11.THREE DIMENSIONAL GEOMETRY

Single Correct Answer Type

1. If $P(3, 2, -4)$, $Q(5, 4, -6)$ and $R(9, 8, -10)$ are collinear, then R divides PQ in the ratio
a) 3 : 2 internally b) 3 : 2 externally c) 2 : 1 internally d) 2 : 1 externally
2. The radius of the circle of $x + 2y + 2z = 15, x^2 + y^2 + z^2 - 2y - 4z = 11$ is
a) 2 b) $\sqrt{7}$ c) 3 d) $\sqrt{5}$
3. Let $A(1, -1, 2)$ and $B(2, 3, -1)$ be two points. If a point P divides AB internally in the ratio 2:3, then the position vector of P is
a) $\frac{1}{\sqrt{5}}(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$
b) $\frac{1}{\sqrt{3}}(\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + \hat{\mathbf{k}})$
c) $\frac{1}{\sqrt{3}}(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$
d) $\frac{1}{5}(7\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$
4. The length of the perpendicular from the origin to the plane passing through the point $\vec{a}$ and containing the line $\vec{r} = \vec{b} + \lambda\, \vec{c}$ is
a) $\frac{[\vec{a}\ \vec{b}\ \vec{c}]}{|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}$
b) $\frac{[\vec{a}\ \vec{b}\ \vec{c}]}{|\vec{a} \times \vec{b} + \vec{b} \times \vec{c}|}$
c) $\frac{[\vec{a}\ \vec{b}\ \vec{c}]}{|\vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}$
d) $\frac{[\vec{a}\ \vec{b}\ \vec{c}]}{|\vec{c} \times \vec{a} + \vec{a} \times \vec{b}|}$
5. The line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ meets the plane $2x + 3y - z = -4$ in the point
a) $(1, 2, 3)$ b) $(-1, -1, -1)$ c) $(2, 1, 3)$ d) $(1, 1, 1)$
6. If the plane $2x - y + z = 0$ is parallel to the line
$\frac{2x-1}{2} = \frac{2-y}{2} = \frac{z+1}{a}$, then the value of a is
a) 4 b) -4 c) 2 d) -2
7. A line AB in three-dimensional space makes angle 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals
a) 30° b) 45° c) 60° d) 75°
8. The locus of a point which moves so that the difference of the squares of its distances from two given points is constant, is a
a) Straight line b) Plane c) Sphere d) None of these
9. If lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ are mutually perpendicular, then k is equal to
a) $-\frac{10}{7}$ b) $-\frac{7}{10}$ c) -10 d) -7
10. The equation of the plane passing through a point $A(2, -1, 3)$ and parallel to the vectors $\vec{\mathbf{a}} = (3, 0, -1)$ and $\vec{\mathbf{b}} = (-3, 2, 2)$ is
a) $2x - 3y + 6z - 25 = 0$ b) $2x - 3y + 6z + 25 = 0$
c) $3x - 2y + 6z - 25 = 0$ d) $3x - 2y + 6z + 25 = 0$
11. The plane $2x - (1 + \lambda)y + 3\lambda z = 0$ passes through the intersection of the planes

a) $2x - y = 0$ and $y + 3z = 0$ b) $2x - y = 0$ and $y - 3z = 0$
c) $2x + 3z = 0$ and $y = 0$ d) None of the above

12. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is
a) $30°$ b) $45°$ c) $90°$ d) $0°$

13. A And B are two give points. Let C divides AB internally and D divides AB externally in the same ratio. Then AC, AB, AD are in
a) AP b) GP c) HP d) None of these

14. The image (or reflection) of the point $(1, 2, -1)$ in the plane $\vec{r} \cdot (3\hat{\imath} - 5\hat{\jmath} + 4\hat{k}) = 5$ is
a) $(73/25, -6/5, 39/25)$
b) $(73/25, 6/5, 39/25)$
c) $(-1, -2, 1)$
d) None of these

15. The equation of the plane containing the two lines $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{3}$ and $\frac{x}{-1} = \frac{y-2}{3} = \frac{z+1}{-1}$ is
a) $8x + y - 5z - 7 = 0$ b) $8x + y + 5z - 7 = 0$ c) $8x - y - 5z - 7 = 0$ d) None of these

16. A variable plane is at a distance, k from the origin and meets the coordinate axes in A, B, C. Then, the locus of the centroid of ΔABC is
a) $x^{-2} + y^{-2} + z^{-2} = k^{-2}$
b) $x^{-2} + y^{-2} + z^{-2} = 4k^{-2}$
c) $x^{-2} + y^{-2} + z^{-2} = 16k^{-2}$
d) $x^{-2} + y^{-2} + z^{-2} = 9k^{-2}$

17. If α, β, γ are the angles which a directed line makes with the positive directions of the coordinate axes, then $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$ is equal to
a) 1 b) 2 c) 3 d) 4

18. The xy-plane divides the line joining the points $(-1,3,4)$and $(2, -5,6)$
a) Internally in the ratio 2 : 3 b) Externally in the ratio 2 : 3
c) Internally in the ratio 3 : 2 d) Externally in the ratio 3 : 2

19. The length of the perpendicular drawn from (1, 2, 3) to the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$ is
a) 4 b) 5 c) 6 d) 7

20. The angle between the lines $\frac{x+4}{1} = \frac{y-3}{2} = \frac{z+2}{3}$ and $\frac{x}{3} = \frac{y-1}{-2} = \frac{z}{1}$, is
a) $\sin^{-1}\frac{1}{7}$ b) $\cos^{-1}\frac{2}{7}$ c) $\cos^{-1}\frac{1}{7}$ d) None of these

21. Under what condition does a straight line does $\frac{x-x_0}{l} = \frac{y-y_0}{m} = \frac{z-z_0}{n}$ is parallel to the xy-plane?
a) $l = 0$ b) $m = 0$ c) $n = 0$ d) $l = 0, m = 0$

22. The value of k such that $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies in the plane $2x - 4y + z = 7$, is
a) 7 b) -7 c) No real value d) 4

23. The points $A(4, 5,1), B(0, -1, -1), C(3, 9, 4)$ and $D(-4, 4,4)$ are
a) Collinear b) Coplanar c) Non-coplanar d) Non-colinear

24. The foot of perpendicular from point $P(1,3,4)$ in the plane $2x - y + z + 3 = 0$ is
a) $(3, 5, -2)$ b) $(-3, 5, 2)$ c) $(3, -5, 2)$ d) $(-1, 4, 3)$

25. The length of perpendicular from $Q(1,6,3)$ to the line
$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ is
a) 3 b) $\sqrt{11}$ c) $\sqrt{13}$ d) 5

26. The angle between
$\vec{\mathbf{r}} = (1 + 2\mu)\hat{\mathbf{i}} + (2 + \mu)\hat{\mathbf{j}} + (2\mu - 1)\hat{\mathbf{k}}$
and the plane $3x - 2y + 6z = 0$ (where μ is a scalar) is
a) $\sin^{-1}\left(\frac{15}{21}\right)$ b) $\cos^{-1}\left(\frac{16}{21}\right)$ c) $\sin^{-1}\left(\frac{16}{21}\right)$ d) $\frac{\pi}{2}$

27. The equation of sphere which passes through the circle $x^2 + y^2 + z^2 = 0$, the plane $2x + 3y + 4z = 5$ and

point (1, 2, 3) is

a) $3(x^2+y^2+z^2)-2x-3y-4z-22=0$
b) $(x^2+y^2+z^2)-2x-3y-4z-22=0$
c) $3(x^2+y^2+z^2)+2x+3y+4z-22=0$
d) $3(x^2+y^2+z^2)-2x-3y-4z+9022=0$

28. The angle between the lines $x=1, y=2$ and $y=-1, z=0$ is

a) 30°
b) 60°
c) 90°
d) 0°

29. The point of intersection of the lines $\frac{x-5}{3}=\frac{y-7}{-1}=\frac{z+2}{1}, \frac{x+3}{-36}=\frac{y-3}{2}=\frac{z-6}{4}$ is

a) (2, 10, 4)
b) $\left(21, \frac{5}{3}, \frac{10}{3}\right)$
c) (5, 7, −2)
d) (−3, 3, 6)

30. A parallelopiped is formed by planes drown through the points (2, 3, 5) and (5, 9, 7) parallel to the coordinate planes. The length of a diagonal of the parallelopiped is

a) 7
b) $\sqrt{38}$
c) $\sqrt{155}$
d) None of these

31. If direction cosines of two lines are proportional to (2, 3, −6) and (3, −4, 5), then the acute angle between them is

a) $\cos^{-1}\left(\frac{49}{36}\right)$
b) $\cos^{-1}\left(\frac{18\sqrt{2}}{35}\right)$
c) 96°
d) $\cos^{-1}\left(\frac{18}{35}\right)$

32. The distance of the point $P(1,2,3)$ from the line which passes through the point $A(4,2,2)$and parallel to the vector $2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+6\hat{\mathbf{k}}$ is

a) $\sqrt{10}$
b) $\sqrt{7}$
c) $\sqrt{5}$
d) 1

33. The angle between the lines $\frac{x-2}{3}=\frac{y+1}{-2}, z=2$ and $\frac{x-1}{1}=\frac{2y+3}{3}=\frac{z+5}{2}$ is

a) $\pi/2$
b) $\pi/3$
c) $\pi/6$
d) None of these

34. The equation of the plane containing the line $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ is $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$, where

a) $a\,x_1+by_1+cz_1=0$
b) $al+bm+cn=0$
c) $a/l=b/m=c/n$
d) $l\,x_1+m\,y_1+n\,z_1=0$

35. If the lines $\frac{1-x}{3}=\frac{y-2}{2\alpha}=\frac{z-3}{2}$

and

$\frac{x-1}{3\alpha}=y-1=\frac{6-z}{5}$

are perpendicular, then the value of α is

a) $\frac{-10}{7}$
b) $\frac{10}{7}$
c) $\frac{-10}{11}$
d) $\frac{10}{11}$

36. The perpendicular distance of the point (6, 5, 8) from y-axis is

a) 5 units
b) 6 units
c) 8 units
d) 10 units

37. The vectors of magnitude $a, 2a, 3a$ meet at a point and their directions are along the diagonals of three adjacent faces of a cube. Then, the magnitude of their resultant is

a) $5a$
b) $6a$
c) $10a$
d) $9a$

38. Cartesian form of the equation of line $\vec{\mathbf{r}}=3\hat{\mathbf{i}}-5\hat{\mathbf{j}}+7\hat{\mathbf{k}}+\lambda(2\hat{\mathbf{i}}+\hat{\mathbf{j}}-3\hat{\mathbf{k}})$ is

a) $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+3}{7}$
b) $\frac{x-3}{2}=\frac{y+5}{1}=\frac{z-7}{-3}$
c) $\frac{x-2}{1}=\frac{y-1}{1}=\frac{z-7}{5}$
d) None of the above

39. The equation of the plane containing the lines $\vec{r}=\vec{a_1}+\lambda\,\vec{a_2}$ and $\vec{r}=\vec{a_2}+\lambda\,\vec{a_1}$, is

a) $[\vec{r}\,\vec{a_1}\,\vec{a_2}]=0$
b) $[\vec{r}\,\vec{a_1}\,\vec{a_2}]=\vec{a_1}\cdot\vec{a_2}$
c) $[\vec{r}\,\vec{a_2}\,\vec{a_1}]=\vec{a_1}\cdot\vec{a_2}$
d) None of these

40. The points $A(4,5,1), B(0,-1,-1), C(3,9,4)$and $D(-4,4,4)$ are

a) Collinear
b) Coplanar
c) Non-coplanar
d) Non-collinear

41. The direction ratio of the line $x-y+z-5=0=x-3y-6$ are

a) $3, 1, -2$ b) $2, -4, 1$ c) $\frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{-2}{\sqrt{14}}$ d) $\frac{2}{\sqrt{41}}, \frac{-4}{\sqrt{41}}, \frac{1}{\sqrt{41}}$

42. The value of k such that $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies in the plane $2x - 4y + z = 7$, is
a) 7 b) -7 c) No real value d) 4

43. If M denotes the mid point of the line joining $A(4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 10\hat{\mathbf{k}})$ and $B(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}})$, then equation of the plane through M and perpendicular to AB is
a) $\vec{\mathbf{r}} \cdot (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 11\hat{\mathbf{k}}) - \frac{135}{2} = 0$
b) $\vec{\mathbf{r}} \cdot \left(\frac{3}{2}\hat{\mathbf{i}} + \frac{7}{2}\hat{\mathbf{j}} - \frac{9}{2}\hat{\mathbf{k}}\right) + \frac{135}{2} = 0$
c) $\vec{\mathbf{r}} \cdot (4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 10\hat{\mathbf{k}}) + 4 = 0$
d) $\vec{\mathbf{r}} \cdot (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) + 4 = 0$

44. The plane passing through the point (5, 1, 2) perpendicular to the line $2(x-2) = y - 4 = z - 5$ will meet the line in the point
a) (1, 2, 3) b) (2, 3, 1) c) (1, 3, 2) d) (3, 2, 1)

45. The vector equation of the plane through the point $\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and perpendicular to the line of intersection of the plane $\vec{r} \cdot (3\hat{\imath} - \hat{\jmath} + \hat{k}) = 1$ and $\vec{r} \cdot (\hat{\imath} + 4\hat{\jmath} - 2\hat{k}) = 2$, is
a) $\vec{r} \cdot (2\hat{\imath} + 7\hat{\jmath} - 13\hat{k}) = 1$
b) $\vec{r} \cdot (2\hat{\imath} - 7\hat{\jmath} - 13\hat{k}) = 1$
c) $\vec{r} \cdot (2\hat{\imath} + 7\hat{\jmath} + 13\hat{k}) = 0$
d) None of these

46. A plane which passes through the point (3, 2, 0) and the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$ is
a) $x - y + z = 1$ b) $x + y + z = 5$ c) $x + 2y - z = 0$ d) $2x - y + z = 5$

47. The equation $|x| = p, |y| = p, |z| = p$ in xyz space represent
a) Cube b) Rhombus c) Sphere of radius p d) Point (p, p, p)

48. The centre of the sphere passing through the origin and through the intersection points of the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ with axes is
a) $\left(\frac{a}{2}, 0, 0\right)$ b) $\left(0, \frac{a}{2}, 0\right)$ c) $\left(0, 0, \frac{a}{2}\right)$ d) $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$

49. The shortest distance between the straight lines $\frac{x-6}{1} = \frac{2-y}{2} = \frac{z-2}{2}$ and $\frac{x+4}{3} = \frac{y}{-2} = \frac{1-z}{2}$ is
a) 9 b) $\frac{25}{3}$ c) $\frac{16}{3}$ d) 4

50. The equation of the plane containing the line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ is
a) $ax_1 + by_1 + cz_1 = 0$ b) $al + bm + cn = 0$ c) $\frac{a}{l} = \frac{b}{m} = \frac{c}{n}$ d) $lx_1 + my_1 + nz_1 = 0$

51. The coordinate the point of intersection of the line $\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-2}{2}$ with the plane $3x + 4y + 5z - 25 = 0$ is
a) $(5, 10, 6)$ b) $(10, 5, 6)$ c) $(5, 5, -6)$ d) $(5, 10, -6)$

52. Equation of the plane which passes through the line of intersection of the planes $P = ax + by + cz + d = 0, P' = a'x + b'y + c'z + d' = 0$ and parallel to x-axis, is
a) $Pa - P'a' = 0$ b) $P/a = P'/a' = 0$ c) $Pa + P'a' = 0$ d) $P/a = P'/a'$

53. The distance of the point $A(-2, 3, 1)$ from the line PQ through $P(-3, 5, 2)$ which make equal angles with the axes is
a) $\frac{2}{\sqrt{3}}$ b) $\sqrt{\frac{14}{3}}$ c) $\frac{16}{\sqrt{3}}$ d) $\frac{5}{\sqrt{3}}$

54. A line with positive direction cosines passes through the point $P(2, -1, 2)$ and makes equal angles with the

coordinate axes. The line meets the plane $2x + y + z = 9$ at point Q. The length of the line segment PQ equals

a) 1 b) $\sqrt{2}$ c) $\sqrt{3}$ d) 2

55. A line with direction ratios proportional to 2, 1, 2 meets each of the lines $x = y + a = z$ and $x + a = 2y = 2z$. The coordinates of the points of intersection are given by

a) $(3a, 3a, 3a), (a, a, a)$ b) $(3a, 2a, 3a), (a, a, a)$ c) $(3a, 2a, 3a), (a, a, 2a)$ d) $(2a, 3a, 3a), (2a, a, a)$

56. The position vectors of two points P and Q are $3\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\hat{\imath} - 2\hat{\jmath} - 4\hat{k}$ respectively. The equation of the plane through Q and perpendicular to PQ, is

a) $\vec{r} \cdot (2\,\hat{\imath} + 3\,\hat{\jmath} + 6\,\hat{k}) = 28$

b) $\vec{r} \cdot (2\,\hat{\imath} + 3\,\hat{\jmath} + 6\,\hat{k}) = 32$

c) $\hat{r} \cdot (2\,\hat{\imath} + 3\,\hat{\jmath} + 6\,\hat{k}) + 28 = 0$

d) None of these

57. The equation of the line of intersection of the planes $x + 2y + z = 3$ and $6x + 8y + 3z = 13$ can be written as

a) $\dfrac{x-2}{2} = \dfrac{y+1}{-3} = \dfrac{z-3}{4}$ b) $\dfrac{x-2}{2} = \dfrac{y+1}{3} = \dfrac{z-3}{4}$

c) $\dfrac{x+2}{2} = \dfrac{y-1}{-3} = \dfrac{z-3}{4}$ d) $\dfrac{x+2}{2} = \dfrac{y+2}{3} = \dfrac{z-3}{4}$

58. If the foot of the perpendicular from (0, 0, 0) to a plane is (1, 2, 2), then the equation of the plane is

a) $-x + 2y + 8z - 9 = 0$ b) $x + 2y + 2z - 9 = 0$

c) $x + y + z - 5 = 0$ d) $x + 2y - 3z + 1 = 0$

59. A plane pass through a fixed point (p, q) and cut the axes in A, B, C. Then, the locus of the centre of the sphere $OABC$ is

a) $\dfrac{p}{x} + \dfrac{q}{y} + \dfrac{r}{z} = 2$ b) $\dfrac{p}{x} + \dfrac{q}{y} + \dfrac{r}{z} = 1$ c) $\dfrac{p}{x} + \dfrac{q}{y} + \dfrac{r}{z} = 3$ d) None of these

60. The equation of the plane containing the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and the point $(0, 7, -7)$, is

a) $x + y + z = 1$ b) $x + y + z = 2$ c) $x + y + z = 0$ d) None of these

61. The equation of the plane passing through the origin and containing the line $\dfrac{x-1}{5} = \dfrac{y-2}{4} = \dfrac{z-3}{5}$ is

a) $x + 5y - 3z = 0$ b) $x - 5y + 3z = 0$ c) $x - 5y - 3z = 0$ d) $3x - 10y + 5z = 0$

62. The distance of the point $(2, 3, -5)$ from the plane $x + 2y - 2z = 9$ is

a) 4 b) 3 c) 2 d) 1

63. The ratio in which yz-palne divides the line segment joining $(-3,4,-2)$ and $(2,1,3)$is

a) $-4:1$ b) $3:2$ c) $-2:3$ d) $1:4$

64. If the direction cosines of a line are $\left(\frac{1}{c}, \frac{1}{c}, \frac{1}{c}\right)$, then

a) $0 < c < 1$ b) $c > 2$ c) $c = \pm\sqrt{2}$ d) $c = \pm\sqrt{3}$

65. If a line makes an angle of $\frac{\pi}{4}$ with the positive direction of x-axis and y-axis, then the angle that the line makes with the positive direction of the z-axis is

a) $\dfrac{\pi}{6}$ b) $\dfrac{\pi}{3}$ c) $\dfrac{\pi}{4}$ d) $\dfrac{\pi}{2}$

66. A plane makes intercepts a, b, c at A, B, C on the coordinate axes respectively. If the centroid of the ΔABC is at (3,2,1) then the equation of the plane is

a) $x + 2y + 3z = 9$ b) $2x - 3y - 6z = 18$ c) $2x + 3y + 6z = 18$ d) $2x + y + 6z = 18$

67. The equation of the plane through the point, (1, 2, 3) and parallel to the plane $x + 2y + 5z = 0$ is

a) $(x - 1) + 2(y - 2) + 5(z - 3) = 0$ b) $x + 2y + 5z = 14$

c) $x + 2y + 5z = 6$ d) None of the above

68. If vertices of a triangle are $A(1,-1,2), B(2,0,-1)$ and $C(0,2,1)$, then the area f a triangle is

a) $\sqrt{6}$ b) $2\sqrt{6}$ c) $3\sqrt{6}$ d) $4\sqrt{6}$

69. The equation of the plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$

and parallel to x-axis is

a) $y-3z+6=0$ b) $3y-z+6=0$ c) $y+3z+6=0$ d) $3y-2z+6=0$

70. The angle between two planes $2x-y+z=6$ and $x+2y+3z=3$ is

a) $\cos^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{7}}\right)$ b) $\cos^{-1}\left(\frac{1}{2}\sqrt{\frac{2}{7}}\right)$ c) $\cos^{-1}\left(\frac{1}{2}\sqrt{\frac{3}{7}}\right)$ d) $\cos^{-1}\left(\frac{1}{2}\sqrt{\frac{4}{7}}\right)$

71. The projection of the line segment joining the points $(-1,0,3)$ and $(2,5,1)$ on the line whose direction ratios are 6,2,3 is

a) $\frac{10}{7}$ b) $\frac{22}{7}$ c) $\frac{18}{7}$ d) None of these

72. If the foot of the perpendicular from the origin to a plane is (a,b,c), then equation of the plane is

a) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ b) $ax+by+cz=1$

c) $ax+by+cz=a^2+b^2+c^2$ d) $ax+by+cz=0$

73. A variable plane which remains at a constant distance p from the origin cuts the coordinate axes in A,B,C. The locus of the centroid of the tetrahedron $OABC$ is $y^2z^2+z^2x^2+x^2y^2=k\,x^2y^2z^2$, where k is equal to

a) $9p^2$ b) $\frac{9}{p^2}$ c) $\frac{7}{p^2}$ d) $\frac{16}{p^2}$

74. The equation of the plane through the intersection of the planes $x+y+z=1$ and $2x+3y-z+4=0$ and parallel to x-axis, is

a) $y-3z+6=0$ b) $3y-z+6=0$ c) $y+3z+6=0$ d) $3y-2z+6=0$

75. The plane $2x+3y+4z=1$ meets the coordinate axes in A,B,C The centroid of the triangle ABC is

a) $(2,3,4)$ b) $\left(\frac{1}{2},\frac{1}{3},\frac{1}{4}\right)$ c) $\left(\frac{1}{6},\frac{1}{9},\frac{1}{12}\right)$ d) $\left(\frac{3}{2},\frac{3}{3},\frac{3}{4}\right)$

76. The equation of the plane passing through the line $\frac{x-1}{5}=\frac{y+2}{6}=\frac{z-3}{4}$ and the point (4,3,7) is

a) $4x+8y+7z=41$ b) $4x-8y+7z=41$ c) $4x-8y-7z=41$ d) $4x-8y+7z=39$

77. The distance of the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane $x-y+z=5$ from the point $(-1,-5,-10)$is

a) 13 b) 12 c) 11 d) 8

78. A variable plane passes through a fixed point (a,b,c) and meets the coordinate axes in P,Q,R. The locus of the point of intersection of the planes through P,Q,R parallel to the coordinate planes is

a) $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=1$ b) $ax+by+cz=1$ c) $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=-1$ d) $ax+by+cz=-1$

79. The line passing the points $6\vec{a}-4\vec{b}+4\vec{c},-4\vec{c}$ and the line joining the points $-\vec{a}-2\vec{b}-3\vec{c},\vec{a}+2\vec{b}-5\vec{c}$ intersect at

a) $-4\vec{a}$ b) $4\vec{a}-\vec{b}-\vec{c}$ c) $4\vec{c}$ d) None of these

80. The vector equation of the plane through the point $(2,1,-1)$ and passing through the line of intersection of the plane $\vec{r}\cdot(\hat{\imath}+3\,\hat{\jmath}-\hat{k})=0$ and $\vec{r}\cdot(\hat{\jmath}+2\,\hat{k})=0$, is

a) $\vec{r}\cdot(\hat{\imath}+9\,\hat{\jmath}+11\,\hat{k})=0$

b) $\hat{r}\cdot(\hat{\imath}+9\,\hat{\jmath}+11\,\hat{k})=6$

c) $\hat{r}\cdot(\hat{\imath}-3\,\hat{\jmath}-13\,\hat{k})=0$

d) None of these

81. The distance between the planes given by $\vec{r}\cdot(\hat{\imath}+2\hat{\jmath}-2\hat{k})+5=0$ and $\vec{r}\cdot(\hat{\imath}+2\hat{\jmath}-2\hat{k})-8=0$ is

a) 1 unit b) $\frac{13}{3}$ units c) 13 units d) None of these

82. A line joining points $(4,-1,2)$ and $(-3,2,3)$ meets the plane at the point $(-10,5,4)$ at $90°$, then equation of the plane is

a) $7x-3y-z+89=0$ b) $7x+3y+z+89=0$

c) $7x-3y+z+89=0$ d) None of these

83. Two systems of rectangular axes have the same origin. If a plane cuts them at distance a,b,c and a′, b′, c′

from the origin, then

a) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}=0$
b) $\frac{1}{a^2}+\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}-\frac{1}{c'^2}=0$
c) $\frac{1}{a^2}-\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$
d) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$

84. If $(2,-1,3)$ is the foot of the perpendicular drown from the origin to the plane, then the equation of the plane is
a) $2x+y-3z+6=0$ b) $2x-y+3z-14=0$ c) $2x-y+3z-13=0$ d) $2x+y+3z-10=0$

85. If projection of a line on x, y and z-axes are 6,2 and 3 respectively, then direction cosines of the line is
a) $\left(\frac{6}{7},\frac{2}{7},\frac{3}{7}\right)$ b) $\left(\frac{7}{6},\frac{7}{2},\frac{7}{3}\right)$ c) $\left(\frac{6}{11},\frac{2}{11}.\frac{3}{11}\right)$ d) None of these

86. If the plane $3x-2y-z-18=0$ meets the coordinate axes in A, B, C then the centroid of ΔABC is
a) $(2,3,-6)$ b) $(2,-3,6)$ c) $(-2,-3,6)$ d) $(2,-3,-6)$

87. If for a plane, the intercepts on the coordinate axes are 8, 4, 4 then the length of the perpendicular from the origin on to the plane is
a) $\frac{8}{3}$ b) $\frac{3}{8}$ c) 3 d) $\frac{4}{3}$

88. The equation of the sphere concentric with the sphere
$2x^2+2y^2+2z^2-6x+2y-4z=1$ and double its radius is
a) $x^2+y^2+z^2-x+y-z=1$
b) $x^2+y^2+z^2-6x+2y-4z=1$
c) $2x^2+2y^2+2z^2-6x+2y-4z-15=0$
d) $2x^2+2y^2+2z^2-6x+2y-4z-25=0$

89. The angle between the lines $\vec{\mathbf{r}}=(4\hat{\mathbf{i}}-\hat{\mathbf{j}})+s(2\hat{\mathbf{i}}+\hat{\mathbf{j}}-3\hat{\mathbf{k}})$ and $\vec{\mathbf{r}}=(\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}})+t(\hat{\mathbf{i}}+3\hat{\mathbf{j}}+2\hat{\mathbf{k}})$ is
a) $3\pi/2$ b) $\pi/3$ c) $2\pi/3$ d) $\pi/6$

90. Equation of the plane, passing through the line of intersection of the plane $P\equiv ax+by+cz+d=0, P'\equiv a'x+b'y+c'z+d'=0$ and parallel to x-axis is
a) $Pa-P'a'=0$ b) $P/a+P'/a'=0$ c) $Pa+P'a'=0$ d) $P/a=P'/a'$

91. Equation of the plane through three points A, B, C with position vectors $-6\hat{\mathbf{i}}+3\hat{\mathbf{j}}+2\hat{\mathbf{k}}, 3\hat{\mathbf{i}}-2\hat{\mathbf{j}}+4\hat{\mathbf{k}}, 5\hat{\mathbf{i}}+7\hat{\mathbf{j}}+3\hat{\mathbf{k}}$ is
a) $\vec{\mathbf{r}}\cdot(\hat{\mathbf{i}}-\hat{\mathbf{j}}-7\hat{\mathbf{k}})+23=0$
b) $\vec{\mathbf{r}}\cdot(\hat{\mathbf{i}}+\hat{\mathbf{j}}+7\hat{\mathbf{k}})=23$
c) $\vec{\mathbf{r}}\cdot(\hat{\mathbf{i}}+\hat{\mathbf{j}}-7\hat{\mathbf{k}})+23=0$
d) $\vec{\mathbf{r}}\cdot(\hat{\mathbf{i}}-\hat{\mathbf{j}}-7\hat{\mathbf{k}})=23$

92. The equation of the plane in which the lines $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$ and $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$ lie, is
a) $17x-47y-24z+172=0$
b) $17x+47y-24z+172=0$
c) $17x+47y+24z+172=0$
d) $17x-47y+24z+172=0$

93. The length of the perpendicular from the origin to the plane passing through three non-collinear points $\vec{a}, \vec{b}, \vec{c}$ is
a) $\frac{[\vec{a}\,\vec{b}\,\vec{c}]}{|\vec{a}\times\vec{b}+\vec{c}\times\vec{a}+\vec{b}\times\vec{c}|}$
b) $\frac{2[\vec{a}\,\vec{b}\,\vec{c}]}{|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}$
c) $[\vec{a}\,\vec{b}\,\vec{c}]$
d) None of these

94. The shortest distance from the point $(1, 2, -1)$ to the surface of the sphere $x^2+y^2+z^2=24$ is
a) $3\sqrt{6}$ b) $\sqrt{6}$ c) $2\sqrt{6}$ d) 2

95. A plane π makes intercepts 3 and 4 respectively on z-axis and x-axis. If π is parallel to y-axis, then its equation is
a) $3x+4z=12$ b) $3z+4x=12$ c) $3y+4z=12$ d) $3z+4y=12$

96. A line passes through two points $A(2,-3,-1)$ and $B(8,-1,2)$. The coordinates of a point on this line at a distance of 14 units from A are
a) $(14, 1, 5)$ b) $(-10,-7,7)$ c) $(86, 25, 41)$ d) None of these

97. The equation of the perpendicular from the point (α, β, γ) to the plane $ax+by+cz+d=0$ is

a) $\frac{x-a}{a\alpha}=\frac{y-b}{b\beta}=\frac{z-c}{c\gamma}$ b) $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$ c) $\frac{x-\alpha}{a}=\frac{y-\beta}{b}=\frac{z-\gamma}{c}$ d) $\frac{x}{\alpha}=\frac{y}{\beta}=\frac{z}{\gamma}$

98. The equation of the plane through the points (2, 2, 1) and (9, 3, 6) and perpendicular to the plane $2x+6y+6z-1=0$, is

a) $3x+4y+5z=9$ b) $3x+4y-5z=9$ c) $3x+4y-5z-9=0$ d) None of these

99. The perimeter of the triangle with verities at (1,0,0).(0,1,0) and (0,0,1) is

a) 3 b) 2 c) $2\sqrt{2}$ d) $3\sqrt{2}$

100. If the distance of the point $P(1,-2,1)$ from the plane $x+2y-2z=\alpha$, where α>0, is 5, then the foot of the perpendicular from P to the plane is

a) $\left(\frac{8}{3},\frac{4}{3},-\frac{7}{3}\right)$ b) $\left(\frac{4}{3},-\frac{4}{3},\frac{1}{3}\right)$ c) $\left(\frac{1}{3},\frac{2}{3},\frac{10}{3}\right)$ d) $\left(\frac{2}{3},-\frac{1}{3},\frac{5}{2}\right)$

101. If the lines

$\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$

and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$

interested, then the value of k is

a) $\frac{3}{2}$ b) $\frac{9}{2}$ c) $-\frac{2}{9}$ d) $-\frac{3}{2}$

102. A point on x-axis which is equidistance from both the points (1, 2, 3) and (3, 5, −2) is

a) (−6, 0, 0) b) (5, 0, 0) c) (−5, 0, 0) d) (6, 0, 0)

103. The angle between the line

$\frac{x+4}{1}=\frac{y-3}{2}=\frac{z+2}{3}$ and $\frac{x}{3}=\frac{y-1}{-2}=\frac{z}{1}$ is

a) $\sin^{-1}\left(\frac{1}{7}\right)$ b) $\cos^{-1}\left(\frac{2}{7}\right)$ c) $\cos^{-1}\left(\frac{1}{7}\right)$ d) None of these

104. The points (5, 2, 4), (6, −1, 2) and (8, −7, k) are collinear, if k is equal to

a) −2 b) 2 c) 3 d) −1

105. The equation of the plane through the point (2, 5, −3) perpendicular to the planes $x+2y+2z=1$ and $x-2y+3z=4$ is

a) $3x-4y+2z-20=0$ b) $7x-y+5z=30$

c) $x-2y+z=11$ d) $10x-y-4z=27$

106. The direction cosines of the line $4x-4=1-3y=2z-1$ are

a) $\frac{3}{\sqrt{56}},\frac{-4}{\sqrt{56}},\frac{6}{\sqrt{56}}$ b) $\frac{3}{\sqrt{29}},\frac{-4}{\sqrt{29}},\frac{6}{\sqrt{29}}$ c) $\frac{3}{\sqrt{61}},\frac{-4}{\sqrt{61}},\frac{6}{\sqrt{61}}$ d) 4, −3,2

107. Equation of the plane passing through the intersection of the planes $x+y+z=6$ and $2x+3y+4z+5=0$ and the point (1, 1, 1) is

a) $20x+23y+26z-69=0$ b) $31x+45y+49z+52=0$

c) $8x+5y+2z-69=0$ d) $4x+5y+6z-7=0$

108. The equation of the plane through the point (0, −4, −6) and (−2, 9, 3) and perpendicular to the plane $x-4y-2z=8$ is

a) $3x+3y-2z=0$ b) $x-2y+z=2$ c) $2x+y-z=2$ d) $5x-3y+2z=0$

109. If a line makes angle $\frac{\pi}{3}$ and $\frac{\pi}{4}$ with the x and y-axes respectively, then the angle made by the line and z-axis is

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{5\pi}{12}$

110. Let (3, 4, −1)and (−1, 2, 3) are the end points of a diameter of sphere. Then the radius of the sphere is equal to

a) 1 b) 2 c) 3 d) 9

111. The points $(5,-4,2),(4,-3,1),(7,-6,4)$ and $(8,-7,5)$ are the vertices of
a) A rectangle
b) A square
c) A parallelogram
d) None of these

112. The equation of the plane containing the lines $\vec{r}=\overrightarrow{a_1}+\lambda\,\vec{b}$ and $\vec{r}=\overrightarrow{a_2}+\mu\,\vec{b}$, is
a) $\vec{r}\cdot(\overrightarrow{a_1}-\overrightarrow{a_2})\times\vec{b}=[\overrightarrow{a_1}\,\overrightarrow{a_2}\,\vec{b}]$
b) $\vec{r}\cdot(\overrightarrow{a_2}-\overrightarrow{a_1})\times\vec{b}=[\overrightarrow{a_1}\,\overrightarrow{a_2}\,\vec{b}]$
c) $\vec{r}\cdot(\overrightarrow{a_1}+\overrightarrow{a_2})\times\vec{b}=[\overrightarrow{a_2}\,\overrightarrow{a_1}\,\vec{b}]$
d) None of these

113. If $\left(\frac{1}{2},\frac{1}{3},n\right)$ are the direction cosines of a line, then the value of n is
a) $\dfrac{\sqrt{23}}{6}$
b) $\dfrac{23}{6}$
c) $\dfrac{2}{3}$
d) $\dfrac{3}{2}$

114. The vector equation of the plane passing through the origin and the line of intersection of the plane $\vec{r}\cdot\vec{a}=\lambda$ and $\vec{r}\cdot\vec{b}=\mu$ is
a) $\vec{r}\cdot(\lambda\,\vec{a}-\mu\,\vec{b})=0$
b) $\vec{r}\cdot(\lambda\,\vec{b}-\mu\,\vec{a})=0$
c) $\vec{r}\cdot(\lambda\,\vec{a}+\mu\,\vec{b})=0$
d) $\vec{r}\cdot(\lambda\,\vec{b}+\mu\,\vec{a})=0$

115. If l_1,m_1,n_1 and l_2,m_2,n_2 are direction cosines of the two lines inclined to each other at an angle , then the direction cosines of the external bisector of the angle between the lines are
a) $\dfrac{l_1+l_2}{2\sin\theta/2},\dfrac{m_1+m_2}{2\sin\theta/2},\dfrac{n_1+n_2}{2\sin\theta/2}$
b) $\dfrac{l_1+l_2}{2\cos\theta/2},\dfrac{m_1+m_2}{2\cos\theta/2},\dfrac{n_1+n_2}{2\cos\theta/2}$
c) $\dfrac{l_1-l_2}{2\sin\theta/2},\dfrac{m_1-m_2}{2\sin\theta/2},\dfrac{n_1-n_2}{2\sin\theta/2}$
d) $\dfrac{l_1-l_2}{2\cos\theta/2},\dfrac{m_1-m_2}{2\cos\theta/2},\dfrac{n_1-n_2}{2\cos\theta/2}$

116. The direction ratios of the normal to the plane passing through the points $(1,-2,3),(-1,2,-1)$ and parallel to the line $\frac{x-2}{2}=\frac{y+1}{3}=\frac{z}{4}$ are proportional to
a) 2, 3, 4
b) 4, 0, 7
c) $-2,0,-1$
d) $2,0,-1$

117. The position vector of a point at a distance of $3\sqrt{11}$ units from $\hat{\imath}-\hat{\jmath}+2\hat{k}$ on a line passing through the points $\hat{\imath}-\hat{\jmath}+2\hat{k}$ and $3\hat{\imath}+\hat{\jmath}+\hat{k}$ is
a) $10\hat{\imath}+2\hat{\jmath}-5\hat{k}$
b) $-8\hat{\imath}-4\hat{\jmath}-\hat{k}$
c) $8\hat{\imath}+4\hat{\jmath}+\hat{k}$
d) $-10\hat{\imath}-2\hat{\jmath}-5\hat{k}$

118. The centre and radius of the sphere $x^2+y^2+z^2+3x-4z+1=0$ are
a) $\left(-\frac{3}{2},0,-2\right),\frac{\sqrt{21}}{2}$
b) $\left(\frac{3}{2},0,2\right),\sqrt{21}$
c) $\left(-\frac{3}{2},0,2\right)\cdot\frac{\sqrt{21}}{2}$
d) $\left(-\frac{3}{2},2,0\right),\frac{21}{2}$

119. The direction cosines of the line $6\,x-2=3\,y+1=2\,z-2$ are
a) $\dfrac{1}{\sqrt{3}},\dfrac{1}{\sqrt{3}},\dfrac{1}{\sqrt{3}}$
b) $\dfrac{1}{\sqrt{14}},\dfrac{2}{\sqrt{14}},\dfrac{3}{\sqrt{14}}$
c) 1, 2, 3
d) None of these

120. The cartesian equation of the plane perpendicular to the line $\frac{x-1}{2}=\frac{y-3}{-1}=\frac{z-4}{2}$ and passing through the origin is
a) $2x-y+2z-7=0$
b) $2x+y+2z=0$
c) $2x-y+2z=0$
d) $2x-y-z=0$

121. The point of intersection of the lines
$$\frac{x-5}{3}=\frac{y-7}{-1}=\frac{z+2}{1},\frac{x+3}{-36}=\frac{y-3}{2}=\frac{z-6}{4}$$
is
a) $(2,10,-4)$
b) $\left(21,\frac{5}{3},\frac{10}{3}\right)$
c) $(5,-7,-2)$
d) $(-3,3,6)$

122. If the position vectors of the points A and B are $3\hat{\imath}+\hat{\jmath}+2\hat{k}$ and $\hat{\imath}-2\hat{\jmath}-4\hat{k}$ respectively, then the equation of the plane through B and perpendicular to AB is
a) $2x+3y+6z+28=0$
b) $3x+2y+6z=28$

c) $2x - 3y + 6z + 28 = 0$
d) $3x - 2y + 6z = 28$

123. The point equidistant from the point $(a, 0, 0)$, $(0, b, 0)$, $(0, 0, c)$ ad $(0, 0, 0)$ is
a) $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$ b) (a, b, c) c) $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$ d) None of these

124. If a plane meets the coordinate axes at A, B and C such that the centroid of the triangle is (1,2,4), then the equation of the plane is
a) $x + 2y + 4z = 12$ b) $4x + 2y + z = 12$ c) $x + 2y + 4z = 3$ d) $4x + 2y + z = 3$

125. If the coordinates of the vertices of a $\Delta\, ABC$ are $A(-1, 3, 2)$, $B(2, 3, 5)$ and $C(3, 5, -2)$, then $\angle A$ is equal to
a) 45° b) 60° c) 90° d) 30°

126. The distance between the line
$\vec{\mathbf{r}} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}} + \lambda(\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ and the plane $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + \hat{\mathbf{k}}) = 5$ is
a) $\frac{10}{3}$ b) $\frac{3}{10}$ c) $\frac{10}{3\sqrt{3}}$ d) $\frac{10}{9}$

127. A point on XOZ- plane divides the join of $(5, -3, -2)$ and $(1, 2, -2)$ at
a) $\left(\frac{13}{5}, 0, -2\right)$
b) $\left(\frac{13}{5}, 0, 2\right)$
c) (5,0,2)
d) $(5, 0, -2)$

128. A plane makes intercepts $-6, 3, 4$ upon the coordinate axes. Then, the length of the perpendicular from the origin on it is
a) $\frac{2}{\sqrt{29}}$ b) $\frac{3}{\sqrt{29}}$ c) $\frac{4}{\sqrt{29}}$ d) $\frac{12}{\sqrt{29}}$

129. The equation of the plane counting the lines
$\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{3}$ and $\frac{x}{2} = \frac{y-2}{-1} = \frac{z+1}{3}$ is
a) $8x - y + 5z - 8 = 0$ b) $8x + y - 5z - 7 = 0$ c) $x - 8y + 3z + 6 = 0$ d) $8x + y - 5z + 7 = 0$

130. The equation of the plane which bisects the line joining $(2, 3, 4)$ and $(6, 7, 8)$, is
a) $x - y - z - 15 = 0$ b) $x - y + z - 15 = 0$ c) $x + y + z - 15 = 0$ d) $x + y + z + 15 = 0$

131. The distance between the points (1,4,5) and (2,2,3) is
a) 5 b) 4 c) 3 d) 2

132. The equation of the plane through the points $(1, 2, 3)$, $(-1, 4, 2)$and $(3, 1, 1)$ is
a) $5x + y + 12z - 23 = 0$ b) $5x + 6y + 2z - 23 = 0$
c) $x + 6y + 2z - 13 = 0$ d) $x + y + z - 13 = 0$

133. Equation of the plane parallel to the planes
$x + 2y + 3z - 5 = 0$, $x + 2y + 3z - 7 = 0$ and equidistant from them is
a) $x + 2y + 3z - 6 = 0$ b) $x + 2y + 3z - 1 = 0$
c) $x + 2y + 3z - 8 = 0$ d) $x + 2y + 3z - 3 = 0$

134. The image of the point $(1, 2, 3)$ in lie $\frac{x}{2} = \frac{y-1}{3} = \frac{z-1}{3}$ is
a) $\left(1, \frac{5}{2}, \frac{5}{2}\right)$ b) $\left(1, \frac{9}{4}, \frac{11}{4}\right)$ c) $(1, 3, 2)$ d) $(3, 1, 2)$

135. If O is the origin and $OP = 3$ with direction ratios $-1, 2, -2$, then coordinates of P are
a) $(1, 2, 2)$ b) $(-1, 2, -2)$ c) $(-3, 6, -9)$ d) $(-1/3, 2/3, -2/3)$

136. The equation of the sphere touching the three coordinate planes is
a) $x^2 + y^2 + z^2 + 2a(x + y + z) + 2a^2 = 0$ b) $x^2 + y^2 + z^2 - 2a(x + y + z) + 2a^2 = 0$
c) $x^2 + y^2 + z^2 \pm 2a(x + y + z) + 2a^2 = 0$ d) $x^2 + y^2 + z^2 \pm 2ax \pm 2ay \pm 2az + 2a^2 = 0$

137. The angle between the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and the plane $3x + 2y - 3z = 4$ is
a) 45° b) 0° c) $\cos^{-1}\left(\frac{24}{\sqrt{29}\sqrt{22}}\right)$ d) 90°

138. The equation of a line of intersection of planes $4x + 4y - 5z = 12$ and $8x + 12y - 13z = 32$ can be written as

a) $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z}{4}$ b) $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z}{4}$ c) $\frac{x}{2} = \frac{y+1}{3} = \frac{z-2}{4}$ d) $\frac{x}{2} = \frac{y}{3} = \frac{z-2}{4}$

139. Let the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lies in the plane $x + 3y - \alpha z + \beta = 0$ Then (α, β) equals

a) $(6, -17)$ b) $(-6, 7)$ c) $(5, -15)$ d) $(-5, 15)$

140. If a line makes angles α, β, γ and δ with four diagonals of a cube, then the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta$, is

a) $\frac{4}{3}$ b) $\frac{8}{3}$ c) $\frac{7}{3}$ d) 1

141. Find the direction ratio of

$$\frac{3-x}{1} = \frac{y-2}{5} = \frac{2z-3}{1}$$

a) $1:5:\frac{1}{2}$ b) $-1:5:1$ c) $-1:5:\frac{1}{2}$ d) $1:5:1$

142. If l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines of the two lines inclined to each other at an angle θ, then the direction cosines of internal bisector of the angle between these lines are

a) $\frac{l_1+l_2}{2\sin\frac{\theta}{2}}, \frac{m_1+m_2}{2\sin\frac{\theta}{2}}, \frac{n_1+n_2}{2\sin\frac{\theta}{2}}$ b) $\frac{l_1+l_2}{2\cos\frac{\theta}{2}}, \frac{m_1+m_2}{2\cos\frac{\theta}{2}}, \frac{n_1+n_2}{2\cos\frac{\theta}{2}}$

c) $\frac{l_1-l_2}{2\sin\frac{\theta}{2}}, \frac{m_1-m_2}{2\sin\frac{\theta}{2}}, \frac{n_1-n_2}{2\sin\frac{\theta}{2}}$ d) $\frac{l_1-l_2}{2\cos\frac{\theta}{2}}, \frac{m_1-m_2}{2\cos\frac{\theta}{2}}, \frac{n_1-n_2}{2\cos\frac{\theta}{2}}$

143. The plane $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$, cuts the axes in A, B, C, then the area of the ΔABC, is

a) $\sqrt{29}$ sq units b) $\sqrt{41}$ sq units c) $\sqrt{61}$ sq units d) None of these

144. If the planes $\vec{r} \cdot (2\hat{\imath} - \lambda\hat{\jmath} + 3\hat{k}) = 0$ and $\vec{r} \cdot (\lambda\hat{\imath} + 5\hat{\jmath} - \hat{k}) = 5$ are perpendicular to each other, then value of $\lambda^2 + \lambda$, is

a) 0 b) 2 c) 3 d) 1

145. If a sphere of radius r passes through the origin, then the extremities of the diameter parallel to x-axis lie on each of the spheres

a) $x^2 + y^2 + z^2 \pm 2rx = 0$ b) $x^2 + y^2 + z^2 \pm 2ry = 0$

c) $x^2 + y^2 + z^2 \pm 2rz = 0$ d) $x^2 + y^2 + z^2 \pm 2ry \pm 2rz = 0$

146. If the distance of the point (1, 1,1) from the origin is half is distance from the plane $x + y + z + k = 0$, then k is equal to

a) ± 3 b) ± 6 c) $-3, 9$ d) $3, -9$

147. XOZ plane divides the join of (2, 3, 1) and (6, 7, 1) in the ratio

a) 3 : 7 b) 2 : 7 c) $-3 : 7$ d) $-2 : 7$

148. The point on the line $\frac{x-2}{1} = \frac{y+3}{-2} = \frac{z+5}{-2}$ at a distance of 6 from the point $(2, -3, -5)$ is

a) $(3, -5, -3)$ b) $(4, -7, -9)$ c) $(0, 2, -1)$ d) $(-3, 5, 3)$

149. The direction ratios of a normal to the plane passing through (0, 0, 1), (0, 1, 2) and (1, 2, 3) are proportional to

a) $0, 1, -1$ b) $1, 0, -1$ c) $0, 0, -1$ d) $1, 0, 0$

150. Ratio in which the xy-plane divides the join of (1, 2, 3) and (4, 2, 1) is

a) 3 : 1 internally b) 3 : 1 externally c) 1 : 2 internally d) 2 : 1 externally

151. A vector $\vec{r}$ is equally inclined with the coordinate axes. If the tip of $\vec{r}$ is in the positive octant and $|\vec{r}| = 6$, then $\vec{r}$ is

a) $2\sqrt{3}(\hat{\imath} - \hat{\jmath} + \hat{k})$ b) $2\sqrt{3}(-\hat{\imath} + \hat{\jmath} + \hat{k})$ c) $2\sqrt{3}(\hat{\imath} + \hat{\jmath} - \hat{k})$ d) $2\sqrt{3}(\hat{\imath} + \hat{\jmath} + \hat{k})$

152. The angle between the planes $2x - y + z = 6$ and $x + y + 2z = 3$ is

a) $\pi/3$ b) $\cos^{-1}(1/6)$ c) $\pi/4$ d) $\pi/6$

153. The vector equation of the plane through the point $2\hat{\imath} - \hat{\jmath} - 4\hat{k}$ and parallel to the plane $\vec{r} \cdot (4\hat{\imath} - 12\hat{\jmath} -$

$3\hat{k}) - 7 = 0$, is

a) $\vec{r}\cdot(4\hat{\imath} - 12\hat{\jmath} - 3\hat{k}) = 0$

b) $\vec{r}\cdot(4\hat{\imath} - 12\hat{\jmath} - 3\hat{k}) = 32$

c) $\vec{r}\cdot(4\hat{\imath} - 12\hat{\jmath} - 3\hat{k}) = 12$

d) None of these

154. An equation of the line passing through $3\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$ and perpendicular to the plane $3x - 4y = 5z = 8$ is

a) $\frac{x-3}{3} = \frac{y+5}{-4} = \frac{z-7}{5}$

b) $\frac{x-3}{3} = \frac{y+4}{-5} = \frac{z-5}{7}$

c) $\vec{\mathbf{r}} = 3\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 7\hat{\mathbf{k}} + \lambda(3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}})$

d) $\vec{\mathbf{r}} = 3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}} + \mu(3\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 7\hat{\mathbf{k}})$
λ, μ are parameters

155. The equation of a line is $6x - 2 = 3y - 1 = 2z - 2$ The direction ratios of the line are

a) 1,2,3

b) 1 ,1, 1

c) $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$

d) $\frac{1}{3}, \frac{-1}{3}, \frac{1}{3}$

156. Angle between the line $\vec{r} = (2\hat{\imath} - \hat{\jmath} + \hat{k}) + \lambda(-\hat{\imath} + \hat{\jmath} + \hat{k})$ and the plane $\vec{r}\cdot(3\hat{\imath} + 2\hat{\jmath} - \hat{k}) = 4$ is

a) $\cos^{-1}\left(\frac{2}{\sqrt{42}}\right)$

b) $\cos^{-1}\left(\frac{-2}{\sqrt{42}}\right)$

c) $\sin^{-1}\left(\frac{2}{\sqrt{42}}\right)$

d) $\sin^{-1}\left(\frac{-2}{\sqrt{42}}\right)$

157. A mirror and a source of light are situated at the origin O and at a point on OX respectively. A ray of light from the source strikes the mirror and is reflected. If the direction ratios of the normal to the plane are proportional to $1, -1, 1$, then direction cosines of the reflected ray are

a) $\frac{1}{2}, \frac{2}{3}, \frac{2}{3}$

b) $-\frac{1}{2}, \frac{2}{3}, \frac{2}{3}$

c) $-\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}$

d) $-\frac{1}{2}, -\frac{2}{3}, \frac{2}{3}$

158. If the direction ratio of two lines are given by $3lm - 4ln + mn = 0$ and $l + 2m + 3n = 0$, then the angle between the line is

a) $\frac{\pi}{6}$

b) $\frac{\pi}{4}$

c) $\frac{\pi}{3}$

d) $\frac{\pi}{2}$

159. The points $A(-1,3,0)$, $B(2,2,1)$ and $C(1,1,3)$ determine a plane. The distance from the plane to the point $D(5,7,8)$ is

a) $\sqrt{66}$

b) $\sqrt{71}$

c) $\sqrt{73}$

d) $\sqrt{76}$

160. The line of intersection of the planes $\vec{r}\cdot(3\hat{\imath} - \hat{\jmath} + \hat{k}) = 1$ and $\vec{r}\cdot(\hat{\imath} + 4\hat{\jmath} - 2\hat{k}) = 2$ is parallel to the vector

a) $-2\hat{\imath} + 7\hat{\jmath} + 13\hat{k}$

b) $2\hat{\imath} + 7\hat{\jmath} - 13\hat{k}$

c) $-2\hat{\imath} - 7\hat{\jmath} + 13\hat{k}$

d) $2\hat{\imath} + 7\hat{\jmath} + 13\hat{k}$

161. The equation of the line of intersection of planes $4x + 4y - 5z = 12, 8x + 12y - 13z = 32$ can be written as

a) $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z}{4}$

b) $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z}{4}$

c) $\frac{x}{2} = \frac{y+1}{3} = \frac{z-2}{4}$

d) $\frac{x}{2} = \frac{y}{3} = \frac{z-2}{4}$

162. The vector equation of the line of intersection of the planes $\vec{r}\cdot(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) = 0$ and $\vec{r}\cdot(3\hat{\imath} + 2\hat{\jmath} + \hat{k}) = 0$, is

a) $\vec{r} = \lambda(\hat{\imath} + 2\hat{\jmath} + \hat{k})$

b) $\vec{r} = \lambda(\hat{\imath} - 2\hat{\jmath} + 3\hat{k})$

c) $\vec{r} = \lambda(\hat{\imath} + 2\hat{\jmath} - 3\hat{k})$

d) None of these

163. Let a plane passes through the point $P(-1,-1,1)$ and also passes through a line joining the points $Q(0,1,1)Q(0,1,1)$and $R(0,0,2)$. Then the distance of the plane from the point $(0,0,0)$ is

a) 3

b) 0

c) $\frac{1}{\sqrt{6}}$

d) $\frac{2}{\sqrt{6}}$

164. The direction cosines of the line passing through $P(2,3,-1)$ and the origin are

a) $\frac{2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{1}{\sqrt{14}}$

b) $\frac{2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{1}{\sqrt{14}}$

c) $\frac{-2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{1}{\sqrt{14}}$

d) $\frac{2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{-1}{\sqrt{14}}$

165. The shortest distance from the point $(1,2,-1)$ to the surface of the sphere $x^2 + y^2 + z^2 = 54$ is

a) $3\sqrt{6}$

b) $2\sqrt{6}$

c) $\sqrt{6}$

d) 2

166. The shortest distance between the lines
$\frac{x-2}{3} = \frac{y+3}{4} = \frac{z-1}{5}$ and

$\dfrac{x-5}{1}=\dfrac{y-1}{2}=\dfrac{z-6}{3}$ is

a) 3 b) 2 c) 1 d) 0

167. The image of the point $P(1,3,4)$ in the plane $2x-y+z+3=0$ is

a) $(3,5,-2)$ b) $(-3,5,2)$ c) $(3,-5,2)$ d) $(-1,4,2)$

168. The vector from of the sphere $2(x^2+y^2+z^2)-4x+6y+8z-5=0$ is

a) $\mathbf{\vec{r}}\cdot[\mathbf{\vec{r}}-(2\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})]=\dfrac{2}{5}$ b) $\mathbf{\vec{r}}\cdot[\mathbf{\vec{r}}-(2\hat{\mathbf{i}}-3\hat{\mathbf{j}}-4\hat{\mathbf{k}})]=\dfrac{1}{2}$

c) $\mathbf{\vec{r}}\cdot[\mathbf{\vec{r}}-(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+4\hat{\mathbf{k}})]=\dfrac{5}{2}$ d) $\mathbf{\vec{r}}\cdot[\mathbf{\vec{r}}-(2\hat{\mathbf{i}}-3\hat{\mathbf{j}}-4\hat{\mathbf{k}})]=\dfrac{5}{2}$

169. The angle between the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z+3}{-2}$ and the plane $x+y+4=0$, is

a) $0°$ b) $30°$ c) $45°$ d) $90°$

170. The equation to the straight line passing through the points $(4,-5,-2)$ and $(-1,5,3)$ is

a) $\dfrac{x-4}{1}=\dfrac{y+5}{-2}=\dfrac{z+2}{-1}$ b) $\dfrac{x+1}{1}=\dfrac{y-5}{2}=\dfrac{z-3}{-1}$

c) $\dfrac{x}{-1}=\dfrac{y}{5}=\dfrac{z}{3}$ d) $\dfrac{x}{4}=\dfrac{y}{-5}=\dfrac{z}{-2}$

171. The reflection of the plane $2x-3y+4z-3=0$ in the plane $x-y+z-3=0$ is the plane

a) $4x-3y+2z-15=0$ b) $x-3y+2z-15=0$

c) $4x+3y-2z+15=0$ d) None of these

172. The equation of the plane passing through the line of intersection of the planes $x+y+z=6$ and $2x+3y+4z+5=0$ and perpendicular to the plane $4x+5y-3z=8$ is

a) $x+7y+13z-96=0$ b) $x+7y+13z+96=0$

c) $x+7y-13z-96=0$ d) $x-7y+13z+96=0$

173. The distance between the line $\vec{r}=2\hat{\imath}-2\hat{\jmath}+3\hat{k}+\lambda(\hat{\imath}-\hat{\jmath}+4\hat{k})$ and the plane $\vec{r}\cdot(\hat{\imath}+5\hat{\jmath}+\hat{k})=5$, is

a) $\dfrac{10}{3\sqrt{3}}$ b) $\dfrac{10}{3}$ c) $10/9$ d) None of these

174. The angle between the line $\frac{x-3}{2}=\frac{y-1}{1}=\frac{z+4}{-2}$ and the plane, $x+y+z+5=0$

Is

a) $\sin^{-1}\left(\dfrac{2}{\sqrt{3}}\right)$ b) $\sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$ c) $\dfrac{\pi}{4}$ d) $\sin^{-1}\left(\dfrac{1}{3\sqrt{3}}\right)$

175. If for a plane, the intercepts on the coordinate axes are 8, 4, 4, then the length of the perpendicular from the origin to the plane is

a) $8/3$ b) $3/8$ c) 3 d) $4/3$

176. A line makes acute angles of α, β and γ with the coordinate axes such that

$\cos\alpha\cos\beta=\cos\beta\cos\gamma=\dfrac{2}{9}$

And $\cos\gamma\cos\alpha=\frac{4}{9}$,

Then $\cos\alpha+\cos\beta+\cos\gamma$ is equal

To

a) $\dfrac{25}{9}$ b) $\dfrac{5}{9}$ c) $\dfrac{5}{3}$ d) $\dfrac{2}{3}$

177. The equation of the plane passing through the mid-point of the line segment of join of the points $P(1,2,3)$ and $Q(3,4,5)$ and perpendicular to it is

a) $x+y+z=9$ b) $x+y+z=-9$ c) $2x+3y+4z=9$ d) $2x+3y+4z=-9$

178. The intersection of the sphere $x^2+y^2+z^2+7x-2y-z=1$ and $x^2+y^2+z^2-3x+3y+4z=-4$ is same as the intersection of one of the spheres and the plane is

a) $2x-y-z=1$ b) $-2x+y+z=1$ c) $2x-y+z=1$ d) $2x+y+z=1$

179. The equation of the plane which passes through the point $(2,-3,1)$ and perpendicular to the line joining points $(3,4,-1)$ and $(2,-1,5)$, is

a) $x+5y-6z+19=0$ b) $x-5y+6z-19=0$
c) $x+5y+6z+19=0$ d) $x-5y-6z-19=0$

180. The vector equation of the straight line
$\frac{1-x}{3}=\frac{y+1}{-2}=\frac{3-z}{-1}$ is
a) $\vec{\mathbf{r}}=(\hat{\mathbf{i}}-\hat{\mathbf{j}}+3\hat{\mathbf{k}})+\lambda(3\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}})$ b) $\vec{\mathbf{r}}=(\hat{\mathbf{i}}-\hat{\mathbf{j}}+3\hat{\mathbf{k}})+\lambda(3\hat{\mathbf{i}}-2\hat{\mathbf{j}}-\hat{\mathbf{k}})$
c) $\vec{\mathbf{r}}=(3\hat{\mathbf{i}}-2\hat{\mathbf{j}}-\hat{\mathbf{k}})+\lambda(\hat{\mathbf{i}}-\hat{\mathbf{j}}+3\hat{\mathbf{k}})$ d) $\vec{\mathbf{r}}=(3\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}})+\lambda(\hat{\mathbf{i}}-\hat{\mathbf{j}}+3\hat{\mathbf{k}})$

181. The plane $\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=1$ cuts the coordinate axes in A,B,C then the area of the ΔABC is
a) $\sqrt{29}$ sq units b) $\sqrt{41}$ sq units c) $\sqrt{61}$ sq units d) None of these

182. A line makes acute angles of α,β and γ with the coordinate axes such that $\cos\alpha\cos\beta=\cos\beta\cos\gamma=\frac{2}{9}$ and $\cos\gamma\cos\alpha=\frac{4}{9}$, then $\cos\alpha+\cos\beta+\cos\gamma$ is equal to
a) $\frac{25}{9}$ b) $\frac{5}{9}$ c) $\frac{5}{3}$ d) $\frac{2}{3}$

183. The equation of the plane containing the line $\vec{r}=\hat{\imath}+\hat{\jmath}+\lambda(2\hat{\imath}+\hat{\jmath}+4\hat{k})$, is
a) $\vec{r}\cdot(\hat{\imath}+2\hat{\jmath}-\hat{k})=3$ b) $\vec{r}\cdot(\hat{\imath}+2\hat{\jmath}-\hat{k})=6$ c) $\vec{r}\cdot(-\hat{\imath}-2\hat{\jmath}+\hat{k})$ d) None of these

184. If OA is equally inclined to OX,OY and OZ and if A is $\sqrt{3}$ units from the origin, then A is
a) (3, 3, 3) b) $(-1,1,-1)$ c) $(-1,1,1)$ d) (1, 1, 1)

185. If a plane meets the coordinate axes at A,B and C such that the centroid of the triangle is (1, 2, 4) then the equation of the plane is
a) $x+2y+4z=12$ b) $4x+2y+z=12$ c) $x+2y+4z=3$ d) $4x+2y+z=3$

186. The equation of the plane passing through the midpoint of the line of join of the points (1, 2, 3) and (3, 4, 5) and perpendicular to it is
a) $x+y+z=9$ b) $x+y+z=-9$ c) $2x+3y+4z=9$ d) $2x+3y+4z=-9$

187. The line passing through the points $(5,1,a)$ and $(3,b,1)$ crosses the yz-plane at the point $\left(0,\frac{17}{2},-\frac{13}{2}\right)$. Then,
a) $a=8,b=2$ b) $a=2,b=8$ c) $a=4,b=6$ d) $a=6,b=4$

188. The image of the point (5, 4, 6) in the plane $x+y+2z-15=0$ is
a) (3, 2, 2) b) (2, 3, 2) c) (2, 2, 3) d) $(-5,-4,-6)$

189. If α, β, γ be the angle which a line makes with the coordinate axes, then
a) $\sin^2\alpha+\cos^2\beta+\sin^2\gamma=1$ b) $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=1$
c) $\cos^2\alpha+\cos^2\beta+\sin^2\gamma=1$ d) $\sin^2\alpha+\cos^2\beta+\sin^2\gamma=1$

190. The angle between the lines whose direction cosines are given by the equation $l^2+m^2-n^2=0, l+m+n=0$, is
a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

191. Equation of plane passing through the points (2, 2, 1), (9, 3, 6) and perpendicular to the plane $2x+6y+6z-1=0$, is
a) $3x+4y+5z=9$ b) $3x+4y-5z+9=0$ c) $3x+4y-5z-9=0$ d) None of these

192. The vector equation of the plane passing through the origin and the line of intersection of the plane $\vec{\mathbf{r}}\cdot\vec{\mathbf{a}}=\lambda$ and $\vec{\mathbf{r}}\cdot\vec{\mathbf{b}}=\mu$, is
a) $\vec{\mathbf{r}}\cdot(\lambda\vec{\mathbf{a}}-\mu\vec{\mathbf{b}})=0$ b) $\vec{\mathbf{r}}\cdot(\lambda\vec{\mathbf{b}}-\mu\vec{\mathbf{a}})=0$ c) $\vec{\mathbf{r}}\cdot(\lambda\vec{\mathbf{a}}+\mu\vec{\mathbf{b}})=0$ d) $\vec{\mathbf{r}}\cdot(\lambda\vec{\mathbf{b}}+\mu\vec{\mathbf{a}})=0$

193. The shortest distance between the lines $\vec{r}=(\hat{\imath}+2\hat{\jmath}+\hat{k})+\lambda(2\hat{\imath}+\hat{\jmath}+2\hat{k})$ and, $\vec{r}=2\hat{\imath}-\hat{\jmath}-\hat{k}+\mu(2\hat{\imath}+\hat{\jmath}+2\hat{k})$, is
a) 0 b) $\sqrt{101}/3$ c) 101/3 d) None of these

194. A straight line which makes an angle of 60° with each of y and z-axes, this line makes with x-axis at an angle
a) 30° b) 60° c) 75° d) 45°

195. If the straight lines $\frac{x-1}{k}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x-2}{3}=\frac{y-3}{k}=\frac{z-1}{3}$

Intersect at a point, then the integer k is equal to

a) -2 b) -5 c) 5 d) 2

196. The cosine of the angle A of the triangle with verities $A(1,-1,2), B(6,11,2), C(1,2,6)$ is

a) 63/65 b) 36/65 c) 16/65 d) 13/64

197. The angle between a line whose direction ratios are in the ratio 2:2:1 and a line joining (3, 1, 4) to (7, 2, 12) is

a) $\cos^{-1}(2/3)$ b) $\cos^{-1}(-2/3)$ c) $\tan^{-1}(2/3)$ d) None of these

198. The equation of the plane passing through (1, 1, 1) and (1, −1, −1) and perpendicular to $2x-y+z+5=0$ is

a) $2x+5y+z-8=0$ b) $x+y-z-1=0$ c) $2x+5y+z+4=0$ d) $x-y+z-1=0$

199. The distance of origin from the point of intersection of the line

$\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and the plane $2x+y-z=2$ is

a) $\sqrt{120}$ b) $\sqrt{83}$ c) $2\sqrt{19}$ d) $\sqrt{78}$

200. Consider the following statements:

1. Line joining (1, 2, 5); (4, 3, 2) is parallel to the line joining (5, 1, −11), (8, 2,−8)

2. Three concurrent lines with DC$'s(l_i, m_i, n_i)i=1,2,3$ are

Coplanar, if $\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}=0$

3. The plane $x-2y+z=21$ and the line $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-1}{3}$ are parallel

Which of these is/are correct?

a) (1)and (2) b) (2) and (3) c) (3) and (1) d) (1), (2) and (3)

201. The distance of the point $P(2,3,4)$ from the line

$1-x=\frac{y}{2}=\frac{1}{3}(1+z)$is

a) $\frac{1}{7}\sqrt{35}$ b) $\frac{4}{7}\sqrt{35}$ c) $\frac{2}{7}\sqrt{35}$ d) $\frac{3}{7}\sqrt{35}$

202. The equation of the line of intersection of the planes $x+2y+z=3$ and $6x+8y+3z=13$ can be written as

a) $\frac{x-2}{2}=\frac{y+1}{-3}=\frac{z-3}{4}$

b) $\frac{x-2}{2}=\frac{y+1}{3}=\frac{z-3}{4}$

c) $\frac{x+2}{2}=\frac{y-1}{-3}=\frac{z-3}{4}$

d) $\frac{x+2}{2}=\frac{y+2}{3}=\frac{z-3}{4}$

203. A line makes an obtuse angle with the positive x-axis and angles $\frac{\pi}{4}$ and $\frac{\pi}{3}$ with the positive y and z-axes respectively. Its direction cosine are

a) $-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},\frac{1}{2}$ b) $\frac{1}{\sqrt{2}},-\frac{1}{2},\frac{1}{2}$ c) $-\frac{1}{2},\frac{1}{\sqrt{2}},\frac{1}{2}$ d) $\frac{1}{2},\frac{1}{\sqrt{2}},\frac{1}{2}$

204. If $\vec{\mathbf{a}}$ is a constant vector and p is a real constant with $|\vec{\mathbf{a}}|^2>p$, then the locus of a point with position vector $\vec{\mathbf{r}}$ such that $|\vec{\mathbf{r}}|^2-2\vec{\mathbf{r}}\cdot\vec{\mathbf{a}}+p=0$is

a) A sphere b) An ellipse c) A circle d) A plane

205. The image of the point $P(1,3,4)$ in the plane $2x-y+z+3=0$ is

a) $(3,5,-2)$ b) $(-3,5,2)$ c) $(3,-5,2)$ d) $(-1,4,2)$

206. The distance of the point $(1,-2,3)$ from the planes $x-y+z=5$ measured along the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$ is

a) 1 b) $\frac{6}{7}$ c) $\frac{7}{6}$ d) None of these

207. The coordinates of the foot of the perpendicular drawn from the point $A(1, 0, 3)$ to the join of the points $B(4, 7, 1)$ and $C(3, 5, 3)$ are

a) $(5/3, 7/3, 17/3)$ b) $(5, 7, 17)$ c) $(5/7, -7/3, 17/3)$ d) $(-5/3, 7/3, -17/3)$

208. The symmetric equation of lines $3x + 2y + z - 5 = 0$ and $x + y - 2z - 3 = 0$, is

a) $\frac{x-1}{5} = \frac{y-4}{7} = \frac{z-0}{1}$ b) $\frac{x+1}{5} = \frac{y+4}{7} = \frac{z-0}{1}$

c) $\frac{x+1}{-5} = \frac{y-4}{7} = \frac{z-0}{1}$ d) $\frac{x-1}{-5} = \frac{y-4}{7} = \frac{z-0}{1}$

209. The distance of the point $P(a, b, c)$ from x-axis is

a) $\sqrt{b^2 + c^2}$ b) $\sqrt{a^2 + b^2}$ c) $\sqrt{a^2 + c^2}$ d) None of these

210. P, Q, R, S Are four coplanar points on the sides AB, BC, CD, DA of a skew quadrilateral. The product $\frac{AP}{PB} \cdot \frac{BQ}{QC} \cdot \frac{CR}{RD} \cdot \frac{DS}{SA}$ equals

a) -2 b) -1 c) 2 d) 1

211. The shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$, is

a) $1/\sqrt{6}$ b) $1/6$ c) $1/3$ d) $1/\sqrt{3}$

212. A plane $x + y + z = -a\sqrt{3}$ touches the sphere $2x^2 + 2y^2 + 2z^2 - 2x + 4y - 4z + 3 = 0$, then the value of a is

a) $\frac{1}{\sqrt{3}}$ b) $\frac{1}{2\sqrt{3}}$ c) $1 - \frac{1}{\sqrt{3}}$ d) $1 + \frac{1}{\sqrt{3}}$

213. $A(3, 2, 0), B(5, 3, 2)$ and $C(-9, 6, -3)$ are the vertices of a triangle ABC. If the bisector of $\angle ABC$ meets BC at D, then coordinates of D are

a) $(19/8, 57/16, 17/16)$

b) $(-19/8, 57/16, 17/16)$

c) $(19/8, -57/16, 17/16)$

d) None of these

214. The equation of the plane through the line of intersection of planes $ax + by + cz + d = 0, a'x + b'y + c'z + d' = 0$ and parallel to the line $y = 0, z = 0$ is

a) $(ab' - a'b)x + (bc' - b'c)y + (ad' - a'd) = 0$

b) $(ab' - a'b)x + (bc' - b'c)y + (ad' - a'd)z = 0$

c) $(ab' - a'b)y + (ac' - a'c)z + (ad' - a'd) = 0$

d) None of these

215. If P is a point in space such that $\vec{O}P$ is inclined to OX at 45° and OY to 60°, then $\vec{O}P$ is inclined to OZ at

a) 75° b) 60° or 120° c) 75° or 105° d) 255°

216. A variable plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ at a unit distance from origin cuts the coordinate axes at A, B and C.Centriod (x, y, z)satisfies the equation $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = k$.The value of k is

a) 9 b) 3 c) $\frac{1}{9}$ d) $\frac{1}{3}$

217. Equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is

a) $x + 2y - 2z = 0$ b) $3x + 2y - 2z = 0$ c) $x - 2y + z = 0$ d) $5x + 2y - 4z = 0$

218. The equation of the plane passing through the points $(0, 1, 2)$ and $(-1, 0, 3)$ and perpendicular to the plane $2x + 3y + z = 5$ is

a) $3x - 4y + 18z + 32 = 0$ b) $3x + 4y - 18z + 32 = 0$

c) $4x + 3y - 17z + 31 = 0$ d) $4x - 3y + z + 1 = 0$

219. The shortest distance between the skew lines $l_1: \vec{\mathbf{r}} = \vec{\mathbf{a}}_1 + \lambda \vec{\mathbf{b}}_1$ $l_2: \vec{\mathbf{r}} = \vec{\mathbf{a}}_2 + \mu \vec{\mathbf{b}}_2$ is

a) $\frac{|\vec{\mathbf{a}}_2 - \vec{\mathbf{a}}_1) \cdot \vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2|}{|\vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2|}$ b) $\frac{|\vec{\mathbf{a}}_2 - \vec{\mathbf{a}}_1) \cdot \vec{\mathbf{a}}_2 \times \vec{\mathbf{b}}_2|}{|\vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2|}$ c) $\frac{|\vec{\mathbf{a}}_2 - \vec{\mathbf{b}}_2) \cdot \vec{\mathbf{a}}_1 \times \vec{\mathbf{b}}_1|}{|\vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2|}$ d) $\frac{|\vec{\mathbf{a}}_1 - \vec{\mathbf{b}}_2) \cdot \vec{\mathbf{b}}_1 \times \vec{\mathbf{a}}_2|}{|\vec{\mathbf{b}}_1 \times \vec{\mathbf{a}}_2|}$

220. The perpendicular distance from the origin to the plane through the point $(2, 3, -1)$ and perpendicular to

the vector $3\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$

a) $\frac{13}{\sqrt{74}}$ b) $\frac{-13}{\sqrt{74}}$ c) 13 d) None of these

221. The lines $\vec{r} = \vec{a} + \lambda(\vec{b} \times \vec{c})$ and $\vec{r} = \vec{b} + \mu(\vec{c} \times \vec{a})$ will intersect, if

a) $\vec{a} \times \vec{c} = \vec{b} \times \vec{c}$ b) $\vec{a} \cdot \vec{c} = \vec{b} \cdot \vec{c}$ c) $\vec{b} \times \vec{a} = \vec{c} \times \vec{a}$ d) None of these

222. The radius of the circle in which the sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ is cut by the plane $x + 2y + 2z + 7 = 0$ is

a) 1 b) 2 c) 3 d) 4

223. A line passes through the points $(6, -7, -1)$ and $(2, -3, 1)$. The direction cosines of the line so directed that the angle made by it with the positive direction of x-axis is acute, are

a) $\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}$ b) $-\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$ c) $\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}$ d) $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$

224. If the angle between the line $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda}z + 4 = 0$ is such thatsin $\theta =$ 13 Then, value of λ is

a) $-\frac{4}{3}$ b) $\frac{3}{4}$ c) $\frac{-3}{5}$ d) $\frac{5}{3}$

225. The equation of the line passing through the point $(3, 0, -4)$ and perpendicular to the plane $2x - 3y + 5z - 7 = 0$ is

a) $\frac{x-2}{3} = \frac{y}{-3} = \frac{z+4}{5}$ b) $\frac{x-3}{2} = \frac{y}{-3} = \frac{z-4}{5}$ c) $\frac{x-3}{2} = \frac{-y}{3} = \frac{z+4}{5}$ d) $\frac{x+3}{2} = \frac{y}{3} = \frac{z-4}{5}$

226. If the plane $3x + y + 2z + 6 = 0$ is parallel to the line $\frac{3x-1}{2b} = 3 - y = \frac{z-1}{a}$, then the value of $3a + 3b$ is

a) $\frac{1}{2}$ b) $\frac{3}{2}$ c) 3 d) 4

227. The equation of the plane which meets the axes in A, B, C such that the triangle ABC is $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ is given by

a) $x + y + z = 1$ b) $x + y + z = 2$ c) $\frac{x}{3} + \frac{y}{3} + \frac{z}{3} = 3$ d) $x + y + z = \frac{1}{3}$

228. The equation to the plane through the points $(2, 3, 1)$ and $(4, -5, 3)$ parallel to x-axis is

a) $x + y + 4z = 7$ b) $x + 4z = 7$ c) $y - 4z = 7$ d) $y + 4z = -7$

229. The equation of the plane passing through the intersection of the planes $x + 2y + 3z + 4 = 0$ and $4x + 3y + 2z + 1 = 0$ and the origin, is

a) $3x + 2y + z + 1 = 0$ b) $3x + 2y + z = 0$ c) $2x + 3y + z = 0$ d) $x + y + z = 0$

230. The vector equation of a plane which contains the line $\vec{r} = 2\hat{\imath} + \lambda(\hat{\jmath} - \hat{k})$ and perpendicular to the plane $\vec{r} \cdot (\hat{\imath} + \hat{k}) = 3$, is

a) $\vec{r} \cdot (\hat{\imath} - \hat{\jmath} - \hat{k}) = 2$ b) $\vec{r} \cdot (\hat{\imath} + \hat{\jmath} - \hat{k}) = 2$ c) $\vec{r} \cdot (\hat{\imath} + \hat{\jmath} + \hat{k}) = 2$ d) None of these

231. The ratio in which the line joining (2,4,5), $(3,5,-4)$ is divided by the yz-plane is

a) $2:3$ b) $3:2$ c) $-2:3$ d) $4:-3$

232. Distance between two parallel planes $4x + 2y + 4z + 5 = 0$ and $2x + y + 2z - 8 = 0$ is

a) $\frac{7}{2}$ b) $\frac{2}{7}$ c) $-\frac{7}{2}$ d) $-\frac{2}{7}$

233. The shortest distance between the lines $\vec{r} = (5\hat{\imath} + 7\hat{\jmath} + 3\hat{k}) + \lambda(5\hat{\imath} - 16\hat{\jmath} + 7\hat{k})$ and, $\vec{r} = 9\hat{\imath} + 13\hat{\jmath} + 15\hat{k} + \mu(3\hat{\imath} + 8\hat{\jmath} - 5\hat{k})$, is

a) 10 units b) 12 units c) 14 units d) None of these

234. A plane which passes through the point (3, 2, 0) and the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$, is

a) $x - y + z = 1$ b) $x + y + z = 5$ c) $x + 2y - z = 0$ d) $2x - y + z = 5$

235. The equation of the plane through the point (1, 2,3) and parallel to the plane $x + 2y + 5z = 0$ is

a) $(x-1)+2(y-2)+5(z-3)=0$ b) $x+2y+5z=14$
c) $x+2y+5z=6$ d) None of the above

236. Radius of the circle $\overrightarrow{\mathbf{r}^2}+\vec{\mathbf{r}}\cdot(2\hat{\mathbf{i}}-2\hat{\mathbf{j}}-4\hat{\mathbf{k}})-19=0$ and $\vec{\mathbf{r}}\cdot(\hat{\mathbf{i}}-2\hat{\mathbf{j}}+2\hat{\mathbf{k}})+8=0$
a) 5 b) 4 c) 3 d) 2

237. If P be the point (2, 6, 3), then the equation of the plane through P at right angle to OP, O being the origin, is
a) $2x+6y+3z=7$ b) $2x-6y+3z=7$ c) $2x+6y-3z=49$ d) $2x+6y+3z=49$

238. If the coordinate of the verities of a triangle ABC be $A(-1,3,2)$, $B(2,3,5)$and $C(3,5,-2)$, then $\angle A$ is equal to
a) 45° b) 60° c) 90° d) 30°

239. The position vector of the point in which the line joining the points $\hat{\imath}-2\hat{\jmath}+\hat{k}$ and $3\hat{k}-2\hat{\jmath}$ cuts the plane through the origin and the points $4\hat{\jmath}$ and $2\hat{\jmath}+\hat{k}$, is
a) $6\hat{\imath}-10\hat{\jmath}+3\hat{k}$ b) $\frac{1}{5}(6\hat{\imath}-10\hat{\jmath}+3\hat{k})$ c) $-6\hat{\imath}+10\hat{\jmath}-3\hat{k}$ d) None of these

240. The plane of intersection of spheres $x^2+y^2+z^2+2x+2y+2z=2$ and $2x^2+2y^2+2z^2+4x+2y+4z=0$ is
a) Parallel to xz-plane b) Parallel to y-axis c) $y=0$ d) None of these

241. The direction cosines of a line equally inclined to three mutually perpendicular lines having direction cosines as $l_1, m_1, n_1; l_2, m_2, n_2; l_3, m_3, n_3$ are
a) $l_1+l_2+l_3, m_1+m_2+m_3, n_1+n_2+n_3$
b) $\frac{l_1+l_2+l_3}{\sqrt{3}}, \frac{m_1+m_2+m_3}{\sqrt{3}}, \frac{n_1+n_2+n_3}{\sqrt{3}}$
c) $\frac{l_1+l_2+l_3}{3}, \frac{m_1+m_2+m_3}{3}, \frac{n_1+n_2+n_3}{3}$
d) None of these

242. A line makes angles of 45° and 60° with the x-axis and the z-axis respectively. The angle made by it with y-axis is
a) 30° or 150° b) 60° or 120° c) 45° or 135° d) 90°

243. If the direction cosines of two lines are such that $l+m+n=0, l^2+m^2+n^2=0$, then the angle between them is
a) π b) $\pi/3$ c) $\pi/4$ d) $\pi/6$

244. The value of λ for which the lines $\frac{x-1}{1}=\frac{y-2}{\lambda}=\frac{z+1}{-1}$ and $\frac{x+1}{-\lambda}=\frac{y+1}{2}=\frac{z-2}{1}$ are perpendicular to each other is
a) 0 b) 1 c) -1 d) None of these

245. In a three dimensional xyz-space, the equation $x^2-5x+6=0$ represents
a) Points b) Plane c) Curves d) Pair of straight lines

246. Angle between the line $\frac{x+1}{1}=\frac{y}{2}=\frac{y-1}{1}$ and a normal to the plane $x-y+z=0$ is
a) 0° b) 30° c) 45° d) 90°

247. The angle between the line $\frac{3x-1}{3}=\frac{y+3}{-1}=\frac{5-2z}{4}$ and the plane $3x-3y-6z=10$ is equal to
a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

248. The foot of the perpendicular drawn from a point with position vector $\hat{\imath}+4\hat{k}$ on the line joining the points having position vectors as $-11\hat{\jmath}+3\hat{k}$ and $2\hat{\imath}-3\hat{\jmath}+\hat{k}$ has the position vector
a) $4\hat{\imath}+5\hat{\jmath}+5\hat{k}$ b) $4\hat{\imath}+5\hat{\jmath}-5\hat{k}$ c) $5\hat{\imath}+4\hat{\jmath}-5\hat{k}$ d) $4\hat{\imath}-5\hat{\jmath}+5\hat{k}$

249. What are the DR's of vector parallel to $(2,-1,1)$ and $(3,4,-1)$?
a) $(1,5-2)$ b) $(-2,-5,2)$ c) $(-1,5,2)$ d) $(-1,-5,-2)$

250. The point of intersection of the line $\frac{x-1}{3}=\frac{y+2}{4}=\frac{z-3}{-2}$ and plane $2x-y+3z-1=0$ is
a) $(10,-10,3)$ b) $(10,10,-3)$ c) $(-10,10,3)$ d) None of these

251. The equation of the plane containing the line

$\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ and the point $(0,7,-7)$is

a) $x+y+z=1$ b) $x+y+z=2$ c) $x+y+z=0$ d) None of these

252. Equation of the plane passing through the point (1, 1, 1) and perpendicular to each of the planes $x+2y+3z=7$ and $2x-3y+4z=0$, is

a) $17x-2y+7z=12$ b) $17x+2y-7z=12$ c) $17x+2y+7z=12$ d) $17x-2y-7z=12$

253. The equation of the plane passing through (1, 1, 1) and $(1,-1,-1)$ and perpendicular to $2x-y+z+5=0$ is

a) $2x+5y+z-8=0$ b) $x+y-z-1=0$ c) $2x+5y+z+4=0$ d) $x-y+z-1=0$

254. If $\vec{r}$ is a vector of magnitude 21 and has direction ratios proportional to $2,-3,6$, then $\vec{r}$ is equal to

a) $6\hat{\imath}-9\hat{\jmath}+18\hat{k}$ b) $6\hat{\imath}+9\hat{\jmath}+18\hat{k}$ c) $6\hat{\imath}-9\hat{\jmath}-18\hat{k}$ d) $6\hat{\imath}+9\hat{\jmath}-18\hat{k}$

255. The line perpendicular to the plane $2x-y+5z=4$ passing through the point $(-1,0,1)$ is

a) $\frac{x+1}{2}=y=\frac{z-1}{-5}$ b) $\frac{x+1}{-2}=y=\frac{z-1}{-5}$ c) $\frac{x+1}{2}=-y=\frac{z-1}{5}$ d) $\frac{x+1}{2}=y=\frac{z-1}{5}$

256. The equation of the sphere whose centre is $(6,-1,2)$ and which touches the plane $2x-y+2z-2=0$, is

a) $x^2+y^2+z^2-12x+2y-4z-16=0$ b) $x^2+y^2+z^2-12x+2y-4z=0$
c) $x^2+y^2+z^2-12x+2y-4z+16=0$ d) $x^2+y^2+z^2-12x+2y-4z+6=0$

257. The equation of the plane passing through three non-collinear points with position vectors $\vec{\mathbf{a}},\vec{\mathbf{b}},\vec{\mathbf{c}}$ is

a) $\vec{\mathbf{r}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}}+\vec{\mathbf{c}}\times\vec{\mathbf{a}}+\vec{\mathbf{a}}\times\vec{\mathbf{b}})=0$ b) $\vec{\mathbf{r}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}}+\vec{\mathbf{c}}\times\vec{\mathbf{a}}+\vec{\mathbf{a}}\times\vec{\mathbf{b}})=[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$
c) $\vec{\mathbf{r}}\cdot\left(\vec{\mathbf{a}}\times(\vec{\mathbf{b}}+\vec{\mathbf{c}})\right)=[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$ d) $\vec{\mathbf{r}}\cdot(\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}})=0$

258. If the planes $x=cy+bz$, $y=az+cx, z=bx+ay$ pass through a line, then $a^2+b^2+c^2+2abc$ is

a) 0 b) 1 c) 2 d) 3

259. The equation of the plane, which makes with coordinate axes, a triangle with its centroid $(\alpha,\beta,\gamma)$is

a) $\alpha x+\beta y+\gamma z=3$ b) $\alpha x+\beta y+\gamma z=1$ c) $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=3$ d) $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1$

260. The equation of the plane perpendicular to the line $\frac{x-1}{1}=\frac{y-2}{-1}=\frac{z+1}{2}$ and passing through the point (2, 3, 1), is

a) $\vec{r}\cdot(\hat{\imath}+\hat{\jmath}+2k)=1$ b) $\vec{r}\cdot(\hat{\imath}-\hat{\jmath}+2k)=1$ c) $\vec{r}\cdot(\hat{\imath}-\hat{\jmath}+2k)=7$ d) $\vec{r}\cdot(\hat{\imath}+\hat{\jmath}-2k)=10$

261. Equation of a line passing through $(1,-2,3)$ and parallel to the plane $2x+3y+z+5=0$ is

a) $\frac{x-1}{-1}=\frac{y+2}{1}=\frac{z-3}{-1}$
b) $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{1}$
c) $\frac{x+1}{-1}=\frac{y-2}{1}=\frac{z-3}{-1}$
d) None of these

262. The shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ is

a) $\sqrt{30}$ b) $2\sqrt{30}$ c) $5\sqrt{30}$ d) $3\sqrt{30}$

263. The point in the xy −plane which is equidistant from the point (2, 0, 3) and (0, 3, 2) and (0, 0, 1), is

a) (1, 2, 3) b) $(-3,2,0)$ c) $(3,-2,0)$ d) (3, 2, 0)

264. A sphere of constant radius $2k$ passes through the origin and meets the axes in A,B,C. The locus of the centroid of the tetrahedron ABC is

a) $x^2+y^2+z^2=4k^2$ b) $x^2+y^2+z^2=k^2$
c) $2(x^2+y^2+z^2)=k^2$ d) None of these

265. The direction cosines of the line which is perpendicular to the lines whose direction cosines are proportional to
$(1,-1,2)$and $(2,1,-1)$ are

a) $\frac{-2}{\sqrt{35}},\frac{5}{\sqrt{35}},\frac{3}{\sqrt{35}}$ b) $-\frac{1}{\sqrt{35}},\frac{5}{\sqrt{35}},\frac{3}{\sqrt{35}}$ c) $-\frac{1}{\sqrt{35}},\frac{5}{\sqrt{35}},\frac{3}{\sqrt{35}}$ d) None of these

266. The shortest distance between the lines $1+x=2y=-12z$ and $x=y+2=6z-6$ is

a) 1 b) 2 c) 3 d) 4

267. The position vectors of points A and B are $\hat{\imath} - \hat{\jmath} + 3\,\hat{k}$ and $3\,\hat{\imath} + 3\,\hat{\jmath} + 3\,\hat{k}$ respectively. The equation of a plane is $\vec{r} \cdot (5\,\hat{\imath} + 2\,\hat{\jmath} - 7\,\hat{k}) + 9 = 0$. The points A and B

a) Lie on the plane

b) Are on the same side of the plane

c) Are on the opposite side of the plane

d) None of these

268. The distance of the point $(-1, -5, -10)$ from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 5$ is

a) $\frac{14}{5}$ b) 11 c) 13 d) 15

269. The line segment adjoining the points A, B makes projection 1, 4, 3 on x, y, z-axes respectively. Then, the direction cosines of AB are

a) 1, 4, 3 b) $1/\sqrt{26}, 4/\sqrt{26}, 3/\sqrt{26}$

c) $-1/\sqrt{26}, 4/\sqrt{26}, 3/\sqrt{26}$ d) $1/\sqrt{26}, -4/\sqrt{26}, 3/\sqrt{26}$

270. If the direction ratios of two lines are given by $3lm + 4ln + mn = 0$ and $l + 2m + 3n = 0$, then the angle between the lines is

a) $\pi/2$ b) $\pi/3$ c) $\pi/4$ d) $\pi/6$

271. If $(2, 3, 5)$ is one end of a diameter of the sphere $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$, then the coordinates of the other end of the diameter are

a) $(4, 9, -3)$ b) $(4, -3, 3)$ c) $(4, 3, 5)$ d) $(4, 3, -3)$

272. If Q is the image of the point $P(2, 3, 4)$ under the reflection in the plane $x - 2y + 5z = 6$, then the equation of the line PQ is

a) $\frac{x-2}{-1} = \frac{y-3}{2} = \frac{z-4}{5}$ b) $\frac{x-2}{1} = \frac{y-3}{-2} = \frac{z-4}{5}$ c) $\frac{x-2}{-1} = \frac{y-3}{-2} = \frac{z-4}{5}$ d) $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{5}$

273. There is point $P(a, a, a)$ on the line passing through the origin and equally inclined with axes. The equation of plane perpendicular to OP and passing through P cuts the intercepts on axes. The sum of whose reciprocals is

a) a b) $\frac{3}{2a}$ c) $\frac{3a}{2}$ d) $\frac{1}{a}$

274. If P is a point in space such that $OP = 12$ and $\vec{O}P$ is inclined at angles of 45° and 60° with OX and OY respectively, then the position vector of P is

a) $6\hat{\imath} + 6\hat{\jmath} \pm 6\sqrt{2}\,\hat{k}$ b) $6\hat{\imath} + 6\sqrt{2}\hat{\jmath} \pm 6\,\hat{k}$ c) $6\sqrt{2}\hat{\jmath} + 6\hat{\jmath} \pm 6\,\hat{k}$ d) None of these

275. Equation of plane containing the lines

$\vec{\mathbf{r}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}} + \lambda(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$

And $\vec{\mathbf{r}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}} + \mu(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ is

a) $\vec{\mathbf{r}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}} + \lambda(\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ b) $\vec{\mathbf{r}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}} + \lambda(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) + \mu(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}})$

c) $\vec{\mathbf{r}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}} + \lambda(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) + \mu(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}})$ d) $\vec{\mathbf{r}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}} + \lambda(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) + \mu(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

276. Cosine of the angle between two diagonals of cube is equal to

a) $\frac{2}{\sqrt{6}}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) None of these

277. The equation of the plane which bisects the line joining $(2, 3, 4)$ and $(6, 7, 8)$ is

a) $x - y - z - 15 = 0$ b) $x - y + z - 15 = 0$ c) $x + y + z - 15 = 0$ d) $x + y + z + 15 = 0$

278. The distance of the point $(3, 8, 2)$from the line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-2}{3}$ measured parallel to the plane $3x + 2y - 2z = 0$ is

a) 2 b) 3 c) 6 d) 7

279. The direction cosines l, m, n of two lines are connected by the relations $l + m + n = 0, lm = 0$, then the angle between them is

a) π/3 b) π/4 c) π/2 d) 0

280. If a line lies in the octant $OXYZ$ and it makes equal angles with the axes, then

a) $l = m = n = \frac{1}{\sqrt{3}}$
b) $l = m = n \pm \frac{1}{\sqrt{3}}$
c) $l = m = n = -\frac{1}{\sqrt{3}}$
d) $l = m = n = \pm\frac{1}{\sqrt{3}}$

281. The line joining the points $(1, 1, 2)$ and $(3, -2, 1)$ meets the plane $3x + 2y + z = 6$ at the point

a) $(1, 1, 2)$
b) $(3, -2, 1)$
c) $(2, -3, 1)$
d) $(3, 2, 1)$

282. The points $A(5, -1, 1), B(7, -4, 7), C(1, -6, 10)$ and $D(-1, -3, 4)$ are vertices of a

a) Square
b) Rhombus
c) Rectangle
d) None of these

283. If $P(x, y, z)$ is a point on the line segment joining $Q(2,24)$ and $R(3,5,6)$ such that the projections of OP on the axes are $\frac{13}{5}, \frac{19}{5}$ and $\frac{26}{5}$ respectively, then P divides QR in the ratio

a) 1:2
b) 3:2
c) 2:3
d) 1:3

284. If direction cosines of two lines are proportional to $(2,3-6)$and $(3, -4, 5)$ then the acute angle between then is

a) $\cos^{-1}\left(\frac{49}{36}\right)$
b) $\cos^{-1}\left(\frac{18\sqrt{2}}{35}\right)$
c) $96°$
d) $\cos^{-1}\left(\frac{18}{35}\right)$

285. The cartesian equation of the plane $\vec{r} = (s - 2t)\hat{\imath} + (3 - t)\hat{\jmath} + (2s + t)\hat{k}$, is

a) $2x - 5y - z - 15 = 0$
b) $2x - 5y + z - 15 = 0$
c) $2x - 5y - z + 15 = 0$
d) $2x + 5y - z + 15 = 0$

286. The plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ at the point

a) $(1, -4, -2)$
b) $(-1, 4, -2)$
c) $(-1, -4, 2)$
d) $(1, 4, -2)$

287. If θ is the angle between the planes $2x - y + z - 1 = 10$ and $x - 2y + z + 2 = 0$ Then $\cos\theta$ is equal to

a) 2/3
b) 3/4
c) 4/5
d) 5/6

288. Let $(3, 4, -1)$ and $(-1, 2, 3)$ are the end points of a diameter of sphere. Then, the radius of the sphere is equal to

a) 1
b) 2
c) 3
d) 9

289. Let $A(4, 7, 8), B(2, 3, 4)$ and $C(2, 5, 7)$ be the position vectors of the vertices of a $\Delta\, ABC$. The length of the internal bisector of the angle of A is

a) $\frac{3}{2}\sqrt{34}$
b) $\frac{2}{3}\sqrt{34}$
c) $\frac{1}{2}\sqrt{34}$
d) $\frac{1}{3}\sqrt{34}$

290. The distance of the plane $6x - 3y + 2z - 14 = 0$ from the origin is

a) 2
b) 1
c) 14
d) 8

291. In ΔABC and mid points of the sides AB, BC and CA are respectively $(1,0,0)$, $(0, m, 0)$ and $(0,0, n)$ Then,

$\frac{AB^2 + BC^2 + CA^2}{(l^2 + m^2 + n^2)}$ is equal to

a) 2
b) 4
c) 8
d) 16

292. The angle between the straight line $\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4}$ and $\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-3}{-3}$ is

a) $45°$
b) $30°$
c) $60°$
d) $90°$

293. A plane passes through $(1, -2, 1)$ and is perpendicular to two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$, then the distance of the plane from the point $(1, 2, 2)$is

a) 0
b) 1
c) $\sqrt{2}$
d) $2\sqrt{2}$

294. The line through $\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$ and perpendicular to the lines $\vec{r} = (\hat{\imath} + 2\hat{\jmath} - \hat{k}) + \lambda(2\hat{\imath} + \hat{\jmath} + \hat{k})$ and, $\vec{r} = (2\hat{\imath} + 6\hat{\jmath} + \hat{k}) + \mu(\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$ is

a) $\vec{r} = (\hat{\imath} + 2\hat{\jmath} - \hat{k}) + \lambda(-\hat{\imath} + 5\hat{\jmath} - 3\hat{k})$
b) $\vec{r} = \hat{\imath} + 3\hat{\jmath} + 2\hat{k} + \lambda(\hat{\imath} - 5\hat{\jmath} + 3\hat{k})$

c) $\vec{r} = \hat{\imath} + 3\hat{\jmath} + 2\hat{k} + \lambda(\hat{\imath} + 5\hat{\jmath} + 3\hat{k})$
d) $\vec{r} = \hat{\imath} + 3\hat{\jmath} + 2\hat{k} + \lambda(-\hat{\imath} - 5\hat{\jmath} - 3\hat{k})$

295. Let O be the origin and P be the point at a distance 3 units from origin. If direction ratios of OP are $(1, -2, -2)$, then coordinates of P is given by
a) $(1, -2, -2)$ b) $(3, -6, -6)$ c) $(1/3, -2/3, -2/3)$ d) $(1/9, -2/9, -2/9)$

296. The direction cosines l, m, n of two lines are connected by the relation $l + m + n = 0$, $lm = 0$, then the angles between them is
a) $\pi/3$ b) $\pi/4$ c) $\pi/2$ d) 0

297. Equation of plane containing the line $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and parallel to the line $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is
a) $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = 0$
b) $\begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ a_2 & b_2 & c_2 \\ x_1 & y_1 & z_1 \end{vmatrix} = 0$
c) $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$
d) None of the above

298. The plane $2x - (1 + \lambda)y + 3z = 0$ passes through the intersection of the planes
a) $2x - y = 0$ and $y + 3z = 0$ b) $2x - y = 0$ and $y - 3z = 0$
c) $2x + 3z = 0$ and $y = 0$ d) None of the above

299. The radius of the sphere $x^2 + y^2 + z^2 = x + 2y + 3z$ is
a) $\frac{\sqrt{14}}{2}$ b) $\sqrt{7}$ c) $\frac{7}{2}$ d) $\frac{\sqrt{7}}{2}$

300. If a plane meets the coordinate axes at A, B and C in such a way that the centroid of ΔABC is at the point (1, 2, 3) the equation of the plane is
a) $\frac{x}{1} + \frac{y}{2} + \frac{z}{3} = 1$ b) $\frac{x}{3} + \frac{y}{6} + \frac{z}{9} = 1$ c) $\frac{x}{1} + \frac{y}{2} + \frac{z}{3} = \frac{1}{3}$ d) None of these

301. The triangle formed by the points $(0, 7, 10)$, $(-1, 6, 6)$, $(-4, 9, 6)$ is
a) Equilateral b) Isosceles c) Right angled d) Right angled isosceles

302. The vector equation of plane passing through three non-collinear points having position vectors $\vec{a}, \vec{b}, \vec{c}$ is
a) $\vec{r} \cdot (\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}) = 0$
b) $\vec{r} \times (\vec{a} \times \vec{b} + \vec{b} \times \vec{c}) = [\vec{a}\ \vec{b}\ \vec{c}]$
c) $\vec{r} \cdot (\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}) + [\vec{a}\ \vec{b}\ \vec{c}] = 0$
d) None of these

303. Let L be the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$. If L makes an angle α with the positive x-axis, then $\cos\alpha$ equals
a) $\frac{1}{\sqrt{3}}$ b) $\frac{1}{2}$ c) 1 d) $\frac{1}{\sqrt{2}}$

304. The area of triangle whose vertices are $(1, 2, 3)$, $(2, 5, -1)$ and $(-1, 1, 2)$ is
a) 150 sq unit b) 145 sq unit c) $\frac{\sqrt{155}}{2}$ sq unit d) $\frac{155}{2}$ sq unit

305. The angles between two planes $x + 2y + 2z = 3$ and $-5x + 3y + 4z = 9$ is
a) $\cos^{-1}\frac{9\sqrt{2}}{20}$ b) $\cos^{-1}\frac{3\sqrt{2}}{5}$ c) $\cos^{-1}\frac{3\sqrt{2}}{10}$ d) $\cos^{-1}\frac{19\sqrt{2}}{30}$

306. The projection of the line joining the points $(3, 4, 5)$ and $(4, 6, 3)$ on the line joining the points $(-1, 2\ 4)$ and $(1, 0, 5)$ is
a) $4/3$ b) $2/3$ c) $-4/3$ d) $1/2$

307. If the straight lines $x = 1 + s, y = -3 - \lambda s, z = 1 + \lambda s$ and
$x = \frac{t}{2}, y = 1 + t, z = 2 - t$ with parameters s and t respectively, are coplanar, then λ
Equals
a) -2 b) -1 c) $-\frac{1}{2}$ d) 0

308. The angle between the planes $x + 2y + 2z = 3$ and$-5x + 3y + 4z = 9$ is

a) $\cos^{-1}\frac{9\sqrt{2}}{20}$ b) $\cos^{-1}\frac{3\sqrt{2}}{5}$ c) $\cos^{-1}\frac{3\sqrt{2}}{10}$ d) $\cos^{-1}\frac{19\sqrt{2}}{30}$

309. The equation of the plane through the point (2, 3, 1) and (4, −5, 3) and parallel to x-axis is

a) $y - 4z = 7$ b) $y + 4z = 7$ c) $y + 4z = -7$ d) $x + 4z = 7$

310. If the direction ratio of two lines are given by $l + m + n = 0, mn - 2ln + lm = 0$, then the angle between the line is

a) $\frac{\pi}{4}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{2}$ d) 0

311. If a line in the space makes angle α, β and γ with the coordinate axes, then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma + \sin^2\alpha + \sin^2\beta + \sin^2\gamma$ equals

a) −1 b) 0 c) 1 d) 2

312. The equation of the plane through the point (2, −1, −3) and parallel to the lines $\frac{x-1}{3} = \frac{y+2}{2} = \frac{z}{-4}$ and $\frac{x}{2} = \frac{y-1}{-3} = \frac{z-2}{2}$ is

a) $8x + 14y + 13z + 37 = 0$ b) $8x - 14y + 13z + 37 = 0$

c) $8x + 14y - 13z + 37 = 0$ d) $8x + 14y + 13z - 37 = 0$

313. If A, B, C, D are the points (2,3,−1), (3,5,−3), (1,2,3), (3,5,7) respectively, then the angle between AB and $CD$is

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$

314. A line makes the same angle θ with each of the x and z axes. If the angle β, which it makes with y-axis, is such that $\sin^2\theta$, then $\cos^2\theta$ equals

a) 2/3 b) 1/5 c) 3/5 d) 2/5

315. The equation of the plane passing through the point (1, 1, 1) and containing the line of intersection of the planes $x + y + z = 6$ and $2x + 3y + 4z = 12$ is

a) $x + y + z = 3$ b) $x + 2y + 3z = 6$ c) $2x + 3y + 4z = 9$ d) $3x + 4y + 5z = 18$

316. The point in the xy-plane which is equidistant from the points (2,0,3), (0,3,2) and (0,0,1) is

a) (1,2,3) b) (−3,2,0) c) (3, −2,0) d) (3,2,0)

317. Let $P(-7,1,-5)$ be a point on a plane and let Obe the origin. If OP is normal to the plane, then the equation of the plane is

a) $7x - y + 5z + 75 = 0$ b) $7x + y - 5z + 73 = 0$ c) $7x + y + 5z + 73 = 0$ d) $7x - y - 5z + 75 = 0$

318. The equation of the plane through the points (1, 2, 3), (−1, 4, 2) and (3, 1, 1) is

a) $5x + y + 12z - 23 = 0$ b) $5x + 6y + 2z - 23 = 0$

c) $x + 6y + 2z - 13 = 0$ d) $x - y + z - 13 = 0$

319. The line drown from (4, −1,2) the point (−3,2,3) meets a plane at right angle at the point (−10,5,4), then the equation of plane is

a) $7x + 3y + z + 89 = 0$ b) $7x - 3y - z + 89 = 0$

c) $7x - 3y + z + 89 = 0$ d) None of these

320. The equation of the plane perpendicular to the line $\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z+1}{2}$ and passing through the point (2, 3, 1), is

a) $\vec{r} \cdot (\hat{\imath} + \hat{\jmath} + 2\hat{k}) = 1$ b) $\vec{r} \cdot (\hat{\imath} - \hat{\jmath} + 2\hat{k}) = 1$ c) $\vec{r} \cdot (\hat{\imath} - \hat{\jmath} + 2\hat{k}) = 7$ d) None of these

321. The acute angle between the line joining the points (2,1,−3), (−3,1,7) and a line parallel to $\frac{x-1}{3} = \frac{y}{4} = \frac{z+3}{5}$ through the point (-1,0,4)is

a) $\cos^{-1}\left(\frac{1}{\sqrt{10}}\right)$ b) $\cos^{-1}\left(\frac{1}{5\sqrt{10}}\right)$ c) $\cos^{-1}\left(\frac{7}{5\sqrt{10}}\right)$ d) $\cos^{-1}\left(\frac{3}{5\sqrt{10}}\right)$

322. The shortest distance from the plane $12x + 4y + 3z = 327$ to the sphere $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$ is

a) 26 b) $11\frac{4}{13}$ c) 13 d) 39

323. The projection of a directed line segment on the coordinate axes are 12, 4, 3. The DC's of the line are

a) $\frac{12}{13}, -\frac{4}{13}, \frac{3}{13}$ b) $-\frac{12}{13}, -\frac{4}{13}, \frac{3}{13}$ c) $\frac{12}{13}, \frac{4}{13}, \frac{3}{13}$ d) None of these

324. The radius of the circle $x^2 + y^2 + z^2 - 2y - 4z - 11 = 0, x + 2y + 2z - 15 = 0$ is

a) $\sqrt{3}$ b) $\sqrt{5}$ c) $\sqrt{7}$ d) 3

325. The coordinates of the foot of perpendicular drawn from point $P(1, 0, 3)$ to the join of points $A(4, 7, 1)$ and $B(3, 5, 3)$ is

a) $(5, 7, 1)$ b) $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$ c) $\left(\frac{2}{3}, \frac{5}{3}, \frac{7}{3}\right)$ d) $\left(\frac{5}{3}, \frac{2}{3}, \frac{7}{3}\right)$

326. The position vector of the point where the line $\vec{\mathbf{r}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}} + t(\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}})$ meets the plane $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 5$ is

a) $5\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ b) $5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ c) $2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ d) $5\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

327. If O is the origin and A is the point (a, b, c) then the equation of the plane through A and at right angles to OA is

a) $a(x - a) - b(y - b) - c(z - c) = 0$ b) $a(x + a) + b(y + b) + c(z + c) = 0$

c) $a(x - a) + b(y - b) + c(z - c) = 0$ d) None of these above

328. Equation of a line passing through $(-1, 2, -3)$ and perpendicular to the plane $2x + 3y + z + 5 = 0$ is

a) $\frac{x-1}{-1} = \frac{y+2}{1} = \frac{z-3}{-1}$

b) $\frac{x+1}{-1} = \frac{y-2}{1} = \frac{z+3}{1}$

c) $\frac{x+1}{2} = \frac{y-2}{3} = \frac{z+3}{1}$

d) None of these

329. The foot of the perpendicular from $(2, 4, -1)$ to the line $x + 5 = \frac{1}{4}(y + 3) = -\frac{1}{9}(z - 6)$ is

a) $(-4, 1, -3)$ b) $(4, -1, -3)$ c) $(-4, -1, 3)$ d) $(-4, -1, -3)$

330. The angle between the lines whose direction cosines are $\left(\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{\sqrt{3}}{2}\right)$ and $\left(\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{-\sqrt{3}}{2}\right)$ is

a) π b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{4}$

331. The value of k so that the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and, $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ may be perpendicular is given by

a) $-7/10$ b) $-10/7$ c) -10 d) $10/7$

332. The plane $x + 2y - z = 4$ cut the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle of radius

a) $\sqrt{2}$ b) 2 c) 1 d) 3

333. The angle between $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and the plane $3x + 2y - 3z = 4$, is

a) $45°$ b) $0°$ c) $\cos^{-1}\left(\frac{24}{\sqrt{29}\sqrt{22}}\right)$ d) $90°$

334. The lines

$\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$

And

$\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$

are coplanar, if

a) $k = 0$ or -1 b) $k = 1$ or -1 c) $k = 0$ or -3 d) $k = 3$ or -3

335. Equation of the plane passing through line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{4}$ and perpendicular to the plane $x + 2y + z =$

12 is given by $ax + by + cz + 4 = 0$, then

a) $a = -8, b = 2, c = -5$
b) $a = -9, b = -2, c = -5$
c) $a = 9, b = -2, c = -5$
d) None of the above

336. The intercepts of the plane $2x - 3y + 4z = 12$ on the coordinate axes are given by

a) $3, -2, 15$
b) $6, -4, 3$
c) $6, -4, -3$
d) $2, -3, 4$

337. The equation of the straight line passing through the points $(4, -5, -2)$and $(-1,5,3)$ is

a) $\frac{x-4}{1} = \frac{y+5}{-2} = \frac{z+2}{-1}$
b) $\frac{x+1}{1} = \frac{y-5}{2} = \frac{z-3}{-1}$
c) $\frac{x}{-1} = \frac{y}{5} = \frac{z}{3}$
d) $\frac{x}{4} = \frac{y}{-5} = \frac{z}{-2}$

338. The length of the perpendicular from the origin to the plane $3x + 4y + 12z = 52$ is

a) 3
b) -4
c) 5
d) None of these

339. If from a point $P(a, b, c)$ perpendiculars PA, PB are drown to yz and zx plane, then the equation of the plane OAB is

a) $bcx + cay + abz = 0$
b) $bcx + cay - abz = 0$
c) $bcx - cay + abz = 0$
d) $-bcx + cay + abz = 0$

340. The smallest radius of the sphere passing through (1, 0, 0), (0, 1, 0) and (0, 0, 1) is

a) $\sqrt{\frac{3}{5}}$
b) $\sqrt{\frac{3}{8}}$
c) $\sqrt{\frac{2}{3}}$
d) $\sqrt{\frac{5}{12}}$

341. The cartesian equation of the plane $\vec{r} = (1 + \lambda - \mu)\hat{\imath} + (2 - \lambda)\hat{\imath} + (3 - 2\lambda + 2\mu)\hat{k}$, is

a) $2x + y = 5$
b) $2x - y = 5$
c) $2x + z = 5$
d) $2x - z = 5$

342. Foot of the perpendicular from $B(-2, 1, 4)$ to the plane is $(3, 1, 2)$. Then, the equation of the plane is

a) $4x - 2y = 11$
b) $5x - 2y = 10$
c) $5x - 2z = 11$
d) $5x + 2z = 11$

343. A straight line $\vec{r} = \vec{a} + \lambda\vec{b}$ meets the plane $\vec{r} \cdot \vec{n} = 0$ in P. The position vector of P is

a) $\vec{a} + \frac{\vec{a} \cdot \vec{n}}{\vec{b} \cdot \vec{n}}\vec{b}$
b) $\vec{a} - \frac{\vec{a} \cdot \vec{n}}{\vec{b} \cdot \vec{n}}\vec{b}$
c) $\vec{a} - \frac{\vec{a} \cdot \vec{n}}{\vec{b} \cdot \vec{n}}\vec{b}$
d) None of these

344. A equation of the plane passing through the points $(3, 2, -1)$, $(3, 4, 2)$ and $(7, 0, 6)$ is $5x + 3y - 2z = \lambda$, where λ is

a) 23
b) 21
c) 19
d) 27

345. The lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$ are coplanar if

a) $\overrightarrow{a_1} \times \overrightarrow{a_2} = \vec{0}$
b) $\overrightarrow{b_1} \times \overrightarrow{b_2} = 0$
c) $(\overrightarrow{a_2} - \overrightarrow{a_1}) \times (\overrightarrow{b_1} \times \overrightarrow{b_2}) = 0$
d) $[\overrightarrow{a_1}\,\overrightarrow{b_1}\,\overrightarrow{b_2}] = [\overrightarrow{a_2}\,\overrightarrow{b_1}\,\overrightarrow{b_2}]$

346. The point of intersection of the lines
$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and
$\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ is

a) $\left(\frac{1}{2}, \frac{1}{2} - \frac{3}{2}\right)$
b) $\left(-\frac{1}{2}, -\frac{1}{2}, \frac{3}{2}\right)$
c) $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$
d) $\left(-\frac{1}{2}, \frac{1}{2}, \frac{3}{2}\right)$

347. If a line makes angles α, β, γ with the coordinate axe, then

a) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma - 1 = 0$
b) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma - 2 = 0$
c) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma + 1 = 0$
d) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma + 1 = 0$

348. The centre of sphere passes through four points (0, 0, 0), (0, 2, 0), (1, 0, 0) and (0,0,4) is

a) $\left(\frac{1}{2}, 1, 2\right)$
b) $\left(-\frac{1}{2}, 1, 2\right)$
c) $\left(\frac{1}{2}, 1, -2\right)$
d) $\left(1, \frac{1}{2}, 2\right)$

349. A variable plane moves so that sum of the reciprocals of its intercepts on the coordinate axes is 1/2 Then, the plane passes through

a) $\left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right)$ b) $(-1, 1, 1)$ c) $(2, 2, 2)$ d) $(0, 0, 0)$

350. The distance from the point $-\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$ to the straight line through the point $(2, 3, -4)$ and parallel to the vector $6\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$, is

a) 7 b) 10 c) 9 d) None of these

351. The equation of the plane passing through the points $(a, 0, 0), (0, b, 0)$and $(0, 0, c)$ is

a) $ax + by + cz = 0$ b) $ax + by + cz = 1$ c) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ d) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$

352. The distance between the planes $2x - 2y + z + 3 = 0$ and $4x - 4y + 2z + 5 = 0$ is

a) 3 b) 6 c) $\frac{1}{6}$ d) $\frac{1}{3}$

353. If $P = (0,1,2), Q = (4, -2,1), O = (0,0,0)$, then $\angle POQ$ is equal to

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

354. The point of intersection of the line $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{4}$ and the plane $2x + 4y - z + 1 = 0$ is

a) $\left(-\frac{10}{3}, \frac{3}{2}, -\frac{5}{3}\right)$ b) $\left(-\frac{10}{3}, -\frac{3}{2}, \frac{5}{3}\right)$ c) $\left(\frac{10}{3}, \frac{3}{2}, -\frac{5}{3}\right)$ d) $\left(\frac{10}{3}, -\frac{3}{2}, \frac{5}{3}\right)$

355. The point of intersection of the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

and

$$\frac{x-4}{5} = \frac{y-1}{2} z \text{ is}$$

a) $(0, 0, 0)$ b) $(1, 1, 1)$ c) $(-1, -1, -1)$ d) $(1, 2, 3)$

356. Which of the following is an equation of a sphere?

a) $x^2 + y^2 + z^2 - 2xy - 2yz - 6zx = 4$ b) $x^2 + y^2 + z^2 + 1 = 0$

c) $x^2 + y^2 + z^2 - 2x - 2y - 2z + 2 = 0$ d) $x^2 + y^2 + z^2 - 4x - 4y - 4z + 25 = 0$

357. The angle between the line $\frac{x}{1} = \frac{y}{0} = \frac{z}{-1}$ and $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$ is equal to

a) $\pi - \cos^{-1}\left(\frac{1}{5}\right)$ b) $\cos^{-1}\left(\frac{1}{3}\right)$ c) $\cos^{-1}\left(\frac{1}{2}\right)$ d) $\cos^{-1}\left(\frac{1}{4}\right)$

358. A point moves such that the sum of its distance from points $(4, 0, 0)$ and $(-4, 0, 0)$ is 10, then the locus of the point is

a) $9x^2 - 25y^2 + 25z^2 = 225$ b) $9x^2 + 25y^2 + 25z^2 = 225$

c) $9x^2 + 25y^2 - 25z^2 = 225$ d) $9x^2 + 25y^2 + 25z^2 = 225 = 0$

359. A plane makes intercepts 3 and 4 respectively on z-axis and x-axis. If plane is parallel to y-axis, then its equation is

a) $3x + 4z = 12$ b) $3z + 4x = 12$ c) $3y + 4z = 12$ d) $3z + 4y = 12$

360. If x coordinate of a point P of line joining the points $Q(2, 2,1)$ and $R(5, 2, -2)$ is 4, then the z coordinate of P is

a) -2 b) -1 c) 1 d) 2

361. The two lines $x = ay + b, z = cy + d$ and $x = a'\, y + b', z = c'\, y + d'$ are perpendicular to each other, if

a) $aa' + cc' = 1$ b) $\frac{a}{a'} + \frac{c}{c'} = -1$ c) $\frac{a}{a'} + \frac{c}{c'} = 1$ d) $aa' + cc' = -1$

362. The vector equation of the plane containing the lines $\vec{r} = (\hat{\imath} + \hat{\jmath}) + \lambda(\hat{\imath} + 2\hat{\jmath} - \hat{k})$ & $\vec{r} = (\hat{\imath} + \hat{\jmath}) + \mu(-\hat{\imath} + \hat{\jmath} - 2\hat{k})$, is

a) $\vec{r} \cdot (\hat{\imath} + \hat{\jmath} + \hat{k}) = 0$ b) $\vec{r} \cdot (\hat{\imath} - \hat{\jmath} - \hat{k}) = 0$ c) $\vec{r} \cdot (\hat{\imath} + \hat{\jmath} + \hat{k}) = 3$ d) None of these

363. The equation of the plane passing through $(2, 3, 4)$ and parallel to the plane $5x - 6y + 7z = 3$ is

a) $5x - 6y + 7z + 20 = 0$ b) $5x - 6y + 7z - 20 = 0$

c) $-5x + 6y - 7z + 3 = 0$ d) $5x - 6y + 7z + 3 = 0$

364. If the plane $2ax - 3ay + 4az + 6 = 0$ passes through the mid point of the line joining the centres of the spheres $x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and
$x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$, then a equals

a) -2 b) 2 c) -1 d) 1

365. The centre of sphere passes through four points $(0, 0, 0), (0,2,0), (1,0, 0)$and $(0, 0, 4)$ is

a) $\left(-\frac{1}{2}, 1,2\right)$ b) $\left(1, \frac{1}{2}, 2\right)$ c) $\left(\frac{1}{2}, 1, 2\right)$ d) $\left(\frac{1}{2}, 1, -2\right)$

366. If the planes $x + 2y + kz = 0$ and $2x + y - 2z = 0$, are at right angles, then the value of k is

a) 2 b) -2 c) $\frac{1}{2}$ d) $-\frac{1}{2}$

: ANSWER KEY :

1)	b	2)	b	3)	d	4)	c
5)	b	6)	b	7)	c	8)	b
9)	a	10)	a	11)	b	12)	c
13)	c	14)	a	15)	a	16)	d
17)	b	18)	b	19)	d	20)	c
21)	c	22)	a	23)	b	24)	d
25)	c	26)	c	27)	a	28)	c
29)	b	30)	a	31)	b	32)	a
33)	a	34)	b	35)	a	36)	d
37)	a	38)	b	39)	a	40)	b
41)	a	42)	a	43)	a	44)	a
45)	b	46)	a	47)	a	48)	d
49)	b	50)	b	51)	d	52)	d
53)	b	54)	c	55)	b	56)	c
57)	c	58)	b	59)	a	60)	c
61)	b	62)	b	63)	b	64)	d
65)	d	66)	c	67)	a	68)	b
69)	a	70)	c	71)	b	72)	c
73)	d	74)	a	75)	c	76)	b
77)	a	78)	a	79)	d	80)	a
81)	b	82)	a	83)	d	84)	b
85)	a	86)	d	87)	a	88)	d
89)	b	90)	d	91)	a	92)	a
93)	a	94)	b	95)	a	96)	a
97)	c	98)	b	99)	d	100)	a
101)	b	102)	d	103)	c	104)	a
105)	d	106)	c	107)	a	108)	c
109)	b	110)	c	111)	c	112)	b
113)	a	114)	b	115)	c	116)	d
117)	b	118)	c	119)	b	120)	c
121)	b	122)	a	123)	c	124)	b
125)	c	126)	c	127)	a	128)	d
129)	b	130)	c	131)	c	132)	b
133)	a	134)	c	135)	b	136)	b
137)	b	138)	b	139)	b	140)	b
141)	c	142)	b	143)	c	144)	d
145)	a	146)	d	147)	c	148)	b
149)	a	150)	b	151)	d	152)	a
153)	b	154)	a	155)	a	156)	d
157)	d	158)	d	159)	a	160)	a
161)	b	162)	b	163)	d	164)	c
165)	b	166)	d	167)	b	168)	d
169)	c	170)	a	171)	a	172)	b
173)	a	174)	d	175)	a	176)	c
177)	a	178)	a	179)	a	180)	a
181)	c	182)	c	183)	a	184)	d
185)	b	186)	a	187)	d	188)	a

189)	c	190)	c	191)	c	192)	b
193)	b	194)	d	195)	b	196)	b
197)	a	198)	b	199)	d	200)	d
201)	d	202)	a	203)	c	204)	a
205)	b	206)	c	207)	a	208)	c
209)	a	210)	d	211)	a	212)	a
213)	a	214)	c	215)	b	216)	a
217)	c	218)	d	219)	a	220)	a
221)	b	222)	c	223)	a	224)	d
225)	c	226)	b	227)	a	228)	c
229)	b	230)	a	231)	c	232)	a
233)	c	234)	a	235)	a	236)	b
237)	d	238)	c	239)	b	240)	d
241)	b	242)	b	243)	b	244)	b
245)	b	246)	a	247)	d	248)	b
249)	a	250)	b	251)	c	252)	b
253)	b	254)	a	255)	c	256)	c
257)	b	258)	b	259)	c	260)	b
261)	a	262)	d	263)	d	264)	b
265)	b	266)	b	267)	c	268)	c
269)	b	270)	a	271)	a	272)	b
273)	d	274)	c	275)	d	276)	b
277)	c	278)	d	279)	a	280)	a
281)	b	282)	b	283)	b	284)	b
285)	c	286)	b	287)	d	288)	c
289)	b	290)	a	291)	c	292)	d
293)	d	294)	d	295)	a	296)	a
297)	c	298)	b	299)	a	300)	b
301)	d	302)	d	303)	a	304)	c
305)	c	306)	c	307)	a	308)	c
309)	b	310)	c	311)	c	312)	a
313)	a	314)	c	315)	b	316)	d
317)	a	318)	b	319)	b	320)	b
321)	c	322)	c	323)	c	324)	c
325)	b	326)	b	327)	c	328)	c
329)	a	330)	c	331)	b	332)	c
333)	b	334)	c	335)	c	336)	b
337)	a	338)	d	339)	b	340)	c
341)	c	342)	c	343)	c	344)	a
345)	d	346)	c	347)	c	348)	a
349)	c	350)	a	351)	c	352)	c
353)	d	354)	d	355)	c	356)	c
357)	a	358)	b	359)	a	360)	d
361)	d	362)	b	363)	b	364)	a
365)	c	366)	a				

: HINTS AND SOLUTIONS :

1 **(b)**

Suppose R divides PQ in the ratio $\lambda : 1$. Then, the coordinates of R are

$$\left(\frac{5\lambda+3}{\lambda+1}, \frac{4\lambda+2}{\lambda+1}, \frac{-6\lambda-4}{\lambda+1}\right)$$

But, the coordinates of R are given as $(9, 8, -10)$

$$\therefore \frac{5\lambda+3}{\lambda+1} = 9, \frac{4\lambda+2}{\lambda+1} = 8 \text{ and } \frac{-6\lambda-4}{\lambda+1} = -10$$

$$\Rightarrow \lambda = -\frac{3}{2}$$

Hence, R divides PQ externally in the ratio 3 : 2

2 **(b)**

The centre and radius of given sphere are

$C(0, 1, 2)$ and $R = \sqrt{0+1+4+11} = 4$

Now, perpendicular distance from centre to the plane,

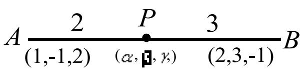
$$d = \frac{|0+2+4-15|}{\sqrt{1+4+4}} = 3$$

$\therefore$ Radius of circle $= \sqrt{R^2 - d^2} = \sqrt{16-9} = \sqrt{7}$

3 **(d)**

$$\alpha = \frac{2\cdot 2 + 3\cdot 1}{2+3} = \frac{7}{5}$$

$$\beta = \frac{2\cdot 3 + 3\cdot(-1)}{2+3} = \frac{3}{5}$$



and $\gamma = \dfrac{2(-1) + 3\cdot 2}{2+3} = \dfrac{4}{5}$

$$\therefore \overrightarrow{\mathbf{OP}} = \frac{1}{5}(7\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$$

4 **(c)**

The given plane passes through $\vec{a}$ and is parallel to the vectors $(\vec{b} - \vec{a})$ and $\vec{c}$. So it is normal to $(\vec{b} - \vec{a}),\times \vec{c}$

$$(\vec{r} - \vec{a}) \cdot \left((\vec{b} - \vec{a}) \times \vec{c}\right) = 0$$

$$\Rightarrow \vec{r} \cdot (\vec{b} \times \vec{c} + \vec{c} \times \vec{a}) = [\vec{a}\, \vec{b}\, \vec{c}]$$

The length of the perpendicular from the origin to this plane is

$$\frac{[\vec{a}\, \vec{b}\, \vec{c}]}{|\vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}$$

5 **(b)**

Any point on the line

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = k \quad [\text{say}]$$

is $(2k+1, 3k+2, 4k+3)$

which lies on the plane $2x + 3y - z = -4$

$\therefore 2(2k+1) + 3(3k+2) - 4(4k+3) = -4$

$\Longrightarrow k = -1$

$\therefore$ Required point is $(-1, -1, -1)$

6 **(b)**

Given line can be rewritten as

$$\frac{x - \frac{1}{2}}{1} = \frac{y-2}{-2} = \frac{z+1}{a}$$

If any line parallel to plane, then $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

Here, $(a_1, b_1, c_1) = (2, -1, 1)$

and $(a_2, b_2, c_2) = (1, -2, a)$

$\therefore 2(1) - 2(-1) + a(1) = 0$

$\Longrightarrow a = -4$

7 **(c)**

$\cos^2 45° + \cos^2 120° + \cos^2 \theta = 1$

$$\Longrightarrow \frac{1}{2} + \frac{1}{4} + \cos^2\theta = 1 \Longrightarrow \cos^2\theta = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\Longrightarrow \cos\theta = \frac{1}{2} \qquad (\because\ \theta \text{ is acute})$$

$\Longrightarrow\ \theta = 60°$

8 **(b)**

Let the position vectors of the given points A and B be $\vec{a}$ and $\vec{b}$ respectively and that of the variable point P be $\vec{r}$. It is given that

$PA^2 - PB^2 = k$ (Constant)

$$\Rightarrow |\overrightarrow{AP}|^2 - |\overrightarrow{BP}|^2 = k$$

$$\Rightarrow |\vec{r} - \vec{a}|^2 - |\vec{r} - \vec{b}|^2 = k$$

$$\Rightarrow \{|\vec{r}|^2 + |\vec{a}|^2 - 2\,\vec{r}\cdot\vec{a}\} - \left\{|\vec{r}|^2 + |\vec{b}|^2 - 2\,\vec{r}\cdot\vec{b}\right\} = k$$

$$\Rightarrow 2\vec{r}\cdot(\vec{b} - \vec{a}) = k + |\vec{b}|^2 - |\vec{a}|^2$$

$$\Rightarrow \vec{r}\cdot(\vec{b} - \vec{a}) = \lambda, \text{ where, } \lambda = \tfrac{1}{2}\left\{k + |\vec{b}|^2 - |\vec{a}|^2\right\}$$

Clearly, it represents a plane

9 **(a)**

Equation of lines are

$\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ ...(i)

And $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ ...(ii)

These line are perpendicular to each other

$\therefore\ -3(3k) + 2k + 2(-5) = 0$

$\Rightarrow\ -7k - 10 = 0$

$$\Rightarrow k = -\frac{10}{7}$$

10 **(a)**

The equation of any plane through $(2, -1, 3)$ is

$a(x-2) + b(y+1) + c(z-3) = 0$ ...(i)

Since, Eq. (i), is parallel to $\vec{a}$ and $\vec{b}$

$\therefore 3a + 0b - c = 0$

and $-3a + 2b + 2c = 0$

$\Rightarrow \frac{a}{2} = \frac{b}{3-6} = \frac{c}{6} = k$ [say]

$\Rightarrow a = 2k, \quad b = -3k, \quad c = 6k$

On putting the values of a, b and c in Eq. (i), we get

$2k(x-2) - 3k(y+1) + 6k(z-3) = 0$

$\Rightarrow 2x - 3y + 6z - 25 = 0$

11 **(b)**

We know that the equation of a plane passing through the intersection of the planes

$a_1x + b_1y + c_1z + d_1 = 0$

And $a_2x + b_2y + c_2z + d_2 = 0$ is

$$(a_1x + b_1y + c_1z + d_1) + \lambda(a_2x + b_2y + c_2z + d_2) = 0$$

Where λ is constant

Thus, the equation of plane $2x - (1+\lambda)y + 3\lambda z = 0$ can be rewritten as

$(2x - y) + \lambda(-y + 3z) = 0$

So, it is clear that the equation of plane passes through the intersection of the planes

$2x - y = 0$ and $y - 3z = 0$

12 **(c)**

The given lines can be rewritten as

$\frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$ and $\frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$

$\therefore$ Angle between the lines is

$$\theta = \cos^{-1}\left(\frac{3 \times 2 + 2(-12) - 6(-3)}{\sqrt{3^2 + 2^2 + (-6)^2}\sqrt{(2)^2 + (-12)^2 + (-3)^2}}\right)$$

$= 0$

$\Rightarrow \theta = 90°$

14 **(a)**

We know that the image of the point (x_1, y_1, z_1) in the plane $ax + by + cz + d = 0$ is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = -2\left(\frac{ax_1 + by_1 + cz_1 + d}{a^2 + b^2 + c^2}\right)$$

The equation of the plane is

$\vec{r} \cdot (3\hat{\imath} - 5\hat{\jmath} + 4\hat{k}) = 5$ or, $3x - 5y + 4z = 5$

The image of $(1, 2, -1)$ in this plane is given by

$$\frac{x-1}{3} = \frac{y-2}{-5} = \frac{z+1}{4} = -2\left(\frac{3 - 10 - 4 - 5}{\sqrt{9 + 25 + 16}}\right)$$

$\Rightarrow x = \frac{73}{25}, y = -\frac{6}{5}, z = \frac{39}{25}$

15 **(a)**

We know that the equation of the plane containing the lines

$\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$ is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

So, the equation of the plane containing the given lines is

$$\begin{vmatrix} x - 1 & y + 1 & z \\ 2 & -1 & 3 \\ -1 & 3 & -1 \end{vmatrix} = 0 \Rightarrow 8x + y - 5z - 7 = 0$$

16 **(d)**

Let the equation of the variable plane be

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

This meets the coordinate axes at

$A(a, 0, 0), B(0, b, 0)$ and $C(0, 0, c)$

Let (α, β, γ) be the coordinates of the centroid of ΔABC. Then,

$\alpha = \frac{a}{3}, \beta = \frac{b}{3}, \gamma = \frac{c}{3} \Rightarrow a = 3\alpha, b = 3\beta, c = 3\gamma$ ... (i)

The plane is at a distance, k from the origin

$$\therefore \left|\frac{\frac{0}{a} + \frac{0}{b} + \frac{0}{c} - 1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}\right| = k$$

$\Rightarrow \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{k^2} \Rightarrow \alpha^{-2} + \beta^{-2} + \gamma^{-2} = 9k^{-2}$

Hence, the locus of (α, β, γ) is $x^{-2} + y^{-2} + z^{-2} = 9k^{-2}$

17 **(b)**

The direction cosines of the line are

$l = \cos\alpha, m = \cos\beta, n = \cos\gamma$

Now,

$l^2 + m^2 + n^2 = 1$

$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow 1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

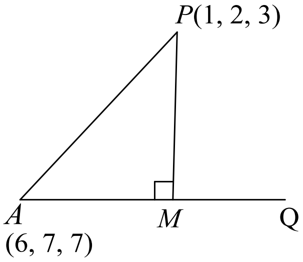
18 **(b)**

Suppose xy-plane divides at the line joining the given points in the ratio $\lambda : 1$. The coordinate of the points of division are $\left[\frac{2\lambda-1}{\lambda+1}, \frac{-5\lambda+3}{\lambda+1}, \frac{6\lambda+4}{\lambda+1}\right]$ Since the point lies on the xy-plane

$\therefore \frac{6\lambda + 4}{\lambda + 1} = 0 \Rightarrow \lambda = \frac{-2}{3}$

19 **(d)**

Direction cosines of given line are $\frac{3}{\sqrt{17}}, \frac{2}{\sqrt{17}}, -\frac{2}{\sqrt{17}}$



$\therefore AM = \left|6-1\cdot\frac{3}{\sqrt{17}}+(7-2)\cdot\frac{2}{\sqrt{17}}+(7-3)\cdot-\frac{2}{\sqrt{17}}\right|$

$=\sqrt{17}$

$AP=\sqrt{(6-1)^2+(7-2)^2+(7-3)^2}$

$=\sqrt{25+25+16}=\sqrt{66}$

$\therefore$ Length of perpendicular

$PM=\sqrt{AP^2-AM^2}$

$=\sqrt{66-17}=\sqrt{49}=7$

20 **(c)**

Let θ be the angle between the given linear. Then,

$\cos\theta=\frac{1\times3+(-2)\times2+1\times3}{\sqrt{1+4+9}\sqrt{9+4+1}}=\frac{1}{7}\Rightarrow\theta = \cos^{-1}\left(\frac{1}{7}\right)$

21 **(c)**

Since, the given line is parallel to the xy-plane, it means that the normal line is perpendicular to z-axis

$\therefore$ Dr's of z coordinate is zero

$ie, n=0$

22 **(a)**

Since, the line lie in the plane, therefore its point $(4,2,k)$ should lie in the given plane

$\Rightarrow 2(4)-4(2)+1(k)=7 \Rightarrow k=7$

24 **(d)**

The foot of the point (x_1,y_1,z_1) in the plane $ax+by+cz+d=0$ is given by

$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$

$=-\frac{(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}$

$\frac{x-1}{2}=\frac{y-3}{(-1)}=\frac{z-4}{1}=\frac{2-3+4+3}{6}$

$\Rightarrow x=-1, y=4, z=3$

25 **(c)**

Given, $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}=\lambda$ [say]

Any point on the line is $P(\lambda, 2\lambda, +1, 3\lambda+2)$

Therefore, direction ratios of PQ are $\lambda-1, 2\lambda-5, 3\lambda-1$

$\because PQ$ is perpendicular to the given line

Therefore, $1(\lambda-1)+2(2\lambda-5)+3(3\lambda-1)=0$

$\Rightarrow \lambda=1$

$\therefore$The coordinate of P are$(1,3,5)$

$\therefore$Length of perpendicular

$=\sqrt{(1-1)^2+(3-6)^2+(5-3)^2}$

$=\sqrt{13}$

26 **(c)**

The given line is

$\vec{\mathbf{r}}=(1+2\mu)\hat{\mathbf{i}}+(2+\mu)\hat{\mathbf{j}}+(2\mu-1)\hat{\mathbf{k}}$

$=(\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}})+\mu(2\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}})$

Vector equation of line written in cartesian from is

$\frac{x-1}{2}=\frac{y-2}{1}=\frac{z+1}{2}$

$\therefore$ Angle between line and a plane is given by

$\therefore \sin\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

$=\frac{2\times3+1\times(-2)+2\times6}{\sqrt{4+1+4}\sqrt{9+4+36}}=\frac{16}{21}$

$\Rightarrow \theta=\sin^{-1}\left(\frac{16}{21}\right)$

27 **(a)**

The equation of circle and plane are

$x^2+y^2+z^2=9$ ...(i)

And $2x+3y+4z-5=0$ ...(ii)

Respectively.

$\therefore$ Equation of sphere is

$x^2+y^2+z^2-9+\lambda(2x+3y+4z-5)=0$ ...(iii)

Which passes through (1, 2, 3)

$\therefore 1+4+9-9+\lambda(2+6+12-5)=0$

$\Rightarrow 5+\lambda(15)=0$

$\Rightarrow \lambda=-\frac{1}{3}$

$\therefore$ From Eq. (iii)

$\therefore x^2+y^2+z^2-9-\frac{1}{3}(2x-3y+4z-5)=0$

$\Rightarrow 2(x^2+y^2+z^2)-2x-3y-4z-22=0$

28 **(c)**

Given lines are $\frac{x-1}{0}=\frac{y-2}{0}=\frac{z}{1}$

and $\frac{x}{1}=\frac{y+1}{0}=\frac{z}{0}$

$\therefore \cos\theta=0\cdot1+0\cdot0+1\cdot0=0$

$\Rightarrow \theta=90°$

29 **(b)**

We have, equation of lines are

$\frac{x-5}{3}=\frac{y-7}{-1}=\frac{z+2}{1}$ ...(i)

And $\frac{x+3}{-36}=\frac{y-3}{2}=\frac{z-6}{4}$ ...(ii)

$\therefore$ Any point on line $\frac{x-5}{3}=\frac{y-7}{-1}=\frac{z+2}{1}=k$ is $(3k+5, 7-k, k-2)$

It should lie on

$$\frac{x+3}{-36}=\frac{y-3}{2}=\frac{z-6}{4}$$

$$\Rightarrow \frac{3k+5+3}{-36}=\frac{7-k-3}{2}=\frac{k-2-6}{4}$$

On solving, we get $k=\frac{16}{3}$

$\therefore x=16+5=21, y=7-\frac{16}{3}=\frac{5}{3}$

And $z=\frac{16}{3}-2=\frac{10}{3}$

$\therefore$ Coordinate of point are (21, 5/3, 10/3)

30 **(a)**

The length of the edges are given by

$a=5-2=3$

$b=9-3=6$

$c=7-5=2$

So, length of the diagonal $=\sqrt{9+36+4}=7$

31 **(b)**

We know, $\cos\theta=\frac{|a_1a_2+b_1b_2+c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

$$=\frac{|(2)(3)+(3)(-4)+(-6)(5)|}{\sqrt{2^2+3^2+(-6)^2}\sqrt{3^2+(-4)^2+(5)^2}}$$

$$=\frac{|6-12-30|}{\sqrt{4+9+36}\sqrt{9+16+25}}$$

$$=\frac{36}{7\cdot 5\sqrt{2}}=\frac{18\sqrt{2}}{35}$$

$$\Rightarrow \theta=\cos^{-1}\left(\frac{18\sqrt{2}}{35}\right)$$

32 **(a)**

The equation of line which passes through the point $A(4,2,2)$ and parallel to the vector $2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+6\hat{\mathbf{k}}$ is

$$\frac{x-4}{2}=\frac{y-2}{3}=\frac{z-2}{6}$$

Distance of point P from the line

$$=\sqrt{\sum(x_1-x_2)^2-\left(\sum l(x_1-x_2)\right)^2}$$

$=\sqrt{(1-4)^2+(2-2)^2+(3-2)^2-\{2(1-4)+3(2-2)+}$

$=\sqrt{9+0+1-(-6+0+6)^2}$

$=\sqrt{10}$

34 **(b)**

If the given plane contains the given line, then normal to the plane must be perpendicular to the line and the condition for the same is

$al+bm+cn=0$

35 **(a)**

Given lines can be rewritten as

$$\frac{x-1}{-3}=\frac{y-2}{2\alpha}=\frac{z-3}{2}$$

and $\frac{x-1}{3\alpha}=\frac{y-1}{1}=\frac{z-6}{-5}$

since, lines are perpendicular.

$\therefore a_1a_2+b_1b_2+c_1c_2=0$

$\Rightarrow (-3)(3\alpha)+2\alpha(1)+2(-5)=0$

$\Rightarrow -9\alpha+2\alpha-10=0$

$\Rightarrow \alpha=-\frac{10}{7}$

36 **(d)**

Perpendicular distance of the point (6, 5, 8) from y-axis $=\sqrt{6^2+8^2}=10$ units

37 **(a)**

Let the sides of the cube be along the axes, then diagonals have direction cosine as

$\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0\right),\left(\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}},\right)$ and $\left(0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$

$\therefore$ Resultant vector is

$$\frac{a}{\sqrt{2}}(\hat{\mathbf{i}}+\hat{\mathbf{j}})+\frac{2a}{\sqrt{2}}(\hat{\mathbf{i}}+\hat{\mathbf{k}})+\frac{3a}{\sqrt{2}}(\hat{\mathbf{j}}+\hat{\mathbf{k}})$$

$$=\frac{a}{\sqrt{2}}(3\hat{\mathbf{i}}+4\hat{\mathbf{j}}+5\hat{\mathbf{k}})$$

$\Rightarrow$ Magnitude of the resultant

$$=\frac{a}{\sqrt{2}}\sqrt{9+16+25}=\frac{a}{\sqrt{2}}\cdot\sqrt{50}=5a$$

38 **(b)**

Line is passing through $(3,-5,7)$ and parallel to $(2,1,-3)$, then equation of line is $\frac{x-3}{2}=\frac{y+5}{1}=\frac{z-7}{-3}$

39 **(a)**

The required plane passes through a point having position vector $\overrightarrow{a_1}$ and is parallel to the vectors $\overrightarrow{a_1}$ and $\overrightarrow{a_2}$. So, it is normal to $\overrightarrow{a_1}\times\overrightarrow{a_2}$

Thus, the equation on the plane is

$(\vec{r}-\overrightarrow{a_1})\cdot(\overrightarrow{a_1}\times\overrightarrow{a_2})=0$

$\Rightarrow [\vec{r}\,\overrightarrow{a_1}\,\overrightarrow{a_2}]=[\overrightarrow{a_1}\,\overrightarrow{a_1}\,\overrightarrow{a_2}]$

$\Rightarrow [\vec{r}\,\overrightarrow{a_1}\,\overrightarrow{a_2}]=0$

Hence, the required plane is $[\vec{r}\,\overrightarrow{a_1}\,\overrightarrow{a_2}]=0$

40 **(b)**

The equation of any plane through $A(4,5,1)$ is

$a(x-4)+b(y-5)+c(z-1)=0$ ...(i)

The points $B(0,-1,-1)$and $C(3,9,4)$ lies on Eq. (i)

$\Rightarrow a(0-4)+b(-1-5)+c(-1-1)=0$

$\Rightarrow 2a+3b+c=0$ ....(ii)

and $a(3-4)+b(9-5)+c(4-1)=0$

$\Rightarrow a-4b-3c=0$ ....(iii)

On solving Eqs, (i) and (iii), we get

$$\frac{a}{5}=\frac{b}{-7}=\frac{c}{11}$$

$\therefore$ Equation of plane is

$5(x-4)-7(y-5)+11(z-1)=0$

$\Rightarrow 5x-7y+11z+4=0$

Also, point $D(-4,4,4)$ lies on it, then
$-20-28+44+4=0 \Rightarrow 0=0$
Hence, points A, B, C and D are coplanar.
Alternate
DR's of $AB(-4,-6,-2), AC=(-1,4,3)$
and $AD(-8,-1,3)$,
Now $\begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = -4(15)+6(21)-2(33)$
$= 0$

41 **(a)**
If l, m, n are the direction cosines of the line, then
$1 \cdot l - 1 \cdot m + 1 \cdot n = 0$
and $1 \cdot l - 3 \cdot m + 0 \cdot n = 0$
$\therefore \frac{l}{0+3} = \frac{m}{1-0} = \frac{n}{-3+1}$
Hence, the direction ratios of the line are $3, 1, -2$

42 **(a)**
Since, line lies in a plane, it means point $(4,2,k)$ lies in a plane.
$\therefore 8-8+k=7$
$\Rightarrow k=7$

43 **(a)**
Since, M is the mid point of $A(4\hat{\mathbf{i}}+5\hat{\mathbf{j}}-10\hat{\mathbf{k}})$ and $B(-\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}})$
$\therefore$ Coordinate of point M are $\left(\frac{3}{2}, \frac{7}{2}, -\frac{9}{2}\right)$
$\left(\vec{\mathbf{r}} - \left(\frac{3}{2}\hat{\mathbf{i}} + \frac{7}{2}\hat{\mathbf{j}} - \frac{9}{2}\hat{\mathbf{k}}\right)\right) \cdot \overrightarrow{\mathbf{AB}} = 0$
$\left(\vec{\mathbf{r}} - \left(\frac{3}{2}\hat{\mathbf{i}} + \frac{7}{2}\hat{\mathbf{j}} - \frac{9}{2}\hat{\mathbf{k}}\right)\right) \cdot (5\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 11\hat{\mathbf{k}}) = 0$
$\vec{\mathbf{r}} \cdot (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 11\hat{\mathbf{k}}) - \frac{135}{2} = 0$

44 **(a)**
Equation of the plane through (5, 1, 2) is
$a(x-5)+b(y-1)+c(z-2)=0$ ...(i)
Given plane (i) is perpendicular to the line
$\frac{x-2}{1/2} = \frac{y-4}{1} = \frac{z-5}{1}$
$\therefore$ Equation of normal of Eq. (i) and straight line (ii) are parallel
$ie, \frac{a}{1/2} = \frac{b}{1} = \frac{c}{1} = k$ (say)
$\therefore a = \frac{k}{2}, b = k, c = k$
From Eq. (i),
$\frac{k}{2}(x-5) + k(y-1) + k(z-2) = 0$
Or $x+2y+2z=11$
Any point on Eq. (ii) is $\left(2+\frac{\lambda}{2}, 4+\lambda, 5+\lambda\right)$
Which lies on Eq. (iii), then $\lambda = -2$
$\therefore$ Required point is (1, 2, 3)

45 **(b)**
The line of intersection of the plane
$\vec{r} \cdot (3\hat{\imath} - \hat{\jmath} + \hat{k}) = 1$ and $\vec{r} \cdot (\hat{\imath} + 4\hat{\jmath} - 2\hat{k}) = 2$
is common to both the planes. Therefore, it is perpendicular to normal to the two planes i.e.,
$\overrightarrow{n_1} = 3\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\overrightarrow{n_2} = \hat{\imath} + 4\hat{\jmath} - 2\hat{k}$
Hence, it is parallel to the vector $\overrightarrow{n_1} \times \overrightarrow{n_2} = -2\hat{\imath} + 7\hat{\jmath} + 13\hat{k}$. Thus, we have to find the equation of the plane passing through $\vec{a} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$ and normal to the vector $\vec{n} = \overrightarrow{n_1} \times \overrightarrow{n_2}$
The equation of the required plane is
$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$
$\Rightarrow \vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$
$\Rightarrow \vec{r} \cdot (-2\hat{\imath} + 7\hat{\jmath} + 13\hat{k})$
$= (\hat{\imath} + 2\hat{\jmath} - \hat{k}) \cdot (-2\hat{\imath} + 7\hat{\jmath} + 13\hat{k})$
$\Rightarrow \vec{r} \cdot (2\hat{\imath} - 7\hat{\jmath} - 13\hat{k}) = 1$

46 **(a)**
Any plane passing through (3, 2, 0) is
$a(x-3)+b(y-2)+c(z-0)$ ...(i)
Plane is passing through the line
$\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$
$\therefore a(3-3)+b(6-2)+c(4-0)=0$
$\Rightarrow 0a+4b+4c=0$ ...(ii)
Since, the given plane is passing through the line, therefore the DR's of the normal is perpendicular to the line,
$\therefore a+5b+4c=0$ ...(iii)
On solving Eqs. (ii) and (iii), we get
$\frac{a}{16-20} = \frac{b}{4-0} = \frac{c}{0-4}$
$\Rightarrow \frac{a}{-1} = \frac{b}{1} = \frac{c}{-1}$
On putting the values of a, b and c in Eq. (i), we get
$x-y+z=1$

47 **(a)**
Since, we are given the equal intercept of the coordinate axes $ie, |x|=|y|=|z|=p$
Therefore, it make a cube

48 **(d)**
Let the equation of sphere passing through origin be
$x^2+y^2+z^2+2ux+2vy+2wz=0$
Also, it passes through $(a,0,0), (0,b,0), (0,0,c)$
$\Rightarrow a^2+2ua=0 \Rightarrow u = -\frac{a}{2}$
Similarly, $v = -\frac{b}{2}, w = -\frac{c}{2}$
$\therefore$ Centre $(-u,-v,-w) = \left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$

49 **(b)**

Given lines can be rewritten as

$\frac{x-6}{1}=\frac{y-2}{-2}=\frac{z-2}{2}$ and

$\frac{x+4}{3}=\frac{y}{-2}=\frac{z-1}{-2}$

Here, $x_1=6, y_1=2, z_1=2$

$x_2=-4, y_2=0, z_2=1$

$a_1=1, b_1=-2, c_1=2$

and $a_2=3, b_2=-2, c_2=-2$

Now, $\begin{vmatrix} x_2-y_2 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$

$=\begin{vmatrix} -10 & -2 & -1 \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix}$

$=-10(4+4)+2(-2-6)-1(-2+6)$

$=-100$

and $\sqrt{(b_1c_2-c_1b_2)^2+(c_1a_2-a_1c_2)^2+(a_1b_2-a_2b_1)^2}$

$=\sqrt{(4+4)^2+(6+2)^2+(-2+6)^2}$

$=\sqrt{64+64+16}=12$

$\therefore$ Required shortest distance $= l\frac{-100}{12}=\frac{25}{3}$

[neglect(-ve)sign]

50 **(b)**

Required plane contains the given line, so normal to the plane must be perpendicular to the line and the condition for the same is $al+bm+cn=0$.

51 **(d)**

Given line is

$\frac{x-1}{1}=\frac{y+2}{3}=\frac{z-2}{-2}=r$ [say]

$\therefore x=r+1, \quad y=3r-2, \quad z=-2r+2$

These values of x and z will satisfy the plane

$3x+4y+5z-25=0$

$\therefore 3(r+1)+4(3r-2)+5(-2r+2)-25=0$

$\Longrightarrow 3r+3+12r-8-10r+10-25=0$

$\Longrightarrow r=4$

$\therefore x=5, \quad y=10$ and $z=-6$

52 **(d)**

Given that equation of planes are

$P \equiv ax+by+cz+d=0$ ...(i)

And $P' \equiv a'x+b'y+c'z+d'=0$ ...(ii)

Equation of intersection of planes is

$P+\lambda P'=0$ ....(iii)

$\Rightarrow ax+by+cz+d+\lambda(a'x+b'y+c'z+d)=0$

$\Rightarrow a+\lambda a'=0$

$\Rightarrow \lambda=-\frac{a}{a'}$

$\therefore$ From Eq. (iii), we get

$P-\frac{a}{a'}P'=0 \Rightarrow \frac{P}{a}=\frac{P'}{a'}$

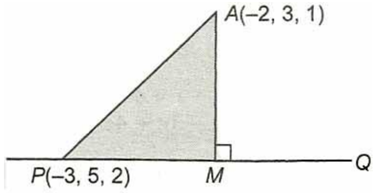
53 **(b)**

Here, $\alpha=\beta=\gamma$

$\because \cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$\therefore \cos\alpha=\frac{1}{\sqrt{3}}$

DC' of PQ are $\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$



$PM=$ Projection of AP on PQ

$=\left|(-2+3)\frac{1}{\sqrt{3}}+(3-5)\cdot\frac{1}{\sqrt{3}}+(1-2)\cdot\frac{1}{\sqrt{3}}\right|$

$=\frac{2}{\sqrt{3}}$

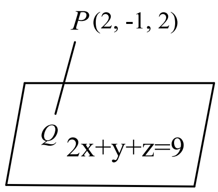
And $AP=\sqrt{(-2+3)^2+(3-5)^2+(1-2)^2}=\sqrt{6}$

$AM=\sqrt{(AP)^2-(PM)^2}$

$=\sqrt{6-\frac{4}{3}}=\sqrt{\frac{14}{3}}$

54 **(c)**

Since, $l=m=n=\frac{1}{\sqrt{3}}$



$\therefore$ Equation of line is $\frac{x-2}{1/\sqrt{3}}=\frac{y+1}{1/\sqrt{3}}=\frac{z-2}{1/\sqrt{3}}$

$\Longrightarrow x-2=y+1=z-2=r$ [say]

$\therefore$ Any point on the line is

$Q=(r+2, r-1, r+2)$

$\because Q$ lies on the plane $2x+y+z=9$

$\therefore 2(r+2)+(r-1)+(r+2)=9$

$\Longrightarrow 4r+5=9 \Longrightarrow r=1$

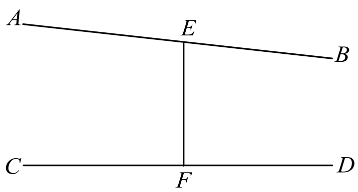
$\therefore$ Coordinate $Q(3,0,3)$

$\therefore \quad PQ=\sqrt{(3-2)^2+(0+1)^2+(3-2)^2}=\sqrt{3}$

55 **(b)**

Let the equation of line AB be

$\frac{x-0}{1}=\frac{y+a}{1}=\frac{z-0}{1}=k$ [say]



$\therefore$ Coordinate of E are $(k, k-a, k)$.

Also, the equation of the other line CD is

$$\frac{x+a}{2}=\frac{y-0}{1}=\frac{z-0}{1}=\lambda \text{ [say]}$$
$\therefore$ Coordinates of F are $(2\lambda-a,\lambda,\lambda)$
Direction ratio of FE are $\{(k-2\lambda+a),(k-\lambda-a,k-\lambda$
$$\therefore \frac{k-2\lambda+a}{2}=\frac{k-\lambda-a}{1}=\frac{k-\lambda}{2}$$
From Ist and IInd terms,
$k-2\lambda+a=2k-2\lambda-2a$
$\Rightarrow k=3a$
From IInd and IIIrd terms,
$2k-2\lambda-2a=k-\lambda$
$\Rightarrow \lambda=k-2a=3a-2a$
$\Rightarrow \lambda=a$
$\therefore$ Coordinate of $E=(3a,2a,3a)$ and coordinate of $F=(a,a,a)$

57 **(c)**
Let the DR's of a required line be a,b and c Since, the normal to the given planes $x+2y+z=3$ and $6x+8y+3z=13$ are perpendicular to the line.
$\therefore a+2b+c=0$
and $6a+8b+3c=0$
$$\Rightarrow \frac{a}{6-8}=\frac{b}{6-3}=\frac{c}{8-12}$$
$$\Rightarrow \frac{a}{2}=\frac{b}{-3}=\frac{c}{4}$$
Hence, option (c) is the required solution.

58 **(b)**
Let $A(1,2,2)$ be the foot of the perpendicular from $O(0,0,0)$ on the plane, then direction ratios of OA are (1, 2, 2),
$\therefore$ Equation of the plane is
$1(x-1)+2(y-2)+2(z-2)=0$
$\Rightarrow x+2y+2z-9=0$

59 **(a)**
Let the coordinates of A,B and C be $(a,0,0)$, $(0,b,0)$, $(0,0,c)$ respectively
Then, equation of the plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$
Also, it passes through (p,q,r)
$$\therefore \frac{p}{a}+\frac{q}{b}+\frac{r}{c}=1$$
Also, equation of sphere passes through A,B,C will be
$x^2+y^2+z^2-ax-by-cz=0$
If its centre (x_1,y_1,z_1), then
$$x_1=\frac{a}{2},y_1=\frac{b}{2},z_1=\frac{c}{2}$$
$\Rightarrow a=2x_1,b=2y_1,c=2z_1$
$\therefore$ Locus of centre of sphere is
$$\frac{p}{x}+\frac{q}{y}+\frac{r}{z}=2$$

60 **(c)**
The position vector of any point on the given line is
$\hat{\imath}+\hat{\jmath}+\lambda(2\hat{\imath}+\hat{\jmath}+4\hat{k})$
$=(2\lambda+1)\hat{\imath}+(\lambda+1)\hat{\jmath}+4\lambda\hat{k}$
Clearly, this point lies on the plane $\vec{r}\cdot(\hat{\imath}+2\hat{\jmath}-\hat{k})=3$
Hence, the plane $\vec{r}\cdot(\hat{\imath}+2\hat{\jmath}-\hat{k})=3$ contains the given line

61 **(b)**
The equation of the plane through given line is
$a(x-1)+b(y-2)+c(z-3)=0$ ...(i)
Since, the straight line lie on the plane.
$\therefore$ DR's of the plane is perpendicular to the line *ie*,
$5a+4b+5c=0$ ...(ii)
Since, the plane passes through (0, 0, 0), we get
$-a-2b-3c=0$
$\Rightarrow a+2b+3c=0$ ....(iii)
On solving Eqs. (ii) and (ii), we get
$$\frac{a}{2}=\frac{b}{-10}=\frac{c}{6}$$
From Eq. (i),
$2(x-1)-10(y-2)+6(z-3)=0$
$\Rightarrow 2x-10y+6z=0$
$\Rightarrow x-5y+3z=0$

62 **(b)**
The distance of the point (2, 3, −5) from the plane $x+2y-2z=9$ is
$$D=\frac{|2(1)+2(3)-2(-5)-9|}{\sqrt{1^2+2^2+(-2)^2}}$$
$$=\frac{|2+6+10-9|}{\sqrt{1+4+4}}=3$$

63 **(b)**
The ratio in which yz-plane divide the line segment
$=x_1:x_2=-(-3):2=3:2$

64 **(d)**
Since, DC'sof a line are $\left(\frac{1}{c},\frac{1}{c},\frac{1}{c}\right)$
$$\therefore \left(\frac{1}{c}\right)^2+\left(\frac{1}{c}\right)^2+\left(\frac{1}{c}\right)^2=1$$
$\Rightarrow c^2=3 \Rightarrow c=\pm\sqrt{3}$

65 **(d)**
Let $\alpha=\frac{\pi}{4},\beta=\frac{\pi}{4}$
We know, $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$
$$\therefore \cos^2\frac{\pi}{4}+\cos^2\frac{\pi}{4}+\cos^2\gamma=1$$
$$\Rightarrow \frac{1}{2}+\frac{1}{2}+\cos^2\gamma=1$$

$\Rightarrow \cos^2\gamma = 0 \Rightarrow \gamma = \dfrac{\pi}{2}$

66 **(c)**

Equation of plane is

$\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$

Also, $3 = \dfrac{a+0+0}{3}$

$\Rightarrow a = 9$ and similarly $b = 6$ and $c = 3$

$\therefore$ Equation of required plane is

$\dfrac{x}{9} + \dfrac{y}{6} + \dfrac{z}{3} = 1$

$\Rightarrow 2x + 3y + 6z = 18$

67 **(a)**

Equation of any plane passing through the point (1, 2, 3) is

$a(x-1) + b(y-2) + c(z-3) = 0$

Since, the above plane is parallel to $x + 2y + 5z = 0$

$\therefore 1(x-1) + 2(y-2) + 5(z-3) = 0$

68 **(b)**

If we have two vectors $\overrightarrow{\mathbf{AB}}$ and $\overrightarrow{\mathbf{AC}}$, then area of triangle

$\Delta = \dfrac{1}{2}\,|\overrightarrow{\mathbf{AB}} \times \overrightarrow{\mathbf{AC}}|$

$= \dfrac{1}{2}\left\|\begin{matrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{matrix}\right\|$

$= \dfrac{1}{2}\left\|\begin{matrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & -3 \\ -1 & 3 & -1 \end{matrix}\right\|$

$= \dfrac{1}{2}\left|\hat{\mathbf{i}}(-1+9) - \hat{\mathbf{j}}(-1-3) + \hat{\mathbf{k}}(3+1)\right|$

$= \dfrac{1}{2}\sqrt{8^2 + 4^2 + 4^2} = \dfrac{1}{2}\sqrt{64 + 16 + 16}$

$= \dfrac{\sqrt{96}}{2} = \dfrac{4\sqrt{6}}{2} = 2\sqrt{6}$

69 **(a)**

The equation of the plane through the intersection of given planes is

$(x + y + z - 1) + k(2x + 3y - z + 4) = 0$ ....(i)

$\Rightarrow x(1 + 2k) + y(1 + 3k) + z(1 - k) - 1(1 - 4k) = 0$

$\because$ Plane is parallel to x-axis $x(1 + 2k) = 0$

$\Rightarrow k = -\dfrac{1}{2}$

Put $k = -1/2$ in Eq. (i) we get the required equation of plane which is

$2(x + y + z - 1) - 2x - 3y + z - 4 = 0$

$\Rightarrow y - 3z + 6 = 0$

70 **(c)**

Direction ratios of given planes are

$a_1 = 2, b_1 = -1, c_1 = 1$

and $a_2 = 1, b_2 = 2, c_2 = 3$

$\therefore \cos\theta = \dfrac{2(1) - 1(2) + 1(3)}{\sqrt{2^2 + 1^2 + 1^2}\,\sqrt{1^2 + 2^2 + 3^2}}$

$= \dfrac{3}{\sqrt{6}\,\sqrt{14}}$

$\Rightarrow \theta = \cos^{-1}\left(\dfrac{1}{2}\sqrt{\dfrac{3}{7}}\right)$

71 **(b)**

Projection

$= [2 - (-1)]\dfrac{6}{7} + [5 - 0]\dfrac{2}{7} + [1 - 3]\dfrac{3}{7}$

$= \dfrac{18 + 10 - 6}{7} = \dfrac{22}{7}$

72 **(c)**

Let P be the foot of the perpendicular from the origin on the plane, then direction ratios of OP, the normal to the plane are $a - 0, b - 0, c - 0$ *ie*, a, b, c. Also, since, it passes through (a, b, c), the equation of the plane is

$a(x - a) + b(y - b) + c(z - c) = 0$

$\Rightarrow ax + by + cz = a^2 + b^2 + c^2$

73 **(d)**

Let equation of plane is

$lx + my + nz = p$

Or $\dfrac{x}{\left(\frac{p}{l}\right)} + \dfrac{y}{\left(\frac{p}{m}\right)} + \dfrac{z}{\left(\frac{p}{n}\right)} = 1$

Coordinate of A, B, C are $\left(\frac{p}{l}, 0, 0\right), \left(0, \frac{p}{m}, 0\right)$ and $\left(0, 0, \frac{p}{n}\right)$ respectively

$\therefore$ Centroid of $OABC$ is $\left(\frac{p}{4l}, \frac{p}{4m}, \frac{1}{4n}\right)$

$x_1 = \dfrac{p}{4l}, y_1 = \dfrac{p}{4m}, z_1 = \dfrac{p}{4n}$

$\because l^2 + m^2 + n^2 = 1$

$\Rightarrow \dfrac{p^2}{16x_1^2} + \dfrac{p^2}{16y_1^2} + \dfrac{p^2}{16z_1^2} = 1$

Or $x_1^2 y_1^2 + y_1^2 z_1^2 + z_1^2 x^2 = \frac{16}{p^2} x_1^2 y_1^2 z_1^2$

$\therefore$ Locus is $x^2y^2 + y^2z^2 + z^2x^2 = \frac{16}{p^2} x^2y^2z^2$

Hence, $k = \frac{16}{p^2}$

74 **(a)**

The equation of any plane through the intersection of the plane $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ is

$(x + y + z - 1) + \lambda(2x + 3y - z + 4) = 0$

$\Rightarrow (1 + 2\lambda)x + (1 + 3\lambda)y + (1 - \lambda)z + 4\lambda - 1 = 0$

Since, the plane parallel to x-axis

Therefore, DR's of the above plane *ie*, the coefficient of x is zero

$\therefore 1+2\lambda=0 \Rightarrow \lambda=-\dfrac{1}{2}$

Hence, the required equation will be $y-3z+6=0$

75 **(c)**

Given equation can be rewritten as

$\dfrac{x}{\frac{1}{2}}+\dfrac{y}{\frac{1}{3}}+\dfrac{z}{\frac{1}{4}}=1$

$\therefore$ The coordinate of ΔABC is

$A\left(\dfrac{1}{2},0,0\right), B\left(0,\dfrac{1}{3},0\right), C\left(0,0,\dfrac{1}{4}\right)$

Centroid of traingle

$=\left(\dfrac{\frac{1}{2}+0+0}{3},\dfrac{0+\frac{1}{3}+0}{3},\dfrac{0+0+\frac{1}{4}}{3}\right)$

$=\left(\dfrac{1}{6},\dfrac{1}{9},\dfrac{1}{12}\right)$

76 **(b)**

Lane through given line is

$A(x-1)+B(y+2)+C(z-3)=0$ ...(i)

Where A, B and C are the DR's of the normal to the plane. Since the straight line lie on the plane

$\therefore$ DR's of plane is perpendicular to the line *ie*,

$5A+6B+4C=0$ ...(ii)

Since, it passes through (4, 3, 7), we get

$3A+5B+4C=0$ ...(iii)

On solving Eqs. (ii) and (iii), we get

$\dfrac{A}{4}=\dfrac{B}{-8}=\dfrac{C}{7}$

$\therefore$ Equation of required plane is

$4x-8y+7z=41$

77 **(a)**

Given line is

$\dfrac{x-2}{3}=\dfrac{y+1}{4}=\dfrac{z-2}{12}=k$ (say)

Any point on the line is $(3k+2, 4k-1, 12k+2)$

This point lies on the plane $x-y+z=5$

$\therefore 3k+2-(4k-1)+12k+2=5$

$\Longrightarrow 11k=0 \Longrightarrow k=0$

$\therefore$ Intersection point is $(2,-1,2)$

$\therefore$ Distance, between points$(2-1,2)$ and $(-1,-5,-10)$

$=\sqrt{(-1-2)^2+(-5+1)^2+(-10-2)^2}$

$=\sqrt{9+16+144}=13$

79 **(d)**

The equations of the lines joining

$6\vec{a}-4\vec{b}+4\vec{c}, -4\vec{c}$ and $-\vec{a}-2\vec{b}-3\vec{c}, \vec{a}+2\vec{b}-5\vec{c}$ are respectively

$\vec{r}=6\vec{a}-4\vec{b}+4\vec{c}+m(-6\vec{a}-4\vec{b}-8\vec{c})$ ...(i)

and, $\vec{r}=-\vec{a}-2\vec{b}-3\vec{c}+n(2\vec{a}+4\vec{b}-2\vec{c})$ ...(ii)

For the point of intersection, the equations (i) and (ii) should give the same value of $\vec{r}$

Hence, equating the coeff. of vectors $\vec{a}, \vec{b}$ and $\vec{c}$ in the two expressions for $\vec{r}$, we get

$6m+2n=7, 2m-2n=1$ and $8m-2n=7$

Solving first two equations, we get $m=1, n=1/2$

These values of m and n also satisfy the third equation

Hence, the lines intersect

Putting the value of m in (i), we obtain that the position vector of the point of intersection as $-4\vec{c}$

80 **(a)**

The vector equation of a plane through the line of intersection of the planes $\vec{r}\cdot(\hat{\imath}+3\hat{\jmath}-\hat{k})=0$ and $\vec{r}\cdot(\hat{\jmath}+2\hat{k})=0$ can be written as

$\{\vec{r}\cdot(\hat{\imath}+3\hat{\jmath}+-\hat{k})\}+\lambda\{\vec{r}\cdot(\hat{\jmath}+2\hat{k})\}=0$ ...(i)

This passes through $2\hat{\imath}+\hat{\jmath}-\hat{k}$

$\therefore (2\hat{\imath}+\hat{\jmath}-\hat{k})\cdot(\hat{\imath}+3\hat{\jmath}-\hat{k})+\lambda(2\hat{\imath}+\hat{\jmath}-\hat{k})\cdot(\hat{\jmath}+2\hat{k})=0$

$\Rightarrow (2+3+1)+\lambda(0+1-2)=0 \Rightarrow \lambda=6$

Putting the value of λ in (i), we get the equation of the required plane as

$\vec{r}\cdot(\hat{\imath}+9\hat{\jmath}+11\hat{k})=0$

81 **(b)**

We know that the distance between the parallel planes $\vec{r}\cdot\vec{n}=d_1$ and $\vec{r}\cdot\vec{n}=d_2$ is given by $\dfrac{|d_1-d_2|}{|\vec{n}|}$

Given planes are $\vec{r}\cdot(\hat{\imath}+2\hat{\jmath}-2\hat{k})=-5$ and $\vec{r}\cdot(\hat{\imath}+2\hat{\jmath}-2\hat{k})=8$

$\therefore$ Required distance $=\dfrac{|-5-8|}{\sqrt{1+4+4}}=\dfrac{13}{3}$

82 **(a)**

$\because$ A line joining points $(4,-1,2)$ and $(-3,2,3)$ meets the plane at 90°, then this line is normal to the plane

Also, DR's of normal are $<-7,3,1>$

$\therefore$ DR's of plane are <-7,3,1>and point $(-10,5,4)$ lies on the plane

Hence, equation of plane is

$-7(x+10)+3(y-5)+1(z-4)=0$

$\Rightarrow 7x-3y-z+89=0$

83 **(d)**

Consider OX, OY, OZ and Ox, Oy, Oz are two system of rectangular axes.

Let their corresponding equation of plane be

$\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$ .... (i)

and $\dfrac{x}{a'}+\dfrac{y}{b'}+\dfrac{z}{c'}=1$ .... (ii)

Length of perpendicular from origin to Eqs. (i) and (ii) must be same

$$\therefore \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}=\frac{1}{\sqrt{\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}}}$$

$$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}=0$$

84 **(b)**

Let the equation of any plane passing through $P(2,-1,3)$ is

$a(x-2)+b(y+1)+c(z-3)=0$ .....(i)

$\therefore$ DR's of $OP=2,-1,3$

Since, the line OP is perpendicular to the plane, therefore the DR's of the normal to the plane is proportional to the DR's of OP.

$\therefore$ Required equation of plane is

$2(x-2)-1(y+1)+3(z-3)=0$

$\Rightarrow 2x-y+3z-14=0$

85 **(a)**

Direction cosines

$$=\left(\frac{6}{\sqrt{36+4+9}},\frac{2}{\sqrt{36+4+9}},\frac{3}{\sqrt{36+4+9}}\right)$$

$$=\left(\frac{6}{7},\frac{2}{7},\frac{3}{7}\right)$$

86 **(d)**

Given, equation can be rewritten as

$$\frac{x}{18/3}-\frac{y}{18/2}-\frac{z}{18}=1$$

$$\Rightarrow \frac{x}{6}-\frac{y}{9}-\frac{z}{18}=1$$

$\therefore$ Points of coordinates axes are

$A(6,0,0),B(0,-9,0)$and $C(0,0,-18)$

$\therefore$ Centroid of a triangle

$$=\left(\frac{6+0+0}{3},\frac{0-9+0}{3},\frac{0+0-18}{3}\right)$$

$=(2,-3,-6)$

87 **(a)**

Equation of plane is$\frac{x}{8}+\frac{y}{4}+\frac{z}{4}=1$

$\Rightarrow x+2y+2z=8$

Length of perpendicular from origin to

$x+2y+2z-8=0$

$$=\left|\frac{-8}{\sqrt{1+4+4}}\right|=\frac{8}{3}$$

88 **(d)**

Given equation of sphere is

$$x^2+y^2+z^2-3x+y-2z-\frac{1}{2}=0$$

where centre is $\left(\frac{3}{2},-\frac{1}{2},1\right)$

and radius of sphere is $\sqrt{\frac{9}{4}+\frac{1}{4}+1+\frac{1}{2}}=2$

equation of family of concentric sphere is

$x^2+y^2+z^2-3x+y-2z+\lambda=0$ ....(i)

$\therefore$ According to question,

$$\sqrt{\frac{9}{4}+\frac{1}{4}+1-\lambda}=4$$

$\Rightarrow \frac{14}{4}-\lambda=16$

$$\Rightarrow \lambda=-\frac{25}{2}$$

$\therefore$ From Eq. (i),

$$x^2+y^2+z^2-3x+y-2z-\frac{25}{2}=0$$

$\Rightarrow 2x^2+2y^2+2z^2-6x+2y-4z-25=0$

89 **(b)**

Equation of first line is $\frac{x-4}{2}=\frac{y+1}{1}=\frac{z}{-3}$ and second line is $\frac{x-1}{1}=\frac{y+1}{-3}=\frac{z-2}{2}$

Angle between the lines

$$\theta=\cos^{-1}\left(\left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|\right)$$

$$=\cos^{-1}\left(\left|\frac{2-3-6}{\sqrt{14}\sqrt{14}}\right|\right)=\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$$

90 **(d)**

Since, $P+\lambda P'=0$ ....(i)

$$\Rightarrow ax+by+cz+d+\lambda(a'x+b'y+c'z+d')=0$$

For parallel to $x-axis$, coefficient of $x=0$

$$\Rightarrow a+\lambda a'=0 \Rightarrow \lambda=-\frac{a}{a'}$$

$\therefore$ From Eq. (i), we get

$$P-\frac{a}{a'}P'=0$$

$$\Rightarrow \frac{P}{a}=\frac{P'}{a'}$$

91 **(a)**

Equation of the plane passing through three points A,B,C with position vectors a,b,c is

$\vec{\mathbf{r}}\cdot(\vec{\mathbf{a}}\times\vec{\mathbf{b}}+\vec{\mathbf{b}}\times\vec{\mathbf{c}}+\vec{\mathbf{c}}\times\vec{\mathbf{a}})=\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}\times\vec{\mathbf{c}}$

So, that if a,b,c represents the given vectors, then

$(\vec{\mathbf{a}}\times\vec{\mathbf{b}}+\vec{\mathbf{b}}\times\vec{\mathbf{c}}+\vec{\mathbf{c}}\times\vec{\mathbf{a}})$

$$=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ -6 & 3 & 2\\ 3 & -2 & 4\end{vmatrix}+\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 3 & -2 & 4\\ 5 & 7 & 3\end{vmatrix}+\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 5 & 7 & 3\\ -6 & 3 & 2\end{vmatrix}$$

$$=[\hat{\mathbf{i}}(12+4-6-28+14-9) -\hat{\mathbf{j}}(-24-6+9-20+10+18) +\hat{\mathbf{k}}(12-9+21+10+15+42)]$$

$=-13\hat{\mathbf{i}}+13\hat{\mathbf{j}}+91\hat{\mathbf{k}}$

$$\text{and } \vec{\mathbf{a}}\cdot\vec{\mathbf{b}}\times\vec{\mathbf{c}}=\begin{vmatrix}-6 & 3 & 2\\ 3 & -2 & 4\\ 5 & 7 & 3\end{vmatrix}=299$$

so, the required equation of the plane is

$\vec{\mathbf{r}}\cdot(-13\hat{\mathbf{i}}-13\hat{\mathbf{j}}+91\hat{\mathbf{k}})=299$

Or $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}} - 7\hat{\mathbf{k}}) = -23$

Or $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}} - 7\hat{\mathbf{k}}) + 23 = 0$

92 **(a)**

The equation of plane, in which the line $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ lies is

$A(x-5) + B(y-7) + C(z+3) = 0$ ...(i)

Where A, B and C are the direction ratios of the plane. Since, the first line lie on the plane

$\therefore$ Direction ratios of normal to the plane is perpendicular to the direction ratios of line *ie*,

$4A + 4B - 5C = 0$ ....(ii)

Since, line $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$ lies in this plane. The direction ratios is also perpendicular to this line

$\therefore 7A + B + 3C = 0$ ...(iii)

From Eqs. (ii) and (iii), we get

$$\frac{A}{17} = \frac{B}{-47} = \frac{C}{-24}$$

$\therefore$ The required equation of plane is

$17(x-5) - 47(y-7) + (-24)(z+3) = 0$

$\Rightarrow 17x - 47y - 24z + 172 = 0$

93 **(a)**

The vector equation of the plane passing through points $\vec{a}, \vec{b}, \vec{c}$ is

$\vec{r} \cdot (\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}) = [\vec{a}\ \vec{b}\ \vec{c}]$

Therefore, the length of the perpendicular from the origin to this plane is given by

$$\frac{[\vec{a}\ \vec{b}\ \vec{c}]}{|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}$$

94 **(b)**

$\because$ Centre of sphere is $(0, 0, 0)$, then the shortest distance between $(1, 2, -1)$ and surface of sphere

$= \sqrt{24} - \sqrt{6} = 2\sqrt{6} - \sqrt{6} = \sqrt{6}$

95 **(a)**

Given, $a = 4, c = 3$

Equation of the plane π is

$$\frac{x}{4} + \frac{y}{b} + \frac{z}{3} = 1$$

Since, π is parallel to y-axis

$\therefore$ Coefficient of $y = 0$ *ie*, $\frac{1}{b} = 0$

Thus, the equation of plane π is

$$\frac{x}{4} + \frac{z}{3} = 1$$

$\Rightarrow 3x + 4z - 12 = 0$

96 **(a)**

The equation of a line passing through $A(2, -3, -1)$ and $B(8, -1, 2)$ is

$$\frac{x-2}{6} = \frac{y+3}{2} = \frac{z-1}{3} \Rightarrow \frac{x-2}{\frac{6}{7}} = \frac{y+3}{\frac{2}{7}} = \frac{z-1}{\frac{3}{7}}$$

The coordinates of points on this line at a distance r from A are given by $\left(2 \pm \frac{6r}{7}, -3 \pm \frac{2r}{7}, 1 \pm \frac{3r}{7}\right)$

Putting $r = 14$, we get the required points as $(4, 1, 5)$ and $(-10, -7, -7)$

97 **(c)**

Equation of line which is passing through (α, β, γ) and perpendicular to plane

$ax + by + cz + d = 0$

is $\frac{x-\alpha}{a} = \frac{y-\beta}{b} = \frac{z-\gamma}{c}$

99 **(d)**

Let $A = (1, 0, 0) B = (0, 1, 0)$ and $C = (0, 0, 1)$

Now, $AB = \sqrt{(0-1)^2 + (1-0)^2 + 0^2} = \sqrt{2}$

$BC = \sqrt{0^2 + (0-1)^2 + (1-0)^2} = \sqrt{2}$

and $CA = \sqrt{(1-0)^2 + 0^2 + (0-1)^2} = \sqrt{2}$

$\therefore$Perimeter of trangle$= AB + BC + CA$

$= \sqrt{2} + \sqrt{2} + \sqrt{2} = 3\sqrt{2}$

100 **(a)**

Distance of point P from plane=5

$\therefore 5 \left|\frac{1-4-2-\alpha}{3}\right|$

$\alpha = 10$

Foot perpendicular

$$\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-1}{-2} - \frac{(1-4-2-10)}{1+4+4} = \frac{5}{3}$$

$\Rightarrow x = \frac{8}{3}, y = \frac{4}{3}, z - \frac{7}{3}$

Thus, the foot of the perpendicular is

$A\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$

101 **(b)**

Given, $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = \lambda$

and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1} = \mu$

$\Rightarrow x = 2\lambda + 1, y = 3\lambda - 1, z = 4\lambda + 1$

and $x = \mu + 3, y = 2\mu + k, z = \mu$

As the lines intersect they must have a point in common.

$\therefore 2\lambda + 1 = \mu + 3, 3\lambda - 1 = 2\mu + k, 4\lambda + 1 = \mu$

$\Rightarrow \lambda = -\frac{3}{2}$ and $\mu = -5$

$\therefore k = 3\lambda - 2\mu - 1$

$\Rightarrow k = 3\left(-\frac{3}{2}\right) - 2(-5) - 1$

$\Rightarrow k = \frac{9}{2}$

102 **(d)**

Let the point on x-axis is $A(x, 0, 0)$

Given, $B = (1, 2, 3,)$ and $C = (3, 5, -2)$

Since, $|AB| = |AC|$

$\Rightarrow \sqrt{(x-1)^2+(0-2)^2+(0-3)^2}$
$= \sqrt{(x-3)^2+(0-5)^2+(0+2)^2}$
$\Rightarrow x^2+1-2x+4+9 = x^2+9-6x+25+4$
$\Rightarrow x = 6$
$\therefore$ Required point is $(6,0,0)$

103 **(c)**
Angle between two lines given by

$$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\therefore \cos\theta \frac{1\times 3+2\times -2+3\times 1}{\sqrt{1^2+2^2+3^2}\sqrt{3^2+(-2)^2+1^2}}$$

$$= \frac{2}{\sqrt{14}\sqrt{14}}$$

$\therefore\ \theta = \cos^{-1}\left(\frac{1}{7}\right)$

104 **(a)**
Let the give points are A, B and C respectively
$\therefore$ Direction ratios of AB and BC are $1, -3, -2$ and $2, -6, K-2$ respectively
Since given points are collinear

$$\therefore \frac{2}{1} = \frac{-6}{-3} = \frac{K-2}{-2}$$

$\Rightarrow K-2 = -4$
$\Rightarrow K = -2$

105 **(d)**
Equation of plane through $(2, 5, -3)$ is
$a(x-2)+b(y-5)+c(z+3) = 0$ ...(i)
Which is perpendicular to
$x+2y+2z = 1$
and $x-2y+3z = 4$
then $a+2b+2c = 0$ ...(ii)
and $a-2b+3c = 0$ ...(iii)
On eliminating a, b, c from Eqs.(i), (ii) and (iii), we get

$$\begin{vmatrix} x-2 & y-5 & z+3 \\ 1 & 2 & 2 \\ 1 & -2 & 3 \end{vmatrix} = 0$$

$\Rightarrow 10x-y-4z-27 = 0$

106 **(c)**
Given line can be rewritten as

$$\frac{x-1}{\frac{1}{4}} = \frac{y-\frac{1}{3}}{-\frac{1}{3}} = \frac{z-\frac{1}{2}}{\frac{1}{2}}$$

$\therefore$ Direction cosines are

$$\frac{\frac{1}{4}}{\sqrt{\frac{1}{16}+\frac{1}{9}+\frac{1}{4}}}, \frac{\frac{-1}{3}}{\sqrt{\frac{1}{16}+\frac{1}{9}+\frac{1}{4}}}, \frac{\frac{1}{2}}{\sqrt{\frac{1}{16}+\frac{1}{9}+\frac{1}{4}}}$$

$$= \frac{3}{\sqrt{16}}, \frac{-4}{\sqrt{16}}, \frac{6}{\sqrt{16}}$$

107 **(a)**
Equation of required plane is
$(x+y+z-6)+\lambda(2x+3y+4z+5) = 0$
Which is passing through $(1,1,1)$
$\Rightarrow -3+14\lambda = 0$
$\Rightarrow \lambda = \frac{3}{14}$
$\therefore$Required plane is $20x+23y+26z = 69$

108 **(c)**
Equation of plane through $(0,-4,-6)$ is
$a(x-0)+b(y+4)+c(z+6) = 0$ ...(i)
Point $(-2, 9, 3)$ lies on Eq. (i), then
$-2a+13b+9c = 0$ ...(ii)
Also required plane is perpendicular to
$x-4y-2z = 8$
$\therefore a-4b-2c = 0$ ...(iii)
From Eqs. (i), (ii), (iii) we get

$$\begin{vmatrix} x & y+4 & z+6 \\ -2 & 13 & 9 \\ 1 & -4 & -2 \end{vmatrix} = 0$$

$ie, 2x+y-z-2 = 0$

109 **(b)**
Let α, β, γ be the angles with x-axis, z-axis respectively, then direction cosines are
$\cos\alpha, \cos\beta,$ and $\cos\gamma$
Given, $\alpha = \frac{\pi}{3}, \quad \beta = \frac{\pi}{4}$
$\therefore l = \cos\frac{\pi}{3} = \frac{1}{2},$

$$m = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} \text{ and } n = \cos\gamma$$

Using $l^2+m^2+n^2 = 1$
$\Rightarrow \frac{1}{4}+\frac{1}{2}+n^2 = 1 \Rightarrow n = \frac{1}{2}$
$\therefore \cos\gamma = \frac{1}{2} \Rightarrow \gamma = \frac{\pi}{3}$

110 **(c)**
Given $(3,4,-1)$and $(-1,2,3)$ are the end points of diameter of sphere
$\therefore$ Radius $= \frac{1}{2}$(length of the diameter)
$= \frac{1}{2}\sqrt{(3+1)^2+(4-2)^2+(-1-3)^2}$
$= 3$

111 **(c)**

Let $A(5,-4,2), B(4,-3,1), C(7,-6,4)$ and $D(8,-7,5)$

Then, $AB=\sqrt{(4-5)^2+(-3+4)^2+(1-2)^2}$

$=\sqrt{1+1+1}=\sqrt{3}$

$BC\sqrt{(7-4)^2+(-6+3)^2+(4-1)^2}$

$=\sqrt{9+9+9}=3\sqrt{3}$

$CD=\sqrt{(8-7)^2+(-7+6)^2+(5-4)^2}$

$=\sqrt{1+1+1}=\sqrt{3}$

$AD=\sqrt{(8-5)^2+(-7+4)^2+(5-2)^2}$

$=\sqrt{9+9+9}=3\sqrt{3}$

Position vector f $\overrightarrow{\mathbf{AB}}=(4-5)\hat{\mathbf{i}}+(-3+4)\hat{\mathbf{j}}+(1-2)\hat{\mathbf{k}}$

$=-\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$

And position vector of $\overrightarrow{\mathbf{BC}}=(7-4)\hat{\mathbf{i}}+(-6+3)\hat{\mathbf{j}}+(4-1)\hat{\mathbf{k}}$

$=3\hat{\mathbf{i}}-3\hat{\mathbf{j}}+3\hat{\mathbf{k}}$

Now, $\overrightarrow{\mathbf{AB}}\cdot\overrightarrow{\mathbf{BC}}=\left(-\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}\right).(3\hat{\mathbf{i}}-3\hat{\mathbf{j}}+3\hat{\mathbf{k}})$

$=-3-3-3\neq 0$

$\therefore \square ABCD$ is parallelogram

112 **(b)**

The required plane passes through the points having position vectors $\overrightarrow{a_1}$ and $\overrightarrow{a_2}$ and is parallel to the vector $\vec{b}$. Therefore, it is normal to the vector $(\overrightarrow{a_2}-\overrightarrow{a_1})\times\vec{b}$

So, the equation of the required plane is

$(\vec{r}-\overrightarrow{a_1})\cdot\{(\overrightarrow{a_2}-\overrightarrow{a_1})\times\vec{b}\}=0$

$\Rightarrow \vec{r}\cdot(\overrightarrow{a_2}-\overrightarrow{a_1})\times\vec{b}-\overrightarrow{a_1}\cdot\left(\overrightarrow{a_2}\times\vec{b}\right)=0$

$\Rightarrow \vec{r}\cdot(\overrightarrow{a_2}-\overrightarrow{a_1})\times\vec{b}=\left[\overrightarrow{a_1}\,\overrightarrow{a_2}\,\vec{b}\right]$

113 **(a)**

If $\left(\frac{1}{2},\frac{1}{3},n\right)$ are the DC's of line, then using the relation $l^2+m^2+n^2=1$, we get

$\left(\frac{1}{2}\right)^2+\left(\frac{1}{3}\right)^2+n^2=1$

$\Rightarrow n^2=1-\frac{1}{4}-\frac{1}{9}$

$\Rightarrow n^2=\frac{23}{36}$

$\Rightarrow n=\frac{\sqrt{23}}{6}$

114 **(b)**

The equation of a plane through the line of intersection of the planes $\vec{r}\cdot\vec{a}=\lambda$ and $\vec{r}\cdot\vec{b}=\mu$ can be written as

$(\vec{r}\cdot\vec{a}-\lambda)+k\left(\vec{r}\cdot\vec{b}-\mu\right)=0$

Or, $\vec{r}\cdot\left(\vec{a}+k\vec{b}\right)=\lambda+k\mu$ ... (i)

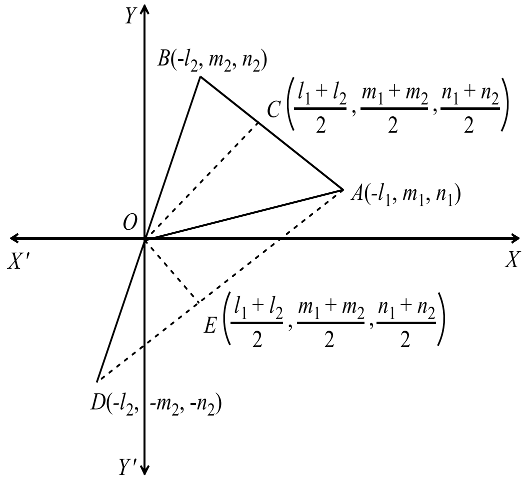
This passes through the origin

$\therefore \vec{0}\cdot\left(\vec{a}+k\vec{b}\right)=\lambda+\mu\,k\Rightarrow k=\frac{-\lambda}{\mu}$

Putting the value of k in (i), we get the equation of the required plane as

$\vec{r}\cdot\left(\mu\vec{a}-\lambda\vec{b}\right)=0\Rightarrow\vec{r}\cdot\left(\lambda\vec{b}-\mu\vec{a}\right)=0$

115 **(c)**



In Fig. OE is the external bisector

The co-ordinates of E are $\left(\frac{l_1-l_2}{2}, \frac{m_1-m_2}{2}, \frac{n_1-n_2}{2}\right)$

Therefore, direction ratios of OE are proportional to

$\frac{l_1 - l_2}{2}, \frac{m_1 - m_2}{2}, \frac{n_1 - n_2}{2}$

116 **(d)**

The equation of a plane passing through $(1, -2, 3)$ is

$a(x-1) + b(y+2) + c(z-3) = 0$

It passes through $(-1, 2, -1)$ and is parallel to the given line

$\therefore a(-2) + b(4) + c(-4) = 0$ and, $2a + 3b + 4c = 0$

$\Rightarrow \frac{a}{28} = \frac{b}{0} = \frac{c}{-14} \Rightarrow \frac{a}{2} = \frac{b}{0} = \frac{c}{-1}$

Hence, $a : b : c = 2 : 0 : -1$

ALITER Let $P(1, -2, 3)$ and $Q(-1, 2, -1)$ be the given points

Given line is parallel to the vector $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$

$\therefore$ Normal to the plane is parallel to the vector

$\vec{P}Q \times \vec{b} = 28\hat{\imath} - 14\hat{k} = 14(2\hat{\imath} + 0\hat{\jmath} - \hat{k})$

117 **(b)**

The equation of a line passing through the points $A(\hat{\imath} - \hat{\jmath} + 2\hat{k})$ and $B(3\hat{\imath} + \hat{\jmath} + \hat{k})$ is given by

$\vec{r} = (\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \lambda(3\hat{\imath} + \hat{\jmath} + \hat{k})$

The position vector of a variable point P on the line, is $(\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \lambda(3\hat{\imath} + \hat{\jmath} + \hat{k})$

$\therefore \vec{A}P = \lambda(3\hat{\imath} + \hat{\jmath} + \hat{k}) \Rightarrow |\vec{A}P| = |\lambda|\sqrt{11}$

Now, $|\lambda|\sqrt{11} = 3\sqrt{11}, \Rightarrow \lambda = \pm 3$

Thus, the position vectors of P are

$10\hat{\imath} + 2\hat{\jmath} + 5\hat{k}$ and $-8\hat{\imath} - 4\hat{\jmath} - \hat{k}$

118 **(c)**

The given equation of sphere is

$x^2 + y^2 + z^2 + 3x - 4z + 1 = 0$

$\therefore$ Coordinates of centre of sphere$= \left(-\frac{3}{2}, 0, 2\right)$

and radius of sphere $= \sqrt{u^2 + v^2 + w^2 - d}$

$= \sqrt{\frac{9}{4} + 4 - 1} = \frac{\sqrt{21}}{2}$

120 **(c)**

It is given that the line

$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{2}$

Is perpendicular to the required. This means that the normal to the plane is parallel to the line. So, its direction ratios are proportional to $2, -1, 2$

The plane passes through the origin

Hence, its equation is

$2(x-0) - (y-0) + 2(z-0) = 0 \Rightarrow 2x - y + 2z = 0$

121 **(b)**

Given equation of lines are

$\frac{x-5}{3} = \frac{y-7}{-1} = \frac{z+2}{1} = k$ [say] …..(i)

and $\frac{x+3}{-36} = \frac{y-3}{2} = \frac{z-6}{4}$ (ii)

Any point on the line (i) is $P(3k+5, -k+7, k-2$

This point is satisfied the Eq. (ii),

$\therefore \frac{3k+5+3}{-36} = \frac{-k+7-3}{2} = \frac{k-2-6}{4}$

$\Longrightarrow \frac{3k+8}{-36} = \frac{-k+4}{2} = \frac{k-8}{4}$

$\Longrightarrow 3k + 8 = 18k - 72 \Longrightarrow k = \frac{16}{3}$

$\therefore P\left(16 + 5, -\frac{16}{3} + 7, \frac{16}{3} - 2\right)$

$ie, P\left(21, \frac{5}{3}, \frac{10}{3}\right)$

122 **(a)**

We have, $\vec{AB} = -2\hat{\imath} - 3\hat{\jmath} - 6\hat{k}$

So, vector equation of the plane is

$\{\vec{r} - (\hat{\imath} - 2\hat{\jmath} - 4\hat{k})\} \cdot \vec{AB} = 0$

$\Rightarrow \vec{r} \cdot (-2\hat{\imath} - 3\hat{\jmath} - 6\hat{k})$

$= (\hat{\imath} - 2\hat{\jmath} - 4\hat{k}) \cdot (-2\hat{\imath} - 3\hat{\jmath} - 6\hat{k})$

$\Rightarrow -2x - 3y - 6z = -2 + 6 + 24$

$\Rightarrow 2x + 3y + 6x + 28 = 0$

123 **(c)**

Let point is (α, β, γ)

$\therefore (\alpha - \alpha)^2 + \beta^2 + \gamma^2 = \alpha^2 + (\beta - b)^2 + \gamma^2$

$= \alpha^2 + \beta^2 + (\gamma - c)^2$

$= \alpha^2 + \beta^2 + \gamma^2$

We get, $\alpha = \frac{a}{2}, \beta = \frac{b}{2}$ and $\gamma = \frac{c}{2}$

$\therefore$ Required point is $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$

124 **(b)**

Let equation of plane is $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 1$, then

$A(\alpha, 0, 0), B(0, \beta, 0)$ and $C(0,0,\gamma)$ are the points on coordinate axes

Since, the centroid of a triangle is (1, 2, 4)

Now, $\frac{\alpha}{3} = 1$

$\therefore \alpha = 3, \frac{\beta}{3} = 2 \Rightarrow \beta = 6$

And $\frac{\gamma}{3} = 4 \Rightarrow \gamma = 12$

$\therefore$ Equation of plane is

$\frac{x}{3} + \frac{y}{6} + \frac{z}{12} = 1$

$\Rightarrow 4x + 2y + z = 12$

125 **(c)**

$\because$ Vertices of ΔABC are $A(-1, 3, 2), B(2, 3, 5)$ and $C(3, 5, -2)$

$\Rightarrow AB = \sqrt{9 + 0 + 9} = \sqrt{18}$

$CA = \sqrt{16 + 4 + 16} = 6$

And $BC = \sqrt{1 + 4 + 49} = \sqrt{54}$

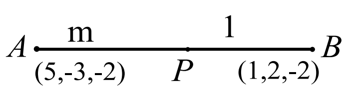
$\because AB^2 + CA^2 = BC^2$

$\Delta\, ABC$ is right angled triangle at A

$\therefore \angle A = 90°$

127 **(a)**

Let the point $P(x, y, z)$ divides the line joining the points A and B in the ratio m: 1.



Since, point P is on XOZ-plane

$\therefore y$ coordinate $= 0$

$\Rightarrow \frac{2m - 3}{m + 1} = 0 \Rightarrow m = \frac{3}{2}$

Now, $x = \frac{3 + 2 \times 5}{3 + 2} = \frac{13}{5}$

and $z = \frac{3 \times (-2) + 2 \times (-2)}{5} = -2$

$\therefore$ Required points is $\left(\frac{13}{5}, 0, -2\right)$

128 **(d)**

Let the equation of plane is $-\frac{x}{6} + \frac{y}{3} + \frac{z}{4} = 1$

$\therefore$ The perpendicular distance from origin to the above plane

$= \frac{|0 + 0 + 0 - 1|}{\sqrt{\left(\frac{1}{6}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{4}\right)^2}}$

$= \frac{1}{\sqrt{\frac{4+16+9}{144}}}$

$= \frac{12}{\sqrt{29}}$

129 **(b)**

Equation of plane is $a(x - 1) + b((y + 1) + cz = 0$

($\because$ plane is passing through$(1,2,-1)$)

Above plane also passing through $(0, 2, -1)$

$\therefore -a + 3b - c = 0$

Also $2a - b + 3c = 0$

$\Rightarrow \frac{a}{8} = \frac{b}{1} = \frac{c}{-5}$

Hence, equation of plane is

$8x + y - 5z - 7 = 0$

130 **(c)**

$\because$ Mid point of line joining (2, 3, 4) and (6, 7, 8) is (4, 5, 6). This point satisfied the equation

$x + y + z - 15 = 0$

$\therefore x + y + z - 15 = 0$ is required equation of plane

131 **(c)**

The distance between given points

$= \sqrt{(2 - 1)^2 + (2 - 4)^2 + (3 - 5)^2}$

$= \sqrt{1 + 4 + 4} = 3$

132 **(b)**

Equation of plane through (1, 2, 3) is

$a(x - 1) + b(y - 2) + c(z - 3) = 0$ ....(i)

$\because$ It passes through $(-1, 4,2)$ and $(3, 1, 1)$

$\therefore -2a + 2b - c = 0$ and $2a - b - 2c = 0$

$\Rightarrow \frac{a}{-5} = \frac{b}{-6} = \frac{c}{-2}$

$\therefore$ Equation of plane is

$-5x - 6y - 2z + 5 + 12 + 6 = 0$

$\Rightarrow 5x + 6y + 2z - 23 = 0$

Alternate

Equation plane is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & y-2 & z-3 \\ -2 & 2 & -1 \\ 2 & -1 & -2 \end{vmatrix} = 0$$

$\Rightarrow (x-1)(-4-1)-(y-2)(4+2)$
$\qquad + (z-3)(2-4) = 0$

$\Rightarrow -5x+5-6y+12-2z+6=0$

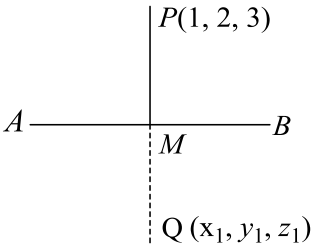
$\Rightarrow 5x+6y+2z-23=0$

133 **(a)**

Given planes are parallel to each other but only $x+y+3z-6=0$ is equidistant from $x+2y+3z-5=0$ and $x+2y+3z-7=0$ having distance $\frac{1}{\sqrt{14}}$

134 **(c)**

Equation of given line is $\frac{x}{2}=\frac{y-1}{3}=\frac{z-1}{3}=k$ (say)



Any point on the line is $M(2k, 3k+1, 3k+1)$

Direction ratio of PM are $(2k-1, 3k-1, 3k-2)$

since, the line PMis perpendicular to AB

$\therefore 2(2k-1)+3(3k-1)+3(3k-2)=0$

$\Rightarrow 22k-11=0$

$\Rightarrow k=\frac{1}{2}$

$\therefore$ Point M is $\left(1, \frac{5}{2}, \frac{5}{2}\right)$

Let the image of P about the line AB is Q, where M is the mid point of PQ

$\therefore \frac{x_1+1}{2}=1, \frac{y_1+2}{2}=\frac{5}{2}, \frac{z_1+3}{2}=\frac{5}{2}$

$\Rightarrow x_1=1, y_1=3, z_1=2$

135 **(b)**

The equation of straight line passing through origin and direction cosine (l, m, n) is

$\frac{x}{l}=\frac{y}{m}=\frac{z}{n}=r$ (say)

Coordinates of any point P are (lr, mr, nr)

Here, $l=\frac{-1}{\sqrt{1^2+2^2+2^2}}=\frac{-1}{3}, m=\frac{2}{3}, n=\frac{-2}{3}$

and $r=3$ (given)

$\therefore$ Coordinates of P are $(-1, 2, -2)$

136 **(b)**

Since, the given sphere touching the three coordinates planes. So, it is clear that centre is (a, a, a) and radius is a

$\therefore$ The equation of sphere at the centre (a, a, a) and radius a is

$(x-a)^2+(y-a)^2+(z-a)^2=a^2$

$\Rightarrow x^2+y^2+z^2-2ax-2ay-2az+3a^2=a^2$

$\therefore x^2+y^2+z^2-2a(x+y+z)+2a^2=0$ is the required equation of sphere

137 **(b)**

Angle between the plane and line is given by

$$\sin\theta = \frac{aa'+bb'+cc'}{\sqrt{a^2+b^2+c^2}\sqrt{a'^2+b'^2+c'^2}}$$

$$\therefore \sin\theta = \frac{2\times\frac{3}{4}+3\times\frac{2}{4}-4\times\frac{3}{4}}{\sqrt{2^2+3^2+4^2}\sqrt{\left(\frac{3}{4}\right)^2+\left(\frac{2}{4}\right)^2+\left(\frac{-3}{4}\right)^2}}$$

$$= \frac{\frac{6}{4}+\frac{6}{4}-\frac{12}{4}}{\sqrt{4+9+16}\sqrt{\frac{9}{16}+\frac{4}{16}+\frac{9}{16}}} = 0$$

$\therefore \sin\theta = \sin 0°$

$\Rightarrow \theta = 0°$

138 **(b)**

Given that equation of planes are,

$4x+4y-5z=12$ ...(i)

And $8x+12y-13z=32$ ...(ii)

Let direction ratios of the line are (l, m, n)

$\therefore$ Eqs. (i) and (ii) becomes

$4l+4m-5n=0$ ...(iii)

And $8l+12m-13n=0$ ...(iv)

$\Rightarrow \frac{l}{8}=\frac{m}{12}=\frac{n}{16} \Rightarrow \frac{l}{2}=\frac{m}{3}=\frac{n}{4}$

Now, we take intersection point with $z=0$ given by

$4x+4y=12$ ...(v)

and $8x+12y=32$ ...(vi)

On solving Eqs. (v) and (vi), we get (1, 2, 0)

$\therefore$ Required line is $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z}{4}$

139 **(b)**

DR's of given line are $(3, -5, 2)$

DR's of normal to the plane $=(1, 3, -\alpha)$

$\therefore$ Line is perpendicular to the normal

$\Rightarrow 3(1)-5(3)+2(-\alpha)=0$

$\Rightarrow 3-15-2\alpha=0$

$\Rightarrow 2\alpha=-12$

$\Rightarrow \alpha=-6$

Also point $(2, 1, -2)$ lies on the plane

$2+3+6(-2)+\beta=0$

$\Rightarrow \beta=7$

$\therefore (\alpha, \beta)=(-6, 7)$

140 **(b)**

We know

$\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3}$

where α, β, γ and δ are the angles with diagonals of cube.

$\therefore 1-\sin^2\alpha+1-\sin^2\beta+1-\sin^2\gamma+1-\sin^2\delta$

$$=\frac{4}{3}$$

$\Rightarrow \sin^2\alpha+\sin^2\beta+\sin^2\gamma+\sin^2\delta=\frac{8}{3}$

141 **(c)**

Given equation of line is

$$\frac{3-x}{1}=\frac{y-2}{5}=\frac{2z-3}{1}$$

$$\Rightarrow \frac{x-3}{-1}=\frac{y-2}{5}=\frac{z-\frac{3}{2}}{\frac{1}{2}}$$

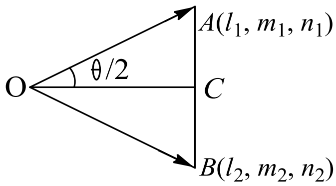
$\therefore$ Direction ratios of line are $-1,5,\frac{1}{2}$

142 **(b)**

$\because \overrightarrow{\mathbf{OC}}=\left(\frac{l_1+l_2}{2},\frac{m_1+m_2}{2},\frac{n_1+n_2}{2}\right)$

And $|\overrightarrow{\mathbf{OC}}|=\cos\frac{\theta}{2}$

So, direction cosines of internal angle bisector are



$$\frac{l_1+l_2}{2\cos\frac{\theta}{2}},\frac{m_1+m_2}{2\cos\frac{\theta}{2}},\frac{n_1+n_2}{2\cos\frac{\theta}{2}}$$

143 **(c)**

The given equation of plane is $\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=1$

On comparing with $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, we get

$a=2, b=3, c=4$

Area of $\Delta\, ABC=\frac{1}{2}\sqrt{a^2b^2+b^2c^2+c^2a^2}$

$$\Delta=\frac{1}{2}\sqrt{4\times 9+9\times 16+16\times 4}$$

$$=\frac{1}{2}\sqrt{36+144+64}=\frac{1}{2}\sqrt{244}=\sqrt{61}$$

145 **(a)**

Let (u,v,w) be the centre of the sphere with radius r. Since, it passes through the origin

$\therefore u^2+v^2+w^2=r^2$. Equation of the diameter parallel to x-axis is

$\frac{x-u}{1}=\frac{y-v}{0}=\frac{z-w}{0}$ ...(i)

As it passes through u,v,w and direction ratios of x- axis are 1, 0, 0

The extremities of diameter are the points on Eq. (i) at a distance r from the centre (u,v,w)

$\therefore$ The required extremities are $P(r+u,v,w)$ and $Q(-r+u,v,w)$

P lies on the sphere $x^2+y^2+z^2-2rx=0$ as

$(r+u)^2+v^2+w^2-2r(r+u)=0$

Because $u^2+v^2+w^2=r^2$

and similarly Q lies on the sphere $x^2+y^2+z^2+2rx=0$

146 **(d)**

Distance of a point (1,1,1) from $x+y+z+k=0$ is

$$\left|\frac{1+1+1+k}{\sqrt{3}}\right|=\left|\frac{3+k}{\sqrt{3}}\right|$$

According to question

$$\left|\frac{3+k}{\sqrt{3}}\right|=\pm 2\sqrt{3} \Rightarrow k=3,-9$$

147 **(c)**

Since, given points divide the XOZ-plane.

$\therefore$ Required ratio $=-y_1:y_2=-3:7$

148 **(b)**

DC's of the given line are $\frac{1}{3},-\frac{2}{3},-\frac{2}{3}$

Hence, the equation of line can be point in the form

$$\frac{x-2}{1/3}=\frac{y+3}{-2/3}=\frac{z+5}{-2/3}=r$$

$\therefore$ Point is $\left(2+\frac{r}{3},-3-\frac{2r}{3},-5-\frac{2r}{3}\right)$

$\therefore r=\pm 6$

Points are $(4,-7,-9)$ and $(0,1,-1)$

149 **(a)**

The plane passes through $A(0,0,1), B(0,1,2)$ and $C(1,2,3)$. Therefore, a vector normal to the plane is given by

$$\vec{n}=\vec{A}B\times\vec{A}C=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 0 & 1 & 1\\ 1 & 2 & 2\end{vmatrix}=0\,\hat{\imath}+\hat{\jmath}-\hat{k}$$

Hence, direction ratios of normal to the plane are proportional to $0,1,-1$

150 **(b)**

Suppose xy-plane divides the join of (1, 2, 3) and (4, 2, 1) in the ratio $\lambda : 1$. Then, the coordinates of the point of division are

$$\left(\frac{4\,\lambda+1}{\lambda+1},\frac{2\,\lambda+2}{\lambda+1},\frac{\lambda+3}{\lambda+1}\right)$$

This point lies on xy-plane

$\therefore z$-coordinate $=0 \Rightarrow \frac{\lambda+3}{\lambda+1}=0 \Rightarrow \lambda=-3$

Hence, xy-plane divides the join of (1, 2, 3) and (4, 2, 1) externally in the ratio 3 : 1

ALTER We know that the XY-plane divides the segment joining $P(x_1,y_1,z_1)$ and $Q(x_2,y_2,z_2)$ in the ratio $(-z_1):z_2$

$\therefore XY$-plane divides the join of (1, 2, 3) and (4, 2, 1) in the ratio $-3:1$ i.e. $3:1$ externally

151 **(d)**

Let l,m,n be the direction cosines of $\vec{r}$. Then,

$l=m=n$ [Given]

$\therefore l^2 + m^2 + n^2 = 1 \Rightarrow 3\, l^2 = 1 \Rightarrow l = \frac{1}{\sqrt{3}} = m$
$= n$

Now, $\vec{r} = |\vec{r}|(l\,\hat{\imath} + m\,\hat{\jmath} + n\,\hat{k})$

$\Rightarrow \vec{r} = 6\left(\frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} + \frac{1}{\sqrt{3}}\hat{k}\right) = 2\sqrt{3}(\hat{\imath} + \hat{\jmath} + \hat{k})$

152 **(a)**

Since, direction ratio of given planes are $(2, -1, 1)$ and $(1, 1, 2)$

$\therefore\ \theta \cos^{-1}\left(\frac{2 \times 1 - 1 \times 1 + 1 \times 2}{\sqrt{4+1+1}\sqrt{1+1+4}}\right)$

$= \cos^{-1}\left(\frac{3}{\sqrt{6}\sqrt{6}}\right)$

$= \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

153 **(b)**

The equation of a plane parallel to the plane

$\vec{r} \cdot (4\hat{\imath} - 12\hat{\jmath} - 3\hat{k}) - 7 = 0$ is,

$\vec{r} \cdot (4\hat{\imath} - 12\hat{\jmath} - 3\hat{k}) + \lambda = 0$

This passes through $2\hat{\imath} - \hat{\jmath} - 4\hat{k}$

$\therefore (2\hat{\imath} - \hat{\jmath} - 4\hat{k}) \cdot (4\hat{\imath} - 12\hat{\jmath} - 3\hat{k}) + \lambda = 0$

$\Rightarrow 8 + 12 + 12 + \lambda = 0$

$\Rightarrow \lambda = -32$

So, the required plane is $\vec{r} \cdot (4\hat{\imath} - 12\hat{\jmath} - 3\hat{k}) - 32 = 0$

154 **(a)**

Equation of the give plane can be writer as

$(3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) \cdot (x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\hat{\mathbf{k}}) = 8$

So, that the normal to the given plane is $3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and the required line being perpendicular to the plane is parallel to this normal and since, it passes through $3\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$, its equation is

$\vec{\mathbf{r}} = 3\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 7\hat{\mathbf{k}} + \lambda(3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$

Where λ is a parameter

Since, this lie passes through the vector $3\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$ *ie*, the point $(3, -5, 7)$ and is parallel to $3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$, its direction ratios are $3, -4, 5$

Its cartesian equation is $\frac{x-3}{3} = \frac{y+5}{-4} = \frac{z-7}{5}$

155 **(a)**

Given lines can be rewritten as

$\frac{x - \frac{1}{3}}{1} = \frac{y - \frac{1}{3}}{2} = \frac{z - 1}{3}$

This shows that DR's of given equation are (1, 2, 3).

156 **(d)**

Given line is parallel to $\vec{b} = -\hat{\imath} + \hat{\jmath} + \hat{k}$ and the given plane is normal to $\vec{n} = 3\hat{\imath} + 2\hat{\jmath} - \hat{k}$

Let θ be the angle between the given line and given plane. Then,

$\sin\theta = \frac{\vec{b} \cdot \vec{n}}{|\vec{b}|\,|\vec{n}|}$

$\Rightarrow \sin\theta = \frac{-3 + 2 - 1}{\sqrt{3}\sqrt{14}} \Rightarrow \theta = \sin^{-1}\left(\frac{-2}{\sqrt{42}}\right)$

157 **(d)**

Let the source of light be situated at $A(a, 0, 0)$, where, $a \neq 0$

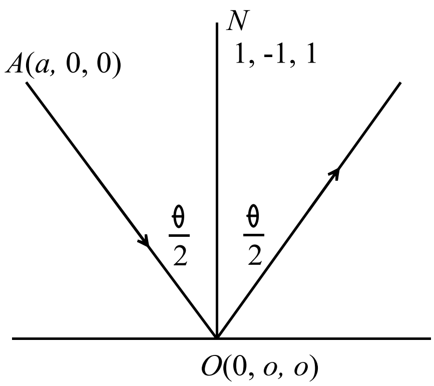
Let OA be the incident ray, OB be the reflected ray and ON be the normal to the mirror at O

$\therefore \angle AON = \angle NOB = \frac{\theta}{2}$ (say)

Direction ratios of $\vec{OA}$ are proportional to $a, 0, 0$ and so its direction cosines are $1, 0, 0$

Direction cosines of ON are $\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

$\therefore \cos\frac{\theta}{2} = \frac{1}{\sqrt{3}}$



Let l, m, n be the direction cosines of the reflected ray OB. Then,

$\frac{l + 1}{2\cos\theta/2} = \frac{1}{\sqrt{3}}, \frac{m + 0}{2\cos\theta/2} = -\frac{1}{\sqrt{3}}$ and , $\frac{n + 0}{2\cos\theta/2} = \frac{1}{\sqrt{3}}$

$\Rightarrow l = \frac{2}{3} - 1, m = -\frac{2}{3}, n = \frac{2}{3}$

$\Rightarrow l = -\frac{1}{3}, m = -\frac{2}{3}, n = \frac{2}{3}$

Hence, direction cosines of the reflected ray are $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$

158 **(d)**

Given, $3lm - 4ln + mn = 0$ .....(i)

and $l + 2m + 3n = 0$ ...(ii)

From Eq. (ii), $l = -(2m + 3n)$ putting in Eq. (i)

$-3(2m + 3n)m + 4(2m + 3n)n + mn = 0$

$\Longrightarrow -6m^2 + 12n^2 = 0$

$\Longrightarrow m = \pm\sqrt{2}n$

Now, $m = \sqrt{2}n$

$\Longrightarrow l = -(2\sqrt{2}n + 3n) = -(2\sqrt{2} + 3)n$

$\therefore l : m : n = -(3 + 2\sqrt{2})n : \sqrt{2}n : n$

$= -(3+2\sqrt{2}):\sqrt{2}:1$

Also, $\quad m = -\sqrt{2}n \Rightarrow l = -(-2\sqrt{2}+3)n$

$\therefore l:m:n = -(3-2\sqrt{2})n : -\sqrt{2}:n$

$= -(3-2\sqrt{2}):-\sqrt{2}:1$

$= \cos\theta$

$$= \frac{(3+2\sqrt{2})(3-2\sqrt{2})+(\sqrt{2})(-\sqrt{2})+1\cdot 1}{\sqrt{(3+2\sqrt{2})^2+(\sqrt{2})^2+1^2}\ \sqrt{(3-2\sqrt{2})^2+(-\sqrt{2})^2+1^2}}$$

$= 0$

$\Rightarrow \theta = \frac{\pi}{2}$

159 **(a)**

Equation of plane passing through $(-1,3,0)$ is

$A(x+1)+B(y-3)+C(z-0)=0$ ....(i)

Also, plane (i) is passing through the points (2, 2, 1) and (1,1,3)

$3A-B+C=0$ ...(ii)

And $2A-2B+3C=0$ ...(iii)

On solving Eqs. (i) and (iii), we get

$$\frac{A}{-3+2}=\frac{B}{2-9}=\frac{C}{-6+2}$$

$\therefore A:B:C = -1:-7:-4$

$\Rightarrow A:B:C = 1:7:4$

From Eq. (i), $1(x+1)+7(y-3)+4(z)=0$

$\Rightarrow x+7y+4z-20=0$

$\therefore$ Distance from the plane to the point (5, 7, 8)

$$= \frac{1\times 5+7\times 7+4\times 8-20}{\sqrt{1^2+7^2+4^2}}$$

$$= \frac{5+49+32-20}{\sqrt{66}} = \frac{66}{\sqrt{66}} = \sqrt{66}$$

160 **(a)**

The line of intersection of the plane $\vec{r}\cdot(3\hat{\imath}-\hat{\jmath}+\hat{k})=1$ and $\vec{r}\cdot(\hat{\imath}+4\hat{\jmath}-2\hat{k})=2$ is perpendicular to each of the normal vectors $\overrightarrow{n_1}=3\hat{\imath}-\hat{\jmath}+\hat{k}$ and $\overrightarrow{n_2}=\hat{\imath}+4\hat{\jmath}-2\hat{k}$ and hence it is parallel to the vector

$$\overrightarrow{n_1}\times\overrightarrow{n_2}=(3\hat{\imath}-\hat{\jmath}+\hat{k})\times(\hat{\imath}+4\hat{\jmath}-2\hat{k})$$

$$= -2\hat{\imath}+7\hat{\jmath}+13\hat{k}$$

161 **(b)**

Let DR's of line be (l,m,n), Also, normal to the plane are perpendicular to the required line.

$\therefore 4l+4m-5n=0$

and $8l+12m-13n=0$

$$\Rightarrow \frac{l}{8}=\frac{m}{12}=\frac{n}{16} \quad \Rightarrow \frac{l}{2}=\frac{m}{3}=\frac{n}{4}$$

Intersection point with $z=0$ is given by

$4x+4y=12$ ....(i)

and $8x+12y=32$ ...(ii)

on solving Eqs. (i) and (ii), we get (1,2, 0)

$\therefore$Required lines is$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z}{4}$

162 **(b)**

The line of intersection of the planes $\vec{r}\cdot(\hat{\imath}+2\hat{\jmath}+3\hat{k})=0$ and $\vec{r}\cdot(3\hat{\imath}+2\hat{\jmath}+\hat{k})=0$ is parallel to the vector

$(\hat{\imath}+2\hat{\jmath}+3\hat{k})\times(3\hat{\imath}+2\hat{\jmath}+\hat{k})=-4\hat{\imath}+8\hat{\jmath}-4\hat{k}$

Since both the planes $\vec{r}\cdot(\hat{\imath}+2\hat{\jmath}+3\hat{k})=0$ and $\vec{r}\cdot(3\hat{\imath}+2\hat{\jmath}+\hat{k})=0$ pass through the origin. Therefore, their line of intersection will also pass through the origin. Thus, the required line passes through the origin and is parallel to the vector $-4\hat{\imath}+8\hat{\jmath}-4\hat{k}$

Hence, its equation is

$\vec{r}=\vec{0}+\lambda'(=-4\hat{\imath}+8\hat{\jmath}-4\hat{k})$

$\Rightarrow \vec{r}=\lambda(\hat{\imath}-2\hat{\jmath}+\hat{k})$, where, $\lambda=-4\lambda'$

163 **(d)**

Let the equation of plane passing through the point $P(-1,-1,1)$ is

$a(x+1)+b(y+1)+c(z-1)=0$ ... (i)

Which passes through the points $Q(0,1,1)$and $R(0,0,2)$

$\therefore a+2b+0c=0$

and $a+b+c=0$

$$\Rightarrow \frac{a}{2-0}=-\frac{b}{1-0}=\frac{c}{1-2}$$

$$\Rightarrow \frac{a}{2}=\frac{b}{-1}=\frac{c}{-1}$$

From Eq. (i)

$2(x+1)-1(y+1)-1(z-1)=0$

$\Rightarrow 2x-y-z+2=0$

$\therefore$ Distance of plane from point (0, 0, 0)

$$= \frac{0+0+0+2}{\sqrt{2^2+(-1)^2+(-1)^2}}$$

$$= \frac{2}{\sqrt{6}}$$

164 **(c)**

The direction cosines of PO

$$= \left(\frac{2}{\sqrt{4+9+1}},\frac{3}{\sqrt{4+9+1}},\frac{1}{\sqrt{4+9+1}}\right)$$

or $\left(\frac{-2}{\sqrt{4+9+1}},\frac{-3}{\sqrt{4+9+1}},\frac{1}{\sqrt{4+9+1}}\right)$

$$= \left(\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}},\frac{1}{\sqrt{14}}\right) \text{ or } \left(\frac{-2}{\sqrt{14}},\frac{-3}{\sqrt{14}},\frac{1}{\sqrt{14}}\right)$$

165 **(b)**

The centre and radius of given sphere are (0, 0, 0) and $\sqrt{54}$ ie, $3\sqrt{6}$.

Distance between $(1,2,-1)$ and(0,0,0) is $\sqrt{6}$

$\therefore$ Shortest distance between point $(1,2,-1)$ and surface of the sphere

$= 3\sqrt{6} - \sqrt{6} = 2\sqrt{6}$

166 **(d)**

Shortest distance

$$= \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{\Sigma(m_1 n_2 - m_2 n_1)^2}}$$

Now, $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$

$= \begin{vmatrix} 5-2 & 1+3 & 6-1 \\ 3 & 4 & 5 \\ 1 & 2 & 3 \end{vmatrix} = \begin{vmatrix} 3 & 4 & 5 \\ 3 & 4 & 5 \\ 1 & 2 & 3 \end{vmatrix}$

$= 0$ [$\because$ two rows are identical]

$\therefore$ Shortest distance $= 0$

167 **(b)**

Here, $(x_1, y_1, z_1) = (1, 2, 3)$

and $a = 2, b = -1, c = 1, d = 3$

$\therefore \frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1}$

$= -2\left(\frac{2-3+4+3}{2^2 + (-1)^2 + (1)^2}\right) = -2$

$\Rightarrow x = -3, \quad y = 5$ and $z = 2$

168 **(d)**

Given, equation can be rewritten as

$x^2 + y^2 + z^2 - 2x + 3y + 4z - \frac{5}{2} = 0$

Let $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

$\therefore$ Given equation written in vector form is

$\vec{\mathbf{r}} \cdot [\vec{\mathbf{r}} - (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 4\hat{\mathbf{k}})] = \frac{5}{2}$

169 **(c)**

Direction ratio of the line and the normal to the plane are $2, 1, -2$ and $1, 1, 0$ respectively

$\therefore$Their direction cosines are

$\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}$ and $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$

If θ is the angle between the line and the plane, then

$\cos(90° - \theta) = \frac{2}{3} \cdot \frac{1}{\sqrt{2}} + \frac{1}{3} \cdot \frac{1}{\sqrt{2}} + \left(-\frac{2}{3}\right) \times 0$

$\Rightarrow \sin\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = 45°$

170 **(a)**

We know, if the line is passing through (x_1, y_1, z_1) and (x_2, y_2, z_2), then equation of line is

$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$

Since, the line passing through $(4, -5, -2)$ and $(-1, 5, 3)$

$\therefore$ The equation of straight line is

$\frac{x-4}{1} = \frac{y+5}{-2} = \frac{z+2}{-1}$

Which is the required straight line

171 **(a)**

Reflection of plane $2x - 3y + 4z - 3 = 0$ in the plane

$x - y + z - 3 = 0$ is

$2(2 + 3 + 4)(x - y + z - 3)$

$= 3(2x - 3y + 4z - 3)$

$\Rightarrow 4x - 3y + 2z - 15 = 0$

172 **(b)**

The intersection of two planes is

$(x + y + z - 6) + \lambda(2x + 3y + 4z + 5) = 0$

$\Rightarrow (1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z + (-6 + 5\lambda) = 0$ ...(i)

Since, this plane is perpendicular to the plane

$4x + 5y - 3z - 8 = 0$

$\therefore (1 + 2\lambda)4 + (1 + 3\lambda)5 + (1 + 4\lambda)(-3) = 0$

$\Rightarrow \lambda = -\frac{6}{11}$

On putting the value of λ in Eq. (i), we get

$\left(-\frac{1}{11}\right)x + \left(-\frac{7}{11}\right)y + \left(-\frac{13}{11}\right)z + \left(-\frac{96}{11}\right) = 0$

$\Rightarrow x + 7y + 13z + 96 = 0$

173 **(a)**

The given line is $\vec{r} = 2\hat{\imath} - 2\hat{\jmath} + 3\hat{k} + \lambda(\hat{\imath} - \hat{\jmath} + 4\hat{k})$

or, $\vec{r} = \vec{a} + \lambda\vec{b}$, where $\vec{a} = 2\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = \hat{\imath} - \hat{\jmath} - 4\hat{k}$

The given plane is $\vec{r} \cdot (\hat{\imath} + 5\hat{\jmath} + \hat{k}) = 5$

We have, $\vec{b} \cdot \vec{n} = (\hat{\imath} - \hat{\jmath} + 4\hat{k}) \cdot (\hat{\imath} + 5\hat{\jmath} + \hat{k}) = 1 - 5 + 4 = 0$

Therefore, the line is parallel to the plane. Thus, the distance between the line and the plane is equal to the length of the perpendicular from a point $\vec{a} = 2\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ on the line to the given plane

Hence,

Required distance

$= \left|\frac{(2\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) \cdot (\hat{\imath} + 5\hat{\jmath} + \hat{k}) - 5}{\sqrt{1 + 25 + 1}}\right|$

$\Rightarrow$ Required distance $= \left|\frac{2-10+3-5}{\sqrt{27}}\right| = \frac{10}{3\sqrt{3}}$

174 **(d)**

$\because$ Direction ratios of lines and planes are

$(a_1, b_1, c_1) = (2, 1, -2)$ and $(a_2, b_2, c_2) = (1, 1, 1)$

$\therefore \sin\theta = \frac{2 + 1 - 2}{\sqrt{4 + 1 + 4}\sqrt{1 + 1 + 1}}$

$\Rightarrow \theta = \sin^{-1}\left(\frac{1}{3\sqrt{3}}\right)$

175 **(a)**

Equation of plane is $\frac{x}{8}+\frac{y}{4}+\frac{z}{4}=1$
$\Rightarrow x+2y+2z=8$
Length of perpendicular from origin to the plane
$x+2y+2z-8=0$ is
$\left|\frac{-8}{\sqrt{1+4+4}}\right|=\frac{8}{3}$

176 **(c)**
Given, $\cos\alpha\cos\beta\cos\gamma=\frac{2}{9}$
and $\cos\gamma\cos\alpha=\frac{4}{9}$
Then, $\cos\alpha=\frac{2}{3},\cos\beta=\frac{1}{3}$ and $\cos\gamma=\frac{2}{3}$
$\therefore \cos\alpha+\cos\beta+\cos\gamma=\frac{2}{3}+\frac{1}{3}+\frac{2}{3}=\frac{5}{3}$

177 **(a)**
The coordinates of the mid-point of PQ are (2, 3, 4). The direction ratios of PQ are proportional to $3-1, 4-2, 5-3$ i.e. 1, 1, 1
So, equation of the required plane is
$1\times(x-2)+1\times(y-3)+1\times(z-4)=0$ or,
$x+y+z=9$

178 **(a)**
Given sphere are
$S_1\equiv x^2+y^2+z^2+7x-2y-z=1$
and $S_2\equiv x^2+y^2+z^2-3x+3y+4z=-4$
required equation of plane is
$(x^2+y^2+z^2+7x-2y-z-1)-(x^2+y^2+$
z2−3x+3y+4z+4=0
$[\because S_1-S_2=0]$
$\Longrightarrow 10x-5y-5z=5$
$\Longrightarrow 2x-y-z=1$

180 **(a)**
Given line is
$\frac{x-1}{-3}=\frac{y+1}{-2}=\frac{z-3}{1}$
$\Longrightarrow$ Line is passing through $(1,-1,3)$ and having direction ratios $-3,-2,1$ *ie*, $3,2,-1$
$\therefore$ Vector equation of the line is
$\vec{\mathbf{r}}=(\hat{\mathbf{i}}-\hat{\mathbf{j}}+3\hat{\mathbf{k}})+\lambda(3\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}})$

181 **(c)**
Equation of the plane is $\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=1$
Here, $a=2,\quad b=3,\quad c=4$
$\therefore$ Area of $\Delta ABC=\frac{1}{2}\sqrt{a^2b^2+b^2c^2+c^2a^2}$
$=\frac{1}{2}\sqrt{2^2\cdot3^2+3^2\cdot4^2+4^2\cdot2^2}$
$=\frac{1}{2}\sqrt{244}=\sqrt{61}$ sq units

182 **(c)**
$\because \cos\alpha\cos\beta=\cos\beta\cos\gamma=\frac{2}{9}$ and $\cos\gamma\cos\alpha=\frac{4}{9}$,
Then $\cos\alpha=\frac{2}{3},\cos\beta=\frac{1}{3}$ and $\cos\gamma=\frac{2}{3}$
$\therefore \cos\alpha+\cos\beta+\cos\gamma=\frac{2}{3}+\frac{1}{3}+\frac{2}{3}=\frac{5}{3}$

183 **(a)**
The equation of a plane passing through (2, 2, 1) is $a(x-2)+b(y-2)+c(z-1)=0$
This passes through (9, 3, 6) and is perpendicular to the plane
$2x+6y+6z-1=0$
$\therefore 7a+1\cdot b+5c=0$ and $2a+6b+6c=0$
$\Rightarrow \frac{a}{-24}=\frac{b}{-32}=\frac{c}{40}\Rightarrow\frac{a}{3}=\frac{b}{4}=\frac{c}{-5}$
So, equation of the required plane is
$3(x-2)+4(y-2)-5(z-1)=0$ or,
$3x+4y-5z=9$

184 **(d)**
Since, OA is equally inclined to OX, OY and OZ
So, coordinate of A are (a,a,a)
Also, $OA=\sqrt{3}$
$\therefore \sqrt{(a-0)^2+(a-0)^2+(a-0)^2}=\sqrt{3}$
$\Longrightarrow \sqrt{3a^2}=\sqrt{3}\Longrightarrow a=\pm1$
$\therefore$ Coordinate of A are (1, 1, 1) or $(-1,-1,-1)$.

185 **(b)**
Let equation of plane is
$\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=$
1, then $A(\alpha,0,0), B(0,\beta,0)$and $C(0,0,\gamma)$are the points on coordinate
axes.
$\therefore$Centroid of $\Delta ABC=\left(\frac{\alpha}{3},\frac{\beta}{3},\frac{\gamma}{3}\right)$
But $\frac{\alpha}{3}=1$
$\Longrightarrow \alpha=3,\frac{\beta}{3}=2$
$\Longrightarrow \beta=6$ and $\frac{\gamma}{3}=4$
$\Longrightarrow \gamma=12$
$\therefore$ Equation of plane is
$\frac{x}{3}+\frac{y}{6}+\frac{z}{12}=1$
$\Longrightarrow 4x+2y+z=12$

186 **(a)**
The DR's of the joining of the points (1, 2, 3) and (3, 4, 5) are (2, 2, 2)
Also, the midpoint of the join of the points (1, 2, 3) and (3, 4, 5) is (2, 3, 4)
$\therefore$ Equation of plane is
$2(x-2)+2(y-3)+2(z-4)=0$
$\Longrightarrow x+y+z=9$

187 **(d)**
Equation of the line passing through

$(5, 1, a)$ and $(3, b, 1)$ is

$$\frac{x-3}{5-3}=\frac{y-b}{1-b}=\frac{z-1}{a-1} \quad \ldots..(i)$$

Also, point $\left(0, \frac{17}{2}, -\frac{13}{2}\right)$ satisfies Eq. (i), we get

$$-\frac{3}{2}=\frac{\frac{17}{2}-b}{1-b}=\frac{-\frac{13}{2}-1}{a-1}$$

From Ist and IIIrd terms $a-1=\frac{\left(-\frac{15}{2}\right)}{\left(-\frac{3}{2}\right)} \Rightarrow a=6$

From Ist and IIed terms $-3(1-b)=2\left(\frac{17}{2}-b\right)$

$$\Rightarrow b=4$$

188 **(a)**

The image (x, y, z) of a point (x_1, y_1, z_1) in a plane $ax+by+cz+d=0$ is

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$
$$=\frac{-2(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}$$

Here, $(x_1, y_1, z_1)=(5,4,6)$

$a=1, \ b=1, c=2, d=-15$

$$\therefore \frac{x-5}{1}=\frac{y-4}{1}=\frac{z-6}{2}$$
$$=\frac{-2(5+4+12-15)}{1+1+4}=-2$$
$$\Rightarrow x=3, y=2, z=2$$

189 **(c)**

If α, β, γ are the angles which the line makes with coordinate axes, then

$l=\cos\alpha, m=\cos\beta, \quad n=\cos\gamma$

$\therefore l^2+m^2+n^2=\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

190 **(c)**

We are given by $l^2+m^2-n^2=0$ and

$l+m+n=0$ and we have $l^2+m^2+n^2=1$

So that, $2n^2=1$

$\Rightarrow n=\pm\frac{1}{\sqrt{2}}$

And $l+m=-n$

$\Rightarrow (l+m)^2=n^2=l^2+m^2$

$\Rightarrow 2lm=0$

$\Rightarrow$ Either $l=0$ or $m=0$, if $l=0, m+n=0$

$\Rightarrow m=-n=\mp\frac{1}{\sqrt{2}}$

So, the direction cosines of one of the lines are $0, \mp\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}$ and if $m=0, l+n=0 \Rightarrow l=-n=\mp\frac{1}{\sqrt{2}}$ and the direction cosines of the other line are $\mp\frac{1}{\sqrt{2}}, 0, \pm\frac{1}{\sqrt{2}}$

Hence, the required angle is

$$\cos^{-1}\left[0\times\mp\frac{1}{\sqrt{2}}+\mp\frac{1}{\sqrt{2}}\times 0+\left(\mp\frac{1}{\sqrt{2}}\right)\left(\pm\frac{1}{\sqrt{2}}\right)\right]$$
$$=\cos^{-1}\frac{1}{2}=\frac{\pi}{3}$$

191 **(c)**

Equation of a plane passing through (2, 2, 1) is

$a(x-2)+b(y-2)+c(z-1)=0 \quad \ldots$ (i)

This passes through (9, 3, 6) and is perpendicular to $2x+6y+6z-1=0$

$\therefore 7a+b+5c=0$ and, $2a+6b+6c=0$

Solving these two by cross-multiplication, we get

$$\frac{a}{-24}=\frac{b}{-32}=\frac{c}{40} \Rightarrow \frac{a}{-3}=\frac{b}{-4}=\frac{c}{5}$$

Substituting the values of a, b, c in (i), we get

$3x+4y-5z-9=0$ as the required plane

192 **(b)**

The equation of a plane through the line of intersection of the planes $\vec{\mathbf{r}}\cdot\vec{\mathbf{a}}=\lambda$ and $\vec{\mathbf{r}}\cdot\vec{\mathbf{b}}=\mu$ can be written as

$(\vec{\mathbf{r}}\cdot\vec{\mathbf{a}}=\lambda)+k(\vec{\mathbf{r}}\cdot\vec{\mathbf{b}}=\mu)=0$

$\Rightarrow \vec{\mathbf{r}}\cdot(\vec{\mathbf{a}}+k\vec{\mathbf{b}})=\lambda+k\mu \quad \ldots$(i)

This plane passes through the origin, therefore

$\vec{\mathbf{O}}\cdot(\vec{\mathbf{a}}+k\vec{\mathbf{b}})=\lambda+\mu k$

$\Rightarrow k=-\frac{\lambda}{\mu}$

On putting the value of k in Eq. (i), the equation of the required plane is

$\vec{\mathbf{r}}\cdot(\mu\vec{\mathbf{a}}-\lambda\vec{\mathbf{b}})=0$

$\Rightarrow \vec{\mathbf{r}}\cdot(\lambda\vec{\mathbf{b}}-\mu\vec{\mathbf{a}})=0$

194 **(d)**

Clearly, $\cos^2\alpha+\cos^2 60°+\cos^2 60°=1$ where α is the angle which the straight line makes with x-axis

$\therefore \cos^2\alpha=1-\frac{1}{4}-\frac{1}{4}=\frac{1}{2}$

$\Rightarrow \cos\alpha=\frac{1}{\sqrt{2}} \Rightarrow \alpha=45°$

195 **(b)**

Since two lines intersect at a point. Then shortest distance between them is zero.

$$\therefore \begin{vmatrix} k & 2 & 3 \\ 3 & k & 2 \\ 1 & 1 & -2 \end{vmatrix}=0$$

$$\left[\therefore \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \end{vmatrix}=0\right]$$

$\Rightarrow k(-2k-2)-2(-6-2)+3(3-k)=0$

$\Rightarrow 2k^2+5k-25=0$

$\Rightarrow (2k-5)(k+5)=0$

$\Rightarrow k=\frac{5}{2}, -5$

Hence, integer value of k is -5

196 **(b)**

Direction ratio of $AB = (6-1, 11+1, 2-2)$
$= (5, 12, 0)$
Direction ratios of $AC = (1-1, 2+1, 6-2) =$
$(0, 3, 4)$

Now, $\cos A = \dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

$\Rightarrow \cos A = \dfrac{5\times 0 + 12\times 3 + 0\times 4}{\sqrt{25+144+0}\,\sqrt{0+9+16}} = \dfrac{36}{65}$

197 **(a)**

Let $a_1 = 2x, b_1 = 2x, c_1 = x$
And $a_2 = 7-3 = 4, b_2 = 2-1 = 1$
$c_2 = 12-4 = 8$

$\therefore \cos\theta = \dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

$= \dfrac{2x\times 4 + 2x\times 1 + x\times 8}{\sqrt{4x^2+4x^2+x^2}\sqrt{16+1+64}}$

$= \dfrac{18x}{3x\times 9} = \dfrac{2}{3}$

$\Rightarrow \theta = \cos^{-1}\left(\dfrac{2}{3}\right)$

198 **(b)**

Any planes passing through (1,1,1) is
$a(x-1) + b(y-1) + c(y-1) = 0$ ...(i)
Since, it is passing through $(1,-1,1)$, we get
$a\cdot 0 + b(-2) + c(-2) = 0$
$\Rightarrow 0\cdot a - 2b - 2c = 0$
$\Rightarrow 0\cdot a + b + c = 0$ ...(ii)
Eq. (i) is perpendicular to $2x - y + z + 5 = 0$ is
$2a - b + c = 0$ ...(iii)
From Eqs. (ii) and (iii), we get
$a = b = 1, c = -1$
On substituting the value of a, b and c in Eq. (i), we get
$x + y - z - 1 = 0$

199 **(d)**

Given equation of line is

$\dfrac{x}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4} = \lambda$ [say]

Any point on the line is $P(2\lambda,\ 3\lambda+2, 4\lambda+3)$.
Also, this point lies in the plan.
$\therefore 2(2\lambda) + (3\lambda+2) - (4\lambda+3) = 2$
$\Rightarrow \lambda = 1$
$\therefore$ Coordinate of P are (2, 5, 7)
$\therefore$ Required distance
$= \sqrt{(2-0)^2 + (5-0)^2 + (7-0)^2}$
$= \sqrt{78}$

200 **(d)**

(1) Direction ratio of the joining the points (1, 2, 5) and (4, 3, 2) is (3, 1, −3) and direction ratios of the joining the points (5, 1, −11) and (8, 2, −8) is (3, 1, 3)
$\therefore$ These are parallel
(2) It is true
(3) Direction ratios of the plane $x - 2y + z = 21$ are (1,−2, 1) and direction ratios of the line $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-1}{3}$ are (1, 2, 3). If they are parallel, then
$1(1) - 2(2) + 1(3) = 0$

201 **(d)**

Given, $\dfrac{x-1}{-1} = \dfrac{y-0}{2} = \dfrac{z+1}{3} = r$ [say] ... (i)

Then, coordinate of any point N on the line (i) are
$(-r+1, 2r, 3r-1)$ ....(ii)
Let N be the foot of the perpendicular to line(i)
$\therefore$ Direction ratios of PN are
$(-r+1-2, 2r-3, 3r-1-4) = (-r-1, 2r-$
$3, 3r-5$ ....(iii)
$\because$ PN is perpendicular to line (i)
$\therefore$ Using the condition,
$a_1a_2 + b_1b_2 + c_1c_2 = 0$
$\Rightarrow -1(-r-1) + 2(2r-3) + 3(3r-5) = 0$
$\Rightarrow r + 1 + 4r - 6 + 9r - 15 = 0$
$\Rightarrow r = \dfrac{10}{7}$

Then, from Eq.(ii), we get

$N = \left(-\dfrac{10}{7}+1, \dfrac{20}{7}, \dfrac{30}{7}-1\right) = \left(-\dfrac{3}{7}, \dfrac{20}{7}, \dfrac{23}{7}\right)$

Now, perpendicular distance

$PN = \sqrt{\left(-\dfrac{3}{7}-2\right)^2 + \left(\dfrac{20}{7}-3\right)^2 + \left(\dfrac{23}{7}-4\right)^2}$

$= \dfrac{1}{7}\sqrt{289+1+25}$

$= \dfrac{3}{7}\sqrt{35}$

203 **(c)**

Let, $m = \cos\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}$

and $n = \cos\dfrac{\pi}{3} = \dfrac{1}{2}$

$\because\ l^2 + m^2 + n^2 = 1$

$\Rightarrow l = \sqrt{1 - (m^2+n^2)}$

$= \sqrt{1 - \left(\dfrac{1}{2} + \dfrac{1}{4}\right)}$

$= \sqrt{1 - \dfrac{3}{4}}$

$\Rightarrow l = \pm\dfrac{1}{2}$

Since, line makes an obtuse angle, so we take

$l = -\dfrac{1}{2}$

$\therefore$ Direction cosines are $-\dfrac{1}{2}, \dfrac{1}{\sqrt{2}}, \dfrac{1}{2}$

204 **(a)**

Let $\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

and $\vec{\mathbf{a}} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$, where a, b, c are constant.

Now, $\overrightarrow{|\mathbf{r}|^2} - 2\vec{\mathbf{r}} \cdot \vec{\mathbf{a}} + p = 0$

$\Longrightarrow \quad x^2 + y^2 + z^2 - 2(ax + by + cz) + p = 0$

Which represent a sphere,

Where radius$= \sqrt{a^2 + b^2 + c^2 - p} =$

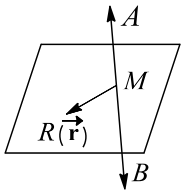
$+$ve $[\because |\vec{\mathrm{a}}|^2 > p]$

205 **(b)**

The image of the point (x_1, y_1, z_1) in the plane $ax + by + cz + d = 0$ is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$$= \frac{-2(ax_1 + by_1 + cz_1 + d)}{a^2 + b^2 + c^2}$$



$$\therefore \frac{x - 1}{2} = \frac{y - 3}{-1} = \frac{z - 4}{1} = -2\left(\frac{2 - 3 + 4 + 3}{6}\right)$$

Therefore, image of the point is $(-3, 5, 2)$

207 **(a)**

Let D be the foot of the perpendicular and let it divide BC in the ratio $\lambda : 1$. Then, the coordinates of D are

$$\left(\frac{3\lambda + 4}{\lambda + 1}, \frac{5\lambda + 7}{\lambda + 1}, \frac{3\lambda + 1}{\lambda + 1}\right)$$

Now, $\vec{A}D \perp \vec{B}C$

$\Rightarrow \vec{A}D \cdot B\vec{C} = 0$

$\Rightarrow -(2\lambda + 3) - 2(5\lambda + 7) - 4 = 0 \Rightarrow \lambda = -7/4$

So, the coordinates of D are $(5/3, 7/3, 17/3)$

208 **(c)**

Let a, b, c be the direction ratios of required line.

$\therefore 3a + 2b + c = 0$ and $a + b - 2c = 0$

$$\Longrightarrow \frac{a}{-4 - 1} = \frac{b}{1 + 6} = \frac{c}{3 - 2}$$

$$\Longrightarrow \frac{a}{-5} = \frac{b}{7} = \frac{c}{1}$$

In order to find a point on the required line we put $z = 0$ in the two given equations to obtain,

$3x + 2y = 5$ and $x + y = 3$

$\therefore$ Coordinate of point on required line are $(-1,4,0)$

Hence, required line is

$$\frac{x + 1}{-5} = \frac{y - 4}{7} = \frac{z - 0}{1}$$

209 **(a)**

Let the coordinate of a point Q on x-axis be $(a, 0, 0)$

$\therefore$ Distance, PQ

$= \sqrt{(a - a)^2 + (b - 0)^2 + (c - 0)^2}$

$\sqrt{b^2 + c^2}$

210 **(d)**

Let the vertices A, B, C, D of quadrilateral be $(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3)$ and (x_4, y_4, z_4) the equation of plane $PQRS$ be

$u \equiv ax + by + cz + d = 0$

Let $u_r = a_r x + b_r y + c_r z + d$

Where $r = 1, 2, 3, 4$

Then, $\frac{AP}{PB} \cdot \frac{BQ}{QC} \cdot \frac{CR}{RD} \cdot \frac{DS}{SA}$

$$= \left(-\frac{u_1}{u_2}\right)\left(-\frac{u_2}{u_3}\right)\left(-\frac{u_3}{u_4}\right)\left(-\frac{u_4}{u_1}\right) = 1$$

211 **(a)**

The vector equations of the given lines are

$\vec{r} = \overrightarrow{a_1} + \lambda \overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu \overrightarrow{b_2}$

Where,

$\overrightarrow{a_1} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}, \overrightarrow{b_1} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$

$\overrightarrow{a_2} = 2\hat{\imath} + 4\hat{\jmath} + 5\hat{k}, \overrightarrow{b_2} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$

$$\therefore \overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = -\hat{\imath} + 2\hat{\jmath} - \hat{k}$$

$(\overrightarrow{a_2} - \overrightarrow{a_1}) \cdot (\overrightarrow{b_1} \times \overrightarrow{b_2})$

$= (\hat{\imath} + 2\hat{\jmath} + 2\hat{k}) \cdot (-\hat{\imath} + 2\hat{\jmath} - \hat{k})$

$= 1$

$$\therefore \text{Required S. D.} = \frac{|(\overrightarrow{a_2} - \overrightarrow{a_1}) \cdot (\overrightarrow{b_1} \times \overrightarrow{b_2})|}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}$$

$$= \frac{1}{\sqrt{1 + 4 + 1}} = \frac{1}{\sqrt{6}}$$

212 **(a)**

Given, equation of sphere is

$$x^2 + y^2 + z^2 - x + 2y - 2z + \frac{3}{2} = 0$$

The centre of sphere is $\left(\frac{1}{2}, -1, 1\right)$.

The plane $x + y + z + a\sqrt{3} = 0$ will touch the sphere, if

$$\left|\frac{\frac{1}{2} - 1 + 1 + a\sqrt{3}}{\sqrt{1 + 1 + 1}}\right| = \sqrt{\frac{1}{4} + 1 + 1 - \frac{3}{2}}$$

$$\Longrightarrow a\sqrt{3} + \frac{1}{2} = \pm\frac{3}{2} \Longrightarrow a\sqrt{3} = 1, -2$$

$$\Longrightarrow a = \frac{1}{\sqrt{3}}, -\frac{2}{\sqrt{3}}$$

213 **(a)**

D divides BC in the ratio $= AB : AC$ i.e. $3 : 13$

Therefore, coordinates of D are

$$\left(\frac{3\times -9+13\times 5}{3+13}, \frac{3\times 6+13\times 3}{3+13}, \frac{3\times -3+13\times 2}{3+13}\right)$$

or, $\left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$

214 **(c)**

The equation of a plane through the line of intersection of the planes

$ax+by+cz+d=0$ and $a'x+b'y+c'z+d'=0$ is

$(ax+by+cz+d)+\lambda(a'x+b'y+c'z+d')=0$

$\Rightarrow x(a+\lambda a')+y(b+\lambda b')+z(c+\lambda c')+d+\lambda d'=0$ ...(i)

This parallel to x-axis i.e., $\frac{x}{1}=\frac{y}{0}=\frac{z}{0}$

$\therefore 1+(a+\lambda a')+0(b+\lambda b')+0(c+\lambda c')=0$

$$\Rightarrow \lambda - \frac{a}{a'}$$

Putting the value of λ in (i), the required plane is

$y(a'b-ab')+z(a'c-ac')+a'd-ad'=0$

215 **(b)**

We have, $\alpha=45°$ and $\beta=60°$

Suppose $\vec{OP}$ makes angle γ with OZ. Then,

$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$\Rightarrow \left(\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{2}\right)^2+\cos^2\gamma=1$

$\Rightarrow \cos^2\gamma=\frac{1}{4} \Rightarrow \cos\gamma=\pm\frac{1}{2} \Rightarrow \gamma=60°, 120°$

216 **(a)**

As $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ cuts the coordinate axes at

$A(a,0,0), B(0,b,0), C(0,0,c)$

Since, distance from origin $=1$

$$\Longrightarrow \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}=1$$

$$\Longrightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=1 \quad \ldots\ldots (i)$$

$\therefore$ Centroid $P(x,y,z)$

$$=\left(\frac{a+0+0}{3}, \frac{0+b+0}{3}, \frac{0+0+c}{3}\right)$$

$$\Longrightarrow x=\frac{a}{3}, y=\frac{b}{3}, z=\frac{c}{3} \quad ..(ii)$$

From Eqs. (i) and (ii),

$$\frac{1}{9x^2}+\frac{1}{9y^2}+\frac{1}{9z^2}=1$$

$$\Longrightarrow \frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=9=\text{k} \quad \text{(given)}$$

$\Longrightarrow k=9$

217 **(c)**

Equation of plane containing the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ is

$a(x-0)+b(y-0)+c(z-0)=0$ ...(i)

and $2a+3b+4c=0$ ....(ii)

Another equation of the plane containing the other two lines is

$a_1(x-0)+b_1(y-0)+c_1(z-0)=0$ ....(iii)

Also, $3a_1+4b_1+2c_1=0$

and $4a_1+2b_1+3c_1=0$

on solving we get

$$\frac{a_1}{8}=\frac{b_1}{-1}=\frac{c}{-10}$$

$\therefore$ Eq. (iii) becomes

$8x-y-10c=0$ ...(iv)

Since, the plane (i) is perpendicular to the plane (ii)

$\therefore$ $8a-b-10c=0$ ...(v)

On solving Eqs. (ii) and (v), we get

$$\frac{a}{-26}=\frac{b}{52}=\frac{c}{-26} \text{ or } \frac{a}{1}=\frac{b}{-2}=\frac{c}{1}$$

$\therefore$ From Eq. (i)

$x-2y+z=0$

Alternate

Let $\vec{\mathbf{a}}=2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+2\hat{\mathbf{k}}$,

$\vec{\mathbf{b}}=3\hat{\mathbf{i}}+4\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and $\vec{\mathbf{c}}=4\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$

$\vec{\mathbf{a}}\times(\vec{\mathbf{b}}\times\vec{\mathbf{c}})=(\vec{\mathbf{a}}\cdot\vec{\mathbf{c}})\vec{\mathbf{b}}-(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}})\vec{\mathbf{c}}$

$=26(-\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}})$

$\Longrightarrow$ Direction ratio of normal to the required plane (passing through origin) is $1,-2,1$

$\Longrightarrow$ Equation of required plane is $x-2y+z=0$

218 **(d)**

Any plane passing through (0, 1, 2) is

$a(x-0)+b(y-1)+c(z-2)=0$

$\Longrightarrow ax+b(y-1)+c(z-2)=0$ ....(i)

Since, it is passing through $(-1,0,3)$, we get

$-a-b+c=0$ .....(ii)

Also, Eq. is perpendicular to $2x+3y+z=5$

$\therefore 2a+3b+c=0$ ....(iii)

On solving Eqs. (ii) and (iii), we get

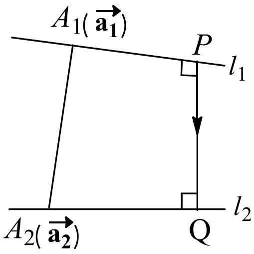
$$\frac{a}{-4}=\frac{b}{3}=\frac{c}{-1}$$

$\therefore$From Eq. (i)

$-4x + 3(y-1) - 1(z-2) = 0$
$\Rightarrow 4x - 3y + z + 1 = 0$

219 **(a)**

Let PQ be the shortest distance vector between l_1 and l_2. Now, l_1 passes through $A_1(\vec{\mathbf{a}}_1)$ and is parallel to $\vec{\mathbf{b}}_1$ and l_2 passes through $A_2(\vec{\mathbf{a}}_2)$ and is parallel to$\vec{\mathbf{b}}_2$. Since, PQ is perpendicular to both l_1 and l_2 it is parallel to $\vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2$



Let $\hat{\mathbf{n}}$ be the unit vector along PQ

Then, $\hat{\mathbf{n}} = \frac{\vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2}{|\vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2|}$

Let d be the shortest distance between the given lines l_1 and l_2

$|\overrightarrow{\mathbf{PQ}}| = d$ and $\overrightarrow{\mathbf{PQ}} = d\,\hat{\mathbf{n}}$

Next PQ being the line of shortest distance between l_1 and l_2 is the projection of the line joining the points $A_1(\vec{\mathbf{a}}_1)$ and $A_2(\vec{\mathbf{a}}_2)$ on $\hat{\mathbf{n}}$

$|\overrightarrow{\mathbf{PQ}}| = |\overrightarrow{\mathbf{A}_1}\overrightarrow{\mathbf{A}_2} \cdot \hat{\mathbf{n}}|$

$$\Rightarrow d = \left|\frac{(\vec{\mathbf{a}}_2 - \vec{\mathbf{a}}_1)\cdot \vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2}{|\vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2|}\right|$$

221 **(b)**

The lines $\vec{r} = \vec{a} + \lambda(\vec{b} \times \vec{c})$ and $\vec{r} = \vec{b} + \mu(\vec{c} \times \vec{a})$ pass through points $\vec{a}$ and $\vec{b}$ respectively and are parallel to vectors $\vec{b} \times \vec{c}$ and $\vec{c} \times \vec{a}$ respectively.

Therefore, they will intersect, if

$\vec{a} - \vec{b}, \vec{b} \times \vec{c}$ and $\vec{c} \times \vec{a}$ are coplanar

$\Rightarrow (\vec{a} - \vec{b}) \cdot \{(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a})\} = 0$

$\Rightarrow (\vec{a} - \vec{b}) \cdot \{[\vec{b}\,\vec{c}\,\vec{a}]\vec{c} - [\vec{b}\,\vec{c}\,\vec{c}]\vec{a}\} = 0$

$\Rightarrow (\vec{a} - \vec{b}) \cdot \vec{c}[\vec{b}\,\vec{c}\,\vec{a}] = 0$

$\Rightarrow \vec{a} \cdot \vec{c} - \vec{b} \cdot \vec{c} = 0 \Rightarrow \vec{a} \cdot \vec{c} = \vec{b} \cdot \vec{c}$

222 **(c)**

The centre and the radius of given sphere are $C(-1, 1, 2)$

and $R = \sqrt{(-1)^2 + (1)^2 + (2)^2 + 19} = 5$

length of perpendicular from centre C on the plan,

$$d = \frac{-1 \times 1 + 1 \times 2 + 2 \times 2 + 7}{\sqrt{1^2 + 2^2 + 2^2}} = 4$$

$\therefore$ Radius of circle $= \sqrt{R^2 - d^2} = \sqrt{25 - 16} = 3$

223 **(a)**

Let l, m, n be the direction cosines of the given line. Then, as it makes an acute angle with x-axis. Therefore, $l > 0$. The lines passes through $(6, -7, -1)$ and $(2, -3, 1)$. Therefore, its direction ratios are

$6 - 2, -7 + 3, -1 - 1$ or, $4, -4, -2$ or, $2, -2, -1$

Hence, direction cosines of the given line are

$\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}$

224 **(d)**

Here DR's of line and a plane are

$a_1 = 1, b_1 = 2, c_1 = 2$ and the plane $a_2 = 2, b_2 = -1$ and $c_2 = \sqrt{\lambda}$.

$$\because \sin\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

$$\Rightarrow \frac{1}{3} = \left|\frac{2 - 2 + 2\sqrt{\lambda}}{\sqrt{1 + 4 + 4}\sqrt{4 + 1 + \lambda}}\right|$$

$\Rightarrow \sqrt{5 + \lambda} = 2\sqrt{\lambda}$

$\Rightarrow \lambda = \frac{5}{3}$

225 **(c)**

Direction ratio of normal to the given plane is $2, -3, 5$ which is the direction ratio of line passing through $(3, 0, -4)$

$\therefore$ Equation of required line

$$\frac{x-3}{2} = \frac{y-0}{-3} = \frac{z+4}{5}$$

$$\Rightarrow \frac{x-3}{2} = \frac{-y}{3} = \frac{z+4}{5}$$

226 **(b)**

Given line can be rewritten as

$$\frac{x - \frac{1}{3}}{\frac{2b}{3}} = \frac{y-3}{-1} = \frac{z-1}{a}$$

Given plane$3x + y + 2z + 6 = 0$ is parallel to the above line

$\therefore \frac{2b}{3} \cdot 3 + 1 \cdot (-1) + 2 \cdot a = 0$

$\Rightarrow 2a + 2b = 1$

$\Rightarrow 3a + 3b = \frac{3}{2}$

227 **(a)**

Let the equation of plane be

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \quad \ldots.(\text{i})$

Then, coordinates of A, B and C are $(a, 0, 0), (0, b, 0)$and $(0, 0, c)$ rspectively.

The centroid of a $\Delta\, ABC$ is $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$ but it is given

$\therefore \frac{a}{3} = \frac{1}{3}, \frac{b}{3} = \frac{1}{3}, \frac{c}{3} = \frac{1}{3}$

$\Rightarrow a = b = c = 1$

$\therefore$ From Eq. (i)

$x + y + z = 1$

228 **(c)**

We know that the equation of a plane parallel to x-axis is

$by + cz + d = 0$

Since, it passes through the points (2, 3, 1) and (4, −5, 3)

$\therefore 3b + c + d = 0$

and $-5b + 3c + d = 0$

$\Rightarrow \frac{b}{1-3} = \frac{c}{-8} = \frac{d}{14}$

$\Rightarrow \frac{b}{-2} = \frac{c}{-8} = \frac{d}{14}$

$\therefore$ Equation of plane is $-2y - 8z + 14 = 0$

$\Rightarrow y + 4z = 7$

229 **(b)**

Equation of plane passing through the intersection of given planes, is

$(x + 2y + 3z + 4) + \lambda(4x + 3y + 2z + 1) = 0$ ...(i)

Plane (i) is passing through the origin *ie*, (0, 0, 0)

$\therefore 4 + \lambda = 0 \Rightarrow \lambda = -4$

On putting the value of λ in Eq. (i), we get

$(x + 2y + 3z + 4) - 4(4x + 3y + 2z + 1) = 0$

$\Rightarrow -15x - 10y - 5z = 0$

$\Rightarrow 3x + 2y + z = 0$

230 **(a)**

Since the required plane contains the line $\vec{r} = 2\hat{\imath} + \lambda(\hat{\jmath} - \hat{k})$ and is perpendicular to the plane $\vec{r} \cdot (\hat{\imath} + \hat{k}) = 3$. Therefore, it passes through the point $\vec{a} = 2\hat{\imath}$ and is parallel to the vectors $\vec{b} = \hat{\jmath} - \hat{k}$ and $\vec{c} = \hat{\imath} + \hat{k}$. Hence, it is perpendicular to the vector

$\vec{n} = \vec{b} \times \vec{c} = (\hat{\jmath} - \hat{k}) \times (\hat{\imath} + \hat{k}) = \hat{\imath} - \hat{\jmath} - \hat{k}$

Therefore, the equation of the required plane is

$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$

$\Rightarrow (\vec{r} - 2\hat{\imath}) \cdot (\hat{\imath} - \hat{\jmath} - \hat{k}) = 0$

$\Rightarrow \vec{r} \cdot (\hat{\imath} - \hat{\jmath} - \hat{k}) = 2$

231 **(c)**

Let the point R divides the line joining the points $P(2, 4, 5)$ and $Q(3, 5, -4)$ in the ratio $m:n$ Then, the coordinate of R is $\left(\frac{3m+2n}{m+n}, \frac{5m+4n}{m+n}, \frac{-4m+}{m+}\right.$

For yz-plane, x-coordinate will be zero.

$\therefore \frac{3m+2n}{m+n} = 0 \Rightarrow \frac{m}{n} = \frac{-2}{3}$

Alternate The ratio in which yz-plane divides the line segment $= -x_1 : x_2 = -2 : 3$

232 **(a)**

Given, planes are $2x + y + 2z + \frac{5}{2} = 0$

and $2x + y + 2z - 8 = 0$

$\therefore$ Distance $= \left| \frac{\frac{5}{2} - (-8)}{\sqrt{2^2 + 1^2 + 2^2}} \right| = \frac{7}{2}$

235 **(a)**

Equation of plane passing through the point (1, 2, 3) is

$A(x-1) + B(y-2) + C(z-3) = 0$ ...(i)

Since, plane (i) is parallel to plane $x + 2y + 5z = 0$

$\Rightarrow A = 1, B = 2, C = 5$

Putting these values in Eq. (i), we get

$(x-1) + 2(y-2) + 5(z-3) = 0$ is the required plane

236 **(b)**

Required circle is intersection of sphere

$x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$

and plane $x - 2y + 2z + 8 = 0$

Centre of sphere is $(-1, 1, 2)$

P =length of the perpendicular from, $(-1, 1, 2)$ to the plane

$= \frac{-1 - 2 + 4 + 8}{\sqrt{1 + 4 + 4}}$

$= \frac{9}{3} = 3$

R =radius of sphere

$= \sqrt{1 + 1 + 4 + 19} = 5$

Radius of the circle $= \sqrt{R^2 - P^2}$

$= \sqrt{25 - 9} = 4$

237 **(d)**

Distance of point $P(2, 6, 3)$ from origin

$OP = \sqrt{(0-2)^2 + (0-6)^2 + (0-3)^2}$

$= \sqrt{4 + 36 + 9} = 7$

Now, DR's of $OP = 2 - 0, 6 - 0, 3 - 0 = 2, 6, 3$

$\therefore$ DC's of OP are $\frac{2}{7}, \frac{6}{7}, \frac{3}{7}$

$\therefore$ Equation of plane in normal form is

$lx + my + nz = p$

$\Rightarrow \frac{2}{7}x + \frac{6}{7}y + \frac{3}{7}z = 7$

$\Rightarrow 2x + 6y + 3z = 49$

238 **(c)**

Now, $AB = \sqrt{3^2 + 0 + 3^2} = \sqrt{18}$

$CA = \sqrt{16 + 4 + 16} = 6$

and $BC = \sqrt{1 + 4 + 49} = \sqrt{54}$

$\because AB^2 + CA^2 = BC^2$

$\therefore \Delta ABC$ is right angled triangle, right angled at, A..

Thus, $\angle A = 90°$

239 **(b)**

The vector equation of the line joining the points $\hat{\imath}-2\hat{\jmath}+\hat{k}$ and $-2\hat{\jmath}+3\hat{k}$ is
$\vec{r}=(\hat{\imath}-2\hat{\jmath}+\hat{k})+\lambda(-\hat{\imath}+2\hat{k})$ ...(i)
Using $\vec{r}\cdot(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})=[\vec{a}\,\vec{b}\,\vec{c}]$ the vector equation of the plane through the origin, $4\hat{\jmath}$ and $2\hat{\imath}+\hat{k}$
$\vec{r}\cdot(4\hat{\imath}-8\hat{k})=0$ ...(ii)
The position vector of any point on (i) is
$(\hat{\imath}-2\hat{\jmath}+\hat{k})+\lambda(-\hat{\imath}+2\hat{k})$
If it lies on (ii), then
$\{(\hat{\imath}-2\hat{\jmath}+\hat{k})+\lambda(-\hat{\imath}+2\hat{k})\}\cdot(4\hat{\imath}-8\hat{k})=0$
$\Rightarrow -4-20\lambda=0\Rightarrow\lambda=-1/5$
Putting the value of λ in $(\hat{\imath}-2\hat{\jmath}+\hat{k})+\lambda(-\hat{\imath}+2\hat{k})$, we get the position vector of the required point as $\frac{1}{5}(6\hat{\imath}-10\hat{\jmath}+3\hat{k})$

240 **(d)**
Given equation of sphere are
$x^2+y^2+z^2+2x+2y+2z=2$
Whose centre is $C_1=(-1,-1,-1)$ and radius $=\sqrt{5}$
And $2x^2+2y^2+2z^2+4x+2y+4z=0$
Whose centre is $C_2=\left(-1,-\frac{1}{2},-1\right)$ and radius $=\sqrt{\frac{9}{4}}=\frac{3}{2}$
Also, $C_1C_2=\sqrt{0+\frac{1}{4}+0}=\frac{1}{2}$
$C_1C_2<|r_1-r_2|$
So, second sphere is completely inside of first sphere

242 **(b)**
$\because\ \cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$
$\Longrightarrow\ \cos^2 45°+\cos^2\beta+\cos^2 60°=1$
$\Longrightarrow\ \cos^2\beta=\frac{1}{4}$
$\Longrightarrow \cos\beta=\pm\frac{1}{2}$
$\Longrightarrow\ \beta=60°$ or $120°$

243 **(b)**
Given, $l+m+n=0$ ....(i)
and $l^2+m^2-n^2=0$ .....(ii)
$\therefore\ l^2+m^2-(-l-m)^2=0$
$\Longrightarrow 2lm=0$
$\Longrightarrow l=0$ or $m=0$
if $l=0, then\ n=-m$
$\Longrightarrow l:m:n=0:1:-1$
and if $m=0$, then $n=-1$
$\Longrightarrow l:m:n=1:0:-1$
$\therefore \cos\theta=\frac{0+0+1}{\sqrt{0+1+1}\sqrt{0+1+1}}=\frac{1}{2}$
$\Longrightarrow\ \theta=\frac{\pi}{3}$

244 **(b)**
The given lines are parallel to the vectors
$\overrightarrow{b_1}=\hat{\imath}+\lambda\hat{\jmath}-\hat{k}$ and $\overrightarrow{b_2}=-\lambda\hat{\imath}+2\hat{\jmath}+\hat{k}$
respectively. The lines will be perpendicular to each other, if
$\overrightarrow{b_1}\cdot\overrightarrow{b_2}=0\Rightarrow-\lambda+2\lambda-1=0\Rightarrow\lambda=1$

245 **(b)**
Given equation is $x^2-5x+6=0$
$\Rightarrow(x-2)(x-3)=0$
$\Rightarrow(x-2)=0$ or $(x-3)=0$
Which represents a plane

246 **(a)**
$\because\ \sin\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$
$=\frac{1\times1+2\times-1+1\times1}{\sqrt{1+4+1}\sqrt{1+1+1}}=0$
$\Longrightarrow\ \theta=0°$

247 **(d)**
Given line and plane can be rewritten as
$\frac{3x-1}{3}=\frac{y+3}{-1}=\frac{5-2z}{4}$
$\frac{x-\frac{1}{3}}{1}=\frac{y+3}{-1}=\frac{\left(z-\frac{5}{2}\right)}{-2}$
and $x-y-2z=0$
here, $a_1=1, b_1=-1. c_1=-2$
and $a_2=1, b_2=-1, c_2=-2$
$\therefore \sin\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$
$=\frac{1\times1+(-1)\times(-1)+(-2)\times(-2)}{\sqrt{1+1+4}\sqrt{1+1+4}}$
$=\frac{6}{\sqrt{6}\sqrt{6}}=1$
$\Longrightarrow\theta=\frac{\pi}{2}$

248 **(b)**
Let AB be the given line and the let its direction cosines of AB be l,m,n. Then,
Projection of AB on x-axis $=AB\ l=12$ (given)
Projection of AB on y-axis $=AB\ m=4$ (given)
Projection of AB on z-axis $=AB\ n=3$ (given)
$\therefore (AB)^2(l^2+m^2+n^2)=12^2+4^2+3^2\Rightarrow AB=13$
Hence, direction cosines of AB are $\frac{12}{13},\frac{4}{13},\frac{3}{13}$

249 **(a)**
Required DR's are $(3-2, 4+1, -1-1)$
$ie, (1,5,-2)$.

250 **(b)**
Any point on the line

$\frac{x-1}{3} = \frac{y+2}{4} + \frac{z-3}{-2} = k$ [say]
is $(3k+1, 4k-2, -2k+3)$.
If the given line intersect the plane $2x - y + 3z - 1 = 0$, then any point on the line lies in the plane.
$\therefore 2(3k+1) - (4k-2) + 3(-2k+3) - 1 = 0$
$\Rightarrow k = 3$
$\therefore$ Point is $(9+1, 12-2, -6+3) ie, (10, 10, -3)$.

251 **(c)**
The equation of plane containing the line
$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ is
$a(x+1) + b(y-3) + c(z+2) = 0$ ....(i)
Alos, $-3a + 2b + c = 0$ ....(ii)
Also, plane passes through $(0, 7, -7)$
$\therefore a + 4b - 5c = 0$ .....(iii)
From Eqs. (ii) and (iii),
$\frac{a}{-14} = \frac{b}{-14} = \frac{c}{-14}$
$\Rightarrow \frac{a}{1} = \frac{b}{1} = \frac{c}{1}$

252 **(b)**
The equation of the plane containing the line
$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ is
$a(x+1) + b(y-3) + c(z+2) = 0$ ...(i)
Where, $-3a + 2b + c = 0$ ...(ii)
This passes through $(0, 7, -7)$
$\therefore a + 4b - 5c = 0$
From (ii) and (iii), we have
$\frac{a}{-14} = \frac{b}{-14} = \frac{c}{-14} \Rightarrow \frac{a}{1} = \frac{b}{1} = \frac{c}{1}$
So, the required plane is $x + y + z = 0$

253 **(b)**
Let the equation of plane passing through (1, 1, 1) is
$a(x-1) + b(y-1) + c(z-1) = 0$ ....(i)
$\because$ It is also passing through $(1, -1, -1)$
$\therefore b + c = 0$ ....(ii)
Since, the Eq. (i) is perpendicular to the plane $2x - y + z + 5 = 0$
$\therefore 2a - b + c = 0$ ....(iii)
Since, Eqs, (ii) and (iii) are identical
$\therefore \frac{a}{1+1} = \frac{b}{2-0} = \frac{c}{-2+0}$
$\Rightarrow \frac{a}{1} = \frac{b}{1} = \frac{c}{-1}$
$\therefore$ Required equation of plane is $x + y - z - 1 = 0$

254 **(a)**
It is given that the direction ratios of $\vec{r}$ are proportional to $2, -3, 6$. Therefore, its direction cosines are
$l = \frac{2}{7}, m = \frac{-3}{7}, n = \frac{6}{7}$
$\therefore \vec{r} = |\vec{r}|(l\,\hat{\imath} + m\hat{\jmath} + n\,\hat{k})$
$\Rightarrow \vec{r} = 21\left(\frac{2}{7}\hat{\imath} - \frac{3}{7}\hat{\jmath} + \frac{6}{7}\hat{k}\right) = 6\hat{\imath} - 9\hat{\jmath} + 18\hat{k}$

255 **(c)**
The line perpendicular to the plane $2x - y + 5z = 4$ and passing through the point $(-1, 0, 1)$ is given by
$\frac{x+1}{2} = \frac{y-0}{-1} = \frac{z-1}{5}$
$\Rightarrow \frac{x+1}{2} = -y = \frac{z-1}{5}$

256 **(c)**
Radius of sphere is perpendicular distance from $(6, -1, 2)$to
$2x - y + 2z - 2 = 0$
$ie, \left|\frac{12+1+4-2}{\sqrt{4+1+4}}\right| = 5$
$\therefore$ Equation of sphere is
$(x-6)^2 + (y+1)^2 + (z-2)^2 = 25$
$\Rightarrow x^2 + y^2 + z^2 - 12x + 2y - 4z + 16 = 0$

257 **(b)**
Let $P(\vec{\mathbf{r}})$ be any point on plane
Clearly $\vec{\mathbf{r}} - \vec{\mathbf{a}}$ will be in linear combination of $\vec{\mathbf{b}} - \vec{\mathbf{a}}$ and $\vec{\mathbf{c}} - \vec{\mathbf{a}}$
$\Rightarrow$ $\vec{\mathbf{r}} - \vec{\mathbf{a}}, \vec{\mathbf{b}} - \vec{\mathbf{a}}, \vec{\mathbf{c}} - \vec{\mathbf{a}}$ will be coplanar
$\Rightarrow (\vec{\mathbf{r}} - \vec{\mathbf{a}}) \cdot \{(\vec{\mathbf{b}} - \vec{\mathbf{a}}) \times (\vec{\mathbf{c}} - \vec{\mathbf{a}})\} = 0$
$\Rightarrow (\vec{\mathbf{r}} - \vec{\mathbf{a}}) \cdot \{\vec{\mathbf{b}} \times \vec{\mathbf{c}} + \vec{\mathbf{a}} \times \vec{\mathbf{b}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}}\} = 0$
$= \vec{\mathbf{r}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}} + \vec{\mathbf{c}} \times \vec{\mathbf{a}} + \vec{\mathbf{a}} \times \vec{\mathbf{b}}) = [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$

258 **(b)**
Since, the given plane are
$x - cy - bz = 0$
$cx - y + az = 0$
and $bx + ay - z = 0$
passes through a line
$\therefore \begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$
$\Rightarrow 1(1 - a^2) + c(-c - ab) - b(ac + b) = 0$
$\Rightarrow 1 - a^2 - c^2 - abc - abc - b^2 = 0$
$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1$

259 **(c)**
Let the verities of triangle be $A(a, 0, 0), B(0, b, 0)$ and $C(0, 0, c)$ and the equation of plane is
$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ ... (i)
$\because$ Centroid of ΔABC is (α, β, γ)
$\therefore \frac{a+0+0}{3} = \alpha$
$\Rightarrow a = 3\alpha$

Similarly, $b = 3\beta$ and $c = 3\gamma$
$\therefore$ From Eq. (i),
$$\frac{x}{3\alpha}+\frac{y}{3\beta}+\frac{z}{3\gamma}=1$$
$$=\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=3$$

261 **(a)**
We observe that the line given in option (a) passes through $(1, -2, 3)$. Also, it is normal to the plane $2x + 3y + z = 0$

262 **(d)**
The shortest distance between the lines
$$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$$
And $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ is given by
$$\text{Shortest distance} = \frac{\left\|\begin{matrix}\alpha-\alpha' & \beta-\beta' & \gamma-\gamma' \\ l & m & n \\ l' & m' & n'\end{matrix}\right\|}{\sqrt{\Sigma(mn'-nm')^2}}$$
$$=\frac{\left\|\begin{matrix}6 & 15 & -3 \\ 3 & -1 & 1 \\ -3 & 2 & 4\end{matrix}\right\|}{\sqrt{(-4-2)^2+(12+3)^2+(6-3)^2}}$$
$$=\frac{270}{\sqrt{270}}=\sqrt{270}=3\sqrt{30}$$

264 **(b)**
Equation of sphere $OABC$ is
$$x^2+y^2+z^2-ax-by-cz=0$$
Where $\sqrt{\frac{a^2+b^2+c^2}{4}}=2k$
$\Rightarrow a^2+b^2+c^2=16k^2$ ...(i)
Let (α, β, γ) be the centroid of the tetrahedron $OABC$, then $\alpha=\frac{a}{4}, \beta=\frac{b}{4}, \gamma=\frac{c}{4}$
From Eq. (i), $\alpha^2+\beta^2+\gamma^2=k^2$
Locus is $x^2+y^2+z^2=k^2$

265 **(b)**
Let DR's of required line be a, b, c
According to given condition,
$a(1)+b(-1)+c(2)=0$
$\Longrightarrow a-b+2c=0$ .....(i)
and $a(2)+b(1)+c(-1)=0$
$\Longrightarrow 2a+b-c=0$ ....(ii)
From Eqs. (i) and (ii),
$$\frac{a}{1-2}=\frac{b}{4+1}=\frac{c}{1+2}$$
$$\Longrightarrow \frac{a}{-1}=\frac{b}{5}=\frac{c}{3}$$
$\therefore$ Required DC's are
$$l=-\frac{1}{\sqrt{1^2+5^2+3^2}},$$
$$m=\frac{5}{\sqrt{1^2+5^2+3^2}}, n=\frac{3}{\sqrt{1^2+5^2+3^2}}$$
$$\Longrightarrow l=-\frac{1}{\sqrt{35}}, m=\frac{5}{\sqrt{35}}, n=\frac{3}{\sqrt{35}}$$

266 **(b)**
The shortest distance, between two lines is
$$d=\frac{\begin{vmatrix}x_2-x_1 & y_2-y_1 & z_1-z_2 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2\end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-a_1c_2)^2+(a_1b_2-b_1a_2)^2}}$$
Given lines are $\frac{x+1}{12}=\frac{y}{6}=\frac{z}{-1}$ and $\frac{x}{6}=\frac{y+2}{6}=\frac{z-1}{1}$
$$\therefore d=\frac{\begin{vmatrix}1 & -2 & 1 \\ 12 & 6 & -1 \\ 6 & 6 & 1\end{vmatrix}}{\sqrt{(6+6)^2+(-6-12)^2+(72-36)^2}}$$
$$=\frac{1(6+6)+2(12+6)+1(72-36)}{\sqrt{144+324+1296}}$$
$$=\frac{84}{42}=2$$

267 **(c)**
The position vectors of two given points are $\vec{a}=\hat{\imath}-\hat{\jmath}+3\hat{k}$ and $\vec{b}=3\hat{\imath}+3\hat{\jmath}+3\hat{k}$ and the equation of the given plane is
$\vec{r}\cdot(5\hat{\imath}+2\hat{\jmath}-7\hat{k})+9=0$ or, $\vec{r}\cdot\vec{n}+d=0$
We have,
$\vec{a}\cdot\vec{n}+d=(\hat{\imath}-\hat{\jmath}+3\hat{k})\cdot(5\hat{\imath}+2\hat{\jmath}-7\hat{k})+9$
$=5-2-21+9<0$
and, $\vec{b}\cdot\vec{n}+d=(3\hat{\imath}+3\hat{\jmath}+3\hat{k})\cdot(5\hat{\imath}+2\hat{\jmath}-7\hat{k})+9$
$=15+6-21+9>0$
So, the points $\vec{a}$ and $\vec{b}$ are on the opposite sides of the plane

268 **(c)**
Clearly point $(2, -1, 2)$ lies on the line as well as plane
$\therefore$ Required distance of point $(-1, -5, -10)$
$=\sqrt{(-1-2)^2+(-5+1)^2+(-10-2)^2}$
$=\sqrt{9+16+144}$
$=\sqrt{169}=13$

269 **(b)**
DC's of AB
$$=\frac{1}{\sqrt{1^2+4^2+3^2}}, \frac{4}{\sqrt{1^2+4^2+3^2}}, \frac{3}{\sqrt{1^2+4^2+3^2}}$$
$$=\frac{1}{\sqrt{26}}, \frac{4}{\sqrt{26}}, \frac{3}{\sqrt{26}}$$

270 **(a)**
By solving two equations, we get
$(l_1, m_1, n_1)=(2\sqrt{2}-3-\sqrt{2}, 1)$
$(l_2, m_2, n_2)=(-2\sqrt{2}-3, \sqrt{2}, 1)$
Now, $\cos\theta=\dfrac{-(2\sqrt{2}-3)(2\sqrt{2}+3)-\sqrt{2}(\sqrt{2})+1(1)}{\left[\begin{matrix}(2\sqrt{2}-3)^2+(-\sqrt{2})^2+1^2 \\ \times\sqrt{(-2\sqrt{2}-3)^2+(\sqrt{2})^2+1^2}\end{matrix}\right]}$

$\Rightarrow \cos\theta = 0°$

$\Rightarrow\ \theta = \dfrac{\pi}{2}$

$\therefore$ The angle between them is $\frac{\pi}{2}$

271 **(a)**

Centre of a given sphere is (3, 6, 1).

Since, one end of diameter are (2, 3, 5) and let the other end of diameter are (α, β, γ), then

$\dfrac{\alpha+2}{2} = 3, \dfrac{\beta+3}{2} = 6, \dfrac{\gamma+5}{2} = 1$

$\Longrightarrow \alpha=4, \beta=9$ and $\gamma=-3$.

272 **(b)**

Since, point Q is the image of P, therefore PQ perpendicular to the plane

$x - 2y + 5z = 6$

$\therefore$ Required equation of line is

$\dfrac{x-2}{1} = \dfrac{y-3}{-2} = \dfrac{z-4}{5}$

273 **(d)**

$\because OP = \sqrt{a^2+a^2+a^2} = \sqrt{3}\,a$

$\therefore$ DC's of OP are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

Equation of plane is

$x + y + z = 3a$

$\Rightarrow \dfrac{x}{3a} + \dfrac{y}{3a} + \dfrac{z}{3a} = 1$

$\therefore$ Intersection on axes are $3a, 3a$ and $3a$ respectively

Sum of their reciprocals $= \frac{1}{3a} + \frac{1}{3a} + \frac{1}{3a} = \frac{1}{a}$

274 **(c)**

Let l, m, n be the direction cosines of $\vec{O}P$. It is given that $l = 45° = \frac{1}{\sqrt{2}}$ and $m = \cos 60° = \frac{1}{2}$

$\therefore l^2 + m^2 + n^2 = 1 \Rightarrow \dfrac{1}{2} + \dfrac{1}{4} + n^2 = 1 \Rightarrow n = \pm\dfrac{1}{2}$

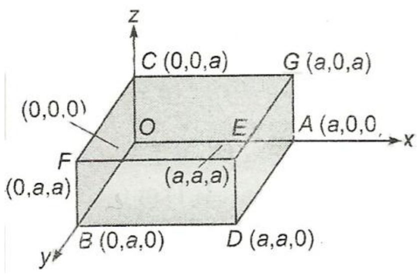
Now, $\vec{r} = |\vec{r}|\left(l\,\hat{\imath} + m\,\hat{\jmath} + n\,\hat{k}\right)$

$\Rightarrow \vec{r} = 12\left(\dfrac{1}{\sqrt{2}}\hat{\imath} + \dfrac{1}{2}\hat{\jmath} \pm \dfrac{1}{2}\hat{k}\right) = 6\sqrt{2}\hat{\imath} + 6\hat{\jmath} \pm 6\hat{k}$

276 **(b)**

Let OA, OB, OC be the sides of a cube such that $OA = OB = OC = a$

$OA = OB = OC = a$



$\therefore$ Direction ratios of OE are $(a-0, a-0$ *ie* a, a, a

$\therefore$Direction cosines of AF are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Similarly, direction of AF are $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.

$\therefore$ Angle between OE and AF is

$\cos^{-1}\left[-\dfrac{1}{\sqrt{3}}\cdot\dfrac{1}{\sqrt{3}} + \dfrac{1}{\sqrt{3}}\cdot\dfrac{1}{\sqrt{3}} + \dfrac{1}{\sqrt{3}}\cdot\dfrac{1}{\sqrt{3}}\right]\cos^{-1}\left(\dfrac{1}{3}\right)$

277 **(c)**

$\because$ Midpoint of line joining (2,3,4) and (6,7,8) is (4, 5, 6). This point is satisfied by one of the option *ie*, $x + y + z - 15 = 0$

278 **(d)**

Equation of the plane passing through $P(3, 8, 2)$ and parallel to

$3x + 2y - 2z + 15 = 0$ is

$3(x-3) + 2(y-8) - 2(z-2) = 0$

$\Longrightarrow 3x + 2y - 2z - 21 = 0$ ....(i)

Given line is

$\dfrac{x-1}{2} = \dfrac{y-3}{4} = \dfrac{z-2}{3} = r$ [say]

Any point of the line is

$Q(2r+1, 4r+3, 3r+2)$

This point is lies on the above plane

$\therefore 3(2r+1) + 2(4r+3) - 2(3r+2) - 21 = 0$

$\Longrightarrow 8r - 16 = 0 \Longrightarrow r = 2$

$\therefore$ Coordinate of $Q(5, 11, 8)$

$\therefore$ Distance between P and Q

$= \sqrt{(5-3)^2 + (11-8)^2 + (8-2)^2}$

$= \sqrt{4 + 9 + 36} = 7$

279 **(a)**

Given, $l + m + n = 0$ ....(i)

and $lm = 0$

$\Longrightarrow$ Either $m = 0$ or $l = 0$

If $l = 0$, then put in Eq. (i), we get $m = -n$

$\therefore$ Direction ratios are $0, -n, n$ *ie*, $0, -1, 1$

If $m = 0$, then put in Eq. (i), we get $l = -n$

$\therefore$ Direction ratios are $-n, 0, n$, *ie*, $-1, 0, 1$

$\therefore \cos\theta = \dfrac{0\times(-1) + (-1)\times 0 + 1\times 1}{\sqrt{0^2+(-1)^2+1^2}\,\sqrt{(-1)^2+0^2+1^2}}$

$= \dfrac{1}{2}$

$\Longrightarrow\ \theta = \dfrac{\pi}{3\pi}$

280 **(a)**

Since, it is given that line makes equal angle with the coordinate axes

$\therefore l = m = n$

We know, $l^2 + m^2 + n^2 = 1$

$\Rightarrow 3l^2 = 1$

$\Rightarrow l^2 = \dfrac{1}{3}$

$\Rightarrow l = \frac{1}{\sqrt{3}}$ (neglect – ve sign)

281 **(b)**

The straight line joining the points $(1,1,2)$ and $(3, -2,1)$ is

$\frac{x-1}{2} = \frac{y-1}{-3} = \frac{z-2}{-1} = r$ (say)

$\therefore$ Point is $(2r + 1, 1 - 3r, 2 - r)$ which lies on $3x + 2y + z = 6$

$\therefore 3(2r + 1) + 2(1 - 3r) + 2 - r = 6$

$\Rightarrow r = 1$

Required points is $(3, -2, 1)$

282 **(b)**

Given that, $A(5, -1, 1), B(7, -4, 7), C(1, -6, 10)$ and $D(-1, -3, 4)$

Now, $AB = \sqrt{(7-5)^2 + (-4+1)^2 + (7-1)^2}$

$= \sqrt{4 + 9 + 36} = 7$

$BC = \sqrt{(1-7)^2 + (-6+4)^2 + (10-7)^2}$

$= \sqrt{36 + 4 + 9} = 7$

$CD = \sqrt{(-1-1)^2 + (-3+6)^2 + (4-10)^2}$

$= \sqrt{4 + 9 + 36} = 7$

$DA = \sqrt{(5+1)^2 + (-1+3)^2 + (1-4)^2}$

$= \sqrt{36 + 4 + 9} = 7$

$\therefore AB = BC = CD = DA = 7,$

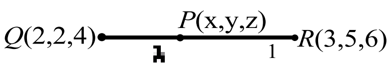
Also, $\overrightarrow{\mathbf{AB}} \cdot \overrightarrow{\mathbf{BC}} \neq 0$ (These are not perpendicular)

$\therefore ABCD$ is not square. It is rhombus

283 **(b)**

The coordinate of P are

$\left(\frac{3\lambda + 2}{\lambda + 1}, \frac{5\lambda + 2}{\lambda + 1}, \frac{6\lambda + 4}{\lambda + 1}\right)$



Since, the projection of OP on x-axis is

$\frac{3\lambda + 2}{\lambda + 1} = \frac{13}{5}$

$\Rightarrow 15\lambda + 10 = 13\lambda + 13$

$\Rightarrow \lambda = \frac{3}{2}$

284 **(b)**

Since, direction cosines of two lines are proportional to $(2, 3, -6)$and $(3, -4,5)$

$\therefore \cos = \frac{|2 \times 3 + 3 \times (-4) - 6 \times 5|}{\sqrt{2^2 + 3^2 + (-6)^2}\sqrt{3^2 + (-4)^2 + 5^2}}$

$= \frac{|6 - 12 - 13|}{\sqrt{49}\sqrt{50}}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{18\sqrt{2}}{35}\right)$

287 **(d)**

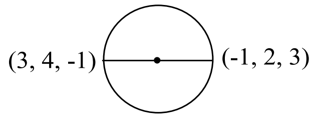
Here, $a_1 = 2,\ b_1 = -1, c_1 = 1$

and $a_2 = 1, b_2 = -2, c_2 = 1$

$\therefore \cos\theta = \left|\frac{(2 \times 1) + (-1 \times -2) + (1 \times 1)}{\sqrt{4 + 1 + 1}\sqrt{1 + 4 + 1}}\right| = \frac{5}{6}$

288 **(c)**

If $(3, 4, -1)$ and $(-1, 2, 3)$ are the end points of a sphere, then the length of diameter



$d = \sqrt{(-1-3)^2 + (2-4)^2 + (3+1)^2}$

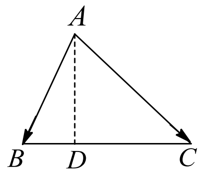
$= \sqrt{16 + 4 + 16}$

$= \sqrt{36} = 6$

So, radius, $r = \frac{d}{2} = \frac{6}{2} = 3$

289 **(b)**

$\overrightarrow{\mathbf{OA}} = 4\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 8\hat{\mathbf{k}}, \overrightarrow{\mathbf{OB}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$



And $\overrightarrow{\mathbf{OC}} = 2\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 7\hat{\mathbf{k}},$

$\therefore \overrightarrow{\mathbf{AB}} = -2\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

$\Rightarrow |\overrightarrow{\mathbf{AB}}| = 6$

And $\overrightarrow{\mathbf{AC}} = -2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$

$\Rightarrow |\overrightarrow{\mathbf{AC}}| = 3$

Now, $\overrightarrow{\mathbf{AD}} = \frac{2\overrightarrow{\mathbf{AC}} + \overrightarrow{\mathbf{AB}}}{3}$

$\Rightarrow \overrightarrow{\mathbf{AD}} = \frac{-4\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 2\hat{\mathbf{k}} - 2\hat{\mathbf{i}}}{3}$

$\Rightarrow \overrightarrow{\mathbf{AD}} = \frac{1}{3}(-6\hat{\mathbf{i}} - 8\hat{\mathbf{j}} - 6\hat{\mathbf{k}})$

Hence, the length of internal bisector of $\angle A = \frac{2}{3}\sqrt{34}$

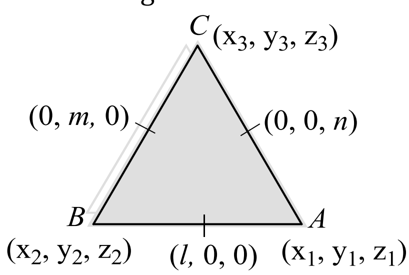
290 **(a)**

The distance from origin $(0, 0, 0)$ to the plane $6x - 3y + 2z - 14 = 0$ is

$d = \frac{|6(0) - 3(0) + 2(0) - 14|}{\sqrt{36 + 9 + 4}} = 2$

291 **(c)**

From the figure



$x_1 + x_2 = 2l, y_1 + y_2 = 0, z_1 + z_2 = 0,$

$x_2 + x_3 = 0, y_2 + y_3 = 2m, z_2 + z_3 = 0,$

and

$x_1 + x_3 = 0, y_1 + y_3 = 0, \quad z_1 + z_3 = 2n$

On solving, we get the coordinate are
$A(l,-m,n), B(l,m,-n)$ and $C(-l,m\,n)$.
$\therefore \dfrac{AB^2+BC^2+CA^2}{l^2+m^2+n^2}$
$= \dfrac{(4m^2+4n^2)+(4l^2+4n^2)+(4l^2+4m^2)}{l^2+m^2+n^2} = 8$

292 **(d)**
The direction ratio of the line are
$a_1 = 2, b_1 = 5, c_1 = 4$
And $a_2 = 1, b_2 = 2, c_2 = -3$
$\therefore \cos\theta = \dfrac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$
$= \dfrac{2.1+5.2+4(-3)}{\sqrt{2^2+5^2+4^2}\sqrt{1^2+2^2+(-3)^2}}$
$\Rightarrow \theta = \cos^{-1}\dfrac{(2+10-12)}{\sqrt{4+25+16}\sqrt{1+4+9}}$
$= \cos^{-1}(0)$
$\Rightarrow\ \theta = 90°$

293 **(d)**
Let the equation of plane be,
$a(x-1)+b(y+2)+c(z-1)=0$
Which is perpendicular to $2x-2y+z=$ 0 and $x-y+2z=4$
$\therefore 2a-2b+c=0$ and $a-b+2c=0$
$\Longrightarrow \dfrac{a}{-3} = \dfrac{b}{-3} = \dfrac{c}{0}$
$\Longrightarrow \dfrac{a}{1} = \dfrac{b}{1} = \dfrac{c}{0}$
$\therefore$ The equation of plane is,
$1(x-1)+1(y+2)+0(z-1)=0$
$\Longrightarrow x+y+1$
$=0$, its distance from the point $(1,2,2)$is $\dfrac{|1+2+}{\sqrt{2}}$
$= 2\sqrt{2}$

294 **(d)**
The required line passes through the point
$\hat{\imath}+3\hat{\jmath}+2\hat{k}$ and is perpendicular to the lines
$\vec{r} = (\hat{\imath}+2\hat{\jmath}-\hat{k})+\lambda(2\hat{\imath}+\hat{\jmath}+\hat{k})$
and, $\vec{r} = (2\hat{\imath}+6\hat{\jmath}+\hat{k})+\mu(\hat{\imath}+2\hat{\jmath}+3\hat{k})$
Therefore, it is parallel to the vector
$\vec{b} = (2\hat{\imath}+\hat{\jmath}+\hat{k})\times(\hat{\imath}+2\hat{\jmath}+3\hat{k}) = (\hat{\imath}+5\hat{\jmath}+3\hat{k})$
Hence, the equation of the required line is
$\vec{r} = (\hat{\imath}+3j+2\hat{k})+\lambda'(\hat{\imath}-5j+3k)$
$\Rightarrow \vec{r} = (\hat{\imath}+3\hat{\jmath}+2\hat{k})+\lambda(-\hat{\imath}+3\hat{\jmath}-3\hat{k})$, where
$\lambda = -\lambda'$

295 **(a)**
$\because$ Direction cosines
of OP are $\left(\frac{1}{3},\frac{-2}{3},\frac{-2}{3}\right)$, also $OP = r = 3$
Now, point P is given by $P(lr, mr, nr)$
$ie, P(x,y,z) = P\left[\frac{1}{3}(3), -\left(\frac{2}{3}\right)3, \left(-\frac{2}{3}\right)3\right]$
$= P(1,-2,-2)$

296 **(a)**
Given that, $l+m+n=0$ ...(i)
And $lm = 0$ ...(ii)
$\therefore$ From Eq. (i) $\Rightarrow l = -(m+n)$
And from Eq.(ii)$\Rightarrow\ -(m+n)m = 0$
$\Rightarrow\ -(m^2+mn) = 0$
$\Rightarrow m^2+mn = 0$
$\Rightarrow m = 0, m+n = 0$
If $m = 0, l+m+n = 0$ [from Eq. (i)]
Then, $\frac{l_1}{-1} = \frac{m_1}{0} = \frac{n_1}{1}$
And if $l+m+n=0$
Then $\frac{l_2}{0} = \frac{m_2}{-1} = \frac{n_2}{1}$
$\therefore (l_1,m_1,n_1) = (-1,0,1)$
And $(l_2,m_2,n_2) = (0,-1,1)$
$\therefore$ Angle between them is given by
$\cos\theta = \dfrac{0+0+1}{\sqrt{1+0+1}\sqrt{0+1+1}} = \dfrac{1}{2} = \dfrac{\pi}{3}$

298 **(b)**
Given equation can be rewritten as
$(2x-y)\lambda+(-y+3z)=0$
So, it is clear that the equation of the plane passes through the intersection of planes $2x-y=$ 0 and $y-3z=0$

299 **(a)**
Given equation of sphere is
$x^2+y^2+z^2-x-2y-3z=0$
$\therefore$ Centre is $\left(\dfrac{1}{2},1,\dfrac{3}{2}\right)$
$\therefore$ Radius $= \sqrt{\left(\dfrac{1}{2}\right)^2+(1)^2+\left(\dfrac{3}{2}\right)^2-0} = \dfrac{\sqrt{14}}{2}$

300 **(b)**
Let the plane$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the coordinate axes at A, B and C, the coordinates of the centroid of ΔABC are $\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)$
Given, $\dfrac{a}{3}=1, \dfrac{b}{3}=2, \dfrac{c}{3}=3$
$\Longrightarrow a=3, b=6, c=9$
Hence, the equation of the plane is
$\dfrac{x}{3}+\dfrac{y}{6}+\dfrac{z}{9}=1$

301 **(d)**

Let $A(0,7,10), B(-1,6,6)$ and $C(-4,9,6)$

Then, $AB=\sqrt{(-1-0)^2+(6-7)^2+(6-10)^2}$

$=\sqrt{1+1+16}=\sqrt{18}=3\sqrt{2}$

$BC=\sqrt{(-4+1)^2+(9-6)^2+(6-6)^2}$

$=\sqrt{9+9+0}=\sqrt{18}=3\sqrt{2}$

$AC=\sqrt{(-4-0)^2+(9-7)^2+(6-10)^2}$

$=\sqrt{16+4+16}=\sqrt{36}=6$

Clearly, $AC^2=AB^2+BC^2$

Hence, triangle is right angled. Also, $AB=BC$

$\therefore$ Triangle is right angled isosceles

303 **(a)**

Let the direction cosines of the line L be l, m, n.

Since, the line intersect the given planes, then the normal to the planes are perpendicular to the line L

$\therefore\ 2l+3m+n=0$ ...(i)

and $l+3m+2n=0$ ....(ii)

From Eqs. (i) and (ii), we get

$\frac{l}{3}=\frac{m}{-3}=\frac{n}{3}=k$ [say]

We, know, $l^2+m^2+n^2=1$

$\therefore\ (3k)^2+(-3k)^2+(3k)^2=1$

$\Rightarrow k=\frac{1}{3\sqrt{3}}$

$\therefore l=\frac{1}{\sqrt{3}} \Rightarrow \cos\alpha=\frac{1}{\sqrt{3}}$

304 **(c)**

Let the vertices of ΔABC are $A(1,2,3), B(2,5,-1)$ and $C(-1,1,2)$

Area of triangle $=\frac{1}{2}|\overrightarrow{\mathbf{AB}}\times\overrightarrow{\mathbf{AC}}|$

$=\frac{1}{2}\left\|\begin{matrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ x_2-x_1 & y_2-y_1 & z_2-z_1\\ x_3-x_1 & y_3-y_1 & z_3-z_1\end{matrix}\right\|$

Here ,$(x_1,y_1,z_1)=(1,2,3),(x_2,y_2,z_2)=(2,5,-1)$

And $(x_3,y_3,z_3)=(-1,1,2)$

$\therefore$Area of triangle $=\frac{1}{2}\left\|\begin{matrix}\hat{\boldsymbol{i}} & \hat{\boldsymbol{j}} & \hat{\boldsymbol{k}}\\ 1 & 2 & -4\\ -2 & -1 & -1\end{matrix}\right\|$

$=\frac{1}{2}|(-7\hat{\mathbf{i}}+9\hat{\mathbf{j}}+5\hat{\mathbf{k}})|$

$=\frac{1}{2}\sqrt{(-7)^2+(9)^2+(5)^2}$

$=\frac{1}{2}\sqrt{49+81+25}$

$=\frac{\sqrt{155}}{2}$ sq unit

305 **(c)**

We know that the angle between two planes

$a_1x+b_1y+c_1z+d_1=0$

And $a_2x+b_2y+c_2z+d_2=0$ is given by

$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$ ...(i)

From the given equations of planes on comparing both with standard equation of plane

ie, $ax+by+cz+d=0$ respectively, we get

$a_1=1, b_1=2, c_1=2$

And $a_2=-5, b_2=3$ and $c_2=4$

On putting these values in Eq. (i), we get

$\cos\theta=\frac{1\times(-5)+2\times 3+2\times 4}{\sqrt{1^2+2^2+2^2}\sqrt{(-5)^2+3^2+4^2}}$

$=\frac{-5+6+8}{\sqrt{9}\sqrt{50}}=\frac{9}{3\sqrt{50}}=\frac{3}{5\sqrt{2}}=\frac{3\sqrt{2}}{10}$

$\Rightarrow \cos^{-1}\left(\frac{3\sqrt{2}}{10}\right)$

306 **(c)**

Let $A=(3,4\,,5), B=(4,6,3), C=(-1,2,4), D\equiv(1,0,5)$

For $AB, x_2-x_1=4-3=1, y_2-y_1=6-4=2$

$z_2-z_1=3-5=-2$

Let l,m,n for CD are $\frac{2}{3},\frac{-2}{3},\frac{1}{3}$.

$\therefore$ Projection of AB on $CD=\sum(x_2-x_1)$

$=\frac{2(1)}{3}+\left(-\frac{2}{3}\right)2+\left(\frac{1}{3}\right)(-2)$

$=-\frac{4}{3}$

307 **(a)**

Given lines can be rewritten as

$\frac{x-1}{1}=\frac{y+3}{-\lambda}=\frac{z-1}{\lambda}=s$

and $\frac{x-0}{1}=\frac{y-1}{2}=\frac{z-2}{-2}=\frac{t}{2}$

Since, two lines are coplanar.

$\therefore\begin{vmatrix}1-0 & -3-1 & 1-2\\ 1 & -\lambda & \lambda\\ 1 & 2 & -2\end{vmatrix}=0$

$\Rightarrow\begin{vmatrix}1 & -4 & -1\\ 1 & -\lambda & \lambda\\ 1 & 2 & -2\end{vmatrix}=0$

$\Rightarrow 1(2\lambda-2\lambda)+4(-2-\lambda)-1(2+\lambda)=0$

$\Rightarrow\ -8-4\lambda-2-\lambda=0$

$\Rightarrow\ \lambda=-2$

308 **(c)**

The direction ratios of given plane are (1,2,2) and $(-5,3,4)$.

The angle between two planes is given by

$\theta=\cos^{-1}\left(\frac{1(-5)+2(3)+2(4)}{\sqrt{1+4+4}\sqrt{25+9+16}}\right)$

$=\cos^{-1}\left(\frac{9}{3\cdot 5\sqrt{2}}\right)$

$\Rightarrow \cos^{-1}\left(\frac{3\sqrt{2}}{10}\right)$

309 **(b)**

The equation of a plane passing through (2, 3, 1) is

$a(x-2)+b(y-3)+c(z-1)=0$ ... (i)

It passes through $(4,-5,3)$ and is parallel to x-axis

$2a-8b+2c=0$

and, $a\times 1+b\times 0+c\times 0=0$

$\therefore \frac{a}{0}=\frac{b}{2}=\frac{c}{8}\Rightarrow\frac{a}{0}=\frac{b}{1}=\frac{c}{4}$

Substituting the values of a, b, c in (i), we get

$y+4z=7$ as the equation of the required plane

310 **(c)**

Given lines are

$l+m+n=0 \Rightarrow l=-(m+n)$ ....(i)

and $mn-2\ln+lm=0$ .....(ii)

$\Rightarrow mn+2(m+n)n-(m+n)m=0$ [from Eq. (i)]

$\Rightarrow mn+2mn+2n^2-m^2-nm=0$

$\Rightarrow 2\left(\frac{n}{m}\right)^2+\frac{2n}{m}-1=0$

This is quadratic equation in $\left(\frac{n}{m}\right)$,

$\therefore \frac{n_1n_2}{m_1m_2}=\frac{-1}{2}$ ... ... (iii)

[where $\frac{n_1}{m_1},\frac{n_2}{m_2}$ are the roots of the equation]

From, Eq, (i)

$m=-(n+l)$

On putting in Eq. (ii), we get

$-(n+l)n-2ln-l(n+l)=0$

$\Rightarrow l^2+4ln+n^2=0$

$\Rightarrow \left(\frac{l}{n}\right)^2+\frac{4l}{n}+1=0$

$\Rightarrow \frac{l_1l_2}{n_1n_2}=1$ (iv)

[where $\frac{l_1}{n_1},\frac{l_2}{n_2}$ are the roots of the equation]

$\therefore$ From Eqs. (iii) and (iv)

$l_1l_2=-\frac{1}{2}m_1m_2=n_1n_2$

$\Rightarrow \frac{l_1l_2}{1}=\frac{m_1m_2}{-2}=\frac{n_1n_2}{1}=k$ [say]

Now, $l_1l_2+m_1m_2=k-2k+k=0$

$\therefore \cos\theta=0 \Rightarrow \theta=90°$

311 **(c)**

$\cos 2\alpha+\cos 2\beta+\cos 2\gamma+\sin^2\alpha+\sin^2\beta+\sin^2\gamma$

$=(\cos^2\alpha-\sin^2\alpha)+(\cos^2\beta-\sin^2\beta)$

$+(\cos^2\gamma-\sin^2\gamma)+\sin^2\alpha+\sin^2\beta+\sin^2\gamma$

$=\cos^2\alpha+\cos^2\beta+\cos^2\gamma$

$=1$

312 **(a)**

Equation of plane passing through $(2,-1,-3)$ is

$a(x-2)+b(y+1)+c(z+3)=0$ ...(i)

Now, given lines are parallel to it.

$\therefore 3a+2b-4c=0$ ....(ii)

and $2a-3b+2c=0$ ...(iii)

Elimination of a, b and c from Eqs. (i), (ii) and (iii), gives

$\begin{vmatrix} x-2 & y+1 & z+3 \\ 3 & 2 & -4 \\ 2 & -3 & 2 \end{vmatrix}=0$

$\Rightarrow (x-2)(4-12)-(y+1)(6+8)$

$\Rightarrow 8x+14y+13z+37=0$

313 **(a)**

DR's of $AB=\{(3-2),(5-3),(-3+1)\}$

$=\{1,2,-2\}$

DR's of $CD=\{(3-1),(5-2),(7-3)\}$

$=\{2,3,4\}$

$\therefore$ Angle between AB and CD is given by

$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

$=\frac{1\times 2+2\times 3+4\times(-2)}{\sqrt{1+4+4}\sqrt{4+9+16}}=0$

$\Rightarrow \theta=\frac{\pi}{2}$

314 **(c)**

Since, $l^2+m^2+n^2=1$

$\Rightarrow \cos^2\theta+\cos^2\beta+\cos^2\theta=1$

$[\because l=\cos\theta, m=\cos\beta, \text{given}]$

$\Rightarrow \cos^2\theta+\cos^2\beta$

But $\sin^2\beta=3\sin^2\theta$

$\therefore 3\sin^2\theta=2\cos^2\theta$

$\Rightarrow 3=5\cos^2\theta$

$\Rightarrow \cos^2\theta=\frac{3}{5}$

315 **(b)**

Equation of plane containing the line of intersection of planes is

$(x+y+z-6)+\lambda(2x+3y+4z-12)=0$

Since, it passes through the point (1, 1, 1),

$\therefore (1+1+1-6)+\lambda(2+3+4-12)=0$

$\Rightarrow -3+\lambda(-3)=0$

$\Rightarrow \lambda=-1$

Hence, required equation of plane is

$(x+y+z-6)-(2x+3y+4z-12)=0$

ie, $x+2y+3z=6$

316 **(d)**

Let the point in xy-plane be $P(x_1,y_1,0)$. Let the

given points are $A(2,0,3)B(0,3,2)$
And $C(0,0,1)$
According to the given condition,
$AP^2 = BP^2 = CP^2$
$\therefore (x_1-2)^2 + y_1^2 + 9 = x_1^2 + (y_1-3)^2 + 4$
$= x_1^2 + y_1^2 + 1$
From Ist and IInd terms,
$x_1^2 + 4 - 4x_1 + y_1^2 + 9 = x_1^2 + y_1^2 - 6y_1 + 9 + 4$
$\Longrightarrow 4x_1 - 6y_1 = 0 \quad \ldots..\text{(i)}$
From IInd and IIIrd terms,
$x_1^2 + y_1^2 + 9 - 6y_1 + 4 = x_1^2 + y_1^2 + 1$
$\Longrightarrow 6y_1 = 12 \implies y_1 = 2$
On putting the value of y_1 in Eq.(i), we get $x_1 = 3$
Hence, required point is (3, 2, 0).

317 **(a)**
Equation of any plane passing through $(-7,1,-5)$ is
$a(x+7) + b(y-1) + c(z+5) = 0$ .....(i)
The DR's of normal to above plane are
$a = -7, \qquad b = 1, \qquad c = -5$
$\therefore$ From Eq. (i) we get
$-7(x+7) + 1(y-1) - 5(z+1) = 0$
$\Longrightarrow 7x - y + 5z + 75 = 0$

318 **(b)**
Equation of plane through (1, 2, 3) is
$a(x-1) + b(y-2) + c(z-3) = 0$ ...(i)
It passes through $(-1,4,2)$ and (3, 1,1), so
$-2a + 2b - c = 0$ ...(ii)
And $2a - b - 2c = 0$ ...(iii)
From Eqs. (ii) and (iii),
$$\frac{a}{-5} = \frac{b}{-6} = \frac{c}{-2}$$
$\therefore$ Equation of plane is
$-5x - 6y - 2z + 5 + 12 + 6 = 0$
$\Rightarrow 5x + 6y + 2z - 23 = 0$

319 **(b)**
Since, the line passing through the points $(4,-1,2)$ and $(-3,2,3)$. So, the DR's of the line is $(4+3, -1-2, 2-3) ie,\ (7,-3,-1)$
Since, the line is perpendicular to the plane therefore DR's of this line is proportional to the normal of the plane.
$\therefore$ Required equation plane is
$7(x+10) - 3(y-5) - 1(z-4) = 0$
$\Longrightarrow 7x - 3y - z + 89 = 0$

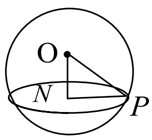
320 **(b)**
The given line is parallel to the vector $\vec{n} = \hat{\imath} - \hat{\jmath} + 2\hat{k}$. The required plane passes through the point (2, 3, 1) i.e. $2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ and is perpendicular to the vector $\vec{n} = \hat{\imath} - \hat{\jmath} + 2\hat{k}$. So, its equation is
$$\{\vec{r} - (2\hat{\imath} + 3\hat{\jmath} + \hat{k})\} \cdot \{\hat{\imath} - \hat{\jmath} + 2\hat{k}\} = 0$$
$$\Rightarrow \vec{r} \cdot (\hat{\imath} + \hat{\jmath} + 2\hat{k}) = 1$$

321 **(c)**
Direction ratio of the line joining the points $(2,1,-3)$ and $(-3,1,7)$are $(a_1, b_1, c_1) ie$, $(-5,0,10)$
Direction ratio of the line parallel to the line$\frac{x-1}{3} = \frac{y}{4} = \frac{z+3}{5}$
are $(a_2, b_2, c_2) ie, (3,4,5)$
Angle between two lines given by
$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$
$$\therefore \cos\theta \frac{(-5\times 3) + (0\times 4) + (10\times 5)}{\sqrt{25+0+100}\sqrt{9+16+25}}$$
$$= \frac{35}{25\sqrt{10}}$$
$$\Rightarrow \theta = \cos^{-1}\left(\frac{7}{5\sqrt{10}}\right)$$

322 **(c)**
The centre of the given sphere is $C(-2,1,3)$. The distance from the centre of sphere to the plane
$$= \left|\frac{-2\times 12 + 4\times 1 + 3\times 3 - 327}{\sqrt{144+16+9}}\right|$$
$$= \left|\frac{-24+4+9-327}{\sqrt{169}}\right| = 26$$
$\therefore$ Shortest distance
$= 26 - \sqrt{4+1+9+155} = 13$

323 **(c)**
DC's of line
$$= \left(\frac{12}{\sqrt{12^2+4^2+3^2}}, \frac{4}{\sqrt{12^2+4^2+3^2}}, \frac{3}{\sqrt{12^2+4^2+3^2}}\right)$$
$$= \left(\frac{12}{13}, \frac{4}{13}, \frac{3}{13}\right)$$

324 **(c)**
Since, the centre of sphere $x^2 + y^2 + z^2 - 2y - 4z - 11 = 0$ is (0, 1, 2) and radius is 4
Distance of a plane $x + 2y + 2z - 15 = 0$ from (0, 1, 2)



$$= \frac{|0+2+4-15|}{\sqrt{1+4+4}} = \frac{9}{3} = 3$$
Now, $NP = \sqrt{OP^2 - ON^2}$
$= \sqrt{4^2 - 3^2} = \sqrt{16-9} = \sqrt{7}$
$\therefore$ Radius of circle $= \sqrt{7}$

325 **(b)**
Let D be the foot of perpendicular drawn from

$P(1, 0, 3)$ on the line AB joining (4,7,1) and (3, 5, 3)
If D divides AB in ratio $\lambda : 1$, then the coordinate of D
$= \left(\frac{3\lambda+4}{\lambda+1}, \frac{5\lambda+7}{\lambda+1}, \frac{3\lambda+1}{\lambda+1}\right)$ ...(i)
DR's of PD are $\frac{2\lambda+3}{\lambda+1}, \frac{5\lambda+7}{\lambda+1}, \frac{-2}{\lambda+1}$
Dr's of AB are $-1, -2, 2$
$\because PD$ is perpendicular to AB
$\therefore -\frac{(2\lambda+3)}{\lambda+1} - \frac{2(5\lambda+7)}{\lambda+1} - \frac{4}{\lambda+1} = 0$
$\Rightarrow \lambda = \frac{-7}{4}$
On putting the value of λ in Eq. (i), we get the point $D\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$

326 **(b)**
$\vec{\mathbf{r}} = (\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}) + t(\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}})$
$= (1+t)\hat{\mathbf{i}} - (1-t)\hat{\mathbf{j}} + (1-t)\hat{\mathbf{k}}$
Also $\vec{\mathbf{r}} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 5$
$\Rightarrow (1+t) - (1-t) + (1-t) = 5$
$\Rightarrow 1 + t = 5 \Rightarrow t = 4$
$\therefore \vec{\mathbf{r}} = (1+4)\hat{\mathbf{i}} - (1-4)\hat{\mathbf{j}} + (1-4)\hat{\mathbf{k}}$
$= 5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

327 **(c)**
Equation of any plane passing through (a, b, c) is
$a'(x-a) + b'(y-b) + c'(z-c) = 0$ ....(i)
DR's of $OA = (a, b, c)$
Since, plane (i) is perpendicular to the line OA, therefore its
DR's is proportional to (a, b, c)
$\therefore$ Required equation of plane is
$a(x-a) + b(y-b) + c(z-c) = 0$

328 **(c)**
The required line passes through $(-1, 2, -3)$ and is perpendicular to the plane $2x + 3y + z + 5 = 0$. Therefore, it is parallel to the normal to the plane whose direction ratios are proportional to 2, 3, 1
Hence, direction ratios of the line are proportional to 2, 3, 1 and so its equation is
$\frac{x+1}{2} = \frac{y-2}{3} = \frac{z+3}{1}$

329 **(a)**
In a given options, only option (a) satisfies the given equation of line.

330 **(c)**
$\cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$
$= \left|\frac{\sqrt{3}}{4} \times \frac{\sqrt{3}}{4} + \frac{1}{4} \times \frac{1}{4} + \frac{\sqrt{3}}{2} \times \left(\frac{-\sqrt{3}}{2}\right)\right|$
$= \left|\frac{3}{16} + \frac{1}{16} - \frac{3}{4}\right|$
$= \left|-\frac{2}{4}\right| = \frac{1}{2}$
$= \theta = \frac{\pi}{3}$

331 **(b)**
Given lines will be perpendicular, if
$-3 \times 3k + 2k \times 1 + 2 \times -5 = 0 \Rightarrow -7k - 10 = 0$
$\Rightarrow k = -\frac{10}{7}$

332 **(c)**
The centre of sphere is $\left(\frac{1}{2}, 0, -\frac{1}{2}\right)$
and radius $= \sqrt{\frac{1}{4} + \frac{1}{4} + 2} = \frac{\sqrt{10}}{2}$
distance from centre of sphere to the given plane
$= \left|\frac{\frac{1}{2} + \frac{1}{2} - 4}{\sqrt{1+4+1}}\right| = \frac{3}{\sqrt{6}}$
So, radius of circle$= \sqrt{\frac{10}{4} - \frac{9}{6}}$
$= 1$

333 **(b)**
$\sin\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$
$= \frac{2 \times 3 + 3 \times 2 - 4 \times 3}{\sqrt{2^2 + 3^2 + 4^2}\sqrt{(3)^2 + (2)^2 + (-3)^2}}$
$= \frac{6 + 6 - 12}{\sqrt{4+9+16}\sqrt{9+4+9}} = 0$
$\Longrightarrow \theta = 0°$

334 **(c)**
If two lines are coplanar, then
$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$
$\therefore \begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$
$\Longrightarrow -1(1+2k) - 1(1+k^2) + 1(2-k) = 0$
$\Longrightarrow -k^2 - 3k = 0$
$\Longrightarrow k = 0 \text{ or } -3$

335 **(c)**
Equation of any plane passing through given line is
$a(x-1) + b(y+1) + c(z-3) = 0$ ...(i)
Above plane is perpendicular to the plane
$x + 2y + z = 12$
$\therefore a + 2b + c = 0$
Also, normal to the plane is perpendicular to the line
$\therefore 2a - b + 4c = 0$
$\Longrightarrow \frac{a}{8+1} = \frac{b}{2-4} = \frac{c}{-1-4}$

$\Rightarrow \dfrac{a}{9} = \dfrac{b}{-2} = \dfrac{c}{-5}$
$\therefore 9(x-1) - 2(y+1) - 5(z-3) = 0$
$\Rightarrow 9x - 2y - 5z + 4 = 0$
$\therefore a = 9, b = -2, c = -5$

336 **(b)**
Plane can be rewritten as
$\dfrac{x}{6} + \dfrac{y}{-4} + \dfrac{z}{3} = 1$
$\therefore$ Intercepts are $6, -4, 3$

337 **(a)**
Equation of straight line passing through $(4, -5, -2)$ and $(-1, 5, 3)$ is
$\dfrac{x-4}{-5} = \dfrac{y+5}{10} = \dfrac{z+2}{5}$
$\Rightarrow \dfrac{x-4}{1} = \dfrac{y+5}{-2} = \dfrac{z+2}{-1}$

338 **(d)**
The length of the perpendicular from origin to the plane is
$p = \left|\dfrac{0+0+0-52}{\sqrt{9+16+144}}\right|$
$= \left|\dfrac{-52}{13}\right| = 4$

339 **(b)**
Since, PA, PB are perpendicular drawn from $P(a, b, c)$ on yz and zx-planes.
$\therefore A(0, b, c)$ and $B(a, 0, c)$ are the points on yz and xz-plances.
The equation of plane passing through (0, 0, 0) is
$Ax + By + Cz = 0$
Which also passes through points A and B
$\therefore A \cdot 0 + B \cdot b + C \cdot c = 0$
and $A \cdot a + B \cdot 0 + C \cdot c = 0$
$\Rightarrow \dfrac{A}{bc - 0} = \dfrac{B}{ac - 0} = \dfrac{C}{0 - ab} = \lambda \quad$ [say]
$\Rightarrow A = \lambda bc, \quad B = \lambda ac, \quad C = -\lambda ab$
$\therefore$ Required equation is
$bcx + acy - abz = 0$

340 **(c)**
Let the equation of the sphere be
$x^2 + y^2 + z^2 + 24x + 2vx + 2wz + d = 0$
Since, above sphere passes through (1, 0, 0), (0, 1, 0) and (0, 0, 1)
$\therefore u = v = w = -\dfrac{d+1}{2}$
Let r be the radius of sphere
$\therefore r^2 = u^2 + v^2 + w^2 - d$
$= 3\left(\dfrac{d+1}{2}\right)^2 - d$
$= \dfrac{3}{4}\left(d^2 + \dfrac{2}{3}d + 1\right)$
$= \dfrac{3}{4}\left[\left(d + \dfrac{1}{3}\right)^2 + \dfrac{8}{9}\right]$
Clearly at $d = -\frac{1}{3}$, r^2 attains minimum and minimum value of $r^2 = \frac{2}{3}$
$\Rightarrow$ Minimum value of $r = \sqrt{\frac{2}{3}}$

341 **(c)**
We have,
$\vec{r} = (1 + \lambda - \mu) + \hat{\imath} + (2 - \lambda)\hat{\imath} + (3 - 2\lambda + 2\mu)\hat{k}$
$\Rightarrow \vec{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(\hat{\imath} - \hat{\jmath} - 2\hat{k}) +$
$\mu(-\hat{\imath} + 2\hat{k})$ which is a plane passing through $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and parallel to the vectors $\vec{b} = \hat{\imath} - \hat{\jmath} - 2\hat{k}$ and $\vec{c} = -\hat{\imath} + 2\hat{k}$
Therefore, it is normal to the vector
$\vec{n} = \vec{b} \times \vec{c} = -2\hat{\imath} - \hat{k}$
Hence, its vector equation is
$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$
$\Rightarrow \vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$
$\Rightarrow \vec{r} \cdot (= -2\hat{\imath} - \hat{k}) = -2 - 3$
$\Rightarrow \vec{r} \cdot (2\hat{\imath} + \hat{k}) = 5$
So, the Cartesian equation of the plane is
$(x\hat{\imath} + y\hat{\jmath} + z\hat{k}) \cdot (2\hat{\imath} + \hat{k}) = 5 \Rightarrow 2x + z = 5$

342 **(c)**
Given, $A(3, 1, 2)$ be the foot of the perpendicular from $B(-2, 1, 4)$on the plane, then direction ratios of BA, which is the normal to plane are $(3+2, 1-1, 2-4)ie$,
$(5, 0, -2)$
$\therefore$ The equation of plane is
$5(x-3) + 0(y-1) - 2(z-2) = 0$
$\Rightarrow 5x - 2z = 11$

343 **(c)**
The straight line $\vec{r} = \vec{a} + \lambda\,\vec{b}$ meets the plane $\vec{r} \cdot \vec{n} = 0$ in P for which λ is given by
$(\vec{a} + \lambda\,\vec{b}) \cdot \vec{n} = 0 \Rightarrow \lambda = -\dfrac{\vec{a} \cdot \vec{n}}{\vec{b} \cdot \vec{n}}$
Thus, the position vector of P is
$\vec{r} = \vec{a}\left(\dfrac{\vec{a} \cdot \vec{n}}{\vec{b} \cdot \vec{n}}\right)\vec{b}$ [Putting the value of λ in $\vec{r}$
$= \vec{a} + \lambda\vec{b}$]

344 **(a)**
Equation of plane through $(3, 2, -1)$ is
$a(x-3) + b(y-2) + c(z+1) = 0 \quad$ ...(i)
Also, (3, 4, 2) and (7, 0, 6) lie on Eq. (i), then
$0 \cdot a + 2b + 3c = 0 \quad$ ...(ii)
And $4a - 2b + 7c = 0 \quad$ ...(iii)
On eliminating a, b, c from Eqs. (i), (ii) and (iii), we get

$$\begin{vmatrix} x-3 & y-2 & z+1 \\ 0 & 2 & 3 \\ 4 & -2 & 7 \end{vmatrix} = 0$$

We get, $5x + 3y - 2z = 23$

$\therefore\ \lambda = 23$

345 **(d)**

Given lines pass through points $P(\overrightarrow{a_1})$ and $Q(\overrightarrow{a_2})$ and are parallel to vectors $\overrightarrow{b_1}$and $\overrightarrow{b_2}$respectively

If the lines are coplanar, then

$\vec{P}Q \perp (\overrightarrow{b_1} \times \overrightarrow{b_2})$

$\Rightarrow \vec{P}Q \cdot (\overrightarrow{b_1} \times \overrightarrow{b_2}) = 0$

$\Rightarrow (\overrightarrow{a_2} - \overrightarrow{a_1}) \cdot (\overrightarrow{b_1} \times \overrightarrow{b_2}) = 0 \Rightarrow [\overrightarrow{a_1}\,\overrightarrow{b_1}\,\overrightarrow{b_2}]$

$= [\overrightarrow{a_2}\,\overrightarrow{b_1}\,\overrightarrow{b_2}]$

346 **(c)**

Let, $\dfrac{x+1}{3} = \dfrac{y+3}{5} = \dfrac{z+5}{7} = \lambda$ ...... (i)

Any point on the line is $(3\lambda - 1, 5\lambda - 3, 7\lambda - 5)$

Again let

$\dfrac{x-2}{1} = \dfrac{y-4}{3} = \dfrac{z-6}{5} = \mu$ ... (ii)

Any point on the line is (μ+2, 3μ+4, 5μ+6)

For intersection, they have a common point.

$\therefore (3\lambda - 1) = (\mu + 2)$,

$(5\lambda - 3) = (3\mu + 4)$,

$(7\lambda - 5) = (5\mu + 6)$

From first two, we have

$\mu = 3\lambda - 3$ ..... (iii)

and $3\mu = 5\lambda - 7$ ... (iv)

From Eqs. (iii), and (iv), we have

$3(3\lambda - 3) = 5\lambda - 7 \Longrightarrow \lambda = \dfrac{1}{2}$

Point of intersection is

$\left(\dfrac{3}{2} - 1, \dfrac{5}{2} - 3, \dfrac{7}{2} - 5\right)$

$= \left(\dfrac{1}{2}, -\dfrac{1}{2}, -\dfrac{3}{2}\right)$

347 **(c)**

Since, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 2$

$[\because\ l^2 + m^2 + n^2 = 1]$

$\Longrightarrow 1 + \cos 2\alpha + 1 + \cos 2\beta + 1 + \cos 2\gamma = 2$

$\Longrightarrow\ \cos 2\alpha + \cos 2\beta + \cos 2\gamma + 1 = 0$

348 **(a)**

Let the equation of sphere passing through origin is

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz = 0$

It passes through $(0, 2, 0)$

$\therefore 4 + 4v = 0\ \Rightarrow v = -1$

Also, it passes through (1, 0, 0)

$\therefore 1 + 2u = 0$

$\Rightarrow u = \dfrac{-1}{2}$

And it passes through (0, 0, 4)

$\therefore 16 + 8w\ \Rightarrow w = -2$

$\therefore$ Centre of sphere is $(-u, -v, -w) = \left(\frac{1}{2}, 1, 2\right)$

349 **(c)**

Let $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$ .... (i)

Then, $\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c} = \dfrac{1}{2}$ ... (ii)

On comparing Eqs. (i) and (ii), we get

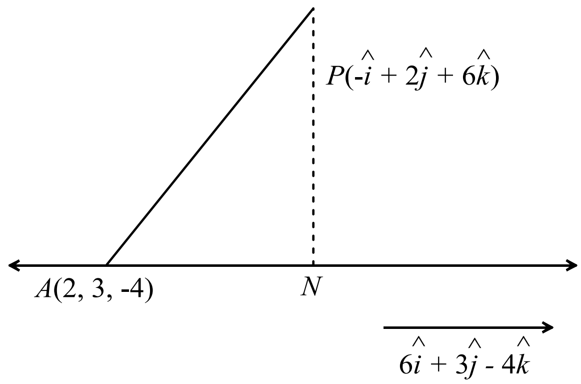
$x = 2, \quad y = 2, \quad z = 2$

350 **(a)**

We have,

$\vec{A}P = -3\hat{\imath} - \hat{\jmath} + 10\hat{k} \Rightarrow |\vec{A}P| = \sqrt{9 + 1 + 100}$

$= \sqrt{110}$



Now,

$AN =$ Projection of $\vec{A}P$ on $6\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$

$\Rightarrow AN = \left|\dfrac{\vec{A}P \cdot (6\hat{\imath} + 3\hat{\jmath} - 4\hat{k})}{|6\hat{\imath} + 3\hat{\jmath} - 4\hat{k}|}\right| = \left|\dfrac{-18 - 3 - 40}{\sqrt{61}}\right|$

$= \sqrt{61}$

$\therefore PN = \sqrt{AP^2 - AN^2} = \sqrt{110 - 61} = 7$

351 **(c)**

Given points on the plane are

$(a, 0, 0), (0, b, 0)$and $(0, 0, c)$

$\therefore$ Length of intercept with x-axis, y-axis and z-axis are a, b and c respectively.

$\therefore$ Equation of the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

352 **(c)**

Since, the planes $2x - 2y + z + 3 = 0$ and $2x - 2y + z + \frac{5}{2} = 0$ are parallel to each other.

$\therefore$Distance between them$= \frac{|c_2 - c_1|}{\sqrt{a_1^2 + b_1^2 + c_1^2}}$

$= \dfrac{\left|\frac{5}{2} - 3\right|}{\sqrt{4 + 4 + 1}}$

$= \dfrac{1}{6}$

353 **(d)**

Direction ratio of OP and OQ are (0, 1, 2) and

$(4, -2, 1)$
Let $\angle POQ = \theta$, then
$$\cos\theta \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$
$$\Rightarrow \cos\theta = \frac{0 - 2\times 1 + 2\times 1}{\sqrt{0+1+4}\sqrt{16+4+1}} = 0$$
$$\Rightarrow \theta = \frac{\pi}{2}$$
354 **(d)**
Given equation line is
$$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{4} = \lambda \quad \text{[say]}$$
Any point on the line is $P(2\lambda + 1, -3\lambda + 2, 4\lambda - 3)$
Since, these point lies on the given plane.
$\therefore 2(2\lambda + 1) + 4(-3\lambda + 2) - (4\lambda - 3) + 1 = 0$
$$\Rightarrow \lambda = \frac{7}{6}$$
$\therefore$ Required point is $P\left(\frac{10}{3}, -\frac{3}{2}, \frac{5}{3}\right)$
355 **(c)**
Given lines are
$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = r(\text{say}) \quad \ldots\ldots (i)$$
and $\dfrac{x-4}{5} = \dfrac{y-1}{2} = z \quad \ldots\ldots (ii)$
Any point on the line (i) is $(2r + 1, 3r + 2, 4r + 3,$
If they intersect, then the point satisfies the second line, we get
$$\frac{2r+1-4}{5} = \frac{3r+2-1}{2} = 4r + 3$$
$$\Rightarrow \frac{2r-3}{5} = \frac{3r+1}{2} \Rightarrow r = -1$$
$\therefore$ Required point is $(-1, -1, -1)$
356 **(c)**
Clearly in option (a), it is not a sphere as it contains xy, yz and zx terms. In options (b) and (d)
$u^2 + v^2 + w^2 - c^2 < 0$
So, option (c) is sphere
357 **(a)**
$$\because \cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$
$$= \frac{1\times 3 + 0\times 4 - 1\times 5}{\sqrt{1+0+1}\sqrt{9+16+25}} = -\frac{1}{5}$$
$$\Rightarrow \theta = \pi - \cos^{-1}\left(\frac{1}{5}\right)$$
358 **(b)**
Let the point is P whose coordinate are (x, y, z) and the given points are $A(4, 0,0)$ and $(-4, 0, 0)$
$\because PA + PB = 10$
$\therefore \sqrt{(x-4)^2 + y^2 + z^2} + \sqrt{(x+4)^2 + y^2 + z^2} = 10 \quad \ldots(i)$
Also, $[(x+4)^2 + y^2 + z^2] + [(x-4)^2 + y^2 + z2=16x$
And
$$\left[\sqrt{(x+4)^2 + y^2 + z^2} + \sqrt{(x-4)^2 + y^2 + z^2}\right]\left[\sqrt{(x+4)^2 + y^2 + z^2} - \sqrt{(x-4)^2 + y^2 + z^2}\right]16x$$
$$\Rightarrow \left[\sqrt{(x+4)^2 + y^2 + z^2} - \sqrt{(x-4)^2 + y^2 + z^2}\right] = \frac{16x}{10} \quad \ldots(ii)$$
On solving Eqs.(i) and (ii), we get
$$2\sqrt{(x+4)^2 + y^2 + z^2} = \frac{16x}{10} + 10$$
$$\Rightarrow (x+4)^2 + y^2 + z^2 = \left(\frac{4x}{5} + 5\right)^2$$
$$\Rightarrow x^2 + 8x + 16 + y^2 + z^2 = \frac{16x^2 + 625 + 200x}{25}$$
$$\Rightarrow [25x^2 + 400 + 200x + 25y^2 + 25z^2] - 16x^2 - 625 - 200x] = 0$$
$$\Rightarrow 9x^2 + 25y^2 + 25z^2 = 225$$
359 **(a)**
Plane intercept on x-axis at $a = 4$
Plane intercept on z-axis at $c = 3$
Required equation is $\frac{x}{4} + \frac{z}{3} = 1$ or $3x + 4z = 12$
360 **(d)**
Suppose P divides QR in the ratio $\lambda: 1$. Then, coordinates of P are $\left(\frac{5\lambda+2}{\lambda+1}, \frac{2\lambda+2}{\lambda+1}, \frac{-2\lambda+1}{\lambda+1}\right)$
Since, the x coordinates of P is 4
$$ie, \frac{5\lambda+2}{\lambda+1} = 4 \Rightarrow \lambda = 2$$
So, z coordinate of P is $\frac{-2\lambda+1}{\lambda+1} = \frac{-4+1}{2+1} = -1$
361 **(d)**
Given lines can be rewritten as
$$\frac{x-b}{a} = \frac{y-0}{1} = \frac{z-d}{c}$$
and $\dfrac{x-b'}{a'} = \dfrac{y-0}{1} = \dfrac{z-d'}{c'}$
These lines will perpendicular, if
$aa' + 1 + cc' = 0$
362 **(b)**
Given two lines $\vec{r} = (\hat{\imath} + \hat{\jmath}) + \lambda(\hat{\imath} + 2\hat{\jmath} - k$ and $r=i+j+\mu(-i+j-2k)$ pass through $a=i+j$ and, are parallel to the vectors $\vec{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\vec{c} = -\hat{\imath} + \hat{\jmath} - 2\hat{k}$ respectively. Therefore, the plane containing them passes through $\vec{a} = \hat{\imath} + \hat{\jmath}$ and is perpendicular to $\vec{n}$ given by

$\vec{n} = \vec{b} \times \vec{c} = (\hat{\imath} + 2\hat{\jmath} - \hat{k}) \times (-\hat{\imath} + \hat{\jmath} - 2\hat{k})$

$= -3\hat{\imath} + 3\hat{\jmath} + 3\hat{k}$

Hence, the equation of the required plane is

$(\vec{r} - \vec{a}) \cdot \vec{n} = 0 \Rightarrow \vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n} \Rightarrow \vec{r} \cdot (\hat{\imath} - \hat{\jmath} - \hat{k})$

$= 0$

363 **(b)**

Equation of any plane passing through (2, 3, 4) is

$A(x-2) + B(y-3) + C(z-4) = 0$ ...(i)

Plane (i) is parallel to $5x - 6y + 7z = 3$

$\therefore$ DR's of this plane is same as the Eq. (i)

$ie, A = 5, B = -6, C = 7$

$\therefore 5(x-2) - 6(y-3) + 7(z-4) = 0$

$\therefore\ 5x - 6y + 7z - 20 = 0$ is the required plane

364 **(a)**

Centre of given sphere are

$C_1(-3, 4, 1)$and $C_2(5, -2, 1)$

So, midpoint of C_1C_2

$\equiv P\left(\frac{5-3}{2}, \frac{4-2}{2}, \frac{1+1}{2}\right) = P(1, 1, 1)$

Now, the plane $2ax - 3ay + 4az + 6 = 0$ passes through the point P.

$\therefore 2a(1) - 3a(1) + 4a(1) + 6 = 0$

$\Longrightarrow a = -2$

365 **(c)**

Let the equation of sphere passing through (0, 0, 0) be

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz = 0$

Also, it passes through (0, 2, 0), (1, 0, 0), (0, 0, 4) respectively are

$4 + 4v = 0$

$\Longrightarrow v = -1$

$1 + 2u = 0 \Longrightarrow u = -\frac{1}{2}$

and $16 + 8w \Longrightarrow w = -2$

$\therefore$ Centre is $(-u, -v, -w) = \left(\frac{1}{2}, 1, 2\right)$

366 **(a)**

Here $a_1 = 1, b_1 = 2, c_2 = k$

and $a_2 = 2\ , b_2 = 1, c_2 = -2$

Since, two planes are perpendicular, then

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Longrightarrow 2 \cdot 1 + 1 \cdot 2 - 2(k) = 0$

$\Longrightarrow k = 2$