3.TRIGONOMETRIC FUNCTIONS

Single Correct Answer Type

1. If $\tan\theta, \cos\theta, \frac{1}{6}\sin\theta$ are in G.P., then general value of θ is
a) $2n\pi \pm \frac{\pi}{3}, n \in Z$ b) $2n\pi \pm \frac{\pi}{6}, n \in Z$ c) $n\pi + (-1)^n \frac{\pi}{3}, n \in Z$ d) $n\pi + \frac{\pi}{3}, n \in Z$
2. $\sin 47° + \sin 61° - \sin 11° - \sin 25°$ is equal to
a) $\sin 7°$ b) $\cos 7°$ c) $\sin 36°$ d) $\cos 36°$
3. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is
a) 6 b) 1 c) 2 d) 4
4. If $\sec\theta \tan\theta = \sqrt{2}$, then $\theta =$
a) $n\pi + (-1)^n \frac{\pi}{4}, n \in Z$ b) $2n\pi \pm \frac{\pi}{3}, n \in Z$ c) $n\pi \pm \frac{2\pi}{3}, n \in Z$ d) $n\pi - \frac{\pi}{4}, n \in Z$
5. If $\cos(\theta + \phi) = m\cos(\theta - \phi)$, then $\tan\theta$ is equal to
a) $\frac{1+m}{1-m}\tan\phi$ b) $\frac{1-m}{1+m}\tan\phi$ c) $\frac{1-m}{1+m}\cot\phi$ d) $\frac{1+m}{1-m}\sec\phi$
6. If $\sin(\pi\cos\theta) = \cos(\pi\sin\theta)$, then which of the following is correct?
a) $\cos\theta = \frac{3}{2\sqrt{2}}$ b) $\cos\left(\theta - \frac{\pi}{2}\right) = \frac{1}{2\sqrt{2}}$ c) $\cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}$ d) $\cos\left(\theta + \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}$
7. $\sec\theta = \frac{a^2+b^2}{a^2-b^2}$, where $a, b, \in R$ gives real values of θ if and only if
a) $a = b \neq 0$ b) $|a| \neq |b| \neq 0$ c) $a + b = 0, a \neq 0$ d) None of these
8. If $\sin A = \frac{1}{\sqrt{10}}$ and $\sin B = \frac{1}{\sqrt{5}}$, where A and B are positive acute angles, then $(A + B)$ is equal to
a) π b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{4}$
9. The number of values of x lying in the interval $(-\pi, \pi)$ which satisfy the equation $8^{(1+|\cos x|+\cos^2 x+|\cos^3 x|+\ldots\infty)} = 4^3$, is
a) 3 b) 4 c) 5 d) 6
10. The value of $\sin\frac{15\pi}{32}\sin\frac{7\pi}{16}\sin\frac{3\pi}{8}$ is
a) $\frac{1}{8\sqrt{2}\cos\left(\frac{15\pi}{32}\right)}$ b) $\frac{1}{8\sin\left(\frac{\pi}{32}\right)}$ c) $\frac{1}{4\sqrt{2}}\text{cosec}\left(\frac{\pi}{16}\right)$ d) None of these
11. If $\alpha, \beta, \gamma \in \left[0, \frac{\pi}{2}\right]$, then the value of $\frac{\sin(\alpha+\beta+\gamma)}{\sin\alpha+\sin\beta+\sin\gamma}$ is
a) < 1 b) $= -1$ c) < 0 d) None of these
12. The expression $\cos\frac{10\pi}{13} + \cos\frac{8\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$ is equal to
a) -1 b) 0 c) 1 d) None of these
13. $\sin 120° \cos 150° - \cos 240° \sin 330°$ is equal to
a) 1 b) -1 c) $\frac{2}{3}$ d) $-\left(\frac{\sqrt{3}+1}{4}\right)$
14. If $A = 30°, a = 7, b = 8$ in ΔABC, then B has
a) One solution b) Two solutions c) No solution d) None of these
15. If $\cos(\theta - \alpha), \cos\theta$ and $\cos(\theta + \alpha)$ are in HP, then $\cos\theta \sec\frac{\alpha}{2}$ is equal to
a) $\pm\sqrt{2}$ b) $\pm\sqrt{3}$ c) $\pm\frac{1}{\sqrt{2}}$ d) None of these
16. The number of values of x for which $\sin 2x + \cos 4x = 2$, is
a) 0 b) 1 c) 2 d) infinite
17. The value of $\text{cosec}^2\frac{\pi}{2} + \text{cosec}^2\frac{2\pi}{7} + \text{cosec}^2\frac{3\pi}{7}$ is
a) $2°$ b) 2 c) 2^2 d) 2^3

18. The maximum value of $4\sin^2 x + 3\cos^2 x$ is

a) 4 b) 3 c) 7 d) 5

19. If $0° < \theta < 180°$, then $\sqrt{2+\sqrt{2+\sqrt{2+\ldots+\sqrt{2(1+\cos\theta)}}}}$, then being n number of 2's, is equal to

a) $2\cos\left(\frac{\theta}{2^n}\right)$ b) $2\cos\left(\frac{\theta}{2^{n-1}}\right)$ c) $2\cos\left(\frac{\theta}{2^{n+1}}\right)$ d) None of these

20. If $S_n = \cos^n\theta + \sin^n\theta$, then the value of $3S_4 - 2S_6$ is given by

a) 4 b) 0 c) 1 d) 7

21. If $\tan 2x = \tan\frac{2}{x}$, then the value of x is

a) $\frac{n\pi \pm \sqrt{n^2\pi^2+16}}{4}$ b) $\frac{n\pi}{4}$ c) $\frac{n\pi \pm \sqrt{n^2\pi^2-16}}{4}$ d) None of these

22. The set of values of x for which $\frac{\tan 3x - \tan 2x}{1+\tan 3x \tan 2x} = 1$ is

a) Φ

b) $\left\{\frac{\pi}{4}\right]$

c) $\left\{n\pi + \frac{\pi}{4}, n = 1,2,3,\ldots\right\}$

d) $\left\{2n\pi + \frac{\pi}{4}, n = 1,2,3,\ldots\right\}$

23. The maximum value of $(\cos\alpha_1)\ (\cos\alpha_2)\ldots(\cos\alpha_n)$ under the restriction $0 \le \alpha_1, \alpha_2, \ldots, \alpha_n \le \frac{\pi}{2}$ and $(\cos\alpha_1)(\cos\alpha_2)\ldots(\cos\alpha_n) = 1$, is

a) $\frac{1}{2^{n/2}}$ b) $\frac{1}{2^n}$ c) $\frac{1}{2n}$ d) 1

24. If A, B, C are angles of a triangle, then the minimum value of $\tan^2\frac{A}{2} + \tan^2\frac{B}{2} + \tan^2\frac{C}{2}$, is

a) 0 b) 1 c) 1/2 d) None of these

25. If the interior angles of a polygon are in A.P. with common difference 5° and the smallest angle 120°, then the number of sides of the polygon is

a) 9 or 16 b) 9 c) 13 d) 16

26. The number of values of x in the interval $[0, 5\pi]$ satisfying the equation $3\sin^2 x - 7\sin x + 2 = 0$ is

a) 0 b) 5 c) 6 d) 10

27. If $\tan(A+B) = p$ and $\tan(A-B) = q$, then the value of $\tan 2A$ is

a) $\frac{p+q}{p-q}$ b) $\frac{p-q}{1+pq}$ c) $\frac{1+pq}{1-p}$ d) $\frac{p+q}{1-pq}$

28. If $\sin\theta = \sqrt{3}\cos\theta, \pi < \theta < 0$, then θ is equal to

a) $-\frac{5\pi}{6}$ b) $-\frac{4\pi}{6}$ c) $\frac{4\pi}{6}$ d) $\frac{5\pi}{6}$

29. The general value of θ satisfying $\sin^2\theta + \sin\theta = 2$ is

a) $n\pi + (-1)^n\frac{\pi}{6}$ b) $2n\pi + \frac{\pi}{4}$ c) $n\pi + (-1)^n\frac{\pi}{2}$ d) $n\pi + (-1)^n\frac{\pi}{3}$

30. The number of solutions of the equation $\tan x + \sec x = 2\cos x$ lying in the interval $[0, 2\pi]$ is

a) 0 b) 1 c) 2 d) 3

31. $\frac{\tan 80° - \tan 10°}{\tan 70°} =$

a) 0 b) 1 c) 2 d) 3

32. If $2\cos^2 x + 3\sin x - 3 = 0, 0 \le x \le 180°$, then the value of x is

a) 30°, 90°, 150° b) 60°, 120°, 180° c) 0°, 30°, 150° d) 45°, 90°, 135°

33. If $\frac{x}{\cos\theta} = \frac{y}{\cos\left(\theta - \frac{2\pi}{3}\right)} = \frac{z}{\cos\left(\theta + \frac{2\pi}{3}\right)}$ then $x + y + z =$

a) 1 b) 0 c) −1 d) 2

34. If $a\sec\alpha - c\tan\alpha = d$ and $b\sec\alpha + d\tan\alpha = c$, then

a) $a^2+c^2=b^2+d^2$ b) $a^2+d^2=b^2+c^2$ c) $a^2+b^2=c^2+d^2$ d) $ab=cd$

35. The value of $\cos\frac{\pi}{65}\cos\frac{2\pi}{65}\cos\frac{4\pi}{65}\dots\cos\frac{32\pi}{65}$ is

a) $\frac{1}{32}$ b) $\frac{1}{64}$ c) $-\frac{1}{32}$ d) $-\frac{1}{64}$

36. The number of solutions of $2\cos^2\left(\frac{x}{2}\right)\sin^2 x=x^2+\frac{1}{x^2}, 0\le x\le\frac{\pi}{2}$ is

a) 0 b) 1 c) Infinite d) None of these

37. If $\sin A=\sin B, \cos A=\cos B$, then the value of A in terms of B is

a) $n\pi+B$ b) $n\pi+(-1)^n B$ c) $2n\pi+B$ d) $2n\pi-B$

38. The value of $\frac{(3+\cot 76°\cot 16°)}{\cot 76°+\cot 16°}$ is

a) $\cot 44°$ b) $\tan 44°$ c) $\tan 2°$ d) $\cot 46°$

39. The largest positive solution of $1+\sin^4 x=\cos^2 3x$ in $[-5\pi/2, 5\pi/2]$ is

a) π b) 2π c) $\frac{5\pi}{2}$ d) None of these

40. If in a $\Delta ABC, \cos A+2\cos B+\cos C=2$, then a, b, c are in

a) A.P. b) H.P. c) G.P d) None of these

41. If $\cos\theta-4\sin\theta=1$, then $\sin\theta+4\cos\theta=$

a) ± 1 b) 0 c) ± 2 d) ± 4

42. Given $\tan A$ and $\tan B$ are the roots of $x^2-ax+b=0$. The value of $\sin^2(A+B)$ is

a) $\frac{a^2}{a^2(1-b)^2}$ b) $\frac{a^2}{a^2+b^2}$ c) $\frac{a^2}{(a+b)^2}$ d) $\frac{b^2}{a^2(a-b)^2}$

43. If $(a-b)\sin(\theta+)=(a+b)\sin(\theta-\phi)$ and $a\tan\frac{\theta}{2}-b\tan\frac{\phi}{2}=c$, then the value of $\sin\phi$ is equal to

a) $\frac{2ab}{a^2-b^2-c^2}$ b) $\frac{2bc}{a^2-b^2-c^2}$ c) $\frac{2bc}{a^2-b^2+c^2}$ d) $\frac{2ab}{a^2-b^2+c^2}$

44. The number of solutions of the equation $\sin x\, cos 3x=\sin 3x\cos 5x$ in $\left[0,\frac{\pi}{2}\right]$ is

a) 3 b) 4 c) 5 d) 6

45. $A+B=C \Rightarrow \cos^2 A+\cos^2 B+\cos^2 C-2\cos A\cos B\cos C$ is equal to

a) 1 b) 2 c) 0 d) 3

46. If $0<x<\pi$ and $\cos x+\sin x=\frac{1}{2}$, then the value of $\tan x$ *is*

a) $\frac{2-\sqrt{7}}{3}$ b) $-\frac{4+\sqrt{7}}{3}$ c) $-\frac{1+\sqrt{7}}{3}$ d) $-\frac{2+\sqrt{7}}{3}$

47. If α and β be between 0 and $\frac{\pi}{2}$ and if $\cos(\alpha+\beta)=\frac{12}{13}$ and $\sin(\alpha-\beta)=\frac{3}{5}$, then $\sin 2\alpha$ is equal to

a) 64/65 b) 56/65 c) 0 d) 16/15

48. The solution of the equation $\sin^{10} 2x=1+\cos^{10} x$ is

a) $x=(2n+1)\frac{\pi}{2}$ b) $x=n\pi$ c) $x=(2n+1)\frac{\pi}{4}$ d) None of these

49. In a $\Delta PQR, \angle R=\frac{\pi}{2}$. If $\tan\frac{P}{2}$ and $\tan\frac{Q}{2}$ are the roots of the equation $ax^2+bx+c=0 (a\ne 0)$, then

a) $a+b=c$ b) $b+c=a$ c) $c+a=b$ d) $b=c$

50. The equation $\sqrt{3}\sin x+\cos x=4$ has

a) Infinity many solutions b) No solution
c) Two solutions d) Only one solution

51. If $\frac{\cos\theta}{a}=\frac{\sin\theta}{b}$, then $\frac{a}{\sec 2\theta}+\frac{b}{\text{cosec } 2\theta}$ is equal to

a) a b) b c) $\frac{a}{b}$ d) $a+b$

52. The maximum value of $(x+\pi/6)+\cos(x+\pi/6)$ in the interval $(0,\pi/2)$ is attained at

a) $\pi/12$ b) $\pi/6$ c) $\pi/3$ d) $\pi/2$

53. If $\tan x=\frac{2b}{a-c}, a\ne c$; and $y=a\cos^2 x+2b\sin x\cos x+c\sin^2 x$
$z=a\sin^2 x-2b\sin x\cos x+c\cos^2 x$, then

a) $y = z$
b) $y + z = a - c$
c) $y - z = a - c$
d) $(y - z) = (a - c)^2 + 4b^2$

54. If $\frac{1}{6}\sin x, \cos x, \tan x$ are in G.P., then x is equal to
a) $n\pi \pm \frac{\pi}{3}, n \in Z$ b) $2n\pi \pm \frac{\pi}{3}, n \in Z$ c) $n\pi + (-1)^n \frac{\pi}{3}, n \in Z$ d) None of these

55. In a $\Delta ABC, a^2 \sin 2C + c^2 \sin 2A =$
a) Δ b) 2Δ c) 3Δ d) 4Δ

56. $e^{\log(\cosh^{-1} 2)}$ is equal to
a) $\log(2 - \sqrt{3})$ b) $\log(\sqrt{3} - 2)$ c) $\log(2 + \sqrt{3})$ d) $\log(2 + \sqrt{5})$

57. If $x + \frac{1}{x} = 2\cos\theta$, then $x^3 + \frac{1}{x^3}$ id equal to
a) $\sin 3\theta$ b) $2\sin 3\theta$ c) $\cos 3\theta$ d) $2\cos 3\theta$

58. If $A + B = \frac{\pi}{4}$, then $(\tan A + 1)(\tan B + 1)$ equals
a) 1 b) $\sqrt{3}$ c) 2 d) $\frac{1}{\sqrt{3}}$

59. The maximum value of $\cos x \left\{\frac{\cos x}{1-\sin x} + \frac{1-\sin x}{\cos x}\right\}$, is
a) 1 b) 3 c) 2 d) 4

60. The value of $\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}$, is
a) 1/128 b) 1/64 c) 1/16 d) None of these

61. The side of a triangle are $a, b, \sqrt{a^2 + b^2 + ab}$, then the greatest angle is
a) 60° b) 90° c) 120° d) 135°

62. If $\alpha, \beta, \gamma, \delta$ are four solutions of the equation $\tan\left(\theta + \frac{\pi}{4}\right) = 3\tan 3\theta$, then $\tan\alpha \tan\beta \tan\gamma \tan\delta$ equals
a) 3 b) 1/3 c) $-\frac{1}{3}$ d) None of these

63. In a ΔABC if $a = 5, b = 4$ and $\tan\frac{C}{2} = \frac{\sqrt{7}}{3}$, then $c =$
a) $\sqrt{6}$ b) $\sqrt{5}$ c) 6 d) 5

64. $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha$ is equal to
a) $\tan 16\alpha$ b) 0 c) $\cot\alpha$ d) None of these

65. The most general value of θ for which $\sin\theta - \cos\theta = \min_{x \in R}\{1, x^2 - 4x + 6\}$ are given by
a) $\theta = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}, n \in Z$
b) $\theta = n\pi + (-1)^n \frac{\pi}{4} + \frac{\pi}{4}, n \in Z$
c) $\theta = 2n\pi + \frac{\pi}{4}, n \in Z$
d) None of these

66. If $\tan\frac{x}{2} = \operatorname{cosec} x - \sin x$, then the value of $\tan^2\frac{x}{2}$, is
a) $2 - \sqrt{5}$ b) $2 + \sqrt{5}$ c) $-2 - \sqrt{5}$ d) $-2 + \sqrt{5}$

67. In a $\Delta ABC, \frac{\cos C + \cos A}{c + a} + \frac{\cos B}{b} =$
a) $\frac{1}{a}$ b) $\frac{1}{b}$ c) $\frac{1}{c}$ d) $\frac{c + a}{b}$

68. $\sin 12° \sin 48° \sin 54°$ is equal to
a) 1/16 b) 1/32 c) 1/8 d) 1/4

69. If $\sin\alpha = \sin\beta$ and $\cos\alpha = \cos\beta$, then

a) $\sin\frac{\alpha+\beta}{2}=0$ b) $\cos\frac{\alpha+\beta}{2}=0$ c) $\sin\frac{\alpha-\beta}{2}=0$ d) $\cos\left(\frac{\alpha-\beta}{2}\right)=0$

70. In triangle $ABC, A=30°, b=8, a=6$, then $B=\sin^{-1}x$, where $x=$
a) 1/2 b) 1/3 c) 2/3 d) 1

71. Consider the following statements :
1. If $\operatorname{cosec} x = 1+\cot x$, then $x=2n\pi+\frac{3\pi}{4}$
2. General value of θ satisfying $\tan^2\theta+\sec 2\theta=1$ is $n\pi+\frac{\pi}{2}$
Which of the statements given above is/are correct?
a) Only (1) b) Only (2) c) Both (1) and (2) d) Neither (1) nor (2)

72. If $\cos A=\frac{3}{4}$, then $32\sin\left(\frac{A}{2}\right)\sin\left(\frac{5A}{2}\right)=$
a) 7 b) 8 c) 11 d) None of these

73. In any ΔABC, the distance of the orthocentre from the vertices A, B, C are in the ratio
a) $\sin A:\sin B:\sin C$ b) $\cos A:\cos B:\cos C$ c) $\tan A:\tan B:\tan C$ d) None of these

74. If $\tan\alpha=(1+2^{-x})^{-1}, \tan\beta=(1+2^{x+1})^{-1}$, then $\alpha+\beta$ equals
a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

75. If $\cos\theta=\cos\alpha\cos\beta$, then $\tan\left(\frac{\theta+\alpha}{2}\right)\tan\left(\frac{\theta-\alpha}{2}\right)$ is equal to
a) $\tan^2\frac{\alpha}{2}$ b) $\tan^2\frac{\beta}{2}$ c) $\tan^2\frac{\theta}{2}$ d) $\cot^2\frac{\beta}{2}$

76. Consider the following statements :
1. $\cot\theta-\tan\theta$, then $\theta=(4n+1)\frac{\pi}{8}$
2. $\sin 2x+\cos 2x+\sin x+\cos x+1=0$ has no solution in the Ist quadrant.
Which of these is/are correct?
a) Only (1) b) Only (2) c) Both of these d) None of these

77. The value of $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$ is equal to
a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{1}{8}$ d) $\frac{1}{16}$

78. $\frac{1-\tan^2(45°-A)}{1+\tan^2(45°-A)}$ is equal to
a) $\sin 2A$ b) $\cos 2A$ c) $\tan 2A$ d) $\cot 2A$

79. If $\tan\alpha=\frac{m}{m+1}$ and $\tan\beta=\frac{1}{2m+1}$, then $\alpha+\beta$ is equal to
a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{6}$ d) None of these

80. If $\sin\left(\frac{\pi}{4}\cot\theta\right)=\cos\left(\frac{\pi}{4}\tan\theta\right)$, then θ is equal to
a) $2n\pi+\frac{\pi}{4}$ b) $2n\pi\pm\frac{\pi}{4}$ c) $2n\pi-\frac{\pi}{4}$ d) $n\pi+\frac{\pi}{4}$

81. If $\frac{\sin(x+y)}{\sin(x-y)}=\frac{a+b}{a-b}$, then $\frac{\tan x}{\tan y}$ is equal to
a) $\frac{a^2}{b^2}$ b) $\frac{a}{b}$ c) $\frac{b}{a}$ d) $\frac{a^2+b^2}{a^2-b^2}$

82. If $\cos\theta-4\sin\theta=1$, then $\sin\theta+4\cos\theta$ is equal to
a) ±1 b) 0 c) ±2 d) ±4

83. If $\frac{\cos(\theta_1-\theta_2)}{\cos(\theta_1+\theta_2)}+\frac{\cos(\theta_3+\theta_4)}{\cos(\theta_3-\theta_4)}=0$, then $\tan\theta_1\tan\theta_2\tan\theta_3\tan\theta_4=$
a) 1 b) 2 c) -1 d) None of these

84. General solution of the equation $\cot\theta-\tan\theta=2$ is
a) $n\pi+\frac{\pi}{4}$ b) $\frac{n\pi}{2}+\frac{\pi}{8}$ c) $\frac{n\pi}{2}\pm\frac{\pi}{8}$ d) None of these

85. The value of $\sin 36°\sin 72°\sin 108°\sin 144°$ is equal to
a) $\frac{1}{4}$ b) $\frac{1}{16}$ c) $\frac{3}{4}$ d) $\frac{5}{16}$

86. The equation $\sin\theta = x + \frac{p}{x}$ for real values of x is possible when

a) $p \geq 0$ b) $p \leq 0$ c) $p \leq \frac{1}{4}$ d) $p \geq \frac{1}{2}$

87. The number of values of x in $[0,5\,\pi]$ satisfying the equation $3\cos 2x - 10\cos x + 7 = 0$, is

a) 5 b) 6 c) 8 d) 10

88. $\sum_{r=1}^{n-1} \cos^2 \frac{r\pi}{n}$ is equal to

a) $\frac{n}{2}$ b) $\frac{n-1}{2}$ c) $\frac{n}{2} - 1$ d) None of these

89. The solution of the equation $1 - \cos\theta = \sin\theta \sin\frac{\theta}{2}$ is

a) $n\pi, n \in Z$ b) $2n\pi, n \in Z$ c) $\frac{n\pi}{2}, n \in Z$ d) None of these

90. The greatest value of $\cos\theta$ for which $\cos 5\theta = 0$, is

a) 0 b) $\frac{1+\sqrt{5}}{4}$ c) $\sqrt{\frac{5+\sqrt{5}}{8}}$ d) $\sqrt{\frac{\sqrt{5}+1}{4}}$

91. The solution of equation $\cos^2\theta + \sin\theta + 1 = 0$ lies in the interval

a) $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$ b) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ c) $\left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$ d) $\left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$

92. The circular wire of diameter 10 cm is cut and placed along the circumference of a circle of diameter 1 m. The angle subtended by the wire at the center of the circle is equal to

a) $\frac{\pi}{4}$rad b) $\frac{\pi}{3}$ rad c) $\frac{\pi}{5}$rad d) $\frac{\pi}{10}$ rad

93. If $\cos A = \tan B$, $\cos B = \tan C$, $\cos C = \tan A$, then $\sin A$ is equal to

a) $\sin 18°$ b) $2\sin 18°$ c) $2\cos 18°$ d) $2\cos 36°$

94. $\cos 1° + \cos 2° + \cos 3° + \cdots + \cos 180° =$

a) 1 b) 0 c) 2 d) -1

95. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has

a) no solution b) two solution c) three solution d) None of these

96. In a triangle ABC, $\cos A + \cos B + \cos C =$

a) $1 + \frac{r}{R}$ b) $1 - \frac{r}{R}$ c) $1 - \frac{R}{r}$ d) $1 + \frac{R}{r}$

97. $\frac{1}{\cos 80°} - \frac{\sqrt{3}}{\sin 80°}$ is equal to

a) $\sqrt{2}$ b) $\sqrt{3}$ c) 2 d) 4

98. If $\sin A + cos\ B = a$ and $\sin B + \cos A = b$, then $\sin(A+B)$ is equal to

a) $\frac{a^2+b^2}{2}$ b) $\frac{a^2-b^2+2}{2}$ c) $\frac{a^2+b^2-2}{2}$ d) None of these

99. If in a $\Delta\, ABC$, $3a = b + c$, then the value of $\cot\frac{B}{2}\cot\frac{C}{2}$ is

a) 1 b) $\sqrt{3}$ c) 2 d) None of these

100. If p_1, p_2, p_3 are respectively the perpendiculars from the vertices of a triangle to the opposite sides, then $\frac{\cos A}{p_1} + \frac{\cos B}{p_2} + \frac{\cos C}{p_3}$ is equal to

a) $\frac{1}{r}$ b) $\frac{1}{R}$ c) $\frac{1}{\Delta}$ d) None of these

101. In a triangle ABC, $\cos A + \cos B + \cos C = \frac{3}{2}$, then the triangle is

a) Isosceles b) Right angled c) Equilateral d) None of these

102. If $\sin A = \frac{336}{625}$ where $450° < A < 540°$, then $\sin\frac{A}{4} =$

a) $\frac{3}{5}$ b) $-\frac{3}{5}$ c) $\frac{4}{5}$ d) $-\frac{4}{5}$

103. If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$, $0 < \theta < \frac{3\pi}{4}$, then $\sin\left(\theta + \frac{\pi}{4}\right)$ equals

a) $\frac{1}{\sqrt{2}}$ b) $\frac{1}{2}$ c) $\frac{1}{2\sqrt{2}}$ d) $\sqrt{2}$

104. The number of ordered pairs (α, β), where $\alpha, \beta \in (-\pi, \pi)$ satisfying $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = \frac{1}{e}$, is

a) 0 b) 1 c) 2 d) 4

105. If $x + \frac{1}{x} = 2\cos\theta$, then $x^n + \frac{1}{x^n}$ is equal to

a) $2\sin n\theta$ b) $2\cos n\theta$ c) $\sin(2n\theta)$ d) $\cos(2n\theta)$

106. In a ΔABC, $\cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2} =$

a) $2 - \frac{r}{R}$ b) $2 - \frac{r}{2R}$ c) $2 + \frac{r}{2R}$ d) None of these

107. The value of $\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}$ is

a) $\frac{1}{2^6}$ b) $\frac{1}{2^7}$ c) $\frac{1}{2^8}$ d) None of these

108. If $\tan A$ and $\tan B$ are the roots of $abx^2 - c^2x + ab = 0$ where a, b, c are the sides of the triangles ABC, then the value of $\sin^2 A + \sin^2 B + \sin^2 C$ is

a) 1 b) 3 c) 4 d) 2

109. If $x = \tan 15°, y = \text{cosec } 75°, z = 4\sin 18°$

a) $x < y < z$ b) $y < z < x$ c) $z < x < y$ d) $x < z < y$

110. If $P = \frac{1}{2}\sin^2\theta + \frac{1}{3}\cos^2\theta$, then

a) $\frac{1}{3} \leq P \leq \frac{1}{2}$ b) $P \geq \frac{1}{2}$ c) $2 \leq P \leq 3$ d) $-\frac{\sqrt{13}}{6} \leq P \leq \frac{\sqrt{13}}{6}$

111. If $|k| = 5$ and $0° \leq \theta \leq 360°$, then the number of different solutions of $3\cos\theta + 4\sin\theta = k$ is

a) Zero b) Two c) One d) Infinite

112. If $p = \cos 55°, q = \cos 65°$ and $r = \cos 175°$, then the value of $\frac{1}{p} + \frac{1}{q} + \frac{r}{pq}$ is

a) 0 b) -1 c) 1 d) None of these

113. If $\sin x + \text{cosec } x = 2$, then $\sin^n x + \text{cosec}^n x$ is equal to

a) 2 b) 2^n c) 2^{n-1} d) 2^{n-2}

114. In a right-angled triangle if the sides are in A.P., then their ratio is

a) 3 : 4 : 5 b) 4 : 5 : 6 c) 3 : 4 : 6 d) None of these

115. If $\alpha, \beta (\alpha \neq \beta)$ satisfies the equation $a\cos\theta + b\sin\theta = c$, then the value of $\tan\left(\frac{\alpha+\beta}{2}\right)$ is

a) b/a b) c/a c) a/b d) c/b

116. In a ΔABC, if $\frac{a}{b^2-c^2} + \frac{c}{b^2-a^2} = 0$, then $\angle B =$

a) $\frac{\pi}{2}$ b) $\frac{\pi}{4}$ c) $\frac{2\pi}{3}$ d) $\frac{\pi}{3}$

117. $\sin^2 5° + \sin^2 10° + \sin^2 15° + \ldots + \sin^2 90°$ is equal to

a) $8\frac{1}{2}$ b) 9 c) $9\frac{1}{2}$ d) $4\frac{1}{2}$

118. In a ΔABC, if $C = 60°$, then $\frac{a}{b+c} + \frac{b}{c+a} =$

a) 2 b) 1 c) 4 d) None of these

119. The value of $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}$ is

a) 1/16 b) 1/8 c) 1/2 d) 1/4

120. The area of a ΔABC is $b^2 - (c-a)^2$. Then, $\tan B =$

a) $\frac{4}{3}$ b) $\frac{3}{4}$ c) $\frac{8}{15}$ d) None of these

121. The value of $\cos^2 76° + \cos^2 16 - \cos 76° \cos 16°$, is

a) 1/2 b) 0 c) $-1/4$ d) 3/4

122. If $\sec\theta + \tan\theta = k$, $\cos\theta$ equals to

a) $\frac{k^2+1}{2k}$ b) $\frac{2k}{k^2+1}$ c) $\frac{k}{k^2+1}$ d) $\frac{k}{k^2-1}$

123. Let n be an odd integer. If $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r \theta$ for all real θ, then

a) $b_0 = 1, b_1 = 3$
b) $b_{0=}0, b_1 = n$
c) $b_0 = -1, b_1 = n$
d) $b_0 = 0, b_1 = n^2 - 3n - 3$

124. $\{x \in R : \cos 2x + 2\cos^2 x = 2\}$ is equal to

a) $\left\{2n\pi + \frac{\pi}{3} : n \in Z\right\}$
b) $\left\{n\pi \pm \frac{\pi}{6} : n \in Z\right\}$
c) $\left\{n\pi + \frac{\pi}{3} : n \in Z\right\}$
d) $\left\{2n\pi - \frac{\pi}{3} : n \in Z\right\}$

125. $1 + \sin x + \sin^2 x + \cdots$ to $\infty = 4 + 2\sqrt{3}$, if

a) $x = \frac{2\pi}{3}$ or, $\frac{\pi}{3}$
b) $x = \frac{7\pi}{6}$
c) $x = \frac{\pi}{6}$
d) $x = \frac{\pi}{4}$

126. The number of solutions of $\sin x = \sin 2x$ between $\frac{-\pi}{2}$ and $\frac{\pi}{2}$ is

a) 3
b) 2
c) 1
d) 0

127. If $\sin(\pi \cot \theta) = \cos(\pi \tan \theta)$, then

a) $\cot 2\theta = \pm\frac{1}{4}, -\frac{3}{4}$
b) $\cot 2\theta = 4, \frac{4}{3}$
c) $\cot 2\theta = -\frac{3}{4}, -\frac{1}{4}$
d) None of these

128. The value of $\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{14\pi}{15}$ is

a) $\frac{1}{16}$
b) $\frac{1}{8}$
c) $\frac{1}{12}$
d) $\frac{1}{4}$

129. The equation $3\sin^2 x + 10\cos x - 6 = 0$ is satisfied, if

a) $x = n\pi \pm \cos^{-1}\left(\frac{1}{3}\right)$
b) $x = 2n\pi \pm \cos^{-1}\left(\frac{1}{3}\right)$
c) $x = n\pi \pm \cos^{-1}\left(\frac{1}{6}\right)$
d) $x = 2n\pi \pm \cos^{-1}\left(\frac{1}{6}\right)$

130. Let α, β be any two positive values of x for which $2\cos x$, $|\cos x|$ and $1 - 3\cos^2 x$ are in GP. The minimum value of $|\alpha - \beta|$ is

a) $\frac{\pi}{3}$
b) $\frac{\pi}{4}$
c) $\frac{\pi}{2}$
d) None of these

131. If $\cos\theta = -\frac{1}{\sqrt{2}}$ and $\tan\theta = 1$, then the general value of θ is

a) $2n\pi + \frac{\pi}{4}$
b) $2(n+1)\pi + \frac{\pi}{4}$
c) $n\pi + \frac{\pi}{4}$
d) $n\pi \pm \frac{\pi}{4}$

132. In any triangle ABC, $\sum \frac{\sin^2 A + \sin A + 1}{\sin A}$ is always greater than

a) 9
b) 3
c) 27
d) None of these

133. If $\alpha + \beta + \gamma = 2\theta$, then $\cos\theta + \cos(\theta - \alpha) + \cos(\theta - \beta) + \cos(\theta - \gamma)$ is equal to

a) $4\sin\frac{\alpha}{2}\cos\frac{\beta}{2}\sin\frac{\gamma}{2}$
b) $4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$
c) $4\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}$
d) $4\sin\alpha\sin\beta\sin\gamma$

134. If the data given to construct a triangle ABC are $a = 5, b = 7, \sin A = 3/4$, then it is possible to construct

a) Only one triangle
b) Two triangles
c) Infinitely many triangles
d) No triangles

135. If in a $\Delta ABC, \angle A = \pi/3$ and AD is a median, then

a) $2\,AD^2 = b^2 + c^2 + bc$
b) $4\,AD^2 = b^2 + c^2 + bc$
c) $6\,AD^2 = b^2 + c^2 + bc$
d) None of these

136. If $\tan\theta = \frac{1}{2}$ and $\tan\phi = \frac{1}{3}$, then the value of $\theta + \phi$, is

a) $\pi/6$
b) π
c) Zero
d) $\pi/4$

137. If $a = \tan 27\theta - \tan\theta$ and $b = \frac{\sin\theta}{\cos 3\theta} + \frac{\sin 3\theta}{\cos 9\theta} + \frac{\sin 9\theta}{\cos 27\theta}$, then

a) $a = b$
b) $a = 2b$
c) $b = 2a$
d) $a + b = 2$

138. If in a triangle ABC, right angled at B, $s - a = 3, s - c = 2$, then the values of a and c are respectively

a) 2, 3
b) 3, 4
c) 4, 3
d) 6, 8

139. If $\frac{\pi}{2} < \alpha < \pi, \pi < \beta < \frac{3\pi}{2}$; $\sin\alpha = \frac{15}{17}$ and $\tan\beta = \frac{12}{5}$, then the value of $\sin(\beta - \alpha)$ is

a) $-\frac{171}{221}$ b) $-\frac{21}{221}$ c) $\frac{21}{221}$ d) $\frac{171}{221}$

140. If $\tan 2\theta \tan\theta = 1$, then $\theta =$

a) $n\pi + \frac{\pi}{6}, n \in Z$ b) $n\pi \pm \frac{\pi}{6}, n \in Z$ c) $2n\pi \pm \frac{\pi}{6}, n \in Z$ d) None of these

141. The value of $\sin\frac{2\pi}{7} + \sin\frac{4\pi}{7} + \sin\frac{8\pi}{7}$, is

a) $\sqrt{7}/8$ b) $1/8$ c) $\sqrt{7}/2$ d) $-\sqrt{7}/2$

142. The perimeter of a triangle is 16 cm. One of the sides is of length 6 cm. If the area of the triangle is 12 cm^2, then the triangle is

a) Right angled b) Isosceles c) Equilateral d) Scalene

143. If $\sin\alpha + \cos\alpha = m$, then $\sin^6\alpha + \cos^6\alpha$ is equal to

a) $\frac{4-3(m^2-1)^2}{4}$ b) $\frac{4+3(m^2-1)^2}{4}$ c) $\frac{3+4(m^2-1)^2}{4}$ d) None of these

144. For $x \in R$,

$\tan x + \frac{1}{2}\tan\frac{x}{2} + \frac{1}{2^2}\tan\frac{x}{2^2} + \cdots + \frac{1}{2^{n-1}}\tan\left(\frac{x}{2^{n-1}}\right)$ is equal to

a) $2\cot 2x - \frac{1}{2^{n-1}}\cot\left(\frac{x}{2^{n-1}}\right)$

b) $\frac{1}{2^{n-1}}\cot\left(\frac{x}{2^{n-1}}\right) - 2\cot 2x$

c) $\cot\left(\frac{x}{2^{n-1}}\right) - \cot 2x$

d) None of these

145. The value of expression $\frac{1}{\cos 290°} + \frac{1}{\sqrt{3}\sin 250°}$ is equal to

a) $\frac{\sqrt{3}}{4}$ b) $\frac{4}{\sqrt{3}}$ c) $\frac{2}{\sqrt{3}}$ d) $\frac{\sqrt{3}}{2}$

146. If $A = \sin^2\theta + \cos^4\theta$, then for all real values of θ

a) $1 \le A \le 2$ b) $\frac{3}{4} \le A \le 1$ c) $\frac{13}{16} \le A \le 1$ d) $\frac{3}{4} \le A \le \frac{13}{16}$

147. If $12\cot^2\theta - 31\,cosec\,\theta + 32 = 0$, then the value of $\sin\theta$ is

a) $\frac{3}{5}$ or 1 b) $\frac{2}{3}$ or $-\frac{2}{3}$ c) $\frac{4}{5}$ or $\frac{3}{4}$ d) $\pm\frac{1}{2}$

148. The value of $\sin 20° (4 + \sec 20°)$ is

a) 0 b) 1 c) $\sqrt{2}$ d) $\sqrt{3}$

149. The angle θ whose cosine equals to its tangent is given by

a) $\cos\theta = 2\cos 18°$ b) $\cos\theta = 2\sin 18°$ c) $\sin\theta = 2\sin 18°$ d) $\sin\theta = 2\cos 18°$

150. The value of $3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right) + \sin^4(3\pi+\alpha)\right] - 2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right) + \sin^6(5\pi-\alpha)\right]$ is equal to

a) 0 b) 1 c) 3 d) $\sin 4\alpha + \sin 6\alpha$

151. The most general value of θ satisfying

$\tan\theta + \tan\left(\frac{3\pi}{4} + \theta\right) = 2\ are$

a) $n\pi \pm \frac{\pi}{3}, n \in Z$ b) $2n\pi + \frac{\pi}{3}, n \in Z$ c) $2n\pi \pm \frac{\pi}{3}, n \in Z$ d) $n\pi + (-1)^n\frac{\pi}{3}, n \in Z$

152. If angle θ be divided into two parts such that the tangent of one part is k times the tangent of the other and ϕ is their difference, then $\sin\theta$ is equal to

a) $\frac{k+1}{k-1}\sin\phi$ b) $\frac{k-1}{k+1}\sin\phi$ c) $\frac{2k-1}{2k+1}\sin\phi$ d) None of these

153. If A,B,C,D are the angles of a cyclic quadrilateral, then $\cos A + \cos B + \cos D$ is equal to

a) $2(\cos A + \cos C)$ b) $2(\cos A + \cos B)$

c) $2(\cos A + \cos D)$ d) 0

154. If $\cos(\theta-\alpha) = a, \cos(\theta-\beta) = b$, then $\sin^2(\alpha-\beta) + 2ab\cos(\alpha-\beta) =$

a) a^2+b^2 b) a^2-b^2 c) b^2-a^2 d) $-a^2-b^2$

155. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is

a) $\frac{(4-\sqrt{7})}{3}$ b) $-\frac{(4+\sqrt{7})}{3}$ c) $\frac{(1+\sqrt{7})}{4}$ d) $\frac{(1-\sqrt{7})}{4}$

156. If the equation $\sec\theta + \operatorname{cosec}\theta = c$ has real roots between 0 and 2π, then

a) $c^2 < 8$ b) $c^2 > 8$ c) $c^2 = 8$ d) None of these

157. The set of values of θ satisfying the inequation $2\sin^2\theta - 5\sin\theta + 2 > 0$, where $0 < \theta < 2\pi$, is

a) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ b) $\left[0, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, 2\pi\right]$ c) $\left[0, \frac{\pi}{3}\right] \cup \left[\frac{2\pi}{3}, 2\pi\right]$ d) None of these

158. In a $\Delta ABC, B = \frac{\pi}{8}$ and $C = \frac{5\pi}{8}$. The altitude from A to the side BC, is

a) $\frac{a}{2}$ b) $2a$ c) $\frac{1}{2}(b+c)$ d) $b+c$

159. The solution set of $(5 + 4\cos\theta)(2\cos\theta + 1) = 0$ in the interval $[0, 2\pi]$ is

a) $\left\{\frac{\pi}{3}, \frac{2\pi}{3}\right\}$ b) $\left\{\frac{\pi}{3}, \pi\right\}$ c) $\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$ d) $\left\{\frac{2\pi}{3}, \frac{5\pi}{3}\right\}$

160. If $\cos 3x + \sin\left(2c - \frac{7\pi}{6}\right) = -2$, then $x =$

a) $\frac{\pi}{3}(6k+1), k \in Z$ b) $\frac{\pi}{3}(6k-1), k \in Z$ c) $\frac{\pi}{3}(2k+1), k \in Z$ d) None of these

161. The maximum value of $\cos^2\left(\frac{\pi}{3} - x\right) - \cos^2\left(\frac{\pi}{3} + x\right)$ is

a) $-\frac{\sqrt{3}}{2}$ b) $\frac{1}{2}$ c) $\frac{\sqrt{3}}{2}$ d) $\frac{3}{2}$

162. If $\alpha, \beta, \gamma \in (0, \pi/2)$, then the value of $\frac{\sin(\alpha+\beta+\gamma)}{\sin\alpha+\sin\beta+\sin\gamma}$, is

a) < 1 b) > 1 c) $= 1$ d) $= -1$

163. If $\tan\theta\tan\left(\frac{\pi}{3}+\theta\right)\tan\left(-\frac{\pi}{3}+\theta\right) = k\tan 3\theta$, then the value of k is

a) 1 b) 1/3 c) 3 d) None of these

164. If $4n\alpha = \pi$, then the value of $\tan\alpha\tan 2\alpha\tan 3\alpha\tan 4\alpha \dots \tan(2n-2)\alpha\tan(2n-1)\alpha$, is

a) 0 b) 1 c) -1 d) None of these

165. If $\alpha, \beta, \gamma \in \left(0, \frac{\pi}{2}\right)$, then the value of $\frac{\sin(\alpha+\beta+\gamma)}{\sin\alpha+\sin\beta+\sin\gamma}$ is

a) < 1 b) > 1 c) 1 d) None of these

166. Total number of solutions of the equation $3x + 2\tan x = \frac{5\pi}{2}$ in $x \in [0, 2\pi]$, is equal to

a) 1 b) 2 c) 3 d) 4

167. If $A + B + C = 180°$, then $\sum \tan\frac{A}{2}\tan\frac{B}{2}$ is

a) 0 b) 1 c) 2 d) 3

168. If $\tan\alpha = \frac{m}{m+1}$ and $\tan\beta = \frac{1}{2m+1}$, then $\alpha + \beta$ is equal to

a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$ c) 0 d) $\frac{\pi}{2}$

169. If $\tan A = 2\tan B + \cot B$, then $2\tan(A - B)$ is equal to

a) $\tan B$ b) $2\tan B$ c) $\cot B$ d) $2\cot B$

170. If $\sin\theta + \operatorname{cosec}\theta = 2$, the value of $\sin^{10}\theta + \operatorname{cosec}^{10}\theta$ is

a) 2 b) 2^{10} c) 2^9 d) 10

171. If $\cos x \neq -\frac{1}{2}$, then the solutions of $\cos x + \cos 2x + \cos 3x = 0$ are

a) $2n\pi \pm \frac{\pi}{4}, n \in Z$ b) $2n\pi \pm \frac{\pi}{3}, n \in Z$ c) $2n\pi \pm \frac{\pi}{6}, n \in Z$ d) $2n\pi \pm \frac{\pi}{2}, n \in Z$

172. If $\cosh^{-1} x = \log(2 + \sqrt{3})$, then x is equal to

a) 2 b) 1 c) 3 d) 5

173. The number of distinct roots of the equation $A\sin^3 x + B\cos^3 x + C = 0$ no two of which differ by 2π is

a) 3 b) 4 c) Infinite d) 6

174. The value of

$\cos(270° + \theta)\cos(90° - \theta) - \sin(270° - \theta)\cos\theta$ is

a) 0 b) -1 c) $1/2$ d) 1

175. If $2\cos\frac{A}{2} = \sqrt{1+\sin A} + \sqrt{1-\sin A}$, then $\frac{A}{2}$ lies between, $(n \in Z)$

a) $2n\pi + \frac{\pi}{4}$ and $2n\pi + \frac{3\pi}{4}$

b) $2n\pi - \frac{\pi}{4}$ and $2n\pi + \frac{\pi}{4}$

c) $2n\pi - \frac{3\pi}{4}$ and $2n\pi - \frac{\pi}{4}$

d) $-\infty$ and $+\infty$

176. If $\tan x = \frac{b}{a}$, then the value of $a\cos 2x + b\sin 2x$ is

a) 1 b) ab c) b d) a

177. The maximum value of $1 + 8\sin^2 x^2 \cos^2 x^2$, is

a) 3 b) -1 c) -8 d) 9

178. If in a ΔABC,

$\sin^3 A + \sin^3 B + \sin^3 C = 3\sin A \sin B \sin C$, then

$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} =$

a) 0

b) $(a+b+c)^3$

c) $(a+b+c)(ab+bc+ca)$

d) None of these

179. The minimum value of $9\tan^2\theta + 4\cot^2\theta$ is

a) 13 b) 9 c) 6 d) 12

180. The maximum value of $4\sin^2 x - 12\sin x + 7$ is

a) 25 b) 4 c) Does not exist d) None of these

181. If $f: R \to S$ defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$, is onto, then the interval of S is

a) $[0, 3]$ b) $[-1, 1]$ c) $[0, 1]$ d) $[-1, 3]$

182. If $y = \frac{\sec^2\theta - \tan\theta}{\sec^2\theta + \tan\theta}$, then

a) $\frac{1}{3} < y < 3$ b) $y \notin [1/3, 3]$ c) $-3 < y < -\frac{1}{3}$ d) None of these

183. If $\sin\theta, \cos\theta$ are the roots of $ax^2 - bx + c = 0$ then

a) $a^2 + b^2 = 2ac$ b) $a^2 - b^2 = 2ac$ c) $a^2 + b^2 = c^2$ d) $b^2 + a^2 = 2ac$

184. If $\sqrt{2}\sec\theta + \tan\theta = 1$, then the general value of θ is

a) $n\pi + \frac{3\pi}{4}$ b) $2n\pi + \frac{\pi}{4}$ c) $2n\pi - \frac{\pi}{4}$ d) $2n\pi \pm \frac{\pi}{4}$

185. The number of ordered pairs (x, y) where $x, y \in [0,10]$ satisfying $\left(\sqrt{\sin^2 - \sin x + \frac{1}{2}}\right) \cdot 2^{\sec^2 y} \leq 1$ is

a) 0 b) 16 c) Infinite d) 12

186. The value of the series

$\cos 12° + \cos 84° + \cos 132° + \cos 156°$ is

a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $-\frac{1}{4}$ d) $-\frac{1}{2}$

187. The equation $\cos^4 x - (\lambda + 2)\cos^2 x - (\lambda + 3) = 0$ possesses a solution, if

a) $\lambda > -3$ b) $\lambda < -2$

c) $-3 \leq \lambda \leq -2$ d) λ is any positive integer

188. If $\sin 5x + \sin 3x + \sin x = 0$, then the value of x other than zero, lying between $0 \leq x \leq \frac{\pi}{2}$ is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{12}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{4}$

189. If $0 \leq x \leq \pi/2$ and $81^{\sin^2 x} + 81^{\cos^2 x} = 30$, then x is equal to

a) $\frac{\pi}{6}, \frac{\pi}{3}$ b) $\frac{\pi}{3}, \frac{5\pi}{2}$ c) $\frac{5\pi}{6}, \frac{\pi}{6}$ d) $\frac{2\pi}{3}, \frac{\pi}{3}$

190. In a ΔABC, if $c = 2, A = 120°, a = \sqrt{6}$, then $C =$

a) 30° b) 60° c) 45° d) None of these

191. If $\sin 6\theta = 32\cos^5\theta \sin\theta - 32\cos^3\theta \sin\theta + 3x$, then x is equal to

a) $\cos\theta$ b) $\cos 2\theta$ c) $\sin\theta$ d) $\sin 2\theta$

192. If $x\cos\theta = y\cos\left(\theta + \frac{2\pi}{3}\right) = z\cos\left(\theta + \frac{4\pi}{3}\right)$, then the value of $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ is equal to

a) 1 b) 2 c) 0 d) $3\cos\theta$

193. Let A and B denote the statements

$A: \cos\alpha + \cos\beta + \cos\gamma = 0$

$B: \sin\alpha + \sin\beta + \sin\gamma = 0$

If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$, then

a) A is true and B is false b) A is false and B is true

c) Both A and B are true d) Both A and B are false

194. If $\sin x \cos x \cos 2x = \lambda$ has a solution, then λ lies in the interval

a) $[-1/4, 1/4]$

b) $[-1/2, 1/2]$

c) $(-\infty, -1/4] \cup [1/4, \infty)$

d) $(-\infty, -1/2] \cup [1/2, \infty)$

195. The value of $1 + \cos 56° + \cos 58° - \cos 66°$ is equal to

a) $2\cos 28° \cos 29° \cos 33°$ b) $4\cos 28° \cos 29° \sin 33°$

c) $4\cos 28° \cos 29° \cos 33°$ d) $2\cos 28° \cos 29° \sin 33°$

196. Consider the following statements:

1.If $\sin A = \sin B$, then we have $\sin 2A = \sin 2B$ always

2.The value of $\cos\frac{\pi}{7}\cos\frac{4\pi}{7}\cos\frac{5\pi}{7}$ is $\frac{1}{4}$

Which of the statements given above is/are correct?

a) Only(1) b) Only (2) c) Both (1)and (2) d) Neither (1)nor(2)

197. $\cos 2x + k\sin x = 2k - 7$ has a solution for

a) $2 \le k \le 6$ b) $1 < k < 7$ c) $4 < k < 7$ d) None of these

198. If $A + B + C = \frac{3\pi}{2}$, then $\cos 2A + \cos 2B + \cos 2C =$

a) $1 - 4\cos A \cos B \cos C$

b) $4\sin A \sin B \sin C$

c) $1 + 2\cos A \cos B \cos C$

d) $1 - 4\sin A \sin B \sin C$

199. The sides of a triangle are 13, 14, 15 then the radius of its in-circle is

a) 67/8 b) 65/4 c) 4 d) 24

200. For $x \in IR$, $3\cos(4x - 5) + 4$ lies in the interval

a) [1, 7] b) [4, 7] c) [0, 7] d) [2, 7]

201. If $\cos x - \sin\alpha \cot\beta \sin x = \cos\alpha$, then $\tan\frac{x}{2}$ is equal to

a) $\cot\frac{\alpha}{2}\tan\frac{\beta}{2}$ b) $-\tan\frac{\alpha}{2}\cot\frac{\beta}{2}$ c) $-\tan\frac{\alpha}{2}\tan\frac{\beta}{2}$ d) $\cot\frac{\alpha}{2}\cot\frac{\beta}{2}$

202. The value of $\sin 20° \sin 40° \sin 60° \sin 80°$ is equal to

a) $-\frac{3}{16}$ b) $\frac{5}{16}$ c) $\frac{3}{16}$ d) $-\frac{5}{16}$

203. The value of $\sin(\pi + \theta)\sin(\pi - \theta)\ \mathrm{cosec}^2\theta$ is equal to

a) -1 b) 0 c) $\sin\theta$ d) None of these

204. If the equation $\sin\theta(\sin\theta + 2\cos\theta) = a$ has a real solution, then the shortest interval containing $'a'$ is

a) $\left[\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right]$ b) $\left(\frac{\sqrt{5}-1}{2}, \frac{\sqrt{5}+1}{2}\right)$ c) $(-1/2, 1/2)$ d) None of these

205. If $\tan\theta, 2\tan\theta+2, 3\tan\theta+3$ are in GP, then the value of $\frac{7-5\cot\theta}{9-4\sqrt{\sec^2\theta-1}}$ is

a) $\frac{12}{5}$ b) $-\frac{33}{28}$ c) $\frac{33}{100}$ d) $\frac{12}{13}$

206. If $\cos A = \frac{3}{4}$, then the value of $\sin\frac{A}{2}\sin\frac{5A}{2}$ is

a) $\frac{1}{32}$ b) $\frac{11}{8}$ c) $\frac{11}{32}$ d) $\frac{11}{16}$

207. If $\cos A + \cos B + \cos C = 0$, then $\cos 3A + \cos 3B + \cos 3C$ is equal to

a) $\cos A \cos B \cos C$ b) $12 \cos A \cos B \cos C$ c) 0 d) $8\cos^3 A \cos^3 B \cos^3 C$

208. The value of $\tan 82\frac{1^\circ}{2}$, is

a) $\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$ b) $(\sqrt{3}+\sqrt{2})(\sqrt{2}-1)$ c) $-(\sqrt{3}+\sqrt{2})(\sqrt{2}+1)$ d) None of these

209. If $x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi$ and $z = r\cos\theta$, then the value of $x^2+y^2+z^2$ is independent of

a) θ, ϕ b) r, θ c) r, ϕ d) r

210. If $ABCD$ is a cyclic quadrilateral such that $12\tan A - 5 = 0$ and $5\cos B + 3 = 0$, then the quadratic equation whose roots are $\cos C$ and $\tan D$, is

a) $39x^2 - 16x - 48 = 0$ b) $39x^2 + 88x + 48 = 0$ c) $39x^2 - 88x + 48 = 0$ d) None of these

211. The ex-radii of a triangle r_1, r_2, r_3 are in harmonic progression, then the sides a, b, c are

a) In H.P. b) In A.P. c) In G.P d) None of these

212. Given both θ and $\emptyset$ are the acute angles $\sin\theta = \frac{1}{2}, \cos\emptyset = \frac{1}{3}$, then the value of $\theta + \emptyset$ belongs to

a) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right]$ b) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ c) $\left(\frac{2\pi}{3}, \frac{5\pi}{6}\right]$ d) $\left(\frac{5\pi}{6}, \pi\right]$

213. If $\sin 4A - \cos 2A = \cos 4A - \sin 2A, \left(0 < A < \frac{\pi}{2}\right)$, then the value of $\tan 4A$ is

a) 1 b) $\frac{1}{\sqrt{3}}$ c) $\sqrt{3}$ d) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

214. If the ex-radii of a triangle are in H.P., then the corresponding sides are in

a) A.P. b) G.P. c) H.P. d) None of these

215. If $1 + \cos 56° + \cos 58° - \cos 66° = \lambda \cos 28° \cos 29° \sin 33°$, then $\lambda =$

a) 2 b) 3 c) 4 d) None of these

216. $\Delta\, ABC$ is right angled at C, then $\tan A + \tan B$ is equal to

a) $\frac{b^2}{ac}$ b) $a + b$ c) $\frac{a^2}{bc}$ d) $\frac{c^2}{ab}$

217. If r is the radius of inscribed circle of a regular polygon of n-sides, then r is equal to

a) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$ b) $\frac{a}{2}\cot\left(\frac{\pi}{n}\right)$ c) $\frac{a}{2}\tan\left(\frac{\pi}{n}\right)$ d) $\frac{a}{2}\cos\left(\frac{\pi}{n}\right)$

218. $\sinh^{-1}(2^{3/2})$ is equal to

a) $\log(3+\sqrt{8})$ b) $\log(3-\sqrt{8})$ c) $\log(2+\sqrt{18})$ d) $\log(\sqrt{8}+\sqrt{27})$

219. If $\tan 2\theta \tan\theta = 1$, then the general value of θ is

a) $\left(n+\frac{1}{2}\right)\frac{\pi}{3}$ b) $\left(n+\frac{1}{2}\right)\pi$ c) $\left(2n \pm \frac{1}{2}\right)\frac{\pi}{3}$ d) None of these

220. The maximum value of $\cos^2 A + \cos^2 B - \cos^2 C$, is

a) 0 b) 1 c) 3 d) 2

221. Value of $\cos^2(A-B) + \cos^2 B - 2\cos(A-B)\cos A \cos B$ is

a) $\sin A$ b) $\sin^2 A$ c) $\cos^2 A$ d) $\cos A$

222. The value of $\sin 36° \sin 72° \sin 108° \sin 144°$ is

a) $\frac{1}{4}$ b) $\frac{1}{16}$ c) $\frac{3}{4}$ d) $\frac{5}{16}$

223. The expression $(1+\tan x+\tan^2 x)(1-\cot x+\cot^2 x)$ has the positive value for x given by

a) $0 \le x \le \frac{\pi}{2}$ b) $0 \le x \le \pi$ c) For all $x \in R$ d) $x \ge 0$

224. The value of the series $x \log_e a + \frac{x^3}{3!}(\log_e a)^3 + \frac{x^5}{5!}(\log_e a)^5 + \ldots$ is

a) $\cosh\ (x \log_e a)$ b) $\coth(x \log_e a)$ c) $\sinh(x \log_e a)$ d) $\tanh(x \log_e a)$

225. If $3\tan(\theta - 15°) = \tan(\theta + 15°), 0 < \theta < \pi$, then $\theta =$

a) $\frac{\pi}{2}$ b) $\frac{\pi}{4}$ c) $\frac{3\pi}{4}$ d) $\frac{\pi}{6}$

226. If $ABCD$ is a convex quadrilateral such that $4\sec A + 5 = 0$ then the quadratic equation whose roots are $\tan A$ and $\operatorname{cosec} A$ is

a) $12x^2 - 29x + 15 = 0$

b) $12x^2 - 11x - 15 = 0$

c) $12x^2 + 11x - 15 = 0$

d) None of these

227. The smallest value of $5\cos\theta + 12$ is

a) 5 b) 12 c) 7 d) 17

228. The values of x satisfying the system of equations $2^{\sin x + \cos y} = 1, 16^{\sin^2 x + \cos^2 y} = 4$ are given by

a) $x = n\pi + (-1)^n \frac{\pi}{6}$ and $y = 2n\pi \pm \frac{\pi}{3}, n \in Z$

b) $x = n\pi + (-1)^{n+1} \frac{\pi}{6}$ and $y = 2n\pi \pm \frac{2\pi}{3}, n \in Z$

c) $x = n\pi + (-1)^n \frac{\pi}{6}$ and $y = 2n\pi \pm \frac{2\pi}{3}, n \in Z$

d) $x = n\pi + (-1)^{n+1} \frac{\pi}{6}$ and $y = 2n\pi \pm \frac{2\pi}{3}, n \in Z$

229. If $\cos\theta = -\frac{\sqrt{3}}{2}$ and $\sin\alpha = -\frac{3}{5}$, where θ does not lie in the third quadrant, then $\frac{2\tan\alpha + \sqrt{3}\tan\theta}{\cot^2\theta + \cos\alpha}$ is equal to

a) $\frac{7}{22}$ b) $\frac{5}{22}$ c) $\frac{9}{22}$ d) $\frac{22}{5}$

230. The expression $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$ is equal to

a) -1 b) 0 c) 1 d) None of these

231. In a ΔABC, if $b = 20, c = 21$ and $\sin A = \frac{3}{5}$, then $a =$

a) 12 b) 13 c) 14 d) 15

232. The value of $\cos^2\frac{\pi}{16} + \cos^2\frac{3\pi}{16} + \cos^2\frac{5\pi}{16} + \cos^2\frac{7\pi}{16}$

a) 2 b) 1 c) 0 d) None of these

233. If $\cos(\alpha + \beta) = \frac{4}{5}$, $\sin(\alpha-\beta)=5/13$ and α, β lies between 0 and $\pi/4$, then $\tan 2\alpha$ is equal to

a) $\frac{16}{63}$ b) $\frac{56}{33}$ c) $\frac{28}{33}$ d) None of these

234. $\cos 2\theta + 2\cos\theta$ is always

a) Greater than $-\frac{3}{2}$

b) Less than or equal to $\frac{3}{2}$

c) Greater than or equal to $-\frac{3}{2}$ and less than or equal to 3

d) None of the above

235. If $A = 130°$ and $x = \sin A + \cos A$, then

a) $x > 0$ b) $x < 0$ c) $x = 0$ d) $x \ge 0$

236. The set of values of x in $(-\pi, \pi)$ satisfying the inequation $|4\sin x - 1| < \sqrt{5}$ is

a) $(-\pi/10, 3\pi/10)$ b) $(-\pi/10, \pi)$ c) $(-\pi, \pi)$ d) $(-\pi, 3\pi/10)$

237. If $\sin A = n\sin B$, then $\frac{n-1}{n+1}\tan\frac{A+B}{2}$ is equal to

a) $\sin\frac{A-B}{2}$ b) $\tan\frac{A-B}{2}$ c) $\cot\frac{A-B}{2}$ d) None of these

238. In a $\Delta ABC, \frac{s}{R} =$

a) $\sin A + \sin B + \sin C$ b) $\cos A + \cos B + \cos C$ c) $\sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2}$ d) None of these

239. Which of the following relations is possible?

a) $\sin\theta = \frac{5}{3}$

b) $\tan\theta = 100^2$

c) $\cos\theta = \frac{1+p^2}{1-p^2}, (p \neq \pm 1)$

d) $\sec\theta = \frac{1}{2}$

240. The value of $\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{3\pi}{7}$, is

a) $\frac{1}{8}$ b) $-\frac{1}{8}$ c) 1 d) 0

241. The number of solutions of the given equation $\tan\theta + \sec\theta = \sqrt{3}$, where $0 < \theta < 2\pi$ is

a) 0 b) 1 c) 2 d) 3

242. The value of $\sin 12° \sin 48° \sin 54°$ is equal to

a) $\frac{1}{16}$ b) $\frac{1}{32}$ c) $\frac{1}{8}$ d) $\frac{1}{4}$

243. If p is the product of the sines of angles of a triangle, and q the product of their cosines, then tangents of the angles are roots of the equation

a) $qx^3 - px^3 + (1+q)x - p = 0$

b) $px^3 - qx^2 + (1+p)x - q = 0$

c) $(1+q)x^3 - px^2 + qx - p = 0$

d) None of these

244. The value of $\cos 480° . \sin 150° + \sin 600° . soc\ 390°$ is equal to

a) 0 b) 1 c) $\frac{1}{2}$ d) -1

245. If $\cos p\,\theta = \cos q\,\theta, p \neq q$, then

a) $\theta = 2\,n\,\pi, n \in Z$ b) $\theta = \frac{2\,n\,\pi}{p \pm q}, n \in Z$ c) $\theta = \frac{n\,\pi}{p+q}, n \in Z$ d) None of these

246. The most general solutions of the equation $\sec x - 1 = (\sqrt{2}-1)\tan x$ are given by

a) $n\pi + \frac{\pi}{8}$ b) $2n\pi, 2n\pi + \frac{\pi}{4}$ c) $2n\pi$ d) None of these

247. If $x = \tan 15°, y = \text{cosec}\ 75°$ and $z = 4\sin 18°$, then

a) $x < y < z$ b) $y < z < x$ c) $z < x < y$ d) $x < z < y$

248. The number of points of intersection of $2y = 1$ and $y = \sin x$, in $-2\pi \leq x \leq 2\pi$ is

a) 1 b) 2 c) 3 d) 4

249. If $A + B = C$, then $\cos^2 A + \cos^2 B + \cos^2 C - 2\cos A \cos B \cos C =$

a) 1 b) 2 c) 0 d) 3

250. The number of solutions of the equation $|\cos x| = 2[x]$, where $[\cdot]$ is the greatest integer, is

a) One b) Two c) Infinite d) nil

251. The number of values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ satisfying the equation $(\sqrt{3})^{\sec^2\theta} = \tan^4\theta + 2\tan^2\theta$ is

a) 1 b) 2 c) 3 d) None of these

252. Let n be a positive integer such that $\sin\frac{\pi}{2n} + \cos\frac{\pi}{2n} = \frac{\sqrt{n}}{2}$. Then

a) $n = 6$ b) $n = 1, 2, 3, \dots, 8$ c) $n = 5$ d) None of these

253. If $\cos 2x = (\sqrt{2}+1)\left(\cos x - \frac{1}{\sqrt{2}}\right), \cos x \neq \frac{1}{2}$, then $x \in I$

a) $\left\{2n\pi \pm \frac{\pi}{3} : n \in Z\right\}$ b) $\left\{2n\pi \pm \frac{\pi}{6} : n \in Z\right\}$ c) $\left\{2n\pi \pm \frac{\pi}{2} : n \in Z\right\}$ d) $\left\{2n\pi \pm \frac{\pi}{4} : n \in Z\right\}$

254. $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha$ is equal to

a) $\tan 16\alpha$ b) 0 c) $\cot\alpha$ d) None of these

255. In a triangle ABC, if $a = 2, B - 60°$ and $C = 75°$, then $b =$

a) $\sqrt{3}$ b) $\sqrt{6}$ c) $\sqrt{9}$ d) $1 + \sqrt{2}$

256. The smallest positive values of x and y which satisfy $\tan(x - y) = 1, \sec(x + y) = \frac{2}{\sqrt{3}}$ are

a) $x = \frac{25\,\pi}{24}, y = \frac{7\,\pi}{24}$ b) $x = \frac{37\,\pi}{24}, y = \frac{19\,\pi}{24}$ c) $x = \frac{\pi}{4}, y = \frac{\pi}{2}$ d) $x = \frac{\pi}{3}, y = \frac{7\,\pi}{12}$

257. If $\sqrt{3}\sin\theta + \cos\theta > 0$, then θ lies in the interval

a) $(-\pi/3, \pi/2)$ b) $(-\pi/6, 5\pi/6)$ c) $(\pi/4, \pi/3)$ d) None of these

258. If $\frac{\cos A}{3} = \frac{\cos B}{4} = \frac{1}{5}, -\frac{\pi}{2} < A < 0, -\frac{\pi}{2} < B < 0$ then the value of $2\sin A + 4\sin B$ is

a) 4 b) -2 c) -4 d) 0

259. The smallest positive root of the equation $\tan x - x = 0$ is in

a) $\left(0, \frac{\pi}{2}\right)$ b) $(\pi, \frac{3\pi}{2})$ c) $\left(\frac{\pi}{2}, \pi\right)$ d) $\left(\frac{3\pi}{2}, 2\pi\right)$

260. The expression $\tan^2\alpha + \cot^2\alpha$, is

a) ≥ 2 b) ≤ 2 c) ≥ -2 d) None of these

261. The most general solution of $\sqrt{3}\cos\theta + \sin\theta = \sqrt{2}$ is

a) $\theta = n\pi \pm \frac{\pi}{4} + \frac{\pi}{6}$ b) $\theta = n\pi \pm \frac{\pi}{4} - \frac{\pi}{6}$ c) $\theta = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{6}$ d) $\theta = 2n\pi \pm \frac{\pi}{4} - \frac{\pi}{6}$

262. The value of $\sin 50° - \sin 70° + \sin 10°$ is

a) 0 b) 1 c) $\frac{1}{2}$ d) $\frac{1}{\sqrt{2}}$

263. In a triangle $ABC, \sin A - \cos B = \cos C$, then angle B is

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$

264. The solution of the equation $4\sin^4 x + \cos^4 x = 1$ is

a) $x = 2n\pi$ b) $x = n\pi + 1$ c) $x = (n + 2)\pi$ d) None of the above

265. In a ΔABC,

$\sin A + \sin B + \sin C = 1 + \sqrt{2}$

and, $\cos A + \cos B + \cos C = \sqrt{2}$

if, the triangle is

a) Equilateral b) Isosceles c) Right angled d) Right angled isosceles

266. $\tan\frac{2\pi}{5} - \tan\frac{\pi}{15} - \sqrt{3}\tan\frac{2\pi}{5}\tan\frac{\pi}{15}$ is equal to

a) $-\sqrt{3}$ b) $\frac{1}{\sqrt{3}}$ c) 1 d) $\sqrt{3}$

267. The general solution of the equation $\tan 3x = \tan 5x$ is

a) $x = \frac{n\,\pi}{2}, n \in Z$ b) $x = n\,\pi, n \in Z$ c) $x = (2n + 1)\pi, n \in Z$ d) None of these

268. If $\tan\theta + \tan 4\,\theta + \tan 7\,\theta = \tan\theta\tan 4\,\theta\tan 7\,\theta$, then $\theta =$

a) $\frac{n\,\pi}{4}, n \in Z$ b) $\frac{n\,\pi}{7}, n \in Z$ c) $\frac{n\,\pi}{12}, n \in Z$ d) $n\,\pi, n \in Z$

269. $\frac{\sin^2 3A}{\sin^2 A} - \frac{\cos^2 3A}{\cos^2 A} =$

a) $\cos 2A$ b) $8\,\cos 2A$ c) $1/8\,\cos 2A$ d) None of these

270. If in a $\Delta ABC, 3\sin A = 6\sin B = 2\sqrt{3}\sin C$, then angle A is

a) $0°$ b) $30°$ c) $60°$ d) $90°$

271. If $x = y\cos\frac{2\pi}{3} = z\cos\frac{4\pi}{3}$, then $xy + yz + zx =$

a) -1 b) 0 c) 1 d) 2

272. If $\alpha+\beta-\gamma=\pi$, then $\sin^2\alpha+\sin^2\beta-\sin^2\gamma$ is equal to

a) $2\sin\alpha\sin\beta\cos\gamma$ b) $2\cos\alpha\cos\beta\cos\gamma$ c) $2\sin\alpha\sin\beta\sin\gamma$ d) None of these

273. If $0<A<\frac{\pi}{6}$ and $\sin A+\cos A=\frac{\sqrt{7}}{2}$, then $\tan\frac{A}{2}=$

a) $\frac{\sqrt{7}-2}{3}$ b) $\frac{\sqrt{7}+2}{3}$ c) $\frac{\sqrt{7}}{3}$ d) None of these

274. If $A+B+C=0$, then the value of $\sum\cot(B+C-A)\cot(C+A-B)$ is equal to

a) 0 b) 1 c) -1 d) 2

275. If $A+B=\frac{\pi}{4}$, then $(\tan A+1)(\tan B+1)$ is equal to

a) 1 b) 2 c) $\sqrt{3}$ d) -1

276. If $\cos x+\cos y+\cos\alpha=0$ and $\sin x+\sin y+\sin\alpha=0$, then $\cot\left(\frac{x+y}{2}\right)$ is equal to

a) $\sin\alpha$ b) $\cos\alpha$ c) $\cot\alpha$ d) $\sin\left(\frac{x+y}{2}\right)$

277. In a ΔABC, if $a=8, b=10$ and $c=12$, then C is equal to

a) $\frac{A}{2}$ b) $2A$ c) $3A$ d) None of these

278. If $-\frac{\pi}{2}<x<\frac{\pi}{2}$, then the value of $\log\sec x$ is

a) $2\coth^{-1}\left(\operatorname{cosec}^2\frac{x}{2}-1\right)$ b) $2\coth^{-1}\left(\operatorname{cosec}^2\frac{x}{2}+1\right)$

c) $2\operatorname{cosech}^{-1}\left(\cot^2\frac{x}{2}-1\right)$ d) $2\operatorname{cosech}^{-1}\left(\cot^2\frac{x}{2}+1\right)$

279. The most general value of θ satisfying the equations $\sin\theta=\sin\alpha$ and $\cos\theta=\cos\alpha$ is

a) $2n\pi+\alpha$ b) $2n\pi-\alpha$ c) $n\pi+\alpha$ d) $n\pi-\alpha$

280. If in a $\Delta\ ABC$, sides a, b, c are in A.P., then $\tan\frac{A}{2}\tan\frac{C}{2}=$

a) 1/4 b) 1/3 c) 3 d) 4

281. The number of solutions of the pair of equations $2\sin^2\theta-\cos 2\theta=0$ and $2\cos^2\theta-3\sin\theta=0$ in the interval $[0, 2\pi]$ is

a) Zero b) One c) Two d) Four

282. If $A=\tan 6°\tan 42°$ and $B=\cot 66°\cot 78°$, then

a) $A=2B$ b) $A=\frac{1}{3}$ c) $A=B$ d) $3A=2B$

283. If in a $\Delta\ ABC, \Delta=a^2-(b-c)^2$, then $\tan A=$

a) 15/16 b) 8/15 c) 8/17 d) 1/2

284. $\sin 47°-\sin 25°+\sin 61°-\sin 11°=$

a) $\cos 7°$ b) $\sin 7°$ c) $2\cos 7°$ d) $2\sin 7°$

285. If $\sin^3 x\sin 3x=\sum_{m=0}^{n}C_m\cos mx$ is an identity in x, where $C_0, C_1, \dots, C_n$ are constants and $C_n\neq 0$, then the value of n equals

a) 2 b) 4 c) 6 d) 8

286. If $\cos x=\tan y$, $\cos y=\tan z$, $\cos z=\tan x$, then the value of $\sin x$ is

a) $2\cos 18°$ b) $\cos 18°$ c) $\sin 18°$ d) $2\sin 18°$

287. The value of $\frac{\cot 54°}{\tan 36°}+\frac{\tan 20°}{\cot 70°}$ is

a) 0 b) 2 c) 3 d) 1

288. The value of $\frac{\cot x-\tan x}{\cot 2x}$ is

a) 1 b) 2 c) -1 d) 4

289. If $3\cos x\neq 2\sin x$, then the general solution of $\sin^2 x-\cos 2x=2-\sin 2x$ is

a) $n\pi+(-1)^n\frac{\pi}{2}, n\in Z$ b) $\frac{n\pi}{2}, n\in Z$

c) $(4n\pm 1)\frac{\pi}{2}, n\in Z$ d) $(2n-1)\pi, n\in Z$

290. $\frac{1+\tanh\frac{x}{2}}{1-\tanh\frac{x}{2}}$ is equal to

a) e^{-x} b) e^{x} c) $2e^{x/2}$ d) $2e^{-x/2}$

291. $2\tanh^{-1}\frac{1}{2}$ is equal to

a) 0 b) log 2 c) log 3 d) log 4

292. If $\cos(\alpha+\beta)\sin(\gamma+\delta)=\cos(\alpha-\beta)\sin(\gamma-\delta)$, then the value of $\cot\alpha\cot\beta\cot\gamma$ is

a) $\cot\alpha$ b) $\cot\beta$ c) $\cot\delta$ d) $\cot(\alpha+\beta+\gamma+\delta)$

293. If $a\cos^3\alpha+3a\cos\alpha\sin^2=m$ and $a\sin^3\alpha+3a\cos^2\alpha\sin\alpha=n$, then $(m+n)^{2/3}+\ (m-n)^{2/3}$ is equal to

a) $2a^2$ b) $2a^{1/3}$ c) $2a^{2/3}$ d) $2a^3$

294. If $x\sin\theta=y\cos\theta=\frac{2z\tan\theta}{1-\tan^2\theta}$, then $4z^2(x^2+y^2)$ is equal to

a) $(x^2+y^2)^3$ b) $(x^2-y^2)^3$ c) $(x^2-y^2)^2$ d) $(x^2+y^2)^2$

295. The solution of $\tan 2\theta\tan\theta=1$ is22

a) $\frac{\pi}{3}$ b) $(6n\pm1)\frac{\pi}{6}$ c) $(4n\pm1)\frac{\pi}{6}$ d) $(2n+\pi)\frac{\pi}{6}$

296. Set of values of x lying in $[0,2\pi]$ satisfying the inequality $|\sin x|>2\sin^2 x$ contains

a) $\left(0,\frac{\pi}{6}\right)\cup\left(\pi,\frac{7\pi}{6}\right)$ b) $\left(0,\frac{7\pi}{6}\right)$ c) $\frac{\pi}{6}$ d) None of these

297. If $\tan\left(\frac{x}{2}\right)=\operatorname{cosec}x-\sin x$, then the value of $\tan^2\left(\frac{x}{2}\right)$ is

a) $2-\sqrt{5}$ b) $2+\sqrt{5}$ c) $-2-\sqrt{5}$ d) $-2+\sqrt{5}$

298. If $5\cos 2\theta+2\cos^2\frac{\theta}{2}+1=0$, when $(0<\theta<\pi)$, then the values of θ are

a) $\frac{\pi}{3}\pm\pi$ b) $\frac{\pi}{3},\cos^{-1}\left(\frac{3}{5}\right)$ c) $\cos^{-1}\left(\frac{3}{5}\right)\pm\pi$ d) $\frac{\pi}{3},\pi-\cos^{-1}\left(\frac{3}{5}\right)$

299. If $\tan\alpha,\tan\beta,\tan\gamma$ are the roots of the equation $x^3-px^2-r=0$, then the value of $(1+\tan^2\alpha)(1+$ tan*2β1+*tan*2γ* is equal to

a) $(p-r)^2$ b) $1+(p-r)^2$ c) $1-(p-r)^2$ d) None of these

300. $\cos^4\theta-\sin^4\theta$ is equal to

a) $1+2\sin^2\frac{\theta}{2}$ b) $2\cos^2\theta-1$ c) $1-2\sin^2\frac{\theta}{2}$ d) $1+2\cos^2\theta$

301. The value of $\sqrt{3}\operatorname{cosec}20°-\sec 20°$ is equal to

a) 2 b) 1 c) 4 d) -4

302. If θ lies in the first quadrant which of the following is not true

a) $\frac{\theta}{2}<\tan\left(\frac{\theta}{2}\right)$ b) $\frac{\theta}{2}<\sin\frac{\theta}{2}$ c) $\theta\cos^2\left(\frac{\theta}{2}\right)<\sin\theta$ d) $\theta\sin\frac{\theta}{2}<2\sin\frac{\theta}{2}$

303. Number of solutions of $|x-1|=\cos x$ is

a) 2 b) 3 c) 4 d) None of these

304. If $5\cos x+12\cos y=13$, then the maximum value of $5\sin x+12\sin y$ is

a) 12 b) $\sqrt{120}$ c) $\sqrt{20}$ d) 13

305. If $\cos\theta=\frac{8}{17}$ and θ lies in the I st quadrant, then the value of $\cos(30°+\theta)+\cos(45°-\theta)+\cos(120°-\theta)$ is

a) $\frac{23}{17}\left(\frac{\sqrt{3}-1}{2}+\frac{1}{\sqrt{2}}\right)$ b) $\frac{23}{17}\left(\frac{\sqrt{3}+1}{2}+\frac{1}{\sqrt{2}}\right)$ c) $\frac{23}{17}\left(\frac{\sqrt{3}-1}{2}-\frac{1}{\sqrt{2}}\right)$ d) $\frac{23}{17}\left(\frac{\sqrt{3}+1}{2}-\frac{1}{\sqrt{2}}\right)$

306. $\cos 1°+\cos 2°+\cos 3°+\ldots+\cos 180°$ is equal to

a) 1 b) 0 c) 2 d) -1

307. In any ΔABC, if $\cot\frac{A}{2},\cot\frac{B}{2},\cot\frac{C}{2}$ are in A.P., then a,b,c are in

a) A. P. b) G. P. c) H. P. d) None of these

308. The expression $(1+\tan x+\tan^2 x)(1-\cot x+\cot^2 x)$ has the positive values for x, given by

a) $0\le x\le\frac{\pi}{2}$ b) $0\le x\le\pi$ c) For all $x\in R$ d) $x\ge0$

309. The value of $\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}$, is

a) 0 b) $\frac{1}{2}$ c) $\frac{3}{2}$ d) 1

310. If $\sin(\pi\cos\theta) = \cos(\pi\sin\theta)$, then $\cos\left(\theta \pm \frac{\pi}{4}\right)$ is equal to
a) $\cos\frac{\pi}{4}$ b) $\frac{1}{2}\cos\frac{\pi}{4}$ c) $\cos\frac{\pi}{8}$ d) None of these

311. If the angles A, B, C of a triangle are in A.P. and sides a, b, c are in G.P, then a^2, b^2, c^2 are in
a) A.P. b) H.P. c) G.P. d) None of these

312. If the complex numbers $\sin x + i\cos 2x$ and $\cos x - i\sin 2x$ are conjugate to each other, then x is equal to
a) $n\pi$ b) $\left(n+\frac{1}{2}\right)\pi, n \in Z$ c) 0 d) None of these

313. The most general solutions of the equation $\sec x - 1 = (\sqrt{2}-1)\tan x$ are given by
a) $n\pi + \frac{\pi}{8}$ b) $2n\pi, 2n\pi + \frac{\pi}{4}$ c) $2n\pi$ d) None of these

314. If $\tan A = 2\tan B + \cot B$, then $2\tan(A-B)$ is equal to
a) $\tan B$ b) $2\tan B$ c) $\cot B$ d) $2\cot B$

315. If $n = 1, 2, 3, \ldots$, then $\cos\alpha\cos 2\alpha\cos 4\alpha \ldots \cos 2^{n-1}\alpha$ is equal to
a) $\frac{\sin 2n\alpha}{2n\sin\alpha}$ b) $\frac{\sin 2^n\alpha}{2^n\sin 2^{n-1}\alpha}$ c) $\frac{\sin 4^{n-1}\alpha}{4^{n-1}\sin\alpha}$ d) $\frac{\sin 2^n\alpha}{2^n\sin\alpha}$

316. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, then $x^2 + y^2$ is
a) 2 b) 0 c) 3 d) 1

317. If $\tan\alpha/2$ and $\tan\beta/2$ are the roots of the equation $8x^2 - 26x + 15 = 0$ then $\cos(\alpha+\beta)$ is equal to
a) $-\frac{627}{725}$ b) $\frac{627}{725}$ c) -1 d) None of these

318. If $0 \le x \le \pi$ and $81^{\sin^2 x} + 81^{\cos^2 x} = 30$, then x is equal to
a) $\frac{\pi}{6}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) $\frac{3\pi}{4}$

319. If $32\tan^8\theta = 2\cos^2\alpha - 3\cos\alpha$ and $3\cos 2\theta = 1$, then $\alpha =$
a) $2n\pi, n \in Z$ b) $2n\pi \pm \frac{2\pi}{3}, n \in Z$ c) $2n\pi \pm \frac{\pi}{3}, n \in Z$ d) $n\pi \pm \frac{\pi}{3}, n \in Z$

320. $\frac{\cos x}{\cos(x-2y)} = \lambda \Rightarrow \tan(x-y)\tan y$ is equal to
a) $\frac{1+\lambda}{1-\lambda}$ b) $\frac{1-\lambda}{1+\lambda}$ c) $\frac{\lambda}{1+\lambda}$ d) $\frac{\lambda}{1-\lambda}$

321. If $\sin 3\theta = 4\sin\theta(\sin^2 x - \sin^2\theta), \theta \ne n\pi, n \in Z$. Then, the set of values of x is
a) $\left\{n\pi \pm \frac{\pi}{3} : n \in Z\right\}$ b) $\left\{n\pi \pm \frac{2\pi}{3} : n \in Z\right\}$ c) $\left\{n\pi \pm \frac{\pi}{2} : n \in Z\right\}$ d) $\left\{n\pi \pm \frac{\pi}{4} : n \in Z\right\}$

322. If in a triangle $ABC, b + c = 3a$, then $\tan\left(\frac{B}{2}\right)\tan\left(\frac{C}{2}\right)$ is equal to
a) 1 b) -1 c) 2 d) None of these

323. If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$ then $\cos\theta - \sin\theta$ is equal to
a) $\sqrt{2}\cos\theta$ b) $\sqrt{2}\sin\theta$ c) $\sqrt{2}(\cos\theta + \sin\theta)$ d) None of these

324. The equation $\sin x + \sin y + \sin z = -3$ for $0 \le x \le 2\pi, 0 \le y \le 2\pi, 0 \le z \le 2\pi$ has
a) One solution b) Two sets of solution c) Four sets of solution d) No solution

325. The value of $\sin\frac{\pi}{16}\sin\frac{3\pi}{16}\sin\frac{5\pi}{16}\sin\frac{7\pi}{16}$ is
a) $\frac{\sqrt{2}}{16}$ b) $\frac{1}{8}$ c) $\frac{1}{16}$ d) $\frac{\sqrt{2}}{32}$

326. The equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$, where x is a variable, has real roots. Then, the interval of p may be any one of the following:
a) $(0, 2\pi)$ b) $(-\pi, 0)$ c) $(-\pi/2, \pi/2)$ d) $(0, \pi)$

327. If $\tan\left(\frac{\theta}{2}\right) = \frac{5}{2}$ and $\tan\left(\frac{\phi}{2}\right) = \frac{3}{4}$, the value of $\cos(\theta + \phi)$ is

a) $-\frac{364}{725}$ b) $-\frac{627}{725}$ c) $-\frac{240}{339}$ d) $-\frac{339}{725}$

328. If in a triangle $ABC, \sin A = \sin^2 B$ and $2\cos^2 A = \cos^2 B$, then the $\Delta\, ABC$ is
a) Right angled b) Obtuse angled c) Isosceles d) Equilateral

329. If α and β are acute angles $\cos 2\alpha = \frac{3\cos 2\beta - 1}{3 - \cos 2\beta}$, then $\tan\alpha \cot\beta =$
a) $\sqrt{3}$ b) $\sqrt{2}$ c) 1 d) None of these

330. If $\sin 2x, \frac{1}{2}$ and $\cos 2x$ are in A.P., then the general values of x are given by
a) $n\,\pi, n\,\pi + \frac{\pi}{2}, n \in Z$ b) $n\,\pi, n\,\pi + \frac{\pi}{4}, n \in Z$ c) $n\,\pi + \frac{\pi}{4}, n \in Z$ d) $n\,\pi, n \in Z$

331. The area of a regular polygon of n sides is
a) $\frac{nR^2}{2}\sin\left(\frac{2\,\pi}{n}\right)$ b) $nr^2\tan\left(\frac{2\,\pi}{2n}\right)$ c) $\frac{nr^2}{2}\sin\left(\frac{2\,\pi}{n}\right)$ d) $n\,R^2\tan\left(\frac{\pi}{n}\right)$

332. The base of a triangle is 80 cm and one of the base angles is 60°. If the sum of the lengths of the other two sides is 90 cm, then the length of the shortest side is
a) 15 cm b) 19 cm c) 21 cm d) 17 cm

333. Total number of solutions of $\sin^4 x + \cos^4 x = \sin x \cdot \cos x$ in $[0, 2\pi]$ is equal to
a) 2 b) 4 c) 6 d) 8

334. If $\alpha, \beta, \gamma, \delta$ are the smallest positive angles in ascending order of magnitude which have their sines equal to the positive quantity k, then the value of
$4\sin\frac{\alpha}{2} + 3\sin\frac{\beta}{2} + 2\sin\frac{\gamma}{2} + \sin\frac{\delta}{2}$ is equal to
a) $2\sqrt{1-k}$ b) $2\sqrt{1+k}$ c) $\frac{\sqrt{1+k}}{2}$ d) $\sqrt{1+k}$

335. Maximum value of $\sin\theta + \cos\theta$ in $\left[0, \frac{\pi}{2}\right]$ is
a) $\sqrt{2}$ b) 2 c) 0 d) $-\sqrt{2}$

336. $\{x \in R: \cos 2x + 2\cos^2 x = 2\}$ is equal to
a) $\left\{2n\pi + \frac{\pi}{3}: n \in Z\right\}$ b) $\left\{n\pi \pm \frac{\pi}{6}: n \in Z\right\}$ c) $\left\{n\pi + \frac{\pi}{3}: n \in Z\right\}$ d) $\left\{2n\pi - \frac{\pi}{3}: n \in Z\right\}$

337. The most general value of θ satisfying the equation $(1 + 2\sin\theta)^2 + (\sqrt{3}\tan\theta - 1)^2 = 0$ are given by
a) $n\pi \pm \frac{\pi}{6}$ b) $n\pi + (-1)^n\frac{7\pi}{6}$ c) $2n\pi + \frac{7\pi}{6}$ d) $2n\pi + \frac{11\pi}{6}$

338. The solution set of $(2\cos x - 1)(3 + 2\cos x) = 0$ in the interval $0 \le x \le 2\,\pi$, is
a) $\left\{\frac{\pi}{3}\right\}$ b) $\left\{\frac{\pi}{3}, \frac{5\,\pi}{3}\right\}$ c) $\left\{\frac{\pi}{3}, \frac{5\,\pi}{3}, \cos^{-1}\left(-\frac{3}{2}\right)\right\}$ d) None of these

339. In a ΔABC, if $B = 90°$, then the value of $\tan\frac{A}{2}$ in terms of the sides is
a) $\sqrt{\frac{b+c}{b-c}}$ b) $\sqrt{\frac{b-c}{b+c}}$ c) $\sqrt{\frac{a+c}{a-c}}$ d) $\sqrt{\frac{a-c}{a+c}}$

340. If $\cos(\theta + \emptyset) = m\cos(\theta - \emptyset)$, then $\tan\theta$ is equal to
a) $[(1+m)/(1-m)]\tan\emptyset$ b) $[(1-m)/(1+m)]\tan\emptyset$
c) $[(1-m)/(1+m)]\cot\emptyset$ d) $[(1+m)/(1-m)]\sec\emptyset$

341. In a triangle $ABC, \angle B = \frac{\pi}{3}$ and $\angle C = \frac{\pi}{4}$. Let D divide BC internally in the ratio 1 : 3. Then, $\frac{\sin\angle BAD}{\sin\angle CAD}$ equals
a) $\frac{1}{\sqrt{6}}$ b) $\frac{1}{3}$ c) $\frac{1}{\sqrt{3}}$ d) $\sqrt{\frac{2}{3}}$

342. If $\tan x = \frac{b}{a}$, then the value of $a\cos 2x + b\sin 2x$ is
a) a b) $a - b$ c) $a + b$ d) b

343. In a $\Delta ABC, b = 2, C = 60°, c = \sqrt{6}$, then $a =$
a) $\sqrt{3} - 1$ b) $\sqrt{3}$ c) $\sqrt{3} + 1$ d) None of these

344. If $A_1A_2A_3A_4A_5$ be a regular pentagon inscribed in a unit circle. Then $(A_1A_2)(A_1A_3)$ is equal to

a) 1 b) 3 c) 4 d) $\sqrt{5}$

345. If $x = \log\left[\cot\left(\frac{\pi}{4} + \theta\right)\right]$, then the value of $\sinh x$ is

a) $\tan 2\theta$ b) $-\tan 2\theta$ c) $\cot 2\theta$ d) $-\cot 2\theta$

346. The angle of a right angled triangle are in A.P. The ratio of the in-radius and the perimeter is

a) $(2-\sqrt{3}) : 2\sqrt{3}$ b) $1 : 8\sqrt{3}\,(2+\sqrt{3})$ c) $(2+\sqrt{3}) : 4\sqrt{3}$ d) None of these

347. The minimum value of $\cos 2\theta + \cos\theta$ for real values of θ, is

a) $-9/8$ b) 0 c) -2 d) None of these

348. $3\,(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x)$ is equal to

a) 12 b) 13 c) 14 d) 11

349. The sides of a triangle are $3x + 4y, 4x + 3y$ and $5x + 5y$, where, $x, y > 0$ then the triangle is

a) Right angled b) Obtuse angled c) Equilateral d) None of these

350. If $\sin\beta$ is the GM between $\sin\alpha$ and $\cos\alpha$, then $\cos 2\beta =$

a) $2\sin^2\left(\frac{3\pi}{4} - \alpha\right)$ b) $2\cos^2\left(\frac{\pi}{4} - \alpha\right)$ c) $\cos^2\left(\frac{\pi}{4} + \alpha\right)$ d) $2\sin^2\left(\frac{\pi}{4} + \alpha\right)$

351. If in a triangle ABC, $\cos A\cos B + \sin A\sin B\sin C = 1$, then the triangle is

a) Isosceles
b) Right angled
c) Isosceles right angled
d) Equilateral

352. The solution of the equation $\cos^2 x - 2\cos x = 4\sin x - \sin 2x (0 \le x \le \pi)$ is

a) $\pi - \cot^{-1}\frac{1}{2}$ b) $\pi - \tan^{-1} 2$ c) $\pi + \tan^{-1}\left(-\frac{1}{2}\right)$ d) None of these

353. The area of the triangle ABC, in which $a = 1, b = 2, \angle C = 60°$, is

a) 4 sq. units b) $\frac{1}{2}$ sq. unit c) $\frac{\sqrt{3}}{2}$ sq. unit d) $\sqrt{3}$ sq. units

354. If $\cos(A - B) = 3/5$ andtan $A\tan B = 2$, then which one of the following is true?

a) $\sin(A + B) = \frac{1}{5}$ b) $\sin(A + B) = -\frac{1}{5}$ c) $\cos(A - B) = \frac{1}{5}$ d) $\cos(A + B) = -\frac{1}{5}$

355. The value of $\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{3\pi}{9}\cos\frac{4\pi}{9}$ is

a) $\frac{1}{8}$ b) $\frac{1}{16}$ c) $\frac{1}{64}$ d) $\frac{1}{4}$

356. If the equation $\sec\theta + \operatorname{cosec}\theta = c$ has four real roots between 0 and 2π, then

a) $c^2 < 8$ b) $c^2 > 8$ c) $c^2 = 8$ d) None of these

357. If $\frac{\tan 3A}{\tan A} = k$, then $\frac{\sin 3A}{\sin A}$ is equal to

a) $\frac{2k}{k-1}, k \in R$ b) $\frac{2k}{k-1}, k \in [1/3, 3]$ c) $\frac{2k}{k-1}, k \notin [1/3, 3]$ d) $\frac{k-1}{2k}, k \notin [1/3, 3]$

358. If $\sin\theta - \cos\theta < 0$, then θ lies between

a) $n\pi - \frac{3\pi}{4}$ and $n\pi + \frac{\pi}{4}, n \in Z$
b) $n\pi - \frac{\pi}{4}$ and $n\pi + \frac{3\pi}{4}, n \in Z$
c) $2n\pi - \frac{3\pi}{4}$ and $2n\pi - \frac{\pi}{4}, n \in Z$
d) $2n\pi - \frac{3\pi}{4}$ and $2n\pi + \frac{\pi}{4}, n \in Z$

359. If $y\tan(A + B + C) = x\tan(A + B - C) = \lambda$, then the $2C =$

a) $\frac{\lambda(x+y)}{\lambda^2 - xy}$ b) $\frac{\lambda(x+y)}{\lambda^2 + xy}$ c) $\frac{\lambda(x-y)}{xy - \lambda^2}$ d) $\frac{\lambda(x-y)}{xy + \lambda^2}$

360. If $\alpha + \beta = \frac{\pi}{2}, \beta + \gamma = \alpha$, then the value of $\tan\alpha$ equals

a) $\tan\beta + \tan\gamma$ b) $2(\tan\beta + \tan\gamma)$ c) $\tan\beta + 2\tan\gamma$ d) $2\tan\beta + \tan\gamma$

361. $\tan\frac{2\pi}{5} - \tan\frac{\pi}{15} - \sqrt{3}\tan\frac{2\pi}{5}\tan\frac{\pi}{15}$ is equal to

a) $-\sqrt{3}$ b) $1/\sqrt{3}$ c) 1 d) $\sqrt{3}$

362. If $\frac{\cos A}{\cos B} = n$ and $\frac{\sin A}{\sin B} = m$, then $(m^2 - n^2)\sin^2 B =$

a) $1 - n^2$ b) $1 + n^2$ c) $1 - n$ d) $1 + n$

363. If $A = 130°$ and $x = \sin A + \cos A$, then

a) $x > 0$ b) $x < 0$ c) $x = 0$ d) $x \geq 0$

364. The value of $\tan 9° - \tan 27° - \tan 63° + \tan 81°$ is

a) 2 b) 3 c) 4 d) 1

365. If $\pi < \alpha < \frac{3\pi}{2}$, then the expression $\sqrt{4\sin^4\alpha + \sin^2 2\alpha} + 4\cos^2\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)$ is equal to

a) $2 + 4\sin\alpha$ b) $2 - 4\sin\alpha$ c) 2 d) None of these

366. If $5\cos 2\theta + 2\cos^2\frac{\theta}{2} + 1 = 0, -\pi < \theta < \pi$, then θ is equal to

a) $\frac{\pi}{3}$ b) $\frac{\pi}{3}, \cos^{-1}\left(\frac{3}{5}\right)$ c) $\cos^{-1}\left(\frac{3}{5}\right)$ d) $\frac{\pi}{3}, \pi - \cos^{-1}\left(\frac{3}{5}\right)$

367. The equation $\sin^4 x + \cos^4 x = \alpha$ has a real solution, if

a) $0 < a \leq 1$ b) $\frac{1}{2} \leq a \leq 1$ c) $\frac{1}{4} \leq a \leq \frac{1}{2}$ d) $-1 \leq a \leq 1$

368. If $x\sin a + y\sin 2a + z\sin 3a = \sin 4a$

$x\sin b + y\sin 2b + z\sin 3b = \sin 4b$

$x\sin c + y\sin 2c + z\sin 3c = \sin 4c$

Then, the roots of the equation

$t^3 - \left(\frac{z}{2}\right)t^2 - \left(\frac{y+2}{4}\right)t + \left(\frac{z-x}{8}\right) = 0, a, b, c \neq n\pi$, are

a) $\sin a, \sin b, \sin c$ b) $\cos a, \cos b, \cos c$

c) $\sin 2a, \sin 2b, \sin 2c$ d) $\cos 2a, \cos 2b, \cos 2c$

369. If α, β are different values of x satisfying $a\cos x + b\sin x = c$, then $\tan\left(\frac{\alpha+\beta}{2}\right)$ is equal to

a) $(a + b)$ b) $(a - b)$ c) $\frac{b}{a}$ d) $\frac{a}{b}$

370. Let AB and C be the angles of a plain triangle and $\tan\frac{A}{2} = \frac{1}{3}, \tan\frac{B}{2} = \frac{2}{3}$. Then, $\tan\frac{C}{2}$ is equal to

a) 7/9 b) 2/9 c) 1/3 d) 2/3

371. If $\tan\theta = \frac{1}{\sqrt{7}}$, then $\frac{(\text{cosec}^2\theta - \sec^2\theta)}{(\text{cosec}^2\theta + \sec^2\theta)}$ is equal to

a) $\frac{1}{2}$ b) $\frac{3}{4}$ c) $\frac{5}{4}$ d) 2

372. The general solution of $\tan 3x = 1$, is

a) $n\pi + \frac{\pi}{4}$ b) $\frac{n\pi}{3} + \frac{\pi}{12}$ c) $n\pi$ d) $n\pi \pm \frac{\pi}{4}$

373. If A lies in the third quadrant and $3\tan A - 4 = 0$, then $5\sin 2A + 3\sin A + 4\cos A =$

a) 0 b) $\frac{-24}{5}$ c) $\frac{24}{5}$ d) $\frac{48}{5}$

374. The number of solutions of $\cos 2\theta = \sin\theta$ in $(0, 2\pi)$ is

a) 1 b) 2 c) 3 d) 4

375. The equation $\sin^4 x - 2\cos^2 x + a^2 = 0$ is solvable for

a) $-\sqrt{3} \leq a \leq \sqrt{3}$ b) $-\sqrt{2} \leq a \leq \sqrt{2}$ c) $-1 \leq a \leq 1$ d) None of these

376. Let A, B and C are the angles of a teiangle andtan $\left(\frac{A}{2}\right) = \frac{1}{3}, \tan\left(\frac{B}{2}\right) = \frac{2}{3}$. Then, $\tan\left(\frac{C}{2}\right)$ is equal to

a) 1/3 b) 2/3 c) 2/9 d) 7/9

377. If $x + \frac{1}{x} = 2\cos\alpha$, then $x^n + \frac{1}{x^n}$ is equal to

a) $2^n\cos\alpha$ b) $2^n\cos n\alpha$ c) $2i\sin n\alpha$ d) $2\cos n\alpha$

378. If $\tan\alpha = \frac{m}{m+1}$ and $\tan\beta = \frac{1}{2m+1}$, then $\alpha + \beta$ is equal to

a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$ c) 0 d) $\frac{\pi}{2}$

379. In a right angled triangle, the hypotenuse is four times as long as the perpendicular drawn to it from the opposite vertex. One of the acute angle is

a) $15°$ b) $30°$ c) $45°$ d) None of these

380. If $\cot\theta\cot 7\theta+\cot\theta\cot 4\theta+\cot 4\theta\cot 7\theta=1$, then $\theta=$

a) $n\pi, n\in Z$ b) $(2n+1)\frac{\pi}{2}, n\in Z$ c) $n\pi+(-1)^n\frac{\pi}{2}, n\in Z$ d) $\frac{n\pi}{12}, n\in Z$

381. If r, r_1, r_2, r_3 have their usual meanings, the value of $\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$, is

a) 1 b) 0 c) $1/r$ d) None of these

382. If $A=35°, B=15°$ and $C=40°$, then $\tan A.\tan B+\tan B.\tan C+\tan C.\tan A$ is equal to

a) 0 b) 1 c) 2 d) 3

383. The value of $\cos 10°-\sin 10°$ is

a) Positive b) Negative c) 0 d) 1

384. The number of all possible triplets (a_1, a_2, a_3) such that $a_1+a_2\cos 2x+a_3\sin^2 x=0$ for all x is

a) 0 b) 1 c) 3 d) None of these

385. The value of $\tan 40°+\tan 20°+\sqrt{3}\tan 20°\tan 40°$ is equal to

a) $\sqrt{12}$ b) $\frac{1}{\sqrt{3}}$ c) 1 d) $\sqrt{3}$

386. The arithmetic mean of the roots of the equation $4\cos^3 x-4\cos^2 x-\cos(315\pi+x)=1$ in the interval $(0, 315)$ is equal to

a) 50π b) 51π c) 100π d) 315π

387. The value of $\sum_{k=1}^{3}\cos^2(2k-1)\frac{\pi}{12}$, is

a) 0 b) $1/2$ c) $-1/2$ d) $3/2$

388. Total number of solutions of $\cos x=\sqrt{1-\sin 2x}$ in $[0, 2\pi]$ is equal to

a) 2 b) 3 c) 5 d) None of these

389. If $\sin 2x\cos 2x\cos 4x=\lambda$ has a solution, then λ lies in the interval

a) $[-1/2, 1/2]$ b) $[-1/4, 1/4]$ c) $[-1/3, 1/3]$ d) None of these

390. If $\sin x+\sin y=3(\cos y-\cos x)$, then the value of $\frac{\sin 3x}{\sin 3y}$ is

a) 1 b) -1 c) 0 d) ± 1

391. If $\tan\alpha=k\cot\beta$, then $\frac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)}$ is equal to

a) $\frac{1+k}{1-k}$ b) $\frac{1-k}{1+k}$ c) $\frac{k+1}{k-1}$ d) $\frac{k-1}{k+1}$

392. If $\theta=\frac{2\sin x}{1+\sin x+\cos x}$, then $\frac{1+\sin x-\cos x}{1+\sin x}$ equals

a) 0 b) $-\theta$ c) θ d) $-\theta/2$

393. In ΔABC, if $\frac{s-a}{\Delta}=\frac{1}{8}, \frac{s-b}{\Delta}=\frac{1}{12}$ and $\frac{s-c}{\Delta}=\frac{1}{24}$, then $b=$

a) 16 b) 20 c) 24 d) 28

394. Solution of the equation $\cos^2\left(\frac{1}{2}px\right)+\cos^2\left(\frac{1}{2}qx\right)=1$ form an arithmetic progression with common difference

a) $\frac{2}{p+q}$ b) $\frac{2}{p-q}$ c) $\frac{\pi}{p+q}$ d) None of these

395. If $\sec^2\theta=\sqrt{2}(1-\tan^2\theta)$, then $\theta=$

a) $n\pi+\frac{\pi}{8}, n\in Z$ b) $n\pi\pm\frac{\pi}{4}, n\in Z$ c) $n\pi\pm\frac{\pi}{8}, n\in Z$ d) None of these

396. If $\tanh^{-1}(x+iy)=\frac{1}{2}\tanh^{-1}\left(\frac{2x}{1+x^2+y^2}\right)+\frac{i}{2}\tan^{-1}\left(\frac{2y}{1-x^2-y^2}\right), x, y\in R$, then $\tanh^{-1}(iy)$ is

a) $i\tanh^{-1}(y)$ b) $-i\tanh^{-1}(y)$ c) $i\tan^{-1}y$ d) $-i\tan^{-1}(y)$

397. If $-\frac{\pi}{2}\leq x\leq\frac{\pi}{2}$, then the two curves $y=\cos x$ and $y=\sin 3x$ intersect at

a) $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$ and $\left(\frac{\pi}{8}, \cos\frac{\pi}{8}\right)$

b) $\left(\frac{-\pi}{4}, \frac{1}{\sqrt{2}}\right)$ and $\left(\frac{-\pi}{8}, \cos\frac{\pi}{8}\right)$

c) $\left(\frac{\pi}{4}, \frac{-1}{\sqrt{2}}\right)$ and $\left(\frac{\pi}{8}, -\cos\frac{\pi}{8}\right)$

d) $\left(\frac{-\pi}{4}, \frac{1}{\sqrt{2}}\right)$

398. If $A = \cos^2\theta + \sin^4\theta$, then for all values of θ,

a) $1 \le A \le 2$ b) $\frac{13}{16} \le A \le 1$ c) $\frac{3}{4} \le A \le \frac{13}{16}$ d) $\frac{3}{4} \le A \le 1$

399. The value of $\sin 10° + \sin 20° + \sin 30° + \cdots + \sin 360°$ is

a) 1 b) 0 c) -1 d) 1/2

400. The maximum value of $5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$ is

a) 5 b) 11 c) 10 d) -1

401. If $A = \left\{x: \frac{\pi}{6} \le x \le \frac{\pi}{3}\right\}$ and $f(x) = \cos x - x(1+x)$, then $f(A)$ is equal to

a) $\left[-\frac{\pi}{3}, -\frac{\pi}{6}\right]$ b) $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

c) $\left[\frac{1}{2} - \frac{\pi}{3}\left(1 + \frac{\pi}{3}\right), \frac{\sqrt{3}}{2} - \frac{\pi}{6}\left(1 + \frac{\pi}{6}\right)\right]$ d) $\left[\frac{1}{2} + \frac{\pi}{3}\left(1 - \frac{\pi}{3}\right), \frac{\sqrt{3}}{2} + \frac{\pi}{6}\left(1 - \frac{\pi}{6}\right)\right]$

402. If $\frac{x}{a}\cos\alpha + \frac{y}{b}\sin\alpha = 1, \frac{x}{a}\cos\beta + \frac{y}{b}\sin\beta = 1$ and $\frac{\cos\alpha\cos\beta}{a^2} + \frac{\sin\alpha\sin\beta}{b^2} = 0$, then

a) $\tan\alpha\tan\beta = \frac{b^2(x^2 - a^2)}{a^2(y^2 - b^2)}$ and $x^2 + y^2 = a^2 - b^2$

b) $\tan\alpha\tan\beta = \frac{a^2}{b^2}$

c) $x^2 + y^2 = a^2 - b^2$

d) None of these

403. The value of $\left(1 + \cos\frac{\pi}{6}\right)\left(1 + \cos\frac{\pi}{3}\right)\left(1 + \cos\frac{2\pi}{3}\right)\left(1 + \cos\frac{7\pi}{6}\right)$ is

a) $\frac{3}{16}$ b) $\frac{3}{8}$ c) $\frac{3}{4}$ d) $\frac{1}{2}$

404. If the solutions for θ, $\cos p\theta + \cos q\theta = 0, p > 0, q > 0$ are in AP, then the numerically smallest common difference of AP is

a) $\frac{\pi}{p+q}$ b) $\frac{2\pi}{p+q}$ c) $\frac{\pi}{2(p+q)}$ d) $\frac{1}{p+q}$

405. The equation $\sin^4\theta + \cos^4\theta = a$ has a real solution if

a) $a \in [1/2, 1]$ b) $a \in [1/4, 1/2]$ c) $a \in [1/3, 1]$ d) None of these

406. Which one of the following equations has no solution?

a) $\operatorname{cosec}\theta - \sec\theta = \operatorname{cosec}\theta . \sec\theta$ b) $\operatorname{cosec}\theta . \sec\theta = 1$

c) $\cos\theta + \sin\theta = \sqrt{2}$ d) $\sqrt{3}\sin\theta - \cos\theta = 2$

407. For $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, $\frac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta}$ lies in the interval

a) $(-\infty, \infty)$ b) $(-2, 2)$ c) $(0, \infty)$ d) $(-1, 1)$

408. The number of integral values of k for which the equation $7\cos\theta + 5\sin\theta = 2k + 1$ has a solution is

a) 4 b) 8 c) 10 d) 12

409. The solution of the inequality $\log_{1/2}\sin x > \log_{1/2}\cos x$ in $(0, 2\pi)$ is

a) $x \in \left(\frac{5\pi}{4}, 2\pi\right)$ b) $x \in \left(0, \frac{\pi}{4}\right)$

c) $x \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right)$ d) None of these

410. The solution set of the inequation $\log_{1/2}\sin x > \log_{1/2}\cos x$ in $[0, 2\pi]$, is

a) $(0, \pi/2)$ b) $(-\pi/4, \pi/4)$ c) $(0, \pi/4)$ d) None of these

411. The value of $\cos^2\left(\frac{\pi}{4}+\theta\right)-\sin^2\left(\frac{\pi}{2}-\theta\right)$ is

a) 0 b) $\cos 2\theta$ c) $\sin 2\theta$ d) $\cos\theta$

412. The solution set of the inequality $\cos^2\theta<\frac{1}{2}$, is

a) $\left\{\theta : (8n+1)\frac{\pi}{4}<\theta<(8n+3)\frac{\pi}{4}, n\in Z\right\}$

b) $\left\{\theta : (8n-3)\frac{\pi}{4}<\theta<(8n-1)\frac{\pi}{4}, n\in Z\right\}$

c) $\left\{\theta : (4n+1)\frac{\pi}{4}<\theta<(4n+3)\frac{\pi}{4}, n\in Z\right\}$

d) None of these

413. $\alpha,\beta(\alpha\neq\beta)$ satisfy the equation $a\cos\theta+b\sin\theta=c$, then the value of $\tan\left(\frac{\alpha+\beta}{2}\right)$, is

a) b/a b) c/a c) a/b d) c/b

414. If the equation $\cos 3x+\cos 2x=\sin\frac{3x}{2}+\sin\frac{x}{2}, 0\leq x\leq 2\pi$, then the number of values of x is

a) 6 b) 7 c) 4 d) 5

415. If the sides of a triangle are the roots of the equation $x^3-2x^2-x-16=0$, then the product of the in-radius and circum-radius of the triangle is

a) 3 b) 6 c) 4 d) 2

416. The values of θ lying between $\theta=0$ and $\theta=\frac{\pi}{2}$ and satisfying the equation

$\begin{vmatrix}1+\cos^2\theta & \sin^2\theta & 4\sin 4\theta\\ \cos^2\theta & 1+\sin^2\theta & 4\sin 4\theta\\ \cos^2\theta & \sin^2\theta & 1+4\sin 4\theta\end{vmatrix}=0$, is

a) $\frac{11\pi}{24},\frac{7\pi}{24}$ b) $\frac{7\pi}{24},\frac{5\pi}{24}$ c) $\frac{5\pi}{24},\frac{\pi}{24}$ d) $\frac{\pi}{24},\frac{11\pi}{24}$

417. The value of $\sqrt{3}\cot 20°-4\cos 20°$ is

a) 1 b) -1 c) 0 d) None of these

418. The general solution of the equation $\cos x\cos 6x=-1$ is

a) $x=(2n+1)\pi, n\in Z$ b) $x=2n\pi, n\in Z$ c) $x=(2n-1)\pi, n\in Z$ d) None of these

419. The smallest angle of the triangle whose sides are $6+\sqrt{12},\sqrt{48},\sqrt{24}$ is

a) $\pi/3$ b) $\pi/4$ c) $\pi/6$ d) None of these

420. The general solution of the equation $\tan 2\theta.\tan\theta=1$ for $n\in Z$ is

a) $(2n+1)\frac{\pi}{4}$ b) $(2n+1)\frac{\pi}{6}$ c) $(2n+1)\frac{\pi}{2}$ d) $\frac{1}{1}(2n+1)\frac{\pi}{3}$

421. If $\cot(\alpha+\beta)=0$, then $\sin(\alpha+2\beta)$ is equal to

a) $\sin\alpha$ b) $\cos\alpha$ c) $\sin\beta$ d) $\cos 2\beta$

422. The value of $6(\sin^6\theta+\cos^6\theta)-9(\sin^4\theta+\cos^4\theta)+4$ is

a) -3 b) 0 c) 1 d) 3

423. If $y=\cos^2 x+\sec^2 x$, then

a) $y\leq 2$ b) $y\leq 1$ c) $y\geq 2$ d) $1<y<2$

424. If $0<x<\frac{\pi}{2}$, then the largest angle of a triangle whose sides are $1,\sin x,\cos x$ is

a) $\frac{\pi}{3}$ b) $\frac{\pi}{2}$ c) x d) $\frac{\pi}{2}-x$

425. In a ΔABC, $\cos\left(\frac{B+2C+3A}{2}\right)+\cos\left(\frac{A-B}{2}\right)$ is equal to

a) -1 b) 0 c) 1 d) 2

426. If $\cos 2\alpha=\frac{3\cos 2\beta-1}{3-\cos 2\beta}$, then $\tan\alpha$ is equal to

a) $\sqrt{2}\tan\beta$ b) $\tan\beta$ c) $\sin 2\beta$ d) $\sqrt{2}\cot\beta$

427. The value of the expression $\sin^6\theta+\cos^6\theta+3\sin^2\theta\cos^2\theta$ equals

a) 0 b) 2 c) 3 d) 1

428. $\tan^6\frac{\pi}{9}-33\tan^4\frac{\pi}{9}+27\tan^2\frac{\pi}{9}=$

a) 0 b) $\sqrt{3}$ c) 3 d) 9

429. General solution of $\sin x+\cos x=\min_{a\in R}\{1,a^2-4a+6\}$ is

a) $\frac{n\pi}{2}+(-1)^n\frac{\pi}{4}$ b) $2n\pi+(-1)^n\frac{\pi}{4}$

c) $n\pi+(-1)^{n+1}\frac{\pi}{4}$ d) $n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{4}$

430. The value of $\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{2\pi}{15}\cos\frac{8\pi}{15}$ is

a) $\frac{1}{16}$ b) $-\frac{1}{16}$ c) 1 d) 0

431. If $A+C=2B$, then $\frac{\cos C-\cos A}{\sin A-\sin C}=$

a) $\cot B$ b) $\cot 2B$ c) $\tan 2B$ d) $\tan B$

432. If $\sec\alpha$ and $\operatorname{cosec}\alpha$ are the roots of the equation $x^2-ax+b=0$, then

a) $a^2=b(b-2)$ b) $a^2=b(b+2)$ c) $a^2+b^2=2b$ d) None of these

433. The number of solutions of the equation $1+\sin x\sin^2\frac{x}{2}=0$ in $[-\pi,\pi]$ is

a) zero b) one c) two d) three

434. The solution of $\sin x+\sin 5x=\sin 3x$ in $\left(0,\frac{\pi}{2}\right)$

a) $\frac{\pi}{4},\frac{\pi}{10}$ b) $\frac{\pi}{6},\frac{\pi}{3}$ c) $\frac{\pi}{4},\frac{\pi}{2}$ d) $\frac{\pi}{8},\frac{\pi}{16}$

435. In a ΔABC, the HM of the ex-radii is equal to

a) $3r$ b) $2R$ c) $R+r$ d) None of these

436. The value of the expression $3(\sin x-\cos x)^4+4(\sin^6 x+\cos^6 x)+6(\sin x+\cos x)^2$ is

a) 10 b) 12 c) 13 d) None of these

437. If $4\cos\theta-3\sec\theta=2\tan\theta$, then θ is equal to

a) $n\pi+(-1)^n\frac{\pi}{10}$ b) $n\pi+(-1)^n\frac{\pi}{6}$ c) $n\pi+(-1)^n\frac{3\pi}{10}$ d) $n\pi$

438. In a $\Delta ABC, AD$ is the altitude from A. Given $b>c, \angle C=23°$ and $AD=\frac{abc}{b^2-c^2}$, then $\angle B$ is equal to

a) 53° b) 113° c) 87° d) None of these

439. If $\cos(x-y)$, $\cos x$ and $\cos(x+y)$ are in H.P., then $\left|\cos x\sec\frac{y}{2}\right|$ equals

a) 1 b) 2 c) $\sqrt{2}$ d) None of these

440. The maximum value of $\frac{1}{3\sin\theta-4\cos\theta+7}$, is

a) $\frac{1}{12}$ b) $\frac{5}{12}$ c) $\frac{7}{12}$ d) $\frac{1}{6}$

441. If $\frac{a^2+1}{2a}=\cos\theta$, then $\frac{a^6+1}{2a^3}=$

a) $\cos^2\theta$ b) $\cos^3\theta$ c) $\cos 2\theta$ d) $\cos 3\theta$

442. The value of $2\cos x-\cos 3x-\cos 5x$ is equal to

a) $16\cos^3 x\sin^2 x$ b) $16\sin^3 x\cos^2 x$ c) $4\cos^3 x\sin^2 x$ d) $4\sin^3 x\cos^2 x$

443. If $\tan\left(\frac{\alpha\pi}{4}\right)=\cot\left(\frac{\beta\pi}{4}\right)$, then

a) $\alpha+\beta=0$

b) $\alpha+\beta=2n$

c) $\alpha+\beta=2n+1$

d) $\alpha+\beta=2(2n+1), n\in Z$

444. The value of x in $\left(0,\frac{\pi}{2}\right)$ satisfying the equation $\sin x\cos x=\frac{1}{4}$ is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{8}$ d) $\frac{\pi}{12}$

445. If $\sin\theta+\operatorname{cosec}\theta=2$, then $\sin^2\theta+\operatorname{cosec}^2\theta$ is equal to

a) 1 b) 4 c) 2 d) None of these

446. If $A+C=2B$, then $\frac{\cos C-\cos A}{\sin A-\sin C}$ is equal to

a) $\cot B$ b) $\cot 2B$ c) $\tan 2B$ d) $\tan B$

447. If the median of ΔABC through A is perpendicular to AB, then

a) $\tan A + \tan B = 0$ b) $2\tan A + \tan B = 0$ c) $\tan A + 2\tan B = 0$ d) None of these

448. In a ΔABC, if $a = (b-c)\sec\theta$, then $\frac{2\sqrt{bc}}{b-c}\sin\frac{A}{2} =$

a) $\cos\theta$ b) $\cot\theta$ c) $\tan\theta$ d) $\sin\theta$

449. The value of $\cot 36° \cot 72°$ is

a) $\frac{1}{5}$ b) $\frac{1}{\sqrt{5}}$ c) 1 d) $\frac{1}{3}$

450. If θ lies in the first quadrant and $5\tan\theta = 4$, then $\frac{5\sin\theta - 3\cos\theta}{\sin\theta + 2\cos\theta}$ is equal to

a) $\frac{5}{14}$ b) $\frac{3}{14}$ c) $\frac{1}{14}$ d) 0

451. Consider the following statements :

1. If $\tan\alpha = \frac{m}{m+1}$, $\tan\beta = \frac{1}{2m+1}$, then $\alpha + \beta = \frac{\pi}{4}$
2. If $3\tan(\theta - 15°) = \tan(\theta + 15°)$, $0 < \theta < \pi$, then $\theta = \frac{\pi}{4}$
3. If $\sin^2 ax - \sin^2(a-1)x = \sin^2 x$, then x is equal to $\frac{n\pi}{a-1}$

Which of the statements given above are correct?

a) (1) and (2) b) (2) and (3) c) (3) and (1) d) All (1), (2) and (3)

452. If $\cos(A-B) = \frac{3}{5}$ and $\tan A \tan B = 2$, then

a) $\cos A \cos B = \frac{1}{5}$ b) $\sin A \sin B = -\frac{2}{5}$ c) $\cos(A+B) = -\frac{1}{5}$ d) None of these

453. If $\sin\theta = -\frac{4}{5}$ and θ lies in the third quadrant, then $\cos\frac{\theta}{2}$ is equal to

a) $\frac{1}{\sqrt{5}}$ b) $-\frac{1}{\sqrt{5}}$ c) $\sqrt{\frac{2}{5}}$ d) $-\sqrt{\frac{2}{5}}$

454. Let α, β be such that $\pi < \alpha - \beta < 3\pi$. If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\frac{(\alpha-\beta)}{2}$ is

a) $-\frac{3}{\sqrt{130}}$ b) $\frac{3}{\sqrt{130}}$ c) $\frac{6}{65}$ d) $-\frac{6}{65}$

455. If $\sin x + \sin^2 x = 1$, then $\cos^8 x + 2\cos^6 x + \cos^4 x =$

a) 0 b) -1 c) 2 d) 1

456. If $\cos(\alpha+\beta)\sin(\gamma+\delta) = \cos(\alpha-\beta)\sin(\gamma-\delta)$, then $\cot\alpha\cot\beta\cot\gamma$ is equal to

a) $\cot\delta$ b) $-\cot\delta$ c) $\tan\delta$ d) $-\tan\delta$

457. If AD, BE and CF are the medians of a ΔABC, then $(AD^2 + BE^2 + CF^2) : (BC^2 + CA^2 + AB^2)$ is equal to

a) 4 : 3 b) 3 : 2 c) 3 : 4 d) 2 : 3

458. The equation $a\cos\theta + b\sin\theta = c$ has a solution, when a, b and c are real numbers such that

a) $a < b < c$ b) $a = b = c$
c) $c^2 \le a^2 + b^2$ d) $a^2 \le a^2 - b^2$

459. If $\tan\left(\frac{\alpha\pi}{4}\right) = \cot\left(\frac{\beta\pi}{4}\right)$, then

a) $\alpha + \beta = 0$ b) $\alpha + \beta = 2n$
c) $\alpha + \beta = 2n + 1$ d) $\alpha + \beta = 2(2n+1)$, $\forall n$ is an integer

460. If $\frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} = x$, then $\frac{1-\cos\alpha-\sin\alpha}{\cos\alpha}$ is equal to

a) $\frac{1}{x}$ b) x c) $1-x$ d) None of these

461. The area of the circle and the area of a regular polygon of n sides and of perimeter equal to that of the circle are in the ratio of

a) $\tan\left(\frac{\pi}{n}\right) : \frac{\pi}{n}$ b) $\cos\left(\frac{\pi}{n}\right) : \frac{\pi}{n}$ c) $\sin\frac{\pi}{n} : \frac{\pi}{n}$ d) $\cot\left(\frac{\pi}{n}\right) : \frac{\pi}{n}$

462. In a triangle $ABC, \frac{a\cos A+b\cos B+c\cos C}{a+b+c}$ is equal to

a) $\frac{r}{R}$ b) $\frac{R}{r}$ c) $\frac{2r}{R}$ d) $\frac{R}{2r}$

463. From the identity $\sin 3x = 3\sin x - 4\sin^3 x$ it follows that if x is real and $|x| < 1$, then

a) $(3x - 4x^3) > 1$ b) $(3x - 4x^3) \leq 1$ c) $(3x - 4x^3) < 1$ d) None of these

464. If $\tan(\cot x) = \cot(\tan x)$, then $\sin 2x$ is equal to

a) $\frac{2}{(2n+1)\pi}$ b) $\frac{4}{(2n+1)\pi}$ c) $\frac{2}{n(n+1)\pi}$ d) $\frac{4}{n(n+1)\pi}$

465. If $\frac{\sin A}{\sin B} = \frac{\sqrt{3}}{2}$ and $\frac{\cos A}{\cos B} = \frac{\sqrt{5}}{2}, 0 < A, B < \frac{\pi}{2}$, then

a) $\tan A = \frac{\sqrt{3}}{\sqrt{5}}$ b) $\tan A = \frac{\sqrt{5}}{\sqrt{3}}$ c) $\tan A = 2$ d) $\tan B = 2$

466. The expression $\cos^2(A-B) + \cos^2 B - 2\cos(A-B)\cos A\cos B$ is

a) dependent on B b) dependent on A and B

c) dependent on A d) Independent of A and B

467. In a $\Delta ABC, \frac{r_1}{bc} + \frac{r_2}{ca} + \frac{r_3}{ab}$ is equal to

a) $\frac{1}{2R} - \frac{1}{r}$ b) $2R - r$ c) $r - 2R$ d) $\frac{1}{r} - \frac{1}{2R}$

468. If $\sin A + \sin B = a$ and $\cos A + \cos B = b$, then $\cos(A+B)$

a) $\frac{a^2+b^2}{b^2-a^2}$ b) $\frac{2ab}{a^2+b^2}$ c) $\frac{b^2-a^2}{a^2+b^2}$ d) $\frac{a^2-b^2}{a^2+b^2}$

469. The number of solutions of the equation $\tan x + \sec x = 2\cos x$ lying in the interval $[0, 2\pi]$ is

a) 0 b) 1 c) 2 d) 3

470. If $A + B + C = \pi, n \in Z$, then $\tan nA + \tan nB + \tan nC$ is equal to

a) 0 b) 1 c) $\tan nA \tan nB \tan nC$ d) None of these

471. If I is the incentre of a ΔABC, then $IA : IB : IC$ is equal to

a) $\operatorname{cosec}\frac{A}{2} : \operatorname{cosec}\frac{B}{2} : \operatorname{cosec}\frac{C}{2}$

b) $\sin\frac{A}{2} : \sin\frac{B}{2} : \sin\frac{C}{2}$

c) $\sec\frac{A}{2} : \sec\frac{B}{2} : \sec\frac{C}{2}$

d) None of these

472. The value of $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$ is

a) 1/16 b) 1/64 c) 1/128 d) 1/32

473. The general solution of : $\sin^2\theta\sec\theta + \sqrt{3}\tan\theta = 0$ is

a) $\theta = n\pi + (-1)^{n+1}\frac{\pi}{3}, \theta = n\pi, n \in Z$

b) $\theta = n\pi, n \in Z$

c) $\theta = n\pi + (-1)^{n+1}\frac{\pi}{3}, n \in Z$

d) $\theta = \frac{n\pi}{2}, n \in Z$

474. If in a $\Delta ABC, 2a = \sqrt{3}b + c$, then

a) $c^2 = a^2 + b^2 - ab$ b) $a^2 = b^2 + c^2$ c) $b^2 = a^2 + c^2 - \sqrt{3}\,ac$ d) None of these

475. The value of $\tan 1° \tan 2° \tan 3° \ldots \tan 89°$ is

a) 1 b) 0 c) ∞ d) 1/2

476. If $\tan\theta = \frac{a}{b}$, then $b\cos 2\theta + a\sin 2\theta =$

a) a b) b c) b/a d) a/b

477. The general solution of $|\sin x| = \cos x$ is (when $n \in I$) given by

a) $n\pi + \frac{\pi}{4}$ b) $2n\pi \pm \frac{\pi}{4}$ c) $n\pi \pm \frac{\pi}{4}$ d) $n\pi - \frac{\pi}{4}$

478. The expression $3\left\{\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi-\alpha)\right\}-2\left\{\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right\}$ is equal to

a) 0 b) 1 c) 3 d) $\sin 4\alpha+\cos 6\alpha$

479. The solution set of the equation
$4\sin\theta\cos\theta-2\cos\theta-2\sqrt{3}\sin\theta+\sqrt{3}=0$ in the interval $(0,2\pi)$ is

a) $\left\{\frac{3\pi}{4},\frac{7\pi}{4}\right\}$ b) $\left\{\frac{\pi}{3},\frac{5\pi}{3}\right\}$ c) $\left\{\frac{3\pi}{4},\frac{7\pi}{4},\frac{\pi}{3},\frac{5\pi}{3}\right\}$ d) $\left\{\frac{\pi}{6},\frac{5\pi}{6},\frac{11\pi}{6}\right\}$

480. $\sum a^3\cos(B-C)=$

a) $3\,abc$ b) $3(a+b+c)$ c) $abc(a+b+c)$ d) 0

481. If $\sin 6\theta+\sin 4\theta+\sin 2\theta=0$, then θ is equal to

a) $\frac{n\pi}{4}$ or $n\pi\pm\frac{\pi}{3}$ b) $\frac{n\pi}{4}$ or $n\pi\pm\frac{\pi}{6}$ c) $\frac{n\pi}{4}$ or $2n\pi\pm\frac{\pi}{6}$ d) None of these

482. Which of the following number is rational?

a) $\sin15°$ b) $\cos15°$ c) $\sin15°\cos15°$ d) $\sin15°\cos75°$

483. The value of $\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8}{15}\cos\frac{14\pi}{15}$ is

a) 1 b) 1/2 c) 1/4 d) 1/16

484. If $\tan\theta\tan(120°-\theta)\tan(120°+\theta)=\frac{1}{\sqrt{3}}$, then $\theta=$

a) $\frac{n\pi}{3}-\frac{\pi}{2}, n\in Z$ b) $\frac{n\pi}{3}-\frac{\pi}{18}, n\in Z$ c) $\frac{n\pi}{3}+\frac{\pi}{18}, n\in Z$ d) $\frac{n\pi}{3}+\frac{\pi}{12}, n\in Z$

485. The maximum value of $\sin\left(x+\frac{\pi}{6}\right)+\cos\left(x+\frac{\pi}{6}\right)$ in the interval $\left(0,\frac{\pi}{2}\right)$ is attained at

a) $x=\frac{\pi}{12}$ b) $x=\frac{\pi}{6}$ c) $x=\frac{\pi}{3}$ d) $x=\frac{\pi}{2}$

486. If $\tan\alpha=\frac{1}{\sqrt{x(x^2+x+1)}}$, $\tan\beta=\frac{\sqrt{x}}{\sqrt{x^2+x+1}}$ and $\tan\gamma=\sqrt{x^{-3}+x^{-2}+x^{-1}}$, then $\alpha+\beta$ is

a) γ b) 2γ c) $-\gamma$ d) None of these

487. ABC is a right angled isosceles triangle with $\angle B=90°$. If D is a point on AB so that $\angle CDB=15°$ and, if $AD=35$ cm, then CD equal to

a) $35\sqrt{2}$ cm b) $70\sqrt{2}$cm c) $\frac{35\sqrt{3}}{2}$ cm d) $35\sqrt{6}$ cm

488. If $4n\alpha=\pi$, then the value of
$\tan\alpha\cdot\tan 2\alpha\cdot\tan 3\alpha\cdot\tan 4\alpha\ldots\tan(2n-2)\alpha\ \tan(2n-1)\,\alpha$ is

a) 0 b) 1 c) -1 d) None of these

489. In a ΔABC if $r_1:r_2:r_3=2:4:6$, then $a:b:c=$

a) $3:5:7$ b) $1:2:3$ c) $5:8:9$ d) None of these

490. If in a ΔABC, $\cos A=\frac{\sin B}{2\sin C}$, then the ΔABC is

a) Equilateral b) Isosceles c) Right angled d) None of these

491. The number of pairs (x,y) satisfying the equations
$\sin x+\sin y=\sin(x+y)$ and $|x|+|y|=1$ is

a) 2 b) 4 c) 6 d) Infinite

492. The number of all possible ordered pairs $(x,y), x,y\in R$ satisfying the system of equations
$x+y=\frac{2\pi}{3}, \cos x+\cos y=\frac{3}{2}$, is

a) 0 b) 1 c) Infinite d) None of these

493. If $p=\sin^2 x+\cos^4 x$, then

a) $\frac{3}{4}\le p\le 1$ b) $\frac{3}{16}\le p\le\frac{1}{4}$ c) $\frac{1}{4}\le p\le 1$ d) None of these

494. $\tan 6°\tan 42°\tan 66°\tan 78°$ is equal to

a) 1 b) $\frac{1}{2}$ c) $\frac{1}{4}$ d) $\frac{1}{8}$

495. If $\cos x+\cos^2 x=1$, then the value of $\sin^{12}x+3\sin^{10}x+3\sin^8 x+\sin^6 x-1$, is equal to

a) 2 b) 1 c) -1 d) 0

496. The value of $\tan 20° + 2\tan 50° - \tan 70°$, is

a) 1 b) 0 c) $\tan 50°$ d) None of these

497. The value of $\tan \alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\alpha$ is equal to

a) $\tan \alpha$ b) $\tan 2\alpha$ c) $\cot \alpha$ d) $\cot 2\alpha$

498. If $\tan^2 \alpha + \tan^2 \beta + \tan^2 \beta + \tan^2 \gamma + \tan^2 \gamma \tan^2 \alpha + 2\tan^2 \alpha \tan^2 \beta \tan^2 \gamma = 1$, then the value of $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$ is

a) 0 b) -1 c) 1 d) ± 1

499. If $\sqrt{\frac{1+\cos A}{1-\cos A}} = \frac{x}{y}$, then the value of $\tan A$ is equal to

a) $\frac{x^2+y^2}{x^2-y^2}$ b) $\frac{2xy}{x^2+y^2}$ c) $\frac{2xy}{x^2-y^2}$ d) $\frac{2xy}{y^2-x^2}$

500. Let ABC be a triangle such that $\angle A = 45°, \angle B = 75°$, then $a + c\sqrt{2}$ is equal to

a) 0 b) b c) $2b$ d) $-b$

501. The minimum value of $f(x) = \sin^4 x + \cos^4 x, 0 \le x \le \frac{\pi}{2}$ is

a) $\frac{1}{2\sqrt{2}}$ b) $\frac{1}{4}$ c) $\frac{-1}{2}$ d) $\frac{1}{2}$

502. If θ is an acute angle and $\tan\theta = \frac{1}{\sqrt{7}}$, then the value of $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$ is

a) 3 /4 b) 1 /2 c) 2 d) 5 /4

503. The value of $\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{16\pi}{15}$, is

a) 0 b) 1 c) -1 d) 1/8

504. If $y = \sec^2\theta + \cos^2\theta, \theta \ne 0$, then

a) $y = 0$ b) $y \le 2$ c) $y \ge -2$ d) $y \ne 2$

505. If $\sec\theta = x + \frac{1}{4x}$, then $\sec\theta + \tan\theta =$

a) $x, \frac{1}{x}$ b) $2x, \frac{1}{2x}$ c) $-2x, \frac{1}{2x}$ d) $-\frac{1}{x}, x$

506. If $\frac{\sin\theta}{6}$, $\cos\theta$ and $\tan\theta$ are in GP, then the general value of θ is

a) $2n\pi \pm \frac{\pi}{3}, n \in I$ b) $2n\pi \pm \frac{\pi}{6}, n \in I$

c) $2n\pi + (-1)^n \frac{\pi}{3}, n \in I$ d) $n\pi + \frac{\pi}{3}, n \in I$

507. The number of roots of the equation $3\sin^2 x = 8\cos x$ in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is

a) 1 b) 2 c) 3 d) 4

508. The solution of the equation
$\log_{\cos x} \sin x + \log_{\sin x} \cos x = 2$ is given by

a) $x = 2n\pi + \frac{\pi}{4}, n \in Z$ b) $x = n\pi + \frac{\pi}{2}, n \in Z$ c) $x = n\pi + \frac{\pi}{8}, n \in Z$ d) $x = 2n\pi + \frac{\pi}{6}, n \in Z$

509. The value of $16\sin 144° \sin 108° \sin 72° \sin 36°$ is equal to

a) 5 b) 4 c) 3 d) 1

510. In $(0, \pi/2)$, $\tan^m x + \cot^m x$ attains
Which one of the above statement is correct?

a) A minimum value which is independent of m

b) A minimum value which is a function of m

c) The minimum value of 2

d) The minimum value at the some point independent of m

511. If $7\cos x - 24\sin x = \lambda\cos(x + \alpha), 0 < \alpha < \frac{\pi}{2}$, be true for all $x \in R$, then

a) $\lambda = 25$ b) $\alpha = \sin^{-1}\frac{21}{25}$ c) $\lambda = -25$ d) $\alpha = \cos^{-1}\frac{17}{25}$

512. Which one of the following is possible?

a) $\sin\theta = \dfrac{a^2+b^2}{a^2-b^2}, (a \neq b)$ b) $\sec\theta = \dfrac{4}{5}$
c) $\tan\theta = 45$ d) $\cos\theta = \dfrac{7}{3}$

513. If $y = (1+\tan A)(1-\tan B)$, where $A - B = \frac{\pi}{4}$, then $(y+1)^{y+1}$ is equal to
a) 9 b) 4 c) 27 d) 81

514. If $a\cos A = b\cos B$, then the triangle is
a) Equilateral
b) Right angled
c) Isosceles
d) Isosceles or right angled

515. If $-\pi \le x \le \pi, -\pi \le y \le \pi$ and $\cos x + \cos y = 2$, then the value of $\cos(x-y)$ is
a) -1 b) 0 c) 1 d) None of these

516. If $A + B + C = \pi$, then $\sin 2A + \sin 2B + \sin 2C =$
a) $4\sin A\sin B\sin C$ b) $4\cos A\cos B\cos C$ c) $4\cos A\cos B\cos C$ d) $2\sin A\sin B\sin C$

517. The maximum value of $4\sin^2 x + 3\cos^2 x + \sin\frac{x}{2} + \cos\frac{x}{2}$ is
a) $4+\sqrt{2}$ b) $3+\sqrt{2}$ c) 9 d) 4

518. If $\sin 3\theta = \sin\theta$, how many solutions exist such that $0 < \theta < 2\pi$?
a) 8 b) 9 c) 5 d) 7

519. The value of $\frac{\pi}{16}\sin\frac{3\pi}{16}\sin\frac{5\pi}{16}\sin\frac{7\pi}{16}$, is
a) $\dfrac{\sqrt{2}}{16}$ b) $\dfrac{1}{8}$ c) $\dfrac{1}{16}$ d) $\dfrac{\sqrt{2}}{32}$

520. $\tan 25° + \tan 20° + \tan 25° \tan 20°$ is equal to
a) 1 b) 2 c) 3 d) 4

521. If $\sin^4 x + \cos^4 y + 2 = 4\sin x\cos y$ and $0 \le x, y \le \frac{\pi}{2}$, then $\sin x + \cos y$ is equal to
a) -2 b) 0 c) 2 d) $\dfrac{3}{2}$

522. In a right angled $\Delta\, ABC$ $\sin^2 A + \sin^2 B + \sin^2 C =$
a) 0 b) 1 c) -1 d) None of these

523. If $x\cos\theta = y\cos\left(\theta + \frac{2\pi}{3}\right) = z\cos\left(\theta + \frac{4\pi}{3}\right)$, then the value of $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ is equal to
a) 1 b) 2 c) 0 d) $3\cos\theta$

524. If $A + B + C = \frac{3\pi}{2}$, then $\cos 2A + \cos 2B + \cos 2C$ is equal to
a) $1 - 4\cos A\cos B\cos C$ b) $4\sin A\sin B\sin C$
c) $1 + 2\cos A\cos B\cos C$ d) $1 - 4\sin A\sin B\sin C$

525. If $A + B + C = \pi (A, B, C > 0)$ and the angle C is obtuse, then
a) $\tan A\tan B > 1$ b) $\tan A\tan B < 1$ c) $\tan A\tan B = 1$ d) None of these

526. If $\sec\theta + \tan\theta = 1$, then root of the equation $(a - 2b + c)x^2 + (b - 2c + a)x + (c - 2a + b) = 0$ is
a) $\sec\theta$ b) $\tan\theta$ c) $\sin\theta$ d) $\cos\theta$

527. The area of the regular polygon of n sides is (where R is the radius of the circumpolygon),
a) $\dfrac{1}{2}R^2\sin\left(\dfrac{2\pi}{n}\right)$ b) $\dfrac{n}{2}R^2\sin\left(\dfrac{\pi}{n}\right)$ c) $\dfrac{n}{2}R\sin\left(\dfrac{2\pi}{n}\right)$ d) $\dfrac{nR^2}{2}\sin\left(\dfrac{2\pi}{n}\right)$

528. The number of all possible 5-tuples $(a_1, a_2, a_3, a_4, a_5)$ such that $a_1 + a_2\sin x + a_3\cos x + a_4\sin 2x + a5\cos 2x = 0$ holds for all x is
a) Zero b) 1 c) 2 d) Infinite

529. The value of $\frac{\tan 70° - \tan 20°}{\tan 50°}$ is equal to
a) 2 b) 1 c) 0 d) 3

530. If $\cos 20° = k$ and $\cos x = 2k^2 - 1$, then the possible values of x between 0° and 360° are
a) 140° and 270° b) 40° and 140° c) 40° and 320° d) 50° and 130°

531. The expression $\tan 9° - \tan 27° - \tan 63° + \tan 81°$ is equal to

a) 4 b) 3 c) 2 d) 1

532. If $\tan(\pi \cos\theta) = \cot(\pi \sin\theta)$, then the value(s) of $\cos\left(\theta - \frac{\pi}{4}\right)$ is,(are)

a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{2}}$ c) $\pm\frac{1}{2\sqrt{2}}$ d) None of these

533. If $b = 3, c = 4$ and $B = \pi/3$, then the number of triangles that can be constructed is

a) Infinite b) Two c) One d) Nil

534. In a $\Delta ABC, \sum(b + c)\tan\frac{A}{2}\tan\left(\frac{B-C}{2}\right) =$

a) a b) b c) c d) 0

535. If the sides of a triangle are 7 cm, $4\sqrt{3}$ cm and $\sqrt{13}$ cm, then the smallest angle of the triangle is

a) 15° b) 45° c) 30° d) None of these

536. Set $a, b \in [-\pi, \pi]$ be such that $\cos(a - b) = 1$ and $\cos(a + b) = \frac{1}{e}$. The number of pairs of a, b satisfying the above system of equations is

a) 0 b) 1 c) 2 d) 4

537. If $\tan(k + 1)\theta = \tan\theta$, then θ belongs to the set

a) $\{n\pi : n \in I\}$ b) $\{n\pi / 2 : n \in I\}$ c) $\{n\pi / k : n \in I\}$ d) $\{n\pi / 2k : n \in I\}$

538. Let $\theta \in (0, \pi/4)$ and $t_1 = (\tan\theta)^{\tan\theta}, t_2 = (\tan\theta)^{\cot\theta}, t_3 = (\cot\theta)^{\tan\theta}$ and $t_4 = (\cot\theta)^{\cot\theta}$. Then,

a) $t_1 > t_2 > t_3 > t_4$ b) $t_4 > t_3 > t_1 > t_2$ c) $t_3 > t_1 > t_2 > t_4$ d) $t_2 > t_3 > t_1 > t_4$

539. The value of $\cos 12° \cos 24° \cos 36° \cos 48° \cos 72° \cos 84°$, is

a) 1/64 b) 1/32 c) 1/16 d) 1/128

540. If $\tan A + \sin A = m$ and $\tan A - \sin A = n$, then $\frac{(m^2-n^2)^2}{mn}$ is equal to

a) 4 b) 3 c) 16 d) 9

541. If $A + B + C = 270°$, then
$\cos 2A + \cos 2B + \cos 2C$ is equal to

a) $4 \sin A \sin B \sin C$ b) $4 \cos A \cos B \cos C$
c) $1 - 4 \sin A \sin B \sin C$ d) $1 - 4 \cos A \cos B \cos C$

542. If $\frac{\sin A - \sin C}{\cos C - \cos A} = \cot B$, then A, B, C are in

a) AP b) GP c) HP d) None of these

543. If $\alpha + \beta + \gamma = 2\pi$, then

a) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

b) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2} + \tan\frac{\gamma}{2}\tan\frac{\alpha}{2} = 1$

c) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = -\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

d) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2} + \tan\frac{\gamma}{2}\tan\frac{\alpha}{2} = 0$

544. In a ΔABC, if the diameter of the incircle is $a + c - b$, then $\angle B =$

a) $\frac{\pi}{4}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{2}$ d) None of these

545. If $\sin^2\theta = \frac{x^2+y^2+1}{2x}$, then x must be

a) -3 b) -2 c) 1 d) None o f these

546. If $1 + \sin\theta + \sin^2\theta + \cdots$ to $\infty = 4 + 2\sqrt{3}, 0 < \theta < \pi, \theta \neq \frac{\pi}{2}$, then $\theta =$

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{3}$ or, $\frac{\pi}{6}$ d) $\frac{\pi}{3}$ or, $\frac{2\pi}{3}$

547. If $\cos 20° - \sin 20° = p$, then $\cos 40°$ is equal to

a) $p^2\sqrt{2 - p^2}$ b) $p\sqrt{2 - p^2}$ c) $p + \sqrt{2 - p^2}$ d) $p - \sqrt{2 - p^2}$

548. If $c^2 = a^2 + b^2, 2s = a + b + c$, then $4s(s - a)(s - b)(s - c) =$

a) s^4 b) b^2c^2 c) c^2a^2 d) a^2b^2

549. The value of $\tan 20° + 2\tan 50° - \tan 70°$ is

a) 1 b) 0 c) $\tan 50°$ d) None of these

550. The solutions of the equation $4\cos^2 x + 6\sin^2 x = 5$ are

a) $x = n\pi \pm \frac{\pi}{4}$ b) $x = n\pi \pm \frac{\pi}{3}$ c) $x = n\pi \pm \frac{\pi}{2}$ d) $x = n\pi \pm \frac{2\pi}{3}$

551. $\frac{\tan A}{1+\sec A} + \frac{1+\sec A}{\tan A}$ is equal to

a) $2\sin A$ b) $2\cos A$ c) $2\operatorname{cosec} A$ d) $2\sec A$

552. In ΔABC, if $\sin^2 A + \sin^2 B + \sin^2 C = 2$, then the triangle is

a) Right angled, but need not be isosceles b) Right angled and isosceles
c) Isosceles, but need not be right angled d) Equilateral

553. If $\cos\theta + \cos 2\theta + \cos 3\theta = 0$, the general value of θ is

a) $\theta = 2n\pi \pm \frac{\pi}{4}$ b) $\theta = n\pi + (-1)^n \frac{2\pi}{3}$ c) $\theta = n\pi + (-1)^n \frac{\pi}{3}$ d) $\theta = 2n\pi \pm \frac{2\pi}{3}$

554. If for real values of x, $\cos\theta = x + \frac{1}{x}$, then

a) θ is an acute angle b) θ is a right angle
c) θ is an obtuse angle d) No value of θ is possible

555. If $\sin x + \cos x = \frac{1}{5}$, then $\tan 2x$ is

a) $\frac{25}{17}$ b) $\frac{24}{7}$ c) $\frac{7}{25}$ d) $\frac{25}{7}$

556. If θ is an acute angle and $\sin\frac{\theta}{2} = \sqrt{\frac{x-1}{2x}}$, then $\tan\theta$ is equal to

a) $x^2 - 1$ b) $\sqrt{x^2 - 1}$ c) $\sqrt{x^2 + 1}$ d) $x^2 + 1$

557. In a ΔABC, a, b, A are given and c_2, c_2 are two values of the third side c. The sum of the areas of two triangles with sides a, b, c_1 and a, b, c_2 is

a) $(1/2)b^2 \sin 2A$ b) $(1/2)a^2 \sin 2A$ c) $b^2 \sin 2A$ d) None of these

558. The value of $\cos 1° \cos 2° \cos 3° \dots \cos 179°$ is

a) $\frac{1}{\sqrt{2}}$ b) 0 c) 1 d) None of these

559. In a right angled triangle the hypotenuse is $2\sqrt{2}$ times the length of perpendicular drawn from the opposite vertex on the hypotenuse, then the other two angles are

a) $\frac{\pi}{3}, \frac{\pi}{6}$ b) $\frac{\pi}{4}, \frac{\pi}{4}$ c) $\frac{\pi}{8}, \frac{3\pi}{8}$ d) $\frac{\pi}{12}, \frac{5\pi}{12}$

560. If $12\cot^2\theta - 31\operatorname{cosec}\theta + 32 = 0$, then the value of $\sin\theta$ is

a) $\frac{3}{5}$ or 1 b) $\frac{2}{3}$ or $-\frac{2}{3}$ c) $\frac{4}{5}$ or $\frac{3}{4}$ d) $\pm\frac{1}{2}$

561. If $T_n = \cos^n\theta + \sin^n\theta$, then $2T_6 - 3T_4 + 1 =$

a) 2 b) 3 c) 0 d) 1

562. The general value of θ satisfying the equation $2\sin^2\theta - 3\sin\theta - 2 = 0$ is

a) $n\pi + (-1)^{n+1}\frac{\pi}{6}$ b) $n\pi + (-1)^n \frac{\pi}{2}$ c) $n\pi + (-1)^n \frac{5\pi}{6}$ d) $n\pi + (-1)^n \frac{7\pi}{6}$

563. Which of the following statement is incorrect

a) $\sin\theta = -1/5$ b) $\cos\theta = 1$ c) $\sec\theta = 1/2$ d) $\tan\theta = 20$

564. The expression $3\left\{\sin^4\left(\frac{3\pi}{2} - \alpha\right) + \sin^4(3\pi - \alpha)\right\} - 2\left\{\sin^6\left(\frac{\pi}{2} + \alpha\right) + \sin^6(5\pi - \alpha)\right\}$ is equal to

a) 0 b) 1 c) 3 d) $\sin 4\alpha + \cos 6\alpha$

565. If $\cot x + \operatorname{cosec} x = \sqrt{3}$, then the principle value of $\left(x - \frac{\pi}{6}\right)$ is

a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{6}$

566. If the equation $\cos 3x \cos^3 x + \sin 3x \sin^3 x = 0$, then x is equal to

a) $(2n+1)\frac{\pi}{4}$ b) $(2n-1)\frac{\pi}{4}$ c) $\frac{n\pi}{4}$ d) None of these

567. In the ambiguous case, if a, b and A are given and c_1, c_2 are two values of the third side c, then $c_1^2 - 2c_1c_2 \cos 2A + c_2^2 =$

a) $4a^2 \cos^2 A$ b) $4a^2 \cos A$ c) $4a \cos^2 A$ d) None of these

568. If $A > 0, B > 0$ and $A + B = \frac{\pi}{3}$, then the maximum value of $\tan A \tan B$, is

a) $\frac{1}{3}$ b) 1 c) ∞ d) $\frac{1}{\sqrt{3}}$

569. The number of points of intersection of the curves $2y = 1$ and $y = \sin x, -2\pi \leq x \leq 2\pi$, is

a) 2 b) 3 c) 4 d) 1

570. The number of roots of the equation $x + 2\tan x = \frac{\pi}{2}$ in the interval $[0, 2\pi]$, is

a) 1 b) 2 c) 3 d) Infinite

571. In a ΔABC id $a = 2, b = \sqrt{6}, c = \sqrt{3} + 1$, then $\cos A =$

a) 30° b) 45° c) 60° d) None of these

572. The value of $\frac{\cot^2 \theta + 1}{\cot^2 \theta - 1}$ is equal to

a) $\sin 2\theta$ b) $\cos 2\theta$ c) $\text{cosec } 2\theta$ d) $\sec 2\theta$

573. The least value of $\text{cosec}^2 x + 25 \sec^2 x$ is

a) 0 b) 26 c) 28 d) 36

574. If $\cos(\theta - \alpha), \cos\theta, \cos(\theta + \alpha)$ are in H.P., then $\cos\theta \sec\left(\frac{\alpha}{2}\right)$ is equal to

a) -1 b) $\pm\sqrt{2}$ c) ± 2 d) ± 3

575. $\tan 5x \tan 3x \tan 2x =$

a) $\tan 5x - \tan 3x - \tan 2x$ b) $\frac{\sin 5x - \sin 3x - \sin 2x}{\cos 5x - \cos 3x - \cos 2x}$ c) 0 d) None of these

576. If $\sin x + \sin^2 x = 1$, then value of $\cos^2 x + \cos^4 x$ is

a) 1 b) 2 c) 1.5 d) None of these

577. The value of the expression $\sin^6 \theta + \cos^6 \theta + 3\sin^2 \theta . \cos^2 \theta$ equals

a) 0 b) 2 c) 3 d) 1

578. If $\cos\theta = \frac{8}{17}$ and θ lies in the first quadrant, then the value of $\cos(30° + \theta) + \cos(45° - \theta) + \cos(120° - \theta)$, is

a) $\frac{23}{17}\left(\frac{\sqrt{3}-1}{2} + \frac{1}{\sqrt{2}}\right)$ b) $\frac{23}{17}\left(\frac{\sqrt{3}+1}{2} + \frac{1}{\sqrt{2}}\right)$ c) $\frac{23}{17}\left(\frac{\sqrt{3}-1}{2} - \frac{1}{\sqrt{2}}\right)$ d) $\frac{23}{17}\left(\frac{\sqrt{3}+1}{2} - \frac{1}{\sqrt{2}}\right)$

579. The number of real solutions of $2\sin(e^x) = 5^x + 5^{-x}$ in $[0, 1]$ is /are

a) 0 b) 1 c) 2 d) 4

580. If the angles of a right angled triangle are in A.P., then the ratio of the in-radius and the perimeter is

a) $(2+\sqrt{3}) : 2\sqrt{3}$ b) $(2+\sqrt{3}) : \sqrt{3}$ c) $(2-\sqrt{3}) : 2\sqrt{3}$ d) $(2-\sqrt{3}) : 4\sqrt{3}$

581. If $2\sin^2 \theta + \sqrt{3}\cos\theta + 1 = 0$, then the value of θ is

a) $\frac{\pi}{6}$ b) $\frac{2\pi}{3}$ c) $\frac{5\pi}{6}$ d) π

582. The general solution of $e^{-1/\sqrt{2}}\left(e^{\sin x} + e^{\cos x}\right) = 2$ is

a) $x = m\pi$ b) $x = \frac{(4m+1)\pi}{4}$ c) $x = \frac{(4m+1)\pi}{2}$ d) None of these

583. The most general value of θ which satisfy both the equations $\cos\theta = -\frac{1}{\sqrt{2}}$ and $\tan\theta = 1$, is

a) $2n\pi + \frac{5\pi}{4}, n \in I$ b) $2n\pi + \frac{\pi}{4}, n \in I$ c) $2n\pi + \frac{3\pi}{4}, n \in I$ d) None of these

584. If $\alpha + \beta + \gamma = 2\theta$, then $\cos\theta + \cos(\theta - \alpha) + \cos(\theta - \beta) + \cos(\theta - \gamma)$ is equal to

a) $4\sin\frac{\alpha}{2}.\cos\frac{\beta}{2}.\sin\frac{\gamma}{2}$ b) $4\cos\frac{\alpha}{2}.\cos\frac{\beta}{2}.\cos\frac{\gamma}{2}$ c) $4\sin\frac{\alpha}{2}.\sin\frac{\beta}{2}.\sin\frac{\gamma}{2}$ d) $4\sin\alpha . \sin\beta . \sin\gamma$

585. The number of solutions of the equation $x^3 + x^2 + 4x + 2\sin x = 0$ in $0 \le x \le 2\pi$ is

a) Zero b) One c) Two d) Four

586. The value of $\frac{\tan 70° - \tan 20°}{\tan 50°} =$

a) 2 b) 1 c) 0 d) 3

587. The equation $\sin x + \sin y + \sin z = -3$ for $0 \le x \le 2\pi, 0 \le y \le 2\pi, 0 \le z \le 2\pi$ has

a) One solution b) Two sets of solutions

c) Four sets of solutions d) No solution

588. If $\tan A = \frac{1-\cos B}{\sin B}$, then

a) $\tan 2A = \tan B$ b) $\tan 2A = \tan^2 B$

c) $\tan 2A = \tan^2 B + 2\tan B$ d) None of the above

589. If α is an acute angle and $\sin\frac{\alpha}{2} = \sqrt{\frac{x-1}{2x}}$, then $\tan\alpha$ is

a) $\sqrt{\frac{x-1}{x+1}}$ b) $\frac{\sqrt{x-1}}{x+1}$ c) $\sqrt{x^2-1}$ d) $\sqrt{x^2+1}$

590. If $\sin\alpha + \sin\beta = a$ and $\cos\alpha + \cos\beta = b$, then $\sin(\alpha+\beta) =$

a) ab b) $a+b$ c) $\frac{2ab}{a^2-b^2}$ d) $\frac{2ab}{a^2+b^2}$

591. In $\tan\theta + \sec\theta = \sqrt{3}, 0 < \theta < \pi$, then θ is equal to

a) $5\pi/6$ b) $2\pi/3$ c) $\pi/6$ d) $\pi/3$

592. If $\pi < \theta < 2\pi$, then $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$ is equal to

a) $\operatorname{cosec}\theta + \cot\theta$ b) $\operatorname{cosec}\theta - \cot\theta$ c) $-\operatorname{cosec}\theta + \cot\theta$ d) $-\operatorname{cosec}\theta - \cot\theta$

593. If $a\sin^2 x + b\cos^2 x = c, b\sin^2 y + a\cos^2 y = d$ and $a\tan x = b\tan y$, then $\frac{a^2}{b^2}$ is equal to

a) $\frac{(b-c)(d-b)}{(a-d)(c-a)}$ b) $\frac{(a-d)(c-a)}{(b-c)(d-b)}$ c) $\frac{(d-a)(c-a)}{(b-c)(d-b)}$ d) $\frac{(b-c)(b-d)}{(a-c)(a-d)}$

594. The value of $\frac{\cos 12° - \sin 12°}{\cos 12° + \sin 12°} + \frac{\sin 147°}{\cos 147°}$ is equal to

a) 1 b) -1 c) 0 d) None of these

595. In a ΔABC, if $A = 30°, b = 2, c = \sqrt{3}+1$, then $\frac{C-B}{2} =$

a) 15° b) 30° c) 45° d) None of these

596. If the expression $\frac{\sin\frac{x}{2} + \cos\frac{x}{2} - i\tan x}{1 + 2i\sin\frac{x}{2}}$ is real, then x is equal to

a) $2n\pi + 2\tan^{-1} k, k \in R, n \in Z$

b) $2n\pi + 2\tan^{-1} k$, where $k \in (0,1), n \in Z$

c) $2n\pi + 2\tan^{-1} k$, where $k \in (1,2), n \in Z$

d) $2n\pi + 2\tan^{-1} k, k \in (2,3), n \in Z$

597. In a $\Delta ABC, a = 5, b = 4$ and $\cos(A-B) = \frac{31}{32}$, then side c is

a) 6 b) 7 c) 9 d) None of these

598. The value of $\tan 20° \tan 40° \tan 60° \tan 80°$ is equal to

a) 1 b) 2 c) 3 d) $\sqrt{\frac{3}{2}}$

599. If $5\cos x + 12\cos y = 13$, then the maximum value of $5\sin x + 12\sin y$ is

a) 12 b) $\sqrt{120}$ c) $\sqrt{20}$ d) 13

600. The minimum value of $f(x) = \sin^4 x + \cos^4 x, 0 \le x \le \frac{\pi}{2}$ is

a) $\frac{1}{2\sqrt{2}}$ b) $\frac{1}{4}$ c) $-\frac{1}{2}$ d) $\frac{1}{2}$

601. $\cot\theta = \sin 2\theta, \theta \ne n\pi, n \in Z$, if θ equals

a) $45°$ or $90°$ b) $45°$ or $60°$ c) $90°$ only d) $45°$ only

602. If $\cos\theta = \cos\alpha\cos\beta$, then $\tan\left(\frac{\theta+\alpha}{2}\right)\tan\left(\frac{\theta-\alpha}{2}\right)$ is equal to

a) $\tan^2\frac{\alpha}{2}$ b) $\tan^2\frac{\beta}{2}$ c) $\tan^2\frac{\theta}{2}$ d) $\cot^2\frac{\beta}{2}$

603. In $\Delta ABC, \angle A = \frac{\pi}{3}$ and $b : c = 2 : 3, \tan\theta = \frac{\sqrt{3}}{5}, 0 < \theta < \frac{\pi}{2}$, then

a) $B = 60° + \theta$ b) $C = 60° + \theta$ c) $B = 60° - \theta$ d) $C = 60° - \theta$

604. The value of $\cot 70° + 4\cos 70°$ is

a) $\frac{1}{\sqrt{3}}$ b) $\sqrt{3}$ c) $2\sqrt{3}$ d) $1/2$

605. The general solution of $\sin x - \cos x = \sqrt{2}$, for any integer n is

a) $n\pi$ b) $2n\pi + \frac{3\pi}{4}$ c) $2n\pi$ d) $(2n+1)\pi$

606. If $0 < \theta < \pi$, then $\sqrt{2+\sqrt{2+\sqrt{2+\cdots+\sqrt{2+2\cos\theta}}}}$ there being n number of 2's is equal to

a) $2\cos\frac{\theta}{2^n}$ b) $2\cos\frac{\theta}{2^{n-1}}$ c) $2\cos\frac{\theta}{2^{n+1}}$ d) None of these

607. If $(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C) = (\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$ then each side is equal to

a) 0 b) 1 c) -1 d) ± 1

608. The value of $\frac{\sin 55° - \cos 55°}{\sin 10°}$ is

a) $\frac{1}{\sqrt{2}}$ b) 2 c) 1 d) $\sqrt{2}$

609. If $\frac{\tan 3A}{\tan A} = a$, then $\frac{\sin 3A}{\sin A}$ is equal to

a) $\frac{2a}{a+1}$ b) $\frac{2a}{a-1}$ c) $\frac{a}{a+1}$ d) $\frac{a}{a-1}$

610. The value of $1 + \cos 56° + \cos 58° - \cos 66°$ is

a) $4\cos 28°\cos 29°\sin 33°$ b) $\cos 28°\cos 29°\sin 33°$ c) $4\cos 28°\sin 29°\cos 33°$ d) $4\cos 28°\sin 29°\sin 33°$

611. If $x\cos\alpha + y\sin\alpha = 2a, x\cos\alpha + y\sin\beta = 2a$ and $2\sin\frac{\alpha}{2}\sin\frac{\beta}{2} = 1$, then

a) $\cos\alpha + \cos\beta = \frac{2ax}{x^2+y^2}$

b) $\cos\alpha\cos\beta = \frac{2a^2-y^2}{x^2+y^2}$

c) $y^2 = 4a(\alpha - x)$

d) $\cos\alpha + \cos\beta = 2\cos\alpha\cos\beta$

612. The value of $\log\tan 1° + \log\tan 2° + \cdots + \log\tan 89°$, is

a) 0 b) -1 c) 1 d) ∞

613. The general value of θ in the equation $\cos\theta = \frac{1}{\sqrt{2}}, \tan\theta = -1$ is

a) $2n\pi \pm \frac{\pi}{6}, n \in I$ b) $2n\pi \pm \frac{7\pi}{4}, n \in I$ c) $n\pi + (-1)^n\frac{\pi}{3}, n \in I$ d) $n\pi + (-1)^n\frac{\pi}{4}, n \in I$

614. The number of solutions of the equation $\sin x = \cos 3x$ *in* $[0, \pi]$, is

a) 1 b) 2 c) 3 d) 4

615. If $\frac{\pi}{2} < \theta < \pi$, then $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} + \sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$ is equal to

a) $2\sec\theta$ b) $-2\sec\theta$ c) $\sec\theta$ d) $-\sec\theta$

616. If $\sin\theta = \sin 15° + \sin 45°$,where $0° < \theta < 90°$, then θ is equal to0

a) $45°$ b) $54°$ c) $60°$ d) $75°$

617. $\sinh^{-1} 2 + \sinh^{-1} 3 = x \Rightarrow \cosh x$ is equal to

a) $\frac{1}{2}(3\sqrt{5}+2\sqrt{10})$ b) $\frac{1}{2}(3\sqrt{5}-2\sqrt{10})$ c) $\frac{1}{2}(12+2\sqrt{50})$ d) $\frac{1}{2}(12-2\sqrt{50})$

618. The values of θ satisfying $\sin 7\theta = \sin 4\theta - \sin\theta$ and $0<\theta<\frac{\pi}{2}$ are
a) $\frac{\pi}{9},\frac{\pi}{4}$ b) $\frac{\pi}{3},\frac{\pi}{9}$ c) $\frac{\pi}{6},\frac{\pi}{9}$ d) $\frac{\pi}{3},\frac{\pi}{4}$

619. In a triangle ABC, the line joining the circumcentre to the incentre is parallel to BC, the $\cos B+\cos C=$
a) 3/2 b) 1 c) 3/4 d) 1/2

620. If $2\sin^2\theta = 3\cos\theta, 0\le\theta\le 2\pi$, then θ is equal to
a) $\frac{\pi}{6},\frac{5\pi}{6}$ b) $\frac{\pi}{3},\frac{2\pi}{3}$ c) $\frac{\pi}{3},\frac{5\pi}{3}$ d) $\frac{\pi}{2},\pi$

621. Solution of the equation $3\tan(\theta-15)=\tan(\theta+15)$ is
a) $\theta=n\pi-\frac{\pi}{3}$ b) $\theta=n\pi+\frac{\pi}{3}$ c) $\theta=n\pi-\frac{\pi}{4}$ d) $\theta=n\pi+\frac{\pi}{4}$

622. Number of solutions of $y=e^x$ and $y=\sin x$ is
a) 0 b) 1 c) 2 d) Infinite

623. The value of $\sin 10°+\sin 20°+\sin 30°+...+\sin 360°$ is equal to
a) 0 b) 1 c) $\sqrt{3}$ d) 2

624. For any angle θ, the expression $\frac{2\cos 8\theta+1}{2\cos\theta+1}$ is equal to
a) $(2\cos\theta+1)(2\cos 2\theta+1)(2\cos 4\theta+1)$ b) $(\cos\theta-1)(\cos 2\theta-1)(\cos 4\theta-1)$
c) $(2\cos\theta-1)(2\cos 2\theta-1)(2\cos 4\theta-1)$ d) $(2\cos\theta+1)(2\cos 2\theta+1)(2\cos 4\theta+1)$

625. If $\sec\alpha$ and $\text{cosec}\,\alpha$ are the root of the equation $x^2-px+q=0$,then
a) $p^2=q(q-2)$ b) $p^2=q(q+2)$ c) $p^2+q^2=2q$ d) None of these

626. The value of $\sin\frac{\pi}{7}\sin\frac{2\pi}{7}\sin\frac{3\pi}{7}$, is
a) 1/8 b) $\sqrt{7}/8$ c) $\sqrt{7}/2$ d) $\sqrt{7}/16$

627. If $\sin x+\sin^2 x=1$, then $\cos^{12}x+3\cos^{10}x+3\cos^8 x+\cos^6 x$ is equal to
a) 1 b) 2 c) 3 d) 0

628. If $A+C=B$, then $\tan A\tan B\tan C=$
a) $\tan A\tan B\tan C$
b) $\tan B-\tan C-\tan A$
c) $\tan A+\tan C-\tan B$
d) $-(\tan A\tan B+\tan C)$

629. If $\cos A+\cos B=m$ and $\sin A+\sin B=n$ where $m,n\ne 0$, then $\sin(A+B)$ is equal to
a) $\frac{mn}{m^2+n^2}$ b) $\frac{2mn}{m^2+n^2}$ c) $\frac{m^2+n^2}{2mn}$ d) $\frac{mn}{m+n}$

630. The solution set of $(5+4\cos\theta)(2\cos\theta+1)=0$ in the interval $[0,2\pi]$ is
a) $\left\{\frac{\pi}{3},\frac{2\pi}{3}\right\}$ b) $\left\{\frac{\pi}{3},\pi\right\}$ c) $\left\{\frac{2\pi}{3},\frac{4\pi}{3}\right\}$ d) $\left\{\frac{2\pi}{3},\frac{5\pi}{3}\right\}$

631. The equation $\sin^6 x+\cos^6 x=\lambda$, has a solution if
a) $\lambda\in[1/2,1]$ b) $\lambda\in[1/4,1]$ c) $\lambda\in[-1,1]$ d) $\lambda\in[0,1/2]$

632. If $y=\frac{\sin 3\theta}{\sin\theta}, \theta\ne n\pi$, then
a) $y\in[-1,3]$ b) $y\in[-\infty,-1]$ c) $y\in(3,\infty)$ d) $y\in[-1,3)$

633. If $\sin^2\theta=\frac{1}{4}$, then the most general value of θ is
a) $2n\pi\pm(-1)^n\frac{\pi}{6}$ b) $\frac{n\pi}{2}\pm(-1)^n\frac{\pi}{6}$ c) $n\pi\pm\frac{\pi}{6}$ d) $2n\pi\pm\frac{\pi}{6}$

634. Let n be odd integer. If $\sin n\theta=\sum_{r=0}^{n} b_r\sin^r\theta$ for every value of θ, then
a) $b_0=1, b_1=3$ b) $b_0=0, b_1=n$
c) $b_0=-1, b_1=n$ d) $b_0=0, b_1=n^2-3n+3$

635. If $k=\sin^6 x+\cos^6 x$, then k belongs to the interval
a) $[7/8,5/4]$ b) $[1/2,5/8]$ c) $[1/4,1]$ d) None of these

636. The value of $\frac{1}{\cos 290°}+\frac{1}{\sqrt{3}\sin 250°}$ is equal to

a) $\sqrt{3}/4$ b) $4/3$ c) $3/4$ d) $4/\sqrt{3}$

637. If $0\le x\le\pi$ and $81^{\sin^2 x}+81^{\cos^2 x}=30$, then x is equal to

a) $\frac{\pi}{6},\frac{\pi}{3}$ b) $\frac{\pi}{3},\frac{\pi}{2}$ c) $\frac{5\pi}{6},\frac{\pi}{3}$ d) $\frac{2\pi}{3},\frac{\pi}{3}$

638. The value of $\cos 9°-\sin 9°$ is

a) $\frac{5+\sqrt{5}}{4}$ b) $\frac{\sqrt{5-\sqrt{5}}}{2}$ c) $-\frac{\sqrt{5-\sqrt{5}}}{2}$ d) None of these

639. $\operatorname{sech}^{-1}\left(\frac{1}{2}\right)$ is

a) $\log(\sqrt{3}+\sqrt{2})$ b) $\log(\sqrt{3}+1)$ c) $\log(2+\sqrt{3})$ d) None of above

640. If a triangle is right angled at B, then the diameter of the incircle of the triangle is

a) $c+a-b$ b) $2(c+a-b)$ c) $c+a-2b$ d) $c+a+2b$

641. Let $\theta\in\left(0,\frac{\pi}{4}\right)$ and $t_1=(\tan\theta)^{\tan\theta}$, $t_2=(\tan\theta)^{\cot\theta}$, $t_3=(\cot\theta)^{\tan\theta}$ and $t_4=(\cot\theta)^{\tan\theta}$, then

a) $t_1>t_2>t_3>t_4$ b) $t_4>t_3>t_1>t_2$ c) $t_3>t_1>t_2>t_4$ d) $t_2>t_3>t_1>t_4$

642. If $\frac{\tan\alpha+\tan\beta}{\cot\alpha+\cot\beta}+\{\cos(\alpha-\beta)\sec(\alpha+\beta)+1\}^{-1}=1$, then $\tan\alpha\tan\beta$ is equal to

a) 1 b) -1 c) 2 d) -2

643. The value of the expression $3(\sin\theta-\cos\theta)^4+6(\sin\theta+\cos\theta)^2+4(\sin^6\theta+\cos^6\theta)$ is

a) 1 b) -1 c) 13 d) 0

644. One root of the equation $\cos\theta-\theta+\frac{1}{2}=0$ lies in the interval

a) $(0,\pi/2)$ b) $(-\pi/2,0)$ c) $(\pi/2,\pi)$ d) $(\pi,3\pi/2)$

645. If $\alpha-22°30'$, then $(1+\cos\alpha)(1+\cos 3\alpha)\times(1+\cos 5\alpha)(1+\cos 7\alpha)$ equals

a) $\frac{1}{8}$ b) $\frac{1}{4}$ c) $\frac{1+\sqrt{2}}{2\sqrt{2}}$ d) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$

646. The value of $\sin A\sin(60°-A)\sin(60°+A)$ is equal to

a) $\sin 3A$ b) $\frac{\sin 3A}{2}$ c) $\frac{\sin 3A}{4}$ d) None of these

647. If $A+B+C=\pi$, then $\sin 2A+\sin 2B+\sin 2C$ is equal to

a) $4\sin A\sin B\sin C$ b) $4\cos A\cos B\cos C$ c) $2\cos A\cos B\cos C$ d) $2\sin A\sin B\sin C$

648. If $x=h+a\sec\theta$ and$y=k+b\operatorname{cosec}\theta$. Then,

a) $\frac{a^2}{(x+h)^2}-\frac{b^2}{(y+k)^2}=1$ b) $\frac{a^2}{(x-h)^2}+\frac{b^2}{(y-k)^2}=1$

c) $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ d) $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

649. The maximum of the function $3\cos x-4\sin x$ is

a) 2 b) 3 c) 4 d) 5

650. The equation $3\sin^2 x+10\cos x-6=0$ is satisfied, if

a) $x=n\pi\pm\cos^{-1}\left(\frac{1}{3}\right)$ b) $x=2n\pi\pm\cos^{-1}\left(\frac{1}{3}\right)$ c) $x=n\pi\pm\cos^{-1}\left(\frac{1}{6}\right)$ d) $x=2n\pi\pm\cos^{-1}\left(\frac{1}{6}\right)$

651. The number of solutions of the equation $1+\sin x\sin^2\frac{x}{2}=0$, in $[-\pi,\pi]$ is

a) Zero b) One c) Two d) Three

652. The root of the equation $1-\cos\theta=\sin\theta.\sin\frac{\theta}{2}$ is

a) $k\pi, k\in I$ b) $2k\,\pi, k\in I$ c) $k\frac{\pi}{2}, k\in I$ d) None of these

653. The maximum value of $\sin\left(x+\frac{\pi}{6}\right)+\cos\left(x+\frac{\pi}{6}\right)$ in the interval $\left(0,\frac{\pi}{2}\right)$ is attained at

a) $x=\frac{\pi}{12}$ b) $x=\frac{\pi}{6}$ c) $x=\frac{\pi}{3}$ d) $x=\frac{\pi}{2}$

654. The values of α for which the equation $\sin^4 x+\cos^4 x+\sin 2x+\alpha=0$ may be valid, are

a) $-\frac{3}{2} \leq \alpha \leq 1$ b) $0 \leq \alpha \leq \frac{1}{2}$ c) $-\frac{3}{2} \leq \alpha \leq \frac{1}{2}$ d) None of these

655. The value of $\sin^2 5° + \sin^2 10° + \sin^2 15° + \cdots + \sin^2 85° + \sin^2 90°$, is

a) $7\frac{1}{2}$ b) $8\frac{1}{2}$ c) $9\frac{1}{2}$ d) None of these

656. If $1 + \cos x = k$, where x is acute, then $\sin\frac{x}{2}$ is

a) $\sqrt{\frac{1-k}{2}}$ b) $\sqrt{2-k}$ c) $\sqrt{\frac{2+k}{2}}$ d) $\sqrt{\frac{2-k}{2}}$

657. If in a triangle $ABC, \frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$, then the triangle is

a) Right angled b) Obtuse angled c) Equilateral d) Isosceles

658. In a ΔABC if $c = (a+b)\sin\theta$ and $\cos\theta = \frac{k\sqrt{ab}}{a+b}$, then $k =$

a) $2\cos\frac{C}{2}$ b) $2\cos\frac{B}{2}$ c) $2\cos\frac{A}{2}$ d) $\cos\frac{C}{2}$

659. The value of $\cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{5\pi}{7} + \cos\frac{6\pi}{7} + \cos\frac{7\pi}{7}$, is

a) 1 b) -1 c) 0 d) -2

660. The number of ordered pairs (x, y) satisfying $y = 2\sin x$ and $y = 5x^2 + 2x + 3$ is

a) 0 b) 1 c) 2 d) ∞

661. If the angle of a triangle are in A.P. with common difference equal $\frac{1}{3}$ of the least angle, then the sides are in the ratio

a) $\sqrt{2} : 2\sqrt{3} : \sqrt{6} + \sqrt{2}$

b) $2\sqrt{2} : \sqrt{3} : \sqrt{6} - \sqrt{2}$

c) $2\sqrt{2} : 2\sqrt{3} : \sqrt{6} - \sqrt{2}$

d) $2\sqrt{2} : 2\sqrt{3} : \sqrt{6} + \sqrt{2}$

662. If $\text{Max}_{x \in R}\ \{5\sin x + 3\sin(x-\theta)\} = 7$, then $\theta =$

a) $2\,n\,\pi \pm \frac{\pi}{3}, n \in Z$ b) $2\,n\pi \pm \frac{2\,\pi}{3}, n \in Z$ c) $\frac{\pi}{3}, \frac{2\,\pi}{3}$ d) None of these

663. If $\sin\theta + \cos\theta = m$ and $\sec\theta + \operatorname{cosec}\theta = n$, then $n(m+1)(m-1)$ is equal to

a) m b) n c) $2m$ d) $2n$

664. If $\tan^2\theta - (1+\sqrt{3})\tan\theta + \sqrt{3} = 0$, then the general value of θ is

a) $n\pi + \frac{\pi}{4}, n\pi + \frac{\pi}{3}$ b) $n\pi - \frac{\pi}{4}, n\pi + \frac{\pi}{3}$ c) $n\pi + \frac{\pi}{4}, n\pi - \frac{\pi}{3}$ d) $n\pi - \frac{\pi}{4}, n\pi - \frac{\pi}{3}$

665. If $\frac{x}{\cos\theta} = \frac{y}{\cos\left(\theta - \frac{2\pi}{3}\right)} = \frac{z}{cos\left(\theta + \frac{2\pi}{3}\right)}$, then $x + y + z$ is equal to

a) 1 b) 0 c) -1 d) None of these

666. The value of
$e^{\log_{10}\tan 1° + \log_{10}\tan 2° + \log_{10} 3° + \cdots + \log_{10}\tan 89°}$, is

a) 0 b) e c) $1/e$ d) 1

667. If $\sin^3 x \sin 3x = \sum_{m=0}^{n} c_m\ \cos mx$, where $c_0, c_1, c_2, \dots, c_n$ are constants and $c_n \neq 0$, then the value of n is

a) 15 b) 6 c) 1 d) 0

668. The value of $\tan 67\frac{1°}{2} + \cot 67\frac{1°}{2}$ is

a) $\sqrt{2}$ b) $\frac{2}{3\sqrt{2}}$ c) $2\sqrt{2}$ d) $2-\sqrt{2}$

669. In any ΔABC, $\Pi\left(\frac{\sin^2 A + \sin A + 1}{\sin A}\right)$ is always greater than

a) 9 b) 3 c) 27 d) None of these

670. The number of solutions of the equation $2\cos(e^x) = 5^x + 5^{-x}$ are

a) No solution b) One solution

c) Two solutions d) Infinity many solutions

671. If A_1, A_2, A_3 denote respectively the areas of an inscribed polygon of $2n$ sides, inscribed polygon of n sides and circumscribed polygon of n sides, then A_2, A_1, A_3 are in

a) A.P. b) G.P. c) H.P. d) None of these

672. The most general values of θ satisfying $\tan\theta + \tan\left(\frac{3\pi}{4} + \theta\right) = 2$ is/are

a) $n\pi \pm \frac{\pi}{3}, n \in I$ b) $2n\pi + \frac{\pi}{3}, n \in I$ c) $2n\pi \pm \frac{\pi}{3}, n \in I$ d) $2n\pi + (-1)^n \frac{\pi}{3}, n \in I$

673. The value of $\frac{\sin(B+A) + \cos(B-A)}{\sin(B-A) + \cos(B+A)}$ is equal to

a) $\frac{\cos B + \sin B}{\cos B - \sin B}$ b) $\frac{\cos A + \sin A}{\cos A - \sin A}$ c) $\frac{\cos A - \sin A}{\cos A + \sin A}$ d) None of these

674. The maximum value of $3\cos x + 4\sin x + 5$ is

a) 5 b) 6 c) 7 d) None of these

675. $\sin A + \sin B = \sqrt{3}(\cos B - \cos A) \Rightarrow \sin 3A + \sin 3B$ is equal to

a) 0 b) 2 c) 1 d) -1

676. $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x)$ is equal to

a) 11 b) 12 c) 13 d) 14

677. If $m\tan(\theta - 30°) = n\tan(\theta + 120°)$, then $\cos 2\theta$ equals

a) $\frac{m+n}{m-n}$ b) $\frac{m-n}{m+n}$ c) $\frac{m-n}{2(m+n)}$ d) $\frac{m+n}{2(m-n)}$

678. $\sqrt{3} cosec\ 20° - \sec 20°$ is equal to

a) 2 b) $2\sin 20°.\text{cosec } 40°$ c) 4 d) $4\sin 20°.\text{cosec } 40°$

679. If $\left|\cos\theta\left\{\sin\theta + \sqrt{\sin^2\theta + \sin^2\alpha}\right\}\right| \le k$, then the value of k is

a) $\sqrt{1+\cos^2\alpha}$ b) $\sqrt{1+\sin^2\alpha}$ c) $\sqrt{2+\sin^2\alpha}$ d) $\sqrt{2+\cos^2\alpha}$

680. General value of θ satisfying the equation $\tan^2\theta + \sec 2\theta = 1$ is

a) $m\pi, n\pi + \frac{\pi}{3}$ b) $m\pi, n\pi \pm \frac{\pi}{3}$ c) $m\pi, n\pi \pm \frac{\pi}{6}$ d) None of these

681. If $2\sec 2\alpha = \tan\beta + \cot\beta$, then one of the values of $\alpha + \beta$ is

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) π d) $n\pi - \frac{\pi}{4}, n \in I$

682. The number of solutions of the equation $\tan x + \sec x = 2\cos x$ and $\cos x \ne 0$ lying in the interval $(0, 2\pi)$ is

a) 2 b) 1 c) 0 d) 3

683. The value of $\tan 1° \tan 2° \tan 3° \dots \tan 89°$ is equal to

a) -1 b) 2 c) $\frac{\pi}{2}$ d) 1

684. The value of $\sin 12° \sin 24° \sin 48° \sin 84°$, is

a) $\cos 20° \cos 40° \cos 60° \cos 80°$

b) $\sin 20° \sin 40° \sin 60° \sin 80°$

c) 3/15

d) None of these

685. The most general solution of the equation

$8\tan^2\frac{\theta}{2} = 1 + \sec\theta$, is

a) $\theta = 2n\pi \pm \cos^{-1}\left(\frac{1}{3}\right)$

b) $\theta = 2n\pi \pm \frac{\pi}{6}$

c) $\theta = 2n\pi \pm \cos^{-1}\left(\frac{-1}{3}\right)$

d) None of these

686. If $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$, then $\frac{\tan x}{\tan y}$ is equal to

a) $\frac{b}{a}$ b) $\frac{a}{b}$ c) ab d) None of these

687. If $\alpha+\beta+\gamma=2\pi$, then

a) $\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}+\tan\frac{\gamma}{2}=\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$
b) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}+\tan\frac{\beta}{2}\tan\frac{\gamma}{2}+\tan\frac{\gamma}{2}\tan\frac{\alpha}{2}=1$
c) $\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}+\tan\frac{\gamma}{2}=-\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$
d) None of the above

688. The most general solution of $\tan\theta=-1, \cos\theta=\frac{1}{\sqrt{2}}$ is

a) $n\,\pi+\frac{7\,\pi}{4}, n\in Z$
b) $n\,\pi+(-1)^n\frac{7\,\pi}{4}, n\in Z$
c) $2n\,\pi+\frac{7\,\pi}{4}, n\in Z$
d) None of these

689. In a ΔABC if $C=60°$, then $\frac{a}{b+c}+\frac{b}{c+a}=$

a) 2 b) 4 c) 3 d) 1

690. If x lies in IInd quadrant, then $\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}$ is equal to

a) $\sin\frac{x}{2}$ b) $\tan\frac{x}{2}$ c) $\sec\frac{x}{2}$ d) $\operatorname{cosec}\frac{x}{2}$

691. If $\cos(\alpha+\beta)=\frac{4}{5}, \sin(\alpha-\beta)=\frac{5}{13}$ and α,β lie between 0 and $\frac{\pi}{4}$, then $\tan 2\alpha=$

a) $\frac{56}{33}$ b) $\frac{33}{56}$ c) $\frac{16}{65}$ d) $\frac{60}{61}$

692. If $\sin\theta+\operatorname{cosec}\theta=2$, then $\sin^2\theta+\operatorname{cosec}^2\theta$ is equal to

a) 1 b) 4 c) 2 d) None of these

693. If $\sin 6\,\theta+\sin 4\,\theta+\sin 2\theta=0$,then the general value of θ *is*

a) $\frac{n\pi}{4}, n\pi\pm\frac{\pi}{3}$ b) $\frac{n\pi}{4}, n\pi\pm\frac{\pi}{6}$ c) $\frac{n\pi}{4}, 2n\pi\pm\frac{\pi}{3}$ d) $\frac{n\pi}{4}, 2n\pi\pm\frac{\pi}{6}$

694. The general solution of $\sin x-3\sin 2x+\sin 3x=\cos x-3\cos 2x+\cos 3x$ is

a) $n\pi+\frac{\pi}{8}$ b) $\frac{n\pi}{2}+\frac{\pi}{8}$ c) $(-1)^n\frac{n\pi}{2}+\frac{\pi}{8}$ d) $2n\pi+\cos^{-1}\frac{3}{2}$

695. The equation $2\cos^2\frac{x}{2}\sin^2 x=x^2+^{-2}, x<\frac{\pi}{9}$ has

a) No real solution
b) One real solution
c) More than one real solutions
d) None of the above

696. If the angles of a triangle are 30° and 45° and the included side is $(\sqrt{3}+1)$cms, then the area of the triangle is

a) $\frac{1}{\sqrt{3}-1}$ b) $\sqrt{3}+1$ c) $\frac{1}{\sqrt{3}+1}$ d) None of these

697. If n is any integer, then the general solution of the equation $\cos\theta-\sin\theta=\frac{1}{\sqrt{2}}$ is

a) $\theta=2n\pi-\frac{\pi}{12}$ or $\theta=2n\pi+\frac{7\pi}{12}$
b) $\theta=n\pi+\frac{\pi}{12}$
c) $\theta=2n\pi+\frac{\pi}{12}$ or $\theta=2n\pi-\frac{7\pi}{12}$
d) $\theta=2n\pi+\frac{\pi}{12}$ or $\theta=2n\pi+\frac{7\pi}{12}$

698. We are given b, c and $\sin B$ such that B is acute and $b<c\sin B$. Then,

a) No triangle is possible
b) One triangle is possible
c) Two triangles are possible
d) A right-angled triangle is possible

699. The value of $\cot^2\frac{\pi}{9}+\cot^2\frac{2\pi}{9}+\cot^2\frac{4\pi}{9}$, is

a) 0 b) 3 c) 9 d) 1/3

700. If $\tan x+\cot x=2$, then $\sin^{2n}x+\cos^{2n}x$ is equal to

a) 2^n b) $-\frac{1}{2}$ c) $\frac{1}{2}$ d) None of these

701. The most general value of θ which satisfies both the equations $\tan\theta = -1$ and $\cos\theta = 1/\sqrt{2}$ will be

a) $n\pi + \frac{7\pi}{4}$ b) $n\pi + (-1)^n \frac{7\pi}{4}$ c) $2n\pi + \frac{7\pi}{4}$ d) None of these

702. If $\sin x + \sin^2 x = 1$, then the value of $\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x - 1$ is equal to

a) 2 b) 1 c) 0 d) -1

703. The side of a triangle are $3x + 4y, 4x + 3y$ and $5x + 5y$ units, where $x, y > 0$. The triangle is

a) Right angled b) Equilateral c) Obtuse angled d) None of these

704. If the sides of a triangle are $x^2 + x + 1, x^2 - 1, 2x + 1$, where $x > 1$, then the largest angle is

a) 120° b) 60° c) 40° d) 30°

705. If p_1, p_2, p_3 are altitudes of a triangle ABC from the vertices A, B, C and Δ, the area of the triangle, then $p_1^{-1} + p_2^{-1} - p_3^{-1}$ is equal to

a) $\frac{s-a}{\Delta}$ b) $\frac{s-b}{\Delta}$ c) $\frac{s-c}{\Delta}$ d) $\frac{s}{\Delta}$

706. In a ΔABC if $a = 26, b = 30$ and $\cos C = \frac{63}{65}$, then $r_2 =$

a) 84 b) 45 c) 48 d) 24

707. The value of $\cos 1° \cos 2° \cos 3° \dots \cos 100°$ is equal to

a) 1 b) -1 c) 0 d) None of these

708. The value of $\sin\frac{\pi}{2} + \sin\frac{2\pi}{7} + \sin\frac{3\pi}{7}$ is

a) $\cot\frac{\pi}{14}$ b) $\frac{1}{2}\cot\frac{\pi}{14}$ c) $\tan\frac{\pi}{14}$ d) $\frac{1}{2}\tan\frac{\pi}{14}$

709. The value of x for the maximum value of $\sqrt{3}\cos x + \sin x$, is

a) 30° b) 45° c) 60° d) 90°

710. $\sin^2 17.5° + \sin^2 72.5°$ is equal to

a) $\cos^2 90°$ b) $\tan^2 45°$ c) $\cos^2 30°$ d) $\sin^2 45°$

711. If in $\Delta\, ABC, a\sin A = b\sin B$, then the triangle is

a) Isosceles b) Right angled c) Equilateral d) None of these

712. $\sin^2\theta = \frac{4xy}{(x+y)^2}$ is true if and only if

a) $x + y \neq 0$ b) $x = y, x \neq 0, y \neq 0$ c) $x = y$ d) $x \neq 0, y \neq 0$

713. If $\cos\theta = \frac{1}{2}\left(x + \frac{1}{x}\right)$, then $\frac{1}{2}\left(x^2 + \frac{1}{x^2}\right)$ is equal to

a) $\sin 2\theta$ b) $\cos 2\theta$ c) $\tan 2\theta$ d) None of these

714. $\text{sech}^{-1}(\sin\theta)$ is equal to

a) $\log\tan\frac{\theta}{2}$ b) $\log\sin\frac{\theta}{2}$ c) $\log\cos\frac{\theta}{2}$ d) $\log\cot\frac{\theta}{2}$

715. The number of solutions of the equation $2^{\cos x} = |\sin x|$ in $[-2\pi, 2\pi]$, is

a) 1 b) 2 c) 3 d) 4

716. If the equation $\cos(\lambda\sin\theta) = \sin(\lambda\cos\theta)$ has a solution in $[0, 2\pi]$, then the smallest positive value of λ is

a) $\frac{\pi}{\sqrt{2}}$ b) $\sqrt{2}\,\pi$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{2\sqrt{2}}$

717. In the ambiguous case, given a, b and A. The difference between the two values of C is

a) $2\sqrt{a^2 - b^2}$ b) $\sqrt{a^2 - b^2\sin^2 A}$ c) $2\sqrt{a^2 - b^2\sin^2 A}$ d) $\sqrt{a^2 - b^2}$

718. If $\tan\alpha = (1 + 2^{-x})^{-1}, \tan\beta = (1 + 2^{x+1})^{-1}$, then $\alpha + \beta$ equals

a) $\pi/6$ b) $\pi/4$ c) $\pi/3$ d) $\pi/2$

719. The maximum value of $f(x) = \sin x(1 + \cos x)$ is

a) $\frac{3\sqrt{3}}{4}$ b) $\frac{3\sqrt{3}}{2}$ c) $3\sqrt{3}$ d) $\sqrt{3}$

720. The value of $\cos\frac{\pi}{11} + \cos\frac{3\pi}{11} + \cos\frac{5\pi}{11} + \cos\frac{7\pi}{11} + \cos\frac{9\pi}{11}$, is

a) 0 b) $\frac{-1}{2}$ c) $\frac{1}{2}$ d) 1

721. $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$ is equal to

a) $\frac{1}{2}$ b) $\cos\frac{\pi}{8}$ c) $\frac{1}{8}$ d) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

722. If $2\sin\frac{A}{2}=\sqrt{1+\sin A}+\sqrt{1-\sin A}$, then $\frac{A}{2}$ lies between

a) $2n\pi+\frac{\pi}{4}$ and $2n\pi+\frac{3\pi}{4}, n\in Z$

b) $2n\pi-\frac{\pi}{4}$ and $2n\pi+\frac{\pi}{4}, n\in Z$

c) $2n\pi-\frac{3\pi}{4}$ and $2n\pi-\frac{\pi}{4}, n\in Z$

d) $-\infty$ and $+\infty$

723. In a ΔABC, if $a\cos^2\frac{C}{2}+c\cos^2\frac{A}{2}=\frac{3b}{2}$, then a, b, c are in

a) A.P. b) G.P. c) H.P. d) None of these

724. The value of $\tan 5\theta$ is

a) $\frac{5\tan\theta-10\tan^3\theta+\tan^5\theta}{1-10\tan^2\theta+5\tan^4\theta}$

b) $\frac{5\tan\theta+10\tan^3\theta-\tan^5\theta}{1+10\tan^2\theta-5\tan^4\theta}$

c) $\frac{5\tan^5\theta-10\tan^3\theta+\tan\theta}{1-10\tan^2\theta+5\tan^4\theta}$

d) None of these

725. If the sides a, b and c of a ΔABC are in A.P., then $\left(\tan\frac{A}{2}+\tan\frac{C}{2}\right):\cot\frac{B}{2}$, is

a) 3 : 2 b) 1 : 2 c) 3 : 4 d) None of these

726. If in a triangle ABC

$$2\frac{\cos A}{a}+\frac{\cos B}{b}+2\frac{\cos C}{c}=\frac{a}{bc}+\frac{b}{ca},$$

then the value of the angle A is

a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{6}$

727. The value of $\tan\alpha+2\tan(2\alpha)+4\tan(4\alpha)+\ldots+2^{n-1}\tan(2^{n-1}\alpha)+2^n\cot(2^n\alpha)$ is

a) $\cot(2^n\alpha)$ b) $2^n\tan(2^n\alpha)$ c) 0 d) $\cot\alpha$

728. The maximum value ofcos$^2\left(\frac{\pi}{3}-x\right)-\cos^2\left(\frac{\pi}{3}+x\right)$ is

a) $-\frac{\sqrt{3}}{2}$ b) $\frac{1}{2}$ c) $\frac{\sqrt{3}}{2}$ d) $\frac{3}{2}$

729. If $a=2, b=3, c=5$ in ΔABC, then $C=$

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{2}$ d) None of these

730. If in a $\Delta ABC, \frac{a}{\cos A}=\frac{b}{\cos B}$, then

a) $2\sin A\sin B\sin C=1$

b) $\sin^2 A+\sin^2 B=\sin^2 C$

c) $2\sin A\cos B=\sin C$

d) None of these

731. If $1+\sin\theta+\sin^2\theta+\ldots\infty=4+2\sqrt{3}, 0<\theta<\pi, \theta\neq\frac{\pi}{2}$, then

a) $\theta=\frac{\pi}{3}$ b) $\theta=\frac{\pi}{6}$ c) $\theta=\frac{\pi}{3}$ or $\frac{\pi}{6}$ d) $\theta=\frac{\pi}{3}$ or $\theta=\frac{2\pi}{3}$

732. In a ΔABC,

$a(b^2+c^2)\cos A+b(c^2+a^2)\cos B+c(a^2+b^2)\cos C$ is equal to

a) abc b) $2abc$ c) $3abc$ d) $4abc$

733. If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$, then the value of $\cos\left(\theta - \frac{\pi}{4}\right)$ is equal to

a) $\frac{1}{2\sqrt{2}}$ b) $\frac{1}{\sqrt{2}}$ c) $\frac{1}{3\sqrt{2}}$ d) $\frac{1}{4\sqrt{2}}$

734. The number of points of intersection of the two curves $y = 2\sin x$ and $y = 5x^2 + 2x + 3$, is

a) 0 b) 1 c) 2 d) ∞

735. If, in a ΔABC, $(a+b+c)(b+c-a) = \lambda\, bc$, then

a) $\lambda < 0$ b) $\lambda > 4$ c) $\lambda > 0$ d) $0 < \lambda < 4$

736. The expression $\text{cosec}^2 A \cot^2 A - \sec^2 A \tan^2 A - (\cot^2 A - \tan^2 A)(\sec^2 A\ \text{cosec}^2 A - 1)$ is equal to

a) 1 b) -1 c) 0 d) 2

737. The sides of a triangle are in A.P. and its area is 3/5 times the area of an equilateral triangle of the same perimeter. Then, the ratio of the sides is

a) 1 : 2 : 3 b) 3 : 5 : 7 c) 1 : 3 : 5 d) None of these

738. If $\tan\alpha = \frac{b}{a}, a > b > 0$ and if $0 < \alpha < \frac{\pi}{4}$, then $\sqrt{\frac{a+b}{a-b}} - \sqrt{\frac{a-b}{a+b}}$ is equal to

a) $\frac{2\sin\alpha}{\sqrt{\cos 2\alpha}}$ b) $\frac{2\cos\alpha}{\sqrt{\cos 2\alpha}}$ c) $\frac{2\sin\alpha}{\sqrt{\sin 2\alpha}}$ d) $\frac{2\cos\alpha}{\sqrt{\sin 2\alpha}}$

739. If $\sin\theta + \cos\theta = x$, then $\sin^6\theta + \cos^6\theta = \frac{1}{4}[4 - 3(x^2-1)^2]$ for

a) all real x b) $x^2 \leq 2$ c) $x^2 > 2$ d) None of these

740. If in a triangle ABC, $\frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$, then

a) a, b, c are in A.P. b) a^2, b^2, c^2 are in A.P. c) a, b, c are in H.P. d) a^2, b^2, c^2 are in H.P

741. In a ΔABC, angles A, B, C are in A.P., then

$\lim_{A\to C} \frac{\sqrt{3 - 4\sin A \sin C}}{|A-C|}$ is equal to

a) 1 b) 2 c) 3 d) 4

742. For all values of θ, the values of $3 - \cos\theta + \cos\left(\theta + \frac{\pi}{3}\right)$ lie in the interval

a) $[-2, 3]$ b) $[-2, 1]$ c) $[2, 4]$ d) $[1, 5]$

743. If $\cos A = m\cos B$ and $\cot\frac{A+B}{2} = \lambda\tan\frac{B-A}{2}$, then λ is

a) $\frac{m}{m-1}$ b) $\frac{m+1}{m}$ c) $\frac{m+1}{m-1}$ d) None of these

744. The value of $\cos^4\left(\frac{\pi}{8}\right) + \cos^4\left(\frac{3\pi}{8}\right) + \cos^4\left(\frac{5\pi}{8}\right) + \cos^4\left(\frac{7\pi}{8}\right)$ is

a) 0 b) $\frac{1}{2}$ c) $\frac{3}{2}$ d) 1

745. If $\sin\theta = \frac{12}{13}$, $\left(0 < \theta < \frac{\pi}{2}\right)$ and $\cos\phi = -\frac{3}{5}\left(\pi < \phi < \frac{3\pi}{2}\right)$, then $\sin(\theta + \phi)$ will be

a) $-56/61$ b) $-56/65$ c) $1/65$ d) -56

746. The quadratic equation whose roots are $\sec^2\theta$ and $\text{cosec}^2\theta$ can be

a) $x^2 - 2x + 2 = 0$ b) $x^2 + 5x + 5 = 0$ c) $x^2 - 4x + 4 = 0$ d) None of these

747. If $\sec\theta = m$ and $\tan\theta = n$, then $\frac{1}{m}\left[(m+n) + \frac{1}{(m+n)}\right]$ is

a) 2 b) $2m$ c) $2n$ d) mn

748. If in a $\Delta\, ABC, \angle C = 90°$, then the maximum value of $\sin A \sin B$ is

a) $\frac{1}{2}$ b) 1 c) 2 d) None of these

749. In a cyclic quadrilateral $ABCD$, he value of $\cos A + \cos B + \cos C + \cos D$, is

a) 1 b) 0 c) -1 d) None of these

750. If the angles of a triangle are in the ratio 1 : 2 : 3, the corresponding sides are in the ratio

a) 2 : 3 : 1 b) $\sqrt{3}$: 2 : 1 c) 2 : $\sqrt{3}$: 1 d) 1 : $\sqrt{3}$: 2

751. If $\sin(\pi\cos\theta) = \cos(\pi\sin\theta)$, then the value of $\cos\left(\theta + \frac{\pi}{4}\right)$ equals

a) $\frac{1}{\sqrt{2}}$ b) $\frac{1}{2\sqrt{2}}$ c) $-\frac{1}{2\sqrt{2}}$ d) $-\frac{1}{\sqrt{2}}$

752. The most general solution of $2^{1+|\cos x|+\cos^2 x+|\cos^3 x|+\cdots\infty} = 4$ is given by

a) $x = n\pi \pm \frac{\pi}{3}, n \in Z$

b) $x = 2n\,\pi \pm \frac{\pi}{3}, n \in Z$

c) $x = 2n\,\pi \pm \frac{2\pi}{3}, n \in Z$

d) None of these

753. If $\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$, then $\cos 2\,\alpha + \cos 2\,\beta =$

a) $-2\sin(\alpha+\beta)$ b) $-2\cos(\alpha+\beta)$ c) $2\sin(\alpha+\beta)$ d) $2\cos(\alpha+\beta)$

754. The value of the expression $1 - \frac{\sin^2 y}{1+\cos y} + \frac{1+\cos y}{\sin y} - \frac{\sin y}{1-\cos y}$ is equal to

a) 0 b) 1 c) $\sin y$ d) $\cos y$

755. In a $\Delta ABC, a = 2b$ and $A = 3B$, the $A =$

a) 90° b) 60° c) 30° d) 45°

756. If in a $\Delta ABC, A = \frac{\pi}{3}$ and AD is the median, then

a) $2\,AD^2 = b^2 + c^2 + bc$

b) $4\,AD^2 = b^2 + c^2 + bc$

c) $6\,AD^2 = b^2 + c^2 + bc$

d) None of these

757. If $\cos(\theta - \alpha) = a, \cos(\theta - \beta) = b$, then $\sin^2(\alpha - \beta) + 2ab\cos(\alpha - \beta)$ is equal to

a) $a^2 + b^2$ b) $a^2 - b^2$ c) $b^2 - a^2$ d) $-a^2 - b^2$

758. If $\cos\frac{x}{2}.\cos\frac{x}{2^2}\ldots.\cos\frac{x}{2^n} = \frac{\sin x}{2^n \sin\frac{x}{2^n}}$, then $\frac{1}{2}\tan\frac{x}{2} + \frac{1}{2^2}\tan\frac{x}{2^2} + \ldots + \frac{1}{2^n}\tan\frac{x}{2^n}$ is

a) $\cot x - \cot\frac{x}{2^n}$

b) $\frac{1}{2^n}\cot\left(\frac{x}{2^n}\right) - \cot x$

c) $\frac{1}{2^n}\tan\left(\frac{1}{2^n}\right) - \tan x$

d) $\frac{1}{2}\cot x - \frac{1}{2^n}\cot\left(\frac{x}{2^n}\right)$

759. In triangles ABC and DEF, $AB = DE, AC = EF$ and $\angle A = 2\,\angle E$. Two triangles will have the same area if angle A is equal to

a) $\pi/3$ b) $\pi/2$ c) $2\,\pi/3$ d) $5\,\pi/6$

760. The value of $\sin\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{18}\right)\sin\left(\frac{7\pi}{18}\right)$, is

a) 1/2 b) 1/4 c) 1/8 d) 1/16

761. If the equation $\sin^4\theta + \cos^4\theta = a$ has a real solution then

a) $a \le \frac{1}{2}$ b) $a \ge \frac{1}{2}$ c) $\frac{1}{2} \le a \le 1$ d) $a \ge 0$

762. The general solution of the equation $(\sqrt{3} - 1)\sin\theta + (\sqrt{3} + 1)\cos\theta = 2$ is

a) $2n\pi \pm \frac{\pi}{4} + \frac{\pi}{12}$

b) $n\pi + (-1)^n\frac{\pi}{4} + \frac{\pi}{12}$

c) $2n\pi \pm \frac{\pi}{4} - \frac{\pi}{12}$

d) $n\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{12}$

763. If $\sin A = \frac{1}{\sqrt{10}}$ and $\sin B = \frac{1}{\sqrt{5}}$, where A and B are positive acute angles, then $A + B$ is equal to

a) π b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{4}$

764. The general solution of $\sin^2\theta\sec\theta+\sqrt{3}\tan\theta=0$ is

a) $\theta=n\pi+(-1)^{n+1}\frac{\pi}{3}, \theta=n\pi, n\in I$
b) $\theta=n\pi, n\in I$
c) $\theta=n\pi+(-1)^{n+1}\frac{\pi}{3}, n\in I$
d) $\theta=\frac{n\pi}{2}, n\in I$

765. If $y+\cos\theta=\sin\theta$ has a real solution, then

a) $-\sqrt{2}\leq y\leq\sqrt{2}$
b) $y>\sqrt{2}$
c) $y\leq-\sqrt{2}$
d) None of these

766. If $\cos(\theta-\alpha)=a, \sin(\theta-\beta)=b$, then $\cos^2(\alpha-\beta)+2ab\sin(\alpha-\beta)$ is equal to

a) $4a^2b^2$
b) a^2-b^2
c) a^2+b^2
d) $-a^2b^2$

767. The equation $8\sec^2\theta-6\sec\theta+1=0$ has

a) Exactly two roots
b) Exactly four roots
c) Infinitely many roots
d) No roots

768. If the sides a, b, c of a triangle ABC are the roots of the equation $x^3-13x^2+54x-72=0$, then the value of $\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}$ is equal to

a) $\frac{169}{144}$
b) $\frac{61}{72}$
c) $\frac{61}{144}$
d) $\frac{169}{72}$

769. $\cos^4\theta-\sin^4\theta$ is equal to

a) $1+2\sin^2\left(\frac{\theta}{2}\right)$
b) $2\cos^2\theta-1$
c) $1-2\sin^2\left(\frac{\theta}{2}\right)$
d) $1+2\cos^2\theta$

770. The value of $\cos 15°\cos 7\frac{1}{2}°\sin 7\frac{1}{2}°$ is

a) $\frac{1}{2}$
b) $\frac{1}{8}$
c) $\frac{1}{4}$
d) $\frac{1}{16}$

771. If θ lies in the second quadrant, then the value of $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}+\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$ is equal to

a) $2\sec\theta$
b) $-2\sec\theta$
c) $2\operatorname{cosec}\theta$
d) None of these

772. The value of $\cos^2 A\,(3-4\cos^2 A)^2+\sin^2 A\,(3-4\sin^2 A)^2$ is equal to

a) $\cos 4A$
b) $\sin 4A$
c) 1
d) None of these

773. Let the angles A, B, C of ΔABC be in A.P. and let

a) 75°
b) 45°
c) 60°
d) 15°

774. If $\tan x=\frac{b}{a}$, then $\sqrt{\frac{a+b}{a-b}}+\sqrt{\frac{a-b}{a+b}}=$

a) $\frac{2\sin x}{\sqrt{\sin 2x}}$
b) $\frac{2\cos x}{\sqrt{\cos 2x}}$
c) $\frac{2\cos x}{\sqrt{\sin 2x}}$
d) $\frac{2\sin x}{\sqrt{\cos 2x}}$

775. If $\sin A+\cos A=m$ and $\sin^3 A+\cos^3 A=n$, then

a) $m^3-3m+n=0$
b) $n^3-3n+2m=0$
c) $m^3-3m+2n=0$
d) $m^3+3m+2n=0$

776. The most general solutions of the equation $\sec x-1=(\sqrt{2}-1)\tan x$ are given by

a) $n\pi+\frac{\pi}{8}$
b) $2n\pi, 2n\pi+\frac{\pi}{4}$
c) $2n\pi$
d) None of these

777. If $\cos(\theta-\alpha)=a, \cos(\theta-\beta)=b$, then $\sin^2(\alpha-\beta)+2ab\cos(\alpha-\beta)$ is equal to

a) a^2+b^2
b) a^2-b^2
c) b^2-a^2
d) $-a^2-b^2$

778. The sum $S=\sin\theta+\sin 2\theta+\ldots+\sin n\theta$ equals

a) $\sin\frac{1}{2}(n+1)\theta\sin\frac{n\theta}{2}/\sin\frac{\theta}{2}$
b) $\cos\frac{1}{2}(n+1)\theta\sin\frac{n\theta}{2}/\sin\frac{\theta}{2}$
c) $\sin\frac{1}{2}(n+1)\theta\cos\frac{n\theta}{2}/\sin\frac{\theta}{2}$
d) $\cos\frac{1}{2}(n+1)\theta\cos\frac{n\theta}{2}/\sin\frac{\theta}{2}$

779. The sides of an equilateral triangle, a square and a regular hexagon circumscribed in a circle are in

a) A.P.
b) G.P.
c) H.P.
d) None of these

780. If $\frac{\tan 3\theta-1}{\tan 3\theta+1}=\sqrt{3}$, then the general value of θ is

a) $\frac{n\pi}{3}-\frac{\pi}{12}$
b) $n\pi+\frac{7\pi}{12}$
c) $\frac{n\pi}{3}+\frac{7\pi}{36}$
d) $n\pi+\frac{\pi}{12}$

781. If $\theta\in[0,5\pi]$ and $r\in R$ such that $2\sin\theta=r^4-2r^2+3$, then the maximum number of values of the pair (r,θ) is

a) 6 b) 8 c) 10 d) None of these

782. In a triangle $ABC, r =$

a) $(s-a)\tan\frac{B}{2}$ b) $(s-b)\tan\frac{B}{2}$ c) $(s-b)\tan\frac{C}{2}$ d) $(s-a)\tan\frac{C}{2}$

783. If p_1, p_2, p_3 are altitude of a triangle ABC from the vertices A, B, C and Δ, the area of the triangle, then $\frac{1}{p_1^2}+\frac{1}{p_2^2}+\frac{1}{p_3^2}=$

a) $\frac{\cot A+\cos B+\cot C}{\Delta}$

b) $\frac{\Delta}{\cot A+\cot B+\cot C}$

c) $\Delta(\cot A+\cot B+\cot C)$

d) None of these

784. Number of solutions of the equation $\sin 2\theta+2=4\sin\theta+\cos\theta$ lying in the interval $[\pi, 5\pi]$, is

a) 0 b) 2 c) 4 d) 5

785. If twice the square of the diameter of a circle is equal to half the sum of the squares of the sides of incribed triangle ABC, then $\sin^2 A+\sin^2 C$ is equal to

a) 1 b) 2 c) 4 d) 8

786. $\tan 9°-\tan 27°-\tan 63°+\tan 81°$ is equal to

a) 0 b) 1 c) -1 d) 4

787. If $\sin 4A-\cos 2A=\cos 4A-\sin 2A, \left(0<A<\frac{\pi}{4}\right)$, then the value of $\tan 4A$ is

a) 1 b) $\frac{1}{\sqrt{3}}$ c) $\sqrt{3}$ d) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

788. In a ΔABC, $\sin A$ and $\sin B$ are the roots of the equation $c^2x^2-c(a+b)x+ab=0$, then $\sin C=$

a) $1/\sqrt{2}$ b) $1/2$ c) 1 d) 0

789. If $\sin(\alpha+\beta)=1, \sin(\alpha-\beta)=1/2;\ \alpha,\beta\in[0,\pi/2]$, then $\tan(\alpha+2\beta)\tan(2\alpha+\beta)$ is equal to

a) 1 b) -1 c) 0 d) 1/2

790. If $a_{n+1}=\sqrt{\frac{1}{2}(1+a_n)}$, then $\cos\left(\frac{\sqrt{1-a_0^2}}{a_1a_2a_3\ldots\text{to }\infty}\right)=$

a) 1 b) -1 c) a_0 d) $1/a_0$

791. If the angles of a triangle are in the ratio 1 : 2 : 7, then the ratio of the greatest side to the least side is

a) $(\sqrt{5}-1):(\sqrt{5}+1)$ b) $(\sqrt{5}+1):(\sqrt{5}-1)$ c) $(\sqrt{5}+2):(\sqrt{5}-2)$ d) $(\sqrt{5}-2):(\sqrt{5}+2)$

792. In a $\Delta ABC, A=\frac{2\pi}{3}, b-c=3\sqrt{3}$ cm and $\Delta=\frac{9\sqrt{3}}{2}$ cm^2. Then, $a=$

a) $6\sqrt{3}$ cm b) 9 cm c) 18 cm d) 12 cm

793. If the radius of the incircle of a triangle with its sides $5k, 6k$, and $5k$ is 6, then k is equal to

a) 3 b) 4 c) 5 d) 6

794. The minimum value of $2^{\sin x}+2^{\cos x}$, is

a) 1 b) 2 c) $2^{-\frac{1}{\sqrt{2}}}$ d) $2^{1-\frac{1}{\sqrt{2}}}$

795. Minimum value of $\frac{1}{3\sin\theta-4\cos\theta+7}$ is

a) $\frac{1}{12}$ b) $\frac{5}{12}$ c) $\frac{7}{12}$ d) $\frac{1}{6}$

796. If $\operatorname{cosec}\theta=\frac{p+q}{p-q}$, then $\cot(\pi/4+\theta/2)=$

a) $\sqrt{\frac{p}{q}}$ b) $\sqrt{\frac{q}{p}}$ c) $\sqrt{pq}$ d) pq

797. Suppose $0<t<\pi$ andsin $t+\cos t=\frac{1}{5}$. Then, $\tan\frac{t}{2}$ is equal to

a) 2 b) 3 c) $\frac{1}{3}$ d) 5

798. For what and only what values of α lying between 0 and π is the inequality $\sin\alpha\cos^3\alpha > \sin^3\alpha\cos\alpha$ valid?

a) $\alpha \in (0, \pi/4)$ b) $\alpha \in (0, \pi/2)$ c) $\alpha \in (\pi/4, \pi/2)$ d) None of these

799. If $\alpha + \beta - \gamma = \pi$, then $\sin^2\alpha + \sin^2\beta - \sin^2\gamma$ is equal to

a) $2\sin\alpha\sin\beta\cos\gamma$ b) $2\cos\alpha\cos\beta\cos\gamma$ c) $2\sin\alpha\sin\beta\sin\gamma$ d) None of these

800. If $\sec x\cos 5x + 1 = 0$, where $0 < x < 2\pi$, then x is equal to

a) $\frac{\pi}{5}, \frac{\pi}{4}$ b) $\frac{\pi}{5}$ c) $\frac{\pi}{4}$ d) None of these

801. If $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$, $\sin\alpha = \frac{4}{5}$ and $\cos(\alpha + \beta) = -\frac{12}{13}$, then $\sin\beta$ is equal to

a) $\frac{63}{65}$ b) $\frac{61}{65}$ c) $\frac{3}{5}$ d) $\frac{5}{13}$

802. The value of $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}$, is

a) 1 b) 1/4 c) 1/8 d) $\sqrt{2}/7$

803. If $\theta_1, \theta_2, \theta_3, \theta_4$ are roots of the equation $\sin(\theta + \alpha) = k\sin 2\theta$ no two of which differ by a multiple of 2π, then $\theta_1 + \theta_2 + \theta_3 + \theta_4$ is equal to

a) $2n\pi, n \in Z$ b) $(2n+1)\pi, n \in Z$ c) $n\pi, n \in Z$ d) None of these

804. The radius of the circle whose arc of length 15π cm makes an angle of $\frac{3\pi}{4}$ radian at the centre is

a) 10 cm b) 20 cm c) $11\frac{1}{4}$ cm d) $22\frac{1}{2}$ cm

805. The value of $\cot\theta - \tan\theta - 2\tan 2\theta - 4\tan 4\theta - 8\cot 8\theta$, is

a) 0 b) 1 c) -1 d) None of these

806. In a triangle ABC, $b = \sqrt{3}$, $c = 1$ and $\angle A = 30°$, then the measure of the largest angle of the triangle is

a) 60° b) 135° c) 90° d) 120°

807. The maximum value of $3\cos\theta + 4\sin\theta$ is

a) 3 b) 4 c) 5 d) None of these

808. If the sides of a triangle are proportional to 2, $\sqrt{6}$ and $\sqrt{3} - 1$, the greatest and the least angles of the triangle are

a) 120°, 15° b) 90°, 15° c) 75°, 45° d) 150°, 15°

809. In a ΔABC if $r_1 = 16, r_2 = 48$ *and* $r_3 = 24$, then its in-radius is

a) 7 b) 8 c) 6 d) None of these

810. The number of values of x in the interval $[0, 5\pi]$ satisfying the equation $3\sin^2 x - 7\sin x + 2 = 0$ is

a) 0 b) 5 c) 6 d) 10

811. If $\cos^2\theta = \cos 2\theta$, then the general value of θ is

a) $n\pi$ b) $2n\pi$ c) $\frac{n\pi}{3}$ d) $\frac{n\pi}{2}$

812. The equation $3^{\sin 2x + 2\cos^2 x} + 3^{1 - \sin 2x + 2\sin^2 x} = 28$ is satisfied for the values of x given by

a) $\cos x = 0, \tan x = -1$ b) $\tan x = -1, \cos x = 1$ c) $\tan x = 1, \cos x = 0$ d) None of these

813. The minimum value of $27^{\cos 2x}\, 81^{\sin 2x}$ is

a) -5 b) $\frac{1}{5}$ c) $\frac{1}{243}$ d) $\frac{1}{27}$

814. Let $0 < x \le \pi/4$, then $(\sec 2x - \tan 2x)$ equals

a) $\tan^2(x + \pi/4)$ b) $\tan(x + \pi/4)$ c) $\tan(\pi/4 - x)$ d) $\tan(x - \pi/4)$

815. The number of solutions of the equation $\sin^5 x - \cos^5 x = \frac{1}{\cos x} - \frac{1}{\sin x}$ $(\sin x \neq \cos x)$ is

a) 0 b) 1 c) Infinite d) None of these

816. Let $\cos(\alpha + \beta) = \frac{4}{5}$ and let $\sin(\alpha - \beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha$ is equal to

a) $\frac{25}{16}$ b) $\frac{56}{33}$ c) $\frac{19}{12}$ d) $\frac{20}{7}$

817. The value of $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$, is

a) 1 b) -1 c) 1/2 d) $-1/2$

818. If in a triangle $a\cos^2\left(\frac{C}{2}\right)+c\cos^2\left(\frac{A}{2}\right)=\frac{3b}{2}$, then the sides of the triangle are in

a) AP b) GP c) HP d) None of these

819. If $\frac{1-\cos 2\theta}{1+\cos 2\theta}=3$, then the general value of θ is

a) $2n\pi\pm\frac{\pi}{6}$ b) $n\pi\pm\frac{\pi}{6}$ c) $2n\pi\pm\frac{\pi}{3}$ d) $n\pi\pm\frac{\pi}{3}$

820. In a ΔABC, if $a=5$ cm, $b=4$ cm and $\cos(A-B)=\frac{31}{32}$, then $\cos C=$

a) 1/4 b) 1/8 c) 1/6 d) 1/2

821. The number of solutions for the equation $\sin 2x+\cos 4x=2$ is

a) 0 b) 1 c) 2 d) ∞

822. If $\sin\theta_1+\sin\theta_2+\sin\theta_3=3$, then $\cos\theta_1+\cos\theta_2+\cos\theta_3=$

a) 3 b) 2 c) 1 d) 0

823. The equation $k\sin x+\cos 2\,x=2\,k-7$ possesses solution, if

a) $k>6$ b) $2\leq k\leq 6$ c) $k>2$ d) None of these

824. If$\cos 2B=\frac{\cos(A+C)}{\cos(A-C)}$, then $\tan A$, $\tan B$, $\tan C$ are in

a) AP b) GP c) HP d) None of these

825. If n is an odd positive integer, then $\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^n+\left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^n=$

a) −1 b) 1 c) 0 d) None of these

826. If α,β are the solutions of $a\tan\theta+b\sec\theta=c$, then $\tan(\alpha+\beta)=$

a) $\frac{2\,ac}{a^2-c^2}$ b) $\frac{2\,ac}{c^2-a^2}$ c) $\frac{2\,ac}{a^2+c^2}$ d) $\frac{ac}{a^2+c^2}$

827. If $\tan\theta+\tan\left(\theta+\frac{\pi}{3}\right)+\tan\left(\theta+\frac{2\pi}{3}\right)=3$, then which of the following is equal to 1?

a) $\tan 2\theta$ b) $\tan 3\theta$ c) $\tan^2\theta$ d) $\tan^3\theta$

828. If $y=1+4\sin^2 x\cos^2 x$, then

a) $1\leq y\leq 2$ b) $-1\leq y\leq 1$ c) $-3\leq y\leq 3$ d) None of these

829. If $\alpha+\beta-\gamma=\pi$, then $\sin^2\alpha+\sin^2\beta-\sin^2\gamma$ is equal to

a) $2\sin\alpha\sin\beta\cos\gamma$ b) $2\cos\alpha\cos\beta\cos\gamma$ c) $2\sin\alpha\sin\beta\sin\gamma$ d) None of the above

830. In a ΔABC, $\frac{\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}}{\cot A+\cot B+\cot C}=$

a) $\frac{(a+b+c)^2}{a^2+b^2+c^2}$ b) $\frac{a^2+b^2+c^2}{(a+b+c)^2}$ c) s d) Δ

831. The expression $\tan^2\alpha+\cot^2\alpha$ is

a) ≥ 2 b) ≤ 2 c) ≥ -2 d) None of these

832. For $m\neq n$, if $\tan m\,\theta=\tan n\,\theta$, the different values of θ are in

a) A.P.
b) H.P.
c) G.P.
d) No particular sequence

833. If in a triangle ABC,
$\sin A:\sin C=\sin(A-B):\sin(B-C)\ then,\ a^2:b^2:c^2\ are\ in$

a) A.P. b) G.P. c) H.P. d) None of these

834. If $\tan\theta=x-\frac{1}{4x}$, then $\sec\theta-\tan\theta$ is equal to

a) $-2x,\frac{1}{2x}$ b) $-\frac{1}{2x},2x$ c) $2x$ d) $2x,\frac{1}{2x}$

835. The number of values of $x\in[0,2\,\pi]$ that satisfy $\cot x-\operatorname{cosec} x=2\sin x$, is

a) 3 b) 2 c) 1 d) 0

836. If R is the radius of circumscribing circle of a regular polygon of n-sides, then $R=$

a) $\frac{a}{2}\sin\left(\frac{\pi}{n}\right)$ b) $\frac{a}{2}\cos\left(\frac{\pi}{n}\right)$ c) $\frac{a}{2}\operatorname{cosec}\left(\frac{\pi}{n}\right)$ d) $\frac{a}{2}\operatorname{cosec}\left(\frac{\pi}{2n}\right)$

837. If $\frac{\sin x}{\sin y}=\frac{1}{2}, \frac{\cos x}{\cos y}=\frac{3}{2}$, where $x, y \in \left(0, \frac{\pi}{2}\right)$, then the value of $\tan(x+y)$ is equal to

a) $\sqrt{13}$ b) $\sqrt{14}$ c) $\sqrt{17}$ d) $\sqrt{15}$

838. If $\sin A+\sin B=\sqrt{3}(\cos B \cos A)$, then $\sin 3A+\sin 3B=$

a) 0 b) 2 c) 1 d) -1

839. If $\tan\beta=\cot\theta\tan\alpha$, then $\cot^2\left(\frac{\theta}{2}\right)$ is equal to

a) $\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$ b) $\frac{\sin(\alpha-\beta)}{\sin(\alpha+\beta)}$ c) $\frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}$ d) $\frac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)}$

840. $\frac{\cos 9°+\sin 9°}{\cos 9°-\sin 9°}$ is equals to

a) $\tan 26°$ b) $\tan 81°$ c) $\tan 51°$ d) $\tan 54°$

841. In a ΔABC, if $A=45°, b=\sqrt{6}, a=2$, then $B=$

a) 30° or 150° b) 60° or 120° c) 45° or 135° d) None of these

842. Two sides of a triangle are $2\sqrt{2}$ cm and $2\sqrt{3}$ cm and the angle opposite to the shorter side of the two is $\frac{\pi}{4}$. The largest possible length of the third side is

a) $(\sqrt{6}+\sqrt{2})$ cm b) $(6+\sqrt{2})$ cm c) $(\sqrt{6}-\sqrt{2})$ cm d) None of these

843. The total number of ordered pairs (r, θ) satisfying $r\sin\theta=3, r=4(1+\sin\theta)$, where $r>0$ and $\theta\in[-\pi,\pi]$ is

a) 0 b) 2 c) 4 d) None of these

844. $\sin 65°+\sin 43°-\sin 29°-\sin 7°$ is equal to

a) $\cos 36°$ b) $\cos 18°$ c) $\cos 9°$ d) None of these

845. If $\sin B=\frac{1}{5}\sin(2A+B)$, then $\frac{\tan(A+B)}{\tan A}$ is equal to

a) 5/3 b) 2/3 c) 3/2 d) 3/5

846. If $A+B+C=\pi$ and $\cos A=\cos B\cos C$, then $\tan B\tan C$ is equal to

a) $\frac{1}{2}$ b) 2 c) 1 d) $-\frac{1}{2}$

847. If $\sin x+\operatorname{cosec} x=2$ then, $\sin^n x+\operatorname{cosec}^n x$ is equal to

a) 2 b) 2^n c) 2^{n-1} d) 2^{n-2}

848. If in a triangle $ABC, \frac{a^2-b^2}{a^2+b^2}=\frac{\sin(A-B)}{\sin(A+B)}$, then the triangle is

a) Right angled or isosceles
b) Right angled and isosceles
c) Equilateral
d) None of these

849. In a $\Delta ABC, \cos A=\cos B\cos C$, then $\cot B\cot C$ is equal to

a) 2 b) 3 c) 1/2 d) 5

850. In a ΔABC if $a=13, b=14$ and $c=15$, then reciprocals of r_1, r_2 and r_3 are in the ratio

a) 6 : 7 : 8 b) 6 : 8 : 7 c) 8 : 7 : 6 d) None of these

851. $\frac{\sin 7\theta+6\sin 5\theta+17\sin 3\theta+12\sin\theta}{\sin 6\theta+5\sin 4\theta+12\sin 2\theta}$ is equal to

a) $2\cos\theta$ b) $\cos\theta$ c) $2\sin\theta$ d) $\sin\theta$

852. In a triangle the angles are in A.P. and the lengths of the two larger sides are 10 and 9 respectively, then the length of the third side can be

a) $5\pm\sqrt{6}$ b) 0.7 c) $\sqrt{5}+6$ d) None of these

853. The general value of x for which $\cos 2x, \frac{1}{2}$ and $\sin 2x$ are in AP, are given by

a) $n\pi, n\pi+\frac{\pi}{2}$ b) $n\pi, n\pi+\frac{\pi}{4}$ c) $n\pi+\frac{\pi}{4}, \frac{3n\pi}{4}$ d) None of these

854. If $a=\frac{\pi}{18}$ rad, then $\cos a+\cos 2a+\ldots+\cos 18a$ is equal to

a) 0 b) -1 c) 1 d) ± 1

855. If $\sin\theta+\cos\theta=1$, then the general value of θ is

a) $2n\pi$ b) $n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{4}$ c) $2n\pi+\frac{\pi}{2}$ d) None of these

856. If $1+\sin x+\sin^2 x+\sin^3 x+\cdots+\cdots\infty$ is equal to $4+2\sqrt{3}, 0<x<\pi$, then $x=$

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ or $\frac{\pi}{6}$ d) $\frac{\pi}{3}$ or $\frac{2\pi}{3}$

857. If $\sin x+\sin y=a$ and $\cos x+\cos y=b$, then $\tan\left(\frac{a+y}{2}\right)$ is equal to

a) $\frac{ab}{a+b}$ b) $\frac{a}{b}$ c) $\frac{b}{a}$ d) None of these

858. If $\sin(\pi\cot\theta)=\cos(\pi\tan\theta)$, then $\cot 2\theta$ is equal to where $n\in Z$

a) $n-\frac{1}{4}$ b) $n+\frac{1}{4}$ c) $4n+1$ d) $4n-1$

859. If the altitudes of a triangle are in AP, then the sides of the triangle are in

a) A.P. b) G.P. c) H.P. d) None of these

860. The value of $\cos\frac{\pi}{5}\cos\frac{2\pi}{5}\cos\frac{4\pi}{5}\cos\frac{8\pi}{5}$ is equal to

a) $\frac{1}{16}$ b) 0 c) $-\frac{1}{8}$ d) $-\frac{1}{16}$

861. $cosec\ 15°+\sec 15°$ is equal to

a) $2\sqrt{2}$ b) $\sqrt{6}$ c) $2\sqrt{6}$ d) $\sqrt{6}+\sqrt{2}$

862. If $\sin A=\frac{4}{5}$ and $\cos B=-\frac{12}{13}$, where A and B lie in first and third quadrant respectively, then $\cos(A+B)$ is equal to

a) $\frac{56}{65}$ b) $-\frac{56}{65}$ c) $\frac{16}{65}$ d) $-\frac{16}{65}$

863. If $\cot\theta+\tan\theta=m$ and $\sec\theta-\cos\theta=n$, then which of the following is correct?

a) $m(mn^2)^{1/3}-n(nm^2)^{1/3}=1$ b) $m(m^2n)^{1/3}-n(mn^2)^{1/3}=1$
c) $n(mn^2)^{1/3}-m(nm^2)^{1/3}=1$ d) $n(m^2n)^{1/3}-m(mn^2)^{1/3}=1$

864. If in a $\Delta\ ABC$,
$(\sin A+\sin B+\sin C)(\sin A+\sin B-\sin C)=3\sin A\sin B$, then

a) $A=60°$ b) $B=60°$ c) $C=60°$ d) None of these

865. Equation $\cos 2x+7=a(2-\sin x)$ can have a real solution for

a) All values of a b) $a\in[2,6]$ c) $a\in(-\infty,2)$ d) $a\in(0,\infty)$

866. In a $\Delta\ ABC, \angle A=\frac{\pi}{2}$, then $\cos^2 B+\cos^2 C$ equals

a) -2 b) -1 c) 1 d) 0

867. In any $\Delta\ ABC, b^2\sin 2C+c^2\sin 2B$

a) Δ b) 2Δ c) 3Δ d) 4Δ

868. In a triangle the length of the two larger sides are 24 and 22, respectively. If the angles are in AP, then the third side is

a) $12+2\sqrt{13}$ b) $12-2\sqrt{13}$ c) $2\sqrt{13}+2$ d) $2\sqrt{13}-2$

869. If in a $\Delta ABC, AD, BE$ and CF are the altitudes and R is the circum-radius, then radius of the circumcircle DEF is

a) $\frac{R}{2}$ b) $2R$ c) R d) $\frac{3}{2}R$

870. If a,b,c denote the sides of a ΔABC and the equations $ax^2+bx+c=0$ and $x^2+\sqrt{2}x+1=0$ have a common root, then $\angle C=$

a) 30° b) 45° c) 90° d) 60°

871. If a circle is inscribed in an equilateral triangle of side a, then area of the square inscribed in the circle is

a) $\frac{a^2}{6}$ b) $\frac{a^2}{3}$ c) $\frac{2a^2}{5}$ d) $\frac{2a^2}{3}$

872. The value of the expression $\cos 1°.\cos 2°\ldots..\cos 179°$ equals

a) 0 b) 1 c) $1/\sqrt{2}$ d) -1

873. The general solution of the equation $2^{\cos 2x}+1=3.2^{-\sin x}$ is

a) $n\pi$ b) $n\pi - \pi$ c) $n\pi + \pi$ d) None of these

874. If $\sin A - \sqrt{6}\cos A = \sqrt{7}\cos A$, then $\cos A + \sqrt{6}\sin A$ is equal to

a) $\sqrt{6}\sin A$ b) $-\sqrt{7}\sin A$ c) $\sqrt{6}\cos A$ d) $\sqrt{7}\cos A$

875. If $y = \frac{\tan x}{\tan 3x}$, then

a) $y \in [1/3, 3]$ b) $y \notin [1/3, 3]$ c) $y \in [-3, -1/3]$ d) $y \notin [-3, -1/3]$

876. If $\frac{3\pi}{4} < \alpha < \pi$, then $\sqrt{\operatorname{cosec}^2\alpha + 2\cot\alpha}$ is equal to

a) $1 + \cot\alpha$ b) $1 - \cot\alpha$ c) $-1 - \cot\alpha$ d) $-1 + \cot\alpha$

877. The equation $a\sin x + b\cos x = c$, where $|c| > \sqrt{a^2 + b^2}$ has

a) A unique solution

b) Infinite no. of solutions

c) No solution

d) None of these

878. The number of solutions of the equation $\tan\theta + \sec\theta = 2\cos\theta$ lying in the interval $[0, 2\pi]$, is

a) 0 b) 1 c) 2 d) 3

879. The least positive non-integral solution of $\sin\pi(x^2 + x) - \sin\pi x^2 = 0$, is

a) Rational

b) Irrational of the form $\sqrt{p}$

c) Irrational of the form $\frac{\sqrt{p}-1}{4}$, when p is an odd integer

d) Irrational of the form $\frac{\sqrt{p}+1}{4}$, where p is an even integer

880. If A and B are acute positive angles satisfying the equations $3\sin^2 A + 2\sin^2 B = 1$ and $3\sin 2A - 2\sin 2B = 0$, then $A + 2B$ is equal to

a) 0 b) $\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{3}$

881. The greatest and least value of $\sin x \cos x$ are respectively

a) $1, -1$ b) $\frac{1}{2}, -\frac{1}{2}$ c) $\frac{1}{4}, -\frac{1}{4}$ d) $2, -2$

882. If $x = X\cos\theta - Y\sin\theta, y = X\sin\theta + Y\cos\theta$ and $x^2 + 4x\,y + y^2 = AX^2 + BY^2, 0 \le \theta \le \frac{\pi}{2}, n \in Z$, then

a) $\theta = \frac{\pi}{6}, A = 3, B = 1$ b) $\theta = \frac{\pi}{2}, A = 3, B = 1$ c) $A = 3, B = -1, \theta = \frac{\pi}{4}$ d) $A = -3, B = 1, \theta = \frac{\pi}{4}$

883. The number of values of x in $[0, 2\pi]$ satisfying the equation $3\cos 2x - 10\cos x + 7 = 0$ is

a) 1 b) 2 c) 3 d) 4

884. $\cos\alpha\sin(\beta - \gamma) + \cos\beta\sin(\gamma - \alpha) + \cos\gamma\sin(\alpha - \beta)$ is equal to

a) 0 b) $\frac{1}{2}$ c) 1 d) $4\cos\alpha\cos\beta\cos\gamma$

885. If $A + B = 45°$, then $(\cot A - 1)(\cot B - 1)$ is equal to

a) 1 b) $\frac{1}{2}$ c) -1 d) 2

886. The solution of the equation $[\sin x + \cos x]^{1+\sin 2x} = 2, -\pi \le x \le \pi$ is

a) $\frac{\pi}{2}$ b) π c) $\frac{\pi}{4}$ d) $\frac{3\pi}{4}$

887. If $\sin x + \sin^2 x = 1$, then the value of $\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x + 2\cos^4 x + \cos^2 x - 2$, is equal to

a) 0 b) 1 c) 2 d) $\sin^2 x$

888. $\sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8}$ is equal to

a) 1 b) 3/2 c) 2 d) 1 /4

889. If $\sin(x + 3\alpha) = 3\sin(\alpha - x)$, then

a) $\tan x = \tan\alpha$ b) $\tan x = \tan^2\alpha$ c) $\tan x = \tan^3\alpha$ d) $\tan x = 3\tan\alpha$

890. $\cos\alpha\sin(\beta - \gamma) + \cos\beta\sin(\gamma - \beta) + \cos\gamma\sin(\alpha - \beta) =$

a) 0 b) 1/2 c) 1 d) $4\cos\alpha\cos\beta\cos\gamma$

891. If $\sin A+\cos A=m$ and $\sin^3 A+\cos^3 A=n$, then

a) $m^3-3m+n=0$ b) $n^3-3n+2m=0$ c) $m^3-3m+2n=0$ d) $m^3+3m+2n=0$

892. If $(\sec\theta-1)=(\sqrt{2}-1)\tan\theta$, then $\theta=$

a) $n\pi+\frac{\pi}{8}, n\in Z$

b) $2n\pi, 2n\pi+\frac{\pi}{4}, n\in Z$

c) $2n\pi, n\in Z$

d) None of these

893. The number of values of θ in the interval $[-\pi,\pi]$ satisfying the equation $\cos\theta+\sin 2\theta=0$ is

a) 1 b) 2 c) 3 d) 4

894. The general solution of $\tan\left(\frac{\pi}{2}\sin\theta\right)=\cot\left(\frac{\pi}{2}\cos\theta\right)$ is

a) $\theta=2r\pi+\frac{\pi}{2}, r\in Z$

b) $\theta=2r\pi, r\in Z$

c) $\theta=2r\pi+\frac{\pi}{2}$ and $\theta=2r\pi, r\in Z$

d) None of these

895. The most general values of θ satisfying $\tan\theta+\tan\left(\frac{3\pi}{4}+\theta\right)=2$ are given by

a) $2n\pi\pm\frac{\pi}{3}, n\in Z$ b) $n\pi+\frac{\pi}{3}, n\in Z$ c) $2n\pi\pm\frac{\pi}{6}, n\in Z$ d) $n\pi\pm\frac{\pi}{6}, n\in Z$

896. If $(1+\tan\theta)(1+\tan\phi)=2$, then $\theta+\phi=$

a) 30° b) 45° c) 60° d) 75°

897. If α and β satisfying $2\sec 2\alpha=\tan\beta+\cot\beta$, then $\alpha+\beta$ is equal to

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) π

898. If $0<\theta<2\pi$, then the intervals of values of θ for which $2\sin^2\theta-5\sin\theta+2>0$, is

a) $\left(0,\frac{\pi}{6}\right)\cup\left(\frac{5\pi}{6},2\pi\right)$ b) $\left(\frac{\pi}{8},\frac{5\pi}{6}\right)$ c) $\left(0,\frac{\pi}{8}\right)\cup\left(\frac{\pi}{6},\frac{5\pi}{6}\right)$ d) $\left(\frac{41\pi}{48},\pi\right)$

899. If $\tan A-\tan B=x$ and $\cot B-\cot A=y$, then $\cot(A-B)$ is equal to

a) $\frac{1}{x}+y$ b) $\frac{1}{xy}$ c) $\frac{1}{x}-\frac{1}{y}$ d) $\frac{1}{x}+\frac{1}{y}$

900. In a ΔABC, if a, c, b are in A.P., then the value of $\frac{a\cos B-b\cos A}{a-b}$, is

a) 3 b) 2 c) 1 d) None of these

901. $\tan 10°+\tan 35°+\tan 10°\tan 35°$ is equal to

a) 0 b) $\frac{1}{2}$ c) -1 d) 1

902. The value of $\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^n+\left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^n$ (where n is an even) is

a) $2\tan^n\left(\frac{A-B}{2}\right)$ b) $2\cot^n\left(\frac{A-B}{2}\right)$ c) 0 d) None of these

903. If $\sin(x-y)=\cos(x+y)=\frac{1}{2}$, the values of x and y lying between 0° and 90° are given by

a) $x=15°, y=25°$ b) $x=65°, y=15°$ c) $x=45°, y=45°$ d) $x=45°, y=15°$

904. If $5\cos 2\theta+2\cos^2\frac{\theta}{2}+1=0, -\pi<\theta<\pi$, then $\theta=$

a) $\frac{\pi}{3}$ b) $\frac{\pi}{3}, \cos^{-1}(3/5)$ c) $\cos^{-1}(3/5)$ d) $\frac{\pi}{3}, \pi-\cos^{-1}(3/5)$

905. The value of $\cos x\cos y\sin(x-y)+\cos y\cos z\sin(y-z)$ $+\cos z\cos x\sin(z-x)+\sin(x-y)\sin(y-z)\sin(z-x)$, is

a) 0 b) 1 c) 2 d) -1

906. In any ΔABC if $2\cos B=\frac{a}{c}$, then the triangle is

a) Right angled b) Equilateral c) Isosceles d) None of these

907. The equation $\sin x \cos x = 2$ has

a) One solution b) Two solutions c) Infinite solutions d) No solution

908. If the equation $\sin^2 \theta - \cos \theta = \frac{1}{4}$, then the value of θ lying in the interval $0 \leq \theta \leq 2\pi$ is

a) $\frac{\pi}{3}, \frac{5\pi}{3}$ b) $\frac{\pi}{3}, \frac{2\pi}{3}$ c) $\frac{4\pi}{3}, \frac{5\pi}{3}$ d) $\frac{3\pi}{5}, \frac{\pi}{5}$

909. If in a triangle $ABC, \frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ then $\cos A$ is equal to

a) 1/5 b) 5/7 c) 19/35 d) None of these

910. If $f(x) = \cos^2 x + \sec^2 x$, its value always is

a) $f(x) < 1$ b) $f(x) = 1$ c) $2 > f(x) > 1$ d) $f(x) \geq 2$

911. The values of x between 0 and 2π which satisfy the equation $\sin x\sqrt{8 \cos^2 x} = 1$ are in AP. The common difference of the AP is

a) $\frac{\pi}{8}$ b) $\frac{\pi}{4}$ c) $\frac{3\pi}{8}$ d) $\frac{5\pi}{8}$

912. The maximum value of $12 \sin \theta - 9 \sin^2 \theta$ is

a) 3 b) 4 c) 5 d) None of these

913. $\tan|x| = |\tan x|$, if

a) $x \in \left(-k\,\pi, (2k-1)\frac{\pi}{2}\right), k \in Z$

b) $x \in \left((2k-1)\frac{\pi}{2}, k\,\pi\right), k \in Z$

c) $x \in \left(-(2k+1)\frac{\pi}{2}, -k\,\pi\right) \cup \left(k\,\pi, (2k+1)\frac{\pi}{2}\right), k \in Z$

d) None of these

914. If $\tan \theta + \tan 2\theta + \sqrt{3} \tan \theta \tan 2\theta = \sqrt{3}$, then

a) $\theta = \frac{(6n+1)\pi}{18}, \forall\, n \in I$ b) $\theta = \frac{(6n+1)\pi}{9}, \forall\, n \in I$

c) $\theta = \frac{(3n+1)\pi}{9}, \forall\, n \in I$ d) None of these

915. If in a ΔABC, we define $x = \tan\frac{B-C}{2}\tan\frac{A}{2}$, $y = \tan\frac{C-A}{2}\tan\frac{B}{2}$ and $z = \tan\frac{A-B}{2}\tan\frac{C}{2}$, then $x + y + z =$

a) xyz b) x^2yz c) $x^2y^2z^2$ d) None of these

916. If $\cos x = 3 \cos y$, then $2 \tan\frac{y-x}{2}$ is equal to

a) $\cot\left(\frac{y-x}{2}\right)$ b) $\cot\left(\frac{x+y}{4}\right)$ c) $\cot\left(\frac{y-x}{4}\right)$ d) $\cot\left(\frac{x+y}{2}\right)$

917. The value of $\frac{\sin 85° - \sin 35°}{\cos 65°}$ is

a) 2 b) −1 c) 1 d) 0

918. $\frac{\tan 80° - \tan 10°}{\tan 70°}$ is equal to

a) 0 b) 1 c) 2 d) 3

919. If $\tan^2 \theta = 2 \tan^2 \phi + 1$, then $\cos 2\theta + \sin^2 \phi$ equals

a) −1 b) 0 c) 1 d) None of these

920. Simplest form of $\frac{2}{\sqrt{2+\sqrt{2+\sqrt{2+2\cos 4x}}}}$ is

a) $\sec\frac{x}{2}$ b) $\sec x$ c) $\text{cosec}\, x$ d) 1

: ANSWER KEY :

1)	a	2)	b	3)	d	4)	a	189)	a	190)	c	191)	d	192)	c
5)	c	6)	c	7)	b	8)	d	193)	c	194)	a	195)	b	196)	d
9)	b	10)	a	11)	a	12)	b	197)	a	198)	d	199)	c	200)	a
13)	b	14)	b	15)	a	16)	a	201)	b	202)	c	203)	a	204)	a
17)	d	18)	a	19)	a	20)	c	205)	c	206)	c	207)	b	208)	a
21)	a	22)	a	23)	a	24)	b	209)	a	210)	a	211)	b	212)	b
25)	b	26)	c	27)	d	28)	b	213)	c	214)	a	215)	c	216)	d
29)	c	30)	c	31)	c	32)	a	217)	b	218)	a	219)	a	220)	d
33)	b	34)	c	35)	b	36)	a	221)	b	222)	d	223)	c	224)	c
37)	c	38)	a	39)	b	40)	a	225)	b	226)	b	227)	c	228)	c
41)	d	42)	a	43)	b	44)	c	229)	b	230)	b	231)	b	232)	a
45)	a	46)	b	47)	b	48)	d	233)	b	234)	c	235)	a	236)	a
49)	a	50)	b	51)	a	52)	a	237)	b	238)	a	239)	b	240)	a
53)	c	54)	b	55)	d	56)	c	241)	c	242)	c	243)	a	244)	d
57)	d	58)	c	59)	c	60)	a	245)	b	246)	b	247)	a	248)	d
61)	c	62)	c	63)	c	64)	c	249)	a	250)	d	251)	b	252)	a
65)	b	66)	d	67)	b	68)	c	253)	d	254)	a	255)	b	256)	a
69)	c	70)	c	71)	d	72)	c	257)	b	258)	c	259)	b	260)	a
73)	b	74)	b	75)	b	76)	c	261)	d	262)	a	263)	a	264)	d
77)	c	78)	a	79)	b	80)	d	265)	d	266)	d	267)	b	268)	c
81)	b	82)	d	83)	c	84)	b	269)	b	270)	d	271)	b	272)	a
85)	d	86)	c	87)	c	88)	c	273)	a	274)	b	275)	b	276)	c
89)	b	90)	c	91)	d	92)	c	277)	b	278)	a	279)	c	280)	b
93)	b	94)	d	95)	a	96)	a	281)	c	282)	c	283)	b	284)	a
97)	d	98)	c	99)	c	100)	b	285)	c	286)	d	287)	b	288)	b
101)	c	102)	c	103)	c	104)	d	289)	c	290)	b	291)	c	292)	c
105)	b	106)	c	107)	b	108)	d	293)	c	294)	c	295)	b	296)	a
109)	a	110)	a	111)	b	112)	a	297)	d	298)	d	299)	b	300)	b
113)	a	114)	a	115)	a	116)	d	301)	c	302)	b	303)	a	304)	b
117)	c	118)	b	119)	b	120)	c	305)	a	306)	d	307)	a	308)	c
121)	d	122)	b	123)	b	124)	b	309)	c	310)	b	311)	a	312)	d
125)	a	126)	a	127)	a	128)	a	313)	b	314)	c	315)	d	316)	d
129)	b	130)	d	131)	b	132)	a	317)	a	318)	a	319)	b	320)	b
133)	b	134)	d	135)	b	136)	d	321)	a	322)	d	323)	b	324)	a
137)	b	138)	c	139)	d	140)	b	325)	a	326)	d	327)	c	328)	b
141)	c	142)	b	143)	a	144)	b	329)	b	330)	b	331)	a	332)	d
145)	b	146)	b	147)	c	148)	d	333)	a	334)	b	335)	a	336)	b
149)	c	150)	b	151)	a	152)	a	337)	c	338)	b	339)	b	340)	c
153)	d	154)	a	155)	b	156)	a	341)	a	342)	a	343)	c	344)	d
157)	a	158)	a	159)	c	160)	a	345)	b	346)	a	347)	a	348)	b
161)	c	162)	a	163)	d	164)	b	349)	b	350)	c	351)	c	352)	c
165)	a	166)	c	167)	b	168)	b	353)	c	354)	d	355)	b	356)	b
169)	c	170)	a	171)	a	172)	a	357)	c	358)	d	359)	d	360)	c
173)	d	174)	d	175)	b	176)	d	361)	d	362)	a	363)	a	364)	c
177)	a	178)	a	179)	d	180)	d	365)	c	366)	d	367)	b	368)	b
181)	d	182)	a	183)	d	184)	c	369)	c	370)	a	371)	b	372)	b
185)	b	186)	d	187)	c	188)	c	373)	a	374)	c	375)	b	376)	d

377)	d	378)	b	379)	a	380)	d	581)	c	582)	b	583)	a	584)	b
381)	c	382)	b	383)	a	384)	d	585)	b	586)	a	587)	a	588)	a
385)	d	386)	b	387)	d	388)	a	589)	c	590)	d	591)	c	592)	d
389)	b	390)	b	391)	a	392)	c	593)	b	594)	c	595)	b	596)	c
393)	a	394)	d	395)	c	396)	c	597)	a	598)	c	599)	b	600)	d
397)	a	398)	d	399)	b	400)	c	601)	a	602)	b	603)	b	604)	b
401)	c	402)	a	403)	a	404)	b	605)	b	606)	a	607)	d	608)	d
405)	a	406)	b	407)	a	408)	b	609)	b	610)	a	611)	c	612)	a
409)	b	410)	c	411)	a	412)	c	613)	b	614)	c	615)	b	616)	d
413)	a	414)	d	415)	c	416)	a	617)	c	618)	a	619)	b	620)	c
417)	a	418)	a	419)	c	420)	b	621)	d	622)	d	623)	a	624)	c
421)	a	422)	c	423)	c	424)	b	625)	b	626)	b	627)	a	628)	b
425)	b	426)	a	427)	d	428)	c	629)	b	630)	c	631)	b	632)	d
429)	d	430)	b	431)	d	432)	b	633)	c	634)	b	635)	c	636)	d
433)	a	434)	b	435)	a	436)	c	637)	a	638)	b	639)	c	640)	a
437)	a	438)	b	439)	c	440)	b	641)	b	642)	a	643)	c	644)	a
441)	d	442)	a	443)	d	444)	d	645)	a	646)	c	647)	a	648)	b
445)	c	446)	d	447)	c	448)	c	649)	d	650)	b	651)	a	652)	b
449)	b	450)	a	451)	a	452)	a	653)	a	654)	a	655)	c	656)	d
453)	b	454)	a	455)	d	456)	a	657)	c	658)	a	659)	b	660)	a
457)	c	458)	c	459)	d	460)	d	661)	d	662)	a	663)	c	664)	a
461)	a	462)	a	463)	d	464)	b	665)	b	666)	d	667)	b	668)	c
465)	a	466)	c	467)	d	468)	a	669)	c	670)	a	671)	b	672)	a
469)	c	470)	c	471)	a	472)	b	673)	b	674)	d	675)	a	676)	c
473)	b	474)	b	475)	a	476)	b	677)	d	678)	c	679)	b	680)	b
477)	b	478)	b	479)	d	480)	a	681)	a	682)	a	683)	d	684)	a
481)	a	482)	c	483)	d	484)	c	685)	a	686)	b	687)	a	688)	c
485)	a	486)	a	487)	a	488)	b	689)	d	690)	b	691)	a	692)	c
489)	c	490)	b	491)	c	492)	a	693)	a	694)	b	695)	a	696)	a
493)	a	494)	a	495)	d	496)	b	697)	c	698)	a	699)	b	700)	d
497)	c	498)	c	499)	c	500)	c	701)	c	702)	c	703)	c	704)	a
501)	d	502)	a	503)	d	504)	d	705)	c	706)	c	707)	c	708)	b
505)	b	506)	a	507)	a	508)	a	709)	a	710)	b	711)	a	712)	b
509)	a	510)	b	511)	a	512)	c	713)	b	714)	d	715)	d	716)	d
513)	c	514)	d	515)	c	516)	a	717)	c	718)	b	719)	a	720)	c
517)	a	518)	c	519)	a	520)	a	721)	c	722)	a	723)	a	724)	a
521)	c	522)	d	523)	c	524)	d	725)	d	726)	c	727)	d	728)	c
525)	b	526)	a	527)	a	528)	b	729)	d	730)	c	731)	d	732)	c
529)	a	530)	c	531)	a	532)	c	733)	a	734)	a	735)	d	736)	c
533)	d	534)	d	535)	c	536)	d	737)	b	738)	a	739)	b	740)	b
537)	c	538)	b	539)	a	540)	c	741)	a	742)	c	743)	c	744)	c
541)	c	542)	a	543)	a	544)	c	745)	b	746)	c	747)	a	748)	a
545)	d	546)	d	547)	b	548)	d	749)	b	750)	d	751)	b	752)	a
549)	b	550)	a	551)	c	552)	a	753)	b	754)	d	755)	a	756)	b
553)	d	554)	d	555)	b	556)	b	757)	a	758)	b	759)	c	760)	c
557)	a	558)	b	559)	c	560)	c	761)	c	762)	a	763)	d	764)	b
561)	c	562)	a	563)	c	564)	b	765)	a	766)	c	767)	d	768)	c
565)	d	566)	a	567)	a	568)	a	769)	b	770)	b	771)	b	772)	c
569)	c	570)	c	571)	b	572)	d	773)	a	774)	b	775)	c	776)	b
573)	d	574)	b	575)	a	576)	a	777)	a	778)	a	779)	c	780)	c
577)	d	578)	a	579)	a	580)	c	781)	a	782)	b	783)	a	784)	c

785)	c	786)	d	787)	c	788)	c
789)	a	790)	c	791)	b	792)	b
793)	b	794)	d	795)	a	796)	b
797)	a	798)	a	799)	a	800)	c
801)	a	802)	c	803)	b	804)	b
805)	a	806)	d	807)	c	808)	a
809)	b	810)	c	811)	a	812)	a
813)	c	814)	c	815)	a	816)	b
817)	d	818)	a	819)	d	820)	b
821)	a	822)	d	823)	b	824)	b
825)	c	826)	a	827)	b	828)	a
829)	a	830)	a	831)	a	832)	a
833)	a	834)	a	835)	d	836)	c
837)	d	838)	a	839)	a	840)	d
841)	b	842)	a	843)	b	844)	d
845)	c	846)	b	847)	a	848)	a
849)	c	850)	c	851)	a	852)	a
853)	b	854)	b	855)	b	856)	d

857)	b	858)	b	859)	c	860)	d
861)	c	862)	d	863)	a	864)	c
865)	b	866)	c	867)	d	868)	a
869)	a	870)	b	871)	a	872)	a
873)	a	874)	b	875)	b	876)	c
877)	c	878)	c	879)	c	880)	b
881)	b	882)	b	883)	d	884)	a
885)	d	886)	c	887)	d	888)	b
889)	c	890)	a	891)	c	892)	b
893)	d	894)	b	895)	b	896)	b
897)	c	898)	a	899)	d	900)	b
901)	d	902)	b	903)	d	904)	d
905)	a	906)	c	907)	d	908)	a
909)	a	910)	d	911)	b	912)	b
913)	c	914)	c	915)	d	916)	d
917)	c	918)	c	919)	c	920)	a

: HINTS AND SOLUTIONS :

1 **(a)**

It is given that $\tan\theta, \cos\theta, \frac{1}{6}\sin\theta$ are in G.P.

$\therefore \cos^2\theta = \tan\theta \times \frac{1}{6}\sin\theta$

$\Rightarrow 6\cos^3\theta = \sin^2\theta$

$\Rightarrow 6\cos^3\theta + \cos^2\theta - 1 = 0$

$\Rightarrow (2\cos\theta - 1)(3\cos^2\theta + 2\cos\theta + 1) = 0$

$\Rightarrow \cos\theta = \frac{1}{2}$ $[\because 3\cos^2\theta + 2\cos\theta + 1 \neq 0$ for real $\theta]$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}, n \in Z$

2 **(b)**

$\sin 47° - \sin 25° + \sin 61° - \sin 11°$

$= 2\cos 36° \sin 11° + 2\cos 36° \sin 25°$

$= 2\cos 36°[\sin 11° + \sin 25°]$

$= 2\cos 36°\left[2\sin\left(\frac{25° + 11°}{2}\right)\cos\left(\frac{25° - 11°}{2}\right)\right]$

$= 4\cos 36° \sin 18° \cos 7°$

$= 4\left(\frac{\sqrt{5}+1}{4}\right)\left(\frac{\sqrt{5}-1}{4}\right)\cos 7° = \frac{5-1}{4}\cos 7°$

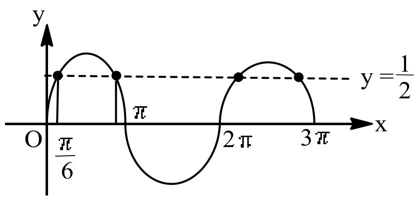
$= \cos 7°$

3 **(d)**

Given, $2\sin^2 x + 5\sin x - 3 = 0$

$\Rightarrow (2\sin x - 1)(\sin x + 3) = 0$

$\Rightarrow \sin x = \frac{1}{2}$ $[\because \sin x \neq -3]$



It is clear from figure that the curve intersect the line at four points in the given interval

Hence, number of solutions are 4

4 **(a)**

We have,

$\sec\theta\tan\theta = \sqrt{2}$

$\Rightarrow \sin\theta = \sqrt{2}\cos^2\theta$

$\Rightarrow \sin\theta = \sqrt{2} - \sqrt{2}\sin^2\theta$

$\Rightarrow \sqrt{2}\sin^2\theta + \sin\theta - \sqrt{2} = 0$

$\Rightarrow (\sqrt{2}\sin\theta - 1)(\sin\theta + \sqrt{2}) = 0$

$\Rightarrow \sqrt{2}\sin\theta - 1 = 0$

$\Rightarrow \sin\theta = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4} \Rightarrow \theta = n\pi + (-1)^n\frac{\pi}{4}, n \in Z$

5 **(c)**

We have,

$\cos(\theta + \phi) = m\cos(\theta - \phi)$

$\Rightarrow \frac{1}{m} = \frac{\cos(\theta - \phi)}{\cos(\theta + \phi)}$

$\Rightarrow \frac{1+m}{1-m} = \frac{2\cos\theta\cos\phi}{2\sin\theta\sin\phi}$

$\Rightarrow \tan\theta\tan\phi = \frac{1-m}{1+m} \Rightarrow \tan\theta = \frac{1-m}{1+m}\cot\phi$

6 **(c)**

We have,

$\sin(\pi\cos\theta) = \cos(\pi\sin\theta)$

$\Rightarrow \sin(\pi\cos\theta) = \sin\left(\frac{\pi}{2} - \pi\sin\theta\right)$

$\Rightarrow \pi\cos\theta = \frac{\pi}{2} - \pi\sin\theta$

$\Rightarrow \cos\theta + \sin\theta = \frac{1}{2}$

$\Rightarrow \frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta = \frac{1}{2\sqrt{2}}$

$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}$

7 **(b)**

Clearly,

$a^2 + b^2 \geq a^2 - b^2$ for all $|a| \neq |b| \neq 0$

$\Rightarrow \frac{a^2+b^2}{a^2-b^2} \geq 1$ or, $\frac{a^2+b^2}{a^2-b^2} \leq -1$

$\therefore \sec\theta = \frac{a^2+b^2}{a^2-b^2}$ is meaningful

Thus, $\sec\theta = \frac{a^2+b^2}{a^2-b^2}$ gives real values of θ if and only if

$|a| \neq |b| \neq 0$

8 **(d)**

Given that, $\sin A = \frac{1}{\sqrt{10}}$ and $\sin B = \frac{1}{\sqrt{5}}$

We know that,

$\sin(A + B) = \sin A\cos B + \sin B\cos A$

$= \frac{1}{\sqrt{10}}\sqrt{1 - \frac{1}{5}} + \frac{1}{\sqrt{5}}\sqrt{1 - \frac{1}{10}}$

$= \frac{1}{\sqrt{10}}\sqrt{\frac{4}{5}} + \frac{1}{\sqrt{5}}\sqrt{\frac{9}{10}}$

$= \frac{1}{\sqrt{50}}(2 + 3) = \sqrt{\frac{5}{\sqrt{50}}} = \frac{1}{\sqrt{2}}$

$\Rightarrow \sin(A + B) = \sin\frac{\pi}{4}$

$\Rightarrow A + B = \frac{\pi}{4}$

9 **(b)**

Now, $1+|\cos x|+\cos^2 x+|\cos^3 x|+\ldots\infty = 11-/\cos x/$

$\therefore\ \frac{1}{8^{1-|\cos x|}}=4^3$

$\Rightarrow\ \frac{3}{2^{1-|\cos x|}}=2^6\ \Rightarrow\ 1=2-2|\cos x|$

$\Rightarrow |\cos x|=\frac{1}{2}$

$\Rightarrow \cos x=\pm\frac{1}{2}$

$\Rightarrow x=\frac{\pi}{3},-\frac{\pi}{3},\frac{2\pi}{3},-\frac{2\pi}{3}$

$\therefore$ Number of solutions = 4

10 **(a)**

We have,

$\sin\frac{15\pi}{32}\sin\frac{7\pi}{16}\sin\frac{3\pi}{8}$

$=\sin\frac{15\pi}{32}\sin\frac{14\pi}{32}\sin\frac{12\pi}{32}$

$=\cos\frac{\pi}{32}\cos\frac{2\pi}{32}\cos\frac{4\pi}{32}$

$=\frac{\sin\left(2^3\times\frac{\pi}{32}\right)}{2^3\sin\frac{\pi}{32}}=\frac{1}{8\sqrt{2}\cos\left(\frac{15\pi}{32}\right)}$

11 **(a)**

$\sin\alpha+\sin\beta+\sin\gamma-\sin(\alpha+\beta+\gamma)$

$=\sin\alpha+\sin\beta+\sin\gamma$
$\quad-\sin\alpha\cos\beta\cos\gamma$
$\quad-\cos\alpha\sin\beta\cos\gamma$
$\quad-\cos\alpha\cos\beta\sin\gamma$
$\quad+\sin\alpha\sin\beta\sin\gamma$

$=\sin\alpha(1-\cos\beta\cos\gamma)+\sin\beta(1-\cos\alpha\cos\gamma)$
$\quad+\sin\gamma(1-\cos\alpha\cos\beta)$
$\quad+\sin\alpha\sin\beta\sin\gamma$

$\therefore \sin\alpha+\sin\beta+\sin\gamma>\sin(\alpha+\beta+\gamma)$

$\Rightarrow\ \frac{\sin(\alpha+\beta+\gamma)}{\sin\alpha+\sin\beta+\sin\gamma}<1$

12 **(b)**

$\left(\cos\frac{10\pi}{13}+\cos\frac{3\pi}{13}\right)+\left(\cos\frac{8\pi}{13}+\cos\frac{5\pi}{13}\right)$

$=2\cos\left(\frac{13\pi}{2\times13}\right).\cos\left(\frac{7\pi}{2\times13}\right)$
$\quad+2\cos\left(\frac{13\pi}{2\times13}\right).\cos\left(\frac{3\pi}{2\times13}\right)$

$=2\cos\frac{\pi}{2}\left(\cos\frac{7\pi}{26}+\cos\frac{3\pi}{26}\right)=0$

13 **(b)**

$\sin 120°\cos 150°-\cos 240°\sin 330°$

$=-\cos 30°\sin 60°-\cos 60°\sin 30°$

$=-\sin(60°+30°)=-1$

14 **(b)**

We have,

$\frac{a}{\sin A}=\frac{b}{\sin B}$

$\Rightarrow \sin B=\frac{b\sin A}{a}=\frac{8\sin 30°}{7}=\frac{4}{7}$

Thus, we have, $b>a>b\sin A$

Hence, angle B has two values given by

$\sin B=4/7$

15 **(a)**

Given, $\cos(\theta-\alpha),\cos\theta$ and $\cos(\theta+\alpha)$ are in HP

$\Rightarrow\ \frac{1}{\cos(\theta-\alpha)},\frac{1}{\cos\theta},\frac{1}{\cos(\theta+\alpha)}$ will be in AP

$\Rightarrow\frac{2}{\cos\theta}=\frac{1}{\cos(\theta-\alpha)}+\frac{1}{\cos(\theta+\alpha)}$

$=\frac{\cos(\alpha+\theta)+\cos(\theta-\alpha)}{\cos^2\theta-\sin^2\alpha}$

$\Rightarrow\frac{2}{\cos\theta}=\frac{2\cos\theta\cos\alpha}{\cos^2\theta-\sin^2\alpha}$

$\Rightarrow\cos^2\theta-\sin^2\alpha=\cos^2\theta\cos\alpha$

$\Rightarrow\cos^2\theta(1-\cos\alpha)=\sin^2\alpha$

$\Rightarrow\cos^2\theta\left(2\sin^2\frac{\alpha}{2}\right)=4\sin^2\frac{\alpha}{2}\cos^2\frac{\alpha}{2}$

$\Rightarrow\cos^2\theta\sec^2\frac{\alpha}{2}=2\ \Rightarrow\cos\theta\sec\frac{\alpha}{2}=\pm\sqrt{2}$

16 **(a)**

$\because\ \sin 2x+\cos 4x=2$

It is possible only when

$\sin 2x=1$ and $\cos 4x=1$

$\Rightarrow 2x=2n\pi+\frac{\pi}{2}$ and $2x=2m\pi$

$\therefore\ x=n\pi+\frac{\pi}{4}$ and $x=m\pi, n\in I$

Then, solution= $\left(n\pi+\frac{\pi}{4},n\in I\right)\cap(m\pi,m\in I)=\phi$

17 **(d)**

We have,

$\text{cosec}^2\theta=\frac{2}{1-\cos 2\theta}$

$\therefore \text{cosec}^2\frac{\pi}{7}+\text{cosec}^2\frac{2\pi}{7}+\text{cosec}^2\frac{3\pi}{7}$

$=\frac{2}{1-\cos\frac{2\pi}{7}}+\frac{2}{1-\cos\frac{4\pi}{7}}+\frac{2}{1-\cos\frac{6\pi}{7}}$

$=\frac{2}{1-a}+\frac{2}{1-b}+\frac{2}{1-c}$, where

$a=\cos\frac{2\pi}{7}, b=\cos\frac{4\pi}{7}, c=\cos\frac{6\pi}{7}$

$$= 2\left\{\frac{3-2(a+b+c)+ab+bc+ca}{1-abc+ab+bc+ca-(a+b+c)}\right\}$$

We know that

$$\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}+\cos\frac{4\pi}{7}\cos\frac{6\pi}{7}+\cos\frac{6\pi}{7}\cos\frac{2\pi}{7}$$

$$=\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}=-\frac{1}{2}$$

and, $\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\cos\frac{6\pi}{7}=18$

i.e. $ab+bc+ca=a+b+c=-\frac{1}{2}$ and $abc=\frac{1}{8}$

$$\therefore \operatorname{cosec}^2\frac{\pi}{7}+\operatorname{cosec}^2\frac{2\pi}{7}+\operatorname{cosec}^2\frac{3\pi}{7}$$

$$=\frac{2\left\{3-2\left(\frac{1}{2}\right)+\frac{-1}{2}\right\}}{1-\frac{1}{8}-\frac{1}{2}+\frac{1}{2}}=8$$

18 **(a)**

$4\sin^2 x+3\cos^2 x=4\sin^2 x+3-3\sin^2 x$

$=\sin^2 x+3$

Maximum value of $\sin x$ is 1 at $x=\frac{\pi}{2}$

Maximum value= $(1)^2+3=4$

19 **(a)**

$\sqrt{2+\sqrt{2+\sqrt{2+\cdots+\sqrt{2(1+\cos\theta)}}}}$ (n numbers of 2"s)

$=\sqrt{2+\sqrt{2+\sqrt{2+\ldots+\sqrt{\left(2+2\cos\frac{\theta}{2}\right)}}}}$ [($n-1$)numbers of 2's]

… … … … …

… … … … …

$=\sqrt{2+2\cos(\theta/2^{n-1})}$

$=\sqrt{2\{1+2\cos^2(\theta/2^n)-1\}}=2\cos(\theta/2^n)$

20 **(c)**

Given, $S_n=\cos^n\theta+\sin^n\theta$

$\therefore 3S_4-2S_6=3[(\cos^4\theta+\sin^4\theta)]-2[\cos^6\theta+\sin^6\theta]$

$=3[(\cos^2\theta+\sin^2\theta)^2-2\sin^2\theta\cos^2\theta]$

$-2[(\cos^2\theta+\sin^2\theta)(\cos^4\theta+\sin^4\theta-\cos^2\theta\sin^2\theta)]$

$=3[1-2\sin^2\theta\cos^2\theta]-2(\cos^2\theta+\sin^2\theta)$

$[(\cos^2\theta+\sin^2\theta)-3\cos^2\theta\sin^2\theta]$

$=3-6\sin^2\theta\cos^2\theta-2+6\cos^2\theta\sin^2\theta$

$=1$

21 **(a)**

Since, $\tan 2x=\tan\frac{2}{x}$

$\Rightarrow 2x=n\pi+\frac{2}{x}\Rightarrow 2x^2-n\pi x-2=0$

$\Rightarrow x=\frac{n\pi\pm\sqrt{n^2\pi^2+16}}{4}$

22 **(a)**

We have,

$\frac{\tan 3x-\tan 2x}{1+\tan 3x\tan 2x}=1$

$\Rightarrow \tan(3x-2x)=1\Rightarrow \tan x=1\Rightarrow x=n\pi+\frac{\pi}{4}$

But, for this value of x, we have

$\tan 2x=\tan(2n\pi+\pi/2)=\infty$

Which does not satisfy the given equation as it reduces to an indeterminate form

23 **(a)**

Since, $(\cot\alpha_1)(\cot\alpha_2)\ldots(\cot\alpha_n)=1$

$\Rightarrow (\cos\alpha_1)(\cos\alpha_1)\ldots(\cos\alpha_n)=(\sin\alpha_1)(\sin\alpha_2)\ldots(\sin\alpha_n)$

$\Rightarrow \cos^2\alpha_1\cos^2\alpha_{2\ldots}\cos^2\alpha_n=\frac{\sin 2\alpha_1\sin 2\alpha_2\ldots\sin 2\alpha_n}{2^n}$

$\Rightarrow \cos\alpha_1\cos\alpha_{2\ldots}\cos\alpha_n=\left(\frac{\sin 2\alpha_1\sin 2\alpha_{2\ldots}\sin 2\alpha_n}{2^n}\right)^{1/2}$

Since, maximum value of $\sin\alpha=1$

$\therefore$ Maximum value of $\cos\alpha_1\ldots\cos\alpha_n=\frac{1}{2^{n/2}}$

24 **(b)**

We have, $A+B+C=\pi$

$\therefore \tan\frac{A}{2}\tan\frac{B}{2}+\tan\frac{B}{2}\tan\frac{C}{2}+\tan\frac{C}{2}+\tan\frac{A}{2}=1$

$\Rightarrow xy+yz+zx=1$

Where $x=\tan\frac{A}{2}, y=\tan\frac{B}{2}, z=\tan\frac{C}{2}$

Now, $(x-y)^2+(y-z)^2+(z-x)^2\geq 0$

$\Rightarrow 2\sum x^2\geq 2\sum xy$

$\Rightarrow \sum x^2\geq\sum xy$

$\Rightarrow \sum x^2\geq 1$ $\quad\left[\because \sum xy=1\ (\text{From (i)})\right]$

$\Rightarrow \tan^2\frac{A}{2}+\tan^2\frac{B}{2}+\tan^2\frac{C}{2}\geq 1$

Thus, the minimum value of $\tan^2\frac{A}{2}+\tan^2\frac{B}{2}+$ $\tan 2C2$ is 1

25 **(b)**

Let a be the first term and d be the common difference of the A.P. Then, $d = 5°, a = 120°$.
Since the sum of all interior angles of a polygon of n sides is
$(2n - 4) \times 90° = (180n - 360°)$
$\therefore \frac{n}{2}\{240 + (n - 1)5\} = 180n - 360$
$\Rightarrow \frac{n}{2}[48 + n - 1] = 36n - 72$
$\Rightarrow n^2 + 47n = 72n - 144$
$\Rightarrow n^2 - 25n + 144 = 0 \Rightarrow n = 16, 9$
For $n = 16$, the last term of the A.P. is more than $180°$. Therefore, $n \neq 16$. Hence, $n = 9$

26 **(c)**
$3\sin^2 x - 7\sin x + 2 = 0$
$\Rightarrow 3\sin^2 x - 6\sin x - \sin x + 2 = 0$
$\Rightarrow 3\sin x(\sin x - 2) - 1(\sin x - 2) = 0$
$\Rightarrow (3\sin x - 1)(\sin x - 2) = 0$
$\Rightarrow \sin x = \frac{1}{2}$ or 2
$\Rightarrow \sin x = \frac{1}{3} \quad (\because \sin x \neq 2)$
Let $\sin^{-1}\frac{1}{3} = \alpha, 0 < \alpha < \frac{\pi}{2}$
Then, $\alpha, \pi - \alpha, 2\pi + \alpha, 3\pi - \alpha, 4\pi + \alpha, 5\pi - \alpha$ are the solutions in $[0, 5\pi]$
$\therefore$ Required number of solutions = 6

27 **(d)**
We have,
$\tan(A + B) = p$ and $\tan(A - B) = q$
$\therefore \tan 2A = \tan\{(A + B) + (A - B)\}$
$\Rightarrow \tan 2A = \frac{\tan(A + B) + \tan(A - B)}{1 - \tan(A + B)\tan(A - B)} = \frac{p + q}{1 - pq}$

28 **(b)**
Given that, $\tan\theta = \sqrt{3} = \tan\frac{\pi}{3} \Rightarrow \theta = n\pi + \frac{\pi}{3}$
For $-\pi < \theta < 0$ put $n = -1$, we get
$\theta = -\pi + \frac{\pi}{3} = \frac{-2\pi}{3}$ or $\frac{-4\pi}{6}$

29 **(c)**
We have, $\sin^2\theta + \sin\theta - 2 = 0$
$\Rightarrow (\sin\theta - 1)(\sin\theta + 2) = 0$
$\Rightarrow \sin\theta = 1, \sin\theta = -2$
But $\sin\theta \neq -2$
$\therefore \sin\theta = 1 = \sin\frac{\pi}{2}$
$\Rightarrow \theta = n\pi + (-1)^n\frac{\pi}{2}$

30 **(c)**
Given, $\tan x + \sec x = 2\cos x$
On multiplying by $\cos x \neq 0$, we get
$\sin x + 1 = 2\cos^2 x$
$\Rightarrow \sin x + 1 = 2(1 - \sin x)(1 + \sin x)$
$\Rightarrow (\sin x + 1)(2\sin x - 1) = 0$
$\Rightarrow \sin x = -1$ and $\sin x = \frac{1}{2}$
$\because \sin x \neq -1 \quad (\because \cos x \neq 0)$
$\therefore \sin x = \frac{1}{2}$
$\Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$

32 **(a)**
We have, $2\cos^2 x + 3\sin x - 3 = 0$
$\Rightarrow 2 - 2\sin^2 x + 3\sin x - 3 = 0$
$\Rightarrow (2\sin x - 1)(\sin x - 1) = 0$
$\Rightarrow \sin x = \frac{1}{2}$ or $\sin x = 1$
$\Rightarrow x = 30°, 150°, 90°$

33 **(b)**
We have,
$$\frac{x}{\cos\theta} = \frac{y}{\cos\left(\theta - \frac{2\pi}{3}\right)} = \frac{z}{\cos\left(\theta + \frac{2\pi}{3}\right)}$$
Therefore, each ratio is equal to
$$\frac{x + y + z}{\cos\theta + \cos\left(\theta - \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{2\pi}{3}\right)} = \frac{x + y + z}{0}$$
$\Rightarrow x + y + z = 0$

34 **(c)**
Given, $a\sec\alpha = d + c\tan\alpha$ ...(i)

and $b\sec\alpha = c - d\tan\alpha$ ... (ii)

On squaring and adding Eqs. (i) and (ii), we get

$$(a^2 + b^2)\sec^2\alpha = d^2 + c^2\tan^2\alpha + 2dc\tan\alpha + c^2 + d^2\tan^2\alpha - 2dc\tan\alpha$$

$$\Rightarrow (a^2 + b^2)\sec^2\alpha = c^2(\tan^2\alpha + 1) + d^2(1 + \tan^2\alpha)$$

$= (c^2 + d^2)\sec^2\alpha$

$\therefore a^2 + b^2 = c^2 + d^2$

35 **(b)**
$$\cos\frac{\pi}{65}\cos\frac{2\pi}{65}\ldots\cos\frac{32\pi}{65} = \cos\frac{\pi}{65}.\cos\frac{2\pi}{65}\ldots\cos\frac{2^5\pi}{65}$$

$$= \frac{\sin\frac{2^6\pi}{65}}{2^6\sin\frac{\pi}{65}} = \frac{\sin\frac{65\pi}{65}}{64\sin\frac{\pi}{65}}$$

$$= \frac{\sin\left(\pi - \frac{\pi}{65}\right)}{64 \sin\frac{\pi}{65}} = \frac{1}{64}$$

36 **(a)**

The LHS of the given equation is less than 2 and RHS is greater than or equal to 2. Therefore, the equation has no solution

37 **(c)**

We have,

$\sin A = \sin B, \cos A = \cos B \Rightarrow A = 2\,n\,\pi + B$

Clearly, this satisfies both the relations for all $n \in Z$

38 **(a)**

We have,

$$\frac{3 + \cot 76° \cot 16°}{\cot 76° + \cot 16°}$$

$$= \frac{3 \sin 76° \sin 16° + \cos 76° \cos 16°}{\cos 76° \sin 16° + \sin 76° \cos 16°}$$

$$= \frac{2 \sin 76° \sin 16° + (\cos 76° \cos 16° + \sin 76° \sin 16°)}{\sin 76° \cos 16° + \cos 76° \sin 16°}$$

$$= \frac{\cos 60° - \cos 92° + \cos 60°}{\sin 92°}$$

$$= \frac{1 - \cos 92°}{\sin 92°} = \frac{2 \sin^2 46°}{2 \sin 46° \cos 46°} = \tan 46°$$

$$= \cot 44°$$

39 **(b)**

We have,

$1 + \sin^4 x = \cos^2 3x$

$\Rightarrow \sin^2 3x + \sin^4 x = 0$

$\Rightarrow \sin 3x = 0$ and $\sin 4x = 0$

$\Rightarrow 3x = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \pm 4\pi, \pm 5\pi, \pm 6\pi, \pm 7\pi$

and,

$4x = \pm n\pi, n = 0,1,2, \dots ,10 \Rightarrow x = 0, \pm\pi, \pm 2\pi$

The largest positive value of x is 2π

40 **(a)**

We have,

$\cos A + 2 \cos B + \cos C = 2$

$\Rightarrow \cos A + \cos C = 2(1 - \cos B)$

$$\Rightarrow 2 \cos\frac{A + C}{2} \cos\frac{A - C}{2} = 4 \sin^2\frac{B}{2}$$

$$\Rightarrow 2 \cos\left(\frac{A - C}{2}\right) = 4 \sin\frac{B}{2}$$

$$\Rightarrow 2 \cos\frac{B}{2} \cos\left(\frac{A - C}{2}\right) = 2\left(2 \sin\frac{B}{2} \cos\frac{B}{2}\right)$$

$$\Rightarrow 2 \sin\left(\frac{A + C}{2}\right) \cos\left(\frac{A - C}{2}\right) = 2\left(2 \sin\frac{B}{2} \cos\frac{B}{2}\right)$$

$\Rightarrow \sin A + \sin C = 2 \sin B$

$\Rightarrow a + c = 2b \Rightarrow a, b, c$ are in A.P.

42 **(a)**

$\tan A + \tan B = a$ andtan $\tan A \tan B = b$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{a}{1 - b}$$

Now, $\sin^2(A + B) = \frac{1}{2}[1 - \cos 2(A + B)]$

$$= \frac{1}{2}\left[1 - \frac{1 - \tan^2(A + B)}{1 + \tan^2(A + B)}\right]$$

$$= \left[\frac{\tan^2(A + B)}{1 + \tan^2(A + B)}\right]$$

$$= \frac{a^2/(1 - b)^2}{\frac{a^2 + (1 - b)^2}{(1 - b)^2}}$$

$$= \frac{a^2}{a^2 + (1 - b)^2}$$

43 **(b)**

Since, $(a - b)\sin(\theta + \phi) = (a + b)\sin(\theta - \phi)$

$\Rightarrow a\{\sin(\theta + \phi) - \sin(\theta - \phi)\}$
$= b\{\sin(\theta - \phi) + \sin(\theta + \phi)\}$

$\Rightarrow 2a \sin\phi \cos\theta = 2b \sin\theta \cos\phi$

$\Rightarrow a \tan\phi = b \tan\theta$

$$\Rightarrow \frac{2a \tan\frac{\phi}{2}}{1 - \tan^2\frac{\phi}{2}} = \frac{2b \tan\frac{\theta}{2}}{1 - \tan^2\frac{\theta}{2}} \quad \dots(i)$$

Since, $a \tan\frac{\theta}{2} - b \tan\frac{\phi}{2} = c$ (given)

$$\Rightarrow \tan\frac{\theta}{2} = \frac{b \tan\frac{\phi}{2} + c}{a} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{a \tan\frac{\phi}{2}}{1 - \tan^2\frac{\phi}{2}} = \frac{b\frac{\left(b \tan\frac{\phi}{2} + c\right)}{a}}{1 - \frac{\left(b \tan\frac{\phi}{2} + c\right)^2}{a^2}}$$

$$\Rightarrow \tan\frac{\phi}{2}(a^2 - b^2 - c^2) = bc\left(1 + \tan^2\frac{\phi}{2}\right)$$

$$\text{Now, } \sin\phi = \frac{2 \tan\frac{\phi}{2}}{1 + \tan^2\frac{\phi}{2}} = \frac{2bc}{a^2 - b^2 - c^2}$$

44 **(c)**

Given equation is

$\sin x \cos 3x = \sin 3x \cos 5x$

$\Rightarrow 2 \sin x \cos 3x - 2 \sin 3x \cos 5x = 0$

$\Rightarrow \sin(3x + x) - \sin(3x - x) - \sin(3x + 5x)$
$+ \sin(5x - 3x) = 0$

$\Rightarrow \sin 4x - \sin 2x - \sin 8x + \sin 2x = 0$

$\sin 4x - \sin 8x = 0$

$$\Rightarrow 2 \cos\left(\frac{4x + 8x}{2}\right) \sin\left(\frac{8x - 4x}{2}\right) = 0$$

$\Rightarrow 2 \cos 6x \sin 2x = 0$

$\Rightarrow \cos 6x = 0$ or $\sin 2x = 0$

$\Rightarrow \quad 6x = (2n+1)\frac{\pi}{2}$ or $x = \frac{n\pi}{2}$

$\Rightarrow \quad x = (2n+1)\frac{\pi}{12}$ or $x = \frac{n\pi}{2}$

$\Rightarrow \quad x = 0, \frac{\pi}{2}, \frac{\pi}{12}, \frac{3\pi}{12}, \frac{5\pi}{12} \in \left[0, \frac{\pi}{2}\right]$

$\therefore$ Number of solutions is 5

45 **(a)**

$\cos^2 A + \cos^2 B + \cos^2 C$

$= \frac{1+\cos 2A}{2} + \frac{1+\cos 2B}{2} + \cos^2 C$

$= 1 + \frac{1}{2}(\cos 2A + \cos 2B) + \cos^2 C$

$= 1 + \frac{2}{2}[\cos(A+B)\cos(A-B)] + \cos^2 C$

$= 1 + \cos C \cos(A-B) + \cos C \cos(A+B)$

$= 1 + \cos C[\cos(A-B) + \cos(A+B)]$

$= 1 + 2\cos C \cos B \cos A$

$\Rightarrow \cos^2 A + \cos^2 B + \cos^2 C - 2\cos A \cos B \cos C = 1$

46 **(b)**

Given, $\cos x + \sin x = \frac{1}{2}$

$\Rightarrow \quad 1 + \sin 2x = \frac{1}{4}$

$\Rightarrow \quad \frac{2\tan x}{1+\tan x} = \frac{-3}{4}$

$\Rightarrow \quad 8\tan x = -3 - 3\tan^2 x$

$\Rightarrow \quad 3\tan^2 x + 8\tan x + 3 = 0$

$\Rightarrow \quad \tan x = \frac{-8 \pm \sqrt{64-36}}{6} = \frac{-8 \pm 2\sqrt{7}}{6}$

$\Rightarrow = -\left(\frac{4 \pm \sqrt{7}}{3}\right)$

47 **(b)**

We have,

$\cos(\alpha+\beta) = \frac{12}{13}$ and $\sin(\alpha-\beta) = \frac{3}{5}$

$\Rightarrow \sin(\alpha+\beta) = \frac{5}{13}$ and $\cos(\alpha-\beta) = \frac{4}{5}$

Now,

$\sin 2\alpha = \sin\{(\alpha+\beta) + (\alpha-\beta)\}$

$\Rightarrow \sin 2\alpha = \sin(\alpha+\beta)\cos(\alpha-\beta) + \cos(\alpha+\beta)\sin(\alpha-\beta)$

$\Rightarrow \sin 2\alpha = \frac{5}{13} \times \frac{4}{5} + \frac{12}{13} \times \frac{3}{5} = \frac{56}{65}$

48 **(d)**

We have, $\sin^{10} 2x = 1 + \cos^{10} x$

Minimum value of RHS = 1 and maximum values of LHS = 1. Therefore, solution is possible only when $\sin^{10} 2x = 1$ and $\cos^{10} x = 0$. But this is not possible. Therefore, it has no solution.

49 **(a)**

We have,

$P + Q + R = \pi$ and $R = \frac{\pi}{2}$

$\therefore P + Q = \frac{\pi}{2}$

$\Rightarrow \frac{P}{2} + \frac{Q}{2} = \frac{\pi}{4}$

$\Rightarrow \tan\left(\frac{P}{2} + \frac{Q}{2}\right) = 1$

$\Rightarrow \tan\frac{P}{2} + \tan\frac{Q}{2} = 1 - \tan\frac{P}{2}\tan\frac{Q}{2}$ ...(i)

It is given that $\tan\frac{P}{2}$ and $\tan\frac{Q}{2}$ are the roots of the equation $ax^2 + bx + c = 0$

$\therefore \tan\frac{P}{2} + \tan\frac{Q}{2} = -\frac{b}{a}$ and $\tan\frac{P}{2}\tan\frac{Q}{2} = \frac{c}{d}$

Substituting these values in (i), we get

$-\frac{b}{a} = 1 - \frac{c}{a} \Rightarrow -b = a - c \Rightarrow a + b = c$

50 **(b)**

We know that,

$-\sqrt{a^2+b^2} \le a\cos\theta + b\sin\theta \le \sqrt{a^2+b^2}$

$\therefore \quad -\sqrt{3+1} \le \sqrt{3}\sin x + \cos x \le \sqrt{3+1}$

$\Rightarrow \quad -2 \le \sqrt{3}\sin x + \cos x \le 2$

But, $\sqrt{3}\sin x + \cos x = 4$

Hence, given equation has no solution

51 **(a)**

We have,

$\frac{\cos\theta}{a} = \frac{\sin\theta}{b}$

$\Rightarrow \frac{\cos\theta}{a} = \frac{\sin\theta}{b} = \sqrt{\frac{\cos^2\theta + \sin^2\theta}{a^2+b^2}}$

$\Rightarrow \cos\theta = \frac{a}{\sqrt{a^2+b^2}}$ and $\sin\theta = \frac{b}{\sqrt{a^2+b^2}}$

$\therefore \frac{a}{\sec 2\theta} + \frac{b}{\operatorname{cosec} 2\theta} = a\cos 2\theta + b\sin 2\theta$

$\Rightarrow \frac{a}{\sec 2\theta} + \frac{b}{\operatorname{cosec} 2\theta} = a(\cos^2\theta - \sin^2\theta) + 2b\sin\theta\cos\theta$

$\Rightarrow \frac{a}{\sec 2\theta} + \frac{b}{\operatorname{cosec} 2\theta} = a\frac{(a^2-b^2)}{a^2+b^2} + \frac{2ab^2}{a^2+b^2} = a$

52 **(a)**

We have,

$y = \sin\left(x + \frac{\pi}{6}\right) + \cos\left(x + \frac{\pi}{6}\right)$

$= \sqrt{2}\cos\left(x + \frac{\pi}{6} - \frac{\pi}{4}\right)$

$\Rightarrow y = \sqrt{2}\cos\left(x - \frac{\pi}{12}\right)$

$\Rightarrow y$ is maximum for $x - \frac{\pi}{12} = 0$ i.e. $x = \frac{\pi}{12}$

53 **(c)**

We have,

$y - z = a(\cos^2 x - \sin^2 x) + 2b\sin 2x + c(\sin^2 x - \cos^2 x)$

$\Rightarrow y - z = a\cos 2x + 2b\sin 2x - c\cos 2x$

$\Rightarrow y-z=(a-c)\cos 2x+2b\sin 2x$

Now,

$$\cos 2x+\frac{1-\tan^2 x}{1+\tan^2 x}=\frac{(a-c)^2-4b^2}{(a-c)^2+4b^2}$$

And,

$$\sin 2x=\frac{2\tan x}{1+\tan^2 x}=\frac{4b(a-c)}{(a-c)^2+4b^2}$$

$$\therefore y-z=\frac{(a-c)[(a-c)^2-4b^2]+8b^2(a-c)}{(a-c)^2+4b^2}$$

$$=a-c$$

54 **(b)**

It is given that $\frac{1}{6}\sin x, \cos x, \tan x$ are in GP

$\therefore \cos^2 x=\frac{1}{6}\sin x\tan x$

$\Rightarrow 6\cos^2 x=\sin x\tan x$

$\Rightarrow 6\cos^3 x+\cos^2 x-1=0$

$\Rightarrow \left(\cos x-\frac{1}{2}\right)(6\cos^2 x+4\cos x+2)=0$

$\Rightarrow \cos x=\frac{1}{2}$ [$\because \cos^2 x + 4\cos x+2 = 0$ has imaginary roots]

$\Rightarrow \cos x=\cos\frac{\pi}{3}$

$\Rightarrow x=2n\pi\pm\frac{\pi}{3}, n\in Z$

55 **(d)**

We have,

$a^2\sin 2C+c^2\sin 2A$

$=2a^2\sin C\cos C+2c^2\sin A\cos A$

$=2(2R\sin A)^2\sin C\cos C + 2(2R\sin C)^2\sin A\cos A$

$=8R^2\sin^2 A\sin C\cos C+8R^2\sin^2 C\sin A\cos A$

$=8R^2\sin A\sin C\sin(A+C)$

$=8R^2\sin A\sin B\sin C$ $[\because A+C=\pi-B]$

$=8R^2\times\frac{a}{2R}\times\frac{b}{2R}\times\frac{c}{2R}$ $\left[\because \sin A=\frac{a}{2R}, \sin B=\frac{b}{2R}, \text{ and } \sin C=\frac{c}{2R}\right]$

$=\frac{abc}{R}=4\Delta$

56 **(c)**

$e^{\log(\cosh^{-1}2)}=\cosh^{-1}(2)=\log(2+\sqrt{2^2-1})$

$=\log(2+\sqrt{3})$

57 **(d)**

We have, $x+\frac{1}{x}=2\cos\theta$

$\Rightarrow \left(x+\frac{1}{x}\right)^3=(2\cos\theta)^3$

$\Rightarrow x^3+\frac{1}{x^3}+3x.\frac{1}{x}\left(x+\frac{1}{x}\right)=8\cos^3\theta$

$\Rightarrow x^3+\frac{1}{x^3}+3.2\cos\theta=8\cos^3\theta$

$\Rightarrow x^3+\frac{1}{x^3}=2(4\cos^3\theta-3\cos\theta)$

$=2\cos 3\theta$

58 **(c)**

$A+B=\frac{\pi}{4} \Rightarrow \tan(A+B)=1$

$\Rightarrow \frac{\tan A+\tan B}{1-\tan A\tan B}=1$

$\Rightarrow \tan A+\tan B+\tan A\tan B=1$

$\Rightarrow (1+\tan A)(1+\tan B)=1+1=2$

59 **(c)**

We have,

$\cos x\left\{\frac{\cos x}{1-\sin x}+\frac{1-\sin x}{\cos x}\right\}$

$=\cos x\left\{\frac{\cos^2 x+(1-\sin x)^2}{(1-\sin x)\cos x}\right\}=\frac{2-2\sin x}{1-\sin x}$

$=2$ for all $x\in R$

Hence, required value $=2$

60 **(a)**

We have,

$\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}$

$=\left\{-\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\right\}\left\{\cos\frac{3\pi}{15}\cos\frac{6\pi}{15}\right\}$ co

$=-\frac{\sin\left(2^4\frac{\pi}{15}\right)}{2^4\sin\frac{\pi}{15}}\times\frac{\sin\left(2^2\times\frac{3\pi}{15}\right)}{2^2\sin\frac{3\pi}{15}}\times\frac{1}{2}$

$=-\frac{\sin\frac{16\pi}{15}}{16\sin\frac{\pi}{15}}\times\frac{\sin\frac{12\pi}{15}}{4\sin\frac{3\pi}{15}}\times\frac{1}{2}=\frac{1}{16}\times\frac{1}{4}\times\frac{1}{2}=\frac{1}{128}$

62 **(c)**

$\frac{1+\tan\theta}{1-\tan\theta}=3\left(\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right)$

On simplification, we get

$3\tan^4\theta-6\tan^2\theta+8\tan\theta-1=0$

$\therefore$ Product of roots $=\tan\alpha\cdot\tan\beta\cdot\tan\gamma\cdot\tan\delta=-13$

63 **(c)**

We have,

$\tan\frac{C}{2}=\frac{\sqrt{7}}{3}$

$\therefore \cos C=\frac{1-\tan^2\frac{C}{2}}{1+\tan^2\frac{C}{2}}\Rightarrow \cos C=\frac{1-\frac{7}{9}}{1+\frac{7}{9}}=\frac{1}{8}$

Now,

$c^2 = a^2 + b^2 - 2ab\cos C = 25 + 16 - 40 \times \frac{1}{8}$

$= 36$

$\Rightarrow c = 6$

64 **(c)**

$\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + \frac{8}{\tan 8\alpha}$

$= \tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + \frac{4(1-\tan^2 4\alpha)}{\tan 4\alpha}$

$\left[\because \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}\right]$

$= \tan\alpha + 2\tan 2\alpha + \frac{4\tan^2 4\alpha + 4 - 4\tan^2 4\alpha}{\tan 4\alpha}$

$= \tan\alpha + 2\tan 2\alpha + \frac{4(1-\tan^2 2\alpha)}{2\tan 2\alpha}$

$= \tan\alpha + \frac{2\tan^2 2\alpha + 2 - 2\tan^2 2\alpha}{\tan 2\alpha}$

$= \tan\alpha + \frac{2(1-\tan^2\alpha)}{2\tan\alpha}$

$= \frac{\tan^2\alpha + 1 - \tan^2\alpha}{\tan\alpha} = \frac{1}{\tan\alpha} = \cot\alpha$

65 **(b)**

We have,

$\sin\theta - \cos\theta = \underset{x\in R}{\text{Min}}\{1, x^2 - 4x + 6\}$

$\Rightarrow \sin\theta - \cos\theta$

$= 1 \quad \left[\begin{matrix}\because x^2 - 4x + 6 \\ = (x-2)^2 + 2 \geq 2 \text{ for all } x\end{matrix}\right]$

$\Rightarrow \sin\left(\theta - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$\Rightarrow \sin\left(\theta - \frac{\pi}{4}\right) = \sin\frac{\pi}{4}$

$\Rightarrow \theta - \frac{\pi}{4} = n\pi + (-1)^n\frac{\pi}{4}, n \in Z$

$\Rightarrow \theta = n\pi + (-1)^n\frac{\pi}{4} + \frac{\pi}{4}, n \in Z$

66 **(d)**

We have,

$\tan\frac{x}{2} = \text{cosec}\, x - \sin x$

$\Rightarrow \tan\frac{x}{2} = \frac{1+\tan^2\frac{x}{2}}{2\tan\frac{x}{2}} - \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$\Rightarrow 2t(1+t) = (1-t)^2$, where $t = \tan^2\frac{x}{2}$

$\Rightarrow t^2 + 4t - 1 = 0 \Rightarrow t$

$= -2 + \sqrt{5} \quad \left[\because t = \tan^2\frac{x}{2} > 0\right]$

67 **(b)**

We have,

$\frac{\cos C + \cos A}{c + a} + \frac{\cos B}{b}$

$= \frac{b\cos C + b\cos A + c\cos B + a\cos B}{(c+a)b}$

$= \frac{(b\cos C + c\cos B) + (a\cos B + b\cos A)}{(c+a)b}$

$= \frac{a+c}{(c+a)b} = \frac{1}{b}$

68 **(c)**

$\sin 12° \sin 48° \sin 54°$

$= \frac{1}{2}[\cos 36° - \cos 60°]\cos 36°$

$= \frac{1}{2}\left[\frac{\sqrt{5}+1}{4} - \frac{1}{2}\right]\left[\frac{\sqrt{5}+1}{4}\right] = \frac{1}{8}$

69 **(c)**

We have,

$\sin\alpha = \sin\beta, \cos\alpha = \cos\beta$

$\Rightarrow \sin\alpha - \sin\beta = \cos\alpha - \cos\beta = 0$

$\Rightarrow 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

$= 0$

$\Rightarrow \sin\frac{\alpha-\beta}{2} = 0$

70 **(c)**

We have,

$\frac{\sin B}{b} = \frac{\sin A}{a} \Rightarrow \sin B = \frac{b\sin A}{a} = \frac{8\sin 30°}{6} = \frac{2}{3}$

72 **(c)**

We have,

$32\sin\frac{A}{2}\sin\frac{5A}{2}$

$= 16(\cos 2A - \cos 3A)$

$= 16(2\cos^2 A - 1 - 4\cos^3 A + 3\cos A)$

$= 16\left(2 \times \frac{9}{16} - 1 - 4 - \frac{27}{64} + 3 \times \frac{3}{4}\right) = 11$

73 **(b)**

We know that the distance of the orthocentre O of $\Delta\, ABC$ from the vertices are given by

$OA = 2R\cos A, OB = 2R\cos B$ and $OC = 2R\cos C$

$\Rightarrow OA : OB : OC = \cos A : \cos B : \cos C$

74 **(b)**

We have, $\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$\because \tan\alpha=\frac{1}{1+2^{-x}}$ and $\tan\beta=\frac{1}{1+2^{x+1}}$

$\therefore \tan(\alpha+\beta)=\frac{\frac{1}{1+\frac{1}{2^x}}+\frac{1}{1+2^{x+1}}}{1-\frac{1}{1+\frac{1}{2^x}}\cdot\frac{1}{1+2^{x+1}}}$

$\Rightarrow \tan(\alpha+\beta)=\frac{2^x+2\cdot 2^{2x}+2^x+1}{1+2^x+2\cdot 2^x+2\cdot 2^{2x}-2^x}$

$\Rightarrow \tan(\alpha+\beta)=1$

$\Rightarrow \alpha+\beta=\frac{\pi}{4}$

75 **(b)**

We have,

$\tan\left(\frac{\theta+\alpha}{2}\right)\tan\left(\frac{\theta-\alpha}{2}\right)$

$=\frac{2\sin\left(\frac{\theta+\alpha}{2}\right)\sin\left(\frac{\theta-\alpha}{2}\right)}{2\cos\left(\frac{\theta+\alpha}{2}\right)\cos\left(\frac{\theta-\alpha}{2}\right)}$

$=-\frac{(\cos\theta-\cos\alpha)}{\cos\theta+\cos\alpha}$

$=-\frac{(\cos\alpha\cos\beta-\cos\alpha)}{\cos\alpha\cos\beta+\cos\alpha}=-\frac{\cos\alpha\,(\cos\beta-1)}{\cos\alpha\,(\cos\beta+1)}$

$=\frac{1-\cos\beta}{1+\cos\beta}=\tan^2\frac{\beta}{2}$

76 **(c)**

(1) $\cot\theta-\tan\theta=2$

$\Rightarrow\ 2\cot 2\theta+2\ \Rightarrow\ \tan 2\theta=1$

$\Rightarrow\ 2\theta=n\pi+\frac{\pi}{4}\ \Rightarrow\ \theta=(4n+1)\frac{\pi}{8}$

(2) The given equation can be written as

$2\sin x\cos x+2\cos^2 x-1+\sin x+\cos x+1=0$

$\Rightarrow\ (2\cos x+1)(\sin x+\cos x)=0$

$\Rightarrow\ \cos x=-\frac{1}{2}$ or $\sin x+\cos x=0$

$\Rightarrow\ \cos x=-\frac{1}{2}$ or $\tan x=-1$

But $\cos x$ and $\tan x$ are positive in Ist quadrant. Therefore, the equation has no solution in the Ist quadrant. Hence, both of statements are correct.

77 **(c)**

Given equation can be written as

$$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)$$

$$=\left(1+\cos\frac{\pi}{8}+\cos\frac{7\pi}{8}+\cos\frac{\pi}{8}\cos\frac{7\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}+\cos\frac{5\pi}{8}+\cos\frac{3\pi}{8}\cos\frac{5\pi}{8}\right)$$

$$=\left(1+\cos\frac{\pi}{8}-\cos\frac{\pi}{8}+\cos\frac{\pi}{8}\cos\frac{7\pi}{8}\right)\left(1-\cos\frac{5\pi}{8}+\cos\frac{5\pi}{8}+\cos\frac{3\pi}{8}\cos\frac{5\pi}{8}\right)$$

$$=\left(1+\cos\frac{\pi}{8}\cos\frac{7\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\cos\frac{5\pi}{8}\right)$$

$$=\frac{1}{4}\left(2+2\cos\frac{\pi}{8}\cos\frac{7\pi}{8}\right)\left(2+2\cos\frac{3\pi}{8}\cos\frac{5\pi}{8}\right)$$

$$=\frac{1}{4}\left(2+\cos\frac{3\pi}{4}+\cos\pi\right)\left(2+\cos\frac{\pi}{4}\cos\pi\right)$$

$$=\frac{1}{4}\left(1+\cos\frac{3\pi}{4}\right)\left(1+\cos\frac{\pi}{4}\right)$$

$$=\frac{1}{4}\left(1-\cos\frac{\pi}{4}\right)\left(1+\cos\frac{\pi}{4}\right)$$

$$=\frac{1}{4}\left(1-\cos^2\frac{\pi}{4}\right)=\frac{1}{4}\left(1-\frac{1}{2}\right)=\frac{1}{8}$$

78 **(a)**

$\frac{1-\tan^2(45°-A)}{1+\tan^2(45°-A)}$

$=\frac{\cos^2(45°-A)-\sin^2(45°-A)}{\cos^2(45°-A)+\sin^2(45°-A)}$

$=\frac{\cos 2(45°-A)}{1}$

$=\sin 2A$

79 **(b)**

We have, $\tan\alpha = \frac{m}{m+1}$ and $\tan\beta = \frac{1}{2m+1}$

We know that, $\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$$= \frac{\frac{m}{m+1}+\frac{1}{2m+1}}{1-\frac{m}{(m+1)}.\frac{1}{(2m+1)}}$$

$$= \frac{2m^2+m+m+1}{2m^2+m+2m+1-m}$$

$$= \frac{2m^2+2m+1}{2m^2+2m+1} = 1$$

$$\Rightarrow \tan(\alpha+\beta) = \tan\frac{\pi}{4}$$

$$\Rightarrow \alpha+\beta = \frac{\pi}{4}$$

80 **(d)**

Given, $\sin\left(\frac{\pi}{4}\cot\theta\right) = \cos\left(\frac{\pi}{4}\tan\theta\right)$

$\Rightarrow \sin\left(\frac{\pi}{4}\cot\theta\right) = \sin\left(\frac{\pi}{2}-\frac{\pi}{4}\tan\theta\right)$

$\Rightarrow \frac{\pi}{4}(\tan\theta+\cot\theta) = \frac{\pi}{2}$

$\Rightarrow (\tan\theta-1)^2 = 0 \quad \Rightarrow \quad \tan\theta = 1 = \tan\frac{\pi}{4}$

$\therefore \quad \theta = n\pi + \frac{\pi}{4}$

81 **(b)**

$\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$

$\Rightarrow \frac{\sin(x+y)+\sin(x-y)}{\sin(x+y)-\sin(x-y)} = \frac{(a+b)+(a-b)}{(a+b)-(a-b)}$

$\Rightarrow \frac{2\sin x\cos y}{2\cos x\sin y} = \frac{2a}{2b}$

$\Rightarrow \frac{\tan x}{\tan y} = \frac{a}{b}$

82 **(d)**

Given, $\cos\theta - 4\sin\theta = 1$ ...(i)

$\cos^2\theta + 16\sin^2\theta - 8\sin\theta\cos\theta = 1$ [on squaring]

$\Rightarrow 15\sin^2\theta - 8\sin\theta\cos\theta = 0$

$\Rightarrow \sin\theta(15\sin\theta - 8\cos\theta) = 0$

$\Rightarrow \sin\theta = 0$ or $\tan\theta = \frac{8}{15}$

But $\tan\theta$ is not satisfy the Eq. (i),

$\therefore \sin\theta = 0 \Rightarrow \theta = 0, \pi$

At $\theta = 0, \sin\theta + 4\cos\theta = 0 + 4 = 4$

At $\theta = \pi, \sin\theta + 4\cos\theta = 0 - 4 = -4$

83 **(c)**

We have,

$$\frac{\cos(\theta_1-\theta_2)}{\cos(\theta_3+\theta_2)} + \frac{\cos(\theta_3+\theta_4)}{\cos(\theta_3-\theta_4)} = 0$$

$$\Rightarrow \frac{\cos(\theta_1-\theta_2)}{\cos(\theta_1+\theta_2)} = \frac{\cos(\theta_3+\theta_4)}{-\cos(\theta_3-\theta_4)}$$

$$\Rightarrow \frac{\cos(\theta_1-\theta_2)-\cos(\theta_1+\theta_2)}{\cos(\theta_1-\theta_2)+\cos(\theta_1+\theta_2)} = \frac{\cos(\theta_3+\theta_4)+\cos(\theta_3-\theta_4)}{\cos(\theta_3+\theta_4)-\cos(\theta_3-\theta_4)}$$

$$\Rightarrow \frac{2\sin\theta_1\sin\theta_2}{2\cos\theta_1\cos\theta_2} = \frac{2\cos\theta_3\cos\theta_4}{-2\sin\theta_3\sin\theta_4}$$

$$\Rightarrow \tan\theta_1\tan\theta_2\tan\theta_3\tan\theta_4 = -1$$

84 **(b)**

We have, $\cot\theta - \tan\theta = 2$

$\Rightarrow \cos^2\theta - \sin^2\theta = 2\sin\theta\cos\theta$

$\Rightarrow \cos 2\theta = \sin 2\theta$

$\Rightarrow \tan 2\theta = \tan\frac{\pi}{4} \Rightarrow 2\theta = n\pi + \frac{\pi}{4}$

$\Rightarrow \theta = \frac{n\pi}{2} + \frac{\pi}{8}$

85 **(d)**

$\sin 36° \sin 72° \sin 108° \sin 144°$

$= \sin^2 36° \sin^2 72°$

$= \frac{1}{4}[(2\sin^2 36°)(2\sin^2 72°)]$

$= \frac{1}{4}[(1-\cos 72°)(1-\cos 144°)]$

$= \frac{1}{4}[(1-\sin 18°)(1+\cos 36°)]$

$= \frac{1}{4}\left[\left(1-\frac{\sqrt{5}-1}{4}\right)\left(1+\frac{\sqrt{5}+1}{4}\right)\right]$

$= \frac{1}{4}\left[1+\left(\frac{\sqrt{5}+1}{4}\right)-\left(\frac{\sqrt{5}-1}{4}\right)-\left(\frac{4}{16}\right)\right]$

$= \frac{1}{4}\left[1+\frac{1}{2}-\frac{1}{4}\right] = \frac{5}{16}$

86 **(c)**

We have,

$\sin\theta = x + \frac{p}{x}$

$\Rightarrow x^2 - x\sin\theta + p = 0$

$\Rightarrow \sin^2\theta - 4p \geq 0$ [$\because x$ is real]

$\Rightarrow 4p \leq \sin^2\theta$

$\Rightarrow 4p \leq 1 \Rightarrow p \leq \frac{1}{4}$ [$\because \sin^2\theta \leq 1$]

87 **(c)**

We have,

$3\cos 2x - 10\cos x + 7 = 0$

$\Rightarrow 3(2\cos^2 x - 1) - 10\cos x + 7 = 0$

$\Rightarrow 6\cos^2 x - 10\cos x + 1 = 0$
$\Rightarrow (3\cos x - 2)(\cos x - 1) = 0$
$\Rightarrow \cos x = \frac{2}{3}$
Now, $\cos x = 1 \Rightarrow x = 0, 2\pi, 4\pi$
and, $\cos x = \frac{2}{3} \Rightarrow x$
$= \cos^{-1}\frac{2}{3}, 2\pi$
$\pm \cos^{-1}\frac{2}{3}, 4\pi \pm \cos^{-1}\frac{2}{3}$
Thus, there are 8 solutions of the given equation $[0, \pi]$

88 **(c)**
We have,
$\sum_{r=1}^{n-1} \cos^2 \frac{r\pi}{n}$
$= \frac{1}{2}\sum_{r=1}^{n-1}\left\{1 + \cos\frac{2r\pi}{n}\right\}$
$= \frac{1}{2}\left(\sum_{r=1}^{n-1} 1\right) + \frac{1}{2}\left(\sum_{r=1}^{n-1}\cos\frac{2r\pi}{n}\right)$
$= \frac{(n-1)}{2} + \frac{1}{2}\left\{\cos\frac{2\pi}{n} + \cos\frac{4\pi}{n} + \dots + \cos\frac{2(n-1)\pi}{n}\right\}$
$= \frac{(n-1)}{2} + \frac{1}{2} \times \frac{\cos\left\{\frac{2\pi}{n} + (n-2)\frac{\pi}{n}\right\}\sin(n-1)\frac{\pi}{n}}{\sin\frac{\pi}{n}}$
$= \frac{(n-1)}{2} + \frac{1}{2}\cos\pi = \left(\frac{n-1}{2}\right) - \frac{1}{2} = \frac{n}{2} - 1$

89 **(b)**
We have,
$1 - \cos\theta = \sin\theta\sin\frac{\theta}{2}$
$\Rightarrow 2\sin^2\frac{\theta}{2} = 2\sin^2\frac{\theta}{2}\cos\frac{\theta}{2}$
$\Rightarrow 2\sin^2\frac{\theta}{2}\left(1 - \cos\frac{\theta}{2}\right) = 0$
$\Rightarrow \sin\frac{\theta}{2} = 0, \cos\frac{\theta}{2} = 1$
$\Rightarrow \frac{\theta}{2} = n\pi$, or, $\frac{\theta}{2} = 2n\pi, n \in Z$
$\Rightarrow \theta = 2n\pi, n \in Z$

90 **(c)**
Given, $\cos 5\theta = 0$

$\Rightarrow 5\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{10}$

$\therefore \cos\theta = \cos\left(\frac{\pi}{10}\right) = \sqrt{\frac{10 + 2\sqrt{5}}{16}}$

$= \sqrt{\frac{5 + \sqrt{5}}{8}}$

91 **(d)**
Given, $\cos^2\theta + \sin^2\theta + 1 = 0$
$\Rightarrow \sin^2\theta - \sin\theta - 2 = 0$
$\Rightarrow (\sin\theta + 1)(\sin\theta - 2) = 0$
$\Rightarrow \sin\theta = -1 = \sin\frac{3\pi}{2}$ $\quad [\because \sin\theta \geq 1]$
$\therefore \theta = \frac{3\pi}{2} \in \left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$

92 **(c)**
Given that, diameter of circular wire=10 cm

$\therefore$ Length of wire $= 10\pi$

Hence, required angle $= \frac{\text{length of arc}}{\text{radius of big circle}}$

$= \frac{10\pi}{50} = \frac{\pi}{5}$ rad

93 **(b)**
Let $\sin A = x$ ...(i)
Then, $\cos A = \tan B$
$\Rightarrow \sqrt{1 - \sin^2 A} = \tan B$
$\Rightarrow \sqrt{1 - x^2} = \tan B$
and,
$\cos B = \tan C$
$\Rightarrow \frac{1}{\sqrt{1 + \tan^2 B}} = \tan C$
$\Rightarrow \frac{1}{\sqrt{2 - x^2}} = \tan C$
$\Rightarrow \cos C = \frac{1}{\sqrt{1 + \tan^2 C}} = \sqrt{\frac{2 - x^2}{3 - x^2}}$ ...(ii)
Now,
$\cos C = \tan A$
$\Rightarrow \sqrt{\frac{2 - x^2}{3 - x^2}} = \frac{x}{\sqrt{1 - x^2}}$ [From (i) and (ii)]
$\Rightarrow x^2 = \frac{(1 \pm \sqrt{5})^2}{4} \Rightarrow x = \frac{\sqrt{5} - 1}{2} = 2\sin 18°$

94 **(d)**
We have,
$\cos 1° + \cos 2° + \cos 3° + \dots + \cos 180°$
$= \sum_{\theta=1}^{90°}\{\cos\theta + \cos(180°) - \theta\} + \cos 180°$
$= \sum_{\theta=1}^{90°}(\cos\theta - \cos\theta) - 1 = -1$

95 **(a)**
Given, $e^{\sin x} - e^{-\sin x} - 4 = 0$
$\Rightarrow e^{2\sin x} - 4e^{\sin x} - 1 = 0$

$\Rightarrow\ e^{\sin x}=\frac{4\pm\sqrt{16+4}}{2}=2+\sqrt{5}$

$\Rightarrow\ \sin x=\log(2+\sqrt{5})$ [$\because\ \log(2-\sqrt{5})$ is not defined]

Since, $2+\sqrt{5}>e\ \Rightarrow\ (2+\sqrt{5})>1$

$\Rightarrow \sin x>1$, which is not possible

Hence, no solution exist

96 **(a)**

We have,

$\cos A+\cos B+\cos C=4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}+1$

$\Rightarrow \cos A+\cos B+\cos C=\frac{r}{R}+1\left[\because r=4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right]$

97 **(d)**

$\frac{2\left[\frac{1}{2}\sin 80°-\frac{\sqrt{3}}{2}\cos 80°\right]}{\sin 80°\cos 80°}=\frac{4[\sin(80°-60°)]}{2\sin 80°\cos 80°}$

$=\frac{4\sin 20°}{\sin 160°}$

$=\frac{4\sin 20°}{\sin(180°-20°)}=4$

98 **(c)**

On squaring and adding the given equations, we get

$\sin^2 A+cos^2B+2\sin A cos B+\sin^2 B+\cos^2 A+2\sin B\cos A=a^2+b^2$

$\Rightarrow\ 2\sin(A+B)+2=a^2+b^2$

$\Rightarrow\ \sin(A+B)=\frac{a^2+b^2-2}{2}$

99 **(c)**

We have,

$\cot\frac{B}{2}\cot\frac{C}{2}$

$=\sqrt{\frac{s(s-b)}{(s-a)(s-c)}}\times\sqrt{\frac{s(s-c)}{(s-b)(s-a)}}$

$=\frac{s}{s-a}=\frac{2s}{2s-2a}=\frac{a+b+c}{b+c-a}=\frac{4a}{2a}=2$ [$\because b+c=3a$]

100 **(b)**

We have,

$\frac{\cos A}{p_1}+\frac{\cos B}{p_2}+\frac{\cos C}{p_3}$

$=\frac{1}{2\,\Delta}(a\cos A+b\cos B+c\cos C)$

$=\frac{R}{\Delta}(\sin A\cos A+\sin B\cos B+\sin C\cos C)$

$=\frac{R}{2\,\Delta}(\sin 2A+\sin 2B+\sin 2C)$

$=R\frac{4\sin A\sin B\sin C}{2\,\Delta}=\frac{2\,R\sin A\sin B\sin C}{\Delta}$

$=\frac{2R}{\Delta}\times\frac{2\,\Delta}{bc}\times\frac{2\,\Delta}{ca}\times\frac{2\,\Delta}{ab}=\frac{16R\,\Delta^2}{a^2b^2c^2}=\frac{16R\,\Delta^2}{(4R\Delta)^2}=\frac{1}{R}$

101 **(c)**

We have,

$\cos A+\cos B+\cos C=\frac{3}{2}$

$\Rightarrow \frac{b^2+c^2-a^2}{2bc}+\frac{c^2+a^2-b^2}{2ac}+\frac{a^2+b^2-c^2}{2ab}-\frac{3}{2}=0$

$\Rightarrow a(b^2+c^2)+b(c^2+a^2)+c(a^2+b^2)=a^3+b^3+c^3+3abc$

$\Rightarrow a(b-c)^2+b(c-a)^2+c(a-b)^2=a^3+b^3+c^3-3abc$

$\Rightarrow a(b-c)^2+b(c-a)^2+c(a-b)^2$

$=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$\Rightarrow 2a(b-c)^2+2b(c-a)^2+2c(a-b)^2$

$=(a+b+c)\{(b-c)^2+(c-a)^2+(a-b)^2\}$

$\Rightarrow (b-c)^2(b+c-a)+(c-a)^2(a+c-b)+(a-b)^2(a+b-c)=0$

$\Rightarrow (b-c)^2(b+c-a)=0,(c-a)^2(a+c-b)=0,(a-b)^2(a+b-c)=0$

$\Rightarrow a=b=c$

Hence, the triangle is an equilateral triangle

102 **(c)**

We have,

$\sin A=\frac{336}{625}$

$\Rightarrow \cos A=-\sqrt{1-\sin^2 A}$

$=-\sqrt{1-\left(\frac{336}{625}\right)^2}$ [$\because$ A is in II quad.]

Now,

$\cos\frac{A}{2}=-\sqrt{\frac{1+\cos\ A}{2}}$

$=-\frac{7}{25}$ $\left[\because \frac{A}{2}\text{ is in II quad.}\right]$

$\therefore \sin\frac{A}{4}=+\sqrt{\frac{1-\cos\frac{A}{2}}{2}}$ $\left[\because \frac{A}{4}\text{ is in II, quad.}\right]$

$\Rightarrow \sin\frac{A}{4}=\sqrt{\frac{1+\frac{7}{25}}{2}}=\frac{4}{5}$

103 **(c)**

We have,

$\tan(\pi\cos\theta)=\cot(\pi\sin\theta)$

$\Rightarrow \tan(\pi\cos\theta)=\tan\left(\frac{\pi}{2}-\pi\sin\theta\right)$

$\Rightarrow \pi\cos\theta=\frac{\pi}{2}-\pi\sin\theta$

$\Rightarrow \sin\theta + \cos\theta = \frac{1}{2}$

$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{1}{2\sqrt{2}} \Rightarrow \sin\left(\theta + \frac{\pi}{4}\right)$

$= \frac{1}{2\sqrt{2}}$

104 **(d)**

We have,

$\cos(\alpha - \beta) = 1$

$\Rightarrow \alpha - \beta = 0 \quad [\because \alpha, \beta \in (-\pi, \pi) \Rightarrow -2\pi < \alpha - \beta < 2\pi]$

$\Rightarrow \alpha = \beta$

Now, $\cos(\alpha+\beta) = \frac{1}{e} \Rightarrow \cos 2\alpha = \frac{1}{e}$

Clearly, there are 4 values of $\alpha \in (-2\pi, 2\pi)$ satisfying $\cos 2\alpha = \frac{1}{e}$

Hence, there are four ordered pairs (α, β) satisfying the given conditions

105 **(b)**

$x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = 4\cos^2\theta - 2$

$\Rightarrow x^2 + \frac{1}{x^2} = 2\cos 2\theta$

Again $x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 8\cos^3\theta$

$x^2 + \frac{1}{x^3} = 8\cos^3\theta - 6\cos\theta = \cos 3\theta$

Similarly, $x^n + \frac{1}{x^n} = 2\cos n\theta$

106 **(c)**

We have,

$\cos^2\frac{A}{2}\cos^2\frac{B}{2} + \cos^2\frac{C}{2}$

$= \frac{1}{2}\{(1+\cos A) + (1+\cos B) + (1+\cos C)\}$

$= \frac{1}{2}\{3 + \cos A + \cos B + \cos C\}$

$= \frac{1}{2}\left\{3 + 1 + \frac{r}{R}\right\} \quad \left[\because \cos A + \cos B + \cos C = 1 + \frac{r}{R}\right]$

$= 2 + \frac{r}{2R}$

107 **(b)**

We have,

$\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}$

$= \left(\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{7\pi}{15}\right) \times \left(\cos\frac{3\pi}{15}\cos\frac{6\pi}{15}\right) \times \cos\frac{5\pi}{15}$

$= \left\{\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{7\pi}{15}\left(\pi - \frac{8\pi}{15}\right)\right\}\left(\cos\frac{3\pi}{15}\cos\frac{6\pi}{15}\right) \times \cos\frac{\pi}{3}$

$= \left(-\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\right) \times \left(\cos\frac{3\pi}{15}\cos\frac{6\pi}{15}\right) \times \frac{1}{2}$

$= -\frac{\sin\left(2^4 \times \frac{\pi}{15}\right)}{2^4\sin\frac{\pi}{15}} \times \frac{\sin\left(2^2 \times \frac{3\pi}{15}\right)}{2^2\sin\frac{3\pi}{15}} \times \frac{1}{2}$

$= -\frac{\sin\frac{16\pi}{15}}{16\sin\frac{\pi}{15}} \times \frac{\sin\left(\frac{12\pi}{15}\right)}{4\sin\frac{3\pi}{15}} \times \frac{1}{2} = \frac{1}{16} \times \frac{1}{4} \times \frac{1}{2} = \frac{1}{2^7}$

108 **(d)**

Given, $\tan A$ and $\tan B$ are the roots of the equation

$abx^2 - c^2x + ab = 0$

$\therefore \tan A + \tan B = \frac{c^2}{ab}, \tan A \tan B = 1$

Now, $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{c^2}{ab}}{1-1} = \infty$

$\Rightarrow A + B = \frac{\pi}{2} \Rightarrow C = \frac{\pi}{2}$

$\therefore \sin^2 A + \sin^2 B + \sin^2 C$

$= \sin^2\left(\frac{\pi}{2} - B\right) + \sin^2 B + \sin^2\frac{\pi}{2}$

$= \cos^2 B + \sin^2 B + 1 = 2$

110 **(a)**

Given, $P = \frac{1}{2}\sin^2\theta + \frac{1}{3}\cos^2\theta$

$= \frac{1}{2}(1 - \cos^2\theta) + \frac{1}{3}\cos^2\theta$

$= \frac{1}{2} - \frac{1}{6}\cos^2\theta$

Since, $0 \le \cos^2\theta \le 1$

$\Rightarrow -\frac{1}{6} \le -\frac{1}{6}\cos^2\theta \le 0$

$\Rightarrow \frac{1}{3} \le \frac{1}{2} - \frac{1}{6}\cos^2\theta \le \frac{1}{2}$

$\Rightarrow \frac{1}{3} \le P \le \frac{1}{2}$

111 **(b)**

We have,

$3\cos\theta + 4\sin\theta$

$= 5\left\{\frac{3}{5}\cos\theta + \frac{4}{5}\sin\theta\right\}$

$= 5\cos(\theta - \alpha)$, where $\cos\alpha = \frac{3}{5}, \sin\alpha = \frac{4}{5}$

$\therefore 3\cos\theta + 4\sin\theta = k$

$\Rightarrow 5\cos(\theta - \alpha) = k$

$\Rightarrow \cos(\theta - \alpha) = \pm 1 \quad [\because k = 5]$

$\Rightarrow \theta - \alpha = 0°, 180° \Rightarrow \theta = \alpha, 180° + \alpha$

112 **(a)**

$\frac{1}{p} + \frac{1}{q} + \frac{r}{pq} = \frac{p+q+r}{pq}$

$= \frac{\cos 55° + \cos 65° + \cos 175°}{\cos 55° \cos 65°}$

$$= \frac{\cos 55° + 2\cos\frac{175°+65°}{2}\cos\frac{175°-65°}{2}}{\cos 55° \cos 65°}$$

$$= \frac{\cos 55° + 2\cos 120° \cos 55°}{\cos 55° \cos 65°} = \frac{1 - 2 \times \frac{1}{2}}{\cos 65°} = 0$$

113 **(a)**

Since, $\sin x + \operatorname{cosec} x = 2$

$\Rightarrow \sin x + \frac{1}{\sin x} = 2$

$\Rightarrow \sin^2 x - 2\sin x + 1 = 0$

$\Rightarrow (\sin x - 1)^2 = 0 \Rightarrow \sin x = 1$

Now, $\sin^n x + \operatorname{cosec}^n x = \sin^n x + \frac{1}{\sin^n x}$

$= 1 + 1 = 2$

114 **(a)**

Let ABC be a right angled triangle such that the sides a, b, c are in A.P. Then,

$2b = a + c$ ...(i)

Let c be the largest side. Then,

$c^2 = a^2 + b^2$ ...(ii)

From (i) and (ii), we have

$(2b - a)^2 = a^2 + b^2$

$\Rightarrow 3b^2 - 4ab = 0 \Rightarrow 3b = 4a \Rightarrow \frac{a}{3} = \frac{b}{4}$ ... (iii)

From (i) and (iii), we get

$5b = 4c$ i.e. $\frac{b}{4} = \frac{c}{5}$

$\therefore \frac{a}{3} = \frac{b}{4} = \frac{c}{5} \Rightarrow a : b : c = 3 : 4 : 5$

115 **(a)**

$a\cos\theta + b\sin\theta = c$ ...(i)

$\because$ α and β $(\alpha \neq \beta)$ satisfy the Eq. (i)

$\Rightarrow a\cos\alpha + b\sin\alpha = c$ ...(ii)

And $a\cos\beta + b\sin\beta = c$ ...(iii)

On, subtracting Eq. (iii) from Eq.; (ii), we get

$a\cos\alpha + b\sin\alpha - a\cos\beta - b\sin\beta = 0$

$\Rightarrow a(\cos\alpha - \cos\beta) = -b(\sin\alpha - \sin\beta)$

$\Rightarrow a\sin\frac{\alpha+\beta}{2} = -b\left[-\cos\left(\frac{\alpha+\beta}{2}\right)\right]$

$\Rightarrow \tan\left(\frac{\alpha+\beta}{2}\right) = \frac{b}{a}$

116 **(d)**

We have,

$\frac{a}{b^2 - c^2} + \frac{c}{b^2 - a^2} = 0$

$\Rightarrow ab^2 - a^3 + b^2c - c^3 = 0$ $[\because a \neq b \neq c]$

$\Rightarrow (a + c)b^2 - (a^3 + c^3) = 0$

$\Rightarrow (a + c)(b^2 - a^2 - c^2 + ac) = 0$

$\Rightarrow b^2 - a^2 - c^2 + ac = 0$ $[\because a + c \neq 0]$

$\Rightarrow a^2 + c^2 - ac = b^2$

$\Rightarrow a^2 + c^2 - ac = c^2 + a^2 - 2ac\cos B$

$\Rightarrow \cos B = \frac{1}{2} \Rightarrow B = \frac{\pi}{3}$

117 **(c)**

$\sin^2 5° + \sin^2 10° + \sin^2 15° + \ldots + \sin^2 90°$

$= \sin^2 5° + \sin^2 10° + \ldots + \sin^2 45° + \ldots + \sin^2 80° + \sin^2 85° + \sin^2 90°$

$= \sin^2 5° + \sin^2 10° + \ldots + \frac{1}{2} + \ldots + \cos^2 10° + \cos^2 5° + \sin^2 90°$

$= (\sin^2 5° + \cos^2 5°) + (\sin^2 10° + \cos^2 10°) + \cdots + (\sin^2 40° + \cos^2 40°) + \sin^2 45° + \sin^2 90°$

$= 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + \frac{1}{2} + 1 = 9\frac{1}{2}$

118 **(b)**

We have,

$C = 60°$

$\Rightarrow \cos C = \frac{1}{2} \Rightarrow \frac{a^2 + b^2 - c^2}{2ab} = \frac{1}{2} \Rightarrow a^2 + b^2 - c^2 = ab$ ... (i)

Now,

$\frac{a}{b + c} + \frac{b}{c + a}$

$= \frac{ac + a^2 + b^2 + bc}{bc + ab + c^2 + ac} = \frac{c^2 + ac + bc + ab}{c^2 + ac + bc + ab}$

$= 1$ [Using : (i)]

120 **(c)**

We have,

$\Delta = b^2 - (c - a)^2$

$\Rightarrow \Delta = (b - c + a)(b + c - a)$

$\Rightarrow \sqrt{s(s - a)(s - b)(s - c)} = (2s - 2c)(2s - 2a)$

$\Rightarrow \sqrt{\frac{(s - a)(s - c)}{s(s - b)}} = \frac{1}{4} \Rightarrow \tan\frac{B}{2} = \frac{1}{4}$

$\therefore \tan B = \frac{2\tan\frac{B}{2}}{1 - \tan^2\frac{B}{2}} = \frac{2/4}{1 - \frac{1}{16}} = \frac{8}{15}$

121 **(d)**

We have,

$\cos^2 76° + \cos^2 16° - \cos 76° \cos 16°$

$= \frac{1}{2}|(1 + \cos 152°) + (1 + \cos 32°) - (\cos 92° + \cos 60°)|$

$= \frac{1}{2}\left\{\frac{3}{2} + \cos 152° + \cos 32° - \cos 92°\right\}$

$= \frac{1}{2}\left\{\frac{3}{2} + 2\cos 92° \cos 60° - \cos 92°\right\} = \frac{3}{4}$

122 **(b)**

$\sec\theta + \tan\theta = k$ ...(i)

$\Rightarrow \dfrac{\sec^2\theta - \tan^2\theta}{\sec\theta - \tan\theta} = k$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{k}$ ...(ii)

On adding Eqs. (i) and (ii), we get

$2\sec\theta = k + \dfrac{1}{k} = \dfrac{k^2+1}{k}$

$\Rightarrow \cos\theta = \dfrac{2k}{k^2+1}$

123 **(b)**

Given, $\sin n\,\theta = \sum_{r=0}^{n} b_r \sin^r\theta = b_0 + b_1\sin\theta + b2\sin 2\theta + \ldots + bn\sin n\theta$ ...(i)

Putting $\theta = 0$ in Eq.(i), we get $0 = b_0$

Again, Eq. (i)can be written as $\sin n\,\theta = \sum_{r=0}^{n} b_r \sin^r\theta$

$$\frac{\sin n\theta}{\sin\theta} = \sum_{r=1}^{n} b_r \sin^{r-1}\theta$$

Taking limit as $\theta \to 0$, we get $\lim_{\theta\to 0}\frac{\sin n\theta}{\sin\theta} = b_1$

$\Rightarrow \lim_{\theta\to 0} n\left(\dfrac{\sin n\theta}{n\theta}\right)\left(\dfrac{\theta}{\sin\theta}\right) = b_1$

$\Rightarrow n = b_1$ Hence, $b_{0=}0$; $b_1 = n$

124 **(b)**

We have,

$\cos 2x + 2\cos^2 x = 2$

$\Rightarrow 4\cos^2 x = 3 \Rightarrow \cos^2 x = \cos^2\dfrac{\pi}{6} \Rightarrow x$

$= n\pi \pm \dfrac{\pi}{6}, n \in Z$

125 **(a)**

We have,

$1 + \sin x + \sin^2 x + \cdots + \text{to } \infty = \left(\sqrt{3}+1\right)^2$

$\Rightarrow \dfrac{1}{1-\sin x} = \left(\sqrt{3}+1\right)^2$

$\Rightarrow 1 - \sin x = \dfrac{1}{\left(\sqrt{3}-1\right)^2}$

$\Rightarrow 1 - \sin x = \dfrac{\left(\sqrt{3}-1\right)^2}{4}$

$\Rightarrow \sin x = 1 - \left(\dfrac{4-2\sqrt{3}}{4}\right)$

$\Rightarrow \sin x = \dfrac{\sqrt{3}}{2} \Rightarrow x = \dfrac{\pi}{3}$ or, $\dfrac{2\pi}{3}$

126 **(a)**

Given, $\sin x = 2\sin x\cos x$

$\Rightarrow \sin x(1 - 2\cos x) = 0$

$\Rightarrow \sin x = 0$ or $\cos x = \frac{1}{2}$

$\Rightarrow x = 0°$ or $x = 60°, -60°$

Hence, number of solution is 3

127 **(a)**

We have,

$\sin(\pi\cot\theta) = \cos(\pi\tan\theta)$

$\Rightarrow \sin(\pi\cot\theta) = \sin\left(\dfrac{\pi}{2}\pi\tan\theta\right)$

or, $\cos(\pi\tan\theta) = \cos\left(\frac{3\pi}{2} + \pi\cot\theta\right)$

$\Rightarrow \pi\cot\theta = \dfrac{\pi}{2} + \pi\tan\theta$ or, $\pi\tan\theta = \dfrac{3\pi}{2} + \pi\cot\theta$

$\Rightarrow \cot\theta - \tan\theta = \dfrac{1}{2}$ or, $\cot\theta - \tan\theta = -\dfrac{3}{2}$

$\Rightarrow \dfrac{1-\tan^2\theta}{2\tan\theta} = \dfrac{1}{4}$ or, $\dfrac{1-\tan^2\theta}{2\tan\theta} = -\dfrac{3}{4}$

$\Rightarrow \cot 2\theta = \dfrac{1}{4}$ or, $\cot 2\theta = -\dfrac{3}{4}$

128 **(a)**

Given, $\cos 24° \cos 48° \cos 96° \cos 168°$

$= -\cos 12° \cos 24° \cos 48° \cos 96°$

$= -\dfrac{16\sin 12°}{16\sin 12°}\cos 12° \cos 24° \cos 48° \cos 96°$

$= \dfrac{-8\sin 24° \cos 24° \cos 48° \cos 96°}{16\sin 12°}$

$= \dfrac{-4\sin 48° \cos 48° \cos 96°}{16\sin 12°} = -\dfrac{\sin 192°}{16\sin 12°}$

$= \dfrac{\sin 12°}{16\sin 12°} = \dfrac{1}{16}$

129 **(b)**

We have, $3\sin^2 x + 10\cos x - 6 = 0$

$\Rightarrow (\cos x - 3)(3\cos x - 1) = 0$

$\Rightarrow \cos x \neq 3$ or $\cos x = \dfrac{1}{3}$

$\Rightarrow x = 2n\pi \pm \cos^{-1}\left(\dfrac{1}{3}\right)$

130 **(d)**

$\because 2\cos x, |\cos x|, 1 - 3\cos^2 x$ are in GP

$\therefore \cos^2 x = 2\cos x \cdot (1 - 3\cos^2 x)$

$\Rightarrow 6\cos^3 x + \cos^2 x - 2\cos x = 0$

$\therefore \cos x = 0, \dfrac{1}{2}, -\dfrac{2}{3}$

$\therefore x = \frac{\pi}{2}, \frac{\pi}{3}, \cos^{-1}\left(-\frac{2}{3}\right)$ ($\because \alpha, \beta$ are positive)

If $\alpha = \frac{\pi}{2}, \beta = \frac{\pi}{3}$

Then, $|\alpha - \beta| = \frac{\pi}{6}$

131 **(b)**

$\because \cos\theta = -\dfrac{1}{\sqrt{2}} \Rightarrow \theta = \dfrac{3\pi}{4}, \dfrac{5\pi}{4}$

and $\tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}, \frac{5\pi}{4}$

$\therefore$ The general value is $2n\pi + \frac{5\pi}{4}$ or $(2n+1)\pi + \frac{\pi}{4}$

132 **(a)**

We have,

$$\frac{\sin^2 A + \sin A + 1}{\sin A}$$

$$= \sin A + 1 + \frac{1}{\sin A}$$

$$= \left(\sin A + \frac{1}{\sin A}\right) + 1 \geq 2 + 1 = 3 \quad \left[\because x + \frac{1}{x} \geq 2\right]$$

$$\therefore \sum \frac{\sin^2 A + \sin A + 1}{\sin A} \geq 3 + 3 + 3 = 9$$

134 **(d)**

We have,

$$\frac{\sin A}{a} = \frac{\sin B}{b} \Rightarrow \frac{3/4}{5} = \frac{\sin B}{7} \Rightarrow \sin B = 21/20 > 1$$

Which is impossible. Hence, no triangle is possible

135 **(b)**

We have,

$$AD^2 = \frac{1}{4}(b^2 + c^2 + 2\,bc \cos A)$$

$$\Rightarrow 4\,AD^2 = b^2 + c^2 + 2bc \cos \pi/3$$

$$\Rightarrow 4\,AD^2 = b^2 + c^2 + bc$$

136 **(d)**

We have,

$$\tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta \tan\phi} = \frac{1/2 + 1/3}{1 - 1/2 \times 1/3} = 1$$

$$\Rightarrow \theta + \phi = \frac{\pi}{4}$$

137 **(b)**

We have,

$$a = \tan 27\theta - \tan\theta$$

$$\Rightarrow a = \tan 27\theta - \tan 9\theta + \tan 9\theta - \tan 3\theta + \tan 3\theta - \tan\theta$$

$$\Rightarrow a = \frac{\sin 18\theta}{\cos 27\theta \cos 9\theta} + \frac{\sin 6\theta}{\cos 9\,\theta \cos 3\theta} + \frac{\sin 2\theta}{\cos 3\theta \cos 2\theta}$$

$$\Rightarrow a = 2\left(\frac{\sin 9\theta}{\cos 27\theta} + \frac{\sin 3\theta}{\cos 9\theta} + \frac{\sin\theta}{\cos 3\theta}\right) \Rightarrow a = 2b$$

138 **(c)**

We have,

$s - a = 3 \Rightarrow b + c - a = 6$ ...(i)

and, $s - c = 2 \Rightarrow a + b - c = 4$ ...(ii)

Adding these two equations, we get $b = 5$

Since B is a right angle

$\therefore b^2 = a^2 + c^2 \Rightarrow a^2 + c^2 = 25$ ...(iii)

Multiplying (i) and (ii), we get

$$[(b - c) + a][(b + c) - a] = 24$$

$$\Rightarrow b^2 - c^2 + 2ac - a^2 = 24$$

$\Rightarrow a^2 + 2ac - a^2 = 24$ $[\because b^2 = a^2 + c^2]$

$\Rightarrow ac = 12$ ...(iv)

From (iii) and (iv), we have

$a + c = 7$ and $a - c = 1 \Rightarrow a = 4, c = 3$

139 **(d)**

Given, $\sin\alpha = \frac{15}{17}, \tan\beta = \frac{12}{5}$

Since, $\frac{\pi}{2} < \alpha < \pi, \pi < \beta < \frac{3\pi}{2}$

$$\therefore \cos\alpha = -\frac{8}{17}, \sin\beta = -\frac{12}{13}$$

and $\cos\beta = -\frac{5}{13}$

Now, $\sin(\beta - \alpha) = \sin\beta\cos\alpha - \cos\beta\sin\alpha$

$$= -\frac{12}{13}\left(\frac{-8}{17}\right) - \left(\frac{-5}{13}\right)\left(\frac{15}{17}\right)$$

$$= \frac{96}{221} + \frac{75}{221} = \frac{171}{221}$$

140 **(b)**

We have,

$$\tan 2\,\theta \tan\theta = 1$$

$$\Rightarrow \frac{2\tan^2\theta}{1 - \tan^2\theta} 1$$

$$\Rightarrow \tan^2\theta = \frac{1}{3} \Rightarrow \tan^2\theta = \tan^2\frac{\pi}{6} \Rightarrow \theta = n\,\pi \pm \frac{\pi}{6}, n \in Z$$

142 **(b)**

Let a, b, c be the lengths of the sides of ΔABC such that $a = 6$ cm

We have,

$$2s = 16 \Rightarrow a + b + c = 16 \Rightarrow 6 + b + c = 16 \Rightarrow b + c = 10$$

Also,

$$\Delta = 12 \text{ cm}^2$$

$$\Rightarrow s(s - a)(s - b)(s - c) = 12^2$$

$$\Rightarrow 8(8 - 6)(8 - b)(8 - c) = 144$$

$$\Rightarrow 64 - 8(b + c) + bc = 9$$

$$\Rightarrow 64 - 80 + bc = 9$$

$$\Rightarrow bc = 25$$

$$\therefore (b - c)^2 = (b + c)^2 - 4bc = 100 - 100 = 0 \Rightarrow b = c$$

Hence, ΔABC is isosceles

143 **(a)**

We have,

$$\sin\alpha + \cos\alpha = m \Rightarrow 1 + \sin 2\alpha = m^2 \Rightarrow \sin 2\alpha = m^2 - 1$$

Now,

$$\sin^6\alpha + \cos^6\alpha$$

$$= (\sin^2\alpha + \cos^2\alpha)^3 - 3\sin^2\alpha\cos^2\alpha\,(\sin^2\alpha + \cos^2\alpha)$$

$$= 1 - 3\,\frac{(-1 + m^2)^2}{4} = \frac{4 - 3(m^2 - 1)^2}{4}$$

144 **(b)**

We have,

$\tan x = \cot x - 2\cot 2x$

$\Rightarrow \frac{1}{2}\tan\frac{x}{2} = \frac{1}{2}\cot\frac{x}{2} - \cot x$

$\Rightarrow \frac{1}{2}\tan\frac{x}{2^2} = \frac{1}{2}\cot\left(\frac{x}{2^2}\right) - \cot\left(\frac{x}{2}\right)$

$\Rightarrow \frac{1}{2^2}\tan\left(\frac{x}{2^2}\right) = \frac{1}{2^2}\cot\left(\frac{x}{2^2}\right) - \frac{1}{2}\cot\left(\frac{x}{2}\right)$

Similarly, we have

$\frac{1}{2^3}\tan\left(\frac{x}{2^3}\right) = \frac{1}{2^3}\cot\left(\frac{x}{2^3}\right) - \frac{1}{2^2}\cot\left(\frac{x}{2^2}\right)$

……………………………………………

……………………………………………

$\frac{1}{2^{n-1}}\tan\left(\frac{x}{2^{n-1}}\right)$

$= \frac{1}{2^{n-1}}\cot\left(\frac{x}{2^{n-1}}\right) - \frac{1}{2^{n-2}}\cot\left(\frac{x}{2^{n-2}}\right)$

Adding all the above results, we get

$\tan x + \frac{1}{2}\tan\frac{x}{2} + \frac{1}{2^2}\tan\left(\frac{x}{x^2}\right) + \cdots + \frac{1}{2^{n-1}}\tan\left(\frac{x}{2^{n-1}}\right)$

$= \frac{1}{2^{n-1}}\cot\left(\frac{x}{2^{n-1}}\right) - 2\cot 2x$

145 **(b)**

$\frac{1}{\cos 290°} + \frac{1}{\sqrt{3}\sin 250°}$

$= \frac{1}{\cos 70°} - \frac{1}{\sqrt{3}\sin 110°}$

$= \frac{\sqrt{3}\sin 110° - \cos 70°}{\sqrt{3}\sin 110°\cos 70°}$

$= \frac{\sqrt{3}\sin(180° - 70°) - \cos 70°}{\sqrt{3}\sin(180° - 70°)\cos 70°}$

$= \frac{\frac{\sqrt{3}}{2}\sin 70° - \frac{1}{2}\cos 70°}{\frac{\sqrt{3}}{2}\sin 70°\cos 70°}$

$= \frac{\cos 30°\sin 70° - \sin 30°\cos 70°}{\frac{\sqrt{3}}{2}.\frac{1}{2}\sin 140°}$

$= \frac{\sin(70° - 30°)}{\frac{\sqrt{3}}{2}\sin(180° - 40°)}$

$= \frac{\sin 40°}{\frac{\sqrt{3}}{4}\sin 40°} = \frac{4}{\sqrt{3}}$

146 **(b)**

We have, $A = \sin^2\theta + \cos^4\theta$

$= \sin^2\theta + \cos^2\theta\cos^2\theta$

$\leq \sin^2\theta + \cos^2\theta$ (since, $\cos^2\theta \leq 1$)

$\Rightarrow \sin^2\theta + \cos^4\theta \leq 1 \Rightarrow A \leq 1$

Again, $\sin^2\theta + \cos^4\theta = 1 - \cos^2\theta +$ cos2θ+cos4θ

$= \cos^4\theta - \cos^2\theta + \cos^4\theta$

$= \cos^4\theta - \cos^2\theta + 1$

$= \left(\cos^2\theta - \frac{1}{2}\right)^2 + \frac{3}{4} \geq \frac{3}{4}$

Hence, $\frac{3}{4} \leq A \leq 1$

147 **(c)**

$12\cot^2\theta - 31\operatorname{cosec}\theta + 32 = 0$

$\Rightarrow\ 12\cos^2\theta - 31\sin\theta + 32\sin^2\theta = 0$

$\Rightarrow 20\sin^2\theta - 31\sin\theta + 12 = 0\quad [\because \cos^2\vartheta = 1 - \sin^2\theta]$

$\therefore\ \sin 3\theta = \frac{31 \pm \sqrt{31^2 - 4.20.12}}{2.20}$

$= \frac{31 \pm \sqrt{961 - 960}}{40} = \frac{31 \pm 1}{40}$

$\Rightarrow\ \sin\theta = \frac{4}{5}, \frac{3}{4}$

148 **(d)**

$\sin 20°\left(4 + \frac{1}{\cos 20°}\right) = \frac{4\sin 20°\cos 20° + \sin 20°}{\cos 20°}$

$= \frac{\sin 40° + \sin 40° + \sin 20°}{\cos 20°}$

$= \frac{\sin 40° + 2\sin 30°\cos 10°}{\cos 20°}$

$= \frac{\cos 50° + \cos 10°}{\cos 20°}$

$[\because \sin 40° = \cos(90° - 40°)]$

$= \frac{2\cos 30°\cos 20°}{\cos 20°} = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$

149 **(c)**

Let θ be the required angle. Then,

$\cos\theta = \tan\theta$

$\Rightarrow \cos^2\theta = \sin\theta$

$\Rightarrow \sin^2\theta + \sin\theta - 1 = 0 \Rightarrow \sin\theta = \frac{\sqrt{5} - 1}{2}$

$= 2\sin 18°$

150 **(b)**

We have, $3\left[\sin^4\left(\frac{3\pi}{2} - \alpha\right) + \sin^4(3\pi + \alpha)\right] -$

$2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right]$

$= 3[(-\cos\alpha)^4+(-\sin\alpha)^4]-2[\cos^6\alpha+\sin^6\alpha]$

$= 3[(\cos^2\alpha+\sin^2\alpha)^2-2\sin^2\alpha\cos^2\alpha]$
$\quad -2[(\cos^2\alpha+\sin^2\alpha)^3$
$\quad -3\cos^2\alpha\sin^2\alpha(\cos^2\alpha+\sin^2\alpha)]$

$= 3-6\sin^2\alpha\cos^2\alpha-2+6\sin^2\alpha\cos^2\alpha$

$= 3-2=1$

151 **(a)**

We have,

$\tan\theta+\tan\left(\theta+\frac{3\pi}{4}\right)=2$

$\Rightarrow \tan\theta+\frac{\tan\theta-1}{1+\tan\theta}=2$

$\Rightarrow \tan^2\theta+2\tan\theta-1=2+2\tan\theta$

$\Rightarrow \tan^2\theta=3$

$\Rightarrow \tan^2\theta=\tan^2\frac{\pi}{3}\Rightarrow\theta=n\pi\pm\frac{\pi}{3}, n\in Z$

152 **(a)**

Let $A+B=\theta$ and $A-B=\phi$

Then, $\tan A=k\tan B \Rightarrow \frac{\tan A}{\tan B}=\frac{k}{1}$

$\Rightarrow \frac{k}{1}=\frac{\sin A\cos B}{\cos A\sin B}$

Applying componendo and dividend rule, we get

$\Rightarrow \frac{k+1}{k-1}=\frac{\sin A\cos B+\cos A\sin B}{\sin A\cos B-\cos A\sin B}$

$=\frac{\sin(A+B)}{\sin(A-B)}=\frac{\sin\theta}{\sin\phi}$

$\Rightarrow \sin\theta=\frac{k+1}{k-1}\sin\phi$

153 **(d)**

Given that, $ABCD$ is a cyclic quadrilateral

So, $A+C=180° \Rightarrow A=180°-C$

$\Rightarrow \cos A=\cos(180°-C)=-\cos C$

$\Rightarrow \cos A+\cos C=0$ ...(i)

Similarly, $\cos B+\cos D=0$ ... (ii)

On adding Eqs. (i) and (ii), we get

$\cos A+\cos B+\cos C+\cos D=0$

154 **(a)**

We have,

$\alpha-\beta=(\theta-\beta)-(\theta-\alpha)$

$\therefore \cos(\alpha-\beta)=\cos(\theta-\beta)\cos(\theta-\alpha)$
$\quad +\sin(\theta-\beta)\sin(\theta-\alpha)$

$\Rightarrow \cos(\alpha-\beta)=ab+\sqrt{1-a^2}\sqrt{1-b^2}$

$\Rightarrow \sin(\alpha-\beta)=\pm\left\{a\sqrt{1-b^2}-b\sqrt{1-a^2}\right\}$

$\Rightarrow \sin^2(\alpha-\beta)=a^2+b^2-2a^2b^2$
$\quad -2ab\sqrt{1-a^2}\sqrt{1-b^2}$

$\Rightarrow \sin^2(\alpha-\beta)=a^2+b^2-2a^2b^2$
$\quad -2a\,b\{\cos(\alpha-\beta)-ab\}$

$\Rightarrow \sin^2(\alpha-\beta)=a^2+b^2-2ab\cos(\alpha-\beta)$

$\Rightarrow \sin^2(\alpha-\beta)+2ab\cos(\alpha-\beta)=a^2+b^2$

155 **(b)**

Given, $\cos x+\sin x=\frac{1}{2}$

$\Rightarrow \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}+\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}=\frac{1}{2}$

Let $\tan\frac{x}{2}=t$, then

$\frac{1-t^2}{1+t^2}+\frac{2t}{1+t^2}=\frac{1}{2}$

$\Rightarrow 3t^2-4t-1=0 \Rightarrow t=\frac{2\pm\sqrt{7}}{3}$

$\Rightarrow t=\tan\frac{x}{2}=\frac{2+\sqrt{7}}{3}$ $\left[\because 0<\frac{x}{2}<\frac{\pi}{2}, \tan\frac{x}{2} \text{ is positive}\right]$

Now, $\tan x=\frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$

$=\frac{2\left(\frac{2+\sqrt{7}}{3}\right)}{1-\left(\frac{2+\sqrt{7}}{3}\right)^2}=-\frac{3(2+\sqrt{7})}{1+2\sqrt{7}}\times\frac{1-2\sqrt{7}}{1-2\sqrt{7}}$

$\Rightarrow \tan x=-\left(\frac{4+\sqrt{7}}{3}\right)$

156 **(a)**

We have,

$\sec\theta+\operatorname{cosec}\theta=c$

$\Rightarrow \sqrt{1+\tan^2\theta}+\sqrt{1+\cot^2\theta}=c$

$\Rightarrow \sqrt{1+t^2}+\sqrt{1+\frac{1}{t^2}}=c$, where $\tan\theta=t$

$\Rightarrow \sqrt{1+t^2}\frac{(t+1)}{t}=c$

$\Rightarrow (t^2+1)(t^2+2t+1)=c^2t^2$

$\Rightarrow (t^2+t+1)^2=(c^2+1)\,t^2$

$\Rightarrow t^2+t+1\mp t\sqrt{(c^2+1)}=0$

$\Rightarrow t^2+t+1+t\sqrt{c^2+1}$
$\quad =0$ or, $t^2+t+1-t\sqrt{c^2+1}=0$

Now, discriminant of

$t^2 + t\left(1 - \sqrt{c^2+1}\right) + 1 = 0$ is D_1

$$= \left\{1 - \sqrt{c^2+1}\right\}^2 - 4$$

and, discriminant of

$t^2 + t\left(1 + \sqrt{c^2+1}\right) + 1 = 0$ is D_2

$$= \left\{1 + \sqrt{c^2+1}\right\}^2 - 4$$

Now, $D_1 > 0$

$\Rightarrow \left(1 - \sqrt{c^2+1}\right)^2 > 4 \Rightarrow 1 + c^2 + 1 - 2\sqrt{c^2+1} > 4$

$\Rightarrow c^2 - 2 > 2\sqrt{c^2+1} \Rightarrow c^4 - 4c^2 + 4 > 4c^2 + +4$

$\Rightarrow c^4 - 8c^2 > 0 \quad \Rightarrow c^2 > 8$

Similarly, we have $D_2 > 0$

$\therefore c^2 > 8$

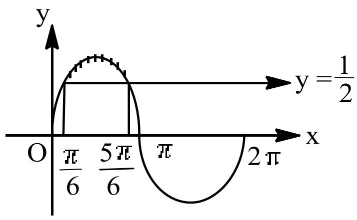
Thus, the equation has two real roots, if $c^2 > 8$

157 **(a)**

Given, $2\sin^2\theta - 5\sin\theta + 2 > 0$

$\Rightarrow \quad (2\sin\theta - 1)(\sin\theta - 2) > 0$

[where, $(\sin\theta - 2) < 0$ for all $\theta \in R$]



$(2\sin\theta - 1) < 0 \quad \Rightarrow \sin\theta < \frac{1}{2}$

It is clear from the figure

$\theta \in \left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$

158 **(a)**

We have,

$B = \frac{\pi}{8}, c = \frac{5\pi}{8} \Rightarrow A = \pi - \left(\frac{\pi}{8} + \frac{5\pi}{8}\right) = \frac{\pi}{4}$

Now,

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\Rightarrow \frac{a}{\sin\frac{\pi}{4}} = \frac{b}{\sin\frac{\pi}{8}} = \frac{c}{\sin\frac{5\pi}{8}}$

$\Rightarrow b = \sqrt{2}a\sin\frac{\pi}{8}$ and $c = \sqrt{2}a\sin\frac{5\pi}{8}$

$\therefore \Delta = \frac{1}{2}bc\sin A$

$\Rightarrow \Delta = \frac{1}{2}\left(\sqrt{2}a\sin\frac{\pi}{8}\right)\left(\sqrt{2}a\sin\frac{5\pi}{8}\right)\sin\frac{\pi}{4}$

$\Rightarrow \Delta = \frac{a^2}{\sqrt{2}}\sin\frac{\pi}{8}\sin\frac{5\pi}{8}$

$\Rightarrow \Delta = \frac{a^2}{2\sqrt{2}}\left(\cos\frac{\pi}{2} - \cos\frac{6\pi}{8}\right)$

$\Rightarrow \Delta = -\frac{a^2}{2\sqrt{2}}\cos\frac{3\pi}{4} = \frac{a^2}{4}$

Also,

$\Delta = \frac{1}{2}a \times$ (Altitude from A to BC)

$\Rightarrow \frac{a^2}{4} = \frac{a}{2} \times$ (Altitude from A to BC)

$\Rightarrow$ Altitude from A to $Bc = \frac{a}{2}$

159 **(c)**

We have, $(5 + 4\cos\theta)(2\cos\theta + 1) = 0$

$\Rightarrow \cos\theta = -\frac{5}{4}$ which is not possible

$\therefore\ 2\cos\theta + 1 = 0 \ \Rightarrow\ \cos\theta = -\frac{1}{2}$

$\Rightarrow\ \theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

$\therefore$ Solution set is $\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\} \in [0, 2\pi]$

160 **(a)**

We have,

$\cos 3x + \sin\left(2x - \frac{7\pi}{6}\right) = -2$

$\Rightarrow 1 + \cos 3x + 1 + \sin\left(2x - \frac{7\pi}{6}\right) = 0$

$\Rightarrow (1 + \cos 3x) + 1 - \cos\left(2x - \frac{2\pi}{3}\right) = 0$

$\Rightarrow 2\cos^2\frac{3x}{2} + 2\sin^2\left(x - \frac{\pi}{3}\right) = 0$

$\Rightarrow \cos\frac{3x}{2} = 0$ and $\sin\left(x - \frac{\pi}{3}\right) = 0$

$\Rightarrow \frac{3x}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$ and $x - \frac{\pi}{3} = 0, \pi, 2\pi \ldots$

$\Rightarrow x = \frac{\pi}{3}$

Therefore, the general solution is given by

$x = 2k\pi + \frac{\pi}{3} = \frac{\pi}{3}(6k + 1)$, where $k \in Z$

161 **(c)**

$\cos^2\left(\frac{\pi}{3} - x\right) - \cos^2\left(\frac{\pi}{3} + x\right)$

$= \sin\left(\frac{\pi}{3} - x + \frac{\pi}{3} + x\right)\left(\frac{\pi}{3} + x - \frac{\pi}{3} + x\right)$

$= \sin\frac{2\pi}{3}\sin 2x = \frac{\sqrt{3}}{2}\sin 2x$

[since, maximum value of $\sin 2x$ is 1]

Its maximum value is $\frac{\sqrt{3}}{2}$

162 **(a)**

We have,

$\sin\alpha + \sin\beta + \sin\gamma - \sin(\alpha + \beta + \gamma)$

$= \sin\alpha + \sin\beta + \sin\gamma - \sin\alpha\cos\beta\cos\gamma$

$- \cos\alpha\sin\beta\cos\gamma - \cos\alpha\cos\beta\sin\gamma$

$+ \sin\alpha\sin\beta\sin\gamma$

$= \sin\alpha\,(1 - \cos\beta\cos\gamma) + \sin\beta\,(1 - \cos\alpha\cos\gamma)$

$+ \sin\gamma\,(1 - \cos\alpha\cos\beta) + \sin\alpha\sin\beta\sin\gamma$

$\therefore \sin\alpha + \sin\beta + \sin\gamma > \sin(\alpha+\beta+\gamma)$

$\Rightarrow \dfrac{\sin(\alpha+\beta+\gamma)}{\sin\alpha + \sin\beta + \sin\gamma} < 1$

163 **(d)**

We have,

$\tan\theta \tan\left(\frac{\pi}{3}+\theta\right)\tan\left(-\frac{\pi}{3}+\theta\right) = k\tan 3\theta$

$\Rightarrow \tan\theta\left(\dfrac{\tan\theta+\sqrt{3}}{1-\sqrt{3}\tan\theta}\right)\left(\dfrac{\tan\theta-\sqrt{3}}{1+\sqrt{3}\tan\theta}\right) = k\tan 3\theta$

$\Rightarrow \dfrac{\tan^3\theta - 3\tan\theta}{1-3\tan^2\theta} = k\tan 3\theta$

$\Rightarrow -\tan 3\theta = k\tan 3\theta \Rightarrow k = -1$

164 **(b)**

We have,

$\tan\alpha\tan 2\alpha\tan 4\alpha \dots .\tan(2n-2)\,\alpha\tan(2n-1)\,\alpha$

$= \{\tan\alpha\tan(2n-1)\alpha\}\{\tan 2\alpha\tan(2n-2)\alpha\}\dots..$

$\dots..\{\tan(n-1)\alpha\tan(n+1)\alpha\}.\tan n\alpha$

$= \left\{\tan\alpha\tan\left(\frac{\pi}{2}-\alpha\right)\right\}\left\{\tan 2\alpha\tan\left(\frac{\pi}{2}-2\alpha\right)\right\}\dots\tan\frac{\pi}{4} = 1$

165 **(a)**

$\sin(\alpha+\beta+\gamma) = \sin\alpha\cos\beta\cos\gamma + \cos\alpha\sin\beta\cos\gamma + \cos\alpha\cos\beta\sin\gamma - \sin\alpha\sin\beta\sin\gamma$

$\Rightarrow \sin(\alpha+\beta+\gamma) - \sin\alpha - \sin\beta - \sin\gamma$

$= \sin\alpha\,(\cos\beta\cos\gamma - 1) + \sin\beta\,(\cos\alpha\cos\gamma - 1) + \sin\gamma(\cos\alpha\cos\beta - 1)\sin\alpha\sin\beta\sin\gamma$

$\Rightarrow \sin(\alpha+\beta+\gamma) - \sin\alpha - \sin\beta - \sin\gamma < 0$

$\Rightarrow \sin\,(\alpha+\beta+\gamma) < sin\,\alpha + \sin\beta - \sin\gamma$

$\Rightarrow \dfrac{\sin(\alpha+\beta+\gamma)}{\sin\alpha + \sin\beta + \sin\gamma} < 1$

167 **(b)**

Given, $A + B = 180° - C$

$\Rightarrow \frac{A}{2} + \frac{B}{2} = 90° - \frac{C}{2}$

$\Rightarrow \tan\left(\frac{A}{2}+\frac{B}{2}\right) = \tan\left(90° - \frac{C}{2}\right)$

$\Rightarrow \dfrac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}\tan\frac{B}{2}} = \cot\frac{C}{2}$

$\Rightarrow \left(\tan\frac{A}{2}+\tan\frac{B}{2}\right)\tan\frac{C}{2} = 1 - \tan\frac{A}{2}\tan\frac{B}{2}$

$\Rightarrow \tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{A}{2}\tan\frac{C}{2} = 1$

168 **(b)**

$\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$= \dfrac{\frac{m}{m+1}+\frac{1}{2m+1}}{1-\frac{m}{m+1}.\frac{1}{2m+1}}$

$= \dfrac{2m^2+m+m+1}{2m^2+3m+1-m} = 1 = \tan\frac{\pi}{4}$

$\therefore \quad \alpha+\beta = \frac{\pi}{4}$

169 **(c)**

Given that,

$\tan A = 2\tan B + \cot B$ ....(i)

Now, $2\tan(A-B) = 2\left(\frac{\tan A - \tan B}{1+\tan A\tan B}\right)$

$= 2\,\frac{(2\tan B + \cot B - \tan B)}{1+(2\tan B+\cot B)\tan B}$ [from Eq.(i)]

$= 2\,\dfrac{\tan B + \cot B}{2(1+\tan^2 B)} = \dfrac{\cot B(\tan^2 B + 1)}{(1+\tan^2 B)} = \cot B$

170 **(a)**

Given, $\sin\theta + \mathrm{cosec}\,\theta = 2$ ...(i)

$\sin^2\theta + \mathrm{cosec}^2\theta + 2 = 4$

$\Rightarrow \sin^2\theta + \mathrm{cosec}^2\theta = 2$ ...(ii)

$\therefore \sin^4\theta + \mathrm{cosec}^4\theta = 2$ ...(iii)

And $(\sin^2\theta + \mathrm{cosec}^2\theta)^3 = 2^3$

$\Rightarrow \sin^6\theta + \mathrm{cosec}^6\theta + 3\sin^2\theta\mathrm{cosec}^2\theta(\sin^2\theta + \mathrm{cosec}^2\theta) = 8$

$\Rightarrow \sin^6 + \mathrm{cosec}^6\theta + 3.2 = 8$

$\Rightarrow \sin^6\theta + \mathrm{cosec}^6\theta = 2$ ...(iv)

On multiplying Eqs. (iv) and (iii), we get

$(\sin^4\theta + \mathrm{cosec}^4\theta)(\sin^6\theta + \mathrm{cosec}^6\theta) = 4$

$\Rightarrow \sin^{10}\theta + \sin^4\theta\mathrm{cosec}^4\theta(\sin^2\theta + \mathrm{cosec}^2\theta) + \mathrm{cosec}^{10}\theta = 4$

$\Rightarrow \sin^{10}\theta + \mathrm{cosec}^{10}\theta = 4 - 2 = 2$

171 **(a)**

Given, $\cos x + \cos 3x + \cos 2x = 0$

$\Rightarrow 2\cos 2x\cos x + \cos 2x = 0$

$\Rightarrow \cos 2x\,(2\cos x + 1) = 0$

$\Rightarrow \cos 2x = 0 \quad \left[\because \cos x \neq -\frac{1}{2}\right]$

$\Rightarrow \quad x = \frac{\pi}{4}$

$\therefore$ General value is $2n\pi \pm \frac{\pi}{4},\ n \in Z$

172 **(a)**

$\cosh^{-1}x = \log\left(x + \sqrt{x^2-1}\right) = \log(2+\sqrt{3})$

$\therefore \quad x = 2$

173 **(d)**

If $\tan\frac{x}{2} = t$, the given equation becomes

$$A\left(\frac{2t}{1+t^2}\right)^2 + B\left(\frac{1-t^2}{1+t^2}\right)^3 + C = 0$$

$$\Rightarrow t^6(C-B) + 3t^4(B+C) + 8At^3 + 3t^2(C-B) + (C+B) = 0$$

This is an equation with six different roots

174 **(d)**

$\cos(270° + \theta)(\cos 90° - \theta) - \sin(270° - \theta)\cos\theta$

$= \sin\theta.\sin\theta + \cos\theta.\cos\theta$

$= \sin^2\theta + \cos^2\theta = 1$

175 **(b)**

We have,

$$2\sin\frac{A}{2} = \sqrt{1+\sin A} + \sqrt{1-\sin A}$$

$$\Rightarrow 2\cos\frac{A}{2} = \sqrt{\left(\cos\frac{A}{2}+\sin\frac{A}{2}\right)^2} + \sqrt{\left(\cos\frac{A}{2}-\sin\frac{A}{2}\right)^2}$$

$$\Rightarrow 2\cos\frac{A}{2} = \left|\cos\frac{A}{2}+\sin\frac{A}{2}\right| + \left|\cos\frac{A}{2}-\sin\frac{A}{2}\right|$$

$$\Rightarrow \cos\frac{A}{2}+\sin\frac{A}{2} \geq 0 \text{ and } \cos\frac{A}{2}-\sin\frac{A}{2} \geq 0$$

$$\Rightarrow -\frac{3\pi}{4} \leq \frac{A}{2} \leq \frac{3\pi}{4} \text{ and } -\frac{\pi}{4} \leq \frac{A}{2} \leq \frac{\pi}{4}$$

$$\Rightarrow -\frac{\pi}{4} \leq \frac{A}{2} \leq \frac{\pi}{4}$$

$$\Rightarrow 2n\pi - \frac{\pi}{4} \leq \frac{A}{2} \leq 2n\pi + \frac{\pi}{4}, n \in Z$$

176 **(d)**

Given, $\tan x = \frac{b}{a}$

$\therefore a\cos 2x = b\sin 2x$

$$= a \times \frac{1-\tan^2 x}{1+\tan^2 x} + b \times \frac{2\tan x}{1+\tan^2 x}$$

$$= a \times \frac{1-\frac{b^2}{a^2}}{1+\frac{b^2}{a^2}} + b \times \frac{2\frac{b}{a}}{1+\frac{b^2}{a^2}}$$

$$= \frac{a(a^2-b^2)}{a^2+b^2} + \frac{2ab^2}{a^2+b^2}$$

$$= \frac{a(a^2+b^2)}{a^2+b^2} = a$$

177 **(a)**

We have,

$1 + 8\sin^2 x^2 \cos^2 x^2$

$= 1 + 2(2 + \sin x^2 \cos x^2)^2$

$= 1 + 2(\sin 2x^2)^2$

$= 1 + 2\sin^2 2x^2 = 1 + (1 - \cos 4x^2)$

$= 2 - \cos 4x^2$

Now,

$-1 \leq \cos 4x^2 \leq 1$

$\Rightarrow 1 \leq 2 - \cos 4x^2 \leq 3$

$\Rightarrow 1 \leq 1 + 8\sin^2 x^2 \cos x^2 \leq 3$

Hence, the required maximum value is 3

178 **(a)**

We have,

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix}$$ Applying $R_1 \to R_1 + R_2 + R_3$ and tl taking $(a+b+c)$common from

$= (a+b+c)(ab+bc+ca-a^2-b^2-c^2)$

$= -(a^3+b^3+c^3-3abc)$

$= -8R^3(\sin^3 A + \sin^3 B + \sin^3 C - 3\sin A\sin B\sin C)$

$= -8R^3 \times 0 = 0$

179 **(d)**

We have,

$AM \geq GM$

$$\Rightarrow \frac{9\tan^2\theta + 4\cot^2\theta}{2} \geq \sqrt{4\cot^2\theta\,.\,9\tan^2\theta}$$

$\Rightarrow 9\tan^2\theta + 4\cot^2\theta \geq 12$

Hence, the minimum value is 12

180 **(d)**

Given, $4(\sin^2 x - 3\sin x) + 7$

$$= 4\left[\left(\sin x - \frac{3}{2}\right)^2 - \frac{9}{4}\right] + 7$$

$$= 4\left(\sin x - \frac{3}{2}\right)^2 - 2$$

Now, $-1 \leq \sin x \leq 1$

$$\Rightarrow -\frac{5}{2} \leq \sin x - \frac{3}{2} \leq -\frac{1}{2}$$

$$\Rightarrow \frac{1}{4} \leq \left(\sin x - \frac{3}{2}\right)^2 \leq \frac{25}{4}$$

$$\Rightarrow 1 \leq 4\left(\sin x - \frac{3}{2}\right)^2 \leq 25$$

$$\Rightarrow -1 \leq 4\left(\sin x - \frac{3}{2}\right)^2 - 2 \leq 23$$

181 **(d)**

Since, $-2 \leq \sin x - \sqrt{3}\cos x \leq 2$

$\Rightarrow -1 \leq \sin x - \sqrt{3}\cos x + 1 \leq 3$

$\therefore$ Range of $f(x) = [-1, 3]$

182 **(a)**

We have,

$$\frac{\sec^2\theta - \tan\theta}{\sec^2\theta + \tan\theta} = y$$

$$\Rightarrow \frac{1+x^2-x}{1+x^2+x} = y, \text{where } \tan\theta = x$$

$\Rightarrow x^2(y-1) + x(y+1) + y - 1 = 0$

$\Rightarrow (y+1)^2 - 4(y-1)^2$

$\geq 0 \quad \begin{bmatrix} \because x = \tan\theta \text{ is real} \\ \therefore \text{Disc} \geq 0 \end{bmatrix}$

$\Rightarrow -3y^2 + 10y - 3 \geq 0$

$\Rightarrow 3y^2 - 10y + 3 \leq 0 \Rightarrow y \in (1/3, 3)$

183 **(d)**

Since $\sin\theta, \cos\theta$ are the roots of $ax^2 - bx + c = 0$

$\therefore \sin\theta + \cos\theta = \dfrac{b}{a}$

and $\sin\theta\cos\theta = \frac{c}{a}$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \dfrac{b^2}{a^2}$

and $\sin\theta\cos\theta = \frac{\text{c}}{\text{a}}$

$\Rightarrow 1 + 2\left(\dfrac{c}{a}\right) = \dfrac{b^2}{a^2}$

$\Rightarrow b^2 - a^2 = 2ac$

184 **(c)**

We have, $\sqrt{2}\sec\theta + \tan\theta = 1$

$\Rightarrow \dfrac{\sqrt{2}}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta} = 1$

$\Rightarrow \sin\theta - \cos\theta = -\sqrt{2}$

$\Rightarrow \dfrac{1}{\sqrt{2}}\sin\theta - \dfrac{1}{\sqrt{2}}\cos\theta = -1$

$\Rightarrow \dfrac{1}{\sqrt{2}}\cos\theta - \dfrac{1}{\sqrt{2}}\sin\theta = 1$

$\Rightarrow \cos\dfrac{\pi}{4}\cos\theta - \sin\dfrac{\pi}{4}\sin\theta = 1$

$\Rightarrow \cos\left(\theta + \dfrac{\pi}{4}\right) = \cos\theta$

$\Rightarrow \theta + \dfrac{\pi}{4} = 2n\pi \pm 0$

$\Rightarrow \theta = 2n\pi - \dfrac{\pi}{4}$

185 **(b)**

$\sqrt{\sin^2 x - \sin x + \dfrac{1}{2}} = \sqrt{\left(\sin x - \dfrac{1}{2}\right)^2 + \dfrac{1}{4}} \geq \dfrac{1}{2}, \forall x$

and $\sec^2 y \geq 1, \forall y$, so $2^{\sec^2 y} \geq 2$. Hence, the above inequality holds only for those values of x and y for which $\sin x = \frac{1}{2}$ and $\sec^2 y = 1$. Hence, $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$ and $y = 0, \pi, 2\pi, 3\pi$. Hence, required number of ordered pairs are 16

186 **(d)**

$\cos 132° + \cos 12° + \cos 156° + \cos 84°$

$= 2\cos 72° \cos 60° + 2\cos 120° \cos 36°$

$= 2\left(\dfrac{\sqrt{5}-1}{4}\right)\dfrac{1}{2} + 2\left(\dfrac{-1}{2}\right)\left(\dfrac{\sqrt{5}+1}{4}\right) = \dfrac{-1}{2}$

187 **(c)**

$\cos^4 x - (\lambda+2)\cos^2 x - (\lambda+3) = 0$

$\Rightarrow (\cos^2 x)^2 - (\lambda+2)\cos^2 x - (\lambda+3) = 0$

$\therefore \cos^2 x = \dfrac{(\lambda+2) \pm \sqrt{(\lambda+2)^2 + 4(\lambda+3)}}{2}$

$= \dfrac{(\lambda+2) \pm (\lambda+4)}{2}$

$= \lambda + 3, -1$

$\Rightarrow \cos^2 x = \lambda + 3 \;\; (\because \cos^2 x \neq -1)$

But $0 \leq \cos^2 x \leq 1$

$\Rightarrow 0 \leq \lambda + 3 \leq 1$

$\Rightarrow -3 \leq \lambda \leq -2$

188 **(c)**

We have,

$\sin 5x + \sin 3x + \sin x = 0$

$\Rightarrow (\sin 5x + \sin x) + \sin 3x = 0$

$\Rightarrow 2\sin 3x \cos x + \sin 3x = 0$

$\Rightarrow \sin 3x(2\cos 2x + 1) = 0$

$\Rightarrow \sin 3x = 0, \cos 2x = -\dfrac{1}{2}$

$\Rightarrow 3x = n\pi, 2x = 2n\pi \pm \dfrac{2\pi}{3}$

$\Rightarrow x = \dfrac{n\pi}{3}, x = n\pi \pm \dfrac{\pi}{3}$

The value of x given by the above expressions and lying between 0 and $\frac{\pi}{2}$ is $\frac{\pi}{3}$

189 **(a)**

Let $81^{\sin^2 x} = y$. Then,

$81^{\cos^2 x} - 81^{1-\sin^2 x} = 81y^{-1}$

$\Rightarrow y^2 - 30y + 81 = 0$

$\Rightarrow y = 3 \text{ or}, y = 27$

$\Rightarrow 81^{\sin^2 x} = 3 \text{ or}, 27$

$\Rightarrow 3^{4\sin^2 x} = 3^1 \text{ or}, 3^3$

$\Rightarrow 4\sin^2 x = 1 \text{ or}, 3$

$\Rightarrow \sin^2 x = \dfrac{1}{4} \text{ or}, \dfrac{3}{4}$

$\Rightarrow \sin x = \pm\dfrac{1}{2} \text{ or}, \pm\dfrac{\sqrt{3}}{2} \Rightarrow x = \dfrac{\pi}{6} \text{ or}, \dfrac{\pi}{3}$

190 **(c)**

We have,

$\dfrac{c}{\sin C} = \dfrac{a}{\sin A}$

$\Rightarrow \sin C = \dfrac{c\sin A}{a} = \dfrac{2\sin 120°}{\sqrt{6}} = \dfrac{1}{\sqrt{2}}$

$\Rightarrow C = 45° \text{ or } 135°$

But, in a triangle there cannot be two obtuse angle

$\therefore C = 45°$

191 **(d)**

$\sin 6\theta = 3\sin 2\theta - 4\sin^3 2\theta$

$= 3\sin 2\theta - 4.8\cos^3\theta\sin^3\theta$
$= 3\sin 2\theta - 32\cos^3\theta\sin\theta(1-\cos^2\theta)$
$\Rightarrow\ \sin 6\theta = 32\cos^5\theta.\sin\theta - 32\cos^3\theta\sin\theta + 3\sin 2\theta$ ...(i)
But $\sin 6\theta = 32\cos^5\theta\sin\theta - 32\cos^3\theta\sin\theta + 3x$ [given]
On comparing Eqs. (i) and (ii), we get
$3x = 3\sin 2\theta \Rightarrow x = \sin 2\theta$

192 **(c)**

$x\cos\theta = y\cos\left(\theta + \frac{2\pi}{3}\right)$
$= z\cos\left(\theta+\frac{4\pi}{3}\right) = k$ (say)
$\Rightarrow\ \cos\theta = \frac{k}{x}, \cos\left(\theta+\frac{2\pi}{3}\right) = \frac{k}{y}$
And $\cos\left(\theta+\frac{4\pi}{3}\right) = \frac{k}{z}$
Hence, $\frac{k}{x}+\frac{k}{y}+\frac{k}{z} = \cos\theta + \cos\left(\theta+\frac{2\pi}{3}\right) + \cos\theta + 4\pi 3$
$= \cos\theta - \cos\left(\frac{\pi}{3}-\theta\right) - \cos\left(\frac{\pi}{3}+\theta\right)$
$= \cos\theta - 2\cos\frac{\pi}{3}\cos\theta = 0$

193 **(c)**

$\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta) = -\frac{3}{2}$
$\Rightarrow\ 2[\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)]+3 = 0$
$\Rightarrow\ 2[\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)] + \sin^2\alpha+\cos^2\alpha+\sin^2\beta+\cos^2\beta + \sin 2\gamma + \cos 2\gamma = 0$
$\Rightarrow\ (\sin\alpha+\sin\beta+\sin\gamma)^2 + (\cos\alpha+\cos\beta+\cos\gamma)^2 = 0$

194 **(a)**

We have,
$\sin x\cos x\cos 2x = \lambda \Rightarrow \sin 4x = 4\lambda$
Clearly, this equation will have a solution if
$|4\lambda| \le 1 \Rightarrow \lambda \in [-1/4, 1/4]$

195 **(b)**

$1+\cos 56° + \cos 58° - \cos 66°$

$= 2\cos^2 28° + 2\sin 62°\sin 4°$

$= 2\cos^2 28° + 2\cos 28°\cos 86°$

$= 2\cos 28°(\cos 28° + \cos 86°)$

$= 2\cos 28°(2\cos 57°\cos 29°)$

$= 4\cos 28°\cos 29°\sin 33°$

196 **(d)**

(1)We have, $\sin A = \sin B$
$\Rightarrow A = B$ or $A = \pi - B$

Now, $\sin 2A = \sin 2B$ is satisfied by $A = B$ but it is not satisfied by $A = \pi - B$

(2)$\cos\frac{\pi}{7}\cos\frac{4\pi}{7}\cos\frac{5\pi}{7} = \cos\frac{\pi}{7}\cos\frac{4\pi}{7}\cos\left(\pi - \frac{2\pi}{7}\right)$

$= -\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}$

$= -\frac{\sin\left(2^3\frac{\pi}{7}\right)}{2^3\sin\frac{\pi}{7}} = -\frac{\sin\frac{8\pi}{7}}{8\sin\frac{\pi}{7}} = \frac{1}{8}$

197 **(a)**

Given, $\cos 2x + k\sin x = 2k - 7$
$\Rightarrow\ 1 - 2\sin^2 x + k\sin x = 2k - 7$
$\Rightarrow\ 2\sin^2 x - k\sin x + 2k - 8 = 0$
$\Rightarrow\ \sin x = \frac{+k \pm \sqrt{k^2 - 8(2k-8)}}{4}$
$\Rightarrow\ \sin x = \frac{k \pm (k-8)}{4}$
$\Rightarrow\ \sin x = \frac{k-4}{2}$
($\because$ for $'-'$sign $\sin x = 2$, which is not possible)
$\because\ -1 \le \sin x \le 1 \ \Rightarrow\ -1 \le \frac{k-4}{2} \le 1$
$\Rightarrow\ -2 \le k - 4 \le 2 \ \Rightarrow\ 2 \le k \le 6$

198 **(d)**

We have,
$\cos 2A + \cos 2B + \cos 2C$
$= 2\cos(A+B)\cos(A-B) + \cos 2C$
$= 2\cos\left(\frac{3\pi}{2} - C\right)\cos(A-B) + \cos 2C$
$= -2\sin C\cos(A-B) + 1 - 2\sin^2 C$
$= 1 - 2\sin C\{\cos(A-B) + \sin C\}$
$= 1 - 2\sin C\{\cos(A-B) + \sin(3\pi/2 - (A+B))\}$
$= 1 - 2\sin C\{\cos(A-B) - \cos(A+B)\}$
$= 1 - 4\sin A\sin B\sin C$

200 **(a)**

Since, $-1 \le \cos\theta \le 1$
$\therefore\ -1 \le \cos(4x-5) \le 1$
$\Rightarrow\ -3 \le 3\cos(4x-5) \le 3$
$\Rightarrow\ 4 - 3 \le 3\cos(4x-5) + 4 \le 3 + 4$
$\Rightarrow\ 1 \le 3\cos(4x-5) + 4 \le 7$

201 **(b)**

We have,
$\cos x = \sin\alpha\cot\beta\sin x = \cos\alpha$
$\Rightarrow \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} - \sin\alpha\cot\beta\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \cos\alpha$
$\Rightarrow \tan^2\frac{x}{2} + \frac{2\sin\alpha\cot\beta}{1+\cos\alpha}\tan\frac{x}{2} - \frac{1-\cos\alpha}{1+\cos\alpha} = 0$

$$\Rightarrow \tan^2\frac{x}{2} + 2\tan\frac{\alpha}{2}\cot\beta\tan\frac{x}{2} - \tan^2\frac{\alpha}{2} = 0$$

$$\Rightarrow \tan^2\frac{x}{2} + \left\{\cot\frac{\beta}{2} - \tan\frac{\beta}{2}\right\}\tan\frac{\alpha}{2}\tan\frac{x}{2} - \tan^2\frac{\alpha}{2} = 0$$

$$\Rightarrow \left(\tan\frac{x}{2} + \cot\frac{\beta}{2}\tan\frac{\alpha}{2}\right)\left(\tan\frac{x}{2} - \tan\frac{\beta}{2}\tan\frac{\alpha}{2}\right) = 0$$

$$\Rightarrow \tan\frac{x}{2} = -\cot\frac{\beta}{2}\tan\frac{\alpha}{2}, \tan\frac{\alpha}{2}\tan\frac{\beta}{2}$$

202 **(c)**

$\sin 20° \sin 40° \sin 60° \sin 80°$

$$= \frac{1}{2}\sin 20° \sin 60°(2\sin 40° \sin 80°)$$

$$= \frac{1}{2}\sin 20° \sin 60°(\cos 40° - \cos 120°)$$

$$= \frac{1}{2}\cdot\frac{\sqrt{3}}{2}\sin 20°\left(1 - 2\sin^2 20° + \frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{4}\sin 20°\left(\frac{3}{2} - 2\sin^2 20°\right)$$

$$= \frac{\sqrt{3}}{8}(3\sin 20° - 4\sin^3 20°)$$

$$= \frac{\sqrt{3}}{8}\sin 60° = \frac{\sqrt{3}}{8}\cdot\frac{\sqrt{3}}{2} = \frac{3}{16}$$

203 **(a)**

We have,

$\sin(\pi+\theta)\sin(\pi-\theta)\ \text{cosec}^2\theta$

$= -\sin\theta\sin\theta\,\text{cosec}^2\theta = -1$

204 **(a)**

We have,

$\sin\theta(\sin\theta + 2\cos\theta) = a$

$\Rightarrow 1 - \cos 2\theta + 2\sin 2\theta = 2a$

$\Rightarrow 2\sin 2\theta - \cos 2\theta = 2a - 1$

This equation will have a solution if

$|2a - 1| \le \sqrt{2^2 + (-1)^2}$

$$\Rightarrow 1 - \sqrt{5} \le 2a \le 1 + \sqrt{5} \Rightarrow a \in \left[\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right]$$

205 **(c)**

Since, $(2\tan\theta + 2)^2 = \tan\theta(3\tan\theta + 3)$

$\Rightarrow 4\tan^2\theta + 8\tan\theta + 4 = 3\tan^2\theta + 3\tan\theta$

$\Rightarrow \tan^2\theta + 5\tan\theta + 4 = 0$

$\Rightarrow (\tan\theta + 4)(\tan\theta + 1) = 0$

$\Rightarrow \tan\theta = -4\ \ (\because \tan\theta \ne -1)$

$$\therefore \frac{7 - 5\cot\theta}{9 - 4\sqrt{\tan^2\theta}} = \frac{7 + \frac{5}{4}}{9 - 4(-4)} = \frac{33}{100}$$

206 **(c)**

We have,

$$\sin\frac{A}{2}\sin\frac{5A}{2}$$

$$= \frac{1}{2}\left(2\sin\frac{A}{2}\sin\frac{5A}{2}\right)$$

$$= \frac{1}{2}(\cos 2A - \cos 3A)$$

$$= \frac{1}{2}\{2\cos^2 A - 1 - 4\cos^3 A + 3\cos A\}$$

$$= \frac{1}{2}\left\{2\times\frac{9}{16} - 1 - 4\times\frac{27}{64} + 3\times\frac{3}{4}\right\} = \frac{11}{32}$$

207 **(b)**

We have,

$\cos A + \cos B + \cos C = 0$

$\Rightarrow \cos^3 A + \cos^3 B + \cos^3 C = 3\cos A\cos B\cos C$

$$\Rightarrow \frac{\cos 3A + 3\cos A}{4} + \frac{\cos 3B + 3\cos B}{4} + \frac{\cos 3C + 3\cos C}{4}$$

$= 3\cos A\cos B\cos C$

$\Rightarrow \cos 3A + \cos 3B + \cos 3C = 12\cos A\cos B\cos C$

208 **(a)**

We have,

$$\tan 82\frac{1°}{2} = \cot 7\frac{1}{2°} = \frac{\cos 7\frac{1°}{2}}{\sin 7\frac{1°}{2}}$$

$$= \frac{2\cos^2 7\frac{1°}{2}}{2\sin 7\frac{1°}{2}\cos 7\frac{1°}{2}} = \frac{1 + \cos 15°}{\sin 15°}$$

$$= \frac{1 + \frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}}$$

$$= \frac{2\sqrt{2} + \sqrt{3} + 1}{\sqrt{3} - 1}$$

$$= \frac{(2\sqrt{2} + \sqrt{3} + 1)(\sqrt{3} + 1)}{(\sqrt{3} + 1)(\sqrt{3} - 1)}$$

$$= \frac{2\sqrt{6} + 2\sqrt{2} + 3 + \sqrt{3} + \sqrt{3} + 1}{2}$$

$= \sqrt{6} + \sqrt{2} + \sqrt{4} + \sqrt{3} = \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6}$

209 **(a)**

$x^2 + y^2 + z^2 = r^2 \sin^2 \theta \cos^2 \phi + r^2 \sin^2 \theta \sin^2 \phi + r^2 \cos^2 \theta$

$= r^2 \sin^2 \theta(\cos^2 \phi + \sin^2 \phi) + r^2 \cos^2 \theta$

$= r^2 \sin^2 \theta + r^2 \cos^2 \theta$

$= r^2$

210 **(a)**

Since $ABCD$ is a cyclic quadrilateral. Therefore,
$A + C = \pi$ and $B + D = \pi$

Now,
$12 \tan A - 5 = 0$ and $5 \cos B + 3 = 0$

$\Rightarrow \tan A = \frac{5}{12}$ and $\cos B = -\frac{3}{5}$

$\Rightarrow \tan C = -\frac{5}{12}$ and $\cos D = \frac{3}{5}$ [$\because A = \pi - C$ and $B = \pi - D$

$\Rightarrow \cos C = -\frac{12}{13}$ and $\tan D = \frac{4}{3}$

$\begin{bmatrix} \tan A > 0 \quad \therefore 0 < A < \frac{\pi}{2} \Rightarrow \frac{\pi}{2} < C < \pi \\ \cos B < 0 \quad \therefore \frac{\pi}{2} < B < \pi \Rightarrow 0 < D < \frac{\pi}{2} \end{bmatrix}$

The equation having $\cos C$ and $\tan D$ as its roots is
$x^2 - x(\cos C + \tan D) + \cos C \tan D = 0$

or, $x^2 - x\left(-\frac{12}{13} + \frac{4}{3}\right) + \left(-\frac{12}{13} \times \frac{4}{3}\right) = 0$

or, $39x^2 - 16x - 48 = 0$

211 **(b)**

It is given that
r_1, r_2, r_3 are in H.P.

$\Rightarrow \frac{2}{r_2} = \frac{1}{r_1} + \frac{1}{r_3}$

$\Rightarrow \frac{2(s-b)}{\Delta} = \frac{s-a}{\Delta} + \frac{s-c}{\Delta}$

$\Rightarrow 2s - 2b = 2s - a - c \Rightarrow 2b = a + c \Rightarrow a, b, c$ are in A.P.

212 **(b)**

As $\sin \theta = \frac{1}{2}$ and $\cos \phi = \frac{1}{3}$

$\Rightarrow \theta = \frac{\pi}{6}$ and $0 < \left(\cos \phi = \frac{1}{3}\right) < \frac{1}{2}$

$\left[\text{as}, 0 < \frac{1}{3} < \frac{1}{2}\right]$

$\Rightarrow \theta = \frac{\pi}{6}$ and $\cos^{-1}(0) > \phi > \cos^{-1}\left(\frac{1}{2}\right)$

[the sign changed as $\cos x$ is decreasing between (

$\Rightarrow \theta = \frac{\pi}{6}$ and $\frac{\pi}{3} < \phi < \frac{\pi}{2}$

$\Rightarrow \frac{\pi}{2} < \theta + \phi < \frac{2\pi}{3}$

$\therefore \phi + \theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

213 **(c)**

$\sin 4A - \cos 4A = \cos 2A - \sin 2A$

On squaring, we get

$1 - 2 \sin 4A \cos 4A = 1 - 2 \sin 2A \cos 2A$

$\Rightarrow \cos 4A = \frac{1}{2}$

$\Rightarrow \tan 4A = \sqrt{3}$

Alternate

Let $\tan 4A = \sqrt{3} = \tan \frac{\pi}{3}$

$\Rightarrow A = \frac{\pi}{12}$

$\therefore \sin 4A - \cos 2A = \sin \frac{\pi}{3} - \cos \frac{\pi}{6} = 0$

And $\cos 4A - \sin 2A = \cos \frac{\pi}{3} - \sin \frac{\pi}{6} = 0$

$\therefore \sin 4A - \cos 2A = \cos 4A - \sin 2A$

Hence, our assumption is true.

214 **(a)**

It is given that r_1, r_2, r_3 are in H.P.

$\therefore \frac{\Delta}{s-a}, \frac{\Delta}{s-b}, \frac{\Delta}{s-c}$ are in H. P.

$\Rightarrow \frac{s-a}{\Delta}, \frac{s-b}{\Delta}, \frac{s-c}{\Delta}$ are in A. P.

$\Rightarrow s-a, s-b, s-c$ are in A.P.

$\Rightarrow a, b, c$ are in A.P.

215 **(c)**

We have,

$1 + \cos 56° + \cos 58° - \cos 66° = \lambda \cos 28° \cos 29° \sin 33°$

$\Rightarrow (1 - \cos 66°) + (\cos 56° + \cos 58°) = \lambda \cos 28° \cos 29° \sin 33°$

$\Rightarrow 2 \sin^2 33° + 2 \cos 57° \cos 1° = \lambda \cos 28° \cos 29° \sin 33°$

$\Rightarrow 2 \sin 33° (\sin 33° + \sin 89°) = \lambda \cos 28° \cos 29° \sin 33°$

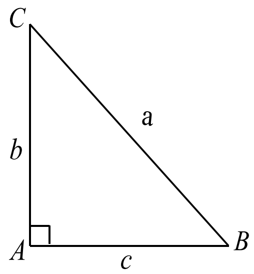
$\Rightarrow 2 \sin 33° \times 2 \sin 61° \cos 28° = \lambda \cos 28° \cos 29° \sin 33°$

$\Rightarrow 4 \cos 28° \cos 29° \sin 33° = \lambda \cos 28° \cos 29° \sin 33°$

$\Rightarrow \lambda = 4$

216 **(d)**

$\tan A = \frac{a}{b}, \tan B = \frac{b}{a}$



$\therefore \tan A + \tan B = \frac{a^2 + b^2}{ab}$

Since, $a^2 + b^2 = c^2$

$\therefore\ \tan A+\tan B=\dfrac{c^2}{ab}$

218 **(a)**

$\sinh^{-1}(2)^{3/2}=\log(2^{3/2}+\sqrt{(2^{3/2})^2+1})$

$=\log(\sqrt{8}+\sqrt{8+1})$

$=\log(3+\sqrt{8})$

219 **(a)**

We have, $\tan 2\theta\tan\theta=1\ \Rightarrow\ \tan 2\theta=\cot\theta$

$=\tan\left(\dfrac{\pi}{2}-\theta\right)$

$\Rightarrow\ 2\theta=n\pi+\dfrac{\pi}{2}-\theta$

$\Rightarrow\ \theta=\dfrac{n\pi}{3}+\dfrac{\pi}{6}$

220 **(d)**

For varying values of A, B and C the expression will attain the maximum value when $\cos^2 A, \cos^2 B$ attain their maximum values each equal to 1 and $\cos^2 C$ is least i.e. 0

Hence, required maximum value $=1+1-0=2$

221 **(b)**

$\cos^2(A-B)+\cos^2 B-2\cos(A-B)\cos A\cos B$

$=\cos^2(A-B)+\cos^2 B$
$\quad-\cos(A-B)[\cos(A-B)$
$\quad+\cos(A+B)]$

$=\cos^2 B-\cos(A-B)\cos(A+B)$

$=\cos^2 B-(\cos^2 A$
$\quad-\sin^2 B)=1-\cos^2 A=\sin^2 A$

222 **(d)**

$\sin 36°\sin 72°\sin 108°\sin 144°$

$=\sin^2 36°\sin^2 72°$

$=\dfrac{1}{4}[(2\sin^2 36°)(2\sin^2 72°)]$

$=\dfrac{1}{4}[(1-\cos 72°)(1-\cos 144°)]$

$=\dfrac{1}{4}[(1-\sin 18°)(1+\cos 36°)]$

$=\dfrac{1}{4}\left[\left(1-\dfrac{\sqrt{5}-1}{4}\right)\left(1+\dfrac{\sqrt{5}+1}{4}\right)\right]=\dfrac{5}{16}$

223 **(c)**

$\dfrac{1+\tan x+\tan^2 x)(1+\tan^2 x-\tan x)}{\tan^2 x}$

$=\dfrac{(1+\tan^2 x)^2-\tan^2 x}{\tan^2 x}$

Since, $1+\tan^2 x\geq\tan^2 x, \forall x$. Hence, it is positive for all values of x

224 **(c)**

$x\log_e a+\dfrac{x^3}{3!}(\log_e a)^3+\dfrac{x^5}{5!}(\log_e a)^5+\ldots$

$=\dfrac{e^x\log_e a-e^{-x}\log_e a}{2}\quad\left[\because\ \dfrac{e^x-e^{-x}}{2}=x+\dfrac{x^3}{3!}+\dfrac{x^5}{5!}+\ldots\right]$

$=\sinh(x\log_e a)$

225 **(b)**

We have,

$\dfrac{\tan(\theta+15°)}{\tan(\theta-15°)}=\dfrac{1}{3}$

$\Rightarrow\dfrac{\tan(\theta+15°)+\tan(\theta-15°)}{\tan(\theta+15°)-\tan(\theta-15°)}=\dfrac{3+1}{3-1}$

$\Rightarrow\dfrac{\sin\{(\theta+15°)+(\theta-15°)\}}{\sin\{(\theta+15°)-(\theta-15°)\}}=2$

$\Rightarrow\sin 2\theta=2\sin 30°=1\Rightarrow 2\theta=\dfrac{\pi}{2}\Rightarrow\theta=\dfrac{\pi}{4}$

226 **(b)**

In a convex quadrilateral each angle is less than 180°

We have,

$4\sec A+5=0\Rightarrow\sec A=-\dfrac{5}{4}\Rightarrow\dfrac{\pi}{2}<A<\pi$

$\therefore\tan A=-\frac{3}{4}$ and $\operatorname{cosec} A=\frac{5}{3}$

The quadratic equation having $\tan A$ and $\operatorname{cosec} A$ as its roots is

$x^2-x\left(-\dfrac{3}{4}+\dfrac{5}{3}\right)+\left(-\dfrac{3}{4}\times\dfrac{5}{3}\right)=0$

$\Rightarrow 12x^2-11x-15=0$

227 **(c)**

Since, $-1\leq\cos\theta\leq 1$

$\Rightarrow\ -5\leq 5\cos\theta\leq 5$

$\Rightarrow\ -5+12\leq 5\cos\theta+12<5+12$

$\Rightarrow\ \ 7\leq 5\cos\theta+12<17$

Hence, minimum value is 7.

Alternate

For minimum value of given expression, we take $\cos\theta=-1$

228 **(c)**

We have,

$2^{\sin x+\cos y}=1=2^0 \Rightarrow \sin x+\cos y=0$ ...(i)

It is given that

$16^{\sin^2 x+\cos^2 y}=4=16^{1/2}$

$\Rightarrow \sin^2 x+\cos^2 y=\frac{1}{2}$ ... (ii)

Eliminating $\cos y$ from (i) and (ii), we get

$2\sin^2 x=\frac{1}{2} \Rightarrow \sin x=\pm\frac{1}{2}$

Now, $\sin x=\frac{1}{2}$

$\Rightarrow \cos y=-\frac{1}{2}$

$\Rightarrow x=n\pi+(-1)^n\frac{\pi}{6}$ and $y=2n\pi\pm\frac{2\pi}{3}, n\in Z$

and, $\sin x=-\frac{1}{2}$

$\Rightarrow \cos y=\frac{1}{2}$

$\Rightarrow x=n\pi+(-1)^{n+1}\frac{\pi}{6}$ and $y=2n\pi\pm\frac{\pi}{3}, n\in Z$

229 **(b)**

Given, $\cos\theta=-\frac{\sqrt{3}}{2}<0$ and θ does not lie in third quadrant.

$\therefore$ θ must be lying in 2nd quadrant

$\Rightarrow$ $\tan\theta=-\frac{1}{\sqrt{3}}$ and $\cot\theta=-\sqrt{3}$ ...(i)

Also, α lies in 3rd quadrant and $\sin\alpha=-\frac{3}{5}$

$\therefore$ $\tan\alpha=\frac{3}{4}$ and $\cos\alpha=-\frac{4}{5}$ ...(ii)

$\therefore$ $\frac{2\tan\alpha+\sqrt{3}\tan\theta}{\cot^2\theta+\cos\alpha}=\frac{2.\frac{3}{4}-\sqrt{3}-\frac{1}{\sqrt{3}}}{3-\frac{4}{5}}=\frac{5}{22}$

230 **(b)**

$2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\frac{4\pi}{13}\cos\frac{\pi}{13}$

$=2\cos\frac{\pi}{13}\left[\cos\frac{9\pi}{13}+\cos\frac{4\pi}{13}\right]$

$=2\cos\frac{\pi}{13}\left[2\cos\frac{\pi}{2}.\cos\frac{5\pi}{26}\right]$

$=0$

231 **(b)**

We have, $\sin A=\frac{3}{5}$

$\therefore \cos A=\frac{4}{5}$

$\Rightarrow \frac{b^2+c^2-a^2}{2bc}=\frac{4}{5}$

$\Rightarrow \frac{400+441-a^2}{2\times 20\times 21}=\frac{4}{5}$

$\Rightarrow 841-a^2=32\times 21 \Rightarrow a^2=841-672=169$

$\Rightarrow a=13$

232 **(a)**

$\cos^2\frac{\pi}{16}+\cos^2\frac{3\pi}{16}+\cos^2\frac{5\pi}{16}+\cos^2\frac{7\pi}{16}$

$=\cos^2\frac{\pi}{16}+\cos^2\frac{3\pi}{16}+\cos^2\left(\frac{\pi}{2}-\frac{3\pi}{16}\right)+\cos^2\left(\frac{\pi}{2}-\frac{\pi}{16}\right)$

$=\cos^2\frac{\pi}{16}+\cos^2\frac{3\pi}{16}+\sin^2\frac{3\pi}{16}+\sin^2\frac{\pi}{16}=1+1=2$

233 **(b)**

We have, $\cos(\alpha+\beta)=\frac{4}{5}$ and $\sin(\alpha-\beta)=\frac{5}{13}$

$\Rightarrow \sin(\alpha+\beta)=\frac{3}{5}$ and $\cos(\alpha-\beta)=\frac{12}{13}$

$\Rightarrow (\alpha+\beta)=\sin^{-1}\frac{3}{5}$ and $(\alpha-\beta)=\sin^{-1}\frac{5}{13}$

$\therefore 2\alpha=\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{5}{13}$

$=\sin^{-1}\left[\frac{3}{5}\sqrt{1-\frac{25}{169}}+\frac{5}{13}\sqrt{1-\frac{9}{25}}\right]$

$=\sin^{-1}\left(\frac{3}{5}\times\frac{12}{13}+\frac{5}{13}\times\frac{4}{5}\right)=\sin^{-1}\left(\frac{36}{65}+\frac{20}{65}\right)$

$\Rightarrow \sin 2\alpha=\frac{56}{65}$

$\therefore \tan 2\alpha=\frac{56}{33}$

234 **(c)**

$\cos 2\theta+2\cos\theta=2\cos^2\theta-1+2\cos\theta$

$=2\left(\cos\theta+\frac{1}{2}\right)^2-\frac{3}{2}$

$\geq -\frac{3}{2}$ $\left[\because 2\left(\cos\theta+\frac{1}{2}\right)^2\geq 0, \forall\theta\right]$

Then maximum value of $\cos 2\theta+2\cos\theta$ is 3

235 **(a)**

We have,

$x=\sin 130°+\cos 130°$

$=\sin(180°-50°)+\cos(90°+40°)$

$\Rightarrow x=\sin 50°-\sin 40°>0$ $[\because \sin 50°>\sin 40°]$

236 **(a)**

We have,

$|4\sin x-1|<\sqrt{5}$

$\Rightarrow 1-\sqrt{5}<4\sin x<1+\sqrt{5}$

$\Rightarrow -\frac{\sqrt{5}-1}{4}<\sin x<\frac{\sqrt{5}+1}{4}$

$\Rightarrow -\sin\frac{\pi}{10} < \sin x < \cos\frac{\pi}{10}$

$\Rightarrow \sin\left(-\frac{\pi}{10}\right) < \sin x < \sin\left(\frac{\pi}{2}-\frac{\pi}{10}\right)$

$\Rightarrow \sin\left(-\frac{\pi}{10}\right) < \sin x < \sin\frac{3\pi}{10}$

$\Rightarrow -\frac{\pi}{10} < x < \frac{3\pi}{10}$ $\left[\because \sin x \text{ is increasing on } (-\pi/2, \pi/2)\right]$

$\Rightarrow x \in (-\pi/10, 3\pi/10)$

237 **(b)**

Given, $\sin A = n\sin B \quad \Rightarrow \frac{n}{1} = \frac{\sin A}{\sin B}$

Applying componendo and dividend, we get

$\frac{n-1}{n+1} = \frac{\sin A - \sin B}{\sin A + \sin B}$

$= \frac{2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}$

$\Rightarrow \frac{n-1}{n+1} = \tan\left(\frac{A-B}{2}\right)\cot\left(\frac{A+B}{2}\right)$

$\Rightarrow \frac{n-1}{n+1}\tan\left(\frac{A+B}{2}\right) = \tan\left(\frac{A-B}{2}\right)$

238 **(a)**

We have,

$\frac{s}{R} = \frac{a+b+c}{2R} = \frac{a}{2R} + \frac{b}{2R} + \frac{c}{2R}$

$= \sin A + \sin B + \sin C$

239 **(b)**

(a)$\sin\theta = \frac{5}{3}$ is not possible, because the value of $\sin\theta$ lies in $[-1,1]$

(b)$\tan\theta = 100^2$. This is possible

(c)$\cos = \frac{1+p^2}{1-p^2}, [p \neq \pm 1]$ this is not possible, because here, $\cos\theta$ is greater than one

(d)$\sec\theta = \frac{1}{2}$, this is not possible because $\sec\theta$ is not less than one

$\therefore$ Option (b) is true

240 **(a)**

We have,

$\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{3\pi}{7}$

$= \cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\left(\pi - \frac{4\pi}{7}\right)$

$= -\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}$

$= -\left\{\frac{\sin\left(2^3 \times \frac{\pi}{7}\right)}{2^3\sin\frac{\pi}{7}}\right\} = -\frac{\sin\frac{8\pi}{7}}{8\sin\frac{\pi}{7}} = \frac{1}{8}$

241 **(c)**

$\because \sec\theta + \tan\theta = \sqrt{3}$ ...(i)

Also, we have

$\sec^2\theta - \tan^2\theta = 1$ ...(ii)

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{\sqrt{3}}$ ...(iii)

From Eqs. (i) and (iii), we get

$\tan\theta = \frac{1}{2}\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}} = \tan\frac{\pi}{6}$

$\Rightarrow \theta = n\pi + \frac{\pi}{6}$

$\therefore$ Solutions for $0 < \theta < 2\pi$ are $\frac{\pi}{6}$ and $\frac{7\pi}{6}$

Hence, there are two solutions

242 **(c)**

Now, $\sin 12° \sin 48° \sin 54°$

$= \frac{1}{2}(\cos 36° - \cos 60°)\cos 36°$

$= \frac{1}{2}\left[\frac{\sqrt{5}+1}{4} - \frac{1}{2}\right]\left[\frac{\sqrt{5}+1}{4}\right]$

$= \frac{1}{2}\left[\frac{\sqrt{5}-1}{4}\right]\left[\frac{\sqrt{5}+1}{4}\right]$

$= \frac{5-1}{32} = \frac{4}{32} = \frac{1}{8}$

243 **(a)**

We have,

$\sin A \sin B \sin C = p$ and, $\cos A \cos B \cos C = q$

$\Rightarrow \tan A \tan B \tan C = \frac{p}{q} \Rightarrow S_3 = \frac{p}{q}$

In a triangle ABC, we have

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$

$\therefore \tan A + \tan B + \tan C = \frac{p}{q} \Rightarrow S_1 = \frac{p}{q}$

Now,

$S_2 = \tan A \tan B + \tan B \tan C + \tan C \tan A$

$\Rightarrow S_2 = \frac{-\cos(A+B+C) + \cos A \cos B \cos C}{\cos A \cos B \cos C}$

$= \frac{1+q}{q}$

Hence, $\tan A, \tan B, \tan C$ are roots of

$x^3 - S_1x^2 + S_2x - S_3 = 0$

or, $x^3 - \frac{p}{q}x^2 + \frac{1+q}{q}x - \frac{p}{q} = 0$

244 **(d)**

$\cos 480°. \sin 150° + \sin 600°. \cos 390°$

$= [\cos(3\pi - 60°)\sin(\pi - 30°) + \sin(3\pi + 60°) \times \cos(2\pi + 30°)]$

$= -\cos 60° \sin 30° + (-\sin 60°)\cos 30°$

$= -\frac{1}{2}.\frac{1}{2} + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= -\frac{1}{4} - \frac{3}{4} = -1$

245 **(b)**

We have,

$\cos p\,\theta = \cos q\,\theta$

$\Rightarrow P\,\theta = 2\,n\,\pi \pm q\,\theta$, where $n \in Z$

$\Rightarrow \theta = \frac{2\,n\,\pi}{p \pm q}, n \in Z$

246 **(b)**

Given, $\sec x - 1 = \left(\sqrt{2} - 1\right) \tan x$

$\Rightarrow \frac{1 - \cos x}{\cos x} = (\sqrt{2} - 1)\frac{\sin x}{\cos x}$

$\Rightarrow 2\sin^2\frac{x}{2} - \left(\sqrt{2} - 1\right)2\sin\frac{x}{2}\cos\frac{x}{2} = 0$

$\Rightarrow \sin\frac{x}{2}\left[\sin\frac{x}{2} - (\sqrt{2} - 1)\cos\frac{x}{2}\right] = 0$

$\Rightarrow \sin\frac{x}{2} = 0$ or $\sin\frac{x}{2} - \left(\sqrt{2} - 1\right)\cos\frac{x}{2} = 0$

$\Rightarrow \frac{x}{2} = n\pi$ or $\tan\frac{x}{2} = \left(\sqrt{2} - 1\right) = \tan\frac{45°}{2}$

$\Rightarrow x = 2n\pi$ or $\frac{x}{2} = \frac{45°}{2} + n\pi$

$\Rightarrow x = 2n\pi$ or $2n\pi + \frac{\pi}{4}$

247 **(a)**

$x = \tan 15°$

$= \frac{\tan 45° - \tan 30°}{\tan 45°.\tan 30°}$

$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \frac{\left(\sqrt{3} - 1\right)^2}{3 - 1} = 2 - \sqrt{3}$

And $y = \text{cosec}\ 75° = \frac{1}{\sin(45° + 30°)}$

$= \frac{1}{\sin 45° \cos 30° + \cos 45° \sin 30°}$

$= \frac{1}{\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}} = \frac{2\sqrt{2}}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1} = \sqrt{6} - \sqrt{2}$

And $z = 4\sin 18° = 4\left(\frac{\sqrt{5} - 1}{4}\right) = \sqrt{5} - 1$

It is clear from above that

$\left(2 - \sqrt{3}\right) < \left(\sqrt{6} - \sqrt{2}\right) < (\sqrt{5} - 1)$

$\Rightarrow \quad x < y < z$

248 **(d)**

$\sin x = \frac{1}{2}$

$\Rightarrow \sin x = \sin\frac{\pi}{6}$

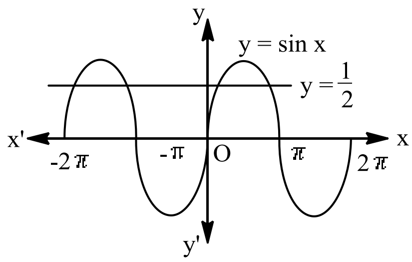
$\Rightarrow x = n\pi + (-1)^n\frac{\pi}{6}$

For $\quad -2\pi \le x \le 2\pi$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, -\frac{7\pi}{6}, -\frac{11\pi}{6}$

$\therefore$ Number of points of intersection of two given curves= 4

Alternate



The number of points of intersection are 4

252 **(a)**

$\because \sin\left(\frac{\pi}{2n}\right) + \cos\left(\frac{\pi}{2n}\right) = \frac{\sqrt{n}}{2}$

On squaring both sides, we get

$\sin^2\left(\frac{\pi}{2n}\right) + \cos^2\left(\frac{\pi}{2n}\right) + \sin\left(\frac{\pi}{n}\right) = \frac{n}{4}$

$\Rightarrow \sin\left(\frac{\pi}{n}\right) = \frac{n}{4} - 1$

$\Rightarrow \sin\left(\frac{\pi}{n}\right) = \frac{n - 4}{4}$

$\Rightarrow n = 6$ only

253 **(d)**

Given, $\cos 2x = \sqrt{2}\cos x - 1 + \cos x - \frac{1}{\sqrt{2}}$

$\Rightarrow 1 + \cos 2x = \cos x\left(\sqrt{2} + 1\right) - \frac{1}{\sqrt{2}}$

$\Rightarrow 2\cos^2 x - \cos x\left(\sqrt{2} + 1\right) + \frac{1}{\sqrt{2}} = 0$

$\Rightarrow \cos x = \frac{(\sqrt{2} + 1) \pm \sqrt{\left(\sqrt{2} + 1\right)^2 - \frac{8}{\sqrt{2}}}}{2(2)}$

$= \frac{(\sqrt{2} + 1) \pm \sqrt{3 + 2\sqrt{2} - 4\sqrt{2}}}{4}$

$= \frac{\sqrt{2} + 1 \pm (\sqrt{2} - 1)}{4}$

$\Rightarrow \cos x = \frac{\sqrt{2} + 1 + \sqrt{2} - 1}{4} = \frac{1}{\sqrt{2}} \quad \left[\because \cos x \ne \frac{1}{2}\right]$

$\Rightarrow x = 2n\pi \pm \frac{\pi}{4}, \forall\, n \in Z$

254 **(a)**

We have,

$\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha$

$= \cot\alpha - \{\cot\alpha - \tan\alpha - 2\tan 2\alpha - 4\tan 4\alpha - 8\cot 8\alpha\}$

$= \cot\alpha - \{2\cot 2\alpha - 2\tan 2\alpha - 4\tan 4\alpha - 8\cot 8\alpha\}$

$= \cot\alpha - \{4\cot 4\alpha - 4\tan 4\alpha - 8\cot 8\alpha\}$

$= \cot\alpha - \{8\cot 8\alpha - 8\cot 8\alpha\} = \cot\alpha$

255 **(b)**

We have,

$B = 60°, C = 75° \Rightarrow A = 180 - 60° - 75° = 45°$

Now, $\frac{b}{\sin B} = \frac{a}{\sin A} \Rightarrow \frac{b}{\sin 60°} = \frac{2}{\sin 45°} \Rightarrow b = \sqrt{6}$

256 **(a)**

We have,

$\tan(x-y)=1 \Rightarrow x-y=\frac{\pi}{4},\frac{5\pi}{4}$

$\sec(x+y)=\frac{2}{\sqrt{3}} \Rightarrow \cos(x+y)=\frac{\sqrt{3}}{2}$

$\Rightarrow x+y=\frac{\pi}{6},\frac{11\pi}{6}$

Since x, y are positive. Therefore, $x+y>x-y$

Thus, we have

$x+y=\frac{11\pi}{6}$ and $x-y=\frac{\pi}{4}$

or

$x+y=\frac{11\pi}{6}$ and $x-y=\frac{5\pi}{4}$

Solving these two systems of equations, we get

$x=\frac{25\pi}{4}$ and $y=\frac{19\pi}{24}$ or, $x=\frac{37\pi}{24}$ and $y=\frac{7\pi}{24}$

257 **(b)**

We have,

$\sqrt{3}\sin\theta+\cos\theta>0$

$\Rightarrow \frac{\sqrt{3}}{2}\sin\theta+\frac{1}{2}\cos\theta>0$

$\Rightarrow \sin\theta\cos\frac{\pi}{6}+\cos\theta\sin\frac{\pi}{6}>0$

$\Rightarrow \sin\left(\theta+\frac{\pi}{6}\right)>0 \Rightarrow 0<\theta+\pi/6<\pi \Rightarrow -\frac{\pi}{6}<\theta<\frac{5\pi}{6}$

258 **(c)**

$\cos A=\frac{3}{5}, \cos B=\frac{4}{5}$

$\because$ $\angle A$ and $\angle B$ lie on 4th quadrant

$\therefore \sin A=-\sqrt{1-\frac{9}{25}}, \sin B=-\sqrt{1-\frac{16}{25}}$

$\Rightarrow \sin A=-\frac{4}{5}, \sin B=-\frac{3}{5}$

$\therefore 2\sin A+4\sin B$

$=2\left(-\frac{4}{5}\right)+4\left(-\frac{3}{5}\right)$

$=-\frac{8}{5}-\frac{12}{5}$

$=-\frac{20}{5}=-4$

260 **(a)**

We have,

$\tan^2\alpha+\cot^2\alpha=(\tan\alpha-\cot\alpha)^2+2\geq 2$

261 **(d)**

Given equation can be rewritten as

$\frac{\sqrt{3}}{2}\cos\theta+\frac{1}{2}\sin\theta=\frac{\sqrt{2}}{2}$

$\Rightarrow \cos\left(\theta+\frac{\pi}{6}\right)=\frac{1}{\sqrt{2}}=\cos\frac{\pi}{4}$

$\Rightarrow \theta=2n\pi\pm\frac{\pi}{4}-\frac{\pi}{6}$

262 **(a)**

$\sin 50°+\sin 10°-\sin 70°$

$=2\sin 30°\cos 20°-\cos 20°$

$=\cos 20°\left(2\times\frac{1}{2}-1\right)=0$

263 **(a)**

Given, $\sin A-\cos B=\cos C$

$\Rightarrow \sin A=\cos B+\cos C$

$\Rightarrow 2\sin\frac{A}{2}\cos\frac{A}{2}=2\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)$

$\Rightarrow 2\sin\frac{A}{2}\cos\frac{A}{2}=2\sin\frac{A}{2}\cos\left(\frac{B-C}{2}\right)$

$\Rightarrow \cos\frac{A}{2}=\cos\left(\frac{B-C}{2}\right) \quad \left[\because \sin\left(\frac{A}{2}\right)\neq 0\right]$

$\Rightarrow \frac{A}{2}=\frac{B-C}{2} \Rightarrow A=B-C$

But $A+B+C=\pi$, therefore

$2B=\pi$

$\Rightarrow B=\frac{\pi}{2}$

264 **(d)**

Given, $4\sin^4 x+\cos^4 x-1=0$

$\Rightarrow 4\sin^4 x+(\cos^2 x-1)(\cos^2 x+1)=0$

$\Rightarrow 4\sin^4 x-\sin^2 x(1-\sin^2+1)=0$

$\Rightarrow \sin^2 x(5\sin^2 x-2)=0$

$\Rightarrow \sin x=0$ or $\pm\sqrt{\frac{2}{5}}$

Hence, $x=n\pi$ is the required solution

265 **(d)**

If the triangle is equilateral

$\sin A+\sin B+\sin C=\frac{3\sqrt{3}}{2}$

If the triangle is isosceles, let

$A=30°, B=30°, C=120°$

Then, $\sin A+\sin B+\sin C=1+\frac{\sqrt{3}}{2}$

If the triangle is right angled, let $A=90°, B=30°, C=60°$

Then,

$\sin A+\sin B+\sin C=\frac{3+\sqrt{3}}{2}$

If the triangle is right angled isosceles, then one of the angles is 90° and the remaining two are 45° each, so that

$\sin A+\sin B+\sin C=1+\sqrt{2}$

and, $\cos A+\cos B+\cos C=\sqrt{2}$

266 **(d)**

Now, $\tan\frac{\pi}{3}=\tan\left(\frac{6\pi}{15}-\frac{\pi}{15}\right)=\frac{\tan\frac{6\pi}{15}-\tan\frac{\pi}{15}}{1+\tan\frac{6\pi}{15}\tan\frac{\pi}{15}}$

$$\Rightarrow \tan\frac{6\pi}{15} - \tan\frac{\pi}{15} = \sqrt{3} + \sqrt{3}\tan\frac{6\pi}{15}\tan\frac{\pi}{15}$$

$$\Rightarrow \tan\frac{6\pi}{15} - \tan\frac{\pi}{15} - \sqrt{3}\tan\frac{6\pi}{15}\tan\frac{\pi}{15} = \sqrt{3}$$

$$= \tan\frac{2\pi}{5} - \tan\frac{\pi}{15} - \sqrt{3}\tan\frac{2\pi}{5}\tan\frac{\pi}{15} = \sqrt{3}$$

267 **(b)**

We have,

$\tan 3x = \tan 5x$

$\Rightarrow 5x = n\pi + 3x, n \in Z$

$\Rightarrow x = \frac{n\pi}{2}, n \in Z$

If n is odd, then $x = \frac{n\pi}{2}$ gives extraneous solutions

Thus, the solution of the given equation will be given by $x = \frac{n\pi}{2}$, where n is even, say

$n = 2m, m \in Z$

Hence, the required solution is $x = m\pi, m \in Z$

268 **(c)**

We have,

$\tan\theta + \tan 4\theta + \tan 7\theta = \tan\theta\tan 4\theta\tan 7\theta$

$\Rightarrow \tan\theta + \tan 4\theta = -\tan 7\theta(1 - \tan\theta\tan 4\theta)$

$\Rightarrow \frac{\tan\theta + \tan 4\theta}{1 - \tan\theta\tan 4\theta} = \tan(-7\theta)$

$\Rightarrow \tan 5\theta = \tan(-7\theta)$

$\Rightarrow 5\theta = n\pi + (-7\theta), n \in Z \Rightarrow \theta = \frac{n\pi}{12}, n \in Z$

269 **(b)**

We have,

$\frac{\sin^2 3A}{\sin^2 A} - \frac{\cos^2 3A}{\cos^2 A}$

$= \frac{\sin^2 3A\cos^2 A - \cos^2 3A\sin^2 A}{\sin^2 A\cos^2 A}$

$= \frac{\sin^2 3A\,(1 - \sin^2 A) - \cos^2 3A\sin^2 A}{\sin^2 A\cos^2 A}$

$= \frac{\sin^2 3A - \sin^2 A\,(\cos^2 3A\sin^2 3A)}{\sin^2 A\cos^2 A}$

$= \frac{\sin(3A + A)\sin(3A - A)}{\sin^2 A\cos^2 A}$

$= \frac{(4\sin A\cos A\cos 2A)(2\sin A\cos A)}{\sin^2 A\cos^2 A} = 8\cos 2A$

270 **(d)**

We have,

$3\sin A = 6\sin B = 2\sqrt{3}\sin C$

$\Rightarrow \frac{\sin A}{2} = \frac{\sin B}{1} = \frac{\sin C}{\sqrt{3}}$

$\Rightarrow \frac{\sin A}{1} = \frac{\sin B}{\frac{1}{2}} = \frac{\sin C}{\frac{\sqrt{3}}{2}} \Rightarrow A = \frac{\pi}{2}, B = \frac{\pi}{6}$ and $C = \frac{\pi}{3}$

271 **(b)**

We have,

$x = y\cos\frac{2\pi}{3} = z\cos\frac{4\pi}{3}$

$\Rightarrow x = -\frac{y}{2} = -\frac{z}{2}$

$\Rightarrow \frac{x}{1} = \frac{y}{-2} = \frac{z}{-2} = \lambda$ (say)

$\Rightarrow x = \lambda, y = -2\lambda, z = -2\lambda$

$\Rightarrow xy + yz + zx = -2\lambda^2 + 4\lambda^2 - 2\lambda^2 = 0$

272 **(a)**

$\sin^2\alpha + \sin^2\beta - \sin^2\gamma$

$= \sin^2\alpha + \sin(\beta - \gamma)\sin(\beta + \gamma)$

$= \sin^2\alpha + \sin(\pi - \alpha)\sin(\beta + \gamma)$ $[\because \alpha + \beta + \gamma = \pi]$

$= \sin^2\alpha\{\sin\alpha + \sin(\beta + \gamma)\}$

$= \sin\alpha\{\sin(\beta - \gamma) + \sin(\beta + \gamma)\}$ $[\because \alpha = \pi - (\beta - \gamma)]$

$= 2\sin\alpha\sin\beta\cos\gamma$

273 **(a)**

We have,

$\sin A + \cos A = \frac{\sqrt{7}}{2}$

$\Rightarrow \frac{2\tan\frac{A}{2}}{1 + \tan^2\frac{A}{2}} + \frac{1 - \tan^2\frac{A}{2}}{1 + \tan^2\frac{A}{2}} = \frac{\sqrt{7}}{2}$

$\Rightarrow 4\tan\frac{A}{2} + 2 - 2\tan^2\frac{A}{2} = \sqrt{7} + \sqrt{7}\tan^2\frac{A}{2}$

$\Rightarrow (\sqrt{7} + 2)\tan^2\frac{A}{2} - 4\tan\frac{A}{2} + (\sqrt{7} - 2) = 0$

$\Rightarrow \tan\frac{A}{2} = \frac{4 \pm \sqrt{16 - 4(\sqrt{7} + 2)(\sqrt{7} - 2)}}{2(\sqrt{7} + 2)}$

$\Rightarrow \tan\frac{A}{2} = \frac{4 \pm 2}{2(\sqrt{7} + 2)}$

$\Rightarrow \tan\frac{A}{2} = \frac{3}{\sqrt{7} + 2}, \frac{1}{\sqrt{7} + 2}$

$\Rightarrow \tan\frac{A}{2} = \sqrt{7} - 2, \frac{\sqrt{7} - 2}{3}$

$\Rightarrow \tan\frac{A}{2} = \frac{\sqrt{7} - 2}{3}$ $\left[\because 0 < A < \pi/6 \;\therefore \tan\frac{A}{2} < 1\right]$

274 **(b)**

We have,

$\sum \cot(B+C-A)\cot(C+A-B)$

$= \sum \cot 2A \cot 2B \quad [\because A+B+C=0]$

$= \cot 2A \cot 2B + \cot 2B \cot 2C + \cot 2C \cot 2A$

Now,

$A+B+C=0$

$\Rightarrow 2A+2B+2C=0$

$\Rightarrow \tan(2A+2B+2C)=0$

$\Rightarrow \tan 2A + \tan 2B + \tan 2C$

$= \tan 2A \tan 2B \tan 2C$

$\Rightarrow \cot 2A \cot 2B + \cot 2B \cot 2C + \cot 2C \cot 2A$

$= 1$

$\Rightarrow \sum \cot(B+C-A)\cot(C+A-B) = 1$

275 **(b)**

We have,

$A+B=\frac{\pi}{4}$

$\Rightarrow \frac{\tan A + \tan B}{1-\tan A \tan B} = 1$

$\Rightarrow \tan A + \tan B + \tan A \tan B = 1$

$\Rightarrow (1+\tan A)(1+\tan B) = 1+1 = 2$

276 **(c)**

Given equations may be written as

$\cos x + \cos y = -\cos\alpha$

and $\sin x + \sin y = -\sin\alpha$

$\Rightarrow 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = -\cos\alpha$ ...(i)

and $2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = -\sin\alpha$ ...(ii)

From Eqs.(i)and (ii),we get

$$\frac{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)} = \frac{\cos\alpha}{\sin\alpha}$$

$\Rightarrow \cot\left(\frac{x+y}{2}\right) = \cot\alpha$

277 **(b)**

We have,

$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{8^2+10^2-12^2}{2\times 8\times 10} = \frac{1}{8}$

And,

$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{10^2+12^2-8^2}{2\times 10\times 12} = \frac{3}{4}$

$\therefore \cos 2A = 2\cos^2 A - 1 = 2\times\frac{9}{16} - 1 = \frac{1}{8}$

Thus, we have

$\cos 2A = \cos C \Rightarrow 2A = C$

278 **(a)**

Let $\log \sec x = y$

$\Rightarrow \frac{1}{\cos x} = e^y = e^{y/2+y/2} = e^{y/2.e^y/2}$

$\therefore \frac{1}{\cos x} = \frac{e^{y/2}}{e^{-y/2}}$

By componendo and Dividendo rule

$\frac{1+\cos x}{1-\cos x} = \frac{e^{y/2}+e^{-y/2}}{e^{y/2}-e^{-y/2}}$

$\Rightarrow \cot^2\left(\frac{x}{2}\right) = \coth\left(\frac{y}{2}\right)$

$\Rightarrow y = 2\coth^{-1}\left(\operatorname{cosec}^2\frac{x}{2} - 1\right)$

279 **(c)**

Since, $\sin\theta = \sin\alpha$ ...(i)

And $\cos\theta = \cos\alpha$ ...(ii)

[divided Eq. (i) by Eq. (ii)]

$\therefore \tan\theta = \tan\alpha$

$\Rightarrow \theta = n\pi + \alpha$

280 **(b)**

We have,

$2b = a+c \Rightarrow a+b+c = 3b \Rightarrow 2s = 3b$

Now,

$\tan\frac{A}{2}\tan\frac{C}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \times \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

$\Rightarrow \tan\frac{A}{2}\tan\frac{C}{2} = \frac{s-b}{s} = \frac{2s-2b}{2s} = \frac{3b-2b}{3b} = \frac{1}{3}$

281 **(c)**

$2\sin^2\theta - \cos 2\theta = 0 \Rightarrow \sin^2\theta = \frac{1}{4} \Rightarrow$

$\sin\theta = \pm 12$ ...(i)

Also, $2\cos^2\theta = 3\sin\theta \Rightarrow 2\sin^2\theta +$

$3\sin\theta - 2 = 0$

$\Rightarrow \sin\theta = \frac{1}{2}$ ...(ii)

From Eqs. (i) and (ii), $\sin\theta = \frac{1}{2}$

Two solutions exist in $[0, 2\pi]$

282 **(c)**

We have,

$\frac{A}{B} = \frac{\tan 6° \tan 42°}{\cot 66° \cot 78°}$

$\Rightarrow \frac{A}{B} = \tan 6° \tan 42° \tan 66° \tan 78°$

$\Rightarrow \frac{A}{B} = \frac{\sin 6° \sin 42° \sin 66° \sin 78°}{\cos 6° \cos 42° \cos 66° \cos 78°} = 1 \Rightarrow A = B$

283 **(b)**

We have,

$\Delta = a^2 - (b-c)^2$

$\Rightarrow \Delta = (a+b-c)(a-b+c)$

$\Rightarrow \Delta = (2s-2c)(2s-2b)$

$\Rightarrow \sqrt{s(s-a)(s-b)(s-c)} = 4(s-b)(s-c)$

$\Rightarrow \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \frac{1}{4} \Rightarrow \tan\frac{A}{2} = \frac{1}{4}$

Now,

$\tan A = \frac{2\tan A/2}{1-\tan^2 A/2} \Rightarrow \tan A = \frac{1/2}{1-1/16} = \frac{8}{15}$

284 **(a)**

We have,

$\sin 47° - \sin 25° + \sin 61° - \sin 11°$

$= 2\sin 11° \cos 36° + 2\sin 25° \cos 36°$

$= 2\cos 36° (\sin 25° + \sin 11°)$

$= 2\cos 36° \times 2\sin 18° \cos 7°$

$= 4\left(\frac{\sqrt{5}+1}{4}\right)\left(\frac{\sqrt{5}-1}{4}\right)\cos 7° = \cos 7°$

285 **(c)**

Given, $\sin^3 x \sin 3x = \sum_{m=0}^{n} C_m \cos mx$

$\Rightarrow \left(\frac{3\sin x - \sin 3x}{4}\right)\sin 3x = \sum_{m=0}^{n} C_m \cos mx$

$\Rightarrow \frac{3}{8}(2\sin 3x \sin x) - \frac{1}{8}(2\sin^2 3x)$

$= \sum_{m=0}^{n} C_m \cos mx$

$\Rightarrow \frac{3}{8}(\cos 2x - \cos 4x) - \frac{1}{8}(1-\cos 6x)$

$= \sum_{m=0}^{n} C_m \cos mx$

$\Rightarrow \frac{1}{8}\cos 6x + \frac{3}{8}(\cos 2x - \cos 4x) - \frac{1}{8}$

$= C_0 \cos 0 + C_1 \cos x + C_2 \cos 2x + \ldots + C_n \cos nx$

$\therefore \quad n = 6$

286 **(d)**

We have,

$\cos x = \tan y$

$\Rightarrow \cos^2 x = \tan^2 y$

$\Rightarrow \cos^2 x = \sec^2 y - 1$

$\Rightarrow \cos^2 x = \cot^2 z$

$- 1 \quad [\because \cos y = \tan z \ \therefore \sec y$

$= \cot z]$

$\Rightarrow 1 + \cos^2 x = \cot^2 z$

$\Rightarrow 1 + \cos^2 x = \frac{\tan^2 x}{1-\tan^2 x} \quad [\because \cos z = \tan x]$

$\Rightarrow 1 + \cos^2 x = \frac{\sin^2 x}{\cos^2 x - \sin^2 x}$

$\Rightarrow 2\sin^4 x - 6\sin^2 x + 2 = 0$

$\Rightarrow \sin^2 x = \frac{3-\sqrt{5}}{2}$

$\Rightarrow \sin^2 x = \left(\frac{\sqrt{5}-1}{2}\right)^2 \Rightarrow \sin x = \frac{\sqrt{5}-1}{2}$

$= 2\sin 18°$

287 **(b)**

$\frac{\cot 54°}{\tan 36°} + \frac{\tan 20°}{\cot 70°}$

$= \frac{\tan 36°}{\tan 36°} + \frac{\tan 20°}{\tan 20°}$

$= 1 + 1 = 2$

288 **(b)**

$\frac{\cot x - \tan x}{\cot 2x}$

$= \tan 2x\left(\cot x - \frac{1}{\cot x}\right)$

$= \tan 2x\left(\frac{\cot^2 x - 1}{\cot x}\right)$

$= \tan 2x\left(\frac{\cot^2 x - 1}{2\cot x}\right).2$

$= \tan 2x. \cot 2x. 2$

$= 2$

289 **(c)**

$\sin^2 x - \cos 2x = 2 - \sin 2x$

$\Rightarrow \ 1 - \cos^2 x - (2\cos^2 x - 1) = 2 - 2\sin x \cos x$

$\Rightarrow \ -3\cos^2 x + 2\sin x \cos x = 0$

$\Rightarrow \ \cos x(2\sin x - 3\cos x) = 0$

$\Rightarrow \ \cos x = 0, \quad (\because \ 2si9n\, x - 3\cos x \neq 0)$

$\Rightarrow \ x = 2n\pi \pm \frac{\pi}{2}$

$\Rightarrow \ x = (4n \pm 1)\frac{\pi}{2}$

290 **(b)**

$\frac{1+\tanh\frac{x}{2}}{1-\tanh\frac{x}{2}} = \frac{\cosh\frac{x}{2} + \sinh\frac{x}{2}}{\cosh\frac{x}{2} - \sinh\frac{x}{2}} = \frac{e^{x/2}}{e^{-x/2}} = e^x$

291 **(c)**

$2\tanh^{-1}\left(\frac{1}{2}\right) = \tanh^{-1}\frac{2\left(\frac{1}{2}\right)}{1+\left(\frac{1}{2}\right)^2} = \tanh^{-1}\frac{4}{5}$

$\left[\because \ 2\tanh^{-1} x = \tanh^{-1}\frac{2x}{1+x^2}\right]$

$= \frac{1}{2}\log\left(\frac{1+\frac{4}{5}}{1-\frac{4}{5}}\right) = \frac{1}{2}\log 3^2$

$= \log 3$

292 **(c)**

We have,

$\cos(\alpha+\beta)\sin(\gamma+\delta) = \cos(\alpha-\beta)\sin(\gamma-\delta)$

$\Rightarrow \frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)} = \frac{\sin(\gamma-\delta)}{\sin(\gamma+\delta)}$

$\Rightarrow \dfrac{\cos(\alpha+\beta)-\cos(\alpha-\beta)}{\cos(\alpha+\beta)+\cos(\alpha-\beta)}$

$= \dfrac{\sin(\gamma-\delta)-\sin(\gamma+\delta)}{\sin(\gamma-\delta)+\sin(\gamma+\delta)}$

$\Rightarrow \dfrac{-2\sin\alpha\sin\beta}{2\cos\alpha\cos\beta} = \dfrac{-2\cos\gamma\sin\delta}{2\sin\gamma\cos\delta}$

$\Rightarrow -\tan\alpha\tan\beta = -\cot\gamma\tan\delta \Rightarrow \cot\alpha\cot\beta\cot\gamma = \cot\delta$

293 **(c)**

Given, $a\cos^3\alpha + 3a\cos\alpha\sin^2\alpha = m$

and $a\sin^3\alpha + 3a\cos^2\alpha\sin^2\alpha = n$

$\therefore (m+n) = a\cos^3\alpha + 3a\cos\alpha\sin^2 + 3a\cos^2\alpha\sin\alpha + a\sin^3\alpha$

$= a(\cos\alpha+\sin\alpha)^3$

and similarly, $(m-n) = a(\cos\alpha-\sin\alpha)^3$

$\therefore (m+n)^{2/3} + (m-n)^{2/3}$

$= a^{2/3}\{(\cos\alpha+\sin\alpha)^2 + (\cos\alpha-\sin\alpha)^2\}$

$= a^{2/3}\{2(\cos^2\alpha+\sin^2\alpha)\} = 2a^{2/3}$

294 **(c)**

Given, $\frac{x}{\operatorname{cosec}\theta} = \frac{y}{\sec\theta} = \frac{z}{\cot 2\theta} = k$ [say]

$\therefore\ 4z^2(x^2+y^2)$

$= 4k^2\cot^2 2\theta(k^2\operatorname{cosec}^2\theta + k^2\sec^2\theta)$

$= 4k^4\cot^2 2\theta\left(\frac{1}{\sin^2\theta\cos^2\theta}\right)$ [$\because \sin^2\theta + \cos 2\theta = 1$]

$= \dfrac{4k^4}{4}\left(\dfrac{\cos^2\theta-\sin^2\theta}{\sin^2\theta\cos^2\theta}\right)^2$

$= (k^2\operatorname{cosec}^2\theta - k^2\sec^2\theta)^2$

$= (x^2-y^2)^2$

295 **(b)**

Given, $\tan 2\theta\tan\theta = 1$

$\therefore\ \dfrac{2\tan^2\theta}{1-\tan^2\theta} = 1 \Rightarrow \tan^2\theta = \dfrac{1}{3}$

$\Rightarrow \tan^2\theta = \tan^2\dfrac{\pi}{6} \Rightarrow \theta = n\pi \pm \dfrac{\pi}{6}$

296 **(a)**

$|\sin x| > 2\sin^2 x$

$\Rightarrow |\sin x|(2|\sin x|-1) < 0$

$\Rightarrow 0 < |\sin x| < \dfrac{1}{2}$

$\Rightarrow x \in \left(0,\dfrac{\pi}{6}\right) \cup \left(\dfrac{5\pi}{6},\pi\right) \cup \left(\pi,\dfrac{7\pi}{6}\right) \cup \left(\dfrac{11\pi}{6}, 2x\right)$

297 **(d)**

Given, $\tan\left(\frac{x}{2}\right) = \operatorname{cosec} x - \sin x$

$= \dfrac{1+\tan^2\frac{x}{2}}{2\tan\frac{x}{2}} - \dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$= \dfrac{\left(1+\tan^2\frac{x}{2}\right) - 4\tan^2\frac{x}{2}}{2\tan\frac{x}{2}\left(1+\tan^2\frac{x}{2}\right)}$

$\Rightarrow 2\tan^2\left(\dfrac{x}{2}\right)\left(1+\tan^2\dfrac{x}{2}\right) = \left(1-\tan^2\dfrac{x}{2}\right)^2$

$\Rightarrow \tan^4\dfrac{x}{2} + 4\tan^2\dfrac{x}{2} - 1 = 0$

$\Rightarrow \tan^2\dfrac{x}{2} = \dfrac{-4\pm\sqrt{16+4}}{2\times 1} = -2\pm\sqrt{5}$

$\Rightarrow \tan^2\dfrac{x}{2} = -2+\sqrt{5}$

$\left(\because \tan^2\dfrac{x}{2} \neq -2-\sqrt{5}\right)$

298 **(d)**

Given equation is

$5\cos 2\theta + 2\cos^2\dfrac{\theta}{2} + 1 = 0$

$\Rightarrow 5(2\cos^2\theta - 1) + 1 + \cos\theta + 1 = 0$

$\Rightarrow 10\cos^2\theta + \cos\theta - 3 = 0$

$\Rightarrow (2\cos\theta - 1)(5\cos\theta + 3) = 0$

$\Rightarrow \cos\theta = \frac{1}{2}$ or $\cos\theta = -\frac{3}{5}$

$\Rightarrow \theta = \frac{\pi}{3}$ or $\theta = \pi - \cos^{-1}\left(\frac{3}{5}\right)$

299 **(b)**

From the given equations we have $\sum\tan\alpha = p$

$\Sigma\tan\alpha\tan\beta = 0$ and $\tan\alpha\tan\beta\tan\gamma = r$

$\therefore\ (1+\tan^2\alpha)(1+\tan^2\beta)(1+\tan^2\gamma)$

$= 1 + \Sigma\tan^2\alpha + \Sigma\tan^2\alpha\tan^2\beta + \tan^2\alpha\tan^2\beta\tan^2\gamma$

$= 1 + (\Sigma\tan\alpha)^2 - 2\Sigma\tan\alpha\tan\beta + (\Sigma\tan\alpha\tan\beta)^2 - 2\tan\alpha\tan\beta\tan\gamma\,\Sigma\tan\alpha + \tan^2\alpha\tan^2\beta\tan^2\gamma$

$= 1 + p^2 - 2pr + r^2 = 1 + (p-r)^2$

300 **(b)**

We have,

$\cos^4\theta - \sin^4\theta = (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$

$= \cos 2\theta = 2\cos^2\theta - 1$

301 **(c)**

We have,

$\sqrt{3}\operatorname{cosec} 20° - \sec 20°$

$= \tan 60°\ \operatorname{cosec} 20° - \sec 20°$

$= \frac{\sin 60° \cos 20° - \cos 60° \sin 20°}{\cos 60° \sin 20° \cos 20°}$

$= \frac{\sin 40°}{\cos 60° \sin 20° \cos 20°} = \frac{2 \sin 20° \cos 20°}{\frac{1}{2} \sin 20° \cos 20°} = 4$

302 **(b)**

Consider the function $f(\theta)$ given by

$f(\theta) = \frac{\theta}{2} - \sin\frac{\theta}{2}$, where $0 \leq \theta \leq \frac{\pi}{2}$

We have,

$f'(\theta) = \frac{1}{2}\left(1 - \cos\frac{\theta}{2}\right) > 0 \quad \left[\because 0 \leq \theta \leq \frac{\pi}{2}\right]$

$\Rightarrow f(\theta)$ is increasing on $[0, \pi/2]$

$\Rightarrow f(\theta) > f(0)$ for $0 \leq \theta \leq \frac{\pi}{2}$

$\Rightarrow \frac{\theta}{2} - \sin\frac{\theta}{2} > 0$ for $0 \leq \theta \leq \frac{\pi}{2}$

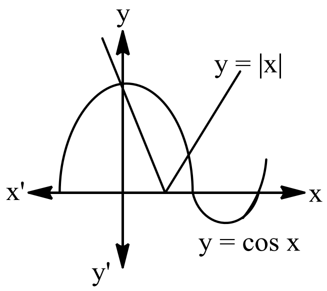
$\Rightarrow \frac{\theta}{2} > \sin\frac{\theta}{2}$ for $0 \leq \theta \leq \frac{\pi}{2}$

On the same lines it can be seen that all other conditions are true except condition given in option (b)

303 **(a)**

Let $y = |x - 1| = \cos x$

It is clear from the graph that two curves intersect at two points.



Hence, number of solutions are 2.

304 **(b)**

Since, $5 \cos x + 12 \cos y = 13$

$\Rightarrow (5 \cos x + 12 \cos y)^2 + (5 \sin x + 12 \sin y)^2$

$= (13)^2 + (5 \sin x + 12 \sin y)^2$

$\Rightarrow 25 + 144 + 120(\sin x \sin y + \cos x \cos y)$

$= 169 + (5 \sin x + 12 \sin y)^2$

$\Rightarrow (5 \sin x + 12 \sin y)^2 = 120 \cos(x - y)$

$\because -1 \leq \cos(x - y) \leq 1$

$\Rightarrow -120 \leq 120 \cos(x - y) \leq 120$

$\therefore$ Maximum value of $5 \sin x + 12 \sin y = \sqrt{120}$

305 **(a)**

Since, $\cos\theta = \frac{8}{17}$ and $0 < \theta < \frac{\pi}{2}$

$\Rightarrow \sin\theta = \sqrt{1 - \frac{8^2}{17^2}} = \frac{15}{17}$

Now, $\cos(30° + \theta) + \cos(45° - \theta) + \cos(120° - \theta)$

$= \cos 30° \cos\theta - \sin 30° \sin\theta + \cos 45° \cos\theta + \sin 45° \sin\theta + \cos 120° \cos\theta + \sin 120° \sin\theta$

$= \cos\theta\left(\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} - \frac{1}{2}\right) - \sin\theta\left(\frac{1}{2} - \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}\right)$

$= \frac{8}{17}\left(\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} - \frac{1}{2}\right) + \frac{15}{17}\left(\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} - \frac{1}{2}\right)$

$= \frac{23}{17}\left(\frac{\sqrt{3} - 1}{2} + \frac{1}{\sqrt{2}}\right)$

306 **(d)**

$\cos 1° + \cos 2° + \cos 3° + \ldots + \cos 89° + \cos 90° + \cos 91° + \cos 92° + \cos 93° + \ldots + \cos 179° + \cos 180°$

$= \cos 1° + \cos 2° + \cos 3° + \ldots + \cos 89° + 0 + \cos(180° - 89°)$
$+ \cos(180° - 88°) + \ldots + \cos(180° - 1°) - 1$

$= \cos 1° + \cos 2° + \cos 3° + \ldots \cos 89° - \cos 89° - \cos 88° - \ldots - \cos 1° - 1$

$= -1$

307 **(a)**

We have,

$\cot\frac{A}{2}, \cot\frac{B}{2}, \cot\frac{C}{2}$ are in $A.P.$

$\Rightarrow 2 \cot\frac{B}{2} = \cot\frac{A}{2} + \cot\frac{C}{2}$

$\Rightarrow 2\sqrt{\frac{s(s-b)}{(s-a)(s-c)}} = \sqrt{\frac{s(s-a)}{(s-b)(s-c)}} + \sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$

$\Rightarrow 2(s - b) = s - a + s - c \Rightarrow 2\,b = a + c \Rightarrow$ a, b, c are in $A.P.$

308 **(c)**

$(1 + \tan x + \tan^2 x)(1 - \cot x + \cot^2 x)$

$= \frac{(1 + \tan x + \tan^2 x)(1 + \tan^2 x - \tan x)}{\tan^2 x}$

$= \frac{(1 + \tan^2 x)^2 - \tan^2 x}{\tan^2 x}$

Obviously, $1 + \tan^2 x \geq \tan^2 x, \forall\, x \in R$

309 **(c)**

We have,

$$\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}$$

$$=2\cos^4\frac{\pi}{8}+2\cos^4\frac{3\pi}{8}\left[\because \cos\frac{5\pi}{8}=-\cos\frac{3\pi}{8}, \cos\frac{7\pi}{8}=-\cos\frac{\pi}{8}\right]$$

$$=\frac{1}{2}\left\{\left(2\cos^2\frac{\pi}{8}\right)^2\left(2\cos^2\frac{3\pi}{8}\right)^2\right\}$$

$$=\frac{1}{2}\left\{\left(1+\cos\frac{\pi}{4}\right)^2+\left(1+\cos\frac{3\pi}{8}\right)^2\right\}$$

$$=\frac{1}{2}\left\{\left(1+\frac{1}{\sqrt{2}}\right)^2+\left(1-\frac{1}{\sqrt{2}}\right)^2\right\}=\frac{3}{2}$$

310 **(b)**

We have,

$\sin(\pi\cos\theta)=\cos(\pi\sin\theta)$

$\Rightarrow \sin(\pi\cos\theta)=\sin\left(\frac{\pi}{2}\pm\pi\sin\theta\right)$

$\Rightarrow \pi\cos\theta=\frac{\pi}{2}\pm\pi\sin\theta$

$\Rightarrow \cos\theta\mp\sin\theta=\frac{1}{2}$

$\Rightarrow \frac{1}{\sqrt{2}}\cos\theta\mp\frac{1}{\sqrt{2}}\sin\theta=\frac{1}{2\sqrt{2}}$

$\Rightarrow \cos\left(\theta\pm\frac{\pi}{4}\right)=\frac{1}{2\sqrt{2}}\Rightarrow\cos\left(\theta\pm\frac{\pi}{4}\right)=\frac{1}{2}\cos\frac{\pi}{4}$

311 **(a)**

Since the angle of ΔABC are in A.P.

$2B=A+C$

$\Rightarrow 3B=A+B+C$

$\Rightarrow 3B=180°$

$\Rightarrow B=60°$

$\Rightarrow \cos B=\frac{1}{2}$

$\Rightarrow \frac{c^2+a^2-b^2}{2\,ac}=\frac{1}{2}$

$\Rightarrow c^2+a^2-b^2=ac$

$\Rightarrow c^2+a^2-b^2=b^2$ $[\because a,b,c$ are in G. P. $\therefore b^2=ac]$

$\Rightarrow c^2+a^2=2b^2$

$\Rightarrow a^2,b^2,c^2$ are in A.P.

312 **(d)**

Since $\sin x+i\cos 2x$ and $\cos x-i\sin 2x$ are conjugate to each other

$\therefore \sin x+i\cos 2x=\cos x+i\sin 2x$

$\Rightarrow \sin x=\cos x$ and $\cos 2x=\sin 2x$

$\Rightarrow \sin x=\cos x$ and $2\cos^2 x-1=2\sin x\cos x$

$\Rightarrow 2\cos^2 x-1=2\cos^2 x$ $\quad[\because \sin x=\cos x]$

This is an absurd result. Therefore, no value of x satisfy these two equations

313 **(b)**

Given, $1-\cos x=(\sqrt{2}-1)\sin x$

$\Rightarrow 2\sin\frac{x}{2}\left(\sin\frac{x}{2}-(\sqrt{2}-1)\cos\frac{x}{2}\right)=0$

$\Rightarrow \sin\frac{x}{2}=0$ or $\tan\frac{x}{2}=\sqrt{2}-1=\tan\frac{45°}{2}$

$\Rightarrow \frac{x}{2}=n\pi,\quad \frac{x}{2}=n\pi+\frac{\pi}{8}$

$\Rightarrow x=2n\pi,\quad 2n\pi+\frac{\pi}{4}$

314 **(c)**

$2\tan(A-B)=2\left(\frac{\tan A-\tan B}{1+\tan A\tan B}\right)$

$=2\left(\frac{2\tan B+\cot B-\tan B}{1+(2\tan B+\cot B)\tan B}\right)$

$[\because \tan A=2\tan B+\cot B]$

$=\frac{2(\tan B+\cot B)}{2(1+\tan^2 B)}=\cot B$

315 **(d)**

$\cos\alpha\cos 2\alpha\cos 4\alpha\ldots\cos 2^{n-1}\alpha$

$=\frac{1}{2\sin\alpha}[2\sin\alpha\cos\alpha \times\cos 2\alpha\cos 4\alpha\ldots\cos 2^{n-1}\alpha]$

$=\frac{1}{2\sin\alpha}\left[\frac{2}{2}\sin 2\alpha\cos 2\alpha\cos 4\alpha\ldots\cos 2^{n-1}\alpha\right]$

$=\frac{1}{2^2\sin\alpha}\left[\frac{2}{2}\sin 4\alpha\cos 4\alpha\ldots\cos 2^{n-1}\alpha\right]$

$=\frac{1}{2^3\sin\alpha}[\sin 8\alpha\cos 8\alpha\ldots\cos 2^{n-1}\alpha]$

Similarly, we can write

$=\frac{\sin 2^n\alpha}{2^n\sin\alpha}$

316 **(d)**

Given, $x\sin^3\theta+ycos^3\theta=\sin\theta\cos\theta$

$\Rightarrow\ y\cos\theta\sin^2\theta+y\cos^3\theta=\sin\theta\cos\theta$

$[\because\ x\sin\theta=y\cos\theta]$

$\Rightarrow\ y\cos\theta\,(\sin^2\theta+\cos^2\theta)=\sin\theta\cos\theta$

$\Rightarrow\ y=\sin\theta$

Now, $x\sin\theta=\sin\theta\cos\theta\ \Rightarrow\ x=\cos\theta$

$\therefore\ x^2+y^2=\cos^2\theta+\sin^2\theta=1$

317 **(a)**

$\because \tan\alpha/2+\tan\beta/2=\frac{26}{8}=\frac{13}{4}$

and $\tan\alpha/2\tan\beta/2=\frac{15}{8}$

$\therefore \tan\left(\frac{\alpha+\beta}{2}\right)=\frac{\tan\alpha/2+\tan\beta/2}{1-\tan\alpha/2\tan\beta/2}$

$=\frac{\frac{13}{4}}{1-\frac{15}{8}}=-\frac{26}{7}$

Now, $\cos(\alpha+\beta)=\frac{1-\tan^2\left(\frac{\alpha+\beta}{2}\right)}{1+\tan^2\left(\frac{\alpha+\beta}{2}\right)}$

$$=\frac{1-\left(-\frac{26}{7}\right)^2}{1+\left(-\frac{26}{7}\right)^2}=\frac{49-676}{49+676}$$

$$=-\frac{627}{725}$$

318 **(a)**

Given, $81^{\sin^2 x}+81^{\cos^2 x}=30$

$\Rightarrow\ 81^{\sin^2 x}+81^{1-\sin^2 x}=30$

$\Rightarrow\ 81^{\sin^2 x}+\frac{81}{81^{\sin^2 x}}=30$

$\Rightarrow\ y+\frac{81}{y}=30$ [put $81^{\sin^2 x}=y$]

$\Rightarrow\ y^2-30y+81=0$

$\Rightarrow\ (y-27)(y-3)=0$

$\Rightarrow\ 81^{\sin^2 x}=27$ or $81^{\sin^2 x}=3$

$\Rightarrow 3^{4\sin^2 x}=3^3$ or $3^{4\sin^2 x}=3$

$\Rightarrow \sin^2 x=\frac{3}{4}$ or $\sin^2 x=\frac{1}{4}$

$\Rightarrow \sin x=\frac{\sqrt{3}}{2}$ or $\sin x=\frac{1}{2}$

$\Rightarrow x=\frac{\pi}{3},\frac{2\pi}{3}$ or $x=\frac{\pi}{6},\frac{5\pi}{6}$

319 **(b)**

We have,

$32\tan^8\theta=2\cos^2\alpha-3\cos\alpha$ and $\cos 2\theta=\frac{1}{3}$

Now,

$\cos 2\theta=\frac{1}{3}\Rightarrow\frac{1-\tan^2\theta}{1+\tan^2\theta}=\frac{1}{3}\Rightarrow\tan^2\theta=\frac{1}{2}$

$\therefore 32\tan^8\theta=2\cos^2\alpha-3\cos\alpha$

$\Rightarrow 32\times\frac{1}{16}=2\cos^2\alpha-3\cos\alpha$

$\Rightarrow 2=2\cos^2\alpha-3\cos\alpha$

$\Rightarrow 2\cos^2\alpha-3\cos\alpha-2=0$

$\Rightarrow (2\cos\alpha+1)(\cos\alpha-2)$

$=0$ [$\because \cos\alpha-2\neq 0$]

$\Rightarrow 2\cos\alpha+1=0$

$\Rightarrow \cos\alpha=-\frac{1}{2}\Rightarrow\cos\alpha=\cos\frac{2\pi}{3}\Rightarrow\alpha$

$=2n\pi\pm\frac{2\pi}{3},n\in Z$

320 **(b)**

Now, $\tan(x-y)\tan y=\frac{\sin(x-y)\sin y}{\cos(x-y)\cos y}\times\frac{2}{2}$

$$=\frac{\cos(x-2y)-\cos(x)}{\cos(x-2y)+\cos(x)}=\frac{1-\frac{\cos x}{\cos(x-2y)}}{1+\frac{\cos(x)}{\cos(x-2y)}}$$

$=\frac{1-\lambda}{1+\lambda}$ $\left[\text{Given}, \lambda=\frac{\cos x}{\cos(x-2y)}\right]$

321 **(a)**

We have,

$\sin 3\theta=4\sin\theta\sin^2 x-4\sin^3\theta$

$\Rightarrow 3\sin\theta-4\sin^3\theta=4\sin\theta\sin^2 x-4\sin^3\theta$

$\Rightarrow 3\sin\theta=4\sin\theta\sin^2 x$

$\Rightarrow \sin^2 x=\frac{3}{4}$ [$\because \theta\neq n\pi\ \therefore \sin\theta\neq 0$]

$\Rightarrow \sin^2 x=\sin^2\frac{\pi}{3}\Rightarrow x=n\pi\pm\frac{\pi}{3},n\in Z$

322 **(d)**

We have,

$b+c=3a$

$\Rightarrow \sin B+\sin C=3\sin A$

$\Rightarrow 2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)=6\sin\frac{A}{2}\cos\frac{A}{2}$

$\Rightarrow \cos\left(\frac{B-C}{2}\right)=3\cos\left(\frac{B+C}{2}\right)$

$\Rightarrow \cos\frac{B}{2}\cos\frac{C}{2}=2\sin\frac{B}{2}\sin\frac{C}{2}$

$\Rightarrow \cot\frac{B}{2}\cot\frac{C}{2}=2\Rightarrow\tan\frac{B}{2}\tan\frac{C}{2}=\frac{1}{2}$

323 **(b)**

We have,

$\sin\theta+\cos\theta=\sqrt{2}\cos\theta$

$\Rightarrow 1+\sin 2\theta=2\cos^2\theta$

$\Rightarrow 1-\sin 2\theta=2-2\cos^2\theta$

$\Rightarrow (\cos\theta-\sin\theta)^2=2\sin^2\theta$

$\Rightarrow \cos\theta-\sin\theta=\sqrt{2}\sin\theta$

324 **(a)**

Given, $\sin x+\sin y+\sin z=-3$ and

$x,y,z\in[0,2\pi]$

$\because$ The minimum value of sin is -1

$\therefore$ In between 0 to 2π, the given equation is satisfied at $x=\frac{3\pi}{2}$

$y=\frac{3\pi}{2},z=\frac{3\pi}{2}$ and having only one solution

325 **(a)**

$\sin\frac{\pi}{16}.\sin\frac{3\pi}{16}.\sin\frac{5\pi}{16}.\sin\frac{7\pi}{16}$

$=\frac{1}{2}\left[2\sin\frac{5\pi}{16}\sin\frac{3\pi}{16}\right]\times\frac{1}{2}\left[2\sin\frac{7\pi}{16}\sin\frac{\pi}{16}\right]$

$=\frac{1}{4}\left[\left(\cos\frac{\pi}{8}-\cos\frac{\pi}{2}\right)\left(\cos\frac{3\pi}{8}-\cos\frac{\pi}{2}\right)\right]$

$=\frac{1}{4\times 2}\left(\cos\frac{\pi}{2}+\cos\frac{\pi}{4}\right)$

$=\frac{1}{8\sqrt{2}}=\frac{\sqrt{2}}{16}$ $\left[\because \cos\frac{\pi}{2}=0\right]$

326 **(d)**

For the quadratic equation to have real roots, we must have

$\cos^2 p-4\sin p(\cos p-1)\geq 0$

$\Rightarrow (\cos p-2\sin p)^2-4\sin^2 p+4\sin p\geq 0$

$\Rightarrow (\cos p-2\sin p)^2+4\sin p(1-\sin p)\geq 0$

Now, $0<p<\pi$

$\Rightarrow 4\sin p\,(1-\sin p)>0$ and, $(\cos p-2\sin p)^2\geq 0$

Thus, $(\cos p - 2\sin p)^2 + 4\sin p(1-\sin p) \geq 0$ for $0 < p < \pi$

Hence, the equation has real roots for $0 < p < \pi$

327 **(c)**

We have,

$$\cos(\theta+\phi) = \frac{1-\tan^2\left(\frac{\theta+\phi}{2}\right)}{1+\tan^2\left(\frac{\theta+\phi}{2}\right)}$$

Also,

$$\tan\left(\frac{\theta+\phi}{2}\right) = \frac{\tan\frac{\theta}{2}+\tan\frac{\phi}{2}}{1-\tan\frac{\theta}{2}\tan\frac{\phi}{2}} = \frac{\frac{5}{2}+\frac{3}{4}}{1-\frac{15}{8}} = -\frac{26}{7}$$

$$\therefore \cos(\theta+\phi) = \frac{1-\frac{676}{49}}{1+\frac{676}{49}} = -\frac{627}{425}$$

328 **(b)**

We have,

$2\cos^2 A = 3\cos^2 B$

$\Rightarrow 2(1-\sin^2 A) = 3(1-\sin^2 B)$

$\Rightarrow 2 - 2\sin^2 A = 3(1-\sin A)$ $[\because \sin^2 B = \sin A]$

$\Rightarrow 2\sin^2 A - 3\sin A + 1 = 0 \Rightarrow \sin A = \frac{1}{2}, 1$

Now,

$\sin A = 1 \Rightarrow \sin B = 1$, which is not possible

$\therefore \sin A = \frac{1}{2}$ and $\sin B = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow A = 30°, B = 135°, C = 15°$

or, $A = 30°, B = 45°, C = 105°$

In each case the triangle ABC is an obtuse angled triangle

329 **(b)**

We have,

$$\cos 2\alpha = \frac{3\cos 2\beta - 1}{3 - \cos 2\beta}$$

$$\Rightarrow \frac{1-\cos 2\alpha}{1+\cos 2\alpha} = \frac{3-\cos 2\beta - 3\cos 2\beta + 1}{3 - \cos 2\beta + 3\cos 2\beta - 1}$$

[Applying componendo and dividendo]

$$\Rightarrow \frac{2\sin^2\alpha}{2\cos^2\alpha} = \frac{4(1-\cos 2\beta)}{2(1+\cos 2\beta)}$$

$$\Rightarrow \tan^2\alpha = \frac{2\times 2\sin^2\beta}{2\cos^2\beta}$$

$\Rightarrow \tan^2\alpha = 2\tan^2\beta \Rightarrow \tan\alpha\cot\beta = \sqrt{2}$

330 **(b)**

It is given that

$\sin 2x, \frac{1}{2}$ and $\cos 2x$ are in A. P.

$\therefore 1 = \sin 2x + \cos 2x$

$\Rightarrow \cos\left(2x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$\Rightarrow 2x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$

$\Rightarrow 2x = 2n\pi, 2n\pi + \frac{\pi}{2}$

$\Rightarrow x = n\pi, n\pi + \frac{\pi}{4}, n \in Z$

332 **(d)**

Let ABC be a triangle such that $BC = 80$ cm, $\angle B = 60°$ and $b + c = 90$ cm

Now,

$b^2 = c^2 + a^2 = 2ac\cos B$

$\Rightarrow (90-c)^2 = c^2 + 80^2 - 2\times 80 \times (90-c)\cos 60°$

$\Rightarrow 8100 - 180c + c^2 = c^2 + 6400 - 7200 + 80c$

$\Rightarrow c = 17$

$\therefore b + c = 90 \Rightarrow b = 73$

Hence, the length of the shortest side is 17 cm

333 **(a)**

Given, $\sin^4 x + \cos^4 x = \sin x \cdot \cos x$

$\Rightarrow (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cdot \cos^2 x = \sin x \cdot \cos x$

$\Rightarrow 1 - \frac{\sin^2 2x}{2} = \frac{\sin 2x}{2}$

$\Rightarrow \sin^2 2x + \sin 2x - 2 = 0$

$\Rightarrow (\sin 2x + 2)(\sin 2x - 1) = 0$

$\Rightarrow \sin 2x = 1$ $(\because \sin 2x \geq -1)$

$\therefore 2x = (4n+1)\frac{\pi}{2}$

$\Rightarrow x = (4n+1)\frac{\pi}{4}$

$\Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$

Hence, two solutions exist

334 **(b)**

Given, $\alpha < \beta < \gamma < \delta$

Also, $\sin\alpha = \sin\beta = \sin\gamma = \sin\delta = k$

$\therefore \beta = \pi - \alpha, \quad \gamma = 2\pi + \alpha, \quad \delta = 3\pi - \alpha$

Now, $4\sin\frac{\alpha}{2} + 3\sin\frac{\beta}{2} + 2\sin\frac{\gamma}{2} + \sin\frac{\delta}{2}$

$4\sin\frac{\alpha}{2} + 3\sin\left(\frac{\pi-\alpha}{2}\right) + 2\sin\left(\frac{2\pi+\alpha}{2}\right) +$ sin*(3π−α)2*

$= 4\sin\frac{\alpha}{2} + 3\cos\frac{\alpha}{2} - 2\sin\frac{\alpha}{2} - \cos\frac{\alpha}{2}$

$= 2\sin\frac{\alpha}{2} + 2\cos\frac{\alpha}{2}$

$= 2\sqrt{\left(\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}\right)^2}$

$= 2\sqrt{\sin^2\frac{\alpha}{2} + \cos^2\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}$

$= 2\sqrt{1+\sin\alpha} = 2\sqrt{1+k}$

335 **(a)**

Maximum value of $\sin\theta + \cos\theta = \sqrt{1+1} = \sqrt{2}$

336 **(b)**

Given, $2\cos^2 x - 1 + 2\cos^2 x = 2$

$\Rightarrow \quad \cos x = \pm\frac{\sqrt{3}}{2} \quad \therefore x = n\pi \pm \frac{\pi}{6} : n \in Z$

337 **(c)**

$(1 + 2\sin\theta)^2 + (\sqrt{3}\tan\theta - 1)^2 = 0$

$\Rightarrow 1 + 2\sin\theta = 0$ and $\sqrt{3}\tan\theta - 1 = 0$

$\therefore \sin\theta = -\frac{1}{2} \Rightarrow \theta = m\pi + (-1)^m\left(-\frac{\pi}{6}\right)$

and $\tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = m\pi + \frac{\pi}{6}$

For common values, m must be odd

$ie, m = 2n + 1$

$\Rightarrow \theta = 2n\pi + \frac{7\pi}{6}$

338 **(b)**

We have,

$(2\cos x - 1)(3 + 2\cos x) = 0$

$\Rightarrow 2\cos x - 1 = 0 \quad [\because \cos x \neq -3/2]$

$\Rightarrow \cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3} \in [0, 2\pi]$

339 **(b)**

We have,

$B = 90°$

$\therefore A + B + C = 180°$

$\Rightarrow A + C = 90° \Rightarrow B = A + C \Rightarrow B - C = A$

Now,

$\tan\frac{B - C}{2} = \frac{b - c}{b + c}\cot\frac{A}{2}$

$\Rightarrow \tan\frac{A}{2} = \frac{b - c}{b + c}\cot\frac{A}{2} = \sqrt{\frac{b - c}{b + c}}$

340 **(c)**

$\cos(\theta + \phi) = m\cos(\theta - \phi)$

$\Rightarrow \cos\theta\cos\phi - \sin\theta\sin\phi$

$= \mathrm{m}\cos\theta\cos\phi + \mathrm{m}\sin\theta\sin\phi$

$\Rightarrow \quad \cos\theta\cos\phi\,(1 - \mathrm{m}) = \sin\theta\sin\phi(1 + \mathrm{m})$

$\Rightarrow \tan\theta = \left[\frac{1 - m}{1 + m}\right]\cot\phi$

341 **(a)**

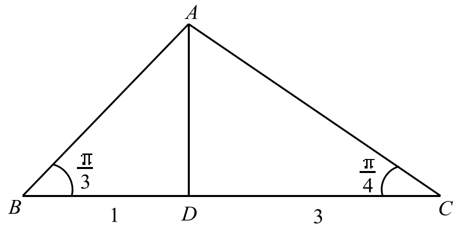
In triangles ABD and ACD, we have

$\frac{AB}{\sin B} = \frac{BD}{\sin\angle BAD}$ and $\frac{AD}{\sin C} = \frac{CD}{\sin\angle CAD}$

$\Rightarrow \frac{\sin C}{\sin B} = \frac{\sin\angle CAD}{\sin\angle BAD} \times \frac{BD}{CD}$

$\Rightarrow \frac{\sin\pi/4}{\sin\pi/3} = \frac{\sin\angle CAD}{\sin\angle BAD} \times \frac{1}{3} \Rightarrow \frac{\sin\angle BAD}{\sin\angle CAD}$

$= \frac{1}{3} \times \frac{\sqrt{3}/2}{1/\sqrt{2}} = \frac{1}{\sqrt{6}}$



342 **(a)**

$a\cos 2x + b\sin 2x$

$= a.\frac{1 - \tan^2 x}{1 + \tan^2 x} + b.\frac{2\tan x}{1 + \tan^2 x}$

$= a.\frac{1 - \frac{b^2}{a^2}}{1 + \frac{b^2}{a^2}} + b.\frac{2.\frac{b}{a}}{1 + \frac{b^2}{a^2}} \quad \left[\because \tan x = \frac{b}{a}\right]$

$= \frac{a(a^2 - b^2)}{(a^2 + b^2)} + \frac{2b^2 a}{a^2 + b^2}$

$= \frac{a^3 - ab^2 + 2ab^2}{a^2 + b^2} = \frac{a^3 + ab^2}{a^2 + b^2} = a$

343 **(c)**

We have,

$\frac{b}{\sin B} = \frac{c}{\sin C}$

$\Rightarrow \sin B = \frac{b\sin C}{c}$

$\Rightarrow \sin B = \frac{2\sin 60°}{\sqrt{6}} = \frac{2}{\sqrt{6}} \cdot \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{2}}$

$\Rightarrow B = 45° \quad [\because B \neq 135°]$

$\therefore A = 180° - (B + C) = 75°$

Now,

$\frac{\sin A}{a} = \frac{\sin B}{b}$

$\Rightarrow a = \frac{b\sin A}{\sin B} = \frac{2\sin 75°}{\sin 45°} = \sqrt{3} + 1$

344 **(d)**

Let O be the centre of the pentagon. Then,

$\angle A_1OA_2 = \angle A_2OA_3 = \cdots \angle A_5OA_1 = \frac{360°}{5} = 72°$

In ΔA_1OA_2, we have,

$A_1A_2^2 = 1^2 + 1^2 - 2 \times 1 \times 1 \times \cos 72°$

In ΔA_1OA_3, we have

$A_1A_3^2 = 1^2 + 1^2 - 2 \times 1 \times 1 \times \cos 144°$

$\therefore (A_1A_2)(A_1A_3)$

$= \sqrt{2 - 2\cos 72°} \times \sqrt{2 - 2\cos 144°}$

$= 2\sqrt{1 - \sin 18°} \times \sqrt{1 - \cos 36°}$

$= 2\sqrt{1 - \frac{\sqrt{5} - 1}{4}} \times \sqrt{1 - \frac{\sqrt{5} + 1}{4}} = 5$

345 **(b)**

Given, $x = \log\left[\cot\left(\frac{\pi}{4} + \theta\right)\right]$

$\Rightarrow \quad e^x = \left[\cot\left(\frac{\pi}{4} + \theta\right)\right] \quad$ ...(i)

And $e^{-x} = \frac{1}{\cot\left(\frac{\pi}{4} + \theta\right)} = \tan\left(\frac{\pi}{4} + \theta\right) \quad$ ...(ii)

Now, $\sinh x = \frac{e^x - e^{-x}}{2}$

$$= \frac{\cot\left(\frac{\pi}{4}+\theta\right) - \tan\left(\frac{\pi}{4}+\theta\right)}{2}$$

$$= \frac{1-\tan^2\left(\frac{\pi}{4}+\theta\right)}{2\tan\left(\frac{\pi}{4}+\theta\right)} = \frac{1}{\tan 2\left(\frac{\pi}{4}+\theta\right)}$$

$$= -\frac{1}{\cot 2\theta} = -\tan 2\theta$$

346 **(a)**

Let ABC be the right angled triangle whose angles are in A.P. Then,

$2B = A + C$

Now, $A + B + C = 180° \Rightarrow 3B = 180° \Rightarrow B = 60°$

So, let the angles be $A = 30°, B = 60°$ and $C = 90°$

$$\therefore \frac{a}{\sin 30°} = \frac{b}{\sin 60°} = \frac{c}{\sin 90°} = 2R$$

$\Rightarrow a = R, b = \sqrt{3}R$ and $c = 2R$

Also,

$$\Delta = \frac{1}{2}ab\sin 90° = \frac{1}{2}ab = \frac{\sqrt{3}}{2}R^2$$

$$\therefore \frac{r}{s} = \frac{\Delta}{s^2}$$

$$\Rightarrow \frac{r}{s} = \frac{\frac{\sqrt{3}}{2}R^2}{\left(\frac{R+\sqrt{3}R+2R}{2}\right)^2} = \frac{\sqrt{3}}{2} \times \frac{4}{\left(\sqrt{3}+3\right)^2}$$

$$= \frac{2\sqrt{3}}{\left(\sqrt{3}+3\right)^2}$$

$$\Rightarrow \frac{r}{s} = \frac{2\sqrt{3}\left(\sqrt{3}-3\right)^2}{(9-3)^2} = \frac{6\sqrt{3}\left(\sqrt{3}-1\right)^2}{36}$$

$$\Rightarrow \frac{r}{s} = \frac{\sqrt{3}(4-2\sqrt{3})}{6} = \frac{2-\sqrt{3}}{\sqrt{3}} \Rightarrow \frac{r}{2s} = \frac{2-\sqrt{3}}{2\sqrt{3}}$$

347 **(a)**

Let $x = \cos 2\theta + \cos\theta$. Then,

$x = 2\cos^2\theta + \cos\theta - 1$

$$\Rightarrow x = -1 + 2\left(\cos^2\theta + \frac{1}{2}\cos\theta\right)$$

$$\Rightarrow x = -1 + 2\left\{\left(\cos\theta + \frac{1}{4}\right)^2 - \frac{1}{16}\right\}$$

$$\Rightarrow x = -\frac{9}{8} + 2\left(\cos\theta + \frac{1}{4}\right)^2$$

$$\Rightarrow x \geq -\frac{9}{8} \quad \left[\because 2\left(\cos\theta + \frac{1}{4}\right)^2 \geq 0\right]$$

Hence, the minimum value of x is $-\frac{9}{8}$

348 **(b)**

$3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2$
$\quad + 4(\sin^6 x + \cos^6 x)$

$= 3(1 - 2\sin x\cos x)^2 + 6(1 + 2\sin x\cos x)$
$+4(\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x$
$\quad - \sin^2 x\cos^2 x)$

$= 3[1 + 4\sin^2 x\cos^2 x - 4\sin x\cos x]$
$+6 + 12\sin x\cos x + 4[(\sin^2 x + \cos^2 x)^2$
$-2\sin^2 x\cos^2 x - \sin^2 x\cos^2 x]$

$= 3 + 12\sin^2 x\cos^2 x + 6 + 4 - 12\sin^2 x\cos^2 x$
$= 13$

349 **(b)**

Let $a = 3x + 4y, b = 4x + 3y$ and $c = 5x + 5y$. Then,

$c - a = 2x + y > 0, c - b = x + 2y > 0$

$\Rightarrow c > a$ and $c > b$

$\Rightarrow$ Side c is the largest side

Now,

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\Rightarrow \cos C = \frac{(3x+4y)^2(4x+3y)^2 - (5x+5y)^2}{2(3x+4y)(4x+3y)}$$

$$\Rightarrow \cos C = -\frac{xy}{(3x+4y)(4x+3y)} < 0$$

$\Rightarrow C$ is an obtuse angle

Hence, the triangle is obtuse angled triangle

350 **(c)**

We have,

$\sin\beta = \sqrt{\sin\alpha\cos\alpha} \Rightarrow \sin^2\beta = (1/2)\sin 2\alpha$

Now,

$\Rightarrow \cos 2\beta = 1 - 2\sin^2\beta$

$\Rightarrow \cos 2\beta = 1 - \sin 2\alpha$

$$\Rightarrow \cos 2\beta = 1 + \cos\left(\frac{\pi}{2} + 2\alpha\right) = 2\cos^2\left(\frac{\pi}{4} + \alpha\right)$$

Again,

$\Rightarrow \cos 2\beta = 1 - \sin 2\alpha$

$$\Rightarrow \cos 2\beta = 1 - \cos\left(\frac{\pi}{2} - 2\alpha\right) = 2\sin^2\left(\frac{\pi}{4} - \alpha\right)$$

351 **(c)**

We have,

$\cos A\cos B + \sin A\sin B\sin C = 1$

$\Rightarrow 2\cos A\cos B + 2\sin A\sin B\sin C = 2$

$\Rightarrow 2\cos A\cos B + 2\sin A\sin B\sin C$
$\quad = \cos^2 A$
$\quad + \sin^2 A + \cos^2 B + \sin^2 B$

$\Rightarrow (\cos A - \cos B)^2 + (\sin A - \sin B)^2$
$\quad + 2\sin A\sin B(1 - \sin C) = 0$

$\Rightarrow \cos A - \cos B = 0, \sin A - \sin B = 0$ and
$1 - \sin C = 0$

$\Rightarrow A = B$ and $C = 90°$ $a = b$ and $C = 90°$

Hence, the triangle is an isosceles right angled triangle

352 **(c)**

We have,

$\cos^2 x - 2\cos x = 4\sin x - \sin 2x$

$\Rightarrow \cos x(\cos x - 2) = -2\sin x(\cos x - 2)$

$\Rightarrow \cos x = -2\sin x \qquad [\because \cos x - 2 \neq 0]$

$\Rightarrow \tan x = -\frac{1}{2} \Rightarrow x = \pi + \tan^{-1}\left(-\frac{1}{2}\right)$

353 **(c)**

Area of

$\Delta ABC = \frac{1}{2}ab\sin C = \frac{1}{2}\times 1\times 2\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

354 **(d)**

$\tan A\tan B = 2$

$\Rightarrow \frac{\sin A\sin B}{\cos A\cos B} = 2$

Using componendo and dividendo, we get

$\frac{\sin A\sin B + \cos A\cos B}{\sin A\sin B - \cos A\cos B} = \frac{2+1}{2-1}$

$\Rightarrow \frac{\cos(A-B)}{-\cos(A+B)} = \frac{3}{1}$

$\Rightarrow \frac{3/5}{-\cos(A+B)} = \frac{3}{1} \qquad \left[\because \cos(A-B) = \frac{3}{5}, \text{given}\right]$

$\Rightarrow \cos(A+B) = -\frac{1}{5}$

355 **(b)**

We have,

$\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{3\pi}{9}$

$= \left\{\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{4\pi}{9}\right\}\times\cos\frac{3\pi}{9}$

$= \frac{\sin(2^3\pi/9)}{2^3\times\sin\pi/9}\times\cos\pi/3 = \frac{\sin 8\pi/9}{8\sin\pi/9}\times\frac{1}{2} = \frac{1}{8}\times\frac{1}{2}$

$= \frac{1}{16}$

357 **(c)**

We have,

$\frac{\tan 3A}{\tan A} = k \Rightarrow \frac{3 - 1\tan^2 A}{1 - 3\tan^2 A} = k \Rightarrow \tan^2 A = \frac{k-3}{3k-1}$

Now,

$\frac{\sin 3A}{\sin A} = 3 - 4\sin^2 A = 3 - \frac{4}{1+\cot^2 A}$

$= 3 - \frac{4}{1+\frac{3k-1}{k-3}} = \frac{2k}{k-1}$

Again, $\frac{\sin 3A}{\sin A} = 3 - 4\sin^2 A$

$\Rightarrow \frac{2K}{k-1} = 3 - 4\sin^2 A$

$\Rightarrow 4\sin^2 A = 3 - \frac{2k}{k-1}$

$\Rightarrow \sin^2 A = \frac{k-3}{4(k-1)}$

$\Rightarrow 0 \leq \frac{k-3}{4(k-1)} \leq 1 \quad [\because 0 \leq \sin^2 A \leq 1]$

$\Rightarrow k < \frac{1}{3}$ or, $k > 3$

Hence, $\frac{\sin 3A}{\sin A} = \frac{2k}{k-1}$, where $k < \frac{1}{3}$ or, $k > 3$

358 **(d)**

We have,

$\sin\theta - \cos\theta = \sqrt{2}\sin\left(\theta - \frac{\pi}{4}\right)$

$\therefore \sin\theta - \cos\theta < 0$

$\Rightarrow \sin\left(\theta - \frac{\pi}{4}\right) < 0$

$\Rightarrow 2n\pi - \pi < \theta - \frac{\pi}{4} < 2n\pi, n \in Z$

$\Rightarrow 2n\pi - \frac{3\pi}{4} < \theta < 2n\pi + \frac{\pi}{4}, n \in Z$

359 **(d)**

We have,

$\tan 2C = \tan\{(A+B+C) - (A+B-C)\}$

$\Rightarrow \tan 2C = \frac{\tan(A+B+C) - \tan(A+B-C)}{1+\tan(A+B+C)\tan(A+B-C)}$

$\Rightarrow \tan 2C = \frac{\frac{\lambda}{y} - \frac{\lambda}{x}}{1+\frac{\lambda}{y}\times\frac{\lambda}{x}} = \frac{\lambda(x-y)}{\lambda^2 + xy}$

360 **(c)**

Given, $\alpha + \beta = \frac{\pi}{2}$, $\beta + \gamma = \alpha$

$\Rightarrow \beta = \frac{\pi}{2} - \alpha, \qquad \beta + \gamma = \alpha$

$\Rightarrow \tan\beta = \tan\left(\frac{\pi}{2} - \alpha\right)$ and $\tan(\beta+\gamma) = \tan\alpha$

$\Rightarrow \tan\beta = \cot\alpha \quad$ ...(i)

And $\frac{\tan\beta + \tan\gamma}{1 - \tan\beta\tan\gamma} = \tan\alpha$

$\Rightarrow \frac{\tan\beta + \tan\gamma}{1 - \cot\alpha\tan\gamma} = \frac{\tan\alpha}{1} \quad$ [from Eq. (i)]

$\Rightarrow \tan\beta + \tan\gamma = \tan\alpha - \tan\gamma$

$\Rightarrow \tan\alpha = \tan\beta + 2\tan\gamma$

361 **(d)**

We have,

$\frac{\tan\frac{6\pi}{15} - \tan\frac{\pi}{15}}{1+\tan\frac{6\pi}{15}\tan\frac{\pi}{15}} = \tan\frac{\pi}{3}$

$\Rightarrow \tan\frac{6\pi}{15} - \tan\frac{\pi}{15} = \sqrt{3} + \sqrt{3}\tan\frac{6\pi}{15}\tan\frac{\pi}{15}$

$\Rightarrow \tan\frac{6\pi}{15} - \tan\frac{\pi}{15} - \sqrt{3}\tan\frac{6\pi}{15}\tan\frac{\pi}{15} = \sqrt{3}$

362 **(a)**

$\frac{\cos A}{\cos B} = n$ and $\frac{\sin A}{\sin B} = m$

$\therefore m^2 - n^2 = (m+n)(m-n)$

$= \frac{\sin(A+B)\sin(A-B)}{\cos^2 B\sin^2 B}$

$\Rightarrow m^2 - n^2 = \frac{\sin^2 A - \sin^2 B}{\cos^2 B\sin^2 B}$

$$\Rightarrow (m^2-n^2)\sin^2 B = \frac{\sin^2 A - \sin^2 B}{\cos^2 B}$$
$$= \frac{\cos^2 B - \cos^2 A}{\cos^2 B}$$
$$\Rightarrow (m^2-n^2)\sin^2 B = 1 - \frac{\cos^2 A}{\cos^2 B} = 1 - n^2$$

363 **(a)**

$x = \sin 130° + \cos 130°$

$= \sin 50° - \sin 40° > 0$

$\left[\because \sin x \text{ is increasing for } 0 < x < \frac{\pi}{2}\right]$

364 **(c)**

We have,

$\tan 9° - \tan 27° - \tan 63° + \tan 81°$

$= (\tan 9° + \tan 81°) - (\tan 27° + \tan 63°)$

$$= \frac{1}{\cos 9° \cos 81°} - \frac{1}{\cos 27° \cos 63°}$$
$$= \frac{1}{\sin 9° \cos 9°} - \frac{1}{\sin 27° \cos 27°}$$
$$= \frac{2}{\sin 18°} - \frac{2}{\sin 54°}$$
$$= 2\left\{\frac{\sin 54 - \sin 18°}{\sin 54° \sin 18°}\right\} = 2\left\{\frac{2\cos 36° \sin 18°}{\sin 18° \cos 36°}\right\} = 4$$

365 **(c)**

We have,

$$\sqrt{4\sin^4\alpha + \sin^2 2\alpha} + 4\cos^2\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)$$
$$= \sqrt{4\sin^2\alpha(\sin^2\alpha + \cos^2\alpha)} + 4\left\{\frac{1+\cos\left(\frac{\pi}{2} - \alpha\right)}{2}\right\}$$
$$= 2|\sin\alpha| + 2(1+\sin\alpha)$$
$$= -2\sin\alpha + 2(1+\sin\alpha) = 2 \quad \left[\begin{matrix}\because \sin\alpha < 0 \\ \text{for } \alpha \in 3\pi/2\end{matrix}\right]$$

366 **(d)**

$$5\cos 2\theta + 2\cos^2\frac{\theta}{2} + 1 = 0$$
$$\Rightarrow \ 5(2\cos^2\theta - 1) + (1+\cos\theta) + 1 = 0$$
$$\Rightarrow \ 10\cos^2\theta + \cos\theta - 3 = 0$$
$$\Rightarrow \ (5\cos\theta + 3)(2\cos\theta - 1) = 0$$
$$\Rightarrow \ \cos\theta = \frac{1}{2}, \cos\theta = -\frac{3}{5} \ \Rightarrow \ \theta = \frac{\pi}{3}, \pi - \cos^{-1}\left(\frac{3}{5}\right)$$

367 **(b)**

Given, $\sin^4 x + \cos^4 x = a$

$\Rightarrow \sin^4 x + (1-\sin^2 x)^4 = a$

$2\sin^4 x - 2\sin^2 x + (1-a) = 0$

For real solution, $D \geq 0$

$\Rightarrow (-2)^2 - 4 \times 2(1-a) \geq 0$

$\Rightarrow 1 - 2 + 2a \geq 0$

$\Rightarrow a \geq \frac{1}{2}$

Hence, option (b) is true

368 **(b)**

Equation first can be written as

$$x\sin a + y \times 2\sin a\cos a + z \times \sin a(3 - 4\sin^2 a) = 2 \times 2\sin a\cos a\cos 2a$$
$$\Rightarrow x + 2y\cos a + z(3 + 4\cos^2 a - 4) = 4\cos a(2\cos^2 a - 1) \text{ as } \sin a \neq 0$$
$$\Rightarrow 8\cos^3 a - 4z\cos^2 a - (2y+4)\cos a + (z-x) = 0$$
$$\Rightarrow \cos^3 a - \left(\frac{z}{2}\right)\cos^2 a - \left(\frac{y+2}{4}\right)\cos a + \left(\frac{z-x}{8}\right) = 0$$

Which shows that $\cos a$ is a root of the equation

$$t^3 - \left(\frac{z}{2}\right)t^2 - \left(\frac{y+2}{4}\right)t + \left(\frac{z-x}{8}\right) = 0$$

Similarly, from second and third equation we can verify that $\cos b$ and $\cos c$ are the roots of the given equation

369 **(c)**

Since, $a\cos x + b\sin x = c$

$$\therefore \ a\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} + b\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = c$$
$$\Rightarrow \ a - a\tan^2\frac{x}{2} + 2b\tan\frac{x}{2} = c\left(1+\tan^2\frac{x}{2}\right)$$
$$\Rightarrow \ (c+a)\tan^2\frac{x}{2} - 2b\tan\frac{x}{2} + c - a = 0$$

Since, α, β are both roots of the given equation

$$\therefore \ \tan\frac{\alpha}{2} + \tan\frac{\beta}{2} = \frac{2b}{c+a}$$

and $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} = \frac{c-a}{c+a}$

Now, $\tan\left(\frac{\alpha+\beta}{2}\right) = \frac{\tan\frac{\alpha}{2} + \tan\frac{\beta}{2}}{1 - \tan\frac{\alpha}{2}\cdot\tan\frac{\beta}{2}}$

$$\Rightarrow \ \tan\left(\frac{\alpha+\beta}{2}\right) = \frac{\frac{2b}{c+a}}{1 - \frac{c-a}{c+a}}$$
$$\Rightarrow \ \tan\left(\frac{\alpha+\beta}{2}\right) = \frac{b}{a}$$

370 **(a)**

In a $\Delta\, ABC$, we have

$$\tan\frac{A}{2}\tan\frac{B}{2}+\tan\frac{B}{2}\tan\frac{C}{2}+\tan\frac{C}{2}\tan\frac{A}{2}=1$$

$$\Rightarrow \frac{1}{3}\times\frac{2}{3}+\frac{2}{3}\tan\frac{C}{2}+\frac{1}{3}\tan\frac{C}{2}=1 \Rightarrow \tan\frac{C}{2}=\frac{7}{9}$$

371 **(b)**

Given, $\tan\theta=\frac{1}{\sqrt{7}} \quad \Rightarrow \quad \cot\theta=\sqrt{7}$

Now, $\frac{(\operatorname{cosec}^2\theta-\sec^2\theta)}{(\operatorname{cosec}^2+\sec^2\theta)}=\frac{(1+\cot^2\theta-1-\tan^2\theta}{1+\cot^2\theta+1+\tan^2\theta}$

$$=\frac{\cot^2\theta-\tan^2\theta}{2+\cot^2\theta+\tan^2\theta}$$

$$=\frac{(\sqrt{7})^2-\left(\frac{1}{\sqrt{7}}\right)^2}{2+(\sqrt{7})^2+\left(\frac{1}{\sqrt{7}}\right)^2}$$

$$=\frac{49-1}{7}\times\frac{7}{63+1}=\frac{48}{64}=\frac{3}{4}$$

372 **(b)**

We have,

$\tan 3\,x=1$

$$\Rightarrow \tan 3\,x=\tan\frac{\pi}{4}$$

$$\Rightarrow 3\,x=n\,\pi+\frac{\pi}{4}\Rightarrow x=\frac{n\,\pi}{3}+\frac{\pi}{12}, n\in Z$$

373 **(a)**

We have,

$3\tan A-4=0$

$$\Rightarrow \tan A=\frac{4}{3}$$

$$\Rightarrow \sin A=-\frac{4}{5}, \cos\; A=-\frac{3}{5} \quad \left[\because \pi<A<\frac{3\pi}{2}\right]$$

$\therefore$ $5\sin 2A+3\sin A+4\cos A$

$=10\sin A\cos A+3\sin A+4\cos A$

$$=10\left(\frac{12}{25}\right)-\frac{12}{5}-\frac{12}{5}=0$$

374 **(c)**

$\cos 2\theta=\sin\theta$

$\Rightarrow\; 1-2\sin^2\theta=\sin\theta$

$\Rightarrow\; 2\sin^2\theta+\sin\theta-1=0$

$\Rightarrow\; 2\sin^2\theta+2\sin\theta-\sin\theta-1=0$

$\Rightarrow\; 2\sin\theta(\sin\theta+1)-(\sin\theta+1)=0$

$\Rightarrow\; (\sin\theta+1)(2\sin\theta-1)=0$

$$\Rightarrow\; \sin\theta=-1, \qquad \sin\theta=\frac{1}{2}$$

$$\Rightarrow\; \sin\theta=\sin\frac{3\pi}{2}, \sin\theta=\sin\frac{\pi}{6}$$

$$\Rightarrow\; \theta=n\pi+(-1)^n\frac{3\pi}{2},$$

$$\Rightarrow\; \theta=m\pi+(-1)^m\frac{\pi}{6}$$

For $\theta\in(0,2\pi)$

$$\theta=\frac{3\pi}{2},\frac{\pi}{6},\frac{5\pi}{6}$$

Hence number of solutions= 3

375 **(b)**

We have,

$\sin^4 x-2\cos^2 x+a^2=0$

$\Rightarrow y^2-2(1-y)+a^2=0$, where $\sin^2 x=y$

$\Rightarrow y^2+2y+a^2-2=0$

$\Rightarrow y=-1\pm\sqrt{3-a^2}$

For y to be real, we must have

Disc. $\geq 0\Rightarrow 4-4(a^2-2)\geq 0\Rightarrow a^2\leq 3$ ...(i)

But, $\sin^2 x=y$. Therefore,

$0\leq y\leq 1$

$\Rightarrow 0\leq -1+\sqrt{3-a^2}\leq 1$

$\Rightarrow 1\leq\sqrt{3-a^2}\leq 2$

$\Rightarrow 1\leq 3-a^2\leq 4$

$\Rightarrow 2-a^2\geq 0\Rightarrow a^2\leq 2$ ...(ii)

From (i) and (ii), we have

$a^2\leq 2\Rightarrow -\sqrt{2}\leq a\leq\sqrt{2}$

376 **(d)**

Given, $A+B+C=\pi$

$$\Rightarrow \quad \frac{A+B}{2}=\frac{\pi}{2}-\frac{C}{2}$$

$$\Rightarrow\; \tan\left(\frac{A+B}{2}\right)=\tan\left(\frac{\pi}{2}-\frac{C}{2}\right)=\cot\frac{C}{2}$$

$$\Rightarrow\; \frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}\tan\frac{B}{2}}=\cot\frac{C}{2}$$

$$\Rightarrow\; \frac{\frac{1}{3}+\frac{2}{3}}{1-\frac{1}{3}\times\frac{2}{3}}=\cot\frac{C}{2}$$

$$\left[\because\; \tan\frac{A}{2}=\frac{1}{3}, \tan\frac{B}{2}=\frac{2}{3}\ \text{(given)}\right]$$

$$\Rightarrow\; \cot\frac{C}{2}=\frac{9}{7}$$

$$\Rightarrow\; \tan\frac{C}{2}=\frac{7}{9}$$

377 **(d)**

Given, $x+\frac{1}{x}=2\cos\alpha$

$\Rightarrow\; x^2-2x\cos\alpha+1=0$

$$\Rightarrow\; x=\frac{2\cos\alpha\pm\sqrt{4\cos^2\alpha-4}}{2}$$

$\Rightarrow\; x=\cos\alpha+i\sin\alpha$

Now, $x^n=(\cos\alpha+i\sin\alpha)^n=\cos n\alpha+i\sin n\alpha$

And $\frac{1}{x^n}=(\cos\alpha-i\sin\alpha)^n=\cos n\alpha-i\sin n\alpha$

$$\therefore\; x^n+\frac{1}{x^n}=\cos n\alpha+i\sin n\alpha+\cos n\alpha-i\sin n\alpha$$

$=2\cos n\alpha$

378 **(b)**

We have,

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{\frac{m}{m+1}+\frac{1}{2m+1}}{1-\frac{m}{m+1}\times\frac{1}{2m+1}}$$

$\Rightarrow \tan(\alpha+\beta) = \dfrac{2m^2+2m+1}{2m^2+2m+1} = 1 \Rightarrow \alpha+\beta = \dfrac{\pi}{4}$

379 **(a)**

Let AM be perpendicular from A on BC such that $AM = p$. Then, $BC = 4p$. Let $AB = x$ and $AC = y$

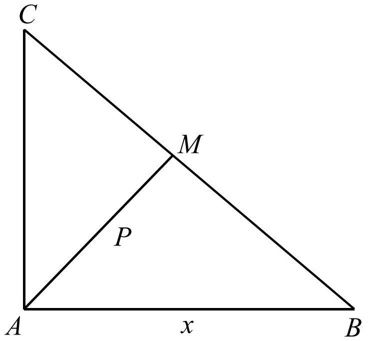
Then,

$\Rightarrow x^2 + y^2 = (4p)^2$

In ΔABM, we have

$p^2 + BM^2 = x^2$

$\Rightarrow p^2 + (49-k)^2 = x^2$, where $k = CM$ ...(i)



In ΔACM, we have

$p^2 = CM^2 = y^2 \Rightarrow p^2 + k^2 = y^2$ ...(ii)

Adding (i) and (ii), we get

$2p^2 + (4p-k)^2 = x^2 + y^2$

$\Rightarrow 2p^2 + (4p-k)^2 = (4p)^2 \quad [\because x^2+y^2=(4p)^2]$

$\Rightarrow k = 2p - \sqrt{3}p$

$\Rightarrow BM = BC - CM \Rightarrow BM = 4p - (2p - \sqrt{3}p)$

$= 2p + \sqrt{3}p$

$\therefore \tan B = \dfrac{AM}{BM} \Rightarrow \tan B = \dfrac{p}{(2+\sqrt{3})p} = 2 - \sqrt{3}$

$\Rightarrow B = 15°$

380 **(d)**

We have,

$\cot\theta \cot 7\theta + \cot\theta \cot 4\theta + \cot 4\theta \cot 7\theta = 1$

$\Rightarrow \cos\theta \cos 4\theta \sin 7\theta + \cos 4\theta \cos 7\theta \sin\theta$

$+ \cos 7\theta \cos\theta \sin 4\theta - \sin\theta \sin 4\theta \sin 7\theta = 0$

$\Rightarrow \sin(\theta + 4\theta + 7\theta) = 0$

$\Rightarrow \sin 12\theta = 0 \Rightarrow 12\theta = n\pi, n \in Z \Rightarrow \theta = \dfrac{n\pi}{12}, n$

$\in Z$

381 **(c)**

We have

$\dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3} = \dfrac{s-a}{\Delta} + \dfrac{s-b}{\Delta} + \dfrac{s-c}{\Delta} = \dfrac{s}{\Delta} = \dfrac{1}{r}$

382 **(b)**

$\tan(A+B+C)$

$= \dfrac{[\tan A + \tan B + \tan C - \tan A \tan B \tan C]}{[1 - \tan A \tan B - \tan B \tan C - \tan C \tan A]}$

$\Rightarrow \tan(90°)$

$= \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$

$\Rightarrow \tan A \tan B + \tan B \tan C + \tan C \tan A = 1$

383 **(a)**

We have,

$\cos x > \sin x$ for $0 < x < \pi/4$

$\Rightarrow \cos 10° > \sin 10° \Rightarrow \cos 10° - \sin 10° > 0$

384 **(d)**

We have, $a_1 + a_2 \cos 2x + a_3 \sin^2 x = 0$, for all x

$\Rightarrow a_1 + a_2 \cos 2x + a_3\left(\frac{1-\cos 2x}{2}\right) = 0$, for all x

$\Rightarrow \left(a_1 + \dfrac{a_3}{2}\right) + \left(a_2 - \dfrac{a_3}{2}\right)\cos 2x = 0, \forall x$

$\Rightarrow a_1 + \frac{a_3}{2} = 0$ and $a_2 - \frac{a_3}{2} = 0$

$\Rightarrow a_1 = -\frac{k}{2}, a_2 = \frac{k}{2}, a_3 = k$, where, $k \in R$

Hence, the solutions are $\left(-\frac{k}{2}, \frac{k}{2}, k\right)$, where k is any real number

Thus, the number of triplets is infinite

385 **(d)**

$\tan 60° = \tan(40° + 20°)$

$\Rightarrow \sqrt{3} = \dfrac{\tan 40° + \tan 20°}{1 - \tan 40°.\tan 20°}$

$\Rightarrow \sqrt{3} - \sqrt{3}\tan 40° \tan 20° = \tan 40° + \tan 20°$

$\Rightarrow \tan 40° + \tan 20° + \sqrt{3}\tan 40° \tan 20° = \sqrt{3}$

386 **(b)**

$\because \cos(315\pi + x) = (-1)^{315}\cos x = -\cos x$

$\therefore 4\cos^3 x - 4\cos^2 x - \cos(315\pi + x) = 1$

$\Rightarrow 4\cos^3 x - 4\cos^2 x + \cos x - 1 = 0$

$\Rightarrow (4\cos^2 x + 1)(\cos x - 1) = 0$

$\Rightarrow \cos x = 1, 4\cos^2 x + 1 \neq 0$

$\Rightarrow \cos x = \cos 0$

$\Rightarrow x = 2n\pi, n \in I$

$\therefore x = 2\pi, 4\pi, 6\pi, 8\pi, \dots, 100\pi \ (\because 0 < x < 315)$

$(ie, 100\pi < 315 < 101\pi)$

Required arithmetic mean

$= \dfrac{2\pi + 4\pi + 6\pi + 8\pi + \dots + 100\pi}{50}$

$= \dfrac{2\pi(1+2+3+4+\dots+50)}{50}$

$= \dfrac{2\pi \cdot \frac{50}{2} \cdot 51}{50} = 51\pi$

387 **(d)**

We have,

$\displaystyle\sum_{k=1}^{3} \cos^2(2k-1)\frac{\pi}{12}$

$= \cos^2\dfrac{\pi}{12} + \cos^2\dfrac{3\pi}{12} + \cos^2\dfrac{5\pi}{12}$

$= \sin^2\left(\dfrac{\pi}{2} - \dfrac{\pi}{12}\right) + \cos^2\dfrac{5\pi}{12} + \cos^2\dfrac{\pi}{4}$

$= \sin^2\dfrac{5\pi}{12} + \cos^2\dfrac{5\pi}{12} + \dfrac{1}{2} = 1 + \dfrac{1}{2} = \dfrac{3}{2}$

388 **(a)**

$\because \cos x = \sqrt{1 - \sin 2x}$

$\Rightarrow\ \cos x = |\sin x - \cos x|$
There are two cases arise.
Case I $\sin x \le \cos x$
$\Rightarrow \cos x = \cos x - \sin x$
$\Rightarrow \sin x = 0$
where, $x \in \left[0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right]$
$\Rightarrow x = 2\pi$, neglecting $x = \pi$
Case II $\sin x > \cos x$
$\Rightarrow\ \tan x = 2$
where, $x \in \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$
$\because\ \tan x = 2$
$\Rightarrow\ x = \tan^{-1}(2)$
Thus, the given equation has two solutions

389 **(b)**
We have,
$\sin 2x \cos 2x \cos 4x = \lambda \Rightarrow \sin 8x = 4\lambda$
This equation will have a solution if
$|4\lambda| \le 1 \Rightarrow \lambda \in [-1/4, 1/4]$

390 **(b)**
We have,
$\sin x + \sin y = 3(\cos y - \cos x)$
$\Rightarrow \sin x + 3\cos x = 3\cos y - \sin y$ ...(i)
$\Rightarrow r\cos(x-\alpha) = r\cos(y+\alpha)$, where
$r = \sqrt{10}, \tan\alpha = \frac{1}{3}$
$\Rightarrow x - \alpha = \pm(y+\alpha)$
$\Rightarrow x = -y$ or $x - y = 2\alpha$
Clearly, $x = -y$ satisfies equation (i).
$\therefore \dfrac{\sin 3x}{\sin 3y} = -\dfrac{\sin 3y}{\sin 3y} = -1$

391 **(a)**
Since, $\tan\alpha = k\cot\beta$ or $\tan\alpha\tan\beta = k$
Now, $\frac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)} = \frac{\cos\alpha\cos\beta+\sin\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}$
$= \dfrac{1+\tan\alpha\tan\beta}{1-\tan\alpha\tan\beta} = \dfrac{1+k}{1-k}$

392 **(c)**
Given, $\theta = \frac{2\sin x}{1+\sin x+\cos x}$
$\Rightarrow\ \theta = \dfrac{4\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}$
$\Rightarrow\ \theta = \dfrac{2\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}} \times \dfrac{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)}{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)}$
$\Rightarrow\ \theta = \dfrac{1-\cos x+\sin x}{1+\sin x}$

393 **(a)**
We have,
$\dfrac{s-a}{\Delta} = \dfrac{1}{8}, \dfrac{s-b}{\Delta} = \dfrac{1}{12}$ and $\dfrac{s-c}{\Delta} = \dfrac{1}{24}$
$\Rightarrow r_1 = 8, r_2 = 12$ and $r_3 = 24$
$\therefore r = \dfrac{\sum r_1r_2}{r_1r_2r_3} \Rightarrow r = \dfrac{1}{r} = \dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3} \Rightarrow r = 4$
Now, $b = \sqrt{(r_2-r)(r_1+r_3)} \Rightarrow b$
$= \sqrt{(12-8)\times(8\times 24)} = 16$

394 **(d)**
We have,
$\cos^2\left(\dfrac{1}{2}p\,x\right) + \cos^2\left(\dfrac{1}{2}\,q\,x\right) = 1$
$\Rightarrow 1 + \cos px + 1 + \cos qx = 2$
$\Rightarrow \cos px + \cos qx = 0$
$\Rightarrow \cos px = \cos(\pi - qx)$
$\Rightarrow p\,x = 2\,n\,\pi \pm \pi - qx, n \in Z$
$\Rightarrow x = \dfrac{(2n+1)\,\pi}{p+q}, \dfrac{(2n-1)\,\pi}{p-q}, n \in Z$
Clearly, the values given by $x = \frac{(2n+1)\,\pi}{p+q}, n \in Z$ form an A.P. with common difference $\frac{2\,\pi}{p+q}$ and the values given by $x = \frac{(2n-1)\,\pi}{p-q}, n \in Z$ form an A.P. with common difference $\frac{2\,\pi}{p-q}$

395 **(c)**
We have,
$\sec^2\theta = \sqrt{2}(1-\tan^2\theta)$
$\Rightarrow (1+\tan^2\theta) = \sqrt{2}(1-\tan^2\theta)$
$\Rightarrow \cos 2\theta = \dfrac{1}{\sqrt{2}}$
$\Rightarrow \cos 2\theta = \cos\dfrac{\pi}{4}$
$\Rightarrow 2\theta = 2\,n\,\pi \pm \dfrac{\pi}{4}, n \in Z \Rightarrow \theta = n\,\pi \pm \dfrac{\pi}{8}, n \in Z$

396 **(c)**
Given, $\tanh^{-1}(x+iy) = \frac{1}{2}\tanh^{-1}\left(\frac{2x}{1+x^2+y^2}\right) + \frac{i}{2}\tan^{-1}\left(\frac{2y}{1-x^2-y^2}\right); x, y \in R$
Put $x = 0$,
$\tanh^{-1}(iy) = \dfrac{1}{2}\tanh^{-1}(0) + \dfrac{i}{2}\tan^{-1}\left(\dfrac{2y}{1-y^2}\right)$
$= 0 + \dfrac{i}{2}\tan^{-1}\left(\dfrac{2\tan\theta}{1-\tan^2\theta}\right)$ (put $y = \tan\theta$)
$= \dfrac{i}{2}\tan^{-1}(\tan 2\theta)$
$= \dfrac{i}{2}\ 2\theta$
$= i\tan^{-1} y$

397 **(a)**
At the intersection point of $y = \cos x$ and $y = \sin 3x$, we have
$\cos x = \sin 3x$
$\Rightarrow \cos x = \cos\left(\dfrac{\pi}{2} - 3x\right)$
$\Rightarrow x = 2n\,\pi \pm \left(\dfrac{\pi}{2} - 3x\right)$

$\Rightarrow x = \frac{\pi}{4}, \frac{\pi}{8}$ $[\because -\pi/2 \leq x \leq \pi/2]$

So, $y = \cos\frac{\pi}{4}$ at $x = \frac{\pi}{4}$ and $y = \cos\frac{\pi}{8}, at\ x = \frac{\pi}{8}$

Thus, the points are $(\pi/4, 1/\sqrt{2})$ and $(\pi/8, \cos\pi/8)$

398 **(d)**

We have,

$A = \cos^2\theta + \sin^4\theta$

$\Rightarrow A = \cos^2\theta + \sin^2\theta.\sin^2\theta$

$\Rightarrow A \leq \cos^2\theta + \sin^2\theta \Rightarrow A \leq 1$ $[\because \sin^2\theta \leq 1]$

Again,

$A = \cos^2\theta + \sin^4\theta = (1 - \sin^2\theta) + \sin^4\theta$

$\Rightarrow A = \left(\sin^2\theta - \frac{1}{2}\right)^2 + \frac{3}{4}$

$\geq \frac{3}{4}$ $[\because (\sin^2\theta - 1/2)^2 \geq 0]$

Hence, $\frac{3}{4} \leq A \leq 1$

399 **(b)**

We have, $\sin(\pi + \theta) = -\sin\theta$

$\therefore \sin 190° = -\sin 10°, \sin 200° = -\sin 20°,$

$\sin 210° = -\sin 30°, \sin 360° = \sin 180° = 0$

Thus, all the terms in the given series cancel with each other. Consequently, the sum is zero

400 **(c)**

$5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$

$= 5\cos\theta + 3\left(\cos\theta\cos\frac{\pi}{3} - \sin\theta\sin\frac{\pi}{3}\right) + 3$

$= 5\cos\theta + \frac{3}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$

$= \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$

$\therefore$ maximum value$= 3 + \sqrt{\left(\frac{13}{2}\right)^2 + \left(-\frac{3\sqrt{3}}{2}\right)^2}$

$= 3 + \sqrt{\frac{196}{4}} = 3 + 7 = 10$

401 **(c)**

Since, $f(x)$ is a continuous decreasing function on $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

$\therefore$ $f(x)$ attains every value between $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

Its minimum value

$f\left(\frac{\pi}{3}\right) = \frac{1}{2} - \frac{\pi}{3}\left(1 + \frac{\pi}{3}\right)$

And maximum value

$f\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} - \frac{\pi}{6}\left(1 + \frac{\pi}{6}\right)$

402 **(a)**

We have,

$\frac{x}{a}\cos\alpha + \frac{y}{b}\sin\alpha = 1$

$\frac{x}{a}\cos\beta + \frac{y}{b}\sin\beta = 1$

By cross-multiplication, we have

$\frac{\frac{x}{a}}{\sin\beta - \sin\alpha} = \frac{\frac{y}{b}}{\cos\alpha - \cos\beta} = \frac{1}{\sin(\beta - \alpha)}$

$= \frac{x}{a} = \frac{\sin\beta - \sin\alpha}{\sin(\beta - \alpha)}$ and $\frac{y}{b} = \frac{\cos\alpha - \cos\beta}{\sin(\beta - \alpha)}$

$\Rightarrow \frac{x}{a} = \frac{\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\beta-\alpha}{2}\right)}$ and $\frac{y}{b} = \frac{\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\beta-\alpha}{2}\right)}$

$\Rightarrow \frac{x^2}{a^2} - 1 = \frac{\cos^2\left(\frac{\alpha+\beta}{2}\right) - \cos^2\left(\frac{\beta-\alpha}{2}\right)}{\cos^2\left(\frac{\beta-\alpha}{2}\right)}$

And,

$\frac{y^2}{b^2} - 1 = \frac{\sin^2\left(\frac{\alpha+\beta}{2}\right) - \cos^2\left(\frac{\beta-\alpha}{2}\right)}{\cos^2\left(\frac{\beta-\alpha}{2}\right)}$

$\Rightarrow \frac{x^2}{a^2} - 1 = \frac{\sin^2\left(\frac{\alpha-\beta}{2}\right) - \sin^2\left(\frac{\alpha+\beta}{2}\right)}{\cos^2\left(\frac{\alpha-\beta}{2}\right)}$

And,

$\frac{y^2}{b^2} - 1 = \frac{\cos^2\left(\frac{\alpha-\beta}{2}\right) - \sin^2\left(\frac{\alpha+\beta}{2}\right)}{\cos^2\left(\frac{\alpha-\beta}{2}\right)}$

$\Rightarrow \frac{x^2}{a^2} - 1 = \frac{-\sin\alpha\sin\beta}{\cos^2\left(\frac{\alpha-\beta}{2}\right)}$ and $\frac{y^2}{b^2} - 1$

$= -\frac{\cos\alpha\cos\beta}{\cos^2\left(\frac{\alpha-\beta}{2}\right)}$

$\Rightarrow \frac{\frac{x^2}{a^2} - 1}{\frac{y^2}{b^2} - 1} = \tan\alpha\tan\beta$

$\Rightarrow \frac{b^2(x^2 - a^2)}{a^2(y^2 - b^2)} = \frac{\frac{\sin\alpha\sin\beta}{b^2}}{\frac{\cos\alpha\cos\beta}{a^2}}$

$\Rightarrow \frac{x^2 - a^2}{y^2 - b^2} = -1$ $\left[\because \frac{\cos\alpha\cos\beta}{a^2} + \frac{\sin\alpha\sin\beta}{b^2} = 0\right]$

$\Rightarrow x^2 + y^2 = a^2 + b^2$

Hence, option (a) is correct

403 **(a)**

$\left(1 + \cos\frac{\pi}{6}\right)\left(1 + \cos\frac{\pi}{3}\right)\left(1 + \cos\frac{2\pi}{3}\right)\left(1 + \cos\frac{7\pi}{6}\right)$

$= \left(1 + \frac{\sqrt{3}}{2}\right)\left(1 + \frac{1}{2}\right)\left(1 - \frac{1}{2}\right)\left(1 - \frac{\sqrt{3}}{2}\right)$

$= \left(1 - \frac{3}{4}\right)\left(1 - \frac{1}{4}\right) = \frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$

404 **(b)**

Given, $\cos p\theta = -\cos q\theta = \cos(\pi + q\theta)$

$\Rightarrow\ p\theta = 2n\pi \pm (\pi + q\theta), n \in I$

$\Rightarrow\ \theta = \dfrac{(2n+1)\pi}{p-q}$ or $\dfrac{(2n-1)\pi}{p+q}, n \in I$

Angle $\theta = \frac{(2n+1)\pi}{p-q}$ gives an AP with common difference $\frac{2\pi}{p-q}$ and $\theta = \frac{(2n-1)\pi}{p-q}$ gives also an AP with common difference $\frac{2\pi}{p+q}$

Certainly, $\frac{2\pi}{p+q} < \left|\frac{2\pi}{p-q}\right|$

$\therefore$ The smallest common difference is $\frac{2\pi}{p+q}$

405 **(a)**

We have,

$\sin^4\theta + \cos^4\theta = a$

$\Rightarrow (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta = a$

$\Rightarrow 1 - \dfrac{1}{2}\sin^2 2\theta = a$

$\Rightarrow 1 - \dfrac{1}{2}\left(\dfrac{1-\cos 4\theta}{2}\right) = a$

$\Rightarrow \dfrac{3}{4} + \dfrac{1}{4}\cos 4\theta = a$

$\Rightarrow \cos 4\theta = 4a - 3$

Now,

$-1 \le \cos 4\theta \le 1 \Rightarrow -1 \le 4a - 3 \le 1 \Rightarrow 2 \le 4a \le 4 \Rightarrow \dfrac{1}{2} \le a \le 1$

406 **(b)**

1. Given, $\operatorname{cosec}\theta - \sec\theta = \operatorname{cosec}\theta . \sec\theta$

$\Rightarrow\ \dfrac{\cos\theta - \sin\theta}{\cos\theta\sin\theta} = \dfrac{1}{\cos\theta\sin\theta}$

$\Rightarrow\ \cos\theta - \sin\theta = 1 \ \Rightarrow\ \cos\left(\dfrac{\pi}{4} + \theta\right) = \dfrac{1}{\sqrt{2}}$

$\Rightarrow\ \frac{\pi}{4} + \theta = 2n\pi \pm \frac{\pi}{4} \ \Rightarrow$ Solution exist

2. $\operatorname{cosec}\theta . \sec\theta = 1$

$\Rightarrow\ \sin\theta\cos\theta = 1$

$\Rightarrow\ 2\sin\theta\cos\theta = 2$

$\Rightarrow\ \sin 2\theta = 2$

As we know $\sin\theta$ is not greater than 1

$\therefore$ The above equation has no solution exist

407 **(a)**

We have,

$$y = \frac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta} = \frac{\sin\theta\,(1 + 2\cos\theta)}{\cos\theta\,(1 + 2\cos\theta)} = \tan\theta$$

$\therefore y \in (-\infty, \infty)$

408 **(b)**

The equation $7\cos\theta + 5\sin\theta = 2k + 1$ possesses a solution, if

$-\sqrt{7^2 + 5^2} \le 2k + 1 \le \sqrt{7^2 + 5^2}$

$\Rightarrow -\sqrt{74} \le 2k + 1 \le \sqrt{74}$

$\Rightarrow -8 \le 2k + 1 \le 8$ [For integral values of k]

$\Rightarrow -4 \le k \le 3 \Rightarrow k = -4, \pm 3, \pm 2, \pm 1, 0$

409 **(b)**

$\sin x > 0 \ \Rightarrow x \in (0, \pi)$ ...(i)

$\cos x > 0 \ \Rightarrow x \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$ ...(ii)

From relations (i) and (ii) , we get

$x \in \left(0, \frac{\pi}{2}\right)$ ...(iii)

Now, $\log_{1/2}\sin x > \log_{1/2}\cos x$

$\Rightarrow\ \sin x < \cos x$ in $x \in \left(0, \frac{\pi}{4}\right)$ ...(iv)

From relations Eqs. (iii) and (iv), we get

$x \in \left(0, \dfrac{\pi}{4}\right)$

410 **(c)**

We have,

$\log_{1/2}\sin x > \log_{1/2}\cos x$

$\Rightarrow \sin x < \cos x$

$\Rightarrow x \in (0, \pi 4) \cup (3\pi 2, 2\pi)$ $\left[\begin{array}{c}\text{Draw graphs of } y = \sin x \\ \text{and } y = \cos x \text{ and compare}\end{array}\right]$

411 **(a)**

$\cos^2\left(\dfrac{\pi}{4} + \theta\right) - \sin^2\left(\dfrac{\pi}{4} - \theta\right)$

$= \cos\left(\dfrac{\pi}{4} + \theta + \dfrac{\pi}{4} - \theta\right)\cos\left(\dfrac{\pi}{4} + \theta - \dfrac{\pi}{4} + \theta\right)$

$= \cos\left(\dfrac{\pi}{2}\right)\cos(2\theta) = 0$

413 **(a)**

It is given that α and β are the roots of the equation $a\cos\theta + b\sin\theta = c$

or, $a\left(\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right) + \frac{2b\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = c$

or, $\tan^2\dfrac{\theta}{2}(a + c) - 2b\tan\dfrac{\theta}{2} + (c - a) = 0$

This equation has

$\tan\frac{\alpha}{2}$ and $\tan\frac{\beta}{2}$ as its roots

$\therefore \tan\frac{\alpha}{2} + \tan\frac{\beta}{2} = \frac{2b}{a+c}$ and, $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \frac{c-a}{c+a}$

$\Rightarrow \tan\left(\dfrac{\alpha + \beta}{2}\right) = \dfrac{2b}{(c + a) - (c - a)} = \dfrac{b}{a}$

414 **(d)**

We have, $\cos 3x + \cos 2x = \sin\frac{3x}{2} + \sin\frac{x}{2}$

$\Rightarrow\ 2\cos\frac{5x}{2}\cos\frac{x}{2} = 2\sin x\cos\frac{x}{2}$

Either $\cos\frac{x}{2} = 0$

$\Rightarrow\ \frac{x}{2} = (2n+1)\frac{\pi}{2}$

$\Rightarrow\ x = (2n+1)\pi$

or $\cos\frac{5x}{2} = \sin x$

$\Rightarrow\ \cos\frac{5x}{2} = \cos\left(\frac{\pi}{2} - x\right)$

$\Rightarrow\ \frac{5x}{2} = 2n\pi \pm \left(\frac{\pi}{2} - x\right)$

Taking the +ve sign $\frac{7x}{2} = 2n\pi + \frac{\pi}{2}$

$\Rightarrow\ x = \frac{4n\pi}{7} + \frac{\pi}{7}$

Taking –ve sign

$\frac{3x}{2} = 2n\pi - \frac{\pi}{2}\ \Rightarrow\ x = \frac{4n\pi}{3} - \frac{\pi}{3}$

For $0 \le x \le 2\pi$

$x = \frac{\pi}{7}, \frac{5\pi}{7}, \frac{9\pi}{7}, \frac{13\pi}{7}, \pi$

Thus, number of solutions = 5

415 **(c)**

Let a, b, c be the lengths of the sides of $\Delta\, ABC$. It is given that a, b, c are the roots of the equation

$x^3 - 2x^2 - x - 16 = 0$

$\therefore a + b + c = 2$ and $abc = 16$

Now,

$Rr = \frac{abc}{4\,\Delta} \times \frac{\Delta}{s} = \frac{abc}{4s} = \frac{abc}{2(a+b+c)} = \frac{16}{2 \times 2} = 4$

416 **(a)**

Applying $R_3 \to R_3 - R_2$ and $R_2 \to R_2 - R_1$, we get

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\sin 4\theta \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 2 & \sin^2\theta & 4\sin 4\theta \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{vmatrix} = 0,$$

[Applying $C_1 \to C_1 + C_2$]

$\Rightarrow 2 + 4\sin 4\theta = 0$

$\Rightarrow \sin 4\theta = -\frac{1}{2}$

$\Rightarrow 4\theta = n\pi + (-1)^n\left(-\frac{\pi}{6}\right), n \in Z$

$\Rightarrow \theta = \frac{n\pi}{4} + (-1)^{n+1}\frac{\pi}{24}, n \in Z$

Clearly, $\theta = \frac{7\pi}{24}, \frac{11\pi}{24}$ are two values of θ lying between 0 and $\frac{\pi}{2}$ given by the above relation

417 **(a)**

We have,

$\sqrt{3}\cot 20° - 4\cos 20°$

$= \frac{\sqrt{3}\cot 20°}{\sin 20°} - 4\cos 20°$

$= \frac{\sqrt{3}\cot 20° - 4\sin 20°\cos 20°}{\sin 20°}$

$= \frac{2\sin 60°\cos 20° - 2\sin 40°}{\sin 20°}$

$= \frac{\sin 80° + \sin 40° - 2\sin 40°}{\sin 20°} = \frac{\sin 80° - \sin 40°}{\sin 20°}$

$= \frac{2\cos 60°\sin 20°}{\sin 20°} = 1$

418 **(a)**

We have,

$\cos x\cos 6x = -1$

$\Rightarrow 2\cos x\cos 6x = -2$

$\Rightarrow \cos 7x + \cos 5x = -2 \Rightarrow \cos 7x = -1$ and $\cos 5x = -1$

The value of x satisfying these two equations simultaneously and lying between 0 and 2π is π.

Therefore, the general solution is given by

$x = 2n\pi + \pi, n \in Z \Rightarrow x = (2n+1)\pi, n \in Z$

419 **(c)**

Let $a = 6 + \sqrt{12}, b = \sqrt{48}, c = \sqrt{24}$

Clearly, c is the smallest side. Therefore, the smallest angle C is given by

$\cos C = \frac{a^2 + b^2 - c^2}{2\,ab} = \frac{\sqrt{3}}{2} \Rightarrow C = \frac{\pi}{6}$

420 **(b)**

Given, $\tan 2\theta = \frac{1}{\tan\theta}$

$\Rightarrow\ \tan 2\theta = \tan\left(\frac{\pi}{2} - \theta\right)$

$\Rightarrow\ 2\theta = n\pi + \frac{\pi}{2} - \theta$

$\Rightarrow\ \theta = \frac{\pi}{6}(2n+1)$

421 **(a)**

Given, $\cot(\alpha + \beta) = 0\ \Rightarrow\ \cos(\alpha + \beta) = 0$

$\Rightarrow \alpha + \beta = (2n+1)\frac{\pi}{2}, n \in I$

$\therefore\ \sin(\alpha + 2\beta) = \sin(2\alpha + 2\beta - \alpha)$

$= \sin[(2n+1)\pi - \alpha]$

$= \sin(2n\pi + \pi - \alpha)$

$= \sin(\pi - \alpha) = \sin\alpha$

422 **(c)**

$6(\sin^6\theta + \cos^6\theta) - 9(\sin^4\theta + \cos^4\theta) + 4$

$= 6[(\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)] - 9[(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta] + 4$

$= -6[1 - 3\sin^2\theta\cos^2\theta] - 9(1 - 2\sin^2\theta\cos^2\theta) + 4$

$= 6 - 9 + 4 = 1$

423 **(c)**

We have, $y = \cos^2 x + \sec^2 x$

$\Rightarrow \ y = (\cos x - \sec x)^2 + 2 \geq 2$

$\Rightarrow \ y \geq 2$

424 **(b)**

Since $0 < \sin x < 1$ and $0 < \cos x < 1$ for all $x \in (0, \pi/2)$. Therefore, angle opposite to the side of one unit length is the largest angle and is given by

$\cos\theta = \dfrac{\sin^2 x + \cos^2 x - 1}{2\sin x \cos x} = 0 \Rightarrow \theta = \dfrac{\pi}{2}$

425 **(b)**

In a ΔABC

$A + B + C = \pi$

$\therefore \ \cos\left(\dfrac{B + 2C + 3A}{2}\right) + \cos\left(\dfrac{A - B}{2}\right)$

$= 2\cos\left(\dfrac{2C + 4A}{4}\right)\cos\left(\dfrac{2A + 2B + 2C}{4}\right)$

$= 2\cos\left(\dfrac{C + 2A}{2}\right)\cos\left(\dfrac{\pi}{2}\right) = 0$

426 **(a)**

Given, $\ \cos 2\alpha = \frac{2\cos 2\beta - 1}{3 - \cos 2\beta}$

$\Rightarrow \dfrac{1 - \tan^2\alpha}{1 + \tan^2\alpha} = \dfrac{3\left(\frac{1 - \tan^2\beta}{1 + \tan^2\beta}\right) - 1}{3 - \left(\frac{1 - \tan^2\beta}{1 + \tan^2\beta}\right)}$

$\Rightarrow \dfrac{1 - \tan^2\alpha}{1 + \tan^2\alpha} = \dfrac{2 - 4\tan^2\beta}{2 + 4\tan^2\beta} = \dfrac{1 - 2\tan^2\beta}{1 + 2\tan^2\beta}$

Applying componendo and dividendo, we get

$\dfrac{1}{\tan^2\alpha} = \dfrac{1}{2\tan^2\beta}$

$\Rightarrow \tan\alpha = \sqrt{2}\tan\beta$

427 **(d)**

We have,

$\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3$

$\quad + 3\sin^2\theta\cos^2\theta\,(\sin^2\theta + \cos^2\theta)$

$= (\sin^2\theta + \cos^2\theta)^3 = 1$

428 **(c)**

Putting $\theta = \frac{\pi}{9}$, in $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

We get

$\tan\dfrac{\pi}{3} = \dfrac{3\tan\frac{\pi}{9} - \tan^3\frac{\pi}{9}}{1 - 3\tan^2\frac{\pi}{9}}$

$\Rightarrow 3\left(1 - 3\tan^2\dfrac{\pi}{9}\right)^2 = \left(3\tan\dfrac{\pi}{9} - \tan^3\dfrac{\pi}{9}\right)^2$

$\Rightarrow \tan^6\dfrac{\pi}{9} - 33\tan^4\dfrac{\pi}{9} + 27\tan^2\dfrac{\pi}{9} = 3$

429 **(d)**

Given that,

$\sin x + \cos x = \min_{a \in R}\{1, a^2 - 4a + 6\}$

Now, $\ a^2 - 4a + 6 = (a - 2)^2 + 2$

$\therefore \ \min_{a \in R}\{1, a^2 - 4a + 6\} = \min\{1, 2\} = 1$

$\therefore \ \sin x + \cos = 1$

$\Rightarrow \ \sin\left(x + \dfrac{\pi}{4}\right) = \dfrac{1}{\sqrt{2}}$

$\Rightarrow \ x + \dfrac{\pi}{4} = n\pi + (-1)^n.\dfrac{\pi}{4}$

$\Rightarrow \ x = n\pi + (-1)^n\dfrac{\pi}{4} - \dfrac{\pi}{4}$

430 **(b)**

$\cos\dfrac{\pi}{15}.\cos\dfrac{2\pi}{15}.\cos\dfrac{4\pi}{15}.\cos\dfrac{8\pi}{15}$

$= \dfrac{1}{4}\left(2\cos\dfrac{4\pi}{15}\cos\dfrac{\pi}{15}\right)\left(2\cos\dfrac{8\pi}{15}\cos\dfrac{2\pi}{15}\right)$

$= \dfrac{1}{4}(\cos 60° + \cos 36°)(\cos 120° + \cos 72°)$

$= \dfrac{1}{4}\left(\dfrac{1}{2} + \dfrac{\sqrt{5} + 1}{4}\right)\left(-\dfrac{1}{2} + \dfrac{\sqrt{5} - 1}{4}\right)$

$= \dfrac{1}{4}\left[-\dfrac{1}{4} + \dfrac{1}{2}\left(\dfrac{\sqrt{5} - 1}{4} - \dfrac{\sqrt{5} + 1}{4}\right) + \dfrac{5 - 1}{16}\right] = -\dfrac{1}{16}$

432 **(b)**

Since $\sec\alpha$ and $\operatorname{cosec}\alpha$ are the roots of the equation

$x^2 - ax + b = 0$

$\therefore \sec\alpha + \operatorname{cosec}\alpha = a$ and $\sec\alpha\ \operatorname{cosec}\alpha = b$

$\Rightarrow \sin\alpha + \cos\alpha = a\sin\alpha\cos\alpha$ and $\sin\alpha\cos\alpha = \frac{1}{b}$

$\Rightarrow \sin\alpha + \cos\alpha = \frac{a}{b}$ and $\sin\alpha\cos\alpha = \frac{1}{b}$

Now,

$(\sin\alpha + \cos\alpha)^2 = 1 + 2\sin\alpha\cos\alpha$

$\Rightarrow \dfrac{a^2}{b^2} = 1 + \dfrac{2}{3} \Rightarrow a^2 = b(b + 2)$

433 **(a)**

Since, $1 + \sin x\sin^2\frac{x}{2} = 0$

$\therefore \ 1 + \sin x\left(\dfrac{1 - \cos x}{2}\right) = 0$

$\Rightarrow \ 2 + \sin x - \sin x\cos x = 0$

$\Rightarrow \ \sin 2x - 2\sin x = 4$

Which is not possible for any x in $[-\pi, \pi]$

434 **(b)**

Given, $\sin x + \sin 5x = \sin 3x$

$\Rightarrow \ 2\sin 3x + \cos 2x = \sin 3x$

$\Rightarrow \ \sin 3x(2\cos 2x - 1) = 0$

$\Rightarrow \ \sin 3x = 0$

Or $\ 2\cos 2x - 1 = 0$

$\Rightarrow \ 3x = 0, \pi \ $ or $2x = \frac{\pi}{3}$

$\Rightarrow \ x = 0, \ x = \frac{\pi}{3}$ or $x = \frac{\pi}{6}$

$\therefore$ Solutions in $\left(0, \frac{\pi}{2}\right)$ are $\frac{\pi}{3}, \frac{\pi}{6}$

435 **(a)**

We have,

H.M. of ex-radii

$$= \frac{3}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}} = \frac{3\,\Delta}{s-a+s-b+s-c} = \frac{3\,\Delta}{s} = 3r$$

436 **(c)**

We have,

$$3(\sin x - \cos x)^4 + 4(\sin^6 x + \cos^6 x) + 6(\sin x + \cos x)^2$$

$$= 3\{(\sin x - \cos x)^2\}^2 + 4\{(\sin^2 x)^3 + (\cos^2 x)^3\} + 6(1 + \sin 2x)$$

$$= 3(1 - \sin 2x)^2 + 4(\sin^4 x + \cos^4 x - \sin^2 x \cos^2 x) + 6(1 + \sin 2x)$$

$$= 3(1 - 2\sin 2x + \sin^2 2x) + 4\left(1 - \frac{3}{4}\sin^2 2x\right) + 6(1 + \sin 2x)$$

437 **(a)**

Since, $4\cos\theta - 3\sec\theta = 2\tan\theta$

$\Rightarrow\ 4\cos\theta - \dfrac{3}{\cos\theta} = 2\dfrac{\sin\theta}{\cos\theta}$

$\Rightarrow\ 4\cos^2\theta - 3 = 2\sin\theta$

$\Rightarrow\ 4 - 4\sin^2\theta - 3 = 2\sin\theta$

$\Rightarrow\ 4\sin^2\theta + 2\sin\theta - 1 = 0$

$\Rightarrow\ \sin\theta = \dfrac{-1 \pm \sqrt{5}}{4}$

Either $\sin\theta = \frac{-1+\sqrt{5}}{4}$ or $\sin\theta = \frac{-1-\sqrt{5}}{4}$

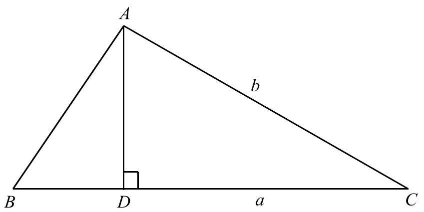
$\Rightarrow\ \sin\theta = \sin\dfrac{\pi}{10}$ or $\sin\theta = \sin\left(-\dfrac{3\pi}{10}\right)$

$\Rightarrow\ \theta = n\pi + (-1)^n\dfrac{\pi}{10}$ or $\theta = n\pi + (-1)^n\left(-\dfrac{3\pi}{10}\right)$

438 **(b)**

In ΔADC, we have

$\sin C = \dfrac{AD}{b} \Rightarrow AD = b\sin C$



Also, $AD = \dfrac{abc}{b^2 - c^2}$ [Given]

$\therefore \dfrac{abc}{b^2 - c^2} = b\sin C$

$\Rightarrow \dfrac{ac}{b^2 - c^2} = \sin C$

$\Rightarrow \dfrac{\sin A \sin C}{\sin^2 B - \sin^2 C} = \sin C$ [Using Sine rule]

$\Rightarrow \dfrac{\sin A \sin C}{\sin(B + C)\sin(B - C)} = \sin C$

$\Rightarrow \sin(B - C) = 1 \Rightarrow B - C = 90°$

$\Rightarrow B = 90° + C \Rightarrow B = 113°$

439 **(c)**

It is given that

$\cos(x - y)$, $\cos x$ and $\cos(x + y)$ are in H.P.

$\therefore \dfrac{2}{\cos x} = \dfrac{1}{\cos(x - y)} + \dfrac{1}{\cos(x + y)}$

$\Rightarrow \dfrac{2}{\cos x} = \dfrac{2\cos x \cos y}{\cos^2 x - \sin^2 y}$

$\Rightarrow \cos^2 x \cos y = \cos^2 x - \sin^2 y$

$\Rightarrow \cos^2 x\,(1 - \cos y) = \sin^2 y$

$\Rightarrow 2\cos^2 x \sin^2\dfrac{y}{2} = 4\sin^2\dfrac{y}{2}\cos^2\dfrac{y}{2}$

$\Rightarrow \cos^2 x \sec^2\dfrac{y}{2} = 2$

$\Rightarrow \left|\cos x \sec\dfrac{y}{2}\right| = \sqrt{2}$

440 **(b)**

We have,

$-5 \le 3\sin\theta - 4\cos\theta \le 5$ for all θ

$\Rightarrow 2 \le 3\sin\theta - 4\cos\theta + 7 \le 12$ for all θ

$\Rightarrow \frac{1}{12} \le \frac{1}{3\sin\theta - 4\cos\theta + 7} \le \frac{1}{2}$ for all θ

441 **(d)**

We have,

$\dfrac{a^2 + 1}{2a} = \cos\theta$

$\Rightarrow a + \dfrac{1}{a} = 2\cos\theta$

$\Rightarrow \left(a + \dfrac{1}{a}\right)^3 = 8\cos^3\theta$

$\Rightarrow a^3 + \dfrac{1}{a^3} + 3\left(a + \dfrac{1}{a}\right) = 8\cos^3\theta$

$\Rightarrow a^3 + \dfrac{1}{a^3} + 6\cos\theta = 8\cos^3\theta$

$\Rightarrow \dfrac{a^6 + 1}{2a^3} = 4\cos^3\theta - 3\cos\theta \Rightarrow \dfrac{a^6 + 1}{2a^3} = \cos 3\theta$

442 **(a)**

$2\cos x - \cos 3x - \cos 5x = 2\cos x - 2\cos x \cos 4x$

$= 2\cos x\,(1 - \cos 4x)$

$= 2\cos x\, 2\sin^2 2x$

$= 4\cos x(2\sin x \cos x)^2$

$= 16\sin^2 x \cos^3 x$

443 **(d)**

We have,

$\tan\left(\dfrac{\alpha\pi}{4}\right) = \cot\left(\dfrac{\beta\pi}{4}\right)$

$\Rightarrow \tan\left(\dfrac{\alpha\pi}{4}\right) = \tan\left(\dfrac{\pi}{2} - \dfrac{\beta\pi}{4}\right)$

$\Rightarrow \alpha\dfrac{\pi}{4} = n\pi + \left(\dfrac{\pi}{2} - \beta\dfrac{\pi}{4}\right)$

$\Rightarrow \alpha = 2(2n+1) - \beta \Rightarrow \alpha + \beta = 2(2n+1)$

444 **(d)**

$2\sin x\cos x = \dfrac{1}{2}$

$\Rightarrow \sin 2x = \dfrac{1}{2} = \sin\dfrac{\pi}{6}$

$\Rightarrow 2x = n\pi + (-1)^n\dfrac{\pi}{6}$

$\Rightarrow x = \dfrac{n\pi}{2} + (-1)^n\dfrac{\pi}{12}$

For $x \in \left(0, \frac{\pi}{2}\right)$

$x = \frac{\pi}{12} \qquad (n = 0)$

445 **(c)**

Given that, $\sin\theta + \operatorname{cosec}\theta = 2$

On squaring both sides, we get

$\sin^2\theta + \operatorname{cosec}^2\theta + 2 = 4$

$\Rightarrow \sin^2\theta + \operatorname{cosec}^2\theta = 2$

446 **(d)**

$\dfrac{\cos C - \cos A}{\sin A - \sin C} = \dfrac{2\sin\left(\frac{A+C}{2}\right)\sin\left(\frac{A-C}{2}\right)}{2\cos\left(\frac{A+C}{2}\right)\sin\left(\frac{A-C}{2}\right)}$

$= \dfrac{2\sin B}{2\cos B} = \tan B \quad [\because A + C = 2B, \text{given}]$

447 **(c)**

We have, $BD = DC$ and $\angle DAB = 90°$

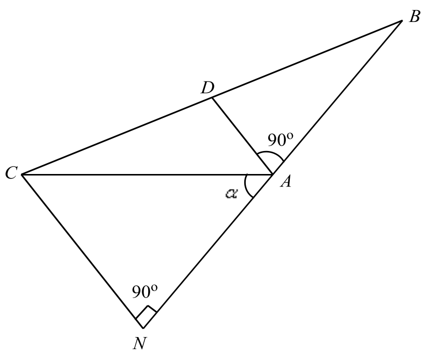
Draw $CN \perp$ to BA produced. Then, in ΔBCN, we have

$DA = \dfrac{1}{2}CN$ and $AB = AN$

Let $\angle CAN = \alpha$

$\therefore \tan A = \tan(\pi - \alpha)$

$\Rightarrow \tan A = -\tan\alpha$



$\Rightarrow \tan A = -\dfrac{CN}{NA} = -2\dfrac{AD}{AB} = -2\tan B$

$\Rightarrow \tan A + 2\tan B = 0$

448 **(c)**

We have,

$a = (b-c)\sec\theta$

$\Rightarrow a^2 = (b-c)^2\sec^2\theta$

$\Rightarrow b^2 + c^2 - 2bc\cos A = (b^2 + c^2 - 2bc)(1 + \tan^2\theta)$

$\Rightarrow 2bc(1 - \cos A) = (b^2 + c^2 - 2bc)\tan^2\theta$

$\Rightarrow 4bc\sin^2\dfrac{A}{2} = (b^2 + c^2 - 2bc)\tan^2\theta$

$\Rightarrow \dfrac{4bc\sin^2\frac{A}{2}}{(b-c)^2} = \tan^2\theta \Rightarrow \dfrac{2\sqrt{bc}}{b-c}\sin\dfrac{A}{2} = \tan\theta$

449 **(b)**

We have,

$\cot^2 36°\cot^2 72°$

$= \dfrac{\cos^2 36°\cos^2 72°}{\sin^2 36°\sin^2 72°}$

$= \dfrac{(1+\cos 72°)(1+\cos 144°)}{(1-\cos 72°)(1-\cos 144°)}$

$= \dfrac{(1+\cos 72°)(1-\cos 36°)}{(1-\cos 72°)(1+\cos 36°)}$

$= \dfrac{1+\cos 72° - \cos 36° - \cos 72°\cos 36°}{1-\cos 72° + \cos 36° - \cos 72°\cos 36°}$

$= \dfrac{1-(\cos 36° - \cos 72°) - \cos 72°\cos 36°}{1+\cos 36° - \cos 72° - \cos 72°\cos 36°}$

$= \dfrac{1-\frac{1}{2}-\frac{1}{4}}{1+\frac{1}{2}-\frac{1}{4}} = \dfrac{1}{5} \quad \left[\begin{array}{l}\because \cos 36° - \cos 72° = \frac{1}{2} \\ \text{and}, \cos 36°\cos 72° = \frac{1}{4}\end{array}\right]$

$\Rightarrow \cot^2 36°\cot^2 72° = \dfrac{1}{5} \Rightarrow \cot 36°\cot 72° = \dfrac{1}{\sqrt{5}}$

450 **(a)**

$\tan\theta = \dfrac{4}{5}$

$\therefore \sin\theta = \dfrac{4}{\sqrt{41}}, \cos\theta = \dfrac{5}{\sqrt{41}}$

Now, $\frac{5\sin\theta - 3\cos\theta}{\sin\theta + 2\cos\theta} = \frac{5\times\frac{4}{\sqrt{41}} - \frac{3\times 5}{\sqrt{41}}}{\frac{4}{\sqrt{41}} + 2\times\frac{5}{\sqrt{41}}} = \frac{5}{14}$

451 **(a)**

(1) $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= \dfrac{\frac{m}{m+1} + \frac{1}{2m+1}}{1 - \frac{m}{m+1}\cdot\frac{1}{2m+1}}$

$= \dfrac{2m^2 + 2m + 1}{2m^2 + 2m + 1} = 1$

$\Rightarrow \alpha + \beta = \dfrac{\pi}{4}$

(2) At $\theta = \frac{\pi}{4}$,

LHS $= 3\tan(45° - 15°) = 3\tan 30° = \sqrt{3}$

RHS $= \tan(45° + 15°) = \tan 60° = \sqrt{3}$

$\therefore$ LHS = RHS

(3) Given $\sin^2 ax - \sin^2(a-1)x = \sin^2 x$

$\Rightarrow \sin(2a-1)x\sin(x) = \sin^2 x$

$\Rightarrow \sin x = 0$ and $\sin(2a-1)x = \sin x$

$\Rightarrow x = n\pi$ and $(2a-1)x = n\pi + (-1)^n x$

Hence, option (a) is correct

452 **(a)**

We have,

$\tan A \tan B = 2$ ...(i)

and, $\cos(A - B) = \frac{3}{5}$ ...(ii)

From (ii), we have

$\Rightarrow \tan(A - B) = \frac{4}{3}$

$\Rightarrow \frac{\tan A - \tan B}{1 + \tan A \tan B} = \frac{4}{3}$

$\Rightarrow \frac{\tan A - \tan B}{1+2} = \frac{4}{3}$ [Using (i)]

$\Rightarrow \tan A - \tan B = 4$ ...(iii)

$\Rightarrow (\tan A + \tan B)^2 = 16 + 8$

$\Rightarrow \tan A + \tan B = 2\sqrt{6}$ ...(iv)

From (iii) and (iv), we get

$\tan A = 2 + \sqrt{6}, \tan B = \sqrt{6} - 2$

$\Rightarrow \cos A = \frac{1}{\sqrt{1+4\sqrt{6}}}$ and, $\sin A = \frac{\sqrt{6}-2}{\sqrt{11+4\sqrt{6}}}$

$\cos B = \frac{1}{\sqrt{11-4\sqrt{6}}}$ and, $\sin B = \frac{\sqrt{6}-2}{\sqrt{11-4\sqrt{6}}}$

$\Rightarrow \cos A \cos B = \frac{1}{5}$ and, $\sin A \sin B = \frac{2}{5}$

$\Rightarrow \cos(A + B) = -\frac{1}{5}$

453 **(b)**

Given that, $\sin \theta = -\frac{4}{5}$ and θ lies in the IIIrd quadrant

$\Rightarrow \cos \theta - \sqrt{1 - \frac{16}{25}} = -\frac{3}{5}$

Now, $\cos \frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} = \pm\sqrt{\frac{1-\frac{3}{5}}{2}} = \pm\sqrt{\frac{1}{5}}$

But we take

$\cos \frac{\theta}{2} =$

-15.Since, if θ lies in IIIrd quadrant, then θ2 will be in IInd quadrant

Hence, $\cos \frac{\theta}{2} = -\frac{1}{\sqrt{5}}$

454 **(a)**

On squaring an adding given equations, we get

$(\sin \alpha + \sin \beta)^2 + (\cos \alpha + \cos \beta)^2$

$$= \left(-\frac{21}{65}\right)^2 + \left(-\frac{27}{65}\right)^2$$

$$\Rightarrow \sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta + \cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta = \frac{1170}{4225}$$

$$\Rightarrow 2 + 2\cos(\alpha - \beta) = \frac{1170}{4225}$$

$$\Rightarrow 2\left[2\cos^2\left(\frac{\alpha - \beta}{2}\right)\right] = \frac{1170}{4225}$$

$$\Rightarrow \cos^2\left(\frac{\alpha - \beta}{2}\right) = \frac{1170}{4 \times 4225} = \frac{9}{130}$$

$$\Rightarrow \cos\left(\frac{\alpha - \beta}{2}\right) = -\frac{3}{\sqrt{130}} \quad [\because \pi < \alpha - \beta < 3\pi]$$

455 **(d)**

We have,

$\sin x + \sin^2 x = 1 \Rightarrow \sin x = 1 - \sin^2 x \Rightarrow \sin x = \cos^2 x$

$\therefore \cos^8 x + 2\cos^6 x + \cos^4 x$

$= \sin^4 x + 2\sin^3 x + \sin^2 x = (\sin x + \sin^2 x)^2 = 1$

456 **(a)**

We have,

$\cos(\alpha + \beta)\sin(\gamma + \delta) = \cos(\alpha - \beta)\sin(\gamma - \delta)$

$$\Rightarrow \frac{\cos(\alpha + \beta)}{\sin(\alpha - \beta)} = \frac{\sin(\gamma - \delta)}{\sin(\gamma + \delta)}$$

$$\Rightarrow \frac{\cos(\alpha + \beta) + \cos(\alpha - \beta)}{\cos(\alpha + \beta) - \cos(\alpha - \beta)} = \frac{\sin(\gamma - \delta) + \sin(\gamma + \delta)}{\sin(\gamma - \delta) - \sin(\gamma + \delta)}$$

$$\Rightarrow \frac{2\cos\alpha\cos\beta}{-2\sin\alpha\sin\beta} = \frac{2\sin\gamma\cos\delta}{-2\sin\delta\cos\gamma}$$

$\Rightarrow \cot\alpha\cot\beta = \tan\gamma\cot\delta$

$\Rightarrow \cot\alpha\cot\beta\cot\gamma = \cot\delta$

457 **(c)**

We have,

$AD^2 + BE^2 + CF^2$

$= \frac{1}{4}(2b^2 + 2c^2 - a^2 + 2c^2 + 2a^2 - b^2 + 2a^2 + 2b^2 - c^2)$

$= \frac{3}{4}(a^2 + b^2 + c^2) = \frac{3}{4}(BC^2 + CA^2 + AB^2)$

$\therefore (AD^2 + BE^2 + CF^2) : (BC^2 + CA^2 + AB^2) = 3 : 4$

458 **(c)**

The given equation is

$a\cos\theta + b\sin\theta = c$

Since, $\sqrt{a^2 - b^2} \leq a\cos\theta + b\sin\theta \leq \sqrt{a^2 + b^2}$

$\Rightarrow c^2 \leq a^2 + b^2$

459 **(d)**

Given, $\tan\left(\frac{\alpha\pi}{4}\right) = \tan\left(\frac{\pi}{2} - \frac{\beta\pi}{4}\right)$

$$\Rightarrow \frac{\alpha\pi}{4} = n\pi + \frac{\pi}{2} - \frac{\beta\pi}{4}$$

$$\Rightarrow (\alpha + \beta)\frac{\pi}{4} = \left(\frac{2n+1}{2}\right)\pi$$

$$\Rightarrow \alpha + \beta = 2(2n + 1), \forall n \in I$$

460 **(d)**

Given, $\frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} = x$

$\Rightarrow \dfrac{2\sin\alpha(1-\cos\alpha-\sin\alpha)}{(1+\cos\alpha+\sin\alpha)\,(1-\cos\alpha-\sin\alpha)}=x$

$\Rightarrow \dfrac{2\sin\alpha\,(1-\cos\alpha-\sin\alpha)}{1-\sin^2\alpha-\cos^2\alpha-2\sin\alpha\cos\alpha}=x$

$\Rightarrow \dfrac{1-\cos\alpha-\sin\alpha}{\cos\alpha}=-x$

461 **(a)**

Let r be the radius of the circle and A_1 be its area. Then, $A_1=\pi\,r^2$

Since the perimeter of the circle is same as the perimeter of a regular polygon of n sides

$\therefore 2\,\pi\,r=n\,a$, when $'a'$ is the length of one side of the regular polygon

$\Rightarrow a=\dfrac{2\,\pi\,r}{n}$

Let A_2 be the area of the polygon. Then,

$A_2=\dfrac{1}{4}\,\pi\,a^2\cot\left(\dfrac{\pi}{n}\right)=\dfrac{\pi^2r^2}{n}\cot\left(\dfrac{\pi}{n}\right)$

$\therefore A_1:A_2=\pi\,r^2:\dfrac{\pi^2\,r^2}{n}\cot\left(\dfrac{\pi}{n}\right)=\tan\left(\dfrac{\pi}{n}\right):\dfrac{\pi}{n}$

462 **(a)**

We have

$\dfrac{a\cos A+b\cos B+c\cos C}{a+b+c}$

$=\dfrac{R(\sin 2A+\sin 2B+\sin 2C)}{2R(\sin A+\sin B+\sin C)}$

$=\dfrac{4\sin A\sin B\sin C}{2\left(4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}\right)}=4\sin\dfrac{A}{2}\sin\dfrac{B}{2}\sin\dfrac{C}{2}=\dfrac{r}{R}$

464 **(b)**

We have,

$\tan(\cot x)=\cot(\tan x)$

$\Rightarrow \tan(\cot x)=\tan\left(\dfrac{\pi}{2}-\tan x\right)$

$\Rightarrow \cot x=n\pi+\dfrac{\pi}{2}-\tan x\,,n\in Z$

$\Rightarrow \cot x+\tan x=n\pi+\dfrac{\pi}{2}$

$\Rightarrow \dfrac{1}{\sin x\cos x}=(2n+1)\dfrac{\pi}{2}$

$\Rightarrow \sin 2x=\dfrac{4}{(2n+1)\pi},n\in Z$

465 **(a)**

Given that, $2\sin A=\sqrt{3}\sin B$

$\Rightarrow 2\sqrt{5}\sin A=\sqrt{15}\sin B$ ...(i)

and $2\cos A=\sqrt{5}\cos B$

$\Rightarrow 2\sqrt{3}\cos A=\sqrt{15}\cos B$ ...(ii)

On squaring and adding Eqs. (i) and (ii), we get

$20\sin^2 A+12\cos^2 A=15$

$\Rightarrow 8\sin^2 A=3\ \Rightarrow \sin^2 A=\dfrac{3}{8}$

$\Rightarrow \cos^2 A=\dfrac{5}{8}$

$\therefore \dfrac{\sin^2 A}{\cos^2 A}=\dfrac{3}{5}\Rightarrow \tan A=\sqrt{\dfrac{3}{5}}$

466 **(c)**

We have,

$\cos^2(A-B)+\cos^2 B-2\cos(A-B)\cos A\cos B$

$=\cos^2(A-B)+\cos^2 B$

$\quad -\cos(A-B)$

$\quad \times[\cos(A-B)+\cos(A+B)]$

$=\cos^2 B-\cos(A-B)\cos(A+B)$

$=\cos^2 B-(\cos^2 A-\sin^2 B)$

$=1-\cos^2 A=\sin^2 A$

Hence, it depends on A

467 **(d)**

We have,

$r_1 = 4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}, b = 2R\sin B$ and $c = 2R\sin C$

$\therefore \frac{r_1}{bc} = \frac{4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}{4R^2\sin B\sin C}$

$\Rightarrow \frac{r_1}{bc} = \frac{\sin\frac{A}{2}}{4R\sin\frac{B}{2}\sin\frac{C}{2}} \Rightarrow \frac{r_1}{bc} = \frac{\sin^2\frac{A}{2}}{4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}$

$\Rightarrow \frac{r_1}{bc} = \frac{\sin^2\frac{A}{2}}{r}$

Similarly, $\frac{r_2}{ca} = \frac{\sin^2\frac{B}{2}}{r}$ and $\frac{r_3}{ab} = \frac{\sin^2\frac{C}{2}}{r}$

$\therefore \frac{r_1}{bc} + \frac{r_2}{ca} + \frac{r_3}{ab} = \frac{1}{r}\left\{\sin^2\frac{A}{2} + \sin^2\frac{B}{2} + \sin^2\frac{C}{2}\right\}$

$\Rightarrow \frac{r_1}{bc} + \frac{r_2}{ca} + \frac{r_3}{ab} = \frac{1}{2r}\{1 - \cos A + 1 - \cos B + 1 - \cos C\}$

$\Rightarrow \frac{r_1}{bc} + \frac{r_2}{ca} + \frac{r_3}{ab} = \frac{1}{2r}\{3 - (\cos A + \cos B + \cos C)\}$

$\Rightarrow \frac{r_1}{bc} + \frac{r_2}{ca} + \frac{r_3}{ab} = \frac{1}{2r}\left\{3 - \left(1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)\right\}$

$\Rightarrow \frac{r_1}{bc} + \frac{r_2}{ca} + \frac{r_3}{ab} = \frac{1}{2r}\left(2 - \frac{r}{R}\right) = \frac{1}{r} - \frac{1}{2R}$

468 **(a)**

We have,

$a = \sin A + \sin B, b = \cos A + \cos B$

$\Rightarrow a^2 + b^2 = 2 + 2\cos(A - B)$ ...(i)

and,

$b^2 - a^2 = \cos 2A + \cos 2B + 2\cos(A + B)$

$\Rightarrow b^2 - a^2 = 2\cos(A + B)\{\cos(A - B) + 1\}$

$\Rightarrow b^2 - a^2 = 2\cos(A + B)\left(\frac{a^2+b^2}{2}\right)$ [Using (i)]

$\Rightarrow \cos(A + B) = \frac{b^2 - a^2}{a^2 + b^2}$

469 **(c)**

Clearly, the given equation is not meaningful at odd multiples of $\frac{\pi}{2}$

We have,

$\tan x + \sec x = 2\cos x$

$\Rightarrow 1 + \sin x = 2(1 - \sin^2 x)$

$\Rightarrow 2\sin^2 x + \sin x - 1 = 0$

$\Rightarrow \sin x = \frac{1}{2}, -1 \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$

470 **(c)**

We have,

$A + B + C = \pi \Rightarrow nA + nB + nC = n\pi$

$\therefore \tan(nA + nB + nC) = \tan n\pi \Rightarrow \frac{S_1 - S_3}{1 - S_2} = 0$

$\Rightarrow S_1 = S_3$

$\Rightarrow \tan nA + \tan nB + \tan nC = \tan nA \tan nB \tan nC$

471 **(a)**

We know that

$IA = \frac{r}{\sin A/2}, IB = \frac{r}{\sin B/2}$ and $IC = \frac{r}{\sin C/2}$

$\therefore IA : IB : IC = \text{cosec}\frac{A}{2} : \text{cosec}\frac{B}{2} : \text{cosec}\frac{C}{2}$

472 **(b)**

We have,

$\sin\frac{13\pi}{14} = \sin\left(\pi - \frac{\pi}{14}\right) = \sin\frac{\pi}{14}$

$\sin\frac{11\pi}{14} = \sin\left(\pi - \frac{3\pi}{14}\right) = \sin\frac{3\pi}{14}$

$\sin\frac{9\pi}{14} = \sin\left(\pi - \frac{5\pi}{14}\right) = \sin\frac{5\pi}{14}\sin\frac{7\pi}{14} = \sin\frac{\pi}{2} = 1$

$\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$

$= \left\{\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\right\}^2$

$= \left\{\cos\left(\frac{\pi}{2} - \frac{\pi}{14}\right)\cos\left(\frac{\pi}{2} - \frac{3\pi}{2}\right)\cos\left(\frac{\pi}{2} - \frac{5\pi}{14}\right)\right\}^2$

$= \left\{\cos\frac{6\pi}{14}\cos\frac{4\pi}{14}\cos\frac{2\pi}{14}\right\}^2$

$= \left\{\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{3\pi}{7}\right\}^2$

$= \left\{-\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\right\}^2$

$= \left\{\frac{-\sin(2^3\pi/7)}{2^3\sin\pi/7}\right\}^2 = \left\{\frac{-\sin 8\pi/7}{8\sin\pi/7}\right\}^2 = \left(\frac{1}{8}\right)^2 = \frac{1}{64}$

473 **(b)**

The given equation can be written as

$(\sin\theta + \sqrt{3})\tan\theta = 0$

$\Rightarrow \tan\theta = 0 \Rightarrow \theta = n\pi, \quad n \in Z$

474 **(b)**

We have,

$2a = \sqrt{3}b + c$

$\Rightarrow 2\sin A = \sqrt{3}\sin B + \sin C \quad \left[\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}\right]$

$\Rightarrow 2\sin(B + C) = \sqrt{3}\sin B + \sin C$

$\Rightarrow \sin B\cos C + \cos B\sin C = \frac{\sqrt{3}}{2}\sin B + \frac{1}{2}\sin C$

$\Rightarrow \cos C = \frac{\sqrt{3}}{2}$ and $\cos B = \frac{1}{2}$ [By comparing two sides]

$\Rightarrow C = \frac{\pi}{6}$ and $B = \frac{\pi}{3} \Rightarrow A = \frac{\pi}{2} \Rightarrow a^2 = b^2 + c^2$

475 **(a)**

We have,

$\tan 89° = \tan(90° - 1°) = \cot 1°$

$\tan 88° = \tan(90° - 2°) = \cot 2°$ etc

$\tan 1° \tan 2° \tan 3° \dots \tan 88° \tan 89°$

$= \tan 1° \tan 2° \tan 3° \dots . \tan 45° \cot 44° \cot 43° \dots$ co

$= 1$

476 **(b)**

We have,

$b \cos 2\theta + a \sin 2\theta$

$= b\left(\frac{1 - \tan^2\theta}{1 + \tan^2\theta}\right) + \frac{2a \tan\theta}{1 + \tan^2\theta}$

$= b\frac{(1 - a^2/b^2)}{(1 + a^2/b^2)} + \frac{2a(a/b)}{1 + (a^2/b^2)}$

$= \frac{b(b^2 - a^2) + 2a^2 b}{a^2 + b^2} = b$

477 **(b)**

Given, $|\sin x| = \cos x$

$\therefore \quad \sin^2 x = \cos^2 x$

$\Rightarrow \quad 2\cos^2 x = 1$

$\Rightarrow \quad \cos x = +\frac{1}{\sqrt{2}}$ $\quad$ [$\because \cos x$ cannot be negative]

$\Rightarrow \quad x = 2n\pi \pm \frac{\pi}{4}$

478 **(b)**

We have,

$3\left\{\sin^4\left(\frac{3\pi}{2} - \alpha\right) + \sin^4(3\pi - \alpha)\right\}$

$- 2\left\{\sin^6\left(\frac{\pi}{2} + \alpha\right) + \sin^6(5\pi - \alpha)\right\}$

$= 3[\cos^4\alpha + \sin^4\alpha] - 2[\cos^6\alpha + \sin^6\alpha]$

$= 3[1 - 2\sin^2\alpha\cos^2\alpha] - 2[1 - 3\sin^2\alpha\cos^2\alpha]$

$= 3 - 6\sin^2\alpha\cos^2\alpha - 2 + 6\sin^2\alpha\cos^2\alpha = 1$

479 **(d)**

We have,

$4\sin\theta\cos\theta - 2\cos\theta - 2\sqrt{3}\sin\theta + \sqrt{3} = 0$

$\Rightarrow 2\sin\theta(2\cos\theta - \sqrt{3}) - 1(2\cos\theta - \sqrt{3}) = 0$

$\Rightarrow (2\cos\theta - \sqrt{3})(2\sin\theta - 1) = 0$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$ or, $\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$

480 **(a)**

We have,

$\sum a^3 \cos(B - C)$

$= \sum k^3 \sin^3 A \cos(B - C)$

$= k^3 \sum \sin^2 A \sin(B + C)\cos(B - C)$

$= \frac{k^3}{2}\sum \sin^2 A(\sin 2B + \sin 2C)$

$= \frac{k^3}{2}\sum[\sin^2 A(\sin 2B + \sin 2C)$

$+ \sin^2 B(\sin 2C + \sin 2A)$

$+ \sin^2 C(\sin 2A + \sin 2B)$

$= k^3 \sum[\sin^2 A \sin B \cos B + \sin^2 B \sin A \cos A]$

$= k^3 \sum \sin A \sin B \sin(A + B)$

$= k^3[\sin A \sin B \sin C + \sin B \sin C \sin A$

$+ \sin C \sin A \sin B]$

$= 3(k \sin A)(k \sin B)(k \sin C) = 3\,abc$

481 **(a)**

$\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$

$\Rightarrow \sin 6\theta + \sin 2\theta + \sin 4\theta = 0$

$\Rightarrow 2\sin 4\theta\cos 2\theta + \sin 4\theta = 0$

$\Rightarrow \sin 4\theta(2\cos 2\theta + 1) = 0$

$\Rightarrow 2\cos 2\theta = -1 \Rightarrow \cos 2\theta = -\frac{1}{2}$

$\Rightarrow \cos 2\theta = \cos\frac{2\pi}{3}$

$\Rightarrow 2\theta = 2n\pi \pm \frac{2\pi}{3} \Rightarrow \theta = n\pi \pm \frac{\pi}{3}$

and $\sin 4\theta = 0 \Rightarrow 4\theta = n\pi \Rightarrow \theta = \frac{n\pi}{4}$

$\Rightarrow \theta = \frac{n\pi}{4}$ or $n\pi \pm \frac{\pi}{3}$

482 **(c)**

Now, on taking option one by one, we get

(a)$\sin 15° = \sin(45° - 30°) = \frac{\sqrt{3}-1}{2\sqrt{2}}$ = irrational

(b)$\cos 15° = \cos(45° - 30°) = \frac{\sqrt{3}+1}{2\sqrt{2}}$ = irrational

(c)$\sin 15°\cos 15° = \frac{1}{2}(2\sin 15°\cos 15°)$

$= \frac{1}{2}\sin 30° = \frac{1}{4}$ = rational

(d)$\sin 15°\cos 75° = \sin 15°\sin 15° = \sin^2 15°$

$= \left(\frac{\sqrt{3} - 1}{2\sqrt{2}}\right)^2 = \frac{4 - 2\sqrt{3}}{8}$ = irrational

483 **(d)**

We have,

$\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8}{15}\cos\frac{14\pi}{15}$

$= -\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}$

$= -\frac{\sin\left(2^4.\frac{\pi}{15}\right)}{2^4\sin\frac{\pi}{15}} = -\frac{\sin\frac{16\pi}{15}}{16\sin\frac{\pi}{15}} = -\frac{\sin\frac{16\pi}{15}}{16\sin\frac{\pi}{15}} = \frac{1}{16}$

484 **(c)**

We have,

$$\tan\theta\tan(120°-\theta)\tan(120°+\theta)=\frac{1}{\sqrt{3}}$$
$$\Rightarrow \tan\theta\tan(60°+\theta)\tan(60°-\theta)=\frac{1}{\sqrt{3}}$$
$[\because \tan\theta\tan(60°-\theta)\tan(60°+\theta)=\tan 3\theta]$
$$\Rightarrow \tan 3\theta=\frac{1}{\sqrt{3}}$$
$$\Rightarrow \tan 3\theta=\tan\frac{\pi}{6}$$
$$\Rightarrow 3\theta=n\pi+\frac{\pi}{6}, n\in Z \Rightarrow \theta=\frac{n\pi}{3}+\frac{\pi}{18}, n\in Z$$

485 **(a)**

$$\cos\left(x+\frac{\pi}{6}\right)+\sin\left(x+\frac{\pi}{6}\right)$$
$$=\sqrt{2}\left[\frac{1}{\sqrt{2}}\cos\left(x+\frac{\pi}{6}\right)+\frac{1}{\sqrt{2}}\sin\left(x+\frac{\pi}{6}\right)\right]$$
$$=\sqrt{2}\cos\left(x+\frac{\pi}{6}-\frac{\pi}{4}\right)=\sqrt{2}\cos\left(x-\frac{\pi}{12}\right)$$
$\therefore$ For maximum value $x=\frac{\pi}{12}$

486 **(a)**

We have, $\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$$=\frac{\frac{1}{\sqrt{x(x^2+x+1)}}+\frac{\sqrt{x}}{\sqrt{x^2+x+1}}}{1-\frac{1}{\sqrt{x(x^2+x+1)}}\cdot\frac{\sqrt{x}}{\sqrt{x^2+x+1}}}$$

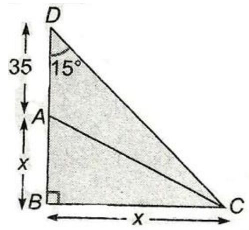
$$=\frac{(1+x)\sqrt{x^2+x+1}}{\sqrt{x}.x(x+1)}$$

$$=\sqrt{x^{-3}+x^{-2}+x^{-1}}=\tan\gamma \text{ (given)}$$

$\therefore\ \alpha+\beta=\gamma$

487 **(a)**

In ΔBCD, $\tan 15°=\frac{BC}{BD}$
$$\Rightarrow \frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}=\frac{x}{x+35}$$
$$\Rightarrow \frac{\sqrt{3}-1}{\sqrt{3}+1}=\frac{x}{x+35}$$
$$\Rightarrow \sqrt{3}x+35\sqrt{3}-x-35=\sqrt{3}x+x$$
$$\Rightarrow x=\frac{35(\sqrt{3}-1)}{2}$$
$$\therefore CD=\sqrt{\left(\frac{35}{2}\right)^2\{(\sqrt{3}+1)^2+(\sqrt{3}-1)^2\}}$$
$$=\frac{35}{2}\times 2\sqrt{2}=35\sqrt{2}\text{ cm}$$



488 **(b)**

$\tan\alpha\tan 2\alpha\ldots\tan(2n-1)\alpha\tan(2n-1)\alpha$

$$=\{\tan\alpha\tan(2n-1)\alpha\}\{\tan 2\alpha\tan(2n-2)\alpha\}\ldots\{\tan(n-1)\alpha\tan(n+1)\alpha\}\tan n\alpha$$

$$=\left\{\tan\alpha\tan\left(\frac{\pi}{2}-\alpha\right)\right\}\left\{\tan 2\alpha\tan\left(\frac{\pi}{2}-2\alpha\right)\right\}\ldots\tan\frac{\pi}{4}\ \left(\because n\alpha=\frac{\pi}{4}\right)$$

$=1$

489 **(c)**

We have,
$r_1:r_2:r_3=2:4:6$
$$\Rightarrow \frac{\Delta}{s-a}:\frac{\Delta}{s-b}:\frac{\Delta}{s-c}=2:4:6$$
$$\Rightarrow s-a:s-b:s-c=\frac{1}{2}:\frac{1}{4}:\frac{1}{6}$$
$$\Rightarrow s-a=\frac{\lambda}{2}, s-b=\frac{\lambda}{4} \text{ and } s-c=\frac{\lambda}{6}$$
Now,
$$a=(s-b)+(s-c)=\frac{\lambda}{4}+\frac{\lambda}{6}=\frac{5\lambda}{12}$$
$$b=(s-c)+(s-a)=\frac{\lambda}{6}+\frac{\lambda}{2}=\frac{8\lambda}{12}$$
$$c=(s-a)+(s-b)=\frac{\lambda}{2}+\frac{\lambda}{4}=\frac{9\lambda}{12}$$
$\therefore a:b:c=5:8:9$

490 **(b)**

We have,
$$\cos A=\frac{\sin B}{2\sin C}$$
$$\Rightarrow \frac{b^2+c^2-a^2}{2bc}=\frac{b}{2c}\Rightarrow c^2=a^2\Rightarrow c=a$$
So, the triangle is an isosceles triangle

491 **(c)**

We have,
$\sin x+\sin y=\sin(x+y)$
$$\Rightarrow 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)-2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x+y}{2}\right)=0$$
$$\Rightarrow 2\sin\left(\frac{x+y}{2}\right)\left\{\cos\left(\frac{x-y}{2}\right)-\cos\left(\frac{x+y}{2}\right)\right\}=0$$
$$\Rightarrow 4\sin\left(\frac{x+y}{2}\right)\sin\frac{x}{2}\sin\frac{y}{2}=0$$

$\Rightarrow \sin\left(\frac{x+y}{2}\right)=0$ or, $\sin\frac{x}{2}=0, \sin\frac{y}{2}=0$ ... (i)

Now, $|x|+|y|=1 \Rightarrow |x|\le 1$ and $|y|\le 1$

Hence, the only solution of equations in (i), can be taken as

$x+y=0, x=0, y=0$

Putting $x=0$ in $|x|+|y|=1$, we get $y=\pm 1$

Putting $y=0$ in $|x|+|y|=1$, we obtain $x=\pm 1$

Finally, putting $x+y=0$ i.e. $y=-x$ in $|x|+|y|=1$, we obtain

$2|x|=1 \Rightarrow |x|=\frac{1}{2} \Rightarrow c=\pm\frac{1}{2}$

Hence, we obtain the following six pairs of (x, y) i.e. $(0, \pm 1), (\pm 1, 0), (1/2, -1/2), (-1/2, 1/2)$

492 **(a)**

We have,

$\cos x+\cos y=\frac{3}{2}$

$\Rightarrow 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)=\frac{3}{2}$

$\Rightarrow 2\cos\frac{\pi}{3}\cos\left(\frac{x-y}{2}\right)=\frac{3}{2} \quad \left[\text{Using}: x+y=\frac{2\pi}{3}\right]$

$\Rightarrow \cos\left(\frac{x-y}{2}\right)=\frac{3}{2}$, which is not possible

Hence, the system of equations has no solution

493 **(a)**

$p=\sin^2 x+\cos^2 x(1-\sin^2 x)$

$\Rightarrow p=(\sin^2 x+\cos^2 x)-\sin^2 x\cos^2 x$

$=1-\sin^2 x\cos^2 x$ ...(i)

Which shows $p\le 1$

Again, $p=1-\cos^2 x+\cos^4 x$

$p=\left(\cos^2 x-\frac{1}{2}\right)^2+\frac{3}{4}$

Which shows

$p\ge\frac{3}{4}$ ...(ii)

From Eqs. (i) and (ii), we get

$\frac{3}{4}\le p\le 1$

494 **(a)**

$\tan 6°\tan 42°\tan 66°\tan 78°$

$=\tan 6°\tan(60°-18°)\tan(60°+6°)\tan(60°+18°)$

$=\frac{\tan 6°\tan(60°+6°)\tan 18°\tan(60°-18°)\tan(60°+18°)}{\tan 18°}$

$=\frac{\tan 6°\tan(60°+6°)\tan(3\times 18°)}{\tan 18°}$

$=\frac{\tan 6°\tan(60°-6°)\tan(60°+6°)}{\tan 18°}$

$=\frac{\tan 18°}{\tan 18°}=1$

495 **(d)**

$\cos x+\cos^2 x=1 \Rightarrow \cos x=\sin^2 x$

Now, $\sin^{12}x+3\sin^{10}x+3\sin^8 x+\sin^6 x-1$

$=\cos^6 x+3\cos^5 x+3\cos^4 x+\cos^3 x-1$

$=(\cos^2 x+\cos x)^3-1=1-1=0$

496 **(b)**

We have,

$\tan 20°+2\tan 50°-\tan 70°$

$=2\tan 50°-(\tan 70°-\tan 20°)$

$=2\tan 50°-\frac{\sin 50°}{\cos 70°\cos 20°}$

$=2\tan 50°-\frac{2\sin 50°}{2\sin 20°\cos 20°}$

$=2\tan 50°-\frac{2\sin 50°}{\sin 40°}$

$=2\tan 50°-\frac{2\sin 50°}{\cos 50°}=2\tan 50°-2\tan 50°$

$=0$

497 **(c)**

$\tan\alpha+2\tan 2\alpha+4\tan 4\alpha+8\cot 8\alpha$

$=\tan\alpha+2\tan 2\alpha+4\left[\frac{\sin 4\alpha}{\cos 4\alpha}+2\frac{\cos 8\alpha}{\sin 8\alpha}\right]$

$=\tan\alpha+2\tan 2\alpha+4\left[\frac{\cos 4\alpha\cos 8\alpha+\sin 4\alpha\sin 8\alpha+\cos 4\alpha\cos 8\alpha}{\sin 8\alpha\cos 4\alpha}\right]$

$=\tan\alpha+2\tan 2\alpha+4\left[\frac{\cos 4\alpha+\cos 4\alpha\cos 8\alpha}{\sin 8\alpha\cos 4\alpha}\right]$

$=\tan\alpha+2\tan 2\alpha+4\left[\frac{\cos 4\alpha(1+\cos 8\alpha)}{\cos 4\alpha\sin 8\alpha}\right]$

$=\tan\alpha+2\tan 2\alpha+4\left[\frac{2\cos^2 4\alpha}{2\sin 4\alpha\cos 4\alpha}\right]$

$=\tan\alpha+2(\tan 2\alpha+2\cot 4\alpha)$

$=\tan\alpha+2\left[\frac{\sin 2\alpha}{\cos 2\alpha}+2\frac{\cos 4\alpha}{\sin 4\alpha}\right]$

$=\tan\alpha+2\left[\frac{\sin 2\alpha\sin 4\alpha+\cos 4\alpha\cos 2\alpha+\cos 4\alpha\cos 2\alpha}{\sin 4\alpha\cos 2\alpha}\right]$

$=\tan\alpha+2\left[\frac{\cos 2\alpha+\cos 2\alpha\cos 4\alpha}{\sin 4\alpha\cos 2\alpha}\right]$

$=\tan\alpha+2\left[\frac{\cos 2\alpha(1+\cos 4\alpha)}{\sin 4\alpha\cos 2\alpha}\right]$

$=\frac{\sin\alpha}{\cos\alpha}+\frac{2\cos 2\alpha}{\sin 2\alpha}$

$$= \frac{\cos\alpha + \cos\alpha\cos 2\alpha}{\sin 2\alpha \cos\alpha}$$

$$= \frac{1+\cos 2\alpha}{\sin 2\alpha}$$

$$= \frac{2\cos^2\alpha}{2\sin\alpha\cos\alpha} = \cot\alpha$$

498 **(c)**

We have,

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$= \frac{\tan^2\alpha}{1+\tan^2\alpha} + \frac{\tan^2\beta}{1+\tan^2\beta} + \frac{\tan^2\gamma}{1+\tan^2\gamma}$

$= \frac{x}{1+x} + \frac{y}{1+y} + \frac{z}{1+z}$, $\left[\begin{array}{c}\text{where } x = \tan^2\alpha, \\ y = \tan^2\beta, z = \tan^2\gamma\end{array}\right]$

$= \frac{(x+y+z)(xy+yz+zx+2xyz)+xy+yz+.}{(1+x)(1+y)(1+z)}$

$= \frac{1+x+y+z+xy+yz+zx}{(1+x)(1+y)(1+z)}$

$= 1 \quad [\because xy+yz+zx+2xyz = 1]$

499 **(c)**

$$\sqrt{\frac{1+\cos A}{1-\cos A}} = \sqrt{\frac{2\cos^2\frac{A}{2}}{2\sin^2\frac{A}{2}}} = \frac{x}{y}$$

$\Rightarrow \tan\frac{A}{2} = \frac{y}{x}$

Now, $\tan A = \frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}$

$= \frac{2xy}{x^2-y^2}$

500 **(c)**

We have, $\angle A = 45°, \angle B = 75°$

$\therefore \angle C = 180° - (45° + 75°) = 60°$

Now,

$a + c\sqrt{2} = k(\sin A + \sqrt{2}\sin C)$

$\Rightarrow a + c\sqrt{2} = k(\sin 45° + \sqrt{2}\sin 60°)$

$= k\left(\frac{\sqrt{3}+1}{\sqrt{2}}\right) \ldots$ (i)

And,

$b = k\sin B$

$\Rightarrow b = k\sin 75° = k\frac{(\sqrt{3}+1)}{2\sqrt{2}}$

$\Rightarrow 2b = k\frac{(\sqrt{3}+1)}{\sqrt{2}} \quad \ldots$ (ii)

From (i) and (ii), we get

$a + c\sqrt{2} = 2b$

501 **(d)**

Given, $f(x) = \sin^4 x + \cos^4 x$

$= (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x$

$\Rightarrow f(x) = 1 - \frac{1}{2}\sin^2 2x$

Also, $0 \le \sin^2 2x \le 1$

$\therefore$ Minimum value of $f(x)$ is $1 - \frac{1}{2} = \frac{1}{2}$

502 **(a)**

Given, $\tan\theta = \frac{1}{\sqrt{7}}$

$\therefore \frac{\text{cosec}^2 - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \frac{(1+\cot^2\theta)-(1+\tan^2\theta)}{(1+\cot^2\theta)+(1+\tan^2\theta)}$

$= \frac{(\cot^2\theta - \tan^2\theta)}{2+\tan^2\theta+\cot^2\theta}$

$= \frac{7-\frac{1}{7}}{2+\frac{1}{7}+7} = \frac{48}{64} = \frac{3}{4}$

504 **(d)**

Since the sum of a positive number and its reciprocal is always greater than or equal to 2. Therefore, $y \ge 2$. But, $y = 2$ only when $\theta = 0$. Hence, $y > 2$

505 **(b)**

We have, $\sec\theta = x + \frac{1}{4x}$

Let $\sec\theta + \tan\theta = \lambda \quad$ ...(i)

Then,

$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{\lambda} \quad$ ...(ii)

Adding (i) and (ii), we get

$2\sec\theta \Rightarrow \lambda + \frac{1}{\lambda} \Rightarrow 2x + \frac{1}{2x} = \lambda + \frac{1}{\lambda} \Rightarrow \lambda = 2x, \frac{1}{2x}$

506 **(a)**

Since, $\cos^2\theta = \frac{1}{6}\sin\theta\cdot\tan\theta$

$\Rightarrow 6\cos^3\theta + \cos^2\theta - 1 = 0$

As $\cos\theta = \frac{1}{2}$ satisfied the equation.

$\therefore (2\cos\theta - 1)(3\cos^2\theta + 2\cos\theta + 1) = 0$

$\Rightarrow \cos\theta = \frac{1}{2}$ (other values of $\cos\theta$ are imaginary)

$\therefore \theta = 2n\pi \pm \frac{\pi}{3}, n \in I$

507 **(a)**

Given, $3\sin^2 x = 8\cos x$

$\Rightarrow 3(1-\cos^2 x) = 8\cos x$

$\Rightarrow 3\cos^2 x - 8\cos x - 3 = 0$

$\Rightarrow (3\cos x + 1)(\cos x - 3) = 0$

$\Rightarrow \cos x = -\frac{1}{3} \quad (\because \cos x \ngeq 1)$

In the given interval only one value of x is exist

508 **(a)**

We have,

$\log_{\cos x} \sin x + \log_{\sin x} \cos x = 2$

$\Rightarrow \log_{\cos x} \sin x + \dfrac{1}{\log_{\cos x} \sin x} = 2$

Clearly, this equation is meaningful for $0 < \sin x < 1$ and $0 < \cos x < 1$ i.e. for $0 < x < \pi/2$

Now,

$\log_{\cos x} \sin x + \dfrac{1}{\log_{\cos x} \sin x} = 2$

$\Rightarrow \log_{\cos x} \sin x = 1$

$\Rightarrow \sin x = \cos x$

$\Rightarrow \tan x = 1$

$\Rightarrow \tan x = \tan \dfrac{\pi}{4}$

$\Rightarrow x \geq 2n\pi + \dfrac{\pi}{4}, n \in Z \quad [\because \sin x > 0 \text{ and} \cos x > 0]$

509 **(a)**

We have,

$16 \sin 144° \sin 108° \sin 72° \sin 36°$

$= 16 \sin 36° \cos 18° \cos 18° \sin 36°$

$= 16 \cos^2 18° \sin^2 36° = 16(\sin 36° \cos 18°)^2$

$= 16 \left\{\dfrac{\sqrt{10-2\sqrt{5}}}{4} \times \dfrac{\sqrt{10+2\sqrt{5}}}{4}\right\}^2 = 5$

510 **(b)**

We have,

$f(x) = \tan^m x + \cot^m x$

$= \left(\tan^{m/2} x - \cot^{m/2} x\right)^2 + 2 \geq 2$

Thus, $f(x)$ attains the minimum value of 2 at points given by $\tan^{m/2} x = \cot^{m/2} x$ $i.e.$ at $x = \frac{\pi}{4}$

<u>ALITER</u> Using A.M. ≥ G.M., we have

$\dfrac{\tan^m x + \cot^m x}{2} \geq \sqrt{\tan^m x \times \cot^m x}$

$\Rightarrow \tan^m x + \cot^m x \geq 2$

511 **(a)**

Given, $7 \cos x - 24 \sin x = \lambda \cos(x + \alpha)$

$\Rightarrow 25\left(\dfrac{7}{25} \cos x - \dfrac{24}{25} \sin x\right) = \lambda \cos(x + \alpha)$

$\Rightarrow \; 25[\cos(\beta + x) = \lambda \cos(x + \alpha)$

Where $\cos \beta = \frac{7}{25}$

$\Rightarrow \; \lambda = 25$

512 **(c)**

3. Suppose $a = 2, b = 1$

$\sin \theta = \frac{2^2+1^2}{2^2-1} = \frac{5}{3} > 1$, which is not possible

4. $\sec \theta = \frac{4}{5} < 1$, which is not possible

5. $\tan \theta = 45$, which is possible

6. $\cos \theta = \frac{7}{3} > 1$, which is not possible

513 **(c)**

$\tan(A - B) = \tan \dfrac{\pi}{4} = 1$

$\Rightarrow \; \dfrac{\tan A - \tan B}{1 + \tan A \tan B} = 1$

$\Rightarrow \tan A - \tan B - \tan A \tan B = 1 \quad$ ...(i)

Now, $y = (1 + \tan A)(1 - \tan B)$

$= (1 - \tan B + \tan A - \tan A \tan B)$

$= (1 + 1) = 2 \quad$ [from Eq. (i)]

$\therefore \quad (y + 1)^{y+1} = (2 + 1)^{2+1} = 3^3 = 27$

514 **(d)**

We have,

$a \cos A = b \cos B$

$\Rightarrow a\left(\dfrac{b^2 + c^2 - a^2}{2\,bc}\right) = b\left(\dfrac{c^2 + a^2 - b^2}{2\,ac}\right)$

$\Rightarrow a^2b^2 + a^2c^2 - a^4 = b^2c^2 + b^2a^2 - b^4$

$\Rightarrow c^2(a^2 - b^2) - (a^4 - b^4) = 0$

$\Rightarrow a = b$ or, $c^2 = a^2 + b^2$

$\Rightarrow \Delta ABC$ is isosceles or right angled

515 **(c)**

Maximum value of $\cos \theta = 1$

So, the equation can have solution only when $\cos x = 1, \cos y = 1$

$\Rightarrow \; x = 0, y = 0$

$\Rightarrow \; \cos(x - y) = \cos 0 = 1$

516 **(a)**

$\sin 2A + \sin 2B + \sin 2C$

$= 2 \sin(A + B) \cos(A - B) + 2 \sin C \cos C$

$= 2 \sin(\pi - C) \cos(A - B) + 2 \sin C \cos\{\pi - (A + B)\}$

$= 2 \sin C \{\cos(A - B) - \cos(A + B)\}$

$= 4 \sin A \sin B \sin C$

517 **(a)**

Maximum value of $4 \sin^2 x + 3 \cos^2 x$ ie, $\sin^2 x + 3$ is 4 and that of sin*x2+*cos*x2* is *12+12=2*, both attained at $x = \frac{\pi}{2}$. Hence, the given function has maximum value $4 + \sqrt{2}$

518 **(c)**

Given, $\sin 3\theta - \sin \theta = 0$

$\Rightarrow \; 2 \cos\left(\dfrac{3\theta + \theta}{2}\right) \sin\left(\dfrac{3\theta - \theta}{2}\right) = 0$

$\Rightarrow \; \cos 2\theta . \sin \theta = 0$

$\Rightarrow \; \cos 2\theta = 0 \;$ or $\sin \theta = 0, \pi, 2\pi$

$\Rightarrow \; 2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$ or $\theta = \pi \quad (\because \; \theta \in (0, 2\pi))$

$\Rightarrow \; \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ or $\theta = \pi$

Hence, total number of solutions are 5

519 **(a)**

We have,

$$\sin\frac{\pi}{16}\sin\frac{3\pi}{16}\sin\frac{5\pi}{16}\sin\frac{7\pi}{16}$$
$$=\frac{1}{4}\left\{\left(2\sin\frac{7\pi}{16}\sin\frac{\pi}{16}\right)\left(2\sin\frac{5\pi}{16}\sin\frac{3\pi}{16}\right)\right\}$$
$$=\frac{1}{4}\left\{\left(\cos\frac{3\pi}{8}-\cos\frac{\pi}{2}\right)\left(\cos\frac{\pi}{8}-\cos\frac{\pi}{2}\right)\right\}$$
$$=\frac{1}{8}\left(2\cos\frac{3\pi}{8}\cos\frac{\pi}{8}\right)=\frac{1}{8}\left(\cos\frac{\pi}{2}+\cos\frac{\pi}{4}\right)=\frac{1}{8\sqrt{2}}$$
$$=\frac{\sqrt{2}}{16}$$

520 **(a)**

$\tan 45° = \tan(25° + 20°)$

$$\Rightarrow\ 1=\frac{\tan 25°+\tan 20°}{1-\tan 25°\tan 20°}$$

$\Rightarrow\ \tan 25° + \tan 20° + \tan 25° \tan 20° = 1$

521 **(c)**

The given equation is

$\sin^4 x + \cos^4 y + 2 = 4\sin x\cos y$

$$\Rightarrow (\sin^2 x-1)^2+(\cos^2 y-1)^2 + 2\sin^2 x + 2\cos^2 y - 4\sin x\cos y = 0$$

$$\Rightarrow\ (\sin^2 x-1)^2+(\cos^2 y-1)^2 + 2(\sin x-\cos y)^2 = 0$$

Which is possible only when

$\sin^2 x - 1 = 0, \cos^2 y - 1 = 0, \sin x - \cos y = 0$

$\Rightarrow \sin^2 x = 1, \cos^2 y = 1, \sin x = \cos y$

As $0 \le x, y \le \frac{\pi}{2}$

We get $\sin x = \cos y = 1$

$\therefore \sin x + \cos y = 1 + 1 = 2$

522 **(d)**

Let $C = 90°$. Then,

$\sin^2 A + \sin^2 B + \sin^2 C$

$= \sin^2 A + \sin^2 B + 1$

$$= \sin^2 A + \sin^2\left(\frac{\pi}{2}-A\right)+1$$
$$= \sin^2 A + \cos^2 A + 1 = 2$$

523 **(c)**

We have, $x\cos\theta = y\cos\left(\theta+\frac{2\pi}{3}\right)$

$= z\cos\left(\theta+\frac{4\pi}{3}\right) = k$ (say)

$$\Rightarrow \cos\theta=\frac{k}{x}, \cos\left(\theta+\frac{2\pi}{3}\right)=\frac{k}{y}$$

and $\cos\left(\theta+\frac{4\pi}{3}\right)=\frac{k}{z}$

$$\therefore\ \frac{k}{x}+\frac{k}{y}+\frac{k}{z}=\cos\theta + \cos\left(\theta+\frac{2\pi}{3}\right)+\cos\left(\theta+\frac{4\pi}{3}\right)$$

$$=\cos\theta-\cos\left(\frac{\pi}{3}-\theta\right)-\cos\left(\frac{\pi}{3}+\theta\right)$$

$$=\cos\theta-2\cos\frac{\pi}{3}\cos\theta=0$$

$$\Rightarrow\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$$

524 **(d)**

Since, $A+B+C=\frac{3\pi}{2}$

$\therefore \cos 2A + \cos 2B + \cos 2C$

$= 2\cos(A+B)\cos(A-B) + \cos 2C$

$$=2\cos\left(\frac{3\pi}{2}-C\right)\cos(A-B)+\ 1-2\sin^2 C$$

$$=1-2\sin C\left[\cos(A-B)+\sin\left(\frac{3\pi}{2}-(A+B)\right)\right]$$

$= 1 - 2\sin C[\cos(A-B) - \cos(A+B)]$

$= 1 - 4\sin A\sin B\sin C$

525 **(b)**

We have,

$A + B + C = \pi$

$\Rightarrow A + B = \pi - C$

$\Rightarrow \tan(A+B) = \tan(\pi - C)$

$$\Rightarrow \frac{\tan A+\tan B}{1-\tan A\tan B}=\tan(\pi-C)\Rightarrow\frac{\tan A+\tan B}{1-\tan A\tan B} = -\tan C$$

Now,

C is an obtuse angle

$\Rightarrow \tan C < 0$

$\Rightarrow -\tan C > 0$

$$\Rightarrow\frac{\tan A+\tan B}{1-\tan A\tan B}>0$$

$\Rightarrow 1 - \tan A\tan B > 0$ $\left[\begin{array}{l}\because A, B \text{ are acute angles}\\ \therefore \tan A > 0, \tan B > 0\end{array}\right]$

$\Rightarrow \tan A\tan B < 1$

526 **(a)**

The given equation is

$$(a-2b+c)x^2+(b-2c+a)x+(c-2a+b) = 0$$

$\because \sum(a-2b+c)=0$

∴ One root of this equation is 1

Now, $\sec\theta + \tan\theta = 1$ ... (i)

We know that, $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \sec\theta - \tan\theta = 1$ ....(ii)

On solving Eqs.(i) and (ii), we get

$\sec\theta = 1$

∴ One root of given equation is $\sec\theta$

528 **(b)**
The equation
$a_1 + a_2 \sin x + a_3 \cos x + a_4 \sin 2x + a_5 \cos 2x = 0$ holds for all values of x. Therefore,
$a_1 + a_3 + a_5 = 0$ [On putting $x = 0$]
$a_1 - a_3 + a_5 = 0$ [On putting $x = \pi$]
$\Rightarrow a_3 = 0$ and $a_1 + a_5 = 0$ ...(i)
Putting $x = \frac{\pi}{2}$ and $\frac{3\pi}{2}$, we get
$a_1 + a_2 - a_5 = 0$ and $a_1 - a_2 - a_5 = 0$
$\Rightarrow a_2 = 0$ and $a_1 - a_5 = 0$ ....(ii)
Equations (i) and (ii) give
$a_1 = a_2 = a_3 = a_5 = 0$
The given equation reduces to $a_4 \sin 2x = 0$. This is true for all values of x. Therefore, $a_4 = 0$
Hence, $a_1 = a_2 = a_3 = a_4 = a_5 = 0$
Thus, the number of 5-tuples is one

529 **(a)**
$\tan(70°) = \tan(50° + 20°) = \frac{\tan 50° + \tan 20°}{1 - \tan 50° \tan 20°}$
$\Rightarrow \tan 70° - (\tan 50° \tan 20°)\tan 70° = \tan 50° + \tan 20°$
$\Rightarrow \tan 70° - \cot 20° \tan 20° \tan 50° = \tan 50° + \tan 20°$
[using, $\tan(90° - \theta = \cot\theta)$]
$\Rightarrow \tan 70° - \tan 50° = \tan 50° + \tan 20°$
$\Rightarrow \tan 70° - \tan 20° = 2\tan 50°$
$\Rightarrow \frac{\tan 70° - \tan 20°}{\tan 50°} = 2$

530 **(c)**
Given, $k = \cos 20°$
And $2k^2 - 1 = \cos x$
$\therefore 2\cos^2 20° - 1 = \cos x$
$\Rightarrow \cos x = \cos 40°$
$\Rightarrow x = 40°$
or $x = 360° - 40° = 320°$

531 **(a)**
$\tan 9° - \tan 27° - \cot 27° + \cot 9°$
$= \left(\frac{\sin^2 9° + \cos^2 9°}{\cos 9° \sin 9°}\right) - \left(\frac{\sin^2 27° + \cos^2 27°}{\cos 27° \sin 27°}\right)$
$= \frac{2}{\sin 18°} - \frac{2}{\sin 54°}$
$= \frac{2}{\frac{\sqrt{5}-1}{4}} - \frac{2}{\frac{\sqrt{5}+1}{4}}$
$= \frac{16}{5-1} = 4$

532 **(c)**
We have,
$\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$
$\Rightarrow \tan(\pi\cos\theta) = \tan\left(\frac{\pi}{2} - \pi\sin\theta\right)$
$\Rightarrow \pi\cos\theta = \left(\frac{\pi}{2} - \pi\sin\theta\right) + n\pi, n \in Z$
$\Rightarrow \cos\theta + \sin\theta = \frac{1}{2} + n, n \in Z$
$\Rightarrow \frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta = \frac{2n+1}{2\sqrt{2}}, n \in Z$
$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \frac{2n+1}{2\sqrt{2}}, n \in Z$
$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \pm\frac{1}{2\sqrt{2}}$ [For $n = 0$ and $n = -1$]

533 **(d)**
We have,
$\frac{\sin B}{b} = \frac{\sin C}{c} \Rightarrow \frac{\sin \pi/3}{3} = \frac{\sin C}{4}$
$\Rightarrow \sin c = \frac{2}{\sqrt{3}} > 1$, which is impossible
Hence, no triangle is possible

535 **(c)**
Let $a = 7$ cm, $b = 4\sqrt{3}$ cm and $c = \sqrt{13}$ cm. Since c is the smallest side. Therefore, the smallest angle is C and is given by
$\cos C = \frac{a^2 + b^2 - c^2}{2\,ab} = \frac{\sqrt{3}}{2} \Rightarrow C = 30°$

537 **(c)**
Given, $\tan(k+1)\theta = \tan\theta$
$\Rightarrow (k+1)\theta = n\pi + \theta \Rightarrow k\theta = n\pi$
$\Rightarrow \theta = \frac{n\pi}{k}$ $\therefore \theta \in \left\{\frac{n\pi}{k} : n \in I\right\}$

538 **(b)**
It is given that $\theta \in (0, \pi/4)$. Therefore,
$0 < \tan\theta < 1$ and $\cot\theta > 1$. Let $\tan\theta = 1 - a$ and $\cot\theta = 1 + b$ where $0 < a < 1$ and $b > 1$
$\therefore t_1 = (1-a)^{1-a}, t_2 = (1-a)^{1+b}, t_3 = (1+b)^{1-a}$ and, $t_4 = (1+b)^{1+b}$
Now,
$1 - a < 1 + b$ and $0 < 1 - a < 1$
$\therefore (1-a)^{1-a} > (1-a)^{1+b}$ and $(1+b)^{1+b} > (1+b)^{1-a}$

$\Rightarrow t_1 > t_2$ and $t_4 > t_3$ ...(i)

Also, $(1+b)^{1-a} > (1-a)^{1-a}$

$\Rightarrow t_3 > t_1$ ...(ii)

From (i) and (ii), we get $t_4 > t_3 > t_1 > t_2$

539 **(a)**

We have,

$\cos 12° \cos 24° \cos 36° \cos 48° \cos 72° \cos 84°$

$= \{-\cos 12° \cos 24° \cos 48° \cos 96°\}\{\cos 36° \cos 72$

$= -\frac{\sin 2^4 \times 12°}{2^4 \sin 12°} \times (\cos 36° \sin 18°)$

$= -\frac{\sin 192°}{16 \sin 12°} \times \left(\frac{\sqrt{5}+1}{4} \times \frac{\sqrt{5}-1}{4}\right) = \frac{1}{16} \times \frac{1}{4}$

$= \frac{1}{64}$

540 **(c)**

Since, $\tan A + \sin A = m$

and $\tan A - \sin A = n$

$\therefore m + n = 2\tan A$

and $m - n = 2\sin A$

Also, $mn = (\tan A + \sin A)(\tan A - \sin A) =$ $\tan 2A - \sin 2A$

Now, $\frac{(m^2-n^2)^2}{mn} = \frac{(m+n)^2(m-n)^2}{mn}$

$= \frac{(2\tan A)^2 (2\sin A)^2}{\tan^2 A - \sin^2 A}$

$= \frac{16\tan^2 A \sin^2 A}{\sin^2 A \tan^2 A} = 16$

541 **(c)**

$\cos 2A + \cos 2B + \cos 2C$

$= 2\cos(A+B)\cos(A-B) + 1 - 2\sin^2 C$

$= 2\, cos\left(\frac{3\pi}{2} - C\right)\cos(A-B) + 1 - 2\sin^2 C$

$\left[\because A + B + C = 270° \Rightarrow B + A = \frac{3\pi}{2} - C\right]$

$= 1 - 2\sin C[\cos(A-B) + \sin C]$

$= 1 - 2\sin C[\cos(A-B) - \cos(A+B)]$

$= 1 - 4\sin A \sin B \sin C$

542 **(a)**

Given, $\frac{\sin A - \sin c}{\cos C - \cos A} = \cot B$

$\Rightarrow \frac{2\cos\left(\frac{A+C}{2}\right)\sin\left(\frac{A-C}{2}\right)}{2\sin\left(\frac{A+C}{2}\right)\sin\left(\frac{A-C}{2}\right)} = \cot B$

$\Rightarrow \cot\left(\frac{A+C}{2}\right)\cot B$

$\Rightarrow B = \frac{A+C}{2}$

Hence, A, B and C will be in AP

543 **(a)**

We have,

$\alpha + \beta + \gamma = 2\pi$

$\Rightarrow \frac{\alpha}{2} + \frac{\beta}{2} + \frac{\gamma}{2} = \pi$

$\Rightarrow \tan\left(\frac{\alpha}{2} + \frac{\beta}{2} + \frac{\gamma}{2}\right) = \tan\pi = 0$

$\Rightarrow \frac{\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} - \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}}{1 - \tan\frac{\alpha}{2}\tan\frac{\beta}{2} - \tan\frac{\beta}{2}\tan\frac{\gamma}{2} - \tan\frac{\gamma}{2}\tan\frac{\alpha}{2}}$

$\Rightarrow \tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

544 **(c)**

We have,

$2r = a + c - b$

$\Rightarrow 2r = 2s - 2b$

$\Rightarrow r = s - b$

$\Rightarrow \frac{\Delta}{s} = s - b$

$\Rightarrow \Delta = s(s-b)$

$\Rightarrow \sqrt{s(s-a)(s-b)(s-c)} = s(s-b)$

$\Rightarrow \sqrt{\frac{(s-a)(s-c)}{s(s-b)}} = 1 \Rightarrow \tan\frac{B}{2} = \tan\frac{\pi}{4} \Rightarrow B = \frac{\pi}{2}$

545 **(d)**

Since, $\sin^2\theta \le 1$

$\Rightarrow \frac{x^2+y^2+1}{2x} \le 0$

$\Rightarrow (x-1)^2 + y^2 \le 0$

Which is possible only when $x = 1, y = 0$

Hence, it also depends on value of y.

546 **(d)**

We have,

$1 + \sin\theta + \sin^2\theta + \cdots \infty = 4 + 2\sqrt{3}$

$\Rightarrow \frac{1}{1-\sin\theta} = 4 + 2\sqrt{3}$

$\Rightarrow 1 - \sin\theta = \frac{1}{2(2+\sqrt{3})}$

$\Rightarrow 1 - \sin\theta = \frac{1}{2}(2-\sqrt{3})$

$\Rightarrow \sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}, \frac{2\pi}{3}$

547 **(b)**

Given, $\cos 20° - \sin 20° = p$

$\Rightarrow \cos^2 20° + \sin^2 20° - 2\sin 20° \cos 20° = p^2$

$\Rightarrow 1 - p^2 = \sin 40°$

$\Rightarrow 1 - p^2 = \sqrt{1 - \cos^2 40°}$

$\Rightarrow (1-p^2)^2 = 1-\cos^2 40°$

$\Rightarrow \cos^2 40° = 1-(1+p^4-2p^2)$

$\Rightarrow \cos 40° = \sqrt{2p^2-p^4}$

$\Rightarrow \cos 40° = p\sqrt{2-p^2}$

548 **(d)**

We have,

$c^2 = a^2+b^2 \Rightarrow \angle C = \frac{\pi}{2}$

$\therefore \Delta = \frac{1}{2}ab\sin C = \frac{1}{2}ab$

$\Rightarrow \sqrt{s(s-a)(s-b)(s-c)} = \frac{1}{2}ab$

$\Rightarrow 4\, s(s-a)(s-b)(s-c) = a^2b^2$

549 **(b)**

$\tan(70°-20°) = \frac{\tan 70° - \tan 20°}{1+\tan 70° \tan 20°}$

$\Rightarrow \tan 50°(1+\tan 70° \tan 20°) = \tan 70° - \tan 20°$ ...(i)

Now, $\tan(70°+20°) = \frac{\tan 70° + \tan 20°}{1-\tan 20° \tan 70°}$

$\Rightarrow \tan 20° \tan 70° = 1 \quad [\because \tan 90° = \infty]$

On putting in Eq. (i), we get

$\tan 50°(1+1) = \tan 70° - \tan 20°$

$\Rightarrow \tan 20° + 2\tan 50° - \tan 70° = 0$

550 **(a)**

Given, $4\cos^2 x + 6\sin^2 x = 5$

$\Rightarrow 4(\cos^2 x + \sin^2 x) + 2\sin^2 x = 5$

$\Rightarrow 2\sin^2 x = 5-4$

$\Rightarrow \sin x = \pm\frac{1}{\sqrt{2}}$

$\therefore x = n\pi \pm \frac{\pi}{4}$

551 **(c)**

$\frac{\tan A}{1+\sec A} + \frac{1+\sec A}{\tan A}$

$= \frac{\tan^2 A + 1 + \sec^2 A + 2\sec A}{\tan A(1+\sec A)}$

$= \frac{2\sec^2 A + 2\sec A}{\tan A(1+\sec A)} = \frac{2\sec A}{\tan A}$

$= \frac{2\cos A}{\cos A.\sin A} = 2\text{cosec A}$

552 **(a)**

Given, $\sin^2 A + \sin^2 B + \sin^2 C = 2$

$\Rightarrow 1-\cos^2 A + 1 - \cos^2 B + 1 - \cos^2 C = 2$

$\Rightarrow 1 = \cos^2 A + \cos^2 B + \cos^2 C$

$\Rightarrow 1 = 1 - 2\cos A \cos B \cos C$

$\Rightarrow \cos A cos\, B \cos C = 0$

At least one should be 90° and sum of two angles should be 90°

553 **(d)**

Given, $(\cos\theta + \cos 3\theta) + \cos 2\theta = 0$

$\Rightarrow 2\cos 2\theta \cos\theta + \cos 2\theta = 0$

$\Rightarrow \cos 2\theta(2\cos\theta + 1) = 0$

$\Rightarrow \cos 2\theta = 0$ or $2\cos\theta + 1 = 0$

$\Rightarrow 2\theta = (2n+1)\frac{\pi}{2}$ or $\theta = 2n\pi \pm \frac{2\pi}{3}$

$\Rightarrow \theta = (2n+1)\frac{\pi}{4}$ or $\theta = 2n\pi \pm \frac{2\pi}{3}$

554 **(d)**

The quadratic equation is $x^2 - x\cos\theta + 1 = 0$

Since, x is real, therefore discriminant ≥ 0

$\Rightarrow B^2 4\, AC \geq 0 \Rightarrow \cos^2\theta \geq 4(1)(1) \Rightarrow \cos^2\theta \geq 4$

Which is impossible because $\cos^2\theta$ is not greater than 1

555 **(b)**

$(\sin x + \cos x)^2 = \frac{1}{25}$

$\Rightarrow \sin^2 x + \cos^2 x + 2\sin x \cos x = \frac{1}{25}$

$\Rightarrow \sin 2x = \frac{1}{25} - 1 = -\frac{24}{25}$ ...(i)

$\Rightarrow \cos 2x = \sqrt{1-\sin^2 2x} = -\frac{\sqrt{49}}{25}$ ...(ii)

Now, $\tan 2x = \frac{\sin 2x}{\cos 2x} = -\frac{24}{25} \times \left(-\frac{25}{\sqrt{49}}\right) = \frac{24}{7}$

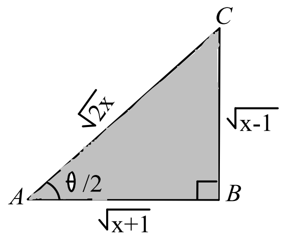
556 **(b)**

Given, $\sin\frac{\theta}{2} = \sqrt{\frac{x-1}{2x}}$

$\therefore \tan\frac{\theta}{2} = \sqrt{\frac{x-1}{x+1}}$

$\therefore \tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$

$= \frac{2\sqrt{\frac{x-1}{x+1}}}{1-\frac{x-1}{x+1}} = \frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{2}{x+1}} = \sqrt{x^2-1}$



558 **(b)**

We have,

$\cos 1° \cos 2° \cos 3° \dots \cos 179°$

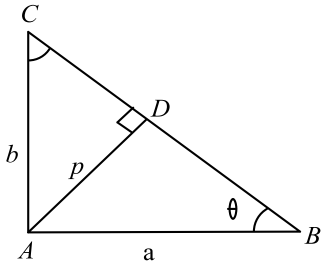
$= \cos 1° \cos 2° \cos 3° \dots \cos 90° \dots . \cos 179°$

$= 0 \quad [\because \cos 90° = 0]$

559 **(c)**

Given, $AD = p$ and $BC = 2\sqrt{2}\,p$

Clearly, $p = \text{a}\sin\theta = b\cos\theta$



Since, $a^2 + b^2 = (2\sqrt{2}p)^2$

$\Rightarrow p^2\left[\frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}\right] = 8p^2$

$\Rightarrow\ 1 = 2\sin^2 2\theta \quad \Rightarrow\ \sin 2\theta = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow \sin 2\theta = \frac{1}{\sqrt{2}}\ \Rightarrow\ 2\theta = \frac{\pi}{4} \Rightarrow\ \theta = \frac{\pi}{8}$

So, the other angle is $\frac{\pi}{2} - \theta = \frac{\pi}{2} - \frac{\pi}{8} = \frac{3\pi}{8}$

560 **(c)**

$12\cot^2\theta - 31\text{cosec}\,\theta + 32 = 0$

$\Rightarrow\ 12\cos^2\theta - 31\sin\theta + 32\sin^2\theta = 0$

$\Rightarrow\ 20\sin^2\theta - 31\sin\theta + 12 = 0 \quad [\because \cos^2\theta = 1 - \sin^2\theta]$

This is a quadratic equation in $\sin\theta$

$\therefore\ \sin\theta = \frac{31 \pm \sqrt{31^2 - 4.20.12}}{2.20} = \frac{31 \pm 1}{40}$

$\Rightarrow \sin\theta = \frac{4}{5}, \frac{3}{4}$

561 **(c)**

We have,

$T_n = \cos^n\theta + \sin^n\theta$

$\therefore T_{n+2} = \cos^{n+2}\theta + \sin^{n+2}\theta$

$\Rightarrow T_n - T_{n+2} = (\cos^n\theta + \sin^n\theta) - (\cos^{n+2}\theta + \sin^{n+2}\theta)$

$\Rightarrow T_n - T_{n+2} = \cos^n\theta\,(1 - \cos^2\theta) + \sin^n\theta\,(1 - \sin^2\theta)$

$\Rightarrow T_n - T_{n+2} = \cos^n\theta\sin^2\theta + \sin^n\theta\cos^2\theta$

$\Rightarrow T_n - T_{n+2} = \cos^2\theta\sin^2\theta\,(\cos^{n-2}\theta + \sin^{n-2}\theta)$

$\Rightarrow T_n - T_{n+2} = \cos^2\theta\sin^2\theta\, T_{n-2}$

Now,

$(2T_6 - 3T_4 + 1)$

$= 2(T_6 - T_4) - T_4 + T_2$

$= -2(T_4 - T_6) + (T_2 - T_4)$

$= -2\cos^2\theta\sin^2\theta\, T_2 + \cos^2\theta\sin^2\theta\, T_0$

$= -2\cos^2\theta\sin^2\theta + 2\cos^2\theta\sin^2\theta$

$= 0\ [\because T_2 = 1\ \&T_0 = 2]$

562 **(a)**

$2\sin^2\theta - 3\sin\theta - 2 = 0$

$\Rightarrow\ (2\sin\theta + 1)(\sin\theta - 2) = 0$

$\Rightarrow\ \sin\theta = -\frac{1}{2} \qquad [\because \sin\theta \neq 2]$

$\Rightarrow\ \sin\theta = \sin\left(-\frac{\pi}{6}\right)$

$\Rightarrow\quad \theta = n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$

$\Rightarrow\ \theta = n\pi + (-1)^{n+1}\ \frac{\pi}{6}$

563 **(c)**

We have,

$\alpha + \beta + \gamma = \pi$

$\therefore \sin\alpha + \sin\beta + \sin\gamma = 4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$

Clearly, $\cos\frac{\alpha}{2} > 0, \cos\frac{\beta}{2} > 0, \cos\frac{\gamma}{2} > 0$

$\therefore 4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2} > 0$

$\Rightarrow$ Minimum value of $\sin\alpha + \sin\beta + \sin\gamma$ is positive

564 **(b)**

We have,

$3\left\{\sin^4\left(\frac{3\pi}{2} - \alpha\right) + \sin^4(3\pi - \alpha)\right\} - 2\left\{\sin^6\left(\frac{\pi}{2} + \alpha\right) + \sin^6(5\pi - \alpha)\right\}$

$= 3\{\cos^4\alpha + \sin^4\alpha\} - 2\{\cos^6\alpha + \sin^6\alpha\}$

$= 3\{(\cos^2\alpha + \sin^2\alpha)^2 - 2\sin^2\alpha\cos^2\alpha\}$

$-2\{(\cos^2\alpha + \sin^2\alpha)^3 - 3\cos^2\alpha\sin^2\alpha\,(\cos^2\alpha + \sin^2\alpha)\}$

$= 3\{1 - 2\sin^2\alpha\cos^2\alpha\} - 2\{1 - 3\cos^2\alpha\sin^2\alpha\}$

$= 1$

565 **(d)**

Given, $\cot x + \text{cosec}\, x = \sqrt{3}$

$\Rightarrow\ \frac{\cos x + 1}{\sin x} = \sqrt{3}$

$\Rightarrow\ \frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \sqrt{3}\ \Rightarrow\ \tan\frac{x}{2} = \frac{1}{\sqrt{3}}$

$\therefore\ \frac{x}{2} = n\pi + \frac{\pi}{6}\ \Rightarrow\ x = 2n\pi + \frac{\pi}{3}$

$\Rightarrow\ x - \frac{\pi}{6} = 2n\pi + \frac{\pi}{6}$

For $n = 0$, principle value of $x - \frac{\pi}{6}$ is $\frac{\pi}{6}$

566 **(a)**

$\because\ \sin 3x = 3\sin x - 4\sin^3 x$

$\therefore\ \sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)$

and $\cos 3x = 4\cos^3 x - 3\cos x$

$\Rightarrow\ \cos^3 x = \frac{1}{4}(\cos 3x + 3\cos x)$

$\therefore\ \cos 3x\cos^3 x + \sin 3x\sin^3 x$

$= \frac{1}{4}[\cos^2 3x + 3\cos x\cos 3x + 3\sin x\sin 3x - \sin^2 3x]$

$= \frac{1}{4}[3\cos 2x + \cos 6x] = \cos^3 2x$

$\Rightarrow\ \cos 2x = 0\ \Rightarrow\ 2x = (2n + 1)\frac{\pi}{2}$

$\Rightarrow\ x = (2n+1)\dfrac{\pi}{4}$

567 **(a)**

We have,

$c_1 + c_2 = 2b\cos A$ and $c_1c_2 = b^2 - a^2$

$\therefore c_1^2 - 2c_1c_2\cos 2A + c_2^2$

$= (c_1+c_2)^2 - 2c_1c_2(1+\cos 2A)$

$= 4b^2\cos^2 A - 4(b^2-a^2)\cos^2 A = 4a\cos^2 A$

568 **(a)**

Let $y = \tan A \tan B$. Then,

$A + B = \dfrac{\pi}{3}$

$\Rightarrow y = \tan A \tan\left(\dfrac{\pi}{3} - A\right)$

$\Rightarrow y = \dfrac{\tan A(\sqrt{3} - \tan A)}{1+\sqrt{3}\tan A} = \dfrac{\sqrt{3}x - x^2}{1+\sqrt{3}x}$, where $x = \tan A$

For maximum or minimum values of y, we must have

$\dfrac{dy}{dx} = 0 \Rightarrow x = \dfrac{1}{\sqrt{3}}$ or, $x = -\sqrt{3}$

But, $x = \tan A > 0$. Therefore, $x = \frac{1}{\sqrt{3}}$

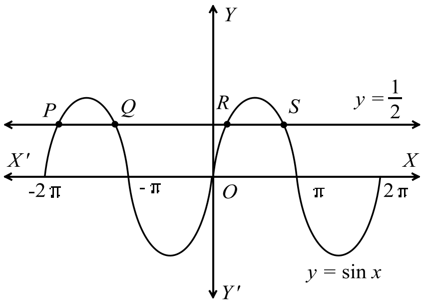
For this value of x, we have $y = \frac{1}{3}$

569 **(c)**

We have,

$2y = 1$ and $y = \sin x$

$\Rightarrow y = \dfrac{1}{2}$ and $y = \sin x$



Clearly, these two curves intersect at 4 points in $[-2\pi, 2\pi]$

570 **(c)**

We have,

$x + 2\tan x = \dfrac{\pi}{2} \Rightarrow \tan x = \dfrac{\pi}{4} - \dfrac{x}{2}$

It can be easily seen from the graphs of the curves $y = \tan x$ and $y = \frac{\pi}{4} - \frac{x}{2}$, in the interval $[0, 2\pi]$, that they intersect at three points. The abscissa of these three points are roots of the equation

572 **(d)**

$\dfrac{\cot^2\theta + 1}{\cot^2\theta - 1} = \dfrac{1+\tan^2\theta}{1-\tan^2\theta} = \dfrac{1}{\cos^2\theta - \sin^2\theta}$

$= \dfrac{1}{\cos 2\theta} = \sec 2\theta$

573 **(d)**

$\text{cosec}^2 x + 25\sec^2 x = 26 + \cot^2 x + 25\tan^2 x$

$= 26 + 10 + (\cot x - 5\tan x)^2 \geq 36$

574 **(b)**

We are given that

$\cos\theta = \dfrac{2\cos(\theta-\alpha)\cos(\theta+\alpha)}{\cos(\theta-\alpha)+\cos(\theta+\alpha)}$

$\Rightarrow \cos\theta = \dfrac{2(\cos^2\theta - \sin^2\alpha)}{2\cos\theta\cos\alpha}$

$\Rightarrow \cos^2\theta\cos\alpha = \cos^2\theta - \sin^2\alpha$

$\Rightarrow \cos^2\theta = \dfrac{\sin^2\alpha}{1-\cos\alpha}$

$\Rightarrow \cos\theta = \dfrac{4\sin^2\frac{\alpha}{2}\cos^2\frac{\alpha}{2}}{2\sin^2\frac{\alpha}{2}}$

$\Rightarrow \cos^2\theta\sec^2\dfrac{\alpha}{2} = 2 \Rightarrow \cos\theta\sec\dfrac{\alpha}{2} = \pm\sqrt{2}$

575 **(a)**

We have,

$5x = 3x + 2x$

$\Rightarrow \tan 5x = \tan(3x+2x)$

$\Rightarrow \tan 5x = \dfrac{\tan 3x + \tan 2x}{1 - \tan 3x\tan 2x}$

$\Rightarrow \tan 5x - \tan 5x\tan 3x\tan 2x = \tan 3x + \tan 2x$

$\Rightarrow \tan 5x\tan 3x\tan 2x = \tan 5x - \tan 3x - \tan 2x$

576 **(a)**

We have,

$\sin x + \sin^2 x = 1$

$\Rightarrow \sin x = 1 - \sin^2 x \Rightarrow \sin x = \cos^2 x$

$\therefore \cos^2 x + \cos^4 x = \sin x + \sin^2 x = 1$

577 **(d)**

$(\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta\cos^2\theta$

$= (\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta) + 3\sin^2\theta\cos^2\theta$

$= 1[(\sin^2\theta+\cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta] + 3\sin^2\theta\cos^2\theta = 1$

$= 1 - 3\sin^2\theta\cos^2\theta + 3\sin^2\theta\cos^2\theta = 1$

578 **(a)**

We have,

$\cos\theta = \dfrac{8}{17} \Rightarrow \sin\theta = \dfrac{15}{17} \quad \left[\because \theta < \dfrac{\pi}{2} < \theta\right]$

$\therefore \cos(30° + \theta) + \cos(45° - \theta) + \cos(120° - \theta)$

$= (\cos 30° + \cos 45° + \cos 120°) \cos\theta + (-\sin 30° + \sin 45° + \sin 120°) \sin\theta$

$= \frac{8}{17}\left(\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} - \frac{1}{2}\right) + \left(-\frac{1}{2} + \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\right)\frac{15}{17}$

$= \frac{23}{17}\left(\frac{\sqrt{3}-1}{2} + \frac{1}{\sqrt{2}}\right)$

579 **(a)**

Since, $\frac{5^x+5^{-x}}{2} \ge \sqrt{5^x.5^{-x}}$

$\Rightarrow\ 5^x + 5^{-x} \ge 2$

But $\sin(e^x) \le 1 \Rightarrow 2\sin(e^x) \le 2$

At $x = 0$, $5^x + 5^{-x} = 2$

Bit $2\sin(e^0) \ne 2$

Hence, no solution exist

580 **(c)**

Let ABC be a right angled triangle right angled at B.

Let the other angles be $A = 90° - d$ and $C = 90° - 2d$. Then,

$A + B + C = 180° \Rightarrow 270° - 3d = 180° \Rightarrow d = 30°$

$\therefore A = 60°$ and $C = 30°$

Let $AC = b$. Then,

$c = AB = AC\cos 60° = \frac{b}{2}$ and $a = BC$

$= b\cos 30° = \frac{\sqrt{3}b}{2}$

$\therefore 2s = a + b + c = \frac{\sqrt{3}b}{2} + b + \frac{b}{2} = \frac{(3+\sqrt{3})b}{2}$

Also,

$\Delta = \frac{1}{2}(BC \times AB) = \frac{1}{2}\left(\frac{\sqrt{3}b}{2} \times \frac{b}{2}\right) = \frac{\sqrt{3}b^2}{8}$

$\therefore$ Required ratio $= \frac{r}{2s} = \frac{\Delta}{2s^2}$

$= \frac{\frac{\sqrt{3}b^2}{8}}{2\left\{\frac{(3+\sqrt{3})^2}{4}b^2\right\}} = \frac{\sqrt{3}}{(3+\sqrt{3})^2} = \frac{\sqrt{3}}{3(\sqrt{3}+1)^2}$

$= \frac{(\sqrt{3}-1)^2}{4\sqrt{3}} = \frac{4-2\sqrt{3}}{4\sqrt{3}} = \frac{2-\sqrt{3}}{2\sqrt{3}}$

581 **(c)**

Given, $2\sin^2\theta + \sqrt{3}\cos\theta + 1 = 0$

$\Rightarrow\ 2\cos^2\theta - \sqrt{3}\cos\theta - 3 = 0$

$\therefore\ \cos\theta = \frac{\sqrt{3} \pm \sqrt{3 + 4\times 3\times 2}}{2\times 2}$

$= \frac{\sqrt{3} \pm 3\sqrt{3}}{4}$

$\Rightarrow\ \cos\theta = -\frac{\sqrt{3}}{2}$ $[\because \cos\theta \ne \sqrt{3}]$

$\Rightarrow\ \theta = \frac{5\pi}{6}$

582 **(b)**

$e^{\sin x} + e^{\cos x} = 2e^{1/2}$ ...(i)

$\Rightarrow\ e^{\sin x} > 0$ and $e^{\cos x} > 0$

$\therefore e^{\sin x} + e^{\cos x} \ge 2\sqrt{e^{\sin x + \cos x}}$ $(\because \text{AM} \ge \text{GM})$

$\Rightarrow\ e^{\sin x} + e^{\cos x} \ge 2e^{1/\sqrt{2}}$ ...(ii)

Since, equality holds

$\Rightarrow\ e^{\sin x} = e^{\cos x}$

$\Rightarrow \sin x = \cos x$

$\Rightarrow \tan x = 1 \Rightarrow x = m\pi + \frac{\pi}{4}$

$\Rightarrow x = (4m+1)\frac{\pi}{4}$

583 **(a)**

Since, $\cos\theta$ is negative and $\tan\theta$ is positive which lies in IIIrd quadrant

$\therefore\ \theta = \frac{5\pi}{4}$ satisfies

Hence, general value of θ is $2n\pi + \frac{5\pi}{4}$

584 **(b)**

$\cos(\theta-\alpha) + \cos(\theta-\beta) + \cos\theta + cos(\theta-\gamma)$

$= 2\cos\left(\theta - \left(\frac{\alpha+\beta}{2}\right)\right)\cos\left(\frac{\beta-\alpha}{2}\right) + 2\cos\left(\frac{\gamma}{2}\right)\cos\left(\theta - \frac{\gamma}{2}\right)$

$= 2\cos\left(\frac{\gamma}{2}\right)\cos\left(\frac{\beta-\alpha}{2}\right) + 2\cos\left(\frac{\gamma}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$

$= [\because 2\theta = \alpha+\beta+\gamma]$

$= 2\cos\left(\frac{\gamma}{2}\right)\left[2\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\right]$

$= 4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$

585 **(b)**

$\because\ x^3 + x^2 + 4x + 2\sin x = 0$

$\Rightarrow x^3 + (x+2)^2 + 2\sin x = 4$

$x = 0$, satisfies this equation.

Now, in $0 < x \le \pi, x^3 + (x+2)^2 + 2\sin x > 4$ and in $\pi < x \le 2\pi, x^3 + (x+2)^2 + 2\sin x > 27 + 25 - 2 = 50$

Hence, $x = 0$ is the only solution

586 **(a)**

We have,

$\frac{\tan 70° - \tan 20°}{\tan 50°}$

$= \frac{\sin(70° - 20°)}{\cos 70° \cos 20°} \times \frac{\cos 50°}{\sin 50°}$

$= \frac{2\cos 50°}{2\cos 70°\cos 20°} = \frac{2\cos 50°}{\cos 90° + \cos 50°} = 2$

587 **(a)**

Given equation, $\sin x + \sin y + \sin z = -3$ is satisfied only when $x = y = z = \frac{3\pi}{2}$ for

$x, y, z \in [0, 2\pi]$

588 **(a)**

$$\tan A = \frac{1-\cos B}{\sin B}$$

$$= \frac{2\sin^2(B/2)}{2\sin\left(\frac{B}{2}\right)\cos(B/2)}$$

$$\Rightarrow \tan A = \tan\frac{B}{2}$$

Now, $\tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{2\tan(B/2)}{1-\tan^2(B/2)}$

$$= \frac{2\sin(B/2)\cos(B/2)}{\cos^2(B/2)-\sin^2(B/2)}$$

$$= \frac{\sin B}{\cos B}$$

$$\Rightarrow \tan 2A = \tan B$$

589 **(c)**

We have,

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{1-2\sin^2\frac{\alpha}{2}}$$

$$= \frac{2\sqrt{\frac{x-1}{2x}}\sqrt{1-\left(\frac{x-1}{2x}\right)^2}}{1-2\left(\frac{x-1}{2x}\right)} = \sqrt{x^2-1}$$

590 **(d)**

We have,

$\sin\alpha + \sin\beta = a$ and $\cos\alpha + \cos\beta = b$

$$\Rightarrow 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = a$$

and,

$$2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = b$$

$$\Rightarrow \tan\left(\frac{\alpha+\beta}{2}\right) = \frac{a}{b}$$

$$\therefore \sin(\alpha+\beta) = \frac{2\tan\left(\frac{\alpha+\beta}{2}\right)}{1+\tan^2\left(\frac{\alpha+\beta}{2}\right)}$$

$$\Rightarrow \sin(\alpha+\beta) = \frac{\frac{2a}{b}}{1+\frac{a^2}{b^2}} = \frac{2ab}{a^2+b^2}$$

591 **(c)**

We have,

$\tan\theta + \sec\theta = \sqrt{3}$, where $0 < \theta < \pi$

$$\Rightarrow \sec\theta - \tan\theta = \frac{1}{\sqrt{3}} \quad \left[\because \sec\theta - \tan\theta = \frac{1}{\sec\theta+\tan\theta}\right]$$

$$\therefore 2\tan\theta = \sqrt{3} - \frac{1}{\sqrt{3}}$$

$$\Rightarrow 2\tan\theta = \frac{2}{\sqrt{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \pi/6$$

592 **(d)**

We have,

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}}$$

$$\Rightarrow \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \frac{1+\cos\theta}{|\sin\theta|}$$

$$\Rightarrow \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \frac{1+\cos\theta}{-\sin\theta} \quad [\because \pi < \theta < 2\pi \Rightarrow \sin\theta < 0]$$

$$\Rightarrow \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = -\operatorname{cosec}\theta - \cot\theta$$

593 **(b)**

We have,

$a\sin^2 x + b\cos^2 x = c$

$\Rightarrow (b-a)\cos^2 x = c - a$

$\Rightarrow (b-a) = (c-a)(1+\tan^2 x)$

Now,

$b\sin^2 y + a\cos^2 y = d$

$\Rightarrow (a-b)\cos^2 y = d - b$

$\Rightarrow (a-b) = (d-b)(1+\tan^2 y)$

$$\Rightarrow \tan^2 y = \frac{a-d}{d-b}$$

$$\therefore \tan^2 x = \frac{b-c}{c-a} \text{ and } \tan^2 y = \frac{a-d}{d-b}$$

$$\Rightarrow \frac{\tan^2 x}{\tan^2 y} = \frac{(b-c)(d-b)}{(c-a)(a-d)} \quad ...(i)$$

But, $a\tan x = b\tan y \Rightarrow \frac{\tan x}{\tan y} = \frac{b}{a}$ ...(ii)

From (i) and (ii), we get

$$\frac{b^2}{a^2} = \frac{(b-c)(d-b)}{(c-a)(a-d)} \Rightarrow \frac{a^2}{b^2} = \frac{(c-a)(a-d)}{(b-c)(d-b)}$$

594 **(c)**

$$\frac{\cos 12° - \sin 12°}{\cos 12° + \sin 12°} + \frac{\sin 147°}{\cos 147°}$$

$$= \frac{1-\tan 12°}{1+\tan 12°} + \tan 147°$$

$$= \tan(45° - 12°) + \tan(180° - 33°)$$

$$= \tan 33° - \tan 33° = 0$$

595 **(b)**

We have,

$$\tan\frac{C-B}{2} = \frac{c-b}{c+b}\cot\frac{A}{2}$$

$$\therefore \tan\left(\frac{C-B}{2}\right) = \frac{\sqrt{3}+1-2}{\sqrt{3}+1+2}\cot 15°$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1}\frac{1}{\tan(45°-30°)}$$

$$\Rightarrow \tan\left(\frac{C-B}{2}\right)=\frac{\sqrt{3}-1}{\sqrt{3}+3}\cdot\frac{\sqrt{3}+1}{\sqrt{3}-1}=\frac{1}{\sqrt{3}}=\tan 30°$$

$$\Rightarrow \frac{C-B}{2}=30°$$

596 **(c)**

We have,

$$\frac{\sin\frac{x}{2}+\cos\frac{x}{2}-i\tan x}{1+2\,i\sin\frac{x}{2}}$$

$$=\frac{\left(\sin\frac{x}{2}+\cos\frac{x}{2}-i\tan x\right)\left(1-2\,i\sin\frac{x}{2}\right)}{1+4\sin^2\frac{x}{2}}$$

$$=\frac{\left(\sin\frac{x}{2}+\cos\frac{x}{2}-2\sin\frac{x}{2}\tan x\right)+i\left(-\tan x-2\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}\right)}{1+4\sin^2\frac{x}{2}}$$

This will be real iff

$$\frac{-\tan x-2\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}{1+4\sin^2\frac{x}{2}}=0$$

$$\Rightarrow -\tan x-2\sin\frac{2x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}=0$$

$$\Rightarrow \sin x+2\cos x\sin\frac{x}{2}\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)=0$$

$$\Rightarrow 2\sin\frac{x}{2}\left\{\cos\frac{x}{2}+\cos x\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)\right\}=0$$

$$\Rightarrow 2\sin\frac{x}{2}\left\{\cos\frac{x}{2}\left(\cos^2\frac{x}{2}+\sin^2\frac{x}{2}\right)+\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)\right\}=0$$

$$\Rightarrow \sin\frac{x}{2}=0$$

or, $\cos\frac{x}{2}\left(\cos^2\frac{x}{2}+\sin^2\frac{x}{2}\right)+\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)=0$

Now, $\sin\frac{x}{2}=0\Rightarrow\frac{x}{2}=n\,\pi\Rightarrow x=2\,n\,\pi, n\in Z$

and,

$$\cos\frac{x}{2}\left(\cos^2\frac{x}{2}+\sin^2\frac{x}{2}\right)+\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)=0$$

$$\Rightarrow \left(1+\tan^2\frac{x}{2}\right)+\left(1-\tan^2\frac{x}{2}\right)\left(1+\tan\frac{x}{2}\right)=0$$

[On dividing by $\cos^3 x/2$]

$$\Rightarrow \tan^3\frac{x}{2}-\tan^2\frac{x}{2}-2=0$$

$\Rightarrow t^3-t^2-2=0$, where $t=\tan x/2$

Let $f(t)=t^3-t^2-2$. Then,

$f(1)<0$ and $f(2)>0$

Therefore, a root of $f(t)=0$ lies between 1 and 2

Let the root be k. Then,

$1<k<2$ and $\tan\frac{x}{2}=k$

$\Rightarrow \frac{x}{2}=n\,\pi+\tan^{-1}k, n\in Z$

$\Rightarrow x=2n\,\pi+2\tan^{-1}k, n\in Z$ and $1<k<2$

597 **(a)**

We have,

$$\tan\left(\frac{A-B}{2}\right)=\sqrt{\frac{1-\cos(A-B)}{1+\cos(A-B)}}=\sqrt{\frac{1-31/32}{1+31/32}}=\frac{1}{\sqrt{63}}$$

$$\Rightarrow \frac{a-b}{a+b}\cot\frac{C}{2}=\frac{1}{\sqrt{63}}\qquad\left[\because \tan\frac{A-B}{2}=\frac{a-b}{a+b}\cot\frac{C}{2}\right]$$

$$\Rightarrow \frac{1}{9}\cot\frac{C}{2}=\frac{1}{\sqrt{63}}\Rightarrow \tan\frac{C}{2}=\frac{\sqrt{7}}{3}$$

Now,

$$\cos C=\frac{1-\tan^2 C/2}{1+\tan^2 C/2}\Rightarrow \cos C=\frac{1-7/9}{1+7/9}=\frac{1}{8}$$

$\therefore c^2=a^2+b^2-2\,ab\cos C$

$\Rightarrow c^2=25+16-40\times 1/8=36\Rightarrow c=6$

598 **(c)**

We have, $\tan 20°\tan 40°\tan 60°\tan 80°=$ sin20°sin40°sin80°cos20°cos40°cos80°tan60°

Here, numerator $=(\sin 20°\sin 40°\sin 80°)$

$$=\frac{\sin 20°}{2}(2\sin 40°\sin 80°)$$

$$=\frac{\sin 20°}{2}(\cos 40°-\cos 120°)$$

$$=\frac{1}{2}\sin 20°\left(1-2\sin^2 20°+\frac{1}{2}\right)$$

$$=\frac{1}{2}\sin 20°\left(\frac{3}{2}-2\sin^2 20°\right)$$

$$=\frac{1}{4}[3\sin 20°-4\sin^3 20°]$$

$$=\frac{\sin 60°}{4}=\frac{\sqrt{3}}{8}$$

Now, denominator $=\cos 20°\cos 40°\cos 80°$

$$=\frac{\sin 2^3 20°}{2^3\sin 20°}=\frac{\sin 160°}{8\sin 20°}$$

$$=\frac{\sin 20°}{8\sin 20°}=\frac{1}{8}$$

Hence, $\tan 20° \tan 40° \tan 80° = \frac{\frac{\sqrt{3}}{8}}{\frac{1}{8}} = \sqrt{3}$

$\Rightarrow \tan 20° \tan 40° \tan 60° \tan 80° = \sqrt{3} \cdot \sqrt{3} = 3$

600 **(d)**

We have,

$f(x) = \sin^4 x + \cos^4 x, 0 \leq x \leq \frac{\pi}{2}$

$\Rightarrow f(x) = (\sin^2 x + \cos^2 x)^2 - \frac{1}{2}\sin^2 2x$

$\Rightarrow f(x) = 1 - \frac{1}{4}(1 - \cos 4x)$

$\Rightarrow f(x) = \frac{3}{4} + \frac{1}{4}\cos 4x$

$\because -1 \leq \cos 4x \leq 1$ for $x \in [0, \pi/2]$

$\therefore -\frac{1}{4} \leq \frac{1}{4}\cos 4x \leq \frac{1}{4}$ for all $x \in [0, \pi/2]$

$\Rightarrow \frac{1}{2} \leq \frac{3}{4} + \frac{1}{4}\cos 4x \leq 1$ for all $x \in [0, \pi/2]$

$\Rightarrow \frac{1}{2} \leq f(x) \leq 1$ for all $x \in [0, \pi/2]$

601 **(a)**

We have,

$\cot\theta = \sin 2\theta$

$\Rightarrow \cos\theta = 2\sin^2\theta\cos\theta$

$\Rightarrow \cos\theta(1 - 2\sin^2\theta) = 0$

$\Rightarrow \cos\theta = 0$ or, $1 - 2\sin^2\theta = 0$

$\Rightarrow \cos\theta = 0$ or, $\sin^2\theta = \left(\frac{1}{\sqrt{2}}\right)^2$

$\Rightarrow \theta = 90°$ or $\theta = 45°$

602 **(b)**

$\tan\left(\frac{\theta+\alpha}{2}\right).\tan\left(\frac{\theta-\alpha}{2}\right)$

$= \frac{\tan^2\frac{\theta}{2} - \tan^2\frac{\alpha}{2}}{1 - \tan^2\frac{\theta}{2}\tan^2\frac{\alpha}{2}}$

$= \frac{\sin^2\frac{\theta}{2}\cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}\cos^2\frac{\theta}{2}}{\cos^2\frac{\alpha}{2}\cos^2\frac{\theta}{2} - \sin^2\frac{\alpha}{2}\sin^2\frac{\theta}{2}}$

$= \frac{\cos^2\frac{\alpha}{2} - \cos^2\frac{\theta}{2}\cos^2\frac{\alpha}{2} - \cos^2\frac{\theta}{2} + \cos^2\frac{\alpha}{2}\cos^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2} - \sin^2\frac{\alpha}{2}\cos^2\frac{\theta}{2} - \sin^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2}\cos^2\frac{\theta}{2}}$

$= \frac{\cos^2\frac{\alpha}{2} - \cos^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2} - \sin^2\frac{\alpha}{2}} = \frac{(\cos\alpha - \cos\theta)}{(\cos\alpha + \cos\theta)}$

$= \frac{\cos\alpha(1 - \cos\beta)}{\cos\alpha(1 + \cos\beta)} = \tan^2\frac{\beta}{2}$

603 **(b)**

We have,

$\angle A = \frac{\pi}{3}, b : c = 2 : 3$ and $\tan\theta = \frac{\sqrt{3}}{5}$

Using Napier's analogy, we have

$\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c} = \cot\frac{A}{2}$

$\Rightarrow \tan\left(\frac{B-C}{2}\right) = -\frac{1}{5}\cot\frac{\pi}{6} = -\frac{\sqrt{3}}{5}$

$\Rightarrow \tan\left(\frac{B-C}{2}\right) = -\tan\theta$

$\Rightarrow \frac{B-C}{2} = -\theta$

$\Rightarrow C - B = 2\theta$

But, $C + B = 120°$ $\quad$ [$\because A = 60°$ (given)]

$\therefore 2C = 120° + 2\theta \Rightarrow C = 60° + \theta$

604 **(b)**

$\frac{\cos 70°}{\sin 70°} + 4\cos 70° = \frac{\cos 70° + 4\sin 70°\cos 70°}{\sin 70°}$

$= \frac{\cos 70° + 2\sin 140°}{\sin 70°}$

$= \frac{\sin 20° + 2\sin 40°}{\sin 70°}$

$= \frac{2\sin 30°\cos 10° + \sin 40°}{\sin 70°}$

$\frac{\sin 80° + \sin 40°}{\sin 70°}$

$= \frac{2\sin 60°\cos 20°}{\sin 70°}$

$= \frac{2\left(\frac{\sqrt{3}}{2}\right)\sin 70°}{\sin 70°} = \sqrt{3}$

605 **(b)**

Given, $\frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x = 1$

$\Rightarrow \cos\left(x + \frac{\pi}{4}\right) = -1$

$\Rightarrow x + \frac{\pi}{4} = 2n\pi + \pi \Rightarrow x = 2n\pi + \frac{3\pi}{4}$

606 **(a)**

Applying $1 + \cos\theta = 2\cos^2\frac{\theta}{2}$ n times, we get

$\sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots + \sqrt{2 + 2\cos\theta}}}} = 2\cos\frac{\theta}{2^n}$

607 **(d)**

We have,

$(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$
$= (\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$

$\Rightarrow (\sec^2 A - \tan^2 A)(\sec^2 B - \tan^2 B)(\sec^2 C - \tan^2 C)$

$= \{(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)\}^2$

$\Rightarrow 1 = \{(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)\}^2$

$\Rightarrow (\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) = \pm 1$

Hence, LHS = RHS = ± 1

608 **(d)**

$$\frac{\sin 55° - \cos 55°}{\sin 10°} = \frac{\sin 55° - \sin 35°}{\sin 10°}$$
$$= \frac{2\cos 45°.\sin 10°}{\sin 10°}$$
$$= \sqrt{2}$$

609 **(b)**

Given, $\frac{\tan 3A}{\tan A} = a$

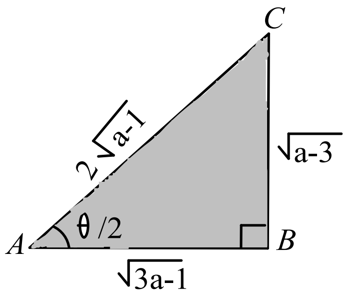
$\Rightarrow \frac{3\tan A - \tan^3 A}{\tan A(1 - 3\tan^2 A)} = a$

$\Rightarrow 3 - \tan^2 A = a - 3a\tan^2 A$

$\Rightarrow \tan^2 A(3a - 1) = a - 3$

$\Rightarrow \tan A = \pm\sqrt{\frac{a-3}{3a-1}}$

Now, $\frac{\sin 3A}{\sin A} = \frac{3\sin A - 4\sin^3 A}{\sin A}$



$= 3 - 4\sin^2 A = 3 - 4\left(\frac{a-3}{4(a-1)}\right)$

$= \frac{3a - 3 - a + 3}{(a-1)} = \frac{2a}{(a-1)}$

610 **(a)**

Using the relation

$\cos A + \cos B - \cos C = -1 + 4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

Put $A = 56°$, $B = 58°$, $C = 66°$

$\therefore \cos 56° + \cos 58° - \cos 66°$

$= -1 + 4\cos 28° \cos 29° \sin 33°$

$\Rightarrow 1 + \cos 56° + \cos 58° - \cos 66°$

$= 4\cos 28° \cos 29° \sin 33°$

611 **(c)**

From the given relations, we can say that α and β are roots of the equation

$x\cos\theta + y\sin\theta = 2a$

$\Rightarrow 2a - x\cos\theta = y\sin\theta$

$\Rightarrow (2a - x\cos\theta)^2 = y^2\sin^2\theta$

$\Rightarrow (x^2 + y^2)\cos^2\theta - 4ax\cos\theta + 4a^2 - y^2 = 0$

$\therefore \cos\alpha + \cos\beta = \frac{4ax}{x^2 + y^2}$, and $\cos\alpha\cos\beta$

$= \frac{4a^2 - y^2}{x^2 + y^2}$

Now,

$2\sin\frac{\alpha}{2}\sin\frac{\beta}{2} = 1$

$\Rightarrow 4\sin^2\frac{\alpha}{2}\sin^2\frac{\beta}{2} = 1$

$\Rightarrow (1 - \cos\alpha)(1 - \cos\beta) = 1$

$\Rightarrow \cos\alpha + \cos\beta = \cos\alpha\cos\beta$

$\Rightarrow \frac{4ax}{x^2 + y^2} = \frac{4a^2 - y^2}{x^2 + y^2}$

$\Rightarrow y^2 = 4a(a - x)$

613 **(b)**

Since, $\cos\theta$ is positive and $\tan\theta$ is negative, which lies in IVth quadrant.

$\therefore \quad \theta = 315° = \frac{7\pi}{4}$

$\therefore$ The general value of θ is $2n\pi + \frac{7\pi}{4},\ n \in I$

614 **(c)**

We have,

$\sin x = \cos 3x$

$\Rightarrow \sin x = 4\cos^3 x - 3\cos x$

$\Rightarrow \tan x \sec^2 x = 4 - 3\sec^2 x$

$\Rightarrow \tan x(1 + \tan^2 x) = 4 - 3(1 + \tan^2 x)$

$\Rightarrow \tan^3 x + 3\tan^2 x + \tan x - 1 = 0$

$\Rightarrow (\tan x + 1)(\tan^2 x + 2\tan x - 1) = 0$

$\Rightarrow \tan x + 1 = 0$ or, $1 - \tan^2 x = 2\tan x$

$\Rightarrow \tan x = -1$ or, $\tan 2x = 1$

$\Rightarrow x = \frac{3\pi}{4}, \frac{\pi}{8}, \frac{5\pi}{8}$

ALITER Graphs of $y = \sin x$ and $y = \cos 3x$ intersect at three points between 0 and π

615 **(b)**

We have,

$\sqrt{\frac{1 - \sin\theta}{1 + \sin\theta}} + \sqrt{\frac{1 + \sin\theta}{1 - \sin\theta}}$

$= \frac{1 - \sin\theta + 1 + \sin\theta}{\sqrt{1 - \sin^2\theta}}$

$= \frac{2}{|\cos\theta|} = \frac{2}{-\cos\theta} = -2\sec\theta \quad \left[\begin{matrix}\because \pi/2 < \theta < \pi \\ \therefore \cos\theta < 0\end{matrix}\right]$

616 **(d)**

$\sin\theta = \sin 15° + \sin 45°$

$= 2\sin 30°\cos 15° = 2 \times \frac{1}{2} \times \cos(90° - 75°)$

$\Rightarrow\ \sin\theta = \sin 75° \ \Rightarrow\ \theta = 75°$

617 **(c)**

Given, $\sinh^{-1} 2 + \sinh^{-1} 3 = x$

$\Rightarrow \cosh(\sinh^{-1} 2 + \sinh^{-1} 3) = \cosh x$

$\Rightarrow\ \cosh(\sinh^{-1} 2)\cosh(\sinh^{-1} 3)$
$\qquad + \sinh(\sinh^{-1} 2)\sinh(\sinh^{-1} 3)$

$= \cosh x$

$\Rightarrow\ \cosh x = \cosh(\cosh^{-1}\sqrt{1+2^2})$
$\qquad \times \cosh\left(\cosh^{-1}\sqrt{1+3^2}\right) + 2 \times 3$

$\Rightarrow\ \cosh x = (\sqrt{5}\sqrt{10} + 6) \times \frac{2}{2}$

$= \frac{1}{2}(12 + 2\sqrt{50})$

618 **(a)**

We have,

$\sin 7\,\theta + \sin\theta - \sin 4\,\theta = 0$

$\Rightarrow 2\sin 4\,\theta\cos 3\,\theta - \sin 4\,\theta = 0$

$\Rightarrow \sin 4\,\theta(2\cos 3\,\theta - 1) = 0$

$\Rightarrow \sin 4\,\theta = 0, \cos 3\,\theta = \frac{1}{2}$

Now, $\sin 4\,\theta = 0 \Rightarrow 4\,\theta = \pi \Rightarrow \theta = \frac{\pi}{4}$

and, $\cos 3\,\theta = \frac{1}{2} \Rightarrow 3\,\theta = \frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{9}$

619 **(b)**

We have,

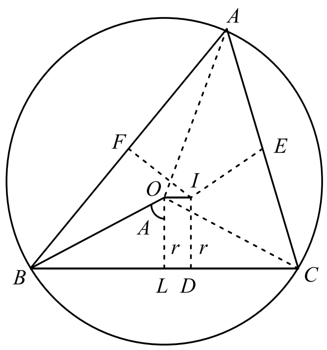
$OI \parallel BC \Rightarrow OL = ID \Rightarrow OL = r \quad [\because ID = r]$

In ΔBLO, we have

$\cos A = \frac{OL}{OB} \Rightarrow \cos A = \frac{r}{R}$

We know that

$\cos A + \cos B + \cos C = 1 + \frac{r}{R}$



$\Rightarrow \frac{r}{R} + \cos B + \cos C = 1 + \frac{r}{R}$

$\Rightarrow \cos B + \cos C = 1 \qquad \left[\because \cos A = \frac{r}{R}\right]$

620 **(c)**

Since, $2\sin^2\theta = 3\cos\theta$

$\Rightarrow\ 2 - 2\cos^2\theta = 3\cos\theta$

$\Rightarrow\ 2\cos^2\theta + 3\cos\theta - 2 = 0$

$\Rightarrow\ (2\cos\theta - 1)(\cos\theta + 2) = 0$

$\Rightarrow\ \cos\theta = \frac{1}{2} \quad (\because \cos\theta \neq -2)$

$\therefore\ \theta = \frac{\pi}{3}, \frac{5\pi}{3} \quad (\because 0 \leq \theta \leq 2\pi)$

621 **(d)**

Given, $3\tan(\theta - 15) = \tan(\theta + 15)$

$\frac{\tan A}{\tan B} = \frac{3}{1}$, where $A = \theta + 15°, B = \theta - 15°$

$\Rightarrow\ \frac{\tan A + \tan B}{\tan A - \tan B} = \frac{3+1}{3-1}$

(Applying componendo and dividendo)

$\Rightarrow\ \frac{\sin(A+B)}{\sin(A-B)} = 2$

$\Rightarrow\ \sin 2\theta = 2\sin 30°$

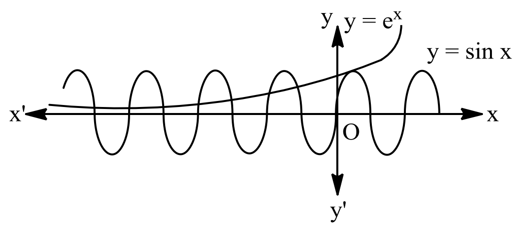
$\Rightarrow\ \sin 2\theta = \frac{2.1}{2} = 1 = \sin\frac{\pi}{2}$

$\Rightarrow\ 2\theta = 2n\pi + \frac{\pi}{2}$

$\Rightarrow\ \theta = n\pi + \frac{\pi}{4}$

622 **(d)**

Given equation of curves are $y = e^x$ and $y = \sin x$



It is clear from the figure that two curves intersect at infinite number of points.

623 **(a)**

Let $S = \sin 10° + \sin 20° + \sin 30° + \ldots + \sin 360°$

Here, the angles 10°, 20°, 30°, ... ,360° are in AP.

Where first term $(\alpha) = 10°$, Common difference $(\beta) = 10°$

Let number of terms be n

$\therefore\ 360° = 10° + (n-1)10°$

$\Rightarrow\ n - 1 = 35 \quad \Rightarrow \quad n = 36$

$\therefore\ S = \frac{\sin\frac{360°}{2}}{\sin 5°}\sin[10° + (36-1)5°]$

$= \frac{\sin 180°}{\sin 5°} \times \sin(180° + 5°) = -\sin 180° = 0$

624 **(c)**

$\frac{2\cos 8\theta + 1}{2\cos\theta + 1} = \frac{2(2\cos^2 4\theta - 1) + 1}{(2\cos\theta + 1)}$

$= \frac{(2\cos 4\theta - 1)(2\cos 4\theta + 1)}{(2\cos\theta + 1)}$

$= \frac{(2\cos 4\theta - 1)(2\cos 2\theta - 1)(2\cos 2\theta + 1)}{(2\cos\theta + 1)}$

$= \frac{[(2\cos 4\theta - 1)(2\cos 2\theta - 1)(2\cos\theta - 1)(2\cos}{(2\cos\theta + 1)}$

$= (2\cos 4\theta - 1)(2\cos 2\theta - 1)(2\cos\theta - 1)$

625 **(b)**

Since,sec α, cosec α are the roots of the equation

$x^2 - px + q = 0$

$\therefore \sec\alpha + \operatorname{cosec}\alpha = p, \sec\alpha \cdot \operatorname{cosec}\alpha = q$

$\Rightarrow \frac{\sin\alpha + \cos\alpha}{\sin\alpha\cos\alpha} = p, \sin\alpha\cos\alpha = \frac{1}{q}$

$\Rightarrow \sin\alpha + \cos\alpha = \frac{p}{q}$

$\Rightarrow \sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha = \frac{p^2}{q^2}$

$\Rightarrow 1 + \frac{2}{q} = \frac{p^2}{q^2} \Rightarrow q(q+2) = p^2$

626 **(b)**

Let $S = \sin\frac{\pi}{7}\sin\frac{2\pi}{7}\sin\frac{3\pi}{7}$. Then,

$S^2 = \sin\frac{2\pi}{7}\sin^2\frac{2\pi}{7}\sin^2\frac{3\pi}{7}$

$\Rightarrow S^2 = \frac{1}{8}\left(1 - \cos\frac{2\pi}{7}\right)\left(1 - \cos\frac{4\pi}{7}\right)\left(1 - \cos\frac{6\pi}{7}\right)$

$\Rightarrow S^2 = \left\{\left(1 - \cos\frac{2\pi}{7}\right)\left(1 + \cos\frac{3\pi}{7}\right)\left(1 + \cos\frac{\pi}{7}\right)\right\}$

$$\Rightarrow S^2 = \frac{1}{8}\left\{1 + \left(\cos\frac{\pi}{7} + \cos\frac{3\pi}{7} - \cos\frac{2\pi}{7}\right) - \cos\frac{\pi}{7}\cos\frac{3\pi}{7} - \cos\frac{\pi}{7}\cos\frac{2\pi}{7} - \cos\frac{\pi}{7}\cos\frac{2\pi}{7} - \cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{3\pi}{7}\right\}$$

$$\Rightarrow S^2 = \frac{1}{8}\left\{1 + \cos\frac{\pi}{7} + \cos\frac{3\pi}{7} - \cos\frac{2\pi}{7} - \cos\frac{\pi}{7} - \cos\frac{3\pi}{7} + \cos\frac{2\pi}{7} - \frac{1}{8}\right\}$$

$\Rightarrow S^2 = \frac{7}{64}$

Hence, $S = \frac{\sqrt{7}}{8}$

627 **(a)**

$\because \sin x + \sin^2 x = 1 \Rightarrow \sin x = \cos^2 x$

Now, $\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x$

$= \sin^6 x + 3\sin^5 x + 3\sin^4 x + \sin^3 x$

$= (\sin^2 x + \sin x)^3 = 1$

628 **(b)**

We have,

$B = A + C$

$\Rightarrow \tan B = \tan(A + C)$

$\Rightarrow \tan B = \frac{\tan A + \tan C}{1 - \tan A\tan C}$

$\Rightarrow \tan A\tan B\tan C = \tan B - \tan A - \tan C$

629 **(b)**

We have,

$mn = (\cos A + \cos B)(\sin A + \sin B)$

$\Rightarrow mn = \cos A\sin A + \sin(A + B) + \sin B\cos B$

$\Rightarrow 2mn = \sin 2A + \sin 2B + \sin(A + B)$

$\Rightarrow 2mn = 2\sin(A + B)\cos(A - B) + \sin(A + B)$ ...(i)

Also, we have,

$m^2 + n^2 = 2 + 2\cos(A - B)$ ...(ii)

From (i) and (ii), we have,

$\sin(A + B) = \frac{2mn}{m^2 + n^2}$

630 **(c)**

Given, $(5 + 4\cos\theta)(2\cos\theta + 1) = 0$ ...(i)

Since, $\cos\theta = \frac{1 - \tan^2\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}} = \frac{1 - t^2}{1 + t^2}$ $\left[\text{put} \tan\frac{\theta}{2} = t\right]$

$\therefore$ From Eq. (i),

$\left[5 + 4\left(\frac{1 - t^2}{1 - t^2}\right)\right]\left[2\left(\frac{1 - t^2}{1 + t^2}\right) + 1\right] = 0$

$\Rightarrow [5 + 5t^2 + 4 - 4t^2][2 - 2t^2 + 1 + t^2] = 0$

$\Rightarrow (t^2 + 9)(3 - t^2) = 0 \Rightarrow t = \pm\sqrt{3}$

$\therefore \tan\frac{\theta}{2} = \sqrt{3}$ or $\tan\frac{\theta}{2} = -\sqrt{3}$

$\Rightarrow \frac{\theta}{2} = \frac{\pi}{3}$ or $\frac{\theta}{2} = \frac{2\pi}{3}$

$\therefore \theta = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$

631 **(b)**

We have,

$\sin^6 x + \cos^6 x = \lambda$

$\Rightarrow (\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x - \sin^2 x\cos^2 x) = \lambda$

$\Rightarrow [(\sin^2 x + \cos^2 x)^2 - 3\sin^2 x\cos^2 x] = \lambda$

$\Rightarrow 1 - \frac{3}{4}\sin^2 2x = \lambda$

$\Rightarrow \sin 2x = \pm 2\sqrt{\frac{1 - \lambda}{3}}$

This equation has a solution if

$1 - \lambda \geq 0$ and $-1 \leq 2\sqrt{\frac{1 - \lambda}{3}} \leq 1$

$\Rightarrow \lambda \leq 1$ and $\frac{4}{3}(1 - \lambda) \leq 1$

$\Rightarrow \lambda \leq 1$ and $\lambda \geq \frac{1}{4} \Rightarrow \lambda \in [1/4, 1]$

632 **(d)**

We have,

$$y = \frac{\sin 3\theta}{\sin\theta} \Rightarrow y = 3 - 4\sin^2\theta \Rightarrow \sin^2\theta = \frac{3-y}{4}$$
Now,
$0 < \sin^2\theta \le 1 \quad [\because \theta \ne n\pi]$
$\Rightarrow 0 < \frac{3-y}{4} \le 1$
$\Rightarrow 0 < 3 - y \le 4$
$\Rightarrow -3 < -y \le 1 \Rightarrow -1 \le y < 3 \Rightarrow y \in [-1, 3)$

633 **(c)**
Since, $\sin^2\frac{1}{4} \Rightarrow \sin^2\theta = \sin^2\frac{\pi}{6}$
$\Rightarrow \theta = n\pi \pm \frac{\pi}{6}$

634 **(b)**
Given, $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r\theta$
$\Rightarrow \sin n\theta = b_0.\sin^0\theta + b_1\sin^1\theta + b_2$
$\sin^2\theta + \ldots + b_n\sin^n\theta$
$\Rightarrow \sin n\theta = b_0 + b_1\sin\theta + \ldots b_n\sin^n\theta$
$\because \sin n\theta = {}^nC_1\sin\theta\cos^{n-1}\theta - {}^nC_3$
$\sin^3\theta\cos^{n-3}\theta + \ldots$
$= {}^nC_1\sin\theta(1-\sin^2\theta)^{\frac{n-1}{2}} - {}^nC_3$
$\sin^3\theta(1-\sin^2\theta)^{(n-3)/2} + \ldots$
$\therefore b_0 = 0$
$b_1 =$ Coefficient of $\sin\theta = {}^nC_1 = n$
$[\because n-1, n-3$ are all even integer$]$

Alternate
$\sin n\theta = b_0 + b_1\sin\theta + b_2\sin^2\theta + \ldots + b_n\sin^n\theta$
Put $\theta = 0$, we get $b_0 = 1$
Again, $\frac{\sin n\theta}{\sin\theta} = \sum_{r=1}^{n} b_r\sin^{r-1}\theta$
Taking limit as $\theta \to 0$, we get
$\lim_{\theta\to 0}\frac{\sin n\theta}{\sin\theta} = b_1 + 0$
$\Rightarrow \quad n = b_1$

635 **(c)**
We have,
$k = \sin^6 x + \cos^6 x$
$\Rightarrow k = (\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x - \sin^2 x\cos^2 x)$
$\Rightarrow k = (1 - 3\sin^2 x\cos^2 x)$
$\Rightarrow k = \left(1 - \frac{3}{4}\sin^2 2x\right)$
Now,
$0 \le \frac{3}{4}\sin^2 2x \le \frac{3}{4}$, for all x
$\Rightarrow -\frac{3}{4} \le -\frac{3}{4}\sin^2 2x \le 0$, for all x
$\Rightarrow 1 - \frac{3}{4} \le 1 - \frac{3}{4}\sin^2 2x \le 1$, for all x
$\Rightarrow \frac{1}{4} \le 1 - \frac{3}{4}\sin^2 2x \le 1$, for all x
$\Rightarrow \frac{1}{4} \le k \le 1$

636 **(d)**
We have,
$\frac{1}{\cos 290°} + \frac{1}{\sqrt{3}\sin 250°}$
$= \frac{\sqrt{3}\sin 250° + \cos 290°}{\sqrt{3}\sin 250°\cos 290°}$
$= \frac{-\sqrt{3}\cos 20° + \sin 20°}{-\sqrt{3}\cos 20°\sin 20°}$
$= \frac{2(\sin 20° - \tan 60°\cos 20°)}{-\sqrt{3}(2\sin 20°\cos 20°)}$
$= \frac{2(\sin 20°\cos 60° - \cos 20°\sin 60°)}{-\sqrt{3}\sin 40°\cos 60°}$
$= \frac{2\sin(-40°)}{-\sqrt{3}/2\sin 40°} = \frac{4}{\sqrt{3}}$

637 **(a)**
Let $81^{\sin^2 x} = y$. Then,
$81^{\cos^2 x} = 81^{1-\sin^2 x} = 81\,y^{-1}$
Now,
$81^{\sin^2 x} + 81^{\cos^2 x} = 30$
$\Rightarrow y + \frac{81}{y} = 30$
$\Rightarrow y^2 - 30y + 81 = 0$
$\Rightarrow y = 3$ or, $y = 27$
$\Rightarrow 81^{\sin^2 x} = 3$ or, $81^{\sin^2 x} = 27$
$\Rightarrow 3^{4\sin^2 x} = 3^1$ or, $3^{4\sin^2 x} = 3^3$
$\Rightarrow 4\sin^2 x = 1, 4\sin^2 x = 3$
$\Rightarrow \sin x = \pm\frac{1}{\sqrt{2}}$ or, $\sin x = \pm\frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{6}$ or $\frac{\pi}{3}$

638 **(b)**
We have,
$\cos 9° - \sin 9°$
$= \sqrt{(\cos 9° - \sin 9°)^2} \quad [\because \cos 9° > \sin 9°]$
$= \sqrt{1 - \sin 18°} = \sqrt{1 - \left(\frac{\sqrt{5}-1}{4}\right)} = \sqrt{\frac{5-\sqrt{5}}{2}}$

639 **(c)**
$\text{sech}^{-1}\left(\frac{1}{2}\right) = \cosh^{-1}(2)$
$= \log\left(2 + \sqrt{2^2 - 1}\right) = \log(2 + \sqrt{3})$

640 **(a)**
Since the triangle ABC is right angled at B
$\therefore \tan\frac{B}{2} = 1$
$\Rightarrow \sqrt{\frac{(s-c)(s-c)}{s(s-b)}} = 1 \Rightarrow (s-c)(s-a)$
$= s(s-b) \ldots \text{(i)}$
Now,
$r = \frac{\Delta}{s}$

$$\Rightarrow r=\frac{\sqrt{s(s-a)(s-b)(s-c)}}{s}$$

$$\Rightarrow r=\frac{s(s-b)}{s} \quad \text{[Using : (i)]}$$

$$\Rightarrow 2r=2s-2b \Rightarrow 2r=a+c-b$$

641 **(b)**

When, $\theta \in \left(0,\frac{\pi}{4}\right)$

$\tan\theta < \cot\theta$

Since, $\tan\theta<1$ and $\cot\theta>1$

$\therefore$ $(\tan\theta)^{\cot\theta}<1$ and $(\cot\theta)^{\tan\theta}>1$

$\therefore$ $t_4>t_1$, which only holds in (b)

642 **(a)**

We have,

$$\frac{\tan\alpha+\tan\beta}{\cot\alpha+\cot\beta}+\{\cos(\alpha-\beta)\sec(\alpha+\beta)+1\}^{-1}=1$$

$$\Rightarrow \frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}\times\frac{\sin\alpha\sin\beta}{\sin(\alpha+\beta)}+\frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)+\cos(\alpha+\beta)}=1$$

$$\Rightarrow \tan\alpha\tan\beta+\frac{\cos\alpha\cos\beta-\sin\alpha\sin\beta}{2\cos\alpha\cos\beta}=1$$

$$=\frac{1}{2}\tan\alpha\tan\beta+\frac{1}{2}=1\Rightarrow\tan\alpha\tan\beta=1$$

643 **(c)**

We have,

$$3(\sin\theta-\cos\theta)^4+6(\sin\theta+\cos\theta)^2+4(\sin^6\theta+\cos^6\theta)$$

$$=3\{\sin^4\theta+\cos^4\theta-4\sin^3\theta\cos\theta+6\sin^2\theta\cos^2\theta-4\sin\theta\cos^3\theta\}$$
$$+6[1+2\sin\theta\cos\theta]+4[\sin^4\theta+\cos^4\theta-\sin^2\theta\cos^2\theta]$$

$$=7[\sin^4\theta+\cos^4\theta]+14\sin^2\theta\cos^2\theta-12\sin\theta\cos\theta+6+12\sin\theta\cos\theta$$

$$=7(\sin^2\theta+\cos^2\theta)^2+6=13$$

644 **(a)**

Let $f(\theta)=\cos\theta-\theta+\frac{1}{2}$. Then,

$f(0)=1+\dfrac{1}{2}>0$ and $f\left(\dfrac{\pi}{2}\right)=\dfrac{1-\pi}{2}<0$

Clearly, $f(\theta)$ is a continuous function on $(0,\pi/2)$

Hence, a root of $f(\theta)=0$ lies in the interval $(0,\pi/2)$

645 **(a)**

We know that, $\sin 22\frac{1^\circ}{2}=\frac{1}{2}\sqrt{2-\sqrt{2}}$

and $\cos 22\frac{1^\circ}{2}=\frac{1}{2}\sqrt{2+\sqrt{2}}$

Since, $\alpha=22°30'=22\frac{1^\circ}{2}$

$$\therefore \left(1+\cos 22\frac{1^\circ}{2}\right)\left(1+\cos 67\frac{1^\circ}{2}\right)\times\left(1+\cos 112\frac{1^\circ}{2}\right)\left(1+\cos 157\frac{1^\circ}{2}\right)$$

$$=\left(1+\frac{1}{2}\sqrt{2+\sqrt{2}}\right)\left(1+\frac{1}{2}\sqrt{2-\sqrt{2}}\right)\times\left(1-\frac{1}{2}\sqrt{2-\sqrt{2}}\right)\left(1-\frac{1}{2}\sqrt{2+\sqrt{2}}\right)$$

$$=\left[1-\frac{1}{4}(2+\sqrt{2})\right]\left[1-\frac{1}{4}(2-\sqrt{2})\right]$$

$$=\frac{(2-\sqrt{2})(2+\sqrt{2})}{16}$$

$$=\frac{4-2}{16}=\frac{1}{8}$$

646 **(c)**

$$\sin A\sin(60°-A)\sin(60°+A)$$

$$=\sin A(\sin^2 60°-\sin^2 A)$$

$$=\sin A\left(\frac{3}{4}-\sin^2 A\right)$$

$$=\frac{3\sin A-4\sin^3 A}{4}=\frac{\sin 3A}{4}$$

647 **(a)**

$B+C=\pi-A$

$\Rightarrow$ $\sin(B+C)=\sin(\pi-A)=\sin A$

$\therefore$ $\sin 2A+\sin 2B+\sin 2C$

$=2\sin A\cos A+2\sin(B+C)\cos(B-C)$

$=2\sin A[\cos A+\cos(B-C)]$

$=2\sin A[\cos(B-C)-\cos(B+C)]$

$=2\sin A[2\sin B\sin C]$

$=4\sin A\sin B\sin C$

648 **(b)**

Given equations can be rewritten as

$$\cos\theta=\frac{a}{x-h},\qquad \sin\theta=\frac{b}{y-k}$$

Now, $\frac{a^2}{(x-h)^2}+\frac{b^2}{(y-k)^2}=1$ $[\because \cos^2\theta+\sin^2\theta=1]$

649 **(d)**

Here, $a=3$, and $b=4$

$\therefore$ Maximum value$=\sqrt{3^2+4^2}=5$

650 **(b)**

Given, $3\sin^2 x + 10\cos x - 6 = 0$

$\Rightarrow\ 3(1-\cos^2 x) + 10\cos x - 6 = 0$

$\Rightarrow\ -3\cos^2 x + 10\cos x - 3 = 0$

$\Rightarrow\ (\cos x - 3)(1 - 3\cos x) = 0$

$\Rightarrow\ \cos x \neq 3$ or $\cos x = \frac{1}{3}$

$\Rightarrow\ x = 2n\pi \pm \cos^{-1}\left(\frac{1}{3}\right)$

651 **(a)**

Given, $1 + \sin x\left(\frac{1-\cos x}{2}\right) = 0$

$\Rightarrow\ \sin 2x - 2\sin x = 4$

Since, the maximum values of $\sin x$ and $\sin 2x$ are 1, which is not possible for any x in $[-\pi, \pi]$

652 **(b)**

Given, $2\sin^2\frac{\theta}{2} = 2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}.\sin\frac{\theta}{2}$

$\Rightarrow\ 2\sin^2\frac{\theta}{2}\left[1-\cos\frac{\theta}{2}\right] = 0$

$\Rightarrow\ \sin\frac{\theta}{2} = 0$ or $2\sin^2\frac{\theta}{4} = 0$

$\Rightarrow\ \frac{\theta}{2} = k\pi$ or $\frac{\theta}{4} = k\pi$

Hence, $\theta = 2k\pi$ or $\theta = 4k\pi,\ k \in I$

653 **(a)**

We have, $\sin\left(x+\frac{\pi}{6}\right) + \cos\left(x+\frac{\pi}{6}\right)$

$= \sqrt{2}\left[\frac{1}{\sqrt{2}}\sin\left(x+\frac{\pi}{6}\right) + \frac{1}{\sqrt{2}}\cos\left(x+\frac{\pi}{6}\right)\right]$

$= \sqrt{2}\cos\left[x + \frac{\pi}{6} - \frac{\pi}{4}\right]$

$= \sqrt{2}\cos\left(x - \frac{\pi}{12}\right)$

Hence, maximum value will be at $x = \frac{\pi}{12}$

654 **(a)**

We have,

$\sin^4 x + \cos^4 x + \sin 2x + \alpha = 0$

$\Rightarrow (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x + \sin 2x + \alpha = 0$

$\Rightarrow 1 - \frac{1}{2}\sin^2 2x + \sin 2x + \alpha = 0$

$\Rightarrow \sin^2 2x - 2\sin 2x - 2 - 2\alpha = 0$

$\Rightarrow (\sin 2x - 1)^2 = 3 + 2\alpha$

$\Rightarrow \sin 2x = 1 \pm \sqrt{3+2\alpha}$

This equation is meaningful if

$-1 \leq 1 \pm \sqrt{3+2\alpha} \leq 1$

$\Rightarrow -2 \leq \pm\sqrt{3+2\alpha} \leq 0$

$\Rightarrow 0 \leq 3 + 2\alpha \leq 4$ and $3 + 2\alpha \geq 0$

$\Rightarrow -3 \leq 2\alpha \leq 1$ and $\alpha \geq -\frac{3}{2} \Rightarrow -\frac{3}{2} \leq \alpha \leq \frac{1}{2}$

655 **(c)**

We have,

$\sin^2 5° + \sin^2 10° + \sin^2 15° + \cdots + \sin^2 85° + \sin^2 90°$

$= (\sin^2 5° + \sin^2 85°) + (\sin^2 10° + \sin^2 80°) + (\sin^2 15° + \sin^2 75°) + \cdots + (\sin^2 40° + \sin^2 50°) + (\sin^2 45° + \sin^2 90°)$

$= 8 + \frac{1}{2} + 1 = 9\frac{1}{2}$

656 **(d)**

$1 + \cos x = k$

$\Rightarrow\ 1 + 1 - 2\sin^2\frac{x}{2} = k$

$\Rightarrow\ 1 - \sin^2\frac{x}{2} = \frac{k}{2}$

$\Rightarrow\ \sin^2\frac{x}{2} = 1 - \frac{k}{2}$

$\Rightarrow\ \sin\frac{x}{2} = \sqrt{\frac{2-k}{2}}$

657 **(c)**

We have,

$\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$

$\Rightarrow \frac{\cos A}{R\sin A} = \frac{\cos B}{R\sin B} = \frac{\cos C}{R\sin C}$ [Using : Sine rule]

$\Rightarrow \cot A = \cot B = \cot C$

$\Rightarrow A = B = C \Rightarrow \Delta ABC$ is equilateral

658 **(a)**

We have,

$c = (a+b)\sin\theta$

$\Rightarrow \sin\theta = \frac{c}{a+b}$

$\Rightarrow \cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1 - \frac{c^2}{(a+b)^2}}$

$\Rightarrow \cos\theta = \sqrt{\frac{(a+b)^2 - c^2}{(a+b)^2}}$

$\Rightarrow \cos\theta = \frac{\sqrt{(a+b+c)(a+b-c)}}{a+b}$

$\Rightarrow \cos\theta = \sqrt{\frac{2s(2s-2c)}{(a+b)^2}}$

$\Rightarrow \cos\theta = 2\sqrt{\frac{s(s-c)}{ab}} \times \frac{\sqrt{ab}}{a+b}$

$\Rightarrow \frac{k\sqrt{ab}}{a+b} = 2\cos\frac{C}{2} \times \frac{\sqrt{ab}}{a+b} \Rightarrow k = 2\cos\frac{C}{2}$

659 **(b)**

We have,

$$\cos\frac{\pi}{7}+\cos\frac{2\pi}{7}+\cos\frac{3\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{5\pi}{7}+\frac{6\pi}{7}+\cos\frac{7\pi}{7}$$

$$=\left(\cos\frac{\pi}{7}+\cos\frac{6\pi}{7}\right)+\left(\cos\frac{2\pi}{7}+\cos\frac{5\pi}{7}\right)+\left(\cos\frac{3\pi}{7}+\cos\frac{4\pi}{7}\right)+\cos\pi$$

$$=\left(\cos\frac{\pi}{7}-\cos\frac{\pi}{7}\right)+\left(\cos\frac{2\pi}{7}-\cos\frac{2\pi}{7}\right)+\left(\cos\frac{3\pi}{7}-\cos\frac{3\pi}{7}\right)+\cos\pi$$

$=\cos\pi=-1$

ALITER This can be done by using the fact that the sum of the roots of $x^7-1=0$ is zero

660 **(a)**

$2\sin x=5x^2+2x+3$

$\Rightarrow\ 2\sin x=4x^2+(x+1)^2+2$

But $2\sin x\le 2$

and $4x^2+(x+1)^2+2>2$, so it has no solution

661 **(d)**

Let ABC be the triangle with A as the least angle.

Then, the other angles are

$B=A+\frac{A}{3}$ and $c=A+\frac{2A}{3}$

Now,

$A+B+C=180°$

$\Rightarrow A+\left(A+\frac{A}{3}\right)+\left(A+\frac{2A}{3}\right)=180°\Rightarrow A=45°$

Thus, we have

$A=45°, B=60°$ and $C=75°$

Now,

$a:b:c=\sin A:\sin B:\sin C$

$$\Rightarrow a:b:c=\frac{1}{\sqrt{2}}:\frac{\sqrt{3}}{2}:\frac{\sqrt{3}+1}{2\sqrt{2}}=2\sqrt{2}:2\sqrt{3}:\sqrt{2}+\sqrt{6}$$

662 **(a)**

We have,

$5\sin x+3\sin(x-\theta)$

$=(5+3\cos\theta)\sin x-3\sin\theta\cos x$

$\le\sqrt{(5+3\cos\theta)^2+9\sin^2\theta}$

$\therefore$ Max$\{5\sin x+3\sin(x-\theta)\}$

$=\sqrt{(5+3\cos\theta)^2+9\sin^2\theta}$

$\Rightarrow 7=\sqrt{34+30\cos\theta}$

$\Rightarrow 34+30\cos\theta=49\Rightarrow\cos\theta=\frac{1}{2}\Rightarrow\theta=2n\pi\pm\frac{\pi}{3}, n\in Z$

663 **(c)**

Given that, $\sin\theta+\cos\theta=m$ ... (i)

and $\sec\theta+\operatorname{cosec}\theta=n$ ... (ii)

Now, $n(m+1)(m-1)=n(m^2-1)$

$=(\sec\theta+\operatorname{cosec}\theta)2\sin\theta\cos\theta$ $(\because m^2=1+2\sin\theta\cos\theta)$

$$=\frac{\sin\theta+\cos\theta}{\sin\theta\cos\theta}2\sin\theta\cos\theta$$

$=2m$ [from Eq.(i)]

664 **(a)**

Given that, $\tan^2\theta-\tan\theta-\sqrt{3}\tan\theta+\sqrt{3}=0$

$\Rightarrow\ \tan\theta(\tan\theta-1)-\sqrt{3}(\tan\theta-1)=0$

$\Rightarrow(\tan\theta-\sqrt{3})\,(\tan\theta-1)=0$

$\Rightarrow\ \theta=n\pi+\frac{\pi}{3}, n\pi+\frac{\pi}{4}$

665 **(b)**

We have, $\frac{x}{\cos\theta}=\frac{y}{\cos\left(\theta-\frac{2\pi}{3}\right)}=\frac{z}{\cos\left(\theta+\frac{2\pi}{3}\right)}$

Therefore, each ratio is equal to $\frac{x+y+z}{\cos\theta+\cos\left(\theta-\frac{2\pi}{3}\right)+\cos\left(\theta+\frac{2\pi}{3}\right)}$

$$=\frac{x+y+z}{\cos\theta+2\cos\theta\cos\frac{2\pi}{3}}$$

$$=\frac{x+y+z}{0}$$

$\Rightarrow x+y+z=0$

666 **(d)**

We have,

$e^{\log_{10}\tan 1°+\log_{10}\tan 2°+\log_{10}3°+\cdots+\log_{10}\tan 89°}$

$=e^{\log_{10}(\tan 1°\tan 2°\tan 3°\ldots\tan 89°)}=e^{\log_{10}1}=e^0=1$

667 **(b)**

We have, $\sin^3 x\sin 3x=\sum_{m=0}^{n}c_m\cos mx$

Now, $\sin^3 x\sin 3x=\frac{1}{4}(3\sin x-\sin 3x)\sin 3x$

$$=\frac{3}{8}\cdot 2\sin x\sin 3x-\frac{1}{8}\cdot 2\sin^2 3x$$

$$=\frac{3}{8}(\cos 2x-\cos 4x)-\frac{1}{8}(1-\cos 6x)$$

$$=-\frac{1}{8}+\frac{3}{8}\cos 2x-\frac{3}{8}\cos 4x+\frac{1}{8}\cos 6x\ \ldots\text{(i)}$$

$$\text{RHS}=\sum_{m=0}^{n}c_m\cos mx$$

$=$
$c_0 +$
$c_1 \cos x + c_2 \cos 2x + c_3 \cos 3x + \ldots + c_n \cos nx$
...(ii)

On comparing Eqs.(i)and (ii), we get $n = 6$

668 **(c)**

Given, $\tan\left(90° - 22\frac{1}{2}°\right) + \cot\left(90° - 22\frac{1}{2}°\right)$

$= \tan 22\frac{1}{2}° + \cot 22\frac{1}{2}° = \sqrt{2} - 1 + \sqrt{2} + 1 = 2\sqrt{2}$

669 **(c)**

We have

$\frac{\sin^2 A + \sin A + 1}{\sin A}$

$= \sin A + 1 + \frac{1}{\sin A}$

$= \left(\sin A + \frac{1}{\sin A}\right) + 1 \geq 2 + 1 = 3 \quad \left[\because x + \frac{1}{x} \geq 2\right]$

$\frac{\sin^2 A + \sin A + 1}{\sin A} \geq 3$

$\therefore \prod \frac{\sin^2 A + \sin A + 1}{\sin A} \geq 3 \times 3 \times 3 = 27$

670 **(a)**

Given, $2\cos(e^x) = 5^x + 5^{-x}$

Since, $\cos e^x \leq 1 \Rightarrow 2\cos e^x \leq 2$ ...(i)

And $\frac{5^x + 5^{-x}}{2} \geq \sqrt{5^x . 5^{-x}}$

$\Rightarrow 5^x + 5^{-x} \geq 2$

$\therefore$ LHS ≤ 2, RHS≥ 2

Now, $5^x + \frac{1}{5^x} = 2$ at $x = 0$

But, at $x = 0$

$2\cos e^x \neq 2$

Hence, no solution will exist

671 **(b)**

Let r be the radius of the circle. Then,

$A_1 = nr^2 \sin\frac{\pi}{n}, A_2 = \frac{n}{2} r^2 \sin^2\frac{\pi}{n}$ and $A_3 = nr^2 \tan\frac{\pi}{n}$

Now, $A_2 A_3 = \frac{n^2}{2} r^4 \sin\frac{2\pi}{n} \frac{\sin\frac{\pi}{n}}{\cos\frac{\pi}{n}}$

$\Rightarrow A_2 A_3 = \frac{n^2}{2} r^4 \left(2\sin^2\frac{\pi}{n}\right) = \left(nr^2 \sin\frac{\pi}{n}\right)^2 = A_1^2$

$\Rightarrow A_2, A_2, A_3$ are in G. P.

672 **(a)**

Since, $\tan\theta + \tan\left(\frac{3\pi}{4} + \theta\right) = 2$

$\therefore \tan\theta + \frac{-1 + \tan\theta}{1 + \tan\theta} = 2$

$\Rightarrow \tan\theta + \tan^2\theta - 1 + \tan\theta = 2 + 2\tan\theta$

$\Rightarrow \tan^2\theta = 3$

$\Rightarrow \tan^2\theta = \left(\sqrt{3}\right)^2 = \tan^2\frac{\pi}{3}$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{3}, n \in I$

673 **(b)**

$\frac{\sin(B + A) + \cos(B - A)}{\sin(B - A) + \cos(B + A)}$

$= \frac{\sin(B + A) + \sin(90° - \overline{B - A})}{\sin(B - A) + \sin(90° - \overline{A + B})}$

$= \frac{2\sin(A + 45°)\cos(45° - B)}{2\sin(45° - A)\cos(45° - B)}$

$= \frac{\sin(A + 45°)}{\sin(45° - A)} = \frac{\frac{1}{\sqrt{2}}\sin A + \frac{1}{\sqrt{2}}\cos A}{\frac{1}{\sqrt{2}}\cos A - \frac{1}{\sqrt{2}}\sin A}$

$= \frac{\cos A + \sin A}{\cos A - \sin A}$

674 **(d)**

Let $f(x) = 3\cos x + 4\sin x + 5$

Since, $-\sqrt{3^2 + 4^2} \leq 3\cos x + 4\sin x \leq \sqrt{3^2 + 4^2}$

$\Rightarrow -5 \leq 3\cos x + 4\sin x \leq 5$

$\Rightarrow -5 + 5 \leq 3\cos x + 4\sin x + 5 \leq 5 + 5$

$\Rightarrow 0 \leq f(x) \leq 10$

Hence, maximum value of $f(x)$ is 10

675 **(a)**

$\sin A + \sqrt{3}\cos A = \sqrt{3}\cos B - \sin B$

$\Rightarrow \frac{1}{2}\sin A + \frac{\sqrt{3}}{2}\cos A = \frac{\sqrt{3}}{2}\cos B - \frac{1}{2}\sin B$

$\Rightarrow \cos\frac{\pi}{3}\sin A + \sin\frac{\pi}{3}\cos A = \sin\frac{\pi}{3}\cos B - \cos\frac{\pi}{3}\sin B$

$\Rightarrow \sin\left(A + \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3} - B\right)$

$\Rightarrow A + \frac{\pi}{3} = \frac{\pi}{3} - B$

$\Rightarrow A = -B$

Now, $\sin 3(A) + \sin 3B = \sin(-3B) + \sin 3B$

$= -\sin 3B + \sin 3B = 0$

676 **(c)**

$3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x)$

$= 3(1 - \sin 2x)^2 + 6(1 + \sin 2x) + 4\{(\sin^2 x + \cos^2 x)^3 - 3\sin^2 x \cos^2 x \cdot (\sin^2 x + \cos^2 x)\}$

$= 3(1 - 2\sin 2x + \sin^2 2x) + 6 + 6\sin 2x + 4\{1 - 3\sin^2 x \cos^2 x\}$

$= 3\{1 - 2\sin 2x + \sin^2 2x + 2 + 2\sin 2x\} + 4\left\{1 - \frac{3}{4}\sin^2 2x\right\}$

$= 13 + 3\sin^2 2x - 3\sin^2 2x = 13$

677 **(d)**

$m\tan(\theta - 30°) = n\tan(\theta + 120°)$

$\Rightarrow\ m\left(\dfrac{\tan\theta - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}\tan\theta}\right) = -n\left(\dfrac{-\tan\theta + \sqrt{3}}{1 + \sqrt{3}\tan\theta}\right)$

$\Rightarrow\ m\left[\left(\sqrt{3}\tan\theta\right)^2 - 1\right] = -n(-\tan^2\theta + 3)$

$\Rightarrow\ 3m\tan^2\theta - m = n\tan^2\theta - 3n$

$\Rightarrow\ \tan^2\theta = \dfrac{m - 3n}{3m - n}$

Now, $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$= \dfrac{1 - \frac{m-3n}{3m-n}}{1 + \frac{m-3n}{3m-n}} = \dfrac{3m - n - m + 3n}{3m - n + m - 3n}$

$= \dfrac{2(m+n)}{4(m-n)} = \dfrac{m+n}{2(m-n)}$

678 **(c)**

$\sqrt{3}\text{cosec } 20° - \sec 20°$

$= \dfrac{\tan 60°}{\sin 20°} - \dfrac{1}{\cos 20°}$

$= \dfrac{\sin 60°\cos 20° - \sin 20°\cos 60°}{\cos 60°\sin 20°\cos 20°}$

$= \dfrac{2\sin 20°\cos 20°}{\frac{1}{2}(\sin 20°\cos 20°)} = 4$

679 **(b)**

Let $u = \cos\theta\left\{\sin\theta + \sqrt{\sin^2\theta + \sin^2\alpha}\right\}$

$\Rightarrow (u - \sin\theta\cos\theta)^2 = \cos^2\theta\,(\sin^2\theta + \sin^2\alpha)$

$\Rightarrow u^2\tan^2\theta - 2u\tan\theta + u^2 - \sin^2\alpha = 0$

$\Rightarrow 4u^2 - 4u^2(u^2 - \sin^2\alpha)$

≥ 0 [$\because \tan\theta$ is real $\therefore$ Disc ≥ 0]

$\Rightarrow u^2 - (1 + \sin^2\alpha) \geq 0 \Rightarrow |u| \leq \sqrt{1 + \sin^2\alpha}$

680 **(b)**

$\because\ \sec 2\theta = \dfrac{1}{\cos 2\theta} = \dfrac{1 + \tan^2\theta}{1 - \tan^2\theta}$

$\therefore\ \tan^2\theta + \sec 2\theta = 1$ (given)

$\Rightarrow\ \tan^2\theta + \dfrac{1 + \tan^2\theta}{1 - \tan^2\theta} = 1$

$\Rightarrow\ \tan^2\theta(1 - \tan^2\theta) + 1 + \tan^2\theta = 1 - \tan^2\theta$

$\Rightarrow\ 3\tan^3\theta - \tan^4\theta = 0$

$\Rightarrow\ \tan^2\theta(3 - \tan^2\theta) = 0$

$\Rightarrow\ \tan\theta = 0$ or $\tan\theta = \pm\sqrt{3}$

Now, $\tan\theta = 0\ \Rightarrow\ \theta = m\pi$

where m is an integer

And $\tan\theta = (\pm\sqrt{3}) = \tan\left(\pm\frac{\pi}{3}\right)$

$\Rightarrow\ \theta = n\pi \pm \dfrac{\pi}{3}$

where n is an integer

Thus, $\theta = m\pi, n\pi \pm \frac{\pi}{3}$

681 **(a)**

Given, $2\sec 2\alpha = \tan\beta + \cot\beta$

$\Rightarrow\ 2\sec 2\alpha = \dfrac{1 + \tan^2\beta}{\tan\beta} = \dfrac{\sec^2\beta}{\tan\beta}$

$= \dfrac{2}{2\cos\beta.\sin\beta} = 2\text{ cosec } 2\beta$

$\therefore\ \sec 2\alpha = \sec\left(\dfrac{\pi}{2} - 2\beta\right)$

$\Rightarrow\ 2\alpha = 2n\pi \pm \left(\dfrac{\pi}{2} - 2\beta\right)$

Taking +ve sing, we have

$2(\alpha + \beta) = 2n\pi + \dfrac{\pi}{2}$

$\Rightarrow\ \alpha + \beta = n\pi + \dfrac{\pi}{4},\qquad n \in I$

For, $\quad n = 0, \alpha + \beta = \frac{\pi}{4}$

682 **(a)**

Given, $\tan x + \sec x = 2\cos x$

$\Rightarrow 1 + \sin x = 2 - 2\sin^2 x$

$\Rightarrow\ (2\sin x - 1)(\sin x + 1) = 0$

$\Rightarrow\ \sin x = -1, \dfrac{1}{2}$

$\Rightarrow\ x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{3\pi}{2}$

But at $x = \frac{3\pi}{2}$ given equation does not exist

683 **(d)**

$\tan 1° \tan 2° \tan 3° \ldots \tan 89°$

$= \tan 1° \tan 2° \tan 3° \ldots \tan 45° \cot 44° \ldots \cot 2° \cot$

$= (\tan 1° \cot 1°)(\tan 2° \cot 2°) \ldots \tan 45°$

$= 1.1 \ldots .1 = 1$

684 **(a)**

We have,

$\sin 12° \sin 24° \sin 48° \sin 84°$

$= \dfrac{1}{4}(2\sin 12° \sin 48°)(2\sin 24° \sin 48°)$

$= \dfrac{1}{2}(\cos 36° - \cos 60°)(\cos 60° - \cos 108°)$

$= \dfrac{1}{4}\left(\cos 36° - \dfrac{1}{2}\right)\left(\dfrac{1}{2} + \sin 18°\right)$

$= \dfrac{1}{4}\left\{\dfrac{1}{4}(\sqrt{5} + 1) - \dfrac{1}{2}\right\}\left\{\dfrac{1}{2} + \dfrac{1}{4}(\sqrt{5} - 1)\right\} = \dfrac{1}{16}$

and, $\cos 20° \cos 40° \cos 60° \cos 80°$

$= \dfrac{1}{2}[\cos(60° - 20°)\cos 20° \cos(60° + 20°)]$

$= \dfrac{1}{2}\left\{\dfrac{1}{4}\cos 3(20°)\right\} = \dfrac{1}{8}\cos 60° = \dfrac{1}{2} \times \dfrac{1}{8} = \dfrac{1}{16}$

685 **(a)**

We have,

$8\tan^2\dfrac{\theta}{2} = 1 + \sec\theta$

$\Rightarrow 8\left(\dfrac{1 - \cos\theta}{1 + \cos\theta}\right) = \dfrac{1 + \cos\theta}{\cos\theta}$

$\Rightarrow 8\cos\theta(1 - \cos\theta) = (1 + \cos\theta)^2$

$\Rightarrow 9\cos^2\theta - 6\cos\theta + 1 = 0$

$\Rightarrow \cos\theta = \frac{1}{3} \Rightarrow \theta = 2n\pi \pm \cos^{-1}\left(\frac{1}{3}\right), n \in Z$

686 **(b)**

We have,

$\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$

$\Rightarrow \frac{\sin(x+y)+\sin(x-y)}{\sin(x-y)-\sin(x-y)} = \frac{(a+b)+(a-b)}{(a+b)-(a-b)}$

$\Rightarrow \frac{2\sin x\cos y}{2\cos x\sin y} = \frac{2a}{2b}$

$\Rightarrow \frac{\tan x}{\tan y} = \frac{a}{b}$

687 **(a)**

We have, $\alpha + \beta + \gamma = 2\pi$

$\Rightarrow \frac{\alpha}{2} + \frac{\beta}{2} + \frac{\gamma}{2} = \pi$

$\Rightarrow \frac{\alpha}{2} + \frac{\beta}{2} = \pi - \frac{\gamma}{2}$

$\Rightarrow \tan\left(\frac{\alpha}{2} + \frac{\beta}{2}\right) = \tan\left(\pi - \frac{\gamma}{2}\right)$

$\Rightarrow \frac{\tan\frac{\alpha}{2} + \tan\frac{\beta}{2}}{1 - \tan\frac{\alpha}{2}\tan\frac{\beta}{2}} = -\tan\frac{\gamma}{2}$

$\Rightarrow \tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

688 **(c)**

We have,

$\tan\theta = -1$ and $\cos\theta = \frac{1}{\sqrt{2}}$

The value of θ lying between 0 and 2π and satisfying these two is $\frac{7\pi}{4}$. Therefore, the most general solution is

$\theta - 2n\pi + \frac{7\pi}{4}$, where $n \in Z$

689 **(d)**

We have,

$c^2 = a^2 + b^2 - 2ab\cos C \Rightarrow c^2 = a^2 + b^2 - ab$ [$\because C = 60°$]

Now,

$\frac{a}{b+c} + \frac{b}{c+a} = \frac{ac + a^2 + b^2 + bc}{bc + ba + ca + c^2}$

$\Rightarrow \frac{a}{b+c} + \frac{b}{c+a} = \frac{ac + bc + (c^2 + ab)}{bc + ba + ca + c^2} = 1$ [$\because a^2 + b^2 = c^2 + ab$]

690 **(b)**

We have, $\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}$

$= \frac{\sqrt{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2} + \sqrt{\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)^2}}{\sqrt{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2} - \sqrt{\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)^2}}$

$= \frac{\cos\frac{x}{2} + \sin\frac{x}{2} + \sin\frac{x}{2} - \cos\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2} - \sin\frac{x}{2} + \cos\frac{x}{2}} = \tan\frac{x}{2}$

691 **(a)**

We have,

$\cos(\alpha+\beta) = \frac{4}{5}$ and $\sin(\alpha-\beta) = \frac{5}{13}$

$\Rightarrow \sin(\alpha+\beta) = \frac{3}{5}$ and $\cos(\alpha-\beta) = \frac{12}{13}$

$\Rightarrow \tan(\alpha+\beta) = \frac{3}{4}$ and $\tan(\alpha-\beta) = \frac{5}{12}$

Now,

$\tan 2\alpha = \tan\{(\alpha+\beta) + (\alpha-\beta)\}$

$\Rightarrow \tan 2\alpha = \frac{\tan(\alpha+\beta) + \tan(\alpha-\beta)}{1 - \tan(\alpha+\beta)\tan(\alpha-\beta)}$

$= \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \times \frac{5}{12}} = \frac{56}{33}$

692 **(c)**

We have,

$\sin\theta + \operatorname{cosec}\theta = 2$

$\Rightarrow (\sin\theta + \operatorname{cosec}\theta)^2 = 4$

$\Rightarrow \sin^2\theta + \operatorname{cosec}^2\theta + 2 = 4 \Rightarrow \sin^2\theta + \operatorname{cosec}^2\theta = 2$

693 **(a)**

$\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$

$\Rightarrow (\sin 6\theta + \sin 2\theta) + \sin 4\theta = 0$

$\Rightarrow 2\sin 4\theta\cos 2\theta + \sin 4\theta = 0$

$\Rightarrow \sin 4\theta(2\cos 2\theta + 1) = 0$

$\therefore$ Either $\sin 4\theta = 0$ or $\cos 2\theta = -\frac{1}{2}$

When $\sin 4\theta = 0$

$\Rightarrow 4\theta = n\pi$

$\Rightarrow \theta = \frac{n\pi}{4}$

And when $\cos 2\theta = -\frac{1}{2} = \cos\frac{2\pi}{3}$

$\Rightarrow 2\theta = 2n\pi \pm \frac{2\pi}{3}$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{3}$

694 **(b)**

We have, $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$

$\Rightarrow \sin x + \sin 3x - 3\sin 2x = \cos x + \cos 3x - 3\cos 2x$

$\Rightarrow 2\sin 2x \cos x - 3\sin 2x - 2\cos 2x \cos x + 3\cos 2x = 0$

$\Rightarrow \sin 2x(2\cos x - 3) - \cos 2x(2\cos x - 3) = 0$

$\Rightarrow (\sin 2x - \cos 2x)(2\cos x - 3) = 0$

$\Rightarrow \sin 2x = \cos 2x \quad \left(\because \cos x \neq \frac{3}{2}\right)$

$\Rightarrow 2x = 2n\pi \pm \left(\frac{\pi}{2} - 2x\right)$

Taking +ve sign

$x = \frac{n\pi}{2} + \frac{\pi}{8}$

695 **(a)**

Since, $2\cos^2\frac{x}{2}\sin^2 x < 2$

But $x^2 + \frac{1}{x^2} \geq 2$

Thus, the equation has no solution

696 **(a)**

Using sine formula, we have

$\frac{\sqrt{3}+1}{\sin 105°} = \frac{b}{\sin 30°} = \frac{c}{\sin 45°}$

$\Rightarrow 2\sqrt{2} = 2\,b = \sqrt{2}\,c \Rightarrow b = \sqrt{2}, c = 2$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}bc\sin A$

$\Rightarrow$ Area of $\Delta ABC = \frac{1}{2} \times \left(2\sqrt{2}\sin 105°\right) = \frac{\sqrt{3}+1}{2}$

$= \frac{1}{\sqrt{3}-1}$

697 **(c)**

Given, $\cos\theta - \sin\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta = \frac{1}{2}$

$\Rightarrow \cos\left(\theta + \frac{\pi}{4}\right) = \cos\frac{\pi}{3}$

$\Rightarrow \theta + \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{3}$

$\Rightarrow \theta = 2n\pi - \frac{7\pi}{12}$ or $2n\pi + \frac{\pi}{12}$

698 **(a)**

We have,

$\frac{\sin B}{b} = \frac{\sin C}{c} \Rightarrow \sin C = \frac{c}{b}\sin B > 1 \quad [\because b < \sin B \text{ (Given)}]$,

Which is impossible

Hence, no triangle is possible

699 **(b)**

We have,

$\cot^2\frac{\pi}{9} + \cot^2\frac{2\pi}{9} + \cot^2\frac{4\pi}{9}$

$= \text{cosec}^2\frac{\pi}{9} + \text{cosec}^2\frac{2\pi}{9} + \text{cosec}^2\frac{4\pi}{9} - 3$

$= \frac{1}{1-\cos\frac{2\pi}{9}} + \frac{1}{1-\cos\frac{4\pi}{9}} + \frac{1}{1-\cos\frac{8\pi}{9}} - 3 \quad$ ...(i)

Let $a = \cos\frac{2\pi}{9}, b = \cos\frac{4\pi}{9}, c = \cos\frac{8\pi}{9}$. Then,

$\frac{1}{1-\cos\frac{2\pi}{9}} + \frac{1}{1-\cos\frac{4\pi}{9}} + \frac{1}{1-\cos\frac{8\pi}{9}}$

$= \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$

$= \frac{3+(ab+bc+ca)-2(a+b+c)}{1-(a+b+c)+(ab+bc+ca)-abc} \quad$ ...(ii)

Now,

$a + b + c = \cos\frac{2\pi}{9} + \cos\frac{4\pi}{9} + \cos\frac{8\pi}{9}$

$\Rightarrow a + b + c = 2\cos\frac{\pi}{3}\cos\frac{\pi}{9} + \cos\frac{8\pi}{9}$

$\Rightarrow a + b + c = \cos\frac{\pi}{9} + \cos\left(\pi - \frac{\pi}{9}\right)$

$\Rightarrow a + b + c = \cos\frac{\pi}{9} - \cos\frac{\pi}{9} = 0$,

$abc = \cos\frac{2\pi}{9}\cos\frac{4\pi}{9}\cos\frac{8\pi}{9}$

$\Rightarrow abc = \frac{1}{2}\left\{\cos\frac{2\pi}{3} + \cos\frac{2\pi}{9}\right\}\cos\frac{8\pi}{9}$

$\Rightarrow abc = \frac{1}{2}\left\{-\frac{1}{2}\cos\frac{8\pi}{9} + \cos\frac{8\pi}{9}\cos\frac{2\pi}{9}\right\}$

$\Rightarrow abc = \frac{1}{4}\left\{-\cos\frac{8\pi}{9} + \cos\frac{10\pi}{9} + \cos\frac{2\pi}{3}\right\} = -\frac{1}{8}$,

and, $ab + bc + ca$

$= \cos\frac{2\pi}{9}\cos\frac{4\pi}{9} + \cos\frac{4\pi}{9}\cos\frac{8\pi}{9} + \cos\frac{8\pi}{9}\cos\frac{2\pi}{9}$

$= \frac{1}{2}\left\{\cos\frac{2\pi}{3} + \cos\frac{2\pi}{9} + \cos\frac{4\pi}{3} + \cos\frac{4\pi}{9} + \cos\frac{2\pi}{3} + \cos\frac{10\pi}{9}\right\}$

$= \frac{1}{2}\left\{-\frac{3}{2} + \cos\frac{2\pi}{9} + \cos\frac{4\pi}{9} + \cos\frac{10\pi}{9}\right\}$

$= \frac{1}{2}\left\{-\frac{3}{2} + 2\cos\frac{\pi}{3}\cos\frac{\pi}{9} - \cos\frac{\pi}{9}\right\} = -\frac{3}{4}$

$\therefore \frac{1}{1-\cos\frac{2\pi}{9}} + \frac{1}{1-\cos\frac{4\pi}{9}} + \frac{1}{1-\cos\frac{8\pi}{9}} = \frac{7-\frac{3}{4}}{1-\frac{3}{4}+\frac{1}{8}}$

$= \frac{\frac{9}{4}}{\frac{3}{8}} = 6$

$\Rightarrow \cot^2\frac{\pi}{9} + \cot^2\frac{2\pi}{9} + \cot^2\frac{4\pi}{9} = 6 - 3 = 3$

700 **(d)**

Since, $\tan x + \frac{1}{\tan x} = 2$

$\Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}$

$\therefore \sin x = \frac{1}{\sqrt{2}}$ and $\cos x = \frac{1}{\sqrt{2}}$

Hence, $\sin^{2n} x + \cos^{2n} x = \frac{1}{2^n} + \frac{1}{2^n} = \frac{1}{2^{n-1}}$

702 **(c)**

We have,

$\sin x + \sin^2 x = 1 \Rightarrow \sin x = \cos^2 x$

Now,

$\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x - 1$

$= \cos^6 x(\cos^6 x + 3\cos^4 x + 3\cos^2 x + 1) - 1$

$= \cos^6 x(\cos^2 x + 1)^3 - 1$

$= \sin^3 x(\sin x + 1)^3 - 1$

$= (\sin^2 x + \sin x)^3 - 1$

$= (\sin^2 x + \cos^2 x)^3 - 1 \quad [\because \sin x = \cos^2 x]$

$= 1 - 1 = 0$

703 **(c)**

Let $a = 3x + 4y, b = 4x + 3y$ and $c = 5x + 5y$.

Clearly, c is the largest side and thus the largest angle C is given by

$$\cos C \frac{a^2 + b^2 - c^2}{2ab} = \frac{-2xy}{2(12x^2 + 25xy + 12y^2)} < 0$$

$\Rightarrow C$ is an obtuse angle

704 **(a)**

Let $a = x^2 + x + 1, b = x^2 - 1$ and $c = 2x + 1$.

Then,

$a - b = x + 2 > 0 \qquad [\because x > 1]$

$a - c = x^2 - x > 0 \qquad [\because x > 1]$

So, a is the largest side

Hence, the largest angle is given by

$$\cos\theta = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\Rightarrow \cos\theta = \frac{(x^2-1)^2 + (2x+1)^2 - (x^2+x+1)^2}{2(x^2-1)(2x+1)} = -\frac{1}{2}$$

$\Rightarrow \theta = 2\pi/3 = 120°$

705 **(c)**

We have,

$\frac{1}{2} a p_1 = \Delta, \frac{1}{2} b p_2 = \Delta, \frac{1}{2} c p_3 = \Delta$

$\Rightarrow p_1 = \frac{2\Delta}{a}, p_2 = \frac{2\Delta}{b}, p_3 = \frac{2\Delta}{c}$

$\therefore \frac{1}{p_1^2} + \frac{1}{p_2^2} + \frac{1}{p_3^2} = \frac{a^2 + b^2 + c^2}{4\Delta^2}$

$$\frac{1}{p_1} + \frac{1}{p_2} - \frac{1}{p_3} = \frac{a}{2\Delta} + \frac{b}{2\Delta} - \frac{c}{2\Delta} = \frac{a + b - c}{2\Delta} = \frac{2(s-c)}{2\Delta} = \frac{s-c}{\Delta}$$

706 **(c)**

We have,

$$\cos C = \frac{63}{65} \Rightarrow \frac{a^2 + b^2 - c^2}{2ab} = \frac{63}{65} \Rightarrow \frac{26^2 + 30^2 - c^2}{2 \times 26 \times 30} = \frac{63}{65}$$

$\Rightarrow 676 + 900 - c^2 = 1260 \Rightarrow c^2 = 64 \Rightarrow c = 8$

Thus, we have

$a = 26, b = 30$ and $c = 8$

$\therefore 2s = a + b + c \Rightarrow 2s = 26 + 30 + 8 = 64 \Rightarrow s = 32$

Also,

$\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{32 \times 6 \times 2 \times 24} = 96$

Hence, $r_2 = \frac{\Delta}{s-b} = \frac{96}{32-30} = 48$

707 **(c)**

$\cos 1° \cos 2° \cos 3° \ldots \cos 90° \ldots \cos 100°$

$= \cos 1° \cos 2° \cos 3° \ldots 0 \ldots \cos 100° = 0$

708 **(b)**

We have,

$\sin\frac{\pi}{2} + \sin\frac{2\pi}{7} + \sin\frac{3\pi}{7}$

$$= \frac{1}{2\sin\frac{\pi}{7}}\left\{2\sin^2\frac{\pi}{7} + 2\sin\frac{\pi}{7}\sin\frac{2\pi}{7} + 2\sin\frac{\pi}{7}\sin\frac{3\pi}{7}\right\}$$

$$= \frac{1}{2\sin\left(\frac{\pi}{7}\right)}\left\{1 - \cos\frac{2\pi}{7} + \cos\frac{\pi}{7} - \cos\frac{3\pi}{7} + \cos\frac{2\pi}{7} - \cos\frac{4\pi}{7}\right\}$$

$$= \frac{1}{2\sin\frac{\pi}{7}}\left\{1 + \cos\frac{\pi}{7}\right\} = \frac{2\cos^2\frac{\pi}{14}}{4\sin\frac{\pi}{14}\cos\frac{\pi}{14}} = \frac{1}{2}\cot\frac{\pi}{14}$$

709 **(a)**

Let $f(x) = \sqrt{3}\cos x + \sin x$

$\Rightarrow f(x) = 2\left(\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x\right) = 2\sin\left(x + \frac{\pi}{3}\right)$

Since, $-1 \le \sin\left(x + \frac{\pi}{3}\right) \le 1$

Hence, $f(x)$is maximum, if $x + \frac{\pi}{3} = \frac{\pi}{2}$

$\Rightarrow x = \frac{\pi}{6} = 30°$

710 **(b)**

$\sin^2 17.5° + \sin^2 72.5°$
$= \sin^2 17.5° + \cos^2 17.5°$ [$\because \sin(9\theta - \theta) = \cos\theta$]
$= 1 = \tan^2 45°$

711 **(a)**
We have,
$a \sin A = b \sin B$
$\Rightarrow a \cdot ak = b \cdot bk \Rightarrow a = b \Rightarrow \Delta ABC$ is isosceles

712 **(b)**
We know that $\sin^2\theta \geq 1$
$\Rightarrow \dfrac{4xy}{(x+y)^2} \geq 1$
$\Rightarrow 4xy \geq (x+y)^2$
$\Rightarrow (x-y)^2 \leq 0$
$\Rightarrow x - y = 0 \Rightarrow y = x$
And $x \neq 0, \ y \neq 0$

713 **(b)**
Given that, $\cos\theta = \frac{1}{2}\left(x + \frac{1}{x}\right)$

$\Rightarrow x + \frac{1}{x} = 2\cos\theta$ ....(i)

We know that, $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$

$= (2\cos\theta)^2 - 2 = 4\cos^2\theta - 2$

$= 2\cos 2\theta$ [from Eq.(i)]

$\therefore \dfrac{1}{2}\left(x^2 + \dfrac{1}{x^2}\right) = \dfrac{1}{2} \times 2\cos 2\theta = \cos 2\theta$

714 **(d)**
$\text{sech}^{-1}(\sin\theta)$
$= \cosh^{-1}(\text{cosec}\,\theta)$
$= \log\left[\text{cosec}\,\theta + \sqrt{(\text{cosec}^2\theta - 1)}\right]$
$= \log\left[\dfrac{1}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta}\right] = \log\cot\dfrac{\theta}{2}$

715 **(d)**
Consider the curves $y = 2^{\cos x}$ and $y = |\sin x|$.
Clearly, both the curves are symmetrical about y-axis as $\cos x$ and $|\sin x|$ are even functions
Also, $y = 2^{\cos x}$ and $y = |\sin x|$ intersect at two points in $[0, 2\pi]$
Hence, there are four solutions of the given equation

716 **(d)**
We have,
$\cos(\lambda\sin\theta) = \sin(\lambda\cos\theta)$
$\Rightarrow \cos(\lambda\sin\theta) = \cos\left(\dfrac{\pi}{2} - \lambda\cos\theta\right)$
$\Rightarrow \lambda\sin\theta = \dfrac{\pi}{2} - \lambda\cos\theta \Rightarrow \cos\theta + \sin\theta = \dfrac{\pi}{2\lambda}$
This equation will have a solution if
$\left|\dfrac{\pi}{2\lambda} \leq \sqrt{2}\right|$ $\left[\because |a\cos\theta + b\sin\theta| \leq \sqrt{a^2+b^2}\right]$
$\Rightarrow \dfrac{\pi}{2\lambda} \leq \sqrt{2} \Rightarrow \lambda \geq \dfrac{\pi}{2\sqrt{2}}$ $[\because \lambda > 0]$

717 **(c)**
We have,
$c_1 + c_2 = 2b\cos A$ and $c_1c_2 = b^2 - a^2$
$\therefore c_1 - c_2 = \sqrt{(c_1+c_2)^2 - 4c_1c_2}$
$\Rightarrow c_1 - c_2 = \sqrt{4b^2\cos^2 A - 4(b^2 - a^2)}$
$= 2\sqrt{a^2 - b^2\sin^2 A}$

718 **(b)**
We have,
$\tan\alpha = (1 + 2^{-x})^{-1} = \dfrac{2^x}{2^x + 1}$ and $\tan\beta = \dfrac{1}{2^{x+1}+1}$
$\therefore \tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$
$\Rightarrow \tan(\alpha+\beta) = \dfrac{2^x(2^{x+1}+1) + (2^x+1)}{(2^x+1)(2^{x+1}+1) - 2^x}$
$\Rightarrow \tan(\alpha+\beta) = \dfrac{2(2^x)^2 + 2.2^x + 1}{2(2^x)^2 + 2.2^x + 1} = 1 \Rightarrow \alpha + \beta = \pi/4$

719 **(a)**
Given, $f(x) = \sin x(1 + \cos x)$
It is minimum at $x = \frac{\pi}{3}$
$\therefore f\left(\dfrac{\pi}{3}\right) = \sin\left(\dfrac{\pi}{3}\right)\left(1 + \cos\dfrac{\pi}{3}\right)$
$= \dfrac{\sqrt{3}}{2}\left(1 + \dfrac{1}{2}\right) = \dfrac{3\sqrt{3}}{4}$

720 **(c)**
We have,
$\cos\dfrac{\pi}{11} + \cos\dfrac{3\pi}{11} + \cos\dfrac{5\pi}{11} + \cos\dfrac{7\pi}{11}\cos\dfrac{9\pi}{11}$
$= \dfrac{\cos\left\{\frac{\pi}{11} + \left(\frac{5-1}{2}\right)\frac{2\pi}{11}\right\}\sin\left(\frac{5\pi}{11}\right)}{\sin\left(\frac{\pi}{11}\right)}$
$= \dfrac{\cos\frac{5\pi}{11}\sin\frac{5\pi}{11}}{\sin\frac{\pi}{11}} = \dfrac{1}{2}\dfrac{\sin\left(\frac{10\pi}{11}\right)}{\sin\frac{\pi}{11}} = \dfrac{1}{2}$

721 **(c)**
$\left(1 + \cos\dfrac{\pi}{8}\right)\left(1 + \cos\dfrac{3\pi}{8}\right)\left(1 + \cos\dfrac{5\pi}{8}\right)\left(1 + \cos\dfrac{7\pi}{8}\right)$
$= \left(1 + \cos\dfrac{\pi}{8}\right)\left(1 + \cos\dfrac{3\pi}{8}\right)\left(1 - \cos\dfrac{3\pi}{8}\right) \times \left(1 - \cos\dfrac{\pi}{8}\right)$

$= \left(1-\cos^2\frac{\pi}{8}\right)\left(1-\cos^2\frac{3\pi}{8}\right)$

$= \sin^2\frac{\pi}{8}.\sin^2\frac{3\pi}{8} = \frac{1}{4}\left[2\sin\frac{\pi}{8}\sin\frac{3\pi}{8}\right]^2$

$= \frac{1}{4}\left[\cos\frac{\pi}{4}-\cos\frac{\pi}{2}\right]^2 = \frac{1}{4}\left[\frac{1}{\sqrt{2}}-0\right]^2 = \frac{1}{8}$

722 **(a)**

We have,

$2\sin\frac{A}{2} = \sqrt{1+\sin A}+\sqrt{1-\sin A}$

$\Rightarrow 2\sin\frac{A}{2} - \sqrt{(\cos A/2+\sin A/2)^2}$
$+ \sqrt{(\cos A/2-\sin A/2)^2}$

$\Rightarrow 2\sin A/2 = |\cos A/2+\sin A/2|$
$+ |\sin A2/ - \sin A/2|$

$\Rightarrow \cos A/2 + \sin A\,/2 \geq 0$ and $\cos A/2 - \sin A/2 \leq 0$

$\Rightarrow \pi/4 \leq A/2 \leq 3\pi/4$ and $\pi/4 \leq A \leq 5\pi/4$

$\Rightarrow \pi/4 \leq A/2 \leq 3\pi/4$

$\Rightarrow 2n\,\pi + \pi/4 \leq A/2 \leq 2n\,\pi + 3\pi/4, n \in Z$

723 **(a)**

We have,

$a\cos^2\frac{C}{2} + c\cos^2\frac{A}{2} = \frac{3b}{2}$

$\Rightarrow a\left\{\frac{s(s-c)}{ab}\right\} + c\left\{\frac{s(s-a)}{bc}\right\} = \frac{3b}{2}$

$\Rightarrow \frac{s}{b}(2s-a-c) = \frac{3b}{2}$

$\Rightarrow 2s = 3b \Rightarrow a+c = 2b \Rightarrow a, b, c$ are in A.P.

724 **(a)**

We have,

$\tan(\theta_1+\theta_2+\cdots+\theta_n) = \frac{S_1-S_3+S_5-S_7+\cdots}{1-S_2+S_4-S_6+\cdots}$

$\therefore \tan 5\theta = \frac{{}^5C_1\tan\theta - {}^5C_3\tan^3\theta + {}^5C_5\tan^5\theta}{1-{}^5C_2\tan^2\theta + {}^5C_4\tan^4\theta}$

725 **(d)**

It is given that a, b, c are in A.P.

$\therefore 2b = a+c$

Now,

$\frac{\tan\frac{A}{2}+\tan\frac{C}{2}}{\cot\frac{B}{2}} = \left(\tan\frac{A}{2}+\tan\frac{C}{2}\right)\tan\frac{B}{2}$

$\Rightarrow \frac{\tan\frac{A}{2}+\tan\frac{C}{2}}{\cot\frac{B}{2}} = \left\{\frac{\Delta}{s(s-a)}+\frac{\Delta}{s(s-c)}\right\}\frac{\Delta}{s(s-b)}$

$\Rightarrow \frac{\tan\frac{A}{B}+\tan\frac{C}{2}}{\cot\frac{B}{2}} = \frac{\Delta^2}{s^2(s-b)}\left\{\frac{1}{s-a}+\frac{1}{s-c}\right\}$

$\Rightarrow \frac{\tan\frac{A}{2}+\tan\frac{C}{2}}{\cot\frac{B}{2}} = \frac{\Delta^2 b}{s\,\Delta^2}$

$\Rightarrow \frac{\tan\frac{A}{2}+\tan\frac{C}{2}}{\cot\frac{B}{2}} = \frac{2b}{2s} = \frac{2b}{a+b+c} = \frac{2b}{3b} = \frac{2}{3}$ [$\because a+c = 2b$]

726 **(c)**

We have,

$2\frac{\cos A}{a} + \frac{\cos B}{b} + 2\frac{\cos C}{c} = \frac{a}{bc}+\frac{b}{ac}$

$\Rightarrow 2\left(\frac{b^2+c^2-a^2}{2abc}\right) + \frac{c^2+a^2-b^2}{2abc}$
$+ 2\left(\frac{a^2+b^2-c^2}{2abc}\right) = \frac{a^2+b^2}{abc}$

$\Rightarrow b^2+c^2 = a^2 \Rightarrow A = \frac{\pi}{2}$

727 **(d)**

$2^{n-1}\tan(2^{n-1}\alpha) + 2^n\cot(2^n\alpha)$

$= 2^{n-1}\left[\frac{\sin 2^{n-1}\alpha}{\cos 2^{n-1}\alpha} + 2\frac{\cos 2^n\alpha}{\sin 2^n\alpha}\right]$

$= 2^{n-1}\left[\frac{\cos 2^n\alpha\cos 2^{n-1}\alpha + \sin 2^n\alpha \; \sin 2^{n-1}\alpha + \cos 2^n\alpha\cos 2^{n-1}\alpha}{\sin 2^n\alpha\cos 2^{n-1}\alpha}\right]$

$= 2^{n-1}\left[\frac{\cos 2^{n-1}\alpha(1+\cos 2^n\alpha)}{\sin 2^n\alpha\cos 2^{n-1}\alpha}\right]$

$= 2^{n-1}\cot 2^{n-1}\alpha$

Proceeding in similar way in last, we get

$\tan\alpha + 2\cot 2\alpha$

$= \frac{\sin\alpha}{\cos\alpha} + 2\frac{\cos 2\alpha}{\sin 2\alpha}$

$= \frac{\cos 2\alpha\cos\alpha + \sin 2\alpha\sin\alpha + \cos 2\alpha\cos\alpha}{\sin 2\alpha\cos\alpha}$

$= \frac{\cos\alpha(1+\cos 2\alpha)}{2\sin\alpha\cos^2\alpha}$

$= \frac{2\cos^2\alpha}{2\sin\alpha}$

$= \frac{\cos\alpha}{\sin\alpha} = \cot\alpha$

728 **(c)**

$\cos^2\left(\frac{\pi}{3}-x\right) - \cos^2\left(\frac{\pi}{3}+x\right)$

$= \left[\cos\left(\frac{\pi}{3}-x\right)+\cos\left(\frac{\pi}{3}+x\right)\right]\left[\cos\left(\frac{\pi}{3}-x\right)\right.$
$\left. - \cos\left(\frac{\pi}{3}+x\right)\right]$

$= \left(2\cos\frac{\pi}{3}\cos x\right)\left(2\sin\frac{\pi}{3}\sin x\right)$

$= \sin\frac{2\pi}{3}\sin 2x = \frac{\sqrt{3}}{2}\sin 2x$

Hence, maximum value of given expression is $\frac{\sqrt{3}}{2}$

729 **(d)**

We have,

$$\cos C=\frac{a^2+b^2-c^2}{2\,ab}\Rightarrow \cos C=-1\Rightarrow C=\pi$$

Which is impossible in a triangle

730 **(c)**

We have,

$$\frac{a}{\cos A}=\frac{b}{\cos B}$$

$\Rightarrow 2R\sin A\cos B=2R\sin B\cos A$

$\Rightarrow \sin(A-B)=0\Rightarrow A=B$

$\therefore 2\sin A\cos B=\sin 2\,A=\sin(180°-C)$ [$\because 2A+C=180°$]

$\Rightarrow 2\sin A\cos B=\sin C$

731 **(d)**

Given, $1+\sin\theta+\sin^2\theta+\cdots\infty=4+2\sqrt{3}$

$\Rightarrow \dfrac{1}{1-\sin\theta}=4+2\sqrt{3}\quad [\because 0<\sin\theta<1]$

$\Rightarrow 1-\sin\theta=\dfrac{4-2\sqrt{3}}{16-12}=1-\dfrac{\sqrt{3}}{2}$

$\Rightarrow \sin\theta=\dfrac{\sqrt{3}}{2}$

$\Rightarrow \theta=\frac{\pi}{3}$ or $\frac{2\pi}{3}$

732 **(c)**

We have,

$a(b^2+c^2)\cos A+b(c^2+a^2)\cos B+c(a^2+b^2)\cos C$

$=(ab^2\cos A+ba^2\cos B)+(ac^2\cos A+ca^2\cos C)+(bc^2\cos B+cb^2\cos C)$

$=ab(b\cos A+a\cos B)+ca(c\cos A+a\cos C)+bc(c\cos B+b\cos C)$

$=abc+abc+abc=3abc$

733 **(a)**

We have, $\tan(\pi\cos\theta)=\tan\left(\frac{\pi}{2}-\pi\sin\theta\right)$

$\therefore \sin\theta+\cos\theta=\dfrac{1}{2}$

$\Rightarrow \dfrac{1}{\sqrt{2}}\cos\theta+\dfrac{1}{\sqrt{2}}\sin\theta=\dfrac{1}{2\sqrt{2}}$

$\Rightarrow \cos\left(\theta-\dfrac{\pi}{4}\right)=\dfrac{1}{2\sqrt{2}}$

734 **(a)**

We have,

$y=5x^2+2x+3$

Clearly, it represents an upward opening parabola having its vertex at $(-1/5,14/5)$

$\therefore y\geq\dfrac{14}{5}>2$

Now, $y=2\sin x\leq 2$

Thus, the two curves do not intersect. Hence, there is no common point in the two curves

735 **(d)**

We have,

$(a+b+c)(b+c-a)=\lambda\,bc$

$\Rightarrow 2\,s(2s-a)=\lambda\,bc$

$\Rightarrow \dfrac{s(s-a)}{bc}=\dfrac{\lambda}{4}$

$\Rightarrow \cos^2\dfrac{A}{2}=\dfrac{\lambda}{4}$

$\Rightarrow 0<\dfrac{\lambda}{4}<1\Rightarrow 0<\lambda<4\quad\left[\because \cos^2\dfrac{A}{2}\leq 1\right]$

736 **(c)**

The given expression can be written as

$(1+\cot^2 A)\cot^2 A-(1+\tan^2 A)\tan^2 A-(\cot^2 A-\tan^2 A)\{(1+\tan^2 A)(1+\cot^2 A)-1\}$

$=\cot^2 A+\cot^4 A-\tan^2 A-\tan^4 A$
$-(\cot^2 A-\tan^2 A)(\cot^2 A+\tan^2 A+1)$

$=\cot^2 A+\cot^4 A-\tan^2 A-\tan^4 A-(\cot^2 A-\tan^2 A)-(\cot^4 A-\tan^4 A)$

$=0$

737 **(b)**

We have, $2b=a+c$

And,

$$\Delta=\frac{3}{5}\times\frac{\sqrt{3}}{4}\left(\frac{a+b+c}{3}\right)^2$$

$\Rightarrow \Delta = \frac{3\sqrt{3}}{20} b^2$

$\Rightarrow s(s-a)(s-b)(s-c) = \frac{27}{400} b^4$

$\Rightarrow \left(\frac{a+b+c}{2}\right)\left(\frac{b+c-a}{2}\right)\left(\frac{c+a-b}{2}\right)\left(\frac{a+b-c}{2}\right) = \frac{27}{400} b^4$

$\Rightarrow \left(\frac{3b}{2}\right) \times \left(\frac{b+c-2b+c}{2}\right)\left(\frac{b}{2}\right)\left(\frac{2b-c+b-c}{2}\right) = \frac{27}{400} b^4$

$[\because 2b = a+c]$

$\Rightarrow \frac{3b}{2} \times \left(\frac{2c-b}{2}\right) \times \frac{b}{2} \times \left(\frac{3b-2c}{2}\right) = \frac{27}{400} b^4$

$\Rightarrow (2c-b)(3b-2c) = \frac{9b^2}{25}$

$\Rightarrow (6bc - 4c^2 - 3b^2 + 2bc) = \frac{9b^2}{25}$

$\Rightarrow 8bc - 4c^2 - 3b^2 = \frac{9b^2}{25}$

$\Rightarrow \frac{84}{25} b^2 - 8\,bc + 4c^2 = 0$

$\Rightarrow 21b^2 - 50bc + 25c^2 = 0$

$\Rightarrow (7b - 5c)(3b - 5c) = 0$

$\Rightarrow 7b = 5c$ or, $3b = 5c \Rightarrow \frac{b}{c} = \frac{5}{7}, \frac{5}{3}$

Now,

$2b = a + c \Rightarrow \frac{2b}{c} = \frac{a}{c} + 1 \Rightarrow \frac{a}{c} = \frac{3}{7}, \frac{7}{3}$

Hence, $a : b : c = 3 : 5 : 7$

738 **(a)**

$\sqrt{\frac{a+b}{a-b}} - \sqrt{\frac{a-b}{a+b}}$

$= \sqrt{\frac{1+\frac{b}{a}}{1-\frac{b}{a}}} - \sqrt{\frac{1-\frac{b}{a}}{1+\frac{b}{a}}}$

$= \sqrt{\frac{1+\tan\alpha}{1-\tan\alpha}} - \sqrt{\frac{1-\tan\alpha}{1+\tan\alpha}}$

$= \frac{(1+\tan\alpha) - (1-\tan\alpha)}{\sqrt{1-\tan^2\alpha}}$

$= \frac{2\tan\alpha}{\sqrt{1-\tan^2\alpha}} = \frac{2\sin\alpha}{\sqrt{\cos 2\alpha}}$

739 **(b)**

Since, $\sin\theta + \cos\theta = x$ ...(i)

and $\sin^6\theta + \cos^6\theta = \frac{1}{4}[4 - 3(x^2-1)^2]$

On equation Eq (i), we get

$\sin 2\theta = x^2 - 1 \leq 1 \quad (\because \sin 2\theta \leq 1)$

$\Rightarrow x^2 \leq 2 \Rightarrow -\sqrt{2} \leq x \leq \sqrt{2}$

Now, $\sin^6\theta + \cos^6\theta = (\sin^2\theta + \cos^2\theta)^3 -$ $3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)$

$= 1 - 3\sin^2\theta\cos^2\theta = 1 - \frac{3}{4}\sin^2 2\theta$

$= 1 - \frac{3}{4}(x^2 -)^2 = \frac{1}{4}[4 - 3(x^2-1)^2]$

Thus, the given result will hold true only when $x^2 \leq 2$ and not for all real values of x

740 **(b)**

We have,

$\frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$

$\Rightarrow \sin(B+C)\sin(B-C) = \sin(A+B)\sin(A-B)$

$\Rightarrow \sin^2 B - \sin^2 C = \sin^2 A - \sin^2 B$

$\Rightarrow b^2 - c^2 = a^2 - b^2 \Rightarrow a^2, b^2, c^2$ are in A.P.

741 **(a)**

It is given that A, B, C are in A.P.

$\therefore 2B = A + C \Rightarrow 3B = A + B + C \Rightarrow 3B = 180°$

$\Rightarrow B = 60°$

$\Rightarrow \cos B = \frac{1}{2}$

$\Rightarrow \frac{c^2 + a^2 - b^2}{2ac} = \frac{1}{2}$

$\Rightarrow c^2 + a^2 - b^2 = ac$

$\Rightarrow (a-c)^2 = b^2 - ac$

$\Rightarrow |a-c| = \sqrt{b^2 - ac}$

$\Rightarrow |\sin A - \sin C| = \sqrt{\sin^2 B - \sin A \sin C}$

$\Rightarrow 2\left|\sin\frac{A-C}{2}\right|\cos\frac{A+C}{2} = \sqrt{\frac{3}{4} - \sin A\sin C}$

$\Rightarrow 2\left|\sin\frac{A-C}{2}\right| = \sqrt{3 - 4\sin A\sin C}$

$\Rightarrow \frac{\sqrt{3-4\sin A\sin C}}{|A-C|} = \frac{2\left|\sin\frac{A-C}{2}\right|}{|A-C|}$

$\Rightarrow \lim_{A\to C}\frac{\sqrt{3-4\sin A\sin C}}{|A-C|} = \lim_{A\to C}\left|\frac{\frac{\sin\left(\frac{A-C}{2}\right)}{\frac{A-C}{2}}}{2}\right| = 1$

742 **(c)**

$3 - \cos\theta + \cos\left(\theta + \frac{\pi}{3}\right)$

$= 3 - \cos\theta + \frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta$

$= 3 - \frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta = 3 - \sin\left(\theta + \frac{\pi}{6}\right)$

Since, $-1 \le \sin\theta \le 1$

Hence, the value of expression lies in [2, 4]

743 **(c)**

We have, $\cos A = m\cos B$

$\Rightarrow \frac{\cos A}{\cos B} = \frac{m}{1}$

$\Rightarrow \frac{\cos A + \cos B}{\cos A - \cos B} = \frac{m+1}{m-1}$

$\Rightarrow \frac{2\cos\frac{A+B}{2}\cos\frac{B-A}{2}}{2\sin\frac{A+B}{2}\sin\frac{B-A}{2}} = \frac{m+1}{m-1}$

$\Rightarrow \cot\frac{A+B}{2} = \left(\frac{m+1}{m-1}\right)\tan\frac{B-A}{2}$

But $\cot\frac{A+B}{2} = \lambda\tan\frac{B-A}{2}$

$\therefore\ \lambda = \frac{m+1}{m-1}$

744 **(c)**

$\cos^4\frac{\pi}{8} + \cos^4\frac{7\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8}$

$= \cos^4\frac{\pi}{8} + \cos^4\frac{\pi}{8} + \cos^4\left(\frac{\pi}{2} - \frac{\pi}{8}\right) + \cos^4\left(\frac{\pi}{2} + \frac{\pi}{8}\right)$

$= 2\left[\cos^4\frac{\pi}{8} + \sin^4\frac{\pi}{8}\right]$

$= 2\left[\left(\cos^2\frac{\pi}{8} + \sin^2\frac{\pi}{8}\right)^2 - 2\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right]$

$= 2\left[1 - \frac{1}{2}\left(\sin\frac{\pi}{4}\right)^2\right]$

$= 2\left[1 - \frac{1}{4}\right] = \frac{3}{2}$

745 **(b)**

Given, $\sin\theta = \frac{12}{13}$ and $\cos\phi = -\frac{3}{5}$

$\therefore\ \cos\theta = \frac{5}{13}$ and $\sin\phi = -\frac{4}{5}$

$\therefore\ \sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

$= \frac{12}{13}\times\left(-\frac{3}{5}\right) + \frac{5}{13}\times\left(-\frac{4}{5}\right)$

$= \frac{-36}{65} + \frac{(-20)}{65} = -\frac{56}{65}$

746 **(c)**

We have,

$\sec^2\theta\,\text{cosec}^2\theta = \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta} - \frac{4}{\sin^2 2\theta} \ge 4$

and, $\sec^2\theta\,\text{cosec}^2\theta = \frac{4}{\sin^2 2\theta} \ge 4$

Thus, the required equation is

$x^2 - \lambda x + \lambda = 0$, where $\lambda \ge 4$

747 **(a)**

$\frac{1}{m}\left[(m+n) + \frac{1}{(m+n)}\right]$

$= \frac{1}{\sec\theta}\left[\sec\theta + \tan\theta + \frac{1}{\sec\theta + \tan\theta}\right]$

$= \frac{[\sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta + 1]}{\sec\theta(\sec\theta + \tan\theta)}$

$= \frac{2\sec^2\theta + 2\sec\theta\tan\theta}{\sec\theta(\sec\theta + \tan\theta)}$

$= 2$

748 **(a)**

$\because \sin A\sin B = \frac{1}{2}\times\ 2\sin A\sin B$

$= \frac{1}{2}[\cos(A-B) - \cos(A+B)]$

$= \frac{1}{2}[\cos(A-B) - \cos 90°]$ ($\because A + B + C = 180°$ and $\angle C = 90°$, given)

$= \frac{1}{2}\cos(A-B) \le \frac{1}{2}$

$\therefore$ Maximum value of $\sin A\sin B = \frac{1}{2}$

749 **(b)**

In cyclic quadrilateral $ABCD$, we have

$A + C = \pi$ and $B + D = \pi$

$\therefore \cos A = -\cos C$ and $\cos B = -\cos D$

$\Rightarrow \cos A + \cos B + \cos C + \cos D = 0$

750 **(d)**

Let $A = \theta, B = 2\,\theta$ and $C = 3\,\theta$. Then,

$A + B + C = 180° \Rightarrow 6\,\theta = 180° \Rightarrow \theta = 30°$

$\therefore A = 30°, B = 60°$ and $C = 90°$

Now,
$a : b : c = \sin A : \sin B : \sin C$,
$\Rightarrow a : b : c = \frac{1}{2} : \frac{\sqrt{3}}{2} : 1 \Rightarrow a : b : c = 1 : \sqrt{3} : 2$

751 **(b)**
We have,
$\sin(\pi \cos\theta) = \cos(\pi \sin\theta)$
$\Rightarrow \sin(\pi\cos\theta) = \sin\left(\frac{\pi}{2} + \pi\sin\theta\right)$
$\Rightarrow \pi\cos\theta = \frac{\pi}{2} + \pi\sin\theta$
$\Rightarrow \pi\cos\theta - \pi\sin\theta = \frac{\pi}{2}$
$\Rightarrow \cos\theta - \sin\theta = \frac{1}{2}$
$\Rightarrow \frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta = \frac{1}{2\sqrt{2}} \Rightarrow \cos\left(\theta + \frac{\pi}{4}\right)$
$= \frac{1}{2\sqrt{2}}$

752 **(a)**
The given equation is not meaningful for
$|\cos x| = 1$
So, let $|\cos x| \neq 1$
Now,
$2^{1+|\cos x|+\cos^2 x+|\cos x|^3+\cdots+\text{to } \infty} = 4$
$\Rightarrow \frac{1}{2^{1-|\cos x|}} = 2^2$
$\Rightarrow \frac{1}{1-|\cos x|} = 2$
$\Rightarrow 2 - 2|\cos x| = 1$
$\Rightarrow |\cos x| = \frac{1}{2}$
$\Rightarrow \cos x = \pm\frac{1}{2}$
$\Rightarrow \cos x = \cos\frac{\pi}{3}, \cos x = \cos\frac{2\pi}{3}$
$\Rightarrow x = 2n\pi \pm \frac{\pi}{3}, x = 2n\pi \pm \frac{2\pi}{3}, n \in Z$
$\Rightarrow x = 2n\pi \pm \frac{\pi}{3}, x = (2n \pm 1)\pi \pm \frac{\pi}{3}$
$\Rightarrow x = n\pi \pm \frac{\pi}{3}, n \in Z$

753 **(b)**
We have,
$(\cos\alpha + \cos\beta)^2 - (\sin\alpha + \sin\beta)^2 = 0$
$\Rightarrow (\cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta)$
$- (\sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta)$
$= 0$
$\Rightarrow \cos 2\alpha + \cos 2\beta = -2(\cos\alpha\cos\beta - \sin\alpha\sin\beta)$
$\Rightarrow \cos 2\alpha + \cos 2\beta = -2\cos(\alpha + \beta)$

754 **(d)**
The given expression can be written as
$\frac{1+\cos y - \sin^2 y}{1+\cos y} + \frac{(1-\cos^2 y) - \sin^2 y}{\sin y(1-\cos y)}$
$= \frac{\cos y\,(1+\cos y)}{1+\cos y} + 0 = \cos y$

755 **(a)**
We have,
$a = 2b$
$\Rightarrow 2R\sin A = 4R\sin B$
$\Rightarrow \sin A = 2\sin B$
$\Rightarrow \sin 3B = 2\sin B \quad [\because A = 3B]$
$\Rightarrow 3\sin B - 4\sin^3 B = 2\sin B$
$\Rightarrow \sin B - 4\sin^3 B = 0$
$\Rightarrow 1 - 4\sin^2 B = 0 \Rightarrow \sin B = \frac{1}{2} \Rightarrow B = \frac{\pi}{6}$
$\therefore A = 3B = \frac{\pi}{2}$

756 **(b)**
We know that
$AD^2 = \frac{1}{4}(b^2 + c^2 + 2bc\cos A)$
$\therefore 4AD^2 = b^2 + c^2 + 2bc\cos\frac{\pi}{3} \Rightarrow 4AD^2$
$= b^2 + c^2 + bc$

757 **(a)**
We know that,

$\alpha - \beta = (\theta - \beta) - (\theta - \alpha)$

$\therefore \cos(\alpha - \beta) = \cos(\theta - \beta)\cos(\theta - \alpha)$
$+ \sin(\theta - \beta)\sin(\theta - \alpha)$

$= ab + \sqrt{1-a^2}\sqrt{1-b^2}$

and $\sin(\alpha - \beta) = \pm(a\sqrt{1-b^2} - b\sqrt{1-a^2})$

$\Rightarrow \sin^2(\alpha - \beta) = a^2 + b^2 - 2a^2b^2$
$- 2ab\sqrt{1-a^2}\sqrt{1-b^2}$

$\Rightarrow \sin^2(\alpha - \beta) = a^2 + b^2 - 2a^2b^2$
$- 2ab[\cos(\alpha - \beta) - ab]$

$\therefore \sin^2(\alpha - \beta) - a^2 + b^2 - 2ab\cos(\alpha - \beta)$

$\Rightarrow \sin^2(\alpha - \beta) + 2ab\cos(\alpha - \beta) = a^2 + b^2$

758 **(b)**
$\frac{1}{2}\tan\frac{x}{2} = \frac{1}{2}\cot\frac{x}{2} - \cot x \qquad \left[\because \cot x = \frac{1-\tan^2\frac{x}{2}}{2\tan\frac{x}{2}}\right]$
And $\frac{1}{2^2}\tan\frac{x}{2^2} = \frac{1}{2^2}\cot\left(\frac{x}{2^2}\right) - \frac{1}{2}\cot\left(\frac{x}{2}\right)$
Similarly, $\frac{1}{2^3}\tan\left(\frac{x}{2^3}\right) = \frac{1}{2^3}\cot\left(\frac{x}{2^3}\right) - \frac{1}{2^2}\cot\ldots\left(\frac{x}{2^2}\right)$
$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$

$$\frac{1}{2^n}\tan\left(\frac{x}{2^n}\right)=\frac{1}{2^n}\cot\left(\frac{x}{2^n}\right)-\frac{1}{2^{n-1}}\cot\left(\frac{x}{2^{n-1}}\right)$$

Om adding all the above results, we get

$$\frac{1}{2}\tan\frac{x}{2}+\frac{1}{2^2}\tan\left(\frac{x}{2^2}\right)+\ldots+\frac{1}{2^n}\tan\left(\frac{x}{2^n}\right)$$
$$=\frac{1}{2^n}\cot\left(\frac{x}{2^n}\right)-\cot x$$

759 **(c)**

It is given that

Area of ΔABC = Area of ΔDEF

$\Rightarrow \frac{1}{2}AB\cdot AC\sin A=\frac{1}{2}\ CE\cdot EF\sin E$

$\Rightarrow \sin A=\sin E$

$\Rightarrow \sin 2E=\sin E$

$\Rightarrow 2E=\pi-E\Rightarrow E=\frac{\pi}{3}\Rightarrow A=2E=\frac{2\,\pi}{3}$

760 **(c)**

We have,

$\sin\frac{\pi}{18}\sin\frac{5\pi}{18}\sin\frac{7\pi}{18}$

$=\cos\left(\frac{\pi}{2}-\frac{\pi}{18}\right)\cos\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)\cos\left(\frac{\pi}{2}-\frac{7\pi}{18}\right)$

$=\cos\frac{8\pi}{18}\cos\frac{4\pi}{18}\cos\frac{2\pi}{8}$

$=\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{4\pi}{9}=\frac{\sin(2^3.\pi/9)}{2^3\sin\pi/9}=\frac{1}{2^3}=\frac{1}{8}$

761 **(c)**

We have,

$a=\sin^4\theta+\cos^4\theta\le\sin^2\theta+\cos^2\theta\le 1$

Also,

$a=\sin^4\theta+\cos^4\theta$
$=(\sin^2\theta+\cos^2\theta)^2-2\sin^2\theta+\cos^2\theta$

$\Rightarrow a=\sin^4\theta+\cos^4\theta=1-\frac{1}{2}\sin^2 2\theta$

$\Rightarrow \sin^2 2\theta=2(1-a)\Rightarrow 2(1-a)\le 1\Rightarrow a\ge\frac{1}{2}$

Hence, $\frac{1}{2}\le a\le 1$

762 **(a)**

Let $\sqrt{3}+1=r\cos\alpha$ and $\sqrt{3}-1=r\sin\alpha$, then

$r=\sqrt{\left(\sqrt{3}+1\right)^2+\left(\sqrt{3}-1\right)^2}$

$=\sqrt{3+1+2\sqrt{3}+3+1-2\sqrt{3}}=2\sqrt{2}$

and $\tan\alpha=\frac{\sqrt{3}-1}{\sqrt{3}+1}=\frac{1-\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)}=\tan\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$

$\Rightarrow\ \alpha=\frac{\pi}{12}$

The given equation reduces to

$2\sqrt{2}\cos(\theta-\alpha)=2$

$\Rightarrow\ \cos\left(\ \theta-\frac{\pi}{12}\right)=\cos\frac{\pi}{4}$

$\Rightarrow\ \theta-\frac{\pi}{12}=2n\pi\pm\frac{\pi}{4}$

$\Rightarrow\ \theta=2n\pi\pm\frac{\pi}{4}+\frac{\pi}{12}$

763 **(d)**

$\sin(A+B)=\sin A\cos B+\sin B\cos A$

$=\frac{1}{\sqrt{10}}.\sqrt{1-\frac{1}{5}}+\frac{1}{\sqrt{5}}\sqrt{1-\frac{1}{10}}$

$\left[\because\sin A=\frac{1}{\sqrt{10}},\sin B=\frac{1}{\sqrt{5}}\right]$

$=\frac{1}{\sqrt{10}}\sqrt{\frac{4}{5}}+\frac{1}{\sqrt{5}}\sqrt{\frac{9}{10}}=\frac{5}{\sqrt{50}}=\frac{1}{\sqrt{2}}=\sin\frac{\pi}{4}$

$\Rightarrow\ A+B=\frac{\pi}{4}$

764 **(b)**

The given equation can be rewritten as

$\tan\theta(\sin\theta+\sqrt{3})=0$

$\Rightarrow\tan\theta=0$, but $\sin\theta+\sqrt{3}\ne 0$

$\Rightarrow\tan\theta=0\Rightarrow\ \theta=n\pi, n\in I$

765 **(a)**

We have, $y=\sin\theta-\cos\theta$ and $\sin\theta-\cos\theta$ lies between $-\sqrt{2}$ and $+\sqrt{2}$

$\therefore -\sqrt{2}\le y\le\sqrt{2}$

766 **(c)**

Now, $\sin(\alpha-\beta)=\sin(\theta-\beta-(\theta-\alpha))$

$=\sin(\theta-\beta)=\cos(\theta-\alpha)$
$-\cos(\theta-\beta)\sin(\theta-\alpha)$

$=ba-\sqrt{1-b^2}\sqrt{1-a^2}$

and $\cos\ (\alpha-\beta)=\cos(\theta-\beta-(\theta-\alpha))$

$=\cos(\theta-\beta)\cos(\theta-\alpha)+\sin(\theta-\beta)\sin(\theta-\alpha)$

$=a\sqrt{1-b^2}+b\sqrt{1-a^2}$

$\therefore\cos^2(\alpha-\beta)+2ab\sin(\alpha-\beta)$

$=(a\sqrt{1-b^2}+b\sqrt{1-a^2})^2+2ab(ab$
$-\sqrt{1-a^2}\sqrt{1-b^2})$

$=a^2+b^2$

767 **(d)**

We have,

$8\sec^2\theta-6\sec\theta+1=0$

$\Rightarrow(4\sec\theta-1)(2\sec\theta-1)=0$

$\Rightarrow\sec\theta=\frac{1}{4},\sec\theta=\frac{1}{2}$

But, this is not possible as $|\sec\theta|\ge 1$

768 **(c)**

We have,
$x^3 - 13x^2 + 54x - 72 = 0$
$\Rightarrow (x-3)(x^2 - 10x + 24) = 0$
$\Rightarrow (x-3)(x-4)(x-6) = 0 \Rightarrow x = 3,4,6$
Let $a = 3, b = 4$ and $c = 6$
$\therefore \frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2+b^2+c^2}{2abc} = \frac{61}{144}$

769 **(b)**
$\cos^4\theta - \sin^4\theta = (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$
$= \cos 2\theta = 2\cos^2\theta - 1$

770 **(b)**
$\cos 15° \cos 7\frac{1}{2}° \sin 7\frac{1}{2}°$
$= \frac{1}{2}\cos 15° \sin 15° = \frac{1}{4}\sin 30°$
$= \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

771 **(b)**
$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} + \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \frac{1-\sin\theta+1+\sin\theta}{\sqrt{1-\sin^2\theta}}$
$= \frac{2}{\sqrt{\cos^2\theta}} = \frac{2}{|\cos\theta|}$
$= -\frac{2}{\cos\theta} = -2\sec\theta \left(\because \frac{\pi}{2} < \theta < \pi\right)$

772 **(c)**
$\cos^2 A(3-4\cos^2 A)^2 + \sin^2 A(3-4\sin^2 A)^2$
$= (3\cos A - 4\cos^3 A)^2 + (3\sin A - 4\sin^3 A)^2$
$= (-\cos 3A)^2 + (\sin 3A)^2 = 1$

773 **(a)**
It is given that A, B, C are in A.P.
$\therefore 2B = A + C$
$\Rightarrow 3B = A + B + C \Rightarrow 3B = 180° \Rightarrow B = 60°$
Also,
$b : c = \sqrt{3} : \sqrt{2}$
$\Rightarrow \frac{\sin B}{\sin C} = \frac{\sqrt{3}}{\sqrt{2}}$
$\Rightarrow \frac{\sqrt{3}}{2\sin C} = \frac{\sqrt{3}}{\sqrt{2}} \Rightarrow \sin C = \frac{1}{\sqrt{2}} \Rightarrow C = 45°$
$\therefore A = 180° - (60° + 45°) = 75°$

774 **(b)**
We have, $\tan x = \frac{b}{a}$
$\therefore \sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$
$= \sqrt{\frac{1+\frac{b}{a}}{1-\frac{b}{a}}} + \sqrt{\frac{1-\frac{b}{a}}{1+\frac{b}{a}}}$
$= \sqrt{\frac{1+\tan x}{1-\tan x}} + \sqrt{\frac{1-\tan x}{1+\tan x}}$
$= \sqrt{\frac{\cos x + \sin x}{\cos x - \sin x}} + \sqrt{\frac{\cos x - \sin x}{\cos x + \sin x}}$
$= \frac{\cos x + \sin x + \cos x - \sin x}{\sqrt{\cos^2 x - \sin^2 x}} = \frac{2\cos x}{\sqrt{\cos 2x}}$

775 **(c)**
We have,
$\sin A + \cos A = m$ and $\sin^3 A + \cos^3 A = n$
Now, $\sin A + \cos A = m$
$\Rightarrow (\sin A + \cos A)^3 = m^3$
$\Rightarrow \sin^3 A + \cos^3 A + 3\sin A\cos A(\sin A + \cos A) = m^3$
$\Rightarrow n + 3\sin A\cos A \; m = m^3$ ...(i)
Again,
$\sin A + \cos A = m$
$\Rightarrow \sin^2 A + \cos^2 A + 2\sin A\cos A = m^2$
$\Rightarrow \sin A\cos A = \frac{m^2-1}{2}$ ...(ii)
From (i) and (ii), we have
$n + 3m\frac{(m^2-1)}{2} = m^3$
$\Rightarrow 2n + 3m^3 - 3m = 2m^3 \Rightarrow m^3 - 3m + 2n = 0$

776 **(b)**
$\because \sec x - 1 = (\sqrt{2}-1)\tan x$
$\Rightarrow 1 - \cos x = (\sqrt{2}-1)\sin x$
$\Rightarrow \sin\frac{x}{2}\left\{\sin\frac{x}{2} - (\sqrt{2}-1)\cos\frac{x}{2}\right\} = 0$
$\Rightarrow \sin\frac{x}{2} = 0$ or $\tan\frac{x}{2} = \sqrt{2} - 1 = \tan\frac{\pi}{8}$
$\Rightarrow \frac{x}{2} = n\pi$ or $\frac{x}{2} = n\pi + \frac{\pi}{8}$
$\therefore x = 2n\pi, 2n\pi + \frac{\pi}{4}$

777 **(a)**
$\alpha - \beta = (\theta - \beta) - (\theta - \alpha)$
$\therefore \cos(\alpha - \beta) = \cos(\theta-\beta)\cos(\theta-\alpha) + \sin(\theta-\beta)\sin(\theta-\alpha)$
And $\sin(\alpha-\beta) = \sin(\theta-\beta)\cos(\theta-\alpha) - \sin(\theta-\alpha)\cos(\theta-\beta)$
$\Rightarrow \cos(\alpha-\beta) = b.a + \sqrt{1-a^2}\sqrt{1-b^2}$
And $\sin(\alpha-\beta) = (a\sqrt{1-b^2}) - (b\sqrt{1-b^2})$
Now,
$\sin^2(\alpha-\beta) = (a\sqrt{1-b^2})^2 + (b\sqrt{1-b^2})^2 -$
2ab1−a21−b2

$\Rightarrow \sin^2(\alpha-\beta) = a^2(1-b^2) + b^2(1-a^2) - 2ab\{\cos(\alpha-\beta) - ab\}$

$\Rightarrow \sin^2(\alpha-\beta) + 2ab\cos(\alpha-\beta) = a^2 - a^2b^2 + b^2 - b^2a^2 + 2a^2b^2$

$\Rightarrow \sin^2(\alpha-\beta) + 2ab\cos(\alpha-\beta) = a^2 + b^2$

778 **(a)**

We have, $S = \sin\theta + \sin 2\theta + \sin 3\theta + \ldots + \sin n\theta$

We know that,

$\sin\theta + \sin(\theta+\beta) + \sin(\theta+2\beta) + \ldots n$ terms

$= \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} \sin\left[\frac{\theta+\theta+(n-1)\beta}{2}\right]$

Put, $\beta = \theta$

$\therefore S = \frac{\sin\frac{n\theta}{2} \cdot \sin\frac{(n+1)\theta}{2}}{\sin\frac{\theta}{2}}$

780 **(c)**

Given, $\frac{\tan 3\theta - 1}{\tan 3\theta + 1} = \sqrt{3}$

$\Rightarrow \tan 3\theta - 1 - \sqrt{3}\tan 3\theta - \sqrt{3} = 0$

$\Rightarrow \tan 3\theta = \frac{1+\sqrt{3}}{1-\sqrt{3}} = \tan(45° + 60°)$

$\Rightarrow \tan 3\theta = \tan\frac{7\pi}{12}$

$\Rightarrow 3\theta = n\pi + \frac{7\pi}{12}$

$\Rightarrow \theta = \frac{n\pi}{3} + \frac{7\pi}{12}$

781 **(a)**

We have,

$2\sin\theta = r^4 - 2r^2 + 3$

$\Rightarrow 2\sin\theta = (r^2-1)^2 + 2$

Clearly, LHS ≤ 2 and RHS ≥ 2

So, the equation is meaningful if each side is equal to 2

Clearly, RHS $= 2$ for $r^2 = 1$

For $r^2 = 1$, we have

$2\sin\theta = 2$

$\Rightarrow \sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$ $[\because 0 \leq \theta \leq 5\pi]$

Also, $r^2 = 1 \Rightarrow r = \pm 1$

Hence, the total number of pairs of the form (r, θ) is $2 \times 3 = 6$

783 **(a)**

We have,

$\frac{1}{p_1^2} + \frac{1}{p_2^2} + \frac{1}{p_3^2} = \frac{a^2+b^2+c^2}{4\Delta^2}$

Also,

$\cot A + \cos B + \cot C = \frac{2R}{abc}(b^2+c^2-a^2+c^2 + a^2-b^2+a^2+b^2-c^2)$

$\Rightarrow \cot A + \cos B + \cot C = \frac{R(a^2+b^2+c^2)}{abc}$

$= \frac{a^2+b^2+c^2}{4\Delta}$

Hence, $\frac{1}{p_1^2} + \frac{1}{p_2^2} + \frac{1}{p_3^2} = \frac{\cot A + \cot B + \cot C}{\Delta}$

784 **(c)**

We have,

$\sin 2\theta + 2 = 4\sin\theta + \cos\theta$

$\Rightarrow 2\sin\theta\cos\theta - \cos\theta + 2 - 4\sin\theta = 0$

$\Rightarrow \cos\theta(2\sin\theta - 1) - 2(2\sin\theta - 1) = 0$

$\Rightarrow (2\sin\theta - 1)(\cos\theta - 2) = 0$

$\Rightarrow 2\sin\theta - 1 = 0$ $[\because \cos\theta - 2 \neq 0]$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = 2\pi + \frac{\pi}{6}, 2\pi + \frac{5\pi}{6}, 4\pi + \frac{\pi}{6}, 4\pi + \frac{5\pi}{6}$

Hence, the equation has 4 solutions

ALITER The curves $y = \sin x$ and $y = \frac{1}{2}$ intersect at 4 points in $[\pi, 5\pi]$. So, the equation has 4 solutions

785 **(c)**

For a triangle inscribed in a circle, we have

$\frac{a}{2\sin A} = \frac{b}{2\sin B} = \frac{c}{2\sin C} = R$

$\therefore \sin^2 A + \sin^2 B + \sin^2 C$

$= \frac{a^2}{4R^2} + \frac{b^2}{4R^2} + \frac{c^2}{4R^2}(a^2+b^2+c^2)$

It is given that

$\frac{a^2+b^2+c^2}{2} = 2(2R)^2 \Rightarrow a^2+b^2+c^2 = 16R^2$

$\therefore \sin^2 A + \sin^2 B + \sin^2 C = \frac{1}{4R^2}(16R^2) = 4$

786 **(d)**

We have,

$\tan 9° - \tan 27° - \tan 63° + \tan 81°$

$= (\tan 9° + \tan 81°) - (\tan 27° + \tan 63°)$

$= (\tan 9° + \cot 9°) - (\tan 27° + \cot 27°)$

$= \frac{1}{\sin 9° \cos 9°} - \frac{1}{\sin 27° \cos 27°}$

$= \frac{2}{\sin 18°} - \frac{2}{\sin 54°}$

$= 2\frac{\sin 54° - \sin 18°}{\sin 54° \sin 18°} = 2\frac{\cos 36° - \sin 18°}{\sin 18° \cos 36°} = 4$

787 **(c)**

Given, $\sin 4A + \sin 2A = \cos 4A + \cos 2A$

$\Rightarrow 2\sin 3A\cos A = 2\cos 3A\cos A$

$\therefore \tan 3A = 1$ and $\cos A = 0$

$\Rightarrow A = \frac{\pi}{12}$ and $A = \frac{\pi}{2} \notin \left(0, \frac{\pi}{4}\right)$

$\therefore\ \tan 4A = \tan\frac{\pi}{3} = \sqrt{3}$

788 **(c)**

We have,

$\sin A + \sin B = \frac{a+b}{c}$

$\Rightarrow \sin A + \sin B = \frac{\sin A + \sin B}{\sin C} \Rightarrow \sin C = 1$

789 **(a)**

We have,

$\sin(\alpha+\beta) = 1, \sin(\alpha-\beta) = \frac{1}{2}$

$\Rightarrow \alpha+\beta = \frac{\pi}{2}$ and, $\alpha-\beta = \frac{\pi}{6}$

$\Rightarrow \alpha = \frac{\pi}{3}, \beta = \frac{\pi}{6}$

$\therefore \tan(\alpha+2\beta)\tan(2\alpha+\beta)$

$= \tan\left(\frac{2\pi}{3}\right)\tan\frac{5\pi}{6} = \left(-\cot\frac{\pi}{6}\right)\left(-\cot\frac{\pi}{3}\right) = 1$

790 **(c)**

Let $a_0 = \cos\theta$. Then,

$a_1 = \sqrt{\frac{1}{2}(1+a_0)} = \sqrt{\frac{1}{2}(1+\cos\theta)} = \cos\frac{\theta}{2}$

$a_2 = \sqrt{\frac{1}{2}(1+a_1)} = \sqrt{\frac{1}{2}\left(1+\cos\frac{\theta}{2}\right)} = \cos\left(\frac{\theta}{2^2}\right)$

and so on

Now, $\frac{1-a_0^2}{a_1a_2a_3 \ldots \text{to } \infty}$

$= \frac{\sin\theta}{\cos\frac{\theta}{2}\cos\frac{\theta}{2^2}\cos\frac{\theta}{2^3}\ldots \text{to } \infty}$

$= \lim_{n\to\infty}\frac{\sin\theta}{\cos\frac{\theta}{2}\cos\frac{\theta}{2^2}\cos\frac{\theta}{2^3}\ldots \text{to } \infty}$

$= \lim_{n\to\infty}\frac{\{2^n\sin(\theta/2^n)\}\sin\theta}{\sin(2^n\times\theta/2^n)} = \lim_{n\to\infty}\frac{\sin(\theta/2^n).\theta}{(\theta/2^n)}$

$= \theta = a_0$

791 **(b)**

Le the angles of triangle ABC be $A=\theta, B = 2\,\theta$ and $C = 7\,\theta$. Then,

$A+B+C = 180° \Rightarrow 10\,\theta = 180° \Rightarrow \theta = 18°$

$\therefore A = 18°, B = 36°$ and $C = 126°$

Clearly, c is the greatest side and a is the smallest side.

Now,

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\Rightarrow \frac{c}{a} = \frac{\sin C}{\sin A} = \frac{\sin 126°}{\sin 18°} = \frac{\cos 36°}{\sin 18°} = \frac{\sqrt{5}+1}{\sqrt{5}-1}$

792 **(b)**

We have,

$A = \frac{2\,\pi}{3}$ and $\Delta = \frac{9\sqrt{3}}{2}\text{cm}^2$

$\therefore \Delta = \frac{1}{2}bc\sin A \Rightarrow \frac{9\sqrt{3}}{2} = \frac{1}{2}\,bc\sin\frac{2\,\pi}{3} \Rightarrow bc = 18$

Also,

$\cos A = \frac{b^2+c^2-a^2}{2bc}$

$\Rightarrow \cos\frac{2\,\pi}{3} = \frac{(b-c)^2+2bc-a^2}{2bc}$

$\Rightarrow -\frac{1}{2} = \frac{27+36-a^2}{36} \Rightarrow a^2 = 81 \Rightarrow a = 9$ cm

793 **(b)**

We have,

$s = 8k$ and, $\Delta = \sqrt{8k\times 3k\times 2k\times 3k} = 12\,k^2$

$\therefore r = \frac{\Delta}{s} \Rightarrow \frac{12\,k^2}{8k} = 6 \Rightarrow k = 4$

794 **(d)**

We have,

$\frac{2^{\sin x}+2^{\cos x}}{2} \geq \sqrt{2^{\sin x}2^{\cos x}}$ $\quad [\because AM \geq GM]$

$\Rightarrow 2^{\sin x}+2^{\cos x} \geq \sqrt{2^{\sin x+\cos x}}$

$\Rightarrow 2^{\sin x}+2^{\cos x}$

$\geq 2\sqrt{2^{-\sqrt{2}}}$ $\quad [\because -\sqrt{2} \leq \sin x + \cos x \leq \sqrt{2}]$

$\Rightarrow 2^{\sin x}+2^{\cos x} \geq 2^{1-\frac{1}{\sqrt{2}}}$

795 **(a)**

Let $A = \frac{1}{3\sin\theta - 4\cos\theta+7}$

Now, A will be minimum when $3\sin\theta - 4\cos\theta + 7$ is maximum

$\therefore$ Maximum value of

$3\sin\theta - 4\cos\theta + 7 = \sqrt{3^2+4^2}+7 = 12$

$\therefore$ Minimum value of $\frac{1}{3\sin\theta-4\cos\theta+7}$ is $\frac{1}{12}$

796 **(b)**

We have,

$\operatorname{cosec}\theta = \frac{p+q}{p-q}$

Now,

$\cos\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = \frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}} = \frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}$

$\Rightarrow \cos\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = \sqrt{\left(\frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}\right)^2} = \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

$\Rightarrow \cot\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = \sqrt{\frac{\operatorname{cosec}\theta-1}{\operatorname{cosec}\theta+1}} = \sqrt{\frac{\frac{p+q}{p-q}-1}{\frac{p+q}{p-q}+1}} = \sqrt{\frac{q}{p}}$

797 **(a)**

$\sin t + \cos t = \frac{1}{5}$

$$\Rightarrow \frac{2\tan\frac{t}{2}}{1+\tan^2\frac{t}{2}}+\frac{1-\tan^2\frac{t}{2}}{1+\tan^2\frac{t}{2}}=\frac{1}{5}$$
$$\Rightarrow 10\tan\frac{t}{2}+5-5\tan^2\frac{t}{2}=1+\tan^2\frac{t}{2}$$
$$\Rightarrow 6\tan^2\frac{t}{2}-10\tan\frac{t}{2}-4=0$$
$$\Rightarrow \left(6\tan\frac{t}{2}+2\right)\left(\tan\frac{t}{2}-2\right)=0$$
$\Rightarrow \tan\frac{t}{2}=\frac{-1}{3}, 2$ for , $0<t<\pi$

$\tan\frac{t}{2}=2$

798 **(a)**

We have,

$\sin\alpha\cos^3\alpha>\sin^3\alpha\cos\alpha$

$\Rightarrow \sin\alpha\cos\alpha\,(\cos^2\alpha-\sin^2\alpha)>0$

$\Rightarrow \cos\alpha\,(1-\tan^2\alpha)$

>0 $[\because \sin\alpha>0 \text{ for } 0<\alpha<\pi]$

$\Rightarrow \cos\alpha>0$ and $1-\tan^2\alpha>0$

$\Rightarrow \cos\alpha<0$ and $1-\tan^2\alpha<0$

$\Rightarrow \alpha\in(0,\pi/4)$ or, $\alpha\in(3\pi/4,\pi)$

799 **(a)**

We have, $\alpha+\beta+\gamma=\pi$

Now, $\sin^2\alpha+\sin^2\beta+\sin^2\gamma$

$=\sin^2\alpha+\sin^2(\beta-\gamma)\sin(\beta+\gamma)$

$=\sin^2\alpha+\sin(\pi-\alpha)\sin(\beta+\gamma)$ $(\because \alpha+\beta-\gamma=\pi)$

$=\sin^2\alpha+\sin\alpha\ \sin(\beta+\gamma)$

$=\sin\alpha[\sin\alpha+\sin(\beta+\gamma)]$

$=\sin\alpha[\sin(\pi-(\beta-\gamma))+\sin(\beta+\gamma)]$

$=\sin\alpha[\sin(\beta-\gamma)+\sin(\beta+\gamma)]$

$=\sin\alpha\,[2\sin\beta\cos\gamma]$

$=2\sin\alpha\sin\beta\cos\gamma$

800 **(c)**

$\sec x\cos 5x=-1$

$\Rightarrow \cos 5x=-\cos x$

$\Rightarrow 5x=2n\pi\pm(\pi-x)$

$\Rightarrow x=\frac{(2n+1)\pi}{6}$ or $\frac{(2n-1)\pi}{6}$

The possible values of x which lies in the interval $(0,2\pi)$ are $\frac{\pi}{4},\frac{\pi}{6},\frac{\pi}{2},\frac{3\pi}{4},\frac{5\pi}{6},\frac{5\pi}{4},\frac{7\pi}{6},\frac{7\pi}{4},\frac{9\pi}{6}$ and $\frac{11\pi}{6}$

801 **(a)**

$\cos(\alpha+\beta)=-\frac{12}{13}$

Here, $0<(\alpha+\beta)<\pi$

$\therefore \sin(\alpha+\beta)=\sqrt{1-\cos^2(\alpha+\beta)}$

$=\sqrt{1-\frac{144}{169}}$

$=\frac{5}{13}$

Now, $\sin\beta=\sin[(\alpha+\beta)-\alpha]$

$=\sin(\alpha+\beta)\cos\alpha-\cos(\alpha+\beta)\sin\alpha$

$=\frac{5}{13}.\frac{3}{5}-\left(-\frac{12}{13}\right).\frac{4}{5}$

$=\frac{15}{65}+\frac{48}{65}$

$=\frac{63}{65}$

802 **(c)**

We have,

$\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}$

$=\sin\left(\frac{\pi}{2}-\frac{3\pi}{7}\right)\sin\left(\frac{\pi}{2}-\frac{2\pi}{7}\right)\sin\left(\frac{\pi}{2}-\frac{\pi}{7}\right)\sin\frac{\pi}{2}$

$=\cos\frac{3\pi}{7}\cos\frac{2\pi}{7}\cos\frac{\pi}{7}$

$=-\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}$

$=-\frac{\sin(2^3\pi/7)}{2^3\sin\pi/7}=-\frac{\sin(8\pi/7)}{8\sin\pi/7}=\frac{1}{8}$

803 **(b)**

We have,

$\sin\theta\cos\alpha+\cos\theta\sin\alpha=2\,k\sin\theta\cos\theta$

$\Rightarrow \cos\alpha\frac{2\,t}{1+t^2}+\sin\alpha\frac{1-t^2}{1+t^2}=2k\frac{2t}{1+t^2}\times\frac{1-t^2}{1+t^2}$

where $t=\tan\frac{\theta}{2}$

$\Rightarrow \sin\alpha\,t^4-(2\cos\alpha+4k)t^3+t(4k-2\cos\alpha)$
$-\sin\alpha=0$

$\Rightarrow S_1=2\cos\alpha+4k, S_2=0$

$S_3=2\cos\alpha-4k, S_4=-1$

where S_r denotes the sum of the product of roots taken r at a time

Now,

$\tan\left(\frac{\theta_1}{2}+\frac{\theta_2}{2}+\frac{\theta_3}{2}+\frac{\theta_4}{2}\right)=\frac{S_1-S_3}{1-S_2+S_4}=\infty$

$=\tan\left(\frac{\pi}{2}\right)$

$\Rightarrow \frac{\theta_1}{2}+\frac{\theta_2}{2}+\frac{\theta_3}{2}+\frac{\theta_4}{2}=n\,\pi+\frac{\pi}{2}, n\in Z$

$\Rightarrow \theta_1+\theta_2+\theta_3+\theta_4=(2n+1)\,\pi, n\in Z$

804 **(b)**

Let r be the radius of the circle. Then,

$\frac{3\pi}{4}=\frac{15\pi}{r}\Rightarrow r=20$ cm

805 **(a)**

We know that

$\cot\alpha - \tan\alpha = 2\cot 2\alpha$

$\therefore \cot\theta - \tan\theta - 2\tan 2\theta - 4\tan 4\theta - 8\cot 8\theta$

$= 2\cot 2\theta - 2\tan 2\theta - 4\tan 4\theta - 8\cot 8\theta$

$= 2(2\cot 4\theta) - 4\tan 4\theta - 8\cot 8\theta$

$= 4\cot 4\theta - 4\tan 4\theta - 8\cot 8\theta$

$= 4(\cot 4\theta - \tan 4\theta) - 8\cot 8\theta$

$= 4\times 2\cot 8\theta - 8\cot 8\theta = 0$

806 **(d)**

We have,

$b = \sqrt{3}, c = 1$ and $A = 30°$

$\therefore \cos A = \frac{b^2+c^2-a^2}{2bc} \Rightarrow \frac{\sqrt{3}}{2} = \frac{4-a^2}{2\sqrt{3}} \Rightarrow a = 1$

$\therefore \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\Rightarrow 2 = \frac{\sqrt{3}}{\sin B} = \frac{1}{\sin C}$

$\Rightarrow \sin B = \frac{\sqrt{3}}{2}$ and $\sin C = \frac{1}{2}$

$\Rightarrow B = 120°$ and $C = 30°$ $[\because b > c \therefore B > C]$

807 **(c)**

Here, $a = 3,\ b = 4$

$\therefore$ maximum value$= \sqrt{3^2+4^2} = 5$

808 **(a)**

Let ABC be the triangle such that $a = 2, b = \sqrt{6}$ and $c = \sqrt{3}-1$

Clearly, $b > a > c$

So, B is the greatest angle and C is the smallest angle

Now,

$\cos B = \frac{c^2+a^2-b^2}{2ac}$

$\Rightarrow \cos B = \frac{(\sqrt{3}-1)^2+4-6}{4(\sqrt{3}-1)^2} = -\frac{1}{2} \Rightarrow B = 120°$

And,

$\cos C = \frac{a^2+b^2-c^2}{2ab}$

$\Rightarrow \cos C = \frac{4+6-(\sqrt{3}-1)^2}{4\sqrt{6}} = \frac{\sqrt{3}+1}{2\sqrt{2}} \Rightarrow C$
$= 15°$

809 **(b)**

We have,

$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$

$\Rightarrow \frac{1}{r} = \frac{1}{16} + \frac{1}{48} + \frac{1}{24} \Rightarrow r = 8$

810 **(c)**

Given, $3\sin^2 x - 7\sin x + 2 = 0$

$(3\sin x - 1)(\sin x - 2) = 0$

$\Rightarrow \sin x = \frac{1}{3}$ or 2

$\Rightarrow \sin x = \frac{1}{3}$ $[\because \sin x \neq 2]$

Let $\sin^{-1}\frac{1}{3} = \alpha, 0 < \alpha < \frac{\pi}{2}$, then $\alpha, \pi-\alpha, 2\pi+\alpha, 3\pi-\alpha, 4\pi+\alpha, 5\pi-\alpha$ are the solution in $[0, 5\pi]$

Hence, required number of solutions are 6

811 **(a)**

We have, $\cos^2\theta = \cos 2\theta$

$\Rightarrow \cos^2\theta = 2\cos^2\theta - 1$

$\Rightarrow \cos^2\theta = 1 \Rightarrow \theta = n\pi$

812 **(a)**

The given equation is

$3^{\sin 2x + 2\cos^2 x} + 3^{1-\sin 2x + 2\sin^2 x} = 28$

$\Rightarrow 3^{\sin 2x+2\cos^2 x} + 3^{3-(\sin 2x + 2\cos^2 x)} = 28$

$\Rightarrow y + \frac{27}{y} = 28$, where $y = 3^{\sin 2x + 2\cos^2 x}$

$\Rightarrow y^2 - 28y + 27 = 0 \Rightarrow y = 27$ or, $y = 1$

If $y = 27$, then

$3^{\sin 2x+2\cos^2 x} = 3^3$

$\Rightarrow \sin 2x + 2\cos^2 x = 3$

$\Rightarrow \sin 2x + 2\cos 2x = 2$

$\Rightarrow \sin 2x = 2\cos 2x = 1$

$\Rightarrow \sin x = 0$ or, $\tan x = \frac{1}{2}$

If $y = 1$

$\Rightarrow 3^{\sin 2x + 2\cos^2 x} = 1$

$\Rightarrow \sin 2x + 2\cos^2 x = 0$

$\Rightarrow 2\cos x(\sin x + \cos x) = 0$

$\Rightarrow \cos x = 0$ or, $\tan x = -1$

813 **(c)**

Let $f(x) = 27^{\cos 2x} 81^{\sin 2x} = 3^{3\cos 2x + 4\sin 2x}$

$= 3^{5\left(\frac{3}{5}\cos 2x + \frac{4}{5}\sin 2x\right)}$

Let $\frac{3}{5} = \sin\phi$

$\Rightarrow \frac{4}{5} = \cos\phi$

Then, $f(x) = 3^{5(\sin\phi\cos 2x + cos\phi\sin 2x)}$

$= 3^{5(\sin(\phi+2x))}$

For minimum value of given function, $\sin(\phi+2x)$ will be minimum

ie, $\sin(\phi + 2x) = -1$

$\therefore\ f(x) = 3^{5(-1)} = \frac{1}{243}$

814 **(c)**

We have,

$\sec 2x - \tan 2x = \frac{1-\sin 2x}{\cos 2x}$

$$\Rightarrow \sec 2x - \tan 2x = \frac{1-\cos\left(\frac{\pi}{2}-2x\right)}{\sin\left(\frac{\pi}{2}-2x\right)}$$

$$\Rightarrow \sec 2x - \tan 2x = \frac{2\sin^2(\pi/4 - x)}{2\sin(\pi/4-x)\cos(\pi/4-x)}$$

$$= \tan\left(\frac{\pi}{4}-x\right)$$

815 **(a)**

$$\because\ \sin^5 x - \cos^5 x = \frac{\sin x - \cos x}{\sin x \cos x}$$

$$\Rightarrow\ \sin x\cos x\left[\frac{\sin^5 x - \cos^5 x}{\sin x - \cos x}\right] = 1$$

$$\Rightarrow\ \frac{1}{2}\sin 2x[\sin^4 x + \sin^3 x\cos x + \sin^2 x\cos^2 x + \sin x\cos^2 x + \cos^4 x] = 1$$

$$\Rightarrow\ \sin 2x[(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x + \sin x\cos x(\sin^2 x + \cos^2 x) + \sin^2 x + \cos^2 x] = 2$$

$$\Rightarrow \sin 2x[1 - \sin^2 x\cos^2 x + \sin x\cos x] = 2$$

$$\Rightarrow \sin^3 2x - 2\sin^2 2x - 4\sin 2x + 8 = 0$$

$$\Rightarrow (\sin 2x - 2)^2(\sin 2x + 2) = 0$$

$\Rightarrow \sin 2x = \pm 2$, which is not possible for any x

816 **(b)**

$\cos(\alpha+\beta) = \frac{4}{5} \Rightarrow \alpha+\beta \in$ 1st quadrant and

$\sin(\alpha-\beta) = \frac{5}{13}$

$\Rightarrow\ \alpha-\beta \in$ 1st quadrant

$\Rightarrow\ 2\alpha = (\alpha+\beta) + (\alpha-\beta)$

$$\therefore\ \tan 2\alpha = \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$= \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}.\frac{5}{12}} = \frac{56}{33}$$

817 **(d)**

We have,

$$\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$$

$$= \frac{1}{2\sin\frac{\pi}{7}}\left\{2\sin\frac{\pi}{7}\cos\frac{2\pi}{7} + 2\sin\frac{\pi}{7}\cos\frac{4\pi}{7} + 2\sin\frac{\pi}{7}\cos\frac{6\pi}{7}\right\}$$

$$= \frac{1}{2\sin\frac{\pi}{7}}\left\{\sin\frac{3\pi}{7} - \sin\frac{\pi}{7} + \sin\frac{5\pi}{7} - \sin\frac{3\pi}{7} + \sin\pi - \sin\frac{5\pi}{7}\right\}$$

$$= -\frac{1}{2}$$

818 **(a)**

We have,

$$2a\cos^2\left(\frac{C}{2}\right) + 2c\cos^2\left(\frac{A}{2}\right) = 3b$$

$$\Rightarrow a(1+\cos C) + c(1+\cos A) = 3b$$

$$\Rightarrow a + c + (a\cos C + c\cos A) = 3b$$

$\Rightarrow a + c + b = 3\,b \Rightarrow a + c = 2b \Rightarrow a, b, c$ are in A.P.

819 **(d)**

Given that, $\frac{1-\cos 2\theta}{1+\cos 2\theta} = 3$

$$\Rightarrow\ \frac{2\sin^2\theta}{2\cos^2\theta} = 3$$

$$\Rightarrow\ \tan^2\theta = (\sqrt{3})^2$$

$$\Rightarrow\ \tan^2\theta = \tan^2\frac{\pi}{3}$$

$$\Rightarrow\ \theta = n\pi \pm \frac{\pi}{3}$$

820 **(b)**

We have,

$a = 5$ cm, $b = 4$ cm and $\cos(A-B) = \frac{31}{32}$

$$\therefore \tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$$

$$\Rightarrow \sqrt{\frac{1-\cos(A-B)}{1+\cos(A-B)}} = \frac{a-b}{a+b}\sqrt{\frac{1+\cos C}{1-\cos C}}$$

$$\Rightarrow \frac{1-\frac{31}{32}}{1+\frac{31}{32}} = \left(\frac{5-4}{5+4}\right)^2\left(\frac{1+\cos C}{1-\cos C}\right)$$

$$\Rightarrow \frac{81}{63} = \frac{1+\cos C}{1-\cos C} \Rightarrow \cos C = \frac{1}{8}$$

821 **(a)**

Given, $\sin 2x + \cos 4x = 2$

$$\Rightarrow\ \sin 2x + 1 - 2\sin^2 2x = 2$$

$$\Rightarrow\ 2\sin^2 2x - \sin 2x + 1 = 0$$

Now, Discriminant, $D = (-1)^2 - 4.2.1 = -7 < 0$

Hence, it is an imaginary equation, so the real root does not exist.

822 **(d)**

We have,

$$\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$$

$$\Rightarrow \sin\theta_1 = \sin\theta_2 = \sin\theta_3 = 1\quad [\because -1 \le \sin x \le 1]$$

$$\Rightarrow \theta_1 = \theta_2 = \theta_3 = \frac{\pi}{2} \Rightarrow \cos\theta_1 + \cos\theta_2 + \cos\theta_3 = 0$$

823 **(b)**

We have,

$$k\sin x + (1 - 2\sin^2 x) = 2k - 7$$

$$\Rightarrow 2\sin^2 x - k\sin x + 2(k-4) = 0$$

$$\Rightarrow \sin x = \frac{k \pm \sqrt{k^2 - 16\,k + 64}}{4} = \frac{k \pm (k-8)}{4}$$

$$= \frac{1}{2}(k-4), 2$$

$\Rightarrow \sin x = \frac{1}{2}(k-4) \quad [\because \sin x \neq 2]$

Now, $-1 \leq \sin x \leq 1 \Rightarrow -1 \leq \frac{k-4}{2} \leq 1 \Rightarrow 2 \leq k \leq 6$

824 **(b)**

Given that, $\cos 2B = \frac{\cos(A+C)}{\cos(A-C)}$

$= \frac{\cos A \cos C - \sin A \sin C}{\cos A \cos C + \sin A \sin C}$

$\Rightarrow \frac{1-\tan^2 B}{1+\tan^2 B} = \frac{1-\tan A \tan C}{1+\tan A \tan C}$

$\Rightarrow 1+\tan^2 B - \tan A \tan C - \tan A \tan C \tan^2 B$

$= 1-\tan^2 B + \tan A \tan C - \tan A \tan C \tan^2 B$

$\Rightarrow 2\tan^2 B = 2\tan A \tan C$

$\Rightarrow \tan^2 B = \tan A \tan C$

Hence, $\tan A, \tan B$ and $\tan C$ will be in GP

825 **(c)**

We have,

$\left(\frac{\cos A + \cos B}{\sin A - \sin B}\right)^n + \left(\frac{\sin A + \sin B}{\cos A - \cos B}\right)^n$

$= \left(\cot\frac{A-B}{2}\right)^n + \left(-\cot\frac{A-B}{2}\right)^n$

$= \{1+(-1)^n\}\cot^n\left(\frac{A-B}{2}\right)$

$= 0 \times \cot^n\left(\frac{A-B}{2}\right) = 0 \quad [\because n \text{ is odd}]$

826 **(a)**

We have,

$a\tan\theta + b\sec\theta = c$

$\Rightarrow b\sec\theta = c - a\tan\theta$

$\Rightarrow b^2\sec^2\theta = c^2 + a^2\tan^2\theta - 2\,ac\tan\theta$

$\Rightarrow b^2(1+\tan^2\theta) = c^2 + a^2\tan^2\theta - 2\,ac\tan\theta$

$\Rightarrow \tan^2\theta(b^2-a^2) + 2\,ac\tan\theta + b^2 - c^2 = 0$

Since $\tan\alpha$ and $\tan\beta$ are roots of this equation

$\therefore \tan\alpha + \tan\beta = \frac{-2\,ac}{b^2-a^2}$ and $\tan\alpha\tan\beta = \frac{b^2-c^2}{a^2-c^2}$

Now,

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} = \frac{-\frac{2\,ac}{b^2-a^2}}{1-\frac{b^2-c^2}{b^2-a^2}}$

$= \frac{2\,ac}{a^2-c^2}$

827 **(b)**

$\tan\theta + \frac{\tan\theta+\sqrt{3}}{1-\sqrt{3}\tan\theta} + \frac{\tan\theta-\sqrt{3}}{1+\sqrt{3}\tan\theta} = 3$

$\Rightarrow \tan\theta + \frac{8\tan\theta}{1-3\tan^2\theta} = 3$

$\Rightarrow \frac{9\tan\theta - 3\tan^3\theta}{1-3\tan^2\theta} = 3$

$\Rightarrow 3\tan 3\theta = 3 \Rightarrow \tan 3\theta = 1$

828 **(a)**

Since, $y = 1 + 4\sin^2 x\cos^2 x$

$\Rightarrow y = 1 + \sin^2 2x$

We know that, $0 \leq \sin^2 2x \leq 1$

$\Rightarrow 1 \leq 1 + \sin^2 2x \leq 2$

$\Rightarrow 1 \leq y \leq 2$

829 **(a)**

$\sin^2\alpha + \sin^2\beta - \sin^2\gamma$

$= \sin^2\alpha + \sin(\beta-\gamma)\sin(\beta+\gamma)$

$= \sin^2\alpha\sin(\pi-\alpha)\sin(\beta+\gamma) \quad [\because \alpha+\beta-\gamma = \pi]$

$= \sin\alpha[\sin\alpha + \sin(\beta+\gamma)]$

$= \sin\alpha[\sin\{\pi-(\beta-\gamma)\} + \sin(\beta+\gamma)]$

$= \sin\alpha[\sin(\beta-\gamma) + \sin(\beta+\gamma)]$

$= \sin\alpha[2\sin\beta\cos\gamma]$

$= 2\sin\alpha\sin\beta\cos\gamma$

830 **(a)**

We have,

$\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2}$

$= \frac{s(s-a)}{\Delta} + \frac{s(s-b)}{\Delta} + \frac{s(s-c)}{\Delta} = \frac{s}{\Delta}(3s-2s)$

$= \frac{s^2}{\Delta}$

And,

$\cot A + \cot B + \cot C$

$= \frac{\cos A}{\sin A} + \frac{\cos B}{\sin B} + \frac{\cos C}{\sin C}$

$= \frac{b^2+c^2-a^2}{2\,bc\sin A} + \frac{c^2+a^2-b^2}{2ac\sin B} + \frac{a^2+b^2-c^2}{2ab\sin C}$

$= \frac{b^2+c^2-a^2}{4\,\Delta} + \frac{c^2+a^2-b^2}{4\,\Delta} + \frac{a^2+b^2-c^2}{4\,\Delta}$

$= \frac{a^2+b^2+c^2}{4\,\Delta}$

$\therefore \frac{\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2}}{\cot A + \cot B + \cot C} = \frac{\frac{s^2}{\Delta}}{\frac{a^2+b^2+c^2}{4\,\Delta}}$

$= \frac{(2s)^2}{a^2+b^2+c^2} = \frac{(a+b+c)^2}{a^2+b^2+c^2}$

831 **(a)**

$\because \quad (\tan\alpha - \cot\alpha)^2 \geq 0$

$\Rightarrow \tan^2\alpha + \cot^2\alpha - 2 \geq 0$

$\Rightarrow \tan^2\alpha + \cot^2\alpha \geq 2$

832 **(a)**

We have,

$\tan m\,\theta = \tan n\,\theta$

$\Rightarrow m\,\theta = t\,\pi + n\,\theta$, where $r \in Z$

$\Rightarrow \theta = \dfrac{r\,\pi}{m-n}, r \in Z$

Clearly, these values of θ from an A.P. with common difference $\frac{\pi}{m-n}$

833 **(a)**

We have,

$\dfrac{\sin A}{\sin C} = \dfrac{\sin(A-B)}{\sin(B-C)}$

$\Rightarrow \dfrac{\sin(B+C)}{\sin(a+B)} = \dfrac{\sin(A-B)}{\sin(B-C)}$

$\Rightarrow \sin^2 B - \sin^2 C = \sin^2 A - \sin^2 B$

$\Rightarrow b^2 - c^2 = a^2 - b^2$

$\Rightarrow a^2, b^2, c^2$ are in A.P.

834 **(a)**

Let $\sec\theta - \tan\theta = \lambda$ ...(i)

Then,

$(\sec\theta + \tan\theta) = \dfrac{1}{\sec\theta - \tan\theta}$

$\Rightarrow \sec\theta + \tan\theta = \frac{1}{\lambda}$ ...(ii)

$\therefore 2\tan\theta = \frac{1}{\lambda} + \lambda$ [On subtracting (i) from (ii)]

$\Rightarrow 2x - \dfrac{1}{2x} = \dfrac{1}{\lambda} - \lambda$

$\Rightarrow \lambda = \dfrac{1}{2x}, -2x \Rightarrow \sec\theta - \tan\theta = \dfrac{1}{2x}, -2x$

835 **(d)**

We observe that the LHS of the given equation is not defined for $x = n\,\pi, n \in Z$

Now,

$\cot x - \operatorname{cosec} x = 2\sin x$

$\Rightarrow \cot x - 1 = 2\sin^2 x$

$\Rightarrow 2\cos^2 x + \cos x - 3 = 0$

$\Rightarrow (2\cos x + 3)(\cos x - 1) = 0$

$\Rightarrow \cos x = 1$ [$\because 2\cos x + 3 \neq 0$]

$\Rightarrow x = 0, 2\,\pi$

But, $x \neq n\,\pi, n \in Z$

Hence, the given equation has no solution

837 **(d)**

Given, $\frac{\sin x}{\sin y} = \frac{1}{2}, \frac{\cos x}{\cos y} = \frac{3}{2}$

$\Rightarrow \dfrac{\tan x}{\tan y} = \dfrac{1}{3}$

$\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y} = \dfrac{4\tan x}{1 - 3\tan^2 x}$

Also, $\sin y = 2\sin x, \cos y = \frac{2}{3}\cos x$

$\Rightarrow \sin^2 y + \cos^2 y = 4\sin^2 x + \dfrac{4\cos^2 x}{9} = 1$

$\Rightarrow 36\tan^2 x + 4 = 9\sec^2 x = 9(1 + \tan^2 x)$

$\Rightarrow 27\ \tan^2 x = 5$

$\Rightarrow \tan x = \dfrac{\sqrt{5}}{3\sqrt{3}}$

$\Rightarrow \tan(x+y) = \dfrac{\frac{4\sqrt{5}}{3\sqrt{3}}}{1 - \frac{15}{27}} = \sqrt{15}$

840 **(d)**

$\dfrac{\cos 9° + \sin 9°}{\cos 9° - \sin 9°} = \dfrac{1 + \tan 9°}{1 - \tan 9°}$

$= \tan(45° + 9°)$

$= \tan 54°$

842 **(a)**

Let ABC be the triangle such that $a = 2\sqrt{2}$ cm, $b = 2\sqrt{3}$ cm and $\angle A = \frac{\pi}{4}$

$\therefore \cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\Rightarrow \dfrac{1}{\sqrt{2}} = \dfrac{12 + c^2 - 8}{4\sqrt{3}\,c}$

$\Rightarrow 4 + c^2 = 2\sqrt{6}\,c$

$\Rightarrow c^2 - 2\sqrt{6}\,c + 4 = 0$

$\Rightarrow c = \dfrac{2\sqrt{6} \pm \sqrt{24 - 16}}{2}$

$\Rightarrow c = \sqrt{6} \pm \sqrt{2} \Rightarrow c = \sqrt{6} + \sqrt{2}$ [$\because c$ is the largest side]

843 **(b)**

We have,

$r\sin\theta = 3$ and $r = 4(1 + \sin\theta)$

$\Rightarrow r = 4 + \dfrac{12}{r}$

$\Rightarrow r^2 - 4r - 12 = 0$

$\Rightarrow (r-6)(r+2) = 0$

$\Rightarrow r = 6$ [$\because r > 0$]

$\therefore r\sin\theta = 3 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Hence, the total number of ordered pairs of the form (r, θ) is $1 \times 2 = 2$

844 **(d)**

We have,

$\sin 65° + \sin 43° - \sin 29° - \sin 7°$

$= (\sin 65° + \sin 43°) - (\sin 29° + \sin 7°)$

$= 2\sin 54° \cos 11° - 2\sin 18° \cos 11°$

$= 2\cos 11° (\cos 36° - \sin 18°)$

$= 2\cos 11° \left(\frac{\sqrt{5}+1}{4} - \frac{\sqrt{5}-1}{4}\right) = \cos 11°$

845 **(c)**

We have,

$\sin B = \frac{1}{5}\sin(2A+B)$

$\Rightarrow \frac{\sin(2A+B)}{\sin B} = \frac{5}{1}$

$\Rightarrow \frac{\sin(2A+B)+\sin B}{\sin(2A+B)-\sin B} = \frac{5+1}{5-1}$

$\Rightarrow \frac{2\sin(A+B)\cos A}{2\sin A\cos(A+B)} = \frac{3}{2}$

$\Rightarrow \frac{\tan(A+B)}{\tan A} = \frac{3}{2}$

846 **(b)**

Since, $A + B + C = \pi$

$\Rightarrow a = \pi - (B + C)$

We have, $\cos A = \cos B \cos C$

$\Rightarrow \cos[\pi - (B + C)] = \cos B \cos C$

$\Rightarrow -\cos(B + C) = \cos B \cos C$

$\Rightarrow -[\cos B \cos C - \sin B \sin C] = \cos B \cos C$

$\Rightarrow \sin B \sin C = 2\cos B \cos C$

$\Rightarrow \tan B \tan C = 2$

847 **(a)**

We have,

$\sin x + \operatorname{cosec} x = 2 \Rightarrow (\sin x - 1)^2 = 0 \Rightarrow \sin x = 1$

$\therefore \sin^n x + \operatorname{cosec}^n x = 1 + 1 = 2$

848 **(a)**

We have,

$\frac{a^2 - b^2}{a^2 + b^2} = \frac{\sin(A-B)}{\sin(A+B)}$

$\Rightarrow \frac{\sin^2 A - \sin^2 B}{\sin^2 A + \sin^2 B} = \frac{\sin^2 A - \sin^2 B}{\sin^2(A+B)}$

$\Rightarrow (\sin^2 A - \sin^2 B)(\sin^2 A + \sin^2 B - \sin^2 C) = 0$

$\Rightarrow \sin(A+B)\sin(A-B)(\sin^2 A + \sin^2 B - \sin^2 C) = 0$

$\Rightarrow \sin(A-B) = 0$ or, $\sin^2 A + \sin^2 B = \sin^2 C$

$\Rightarrow A = B$ or, $a^2 + b^2 = c^2$

$\Rightarrow \Delta ABC$ is either right angled or isosceles

849 **(c)**

We have,

$\cos A = \cos B \cos C$

$\Rightarrow \cos\{\pi - (B + C)\} = \cos B \cos C$

$\Rightarrow -\cos(B + C) = \cos B \cos C$

$\Rightarrow 2\cos B \cos C = \sin B \sin C$

$\Rightarrow \cot B \cot C = \frac{1}{2}$

850 **(c)**

We have,

$2s = a + b + c = 13 + 14 + 15$

$\Rightarrow s = 21$

$\Rightarrow s - a = 8, s - b = 7$ and $s - c = 6$

Now,

$\frac{1}{r_1} : \frac{1}{r_2} : \frac{1}{r_3} = \frac{s-a}{\Delta} : \frac{s-b}{\Delta} : \frac{s-c}{\Delta}$

$\Rightarrow \frac{1}{r_1} : \frac{1}{r_2} : \frac{1}{r_3} = s - a : s - b : s - c = 8 : 7 : 6$

851 **(a)**

We have,

$\frac{\sin 7\theta + 6\sin 5\theta + 17\sin 3\theta + 12\sin\theta}{\sin 6\theta + 5\sin 4\theta + 12\sin 2\theta}$

$= \frac{(\sin 7\theta + \sin 5\theta) + 5(\sin 5\theta + \sin 3\theta) + 12(\sin 3\theta + \sin\theta)}{\sin 6\theta + 5\sin 4\theta + 12\sin 2\theta}$

$$= \frac{2\sin 6\theta \cos\theta + 10\sin 4\theta\cos\theta + 24\sin 2\theta\cos\theta}{\sin 6\theta + 5\sin 4\theta + 12\sin 2\theta}$$

$$= \frac{2\cos\theta\,(\sin 6\theta + 5\sin 4\theta + 12\sin 2\theta)}{\sin 6\theta + 5\sin 4\theta + 12\sin 2\theta}$$

$$= 2\cos\theta$$

852 **(a)**

Let the angles be $A = x - d, B = x, C = x + d$.

Then,

$x - d + x + x + d = 180° \Rightarrow x = 60°$

Therefore, two larger angles are $B = 60°$ and C

Hence, $b = 9$ and $c = 10$

Now,

$$\cos B = \frac{c^2 + a^2 - b^2}{2\,ac}$$

$$\Rightarrow \frac{1}{2} = \frac{100 + a^2 - 81}{20\,a} \Rightarrow a^2 - 10\,a + 19 = 0 \Rightarrow a = 5 \pm \sqrt{6}$$

853 **(b)**

Since, $\cos 2x, \frac{1}{2}, \sin 2x$ are in AP

$\Rightarrow \cos 2x + \sin 2x = 1$

$\Rightarrow \sin 2x = 1 - \cos 2x = 2\sin^2 x$

$\Rightarrow 2\sin x(\cos x - \sin x) = 0$

$\Rightarrow \sin x = 0$ or $\cos x - \sin x = 0$

$\Rightarrow x = n\pi$ or $\tan x = 1$

$\Rightarrow x = n\pi$ or $x = n\pi + \frac{\pi}{4}$

Thus, required values of x are $n\pi$ and $n\pi + \frac{\pi}{4}$

854 **(b)**

$$\cos\frac{\pi}{18} + \cos\frac{2\pi}{18} + \ldots + \cos\frac{16\pi}{18} + \cos\frac{17\pi}{18} + \cos\pi$$

$$= \cos\frac{\pi}{18} + \cos\frac{2\pi}{18} + \ldots - \cos\frac{2\pi}{18} - \cos\frac{\pi}{18} + \cos\pi$$

$$= \cos\pi = -1$$

855 **(b)**

Given that, $\sin\theta + \cos\theta = 1$

$$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$$

$$\Rightarrow \theta + \frac{\pi}{4} = n\pi + (-1)^n\frac{\pi}{4}$$

$$\Rightarrow \theta = n\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{4}$$

856 **(d)**

We have,

$0 < x < \pi \Rightarrow \sin x > 0$

Now,

$1 + \sin x + \sin^2 x + \cdots\infty = 4 + 2\sqrt{3}$

$$\Rightarrow \frac{1}{1 - \sin x} = 4 + 2\sqrt{3}$$

$$\Rightarrow \sin x = 1 - \frac{1}{4 + 2\sqrt{3}}$$

$$\Rightarrow \sin x = \frac{3 + 2\sqrt{3}}{4 + 2\sqrt{3}} = \frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{3} \text{ or, } \frac{2\pi}{3}$$

857 **(b)**

$$\frac{\sin x + \sin y}{\cos x + \cos y} = \frac{a}{b}$$

$$\Rightarrow \frac{2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)} = \frac{a}{b}$$

$$\Rightarrow \tan\left(\frac{x+y}{2}\right) = \frac{a}{b}$$

858 **(b)**

The given equation can be written as

$$\cos(\pi\tan\theta) = \cos\left(\frac{\pi}{2} - \pi\cot\theta\right)$$

$$\Rightarrow \pi\tan\theta = 2n\,\pi \pm \left(\frac{\pi}{2} - \pi\cot\theta\right), n \in Z$$

$$\Rightarrow \tan\theta = 2n \pm \left(\frac{1}{2} - \cot\theta\right), n \in Z$$

$$\Rightarrow \tan\theta - \cot\theta = 2n - \frac{1}{2}, n \in Z \quad \text{[Taking negative sign]}$$

$$\Rightarrow \frac{\tan^2\theta - 1}{\tan\theta} = 2n - \frac{1}{2}$$

$$\Rightarrow \frac{\tan^2\theta - 1}{2\tan\theta} = n - \frac{1}{4}$$

$$\Rightarrow \frac{1 - \tan^2\theta}{2\tan\theta} = -n + \frac{1}{4}$$

$$\Rightarrow \cot 2\,\theta = m + \frac{1}{4}, \text{ where } m = -n \in Z$$

859 **(c)**

From Questions 47, we have

$$\Delta = \frac{1}{2}ap_1, \Delta = \frac{1}{2}bp_2, \Delta = \frac{1}{2}cp_3$$

Now,

p_1, p_2, p_3 are in A.P.

$\Rightarrow \frac{2\,\Delta}{a}, \frac{2\,\Delta}{b}, \frac{2\,\Delta}{c}$ are in A. P.

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A. P. $\Rightarrow a, b, c$ are in H. P.

860 **(d)**

$$\cos\frac{\pi}{5}\cos\frac{2\pi}{5}\cos\frac{4\pi}{5}\cos\frac{8\pi}{5}$$

$$= \frac{\sin 2^4\frac{\pi}{5}}{2^4\sin\frac{\pi}{5}}\left(\because \cos x\cos 2x\cos 4x \ldots \cos 2nx = \frac{\sin 2^n x}{2^n \sin x}\right)$$

$$= \frac{\sin\frac{16\pi}{5}}{16\sin\frac{\pi}{5}} = \frac{\sin\left(3\pi + \frac{\pi}{5}\right)}{16\sin\frac{\pi}{5}}$$

$$= \frac{-\sin\frac{\pi}{5}}{16\sin\frac{\pi}{5}} = -\frac{1}{16}$$

861 **(c)**

$$\text{cosec } 15° + \sec 15° = \frac{2(\sin 15° + \cos 15°)}{2\sin 15° \cos 15°}$$

$$= 2\left[\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right] / \sin 30°$$

$$= \frac{4\sqrt{3}}{\sqrt{2}} = 2\sqrt{6}$$

862 **(d)**

We have, $\sin A = \frac{4}{5}$ and $\cos B = -\frac{12}{13}$

Now, $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$= \sqrt{1 - \frac{16}{25}}\left(-\frac{12}{13}\right) - \frac{4}{5}\sqrt{1 - \frac{144}{169}}$$

$$= -\frac{3}{5} \times \frac{12}{13} - \frac{4}{5}\left(-\frac{5}{13}\right)$$

$$= -\frac{36}{65} + \frac{20}{65} = -\frac{16}{65}$$

863 **(a)**

Given that, $\frac{1}{\tan\theta} + \tan\theta = m$

$\Rightarrow 1 + \tan^2\theta = m\tan\theta$

$\Rightarrow \sec^2\theta = m\tan\theta$ ...(i)

and $\sec\theta - \cos\theta = n$

$\Rightarrow \sec^2\theta - 1 = n\sec\theta$

$\Rightarrow \tan^2\theta = n\sec\theta$

$\Rightarrow \tan^4\theta = n^2\sec^2\theta = n^2 \cdot m\tan\theta$ [from Eq.(i)]

$\Rightarrow \tan^3\theta = n^2 m$ $(\because \tan\theta \neq 0)$

$\Rightarrow \tan\theta = (n^2 m)^{1/3}$ ...(ii)

From Eq. (i), we get

$\sec^2\theta = m\,(n^2 m)^{1/3}$

As we know that, $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow m(mn^2)^{1/3} - (n^2 m)^{2/3} = 1$

$\Rightarrow m(mn^2)^{1/3} - n(nm^2)^{1/3} = 1$

864 **(c)**

We have,

$(\sin A + \sin B + \sin C)(\sin A + \sin B - \sin C)$

$= 3\sin A \sin B$

$\Rightarrow (\sin A + \sin B)^2 - \sin^2 C = 3\sin A \sin B$

$\Rightarrow \sin^2 A + \sin^2 B - \sin^2 C = \sin A \sin B$

$\Rightarrow \sin^2 A + \sin(B+C)\sin(B-C) = \sin A \sin B$

$\Rightarrow \sin^2 A + \sin A \sin(B-C) = \sin A \sin B$

$\Rightarrow \sin A[\sin(B+C) + \sin(B-C)] = \sin A \sin B$

$\Rightarrow 2\sin A \sin B \cos C = \sin A \sin B$

$\Rightarrow \cos C = 1/2$ $[\because \sin A \sin B \neq 0]$

$\Rightarrow C = 60°$

865 **(b)**

Given, $\cos 2x + 7 = a\,(2 - \sin x)$

$\Rightarrow 1 - 2\sin^2 x + 7 = 2a - a\sin x$

$\Rightarrow 2\sin^2 x - a\sin x + (2a - 8) = 0$

$$\therefore \sin x = \frac{a \pm \sqrt{(-a)^2 - 8(2a-8)}}{2 \times 2}$$

$$= \frac{a \pm (a-8)}{4}$$

For (+) sign,

$$\sin x = \frac{a-4}{2}$$

For (−) sign,

$\sin x = 2$ which is not possible

We know $-1 \leq \sin x \leq 1$

$$\therefore -1 \leq \frac{a-4}{2} \leq 1 \Rightarrow 2 \leq a \leq 6$$

866 **(c)**

$$\cos^2 B + \cos^2 C = \cos^2 B + \cos^2\left(\frac{\pi}{2} - B\right)$$

$= \cos^2 B + \sin^2 B = 1$

867 **(d)**

We have,

$b^2 \sin 2C + c^2 \sin 2B$

$= b^2 \cdot (2\sin C \cos C) + c^2 \cdot (2\sin B \cos B)$

$= 2(b\sin C)(b\cos C) + 2(c\sin B)(c\cos B)$

$= 2(c\sin B)(b\cos c) + 2(c\sin B)(c\cos B)$

$\left[\because \frac{b}{\sin B} = \frac{c}{\sin C}\right]$

$= 2c\sin B(b\cos C + c\cos B) = 2ac\sin B = 4\Delta$

868 **(a)**

Since the angles of ΔABC are in A.P.

$\therefore 2B = A + C \Rightarrow 3B = A + B + C \Rightarrow 3B = 180°$

$\Rightarrow B = 60°$

Now, $\frac{\sin A}{a} = \frac{\sin B}{b}$

$$\Rightarrow \sin A = \frac{a}{b}\sin B = \frac{24}{22}\sin 60° = \frac{6\sqrt{3}}{11}$$

$$\Rightarrow \cos A = \frac{\sqrt{13}}{11}$$

We have,

$\sin C = \sin\{180° - (A+B)\}$

$\Rightarrow \sin C = \sin(A+B)$

$\Rightarrow \sin C = \sin A \cos B + \cos A \sin B$

$\Rightarrow \sin C = \left(\frac{6\sqrt{3}}{11}\right)\left(\frac{1}{2}\right) + \frac{\sqrt{13}}{11}\left(\frac{\sqrt{3}}{2}\right) = \frac{6\sqrt{3}+\sqrt{39}}{22}$

$\therefore c = \frac{b \sin C}{\sin B} \Rightarrow c = 12 + 2\sqrt{13}$

869 **(a)**

We have, $\angle BFC = \frac{\pi}{2} = \angle BEC$

So, the circle with BC as diameter will pass through E and F. Clearly, the circle with BC as diameter is the circumcircle of ΔBEF such that $\angle FBE = 90° - A$

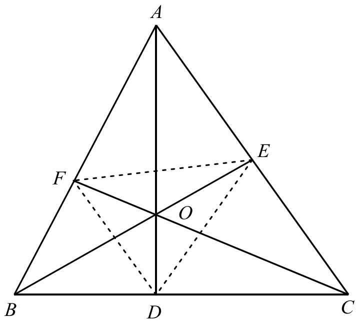
$\therefore FE = 2\left(\frac{a}{2}\right) \sin \angle FBE$ [Using : $a = 2R \sin A$]

$\Rightarrow FE = a \cos A$

Let R_1 be the radius of the circumcircle of ΔDEF. Then,

$R_1 = \frac{FE}{2 \sin \angle FDE} = \frac{a \cos A}{2 \sin(180° - 2A)}$

$\Rightarrow R_1 = \frac{a \cos A}{4 \sin A \cos A} = \frac{a}{4 \sin A} = \frac{R}{2}$



870 **(b)**

Clearly, the equation $x^2 + \sqrt{2}x + 1 = 0$ has imaginary roots. So, the two equations have both common roots

$\therefore \frac{a}{1} = \frac{b}{\sqrt{2}} = \frac{c}{1}$

$\Rightarrow \frac{\sin A}{1} = \frac{\sin B}{\sqrt{2}} = \frac{\sin C}{1}$

$\Rightarrow \frac{\sin A}{1/\sqrt{2}} = \frac{\sin B}{1} = \frac{\sin C}{1/\sqrt{2}}$

$\Rightarrow A = \frac{\pi}{4}, B = \frac{\pi}{2}, C = \frac{\pi}{4}$

871 **(a)**

We have,

$s = \frac{3a}{2}$ and $\Delta = \frac{\sqrt{3}}{4}a^2 \ \therefore r = \frac{\Delta}{s} = \frac{a}{2\sqrt{3}}$

Let the length of each side of the square inscribed in the incircle be x. Then,

$x^2 + x^2 = (\text{Diameter})^2$

$\Rightarrow 2x^2 = \frac{a^2}{3} \Rightarrow x^2 = \frac{a^2}{6} \Rightarrow$ Area of the square $= \frac{a^2}{6}$

872 **(a)**

$\cos 1°. \cos 2° \dots\dots \cos 179°$

$= \cos 1°. \cos 2° \dots. \cos 90°. \cos 179°$

$= 0 \quad [\because \cos 90° = 0]$

873 **(a)**

Given equation is

$2^{\cos 2x} + 1 = 3.2^{-\sin x}$

By taking option (a)

Let $x = n\pi$

When, $n = 1, \ x = \pi$

$\therefore \ 2^{\cos 2\pi} + 1 = 3.2^{-\sin \pi}$

$\Rightarrow \ 2 + 1 = 3.2^{\circ} \ \Rightarrow \ 3 = 3$

When $n = 2, x = 2\pi$

$\therefore \ 2^{\cos 4\pi} + 1 = 3.2^{-\sin 2\pi}$

$\Rightarrow \ 2^1 + 1 = 3.2^{\circ}$

$\Rightarrow \ 3 = 3$

874 **(b)**

On squaring given equation, we get

$\sin^2 A + 6\cos^2 A - 2\sqrt{6} \sin A \cos A = 7 \cos^2 A$

$\Rightarrow \sin^2 A + 6(1 - \sin^2 A) = \cos^2 A + 6\cos^2 A + 2\sqrt{6} \sin A \cos A$

$\Rightarrow \sin^2 A - 6\cos^2 A + 6 = \cos^2 A + 6 \sin^2 A + 2\sqrt{6} \sin A \cos A$

$\Rightarrow 7 \sin^2 A = (\cos A + \sqrt{6} \sin A)^2$

$\Rightarrow \pm\sqrt{7} \sin A = \cos A + \sqrt{6} \sin A$

Alternate

Given, $\sin A - \sqrt{6} \cos A = \sqrt{7} \cos A$

Replacing A by $90° + A$, we get

$\sin(90° + A) - \sqrt{6}\cos(90° + A) = \sqrt{7}\cos(90° + A)$

$\Rightarrow \cos A + \sqrt{6} \sin A = -\sqrt{7} \sin A$

875 **(b)**

We have,

$y = \frac{\tan x}{\tan 3x}$

$\Rightarrow y = \frac{1 - 3\tan^2 x}{3 - \tan^2 x}$

$\Rightarrow 3y - y\tan^2 x = 1 - 3\tan^2 x$

$\Rightarrow \tan^2 x (y - 3) = 1 - 3y$

$\Rightarrow \tan^2 x = \frac{y-3}{1-3y}$

$\Rightarrow -\frac{y-3}{3y-1} \geq 0 \ [\because \tan^2 x \geq 0]$

$\Rightarrow \frac{y-3}{3y-1} \leq 0 \Rightarrow \frac{1}{3} \leq y < 3 \Rightarrow y \in [1/3, 3]$

876 **(c)**

We have, $\sqrt{\text{cosec}^2\alpha + 2\cot\alpha}$

$= \sqrt{1 + \cot^2 \alpha + 2\cot\alpha} = |1 + \cot\alpha|$

But $\frac{3\pi}{4} < \alpha < \pi$

$\Rightarrow \cot\alpha < -1 \Rightarrow 1 + \cot\alpha < 0$

Hence, $|1 + \cot\alpha| = -(1 + \cot\alpha)$

877 **(c)**

Since $-\sqrt{a^2 + b^2} \le a\sin x + b\cos x \le \sqrt{a^2 + b^2}$. Therefore, $a\sin x + b\cos x = c$ has no solution for $|c| > \sqrt{a^2 + b^2}$

878 **(c)**

We have,

$\tan\theta + \sec\theta = 2\cos\theta$

$\Rightarrow 1 + \sin\theta = 2\cos^2\theta$

$\Rightarrow 1 + \sin\theta = 2 - 2\sin^2\theta$

$\Rightarrow 2\sin^2\theta + \sin\theta - 1 = 0$

$\Rightarrow (2\sin\theta - 1)(\sin\theta + 1) = 0$

$\Rightarrow \sin\theta = \frac{1}{2}, \sin\theta = -1$

$\Rightarrow \sin\theta = \frac{1}{2}$ $\left[\begin{array}{c}\because \sin\theta = -1 \Rightarrow \theta = \frac{3\pi}{2} \\ \text{for which the equation is not defined}\end{array}\right]$

$\Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$ $[\because \theta \in [0, 2\pi]]$

Hence, the given equation has two solutions in $[0, 2\pi]$

879 **(c)**

Given, $\sin\pi(x^2 + x) - \sin\pi x^2 = 0$

$\Rightarrow 2\cos\pi\left(\frac{2x^2 + x}{2}\right)\sin\frac{\pi x}{2} = 0$

$\Rightarrow \pi\left(\frac{2x^2 + x}{2}\right) = n\pi + \frac{\pi}{2}$

$\Rightarrow 2x^2 + x = 2n + 1$

$\Rightarrow 2x^2 + x - p' = 0$, where $p' = 2n + 1$, is an odd integer

$\therefore x = \frac{-1 \pm \sqrt{1 + 8p'}}{4}$ [put $1 + 8p' = p$]

$\therefore x = \frac{-1 \pm \sqrt{p}}{4} \Rightarrow x = \frac{\sqrt{p} - 1}{4}$ $\left[\text{neglect } x = \frac{-1 - \sqrt{p}}{4}\right]$

880 **(b)**

Given that, $3\sin^2 A + 2\sin^2 B = 1$ ...(i)

and $3\sin 2A - 2\sin 2B = 0$ ...(ii)

From Eq. (i)

$3\left(\frac{1 - \cos 2A}{2}\right) + 2\left(\frac{1 - \cos 2B}{2}\right) = 1$

$\Rightarrow 3\cos 2A + 2\cos 2B = 3$ ...(iii)

$\Rightarrow 3\cos 2A = 3 - 2\cos 2B$

$\Rightarrow 9\cos^2 2A = 9 + 4\cos^2 2B - 12\cos 2B$

$\Rightarrow 9(1 - \sin^2 2A) = 9 + 4\cos^2 2B - 12\cos 2B$

$\Rightarrow 9 - 4\sin^2 2B = 9 + 4\cos^2 2B - 12\cos 2B$ [from Eq. (ii)]

$\Rightarrow -4(1 - \cos^2 2B) = 4\cos^2 2B - 12\cos 2B$

$\Rightarrow -4 = -12\cos 2B$

$\Rightarrow \cos 2B = \frac{1}{3}$

Now, from Eq. (iii)

$\cos 2A = \frac{7}{9} \Rightarrow 2\cos^2 A - 1 = \frac{7}{9}$

$\Rightarrow \cos A = \frac{2\sqrt{2}}{3}$

$\therefore A + 2B = \cos^{-1}\frac{2\sqrt{2}}{3} + \cos^{-1}\frac{1}{3}$

$= \cos^{-1}\left(\frac{2\sqrt{2}}{3}\cdot\frac{1}{3} - \sqrt{1 - \frac{8}{9}}\sqrt{1 - \frac{1}{9}}\right)$

$= \cos^{-1}\left(\frac{2\sqrt{2}}{9} - \frac{2\sqrt{2}}{9}\right)$

$= \cos^{-1}(0) = \frac{\pi}{2}$

881 **(b)**

Let $f(x) = \sin x\cos x = \frac{1}{2}\sin 2x$

We know that, $-1 \le \sin 2x \le 1$

$\Rightarrow -\frac{1}{2} \le \frac{1}{2}\sin 2x \le \frac{1}{2}$

Thus, the greatest and least value of $f(x)$ are $\frac{1}{2}$ and $\frac{1}{2}$ respectively

882 **(b)**

We have,

$x^2 + 4xy + y^2$

$= (X\cos\theta - Y\sin\theta)^2 + 4(X\cos\theta - Y\sin\theta)(X\sin\theta + Y\cos\theta) + (X\sin\theta + Y\cos\theta)^2$

$= (1 + 4\sin\theta\cos\theta)X^2 + 4(\cos^2\theta - \sin^2\theta)XY + (1 - 4\sin\theta\cos\theta)Y^2$

$\therefore x^2 + 4xy + y^2 = AX^2 + BY^2$

$\Rightarrow (1 + 2\sin 2\theta)X^2 + 4\cos 2\theta\, XY + (1 - 2\sin 2\theta)Y^2 = AX^2 + BY^2$

$\Rightarrow \cos 2\theta = 0, A = 1 + 2\sin 2\theta, B = 1 - 2\sin 2\theta$

$\Rightarrow \theta = \frac{\pi}{4}$ and $A = 1 + 2 = 3, B = 1 - 2 = -1$

883 **(d)**

Given, $3\cos 2x - 10\cos x + 7 = 0$

$\Rightarrow\ 6\cos^2 x - 10\cos x + 4 = 0$

$[\because \cos 2x = 2\cos^2 x - 1]$

$\Rightarrow\ 2(3\cos x - 2)(\cos x - 1) = 0$

$\Rightarrow\ \cos x = 1$ or $\cos x = \frac{2}{3}$

Since, $\cos x$ is positive in Ist and IIIrd quadrant.

Hence, total number of solutions are 4

884 **(a)**

$\cos\alpha\sin(\beta-\gamma) + \cos\beta\sin(\gamma-\alpha) + \cos\gamma\sin(\alpha-\beta)$

$=\cos\alpha[\sin\beta\cos\gamma - \cos\beta\sin\gamma] + \cos\beta[\sin\gamma\cos\alpha - \cos\gamma\sin\alpha] + \cos\gamma[\sin\alpha\cos\beta - \cos\alpha\sin\beta]$

$= 0$

885 **(d)**

Given, $A + B = 45°$

$\Rightarrow\ \cot(A+B) = 1$

$\Rightarrow\ \dfrac{\cot A\cot B - 1}{\cot A + \cot B} = 1$

$\Rightarrow\ \cot A\cot B - (\cot A + \cot B) = 1$ ...(i)

Now, $(\cot A - 1)(\cot B - 1) = \cot A\cot B - \cot A + \cot B + 1$

$= 1 + 1 = 2$ [from Eq. (i)]

886 **(c)**

Let $I = [\sin x + \cos x]^{1+\sin 2x}$

$= \left[\sqrt{2}\sin\left(\frac{\pi}{4} + x\right)\right]^{1+\sin 2x}$

At $x = \frac{\pi}{4}$,

$I = \left[\sqrt{2}\sin\left(\frac{\pi}{4} + \frac{\pi}{4}\right)\right]^{1+\sin\frac{2\pi}{4}}$

$= (\sqrt{2})^2 = 2$

887 **(d)**

The given expression can be written as

$\cos^6 x(\cos^6 x + 3\cos^4 x + 3\cos^2 x + 1) + 2\cos^4 x + \cos^2 x - 2$

$= \sin^3 x(\cos^2 x + 1)^3 + 2\cos^4 x + \cos^2 x - 2$

$= \sin^3 x(\sin x + 1)^3 + 2\sin^2 x + \cos^2 x - 2$

$[\because \sin x + \sin^2 x = 1 \Rightarrow \sin x = \cos^2 x]$

$= (\sin x + \sin^2 x)^3 + \sin^2 x + (\sin^2 x + \cos^2 x) - 2$

$= 1^3 + \sin^2 x + 1 - 2 = \sin^2 x$

888 **(b)**

$\sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8}$

$= \frac{1}{4}\left[\left(2\sin^2\frac{\pi}{8}\right)^2 + \left(2\sin^2\frac{3\pi}{8}\right)^2\right] + \frac{1}{4}\left[\left(2\sin^2\frac{\pi}{8}\right)^2 + \left(2\sin^2\frac{3\pi}{8}\right)^2\right]$

$= \frac{1}{4}\left[\left(1 - \cos\frac{\pi}{4}\right)^2 + \left(1 - \cos\frac{3\pi}{4}\right)^2\right] + \frac{1}{4}\left[\left(1 - \cos\frac{\pi}{4}\right)^2 + \left(1 - \cos\frac{3\pi}{4}\right)^2\right]$

$= \frac{1}{4}\left[\left(1 - \frac{1}{\sqrt{2}}\right)^2 + \left(1 + \frac{1}{\sqrt{2}}\right)^2\right] + \frac{1}{4}\left[\left(1 - \frac{1}{\sqrt{2}}\right)^2 + \left(1 + \frac{1}{\sqrt{2}}\right)^2\right]$

$= \frac{1}{4}(3) + \frac{1}{4}(3) = \frac{3}{2}$

889 **(c)**

Given, $\frac{\sin(x+3\alpha)}{\sin(\alpha - x)} = 3$

Applying componendo and dividendo, we get

$\dfrac{\sin(x+3\alpha) + \sin(\alpha - x)}{\sin(x+3\alpha) - \sin(\alpha - x)} = \dfrac{3+1}{3-1}$

$\Rightarrow\ \dfrac{2\sin 2\alpha\cos(\alpha+x)}{2\cos 2\alpha\sin(\alpha+x)} = 2$

$\Rightarrow\ \dfrac{\tan 2\alpha}{\tan(\alpha+x)} = 2$

$\Rightarrow\ \dfrac{2\tan\alpha}{1-\tan^2\alpha} \times \dfrac{(1 - \tan\alpha\tan x)}{(\tan\alpha + \tan x)} = 2$

$\Rightarrow\ \tan\alpha - \tan^2\alpha\tan x = \tan\alpha + \tan x - \tan^3\alpha - \tan^2\alpha\tan x$

$\Rightarrow\ \tan x = \tan^3\alpha$

890 **(a)**

We have,

$\cos\alpha\sin(\beta-\gamma) + \cos\beta\sin(\gamma-\beta) + \cos\gamma\sin(\alpha-\beta)$

$= \frac{1}{2}\{\sin(\alpha+\beta-\gamma) + \sin(\beta-\gamma-\alpha) + \sin(\gamma-\alpha+\beta) + \sin(\gamma-\alpha-\beta) + \sin(\alpha-\beta+\gamma) + \sin(\alpha-\beta-\gamma)\}$

$= \frac{1}{2}\{\sin(\alpha+\beta-\gamma) - \sin(\alpha-\beta+\gamma)$
$- \sin(\alpha-\beta-\gamma) - \sin(\alpha+\beta-\gamma)$
$+ \sin(\alpha-\beta+\gamma)$
$+ \sin(\alpha-\beta-\gamma)\}$

$= \frac{1}{2} \times 0 = 0$

891 **(c)**

$\sin A + \cos A = m$ [given]

$\Rightarrow \sin^3 A + \cos^3 A + 3\cos A \sin A$

$(\sin A + \cos A) = m^3$

$\Rightarrow n + 3m \sin A \cos A = m^3$ ....(i)

$[\because \sin^3 A + \cos^3 A = n]$

Again, $\sin A + \cos A = m$

$\Rightarrow \sin^2 A + \cos^2 A + 2sin\, A \cos A = m^2$

$\Rightarrow \sin A \cos A = \frac{m^2-1}{2}$ ...(ii)

From Eqs. (i) and (ii), we get

$n + 3m\frac{(m^2-1)}{2} = m^3$

$\Rightarrow 2n + 3m^3 - 3m = 2m^3$

$\Rightarrow m^3 - 3m + 2n = 0$

892 **(b)**

We have,

$(\sec\theta - 1) = (\sqrt{2}-1)\tan\theta$

$\Rightarrow 1 - \cos\theta = (\sqrt{2}-1)\sin\theta$

$\Rightarrow 2\sin^2\frac{\theta}{2} = 2(\sqrt{2}-1)\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$\Rightarrow \sin\frac{\theta}{2} = 0$ or, $\tan\frac{\theta}{2} = \sqrt{2}-1 = \tan\frac{\pi}{8}$

$\Rightarrow \frac{\theta}{2} = n\,\pi$ or, $\frac{\theta}{2} = n\,\pi + \frac{\pi}{8}, n \in Z$

$\Rightarrow \theta = 2\,n\,\pi, \theta = 2\,n\,\pi + \frac{\pi}{4}, n \in Z$

893 **(d)**

Given, $\cos\theta + \sin 2\theta = 0$

$\Rightarrow \cos\theta + 2\sin\theta\cos\theta = 0$

$\Rightarrow \cos\theta(1 + 2\sin\theta) = 0$

$\Rightarrow \cos\theta = 0$ or $\sin\theta = -\frac{1}{2}$

For $\theta \in [-\pi, \pi]$

$\theta = \frac{\pi}{2}, -\frac{\pi}{2}$

Or $\theta = -\frac{\pi}{6}, -\frac{5\pi}{6}$

894 **(b)**

We have,

$\tan\left(\frac{\pi}{2}\sin\theta\right) = \cot\left(\frac{\pi}{2}\cos\theta\right)$

$\Rightarrow \tan\left(\frac{\pi}{2}\sin\theta\right) = \tan\left(\frac{\pi}{2} - \frac{\pi}{2}\cos\theta\right)$

$\Rightarrow \frac{\pi}{2}\sin\theta = r\,\pi + \frac{\pi}{2} - \frac{\pi}{2}\cos\theta,\ r \in Z$

$\Rightarrow \sin\theta + \cos\theta = (2r+1), r \in Z$

$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{2r+1}{\sqrt{2}}, r \in Z$

$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \frac{2r+1}{\sqrt{2}}, r \in Z$

$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}}$ [For $r = 0, -1$]

$\Rightarrow \theta - \frac{\pi}{4} = 2\,r\,\pi \pm \frac{\pi}{4}, r \in Z$

$\Rightarrow \theta = 2\,r\,\pi \pm \frac{\pi}{4} + \frac{\pi}{4}, r \in Z$

$\Rightarrow \theta = 2\,r\,\pi, 2\,r\,\pi + \frac{\pi}{2}, r \in Z$

But, $\theta = 2\,r\,\pi + \frac{\pi}{2}, r \in Z$ gives extraneous roots as it does not satisfy the given equation. Therefore, $\theta = 2\,r\,\pi, r \in Z$

895 **(b)**

$\tan\theta + \tan\left(\frac{3\pi}{4} + \theta\right) = 2$

$\Rightarrow \tan\theta + \tan\left(\frac{\pi}{2} + \left(\frac{\pi}{4} + \theta\right)\right) = 2$

$\Rightarrow \tan\theta - \cot\left(\frac{\pi}{4} + \theta\right) = 2$

$\Rightarrow \tan\theta - \frac{\cot\frac{\pi}{4}\cot\theta - 1}{\cot\frac{\pi}{4} + \cot\theta} = 2$

$\Rightarrow \tan\theta - \frac{\cot\theta - 1}{1 + \cot\theta} = 2$

$\Rightarrow \tan\theta - \frac{1 - \tan\theta}{1 + \tan\theta} = 2$

$\Rightarrow \tan\theta + \tan^2\theta - 1 + \tan\theta = 2 + 2\tan\theta$

$\Rightarrow \tan^2\theta = 3$

$\Rightarrow \tan\theta = \pm\sqrt{3} = \pm\tan\frac{\pi}{3}$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{3}, n \in Z$

896 **(b)**

We have,

$(1 + \tan\theta)(1 + \tan\phi) = 2$

$\Rightarrow 1 + \tan\theta + \tan\phi + \tan\theta\tan\phi = 2$

$\Rightarrow \tan\theta + \tan\phi = 1 - \tan\theta\tan\phi$

$\Rightarrow \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi} = 1$

$\Rightarrow \tan(\theta + \phi) = 1 \Rightarrow \theta + \phi = \frac{\pi}{4}, n \in Z$

897 **(c)**

Given, $2\sec 2\alpha = \tan\beta + \cot\beta$

$\Rightarrow 2\sec 2\alpha = \frac{\sin^2\beta + \cos^2\beta}{\sin\beta\cos\beta}$

$\Rightarrow \frac{2}{\cos 2\alpha} = \frac{1}{\sin\beta\cos\beta}$

$\Rightarrow \sin 2\beta = \cos 2\alpha$

$\Rightarrow \quad \alpha+\beta=\frac{\pi}{4}$

898 **(a)**

We have,

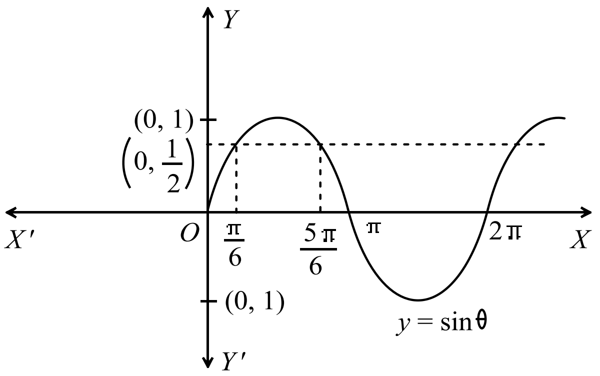
$2\sin^2\theta-5\sin\theta+2>0$

$\Rightarrow (\sin\theta-2)(2\sin\theta-1)>0$

$\Rightarrow 2\sin\theta-1<0 \quad [\because -1\le\sin\theta\le 1 \ \therefore \sin\theta-2<0]$

$\Rightarrow \sin\theta<\frac{1}{2}$

$\Rightarrow \theta\in(0,\pi/6)\cup(5\pi/6,\pi)$



899 **(d)**

Given that,

$\tan A-\tan B=x \quad \dots$(i)

and $\cot B-\cot A=y \quad \dots$(ii)

Now, $\cot(A-B)=\frac{1}{\tan(A-B)}$

$=\frac{1+\tan A\tan B}{\tan A-\tan B}$

$=\frac{1}{\tan A-\tan B}+\frac{\tan A\tan B}{\tan A-\tan B}$

$=\frac{1}{x}+\frac{1}{y}$ [from Eqs.(i)and(ii)]

900 **(b)**

We have,

$\frac{a\cos B-b\cos A}{a-b}$

$=\frac{a\left(\frac{c^2+a^2-b^2}{2ac}\right)-b\left(\frac{b^2+c^2-a^2}{2bc}\right)}{a-b}$

$=\frac{2(a^2-b^2)}{2c(a-b)}=\frac{a+b}{c}=\frac{2c}{c}$

$=2 \quad \left[\begin{array}{c}\because a,c,b \text{ are in A. P.}\\ \therefore a+b=2c\end{array}\right]$

901 **(d)**

$\tan 45°=\frac{\tan 10°+\tan 35°}{1-\tan 10°\tan 35°}$

$\Rightarrow \quad 1-\tan 10°\tan 35°=\tan 10°+\tan 35°$

$\Rightarrow \quad \tan 10°+\tan 35°+\tan 10°\tan 35°=1$

902 **(b)**

$\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^n+\left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^n$

$\left[\frac{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}\right]^n$

$+\left[\frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)}\right]^n$

$=\cot^n\left(\frac{A-B}{2}\right)+\cot^n\left(\frac{B-A}{2}\right)$

$=\cot^n\left(\frac{A-B}{2}\right)+(-1)^n\cot\left(\frac{A-B}{2}\right)$

$=2\cot^n\left(\frac{A-B}{2}\right) \quad (\because n$ is even)

903 **(d)**

We have,

$\sin(x-y)=\frac{1}{2}$ and $\cos(x+y)=\frac{1}{2}$

$\Rightarrow (x-y=30°)$ and $(x+y=60°)$ $[\because 0<x,y<$ *90°*

$\Rightarrow x=45°, y=15°$

904 **(d)**

We have,

$5\cos 2\theta+2\cos^2\frac{\theta}{2}+1=0$

$\Rightarrow 5(2\cos^2\theta-1)+(1+\cos\theta)+1=0$

$\Rightarrow 10\cos^2\theta+\cos\theta-3=0$

$\Rightarrow (5\cos\theta+3)(2\cos\theta-1)=0$

$\Rightarrow \cos\theta=\frac{1}{2},\cos\theta=-\frac{3}{5}\Rightarrow\theta=\frac{\pi}{3},\pi-\cos^{-1}\left(\frac{3}{5}\right)$

905 **(a)**

We have,

$\cos x\cos y\sin(x-y)$

$=\frac{1}{2}[2\cos x\cos y\sin(x-y)]$

$=\frac{1}{2}[\cos(x+y)+\cos(x-y)]\sin(x-y)$

$=\frac{1}{4}[2\sin(x-y)\cos(x+y)$

$+2\sin(x-y)\cos(x-y)]$

$=\frac{1}{4}[\sin 2x-\sin 2y+\sin(2x-2y)]$

Similarly, we have

$\cos y\cos z\sin(y-z)$

$=\frac{1}{4}[\sin 2y-\sin 2z$

$+\sin(2y-2z)]$

and,

$\cos z \cos x \sin(z-x)$

$$= \frac{1}{4}[\sin 2z - \sin 2x + \sin(2z-2x)]$$

Also,

$\sin(x-y)\sin(y-z)\sin(z-x)$

$$= -\frac{1}{4}\{\sin(2x-2y) + \sin(2y-2z) + \sin(2z-2x)\}$$

On adding the above results, we find that the value of the given expression is zero

906 **(c)**

We have,

$2\cos B = \frac{a}{c}$

$\Rightarrow 2\left(\frac{c^2+a^2-b^2}{2\,ac}\right) = ac$

$\Rightarrow c^2 = b^2 \Rightarrow c = b \Rightarrow \Delta ABC$ is isosceles

907 **(d)**

We have, $\sin x \cos x = 2$

$\Rightarrow \sin 2x = 4$

Which is impossible because the value of $\sin x$ is not greater than one

Thus, given equation has no solution

908 **(a)**

The given equation can be rewritten as

$1 - \cos^2\theta - \cos\theta = \frac{1}{4}$

$\Rightarrow \cos^2\theta + \cos\theta - \frac{3}{4} = 0$

$\Rightarrow 4\cos^2\theta + 4\cos\theta - 3 = 0$

$\Rightarrow \cos\theta = \frac{-4 \pm \sqrt{16+48}}{8} = \frac{1}{2}, -\frac{3}{2}$

Since, $\cos\theta = -\frac{3}{2}$ is not possible, so we take

$\cos\theta = \frac{1}{2} = \cos\frac{\pi}{3}$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}$ ...(i)

For the given interval, put $n = 0$ and $n = 1$ in Eq. (i) we get

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

909 **(a)**

We have,

$\frac{b+c}{11} = \frac{a+c}{12} = \frac{a+b}{13} = K$ (say)

$\Rightarrow b+c = 11\,K, c+a = 12\,K, a+b = 13\,K$

$\Rightarrow 2(a+b+c) = 36\,K \Rightarrow a+b+c = 18\,K$

$\Rightarrow a = 7K, b = 6\,K, c = 5\,K$

$\therefore \cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{36+25-49}{2\times 6\times 5} = \frac{1}{5}$

910 **(d)**

We have,

$f(x) = (\sec x - \cos x)^2 + 2 \Rightarrow f(x) \geq 2$ for all x

911 **(b)**

Given that, $\sin x\sqrt{8\cos^2 x} = 1$

$\Rightarrow 2\sin x|\cos x| = \frac{1}{\sqrt{2}}$

If $\cos x > 0$, then $\sin 2x = \frac{1}{\sqrt{2}}$

$\Rightarrow x = \frac{\pi}{8}, \frac{3\pi}{8}$

And if $\cos x < 0$, then

$\sin 2x = -\frac{1}{\sqrt{2}} \Rightarrow x = \frac{5\pi}{8}, \frac{7\pi}{8}$

So, the required values of x are $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$, which form an AP with common difference $\frac{\pi}{4}$

912 **(b)**

Given, $z = 12\sin\theta - 9\sin^2\theta + 4 - 4$

$\Rightarrow z = -4 - (2 - 3\sin\theta)^2 \leq 4$

$\therefore$ maximum value of $12\sin\theta - 9\sin^2\theta$ is 4

913 **(c)**

We have,

$\tan|x| = |\tan x|$

$\Rightarrow \tan|x| \geq 0$ and $x \neq (2n+1)\frac{\pi}{2}, n \in Z\ [\because |\tan x| \geq 0]$

$\Rightarrow x$ lies in the third quadrant

$\Rightarrow x \in \left(-(2k+1)\frac{\pi}{2}, k\,\pi\right) \cup \left(k\,\pi, (2k+1)\frac{\pi}{2}\right)$

914 **(c)**

$\because \tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$

$\Rightarrow \tan\theta + \tan 2\theta = \sqrt{3}(1 - \tan\theta\tan 2\theta)$

$\Rightarrow \frac{\tan\theta + \tan 2\theta}{1 - \tan\theta\tan 2\theta} = \sqrt{3} \Rightarrow \tan 3\theta = \tan\frac{\pi}{3}$

$\Rightarrow 3\theta = n\pi + \frac{\pi}{3} \Rightarrow \theta = \frac{(3n+1)\pi}{9}, n \in I$

915 **(d)**

We have,

$x = \tan\frac{B-C}{2}\tan\frac{A}{2}$

$\Rightarrow x = \frac{b-c}{c+a}$ $\left[\because \tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}\right]$

Similarly, we have

$y = \frac{c-a}{c+a}$ and $x = \frac{a-b}{a+b}$

Now,

$x = \frac{b-c}{b+c} \Rightarrow \frac{x+1}{x-1} = \frac{b}{-c} \Rightarrow \frac{b}{c} = \frac{1+x}{1-x}$

Similarly, we have

$\frac{c}{a} = \frac{1+y}{1-y}$ and $\frac{a}{b} = \frac{1+z}{1-z}$

Now,

$\frac{a}{b}\cdot\frac{b}{c}\cdot\frac{c}{a}=1$

$\Rightarrow \frac{1+z}{1-z}\times\frac{1+x}{1-x}\times\frac{1+y}{1-y}=1$

$\Rightarrow (1+x)(1+y)(1+z)=(1-x)(1-y)(1-z)$

$\Rightarrow 2(x+y+z)=-2\,xyz$

$\Rightarrow x+y+z=-xyz$

916 **(d)**

Given, $\cos x=3\cos y$

$\Rightarrow \frac{3}{1}=\frac{\cos x}{\cos y}$

Applying componendo and dividend, we get

$\frac{3+1}{3-1}=\frac{\cos x+\cos y}{\cos x-\cos y}$

$\Rightarrow\ 2=\frac{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{y-x}{2}\right)}$

$\Rightarrow\ 2=\cot\left(\frac{x+y}{2}\right)\cot\left(\frac{y-x}{2}\right)$

$\Rightarrow\ 2\tan\left(\frac{y-x}{2}\right)=\cot\left(\frac{x+y}{2}\right)$

917 **(c)**

$\frac{\sin 85°-\sin 35°}{\cos 65°}=\frac{2\cos\frac{85°+35°}{5}\sin\frac{85°-35°}{2}}{\cos(90°-25°)}$

$\frac{2\cos 60°\sin 25°}{\cos(90°-25°)}=\frac{2.\frac{1}{2}.\sin 25°}{\sin 25°}=1$

918 **(c)**

$\frac{\tan 80°-\tan 10°}{\tan(80°-10°)}$

$=\frac{\tan 80°-\tan 10°}{\tan 80°-\tan 10°}\times(1+\tan 80°\tan 10°)$

$=1+\tan 80°\tan 10°=2$

919 **(c)**

We have,

$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$

$=\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{\pi}{8}\right)$

$=\left(1-\cos^2\frac{\pi}{8}\right)\left(1-\cos^2\frac{3\pi}{8}\right)$

$=\frac{1}{4}\left(2-1-\cos\frac{\pi}{4}\right)\left(2-1-\cos\frac{3\pi}{4}\right)$

$=\frac{1}{4}\left(1-\cos\frac{\pi}{4}\right)\left(1-\cos\frac{3\pi}{4}\right)$

$=\frac{1}{4}\left(1-\frac{1}{\sqrt{2}}\right)\left(1+\frac{1}{\sqrt{2}}\right)=\frac{1}{4}\left(1-\frac{1}{2}\right)=\frac{1}{8}$

920 **(a)**

$\frac{2}{\sqrt{2+\sqrt{2+\sqrt{2+2\cos 4x}}}}$

$=\frac{2}{\sqrt{2+\sqrt{2+\sqrt{2.2\cos^2 2x}}}}$

$=\frac{2}{\sqrt{2+\sqrt{2+2\cos 2x}}}$

$=\frac{2}{\sqrt{2+2\cos x}}=\frac{2}{2\cos\frac{x}{2}}=\sec\frac{x}{2}$