10.VECTOR ALGEBRA

Single Correct Answer Type

1. A unit vector in xy-plane that makes an angle 45° with the vector $(\hat{\imath}+\hat{\jmath})$ and an angle of 60° with the vector $(3\hat{\mathbf{i}}-4\hat{\mathbf{j}})$, is
   a) $\hat{\mathbf{i}}$ b) $\frac{1}{\sqrt{2}}(\hat{\mathbf{i}}-\hat{\mathbf{j}})$ c) $\frac{1}{\sqrt{2}}(\hat{\mathbf{i}}+\hat{\mathbf{j}})$ d) None of these
2. Let $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ be unit vectors inclined at an angle $2\alpha(0\leq\alpha\leq\pi)$ each other, then $|\vec{\mathbf{a}}+\vec{\mathbf{b}}|<1$, if
   a) $\alpha=\frac{\pi}{2}$ b) $\alpha<\frac{\pi}{3}$ c) $\alpha>\frac{2\pi}{3}$ d) $\frac{\pi}{3}<\alpha<\frac{2\pi}{3}$
3. The cartesian from of the plane $\vec{\mathbf{r}}=(s-2t)\hat{\mathbf{i}}+(3-t)\hat{\mathbf{j}}+(2s+t)\hat{\mathbf{k}}$ is
   a) $2x-5y-z-15=0$ b) $2x-5y+z-15=0$
   c) $2x-5y-z+15=0$ d) $2x+5y-z+15=0$
4. If $\vec{\mathbf{a}}=4\hat{\mathbf{i}}+6\hat{\mathbf{j}}$ and $\vec{\mathbf{b}}=3\hat{\mathbf{j}}+4\hat{\mathbf{k}}$, the vector form of the component of $\vec{\mathbf{a}}$ along $\vec{\mathbf{b}}$ is
   a) $\frac{18}{5}(3\hat{\mathbf{i}}+4\hat{\mathbf{k}})$ b) $\frac{18}{25}(3\hat{\mathbf{j}}+4\hat{\mathbf{k}})$ c) $\frac{36}{25}(3\hat{\mathbf{j}}+4\hat{\mathbf{k}})$ d) $\frac{19}{18}(2\hat{\mathbf{i}}+3\hat{\mathbf{j}})$
5. A force $\vec{\mathbf{F}}=2\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$ acts at a point A, whose position vectors is $2\hat{\mathbf{i}}-\hat{\mathbf{j}}$. The moment of $\vec{\mathbf{F}}$ about the origin is
   a) $\hat{\mathbf{i}}+2\hat{\mathbf{j}}-4\hat{\mathbf{k}}$ b) $\hat{\mathbf{i}}-2\hat{\mathbf{j}}-4\hat{\mathbf{k}}$ c) $\hat{\mathbf{i}}+2\hat{\mathbf{j}}+4\hat{\mathbf{k}}$ d) $\hat{\mathbf{i}}-2\hat{\mathbf{j}}+4\hat{\mathbf{k}}$
6. If $\vec{a},\vec{b},\vec{c}$ are linearly independent vectors, then
   $\frac{(\vec{a}+2\vec{b})\times(2\vec{b}+\vec{c})\cdot(5\vec{c}+\vec{a})}{\vec{a}\cdot(\vec{b}\times\vec{c})}$ is equal to
   a) 10 b) 14 c) 18 d) 12
7. If $\vec{\mathbf{a}},\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are perpendicular to $\vec{\mathbf{b}}+\vec{\mathbf{c}},\vec{\mathbf{c}}+\vec{\mathbf{a}}$ and $\vec{\mathbf{a}}+\vec{\mathbf{b}}$ respectively and if $|\vec{\mathbf{a}}+\vec{\mathbf{b}}|=6$, $|\vec{\mathbf{b}}+\vec{\mathbf{c}}|=8$ and $|\vec{\mathbf{c}}+\vec{\mathbf{a}}|=10$, then $|\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}|$ is equal to
   a) $5\sqrt{5}$ b) 50 c) $10\sqrt{2}$ d) 10
8. If $\vec{a},\vec{b},\vec{c}$ are three mutually perpendicular vectors of equal magnitude, then the angle θ which $\vec{a}+\vec{b}+\vec{c}$ makes with any one of three given vectors is given by
   a) $\cos^{-1}\frac{1}{\sqrt{3}}$ b) $\cos^{-1}\frac{1}{3}$ c) $\cos^{-1}\frac{2}{\sqrt{3}}$ d) None of these
9. Forces $3\ \vec{OA}, 5\ \vec{OB}$ act along OA and OB. If their resultant passes through C on AB, then
   a) C is a mid-point of AB
   b) C divides AB in the ratio 2 : 1
   c) $3\ AC=5\ CB$
   d) $2\ AC=3\ CB$
10. The centre of the circle given by $\vec{\mathbf{r}}\cdot(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+2\hat{\mathbf{k}})=15$ and $\vec{\mathbf{r}}-(\hat{\mathbf{j}}+2\hat{\mathbf{k}})=4$ is
    a) (1,2,4) b) (3,1,4) c) (1,3,4) d) None of these
11. Consider a tetrahedron with faces F_1,F_2,F_3,F_4. Let $\vec{v}_1,\vec{v}_2,\vec{v}_3,\vec{v}_4$ be the vectors whose magnitudes are respectively equal to areas of F_1,F_2,F_3,F_4 and whose directions are perpendicular to these faces in outward direction. Then, $|\vec{v}_1+\vec{v}_2+\vec{v}_3+\vec{v}_4|$ equals
    a) 1 b) 4 c) 0 d) None of these
12. The volume of the tetrahedron having the edges $\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}},\ \hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}},\hat{\mathbf{i}}-\hat{\mathbf{j}}+\lambda\hat{\mathbf{k}}$ as coterminous is $\frac{2}{3}$ cu unit. Then, λ equals
    a) 1 b) 2 c) 3 d) 4
13. If $\vec{\mathbf{a}},\vec{\mathbf{b}},\vec{\mathbf{c}}$ are three non-coplanar vectors then the vector equation
    $\vec{\mathbf{r}}=(1-p-q)\vec{\mathbf{a}}+p\vec{\mathbf{b}}+q\vec{\mathbf{c}}$ represent a
    a) Straight line b) Plane

c) Plane passing through the origin d) Sphere

14. A force of magnitude 5 units acting along the vector $2\hat{\imath} - 2\hat{\jmath} + \hat{k}$ displaces the point of application from the point (1, 2, 3) to the point (5, 3, 7), then the work done by the force is

a) $\frac{50}{7}$ units b) $\frac{50}{3}$ units c) $\frac{25}{3}$ units d) $\frac{25}{4}$ units

15. If $\vec{\alpha} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\vec{\beta} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$, $\vec{\gamma} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, then what is the value of $(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \cdot (\vec{\alpha} \times \vec{\gamma})$?

a) 47 b) 74 c) -74 d) None of these

16. The line of intersection of the planes $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}) = 1$ and $\vec{\mathbf{r}} \cdot (2\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) = 2$ is parallel to the vector

a) $-4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$ b) $4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$ c) $-4\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$ d) $-4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 11\hat{\mathbf{k}}$

17. $ABCDEF$ is a regular hexagon with centre at the origin such that $\overrightarrow{\mathbf{AD}} + \overrightarrow{\mathbf{EB}} + \overrightarrow{\mathbf{FC}} = \lambda\, \overrightarrow{\mathbf{ED}}$. Then, λ is equal to

a) 2 b) 4 c) 6 d) 3

18. If $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are two non-zero, non-collinear vectors, then $2[\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \hat{\mathbf{i}}]\hat{\mathbf{i}} + 2[\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \hat{\mathbf{j}}]\hat{\mathbf{j}} + 2[\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \hat{\mathbf{k}}]\hat{\mathbf{k}} + [\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{a}}]$ is equal to

a) $2(\vec{\mathbf{a}} \times \vec{\mathbf{b}})$ b) $\vec{\mathbf{a}} \times \vec{\mathbf{b}}$ c) $\vec{\mathbf{a}} + \vec{\mathbf{b}}$ d) None of these

19. If $\vec{\mathbf{a}} = (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$, $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 1$ and $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \hat{\mathbf{j}} - \hat{\mathbf{k}}$, then $\vec{\mathbf{b}}$ is

a) $\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ b) $2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ c) $\hat{\mathbf{i}}$ d) $2\hat{\mathbf{i}}$

20. Let $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ be three non-coplanar vectors and let $\vec{\mathbf{p}}, \vec{\mathbf{q}}$ and $\vec{\mathbf{r}}$ be vector defined by the relations. $\vec{\mathbf{p}} = \frac{\vec{\mathbf{b}} \times \vec{\mathbf{c}}}{[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]}, \vec{\mathbf{q}} = \frac{\vec{\mathbf{c}} \times \vec{\mathbf{a}}}{[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]}$ and $\vec{\mathbf{r}} = \frac{\vec{\mathbf{a}} \times \vec{\mathbf{b}}}{[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]}$. Then, the value of the expression $(\vec{\mathbf{a}} + \vec{\mathbf{b}}) \cdot \vec{\mathbf{p}} + (\vec{\mathbf{b}} + \vec{\mathbf{c}}) \cdot \vec{\mathbf{q}} + (\vec{\mathbf{c}} + \vec{\mathbf{a}}) \cdot \vec{\mathbf{r}}$ is equal to

a) 0 b) 1 c) 2 d) 3

21. If m_1, m_2, m_3 and m_4 are respectively the magnitudes of the vectors $\vec{\mathbf{a_1}} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}, \vec{\mathbf{a_2}} = 3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}}, \vec{\mathbf{a_3}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\vec{\mathbf{a_4}} = \widehat{-\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$, then the correct order of m_1, m_2, m_3 and m_4 is

a) $m_3 < m_1 < m_4 < m_2$ b) $m_3 < m_1 < m_2 < m_4$ c) $m_3 < m_4 < m_1 < m_2$ d) $m_3 < m_4 < m_2 < m_1$

22. If $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are non-coplanar vectors and λ is a real number, then $[\lambda(\vec{\mathbf{a}} + \vec{\mathbf{b}})\lambda^2\ \vec{\mathbf{b}}\ \lambda\ \vec{\mathbf{c}}] = [\vec{\mathbf{a}}\,\vec{\mathbf{b}} + \vec{\mathbf{c}}\,\vec{\mathbf{b}}]$ for

a) exactly two values of λ b) exactly three values of λ

c) no real values of λ d) exactly one values of λ

23. Let a, b and c be distinct non-negative numbers. If the vectors $a\hat{\mathbf{i}} + a\hat{\mathbf{j}} + c\hat{\mathbf{k}}, \hat{\mathbf{i}} + \hat{\mathbf{k}}$ and $c\hat{\mathbf{i}} + c\hat{\mathbf{j}} + b\hat{\mathbf{k}}$ lie in a plane, then c is

a) The harmonic mean of a and b b) Equal to zero

c) The arithmetic mean of a and b d) The geometric mean of a and b

24. In a trapezium $ABCD$ the vector $\vec{BC} = \lambda\, \vec{AD}$. If $\vec{P} = \vec{AC} + \vec{BD}$ is collinear with $\vec{AD}$ such that $\vec{p} = \mu\, \vec{AD}$, then

a) $\mu = \lambda + 1$ b) $\lambda = \mu + 1$ c) $\lambda + \mu = 1$ d) $\mu = 2 + \lambda$

25. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 2, |\vec{b}| = 3, |\vec{c}| = 4$, then the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ is equal to

a) 29 b) -29 c) 29/2 d) $-29/2$

26. If $|\vec{\mathbf{A}}| = 3, |\vec{\mathbf{b}}| = 4$, then a value of λ for which $\vec{\mathbf{a}} + \lambda\vec{\mathbf{b}}$ is perpendicular to $\vec{\mathbf{a}} - \lambda\vec{\mathbf{b}}$ is

a) $\frac{9}{16}$ b) $\frac{3}{4}$ c) $\frac{3}{2}$ d) $\frac{4}{3}$

27. $\vec{\mathbf{u}} = \hat{\mathbf{i}} \times (\vec{\mathbf{a}} \times \hat{\mathbf{i}}) + \hat{\mathbf{j}} \times (\vec{\mathbf{a}} \times \hat{\mathbf{j}}) + \hat{\mathbf{k}} \times (\vec{\mathbf{a}} \times \hat{\mathbf{k}})$ is equal

a) $\vec{\mathbf{a}}$ b) $2\vec{\mathbf{a}}$ c) $3\vec{\mathbf{a}}$ d) None of these

28. The locus of a point equidistant from two points whose position vectors are $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$, is

a) $\left\{\vec{\mathbf{r}} - \frac{1}{2}(\vec{\mathbf{a}} + \vec{\mathbf{b}})\right\}(\vec{\mathbf{a}} - \vec{\mathbf{b}}) = 0$ b) $\{\vec{\mathbf{r}} - (\vec{\mathbf{a}} + \vec{\mathbf{b}})\} \cdot \vec{\mathbf{b}} = 0$

c) $\left\{\vec{\mathbf{r}} - \frac{1}{2}(\vec{\mathbf{a}} + \vec{\mathbf{b}})\right\} \cdot \vec{\mathbf{a}} = 0$ d) $\left\{\vec{\mathbf{r}} - \frac{1}{2}(\vec{\mathbf{a}} - \vec{\mathbf{b}})\right\} \cdot (\vec{\mathbf{a}} + \vec{\mathbf{b}}) = 0$

29. If $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are two vectors such that $|\vec{\mathbf{a}}| + 3\sqrt{3}, \vec{\mathbf{b}} = 4$ and $|\vec{\mathbf{a}} + \vec{\mathbf{b}}| = \sqrt{7}$, then the angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is

a) $120°$ b) $60°$ c) $30°$ d) $150°$

30. If $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\vec{\mathbf{b}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$, $\vec{\mathbf{c}} = 3\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - \hat{\mathbf{k}}$, then a vector perpendicular to $\vec{\mathbf{a}}$ and in the plane containing $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ is

a) $-17\hat{\mathbf{i}} + 21\hat{\mathbf{j}} - 97\hat{\mathbf{k}}$ b) $17\hat{\mathbf{i}} + 21\hat{\mathbf{j}} - 123\hat{\mathbf{k}}$ c) $-17\hat{\mathbf{i}} - 21\hat{\mathbf{j}} + 97\hat{\mathbf{k}}$ d) $-17\hat{\mathbf{i}} - 21\hat{\mathbf{j}} - 97\hat{\mathbf{k}}$

31. If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular vectors each of magnitude unity, then $|\vec{a} + \vec{b} + \vec{c}|$ is equal to

a) 3 b) 1 c) $\sqrt{3}$ d) None of these

32. $(\vec{\mathbf{a}} \cdot \hat{\mathbf{i}})\hat{\mathbf{i}} + (\vec{\mathbf{a}} \cdot \hat{\mathbf{j}})\hat{\mathbf{j}} + (\vec{\mathbf{a}} \cdot \hat{\mathbf{k}})\hat{\mathbf{k}}$ is equal to

a) $\vec{\mathbf{a}}$ b) $2\,\vec{\mathbf{a}}$ c) $3\,\vec{\mathbf{a}}$ d) $\vec{\mathbf{0}}$

33. If $\vec{\mathbf{a}} \cdot \hat{\mathbf{i}} = \vec{\mathbf{a}} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}}) = \vec{\mathbf{a}} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$, then $\vec{\mathbf{a}}$ is equal to

a) $\hat{\mathbf{i}}$ b) $\hat{\mathbf{k}}$ c) $\hat{\mathbf{j}}$ d) $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

34. Let $\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}, \vec{\mathbf{b}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\vec{\mathbf{c}}$ be a unit victor perpendicular to $\vec{\mathbf{a}}$ and coplanar with $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$, then $\vec{\mathbf{c}}$ is

a) $\frac{1}{\sqrt{2}}(\hat{\mathbf{j}} + \hat{\mathbf{k}})$ b) $\frac{1}{\sqrt{2}}(\hat{\mathbf{j}} - \hat{\mathbf{k}})$ c) $\frac{1}{\sqrt{6}}(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}})$ d) $\frac{1}{\sqrt{6}}(2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})$

35. The plane through the point $(-1,-1,-1)$ and containing the line of intersection of the planes $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}) = 0$ and $\vec{\mathbf{r}} \cdot (\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 0$ is

a) $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) = 0$ b) $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + \hat{\mathbf{k}}) = 0$ c) $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 5\hat{\mathbf{k}}) = 0$ d) $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}) = 0$

36. If a parallelogram is constructed on the vectors $\vec{a} = 3\vec{\mu} - \vec{v}, \vec{b} = \vec{u} + 3\vec{v}$ and $|\vec{u}| = |\vec{v}| = 2$ and the angle between $\vec{u}$ is $\pi/3$, then the ratio of the lengths of the sides is

a) $\sqrt{7}:\sqrt{13}$ b) $\sqrt{6}:\sqrt{2}$ c) $\sqrt{3}:\sqrt{5}$ d) None of these

37. Let $\vec{a}, \vec{b}, \vec{c}$ be the position vectors of the vertices A, B, C respectively of ΔABC. The vector area of ΔABC is

a) $\frac{1}{2}\{\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b})\}$

b) $\frac{1}{2}(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a})$

c) $\frac{1}{2}(\vec{a} + \vec{b} + \vec{c})$

d) $\frac{1}{2}\{(\vec{b}.\vec{c})\vec{a} + (\vec{c}.\vec{a})\vec{b} + (\vec{a}.\vec{b})\vec{c}\}$

38. The work done in moving an object along a vector $\vec{d} = 3\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$ if the applied force is $\vec{F} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$ is

a) 12 units b) 11 units c) 10 units d) 9 units

39. $\vec{\mathbf{a}} \times [\vec{\mathbf{a}} \times (\vec{\mathbf{a}} \times \vec{\mathbf{b}})]$ is equal to

a) $(\vec{\mathbf{a}} \times \vec{\mathbf{a}}) \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{a}})$ b) $\vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{a}}) - \vec{\mathbf{b}}(\vec{\mathbf{a}} \times \vec{\mathbf{b}})$

c) $[\vec{\mathbf{a}} \cdot (\vec{\mathbf{a}} \times \vec{\mathbf{b}})]\vec{\mathbf{a}}$ d) $(\vec{\mathbf{a}} \cdot \vec{\mathbf{a}})(\vec{\mathbf{b}} \times \vec{\mathbf{a}})$

40. If $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are non-coplanar vectors, then $\begin{vmatrix} \vec{\mathbf{a}} \cdot \vec{\mathbf{a}} & \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} & \vec{\mathbf{a}} \cdot \vec{\mathbf{c}} \\ \vec{\mathbf{b}} \cdot \vec{\mathbf{a}} & \vec{\mathbf{b}} \cdot \vec{\mathbf{b}} & \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} \\ \vec{\mathbf{c}} \cdot \vec{\mathbf{a}} & \vec{\mathbf{c}} \cdot \vec{\mathbf{b}} & \vec{\mathbf{c}} \cdot \vec{\mathbf{c}} \end{vmatrix}$ is equal to

a) $[\,\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}\,]^2$ b) $[\,\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}\,]$ c) $[\,\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}\,]^{1/3}$ d) None of these

41. If $|\vec{\mathbf{a}}| = 10, |\vec{\mathbf{b}}| = 2$ and $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 12$ then $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}|$ is equal to

a) 12 b) 14 c) 16 d) 18

42. If $|\vec{a}| = 4, |\vec{b}| = 4$ and $|\vec{c}| = 5$ such that $\vec{a} \perp (\vec{b} + \vec{c}), \vec{b} \perp (\vec{c} + \vec{a})$ and $\vec{c} \perp (\vec{a} + \vec{b})$, then $|\vec{a} + \vec{b} + \vec{c}|$ is

a) 7 b) 5 c) 13 d) $\sqrt{57}$

43. The summation of two unit vectors is a third unit vector, then the modulus of the difference of the unit vectors is

a) $\sqrt{3}$ b) $1 - \sqrt{3}$ c) $1 + \sqrt{3}$ d) $-\sqrt{3}$

44. If θ is the angle between vectors $\vec{a}$ and $\vec{b}$ such that $\vec{a}.\vec{b} \geq 0$, then

a) $0 \leq \theta \leq \pi$ b) $\frac{\pi}{2} \leq \theta \leq \pi$ c) $0 \leq \theta \leq \frac{\pi}{2}$ d) $0 < \theta < \frac{\pi}{2}$

45. The vectors $2\hat{\imath} + 3\hat{\jmath}, 5\hat{\imath} + 6\hat{\jmath}$ and $8\hat{\imath} + \lambda\hat{\jmath}$ have their initial points at $(1, 1)$. The value of λ so that the vectors terminate on one straight line, is

a) 0 b) 3 c) 6 d) 9

46. If pth, qth, rth term of a GP are the positive numbers a, b, c then angle between the vectors $\log a^3\hat{\mathbf{i}} +$ $\log b3\mathbf{j} + \log c3\mathbf{k}$ and $q - r\mathbf{i} + r - p\mathbf{j} + (p - q)\mathbf{k}$ is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{2}$

c) $\frac{\pi}{3}$ d) $\sin^{-1}\left(\frac{1}{\sqrt{a^2 + b^2 + c^2}}\right)$

47. The value of λ, for which the four points $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}, \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}, 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 2\hat{\mathbf{k}}, \hat{\mathbf{i}} - \lambda\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ are coplanar, is

a) -2 b) 8 c) 6 d) 0

48. Given that $|\vec{\mathbf{a}}| = 3, |\vec{\mathbf{b}}| = 4, |\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = 10$, then $|\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}|^2$ equals

a) 88 b) 44 c) 22 d) None of these

49. If the diagonals of a parallelogram are $3\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$, then the lengths of its sides are

a) $\sqrt{8}, \sqrt{10}$ b) $\sqrt{6}, \sqrt{14}$ c) $\sqrt{5}, \sqrt{12}$ d) None of these

50. If $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{c}}$ and $\vec{\mathbf{b}} \times \vec{\mathbf{c}} = \vec{\mathbf{a}}$, then

a) $|\vec{\mathbf{a}}| = 1, |\vec{\mathbf{b}}| = |\vec{\mathbf{c}}|$ b) $|\vec{\mathbf{c}}| = 1, |\vec{\mathbf{a}}| = 1$ c) $|\vec{\mathbf{b}}| = 2, |\vec{\mathbf{b}}| = 2|\vec{\mathbf{a}}|$ d) $|\vec{\mathbf{b}}| = 1, |\vec{\mathbf{c}}| = |\vec{\mathbf{a}}|$

51. Let $\vec{\mathbf{a}} = \hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\vec{\mathbf{c}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$. Then the vectors $\vec{\mathbf{b}}$ satisfying $\vec{\mathbf{a}} \times \vec{\mathbf{b}} + \vec{\mathbf{c}} = \vec{\mathbf{0}}$ and $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 3$ is

a) $-\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ b) $2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ c) $\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ d) $\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

52. The value of c so that for all real x, the vectors $cx\,\hat{\imath} - 6\hat{\jmath} + 3\hat{k}, x\hat{\imath} + 2\hat{\jmath} + 2cx\,\hat{k}$ make an obtuse angle are

a) $c < 0$ b) $0 < c < 4/3$ c) $-4/3 < c < 0$ d) $c > 0$

53. If $\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}} = \vec{\mathbf{0}}$ and $|\vec{\mathbf{a}}| = 3, |\vec{\mathbf{b}}| = 5, \vec{\mathbf{a}} + |\vec{\mathbf{c}}| = 7$, then angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{2}$ d) π

54. If $\vec{\mathbf{p}} = \hat{\mathbf{i}} + \hat{\mathbf{j}}, \vec{\mathbf{q}} = 4\hat{\mathbf{k}} - \hat{\mathbf{j}}$ and $\vec{\mathbf{r}} = \hat{\mathbf{i}} + \hat{\mathbf{k}}$,then the unit vector in the direction of $3\vec{\mathbf{p}} + \vec{\mathbf{q}} - 2\vec{\mathbf{r}}$ is

a) $\frac{1}{3}(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ b) $\frac{1}{3}(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$ c) $\frac{1}{3}(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ d) $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

55. The vectors $\vec{X}$ and $\vec{Y}$ satisfy the equations $2\vec{X} + \vec{Y} = \vec{p}$ and $\vec{X} + 2\vec{Y} = \vec{q}$, where $\vec{p} = \vec{\imath} + \vec{\jmath}$ and $\vec{q} = \hat{\imath} - \hat{\jmath}$. If θ is the angle between $\vec{X}$ and $\vec{Y}$, then

a) $\cos\theta = \frac{4}{5}$ b) $\sin\theta = \frac{1}{\sqrt{2}}$ c) $\cos\theta = -\frac{4}{5}$ d) $\cos\theta = -\frac{3}{5}$

56. If $\vec{\mathbf{p}}, \vec{\mathbf{q}}$ and $\vec{\mathbf{r}}$ are perpendicular to $\vec{\mathbf{q}} + \vec{\mathbf{r}}, \vec{\mathbf{r}} + \vec{\mathbf{p}}$ and $\vec{\mathbf{p}} + \vec{\mathbf{q}}$ respectively and if $|\vec{\mathbf{p}} + \vec{\mathbf{q}}| = 6, |\vec{\mathbf{q}} + \vec{\mathbf{r}}| = 4\sqrt{3}$ and $|\vec{\mathbf{r}} + \vec{\mathbf{p}}| = 4$, then $|\vec{\mathbf{p}} + \vec{\mathbf{q}} + \vec{\mathbf{r}}|$ is

a) $5\sqrt{2}$ b) 10 c) 15 d) 5

57. Let $ABCD$ be a parallelogram and M be the point of intersection of the diagonals. If O is any point, then $\vec{O}A + \vec{O}B + \vec{O}C + \vec{O}D =$

a) $3\,\vec{O}M$ b) $4\,\vec{O}M$ c) $2\,\vec{O}M$ d) $\vec{O}M$

58. The work done by the force $\vec{F} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$ in moving an object along the vector $3\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$ is

a) -9 units b) 15 units c) 9 units d) None of these

59. $[\hat{\mathbf{i}}\,\hat{\mathbf{k}}\,\hat{\mathbf{j}}] + [\hat{\mathbf{k}}\,\hat{\mathbf{j}}\,\hat{\mathbf{i}}] + [\hat{\mathbf{j}}\,\hat{\mathbf{k}}\,\hat{\mathbf{i}}]$ is equal to

a) 1 b) 3 c) -3 d) -1

60. If $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are the unit vectors such that $\vec{\mathbf{a}}$ is perpendicular to the plane $\vec{\mathbf{b}}, \vec{\mathbf{c}}$ and the angle between $\vec{\mathbf{b}}, \vec{\mathbf{c}}$ is $\frac{\pi}{3}$ then $|\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}|$ is equal to

a) 0 b) ± 1 c) ± 2 d) ± 3

61. A parallelogram is constructed on the vectors $\vec{\mathbf{a}} = 3\vec{\alpha} - \vec{\beta}, \vec{\mathbf{b}} = \vec{\alpha} + 3\vec{\beta}$, if $|\vec{\alpha}| = |\vec{\beta}| = 2$ and angle between $\vec{\alpha}$ and $\vec{\beta}$ is $\frac{\pi}{3}$, then length of diagonal of the parallelogram is

a) $4\sqrt{7}$ b) $4\sqrt{3}$ c) $4\sqrt{17}$ d) None of these

62. If $|\vec{\mathbf{a}}| = |\vec{\mathbf{b}}|$, then $(\vec{\mathbf{a}} + \vec{\mathbf{b}}) \cdot (\vec{\mathbf{a}} - \vec{\mathbf{b}})$ is

a) Positive b) Negative c) Zero d) None of these

63. If $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}$ and $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}, \vec{a}\neq 0$, then

a) $\vec{b}=\vec{c}$ b) $\vec{b}-\vec{c}||\vec{a}$ c) $\vec{b}-\vec{c}\perp\vec{a}$ d) None of these

64. If the volume of the tetrahedron whose vertices are $(1,-6,10),(-1,-3,7),(5,-1,\lambda)$ and $(7,-4,7)$ is 11 cubic units, then $\lambda=$

a) 2, 6 b) 3, 4 c) 1, 7 d) 5, 6

65. The vector $\frac{1}{3}\left(2\hat{\imath}-2\hat{\jmath}+\hat{k}\right)$ is

a) Unit vector

b) Parallel to the vector $\hat{\imath}+\hat{\jmath}-1/2\hat{k}$

c) Perpendicular to the vector $3\hat{\imath}+2\hat{\jmath}-2\hat{k}$

d) All the above

66. If $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{r}.\vec{a}=0$ where $\vec{a}=2\hat{\imath}+3\hat{\jmath}-\hat{k}, \vec{b}=3\hat{\imath}-\hat{\jmath}+\hat{k}$ and $\vec{c}=\hat{\imath}+\hat{\jmath}+3\hat{k}$, then $\vec{r}=$

a) $\frac{1}{2}\left(\hat{\imath}+\hat{\jmath}+\hat{k}\right)$ b) $2\left(\hat{\imath}+\hat{\jmath}+\hat{k}\right)$ c) $2\left(-\hat{\imath}+\hat{\jmath}+\hat{k}\right)$ d) $\frac{1}{2}\left(\hat{\imath}-\hat{\jmath}+\hat{k}\right)$

67. $(\vec{\mathbf{a}}\cdot\hat{\mathbf{i}})\hat{\mathbf{i}}+(\vec{\mathbf{a}}\cdot\hat{\mathbf{j}})\hat{\mathbf{j}}+(\vec{\mathbf{a}}\cdot\hat{\mathbf{k}})\hat{\mathbf{k}}$ is equal to

a) $\vec{\mathbf{a}}$ b) $2\vec{\mathbf{a}}$ c) $3\vec{\mathbf{a}}$ d) $\vec{\mathbf{0}}$

68. If $|\vec{a}|=3, |\vec{b}|=4$ and $|\vec{a}+\vec{b}|=5$, then $|\vec{a}-\vec{b}|=$

a) 6 b) 5 c) 4 d) 3

69. If $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are any two non-collinear unit vectors and $\vec{\mathbf{a}}$ is any vector, then

$$(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}})+\vec{\mathbf{b}}(\vec{\mathbf{a}}\cdot\vec{\mathbf{c}})\vec{\mathbf{c}}+\frac{\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})}{|\vec{\mathbf{b}}\times\vec{\mathbf{c}}|}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})$$

is equal to

a) $\vec{\mathbf{0}}$ b) $\vec{\mathbf{a}}$ c) $\vec{\mathbf{b}}$ d) $\vec{\mathbf{c}}$

70. The unit vector perpendicular to $\hat{\mathbf{i}}-\hat{\mathbf{j}}$ and coplanar with $\hat{\mathbf{i}}+2\hat{\mathbf{j}}$ and $2\hat{\mathbf{i}}+3\hat{\mathbf{j}}$ is

a) $\frac{2\hat{\mathbf{i}}-5\hat{\mathbf{j}}}{\sqrt{2a}}$ b) $2\hat{\mathbf{i}}+5\hat{\mathbf{j}}$ c) $\frac{1}{\sqrt{2}}(\hat{\mathbf{i}}+\hat{\mathbf{j}})$ d) $\hat{\mathbf{i}}+\hat{\mathbf{j}}$

71. Unit vector which is perpendicular to both the vectors $3\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and $2\hat{\mathbf{i}}-2\hat{\mathbf{j}}+4\hat{\mathbf{k}}$ is

a) $\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}}{\sqrt{3}}$ b) $\frac{\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}}{\sqrt{3}}$ c) $\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}}{\sqrt{3}}$ d) $\frac{\hat{\mathbf{i}}-\hat{\mathbf{j}}-\hat{\mathbf{k}}}{\sqrt{3}}$

72. If the position vector of the vertices, A, B, C of a ΔABC are $7\hat{\mathbf{j}}+10\hat{\mathbf{k}}, -\hat{\mathbf{i}}+6\hat{\mathbf{j}}+6\hat{\mathbf{k}}$ and $-4\hat{\mathbf{i}}+9\hat{\mathbf{j}}+6\hat{\mathbf{k}}$ respectively, then triangle is

a) Equilateral b) Isosceles

c) Scalene d) Right angled and isosceles also

73. If three points A, B and C have position vectors $\hat{\imath}+x\hat{\jmath}+3\hat{k}, 3\hat{\imath}+4\hat{\jmath}+7\hat{k}$ and $y\hat{\imath}-2\hat{\jmath}-5\hat{k}$ respectively are collinear, then $(x,y)=$

a) $(2,-3)$ b) $(-2,3)$ c) $(-2,-3)$ d) $(2,3)$

74. The vectors $\vec{\mathbf{a}}(x)=\cos x\,\hat{\mathbf{i}}+(\sin x)\hat{\mathbf{j}}$ and $\vec{\mathbf{b}}(x)=x\,\hat{\mathbf{i}}+\sin x\,\hat{\mathbf{j}}$ are collinear for

a) Unique value of $x, 0<x<\frac{\pi}{6}$ b) Unique value of $x, \frac{\pi}{6}<x<\frac{\pi}{3}$

c) No value of x d) Infinitely many values of $x, 0<x<\frac{\pi}{2}$

75. A unit vector in xy-plane makes an angle of 45° with the vector $\hat{\mathbf{i}}+\hat{\mathbf{j}}$ and an angle of 60° with the vector $3\hat{\mathbf{i}}-4\hat{\mathbf{j}}$ is

a) $\hat{\mathbf{i}}$ b) $\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}$ c) $\frac{\hat{\mathbf{i}}-\hat{\mathbf{j}}}{\sqrt{2}}$ d) None of these

76. The vector $\vec{a}$ lies in the plane of vectors $\vec{b}$ and $\vec{c}$, which of the following is correct

a) $\vec{a}.(\vec{b}\times\vec{c})=0$ b) $\vec{a}.\vec{b}\times\vec{c}=1$ c) $\vec{a}.\vec{b}\times\vec{c}=-1$ d) $\vec{a}.\vec{b}\times\vec{c}=3$

77. If the volume of parallelopiped with coterminous $4\hat{\mathbf{i}}+5\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $3\hat{\mathbf{i}}-9\hat{\mathbf{j}}+p\hat{\mathbf{k}}$ is 34 cu units, then p is

equal to

a) 4 b) -13 c) 13 d) 6

78. The value of $\dfrac{(\vec{\mathbf{a}}\times\vec{\mathbf{b}})^2+(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}})^2}{2\vec{\mathbf{a}}^2\vec{\mathbf{b}}^2}$ is

a) $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}$ b) 1 c) 0 d) $\frac{1}{2}$

79. The magnitude of cross product of two vectors is $\sqrt{3}$ times the dot product. The angle between the vectors is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$

80. If G is the intersection of diagonals of a parallelogram $ABCD$ and O is any point, then $O\vec{A}+O\vec{B}+O\vec{C}+O\vec{D}=$

a) $2\,\vec{O}G$ b) $4\,\vec{O}G$ c) $5\,\vec{O}G$ d) $3\,\vec{O}G$

81. If $\vec{a}=(-1,1,1)$ and $\vec{b}=(2,0,1)$, then the vector $\vec{X}$ satisfying the conditions

(i) that it is coplanar with $\vec{a}$ and $\vec{b}$

(ii) that it is perpendicular to $\vec{b}$, (iii) that $\vec{a}.\vec{X}=7$ is,

a) $-3\hat{\imath}+4\hat{\jmath}+6\hat{k}$ b) $-\frac{3}{2}\hat{\imath}+\frac{5}{2}\hat{\jmath}+3\hat{k}$ c) $3\hat{\imath}+16\hat{\jmath}-6\hat{k}$ d) None of these

82. If $ABCDEF$ is a regular hexagon, then $\vec{A}C+\vec{A}D+\vec{E}A+\vec{F}A=$

a) $2\vec{A}B$ b) $3\vec{A}B$ c) $\vec{A}B$ d) $\vec{0}$

83. $[(\vec{a}\times\vec{b})\times(\vec{b}\times\vec{c})\;(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})\;(\vec{c}\times\vec{a})\times(\vec{a}\times\vec{b})]$ is equal to

a) $[\vec{a}\vec{b}\vec{c}]^2$ b) $[\vec{a}\vec{b}\vec{c}]^3$ c) $[\vec{a}\vec{b}\vec{c}]^4$ d) None of these

84. Suppose $\vec{\mathbf{a}}=\lambda\,\hat{\mathbf{i}}-7\hat{\mathbf{j}}+3\hat{\mathbf{k}},\vec{\mathbf{b}}=\lambda\,\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\lambda\,\hat{\mathbf{k}}$. If the angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is grater than 90°, then λ satisfies the inequality

a) $-7<\lambda<1$ b) $\lambda>1$ c) $1<\lambda<7$ d) $-5<\lambda<1$

85. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-coplanar vectors such that $\vec{r_1}=\vec{a}-\vec{b}+\vec{c},\vec{r_2}=\vec{b}+\vec{c}-\vec{a},\vec{r_3}=\vec{c}+\vec{a}+\vec{b},\vec{r}=2\vec{a}-3\vec{b}+4\vec{c}$

If $\vec{r}=\lambda_1\vec{r_1}+\lambda_2\vec{r_2}+\lambda_3\vec{r_3}$, then

a) $\lambda_1=7$ b) $\lambda_1+\lambda_3=3$ c) $\lambda_1+\lambda_2+\lambda_3=3$ d) $\lambda_3+\lambda_2=2$

86. If $\vec{a},\vec{b},\vec{c},\vec{d}$ are the position vectors of points A,B,C and D respectively such that $(\vec{a}-\vec{d})\cdot(\vec{b}-\vec{c})=(\vec{b}-\vec{a})\cdot(\vec{c}-\vec{a})=0$, then D is the

a) Centroid of ΔABC

b) Circumcentre of ΔABC

c) Orthocenter of ΔABC

d) None of these

87. A,B,C,D,E,F in that order, are the vertices of a regular hexagon with center origin. If the position vectors A and B are respectively, $4\,\hat{\mathbf{i}}+3\hat{\mathbf{j}}-\hat{\mathbf{k}}$ and $-3\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$, then $\overrightarrow{\mathbf{DE}}$ is equal to

a) $7\hat{\mathbf{i}}+2\hat{\mathbf{j}}-2\hat{\mathbf{k}}$ b) $-7\hat{\mathbf{i}}-2\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ c) $3\hat{\mathbf{i}}-\hat{\mathbf{j}}-\hat{\mathbf{k}}$ d) $-4\hat{\mathbf{i}}-3\hat{\mathbf{j}}+2\hat{\mathbf{k}}$

88. If $|\vec{\mathbf{a}}\times\vec{\mathbf{b}}|=|\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}|$, then the angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is

a) Π b) $\frac{2\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{2}$

89. The ratio in which $\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ divides the join of $-2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+5\hat{\mathbf{k}}$ and $7\hat{\mathbf{i}}-\hat{\mathbf{k}}$ is

a) 2:1 b) 2:3 c) 3:4 d) 1:4

90. The values of x for which the angle between the vectors $\vec{a}=x\hat{\imath}-3\hat{\jmath}-\hat{k}$ and $\vec{b}=2x\hat{\imath}-x\hat{\jmath}-\hat{k}$ is acute and angle between $\vec{b}$ and y-axis lies between $\pi/2$ and π are

a) -1 b) All $x>0$ c) 1 d) All $x<0$

91. The moment about the point $M(-2,4,-6)$ of the force represented in magnitude and position $\overrightarrow{\mathbf{AB}}$ where

the points A and B have the coordinates $(1, 2, -3)$ and $(3, -4, 2)$ respectively, is

a) $8\hat{\mathbf{i}} - 9\hat{\mathbf{j}} - 14\hat{\mathbf{k}}$ b) $2\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ c) $-3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ d) $-5\hat{\mathbf{i}} - 8\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$

92. The angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is $\frac{5\pi}{6}$ and the projection of $\vec{\mathbf{a}}$ in the direction of $\vec{\mathbf{b}}$ is $\frac{-6}{\sqrt{3}}$, then $|\vec{\mathbf{a}}|$ is equal to

a) 6 b) $\frac{\sqrt{3}}{2}$ c) 12 d) 4

93. The equation of the line passing through the points $a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$ and $b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + b_3\hat{\mathbf{k}}$ is

a) $(a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}) + t(b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + b_3\hat{\mathbf{k}})$ b) $(a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}) - t(b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + b_3\hat{\mathbf{k}})$

c) $a_1(1-t)\hat{\mathbf{i}} + a_2(1-t)\hat{\mathbf{j}} + a_3(1-t)\hat{\mathbf{k}} + (b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + b_3\hat{\mathbf{k}})t$ d) None of the above

94. The vector $\vec{b} = 3\hat{\imath} + 4\hat{k}$ is to be written as the sum of a vector $\vec{\alpha} = \hat{\imath} + \hat{\jmath}$ and a vector $\vec{\beta}$ perpendicular to $\vec{\alpha}$. Then $\vec{\alpha} =$

a) $\frac{3}{2}(\hat{\imath} + \hat{\jmath})$ b) $\frac{2}{3}(\hat{\imath} + \hat{\jmath})$ c) $\frac{1}{2}(\hat{\imath} + \hat{\jmath})$ d) $\frac{1}{3}(\hat{\imath} + \hat{\jmath})$

95. A parallelogram is constructed on $3\vec{\mathbf{a}} + \vec{\mathbf{b}}$ and $\vec{\mathbf{a}} - 4\vec{\mathbf{b}}$, where $|\vec{\mathbf{a}}| = 6$ and $|\vec{\mathbf{b}}| = 8$ and $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are anti-parallel, then the length of the longer diagonal is

a) 40 b) 64 c) 42 d) 48

96. If the vectors $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ from the sides BC, CA and AB respectively of a triangle ABC then

a) $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} = \vec{\mathbf{c}} \cdot \vec{\mathbf{b}} = 0$ b) $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{b}} \times \vec{\mathbf{c}} = \vec{\mathbf{c}} \times \vec{\mathbf{a}} = 0$

c) $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} = \vec{\mathbf{c}} \cdot \vec{\mathbf{a}} = 0$ d) $\vec{\mathbf{a}} \times \vec{\mathbf{a}} + \vec{\mathbf{a}} \times \vec{\mathbf{c}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}} = 0$

97. The vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ of equal magnitude 5 originating from a point and directs respectively towards north-east and north-west. Then, the magnitude of $\vec{\mathbf{a}} - \vec{\mathbf{b}}$ is

a) $3\sqrt{2}$ b) $2\sqrt{3}$ c) $2\sqrt{5}$ d) $5\sqrt{2}$

98. If the vectors $\vec{\mathbf{a}} = \hat{\mathbf{i}} + a\hat{\mathbf{j}} + a^2\hat{\mathbf{k}}, \vec{\mathbf{b}} = \hat{\mathbf{i}} + b\hat{\mathbf{j}} + b^2\hat{\mathbf{k}}$ and $\vec{\mathbf{c}} = \hat{\mathbf{i}} + c\hat{\mathbf{j}} + c^2\hat{\mathbf{k}}$ are three non-coplanar vectors and $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$, then the value of abc is

a) 0 b) 1 c) 2 d) -12

99. If $\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \vec{\mathbf{b}} = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and $\vec{\mathbf{c}} = 7\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$ then the area of parallelogram having diagonals $\vec{\mathbf{a}} + \vec{\mathbf{b}}$ and $\vec{\mathbf{b}} + \vec{\mathbf{c}}$ is

a) $4\sqrt{6}$ sq units b) $\frac{1}{2}\sqrt{21}$ sq units c) $\frac{\sqrt{6}}{2}$ sq units d) $\sqrt{6}$ sq units

100. Let $\vec{a} = \hat{\imath} + \hat{\jmath} - \hat{k}, \vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{c}$ be a unit vector perpendicular to $\vec{a}$ and coplanar with $\vec{a}$ and $\vec{b}$, then it is given by

a) $\frac{1}{\sqrt{6}}(2\hat{\imath} - \hat{\jmath} + \hat{k})$ b) $\frac{1}{\sqrt{2}}(\hat{\jmath} + \hat{k})$ c) $\frac{1}{\sqrt{6}}(\hat{\imath} - 2\hat{\jmath} + \hat{k})$ d) $\frac{1}{2}(\hat{\jmath} - \hat{k})$

101. If $\vec{a}.\hat{\imath} = 4$, then $(\vec{a} \times \hat{\jmath}).(2\hat{\jmath} - 3\hat{k}) =$

a) 12 b) 2 c) 0 d) -12

102. If $\vec{\mathbf{a}} + 2\vec{\mathbf{b}} + 4\vec{\mathbf{c}} = \vec{\mathbf{0}}$ and $(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) + (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) + (\vec{\mathbf{c}} \times \vec{\mathbf{a}}) = \lambda(\vec{\mathbf{b}} \times \vec{\mathbf{c}})$, then λ is equal to

a) 4 b) 7 c) 8 d) 9

103. Forces acting on a particle have magnitude 5, 3 and 1 unit and act in the direction of the vectors $6\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}, 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ and $2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 6\hat{\mathbf{k}}$ respectively. They remain constant while the particle is displaced from the points $A(2, -1, -3)$ to $B(5, -1, 1)$. The work done is

a) 11 unit b) 33 unit c) 10 unit d) 30 unit

104. For any vector $\vec{r}$, the value of $\hat{\imath} \times (\vec{r} \times \hat{\imath}) + \hat{\jmath} \times (\vec{r} \times \hat{\jmath}) + \hat{k} \times (\vec{r} \times \hat{k})$, is

a) $\vec{0}$ b) $2\,\vec{r}$ c) $-2\,\vec{r}$ d) None of these

105. The vector equation of the plane passing through the origin and the line of intersection of the planes

$\vec{\mathbf{r}}\cdot\vec{\mathbf{a}}=\lambda$ and $\vec{\mathbf{r}}\cdot\vec{\mathbf{b}}=\mu$, is

a) $\vec{\mathbf{r}}\cdot(\lambda\vec{\mathbf{a}}-\mu\vec{\mathbf{b}})=0$ b) $\vec{\mathbf{r}}\cdot(\lambda\vec{\mathbf{b}}-\mu\vec{\mathbf{a}})=0$ c) $\vec{\mathbf{r}}\cdot(\lambda\vec{\mathbf{a}}+\mu\vec{\mathbf{b}})=0$ d) $\vec{\mathbf{r}}\cdot(\lambda\vec{\mathbf{b}}+\mu\vec{\mathbf{a}})=0$

106. If $\vec{\mathbf{a}},\vec{\mathbf{b}},\vec{\mathbf{c}}$ are non-coplanar and $[\vec{\mathbf{a}}+\ \vec{\mathbf{b}}\ \vec{\mathbf{b}}+\vec{\mathbf{c}}\ \ \vec{\mathbf{c}}+\vec{\mathbf{a}}]=k[\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}]$, then k is equal to

a) 0 b) 1 c) 2 d) 3

107. If $\vec{\mathbf{a}}=(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}),\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=1$ and $\vec{\mathbf{a}}\times\vec{\mathbf{b}}=\hat{\mathbf{j}}-\hat{\mathbf{k}}$, then $\vec{\mathbf{b}}$ is

a) $\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$ b) $2\hat{\mathbf{j}}-\hat{\mathbf{k}}$ c) $\hat{\mathbf{i}}$ d) $2\hat{\mathbf{i}}$

108. A tetrahedron has vertices at $O(0,0,0)$, $A(1,2,1)$, $B(2,1,3)$and $C(-1,1,2)$. Then, the angle between the faces OAB and ABC will be

a) $\cos^{-1}\left(\frac{19}{35}\right)$ b) $\cos^{-1}\left(\frac{17}{31}\right)$ c) 30° d) 90°

109. If $2\,\vec{\mathbf{a}}+3\,\vec{\mathbf{b}}-5\,\vec{\mathbf{c}}=\vec{\mathbf{0}}$, then ratio in which $\vec{\mathbf{c}}$ divides $\overrightarrow{\mathbf{AB}}$ is

a) 3:2 internally b) 3:2 externally c) 2:3 internally d) 2:3 externally

110. The perimeter of the triangle whose vertices have the position vectors $\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$, $5\hat{\mathbf{i}}+3\hat{\mathbf{j}}-3\hat{\mathbf{k}}$ and $2\hat{\mathbf{i}}+5\hat{\mathbf{j}}+9\hat{\mathbf{k}}$ is given by

a) $15+\sqrt{157}$ b) $15-\sqrt{157}$ c) $\sqrt{15}+\sqrt{157}$ d) $\sqrt{15}-\sqrt{157}$

111. If $\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}=\vec{\mathbf{0}}$ and $|\vec{\mathbf{a}}|=5$, $|\vec{\mathbf{b}}|=4$ and $|\vec{\mathbf{c}}|=3$, then the value of $|\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}+\vec{\mathbf{b}}\cdot\vec{\mathbf{c}}+\vec{\mathbf{c}}\cdot\vec{\mathbf{a}}|$ is

a) 25 b) 50 c) -25 d) 20

112. If $\vec{a}$ is any vector, then $(a\times\hat{\imath})^2+(a\times\hat{\jmath})^2+(a\times\hat{k})^2=$

a) $\vec{a}^2$ b) $2\vec{a}^2$ c) $3\vec{a}^2$ d) $4\vec{a}^2$

113. If $\vec{a},\vec{b},\vec{c}$ are three non-zero vectors (no two of which are collinear), such that the pairs of vectors $(\vec{a}+\vec{b},\vec{c})$ and $(\vec{b}+\vec{c},\vec{a})$ are collinear, then $\vec{a}+\vec{b}+\vec{c}=$

a) $\vec{a}$ b) $\vec{b}$ c) $\vec{c}$ d) $\vec{0}$

114. Let $\vec{\mathbf{a}},\vec{\mathbf{b}},\vec{\mathbf{c}}$ be three non-coplanar vectors and $\vec{\mathbf{r}}$ be any vector in space such that $\vec{\mathbf{r}}\cdot\vec{\mathbf{a}}=1,\vec{\mathbf{r}}\cdot\vec{\mathbf{b}}=2$ and $\vec{\mathbf{r}}\cdot\vec{\mathbf{c}}=3$. If $[\,\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}\,]=1$, then $\vec{\mathbf{r}}$ is equal to

a) $\vec{\mathbf{a}}+2\vec{\mathbf{b}}+3\vec{\mathbf{c}}$ b) $\vec{\mathbf{b}}\times\vec{\mathbf{c}}+2\vec{\mathbf{c}}\times\vec{\mathbf{a}}+3\vec{\mathbf{a}}\times\vec{\mathbf{b}}$

c) $(\vec{\mathbf{b}}\cdot\vec{\mathbf{c}})\vec{\mathbf{a}}+2(\vec{\mathbf{c}}\cdot\vec{\mathbf{a}})\vec{\mathbf{b}}+3(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}})+\vec{\mathbf{c}}$ d) None of these

115. If $\vec{\mathbf{x}}+\vec{\mathbf{y}}+\vec{\mathbf{z}}=\vec{\mathbf{0}},|\vec{\mathbf{x}}|=|\vec{\mathbf{y}}|+|\vec{\mathbf{z}}|=2$, and θ is angle between $\vec{\mathbf{y}}$ and $\vec{\mathbf{z}}$, then the value of $\operatorname{cosec}^2\theta+\cot^2\theta$ is equal to

a) 4/3 b) 5/3 c) 1/3 d) 1

116. If $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=-|\vec{\mathbf{a}}||\vec{\mathbf{b}}|$,then the angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is

a) 45° b) 180° c) 90° d) 60°

117. Let $\vec{\mathbf{a}}=2\hat{\mathbf{i}}+\hat{\mathbf{k}},\vec{\mathbf{b}}=\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $\vec{\mathbf{c}}=4\hat{\mathbf{i}}-3\hat{\mathbf{j}}+7\hat{\mathbf{k}}$. If $\vec{\mathbf{r}}$ is a vector such that $\vec{\mathbf{r}}\times\vec{\mathbf{b}}=\vec{\mathbf{c}}\times\vec{\mathbf{b}}$ and $\vec{\mathbf{r}}\cdot\vec{\mathbf{a}}=0$, then value of $\vec{\mathbf{r}}\cdot\vec{\mathbf{b}}$ is

a) 7 b) -7 c) -5 d) 5

118. If the vectors $\vec{\mathbf{a}}+\lambda\,\vec{\mathbf{b}}+3\vec{\mathbf{c}}.\,-2\vec{\mathbf{a}}+3\,\vec{\mathbf{b}}-4\,\vec{\mathbf{c}}$ and $\vec{\mathbf{a}}-3\vec{\mathbf{b}}+5\,\vec{\mathbf{c}}$ are coplanar, then the value of λ is

a) 2 b) -1 c) 1 d) -2

119. $\vec{\mathbf{A}},\vec{\mathbf{B}},\vec{\mathbf{C}}$ are three non-zero vectors, no two of them are parallel. If $\vec{\mathbf{A}}+\vec{\mathbf{B}}$ is collinear to $\vec{\mathbf{C}}$ and $\vec{\mathbf{B}}+\vec{\mathbf{C}}$ is collinear to $\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}$ is equal to

a) $\vec{\mathbf{A}}$ b) $\vec{\mathbf{B}}$ c) $\vec{\mathbf{C}}$ d) $\vec{\mathbf{0}}$

120. Consider points A,B,C and D with position vectors $7\hat{\mathbf{i}}-4\hat{\mathbf{j}}+7\hat{\mathbf{k}},\hat{\mathbf{i}}-6\hat{\mathbf{j}}+10\hat{\mathbf{k}},-\hat{\mathbf{i}}+3\hat{\mathbf{j}}+4\hat{\mathbf{k}}$ and $5\hat{\mathbf{i}}-\hat{\mathbf{j}}+5\hat{\mathbf{k}}$ respectively. Then, $ABCD$ is a

a) Square b) Rhombus c) Rectangle d) None of these

121. If $|\vec{a}|=7,|\vec{b}|=11,|\vec{a}+\vec{b}|=10\sqrt{3}$, then $|\vec{a}-\vec{b}|$ equals

a) 10 b) $\sqrt{10}$ c) $2\sqrt{10}$ d) 20

122. In a parallelogram $ABCD$, $|\vec{AB}|=a,|\vec{AD}|=b$ and $|\vec{AC}|=c$. The value of $\vec{DB}.\vec{AB}$ is

a) $\frac{3a^2+b^2-c^2}{2}$ b) $\frac{a^2+3b^2-c^2}{2}$ c) $\frac{a^2-b^2+3c^2}{2}$ d) $\frac{a^2+3b^2+c^2}{2}$

123. If θ be the angle between the vectors $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\vec{\mathbf{b}} = 6\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$, then

a) $\cos\theta = \frac{4}{21}$ b) $\cos\theta = \frac{3}{19}$ c) $\cos\theta = \frac{2}{19}$ d) $\cos\theta = \frac{5}{21}$

124. Let $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ be three non-zero vectors such that no two these are collinear. If the vector $\vec{\mathbf{a}}+2\vec{\mathbf{b}}$ is collinear with $\vec{\mathbf{c}}$ and $\vec{\mathbf{b}} + 3\vec{\mathbf{c}}$ is collinear with $\vec{\mathbf{a}}$(λ being some non-zero scalar). Then $\vec{\mathbf{a}} + 2\vec{\mathbf{b}} + 6\vec{\mathbf{c}}$ equals

a) $\lambda\,\vec{\mathbf{a}}$ b) $\lambda\,\vec{\mathbf{b}}$ c) $\lambda\,\vec{\mathbf{c}}$ d) $\vec{\mathbf{0}}$

125. Let ABC be a triangle the position vectors of whose vertices are respectively $7\hat{j} + 10\hat{k}, -\hat{\imath} + 6\hat{j} + 6\hat{k}$ and $-4\hat{\imath} + 9\hat{j} + 6\hat{k}$. Then, ΔABC is

a) Isosceles and right angled

b) Equilateral

c) Right angled but not isosceles

d) None of these

126. $(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a}) =$

a) $[\vec{a}\vec{b}\vec{c}]\vec{c}$ b) $[\vec{a}\vec{b}\vec{c}]\vec{b}$ c) $[\vec{a}\vec{b}\vec{c}]\vec{a}$ d) $a \times (\vec{b} \times \vec{c})$

127. If $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are non-coplanar vectors and λ be a real number, then the vectors $\vec{\mathbf{a}} + 2\vec{\mathbf{b}} + 3\vec{\mathbf{c}}, \lambda\,\vec{\mathbf{b}} + 4\,\vec{\mathbf{c}}$ and $(2\lambda - 1)\,\vec{\mathbf{c}}$ are non-coplanar for

a) All values of λ b) All except one value of λ

c) All except two values of λ d) No value of λ

128. The position vectors of P and Q are respectively $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ .If R is a point on $\overrightarrow{\mathbf{PQ}}$ such that $\overrightarrow{\mathbf{PR}} = 5\overrightarrow{\mathbf{PQ}}$, then the position vector of R is

a) $5\vec{\mathbf{b}} - 4\vec{\mathbf{a}}$ b) $5\vec{\mathbf{b}} + 4\vec{\mathbf{a}}$ c) $4\vec{\mathbf{b}} - 5\vec{\mathbf{a}}$ d) $4\vec{\mathbf{b}} + 5\,\vec{\mathbf{a}}$

129. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors. Suppose $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}\,\frac{\pi}{6}$. Then, $\vec{a} =$

a) $\pm 2(\vec{b} \times \vec{c})$ b) $-2(\vec{b} \times \vec{c})$ c) $2(\vec{b} \times \vec{c})$ d) $(\vec{b} \times \vec{c})$

130. If $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ are the position vectors of the points A and B, then the position vector of the points of trisection of AB are

a) $\frac{4}{3}\hat{\mathbf{i}} + \hat{\mathbf{j}} + \frac{10}{3}\hat{\mathbf{k}}, \frac{5}{3}\hat{\mathbf{i}} + \frac{11}{3}\hat{\mathbf{k}}$

b) $-\frac{4}{3}\hat{\mathbf{i}} - \hat{\mathbf{j}} - \frac{10}{3}\hat{\mathbf{k}}, -\frac{5}{3}\hat{\mathbf{i}} - \frac{11}{3}\hat{\mathbf{k}}$

c) $\frac{4}{3}\hat{\mathbf{i}} - \hat{\mathbf{j}} - \frac{10}{3}\hat{\mathbf{k}}, -\frac{5}{3}\hat{\mathbf{i}} - \frac{11}{3}\hat{\mathbf{k}}$

d) $-\frac{4}{3}\hat{\mathbf{i}} + \hat{\mathbf{j}} - \frac{10}{3}\hat{\mathbf{k}}, \frac{5}{3}\hat{\mathbf{i}} - \frac{11}{3}\hat{\mathbf{k}}$

131. D, E and F are the mid-points of the sides BC, CA and AB respectively of ΔABC and G is the centroid of the triangle, then $\overrightarrow{GD} + \overrightarrow{GE} + \overrightarrow{GF} =$

a) $\vec{0}$ b) $2\,\overrightarrow{AB}$ c) $2\,\overrightarrow{GA}$ d) $2\,\overrightarrow{GC}$

132. If D, E and F are respectively the mid points of AB, AC and BC in ΔABC, then $\overrightarrow{\mathbf{BE}} + \overrightarrow{\mathbf{AF}}$ is equal to

a) $\overrightarrow{\mathbf{DC}}$ b) $\frac{1}{2}\overrightarrow{\mathbf{BF}}$ c) $2\,\overrightarrow{\mathbf{BF}}$ d) $\frac{3}{2}\overrightarrow{\mathbf{BF}}$

133. If $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} + \vec{b}$ makes an angle of 30° with $\vec{a}$, then

a) $|\vec{b}| = 2|\vec{a}|$ b) $|\vec{a}| = 2|\vec{b}|$ c) $|\vec{a}| = \sqrt{3}|\vec{b}|$ d) None of these

134. If $\vec{\mathbf{a}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$, and

$\vec{\mathbf{b}} = \hat{\mathbf{i}} \times (\vec{\mathbf{a}} \times \hat{\mathbf{i}}) + \hat{\mathbf{j}} \times (\vec{\mathbf{a}} \times \hat{\mathbf{j}}) + \hat{\mathbf{k}} \times (\vec{\mathbf{a}} \times \hat{\mathbf{k}})$

Then length of $\vec{\mathbf{b}}$ is equal to

a) $\sqrt{12}$ b) $2\sqrt{12}$ c) $3\sqrt{14}$ d) $2\sqrt{14}$

135. If a, b, c are different real numbers and $a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$, $b\hat{\mathbf{i}} + c\hat{\mathbf{j}} + a\hat{\mathbf{k}}$ and $c\hat{\mathbf{i}} + a\hat{j} + b\widehat{\boldsymbol{k}}$ are position vectors of three non-collinear points, then

a) centroid of ΔABC is $\dfrac{a+b+c}{3}(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$

b) $(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$ is not really inclined to three vectors

c) Triangle ABC is a scalene triangle

d) Perpendicular from the origin to the plane of the triangle does not meet it at the centroid

136. If $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are unit vectors and $|\vec{\mathbf{a}} + \vec{\mathbf{b}}| = 1$, then $|\vec{\mathbf{a}} + \vec{\mathbf{b}}|$is equal to

a) $\sqrt{2}$ b) 1 c) $\sqrt{5}$ d) $\sqrt{3}$

137. If $\vec{\mathbf{a}} = (1, -1)$ and $\vec{\mathbf{b}} = (-2, m)$ are two collinear vectors, then m is equal to

a) 2 b) 4 c) 3 d) 0

138. If O is origin of C is the mid point of $A(2, -1)$ and $B(-4, 3)$. Then, the value of $\overrightarrow{\mathbf{OC}}$ is

a) $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ b) $\hat{\mathbf{i}} - \hat{\mathbf{j}}$ c) $-\hat{\mathbf{i}} + \hat{\mathbf{j}}$ d) $-\hat{\mathbf{i}} - \hat{\mathbf{j}}$

139. The values of x for which the angle between the vectors $\vec{a} = x\hat{\imath} - 3\hat{\jmath} - \hat{k}$ and $\vec{b} = 2x\hat{\imath} + x\hat{\jmath} - \hat{k}$ is acute and the angle between the vector $\vec{b}$ and the y-axis lies between $\frac{\pi}{2}$ and π are

a) 1, 2 b) $-2, -3$ c) All $x < 0$ d) All $x > 0$

140. If $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are position vectors of the vertices of the triangle ABC, then $\dfrac{|(\vec{\mathbf{a}} - \vec{\mathbf{c}}) \times (\vec{\mathbf{b}} - \vec{\mathbf{a}})|}{(\vec{\mathbf{c}} - \vec{\mathbf{a}}) \cdot (\vec{\mathbf{b}} - \vec{\mathbf{a}})}$ is equal to

a) $\cot A$ b) $\cot C$ c) $-\tan C$ d) $\tan A$

141. $\vec{\mathbf{a}} \cdot \hat{\mathbf{i}} = \vec{\mathbf{a}} \cdot (2\hat{\mathbf{i}} + \hat{\mathbf{j}}) = \vec{\mathbf{a}} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}) = 1$, then $\vec{\mathbf{a}}$ is equal to

a) $\hat{\mathbf{i}} - \hat{\mathbf{k}}$ b) $(3\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}})/3$ c) $(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})/3$ d) $(3\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}})/3$

142. If $\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \vec{\mathbf{b}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\vec{\mathbf{c}} = x\,\hat{\mathbf{i}} + (x-2)\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and if the vector $\vec{\mathbf{c}}$ lies in the plane of vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$, then x equals

a) 0 b) 1 c) -2 d) 2

143. The figure formed by the four points $\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}, 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}, 5\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ and $\hat{\mathbf{k}} - \hat{\mathbf{j}}$ is

a) Trapezium b) Rectangle c) Parallelogram d) None of these

144. If $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{b} = 3\hat{\imath} + 6\hat{\jmath} + 2\hat{k}$, then the vector in the direction of $\vec{a}$ and having magnitude as $|\vec{b}|$, is

a) $7(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$ b) $\frac{7}{9}(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$ c) $\frac{7}{3}(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$ d) None of these

145. If I is incentre of $\Delta\ ABC$, then I is

a) $\dfrac{a\vec{\mathbf{a}} + b\vec{\mathbf{b}} + c\,\vec{\mathbf{c}}}{a+b+c}$ b) $\dfrac{a\vec{\mathbf{a}} + b\vec{\mathbf{b}} + c\,\vec{\mathbf{c}}}{\sqrt{a^2+b^2+c^2}}$ c) $\frac{1}{3}[\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}]$ d) $\dfrac{\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}}{a+b+c}$

146. If $\vec{a}$ and $\vec{b}$ are unit vectors, then which of the following values of $\vec{a}.\vec{b}$ is not possible?

a) $\sqrt{3}$ b) $\sqrt{3}/2$ c) $1/\sqrt{2}$ d) $-1/2$

147. The two vectors $\{\vec{\mathbf{a}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}, \vec{\mathbf{b}} = 4\hat{\mathbf{i}} - \lambda\hat{\mathbf{j}} + 6\hat{\mathbf{k}}\}$ are parallel, if λ is equal to

a) 2 b) -3 c) 3 d) -2

148. Force acting on a particle have magnitude 5,3 and 1 unit act in the direction of the vectors $6\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$, $3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ and $2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 6\hat{\mathbf{k}}$ respectively. They remain constant while the particle is displaced from the point $A(2, -1, -3)$ to $B(5 - 1, 1)$. The work done is

a) 11 units b) 33 units c) 10 units d) 30 units

149. If $\vec{a}.\vec{b} = \vec{a}.\vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, then

a) Either $\vec{a} = \vec{0}$ or $\vec{b} = \vec{c}$ b) $\vec{a} || (\vec{b} - \vec{c})$ c) $\vec{a} \perp (\vec{b} - \vec{c})$ d) None of these

150. The two vectors $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}, \vec{b} = 4\hat{\imath} - \lambda\hat{\jmath} + 6\hat{k}$ are parallel if $\lambda =$

a) 2 b) -3 c) 3 d) -2

151. If $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are unit coplanar vectors, then $[2\vec{\mathbf{a}} - \vec{\mathbf{b}}\ 2\,\vec{\mathbf{b}} - \vec{\mathbf{c}}\ 2\,\vec{\mathbf{c}} - \vec{\mathbf{a}}]$ is equal to

a) 1 b) 0 c) $-\sqrt{3}$ d) $\sqrt{3}$

152. The angle between the vectors $\vec{\mathbf{a}}+\vec{\mathbf{b}}$ and $\vec{\mathbf{a}}-\vec{\mathbf{b}}$ when $\vec{\mathbf{a}}=(1,1,4)$and $\vec{\mathbf{b}}=(1,-1,4)$ is

a) 45° b) 90° c) 15° d) 30°

153. Let $P(3,2,6)$ be a point in space and Qbe a point on the line $\vec{\mathbf{r}}=(\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}})+\mu(-3\hat{\mathbf{i}}+\hat{\mathbf{j}}+5\hat{\mathbf{k}})$. Then, the value of μ for which the vector $\overrightarrow{\mathbf{PQ}}$ is parallel to the plane $x-4y+3z=1$ is

a) $\frac{1}{4}$ b) $-\frac{1}{4}$ c) $\frac{1}{8}$ d) $-\frac{1}{8}$

154. The area of triangle having verities as $\hat{\mathbf{i}}-2\hat{\mathbf{j}}+3\hat{\mathbf{k}},-2\hat{\mathbf{i}}+3\hat{\mathbf{j}}-\hat{\mathbf{k}},4\hat{\mathbf{i}}-7\hat{\mathbf{j}}+7\hat{\mathbf{k}}$ is

a) 36 sq units b) 0 sq units c) 39 sq units d) 11 sq units

155. If $\vec{r}\times\vec{a}=\vec{b}\times\vec{a}$; $\vec{r}\times\vec{b}=\vec{a}\times\vec{b}$; $\vec{a}\neq 0$; $\vec{b}\neq 0$; $\vec{a}\neq\lambda\vec{b}$, $\vec{a}$ is not perpendicular to $\vec{b}$, then $\vec{r}=$

a) $\vec{a}-\vec{b}$ b) $\vec{a}+\vec{b}$ c) $\vec{a}\times\vec{b}+\vec{a}$ d) $\vec{a}\times\vec{b}+\vec{b}$

156. If $\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}$ are three unit vectors such that $\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}=0$, where $\vec{\mathbf{0}}$ is null vector, then $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}+\vec{\mathbf{b}}\cdot\vec{\mathbf{c}}+\vec{\mathbf{c}}\cdot\vec{\mathbf{a}}$ is

a) -3 b) -2 c) $-\frac{3}{2}$ d) 0

157. The edges of a parallelopiped are unit length and are parallel to non-coplanar unit vectors $\vec{\mathbf{a}},\vec{\mathbf{b}},\vec{\mathbf{c}}$ such that $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=\vec{\mathbf{b}}\cdot\vec{\mathbf{c}}=\vec{\mathbf{c}}\cdot\vec{\mathbf{a}}=\frac{1}{2}$ Then, the volume of the parallelopiped is

a) $\frac{1}{\sqrt{2}}$cu unit b) $\frac{1}{2\sqrt{2}}$ cu unit c) $\frac{\sqrt{3}}{2}$cu unit d) $\frac{1}{\sqrt{3}}$cu unit

158. If $\vec{\mathbf{a}}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ and $\vec{\mathbf{b}}=\hat{\mathbf{i}}\times(\vec{\mathbf{a}}\times\hat{\mathbf{i}})+\hat{\mathbf{j}}\times(\vec{\mathbf{a}}\times\hat{\mathbf{j}})+\hat{\mathbf{k}}\times(\vec{\mathbf{a}}\times\hat{\mathbf{k}})$, then length of $\vec{\mathbf{b}}$ is equal to

a) $\sqrt{12}$ b) $2\sqrt{12}$ c) $3\sqrt{14}$ d) $2\sqrt{14}$

159. A vector $\vec{\mathbf{a}}$ has components 2 p and 1 with respect to a rectangular cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense, if this respect to new system $\vec{\mathbf{a}}$ has components $p+1$ and 1, then

a) $p=0$ b) $p=1$ or $p=\frac{-1}{2}$ c) $p=-1$ d) $p=1$ or $p=-1$

160. If the vectors $\overrightarrow{r_1}=a\hat{\imath}+\hat{\jmath}+\hat{k},\overrightarrow{r_2}=\hat{\imath}+b\hat{\jmath}+\hat{k},\overrightarrow{r_3}=\hat{\imath}+\hat{\jmath}+c\hat{k}(a\neq 1,b\neq 1,c\neq 1)$ are coplanar, then the value of $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$, is

a) -1 b) 0 c) 1 d) None of these

161. A non-zero vectors $\vec{\mathbf{a}}$ is such that its projection along the vectors $\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}$ and $\frac{-\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}$ and $\hat{\mathbf{k}}$ are equal, then unit vector along $\vec{\mathbf{a}}$ is

a) $\frac{\sqrt{2}\hat{\mathbf{j}}-\hat{\mathbf{k}}}{\sqrt{3}}$ b) $\frac{\hat{\mathbf{j}}-\sqrt{2}\hat{\mathbf{k}}}{\sqrt{3}}$ c) $\frac{\sqrt{2}}{\sqrt{3}}\hat{\mathbf{j}}+\frac{\hat{\mathbf{k}}}{\sqrt{3}}$ d) $\frac{\hat{\mathbf{j}}-\hat{\mathbf{k}}}{\sqrt{2}}$

162. Let P,Q,R and S be the points on the plane with position vectors $-2\hat{\mathbf{i}}-\hat{\mathbf{j}}$,$4\hat{\mathbf{i}}$, $3\hat{\mathbf{i}}+3\hat{\mathbf{j}}$ and $-3\hat{\mathbf{i}}+2\hat{\mathbf{j}}$ respectively. The quadrilateral $PQRS$ must be

a) Parallelogram, which is neither a rhombus nor a rectangle
b) Square
c) Rectangle, but not a square
d) Rhombus, but not a square

163. If $\vec{a},\vec{b},\vec{c}$ are linearly independent vectors and $\Delta=\begin{vmatrix}\vec{a} & \vec{b} & \vec{c}\\ \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c}\\ \vec{a}.\vec{c} & \vec{b}.\vec{c} & \vec{c}.\vec{c}\end{vmatrix}$, then

a) $\Delta=0$
b) $\Delta=1$
c) $\Delta=$ any non-zero value
d) None of these

164. If $\vec{a}=-2\hat{\imath}+\hat{\jmath}+\hat{k},\vec{b}=\hat{\imath}+5\hat{\jmath}$ and $\vec{c}=4\hat{\imath}+4\hat{\jmath}-2\hat{k}$, then the projection of $3\vec{a}-2\vec{b}$ on the axis of the vector $\vec{c}$ is

a) 11 b) -11 c) 33 d) -33

165. A tetrahedron has vertices at O $(0,0)$, $A(1,2,1)$, $B(2,1,3)$ and $C(-1,1,2)$. Then, the angle between the faces OAB and ABC will be

a) $\cos^{-1}\left(\frac{19}{35}\right)$ b) $\cos^{-1}\left(\frac{7}{31}\right)$ c) $30°$ d) $90°$

166. If $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$ and $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ is equal to $\lambda(\vec{b}\times\vec{c})$, then $\lambda=$

a) 3 b) 4 c) 5 d) None of these

167. $\vec{\mathbf{a}}\times[\vec{\mathbf{a}}\times(\vec{\mathbf{a}}\times\vec{\mathbf{b}})]$ is equal to

a) $(\vec{\mathbf{a}}\times\vec{\mathbf{a}})\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{a}})$ b) $\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{a}})-\vec{\mathbf{b}}\cdot(\vec{\mathbf{a}}\times\vec{\mathbf{b}})$

c) $[\vec{\mathbf{a}}\cdot(\vec{\mathbf{a}}\times\vec{\mathbf{b}})]\vec{\mathbf{a}}$ d) $(\vec{\mathbf{a}}\cdot\vec{\mathbf{a}})(\vec{\mathbf{b}}\times\vec{\mathbf{a}})$

168. If $\vec{a}$ is a unit vector such that $\vec{a}\times(\hat{\imath}+2\hat{\jmath}+\hat{k})=\hat{\imath}-\hat{k}$, then $\vec{a}=$

a) $-\frac{1}{3}(2\hat{\imath}+\hat{\jmath}+2\hat{k})$ b) $\hat{\jmath}$ c) $\frac{1}{3}(\hat{\imath}+2\hat{\jmath}+2\hat{k})$ d) $\hat{\imath}$

169. The medium AD of the triangle ABC is bisected at E, BE meets AC in F, then $AF:AC=$

a) 3/4 b) 1/3 c) 1/2 d) 1/4

170. Vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are inclined at an angle $\theta=120°$. If $|\vec{\mathbf{a}}|=1$, $|\vec{\mathbf{b}}|=2$, then $[(\vec{\mathbf{a}}+3\vec{\mathbf{b}})\times(3\vec{\mathbf{a}}+\vec{\mathbf{b}}]^2$ is equal to

a) 190 b) 275 c) 300 d) 192

171. If $\vec{\mathbf{a}},\vec{\mathbf{b}},\vec{\mathbf{c}}$ are three non-coplanar vectors, then $(\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}})\cdot[(\vec{\mathbf{a}}+\vec{\mathbf{b}})\times(\vec{\mathbf{a}}\times\vec{\mathbf{c}})]$ is

a) 0 b) $2[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$ c) $-[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$ d) $[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$

172. If $\vec{a}=\hat{\imath}+\hat{\jmath}-\hat{k}$, $\vec{b}=\hat{\imath}-\hat{\jmath}+\hat{k}$ and $\vec{c}$ is a unit vector perpendicular to the vector $\vec{a}$ and coplanar with $\vec{a}$ and $\vec{b}$, then a unit vector $\vec{d}$ perpendicular to both $\vec{a}$ and $\vec{c}$ is

a) $\frac{1}{\sqrt{6}}(2\hat{\imath}-\hat{\jmath}+\hat{k})$ b) $\frac{1}{\sqrt{2}}(\hat{\jmath}+\hat{k})$ c) $\frac{1}{\sqrt{2}}(\hat{\imath}+\hat{\jmath})$ d) $\frac{1}{\sqrt{2}}(\hat{\imath}+\hat{k})$

173. If G is the centroid of the $\Delta\,ABC$, then $\overrightarrow{\mathbf{GA}}+\overrightarrow{\mathbf{BG}}+\overrightarrow{\mathbf{GC}}$ is equal to

a) $2\overrightarrow{\mathbf{GB}}$ b) $2\overrightarrow{\mathbf{GA}}$ c) $\vec{\mathbf{0}}$ d) $2\overrightarrow{\mathbf{BG}}$

174. A non-zero vector $\vec{\mathbf{a}}$ is parallel to the line of intersection of the plane determined by vectors $\hat{\mathbf{i}},\hat{\mathbf{i}}-\hat{\mathbf{j}}$ and the plane determined by the vectors $\hat{\mathbf{i}}+\hat{\mathbf{j}}$, $\hat{\mathbf{i}}-\hat{\mathbf{k}}$. The angle between $\vec{\mathbf{a}}$ and $\hat{\mathbf{i}}+2\hat{\mathbf{j}}-2\hat{\mathbf{k}}$ is

a) $\frac{\pi}{3}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{4}$ d) None of these

175. If $|\vec{\mathbf{a}}|=5$, $|\vec{\mathbf{b}}|=6$ and $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=-25$, then $|\vec{\mathbf{a}}\times\vec{\mathbf{b}}|$ is equal to

a) 25 b) $6\sqrt{11}$ c) $11\sqrt{5}$ d) $5\sqrt{11}$

176. If $ABCDE$ is a pentagon, then
$\vec{AB}+\vec{AE}+\vec{BC}+\vec{DC}+\vec{ED}+\vec{AC}$ is equal to

a) $4\,\vec{AC}$ b) $2\,\vec{AC}$ c) $3\,\vec{AC}$ d) $5\,\vec{AC}$

177. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, then

a) $(\vec{a}\pm\vec{d})=\lambda(\vec{b}\pm\vec{c})$ b) $\vec{a}+\vec{c}=\lambda(\vec{b}+\vec{d})$ c) $(\vec{a}-\vec{c})=\lambda(\vec{c}+\vec{d})$ d) None of these

178. $\vec{a}\times(\vec{a}\times(\vec{a}\times\vec{b}))$ equals

a) $(\vec{a}\cdot\vec{a})(\vec{a}\times\vec{b})$ b) $(\vec{a}\cdot\vec{a})(\vec{b}\times\vec{a})$ c) $(\vec{b}\cdot\vec{b})(\vec{a}\times\vec{b})$ d) $(\vec{b}\cdot\vec{b})(\vec{b}\times\vec{a})$

179. In a quadrilateral $ABCD$, $\vec{AB}+\vec{DC}=$

a) $\vec{AB}+\vec{CB}$ b) $\vec{AC}+\vec{BD}$ c) $\vec{AC}+\vec{DB}$ d) $\vec{AD}-\vec{CB}$

180. Let $\vec{a}=x\hat{\imath}+y\hat{\imath}+z\hat{k}$, $\vec{b}=\hat{\jmath}$. The value of $\vec{c}$ for which $\vec{a},\vec{b},\vec{c}$ form a right handed system is

a) $y\hat{\imath}$ b) $-3\hat{\imath}+x\hat{k}$ c) $\vec{0}$ d) $3\hat{\imath}-x\hat{k}$

181. If the position vector of a point $\vec{a}+2\vec{b}$ and $\vec{a}$ divides AB in the ratio 2 : 3, then the position vector of B, is

a) $2\vec{a}-\vec{b}$ b) $\vec{b}-2\vec{a}$ c) $\vec{a}-3\vec{b}$ d) $\vec{b}$

182. The value of a so that the volume of parallelopiped formed by $\hat{\mathbf{i}}+a\hat{\mathbf{j}}+\hat{\mathbf{k}}$, $\hat{\mathbf{j}}+a\hat{\mathbf{k}}$ and $a\hat{\mathbf{i}}+\hat{\mathbf{k}}$ becomes minimum is

a) -3 b) 3 c) $1/\sqrt{3}$ d) $\sqrt{3}$

183. A vector of magnitude 12 units perpendicular to the plane containing the vectors $4\hat{\mathbf{i}}+6\hat{\mathbf{j}}-\hat{\mathbf{k}}$ and $3\hat{\mathbf{i}}+8\hat{\mathbf{j}}+\hat{\mathbf{k}}$ is

a) $-8\hat{\mathbf{i}}+4\hat{\mathbf{j}}+8\hat{\mathbf{k}}$ b) $8\hat{\mathbf{i}}+4\hat{\mathbf{j}}+8\hat{\mathbf{k}}$ c) $8\hat{\mathbf{i}}-4\hat{\mathbf{j}}+8\hat{\mathbf{k}}$ d) $8\hat{\mathbf{i}}-4\hat{\mathbf{j}}-8\hat{\mathbf{k}}$

184. Let the unit vectors $\vec{\mathbf{a}}$and $\vec{\mathbf{b}}$ be perpendicular to each other and the unit vector $\vec{\mathbf{c}}$ be inclined at an angle θ to both $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$. If $\vec{\mathbf{c}}=\alpha,\vec{\mathbf{a}}+\beta,\vec{\mathbf{b}}+\gamma(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}})$, where α,β,γ are scalars, then

a) $\alpha=\cot\theta,\beta=\sin\theta,\gamma^2=\cos 2\theta$ b) $\alpha=\cos\theta,\beta=\cos\theta,\gamma^2=\cos 2\theta$

c) $\alpha=\cos\theta,\beta=\sin\theta,\gamma^2=\cos 2\theta$ d) $\alpha=\sin\theta,\beta=\cos\theta,\gamma^2=\cos 2\theta$

185. If the volume of the parallelopiped with $\vec{\mathbf{a}},\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ as coterminous edges is 40 cu units, then the volume of the parallelopiped having $\vec{\mathbf{b}}+\vec{\mathbf{c}},\vec{\mathbf{c}}+\vec{\mathbf{a}}$ and $\vec{\mathbf{a}}+\vec{\mathbf{b}}$ as coterminous edges inn cubic units is

a) 80 b) 120 c) 160 d) 40

186. Let two non-collinear unit vectors $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ from and acute angle. A point P moves so that at any time t the position vector $\overrightarrow{\mathbf{OP}}$ (where O is the origin) is given by $\hat{\mathbf{a}}\cos t+\hat{\mathbf{b}}\sin t$. When P is farthest form origin O, let M be the length of $\overrightarrow{\mathbf{OP}}$ and $\hat{\mathbf{u}}$ be the unit vector along $\overrightarrow{\mathbf{OP}}$ Then,

a) $\hat{\mathbf{u}}=\dfrac{\hat{\mathbf{a}}+\hat{\mathbf{b}}}{|\hat{\mathbf{a}}+\hat{\mathbf{b}}|}$ and $M=(1+\hat{\mathbf{a}}\cdot\hat{\mathbf{b}})^{1/2}$ b) $\hat{\mathbf{u}}=\dfrac{\hat{\mathbf{a}}-\hat{\mathbf{b}}}{|\hat{\mathbf{a}}-\hat{\mathbf{b}}|}$ and $M=(1+\hat{\mathbf{a}}\cdot\hat{\mathbf{b}})^{1/2}$

c) $\hat{\mathbf{u}}=\dfrac{\hat{\mathbf{a}}+\hat{\mathbf{b}}}{|\hat{\mathbf{a}}+\hat{\mathbf{b}}|}$ and $M=(1+2\hat{\mathbf{a}}\cdot\hat{\mathbf{b}})^{1/2}$ d) $\hat{\mathbf{u}}=\dfrac{\hat{\mathbf{a}}-\hat{\mathbf{b}}}{|\hat{\mathbf{a}}-\hat{\mathbf{b}}|}$ and $M=(1+2\hat{\mathbf{a}}\cdot\hat{\mathbf{b}})^{1/2}$

187. The position vector of midpoint lying on the line joining the points whose position vectors are $\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$ and $\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$, is

a) $\hat{\mathbf{j}}$ b) $\hat{\mathbf{i}}$ c) $\hat{\mathbf{k}}$ d) $\vec{\mathbf{0}}$

188. If A,B,C are vertices of a triangle whose position vectors are $\vec{a},\vec{b}$ and $\vec{c}$ respectively and G is the centroid of ΔABC, then $\vec{G}A+\vec{G}B+\vec{G}C$, is

a) $\vec{0}$ b) $\vec{a}+\vec{b}+\vec{c}$ c) $\dfrac{\vec{a}+\vec{b}+\vec{c}}{3}$ d) $\dfrac{\vec{a}-\vec{b}-\vec{c}}{3}$

189. A non-zero vector $\vec{\mathbf{a}}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{\mathbf{i}},\hat{\mathbf{i}}+\hat{\mathbf{j}}$ and the plane determined by the vectors $\hat{\mathbf{i}}-\hat{\mathbf{j}},\hat{\mathbf{i}}+\hat{\mathbf{k}}$. The angle between $\vec{\mathbf{a}}$ and $\hat{\mathbf{i}}-2\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ is

a) $\dfrac{\pi}{2}$ b) $\dfrac{\pi}{3}$ c) $\dfrac{\pi}{6}$ d) $\dfrac{\pi}{4}$

190. If the planes $\vec{\mathbf{r}}\cdot(2\hat{\mathbf{i}}-\lambda\hat{\mathbf{j}}+3\hat{\mathbf{k}})=0$ and $\vec{\mathbf{r}}\cdot(\lambda\,\hat{\mathbf{i}}+5\hat{\mathbf{j}}-\hat{\mathbf{k}})=5$ are perpendicular to each other, then the value of $\lambda^2+\lambda$ is

a) 0 b) 2 c) 1 d) 3

191. In ΔABC, if $2\,\overrightarrow{\mathbf{AC}}=3\overrightarrow{\mathbf{CB}}$, then $2\overrightarrow{\mathbf{OA}}+3\overrightarrow{\mathbf{OB}}$ equals

a) $5\,\overrightarrow{\mathbf{OC}}$ b) $-\overrightarrow{\mathbf{OC}}$ c) $\overrightarrow{\mathbf{OC}}$ d) $4\,\overrightarrow{\mathbf{OC}}$

192. If the vectors $(\sec^2 A)\hat{\imath}+\hat{\jmath}+\hat{k},\hat{\imath}+(\sec^2 B)\hat{\jmath}+\hat{k},\hat{\imath}+\hat{\jmath}+(\sec^2 c)\hat{k}$ are coplanar, then the value of $\operatorname{cosec}^2 A+\operatorname{cosec}^2 B+\operatorname{cosec}^2 C$ is

a) 1 b) 2 c) 3 d) None of these

193. If the points whose position vectors are $2\hat{\imath}+\hat{\jmath}+\hat{k},6\hat{\imath}-\hat{\jmath}+2\hat{k}$ and $14\hat{\imath}-5\hat{\jmath}+p\hat{k}$ are collinear, then $p=$

a) 2 b) 4 c) 6 d) 8

194. If $\vec{\alpha}+\vec{\beta}+\vec{\gamma}=a\vec{\delta}$ and $\vec{\beta}+\vec{\gamma}+\vec{\delta}=b\vec{\alpha}$ and $\vec{\alpha},\vec{\beta},\vec{\gamma}$ are non-coplanar and $\vec{\alpha}$ is not parallel to $\vec{\delta}$, then $\vec{\alpha}+\vec{\beta}+\vec{\gamma}$ $+\vec{\delta}$ equals

a) $a\vec{\alpha}$ b) $b\vec{\delta}$ c) 0 d) $(a+b)\vec{\gamma}$

195. If the points with position vectors $20\hat{\imath}+p\hat{\jmath},5\hat{\imath}-\hat{\jmath}$ and $10\hat{\imath}-13\hat{\jmath}$ are collinear, then $p=$

a) 7 b) -37 c) -7 d) 37

196. If $[\vec{a}\vec{b}\vec{c}]=3$, then the volume (in cubic units) of the parallelopiped with $2\vec{a}+\vec{b},2\vec{b}+\vec{c}$ and $2\vec{c}+\vec{a}$ as coterminus edges is

a) 15 b) 22 c) 25 d) 27

197. Le the vectors $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ and $\vec{\mathbf{d}}$ be such that $(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times (\vec{\mathbf{c}} \times \vec{\mathbf{d}}) = 0$. Let P_1 and P_2 be planes determined by pair of vectors $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}, \vec{\mathbf{d}}$ respectively. Then, the angle between P_1 and P_2 is

a) 0 b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

198. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors then $\frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a})\cdot\vec{b}} + \frac{\vec{b}\cdot(\vec{a}\times\vec{c})}{\vec{c}\cdot(\vec{a}\times\vec{b})}$ is equal to

a) 0 b) 2 c) 1 d) None of these

199. Let $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$, $\vec{\mathbf{b}} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ If $\vec{\mathbf{c}}$ is a vector such that $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}\ |\vec{\mathbf{c}}|$ and $|\vec{\mathbf{c}} - \vec{\mathbf{a}}| = 2\sqrt{2}$ and angle between $\vec{\mathbf{a}} \times \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ is 30°, then $|(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times \vec{\mathbf{c}}|$ is

a) $\frac{3}{2}$ b) $\frac{2}{3}$ c) 2 d) $\frac{\sqrt{3}}{2}$

200. The area of the parallelogram whose diagonals are the vectors $2\vec{a} - \vec{b}$ and $4\vec{a} - 5\vec{b}$ where $\vec{a}$ and $\vec{b}$ are the unit vectors forming an angle of 45°, is

a) $3\sqrt{2}$ b) $3/\sqrt{2}$ c) $\sqrt{2}$ d) None of these

201. In a quadrilateral $ABCD$, the point P divides DC in the ratio 1:2 Q is the mid point of AC. If $\overrightarrow{\mathbf{AB}} + 2\ \overrightarrow{\mathbf{AD}} + \overrightarrow{\mathbf{BC}} - 2\,\overrightarrow{\mathbf{DC}} = k\,\overrightarrow{\mathbf{PQ}}$, then k is equal to

a) -6 b) -4 c) 6 d) 4

202. If $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = 4$ and $|\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}| = 2$, then $|\vec{\mathbf{a}}|^2\ |\vec{\mathbf{b}}|^2$ is equal to

a) 6 b) 2 c) 20 d) 8

203. If $\vec{a}\vec{b}\vec{c}$ and $\vec{p}, \vec{q}, \vec{r}$ are reciprocal system of vectors, then $\vec{a} \times \vec{p} + \vec{b} \times \vec{q} + \vec{c} \times \vec{r}$ equals

a) $[\vec{a}\vec{b}\vec{c}]$ b) $(\vec{p} + \vec{q} + \vec{r})$ c) $\vec{0}$ d) $\vec{a} + \vec{b} + \vec{c}$

204. If the vectors $\vec{\mathbf{a}} = (c\log_2 x)\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\vec{\mathbf{b}} = (\log_2 x)\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + (2c\log_2 x)\hat{\mathbf{k}}$ make an abtuse angle for any $x \in (0, \infty)$, then the interval of which c belongs

a) $\left(\frac{4}{3}, 0\right)$ b) $\left(-\infty, -\frac{4}{3}\right)$ c) $\left(\frac{3}{4}, 0\right)$ d) $\left(-\frac{3}{4}, 0\right)$

205. Let $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\vec{c} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$ be three vectors. A vector in the plane of $\vec{b}$ and $\vec{c}$ whose projection on $\vec{a}$ is of magnitude $\sqrt{2/3}$ is

a) $2\hat{\imath} + 3\hat{\jmath} - 3\hat{k}$ b) $2\hat{\imath} + 3\hat{\jmath} + 3\hat{k}$ c) $-2\hat{\imath} + 5\hat{\jmath} + 5\hat{k}$ d) $2\hat{\imath} + \hat{\jmath} + 5\hat{k}$

206. The angle between the straight lines $\vec{\mathbf{r}} = (2 - 3t)\hat{\mathbf{i}} + (1 + 2t)\hat{\mathbf{j}} + (2 + 6t)\hat{\mathbf{k}}$ and $\vec{\mathbf{r}} = (1 + 4s)\hat{\mathbf{i}} + (2 - s)\hat{\mathbf{j}} + (8s - 1)\hat{\mathbf{k}}$ is

a) $\cos^{-1}\left(\frac{\sqrt{41}}{34}\right)$ b) $\cos^{-1}\left(\frac{21}{34}\right)$ c) $\cos^{-1}\left(\frac{43}{63}\right)$ d) $\cos^{-1}\left(\frac{34}{63}\right)$

207. A vector which makes equal angles with the vectors $\frac{1}{3}(\hat{\imath} - 2\hat{\jmath} + 2\hat{k}), \frac{1}{5}(-4\hat{\imath} - 3\hat{k})$ and $\hat{\jmath}$ is

a) $5\hat{\imath} + \hat{\jmath} + 5\hat{k}$ b) $-5\hat{\imath} + \hat{\jmath} + 5\hat{k}$ c) $5\hat{\imath} - \hat{\jmath} + 5\hat{k}$ d) $5\hat{\imath} + \hat{\jmath} - 5\hat{k}$

208. In a ΔABC, if $\vec{A}B = \hat{\imath} - 7\hat{\jmath} + \hat{k}$ and $\vec{B}C = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$, then $|\vec{C}A| =$

a) $\sqrt{61}$ b) $\sqrt{52}$ c) $\sqrt{51}$ d) $\sqrt{41}$

209. If $\hat{\imath}, \hat{\jmath}, \hat{k}$ are unit orthonormal vectors and $\vec{a}$ is a vector, if $\vec{a} \times \vec{r} = \hat{\jmath}$, then $\vec{a}.\vec{r}$ is

a) 0 b) 1 c) -1 d) Arbitrary scalar

210. If the scalar product of the vector $\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ with the unit vector along $m\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ is equal to 2, then one of the value of m is

a) 3 b) 4 c) 5 d) 6

211. Let $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are non-collinear vectors. If there exists scalars α, β such that $\alpha\vec{\mathbf{a}} + \beta\vec{\mathbf{b}} = \vec{\mathbf{0}}$, then

a) $\alpha = \beta \neq 0$ b) $\alpha + \beta = 0$ c) $\alpha = \beta = 0$ d) $\alpha \neq \beta$

212. The vector $\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + m\hat{\mathbf{k}}$, $\vec{\mathbf{b}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + (m + 1)\hat{\mathbf{k}}$ and $\vec{\mathbf{c}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + m\hat{\mathbf{k}}$ are coplanar, if m is equal to

a) 1
b) 4
c) 3

d) No value of m for which vectors are coplanar

213. The unit vector in XOY plane and making angles 45° and 60° respectively with $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $b = 0\hat{\imath} + \hat{\jmath} - \hat{k}$, is

a) $-\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{k}$

b) $\frac{1}{\sqrt{2}}\hat{\imath} - \frac{1}{\sqrt{2}}\hat{k}$

c) $\frac{1}{3\sqrt{2}}\hat{\imath} + \frac{4}{3\sqrt{2}}\hat{\jmath} + \frac{1}{3\sqrt{2}}\hat{k}$

d) None of these

214. The value of λ, for which the four points $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$, $3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$, $\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + \lambda\hat{\mathbf{k}}$ are coplanar, is

a) 2 b) 4 c) 6 d) 8

215. If $|\vec{a}| = |\vec{b}|$, then

a) $(\vec{a} + \vec{b})$ is parallel to $\vec{a} - \vec{b}$

b) $\vec{a} + \vec{b}$ is $\perp$ to $\vec{a} - \vec{b}$

c) $(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 2|\vec{a}|^2$

d) None of these

216. The area of a parallelogram whose adjacent sides are given by the vectors $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $-3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ (in sq unit), is

a) $\sqrt{180}$ sq unit b) $\sqrt{140}$ sq unit c) $\sqrt{80}$ sq unit d) $\sqrt{40}$ sq unit

217. If P is any point with in a triangle ABC, then $\overrightarrow{\mathbf{PA}} + \overrightarrow{\mathbf{CP}}$ is equal to

a) $\overrightarrow{\mathbf{AC}} + \overrightarrow{\mathbf{CB}}$ b) $\overrightarrow{\mathbf{BC}} + \overrightarrow{\mathbf{BA}}$ c) $\overrightarrow{\mathbf{CB}} + \overrightarrow{\mathbf{AB}}$ d) $\overrightarrow{\mathbf{CB}} + \overrightarrow{\mathbf{BA}}$

218. Let the unit vectors $\vec{a}$ and $\vec{b}$ be perpendicular to each other and the unit vector $\vec{c}$ be inclined at an angle θ to both $\vec{a}$ and $\vec{b}$. If $\vec{c} = x\vec{a} + y\vec{b} + \vec{c}(\vec{a} \times \vec{b})$, then

a) $x = \cos\theta, y = \sin\theta, z = \cos 2\theta$

b) $x = \sin\theta, y = \cos\theta, z = -\cos 2\theta$

c) $x = y = \cos\theta, z^2 = \cos 2\theta$

d) $x = y = \cos\theta, z^2 = -\cos 2\theta$

219. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\vec{a}.\vec{b} = 0$ and $\vec{a} + \vec{b} = \vec{c}$, then

a) $|\vec{a}|^2 + |\vec{b}|^2 = |\vec{c}|^2$ b) $|\vec{a}|^2 = |\vec{b}|^2 + |\vec{c}|^2$ c) $|\vec{b}|^2 = |\vec{a}|^2 = |\vec{c}|^2$ d) None of these

220. If $OACB$ is a parallelogram with $\vec{O}C = \vec{a}$ and $\vec{A}B = \vec{b}$, then $\vec{O}A =$

a) $\vec{a} + \vec{b}$ b) $\vec{a} - \vec{b}$ c) $\frac{1}{2}(\vec{b} - \vec{a})$ d) $\frac{1}{2}(\vec{a} - \vec{b})$

221. Five points given by A, B, C, D, E are in plane. Three forces $\overrightarrow{\mathbf{AC}}$, $\overrightarrow{\mathbf{AD}}$ and $\overrightarrow{\mathbf{AE}}$ act a A and three forces $\overrightarrow{\mathbf{CB}}, \overrightarrow{\mathbf{DB}}, \overrightarrow{\mathbf{EB}}$ act at B. Then, their resultant is

a) $2\,\overrightarrow{\mathbf{AC}}$ b) $3\overrightarrow{\mathbf{AB}}$ c) $3\,\overrightarrow{\mathbf{DB}}$ d) $2\,\overrightarrow{\mathbf{BC}}$

222. The vector $\vec{\mathbf{a}} = \alpha\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \beta\hat{\mathbf{k}}$ lies in the plane of the vectors $\vec{\mathbf{b}} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\vec{c} = \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and bisects the angle between $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ Then, which one of the following gives possible value of α and β?

a) α=1, β = 1 b) α=2, β = 2 c) α = 1, β = 2 d) α=2, β = 1

223. A unit vector perpendicular to the plane of $\vec{a} = 2\hat{i} - 6\hat{j} - 3\hat{k}, \vec{b} = 4\hat{i} + 3\hat{j} - \hat{k}$ is

a) $\frac{4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}}{\sqrt{26}}$ b) $\frac{2\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 3\hat{\mathbf{k}}}{7}$ c) $\frac{3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}}}{7}$ d) $\frac{2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 6\hat{\mathbf{k}}}{7}$

224. Vectors $\vec{a}$ and $\vec{b}$ are inclined at angle $\theta = 120°$. If $|\vec{a}| = 1, |\vec{b}| = 2$, then $[(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b})]^2$ is equal to

a) 300 b) 325 c) 275 d) 225

225. If $\vec{\mathbf{a}} \cdot \hat{\mathbf{i}} = 4$ then $(\vec{\mathbf{a}} \times \hat{\mathbf{j}}) \cdot (2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})$ is equal to

a) 12 b) 2 c) 0 d) -12

226. The volume (in cubic unit) of the tetrahedron with edges $\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$, $\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}$ is

a) 4 b) $\frac{2}{3}$ c) $\frac{1}{6}$ d) $\frac{1}{3}$

227. If $|\vec{a}\times\vec{b}|=4, |\vec{a}.\vec{b}|=2$, then $|\vec{a}|^2+|\vec{b}|^2=$

a) 6 b) 2 c) 20 d) 8

228. If $\vec{\mathbf{a}},\vec{\mathbf{b}},\vec{\mathbf{c}}$ be three unit vectors such that $\vec{\mathbf{a}}\times(\vec{\mathbf{b}}\times\vec{\mathbf{c}})=\frac{1}{2}\vec{\mathbf{b}},\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ being non-parallel. If θ_1 is the angle between $\vec{\mathbf{a}}$ and θ_2 is the angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{c}}$, then

a) $\theta_1=\frac{\pi}{6},\theta_2=\frac{\pi}{3}$ b) $\theta_1=\frac{\pi}{3},\theta_2=\frac{\pi}{6}$ c) $\theta_1=\frac{\pi}{2},\theta_2=\frac{\pi}{3}$ d) $\theta_1=\frac{\pi}{3},\theta_2=\frac{\pi}{2}$

229. If P,Q,R are the mid-points of the sides AB,BC and CA of ΔABC are O is a point within the triangle, then $\vec{O}A+\vec{O}B+\vec{O}C=$

a) $2(\vec{O}P+\vec{O}Q+\vec{O}R)$ b) $\vec{O}P+\vec{O}Q+\vec{O}R$ c) $4(\vec{O}P+\vec{O}Q+\vec{O}R)$ d) $6(\vec{O}P+\vec{O}Q+\vec{O}R)$

230. $(\vec{\mathbf{a}}\times\vec{\mathbf{b}})^2+(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}})^2$ is equal to

a) $\vec{\mathbf{a}}^2\vec{\mathbf{b}}^2$ b) $\vec{\mathbf{a}}^2+\vec{\mathbf{b}}^2$ c) 1 d) $2\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}$

231. If $\vec{\mathbf{a}}$ is a vector of magnitude 50, collinear with the vector $\vec{\mathbf{b}}=6\hat{\mathbf{i}}-8\hat{\mathbf{j}}-\frac{15}{2}\hat{\mathbf{k}}$ and makes an acute angle with the positive direction of z-axis, then $\vec{\mathbf{a}}$ is equal to

a) $-24\hat{\mathbf{i}}+32\hat{\mathbf{j}}+30\hat{\mathbf{k}}$ b) $24\hat{\mathbf{i}}-32\hat{\mathbf{j}}-30\hat{\mathbf{k}}$ c) $12\hat{\mathbf{i}}-16\hat{\mathbf{j}}-15\hat{\mathbf{k}}$ d) $-12\hat{\mathbf{i}}+16\hat{\mathbf{j}}-15\hat{\mathbf{k}}$

232. If $ABCDEF$ is a regular hexagon with $\vec{A}B=\vec{a}$ and $\vec{B}C=\vec{b}$, then $\vec{C}E$ equals

a) $\vec{b}-\vec{a}$ b) $-\vec{b}$ c) $\vec{b}-2\vec{a}$ d) $\vec{b}+\vec{a}$

233. If the vectors $2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+4\hat{\mathbf{k}}$ and $\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}$ and $m\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ are coplanar, then the value of m is

a) $\frac{5}{8}$ b) $\frac{8}{5}$ c) $-\frac{7}{4}$ d) $\frac{2}{3}$

234. If $\vec{\mathbf{a}},\vec{\mathbf{b}},\vec{\mathbf{c}}$ are the three vectors mutually perpendicular to each other to form a right handed system and $|\vec{\mathbf{a}}|=1, |\vec{\mathbf{b}}|=3$ and $|\vec{\mathbf{c}}|=5$, then $[\,\vec{\mathbf{a}}-2\vec{\mathbf{b}}\;\vec{\mathbf{b}}-3\vec{\mathbf{c}}\;\vec{\mathbf{c}}-4\vec{\mathbf{a}}\,]$ is equal to

a) 0 b) -24 c) 3600 d) -215

235. The value of $\hat{\mathbf{i}}\times(\hat{\mathbf{j}}\times\hat{\mathbf{k}})+\hat{\mathbf{j}}\times(\hat{\mathbf{k}}\times\hat{\mathbf{i}})+\hat{\mathbf{k}}\times(\hat{\mathbf{i}}\times\hat{\mathbf{j}})$ is

a) $\vec{\mathbf{0}}$ b) $\hat{\mathbf{i}}$ c) $\hat{\mathbf{j}}$ d) $\hat{\mathbf{k}}$

236. The number of the distinct real values of λ, for which the vectors $-\lambda^2\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$, $\hat{\mathbf{i}}-\lambda^2\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $\hat{\mathbf{i}}+\hat{\mathbf{j}}-\lambda^2\hat{\mathbf{k}}$ are coplanar, is

a) Zero b) One c) Two d) Three

237. A particle is acted on by a force of 6 units in the direction $9\hat{\mathbf{i}}+6\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and is displaced from the point $3\hat{\mathbf{i}}+4\hat{\mathbf{j}}-15\hat{\mathbf{k}}$ to the point $7\hat{\mathbf{i}}-6\hat{\mathbf{j}}+8\hat{\mathbf{k}}$. The work done is

a) 18 b) 15 c) 12 d) 9

238. If $\hat{\mathbf{u}}$ and $\hat{\mathbf{v}}$ unit vectors and θ is the acute angle between them, then $2\hat{\mathbf{u}}\times 3\hat{\mathbf{v}}$ is a unit vector for

a) Exactly two values of θ b) More than two values of θ
c) No value of θ d) Exactly one value of θ

239. The total work done by two forces $\vec{\mathbf{F}}_1=2\hat{\mathbf{i}}-\hat{\mathbf{j}}$ and $\vec{\mathbf{F}}_2=3\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}$ acting on a particle when it is displaced from the point $3\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}$ to $5\hat{\mathbf{i}}+5\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ is

a) 8 units b) 9 units c) 10 units d) 11 units

240. In a regular hexagon $ABCDEF, \vec{AB}=\vec{a}, \vec{BC}=\vec{b}$ and $\vec{CD}=\vec{c}$. Then, $\vec{AE}=$

a) $\vec{a}+\vec{b}+\vec{c}$ b) $2\vec{a}+\vec{b}+\vec{c}$ c) $\vec{b}+\vec{c}$ d) $\vec{a}+2\vec{b}+2\vec{c}$

241. If $\vec{a}=2\hat{\imath}+2\hat{\jmath}+3\hat{k}, \vec{b}=-\hat{\imath}+2\hat{\jmath}+\hat{k}$ and $\vec{c}=3\hat{\imath}+\hat{\jmath}$, then $\vec{a}+t\,\vec{b}$ is perpendicular to $\vec{c}$, if t is equal to

a) 8 b) 4 c) 6 d) 2

242. Let $\vec{\mathbf{a}},\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ be three non-coplanar vectors, and let $\vec{\mathbf{p}},\vec{\mathbf{q}}$ and $\vec{\mathbf{r}}$ be vectors defined by the relations

$\vec{\mathbf{p}} = \dfrac{\vec{\mathbf{b}} \times \vec{\mathbf{c}}}{[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]}$, $\vec{\mathbf{q}} = \dfrac{\vec{\mathbf{c}} \times \vec{\mathbf{a}}}{[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]}$ and $\vec{\mathbf{r}} = \dfrac{\vec{\mathbf{a}} \times \vec{\mathbf{b}}}{[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]}$

Then, the value of the expression

$(\vec{\mathbf{a}} + \vec{\mathbf{b}}) \cdot \vec{\mathbf{p}} + (\vec{\mathbf{b}} + \vec{\mathbf{c}}) \cdot \vec{\mathbf{q}} + (\vec{\mathbf{c}} + \vec{\mathbf{a}}) \cdot \vec{\mathbf{r}}$ is equal to

a) 0 b) 1 c) 2 d) 3

243. If $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are three non-zero, non-coplanar vectors and

$\vec{\mathbf{b}}_1 + \vec{\mathbf{b}} - \dfrac{\vec{\mathbf{b}} \cdot \vec{\mathbf{a}}}{|\vec{\mathbf{a}}|^2}\vec{\mathbf{a}}$, $\vec{\mathbf{b}}_2 + \vec{\mathbf{b}} - \dfrac{\vec{\mathbf{b}} \cdot \vec{\mathbf{a}}}{|\vec{\mathbf{a}}|^2}\vec{\mathbf{a}}$

And

$\vec{\mathbf{c}}_1 + \vec{\mathbf{b}} - \dfrac{\vec{\mathbf{c}} \cdot \vec{\mathbf{a}}}{|\vec{\mathbf{a}}|^2}\vec{\mathbf{a}} + \dfrac{\vec{\mathbf{c}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{b}}|^2}\vec{\mathbf{b}}_1$

$\vec{\mathbf{c}}_2 + \vec{\mathbf{c}} - \dfrac{\vec{\mathbf{c}} \cdot \vec{\mathbf{a}}}{|\vec{\mathbf{a}}|^2}\vec{\mathbf{a}} - \dfrac{\vec{\mathbf{c}} \cdot \vec{\mathbf{b}}_1}{|\vec{\mathbf{b}}_1|^2}\vec{\mathbf{b}}_1$,

$\vec{\mathbf{c}}_3 = \vec{\mathbf{c}} - \dfrac{\vec{\mathbf{c}} \cdot \vec{\mathbf{a}}}{|\vec{\mathbf{c}}|^2}\vec{\mathbf{a}} - \dfrac{\vec{\mathbf{c}} \cdot \vec{\mathbf{b}}_2}{|\vec{\mathbf{c}}|^2}\vec{\mathbf{b}}_1$,

$\vec{\mathbf{c}}_4 = \vec{\mathbf{c}} - \dfrac{\vec{\mathbf{c}} \cdot \vec{\mathbf{a}}}{|\vec{\mathbf{c}}|^2}\vec{\mathbf{a}} - \dfrac{\vec{\mathbf{b}} \cdot \vec{\mathbf{c}}}{|\vec{\mathbf{b}}|^2}\vec{\mathbf{b}}_1$

Then, which of the following is a set of mutually orthogonal vectors?

a) $\{\vec{\mathbf{a}}, \vec{\mathbf{b}}_1, \vec{\mathbf{c}}_1\}$ b) $\{\vec{\mathbf{a}}, \vec{\mathbf{b}}_1, \vec{\mathbf{c}}_2\}$ c) $\{\vec{\mathbf{a}}, \vec{\mathbf{b}}_2, \vec{\mathbf{c}}_3\}$ d) $\{\vec{\mathbf{a}}, \vec{\mathbf{b}}_2, \vec{\mathbf{c}}_4\}$

244. If $\vec{\mathbf{a}}$ is vector perpendicular to both $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ then

a) $\vec{\mathbf{a}} + (\vec{\mathbf{b}} + \vec{\mathbf{c}}) = \vec{\mathbf{0}}$ b) $\vec{\mathbf{a}} \times (\vec{\mathbf{b}} + \vec{\mathbf{c}}) = \vec{\mathbf{0}}$ c) $\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = \vec{\mathbf{0}}$ d) $\vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = \vec{\mathbf{0}}$

245. If G is the centroid of ΔABC and G' is the centroid of $\Delta A'B'C'$, then $A\vec{A}' + B\vec{B}' + C\vec{C}' =$

a) $2G\vec{G}'$ b) $3G\vec{G}'$ c) $G\vec{G}'$ d) $4G\vec{G}'$

246. If $\vec{u} = \vec{a} - \vec{b}, \vec{v} = \vec{a} + \vec{b}$ and $|\vec{a}| = |\vec{b}| = 2$, then $|\vec{u} \times \vec{v}|$ is

a) $2\sqrt{16 - (\vec{a}.\vec{b})^2}$ b) $2\sqrt{4 - (\vec{a}.\vec{b})^2}$ c) $\sqrt{16 - (\vec{a}.\vec{b})^2}$ d) $\sqrt{4 - (\vec{a}.\vec{b})^2}$

247. If the vectors $\vec{a} = (2\log_3 x, a)$ and $\vec{b} = (-3, a\log_3 x, \log_3 x)$ are inclined at an acute angle, then

a) $a = 0$ b) $a < 0$ c) $a > 0$ d) None of these

248. Let $\vec{\mathbf{a}} = \hat{\mathbf{i}} - \hat{\mathbf{j}}, \vec{\mathbf{b}} = \hat{\mathbf{j}} - \hat{\mathbf{k}}, \vec{\mathbf{c}} = \hat{\mathbf{k}} - \hat{\mathbf{i}}$. If $\vec{\mathbf{d}}$ is a unit vectors such that $\vec{\mathbf{a}} \cdot \vec{\mathbf{d}} = 0 = [\vec{\mathbf{b}}\ \vec{\mathbf{c}}\ \vec{\mathbf{d}}]$, then $\vec{\mathbf{d}}$ is (are)

a) $\pm\dfrac{\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}}{\sqrt{3}}$ b) $\pm\dfrac{\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}}{\sqrt{6}}$ c) $\pm\dfrac{\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{3}}$ d) $\pm\hat{\mathbf{k}}$

249. If $\vec{a}$ is a vector of magnitude 50 collinear with the vector $\vec{b} = 6\hat{\imath} - 8\hat{\jmath} - \frac{15}{2}\hat{k}$ and makes an acute angle with the positive direction of z-axis, then $\vec{a} =$

a) $24\hat{\imath} - 32\hat{\jmath} - 30\hat{k}$ b) $-24\hat{\imath} + 32\hat{\jmath} + 30\hat{k}$ c) $12\hat{\imath} - 16\hat{\jmath} - 15\hat{k}$ d) None of these

250. The work done by the force $\vec{F} = 2\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ in moving a particle from $A(3, 4, 5)$ to $B(1, 2, 3)$ is

a) 0 b) 3/2 c) -4 d) -2

251. Let the pairs, $\vec{a}, \vec{b}$ and $\vec{c}, \vec{d}$ each determines a plane. Then the planes are parallel, if

a) $(\vec{a} \times \vec{c}) \times (\vec{b} \times \vec{d}) = 0$ b) $(\vec{a} \times \vec{c}).(\vec{b} \times \vec{d}) = 0$ c) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = 0$ d) $(\vec{a} \times \vec{b}).(\vec{c} \times \vec{d}) = 0$

252. Magnitude of vectors $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are 3,4,5 respectively. If $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}} + \vec{\mathbf{c}}$, $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}} + \vec{\mathbf{a}}$, $\vec{\mathbf{c}}$ and $\vec{\mathbf{a}} + \vec{\mathbf{b}}$ are mutually perpendicular, then magnitude of $\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}$ is

a) $4\sqrt{2}$ b) $3\sqrt{2}$ c) $5\sqrt{2}$ d) $3\sqrt{3}$

253. If $ABCD$ be a parallelogram and M be the point of intersection of the diagonals. If O is any point, then $\overrightarrow{\mathbf{OA}} + \overrightarrow{\mathbf{OB}} + \overrightarrow{\mathbf{OC}} + \overrightarrow{\mathbf{OD}}$ is

a) $3\overrightarrow{\mathbf{OM}}$ b) $4\overrightarrow{\mathbf{OM}}$ c) $\overrightarrow{\mathbf{OM}}$ d) $2\overrightarrow{\mathbf{OM}}$

254. The position vectors of the point A and B with respect to O are $2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$. The length of

the internal bisector of $\angle BOA$ of ΔAOB is

a) $\frac{\sqrt{136}}{9}$ b) $\frac{\sqrt{136}}{3}$ c) $\frac{20}{3}$ d) $\frac{\sqrt{217}}{9}$

255. Let $\vec{A} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \vec{\mathbf{B}} = \hat{\mathbf{i}}, \vec{\mathbf{C}} = c_1\hat{\mathbf{i}} + c_2\hat{\mathbf{j}} + c_3\hat{\mathbf{k}}$. If $c_2 = -1$ and $c_3 = 1$, then to make three vectors coplanar

a) $c_1 = 0$ b) $c_1 = 1$

c) $c_1 = 2$ d) No value of c_1 can be found

256. If, in a right triangle ABC, the hypotenuse $AB = p$, then $\vec{AB} \cdot \vec{AC} + \vec{BC} \cdot \vec{BA} + \vec{CA} \cdot \vec{CB}$ is equal to

a) $2p^2$ b) $\frac{p^2}{2}$ c) p^2 d) None of these

257. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices of an equilateral triangle whose orthocenter is at the origin, then

a) $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ b) $|\vec{a}|^2 = |\vec{b}|^2 + |\vec{c}|^2$ c) $\vec{a} + \vec{b} = \vec{c}$ d) None of these

258. If $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = 4$ and $|\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}| = 2$, then $|\vec{\mathbf{a}}|^2 |\vec{\mathbf{b}}|^2$ is equal to

a) 2 b) 6 c) 8 d) 20

259. If $ABCDEF$ is a regular hexagon, then $\vec{AD} + \vec{EB} + \vec{FC}$ equals

a) $2\vec{AB}$ b) $\vec{0}$ c) $3\vec{AB}$ d) $4\vec{AB}$

260. If $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}, \vec{\mathbf{b}} = 3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and $\vec{\mathbf{c}} = 5\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$, then the volume of the parallelopiped with coterminous edges $\vec{\mathbf{a}} + \vec{\mathbf{b}}, \vec{\mathbf{b}} + \vec{\mathbf{c}}, \vec{\mathbf{c}} + \vec{\mathbf{a}}$ is

a) 4 b) 5 c) 63 d) 8

261. If $\vec{\mathbf{a}} \cdot \hat{\mathbf{i}} = \vec{\mathbf{a}} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}}) = \vec{\mathbf{a}} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 1$, then $\vec{\mathbf{a}}$ is equal to

a) $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ b) $\hat{\mathbf{i}} - \hat{\mathbf{k}}$ c) $\hat{\mathbf{i}}$ d) $\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$

262. If $\vec{a}$ and $\vec{b}$ are unit vectors, then the greatest value of $\sqrt{3}|\vec{a} + \vec{b}| + |\vec{a} - \vec{b}|$ is

a) 2 b) $2\sqrt{2}$ c) 4 d) None of these

263. If A, B, C, D, E are five coplanar points, then $\vec{\mathbf{DA}} + \vec{\mathbf{DB}} + \vec{\mathbf{DC}} + \vec{\mathbf{AE}} + \vec{\mathbf{BE}} + \vec{\mathbf{CE}}$ is equal to

a) $\vec{\mathbf{OE}}$ b) $3\ \vec{\mathbf{DE}}$ c) $2\ \vec{\mathbf{DE}}$ d) $4\ \vec{\mathbf{ED}}$

264. If $\vec{a}.\hat{\imath} = \vec{a}(\hat{\imath} + \hat{\jmath}) = \vec{a}.(\hat{\imath} + \hat{\jmath} + \hat{k}) = 1$, then $\vec{a} =$

a) $\vec{0}$ b) $\hat{\imath}$ c) $\hat{\jmath}$ d) $\hat{\imath} + \hat{\jmath} + \hat{k}$

265. If the position vectors of the vertices of $\Delta\ ABC$ are $3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$, $\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$ and $-2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$, then the triangle ABC is

a) Right angled and isosceles b) Right angled, but not isosceles

c) Isosceles but not right angled d) Equilateral

266. The volume of the parallelopiped whose coterminous edges are $\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $2\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and $3\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$, is

a) 4 cu unit b) 3 cu unit c) 2 cu unit d) 8 cu unit

267. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then

a) $\vec{a}$ is parallel to $\vec{b}$ b) $\vec{a} \perp \vec{b}$ c) $|\vec{a}| = |\vec{b}|$ d) None of these

268. If $|\vec{\mathbf{a}} + \vec{\mathbf{b}}| = |\vec{\mathbf{a}} - \vec{\mathbf{b}}|$, then angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is ($\vec{\mathbf{a}} \neq \vec{\mathbf{0}}, \vec{\mathbf{b}} \neq \vec{\mathbf{0}}$)

a) $\frac{\pi}{3}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{2}$

269. If the vectors $\alpha\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\hat{\mathbf{i}} + \beta\hat{\mathbf{j}} + \hat{\mathbf{k}}, \hat{\mathbf{i}} + \hat{\mathbf{j}} + \gamma\hat{\mathbf{k}} (\alpha, \beta, \gamma \neq 1)$ are coplanar, then the value of $\frac{1}{1-\alpha} + \frac{1}{1-\beta} - \frac{1}{1-\gamma}$ is

a) -1 b) 0 c) 1 d) 1/2

270. The unit vector in ZOX plane and making angle 45° and 60° respectively with $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\vec{\mathbf{b}} = 0\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$, is

a) $-\frac{1}{\sqrt{2}}\hat{\mathbf{i}}+\frac{1}{\sqrt{2}}\hat{\mathbf{k}}$
b) $\frac{1}{\sqrt{2}}\hat{\mathbf{i}}-\frac{1}{\sqrt{2}}\hat{\mathbf{k}}$
c) $\frac{1}{3\sqrt{2}}\hat{\mathbf{i}}+\frac{4}{3\sqrt{2}}\hat{\mathbf{j}}+\frac{1}{3\sqrt{2}}\hat{\mathbf{k}}$
d) None of these above

271. If the vectors
$\vec{a}=\hat{\imath}+a\hat{\jmath}+a^2\hat{k}, \vec{b}=\hat{\imath}+b\hat{\jmath}+b^2\hat{k}, \vec{c}=\hat{\imath}+c\hat{\jmath}+c^2\hat{k}$
are three non-coplanar vectors and $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=0$, then the value of abc is
a) 0
b) 1
c) 2
d) -1

272. Let $\hat{\mathbf{u}}$ and $\hat{\mathbf{v}}$ are unit vectors such that $\hat{\mathbf{u}}\cdot\hat{\mathbf{v}}=0$ If $\hat{\mathbf{r}}$ is any vector coplanar with $\hat{\mathbf{u}}$ and $\hat{\mathbf{v}}$, then the magnitude of the vector $\vec{\mathbf{r}}\times(\hat{\mathbf{u}}\times\hat{\mathbf{v}})$ is
a) 0
b) 1
c) $|\vec{\mathbf{r}}|$
d) $2|\vec{\mathbf{r}}|$

273. The projection of the vector $\hat{\imath}-2\hat{\jmath}+\hat{k}$ on the vector $4\hat{\imath}-4\hat{\jmath}+7\hat{k}$, is
a) $\frac{5\sqrt{6}}{10}$
b) $\frac{19}{9}$
c) $\frac{9}{19}$
d) $\frac{\sqrt{6}}{19}$

274. $\frac{\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})}{\vec{\mathbf{b}}\cdot(\vec{\mathbf{c}}\times\vec{\mathbf{a}})}+\frac{\vec{\mathbf{b}}\cdot(\vec{\mathbf{a}}\times\vec{\mathbf{b}})}{\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})}$ is equal to
a) 1
b) 2
c) 0
d) ∞

275. If $\vec{\mathbf{u}}_1$ and $\vec{\mathbf{u}}_2$ be vectors of unit length and θ be the angle between them, then
$\frac{1}{2}|\vec{\mathbf{u}}_2-\vec{\mathbf{u}}_1|$is
a) $\sin\theta$
b) $\sin\frac{\theta}{2}$
c) $\cos\theta$
d) $\cos\frac{\theta}{2}$

276. Let $\vec{b}=4\hat{\imath}+3\hat{\jmath}$ be two vectors perpendicular to each other in the xy-plane. Then, a vector in the same plane having projections 1 and 2 along $\vec{b}$ and $\vec{c}$, respectively, is
a) $\hat{\imath}+2\hat{\jmath}$
b) $2\hat{\imath}-\hat{\jmath}$
c) $2\hat{\imath}+\hat{\jmath}$
d) None of these

277. Find the equation of the perpendicular drown from the origin to the plane $2x+4y-5z=10$
a) $\vec{\mathbf{r}}=(2k,5k,4k)k\in R$
b) $\vec{\mathbf{r}}=(2k,4k,-5k)k\in R$
c) $\vec{\mathbf{r}}=(2k,4k,5k)k\in R$
d) None of these

278. The vector $\vec{a}$ coplanar with the vectors $\hat{\imath}$ and $\hat{\jmath}$, perpendicular to the vector $\vec{b}=4\hat{\imath}-3\hat{\jmath}+5\hat{k}$ such that $|\vec{a}|=|\vec{b}|$ is
a) $\sqrt{2}(3\hat{\imath}+4\hat{\jmath})$ or, $-\sqrt{2}(3\hat{\imath}+4\hat{\jmath})$
b) $\sqrt{2}(4\hat{\imath}+3\hat{\jmath})$ or, $-\sqrt{2}(4\hat{\imath}+3\hat{\jmath})$
c) $\sqrt{3}(4\hat{\imath}+5\hat{\jmath})$ or, $-\sqrt{3}(4\hat{\imath}+5\hat{\jmath})$
d) $\sqrt{3}(5\hat{\imath}+4\hat{\jmath})$ or, $-\sqrt{3}(5\hat{\imath}+4\hat{\jmath})$

279. Let $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ be vectors with magnitude 3,4 and 5 respectively and $\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}=\vec{\mathbf{0}}$, then the value of $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}+\vec{\mathbf{b}}\cdot\vec{\mathbf{c}}+\vec{\mathbf{c}}\cdot\vec{\mathbf{a}}$ is
a) 47
b) 25
c) 50
d) -25

280. If $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are the position vectors of the vertices of an equilateral triangle, whose orthocenter is at the origin, then
a) $\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}=\vec{\mathbf{0}}$
b) $\vec{\mathbf{a}}^2=\vec{\mathbf{b}}^2+\vec{\mathbf{c}}^2$
c) $\vec{\mathbf{a}}+\vec{\mathbf{b}}=\vec{\mathbf{c}}$
d) None of these

281. If $4\hat{\mathbf{i}}+7\hat{\mathbf{j}}+8\hat{\mathbf{k}}, 2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+4\hat{\mathbf{k}}$ and $2\hat{\mathbf{i}}+5\hat{\mathbf{j}}+7\hat{\mathbf{k}}$ are the position vectors of the vertices A, B and C respectively of triangle ABC. The position vector of the point where the bisector of angle A meets BC is
a) $\frac{1}{2}(6\hat{\mathbf{i}}+13\hat{\mathbf{j}}+18\hat{\mathbf{k}})$
b) $\frac{2}{3}(6\hat{\mathbf{i}}+12\hat{\mathbf{j}}-8\hat{\mathbf{k}})$
c) $\frac{1}{3}(-6\hat{\mathbf{i}}-8\hat{\mathbf{j}}-9\hat{\mathbf{k}})$
d) $\frac{2}{3}(-6\hat{\mathbf{i}}-12\hat{\mathbf{j}}+8\hat{\mathbf{k}})$

282. If the vectors $\vec{a}=2\hat{\imath}+3\hat{\jmath}+6\hat{k}$ are collinear and $|\vec{b}|=21$, then $\vec{b}=$
a) $\pm3(2\hat{\imath}+3\hat{\jmath}+6\hat{k})$
b) $\pm(2\hat{\imath}+3\hat{\jmath}-6\hat{k})$
c) $\pm21(2\hat{\imath}+3\hat{\jmath}+6\hat{k})$
d) $\pm21(\hat{\imath}+\hat{\jmath}+\hat{k})$

283. The value of $[\vec{a}-\vec{b}, \vec{b}-\vec{c}, \vec{c}-\vec{a}]$, where $|\vec{a}|=1, |\vec{b}|=5, |\vec{c}|=3$, is

a) 0 b) 1 c) 6 d) None of these

284. The distance between the line $\vec{\mathbf{r}} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}} + \lambda\,(\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})$ and the plane $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + \hat{\mathbf{k}}) = 5$ is

a) $\frac{10}{9}$ b) $\frac{3}{10}$ c) $\frac{10}{3\sqrt{3}}$ d) $\frac{10}{9}$

285. In a parallelogram $ABCD$, $|\vec{AB}| = a$, $|\vec{AD}| = b$ and $|\vec{AC}| = c$. Then, $\vec{DB}.\vec{AB}$ has the value

a) $\frac{3a^2+b^2-c^2}{2}$ b) $\frac{a^2+3b^2-c^2}{2}$ c) $\frac{a^2-b^2+3c^2}{2}$ d) $\frac{a^2+3b^2+c^2}{2}$

286. If $\vec{\mathbf{a}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $\vec{\mathbf{b}} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$, then the angle between the vectors $\vec{\mathbf{a}} + \vec{\mathbf{b}}$ and $\vec{\mathbf{a}} - \vec{\mathbf{b}}$ is

a) 60° b) 90° c) 45° d) 55°

287. If $\vec{a} = \hat{\imath} + \hat{\jmath} - \hat{k}$, $\vec{b} = -\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{c} = -\hat{\imath} + 2\hat{\jmath} - \hat{k}$, then a unit vector normal to the vectors $\vec{a} + \vec{b}$ and $\vec{b} - \vec{c}$ is

a) $\hat{\imath}$ b) $\hat{\jmath}$ c) $\hat{k}$ d) None of these

288. If $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ and three vectors such that $\vec{\mathbf{a}} = \vec{\mathbf{b}} + \vec{\mathbf{c}}$ and the angle between $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ is $\frac{\pi}{2}$ then

a) $a^2 = b^2 + c^2$ b) $b^2 = c^2 + a^2$ c) $c^2 = a^2 + b^2$ d) $2a^2 - b^2 = c^2$

289. If the position vector of A with respect to O is $3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ and $\overrightarrow{\mathbf{AB}} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$

Then the position vector of B with respect to O is

a) $-\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ b) $6\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ c) $\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ d) $\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

290. If $\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\vec{\mathbf{b}} = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and $\vec{\mathbf{c}} = 7\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$, then the area of the parallelogram having diagonals $\vec{\mathbf{a}} + \vec{\mathbf{b}}$ and $\vec{\mathbf{b}} + \vec{\mathbf{c}}$ is

a) $4\sqrt{6}$ b) $\frac{1}{2}\sqrt{21}$ c) $\frac{\sqrt{6}}{2}$ d) $\sqrt{6}$

291. The angle between the vectors $\vec{\mathbf{a}} + \vec{\mathbf{b}}$ and $\vec{\mathbf{a}} - \vec{\mathbf{b}}$, where $\vec{\mathbf{a}} = (1,1,4)$ and $\vec{\mathbf{b}} = (1,-1,4)$ is

a) 90° b) 45° c) 30° d) 15°

292. Area of rhombus is ……., where diagonals are $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and $\vec{\mathbf{b}} = -\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

a) $\sqrt{21.5}$ b) $\sqrt{31.5}$ c) $\sqrt{28.5}$ d) $\sqrt{38.5}$

293. If the vectors $\hat{\imath} - 2x\,\hat{\jmath} - 3y\,\hat{k}$ and $\hat{\imath} + 3x\,\hat{\jmath} + 2y\,\hat{k}$ are orthogonal to each other, then the locus of the point (x, y) is

a) A circle b) An ellipse c) A parabola d) A straight line

294. If the position vectors of the vertices of a triangle are $2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ and $3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$, then the triangle is

a) Equilateral b) Isosceles c) Right angled isosceles d) Right angled

295. The two variable vectors $3x\hat{\mathbf{i}} + y\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $x\hat{\mathbf{i}} - 4\,y\,\hat{\mathbf{j}} + 4\,\hat{\mathbf{k}}$ are orthogonal to each other, then the locus of $(x, y)$is

a) Hyperbola b) Circle c) Straight line d) Ellipse

296. If $|\vec{a}| = |\vec{b}| = |\vec{a} + \vec{b}| = 1$, then $|\vec{a} - \vec{b}|$ is equal to

a) 1 b) $\sqrt{2}$ c) $\sqrt{3}$ d) None of these

297. The angle between the vectors $2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ and $2\hat{\imath} - \hat{\jmath} - \hat{k}$ is

a) $\pi/2$ b) $\pi/4$ c) $\pi/3$ d) None of these

298. A unit vector coplanar with $\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and perpendicular to $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ is

a) $\left(\frac{\hat{\mathbf{j}} - \hat{\mathbf{k}}}{\sqrt{2}}\right)$ b) $\left(\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{3}}\right)$ c) $\left(\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}}{\sqrt{6}}\right)$ d) $\left(\frac{\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{6}}\right)$

299. The length of the longer diagonal of the parallelogram constructed on $5\vec{a} + 2\vec{b}$ and $\vec{a} - 3\vec{b}$ if it is given that $|\vec{a}| = 2\sqrt{2}$, $|\vec{b}| = 3$ and angle between $\vec{a}$ and $\vec{b}$ is $\pi/4$, is

a) 15 b) $\sqrt{113}$ c) $\sqrt{593}$ d) $\sqrt{369}$

300. The position vector of the point where the line $\vec{\mathbf{r}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}} + t(\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}})$ meets the plane $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 5$ is

a) $5\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$ b) $5\hat{\mathbf{i}}+3\hat{\mathbf{j}}-3\hat{\mathbf{k}}$ c) $2\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ d) $5\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$

301. If $\vec{a}+\vec{b}+\vec{c}=0, |\vec{a}|=3, |\vec{b}|=5, |\vec{c}|=7$, then the angle between $\vec{a}$ and $\vec{b}$ is
a) $\pi/6$ b) $2\pi/3$ c) $5\pi/3$ d) $\pi/3$

302. If $\vec{\mathbf{a}}$ is perpendicular to $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}|\vec{\mathbf{a}}|=2, |\vec{\mathbf{b}}|=3, |\vec{\mathbf{c}}|=4$ and the angle between $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ is $\frac{2\pi}{3}$, then $[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$is equal to
a) $4\sqrt{3}$ b) $6\sqrt{3}$ c) $12\sqrt{3}$ d) $18\sqrt{3}$

303. The position vectors of the points A, B, C are $(2\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}), (3\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}})$ and $(\hat{\mathbf{i}}+4\hat{\mathbf{j}}-3\hat{\mathbf{k}})$ respectively. These points
a) Form an isosceles triangle
b) Form a right angled triangle
c) Are collinear
d) Form a scalene triangle

304. If $\vec{a}=4\hat{\imath}+6\hat{\jmath}$ and $\vec{b}=3\hat{\jmath}+4\hat{k}$, then the vector form of component of $\vec{a}$ along $\vec{b}$ is
a) $\frac{18}{10\sqrt{3}}(3\hat{\jmath}+4\hat{k})$ b) $\frac{18}{25}(3\hat{\jmath}+4\hat{k})$ c) $\frac{18}{\sqrt{3}}(3\hat{\jmath}+4\hat{k})$ d) $3\hat{\jmath}+4\hat{k}$

305. Two vectors $\vec{a}$ and $\vec{b}$ are non-collinear. If vectors $\vec{c}=(x-2)\vec{a}+\vec{b}$ and $\vec{d}=(2x+1)\vec{a}-\vec{b}$ are collinear, then $x=$
a) $1/3$ b) $1/2$ c) 1 d) 0

306. Through the point $P(\alpha,\beta,\gamma)$ a plane is drawn at right angles to OP to meet the coordinate axes are A, B, C respectively. If $OP=p$ then equation of plane $\overrightarrow{A, B, C}$ is
a) $\alpha x+\beta y+\gamma z=p$
b) $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=p$
c) $2\alpha x+2\beta y+2\gamma z=p^2$
d) $\alpha x+\beta y+\gamma z=p^2$

307. If $ABCDEF$ is a regular hexagon with $\overrightarrow{\mathbf{AB}}=\vec{\mathbf{a}}$ and $\overrightarrow{\mathbf{BC}}=\vec{\mathbf{b}}$, then $\overrightarrow{\mathbf{CE}}$ equals
a) $\vec{\mathbf{b}}-\vec{\mathbf{a}}$ b) $-\vec{\mathbf{b}}$ c) $\vec{\mathbf{b}}-2\vec{\mathbf{a}}$ d) None of these

308. A unit vector perpendicular to both $\hat{\imath}+\hat{\jmath}$ and $\hat{\jmath}+\hat{k}$, is
a) $\hat{\imath}-\hat{\jmath}+\hat{k}$ b) $\hat{\imath}+\hat{\jmath}+\hat{k}$ c) $\frac{\hat{\imath}+\hat{\jmath}+\hat{k}}{\sqrt{3}}$ d) $\frac{\hat{\imath}-\hat{\jmath}+\hat{k}}{\sqrt{3}}$

309. Let $ABCD$ be the parallelogram whose sides AB and AD are represented by the vectors $2\hat{\mathbf{i}}+4\hat{\mathbf{j}}-5\hat{\mathbf{k}}$ and $\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ respectively. Then, if $\vec{\mathbf{a}}$ is a unit vector parallel to $\overrightarrow{\mathbf{AC}}$, then $\vec{\mathbf{a}}$ equal to
a) $\frac{1}{3}(3\hat{\mathbf{i}}-6\hat{\mathbf{j}}-2\hat{\mathbf{k}})$ b) $\frac{1}{3}(3\hat{\mathbf{i}}+6\hat{\mathbf{j}}+2\hat{\mathbf{k}})$ c) $\frac{1}{7}(3\hat{\mathbf{i}}-6\hat{\mathbf{j}}-3\hat{\mathbf{k}})$ d) $\frac{1}{7}(3\hat{\mathbf{i}}+6\hat{\mathbf{j}}-2\hat{\mathbf{k}})$

310. The value of b such that the scalar product of the vector $\hat{\imath}+\hat{\jmath}+\hat{k}$ with the unit vector parallel to the sum of the vectors $2\hat{\imath}+4\hat{\jmath}-5\hat{k}$ and $b\hat{\imath}+2\hat{\jmath}+3\hat{k}$ is one, is
a) -2 b) -1 c) 0 d) 1

311. If $\vec{a},\vec{b},\vec{c}$ are non-coplanar vectors and $x\vec{a}+y\vec{b}+z\vec{c}=0$, then
a) At least one of x, y, z is zero
b) x, y, z are necessarily zero
c) None of them are zero
d) None of these

312. The ratio in which $\hat{\imath}+2\hat{\jmath}+3\hat{k}$ divides the join of $-2\hat{\imath}+3\hat{\jmath}+5\hat{k}$ and $7\hat{\imath}-\hat{k}$, is
a) $1:2$ b) $2:3$ c) $3:4$ d) $1:4$

313. For any three vectors $\vec{a},\vec{b},\vec{c}$ the expression $(\vec{a}-\vec{b})\cdot\{(\vec{b}-\vec{c})\times(\vec{c}-\vec{a})\}$ equals
a) $[\vec{a}\vec{b}\vec{c}]$ b) $2[\vec{a}\vec{b}\vec{c}]$ c) $[\vec{a}\vec{b}\vec{c}]^2$ d) None of these

314. The point of intersection of the lines $\vec{\mathbf{r}}=7\hat{\mathbf{i}}+10\hat{\mathbf{j}}+3\hat{\mathbf{k}}+s(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+4\hat{\mathbf{k}})$ and $\vec{\mathbf{r}}=3\hat{\mathbf{i}}+5\hat{\mathbf{j}}+7\hat{\mathbf{k}}+t(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}})$ is
a) $\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$ b) $2\hat{\mathbf{i}}-\hat{\mathbf{j}}+4\hat{\mathbf{k}}$ c) $\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$ d) $\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$

315. let $\vec{p}$ and $\vec{q}$ be the position vectors of P and Q respectively, with respect to O and $|\vec{p}|=p, |\vec{q}|=q$. The points R and S divide PQ internally and externally in the ratio $2:3$ respectively. If $\overrightarrow{OR}$ and $\overrightarrow{OS}$ are

perpendicular, then

a) $9p^2 = 4q^2$ b) $4p^2 = 9q^2$ c) $9p = 4q$ d) $4p = 9q$

316. If $\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\vec{\mathbf{b}} = 2\hat{\mathbf{i}} - \hat{\mathbf{k}}$ are two vectors, then the point of intersection of two lines $\vec{\mathbf{r}} \times \vec{\mathbf{a}} = \vec{\mathbf{b}} \times \vec{\mathbf{a}}$ and $\vec{\mathbf{r}} \times \vec{\mathbf{b}} = \vec{\mathbf{a}} \times \vec{\mathbf{b}}$ is

a) $\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ b) $\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ c) $3\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ d) $3\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$

317. If $\vec{\mathbf{A}} \times (\vec{\mathbf{B}} \times \vec{\mathbf{C}}) = \vec{\mathbf{B}} \times (\vec{\mathbf{C}} \times \vec{\mathbf{A}})$ and $[\vec{\mathbf{A}}\,\vec{\mathbf{B}}\,\vec{\mathbf{C}}] \neq 0$, then $\vec{\mathbf{A}} \times (\vec{\mathbf{B}} \times \vec{\mathbf{C}})$is equal to

a) $\vec{\mathbf{0}}$ b) $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$ c) $\vec{\mathbf{B}} \times \vec{\mathbf{C}}$ d) $\vec{\mathbf{C}} \times \vec{\mathbf{A}}$

318. If $\vec{a}$ and $\vec{b}$ are two vectors, then the equality $|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$ holds

a) Only if $\vec{a} = \vec{b} = \vec{0}$

b) For all $\vec{a}, \vec{b}$

c) Only if $\vec{a} = \lambda\vec{b}, \lambda > 0$ or $\vec{a} = \vec{b} = \vec{0}$

d) None of these

319. Let $\vec{\mathbf{a}} = \hat{\mathbf{i}} - \hat{\mathbf{k}}, \vec{\mathbf{b}} = x\hat{\mathbf{i}} + \hat{\mathbf{j}} + (1-x)\hat{\mathbf{k}}$ and $\vec{\mathbf{c}} = y\hat{\mathbf{i}} + x\hat{\mathbf{j}} + (1+x-y)\hat{\mathbf{k}}$. Then $[\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}]$ depends on

a) neither x nor y b) both x and y c) only x d) only y

320. If the position vectors of three points A, B, C are respectively $\hat{\imath} + \hat{\jmath} + \hat{k}, 2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$ and $7\hat{\imath} + 4\hat{\jmath} + 9\hat{k}$, then the unit vector perpendicular to the plane of triangle ABC is

a) $31\hat{\imath} - 18\hat{\jmath} - 9\hat{k}$ b) $\dfrac{31\hat{\imath} - 38\hat{\jmath} - 9\hat{k}}{\sqrt{2486}}$ c) $\dfrac{31\hat{\imath} + 38\hat{\jmath} + 9\hat{k}}{\sqrt{2486}}$ d) None of these

321. For any three vectors $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$, $(\vec{\mathbf{a}} - \vec{\mathbf{b}}) \cdot (\vec{\mathbf{b}} + \vec{\mathbf{c}}) \times (\vec{\mathbf{c}} + \vec{\mathbf{a}})$ is equal to

a) $2\vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}})$ b) $[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$ c) $[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]^2$ d) 0

322. If $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are unit coplanar vectors, then $[2\vec{\mathbf{a}} - \vec{\mathbf{b}}\;\; 2\vec{\mathbf{b}} - \vec{\mathbf{c}}\;\; 2\vec{\mathbf{c}} - \vec{\mathbf{a}}]$ is equal to

a) 1 b) 0 c) $-\sqrt{3}$ d) $\sqrt{3}$

323. If $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are two unit vectors inclined to x-axis at anlges 30° and 120°, then $|\vec{\mathbf{a}} + \vec{\mathbf{b}}|$ equals

a) $\sqrt{\dfrac{2}{3}}$ b) $\sqrt{2}$ c) $\sqrt{3}$ d) 2

324. If the vectors $\hat{\imath} - 2x\,\hat{\jmath} + 3y\,\hat{k}$ and $\hat{\imath} + 2x\hat{\jmath} - 3y\hat{k}$ perpendicular, then the locus of (x, y) is

a) A circle b) An ellipse c) A hyperbola d) None of these

325. Let $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ be non-zero vectors such that $(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times \vec{\mathbf{c}} = -\frac{1}{4}|\vec{\mathbf{b}}||\vec{\mathbf{c}}|\vec{\mathbf{a}}$. If θ is the acute angle between vectors $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$, then the angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{c}}$ is equal to

a) $\dfrac{2\pi}{3}$ b) $\dfrac{\pi}{4}$ c) $\dfrac{\pi}{3}$ d) $\dfrac{\pi}{2}$

326. A vector perpendicular to both the vectors $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ is

a) $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ b) $\hat{\mathbf{i}} - \hat{\mathbf{j}}$ c) $c(\hat{\mathbf{i}} - \hat{\mathbf{j}}), c$ is a scalar d) None of these

327. If $\vec{a}, \vec{b}, \vec{c}$ are non-collinear vectors such that $\vec{a} + \vec{b}$ is parallel to $\vec{c}$ and $\vec{c} + \vec{a}$ is parallel to $\vec{b}$, then

a) $\vec{a} + \vec{b} = \vec{c}$

b) $\vec{a}, \vec{b}, \vec{c}$ taken in order from the sides of a triangle

c) $\vec{b} + \vec{c} = \vec{a}$

d) None of these

328. A force of magnitude $\sqrt{6}$ acting along the line joining the points $A(2, -1, 1)$ and $B(3, 1, 2)$ displaces a particle from A to B. The work done by the force is

a) 6 b) $6\sqrt{6}$ c) $\sqrt{6}$ d) 12

329. A unit vector $\vec{\mathbf{a}}$ makes an angle $\frac{\pi}{4}$ with z-axis, if $\vec{\mathbf{a}} + \hat{\mathbf{i}} + \hat{\mathbf{j}}$ is a unit vector, then $\vec{\mathbf{a}}$ is equal to

a) $\dfrac{\hat{\mathbf{i}}}{2} + \dfrac{\hat{\mathbf{j}}}{2} + \dfrac{\hat{\mathbf{k}}}{2}$ b) $\dfrac{\hat{\mathbf{i}}}{2} + \dfrac{\hat{\mathbf{j}}}{2} - \dfrac{\hat{\mathbf{k}}}{\sqrt{2}}$ c) $-\dfrac{\hat{\mathbf{i}}}{2} - \dfrac{\hat{\mathbf{j}}}{2} + \dfrac{\hat{\mathbf{k}}}{\sqrt{2}}$ d) $\dfrac{\hat{\mathbf{i}}}{2} - \dfrac{\hat{\mathbf{j}}}{2} - \dfrac{\hat{\mathbf{k}}}{\sqrt{2}}$

330. If $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}|^2 + |\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}|^2 = 144$ and $|\vec{\mathbf{a}}| = 4$ then $|\vec{\mathbf{b}}|$ is equal to

a) 12 b) 3 c) 8 d) 4

331. If $\vec{\mathbf{a}}$ is non-zero vector of modulus $|\vec{\mathbf{a}}|$ and m is a non-zero scalar, then $m\,\vec{\mathbf{a}}$ is a unit vector, if

a) $m = \pm 1$ b) $m = |\vec{\mathbf{a}}|$ c) $m = \dfrac{1}{|\vec{\mathbf{a}}|}$ d) $m = \pm\, 2$

332. If the constant forces $2\hat{\imath} - 5\hat{\jmath} + 6\hat{k}$ and $-\hat{\imath} + 2\hat{\jmath} - \hat{k}$ act on a particle due to which it is displaced from a point $A(4, -3, -2)$ to a point $B(6, 1, -3)$, then the work done by the forces is

a) 15 units b) -15 units c) 9 units d) -9 units

333. If P, Q, R are three points with respective position vectors $\hat{\imath} + \hat{\jmath}, \hat{\imath} - \hat{\jmath}$ and $a\hat{\imath} + b\hat{\jmath} + c\hat{k}$. The points P, Q, R are collinear, if

a) $a = b = c = 1$ b) $a = b = c = 0$ c) $a = 1, b, c \in R$ d) $a = 1, c = 0, b \in R$

334. The projection of the vector $\vec{a} = 4\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ on the axis making equal acute angles with the coordinate axes is

a) 3 b) $\sqrt{3}$ c) $\dfrac{3}{\sqrt{3}}$ d) None of these

335. The value of $[2\,\hat{\mathbf{i}}\; 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}\,]$ is equal to

a) -30 b) -25 c) 0 d) 11

336. $(\vec{a} \times \vec{b}) \times (\vec{a} \times \vec{c}) \cdot \vec{d}$ equals

a) $[\vec{a}\vec{b}\vec{c}](\vec{b} \cdot \vec{d})$ b) $[\vec{a}\vec{b}\vec{c}](\vec{a} \cdot \vec{d})$ c) $[\vec{a}\vec{b}\vec{c}](\vec{c} \cdot \vec{d})$ d) None of these

337. If the constant force $2\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ and $-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ act on a particle due to which it is displaced from a point $A(4, -3, -2)$ to a point $B(6,1,-3)$ then the work done by the force is

a) 10 units b) -10 units c) 9 units d) None of these

338. If forces of magnitudes 6 and 7 units acting in the directions $\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ and $2\hat{\imath} - 3\hat{\jmath} - 6\hat{k}$ respectively act on a particle which is displaced from the point $P(2, -1, -3)$ to $Q(5, -1, 1)$, then the work done by the forces is

a) 4 units b) -4 units c) 7 units d) -7 units

339. $[\vec{\mathbf{b}} \times \vec{\mathbf{c}}\;\; \vec{\mathbf{c}} \times \vec{\mathbf{a}}\;\; \vec{\mathbf{a}} \times \vec{\mathbf{b}}]$ is equal to

a) $[\,\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}\,]$ b) $2[\,\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}\,]$ c) $[\,\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}\,]^2$ d) $\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}})$

340. $ABCD$ is a quadrilateral , P, Q are the mid points of $\overrightarrow{\mathbf{BC}}$ and $\overrightarrow{\mathbf{AD}}$, then $\overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{DC}}$ is equal to

a) $3\,\overrightarrow{\mathbf{QP}}$ b) $\overrightarrow{\mathbf{QP}}$ c) $4\overrightarrow{\mathbf{QP}}$ d) $2\overrightarrow{\mathbf{QP}}$

341. If D, E, F are respectively the mid-points of AB, AC and BC respectively in a ΔABC, then $\overrightarrow{BE} + \overrightarrow{AF} =$

a) $\overrightarrow{DC}$ b) $\dfrac{1}{2}\overrightarrow{BF}$ c) $2\overrightarrow{BF}$ d) $\dfrac{3}{2}\overrightarrow{BF}$

342. $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are mutually perpendicular unit vectors, then $|\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}|$ is equal to

a) $\sqrt{3}$ b) 3 c) 1 d) 0

343. Let $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}, \vec{b} = 3\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $\vec{c} = d\hat{\imath} + \hat{\jmath} + (2d - 1)\hat{k}$. If $\vec{c}$ is parallel to the plane of the vectors $\vec{a}$ and $\vec{b}$, then $11d =$

a) 2 b) 1 c) -1 d) 0

344. If $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are three non-coplanar vectors and $\vec{\mathbf{p}}, \vec{\mathbf{q}}, \vec{\mathbf{r}}$, are reciprocal vectors, then $(l\vec{\mathbf{a}} + m\vec{\mathbf{b}} + n\vec{\mathbf{c}}) \cdot (l\vec{\mathbf{p}} + m\vec{\mathbf{q}} + n\vec{\mathbf{r}})$ is

a) $l + m + n$ b) $l^3 + m^3 + n^3$ c) $l^2 + m^2 + n^2$ d) None of these

345. If $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \cdot \vec{\mathbf{c}}$ are unit vectors, then $|\,\vec{\mathbf{a}} - \vec{\mathbf{b}}\,|^2 + |\vec{\mathbf{b}} - \vec{\mathbf{c}}\,|^2 + |\vec{\mathbf{c}} - \vec{\mathbf{a}}|^2$ does not exceed

a) 4 b) 9 c) 8 d) 6

346. A constant force $\vec{\mathbf{F}} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ is acting on a particle such that the particle is displaced from the point(1,2,3) to the point (3,4,5). The work done by the force is

a) 2 b) 3 c) 4 d) 5

347. The value of a, for which the points A, B, C with position vectors $2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}, \hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$ and $a\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$ respectively are the vertices of a right

with $C = \frac{\pi}{2}$ are

a) -2 and -1 b) -2 and 1 c) 2 and -1 d) 2 and 1

348. If $(\vec{a} \times \vec{b}) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$, then

a) $\vec{b} \times (\vec{c} \times \vec{a}) = \vec{0}$ b) $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{0}$ c) $\vec{c} \times \vec{a} = \vec{a} \times \vec{b}$ d) $\vec{c} \times \vec{b} = \vec{b} \times \vec{a}$

349. If $\vec{a} + \vec{b} \neq 0$ and $\vec{c}$ is a non-zero vector, then $(\vec{a} + \vec{b}) \times \{\vec{c} - (\vec{a} + \vec{b})\}$ is equal to

a) $\vec{a} + \vec{b}$ b) $(\vec{a} + \vec{b}) \times \vec{c}$ c) $\lambda\, \vec{c}$, where $\lambda \neq 0$ d) $\lambda(\vec{a} \times \vec{b}), \lambda \neq 0$

350. If a force $\vec{\mathbf{F}} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ is acting at the point $P(1, -1, 2)$ then the magnitude of moment of $\vec{\mathbf{F}}$ about the point $Q(2, -1, 3)$ is

a) $\sqrt{57}$ b) $\sqrt{39}$ c) 12 d) 17

351. If $|\vec{\mathbf{a}}| = |\vec{\mathbf{b}}| = 1$ and $|\vec{\mathbf{a}} + \vec{\mathbf{b}}| = \sqrt{3}$, then the value of $(3\,\vec{\mathbf{a}} - 4\vec{\mathbf{b}}) \cdot (2\vec{\mathbf{a}} + 5\vec{\mathbf{b}})$is

a) -21 b) $-\frac{21}{2}$ c) 21 d) $\frac{21}{2}$

352. If $\hat{a}, \hat{b}, \hat{c}$ are three unit vectors such that $\hat{b}$ and $\hat{c}$ are non-parallel and $\hat{a} \times (\hat{b} \times \hat{c}) = \frac{1}{2}\hat{b}$, then the angle between $\hat{a}$ and $\hat{c}$ is

a) 30° b) 45° c) 60° d) 90°

353. If the vectors $3\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$ and $2\hat{\imath} - \hat{\jmath} + 8\hat{k}$ are perpendicular, then λ is equal to

a) -14 b) 7 c) 14 d) 1/7

354. The equation of the plane perpendicular to the line $\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z+1}{2}$ and passing through the point(2,3,1) is

a) $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 1$ b) $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 1$ c) $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 7$ d) $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}) = 10$

355. $(\vec{\mathbf{a}} - \vec{\mathbf{b}}) \cdot \{(\vec{\mathbf{b}} - \vec{\mathbf{c}}) \times (\vec{\mathbf{c}} - \vec{\mathbf{a}})\}$ is equal to

a) $2\,\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \times \vec{\mathbf{c}}$ b) $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \times \vec{\mathbf{c}}$ c) 0 d) $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}$

356. If $\hat{n}_1, \hat{n}_2$ are two unit vectors and θ is the angle between them, then $\cos\theta/2 =$

a) $\frac{1}{2}|\hat{n}_1 + \hat{n}_2|$ b) $\frac{1}{2}|\hat{n}_1 - \hat{n}_2|$ c) $\frac{1}{2}(\hat{n}_1.\hat{n}_2)$ d) $\frac{|\hat{n}_1 \times \hat{n}_2|}{2|\hat{n}_1|\,|\hat{n}_2|}$

357. Let $ABCD$ be the parallelogram whose sides AB and AD are represented by the vectors $2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$ and $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ respectively. Then if $\vec{\mathbf{a}}$ is a unit vector parallel to $\overrightarrow{\mathbf{AC}}$, then $\vec{\mathbf{a}}$ is equal to

a) $(3\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}})/3$ b) $(3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 2\hat{\mathbf{k}})/3$ c) $(3\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 3\hat{\mathbf{k}})/7$ d) $(3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}})/7$

358. If the points with position vectors $60\hat{\mathbf{i}} + 3\hat{\mathbf{j}}, 40\hat{\mathbf{i}} - 8\hat{\mathbf{j}}$ and $a\hat{\mathbf{i}} - 52\hat{\mathbf{j}}$ are collinear, then a is equal to

a) -40 b) -20 c) 20 d) 40

359. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors such that $\vec{a} + \vec{b} + \vec{c} = \alpha\vec{d}$ and $\vec{b} + \vec{c} + \vec{d} = \beta\vec{a}$, then $\vec{a} + \vec{b} + \vec{c} + \vec{d}$ is equal to

a) $\vec{0}$ b) $\alpha\vec{a}$ c) $\beta\vec{b}$ d) $(\alpha + \beta)\vec{c}$

360. The unit vector perpendicular to $\hat{\mathbf{i}} - \hat{\mathbf{j}}$ and coplanar with $\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ and $\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ is

a) $\frac{2\hat{\mathbf{i}} - 5\hat{\mathbf{j}}}{\sqrt{29}}$ b) $2\hat{\mathbf{i}} + 5\hat{\mathbf{j}}$ c) $\frac{1}{\sqrt{2}}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$ d) $\hat{\mathbf{i}} + \hat{\mathbf{j}}$

361. If $\vec{r} \cdot \vec{a} = \vec{r} \cdot \vec{b} = \vec{r} \cdot \vec{c} = 0$ for some non-zero vector $\vec{r}$, then the value of $[\vec{a}\vec{b}\vec{c}]$, is

a) 2 b) 3 c) 0 d) None of these

362. If the angle between $\hat{\mathbf{i}} + \hat{\mathbf{k}}$ and $\hat{\mathbf{i}} + \hat{\mathbf{j}} + a\hat{\mathbf{k}}$ is $\frac{\pi}{3}$, then the value of a is

a) 0 or 2 b) -4 or 0 c) 0 or -2 d) 2 or -2

363. A vector which makes equal angles with the vectors $\frac{1}{3}(\hat{\imath} - 2\hat{\jmath} + 2\hat{k}), \frac{1}{5}(-4\hat{\imath} - 3\hat{k})$, and $\hat{\jmath}$, is

a) $5\hat{\imath} + \hat{\jmath} + 5\hat{k}$ b) $-5\hat{\imath} + \hat{\jmath} + 5\hat{k}$ c) $-5\hat{\imath} + \hat{\jmath} + 5\hat{k}$ d) $5\hat{\imath} + \hat{\jmath} - 5\hat{k}$

364. Which one of the following vectors is of magnitude 6 and perpendicular to both $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and$\vec{\mathbf{b}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$?

a) $2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ b) $2(2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ c) $3(2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}})$ d) $2(2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

365. In a right angled triangle ABC, the hypotenuse $Ab = p$, then $\vec{AB}.\vec{AC} + \vec{BC}.\vec{BA} + \vec{CA}.\vec{CB}$ is equal to

a) $2p^2$ b) $\frac{p^2}{2}$ c) p^2 d) None of these

366. Which one of the following is not correct?

a) If $\vec{\mathbf{p}} \cdot \vec{\mathbf{a}} = \vec{\mathbf{p}} \cdot \vec{\mathbf{b}} = \vec{\mathbf{p}} \cdot \vec{\mathbf{c}}$ for some non-zero vector $\vec{\mathbf{p}}$ then $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are coplanar

b) The vectors $\hat{\mathbf{i}} + 3\hat{\mathbf{j}}, 2\hat{\mathbf{i}} + \hat{\mathbf{k}}$ and $\hat{\mathbf{j}} + \hat{\mathbf{k}}$ are coplanar

c) The vector $\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}})$ is coplanar with $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$

d) If $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ are unit vectors and angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is $\frac{\pi}{3}$, then $|\vec{\mathbf{a}} + \vec{\mathbf{b}}| < 1$

367. The length of the shortest distance between the two lines
$\vec{\mathbf{r}} = (-3\hat{\mathbf{i}} + 6\hat{\mathbf{j}}) + s(-4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ and $\vec{\mathbf{r}} = (-2\hat{\mathbf{i}} + 7\hat{\mathbf{k}}) + t(-4\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$ is

a) 7 units b) 13 units c) 8 units d) 9 units

368. A vector perpendicular to the plane containing the points $A(1.-1,2), B(2,0,-1), C(0,2,1)$ is

a) $4\hat{\mathbf{i}} + 8\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ b) $8\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ c) $3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ d) $\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$

369. If $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are unit vectors such that $[\vec{\mathbf{a}}\, \vec{\mathbf{b}}\, \vec{\mathbf{a}} \times \vec{\mathbf{b}}] = \frac{1}{4}$, then angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is

a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{2}$

370. If $|\vec{\mathbf{a}}| = 3, |\vec{\mathbf{b}}| = 4$, then a value of λ for which $\vec{\mathbf{a}} + \lambda\vec{\mathbf{b}}$ is perpendicular to $\vec{\mathbf{a}} - \lambda\vec{\mathbf{b}}$, is

a) $\frac{9}{16}$ b) $\frac{3}{4}$ c) $\frac{3}{2}$ d) $\frac{4}{3}$

371. $(\vec{\mathbf{x}} - \vec{\mathbf{y}}) \times (\vec{\mathbf{x}} + \vec{\mathbf{y}}) =$....... where $\vec{\mathbf{x}}, \vec{\mathbf{y}} \in R^3$

a) $2(\vec{\mathbf{x}} \times \vec{\mathbf{y}})$ b) $|\vec{\mathbf{x}}|^2 - |\vec{\mathbf{y}}|^2$ c) $\frac{1}{2}(\vec{\mathbf{x}} \times \vec{\mathbf{y}})$ d) None of these

372. If the vectors $\vec{\mathbf{a}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}, \vec{\mathbf{b}} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\vec{\mathbf{c}} = \lambda\hat{\mathbf{i}} + \hat{\mathbf{j}} + \mu\hat{\mathbf{k}}$ are mutually orthogonal, then (λ, μ) is equal to

a) $(-3,2)$ b) $(2,-3)$ c) (-2.3) d) $(3,-2)$

373. Given that $\vec{a} = (1,1,1), \vec{c} = (0,1,-1)$ and $\vec{a}.\vec{b} = 3$. If $\vec{a} \times \vec{b} = \vec{c}$, then $\vec{b} =$

a) $\left(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right)$ b) $\left(\frac{2}{3}, \frac{2}{3}, \frac{4}{3}\right)$ c) $\left(\frac{5}{3}, \frac{2}{3}, \frac{2}{3}\right)$ d) None of these

374. If $\hat{a}, \hat{b}$ and $\hat{c}$ are three unit vectors such that $\hat{a} + \hat{b} + \hat{c}$ is also a unit vector and θ_1, θ_2 and θ_3 are the angles between the vectors $\hat{a}, \hat{b}; \hat{b}, \hat{c}$ and $\hat{c}, \hat{a}$ respectively, then among θ_1, θ_2 and θ_3

a) All are acute angles
b) All are right angles
c) At least one is obtuse angle
d) None of these

375. Given vectors $\vec{x} = 3\hat{\imath} - 6\hat{\jmath} - \hat{k}, \vec{y} = \hat{\imath} + 4\hat{\jmath} - 3\hat{k}$ and $\vec{z} = 3\hat{\imath} + 4\hat{\jmath} + 12\hat{k}$, then the projection of $\vec{x} \times \vec{y}$ on vector $\vec{z}$ is

a) 14 b) -14 c) 12 d) 15

376. If the vectors $\vec{a}$ and $\vec{b}$ are mutually perpendicular, then $\vec{a} \times \left\{\vec{a} \times \left\{\vec{a} \times \left(\vec{a} \times \vec{b}\right)\right\}\right\}$ is equal to

a) $|\vec{a}|^2\vec{b}$ b) $|\vec{a}|^3\vec{b}$ c) $|\vec{a}|^4\vec{b}$ d) None of these

377. Let G be the centroid of ΔABC. If $\vec{AB} = \vec{a}, \vec{AC} = \vec{b}$, then the $\vec{AG}$, in terms of $\vec{a}$ and $\vec{b}$ is

a) $\frac{2}{3}(\vec{a} + \vec{b})$ b) $\frac{1}{6}(\vec{a} + \vec{b})$ c) $\frac{1}{3}(\vec{a} + \vec{b})$ d) $\frac{1}{2}(\vec{a} + \vec{b})$

378. The moment of the couple formed by the forces $5\hat{\imath} + \hat{k}$ and $-5\hat{\imath} - \hat{k}$ acting at the point $(9,-1,2)$ and $(3,-2,1)$ respectively is

a) $-\hat{\imath} + \hat{\jmath} + 5\hat{k}$ b) $\hat{\imath} - \hat{\jmath} - 5\hat{k}$ c) $2\hat{\imath} - 2\hat{\jmath} - 10\hat{k}$ d) $-2\hat{\imath} + 2\hat{\jmath} + 10\hat{k}$

379. The value of c so that for all real x, then vectors o$cx\,\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 3\hat{\mathbf{k}}, x\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2cx\hat{\mathbf{k}}$ make an obtuse angle are

a) $c < 0$ b) $0 < c < \frac{4}{3}$ c) $-\frac{4}{3} < c < 0$ d) $c > 0$

380. If θ be the angle between the vectors $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\vec{\mathbf{b}} = 6\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$, then

a) $\cos\theta = \dfrac{4}{21}$ b) $\cos\theta = \dfrac{3}{19}$ c) $\cos = \dfrac{2}{19}$ d) $\cos\theta = \dfrac{5}{21}$

381. The vectors $2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$ and $a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ are perpendicular when

a) $a = 2, b = 3, c = -4$ b) $a = 4, b = 4, c = 5$ c) $a = 4, b = 4, c = -2$ d) None of these

382. If $\vec{\boldsymbol{\alpha}} = x(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) + y(\vec{\mathbf{b}} \times \vec{\mathbf{b}}) + z(\vec{\mathbf{c}} \times \vec{\mathbf{a}})$ and $[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}] = \frac{1}{8}$, then x + y + z is equal to

a) $8\,\vec{\alpha} \cdot (\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}})$ b) $\vec{\alpha} \cdot (\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}})$ c) $8\,(\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}})$ d) None of these

383. If vectors $3\hat{\mathbf{i}} + \hat{\mathbf{j}} - 5\hat{\mathbf{k}}$ and $a\hat{\mathbf{i}} + b\hat{\mathbf{j}} - 15\hat{\mathbf{k}}$ are collinear, then

a) $a = 3, b = 1$ b) $a = 9, b = 1$ c) $a = 3, b = 3$ d) $a = 9, b = 3$

384. Let $\vec{\mathbf{a}}$and $\vec{\mathbf{b}}$ be two unit vectors such that angle between them is 60°. Then, $|\vec{\mathbf{a}} - \vec{\mathbf{b}}|$is equal to

a) $\sqrt{5}$ b) $\sqrt{3}$ c) 0 d) 1

385. The point collinear with $(1, -2, -3)$and $(2,0,0)$ among the following is

a) $(0,4,6)$ b) $(0, -4, -5)$ c) $(0, -4, -6)$ d) $(0, -4,6)$

386. If $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are unit vectors, then the vectors $(\vec{\mathbf{a}} + \vec{\mathbf{b}}) \times (\vec{\mathbf{a}} \times \vec{\mathbf{b}})$ is parallel to the vector

a) $\vec{\mathbf{a}} - \vec{\mathbf{b}}$ b) $\vec{\mathbf{a}} + \vec{\mathbf{b}}$ c) $2\,\vec{\mathbf{a}} - \vec{\mathbf{b}}$ d) $2\,\vec{\mathbf{a}} + \vec{\mathbf{b}}$

387. If θ is the angle between the lines AB and AC where A, B and C are the three points with coordinates $(1,2,-1), (2,0,3), (3,-1,2)$ respectively, then $\sqrt{462}\cos\theta$ is equal to

a) 20 b) 10 c) 30 d) 40

388. Let $\vec{\mathbf{v}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\vec{\mathbf{w}} = \hat{\mathbf{i}} + 3\hat{\mathbf{k}}$, If $\vec{\mathbf{u}}$ is a unit vector, then maximum value of the scalar triple product $[\,\vec{\mathbf{u}}\,\vec{\mathbf{v}}\,\vec{\mathbf{w}}\,]$ is

a) -1 b) $\sqrt{10} + \sqrt{6}$ c) $\sqrt{59}$ d) $\sqrt{60}$

389. Each of the angle between vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is equal to 60°. If $|\vec{a}| = 4, |\vec{b}| = 2$ and $|\vec{c}| = 6$, then the modulus of $\vec{a} + \vec{b} + \vec{c}$, is

a) 10 b) 15 c) 12 d) None of these

390. A force of magnitude 5 unit acting along the vector $2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ displaces the point of applications from (1,2,3) to (5,3,7) then the work done is

a) 50/7 unit b) 50/3 unit c) 25/3 unit d) 25/4 unit

391. The equation of the plane passing through three non-collinear points $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ is

a) $\vec{\mathbf{r}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}} + \vec{\mathbf{a}} \times \vec{\mathbf{b}}) = 0$ b) $\vec{\mathbf{r}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}} + \vec{\mathbf{a}} \times \vec{\mathbf{b}}) = [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$

c) $\vec{\mathbf{r}} \cdot \left(\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}})\right) = [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$ d) $\vec{\mathbf{r}} \cdot (\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}) = 0$

392. If a vector $\vec{r}$ of magnitude $3\sqrt{6}$ is directed along the bisector of the angle between the vectors $\vec{a} = 7\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$ and $\vec{b} = -2\hat{\imath} - \hat{\jmath} + 2\hat{k}$, then $\vec{r} =$

a) $\hat{\imath} - 7\hat{\jmath} + 2\hat{k}$ b) $\hat{\imath} + 7\hat{\jmath} - 2\hat{k}$ c) $-\hat{\imath} + 7\hat{\jmath} + 2\hat{k}$ d) $\hat{\imath} - 7\hat{\jmath} - 2\hat{k}$

393. If the point whose position vectors are $2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $6\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $14\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + p\hat{\mathbf{k}}$ are collinear, then the value of p is

a) 2 b) 4 c) 6 d) 8

394. Let $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ be non-zero vectors such that

$$(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times \vec{\mathbf{c}} = \frac{1}{3}|\vec{\mathbf{b}}|\,|\vec{\mathbf{c}}|\vec{\mathbf{a}}$$

If θ is the acute angle between the vectors $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ then $\sin\theta$ equals

a) $\dfrac{1}{3}$ b) $\dfrac{\sqrt{2}}{3}$ c) $\dfrac{2}{3}$ d) $\dfrac{2\sqrt{2}}{3}$

395. Let ABC be a triangle, the position vectors of whose vertices are respectively $7\hat{\mathbf{i}} + 10\hat{\mathbf{k}}, -\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ and $-4\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ Then, the ΔABC is

a) Isosceles b) Equilateral

c) Right angled isosceles d) None of these

396. If C is the middle point of AB and P is any point outside AB, then

a) $\vec{PA} + \vec{PB} = \vec{PC}$ b) $\vec{PA} + \vec{PB} = 2\vec{PC}$ c) $\vec{PA} + \vec{PB} + \vec{PC} = \vec{0}$ d) $\vec{PA} + \vec{PB} + 2\vec{PC} = \vec{0}$

397. If $\vec{a},\vec{b}$ are any two vwctors, then $(2\vec{a}+3\vec{b})\times(5\vec{a}+7\vec{b})+\vec{a}\times\vec{b}$ is equal to
a) $\vec{0}$ b) 0 c) $\vec{a}\times\vec{b}$ d) $\vec{b}\times\vec{a}$

398. The moment about the point $M(-2,4,-6)$ of the force represented in magnitude and position by AB where the points A and B have the coordinates $(1,2,-3)$ and $(3,-4,2)$ respectively is
a) $8\hat{\imath}-9\hat{\jmath}-14\hat{k}$ b) $2\hat{\imath}-6\hat{\jmath}+5\hat{k}$ c) $-3\hat{\imath}+2\hat{\jmath}-3\hat{k}$ d) $-5\hat{\imath}+8\hat{\jmath}-8\hat{k}$

399. If the position vectors of A,B and $C$are respectively $2\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}},\hat{\mathbf{i}}-3\hat{\mathbf{j}}-5\hat{\mathbf{k}}$ and $3\hat{\mathbf{i}}-4\hat{\mathbf{j}}-4\hat{\mathbf{k}}$ then $\cos^2 A$ is equal to
a) 0 b) $\frac{6}{41}$ c) $\frac{35}{41}$ d) 1

400. If $\vec{r}\cdot\vec{a}=\vec{r}\cdot\vec{b}=\vec{r}\cdot\vec{c}=0$ where $\vec{a},\vec{b},\vec{c}$ are non-coplanar, then
a) $\vec{r}\perp\vec{c}\times\vec{a}$ b) $\vec{r}\perp\vec{a}\times\vec{b}$ c) $\vec{r}\perp\vec{b}\times\vec{c}$ d) $\vec{r}=\vec{0}$

401. If $\vec{a},\vec{b},\vec{c}$ be three non-coplanar vectors and $\vec{p},\vec{q},\vec{r}$ constitute the corresponding reciprocal system of vectors then for any arbitrary vector $\vec{\alpha}$
a) $\vec{\alpha}=(\vec{\alpha}\cdot\vec{a})\vec{a}+(\vec{\alpha}\cdot\vec{b})\vec{b}+(\vec{\alpha}\cdot\vec{c})\vec{c}$
b) $\vec{\alpha}=(\vec{\alpha}\cdot\vec{p})\vec{p}+(\vec{\alpha}\cdot\vec{q})\vec{q}+(\vec{\alpha}\cdot\vec{r})\mathbf{r}$
c) $\vec{\alpha}=(\vec{\alpha}\cdot\vec{p})\vec{a}+(\vec{\alpha}\cdot\vec{q})\vec{b}+(\vec{\alpha}\cdot\vec{r})\vec{c}$
d) None of the above

402. The vector $\vec{a}\times(\vec{b}\times\vec{c})$ is coplanar with the vectors
a) $\vec{b},\vec{c}$ b) $\vec{a},\vec{b}$ c) $\vec{a},\vec{c}$ d) $\vec{a},\vec{b},\vec{c}$

403. If $\vec{b}$ is a unit vector, then $(\vec{a}\cdot\vec{b})\vec{b}+\vec{b}\times(\vec{a}\times\vec{b})$is
a) $|\vec{a}|^2\vec{b}$ b) $|\vec{a}\cdot\vec{b}|\vec{a}$ c) $\vec{a}$ d) $\vec{b}$

404. If $\sum_{i=1}^{n}|\vec{a_i}|=\vec{0}$, where $|\vec{a_i}|=1\forall i$, then the value of $\sum_{1\le i<}\sum_{j\le n}\vec{a_i}\cdot\vec{a}_j$ is
a) n^2 b) $-n^2$ c) n d) $-\frac{n}{2}$

405. If the vector $3\hat{\mathbf{i}}-2\hat{\mathbf{j}}-5\hat{\mathbf{k}}$ is perpendicular to $c\hat{\mathbf{k}}-\hat{\mathbf{j}}+6\hat{\mathbf{i}}$ then c is equal to
a) 3 b) 4 c) 5 d) 6

406. If $\vec{a}\times\vec{b}=\vec{0}$ and $\vec{a}\cdot\vec{b}=0$, then
a) $\vec{a}\perp\vec{b}$ b) $\vec{a}||\vec{b}$ c) $\vec{a}=\vec{0}$ and $\vec{b}=\vec{0}$ d) $\vec{a}=\vec{0}$ or $\vec{b}=\vec{0}$

407. If $2\hat{\mathbf{i}}+4\hat{\mathbf{j}}-5\hat{\mathbf{k}}$ and $\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ are adjacent side of a parallelogram, then the lengths of its diagonals are
a) $7,\sqrt{69}$ b) $6,\sqrt{59}$ c) $5,\sqrt{65}$ d) $5,\sqrt{55}$

408. Let $\vec{a},\vec{b},\vec{c}$ be unit vectors such that $\vec{a}+\vec{b}+\vec{c}=0$. Which of the following is correct?
a) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}=\vec{0}$
b) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}\neq\vec{0}$
c) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{a}\times\vec{c}=\vec{0}$
d) $\vec{a}\times\vec{b},\vec{b}\times\vec{c},\vec{c}\times\vec{a}$ are mutually perpendicular

409. If G is the centre of a regular hexagon $ABCDEF$, then $\vec{AB}+\vec{AC}+\vec{AD}+\vec{AE}+\vec{AF}=$
a) $3\vec{AG}$ b) $2\vec{AG}$ c) $6\vec{AG}$ d) $4\vec{AG}$

410. **I.** Two non-zero. Non-collinear vectors are linearly independent .
II. Any three coplanar vectors are linearly dependent. Which of the above statements is /are true?
a) Only **I** b) Only **II** c) Both **I** and **II** d) Neither **I** nor **II**

411. If $\vec{a},\vec{b}$ and $\vec{c}$ are unit coplanar vectors, then
$[2\vec{a}-3\vec{b}\ 7\vec{b}-9\vec{c}\ 12\vec{c}-23\vec{a}]$ is equal ro
a) 0 b) 1/2 c) 24 d) 32

412. $[\vec{a}+\vec{b}\ \vec{b}+\vec{c}\ \vec{c}+\vec{a}]=[\vec{a}\ \vec{b}\ \vec{c}]$, then
a) $[\vec{a}\ \vec{b}\ \vec{c}]=1$
b) $\vec{a},\vec{b},\vec{c}$ are coplanar
c) $[\vec{a}\ \vec{b}\ \vec{c}]=-1$
d) $\vec{a},\vec{b},\vec{c}$ are mutually perpendicular

413. If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=\sqrt{37},|\vec{b}|=3,|\vec{c}|=4$, then the angle between $\vec{b}$ and $\vec{c}$
a) 30° b) 45° c) 60° d) 90°

414. A unit vector coplanar with $\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and $\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}$, and perpendicular to $\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$ is

a) $\left(\frac{\hat{\mathbf{j}}-\hat{\mathbf{k}}}{\sqrt{2}}\right)$ b) $\left(\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}}{\sqrt{3}}\right)$ c) $\left(\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}}}{\sqrt{6}}\right)$ d) $\left(\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}}{\sqrt{6}}\right)$

415. The projection of the vector $\hat{\imath}+\hat{\jmath}+\hat{k}$ along the vector of $\hat{\jmath}$, is
a) 1 b) 0 c) 2 d) -1

416. Volume of the parallelopiped having vertices at $O \equiv (0,0,0)$, $A \equiv (2,-2,4)$, $B \equiv (5,-4,4)$ and $C \equiv (1,-2,4)$
a) 5 cu units b) 10 cu units c) 15 cu units d) 20 cu units

417. The area of parallelogram constructed on the vectors $\vec{a}=\vec{p}+2\vec{q}$ and $\vec{b}=2\vec{p}+\vec{q}$, where $\vec{p}$ and $\vec{q}$ are unit vectors forming an angle of 30° is
a) 3/2 b) 5/2 c) 7/2 d) None of these

418. If $\vec{a}$ is a vector perpendicular to the vectors $\vec{b}=\hat{\imath}+2\hat{\jmath}+3\hat{k}$ and $c=-2\hat{\imath}+4\hat{\jmath}+\hat{k}$ and satisfies the condition $\vec{a}.\left(\hat{\imath}-2\hat{\jmath}+\hat{k}\right)=-6$, then $\vec{a}=$
a) $5\hat{\imath}+\frac{7}{2}\hat{\jmath}-4\hat{k}$ b) $10\hat{\imath}+7\hat{\jmath}-8\hat{k}$ c) $5\hat{\imath}-\frac{7}{2}\hat{\jmath}+4\hat{k}$ d) None of these

419. The projection of $\vec{\mathbf{a}}=3\hat{\mathbf{i}}-\hat{\mathbf{j}}+5\hat{\mathbf{k}}$ on $\vec{\mathbf{b}}=2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+\hat{\mathbf{k}}$ is
a) $\frac{8}{\sqrt{35}}$ b) $\frac{9}{\sqrt{39}}$ c) $\frac{8}{\sqrt{14}}$ d) $\sqrt{14}$

420. Let $ABCDEF$ be a regular hexagon and $\overrightarrow{\mathbf{AB}}=\vec{\mathbf{a}}, \overrightarrow{\mathbf{BC}}=\vec{\mathbf{b}}, \overrightarrow{\mathbf{CD}}=\vec{\mathbf{c}}$, then $\overrightarrow{\mathbf{AE}}$ is equal to
a) $\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}$ b) $\vec{\mathbf{b}}+\vec{\mathbf{c}}$ c) $\vec{\mathbf{a}}+\vec{\mathbf{b}}$ d) $\vec{\mathbf{a}}+\vec{\mathbf{c}}$

421. Three vectors $7\hat{\mathbf{i}}-11\hat{\mathbf{j}}+\hat{\mathbf{k}}, 5\hat{\mathbf{i}}+3\hat{\mathbf{j}}-2\hat{\mathbf{k}}$ and $12\hat{\mathbf{i}}-8\hat{\mathbf{j}}-\hat{\mathbf{k}}$ from
a) an equilateral triangle b) an isosceles triangle
c) a right angled triangle d) Collinear

422. If $|\vec{\mathbf{a}}|=2, |\vec{\mathbf{b}}|=3$, and $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ are mutually perpendicular, then the area of triangle whose vertices are $\vec{\mathbf{0}}, \vec{\mathbf{a}}+\vec{\mathbf{b}}, \vec{\mathbf{a}}-\vec{\mathbf{b}}$ is
a) 5 b) 1 c) 6 d) 8

423. If V is the volume of the parallelopiped having three coterminus edges as $\vec{a}, \vec{b}$ and $\vec{c}$, then the volume of the parallelopiped having three coterminus edges as
$\vec{\alpha}=(\vec{a}\cdot\vec{a})\vec{a}+\left(\vec{a}\cdot\vec{b}\right)\vec{b}+(\vec{a}\cdot\vec{c})\vec{c}$
$\vec{\beta}=\left(\vec{a}\cdot\vec{b}\right)\vec{a}+\left(\vec{b}\cdot\vec{b}\right)\vec{b}+\left(\vec{b}\cdot\vec{c}\right)\vec{c}$
$\vec{\gamma}=(\vec{a}\cdot\vec{c})\vec{a}+\left(\vec{b}\cdot\vec{c}\right)\vec{b}+(\vec{c}\cdot\vec{c})\vec{c}$, is
a) V^3 b) $3V$ c) V^2 d) $2V$

424. The unit vectors orthogonal to the vector $-\hat{\imath}+2\hat{\jmath}+2\hat{k}$ and making equal angles with the X and Y axes is (are)
a) $\pm\frac{1}{3}\left(2\hat{\imath}+2\hat{\jmath}-\hat{k}\right)$ b) $\pm\frac{1}{3}\left(\hat{\imath}+\hat{\jmath}-\hat{k}\right)$ c) $\pm\frac{1}{3}\left(2\hat{\imath}-2\hat{\jmath}-\hat{k}\right)$ d) None of these

425. The unit vector perpendicular to vectors $\hat{\imath}-\hat{\jmath}$ and $\hat{\imath}+\hat{\jmath}$ forming a right handed system is
a) $\hat{k}$ b) $-\hat{k}$ c) $\frac{1}{\sqrt{2}}(\hat{\imath}-\hat{\jmath})$ d) $\frac{1}{\sqrt{2}}(\hat{\imath}+\hat{\jmath})$

426. Given, $\vec{\mathbf{p}}=3\hat{\mathbf{i}}+2\hat{\mathbf{j}}+4\hat{\mathbf{k}}, \vec{\mathbf{a}}=\hat{\mathbf{i}}+\hat{\mathbf{j}}, \vec{\mathbf{b}}=\hat{\mathbf{j}}+\hat{\mathbf{k}}, \vec{\mathbf{c}}=\hat{\mathbf{i}}+\hat{\mathbf{k}}$ and $\vec{\mathbf{p}}=x\vec{\mathbf{a}}+y\vec{\mathbf{b}}+z\vec{\mathbf{c}}$, then x, y, z are respectively
a) $\frac{3}{2}, \frac{1}{2}, \frac{5}{2}$ b) $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ c) $\frac{5}{2}, \frac{3}{2}, \frac{1}{2}$ d) $\frac{1}{2}, \frac{5}{2}, \frac{3}{2}$

427. If S is the circumcentre, O is the orthocentre of ΔABC, then $\overrightarrow{\mathbf{SA}}+\overrightarrow{\mathbf{SB}}+\overrightarrow{\mathbf{SC}}$ is equal to
a) $\overrightarrow{\mathbf{SO}}$ b) $2\,\overrightarrow{\mathbf{SO}}$ c) $\overrightarrow{\mathbf{OS}}$ d) $2\,\overrightarrow{\mathbf{OS}}$

428. If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a}.\vec{b}=0$ and $\vec{a}\times\vec{b}=\vec{0}$, then
a) $\vec{a}||\vec{b}$ b) $\vec{a}\perp\vec{b}$ c) Either $\vec{a}$ and $\vec{b}$ is a null vector d) None of these

429. If a tetrahedron has vertices at $O(0,0,0), A(1,2,1), B(2,1,3)$ and $C(-1,1,2)$. Then, the angle between the faces OAB and ABC will be

a) $\cos^{-1}\left(\frac{19}{35}\right)$ b) $\cos^{-1}\left(\frac{17}{31}\right)$ c) $30°$ d) $90°$

430. If $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are vectors such that the $|\vec{\mathbf{a}}+\vec{\mathbf{b}}| = |\vec{\mathbf{a}}-\vec{\mathbf{b}}|$, then the angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is
a) $120°$ b) $60°$ c) $90°$ d) $30°$

431. If $\vec{a}$ and $\vec{b}$ are not perpendicular to each other and $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}, \vec{r}.\vec{c} = 0$, then $\vec{r}$ is equal to
a) $\vec{a} - \vec{c}$
b) $\vec{b} + x\,\vec{a}$ for all scalars x
c) $\vec{b} - \frac{(\vec{b}.\vec{c})}{(\vec{a}.\vec{c})}\vec{a}$
d) None of these

432. Let $\vec{\alpha}, \vec{\beta}$ and $\vec{\gamma}$ be the unit vectors such that $\vec{\alpha}$ and $\vec{\beta}$ are mutually perpendicular and $\vec{\gamma}$ is equally inclined to $\vec{\alpha}$ and $\vec{\beta}$ at an angle θ. If $\vec{\gamma} = x\vec{\alpha} + y\vec{\beta} + z(\vec{\alpha} \times \vec{\beta})$, then which one of the following is incorrect?
a) $z^2 = 1 - 2x^2$ b) $z^2 = 1 - 2y^2$ c) $z^2 = 1 - x^2 - y^2$ d) $x^2 + y^2 = 1$

433. If $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{c}} \times \vec{\mathbf{d}}$ and $\vec{\mathbf{a}} \times \vec{\mathbf{c}} = \vec{\mathbf{b}} \times \vec{\mathbf{d}}$, then
a) $(\vec{\mathbf{a}} - \vec{\mathbf{d}}) = \lambda(\vec{\mathbf{b}} - \vec{\mathbf{c}})$ b) $(\vec{\mathbf{a}} + \vec{\mathbf{d}}) = \lambda(\vec{\mathbf{b}} + \vec{\mathbf{c}})$ c) $(\vec{\mathbf{a}} - \vec{\mathbf{b}}) = \lambda(\vec{\mathbf{c}} + \vec{\mathbf{d}})$ d) $(\vec{\mathbf{a}} + \vec{\mathbf{b}}) = \lambda(\vec{\mathbf{c}} - \vec{\mathbf{d}})$

434. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar mutually perpendicular unit vectors, then $[\vec{a}\vec{b}\vec{c}]$, is
a) $+1$ b) 0 c) -2 d) 2

435. If P, Q, R and S are four points in space, then $|\overrightarrow{\mathbf{PQ}} \times \overrightarrow{\mathbf{RS}} + \overrightarrow{\mathbf{QR}} \times \overrightarrow{\mathbf{SP}} + \overrightarrow{\mathbf{RS}} \times \overrightarrow{\mathbf{QS}}| = k$ (area of ΔPQR). The value of k is
a) 0 b) 2 c) 4 d) 3

436. In a ΔABC, if $\vec{AB} = 3\hat{\imath} + 4\hat{k}, \vec{AC} = 5\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$, then the length of median through A, is
a) $3\sqrt{2}$ b) $6\sqrt{2}$ c) $5\sqrt{2}$ d) $\sqrt{33}$

437. The vectors $\overrightarrow{\mathbf{AB}} = 3\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ and $\overrightarrow{\mathbf{AC}} = 5\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ are the sides of a triangle ABC. The length of the median through A is
a) $\sqrt{13}$ units b) $2\sqrt{5}$ units c) 5 units d) 10 units

438. If $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are non-coplanar vectors and $(\vec{\mathbf{a}} - \lambda\,\vec{\mathbf{b}}) \cdot (\vec{\mathbf{b}} - 2\vec{\mathbf{c}}) \times (\vec{\mathbf{c}} + 2\vec{\mathbf{a}}) = 0$, then λ is equal to
a) 1 b) $1/4$ c) 0 d) $-1/4$

439. If $\vec{a}$ is perpendicular to $\vec{b}$ and $\vec{r}$ is a non-zero vector such that, $p\vec{r} + (\vec{r}.\vec{b})\vec{a} = \vec{c}$, then $\vec{r} =$
a) $\frac{\vec{c}}{p} - \frac{(\vec{b}.\vec{c})\vec{a}}{p^2}$ b) $\frac{\vec{a}}{p} - \frac{(\vec{c}.\vec{a})\vec{b}}{p^2}$ c) $\frac{\vec{b}}{p} - \frac{(\vec{a}.\vec{b})\vec{c}}{p^2}$ d) $\frac{\vec{c}}{p^2} - \frac{(\vec{b}.\vec{c})\vec{a}}{p}$

440. Constant forces $\vec{\mathbf{P}}_1 = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}, \vec{\mathbf{P}}_2 = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\vec{\mathbf{P}}_3 = \hat{\mathbf{j}} - \hat{\mathbf{k}}$ act on a particle at point A. The work done when the particle is displaced from the point A to B where $\vec{\mathbf{A}} = 4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = 6\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ is
a) 3 b) 9 c) 20 d) None of these

441. The point of intersection of $\vec{\mathbf{r}} \times \vec{\mathbf{a}} = \vec{\mathbf{b}} \times \vec{\mathbf{a}}$ and $\vec{\mathbf{r}} \times \vec{\mathbf{b}} = \vec{\mathbf{a}} \times \vec{\mathbf{b}}$, where $\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\vec{\mathbf{b}} = \hat{\mathbf{i}} - \hat{\mathbf{k}}$ is
a) $3\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ b) $3\hat{\mathbf{i}} - \hat{\mathbf{k}}$ c) $3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ d) None of these

442. If the non-zero vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other, then the solution of the equation, $\vec{r} \times \vec{a} = \vec{b}$ is given by
a) $\vec{r} = x\vec{a} + \frac{\vec{a} \times \vec{b}}{|\vec{a}|^2}$ b) $\vec{r} = x\vec{b} - \frac{\vec{a} \times \vec{b}}{|\vec{b}|^2}$ c) $\vec{r} = x(\vec{a} \times \vec{b})$ d) $\vec{r} = x(\vec{b} \times \vec{a})$

443. If $\vec{a}, \vec{b}, \vec{c}$ are position vectors of the vertices of a triangle ABC, then a unit vector perpendicular to its plane is
a) $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ b) $\frac{\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}}{|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}$ c) $\frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$ d) None of these

444. If $\vec{\mathbf{u}}, \vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ are three non-coplanar vectors, then $(\vec{\mathbf{u}} + \vec{\mathbf{v}} - \vec{\mathbf{w}}) \cdot [(\vec{\mathbf{u}} - \vec{\mathbf{v}}) \times (\vec{\mathbf{v}} - \vec{\mathbf{w}})]$ equals
a) 0 b) $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} \times \vec{\mathbf{w}}$ c) $\vec{\mathbf{u}} \cdot \vec{\mathbf{w}} \times \vec{\mathbf{v}}$ d) $3\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} \times \vec{\mathbf{w}}$

445. The resultant of $(\vec{\mathbf{p}}-2\vec{\mathbf{q}})$ where. $\vec{\mathbf{p}}=7\hat{\mathbf{i}}-2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ and $\vec{\mathbf{q}}=3\hat{\mathbf{i}}+\hat{\mathbf{j}}+5\hat{\mathbf{k}}$ is

a) $\sqrt{29}$ b) 4 c) $\sqrt{62}-2\sqrt{35}$ d) $\sqrt{66}$

446. If $\vec{a},\vec{b},\vec{c}$ are three non-zero vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $m=\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$, then

a) $m<0$ b) $m>0$ c) $m=0$ d) $m=3$

447. If $\vec{\mathbf{a}}=\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$, $\vec{\mathbf{b}}=\hat{\mathbf{i}}+\hat{\mathbf{j}}$, $\vec{\mathbf{c}}=\hat{\mathbf{i}}$ and $(\vec{\mathbf{a}}\times\vec{\mathbf{b}})\times\vec{\mathbf{c}}=\lambda\,\vec{\mathbf{a}}+\mu\,\vec{\mathbf{b}}$, then $\lambda+\mu$ is equal to

a) 0 b) 1 c) 2 d) 3

448. If $\vec{a}=2\hat{\imath}+2\hat{\jmath}+3\hat{k},\vec{b}=-\hat{\imath}+2\hat{\jmath}+\hat{k},\vec{c}=3\hat{\imath}+\hat{\jmath}$ and $\vec{a}+t\vec{b}$ is normal to the vector $\vec{c}$, then the vector of t is

a) 8 b) 4 c) 6 d) 2

449. If $\vec{a},\vec{b}$ represent the diagonals of a rhombus, then

a) $\vec{a}\times\vec{b}=\vec{0}$ b) $\vec{a}.\vec{b}=\vec{0}$ c) $\vec{a}\times\vec{b}=1$ d) $\vec{a}\times\vec{b}=\vec{a}$

450. Three vectors $\vec{a},\vec{b},\vec{c}$ are such that $\vec{a}\times\vec{b}=2\vec{a}\times\vec{c},|\vec{a}|=|\vec{c}|=1$ and $|\vec{b}|=4$. If the angle between $\vec{b}$ and $\vec{c}$ is $\cos^{-1}\left(\frac{1}{4}\right)$, then $\vec{b}-2\vec{c}$ is equal to

a) $\pm4\vec{a}$ b) $\pm3\vec{a}$ c) $\pm5\vec{a}$ d) $\pm4\vec{a}$

451. $\hat{\imath}.(\hat{\jmath}\times\hat{k})+\hat{\jmath}.(\hat{k}\times\hat{\imath})+\hat{k}.(\hat{\imath}\times\hat{\jmath})=$

a) 1 b) 3 c) -3 d) 0

452. If $\vec{a}=\hat{\imath}+2\hat{\jmath}-3\hat{k}$ and $\vec{b}=3\hat{\imath}-\hat{\jmath}+2\hat{k}$, then the angle between the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, is

a) 30° b) 60° c) 90° d) 0°

453. If $\vec{a},\vec{b},\vec{c}$ are three non-zero vectors such that $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}$, then

a) $\vec{b}=\vec{c}$

b) $\vec{a}\perp\vec{b},\vec{c}$

c) $\vec{a}\perp(\vec{b}-\vec{c})$

d) Either $\vec{a}\perp(\vec{b}-\vec{c})$ or $\vec{b}=\vec{c}$

454. The length of longer diagonal of the parallelogram constructed on $5\vec{\mathbf{a}}+2\vec{\mathbf{b}}$ and $\vec{\mathbf{a}}-3\vec{\mathbf{b}}$. If it is given that $|\vec{\mathbf{a}}|=2\sqrt{2},|\vec{\mathbf{b}}|=3$ and angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is $\frac{\pi}{4}$, is

a) 15 b) $\sqrt{113}$ c) $\sqrt{593}$ d) $\sqrt{369}$

455. If the projection of the vector $\vec{\mathbf{a}}$ on $\vec{\mathbf{b}}$ is $|\vec{\mathbf{a}}\times\vec{\mathbf{b}}|$ and if $3\vec{\mathbf{b}}=\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$, then the angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is

a) $\frac{\pi}{3}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$

456. The unit vector perpendicular to the plane passing through points $P(\hat{\imath}-\hat{\jmath}+2\hat{k}),Q(2\hat{\imath}-\hat{k})$ and $R(2\hat{\jmath}+\hat{k})$ is

a) $2\hat{\imath}+\hat{\jmath}+\hat{k}$ b) $\sqrt{6}(2\hat{\imath}+\hat{\jmath}+\hat{k})$ c) $\frac{1}{\sqrt{6}}(2\hat{\imath}+\hat{\jmath}+\hat{k})$ d) $\frac{1}{6}(2\hat{\imath}+\hat{\jmath}+\hat{k})$

457. Let $\vec{\mathbf{a}},\vec{\mathbf{b}},\vec{\mathbf{c}}$ be three non-zero vectors such that no two of these are collinear. If the vector $\vec{\mathbf{a}}+2\,\vec{\mathbf{b}}$ is collinear with $\vec{\mathbf{c}}$,then $\vec{\mathbf{a}}+2\vec{\mathbf{b}}+6\,\vec{\mathbf{c}}$ equals

a) $\lambda\,\vec{\mathbf{a}}$ ($\lambda\neq0$, a scalar) b) $\lambda\,\vec{\mathbf{b}}$ ($\lambda\neq0$, a scalar) c) $\lambda\,\vec{\mathbf{c}}$ ($\lambda\neq0$, a scalar) d) 0

458. Let $\vec{\mathbf{u}}=\hat{\mathbf{i}}+\hat{\mathbf{j}},\vec{\mathbf{v}}=\hat{\mathbf{i}}-\hat{\mathbf{j}}$ and $\vec{\mathbf{w}}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$. If $\hat{\mathbf{n}}$ is a unit vector such that $\vec{\mathbf{u}}.\hat{\mathbf{n}}=0$ and $\vec{\mathbf{v}}.\hat{\mathbf{n}}=0$, then $|\vec{w}.\hat{\mathbf{n}}|$ is equal to

a) 0 b) 1 c) 2 d) 3

459. If position vector of point A is $\vec{\mathbf{a}}+2\vec{\mathbf{b}}$ and any point $P(\vec{\mathbf{a}})$ divides $\overrightarrow{AB}$ in the ratio of 2 : 3, then position vector of B is

a) $2\,\vec{\mathbf{a}}-\vec{\mathbf{b}}$ b) $\vec{\mathbf{b}}-2\vec{\mathbf{a}}$ c) $\vec{\mathbf{a}}-3\vec{\mathbf{b}}$ d) $\vec{\mathbf{b}}$

460. If $\vec{\mathbf{A}}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$, $\vec{\mathbf{B}}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $\vec{\mathbf{C}}=3\hat{\mathbf{i}}+\hat{\mathbf{j}}$, evaluate t, if the vector $(\vec{\mathbf{A}}+t\vec{\mathbf{B}})$ and $\vec{\mathbf{C}}$ are mutually perpendicular.

a) 5 b) 4 c) 1 d) 2

461. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them then $\left|\frac{\vec{a}-\vec{b}}{2}\right|$, is

a) $\sin\frac{\theta}{2}$ b) $\sin\theta$ c) $2\sin\theta$ d) $\sin 2\theta$

462. If $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are two non-collinear vectors and $x\,\vec{\mathbf{a}} + y\,\vec{\mathbf{b}} = 0$

a) $x = 0$, but y is not necessarily zero b) $y = 0$, but x is not necessarily zero

c) $x = 0, y = 0$ d) None of the above

463. Two adjacent sides of a parallelogram $ABCD$ are given by $\overrightarrow{\mathbf{AB}} = 2\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$ and $\overrightarrow{\mathbf{AD}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angle α is given by

a) $\frac{8}{9}$ b) $\frac{\sqrt{17}}{9}$ c) $\frac{1}{9}$ d) $\frac{4\sqrt{5}}{9}$

464. If the scalar projection of the vector $x\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ on the vector $2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ is $\frac{1}{\sqrt{30}}$ then the value of x is

a) $-3/2$ b) 6 c) -6 d) 3

465. If $\vec{\mathbf{a}} = -\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$, $\vec{\mathbf{b}} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\vec{\mathbf{c}} = -2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$, then the angle between $2\vec{\mathbf{a}} - \vec{\mathbf{c}}$ and $\vec{\mathbf{a}} + \vec{\mathbf{b}}$ is

a) $\frac{\pi}{4}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{2}$ d) $\frac{3\pi}{2}$

466. Let $\vec{a}, \vec{b}, \vec{c}$ three non-zero vectors such that no two of which are collinear and the vector $\vec{a} + \vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + \vec{c}$ is collinear with $\vec{a}$. Then, $\vec{a} + \vec{b} + \vec{c} =$

a) $\vec{a}$ b) $\vec{b}$ c) $\vec{c}$ d) $\vec{0}$

467. The value of $[\,\vec{\mathbf{a}}\,\vec{\mathbf{b}} + \vec{\mathbf{c}}\,\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}]$ is

a) $[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$ b) 0 c) $2[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$ d) $\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}})$

468. If the points with position vectors $60\hat{\imath} + 3\hat{\jmath}, 40\hat{\imath} - 8\hat{\jmath}$ and $a\hat{\imath} - 52\hat{\jmath}$ are collinear, then $a =$

a) -40 b) 40 c) 20 d) 30

469. Let $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \vec{\mathbf{b}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and a unit vector $\vec{\mathbf{c}}$ be coplanar. If $\vec{\mathbf{c}}$ is perpendicular to $\vec{\mathbf{a}}$ and $\vec{\mathbf{c}}$ is equal to

a) $\pm\frac{1}{\sqrt{2}}(-\hat{\mathbf{j}} + \hat{\mathbf{k}})$ b) $\pm\frac{1}{\sqrt{3}}(-\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})$ c) $\pm\frac{1}{\sqrt{5}}(\hat{\mathbf{i}} - 2\,\hat{\mathbf{j}})$ d) $\pm\frac{1}{\sqrt{3}}(\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})$

470. If the vectors $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 4\hat{\mathbf{k}}, \vec{\mathbf{b}} = 4\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\vec{\mathbf{c}} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - \lambda\hat{\mathbf{k}}$ are coplanar, then the value of λ is equal to

a) 2 b) 1 c) 3 d) -1

471. The vectors

$\vec{u} = (al + a_1l_1)\hat{\imath} + (am + a_1m_1)\hat{\jmath} + (an + a_1n_1)\hat{k}$,

$\vec{v} = (bl + b_1l_1)\hat{\imath} + (bm + b_1m_1)\hat{\jmath} + (bn + b_1n_1)\hat{k}$,

$\vec{w} = (cl + c_1l_1)\hat{\imath} + (cm + c_1m_1)\hat{\jmath} + (cn + c_1n_1)\hat{k}$

a) Form an equilateral triangle

b) Are coplanar

c) Are collinear

d) Are mutually perpendicular

472. If A, B, C, D are any four points in space, then $\left|\vec{AB} \times \vec{CD} + \vec{BC} \times \vec{AD} + \vec{CA} \times \vec{BD}\right|$ is equal to

a) 2Δ b) 4Δ c) 3Δ d) 5Δ

473. If $\vec{a}$ lies in the plane of vectors $\vec{b}$ and $\vec{c}$, then which of the following is correct?

a) $[\vec{a}\vec{b}\vec{c}] = 0$ b) $[\vec{a}\vec{b}\vec{c}] = 1$ c) $[\vec{a}\vec{b}\vec{c}] = 3$ d) $[\vec{b}\vec{c}\vec{a}] = 1$

474. What is the value of $(\,\vec{\mathbf{d}} + \vec{\mathbf{a}}\,] \cdot [\,\vec{\mathbf{a}} \times \{\vec{\mathbf{b}} \times (\,\vec{\mathbf{c}} \times \vec{\mathbf{d}}\,)\}]$?

a) $(\vec{\mathbf{d}} \cdot \vec{\mathbf{a}}) \cdot [\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}\,\vec{\mathbf{d}}\,]$ b) $(\vec{\mathbf{a}} \cdot \vec{\mathbf{d}}) \cdot [\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}\,\vec{\mathbf{d}}\,]$ c) $(\vec{\mathbf{b}} \cdot \vec{\mathbf{d}}) \cdot [\,\vec{\mathbf{a}}\,\vec{\mathbf{c}}\,\vec{\mathbf{d}}\,]$ d) $(\vec{\mathbf{b}} \cdot \vec{\mathbf{d}}) \cdot [\,\vec{\mathbf{a}}\,\vec{\mathbf{d}}\,\vec{\mathbf{c}}\,]$

475. A parallelogram is constructed on the vectors $\vec{a} = 3\vec{\alpha} - \vec{\beta}, \vec{b} = \vec{\alpha} + 3\vec{\beta}$. If $|\vec{\alpha}| = |\vec{\beta}| = 2$ and the angle

between $\vec{\alpha}$ and $\vec{\beta}$ is $\frac{\pi}{3}$, then the angle of a diagonal of the parallelogram are

a) $4\sqrt{5}, 4\sqrt{3}$ b) $4\sqrt{3}, 4\sqrt{7}$ c) $4\sqrt{7}, 4\sqrt{5}$ d) None of these

476. If the vectors $\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$, $-2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}}, \lambda\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ are linearly dependent, then the value of λ is equal to

a) 0 b) 1 c) 2 d) 3

477. For any vector $\vec{\mathbf{a}}$, the value of $(\vec{\mathbf{a}} \times \hat{\mathbf{i}})^2 + (\vec{\mathbf{a}} \times \hat{\mathbf{j}})^2 + (\vec{\mathbf{a}} \times \hat{\mathbf{k}})^2$ is equal to

a) $4\vec{\mathbf{a}}^2$ b) $2\vec{\mathbf{a}}^2$ c) $\vec{\mathbf{a}}^2$ d) $3\vec{\mathbf{a}}^2$

478. If $\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \vec{\mathbf{b}} = 2\hat{\mathbf{i}} - 4\hat{\mathbf{k}}, \vec{\mathbf{c}} = \hat{\mathbf{i}} + \lambda\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ are coplanar, then the value of λ is

a) $\frac{5}{2}$ b) $\frac{3}{5}$ c) $\frac{7}{3}$ d) None of these

479. If the position vectors of P and Q are $\hat{\imath} + 3\hat{\jmath} - 7\hat{k}$ and $5\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$ then the cosine of the angle between $\vec{P}Q$ and y-axis is

a) $\frac{5}{\sqrt{162}}$ b) $\frac{4}{\sqrt{162}}$ c) $-\frac{5}{\sqrt{162}}$ d) $\frac{11}{\sqrt{162}}$

480. The value of `a' so that volume of parallelopiped formed by $\hat{\mathbf{i}} + a\hat{\mathbf{j}} + \hat{\mathbf{k}}, \hat{\mathbf{j}} + a\hat{\mathbf{k}}$ and $a\hat{\mathbf{i}} + \hat{\mathbf{k}}$ becomes minimum, is

a) -3 b) 3 c) $1/\sqrt{3}$ d) $\sqrt{3}$

481. If C is the mid point of AB and P is any point outside AB, then

a) $\overrightarrow{\mathbf{PA}} + \overrightarrow{\mathbf{PB}} = \overrightarrow{\mathbf{PC}}$ b) $\overrightarrow{\mathbf{PA}} + \overrightarrow{\mathbf{PB}} + 2\overrightarrow{\mathbf{PC}} = \vec{0}$ c) $\overrightarrow{\mathbf{PA}} + \overrightarrow{\mathbf{PB}} - 2\overrightarrow{\mathbf{PC}} = \vec{0}$ d) $\overrightarrow{\mathbf{PA}} + \overrightarrow{\mathbf{PB}} + 2\overrightarrow{\mathbf{PC}} = \vec{0}$

482. The vector equation of the line passing through the points (3,2,1) and (−2,1,3) is

a) $\vec{\mathbf{r}} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}} + \lambda(-5\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ b) $\vec{\mathbf{r}} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}} + \lambda(-5\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$

c) $\vec{\mathbf{r}} = -2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}} + \lambda(5\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ d) $\vec{\mathbf{r}} = -2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}} + \lambda(5\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

483. The angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is $\frac{5\pi}{6}$ and the projection of $\vec{\mathbf{a}}$ in the direction of $\vec{\mathbf{b}}$ is $\frac{-6}{\sqrt{3}}$ then $|\vec{\mathbf{a}}|$ is equal to

a) 6 b) $\sqrt{3}/2$ c) 12 d) 4

484. When a right handed rectangular cartesian system $OXYZ$ rotated about z-axis through $\pi/4$ in the counter-clock-wise sense it is found that a vector $\vec{r}$ has the components $2\sqrt{2}, 3\sqrt{2}$ and 4. The components of $\vec{a}$ in the $OXYZ$ coordinate system are

a) $5, -1, 4$ b) $5, -1, 4\sqrt{2}$ c) $-1, -5, 4\sqrt{2}$ d) None of these

485. If $\vec{\mathbf{x}} \cdot \vec{\mathbf{a}} = \vec{\mathbf{x}} \cdot \vec{\mathbf{b}} = \vec{\mathbf{x}} \cdot \vec{\mathbf{c}} = 0$ where $\vec{\mathbf{x}}$ is a non-zero vector. Then, $[\vec{\mathbf{a}} \times \vec{\mathbf{b}}\ \ \vec{\mathbf{b}} \times \vec{\mathbf{c}}\ \ \vec{\mathbf{c}} \times \vec{\mathbf{a}}]$ is equal to

a) $[\vec{\mathbf{x}}\,\vec{\mathbf{a}}\,\vec{\mathbf{b}}]^2$ b) $[\vec{\mathbf{x}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]^2$ c) $[\vec{\mathbf{x}}\,\vec{\mathbf{c}}\,\vec{\mathbf{a}}]^2$ d) 0

486. If $ABCDEF$ is regular hexagon, then $\overrightarrow{\mathbf{AD}} + \overrightarrow{\mathbf{EB}} + \overrightarrow{\mathbf{FC}}$ is equal to

a) 0 b) $2\,\overrightarrow{\mathbf{AB}}$ c) $3\,\overrightarrow{\mathbf{AB}}$ d) $4\,\overrightarrow{\mathbf{AB}}$

487. The shortest distance between the straight lines through the points $A_1 = (6,2,2)$ and $A_2 = (-4,0,-1)$ in the directions of $(1,-2,2)$ and $(3,-2,-2)$ is

a) 6 b) 8 c) 12 d) 9

488. A unit vector perpendicular to the plane of $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $\vec{\mathbf{b}} = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$ is

a) $\frac{4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}}{\sqrt{26}}$ b) $\frac{2\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 3\hat{\mathbf{k}}}{7}$ c) $\frac{3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}}}{7}$ d) $\frac{2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 6\hat{\mathbf{k}}}{7}$

489. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are the position vectors of points A, B, C, D such that no three of them are collinear and $\vec{a} + \vec{c} = \vec{b} + \vec{d}$, then $ABCD$ is a

a) Rhombus b) Rectangle c) Square d) Parallelogram

490. If D, E, F are respectively the mid point of AB, AC and BC in $\Delta\, ABC$, then $\overrightarrow{\mathbf{BE}} + \overrightarrow{\mathbf{AF}}$ is equal to

a) $\overrightarrow{\mathbf{DC}}$ b) $\frac{1}{2}\overrightarrow{\mathbf{BF}}$ c) $2\,\overrightarrow{\mathbf{BF}}$ d) $\frac{3}{2}\overrightarrow{\mathbf{BF}}$

491. Let $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ be two unit vectors such that angle between them is 60°. Then, $|\vec{\mathbf{a}} - \vec{\mathbf{b}}|$ is equal to

a) $\sqrt{5}$ b) $\sqrt{3}$ c) 0 d) 1

492. If $2\vec{\mathbf{a}} + 3\vec{\mathbf{b}} + \vec{\mathbf{c}} = \vec{\mathbf{0}}$, then $\vec{\mathbf{a}} \times \vec{\mathbf{b}} + \vec{\mathbf{b}} \times \vec{\mathbf{c}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}}$ is equal to

a) $6(\vec{\mathbf{b}} \times \vec{\mathbf{c}})$ b) $3(\vec{\mathbf{b}} \times \vec{\mathbf{c}})$ c) $2(\vec{\mathbf{b}} \times \vec{\mathbf{c}})$ d) $\vec{\mathbf{0}}$

493. If $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are the three vectors mutually perpendicular to each other and $|\vec{\mathbf{a}}| = 1, |\vec{\mathbf{b}}| = 3$ and $|\vec{\mathbf{c}}| = 5$, then $[\vec{\mathbf{a}} - 2\,\vec{\mathbf{b}}\;\vec{\mathbf{b}} - 3\vec{\mathbf{c}}\;\;\vec{\mathbf{c}} - 4\vec{\mathbf{a}}\,]$ is equal to

a) 0 b) -24 c) 3600 d) -215

494. If the area of the parallelogram with $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ as two adjacent side is 15 sq units, then the area of the parallelogram having $3\vec{\mathbf{a}} + 2\vec{\mathbf{b}}$ and $\vec{\mathbf{a}} + 3\vec{\mathbf{b}}$ as two adjacent sides in sq units is

a) 120 b) 105 c) 75 d) 45

495. If $\left(\vec{a} \times \vec{b}\right) + \left(\vec{a}.\vec{b}\right)^2 = 144$ and $|\vec{a}| = 4$, then $|\vec{b}| =$

a) 16 b) 8 c) 3 d) 12

496. If the vectors $\vec{c}, \vec{a} = x\hat{\imath} + y\hat{\imath} + z\hat{k}$ and $\vec{b} = \hat{\jmath}$ are such that $\vec{a}, \vec{c}$ and $\vec{b}$ form a right handed system, then $\vec{c}$ is

a) $z\hat{\imath} - x\hat{k}$ b) $\vec{0}$ c) $y\hat{\imath}$ d) $-z\hat{\imath} + x\hat{k}$

497. The vectors $2\hat{\imath} - m\hat{\jmath} + 3m\hat{k}$ and $(1 + m)\hat{\imath} - 2m\hat{\jmath} + \hat{k}$ include an acute angle for

a) $m = -1/2$

b) $m \in [-2, -1/2]$

c) $m \in R$

d) $m \in (-\infty, -2) \cup (-1/2, \infty)$

498. If $|\vec{\mathbf{a}}| + 3, |\vec{\mathbf{a}}| = 4, |\vec{\mathbf{c}}| = 5$ and $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are such that each is perpendicular to the saum of other two, then $|\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}|$ is

a) $5\sqrt{2}$ b) $\dfrac{5}{\sqrt{2}}$ c) $10\sqrt{2}$ d) $10\sqrt{3}$

499. For any three vectors $\vec{a}, \vec{b}, \vec{c}$, the vector $(\vec{b} \times \vec{c}) \times \vec{a}$ equals

a) $\left(\vec{a} \cdot \vec{b}\right)\vec{c} - (\vec{b} \cdot \vec{c})\vec{a}$ b) $\left(\vec{a} \cdot \vec{b}\right)\vec{c} - (\vec{a} \cdot \vec{c})\vec{b}$ c) $\left(\vec{b} \cdot \vec{a}\right)\vec{c} - (\vec{c} \cdot \vec{a})\vec{b}$ d) None of these

500. The vector $\cos\alpha \cos\beta\,\hat{\imath} + \cos\alpha \sin\beta\,\hat{\jmath} + \sin\alpha\,\hat{k}$ is a

a) Null vector b) Unit vector c) Constant vector d) None of these

501. Let $\vec{\mathbf{u}}, \vec{\mathbf{v}}, \vec{\mathbf{w}}$ be such that $|\vec{\mathbf{u}}| = 1, |\vec{\mathbf{v}}| = 2, \vec{\mathbf{w}} = 3$. If the projection $\vec{\mathbf{v}}$ along $\vec{\mathbf{u}}$ is equal to that of $\vec{\mathbf{w}}$ along $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}, \vec{\mathbf{w}}$ are perpendicular to each other, then $|\vec{\mathbf{u}} - \vec{\mathbf{v}} + \vec{\mathbf{w}}|$ are equals

a) 2 b) $\sqrt{7}$ c) $\sqrt{14}$ d) 14

502. Let $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ be the position vectors of the vertices A, B, C respectively of $\Delta\,ABC$. The vector area of $\Delta\,ABC$ is

a) $\frac{1}{2}\{\vec{\mathbf{a}} \times \left(\vec{\mathbf{b}} \times \vec{\mathbf{c}}\right) + \vec{\mathbf{b}} \times (\vec{\mathbf{c}} \times \vec{\mathbf{a}}) + \vec{\mathbf{c}} \times \left(\vec{\mathbf{a}} \times \vec{\mathbf{b}}\right)\}$ b) $\frac{1}{2}\{\vec{\mathbf{a}} \times \vec{\mathbf{b}} + \vec{\mathbf{b}} \times \vec{\mathbf{c}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}}\}$

c) $\frac{1}{2}\{\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}\}$ d) $\frac{1}{2}\left(\vec{\mathbf{b}} \cdot \vec{\mathbf{c}}\right)\vec{\mathbf{a}} + (\vec{\mathbf{c}} \cdot \vec{\mathbf{a}})\,\vec{\mathbf{b}} + (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{c}}$

503. IF $\vec{\mathbf{a}} \times \left(\vec{\mathbf{b}} \times \vec{\mathbf{c}}\right) = (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times \vec{\mathbf{c}}$, where $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are any three vectors such that $\vec{\mathbf{a}}.\vec{\mathbf{b}} \neq 0$, $\vec{\mathbf{b}} \cdot \vec{\mathbf{c}} \neq 0$, then $\vec{\mathbf{a}}$ and $\vec{\mathbf{c}}$ are

a) inclined at angle of $\frac{\pi}{6}$ between them b) Perpendicular

c) Parallel d) inclined at an angle of $\frac{\pi}{3}$ between them

504. A unit vector in the plane of $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and perpendicular to $2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ is

a) $\hat{\mathbf{j}} - \hat{\mathbf{k}}$ b) $\dfrac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}$ c) $\dfrac{\hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{2}}$ d) $\dfrac{\hat{\mathbf{j}} - \hat{\mathbf{k}}}{\sqrt{2}}$

505. The unit vectors $\vec{a}$ and $\vec{b}$ are perpendicular, and the unit vector $\vec{c}$ is inclined at an angle θ to both $\vec{a}$ and $\vec{b}$. If $\vec{c} = \alpha\vec{a} + \beta\vec{b} + \gamma\left(\vec{a} \times \vec{b}\right)$, then which one of the following is incorrect?

a) $\alpha \neq \beta$ b) $\gamma^2 = 1 - 2\,\alpha^2$ c) $\gamma^2 = -\cos 2\,\theta$ d) $\beta^2 = \dfrac{1 + \cos 2\,\theta}{2}$

506. A vector $\vec{c}$ of magnitude $5\sqrt{6}$ directed along the bisector of the angle between $\vec{a} = 7\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$ and $\vec{b} = -2\hat{\imath} - \hat{\jmath} + 2\hat{k}$, is

a) $\pm\frac{5}{3}\left(2\hat{\imath} + 7\hat{\jmath} + \hat{k}\right)$ b) $\pm\frac{3}{5}\left(\hat{\imath} + 7\hat{\jmath} + 2\hat{k}\right)$ c) $\pm\frac{5}{3}\left(\hat{\imath} - 2\hat{\jmath} + 7\hat{k}\right)$ d) $\pm\frac{5}{3}\left(\hat{\imath} - 7\hat{\jmath} + 2\hat{k}\right)$

507. If the vectors $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ and $\vec{\mathbf{b}}$ are collinear and $|\vec{\mathbf{b}}| = 21$, then $\vec{\mathbf{b}}$ is equal to

a) $\pm(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+6\hat{\mathbf{k}})$ b) $\pm3(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+6\hat{\mathbf{k}})$ c) $(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})$ d) $\pm21(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+6\hat{\mathbf{k}})$

508. A parallelogram is constructed on the vectors $\vec{a}=3\vec{p}-\vec{q}$, $\vec{b}=\vec{p}+3\vec{q}$ and also given that $|\vec{p}|=|\vec{q}|=2$. If the vectors $\vec{p}$ and $\vec{q}$ are inclined at an angle $\pi/3$, then the ratio of the lengths of the diagonals of the parallelogram is

a) $\sqrt{6}:\sqrt{2}$ b) $\sqrt{3}:\sqrt{5}$ c) $\sqrt{7}:\sqrt{3}$ d) $\sqrt{6}:\sqrt{5}$

509. If $[2\vec{a}+4\vec{b}\vec{c}\vec{d}]=\lambda[\vec{a}\vec{c}\vec{d}]+\mu[\vec{b}\vec{c}\vec{d}]$, then $\lambda+\mu=$

a) 6 b) -6 c) 10 d) 8

510. If A, B and C are the vertices of a triangle whose position vectors are $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ respectively G is the centroid of the ΔABC, then $\overrightarrow{\mathbf{GA}}+\overrightarrow{\mathbf{GB}}+\overrightarrow{\mathbf{GC}}$ is

a) $\vec{\mathbf{0}}$ b) $\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}$ c) $\frac{\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}}{3}$ d) $\frac{\vec{\mathbf{a}}-\vec{\mathbf{b}}-\vec{\mathbf{c}}}{3}$

511. A, B have position vectors $\vec{a}, \vec{b}$ relative to the origin O and X, Y divide $\overrightarrow{AB}$ internally and externally respectively in the ratio 2 : 1. Then, $\overrightarrow{XY}=$

a) $\frac{3}{2}(\vec{b}-\vec{a})$ b) $\frac{4}{3}(\vec{a}-\vec{b})$ c) $\frac{5}{6}(\vec{b}-\vec{a})$ d) $\frac{4}{3}(\vec{b}-\vec{a})$

512. If $\vec{\mathbf{a}}=(2,1,-1), \vec{\mathbf{b}}=(1,-1,0), \vec{\mathbf{c}}=(5-1,1)$, then unit vector parallel to $\vec{\mathbf{a}}+\vec{\mathbf{b}}-\vec{\mathbf{c}}$ but in opposite direction is

a) $\frac{1}{3}(2\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}})$ b) $\frac{1}{2}(2\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}})$ c) $\frac{1}{3}(2\hat{\mathbf{i}}-\hat{\mathbf{j}}-2\hat{\mathbf{k}})$ d) None of these

513. The number of vectors of unit length perpendicular to the two vectors $\vec{\mathbf{a}}=(1,1,0)$ and $\vec{\mathbf{b}}=(0,1,1)$ is

a) One b) Two c) Three d) Infinite

514. A vector which is a linear combination of the vectors $3\hat{\mathbf{i}}+4\hat{\mathbf{j}}+5\hat{\mathbf{k}}$ and $6\hat{\mathbf{i}}-7\hat{\mathbf{j}}-3\hat{\mathbf{k}}$ and is perpendicular to the vector $\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$ is

a) $3\hat{\mathbf{i}}-11\hat{\mathbf{j}}-8\hat{\mathbf{k}}$ b) $-3\hat{\mathbf{i}}+11\hat{\mathbf{j}}+87\hat{\mathbf{k}}$ c) $-9\hat{\mathbf{i}}+3\hat{\mathbf{j}}-2\hat{\mathbf{k}}$ d) $9\hat{\mathbf{i}}-3\hat{\mathbf{j}}+2\hat{\mathbf{k}}$

515. If $\vec{\mathbf{x}}$ and $\vec{\mathbf{y}}$ are unit vectors and $\vec{\mathbf{x}}\cdot\vec{\mathbf{y}}=0$, then

a) $|\vec{\mathbf{x}}+\vec{\mathbf{y}}|=1$ b) $|\vec{\mathbf{x}}+\vec{\mathbf{y}}|=\sqrt{3}$ c) $|\vec{\mathbf{x}}+\vec{\mathbf{y}}|=2$ d) $|\vec{\mathbf{x}}+\vec{\mathbf{y}}|=\sqrt{2}$

516. If the volume of a parallelopiped with $\vec{\mathbf{a}}\times\vec{\mathbf{b}}, \vec{\mathbf{b}}\times\vec{\mathbf{c}}, \vec{\mathbf{c}}\times\vec{\mathbf{a}}$ as coterminous edges is 9 cu units, then the volume of the parallelopiped with $(\vec{\mathbf{a}}\times\vec{\mathbf{b}})\times(\vec{\mathbf{b}}\times\vec{\mathbf{c}}), (\vec{\mathbf{b}}\times\vec{\mathbf{c}})\times(\vec{\mathbf{c}}\times\vec{\mathbf{a}}), (\vec{\mathbf{c}}\times\vec{\mathbf{a}})\times(\vec{\mathbf{a}}\times\vec{\mathbf{b}})$ as coterminous edges is

a) 9 cu units b) 729 cu units c) 81 cu units d) 27 cu units

517. The non-zero vectors $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are related by $\vec{\mathbf{a}}=8\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}=-7\vec{\mathbf{b}}$.Then, the angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{c}}$ is

a) π b) 0 c) $\frac{\pi}{4}$ d) $\frac{\pi}{2}$

518. For any three non-zero vectors $\vec{\mathbf{r}}_1\vec{\mathbf{r}}_2$ and $\vec{\mathbf{r}}_3$, $\begin{vmatrix}\vec{\mathbf{r}}_1\cdot\vec{\mathbf{r}}_1 & \vec{\mathbf{r}}_1\cdot\vec{\mathbf{r}}_2 & \vec{\mathbf{r}}_1\cdot\vec{\mathbf{r}}_3\\ \vec{\mathbf{r}}_2\cdot\vec{\mathbf{r}}_1 & \vec{\mathbf{r}}_2\cdot\vec{\mathbf{r}}_2 & \vec{\mathbf{r}}_2\cdot\vec{\mathbf{r}}_3\\ \vec{\mathbf{r}}_3\cdot\vec{\mathbf{r}}_1 & \vec{\mathbf{r}}_3\cdot\vec{\mathbf{r}}_2 & \vec{\mathbf{r}}_3\cdot\vec{\mathbf{r}}_3\end{vmatrix}=0$, Then, which of the following is false?

a) All the three vectors are parallel to one and the same plane
b) All the three vectors are linearly dependent
c) This system of equation has a non-trivial solution
d) All the three vectors are perpendicular to each other

519. If $\vec{\mathbf{a}}=\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}, \vec{\mathbf{b}}=\hat{\mathbf{i}}+\hat{\mathbf{j}}, \vec{\mathbf{c}}=\hat{\mathbf{i}}$ and $(\vec{\mathbf{a}}\times\vec{\mathbf{b}})\times\vec{\mathbf{c}}=\lambda\vec{\mathbf{a}}+\mu\vec{\mathbf{b}}$, then $\lambda+\mu$ is equal to

a) 0 b) 1 c) 2 d) 3

520. Let $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ be three vector such that $\vec{\mathbf{a}}\neq\vec{\mathbf{0}}$ and $\vec{\mathbf{a}}\times\vec{\mathbf{b}}=2\vec{\mathbf{a}}\times\vec{\mathbf{c}}, |\vec{\mathbf{a}}|=|\vec{\mathbf{c}}|=1, |\vec{\mathbf{b}}|=4$ and $|\vec{\mathbf{b}}\times\vec{\mathbf{c}}|=\sqrt{15}$. If $\vec{\mathbf{b}}-2\vec{\mathbf{c}}=\lambda\vec{\mathbf{a}}$, then λ is equal to

a) 1 b) ±4 c) 3 d) -2

521. If $\vec{\mathbf{r}}\cdot\vec{\mathbf{a}}=0, \vec{\mathbf{r}}\cdot\vec{\mathbf{b}}=0$ and $\vec{\mathbf{r}}\cdot\vec{\mathbf{c}}=0$ for some non-zero vector $\vec{\mathbf{r}}$. Then, the value of $[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$ is

a) 0 b) $\frac{1}{2}$ c) 1 d) 2

522. If $\vec{a}, \vec{b}, \vec{c}$ are any three mutually perpendicular vectors of equal magnitude a, then $|\vec{a}+\vec{b}+\vec{c}|$ is equal to

a) a b) $\sqrt{2}\,a$ c) $\sqrt{3}\,a$ d) $2a$

523. A unit vector perpendicular to both the vectors $\hat{\mathbf{i}}+\hat{\mathbf{j}}$ and $\hat{\mathbf{j}}+\hat{\mathbf{k}}$ is

a) $\frac{-\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}}{\sqrt{3}}$ b) $\frac{-\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}}{3}$ c) $\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}}{\sqrt{3}}$ d) $\frac{\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}}{\sqrt{3}}$

524. Let, $\vec{\mathbf{a}}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}$, $\vec{\mathbf{b}}=\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$, $\vec{\mathbf{c}}=\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$. A vector coplanar to $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ has a projection along $\vec{\mathbf{c}}$ of magnitude $\frac{1}{\sqrt{3}}$, then the vector is

a) $4\hat{\mathbf{i}}-\hat{\mathbf{j}}+4\hat{\mathbf{k}}$ b) $4\hat{\mathbf{i}}+\hat{\mathbf{j}}-4\hat{\mathbf{k}}$ c) $2\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$ d) None of these

525. Let $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ are unit vectors such that $\vec{\mathbf{u}}\times\vec{\mathbf{v}}+\vec{\mathbf{u}}=\vec{\mathbf{w}}$ and $\vec{\mathbf{w}}\times\vec{\mathbf{u}}=\vec{\mathbf{v}}$, then the value of $[\,\vec{\mathbf{u}}\;\vec{\mathbf{v}}\;\vec{\mathbf{w}}\,]$ is

a) 1 b) -1 c) 0 d) None of these

526. The position vectors of the points A, B, C are $2\hat{\imath}+\hat{\jmath}-\hat{k}$, $3\hat{\imath}-2\hat{\jmath}+\hat{k}$ and $\hat{\imath}+4\hat{\jmath}-3\hat{k}$ respectively. These points

a) Form an isosceles triangle
b) Form a right triangle
c) Are collinear
d) Form a scalene triangle

527. If $\vec{\mathbf{a}}=\hat{\mathbf{i}}-\hat{\mathbf{j}}-\hat{\mathbf{k}}$ and $\vec{\mathbf{b}}=\lambda\hat{\mathbf{i}}-3\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and the orthogonal projection of $\vec{\mathbf{b}}$ on $\vec{\mathbf{a}}$ is $\frac{4}{3}(\hat{\mathbf{i}}-\hat{\mathbf{j}}-\hat{\mathbf{k}})$then λ is equal to

a) 0 b) 2 c) 12 d) -1

528. If three points A, B and C have position vectors $(1, x, 3)$, $(3,4,7)$ and $(y, -2, -5)$ respectively and, if they are collinear, then (x, y) is equal to

a) $(2, -3)$ b) $(-2, 3)$ c) $(2, 3)$ d) $(-2, -3)$

529. $\overrightarrow{\mathbf{OA}}$ and $\overrightarrow{\mathbf{BO}}$ are two vectors of magnitude 5 and 6 respectively. If $\angle BOA=60°$, then $\overrightarrow{\mathbf{OA}}\cdot\overrightarrow{\mathbf{OB}}$ is equal to

a) 0 b) 15 c) -15 d) $15\sqrt{3}$

530. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at an angle θ such that $\vec{a}+\vec{b}$ is a unit vector, then θ is equal to

a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) $\frac{2\pi}{3}$

531. $\overrightarrow{\mathbf{AB}}\times\overrightarrow{\mathbf{AC}}=2\hat{\mathbf{i}}-4\hat{\mathbf{j}}+4\hat{\mathbf{k}}$, then the area of $\Delta\, ABC$ is

a) 3 sq units b) 4 sq units c) 16 sq units d) 9 sq units

532. If the vectors $\vec{\mathbf{c}}, \vec{\mathbf{a}}=x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}}$ and $\vec{\mathbf{b}}=\hat{\mathbf{j}}$ are such that $\vec{\mathbf{a}}, \vec{\mathbf{c}}$ and $\vec{\mathbf{b}}$ from a right handed system, then $\vec{\mathbf{c}}$ is

a) $z\hat{\mathbf{i}}-x\hat{\mathbf{k}}$ b) $\vec{\mathbf{0}}$ c) $y\hat{\mathbf{j}}$ d) $-z\hat{\mathbf{i}}-x\hat{\mathbf{k}}$

533. Let $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ be the vectors such that $\vec{\mathbf{a}}\neq 0$ and $\vec{\mathbf{a}}\times\vec{\mathbf{b}}=2\vec{\mathbf{a}}\times\vec{\mathbf{c}}$, $|\vec{\mathbf{a}}|=|\vec{\mathbf{c}}|=1$, $|\vec{\mathbf{b}}|=4$ and $|\vec{\mathbf{b}}\times\vec{\mathbf{c}}|=\sqrt{15}$. If $\vec{\mathbf{b}}-2\vec{\mathbf{c}}=\lambda\,\vec{\mathbf{a}}$, then λ is equal to

a) 1 b) -4 c) 3 d) -2

534. The position vectors of P and Q are respectively $\vec{a}$ and $\vec{b}$. If R is a point on $\vec{PQ}$ such that $\vec{PR}=5\,\vec{PQ}$, then the position vector of R, is

a) $5\vec{b}-4\vec{a}$ b) $5\vec{b}+4\vec{a}$ c) $4\vec{a}-5\vec{b}$ d) $4\vec{b}+5\vec{a}$

535. The vector $\vec{c}$ is perpendicular to the vectors $\vec{a}=(2,-3,1)$, $\vec{b}=(1,-2,3)$ and satisfies the condition $\vec{c}.(\hat{\imath}+2\hat{\jmath}-7\hat{k})$. Then, $\vec{c}=$

a) $7\hat{\imath}+5\hat{\jmath}+\hat{k}$ b) $-7\hat{\imath}-5\hat{\jmath}-\hat{k}$ c) $\hat{\imath}+\hat{\jmath}-\hat{k}$ d) None of these

536. If $ABCD$ is a quadrilateral, then $\vec{BA}+\vec{BC}+\vec{CD}+\vec{DA}=$

a) $2\vec{BA}$ b) $2\vec{AB}$ c) $2\vec{AC}$ d) $2\vec{BC}$

537. The vector equation of the sphere whose centre is the point (1,0,1)and radius is 4, is

a) $|\vec{\mathbf{r}}-(\hat{\mathbf{i}}+\hat{\mathbf{k}})|=4$ b) $|\vec{\mathbf{r}}+(\hat{\mathbf{i}}+\hat{\mathbf{k}})|=4^2$ c) $|\vec{\mathbf{r}}\cdot(\hat{\mathbf{i}}+\hat{\mathbf{k}})|=4$ d) $|\vec{\mathbf{r}}\cdot(\hat{\mathbf{i}}+\hat{\mathbf{k}})|=4^2$

538. If three concurrent edges of a parallelopiped of volume V represent vectors $\vec{a}, \vec{b}, \vec{c}$ then the volume of the parallelopiped whose three concurrent edges are the three concurrent diagonals of the three faces of the given parallelopiped, is

a) V b) $2V$ c) $3V$ d) None of these

539. A unit vector in xy-plane makes an angle of 45° with the vector $\hat{\imath}+\hat{\jmath}$ and an angle of 60° with the vector $3\hat{\imath}-4\hat{\jmath}$ is

a) $\hat{\imath}$ b) $\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}$ c) $\frac{\hat{\imath}-\hat{\jmath}}{\sqrt{2}}$ d) None of these

540. The equation $\vec{\mathbf{r}}^2 - 2\,\vec{\mathbf{r}}\cdot\vec{\mathbf{c}}+h=0, |\vec{\mathbf{c}}|>\sqrt{h}$, represent

a) Circle b) Ellipse c) Cone d) Sphere

541. The points with position vectors $10\hat{\imath}+3\hat{\jmath}, 12\hat{\imath}-5\hat{\jmath}$ and $a\hat{\imath}+11\hat{\jmath}$ are collinear if the value of a is

a) -8 b) 4 c) 8 d) 12

542. If $\vec{a}\times(\vec{a}\times\vec{b})=\vec{b}\times(\vec{b}\times\vec{c})$ and $\vec{a}.\vec{b}\neq 0$, then $[\vec{a}\vec{b}\vec{c}]=$

a) 0 b) 1 c) 2 d) 3

543. $[\vec{a}\ \vec{b}\ \vec{a}\times\vec{b}]+(\vec{a}.\vec{b})^2=$

a) $|\vec{a}|^2\,|\vec{b}|^2$ b) $|\vec{a}+\vec{b}|^2$ c) $|\vec{a}|^2+|\vec{a}|^2$ d) None of these

544. If $\vec{\mathbf{u}}, \vec{\mathbf{v}}, \vec{\mathbf{w}}$ are non-coplanar vectors and p, q are real numbers, then the equality $[3\vec{\mathbf{u}}\ p\,\vec{\mathbf{v}}\ p\,\vec{\mathbf{w}}]-[p\,\vec{\mathbf{v}}\ \vec{\mathbf{w}}\ q\,\vec{\mathbf{u}}]-[2\vec{\mathbf{w}}\ q\,\vec{\mathbf{v}}\ q\,\vec{\mathbf{u}}]=0$ holds for

a) Exactly two value of (p,q) b) More than two but not all values of (p,q)

c) All values of $(p.q)$ d) Exactly one value of (p,q)

545. $\vec{\mathbf{a}}\cdot[(\vec{\mathbf{b}}+\vec{\mathbf{c}})\times(\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}})]$ equals

a) 0 b) $\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}$ c) $\vec{\mathbf{a}}$ d) $\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}+\vec{\mathbf{c}})$

546. If the vectors $\hat{\mathbf{i}}-3\hat{\mathbf{j}}+2\hat{\mathbf{k}}, -\hat{\mathbf{i}}+2\hat{\mathbf{j}}$ represent the diagonals of a parallelogram, them its area will be

a) 21 b) $\frac{\sqrt{21}}{2}$ c) $2\sqrt{21}$ d) $\frac{\sqrt{21}}{4}$

547. Given $\vec{\mathbf{a}}\perp\vec{\mathbf{b}}, |\vec{\mathbf{a}}|=1$ and if $(\vec{\mathbf{a}}+3\vec{\mathbf{b}})\cdot(2\vec{\mathbf{a}}-\vec{\mathbf{b}})=-10$ then $|\vec{\mathbf{b}}|$ is equal to

a) 1 b) 3 c) 2 d) 4

548. If $\vec{a}=\hat{\imath}+\hat{\jmath}+\hat{k}, \vec{b}=\hat{\imath}+\hat{\jmath}, \vec{c}=\hat{\imath}$ and $(\vec{a}\times\vec{b})\times\vec{c}=\lambda\vec{a}+\mu\vec{b}$, then $\lambda+\mu=$

a) 0 b) 1 c) 2 d) 3

549. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a}=\vec{b}+\vec{c}$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{2}$, then

a) $a^2=b^2+c^2$ b) $b^2=c^2+a^2$ c) $c^2=a^2+b^2$ d) $2a^2-b^2=c^2$

550. If $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ and $\vec{\mathbf{d}}$ are the unit vectors such that $(\vec{\mathbf{a}}\times\vec{\mathbf{b}})\cdot(\vec{\mathbf{c}}\times\vec{\mathbf{d}})=1$ and $\vec{\mathbf{a}}\cdot\vec{\mathbf{c}}=\frac{1}{2}$, then

a) $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are non-coplanar b) $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{d}}$ are non-coplanar

c) $\vec{\mathbf{b}}, \vec{\mathbf{d}}$ are non-parallel d) $\vec{\mathbf{a}}, \vec{\mathbf{d}}$ are parallel and $\vec{\mathbf{b}}, \vec{\mathbf{c}}$ are parallel

551. The projection of the vector $2\hat{\imath}+3\hat{\jmath}-2\hat{k}$ on the vector $\hat{\imath}+2\hat{\jmath}+3\hat{k}$, is

a) $\frac{2}{\sqrt{14}}$ b) $\frac{1}{\sqrt{14}}$ c) $\frac{3}{\sqrt{14}}$ d) None of these

552. If unit vector $\vec{\mathbf{c}}$ makes an angle $\frac{\pi}{3}$ with $\hat{\mathbf{i}}+\hat{\mathbf{j}}$, then minimum and maximum values of $(\hat{\mathbf{i}}\times\hat{\mathbf{j}})\cdot\vec{\mathbf{c}}$ respectively are

a) $0, \frac{\sqrt{3}}{2}$ b) $-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}$ c) $-1, \frac{\sqrt{3}}{2}$ d) None of these

553. $\hat{a}$ and $\hat{b}$ are two mutually perpendicular unit vectors. If the vectors $x\,\hat{a}+x\hat{b}+z(\hat{a}\times\hat{b}), \hat{a}+(\hat{a}\times\hat{b})$ and $z\hat{a}+z\hat{b}+y(\hat{a}\times\hat{b})$ lie in a plane, then z is

a) A.M. of x and y b) G.M. of x and y c) H.M. of x and y d) Equal to zero

554. If $\vec{\mathbf{a}}=(1,p,1),\ \vec{\mathbf{b}}=(q,2,2),\ \vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=r$ and $\vec{\mathbf{a}}\times\vec{\mathbf{b}}=(0,-3,3)$, then p, q, r are in that order

a) 1,5,9 b) 9,5,1 c) 5,1,9 d) None of these

555. The vectors $3\hat{\imath}-2\hat{\jmath}+\hat{k}, \hat{\imath}-3\hat{\jmath}+5\hat{k}$ and $2\hat{\imath}+\hat{\jmath}-4\hat{k}$ form the sides of a triangle. This triangle is
a) An acute angled triangle
b) An obtuse angled triangle
c) A right angled triangle
d) An equilateral triangle

556. The vector $\hat{\mathbf{i}}+x\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ is rotated through an angle θ and doubled in magnitude, then it becomes $4\hat{\mathbf{i}}+(4x-2)\hat{\mathbf{j}}+2\hat{\mathbf{k}}$. The values of $x$is
a) $\left\{-\frac{2}{3},2\right\}$ b) $\left\{\frac{1}{3},2\right\}$ c) $\left\{\frac{2}{3},0\right\}$ d) $\{2,7\}$

557. If $\vec{a}=2\hat{\imath}-3\hat{\jmath}+5\hat{k}, \vec{b}=3\hat{\imath}-4\hat{\jmath}+5\hat{k}$ and $\vec{c}=5\hat{\imath}-3\hat{\jmath}-2\hat{k}$, then the volume of the parallelopiped with coterminus edges $a+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}$ is
a) 2 b) 1 c) -1 d) 0

558. Image of the point P with position vector $7\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ in the line whose vector equation is $\vec{\mathbf{r}}=\big(9\hat{\mathbf{i}}+5\hat{\mathbf{j}}+5\mathbf{k}+\lambda(\mathbf{i}+3\mathbf{j}+5\mathbf{k})$ has position vector
a) $-9\hat{\mathbf{i}}+5\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ b) $9\hat{\mathbf{i}}+5\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ c) $9\hat{\mathbf{i}}+5\hat{\mathbf{j}}-2\hat{\mathbf{k}}$ d) $9\hat{\mathbf{i}}-5\hat{\mathbf{j}}-2\hat{\mathbf{k}}$

559. If $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are the pth, qth, nth terms of an HP and $\vec{u}=(q-r)\hat{\mathbf{i}}+(r-p)\hat{\mathbf{j}}+(p-q)\hat{\mathbf{k}}$ and $\vec{v}=\frac{\hat{\mathbf{i}}}{a}+\frac{\hat{\mathbf{j}}}{b}+\frac{\hat{\mathbf{k}}}{c}$, then
a) $\vec{\mathbf{u}}, \vec{\mathbf{v}}$ are parallel vectors b) $\vec{\mathbf{u}}, \vec{\mathbf{v}}$ are orthogonal vectors
c) $\vec{\mathbf{u}}\cdot\vec{\mathbf{v}}=1$ d) $\vec{\mathbf{u}}\times\vec{\mathbf{v}}=\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$

560. If $\hat{\mathbf{i}}-\hat{\mathbf{k}}, \lambda\hat{\mathbf{i}}+\hat{\mathbf{j}}+(1-\lambda)\hat{\mathbf{k}}$ and $\mu\hat{\mathbf{i}}+\lambda\hat{\mathbf{j}}+(1+\lambda\hat{\mathbf{j}}-\mu)\hat{\mathbf{k}}$ are three coterminal edges of a parallelopiped, then its volume depends on
a) only λ b) Only μ c) Both λ and μ d) Neither λ nor μ

561. The vector $\vec{\mathbf{c}}\cdot\left(\vec{\mathbf{b}}+\vec{\mathbf{c}}\right)\times(\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}})$ is equal to
a) $\vec{\mathbf{c}}\cdot\ \vec{\mathbf{b}}\times\vec{\mathbf{a}}$ b) $\vec{\mathbf{0}}$ c) $\vec{\mathbf{a}}\cdot\vec{\mathbf{a}}\times\vec{\mathbf{b}}$ d) $\vec{\mathbf{a}}\cdot\vec{\mathbf{c}}\times\vec{\mathbf{b}}$

562. If $ABCD$ is a parallelogram, then $\vec{A}C-\vec{B}D=$
a) $4\vec{A}B$ b) $3\vec{A}B$ c) $2\vec{A}B$ d) $\vec{A}B$

563. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=0$, then the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$, is
a) 1 b) 3 c) $-3/2$ d) None of these

564. If $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are vectors such that $\vec{\mathbf{c}}=\vec{\mathbf{a}}+\vec{\mathbf{b}}$ and $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=0$, then
a) $a^2+b^2+c^2=0$ b) $a^2-b^2=c^2$ c) $a^2+b^2=c^2$ d) $\vec{\mathbf{c}}=\vec{\mathbf{a}}\times\vec{\mathbf{b}}$

565. If $\vec{\mathbf{a}}=2\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and $\vec{\mathbf{b}}=5\hat{\mathbf{i}}-3\hat{\mathbf{j}}+\hat{\mathbf{k}}$, then the projection of $\vec{\mathbf{b}}$ on $\vec{\mathbf{a}}$ is
a) 3 b) 4 c) 5 d) 6

566. Forces of magnitudes 3 and 4 units acting along $6\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ and $3\hat{\mathbf{i}}-2\hat{\mathbf{j}}+6\hat{\mathbf{k}}$ respectively act on a particle and displace it from $(2,2-1)$ to $(4,3,1)$. The work done is
a) 124/7 b) 120/7 c) 125/7 d) 121/7

567. The value of $[\vec{\mathbf{a}}\,\vec{\mathbf{b}}+\vec{\mathbf{c}}\,\vec{\mathbf{a}}+\vec{\mathbf{b}}+\ \vec{\mathbf{c}}]$ is
a) $[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$ b) 0 c) $2[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$ d) $\vec{\mathbf{a}}\times(\vec{\mathbf{b}}\times\vec{\mathbf{c}})$

: ANSWER KEY :

1)	d	2)	d	3)	c	4)	b	189)	d	190)	a	191)	a	192)	b
5)	c	6)	d	7)	d	8)	a	193)	b	194)	c	195)	b	196)	d
9)	c	10)	c	11)	c	12)	a	197)	a	198)	a	199)	a	200)	b
13)	b	14)	b	15)	c	16)	b	201)	a	202)	c	203)	c	204)	b
17)	b	18)	a	19)	c	20)	d	205)	a	206)	d	207)	b	208)	a
21)	a	22)	c	23)	d	24)	a	209)	d	210)	d	211)	c	212)	d
25)	d	26)	b	27)	b	28)	a	213)	b	214)	c	215)	b	216)	a
29)	d	30)	d	31)	c	32)	a	217)	d	218)	d	219)	a	220)	d
33)	a	34)	d	35)	a	36)	a	221)	b	222)	a	223)	c	224)	a
37)	b	38)	d	39)	d	40)	a	225)	d	226)	b	227)	c	228)	c
41)	c	42)	d	43)	a	44)	c	229)	b	230)	a	231)	a	232)	c
45)	d	46)	b	47)	a	48)	b	233)	b	234)	d	235)	a	236)	c
49)	b	50)	d	51)	a	52)	c	237)	c	238)	d	239)	d	240)	c
53)	b	54)	a	55)	c	56)	a	241)	a	242)	d	243)	b	244)	c
57)	b	58)	c	59)	d	60)	c	245)	b	246)	a	247)	d	248)	b
61)	b	62)	c	63)	a	64)	a	249)	b	250)	d	251)	c	252)	c
65)	d	66)	c	67)	a	68)	b	253)	b	254)	b	255)	d	256)	c
69)	b	70)	c	71)	d	72)	d	257)	a	258)	d	259)	d	260)	d
73)	a	74)	b	75)	d	76)	a	261)	c	262)	c	263)	b	264)	b
77)	b	78)	d	79)	b	80)	b	265)	d	266)	d	267)	b	268)	d
81)	b	82)	b	83)	c	84)	a	269)	c	270)	b	271)	d	272)	c
85)	b	86)	c	87)	a	88)	c	273)	b	274)	a	275)	b	276)	b
89)	a	90)	d	91)	a	92)	d	277)	b	278)	a	279)	d	280)	a
93)	c	94)	a	95)	d	96)	b	281)	a	282)	a	283)	a	284)	c
97)	d	98)	d	99)	a	100)	a	285)	a	286)	b	287)	a	288)	a
101)	d	102)	b	103)	b	104)	b	289)	b	290)	a	291)	a	292)	c
105)	b	106)	c	107)	c	108)	a	293)	a	294)	d	295)	a	296)	c
109)	a	110)	a	111)	c	112)	b	297)	a	298)	a	299)	c	300)	b
113)	d	114)	b	115)	b	116)	b	301)	d	302)	c	303)	a	304)	b
117)	b	118)	d	119)	d	120)	d	305)	a	306)	a	307)	c	308)	c
121)	c	122)	a	123)	a	124)	d	309)	d	310)	d	311)	b	312)	a
125)	a	126)	a	127)	c	128)	a	313)	d	314)	d	315)	a	316)	c
129)	a	130)	a	131)	a	132)	a	317)	a	318)	c	319)	a	320)	b
133)	c	134)	d	135)	a	136)	d	321)	d	322)	b	323)	b	324)	b
137)	a	138)	c	139)	c	140)	d	325)	d	326)	c	327)	b	328)	a
141)	d	142)	c	143)	d	144)	c	329)	c	330)	b	331)	c	332)	b
145)	a	146)	a	147)	d	148)	b	333)	d	334)	b	335)	a	336)	b
149)	a	150)	d	151)	b	152)	b	337)	d	338)	a	339)	c	340)	d
153)	a	154)	b	155)	b	156)	c	341)	a	342)	a	343)	c	344)	c
157)	a	158)	d	159)	b	160)	c	345)	b	346)	a	347)	d	348)	a
161)	c	162)	a	163)	c	164)	b	349)	b	350)	a	351)	b	352)	c
165)	a	166)	d	167)	d	168)	a	353)	c	354)	b	355)	c	356)	a
169)	b	170)	d	171)	c	172)	b	357)	d	358)	a	359)	a	360)	c
173)	d	174)	c	175)	d	176)	c	361)	c	362)	b	363)	b	364)	d
177)	a	178)	b	179)	c	180)	b	365)	c	366)	d	367)	d	368)	b
181)	c	182)	c	183)	c	184)	b	369)	c	370)	b	371)	a	372)	a
185)	a	186)	a	187)	b	188)	a	373)	c	374)	c	375)	b	376)	c

377)	a	378)	b	379)	c	380)	a
381)	b	382)	a	383)	d	384)	d
385)	c	386)	a	387)	a	388)	c
389)	a	390)	b	391)	b	392)	a
393)	b	394)	d	395)	c	396)	b
397)	a	398)	a	399)	c	400)	d
401)	c	402)	a	403)	c	404)	d
405)	b	406)	d	407)	a	408)	b
409)	c	410)	c	411)	a	412)	b
413)	c	414)	a	415)	a	416)	b
417)	a	418)	a	419)	c	420)	b
421)	d	422)	c	423)	a	424)	a
425)	a	426)	b	427)	a	428)	c
429)	a	430)	c	431)	c	432)	d
433)	a	434)	a	435)	c	436)	d
437)	c	438)	d	439)	a	440)	d
441)	a	442)	a	443)	b	444)	b
445)	d	446)	a	447)	a	448)	a
449)	b	450)	a	451)	b	452)	c
453)	d	454)	c	455)	a	456)	c
457)	c	458)	d	459)	c	460)	a
461)	a	462)	c	463)	b	464)	a
465)	b	466)	d	467)	b	468)	a
469)	a	470)	b	471)	b	472)	b
473)	a	474)	c	475)	b	476)	a
477)	b	478)	d	479)	c	480)	c
481)	c	482)	a	483)	d	484)	d
485)	d	486)	d	487)	d	488)	c
489)	d	490)	a	491)	d	492)	b
493)	d	494)	b	495)	c	496)	a
497)	d	498)	a	499)	b	500)	b
501)	c	502)	b	503)	c	504)	d
505)	a	506)	d	507)	b	508)	a
509)	a	510)	a	511)	d	512)	a
513)	b	514)	b	515)	d	516)	c
517)	a	518)	a	519)	a	520)	b
521)	a	522)	c	523)	d	524)	a
525)	a	526)	a	527)	b	528)	a
529)	b	530)	d	531)	a	532)	a
533)	b	534)	a	535)	a	536)	a
537)	a	538)	b	539)	b	540)	d
541)	c	542)	a	543)	a	544)	d
545)	a	546)	b	547)	c	548)	a
549)	a	550)	c	551)	a	552)	b
553)	b	554)	d	555)	c	556)	a
557)	d	558)	c	559)	b	560)	d
561)	a	562)	c	563)	c	564)	c
565)	a	566)	a	567)	b		

: HINTS AND SOLUTIONS :

1 **(d)**

Let the unit vector in xy-plane be $\vec{\mathbf{a}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$.

$\therefore \cos 45° = \dfrac{(x\hat{\mathbf{i}} + y\hat{\mathbf{j}})(\hat{\mathbf{i}} + \hat{\mathbf{j}})}{\sqrt{x^2+y^2}\sqrt{1^2+1^2}}$

$\Rightarrow \dfrac{1}{\sqrt{2}} = \dfrac{x+y}{\sqrt{2}\sqrt{x^2+y^2}}$

$\Rightarrow 1 = \dfrac{x+y}{\sqrt{x^2+y^2}}$

$\Rightarrow x + y = \sqrt{x^2+y^2}$

Since, $\vec{a}$ is a unit vector.

$\therefore |\vec{\mathbf{a}}| = \sqrt{x^2+y^2} = 1$

$\Rightarrow x + y = 1$ ...(i)

Again $\cos 60° = \frac{(x\hat{\mathbf{i}}+y\hat{\mathbf{j}})\cdot(3\hat{\mathbf{i}}-4\hat{\mathbf{j}})}{\sqrt{x^2+y^2}\sqrt{3^2+4^2}}$

$\Rightarrow \dfrac{1}{2} = \dfrac{3x-4y}{1\cdot 5} \Rightarrow \dfrac{5}{2} = 3x - 4y$

$5 = 6x - 8y$ .....(ii)

On solving Eqs. (i) and (ii), we get

$x = \dfrac{13}{14}, y = \dfrac{1}{14}$

$\therefore \vec{\mathbf{a}} = \dfrac{1}{14}(13\hat{\mathbf{i}} + \hat{\mathbf{j}})$

No value in the given options satisfies the above relations.

Thus, option (d) is correct.

2 **(d)**

Given, $|\vec{\mathbf{a}} + \vec{\mathbf{b}}| < 1$

$\Rightarrow \sqrt{1+1+2\cos 2\alpha} < 1$

$\Rightarrow \sqrt{2(1+\cos 2\alpha)} < 1$

$\Rightarrow \sqrt{4\cos^2\alpha} < 1$

$\Rightarrow |\cos\alpha| < \dfrac{1}{2}$

$\Rightarrow \dfrac{\pi}{3} < \alpha < \dfrac{2\pi}{3}$ $(\because 0 \le \alpha \le \pi)$

3 **(c)**

Given equation can be rewritten as

$\vec{\mathbf{r}} = 3\hat{\mathbf{j}} + (\hat{\mathbf{i}} + 2\hat{\mathbf{k}})s + (-2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})t$

which is a plane passing through $\vec{\mathbf{a}} = 3\hat{\mathbf{j}}$ and parallel to the vectors $\vec{\mathbf{b}} = \hat{\mathbf{i}} + 2\hat{\mathbf{k}}$ and $\vec{\mathbf{c}} = -2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$.

Therefore, it is perpendicular to the vector

$\vec{\mathbf{n}} = \vec{\mathbf{b}} \times \vec{\mathbf{c}} = 2\hat{\mathbf{i}} - 5\hat{\mathbf{j}} - \hat{\mathbf{k}}$

Hence, its vector equation is $(\vec{\mathbf{r}} - \vec{\mathbf{a}}) \cdot \vec{\mathbf{n}} = 0$

$\Longrightarrow \vec{\mathbf{r}} \cdot \vec{\mathbf{n}} = \vec{\mathbf{a}} \cdot \vec{\mathbf{n}}$

$\Longrightarrow (x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}) \cdot (2\hat{\mathbf{i}} - 5\hat{\mathbf{j}} - \hat{\mathbf{k}})$

$= 3\hat{\mathbf{j}} \cdot (2\hat{\mathbf{i}} - 5\hat{\mathbf{j}} - \hat{\mathbf{k}})$

$\Longrightarrow 2x - 5y - z + 15 = 0$

4 **(b)**

$\because \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 18$ and $|\vec{\mathbf{b}}| = 5$

$\therefore$ Vector component of $\vec{\mathbf{a}}$ along $\vec{\mathbf{b}}$

$= \left(\dfrac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{b}}|^2}\right)\vec{\mathbf{b}} = \dfrac{18}{25}(3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$

5 **(c)**

Given that, $(\vec{\mathbf{F}}) = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ and its position vector $2\hat{\mathbf{i}} - \hat{\mathbf{j}}$.

The position vector of a force about origin $(\vec{\mathbf{r}}) = (2\hat{\mathbf{i}} - \hat{\mathbf{j}})$.

$\therefore$ Moment of the force about origin

$= \vec{\mathbf{r}} \times \vec{\mathbf{F}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & -1 & 0 \\ 2 & 1 & -1 \end{vmatrix} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

7 **(d)**

Since, $\vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} + \vec{\mathbf{c}}) = 0$

$\Longrightarrow \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{a}} \cdot \vec{\mathbf{c}} = 0$

Similarly, $\vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{a}} = 0$

and $\vec{\mathbf{c}} \cdot \vec{\mathbf{a}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{b}} = 0$

$\therefore \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{a}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} = 0$ ...(i)

Given, $|\vec{\mathbf{a}} + \vec{\mathbf{b}}| = 6$

$\Longrightarrow |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + 2\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 36$ ....(ii)

Similarly, $|\vec{\mathbf{b}}|^2 + |\vec{\mathbf{c}}|^2 + 2\vec{\mathbf{b}} \cdot \vec{\mathbf{c}} = 64$ ...(iii)

and $|\vec{\mathbf{c}}|^2 + |\vec{\mathbf{a}}|^2 + 2\vec{\mathbf{c}} \cdot \vec{\mathbf{a}} = 100$ ....(iv)

On adding Eqs. (ii),(iii) and (iv), we get

$2|\vec{\mathbf{a}}|^2 + 2|\vec{\mathbf{b}}|^2 + 2|\vec{\mathbf{c}}|^2 + 2(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{a}})$

$= 200$

$\Longrightarrow |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + |\vec{\mathbf{c}}|^2 = 100$...(v)[from Eqs. (i)]

Now, $|\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}|^2 = |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + |\vec{\mathbf{c}}|^2 + 2(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{a}})$

$\Longrightarrow |\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}|^2 = 100$ [from Eqs. (i) and (v)]

$\Longrightarrow |\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}| = 10$

8 **(a)**

It is given that $|\vec{a}| = |\vec{b}| = |\vec{c}| = \lambda$ (say) and $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors. Therefore,

$|\vec{a} + \vec{b} + \vec{c}| = \sqrt{3}\,\lambda$

Let θ be the angle which $\vec{a} + \vec{b} + \vec{c}$ makes with $\vec{a}$. Then,

$$\cos\theta = \frac{\vec{a}(\vec{a}+\vec{b}+\vec{c})}{|\vec{a}||\vec{a}+\vec{b}+\vec{c}|} = \frac{|\vec{a}|^2}{|\vec{a}||\vec{a}+\vec{b}+\vec{c}|}$$

$$\Rightarrow \cos\theta = \frac{\lambda^2}{\lambda(\sqrt{3}\lambda)} = \frac{1}{\sqrt{3}} \Rightarrow \theta = \cos^{-1}(1/\sqrt{3})$$

9 **(c)**

The resultant of forces $3\,\vec{O}A$ and $5\,\vec{O}B$ is $8\,\vec{O}B$, where C divides AB in the ratio 5 : 3 i.e.

$3AC = 5CB$

10 **(c)**

The equation of a line passing through the centre $(\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ and normal to the given plane is

$\vec{\mathbf{r}} = \hat{\mathbf{j}} + 2\hat{\mathbf{k}} + \lambda(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ .....(i)

This meets the plane at a point for which we must have

$[(\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) + \lambda(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})].(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 15$

$\Longrightarrow 6 + \lambda(9) = 15 \Longrightarrow \lambda = 1$

∴ From Eq. (i),

$\vec{\mathbf{r}} = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

∴ Coordinates of the centre of the circle are (1,3,4)

12 **(a)**

Let $\vec{\mathbf{a}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}, \vec{\mathbf{b}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

and $\vec{\mathbf{c}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \lambda\hat{\mathbf{k}}$

Since, volume of tetrahedron $= \frac{1}{6}[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$

$$\Longrightarrow \frac{2}{3} = \frac{1}{6}\begin{vmatrix} 1 & 2 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & \lambda \end{vmatrix}$$

$\Longrightarrow \frac{2}{3} = \frac{1}{6}[1(\lambda+1) - 2(\lambda-1) - 1(-1-1)]$

$\Longrightarrow 4 = [-\lambda + 5]$

$\Longrightarrow \lambda = 1$

13 **(b)**

Given equation represents a plane.

15 **(c)**

$$\because \vec{\alpha} \times \vec{\beta} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 3 & -1 \\ -1 & 2 & -4 \end{vmatrix} = -10\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$$

$$\therefore (\vec{\alpha} \times \vec{\beta}) \cdot (\vec{\alpha} \times \vec{\gamma}) = \begin{vmatrix} -10 & 9 & 7 \\ 2 & 3 & -1 \\ 1 & 1 & 1 \end{vmatrix}$$

$= -10(3+1) - 9(2+1) + 7(2-3)$

$= -74$

Alternate

$$(\vec{\alpha} \times \vec{\beta}) \cdot (\vec{\alpha} \times \vec{\gamma}) = \begin{vmatrix} \vec{\alpha}\cdot\vec{\alpha} & \vec{\alpha}\cdot\vec{\gamma} \\ \vec{\beta}\cdot\vec{\alpha} & \vec{\beta}.\vec{\gamma} \end{vmatrix}$$

$= \begin{vmatrix} 14 & 4 \\ 8 & -3 \end{vmatrix} = -42 - 32$

$= -74$

16 **(b)**

Given planes are

$\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}) = 1$ .....(i)

and $\vec{\mathbf{r}} \cdot (2\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) = 2$ ....(ii)

Now,

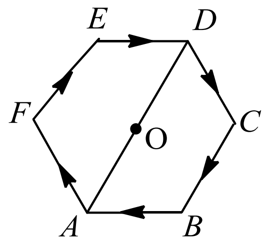
$$(\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}) \times (2\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -3 & 1 \\ 2 & 5 & -3 \end{vmatrix}$$

$= 4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$

Hence, line of intersection of the planes is parallel to the vector $4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$.

17 **(b)**

Given, $\overrightarrow{\mathbf{AD}} + \overrightarrow{\mathbf{EB}} + \overrightarrow{\mathbf{FC}} = \lambda\overrightarrow{\mathbf{ED}}$



$\Longrightarrow (\overrightarrow{\mathbf{AE}} + \overrightarrow{\mathbf{ED}}) + (\overrightarrow{\mathbf{ED}} + \overrightarrow{\mathbf{DB}}) + 2\overrightarrow{\mathbf{ED}} = \lambda\overrightarrow{\mathbf{ED}}$

$\Longrightarrow 4\overrightarrow{\mathbf{ED}} + (\overrightarrow{\mathbf{AE}} + \overrightarrow{\mathbf{DB}}) = \lambda\overrightarrow{\mathbf{ED}}$

$\Longrightarrow \quad 4\overrightarrow{\mathbf{ED}} = \lambda\overrightarrow{\mathbf{ED}} \qquad (\because \overrightarrow{\mathbf{AE}} = -\overrightarrow{\mathbf{DB}})$

Alternate

Now, $\overrightarrow{\mathbf{AD}} + \overrightarrow{\mathbf{EB}} + \overrightarrow{\mathbf{FC}} = 2(\overrightarrow{\mathbf{OD}} + \overrightarrow{\mathbf{EO}} + \overrightarrow{\mathbf{ED}})$

$= 2(\overrightarrow{\mathbf{ED}} + \overrightarrow{\mathbf{ED}}) = 4\overrightarrow{\mathbf{ED}} \;\therefore \lambda = 4$

18 **(a)**

Let $\vec{\mathbf{a}} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$

and $\vec{\mathbf{b}} = b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + b_3\hat{\mathbf{k}}$

$$\text{Now, } [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\hat{\mathbf{i}}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ 1 & 0 & 0 \end{vmatrix}$$

$= a_1(0-0) - a_2(0-b_3) + a_3(0-b_2)$

$= a_2b_3 - a_3b_2$

$\therefore 2[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\hat{\mathbf{i}}]\hat{\mathbf{i}} = 2[a_2b_3 - a_3b_2]\hat{\mathbf{i}}$

Similarly, $2[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\hat{\mathbf{j}}]\hat{\mathbf{j}} = 2[a_3b_1 - a_1b_3]\hat{\mathbf{j}}$

and $2[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\hat{\mathbf{k}}]\hat{\mathbf{k}} = 2[a_1b_2 - a_2b_1]\hat{\mathbf{k}}$

$\therefore 2[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\hat{\mathbf{i}}]\hat{\mathbf{i}} + 2[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\hat{\mathbf{j}}]\hat{\mathbf{j}} + 2[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\hat{\mathbf{k}}]\hat{\mathbf{k}} + [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{a}}]$

$= 2[(a_2b_3 - a_3b_2)\hat{\mathbf{i}} + (a_3b_1 - a_1b_3)\hat{\mathbf{j}}$

$+ (a_1b_2 - a_2b_1)\hat{\mathbf{k}}]$

$= (\vec{\mathbf{a}} \times \vec{\mathbf{b}})$

19 **(c)**

Given that, $\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 1$ and

$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \hat{\mathbf{j}} - \hat{\mathrm{k}}$

As we know $\vec{\mathbf{a}}(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) = (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{a}} - (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{b}}$

$\Rightarrow (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) \times (\hat{\mathbf{j}} - \hat{\mathbf{k}}) = (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) - (\sqrt{3})^2\vec{\mathbf{b}}$

$\Rightarrow -2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}} - 3\vec{\mathbf{b}}$

$\Rightarrow 3\vec{\mathbf{b}} = 3\hat{\mathbf{i}}$

$\Rightarrow \vec{\mathbf{b}} = \hat{\mathbf{i}}$

20 **(d)**

Given, $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors and $\vec{p}, \vec{q}, \vec{r}$ defined by the relations

$\vec{\mathbf{p}} = \frac{\vec{b}\times\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}, \vec{\mathbf{q}} = \frac{\vec{c}\times\vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]}$ and $\vec{\mathbf{r}} = \frac{\vec{a}\times\vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]}$

$$\therefore\ \vec{\mathbf{a}}\cdot\vec{\mathbf{p}} = \frac{\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}\times\vec{\mathbf{c}}}{[\vec{\mathbf{a}}\vec{\mathbf{b}}\ \vec{\mathbf{c}}]} = \frac{\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})}{[\vec{\mathbf{a}}\vec{\mathbf{b}}\ \vec{\mathbf{c}}]} = 1$$

and $\vec{\mathbf{a}}\cdot\vec{\mathbf{q}} = \vec{\mathbf{a}}\cdot\frac{\vec{c}\times\vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]} = \frac{\vec{a}\cdot(\vec{c}\times\vec{a})}{[\vec{a}\,\vec{b}\,\vec{c}]} = 0$

Similarly, $\vec{\mathbf{b}}\cdot\vec{\mathbf{q}} = \vec{\mathbf{c}}\cdot\vec{\mathbf{r}} = 1$

and $\vec{\mathbf{a}}\cdot\vec{\mathbf{r}} = \vec{\mathbf{b}}\cdot\vec{\mathbf{p}} = \vec{\mathbf{c}}\cdot\vec{\mathbf{q}} = \vec{\mathbf{c}}\cdot\vec{\mathbf{p}} = \vec{\mathbf{b}}\cdot\vec{\mathbf{r}} = 0$

$\therefore \left(\vec{\mathbf{a}}+\vec{\mathbf{b}}\right)\cdot\vec{\mathbf{p}} + \left(\vec{\mathbf{b}}+\vec{\mathbf{c}}\right)\cdot\vec{\mathbf{q}} + (\vec{\mathbf{c}}+\vec{\mathbf{a}})\cdot\vec{\mathbf{r}}$

$= \vec{\mathbf{a}}\cdot\vec{\mathbf{p}} + \vec{\mathbf{b}}\cdot\vec{\mathbf{p}} + \vec{\mathbf{b}}\cdot\vec{\mathbf{q}} + \vec{\mathbf{c}}\cdot\vec{\mathbf{q}} + \vec{\mathbf{c}}\cdot\vec{\mathbf{r}} + \vec{\mathbf{a}}\cdot\vec{\mathbf{r}}$

$= 1 + 1 + 1 = 3$

21 **(a)**

Given, $m_1 = |\vec{\mathbf{a}_1}| = \sqrt{2^2 + (-1)^2 + (1)^2} = \sqrt{6}$

$m_2 = |\vec{\mathbf{a}_2}| = \sqrt{3^2 + (-4)^2 + (-4)^2} = \sqrt{41}$

$m_3 = |\vec{\mathbf{a}_3}| = \sqrt{1^2 + 1^2 + (-1)^2 = \sqrt{3}}$

and $m_4 = |\vec{\mathbf{a}_4}| = \sqrt{(-1)^2 + (3)^2 + (1)^2} = \sqrt{11}$

$\therefore\ m_3 < m_1 < m_4 < m_2$

22 **(c)**

Given, $[\lambda(\vec{\mathbf{a}}+\vec{\mathbf{b}})\ \lambda^2\vec{\mathbf{b}}\ \ \lambda\vec{\mathbf{c}}] = [\vec{\mathbf{a}}\ \vec{\mathbf{b}}+\vec{\mathbf{c}}\ \vec{\mathbf{b}}]$

$$\Rightarrow \begin{vmatrix} \lambda(a_1+b_1) & \lambda(a_2+b_2) & \lambda(a_3+b_3) \\ \lambda^2 b_1 & \lambda^2 b_2 & \lambda^2 b_3 \\ \lambda c_1 & \lambda c_2 & \lambda c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1+c_1 & b_2+c_2 & b_3+c_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$\Rightarrow \lambda^4 \begin{vmatrix} a_1+b_1 & a_2+b_2 & a_3+b_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1+c_1 & b_2+c_2 & b_3+c_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

[applying $R_1 \to R_1 - R_2$ in LHS and $R_2 \to R_2 - R_3$ in RHS]

$$\Rightarrow \lambda^4 \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = -\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$\Rightarrow \lambda^4 = -1$

Hence, no real value of λ exists.

23 **(d)**

Since the given points lie in a plane.

$$\therefore \begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$

Applying $C_1 \to C_1 - C_2$

$$\Rightarrow \begin{vmatrix} 0 & a & c \\ 1 & 0 & 1 \\ 0 & c & b \end{vmatrix} = 0$$

$\Rightarrow -1(ab - c^2) = 0$

$\Rightarrow c^2 = ab$

Hence, c is GM of a and b.

24 **(a)**

We have,

$\vec{P} = \vec{AC} + \vec{BD}$

$\Rightarrow \vec{p} = \vec{AC} + \vec{BC} + \vec{CD}$

$\Rightarrow \vec{p} = \vec{AC} + \lambda\,\vec{AD} + \vec{CD}$

$\Rightarrow \vec{p} = \lambda\,\vec{AD} + (\vec{AC} + \vec{CD})$

$\Rightarrow \vec{p} = \lambda\,\vec{AD} + \vec{AD} = (\lambda+1)\vec{AD}$

$\therefore \vec{p} = \mu\,\vec{AD} \Rightarrow \mu = \lambda + 1$

25 **(d)**

We have,

$\vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow \left|\vec{a} + \vec{b} + \vec{c}\right|^2 = 0$

$\Rightarrow \left(\vec{a} + \vec{b} + \vec{c}\right)\cdot\left(\vec{a} + \vec{b} + \vec{c}\right) = 0$

$\Rightarrow 2\left(\vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{c}\right) = -\left\{|\vec{a}|^2 + \left|\vec{b}\right|^2 + |\vec{c}|^2\right\}$

$\Rightarrow \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = -\dfrac{29}{2}$

26 **(b)**

Since, $\left(\vec{\mathbf{a}} + \lambda\vec{\mathbf{b}}\right)\cdot\left(\vec{\mathbf{a}} - \lambda\vec{\mathbf{b}}\right) = 0$

$\Rightarrow (\vec{\mathbf{a}})^2 - \lambda^2(\vec{\mathbf{b}})^2 = 0$

$\Rightarrow\ \lambda^2\dfrac{(\vec{\mathbf{a}})^2}{(\vec{\mathbf{b}})^2} = \left(\dfrac{3}{4}\right)^2$

$\Rightarrow \lambda = \dfrac{3}{4}$

27 **(b)**

Let $\vec{\mathbf{a}} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$

$\therefore \vec{\mathbf{u}} = \hat{\mathbf{i}}\times\left(\vec{\mathbf{a}}\times\hat{\mathbf{i}}\right) + \hat{\mathbf{j}}\times\left(\vec{\mathbf{a}}\times\hat{\mathbf{j}}\right) + \hat{\mathbf{k}}\times\left(\vec{\mathbf{a}}\times\hat{\mathbf{k}}\right)$

$= \hat{\mathbf{i}}\times\left(-a_2\hat{\mathbf{k}} + a_3\hat{\mathbf{j}}\right) + \hat{\mathbf{j}}\times\left(a_1\hat{\mathbf{k}} - a_3\hat{\mathbf{i}}\right) + \hat{\mathbf{k}} \times\left(-a_1\hat{\mathbf{j}} + a_2\hat{\mathbf{i}}\right)$

$= a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}} + a_1\hat{\mathbf{i}} + a_3\hat{\mathbf{k}} + a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}}$

$= 2\vec{\mathbf{a}}$

29 **(d)**

Since, $|\vec{\mathbf{a}} + \vec{\mathbf{b}}|^2 = |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + 2\,|\vec{\mathbf{a}}||\vec{\mathbf{b}}|\cos\theta$

$\Rightarrow (\sqrt{7})^2 = (3\sqrt{3})^2 + 4^2 + 2(3\sqrt{3})^2(4)\cos\theta$

$\Rightarrow 7 = 27 + 16 + 24\sqrt{3}\cos\theta$

$\Rightarrow \cos\theta = -\sqrt{3}/2$

$\Rightarrow\ \theta = 150°$

30 **(d)**

$$\because \vec{\mathbf{b}}\times\vec{\mathbf{c}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 2 & -5 \\ 3 & 5 & -1 \end{vmatrix} = 23\hat{\mathbf{i}} - 14\hat{\mathbf{j}} - \hat{\mathbf{k}}$$

$$\therefore \vec{\mathbf{a}}\times\left(\vec{\mathbf{b}}\times\vec{\mathbf{c}}\right) = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 3 & -1 \\ 23 & -14 & -1 \end{vmatrix} = -17\hat{\mathbf{i}} - 21\hat{\mathbf{j}} - 97\hat{\mathbf{k}}$$

31 **(c)**

We have,

$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$ and $\vec{a} \perp \vec{b} \perp \vec{c}$

$\Rightarrow \vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

$\therefore \left|\vec{a} + \vec{b} + \vec{c}\right|^2 = \left(\vec{a} + \vec{b} + \vec{c}\right).\left(\vec{a} + \vec{b} + \vec{c}\right)$

$= |\vec{a}|^2 + \left|\vec{b}\right|^2 + |\vec{c}|^2 + 2\vec{a}.\vec{b} + 2\vec{b}.\vec{c} + 2\vec{c}.\vec{a} = 3$

$\Rightarrow \left|\vec{a} + \vec{b} + \vec{c}\right| = \sqrt{3}$

32 **(a)**

Let $\vec{\mathbf{a}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

$\therefore\ (\vec{\mathbf{a}} \cdot \hat{\mathbf{i}})\hat{\mathbf{i}} = \left[\left(x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}\right) \cdot \hat{\mathbf{i}}\right]\hat{\mathbf{i}} = x\hat{\mathbf{i}}$

Similarly, $(\vec{\mathbf{a}} \cdot \hat{\mathbf{j}})\hat{\mathbf{i}} = y\hat{\mathbf{j}}, (\vec{\mathbf{a}} \cdot \hat{\mathbf{k}})z\hat{\mathbf{k}}$

$\therefore (\vec{\mathbf{a}} \cdot \hat{\mathbf{i}})\hat{\mathbf{i}} + (\vec{\mathbf{a}} \cdot \hat{\mathbf{j}})\hat{\mathbf{j}} + (\vec{\mathbf{a}} \cdot \hat{\mathbf{k}})\hat{\mathbf{k}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}} = \vec{\mathbf{a}}$

33 **(a)**

Let $\vec{\mathbf{a}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

$\therefore \vec{\mathbf{a}} \cdot\ \hat{\mathbf{i}} = \left(x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}\right) \cdot \hat{\mathbf{i}} = x$

$\vec{\mathbf{a}} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}}) = \left(x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}\right) \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}}) = x + y$

and $\vec{\mathbf{a}}\left(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}\right) = \left(x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}\right) \cdot \left(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}\right) = x + y + z$

$\because$ Given that, $\vec{\mathbf{a}} \cdot \hat{\mathbf{i}} = \vec{\mathbf{a}} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}}) = \vec{\mathbf{a}} \cdot \left(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}\right)$

$\Rightarrow x = x + y = x + y + z$

Take $x = x + y\ \Rightarrow y = 0$

and $x + y = x + y + z\ \Rightarrow\ z = 0$

$\Rightarrow x$ has any real values.

Now, take $x = 1\ \therefore\ \vec{\mathbf{a}} = \hat{\mathbf{i}}$

34 **(d)**

Let $\vec{\mathbf{c}} = \vec{\mathbf{a}} + \lambda + \vec{\mathbf{b}} = (1 + \lambda)\hat{\mathbf{i}} + (1 - \lambda)\hat{\mathbf{j}} + (\lambda - 1)\hat{\mathbf{k}}$

Also, $\vec{\mathbf{c}} \cdot \vec{\mathbf{a}} = 0$

$\Longrightarrow \left[(1 + \lambda)\hat{\mathbf{i}} + (1 - \lambda)\hat{\mathbf{j}} + (\lambda - 1)\hat{\mathbf{k}}\right] \cdot \left[\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}\right] = 0$

$\Longrightarrow 1 + \lambda + 1 - \lambda - \lambda + 1 = 0$

$\Longrightarrow\ \lambda = 3$

$\therefore\ \ \vec{\mathbf{c}} = 4\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\Longrightarrow\ \ \vec{\mathbf{c}} = \pm\dfrac{2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{6}}$

35 **(a)**

The cartesian form of an equation of planes are

$x + 3y - z = 0$ and $y + 2z = 0$

The line of intersection of two planes is

$(x + 3y - z) + \lambda(y + 2z) = 0$ ....(i)

Since, it is passing through $(-1, -1, -1)$

$\therefore (-1 - 3 + 1) + \lambda(-1 - 2) = 0$

$\Longrightarrow\ \lambda = -1$

On putting the value of λ in Eq. (i), we get

$x + 2y - 3z = 0$

Hence, vector equation of plane is

$\vec{\mathbf{r}} \cdot \left(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}\right) = 0$

39 **(d)**

$\vec{\mathbf{a}} \times \left[\vec{\mathbf{a}} \times \left(\vec{\mathbf{a}} \times \vec{\mathbf{b}}\right)\right] = \vec{\mathbf{a}} \times \left\{\left(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}\right)\vec{\mathbf{a}} - (\vec{\mathbf{a}} \cdot \vec{\mathbf{a}})\vec{\mathbf{b}}\right\}$

$= 0 - (\vec{\mathbf{a}} \cdot \vec{\mathbf{a}})(\vec{\mathbf{a}} \times \vec{\mathbf{b}})$

$= (\vec{\mathbf{a}} \cdot \vec{\mathbf{a}})(\vec{\mathbf{b}} \times \vec{\mathbf{a}})$

40 **(a)**

$$\begin{vmatrix} \vec{\mathbf{a}} \cdot \vec{\mathbf{a}} & \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} & \vec{\mathbf{a}} \cdot \vec{\mathbf{c}} \\ \vec{\mathbf{b}} \cdot \vec{\mathbf{a}} & \vec{\mathbf{b}} \cdot \vec{\mathbf{b}} & \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} \\ \vec{\mathbf{c}} \cdot \vec{\mathbf{a}} & \vec{\mathbf{c}} \cdot \vec{\mathbf{b}} & \vec{\mathbf{c}} \cdot \vec{\mathbf{c}} \end{vmatrix}$$

$$= \begin{vmatrix} a_1^2 + a_2^2 + a_3^2 & a_1b_1 + a_2b_2 + a_3b_3 & a_1c_1 \\ a_1b_1 + a_2b_2 + a_3b_3 & b_1^2 + b_2^2 + b_3^2 & b_1c_1 \\ a_1c_1 + a_2c_2 + a_3c_3 & b_1c_1 + b_2c_2 + b_3c_3 & c_1^2 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$= \left[\,\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}\,\right]^2$

41 **(c)**

Given, $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 12$

$\Longrightarrow |\vec{\mathbf{a}}||\vec{\mathbf{b}}| \cos\theta\ = 12$

$\Longrightarrow 10 \times 2 \times \cos\theta = 12$

$\Longrightarrow \cos\theta = \dfrac{3}{5}$

$\therefore \sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \dfrac{9}{25}} = \dfrac{4}{5}$

Now,

$|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = |\vec{\mathbf{a}}||\vec{\mathbf{b}}| \sin\theta = 10 \times 2 \times \dfrac{4}{5} = 16$

42 **(d)**

We have,

$\left|\vec{a} + \vec{b} + \vec{c}\right|^2$

$= |\vec{a}|^2 + \left|\vec{b}\right|^2 + |\vec{c}|^2 + 2\left(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}\right)$ ....(i)

It is given that $\vec{a} \perp \left(\vec{b} + \vec{c}\right), \vec{b} \perp (\vec{c} + \vec{a})$ and $\vec{c} \perp \left(\vec{a} + \vec{b}\right)$

$\therefore \vec{a} \cdot \left(\vec{b} + \vec{c}\right) = 0, \vec{b} \cdot (\vec{c} + \vec{a}) = 0$ and $\vec{c} \cdot \left(\vec{a} + \vec{b}\right) = 0$

$\Rightarrow 2\left(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}\right) = 0$

$\Rightarrow \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = 0$

$\therefore \left|\vec{a} + \vec{b} + \vec{c}\right|^2 = 16 + 16 + 25 + 0$ [From (i)]

$\Rightarrow \left|\vec{a} + \vec{b} + \vec{c}\right| = \sqrt{57}$

43 **(a)**

Since, $\vec{\mathbf{a}} + \vec{\mathbf{b}} = \vec{\mathbf{c}}\ \Longrightarrow (\vec{\mathbf{a}} + \vec{\mathbf{b}})^2 = \vec{\mathbf{c}}^2$

$\Longrightarrow\ |\vec{\mathbf{a}}|^2 + \left|\vec{\mathbf{b}}\right|^2 + 2|\vec{\mathbf{a}}|\left|\vec{\mathbf{b}}\right| \cos\theta = |\vec{\mathbf{c}}|^2$

$\Longrightarrow 2(1 + \cos\theta) = 1\ \Longrightarrow \cos\theta = -\dfrac{1}{2}$ $[\because |\vec{\mathbf{a}}| = \left|\vec{\mathbf{b}}\right| = |\vec{\mathbf{c}}| = 1, \text{given}]$

Now, $|\vec{\mathbf{a}} + \vec{\mathbf{b}}|^2 = |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 - 2|\vec{\mathbf{a}}||\vec{\mathbf{b}}| \cos\theta$

$= 1 + 1 + 2 \cdot \dfrac{1}{2} = 3$

$\Rightarrow |\vec{\mathbf{a}} - \vec{\mathbf{b}}| = \sqrt{3}$

44 **(c)**

We have,

$\vec{a}.\vec{b} \geq 0 \Rightarrow |\vec{a}||\vec{b}| \cos\theta \geq 0 \Rightarrow 0 \leq \theta \leq \frac{\pi}{2}$

45 **(d)**

Since the vectors $2\hat{\imath} + 3\hat{\jmath}$ and $5\hat{\imath} + 6\hat{\jmath}$ have (1, 1) as initial point. Therefore, their terminal points are (3, 4) ad (6, 7) respectively. The equation of the line joining these two points is $x - y + 1 = 0$. The terminal point of $8\hat{\imath} + \lambda\hat{\jmath}$ is $(9, \lambda + 1)$. Since the vectors terminate on the same straight line.

Therefore, point $(9, (\lambda + 1))$ lies on $x - y + 1 = 0$

$\Rightarrow 9 - (\lambda + 1) + 1 = 0 \Rightarrow \lambda = 9$

47 **(a)**

Let $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$ ....(i)

$\vec{\mathbf{B}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ ....(ii)

$\vec{\mathbf{C}} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ ...(iii)

$\vec{\mathbf{D}} = \hat{\mathbf{i}} - \lambda\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ .....(iv)

From Eq. (i) and (ii), we get

$\overrightarrow{\mathbf{AB}} = -\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

∴ From Eq. (i) and (iii), we get

$\overrightarrow{\mathbf{AC}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$

Similarly, from Eqs.(i) and (iv), we get

$\overrightarrow{\mathbf{AD}} = -\hat{\mathbf{i}} - (\lambda - 3)\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$

Now, using condition of coplanarity

$$\begin{vmatrix} -1 & -1 & 4 \\ 1 & 1 & -1 \\ -1 & -(\lambda+3) & 7 \end{vmatrix} = 0$$

Applying $R_1 \to R_1 + R_2$, we get

$$\begin{vmatrix} 0 & 0 & 3 \\ 1 & 1 & -1 \\ -1 & -(\lambda+3) & 7 \end{vmatrix} = 0$$

$\Rightarrow -\lambda - 2 = 0 \Rightarrow \lambda = -2$

48 **(b)**

Since, $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}|^2 + |\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}|^2 = |\vec{\mathbf{a}}|^2 \, |\vec{\mathbf{b}}|^2$

$\Longrightarrow (10)^2 + |\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}|^2 = (3)^2 \cdot (4)^2$

$\Longrightarrow |\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}|^2 = 44$

49 **(b)**

Let $\vec{a}, \vec{b}$ be the sides of the given parallelogram.

Then, its diagonals are $\vec{a} + \vec{b}$ and $\pm(\vec{a} - \vec{b})$

We have,

$\vec{a} + \vec{b} = 3\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\vec{a} - \vec{b} = \pm(\hat{\imath} - 3\hat{\jmath} + 4\hat{k})$

$\Rightarrow \vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$

or $\vec{a} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{b} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$\Rightarrow |\vec{a}| = \sqrt{6}, |\vec{b}| = \sqrt{14}$ or $|\vec{a}| = \sqrt{14}, |\vec{b}| = \sqrt{6}$

50 **(d)**

We have, $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{c}}$

$\Rightarrow$ $\vec{\mathbf{c}}$ is perpendicular to $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ and $\vec{\mathbf{b}} \times \vec{\mathbf{c}} = \vec{\mathbf{a}}$.

$\Rightarrow$ $\vec{\mathbf{a}}$ is perpendicular to $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$.

$\Rightarrow$ $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are mutually perpendicular.

Again $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{c}}$

$\Rightarrow |\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = |\vec{\mathbf{c}}|$

$= |\vec{\mathbf{a}}||\vec{\mathbf{b}}| \cdot \sin 90° = |\vec{\mathbf{c}}|$

$\Rightarrow |\vec{\mathbf{a}}||\vec{\mathbf{b}}| = |\vec{\mathbf{c}}|$ ...(i)

Also, $\vec{\mathbf{b}} \times \vec{\mathbf{c}} = |\vec{\mathbf{a}}|$

$|\vec{\mathbf{b}}||\vec{\mathbf{c}}| \cdot \sin 90° = |\vec{\mathbf{a}}|$

$|\vec{\mathbf{b}}||\vec{\mathbf{c}}| = |\vec{\mathbf{a}}|$ ....(ii)

From Eqs. (i) and (ii), we get

$|\vec{\mathbf{b}}|^2 |\vec{\mathbf{c}}| = |\vec{\mathbf{c}}|$

$\therefore |\vec{\mathbf{b}}|^2 = 1$ $(\because |\vec{\mathbf{c}}| \neq 0)$

$\Rightarrow |\vec{\mathbf{b}}| = 1$

$\Rightarrow |\vec{\mathbf{a}}| = |\vec{\mathbf{c}}|$

51 **(a)**

$$\vec{\mathbf{a}} \times \vec{\mathbf{c}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 1 & -1 \\ 1 & -1 & -1 \end{vmatrix}$$

$= \hat{\mathbf{i}}(-1 - 1) - \hat{\mathbf{j}}(0 + 1) + \hat{\mathbf{k}}(0 - 1)$

$= -2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$

Given,

$\vec{\mathbf{a}} \times \vec{\mathbf{b}} + \vec{\mathbf{c}} = \vec{\mathbf{0}}$

$\Longrightarrow \vec{\mathbf{a}} \times (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) + \vec{\mathbf{a}} \times \vec{\mathbf{c}} = \vec{\mathbf{0}}$

$\Longrightarrow (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{a}} - (\vec{\mathbf{a}} \cdot \vec{\mathbf{a}})\vec{\mathbf{b}} = -\vec{\mathbf{a}} \times \vec{\mathbf{c}}$

$\Longrightarrow 3\vec{\mathbf{a}} - 2\vec{\mathbf{b}} = -\vec{\mathbf{a}} \times \vec{\mathbf{c}}$

$\Longrightarrow \vec{\mathbf{b}} = \dfrac{3\vec{\mathbf{a}} + \vec{\mathbf{a}} \times \vec{\mathbf{c}}}{2}$

$\Longrightarrow \vec{\mathbf{b}} = \dfrac{3\hat{\mathbf{j}} - 3\hat{\mathbf{k}} - 2\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}}{2}$

$= \dfrac{-2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}}{2} = -\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

53 **(b)**

Given $\vec{\mathbf{a}} + \vec{\mathbf{b}} = -\vec{\mathbf{c}}$

$\Longrightarrow |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + 2\,|\vec{\mathbf{a}}||\vec{\mathbf{b}}|\cos\theta == |\vec{\mathbf{c}}|^2$

$\Longrightarrow 9 + 25 + 2 \cdot 3 \cdot 5 \cos\theta = 49$

$\Longrightarrow \cos\theta = \dfrac{1}{2}$

$\Longrightarrow \theta = \dfrac{\pi}{3}$

54 **(a)**

$3\vec{\mathbf{p}} + \vec{\mathbf{q}} - 2\vec{\mathbf{r}} = 3(\hat{\mathbf{i}} + \hat{\mathbf{j}}) + (4\hat{\mathbf{k}} - \hat{\mathbf{j}}) - 2(\hat{\mathbf{i}} + \hat{\mathbf{k}})$

$= \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

∴ Unit vector in the direction of $3\vec{\mathbf{p}} + \vec{\mathbf{q}} - 2\vec{\mathbf{r}}$

$= \dfrac{1}{3}(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

55 **(c)**

Solving the two equations for $\vec{X}$ and $\vec{Y}$, we get

$\vec{X} = \frac{1}{3}(\hat{\imath} + 3\hat{\jmath})$ and $\vec{Y} = \frac{1}{3}(\hat{\imath} - 3\hat{\jmath})$

$\therefore \cos\theta = \frac{\vec{X}\cdot\vec{Y}}{|\vec{X}||\vec{Y}|} \Rightarrow \cos\theta = -\frac{4}{5}$

56 **(a)**

$|\vec{\mathbf{p}} + \vec{\mathbf{q}}| = 6$

$\Longrightarrow \quad |\vec{\mathbf{p}} + \vec{\mathbf{q}}|^2 = 36$

$\Longrightarrow \; p^2 + q^2 + 2\,\vec{\mathbf{p}}\cdot\vec{\mathbf{q}} = 36$

Similarly, $q^2 + r^2 + 2\,\vec{\mathbf{q}}\cdot\vec{\mathbf{r}} = 48$

and $r^2 + p^2 + 2\,\vec{\mathbf{r}}\cdot\vec{\mathbf{p}} = 16$

adding all, we get

$2(p^2 + q^2 + r^2 + \vec{\mathbf{p}}\cdot\vec{\mathbf{q}} + \vec{\mathbf{q}}\cdot\vec{\mathbf{r}} + \vec{\mathbf{r}}\cdot\vec{\mathbf{p}})$

$\Longrightarrow 2(p^2 + q^2 + r^2)$

$= 100 \quad (\because \vec{\mathbf{p}}\cdot\vec{\mathbf{q}} + \vec{\mathbf{q}}\cdot\vec{\mathbf{r}} + \vec{\mathbf{r}}\cdot\vec{\mathbf{p}} = 0)$

$\Longrightarrow p^2 + q^2 + r^2 = 50$

$\Longrightarrow |\vec{\mathbf{p}} + \vec{\mathbf{q}} + \vec{\mathbf{r}}|^2 = 50$

$\Longrightarrow |\vec{\mathbf{p}} + \vec{\mathbf{q}} + \vec{\mathbf{r}}| = 5\sqrt{2}$

57 **(b)**

In triangles OAC and OBD, we have

$\vec{O}A + \vec{O}C = 2\,\vec{O}M$ and $\vec{O}B + \vec{O}D = 2\,\vec{O}M$

$\Rightarrow \vec{O}A + \vec{O}B + \vec{O}C + \vec{O}D = 4\,\vec{O}M$

58 **(c)**

The work done is given by

$W = \vec{F}\cdot\vec{d} = (2\hat{\imath} - \hat{\jmath} - \hat{k})\cdot(3\hat{\imath} + 2\hat{\jmath} - 5\hat{k})$

$= 9$ units

59 **(d)**

$[\,\hat{\mathbf{i}}\,\hat{\mathbf{k}}\,\hat{\mathbf{j}}\,] + [\,\hat{\mathbf{k}}\,\hat{\mathbf{j}}\,\hat{\mathbf{i}}\,] + [\,\hat{\mathbf{j}}\,\hat{\mathbf{k}}\,\hat{\mathbf{i}}\,]$

$= [\,\hat{\mathbf{i}}\,\hat{\mathbf{k}}\,\hat{\mathbf{j}}\,] + [\,\hat{\mathbf{i}}\,\hat{\mathbf{k}}\,\hat{\mathbf{j}}\,] - [\,\hat{\mathbf{i}}\,\hat{\mathbf{k}}\,\hat{\mathbf{j}}\,]$

$= [\,\hat{\mathbf{i}}\,\hat{\mathbf{k}}\,\hat{\mathbf{j}}\,] = \hat{\mathbf{i}}\cdot(\hat{\mathbf{k}} \times \hat{\mathbf{j}})$

$= \hat{\mathbf{i}}\cdot(-\hat{\mathbf{i}}) = -1$

60 **(c)**

Given, $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = \vec{\mathbf{a}}\cdot\vec{\mathbf{c}} = 0$

Now, $|\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}|^2 = |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + |\vec{\mathbf{c}}|^2 + 2(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} + \vec{\mathbf{b}}\cdot\vec{\mathbf{c}} + \vec{\mathbf{c}}\cdot\vec{\mathbf{a}})$

$= (1)^2 + (1)^2 + (1)^2 + 2\left(0 + |\vec{\mathbf{b}}||\vec{\mathbf{c}}|\cos\frac{\pi}{3} + 0\right)$

$= 3 + 2 \times 1 \times 1 \times \frac{1}{2} = 4$

$\Longrightarrow |\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}| = \pm 2$

61 **(b)**

Let $\overrightarrow{\mathbf{AB}} = \vec{\mathbf{a}} = 3\,\vec{\alpha} - \vec{\beta}, \overrightarrow{\mathbf{BC}} = \vec{\mathbf{b}} = \vec{\alpha} + 3\vec{\beta}$

Diagonal $\overrightarrow{\mathbf{AC}} = \overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{BC}} = \vec{\mathbf{a}} + \vec{\mathbf{b}}$

$\Longrightarrow |\overrightarrow{\mathbf{AC}}| = |\vec{\mathbf{a}} + \vec{\mathbf{b}}|$

$\Longrightarrow |\overrightarrow{\mathbf{AC}}| = |4\,\vec{\alpha} + 2\vec{\beta}|$

$\Longrightarrow |\overrightarrow{\mathbf{AC}}|^{\mathbf{2}} = 16\vec{\alpha}^2 + 4\vec{\beta}^2 + 16\vec{\alpha}\cdot\vec{\beta}$

$\Longrightarrow |\overrightarrow{\mathbf{AC}}|^{\mathbf{2}} = 64 + 16 + 16|\vec{\alpha}||\vec{\beta}|\cos\frac{\pi}{3}$

$\Longrightarrow |\overrightarrow{\mathbf{AC}}|^{\mathbf{2}} = 80 + 16 \times 4 \times \frac{1}{2} = 112$

$\Longrightarrow \; |\overrightarrow{\mathbf{AC}}| = 4\sqrt{7}$

Other diagonal is $|\overrightarrow{\mathbf{BD}} = |\vec{\mathbf{a}} - \vec{\mathbf{b}}|$

$\Longrightarrow |\overrightarrow{\mathbf{BD}}|^{\mathbf{2}} = |2\vec{\alpha} - 4\vec{\beta}|^2$

$= 4|\vec{\alpha}|^2 + 16|\vec{\beta}|^2 - 16\,|\vec{\alpha}||\vec{\beta}|\cos\frac{\pi}{3}$

$= 64 + 16 - 16 \times 4 \times \frac{1}{2} = 48$

$\Longrightarrow |\overrightarrow{\mathbf{BD}} = \sqrt{48} = 4\sqrt{3}$

62 **(c)**

Given that, $|\vec{\mathbf{a}}| = |\vec{\mathbf{b}}|$

Now, $(\vec{\mathbf{a}} + \vec{\mathbf{b}})\cdot(\vec{\mathbf{a}} - \vec{\mathbf{b}}) = \vec{\mathbf{a}}\cdot\vec{\mathbf{a}} + \vec{\mathbf{b}}\cdot\vec{\mathbf{a}} - \vec{\mathbf{b}}\,\vec{\mathbf{a}} - \vec{\mathbf{b}}\,\vec{\mathbf{b}}$

$= 0 \;\; (\because \; |\overrightarrow{\mathbf{a}}| = |\overrightarrow{\mathbf{b}}|)$

63 **(a)**

We have,

$\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$

$\Rightarrow \vec{a}\cdot(\vec{b} - \vec{c}) = 0$ and $\vec{a} \times (\vec{b} - \vec{c}) = 0$

$\Rightarrow (\vec{b} - \vec{c} = 0$ or, $\vec{b} - \vec{c} \perp \vec{a})$ and $(\vec{b} - \vec{c}$

$= 0$ or, $\vec{b} - \vec{c}||\vec{a})$

$\Rightarrow \vec{b} - \vec{c} = 0 \Rightarrow \vec{b} = \vec{c}$

67 **(a)**

We know that, any vector $\vec{\mathbf{a}}$ can be uniquely expressed in terms of three non-coplanar vectors as $\vec{\mathbf{a}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ multiply in succession by $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$, we get

$x = \vec{\mathbf{a}}\cdot\hat{\mathbf{i}}, y = \vec{\mathbf{a}}\cdot\hat{\mathbf{j}}, z = \vec{\mathbf{a}}\cdot\hat{\mathbf{k}}$

$\therefore (\vec{\mathbf{a}}\cdot\hat{\mathbf{i}})\hat{\mathbf{i}} + (\vec{\mathbf{a}}\cdot\hat{\mathbf{j}})\hat{\mathbf{j}} + (\vec{\mathbf{a}}\cdot\hat{\mathbf{k}})\hat{\mathbf{k}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}} = \vec{\mathbf{a}}$

69 **(b)**

Let $\vec{\mathbf{b}} = \hat{\mathbf{i}}$ and $\vec{\mathbf{c}} = \hat{\mathbf{j}}$

$\therefore |\vec{\mathbf{b}} \times \vec{\mathbf{c}}| = |\hat{\mathbf{k}}| = 1$

Let $\vec{\mathbf{a}} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$

Now, $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = \vec{\mathbf{a}}\,\hat{\mathbf{i}} = a_1, \vec{\mathbf{a}}\cdot\vec{\mathbf{c}} = \vec{\mathbf{a}}\cdot\hat{\mathbf{j}} = a_2$

and $\vec{\mathbf{a}}\cdot\frac{\vec{\mathbf{b}} \times \vec{\mathbf{c}}}{|\vec{\mathbf{b}} \times \vec{\mathbf{c}}|} = \vec{\mathbf{a}}\cdot\hat{\mathbf{k}} = a_3$

$\therefore (\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}) + \vec{\mathbf{b}} + (\vec{\mathbf{a}}\cdot\vec{\mathbf{c}})\vec{\mathbf{c}} + \frac{\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}} \times \vec{\mathbf{c}})}{|\vec{\mathbf{b}} \times \vec{\mathbf{c}}|}\cdot(\vec{\mathbf{b}} \times \vec{\mathbf{c}})$

$= a_1\vec{\mathbf{b}} + a_2\vec{\mathbf{c}} + a_3(\vec{\mathbf{b}} \times \vec{\mathbf{c}})$

$= a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}} = \vec{\mathbf{a}}$

70 **(c)**

Let the unit vector $\frac{\hat{\mathbf{i}} - \hat{\mathbf{j}}}{\sqrt{2}}$ is perpendicular to $\hat{\mathbf{i}} - \hat{\mathbf{j}}$, then we get

$\frac{(\hat{\mathbf{i}} + \hat{\mathbf{j}})\cdot(\hat{\mathbf{i}} - \hat{\mathbf{j}})}{\sqrt{2}} = \frac{1 - 1}{\sqrt{2}} = 0$

$\therefore \frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}$ is the required unit vector.

71 **(d)**

Let the unit vector be $\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

Since, $\vec{\mathbf{r}} \cdot (3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 0$ and $\vec{\mathbf{r}} \cdot (2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\mathbf{k} = 0$

$\Rightarrow 3x + y + 2z = 0$ and $2x - 2y + 4z = 0$

On solving, we get $x = 1, y = -1$ and $z = -1$

$\therefore$ Required unit vector $= \dfrac{\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}}{\sqrt{1^2 + 1^2 + 1^2}}$

$= \dfrac{\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}}{\sqrt{3}}$

72 **(d)**

The position vector of the vertices A, B, C of ΔABC are $7\hat{\mathbf{j}} + 10\hat{\mathbf{k}}, -\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ and $-4\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ respectively.

$\therefore \overrightarrow{\mathbf{AB}} = -\hat{\mathbf{i}} - \hat{\mathbf{j}} - 4\hat{\mathbf{k}}, \overrightarrow{\mathbf{BC}} = -3\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$

And $\overrightarrow{\mathbf{CA}} = 4\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

$\Rightarrow |\overrightarrow{\mathbf{AB}}| = \sqrt{(-1)^2 + (-1)^2 + (-4)^2} = \sqrt{18}$

$= 3\sqrt{2}$

$|\overrightarrow{\mathbf{BC}}| = \sqrt{(-3)^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$

and $|\overrightarrow{\mathbf{CA}}| = \sqrt{4^2 + (-2)^2 + (-4)^2} = \sqrt{36} = 6$

It is clear from these values that

$|\overrightarrow{\mathbf{AB}}|^2 + |\overrightarrow{\mathbf{BC}}|^2 = |\overrightarrow{\mathbf{CA}}|^2$

Hence, $\Delta\, ABC$ is right angled and isosceles also.

74 **(b)**

For collinearity, $\cos x\, \hat{\mathbf{i}} + \sin x\, \hat{\mathbf{j}} = \lambda(x\hat{\mathbf{i}} + \sin x\, \hat{\mathbf{j}})$

$\Rightarrow \cos x = x$

Let $f(x) = \cos x - x$

$\Rightarrow f'(x) = -\sin x - 1 < 0$

$f(x)$ is decreasing function and for $x \geq \frac{\pi}{3}, f(x) <$ 0 and for $\frac{\pi}{3} < x < \frac{\pi}{6}, f(x) > 0$.

Hence, unique solution exist.

75 **(d)**

Let the required unit vector be $\vec{\mathbf{r}} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$

Then, $|\vec{\mathbf{r}}| = 1$

$\Rightarrow a^2 + b^2 = 1$ ...(i)

Since, $\vec{\mathbf{r}}$ makes an angle of 45° with $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ and an angle of 60° with$3\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$, therefore

$\cos\dfrac{\pi}{4} = \dfrac{\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}})}{|\vec{\mathbf{r}}||\hat{\mathbf{i}} + \hat{\mathbf{j}}|}$

and $\cos\dfrac{\pi}{3} = \dfrac{\vec{\mathbf{r}} \cdot (3\hat{\mathbf{i}} - 4\hat{\mathbf{j}})}{|\vec{\mathbf{r}}||3\hat{\mathbf{i}} - 4\hat{\mathbf{j}}|}$

$\Rightarrow \dfrac{1}{\sqrt{2}} = \dfrac{a + b}{\sqrt{2}}$

and $\dfrac{1}{2} = \dfrac{3a - 4b}{5}$

$\Rightarrow a + b = 1$

and $3a - 4b = \dfrac{5}{2}$

$\Rightarrow a = \dfrac{13}{14}, b = \dfrac{1}{14}$

$\therefore \vec{\mathbf{r}} = \dfrac{13}{14}\hat{\mathbf{i}} + \dfrac{1}{14}\hat{\mathbf{j}}$

77 **(b)**

Since, volume of parallelopiped $= 34$

$\therefore \begin{vmatrix} 4 & 5 & 1 \\ 0 & -1 & 1 \\ 3 & 9 & p \end{vmatrix} = 34$

$\Rightarrow 4(-p - 9) - 5(-3) + 1(3) = 34$

$\Rightarrow -4p - 36 + 15 + 3 = 34$

$\Rightarrow 4p = -52$

$\Rightarrow p = -13$

78 **(d)**

$\dfrac{(\vec{\mathbf{a}} \times \vec{\mathbf{b}})^2 + (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})^2}{2\vec{\mathbf{a}}^2\vec{\mathbf{b}}^2} = \dfrac{\vec{\mathbf{a}}^2\vec{\mathbf{b}}^2\sin^2\theta + \vec{\mathbf{a}}^2\vec{\mathbf{b}}^2\cos^2\theta}{2\vec{\mathbf{a}}^2\vec{\mathbf{b}}^2}$

$= \dfrac{\cos^2\theta + \sin^2\theta}{2} = \dfrac{1}{2}$

79 **(b)**

Let two vectors are $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$

Given, $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = \sqrt{3}|\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}|$

$\Rightarrow |\vec{\mathbf{a}}| \cdot |\vec{\mathbf{b}}| \sin\theta = \sqrt{3}\, |\vec{\mathbf{a}}||\vec{\mathbf{b}}| \cos\theta$

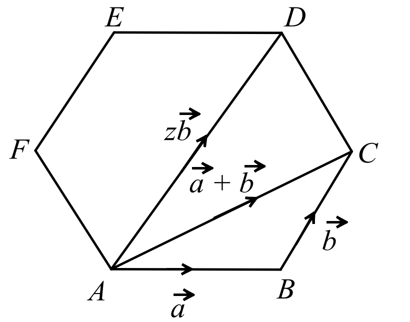
$\Rightarrow \tan\theta = \sqrt{3}$

$\Rightarrow \theta = \dfrac{\pi}{3}$

82 **(b)**

We have,

$\vec{AC} = \vec{a} + \vec{b}, \vec{AD} = 2\vec{b}$



In ΔADE, we have

$\vec{AD} = \vec{DE} = \vec{AE} \Rightarrow 2\vec{b} - \vec{a} = \vec{AE} \Rightarrow \vec{E}A = \vec{a} - 2\,\vec{b}$

In ΔACD, we have

$\vec{AC} + \vec{CD} = \vec{AD} \Rightarrow \vec{a} + \vec{b} + \vec{CD} = 2\,\vec{b} \Rightarrow \vec{CD}$

$= \vec{b} - \vec{a}$

$\therefore \vec{F}A = -\vec{CD} = \vec{a} - \vec{b}$

Hence, $\vec{AC} + \vec{AD} + \vec{E}A + \vec{F}A$

$= \vec{a} + \vec{b} + 2\,\vec{b} + \vec{a} - 2\,\vec{b} + \vec{a} - \vec{b} = 3\,\vec{a} = 3\,\vec{AB}$

83 **(c)**

We have,

$(\vec{a} \times \vec{b}) \times (\vec{b} \times \vec{c}) = \{(\vec{a} \times \vec{b}) \cdot \vec{c}\}\vec{b} - \{(\vec{a} \times \vec{b}) \cdot \vec{b}\}\vec{c}$

$= [\vec{a}\,\vec{b}\,\vec{c}]\vec{b}$

$(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})$
$= \{(\vec{b}\times\vec{c})\cdot\vec{a}\}\cdot\vec{c} - \{(\vec{b}\times\vec{c})\cdot\vec{c}\}\vec{a}$
$= [\vec{b}\ \vec{c}\ \vec{a}]\vec{c}$
and,
$(\vec{c}\times\vec{a})\times(\vec{a}\times\vec{b}) = \{(\vec{c}\times\vec{a})\cdot\vec{b}\}\vec{a} - \{(\vec{c}\times\vec{a})\cdot\vec{a}\}\vec{b}$
$= [\vec{c}\ \vec{a}\ \vec{b}]\vec{a}$
$\therefore [(\vec{a}\times\vec{b})\times(\vec{b}\times\vec{c})(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})(\vec{c}\times\vec{a}) \times(\vec{a}\times\vec{b})]$
$= \left[[\vec{a}\ \vec{b}\ \vec{c}]\vec{a}\ [\vec{a}\ \vec{b}\ \vec{c}]\vec{b}[\vec{a}\ \vec{b}\ \vec{c}]\vec{c}\right]$
$= [\vec{a}\ \vec{b}\ \vec{c}]^3[\vec{a}\ \vec{b}\ \vec{c}] = [\vec{a}\ \vec{b}\ \vec{c}]^4$

84 **(a)**
Given, $\vec{\mathbf{a}} = \lambda\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 3\hat{\mathbf{k}}, \vec{\mathbf{b}} = \lambda\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\lambda\hat{\mathbf{k}}$
$\therefore \cos\theta = \dfrac{\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}}{|\vec{\mathbf{a}}||\vec{\mathbf{b}}|}$
$= \dfrac{\lambda^2 - 7 + 6\lambda}{\sqrt{\lambda^2+49+9}\sqrt{\lambda^2+1+4\lambda^2}} < 0$
$\Longrightarrow (\lambda+7)(\lambda-1) < 0$
$\Longrightarrow -7 < \lambda < 1$

85 **(b)**
We have,
$\vec{r} = \lambda_1\vec{r_1} + \lambda_2\vec{r_2} + \lambda_3\vec{r_3}$
$\Rightarrow 2\vec{a} - 3\vec{b} + 4\vec{c}$
$= (\lambda_1 - \lambda_2 + \lambda_3)\vec{a} + (-\lambda_1 + \lambda_2 - \lambda_3)\vec{b} + (\lambda_1 + \lambda_2 + \lambda_3)\vec{c}$
$\Rightarrow \lambda_1 - \lambda_2 + \lambda_3 = 2, -\lambda_1 + \lambda_2 - \lambda_3 = -3, \lambda_1 + \lambda_2 + \lambda_3 = 4$
$[\because \vec{a}, \vec{b}, \vec{c}$ are non − coplanar$]$
$\Rightarrow \lambda_1 = \dfrac{7}{2}, \lambda_2 = 1, \lambda_3 = -\dfrac{1}{2}$

86 **(c)**
We have,
$(\vec{a}-\vec{d})\cdot(\vec{b}-\vec{c}) = (\vec{b}-\vec{d})\cdot(\vec{c}-\vec{a}) = 0$
$\Rightarrow \vec{DA}\cdot\vec{BC} = 0$ and $\vec{DB}\cdot\vec{AC} = 0$
$\Rightarrow AD \perp BC$ and $DB \perp AC$
$\Rightarrow D$ is the orthocenter of ΔABD

87 **(a)**
Given $\overrightarrow{\mathbf{OA}} = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$
$\overrightarrow{\mathbf{OB}} = -3\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$
$\therefore\ \overrightarrow{\mathbf{AB}} = \overrightarrow{\mathbf{OB}} - \overrightarrow{\mathbf{OA}} = -7\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$
$\therefore\ \overrightarrow{\mathbf{DE}} = -\overrightarrow{\mathbf{AB}} = 7\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

88 **(c)**
Given, $|\vec{\mathbf{a}}\times\vec{\mathbf{b}}| = |\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}|$
$\Longrightarrow |\vec{\mathbf{a}}|\cdot|\vec{\mathbf{b}}|\sin\theta = |\vec{\mathbf{a}}||\vec{\mathbf{b}}|\cos\theta$
$\Longrightarrow \sin\theta = \cos\theta \Longrightarrow \theta = \dfrac{\pi}{4}$

89 **(a)**
Let the line joining the points with position vectors $-2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and $7\hat{\mathbf{i}} - \hat{\mathbf{k}}$ be
Divide in the ratio λ:1 by $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$
$\therefore \dfrac{\lambda(7\hat{\mathbf{i}} - \hat{\mathbf{k}}) + (-2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}})}{\lambda+1} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$
$\Longrightarrow (7\lambda-2)\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + (5-\lambda)\hat{\mathbf{k}} = (\lambda+1)\hat{\mathbf{i}} + 2(\lambda+1)\hat{\mathbf{j}} + 3(\lambda+1)\hat{\mathbf{k}}$
On equating the coefficient of $\hat{\mathbf{i}}$, we get
$7\lambda - 2 = \lambda + 1 \Longrightarrow \lambda = 2$
Hence, required ratio $= \lambda : 1 = 2 : 1$

91 **(a)**
Force $\vec{\mathbf{F}} = \overrightarrow{\mathbf{AB}} = (3-1)\hat{\mathbf{i}} + (-4-2)\hat{\mathbf{j}} + (2+3)\hat{\mathbf{k}}$
$= 2\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$
Moment of force $\vec{\mathbf{F}}$ with respect to $M = \overrightarrow{\mathbf{MA}}\times\vec{\mathbf{F}}$
$\because \overrightarrow{\mathbf{MA}} = (1+2)\hat{\mathbf{i}} + (2-4)\hat{\mathbf{j}} + (-3+6)\hat{\mathbf{k}}$
$= 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$
Now, $\overrightarrow{\mathbf{MA}}\times\vec{\mathbf{F}} = \begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 3 & -2 & 3\\ 2 & -6 & 5\end{vmatrix}$
$= \hat{\mathbf{i}}(-10+18) + \hat{\mathbf{j}}(6-15) + \hat{\mathbf{k}}(-18+4)$
$= 8\hat{\mathbf{i}} - 9\hat{\mathbf{j}} - 14\hat{\mathbf{k}}$

92 **(d)**
$\because\ \vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = |\vec{\mathbf{a}}||\vec{\mathbf{b}}|\cos\frac{5\pi}{6} = -\frac{|\vec{\mathbf{a}}||\vec{\mathbf{b}}|\sqrt{3}}{2}$
$\therefore\ -\frac{6}{\sqrt{3}} = -\frac{|\vec{\mathbf{a}}||\vec{\mathbf{b}}|\sqrt{3}}{2|\vec{\mathbf{b}}|}$ (given condition)
$\Rightarrow |\vec{\mathbf{a}}| = \dfrac{6\times2}{3} = 4$

93 **(c)**
Equation of straight line passing through the points
$a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$ and $b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + b_3\hat{\mathbf{k}}$ is
$a_1(1-t)\hat{\mathbf{i}} + a_2(1-t)\hat{\mathbf{j}} + a_3(1-t)\hat{\mathbf{k}} + (b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + b_3\hat{\mathbf{k}})t$

95 **(d)**
$(3\vec{\mathbf{a}} + \vec{\mathbf{b}})\cdot(\vec{\mathbf{a}} - 4\vec{\mathbf{b}}) = 3|\vec{\mathbf{a}}|^2 - 11\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} - 4|\vec{\mathbf{b}}|^2$
$= 3\cdot36 - 11\cdot6\cdot8\cos\pi - 4\cdot64 > 0$
$\therefore$ Angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is acute angle.
$\therefore$ The longer diagonal is given by
$\vec{\alpha} = (3\vec{\mathbf{a}} + \vec{\mathbf{b}}) + (\vec{\mathbf{a}} - 4\vec{\mathbf{b}}) = 4\vec{\mathbf{a}} - 3\vec{\mathbf{b}}$
Now, $|\vec{\alpha}|^2 = |4\vec{\mathbf{a}} - 3\vec{\mathbf{b}}|^2$
$= 16|\vec{\mathbf{a}}|^2 + 9|\vec{\mathbf{b}}|^2 - 24\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}$
$= 16\cdot36 + 9\cdot64 - 24\cdot6\cdot8\cos\pi$
$= 16\times144$
$|4\vec{\mathbf{a}} - 3\vec{\mathbf{b}}| = 48$

96 **(b)**

Given, $\vec{\mathbf{a}}+\vec{\mathbf{b}}=-\vec{\mathbf{c}}$

$\Longrightarrow\ \vec{\mathbf{a}}\times\vec{\mathbf{c}}+\vec{\mathbf{b}}\times\vec{\mathbf{c}}=\vec{\mathbf{0}}\Longrightarrow\vec{\mathbf{b}}\times\vec{\mathbf{c}}=\vec{\mathbf{c}}\times\vec{\mathbf{a}}$

Similarly, $\vec{\mathbf{a}}\times\vec{\mathbf{b}}=\vec{\mathbf{b}}\times\vec{\mathbf{c}}$

Hence, $\vec{\mathbf{a}}\times\vec{\mathbf{b}}=\vec{\mathbf{b}}\times\vec{\mathbf{c}}=\vec{\mathbf{c}}\times\vec{\mathbf{a}}$

97 **(d)**

$|\vec{\mathbf{a}}-\vec{\mathbf{b}}|^2=|\vec{\mathbf{a}}|^2+|\vec{\mathbf{b}}|^2-2|\vec{\mathbf{a}}||\vec{\mathbf{b}}|\cos 90°$

$25+25-2\times 0=50$

$\Longrightarrow |\vec{\mathbf{a}}-\vec{\mathbf{b}}|=5\sqrt{2}$

98 **(d)**

Given vectors are non-coplanar, if

$\Delta=\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}\neq 0$

Now, $\begin{vmatrix}a & a^2 & 1+a^3\\ b & b^2 & 1+b^3\\ c & c^2 & 1+c^3\end{vmatrix}=0$

$\Longrightarrow \begin{vmatrix}a & a^2 & 1\\ b & b^2 & 1\\ c & c^2 & 1\end{vmatrix}+\begin{vmatrix}a & a^2 & a^3\\ b & b^2 & b^3\\ c & c^2 & c^3\end{vmatrix}=0$

$\Longrightarrow\ \Delta(1+abc)=0\ \Longrightarrow abc=-1$

99 **(a)**

Let $\vec{\mathbf{A}}=\vec{\mathbf{a}}+\vec{\mathbf{b}}=(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})+(\hat{\mathbf{i}}+3\hat{\mathbf{j}}+5\hat{\mathbf{k}})$

$=2\hat{\mathbf{i}}+4\hat{\mathbf{j}}+6\hat{\mathbf{k}}$

and $\vec{\mathbf{B}}=\vec{\mathbf{b}}+\vec{\mathbf{c}}=(\hat{\mathbf{i}}+3\hat{\mathbf{j}}+5\hat{\mathbf{k}})+(7\hat{\mathbf{i}}+9\hat{\mathbf{j}}+11\hat{\mathbf{k}})$

$=8\hat{\mathbf{i}}+12\hat{\mathbf{j}}+16\hat{\mathbf{k}}$

$\therefore$ Area of parallelogram $=\frac{1}{2}||\vec{\mathbf{A}}\times\vec{\mathbf{B}}||$

$=\frac{1}{2}\left\|\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 2 & 4 & 6\\ 8 & 12 & 16\end{vmatrix}\right\|$

$=\frac{1}{2}|-8\hat{\mathbf{i}}+16\hat{\mathbf{j}}-8\hat{\mathbf{k}}|$

$=\sqrt{(-4)^2+(8)^2+(-4)^2}$

$=4\sqrt{6}$ sq units

100 **(a)**

Clearly, $\vec{c}$ is a unit vector parallel to the vector $\vec{a}\times(\vec{a}\times\vec{b})$

i. e. $\vec{c}=\pm\frac{\vec{a}\times(\vec{a}\times\vec{b})}{|\vec{a}\times(\vec{a}\times\vec{b})|}$

We have,

$\vec{a}=\hat{\imath}+\hat{\jmath}-\hat{k}$ and $\vec{b}=\hat{\imath}-\hat{\jmath}+\hat{k}$

$\therefore \vec{a}\times(\vec{a}\times\vec{b})=(\vec{a}\cdot\vec{b})\vec{a}-(\vec{a}\cdot\vec{a})\vec{b}$

$\Rightarrow \vec{a}\times(\vec{a}\times\vec{b})=-\vec{a}-3\vec{b}=-4\hat{\imath}+2\hat{\jmath}-2\hat{k}$

$\therefore \vec{c}=\pm\frac{(-4\hat{\imath}+2\hat{\jmath}-2\hat{k})}{\sqrt{16+4+4}}=\pm\frac{1}{\sqrt{6}}(-2\hat{\imath}+\hat{\jmath}-\hat{k})$

102 **(b)**

Given, $\vec{\mathbf{a}}+2\vec{\mathbf{b}}+4\vec{\mathbf{c}}=\vec{\mathbf{0}}$

Now, $\vec{\mathbf{a}}\times(\vec{\mathbf{a}}+2\vec{\mathbf{b}}+4\vec{\mathbf{c}})=\vec{\mathbf{0}}$

$\Longrightarrow 2(\vec{\mathbf{a}}\times\vec{\mathbf{b}})+4(\vec{\mathbf{a}}\times\vec{\mathbf{c}})=\vec{\mathbf{0}}$

$\Longrightarrow\ \frac{(\vec{\mathbf{a}}\times\vec{\mathbf{b}})}{4}=\frac{(\vec{\mathbf{c}}\times\vec{\mathbf{a}})}{2}\quad \ldots.(i)$

Again, $\vec{\mathbf{b}}\times(\vec{\mathbf{a}}+2\vec{\mathbf{b}}+4\vec{\mathbf{c}})=\vec{\mathbf{0}}$

$\Longrightarrow\ \vec{\mathbf{b}}\times\vec{\mathbf{a}}+4(\vec{\mathbf{b}}\times\vec{\mathbf{c}})=\vec{\mathbf{0}}$

$\Longrightarrow\ \vec{\mathbf{b}}\times\vec{\mathbf{c}}=(\vec{\mathbf{a}}\times\vec{\mathbf{b}})/4$ ...(ii)

From Eqs. (i) and (ii)

$(\vec{\mathbf{a}}\times\vec{\mathbf{b}})/4=\vec{\mathbf{b}}\times\vec{\mathbf{c}}=(\vec{\mathbf{c}}\times\vec{\mathbf{a}})/2=\vec{\mathbf{p}}$

$\therefore\ \vec{\mathbf{a}}\times\vec{\mathbf{b}}=4\vec{\mathbf{p}},\vec{\mathbf{b}}\times\vec{\mathbf{c}}=\vec{\mathbf{p}}$

and $\vec{\mathbf{c}}\times\vec{\mathbf{a}}=2\vec{\mathbf{p}}$

$\therefore (\vec{\mathbf{a}}\times\vec{\mathbf{b}})+(\vec{\mathbf{b}}\times\vec{\mathbf{c}})+(\vec{\mathbf{c}}\times\vec{\mathbf{a}})=4\vec{\mathbf{p}}+\vec{\mathbf{p}}+2\vec{\mathbf{p}}$

$=7\vec{\mathbf{p}}=7(\vec{\mathbf{b}}\times\vec{\mathbf{c}})$

$\therefore \lambda=7$

103 **(b)**

$\because\ \vec{\mathbf{F}}_1=\frac{5(6\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}})}{7},\vec{\mathbf{F}}_2=\frac{3(3\hat{\mathbf{i}}-2\hat{\mathbf{j}}+6\hat{\mathbf{k}})}{7}$

$\vec{\mathbf{F}}_3=\frac{1(2\hat{\mathbf{i}}-3\hat{\mathbf{j}}-6\hat{\mathbf{k}})}{7}$

And $\vec{\mathbf{F}}=\vec{\mathbf{F}}_1+\vec{\mathbf{F}}_2+\vec{\mathbf{F}}_3$

$=\frac{1}{7}(30\hat{\mathbf{i}}+10\hat{\mathbf{j}}+15\hat{\mathbf{k}}+9\hat{\mathbf{i}}-6\hat{\mathbf{j}}+18\hat{\mathbf{k}}+2\hat{\mathbf{i}}-3\hat{\mathbf{j}}-6\hat{\mathbf{k}})$

$=\frac{1}{7}(41\hat{\mathbf{i}}+\hat{\mathbf{j}}+27\hat{\mathbf{k}})$

and $\overrightarrow{\mathbf{AB}}=5\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}-2\hat{\mathbf{i}}+\hat{\mathbf{j}}+3\hat{\mathbf{k}}$

$=3\hat{\imath}+4\hat{\mathbf{k}}$

$\therefore$ Work done $=\frac{1}{7}\left[41\hat{\mathbf{i}}+\hat{\mathbf{j}}+27\hat{\mathbf{k}}\right]\cdot\left[3\hat{\mathbf{i}}+4\hat{\mathbf{k}}\right]$

$=\frac{1}{7}[123+108]=33$ unit

104 **(b)**

Let $\vec{r}=x\hat{\imath}+y\hat{\jmath}+z\hat{k}$. Then,

$\hat{\imath}\times(\vec{r}\times\hat{\imath})+\hat{\jmath}\times(\vec{r}\times\hat{\jmath})+\hat{k}\times(\vec{r}\times\hat{k})$

$=(\hat{\imath}\cdot\hat{\imath})\vec{r}-(\hat{\imath}\cdot\vec{r})\hat{\imath}+(\hat{\jmath}\cdot\hat{\jmath})\vec{r}-(\hat{\jmath}\cdot\vec{r})\hat{\jmath}+(\hat{k}\cdot\hat{k})\vec{r}-(\hat{k}\cdot\vec{r})\hat{k}$

$=\vec{r}-x\hat{\imath}+\vec{r}-y\hat{\jmath}+\vec{r}-z\hat{k}$

$=3\vec{r}-(x\hat{\imath}+y\hat{\jmath}+z\hat{k})=3\vec{r}-\vec{r}=2\vec{r}$

105 **(b)**

The equation of the plane through the line of intersection of given plane is

$(\vec{\mathbf{r}}\cdot\vec{\mathbf{a}}-\lambda)+k(\vec{\mathbf{r}}\cdot\vec{\mathbf{b}}-\mu)=0$

or $\vec{\mathbf{r}}\cdot(\vec{\mathbf{a}}+k\vec{\mathbf{b}})=\lambda+k\mu$ .....(i)

this passes through the origin, therefore

$0\cdot(\vec{\mathbf{a}}+k\vec{\mathbf{b}})=\lambda+k\mu$

$\Longrightarrow k=-\frac{\lambda}{\mu}$

On putting the value of k in Eq. (i), we get the equation of the required plane as

$\vec{\mathbf{r}} \cdot (\mu\vec{\mathbf{a}} - \lambda\vec{\mathbf{b}}) = 0$

$0 \Longrightarrow \vec{\mathbf{r}} \cdot (\lambda\vec{\mathbf{b}} - \mu\vec{\mathbf{a}}) = 0$

106 **(c)**

By the properties of scalar triple product

$[\vec{\mathbf{a}} + \vec{\mathbf{b}}\ \vec{\mathbf{b}} + \vec{\mathbf{c}}\ \vec{\mathbf{c}} + \vec{\mathbf{a}}] = 2[\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}]$

$\therefore k = 2$

107 **(c)**

$\vec{\mathbf{a}} \cdot \vec{\mathbf{a}} = 1 + 1 + 1 = 3$

Using,

$\vec{\mathbf{a}} \times (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) = (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{a}} - (\vec{\mathbf{a}} \cdot \vec{\mathbf{a}})\vec{\mathbf{b}}$

$\therefore (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) \times (\hat{\mathbf{j}} - \hat{\mathbf{k}}) = (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) - 3\vec{\mathbf{b}}$

$\Longrightarrow -2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}} - 3\vec{\mathbf{b}}$

$\Longrightarrow \vec{\mathbf{b}} = \hat{\mathbf{i}}$

108 **(a)**

Vector perpendicular to face OAB is $\vec{\mathbf{n}}_1$

$$= \overrightarrow{\mathbf{OA}} \times \overrightarrow{\mathbf{OB}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix}$$

$= 5\hat{\mathbf{i}} - \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

Vector perpendicular to face ABC is $\vec{\mathbf{n}}_2$

$$= \overrightarrow{\mathbf{AB}} \times \overrightarrow{\mathbf{AC}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -1 & 2 \\ -2 & -1 & 1 \end{vmatrix}$$

$= \hat{\mathbf{i}} - 5\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

$$\therefore \cos\theta = \frac{\vec{\mathbf{n}}_1 \cdot \vec{\mathbf{n}}_2}{|\vec{\mathbf{n}}_1||\vec{\mathbf{n}}_2|}$$

$$= \frac{5 \times 1 + (-1) \times (-5) + (-3) \times (-3)}{\sqrt{5^2 + (-1)^2 + (-3)^2}\sqrt{1^2 + (-5)^2 + (-3)^2}}$$

$$= \frac{5 + 5 + 9}{\sqrt{35}\sqrt{35}} = \frac{19}{35}$$

$$\Longrightarrow \theta = \cos^{-1}\left(\frac{19}{35}\right)$$

109 **(a)**

Given, $2\,\vec{\mathbf{a}} + 3\vec{\mathbf{b}} - 5\vec{\mathbf{c}} = \vec{\mathbf{0}}$

$$\Longrightarrow \frac{2\vec{\mathbf{a}} + 3\vec{\mathbf{b}}}{5} = \vec{\mathbf{c}}$$

$$\Longrightarrow \frac{2\vec{\mathbf{a}} + 3\vec{\mathbf{b}}}{2 + 3} = \vec{\mathbf{c}}$$

$$\Longrightarrow \frac{\vec{\mathbf{a}} + \frac{3}{2}\vec{\mathbf{b}}}{1 + \frac{3}{2}} = \vec{\mathbf{c}} \quad \ldots\ldots\ldots(i)$$

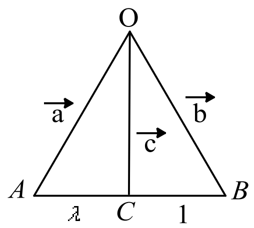
Let $\vec{\mathbf{c}}$divides $\overrightarrow{\mathbf{AB}}$ in the ratio λ:1

Then, $\quad \vec{\mathbf{c}}\dfrac{\vec{\mathbf{a}} + \lambda\,\vec{\mathbf{b}}}{1 + \lambda} \quad \ldots.(\text{ii})$

On comparing Eqs.(i)and (ii), we get

$$\lambda = \frac{3}{2}$$

$\therefore$ Required ratio is 3:2 internally.



110 **(a)**

Let $\overrightarrow{\mathbf{OA}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \overrightarrow{\mathbf{OB}} = 5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $\overrightarrow{\mathbf{OC}} = 2\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 9\hat{\mathbf{k}}$

$\therefore \overrightarrow{\mathbf{AB}} = 4\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}, \overrightarrow{\mathbf{BC}} = -3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 12\hat{\mathbf{k}}$ and $\overrightarrow{\mathbf{AC}} = \hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$

$\Rightarrow AB = 6, BC = \sqrt{157}, AC = 9$

$\therefore$ Perimeter of $\Delta\, ABC = 15 + \sqrt{157}$

111 **(c)**

Given, $\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}} = \vec{\mathbf{0}}$

$\therefore |\,\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + |\vec{\mathbf{c}}|^2 + 2[\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{a}}] = 0$

$\Rightarrow 25 + 16 + 9 + 2[\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{a}}] = 0$

$\Rightarrow 2[\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{a}}] = -50$

$\Rightarrow [\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{a}}] = -25$

113 **(d)**

It is given that $\vec{a} + \vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + \vec{c}$ is collinear with $\vec{a}$

$\therefore \vec{a} + \vec{b} = \lambda\,\vec{c}$ and $\vec{b} + \vec{c} = \mu\,\vec{a}$ for some scalars λ and μ

$\Rightarrow \vec{b} + \vec{c} = \mu\,(\lambda\,\vec{c} - \vec{b})$ [On eliminating $\vec{a}$]

$\Rightarrow (\mu + 1)\vec{b} + (1 - \mu\,\lambda)\vec{c} = \vec{0}$

$\Rightarrow \mu + 1 = 0$ and $\mu\,\lambda = 1$ [$\because \vec{b}$ and $\vec{c}$ are non-collinear]

$\Rightarrow \mu = -1$ and $\lambda = -1$

$\therefore \vec{a} + \vec{b} + \vec{c} = 0$ [Putting $\lambda = -1$ in $\vec{a} + \vec{b} = \lambda\,\vec{c}$]

114 **(b)**

Let $\vec{\mathbf{r}} = l(\vec{\mathbf{b}} \times \vec{\mathbf{c}}) + m(\vec{\mathbf{c}} \times \vec{\mathbf{a}}) + n(\vec{\mathbf{a}} \times \vec{\mathbf{b}})$

$\vec{\mathbf{r}} \cdot \vec{\mathbf{a}} = l[\,\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}\,]$

$\Rightarrow l = 1$

Similarly, $m = 2, n = 3$

115 **(b)**

Given, $|\vec{\mathbf{x}}| = |\vec{\mathbf{y}}| = |\vec{\mathbf{z}}| = 2$

and $\vec{\mathbf{x}} = -\vec{\mathbf{y}} - \vec{\mathbf{z}}$

$\Rightarrow \; |\vec{\mathbf{x}}|^2 = |\vec{\mathbf{y}}|^2 + |\vec{\mathbf{z}}|^2 + 2|\vec{\mathbf{y}}||\vec{\mathbf{z}}|\cos\theta$

$\Rightarrow 4 = 4 + 4 + 2 \times 2 \times 2\cos\theta$

$\Rightarrow \cos\theta = -\frac{1}{2}$

$\Rightarrow \theta = 120°$

Now, $\operatorname{cosec}^2\theta + \cot^2\theta = \operatorname{cosec}^2 120° + \cot^2 120°$

$= \left(\frac{2}{\sqrt{3}}\right)^2 + \left(-\frac{1}{\sqrt{3}}\right)^2 = \frac{5}{3}$

116 **(b)**

Given, $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = -|\vec{\mathbf{a}}||\vec{\mathbf{b}}|$

$\Rightarrow |\vec{\mathbf{a}}||\vec{\mathbf{b}}|\cos\theta = -|\vec{\mathbf{a}}||\vec{\mathbf{b}}|$

$\Rightarrow \cos\theta = -1$

$\Rightarrow \theta = 180°$

117 **(b)**

Let $\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

Given, $\vec{\mathbf{r}} \times \vec{\mathbf{b}} = \vec{\mathbf{c}} \times \vec{\mathbf{b}}$

$\Rightarrow (x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}) \times (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$

$= (4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 7\hat{\mathbf{k}}) \times (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$

$\Rightarrow (y - z)\hat{\mathbf{i}} - (x - z)\hat{\mathbf{j}} + (x - y)\hat{\mathbf{k}}$

$= -10\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$

$\Rightarrow y - z = -10, -(x - z) = 3, x - y = 7$

$\Rightarrow y - z = -10, -x + z = 3, x - y = 7$ ....(i)

and $\vec{\mathbf{r}} \cdot \vec{\mathbf{a}} = 0$

$\Rightarrow (x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}) \cdot (2\hat{\mathbf{i}} + \hat{\mathbf{k}})$

$\Rightarrow 2x + z = 0$ ......(ii)

From Eqs. (i) and (ii), we get

$x = -1, y = -8, z = 2$

$\therefore \; \vec{\mathbf{r}} \cdot \vec{\mathbf{b}} = (-\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$

$= -1 - 8 + 2$

$= -7$

118 **(d)**

Since, given vectors are coplanar so it can be written as

$$\vec{\mathbf{a}} + \lambda\,\vec{\mathbf{b}} + 3\vec{\mathbf{c}} = x(-2\vec{\mathbf{a}} + 3\vec{\mathbf{b}} - 4\vec{\mathbf{c}}) + y(\vec{\mathbf{a}} - 3\vec{\mathbf{b}} + 5\vec{\mathbf{c}})$$

On comparing the coefficient of $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ on both sides, we get

$-2x + y = 1$; $3x - 3y = \lambda$ and $-4x + 5y = 3$

On solving, we get

$x = -\frac{1}{3}, y = \frac{1}{3}, \lambda = -2$

119 **(d)**

Since, $\vec{\mathbf{A}} + \vec{\mathbf{B}}$ is collinear to $\vec{\mathbf{C}}$ and $\vec{\mathbf{B}} + \vec{\mathbf{C}}$ is collinear to $\vec{\mathbf{A}}$

$\therefore \; \vec{\mathbf{A}} + \vec{\mathbf{B}} = \lambda\,\vec{\mathbf{C}}$ and $\vec{\mathbf{B}} + \vec{\mathbf{C}} = \mu\,\vec{\mathbf{A}}$

Where λ and μ are scalars.

$\Rightarrow \; \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = (\lambda + 1)\vec{\mathbf{C}}$

and $\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = (\mu + 1)\vec{\mathbf{A}}$

$\Rightarrow (\lambda + 1)\vec{\mathbf{C}} = (\mu + 1)\vec{\mathbf{A}}$

If $\lambda \neq -1$, then

$\vec{\mathbf{C}} = \frac{\mu + 1}{\lambda + 1}\vec{\mathbf{A}}$

$\Rightarrow \vec{\mathbf{C}}$ and $\vec{\mathbf{A}}$ are collinear.

This is a contradiction to the given condition.

$\therefore \; \lambda = -1$

$\therefore \; \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = \vec{\mathbf{0}}$

120 **(d)**

$|\overrightarrow{\mathbf{AB}}| = \sqrt{(7 - 1)^2 + (-4 + 6)^2 + (7 - 10)^2} = 7$

$|\overrightarrow{\mathbf{BC}}| = \sqrt{(1 + 1)^2 + (-6 + 3)^2 + (10 - 4)^2} = 7$

$|\overrightarrow{\mathbf{CD}}| = \sqrt{(-1 - 5)^2 + (-3 + 1)^2 + (4 - 5)^2}$

$= \sqrt{41}$

and $|\overrightarrow{\mathbf{DA}}| = \sqrt{(5 - 7)^2 + (-1 + 4)^2 + (5 - 7)^2} = \sqrt{17}$

121 **(c)**

We have,

$|\vec{a} + \vec{b}|^2 + |\vec{a} - \vec{b}|^2 = 2\left\{|\vec{a}|^2 + |\vec{b}|^2\right\}$

$\Rightarrow 300 + |\vec{a} - \vec{b}|^2 = 2(49 + 121)$

$\Rightarrow |\vec{a} - \vec{b}| = 2\sqrt{10}$

123 **(a)**

We know, if θ is the angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$, then

$\cos\theta = \frac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{a}}||\vec{\mathbf{b}}|}$

$= \frac{(2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}) \cdot (6\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})}{\sqrt{2^2 + 2^2 + (-1)^2}\sqrt{6^3 + (-3)^2 + 2^2}}$

$= \frac{12 - 6 - 2}{\sqrt{4 + 4 + 1}\sqrt{36 + 9 + 4}}$

$= \frac{4}{\sqrt{9}\sqrt{49}} = \frac{4}{21}$

124 **(d)**

If $\vec{\mathbf{a}}+2\vec{\mathbf{b}}$ is collinear with $\vec{\mathbf{c}}$, then

$\vec{\mathbf{a}}+2\vec{\mathbf{b}}=t\vec{\mathbf{c}}$ ....(i)

Also, if $\vec{\mathbf{b}}+3\vec{\mathbf{c}}$ is collinear with $\vec{\mathbf{a}}$ then

$\vec{\mathbf{b}}+3\vec{\mathbf{c}}=\lambda\vec{\mathbf{a}}$

$\Rightarrow\ \vec{\mathbf{b}}=\lambda\vec{\mathbf{a}}-3\vec{\mathbf{c}}$ ...(ii)

On putting the value of $\vec{\mathbf{b}}$ in Eq. (i), we get

$\vec{\mathbf{a}}+2(\lambda\vec{\mathbf{a}}-3\vec{\mathbf{c}})=t\vec{\mathbf{c}}$

$\Rightarrow(\vec{\mathbf{a}}-6\vec{\mathbf{c}})=t\vec{\mathbf{c}}-2\lambda\,\vec{\mathbf{a}}$

On comparing, we get $1=-2\lambda$ and $-6=t$

$\Rightarrow\ \lambda=-\dfrac{1}{2}$ and $t=-6$

From Eq. (i)

$\vec{\mathbf{a}}+2\vec{\mathbf{b}}=-6\vec{\mathbf{c}}$

$\Rightarrow\ \vec{\mathbf{a}}+2\vec{\mathbf{b}}+6\vec{\mathbf{c}}=\vec{\mathbf{0}}$

125 **(a)**

We have,

$\overrightarrow{AB}=-\hat{\imath}-\hat{\jmath}-4\,\hat{k},\overrightarrow{BC}=-3\hat{\imath}+3\hat{\jmath}$ and,

$\overrightarrow{CA}=4\hat{\imath}-2\hat{\jmath}+4\hat{k}$

$\therefore\ |\overrightarrow{AB}|=|\overrightarrow{BC}|=3\sqrt{2}$ and $|\overrightarrow{CA}|=6$

Clearly, $|\overrightarrow{AB}|^2+|\overrightarrow{BC}|^2=|\overrightarrow{AC}|^2$

Hence, the triangle is right angled isosceles triangle

127 **(c)**

Since, three vectors $(\vec{\mathbf{a}}+2\vec{\mathbf{b}}+3\vec{\mathbf{c}}),(\lambda\,\vec{\mathbf{b}}+4\mathbf{c}$ and $(2\lambda-1)\mathbf{c}$ are non-coplanar

$$\therefore\ \begin{vmatrix}1 & 2 & 3\\ 0 & \lambda & 4\\ 0 & 0 & 2\lambda-1\end{vmatrix}\neq 0$$

$\Rightarrow(2\lambda-1)(\lambda)\neq 0$

$\Rightarrow\ \lambda\neq 0,\dfrac{1}{2}$

Hence, these three vectors are non-coplanar for all except two values of λ.

128 **(a)**

Given $\overrightarrow{\mathbf{PR}}=5\overrightarrow{\mathbf{PQ}}$

It means R divides PQ extrenally in the ratio 5:4

$\therefore$ Position vector of $R\,\dfrac{5\vec{\mathbf{b}}-4\vec{\mathbf{a}}}{5-4}$

$=5\vec{\mathbf{b}}-4\vec{\mathbf{a}}$

130 **(a)**

Let $\overrightarrow{\mathbf{OA}}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ and $\overrightarrow{\mathbf{OB}}=2\hat{\mathbf{i}}-\hat{\mathbf{j}}+4\hat{\mathbf{k}}$

Let point $C(x_1,y_1,z_1)$ divide AB in the ratio 1:2

$\therefore\ x_1=\dfrac{2+2}{1+2}=\dfrac{4}{3},\qquad y_1=\dfrac{-1+4}{1+2}=\dfrac{3}{3}=1$

and $z_1=\dfrac{4+6}{1+2}=\dfrac{10}{3}$

Again let point $D(x_2,y_2,z_2)$ divides AB in the ratio 2:1, then

$x_2=\dfrac{4+1}{2+1}=\dfrac{5}{3},\ \ y_2=\dfrac{-2+2}{2+1}=0$

and $z_2=\dfrac{8+3}{2+1}=\dfrac{11}{3}$

So, position vector of the points of trisection of AB are position vector of

$C=-\dfrac{4}{3}\hat{\mathbf{i}}+\hat{\mathbf{j}}+\dfrac{10}{3}\hat{\mathbf{k}}$

and position vector of

$D=\dfrac{5}{3}\hat{\mathbf{i}}+\dfrac{11}{3}\hat{\mathbf{k}}$

131 **(a)**

Let $\vec{a},\vec{b},\vec{c}$ be the position vectors A,B and C respectively. Then, the position vector of G is $\frac{\vec{a}+\vec{b}+\vec{c}}{3}$ and the position vectors of D,E and F are $\frac{\vec{b}+\vec{c}}{2},\frac{\vec{c}+\vec{a}}{2}$ and $\frac{\vec{a}+\vec{b}}{2}$ respectively

$\therefore\ \vec{G}D+\vec{G}E+\vec{G}F$

$$=\left(\frac{\vec{b}+\vec{c}}{2}-\frac{\vec{a}+\vec{b}+\vec{c}}{3}\right)+\left(\frac{\vec{c}+\vec{a}}{2}-\frac{\vec{a}+\vec{b}+\vec{c}}{3}\right)+\left(\frac{\vec{a}+\vec{b}}{2}-\frac{\vec{a}+\vec{b}+\vec{c}}{3}\right)$$

$=(\vec{a}+\vec{b}+\vec{c})-(\vec{a}+\vec{b}+\vec{c})=\vec{0}$

132 **(a)**

Let $\overrightarrow{\mathbf{OA}}=\vec{a},\overrightarrow{\mathbf{OB}}=\vec{\mathbf{b}}$ and $\overrightarrow{\mathbf{OC}}=\vec{\mathbf{c}}$, then

$\overrightarrow{\mathbf{OD}}=\dfrac{\vec{\mathbf{a}}+\vec{\mathbf{b}}}{2},\overrightarrow{\mathbf{OE}}=\dfrac{\vec{\mathbf{a}}+\vec{\mathbf{c}}}{2},\overrightarrow{\mathbf{OF}}=\dfrac{\vec{\mathbf{b}}+\vec{\mathbf{c}}}{2}$

Now, $\overrightarrow{\mathbf{AF}}=\dfrac{1}{2}(\vec{\mathbf{b}}+\vec{\mathbf{c}})-\vec{\mathbf{a}},\overrightarrow{\mathbf{BE}}=\dfrac{1}{2}(\vec{\mathbf{a}}+\vec{\mathbf{c}})-\vec{\mathbf{b}}$

and $\overrightarrow{\mathbf{CD}}=\dfrac{1}{2}(\vec{\mathbf{a}}+\vec{\mathbf{b}})-\vec{\mathbf{c}}$

$\therefore\ \overrightarrow{\mathbf{AF}}+\overrightarrow{\mathbf{BE}}=\dfrac{1}{2}(\vec{\mathbf{b}}+\vec{\mathbf{c}})-\vec{\mathbf{a}}+\dfrac{1}{2}(\vec{\mathbf{a}}+\vec{\mathbf{c}})-\vec{\mathbf{b}}$

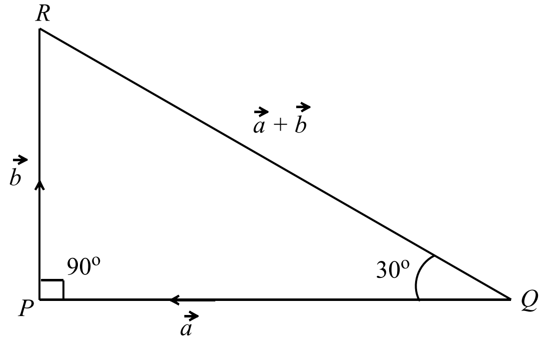
$=\vec{\mathbf{c}}-\dfrac{1}{2}(\vec{\mathbf{a}}+\vec{\mathbf{b}})=\overrightarrow{\mathbf{DC}}$

133 **(c)**

We have,

$\vec{a}\cdot\vec{b}=0\Rightarrow\vec{a}\perp\vec{b}$

So, vectors $\vec{a},\vec{b}$ and $\vec{a}+\vec{b}$ form a right angled triangle



In ΔPQR, we have

$\tan 30° = \frac{|\vec{b}|}{|\vec{a}|} \Rightarrow |\vec{a}| = 3|\vec{b}|$

134 **(d)**

We have, $\vec{\mathbf{a}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

$\therefore\ \vec{\mathbf{b}} = \hat{\mathbf{i}} \times (\vec{\mathbf{a}} \times \hat{\mathbf{i}}) + \hat{\mathbf{j}} \times (\vec{\mathbf{a}} \times \hat{\mathbf{j}}) + \hat{\mathbf{k}} \times (\vec{\mathbf{a}} \times \hat{\mathbf{k}})$ …… (i)

Now, $\hat{\mathbf{i}} \times (\vec{\mathbf{a}} \times \hat{\mathbf{i}}) = (\hat{\mathbf{i}} \cdot \hat{\mathbf{i}})\vec{\mathbf{a}} - (\hat{\mathbf{i}} \cdot \vec{\mathbf{a}})\hat{\mathbf{i}}$

$= 1(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) - (1)\hat{\mathbf{i}}$

$= 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

Similarly, $\hat{\mathbf{j}} \times (\vec{\mathbf{a}} \times \hat{\mathbf{j}}) = \hat{\mathbf{i}} + 3\hat{\mathbf{k}}$

and $\hat{\mathbf{k}} \times (\vec{\mathbf{a}} \times \hat{\mathbf{k}}) = \hat{\mathbf{i}} + 2\hat{\mathbf{j}}$

$\therefore$ From Eq. (i),

$\vec{\mathbf{b}} = 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}} + \hat{\mathbf{i}} + 3\hat{\mathbf{k}} + \hat{\mathbf{i}} + 2\hat{\mathbf{j}}$

$= 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$

$\Longrightarrow |\vec{\mathbf{b}}| = \sqrt{4 + 16 + 36} = 2\sqrt{14}$

135 **(a)**

The centroid of triangle

$= \frac{(a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}) + (b\hat{\mathbf{i}} + c\hat{\mathbf{j}} + a\hat{\mathbf{k}}) + (c\hat{\mathbf{i}} + a\hat{\mathbf{j}} + b\hat{\mathbf{k}}}{3}$

$= \frac{a + b + c}{3}(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$

136 **(d)**

Given, $|\vec{\mathbf{a}} + \vec{\mathbf{b}}| = 1, |\vec{\mathbf{a}}| = |\vec{\mathbf{b}}| = 1$

$\Longrightarrow |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + 2|\vec{\mathbf{a}}||\vec{\mathbf{b}}| = 1$

$\Longrightarrow 2|\vec{\mathbf{a}}||\vec{\mathbf{b}}| = -1$ … (i)

Now, $|\vec{\mathbf{a}} - \vec{\mathbf{b}}|^2 = |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 - 2|\vec{\mathbf{a}}||\vec{\mathbf{b}}|$

$= 1^2 + 1^2 - (-1) = 3$ [from Eq. (i)

$\Longrightarrow |\vec{\mathbf{a}} + \vec{\mathbf{b}}| = \sqrt{3}$

137 **(a)**

Since, $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are collinear vectors.

$\therefore\ \vec{\mathbf{a}} = \lambda\vec{\mathbf{b}}$

$\Longrightarrow\ \hat{\mathbf{i}} - \hat{\mathbf{j}} = \lambda(-2\hat{\mathbf{i}} + m\hat{\mathbf{j}})$

$\Longrightarrow 1 = -2\lambda, -1 = \lambda m$

$\Longrightarrow\ \lambda = -\frac{1}{2}, m = -\frac{1}{\lambda}$

$\Longrightarrow m = 2$

138 **(c)**

Since, C is the mid point of $A(2, -1)$and $B(-4, 3)$.

$\therefore$ Coordinates of C is $\left(\frac{2-4}{2}, \frac{-1+3}{2}\right) = (-1, 1)$

$\therefore\ \overrightarrow{\mathbf{OC}} = -\hat{\mathbf{i}} + \hat{\mathbf{j}}$

139 **(c)**

According to the given conditions, we have

$\vec{a}.\vec{b} > 0$ and $\vec{b}.\hat{\jmath} < 0$

$\Rightarrow 2x^2 - 3x + 1 > 0$ and $x < 0$

$\Rightarrow (x < 1/2$ or $x > 1)$ and $x < 0 \Rightarrow x < 0$

140 **(d)**

$\frac{|(\vec{\mathbf{a}} - \vec{\mathbf{c}}) \times (\vec{\mathbf{b}} - \vec{\mathbf{a}})|}{(\vec{\mathbf{c}} - \vec{\mathbf{a}}) \cdot (\vec{\mathbf{b}} - \vec{\mathbf{a}})} = \frac{|\overrightarrow{\mathbf{AC}} \times \overrightarrow{\mathbf{BA}}|}{\overrightarrow{\mathbf{AC}} \cdot \overrightarrow{\mathbf{BA}}}$

$= \frac{||\overrightarrow{\mathbf{AC}}||\overrightarrow{\mathbf{BA}}|\text{in } A\hat{\mathbf{n}}|}{|\overrightarrow{\mathbf{AC}}||\overrightarrow{\mathbf{BA}}| \cos A} = \tan A$

141 **(d)**

Let, $\vec{\mathbf{a}} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$

$\because\ \vec{\mathbf{a}} \cdot \hat{\mathbf{i}} = 1 \Longrightarrow a_1 = 1$

Since, $\vec{\mathbf{a}} \cdot (2\hat{\mathbf{i}} + \hat{\mathbf{j}}) = 1$

$\Longrightarrow 2a_1 + a_2 = 1$

$\Longrightarrow\ a_2 = 1 - 2$

$\Longrightarrow\ a_2 = -1$

and $\vec{\mathbf{a}} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}) = 1$

$\Longrightarrow a_1 + a_2 + 3a_3 = 1$

$\Longrightarrow 1 - 1 + 3a_3 = 1$

$\Longrightarrow\ a_3 = \frac{1}{3}$

$\therefore \vec{\mathbf{a}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \frac{1}{3}\hat{\mathbf{k}} = \frac{1}{3}(3\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}})$

142 **(c)**

Given, $\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \vec{\mathbf{b}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

and $\vec{\mathbf{c}} = x\,\hat{\mathbf{i}} + (x - 2)\hat{\mathbf{j}} - \hat{\mathbf{k}}$

Since, $\vec{\mathbf{c}}$ lies in the plane of vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ therefore $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are coplanar.

$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & (x-2) & -1 \end{vmatrix} = 0$

$\Longrightarrow 1(1 - 2x + 4) - 1(-1 - 2x) + 1(x - 2 + x) = 0$

$\Longrightarrow 5 - 2x + 1 + 2x + 2x - 2 = 0$

$\Longrightarrow x = -2$

143 **(d)**

Let $\vec{\mathbf{P}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}},\ \vec{\mathbf{Q}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}},\ \vec{\mathbf{R}} = 5\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

and $\vec{\mathbf{S}} = -\hat{\mathbf{j}} + \hat{\mathbf{k}}$

$\therefore\ \overrightarrow{\mathbf{PQ}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$

$\Longrightarrow |\overrightarrow{\mathbf{PQ}}| = \sqrt{6}$

$\overrightarrow{\mathbf{QR}} = -2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

$\Longrightarrow |\overrightarrow{\mathbf{QR}}| = \sqrt{12}$

and $\overrightarrow{\mathbf{RS}} = -6\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

$\Longrightarrow |\overrightarrow{\mathbf{RS}}| = \sqrt{45}$

and $\overrightarrow{\mathbf{SP}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ $\quad \Longrightarrow |\overrightarrow{\mathbf{SP}}| = 3$

Which are not satisfied the conditions of any of the following. Trapezium, rectangle and parallelogram.

144 **(c)**

Clearly,

Required vector $= |\vec{b}|\hat{a} = \frac{|\vec{b}|}{|\vec{a}|}\vec{a} = \frac{7}{3}(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$

145 **(a)**

If I is incentre of $\Delta\, ABC$. Then ,

I is $\dfrac{a\vec{\mathbf{a}} + b\vec{\mathbf{b}} + c\vec{\mathbf{c}}}{a+b+c}$

147 **(d)**

For a parallel $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = 0$

$$\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 1 & 3 \\ 4 & -\lambda & 6 \end{vmatrix} = 0$$

$\Rightarrow \hat{\mathbf{i}}(6+3\lambda) - \hat{\mathbf{j}}(0) + \hat{\mathbf{k}}(-2\lambda - 4) = 0$

$= 0 \cdot \hat{\mathbf{i}} + 0 \cdot \hat{\mathbf{j}} + 0 \cdot \hat{\mathbf{k}}$

$\therefore 6 + 3\lambda = 0 \;\Rightarrow\; \lambda = -2$

148 **(b)**

Total force,

$$\vec{\mathbf{F}} = \frac{5(6\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})}{7} + \frac{3(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}})}{7} + \frac{1(2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 6\hat{\mathbf{k}})}{7}$$

$= \frac{1}{7}(41\hat{\mathbf{i}} + \hat{\mathbf{j}} + 27\hat{\mathbf{k}})$

and $\overrightarrow{\mathbf{AB}} = 5\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}} - 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

$= 3\hat{\mathbf{i}} + 4\hat{\mathbf{k}}$

$\therefore$ Work done $= \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{AB}}$

$= \frac{1}{7}[41\hat{\mathbf{i}} + \hat{\mathbf{j}} + 27\hat{\mathbf{k}}] \cdot [3\hat{\mathbf{i}} + 4\hat{\mathbf{k}}]$

$= \frac{1}{7}[123 + 108] = 33$ units

150 **(d)**

Since vectors $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$ and $\vec{b} = 4\hat{\imath} - \lambda\hat{\jmath} + 6\hat{k}$ are parallel

$\therefore \frac{2}{4} = \frac{1}{-\lambda} = \frac{3}{6} \Rightarrow \lambda = -2$

151 **(b)**

If $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are coplanar vectors, then

$2\vec{\mathbf{a}} - \vec{\mathbf{b}}, 2\vec{\mathbf{b}} - \vec{\mathbf{c}}$ and $2\vec{\mathbf{c}} - \vec{\mathbf{a}}$ are also coplanar.

$\therefore [2\vec{\mathbf{a}} - \vec{\mathbf{b}} 2\,\vec{\mathbf{b}} - \vec{\mathbf{c}}\; 2\,\vec{\mathbf{c}} - \vec{\mathbf{a}}] = 0$

152 **(b)**

Here, $|\vec{\mathbf{a}}| = \sqrt{1 + 1 + (4)^2} = 3\sqrt{2}$

and $|\vec{\mathbf{b}}| = \sqrt{1 + (-1)^2 + (4)^2} = 3\sqrt{2}$

$\therefore |\vec{\mathbf{a}}| = |\vec{\mathbf{b}}|$

Now, $(\vec{\mathbf{a}} + \vec{\mathbf{b}}) \cdot (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}) = |\vec{\mathbf{a}}|^2 - |\vec{\mathbf{b}}|^2 = 0$

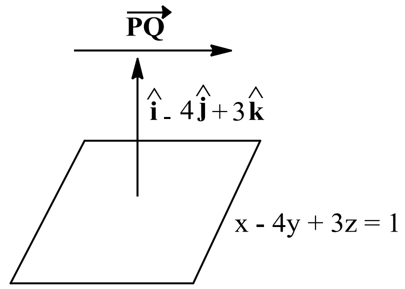
Hence, angle between them is 90°

153 **(a)**

Given,

$\overrightarrow{\mathbf{OQ}} = (1 - 3\mu)\hat{\mathbf{i}} + (\mu - 1)\hat{\mathbf{j}} + (5\mu + 2)\hat{\mathbf{k}}$

$\overrightarrow{\mathbf{OP}} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ (where O is origin)



Now,

$\overrightarrow{\mathbf{PQ}} = (1 - 3\mu - 3)\hat{\mathbf{i}} + (\mu - 1 - 2)\hat{\mathbf{j}} + (5\mu + 2 - 6)\hat{\mathbf{k}}$

$= (-2 - 3\mu)\hat{\mathbf{i}} + (\mu - 3)\hat{\mathbf{j}} + (5\mu - 4)\hat{\mathbf{k}}$

$\because \overrightarrow{\mathbf{PQ}}$ is parallel to the plane $x - 4y + 3z = 1$

$\therefore\; -2 - 3\mu - 4\mu + 12 + 15\mu - 12 = 0$

$\Longrightarrow 8\mu = 2$

$\Longrightarrow\; \mu = \frac{1}{4}$

154 **(b)**

Let $\vec{\mathbf{A}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}},\; \vec{\mathbf{B}} = -2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$

and $\vec{\mathbf{C}} = 4\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$

$\therefore\; \overrightarrow{\mathbf{AB}} = -3\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

and $\overrightarrow{\mathbf{AC}} = 3\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

$\therefore$ Area of $\Delta\, ABC = \frac{1}{2}||\overrightarrow{\mathbf{AB}} \times \overrightarrow{\mathbf{AC}}||$

$$= \frac{1}{2}\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -3 & 5 & -4 \\ 3 & -5 & 4 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -3 & 5 & -4 \\ 0 & 0 & 0 \end{vmatrix}$$

[operating $R_2 \to R_2 + R_3$]

$= \frac{1}{2}[0] = 0$

155 **(b)**

We have,

$\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ and $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$

$\Rightarrow (\vec{r} - \vec{b}) \times \vec{a} = 0$ and $(\vec{r} - \vec{a}) \times \vec{b} = 0$

$\Rightarrow \vec{r} - \vec{b} \parallel \vec{a}$ and $\vec{r} - \vec{a} \parallel \vec{b}$

$\Rightarrow \vec{r} - \vec{b} = \lambda\vec{a}$ and $\vec{r} - \vec{a} = \mu\vec{b}$ for some $\lambda, \mu \in R$

$\Rightarrow \vec{r} = \vec{b} + \lambda\vec{a}$ and $\vec{r} = \vec{a} + \mu\vec{b}$ for some $\lambda, \mu \in R$

$\Rightarrow \vec{b} + \lambda\vec{a} = \vec{a} + \mu\vec{b}$

$\Rightarrow \lambda = \mu = 1 \quad [\because \vec{a}, \vec{b}$ are non − collinear$]$

$\therefore \vec{r} = \vec{a} + \vec{b}$

156 **(c)**

$|\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}|^2$

$= |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + |\vec{\mathbf{c}}|^2 + 2\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + 2\vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + 2\vec{\mathbf{c}} \cdot \vec{\mathbf{a}}$

$\Longrightarrow 0 = 1 + 1 + 1 + 2(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{a}})$

$[\because |\vec{\mathbf{a}}| = |\vec{\mathbf{b}}| = |\vec{\mathbf{c}}| = 1$, given$]$

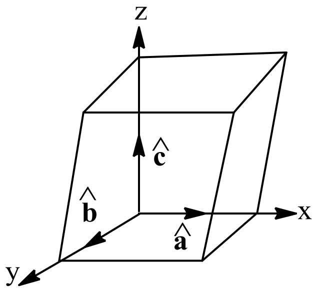
$\therefore\; \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{a}} = -\frac{3}{2}$

157 **(a)**

The volume of the parallelepiped with

coterminous edges as $\hat{\mathbf{a}}, \hat{\mathbf{b}}, \hat{\mathbf{c}}$ is given by

$[\hat{\mathbf{a}}, \hat{\mathbf{b}}, \hat{\mathbf{c}}] = \hat{\mathbf{a}} \cdot (\hat{\mathbf{b}} \times \hat{\mathbf{c}})$



Now, $[\hat{\mathbf{a}}, \hat{\mathbf{b}}, \hat{\mathbf{c}}]^2 = \begin{vmatrix} \hat{\mathbf{a}}\cdot\hat{\mathbf{a}} & \hat{\mathbf{a}}\cdot\hat{\mathbf{b}} & \hat{\mathbf{a}}\cdot\hat{\mathbf{c}} \\ \hat{\mathbf{b}}\cdot\hat{\mathbf{a}} & \hat{\mathbf{b}}\cdot\hat{\mathbf{b}} & \hat{\mathbf{b}}\cdot\hat{\mathbf{c}} \\ \hat{\mathbf{c}}\cdot\hat{\mathbf{a}} & \hat{\mathbf{c}}\cdot\hat{\mathbf{b}} & \hat{\mathbf{c}}\cdot\hat{\mathbf{c}} \end{vmatrix}$

$= \begin{vmatrix} 1 & 1/2 & 1/2 \\ 1/2 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{vmatrix} = \frac{1}{2}$

$[\because |\vec{\mathbf{a}}| = |\vec{\mathbf{b}}| = |\vec{\mathbf{c}}| = 1]$

$\Longrightarrow [\hat{\mathbf{a}}, \hat{\mathbf{b}}, \hat{\mathbf{c}}]^2 = \frac{1}{2}$

Thus, the required volume of the parallelopiped

$= \frac{1}{\sqrt{2}}$ cu unit

158 **(d)**

We have, $\vec{\mathbf{a}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

and $\vec{\mathbf{b}} = \hat{\mathbf{i}} \times (\vec{\mathbf{a}} \times \hat{\mathbf{i}}) + \hat{\mathbf{j}} \times (\vec{\mathbf{a}} \times \hat{\mathbf{j}}) + \hat{\mathbf{k}} \times (\vec{\mathbf{a}} \times \hat{\mathbf{k}})$

$= 3\vec{\mathbf{a}} - \vec{\mathbf{a}} = 2\vec{\mathbf{a}}$

$= 2(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

$\Rightarrow |\vec{\mathbf{b}}| = \sqrt{4 + 16 + 36} = \sqrt{56} = 2\sqrt{14}$

159 **(b)**

Let, $\vec{\mathbf{a}} = 2p\hat{\mathbf{i}} + \hat{\mathbf{j}}$, $\vec{\mathbf{b}} = (p+1)\hat{\mathbf{i}} + \hat{\mathbf{j}}$

Given, $|\vec{\mathbf{a}}| = |\vec{\mathbf{b}}| \Longrightarrow 4p^2 + 1 = (p+1)^2 + 1$

$\Longrightarrow 3p^2 - 2p - 1 = 0 \Longrightarrow p = 1, -\frac{1}{3}$

160 **(c)**

Since $\vec{r_1}, \vec{r_2}, \vec{r_3}$ are coplanar

$\therefore [\vec{r_1}\,\vec{r_2}\,\vec{r_3}] = 0$

$\Rightarrow \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0 \Rightarrow abc = a + b + c - 2 \quad \dots \text{(i)}$

$\therefore \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$

$= \frac{3 - 2(a+b+c) + ab + bc + ca}{1 - (a+b+c) + ab + bc + ca - abc}$

$= \frac{3 - 2(a+b+c) + ab + bc + ca}{1 - (a+b+c) + ab + bc + ca - a - b - c + 2}$

$= \frac{3 - 2(a+b+c) + ab + bc + ca}{3 - 2(a+b+c) + ab + bc + ca} = 1$

161 **(c)**

Let projection be x, then

$\vec{\mathbf{a}} = \frac{x(\hat{\mathbf{i}} + \hat{\mathbf{j}})}{\sqrt{2}} + \frac{x(-\hat{\mathbf{i}} + \hat{\mathbf{j}})}{\sqrt{2}} + x\,\hat{\mathbf{k}}$

$\therefore \vec{\mathbf{a}} = \frac{2x\hat{\mathbf{j}}}{\sqrt{2}} + x\,\hat{\mathbf{k}}$

$\Rightarrow \vec{\mathbf{a}} = \frac{\sqrt{2}}{\sqrt{3}}\hat{\mathbf{j}} + \frac{\hat{\mathbf{k}}}{\sqrt{3}}$

162 **(a)**

$\overrightarrow{\mathbf{PQ}} = 6\hat{\mathbf{i}} + \hat{\mathbf{j}}$l

$\overrightarrow{\mathbf{QR}} = -\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$

$\overrightarrow{\mathbf{RS}} = -6\hat{\mathbf{i}} - \hat{\mathbf{j}}$

$\overrightarrow{\mathbf{SP}} = \hat{\mathbf{i}} - 3\hat{\mathbf{j}}$

$|\overrightarrow{\mathbf{PQ}}| = \sqrt{37} = |\overrightarrow{\mathbf{RS}}|$

$|\overrightarrow{\mathbf{QR}}| = \sqrt{10} = |\overrightarrow{\mathbf{SP}}|$

$\overrightarrow{\mathbf{PQ}} \cdot \overrightarrow{\mathbf{QR}} = -6 + 3 = -3 \neq 0$

$\overrightarrow{\mathbf{PQ}} =$ is not parallel to $\overrightarrow{\mathbf{RS}}$ and their magnitude are equal.

$\Longrightarrow$ Quadrilateral $PQRS$ must be a parallelogram, which is neither a rhombus nor a rectangle.

163 **(c)**

If $\Delta = 0$, then

$\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{a}.\vec{c} & \vec{b}.\vec{c} & \vec{c}.\vec{c} \end{vmatrix} = 0$

$\Rightarrow \lambda\vec{a} + \mu\vec{b} + \nu\vec{c} = 0$

$\Rightarrow \vec{a}, \vec{b}, \vec{c}$ are $L.D.$, which is a contradiction

Hence, Δ can take any non-zero real values

164 **(b)**

We have,

$(3\vec{a} - 2\vec{b}) = -8\hat{\imath} - 7\hat{\jmath} + 3\hat{k}$ and $\vec{c} = \frac{1}{3}(2\hat{\imath} + 2\hat{\jmath} - \hat{k}$

$\therefore$ Required projection $= (3\vec{a} - 2\vec{b}) \cdot \hat{c}$

$= (= -8\hat{\imath} - 7\hat{\jmath} + 3\hat{k}) \cdot \frac{1}{3}(2\hat{\imath} + 2\hat{\jmath} - \hat{k})$

$= \frac{1}{3}(-16 - 14 - 3) = -11$

165 **(a)**

Angle between the faces OAB and ABC is same as angle between normals of faces OAB and ABC.

Vector along the normals of OAB

$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 5\hat{\mathbf{i}} - \hat{\mathbf{j}} - 3\hat{\mathbf{k}} = \vec{\mathbf{a}}$ (let)

Vector along normals of ABC

$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -1 & 2 \\ -2 & -1 & 1 \end{vmatrix} = \hat{\mathbf{i}} - 5\hat{\mathbf{j}} - 3\hat{\mathbf{k}} = \vec{\mathbf{b}}$ (let)

$\therefore \cos\theta = \frac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{a}}||\vec{\mathbf{b}}|} = \frac{5 + 5 + 9}{\sqrt{35}\sqrt{35}}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{19}{35}\right)$

167 **(d)**

$\vec{\mathbf{a}} \times [\vec{\mathbf{a}} \times (\vec{\mathbf{a}} \times \vec{\mathbf{b}})] = \vec{\mathbf{a}} \times \{(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{a}} - (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{b}}\}$
(Expanding by vector triple product)
$= (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})(\vec{\mathbf{a}} \times \vec{\mathbf{a}}) - (\vec{\mathbf{a}} \cdot \vec{\mathbf{a}})(\vec{\mathbf{a}} \times \vec{\mathbf{b}})$
$= (\vec{\mathbf{a}} \cdot \vec{\mathbf{a}})(\vec{\mathbf{b}} \times \vec{\mathbf{a}})$ $(\because (\vec{\mathbf{a}} \times \vec{\mathbf{a}}) = 0)$

169 **(b)**
Taking A as the origin let the position vectors of B and C be $\vec{b}$ and $\vec{c}$ respectively
Equations of lines BF and AC are
$\vec{r} = \vec{b} + \lambda\left(\frac{\vec{b}+\vec{c}}{4} - \vec{b}\right)$ and $\vec{r} = \vec{0} + \mu\vec{c}$ respectively
For the point of intersection F, we have
$$\vec{b} + \lambda\left(\frac{\vec{c} - 3\vec{b}}{4}\right) = \mu\vec{c}$$
$\Rightarrow 1 - \frac{3\lambda}{4} = 0$ and $\frac{\lambda}{4} = \mu \Rightarrow \lambda = \frac{4}{3}$ and $\mu = \frac{1}{3}$
So, the position vector of $\vec{F}$ is $\vec{r} = \frac{1}{3}\vec{c}$
Now, $\vec{AF} = \frac{1}{3}\vec{c} \Rightarrow \vec{AF} = \frac{1}{3}\vec{AC}$
Hence, $AF{:}\,AC = \frac{1}{3}{:}\,1 = \frac{1}{3}$

170 **(d)**
Given, $|\vec{\mathbf{a}}| = 1, |\vec{\mathbf{b}}| = 2$
$\therefore [(\vec{\mathbf{a}} + 3\vec{\mathbf{b}}) \times (3\vec{\mathbf{a}} + \vec{\mathbf{b}})]^2$
$= [0 + \vec{\mathbf{a}} \times \vec{\mathbf{b}} + 9\vec{\mathbf{b}} \times \vec{\mathbf{a}} + 0]^2$
$= [-8\vec{\mathbf{a}} \times \vec{\mathbf{b}}]^2$
$= 64\left[|\vec{\mathbf{a}}|^2\, |\vec{\mathbf{b}}|^2 \sin^2\theta\right]$
$= 64[1 \times 4 \times \sin^2 120°]$
$= 64 \times 4 \times \frac{3}{4} = 192$

171 **(c)**
$(\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}) \cdot [(\vec{\mathbf{a}} + \vec{\mathbf{b}}) \times (\vec{\mathbf{a}} + \vec{\mathbf{c}})]$
$= (\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}) \cdot [\vec{\mathbf{a}} \times \vec{\mathbf{c}} + \vec{\mathbf{b}} \times \vec{\mathbf{a}} + \vec{\mathbf{b}} \times \vec{\mathbf{c}}]$
$0 + 0 + [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}] + [\vec{\mathbf{b}}\,\vec{\mathbf{a}}\,\vec{\mathbf{c}}] + 0 + 0 + 0 + [\vec{\mathbf{c}}\,\vec{\mathbf{b}}\,\vec{\mathbf{a}}] + 0$
$= -[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$

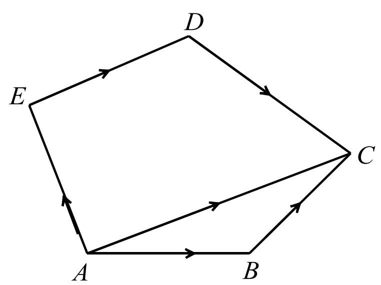
172 **(b)**
Clearly, $\vec{c} = \pm\dfrac{\vec{a} \times (\vec{a} \times \vec{b})}{|\vec{a} \times (\vec{a} \times \vec{b})|}$
Now,
$\vec{a} \times (\vec{a} \times \vec{b}) = (\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b}$
$\Rightarrow \vec{a} \times (\vec{a} \times \vec{b}) = -\hat{\imath} - \hat{\jmath} + \hat{k} - 3(\hat{\imath} - \hat{\jmath} + \hat{k})$
$= -4\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$
$$\therefore \vec{c} = \pm\frac{1}{\sqrt{6}}(2\hat{\imath} - \hat{\jmath} + \hat{k})$$
Since $\vec{d}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{c}$
$$\therefore \vec{d} = \pm\frac{\vec{a} \times \vec{c}}{|\vec{b} \times \vec{c}|}$$
Now, $\vec{a} \times \vec{c} = \pm\dfrac{1}{\sqrt{6}}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & -1 \\ 2 & -1 & 1 \end{vmatrix}$
$= \pm\dfrac{1}{\sqrt{6}}(-3\hat{\jmath} - 3\hat{k})$
$\therefore \vec{d} = \pm\dfrac{1}{\sqrt{2}}(-\hat{\jmath} - \hat{k}) = \pm\dfrac{1}{\sqrt{2}}(\hat{\jmath} + \hat{k})$

173 **(d)**
Since, G is the centroid of a triangle, then
$\overrightarrow{\mathbf{GA}} + \overrightarrow{\mathbf{GB}} + \overrightarrow{\mathbf{GC}} = \vec{\mathbf{0}} \Rightarrow \overrightarrow{\mathbf{GA}} + \overrightarrow{\mathbf{GC}} = -\overrightarrow{\mathbf{GB}}$ .... (i)
Now, $\overrightarrow{\mathbf{GA}} + \overrightarrow{\mathbf{BG}} + \overrightarrow{\mathbf{GC}} = -\overrightarrow{\mathbf{GB}} + \overrightarrow{\mathbf{BG}} = 2\overrightarrow{\mathbf{BG}}$
[from Eq. (i)]

174 **(c)**
Let $\vec{\mathbf{n}}_1$ and $\vec{\mathbf{n}}_2$ be the vectors normal to the plane determined by $\hat{\mathbf{i}}, \hat{\mathbf{i}} - \hat{\mathbf{j}}$ and $\hat{\mathbf{i}} + \hat{\mathbf{j}}, \hat{\mathbf{i}} - \hat{\mathbf{k}}$ respectively
$\therefore\ \vec{\mathbf{n}}_1 = \hat{\mathbf{i}} \times (\hat{\mathbf{i}} - \hat{\mathbf{j}}) = -\hat{\mathbf{k}}$
and $\vec{\mathbf{n}}_2 = (\hat{\mathbf{i}} + \hat{\mathbf{j}}) \times (\hat{\mathbf{i}} - \hat{\mathbf{k}}) = -\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$
Since, $\vec{\mathbf{a}}$ is parallel to the line of intersection of the planes determined by the given planes.
$\therefore\ \vec{\mathbf{a}} || (\vec{\mathbf{n}}_1 \times \vec{\mathbf{n}}_2)$
$\Longrightarrow \vec{\mathbf{a}} = \lambda(\vec{\mathbf{n}}_1 \times \vec{\mathbf{n}}_2) = \lambda(\hat{\mathbf{i}} + \hat{\mathbf{j}})$
Let θ be the angle between $\vec{\mathbf{a}}$ and $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$
$\therefore \cos\theta = \dfrac{\lambda((\hat{\mathbf{i}} + \hat{\mathbf{j}}) \cdot (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}})}{\sqrt{\lambda^2 + \lambda^2}\sqrt{1 + 4 + 4}}$
$= \dfrac{\lambda(1 + 2)}{\sqrt{2}\lambda \times 3} = \dfrac{1}{\sqrt{2}}$
$\Longrightarrow\ \theta = \dfrac{\pi}{4}$

175 **(d)**
$|\vec{\mathbf{a}} \times \vec{\mathbf{b}}|^2 + (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})^2 = |\vec{\mathbf{a}}|^2\ |\vec{\mathbf{b}}|^2$
$\Longrightarrow |\vec{\mathbf{a}} \times \vec{\mathbf{b}}|^2 = 25 \times 36 - (25)^2$
$= 25(36 - 25)$
$= 25 \times 11$
$\Longrightarrow |\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = 5\sqrt{11}$

176 **(c)**
$\vec{AB} + \vec{AE} + \vec{BC} + \vec{DC} + \vec{ED} + \vec{AC}$
$= (\vec{AB} + \vec{BC}) + (\vec{AE} + \vec{ED}) + \vec{DC} + \vec{AC}$
$= \vec{AC} + (\vec{AD} + \vec{DC}) + \vec{AC}$
$= \vec{AC} + \vec{AC} + \vec{AC} = 3\vec{AC}$



177 **(a)**
We have,

$(\vec{a}-\vec{d})\times(\vec{b}-\vec{c})$

$=\vec{a}\times\vec{b}-\vec{a}\times\vec{c}-\vec{d}\times\vec{b}+\vec{d}\times\vec{c}$

$\Rightarrow(\vec{a}-\vec{d})\times(\vec{b}-\vec{c})$

$=\vec{c}\times\vec{d}-\vec{b}\times\vec{d}-\vec{d}\times\vec{b}+\vec{d}\times\vec{c}$

$\Rightarrow(\vec{a}-\vec{d})\times(\vec{b}-\vec{c})$

$=0\ [\because \vec{a}\times\vec{b}=\vec{c}\times\vec{d}, \vec{a}\times\vec{c}$

$=\vec{b}\times\vec{d}\,]$

$\Rightarrow(\vec{a}-\vec{d})||(\vec{b}-\vec{c})$

$\Rightarrow\vec{a}-\vec{d}=\lambda(\vec{b}-\vec{c})$

Similarly, we have

$(\vec{a}+\vec{d})\times(\vec{b}+\vec{c})=\vec{0}\Rightarrow\vec{a}+\vec{d}||\vec{b}+\vec{c}\Rightarrow\vec{a}+\vec{d}$

$=\lambda(\vec{b}+\vec{c})$

178 **(b)**

We have,

$\vec{a}\times\{\vec{a}\times(\vec{a}\times\vec{b})\}=\vec{a}\times\{(\vec{a}\cdot\vec{b})\vec{a}-(\vec{a}\cdot\vec{a})\vec{b}\}$

$\Rightarrow\vec{a}\times\{\vec{a}\times(\vec{a}\times\vec{b})\}=-(\vec{a}\cdot\vec{a})(\vec{a}\times\vec{b})$

$=(\vec{a}\cdot\vec{a})(\vec{b}\times\vec{a})$

179 **(c)**

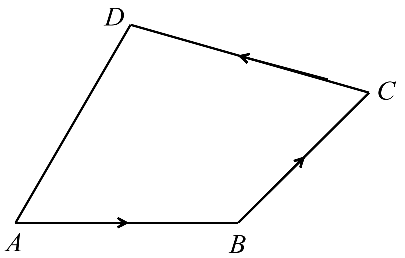
We have,

$\vec{AB}+\vec{DC}=\vec{AB}+\vec{BC}-\vec{BC}+\vec{DC}$

$\Rightarrow\vec{AB}+\vec{DC}=(\vec{AB}+\vec{BC})-\vec{BC}+\vec{CD}$

$\Rightarrow\vec{AB}+\vec{DC}=(\vec{AB}+\vec{BC})-(\vec{BC}+\vec{CD})$

$\Rightarrow\vec{AB}+\vec{DC}=\vec{AC}-\vec{BD}=\vec{AC}+\vec{DB}$



182 **(c)**

Volume of parallelopiped,

$f(a)=\begin{vmatrix}1 & a & 1\\ 0 & 1 & a\\ a & 0 & 1\end{vmatrix}=1+a^3-a$

Now, $f'(a)=3a^2-1$

$\Longrightarrow f''(a)=6a$

Put $f'(a)=0$

$\Longrightarrow a\neq\pm\frac{1}{\sqrt{3}}$

Which shows $f(a)$ is maximum at

$a=\frac{1}{\sqrt{3}}$ and maximum at

$a=-\frac{1}{\sqrt{3}}$

183 **(c)**

Let $\vec{\mathbf{a}}=4\hat{\mathbf{i}}+6\hat{\mathbf{j}}-\hat{\mathbf{k}}$

and $\vec{\mathbf{b}}=3\hat{\mathbf{i}}+8\hat{\mathbf{j}}+\hat{\mathbf{k}}$

$\therefore\ \vec{\mathbf{c}}=\vec{\mathbf{a}}\times\vec{\mathbf{b}}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 4 & 6 & -1\\ 3 & 8 & 1\end{vmatrix}=14\hat{\mathbf{i}}-7\hat{\mathbf{j}}+14\hat{\mathbf{k}}$

$\Longrightarrow\hat{\mathbf{c}}=\frac{14\hat{\mathbf{i}}-7\hat{\mathbf{j}}+14\hat{\mathbf{k}}}{\sqrt{14^2+7^2+14^2}}=\frac{14\hat{\mathbf{i}}-7\hat{\mathbf{j}}+14\hat{\mathbf{k}}}{21}$

$\therefore$Required vector

$=12\cdot\frac{(14\hat{\mathbf{i}}-7\hat{\mathbf{j}}+14\hat{\mathbf{k}})}{21}=8\hat{\mathbf{i}}-4\hat{\mathbf{j}}+8\hat{\mathbf{k}}$

184 **(b)**

Since, $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=0$

Also, $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=\vec{\mathbf{b}}\cdot\vec{\mathbf{c}}=\cos\theta$

Now, $\vec{\mathbf{c}}=\alpha\vec{\mathbf{a}}+\beta\vec{\mathbf{b}}+\gamma(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}})$

$\vec{\mathbf{a}}\cdot\vec{\mathbf{c}}=\alpha\vec{\mathbf{a}}\cdot\vec{\mathbf{a}}+\beta\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}+\gamma\vec{\mathbf{a}}\cdot(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}})$

$\Longrightarrow|\vec{\mathbf{a}}||\vec{\mathbf{c}}|\cos\theta=\alpha+0+0$

$\Longrightarrow\cos\theta=\alpha\quad[\because\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=0]$

and $\vec{\mathbf{b}}\cdot\vec{\mathbf{c}}=\alpha\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}+\beta\vec{\mathbf{b}}\cdot\vec{\mathbf{b}}+\gamma(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}})\cdot\vec{\mathbf{b}}$

$\Longrightarrow|\vec{\mathbf{b}}||\vec{\mathbf{c}}|\cos\theta=\beta\Longrightarrow\cos\theta=\beta$

185 **(a)**

Given volume of parallelopiped

$[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]=40$

$\therefore$ Volume of parallelopiped

$=[\vec{\mathbf{b}}+\vec{\mathbf{c}}\;\vec{\mathbf{c}}+\vec{\mathbf{a}}\;\vec{\mathbf{a}}+\vec{\mathbf{b}}]=2[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$

$=2\times40=80$ cu units

186 **(a)**

Given, $\overrightarrow{\mathbf{OP}}=\hat{\mathbf{a}}\cos t+\hat{\mathbf{b}}\sin t$

$\Longrightarrow\ |\overrightarrow{\mathbf{OP}}|$

$=\sqrt{(\hat{\mathbf{a}}\cdot\hat{\mathbf{a}}\cos^2 t+\hat{\mathbf{b}}\cdot\hat{\mathbf{b}}\sin^2 t+2\hat{\mathbf{a}}\cdot\hat{\mathbf{b}}\sin t\cos t}$

$\Longrightarrow\ |\overrightarrow{\mathbf{OP}}|=\sqrt{1+\hat{\mathbf{a}}\cdot\hat{\mathbf{b}}\sin 2t}$

$\Longrightarrow\ |\overrightarrow{\mathbf{OP}}|_{\text{maxx}}=\sqrt{1+\hat{\mathbf{a}}\cdot\hat{\mathbf{b}}}$

$\left[\text{Max}(\sin 2t)=1\Longrightarrow t=\frac{\pi}{4}\right]$

$\Longrightarrow\ \overrightarrow{\mathbf{OP}}\left(\text{at } t=\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}(\hat{\mathbf{a}}+\hat{\mathbf{b}})$

$\therefore$ Unit vector along $\overrightarrow{\mathbf{OP}}$ at $\left(t=\frac{\pi}{4}\right)=\frac{\hat{\mathbf{a}}+\hat{\mathbf{b}}}{|\hat{\mathbf{a}}+\hat{\mathbf{b}}|}$

187 **(b)**

The position vector of midpoint of line joining the points whose position vector are $\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$ and $\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$

$=\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}+\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}}{2}=\hat{\mathbf{i}}$

188 **(a)**

The position vector of G is $\frac{\vec{a}+\vec{b}+\vec{c}}{3}$

$\therefore\vec{GA}+\vec{GB}+\vec{GC}$

$$= \left(\vec{a} - \frac{\vec{a}+\vec{b}+\vec{c}}{3}\right)\left(\vec{b} - \frac{\vec{a}+\vec{b}+\vec{c}}{3}\right)$$

$$+ \left(\vec{c} - \frac{\vec{a}+\vec{b}+\vec{c}}{3}\right) = \vec{0}$$

189 **(d)**

A vector normal to first plane is $\vec{\mathbf{n}}_1 = \hat{\mathbf{i}} \times (\hat{\mathbf{i}} + \hat{\mathbf{j}}) = \hat{\mathbf{k}}$

A vector normal to second plane is $\vec{\mathbf{n}}_2$

$= (\hat{\mathbf{i}} - \hat{\mathbf{j}}) \times (\hat{\mathbf{i}} + \hat{\mathbf{k}}) = -\hat{\mathbf{j}} + \hat{\mathbf{k}} - \hat{\mathbf{i}}$

Since, $\vec{\mathbf{a}}$ will be parallel to $\vec{\mathbf{n}}_1 \times \vec{\mathbf{n}}_2 = \hat{\mathbf{i}} - \hat{\mathbf{j}}$

Let θ be the angle between $\vec{\mathbf{a}}$ and $\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\therefore \cos\theta \dfrac{(\hat{\mathbf{i}} - \hat{\mathbf{j}}) \cdot (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})}{\sqrt{1^2+1^2}\sqrt{1^2+2^2+2^2}}$

$= \dfrac{1+2}{\sqrt{2}\cdot 3} = \dfrac{1}{\sqrt{2}}$

$\Rightarrow \theta = \dfrac{\pi}{4}$

190 **(a)**

Since, given planes are perpendicular, it means its normal are perpendicular.

$\therefore 2(\lambda) - \lambda(5) + 3(-1) = 0$

$\Rightarrow -3\lambda - 3 = 0$

$\Rightarrow \lambda = -1$

$\therefore \lambda^2 + \lambda = (-1)^2 - 1 = 0$

191 **(a)**

$2\overrightarrow{\mathbf{OA}} + 3\overrightarrow{\mathbf{OB}} = 2(\overrightarrow{\mathbf{OC}} + \overrightarrow{\mathbf{CA}}) + 3(\overrightarrow{\mathbf{OC}} + \overrightarrow{\mathbf{CB}})$

$= 5\overrightarrow{\mathbf{OC}} + 2\overrightarrow{\mathbf{CA}} + 3\overrightarrow{\mathbf{CB}}$

$= 5\overrightarrow{\mathbf{OC}} \quad [\because 2\overrightarrow{\mathbf{CA}} = -3\overrightarrow{\mathbf{CB}}]$

192 **(b)**

If the vectors $(\sec^2 A)\hat{\imath} + \hat{\jmath} + \hat{k}, \hat{\imath} + (\sec^2 B)\hat{\jmath} + \hat{k}$ and $\hat{\imath} + \hat{\jmath} + (\sec^2 C)\hat{k}$ are coplanar, then

$$\begin{vmatrix} \sec^2 A & 1 & 1 \\ 1 & \sec^2 B & 1 \\ 1 & 1 & \sec^2 C \end{vmatrix} = 0$$

$\Rightarrow \sec^2 A \sec^2 B \sec^2 C - \sec^2 A$

$- \sec^2 B - \sec^2 C + 2 = 0$

$\Rightarrow (1 + \tan^2 A)(1 + \tan^2 B)(1 + \tan^2 C)$

$- (1 + \tan^2 A)$

$-(1 + \tan^2 B) - (1 + \tan^2 C) + 2 = 0$

$\Rightarrow \tan^2 A \tan^2 B \tan^2 C + \tan^2 A \tan^2 B$

$+ \tan^2 B \tan^2 C + \tan^2 C \tan^2 A$

$= 0$

$\Rightarrow \cot^2 A + \cot^2 B + \cot^2 C + 1 = 0$

$\Rightarrow \operatorname{cosec}^2 A + \operatorname{cosec}^2 B + \operatorname{cosec}^2 C - 2 = 0$

$\Rightarrow \operatorname{cosec}^2 A + \operatorname{cosec}^2 B + \operatorname{cosec}^2 C = 2$

193 **(b)**

It is given that the points P, Q and R with position vectors $2\hat{\imath} + \hat{\jmath} + \hat{k}, 6\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $14\hat{\imath} - 5\hat{\jmath} + p\,k$ respectively are collinear

$\therefore \vec{P}Q = \lambda \vec{Q}R$ for some scalar λ

$\Rightarrow 4\hat{\imath} - 2\hat{\jmath} + \hat{k} = \lambda\{8\hat{\imath} - 4\hat{\jmath}(p-2)\hat{k}\}$

$\Rightarrow 4 = 8\lambda, -2 = -4\lambda$ and $\lambda(p-2) = 1 \Rightarrow p = 4$

194 **(c)**

Given, $\vec{\alpha} + \vec{\beta} + \vec{\gamma} = a\vec{\delta}$ ....(i)

$\vec{\beta} + \vec{\gamma} + \vec{\delta} = b\vec{\alpha}$ .....(ii)

From Eq. (i)

$\vec{\alpha} + \vec{\beta} + \vec{\gamma} + \vec{\delta} = (a+1)\vec{\delta}$ ...(iii)

From Eq. (ii)

$\vec{\alpha} + \vec{\beta} + \vec{\gamma} + \vec{\delta} = (b+1)\vec{\alpha}$ ...(iv)

From Eq. (iii) and (iv),

$(a+1)\vec{\delta} = (b+1)\vec{\alpha}$ ...(v)

Since, $\vec{\alpha}$ is not parallel to $\vec{\delta}$.

$\therefore$ From Eq. (v),

$a + 1 = 0$ and $b + 1 = 0$

$\therefore$ From Eq. (iii),

$\vec{\alpha} + \vec{\beta} + \vec{\gamma} + \vec{\delta} = \vec{0}$

196 **(d)**

We have,

$$[2\vec{a} + \vec{b}\ 2\vec{b} + \vec{c}\ 2\vec{c} + \vec{a}\,] = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{vmatrix} [\vec{a}\ \vec{b}\ \vec{c}]$$

$= 9 \times 3 = 27$

Hence, required volume $= 27$ cubic units

197 **(a)**

In plane P_1, a vector is perpendicular to $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is $\vec{\mathbf{a}} \times \vec{\mathbf{b}}$.

In plane P_2, a vector is perpendicular to $\vec{\mathbf{c}}$ and $\vec{\mathbf{d}}$ is $\vec{\mathbf{c}} \times \vec{\mathbf{d}}$

$\Rightarrow (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times (\vec{\mathbf{c}} \times \vec{\mathbf{d}}) = 0$

$\Rightarrow (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) || (\vec{\mathbf{c}} \times \vec{\mathbf{d}})$

The angle between the planes is 0.

198 **(a)**

We have,

$$\frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a})\cdot\vec{b}} + \frac{\vec{b}\cdot(\vec{a}\times\vec{c})}{\vec{c}\cdot(\vec{a}\times\vec{b})} = \frac{[\vec{a}\ \vec{b}\ \vec{c}]}{[\vec{b}\ \vec{c}\ \vec{a}]} + \frac{[\vec{b}\ \vec{a}\ \vec{c}]}{[\vec{c}\ \vec{a}\ \vec{b}]}$$

$$= \frac{[\vec{a}\ \vec{b}\ \vec{c}]}{[\vec{a}\ \vec{b}\ \vec{c}]} - \frac{[\vec{a}\ \vec{b}\ \vec{c}]}{[\vec{a}\ \vec{b}\ \vec{c}]} = 1 - 1 = 0$$

199 **(a)**

Given, $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}},\ \vec{\mathbf{b}} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$,

Now, $|\vec{\mathbf{a}}| = \sqrt{4+1+4} = 3$

Since, $|\vec{\mathbf{c}} - \vec{\mathbf{a}}| = 2\sqrt{2}$

$\Rightarrow |\vec{\mathbf{c}}|^2 + |\vec{\mathbf{a}}|^2 - 2\vec{\mathbf{c}}\cdot\vec{\mathbf{a}} = 8$

$\Rightarrow |\vec{\mathbf{c}}|^2 + 9 - 2|\vec{\mathbf{c}}| = 8$

$\Rightarrow |\vec{\mathbf{c}}|^2 - 2|\vec{\mathbf{c}}| + 1 = 0$

$\Rightarrow |\vec{\mathbf{c}}| = 1$

Now, $|(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times \vec{\mathbf{c}}| = |\vec{\mathbf{a}} \times \vec{\mathbf{b}}||\vec{\mathbf{c}}| \sin 30°$ ....(i)

Now, $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix}$

$= \hat{\mathbf{i}}(0+2) - \hat{\mathbf{j}}(0+2) + \hat{\mathbf{k}}(2-1)$

$= 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$

$\Rightarrow |\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = \sqrt{4+4+1} = 3$

$\therefore$ From Eq. (i),

$|(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times \vec{\mathbf{c}}| = 3 \cdot 1 \cdot \frac{1}{2} = \frac{3}{2}$

201 **(a)**

Now, $\overrightarrow{\mathbf{AB}} + 2\overrightarrow{\mathbf{AD}} + \overrightarrow{\mathbf{BC}} - 2\overrightarrow{\mathbf{DC}}$

$= \overrightarrow{\mathbf{AC}} + 2\overrightarrow{\mathbf{AD}} - 2\overrightarrow{\mathbf{DC}}$

$= \overrightarrow{\mathbf{AC}} + 2(\overrightarrow{\mathbf{AC}} + \overrightarrow{\mathbf{CD}}) - 2\overrightarrow{\mathbf{DC}}$

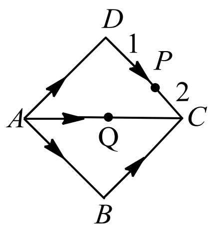
$= 3\overrightarrow{\mathbf{AC}} - 4\overrightarrow{\mathbf{DC}}$

$= 3(2\overrightarrow{\mathbf{QC}}) - 4\left(\frac{3}{2}\overrightarrow{\mathbf{PC}}\right)$

$= 6\overrightarrow{\mathbf{QC}} - 6\overrightarrow{\mathbf{PC}} = 6(\overrightarrow{\mathbf{QC}} + \overrightarrow{\mathbf{CP}})$

$\Rightarrow k\overrightarrow{\mathbf{PQ}} = 6\overrightarrow{\mathbf{QP}} = -6\overrightarrow{\mathbf{PQ}}$ (given)

$\Rightarrow k = -6$



202 **(c)**

Given, $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = 4 \Rightarrow |\vec{\mathbf{a}}||\vec{\mathbf{b}}| \sin\theta = 4$ .....(i)

$\Rightarrow \sin\theta = \frac{4}{|\vec{\mathbf{a}}||\vec{\mathbf{b}}|}$

Alos, $|\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}| = 2 \Rightarrow |\vec{\mathbf{a}}||\vec{\mathbf{b}}| \cos\theta = 2$

$\Rightarrow |\vec{\mathbf{a}}|^2 |\vec{\mathbf{b}}|^2 (1 - \sin^2\theta) = 4$

$\Rightarrow |\vec{\mathbf{a}}|^2 |\vec{\mathbf{b}}|^2 \left(1 - \frac{16}{|\vec{\mathbf{a}}|^2 |\vec{\mathbf{b}}|^2}\right) = 4$ [From Eq. 1]

$\Rightarrow |\vec{\mathbf{a}}|^2 |\vec{\mathbf{b}}|^2 = 20$

203 **(c)**

We have,

$\vec{p} = \frac{\vec{b} \times \vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}, \vec{q} = \frac{\vec{c} \times \vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]}, \vec{r} = \frac{\vec{a} \times \vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]}$

$\therefore \vec{a} \times \vec{p} = \frac{1}{[\vec{a}\,\vec{b}\,\vec{c}]} \vec{a} \times (\vec{b} \times \vec{c})$,

$\vec{b} \times \vec{q} = \frac{1}{[\vec{a}\,\vec{b}\,\vec{c}]} \vec{b} \times (\vec{c} \times \vec{a})$

$\vec{c} \times \vec{r} = \frac{1}{[\vec{a}\,\vec{b}\,\vec{c}]} \vec{c} \times (\vec{a} \times \vec{b})$

$\therefore \vec{a} \times \vec{p} + \vec{b} \times \vec{q} + \vec{c} \times \vec{r}$

$= \frac{1}{[\vec{a}\,\vec{b}\,\vec{c}]} \{\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c}$

$\times (\vec{a} \times \vec{b})\}$

$= \vec{0}$

204 **(b)**

Since, $\cos\theta = \frac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{a}}||\vec{\mathbf{b}}|}$

$= \frac{c(\log_2 x)^2 - 12 + 6c \log_2 x}{[\sqrt{(c \log_2 x)^2 + 36 + 9} \times \sqrt{(\log_2 x)^2 + 4 + 4(c\ \text{lo}}}$

For obtuse angle,

$\cos\theta < 0$

$\Rightarrow c(\log_2 x)^2 - 12 + 6c \log_2 x < 0$

$\Rightarrow c < 0$ and $D < 0$

$\Rightarrow c < 0$ and $(6c)^2 + 48c < 0$

$\Rightarrow c < 0$ and $c < -\frac{4}{3}$

$\therefore c \epsilon \left(-\frac{4}{3}, 0\right)$

206 **(d)**

Given lines can be rewritten as

$\vec{\mathbf{r}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}} + t(-3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$

and $\vec{\mathbf{r}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}} + s(4\hat{\mathbf{i}} - \hat{\mathbf{j}} + 8\hat{\mathbf{k}})$

here, $a_1 = -3, b_1 = 2, c_1 = 6$

and $a_2 = 4, b_2 = -1, c_2 = 8$

$\therefore \cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

$= \frac{-3 \times 4 + 2 \times (-1) + 6 \times 8}{\sqrt{9+4+36}\sqrt{16+1+64}} = \frac{34}{7 \times 9}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{34}{63}\right)$

208 **(a)**

We have,

$\vec{AB} = \hat{\imath} - 7\hat{\jmath} + \hat{k}$ and, $\vec{BC} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$

$\therefore \vec{AC} = \vec{AB} + \vec{BC} = 4\hat{\imath} - 6\hat{\jmath} + 3\hat{k}$

$\Rightarrow |\vec{AC}| = \sqrt{16+36+9} = \sqrt{61}$

210 **(d)**

$(\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \cdot \left(\frac{m\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}}{\sqrt{13 + m^2}}\right) = 2$

$\Rightarrow m + 2 + 6 = 2\sqrt{13 + m^2}$

$\Rightarrow (m+8)^2 = 4(13 + m^2)$

$\Rightarrow m^2 + 16m + 64 = 4m^2 + 52$

$\Rightarrow 3m^2 - 16m - 12 = 0$

$\Rightarrow (3m+2)(m-6) = 0$

$\Rightarrow m = 6, -\frac{2}{3}$

211 **(c)**

If $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are non-zero and non-collinear vectors and there exists α and β such that $\alpha\vec{\mathbf{a}} + \beta\vec{\mathbf{b}} = \vec{\mathbf{0}}$, then $\alpha = \beta = 0$

212 **(d)**

Given vectors are coplanar, if

$$\begin{vmatrix} 1 & 1 & m \\ 1 & 1 & m+1 \\ 1 & -1 & m \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 0 & 0 & -1 \\ 1 & 1 & m+1 \\ 1 & -1 & m \end{vmatrix} = 0 \ [R_1 \to R_1 - R_2]$$

$\Rightarrow -1(-1-1) = 0$

$\Rightarrow 2 \neq 0$

$\therefore$ Now value of m for which vectors are coplanar.

213 **(b)**

Let the required unit vector $\vec{c} = x\hat{\imath} + y\hat{k}$

We have,

$|\vec{c}| = 1 \Rightarrow x^2 + y^2 = 1$ ....(i)

Vectors $\vec{a}$ and $\vec{c}$ are inclined at an angle of 45°

$\therefore \cos 45° = \frac{2x-y}{\sqrt{4+4+1}} \Rightarrow 2x - y = \frac{3}{\sqrt{2}}$ ...(ii)

Vectors $\vec{b}$ and $\vec{c}$ are inclined at an angle of 60°

$\therefore -\frac{y}{\sqrt{2}} = \cos 60° \Rightarrow y = -\frac{1}{\sqrt{2}}$ ....(iii)

From (ii) and (iii), we get $x = 1/\sqrt{2}$

Hence, the required unit vector is $\frac{1}{\sqrt{2}}\hat{\imath} - \frac{1}{\sqrt{2}}\hat{k}$

214 **(c)**

Let $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\vec{\mathbf{B}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$,

$\vec{\mathbf{C}} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ and $\vec{\mathbf{D}} = \hat{\mathbf{i}} - 6\hat{\mathbf{j}} + \lambda\hat{\mathbf{k}}$

Now, $\overrightarrow{\mathbf{AB}} = -\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$, $\overrightarrow{\mathbf{A}C} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$

and $\overrightarrow{\mathbf{AD}} = -\hat{\mathbf{i}} - 9\hat{\mathbf{j}} + (\lambda+1)\hat{\mathbf{k}}$

These will be coplanar, if $[\overrightarrow{\mathbf{AB}}\,\overrightarrow{\mathbf{AC}}\,\overrightarrow{\mathbf{AD}}] = 0$

$$\therefore \begin{vmatrix} -1 & -5 & 4 \\ 1 & 1 & -1 \\ -1 & -9 & (\lambda+1) \end{vmatrix} = 0$$

$\Rightarrow -1(\lambda + 1 - 9) + 5(\lambda + 1 - 1) + 4(-9 + 1) = 0$

$\Rightarrow \lambda = 6$

215 **(b)**

We have,

$|\vec{a}| = |\vec{b}|$

Now,

$\Rightarrow (\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$

$\Rightarrow (\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$ $[\because |\vec{a}| = |\vec{b}|]$

$\Rightarrow (\vec{a} + \vec{b}) \perp (\vec{a} - \vec{b})$

216 **(a)**

Adjacent sides of parallelogram are $\vec{\mathbf{a}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\vec{\mathbf{b}} = -3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$

$$\text{Now, } \vec{\mathbf{a}} \times \vec{\mathbf{b}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 2 & 3 \\ -3 & -2 & 1 \end{vmatrix}$$

$= \hat{\mathbf{i}}(2+6) - \hat{\mathbf{j}}(1+9) + \hat{\mathbf{k}}(-2+6)$

$= 8\hat{\mathbf{i}} - 10\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

Therefore, area of parallelogram

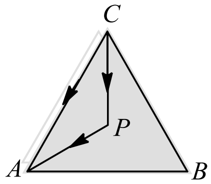
$= |\vec{\mathbf{a}} \times \vec{\mathbf{b}}|$

$= \sqrt{(8)^2 + (-10)^2 + (4)^2}$

$= \sqrt{64 + 100 + 16} = \sqrt{180}$ sq unit

217 **(d)**

$\therefore \overrightarrow{\mathbf{CP}} + \overrightarrow{\mathbf{PA}} + \overrightarrow{\mathbf{BA}}$



By triangle law,

$\overrightarrow{\mathbf{CA}} = \overrightarrow{\mathbf{CB}} + \overrightarrow{\mathbf{BA}}$

$\therefore \overrightarrow{\mathbf{CP}} + \overrightarrow{\mathbf{PA}} = \overrightarrow{\mathbf{CB}} + \overrightarrow{\mathbf{BA}}$

218 **(d)**

We have,

$\vec{c} = x\vec{a} + y\vec{b} + \vec{c}(\vec{a} \times \vec{b})$

$\Rightarrow \vec{c} \cdot \vec{a} = x$ and $\vec{c} \cdot \vec{b} = y \Rightarrow x = y = \cos\theta$

Now,

$\vec{c} \cdot \vec{c} = |\vec{c}|^2$

$\Rightarrow \{x\vec{a} + y\vec{b} + z(\vec{a} \times \vec{b})\} \cdot \{x\vec{a} + y\vec{b} + z(\vec{a} \times \vec{b})\} = |\vec{c}|^2$

$\Rightarrow 2x^2 + x^2|\vec{a} \times \vec{b}|^2 = 1$

$\Rightarrow 2x^2 + z^2\{|\vec{a}|^2|\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2\} = 1$

$\Rightarrow 2x^2 + z^2 = 1$ $[\because |\vec{a}|^2 = 1, |\vec{b}| = 1$ and $\vec{a} \cdot \vec{b} = 0]$

$\Rightarrow z^2 = 1 - 2\cos^2\theta = -\cos 2\theta$

219 **(a)**

We have,

$\vec{a} + \vec{b} = \vec{c}$

$\Rightarrow |\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b}$

$\Rightarrow |\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2 \times 0$ $[\because \vec{a} \cdot \vec{b} = 0]$

$\Rightarrow |\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2$

221 **(b)**

All points A, B, C, D, E are in a plane.

$\therefore$ Resultant $= (\overrightarrow{\mathbf{AC}} + \overrightarrow{\mathbf{AD}} + \overrightarrow{\mathbf{AE}}) + (\overrightarrow{\mathbf{CB}} + \overrightarrow{\mathbf{DB}} + \overrightarrow{\mathbf{EB}}$

$= (\overrightarrow{\mathbf{AC}} + \overrightarrow{\mathbf{CB}}) + (\overrightarrow{\mathbf{AD}} + \overrightarrow{\mathbf{DB}}) + (\overrightarrow{\mathbf{AE}} + \overrightarrow{\mathbf{EB}})$

$= \overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{AB}} = 3\overrightarrow{\mathbf{AB}}$

222 **(a)**

Since, $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are coplanar.

$$\Rightarrow \begin{vmatrix} \alpha & 2 & \beta \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = 0$$

$\Rightarrow \alpha(1-0) - 2(1-0) + \beta(1-0) = 0$

$\Rightarrow \alpha + \beta = 2$ Which is possible for $\alpha = 1, \beta = 1$

223 **(c)**

A unit perpendicular to the plane $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

Now, $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix}$

$= \hat{\mathbf{i}}(6+9) - \hat{\mathbf{j}}(-2+12) + \hat{\mathbf{k}}(6+24)$

$= 15\hat{\mathbf{i}} - 10\hat{\mathbf{j}} + 30\hat{\mathbf{k}}$

and $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = \sqrt{15^2 + (-10)^2 + (30)^2}$

$= \sqrt{1225} = 35$

$\therefore$ Required vector $= \frac{15\hat{i}-10\hat{j}+30\hat{k}}{35} = \frac{3\hat{i}-2\hat{j}+6\hat{k}}{7}$

225 **(d)**

$(\vec{\mathbf{a}} \times \hat{\mathbf{j}}) \cdot (2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) = \vec{\mathbf{a}} \cdot \{\hat{\mathbf{j}} \times (2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})\}$

$= \vec{\mathbf{a}} \cdot \{-3(\hat{\mathbf{j}} \times \hat{\mathbf{k}})\} = -3(\vec{\mathbf{a}} \cdot \hat{\mathbf{i}})$

$= -12 \quad [\because \vec{\mathbf{a}} \cdot \hat{\mathbf{i}} = 4$, given$]$

226 **(b)**

Volume of tetrahedron

$= \frac{1}{6}[\overrightarrow{\mathbf{AB}}\ \overrightarrow{\mathbf{AC}}\ \overrightarrow{\mathbf{AD}}]$

$= \frac{1}{6}\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix}$

$= \frac{1}{6}[-1+2+3] = \frac{2}{3}$ cu unit

228 **(c)**

Since, $\quad \vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = \frac{1}{2}\vec{\mathbf{b}}$

$\Rightarrow (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{b}} - (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{c}} = \frac{1}{2}\vec{\mathbf{b}}$

On comparing both sides, we get

$\vec{\mathbf{a}} \cdot \vec{\mathbf{c}} = \frac{1}{2}$ and $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 0$

Now, $\quad \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 0$

$\Rightarrow |\vec{\mathbf{a}}||\vec{\mathbf{c}}|| \cos\theta_2| = \frac{1}{2} \Rightarrow \cos\theta_2 = \frac{1}{2} \Rightarrow \theta_2 = \frac{\pi}{3}$

and $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 0$

$\Rightarrow |\vec{\mathbf{a}}||\vec{\mathbf{b}}| \cos\theta_1 = 0$

$\Rightarrow \cos\theta_1 = \cos\frac{\pi}{2}$

$\Rightarrow \theta_1 = \frac{\pi}{2}$

229 **(b)**

$\vec{O}A + \vec{O}B + \vec{O}C$

$= \frac{1}{2}(2\,\vec{O}A + 2\,\vec{O}B + 2\,\vec{O}C)$

$= \frac{1}{2}\{(\vec{O}A + \vec{O}B) + (\vec{O}B + \vec{O}C) + (\vec{O}C + \vec{O}A)\}$

$= \frac{1}{2}\{2\,\vec{O}P + 2\,\vec{O}Q + 2\,\vec{O}R\}$

$= \vec{O}P + \vec{O}Q + \vec{O}R$

230 **(a)**

Given, $(\vec{\mathbf{a}} \times \vec{\mathbf{b}})^2 + (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})^2 = \vec{\mathbf{a}}^2\vec{\mathbf{b}}^2\sin^2\theta +$

$\vec{\mathbf{a}}_2\vec{\mathbf{b}}_2\cos^2\theta = \vec{\mathbf{a}}^2\vec{\mathbf{b}}^2$

231 **(a)**

Since, $\vec{\mathbf{a}} = m\vec{\mathbf{b}}$ for some scalar m *ie*,

$\vec{\mathbf{a}} = m\left(6\hat{\mathbf{i}} - 8\hat{\mathbf{j}} - \frac{15}{2}\hat{\mathbf{k}}\right)$

$\Rightarrow |\vec{\mathbf{a}}| = |m|\sqrt{36 + 64 + \frac{225}{4}}$

$\Rightarrow 50 = \frac{25}{2}|m| \Rightarrow |m| = 4$

$\Rightarrow m = \pm 4$

Since, $\vec{\mathbf{a}}$ makes an acute angle with the positive direction of z-axis, so its z componant must be positive and hence, m must be -4

$\therefore \quad \vec{\mathbf{a}} = -4\left(6\hat{\mathbf{i}} - 8\hat{\mathbf{j}} - \frac{15}{2}\hat{\mathbf{k}}\right)$

$= -24\hat{\mathbf{i}} + 32\hat{\mathbf{j}} + 30\hat{\mathbf{k}}$

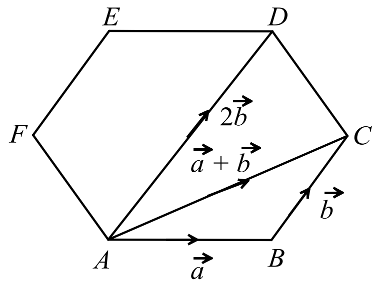
232 **(c)**

In ΔABC, we have

$\vec{AC} = \vec{a} + \vec{b}$

In ΔACD, we have

$\vec{AC} + \vec{CD} = \vec{AD} \Rightarrow \vec{CD} = 2\vec{b} - \vec{a} - \vec{b} = \vec{b} - \vec{a}$



In ΔCDE, we have

$\vec{CD} + \vec{DE} = \vec{CE} \Rightarrow \vec{b} - \vec{a} - \vec{a} = \vec{CE} \Rightarrow \vec{CE}$

$= \vec{b} - 2\vec{a}$

233 **(b)**

Given vectors will be coplanar, if $\begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ m & -1 & 2 \end{vmatrix} = 0$

$\Rightarrow 2(4-1) + 3(2+m) + 4(-1-2m) = 0$

$\Rightarrow m = \frac{8}{5}$

234 **(d)**

Given that, $|\vec{\mathbf{a}}| = 1, |\vec{\mathbf{b}}| = 3$ and $|\vec{\mathbf{c}}| = 5$

$\therefore [\,\vec{\mathbf{a}} - 2\vec{\mathbf{b}}\ \vec{\mathbf{b}} - 3\vec{\mathbf{c}}\ \vec{\mathbf{c}} - 4\vec{\mathbf{a}}\,]$

$= (\vec{\mathbf{a}} - 2\vec{\mathbf{b}}) \cdot \{(\vec{\mathbf{b}} - 3\vec{\mathbf{c}}) \times (\vec{\mathbf{c}} - 4\vec{\mathbf{a}})\}$

$= (\vec{\mathbf{a}} - 2\vec{\mathbf{b}}) \cdot \{\vec{\mathbf{b}} \times \vec{\mathbf{c}} - 4\vec{\mathbf{b}} \times \vec{\mathbf{a}} + 12\vec{\mathbf{c}} \times \vec{\mathbf{a}}\}$

$= (\vec{\mathbf{a}} - 2\vec{\mathbf{b}}) \cdot (\vec{\mathbf{a}} + 4\vec{\mathbf{c}} + 12\vec{\mathbf{b}})$

$= \vec{\mathbf{a}} \cdot \vec{\mathbf{a}} - 24\vec{\mathbf{b}} \cdot \vec{\mathbf{b}} = 1 - 24 \times 9$

$= 1 - 216 = -215$

235 **(a)**

Now, $\hat{\mathbf{i}} \times (\hat{\mathbf{j}} \times \hat{\mathbf{k}}) = \hat{\mathbf{i}} \times \hat{\mathbf{i}} = \vec{\mathbf{0}}$

$\hat{\mathbf{j}} \times (\hat{\mathbf{k}} \times \hat{\mathbf{i}}) = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \vec{\mathbf{0}}$

and $\hat{\mathbf{k}} \times (\hat{\mathbf{i}} \times \hat{\mathbf{j}})\hat{\mathbf{k}} \times \hat{\mathbf{k}} = \vec{\mathbf{0}}$

$\therefore\ \hat{\mathbf{i}} \times (\hat{\mathbf{j}} \times \hat{\mathbf{k}}) + \hat{\mathbf{j}} \times (\hat{\mathbf{k}} \times \hat{\mathbf{i}}) + \hat{\mathbf{k}} \times (\hat{\mathbf{i}} \times \hat{\mathbf{j}}) = \vec{\mathbf{0}}$

236 **(c)**

Given vectors will be coplanar, if

$$\begin{vmatrix} -\lambda^2 & 1 & 1 \\ 1 & -\lambda^2 & 1 \\ 1 & 1 & -\lambda^2 \end{vmatrix} = 0$$

$\Longrightarrow \lambda^6 - 3\lambda^2 - 2 = 0$

$\Longrightarrow (1+\lambda^2)^2(\lambda^2 - 2) = 0 \Longrightarrow \lambda = \pm\sqrt{2}$

237 **(c)**

Here, force $\vec{\mathbf{F}} = 6 \times \frac{(9\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 2\hat{\mathbf{k}})}{\sqrt{81+36+4}}$

$= \frac{6(9\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 2\hat{\mathbf{k}})}{11}$

Displacement vector $\vec{\mathbf{d}}$

$= (7-3)\hat{\mathbf{i}} + (-6-4)\hat{\mathbf{j}} + (8+15)\hat{\mathbf{k}}$

$= 4\hat{\mathbf{i}} - 10\hat{\mathbf{j}} + 23\hat{\mathbf{k}}$

$\therefore$Work done $= \vec{\mathbf{F}} \cdot \vec{\mathbf{d}}$

$= \frac{6}{16}(9\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \cdot (4\hat{\mathbf{i}} - 10\hat{\mathbf{j}} + 23\hat{\mathbf{k}})$

$= \frac{6}{11}(36 - 60 + 46) = 12$

238 **(d)**

Since, $|2\hat{\mathbf{u}} \times 3\hat{\mathbf{v}}| = 1$

$\Longrightarrow 6|\hat{\mathbf{u}}||\hat{\mathbf{v}}||\sin\theta| = 1$

$\Longrightarrow \sin\theta = \frac{1}{6}$ $\qquad [\because |\hat{\mathbf{u}}| = |\hat{\mathbf{u}}| = 1]$

Since, θ is an acute angle, then there is exactly one value of θ for which $(2\hat{\mathbf{u}} \times 3\hat{\mathbf{v}})$ is a unit vector.

239 **(d)**

$\therefore$ Total force , $\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$

$= 5\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$

and displacement,$\vec{\mathbf{d}} = (5-3)\hat{\mathbf{i}} + (5-2)\hat{\mathbf{j}} + (3-1)\hat{\mathbf{k}}$

$= 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\therefore W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}}$

$= (5\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) \cdot (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

$= 11$ units

241 **(a)**

We have,

$\vec{a} + t\,\vec{b} \perp \vec{c}$

$\Rightarrow (\vec{a} + t\,\vec{b}).\vec{c} = 0$

$\Rightarrow \vec{a}.\vec{c} + t\,\vec{b}.\vec{c} = 0 \Rightarrow t = -\frac{\vec{a}.\vec{c}}{\vec{b}.\vec{c}} = -\frac{6+2+0}{-3+2+0}$

$= 8$

242 **(d)**

Given, $\qquad \vec{\mathbf{a}} \cdot \vec{\mathbf{p}} = \frac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \times \vec{\mathbf{c}}}{[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]} = 1$

and $\vec{\mathbf{a}} \cdot \vec{\mathbf{q}} = \vec{\mathbf{a}}\frac{\vec{\mathbf{c}} \times \vec{\mathbf{a}}}{[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]} = 0$

Similarly, $\vec{\mathbf{b}} \cdot \vec{\mathbf{q}} = \vec{\mathbf{c}} \cdot \vec{\mathbf{r}} = 1$,

and $\vec{\mathbf{a}} \cdot \vec{\mathbf{r}} = \vec{\mathbf{b}} \cdot \vec{\mathbf{p}} = \vec{\mathbf{c}} \cdot \vec{\mathbf{q}} = \vec{\mathbf{c}} \cdot \vec{\mathbf{p}} = \vec{\mathbf{b}} \cdot \vec{\mathbf{r}} = 0$

$\therefore (\vec{\mathbf{a}} + \vec{\mathbf{b}}) \cdot \vec{\mathbf{p}}(\vec{\mathbf{b}} + \vec{\mathbf{c}}) \cdot \vec{\mathbf{q}} + (\vec{\mathbf{c}} + \vec{\mathbf{a}}) \cdot \vec{\mathbf{r}}$

$= \vec{\mathbf{a}} \cdot \vec{\mathbf{p}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{p}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{q}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{q}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{r}} + \vec{\mathbf{a}} \cdot \vec{\mathbf{r}}$

$= 1 + 1 + 1 = 3$

243 **(b)**

$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}_1 + \vec{\mathbf{a}} \cdot \left(\vec{\mathbf{b}} - \frac{\vec{\mathbf{b}} \cdot \vec{\mathbf{a}}}{|\vec{\mathbf{a}}|^2}\vec{\mathbf{a}}\right)$

$= \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} - \frac{|\vec{\mathbf{a}}|^2(\vec{\mathbf{b}} \cdot \vec{\mathbf{a}})}{|\vec{\mathbf{a}}|^2}$

$= \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} - \vec{\mathbf{b}} \cdot \vec{\mathbf{a}} = 0$

Similarly, $\vec{\mathbf{a}} \cdot \vec{\mathbf{c}}_2 = \vec{\mathbf{b}}_1 \cdot \vec{\mathbf{c}}_2 = 0$

Hence, $\{\vec{\mathbf{a}}, \vec{\mathbf{b}}_1, \vec{\mathbf{c}}_2\}$ are mutually orthogonal vectors.

244 **(c)**

$\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = (\vec{\mathbf{a}} \cdot \vec{\mathbf{c}})\vec{\mathbf{b}} - (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{c}} = \vec{0}$,

$[\because \vec{\mathbf{a}} \perp \vec{\mathbf{b}}$ and $\vec{\mathbf{a}} \perp \vec{\mathbf{c}}]$

245 **(b)**

Let $\vec{a}, \vec{b}, \vec{c}$ be the position vectors of A, B and C respectively. Then, the position vector of G is $\frac{\vec{a}+\vec{b}+\vec{c}}{3}$

Let the position vectors of A', B' and C' be $\vec{a}, \vec{b}'$ and $\vec{c}$ respectively. Then, the position vectors of G' is $\frac{\vec{a}+\vec{b\prime}+\vec{c}}{3}$

$\therefore A\vec{A}' + B\vec{B}' + C\vec{C}'$

$= (\vec{a} - \vec{a}) + (\vec{b'} - \vec{b}) + (\vec{c} - \vec{c})$

$\Rightarrow A\vec{A}' + B\vec{B}' + C\vec{C}'$

$= (\vec{a'} + \vec{b'} + \vec{c'}) - (\vec{a} + \vec{b} + \vec{c})$

$\Rightarrow A\vec{A}' + B\vec{B}' + C\vec{C}'$

$= 3\left\{\frac{\vec{a'} + \vec{b'} + \vec{c'}}{3} - \frac{\vec{a} + \vec{b} + \vec{c}}{3}\right\}$

$= 3G\vec{G}'$

246 **(a)**

We have,

$\vec{u} = \vec{a} - \vec{b}, \vec{u} = \vec{a} + \vec{b}$

$\Rightarrow \vec{u} \times \vec{v} = (\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$

$\Rightarrow |\vec{u} \times \vec{v}| = 2|\vec{a} \times \vec{b}|$

$\Rightarrow |\vec{u} \times \vec{v}| = 2\sqrt{|\vec{a} \times \vec{b}|}$

$\Rightarrow |\vec{u} \times \vec{v}| = 2\sqrt{|\vec{a}|^2\,|\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2}$

$\Rightarrow |\vec{u} \times \vec{v}| = 2\sqrt{16 - (\vec{a} \cdot \vec{b})^2}$

248 **(b)**

Let $\vec{\mathbf{d}} = d_1\hat{\mathbf{i}} + d_2\hat{\mathbf{j}} + d_3\hat{\mathbf{k}}$

$\vec{\mathbf{a}} \cdot \vec{\mathbf{d}} = d_1 - d_2 = 0 \implies d_1 = d_2$ ....(i)

Also, $\vec{\mathbf{d}}$ is a unit vector.

$\implies d_1^2 + d_2^2 + d_3^2 = 1$ ....(ii)

Also, $[\vec{\mathbf{b}}\ \vec{\mathbf{c}}\ \vec{\mathbf{d}}] = 0 \Rightarrow \begin{vmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ d_1 & d_2 & d_3 \end{vmatrix} = 0$

$\implies -1(-d_3 - d_1) - 1(-d_2) = 0$

$\implies d_1 + d_2 + d_3 = 0 \implies 2d_1 + d_3 = 0$ [from Eq. (i)]

$\implies d_3 = -2d_1$ ....(iii)

Using Eqs. (iii) and (i) in Eq. (ii), we get

$d_1^2 + d_1^2 + 4d_1^2 = 1 \implies d_1 = \pm\frac{1}{\sqrt{6}}$

$\therefore d_2 = \pm\frac{1}{\sqrt{6}}$

and $d_3 = \mp\frac{2}{\sqrt{6}}$

Hence, required vector is

$\pm\frac{1}{\sqrt{6}}(\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

249 **(b)**

Since $\vec{a}$ is collinear to vector $\vec{b}$. Therefore,

$\vec{a} = m\,\vec{b}$ for some scalar m

$\Rightarrow \vec{a} = m\left(6\hat{\imath} - 8\hat{\jmath} - \frac{15}{2}\hat{k}\right)$

$\Rightarrow |\vec{a}| = \frac{25}{2}|m|$

$\Rightarrow 50 = \frac{25}{2}|m| \Rightarrow |m| = 4 \Rightarrow m$
$= \pm 4$ [$\because |\vec{a}| = 50$]

Since $\vec{a}$ makes an acute angle with the positive direction of z-axis. So, its z-component must be positive, and hence $'m'$ must be -4

$\therefore \vec{a} = -4\left(6\hat{\imath} - 8\hat{\jmath} - \frac{15}{2}\hat{k}\right) = -24\hat{\imath} + 32\hat{\jmath} + 30\hat{k}$

251 **(c)**

Since $\vec{a}$ and $\vec{b}$ are coplanar. Therefore, $\vec{a} \times \vec{b}$ is a vector perpendicular to the plane containing $\vec{a}$ and $\vec{b}$

Similarly, $\vec{c} \times \vec{d}$ is a vector perpendicular to the plane containing $\vec{c}$ and $\vec{d}$

Two planes will be parallel if their normal i.e. $\vec{a} \times \vec{b}$ and $\vec{c} \times \vec{d}$ are parallel

$\therefore (\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = 0$

252 **(c)**

Since, $\vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} + \vec{\mathbf{c}}) = 0$

$\implies \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{a}} = 0$ ....(i)

Similarly, $\vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 0$ .....(ii)

and $\vec{\mathbf{c}} \cdot \vec{\mathbf{a}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} = 0$ ....(iii)

On adding Eqs. (i),(ii) and (iii), we get

$2(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{a}}) = 0$

Now, $|\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}|^2 = |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + |\vec{\mathbf{c}}|^2 + 2(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{a}})$

$= |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + |\vec{\mathbf{c}}|^2$

$= 9 + 16 + 25 = 50$

$\implies |\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}| = 5\sqrt{2}$

253 **(b)**

We know that the diagonals of a parallelogram bisect each other. Therefore, M is the mid point of AC and BD both.

$\therefore \overrightarrow{\mathbf{OA}} + \overrightarrow{\mathbf{OC}} = 2\overrightarrow{\mathbf{OM}}$

and $\overrightarrow{\mathbf{OB}} + \overrightarrow{\mathbf{OD}} = 2\,\overrightarrow{\mathbf{OM}}$

$\implies \overrightarrow{\mathbf{OA}} + \overrightarrow{\mathbf{OB}} + \overrightarrow{\mathbf{OC}} + \overrightarrow{\mathbf{OD}} = 4\,\overrightarrow{\mathbf{OM}}$

254 **(b)**

$|\overrightarrow{\mathbf{OA}}| = \sqrt{4 + 4 + 1} = 3$

and $|\overrightarrow{\mathbf{OB}}| = \sqrt{4 + 16 + 16} = 6$

$\therefore$ Required vector $= \lambda(\widehat{\mathbf{OA}} + \widehat{\mathbf{OB}})$

$= \lambda\left[\frac{1}{3}(2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) + \frac{1}{6}(2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}})\right]$

$= \frac{\lambda}{3}(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

$\therefore$ Length of vector $= \frac{\lambda}{3}\sqrt{9 + 16 + 9} = \frac{\lambda}{3}\sqrt{34}$

Take $\lambda = 2$

$\therefore$ Required length of a vector is $\frac{\sqrt{136}}{3}$

255 **(d)**

Given that, $\vec{\mathbf{A}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \vec{\mathbf{B}} = \hat{\mathbf{i}}, \vec{\mathbf{C}} = c_1\hat{\mathbf{i}} + c_2\hat{\mathbf{j}} + c_3\hat{\mathbf{k}}$

Since, $\vec{\mathbf{A}}, \vec{\mathbf{B}}, \vec{\mathbf{C}}$ are coplanar.

$\therefore [\vec{\mathbf{A}}\ \vec{\mathbf{B}}\ \vec{\mathbf{C}}] = 0$

Now, $\vec{\mathbf{B}} \times \vec{\mathbf{C}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 0 & 0 \\ c_1 & c_2 & c_3 \end{vmatrix} = -c_3\hat{\mathbf{j}} + c_2\hat{\mathbf{k}}$

$\therefore \vec{\mathbf{A}} \cdot (\vec{\mathbf{B}} \times \vec{\mathbf{C}}) = (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) \cdot (-c_3\hat{\mathbf{j}} + c_2\hat{\mathbf{k}}) = 0$

$\Rightarrow$ No value of c_1 can be found.

256 **(c)**

We have,

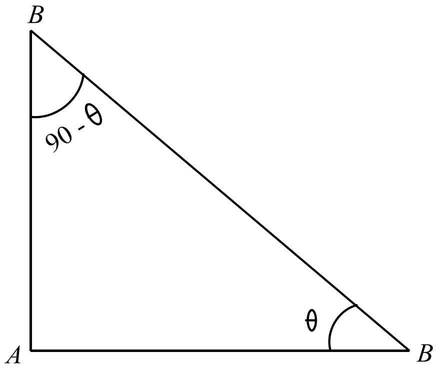
$A\vec{B} \cdot A\vec{C} + B\vec{C} \cdot B\vec{A} + C\vec{A} \cdot C\vec{B}$

$= (AB)(AC)\cos\theta + (BC)(BA)\sin\theta + 0$

$= AB(AC\cos\theta + BC\sin\theta)$

$= AB\left\{\frac{(AC)^2}{AB}+\frac{(BC)^2}{AB}\right\}$ $\left[\because \cos\theta = \frac{AC}{AB}, \sin\theta = \frac{BC}{AB}\right]$

$= AC^2 + BC^2 = AB^2 = p^2$



257 **(a)**

The position vector of the centroid of the triangle is $\frac{\vec{a}+\vec{b}+\vec{c}}{3}$

Since the triangle is an equilateral. Therefore, the orthocenter coincides with the centroid and hence

$\frac{\vec{a}+\vec{b}+\vec{c}}{3} = \vec{0} \Rightarrow \vec{a}+\vec{b}+\vec{c} = \vec{0}$

258 **(d)**

Given, $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = 4$

$\Rightarrow ||\vec{\mathbf{a}}||\vec{\mathbf{b}}| \sin\theta \hat{\mathbf{n}}| = 4$

$\Rightarrow ||\vec{\mathbf{a}}||\vec{\mathbf{b}}| \sin\theta| = 4$ $[\because |\hat{\mathbf{n}}| = 1]$ ....(i)

Also, $|\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}| = 2$

$\Rightarrow ||\vec{\mathbf{a}}||\vec{\mathbf{b}}| \cos\theta| = 2$ ....(ii)

On squaring and then on adding Eqs.(i) and (ii), we get

$|\vec{\mathbf{a}}|^2|\vec{\mathbf{b}}|^2\sin^2\theta + |\vec{\mathbf{a}}|^2|\vec{\mathbf{b}}|^2\cos^2\theta = 4^2 + 2^2$

$\Rightarrow |\vec{\mathbf{a}}|^2|\vec{\mathbf{b}}|^2(\sin^2\theta + \cos^2\theta) = 16 + 4$

$\Rightarrow |\vec{\mathbf{a}}|^2|\vec{\mathbf{b}}|^2 = 20$

260 **(d)**

Given that, $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$, $\vec{\mathbf{b}} = 3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and $\vec{\mathbf{c}} = 5\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

$\therefore$ Volume of parallelopiped where sides are $\vec{\mathbf{a}}+\vec{\mathbf{b}}, \vec{\mathbf{b}}+\vec{\mathbf{c}}$ and $\vec{\mathbf{c}}+\vec{\mathbf{a}}$, is

$[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}] = \begin{vmatrix} 2 & -3 & 5 \\ 3 & -4 & 5 \\ 5 & -3 & -2 \end{vmatrix}$

$= [2(8+15) + 3(-6-25) + 5(-9+20)]$

$= 46 - 93 + 55 = 8$

261 **(c)**

Let $\vec{\mathbf{a}} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$

Given, $\vec{\mathbf{a}} \cdot \hat{\mathbf{i}} = \vec{\mathbf{a}} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}}) = \vec{\mathbf{a}} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 1$

$\therefore a_1 = a_1 + a_2 = a_1 + a_2 + a_3 = 1$

$\Longrightarrow a_1 = 1, a_2 = 0, a_3 = 0$

$\therefore \vec{\mathbf{a}} = \hat{\mathbf{i}}$

263 **(b)**

$\overrightarrow{\mathbf{DA}} + \overrightarrow{\mathbf{DB}} + \overrightarrow{\mathbf{DC}} + \overrightarrow{\mathbf{AE}} + \overrightarrow{\mathbf{BE}} + \overrightarrow{\mathbf{CE}}$

$= (\overrightarrow{\mathbf{DA}} + \overrightarrow{\mathbf{AE}}) + (\overrightarrow{\mathbf{DB}} + \overrightarrow{\mathbf{BE}}) + (\overrightarrow{\mathbf{DC}} + \overrightarrow{\mathbf{CE}})$

$= \overrightarrow{\mathbf{DE}} + \overrightarrow{\mathbf{DE}} + \overrightarrow{\mathbf{DE}}$

$= 3\,\overrightarrow{\mathbf{DE}}$

265 **(d)**

Given vertices are

$A(3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}), B(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 7\hat{\mathbf{k}})$ and $C(-2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$.

Now, $\overrightarrow{\mathbf{AB}} = (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 7\hat{\mathbf{k}}) - (3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

$= -2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

$\therefore |\overrightarrow{\mathbf{AB}}| = \sqrt{4+9+25} = \sqrt{38}$

Similarly, $|\overrightarrow{\mathbf{BC}}| = |\overrightarrow{\mathbf{CA}}| = \sqrt{38}$

$\therefore |\overrightarrow{\mathbf{AB}}| = |\overrightarrow{\mathbf{BC}}| = |\overrightarrow{\mathbf{CA}}| = \sqrt{38}$

$\therefore$ Hence, triangle is an equilateral triangle.

267 **(b)**

We have,

$|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$

$\Rightarrow |\vec{a}+\vec{b}|^2 = |\vec{a}-\vec{b}|^2$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b}$

$\Rightarrow 4\vec{a}.\vec{b} = 0 \Rightarrow \vec{a}.\vec{b} = 0 \Rightarrow \vec{a} \perp \vec{b}$

268 **(d)**

$\because |\vec{\mathbf{a}} + \vec{\mathbf{b}}| = |\vec{\mathbf{a}} - \vec{\mathbf{b}}|$

$\Rightarrow |\vec{\mathbf{a}} + \vec{\mathbf{b}}|^2 = |\vec{\mathbf{a}} - \vec{\mathbf{b}}|^2$

$\Rightarrow a^2 + b^2 + 2\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = a^2 + b^2 - 2\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}$

$\Rightarrow 4\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 0$

$\Rightarrow \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 0$

$\therefore$ Angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is $\frac{\pi}{2}$.

269 **(c)**

Given vectors are coplanar, if $\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \beta & 1 \\ 0 & c & \gamma \end{vmatrix} = 0$

Applying $C_2 \to C_2 - C_1, C_3 \to C_3 - C_2$

$\Rightarrow \begin{vmatrix} \alpha & 1-\alpha & 0 \\ 1 & \beta - 1 & 1-\beta \\ 1 & 0 & \gamma - 1 \end{vmatrix} = 0$

$\Rightarrow (1-\alpha)(1-\beta)(1-\gamma)\begin{vmatrix} \frac{\alpha}{1-\alpha} & 1 & 0 \\ \frac{1}{1-\beta} & -1 & 1 \\ \frac{1}{1-\gamma} & 0 & -1 \end{vmatrix} = 0$

$\Rightarrow (1-\alpha)(1-\beta)(1-\gamma)\left[\frac{\alpha}{1-\alpha}(1) - 1\left(-\frac{1}{1-\beta} - \frac{1}{1-\gamma}\right)\right] = 0$

But $\alpha \neq 1, \beta \neq 1$ and $\gamma \neq 1$

$\therefore \frac{1}{(1-\alpha)} - 1 + \frac{1}{1-\beta} + \frac{1}{1-\gamma} = 0$

$\Rightarrow \frac{1}{1-\alpha} + \frac{1}{1-\beta} + \frac{1}{1-\gamma} = 1$

270 **(b)**

Let the required vector be $\vec{\mathbf{c}} = x\hat{\mathbf{i}} + z\hat{\mathbf{k}}$

Since, $|\vec{\mathbf{c}}| = 1 \Rightarrow x^2 + z^2 = 1$ ....(i)

$\vec{\mathbf{a}}$ and $\vec{\mathbf{c}}$ are inclined at the angle 45°

$\therefore \cos 45° = \frac{2x - z}{\sqrt{4+4+1}} \Rightarrow 2x - z = \frac{3}{\sqrt{2}}$ .....(ii)

$\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are inclined at an angle 60°

$\therefore -\frac{z}{\sqrt{2}} = \cos 60° \Rightarrow z = -\frac{1}{\sqrt{2}}$ ...(iii)

From Eqs. (ii) and (iii), we get $x = \frac{1}{\sqrt{2}}$

Hence, the required ector is $\frac{1}{\sqrt{2}}\hat{\mathbf{i}} - \frac{1}{\sqrt{2}}\hat{\mathbf{k}}$

271 **(d)**

Since $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors. Therefore, $[\vec{a}\ \vec{b}\ \vec{c}] \neq 0$

$\Rightarrow \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0 \Rightarrow \Delta \neq 0$, where $\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

Now,

$\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$

$\Rightarrow \Delta(1 + abc) = 0 \Rightarrow abc = -1$ $[\because \Delta \neq 0]$

272 **(c)**

$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$

$\Rightarrow |\hat{\mathbf{u}}||\hat{\mathbf{v}}| \cos\theta = 0$

$\Rightarrow 1 \times 1 \times \cos\theta = 0$ $(\because |\hat{\mathbf{u}}| = |\hat{\mathbf{v}}| = 1)$

$\Rightarrow \cos\theta = 0$

$\Rightarrow \theta = 90°$

Let $\hat{\mathbf{n}}$ be a unit vector perpendicular to the plane of vectors $\hat{\mathbf{u}}$ and $\hat{\mathbf{v}}$.

$\Rightarrow \hat{\mathbf{u}} \times \hat{\mathbf{v}} = |\hat{\mathbf{u}}||\hat{\mathbf{v}}| \sin 90° \cdot \hat{\mathbf{n}} = \hat{\mathbf{n}}$

Since, $\vec{\mathbf{r}}$ is coplanar with $\hat{\mathbf{u}}$ and $\hat{\mathbf{v}}$

$\therefore$ $\hat{\mathbf{n}}$ is perpendicular to $\vec{\mathbf{r}}$

Let Φ be the angle between $\hat{\mathbf{n}}$ and $\vec{\mathbf{r}}$

$\Rightarrow \Phi = 90°$

$\therefore |\vec{\mathbf{r}} \times (\hat{\mathbf{u}} \times \hat{\mathbf{v}}))| = |\vec{\mathbf{r}} \times \hat{\mathbf{n}}| = |\vec{\mathbf{r}}||\hat{\mathbf{n}}| \sin\Phi$

$= |\vec{\mathbf{r}}| \times 1 \times \sin 90°$

$= |\vec{\mathbf{r}}|$

273 **(b)**

Let $\vec{a} = \hat{\imath} - 2\hat{\jmath} + \hat{k}, \vec{b} = 4\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$. Then,

Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{4+8+7}{\sqrt{16+16+49}} = \frac{19}{9}$

274 **(a)**

$\frac{\vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}})}{\vec{\mathbf{b}} \cdot (\vec{\mathbf{c}} \times \vec{\mathbf{a}})} + \frac{\vec{\mathbf{b}} \cdot (\vec{\mathbf{a}} \times \vec{\mathbf{b}})}{\vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}})}$

$= \frac{[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]}{[\vec{\mathbf{b}}\,\vec{\mathbf{c}}\,\vec{\mathbf{a}}]} + \frac{[\vec{\mathbf{b}}\,\vec{\mathbf{a}}\,\vec{\mathbf{b}}]}{[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]} = 1 + 0 = 1$

275 **(b)**

$\left[\frac{1}{2}|\vec{\mathbf{u}}_2 - \vec{\mathbf{u}}_1|\right]^{-2} = \frac{1}{4}[|\vec{\mathbf{u}}_2|^2 + |\vec{\mathbf{u}}_1|^2 - 2\vec{\mathbf{u}}_2 \cdot \vec{\mathbf{u}}_1]$

$= \frac{1}{4}[1 + 1 - 2|\vec{\mathbf{u}}_2||\vec{\mathbf{u}}_1| \cos\theta]$

$= \frac{1}{4}[2 - 2\cos\theta] = \sin^2\frac{\theta}{2}$

$\Rightarrow \frac{1}{2}|\vec{\mathbf{u}}_2 - \vec{\mathbf{u}}_1| = \sin\frac{\theta}{2}$

276 **(b)**

Let $\vec{c} = x\hat{\imath} + y\hat{\jmath}$. Then,

$\vec{b} \perp \vec{c}$

$\Rightarrow \vec{b} \cdot \vec{c} = 0$

$\Rightarrow 4x + 3y = 0 \Rightarrow \frac{x}{3} = \frac{y}{-4} = \lambda \Rightarrow x = 3\lambda, y = -4\lambda$

$\therefore \vec{c} = \lambda(3\hat{\imath} - 4\hat{\jmath})$

Let the required vector be $\alpha = p\hat{\imath} + q\hat{\jmath}$. Then the projections of $\vec{\alpha}$ on $\vec{b}$ and $\vec{c}$ are $\frac{\vec{\alpha}\cdot\vec{b}}{|\vec{c}|}$ respectively

$\therefore \frac{\vec{\alpha} \cdot \vec{b}}{|\vec{b}|} = 1$ and $\frac{\vec{\alpha} \cdot \vec{c}}{|\vec{c}|} = 2$

$\Rightarrow 4p + 3q = 5$ and $3p - 4q = 10 \Rightarrow p = 2, q = -1$

Hence, the required vector $= 2\hat{\imath} - \hat{\jmath}$

277 **(b)**

Given equation of plane is

$2x + 4y - 5z = 10$

Here, $a = 2, b = 4, c = -5$

Let OP be the perpendicular from O to the plane, then its equation is

$\frac{x-0}{2} = \frac{y-0}{4} = \frac{z-0}{-5}$

Here, direction ratio are $(2,4,-5)$.

Now, equation of line in vector form is

$\vec{\mathbf{r}} = 0 + k(2,4,-5)$

$= (2k, 4k, -5k), k \in R$

$[\because$ equation of line is $\vec{\mathbf{r}} = \vec{\mathbf{a}} + \lambda\,\vec{\mathbf{b}}]$

278 **(a)**

We have,

$\vec{a} = \lambda\{\vec{b} \times (\hat{\imath} \times \hat{\jmath})\} = \lambda\{\vec{b} \cdot \hat{\jmath})\hat{\imath} - (\vec{b} \cdot \hat{\imath})\hat{\jmath}\}$

$= \lambda(-3\hat{\imath} - 4\hat{\jmath})$

Now, $|\vec{a}| = |\vec{b}| \Rightarrow 25\lambda^2 = 16 + 9 + 25 \Rightarrow \lambda = \pm\sqrt{2}$

Hence, $\vec{a} = \pm\sqrt{2}(3\hat{\imath} + 4\hat{\jmath})$

279 **(d)**

Given $\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}} + \vec{\mathbf{0}}$

$\Rightarrow |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + |\vec{\mathbf{c}}|^2 + 2(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{a}}) = 0$

$\Rightarrow 3^2 + 4^2 + 5^2 + 2(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{a}}) = 0$

$\Rightarrow \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{a}} = -25$

280 **(a)**

We know that the position vector of the centroid of the triangle is

$$\frac{\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}}{3}$$

Since, the triangle is an equilateral, therefore the orthocentre coincides

With the centroid and hence,

$$\frac{\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}}{3} = \vec{\mathbf{0}}$$

$\Rightarrow \vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}} = \vec{\mathbf{0}}$

281 **(a)**

$\overrightarrow{\mathbf{AB}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}} - 4\hat{\mathbf{i}} - 7\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$

$= -2\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

and $\overrightarrow{\mathbf{AC}} = 2\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 7\hat{\mathbf{k}} - 4\hat{\mathbf{i}} - 7\hat{\mathbf{j}} - 8\hat{\mathbf{k}} = -2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$

$\therefore |\overrightarrow{\mathbf{AB}}| = 6$ and $|\overrightarrow{\mathbf{AC}}|=3$

$\therefore$ Position vector of required bisector

$$= \frac{6(2\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 7\hat{\mathbf{k}}) + 3(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})}{6+3}$$

$$= \frac{1}{3}(6\hat{\mathbf{i}} + 13\hat{\mathbf{j}} + 18\hat{\mathbf{k}})$$

282 **(a)**

Since $\vec{a}$ and $\vec{b}$ are collinear vectors. Therefore,

$\vec{b} = \lambda\vec{a}$

$\Rightarrow \vec{b} = \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$

$\Rightarrow |\vec{b}| = |\lambda|\sqrt{4+9+36} \Rightarrow 21 = 7|\lambda| \Rightarrow \lambda = \pm 3$

$\therefore \vec{b} = \pm 3\vec{a} = \pm(6\hat{\imath} + 9\hat{\jmath} + 18\hat{k})$

283 **(a)**

We have,

$\vec{a} - \vec{b} + \vec{b} - \vec{c} + \vec{c} - \vec{a} = 0$

$\Rightarrow \vec{a} - \vec{b}, \vec{b} - \vec{c}$ and $\vec{c} - \vec{a}$ are coplanar

$\Rightarrow [\vec{a} - \vec{b}\ \vec{b} - \vec{c}\ \vec{c} - \vec{a}] = 0$

284 **(c)**

Here, $(\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + \hat{\mathbf{k}}) = 0$

It means line is parallel to the plane

General point on the line is $(\lambda + 2, -\lambda - 2, 4\lambda + 3)$

For $\lambda = 0$, point on this line is $(2, -2, 3)$ and distance from

$\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + \hat{\mathbf{k}}) = 5$ is

$$d = \left|\frac{2 + 5(-2) + 3 - 5}{\sqrt{(1)^2 + (5)^2 + (1)^2}}\right| = \frac{10}{3\sqrt{3}}$$

286 **(b)**

$\therefore \vec{\mathbf{a}} + \vec{\mathbf{b}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}} + 3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$= 4\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$

and $\vec{\mathbf{a}} - \vec{\mathbf{b}} = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) - (3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

$= -2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

$$\therefore \cos\theta = \frac{(\vec{\mathbf{a}} + \vec{\mathbf{b}}) \cdot (\vec{\mathbf{a}} - \vec{\mathbf{b}})}{|\vec{\mathbf{a}} + \vec{\mathbf{b}}||\vec{\mathbf{a}} - \vec{\mathbf{b}}|}$$

$$= \frac{(4\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) \cdot (-2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}})}{|4\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}||-2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}|}$$

$$= \frac{-8 + 3 + 5}{\sqrt{16+1+1}\sqrt{4+9+25}} = 0$$

$\Rightarrow \theta = 90°$

288 **(a)**

Given, $\vec{\mathbf{a}} = \vec{\mathbf{b}} + \vec{\mathbf{c}}$

and $\vec{\mathbf{b}} \perp \vec{\mathbf{c}}$

then $|\vec{\mathbf{a}}|^2 = |\vec{\mathbf{b}}|^2 + |\vec{\mathbf{c}}|^2 + 2\vec{\mathbf{b}} \cdot \vec{\mathbf{c}}$

$\Rightarrow a^2 = b^2 + c^2 (\because \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} = 0)$

289 **(b)**

$\therefore \overrightarrow{\mathbf{AB}} = \overrightarrow{\mathbf{OB}} - \overrightarrow{\mathbf{OA}}$

$\therefore \overrightarrow{\mathbf{OB}} = \overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{OA}}$

$= 3\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}} + 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

$= 6\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

290 **(a)**

Given that,

$\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \vec{\mathbf{b}} = \hat{\mathbf{i}} + 3\hat{\jmath} + 5\hat{\mathbf{k}}$

and $\vec{\mathbf{c}} = 7\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$

Let $\vec{\mathbf{A}} = \vec{\mathbf{a}} + \vec{\mathbf{b}} = (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) + (\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$

And $\vec{\mathbf{B}} = \vec{\mathbf{b}} + \vec{\mathbf{c}} = (\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) + (7\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 11\mathbf{k}$=8**i**+12**j**+16**k**

If $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are diagonals, then area of parallelogram

$$= \frac{1}{2}|\vec{\mathbf{A}} \times \vec{\mathbf{B}}| = \frac{1}{2}\left\|\begin{matrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 4 & 6 \\ 8 & 12 & 16 \end{matrix}\right\|$$

$$= \frac{1}{2}|\hat{\mathbf{i}}(64 - 72) - \hat{\mathbf{j}}(32 - 48) + \hat{\mathbf{k}}(24 - 32)|$$

$$= \frac{1}{2}|-8\hat{\mathbf{i}} + 16\hat{\mathbf{j}} - 8\hat{\mathbf{k}}|$$

$= |-4\hat{\mathbf{i}} + 8\hat{\mathbf{j}} - 4\hat{\mathbf{k}}|$

$= \sqrt{(-4)^2 + (8)^2 + (-4)^2}$

$= \sqrt{16 + 64 + 16} = \sqrt{96} = 4\sqrt{6}$

291 **(a)**

Given that, $\vec{\mathbf{a}} = (1,1,4) = \hat{\mathbf{i}} + \hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

and $\vec{\mathbf{b}} = (1, -1, 4) = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{k}$

$\therefore \vec{\mathbf{a}} + \vec{\mathbf{b}} = 2\hat{\mathbf{i}} + 8\hat{\mathbf{k}}$

$\Rightarrow \vec{\mathbf{a}} - \vec{\mathbf{b}} = 2\hat{\mathbf{j}}$

Let θ be the angle between $\vec{\mathbf{a}} + \vec{\mathbf{b}}$ and $\vec{\mathbf{a}} - \vec{\mathbf{b}}$, then

$$\cos\theta = \frac{(\vec{\mathbf{a}}+\vec{\mathbf{b}})\cdot(\vec{\mathbf{a}}-\vec{\mathbf{b}})}{|\vec{\mathbf{a}}+\vec{\mathbf{b}}||\vec{\mathbf{a}}-\vec{\mathbf{b}}|}$$

$$= \frac{(2\hat{\mathbf{i}}+0\hat{\mathbf{j}}+8\hat{\mathbf{k}})\cdot(0\hat{\mathbf{i}}+2\hat{\mathbf{j}}+0\hat{\mathbf{k}})}{\sqrt{2^2+0^2+8^2}\sqrt{0^2+2^2+0^2}}$$

$$= \frac{0+0+0}{\sqrt{4+64}\sqrt{4}} = 0$$

$$\Rightarrow \cos\theta = \cos\theta^\circ \;\Rightarrow\; \theta = \frac{\pi}{2} = 90^\circ$$

292 **(c)**

Area of rhombus $= \frac{1}{2}|\vec{\mathbf{a}}\times\vec{\mathbf{b}}|$

$$= \frac{1}{2}\left|(2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+5\hat{\mathbf{k}})\times(-\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})\right|$$

$$= \frac{1}{2}|-8\hat{\mathbf{i}}-7\hat{\mathbf{j}}-\hat{\mathbf{k}}| = \frac{1}{2}\sqrt{144}$$

$$= \sqrt{28.5}$$

293 **(a)**

It is given that the vectors $\hat{\imath}-2x\,\hat{\jmath}-3y\,\hat{k}$ and $\hat{\imath}+3x\,\hat{\jmath}+2y\,\hat{k}$ are orthogonal

$\therefore (\hat{\imath}-2x\,\hat{\jmath}-3y\,\hat{k})\cdot(\hat{\imath}+3x\,\hat{\jmath}+2y\,\hat{k}) = 0$

$\Rightarrow 1-6x^2-6y^2 = 0 \Rightarrow 6x^2+6y^2 = 1$

Clearly, it represents a circle

295 **(a)**

Given vectors are orthogonal.

$\therefore (3x\hat{\mathbf{i}}+y\hat{\mathbf{j}}-3\hat{\mathbf{k}})\cdot(x\hat{\mathbf{i}}-4y\hat{\mathbf{j}}+4\hat{\mathbf{k}}) = 0$

$\Longrightarrow\; 3x^2-4y^2-12 = 0$

$\Longrightarrow\; \frac{x^2}{4}-\frac{y^2}{3} = 1$

Hence, it represent a hyperbola.

296 **(c)**

We have, $|\vec{a}| = 1, |\vec{b}| = 1$ and $|\vec{a}+\vec{b}| = 1$

Now,

$$|\vec{a}+\vec{b}|^2+|\vec{a}-\vec{b}|^2 = 2\left\{|\vec{a}|^2+|\vec{b}|^2\right\}$$

$$\Rightarrow 1+|\vec{a}-\vec{b}|^2 = 4$$

$$\Rightarrow |\vec{a}-\vec{b}| = \sqrt{3}$$

298 **(a)**

Let unit vector is $a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}}$.

$\because\; a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}}$ is perpendicular to $\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$.

Then, $a+b+c = 0$ ...(i)

and $a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}}$, $(\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}})$ and $(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}})$ are coplanar.

$$\therefore \begin{vmatrix} a & b & c \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow\; -3a+b+c = 0$ ....(ii)

From Eqs. (i) and (ii), we get

$a = 0$ and $c = -b$

$\because\; a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}}$ is a unit vector, then

$a^2+b^2+c^2 = 1$

$\Rightarrow 0+b^2+b^2 = 1$

$\Rightarrow b = \frac{1}{\sqrt{2}}$

$$\therefore\; a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}} = \frac{1}{\sqrt{2}}\hat{\mathbf{j}}-\frac{1}{\sqrt{2}}\hat{\mathbf{k}} = \frac{\hat{\mathbf{j}}-\hat{\mathbf{k}}}{\sqrt{2}}$$

300 **(b)**

Given, $\vec{\mathbf{r}} = (1+t)\hat{\mathbf{i}}-(1-t)\hat{\mathbf{j}}+(1-t)\hat{\mathbf{k}}$

and $\vec{\mathbf{r}}\cdot(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}) = 5$

Since, they intersect, therefore

$(1+t)-(1-t)+(1-t) = 5$

$\Longrightarrow t = 4$

$\therefore \vec{\mathbf{r}} = (1+4)\hat{\mathbf{i}}-(1-4)\hat{\mathbf{j}}+(1-4)\hat{\mathbf{k}}$

$= 5\hat{\mathbf{i}}+3\hat{\mathbf{j}}-3\hat{\mathbf{k}}$

301 **(d)**

We have,

$|\vec{a}| = 3, |\vec{b}| = 5$ and $|\vec{c}| = 7$

Let θ be the angle between $\vec{a}$ and $\vec{b}$

Now, $\vec{a}+\vec{b}+\vec{c} = 0$

$\Rightarrow |\vec{c}|^2 = |\vec{a}+\vec{b}|$

$\Rightarrow |\vec{c}|^2 = |\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}$

$\Rightarrow |\vec{c}|^2 = |\vec{a}|^2+|\vec{b}|^2 = 2|\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow 49 = 9+25+2\times3\times5\cos\theta$

$\Rightarrow 15 = 30\cos\theta \Rightarrow \cos\theta = 1/2 \Rightarrow \theta = \pi/3$

302 **(c)**

$$\therefore\; [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}] = \vec{\mathbf{a}}\cdot\left(|\vec{\mathbf{b}}||\vec{\mathbf{c}}|\sin\frac{2\pi}{3}\hat{\mathbf{n}}\right)$$

$$= |\vec{\mathbf{a}}||\vec{\mathbf{b}}||\vec{\mathbf{c}}|\left(\sin\frac{2\pi}{3}\right)$$

$[\because\; \vec{\mathbf{a}}\cdot\hat{\mathbf{n}} = |\vec{\mathbf{a}}|\hat{\mathbf{n}}|\cos 0^\circ = |\vec{\mathbf{a}}|]$

$$= 2\times3\times4\times\frac{\sqrt{3}}{2} = 12\sqrt{3}$$

303 **(a)**

Given that, $\overrightarrow{\mathbf{OA}} = 2\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}, \overrightarrow{\mathbf{OB}} = 3\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $\overrightarrow{\mathbf{OC}} = \hat{\mathbf{i}}+4\hat{\mathbf{j}}-3\hat{\mathbf{k}}$

$\overrightarrow{\mathbf{AB}} = \overrightarrow{\mathbf{OB}}-\overrightarrow{\mathbf{OA}}$

$= (3-2)\hat{\mathbf{i}}+(-2-1)\hat{\mathbf{j}}+(1+1)\hat{\mathbf{k}}$

$= \hat{\mathbf{i}}-3\hat{\mathbf{j}}+2\hat{\mathbf{k}}$

$|\overrightarrow{\mathbf{AB}}| = \sqrt{1^2+(-3)^2+2^2}$

$= \sqrt{1+9+4} = \sqrt{14}$

$\overrightarrow{\mathbf{BC}} = \overrightarrow{\mathbf{OC}}-\overrightarrow{\mathbf{OB}}$

$= (1-3)\hat{\mathbf{i}}+(4+2)\hat{\mathbf{j}}+(-3-1)\hat{\mathbf{k}}$

$= -2\hat{\mathbf{i}}+6\hat{\mathbf{j}}-4\hat{\mathbf{k}}$

$|\overrightarrow{\mathbf{BC}}| = \sqrt{(-2)^2+6^2+(-4)^2}$

$= \sqrt{4+36+16} = \sqrt{56}$

$\overrightarrow{\mathbf{CA}} = \overrightarrow{\mathbf{OA}}-\overrightarrow{\mathbf{OC}}$

$= (2-1)\hat{\mathbf{i}}+(1-4)\hat{\mathbf{j}}+(-1+3)\hat{\mathbf{k}}$

$= \hat{\mathbf{i}}-3\hat{\mathbf{j}}+2\hat{\mathbf{k}}$

$|\overrightarrow{\mathbf{CA}}| = \sqrt{1^2 + (-3)^2 + (2)^2}$

$= \sqrt{1 + 9 + 4} = \sqrt{14}$

It is clear that two sides of a triangle are equal.

$\therefore$ Points A, B, C from an isosceles triangle.

304 **(b)**

The component of $\vec{a}$ along $\vec{b}$ is given by

$$\left\{\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2}\right\} = \frac{18}{25}(3\hat{j} + 4\hat{k})$$

305 **(a)**

It is given that $\vec{c}$ and $\vec{d}$ are collinear vectors

$\therefore \vec{c} = \lambda\, \vec{d}$ for some scalar λ

$\Rightarrow (x-2)\vec{a} + \vec{b} = \lambda\{(2x+1)\vec{a} - \vec{b}\}$

$\Rightarrow \{x - 2 - \lambda(2x+1)\}\vec{a} + (\lambda + 1)\vec{b} = \vec{0}$

$\Rightarrow \lambda + 1 = 0$ and $x - 2 - \lambda(2x+1) = 0$ [$\because \vec{a}, \vec{b}$ are non-collinear]

$\Rightarrow \lambda = -1$ and $x = \frac{1}{3}$

306 **(a)**

Equation of plane is $\vec{\mathbf{r}} \cdot \hat{\mathbf{n}} = d$,

where d is the perpendicular distance of the plane from origin

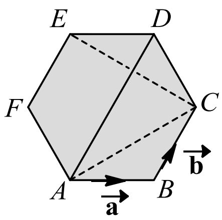
$\therefore$ Required plane is $(\alpha x + \beta y + \gamma z) = p$

307 **(c)**

In $\Delta A\,BC$, $\overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{BC}} + \overrightarrow{\mathbf{AC}}$

$\Longrightarrow \overrightarrow{\mathbf{AC}} = \vec{\mathbf{a}} + \vec{\mathbf{b}}$

AD is parallel to BC and $AD = 2\,BC$



$\therefore\ \overrightarrow{\mathbf{AD}} = 2\vec{\mathbf{b}}$

In ΔACD, $\overrightarrow{\mathbf{AC}} + \overrightarrow{\mathbf{CD}} = \overrightarrow{\mathbf{AD}}$

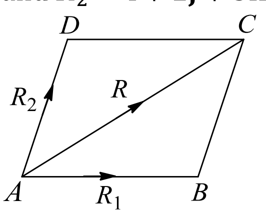
$\Longrightarrow \overrightarrow{\mathbf{CD}} = 2\vec{\mathbf{b}} - (\vec{\mathbf{a}} + \vec{\mathbf{b}}) = \vec{\mathbf{b}} - \vec{\mathbf{a}}$

Now, $\overrightarrow{\mathbf{CE}} = \overrightarrow{\mathbf{CD}} + \overrightarrow{\mathbf{DE}} = \vec{\mathbf{b}} - 2\vec{\mathbf{a}}$

309 **(d)**

Let $\vec{\mathbf{R}}_1 = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

and $\vec{\mathbf{R}}_2 = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$



$\therefore\ \vec{\mathbf{R}}$ (along $\overrightarrow{\mathrm{AC}}$) $= \vec{\mathbf{R}}_1 + \vec{\mathbf{R}}_2 = 3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

$\therefore\ \vec{\mathbf{a}}$ (unit vector angle $\overrightarrow{\mathrm{AC}}$) $= \dfrac{\vec{\mathbf{R}}}{|\vec{\mathbf{R}}|} = \dfrac{3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}}{\sqrt{9 + 36 + 4}}$

$= \frac{1}{7}(3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

311 **(b)**

Since $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors. Therefore, $\vec{a}, \vec{b}, \vec{c}$ are linearly independent vectors

$\therefore x\vec{a} + y\,\vec{b} + z\,\vec{c} = \vec{0} \Rightarrow x = y = z = 0$

312 **(a)**

Suppose point $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ divides the join of points $-2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$ and $7\hat{\imath} - \hat{k}$ in the ratio $\lambda : 1$. Then,

$$\hat{\imath} + 2\hat{\jmath} + 3\hat{k} = \frac{\lambda(7\hat{\imath} - \hat{k}) + (-2\hat{\imath} + 3\hat{\jmath} + 5\hat{k})}{\lambda + 1}$$

$\Rightarrow (\lambda+1)\hat{\imath} + 2(\lambda+1)\hat{\jmath} + 3(\lambda+1)\hat{k}$

$= (7\lambda - 2)\hat{\imath} + 3\hat{\jmath} + (-\lambda + 5)\hat{k}$

$\Rightarrow \lambda + 1 = 7\lambda - 2, 2(\lambda+1) = 3$ and $3(\lambda+1) = -\lambda + 5$

$\Rightarrow \lambda = \dfrac{1}{2}$

Hence, required ratio is 1 : 2

313 **(d)**

Clearly,

$\vec{a} - \vec{b} + \vec{b} - \vec{c} + \vec{c} - \vec{a} = \vec{0}$

$\therefore \vec{a} - \vec{b}, \vec{b} - \vec{c}, \vec{c} - \vec{a}$ are coplanar

$\Rightarrow (\vec{a} - \vec{b}) \cdot \{(\vec{b} \cdot \vec{c}) \times (\vec{c} - \vec{a})\} = 0$

314 **(d)**

Two given lines intersect, if

$7\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 13\hat{\mathbf{k}} + s(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$

$= 3\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 7\hat{\mathbf{k}} + t(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

$\Longrightarrow (7 + 2s)\hat{\mathbf{i}} + (10 + 3s)\hat{\mathbf{j}} + (13 + 4s)\hat{\mathbf{k}}$

$= (3 + t)\hat{\mathbf{i}} + (5 + 2t)\hat{\mathbf{j}} + (7 + 3t)\hat{\mathbf{k}}$

$\Longrightarrow 7 + 2s = 3 + t$

$\Longrightarrow 2s - t = -4$ ...(i)

$10 + 3s = 5 + 2t$

$\Longrightarrow 3s - 2t = -5$ ...(ii)

and $13 + 4s = 7 + 3t$

$\Longrightarrow 4s - 3t = -6$ ...(iii)

On solving Eqs. (i) and (iii), we get

$s = -3, t = -2$

$\therefore$ Required line is

$7\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 13\hat{\mathbf{k}} + (-3)[2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}]$

$\Longrightarrow\ \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ is the required line.

316 **(c)**

Given that, $\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\vec{\mathbf{b}} = 2\hat{\mathbf{i}} - \hat{\mathbf{k}}$

Let $\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$, then

$\vec{\mathbf{r}} \times \vec{\mathbf{a}} = \vec{\mathbf{b}} \times \vec{\mathbf{a}} \Rightarrow (\vec{\mathbf{r}} - \vec{\mathbf{b}}) \times \vec{\mathbf{a}} = \vec{\mathbf{0}}$

Now, $\vec{\mathbf{r}} - \vec{\mathbf{b}} = (x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}) - (2\hat{\mathbf{i}} - \hat{\mathbf{k}})$

$= (x - 2)\hat{\mathbf{i}} + y\hat{\mathbf{j}} + (z + 1)\hat{\mathbf{k}}$

$\therefore\ (\vec{\mathbf{r}}-\vec{\mathbf{b}})\times\vec{\mathbf{a}}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ x-2 & y & z+1\\ 1 & 1 & 0\end{vmatrix}=\vec{\mathbf{0}}$

$\Rightarrow\ -(z+1)\hat{\mathbf{i}}+(z+1)\hat{\mathbf{j}}+(x-2-y)\hat{\mathbf{k}}=\vec{\mathbf{0}}$

On equating the coefficient of $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$, we get

$z=-1, x-y=2$ ....(i)

Now, $\vec{\mathbf{r}}\times\vec{\mathbf{b}}=\vec{\mathbf{a}}\times\vec{\mathbf{b}}\ \Rightarrow\ (\vec{\mathbf{r}}-\vec{\mathbf{a}})\times\vec{\mathbf{b}}=\vec{\mathbf{0}}$

And $\vec{\mathbf{r}}-\vec{\mathbf{a}}=(x-1)\hat{\mathbf{i}}+(y-1)\hat{\mathbf{j}}+z\hat{\mathbf{k}}$

$\therefore (\vec{\mathbf{r}}-\vec{\mathbf{a}})\times\vec{\mathbf{b}}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ x-1 & y-1 & z\\ 2 & 0 & -1\end{vmatrix}=\vec{\mathbf{0}}$

$\Rightarrow (-y+1)\hat{\mathbf{i}}-\hat{\mathbf{j}}(-x+1-2z)+(-2y+2)\hat{\mathbf{k}}=\vec{\mathbf{0}}$

$\Rightarrow y=1, x+2z=1$ ...(ii)

From Eqs. (i) and (ii), we get

$x=3, y=1\ z=-1$

$\therefore \vec{\mathbf{r}}=\ 3\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$

317 **(a)**

Given, $\vec{\mathbf{A}}\times(\vec{\mathbf{B}}\times\vec{\mathbf{C}})=\vec{\mathbf{B}}\times(\vec{\mathbf{C}}\times\vec{\mathbf{A}})$ ....(*i*)

Also, $[\vec{\mathbf{A}}\,\vec{\mathbf{B}}\,\vec{\mathbf{C}}]\neq 0 ie. \vec{\mathbf{A}},\vec{\mathbf{B}},\vec{\mathbf{C}}$ are not coplanar.

$\therefore$From Eq. (i)

$(\vec{\mathbf{A}}\cdot\vec{\mathbf{C}})-(\vec{\mathbf{A}}\cdot\vec{\mathbf{B}})\vec{\mathbf{C}}=(\vec{\mathbf{B}}\cdot\vec{\mathbf{A}})\vec{\mathbf{C}}-(\vec{\mathbf{B}}\cdot\vec{\mathbf{C}})\vec{\mathbf{A}}$

$\Longrightarrow\ (\vec{\mathbf{B}}\cdot\vec{\mathbf{C}})\vec{\mathbf{A}}+(\vec{\mathbf{A}}\cdot\vec{\mathbf{C}})\vec{\mathbf{B}}-[(\vec{\mathbf{A}}\cdot\vec{\mathbf{B}})+(\vec{\mathbf{B}}\cdot\vec{\mathbf{C}})]\vec{\mathbf{C}}=\vec{\mathbf{0}}$

$\Longrightarrow\ \vec{\mathbf{B}}\cdot\vec{\mathbf{C}}=\vec{\mathbf{A}}\cdot\vec{\mathbf{C}}=\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}=\vec{\mathbf{0}}$

$[\because [\vec{\mathbf{A}}\,\vec{\mathbf{B}}\,\vec{\mathbf{C}}]\neq 0]$

Now, consider

$\vec{\mathbf{A}}\times(\vec{\mathbf{B}}\times\vec{\mathbf{C}})=(\vec{\mathbf{A}}\cdot\vec{\mathbf{C}})\vec{\mathbf{B}}-(\vec{\mathbf{A}}\cdot\vec{\mathbf{B}})\vec{\mathbf{C}}$

$=0\cdot\vec{\mathbf{B}}-0\cdot\vec{\mathbf{C}}=\vec{\mathbf{0}}$

319 **(a)**

$[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]=\begin{vmatrix}1 & 0 & -1\\ x & 1 & 1-x\\ y & x & 1+x-y\end{vmatrix}$

Applying $C_3\to C_3+C_1$

$=\begin{vmatrix}1 & 0 & 0\\ x & 1 & 1\\ y & x & 1+x\end{vmatrix}=1[1+x-x]=1$

Hence, $[\vec{\mathbf{a}}\ \vec{\mathbf{b}}\,\vec{\mathbf{c}}]$ does not depend upon neither x nor y.

320 **(b)**

The required vector is given by

$\hat{n}=\dfrac{\vec{AB}\times\vec{AC}}{|\vec{AB}\times\vec{AC}|}=\dfrac{\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}}{|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}$

321 **(d)**

$(\vec{\mathbf{a}}-\vec{\mathbf{b}})\cdot(\vec{\mathbf{b}}+\vec{\mathbf{c}})\times(\vec{\mathbf{c}}+\vec{\mathbf{a}})$

$=(\vec{\mathbf{a}}-\vec{\mathbf{b}})\cdot[\vec{\mathbf{b}}\times\vec{\mathbf{c}}+\vec{\mathbf{b}}\times\vec{\mathbf{a}}+\vec{\mathbf{c}}\times\vec{\mathbf{c}}+\vec{\mathbf{c}}\times\vec{\mathbf{a}}]$

$=\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})+\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{a}})+\vec{\mathbf{a}}\cdot(\vec{\mathbf{c}}\times\vec{\mathbf{a}})-\vec{\mathbf{b}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})-\vec{\mathbf{b}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{a}})-\vec{\mathbf{b}}\cdot(\vec{\mathbf{c}}\times\vec{\mathbf{a}})$

$=\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})-\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})$

$=[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]-[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]=0$

322 **(b)**

$\because \vec{\mathbf{a}},\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are coplanar vectors, so $2\,\vec{\mathbf{a}}-\vec{\mathbf{b}}\ 2\vec{\mathbf{b}}\ -\vec{\mathbf{c}}$ and $2\vec{\mathbf{c}}-\vec{\mathbf{a}}$ are also coplanar. Thus

$[2\,\vec{\mathbf{a}}-\vec{\mathbf{b}}\ 2\vec{\mathbf{b}}\ -\vec{\mathbf{c}}\ \ 2\vec{\mathbf{c}}-\vec{\mathbf{a}}]=0$

323 **(b)**

Clearly, angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}=\frac{\pi}{2}$

$\Rightarrow\ \vec{\mathbf{a}}\ \cdot\ \vec{\mathbf{b}}=0$

$\therefore |\vec{\mathbf{a}}+\ \vec{\mathbf{b}}|^2=a^2+b^2+2\vec{\mathbf{a}}\ \cdot\ \vec{\mathbf{b}}$

$=1+1+0=2$

$\Rightarrow\ |\vec{\mathbf{a}}+\ \vec{\mathbf{b}}|=\sqrt{2}$

325 **(d)**

Given, $(\vec{\mathbf{a}}\times\vec{\mathbf{b}})\times\vec{\mathbf{c}}=-\dfrac{1}{4}|\vec{\mathbf{b}}||\vec{\mathbf{c}}|\vec{\mathbf{a}}$

$\Longrightarrow (\vec{\mathbf{c}}\cdot\vec{\mathbf{a}})\vec{\mathbf{b}}-(\vec{\mathbf{c}}\cdot\vec{\mathbf{b}})\vec{\mathbf{a}}=-\dfrac{1}{4}|\vec{\mathbf{b}}||\vec{\mathbf{c}}|\vec{\mathbf{a}}$

On comparing both sides, we get

$(\vec{\mathbf{c}}\cdot\vec{\mathbf{a}})\vec{\mathbf{b}}=0$

$|\vec{\mathbf{c}}|\vec{\mathbf{a}}|\cos\theta=0$

$\Longrightarrow\ \theta=\dfrac{\pi}{2}$

326 **(c)**

Now, $(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})\times(\hat{\mathbf{i}}+\hat{\mathbf{j}})=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 1 & 1 & 1\\ 1 & 1 & 0\end{vmatrix}$

$=\hat{\mathbf{i}}(-1)+\hat{\mathbf{j}}(1)+\hat{\mathbf{k}}(0)=-\hat{\mathbf{i}}+\hat{\mathbf{j}}$

and $|(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})\times(\hat{\mathbf{i}}+\hat{\mathbf{j}})|=\sqrt{1^2+1^2}=\sqrt{2}$

Vector perpendicular to both of the vectors $\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $\hat{\mathbf{i}}+\hat{\mathbf{j}}$

$=\dfrac{(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})\times(\hat{\mathbf{i}}+\hat{\mathbf{j}})}{|(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})\times(\hat{\mathbf{i}}+\hat{\mathbf{j}})|}$

$=\dfrac{-\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}=\dfrac{-1}{\sqrt{2}}(\hat{\mathbf{i}}-\hat{\mathbf{j}})$

$=c(\hat{\mathbf{i}}-\hat{\mathbf{j}})$, c is a scalar.

327 **(b)**

It is given that $(\vec{a}+\vec{b})||\vec{c}$ and $(\vec{c}+\vec{a})||\vec{b}$

$\therefore (\vec{a}+\vec{b})\times\vec{c}=0$ and $(\vec{c}+\vec{a})\times\vec{b}=0$

$\Rightarrow \vec{a}\times\vec{c}+\vec{b}\times\vec{c}=0$ and $\vec{c}\times\vec{b}+\vec{a}\times\vec{b}=0$

$\Rightarrow \vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

Hence, $\vec{a},\vec{b},\vec{c}$ form the sides of a triangle

328 **(a)**

$\because$ Displacement, $\overrightarrow{\mathbf{AB}}=(3-2)\hat{\mathbf{i}}+(1+1)\hat{\mathbf{j}}+(2-1)\hat{\mathbf{k}}$

$=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}$

and force, $\vec{\mathbf{F}}=\dfrac{\sqrt{6}(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}})}{\sqrt{6}}$

$=(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}})$

$\therefore$ Work done $=\vec{\mathbf{F}}\cdot\overrightarrow{\mathbf{AB}}=(1+2\hat{\mathbf{j}}+\hat{\mathbf{k}})\cdot$

$(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) = 6$

329 **(c)**

let $\vec{\mathbf{a}} = l\hat{\mathbf{i}} + m\hat{\mathbf{j}} + n\hat{\mathbf{k}}$ makes an angle $\frac{\pi}{4}$ with z-axis

Also, $l^2 + m^2 + n^2 = 1$

Here, $n = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}, \qquad l^2 + m^2 = \frac{1}{2} \qquad \dots\dots(i)$

$\therefore\ \vec{\mathbf{a}} = l\hat{\mathbf{i}} + m\hat{\mathbf{j}} + \frac{\hat{\mathbf{k}}}{\sqrt{2}}$

$\Longrightarrow\ \vec{\mathbf{a}} + \hat{\mathbf{i}} + \hat{\mathbf{j}} = (l+1)\hat{\mathbf{i}}(m+1)\hat{\mathbf{j}} + \frac{\hat{\mathbf{k}}}{\sqrt{2}}$

$\Longrightarrow |\vec{\mathbf{a}} + \hat{\mathbf{i}} + \hat{\mathbf{j}}| = \sqrt{(l+1)^2 + (m+1)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$

$\Longrightarrow 1 = l^2 + m^2 + 2 + 2l + 2m + \frac{1}{2}$

$\Longrightarrow l + m = -1$ (From Eq. (i)

$\Longrightarrow\ l^2 + m^2 + 2lm = 1$

$\Longrightarrow 2lm = \frac{1}{2}$

$\Longrightarrow l = m = -\frac{1}{2}$

$\left(\because l = m = \frac{1}{2}\right.$, is not satisfied the given equition$\left.\right)$

$\therefore\ \vec{\mathbf{a}} = -\frac{\hat{\mathbf{i}}}{2} - \frac{\hat{\mathbf{j}}}{2} + \frac{\hat{\mathbf{k}}}{\sqrt{2}}$

330 **(b)**

Given, $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}|^2 + |\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}|^2 = 144$

$\Longrightarrow |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2(\sin^2\theta + \cos^2\theta) = 144$

$\Longrightarrow 16|\vec{\mathbf{b}}|^2 = 144$

$\Longrightarrow |\vec{\mathbf{b}}| = 3$

331 **(c)**

Since, $m\,\vec{\mathbf{a}}$ is a unit vector, if and only, if

$|m\,\vec{\mathbf{a}}| = 1\ \Rightarrow |m\,||\vec{\mathbf{a}}| = 1\ \Rightarrow m|\vec{\mathbf{a}}| = 1$

$\Rightarrow m = \frac{1}{|\vec{\mathbf{a}}|}$

332 **(b)**

Resultant force $\vec{F}$ is given by

$\vec{F} = (2\hat{\imath} - 5\hat{\jmath} + 6\hat{k}) - (-\hat{\imath} + 2\hat{\jmath} - \hat{k}) = \hat{\imath} - 3\hat{\jmath} + 5\hat{k}$

Let $\vec{d}$ be the displacement vector. Then,

$\vec{d} = \overrightarrow{AB}$

$\Rightarrow \vec{d} = (6\hat{\imath} + \hat{\jmath} - 3\hat{k}) - (4\hat{\imath} - 3\hat{\jmath} - 2\hat{k})$
$= 2\hat{\imath} + 4\hat{\jmath} - \hat{k}$

$\therefore W =$ Work done

$\Rightarrow W = \vec{F} \cdot \vec{d}$

$\Rightarrow W = (\hat{\imath} - 3\hat{\jmath} + 5\hat{k}) \cdot (2\hat{\imath} + 4\hat{\jmath} - \hat{k})$

$\Rightarrow W = 2 - 12 - 5 = -15$ units

333 **(d)**

Since, P, Q, R are collinear. Therefore,

$\overrightarrow{PQ} = m\,\overrightarrow{QR}$ for same scalar m

$\Rightarrow -2\hat{\jmath} = m[(a-1)\hat{\imath} + (\vec{b}+1)\hat{\jmath} + c\,\hat{k}]$ for some non-zero scalar m

$\Rightarrow (a-1)m = 0, (b+1)m = -2, cm = 0$

$\Rightarrow a = 1, c = 0, b \in R$

334 **(b)**

The direction cosines of a vector making equal angles with the coordinate axes are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

Therefore, the unit vector along the vector making equal angles with the coordinate axes is

$\vec{b} = \frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} + \frac{1}{\sqrt{3}}\hat{k}$

$\therefore$ Projection of $\vec{a}$ on $\vec{b} = \vec{a} \cdot \vec{b}$

$= (4\hat{\imath} - 3\hat{\jmath} + 2\hat{k}) \cdot \left(\frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} + \frac{1}{\sqrt{3}}\hat{k}\right)$
$= \frac{4 - 3 + 2}{\sqrt{3}} = \sqrt{3}$

335 **(a)**

$[2\,\hat{\mathbf{i}}\ 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}]$

$= -30\,[\,\hat{\mathbf{i}}\ \hat{\mathbf{j}}\ \hat{\mathbf{k}}\,]$

$= -30 \quad (\because [\,\hat{\mathbf{i}}\ \hat{\mathbf{j}}\ \hat{\mathbf{k}}\,] = 1\,)$

336 **(b)**

We have,

$(\vec{a} \times \vec{b}) \times (\vec{a} \times \vec{c}) \cdot \vec{d}$

$= \left\{\left((\vec{a} \times \vec{b}) \cdot \vec{c}\right)\vec{a} - \left((\vec{a} \times \vec{b}) \cdot \vec{a}\right)\vec{c}\right\} \cdot \vec{d}$

$= \{[\vec{a}\,\vec{b}\,\vec{c}]\vec{a} - 0\} \cdot \vec{d} = [\vec{a}\,\vec{b}\,\vec{c}](\vec{a} \cdot \vec{d})$

337 **(d)**

Resultant force $\vec{\mathbf{F}} = (2\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 6\hat{\mathbf{k}}) + (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}})$

$= \hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

and displacement, $\vec{\mathbf{d}} = (6\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}) - (4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

$= 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}}$

$\therefore$ work done $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}}$

$= (\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) \cdot (2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}})$

$= -15$

$= 15$ units [neglecting – ve sign]

338 **(a)**

The resultant force is given by

$\vec{F} = 6\frac{(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})}{\sqrt{1+4+4}} + 7\frac{(2\hat{\imath} - 3\hat{\jmath} - 6\hat{k})}{\sqrt{4+9+36}}$
$= 4\hat{\imath} - 7\hat{\jmath} - 2\hat{k}$

$\vec{d} =$ Displacement $= \overrightarrow{PQ}$

$\vec{d} = (5\hat{\imath} - \hat{\jmath} + \hat{k}) - (2\hat{\imath} - \hat{\jmath} - 3\hat{k}) = 3\hat{\imath} + 4\hat{k}$

$\therefore$ Work done $= \vec{F} \cdot \vec{d} = 12 + 0 - 8 = 4$ units

339 **(c)**

We know, $[\vec{\mathbf{b}} \times \vec{\mathbf{c}}\ \ \vec{\mathbf{c}} \times \vec{\mathbf{a}}\ \ \vec{\mathbf{a}} \times \vec{\mathbf{b}}]$

$= (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) \cdot [(\vec{\mathbf{c}} \times \vec{\mathbf{a}}) \cdot (\vec{\mathbf{a}} \times \vec{\mathbf{b}})]$

$= (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) \cdot \left[\left((\vec{\mathbf{c}} \times \vec{\mathbf{a}}) \cdot \vec{\mathbf{b}}\right)\vec{\mathbf{a}} - ((\vec{\mathbf{c}} \times \vec{\mathbf{a}}) \cdot \vec{\mathbf{a}})\vec{\mathbf{b}} \right]$

$= (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) \cdot ([\vec{\mathbf{c}}\,\vec{\mathbf{a}}\,\vec{\mathbf{b}}]\vec{\mathbf{a}} - [\vec{\mathbf{c}}\,\vec{\mathbf{a}}\,\vec{\mathbf{a}}]\vec{\mathbf{b}})$

$= (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) \cdot \vec{\mathbf{a}}[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\vec{\mathbf{c}}] - 0$

$= [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}][\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$

$= [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]^2$

340 **(d)**

$\because$ $\overrightarrow{\mathbf{QP}}$ is parallel to $\overrightarrow{\mathbf{AB}}$ and $\overrightarrow{\mathbf{DC}}$.

$\therefore$ $\overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{DC}} = \overrightarrow{\mathbf{QP}} + \overrightarrow{\mathbf{QP}} = 2\overrightarrow{\mathbf{QP}}$

341 **(a)**

Taking A as the origin, let the position vectors of B and C be $\vec{b}$ and $\vec{c}$ respectively

$\therefore \vec{B}E + \vec{A}F = \left(\frac{\vec{c}}{2} - \vec{b}\right) + \left(\frac{\vec{b} + \vec{c}}{2} - \vec{0}\right) = \vec{c} - \frac{\vec{b}}{2}$

$= \vec{D}C$

342 **(a)**

Since, $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular unit vectors.

$\Rightarrow |\vec{\mathbf{a}}| = |\vec{\mathbf{b}}| = |\vec{\mathbf{c}}| = 1$

and $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} = \vec{\mathbf{c}} \cdot \vec{\mathbf{a}} = 0$ .....(i)

Now, $|\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}|^2 = (\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}) \cdot (\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}})$

$= |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + |\vec{\mathbf{c}}|^2 + 2(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{c}} \cdot \vec{\mathbf{a}})$

$= 1 + 1 + 1 + 0 = 3$ [from Eq. (i)]

$\Rightarrow |\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}| = \sqrt{3}$

343 **(c)**

Any vector lying in the plane of $\vec{a}$ and $\vec{b}$ is of the from $x\vec{a} + y\vec{b}$

It is given that $\vec{c}$ is parallel to the plane of $\vec{a}$ and $\vec{b}$

$\therefore \vec{c} = \lambda(x\vec{a} + y\vec{b})$ for some scalar λ

$\Rightarrow d\hat{\imath} + \hat{\jmath} + (2d - 1)\hat{k}$

$= \lambda\{x(\hat{\imath} - 2\hat{\jmath} + 3\hat{k})$

$+ y(3\hat{\imath} + 3\hat{\jmath} - \hat{k})\}$

$\Rightarrow d\hat{\imath} + \hat{\jmath} + (2d - 1)\hat{k}$

$= \lambda\{(x + 3y)\hat{\imath} + (-2x + 3y)\hat{\jmath}$

$+ (3x - y)\hat{k}\}$

$\Rightarrow \lambda(x + 3y) = d, \lambda(-2x + 3y) = 1$ and $\lambda(3x - y) = (2d - 1)$

$[\because \hat{\imath}, \hat{\jmath}, \hat{k}$ are non $-$ coplanar$]$

Solving $\lambda(x + 3y) = d$ and $3x - y = 2d - 1$, we get

$x = \frac{7d-3}{10\lambda}$ and $y = \frac{d+1}{10\lambda}$

Substituting these values in $\lambda(x + 3y) = d$, we get $11d = -1$

<u>ALTER</u> *clearly*, $\vec{c}$ *is perpendicular to* $\vec{a} \times \vec{b}$

$\therefore \vec{c} \cdot (\vec{a} \times \vec{b}) = 0$

$\Rightarrow [\vec{c}\vec{a}\vec{b}] = 0 \Rightarrow \begin{vmatrix} d & 1 & 2d-1 \\ 1 & -2 & 3 \\ 3 & 3 & -1 \end{vmatrix} = 0 \Rightarrow 11d$

$= -1$

344 **(c)**

$\because$ $\vec{\mathbf{p}}, \vec{\mathbf{q}}, \vec{\mathbf{r}}$ are reciprocal vectors $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ respectively.

$\therefore \vec{\mathbf{p}} \cdot \vec{\mathbf{a}} = 1, \vec{\mathbf{p}} \cdot \vec{\mathbf{b}} = 0, \vec{\mathbf{p}} \cdot \vec{\mathbf{c}}$ etc.

$\therefore (l\vec{\mathbf{a}} + m\vec{\mathbf{b}} + n\vec{\mathbf{c}}) \cdot (l\vec{\mathbf{p}} + m\vec{\mathbf{q}} + n\vec{\mathbf{r}})$

$= l^2 + m^2 + n^2$

345 **(b)**

Given expression $= 2(1 + 1 + 1) - 2\sum(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})$

$= 6 - 2\sum(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})$ ...(i)

But $(\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}})^2 \geq 0$

$\therefore (1 + 1 + 1) + 2\sum \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \geq 0$

$\therefore 3 \geq -2\sum \vec{\mathbf{a}} \cdot \vec{\mathbf{b}}$ ...(ii)

From relations (i) and (ii), we get

Given expression $\leq 6 + 3 = 9$

346 **(a)**

Let $\overrightarrow{\mathbf{OA}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\overrightarrow{\mathbf{OB}} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

$\therefore$ $\overrightarrow{\mathbf{AB}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\therefore$ work don, $W = \vec{\mathbf{F}} \cdot \overrightarrow{\mathbf{AB}}$

$= (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \cdot (2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

$= 4 - 6 + 4 = 2$

347 **(d)**

$\overrightarrow{\mathbf{AC}} = (a\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}) - (2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}) = (a - 2)\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$

and $\overrightarrow{\mathbf{BC}} = (a\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}) - (\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}) =$ $(a - 1)\hat{\mathbf{i}} + 6\hat{\mathbf{k}}$

Since, the ΔABC is right angled at C, then

$\overrightarrow{\mathbf{AC}} \cdot \overrightarrow{\mathbf{BC}} = 0$

$\Longrightarrow \{(a - 2)\hat{\mathbf{i}} - 2\hat{\mathbf{j}}\} \cdot \{(a - 1)\hat{\mathbf{i}} + 6\hat{\mathbf{k}}\} = 0$

$\Longrightarrow (a - 2)(a - 1) = 0 \Longrightarrow a = 1$and 2

348 **(a)**

We have,

$(\vec{a} \times \vec{b}) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$

$\Leftrightarrow -\vec{c} \times (\vec{a} \times \vec{b}) = \vec{a} \times (\vec{b} \times \vec{c})$

$\Leftrightarrow -\{(\vec{c} \cdot \vec{b})\vec{a} - (\vec{c} \cdot \vec{a})\vec{b}\} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

$\Leftrightarrow (\vec{a} \cdot \vec{b})\vec{c} - (\vec{c} \cdot \vec{b})\vec{a} = 0$

$\Leftrightarrow (\vec{b} \cdot \vec{a})\vec{c} - (\vec{b} \cdot \vec{c})\vec{a} = 0$

$\Leftrightarrow \vec{b} \times (\vec{c} \times \vec{a}) = 0$

349 **(b)**

Clearly,

$(\vec{a} + \vec{b}) \times \{\vec{c} - (\vec{a} + \vec{b})\}$

$= (\vec{a} + \vec{b}) \times \vec{c} - (\vec{a} + \vec{b}) \times (\vec{a} + \vec{b}) = (\vec{a} + \vec{b}) \times \vec{c}$

350 **(a)**

$\overrightarrow{\mathbf{PQ}} = (2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}) - (\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

$= \hat{\mathbf{i}} + \hat{\mathbf{k}}$

and $\vec{\mathbf{F}} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

$\therefore$ Moment $= |\overrightarrow{\mathbf{PQ}} \times \vec{\mathbf{F}}|$

$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 0 & 1 \\ 3 & 2 & -4 \end{vmatrix}$

$= -2\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\therefore$Magnitude of moment$= \sqrt{4+49+4} = \sqrt{57}$

351 **(b)**

Since, $|\vec{\mathbf{a}} + \vec{\mathbf{b}}| = \sqrt{3}$

$\Longrightarrow |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + 2\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 3$

$\Longrightarrow \ \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = \frac{1}{2} \ldots\ldots \text{(i)}$

$\because [|\vec{\mathbf{a}}| = |\vec{\mathbf{b}}| = 1, \text{given}]$

$\therefore (3\vec{\mathbf{a}} - 4\vec{\mathbf{b}}) \cdot (2\vec{\mathbf{a}} + 5\vec{\mathbf{b}}) = 6 + 7\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} - 20$

$= 6 + \frac{7}{2} - 20$

$= -\frac{21}{2}$ [from Eq. (i)]

352 **(c)**

We have,

$\hat{a} \times (\hat{b} \times \hat{c}) = \frac{1}{2}\hat{b}$

$\Rightarrow (\hat{a} \cdot \hat{c})\hat{b} - (\hat{a} \cdot \hat{b})\hat{c} = \frac{1}{2}\hat{b}$

$\Rightarrow \left\{(\hat{a} \cdot \hat{c}) - \frac{1}{2}\right\}\hat{b} - (\hat{a} \cdot \hat{b})\hat{c} = 0$

$\Rightarrow \hat{a} \cdot \hat{c} - \frac{1}{2} = 0$ and $\hat{a} \cdot \hat{b}$

$= 0 \left[\begin{array}{c} \because \hat{b}, \hat{c} \\ \text{are non} - \text{collinear vectors} \end{array}\right]$

$\Rightarrow \cos\theta = \frac{1}{2}$, where θ is the angle between $\hat{a}$ and $\hat{c}$

$\Rightarrow \theta = \pi/3$

354 **(b)**

The given line is parallel to the vector $\vec{\mathbf{n}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$. The required plane passing through the point $(2, 3, 1)$ *ie*, $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and is perpendicular to the vector $\vec{\mathbf{n}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\therefore$ Its equation is

$[(\vec{\mathbf{r}} - (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}})] \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 0$

$\Longrightarrow \vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 1$

355 **(c)**

$(\vec{\mathbf{a}} - \vec{\mathbf{b}}) \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}} - \vec{\mathbf{b}} \times \vec{\mathbf{a}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}})$

$= \vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) - \vec{\mathbf{b}} \times (\vec{\mathbf{c}} \times \vec{\mathbf{a}})$

$= [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}] - [\vec{\mathbf{b}}\,\vec{\mathbf{c}}\,\vec{\mathbf{a}}] = 0$

356 **(a)**

We have,

$|\hat{n}_1 + \hat{n}_2|^2 = |\hat{n}_1| + |\hat{n}_2| + 2\hat{n}_1 \cdot \hat{n}_2$

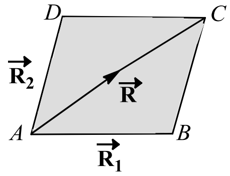
$\Rightarrow |\hat{n}_1 + \hat{n}_2|^2 = |\hat{n}_1|^2 + |\hat{n}_2|^2 + 2|\hat{n}_1| + |\hat{n}_2| \cos\theta$

$\Rightarrow |\hat{n}_1 + \hat{n}_2|^2 = 1 + 1 + 2\cos\theta = 4\cos^2\frac{\theta}{2}$

$\therefore \cos\frac{\theta}{2} = \frac{1}{2}|\hat{n}_1 + \hat{n}_2|$

357 **(d)**

Let $\vec{\mathbf{R}}_1 = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$



and $\vec{\mathbf{R}}_2 = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

$\therefore \ \vec{\mathbf{R}}$ (along $\overrightarrow{\mathbf{AC}}$) $= \vec{\mathbf{R}}_1 + \vec{\mathbf{R}}_2$

$= 3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

$\therefore \ \vec{\mathbf{a}}$ (unit vector along AC) $= \frac{\vec{\mathbf{R}}}{|\vec{\mathbf{R}}|}$

$= \frac{3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}}{\sqrt{9+36+4}}$

$= \frac{1}{7}(3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

358 **(a)**

Let $P(60\hat{\mathbf{i}} + 3\hat{\mathbf{j}}), Q(40\hat{\mathbf{i}} - 8\hat{\mathbf{j}})$and $R(a\hat{\mathbf{i}} - 52\hat{\mathbf{j}})$ be the collinear points. Then $\overrightarrow{\mathbf{PQ}} = \lambda\overrightarrow{\mathbf{QR}}$ for some scalar λ

$\Longrightarrow (-20\hat{\mathbf{i}} - 11\hat{\mathbf{j}}) = \lambda[(a - 40)\hat{\mathbf{i}} - 44\hat{\mathbf{j}}]$

$\Longrightarrow \ \lambda(a - 40) = -20, -44\lambda = -11$

$\Longrightarrow \ \lambda(a - 40) = -20, \lambda = \frac{1}{4}$

$\therefore a - 40 = -20 \times 4 \Longrightarrow a = -40$

359 **(a)**

We have,

$\vec{a} + \vec{b} + \vec{c} = \alpha\vec{d}$ and $\vec{b} + \vec{c} + \vec{d} = \beta\vec{a}$

$\Rightarrow \vec{a} + \vec{b} + \vec{c} + \vec{d} = (\alpha + 1)\vec{d}$ and $\vec{a} + \vec{b} + \vec{c} + \vec{d} = (\beta + 1)\vec{a}$

$\Rightarrow (\alpha + 1)\vec{d} = (\beta + 1)\vec{a}$

If $\alpha \neq -1$, then

$(\alpha + 1)\vec{d} = (\beta + 1)\vec{a} \Rightarrow \vec{d} = \frac{\beta + 1}{\alpha + 1}\vec{a}$

$\therefore \vec{a} + \vec{b} + \vec{c} = \alpha\vec{d}$

$\Rightarrow \vec{a} + \vec{b} + \vec{c} = \alpha\left(\frac{\beta + 1}{\alpha + 1}\right)\vec{a}$

$\Rightarrow \left\{1 - \frac{\alpha(\beta + 1)}{\alpha + 1}\right\}\vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow \vec{a}, \vec{b}, \vec{c}$ are coplanar

It is a contradiction to the given condition

$\therefore \alpha = -1 \Rightarrow \vec{a} + \vec{b} + \vec{c} = 0$

360 **(c)**

Let the unit vector $\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}$ is perpendicular to $\hat{\mathbf{i}} - \hat{\mathbf{j}}$, then we get

$$\frac{(\hat{\mathbf{i}}+\hat{\mathbf{j}})\cdot(\hat{\mathbf{i}}-\hat{\mathbf{j}})}{\sqrt{2}}=\frac{1-1}{\sqrt{2}}=0$$

$\therefore$ $\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}$ is the unit vector

361 **(c)**

We have,

$$\left.\begin{aligned}\vec{r}\cdot\vec{a}=0\Rightarrow\vec{r}\perp\vec{a}\\ \vec{r}\cdot\vec{b}=0\Rightarrow\vec{r}\perp\vec{b}\\ \vec{r}\cdot\vec{c}=0\Rightarrow\vec{r}\perp\vec{c}\end{aligned}\right\}\Rightarrow\vec{a},\vec{b},\vec{c}\text{ are coplanar}$$

Hence, $[\vec{a}\vec{b}\vec{c}]=0$

362 **(b)**

$$\cos\frac{\pi}{3}=\frac{(\hat{\mathbf{i}}+\hat{\mathbf{k}})\cdot(\hat{\mathbf{i}}+\hat{\mathbf{j}}+a\hat{\mathbf{k}})}{\sqrt{2}\sqrt{1+1+a^2}}$$

$$\Rightarrow\frac{1}{2}=\frac{1+a}{\sqrt{2}\sqrt{2+a^2}}$$

$$\Rightarrow\frac{1}{4}=\frac{(1+a)^2}{2(2+a^2)}$$

$\Rightarrow 2+a^2=2(1+a^2+2a)$

$\Rightarrow a^2+4a=0$

$\Rightarrow a=0,-4$

363 **(b)**

Let the required vector be $\vec{a}=x\hat{\imath}+y\hat{\jmath}+z\hat{k}$

It makes equal angles with the unit vectors

$\vec{b}=\frac{1}{3}(\hat{\imath}-2\hat{\jmath}+2\hat{k}),\vec{c}=\frac{1}{5}(-4\hat{\imath}-3\hat{k})$ and $\vec{d}=\hat{\jmath}$

$\therefore\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}=\vec{a}\cdot\vec{d}$ $[\because\vec{b},\vec{c},\vec{d}$ are unit vectors$]$

$\Rightarrow\frac{1}{3}(x-2y+2z)=\frac{1}{5}(-4x-3z)=y$

$\Rightarrow x-2y+2z=3y$ and $-4x-5y-3z=0$

$\Rightarrow x-5y+2z=0$ and $4x+5y+3z=0$

$\Rightarrow\frac{x}{-5}=\frac{y}{1}=\frac{z}{5}=\lambda$ (say)

$\Rightarrow x=-5\lambda,y=\lambda,z=5\lambda$ for some scalar λ

$\Rightarrow\vec{a}=\lambda(-5\hat{\imath}+\hat{\jmath}+5\hat{k})$

Clearly, option (b) is true for $\lambda=1$

364 **(d)**

$$\vec{\mathbf{a}}\times\vec{\mathbf{b}}=\begin{vmatrix}\hat{\mathbf{i}}&\hat{\mathbf{j}}&\hat{\mathbf{k}}\\2&2&1\\1&-2&2\end{vmatrix}$$

$=\hat{\mathbf{i}}(4+2)-\hat{\mathbf{j}}(4-1)+\hat{\mathbf{k}}(-4-2)$

$=6\hat{\mathbf{i}}-3\hat{\mathbf{j}}-6\hat{\mathbf{k}}$

$\Rightarrow|\vec{\mathbf{a}}\times\vec{\mathbf{b}}|=\sqrt{36+9+36}=\sqrt{81}=9$

$\therefore$ Required vectors are

$$\pm6\left|\frac{\vec{\mathbf{a}}\times\vec{\mathbf{b}}}{|\vec{\mathbf{a}}\times\vec{\mathbf{b}}|}\right|$$

$=\pm\frac{6}{9}(6\hat{\mathbf{i}}-3\hat{\mathbf{j}}-6\hat{\mathbf{k}})$

$=\pm2(2\hat{\mathbf{i}}-\hat{\mathbf{j}}-2\hat{\mathbf{k}})$

366 **(d)**

(a) Let $\vec{\mathbf{p}}=x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}}$ where at least one of x,y,z is non-zero. Let

$\vec{\mathbf{a}}=a_1\hat{\mathbf{i}}+a_2\hat{\mathbf{l}}+a_3\hat{\mathbf{k}}$

$\vec{\mathbf{b}}=b_1\hat{\mathbf{i}}+b_2\hat{\mathbf{l}}+b_3\hat{\mathbf{k}}$

$\vec{\mathbf{c}}=c_1\hat{\mathbf{i}}+c_2\hat{\mathbf{l}}+c_3\hat{\mathbf{k}}$

$\therefore$ By given conditions

$a_1x+a_2y+a_3z=0$

$b_1x+b_2y+b_3z=0$

$c_1x+c_2y+c_3z=0$

$$\Rightarrow\begin{vmatrix}a_1&a_2&a_3\\b_1&b_2&b_3\\c_1&c_3&c_3\end{vmatrix}=0$$

$\Rightarrow[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]=0$

$\Rightarrow$ $\vec{\mathbf{a}},\vec{\mathbf{b}},\vec{\mathbf{c}}$ are coplanar.

(b) Vectors are coplanar, if

$$\begin{vmatrix}1&3&0\\2&0&1\\0&1&1\end{vmatrix}=0$$

ie, $-7=0$

Which is not possible.

(c) $\vec{\mathbf{a}}\times(\vec{\mathbf{b}}\times\vec{\mathbf{c}})=(\vec{\mathbf{a}}\cdot\vec{\mathbf{c}})\vec{\mathbf{b}}-(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}})\vec{\mathbf{c}}$

$\Rightarrow$ $\vec{\mathbf{a}}\times(\vec{\mathbf{b}}\times\vec{\mathbf{c}})$ is coplanar with $\vec{\mathrm{b}}$ and $\vec{\mathrm{c}}$.

(d) $|\vec{\mathbf{a}}|=|\vec{\mathbf{b}}|=1$

$\therefore|\vec{\mathbf{a}}+\vec{\mathbf{b}}|^2=(\vec{\mathbf{a}}+\vec{\mathbf{b}})\cdot(\vec{\mathbf{a}}+\vec{\mathbf{b}})$

$=|\vec{\mathbf{a}}|^2+|\vec{\mathbf{b}}|^2+2\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}$

$=1+1=2\cdot1\cdot1\cos\frac{\pi}{3}=3$

$\Rightarrow$ $|\vec{\mathbf{a}}+\vec{\mathbf{b}}|=\sqrt{3}>1$

367 **(d)**

Here, $\vec{\mathbf{a}}_1=3\hat{\mathbf{i}}+6\hat{\mathbf{j}}$, $\vec{\mathbf{a}}_2=-2\hat{\mathbf{i}}+7\hat{\mathbf{k}}$

$\vec{\mathbf{b}}_1=-4\hat{\mathbf{i}}+3\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and $\vec{\mathbf{b}}_2=-4\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$

Now, $\vec{\mathbf{a}}_2-\vec{\mathbf{a}}_1=\hat{\mathbf{i}}-6\hat{\mathbf{j}}+7\hat{\mathbf{k}}$

and

$$\vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2 = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -4 & 3 & 2 \\ -4 & 1 & 1 \end{vmatrix} = \hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$$

$\Rightarrow |\vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2| = \sqrt{1+16+64} = 9$

Now,

$(\vec{\mathbf{a}}_2 - \vec{\mathbf{a}}_1) \cdot (\vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2)$

$= (\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 7\hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 8\hat{\mathbf{k}})$

$= 1 + 24 + 56 = 81$

$\therefore$ Shortest distance,

$$d = \left| \frac{(\vec{\mathbf{a}}_2 - \vec{\mathbf{a}}_1) \cdot (\vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2)}{|\vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2|} \right|$$

$= \frac{81}{9} = 9$ unit

368 **(b)**

We know that a vector perpendicular to the plane containing the points $\vec{\mathbf{A}}, \vec{\mathbf{B}}, \vec{\mathbf{C}}$ is given by

$\vec{\mathbf{A}} \times \vec{\mathbf{B}} + \vec{\mathbf{B}} \times \vec{\mathbf{C}} + \vec{\mathbf{C}} \times \vec{\mathbf{A}}$.

Given, $\vec{\mathbf{A}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}, \vec{\mathbf{B}} = 2\hat{\mathbf{i}} + 0\hat{\mathbf{j}} - \hat{\mathbf{k}}$

and $\vec{\mathbf{C}} = 0\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$

Now,

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -1 & 2 \\ 2 & 0 & -1 \end{vmatrix} = \hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} \times \vec{\mathbf{C}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 0 & -1 \\ 0 & 2 & 1 \end{vmatrix} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$$

$$\vec{\mathbf{C}} \times \vec{\mathbf{A}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 2 & 1 \\ 1 & -1 & 2 \end{vmatrix} = 5\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$$

Thus,

$\vec{\mathbf{A}} \times \vec{\mathbf{B}} + \vec{\mathbf{B}} \times \vec{\mathbf{C}} + \vec{\mathbf{C}} \times \vec{\mathbf{A}}$

$= (\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) + (2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) + (5\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

$= 8\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

369 **(c)**

Given,

$(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \cdot (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) = \frac{1}{4}$

$\Rightarrow (|\vec{\mathbf{a}}||\vec{\mathbf{b}}|\sin\theta)^2 = \frac{1}{4}$

$\Rightarrow \sin^2\theta = \frac{1}{4}$

$\Rightarrow \theta = \frac{\pi}{6}$

370 **(b)**

Given that, $|\vec{\mathbf{a}}| = 3, |\vec{\mathbf{b}}| = 4$ and $\vec{\mathbf{a}} + \lambda\vec{\mathbf{b}}$ is perpendicular to $\vec{\mathbf{a}} - \lambda\vec{\mathbf{b}}$.

$\therefore (\vec{\mathbf{a}} + \lambda\vec{\mathbf{b}}) \cdot (\vec{\mathbf{a}} - \lambda\vec{\mathbf{b}}) = 0$

$\Rightarrow \vec{\mathbf{a}} \cdot \vec{\mathbf{a}} - \vec{\mathbf{a}} \cdot \vec{\mathbf{b}}\lambda + \lambda\vec{\mathbf{b}} \cdot \vec{\mathbf{a}} - \lambda^2\vec{\mathbf{b}} \cdot \vec{\mathbf{b}} = 0$

$\Rightarrow |\vec{\mathbf{a}}|^2 - \lambda^2|\vec{\mathbf{b}}|^2 = 0$

$\Rightarrow \lambda^2 = \frac{|\vec{\mathbf{a}}|^2}{|\vec{\mathbf{b}}|^2} \Rightarrow \lambda = \frac{|\vec{\mathbf{a}}|}{|\vec{\mathbf{b}}|} = \frac{3}{4}$

371 **(a)**

$(\vec{\mathbf{x}} - \vec{\mathbf{y}}) \times (\vec{\mathbf{x}} + \vec{\mathbf{y}})$

$= \vec{\mathbf{x}} \times \vec{\mathbf{x}} + \vec{\mathbf{x}} \times \vec{\mathbf{y}} - \vec{\mathbf{y}} \times \vec{\mathbf{x}} - \vec{\mathbf{y}} \times \vec{\mathbf{y}}$

$= \vec{\mathbf{0}} + \vec{\mathbf{x}} \times \vec{\mathbf{y}} + \vec{\mathbf{x}} \times \vec{\mathbf{y}} - \vec{\mathbf{0}}$

$= 2(\vec{\mathbf{x}} \times \vec{\mathbf{y}})$

372 **(a)**

$\vec{\mathbf{a}} \cdot \vec{\mathbf{c}} = 0$

$\Rightarrow \lambda - 1 + 2\mu = 0$

$\Rightarrow \lambda + 2\mu = 1$ ...(i)

$\vec{\mathbf{b}} \cdot \vec{\mathbf{c}} = 0$

$\Rightarrow 2\lambda + 4 + \mu = 0$

$\Rightarrow 2\lambda + \mu = -4$....(ii)

On solving Eqs. (i) and (ii), we get

$\lambda - 3, \mu = 2$

375 **(b)**

The projection $\vec{x} \times \vec{y}$ on $\vec{z}$ is given by

$$\frac{(\vec{x} \times \vec{y}) \cdot \vec{z}}{|\vec{z}|} = \frac{1}{|\vec{z}|}[\vec{x}\ \vec{y}\ \vec{z}] = \frac{1}{13}\begin{vmatrix} 3 & -6 & -1 \\ 1 & 4 & -3 \\ 3 & -4 & -12 \end{vmatrix}$$

$= -14$

376 **(c)**

We have,

$\vec{a} \times \{\vec{a} \times \{\vec{a} \times (\vec{a} \times \vec{b})\}\}$

$= \vec{a} \times \{\vec{a} \times \{(\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b}\}\}$

$= \vec{a} \times \{\vec{0} - |\vec{a}|^2(\vec{a} \times \vec{b})\}$

$= -|\vec{a}|^2\{\vec{a} \times (\vec{a} \times \vec{b})\} = -|\vec{a}|^2\{(\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b}\}$

$= -|\vec{a}|^2\{0 - |\vec{a}|^2\vec{b}\} = |\vec{a}|^4\vec{b}$

379 **(c)**

For an abtuse angle

$(cx\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) \cdot (x\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2cx\hat{\mathbf{k}}) < 0$

$\Rightarrow cx^2 - 12 + 6cx < 0$

$\Rightarrow cx^2 + 6cx - 12 < 0$

$\therefore (6c)^2 - 4c(-12) < 0 \quad [\because f(x) < 0 \Rightarrow D < 0]$

$\Rightarrow 36c\left(c + \frac{4}{3}\right) < 0$

$\Rightarrow -\frac{4}{3} < c < 0$

380 **(a)**

$$\cos\theta = \frac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{a}}||\vec{\mathbf{b}}|}$$

$$= \frac{(2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}) \cdot (6\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})}{\sqrt{2^2 + 2^2 + (-1)^2}\sqrt{6^2 + (-3)^2 + 2^2}}$$

$$= \frac{12 - 6 - 2}{\sqrt{4+4+1}\sqrt{36+9+4}} = \frac{4}{21}$$

381 **(b)**

Given vectors $2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$ and $a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ will be perpendicular, if

$(2\hat{\imath}+3\hat{\jmath}-4\hat{k}).(a\hat{\imath}+b\hat{\jmath}+c\hat{k})=0 \Rightarrow 2a+3b-c=0$

Clearly, $a=4, b=4, c=5$ satisfy the above equation

382 **(a)**

We have ,$\vec{\alpha}=x(\vec{\mathbf{a}}\times\vec{\mathbf{b}})+y(\vec{\mathbf{b}}\times\vec{\mathbf{c}})+z(\vec{\mathbf{c}}\times\vec{\mathbf{a}})$

Taking dot product with $\vec{\mathbf{a}},\vec{\mathbf{b}},\vec{\mathbf{c}}$ respectively, we get

$\vec{\alpha}\cdot\vec{\mathbf{a}}=y[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}] \Rightarrow y=8(\vec{\alpha}\cdot\vec{\mathbf{a}})$

$\vec{\alpha}\cdot\vec{\mathbf{b}}=z((\vec{\mathbf{c}}\times\vec{\mathbf{a}})\cdot\vec{\mathbf{b}})$

$\Rightarrow \vec{\alpha}\cdot\vec{\mathbf{b}}=z[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}] \Rightarrow z=8(\vec{\alpha}\cdot\vec{\mathbf{b}})$

and $\vec{\alpha}\cdot\vec{\mathbf{c}}=x(\vec{\mathbf{a}}\times\vec{\mathbf{b}}.\vec{\mathbf{c}})$

$\vec{\alpha}\cdot\vec{\mathbf{c}}=x[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}] \Rightarrow x=8(\vec{\alpha}\cdot\vec{\mathbf{c}})$

$\therefore x+y+z=8\vec{\alpha}\cdot(\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}})$

383 **(d)**

Let $\vec{\mathbf{c}}=3\hat{\mathbf{i}}+\hat{\mathbf{j}}-5\hat{\mathbf{k}}$ and $\vec{\mathbf{d}}=a\hat{\mathbf{i}}+b\hat{\mathbf{j}}-15\hat{\mathbf{k}}$

For collinears, $\vec{\mathbf{c}}\times\vec{\mathbf{d}}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 3 & 1 & -5\\ a & b & -15\end{vmatrix}=\vec{\mathbf{0}}$

$\Rightarrow \hat{\mathbf{i}}(-15+5b)-\hat{\mathbf{j}}(-45+5a)+\hat{\mathbf{k}}(3b-a)=\vec{\mathbf{0}}$

$\Rightarrow -15+5b=0, \quad -45+5a=0, \quad 3b-a=0$

$\Rightarrow b=3, a=9$

384 **(d)**

$|\vec{\mathbf{a}}-\vec{\mathbf{b}}|^2=|\vec{\mathbf{a}}|^2+|\vec{\mathbf{b}}|^2-2|\vec{\mathbf{a}}||\vec{\mathbf{b}}|\cos\theta$

$=1^2+1^2 2\cdot 1\cdot 1\cdot\cos 60° \quad [\because |\vec{\mathbf{a}}|=|\vec{\mathbf{b}}|=1]$

$=2-2\cdot\frac{1}{2}=1$

385 **(c)**

Let $\vec{\mathbf{a}}=\hat{\mathbf{i}}-2\hat{\mathbf{j}}-3\hat{\mathbf{k}}$, $\vec{\mathbf{b}}=2\hat{\mathbf{i}}+0\hat{\mathbf{j}}+0\hat{\mathbf{k}}$

Now take option (c).

Let $\vec{\mathbf{c}}=0\hat{\mathbf{i}}-4\hat{\mathbf{j}}-6\hat{\mathbf{k}}$

Now, $\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})=\begin{vmatrix}1 & -2 & -3\\ 2 & 0 & 0\\ 0 & -4 & -6\end{vmatrix}$

$=1(0)+2(-12)-3(-8)=0$

386 **(a)**

$(\vec{\mathbf{a}}+\vec{\mathbf{b}})\times(\vec{\mathbf{a}}\times\vec{\mathbf{b}})$

$=\vec{\mathbf{a}}\times(\vec{\mathbf{a}}\times\vec{\mathbf{b}})+\vec{\mathbf{b}}\times(\vec{\mathbf{a}}\times\vec{\mathbf{b}})$

$=(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}})\vec{\mathbf{a}}-(\vec{\mathbf{a}}\cdot\vec{\mathbf{a}})\vec{\mathbf{b}}+(\vec{\mathbf{b}}\cdot\vec{\mathbf{b}})\vec{\mathbf{a}}-(\vec{\mathbf{b}}\cdot\vec{\mathbf{a}})\vec{\mathbf{b}}$

$=(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}})\vec{\mathbf{a}}-\vec{\mathbf{b}}+\vec{\mathbf{a}}-(\vec{\mathbf{b}}\cdot\vec{\mathbf{a}})\vec{\mathbf{b}}$

$=(\vec{\mathbf{a}}-\vec{\mathbf{b}})(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}-1)$

$\therefore$ Given vector is parallel to $(\vec{\mathbf{a}}-\vec{\mathbf{b}})$.

387 **(a)**

$\overrightarrow{\mathbf{AB}}=(2-1)\hat{\mathbf{i}}+(0-2)\hat{\mathbf{j}}+(3+1)\hat{\mathbf{k}}$

$=\hat{\mathbf{i}}-2\hat{\mathbf{j}}+4\hat{\mathbf{k}}$

and

$\overrightarrow{\mathbf{AC}}=(3-1)\hat{\mathbf{i}}+(-1-2)\hat{\mathbf{j}}+(2+1)\hat{\mathbf{k}}$

$=2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+3\hat{\mathbf{k}}$

$\cos\theta=\frac{(\hat{\mathbf{i}}-2\hat{\mathbf{j}}+4\hat{\mathbf{k}})\cdot(2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+3\hat{\mathbf{k}})}{\sqrt{1+4+16}\sqrt{4+9+9}}$

$=\frac{2+6+12}{\sqrt{21}\sqrt{22}}=\frac{20}{\sqrt{462}}$

$\Rightarrow \sqrt{462}\cos\theta=20$

388 **(c)**

$[\vec{\mathbf{u}}\,\vec{\mathbf{v}}\,\vec{\mathbf{w}}]=|\vec{\mathbf{u}}\cdot(\vec{\mathbf{v}}\times\vec{\mathbf{w}})|$

$=|\vec{\mathbf{u}}\cdot(3\hat{\mathbf{i}}-7\hat{\mathbf{j}}-\hat{\mathbf{k}})|$

$=|\vec{\mathbf{u}}|\sqrt{59}\cos\theta$

$\therefore$ Maximum value of $[\vec{\mathbf{u}}\,\vec{\mathbf{v}}\,\vec{\mathbf{w}}]=\sqrt{59}$ $(\because |\vec{\mathbf{u}}|=1, \cos\theta\le 1)$

390 **(b)**

Given, force $=5\left(\frac{2\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}}}{|2\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}}|}\right)=\frac{5}{3}(2\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}})$

Displacement $=(5\hat{\mathbf{i}}+3\hat{\mathbf{j}}+7\hat{\mathbf{k}})-(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}})$

$=(4\hat{\mathbf{i}}+\hat{\mathbf{j}}+4\hat{\mathbf{k}})$

$\therefore$ Required work done = Force $\cdot$ Displacement

$=\frac{5}{3}[(2\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}})\cdot(4\hat{\mathbf{i}}+\hat{\mathbf{j}}+4\hat{\mathbf{k}})]$

$=\frac{5}{3}[8-2+4]=\frac{50}{3}$ unit

391 **(b)**

We know that the equation of the plane passing through three non-collinear points $\vec{\mathbf{a}},\vec{\mathbf{b}},\vec{\mathbf{c}}$ is

$\vec{\mathbf{r}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}}+\vec{\mathbf{c}}\times\vec{\mathbf{a}}+\vec{\mathbf{a}}\times\vec{\mathbf{b}})=[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$

392 **(a)**

We have,

Required vector $\vec{r}=\lambda(\hat{a}+\hat{b})$, λ is a scalar

$\Rightarrow \vec{r}=\lambda\left\{\frac{1}{9}(7\hat{\imath}-4\hat{\jmath}-4\hat{k})+\frac{1}{3}(-2\hat{\imath}-\hat{\jmath}+2\hat{k})\right\}$

$=\frac{\lambda}{9}(\hat{\imath}-7\hat{\jmath}+2\hat{k})$

Now,

$|\vec{r}|=3\sqrt{6} \Rightarrow |\vec{r}|^2=54 \Rightarrow \frac{\lambda^2}{81}(1+49+4)=54$

$\Rightarrow \lambda=\pm 9$

Hence, required vector $\vec{r}=\pm(\hat{\imath}-7\hat{\jmath}+2\hat{k})$

Clearly, option (a) is true for $\lambda=1$

393 **(b)**

Given vectors are collinear, if $\begin{vmatrix}2 & 1 & 1\\ 6 & -1 & 2\\ 14 & -5 & p\end{vmatrix}=0$

$\Rightarrow 2[-p+10]-1[6p-28]+1[-30+14]=0$

$\Rightarrow -8p+32=0$

$\Rightarrow p=4$

394 **(d)**

Given,

$\frac{1}{3}|\vec{\mathbf{b}}||\vec{\mathbf{c}}|\vec{\mathbf{a}} = (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times \vec{\mathbf{c}}$

$\therefore \frac{1}{3}|\vec{\mathbf{b}}||\vec{\mathbf{c}}|\vec{\mathbf{a}} = (\vec{\mathbf{a}} \cdot \vec{\mathbf{c}})\vec{\mathbf{b}} - (\vec{\mathbf{b}} \cdot \vec{\mathbf{c}})\vec{\mathbf{a}}$

On comparing the coefficient of $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$, we get

$\frac{1}{2}|\vec{\mathbf{b}}||\vec{\mathbf{c}}| = -\vec{\mathbf{b}} \cdot \vec{\mathbf{c}}$ and $\vec{\mathbf{a}} \cdot \vec{\mathbf{c}} = 0$

$\Rightarrow \frac{1}{3}|\vec{\mathbf{b}}||\vec{\mathbf{c}}| = -|\vec{\mathbf{b}}||\vec{\mathbf{c}}|\cos\theta \Rightarrow \cos\theta = -\frac{1}{3}$

$\Rightarrow 1 - \sin^2\theta = \frac{1}{9} \Rightarrow \sin\theta = \frac{2\sqrt{2}}{3}$

395 **(c)**

Let $\vec{\mathbf{A}} = 7\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$, $\vec{\mathbf{B}} = -\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ and $\vec{\mathbf{C}} = -4\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$

Now, $\overrightarrow{\mathbf{AB}} = -\hat{\mathbf{i}} - \hat{\mathbf{j}} - 4\hat{\mathbf{k}}$, $\quad \overrightarrow{\mathbf{BC}} = -3\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$

and $\overrightarrow{\mathbf{CA}} = 4\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

Here, $|\overrightarrow{\mathbf{AB}}| = |\overrightarrow{\mathbf{BC}}| = 3\sqrt{2}$ and $|\overrightarrow{\mathbf{CA}}| = 6$

Now, $|\overrightarrow{\mathbf{AB}}|^2 + |\overrightarrow{\mathbf{BC}}|^2 = |\overrightarrow{\mathbf{AC}}|^2$

Hence, the triangle is right angled isosceles triangle.

396 **(b)**

We know that if A and B are two points and P is any point on AB. Then,

$m\,P\vec{A} + n\,P\vec{B} = (m+n)P\vec{C}$, where C divides AB in the ratio $n:m$

Here, $m = n = 1$

$\therefore P\vec{A} + P\vec{B} = 2P\vec{C}$

397 **(a)**

$(2\vec{\mathbf{a}} + 3\vec{\mathbf{b}}) \times (5\vec{\mathbf{a}} + 7\vec{\mathbf{b}}) + \vec{\mathbf{a}} \times \vec{\mathbf{b}}$

$= \vec{\mathbf{0}} + 14(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) - 15(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) + \vec{\mathbf{0}} + \vec{\mathbf{a}} \times \vec{\mathbf{b}}$

$= \vec{\mathbf{0}}$

399 **(c)**

Let $\overrightarrow{\mathbf{OA}} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}, \overrightarrow{\mathbf{OB}} = \hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

and $\overrightarrow{\mathbf{OC}} = 3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

$\therefore a = |\overrightarrow{\mathbf{OA}}| = \sqrt{6}, b = |\overrightarrow{\mathbf{OB}}| = \sqrt{35}$

and $\vec{\mathbf{c}}|\overrightarrow{\mathbf{OC}}| = \sqrt{41}$

$\therefore \cos A = \frac{b^2 + c^2 + a^2}{2bc}$

$= \frac{(\sqrt{35})^2 + (\sqrt{41})^2 - (\sqrt{6})^2}{2\sqrt{35}\sqrt{41}}$

$\Rightarrow \cos A = \sqrt{\frac{35}{41}}$

$\Rightarrow \sin^2 A = \frac{35}{41}$

400 **(d)**

Let $\vec{p} \neq \vec{0}$. Then,

$\vec{r} \cdot \vec{a} = \vec{r} \cdot \vec{b} = \vec{r} \cdot \vec{c} = 0$

$\Rightarrow \vec{a}, \vec{b}, \vec{c}$ are coplanar, which is a contradiction

Hence, $\vec{r} = \vec{0}$

401 **(c)**

Let $\vec{\alpha} = \lambda\,\vec{\mathbf{a}} + \mu\vec{\mathbf{b}} + t\vec{\mathbf{c}} \quad$ ...(i)

Now, $\vec{\mathbf{a}} \cdot \vec{\mathbf{p}} = \vec{\mathbf{b}} \cdot \vec{\mathbf{q}} = \vec{\mathbf{c}} \cdot \vec{\mathbf{r}} = 1$

$\Rightarrow \vec{\alpha} \cdot \vec{\mathbf{p}} = \lambda(\vec{\mathbf{a}} \cdot \vec{\mathbf{p}}) + 0 + 0$

$\Rightarrow \lambda = \vec{\alpha} \cdot \vec{\mathbf{p}}$

Similarly, $\mu = \vec{\alpha} \cdot \vec{\mathbf{q}}$

and $t = \vec{\alpha} \cdot \vec{\mathbf{r}}$

From Eq. (i), we get

$\vec{\alpha} = (\vec{\alpha} \cdot \vec{\mathbf{p}})\vec{\mathbf{a}} + (\vec{\alpha} \cdot \vec{\mathbf{q}})\vec{\mathbf{b}} + (\vec{\alpha} \cdot \vec{\mathbf{r}})\vec{\mathbf{c}}$

402 **(a)**

Since, $\vec{\mathbf{b}} \times \vec{\mathbf{c}}$ is a vector perpendicular to $\vec{\mathbf{b}}, \vec{\mathbf{c}}$.

Therefore $\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}})$ is a vector again in plane of $\vec{\mathbf{b}}, \vec{\mathbf{c}}$.

403 **(c)**

$(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{b}} + \vec{\mathbf{b}} \times (\vec{\mathbf{a}} \times \vec{\mathbf{b}})$

$= (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{b}} + (\vec{\mathbf{b}} \cdot \vec{\mathbf{b}})\vec{\mathbf{a}} - (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{b}}$

$= \vec{\mathbf{a}} \quad [\because |\vec{\mathbf{b}}| = 1]$

404 **(d)**

$\because \sum_{i=1}^{n} \vec{\mathbf{a}}_i = \vec{\mathbf{0}}$

$\therefore \left(\sum_{i=1}^{n} \vec{\mathbf{a}}_i\right)\left(\sum_{i=1}^{n} \vec{\mathbf{a}}_j\right)$

$= \sum_{i=1}^{n} |\vec{\mathbf{a}}_i|^2 + 2\sum_{1 \le i <}\sum_{j \le n} \vec{\mathbf{a}}_i \cdot \vec{\mathbf{a}}_j$

$\Rightarrow 0 = n + 2\sum_{1 \le i <}\sum_{j \le n} \vec{\mathbf{a}}_i \cdot \vec{\mathbf{a}}_j$

$\therefore \sum_{1 \le i <}\sum_{j \le n} \vec{\mathbf{a}}_i \cdot \vec{\mathbf{a}}_j = -\frac{n}{2}$

405 **(b)**

Since, given vectors are perpendicular.

$\therefore (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 5\hat{\mathbf{k}}) \cdot (6\hat{\mathbf{i}} - \hat{\mathbf{j}} + c\hat{\mathbf{k}}) = 0$

$\Rightarrow 18 + 2 - 5c = 0 \Rightarrow c = 4$

406 **(d)**

Given, $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \vec{\mathbf{0}}$ and $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 0$

$\Rightarrow$ $\vec{\mathbf{a}}$ is parallel to $\vec{\mathbf{b}}$ and $\vec{\mathbf{a}}$ is perpendicular to $\vec{\mathbf{b}}$ which is possible only if

$\vec{\mathbf{a}} = \vec{\mathbf{0}}$ or $\vec{\mathbf{b}} = \vec{\mathbf{0}}$

407 **(a)**

Let $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$, $\vec{\mathbf{b}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

First diagonal, $\vec{\mathbf{a}} + \vec{\mathbf{b}} = 3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

$\Rightarrow |\vec{\mathbf{a}} + \vec{\mathbf{b}}| = 7$

Second diagonal, $\vec{\mathbf{a}} - \vec{\mathbf{b}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$

$\Rightarrow |\vec{\mathbf{a}} - \vec{\mathbf{b}}| = \sqrt{69}$

408 **(b)**

Given $\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}=\vec{\mathbf{0}}$

$\Rightarrow\ \vec{\mathbf{a}}\times\vec{\mathbf{a}}+\vec{\mathbf{a}}\times\vec{\mathbf{b}}+\vec{\mathbf{a}}\times\vec{\mathbf{c}}=0$

$\Rightarrow\ \vec{\mathbf{a}}\times\vec{\mathbf{b}}=\vec{\mathbf{c}}\times\vec{\mathbf{a}}$

Similarly, $\vec{\mathbf{b}}\times\vec{\mathbf{c}}=\vec{\mathbf{c}}\times\vec{\mathbf{a}}$

$\therefore\ \vec{\mathbf{a}}\times\vec{\mathbf{b}}=\vec{\mathbf{b}}\times\vec{\mathbf{c}}=\vec{\mathbf{c}}\times\vec{\mathbf{a}}\neq\vec{\mathbf{0}}$

Alternate: Since, $\vec{\mathbf{a}},\vec{\mathbf{b}},\vec{\mathbf{c}}$ are unit vectors and $\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{a}}+\vec{\mathbf{c}}=\vec{\mathbf{0}}$,

so $\vec{\mathbf{a}},\vec{\mathbf{b}},\vec{\mathbf{c}}$ represent an equilateral triangle.

$\therefore\ \vec{\mathbf{a}}\times\vec{\mathbf{b}}=\vec{\mathbf{b}}\times\vec{\mathbf{c}}=\vec{\mathbf{c}}\times\vec{\mathbf{a}}\neq\vec{\mathbf{0}}$

409 **(c)**

We have,

$\vec{AB}+\vec{AC}+\vec{AD}+\vec{AE}+\vec{AF}$

$=\vec{ED}+\vec{AC}+\vec{AD}+\vec{AE}+\vec{CD}\quad[\because \vec{AB}=\vec{ED}\text{ and }\vec{AF}=\vec{CD}]$

$=\left(\vec{AC}+\vec{CD}\right)+\left(\vec{AE}+\vec{ED}\right)+\vec{AD}$

$=3\vec{AD}=6\vec{AG}\quad[\because \vec{AD}=2\vec{AG}]$

410 **(c)**

I. It is true that non-zero, non-collinear vectors are linearly independent.

II. It is also true that any three coplanar vectors are linearly dependent.

$\therefore$ Both I and II are true.

411 **(a)**

Let $\vec{\alpha}=2\vec{a}-3\vec{b},\vec{\beta}=7\vec{b}-9\vec{c}$ and $\vec{\gamma}=12\vec{c}-23\vec{a}$

Then,

$$[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]=\begin{vmatrix}2&-3&0\\0&7&-9\\-23&0&12\end{vmatrix}[\vec{a}\,\vec{b}\,\vec{c}]$$

$\Rightarrow[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]=(168+3\times-207)[\vec{a}\,\vec{b}\,\vec{c}]$

$\Rightarrow[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]=0\qquad[\because[\vec{a}\,\vec{b}\,\vec{c}]=0]$

$\Rightarrow\vec{\alpha},\vec{\beta},\vec{\gamma}$ are coplanar vectors

412 **(b)**

Given, $[\vec{\mathbf{a}}+\vec{\mathbf{b}}\ \vec{\mathbf{b}}+\vec{\mathbf{c}}\ \vec{\mathbf{c}}+\vec{\mathbf{a}}]=[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$

$\Rightarrow 2[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]=[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$

$=[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]=0$

Hence, $\vec{\mathbf{a}},\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are coplanar.

413 **(c)**

Given, $\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}=\vec{\mathbf{0}}$ and $|\vec{\mathbf{a}}|=\sqrt{37},|\vec{\mathbf{b}}|=3$, and $|\vec{\mathbf{c}}|=4$

Now, $\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}=\vec{\mathbf{0}}$

$\Rightarrow\ \vec{\mathbf{a}}=-\left(\vec{\mathbf{b}}+\vec{\mathbf{c}}\right)$

$\Rightarrow|\vec{\mathbf{a}}|^2=\left|-\left(\vec{\mathbf{b}}+\vec{\mathbf{c}}\right)\right|^2$

$\Rightarrow\ |\vec{\mathbf{a}}|^2=\left|\vec{\mathbf{b}}\right|^2+|\vec{\mathbf{c}}|^2+2\left|\vec{\mathbf{b}}\right||\vec{\mathbf{c}}|\cos\theta$

$=9+16+24\cos\theta$

$\Rightarrow 37=25+24\cos\theta$

$\Rightarrow 24\cos\theta=12\ \Rightarrow\ \theta=60°$

414 **(a)**

Let unit vector be $a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}}$

$\therefore\ a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}}$ is perpendicular to $\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$,

Then $a+b+c=0$ .....(i)

Since, $a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}},\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}}\right),(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}})$ are coplanar

$$\therefore\begin{vmatrix}a&b&c\\1&1&2\\1&2&1\end{vmatrix}=0$$

$\Rightarrow\ -3a+b+c=0$ ....(ii)

From Eqs. (i) and (ii), we get

$a=0$ and $c=-b$

Also, $a^2+b^2+c^2=1$

$\Rightarrow 0+b^2+b^2=1$

$\Rightarrow b=\dfrac{1}{\sqrt{2}}$

$\therefore a\hat{\mathbf{i}}+b\hat{\mathbf{j}}+c\hat{\mathbf{k}}=\dfrac{1}{\sqrt{2}}\hat{\mathbf{j}}-\dfrac{1}{\sqrt{2}}\hat{\mathbf{k}}$

416 **(b)**

Given, $\overrightarrow{\mathbf{OA}}=2\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}}$

$\overrightarrow{\mathbf{OB}}=5\hat{\mathbf{i}}-4\hat{\mathbf{j}}+4\hat{\mathbf{k}}$

and $\overrightarrow{\mathbf{OC}}=\hat{\mathbf{i}}-2\hat{\mathbf{j}}+4\hat{\mathbf{k}}$

volume of parallelopiped

$=[\overrightarrow{\mathbf{OA}}\,\overrightarrow{\mathbf{OB}}\,\overrightarrow{\mathbf{OC}}]$

$$=\begin{vmatrix}2&-2&1\\5&-4&4\\1&-2&4\end{vmatrix}$$

$=2(-16+8)+2(20-4)+1(-10+4)$

$=10$ cu units

418 **(a)**

We have,

$$\vec{a}=\lambda\left(\vec{b}\times\vec{c}\right)=\lambda\begin{vmatrix}\hat{\imath}&\hat{\jmath}&\hat{k}\\1&2&3\\-2&4&1\end{vmatrix}$$

$=\lambda\left(-10\hat{\imath}-7\hat{k}+8\hat{k}\right)$

Now,

$\vec{a}\cdot\left(\hat{\imath}-2\hat{\jmath}+\hat{k}\right)=-6$

$\Rightarrow\lambda(-10+14+8)=-6\Rightarrow\lambda=-\dfrac{1}{2}$

Hence, $\vec{a}=-\frac{1}{2}\left(-10\hat{\imath}-7\hat{k}+8\hat{k}\right)=5\hat{\imath}+\frac{7}{2}\hat{\jmath}-4\hat{k}$

419 **(c)**

The projection of

$\vec{\mathbf{a}}$ on $\vec{\mathbf{b}}=\dfrac{\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}}{|\vec{\mathbf{b}}|}$

$=\dfrac{\left(3\hat{\mathbf{i}}-\hat{\mathbf{j}}+5\hat{\mathbf{k}}\right)\cdot(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+\hat{\mathbf{k}})}{\sqrt{2^2+3^2+1^2}}=\dfrac{8}{\sqrt{14}}$

421 **(d)**

$$\begin{vmatrix} 7 & -11 & 1 \\ 5 & 3 & -2 \\ 12 & -8 & -1 \end{vmatrix}$$

$= 7(-3-16) + 11(-5+24) + 1(-40-36)$

$= -133 + 209 - 76 = 0$

$\therefore$ Vector are collinear.

422 **(c)**

Let the position vectors of the points A, B, C are $\vec{\mathbf{0}}, \vec{\mathbf{a}} + \vec{\mathbf{b}}, \vec{\mathbf{a}} - \vec{\mathbf{b}}$ respectively and θ= 90°

$\therefore$ Area of triangle $= \frac{1}{2}|\overrightarrow{\mathbf{AB}} \times \overrightarrow{\mathbf{AC}}|$

$= \frac{1}{2}|(\vec{\mathbf{a}} + \vec{\mathbf{b}}) \times (\vec{\mathbf{a}} - \vec{\mathbf{b}})|$

$= \frac{1}{2}|2\vec{\mathbf{b}} \times \vec{\mathbf{a}}|$

$= |\vec{\mathbf{b}}|\,|\vec{\mathbf{a}}| \sin\theta = 3 \times 2 \sin 90° = 6$

423 **(a)**

We have, $|[\vec{a}\,\vec{b}\,\vec{c}]| = V$

Let V_1 be the volume of the parallelopiped formed by the vectors $\vec{\alpha}, \vec{\beta}$ and $\vec{\gamma}$. Then,

$V_1 = |[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]|$

Now,

$$[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}] = \begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{a}\cdot\vec{b} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c} \\ \vec{a}\cdot\vec{c} & \vec{b}\cdot\vec{c} & \vec{c}\cdot\vec{c} \end{vmatrix} [\vec{a}\,\vec{b}\,\vec{c}]$$

$\Rightarrow [\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}] = [\vec{a}\,\vec{b}\,\vec{c}]^2[\vec{a}\vec{b}\,\vec{c}]$

$\Rightarrow [\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}] = [\vec{a}\,\vec{b}\,\vec{c}]^3$

$\therefore V_1 = |[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]| = \left|[\vec{a}\,\vec{b}\,\vec{c}]^3\right| = V^3$

424 **(a)**

Let l, m, n be the direction cosines of the required vector. As it makes equal angles with X and Y axes

$\therefore l = m$

$\therefore$ Required vector $\vec{r} = l\hat{\imath} + m\hat{\jmath} + n\hat{k} = l\hat{\imath} + l\hat{\jmath} + n\hat{k}$

Now, $l^2 + m^2 + n^2 = 1 \Rightarrow 2l^2 + n^2 = 1$ ...(i)

Since, $\vec{r}$ is perpendicular to $-\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$

$\therefore \vec{r}\cdot(-\hat{\imath} + 2\hat{\jmath} + 2\hat{k}) = 0 \Rightarrow -l + 2l + 2n = 0 \Rightarrow$

$l + 2n = 0$ ....(ii)

From (i) and (ii), we get $n \mp \frac{1}{3}, l = \mp\frac{2}{3}$

Hence, $\vec{r} = \frac{1}{3}(\pm 2\hat{\imath} \pm 2\hat{\jmath} \mp \hat{k}) = \pm\frac{1}{3}(2\hat{\imath} + 2\hat{\jmath} - \hat{k})$

425 **(a)**

Let the required vector be $\vec{a}$. Then, $\hat{\imath} - \hat{\jmath}, \hat{\imath} + \hat{\jmath}$ and $\vec{a}$ form a right handed system

$\therefore \vec{a} = (\hat{\imath} - \hat{\jmath}) \times (\hat{\imath} + \hat{\jmath}) = \hat{k} + \hat{k} = 2\hat{k}$

Hence, the required unit vector $\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \hat{k}$

426 **(b)**

$\vec{\mathbf{p}} = x\vec{\mathbf{a}} + y\vec{\mathbf{b}} + z\vec{\mathbf{c}}$

$\Longrightarrow 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}} = x(\hat{\mathbf{i}} + \hat{\mathbf{j}}) + y(\hat{\mathbf{j}} + \hat{\mathbf{k}}) + z(\hat{\mathbf{i}} + \hat{\mathbf{k}})$

$\Longrightarrow 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}} = (x+z)\hat{\mathbf{i}} + (x+y)\hat{\mathbf{j}} + (y+z)\hat{\mathbf{k}}$

On comparing both sides the coefficients of $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$, we get

$x + z = 3$ ....(i)

$x + y = 2$ ....(ii)

and $y + z = 4$ (iii)

on solving Eqs. (i), (ii) and (iii), we get

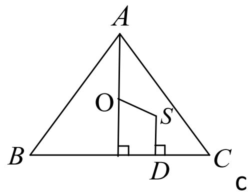
$x = \frac{1}{2}, y = \frac{3}{2}, z = \frac{5}{2}$

427 **(a)**

From geometry

$\overrightarrow{\mathbf{AO}} = 2\overrightarrow{\mathbf{SD}}$

Where D is the mind point of BC



$\therefore \overrightarrow{\mathbf{SA}} + \overrightarrow{\mathbf{SB}} + \overrightarrow{\mathbf{SC}}$

$= \overrightarrow{\mathbf{SA}} + 2\overrightarrow{\mathbf{SD}}$ $(\because \overrightarrow{\mathbf{SB}} + \overrightarrow{\mathbf{SC}} = 2\overrightarrow{\mathbf{SD}})$

$= \overrightarrow{\mathbf{SA}} + \overrightarrow{\mathbf{AO}}$

$= \overrightarrow{\mathbf{SO}}$

428 **(c)**

We have,

$\vec{a}\cdot\vec{b} = 0$ and $\vec{a} \times \vec{b} = \vec{0}$

$\Rightarrow |\vec{a}||\vec{b}|\cos\theta = 0$ and $|\vec{a}||\vec{b}|\sin\theta = 0$

$\Rightarrow (|\vec{a}| = 0 \text{ or}, |\vec{b}| = 0 \text{ or}, \cos\theta = 0)$

And,

$(|\vec{a}| = 0 \text{ or}, |\vec{b}| = 0 \text{ or}, \sin\theta = 0)$

$\Rightarrow |\vec{a}| = 0$ or,

$|\vec{b}| = 0 \quad \begin{bmatrix} \because \cos\theta \text{ and } \sin\theta \\ \text{are not zero zimultaneously} \end{bmatrix}$

430 **(c)**

Given $|\vec{\mathbf{a}} + \vec{\mathbf{b}}|^2 = |\vec{\mathbf{a}} - \vec{\mathbf{b}}|^2$

$\Longrightarrow 4\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = 0 \Longrightarrow \vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = 0$

So, angle between them is 90°

431 **(c)**

We have,

$\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$

$\Rightarrow (\vec{r} - \vec{b}) \times \vec{a} = 0$

$\Rightarrow \vec{r} - \vec{b}$ is parallel to $\vec{a}$

$\Rightarrow \vec{r} - \vec{b} = \lambda\vec{a}$ for some scalar λ

$\Rightarrow \vec{r} - \vec{b} + \lambda\vec{a}$ ....(i)

Now,

$\vec{r} \perp \vec{c}$

$\Rightarrow \vec{r}\cdot\vec{c}\cdot\vec{c} = 0$

$\Rightarrow \vec{b}\cdot\vec{c}+\lambda(\vec{a}\cdot\vec{c})=0 \Rightarrow \lambda=-\dfrac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{c}}$

Putting the value of λ in (i), we get

$\vec{r}=\vec{b}-\left(\dfrac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{c}}\right)\vec{a}$

432 **(d)**

We have, $|\vec{\alpha}|=1=|\vec{\beta}|$ and $\vec{\alpha}\cdot\vec{\beta}=0$

Now,

$\vec{\gamma}=x\vec{\alpha}+y\vec{\beta}+z(\vec{\alpha}\times\vec{\beta})$

$\Rightarrow \vec{\alpha}\cdot\vec{\gamma}=x(\vec{\alpha}\cdot\vec{\alpha})+y(\vec{\alpha}\cdot\vec{\beta})+z\{\vec{\alpha}\cdot(\vec{\alpha}\times\vec{\beta})\}$

$\vec{\beta}\cdot\vec{\gamma}=x(\vec{\beta}\cdot\vec{\alpha})+y(\vec{\beta}\cdot\vec{\beta})+z\{\vec{\beta}\cdot(\vec{\alpha}\times\vec{\beta})\}$

And,

$(\vec{\alpha}\times\vec{\beta})\cdot\vec{\gamma}=x\{\vec{\alpha}\cdot(\vec{\alpha}\times\vec{\beta})+y\{\vec{\beta}\cdot(\vec{\alpha}\times\vec{\beta})\}$
$\quad +z\{(\vec{\alpha}\times\vec{\beta})\cdot(\vec{\alpha}\times\vec{\beta})\}$

$\Rightarrow \cos\theta=x, \cos\theta=y$ and $[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]=z|\vec{\alpha}\times\vec{\beta}|^2$

$\Rightarrow x=\cos\theta, y=\cos\theta$ and $[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]=z$

$[\because |\vec{\alpha}\times\vec{\beta}|=|\vec{\alpha}|\,|\vec{\beta}|\sin 90°=1]$

Now,

$$[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]^2=\begin{vmatrix}\vec{\alpha}\cdot\vec{\alpha} & \vec{\alpha}\cdot\vec{\beta} & \vec{\alpha}\cdot\vec{\gamma}\\ \vec{\beta}\cdot\vec{\alpha} & \vec{\beta}\cdot\vec{\beta} & \vec{\beta}\cdot\vec{\gamma}\\ \vec{\gamma}\cdot\vec{\alpha} & \vec{\gamma}\cdot\vec{\beta} & \vec{\gamma}\cdot\vec{\gamma}\end{vmatrix}$$

$$\Rightarrow [\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]^2=\begin{vmatrix}1 & 0 & \cos\theta\\ 0 & 1 & \cos\theta\\ \cos\theta & \cos\theta & 1\end{vmatrix}=1-2\cos^2\theta$$

$\Rightarrow z^2=1-2x^2$

Also, $z^2=1-2y^2$ and $z^2=1-x^2-y^2$

433 **(a)**

$(\vec{\mathbf{a}}-\vec{\mathbf{d}})\times(\vec{\mathbf{b}}-\vec{\mathbf{c}})$

$=\vec{\mathbf{a}}\times\vec{\mathbf{b}}-\vec{\mathbf{a}}\times\vec{\mathbf{c}}-\vec{\mathbf{d}}\times\vec{\mathbf{b}}+\vec{\mathbf{d}}\times\vec{\mathbf{c}}$

$=\vec{\mathbf{c}}\times\vec{\mathbf{d}}-\vec{\mathbf{b}}\times\vec{\mathbf{d}}-\vec{\mathbf{d}}\times\vec{\mathbf{b}}+\vec{\mathbf{d}}\times\vec{\mathbf{c}}=\vec{\mathbf{0}}$

$[\because \vec{\mathbf{a}}\times\vec{\mathbf{b}}=\vec{\mathbf{c}}\times\vec{\mathbf{d}}, \vec{\mathbf{a}}\times\vec{\mathbf{c}}=\vec{\mathbf{b}}\times\vec{\mathbf{d}}$, given$]$

$\Rightarrow (\vec{\mathbf{a}}-\vec{\mathbf{d}})||(\vec{\mathbf{b}}-\vec{\mathbf{c}})$

$\Rightarrow \vec{\mathbf{a}}-\vec{\mathbf{d}}=\lambda(\vec{\mathbf{b}}-\vec{\mathbf{c}})$

434 **(a)**

Since $\vec{a},\vec{b},\vec{c}$ are non-coplanar unit vectors

$\therefore [\vec{a}\vec{b}\vec{c}]=$ Volume of a parallelopiped whose each edge is of one unit length

$\Rightarrow [\vec{a}\vec{b}\vec{c}]=\pm 1$

436 **(d)**

Let D be the mid-point of BC. Then,

$\vec{AB}+\vec{AC}=2\vec{AD}$

$\Rightarrow 2\,\vec{AD}=8\hat{\imath}+2\hat{\jmath}+8\hat{k}$

$\Rightarrow \vec{AD}=4\hat{\imath}+\hat{\jmath}+4\hat{k}$

$\Rightarrow |\vec{AD}|=\sqrt{16+1+16}=\sqrt{33}$

437 **(c)**

$\therefore$ Median vector through $\vec{\mathbf{A}}=\dfrac{1}{2}(\overrightarrow{\mathbf{AB}}+\overrightarrow{\mathbf{AC}})$

$=\dfrac{1}{2}[(3\hat{\mathbf{i}}+5\hat{\mathbf{j}}+4\hat{\mathbf{k}})+(5\hat{\mathbf{i}}-5\hat{\mathbf{j}}+2\hat{\mathbf{k}})]$

$=4\hat{\mathbf{i}}+3\hat{\mathbf{k}}$

$\therefore$ Length of the median $=\sqrt{4^2+3^2}=5$ units

438 **(d)**

Given, $(\vec{\mathbf{a}}-\lambda\vec{\mathbf{b}})\cdot(\vec{\mathbf{b}}-2\vec{\mathbf{c}})\times(\vec{\mathbf{c}}+2\vec{\mathbf{a}})=0$

$\Rightarrow (\vec{\mathbf{a}}-\lambda\vec{\mathbf{b}})\cdot\{\vec{\mathbf{b}}\times\vec{\mathbf{c}}+\vec{\mathbf{b}}\times 2\vec{\mathbf{a}}-4(\vec{\mathbf{c}}\times\vec{\mathbf{a}})\}=0$

$\Rightarrow \vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})+\vec{\mathbf{a}}.(\vec{\mathbf{b}}\times 2\vec{\mathbf{a}})-\vec{\mathbf{a}}.4(\vec{\mathbf{c}}\times\vec{\mathbf{a}})$
$-\lambda\vec{\mathbf{b}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})-\lambda\vec{\mathbf{b}}\cdot(\vec{\mathbf{b}}\times 2\vec{\mathbf{a}})+4\lambda\,\vec{\mathbf{b}}\cdot(\vec{\mathbf{c}}\times\vec{\mathbf{a}})=0$

$\Rightarrow \vec{\mathbf{a}}(\vec{\mathbf{b}}\times\vec{\mathbf{c}})+4\lambda\vec{\mathbf{b}}.(\vec{\mathbf{c}}\times\vec{\mathbf{a}})=0$

$\Rightarrow \{\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})\}(1+4\lambda)=0$

$\Rightarrow \lambda=-\dfrac{1}{4}\ [\because \vec{\mathbf{a}}.(\vec{\mathbf{b}}\times\vec{\mathbf{c}})\neq 0$, given$]$

440 **(d)**

$\therefore$ Total force $\vec{\mathbf{P}}=\vec{\mathbf{P}}_1+\vec{\mathbf{P}}_2+\vec{\mathbf{P}}_3$

$=\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}-\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}=2\hat{\mathbf{j}}$

and displacement $\overrightarrow{\mathbf{AB}}=6\hat{\mathbf{i}}+\hat{\mathbf{j}}-3\hat{\mathbf{k}}-(4\hat{\mathbf{i}}+3\hat{\mathbf{j}}-2\hat{\mathbf{k}})$

$=2\hat{\mathbf{i}}+4\hat{\mathbf{j}}-\hat{\mathbf{k}}$

$\therefore$Work done $=\vec{\mathbf{P}}\cdot\overrightarrow{\mathbf{AB}}$

$=2\hat{\mathbf{j}}(2\hat{\mathbf{i}}+4\hat{\mathbf{j}}-\hat{\mathbf{k}})=8$

441 **(a)**

The point of intersection of $\vec{\mathbf{r}}\times\vec{\mathbf{a}}=\vec{\mathbf{b}}\times\vec{\mathbf{a}}$ and $\vec{\mathbf{r}}\times\vec{\mathbf{b}}=\vec{\mathbf{a}}\times\vec{\mathbf{b}}$ is $\vec{\mathbf{r}}=\vec{\mathbf{a}}+\vec{\mathbf{b}}$

$\therefore\ \vec{\mathbf{r}}=(\hat{\mathbf{i}}+\hat{\mathbf{j}})+(2\hat{\mathbf{i}}-\hat{\mathbf{k}})=3\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$

442 **(a)**

Since $\vec{a},\vec{b}$ and $a\times\vec{b}$ are non-coplanar vectors

$\therefore \vec{r}=x\vec{a}+y\vec{b}+z(\vec{a}\times\vec{b})$ for some scalars x,y,z ...(i)

Now,

$\vec{b}=\vec{r}\times\vec{a}$

$\Rightarrow \vec{b}=\{x\vec{a}+y\vec{b}+z(\vec{a}\times\vec{b})\}\times\vec{a}$

$\Rightarrow \vec{b}=y(\vec{b}\times\vec{a})+z((\vec{a}\times\vec{b})\times\vec{a})$

$\Rightarrow \vec{b}=y(\vec{b}\times\vec{a})-z(\vec{a}\times(\vec{a}\times\vec{b}))$

$\Rightarrow \vec{b}=y(\vec{b}\times\vec{a})-z\{(\vec{a}\cdot\vec{b})\vec{a}-(\vec{a}\cdot\vec{a})\vec{b}\}$

$\Rightarrow \vec{b}=y(\vec{b}\times\vec{a})+z(\vec{a}\cdot\vec{a})\vec{b}\ \ [\because \vec{a}\cdot\vec{b}=0]$

Comparing the coefficients, we get

$y=0, z=\dfrac{1}{\vec{a}\cdot\vec{a}}=\dfrac{1}{|\vec{a}|^2}$

Putting the values of y and z in (i), we get

$\vec{r}=x\vec{a}+\dfrac{1}{|\vec{a}|^2}(\vec{a}\times\vec{b})$

444 **(b)**

$(\vec{\mathbf{u}}+\vec{\mathbf{v}}-\vec{\mathbf{w}}).[(\vec{\mathbf{u}}-\vec{\mathbf{v}})\times(\vec{\mathbf{v}}-\vec{\mathbf{w}})]$

$= (\vec{\mathbf{u}} + \vec{\mathbf{v}} - \vec{\mathbf{w}}).[\,\vec{\mathbf{u}} \times \vec{\mathbf{v}} - \vec{\mathbf{u}} \times \vec{\mathbf{w}} + \vec{\mathbf{v}} \times \vec{\mathbf{w}}]$

$= \vec{\mathbf{u}} \cdot \vec{\mathbf{v}} \times \vec{\mathbf{w}} - \vec{\mathbf{v}} \cdot \vec{\mathbf{u}} \times \vec{\mathbf{w}} - \vec{\mathbf{w}} \cdot \vec{\mathbf{u}} \times \vec{\mathbf{v}}$

$= \vec{\mathbf{u}} \cdot \vec{\mathbf{v}} \times \vec{\mathbf{w}} + \vec{\mathbf{w}} \cdot \vec{\mathbf{u}} \times \vec{\mathbf{v}} - \vec{\mathbf{w}} \cdot \vec{\mathbf{u}} \times \vec{\mathbf{v}}$

$= \vec{\mathbf{u}} \cdot \vec{\mathbf{v}} \times \vec{\mathbf{w}}$

445 **(d)**

$\therefore\ \vec{\mathbf{p}} - 2\,\vec{\mathbf{q}} = 7\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}} - 2(3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 5\hat{\mathbf{k}})$

$= \hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 7\hat{\mathbf{k}}$

$\Longrightarrow |\vec{\mathbf{p}} - 2\vec{\mathbf{q}}| = \sqrt{1^2 + (-4)^2 + (-7)^2} = \sqrt{66}$

447 **(a)**

$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$

$= -\hat{\mathbf{i}} + \hat{\mathbf{j}}$

$\Longrightarrow (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times \vec{\mathbf{c}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = -\hat{\mathbf{k}}$

Now, $\lambda\,\vec{\mathbf{a}} + \mu\,\vec{\mathbf{b}} = \lambda(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) + \mu(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

$= (\lambda + \mu)\hat{\mathbf{i}} + (\lambda + \mu)\hat{\mathbf{j}} + \lambda\hat{\mathbf{k}}$

$\because\ \lambda\,\vec{\mathbf{a}} + \mu\,\vec{\mathbf{b}} = (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times \vec{\mathbf{c}}$

$\Longrightarrow (\lambda + \mu)\hat{\mathbf{i}} + (\lambda + \mu)\hat{\mathbf{j}} + \lambda\hat{\mathbf{k}} = -\hat{\mathbf{k}}$

On equating the coefficient of $\hat{\mathbf{i}}$ we get $\lambda + \mu = 0$

453 **(d)**

We have,

$\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$

$\Rightarrow \vec{a} \cdot (\vec{b} - \vec{c}) = 0$

$\Rightarrow \vec{a} \perp (\vec{b} - \vec{c})$ or, $\vec{b} - \vec{c} = 0 \Rightarrow \vec{a} \perp (\vec{b} - \vec{c})$ or, $\vec{b} = \vec{c}$

454 **(c)**

Given that, $|\vec{\mathbf{a}}| = 2\sqrt{2}, |\vec{\mathbf{b}}| = 3$

The longer vectors is $5\vec{\mathbf{a}} + 2\vec{\mathbf{b}} + \vec{\mathbf{a}} - 3\vec{\mathbf{b}} = 6\vec{\mathbf{a}} - \vec{\mathbf{b}}$

Length of one diagonal

$= |6\vec{\mathbf{a}} - \vec{\mathbf{b}}|$

$= \sqrt{36\vec{\mathbf{a}}^2 + \vec{\mathbf{b}}^2 - 2 \times 6|\vec{\mathbf{a}}|\,|\vec{\mathbf{b}}| \cos 45^\circ}$

$= \sqrt{36 \times 8 + 9 - 12 \times 2\sqrt{2} \times 3 \times \frac{1}{\sqrt{2}}}$

$= \sqrt{288 + 9 - 12 \times 6} = \sqrt{225} = 15$

Other diagonal is $4\vec{\mathbf{a}} + 5\vec{\mathbf{b}}$.

Its length $= \sqrt{16 \times 8 + 25 \times 9 + 40 \times 6} = \sqrt{593}$

455 **(a)**

Given projection of $\vec{\mathbf{a}}$ on $\vec{\mathbf{b}} = |\vec{\mathbf{a}} \times \vec{\mathbf{b}}|$

$\Longrightarrow \frac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{b}}|} = |\vec{\mathbf{a}} \times \vec{\mathbf{b}}|$

$\Longrightarrow \frac{|\vec{\mathbf{a}}||\vec{\mathbf{b}}| \cos\theta}{|\vec{\mathbf{b}}|} = |\vec{\mathbf{a}}||\vec{\mathbf{b}}| \sin\theta$

$\Longrightarrow \tan\theta = \frac{1}{|\vec{\mathbf{b}}|}$

$\Longrightarrow \tan\theta = \frac{1}{\frac{1}{3}\sqrt{1^2 + 1^2 + 1^2}}$

$\Longrightarrow \tan\theta = \sqrt{3}$

$\Longrightarrow\ \theta = \frac{\pi}{3}$

457 **(c)**

Since, $\vec{\mathbf{a}} + 2\vec{\mathbf{b}} = k\vec{\mathbf{c}}$

$\therefore\ \vec{\mathbf{a}} + 2\vec{\mathbf{b}} + 6\vec{\mathbf{c}} = k\vec{\mathbf{c}} + 6\vec{\mathbf{c}}$

$= (k + 6)\vec{\mathbf{c}} = \lambda\vec{\mathbf{c}}\ \ (\because \lambda \neq 0)$

458 **(d)**

$\vec{\mathbf{u}} \times \vec{\mathbf{v}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix} = -2\hat{\mathbf{k}}$

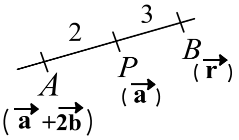
$\therefore |\vec{\mathbf{w}} \cdot \hat{\mathbf{n}}| = \frac{|\vec{\mathbf{w}} \cdot \vec{\mathbf{u}} \times \vec{\mathbf{v}}|}{|\vec{\mathbf{u}} \times \vec{\mathbf{v}}|}$

$\Longrightarrow |\vec{\mathbf{w}} \cdot \hat{\mathbf{n}}| = \frac{|-6\hat{\mathbf{k}}|}{|-2\hat{\mathbf{k}}|} = 3$

459 **(c)**

Let the position of B is $\vec{\mathbf{r}}$.

$\therefore\ \vec{\mathbf{a}} = \frac{2\vec{\mathbf{r}} + 3(\vec{\mathbf{a}} + 2\vec{\mathbf{b}})}{2 + 3}$



$\Rightarrow 5\vec{\mathbf{a}} = 2\vec{\mathbf{r}} + 3\vec{\mathbf{a}} + 6\vec{\mathbf{b}}$

$\Rightarrow\ 2\vec{\mathbf{r}} = 2\vec{\mathbf{a}} - 6\vec{\mathbf{b}}$

$\therefore\ \vec{\mathbf{r}} = \vec{\mathbf{a}} - 3\vec{\mathbf{b}}$

460 **(a)**

Since, $(\vec{\mathbf{A}} + t\vec{\mathbf{B}}) \cdot \vec{\mathbf{C}} = 0$ [given]

$\Longrightarrow [(1 - t)\hat{\mathbf{i}} + (2 + 2t)\hat{\mathbf{j}} + (3 + t)\hat{\mathbf{k}}] \cdot (3\hat{\mathbf{i}} + \hat{\mathbf{j}}) = 0$

$\Longrightarrow 3(1 - t) + (2 + 2t) = 0 \Longrightarrow t = 5$

461 **(a)**

We have,

$|\vec{a}| = 1, |\vec{b}| = 1$ and $\vec{a}.\vec{b} = \cos\theta$

Now, $|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b}$

$\Rightarrow |\vec{a} - \vec{b}|^2 = 1 + 1 - 2|\vec{a}|\,|\vec{b}| \cos\theta$

$\Rightarrow |\vec{a} - \vec{b}|^2 = 4\sin^2\frac{\theta}{2}$

$\Rightarrow \left|\frac{\vec{a} - \vec{b}}{2}\right|^2 = \sin^2\frac{\theta}{2} \Rightarrow \left|\frac{\vec{a} - \vec{b}}{2}\right| = \sin\frac{\theta}{2}$

462 **(c)**

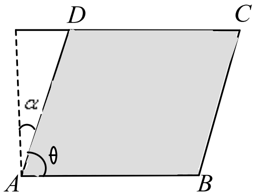
If $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ are two non-zero non-collinear vectors and x, y are two scalars such that $x\vec{\mathbf{a}} + y\vec{\mathbf{b}} = 0$, then $x = 0, y = 0$.

Because otherwise one will be a scalar multiple of the other and hence collinear, which is a contradiction

463 **(b)**

$\overrightarrow{\mathbf{AB}} = 2\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$

$\overrightarrow{\mathbf{AD}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$



$\overrightarrow{\mathbf{AB}} \cdot \overrightarrow{\mathbf{AD}} = -2 + 20 + 22 = 40$

$|\overrightarrow{\mathbf{AB}}| = \sqrt{4 + 100 + 120} = \sqrt{225} = 15$

$|\overrightarrow{\mathbf{AD}}| = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$

$\therefore\ \cos\theta = \frac{40}{45} = \frac{8}{9}$

$\therefore\ \theta + \alpha = 90°$

$\Longrightarrow\ \alpha = 90° - \theta$

$\Longrightarrow \cos\alpha = \sin\theta = \sqrt{1 - \frac{64}{81}} = \frac{\sqrt{17}}{9}$

464 **(a)**

Let $\vec{\mathbf{a}} = x\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\vec{\mathbf{b}} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

Sience, $\frac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{b}}|} = \frac{1}{\sqrt{30}}$

$\Longrightarrow \frac{(x\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) \cdot (2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 5\hat{\mathbf{k}})}{|\sqrt{4 + 1 + 25}|} = \frac{1}{\sqrt{30}}$

$\Longrightarrow 2x - 1 + 5 = 1$

$\Longrightarrow x = -\frac{3}{2}$

465 **(b)**

Now, $2\vec{\mathbf{a}} - \vec{\mathbf{c}} = 2(-\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) - (2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

$= \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

and $\vec{\mathbf{a}} + \vec{\mathbf{b}} = -\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}} + 2\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$

$= \hat{\mathbf{i}} + \hat{\mathbf{k}}$

let θ be the angle between $2\vec{\mathbf{a}} - \vec{\mathbf{c}}$ and $\vec{\mathbf{a}} + \vec{\mathbf{b}}$.

$\therefore \cos\theta = \frac{(\hat{\mathbf{j}} + \hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}})}{\sqrt{1^2 + 1^2}\sqrt{1^2 + 1^2}}$

$\Longrightarrow \cos\theta = \frac{1}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$

$\Longrightarrow\ \theta = \frac{\pi}{3}$

466 **(d)**

Since $\vec{a} + \vec{b}$ and $\vec{b} + \vec{c}$ are collinear with $\vec{c}$ and $\vec{a}$ respectively. Therefore, there exist scalars x, y such that $\vec{a} + \vec{b} = x\vec{c}$ and $\vec{b} + \vec{c} = y\vec{a}$. Now,

$\vec{a} + \vec{b} = x\vec{c} \Rightarrow \vec{a} + \vec{b} + \vec{c} = (x + 1)\vec{c}$ ...(i)

and,

$\vec{b} + c = y\vec{a} \Rightarrow \vec{a} + \vec{b} + \vec{c} = (y + 1)\vec{a}$ ....(ii)

From (i) and (ii), we get

$(x + 1)\vec{c} = (y + 1)\vec{a}$

If $x \neq -1$, then

$(x + 1)\vec{c} = (y + 1)\vec{a} \Rightarrow \vec{c} = \frac{y + 1}{x + 1}\vec{a}$

$\Rightarrow \vec{c}$ and $\vec{a}$ are collinear

This is a contradiction to the given condition.

Therefore, $x = -1$

Putting $x = -1$ in $\vec{a} + \vec{b} = x\vec{c}$, we get

$\vec{a} + \vec{b} + \vec{c} = (-1 + 1)\vec{c} = \vec{0}$

467 **(b)**

We have, $[\,\vec{\mathbf{a}}\ \vec{\mathbf{b}} + \vec{\mathbf{c}}\ \vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}]$

$= \vec{\mathbf{a}} \cdot [\,(\vec{\mathbf{b}} + \vec{\mathbf{c}}) \times (\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}})]$

$= \vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{a}} + \vec{\mathbf{b}} \times \vec{\mathbf{b}} + \vec{\mathbf{b}} \times \vec{\mathbf{c}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}} + \vec{\mathbf{c}} \times \vec{\mathbf{b}} + \vec{\mathbf{c}} \times$
$\mathbf{c}$

$= \vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{a}} + \vec{\mathbf{b}} \times \vec{\mathbf{c}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}} - \vec{\mathbf{b}} \times \vec{\mathbf{c}})$

$= \vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{a}}) + \vec{\mathbf{a}} \cdot (\vec{\mathbf{c}} \times \vec{\mathbf{a}})$

$= [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}] + [\vec{\mathbf{a}}\,\vec{\mathbf{c}}\,\vec{\mathbf{a}}] = 0$

468 **(a)**

It is given that points P, Q and R with position vectors $60\hat{\imath} + 3\hat{\jmath}, 40\hat{\imath} - 8\hat{\jmath}$ and $a\hat{\imath} - 52\hat{\jmath}$ respectively are collinear

$\therefore \vec{P}Q = \lambda\,\vec{Q}R$ for some scalar λ

$\Rightarrow -20\hat{\imath} - 11\hat{\jmath} = \lambda\{(a - 40)\hat{\imath} - 44\hat{\jmath}\}$

$\Rightarrow \lambda(a - 40) = -20, -11 = -44\,\lambda$

$\Rightarrow \lambda = \frac{1}{4}$ and $a = -40$

469 **(a)**

Required unit vector

$\vec{\mathbf{c}} = \frac{\vec{\mathbf{a}} \times (\vec{\mathbf{a}} \times \vec{\mathbf{b}})}{|\vec{\mathbf{a}} \times (\vec{\mathbf{a}} \times \vec{\mathbf{b}})|}$

Now,

$\vec{\mathbf{a}} \times (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) = (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{a}} - (\vec{\mathbf{a}} \cdot \vec{\mathbf{a}})\vec{\mathbf{b}}$

$= 3(2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) - 6(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}})$

$= -9\hat{\mathbf{j}} + 9\hat{\mathbf{k}}$

$\therefore\ \vec{\mathbf{c}} = \frac{-9\hat{\mathbf{j}} + 9\hat{\mathbf{k}}}{\sqrt{9^2 + 9^2}} = \pm\frac{1}{\sqrt{2}}(-\hat{\mathbf{j}} + \hat{\mathbf{k}})$

470 **(b)**

$\begin{vmatrix} 2 & 1 & 4 \\ 4 & -2 & 3 \\ 2 & -3 & -\lambda \end{vmatrix} = 0$

$\Longrightarrow 2(2\lambda + 9) - 1(-4\lambda - 6) + 4(-12 + 4) = 0$

$\Longrightarrow 4\lambda + 18 + 4\,\lambda + 6 - 48 + 16 = 0$

$\Longrightarrow 8\lambda = 8$

$\Longrightarrow\ \lambda = 1$

471 **(b)**

We have,

$[\vec{u}\,\vec{v}\,\vec{w}] = \begin{vmatrix} al + a_1l_1 & am + a_1m_1 & an + a_1n_1 \\ bl + b_1l_1 & bm + b_1m_1 & bn + b_1n_1 \\ cl + c_1l_1 & cm + c_1m_1 & cn + a_1n_1 \end{vmatrix}$

$\Rightarrow [\vec{u}\,\vec{v}\,\vec{w}] = \begin{vmatrix} a & a_1 & 0 \\ b & b_1 & 0 \\ c & c_1 & 0 \end{vmatrix} \begin{vmatrix} l & l_1 & 0 \\ m & m_1 & 0 \\ n & n_1 & 0 \end{vmatrix} = 0$

Hence, the given vectors are coplanar

473 **(a)**

Given that $\vec{a}, \vec{b}, \vec{c}$ are coplanar

$\therefore \vec{a} \perp \vec{b} \times \vec{c} \Rightarrow \vec{a} \cdot (\vec{b} \times \vec{c}) = 0 \Rightarrow [\vec{a}\vec{b}\vec{c}] = 0$

474 **(c)**

$(\vec{\mathbf{d}} + \vec{\mathbf{a}}) \cdot [\vec{\mathbf{a}} \times \{\vec{\mathbf{b}} \times (\vec{\mathbf{c}} \times \vec{\mathbf{d}})\}]$

$= (\vec{\mathbf{d}} + \vec{\mathbf{a}}) \cdot [\vec{\mathbf{a}} \times \{(\vec{\mathbf{b}} \cdot \vec{\mathbf{d}})\vec{\mathbf{c}} - (\vec{\mathbf{b}} \cdot \vec{\mathbf{c}})\vec{\mathbf{d}}\}]$

$= (\vec{\mathbf{b}} \cdot \vec{\mathbf{d}})[\vec{\mathbf{d}} \cdot (\vec{\mathbf{a}} \times \vec{\mathbf{c}})] - (\vec{\mathbf{b}} \cdot \vec{\mathbf{c}})[\vec{\mathbf{d}} \cdot (\vec{\mathbf{a}} \times \vec{\mathbf{d}})]$

$+(\vec{\mathbf{b}} \cdot \vec{\mathbf{d}})[\vec{\mathbf{a}} \cdot (\vec{\mathbf{a}} \times \vec{\mathbf{c}})] - (\vec{\mathbf{b}} \cdot \vec{\mathbf{c}})[\vec{\mathbf{a}} \cdot (\vec{\mathbf{a}} \times \vec{\mathbf{d}})]$

$= (\vec{\mathbf{b}} \cdot \vec{\mathbf{d}})[\vec{\mathbf{d}}\,\vec{\mathbf{a}}\,\vec{\mathbf{c}}] = (\vec{\mathbf{b}} \cdot \vec{\mathbf{d}})[\vec{\mathbf{a}}\,\vec{\mathbf{c}}\,\vec{\mathbf{d}}]$

476 **(a)**

Let $\vec{\mathbf{a}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}, \vec{\mathbf{b}} = -2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

and $\vec{\mathbf{c}} = \lambda\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\therefore\ [\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}] = 0$

$\Longrightarrow \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ \lambda & -1 & 2 \end{vmatrix} = 0$

$\Longrightarrow 1(6-4) + 2(-4+4\lambda) + 3(2-3\lambda) = 0$

$\Longrightarrow \lambda = 0$

477 **(b)**

Let $\vec{\mathbf{a}} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$

$|\vec{\mathbf{a}}|^2 = a_1^2 + a_2^2 + a_3^2$

and $\vec{\mathbf{a}} \times \hat{\mathbf{i}} = (a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}) \times \hat{\mathbf{i}}$

$= -a_2\hat{\mathbf{k}} + a_3\hat{\mathbf{j}}$

$(\vec{\mathbf{a}} \times \hat{\mathbf{i}})^2 = a_2^2 + a_3^2$

Similarly, $(\vec{\mathbf{a}} \times \hat{\mathbf{j}})^2 = a_3^2 + a_1^2$

and $(\vec{\mathbf{a}} \times \hat{\mathbf{k}})^2 = a_1^2 + a_2^2$

Now, $(\vec{\mathbf{a}} \times \hat{\mathbf{i}})^2 + (\vec{\mathbf{a}} \times \hat{\mathbf{j}})^2 + (\vec{\mathbf{a}} \times \hat{\mathbf{k}})^2$

$= a_2^2 + a_3^2 + a_3^2 + a_1^2 + a_1^2 + a_2^2$

$= 2(a_1^2 + a_2^2 + a_3^2) = 2(\vec{\mathbf{a}})^2$

478 **(d)**

Since, $\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \vec{\mathbf{b}} = 2\hat{\mathbf{i}} - 4\hat{\mathbf{k}}, \vec{\mathbf{c}} = \hat{\mathbf{i}} + \lambda\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ are coplanar.

$\therefore [\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}] = 0 \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 2 & 0 & -4 \\ 1 & \lambda & 3 \end{vmatrix} = 0$

$\Rightarrow 4\lambda - 1(6+4) + 2\lambda = 0$

$\Rightarrow 6\lambda = 10 \Rightarrow \lambda = \frac{5}{3}$

480 **(c)**

$\vec{\mathbf{A}}, \vec{\mathbf{B}}$ and $\vec{\mathbf{C}}$ are three vectors, then volume of parallelepiped

$V = [\vec{\mathbf{A}}\ \vec{\mathbf{B}}\ \vec{\mathbf{C}}]$

$= \begin{vmatrix} 1 & a & 1 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 1 + a^3 - a$

$\Rightarrow V = 1 + a^3 - a$

On differentiating with respect to a, we get

$\frac{dV}{da} = 3a^2 - 1 = 0$

For maximum or minimum, put $\frac{dV}{a} = 0$

$\Rightarrow a = \pm\frac{1}{\sqrt{3}}$

$\frac{d^2V}{da^2} = 6a$, positive at $a = \frac{1}{\sqrt{3}}$.

$\therefore V$ is minimum at $a = \frac{1}{\sqrt{3}}$.

481 **(c)**

By the properties of midpoint theorem,

$\overrightarrow{\mathbf{PA}} + \overrightarrow{\mathbf{PB}} = 2\overrightarrow{\mathbf{PC}}$

482 **(a)**

The vector equation of line passing through points $(3, 2, 1)$ and $(-2, 1, 3)$

$\vec{\mathbf{r}} = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}} + \lambda[(-2-3)\hat{\mathbf{i}} + (1-2)\hat{\mathbf{j}} + (3-1)\hat{\mathbf{k}}]$

$= 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}} + \lambda(-5\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

483 **(d)**

$\because\ \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = |\vec{\mathbf{a}}||\vec{\mathbf{b}}| \cos\frac{5\pi}{6}$

$= -\frac{|\vec{\mathbf{a}}||\vec{\mathbf{b}}|\sqrt{3}}{2}$

Since, the projection of $\vec{\mathbf{a}}$ in the direction of $\vec{\mathbf{b}} = -\frac{6}{\sqrt{3}}$

$\Longrightarrow -\frac{|\vec{\mathbf{a}}||\vec{\mathbf{b}}|\sqrt{3}}{2|\vec{\mathbf{b}}|} = -\frac{6}{\sqrt{3}}$

$\Longrightarrow |\vec{\mathbf{a}}| = \frac{6 \times 2}{3} = 4$

484 **(d)**

Let $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ in $OXYZ$ system

Also, let $\vec{r} = X\hat{\imath} + Y\hat{\jmath} + Z\hat{k}$ in the new coordinate system

Since the right handed rectangular system $OXYZ$ is rotated about z-axis through $\frac{\pi}{4}$ in anticlockwise direction. Therefore,

$x = X\cos\theta - Y\sin\theta$ and $y = X\sin\theta + Y\cos\theta$

$\Rightarrow x = X\cos\frac{\pi}{4} - Y\sin\frac{\pi}{4}, y = X\sin\frac{\pi}{4} + Y\cos\frac{\pi}{4}$

and, $z = Z$

It is given that $X = 2\sqrt{2}, Y = 3\sqrt{2}$ and $Z = 4$

$\therefore x = 2 - 3 = -1, y = 5$ and $z = 4$

Hence, $\vec{r} = -\hat{\imath} + 5\hat{\jmath} + 4\hat{k}$

ALITER Let $l_1, m_1, n_1; l_2, m_2, n_2$ and l_3, m_3, n_3 be the direction cosines of the new axes with respect to the old axes. Then,

$l_1 = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}, m_1 = \cos\left(-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}, n_1$
$= \cos\frac{\pi}{2} = 0$

$l_2 = \cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}, m_2 = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}, n_2$
$= \cos\frac{\pi}{2} = 0$

$l_3 = \cos\frac{\pi}{2} = 0, m_3 = \cos\frac{\pi}{2} = 0, n_3 = \cos 0 = 1$

Let x', y', z' and x, y, z be the components of the given vector with respect to new and old axes. Then,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2\sqrt{2} \\ 3\sqrt{2} \\ 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & -3 & +0 \\ 2 & +3 & +0 \\ 0 & 0 & +4 \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \\ 4 \end{bmatrix}$$

Hence, the components of $\vec{a}$ in the $Oxyz$ coordinate system are $-1,5,4$

485 **(d)**

$\because \vec{\mathbf{x}} \cdot \vec{\mathbf{a}} = \vec{\mathbf{x}} \cdot \vec{\mathbf{b}} = \vec{\mathbf{x}} \cdot \vec{\mathbf{c}} = 0$

For non-zero vector $\vec{\mathbf{x}}$

$[\vec{\mathbf{a}}\, \vec{\mathbf{b}}\, \vec{\mathbf{c}}] = 0$ (three vectors $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are coplanar)

and $[\vec{\mathbf{a}} \times \vec{\mathbf{b}}\ \vec{\mathbf{b}} \times \vec{\mathbf{c}}\ \vec{\mathbf{c}} \times \vec{\mathbf{a}}]$

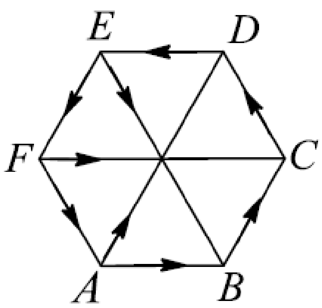
$= [\vec{\mathbf{a}}\, \vec{\mathbf{b}}\, \vec{\mathbf{c}}]^2 = 0$

486 **(d)**

$ABCDEF$ is a regular hexagon. We know from the hexagon that $\overrightarrow{\mathbf{AD}}$ is parallel to $\overrightarrow{\mathbf{BC}}$.

$\Rightarrow \overrightarrow{\mathbf{AD}} = 2\overrightarrow{\mathbf{BC}}$

Similarly, $\overrightarrow{\mathbf{EB}}$ is a parallel to $\overrightarrow{\mathbf{FA}}$



$\Rightarrow \overrightarrow{\mathbf{EB}} = 2\overrightarrow{\mathbf{FA}}$

and $\overrightarrow{\mathbf{FC}}$ is parallel to $\overrightarrow{\mathbf{AB}}$.

$\Rightarrow \overrightarrow{\mathbf{FC}} = 2\overrightarrow{\mathbf{AB}}$

Thus, $\overrightarrow{\mathbf{AD}} + \overrightarrow{\mathbf{EB}} + \overrightarrow{\mathbf{FC}} = 2\overrightarrow{\mathbf{BC}} + 2\overrightarrow{\mathbf{FA}} + 2\overrightarrow{\mathbf{AB}}$

$= 2(\overrightarrow{\mathbf{FA}} + \overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{BC}})$

$= 2(\overrightarrow{\mathbf{FC}}) = 2(2\overrightarrow{\mathbf{AB}}) = 4\overrightarrow{\mathbf{AB}}$

487 **(d)**

Here, $\overrightarrow{\mathbf{a_1}} = 6\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}, \overrightarrow{\mathbf{a_2}} = -4\hat{\mathbf{i}} + 0\hat{\mathbf{j}} - \hat{\mathbf{k}}$,

$\overrightarrow{\mathbf{b_1}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\overrightarrow{\mathbf{b_2}} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

$\therefore$Shortest distance

$= \left|\frac{(\overrightarrow{\mathbf{a_2}} - \overrightarrow{\mathbf{a_1}}) \cdot (\overrightarrow{\mathbf{b_1}} \times \overrightarrow{\mathbf{b_2}})}{|\overrightarrow{\mathbf{b_1}} \times \overrightarrow{\mathbf{b_2}}|}\right|$

$= \left|\frac{(-10\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) \cdot (8\hat{\mathbf{i}} + 8\hat{\mathbf{j}} + 4\hat{\mathbf{k}})}{\sqrt{64 + 64 + 16}}\right|$

$= \left|-\frac{108}{12}\right| = 9$

488 **(c)**

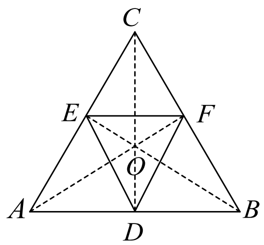
$$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix} = 15\hat{\mathbf{i}} - 10\hat{\mathbf{j}} + 30\hat{\mathbf{k}}$$

and $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = \sqrt{15^2 + (-10)^2 + (30)^2} = 35$

$\therefore$ Required vector $= \frac{3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}}}{7}$

490 **(a)**

Let O be the origin



$\therefore \overrightarrow{\mathbf{BE}} + \overrightarrow{\mathbf{AF}} = \overrightarrow{\mathbf{OE}} - \overrightarrow{\mathbf{OB}} + \overrightarrow{\mathbf{OF}} - \overrightarrow{\mathbf{OA}}$

$= \frac{\overrightarrow{\mathbf{OA}} + \overrightarrow{\mathbf{OC}}}{2} - \overrightarrow{\mathbf{OB}} + \frac{\overrightarrow{\mathbf{OB}} + \overrightarrow{\mathbf{OC}}}{2} - \overrightarrow{\mathbf{OA}}$

$= \frac{\overrightarrow{\mathbf{OC}}}{2} + \frac{\overrightarrow{\mathbf{OC}}}{2} + \frac{\overrightarrow{\mathbf{OA}}}{2} - \overrightarrow{\mathbf{OA}} + \frac{\overrightarrow{\mathbf{OB}}}{2} - \overrightarrow{\mathbf{OB}}$

$= \overrightarrow{\mathbf{OC}} - \frac{\overrightarrow{\mathbf{OA}} + \overrightarrow{\mathbf{OB}}}{2} = \overrightarrow{\mathbf{OC}} - \overrightarrow{\mathbf{OD}} = \overrightarrow{\mathbf{DC}}$

491 **(d)**

$|\vec{\mathbf{a}} - \vec{\mathbf{b}}|^2 = |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 - 2|\vec{\mathbf{a}}||\vec{\mathbf{b}}|\cos\theta$

$\Rightarrow |\vec{\mathbf{a}} - \vec{\mathbf{b}}|^2 = 1 + 1 - 2\cos 60° = 2 - 1$

$\Rightarrow |\vec{\mathbf{a}} - \vec{\mathbf{b}}| = 1$

492 **(b)**

Given, $2\vec{\mathbf{a}}+3\vec{\mathbf{b}}+\vec{\mathbf{c}}=\vec{\mathbf{0}}$
$\Rightarrow 2\vec{\mathbf{a}}+3\vec{\mathbf{b}}=-\vec{\mathbf{c}}$
Taking cross product with $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ respectively, we get
$2(\vec{\mathbf{a}}\times\vec{\mathbf{a}})+3(\vec{\mathbf{a}}\times\vec{\mathbf{b}})=-\vec{\mathbf{a}}\times\vec{\mathbf{c}}$
$\Rightarrow 3(\vec{\mathbf{a}}\times\vec{\mathbf{b}})=-\vec{\mathbf{c}}\times\vec{\mathbf{a}}$ ...(i)
and $2(\vec{\mathbf{b}}\times\vec{\mathbf{a}})+3(\vec{\mathbf{b}}\times\vec{\mathbf{b}})=-\vec{\mathbf{b}}\times\vec{\mathbf{c}}$
$\Rightarrow 2(\vec{\mathbf{a}}\times\vec{\mathbf{b}})=\vec{\mathbf{b}}\times\vec{\mathbf{c}}$ ....(ii)
Now, $\vec{\mathbf{a}}\times\vec{\mathbf{b}}+\vec{\mathbf{b}}\times\vec{\mathbf{c}}+\vec{\mathbf{c}}\times\vec{\mathbf{a}}$
$=\vec{\mathbf{a}}\times\vec{\mathbf{b}}+\vec{\mathbf{b}}\times\vec{\mathbf{c}}+3(\vec{\mathbf{a}}\times\vec{\mathbf{b}})$ [using Eq. (i)]
$=4(\vec{\mathbf{a}}\times\vec{\mathbf{b}})+\vec{\mathbf{b}}\times\vec{\mathbf{c}}$
$=2(\vec{\mathbf{b}}\times\vec{\mathbf{c}})+\vec{\mathbf{b}}\times\vec{\mathbf{c}}$ [using Eq. (ii)]
$=3(\vec{\mathbf{b}}\times\vec{\mathbf{c}})$

493 **(d)**
$[\vec{\mathbf{a}}-2\vec{\mathbf{b}},\vec{\mathbf{b}}-3\vec{\mathbf{c}},\vec{\mathbf{c}}-4\vec{\mathbf{a}}]$
$=(\vec{\mathbf{a}}-2\vec{\mathbf{b}})\cdot\{(\vec{\mathbf{b}}-3\vec{\mathbf{c}})\times(\vec{\mathbf{c}}-4\vec{\mathbf{a}})\}$
$=(\vec{\mathbf{a}}-2\vec{\mathbf{b}})\cdot\{\vec{\mathbf{b}}\times\vec{\mathbf{c}}-4\vec{\mathbf{b}}\times\vec{\mathbf{a}}+12\vec{\mathbf{c}}\times\vec{\mathbf{a}}\}$
$=(\vec{\mathbf{a}}-2\vec{\mathbf{b}})\cdot(\vec{\mathbf{a}}+4\vec{\mathbf{c}}+12\vec{\mathbf{b}})$
$=\vec{\mathbf{a}}\cdot\vec{\mathbf{a}}-24\,\vec{\mathbf{b}}\cdot\vec{\mathbf{b}}$
$=1-24\times 9=1-216=-215$

494 **(b)**
Given , area $=|\vec{\mathbf{a}}\times\vec{\mathbf{b}}|=15$
If the sides are $(3\vec{\mathbf{a}}+2\vec{\mathbf{b}})$and $(\vec{\mathbf{a}}+3\vec{\mathbf{b}})$, then
Area of parallelogram
$=|(3\vec{\mathbf{a}}+2\vec{\mathbf{b}})\times(\vec{\mathbf{a}}+3\vec{\mathbf{b}})|=7|\vec{\mathbf{a}}\times\vec{\mathbf{b}}|$
$=7\times 15=105$ sq units

498 **(a)**
Given, $\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}+\vec{\mathbf{c}})=0\ \Rightarrow \vec{\mathbf{a}}\cdot\vec{\mathbf{b}}+\vec{\mathbf{c}}\cdot\vec{\mathbf{a}}=0$
$\vec{\mathbf{b}}\cdot(\vec{\mathbf{c}}+\vec{\mathbf{a}})=0$
$\Rightarrow\ \vec{\mathbf{b}}\cdot\vec{\mathbf{c}}+\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=0$
and $\vec{\mathbf{c}}\cdot(\vec{\mathbf{a}}+\vec{\mathbf{b}})=0$
$\Rightarrow\ \vec{\mathbf{c}}\cdot\vec{\mathbf{a}}+\vec{\mathbf{b}}\cdot\vec{\mathbf{c}}=0$
$\therefore\ \vec{\mathbf{a}}\cdot\vec{\mathbf{b}}+\vec{\mathbf{b}}\cdot\vec{\mathbf{c}}+\vec{\mathbf{c}}\cdot\vec{\mathbf{a}}=0$
Now, $|\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}|^2=|\vec{\mathbf{a}}|^2+|\vec{\mathbf{b}}|^2+|\vec{\mathbf{c}}|^2+2(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}+\vec{\mathbf{b}}\cdot\vec{\mathbf{c}}+\vec{\mathbf{c}}\cdot\vec{\mathbf{a}})$
$\Rightarrow|\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}|^2=9+16+25+0=50$
$\Rightarrow|\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}|=5\sqrt{2}$

499 **(b)**
We have,
$(\vec{b}\times\vec{c})\times\vec{a}=-\{\vec{a}\times(\vec{b}\times\vec{c})\}$
$\Rightarrow(\vec{b}\times\vec{c})\times\vec{a}=-\{(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}\}$
$=(\vec{a}\cdot\vec{b})\vec{c}-(\vec{a}\cdot\vec{c})\vec{b}$

501 **(c)**
Since, $|\vec{\mathbf{u}}|=1,|\vec{\mathbf{v}}|=2,|\vec{\mathbf{w}}|=3$
The projection of $\vec{\mathbf{v}}$ along $\vec{\mathbf{u}}=\frac{\vec{\mathbf{v}}\cdot\vec{\mathbf{u}}}{|\vec{\mathbf{u}}|}$
and the projection of $\vec{\mathbf{w}}$ along $\vec{\mathbf{u}}=\frac{\vec{\mathbf{w}}\cdot\vec{\mathbf{u}}}{|\vec{\mathbf{u}}|}$
according to given condition,
$\frac{\vec{\mathbf{v}}\cdot\vec{\mathbf{u}}}{|\vec{\mathbf{u}}|}=\frac{\vec{\mathbf{w}}\cdot\vec{\mathbf{u}}}{|\vec{\mathbf{u}}|}\Rightarrow\vec{\mathbf{v}}\cdot\vec{\mathbf{u}}=\vec{\mathbf{w}}\cdot\vec{\mathbf{u}}$ ....(i)
Also, $\vec{\mathbf{v}}\cdot\vec{\mathbf{w}}=0$
Now, $|\vec{\mathbf{u}}-\vec{\mathbf{v}}+\vec{\mathbf{w}}|^2=|\vec{\mathbf{u}}|^2+|\vec{\mathbf{v}}|^2+|\vec{\mathbf{w}}|^2$
$-2\vec{\mathbf{u}}\cdot\vec{\mathbf{v}}-2\vec{\mathbf{v}}\cdot\vec{\mathbf{w}}+2\vec{\mathbf{u}}\cdot\vec{\mathbf{w}}$
$=1+4+9-2\vec{\mathbf{u}}\cdot\vec{\mathbf{v}}+\vec{\mathbf{v}}\cdot\vec{\mathbf{u}}$ [from Eq. (i)
$\Rightarrow|\vec{\mathbf{u}}-\vec{\mathbf{v}}+\vec{\mathbf{w}}|^2=14+0$
$\Rightarrow|\vec{\mathbf{u}}-\vec{\mathbf{v}}+\vec{\mathbf{w}}|=\sqrt{14}$

502 **(b)**
Area of triangle$=\frac{1}{2}\{\vec{\mathbf{a}}\times\vec{\mathbf{b}}+\vec{\mathbf{b}}\times\vec{\mathbf{c}}+\vec{\mathbf{c}}\times\vec{\mathbf{a}}\}$

503 **(c)**
$\because\ (\vec{\mathbf{a}}\times\vec{\mathbf{b}})\times\vec{\mathbf{c}}=\vec{\mathbf{a}}\times(\vec{\mathbf{b}}\times\vec{\mathbf{c}})$
$\Rightarrow(\vec{\mathbf{a}}\cdot\vec{\mathbf{c}})\vec{\mathbf{b}}-(\vec{\mathbf{b}}\cdot\vec{\mathbf{c}})\vec{\mathbf{a}}=(\vec{\mathbf{a}}\cdot\vec{\mathbf{c}})\vec{\mathbf{b}}-(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}})\vec{\mathbf{c}}$
$\Rightarrow(\vec{\mathbf{b}}\cdot\vec{\mathbf{c}})\vec{\mathbf{a}}=(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}})\vec{\mathbf{c}}$
$\Rightarrow\ \vec{\mathbf{a}}$ is parallel to $\vec{\mathbf{c}}$

504 **(d)**
Let $\vec{\mathbf{r}}$ be a unit vector such that
$\vec{\mathbf{r}}=x(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}})+y(\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}})$
$=(x+y)\hat{\mathbf{i}}+(2x+y)\hat{\mathbf{j}}+(x+2y)\hat{\mathbf{k}}$
Since, $\vec{\mathbf{r}}\cdot(2\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})=0$
$\Rightarrow 2x+2y+2x+y+x+2y=0$
$\Rightarrow y=-x$
$\therefore\ \vec{\mathbf{r}}=x\hat{\mathbf{i}}-x\hat{\mathbf{k}}\Rightarrow\vec{\mathbf{r}}=\frac{\hat{\mathbf{i}}-\hat{\mathbf{k}}}{\sqrt{2}}$

505 **(a)**
Since $\vec{a},\vec{b}$ and $\vec{c}$ are unit vectors inclined at an angle θ. Therefore,
$|\vec{a}|=|\vec{b}|=|\vec{c}|=1$ and $\cos\theta=\vec{a}.\vec{c}=\vec{b}.\vec{c}$
Now,
$\vec{c}=\alpha\vec{a}+\beta\vec{b}+\gamma(\vec{a}\times\vec{b})$ ...(i)
$\Rightarrow\vec{a}.\vec{c}=\alpha(\vec{a}.\vec{a})+\beta(\vec{a}\cdot\vec{b})+\gamma\{\vec{a}\cdot(\vec{a}\times\vec{b})\}$
$\Rightarrow\cos\theta=\alpha|\vec{a}|^2\quad[\because\vec{a}.\vec{b}=0,\vec{a}.(\vec{a}\times\vec{b})=0]$
$\Rightarrow\cos\theta=\alpha$
Similarly, by taking dot product on both sides of (i) by $\vec{b}$, we get, $\beta=\cos\theta$
$\therefore\alpha=\beta$
Thus, option (a) is incorrect
Again,
$\vec{c}=\alpha\vec{a}+\beta\vec{b}+\gamma(\vec{a}\times\vec{b})$
$\Rightarrow|\vec{c}|^{\mathbf{2}}=|\alpha\vec{a}+\beta\vec{b}+\gamma(\vec{a}\times\vec{b})|^{\mathbf{2}}$

$\Rightarrow |\vec{c}|^2 = \alpha^2|\vec{a}|^2 + \beta^2|\vec{b}|^2 + \gamma^2|\vec{a}\times\vec{b}|^2$
$+ 2\,\alpha\,\beta\,(\vec{a}.\vec{b}) + 2\,\alpha\,\gamma\{\vec{a}.(\vec{a}\times\vec{b})\}$
$+ 2\,\beta\,\gamma\{\vec{b}.(\vec{a}\times\vec{b})\}$

$\Rightarrow 1 = \alpha^2 + \beta^2 + \gamma^2|\vec{a}\times\vec{b}|^2$

$\Rightarrow 1 = 2\,\alpha^2 + \gamma^2\left\{|\vec{a}|^2|\vec{b}|^2 \sin^2\frac{\pi}{2}\right\}$

$\Rightarrow 1 = 2\,\alpha^2 + \gamma^2$

$\Rightarrow \alpha^2 = \dfrac{1-\gamma^2}{2}$

But, $\alpha = \beta = \cos\theta$

$\therefore 1 = 2\,\alpha^2 + \gamma^2 \Rightarrow \gamma^2 = 1 - 2\cos^2\theta = -\cos 2\,\theta$

$\therefore \alpha^2 = \beta^2 = \dfrac{1-\gamma^2}{2} = \dfrac{1+\cos 2\,\theta}{2}$

Thus, option (b), (c) and (d) are correct

506 **(d)**

Let $\vec{a} = 7\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$ and $\vec{b} = -2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ be the position vectors of points A and B respectively. Then the bisector of $\angle AOB$ divides AB in the ratio $OA : OB$ i.e. 9 : 3or 3 : 1. Therefore, the vector lying along the bisector is

$$\frac{3(-2\hat{\imath} - \hat{\jmath} + 2\hat{k}) + (7\hat{\imath} - 4\hat{\jmath} - 4\hat{k})}{3+1} = \frac{1}{4}(\hat{\imath} - 7\hat{\jmath} + 2\hat{k})$$

$\therefore$ Required vector $= \pm 5\sqrt{6}\left(\frac{(\hat{\imath}-7\hat{\jmath}+2\hat{k})}{\sqrt{54}}\right) = \pm\frac{5}{3}(\hat{\imath} - 7\hat{\jmath} + 2\hat{k})$

507 **(b)**

Since, $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are collinear.

$\therefore\ \vec{\mathbf{b}} = m\vec{\mathbf{a}}$

$\Longrightarrow |\vec{\mathbf{b}}| = m|\vec{\mathbf{a}}|$

$\Longrightarrow |\vec{\mathbf{b}}| = m\sqrt{4+9+36} = \pm 7m$

$\Longrightarrow 21 = \pm 7m \Longrightarrow m = \pm 3$

$\therefore\ \vec{\mathbf{b}} = \pm 3\vec{\mathbf{a}} = \pm(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$

510 **(a)**

Position vectors of vertices A, B and C of the triangle ABC are $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$

$\therefore$Centroid of triangle

$(G) = \dfrac{\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}}{3}$

Now, $\overrightarrow{\mathbf{GA}} + \overrightarrow{\mathbf{GB}} + \overrightarrow{\mathbf{GC}}$

$$= \left(\vec{\mathbf{a}} - \frac{\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}}{3}\right) + \left(\vec{\mathbf{b}} - \frac{\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}}{3}\right) + \left(\vec{\mathbf{c}} - \frac{\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}}{3}\right)$$

$= \vec{\mathbf{0}}$

511 **(d)**

Since X and Y divide $\overrightarrow{AB}$ internally and externally in the ratio 2 : 1. Therefore, the position vectors of X and Y are given by $\frac{2\vec{b}+\vec{a}}{3}$ and $2\vec{b} - \vec{a}$ respectively

Hence, $\overrightarrow{XY} = (2\vec{b} - \vec{a}) - \frac{1}{3}(2\vec{b} + \vec{a}) = \frac{4}{3}(\vec{b} - \vec{a})$

512 **(a)**

Let $\vec{\mathbf{a}} = (2,1,-1), \vec{\mathbf{b}} = (1,-1,0)$and $\vec{\mathbf{c}} = (5,-1,1)$

$\therefore\ \vec{\mathbf{a}} + \vec{\mathbf{b}} - \vec{\mathbf{c}} = (2+1-5)\hat{\mathbf{i}} + (1-1+1)\hat{\mathbf{j}} + (-1+0-1)\hat{\mathbf{k}}$

$= -(2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

$\therefore$ Unit vector of

$(\vec{\mathbf{a}} + \vec{\mathbf{b}} - \vec{\mathbf{c}}) = -\dfrac{(2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}})}{3}$

$\therefore$ Required unit vector of

$(\vec{\mathbf{a}} + \vec{\mathbf{b}} - \vec{\mathbf{c}}) = \dfrac{(2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}})}{3}$

513 **(b)**

$$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$$

$\therefore$ Unit vector

$= \pm\dfrac{\vec{\mathbf{a}} \times \vec{\mathbf{b}}}{|\vec{\mathbf{a}} \times \vec{\mathbf{b}}|} = \pm\dfrac{\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}}{1^2 + 1^2 + 1^2}$

$= \pm\dfrac{\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{3}}$

So, there are two perpendicular vectors of unit length.

514 **(b)**

Let $\vec{\mathbf{r}} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) + b(6\hat{\mathbf{i}} - 7\hat{\mathbf{j}} - 3\hat{\mathbf{k}})$

$= (3+6b)\hat{\mathbf{i}} + (4-7b)\hat{\mathbf{j}} + (5-3b)\hat{\mathbf{k}}$

Since, $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) = 0$

$\Longrightarrow (3+6b)1 + (4-7b)1 - (5-3b)1 = 0$

$\Longrightarrow b = -1$

$\therefore\ \vec{\mathbf{r}} = -3\hat{\mathbf{i}} + 11\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$

515 **(d)**

Given $|\vec{\mathbf{x}}| = |\vec{\mathbf{y}}| = 1$ and $\vec{\mathbf{x}} \cdot \vec{\mathbf{y}} = 0$

$|\vec{\mathbf{x}} + \vec{\mathbf{y}}|^2 = |\vec{\mathbf{x}}|^2 + |\vec{\mathbf{y}}|^2 + 2(\vec{\mathbf{x}} \cdot \vec{\mathbf{y}})$

$\Longrightarrow |\vec{\mathbf{x}} + \vec{\mathbf{y}}|^2 = 1 + 1 + 0$

$\Longrightarrow |\vec{\mathbf{x}} + \vec{\mathbf{y}}| = \sqrt{2}$

516 **(c)**

Let $\vec{\mathbf{A}} = \vec{\mathbf{a}} \times \vec{\mathbf{b}}, \vec{\mathbf{B}} = \vec{\mathbf{b}} \times \vec{\mathbf{c}}, \vec{\mathbf{C}} = \vec{\mathbf{c}} \times \vec{\mathbf{a}}$

Given, $[\vec{\mathbf{A}}\,\vec{\mathbf{B}}\,\vec{\mathbf{C}}] = 9$ cu units

Using the relation $[\vec{\mathbf{A}} \times \vec{\mathbf{B}}\ \vec{\mathbf{B}} \times \vec{\mathbf{C}}\ \vec{\mathbf{C}} \times \vec{\mathbf{A}}] = [\vec{\mathbf{A}}\,\vec{\mathbf{B}}\,\vec{\mathbf{C}}]^2 = (9)^2 = 81$ cu units

517 **(a)**

Since, $\vec{\mathbf{a}} = 8\vec{\mathbf{b}}$ and $\vec{\mathbf{c}} = -7\vec{\mathbf{b}}$

$\therefore\ \vec{\mathbf{a}}$ is parallel to $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ is anti-parallel to $\vec{\mathbf{b}}$

$\Longrightarrow\ \vec{\mathbf{a}}$ and $\vec{\mathbf{c}}$ are anti-parallel

$\Rightarrow$ Angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{c}}$ is π

519 **(a)**

$\vec{\mathbf{a}} \cdot \vec{\mathbf{c}} = (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) \cdot \hat{\mathbf{i}} = 1$

and $\vec{\mathbf{b}} \cdot \vec{\mathbf{c}} = (\hat{\mathbf{i}} + \hat{\mathbf{j}}) \cdot \hat{\mathbf{i}} = 1$

Now, $(\vec{\mathbf{a}} \times \vec{\mathbf{b}})\vec{\mathbf{c}} = (\vec{\mathbf{c}} \cdot \vec{\mathbf{a}})\vec{\mathbf{b}} - (\vec{\mathbf{c}} \cdot \vec{\mathbf{b}})\vec{\mathbf{a}} = \mu\vec{\mathbf{b}} + \lambda\vec{\mathbf{a}}$

$\Rightarrow \mu = \vec{\mathbf{c}} \cdot \vec{\mathbf{a}}$ and $\lambda = -\vec{\mathbf{c}} \cdot \vec{\mathbf{b}}$

$\Rightarrow \mu = 1$ and $\lambda = -1$

$\therefore \mu + \lambda = 1 - 1 = 0$

520 **(b)**

Let angle between $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ is α.

Given, $|\vec{\mathbf{b}} \times \vec{\mathbf{c}}| = \sqrt{15}$

$\Rightarrow |\vec{\mathbf{b}}||\vec{\mathbf{c}}| \sin\alpha = \sqrt{15}$

$\Rightarrow \sin\alpha = \dfrac{\sqrt{15}}{4}$

$\therefore \cos\alpha = \sqrt{1 - \sin^2\alpha} = \sqrt{1 - \dfrac{15}{16}}$

$= \dfrac{1}{4}$

$\because \vec{\mathbf{b}} - 2\vec{\mathbf{c}} = \lambda\vec{\mathbf{a}}$ [given]

$\Rightarrow (\vec{\mathbf{b}} - 2\vec{\mathbf{c}})^2 = \lambda^2(\vec{\mathbf{a}})^2$

$\Rightarrow \vec{\mathbf{b}}^2 + 4\vec{\mathbf{c}}^2 - 4\vec{\mathbf{b}} \cdot \vec{\mathbf{c}} = \lambda^2\vec{\mathbf{a}}^2$

$\Rightarrow 16 + 4 \times 1 - 4(|\vec{\mathbf{b}}|\,|\vec{\mathbf{c}}| \cos\alpha) = \lambda^2 \cdot 1^2$

$\Rightarrow 20 - 4 = \lambda^2$

$\Rightarrow \lambda = \pm 4$

521 **(a)**

The given condition mean that $\vec{\mathbf{r}}$ is perpendicular to all three vectors $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$. This is possible only if they are coplanar.

$\therefore [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}] = 0$

523 **(d)**

Let $\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\vec{\mathbf{b}} = \hat{\mathbf{j}} + \hat{\mathbf{k}}$

Now, $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix}$

$= \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$

and $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}$

$\therefore$*Required unit vector*

$= \dfrac{\vec{\mathbf{a}} \times \vec{\mathbf{b}}}{|\vec{\mathbf{a}} \times \vec{\mathbf{b}}|} = \dfrac{\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{3}}$

Alternate Let $\vec{\mathbf{a}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

Since, $\vec{\mathbf{a}} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}}) = 0$ and $\vec{\mathbf{a}} \cdot (\hat{\mathbf{j}} + \hat{\mathbf{k}}) = 0$

$\Rightarrow x + y = 0$ and $y + z = 0$

Also $x^2 + y^2 + z^2 = 1$

$\Rightarrow x = 1, y = -1$ and $z = 1$

$\therefore \vec{\mathbf{a}} = \dfrac{\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{3}}$

524 **(a)**

Let $\vec{\mathbf{r}} = \vec{\mathbf{a}} + t\vec{\mathbf{b}}$

$\Rightarrow \vec{\mathbf{r}} = \hat{\mathbf{i}}(1 + t) + \hat{\mathbf{j}}(2 - t) + \hat{\mathbf{k}}(1 + t)$

Since, The projection of $\vec{\mathbf{r}}$ on $\vec{\mathbf{c}}$,

$\dfrac{\vec{\mathbf{r}} \cdot \vec{\mathbf{c}}}{|\vec{\mathbf{c}}|} = \dfrac{|1|}{|\sqrt{3}|}$ [given]

$\Rightarrow \dfrac{1 \cdot (1 + t) + 1 \cdot (2 - t) - 1 \cdot (1 + t)}{\sqrt{3}} = \pm\dfrac{1}{\sqrt{3}}$

$\Rightarrow 2 - t = \pm 1$

$\Rightarrow t = 1$ or 3

When, $t = 1, \vec{\mathbf{r}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

When, $t = 3, \vec{\mathbf{r}} = 4\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

525 **(a)**

Given, $\vec{\mathbf{u}} \times \vec{\mathbf{v}} + \vec{\mathbf{u}} = \vec{\mathbf{w}}$ and $\vec{\mathbf{w}} \times \vec{\mathbf{u}} = \vec{\mathbf{v}}$

$\Rightarrow (\vec{\mathbf{u}} \times \vec{\mathbf{v}} + \vec{\mathbf{u}}) \times \vec{\mathbf{u}} = \vec{\mathbf{v}}$

$\Rightarrow (\vec{\mathbf{u}} \times \vec{\mathbf{v}}) \times \vec{\mathbf{u}} = \vec{\mathbf{v}}$

$\Rightarrow \vec{\mathbf{v}} - (\vec{\mathbf{u}} \cdot \vec{\mathbf{v}}) = \vec{\mathbf{v}}$

$\Rightarrow (\vec{\mathbf{u}} \cdot \vec{\mathbf{v}})\vec{\mathbf{u}} = 0$

$\Rightarrow (\vec{\mathbf{u}} \cdot \vec{\mathbf{v}}) = 0$

Now, $[\vec{\mathbf{u}}\,\vec{\mathbf{v}}\,\vec{\mathbf{w}}] = \vec{\mathbf{u}} \cdot (\vec{\mathbf{v}} \times \vec{\mathbf{w}})$

$= \vec{\mathbf{u}} \cdot (\vec{\mathbf{v}} \times (\vec{\mathbf{u}} \times \vec{\mathbf{v}} + \vec{\mathbf{u}}))$

$= \vec{\mathbf{u}} \cdot (\vec{\mathbf{v}}(\vec{\mathbf{u}} \times \vec{\mathbf{v}}) + \vec{\mathbf{v}} + \vec{\mathbf{u}})$

$= \vec{\mathbf{u}} \cdot (\vec{\mathbf{v}}^2 \times \vec{\mathbf{u}} - (\vec{\mathbf{u}} \cdot \vec{\mathbf{v}}) \cdot \vec{\mathbf{v}} + \vec{\mathbf{v}} \times \vec{\mathbf{u}}$

$= \vec{\mathbf{v}}^2\vec{\mathbf{u}}^2 = 1$

527 **(b)**

Given, $\dfrac{(\vec{\mathbf{b}} \cdot \vec{\mathbf{a}}) \cdot \vec{\mathbf{a}}}{|\vec{\mathbf{a}}|^2} = \dfrac{4}{3}(\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})$

$\Rightarrow \dfrac{\{(\lambda\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}).(\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})\}(\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})}{(1 + 1 + 1)}$

$= \dfrac{4}{3}(\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})$

$\Rightarrow (\lambda + 3 - 1)(\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}) = 4(\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})$

$\Rightarrow (\lambda + 2)(\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}) = 4(\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})$

On equating the coefficient of $\hat{\mathbf{i}}$, we get

$\lambda + 2 = 4 \Rightarrow \lambda = 2$

528 **(a)**

Given that, $\overrightarrow{\mathbf{OA}} = \hat{\mathbf{i}} + x\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

$\overrightarrow{\mathbf{OB}} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$

and $\overrightarrow{\mathbf{O}C} = y\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

Since A, B, C are collinear. Then $\vec{\mathbf{A}} = \lambda\,\overrightarrow{\mathbf{BC}}$

$\Rightarrow 2\hat{\mathbf{i}} + (4 - x)\hat{\mathbf{j}} + 4\hat{\mathbf{k}} = \lambda\,[(y - 3)\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 12\hat{\mathbf{k}}]$

On comparing the coefficient of $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$, we get

$2 = (y - 3)\lambda$ ...(i)

$4 - x = -6\lambda$ ....(ii)

and $4 = -12\lambda \Rightarrow \lambda = -\frac{1}{3}$ .....(iii)

On putting the value of λ is Eqs. (i) and (ii),we get

$y = -3$ and $x = 2$

529 **(b)**

Given have magnitude of $\overrightarrow{\mathbf{OA}}$ and $\overrightarrow{\mathbf{OB}}$ are 5 and 6 respectively

and $\angle BOA = 60°$

$\therefore\ \overrightarrow{\mathbf{OA}} \cdot \overrightarrow{\mathbf{OB}} = |\overrightarrow{\mathbf{OA}}||\overrightarrow{\mathbf{OB}}| \cdot \cos 60°$

$\Longrightarrow\ \overrightarrow{\mathbf{OA}} \cdot \overrightarrow{\mathbf{OB}} = 5 \cdot 6 \cos 60°$

$\Longrightarrow \overrightarrow{\mathbf{OA}} \cdot \overrightarrow{\mathbf{OB}} = 5 \times 6 \times \frac{1}{2} = 15$

530 **(d)**

It is given that $|\vec{a}| = |\vec{b}| = |\vec{a} + \vec{b}| = 1$

We have,

$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b}$

$\Rightarrow 1 = 1 + 1 + 2|\vec{a}||\vec{b}| \cos \theta$

$\Rightarrow \cos \theta = -\frac{1}{2} \Rightarrow \theta = \frac{2\pi}{3}$

531 **(a)**

Area of $\Delta\, ABC = \frac{1}{2}|\overrightarrow{\mathbf{AB}} \times \overrightarrow{\mathbf{AC}}|$

$= \frac{1}{2}\sqrt{4 + 16 + 16} = 3$ sq units

532 **(a)**

Since, $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ from a right handed system

$\therefore\ \vec{\mathbf{c}} = \vec{\mathbf{b}} \times \vec{\mathbf{a}}$

$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 1 & 0 \\ x & y & z \end{vmatrix} = z\hat{\mathbf{i}} - x\hat{\mathbf{k}}$

533 **(b)**

Given that, $|\vec{\mathbf{a}}| = |\vec{\mathbf{c}}| = 1, |\vec{\mathbf{b}}| = 4$

Let angle between $\vec{\mathrm{b}}$ and $\vec{\mathrm{c}}$ is α, then

$|\vec{\mathbf{b}} \times \vec{\mathbf{c}}| = \sqrt{15}$ (given)

$\Rightarrow\ |\vec{\mathbf{b}}||\vec{\mathbf{c}}| \sin \alpha = \sqrt{15}$

$\Rightarrow \sin \alpha = \frac{\sqrt{15}}{4 \times 1} = \frac{\sqrt{15}}{4}$

$\therefore \cos \alpha = \sqrt{1 - \sin^2 \alpha} = \frac{1}{4}$

We have, $\vec{\mathbf{b}} = 2\vec{\mathbf{c}} = \lambda\, \vec{\mathbf{a}}$

On squaring both sides, we get

$(\vec{\mathbf{b}} - 2\vec{\mathbf{c}})^2 = \lambda^2(\vec{\mathbf{a}})^2$

$\Rightarrow \vec{\mathbf{b}}^2 + 4\vec{\mathbf{c}}^2 - 4\, \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} = \lambda^2\, \vec{\mathbf{a}}^2$

$\Rightarrow 16 + 4 - 4|\vec{\mathbf{b}}||\vec{\mathbf{c}}| \cos \alpha = \lambda^2$

$\Rightarrow\ 16 + 4 - 4 \times 4 \times 1 \times \frac{1}{4} = \lambda^2$

$\Rightarrow \lambda^2 = 16 + 4 - 4 = 16$

$\Rightarrow\ \lambda = \pm 4$

534 **(a)**

We have,

$PR = 5\, PQ \Rightarrow PQ + QR = 5\, PQ \Rightarrow 4\, PQ = QR$

$\therefore PR : QR = 5 : 4$

$\Rightarrow R$ divides PQ externally in the ratio $5 : 4$

$\Rightarrow$ Position vector of R is $5\, \vec{b} - 4\, \vec{a}$

536 **(a)**

We have,

$\vec{B}A + \vec{B}C + \vec{C}D + \vec{D}A$

$= \vec{B}A + (\vec{B}C + \vec{C}D) + \vec{D}A = \vec{B}A + (\vec{B}D + \vec{D}A)$

$= \vec{B}A + \vec{B}A = 2\vec{B}A$

537 **(a)**

Given centre of sphere=(1, 0, 1) and radius=4

$\therefore$Vector equation of sphere is

$|\vec{\mathbf{r}} - \vec{\mathbf{a}}| = R$ Where $\vec{\mathbf{a}}$ centre of sphere and R radius of sphere.

Hence, the vector equation of sphere is

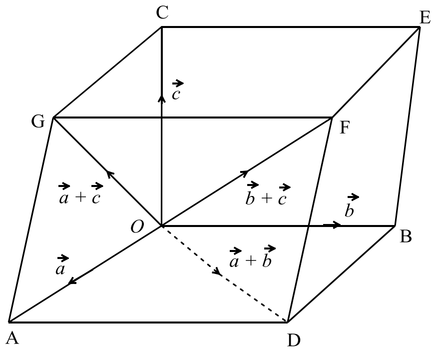
$|\vec{\mathbf{r}} - (\hat{\mathbf{i}} + \hat{\mathbf{k}})| = 4$

538 **(b)**

We have, $|[\vec{a}\ \vec{b}\ \vec{c}]| = V$

Volume V_1 of the parallelopiped having diagonals of the given parallelopiped as three concurrent edges is given by

$V_1 = |[\vec{a} + \vec{b}\ \vec{b} + \vec{c}\ \vec{c} + \vec{a}]| = |2[\vec{a}\ \vec{b}\ \vec{c}]| = 2V$



540 **(d)**

The given equation is

$\overrightarrow{\mathbf{r^2}} - 2\vec{\mathbf{r}} \cdot \vec{\mathbf{c}} + h = 0, |\vec{\mathbf{c}}| > \sqrt{h}$

This is the equation of sphere in diameter form.

$ie, (\vec{\mathbf{r}} - \vec{\mathbf{a}}) \cdot (\vec{\mathbf{r}} - \vec{\mathbf{b}}) = 0$

541 **(c)**

Let the given points be A, B, C respectively.

If A, B, C are collinear, then

$A\vec{B} = \lambda\, B\vec{C}$ for some scalar λ

$\Rightarrow 2\hat{\imath} - 8\hat{\imath} = \lambda\, \{(a! - 12)\hat{\imath} + 16\hat{\jmath}\}$

$\Rightarrow \lambda(a - 12) = 2$ and $16\lambda = -8$

$\Rightarrow a - 12 = -4 \Rightarrow a = 8$

542 **(a)**

We have,

$\vec{a} \times (\vec{a} \times \vec{b}) = \vec{b} \times (\vec{b} \times \vec{c})$

$\Rightarrow (\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b} = (\vec{b} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{b})\vec{c}$

Taking dot product on both sides by $\vec{b} \times \vec{c}$, we get

$\Rightarrow (\vec{a} \cdot \vec{b})\{\vec{a} \cdot (\vec{b} \times \vec{c})\} - (\vec{a} \cdot \vec{a})\{\vec{b} \cdot (\vec{b} \times \vec{c})\}$

$= (\vec{b} \cdot \vec{c})\{\vec{b} \cdot (\vec{b} \times \vec{c})\}$

$- (\vec{b} \cdot \vec{b})\{\vec{c} \cdot (\vec{b} \times \vec{c})\}$

$\Rightarrow (\vec{a}\cdot\vec{b})[\vec{a}\,\vec{b}\,\vec{c}] = 0$

$\Rightarrow [\vec{a}\,\vec{b}\,\vec{c}] = 0 \qquad [\because \vec{a}\cdot\vec{b} \neq 0]$

543 **(a)**

We have,

$[\vec{a}\,\vec{b}\,\vec{a}\times\vec{b}] + (\vec{a}\cdot\vec{b})^2$

$= (\vec{a}\times\vec{b})\cdot(\vec{a}\times\vec{b}) + (\vec{a}\cdot\vec{b})^2$

$\Rightarrow [\vec{a}\,\vec{b}\,\vec{a}\times\vec{b}] + (\vec{a}\cdot\vec{b})^2 = |\vec{a}\times\vec{b}|^2 + (\vec{a}\cdot\vec{b})^2$

$= |\vec{a}|^2|\vec{b}|^2$

544 **(d)**

Since,

$[3\vec{\mathbf{v}}\,p\,\vec{\mathbf{v}}\,p\,\vec{\mathbf{w}}] - [p\,\vec{\mathbf{v}}\,\vec{\mathbf{w}}\,q\,\vec{\mathbf{u}}] - [2\,\vec{\mathbf{w}}\,q\,\vec{\mathbf{v}}\,q\,\vec{\mathbf{u}}] = 0$

$\therefore 3p^2[\vec{\mathbf{u}}\cdot(\vec{\mathbf{v}}\times\vec{\mathbf{w}})] - pq[\vec{\mathbf{v}}\cdot(\vec{\mathbf{w}}\times\vec{\mathbf{u}})]$

$-2q^2[\vec{\mathbf{w}}\cdot(\vec{\mathbf{v}}\times\vec{\mathbf{u}})] = 0$

$\Longrightarrow (3p^2 - pq + 2q^2)[\vec{\mathbf{u}}\cdot(\vec{\mathbf{v}}\times\vec{\mathbf{w}})] = 0$

But $[\vec{\mathbf{u}}\,\vec{\mathbf{v}}\,\vec{\mathbf{w}}] \neq 0$

$\Longrightarrow 3p^2 - pq + 2q^2 = 0$

$\Longrightarrow p = q = 0$

545 **(a)**

$\vec{\mathbf{a}}\cdot[(\vec{\mathbf{b}}+\vec{\mathbf{c}})\times(\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}})]$

$= \vec{\mathbf{a}}\cdot[\vec{\mathbf{b}}\times\vec{\mathbf{a}} + \vec{\mathbf{b}}\times\vec{\mathbf{c}} + \vec{\mathbf{c}}\times\vec{\mathbf{a}} + \vec{\mathbf{c}}\times\vec{\mathbf{b}})]$

$= \vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}}) + \vec{\mathbf{a}}\cdot(\vec{\mathbf{c}}\times\vec{\mathbf{b}})$

$= [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}] + [\vec{\mathbf{a}}\,\vec{\mathbf{c}}\,\vec{\mathbf{b}}] = 0 \qquad [\because [\vec{\mathbf{a}}\,\vec{\mathbf{c}}\,\vec{\mathbf{b}}] = -[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]]$

546 **(b)**

Let, $\vec{\mathbf{a}} = \hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}},\ \vec{\mathbf{b}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$

Now, $\vec{\mathbf{a}}\times\vec{\mathbf{b}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -3 & 2 \\ -1 & 2 & 0 \end{vmatrix} = -4\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$

$\therefore$ Area of parallelogram $= \frac{1}{2}|\vec{\mathbf{a}}\times\vec{\mathbf{b}}|$

$= \frac{1}{2}\sqrt{16+4+1} = \frac{\sqrt{21}}{2}$

547 **(c)**

Since, $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = 0$ ...(i)

Also, $(\vec{\mathbf{a}}+3\vec{\mathbf{b}})\cdot(2\vec{\mathbf{a}}-\vec{\mathbf{b}}) = -10$

$\Longrightarrow 2|\vec{\mathbf{a}}|^2 - \vec{\mathbf{a}}\cdot\vec{\mathbf{b}} + 6\vec{\mathbf{b}}\cdot\vec{\mathbf{a}} - 3|\vec{\mathbf{b}}|^2 = -10$

$\Longrightarrow 2 - 3|\vec{\mathbf{b}}|^2 = -10 \Longrightarrow |\vec{\mathbf{b}}| = 2$ [from Eq. (i)]

548 **(a)**

We have, $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} + \hat{\jmath}, \vec{c} = \hat{\imath}$

$\therefore (\vec{a}\times\vec{b})\times\vec{c} = \lambda\vec{a} + \mu\vec{b}$

$\Rightarrow (\vec{c}\cdot\vec{a})\vec{b} - (\vec{c}\cdot\vec{b})\vec{a} = \lambda\vec{a} + \mu\vec{b}$

$\Rightarrow \vec{b} - \vec{a} = \lambda\vec{a} + \mu\vec{b}$

$\Rightarrow (\lambda+1)\vec{a} + (\mu-1)\vec{b} = \vec{0}$

$\Rightarrow \lambda + 1 = 0$ and $\mu - 1 = 0 \quad [\because \vec{a}, \vec{b}$, are non – collinear

$\Rightarrow \lambda + \mu = 0$

550 **(c)**

Let angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ be θ_1. $\vec{\mathbf{c}}$ and $\vec{\mathbf{d}}$ be θ_2 and $\vec{\mathbf{a}}\times\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}\times\vec{\mathbf{d}}$ be θ

Since, $(\vec{\mathbf{a}}\times\vec{\mathbf{b}})\cdot(\vec{\mathbf{c}}\times\vec{\mathbf{d}}) = 1$

$\Longrightarrow \sin\theta_1\cdot\sin\theta_2$

$\cdot\cos\theta$

$= 1 \quad (\because |\vec{\mathbf{a}}| = |\vec{\mathbf{b}}| = |\vec{\mathbf{c}}| = |\vec{\mathbf{d}}| = 1)$

$\Longrightarrow \theta_1 = 90°\cdot\theta_2 = 90°, \theta = 0°$

$\Longrightarrow \vec{\mathbf{a}} \perp \vec{\mathbf{b}}, \vec{\mathbf{c}} \perp \vec{\mathbf{d}}, (\vec{\mathbf{a}}\times\vec{\mathbf{b}})||(\vec{\mathbf{c}}\times\vec{\mathbf{d}})$

So, $\vec{\mathbf{a}}\times\vec{\mathbf{b}} = k(\vec{\mathbf{c}}\times\vec{\mathbf{d}})$ and $\vec{\mathbf{a}}\times\vec{\mathbf{b}} = k(\vec{\mathbf{c}}\times\vec{\mathbf{d}})$

$\Longrightarrow (\vec{\mathbf{a}}\times\vec{\mathbf{b}})\cdot\vec{\mathbf{c}} = k(\vec{\mathbf{c}}\times\vec{\mathbf{d}})\cdot\vec{\mathbf{c}}$

and $(\vec{\mathbf{a}}\times\vec{\mathbf{b}})\cdot\vec{\mathbf{d}} = k(\vec{\mathbf{c}}\times\vec{\mathbf{d}})\cdot\vec{\mathbf{d}}$

$\Longrightarrow [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}] = 0$ and $[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{d}}] = 0$

$\Longrightarrow$ $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ and $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{d}}$ are coplanar vector so option (A) and (B) are incorrect.

Let $\vec{\mathbf{b}}||\vec{\mathbf{d}} \Longrightarrow \vec{\mathbf{b}} = \pm\vec{\mathbf{d}}$

As $(\vec{\mathbf{a}}\times\vec{\mathbf{b}})\cdot(\vec{\mathbf{c}}\times\vec{\mathbf{d}}) = 1 \Longrightarrow (\vec{\mathbf{a}}\times\vec{\mathbf{b}})\cdot(\vec{\mathbf{c}}\times\vec{\mathbf{b}}) = \pm 1$

$\Longrightarrow [\vec{\mathbf{a}}\times\vec{\mathbf{b}}\,\vec{\mathbf{c}}\,\vec{\mathbf{b}}] = \pm 1$

$\Longrightarrow [\vec{\mathbf{c}}\,\vec{\mathbf{b}}\,\vec{\mathbf{a}}\times\vec{\mathbf{b}}] = \pm 1$

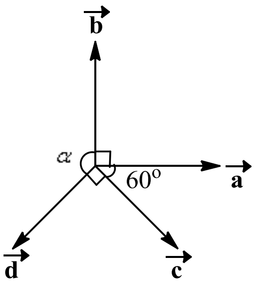
$\Longrightarrow \vec{\mathbf{c}}\cdot[\vec{\mathbf{b}}\times(\vec{\mathbf{a}}\times\vec{\mathbf{b}})] = \pm 1$

$\Longrightarrow \vec{\mathbf{c}}\cdot[\vec{\mathbf{a}} - (\vec{\mathbf{b}}\cdot\vec{\mathbf{a}})\vec{\mathbf{b}}] = \pm 1$

$\Longrightarrow \vec{\mathbf{c}}\cdot\vec{\mathbf{a}} = \pm 1 \quad (\because \vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = 0)$

Which is a contradiction so option (c) is correct.

Let option (d) is correct



$\Longrightarrow \vec{\mathbf{d}} = \pm\vec{\mathbf{a}}$ and $\vec{\mathbf{c}} = \pm\vec{\mathbf{b}}$

As $(\vec{\mathbf{a}}\times\vec{\mathbf{b}})\cdot(\vec{\mathbf{c}}\times\vec{\mathbf{d}}) = 1$

$\Longrightarrow (\vec{\mathbf{a}}\times\vec{\mathbf{b}})\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{a}}) = \pm 1$

Which is a contradiction so option (d) is incorrect.

Alternate Option (c) and (d) may be observed from given in figure.

552 **(b)**

$(\hat{\mathbf{i}}\times\hat{\mathbf{j}})\cdot\vec{\mathbf{c}} \leq |\hat{\mathbf{i}}\times\hat{\mathbf{j}}||\vec{c}|\cos\frac{\pi}{6}$

$\Rightarrow -\frac{\sqrt{3}}{2} \leq (\hat{\mathbf{i}}\times\hat{\mathbf{j}})\cdot\vec{\mathbf{c}} \leq \frac{\sqrt{3}}{2}$

553 **(b)**

It is given that $\hat{a}$ and $\hat{b}$ are mutually perpendicular unit vectors. Therefore, $\hat{a}, \hat{b}$ and $\hat{a}\times\hat{b}$ are non-coplanar vectors.

$\therefore [\hat{a}\ \hat{b}\ \hat{a}\times\hat{b}] \neq 0$

If the vectors $\vec{\alpha} = x\hat{a} + x\hat{b} + z(\hat{a}\times\hat{b}), \vec{\beta} = \hat{a} + (\hat{a}\times\hat{b})$

and, $\vec{\gamma} = z\hat{a} + z\hat{b} + y(\hat{a}\times\hat{b})$ are coplanar, then

$[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}] = 0$

$\Rightarrow \begin{vmatrix} x & x & z \\ 1 & 0 & 1 \\ z & z & y \end{vmatrix} [\hat{a}\ \hat{b}\ \hat{a}\times\hat{b}] = 0$

$\Rightarrow \begin{vmatrix} x & x & z \\ 1 & 0 & 1 \\ z & z & y \end{vmatrix} = 0 \qquad [\because [\hat{a}\ \hat{b}\ \hat{a}\times\hat{b}] \neq 0]$

$\Rightarrow x(0-z) - x(y-z) + z(z-0) = 0$

$\Rightarrow -xz - yx + xz + z^2 = 0$

$\Rightarrow z^2 = xy$

$\Rightarrow z$ is the geometric mean of x and y

554 **(d)**

Given, $\vec{\mathbf{a}} = (1, p, 1), \vec{\mathbf{b}} = (q, 2, 2)$

$\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = r$ and $\vec{\mathbf{a}}\times\vec{\mathbf{b}} = (0, -3, -3)$

Now, $\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = (\hat{\mathbf{i}} + p\hat{\mathbf{j}} + \hat{\mathbf{k}})\cdot(q\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

$\Longrightarrow q + 2p + 2 = r$ [given]....(i)

Now, $\vec{\mathbf{a}}\times\vec{\mathbf{b}} \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & p & 1 \\ q & 2 & 2 \end{vmatrix}$

$\Longrightarrow (2p-2)\hat{\mathbf{i}} + (q-2)\hat{\mathbf{j}} + (2-pq)\hat{\mathbf{k}}$

$= \{0\hat{\mathbf{i}} + (-3)\hat{\mathbf{j}} + (3)\hat{\mathbf{k}}$ [given]

$\Longrightarrow 2p - 2 = 0; q - 2 = -3; 2 - pq = 3$

$\Longrightarrow\ p = 1, q = -1$

From Eqs. (i),

$-1 + 2 + 2 = r$

$= r = 3$

555 **(c)**

We have,

$(3\hat{\imath} - 2\hat{\jmath} + \hat{k})\cdot(2\hat{\imath} + \hat{\jmath} - 4\hat{k}) = 0$

So, the triangle is right angled

556 **(a)**

Since, $2|\hat{\mathbf{i}} + x\hat{\mathbf{j}} + 3\hat{\mathbf{k}}| = |4\hat{\mathbf{i}} + (4x-2)\hat{\mathbf{j}} + 2\hat{\mathbf{k}}|$

$\Longrightarrow 2\sqrt{1 + x^2 + 9} = \sqrt{4^2 + (4x-2)^2 + 2^2}$

$\Longrightarrow\ 12x^2 - 16x - 16 = 0$

$\Longrightarrow (3x+2)(x-2) = 0$

$\Longrightarrow x = 2, -\dfrac{2}{3}$

559 **(b)**

$\because \vec{\mathbf{a}}, \vec{\mathbf{b}}$, and $\vec{\mathbf{c}}$ are the pth, qth, nth terms of an HP respectively.

$\dfrac{1}{a} = A + (p-1)D, \dfrac{1}{b} = A + (q-1)D$ and $\dfrac{1}{c}$

$= A + (r-1)D$

$\therefore q - r = \dfrac{c-b}{bcD}, r - p = \dfrac{a-c}{acD}$

And $q - r = \frac{b-a}{abD}$

$\Rightarrow \dfrac{(q-r)}{a} + \dfrac{(r-p)}{b} + \dfrac{(p-q)}{c} = 0$

$\Rightarrow\ \vec{\mathbf{u}}\cdot\vec{\mathbf{v}} = 0$

560 **(d)**

Given edges are

$\vec{\mathbf{a}} = \hat{\mathbf{i}} - \hat{\mathbf{k}}, \vec{\mathbf{b}} = \lambda\,\hat{\mathbf{i}} + \hat{\mathbf{j}} + (1-\lambda)\hat{\mathbf{k}}$

and $\vec{\mathbf{c}} = \mu\,\hat{\mathbf{i}} + \lambda\,\hat{\mathbf{j}} + (1 + \lambda - \mu)\hat{\mathbf{k}}$

$\therefore$ Volume of parallelopiped

$= [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$

$= \begin{vmatrix} 1 & 0 & -1 \\ \lambda & 1 & 1-\lambda \\ \mu & \lambda & 1+\lambda-\mu \end{vmatrix}$

$= 1(1 + \lambda - \mu - \lambda + \lambda^2) - 0 - 1(\lambda^2 - \mu)$

$= 1 + \lambda^2 - \mu - \lambda^2 + \mu = 1$

Hence, volume depends on neither λ nor μ.

561 **(a)**

$\vec{\mathbf{c}}\cdot(\vec{\mathbf{b}} + \vec{\mathbf{c}})\times(\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}})$

$= \vec{\mathbf{c}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{a}} + \vec{\mathbf{b}}\times\vec{\mathbf{c}} + \vec{\mathbf{c}}\times\vec{\mathbf{a}} + \vec{\mathbf{c}}\times\vec{\mathbf{b}})$

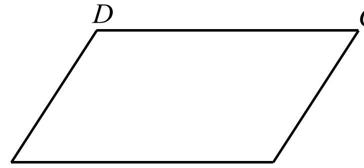
$= \vec{\mathbf{c}}\cdot\vec{\mathbf{b}}\times\vec{\mathbf{a}}$

562 **(c)**

$\vec{A}C - \vec{B}D$

$= (\vec{A}B + \vec{B}C) - (\vec{B}A + \vec{A}D)$

$= \vec{A}B + \vec{B}C + \vec{A}B - \vec{A}D = 2\vec{A}B$



563 **(c)**

We have,

$\vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow |\vec{a} + \vec{b} + \vec{c}|^2 = 0$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 = 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + +\vec{c}\cdot\vec{a})$

$= 0$

$\Rightarrow \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + +\vec{c}\cdot\vec{a}$

$= -\dfrac{3}{2}\ [\because |\vec{a}| = |\vec{b}| = |\vec{c}| = 1]$

565 **(a)**

Given that, $\vec{\mathbf{a}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\vec{\mathbf{b}} = 5\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$

The projection of $\vec{\mathbf{b}}$ on $\vec{\mathbf{a}} = \frac{\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}}{|\vec{\mathbf{a}}|}$

$= \dfrac{(2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}})\cdot(5\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}})}{\sqrt{(2)^2 + (1)^2 + (2)^2}}$

$= \dfrac{10 - 3 + 2}{\sqrt{9}} = \dfrac{9}{3} = 3$

566 **(a)**

Total force,

$\vec{\mathbf{F}} = 3\left(\frac{6\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}}{7}\right) + 4\left(\frac{3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}}}{7}\right)$

$= \frac{(30\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 33\hat{\mathbf{k}})}{7}$

$\therefore\ \vec{\mathbf{d}} = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}} - (2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}})$

$= 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\therefore$ Work done $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}}$

$= \left(\frac{30\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 33\hat{\mathbf{k}}}{7}\right) \cdot (2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

$= \frac{60 - 2 + 66}{7} = \frac{124}{7}$

567 **(b)**

$[\vec{\mathbf{a}}\ \vec{\mathbf{b}} + \vec{\mathbf{c}}\ \vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}] = \vec{\mathbf{a}} \cdot \{(\vec{\mathbf{b}} + \vec{\mathbf{c}}) \times (\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}})\}$

$= \vec{\mathbf{a}} \cdot \{\vec{\mathbf{b}} \times \vec{\mathbf{a}} + \vec{\mathbf{b}} \times \vec{\mathbf{c}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}} + \vec{\mathbf{c}} \times \vec{\mathbf{b}}\}$

$= \vec{\mathbf{a}} \cdot \{\vec{\mathbf{b}} \times \vec{\mathbf{a}} + \vec{\mathbf{b}} \times \vec{\mathbf{c}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}} - \vec{\mathbf{b}} \times \vec{\mathbf{c}}\}$

$= [\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{a}}] + [\vec{\mathbf{a}}\ \vec{\mathbf{c}}\ \vec{\mathbf{a}}] = 0$