are equations for the plane passing through the points $P(1,1,-1), Q(3,0,2)$ and $R(-2,1,0)$?

a) $(2\hat{\imath} - 3\hat{\jmath} + 3\hat{k}) \cdot ((x + 2)\hat{\imath} + (y - 1)\hat{\jmath} + z\hat{k}) = 0$

b) $x = 3 - t, y = -11t, z = 2 - 3t$

c) $(x + 2) + 11(y - 1) = 3z$

d) $(2\hat{\imath} - \hat{\jmath} + 3\hat{k}) \times (-3\hat{\imath} + \hat{k}) \cdot ((x + 2)\hat{\imath} + (y - 1)\hat{\jmath} + z\hat{k}) = 0$

28. The line through $\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$ and $\perp$ to the line $\vec{r} = (\hat{\imath} + 2\hat{\jmath} - \hat{k}) + \lambda(2\hat{\imath} + \hat{\jmath} + \hat{k})$ and $\vec{r} = (2\hat{\imath} + 6\hat{\jmath} + \hat{k}) + \mu(\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$ is

a) $\vec{r} = (\hat{\imath} + 2\hat{\jmath} - \hat{k}) + \lambda(-\hat{\imath} + 5\hat{\jmath} - 3\hat{k})$

b) $\vec{r} = \hat{\imath} + 3\hat{\jmath} + 2\hat{k} + \lambda(\hat{\imath} - 5\hat{\jmath} + 3\hat{k})$

c) $\vec{r} = \hat{\imath} + 3\hat{\jmath} + 2\hat{k} + \lambda(\hat{\imath} + 5\hat{\jmath} + 3\hat{k})$

d) $\vec{r} = \hat{\imath} + 3\hat{\jmath} + 2\hat{k} + \lambda(-\hat{\imath} - 5\hat{\jmath} - 3\hat{k})$

29. Equation of the plane passing through the points (2, 2, 1) and (9, 3, 6) and $\perp$ to the plane $2x + 6y + 6z - 1 = 0$ is

a) $3x + 4y + 5z = 9$ b) $3x + 4y - 5z = 9$ c) $3x + 4y - 5z = 9$ d) None of the above

30. The intersection of the spheres $x^2 + y^2 + z^2 + 7x - 2y - z = 13$ and $x^2 + y^2 + z^2 - 3x + 3y + 4z = 8$ is the same as the intersection of one of the spheres and the plane

a) $x - y - z = 1$ b) $x - 2y - z = 1$ c) $x - y - 2z = 1$ d) $2x - y - z = 1$

31. The length of the perpendicular drawn from (1, 2, 3) to the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$ is

a) 4 b) 5 c) 6 d) 7

32. If angle θ between the line $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda}\, z + 4 = 0$ is such that $\sin\theta = \frac{1}{3}$, the value of λ is

a) $\frac{-3}{5}$ b) $\frac{5}{3}$ c) $\frac{-4}{3}$ d) $\frac{3}{4}$

33. Let L be the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$. If L makes an angle α with the positive x-axis, then $\cos\alpha$ equals

a) $\frac{1}{2}$ b) 1 c) $\frac{1}{\sqrt{2}}$ d) $\frac{1}{\sqrt{3}}$

34. For what value(s) of a, will the two points $(1, a, 1)$ and $(-3,0, a)$ lie on opposite sides of the plane $3x + 4y - 12z + 13 = 0$?

a) $a < -1$ or $a > 1/3$ b) $a = 0$ only c) $0 < a < 1$ d) $-1 < a < 1$

35. The reflection of the point $\vec{a}$ in the plane $\vec{r} \cdot \vec{n} = q$ is

a) $\vec{a} + \frac{(\vec{q} - \vec{a} \cdot \vec{n})}{|\vec{n}|}$ b) $\vec{a} + 2\left(\frac{(\vec{q} - \vec{a} \cdot \vec{n})}{|\vec{n}|^2}\right)\vec{n}$ c) $\vec{a} + \frac{2(\vec{q} + \vec{a} \cdot \vec{n})}{|\vec{n}|}\vec{n}$ d) None of these

36. The point of intersection of the lines $\frac{x-5}{3} = \frac{y-7}{-1} = \frac{z+2}{1}$ and $\frac{x+3}{-36} = \frac{y-3}{2} = \frac{z-6}{4}$ is

a) $\left(21, \frac{5}{3}, \frac{10}{3}\right)$ b) $(2, 10, 4)$ c) $(-3, 3, 6)$ d) $(5, 7, -2)$

37. What is the equation of the plane which passes through the z-axis and is perpendicular to the line $\frac{x-a}{\cos\theta} = \frac{y+2}{\sin\theta} = \frac{z-3}{0}$?

a) $x + y\ \tan\theta = 0$ b) $y + x\tan\theta = 0$ c) $x\cos\theta - y\sin\theta = 0$ d) $x\sin\theta - y\cos\theta = 0$

38. The line $\frac{x+6}{5} = \frac{y+10}{3} = \frac{z+14}{8}$ is the hypotenuse of an isosceles right angled triangle whose opposite vertex is $(7, 2, 4)$. Then which of the following is not the side of the triangle?

a) $\frac{x-7}{2}=\frac{y-2}{-3}=\frac{z-4}{6}$
b) $\frac{x-7}{3}=\frac{y-2}{6}=\frac{z-4}{2}$
c) $\frac{x-7}{3}=\frac{y-2}{5}=\frac{z-4}{-1}$
d) None of these

39. The distance of point $A(-2,3,1)$ from the line PQ through $P(-3,5,2)$, which makes equal angles with the axes is
a) $2/\sqrt{3}$ b) $\sqrt{14/3}$ c) $16/\sqrt{3}$ d) $5/\sqrt{3}$

40. From the point $P(a,b,c)$, let perpendicular PL and PM be drawn to YOZ and ZOX planes, respectively. Then the equation of the plane OLM is
a) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0$ b) $\frac{x}{a}+\frac{y}{b}-\frac{z}{c}=0$ c) $\frac{x}{a}-\frac{y}{b}-\frac{z}{c}=0$ d) $\frac{x}{a}-\frac{y}{b}+\frac{z}{c}=0$

41. If the lines
$\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$
and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$
interested, then the value of k is
a) $\frac{3}{2}$ b) $\frac{9}{2}$ c) $-\frac{2}{9}$ d) $-\frac{3}{2}$

42. In a three-dimensional xyz space, the equation $x^2-5x+6=0$ represnts
a) Points b) Planes c) Curves d) Pair of straight lines

43. The direction ratios of a normal to the plane through (1, 0,0) and (0, 1, 0), which makes an angle of $\frac{\pi}{4}$ with the plane $x+y=3$ are
a) $<1,\sqrt{2},1>$ b) $<1,1,\sqrt{2}>$ c) $<1,1,2>$ d) $<\sqrt{2},1,1>$

44. The value of m for which straight line $3x-2y+z+3=0=4x-3y+4z+1$ is parallel to the plane $2x-y+mz-2=0$ is
a) -2 b) 8 c) -18 d) 11

45. A plane passes through a fixed point (a,b,c). The locus of the foot of the perpendicular to it from the origin is a sphere of radius
a) $\frac{1}{2}\sqrt{a^2+b^2+c^2}$ b) $\sqrt{a^2+b^2+c^2}$ c) $a^2+b^2+c^2$ d) $-\frac{1}{2}(a^2+b^2+c^2)$

46. Line $\vec{r}=\vec{a}+\lambda\vec{b}$ will not meet the plane $\vec{r}\cdot\vec{n}=q$, if
a) $\vec{b}\cdot\vec{n}=0,\vec{a}\cdot\vec{n}=q$ b) $\vec{b}\cdot\vec{n}\neq 0,\vec{a}\cdot\vec{n}\neq q$ c) $\vec{b}\cdot\vec{n}=0,\vec{a}\cdot\vec{n}\neq q$ d) $\vec{b}\cdot\vec{n}\neq 0,\vec{a}\cdot\vec{n}=q$

47. Let $A(\vec{a})$ and $B(\vec{b})$ be points on two skew lines $\vec{r}=\vec{a}+\lambda\,\vec{p}$ and $\vec{r}=\vec{b}+u\vec{q}$ and the shortest distance between the skew lines is 1, where $\vec{p}$ and $\vec{q}$ are unit vectors forming adjacent sides of a parallelogram enclosing an area of $\frac{1}{2}$ units. If an angle between AB and the line of shortest distance is 60°, then $AB=$
a) $\frac{1}{2}$ b) 2 c) 1 d) $\lambda\in \mathrm{R}-\{0\}$

48. Consider triangle AOB in the x-y plane, where $A\equiv(1,0,0)$; $B\equiv(0,2,0)$; and $O\equiv(0,0,0)$. The new position of O, when triangle is rotated about side AB by 90° can be
a) $\left(\frac{4}{5},\frac{3}{5},\frac{2}{\sqrt{5}}\right)$ b) $\left(\frac{-3}{5},\frac{\sqrt{2}}{5},\frac{2}{\sqrt{5}}\right)$ c) $\left(\frac{4}{5},\frac{2}{5},\frac{5}{\sqrt{5}}\right)$ d) $\left(\frac{4}{5},\frac{2}{5},\frac{1}{\sqrt{5}}\right)$

49. Let $A(1,1,1)$, $B(2,3,5)$ and $C(-1,0,2)$ be three points, then equation of a plane parallel to the plane ABC which is at distance 2 is
a) $2x-3y+z+2\sqrt{14}=0$ b) $2x-3y+z-\sqrt{14}=0$
c) $2x-3y+z+2=0$ d) $2x-3y+z-2=0$

50. Value of λ such that the line $\frac{x-1}{2}=\frac{y-1}{3}=\frac{z-1}{\lambda}$ is $\perp$ to normal to the plane $\vec{r}\cdot(2\vec{\imath}+3\vec{\jmath}+4\vec{k})=0$ is

a) $-\frac{13}{4}$ b) $-\frac{17}{4}$ c) 4 d) None of these

51. The shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ is

a) $\sqrt{30}$ b) $2\sqrt{30}$ c) $5\sqrt{30}$ d) $3\sqrt{30}$

52. If a line makes an angle of $\frac{\pi}{4}$ with the positive direction of each of x-axis and y-axis, then the angle that the line makes with the positive direction of the z-axis is

a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{6}$

53. The length of projection of the line segment joining the points $(1,0,-1)$ and $(-1,2,2)$ on the plane $x+3y-5z=6$, is equal to

a) 2 b) $\sqrt{\frac{271}{53}}$ c) $\sqrt{\frac{472}{31}}$ d) $\sqrt{\frac{474}{35}}$

54. Distance of point $P(\vec{p})$ from the plane $\vec{r}\cdot\vec{n}=0$ is

a) $|\vec{p}\cdot\vec{n}|$ b) $\frac{|\vec{p}\times\vec{n}|}{|\vec{n}|}$ c) $\frac{|\vec{p}\cdot\vec{n}|}{|\vec{n}|}$ d) None of these

55. The three planes $4y+6z=5$; $2x+3y+5z=5$ and $6x+5y+9z=10$

a) Meet in a point b) Have a line in common
c) Form a triangular prism d) None of these

56. Equation of the plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is

a) $x+2y-2z=0$ b) $3x+2y-2z=0$ c) $x-2y+z=0$ d) $5x+2y-4z=0$

57. The value of k such that $\frac{x-4}{1}=\frac{y-2}{1}=\frac{z-k}{2}$ lies in the plane $2x-4y+z=7$, is

a) 7 b) -7 c) No real value d) 4

58. If lines $x=y=z$ and $x=\frac{y}{2}=\frac{z}{3}$, and third line passing through $(1,1,1)$ form a triangle of area $\sqrt{6}$ units, then point of intersection of third line with second line will be

a) $(1,2,3)$ b) $(2,4,6)$ c) $\left(\frac{4}{3},\frac{8}{3},\frac{12}{3}\right)$ d) None of these

59. The point on the line $\frac{x-2}{1}=\frac{y+3}{-2}=\frac{z+5}{-2}$ at a distance of 6 from the point $(2,-3,-5)$ is

a) $(3,-5,-3)$ b) $(4,-7,-9)$ c) $(0,2,-1)$ d) $(-3,5,3)$

60. A sphere of constnat radius $2k$ passes through the origin and meets the axes in A,B and C. The locus of a centroid of the tetrahedron $OABC$ is

a) $x^2+y^2+z^2=4k^2$ b) $x^2+y^2+z^2=k^2$
c) $2(k^2+y^2+z)^2=k^2$ d) None of these

61. The radius of the circle in which the sphere $x^2+y^2+z^2+2z-2y-4z-19=0$ is cut by the plane $x+2y+2z+7=0$ is

a) 2 b) 3 c) 4 d) 1

62. The image of the point $(-1,3,4)$ in the plane $x-2y=0$ is

a) $\left(-\frac{17}{3},-\frac{19}{3},4\right)$ b) $(15,11,4)$ c) $\left(-\frac{17}{3},-\frac{19}{3},1\right)$ d) $\left(\frac{9}{5},-\frac{13}{5},4\right)$

63. If the foot of the perpendicular from the origin to a plane is $P(a,b,c)$, the equation of the plane is

a) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=3$
b) $ax+by+cz=3$
c) $ax+by+cz=a^2+b^2+c^2$
d) $ax+by+cz=a+b+c$

64. The equation of the plane passing through the lines $\frac{x-4}{1}=\frac{y-3}{1}=\frac{z-2}{2}$ and $\frac{x-3}{1}=\frac{y-2}{-4}=\frac{z}{5}$ is

a) $11x-y-3z=35$ b) $11x+y-3z=35$ c) $11x-y+3z=35$ d) None of these

65. Find the equation of a straight line in the plane $\vec{r}\cdot\vec{n}=d$ which is parallel to $r=\vec{a}+\lambda\vec{b}$ and passes

through the foot of the perpendicular drawn from point $P(\vec{a})$ to $\vec{r}\cdot\vec{n}=d$ (where $\vec{n}\cdot\vec{b}=0$)

a) $\vec{r}=\vec{a}+\left(\frac{d-\vec{a}\cdot\vec{n}}{n^2}\right)\vec{n}+\lambda\,\vec{b}$ b) $\vec{r}=\vec{a}+\left(\frac{d-\vec{a}\cdot\vec{n}}{n}\right)\vec{n}+\lambda\,\vec{b}$

c) $\vec{r}=\vec{a}+\left(\frac{\vec{a}\cdot\vec{n}-d}{n^2}\right)\vec{n}+\lambda\,\vec{b}$ d) $\vec{r}=\vec{a}+\left(\frac{\vec{a}\cdot\vec{n}-d}{n}\right)\vec{n}+\lambda\,\vec{b}$

66. A plane passes through $(1,-2,1)$ and is perpendicular to two planes $2x-2y+z=0$ and $x-y+2z=4$, then the distance of the plane from the point $(1,2,2)$is

a) 0 b) 1 c) $\sqrt{2}$ d) $2\sqrt{2}$

67. A straight line L on the xy-plane bisects the angle between OX and OY. What are the direction cosines of L?

a) $<(1/\sqrt{2}),(1/\sqrt{2}),0>$ b) $<(1/2),(\sqrt{3}/2),0>$

c) $<0,0,1>$ d) $<(2/3),(2/3),(1/3)>$

68. A unit vector parallel to the intersection of the plane $\vec{r}\cdot(\hat{\imath}-\hat{\jmath}+\hat{k})=5$ and $\vec{r}\cdot(2\hat{\imath}+\hat{\jmath}-3\hat{k})=4$ is

a) $\frac{2\hat{\imath}+5\hat{\jmath}-3\hat{k}}{\sqrt{38}}$ b) $\frac{2\hat{\imath}-5\hat{\jmath}+3\hat{k}}{\sqrt{38}}$ c) $\frac{-2\hat{\imath}-5\hat{\jmath}-3\hat{k}}{\sqrt{38}}$ d) $\frac{-2\hat{\imath}+5\hat{\jmath}-3\hat{k}}{\sqrt{38}}$

69. If the plane $\frac{x}{2}+\frac{y}{3}+\frac{z}{6}=1$ cuts the axes of coordinates at points A,B and C, then find the area of then triangle ABC

a) 18 sq unit b) 36 sq unit c) $3\sqrt{14}$ sq unit d) $2\sqrt{14}$ sq unit

70. The ratio in which the line segment joining the points whose position vectors are $2\hat{\imath}-4\hat{\jmath}-7\hat{k}$ and $-3\hat{\imath}+5\hat{\jmath}-8\hat{k}$ is divided by the plane whose equation is $\hat{r}.(\hat{\imath}-2\hat{\jmath}+3\hat{k})=13$, is

a) 13:12 internally b) 12:25 externally c) 13:25 internally d) 37:25 internally

71. A plane passing through (1, 1, 1) cuts positive direction of co-ordinate axes at A,B and C, then the volume of tetrahedron $OABC$ satisfies

a) $V\leq\frac{9}{2}$ b) $V\geq\frac{9}{2}$ c) $V=\frac{9}{2}$ d) None of these

72. The direction cosines of a line satisfy the relations $\lambda(l+m)=n$ and $mn+nl+lm=0$. The value of λ, for which the two lines are perpendicular to each other, is

a) 1 b) 2 c) 1/2 d) None of these

73. The plane $4x+7y+4z+81=0$ is rotated through a right angle about its line of intersection with the plane $5x+3y+10z=25$. The equation of the plane in its new position is

a) $x-4y+6z=106$ b) $x-8y+13z=103$ c) $x-4y+6z=110$ d) $x-8y+13z=105$

74. A line makes an angle θ with each of the x- and z-axes. If the angle β, which it makes with y-axis, is such that $\sin^2\beta=3\sin^2\theta$, then $\cos^2\theta$ equals

a) $\frac{2}{3}$ b) $\frac{1}{5}$ c) $\frac{3}{5}$ d) $\frac{2}{5}$

75. The equation of the plane which passes through the line of intersection of planes $\vec{r}\cdot\vec{n}_1=q_1,\vec{r}\cdot\vec{n}_2=q_2$ and is parallel to the line of intersection of planes $\vec{r}\cdot\vec{n}_3=q_3$ and $\vec{r}\cdot\vec{n}_4=q_4$, is

a) $[\vec{n}_2\vec{n}_3\vec{n}_4](\vec{r}\cdot\vec{n}_1-q_1)=[\vec{n}_1\vec{n}_3\vec{n}_4](\vec{r}\cdot\vec{n}_2-q_2)$

b) $[\vec{n}_1\vec{n}_2\vec{n}_3](\vec{r}\cdot\vec{n}_4-q_4)=[\vec{n}_4\vec{n}_3\vec{n}_1](\vec{r}\cdot\vec{n}_2-q_2)$

c) $[\vec{n}_4\vec{n}_3\vec{n}_1](\vec{r}\cdot\vec{n}_4-q_4)=[\vec{n}_1\vec{n}_2\vec{n}_3](\vec{r}\cdot\vec{n}_2-q_2)$

d) None of these

76. In a three dimensional co-ordinate system, P,Q and R are images of a point $A(a,b,c)$ in the x-y, y-z and z-x planes, respectively. If G is the centroid of triangle PQR, then area of triangle AOG is (O is the origin)

a) 0 b) $a^2+b^2+c^2$ c) $\frac{2}{3}(a^2+b^2+c^2)$ d) None of these

77. The lines which intercept the skew lines $y=mx,z=z;y=-mx,z=-c$ and the x-axis lie on the surface

a) $cz=mxy$ b) $xy=cmz$ c) $cy=mxz$ d) None of these

78. Equation of a line in the plane $\pi\equiv 2x-y+z-4=0$ which is perpendicular to the line l whose equation is $\frac{x-2}{1}=\frac{y-2}{-1}=\frac{z-3}{-2}$ and which passes through the point of intersection of l and π is

a) $\frac{x-2}{1}=\frac{y-1}{5}=\frac{z-1}{-1}$ b) $\frac{x-1}{3}=\frac{y-3}{5}=\frac{z-5}{-1}$

c) $\frac{x+2}{2}=\frac{y+1}{-1}=\frac{z+1}{1}$ d) $\frac{x-2}{2}=\frac{y-1}{-1}=\frac{z-1}{1}$

79. The Cartesian equation of the plane $\vec{r}=(1+\lambda-\mu)\hat{\imath}+(2-\lambda)\hat{\jmath}+(3-2\lambda+2\mu)\hat{k}$ is

a) $2x+y=5$ b) $2x-y=5$ c) $2x+z=5$ d) $2x-z=5$

80. The coordinates of the point P on the line $\vec{r}=(\hat{\imath}+\hat{\jmath}+\hat{k})+\lambda(-\hat{\imath}+\hat{\jmath}-\hat{k})$ which is nearest to the origin is

a) $\left(\frac{2}{3},\frac{4}{3},\frac{2}{3}\right)$ b) $\left(-\frac{2}{3},-\frac{4}{3},\frac{2}{3}\right)$ c) $\left(\frac{2}{3},\frac{4}{3},-\frac{2}{3}\right)$ d) None of these

81. The lines: $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar if:

a) $k=1$ or -1 b) $k=0$ or -3 c) $k=3$ or -3 d) $k=0$ or -1

82. If $P_1:\vec{r}\cdot\vec{n}_1-d_1=0, P_2:\vec{r}\cdot\vec{n}_2-d_2=0$ and $P_3:\vec{r}\cdot\vec{n}_3-d_3=0$ are three planes and $\vec{n}_1,\vec{n}_2$ and $\vec{n}_3$ are three non-coplanar vectors, then three lines $P_1=0,P_2=0; P_2=0,P_3=0$ and $P_3=0,P_1=0$ are

a) Parallel lines b) Coplanar lines c) Coincident lines d) Concurrent lines

83. The pair of lines whose direction cosines are given by the equations $3l+m+5n=0$ and $6mn-2nl+5lm=0$, are

a) Parallel b) Perpendicular c) Inclined at $\cos^{-1}\left(\frac{1}{6}\right)$ d) None of these

84. The centre of the circle given by: $\vec{r}\cdot(\hat{\imath}+2\hat{\jmath}+2\hat{k})=15$ and $|\vec{r}-(\hat{\jmath}+2\hat{k})|=4$ is

a) $(0,1,2)$ b) $(1,3,4)$ c) $(-1,3,4)$ d) None of these

85. Two systems of rectangular axes have the same origin. If a plane cuts them at distance a,b,c and a',b',c' from the origin, then:

a) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}=0$

b) $\frac{1}{a^2}-\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$

c) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$

d) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}=0$

86. The equation of the plane through the intersection of the planes $x+2y+3z-4=0$ and $4x+3y+2z+1=0$ and passing through the origin is

a) $17x+14y+11z=0$ b) $7x+4y+z=0$

c) $x+14y+11z=0$ d) $17x+y+z=0$

87. The equation of the plane through the line of intersection of the planes $ax+by+cz+d=0$ and $a'x+b'y+c'z+d'=0$ and parallel to the line $y=0$ and $z=0$ is

a) $(ab'-a'b)x+(bc'-b'c)y+(ad'-a'd)=0$

b) $(ab'-a'b)x+(bc'-b'c)y+(ad'-a'd)z=0$

c) $(ab'-a'b)y+(ac'-a'c)z+(ad'-a'd)=0$

d) None of these

88. A variable plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ at a unit distance from origin cuts the coordinate axes at A,B and C.Centriod (x,y,z)satisfies the equation $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=k$.The value of k is

a) 9 b) 3 c) $\frac{1}{9}$ d) $\frac{1}{3}$

89. The plane, which passes through the point (3, 2, 0) and the line $\frac{x-3}{1}=\frac{y-6}{5}=\frac{z-4}{4}$ is:

a) $x-y+z=1$ b) $x+y+z=5$ c) $x+2y-z=1$ d) $2x-y+z=5$

90. The point of intersection of the line passing through (0,0, 1) and intersecting the lines $x+2y+z=1,-x+y-2z=2$ and $x+y=2,x+z=2$ with xy plane is

a) $\left(\frac{5}{3}, -\frac{1}{3}, 0\right)$ b) $(1, 1, 0)$ c) $\left(\frac{2}{3}, -\frac{1}{3}, 0\right)$ d) $\left(-\frac{5}{3}, \frac{1}{3}, 0\right)$

91. The shortest distance from the plane $12x + y + 3z = 327$ to the sphere $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$ is

a) 39 b) 26 c) $41\frac{4}{13}$ d) 13

92. The angle between $\hat{\imath}$ line of the intersection of the plane $\vec{r} \cdot (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) = 0$ and $\vec{r} \cdot (3\hat{\imath} + 3\hat{\jmath} + \hat{k}) = 0$, is

a) $\cos^{-1}\left(\frac{1}{3}\right)$ b) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ c) $\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$ d) None of these

93. The ratio in which the plane $\vec{r} \cdot (\vec{\imath} - 2\vec{\jmath} + 3\vec{k}) = 17$ divides the line joining the points $-2\vec{\imath} + 4\vec{\jmath} + 7\vec{k}$ and $3\vec{\imath} - 5\vec{\jmath} + 8\vec{k}$ is

a) 1:5 b) 1:10 c) 3:5 d) 3:10

Multiple Correct Answers Type

94. Let PM be the perpendicular from the point $P(1, 2, 3)$ to the x-y plane. If $\overrightarrow{OP}$ makes an angle θ with the positive direction of the z-axis and $\overrightarrow{OM}$ makes an angle ϕ with the positive direction of x-axis, where O is the origin and θ and ϕ are acute angles, then

a) $\cos\theta \cos\phi = 1/\sqrt{14}$ b) $\sin\theta \sin\phi = 2/\sqrt{14}$ c) $\tan\phi = 2$ d) $\tan\theta = \sqrt{5}/3$

95. The equation of the line $x + y + z - 1 = 0$ and $4x + y - 2z + 2 = 0$ written in the symmetrical form is

a) $\frac{x-1}{2} = \frac{y+2}{-1} = \frac{z-2}{2}$ b) $\frac{x+(1/2)}{1} = \frac{y-1}{-2} = \frac{z-(1/2)}{1}$

c) $\frac{x}{1} = \frac{y}{-2} = \frac{z-1}{1}$ d) $\frac{x+1}{1} = \frac{y-2}{-2} = \frac{z-0}{1}$

96. Consider a set of points R in the space which is at a distance of 2 units from the line $\frac{x}{1} = \frac{y-1}{-1} = \frac{z+2}{2}$ between the planes $x - y + 2z + 3 = 0$ and $x - y + 2z - 2 = 0$

a) The volume of the bounded figure by points R and the planes is $(10/3\sqrt{3})\pi$ cube units

b) The area of the curved surface formed by the set of points R is $(20\pi/\sqrt{6})$ sq. units

c) The volume of the bounded figure by the set of points R and the planes is $(20\pi/\sqrt{6})$ cubic units

d) The area of the curved surface formed by the set of points R is $(10/\sqrt{3})\pi$ sq. units

97. A rod of length 2 units whose one end is $(1,0,-1)$ and other end touches the plane $x - 2y + 2z + 4 = 0$, then

a) The rod sweeps the figure whose volume is π cubic units

b) The area of the region which the rod traces on the plane is 2π

c) The length of projection of the rod on the plane is $\sqrt{3}$ units

d) The centre of the region which the rod traces on the plane is $\left(\frac{2}{3}, \frac{2}{3}, \frac{-5}{3}\right)$

98. Consider the planes $3x - 6y + 2z + 5 = 0$ and $4x - 12y + 3z = 3$. The plane $67x - 162y + 47z + 44 = 0$ bisects the angle between the given planes which

a) Contains the origin b) Is acute c) Is obtuse d) None of these

99. If α, β, γ are the angles which a line makes with the coordinate axes, then

a) $\sin^2\alpha = \cos^2\beta + \cos^2\gamma$ b) $\cos^2\alpha = \cos^2\beta + \cos^2\gamma$

c) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ d) $\sin^2\alpha + \sin^2\beta = 1 + \cos^2\gamma$

100. If $OABC$ is a tetrahedron such that $OA^2 + BC^2 = OB^2 + CA^2 = OC^2 + AB^2$, then

a) $OA \perp BC$ b) $OB \perp CA$ c) $OC \perp AB$ d) $AB \perp BC$

101. The equations of the plane which passes through $(0, 0, 0)$ and which is equally inclined to the planes $x - y + z - 3 = 0$ and $x + y + z + 4 = 0$ is/are

a) $y = 0$ b) $x = 0$ c) $x + y = 0$ d) $x + z = 0$

102. If the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{\lambda}$ and $\frac{x-1}{\lambda} = \frac{y-4}{2} = \frac{z-5}{1}$ intersect, then

a) $\lambda = -1$ b) $\lambda = 2$ c) $\lambda = -3$ d) $\lambda = 0$

103. If the volume of tetrahedron $ABCD$ is 1 cubic units, where $A(0,1,2)$, $B(-1,2,1)$ and $C(1,2,1)$, then the locus of point D is

a) $x + y - z = 3$ b) $y + z = 6$ c) $y + z = 0$ d) $y + z = -3$

104. The equation of the plane which is equally inclined to the lines $\frac{x-1}{2} = \frac{y}{-2} = \frac{z+2}{-1}$ and $\frac{x+3}{8} = \frac{y-4}{1} = \frac{z}{-4}$ and passing through the origin is/are

a) $14x - 5y - 7z = 0$ b) $2x + 7y - z = 0$ c) $3x - 4y - z = 0$ d) $x + 2y - 5z = 0$

105. The extremities of a diameter of a sphere lie on positive y ad positive x-axes at distance 2 and 4 from the origin, respectively, then

a) Sphere passes through the origin b) Centre of the sphere is $(0, 1, 2)$

c) Radius of the sphere is $\sqrt{5}$ d) Equation of a diameter is $\frac{x}{0} = \frac{y-2}{1} = \frac{z-4}{-2}$

106. The lines $\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$ and $\frac{x-4}{2} = \frac{y+0}{0} = \frac{z+1}{3}$

a) Do not intersect b) Intersect c) Intersect at $(4, 0, -1)$ d) Intersect at $(1, 1, -1)$

107. Let P_1 denote the equation of a plane to which the vector $(\hat{\imath} + \hat{\jmath})$ is normal and which contains the line whose equation is $\vec{r} = \hat{\imath} + \hat{\jmath} + \hat{k} + \lambda(\hat{\imath} - \hat{\jmath} - \hat{k})$ and P_2 denote the equation of the plane containing the line L and a point with position vector $\hat{\jmath}$. Which of the following holds good?

a) The equation of P_1 is $x + y = 2$

b) The equation of P_2 is $\vec{r} \cdot (\hat{\imath} - 2\hat{\jmath} + \hat{k}) = 2$

c) The acute angle between P_1 and P_2 is $\cot^{-1}(\sqrt{3})$

d) The angle between the plane P_2 and the line L is $\tan^{-1}\sqrt{3}$

108. Let $P = 0$ be the equation of a plane passing through the line of intersection of the planes $2x - y = 0$ and $3z - y = 0$ and perpendicular to the plane $4x + 5y - 3z = 8$. Then the points which lie on the plane $P = 0$ is/are

a) $(0, 9, 17)$ b) $(1/7, 2, 1/9)$ c) $(1, 3, -4)$ d) $(1/2, 1, 1/3)$

109. The x-y plane is rotated about its line of intersection with line y-z plane by 45°, then the equation of the new plane is/are

a) $z + x = 0$ b) $z - y = 0$ c) $x + y + z = 0$ d) $z - x = 0$

110. A line with direction cosines proportional to $1, -5$ and -2 meets lines $x = y + 5 = z + 1$ and $x + 5 = 3y = 2z$. The coordinates of each of the point of the intersection are given by

a) $(2, -3, 1)$ b) $(1, 2, 3)$ c) $(0, 5/3, 5/2)$ d) $(3, -2, 2)$

111. The equation of a line passing through the point $\vec{a}$ parallel to the plane $\vec{r}.\vec{n} = q$ and perpendicular to the line $\vec{r} = \vec{b} + \vec{t}c$ is

a) $\vec{r} = \vec{a} + \lambda(\vec{n} \times \vec{c})$ b) $(\vec{r} - \vec{a}) \times (\vec{n} \times \vec{c}) = 0$ c) $\vec{r} = \vec{b} + \lambda(\vec{n} \times \vec{c})$ d) None of these

112. Which of the following lines lie on the plane $x + 2y - z + 4 = 0$?

a) $\frac{x-1}{1} = \frac{y}{-1} = \frac{z-5}{-1}$

b) $x - y + z = 2x + y - z = 0$

c) $\vec{r} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k} + \lambda(3\hat{\imath} + \hat{\jmath} + 5\hat{k})$

d) None of these

113. If the planes $\vec{r} \cdot (\hat{\imath} + \hat{\jmath} + \hat{k}) = q_1$, $\vec{r} \cdot (\hat{\imath} + 2a\hat{\jmath} + \hat{k}) = q_2$ and $\vec{r} \cdot (a\hat{\imath} + a^2\hat{\jmath} + \hat{k}) = q_3$ intersect in a line, then the value of a is

a) 1 b) 1/2 c) 2 d) 0

114. $P(1,1,1)$ and $Q(\lambda, \lambda, \lambda)$ are two points in the space such that $PQ = \sqrt{27}$, the value of λ can be

a) -4 b) -1 c) 2 d) 4

Assertion - Reasoning Type

This section contain(s) 0 questions numbered 115 to 114. Each question contains STATEMENT 1(Assertion) and STATEMENT 2(Reason). Each question has the 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

a) Statement 1 is True, Statement 2 is True; Statement 2 **is** correct explanation for Statement 1

b) Statement 1 is True, Statement 2 is True; Statement 2 **is not** correct explanation for Statement 1

c) Statement 1 is True, Statement 2 is False

d) Statement 1 is False, Statement 2 is True

115

Statement 1: A plane passes through the point $A(2,1,-3)$. If distance of this plane from origin is maximum, then its equation is $2x+y-3z=14$

Statement 2: If the plane passing through the point $A(\vec{a})$ is at maximum distance from origin, then normal to the plane is vector $\vec{a}$

116

Statement 1: Let $A\left(\vec{\imath}+\vec{\jmath}+\vec{k}\right)$ and $B\left(\vec{\imath}-\vec{\jmath}+\vec{k}\right)$ be two points. Then point $P(2\vec{\imath}+3\vec{\jmath}+\vec{k})$ lies exterior to the sphere with AB as its diameter

Statement 2: If A and B are any two points and P is a point in space such that $\overrightarrow{PA}\cdot\overrightarrow{PB}>0$, then point P lies exteriuor to the sphere with AB as its diameter

117

Statement 1: Equation of the polar to the sphere $x^2+y^2+z^2=1$ with respect to the point (1,2, 3) is $x+2y+3z=1$

Statement 2: The point (1,2,3) lies outside the sphere $x^2+y^2+z^2=1$

118

Statement 1: The points $A(2,9,12), B(1,8,8), C(-2,11,8)$ and $D(-1,12,12)$ and the vertices of a rhombus

Statement 2: $AB=BC=CD=DA$ and $AC\neq BD$

119

Statement 1: There exists a unique sphere which passes through the three non-collinear points and which has the least radius

Statement 2: The centre of such a sphere lies on the plane determined by the given three points

120

Statement 1: Lines $\vec{r}=\hat{\imath}-\hat{\jmath}+\lambda\left(\hat{\imath}+\hat{\jmath}-\hat{k}\right)$ and $\vec{r}=2\hat{\imath}-\hat{\jmath}+\mu(\hat{\imath}+\hat{\jmath}-\hat{k})$ do not intersect

Statement 2: Skew lines never intersect

121

Statement 1: If centroid and circumcentre of a triangle are known its othocentre can be found

Statement 2: Centroid, orthocentre and circumcentre of a triangle are collinear

122

Statement 1: Let θ be the angle between the line $\frac{x-2}{2}=\frac{y-1}{-3}=\frac{z+2}{-2}$ and the plane $x+y-z=5$. Then $\theta=\sin^{-1}(1/\sqrt{51})$

Statement 2: The angle between a straight line and a plane is the complement of the angle between the line and the normal to the plane

123

Statement 1: Two spheres radii r_1 and r_2 cut orthogonally, then radius of the common circle is $\frac{r_1 r_2}{\sqrt{(r_1^2+r_2^2)}}$

Statement 2: If two spheres

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$ and

$x^2+y^2+z^2+2u'''''$

$x+2v'$

$y+2w'$

$z+d''=0$ cut

Orthogonally, then $2uu'+2vv'+2ww'=d+d'$

124 Consider the lines

$$L_1: \frac{x+1}{3}=\frac{y+2}{1}=\frac{z+1}{2},$$

$$L_2: \frac{x-2}{1}=\frac{y+2}{2}=\frac{z-3}{3}$$

Statement 1: The distance of the point (1,1,1) from the plane passing through the point $(-1,-2,-1)$ and whose normal is perpendicular to both the lines L_1 and L_2 is $\frac{13}{5\sqrt{3}}$

Statement 2: The unit vector perpendicular to both the lines L_1 and L_2 is $\frac{-\hat{\mathbf{i}}-7\hat{\mathbf{j}}+5\hat{\mathbf{k}}}{5\sqrt{3}}$

125

Statement 1: The lines $\frac{x-1}{1}=\frac{y}{-1}=\frac{z+1}{1}$ and $\frac{x-2}{1}=\frac{y+1}{2}=\frac{z}{3}$ are coplanar and equation of the plane containing them is $5x+2y-3z-8=0$

Statement 2: The line $\frac{x-2}{1}=\frac{y+1}{2}=\frac{z}{3}$ is perpendicular to the plane $3x+6y+9z-8=0$ and parallel to the plane $x+y-z=0$

126

Statement 1: Lines $\vec{r}=\hat{\imath}+\hat{\jmath}-\hat{k}+\lambda(3\hat{\imath}-\hat{\jmath})$ and $\vec{r}=4\hat{\imath}-\hat{k}+\mu(2\hat{\imath}+3\hat{k})$ intersect

Statement 2: If $\vec{b}\times\vec{d}=\vec{0}$, then lines $\vec{r}=\vec{a}+\lambda\vec{b}$ and $\vec{r}=\vec{c}+\lambda\vec{d}$ do not intersect

127

Statement 1: The shortest distance between the lines $\frac{x}{-3}=\frac{y-1}{1}=\frac{z+1}{-1}$ and $\frac{x-2}{1}=\frac{y-3}{2}=\left(\frac{z+(13/7)}{-1}\right)$ is zero

Statement 2: The given lines are perpendicular

128

Statement 1: There exists two points on the line $\frac{x-1}{1}=\frac{y}{-1}=\frac{z+2}{2}$ which are at a distance of 2 units from point $(1,2,-4)$

Statement 2: Perpendicular distance of point $(1,2,-4)$ from the line $\frac{x-1}{1}=\frac{y}{-1}=\frac{z+2}{2}$ is 1 unit

129

Statement 1: The spheres $x^2+y^2+z^2+2ax+c=0$ and $x^2+y^2+z^2+2\,by\,c=0$ touch each other, if $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$

Statement 2: Two spheres with centres C_1 and C_2 and radii r_1, r_2 touch each other if $|r_1 \pm r_2| = |C_1C_2|$

130

Statement 1: The plane $5x + 2z - 8 = 0$ contains the line $2x - y + z - 3 = 0$ and $3x + y + z = 5$ and is perpendicular to $2x - y - 5z - 3 = 0$

Statement 2: The plane $3x + y + z = 5$ meets the line $x - 1 = y + 1 = z - 1$ at the point (1,1,1)

131 The equation of two straight lines are $\frac{x-1}{2} = \frac{y+3}{1} = \frac{z-2}{-3}$ and $\frac{x-2}{1} = \frac{y-1}{-3} = \frac{z+3}{2}$

Statement 1: The given lines are coplanar

Statement 2: The equation $2x_1 - y_1 = 1, x_1 + 3y_1 = 4$ and $3x_1 + 2y_1 = 5$ are consistent

132

Statement 1: The shortest distance between the skew lines $\frac{x+3}{-4} = \frac{y-6}{2} = \frac{z}{2}$ and $\frac{x+2}{-4} = \frac{y}{1} = \frac{z-7}{1}$ is 9

Statement 2: Two lines are skew lines if there exists no plane passing through them

133 Consider the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$

Statement 1: The parametric equations of the line of intersection of the given planes are $x = 3 + 14t, y = 1 + 2t, z = 15t$

Statement 2: The vectors $14\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 15\hat{\mathbf{k}}$ is parallel to the line of intersection of the given planes.

134

Statement 1: Line $\frac{x-1}{1} = \frac{y-0}{2} = \frac{z+2}{-1}$ lies in the plane $2x - 3y - 4z - 10 = 0$

Statement 2: If line $\vec{r} = \vec{a} + \lambda\vec{b}$ lies in the plane $\vec{r} \cdot \vec{c} = n$ (where n is scalar), then $\vec{b} \cdot \vec{c} = 0$

135

Statement 1: The point $A(3,1,6)$ is the mirror image of the point $P(1,3,4)$ in the plane $x - y + z = 5$.

Statement 2: The plane $x - y + z = 5$ bisects the line segment joining $A(3,1,6)$ and $B(1,3,4)$

Matrix-Match Type

This section contain(s) 0 question(s). Each question contains Statements given in 2 columns which have to be matched. Statements (A, B, C, D) in **columns I** have to be matched with Statements (p, q, r, s) in **columns II.**

136.

Column-I	**Column- II**
(A) Image of the point (3, 5, 7) in the plane $2x + y + z = -18$ is	(p) $(-1, -1, -1)$
(B) The point of intersection of the line $\frac{x-2}{-3} = \frac{y-1}{-2} = \frac{z-3}{2}$ and the plane $2x + y - z = 3$ is	(q) $(-21, -7, -5)$
(C) The foot of the perpendicular from the point (1, 1, 2) to the plane $2x - 2y + 4z + 5 = 0$ is	(r) $\left(\frac{5}{2}, \frac{2}{3}, \frac{8}{3}\right)$

(D) The intersection point of the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ is	(s)	$\left(-\frac{1}{12},\frac{25}{12},\frac{-2}{12}\right)$

CODES :

	A	B	C	D
a)	q	r	s	p
b)	r	s	p	q
c)	s	p	q	r
d)	p	q	r	s

137.

	Column-I		Column- II
(A)	Lines $\frac{x-1}{-2}=\frac{y+2}{3}=\frac{z}{-1}$ and $\vec{r}=(3\hat{\imath}-\hat{\jmath}+\hat{k})+t(\hat{\imath}+\hat{\jmath}+\hat{k})$ are	(p)	Intersecting
(B)	Lines $\frac{x+5}{1}=\frac{y-3}{7}=\frac{z+3}{3}$ and $x-y+2z-4=0=2x+y-3z+5=0$ are	(q)	Perpendicular
(C)	Lines $(x=t-3, y=-2t+1, z=-3t-2)$ and $\vec{r}=(t+1)\hat{\imath}+(2t+3)\hat{\jmath}+(-t-9)\,\hat{k}$ are	(r)	Parallel
(D)	Lines $\vec{r}=(\hat{\imath}+3\hat{\jmath}-\hat{k})+\ t(2\hat{\imath}-\hat{\jmath}-\hat{k})$ and $\vec{r}=(-\hat{\imath}-2\hat{\jmath}+5\hat{k})+s(\hat{\imath}-2\hat{\jmath}+\frac{3}{4}\hat{k})$ are	(s)	Skew

CODES :

	A	B	C	D
a)	r	p,q	p	q,s
b)	q,s	r	p,q	p
c)	p,q	p	q,s	r
d)	p	q,s	r	p,q

138.

	Column-I		Column- II
(A)	The distance between the line $\vec{r}=(2\hat{\imath}-2\hat{\jmath}+3\hat{k})+\lambda(\hat{\imath}-\hat{\jmath}+4\hat{k})$ and plane $\vec{r}\cdot(\hat{\imath}+5\hat{\jmath}+\hat{k})=5$	(p)	$\frac{25}{3\sqrt{14}}$
(B)	Distance between parallel planes $\vec{r}\cdot(2\hat{\imath}-\hat{\jmath}+3\hat{k})=4$ and $\vec{r}\cdot(6\hat{\imath}-3\hat{\jmath}+9\hat{k})+13=0$ is	(q)	$13/7$
(C)	The distance of a point $(2,5,-3)$ from the plane $\vec{r}\cdot(6\hat{\imath}-3\hat{\jmath}+2\hat{k})=4$ is	(r)	$\frac{10}{3\sqrt{3}}$
(D)	The distance of the point $(1,0,-3)$ from the plane $x-y-z-9=0$ measured parallel to line $\frac{x-2}{2}=\frac{y+2}{3}=\frac{z-6}{-6}$	(s)	7

CODES :

	A	**B**	**C**	**D**
a)	p	q	s	r
b)	q	s	r	p
c)	s	r	p	q
d)	r	p	q	s

139.

	Column-I		**Column- II**
(A)	A vector perpendicular to the line $x = 2t + 1, y = t + 2$ and $z = -t - 3$	(p)	$7\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$
(B)	A vector parallel to the planes $x + y + z - 3 = 0$ and $2x - y + 3z = 0$	(q)	$4\hat{\imath} - \hat{\jmath} - 3\hat{k}$
(C)	A vector along which the distance between the lines $\frac{x}{2} = \frac{y}{-3} = \frac{z}{-1}$ and $\vec{r} = \left(3\hat{\imath} - \hat{\jmath} + \hat{k}\right) + t(\hat{\imath} + \hat{\jmath} - 2\hat{k})$ is the shortest	(r)	$-11\hat{\imath} + 7\hat{\jmath} + 5\hat{k}$
(D)	A vector normal to the plane $\vec{r} = -\hat{\imath} + 4\hat{\jmath} - 6\hat{k} + \lambda\left(\hat{\imath} + 3\hat{\jmath} - 2\hat{k}\right) + \mu(-\hat{\imath} + 2\hat{\jmath} - 5\hat{k})$	(s)	$\hat{\imath} + 3\hat{\jmath} + \hat{k}$

CODES :

	A	**B**	**C**	**D**
a)	s	q	p	r
b)	q	p	r	s
c)	p	r	s	q
d)	r	s	q	p

140.

	Column-I		**Column- II**
(A)	The coordinates of a point on the line $x = 4y + 5, z = 3y - 6$ at a distance 3 from the point $(5, 3, -6)$ is/are	(p)	$(-1, -2, 0)$
(B)	The plane containing the lines $\frac{x-2}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and parallel to $\hat{\imath} + 4\hat{\jmath} + 7\hat{k}$ has the point	(q)	$(5, 0, -6)$
(C)	A line passes through two points $A(2, -3, -1)$ and $B(8, -1, 2)$. The coordinates of a point on this line nearer to the origin and at a distance of 14 units from A is/are	(r)	$(2,5,7)$
(D)	The coordinates of the foot of the perpendicular from the point $(3, -1, 11)$ on the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is/are	(s)	$(14, 1, 5)$

CODES :

	A	B	C	D
a)	s	r	q	p
b)	r	q	p	s
c)	q	p	s	r
d)	p	s	r	q

Linked Comprehension Type

This section contain(s) 13 paragraph(s) and based upon each paragraph, multiple choice questions have to be answered. Each question has atleast 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 141 to -141

Let any two points in a plane be $A(-2,2,3)$ and $B(13,-3,13)$ and L is a line through A

On the basis of above information, answer the following questions

141. A point P moves in the space such that $3PA = 2\,PB$, then the locus of P is

a) $x^2+y^2+z^2+28x-12y+10z-247=0$

b) $x^2+y^2+z^2-28x+12y+10z-247=0$

c) $x^2+y^2+z^2+28x-12y-10z+247=0$

d) $x^2+y^2+z^2-28x+12y-10z+247=0$

Paragraph for Question Nos. 142 to - 142

Consider the lines

$L_1: \dfrac{x+1}{3}=\dfrac{y+2}{1}=\dfrac{z+1}{2}$,

And $L_2: \frac{x-2}{1}=\frac{y+2}{2}=\frac{z-3}{3}$

142. The unit vector perpendicular to both L_1 and L_2 is

a) $\dfrac{-\hat{\mathbf{i}}+7\hat{\mathbf{j}}+7\hat{\mathbf{k}}}{\sqrt{99}}$ b) $\dfrac{-\hat{\mathbf{i}}-7\hat{\mathbf{j}}+5\hat{\mathbf{k}}}{5\sqrt{3}}$ c) $\dfrac{-\hat{\mathbf{i}}+7\hat{\mathbf{j}}+5\hat{\mathbf{k}}}{5\sqrt{3}}$ d) $\dfrac{7\hat{\mathbf{i}}-7\hat{\mathbf{j}}-\hat{\mathbf{k}}}{\sqrt{99}}$

Paragraph for Question Nos. 143 to - 143

Suppose direction cosines of two lines are given by $ul+vm+wn=0$ and $al^2+bm^2+cn^2=0$, where u,v,w,a,b,c are arbitrary constant and l,m,n are direction cosines of the lines.

On the basis of above information, answer the following questions

143. For $u=v=w=1$, both lines satisfies the relation

a) $(b+c)\left(\frac{n}{l}\right)^2+2b\left(\frac{n}{l}\right)+(a+b)=0$

b) $(c+a)\left(\frac{l}{m}\right)^2+2c\left(\frac{l}{m}\right)+(b+c)=0$

c) $(a+b)\left(\frac{m}{n}\right)^2+2a\left(\frac{m}{n}\right)+(c+a)=0$

d) All of the above

Paragraph for Question Nos. 144 to - 144

Given four points $A(2,1,0), B(1,0,1), C(3,0,1)$ and $D(0,0,2)$. Point D lies on a line L orthogonal to the plane determined by the A, B and C

144. The equation of the plane ABC is
   a) $x + y + z - 3 = 0$ b) $y + z - 1 = 0$ c) $x + z - 1 = 0$ d) $2y + z – 1 = 0$

Paragraph for Question Nos. 145 to - 145

A ray of light comes along the line $L = 0$ and strikes the plane mirror kept along the plane $P = 0$ at B. $A(2, 1, 6)$ is a point on the line $L = 0$ whose image about $P = 0$ is A'. It is given that $L = 0$ is $\frac{x-2}{3} = \frac{y-1}{4} = \frac{z-6}{5}$ and $P = 0$ is $x + y - 2z = 3$

145. The coordinates of A' are
   a) $(6, 5, 2)$ b) $(6, 5, -2)$ c) $(6, -5, 2)$ d) None of these

Paragraph for Question Nos. 146 to - 146

Consider three planes $2x + py + 6z = 8, x + 2y + qz = 5$ and $x + y + 3z = 4$

146. Three planes intersect at a point if
   a) $p = 2, q \neq 3$ b) $p \neq 2, q \neq 3$ c) $p \neq 2, q = 3$ d) $p = 2, q = 3$

Paragraph for Question Nos. 147 to - 147

Consider a plane $x + y - z = 1$ and point $A(1, 2, -3)$. A line L has the equation $x = 1 + 3r, y = 2 - r$ and $z = 3 + 4r$

147. The coordinate of a point B of line L such that AB is parallel to the plane is
   a) $(10, -1, 15)$ b) $(-5, 4, -5)$ c) $(4, 1, 7)$ d) $(-8, 5, -9)$

Integer Answer Type

148. Let A_1, A_2, A_3, A_4 be the areas of the triangular faces of a tetrahedron, and h_1, h_2, h_3, h_4 be the corresponding altitude of the tetrahedron. If volume of tetrahedron is 1/6 cubic units, then find the minimum value of $(A_1 + A_2 + A_3 + A_4)(h_1 + h_2 + h_3 + h_4)$ (in cubic units)
149. Let $P(a, b, c)$ be any point on the plane $3x + 2y + z = 7$, then find the least value of $2(a^2 + b^2 + c^2)$
150. Find the distance of the z-axis from the image of the point $M(2, -3, 3)$ in the plane $x - 2y - z + 1 = 0$
151. The position vectors of the four angular points of a tetrahedron $OABC$ are (0, 0, 0), (0, 0, 2), (0, 4, 0) and (6, 0, 0), respectively. A point P inside the tetrahedron is at the same distance 'r' from the four plane faces of the tetrahedron. Find the value of $9r$
152. Let the equation of the plane containing line $x - y - z - 4 = 0 = x + y + 2z - 4$ and paralllle to the line of intersecting of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$ be $x + Ay + Bz + C = 0$. Then find the value of $|A + B + C - 4|$

153. The plane denoted by $P_1: 4x + 7y + 4z + 81 = 0$ is rotated through a right angle its line of intersection with the plane $P_2: 5x + 3y + 10z = 25$. If the plane in its new position be denoted by P, and the distance of this plane from the origin is d, then find the value of $[k/2]$ (where $[\cdot]$ represents greatest integer less than or equal to k)

154. The distance of the point $P(-2, 3, -4)$ from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$ is d, then find the value of $(2d - 8)$

155. If the length of the projection of the line segment with points $(1, 0, -1)$ and $(-1,2,2)$ to the plane $x + 3y - 5z = 6$ is d, then find the value of $[d/2]$ where $[\cdot]$ represent greatest integer function

156. If the angle between the plane $x - 3y + 2z = 1$ and the line $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-1}{-3}$ is θ, then find the value of $\operatorname{cosec} \theta$

157. Find the number of spheres of radius r touching the coordinate axes

: ANSWER KEY :

1)	**b**	**2)**	**a**	**3)**	**b**	**4)**	**a**		**b,c**	**4)**	**a,c,d**				
5)	**c**	**6)**	**b**	**7)**	**c**	**8)**	**a**	**5)**	**a,b**	**6)**	**a,c,d**	**7)**	**a,b,c**	**8)**	
9)	**c**	**10)**	**d**	**11)**	**c**	**12)**	**b**		**a,c**						
13)	**a**	**14)**	**b**	**15)**	**a**	**16)**	**c**	**9)**	**a,d**	**10)**	**b,c**	**11)**	**a,b**	**12)**	
17)	**a**	**18)**	**a**	**19)**	**c**	**20)**	**c**		**a,b,c**						
21)	**d**	**22)**	**a**	**23)**	**c**	**24)**	**c**	**13)**	**b,c**	**14)**	**a,c**	**15)**	**a,d**	**16)**	
25)	**a**	**26)**	**c**	**27)**	**d**	**28)**	**b**		**a,d**						
29)	**b**	**30)**	**d**	**31)**	**d**	**32)**	**b**	**17)**	**a,b**	**18)**	**a,b**	**19)**	**a,c**	**20)**	
33)	**d**	**34)**	**a**	**35)**	**b**	**36)**	**a**		**a,b**						
37)	**a**	**38)**	**c**	**39)**	**b**	**40)**	**b**	**21)**	**b,d**	**1)**	**a**	**2)**	**a**	**3)**	**b**
41)	**b**	**42)**	**b**	**43)**	**b**	**44)**	**a**		**4)**	**c**					
45)	**a**	**46)**	**c**	**47)**	**b**	**48)**	**c**	**5)**	**b**	**6)**	**b**	**7)**	**b**	**8)**	**a**
49)	**a**	**50)**	**a**	**51)**	**d**	**52)**	**c**	**9)**	**a**	**10)**	**a**	**11)**	**b**	**12)**	**b**
53)	**d**	**54)**	**c**	**55)**	**b**	**56)**	**c**	**13)**	**b**	**14)**	**c**	**15)**	**a**	**16)**	**c**
57)	**a**	**58)**	**b**	**59)**	**b**	**60)**	**b**	**17)**	**a**	**18)**	**b**	**19)**	**d**	**20)**	**b**
61)	**b**	**62)**	**d**	**63)**	**c**	**64)**	**d**	**21)**	**a**	**1)**	**a**	**2)**	**b**	**3)**	**d**
65)	**a**	**66)**	**d**	**67)**	**a**	**68)**	**c**		**4)**	**a**					
69)	**c**	**70)**	**b**	**71)**	**b**	**72)**	**b**	**5)**	**c**	**1)**	**a**	**2)**	**b**	**3)**	**d**
73)	**a**	**74)**	**c**	**75)**	**a**	**76)**	**a**		**4)**	**b**					
77)	**c**	**78)**	**b**	**79)**	**c**	**80)**	**a**	**5)**	**b**	**6)**	**b**	**7)**	**d**	**1)**	**8**
81)	**b**	**82)**	**d**	**83)**	**c**	**84)**	**b**		**2)**	**7**	**3)**	**1**	**4)**	**6**	
85)	**c**	**86)**	**a**	**87)**	**c**	**88)**	**a**	**5)**	**6**	**6)**	**7**	**7)**	**9**	**8)**	**3**
89)	**a**	**90)**	**a**	**91)**	**d**	**92)**	**d**	**9)**	**2**	**10)**	**8**				
93)	**d**	**1)**	**b,c,d**	**2)**	**b,c,d**	**3)**									

: HINTS AND SOLUTIONS :

1 **(b)**

Let $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$

$\Rightarrow (\vec{r} - \vec{b}) \times \vec{a} = \vec{0} \Rightarrow \vec{r} = \vec{b} + t\vec{a}$

Similarly, other line $\vec{r} = \vec{a} + k\vec{b}$, where t and k are scalars

Now $\vec{a} + k\vec{b} = \vec{b} + t\vec{a}$

$\Rightarrow t = 1, k = 1$

(equating the coefficients of $\vec{a}$ and $\vec{b}$)

$\therefore \vec{r} = \vec{a} + \vec{b} = \hat{\imath} + \hat{\jmath} + 2\hat{\imath} - \hat{k} = 3\hat{\imath} + \hat{\jmath} - \hat{k}$

i.e., $(3, 1, -1)$

2 **(a)**

It is obvious that the given line and plane are parallel

Given point on the line is $A(2, -2, 3)$

$B(0, 0, 5)$ is a point on the plane

$\therefore \overrightarrow{AB} = (2-0)\hat{\imath} + (-2-0)\hat{\jmath} + (3-5)\hat{k}$

Then distance of B from the plane = projection of $\overrightarrow{AB}$ on vector $\hat{\imath} + 5\hat{\jmath} + \hat{k}$

$$p = \left|\frac{(2\hat{\imath} - 2\hat{\jmath} - 2\hat{k}) \cdot (\hat{\imath} + 5\hat{\jmath} + \hat{k})}{\sqrt{1 + 25 + 1}}\right|$$

$$= \left|\frac{2 - 10 - 2}{\sqrt{27}}\right| = \frac{10}{3\sqrt{3}}$$

3 **(b)**

We have $\vec{s} - \vec{p} = \lambda\vec{n}$ and $\vec{s} \cdot \vec{n} = q$

$\Rightarrow (\lambda\vec{n} + \vec{p}) \cdot \vec{n} = q$

$$\Rightarrow \lambda = \frac{q - \vec{p} \cdot \vec{n}}{|\vec{n}|^2}$$

$$\Rightarrow \vec{s} = \vec{p} + \frac{(q - \vec{p} \cdot \vec{n})\vec{n}}{|\vec{n}|^2}$$

4 **(a)**

Direction ratios of the line joining points $P(1, 2, 3)$ and $Q(-3, 4, 5)$ are $-4, 2, 2$ which are direction ratios of the normal to the plane

Then, equation of plane is $-4x + 2y + 2z = k$

Also this plane passes through the midpoint of $PQ(-1, 3, 4)$

$\Rightarrow -4(-1) + 2(3) + 2(4) = k$

$\Rightarrow k = 18$

$\Rightarrow$ Equation of plane is $2x - y - z = -9$

Then, intercepts are $(-9/2)$,9 and 9

5 **(c)**

We must have $\vec{b} \cdot \vec{n} = 0$ and $\vec{a} \cdot \vec{n} = q$

6 **(b)**

The equation of a plane through the line of intersection of the planes $\vec{r}.\vec{a} = \lambda$ and $\vec{r}.\vec{b} = \mu$ is $(\vec{r}.\vec{a} - \lambda) + k(\vec{r}.\vec{b} - \mu) = 0$ or $\vec{r}.(\vec{a} + k\vec{b}) = \lambda + k\mu$ (i)

This passes through the origin, therefore

$$\vec{0}(\vec{a} + k\vec{b}) = \lambda + \mu k \Rightarrow k = \frac{-\lambda}{\mu}$$

Putting the value of k in (i). we get the equation of the required plane as

$\vec{r}.(\mu\vec{a} - \lambda\vec{b}) = 0 \Rightarrow \vec{r}.(\lambda\vec{b} - \mu\vec{a}) = 0$

7 **(c)**

We have $z = 0$ for the point, where the line intersects the curve

Therefore, $\frac{x-2}{3} = \frac{y+1}{2} = \frac{0-1}{-1}$

$\Rightarrow \frac{x-2}{3} = 1$ and $\frac{y+1}{2} = 1$

$\Rightarrow x = 5$ and $y = 1$

Putting these values in $xy = c^2$, we get

$5 = c^2 \Rightarrow c = \pm\sqrt{5}$

8 **(a)**

Both the lines pass through origin. Line L_1 is parallel to the vector $\vec{V}_1$

$\vec{V}_1 = (\cos\theta + \sqrt{3})\hat{\imath} + (\sqrt{2}\sin\theta)\hat{\jmath} + (\cos\theta - \sqrt{3})\hat{k}$

and L_2 is parallel to the vector $\vec{V}_2$

$\vec{V}_2 = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

$$\therefore \cos\alpha = \frac{\vec{V}_1.\vec{V}_2}{|\vec{V}_1||\vec{V}_2|}$$

$$= \frac{a(\cos\theta + \sqrt{3}) + (b\sqrt{2})\sin\theta + c(\cos\theta - \sqrt{3})}{\sqrt{a^2 + b^2 + c^2}\sqrt{(\cos\theta + \sqrt{3})^2 + 2\sin^2\theta + (\cos\theta - \sqrt{3})^2}}$$

$$= \frac{(a + c)\cos\theta + b\sqrt{2}\sin\theta + (a - c)\sqrt{3}}{\sqrt{a^2 + b^2 + c^2}\sqrt{2 + 6}}$$

For $\cos\alpha$ to be independent of θ, we get

$a + c = 0$ and $b = 0$

$$\therefore \cos\alpha = \frac{2a\sqrt{3}}{a\sqrt{2}\,2\sqrt{2}} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \alpha = \frac{\pi}{6}$$

9 **(c)**

Let $Q(\vec{q})$ be the foot of altitude drawn from $P(\vec{p})$ to the line $\vec{r} = \vec{a} + \lambda\vec{b}$,

$\Rightarrow (\vec{q} - \vec{p}) \cdot \vec{b} = 0$ and $\vec{q} = \vec{a} + \lambda\vec{b}$

$\Rightarrow (\vec{a} + \lambda\vec{b} - \vec{p}) \cdot \vec{b} = 0$

$\Rightarrow (\vec{a} - \vec{p}) \cdot \vec{b} + \lambda|\vec{b}|^2 = 0$

$$\Rightarrow \lambda = \frac{(\vec{p}-\vec{a})\cdot\vec{b}}{|\vec{b}|^2}$$

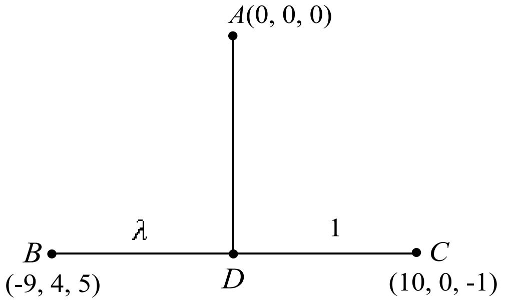
$$\Rightarrow \vec{q}-\vec{p} = \vec{a} + \frac{((\vec{p}-\vec{a})\cdot\vec{b})\vec{b}}{|\vec{b}|^2} - \vec{p}$$

$$\Rightarrow |\vec{q}-\vec{p}| = \left|(\vec{a}-\vec{p}) + \frac{((\vec{p}-\vec{a})\cdot\vec{b})\vec{b}}{|\vec{b}|^2}\right|$$

10 **(d)**

Let AD be the perpendicular and D be the foot of the perpendicular which divides BC in the ratio $\lambda : 1$, then

$$D\left(\frac{10\lambda-9}{\lambda+1}, \frac{4}{\lambda+1}, \frac{-\lambda+5}{\lambda+1}\right)$$



The direction ratios of AD are $\frac{10\lambda-9}{\lambda+1}, \frac{4}{\lambda+1}$ and $\frac{-\lambda+5}{\lambda+1}$ and direction ratios of BC are $19, -4$ and -6

Since $AD \perp BC$, we get

$$19\left(\frac{10\lambda-9}{\lambda+1}\right) - 4\left(\frac{4}{\lambda+1}\right) - 6\left(\frac{-\lambda+5}{\lambda+1}\right) = 0$$

$$\Rightarrow \lambda = \frac{31}{28}$$

Hence, on putting the value of λ in (i), we get required foot of the perpendicular, i.e.,

$\left(\frac{58}{59}, \frac{112}{59}, \frac{109}{59}\right)$

11 **(c)**

(1, 2, 3) satisfies the plane $x - 2y + z = 0$ and also $(\hat{\imath} + 2\hat{\jmath} + 3\hat{k})\cdot(\hat{\imath} - 2\hat{\jmath} + \hat{k}) = 0$

Since the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ both satisfy (0, 0, 0) and (1, 2, 3), both are same. Given line is obviously parallel to the plane $x - 2y + z = 6$

12 **(b)**

Let a point $(3\lambda+1, \lambda+2, 2\lambda+3)$ of the first line also lies on the second line

Then $\frac{3\lambda+1-3}{1} = \frac{\lambda+2-1}{2} = \frac{2\lambda+3-2}{3} \Rightarrow \lambda = 1$

Hence, the point of intersection P of the two lines (4, 3, 5)

Equation of plane perpendicular to OP, where O is (0, 0, 0) and passing through P is

$4x + 3y + 5z = 50$

13 **(a)**

Equation of the plane containing L_1, $A(x-2) + B(y-1) + C(z+1) = 0$

Where $A + 2C = 0; A + B - C = 0$

$\Rightarrow A = -2C, B = 3C, C = C$

$\Rightarrow$ Plane is $-2(x-2) + 3(y-1) + z + 1 = 0$ or $2x - 3y - z - 2 = 0$

Hence, $p = \left|\frac{-2}{\sqrt{14}}\right| = \sqrt{\frac{2}{7}}$

14 **(b)**

The lines $\vec{r} = \vec{a} + \lambda(\vec{b}\times\vec{c})$ and $\vec{r} = \vec{b} + \mu(\vec{c}\times\vec{a})$ pass through $\vec{a}$ and $\vec{b}$, respectively, and are parallel to the vectors $\vec{b}\times\vec{c}$ and $\vec{c}\times\vec{a}$, respectively. Therefore, they intersect if $\vec{a}-\vec{b}, \vec{b}\times\vec{c}$ and $\vec{c}\times\vec{a}$ are coplanar and so

$(\vec{a}-\vec{b}).\{(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})\} = 0$

$\Rightarrow (\vec{a}-\vec{b}).([\vec{b}\vec{c}\vec{a}]\vec{c} - [\vec{b}\vec{c}\vec{c}]\vec{a}) = 0$

$\Rightarrow \left((\vec{a}-\vec{b}).\vec{c}\right)[\vec{b}\vec{c}\vec{a}] = 0$

$\Rightarrow \vec{a}.\vec{c} - \vec{b}.\vec{c} = 0 \Rightarrow \vec{a}.\vec{c} = \vec{b}.\vec{c}$

15 **(a)**

Equation of the plane through $(-1,0,1)$ is

$a(x+1) + b(y-0) + c(z-1) = 0$ (i)

Which is parallel to the given line and perpendicular to the given plane

$-a + 2b + 3c = 0$ (ii)

and $a - 2b + c = 0$ (iii)

From Eqs. (ii) and (iii), we get

$c = 0, a = 2b$

From Eq., $2b(x+1) + by = 0$

$\Rightarrow 2x + y + 2 = 0$

16 **(c)**

Plane meets axes at $A(a, 0,0), B(0, b, 0)$ and $C(0, 0, c)$

Then area of ΔABC,

$= \frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$

$= \frac{1}{2}|(-a\hat{\imath} + b\hat{\jmath})\times(-a\hat{\imath} + c\hat{k})|$

$= \frac{1}{2}\sqrt{(a^2b^2 + b^2c^2 + c^2a^2)}$

17 **(a)**

$\overrightarrow{AB} = \vec{\beta} - \vec{\alpha} = -2\hat{\imath} - 3\hat{\jmath} - 6\hat{k}$

Equation of the plane passing through B and perpendicular to AB is

$(\vec{r} - \overrightarrow{OB})\cdot\overrightarrow{AB} = 0$

$\vec{r}.(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}) + 28 = 0$

Hence the required distance from $\vec{r} = -\hat{\imath} + \hat{\jmath} + \hat{k}$

$$= \left|\frac{\left(-\hat{\imath}+\hat{\jmath}+\hat{k}\right).\left(2\hat{\imath}+3\hat{\jmath}+6\hat{k}\right)+28}{|2\hat{\imath}+3\hat{\jmath}+6\hat{k}|}\right|$$

$$= \left|\frac{-2+3+6+28}{7}\right| = 5 \text{ units}$$

18 **(a)**

Distance of point P from plane=5

$\therefore 5 \left|\frac{1-4-2-\alpha}{3}\right|$

α =10

Foot perpendicular

$$\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-1}{-2} - \frac{(1-4-2-10)}{1+4+4} = \frac{5}{3}$$

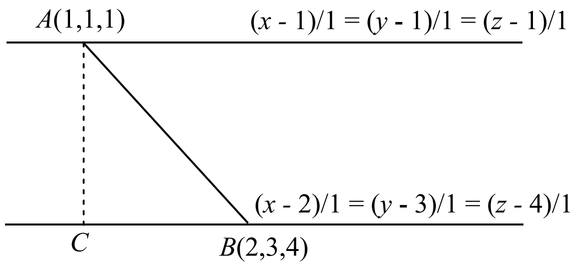
$\Longrightarrow x = \frac{8}{3}, y = \frac{4}{3}, z - \frac{7}{3}$

Thus, the foot of the perpendicular is

$A\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$

19 **(c)**

Since the given lines are parallel



From the figure, we get

$$BC = \frac{(2-1)1}{\sqrt{3}} + \frac{(3-1)1}{\sqrt{3}} + \frac{(4-1)1}{\sqrt{3}} = \frac{1+2+3}{\sqrt{3}}$$

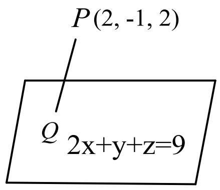
$$= 2\sqrt{3}$$

$AB = \sqrt{1+4+9} = \sqrt{14}$

Shortest distance $= AC = \sqrt{14-12} = \sqrt{2}$

20 **(c)**

Since, $l = m = n = \frac{1}{\sqrt{3}}$



$\therefore$ Equation of line is $\frac{x-2}{1/\sqrt{3}} = \frac{y+1}{1/\sqrt{3}} = \frac{z-2}{1/\sqrt{3}}$

$\Longrightarrow x-2 = y+1 = z-2 = r$ [say]

$\therefore$ Any point on the line is

$Q = (r+2, r-1, r+2)$

$\because Q$ lies on the plane $2x+y+z=9$

$\therefore 2(r+2)+(r-1)+(r+2) = 9$

$\Longrightarrow 4r+5 = 9 \Longrightarrow r = 1$

$\therefore$ Coordinate $Q(3,0,3)$

$\therefore\ PQ = \sqrt{(3-2)^2+(0+1)^2+(3-2)^2} = \sqrt{3}$

21 **(d)**

Vector perpendicular to the face OAB is

$\overrightarrow{OA} \times \overrightarrow{OB} = \left(\hat{\imath}+2\hat{\jmath}+\hat{k}\right) \times (2\hat{\imath}+\hat{\jmath}+3\hat{k})$

Vector perpendicular to face ABC is

$\overrightarrow{AB} \times \overrightarrow{AC} = \left(\hat{\imath}-\hat{\jmath}+2\hat{k}\right) \times (-2\hat{\imath}-\hat{\jmath}+3\hat{k})$

$= \hat{\imath}-5\hat{\jmath}-3\hat{k}$

Since the angle between the face = angle between their normal, therefore

$$\cos\theta = \frac{5+5+9}{\sqrt{35}\sqrt{35}} = \frac{19}{35} \Rightarrow \theta = \cos^{-1}\left(\frac{19}{35}\right)$$

22 **(a)**

$4(2)-2(3)-1(2) = 0$

Also, point $(-3,4,-5)$ does not lie on the plane

Therefore, the line is parallel to the plane

23 **(c)**

The given plane passes through $\vec{a}$ and is parallel to the vectors $\vec{b}-\vec{a}$ and $\vec{c}$. So it is normal to $(\vec{b}-\vec{a}) \times \vec{c}$. Hence, its equation is

$(\vec{r}-\vec{a}) \cdot \left((\vec{b}-\vec{a}) \times \vec{c}\right) = 0$

Or $\vec{r} \cdot \left(\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right) = [\vec{a}\vec{b}\vec{c}]$

The length of the perpendicular from the origin to this plane is

$$\frac{[\vec{a}\vec{b}\vec{c}]}{|\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}$$

24 **(c)**

Let the point be A, B, C and D

The number of planes which have three points on one side and the fourth point on the other side is 4. The number of planes which have two points on each side of the plane is 3

$\Rightarrow$ Number of planes is 7

25 **(a)**

x intercept is say x_1

$\Rightarrow$ Plane passes through it

$\therefore x_1\,\hat{\imath}\cdot\vec{n} = q \Rightarrow x_1 = \frac{q}{\hat{\imath}\cdot\vec{n}}$

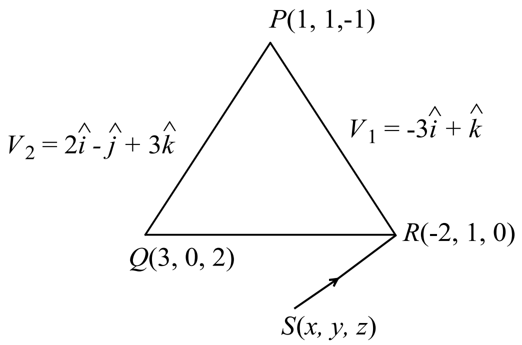
26 **(c)**

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix} = 0$$

27 **(d)**

$\vec{V}_1, \vec{V}_2, \overrightarrow{PS}$ are in the same plane

$$\therefore \left(2\hat{\imath}-\hat{\jmath}+3\hat{k}\right) \times \left(-3\hat{\imath}+\hat{k}\right) \cdot \left((x+2)\hat{\imath}+(y-1)\hat{\jmath}+z\hat{k}\right) = 0$$



28 **(b)**

The required line passes through the point $\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$ and is perpendicular to the lines $\vec{r} = (\hat{\imath} + 2\hat{\jmath} - \hat{k}) + \lambda(2\hat{\imath} + \hat{\jmath} + \hat{k})$ and $\vec{r} = (2\hat{\imath} + 6\hat{\jmath} + \hat{k}) + \mu(\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$; therefore it is parallel to the vector $\vec{b} = (2\hat{\imath} + 6\hat{\jmath} + \hat{k}) \times \mu(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) = (\hat{\imath} - 5\hat{\jmath} + 3\hat{k})$

Hence, the equation of the required line is

$\vec{r} = (\hat{\imath} + 3\hat{\jmath} + 2\hat{k}) + \lambda(\hat{\imath} - 5\hat{\jmath} + 3\hat{k})$

29 **(b)**

Any plane through (2, 2, 1) is

$a(x-2) + b(y-2) + c(z-1) = 0$ (i)

It passes through (9, 3, 6) if $7a + b + 5c = 0$ (ii)

Also (i) is perpendicular to $2x + 6y + 6z - 1 = 0$, we have

$2a + 6b + 6c = 0$

$\therefore\ a + 3b + 3c = 0$ (iii)

$\therefore\ \frac{a}{-12} = \frac{b}{-16} = \frac{c}{20}$ or $\frac{a}{3} = \frac{b}{4} = \frac{c}{-5}$ (from (ii) and (iii))

Therefore, the required plane is $3(x-2) + 4(y-2) - 5(z-1) = 0$ or $3x + 4y - 5z - 9 = 0$

30 **(d)**

The given sphere are

$x^2 + y^2 + z^2 + 7x - 2y - z - 13 = 0$ (i)

and $x^2 + y^2 + z^2 - 3x + 3y + 4z - 8 = 0$ (ii)

Subtracting (ii) from (i), we get

$10x - 5y - 5z - 5 = 0$

$\Rightarrow\ 2x - y - z = 1$

31 **(d)**

Let P be the point (1, 2, 3) and PN be the length of the perpendicular from P on the given line

Coordinates of point N are $(3\lambda + 6, 2\lambda + 7, -2\lambda + 7)$

Now PN is perpendicular to the given line or vector $3\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$

$\Rightarrow 3(3\lambda + 6 - 1) + 2(2\lambda + 7 - 2)$
$- 2(-2\lambda + 7 - 3) = 0$

$\Rightarrow\ \lambda = -1$

Then, point N is (3, 5, 9)

$\Rightarrow PN = 7$

32 **(b)**

The line is $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane is

$2x - y + \sqrt{\lambda}\, z + 4 = 0$

If θ be the angle between the line and the plane, then $90° - \theta$ is the angle between the line and normal to the plane

$$\Rightarrow \cos(90° - \theta) = \frac{(1)(2) + (2)(-1) + (2)(\sqrt{\lambda})}{\sqrt{1+4+4}\sqrt{4+1+\lambda}}$$

$$\Rightarrow \sin\theta = \frac{2 - 2 + 2\sqrt{\lambda}}{3\sqrt{5+\lambda}} \Rightarrow \frac{1}{3} = \frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}}$$

$\Rightarrow \sqrt{5+\lambda} = 2\sqrt{\lambda}$

$\Rightarrow 5 + \lambda = 4\lambda$

$\Rightarrow 3\lambda = 5$

$$\Rightarrow\ \lambda = \frac{5}{3}$$

33 **(d)**

Since line of intersection is perpendicular to both the planes, direction ratios of the line of intersection

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 1 \\ 1 & 3 & 2 \end{vmatrix} = 3\hat{\imath} - 3\hat{\jmath} + 3\hat{k}$$

Hence, $\cos\alpha = \frac{3}{\sqrt{9+9+9}} = \frac{1}{\sqrt{3}}$

34 **(a)**

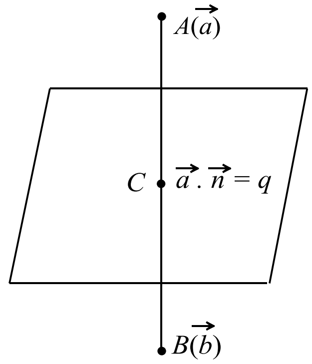
We must have $(3 + 4a - 12 + 13)(-9 - 12a + 13 < 0$

$\Rightarrow (a+1)(12a - 4) > 0$

$\Rightarrow a < -1$ or $a > 1/3$

35 **(b)**

Given plane is $\vec{r} \cdot \vec{n} = q$ (i)



Let the image of $A(\vec{a})$ in the plane be $B(\vec{b})$

Equation of AC is $\vec{r} = \vec{a} + \lambda\vec{n}$ ($\because AC$ is normal to the plane) (ii)

Solving (i) and (ii), we get

$(\vec{a} + \lambda\vec{n}).\vec{n} = q$

$$\Rightarrow\ \lambda = \frac{q - \vec{a}\cdot\vec{n}}{|\vec{n}|^2}$$

$$\therefore \overrightarrow{OC} = \vec{a} + \frac{(q - \vec{a}\cdot\vec{n})}{|\vec{n}|^2}\cdot\vec{n}$$

But $\overrightarrow{OC} = \frac{\vec{a}+\vec{b}}{2}$

$\therefore\ \vec{a} + \frac{(q - \vec{a}\cdot\vec{n})\vec{n}}{|\vec{n}|^2} = \frac{\vec{a}+\vec{b}}{2}$

$\Rightarrow \vec{b} = \vec{a} + 2\left(\frac{q - \vec{a}\cdot\vec{n}}{|\vec{n}|^2}\right)\vec{n}$

36 **(a)**

Given lines are

$\frac{x-5}{3} = \frac{y-7}{-1} = \frac{z+2}{1} = r_1$ (say)

and $\frac{x+3}{-36} = \frac{y-3}{2} = \frac{z-6}{4} = r_2$ (say)

$\therefore x = 3r_1 + 5 = -36r_2 - 3$

$y = -r_1 + 7 = 3 + 2r_2$

and $z = r_1 - 2 = 4r_2 + 6$

On solving, we get

$x = 21, y = \frac{5}{3}, z = \frac{10}{3}$

37 **(a)**

The plane is perpendicular to the line

$\frac{x-a}{\cos\theta} = \frac{y+2}{\sin\theta} = \frac{z-3}{0}$

Hence, the direction ratios of the normal of the plane are $\cos\theta$, $\sin\theta$, and 0 (i)

Now, the required plane passes through the z-axis. Hence the point (0, 0, 0) lies on the plane

From Eqs. (i) and (ii), we get equation of the plane as

$\cos\theta\,(x-0) + \sin\theta\,(y-0) + 0(z-0) = 0$

$\cos\theta\, x + \sin\theta\, y = 0$

$x + y\tan\theta = 0$

38 **(c)**

Given one vertex $A(7,2,4)$ and line $\frac{x+6}{5} = \frac{y+10}{3} = \frac{z+14}{8}$

General point on above line $B \equiv (5\lambda - 6, 3\lambda - 10, 8\lambda - 14)$

Direction ratios of line AB are $< 5\lambda - 13, 3\lambda - 12, 8\lambda - 18 >$

Direction ratios of line BC are <5, 3, 8>

Since angle between AB nad BC is $\pi/4$

$$\cos\frac{\pi}{4} = \frac{(5\lambda-3)5 + 3(3\lambda-12) + 8(8\lambda-18)}{\sqrt{5^2+3^2+8^2}\cdot\sqrt{(5\lambda-13)^2+(3\lambda-12)^2+(8\lambda-18)^2}}$$

Squaring and solving, we have $\lambda = 3, 2$

Hence equation of lines are $\frac{x-7}{2} = \frac{y-2}{-3} = \frac{z-4}{6}$ and $\frac{x-7}{3} = \frac{y-2}{6} = \frac{z-4}{2}$

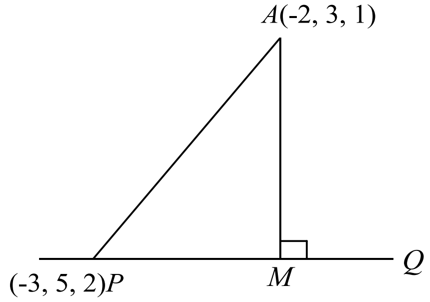
39 **(b)**

Here, $\alpha = \beta = \gamma$

$\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\therefore\ \cos\alpha = \frac{1}{\sqrt{3}}$

DC's of PQ are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$



PM =Projection of AP on PQ

$= \left|(-2+3)\frac{1}{\sqrt{3}} + (3-5)\frac{1}{\sqrt{3}} + (1-2)\frac{1}{\sqrt{3}}\right| = \frac{2}{\sqrt{3}}$

and $AP = \sqrt{(-2+3)^2 + (3-5)^2 + (1-2)^2} = \sqrt{6}$

$AM = \sqrt{(AP)^2 - (PM)^2} = \sqrt{6 - \frac{4}{3}} = \sqrt{\frac{14}{3}}$

40 **(b)**

Coordinates of L and M are $(0, b, c)$ and $(a, 0, c)$, respectively. Therefore, the equation of the plane passing through (0, 0, 0), $(0, b, c)$ and $(a, 0, c)$ is

$\begin{vmatrix} x-0 & y-0 & z-0 \\ 0 & b & c \\ a & 0 & c \end{vmatrix} = 0$ or $\frac{x}{a} + \frac{y}{b} - \frac{z}{c} = 0$

41 **(b)**

Given, $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = \lambda$

and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1} = \mu$

$\Longrightarrow x = 2\lambda + 1, y = 3\lambda - 1, z = 4\lambda + 1$

and $x = \mu + 3, y = 2\mu + k, z = \mu$

As the lines intersect they must have a point in common.

$\therefore 2\lambda + 1 = \mu + 3, 3\lambda - 1 = 2\mu + k, 4\lambda + 1 = \mu$

$\Longrightarrow \lambda = -\frac{3}{2}$ and $\mu = -5$

$\therefore k = 3\lambda - 2\mu - 1$

$\Longrightarrow k = 3\left(-\frac{3}{2}\right) - 2(-5) - 1$

$\Longrightarrow k = \frac{9}{2}$

42 **(b)**

$x^2 - 5x + 6 = 0$

$\Rightarrow x - 2 = 0, x - 3 = 0$

Which represents planes

43 **(b)**

Any plane through (1, 0, 0) is $a(x-1) + by + cz = 0$ (i)

It passes through (0, 1, 0)

$\therefore\ a(0-1) + b(1) + c(0) = 0\ \Rightarrow -a + b = 0$

(ii)

(i) makes an angle of $\frac{\pi}{4}$ with $x+y=3$, therefore

$$\cos\frac{\pi}{4}=\frac{a(1)+b(1)+c(0)}{\sqrt{a^2+b^2+c^2}\sqrt{1+1+0}}$$

$$\Rightarrow\ \frac{1}{\sqrt{2}}=\frac{a+b}{\sqrt{2}\sqrt{a^2+b^2+c^2}}$$

$$\Rightarrow a+b=\sqrt{a^2+b^2+c^2}$$

Squaring, we get

$$a^2+b^2+2ab=a^2+b^2+c^2$$

$$\Rightarrow 2ab=c^2\ \Rightarrow 2a^2=c^2$$

$$\Rightarrow c=\sqrt{2}\,a \qquad \text{(using (ii))}$$

Hence, $a:b:c=a:a:\sqrt{2}a$

$$=1:1:\sqrt{2}$$

44 **(a)**

Vector $\left((3\hat{\imath}-2\hat{\jmath}+\hat{k})\times(4\hat{\imath}-3\hat{\jmath}+4\hat{k})\right)$ is perpendicular to $2\hat{\imath}-\hat{\jmath}+m\hat{k}$

$$\Rightarrow \begin{vmatrix}3 & -2 & 1\\ 4 & -3 & 4\\ 2 & -1 & m\end{vmatrix}=0\ \Rightarrow m=-2$$

45 **(a)**

Let the foot of the perpendicular from the origin on the given plane be $P(\alpha,\beta,\gamma)$. Since the plane passes through $A(a,b,c)$

$AP\perp OP\ \Rightarrow\ \overrightarrow{AP}.\overrightarrow{OP}=0$

$$\Rightarrow [(\alpha-a)\hat{\imath}+(\beta-b)\hat{\jmath}+(\gamma-c)\hat{k}].(\alpha\hat{\imath}+\beta\hat{\jmath}+\gamma\hat{k})=0$$

$\Rightarrow\ \alpha(\alpha-a)+\beta(\beta-b)+\gamma(\gamma-c)=0$

Hence, the locus of (α,β,γ) is

$x(x-a)+y(y-b)+z(z-c)=0$

$x^2+y^2+z^2-ax-by-cz=0$

Which is a sphere of radius $\frac{1}{2}\sqrt{a^2+b^2+c^2}$

46 **(c)**

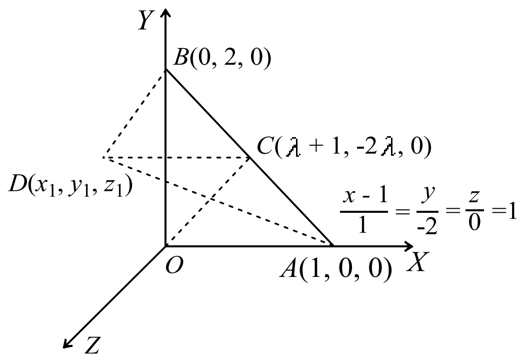
We must have $\vec{b}\cdot\vec{n}=0$ (because the line and the plane must be parallel) and $\vec{a}\cdot\vec{n}\neq q$ (as point $\vec{a}$ on the line should not lie on the plane)

47 **(b)**

$$1=\left|(\vec{b}-\vec{a})\cdot\frac{(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|$$

$\Rightarrow\ |\vec{b}-\vec{a}|\cos 60°=1\ \Rightarrow\ AB=2$

48 **(c)**



Equation of a line AB is $\frac{x-1}{1}=\frac{y}{-2}=\frac{z}{0}=\lambda$

Now $AB\perp OC\Rightarrow\ 1(\lambda+1)+(-2\lambda)(-2)=0\ \Rightarrow$ $5\lambda=-1\ \Rightarrow\ \lambda=-\frac{1}{5}$

$\Rightarrow C$ is $\left(\frac{4}{5},\frac{2}{5},0\right)$. Now

$x_1^2+(y_1-2)^2+z_1^2=4\quad$ (i)

and $(x_1-1)^2+y_1^2+z_1^2=1\quad$ (ii)

Now $OC\perp CD$

$\Rightarrow\ \left(x_1-\frac{4}{5}\right)\frac{4}{5}+\left(y_1-\frac{2}{5}\right)\frac{2}{5}+(z_1-0)0=0\quad$ (iii)

From (i) and (ii), we get

$-4y_1+2x_1=0\ \Rightarrow\ x_1=2y_1$

From (iii), putting $x_1=2y_1\Rightarrow 2y_1=\frac{4}{5}\Rightarrow y_1=\frac{2}{5}\Rightarrow x_1=\frac{4}{5}$. Putting this value of x_1 and y_1 in (i), we get

$$z_1=\pm\frac{2}{\sqrt{5}}$$

49 **(a)**

$A(1,1,1),B(2,3,5),C(-1,0,2)$ direction ratios of AB are $<1,2,4>$

Direction ratios of AC are $<-2,-1,1>$

Therefore, direction ratios of normal to plane ABC are $<2,-3,1>$

As a result, equation of the plane ABC is

$2x-3y+z=0$

Let the equation of the required plane is

$2x-3y+z=k$, then $\left|\frac{k}{\sqrt{4+9+1}}\right|=2$

$k=\pm 2\sqrt{14}$

Hence, equation of the required plane is

$2x-3y+z+2\sqrt{14}=0$

50 **(a)**

Since line is parallel to the plane vector,

$2\vec{\imath}+3\vec{\jmath}+\lambda\vec{k}$ is perpendicular to the normal to the plane

$2\vec{\imath}+3\vec{\jmath}+4\vec{k}$

$\Rightarrow 2\times2+3\times3+4\lambda=0$

$$\Rightarrow\ \lambda=-\frac{13}{4}$$

51 **(d)**

Given lines are $\vec{r}=3\hat{\imath}+8\hat{\jmath}+3\hat{k}+l(3\hat{\imath}-\hat{\jmath}+\hat{k})$

and $\vec{r} = -3\hat{\imath} - 7\hat{\jmath} + 6\hat{k} + m(-3\hat{\imath} + 2\hat{\jmath} + 4\hat{k})$
Required shortest distance

$$= \frac{|(6\hat{\imath} + 15\hat{\jmath} - 3\hat{k}).((3\hat{\imath} - \hat{\jmath} + \hat{k}) \times (-3\hat{\imath} + 2\hat{\jmath} + 4\hat{k}))|}{|(3\hat{\imath} - \hat{\jmath} + \hat{k}) \times (-3\hat{\imath} + 2\hat{\jmath} + 4\hat{k})|}$$

$$= \frac{|(6\hat{\imath} + 15\hat{\jmath} - 3\hat{k}).(-6\hat{\imath} - 15\hat{\jmath} + 3\hat{k})|}{|(-6\hat{\imath} - 15\hat{\jmath} + 3\hat{k})|}$$

$$= \frac{36 + 225 + 9}{\sqrt{36 + 225 + 9}} = \frac{270}{\sqrt{270}} = \sqrt{270} = 3\sqrt{30}$$

52 **(c)**
Here $l = \cos\frac{\pi}{4}, m = \cos\frac{\pi}{4}$
Let the line make an angle 'γ' with z-axis
$\therefore l^2 + m^2 + n^2 = 1$
$\Rightarrow \cos^2\frac{\pi}{4} + \cos^2\frac{\pi}{4} + \cos^2\gamma = 1$
$\Rightarrow \frac{1}{2} + \frac{1}{2} + \cos^2\gamma = 1$
$\Rightarrow 2\cos^2\gamma = 0 \Rightarrow \cos\gamma = 0 \Rightarrow \gamma = \frac{\pi}{2}$

53 **(d)**
Let $A(1, 0, -1), B(-1; 2, 2)$
Direction ratios of segment AB are $< 2, -2, -3 >$

$$\cos\theta = \frac{|2 \times 1 + 3(-2) - 5(-3)|}{\sqrt{1 + 9 + 25}\sqrt{4 + 4 + 9}} = \frac{11}{\sqrt{17}\sqrt{35}} = \frac{11}{\sqrt{595}}$$

Length of projection $= (AB)\sin\theta$

$$= \sqrt{(2)^2 + (2)^2 + (3)^2} \times \sqrt{1 - \frac{121}{595}}$$

$$= \sqrt{17}\frac{\sqrt{474}}{\sqrt{17}\sqrt{35}} = \sqrt{\frac{474}{35}} \text{ units}$$

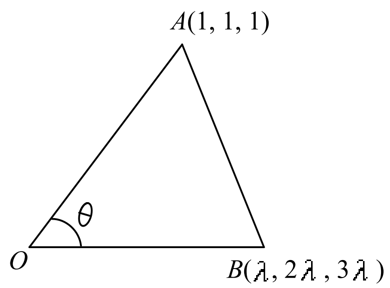
54 **(c)**
Let $Q(\vec{q})$ be the foot of altitude drawn from 'P' to the plane $\vec{r} \cdot \vec{n} = 0$
$\Rightarrow \vec{q} - \vec{p} = \lambda\vec{n} \Rightarrow \vec{q} = \vec{p} + \lambda\vec{n}$
Also $\vec{q} \cdot \vec{n} = 0 \Rightarrow (\vec{p} + \lambda\vec{n}) \cdot \vec{n} = 0$
$\Rightarrow \lambda = -\frac{\vec{p} \cdot \vec{n}}{|\vec{n}|^2} \Rightarrow \vec{q} - \vec{p} = -\frac{(\vec{p} \cdot \vec{n})}{|\vec{n}|^2}\vec{n}$
Thus, required distance $= |\vec{q} - \vec{p}| = \frac{|\vec{p} \cdot \vec{n}|}{|\vec{n}|} = |\vec{p} \cdot \hat{n}|$

55 **(b)**
Plane passing through the line of intersection if planes $4y + 6z = 5$ and $2x + 3y + 5z = 5$ is
$(4y + 6z - 5) + \lambda(2x + 3y + 5z - 5) = 0$, or
$2\lambda x + (3\lambda + 4)y + (5\lambda + 6)z - 5\lambda - 5 = 0$
Clearly, for $\lambda = -3$, we get the plane $6x + 5y + 9z = 10$
Hence, the given three planes have common line of intersection

56 **(c)**
Equation of plane containing the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ is
$a(x - 0) + b(y - 0) + c(z - 0) = 0$ ...(i)
and $2a + 3b + 4c = 0$ ....(ii)
Another equation of the plane containing the other two lines is
$a_1(x - 0) + b_1(y - 0) + c_1(z - 0) = 0$ ....(iii)
Also, $3a_1 + 4b_1 + 2c_1 = 0$
and $4a_1 + 2b_1 + 3c_1 = 0$
on solving we get
$\frac{a_1}{8} = \frac{b_1}{-1} = \frac{c}{-10}$
$\therefore$ Eq. (iii) becomes
$8x - y - 10c = 0$ ...(iv)
Since, the plane (i) is perpendicular to the plane (ii)
$\therefore$ $8a - b - 10c = 0$ ...(v)
On solving Eqs. (ii) and (v), we get
$\frac{a}{-26} = \frac{b}{52} = \frac{c}{-26}$ or $\frac{a}{1} = \frac{b}{-2} = \frac{c}{1}$
$\therefore$ From Eq. (i)
$x - 2y + z = 0$
Alternate
Let $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$,
$\vec{\mathbf{b}} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\vec{\mathbf{c}} = 4\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$
$\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = (\vec{\mathbf{a}} \cdot \vec{\mathbf{c}})\vec{\mathbf{b}} - (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{c}}$
$= 26(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}})$
$\Longrightarrow$ Direction ratio of normal to the required plane (passing through origin) is $1, -2, 1$
$\Longrightarrow$ Equation of required plane is $x - 2y + z = 0$

57 **(a)**
Since, line lies in a plane, it means point $(4,2,k)$ lies in a plane.
$\therefore 8 - 8 + k = 7$
$\Longrightarrow k = 7$

58 **(b)**



Let any point on second line be $(\lambda, 2\lambda, 3\lambda)$
$\cos\theta = \frac{6}{\sqrt{42}}, \sin\theta = \frac{\sqrt{6}}{\sqrt{42}}$
$\Delta_{OAB} = \frac{1}{2}(OA)OB\sin\theta = \frac{1}{2}\sqrt{3}\,\lambda\sqrt{14} \times \frac{\sqrt{6}}{\sqrt{42}} = \sqrt{6}$
$\Rightarrow \lambda = 2$

So B is $(2, 4, 6)$

59 **(b)**

Direction cosines of the given line are $\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}$

Hence, the equation of line can be point in the form $\frac{x-2}{1/3} = \frac{y+3}{-2/3} = \frac{z+5}{-2/3} = r$

Therefore, any point on the line is $\left(2 + \frac{r}{3}, -3 -\right.$ *2r3,−5−2r3*, where *r=±6*

Points are $(4, -7, -9)$ and $(0, 1, -1)$

60 **(b)**

Let the equation of the sphere be $x^2 + y^2 + z^2 - ax - by - cz = 0$. This meets the axes at $A(a, 0, 0), B(0, b, 0)$ and $C(0, 0, c)$

Let (α, β, γ) be the coordinares of the centroid of the tetrahedron $OABC$. Then

$\frac{a}{4} = \alpha, \frac{b}{4} = \beta, \frac{c}{4} = \gamma$

$\Rightarrow a = 4\alpha, b = 4\beta, c = 4\gamma$

Now, radius of the sphere $= 2k$

$\Rightarrow \frac{1}{2}\sqrt{a^2 + b^2 + c^2} = 2k \ \Rightarrow a^2 + b^2 + c^2 = 16k^2$

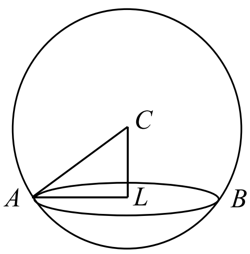
$\Rightarrow 16(\alpha^2 + \beta^2 + \gamma^2) = 16k^2$

Hence, the locus of (α, β, γ) is $(x^2 + y^2 + z^2) = k^2$

61 **(b)**

Centre of the sphere is $(-1, 1, 2)$ and its radius $= \sqrt{1 + 1 + 4 + 19} = 5$

CL, perpendicular distance of C from plane, is $\left|\frac{-1+2+4+7}{\sqrt{1+4+4}}\right| = 4$



Now $AL^2 = CA^2 - CL^2 = 25 - 16 = 9$

Hence, radius of the circle $= \sqrt{9} = 3$

62 **(d)**

Let $P(\alpha, \beta, \gamma)$ be the image of the point $Q(-1, 3, 4)$

Midpoint of PQ lies on $x - 2y = 0$. Then,

$\frac{\alpha - 1}{2} - 2\left(\frac{\beta + 3}{2}\right) = 0$

$\Rightarrow \ \alpha - 1 - 2\beta - 6 = 0 \ \Rightarrow \ \alpha - 2\beta = 7$ (i)

Also PQ is perpendicular to the plane. Then,

$\frac{\alpha+1}{1} = \frac{\beta-3}{-2} = \frac{\gamma-4}{0}$ (ii)

Solving (i) and (ii), we get

$\alpha = \frac{9}{5}, \beta = -\frac{13}{5}, \gamma = 4$

Therefore, image is

$\left(\frac{9}{5}, -\frac{13}{5}, 4\right)$

Alternative method:

For image,

$\frac{\alpha - (-1)}{1} = \frac{\beta - 3}{-2} = \frac{\gamma - 4}{0} = \frac{-2(-1 - 2(3))}{(1)^2 + (-2)^2}$

$\Rightarrow \ \alpha = \frac{9}{5}, \beta = -\frac{13}{5}, \gamma = 4$

63 **(c)**

Direction ratios of OP are (a, b, c)

Therefore, equation of the plane is

$a(x - a) + b(y - b) + c(z - c) = 0$

i.e., $xa + yb + zc = a^2 + b^2 + c^2$

64 **(d)**

Here, the required plane is

$a(x - 4) + b(y - 3) + c(z - 2) = 0$

Also $a + b + 2c = 0$ and $a - 4b + 5c = 0$

Solving, we have

$\frac{a}{5 + 8} = \frac{b}{2 - 5} = \frac{c}{-4 - 1} = k$

$\frac{a}{13} = \frac{b}{-3} = \frac{c}{-5} = k$

Therefore, the required equation of plane is

$-13x + 3y + 5z + 33 = 0$

65 **(a)**

Foot of the perpendicular drawn from point $A(\vec{a})$ on the plane $\vec{r}.\vec{n} = d$ is $\vec{a} + \left(\frac{d - \vec{a}.\vec{n}}{|\vec{n}|^2}\right)\vec{n}$

Therefore, equation of the line parallel to $\vec{r} = \vec{a} + \lambda\vec{b}$ in the plane $\vec{r}.\vec{n} = d$ is given by

$\vec{r} = \vec{a} + \left(\frac{d - \vec{a}.\vec{n}}{|\vec{n}|^2}\right)\vec{n} + \lambda\vec{b}$

66 **(d)**

Let the equation of plane be,

$a(x - 1) + b(y + 2) + c(z - 1) = 0$

Which is perpendicular to $2x - 2y + z = 0$ and $x - y + 2z = 4$

$\therefore 2a - 2b + c = 0$ and $a - b + 2c = 0$

$\Rightarrow \frac{a}{-3} = \frac{b}{-3} = \frac{c}{0}$

$\Rightarrow \frac{a}{1} = \frac{b}{1} = \frac{c}{0}$

$\therefore$ The equation of plane is,

$1(x - 1) + 1(y + 2) + 0(z - 1) = 0$

$\Rightarrow x + y + 1$

$= 0$, its distance from the point $(1, 2, 2)$ is $\frac{|1 + 2 +}{\sqrt{2}}$

$= 2\sqrt{2}$

67 **(a)**

The given line makes angles of $\pi/4, \pi/4,$ and $\pi/2$ with the x-, y- and z-axes, respectively,

⇒ Direction cosines of the given line are $\cos(\pi/4)$, $\cos(\pi/4)$ and $\cos(\pi/2)$, or $(1/\sqrt{2}), (1/\sqrt{2})$ and 0

68 **(c)**

$\hat{a} = \pm \frac{\vec{n}_1 \times \vec{n}_2}{|\vec{n}_1 \times \vec{n}_2|} = \pm \frac{2\hat{\imath}+5\hat{\jmath}+3\hat{k}}{\sqrt{38}}$ (where $\vec{n}_1$ and $\vec{n}_2$ are normal to the planes)

69 **(c)**

Plane meets axes at $A(2,0,0)$, $B(0,3,0)$ and $C(0,0,6)$

Then area of ΔABC is

$\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

$= \frac{1}{2}|(-2\hat{\imath}+3\hat{\jmath}) \times (-2\hat{\imath}+6\hat{\jmath})|$

$= 3\sqrt{14}$ sq units

70 **(b)**

Let P be the point and it divides the line segment in the ratio $\lambda: 1$. Then,

$\overrightarrow{OP} = \vec{r} = \frac{-3\lambda+2}{\lambda+1}\hat{\imath} + \frac{5\lambda-4}{\lambda+1}\hat{\jmath} + \frac{-8\lambda-7}{\lambda+1}\hat{k}$

It satisfies $\vec{r}.(\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) = 13$. So,

$\frac{-3\lambda+2}{\lambda+1} - 2\frac{5\lambda-4}{\lambda+1} + 3\frac{-8\lambda-7}{\lambda+1} = 13$

or $-3\lambda + 2 - 2(5\lambda - 4) + 3(-8\lambda - 7) = 13(\lambda + 1)$

or $-37\lambda - 11 = 13\lambda + 13$ or $50\lambda = -24$ or $\lambda = -\frac{12}{25}$

71 **(b)**

Let the equation of the plane be $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$

⇒ Volume of tetrahedron $OABC = V = \frac{1}{6}(abc)$

Now $(abc)^{1/3} \geq \frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}} \geq 3$ (G.M.≥H.M.)

$\Rightarrow abc \geq 27 \Rightarrow V \geq \frac{9}{2}$

72 **(b)**

Eliminating n, we get

$\lambda(l+m)^2 + lm = 0$

$\Rightarrow \frac{\lambda l^2}{m^2} + (2\lambda+1)\frac{l}{m} + \lambda = 0$

$\Rightarrow \frac{l_1 l_2}{m_1 m_2} = 1$ (product of roots $\frac{l_1}{m_1}$ and $\frac{l_2}{m_2}$)

Where l_1/m_1 and l_2/m_2 are the roots of this equation, further eliminating m, we get

$\lambda l^2 - ln - n^2 = 0$

$\Rightarrow \frac{l_1 l_2}{n_1 n_2} = -\frac{1}{\lambda}$

Since the lines with direction cosines (l_1, m_1, n_1) and (l_2, m_2, n_2) are perpendicular, we have

$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

$\Rightarrow 1 + 1 - \lambda = 0$

$\Rightarrow \lambda = 2$

73 **(a)**

The equation of the plane through the line of intersection of the plane $4x + 7y + 4z + 81 = 0$ and $5x + 3y + 10z = 25$ is $(4x + 7y + 4z + 81+\lambda 5x+3y+10z-25=0$

$\Rightarrow (4+5\lambda)x + (7+3\lambda)y + (4+10\lambda)z + 81 - 25\lambda = 0$ (i)

Which is perpendicular to $4x + 7y + 4z + 81 = 0$

$\Rightarrow 4(4+5\lambda) + 7(7+3\lambda) + 4(4+10\lambda) = 0$

$\Rightarrow 81\lambda + 81 = 0$

$\Rightarrow \lambda = -1$

Hence the place is $x - 4y + 6z = 106$

74 **(c)**

Here $\sin^2\beta = 3\sin^2\theta$ (i)

By the question, $\cos^2\theta + \cos^2\theta + \cos^2\beta = 1$ (ii)

$\Rightarrow \cos^2\beta = 1 - 2\cos^2\theta$ (iii)

Adding (i) and (iii), we get

$1 = 1 + 3\sin^2\theta - 2\cos^2\theta$

$\Rightarrow 1 = 1 + 3(1 - \cos^2\theta) - 2\cos^2\theta$

$\Rightarrow 5\cos^2\theta = 3$

$\Rightarrow \cos^2\theta = \frac{3}{5}$

75 **(a)**

$\vec{r}.\vec{n}_1 + \lambda\vec{r}.\vec{n}_2 = q_1 + \lambda q_2$ (i)

Where λ is a parameter

So, $\vec{n}_1 + \lambda\vec{n}_2$ is normal to plane (i). Now, any plane parallel to the line of intersection of the planes $\vec{r}.\vec{n}_3 = q_3$ and $\vec{r}.\vec{n}_4 = q_4$ is of form $\vec{r}.(\vec{n}_3 \times \vec{n}_4) = d$. Hence we must have

$[\vec{n}_1 + \lambda\vec{n}_2].[\vec{n}_3 \times \vec{n}_4] = 0$

$\Rightarrow [\vec{n}_1\vec{n}_3\vec{n}_4] + \lambda[\vec{n}_2\vec{n}_3\vec{n}_4] = 0$

$\Rightarrow \lambda = \frac{-[\vec{n}_1\vec{n}_3\vec{n}_4]}{[\vec{n}_2\vec{n}_3\vec{n}_4]}$

⇒ On putting this value in Eq. (i), we have the equation of the required plane as

$\vec{r}.\vec{n}_1 - q_1 = \frac{[\vec{n}_1\vec{n}_3\vec{n}_4]}{[\vec{n}_2\vec{n}_3\vec{n}_4]}(r.\vec{n}_2 - q_2)$

$\Rightarrow [\vec{n}_2\vec{n}_3\vec{n}_4](\vec{r}.\vec{n}_1 - q_1) = [\vec{n}_1\vec{n}_3\vec{n}_4](\vec{r}.\vec{n}_2 - q_2)$

76 **(a)**

Point A is (a,b,c) ⇒ Points P, Q, R are $(a,b,-c)$, $(-a,b,c)$ and $(a,-b,c)$, respectively

⇒ Centroid of triangle PQR is $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right) \Rightarrow G \equiv \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

⇒ A, O, G are collinear ⇒ area of triangle AOG is zero

77 **(c)**

Equating of the planes through $y = mx, z = c$ and $y = -mx, z = -c$ are respectively,
$(y - mx) + \lambda_1(z - c) = 0$ (i)
and $(y + mx) + \lambda_2(z + c) = 0$ (ii)
It meets at x-axis, i.e., $y = 0 = z$
$\therefore \lambda_2 = \lambda_1$
From (i) and (ii), $\frac{y-mx}{z-c} = \frac{y+mx}{z+c}$
$\therefore cy = mzx$

78 **(b)**
Let direction ratios of the line be (a, b, c), then $2a - b + c = 0$ and $a - b - 2c = 0$ i.e, $\frac{a}{3} = \frac{b}{5} = \frac{c}{-1}$
Therefore, direction ratios of the line are $(3, 5, -1)$
Any point on the given line is $(2 + \lambda, 2 - \lambda, 3 - 2\lambda)$, it lies on the given plane π if
$2(2 + \lambda) - (2 - \lambda) + (3 - 2\lambda) = 4$
$\Rightarrow 4 + 2\lambda - 2 + \lambda + 3 - 2\lambda = 4 \Rightarrow \lambda = -1$
Therefore, the point of intersection of the line and the plane is (1,3, 5)
Therefore, equation of the required line is
$$\frac{x-1}{3} = \frac{y-3}{5} = \frac{z-5}{-1}$$

79 **(c)**
Given plane is $\vec{r} = (1 + \lambda - \mu)\hat{\imath} + (2 - \lambda)\hat{\jmath} + (3 - 2\lambda + 2\mu)\hat{k}$
$$\Rightarrow \vec{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(\hat{\imath} - \hat{\jmath} - 2\hat{k}) + \mu(-\hat{\imath} + 2\hat{k})$$
Which is a plane passing through $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and parallel to the vectors $\vec{b} = \hat{\imath} - \hat{\jmath} - 2\hat{k}$ and $\vec{c} = -\hat{\imath} + 2\hat{k}$
Therefore, it is perpendicular to the vector
$\vec{n} = \vec{b} \times \vec{c} = -2\hat{\imath} - \hat{k}$
Hence, equation of plane is $-2(x - 1) + (0)(y - 2) - (z - 3) = 0$ or $2x + z = 5$

80 **(a)**
Let the point P be (x, y, z), then the vector $(x\hat{\imath} + y\hat{\jmath} + z\hat{k})$ will lie on the line
$$\Rightarrow (x - 1)\hat{\imath} + (y - 1)\hat{\jmath} + (z - 1)\hat{k} = -\lambda\hat{\imath} + \lambda\hat{\jmath} - \lambda\hat{k}$$
$\Rightarrow x = 1 - \lambda, y = 1 + \lambda$ and $z = 1 - \lambda$
Now point P is nearest to the origin $\Rightarrow D = (1 - \lambda)^2 + (1 + \lambda)^2 + (1 - \lambda)^2$
$$\Rightarrow \frac{dD}{d} = -4(1 - \lambda) + 2(1 + \lambda) = 0 \Rightarrow \lambda = \frac{1}{3}$$
$\Rightarrow$ The point is $\left(\frac{2}{3}, \frac{4}{3}, \frac{2}{3}\right)$

81 **(b)**
The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ (i)
and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ (ii)
are coplanar if $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_1 & n_2 \end{vmatrix} = 0$
or $\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$
$\Rightarrow k^2 + 3k = 0$
$\Rightarrow k = 0$ or -3

82 **(d)**
$P_1 = P_2 = 0, P_2 = P_3 = 0$ and $P_3 = P_1 = 0$ are lines of intersection of the three planes P_1, P_2 and P_3. As $\vec{n}_1, \vec{n}_2$ and $\vec{n}_3$ are non-coplanar, planes P_1, P_2 and P_3 will intersect at unique point. So the given liens will pass through a fixed point

83 **(c)**
$3l + m + 5n = 0$ (i)
$6mn - 2nl + 5ml = 0$ (ii)
Substituting the value of n from Eq. (i) in Eq. (ii), we get
$6l^2 + 9lm - 6m^2 = 0$
$$\Rightarrow 6\left(\frac{l}{m}\right)^2 + 9\left(\frac{l}{m}\right) - 6 = 0$$
$\therefore \frac{l_1}{m_1} = \frac{1}{2}$ and $\frac{l_2}{m_2} = -2$
From Eq. (i), we get
$\frac{l_1}{n_1} = -1$ and $\frac{l_2}{n_2} = -2$
$$\therefore \frac{l_1}{1} = \frac{m_1}{2} = \frac{n_1}{-1} = \sqrt{\frac{l_1^2 + m_1^2 + n_1^2}{1 + 4 + 1}} = \frac{1}{\sqrt{6}}$$
and $\frac{l_2}{2} = \frac{m_2}{-1} = \frac{n_2}{-1} = \frac{\sqrt{l_2^2+m_2^2+n_2^2}}{\sqrt{4+1+1}} = \frac{1}{\sqrt{6}}$
If θ be the angle between the lines, then
$$\cos\theta = \left(\frac{1}{\sqrt{6}}\right)\left(\frac{2}{\sqrt{6}}\right) + \left(\frac{2}{\sqrt{6}}\right)\left(-\frac{1}{\sqrt{6}}\right) + \left(-\frac{1}{\sqrt{6}}\right)\left(-\frac{1}{\sqrt{6}}\right) = \frac{1}{6}$$
$$\therefore \theta = \cos^{-1}\left(\frac{1}{6}\right)$$

84 **(b)**
The equation of the line through the centre $\hat{\jmath} + 2\hat{k}$ and normal to the given plane is
$\vec{r} = \hat{\jmath} + 2\hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$ (i)
This meets the plane for which
$[\hat{\jmath} + 2\hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})] \cdot (\hat{\imath} + 2\hat{\jmath} + 2\hat{k}) = 15$
$\Rightarrow 6 + 9\lambda = 15 \Rightarrow \lambda = 1$
Putting in (i), we get
$\vec{r} = \hat{\jmath} + 2\hat{k} + (\hat{\imath} + 2\hat{\jmath} + 2\hat{k}) = \hat{\imath} + 3\hat{\jmath} + 4\hat{k}$
Hence, centre is (1, 3, 4)

85 **(c)**
The planes are $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and $\frac{x}{a'} + \frac{y}{b'} + \frac{z}{c'} = 1$
Since the perpendicular distance of the origin on

the planes is same, therefore

$$\left|\frac{-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}\right| = \left|\frac{-1}{\sqrt{\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}}}\right|$$

$\Rightarrow \dfrac{1}{a^2}+\dfrac{1}{b^2}+\dfrac{1}{c^2}-\dfrac{1}{a'^2}-\dfrac{1}{b'^2}-\dfrac{1}{c'^2}=0$

86 **(a)**

Any plane through the given planes is

$x+2y+3z-4+\lambda(4x+3y+2z+1)=0$

It passes through $(0,0,0)$. Therefore,

$-4+\lambda=0$

$\therefore\ \lambda=4$

Therefore, the required plane is $x+2y+3z+4(4x+3y+2z)=0$ or $17x+14y+11z=0$

87 **(c)**

The equation of a plane through the line of intersection of the planes $ax+by+cz+d=0$ and $a'x+b'y+c'z+d'=0$ is

$(ax+by+cz+d)+\lambda(a'x+b'y+c'z+d')=0$

or $x(a+\lambda a')+y(b+\lambda b')+z(c+\lambda c')+d+\lambda d'=0$ (i)

This is parallel to x-axis, i.e., $y=0, z=0$. Therefore,

$1(a+\lambda a')+0(b+\lambda b')+0(c+\lambda c')=0$

$\Rightarrow \lambda=-\dfrac{a}{a'}$

Putting the value of λ in (i), the required plane is $y(a'b-ab')+z(ac'-a'c)+a'd-ad'=0$ or $(ab'-a'b)y+(ac'-a'c)z+ad'-a'd=0$

88 **(a)**

As $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ cuts the coordinate axes at $A(a,0,0), B(0,b,0), C(0,0,c)$

Since, distance from origin $=1$

$\Rightarrow \dfrac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}=1$

$\Rightarrow \dfrac{1}{a^2}+\dfrac{1}{b^2}+\dfrac{1}{c^2}=1$ …..(i)

$\therefore$ Centroid $P(x,y,z)$

$=\left(\dfrac{a+0+0}{3},\dfrac{0+b+0}{3},\dfrac{0+0+c}{3}\right)$

$\Rightarrow x=\dfrac{a}{3}, y=\dfrac{b}{3}, z=\dfrac{c}{3}$ ..(ii)

From Eqs. (i) and (ii),

$\dfrac{1}{9x^2}+\dfrac{1}{9y^2}+\dfrac{1}{9z^2}=1$

$\Rightarrow \dfrac{1}{x^2}+\dfrac{1}{y^2}+\dfrac{1}{z^2}=9=\mathrm{k}$ (given)

$\Rightarrow k=9$

89 **(a)**

The required plane is $\begin{vmatrix} x-3 & y-6 & z-4 \\ 3-3 & 2-6 & 0-4 \\ 1 & 5 & 4 \end{vmatrix}=0$

$\Rightarrow \begin{vmatrix} x-3 & y-z-2 & z-4 \\ 0 & 0 & -4 \\ 1 & 1 & 4 \end{vmatrix}=0$ (Operating $C_2\to C_2-C_3$)

$\Rightarrow 4(x-3-y+z+2)=0$

$\Rightarrow x-y+z=1$

90 **(a)**

Equation of line $x+2y+z-1+\lambda(-x+y-2z-2=0$ (i)

$x+y-2+\mu(x+z-2)=0$ (ii)

$(0,0,1)$ lies on it $\Rightarrow \lambda=0, \mu=-2$

For point of intersection, $z=0$ and solve (i) and (ii)

91 **(d)**

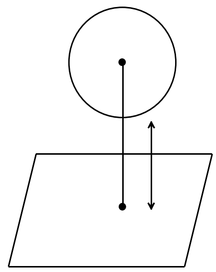
The given sphere is

$x^2+y^2+z^2+4x-2y-6z-155=0$

Its centre is $(-2,1,3)$ and radius

$=\sqrt{4+1+9+155}=\sqrt{169}=13$

Therefore, distance of centre $(-2,1,3)$ from the plane $12x+4y+3z=327$



$=\dfrac{|12(-2)+4(1)+3(3)-327|}{\sqrt{144+16+9}}=26$

Hence, the shortest distance is 13

92 **(d)**

Line of intersection of $\vec{r}\cdot(\hat{\imath}+2\hat{\jmath}+3\hat{k})=0$ and $\vec{r}\cdot(3\hat{\imath}+3\hat{\jmath}+\hat{k})=0$ will be parallel to $(3\hat{\imath}+3\hat{\jmath}+\hat{k})\times(\hat{\imath}+2\hat{\jmath}+3\hat{k})$, i.e., $7\hat{\imath}-8\hat{\jmath}+3\hat{k}$

If the required angle is θ, then

$\cos\theta=\dfrac{7}{\sqrt{49+64+9}}=\dfrac{7}{\sqrt{122}}$

93 **(d)**

Let the plane $\vec{r}\cdot(\vec{\imath}-2\vec{\jmath}+3\vec{k})=17$ divide the line joining the points $-2\vec{\imath}+4\vec{\jmath}+7\vec{k}$ and $3\vec{\imath}-5\vec{\jmath}+8\vec{k}$ in the ratio $t:1$ at point P

Therefore, point P is

$\dfrac{3t-2}{t+1}\vec{\imath}+\dfrac{-5t+4}{t+1}\vec{\jmath}+\dfrac{8t+7}{t+1}\vec{k}$

This lies on the given plane

$\therefore\ \dfrac{3t-2}{t+1}.(1)+\dfrac{-5t+4}{t+1}(-2)+\dfrac{8t+7}{t+1}(3)=17$

Solving, we get

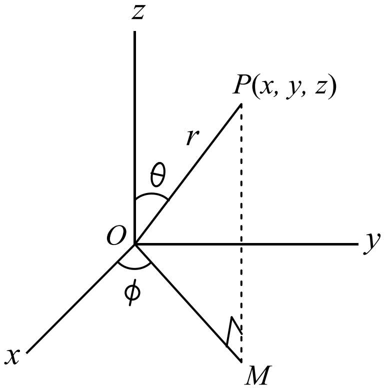
$t = \dfrac{3}{10}$

94 **(b,c,d)**

If P be (x, y, z), then from the figure

$x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$

$1 = r\sin\theta\cos\phi$, $2 = r\sin\theta\sin\phi$ and $3 = r\cos\theta$



$\Rightarrow 1^2 + 2^2 + 3^2 = r^2 \Rightarrow r = \pm\sqrt{14}$

$\therefore\ \sin\theta\cos\phi = \frac{1}{\sqrt{14}}, \sin\theta\sin\phi = \frac{2}{\sqrt{14}}$ and

$\cos\theta = \frac{3}{\sqrt{14}}$

(neglecting negative sign as θ and ϕ are acute)

$\dfrac{\sin\theta\sin\phi}{\sin\theta\cos\phi} = \dfrac{2}{1} \Rightarrow \tan\phi = 2$

Also, $\tan\theta = \sqrt{5}/3$

95 **(b,c,d)**

$x + y + z - 1 = 0$

$4x + y - 2z + 2 = 0$

Therefore, the line is along the vector

$(\hat{\imath} + \hat{\jmath} + \hat{k}) \times (4\hat{\imath} + \hat{\jmath} - 2\hat{k}) = 3\hat{\imath} - 6\hat{\jmath} + 3\hat{k}$

Let $z = k$. Then $x = k - 1$ and $y = 2 - 2k$

Therefore, $(k - 1, 2 - 2k, k)$ is any point on the line

Hence, $(-1, 2, 0)$, $(0, 0, 1)$ and $(-1/2, 1, 1/2)$ are the points on the line

96 **(b,c)**

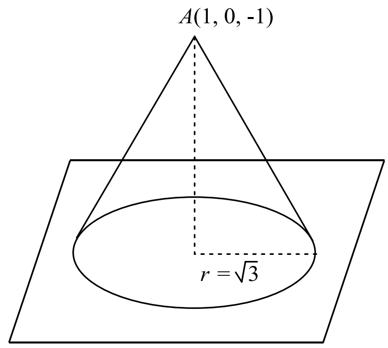
Distance between the planes is $h = 5/\sqrt{6}$

Also the figure formed is cylinder, whose radius is $r = 2$ units

Hence, the volume of the cylinder is $\pi r^2 h = \pi(2)^2.\frac{5}{\sqrt{6}} = \frac{20\pi}{\sqrt{6}}$ cubic units

Also the curved surface area is $2\pi rh = 2\pi(2).\frac{5}{\sqrt{6}} = \frac{20\pi}{\sqrt{6}}$

97 **(a,c,d)**



The rod sweeps out the figure which is a cone

The distance of point $A(1, 0, -1)$ from the plane is $\frac{|1-2+4|}{\sqrt{9}} = 1$ unit

The slant height l of the cone is 2 units

Then the radius of the base of the cone is

$\sqrt{l^2 - 1} = \sqrt{4 - 1} = \sqrt{3}$

Hence, the volume of the cone is $\frac{\pi}{3}\left(\sqrt{3}\right)^2.1 = \pi$ cubic units

Area of the circle on the plane which the of traces is 3π

Also, the centre of the circle is $Q(x, y, z)$. Then

$\frac{x-1}{1} = \frac{y-0}{-2} = \frac{z+1}{2} = \frac{-(1-0-2+4)}{1^2+(-2^2)+2^2}$, or

$Q(x, y, z) \equiv \left(\dfrac{2}{3}, \dfrac{2}{3}, \dfrac{-5}{3}\right)$

98 **(a,b)**

$3x - 6y + 2z + 5 = 0$ (i)

$-4x + 12y - 3z + 3 = 0$ (ii)

Bisectors are $\frac{3x-6y+2z+5}{\sqrt{9+36+4}} = \pm\frac{-4x+12y-3z+3}{\sqrt{16+144+9}}$

The plane which bisects the angle between the planes that contains the origin

$13(3x - 6y + 2z + 5) = 7(-4x + 12y - 3z + 3)$

$67x - 162y + 47z + 44 = 0$ (iii)

Further, $3 \times (-4) + (-6)(12) + 2 \times (-3) < 0$

Hence, the origin lies in acute angle

99 **(a,c,d)**

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ ...(i)

$\Rightarrow 1 - \sin^2\alpha + 1 - \sin^2\beta + \cos^2\gamma = 1$

Or $\sin^2\alpha + \sin^2\beta = 1 + \cos^2\gamma$

Also, from Eq. (i),

$\cos^2\beta + \cos^2\gamma = 1 - \cos^2\alpha$

Or $\cos^2\beta + \cos^2\gamma = \sin^2\alpha$

$\Rightarrow \sin^2\alpha = \cos^2\beta + \cos^2\gamma$

100 **(a,b,c)**

Let $\overrightarrow{\mathbf{OA}} = \vec{\mathbf{a}}, \overrightarrow{\mathbf{OB}} = \vec{\mathbf{b}}, \overrightarrow{\mathbf{OC}} = \vec{\mathbf{c}}$, then

$\vec{\mathbf{a}} \cdot \vec{\mathbf{a}} + (\vec{\mathbf{b}} - \vec{\mathbf{c}}) \cdot (\vec{\mathbf{b}} - \vec{\mathbf{c}})$

$= \vec{\mathbf{b}} \cdot \vec{\mathbf{b}} + (\vec{\mathbf{c}} - \vec{\mathbf{a}}) \cdot (\vec{\mathbf{c}} - \vec{\mathbf{a}})$

$\Rightarrow -2\,\vec{\mathbf{b}} \cdot \vec{\mathbf{c}} = -2\vec{\mathbf{c}} \cdot \vec{\mathbf{a}}$

$\Rightarrow (\vec{\mathbf{a}} - \vec{\mathbf{b}}) \cdot \vec{\mathbf{c}} = 0$ or $\overrightarrow{\mathbf{BA}} \cdot \overrightarrow{\mathbf{OC}} = 0$

Hence, $AB \perp OC$

Similarly, $BC \perp OA$ and $CA \perp OB$

101 **(a,c)**

The required plane is parallel to the bisector of the given planes

Bisectors are $\frac{x-y+z-3}{\sqrt{3}} = \pm\frac{x+y+z+4}{\sqrt{3}}$

or $2y+7=0$ and $2x+2y+1=0$. Hence, the planes are $y=0$ and $x+y=0$

102 **(a,d)**

The given lines intersect if

$$\begin{vmatrix} 2-1 & 3-4 & 4-5 \\ 1 & 1 & \lambda \\ \lambda & 2 & 1 \end{vmatrix} = 0 \Rightarrow \lambda = 0, -1$$

103 **(b,c)**

Volume of tetrahedron $ABCD$ is $\frac{1}{6}\left|\left[\overrightarrow{AB}\overrightarrow{AC}\overrightarrow{AD}\right]\right| = 1$ cubic units

$$\Rightarrow \begin{vmatrix} -1 & 1 & -1 \\ 1 & 1 & -1 \\ x-0 & y-1 & z-2 \end{vmatrix} = \pm 6$$

$\Rightarrow\ -2(y-1)-2(z-2) = \pm 6$

$\Rightarrow y-1+z-2 = \pm 3$

$\Rightarrow y+z=6$ or $y+z=0$

104 **(a,b)**

The plane is equally inclined to the lines. Hence, it is perpendicular to the angle bisector of the vectors $2\hat{\imath}-2\hat{\jmath}-\hat{k}$ and $8\hat{\imath}+\hat{\jmath}-4\hat{k}$

Vector along the angle bisectors of the vectors are

$\frac{2\hat{\imath}-2\hat{\jmath}-\hat{k}}{3} \pm \frac{8\hat{\imath}+\hat{\jmath}-4\hat{k}}{9}$, or

$\frac{14\hat{\imath}-5\hat{\jmath}-7\hat{k}}{9}$ and $\frac{-2\hat{\imath}-7\hat{\jmath}+\hat{k}}{9}$

Hence, the equation of the planes are

$14x-5y-7z=0$ or $2x+7y-z=0$

105 **(a,b,c)**

Extremities of a diameter of the sphere are given as (0, 2, 0) and (0, 0, 4)

$\Rightarrow$ Centre is (0, 1, 2) and radius $=\sqrt{5}$

Equation of the sphere is

$$(x-0)(x-0)+(y-2)(y-0)+(z-0)(z-4) = 0$$

Or $x^2+y^2+z^2-2y-4z=0$

Which passes through the origin

So, option (a), (b), (c) are correct

Now, $\frac{x}{0}=\frac{y-2}{1}=\frac{z-4}{-2}$ represents a diameter, if the centre (0, 1,2) lies on it

$\therefore$ There exists a value of r for which

$(0, r+2, -2r+4) = (0,1,2)$

$\Rightarrow r+2=1$ and $-2r+4=2$

Which is not possible

Hence, option (d) is not correct

106 **(b,c)**

For the given lines

$$\begin{vmatrix} 4-1 & 0-1 & -1-(-1) \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix} = \begin{vmatrix} 3 & -1 & 0 \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix} = 0$$

So, the given lies intersect

Any point on the first line is $(3r_1+1, r_1+1, -1)$ and any point on the second line is $(2r_2+4, 0, 3r_2-1)$

Since, the lines intersect, at the point of intersection

$3r_1+1 = 2r_2+4, -r_1+1=0, -1=3r_2-1$

$\Rightarrow r_1=1, r_2=0$

Hence, the point of intersection is (4, 0, -1)

107 **(a,c)**

Plane P_1 contains the line $\vec{r}=\hat{\imath}+\hat{\jmath}+\hat{k}+\lambda(\hat{\imath}-\hat{\jmath}-\hat{k})$, hence contains the point $\hat{\imath}+\hat{\jmath}+\hat{k}$ and is normal to vector $(\hat{\imath}+\hat{\jmath})$

Hence equation of plane is $\left(\vec{r}-\left(\hat{\imath}+\hat{\jmath}+\hat{k}\right)\right)\cdot(\hat{\imath}+\hat{\jmath})=0$

or $x+y=2$

Plane P_2 contains the line $\vec{r}=\hat{\imath}+\hat{\jmath}+\hat{k}+\lambda\left(\hat{\imath}-\hat{\jmath}-\hat{k}\right)$ and point $\hat{\jmath}$

Hence equation of plane is

$$\begin{vmatrix} x-0 & y-1 & z-0 \\ 1-0 & 1-1 & 1-0 \\ 1 & -1 & -1 \end{vmatrix} = 0$$

or $x+2y-z=2$

If θ is the acute angle between P_1 and P_2, then

$$\cos\theta = \frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|} = \left|\frac{(\hat{\imath}+\hat{\jmath})\cdot\left(\hat{\imath}+2\hat{\jmath}-\hat{k}\right)}{\sqrt{2}\cdot\sqrt{6}}\right|$$

$$= \frac{3}{\sqrt{2}\cdot\sqrt{6}} = \frac{\sqrt{3}}{2}$$

$$\theta = \cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}$$

As L is contained in $P_2\ \Rightarrow\ \theta = 0$

108 **(a,d)**

The equation of the plane passing through the intersection of the planes $2x-y=0$ and $3z-y=0$ is

$2x-y+\lambda(3z-y)=0$ (i)

Or $2x-y(\lambda+1)+3\lambda z=0$

Plane (i) is perpendicular to $4x+5y-3z=8$. Therefore,

$4\times 2-5(\lambda+1)-9\lambda=0$

$\Rightarrow 8-5\lambda-5-9\lambda=0$

$\Rightarrow 3-14\lambda=0$

$\Rightarrow\ \lambda = 3/14$

$\therefore\ \ 2x-y+\frac{3}{14}(3z-y)=0$

$28x-17y+9z=0$

109 **(a,d)**

The equation of a plane passing through the line of intersection of the x-y and y-z planes is

$z + \lambda x = 0, \lambda \in R$

This plane makes an angle 45° with the x-y plane ($z = 0$)

$\Rightarrow \cos 45° = \dfrac{1}{\sqrt{1}\sqrt{\lambda^2 + 1}}$

$\Rightarrow \lambda = \pm 1$

110 **(a,b)**

Let the coordinates of the point(s) be a, b and c

Therefore, the equation of the line passing through (a, b, c) and whose direction ratios are $1, -5$ and -2 is

$\frac{x-a}{1} = \frac{y-b}{-5} = \frac{z-c}{-2}$ (i)

Line (i) intersect the line,

$\frac{x}{1} = \frac{y+5}{1} = \frac{z+1}{1}$ (ii)

Therefore, these are coplanar

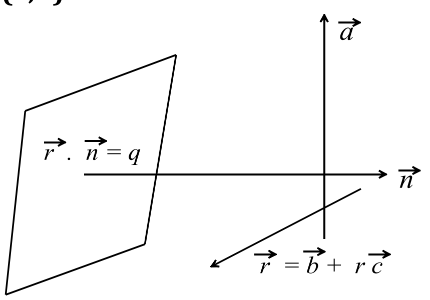
$$\begin{vmatrix} 1 & -5 & -2 \\ 1 & 1 & 1 \\ a & b+5 & c+1 \end{vmatrix} = 0$$

Or $a + b - 2c + 3 = 0$

Also, by using procedure with the second equation, we get the condition

$11a + 15b - 32c + 55 = 0$

111 **(a,b)**



Required line is parallel to $\vec{n} \times \vec{c}$

The equation of line is $\vec{r} = \vec{a} + \lambda(\vec{n} \times \vec{c})$

$\Rightarrow (\vec{r} - \vec{a}) = \lambda(\vec{n} \times \vec{c})$

$\therefore (\vec{r} - \vec{a}) \times (\vec{n} \times \vec{c}) = 0$

112 **(a,c)**

For line $\frac{x-1}{1} = \frac{y}{-1} = \frac{z-5}{-1}$, point (1, 0, 5) lies on the plane. Also, the vector along the line $\hat{\imath} - \hat{\jmath} - \hat{k}$ is perpendicular to the normal $\hat{\imath} + 2\hat{\jmath} - \hat{k}$ to the plane. For line $\vec{r} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k} + \lambda(3\hat{\imath} + \hat{\jmath} + 5\hat{k})$, point $(2, -1, 4)$ lies on the plane and vector $3\hat{\imath} + \hat{\jmath} + 5\hat{k}$ is perpendicular to the normal $\hat{\imath} + 2\hat{\jmath} - \hat{k}$. Line $x - y + z = 2x + y - z = 0$ passes through the origin, which is not on the given plane

113 **(a,b)**

$\vec{r}.\vec{n}_1 = \vec{q}_1$ and $\vec{r}.\vec{n}_2 = \vec{q}_2, \vec{r}.\vec{n}_3 = \vec{q}_3$ intersect in a line if $[\vec{n}_1\vec{n}_2\vec{n}_3] = 0$. So,

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2a & 1 \\ a & a^2 & 1 \end{vmatrix} = 0$$

$\Rightarrow 2a - a^2 - 1 + a + a^2 - 2a^2 = 0$

$\Rightarrow 2a^2 - 3a + 1 = 0$

$\Rightarrow a = 1/2, 1$

114 **(b,d)**

$(PQ)^2 = (\lambda - 1)^2 + (\lambda - 1)^2 + (\lambda - 1)^2$

$= 3(\lambda - 1)^2 = 27$

$\Rightarrow (\lambda - 1)^2 = 9$

$\Rightarrow \lambda - 1 = \pm 3$

$\Rightarrow \lambda = -2$ or 4

115 **(a)**

The direction cosines of segment OA are

$\frac{2}{\sqrt{14}}, \frac{1}{\sqrt{14}}$ and $\frac{-3}{\sqrt{14}}$

$OA = \sqrt{14}$

This means OA will be normal; to the plane and the equation of the plane is $2x + y - 3z = 14$

116 **(a)**

$\overrightarrow{PA}.\overrightarrow{PB} = 9 > 0$. Therefore, P is exterior to the sphere. Statement 2 is also true (standard result)

117 **(b)**

Equation of the polar to the sphere $x^2 + y^2 + z^2 = 1$ with respect to the point (1,2, 3) is

$x \cdot 1 + y \cdot 2 + z \cdot 3 = 1$

$ie, x + 2y + 3z = 1$

Let $f(x, y, z) = x^2 + y^2 + z^2 - 1$

$\therefore f(1, 2, 3) = 1^2 + 2^2 + 3^2 = 1$

$= 13 > 0$

$\therefore$ Point (1, 2, 3) lies outside the sphere $x^2 + y^2 + z^2 = 1$: for polar point may be inside or outside of sphere

118 **(c)**

Thus, statement II is false

Now, $AB = \sqrt{(2-1)^2 + (9-8)^2 + (12-8)^2}$

$= \sqrt{18} = 3\sqrt{2}$

$BC = \sqrt{(1+2)^2 + (8-11)^2 + (8-8)^2}$

$= \sqrt{18} = 3\sqrt{2}$

$CD = \sqrt{\{(-2+1)^2 + (11-12)^2 + (8-12)^2\}}$

$= \sqrt{18} = 3\sqrt{2}$

$DA = \sqrt{(-1-2)^2 + (12-9)^2 + (12-12)^2}$

$= \sqrt{18} = 3\sqrt{2}$

$AC = \sqrt{(2+2)^2 + (9-11)^2 + (12-8)^2}$

$= \sqrt{36} = 6$

$BD = \sqrt{(1+1)^2 + (8-12)^2 + (8-12)^2}$

$= \sqrt{36} = 6$

Hence, $AB = BC = CD = DA$ and $AC = BC$

119 **(b)**
Obviously the answer is (b)

120 **(b)**
Given lines are parallel as both are directed along the same vector $(\hat{\imath} + \hat{\jmath} - \hat{k})$; so they do not intersect. Also Statement 2 is correct by definition of skew lines, but skew lines are those with are neither parallel nor intersecting. Hence, both the statements are true, but Statement 2 is not the correct explanation for Statement 1

121 **(b)**
Since, orthocentre, nine point centre, centroid and circumcentre are collinear and centroid divides orthocenter and circumcentre in the ratio 2 : 1 (internally)

$\therefore\ \alpha = \dfrac{x + 2\gamma}{2+1}$

$\Rightarrow x = 3\alpha - 2\gamma$

And $\beta = \frac{\gamma + 2\delta}{2+1}$

$\Rightarrow y = 3\beta - 2\delta$

$\therefore$ Orthocentre is $(3\alpha - 2\gamma, 3\beta - 2\delta)$

122 **(a)**
$\sin\theta = \left|\dfrac{2-3+2}{\sqrt{4+9+4}\sqrt{3}}\right| = \dfrac{1}{\sqrt{51}}$
Therefore, Statement 1 is true and Statement 2 is also true by definition

123 **(a)**
Let the equation of the common circle be

$x^2 + y^2 = a^2, z = 0$ ...(i)

Its radius is evidently a and we are to evaluate it. Now, let the equations of the two given spheres through this circle be

$(x^2 + y^2 - a^2) + 2\lambda z + z^2 = 0$ ...(ii)

And $(x^2 + y^2 - a^2) + 2\mu\, z + z^2 = 0$ ...(iii)

(Here an extra term z^2 has been introduced in each equation, so that it may represent a sphere)

From Eq. (ii), the radius of the sphere

$= \sqrt{(-\lambda)^2 - (-a)^2} = \sqrt{(\lambda^2 + a^2)} = r_1$ (given)

And similarly from Eq. (iii) the radius of the sphere

$= \sqrt{(\mu^2 + a^2)} = r_2$ (given)

Also, as the sphere (ii) and (iii) cut each other orthogonally, so we have

$2\lambda u = -a^2 - a^2$

Or $\lambda^2\mu^2 = a^4$

Or $(r_1^2 - a^2)(r_2^2 + a^2) = a^4$

$\Rightarrow r_1^2 r_2^2 - a^2(r_1^2 + r_2^2) = 0$

Or $a = \frac{r_1 r_2}{\sqrt{(r_1^2 + r_2^2)}}$

124 **(a)**
Statement II Lines L_1 and L_2 are parallel to the vectors $\vec{\mathbf{a}} = 3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\vec{\mathbf{b}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ respectively. The unit vector perpendicular to both L_1 and L_2 is $\frac{\vec{\mathbf{a}} \times \vec{\mathbf{b}}}{|\vec{\mathbf{a}} \times \vec{\mathbf{b}}|} = \frac{-\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 5\hat{\mathbf{k}}}{\sqrt{1+49+25}}$ using it the plane is

Statement I is $-(x+1) - 7(y+2) + 5(z+1) = 0$ whose distance from (1,1,1) is $\frac{13}{5\sqrt{3}}$

125 **(b)**
The equation of the plane containing them is

$$\begin{vmatrix} x-1 & y & z+1 \\ 1 & -1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = -(5x + 2y - 3z - 8) = 0$$

Statement II Here, $\frac{1}{3} = \frac{2}{6} = \frac{3}{9}$

$\Rightarrow \dfrac{1}{3} = \dfrac{1}{3} = \dfrac{1}{3}$

and $1(1) + 2(1) + 3(-1) = 0$

126 **(b)**

For the given lines, let $\vec{a}_1 = \hat{\imath} + \hat{\jmath} - \hat{k}$, $\vec{a}_2 = 4\hat{\imath} - \hat{k}, \vec{b}_1 = 3\hat{\imath} - \hat{\jmath}$ and $\vec{b}_2 = 2\hat{\imath} - \hat{k}$. Therefore,

$$[\vec{a}_2 - \vec{a}_1 \vec{b}_1 \vec{b}_2] = \begin{vmatrix} 4-1 & 0-1 & -1+1 \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix}$$
$$= \begin{vmatrix} 3 & -1 & 0 \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix} = 0$$

Hence, the lines are coplanar. Also vectors $\vec{b}_1$ and $\vec{b}_2$ along which the lines are not collinear. Hence, the lines intersect. When $\vec{b} \times \vec{d} = \vec{0}$, vectors $\vec{b}$ and $\vec{d}$ are collinear; therefore, lines $\vec{r} = \vec{a} + \lambda\vec{b}$ and $\vec{r} = \vec{c} + \lambda\vec{d}$ are parallel and do not intersect. But this statement is not the correct explanation for Statement 1

127 **(b)**

Direction ratios of the given lines are $(-3, 1, -1)$ and $(1, 2, -1)$. Hence, the lines are perpendicular as $(-3)(1) + (1)(2) + (-1)(-1) = 0$

Also lines are coplanar as

$$\begin{vmatrix} 0-2 & 1-3 & -1+(13/7) \\ -3 & 1 & -1 \\ 1 & 2 & -1 \end{vmatrix} = 0$$

But Statement 2 is not enough reason for the shortest distance to be zero, as two skew lines can also be perpendicular

128 **(c)**

Any point on the line $\frac{x-1}{1} = \frac{y}{-1} = \frac{z+2}{2}$ is

$B(t+1, -t, -2t-2), t \in R$

Also, AB is perpendicular to the line, where A is $(1, 2, -4)$

$\Rightarrow 1(t) - (-t-2) + 2(2t+2) = 0$

$\Rightarrow 6t + 6 = 0$

$\Rightarrow t = -1$

Point B is $(0,1,-4)$

Hence, $AB = \sqrt{1+1+0} = \sqrt{2}$

129 **(a)**

Here, $C_1 \equiv (-a, 0\ 0)$

$r_1 = \sqrt{(a^2 - c)}$

And $C_2 \equiv (0, -b. 0)$

$r_2 = \sqrt{(b^2 - c)}$

$\therefore |C_1C_2| = \sqrt{(a^2 + b^2)}$

For touch

$|C_1C_2| = r_1 \pm r_2$

$\Rightarrow \sqrt{(a^2 + b^2)} = \sqrt{(a^2 - c)} \pm \sqrt{(b^2 - c)}$

On squaring both sides, then

$$a^2 + b^2 = a^2 - c + b^2 - c \pm 2\sqrt{(a^2 - c)}\sqrt{(b^2 - c)}$$

$\Rightarrow c = \pm\sqrt{(a^2 - c)(b^2 - c)}$

Again, on squaring both sides, then

$c^2 = (a^2 - c)(b^2 - c)$

$\Rightarrow c^2 = a^2b^2 - c(a^2 + b^2) + c^2$

$\Rightarrow \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c}$

130 **(c)**

Equation of plane is

$2x - y + z - 3 + \lambda(3x + y + z - 5) = 0$

For $\lambda = 1$, we get

$5x + 2z - 8 = 0$ which is perpendicular to

$2x - y + 5z - 3 = 0$ as $5 \times 2 + 0(-1) + 2(-5) = 0$

131 **(a)**

Any point on the first line is $(2x_1 + 1, x_1 -$ *3, −3x1+2*

Any point on the second line is $(y_1 + 2, -3y_1 +$ *1, 2y1−3*

If two lines are coplanar, then $2x_1 - y_1 = 1, x_1 + 3y_1 = 4$ and $3x_1 + 2y_1 = 5$ are consistent

132 **(b)**

Let l, m, n be the DC's of the line of the common perpendicular (or SD) to the two given lines. Then, we have

$-4l + 3m + 2n = 0$

And $-4l + 1, +1n = 0$

On solving these, we get

$$\frac{l}{3-2} = \frac{m}{-8+4} = \frac{N}{-4+12}$$

Or $\frac{l}{1} = \frac{m}{-4} = \frac{n}{8} = \frac{\sqrt{(l^2+m^2+n^2)}}{\sqrt{(1)^2+(-4)^2+(8)^2}} = \frac{1}{9}$

$\therefore$ DC's of SD are $\frac{1}{9}, \frac{4}{-9}, \frac{8}{9}$

Also , $A(-3, 6, 0)$ is a point on first line and

$B(-2,0,7)$ is a point on second line, then

$$SD = \left|(-2+3)\frac{1}{9} + (0-6)\left(-\frac{4}{9}\right) + (7-0)\left(\frac{8}{9}\right)\right| = 9$$

And two lines are said to be skew lines or non-intersecting lines if they do not lie in the same plane

133 **(d)**

Given planes are $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$

For $z = 0$, we get $x = 3, y = -1$

Direction ratios of given planes are

$< 3, -6, -2 >$ and $< 2, 1, -2 >$

Let a, b and c be the direction ratios of the line of intersection of the given planes. Then,

$3a - 6b - 2c = 0$ and $2a + b - 2c = 0$

∴ The DR's of line of intersection of planes is <14, 2, 15> and line is

$$\frac{x-3}{14} = \frac{y+1}{2} = \frac{z-0}{15} = \lambda \text{ [say]}$$

$\Rightarrow x = 14\lambda + 3, y = 2\lambda - 1, \quad z = 15\lambda$

Hence, statement I is false,

But statement II is true

134 **(b)**

Statement 2 is true as when the line lies in the plane, vector $\vec{b}$ along which the line is directed is perpendicular to the normal $\vec{c}$ of the plane, but it does not explain Statement 1 as for $\vec{b}.\vec{c} = 0$, the line may be parallel to the plane. However, Statement 1 is correct as any point on the line $(t+1, 2t, -t-2)$ lies on the plane for $t \in R$

135 **(a)**

The image of the point (3, 1, 6) with respect to the plane $x - y + z = 5$ is

$$\frac{x-3}{1} = \frac{y-1}{-1} = \frac{z-6}{1} = \frac{-2(3-1+6-5)}{1+1+1} = -2$$

$\Rightarrow \quad x = 3 - 2 = 1,$

$y = 1 + 2 = 3,$

$z = 6 - 2 = 4$

Which show that Statement I is true.

We observe that the line segment joining the points $A(3,1,6)$and $B(1,3,4)$ has direction ratios $2, -2, 2$ which is preoperational to $1, -1, 1$ the direction ratios of the normal to the plane. Hence, Statement II is true

Thus, the Statement I and II are true and Statement II is correct explanation of Statement I.

136 **(a)**

1. If the required image is (x, y, z), then

$\frac{x-3}{2} = \frac{y-5}{1} = \frac{z-7}{1} = -\frac{2(6+5+7+18)}{2^2+1^2+1^2} = -12$ or $(-21, -7, -5)$

2. Any point on the line $\frac{x-2}{-3} = \frac{y-1}{2} = \frac{z-3}{2} = \lambda$ is $(-3\lambda + 2, 2\lambda + 1, 2\lambda + 3)$, which lies on plane $2x + y - z = 3$. Therefore

$-6\lambda + 4 + 2\lambda + 1 - 2\lambda - 3 = 3$

$-6\lambda = 1$

$\lambda = -1/6$

Therefore, the point is $\left(\frac{5}{2}, \frac{2}{3}, \frac{8}{3}\right)$

3. If (x, y, z) is required foot of the perpendicular, then $\frac{x-1}{2} = \frac{y-2}{-2} = \frac{z-2}{4} = -\frac{(2-2+8+5)}{2^2+(-2)^2+4^2}$ or $(x, y, z) \equiv \left(\frac{-1}{12}, \frac{25}{12}, \frac{-2}{12}\right)$

4. Any point on the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$ is $P(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$, which satisfies the line $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$ or $\frac{2\lambda+1-4}{5} = \frac{3\lambda+2-1}{2} = \frac{4\lambda+3}{1}$

$\Rightarrow \lambda = -1$

The required point is $(-1, -1, -1)$

137 **(b)**

1. Line $\frac{x-1}{-2} = \frac{y+2}{3} = \frac{z}{-1}$ is along the vector $\vec{a} = -2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and line $\vec{r} = (3\hat{\imath} - \hat{\jmath} + \hat{k}) + t(\hat{\imath} + \hat{\jmath} + \hat{k})$ is along the vector $\vec{b} = \hat{\imath} + \hat{\jmath} + \hat{k}$. Here $\vec{a} \perp \vec{b}$

Also $\begin{vmatrix} 3-1 & -1-(-2) & 1-0 \\ -2 & 3 & -1 \\ 1 & 1 & 1 \end{vmatrix} \neq 0$

2. The direction ratios of the line $x - y + 2z - 4 = 0 = 2x + y - 3z + 5 = 0$ are $\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 2 \\ 2 & 1 & -3 \end{vmatrix} = \hat{\imath} + 7\hat{\jmath} + 3\hat{k}$. Hence, the given two lines are parallel

3. The given lines are ($x = t - 3, y = -2t + 1, z = -3t - 2$ and $r = t+1i + 2t+3j + -t-9k$, or $\frac{x+3}{1} = \frac{y-1}{-2} = \frac{z+2}{-3}$ and $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z+9}{-1}$.

The lines are perpendicular as $(1)(1) +$

$(-2)(2)+(-3)(-1)=0$

Also $\begin{vmatrix} -3-1 & 1-3 & -2-(9) \\ 1 & -2 & -3 \\ 1 & 2 & -1 \end{vmatrix}=0$

Hence, the lines are intersecting

4. The given lines are $\vec{r}=(\hat{\imath}+3\hat{\jmath}-\hat{k})+t(2\hat{\imath}-\hat{\jmath}-\hat{k})$ and $\vec{r}=(-\hat{\imath}-2\hat{\jmath}+5\hat{k})+s(\hat{\imath}-2\hat{\jmath}+\frac{3}{4}\hat{k})$

$\begin{vmatrix} 1-(-1) & 3-(-2) & -1-5 \\ 2 & -1 & -1 \\ 1 & -2 & 3/4 \end{vmatrix}=0$

Hence, the lines are coplanar and hence intersecting (as the lines are not parallel)

138 **(d)**

1. The given line and plane are $\vec{r}=(2\hat{\imath}-2\hat{\jmath}+3\hat{k})+\lambda(\hat{\imath}-\hat{\jmath}+4\hat{k})$and $\vec{r}.(\hat{\imath}+5\hat{\jmath}+\hat{k}=5$, respectively. Since $\hat{\imath}-\hat{\jmath}+4\hat{k}.(\hat{\imath}+5\hat{\jmath}+\hat{k})=0$ line and plane are parallel

Hence, the required distance = distance of point $(2,-2,3)$ from the plane $x+5y+z-5=0$, which is $\frac{|2-10+3-5|}{\sqrt{1+25+1}}=\frac{10}{3\sqrt{3}}$

2. The distance between two parallel planes $\vec{r}.(2\hat{\imath}-\hat{\jmath}+3\hat{k})=4$ and $\vec{r}.(6\hat{\imath}-3\hat{\jmath}+9\hat{k}+13=0$ is

$$d=\frac{|4-(-13/3)|}{\sqrt{(2)^2+(-1)^2+(3)^2}}=\frac{(25/3)}{\sqrt{14}}=\frac{25}{3\sqrt{14}}$$

3. The perpendicular distance of the point $(2,5,-3)$ from the plane $\vec{r}.(6\hat{\imath}-3\hat{\jmath}+2\hat{k}=4$ or $6x-3y+2z-4=0$ is

$$d=\frac{|12-15-6-4|}{\sqrt{36+9+4}}$$

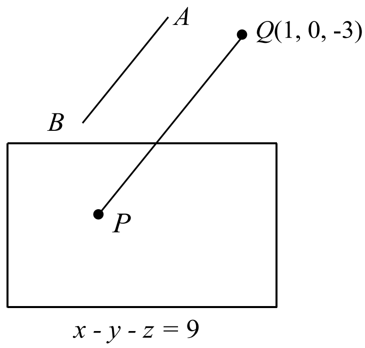
$$=13/\sqrt{49}=13/7$$

4. The equation of the line AB is

$$\frac{x-2}{2}=\frac{y+2}{3}=\frac{z-6}{-6}$$

The equation of line passing through $(1,0,-3)$ an dparallle to AB is

$$\frac{x-1}{2}=\frac{y}{3}=\frac{z+3}{-6}=r \quad \text{(say)}$$



The coordinates of any point on line $P(2r+1, 3r, -6r-3$ which lie on plane

$(2r+1)-(3r)-(-6r-3)=9$

$r=1$

Point $P\equiv(3,3,-9)$

Required distance

$PQ=\sqrt{(3-1)^2+(3-0)^2+(-9+3)^2}=\sqrt{4+9+36}=7$

139 **(a)**

1. Line $x=2t+1, y=t+2, z=-t-3$ or $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z+3}{-1}$, which is along the vector $2\hat{\imath}+\hat{\jmath}-\hat{k}$. Vector $\hat{\imath}+3\hat{\jmath}+5\hat{k}$ is perpendicular to the line

2. Normals to the planes $x+y+z-3=0$ and $2x-y+3z=0$ are $\vec{n}_1=\hat{\imath}+\hat{\jmath}+\hat{k}$ and $\vec{n}_2=2\hat{\imath}-\hat{\jmath}+3\hat{k}$. Then the vector along the line of intersection of planes is

$$\vec{n}_1\times\vec{n}_2=\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{vmatrix}=4\hat{\imath}-\hat{\jmath}-3\hat{k}$$

3. The shortest distance between the lines $\frac{x}{2}=\frac{y}{-3}=\frac{z}{-1}$ and $\vec{r}=(3\hat{\imath}-\hat{\jmath}+\hat{k})+t(\hat{\imath}+\hat{\jmath}-2\hat{k})$ occurs along the vector $(2\hat{\imath}-3\hat{\jmath}-\hat{k})\times(\hat{\imath}+\hat{\jmath}-2\hat{k})=$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -3 & -1 \\ 1 & 1 & -2 \end{vmatrix}=7\hat{\imath}+3\hat{\jmath}+5\hat{k}$$

4. Normal to the plane $\vec{r}=-\hat{\imath}+4\hat{\jmath}-6\hat{k}+\lambda(\hat{\imath}+3\hat{\jmath}-2\hat{k})+\mu(-\hat{\imath}+2\hat{\jmath}-5\hat{k})$ is

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 3 & -2 \\ -1 & 2 & -5 \end{vmatrix}=-11\hat{\imath}+7\hat{\jmath}+5\hat{k}$$

140 **(c)**

a. The given line is $x = 4y + 5, z = 3y - 6$, or

$$\frac{x-5}{4} = y, \frac{z+6}{3} = y$$

or $\frac{x-5}{4} = \frac{y}{1} = \frac{z+6}{3} = \lambda$ (say)

Any point on the line is of the form $(4\lambda + 5, \lambda, 3\lambda - 6$

The distance between $(4\lambda + 5, \lambda, 3\lambda - 6)$ and $(5, 3, -6)$ is 3 units (given). Therefore

$$(4\lambda + 5 - 5)^2 + (\lambda - 3)^2 + (3\lambda - 6 + 6)^2 = 9$$

$$\Rightarrow 16\lambda^2 + \lambda^2 + 9 - 6\lambda + 9\lambda^2 = 9$$

$$\Rightarrow 26\lambda^2 - 6\lambda = 0$$

$$\Rightarrow \lambda = 0, 3/13$$

The point is $(5, 0, -6)$

b. The equation of the plane containing the lines $\frac{x-2}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and parallel to $\hat{\imath} + 4\hat{\jmath} + 7\hat{k}$

$$\begin{vmatrix} x-2 & y+3 & z+5 \\ 1 & 4 & 7 \\ 3 & 5 & 7 \end{vmatrix} = 0$$

$$\Rightarrow x - 2y + z - 3 = 0$$

Point $(-1, -2, 0)$ lies on this plane

c. The line passing through points $A(2, -3, -1)$ and $B(8, -1, 2)$ is $\frac{x-2}{8-2} = \frac{y+3}{-1+3} = \frac{z+1}{2+1}$ or $\frac{x-2}{6} = \frac{y+3}{2} = \frac{z+1}{3} = \lambda$ (say)

Any point on this line is of the form $P(6\lambda + 2, 2\lambda - 3, 3\lambda - 1$, whose distance from point $A(2, -3, -1)$ is 14 units. Therefore,

$$\Rightarrow PA = 14$$

$$\Rightarrow PA^2 = (14)^2$$

$$\Rightarrow (6\lambda^2) + (2\lambda)^2 + (3\lambda)^2 = 196$$

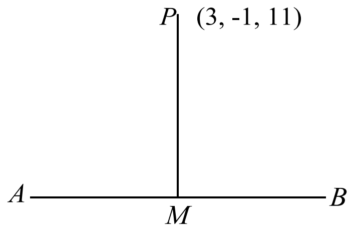
$$\Rightarrow 49\lambda^2 = 196$$

$$\Rightarrow \lambda^2 = 4$$

$$\Rightarrow \lambda = \pm 2$$

Therefore, the required points are (14, 1, 5) and $(-10, -7, -7)$. The point nearer to the origin is (14, 1, 5)

d. Any point on line $AB, \frac{x}{2}\frac{x-2}{3} = \frac{z-3}{4} = \lambda$ is $M(2\lambda, 3\lambda + 2, 4\lambda + 3)$. Therefore the direction ratios of PM are $2\lambda - 3, 3\lambda + 3$ and $4\lambda - 8$



But $PM \perp AB$

$$\therefore 2(2\lambda - 3) + 3(3\lambda + 3) + 4(4\lambda - 8) = 0$$

$$4\lambda - 6 + 9\lambda + 9 + 16\lambda - 32 = 0$$

$$29\lambda - 29 = 0$$

$$\lambda = 1$$

Therefore, foot of the perpendicular is $M(2,5,7)$

141 **(a)**
Let $P(x, y, z)$ be any point on the locus, then
$3PA = 2\,PB$
$\Rightarrow 9[(x+2)^2 + (y-2)^2 + (z-3)^2]$
$= 4[(x-13)^2 + (y+3)^2 + (z-13)^2]$
$\Rightarrow 5(x^2 + y^2 + z^2) + 140x - 60y + 50z - 1235 = 0$
$\Rightarrow x^2 + y^2 + z^2 + 28x - 12y + 10z - 247 = 0$

142 **(b)**
The equation of given lines in vector from may be written as
$L_1: \vec{\mathbf{r}} \cdot (-\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - \hat{\mathbf{k}}) + \lambda(3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$
And $L_2: \vec{\mathbf{r}} \cdot (2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) + \mu(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$
$\therefore$ The vector perpendicular to both L_1 and L_2 is

$$\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = -\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$$

$\therefore$ Required unit vector $= \frac{(-\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 5\hat{\mathbf{k}})}{\sqrt{(1)^2 + (-7)^2 + (5)^2}}$

$$= \frac{1}{5\sqrt{3}}(-\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$$

143 **(d)**
The DC's of the lines are given by
$ul + vm + wn = 0$
And $al^2 + bm^2 + cn^2 = 0$
On eliminating n between them, we get

$$al^2 + bm^2 + c\left\{-\frac{(ul + vm)}{w}\right\}^2 = 0$$

$\Rightarrow (aw^2 + cu^2)l^2 + (bw^2 + cv^2)m^2 + 2cuvlm$
$= 0$

$\Rightarrow (aw^2 + cu^2)\left(\frac{l}{m}\right)^2 + 2cuv\left(\frac{l}{m}\right) + (bw^2 +$
$cv2=0$ ...(i)

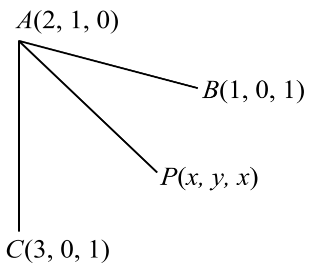
Put $u = v = w = 1$ in Eq. (i), then

$(a + c)\left(\frac{l}{m}\right)^2 + 2c\left(\frac{l}{m}\right) + (b + c) = 0$

Similarly, $(a + b)\left(\frac{m}{n}\right)^2 + 2a\left(\frac{m}{n}\right) + (c + a) = 0$

And $(b + c)\left(\frac{n}{l}\right)^2 + 2b\left(\frac{n}{l}\right) + (a + b) = 0$ ...(ii)

144 **(b)**



$\begin{vmatrix} x-2 & y-1 & z \\ 1-2 & 0-1 & 1-0 \\ 3-2 & 0-1 & 1-0 \end{vmatrix} = 0$

$(x - 2)[(-1) - (-1)] - (y - 1)[(-1) - 1]$
$+ z[1 + 1] = 0$

$2(y - 1) + 2z = 0$

$\Rightarrow y + z - 1 = 0$

The vector normal to the plane is $\vec{r} = 0\hat{\imath} + \hat{\jmath} + \hat{k}$

The equation of the line through (0, 0,2) and parallel to $\vec{n}$ is $\vec{r} = 2\hat{k} + \lambda(\hat{\jmath} + \hat{k})$

The perpendicular distance of $D(0, 0, 2)$ from plane ABC is $\left|\frac{2-1}{\sqrt{1^2+1^2}}\right| = \frac{1}{\sqrt{2}}$

145 **(b)**

Let $Q(x_2, y_2, z_2)$ be the image of $A(2, 1, 6)$ about mirror $x + y - 2z = 3$. Then,

$$\frac{x_2 - 2}{1} = \frac{y_2 - 1}{1} = \frac{z_2 - 6}{-2} = \frac{-2(2 + 1 - 12 - 3)}{1^2 + 1^2 + 2^2}$$
$$= 4$$

$\Rightarrow (x_2, y_2, z_2) \equiv (6, 5, -2)$

146 **(b)**

The given system of equations is

$2x + py + 6z = 8$

$x + 2y + qz = 5$

$x + y + 3z = 4$

$\Delta = \begin{vmatrix} 2 & p & 6 \\ 1 & 2 & q \\ 1 & 1 & 3 \end{vmatrix} = (2 - p)(3 - q)$

By Cramer's rule, if $\Delta \neq 0$, i.e., $p \neq 2$ and $q \neq 3$, the system has a unique solution

If $p = 2$ or $q = 3, \Delta = 0$, then if $\Delta_x = \Delta_y = \Delta_z = 0$, the system has infinite solutions and if any one of Δ_x, Δ_y and $\Delta_z \neq 0$, the system has no solution

Now $\Delta_x = \begin{vmatrix} 8 & p & 6 \\ 5 & 2 & q \\ 4 & 1 & 3 \end{vmatrix}$

$= 30 - 8q - 15p + 4pq = (4q - 15) \cdot (p - 2)$

$\Delta_y = \begin{vmatrix} 2 & 8 & 6 \\ 1 & 5 & q \\ 1 & 4 & 3 \end{vmatrix}$

$= -8q + 8q = 0$

$\Delta_z = \begin{vmatrix} 2 & p & 8 \\ 1 & 2 & 5 \\ 1 & 1 & 4 \end{vmatrix}$

$= p - 2$

Thus, if $p = 2, \Delta_x = \Delta_y = \Delta_z = 0$ for all $q \in R$, the system has infinite solutions

If $p \neq 2, q = 3$ and $\Delta_z \neq 0$, then the system has no solution

Hence the system has (i) no solution if $p \neq 2$ and $q = 3$, (ii) a unique solution if $p \neq 2$ and $q \neq 3$ and (iii) infinite solutions if $p = 2$ and $q \in R$

147 **(d)**

The line $\frac{x-1}{3} = \frac{y-2}{-1} = \frac{z-3}{4} = r$

Any point say $B \equiv (3r + 1, 2 - r, 3 + 4r)$ (on the line L)

$\overrightarrow{AB} = 3r, -r, 4r + 6$

Hence,

$\overrightarrow{AB}$ is parallel to $x + y - z = 1$

$\Rightarrow 3r - r - 4r - 6 = 0$ or $r = -3$

B is $(-8, 5, -9)$

148 **(8)**

Volume (V)$= \frac{1}{3}A_1h_1 \Rightarrow h_1 = \frac{3V}{A_1}$

Similarly $h_2 = \frac{3V}{A_2}, h_3 = \frac{3V}{A_3}$ and $h_4 = \frac{3V}{A_4}$

So $(A_1 + A_2 + A_3 + A_4)(h_1 + h_2 + h_3 + h_4)$

$= (A_1 + A_2 + A_3 + A_4)\left(\frac{3V}{A_1} + \frac{3V}{A_2} + \frac{3V}{A_3} + \frac{3V}{A_4}\right)$

$= 3V(A_1 + A_2 + A_3 + A_4)\left(\frac{1}{A_1} + \frac{1}{A_2} + \frac{1}{A_3} + \frac{1}{A_4}\right)$

Now using A.M.-H.M. inequality in A_1, A_2, A_3, A_4, we get

$$\frac{A_1 + A_2 + A_3 + A_4}{4} \geq \frac{4}{\left(\frac{1}{A_1} + \frac{1}{A_2} + \frac{1}{A_3} + \frac{1}{A_4}\right)}$$

$\Rightarrow (A_1 + A_2 + A_3 + A_4)\left(\frac{1}{A_1} + \frac{1}{A_2} + \frac{1}{A_3} + \frac{1}{A_4}\right) \geq 16$

Hence the minimum value of $(A_1 + A_2 + A_3 +$
$A4h1+h2+h3+h4=3V16=48V=481/6=8$

149 **(7)**

Clearly minimum value of $a^2 + b^2 + c^2$

$= \left(\frac{|(3(0) + 2(0) + (0) - 7|}{\sqrt{(3)^2 + (2)^2 + (1)^2}}\right)^2 = \frac{49}{14} = \frac{7}{2}$ units

150 **(1)**

If image of point $(2,-3,3)$ is the plane $x-2y-z+1=0$ is (a,b,c), then

$$\frac{a-2}{1}=\frac{b+3}{-2}=\frac{c-3}{-1}=\frac{-2(2-2(-3)-3+1)}{(1)^2+(-2)^2+(-1)^2}$$
$$=-2$$

Hence the image is $(0,1,5)$

Obviously distance of image of the point from z-axis is 1

151 **(6)**

The given points are $O(0,0,0), A(0,0,2), B(0,4,0)$ and $C(6,0,0)$

Here three faces of tetrahedron are xy, yz, zx plane

Since point P is equidistance from zx, xy and yz planes, its coiordinates are $P(r,r,r)$

Equation of plane ABC is

$2x+3y+6z=12$ (from intercept form)

P is also at distance r from plane ABC

$$\Rightarrow \frac{|2r+3r+6r-12|}{\sqrt{4+9+36}}=r$$

$\Rightarrow |11r-12|=7r$

$\Rightarrow 11r-12=\pm 7r$

$$\Rightarrow r=\frac{12}{18},3$$

$\therefore\ r=2/3$ (as $r<2$)

152 **(6)**

A plane containing the line of intersection of the given planes is

$x-y-z-4+\lambda(x+y+2z-4)=0$

i.e., $(\lambda+1)x+(\lambda-1)y+(2\lambda-1)z-4(\lambda+1)=0$

vector normal to it

$V=(\lambda+1)\hat{\imath}+(\lambda-1)\hat{\jmath}+(2\lambda-1)\hat{k}S$ (i)

Now the vector along the line of intersection of the planes

$2x+3y+z-1=0$ and $x+3y+2z-2=0$ is given by

$$\vec{n}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 2 & 3 & 1\\ 1 & 3 & 2\end{vmatrix}=3(\hat{\imath}-\hat{\jmath}+\hat{k})$$

As $\vec{n}$ is parallel to the plane (i), therefore

$\vec{n}\cdot\vec{V}=0$

$(\lambda+1)-(\lambda-1)+(2\lambda-1)=0$

$$2+2\lambda-1=0\ \Rightarrow\ \lambda=\frac{-1}{2}$$

Hence the required plane is

$$\frac{x}{2}-\frac{3y}{2}-2z-2=0$$

$x-3y-4z-4=0$

Hence $|A+B+C|=6$

153 **(7)**

$4x+7y+4z+81=0$ (i)

$5x+3y+10z=25$ (ii)

Equation of plane passing through their line of intersection is

$(4x+7y+4z+81)+\lambda(5x+3y+10z-25)=0$

Or $(4+5\lambda)x+(7+3\lambda)y+(4+10\lambda)z+81-25\lambda=0$ (iii)

Plane (iii) $\perp$ to (i), so

$4(4+5\lambda)+7(7+3\lambda)+4(4+10\lambda)=0$

$\therefore\ \lambda=-1$

From (iii), equation of plane is $-x+4y-6z+106=0$ (iv)

Distance of (iv) from $(0,0,0)=\frac{106}{\sqrt{1+16+36}}=\frac{106}{\sqrt{53}}$

154 **(9)**

Line through point $P(-2,3,-4)$ and parallel to the given line $\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$ is $\frac{x+2}{3}=\frac{y+\frac{3}{2}}{2}=\frac{z+\frac{4}{3}}{\frac{5}{3}}=\lambda$

Any point on this line is $Q\left[3\lambda-2, 2\lambda-\frac{3}{2},\frac{5}{3}\lambda-\frac{4}{3}\right]$

Direction ratios of PQ are $\left[3\lambda,\frac{4\lambda-9}{2},\frac{5\lambda+8}{3}\right]$

Now PQ is parallel to the given plane

$4x+12y-3z+1=0$

$\Rightarrow$ line is perpendicular to the normal to the plane

$$\Rightarrow 4(3\lambda)+12\left(\frac{4\lambda-9}{2}\right)-3\left(\frac{5\lambda+8}{3}\right)=0$$

$\Rightarrow\ \lambda=2$

$$\Rightarrow\ Q\left(4,\frac{5}{2},2\right)$$

$$\Rightarrow\ PQ=\sqrt{(6)^2+\left(\frac{5}{2}-3\right)^2+(6)^2}=\frac{17}{2}$$

155 **(3)**

Let $A(1,0,-1), B(-1,2,2)$

Direction ratios of AB are $(2,-2,-3)$

Let θ be the angle between the line and normal to plane, then

$$\cos\theta=\frac{|2.1+3(-2)-5(-3)|}{\sqrt{1+9+25}\sqrt{4+4+9}}=\frac{11}{\sqrt{17}\sqrt{35}}$$
$$=\frac{11}{\sqrt{595}}$$

Length of projection

$=(AB)\sin\theta$

$$=\sqrt{(2)^2+(2)^2+(3)^2}\times\sqrt{1-\frac{121}{595}}$$

$$= \sqrt{\frac{474}{35}} \text{ units}$$

156 **(2)**

Vector normal to the plane is $\vec{n} = \hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ and vector along the line is $\vec{v} = 2\hat{\jmath} + \hat{\jmath} - 3\hat{k}$

Now $\sin\theta = \frac{\vec{x}.\vec{v}}{|\vec{x}||\vec{v}|} = \left|\frac{2-3-6}{\sqrt{14}\sqrt{14}}\right| = \left|\frac{7}{14}\right|$

Hence $\text{cosec}\,\theta = 2$

157 **(8)**

Obviously one in each octant