Single Correct Answer Type

1. The value of $7\log\frac{16}{15}+5\log\frac{25}{24}+3\log\frac{81}{80}$, is
a) $\log 2$ b) $\log 3$ c) $\log 5$ d) None of these
2. The sum of first two terms of an infinite G.P. is 1 and every term is twice the sum of the successive terms. Its first term is
a) 1/3 b) 2/3 c) 3/4 d) 1/4
3. If $\log_8\{\log_2(\log_3(x^2-4x+85))\}=\frac{1}{3}$, then x equals to
a) 5 b) 4 c) 3 d) 2
4. If the sum of first n natural numbers is $\frac{1}{78}$ times the sum of their cubes, then the value of n is
a) 11 b) 12 c) 13 d) 14
5. If n geometric means between a and b be $G_1, G_2, \dots G_n$ and a geometric mean be G, then the true relation is
a) $G_1G_2 \dots G_n = G$ b) $G_1G_2 \dots G_n = G^{1/n}$ c) $G_1G_2 \dots G_n = G^n$ d) $G_1G_2 \dots G_n = G^{2/n}$
6. Consider an infinite geometric series with first term a and common ratio r. If its sum is 4 and the second term is $\frac{3}{4}$, then
a) $a=\frac{4}{7}, r=\frac{3}{7}$ b) $a=2, r=\frac{3}{8}$ c) $a=\frac{3}{2}, r=\frac{1}{2}$ d) $a=3, r=\frac{1}{4}$
7. The sum of n terms of the series $1+(1+x)+(1+x+x^2)+\dots$ will be
a) $\frac{1-x^n}{1-x}$ b) $\frac{x(1-x^n)}{1-x}$
c) $\frac{n(1-x)-x(1-x^n)}{(1-x)^2}$ d) None of these
8. If the mth term of HP be n and nth term be m, then the rth term will be
a) $\frac{r}{mn}$ b) $\frac{mn}{r+1}$ c) $\frac{mn}{r}$ d) $\frac{mn}{r-1}$
9. In the expansion of $\frac{e^{7x}+e^{3x}}{e^{5x}}$, the constant term is
a) 0 b) 1 c) 2 d) None of these
10. The value of $2^{\log_3 7}-7^{\log_3 2}$ is
a) $\log 2$ b) 1 c) 0 d) None of these
11. If x, y, z are in AP, then $\frac{1}{\sqrt{x}+\sqrt{y}}, \frac{1}{\sqrt{z}+\sqrt{x}}, \frac{1}{\sqrt{y}+\sqrt{z}}$ are in
a) AP b) GP c) HP d) AP and HP
12. If $x, 2x+2, 3x+3, \dots$ are in GP, then the fourth term is
a) 27.5 b) $4x+5$ c) -13.5 d) $4x+4$
13. The sum of the infinite series
$\frac{1}{2}\left(\frac{1}{3}+\frac{1}{4}\right)-\frac{1}{4}\left(\frac{1}{3^2}+\frac{1}{4^2}\right)+\frac{1}{6}\left(\frac{1}{3^3}+\frac{1}{4^3}\right)-\dots$ is equal to
a) $\frac{1}{2}\log 2$ b) $\log\frac{3}{5}$ c) $\log\frac{5}{3}$ d) $\frac{1}{2}\log\frac{5}{3}$
14. If a, b, c be in A.P., b, c, d are in G.P., and c, d, e are in H.P., then a, c, e will be in
a) A.P. b) G.P. c) H.P. d) None of these
15. If $\log_y x=\log_z y=\log_x z$, then
a) $x<y<z$ b) $x>y\geq z$ c) $x<y\leq z$ d) $x=y=z$
16. If three numbers are in H.P., then the numbers obtained by subtracting half of the middle number from each of them are in
a) A.P. b) G.P. c) H.P. d) None of these

17. The sum of the series $5.05 + 1.212 + 0.29088 + \ldots \infty$ is

a) 6.93378 b) 6.87342 c) 6.74384 d) 6.64474

18. α, β are the roots of the equation $x^2 - 3x + a = 0$ and γ, δ are the roots of the equation $x^2 - 12x + b = 0$. If $\alpha, \beta, \gamma, \delta$ form an increasing GP, then (a, b) is equal to

a) $(3, 12)$ b) $(12, 3)$ c) $(2, 32)$ d) $(4, 16)$

19. $\sum_{r=0}^{n} \frac{(-1)^r}{{}^nC_r}$ equals

a) $\dfrac{(n+1)}{(n+2)}[1 + (-1)^n]$ b) 0 c) $\dfrac{2(n+1)}{(n+2)}$ d) $\dfrac{n}{n+1}[1 + (-1)^n]$

20. If $\log_{10} 2 = 0.3010$, then $\log_5 64 =$

a) $\dfrac{602}{233}$ b) $\dfrac{233}{602}$ c) $\dfrac{202}{633}$ d) $\dfrac{633}{202}$

21. Suppose a, b, c are in AP and a^2, b^2, c^2 are in GP. If $a < b < c$ and $a + b + c = \frac{3}{2}$, then the value of a is

a) $\dfrac{1}{2\sqrt{2}}$ b) $\dfrac{1}{2\sqrt{3}}$ c) $\dfrac{1}{2} - \dfrac{1}{\sqrt{3}}$ d) $\dfrac{1}{2} - \dfrac{1}{\sqrt{2}}$

22. The sum upto $(2n + 1)$ terms of the series $a^2 - (a + d)^2 + (a + 2d)^2 - (a + 3d)^2 + \cdots$, is

a) $a^2 + 3nd^2$

b) $a^2 + 2nad + n(n - 1)d^2$

c) $a^2 + 3nad + n(n - 1)d^2$

d) $a^2 + 2nad + n(2n + 1)d^2$

23. Let the sequence, $a_1, a_2, a_3, \ldots, a_{2n}$, form an AP, then $a_1^2 - a_2^2 + a_3^2 - \ldots + a_{2n-1}^2 - a_{2n}^2$ is equal to

a) $\dfrac{n}{2n-1}(a_1^2 - a_{2n}^2)$ b) $\dfrac{2n}{n-1}(a_{2n}^2 - a_1^2)$ c) $\dfrac{n}{n+1}(a_1^2 + a_{2n}^2)$ d) None of these

24. Let the positive numbers a, b, c, d be in AP, then abc, abd, acd, bcd are

a) Not in AP/GP/HP b) In AP c) In GP d) In HP

25. Which one of the following is correct? If $a = b = c$, then

a) a, b, c are in HP b) a, b, c are in AP but not in GP

c) a, b, c are in AP as well as in GP d) None of the above

26. If a, b, c be in G.P. and $a + x, b + x, c + x$ in H.P., then the value of x is (a, b, c are distinct numbers)

a) c b) b c) a d) None of these

27. Let a be a positive number such that the arithmetic mean of a and 2 exceeds their geometric mean by 1. Then, the value of a is

a) 3 b) 5 c) 9 d) 8

28. $1 + \frac{3}{2} + \frac{5}{2^2} + \frac{7}{2^3} \ldots . \infty$ is equal to

a) 3 b) 6 c) 9 d) 12

29. Let $S = \frac{8}{5} + \frac{16}{15} + \ldots + \frac{128}{2^{18}+1}$, then

a) $S = \dfrac{1088}{545}$ b) $S = \dfrac{545}{1088}$ c) $S = \dfrac{1056}{545}$ d) $S = \dfrac{545}{1056}$

30. Three non-zero real numbers form an A.P. and the squares of these numbers taken in the same order form a G.P. Then, then number of all possible values of common ratios of the G.P. is

a) 1 b) 2 c) 3 d) None of these

31. The value of $\frac{\sqrt{2}-1}{\sqrt{2}} + \frac{3-2\sqrt{2}}{4} + \frac{5\sqrt{2}-7}{6\sqrt{2}} + \frac{17-12\sqrt{2}}{16} + \cdots + \cdots +$ ad. inf., is

a) $\log_e 2$ b) $\log_e \sqrt{2}$ c) $\log_e 3$ d) $\log_e \sqrt{3}$

32. If $\frac{\log x}{a^2+ab+b^2} = \frac{\log y}{b^2+bc+c^2} = \frac{\log z}{c^2+ca+a^2}$, then $x^{a-b} . y^{b-c} . z^{c-a} =$

a) 0 b) -1 c) 1 d) 2

33. If $S = \frac{1}{1\cdot 2} - \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} - \frac{1}{4\cdot 5} + \cdots + \infty$, then e^S equals

a) $\log_e\left(\dfrac{4}{e}\right)$ b) $\dfrac{4}{e}$ c) $\log_e\left(\dfrac{e}{4}\right)$ d) $\dfrac{e}{4}$

34. Sum of n terms of the following series $1^3 + 3^3 + 5^3 + 7^3 + ..$ is

a) $n^2(2n^2 - 1)$ b) $n^3(n - 1)$ c) $n^3 + 8n + 4$ d) $2n^4 + 3n^2$

35. If the sum of an infinitely decreasing G.P. is 3, and the sum of the squares of its terms is 9/2, the sum of the cubes of the terms is

a) $\frac{105}{13}$ b) $\frac{108}{13}$ c) $\frac{729}{8}$ d) None of these

36. If $x = 1 + a + a^2 + \ldots \infty$ and $y = 1 + b + b^2 + \ldots \infty$ where a and b are proper fractions, then $1 + ab + a^2b^2 + \ldots \infty$ equals

a) $\frac{xy}{y + x - 1}$ b) $\frac{x + y}{x - y}$ c) $\frac{x^2 + y^2}{x - y}$ d) None of these

37. Let the harmonic mean and the geometric mean of two numbers be in the ration 4 : 5. The two numbers are in the ratio

a) 1 : 1 b) 2 : 1 c) 3 : 1 d) 4 : 1

38. If $\log_{x+2}(x^3 - 3x^2 - 6x + 8) = 3$ then x equals to

a) 1 b) 2 c) 3 d) None of these

39. The number of solutions of $\log_2(x - 1) = 2\log_2(x - 3)$ is

a) 2 b) 1 c) 6 d) 7

40. If the interior angles of a polygon are in A.P. with common difference 5° and smallest angle is 120°, then the number of sides of the polygon is

a) 9 or 16 b) 9 c) 16 d) 13

41. If the AM of two numbers be A and GM be G, then the numbers will be

a) $A \pm (A^2 - G^2)$ b) $\sqrt{A} \pm \sqrt{A^2 - G^2}$

c) $A \pm \sqrt{(A + G)(A - G)}$ d) $\frac{A \pm \sqrt{(A + G)(A - G)}}{2}$

42. The determinant $\Delta = \begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ a\alpha + b & b\alpha + c & 0 \end{vmatrix}$ is equal to zero, if

a) a, b, c are in A.P.

b) a, b, c are in G.P.

c) a, b, c are in H.P.

d) α is a root of $ax^2 + bx + c = 0$

43. The value of $0.0\overline{37}$, where $0.0\overline{37}$ stands for the number 0.0373737……………, is

a) 37/1000 b) 37/990 c) 1/37 d) 1/27

44. If roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$ are in A.P., then their common difference will be

a) ± 1 b) ± 2 c) ± 3 d) ± 4

45. The sum of n terms of three AP's whose first term is 1 and common differences are 1, 2, 3 respectively are S_1, S_2, S_3 respectively. The true relation is

a) $S_1 + S_3 = S_2$ b) $S_1 + S_3 = 2S_2$ c) $S_1 + S_2 = 2S_3$ d) $S_1 + S_2 = S_3$

46. If $\log_x\{\log_4(\log_x(5x^2 + 4x^3))\} = 0$, then

a) 2 b) 3 c) 4 d) 5

47. If twice the 11th term of an AP is equal to 7 times its 21st term, then its 25th term is equal to

a) 24 b) 120 c) 0 d) None of these

48. If a, b, c, d are any four consecutive coefficients of any expanded binomial, then $\frac{a+b}{a}, \frac{b+c}{b}, \frac{c+d}{c}$ are in

a) AP b) GP c) HP d) None of these

49. The coefficient of x^{n-2} in the polynomial $(x - 1)(x - 2) \ldots (x - n)$ is

a) $\frac{1}{24}n(n + 1)(n - 1)(3n + 2)$

b) $\frac{1}{24}n(n^2 - 1)(3n + 2)$

c) $\frac{n(n+1)(2n+1)}{6}$

d) None of these

50. If x, y, z be three positive prime numbers. The progression in which $\sqrt{x}, \sqrt{y}, \sqrt{z}$ can be three terms (not necessarily consecutive) is

a) A.P. b) G.P. c) H.P. d) None of these

51. If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in GP with the same common ratio, then the points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3)

a) Lie on a straight line b) Lie on an ellipse

c) Lie on a circle d) Are vertices of a triangle

52. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then $\log(a - bx + cx^2)$ is equal to

a) $\log a + (\alpha+\beta)x + \frac{\alpha^2+\beta^2}{2}x^2 + \frac{\alpha^3+\beta^3}{3}x^3 + \cdots$

b) $\log a + (\alpha+\beta)x - \left(\frac{\alpha^2+\beta^2}{2}\right)x^2 + \frac{\alpha^3+\beta^3}{3}x^3 - \cdots$

c) $\log a - (\alpha+\beta)x - \frac{\alpha^2+\beta^2}{2}x^2 - \frac{\alpha^3+\beta^3}{3}x^3 - \cdots$

d) None of these

53. The sum of n terms of an A.P. is $3n^2 + 5$. The number of term which equals 159 is

a) 13 b) 21 c) 27 d) None of these

54. If the sum of the first n terms of a series be $5n^2 + 2n$, then its second term is

a) $\frac{56}{15}$ b) $\frac{27}{14}$ c) 17 d) 16

55. If the AM and GM between two numbers are in the ratio $m:n$, then the numbers are in the ratio

a) $m + \sqrt{m^2+n^2} : m - \sqrt{m^2+n^2}$ b) $m + \sqrt{n^2-m^2} : m - \sqrt{n^2-m^2}$

c) $m + \sqrt{m^2-n^2} : m - \sqrt{m^2-n^2}$ d) None of the above

56. If a, b, c are in G.P. and $\log_c a, \log_b c, \log_a b$ are in A.P., then the common difference of the A.P. is

a) 3 b) 3/2 c) 1/2 d) 2/3

57. The value of $\frac{\log 49\sqrt{7} + \log 25\sqrt{5} - \log 4\sqrt{2}}{\log 17.5}$ is

a) 5 b) 2 c) $\frac{5}{2}$ d) $\frac{3}{2}$

58. If x, y, z are in G.P. and $x+3, y+3, z+3$ are in H.P., then

a) $y = 2$ b) $y = 3$ c) $y = 1$ d) $y = 0$

59. If a, b, c, d are in HP, then $ab + bc + cd$ is equal to

a) $3\,ad$ b) $(a+b)(c+d)$ c) $3\,ac$ d) None of these

60. The sum of n terms of the series $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \cdots$, is

a) $2^n - n - 1$ b) $1 - 2^{-n}$ c) $n + 2^{-n} - 1$ d) $2^n - 1$

61. If $a^2 + 4b^2 = 12ab$, then $\log(a + 2b) =$

a) $\frac{1}{2}(\log a + \log b - 2)$

b) $\log\frac{a}{2} + \log\frac{b}{2} + \log 2$

c) $\frac{1}{2}(\log a + \log b + 4\log 2)$

d) $\frac{1}{2}(\log a - \log b + 4\log 2)$

62. If $1^2 + 2^2 + 3^2 + \cdots + 2003^2 = (2003)(4007)(334)$ and, $1 \times 2003 + 2 \times 2002 + 3 \times 2001 + \cdots + 2003 \times 1 = (2003)(334)x$, then $x =$

a) 2005 b) 2004 c) 2003 d) 2001

63. The sum of $15^2 + 16^2 + 17^2 + \ldots + 30^2$ is equal to

a) 8840 b) 8440 c) 8540 d) 8450

64. The sum of the products of the numbers $\pm 1, \pm 2, \ldots \pm n$, taken two at a time is

a) $\frac{-n(n+1)}{2}$ b) $\frac{n(n+1)(2n+1)}{6}$ c) $\frac{-n(n+1)(2n+1)}{6}$ d) 0

65. If $(4.2)^x = (0.42)^y = 100$, then $\frac{1}{x} - \frac{1}{y} =$

a) 1 b) 2 c) $\frac{1}{2}$ d) -1

66. The sum of the series $2\{7^{-1} + 3^{-1} \cdot 7^{-3} + 5^{-1} \cdot 7^{-5} + \cdots\}$ is

a) $\log_e\left(\frac{4}{3}\right)$ b) $\log_e\left(\frac{3}{4}\right)$ c) $2\log_e\left(\frac{3}{4}\right)$ d) $2\log_e\left(\frac{4}{3}\right)$

67. Which term of the sequence
$(-8+18i), (-6+15i), (-4+12i), \ldots$ is purely imaginary?

a) 5th b) 7th c) 8th d) 6th

68. If $a^2 + 4b^2 = 12ab$, then $\log(a+2b) =$

a) $\frac{1}{2}(\log a + \log b - \log 2)$

b) $\log\frac{a}{2} + \log\frac{b}{2} + \log 2$

c) $\frac{1}{2}(\log a + \log b + 4\log 2)$

d) $\frac{1}{2}(\log a - \log b + 4\log 2)$

69. If $\frac{1}{\log_x 10} = \frac{2}{\log_a 10} - 2$, then $x =$

a) $\frac{a}{2}$ b) $\frac{a}{100}$ c) $\frac{a^2}{10}$ d) $\frac{a^2}{100}$

70. If x, y, z are in H.P., then $\log(x+z) + \log(x-2y+z)$ is equal to

a) $\log(x-z)$ b) $2\log(x-z)$ c) $3\log(x-z)$ d) $4\log(x-z)$

71. If $\log_e 2 . \log_x 27 = \log_{10} 8 . \log_e 10$, then $x =$

a) 1 b) 3 c) 2 d) 4

72. If $\log_8 x = 2.5$ and $\log_2 y = 5$, then $x =$

a) $y^{3/2}$ b) $2y$ c) y d) $\frac{y}{2}$

73. The sum of the series $1^2 + 1 + 2^2 + 2 + 3^2 + 3 + \cdots + n^2 + n$ is

a) $\frac{n(n+1)}{2}$

b) $\left\{\frac{n(n+1)}{2}\right\}^2$

c) $\frac{n(n+1)(n+2)}{3}$

d) $\frac{n(n+1)(n+2)(n+3)}{4}$

74. For what value of b, will the roots of the equation $\cos x = b, -1 \le b \le 1$ when arranged in ascending order of their magnitudes, form an A.P.?

a) -1 b) $\frac{\sqrt{3}}{2}$ c) $\frac{1}{\sqrt{2}}$ d) 1/2

75. Let a and b be roots of $x^2 - 3x + p = 0$ and let c and d be the roots of $x^2 - 12x + q = 0$, where a, b, c, d form an increasing GP Then the ratio of $q + p : q - p$ is equal to

a) 8 : 7 b) 11 : 10 c) 17 : 15 d) None of these

76. The coefficient of x^n in the expansion of $\frac{(a-bx)}{e^x}$ is

a) $\frac{(-1)^n}{n!}(a+bn)$ b) $\frac{(-1)^n}{n!}(b+an)$ c) $\frac{(-1)^{n+1}}{n!}(a+bn)$ d) None of the above

77. If pth term of an arithmetic progression is q and the qth term is p, then 10th term is
a) $p-q+10$ b) $p+q+11$ c) $p+q-9$ d) $p+q-10$

78. If a, b, c are digits, then the rational number represented by $0.cababab$ ... is
a) $\frac{cab}{990}$ b) $\frac{99c+10a+b}{99}$ c) $\frac{99c+ba}{990}$ d) $\frac{99c+10a+b}{990}$

79. The sum of 24 terms of the following series $\sqrt{2}+\sqrt{8}+\sqrt{18}+\sqrt{32}+\ldots$ is
a) 300 b) $300\sqrt{2}$ c) $200\sqrt{2}$ d) None of these

80. If $\log(2a-3b)=\log a-\log b$, then $a=$
a) $\frac{3b^2}{2b-1}$ b) $\frac{3b}{2b-1}$ c) $\frac{b^2}{2b+1}$ d) $\frac{3b^2}{2b+1}$

81. If $2^x.3^{2x}=100$, then x belongs to
a) $(0,3)$ b) $(1,3)$ c) $(1,2)$ d) $(0,2)$

82. The value of $1-\log_e 2+\frac{(\log_e 2)^2}{2\,!}-\frac{(\log_e 2)^3}{3\,!}+\cdots$, is
a) 2 b) $\frac{1}{2}$ c) $\log_e 3$ d) None of these

83. $\frac{1}{2}-\frac{1}{2.2^2}+\frac{1}{3.2^3}-\frac{1}{4.2^4}+\cdots$ is equal to
a) $\frac{1}{4}$ b) $\log_3\left(\frac{3}{4}\right)$ c) $\log_e\left(\frac{3}{2}\right)$ d) $\log_e\left(\frac{2}{3}\right)$

84. If $\frac{1}{a}+\frac{1}{c}+\frac{1}{a-b}+\frac{1}{c-b}=0$ and $b\neq a+c$, are in
a) G.P. b) H.P. c) A.P. d) None of these

85. If $4a^2+9b^2+16c^2=2(3ab+6bc+4ca)$, where a, b, c are non-zero numbers, then a, b, c are in
a) AP b) GP c) HP d) None of these

86. If the sum of the first n natural numbers is $1/5$ times the sum of their squares, then the value of n is
a) 5 b) 6 c) 7 d) 8

87. Let $a_1, a_2, a_3, \ldots$ be terms of an AP. If $\frac{a_1+a_2+\cdots+a_p}{a_1+a_2+\cdots+a_q}=\frac{p^2}{q^2}, p\neq q$, then $\frac{a_6}{a_{21}}$ equals
a) $\frac{7}{2}$ b) $\frac{2}{7}$ c) $\frac{11}{41}$ d) $\frac{41}{11}$

88. Which of the following statement is correct?
a) If each earn of an AP a number is added or subtracted, then the series so obtained is also an AP.
b) The nth term of geometric series whose first term is a and common ratio r, is ar^{n-1}.
c) If each term of a GP be raised to the same power the resulting terms are in GP.
d) All of the above

89. The sum of 10 terms of the series $\sqrt{2}+\sqrt{6}+\sqrt{18}+\ldots$ is
a) $121(\sqrt{6}+\sqrt{2})$ b) $243(\sqrt{3}+1)$ c) $\frac{121}{\sqrt{3}-1}$ d) $242(\sqrt{3}-1)$

90. If $4a^2+9b^2+16c^2=2(3ab+6bc+4ca)$, where a, b, c are non-zero numbers, then a, b, c are in
a) A.P. b) G.P. c) H.P. d) None of these

91. If a, b, c are in G.P. and $a-b, c-a$ and $b-c$ are in H.P., then $a+4b+c$ is equal to
a) -3 b) 0 c) 3 d) None of these

92. If the number of terms in an AP is $2n+1$, then the ratio of the sum of the odd terms to the sum of even terms is
a) $\frac{n+1}{n}$ b) $\frac{n}{n+1}$ c) $\frac{n^2}{n+1}$ d) $\frac{n+1}{2n}$

93. If a is positive and if A and G are the arithmetic mean and the geometric mean of the roots of $x^2-2ax+a^2=0$ respectively, then
a) $A=G$ b) $A=2G$ c) $2A=G$ d) $A^2=G$

94. The sum of the series $1 \cdot 3 \cdot 5 + 2 \cdot 5 \cdot 8 + 3 \cdot 7 \cdot 11 + \ldots$ upto n term is

a) $\frac{n(n+1)(9n^2+23n+13)}{6}$
b) $\frac{n(n-1)(9n^2+23n+12)}{6}$
c) $\frac{(n+1)(9n^2+23n+13)}{6}$
d) $\frac{n(9n^2+23n+13)}{6}$

95. $1^3 - 2^3 + 3^3 - 4^3 + \cdots + 9^3 =$

a) 425
b) -425
c) 475
d) -475

96. The value of $\log_2 \log_2 \log_4 256 + 2\log_{\sqrt{2}} 2$, is

a) 2
b) 3
c) 5
d) 7

97. If $1 + \sin x + \sin^2 x + \ldots$ upto $\infty = 4 + 2\sqrt{3}, 0 < x < \pi$ and $x \neq \frac{\pi}{2}$, then x is equal to

a) $\frac{\pi}{3}, \frac{5\pi}{6}$
b) $\frac{2\pi}{3}, \frac{\pi}{6}$
c) $\frac{\pi}{3}, \frac{2\pi}{3}$
d) $\frac{\pi}{6}, \frac{\pi}{3}$

98. If the sides of sides of a right angled triangle are in AP, then the sides are proportional to

a) 1 : 2 : 3
b) 2 : 3 : 4
c) 3 : 4 : 5
d) 4 : 5 : 6

99. The values of $3\log\frac{81}{80} + 5\log\frac{25}{24} + 7\log\frac{16}{15}$ is

a) $\log 2$
b) $\log 3$
c) 1
d) 0

100. Consider the following statement :

1. If a_n denotes the nth term of an AP, then

$a_n = \frac{a_{n+k} + a_{n-k}}{2}$

2. In an AP, if the sum of m term is equal to the sum of n terms, then the sum of $(m+n)$ term is always zero.

3. The sum to infinity of the series $\frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \ldots$ is $\frac{1}{2}$.

Which of the statement is given above is/are correct?

a) (1)and (2)
b) (2)and (3)
c) (3)and (1)
d) All (1), (2) and (3)

101. If $\frac{1}{e^{3x}}(e^x + e^{5x}) = a_0 + a_1x + a_2x^2 + \cdots$, then $2a_1 + 2^3a_3 + 2^5a_5 + \cdots$ is equal to

a) e
b) e^{-1}
c) 1
d) 0

102. The value of $a^{\frac{\log_b(\log_b x)}{\log_b a}}$, is

a) $\log_a x$
b) $\log_b x$
c) $\log_x a$
d) $\log_x b$

103. If $a_1, a_2, a_3, \ldots, a_{4001}$ are terms of an AP such that $\frac{1}{a_1a_2} + \frac{1}{a_2a_3} + \ldots + \frac{1}{a_{4000}a_{4001}} = 10$ and $a_2 + a_{4000} = 50$, then $|a_1 - a_{4001}|$ is equal to

a) 20
b) 30
c) 40
d) None of these

104. If p, q, r are in A.P., then pth, qth and rth terms of any G.P. are in

a) A.P.
b) G.P.
c) Reciprocals of these terms are in A.P.
d) None of these

105. The sum $1(1!) + 2(2\,!) + 3(3!) + \ldots + n(n!)$ equal to

a) $3(n!) + n - 3$
b) $(n+1)! - (n-1)!$
c) $(n+1)! - 1!$
d) $2(n!) - 2n - 1$

106. The sixth term of an A.P. is equal to 2 the value of the common difference of the A.P. which makes the product $a_1a_4a_5$ least is given by

a) $x = 8/5$
b) $x = 5/4$
c) $x = 2/3$
d) None of these

107. If the sum to first n terms of the AP 2, 4, 6,… is 240, then the value of n is

a) 14
b) 15
c) 16
d) 17

108. Three numbers whose sum is 15 are in AP. If they are added by 1, 4 and 19 respectively they are in GP. The numbers are

a) 2, 5, 8
b) 26, 5, −16
c) 2, 5, 8 and 26, 5, −16
d) None of these

109. If $1+\cos\alpha+\cos^2\alpha+\ldots\infty=2-\sqrt{2}$, then $\alpha, (0<\alpha<\pi)$ is

a) $\frac{\pi}{8}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{4}$ d) $\frac{3\pi}{4}$

110. Let α,β are the roots of $f(x)=ax^2+bx+c, a\neq 0$ and $\Delta=b^2-4ac$. If $\alpha+\beta, \alpha^2+\beta^2$ and $\alpha^3+\beta^3$ are in GP, then

a) $\Delta\neq 0$ b) $b\Delta=0$ c) $c\Delta=0$ d) $bc\neq 0$

111. If n geometric means be inserted between a and b, then the nth geometric mean will be

a) $a\left(\frac{b}{a}\right)^{\frac{n}{n-1}}$ b) $a\left(\frac{b}{a}\right)^{\frac{n-1}{n}}$ c) $a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$ d) $a\left(\frac{b}{a}\right)^{\frac{1}{n}}$

112. The value of

$(x+y)(x-y)+\frac{1}{2!}(x+y)(x-y)(x^2+y^2)$

$+\frac{1}{3!}(x+y)(x-y)(x^4+y^4+x^2y^2)+\cdots$, is

a) $e^{x^2}+e^{y^2}$ b) $e^{x^2}+e^{y^2}$ c) $e^{x^2-y^2}$ d) $e^{x^2+y^2}$

113. The value of $2.\overline{357}$ is

a) $\frac{2355}{1001}$ b) $\frac{2355}{999}$ c) $\frac{2355}{1111}$ d) None of these

114. If $x, 1, z$ are in A.P. and $x, 2, z$ are in G.P., then $x, 4, z$ are in

a) AP b) G.P. c) H.P. d) None of these

115. The sum of the series

$\frac{1^2\cdot 2}{1!}+\frac{2^2\cdot 3}{2!}+\frac{3^2\cdot 4}{3!}+\frac{4^2\cdot 5}{4!}+\cdots$, is

a) $5e$ b) $3e$ c) $7e$ d) $2e$

116. If $\log_{10}343=2.5353$, then the least positive integer n such that $7^n>10^5$, is

a) 1 b) 6 c) 5 d) 4

117. If a, b, c are three distinct positive real numbers which are in HP, then $\frac{3a+2b}{2a-b}+\frac{3c+2b}{2c-b}$ is

a) Greater than or equal to 10
b) Less than or equal to 10
c) Only equal to 10
d) None of the above

118. If $\frac{\log x}{a-b}=\frac{\log y}{b-c}=\frac{\log z}{c-a}$, then xyz is equal to

a) 0 b) 1 c) -1 d) 2

119. If $a=\sum_{n=0}^{\infty}\frac{x^{3n}}{(3n)!}, b=\sum_{n=1}^{\infty}\frac{x^{3n-2}}{(3n-2)!}$ and $c=\sum_{n=1}^{\infty}\frac{x^{3n-1}}{(3n-1)!}$, then the value of $a^3+b^3+c^3-3abc$ is

a) 1 b) 0 c) -1 d) -2

120. Two sequences $<a_n>$ and $<b_n>$ are defined by $a_n=\log\left(\frac{5^{n+1}}{3^{n-1}}\right), b_n=\left\{\log\left(\frac{5}{3}\right)\right\}^n$, then

a) $<a_n>$ is an A.P. and $<b_n>$ is a G.P.
b) $<a_n>$ and $<b_n>$ both are G.P.
c) $<a_n>$ and $<b_n>$ both are A.P.
d) $<b_n>$ is a G.P. and $<b_n>$ is neither an A.P nor a G.P.

121. Sum to n terms of the series $\frac{1}{1.2.3.4}+\frac{1}{2.3.4.5}+\frac{1}{3.4.5.6}+\cdots$, is

a) $\frac{n^3}{3(n+1)(n+2)(n+3)}$
b) $\frac{n^3+6n^2-3n}{6(n+2)(n+3)(n+4)}$
c) $\frac{15n^2+7n}{4n(n+1)(n+5)}$
d) $\frac{n^3+6n^2+11n}{18(n+1)(n+2)(n+3)}$

122. If the sum of an infinite GP and the sum of square of its term is 3, then the common ratio of the first series is

a) 1 b) $\frac{1}{2}$ c) $\frac{2}{3}$ d) $\frac{3}{2}$

123. The nth term of the series $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\ldots$ will be

a) n^2+2n+1 b) $\frac{n^2+2n+1}{8}$ c) $\frac{n^2+2n+1}{4}$ d) $\frac{n^2-2n+1}{4}$

124. If $\log 2, \log(2^x-1)$ and $\log(2^x+3)$ are in A.P., then $2, 2^x-1, 2^x+3$ are in

a) A.P. b) H.P. c) G.P. d) None of these

125. If $\log_2 a+\log_4 b+\log_4 c=2$
$\log_9 a+\log_3 b+\log_9 c=2$
$\log_{16} a+\log_{16} b+\log_4 c=2$, then

a) $a=\frac{2}{3}, b=\frac{27}{8}, c=\frac{32}{3}$
b) $a=\frac{27}{8}, b=\frac{2}{3}, c=\frac{32}{3}$
c) $a=\frac{32}{3}, b=\frac{27}{8}, c=\frac{2}{3}$
d) $a=\frac{2}{3}, b=\frac{32}{3}, c=\frac{27}{8}$

126. If the pth term of an AP be q and qth term be p, then its rth term will be

a) $p+q+r$ b) $p+q-r$ c) $p+r-q$ d) $p-q-r$

127. The sum of the series $\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\cdots$ is equal to

a) $\frac{(e^2-1)}{2}$ b) $\frac{(e-1)^2}{2e}$ c) $\frac{(e^2-1)}{2e}$ d) $\frac{(e^2-2)}{e}$

128. If $|a|<1$ and $|b|<1$, then the sum of the series $a(a+b)+a^2(a^2+b^2)+a^3(a^3+b^3)+\cdots$ upto ∞, is

a) $\frac{a}{1-a}+\frac{ab}{1-ab}$ b) $\frac{a^2}{1-a^2}+\frac{ab}{1-ab}$ c) $\frac{b}{1-b}+\frac{a}{1-a}$ d) $\frac{b^2}{1-b^2}+\frac{ab}{1-ab}$

129. If $0<y<2^{1/3}$ and $x(y^3-1)=1$, then $\frac{2}{x}+\frac{2}{3x^3}+\frac{2}{5x^5}+\cdots$ is equal to

a) $\log\left(\frac{y^3}{2-y^3}\right)$ b) $\log\left(\frac{y^3}{1-y^3}\right)$ c) $\log\left(\frac{2y^3}{1-y^3}\right)$ d) $\log\left(\frac{y^3}{1-2y^3}\right)$

130. $\{a_n\}$ *and* $\{b_n\}$ be two sequences given by $a_n=(x)^{\frac{1}{2^n}}+(y)^{\frac{1}{2^n}}$ and $b_n=(x)^{\frac{1}{2^n}}-(y)^{\frac{1}{2^n}}$ for all $n\in N$, then $a_1a_2a_3\ldots a_n$ is equal to

a) $x-y$ b) $\frac{x+y}{b_n}$ c) $\frac{x-y}{b_n}$ d) $\frac{xy}{b_n}$

131. The sum of the infinite terms of the series $\frac{5}{3^2+7^2}+\frac{9}{7^2+11^2}+\frac{13}{11^2+15^2}+\ldots$ is

a) $\frac{1}{18}$ b) $\frac{1}{36}$ c) $\frac{1}{54}$ d) $\frac{1}{72}$

132. The first term of a GP is 7, the last term is 448 and sum of all term is 889, then the common ratio is

a) 5 b) 4 c) 3 d) 2

133. Sum of infinite number of terms of a G.P. is 20 and sum of their squares is 100. The common ration of the G.P. is

a) 5 b) 3/5 c) 8/5 d) 1/5

134. If three real numbers a, b, c are in harmonic progression, then which of the following is true?

a) $\frac{1}{a}, b, \frac{1}{c}$ are in AP b) $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in HP
c) ab, bc, ca are in HP d) $\frac{a}{b}, \frac{b}{c}, \frac{c}{a}$ are in HP

135. $i^2+i^4+i^6+\ldots$upto $(2k+1)$terms, $k\epsilon N$ is

a) 0 b) 1 c) -1 d) k

136. If $a^x=b, b^y=c, c^z=a$, then value of xyz is

a) 0 b) 1 c) 2 d) 3

137. If $a_1, a_2, \dots a_n$ are in arithmetic progression, where $a_1 > 0$ for all i.
Then, $\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}$ is equal to

a) $\frac{n^2(n+1)}{2}$ b) $\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$ c) $\frac{n(n-1)}{2}$ d) None of these

138. Let $n(> 1)$ be a positive integer then the largest integer m such that $(n^m + 1)$ divides $(1 + n + n^2 + \dots + n^{127})$, is

a) 32 b) 63 c) 64 d) 127

139. If 1, $\log_3 \sqrt{(3^{1-x} + 2)}$, $\log_3(4.3^x - 1)$ are in AP, then x equals

a) $\log_3 4$ b) $1 - \log_3 4$ c) $1 - \log_4 3$ d) $\log_4 3$

140. The value of $\log_5\left(1 + \frac{1}{5}\right) + \log_5\left(1 + \frac{1}{6}\right) + \log_5\left(1 + \frac{1}{7}\right) + \cdots$
$+ \log_5\left(1 + \frac{1}{624}\right)$ is

a) 5 b) 4 c) 3 d) 2

141. In the expansion of $2 \log_e x - \log_e(x + 1) - \log_e(x - 1)$ the coefficient of x^{-4} is

a) $\frac{1}{2}$ b) -1 c) 1 d) None of these

142. If a, b, c are three unequal positive quantities in H.P., then

a) $a^{3/2} + c^{3/2} > 2b^{1/2}$ b) $a^5 + c^5 > 2b^5$ c) $a^2 + c^2 > 2b^3$ d) None of these

143. If the arithmetic mean of a and b is $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$, then the value of n is

a) -1 b) 0 c) 1 d) None of these

144. If G is the GM of the product of r set of observation with geometric means $G_1, G_2, \dots G_r$ respectively, then G is equal to

a) $\log G_1 + \log G_2 + \dots + \log G_n$ b) $G_1, G_2, \dots, G_r$
c) $\log G_1, \log G_2, \dots, \log G_n$ d) None of the above

145. The sum of all 2 digit odd numbers is

a) 2475 b) 2530 c) 4905 d) 5049

146. The sum of series $\frac{1}{1.2.3} + \frac{1}{3.4.5} + \frac{1}{5.6.7} + \cdots \infty$ is equal to

a) $\log_e 2 - \frac{1}{2}$ b) $\log_e 2$ c) $\log_e 2 + \frac{1}{2}$ d) $\log_e 2 + 1$

147. If $\log_a ab = x$, then the value of $\log_b ab$ is

a) $\frac{x-1}{x}$ b) $\frac{x}{x-1}$ c) $\frac{x}{x+1}$ d) $\frac{x+1}{x}$

148. The value of $1.1! + 2.2! + 3.3! + \dots + n.n!$ is

a) $(n + 1)!$ b) $(n + 1)! + 1$ c) $(n + 1)! - 1$ d) None of these

149. The value of $\log_2[\log_2\{\log_3(\log_3 27^3)\}]$ is

a) 1 b) 0 c) 3 d) 2

150. The sum to n terms of the series $2^2 + 4^2 + 6^2 + \dots$ is

a) $\frac{n(n+1)(2n+1)}{3}$ b) $\frac{2n(n+1)(2n+1)}{3}$ c) $\frac{n(n+1)(2n+1)}{6}$ d) $\frac{n(n+1)(2n+1)}{9}$

151. If arithmetic mean of two positive numbers is A, their geometric mean is G and harmonic mean is H, then H is equal to

a) G^2/A b) A^2/G^2 c) A/G^2 d) G/A^2

152. The sum of the infinite series $1 + \frac{1}{2!} + \frac{1.3}{4!} + \frac{1.3.5}{6!} + \cdots$ is

a) e b) e^2 c) $\sqrt{e}$ d) $\frac{1}{e}$

153. If $a_1, a_2, \dots, a_n$ are in AP with common difference d, then the sum of the series $\sin d\,(\operatorname{cosec} a_1 \operatorname{cosec} a_2 + \operatorname{cosec} a_2 \operatorname{cosec} a_3 + \dots + \operatorname{cosec} a_{n-1} \operatorname{cosec} a_n)$ is

a) $\sec a_1 - \sec a_n$ b) $\cot a_1 - \cot a_n$ c) $\tan a_1 - \tan a_n$ d) $\operatorname{cosec} a_1 = \operatorname{cosec} a_n$

154. The sum of a GP with common ratio 3 is 364 and last term is 243, then the number of terms is

a) 6 b) 5 c) 4 d) 10

155. If a, b, c be respectively the pth, qth and rth terms of a HP., then

$\Delta = \begin{vmatrix} bc & ca & ab \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$ equals

a) 1 b) 0 c) -1 d) None of these

156. If $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P., then

a) a, b, c are in A.P. b) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. c) a, b, c are in H.P. d) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in G.P

157. If $\log(x+z) + \log(x-2y+z) = 2\log(x-z)$, then x, y, z are in

a) H.P. b) G.P. c) A.P. d) None of these

158. If x, y, z are three consecutive positive integers, then

$\frac{1}{2}\log_e x + \frac{1}{2}\log_e z + \frac{1}{2\,x\,z+1} + \frac{1}{3}\left(\frac{1}{2\,x\,z+1}\right)^3 + \cdots$ is equal to

a) $\log_e x$ b) $\log_e y$ c) $\log_e z$ d) None of these

159. The value of $9^{1/3} \times 9^{1/9} \times 9^{1/27} \times \dots \infty$, is

a) 9 b) 1 c) 3 d) None of these

160. The coefficient of x^n in the series $1 + \frac{a+bx}{1!} + \frac{(a+bx)^2}{2!} + \frac{(a+bx)^3}{3!} = \cdots \infty$ is

a) $\frac{(ab)^n}{n!}$ b) $e^b.\frac{a^n}{n!}$ c) $e^a.\frac{b^n}{n!}$ d) $e^{a+b}.\frac{(ab)^n}{n!}$

161. If the 7th term of an H.P. is 1/10 and 12th term is 1/25, then 20th term is

a) $\frac{1}{37}$ b) $\frac{1}{41}$ c) $\frac{1}{45}$ d) $\frac{1}{49}$

162. If $2/3, k, 5/8$ are in AP, then value of k is

a) 15 b) 21 c) 12 d) 31/48

163. The sum of the series $1 + \frac{1}{3}.\frac{1}{4} + \frac{1}{5}.\frac{1}{4^2} + \frac{1}{7}.\frac{1}{4^3} + \cdots \infty$ is

a) $\log_e 1$ b) $\log_e 2$ c) $\log_e 3$ d) $\log_e 4$

164. Let $f(x)$ be a polynomial function of second degree. If $f(1) = f(-1)$ and a, b, c are in AP, then $f'(a), f'(b)$ and $f'(c)$ are in

a) AP b) GP

c) HP d) Arithmetico-Geometric Progression

165. The sum of the series $1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + 3 \cdot 4 \cdot 5 + \dots$ to n terms is

a) $n(n+1)(n+2)$ b) $(n+1)(n+2)(n+3)$

c) $\frac{1}{4}n(n+1)(n+2)(n+3)$ d) $\frac{1}{4}(n+1)(n+2)(n+3)$

166. If AM and GM of x and y are in the ratio $p:q$, then $x:y$ is

a) $p - \sqrt{p^2+q^2} : p + \sqrt{p^2+q^2}$ b) $p + \sqrt{p^2-q^2} : p - \sqrt{p^2-q^2}$

c) $p:q$ d) $p + \sqrt{p^2+q^2} : p - \sqrt{p^2+q^2}$

167. If a and b are two different positive real numbers, then which of the following statements is true?

a) $2\sqrt{ab} > a+b$ b) $2\sqrt{ab} < a+b$ c) $2\sqrt{ab} = a+b$ d) None of these

168. The sum of

$\frac{\frac{1}{2}\cdot\frac{2}{2}}{1^3} + \frac{\frac{2}{2}\cdot\frac{3}{2}}{1^3+2^3} + \frac{\frac{3}{2}\cdot\frac{4}{2}}{1^3+2^3+3^3} + \dots$ upto n terms is equal to

a) $\frac{n-1}{n}$ b) $\frac{n}{n+1}$ c) $\frac{n+1}{n+2}$ d) $\frac{n+1}{n}$

169. $\underset{n-\text{digits}}{(666\ldots.6)^2} + \underset{n-\text{digits}}{(888\ldots.8)}$ is equal to

a) $\frac{4}{9}(10^n - 1)$ b) $\frac{4}{9}(10^{2n} - 1)$ c) $\frac{4}{9}(10^n - 1)^2$ d) None of these

170. If $H_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$, then the value of $S_n = 1 + \frac{3}{2} + \frac{5}{3} + \cdots + \frac{2n-1}{n}$ is

a) $H_n + n$ b) $2n - H_n$ c) $(n-1) + H_n$ d) $H_n + 2n$

171. The length of a side of a square is "a" metre. A second square is formed by joining the middle points of this square. Then a third square is formed by joining the middle points of the sides of the second square and so on. Then, the sum of the areas of squares which carried upto infinity is

a) a^2 b) $2a^2$ c) $3a^2$ d) $4a^2$

172. If the sum of n terms of the series

$1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \cdots$ is $l + \frac{15}{16}\left(1 - \frac{1}{5^{n-1}}\right) - \frac{(3n-2)}{4(5^{n-1})}$, then l is

a) $\frac{4}{5}$ b) $\frac{5}{4}$ c) $\frac{6}{5}$ d) $\frac{5}{6}$

173. 0.5737373 ... is equal to

a) $\frac{284}{497}$ b) $\frac{284}{495}$ c) $\frac{568}{999}$ d) $\frac{567}{990}$

174. If x, y, z are in GP and $a^x = b^y = c^z$, then

a) $\log_a c = \log_b a$ b) $\log_b a = \log_c b$ c) $\log_c b = \log_a c$ d) None of these

175. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then, the common ratio of this progression equals

a) $\frac{1}{2}(1 - \sqrt{5})$ b) $\frac{1}{2}\sqrt{5}$ c) $\sqrt{5}$ d) $\frac{1}{2}(\sqrt{5} - 1)$

176. $\log_3 2, \log_6 2 \log_{12} 2$ are in

a) A.P. b) G.P. c) H.P. d) None of these

177. If p, q, r are in AP and are positive, the roots of the quadratic equation $px^2 + qx + r = 0$ are all real for

a) $\left|\frac{r}{p} - 7\right| \geq 4\sqrt{3}$ b) $\left|\frac{p}{r} - 7\right| < 4\sqrt{3}$ c) All p and r d) No p and r

178. The value of $(0.16)^{\log_{2.5}\left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots\infty\right)}$, is

a) 2 b) 3 c) 4 d) None of these

179. The value of $\frac{\log_a(\log_b x)}{\log_b(\log_a b)}$ is

a) $\log_b a$ b) $\log_a b$ c) $-\log_a b$ d) $-\log_b a$

180. $\frac{1}{1.3} + \frac{1}{2.5} + \frac{1}{3.7} + \frac{1}{4.9} + \ldots$ is equal to

a) $2\log_e 2 - 2$ b) $2 - \log_e 2$ c) $2\log_e 4$ d) $\log_e 4$

181. If S_1, S_2 and S_3 denote the sum of first n_1, n_2 and n_3 terms respectively of an A.P., then $\frac{S_1}{n_1}(n_2 - n_3) + \frac{S_2}{n_2}(n_3 - n_1) + \frac{S_3}{n_3}(n_1 - n_2) =$

a) 0 b) 1 c) $S_1S_2S_3$ d) $n_1n_2n_3$

182. The sum of 100 terms of the series 0.9 + 0.09 + 0.009 ... will be

a) $1 - \left(\frac{1}{10}\right)^{100}$ b) $1 + \left(\frac{1}{10}\right)^{100}$ c) $1 - \left(\frac{1}{10}\right)^{106}$ d) $1 + \left(\frac{1}{10}\right)^{10}$

183. Let $S_n = \frac{1}{1^3} + \frac{1+2}{1^3+2^3} + \ldots + \frac{1+2+\ldots+n}{1^3+2^3+\ldots+n^3}$, $n = 1,2,3, \ldots$. Then S_n is not greater than

a) $\frac{1}{2}$ b) 1 c) 2 d) 4

184. If $\log_4(3x^2 + 11x) > 1$, then x lies in the interval

a) $(-4, 1/3)$ b) $(-4, 2)$ c) $[-4, 1/3]$ d) $(-\infty, -4) \cup (1/3\ \infty)$

185. If pth, qth and rth terms of a G.P. are x, y, z respectively, then $x^{q-r}y^{r-p}z^{p-q}$ is equal to

a) 0 b) 1 c) -1 d) None of these

186. If $5^{3x^2 \log_{10} 2} = 2^{\left(x+\frac{1}{2}\right) \log_{10} 25}$, then x equals to

a) $1, -\frac{1}{3}$ b) 1 c) $1, -\frac{1}{2}$ d) $-\frac{1}{3}, 1$

187. $e^{x-1-\frac{1}{2}(x-1)^2+\frac{1}{3}(x-1)^3-\frac{1}{4}(x-1)^4+\cdots}$ is equal to

a) $\log(x-1)$ b) $\log x$ c) x d) None of these

188. If $e^{e^x} = a_0 + a_1 x + a_2 x^2 + \cdots$, then

a) $a_0 = 1$ b) $a_o = e$ c) $a_0 = e^e$ d) $a_0 = e^2$

189. The coefficient of x^n in the expansion of $\log_e\left(\frac{1}{1+x+x^2+x^3}\right)$, when n is odd, is

a) $-\frac{2}{n}$ b) $-\frac{1}{n}$ c) $\frac{1}{n}$ d) $\frac{2}{n}$

190. The harmonic mean between two numbers is $14\frac{2}{5}$ and the geometric mean is 24. The greater number between them is

a) 72 b) 54 c) 36 d) None of these

191. If the sum of the series 2,5,8,11, ... is 60100, then n, the number of terms, is

a) 100 b) 200 c) 150 d) 250

192. nth term of the series $1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \ldots$ will be

a) $\frac{3n+1}{5^{n-1}}$ b) $\frac{3n-1}{5^n}$ c) $\frac{3n-2}{5^{n-1}}$ d) $\frac{3n+2}{5^{n-1}}$

193. If $|x| < 1$ and $y = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$, then x is equal to

a) $y + \frac{y^2}{2} + \frac{y^3}{3} + \ldots$ b) $y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \ldots$

c) $y + \frac{y^2}{2!} + \frac{y^3}{3!} + \ldots$ d) $y - \frac{y^2}{2!} + \frac{y^3}{3!} - \frac{y^4}{4!} + \ldots$

194. The arithmetic mean of 7 consecutive integers starting with a is m. Then, the arithmetic mean of 11 consecutive integers starting with $a+2$ is

a) $2a$ b) $2m$ c) $a+4$ d) $m+4$

195. The value of $\log 2 + 2\left(\frac{1}{5} + \frac{1}{3} \cdot \frac{1}{5^3} + \frac{1}{5} \cdot \frac{1}{5^5} + \cdots + \infty\right)$ is

a) $\log 2 + \log 3$ b) $\log 2 + 2$ c) $\frac{1}{2} \log 2$ d) $\log 3$

196. If $a_1, a_2, a_3, \ldots, a_{24}$ are in arithmetic progression and $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$, then $a_1 + a_2 + a_3 + \ldots + a_{23} + a_{24}$ is equal to

a) 909 b) 75 c) 750 d) 900

197. If $x > 1, y > 1, z > 1$ are in GP, then $\frac{1}{1+\log x}, \frac{1}{1+\log y}, \frac{1}{1+\log z}$ are in

a) AP b) HP c) GP d) None of these

198. The series expansion of $\log\{(1+x)^{1+x}(1-x)^{1-x}\}$, is

a) $2\left\{\frac{x^2}{1 \cdot 2} + \frac{x^4}{3 \cdot 4} + \frac{x^6}{5 \cdot 6} + \cdots\right\}$

b) $\left\{\frac{x^2}{1 \cdot 2} + \frac{x^4}{3 \cdot 4} + \frac{x^6}{5 \cdot 6} + \cdots\right\}$

c) $2\left\{\frac{x^2}{1 \cdot 2} + \frac{x^4}{2 \cdot 3} + \frac{x^6}{3 \cdot 4} + \cdots\right\}$

d) None of these

199. Fifth term of a G.P. is 2, then the product of its 9 terms is

a) 256 b) 512 c) 1024 d) None of these

200. If $\log_3 x \times \log_x 2x \times \log_{2x} y = \log_x x^2$, then y equals

a) 9 b) 18 c) 27 d) 81

201. If a, b, c are in H.P., then $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ will be in

a) A.P. b) G.P. c) H.P. d) None of these

202. Given that n AM's are inserted between two sets of numbers $a, 2b$ and $2a, b$ where $a, b \in R$. Suppose further that mth mean between these sets of numbers is same, then the ratio $a : b$ equals

a) $(n-m+1):m$ b) $(n-m+1):n$ c) $n:(n-m+1)$ d) $m:(n-m+1)$

203. If $S_n = \frac{1^2 \cdot 2}{1\,!} + \frac{2^2 \cdot 3}{2\,!} + \frac{3^2 \cdot 4}{3\,!} + \cdots + \frac{n^2 \cdot (n+1)}{n\,!}$, then $\lim_{n\to\infty} S_n$ is equal to

a) $3\,e$ b) $5\,e$ c) $7\,e$ d) $9\,e$

204. GM and HM of two numbers are 10 and 8 respectively. The numbers are

a) 5,20 b) 4,25 c) 2,50 d) 1,100

205. Which term of the GP $3, 3\sqrt{3}, 9 \ldots$ is 2187?

a) 15 b) 14 c) 13 d) 19

206. The sum of all two digit natural numbers which leave a remainder 5 when they are divided by 7 is equal to

a) 715 b) 702 c) 615 d) 602

207. The sum of the series $\frac{1^2 \cdot 2^2}{1\,!} + \frac{2^2 \cdot 3^2}{2\,!} + \frac{3^2 \cdot 4^2}{3\,!} + \cdots$, is

a) $27e$ b) $24e$ c) $28e$ d) $25e$

208. The sum of the integers from 1 to 100 which are divisible by 3 and 5, is

a) 2317 b) 2632 c) 315 d) 2489

209. If H be the H.M. between a and b, then the value of $\frac{H}{a} + \frac{H}{b}$, is

a) 2 b) $\frac{ab}{a+b}$ c) $\frac{a+b}{ab}$ d) None of these

210. If $a_1 = a^2 = 2, a_n = a_{n-1} - 1 (n > 2)$, then a_5 is

a) 1 b) -1 c) 0 d) -2

211. The value of 0.234 is

a) $\frac{232}{990}$ b) $\frac{232}{9990}$ c) $\frac{232}{900}$ d) $\frac{232}{9909}$

212. The value of $\sqrt{\log_{0.5}^2 4}$, is

a) -2 b) $\sqrt{-4}$ c) 2 d) None of these

213. In a H.P. p^{th} term is q and q^{th} term is p. Then, $(pq)^{th}$ term is

a) $\frac{p+q}{pq}$ b) 0 c) $\frac{pq}{p+q}$ d) 1

214. The nth term of the sequence 4,14,30,52,80,114, …, is

a) $n^2 + n + 2$ b) $3\,n^2 + n$ c) $3\,n^2 - 5\,n + 2$ d) $(n+1)^2$

215. If $y = 3x + 6x^2 + 10x^3 + \ldots$, then the value of x in terms of y is

a) $1 - (1-y)^{-1/3}$ b) $1 - (1+y)^{1/3}$ c) $1 + (1+y)^{-1/3}$ d) $1 - (1+y)^{-1/3}$

216. The coefficient of x^n in the expansion of $\log_a(1+x)$ is

a) $\frac{(-1)^{n-1}}{n}$ b) $\frac{(-1)^{n-1}}{n}\log_a e$ c) $\frac{(-1)^{n-1}}{n}\log_e a$ d) $\frac{(-1)^n}{n}\log_a e$

217. If $a_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots + \frac{1}{2^n - 1}$, then

a) $a_{100} < 100$ b) $a_{100} > 100$ c) $a_{200} < 100$ d) None of these

218. The sum of the infinite series $\frac{2^2}{2!} + \frac{2^4}{4!} + \frac{2^6}{6!} + \cdots$ is equal to

a) $\frac{e^2+1}{2e}$ b) $\frac{e^4+1}{2e^2}$ c) $\frac{(e^2-1)^2}{2e^2}$ d) $\frac{(e^2+1)^2}{2e^2}$

219. Three numbers are in GP such that their sum is 38 and their product is 1728. The greatest number among them is

a) 18 b) 16 c) 14 d) None of these

220. $\frac{\log_2 a}{3} = \frac{\log_2 b}{4} = \frac{\log_2 c}{5\lambda}$ *and* $a^{-3}b^{-4}c = 1$ then $\lambda =$

a) 3 b) 4 c) 5 d) -5

221. The cubes of the natural numbers are grouped as $1^3, (2^3, 3^3), (4^3, 5^3, 6^3), \ldots$, then the sum of the numbers in the nth group is

a) $\frac{1}{8}n^3(n^2+1)(n^2+3)$

b) $\frac{1}{16}n^3(n^2+16)(n^2+12)$

c) $\frac{n^3}{12}(n^2+2)(n^2+4)$

d) None of these

222. The sum of the series $1 + \frac{1^2+2^2}{2\,!} + \frac{1^2+2^2+3^2}{3\,!} + \frac{1^2+2^2+3^2+4^2}{4\,!} + \cdots$, is

a) $3e$ b) $\frac{17}{6}e$ c) $\frac{13}{6}e$ d) $\frac{19}{6}e$

223. $\sum_{k=1}^{\infty} \frac{1}{k!}\left(\sum_{n=1}^{\infty} 2^{n-1}\right)$ is equal to

a) e b) $e^2 + e$ c) e^2 d) $e^2 - e$

224. If $S_n = \sum_{r=1}^{n} a_r = \frac{1}{6}n(2n^2 + 9n + 13)$, then $\sum_{r=1}^{n} \sqrt{a_r}$ equals

a) $\frac{n(n+1)}{2}$ b) $\frac{n(n+2)}{2}$ c) $\frac{n(n+3)}{2}$ d) $\frac{n(n+5)}{2}$

225. Let a, b, c be in AP and $|a| < 1, |b| < 1, |c| < 1$. If $x = 1 + a + a^2 + \ldots$ to ∞, $y = 1 + b + b^2 + \ldots$ to ∞ and $z = 1 + c + c^2 + \ldots \infty$, then x, y, z are in

a) AP b) GP c) HP d) None of these

226. The 5th term of the sequence $\frac{10}{9}, \frac{1}{3}\frac{\sqrt{20}}{3}, \frac{2}{3}, \ldots$ is

a) $\frac{1}{3}$ b) 1 c) $\frac{2}{5}$ d) $\frac{\sqrt{2}}{3}$

227. If $x, |x+1|, |x-1|$ are first three terms of an A.P., then the sum of its first 20 terms is

a) 360,180 b) 350,180 c) 150,100 d) None of these

228. If three numbers are in G.P., then the numbers obtained by adding the middle number to each of these numbers are in

a) A.P. b) G.P. c) H.P. d) None of these

229. The fourth, seventh and tenth terms of a G.P. are p, q and r respectively, then

a) $p^2 = q^2 + r^2$ b) $p^2 = qr$ c) $q^2 = pr$ d) $r^2 = p^2 + q^2$

230. If three real numbers a, b, c are in H.P., then which one of the following is true?

a) $\frac{1}{a}, b, \frac{1}{c}$ are in A.P. b) $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in H.P. c) ab, bc, ca are in H.P. d) $\frac{a}{b}, \frac{b}{c}, \frac{c}{a}$ are in H.P.

231. If $\log(x+y) = \log 2 + \frac{1}{2}\log x + \frac{1}{2}\log y$, then

a) $x + y = 0$ b) $x - y = 0$ c) $xy = 1$ d) $x^2 + xy + y^2 = 0$

232. If $x^{2\log_{10} x} = 1000x$, then x equals to

a) $10, \sqrt{10}$ b) $10^{-1}, 10\sqrt{10}$ c) $10\sqrt{10}$ d) $\sqrt{10}$

233. The value of $\log_b a \times \log_c b \times \log_a c$, is

a) 0 b) 1 c) $\log abc$ d) 10

234. The number which should be added to the numbers 2, 14, 62, so that the resulting numbers may be in GP, is

a) 1 b) 2 c) 3 d) 4

235. If sum of the first $2n$ terms of an AP series 2, 5, 8,… is equal to the sum of the first n terms of the AP series 57, 59, 61,…, then n equals

a) 10 b) 12 c) 11 d) 13

236. The sum of the series $\frac{1}{3\times7}+\frac{1}{7\times11}+\frac{1}{11\times15}+\dots$ is

a) $\frac{1}{3}$ b) $\frac{1}{6}$ c) $\frac{1}{9}$ d) $\frac{1}{12}$

237. In a G.P. with alternatively positive and negative terms, any term is the A.M. of the next two terms. Then, the common ratio of the G.P. is

a) -1 b) -3 c) -2 d) $-\frac{1}{2}$

238. If the set of natural numbers is partitioned into subsets $S_1=\{1\}$, $S_2=\{2,3)$, $S_3=\{4,5,6\}$ and so on. Then, the sum of the terms in S_{50} is

a) 62525 b) 25625 c) 62500 d) None of these

239. If a,b,c are in H.P., then

a) $\frac{a-b}{b-c}=\frac{a}{c}$ b) $\frac{b-c}{c-a}=\frac{b}{a}$ c) $\frac{c-a}{a-b}=\frac{c}{b}$ d) None of these

240. If three positive real numbers a,b,c are in AP and $abc=4$, then the minimum possible value of b is

a) $2^{3/2}$ b) $2^{2/3}$ c) $2^{1/3}$ d) $2^{5/2}$

241. The GM of roots of the equation $x^2-18x+9=0$ is

a) 3 b) 4 c) 2 d) 1

242. The sum of the series $1+2.2+3.2^2+4.2^3+5.2^4+\dots+100.2^{99}$, is

a) 99×2^{100} b) $99\times2^{100}+1$ c) 100×2^{100} d) None of these

243. The value of $0.2^{\log_{\sqrt{5}}\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\dots\right)}$, is

a) 4 b) $\log 4$ c) $\log 2$ d) None of these

244. The product $(32)(32)^{1/6}(32)^{1/36}\dots$ to ∞ is

a) 16 b) 32 c) 64 d) 0

245. If $\log_4 2+\log_4 4+\log_4 16+\log_4 x=6$, then $x=$

a) 4 b) 64 c) 32 d) 8

246. If $x,1,z$ are in AP and $x,2,z$ are in GP, then $x,4,z$ will be in

a) AP b) GP c) HP d) None of these

247. If in an A.P. $a_1=\log_{10}a$, $a_{n+1}=\log_{10}b$ and $a_{2n+1}=\log_{10}c$, then a,b,c are in

a) A.P. b) G.P. c) H.P. d) None of these

248. $\sum_{n=1}^{10}\sum_{i=1}^{n-1}1$is equal to

a) $n+10$ b) $10n$ c) 55 d) 45

249. If a,b,c are in A.P.; a,x,b are in G.P. and b,y,c are in G.P., then x^2,b^2,y^2 are in

a) H.P. b) G.P. c) A.P. d) None of these

250. If $S=\sum_{n=0}^{\infty}\frac{(\log x)^{2n}}{(2n)!}$, then S equals

a) $x+x^{-1}$ b) $x-x^{-1}$ c) $\frac{1}{2}(x+x^{-1})$ d) None of these

251. The value of $\sum_{r=1}^{n}\log\left(\frac{a^r}{b^{r-1}}\right)$ is

a) $\frac{n}{2}\log\left(\frac{a^n}{b^n}\right)$ b) $\frac{n}{2}\log\left(\frac{a^{n+1}}{b^n}\right)$ c) $\frac{n}{2}\log(\frac{a^{n+1}}{b^{n-1}})$ d) $\frac{n}{2}\log\left(\frac{a^{n+1}}{b^{n+1}}\right)$

252. The sum of the series $1\cdot3^2+2\cdot5^2+3\cdot7^2+\dots$ upto 20 terms is

a) 188090 b) 189080 c) 199080 d) None of these

253. $\log_e\frac{1+3x}{1-2x}$ is equal to

a) $-5x-\frac{5x^2}{2}-\frac{35x^3}{3}-\dots$ b) $-5x+\frac{5x^2}{2}-\frac{35x^3}{3}+\dots$

c) $5x-\frac{5x^2}{2}+\frac{35x^3}{3}-\dots$ d) $5x+\frac{5x^2}{2}+\frac{35x^3}{3}+\dots$

254. If $\log_6\{\log_4(\sqrt{x+4}+\sqrt{x})\}=0$, then $x=$

a) 1 b) $\frac{5}{4}$ c) $\frac{7}{4}$ d) $\frac{9}{4}$

255. If $2^x.9^{2x+3}=7^{x+5}$, then $x=$

a) $\frac{5\log 7+6\log 3}{\log 162-\log 7}$ b) $\frac{5\log 7-6\log 3}{\log 162+\log 7}$ c) $\frac{5\log 7-6\log 3}{\log 162-\log 7}$ d) None of these

256. Fifth term of an GP is 2, then the product of its 9 term is

a) 256 b) 512 c) 1024 d) None of these

257. If $|a|<1, b=\sum_{k=1}^{\infty}\frac{a^k}{k}$, then a is equal to

a) $\sum_{k=1}^{\infty}\frac{(-1)^k b^k}{k}$ b) $\sum_{k=1}^{\infty}\frac{(-1)^{k-1} b^k}{k!}$ c) $\sum_{k=1}^{\infty}\frac{(-1)^k b^k}{(k-1)!}$ d) $\sum_{k=1}^{\infty}\frac{(-1)^{k-1} b^k}{(k+1)!}$

258. The sum of the series $\frac{4}{1\,!}+\frac{11}{2\,!}+\frac{22}{3\,!}+\frac{37}{4\,!}+\frac{56}{5\,!}+\cdots$, is

a) $6\,e$ b) $6\,e-1$ c) $5\,e$ d) $5\,e+1$

259. If $\frac{a+b}{1-ab}, b, \frac{b+c}{1-bc}$ are in A.P., then $a, \frac{1}{b}, c$ are in

a) A.P. b) G.P. c) H.P. d) None of these

260. If $\log 2$, $\log(2^n-1)$ and $\log(2^n+3)$ are in AP, then n is equal to

a) $\frac{5}{2}$ b) $\log_2 5$ c) $\log_3 5$ d) $\frac{3}{2}$

261. The sum of the series $1+\frac{1+a}{2\,!}+\frac{1+a+a^2}{3\,!}+\frac{1+a+a^2+a^3}{4\,!}+\cdots$, is

a) $\frac{e^a-e}{a-1}$ b) $\frac{e^a-e}{a+1}$ c) $\frac{e^{2a}+1}{a-1}$ d) $\frac{e^a+e}{a-1}$

262. If $\frac{5+9+13+\ldots+n \text{ terms}}{7+9+11+\ldots+12 \text{ terms}}=\frac{5}{12}$, then n is equal to

a) 5 b) 6 c) 9 d) 12

263. The sum to the infinity of the series $1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+\ \ldots$ is

a) 3 b) 4 c) 6 d) 2

264. Consider the sequence of numbers 121, 12321, 1234321, ... Each term in the sequence is

a) A prime number b) Square of an odd number
c) Divisible by 11 d) Form a GP

265. If A_1, A_2 are two A.M's, G_1, G_2 are two G.M's and H_1, H_2 are two H.M's between two numbers, then $\frac{A_1+A_2}{H_1+H_2}$ is equal

a) $\frac{H_1H_2}{G_1G_2}$ b) $\frac{G_1G_2}{H_1H_2}$ c) $\frac{H_1H_2}{A_1A_2}$ d) $\frac{G_1G_2}{A_1A_2}$

266. A student read common difference of an AP as -3 instead of 3 and obtained the sum of first 10 terms as -30. Then, the actual sum of first 10 terms is equal to

a) 240 b) 120 c) 300 d) 180

267. If the sum to $2n$ terms of the AP 2, 5, 8, 11,... is equal to the sum to n terms of the AP 57, 59, 61, 63,..., then n is equal to

a) 10 b) 11 c) 12 d) 13

268. The value of n for which $\frac{x^{n+1}+y^{n+1}}{x^n+y^n}$ is the geometric mean of x and y is

a) $n=-\frac{1}{2}$ b) $n=\frac{1}{2}$ c) $n=1$ d) $n=-1$

269. In the sequence {1}, {2,3}, {4,5,6}, {7,8,9,10}, ... of sets, the sum of the elements in 50th set is

a) 62525 b) 65255 c) 56255 d) 55625

270. If $2\log_8 a=x, \log_2 2a=y$ and $y-x=4$, then $x=$

a) 10 b) 16 c) 4 d) 6

271. The product of n positive numbers is unity. Their sum is

a) A positive integer b) Equal to $n + \frac{1}{n}$ c) Divisible by n d) Never less than n

272. The value of $9^{1/3} \times 9^{1/9} \times 9^{1/27} \times \dots \infty$ is

a) 9 b) 1 c) 3 d) None of these

273. The sum to n terms of the infinite series $1.3^2 + 2.5^2 + 3.7^2 + \dots \infty$

a) $\frac{n}{6}(n+1)(6n^2+14n+7)$ b) $\frac{n}{6}(n+1)(2n+1)(3n+1)$

c) $4n^3 + 4n^2 + n$ d) None of the above

274. If $f(x) = \cos^2 x + \sec^2 x$, then

a) $f(x) < 1$ b) $f(x) = 1$ c) $1 < f(x) < 2$ d) $f(x) \geq 2$

275. If $\log_7\{\log_5(\sqrt{x+5} + \sqrt{x})\} = 0$ then $x =$

a) 3 b) 4 c) 2 d) None of these

276. If $a, b, c,$ are in A.P.; b, c, d are in G.P. $c, d, e,$ are in H.P., then $a, c, e,$ are in

a) A.P. b) G.P. c) H.P. d) None of these

277. The sum of the series

$\log_4 2 - \log_8 2 + \log_{16} 2 - \log_{32} 2 + \dots$ is

a) e^2 b) $\log_e 2 + 1$ c) $\log_e 3 - 2$ d) $1 - \log_e 2$

278. If $y = 1 + x + x^2 + x^3 + \dots$, then x is equal to

a) $\frac{y-1}{y}$ b) $\frac{1-y}{y}$ c) $\frac{y}{a-y}$ d) None of these

279. The coefficient of x^{15} in the product $(1-x)(1-2x)(1-2^2x)(1-2^3x)\dots(1-2^{15}x)$ is equal to

a) $2^{105} - 2^{121}$ b) $2^{121} - 2^{105}$ c) $2^{120} - 2^{104}$ d) None of these

280. If $\log_{10} 5 = x$, then $\log_5 1250$ equals to

a) $3 - \frac{1}{x}$ b) $2 + \frac{1}{x}$ c) $3 + \frac{1}{x}$ d) $2 - \frac{1}{x}$

281. If $a^{1/x} = b^{1/y} = c^{1/z}$ and a, b, c are in GP, then x, y, z will be in

a) AP b) GP c) HP d) None of these

282. The sum of series $\sum_{n=1}^{\infty} \frac{2n}{(2n+1)!}$ is

a) e b) e^{-1} c) $2e$ d) None of these

283. If $e^x = y + \sqrt{1+y^2}$, then the value of y is

a) $e^x - e^{-x}$ b) $\frac{1}{2}(e^x - e^{-x})$ c) $e^x + e^{-x}$ d) $\frac{1}{2}(e^x + e^{-x})$

284. The AM, HM and GM between two numbers are $\frac{144}{15}$, 15 and 12, but not necessarily in this order. Then, HM, GM and AM respectively are

a) $15, 12, \frac{144}{15}$ b) $\frac{144}{15}, 12, 15$ c) $12, 15, \frac{144}{15}$ d) $\frac{144}{15}, 15, 12$

285. If $|x| < 1$, then the coefficient of x^3 in the expansion of $\log(1 + x + x^2)$ is ascending powers of x, is

a) $\frac{2}{3}$ b) $\frac{4}{3}$ c) $-\frac{2}{3}$ d) $-\frac{4}{3}$

286. The value of $\frac{2}{3!} + \frac{4}{5!} + \frac{6}{7!} + \cdots$ is

a) $e^{\frac{1}{2}}$ b) e^{-1} c) e d) $e^{-\frac{1}{3}}$

287. If $y + \frac{y^3}{3} + \frac{y^5}{5} + \cdots \infty = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots \infty\right)$, then

a) $y = 2x$ b) $\log y = 2 \log x$ c) $x^2 y = 2\,x - y$ d) None of these

288. The sum of the first and third term of an arithmetic series is 12 and the product of first and second term is 24, then first term is

a) 1 b) 8 c) 4 d) 6

289. If the sum of the roots of the equation $ax^2 + bx + c = 0$ be equal to the sum of the reciprocals of their squares, then bc^2, ca^2, ab^2 will be in

a) AP b) GP c) HP d) None of these

290. If $1+\frac{1+2}{2}+\frac{1+2+3}{3}+\cdots$ to n terms is S, then S is equal to

a) $\frac{n(n+3)}{4}$ b) $\frac{n(n+2)}{4}$ c) $\frac{n(n+1)(n+2)}{6}$ d) n^2

291. Three numbers form a GP. If the 3rd term is decreased by 64, then the three numbers thus obtained will constitute an AP. If the second term of this AP is decreased by 8, a GP will be formed again, then the numbers will be

a) 4, 20, 36 b) 4, 12, 36 c) 4, 20, 100 d) None of these

292. Number of values of x for which $[x]$, sgn x, $\{x\}\{x \neq 0\}$ are in AP, is

a) 0 b) 2 c) 3 d) < 5

293. If $y = 2^{1/\log_x 8}$, then x equal to

a) y b) y^2 c) y^3 d) None of these

294. If a, b, c are in A.P., $b-a, c-b$ and a are in G.P., then $a: b: c$ is

a) 1 : 2 : 3 b) 1 : 3 : 5 c) 2 : 3 : 4 d) 1 : 2 : 4

295. The HM of two numbers is 4. Their AM is A and GM is G. If $2A+G^2=27$, then A is equal to

a) 9 b) $\frac{9}{2}$ c) 18 d) 27

296. If $S_n = 1^3+2^3+\ldots+n^3$ and $T_n = 1+2+\ldots+n$, then

a) $S_n = T_n$ b) $S_n = T_n^4$ c) $S_n = T_n^2$ d) $S_n = T_n^3$

297. If $a_1, a_2, \ldots, a_{n+1}$ are in AP, then $\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+\ldots+\frac{1}{a_na_{n+1}}$ is

a) $\frac{n-1}{a_1a_{n+1}}$ b) $\frac{1}{a_1a_{n+1}}$ c) $\frac{n+1}{a_1a_{n+1}}$ d) $\frac{n}{a_1a_{n+1}}$

298. If d, e, f are in G.P. and the two quadratic equations $ax^2+2\,bx+c=0$ and $dx^2+2\,ex+f=0$ have a common root, then

a) $\frac{d}{a},\frac{e}{b},\frac{f}{c}$ are in H.P. b) $\frac{d}{a},\frac{e}{b},\frac{f}{c}$ are in G.P. c) $abf = aef + cde$ d) $b^2df = ace^2$

299. If a, b, c, d be in HP, then

a) $a^2+c^2 > b^2+d^2$ b) $a^2+d^2 > b^2+c^2$ c) $ac+bd > b^2+c^2$ d) $ac+bd > b^2+d^2$

300. In an AP the sum of any two terms, such that the distance of one of them from the beginning is same as that of the other from the end, is

a) First term b) Sum of first and last terms

c) Last terms d) Half of the sum of the series

301. An infinite GP has first term x and sum 5, then

a) $x < -10$ b) $-10 < x < 0$ c) $0 < x < 10$ d) $x > 10$

302. If $\sum_{r=1}^{n} a_r = \frac{1}{6}n(n+1)(n+2)$ for all $n \geq 1$, then $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{a^r}$ is

a) 2 b) 3 c) 3/2 d) 6

303. Let $S_n = \frac{1}{1^3}+\frac{1+2}{1^3+2^3}+\cdots+\frac{1+2+\cdots+n}{1^3+2^3+\cdots+n^3}; n = 1,2,3, \ldots$

Then, S_n is not greater than

a) $\frac{1}{2}$ b) 1 c) 2 d) 4

304. The sum of the series $\log_4 2 - \log_8 2 + \log_{16} 2 - \cdots$, is

a) e^2 b) $\log_e 2 + 1$ c) $\log_e 3 - 2$ d) $1 - \log_e 2$

305. In a sequence of 21 terms, the first 11 terms are in AP with common difference 2 and the last 11 terms are in GP with common ratio 2. If the middle term of AP be equal to the middle term of the GP, then the middle term of the entire sequence is

a) $-\frac{10}{31}$ b) $\frac{10}{31}$ c) $\frac{32}{31}$ d) $-\frac{31}{32}$

306. If $\log_{12} 27 = a$, then $\log_6 16 =$

a) $\frac{3-a}{3+a}$ b) $4\left(\frac{3-a}{3+a}\right)$ c) $3\left(\frac{4-a}{4+a}\right)$ d) $3\left(\frac{4+a}{4-a}\right)$

307. If $x = \log_3 5,\ y = \log_{17} 25$ which one of the following is correct?

a) $x < y$ b) $x = y$ c) $x > y$ d) None of these

308. If there be n quantities in G.P., whose common ratio is r and S_m denotes the sum of the first m terms, then the sum of their products, taken two by two is

a) $S_m\, S_{m-1}$ b) $\frac{r}{r+1} S_m\, S_{m-1}$ c) $\frac{r}{r-1} S_m\, S_{m-1}$ d) $\frac{r+1}{r} S_m\, S_{m-1}$

309. If the altitudes of a triangle are in AP, then the sides of the triangle are in

a) AP b) HP
c) GP d) Arithmetic- geometric progression

310. If H is the harmonic mean between P and Q, then the value of $\frac{H}{P} + \frac{H}{Q}$ is

a) 2 b) $\frac{PQ}{P+Q}$ c) $\frac{1}{2}$ d) $\frac{P+Q}{PQ}$

311. If $x = \log_a(bc), y = \log_b(ca)$ and $z = \log_c(ab)$, then which of the following is correct?

a) $x + y + z = 1$
b) $\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z} = 1$
c) $xyz = 1$
d) None of these

312. If a, b, c be in GP, then $\log a^n, \log b^n, \log c^n$ will be

a) AP b) GP c) HP d) None of these

313. If the lengths of sides of a right angled triangle are in A.P., then their ratio is

a) $2:3:4$ b) $3:4:5$ c) $4:5:6$ d) None of these

314. If a, b, c are in AP then $10^{ax+10}, 10^{bx+10}, 10^{cx+10} (x \neq 0)$ are in

a) AP b) GP only when $x > 0$ c) GP for all x d) GP only when $x < 0$

315. The set of all possible values of x for which 13 is the A.M. of 5^{1+x} and 5^{1-x}, is

a) $5, \frac{1}{5}$ b) $\{-1,1\}$ c) $\{0, 1\}$ d) None of these

316. The sum of the series $1^2 - 2^3 + 3^2 - 4^2 + 5^2 - 6^2 + \cdots - 2008^2 + 2009^2$ is

a) 2019045 b) 1005004 c) 2000506 d) None of these

317. If the sum of n terms of the series $2^3 + 4^3 + 6^3 + \ldots$ is 3528, then n equals to

a) 10 b) 7 c) 8 d) 6

318. If $\frac{a+b}{2}, b, \frac{b+c}{2}$ are in HP, then a, b, c are in

a) HP b) AP c) GP d) None of these

319. If AM of two numbers is twice of their GM, then the ratio of greatest number to smallest number is

a) $7 - 4\sqrt{3}$ b) $7 + 4\sqrt{3}$ c) 21 d) 5

320. The difference between two numbers is 48 and the difference between their arithmetic mean and their geometric mean is 18. Then, the greater of two numbers is

a) 96 b) 60 c) 54 d) 49

321. If $\log(1 - x + x^2) = a_1x^2 + a_3x^3 + \cdots$, then $a_3 + a_6 + a_9 + \cdots$ is equal to

a) $\log\ 2$ b) $\frac{2}{3}\log\ 2$ c) $\frac{1}{3}\log\ 2$ d) $2\log\ 2$

322. If $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ be the harmonic mean between a and b, then the value of n is

a) 1 b) -1 c) 0 d) 2

323. the sum of n terms of the infinite series
$1 \cdot 3^2 + 2 \cdot 5^2 + 3 \cdot 7^2 + \ldots \infty$ is

a) $\frac{n}{6}(n+1)(6n^2+14n+7)$ b) $\frac{n}{6}(n+1)(2n+1)(3n+1)$
c) $4n^2+4n^2+n$ d) None of the above

324. The sum of the series 5.05 +1.212 + 029088+. . . ∞ is
a) 6.93378 b) 6.87342 c) 6.74384 d) 6.64474

325. The value of $2^{1/4}.4^{1/8}.8^{1/16}\ldots\infty$ is
a) 1 b) 2 c) 3/2 d) 4

326. The value of
$$\left(1+\frac{a^2x^2}{2\,!}+\frac{a^4x^4}{4\,!}+\cdots\right)^2-\left(ax+\frac{a^3x^3}{3\,!}+\frac{a^5x^5}{5\,!}+\cdots\right)^2,\text{ is}$$
a) $e^{a\,x}$ b) $e^{-a\,x}$ c) 0 d) 1

327. $1+\frac{2^3}{2!}+\frac{3^3}{3!}+\frac{4^3}{4!}+\cdots\infty$ equals
a) $5e$ b) $4e$ c) $3e$ d) $2e$

328. $1+\frac{(\log_e n)^2}{2!}+\frac{(\log_e n)^4}{4!}+\ldots$ is equal to
a) n b) $\frac{1}{n}$ c) $\frac{1}{2}(n+n^{-1})$ d) $\frac{1}{2}(e^n+e^{-n})$

329. The sum of first 10 terms of the series
$\left(x+\frac{1}{x}\right)^2+\left(x^2+\frac{1}{x^2}\right)^2+\left(x^3+\frac{1}{x^3}\right)^2+\cdots$ is
a) $\left(\frac{x^{20}-1}{x^2-1}\right)\left(\frac{x^{22}+1}{x^{20}}\right)+20$
b) $\left(\frac{x^{18}-1}{x^2-1}\right)\left(\frac{x^{11}+1}{x^9}\right)+20$
c) $\left(\frac{x^{18}-1}{x^2-1}\right)\left(\frac{x^{11}-1}{x^9}\right)+20$
d) None of these

330. If $x=\sum_{n=0}^{\infty}a^n, y=\sum_{n=0}^{\infty}b^n, z=\sum_{n=0}^{\infty}c^n$, where a,b,c are in A.P. such that $|a|<1,|b|<1$, and $|c|<1$, then x,y,z are in
a) A.P. b) G.P. c) H.P. d) None of these

331. 150 workers were engaged to finish a piece of work in a certain number of days. 4 workers dropped the second day, 4 more workers dropped the third day and so on. It takes eight more days to finish the work now. The number of days in which the work was completed is
a) 15 b) 20 c) 25 d) 30

332. If a,b,c are distinct positive real number and $a^2+b^2+c^2=1$, then $3(a^2b^2c^2)^{1/3}$ is
a) Less than 1 b) Equal to 1 c) Greater than 1 d) Any real number

333. If positive numbers a^{-1},b^{-1},c^{-1} are in AP, then the product of roots of the equation $x^2-kx+2b^{101}-a^{101}-c^{101}=0,(k\in R)$ is
a) >0 b) <0 c) $=0$ d) None of these

334. If $\frac{\log a}{b-c}=\frac{\log b}{c-a}=\frac{\log c}{a-b}$, then $a^ab^bc^c=$
a) 0 b) 1 c) abc d) None of these

335. If $S_n=nP+\frac{1}{2}n(n-1)Q$, where S_n denotes the sum of the first n terms of an AP, then the common difference is
a) $P+Q$ b) $2P+3Q$ c) $2Q$ d) Q

336. If $\{a_n\}$ is a sequence with $a_0=p$ and $a_n-a_{n-1}=ra_{n-1}$ for $n\geq 1$, then the terms of the sequence are in
a) An arithmetic progression b) A geometric progression
c) A harmonic progression d) An arithmetic-geometric progression

337. If a, b, c, d are in H.P., then $ab + bc + cd =$
a) ad b) $2ad$ c) $3ad$ d) None of these

338. If $(1-p)(1+2x+4x^2+8x^3+16x^4+32x^5) = 1-p^6, p \neq 1$ then a value of p/x is
a) 1/2 b) 2 c) 1/4 d) 4

339. If a, b and c are in AP, then $2^{ax+1}, 2^{bx+1}, 2^{cx+1}, x \neq 0$ are in
a) AP b) GP only when $x > 0$
c) GP if $x < 0$ d) GP

340. The coefficients of x^n in the expansion of $\log_a(1+x)$ is
a) $\frac{(-1)^{n-1}}{n}$ b) $\frac{(-1)^{n-1}}{n}\log_a e$ c) $\frac{(-1)^{n-1}}{n}\log_e a$ d) $\frac{(-1)^n}{n}\log_a e$

341. Let $H_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$, then the sum to n terms of the series $\frac{1^2}{1^3} + \frac{1^2+2^2}{1^3+2^3} + \frac{1^2+2^2+3^2}{1^3+2^3+3^3} + \cdots$, is
a) $\frac{4}{3}H_n - 1$ b) $\frac{4}{3}H_n + \frac{1}{n}$ c) $\frac{4}{3}H_n$ d) $\frac{4}{3}H_n - \frac{2}{3}\frac{n}{n+1}$

342. If $a_1, a_2, a_3, \dots, a_n$ are the n arithmetic means between a and b, then $2\sum_{i=1}^{n} a_i$ equals
a) ab b) $n(a+b)$ c) nab d) $\frac{(a+b)}{n}$

343. If m is a root of the given equation $(1-ab)x^2 - (a^2+b^2)x - (1+ab) = 0$ and m harmonic means are inserted between a and b, then the difference between the last and the first of the means equals
a) $b-a$ b) $ab(b-a)$ c) $a(b-a)$ d) $ab(a-b)$

344. If a, b, c are in AP, then $3^a, 3^b, 3^c$ shall be in
a) AP b) GP c) HP d) None of these

345. If $\frac{1}{b-a} + \frac{1}{b-c} = \frac{1}{a} + \frac{1}{c}$ and $b \neq a + c$, then a, b, c are in
a) AP b) GP c) HP d) None of these

346. If S denotes the sum to infinity and S_n the sum of n terms of the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$, such that $S - S_n < \frac{1}{1000}$, then the least value of n is
a) 8 b) 9 c) 10 d) 11

347. Three numbers are in AP such that their sum is 18 and sum of their squares is 158. The greatest number among them is
a) 10 b) 11 c) 12 d) None of these

348. $\frac{1}{1\cdot2\cdot3\cdot4} + \frac{4}{3\cdot4\cdot5\cdot6} + \frac{9}{5\cdot6\cdot7\cdot8} + \frac{16}{7\cdot8\cdot9\cdot10} + \cdots$ to ∞ is equal to
a) $\frac{1}{6}\log_e 2 - \frac{1}{24}$ b) $\frac{5}{2} - \log_e 2$ c) $\frac{3}{2} - \log_e 2$ d) None of these

349. If $a = 1 + \log_x yz,\ b = 1 + \log_y zx, c = 1 + \log_z xy$, then $ab + bc + ca =$
a) 0 b) $2abc$ c) abc d) $a^2+b^2+c^2$

350. Geometric mean of $7, 7^2, 7^3, \dots 7^n$ is
a) $7^{\frac{n+1}{2}}$ b) 7 c) $7^{n/2}$ d) 7^n

351. The sum of series $1 - 3 + 5 - 7 + 9 - 11 + \cdots$ to n terms is
a) $-n$, when n is even b) $2n$, when n is even c) $-n$, when n is odd d) $2n$, when n is odd

352. The sum of integers from 1 to 100 that are divisible by 2 or 5 is
a) 3000 b) 3050 c) 4050 d) None of these

353. If $y = 3^{x-1} + 3^{-x-1}$(x real), then the least value of y is
a) 2 b) 6 c) 2/3 d) None of these

354. The arithmetic mean of first n odd natural number is
a) n^2 b) $2n$ c) n d) $3n$

355. A GP consists an even number of terms. If the sum of all the terms is 5 times the sum of the terms occupying odd places, then the common ratio will be equal to
a) 2 b) 3 c) 4 d) 5

356. If sum of the series $\sum_{n=0}^{\infty} r^n = S$ for $|r| < 1$, then sum of the series $\sum_{n=0}^{\infty} r^{2n}$, is

a) S^2 b) $\frac{S^2}{2S+1}$ c) $\frac{2S}{S^2-1}$ d) $\frac{S^2}{2S-1}$

357. If $T_n = \frac{1}{4}(n+2)(n+3)$ for $n = 1,2,3, \ldots$, then $\frac{1}{T_1} + \frac{1}{T_2} + \cdots + \frac{1}{T_{2003}}$ is equal to

a) $\frac{4006}{3006}$ b) $\frac{4003}{3007}$ c) $\frac{4006}{3008}$ d) $\frac{4006}{3009}$

358. The sum to n terms of the series $\frac{1}{\sqrt{1}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \ldots$ is

a) $\sqrt{2n+1}$ b) $\sqrt{2n+1} - 1$ c) $\frac{1}{2}\sqrt{2n+1}$ d) $\frac{1}{2}(\sqrt{2n+1} - 1)$

359. The sum of all the products of the first n natural numbers taken two at a time is

a) $\frac{1}{24} n(n-1)(n+1)(3n+2)$ b) $\frac{n^2}{48}(n-1)(n-2)$

c) $\frac{1}{6} n(n+1)(n+2)(n+5)$ d) None of the above

360. If $x \neq 0$, then the sum of series $1 + \frac{x}{2!} + \frac{2x^2}{3!} + \frac{3x^3}{4!} + \cdots$ to ∞, is

a) $\frac{e^x + 1}{x}$ b) $\frac{e^x(x-1)}{x}$ c) $\frac{e^x(x-1)+1}{x}$ d) None of these

361. If the sum of first n terms of an AP is cn^2, then the sum of squares of these n terms is

a) $\frac{n(4n^2-1)c^2}{6}$ b) $\frac{n(4n^2+1)c^2}{3}$ c) $\frac{n(4n^2-1)c^2}{3}$ d) $\frac{n(4n^2+1)c^2}{6}$

362. If the roots of equation $x^3 - 12x^2 + 39x - 28 = 0$ are in AP, then their common difference will be

a) ± 1 b) ± 2 c) ± 3 d) ± 4

363. If a, b, c, d are in G.P. and $a^x = b^y = c^z = d^u$, then x, y, z, u are in

a) A.P. b) G.P. c) H.P. d) None of these

364. The value of $\sqrt{e}$ rounded off to three decimal places, is

a) 1.648 b) 1.650 c) 1.652 d) None of these

365. The 4th term of a HP is 3/5 and 8th term is 1/3, then its 6th term is

a) 1/6 b) 3/7 c) 1/7 d) 3/5

366. If a, b, c, d and p are different real numbers such that $(a^2+b^2+c^2)\,p^2 - 2(ab+bc+cd)\,p + (b^2+c^2+d^2) \leq 0$, then a, b, c, d are in

a) AP b) GP c) HP d) $ab = cd$

367. If the 7th term of HP is $\frac{1}{10}$ and the 12th term is $\frac{1}{25}$, then the 20th term is

a) $\frac{1}{41}$ b) $\frac{1}{45}$ c) $\frac{1}{49}$ d) $\frac{1}{37}$

368. The expansion of $\log(1 + 3x + 2x^2)$ is

a) $3x - \frac{5}{4}x^2 + \frac{9}{3}x^3 - \frac{17}{4}x^4 + \ldots\infty$ b) $4x - \frac{5}{4}x^2 + \frac{9}{3}x^3 - \frac{17}{4}x^4 + \ldots\infty$

c) $3x - \frac{5}{2}x^2 + \frac{9}{3}x^3 - \frac{17}{4}x^4 + \ldots\infty$ d) $-3x - \frac{5}{4}x^2 - \frac{9}{3}x^3 - \frac{17}{4}x^4 - \ldots\infty$

369. The 5th term of the series $\frac{10}{9}, \frac{1}{3}\sqrt{\frac{20}{3}}, \frac{2}{3}, \ldots$ is

a) $\frac{1}{3}$ b) 1 c) $\frac{2}{5}$ d) $\sqrt{\frac{2}{3}}$

370. If a and b are two different positive real numbers, then which of the following relations is true

a) $2\sqrt{ab} > (a+b)$ b) $2\sqrt{ab} < (a+b)$ c) $2\sqrt{ab} = (a+b)$ d) None of these

371. If $\log_2 \sin x - \log_2 \cos x - \log_2(1 - \tan^2 x) = -1$, then $x =$

a) $\frac{n\pi}{2}+\frac{\pi}{8}, n \in Z$ b) $n\pi-\frac{\pi}{8}, n \in Z$ c) $\frac{n\pi}{4}+\frac{\pi}{2}, n \in Z$ d) None of these

372. The sum of the series $\frac{x-1}{x+1}+\frac{1}{2}\frac{x^2-1}{(x+1)^2}+\frac{1}{3}\frac{x^3-1}{(x+1)^3}+\cdots$, is equal to

a) $\log_e x$ b) $2\log_e x$ c) $-\log_e(x+1)$ d) None of these

373. $\sum_{n=1}^{\infty}\frac{2n^2+n+1}{n!}$ is equal to

a) $2e-1$ b) $2e+1$ c) $6e-1$ d) $6e+1$

374. In the four numbers first three are in GP and last three are in AP whose common difference is 6. If the first and last numbers are same, then first number will be

a) 2 b) 4 c) 6 d) 8

375. If $(m+1)^{\text{th}}$, $(n+1)^{\text{th}}$ and $(r+1)^{\text{th}}$ terms of an AP are in G.P; and m, n, r are in HP, then the ratio of the first term and common difference of this AP is

a) $n/2$ b) $-n/2$ c) $n/3$ d) $-n/3$

376. $\frac{\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\cdots}{1+\frac{1}{3!}+\frac{1}{5!}+\cdots}$ equals

a) $e+1$ b) $\frac{e-1}{e+1}$ c) $e-1$ d) None of these

377. If an infinite geometric series the first term is a and common ratio is r. If the sum of the series is 4 and the second term is 3/4, then (a, r) is

a) $(4/7, 3/7)$ b) $(2, 3/8)$ c) $(3/2, 1/2)$ d) $(3, 1/4)$

378. $\sum_{k=1}^{5}\frac{1^3+2^3+\ldots+k^3}{1+3+5+\ldots+(2k-1)}$ is equal to

a) 22.5 b) 24.5 c) 28.5 d) 32.5

379. The expansion of $\left(1+\frac{x^2}{2!}+\frac{x^4}{4!}+\cdots\right)^2$ is ascending powers of x, is

a) $1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\cdots$

b) $1+\frac{2^2x^2}{2!}+\frac{2^4x^4}{4!}+\cdots$

c) $1+\frac{2x^2}{2!}+\frac{2^3x^4}{4!}+\frac{2^5+x^5}{6!}+\cdots$

d) None of these

380. If $\tan n\theta = \tan m\theta$, then the different values of θ will be

a) AP b) GP c) HP d) None of these

381. If $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots$ to $\infty = \frac{\pi^2}{6}$, then $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots$ equals

a) $\pi^2/8$ b) $\pi^2/12$ c) $\pi^2/3$ d) $\pi^2/2$

382. If a, b, c are in G.P., then $\log_a \lambda, \log_b \lambda, \log_c \lambda$ are in

a) A.P. b) G.P. c) H.P. d) None of these

383. $2^{\sin\theta}+2^{\cos\theta}$ is greater than

a) 1/2 b) $\sqrt{2}$ c) $2^{\frac{1}{\sqrt{2}}}$ d) $2^{\left(1-\frac{1}{\sqrt{2}}\right)}$

384. After inserting n A.Ms between 2 and 38, the sum of the resulting progression is 200. The value of n is

a) 10 b) 8 c) 9 d) None of these

385. Let $x \in (1, \infty)$ and n be a positive integer greater than 1. If $f_n(x) = \frac{n}{\frac{1}{\log_2 x}+\frac{1}{\log_3 x}+\cdots+\frac{1}{\log_n x}}$, then $(n!)^{f_n(x)}$ equals to

a) n^x b) x^n c) n^n d) n^{nx}

386. The sixth term of an AP is equal to 2. The value of the common difference of the AP which makes the product $T_1T_4T_5$ least, is given by

a) 8/5 b) 5/4 c) 2/3 d) None of these

387. Sum of n terms of series $12+16+24+40+\ldots$ will be

a) $2(2^n-1)+8n$ b) $2(2^n-1)+6n$ c) $3(2^n-1)+8n$ d) $4(2^n-1)+8n$

388. For a sequence $<a_n>, a_1=2$ and $\frac{a_{n+1}}{a_n}=\frac{1}{3}$. Then, $\sum_{r=1}^{20} a_r$ is

a) $\frac{20}{2}[4+19\times 3]$ b) $3\left(1-\frac{1}{3^{20}}\right)$ c) $2(1-3^{20})$ d) None of these

389. If $<a_n>$ and $<b_n>$ be two sequences given by $a_n=(x)^{\frac{1}{2^n}}+(y)^{\frac{1}{2^n}}$ and $b_n=(x)^{\frac{1}{2^n}}-(y)^{\frac{1}{2^n}}$ for all $n\in N$. Then, $a_1a_2a_3\ldots a_n$ is equal to

a) $x-y$ b) $\frac{x+y}{b_n}$ c) $\frac{x-y}{b_n}$ d) $\frac{xy}{b_n}$

390. The coefficient of x^n in the expansion of $\frac{e^{7x}+e^x}{e^{3x}}$, is

a) $\frac{4^{n-1}+(1-2)^n}{n!}$ b) $\frac{4^{n-1}+2^n}{n!}$ c) $\frac{4^{n-1}+(-2)^{n-1}}{n!}$ d) None of these

391. If $|a|<1$, then $1+2a+3a^2+4a^3+\ldots$ is equal to

a) $\frac{1}{1-a}$ b) $\frac{1}{1+a}$ c) $\frac{1}{1+a^2}$ d) $\frac{1}{(1-a)^2}$

392. If a,b,c,d are in G.P., then $(a^3+b^3)^{-1},(b^3+c^3)^{-1},(c^3+d^3)^{-1}$ are in

a) A.P. b) G.P. c) H.P. d) None of these

393. If the sum of 12th and 22nd terms of an AP is 100, then the sum of the first 33 terms of the AP is

a) 1700 b) 1650 c) 3300 d) 3400

394. If arithmetic mean of two positive numbers is A, their geometric mean is G and harmonic mean H, then H is equal to

a) $\frac{G^2}{A}$ b) $\frac{A^2}{G^2}$ c) $\frac{A}{G^2}$ d) $\frac{G}{A^2}$

395. If $10^{x-1}+10^{-x-1}=\frac{1}{3}$, then x equals to

a) $\pm\log_{10}3$ b) $2\log_3 10$ c) $\log_3 3$ d) $\log_2 10$

396. If $x^a=x^{b/2}z^{b/2}=z^c$, then a,b,c are in

a) A.P. b) G.P. c) H.P. d) None of these

397. The sum of the series $\frac{1}{1\cdot 2}+\frac{1\cdot 3}{1\cdot 2\cdot 3\cdot 4}+\frac{1\cdot 3\cdot 5}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6}+\cdots$ to ∞, is

a) $e-1$ b) $e^{1/2}-1$ c) $e^{1/2}+e$ d) None of these

398. If $a_1,a_2,a_3,\ldots,a_n$ are in A.P. and $a_i>0$ for each $i=1,2,3,\ldots,n$, then $\sum_{r=1}^{n-1}\frac{1}{a_{r+1}^{2/3}+a_{r+1}^{1/3}a_r^{1/3}+a_r^{2/3}}$ is equal to

a) $\frac{n+1}{a_{n-1}^{2/3}+a_{n-1}^{1/3}a_1^{1/3}+a_1^{2/3}}$

b) $\frac{n-1}{a_n^{2/3}+a_n^{1/3}+a_1^{2/3}}$

c) $\frac{n-1}{a_n^{2/3}+a_n^{1/3}a_1^{1/3}+a_1^{2/3}}$

d) $\frac{n+1}{a_{n+1}^{2/3}+a_{n+1}^{1/3}a_1^{1/3}+a_1^{2/3}}$

399. The harmonic mean of two numbers is 4 and the arithmetic and geometric means satisfy the relation $2A+G^2=27$, the numbers are

a) 6, 3 b) 5, 4 c) 5, −2.5 d) −3, 1

400. If $3^{2x+1}.4^{x-1}=36$, then $x=$

a) $\log_{36}48$ b) $\log_{48}36$ c) $\log_{24}12$ d) $\log_{12}24$

401. If $p,q,r,s\in N$ and they are four consecutive terms of an A.P., then pth, qth, rth and sth terms of a G.P. are in

a) A.P. b) G.P. c) H.P. d) None of these

402. $\frac{\frac{1}{2}\cdot\frac{2}{2}}{1^3}+\frac{\frac{2}{2}\cdot\frac{3}{2}}{1^3+2^3}+\frac{\frac{3}{2}\cdot\frac{4}{2}}{1^3+2^3+3^3}+\ldots+n$ terms equals

a) $\left(\frac{n}{n+1}\right)^2$ b) $\left(\frac{n}{n+1}\right)^3$ c) $\left(\frac{n}{n+1}\right)$ d) $\left(\frac{1}{n+1}\right)$

403. If $a_1, a_2, a_3, \ldots, a_n$ are in AP, where $a_i > 0$ for all i, then value of $\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots+\frac{1}{a_{n-1}+\sqrt{a_n}}$ is equal to

a) $\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$ b) $\frac{n+1}{\sqrt{a_1}+\sqrt{a_n}}$ c) $\frac{n-1}{\sqrt{a_1}-\sqrt{a_n}}$ d) $\frac{n+1}{\sqrt{a_1}-\sqrt{a_n}}$

404. If $y = 2x^2 - 1$, then $\frac{1}{x^2}+\frac{1}{2x^4}+\frac{1}{3x^6}+\cdots\infty$ equals to

a) $\log_e\left(\frac{y+1}{y-1}\right)$ b) $\log_e\left(\frac{1+y}{1-y}\right)$ c) $\log_e\left(\frac{1-y}{1+y}\right)$ d) $\log\left(\frac{1+2y}{1-2y}\right)$

405. The interior angles of a polygon are in AP. If the smallest angle be $120°$ and the common difference be 5, then the number of side is

a) 8 b) 10 c) 9 d) 6

406. If $\log_x(4x^{\log_5 x}+5) = 2\log_5 x$, then x equals to

a) $4, 5$ b) $-1, 5$ c) $4, -1$ d) $5, \frac{1}{5}$

407. Let a, b, c be in AP. If $0 < a, b, c < 1, x = \sum_{n=0}^{\infty} a^n, y = \sum_{n=0}^{\infty} b^n$ and $z = \sum_{n=0}^{\infty} c^n$, then

a) $2y = x + z$ b) $2x = y + z$ c) $2z = x + y$ d) $2xz = xy + yz$

408. If $x^{\frac{3}{2}(\log_2 x - 3)} = \frac{1}{8}$, then x equals to

a) 2 b) 3 c) 5 d) 6

409. If every terms of a GP with positive terms is the sum of its two previous terms, then the common ratio of the series is

a) 1 b) $\frac{2}{\sqrt{5}}$ c) $\frac{\sqrt{5}-1}{2}$ d) $\frac{\sqrt{5}+1}{2}$

410. If $n_1, n_2, n_3, \ldots, n_{100}$ are positive real numbers such that $n_1 + n_2 + n_3 + \ldots + n_{100} = 20$
And $k = n_1(n_2+n_3+n_4)(n_5+n_6+\ldots+n_9)(n_{10}+\ldots+n_{16})\ldots(\ldots+n_{100})$, then k belongs to

a) $(0, 100]$ b) $(0, 128]$ c) $[0, 144]$ d) None of these

411. If a, b, c are in AP, then the straight line $ax + by + c = 0$ will always pass through the point

a) $(-1, -2)$ b) $(1, -2)$ c) $(-1, 2)$ d) $(1, 2)$

412. If $\frac{e^x}{1-x} = B_0 + B_1x + B_2x^2 + \cdots + B_nx^n + \cdots$, then $B_n - B_{n-1}$ equals

a) $\frac{1}{n!}$ b) $\frac{1}{(n-1)!}$ c) $\frac{1}{n!}-\frac{1}{(n-1)!}$ d) 1

413. If $\frac{a+bx}{a-bx}=\frac{b+cx}{b-cx}=\frac{c+dx}{c-dx}(x \neq 0)$, then a, b, c, d are in

a) AP b) GP c) HP d) None of these

414. If $\sum_{r=1}^{\infty}\frac{1}{(2r-1)^2}=\frac{\pi^2}{8}$, then $\sum_{r=1}^{\infty}\frac{1}{r^2}$ is equal to

a) $\frac{\pi^2}{24}$ b) $\frac{\pi^2}{3}$ c) $\frac{\pi^2}{6}$ d) None of these

415. Jairam purchased a house in Rs 15000 and paid Rs 5000 at once. Rest money he promised to pay in annual installment of Rs 1000 with 10% per annum interest. How much money is to be paid by Jairam?

a) Rs 21555 b) Rs 20475 c) Rs 20500 d) Rs 20700

416. If a, b, c are in A.P., then $a+\frac{1}{bc}, b+\frac{1}{ca}, c+\frac{1}{ab}$ are in

a) A.P. b) G.P. c) H.P. d) None of these

417. The sum of the series
$\frac{12}{2!}+\frac{28}{3!}+\frac{50}{4!}+\frac{78}{5!}+\cdots$, is

a) e b) $3e$ c) $4e$ d) $5e$

418. The sum of the series $\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{3}{4}x^4+\frac{4}{5}x^5+\cdots$ is

a) $\frac{x}{1+x}+\log\ (1+x)$ b) $\frac{x}{1-x}+\log\ (1-x)$ c) $-\frac{x}{1+x}+\log\ (1+x)$ d) None of these

419. The sum of the infinite series $\left(\frac{1}{3}\right)^2+\frac{1}{3}\left(\frac{1}{3}\right)^4+\frac{1}{5}\left(\frac{1}{3}\right)^6+\cdots$ is

a) $\frac{1}{4}\log_e 2$ b) $\frac{1}{2}\log_e 2$ c) $\frac{1}{6}\log_e 2$ d) $\frac{1}{4}\log_e\frac{3}{2}$

420. Let T_r, be rth term of an AP whose first term is a and common difference is d. If for some positive integers $m,n,m\neq n,T_m=\frac{1}{n}$ and $T_n=\frac{1}{m}$, then $a-d$ equals

a) 0 b) 1 c) $\frac{1}{mn}$ d) $\frac{1}{m}+\frac{1}{n}$

421. The sum of
$(x+2)^{n-1}+(x+2)^{n-2}(x+1)+(x+2)^{n-3}(x+1)^2+\ldots+(x+1)^{n-1}$ is equal to

a) $(x+2)^{n-2}-(x+1)^n$ b) $(x+2)^{n-1}-(x+1)^{n-1}$
c) $(x+2)^n-(x+1)^n$ d) None of these

422. If a,b,c are in GP and x,y are arithmetic mean of a,b and b,c respectively, then $\frac{1}{x}+\frac{1}{y}$ is equal to

a) $\frac{2}{b}$ b) $\frac{3}{b}$ c) $\frac{b}{3}$ d) $\frac{b}{2}$

423. The sum of 24 terms of the following series $\sqrt{2}+\sqrt{8}+\sqrt{18}+\sqrt{32}+\ldots$ is

a) 300 b) $200\sqrt{2}$ c) $300\sqrt{2}$ d) $250\sqrt{2}$

424. The sum of the series $1^3+2^3+3^3+\ldots+15^3$ is

a) 22000 b) 10000 c) 14400 d) 15000

425. The value of $a^{\log_b x}$, where $a=0.2, b=\sqrt{5}$,
$x=\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\infty$, is

a) 1 b) 2 c) 1/2 d) 4

426. If the sum of first n natural numbers is 1/5 times the sum of their squares, then the value of n is

a) 5 b) 6 c) 7 d) 8

427. The sum of series $1+\frac{1}{4.2!}+\frac{1}{16.4!}+\frac{1}{64.6!}+\cdots\infty$ is

a) $\frac{e+1}{2\sqrt{e}}$ b) $\frac{e-1}{2\sqrt{e}}$ c) $\frac{e+1}{\sqrt{e}}$ d) $\frac{e-1}{\sqrt{e}}$

428. The sum of the squares of three distinct real numbers which are in G.P. is S^2. If their sum is $\alpha\ S$, then

a) $1<\alpha^2<3$ b) $\frac{1}{3}<\alpha^2<3$ c) $1<\alpha<3$ d) $\frac{1}{3}<\alpha<1$

429. $1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+\cdots$ to ∞ is

a) $\frac{16}{35}$ b) $\frac{11}{8}$ c) $\frac{35}{16}$ d) $\frac{7}{16}$

430. If a,b,c are in H. P., then the value of $\frac{b+a}{b-a}+\frac{b+c}{b-c}$ is

a) 1 b) 2 c) 3 d) None of these

431. The sum of the series $(1+2)+(1+2+2^2)+(1+2+2^2+2^3)+\ldots$ upto n terms is

a) $2^{n+2}-n-4$ b) $2(2^n-1)-n$ c) $2^{n+1}-n$ d) $2^{n+1}-1$

432. The sum of the series $\frac{3}{4.8}-\frac{3.5}{4.8.12}+\frac{3.5.7}{4.8.12.16}-\ldots$

a) $\sqrt{\frac{3}{2}}-\frac{3}{4}$ b) $\sqrt{\frac{2}{3}}-\frac{3}{4}$ c) $\sqrt{\frac{3}{2}}-\frac{1}{4}$ d) $\sqrt{\frac{2}{3}}-\frac{1}{4}$

433. The value of $1-\log 2+\frac{(\log 2)^2}{2!}-\frac{(\log 2)^3}{3!}+\cdots$ is

a) $\log 3$ b) $\log 2$ c) $\frac{1}{2}$ d) None of these

434. If the ratio of the sum of n term of two AP's be $(7n+1):(4n+27)$, then the ratio of their 11th term will be

a) 2 : 3 b) 3 : 4 c) 4 : 3 d) 5 : 6

435. The value of $2.\overline{357}$ is

a) $\frac{2355}{999}$ b) $\frac{2355}{1000}$ c) $\frac{2355}{1111}$ d) None of these

436. The value of $1+\frac{1+2}{2!}+\frac{1+2+3}{3!}+\frac{1+2+3+4}{4!}+\cdots$ is

a) e b) $2e$ c) $\frac{3e}{2}$ d) $\frac{4e}{5}$

437. Sum of first n terms in the series $\cot^{-1} 3+\cot^{-1} 7+\cot^{-1} 13+\cot^{-1} 21+\ldots$ is given by

a) $\tan^{-1}\left(\frac{n}{n+2}\right)$ b) $\cot^{-1}\left(\frac{n+2}{n}\right)$

c) $\tan^{-1}(n+1)-\tan^{-1} 1$ d) All of these

438. Maximum value of n for which $\sum_{-14}^{n} 1>\sum_{1}^{n}\left(n+\frac{1}{2}\right)$ is

a) 4 b) 5 c) 6 d) 7

439. If $x^{18}=y^{21}=z^{28}$, then $3, 3\log_y x, 3\log_z y, 7\log_x z$ are in

a) A.P. b) G.P. c) H.P. d) None of these

440. If, for $0<x<\pi/2$, $y=\exp[(\sin^2 x+\sin^4 x+\sin^6 x+\cdots\infty)\log_e 2]$ is a zero of the quadratic equation $x^2-9x+8=0$, then the value of $\frac{\sin x+\cos x}{\sin x-\cos x}$, is

a) 0 b) $2+\sqrt{3}$ c) $2-\sqrt{3}$ d) None of these

441. Let $a, p, q, r, s \in R \sim \{0\}$.

If $3a^2+2\left(\frac{1}{p}-\frac{1}{s}\right)a+\frac{1}{p^2}+\frac{1}{q^2}+\frac{1}{r^2}-2\left(\frac{1}{pq}+\frac{1}{qr}+\frac{1}{rs}\right)\leq 0$ for some real a, then p, q, r, s are in

a) AP b) GP c) HP d) AGP

442. The sum of series $\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}-\cdots\infty$ is equal to

a) $2\log_e 2$ b) $\log_e 2-1$ c) $\log_e 2$ d) $\log_e\left(\frac{4}{e}\right)$

443. If $x^{\log_x(x^2-4x+5)}=(x-1)$, then $x=$

a) 1 b) 2 c) 4 d) 5

444. If $2(y-a)$ is the H.M. between $y-x$ and $y-z$, then $x-a, y-a, z-a$ are in

a) A.P. b) G.P. c) H.P. d) none of these

445. The sum of the first n terms of the series $1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+\ldots$is $\frac{n(n+1)^2}{2}$ where n is even. When n is odd the sum is

a) $\frac{3n(n+1)}{2}$ b) $\frac{n^2(n+1)}{2}$ c) $\frac{n(n+1)^2}{4}$ d) $\left[\frac{n(n+1)}{2}\right]^2$

446. If $1+\lambda+\lambda^2+\ldots+\lambda^n=(1+\lambda)(1+\lambda^2)(1+\lambda^4)(1+\lambda^8)(1+\lambda^{16})$, then the value of n is (where $n \in N$)

a) 32 b) 16 c) 31 d) 15

447. The solution of the equation

$(x+1)+(x+4)+(x+7)+\ldots+(x+28)=155$ is

a) 1 b) 2 c) 3 d) 4

448. Let a_n be nth term of the GP of positive numbers. Let $\sum_{n=1}^{100} a_{2n}=\alpha$ and $\sum_{n=1}^{100} a_{2n}=\beta$, such that $\alpha\neq\beta$, then the common ratio is

a) $\frac{\alpha}{\beta}$ b) $\frac{\beta}{\alpha}$ c) $\sqrt{\frac{\alpha}{\beta}}$ d) $\sqrt{\frac{\beta}{\alpha}}$

449. 99th term of the series $2+7+14+23+34\ldots$ is

a) 9998 b) 9999 c) 10000 d) 100000

450. If a, b, c, d and p are distinct real number such that $(a^2+b^2+c^2)p^2-2(ab+bc+cd)p+(b^2+c^2+d^2)\leq 0$, then a, b, c, d

a) are in AP b) are in GP c) are in HP d) satisfy $ab=cd$

451. If $2p+3q+4r=15$, then the maximum value of $p^3q^5r^7$ is

a) 2180 b) $\frac{5^4\cdot 3^5}{2^{15}}$ c) $\frac{5^5\cdot 7^7}{2^{17}\cdot 9}$ d) 2285

452. The number 111...1 (91 times) is a/an

a) Even number b) Prime number c) Not prime d) None of these

453. If $|x|<1$, then the sum of the series

$1+2x+3x^2+4x^3+\ldots\infty$ will be

a) $\frac{1}{1-x}$ b) $\frac{1}{1+x}$ c) $\frac{1}{(1+x^2)}$ d) $\frac{1}{(1-x)^2}$

454. The value of $5^{\sqrt{\log_5 7}}7^{\sqrt{\log_7 5}}$ is

a) $\log 2$ b) 1 c) 0 d) None of these

455. If $x_1, x_2, x_3, \ldots x_n$ are in HP

Then, $x_1x_2+x_2x_3+\ldots+x_{n-1}x_n$ is equal to

a) $(n+1)x_1x_n$ b) $(n-1)x_1x_n$ c) $n\,x_1x_n$ d) $(n^2-1)x_1x_n$

456. Let a, b, c are in GP and $4a, 5b, 4c$ are in AP such that $a+b+c=70$, then value of b is

a) 5 b) 10 c) 15 d) 20

457. If three unequal numbers p, q, r are in HP and their squares are in AP, then the ratio $p:q:r$ is

a) $1-\sqrt{3}:2:1+\sqrt{3}$ b) $1:\sqrt{2}:-\sqrt{3}$ c) $1:-\sqrt{2}:\sqrt{3}$ d) $1\mp\sqrt{3}:-2:1\pm\sqrt{3}$

458. If $x=1+2+\frac{4}{2!}+\frac{8}{3!}+\frac{16}{4!}+\cdots$, then x^{-1} is equal to

a) e^{-2} b) e^2 c) $e^{1/2}$ d) None of these

459. It is given that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots+\text{to }\infty=\frac{\pi^4}{90}$. Then, $\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\ldots\infty$ is equal to

a) $\frac{\pi^4}{96}$ b) $\frac{\pi^4}{45}$ c) $\frac{89}{90}\pi$ d) None of these

460. If $|x|<1$ and $|y|<1$, the sum to infinity of the sequence $x+y, (x^2+xy+y^2), (x^3+x^2y+y^3), \ldots$, is

a) $\frac{x+y-xy}{1-x-y+xy}$ b) $\frac{x+y+xy}{1-x-y+xy}$ c) $\frac{x}{1-x}+\frac{y}{1-y}$ d) $\frac{(x-y)(x+y-xy)}{1-x-y+xy}$

461. If H_1, H_2 are two harmonic means between two positive numbers a and b $(a\neq b)$, A and G are the arithmetic and geometric means between a and b, then $\frac{H_2+H_1}{H_2H_1}$ is

a) $\frac{A}{G}$ b) $\frac{2A}{G}$ c) $\frac{A}{2G^2}$ d) $\frac{2A}{G^2}$

462. If the sum of n terms of the series $704+\frac{1}{2}(704)+\frac{1}{4}(704)+\cdots$ and, $1984-\frac{1}{2}(1984)+\frac{1}{4}(1984)\ldots$ are equal, then $n=$

a) 5 b) 3 c) 4 d) 10

463. The sum of series $1+\frac{3}{2}+\frac{7}{4}+\frac{15}{8}+\frac{31}{16}+\ldots$ is equal to

a) $2(n-1)+\frac{1}{2^{n-1}}$ b) $2n-\frac{1}{2^n}$ c) $2+\frac{1}{2^n}$ d) $2n-1+\frac{1}{2^n}$

464. If x, y, z are in HP, then $\log(x+z)+\log(x-2y+z)$ is equal to

a) $\log(x-z)$ b) $2\log(x-z)$ c) $3\log(x-z)$ d) $4\log(x-z)$

465. In a geometric progression (GP) the ratio of the sum of the first three terms and first six terms is 125:152 the common ratio is

a) $\frac{1}{5}$ b) $\frac{2}{5}$ c) $\frac{4}{5}$ d) $\frac{3}{5}$

466. The sum of n terms of the following series $1+(1+x)+(1+x+x^2)+\cdots$ is

a) $\frac{1-x^n}{1-x}$ b) $\frac{x(1-x^n)}{1-x}$ c) $\frac{n(1-x)-x(1-x^n)}{(1-x)^2}$ d) None of these

467. If a, b, c are in GP and $\log a - \log 2b, \log 2b, \log 2b - \log 3c$ and $\log 3c - \log a$ are in AP, then a, b, c are the length of the sides of a triangle which is

a) Acute angled b) Obtuse angled c) Right angled d) Equilateral

468. The sum of the series $\frac{2}{3}+\frac{8}{9}+\frac{26}{27}+\frac{80}{81}+\cdots$ to n terms is

a) $n-\frac{1}{2}(3^{-n}-1)$ b) $n-\frac{1}{2}(1-3^{-n})$ c) $n+\frac{1}{2}(3^n-1)$ d) $n-\frac{1}{2}(3^n-1)$

469. $\sum_{n=0}^{\infty}\frac{(\log_e x)^n}{n\,!}$ is equal to

a) $\log_e x$ b) x c) $\log_x e$ d) None of these

470. If S is the sum of an infinite GP, the first term a, then the common ratio r is given by

a) $\frac{a-S}{S}$ b) $\frac{S-a}{S}$ c) $\frac{a}{1-S}$ d) $\frac{S-a}{a}$

471. The sum of the series $\sum_{n=1}^{\infty}\frac{2\,n}{(2n+1)\,!}$, is

a) e b) e^{-1} c) $2\,e$ d) $2e^{-1}$

472. The sum of the series $1+3x+6x^2+10x^3+\ldots\infty$ will be

a) $\frac{1}{(1-x)^2}$ b) $\frac{1}{1-x}$ c) $\frac{1}{(1+x)^2}$ d) $\frac{1}{(1-x)^3}$

473. $\frac{1}{n!}+\frac{1}{2!(n-2)!}+\frac{1}{4!(n-4)!}+\ldots\infty$ is

a) $\frac{2^{n-1}}{n!}$ b) $\frac{2^n}{(n+1)!}$ c) $\frac{2^n}{n!}$ d) $\frac{2^{n-2}}{(n-1)!}$

474. The sum of 11 terms of an A.P. whose middle term is 30, is

a) 320 b) 330 c) 340 d) 350

475. If $(2.3)^x=(0.23)^y=1000$, then $\frac{1}{x}-\frac{1}{y}$ equals to

a) $\frac{1}{5}$ b) $\frac{1}{4}$ c) $\frac{1}{3}$ d) $\frac{1}{2}$

476. In a G.P. if the $(m+n)^{\text{th}}$ term is p and $(m-n)^{\text{th}}$ term is q, then its m^{th} term is

a) 0 b) pq c) $\sqrt{pq}$ d) $\frac{1}{2}(p+q)$

477. If $\log_6(x+3)-\log_6 x=2$, then $x=$

a) $\frac{1}{35}$ b) $\frac{3}{35}$ c) $\frac{2}{35}$ d) $-\frac{3}{35}$

478. Sum of n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+\ldots$ is

a) 2^{-n} b) $2^{-n}(n-1)$ c) $2^n(n-1)+1$ d) $2^{-n}+n-1$

479. 7th term of an AP is 40. Then, the sum of first 13 terms is

a) 520 b) 53 c) 2080 d) 1040

480. The sum of n terms of two arithmetic progressions are in the ratio $2n+3:6n+5$, then the ratio of their 13th terms is

a) 53 : 155 b) 27 : 87 c) 29 : 83 d) 31 : 89

481. If $\log_x a, a^{x/2}$ and $\log_b x$ are in G.P., then x is equal to

a) $\log_a(\log_b a)$

b) $\log_a(\log_e a)+\log_a(\log_e b)$

c) $-\log_a(\log_a b)$

d) $\log_a(\log_e b)-\log_a(\log_e a)$

482. The sum of $i-2-3i+4$ .... upto 100 terms, where $i=\sqrt{-1}$ is

a) $50(1-i)$ b) $25\,i$ c) $25\,(1+i)$ d) $100\,(1-i)$

483. If $\log_2 x + \log_4 x + \log_{16} x = \frac{21}{4}$, then x equals to

a) 8 b) 4 c) 2 d) 16

484. The sum of all two digit numbers which, when divided by 4, yield unity as a remainder is

a) 1190 b) 1197 c) 1210 d) None of these

485. $\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \ldots$ upto n terms is equal to

a) $\frac{n}{4n+6}$ b) $\frac{1}{6n+4}$ c) $\frac{n}{6n+4}$ d) $\frac{n}{3n+7}$

486. Let $a, b, c > 0$ and $4a^2 + 9b^2 + 16c^2 - 6ab - 12bc - 8ac = 0$, then b is

a) $\leq \sqrt{ac}$ b) $\geq \sqrt{ab}$ c) $\geq \frac{a+c}{2}$ d) $\geq \sqrt{ac}$

487. If 9 AM's and HM's are inserted between the 2 and 3 and if the harmonic mean H is corresponding to arithmetic mean A, then $A + \frac{6}{H}$ is equal to

a) 1 b) 3 c) 5 d) 6

488. If a, b, c are in G.P., then $\log_a x, \log_b x, \log_c x$ are in

a) A.P. b) G.P. c) H.P. d) None of these

489. If $a_1, a_2, a_3 \ldots, a_{20}$ are AM's between 13 and 67, then the maximum value of $a_1, a_2, a_3 \ldots, a_{20}$ is equal to

a) $(20)^{20}$ b) $(40)^{20}$ c) $(60)^{20}$ d) $(80)^{20}$

490. The coefficient of n^{-r} in the expansion of $\log_{10}\left(\frac{n}{n-1}\right)$, is

a) $\frac{1}{r \log_e 10}$ b) $-\frac{1}{r \log_e 10}$ c) $-\frac{1}{r! \log_e 10}$ d) $\frac{\log_e\left(1-\frac{1}{n}\right)}{\log_e^0}$

491. The following consecutive terms $\frac{1}{1+\sqrt{x}}, \frac{1}{1-x}, \frac{1}{1-\sqrt{x}}$ of a series are in

a) H.P. b) G.P. c) A.P. d) A.P., G.P.

492. The value of $\left[(0.16)^{\log_{0.25}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\infty\right)}\right]^{1/2}$ is

a) 1 b) -1 c) 0 d) None of these

493. A person is to count 45000 currency notes. Let a_n denotes the number of notes he counts in the n th minute. If $a_1 = a_2 = \ldots = 10_{10} = 150$ and $a_{10}, a_{11}, \ldots$ are in AP with common difference -2, then the time taken by him to count all notes, is

a) 24 min b) 34 min c) 125 min d) 135 min

494. The minimum number of terms from the beginning of the series $20 + 22\frac{2}{3} + 25\frac{1}{3} + \cdots$ so that the sum may exceed 1568, is

a) 25 b) 27 c) 28 d) 29

495. If pth, qth, rth and sth terms of an A.P. are in G.P., then $p-q, q-r, r-s$ are in

a) A.P. b) G.P. c) H.P. d) None of these

496. If p, q, r are in GP and $\tan^{-1} p, \tan^{-1} q, \tan^{-1} r$ are in AP, then p, q, r satisfies the relation

a) $p = q = r$ b) $p \neq q \neq r$ c) $p + q = r$ d) None of these

497. If S_n denotes the sum of n terms of an A.P. with common difference d, then

a) $d = S_n - S_{n-1} + S_{n-2}$

b) $d = S_n - 2S_{n-1} - S_{n-2}$

c) $d = S_n - 2S_{n-1} + S_{n-2}$

d) None of these

498. If $H_1, H_2, \ldots, H_n$ be n harmonic means between a and b, then $\frac{H_1+a}{H_1-a} + \frac{H_n+b}{H_n-n}$ is equal to

a) 0 b) n c) $2n$ d) 1

499. If S_n denotes the sum of the products of the first n numbers taken two at a time, then $\sum_{n=0}^{\infty} \frac{S_n}{(n+1)!}$ equals

a) $\frac{11\,e}{24}$ b) $\frac{11\,e}{12}$ c) $\frac{13\,e}{24}$ d) None of these

500. Let $\sum_{r=1}^{n} r^4 = f(n)$, then $\sum_{r=1}^{n}(2r-1)^4$ is equal to

a) $f(2n) - 16\,f(n)$ b) $f(2n) - 7f(n)$ c) $f(2n-1) - 8f(n)$ d) None of these

501. If x, y, z are in HP, then the value of expression $\log(x+z) + \log(x-2y+z)$ will be

a) $\log(x-z)$ b) $2\log(x-z)$ c) $3\log(x-z)$ d) $4\log(x-z)$

502. If $< a_n >$ is an arithmetic sequence, then $\Delta = \begin{vmatrix} a_m & a_n & a_p \\ m & n & p \\ 1 & 1 & 1 \end{vmatrix}$ equals

a) 1 b) -1 c) 0 d) None of these

503. $\frac{1}{1!(n-1)!} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + \ldots$ is equal to

a) $\frac{2^{n-4}}{n!}$ for even values of n only b) $\frac{2^{n-4}+1}{n!} - 1$ for odd values of n only

c) $\frac{2^{n-1}}{n!}$ for all values of n d) None of the above

504. The number of solutions of the equation $\log_4(x-1) = \log_2(x-3)$, is

a) 3 b) 1 c) 2 d) 0

505. The sum of the series $(1+2)(1+2+2^2) + (1+2+2^2+2^3) + \cdots$ up to n terms is

a) $2^{n+2} - n - 4$ b) $2(2^n - 1) - n$ c) $2^{n+1} - n$ d) $2^{n+1} - 1$

506. The value of

$$1 + \frac{(\log_e n)^2}{2\,!} + \frac{(\log_e n)^4}{4\,!} + \cdots \text{ is}$$

a) n b) $\frac{1}{n}$ c) $\frac{n+n^{-1}}{2}$ d) $\frac{e^n + e^{-n}}{2}$

507. $\log_e 3 - \frac{\log_e 9}{2^2} + \frac{\log_e 27}{3^2} - \frac{\log_e 81}{4^2} + \ldots$ is

a) $(\log_e 3)(\log_e 2)$ b) $\log_e 3$ c) $\log_e 2$ d) $\frac{\log_e 5}{\log_e 3}$

508. If three positive real numbers a, b, c $(c > a)$ are in H.P., then $\log(a+c) + \log(a-2b+c)$ is equal to

a) $2\log(c-b)$ b) $2\log(a+c)$ c) $2\log(c-a)$ d) $\log a + \log b + \log c$

509. Let T_r be the rth term of an AP for $r = 1, 2, 3, \ldots$ If for some positive integers m, n, we have $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then T_{mn} equals

a) $1/mn$ b) $1/m + 1/n$ c) 1 d) 0

510. If the first, second and last terms of an arithmetic series are a, b and c respectively, then the number of terms is

a) $\frac{b+c-2a}{b-a}$ b) $\frac{b+c+2a}{b-a}$ c) $\frac{b+c-2a}{b+a}$ d) $\frac{b+c+2a}{b+a}$

511. Let $S_1, S_2, \ldots$ be squares such that for each $n \geq 1$, the length of a side of S_n equals the length of a diagonal of S_{n+1}. If the length of a side of S_1 is 10cm, then for which of the following values of n is the area of S_n less than 1 sq cm?

a) 7 b) 6 c) 9 d) None of the above

512. If $x = 1 + a + a^2 + \cdots \infty$ and $y = 1 + b + b^2 + \cdots \infty$ where a and b are proper fractions, then $1 + ab + a^2b^2 + \cdots \infty$ equals

a) $\frac{xy}{y+x-1}$ b) $\frac{x+y}{x-y}$ c) $\frac{x^2+y^2}{x-y}$ d) None of these

513. If $a_1, a_2, \ldots a_n$ are in AP with common difference $d \neq 0$, then $(\sin d)[\sec a_1 \sec a_2 + \sec a_2 \sec a_3 + \ldots + \sec a_{n-1} \sec a_n]$ is equal to

a) $\cot a_n - \cot a_1$ b) $\cot a_1 - \cot a_n$ c) $\tan a_n - \tan a_1$ d) $\tan a_n - \tan a_{n-1}$

514. The H.M. of two numbers is 4 and the arithmetic mean A and geometric mean G satisfy the relation $2\,A + G^2 = 27$, the numbers are

a) 6, 3 b) 5, 4 c) 5, -2.5 d) -3, 1

515. If $a_1, a_2, \ldots, a_n$ are in HP, then the expression $a_1a_2 + a_2a_3 + \ldots + a_{n-1}a_n$ is equal to

a) $(n-1)(a_1-a_n)$ b) na_1a_n c) $(n-1)a_1a_n$ d) $n(a_1-a_n)$

516. If the first term of an A.P. is 2 and common difference is 4, then the sum of its 40 terms is

a) 3200 b) 1600 c) 200 d) 2800

517. If $2^{\log_{10} 3\sqrt{3}} = 3^{k \log_{10}^{2}}$, then $k =$

a) $\frac{1}{2}$ b) $\frac{3}{2}$ c) 3 d) 2

518. Let α, β, γ and δ are four positive real numbers such that their product is unity, then the least value of $(1+\alpha)(1+\beta)(1+\gamma)(1+\delta)$ is

a) 6 b) 16 c) 0 d) 32

519. The sum of the series $6+66+666+\ldots$ upto n term is

a) $\frac{10^{n-1}-9n+10}{81}$ b) $\frac{2(10^{n+1}-9n-10)}{27}$ c) $\frac{2(10^{n}-9n-10)}{27}$ d) None of these

520. The sum to n terms of the series $\frac{4}{3}+\frac{10}{9}+\frac{28}{27}+\ldots$ is

a) $\frac{3^n(2n+1)+1}{2(3^n)}$ b) $\frac{3^n(2n+1)-1}{2(3^n)}$ c) $\frac{3^n\, n-1}{2(3^n)}$ d) $\frac{3^n-1}{2}$

521. If a, b, c, d, e, f are in AP, then the value of $e-c$ will be

a) $2(c-a)$ b) $2(f-d)$ c) $2(d-c)$ d) $d-c$

522. If the pth, $q$the and rth term of a GP and HP are a, b, c, then $a(b-c)\log a + b(c-a)\log b + c(a-b)\log c$ is equal to

a) -1 b) 0 c) 1 d) Does not exist

523. The sum of infinite terms of the series $\frac{1}{(1+a)(2+a)}+\frac{1}{(2+a)(3+a)}+\frac{1}{(3+a)(4+a)}+\ldots+$to ∞, where a is a constant, is

a) $\frac{1}{1+a}$ b) $\frac{2}{1+a}$ c) ∞ d) None of these

524. The number of real solutions of the equation $\log(-x) = 2\log(x+1)$, is

a) 0 b) 1 c) 2 d) 4

525. The value of 0.423, is

a) $\frac{419}{999}$ b) $\frac{419}{990}$ c) $\frac{423}{1000}$ d) None of these

526. The value of the sum $\sum_{r=1}^{n}\sum_{s=1}^{n} S_{rs}\, 2^r 3^s$, where$S_{rs}=0$, if $r \neq s$ and$S_{rs}=1$, if $r=s$, is

a) $\frac{(5^n-1)}{4}$ b) $\frac{6}{5}(6^n-1)$ c) $\frac{5^n 6^n}{n+1}$ d) $\frac{5}{4}(5^n-1)$

527. a, b, c, d, e are five numbers in which the first three are in A.P. and the last three are in H.P. If the three numbers in the middle are in G.P., then the numbers in the odd places are in

a) A.P. b) G.P. c) H.P. d) None of these

528. The sum of the series $2[7^{-1}+3^{-1}.7^{-3}+5^{-1}.7^{-5}+\ldots]$ is

a) $\log_e\left(\frac{4}{3}\right)$ b) $\log_e\left(\frac{3}{4}\right)$ c) $2\log_e\left(\frac{3}{4}\right)$ d) $2\log_e\left(\frac{4}{3}\right)$

529. An AP consists of 23 terms. If the sum of the three terms in the middle is 141 and the sum of the last three terms is 261, then the first term is

a) 6 b) 5 c) 4 d) 3

530. If $2\times 2^2+3\times 2^3+4\times 2^4+\cdots+n\times 2^n = 2^{n+10}$, then $n =$

a) 510 b) 512 c) 513 d) 508

531. An infinite GP has the first term $'x'$ and sum 5, then x belongs to

a) $x<-10$ b) $-10<x<0$ c) $0<x<10$ d) $x<10$

532. If $x=-2$, then the value of $\log_4\left(\frac{x^2}{4}\right) - 2\log_4 4(x^4)$, is

a) 2 b) -4 c) -6 d) 0

533. If $\frac{\log 3}{x-y} = \frac{\log 5}{y-z} = \frac{\log 7}{z-x}$, then $3^{x+y}\, 5^{y+z}\, 7^{z+x} =$

a) 0 b) 2 c) 1 d) None of these

534. $\sum_{n=1}^{n}\sum_{i=1}^{i}\Sigma_{j=1}^{j}$ is equal to

a) $4\frac{n(n+1)(2n+1)}{6}$ b) $\left[\frac{n(n+1)}{2}\right]^2$ c) $\frac{n(n+1)}{2}$ d) $\frac{n(n+1)(n+2)}{6}$

535. $\frac{2}{3!}+\frac{4}{5!}+\frac{6}{7!}+\cdots$ is equal to

a) $e^{1/2}$ b) e^{-1} c) e d) $e^{-1/3}$

536. If $S_n=\frac{1}{6.11}+\frac{1}{11.16}+\frac{1}{16.21}+\ldots$ to n terms, then $6S_n$ equals

a) $\frac{5n-4}{5n+6}$ b) $\frac{n}{(5n+6)}$ c) $\frac{2n-1}{5n+6}$ d) $\frac{1}{(5n+6)}$

537. If $\log(x-y)-\log 5-\frac{1}{2}\log x-\frac{1}{2}\log y=0$, then $\frac{x}{y}+\frac{y}{x}=$

a) 25 b) 26 c) 27 d) 28

538. If $\log_a x, \log_b x, \log_c x$ are in A.P., where $x\neq 1$, then $c^2=$

a) $(ab)^{\log_a b}$ b) $(ac)^{\log_a b}$ c) $(ab)^{\log_b a}$ d) $(ac)^{\log_b a}$

539. If $a^{1/x}=b^{1/y}=c^{1/z}$ and a,b,c are in geometrical progression, then x,y,z are in

a) AP b) GP c) HP d) None of these

540. The coefficient of x^4 in the expansion of $\frac{1-2x-x^2}{e^{-x}}$ is

a) $\frac{1-k-k^2}{k!}$ b) $\frac{k^2+1}{k!}$ c) $\frac{1-k}{k!}$ d) $\frac{1}{k!}$

541. The sum of the series $1.3^2+2.5^2+3.7^2+\ldots$ upto 20 terms is

a) 188090 b) 189080 c) 199080 d) 199089

542. If $\frac{a}{b+c},\frac{b}{c+a},\frac{c}{a+b}$ are in AP, then

a) a,b,c are in AP b) c,a,b are in AP c) a^2,b^2,c^2 are in AP d) a,b,c are in GP

543. The sum of n terms of two arithmetic series are in the ratio $2n+3:6n+5$, then the ratio of their 13th terms is

a) 53: 155 b) 27: 87 c) 29: 83 d) 31: 89

544. If $\frac{x+y}{1-xy},y,\frac{y+z}{1-yz}$ be in A.P., then $x,\frac{1}{y},z$ will be in

a) A.P. b) G.P. c) H.P. d) None of these

545. $x+y+z=15$, if $9,x,y,z,a$ are in AP, while $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{5}{3}$, if $9,x,y,z,a$ are in HP, then value of a will be

a) 1 b) 2 c) 3 d) 9

546. If a,b and c are in AP, then which one of the following is not true?

a) $\frac{k}{a},\frac{k}{b}$ and $\frac{k}{c}$ are in HP b) $a+k,b+k$ and $c+k$ are in AP

c) ka,kb and kc are in AP d) a^2,b^2 and c^2 are in AP

547. The value of $(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\text{to }\infty\right)}$ is

a) 0.16 b) 1 c) 0.4 d) 4

548. If $\frac{3+5+7+\cdots+n\text{ terms}}{5+8+11+\cdots+10\text{ terms}}=7$, then the value of n is

a) 35 b) 36 c) 37 d) 40

549. The sum of the series $1+\frac{1+2}{2!}+\frac{1+2+2^2}{3!}+\frac{1+2+2^2+2^3}{4!}+\cdots$, is

a) e^2 b) e^2+e c) e^2-e d) e^2-e-1

550. If $a_1,a_2,\ldots.a_{50}$ are in GP, then $\frac{a_1-a_3+a_5-\ldots+a_{49}}{a_2-a_4+a_6-\ldots+a_{50}}$ is equal to

a) 0 b) 1 c) $\frac{a_1}{a_2}$ d) $\frac{a_{25}}{a_{24}}$

551. If a,b,c,d are in HP, then

a) $a+b>b+c$ b) $ad>bc$ c) Both (a) and (b) d) None of these

552. If the sum of the series $1+\frac{3}{x}+\frac{9}{x^2}+\frac{27}{x^3}+\cdots$ to ∞ is a finite number, then

a) $x<3$ b) $x>\frac{1}{3}$ c) $x<\frac{1}{3}$ d) $x>3$

553. Consider the following statements :

1.1+3+5+... upto n terms $=n^2$

2.2+4+6+... upto n terms $=n^2+1$

Which of the statement given above is/are correct?

a) Only (1) b) Only (2) c) Both (1) and (2) d) Neither (1) nor (2)

554. If $9a^2+4b^2=18ab$, then $\log(3a+2b)=$

a) $\log 5+\log 3+\log a+\log 5b$

b) $\log 5+\log 3+\log 3a+\log b$

c) $\log 5+\log a+\log b$

d) None of these

555. If $S=\sum_{n=1}^{\infty}\left(\frac{{}^nC_0+{}^nC_1+{}^nC_2+\cdots+{}^nC_n}{{}^nP_n}\right)$, then S equals

a) $2e$ b) $2e-1$ c) $2e+1$ d) None of these

556. The number of divisors of $3\times 7^3, 7\times 11^2$ and 2×61 are in

a) AP b) GP c) HP d) None of these

557. If $\frac{1}{a}+\frac{1}{c}+\frac{1}{a-b}+\frac{1}{c-b}=0$ and $b\neq a+c$, then a,b,c are in

a) G.P. b) H.P. c) A.P. d) None of these

558. If the sum of two extreme numbers of an AP with four terms is 8 and product of remaining two middle terms is 15, then greatest number of the series will be

a) 5 b) 7 c) 9 d) 11

559. The sum of the series $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\cdots+\frac{1}{\sqrt{n^2-1}+\sqrt{n^2}}$ equals

a) $\frac{2n+1}{\sqrt{n}}$ b) $\frac{\sqrt{n}+1}{\sqrt{n}+\sqrt{n-1}}$ c) $\frac{n+\sqrt{n^2-1}}{2\sqrt{n}}$ d) $n-1$

560. If x,y,z are positive integers, then $(x+y)(y+z)(z+x)$ is

a) $=8\,xyz$ b) $>8\,xyz$ c) $<8\,xyz$ d) None of these

561. $\frac{x-y}{x}+\frac{1}{2}\left(\frac{x-y}{x}\right)^2+\frac{1}{3}\left(\frac{x-y}{x}\right)^3+\ldots$ is equal to

a) $\log_e(x-y)$ b) $\log_e(x+y)$ c) $\log_e\left(\frac{x}{y}\right)$ d) $\log_e xy$

562. If x,y,z are three consecutive positive integers, then

$\log_e\sqrt{x}+\log_e\sqrt{z}+\left(\frac{1}{2xz+1}\right)+\frac{1}{3}\left(\frac{1}{2xz+1}\right)^3+\frac{1}{5}\left(\frac{1}{2xz+1}\right)^5+\ldots$is

a) $\log_e\sqrt{y}$ b) $\log_e y$ c) $\log_e y^2$ d) None of these

563. Given that n arithmetic means are inserted between two sets of numbers $a, 2b$ and $2a, b$, where $a,b\in R$. Suppose further that mth mean between these two sets of numbers is same, then the ratio $a:b$ equals

a) $n-m+1:m$ b) $n-m+1:n$ c) $m:n-m+1$ d) $n:n-m+1$

564. An A.P., a G.P. and a H.P. have the same first and last terms and the same odd number of terms. The middle terms of the three series are in

a) A.P. b) G.P. c) H.P. d) None of these

565. The sum to the series $\frac{1}{1.2}+\frac{1.3}{1.2.3.4}+\frac{1.3.5}{1.2.3.4.5.6}+\ldots$ is

a) $e-1$ b) $\sqrt{e}-1$ c) $\sqrt{e}-2$ d) $\sqrt{e}+e$

566. If the arithmetic mean of a and b is $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$, then the value of n is

a) -1 b) 0 c) 1 d) None of these

567. 0.14189189189... can be expressed as a rational number

a) $\frac{7}{3700}$ b) $\frac{7}{50}$ c) $\frac{525}{111}$ d) $\frac{21}{148}$

568. The sum of the series $\log_9 3 + \log_{27} 3 - \log_{81} 3 + \log_{243} 3 - \dots$ is

a) $1 - \log_e 2$ b) $1 + \log_e 2$ c) $\log_e 3$ d) $1 + \log_e 3$

569. If a, b, c be in arithmetic progression, then the value of $(a + 2b - c)(2b + c - a)(a + 2b + c)$ is

a) $16abc$ b) $4abc$ c) $8abc$ d) $3abc$

570. The sum of the series
$1 \cdot n + 1 + 2 \cdot (n - 1) + 3 \cdot (n - 2) \dots + n \cdot 1$ is

a) $\frac{n(n+1)(n+2)}{6}$ b) $\frac{n(n+1)(n+2)}{3}$ c) $\frac{n(n+1)(3n+2)}{6}$ d) $\frac{n(n+1)(2n+2)}{3}$

571. If $a^x = b^y = c^z = d^w$, then $\log_a(bcd)$ equals to

a) $\frac{1}{x}\left(\frac{1}{y} + \frac{1}{z} + \frac{1}{w}\right)$ b) $x\left(\frac{1}{y} + \frac{1}{z} + \frac{1}{w}\right)$ c) $\frac{y+z+w}{x}$ d) None of these

572. If $2^{\frac{3}{\log_3 x}} = \frac{1}{64}$, then $x =$

a) 3 b) $\frac{1}{3}$ c) $\frac{1}{\sqrt{3}}$ d) $-\frac{1}{\sqrt{3}}$

573. The sum of the integers from 1 to 100 which are not divisible by 3 or 5 is

a) 2489 b) 4735 c) 2317 d) 2632

574. If the third term of a GP is P. Then, the product of the first 5 terms of the GP is

a) P^3 b) P^2 c) P^{10} d) P^5

575. The sum of n terms of an A.P. is $a\,n(n-1)$. The sum of the squares of these terms is

a) $a^2\,n^2(n-1)^2$ b) $\frac{a^2}{6}n(n-1)(2n-1)$ c) $\frac{2a^2}{3}n(n-1)(2n-1)$ d) $\frac{2a^2}{3}n(n+1)(2n+1)$

576. If $\log_{30} 3 = x, \log_{30} 5 = y$, then $\log_{30} 8 =$

a) $3(1 - x - y)$ b) $x - y + 1$ c) $1 - x - y$ d) $2(x - y + 1)$

577. The value of $3^{\frac{4}{\log_4 9}} + 27^{\frac{1}{\log_{36} 9}} + 81^{\frac{1}{\log_5 3}}$, is

a) 890 b) 860 c) 857 d) None of these

578. If $a_1, a_2, a_3, \dots, a_n$ be an AP of non-zero terms, then $\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \dots + \frac{1}{a_{n-1} a_n}$ is equal to

a) $\frac{n-1}{a_1 a_n}$ b) $\frac{n}{a_1 a_n}$ c) $\frac{n+1}{a_1 a_n}$ d) None of these

579. If $S = \sum_{n=2}^{\infty} \frac{{}^nC_2}{(n+1)!}$, then S equals

a) $e - 2$ b) $e + 2$ c) $2e$ d) None of these

580. If a, b, c are in H.P., then $\frac{1}{b-a} + \frac{1}{b-c} =$

a) $\frac{1}{a} + \frac{1}{b}$ b) $\frac{1}{a} + \frac{1}{c}$ c) $\frac{1}{b} + \frac{1}{c}$ d) None of these

581. If sum of an infinite geometric series is $\frac{4}{3}$ and its 1st term is $\frac{3}{4}$, then its common ration is

a) $\frac{7}{16}$ b) $\frac{9}{16}$ c) $\frac{1}{9}$ d) $\frac{7}{9}$

582. If $\log_{10}\{98 + \sqrt{x^2 - 12x + 36}\} = 2$, then $x =$

a) 4 b) 8 c) 12 d) 4, 8

583. If $a = \sum_{n=1}^{\infty} \frac{2n}{(2n-1)!}, b = \sum_{n=1}^{\infty} \frac{2n}{(2n+1)!}$, then ab equals

a) 1 b) e^2 c) $\frac{e-1}{e+1}$ d) $\frac{e+1}{e-1}$

584. The sum to n terms of the series $1 + \frac{3}{2} + \frac{7}{4} + \frac{15}{8} + \frac{31}{16} + \cdots$, is

a) $2(n-1) + \frac{1}{2n-1}$ b) $2n - \frac{1}{2^n}$ c) $2 + \frac{1}{2^n}$ d) $2n - 1 + \frac{1}{2^n}$

585. If $0 < \phi < \frac{\pi}{2}, x = \sum_{n=0}^{\infty} \cos^{2n} \phi, y = \sum_{n=0}^{\infty} \sin^{2n} \phi$ and $z = \sum_{n=0}^{\infty} \cos^{2n} \phi \sin^{2n} \phi$, then

a) $xyz = xz + y$ b) $xyz = xy + z$ c) $xyz = x + y + z$ d) $xyz = yz + x$

586. In an arithmetic progression, the 24th term is 100. Then, the sum of the first 47 terms of the arithmetic progression is

a) 2300 b) 2350 c) 2400 d) 4700

587. If a, b, c, d, e, f are A.M.'s between 2 and 12, then $a + b + c + d + e + f$ is equal to

a) 14 b) 42 c) 84 d) None of these

588. If $x = \log_2 3$ and $y = \log_{1/2} 5$, then

a) $x > y$ b) $x < y$ c) $x = y$ d) None of these

589. The sum of n terms of the series $1 + (1 + x) + 1(1 + x + x^2) + \ldots$ will be

a) $\frac{1 - x^n}{1 - x}$ b) $\frac{x(1 - x^n)}{1 - x}$ c) $\frac{n(1 - x) - x(1 - x^n)}{(1 - x)^2}$ d) None of these

590. If $\log_2 7 = x$, then x is:

a) A rational number such that $0 < x < 2$

b) An irrational number such that $2 < x < 3$

c) A rational number such that $2 < x < 3$

d) A prime number of the form $7x + 2$

591. If $4^{\log_9 3} + 9^{\log_2 4} = 10^{\log_x 83}$, then $x =$

a) 4 b) 9 c) 83 d) 10

592. If $\log_a x, \log_b x, \log_c x$ be in HP, then a, b, c are in

a) AP b) HP c) GP d) None of these

593. Consider the following statement :

1. If mth term of HP is n and the $n$the term is m, then the (mn)th term is 1.
2. If a, b, c are in AP and $a, 2b, c$ are in GP, then $a, 4b, c$ are in HP.
3. If any odd number of quantities are in AP, then first middle and last are in AP

Which of the statement give above is/are correct?

a) Only (1) b) Only (2) c) Only (3) d) All of these

594. The first two terms of a geometric progression add upto 12. The sum of the third and the fourth terms is 48. If terms of the geometric progression are alternately positive and negative, then the first term is

a) 4 b) -4 c) -12 d) 12

595. $2\left\{\frac{m-n}{m+n} + \frac{1}{3}\left(\frac{m-n}{m+n}\right)^3 + \frac{1}{5}\left(\frac{m-n}{m+n}\right)^5 + \cdots\right\}$ is equal to

a) $\log\left(\frac{m}{n}\right)$ b) $\log\left(\frac{n}{m}\right)$ c) $\log mn$ d) None of these

596. For any integer $n \geq 1$, the sum $\sum_{k=1}^{n} k\,(k + 2)$ is equal to

a) $\frac{n(n + 1)(n + 2)}{6}$ b) $\frac{n(n + 1)(2n + 1)}{6}$ c) $\frac{n(n + 1)(2n + 7)}{6}$ d) $\frac{n(n + 1)(2n + 9)}{6}$

597. If the sum of first p terms, first q terms and first r terms of an A.P. be x, y and z respectively. Then, $\frac{x}{p}(q - r) + \frac{y}{q}(r - p) + \frac{z}{r}(p - q)$ is

a) 0 b) 2 c) pqr d) $\frac{8\,xyz}{pqr}$

598. If $1, \log_9(3^{1-x} + 2), \log_3(4.3^x - 1)$ are in A.P., then x equals

a) $\log_3 4$ b) $1 - \log_3 4$ c) $1 - \log_4 3$ d) $\log_4 3$

599. If the AM and GM of roots of a quadratic equations are 8 and 5 respectively, then the quadratic equation will be

a) $x^2 - 16x - 25 = 0$ b) $x^2 - 8x + 5 = 0$ c) $x^2 - 16x + 25 = 0$ d) $x^2 + 16x - 25 = 0$

600. The sum of the first n terms of the series $\frac{1}{\sqrt{2}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{8}} + \frac{1}{\sqrt{8}+\sqrt{11}} + \ldots$ is

a) $\frac{1}{3}(\sqrt{3n+2}-\sqrt{2})$ b) $\sqrt{3n+2}-\sqrt{2}$

c) $\sqrt{3n+2}+\sqrt{2}$ d) $\frac{1}{3}(\sqrt{2}-\sqrt{3n+2})$

601. $1+\log_2 2+\frac{(\log_e 2)^2}{2\,!}+\frac{(\log_e 2)^3}{3\,!}+\cdots$ is equal to

a) 2 b) $\frac{1}{2}$ c) $\log_e 3$ d) None of these

602. $x^{1/2}.x^{1/4}.x^{1/8}.x^{1/16}$ ... to ∞ is equal to

a) 0 b) 1 c) x d) ∞

603. If $\frac{1}{b+c},\frac{1}{c+a},\frac{1}{a+b}$ are in A.P., then

a) a,b,c are in A.P. b) a^2,b^2,c^2 are in A.P. c) $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in A.P. d) None of these

604. If a,b,c are in A.P. as well as in G.P. then

a) $a=b\neq c$ b) $a\neq b=c$ c) $a\neq b\neq c$ d) $a=b=c$

605. The coefficient of x^n in the expansion of $\left(1+\frac{x^2}{2\,!}+\frac{x^4}{4\,!}+\cdots\right)^2$, when n is odd, is

a) $\frac{2^n}{n\,!}$ b) $\frac{2^{2n}}{(2n)\,!}$ c) 0 d) $\frac{2^{2n}}{n\,!}$

606. If $A_1,A_2;G_1,G_2$ and H_1,H_2 be two AM's, GM's and HM's between two quantities, then the value of $\frac{G_1G_2}{H_1+H_2}$ is

a) $\frac{A_1+A_2}{H_1+H_2}$ b) $\frac{A_1-A_2}{H_1+H_2}$ c) $\frac{A_1+A_2}{H_1-H_2}$ d) $\frac{A_1-A_2}{H_1-H_2}$

607. $\frac{1}{3!}+\frac{2}{5!}+\frac{3}{7!}+\cdots$ is equal to

a) $\frac{e^{-1}}{2}$ b) e c) $\frac{e}{4}$ d) $\frac{e}{6}$

608. The sum of all odd numbers between 1 and 1000 which are divisible by 3, is

a) 83667 b) 90000

c) 83660 d) None of the above

609. The sum of the series

$\frac{1}{1+1^2+1^4}+\frac{1}{1+2^2+2^4}+\frac{1}{1+3^2+3^4}+\ldots$ to n terms is

a) $\frac{n(n^2+1)}{n^2+n+1}$ b) $\frac{n(n+1)}{2(n^2+n+1)}$ c) $\frac{n(n^2-1)}{2(n^2+n+1)}$ d) None of these

610. Number of identical terms in the sequence 2, 5, 8, 11,... upto 100 terms and 3, 5, 7, 9, 11,... upto 100 terms, are

a) 17 b) 33 c) 50 d) 147

611. The sum of the series $\frac{1^2}{3!}+\frac{2^2}{4!}+\frac{3^2}{5!}+\cdots$ to ∞ equals

a) e b) $2e$ c) $2e-5$ d) None of these

612. The solution of the equation $\log_\pi(\log_2(\log_7 x))=0$, is

a) 7^2 b) π^2 c) 2^2 d) *None of these*

613. $\underset{n\text{ digits}}{(666\ldots6)^2}+\underset{n\text{ digits}}{(888\ldots8)}$ is equal to

a) $\frac{4}{9}(10^n-1)$ b) $\frac{4}{9}(10^{2n}-1)$ c) $\frac{4}{9}(10^n-1)^2$ d) None of these

614. The sum to n terms of the infinite series $1.3^2+2.5^2+3.7^2+\ldots\infty$ is

a) $\frac{n}{6}(n+1)(6n^2+14n+7)$ b) $\frac{n}{6}(n+1)(2n+1)(3n+1)$

c) $4n^3+4n^2+n$ d) None of the above

615. If $S = \sum_{n=2}^{\infty} {}^nC_2 \frac{3^{n-2}}{n!}$, then S equals

a) $e^{3/2}$ b) $\frac{1}{2}e^3$ c) $e^{-3/2}$ d) e^{-3}

616. A G.P. consists of an even number of terms. If terms sum of all the terms is 5 times the sum of the terms occupying odd places, the common ration will be equal to

a) 2 b) 3 c) 4 d) 5

617. If S be the sum, P be the product and R be the sum of the reciprocals of n terms of a GP, then P^2 is equal to

a) $\left(\frac{S}{R}\right)^n$ b) $\frac{S}{R}$ c) $\left(\frac{R}{S}\right)^n$ d) $\frac{R}{S}$

618. If the ratio of AM between two positive real numbers a and b to their HM is $m : n$, then $a : b$ is equal to

a) $\frac{\sqrt{(m-n)}+\sqrt{n}}{\sqrt{m-n}-\sqrt{n}}$ b) $\frac{\sqrt{n}+\sqrt{m-n}}{\sqrt{n}-\sqrt{m-n}}$ c) $\frac{\sqrt{m}+\sqrt{m-n}}{\sqrt{m}-\sqrt{m-n}}$ d) $\frac{\sqrt{(m-n)}+\sqrt{m}}{\sqrt{m-n}-\sqrt{m}}$

619. If $|x| < 1$, then the sum of the series $1 + 2x + 3x^2 + 4x^3 + \ldots \infty$ will be

a) $\frac{1}{1-x}$ b) $\frac{1}{1+x}$ c) $\frac{1}{1+x^2}$ d) $\frac{1}{(1-x)^2}$

620. The product of three geometric means between 4 and $\frac{1}{4}$ will be

a) 4 b) 2 c) -1 d) 1

621. If A_1, A_2 ; G_1, G_2 and H_1, H_2 be two AM's, GM's and HM's is between two quantities, then the value of $\frac{G_1G_2}{H_1H_2}$ is

a) $\frac{A_1+A_2}{H_1+H_2}$ b) $\frac{A_1-A_2}{H_1+H_2}$ c) $\frac{A_1+A_2}{H_1-H_2}$ d) $\frac{A_1-A_2}{H_1-H_2}$

622. The value of $\frac{1}{2!} + \frac{2}{3!} + \ldots + \frac{999}{1000!}$ is equal to

a) $\frac{1000!-1}{1000!}$ b) $\frac{1000!+1}{1000!}$ c) $\frac{999!-1}{999!}$ d) $\frac{999!+1}{999!}$

623. If it is given that $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots$ to $\infty = \frac{\pi^4}{90}$, then the value of $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots$ to ∞ is equal to

a) $\frac{\pi^4}{96}$ b) $\frac{\pi^4}{45}$ c) $\frac{89\pi^4}{90}$ d) None of these

624. If $\frac{\log a}{3} = \frac{\log b}{4} = \frac{\log c}{5}$, then ca equals

a) $2b$ b) b^2 c) $8b$ d) $4b$

625. If a, b, c are in AP, $b-a, c-b$ and a are in GP, then $a : b : c$ is

a) $1:2:3$ b) $1:3:5$ c) $2:3:4$ d) $1:2:4$

626. If the $(p+q)$th term of a geometric series is m and the $(p-q)$th term is n, then the pth term is

a) $(mn)^{1/2}$ b) mn c) $m+n$ d) $m-n$

627. If $\frac{1}{b-c}, \frac{1}{c-a}, \frac{1}{a-b}$ be consecutive terms of an AP, then $(b-c)^2, (c-a)^2, (a-b)^2$ will be in

a) GP b) AP c) HP d) None of these

628. $2\log x - \log(x+1) - \log(x-1)$ is equal to

a) $x^2 + \frac{1}{2}x^4 + \frac{1}{3}x^6 + \cdots$

b) $\frac{1}{x^2} + \frac{1}{2x^4} + \frac{1}{3x^6} + \cdots$

c) $-\left\{\frac{1}{x^2} + \frac{1}{2x^4} + \frac{1}{3x^6} \ldots\right\}$

d) $-\frac{1}{n}(\omega^n + \omega^{2n})$

629. If $\log_3\left\{\log_6\left(\frac{x^2+x}{x-1}\right)\right\} = 0$ then $x =$

a) -1 b) 1 c) 3 d) 4

630. The sum to infinity of the series $1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+\cdots$, is

a) $\frac{16}{35}$ b) $\frac{11}{8}$ c) $\frac{35}{16}$ d) $\frac{8}{6}$

631. Let $a_1, a_2, \dots, a_{10}$ be in AP and $h_1, h_2, \dots, h_{10}$ be in HP. If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$ then $a_4 h_7$ is

a) 2 b) 3 c) 5 d) 6

632. If $\frac{e^{5x}+e^x}{e^{3x}}$ is expand in a series of ascending powers of x and n is an odd natural number, then the coefficient of x^n, is

a) $\frac{2^n}{n!}$ b) $\frac{2^{n+1}}{(2n)!}$ c) $\frac{2^{2n}}{(2n)!}$ d) None of these

633. If $\log_{10} x = y$, then $\log_{10^3} x^2$ equals

a) $\frac{1}{3}y$ b) $\frac{2}{3}y$ c) $\frac{3}{2}y$ d) $3y$

634. If $a, a_1, a_2, \dots, a_{2n}, b$ are in arithmetic progression and $a, g_1, g_2, \dots, g_{2n}, b$ are in geometric progression and h is the harmonic mean of a and b, then $\frac{a_1+a_{2n}}{g_1 g_{2n}}+\frac{a_2+a_{2n-1}}{g_2 g_{2n-1}}+\dots+\frac{a_n+a_{n+1}}{g_n g_{n+1}}$ is equal to

a) $2nh$ b) $\frac{n}{h}$ c) nh d) $\frac{2n}{h}$

635. The sets $S_1, S_2, S_3 \dots$ are given by $S_1=\left\{\frac{2}{1}\right\}, S_2=\left\{\frac{3}{2},\frac{5}{2}\right\}, S_3=\left\{\frac{4}{3},\frac{7}{3},\frac{10}{3}\right\}, S_4=\left\{\frac{5}{4},\frac{9}{4},\frac{13}{4},\frac{17}{4}\right\}, \dots$ Then, the sum of the numbers in the set S_{25} is

a) 320 b) 322 c) 324 d) 326

636. If S_1, S_2, S_3 be the sum of $n, 2n, 3n$ terms respectively of an A.P., then

a) $S_3 = S_1 + S_2$ b) $S_3 = 2(S_1 + S_2)$ c) $S_3 = 3(S_2 - S_1)$ d) None of these

637. If $\alpha \epsilon \left(0, \frac{\pi}{2}\right)$, then $\sqrt{x^2+x}+\frac{\tan^2\alpha}{\sqrt{x^2+x}}$ is always greater than or equal to

a) $2\tan\alpha$ b) 1 c) 2 d) $\sec^2\alpha$

638. The value of $\log_3 e - \log_9 e + \log_{27} e - \log_{81} e + \dots$ ad. inf., is

a) $\log_2 3$ b) $\log_3 2$ c) $\log_{10} e$ d) $\log_e 2$

639. If a, b, c be in arithmetic progression, then the value of $(a+2b-c)(2b+c-a)(a+2b+c)$ is

a) $16\,abc$ b) $4\,abc$ c) $8\,abc$ d) $3\,abc$

640. For any odd integer $n \geq 1, n^3-(n-1)^3+\dots+(-1)^{n-1}1^3$ is equal to

a) $\frac{1}{2}(n-1)^2(2n-1)$ b) $\frac{1}{4}(n-1)^2(2n-1)$ c) $\frac{1}{2}(n+1)^2(2n-1)$ d) $\frac{1}{4}(n+1)^2(2n-1)$

641. The coefficients of x^6 in the expansion of $\log\{(1+x)^{1+x}(1-x)^{1-x}\}$, is

a) $\frac{1}{15}$ b) $\frac{1}{30}$ c) $\frac{1}{10}$ d) $\frac{1}{45}$

642. An infinite GP has first term x and sum 5, then x belongs to

a) $x < -10$ b) $-10 < x < 0$ c) $0 < x < 10$ d) $x > 10$

643. In a GP the sum of three numbers is 14, if 1 is added to first two numbers and subtracted from third number the series becomes AP, then the greatest number is

a) 8 b) 4 c) 24 d) 16

644. If a, b, c are in A.P. and a^2, b^2, c^2 are in H.P., then

a) $a = b = c$ b) $2b = 3a + c$ c) $b^2 = \sqrt{(ac/8)}$ d) None of these

645. If the sum of n terms of a AP is $nA + n^2B$, where A and B are constants, then its common difference will be

a) $A - B$ b) $A + B$ c) $2A$ d) $2B$

646. The coefficient of x^3 in the expansion of 3^x is

a) $\frac{3^3}{6}$ b) $\frac{(\log 3)^3}{3}$ c) $\frac{\log(3)^3}{6}$ d) $\frac{(\log 3)^3}{6}$

647. If a^2, b^2, c^2 are in AP, then which of the following is also an AP?

a) $\sin A, \sin B, \sin C$ b) $\tan A, \tan B, \tan C$ c) $\cot A, \cot B, \cot C$ d) None of these

648. If $\frac{1}{\log_3 \pi} + \frac{1}{\log_4 \pi} > x$, then the greatest integral value of x is

a) 2 b) 3 c) π d) None of these

649. $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)\dots(n+k)}$ is equal to

a) $\frac{1}{(k-1)(k-1)!}$ b) $\frac{1}{kk!}$ c) $\frac{1}{(k-1)k!}$ d) $\frac{1}{k!}$

650. The coefficient of x^n in the series

$1 + \frac{a+b\,x}{1\,!} + \frac{(a+bx)^2}{2\,!} + \frac{(a+bx)^3}{3\,!} + \cdots$, is

a) $\frac{b^n}{n\,!}$ b) $e^b \frac{b^n}{n\,!}$ c) $e^a \frac{b^n}{n\,!}$ d) $\frac{e^b a^n}{n\,!}$

651. If the angles of a quadrilateral are in AP, whose common difference is 10°, then the angles of the quadrilateral are

a) $65°, 85°, 95°, 105°$ b) $75°, 85°, 95°, 105°$ c) $65°, 75°, 85°, 95°$ d) $65°, 95°, 105°, 115°$

652. The sum to n terms of the series $(n^2 - 1^2) + 2(n^2 - 2^2) + 3(n^2 - 3^2) + \cdots$, is

a) $\frac{n^2}{4}(n^2 - 1)$ b) $\frac{n}{4}(n+1)^2$ c) 0 d) $2n(n^2 - 1)$

653. The value of $\frac{3 + \log 343}{2 + \frac{1}{2}\log\left(\frac{49}{4}\right) + \frac{1}{3}\log\left(\frac{1}{125}\right)}$ is

a) 3 b) 2 c) 1 d) $\frac{3}{2}$

654. The sum to infinity of the progression $9 - 3 + 1 - \frac{1}{3} + \dots$ is

a) 9 b) 9/2 c) 27/4 d) 15/2

655. The equation $\log_e x + \log_e(1 + x) = 0$ can be written as

a) $x^2 + x - 1 = 0$ b) $x^2 + x + 1 = 0$ c) $x^2 + x - e = 0$ d) $x^2 + x + e = 0$

656. The value of $\log_{\sqrt{2}} \sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}$, is

a) $\frac{15}{16}$ b) $\frac{7}{16}$ c) $\frac{15}{8}$ d) $\frac{31}{32}$

657. Given two numbers a and b. Let A denote the single A.M. and S denote the sum of n A.M.'s between a and b, then S/A depends on

a) n, a, b b) n, b c) n, a d) n

658. The sum of first n terms of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \cdots$ is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum is

a) $\frac{n(n+1)}{2}$ b) $\frac{n^2(n+1)}{2}$ c) $\frac{n(n+1)^2}{2}$ d) $\left\{\frac{n(n+1)}{2}\right\}^2$

659. If the $m^{\text{th}}, n^{\text{th}}$ and p^{th} terms of an A.P. and G.P. be equal and be respectively x, y, z, then

a) $x^y y^z z^x = x^z y^x z^y$

b) $(x - y)^x (y - z)^y = (z - x)^z$

c) $(x - y)^z (y - z)^x = (z - x)^y$

d) None of these

660. If sum of n terms of an AP is $2n + 3n^2$, then rth term is

a) $2r + 3r^2$ b) $3r^2 - 4r + 1$ c) $6r - 1$ d) $4r + 1$

661. The expression $\left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots \infty\right)^2$ will be represented in ascending power of x as

a) $1 + \frac{2^2 x^2}{2!} + \frac{2^4 x^4}{4!} + \dots \infty$ b) $1 + \frac{(2x)^2}{2!} + \frac{2^2 x^4}{4!} + \dots \infty$

c) $1+\frac{(2x)^2}{2.2!}+\frac{2x^4}{4!}+\ldots\infty$ d) $1+\frac{(2x)^2}{2.2!}+\frac{(2x)^4}{2.4!}+\ldots\infty$

662. If x, y, z are pth, qth and rth terms respectively, of and A.P. and also of G.P., then $x^{y-z}y^{z-x}z^{x-y}$, is equal to

a) xyz b) 0 c) 1 d) None of these

663. If $3+\log_5 x = 2\log_{25} y$, then x equals to

a) $\frac{y}{125}$ b) $\frac{y}{25}$ c) $\frac{y^2}{625}$ d) $3-\frac{y^2}{25}$

664. If a_1, a_2, a_3, a_4, a_5 and a_6 are six arithmetic means between 3 and 31, then $a_6 - a_5$ and $a_1 + a_6$ are respectively equals to

a) 5 and 34 b) 4 and 35 c) 4 and 34 d) 4 and 36

665. If A_1, A_2 are two A · M's between a and b and G_1, G_2 are two GM's between a and b then $\frac{A_1+A_2}{G_1G_2}$ is

a) $\frac{a+b}{ab}$ b) $\frac{a+b}{2}$ c) $\frac{a+b}{a-b}$ d) None of these

666. If $\log_5(\log_5(\log_2 x)) = 0$, then the values of x is

a) 32 b) 125 c) 625 d) 125

667. The correct statement is

a) $0.5+0.55+0.555+\ldots$ to n terms $=\frac{5n}{9}-\frac{5}{81}(1-10^{-n})$

b) $8+88+888+\ldots+$ to n terms$=\frac{80}{81}(10^n-1)-\frac{8n}{9}$

c) $1^2+(1^2+2^2)+(1^2+2^2+3^2)+\ldots$ to n terms $=\frac{n(n+1)^2(n+2)}{12}$

d) All are correct

668. If $\log(1-x+x^2) = a_1x + a_2x^2 + a_3x^3 + \cdots$, and n is not a multiple of 3 then a_n is equal to

a) $\frac{1}{n}$ b) $\frac{(-1)^n}{n}$ c) $\frac{(-1)^{n-1}}{n}$ d) $\frac{(-1)^{n-1}}{n}(\omega^n+\omega^{2n})$

: ANSWER KEY :

1)	a	2)	c	3)	d	4)	b	189)	b	190)	a	191)	b	192)	c
5)	c	6)	d	7)	c	8)	c	193)	c	194)	d	195)	d	196)	d
9)	c	10)	c	11)	a	12)	c	197)	b	198)	a	199)	b	200)	a
13)	d	14)	b	15)	d	16)	b	201)	c	202)	d	203)	c	204)	a
17)	d	18)	c	19)	a	20)	a	205)	c	206)	b	207)	a	208)	c
21)	d	22)	d	23)	a	24)	d	209)	a	210)	b	211)	a	212)	a
25)	c	26)	b	27)	d	28)	d	213)	a	214)	b	215)	d	216)	b
29)	a	30)	c	31)	b	32)	c	217)	a	218)	c	219)	a	220)	c
33)	b	34)	a	35)	b	36)	a	221)	c	222)	b	223)	d	224)	c
37)	a	38)	d	39)	b	40)	b	225)	c	226)	c	227)	b	228)	c
41)	c	42)	b	43)	b	44)	c	229)	b	230)	b	231)	b	232)	b
45)	b	46)	d	47)	c	48)	c	233)	b	234)	b	235)	c	236)	d
49)	b	50)	d	51)	a	52)	b	237)	c	238)	a	239)	a	240)	b
53)	c	54)	c	55)	c	56)	b	241)	a	242)	c	243)	a	244)	c
57)	c	58)	b	59)	a	60)	c	245)	c	246)	c	247)	b	248)	d
61)	c	62)	a	63)	b	64)	c	249)	c	250)	c	251)	c	252)	a
65)	c	66)	a	67)	a	68)	c	253)	c	254)	d	255)	c	256)	b
69)	d	70)	b	71)	b	72)	a	257)	b	258)	b	259)	c	260)	b
73)	c	74)	a	75)	c	76)	a	261)	a	262)	b	263)	a	264)	b
77)	d	78)	d	79)	b	80)	a	265)	b	266)	a	267)	b	268)	a
81)	c	82)	b	83)	c	84)	b	269)	a	270)	d	271)	d	272)	c
85)	c	86)	c	87)	c	88)	d	273)	a	274)	d	275)	b	276)	b
89)	a	90)	c	91)	b	92)	a	277)	d	278)	a	279)	a	280)	c
93)	a	94)	a	95)	a	96)	c	281)	a	282)	b	283)	b	284)	b
97)	c	98)	c	99)	a	100)	d	285)	c	286)	b	287)	c	288)	c
101)	d	102)	b	103)	b	104)	b	289)	a	290)	d	291)	c	292)	b
105)	c	106)	c	107)	b	108)	c	293)	c	294)	a	295)	b	296)	c
109)	d	110)	c	111)	c	112)	a	297)	d	298)	a	299)	c	300)	b
113)	b	114)	c	115)	c	116)	b	301)	c	302)	a	303)	c	304)	d
117)	a	118)	b	119)	a	120)	a	305)	a	306)	b	307)	c	308)	b
121)	d	122)	d	123)	c	124)	c	309)	b	310)	a	311)	b	312)	a
125)	a	126)	b	127)	b	128)	b	313)	b	314)	c	315)	b	316)	a
129)	a	130)	c	131)	d	132)	d	317)	d	318)	c	319)	b	320)	d
133)	b	134)	b	135)	c	136)	b	321)	b	322)	b	323)	a	324)	d
137)	b	138)	c	139)	b	140)	c	325)	b	326)	d	327)	a	328)	c
141)	a	142)	b	143)	c	144)	b	329)	a	330)	c	331)	c	332)	a
145)	a	146)	a	147)	b	148)	c	333)	b	334)	b	335)	d	336)	b
149)	b	150)	b	151)	a	152)	c	337)	c	338)	b	339)	d	340)	b
153)	b	154)	a	155)	b	156)	b	341)	d	342)	b	343)	b	344)	b
157)	a	158)	b	159)	c	160)	c	345)	c	346)	d	347)	b	348)	a
161)	c	162)	d	163)	c	164)	a	349)	c	350)	a	351)	a	352)	b
165)	c	166)	b	167)	b	168)	b	353)	c	354)	c	355)	c	356)	d
169)	b	170)	b	171)	c	172)	b	357)	d	358)	d	359)	a	360)	d
173)	b	174)	b	175)	d	176)	c	361)	c	362)	c	363)	c	364)	a
177)	a	178)	c	179)	c	180)	b	365)	b	366)	b	367)	c	368)	c
181)	a	182)	a	183)	c	184)	d	369)	c	370)	b	371)	a	372)	a
185)	b	186)	a	187)	c	188)	b	373)	c	374)	d	375)	b	376)	b

377)	d	378)	a	379)	c	380)	a
381)	a	382)	c	383)	d	384)	b
385)	b	386)	c	387)	d	388)	b
389)	c	390)	d	391)	d	392)	b
393)	b	394)	a	395)	a	396)	c
397)	b	398)	c	399)	a	400)	a
401)	b	402)	c	403)	a	404)	a
405)	c	406)	d	407)	d	408)	a
409)	d	410)	d	411)	b	412)	a
413)	b	414)	c	415)	c	416)	a
417)	d	418)	b	419)	c	420)	a
421)	c	422)	a	423)	c	424)	c
425)	d	426)	c	427)	a	428)	b
429)	c	430)	b	431)	a	432)	b
433)	c	434)	c	435)	a	436)	c
437)	b	438)	b	439)	a	440)	b
441)	c	442)	d	443)	b	444)	b
445)	b	446)	c	447)	a	448)	a
449)	a	450)	b	451)	c	452)	c
453)	d	454)	c	455)	b	456)	d
457)	d	458)	a	459)	a	460)	a
461)	d	462)	a	463)	a	464)	b
465)	d	466)	c	467)	b	468)	a
469)	b	470)	b	471)	b	472)	d
473)	a	474)	b	475)	c	476)	a
477)	b	478)	d	479)	a	480)	a
481)	a	482)	a	483)	a	484)	c
485)	c	486)	a	487)	c	488)	c
489)	b	490)	a	491)	c	492)	d
493)	b	494)	d	495)	b	496)	a
497)	c	498)	c	499)	a	500)	a
501)	b	502)	c	503)	c	504)	b
505)	a	506)	c	507)	a	508)	b
509)	c	510)	a	511)	c	512)	a
513)	c	514)	a	515)	c	516)	a
517)	b	518)	b	519)	b	520)	b
521)	c	522)	b	523)	a	524)	b
525)	b	526)	b	527)	b	528)	a
529)	d	530)	c	531)	c	532)	c
533)	c	534)	d	535)	b	536)	b
537)	c	538)	b	539)	a	540)	a
541)	a	542)	c	543)	a	544)	c
545)	a	546)	d	547)	d	548)	a
549)	c	550)	c	551)	c	552)	d
553)	a	554)	d	555)	d	556)	a
557)	b	558)	b	559)	d	560)	b
561)	c	562)	b	563)	c	564)	b
565)	b	566)	c	567)	d	568)	c
569)	a	570)	a	571)	b	572)	c
573)	d	574)	d	575)	c	576)	a
577)	c	578)	a	579)	d	580)	b

581)	a	582)	d	583)	a	584)	a
585)	b	586)	d	587)	b	588)	a
589)	c	590)	b	591)	d	592)	c
593)	d	594)	c	595)	a	596)	c
597)	a	598)	b	599)	c	600)	a
601)	a	602)	c	603)	b	604)	d
605)	c	606)	a	607)	a	608)	a
609)	b	610)	b	611)	c	612)	a
613)	b	614)	a	615)	b	616)	c
617)	a	618)	c	619)	d	620)	d
621)	a	622)	a	623)	a	624)	b
625)	a	626)	a	627)	b	628)	b
629)	c	630)	c	631)	d	632)	d
633)	b	634)	d	635)	d	636)	c
637)	a	638)	b	639)	a	640)	d
641)	a	642)	c	643)	a	644)	a
645)	d	646)	d	647)	c	648)	a
649)	c	650)	c	651)	b	652)	a
653)	a	654)	c	655)	a	656)	c
657)	d	658)	b	659)	a	660)	c
661)	d	662)	c	663)	a	664)	c
665)	a	666)	a	667)	d	668)	b

: HINTS AND SOLUTIONS :

1 **(a)**

We have,

$7\log\left(\frac{16}{15}\right)+5\log\left(\frac{25}{24}\right)+3\log\frac{81}{80}$

$=\log\left\{\left(\frac{16}{15}\right)^7\times\left(\frac{25}{24}\right)^5\times\left(\frac{81}{80}\right)^3\right\}$

$=\log\left\{\frac{2^{28}}{3^7\times5^7}\times\frac{5^{10}}{2^{15}\times3^5}\times\frac{3^{12}}{2^{12}\times5^3}\right\}=\log2$

2 **(c)**

Let a be the first term and r be the common ration.

We have,

$a+ar=1$ ....(i)

and,

$ar^{n-1}=2(ar^n+ar^{n+1}+\cdots)$

$\Rightarrow ar^{n-1}=2\frac{ar^n}{1-r}$

$\Rightarrow r^{n-1}-r^n=2r^n$

$\Rightarrow r^{n-1}=3\,r^n$

$\Rightarrow 3\,r=1\ \Rightarrow r=1/3$

Putting $r=1/3$ in (i), we get $a=3/4$

3 **(d)**

We have,

$\log_8[\log_2\{\log_3(x^2-4x+85)\}]=\frac{1}{3}$

$\Rightarrow \log_2\{\log_3(x^2-4x+85)\}=8^{1/3}=2$

$\Rightarrow \log_3(x^2-4x+85)=2^2$

$\Rightarrow x^2-4x+85=3^4$

$\Rightarrow x^2-4x+4=0\Rightarrow(x-2)^2=0\Rightarrow x=2$

4 **(b)**

Since, $\Sigma n=\frac{1}{78}\Sigma n^3$

$\Rightarrow\frac{n(n+1)}{2}=\frac{1}{78}\times\frac{n^2(n+1)^2}{4}$

$\Rightarrow n^2+n-156=0$

$\Rightarrow(n+13)(n-12)=0$

$\Rightarrow n=12\qquad[\because n\neq-13]$

5 **(c)**

Since, G is the geometric mean between a and b.

$\therefore\ G=(ab)^{1/2}$ and n geometric means $G_1,G_2,\dots,G_n$ be inserted between a and b.

$\therefore\ G_1=ar^1,G_2=ar^2,\dots,G_n=ar^n$

Now, $G_1\cdot G_2\cdot G_3\dots G_n=a^nr^{1+2+\dots+n}$

$=a^nr^{n(n+1)/2}$

But $b=ar^{n+1}\ \Rightarrow\cdot r=\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

Therefore, the required product is

$a^n\left(\frac{b}{a}\right)^{\left[\frac{1}{n+1}\right]\frac{n(+1)}{2}}=(ab)^{\frac{n}{2}}$

$=\left\{(ab)^{\frac{1}{2}}\right\}^n=G^n$

6 **(d)**

Given that, sum of geometric series $=4$ second term $=\frac{3}{4}$

$\Rightarrow\frac{a}{1-r}=4,ar=\frac{3}{4}\ \Rightarrow r=\frac{3}{4a}$

$\therefore\ \frac{a}{1-\frac{3}{4a}}=4\ \Rightarrow\ \frac{4a^2}{4a-3}=4$

$\Rightarrow\ (a-1)(a-3)=0\ \Rightarrow a=1$ or 3

When $a=1,r=\frac{3}{4}$

When $a=3,r=\frac{1}{4}$

7 **(c)**

Required sum

$=\frac{1}{1-x}[(1-x)+(1-x^2)+(1-x^3)+\dots$ up to n terms]

$=\frac{1}{1-x}[n-(x+x^2+x^3+\cdots\text{upto } n \text{ terms})]$

$=\frac{1}{1-x}\left[n-x\frac{(1-x^n)}{1-x}\right]$

$=\frac{n(1-x)-x(1-x^n)}{(1-x)^2}$

8 **(c)**

Given, $T_m=n,T_n=m$ in HP, therefore the corresponding AP of mth term and nth term is of AP are $\frac{1}{n}$ and $\frac{1}{m}$ respectively.

Let a *and* d be the first and common difference of an AP, then

$a+(m-1)d=\frac{1}{n}$ ...(i)

$a+(n-1)d=\frac{1}{m}$ ...(ii)

On solving Eqs. (i) and (ii), we get

$a=\frac{1}{mn},d=\frac{1}{mn}$

Now, rth term of AP$=a+(r-1)d$

$=\frac{1}{mn}+(r-1)\frac{1}{mn}$

$=\frac{1+r-1}{mn}=\frac{r}{mn}$

$\therefore r$th term of HP is $\frac{mn}{r}$.

9 **(c)**

$$\frac{e^{7x}+e^{3x}}{e^{5x}}=e^{2x}+e^{-2x}$$
$$=\left[1+\frac{2x}{1!}+\frac{(2x)^2}{2!}+\frac{(2x)^3}{3!}+\ldots\infty\right]$$
$$+\left[1-\frac{2x}{1!}+\frac{(2x)^2}{2!}-\frac{(2x)^3}{3!}+\ldots\infty\right]$$
$$\Rightarrow e^{2x}+e^{-2x}=2\left[1+\frac{(2x)^2}{2!}+\frac{(2x)^4}{4!}+\ldots\infty\right]$$
Hence, the constant term is 2.

10 **(c)**

We have,
$$2^{\log_3 7}-7^{\log_3 2}=7^{\log_3 2}-7^{\log_3 2}$$
$$=0\quad[\because x^{\log_a y}=y^{\log_a x}]$$

11 **(a)**

Let $\frac{1}{\sqrt{x}+\sqrt{y}},\frac{1}{\sqrt{z}+\sqrt{x}},\frac{1}{\sqrt{y}+\sqrt{z}}$ are in AP.

Then, $\frac{1}{\sqrt{z}+\sqrt{x}}-\frac{1}{\sqrt{x}+\sqrt{y}}=\frac{1}{\sqrt{y}+\sqrt{z}}-\frac{1}{\sqrt{z}+\sqrt{x}}$
$$\Rightarrow y-z=x-y\ \Rightarrow\ y=\frac{z+x}{2}$$
$\Rightarrow x,y,z$ are in AP.

Hence, $\frac{1}{\sqrt{x}+\sqrt{y}},\frac{1}{\sqrt{z}+\sqrt{x}},\frac{1}{\sqrt{y}+\sqrt{z}}$ are in AP.

12 **(c)**

Since, $(2x+2)^2=x\times(3x+3)$ $[\because b^2=ac]$
$$\Rightarrow x^2+5x+4=0\ \Rightarrow x=\frac{-5\pm\sqrt{25-16}}{2}$$
$$\Rightarrow x=\frac{-5\pm 3}{2}$$
$$\Rightarrow x=-\tfrac{8}{2}=-4 \text{ and } x=-\tfrac{2}{2}=-1$$
At $x=-1$, second terms become zero, so we neglect that.

At $x=-4, a=-4, r=\frac{3}{2}$
$$\therefore\ T_4=-4\times\left(\frac{3}{2}\right)^3=-\frac{27}{2}=-13.5$$

13 **(d)**
$$\frac{1}{2}\left(\frac{1}{3}+\frac{1}{4}\right)-\frac{1}{4}\left(\frac{1}{3^2}+\frac{1}{4^2}\right)+\frac{1}{6}\left(\frac{1}{3^3}+\frac{1}{4^3}\right)-\cdots$$
$$=\frac{1}{2}\left[\frac{1}{3}-\frac{1}{2}\left(\frac{1}{3^2}\right)+\frac{1}{3}\left(\frac{1}{3^3}\right)-\ldots\right]$$
$$+\frac{1}{2}\left[\frac{1}{4}-\frac{1}{2}\left(\frac{1}{4^2}\right)+\frac{1}{3}\left(\frac{1}{4^3}\right)-\ldots\right]$$
$$=\frac{1}{2}\left[\log\left(1+\frac{1}{3}\right)\right]+\frac{1}{2}\left[\log\left(1+\frac{1}{4}\right)\right]$$
$$=\frac{1}{2}\left[\log\left(\frac{4}{3}\right)\times\left(\frac{5}{4}\right)\right]$$
$$=\frac{1}{2}\log\left(\frac{5}{3}\right)$$

14 **(b)**

We have,
$$2b=a+c, c^2=bd \text{ and } d=\frac{2ce}{c+e}$$
$$\therefore c^2=\left(\frac{a+c}{2}\right)d \text{ and } d=\frac{2ce}{c+e}$$
$$\Rightarrow c^2=\left(\frac{a+c}{2}\right)\left(\frac{2ce}{c+e}\right)$$
$\Rightarrow c^2=ae\ \Rightarrow a,c,e$ are in G.P.

15 **(d)**

We have,

$\log_y x=\log_z y=\log_x z=\lambda$ (Say)
$$\Rightarrow x=y^\lambda, y=z^\lambda, z=x^\lambda$$
$$\Rightarrow xyz=(xyz)^\lambda\Rightarrow(xyz)^{\lambda-1}=1=(xyz)^0\Rightarrow\lambda=1$$
$$\therefore x=y=z$$

16 **(b)**

Let the three numbers in H.P. be a,b,c. Then,
$$b=\frac{2ac}{a+c}$$
Numbers obtained by subtracting $\frac{b}{2}$ (half of the middle number) are
$$a-\frac{b}{2}, b-\frac{b}{2}, c-\frac{b}{2}$$
$$\text{or, } a-\frac{ac}{a+c}\frac{ac}{a+c}, c-\frac{ac}{a+c}$$
$$\text{or, }\frac{a^2}{a+c},\frac{ac}{a+c},\frac{c^2}{a+c}$$
clearly, these numbers are in G.P.

19 **(a)**
$$\therefore\ \frac{(-1)^r}{{}^nC_r}=\frac{n+1}{n+2}\left(\frac{(-1)^r}{{}^{n+1}C_{r+1}}-\frac{(-1)^{r-1}}{{}^{n+1}C_r}\right)$$
$$\Rightarrow\ \sum_{r=0}^{n}a_r=\frac{n+1}{n+2}(1+(-1)^n)$$

20 **(a)**

We have,
$$\log_5 64=\log_5 2^6=6\log_5 2=\frac{6}{\log_2 5}=\frac{6}{\log_2\left(\frac{10}{2}\right)}$$
$$=\frac{6}{\log_2 10-\log_2 2}=\frac{6}{\frac{1}{\log_{10}2}-1}=\frac{6}{\frac{1}{0.3010}-1}$$
$$=\frac{6\times 0.3010}{1-0.3010}$$
$$=\frac{1.8060}{0.699}=\frac{1806}{699}=\frac{602}{233}$$

21 **(d)**

Since, a,b,c are in AP.
$$\therefore b=a+d, c=a+2d,$$
Where d is a common difference, $d>0$

Again, since a^2,b^2,c^2 are in GP.

$\therefore\ a^2,(a+d)^2,(a+2d)^2$ are in GP
$$\Rightarrow(a+d)^4=a^2(a+2d)^2$$

or $(a+d)^2 = \pm a(a+2d)$

$\Rightarrow a^2 + d^2 + 2ad = \pm(a^2 + 2ad)$

Taking (+) sign, $d = 0$ (not possible as $a < b < c$)

Taking (−)sign

$2a^2 + 4ad + d^2 = 0$

$\Rightarrow 2a^2 + 4a\left(\frac{1}{2} - a\right) + \left(\frac{1}{2} - a\right)^2 = 0 \quad \left(\therefore a + b+c=32 \Rightarrow a+d=12\right.$

$\Rightarrow 4a^2 - 4a - 1 = 0$

$\therefore a = \frac{1}{2} \pm \frac{1}{\sqrt{2}}$

Here, $d = \frac{1}{2} - a > 0$

So, $a < \frac{1}{2}$

Hence, $a = \frac{1}{2} - \frac{1}{\sqrt{2}}$

22 **(d)**

Required sum is

$$\{a + (a+d)\}(-d) + \{(a+2d) + (a+3d)(-d)\} + \cdots + \{a + (2n-2)d + a + (2n-1)\,d\}(-d) + (a+2nd)^2$$

$$= -d[a + (a+d) + (a+2) + \cdots + \{a + (2n-1)\,d\}] + (a+2nd)^2$$

$$= -d \times \frac{2n}{2}\{a + a + (2n-1)\,d\} + (a+2nd)^2$$

$$= a^2 + 2nad + n(2n+1)\,d^2$$

23 **(a)**

Since, $a_1, a_2, a_3, \dots, a_n$ form an AP.

$\therefore a_2 - a_1 = a_4 - a_3 = \ldots = a_{2n} - a_{2n-1} = d$

Let $S = a_1^2 - a_2^2 + a_3^2 - a_4^2 + \ldots + a_{2n-1}^2 - a_{2n}^2$

$$= (a_1 - a_2)(a_1 + a_2) + (a_3 - a_4)(a_3 + a_4) + \ldots + (a_{2n-1} - a_{2n})(a_{2n-1} + a_{2n})$$

$$= -d(a_1 + a_2 + \ldots + a_{2n}) = -d\left(\frac{2n}{2}(a_1 + a_{2n})\right)$$

...(i)

Also, we know $a_{2n} = a_1 + (2n-1)d$

$\Rightarrow d = \frac{a_{2n} - a}{2n-1} \quad \Rightarrow -d = \frac{a_1 - a_{2n}}{2n-1}$

On putting the value of d in Eq, (i), we get

$$S = \frac{n(a_1 - a_{2n})(a_1 + a_{2n})}{2n-1} = \frac{n}{2n-1}(a_1^2 - a_{2n}^2)$$

24 **(d)**

a, b, c, d are in AP.

$\Rightarrow \frac{a}{abcd}, \frac{b}{abcd}, \frac{c}{abcd}, \frac{d}{abcd}$ are in AP.

$\Rightarrow \frac{1}{bcd}, \frac{1}{acd}, \frac{1}{abd}, \frac{1}{abc}$ are in AP.

$\Rightarrow bcd, acd, abd, abc$ are in HP.

$\therefore$ In reverse order abc, abd, acd, bcd are in HP.

26 **(b)**

It is given that

$a + x, b + x, c + x$ are in HP

$\Rightarrow b + x = \frac{2(a+x)(c+x)}{(a+x) + (c+x)}$

$\Rightarrow (b+x)(a+c+2x) = 2(a+x)(c+x)$

$\Rightarrow (a+c+2b)x + 2x^2 + ab + bc = 2ac + 2x(a+c) + 2x^2$

$\Rightarrow x(c + a - 2b) = bc + ab - 2ac$

$\Rightarrow x(c + a - 2b) = bc + ab - 2b^2 \quad [\because a, b, c$ are in GP]

$\Rightarrow x(c + a - 2b) = b(c + a - 2b)$

$\Rightarrow x = b$, if $c + a - 2b \neq 0$

If $c + a - 2b = 0$, then a, b, c are in A.P. as well as in G.P.

Therefore,

$a = b = c$

But, we have assumed that a, b, c are distinct

27 **(d)**

$\frac{a+2}{2} = \sqrt{2a} + 1$

$\Rightarrow \frac{a}{2} = \sqrt{2a}$

$\Rightarrow \frac{a^2}{4} = 2a$

$\Rightarrow a = 8$

28 **(d)**

This progression is an arithmetico-geometric series.

$\therefore S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

$= \frac{1}{1 - 1/2} + \frac{2}{(1 - 1/2)^2}$

$= \frac{2}{1/2} + \frac{2}{1/4}$

$= 4 + 8 = 12$

29 **(a)**

$$S = \sum_{r=1}^{16n}\left(\frac{8r}{4r^4 + 1}\right)$$

$$= 2\sum_{r=1}^{16n}\left(\frac{1}{2r^2 - 2r + 1} - \frac{1}{2r^2 + 2r + 1}\right)$$

$$= 2\left(1 - \frac{1}{5} + \frac{1}{5} - \frac{1}{13} + \frac{1}{13} + \ldots + \frac{1}{481} - \frac{1}{545}\right)$$

$$= 2\left(1 - \frac{1}{545}\right) = \frac{1088}{545}$$

30 **(c)**

Let the numbers be $a - d, a, a + d$. Then, it is given that

$(a-s)^2, a^2, (a+d)^2$ are in GP.

$\Rightarrow a^4 = (a-d)^2(a+d)^2$

$\Rightarrow a^4 - 2a^2d^2 = 0 \Rightarrow d = 0, \pm\sqrt{2}\,a$

Hence, d has three values

31 **(b)**

Let $\frac{\sqrt{2}-1}{\sqrt{2}} = x$. Then,

$$\frac{\sqrt{2}-1}{\sqrt{2}} + \frac{3-2\sqrt{2}}{4} + \frac{5\sqrt{2}-7}{6\sqrt{2}} + \frac{17-12\sqrt{2}}{16} + \cdots \text{ad. inf.}$$

$$= x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots$$

$$= -\log_e(1-x) = -\log_e\left(\frac{1}{\sqrt{2}}\right) = \log_e \sqrt{2}$$

32 **(c)**

We have,

$$\frac{\log x}{a^2+ab+b^2} = \frac{\log y}{b^2+bc+c^2} = \frac{\log z}{c^2+ca+a^2} = \lambda(\text{say})$$

$$\Rightarrow \frac{(a-b)\log x}{a^3-b^3} = \frac{(b-c)\log y}{b^3-c^3} = \frac{(c-a)\log z}{c^3-a^3} = \lambda$$

$$\Rightarrow \frac{\log x^{a-b}}{a^3-b^3} = \frac{\log y^{b-c}}{b^3-c^3} = \frac{\log z^{c-a}}{c^3-a^3} = \lambda$$

$$\Rightarrow \log x^{a-b} = \lambda(a^3-b^3), \log y^{b-c} = \lambda(b^3-c^3), \log z^{c-a} = \lambda(c^3-a^3)$$

$$\Rightarrow \log x^{a-b} + \log y^{b-c} + \log z^{c-a} = 0$$

$$\Rightarrow \log\left(x^{a-b}\, y^{b-c}\, z^{c-a}\right) = 0$$

$$\Rightarrow x^{a-b}y^{b-c}z^{c-a} = 1$$

33 **(b)**

We have,

$$S = \frac{1}{1\cdot 2} - \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} - \frac{1}{4\cdot 5} + \cdots\infty$$

$$\Rightarrow S = \left(\frac{1}{1}-\frac{1}{2}\right) - \left(\frac{1}{2}-\frac{1}{3}\right) + \left(\frac{1}{3}-\frac{1}{4}\right) - \left(\frac{1}{4}-\frac{1}{5}\right) + \cdots\infty$$

$$\Rightarrow S = 2\left(1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots\right) - 1$$

$$\Rightarrow S = 2\log(1+1) - \log_e e = \log_e\left(\frac{4}{e}\right)$$

$$\therefore e^S = 4/e$$

34 **(a)**

$$T_n = (2n-1)^3$$

$$= 8n^3 - 1^3 - 3.2n.1(2n-1)$$

$$= 8n^3 - 1 - 12n^2 + 6n$$

$$= 8n^3 - 12n^2 + 6n - 1$$

$$\therefore S_n = \Sigma T_n$$

$$= 8\Sigma n^3 - 12\Sigma n^2 + 6\Sigma n - \Sigma 1$$

$$= 8.\left[\frac{n(n+1)}{2}\right]^2 - 12.\frac{n(n+1)(2n+1)}{6} + 6\frac{n(n+1)}{2} - n$$

$$= 2n^2(n+1)^2 - 2n(n+1)(2n+1) + 3n(n+1) - n$$

$$= n(n+1)[2n(n+1) - 2(2n+1) + 3] - n$$

$$= n(n+1)[2n^2 + 2n - 4n - 2 + 3] - n$$

$$= n(n+1)[2n^2 - 2n + 1] - n$$

$$= n(n+1).2n(n-1) + n(n+1) - n$$

$$= 2n^2(n^2-1) + n^2$$

$$= n^2(2n^2-1)$$

35 **(b)**

Let the G.P. be $a, ar, ar^2, \ldots$, where $0 < r < 1$

It is given that

$a + ar + ar^2 + \cdots = 3$ and $a^2 + a^2\,r^2 + a^2r^4 + \cdots = 9/2$

$$\Rightarrow \frac{a}{1-r} = 3 \text{ and } \frac{a^2}{1-r^2} = \frac{9}{2}$$

$$\Rightarrow \frac{9(1-r)^2}{1-r^2} = \frac{9}{2} \Rightarrow \frac{1-r}{1+r} = \frac{1}{2} \Rightarrow r = \frac{1}{3}$$

Putting $r = \frac{1}{3}$ in $\frac{a}{1-r} = 3$, we get $a = 2$

Now,

Sum of the cubes of the term of the G.P.

$$= a^3 + a^3\,r^3 + a^3\,r^6 + \cdots$$

$$= \frac{a^3}{1-r^3} = \frac{8}{1-(1/27)} = \frac{108}{13}$$

36 **(a)**

Here, $x = \frac{1}{1-a}, y = \frac{1}{1-b}$

$$\Rightarrow a = \frac{x-1}{x},\ b = \frac{y-1}{y}$$

$$\therefore 1 + ab + a^2b^2 + \ldots = \frac{1}{1-ab} = \frac{xy}{x+y-1}$$

38 **(d)**

We have,

$$\log_{x+2}(x^3 - 3x^2 - 6x + 8) = 3$$

$$\Rightarrow x^3 - 3x^2 - 6x + 8 = (x+2)^3$$

$$\Rightarrow x^3 - 3x^2 - 6x + 8 = x^3 + 6x^2 + 12x + 8$$

$$\Rightarrow 9x^2 + 18x = 0 \Rightarrow x = 0, -2$$

$\log_{x+2}(x^3 - 3x^2 - 6x + 8)$ is defined for

$x^3 - 3x^2 - 6x + 8 > 0$ and $x + 2 > 0$

$\therefore x = 0$

39 **(b)**

We have,

$$\log_2(x-1) = 2\log_2(x-3)$$

$$\Rightarrow x - 1 = (x-3)^2 \Rightarrow x^2 - 7x + 10 = 0 \Rightarrow x = 2, 5$$

But, $\log_2(x-1)$ and $\log_2(x-3)$ are defined for $x > 3$

Hence, $x = 5$ is the only solution

40 **(b)**

Let there be n sides of the polygon. Then, the sum of all interior angles is $(2n-4)$ right angles

$$\therefore \frac{n}{2}\{240 + (n-1)5\} = (2n - 4 \times 90)$$

$$\Rightarrow n^2 - 25\,n + 144 = 0 \Rightarrow n - 9, 16$$

But, for $n = 16$, the largest angle $= 120 + (16-1)5 = 195°$; which is impossible

Hence, $n = 9$

41 **(c)**

Let the two numbers be a and b

$\therefore$ AM $= \frac{a+b}{2} = A$ ...(i)

and GM $= \sqrt{ab} \Rightarrow G^2 = ab$ ...(ii)

Now, $(a-b)^2 = (a+b)^2 - 4ab = (2A)^2 - 4G^2$

$= 4(A^2 - G^2)$

$\Rightarrow a - b = \pm 2\sqrt{(A+G)(A-G)}$ ...(iii)

On solving Eqs. (i) and (iii), we get

$a = A \pm \sqrt{(A+G)(A-G)}$

and $b = A \pm \sqrt{(A+G)(A-G)}$

42 **(b)**

We have,

$\Rightarrow (b^2 - ac)(a\,\alpha^2 + 2\,b\,\alpha + c) = 0$

$\Rightarrow$ either a, b, c are in G.P. or, α is a root of the equation $a^2 + 2\,bx + c = 0$

43 **(b)**

$0.0373737 \ldots = 0.037 + 0.00037 + 0.0000037 + \ldots$

$$= \frac{37}{10^3} + \frac{37}{10^5} + \frac{37}{10^7} + \ldots$$

$$= \frac{37}{10^3}\left[1 + \frac{1}{100} + \frac{1}{10000} + \ldots\right]$$

$$= \frac{37}{10^3}\left(\frac{1}{1 - \frac{1}{100}}\right) = \frac{37}{990}$$

Alternate Given value is of the form

$0.\underset{1\text{ term}}{\underline{X}}\ \underset{2\text{ term}}{\underline{X}}$

$$\therefore \quad S = \frac{XY - X}{\underset{1\text{ digits}}{\underline{9}}\ \underset{2\text{ digits}}{\underline{0}}}$$

$$= \frac{037 - 0}{990} = \frac{37}{990}$$

44 **(c)**

Let $a - d, a, a + d$ be the roots of the equation $x^3 - 12\,x^2 + 39\,x - 28 = 0$. Then,

$(a - d) + a + (a + d) = 12$ and $(a - d)a(a + d=28$

$\Rightarrow 3\,a = 12$ and $a(a^2 - d^2) = 28$

$\Rightarrow a = 4$ and $a(a^2 - d^2) = 28$

$\Rightarrow 16 - d^2 = 7 \Rightarrow d = \pm 3$

45 **(b)**

Let a_1, a_2, a_3 and d_1, d_2, d_3 are the first term and common difference of the three AP's respectively.

We have, $a_1 = a_2 = a_3 = 1$ and $d_1 = 1, d_2 = 2, d_3 = 3$

Therefore,

$S_1 = \frac{n}{2}(n + 1)$ ...(i)

$S_2 = \frac{n}{2}(2n)$ ...(ii)

$S_3 = \frac{n}{2}(3n - 1)$ ...(iii)

On adding Eqs. (i) and (iii), we get

$$S_1 + S_3 = \frac{n}{2}[(n + 1) + (3n - 1)]$$

$$= 2\left[\frac{n}{2}(2n)\right] = 2S_2$$

Hence, correct relation is $S_1 + S_3 = 2\,S_2$

46 **(d)**

We have,

$\log_x\{\log_4(\log_x(5x^2 + 4x^3))\} = 0$

$\Rightarrow \log_4(\log_x(5x^2 + 4x^3)) = x^0$

$\Rightarrow \log_x(5x^2 + 4x^3) = 4^1$

$\Rightarrow 5x^2 + 4x^3 = 4^4$

$\Rightarrow x^2(x^2 - 4x - 5) = 0$

$\Rightarrow x^2(x - 5)(x + 1) = 0 \Rightarrow x = 5$ $[\because x \neq 0$ and $x > 0]$

47 **(c)**

Since, $2T_{11} = 7T_{21}$

$\Rightarrow 2(a + 10d) = 7(a + 20d)$

$\Rightarrow 2a + 20d = 7a + 140d$

$\Rightarrow a = -24d$

$\therefore\ T_{25} = -24d + 24d = 0$

48 **(c)**

Let $a = {}^nC_{r-1}, b = {}^nC_r$, and $d = {}^nC_{r+2}$

Substituting these values in given expression

$$\frac{a+b}{a}, \frac{b+c}{b}, \frac{c+d}{c}$$

$$\Rightarrow \frac{{}^nC_{r-1} + {}^nC_r}{{}^nC_{r-1}}, \frac{{}^nC_r + {}^nC_{r+1}}{{}^nC_r}, \frac{{}^nC_{r+1} + {}^nC_{r+2}}{{}^nC_{r+1}}$$

$$\Rightarrow \frac{{}^{n+1}C_r}{{}^nC_{r-1}}, \frac{{}^{n+1}C_{r+1}}{{}^nC_r}, \frac{{}^{n+1}C_{r+2}}{{}^nC_{r+1}},$$

$\Rightarrow$ It is in HP.

49 **(b)**

We have,

$(x - 1)(x - 2)(x - 3) \ldots (x - n)$

$= x^n - S_1\,x^{n-1} + S_2\,x^{n-2} - S_3\,x^{n-3} + \cdots$

$+ (-1)^{n-1}S_{n-1}x + S_n,$

Where S_k = Sum of the products of n natural number taken k at a time

$\therefore$ Coefficient of x^{n-2}

$$= S_2 = \frac{1}{2}\left\{\left(\sum n\right)^2 - \left(\sum n^2\right)\right\}$$

$$= \frac{1}{2}\left[\left\{\frac{n(n+1)}{2}\right\}^2 - \frac{n(n+1)(2n+1)}{6}\right]$$

$$= \frac{n(n+1)(n-1)(3n+2)}{24}$$

50 **(d)**

Let the progression be an A.P. with common difference d, and let $\sqrt{x}, \sqrt{y}, \sqrt{z}$ be mth, $(m+n)th$ and $(m+n+p)th$ terms respectively. Then,

$\sqrt{y} = \sqrt{x} + (n-1)\, d$ and $\sqrt{z} = \sqrt{x} + (n+p-1)\, d$

$\Rightarrow \sqrt{y} - \sqrt{x} = (n-1)\, d$ and $\sqrt{z} - \sqrt{x} = (n+p-1)\, d$

$\Rightarrow \dfrac{\sqrt{y}-\sqrt{x}}{\sqrt{z}-\sqrt{x}} = \dfrac{n-1}{n+p-1}$, a rational number

But, x, y, z are prime integers. Therefore,

$\dfrac{\sqrt{y}-\sqrt{x}}{\sqrt{z}-\sqrt{x}}$ is an irrational number

So, the progression cannot be an A.P.

Similarly, they are not in G.P. or H.P.

51 **(a)**

Since, x_1, x_2, x_3 and y_1, y_2, y_3 are in GP with the same common ratio.

$\therefore\ x_2 = rx_1, x_3 = r^2x_1, y_2 = ry_1, y_3 = r^2y_1$

Area of triangle

$= \dfrac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \dfrac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ rx_1 & ry_1 & 1 \\ r^2x_1 & r^2y_1 & 1 \end{vmatrix}$

$= \frac{1}{2}x_1y_1\begin{vmatrix} 1 & 1 & 1 \\ r & r & 1 \\ r^2 & r^2 & 1 \end{vmatrix} = 0$ ($\because$ two columns are identical)

Hence, three points are in a straight line.

53 **(c)**

Let S_n denote the sum of n terms. Then,

$S_n = 3n^2 + 5$

Now,

$a_n = S_n - S_{n-1}$

$\Rightarrow a_n = (3\,n^2 + 5) - (3(n-1)^2 + 5) = 6n - 3$

$\therefore a_n = 159 \Rightarrow 6n - 3 = 159 \Rightarrow 6n = 162 \Rightarrow n = 27$

54 **(c)**

Let S_n denote the sum of n terms of the given series.

Then,

$S_n = 5n^2 + 2n$

Clearly,

Second term $= S_2 - S_1$

$\Rightarrow$ Second term $= (5 \times 2^2 + 2 \times 2) - (5 \times 1^2 + 2 \times 1) = 24 - 7 = 17$

55 **(c)**

Let the two numbers be a and b. Then,

$\dfrac{AM}{GM} = \dfrac{\frac{a+b}{2}}{\sqrt{ab}} = \dfrac{m}{n}$ [given]

$\Rightarrow \sqrt{\dfrac{a}{b}} + \sqrt{\dfrac{b}{a}} = \dfrac{2m}{n}$

$\Rightarrow \dfrac{a}{b} - 2\sqrt{\dfrac{a}{b}}\dfrac{m}{n} + 1 = 0$

$\Rightarrow x^2 - \dfrac{2m}{n}x + 1 = 0$, where $x = \sqrt{\dfrac{a}{b}}$

$\Rightarrow x = \dfrac{m \pm \sqrt{m^2 - n^2}}{n}$

$\therefore a : b = m + \sqrt{m^2 - n^2} : m - \sqrt{m^2 - n^2}$

56 **(b)**

Let $\log_c a = x, \log_b c = y, \log_a b = z$. Then,

$xyz = 1$ ...(i)

Now,

x, y, z are in AP

$\Rightarrow 2y = x + z$ ...(ii)

$\because a, b, c$ are in GP

$\therefore b^2 = ac$

$\Rightarrow \log_b b^2 = \log_b a + \log_b c$

$\Rightarrow 2\log_b b = \log_b a + \log_b c$ ...(iii)

$\Rightarrow 2 = \dfrac{1}{z} + y$

From (ii) and (iii), we have

$2\left(2 - \dfrac{1}{z}\right) = x + z \Rightarrow x = 4 - \dfrac{2}{z} - z$

Putting the value of x in (i), we get

$\left(4 - \frac{2}{z} - z\right)\left(2 - \frac{1}{z}\right) z = 1 \Rightarrow 2\,z^2 - 7\,z - 2 = 0$ ...(iv)

Now,

Common difference $= z - y = z - \left(2 - \dfrac{1}{z}\right)$

$\Rightarrow$ Common difference $= z - y = \dfrac{z^2 - 2z + 1}{z}$

$\Rightarrow$ Common difference $= z - y = \dfrac{2\,z^2 - 4\,z + 2}{2\,z}$

$= \dfrac{3\,z}{2\,z} = \dfrac{3}{2}$

[Using (iv)]

57 **(c)**

We have,

$\dfrac{\log 49\sqrt{7} + \log 25\sqrt{5} - \log 4\sqrt{2}}{\log 17.5}$

$= \dfrac{\log\left(\frac{7^{5/2} \times 5^{5/2}}{2^{5/2}}\right)}{\log 17.5} = \dfrac{5}{2}\dfrac{\log 17.5}{\log 17.5} = \dfrac{5}{2}$

58 **(b)**

It is given that x, y, z are in G.P.

$\therefore y^2 = xz$

Also, $x+3, y+3, z+3$ are in H.P.

$$\therefore y+3 = \frac{2(x+3)(z+3)}{(x+3)+(z+3)}$$

$$\Rightarrow y+3 = \frac{2\{xz+3(x+2)+9\}}{[(x+z)+6]}$$

$$\Rightarrow y+3 = \frac{2\{y^2+3(x+z)+9\}}{(x+z+6)}$$

Obviously, $y = 3$ satisfies this question

59 **(a)**

Since, a, b, c, d are in H.P.

$\therefore a, b, c$ are in H.P.

$\Rightarrow b$ is the H.M. of a and c

$\Rightarrow b = \frac{2\,ac}{a+c}$ ...(i)

Again, a, b, c, d are in H.P.

$\Rightarrow b, c, d$ are in H.P.

$\Rightarrow c$ is the H.M. of b and d

$\Rightarrow c = \frac{2\,bd}{b+d}$ ...(ii)

From (i) and (ii), we have

$$(a+c)(b+d) = \frac{2\,ac}{b} \cdot \frac{2bd}{c}$$

$$\Rightarrow ab+ad+bc+cd = 4\,ad$$

$$\Rightarrow ab+bc+cd = 3\,ad$$

61 **(c)**

We have,

$$a^2+4b^2 = 12ab$$

$$\Rightarrow (a+2b)^2 = 16ab$$

$$\Rightarrow 2\log(a+2b) = \log 16ab$$

$$\Rightarrow 2\log(a+2b) = \log a + \log b + 4\log 2$$

$$\Rightarrow \log(a+2b) = \frac{1}{2}(\log a + \log b + 4\log 2)$$

62 **(a)**

Let

$$S = 1\times 2003 + 2\times 2002 + 3\times 2001 + \cdots + 2003 \times 1$$

and, $T = 1^2+2^2+3^2+\cdots+2003^2$

$$\therefore S+T = 2004(1+2+3+\cdots+2003)$$

$$\Rightarrow 2003\times 4007\times 334 + 2003\times 334\times x = 2004\times\frac{2003\times(2004)}{2}$$

$$\Rightarrow 2003\times 334(4007+x) = 2004\times 2003\times 1002$$

$$\Rightarrow 4007+x = 2004\times 3 \Rightarrow x = 2005$$

63 **(b)**

$$15^2+16^2+17^2+\ldots+30^2$$

$$= 1^2+2^2+\ldots+30^2-(1^2+2^2+\ldots+14^2)$$

$$= \frac{30(31)(61)}{6} - \frac{14(15)(29)}{6}$$

$$= \frac{56730-6090}{6} = 8440$$

64 **(c)**

Since $(\pm 1 \pm 2 \pm \ldots \pm n)^2$

$= 2[1^2+2^2+\ldots+n^2]$

$+2$[sum of product of two terms]

$\Rightarrow$ 2[sum of product of two terms]

$$= 0 - 2\left[\frac{n(n+1)(2n+1)}{6}\right]$$

$\Rightarrow$ Sum of product of two terms

$$= \frac{-n(n+1)(2n+1)}{6}$$

65 **(c)**

We have,

$$(4.2)^x = (0.42)^y = 100$$

$$\Rightarrow x = \log_{4.2} 100 \text{ and } y = \log_{0.42} 100$$

$$\Rightarrow \frac{1}{x} = \log_{100} 4.2 \text{ and } \frac{1}{y} = \log_{100} 0.42$$

$$\Rightarrow \frac{1}{x} - \frac{1}{y} = \log_{100} 4.2 - \log_{100} 0.42$$

$$\Rightarrow \frac{1}{x} - \frac{1}{y} = \log_{100}\left(\frac{4.2}{0.42}\right) = \log_{100} 10 = \frac{1}{2}$$

66 **(a)**

We have,

$$2\{7^{-1}+3^{-1}7^{-3}+5^{-1}7^{-5}+\cdots\}$$

$$= 2\left\{\frac{1}{7}+\frac{1}{3}\cdot\frac{1}{7^3}+\frac{1}{5}\frac{1}{7^5}+\cdots\right\} = \log_e\left(\frac{1+\frac{1}{7}}{1-\frac{1}{7}}\right)$$

$$= \log_e\left(\frac{8}{6}\right) = \log_e\left(\frac{4}{3}\right)$$

67 **(a)**

Given sequence is $(-8+18i), (-6+15i), (-4+$ *$12i, -2+9i, 0+6i, \ldots$* Hence ,

5th term is purely imaginary.

68 **(c)**

We have,

$$a^2+4b^2 = 12ab$$

$$\Rightarrow a^2+4b^2+4ab = 16ab$$

$$\Rightarrow (a+2b)^2 = 16ab$$

$$\Rightarrow 2\log(a+2b) = \log 16 + \log a + \log b$$

$$\Rightarrow 2\log(a+2b) = 4\log 2 + \log a + \log b$$

$$\Rightarrow \log(a+2b) = \frac{1}{2}(4\log 2 + \log a + \log b)$$

69 **(d)**

We have,

$$\frac{1}{\log_x 10} = \frac{2}{\log_a 10} - 2$$

$$\Rightarrow \log_{10} x = 2\log_{10} a - 2$$

$$\Rightarrow \log_{10} x = \log_{10} a^2 - \log_{10} 100$$

$\Rightarrow \log_{10} x = \log_{10}\left(\frac{a^2}{100}\right) \Rightarrow x = \frac{a^2}{100}$

70 **(b)**
Since x, y, z are in H.P.
$\therefore y = \frac{2\,x\,z}{x+z}$
$\Rightarrow x - 2\,y + z = x + z - \frac{4\,xz}{x+z} = \frac{(x-z)^2}{x+z}$
$\Rightarrow \log(x - 2\,y + z) = 2\log(x-z) - \log(x+z)$
$\Rightarrow \log(x - 2\,y + z) + \log(x+z) = 2\log(x-z)$

71 **(b)**
We have,
$\log_e 2 \times \log_x 27 = \log_{10} 8 \times \log_e 10$
$\Rightarrow \log_e 2 \times 3\log_x 3$
$= 3\log_e 2 \;\Rightarrow \log_x 3 = 1 \Rightarrow x = 3$

72 **(a)**
We have,
$\log_8 x = 2.5$ and $\log_2 y = 5$
$\Rightarrow x = (8)^{5/2}$ and $y = 2^5$
$\Rightarrow x = 2^{15/2}$ and $y = 2^5 \Rightarrow x = y^{3/2}$

73 **(c)**
We have,
$1^2 + 1 + 2^2 + 2 + 3^3 + 3 + \cdots + n^2 + n$
$= (1 + 2 + \cdots + n)(1^2 + 2^2 + \cdots + n^2)$
$= \frac{n(n+1)}{2} + \frac{n(n+1)(2n+1)}{6}$
$= \frac{n(n+1)(n+2)}{3}$

74 **(a)**
When $\cos x = -1$, we have
$x = (2\,n+1)\pi, n \in Z$
$\Rightarrow$ Different values of x form an A.P.

75 **(c)**
Since, a and b are roots of $x^2 - 3\,x + p = 0$
$\therefore a + b = 3, ab = p$
Also, c and d are roots of $x^2 - 12\,x + q = 0$
$\therefore c + d = 12, cd = q$
Now,
a, b, c, d are in G.P.
$\Rightarrow \frac{b}{a} = \frac{d}{c}$
$\Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$
$\Rightarrow \frac{(a-b)^2}{(a+b)^2} = \frac{(c-d)^2}{(c+d)^2}$
$\Rightarrow 1 - \frac{4\,ab}{(a+b)^2} = 1 - \frac{4\,cd}{(c+d)^2}$
$\Rightarrow \frac{ab}{(a+b)^2} = \frac{cd}{(c+d)^2}$
$\Rightarrow \frac{p}{9} = \frac{q}{144}$
$\Rightarrow \frac{p}{1} = \frac{q}{16}$
$\Rightarrow \frac{p}{q} = \frac{1}{16} \Rightarrow \frac{p+q}{q-p} = \frac{17}{15} \;\Rightarrow \frac{q+p}{q-p} = \frac{17}{15}$

76 **(a)**
$(a - bx)e^{-x} = (a - bx)\left[1 - x + \frac{x^2}{2!} - \ldots\right]$
$\therefore$ Coefficient of x^n is $(-1)^n \frac{a}{n!} + \frac{(-b)(-1)^{n-1}}{(n-1)!}$
$= \frac{(-1)^n}{(n-1)!}\,\frac{(a+bn)}{n} = \frac{(-1)^n}{n!}(a+bn)$

77 **(d)**
Since, $T_p = q = a + (p-1)d$ ...(i)
and $T_q = p = a + (q-1)d$ ...(ii)
On solving Eqs. (i) and (ii), we get
$d = -1$ and $a = p + q - 1$
$\therefore\; T_{10} = a + (10-1)d = p + q - 10$

78 **(d)**
Let $x = 0.cababab\ldots$
$10x = c.ababab\ldots$ ...(i)
And $100 \times 10x = cab.abab\ldots$ ...(ii)
On subtracting Eq. (i) from Eq.(ii), we get
$990x = cab - c$
$\Rightarrow\; x = \frac{100c + 10a + b - c}{990}$
$= \frac{99c + 10a + b}{990}$

79 **(b)**
We have, $\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + \ldots$
$= 1\sqrt{2} + 2\sqrt{2} + 3\sqrt{2} + 4\sqrt{2} + \ldots$
$= \sqrt{2}(1 + 2 + 3 + 4 + \ldots$ upto 24 terms)
$= \sqrt{2} \times \frac{24 \times 25}{2}$
$= 300\sqrt{2}$ $\left[\because \sum n = \frac{n(n+1)}{2}\right]$

80 **(a)**
We have,
$\log(2a - 3b) = \log a - \log b$
$\Rightarrow \log(2a - 3b) = \log\left(\frac{a}{b}\right)$
$\Rightarrow 2a - 3b = \frac{a}{b} \Rightarrow 2ab - 3b^2 = a \Rightarrow a = \frac{3b^2}{2b-1}$

81 **(c)**
We have,
$2^x \times 3^{2x} = 100$
$\Rightarrow (2 \times 9)^x = 100$
$\Rightarrow x = \log_{18} 100 \Rightarrow x \in (1, 2)$ $[\because 18^1 = 18$ and $18^2 = 324]$

82 **(b)**
We have,
$1 - \log_e 2 + \frac{(\log_e 2)^2}{2\,!} - \frac{(\log_e 2)^3}{3\,!} + \cdots$

$= e^{-\log_e 2} = 2^{-1} = \frac{1}{2}$

83 **(c)**

On putting $x = \frac{1}{2}$ in $\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots \infty$

We get, $\frac{1}{2} - \frac{1}{2.2^2} + \frac{1}{3.2^3} - \frac{1}{4.2^4} + \ldots =$

$\log_e\left(1+\frac{1}{2}\right) = \log_e\left(\frac{3}{2}\right)$

84 **(b)**

We have,

$$\frac{1}{a} + \frac{1}{c} + \frac{1}{a-b} + \frac{1}{c-b} = 0$$

$$\Rightarrow \frac{1}{a} + \frac{1}{c} + \frac{a+c-2b}{ac+b^2-b(a+c)} = 0$$

$$\Rightarrow (a+c)\big(ac+b^2-b(a+c)\big) + ac(a+c) - 2\,abc = 0$$

$$= 2\,ac(a+c) + b^2(a+c) - b(a+c)^2 - 2\,abc = 0$$

$$\Rightarrow 2\,ac\,[a+c-b] - b(a+c)\{a+c-b\} = 0$$

$$\Rightarrow (a+c-b)(2\,ac - ab - ac) = 0$$

$$\Rightarrow 2\,ac = ab + ac \qquad [\because a+c-b \neq 0 \text{ given}]$$

$\Rightarrow a, b, c$ are in H.P.

85 **(c)**

Since, $4a^2 + 9b^2 + 16c^2 = 2(3ab + 6bc + 4ca)$

$$\Rightarrow 8a^2 + 18b^2 + 32c^2 - 12ab - 24bc - 16ca = 0$$

$$\Rightarrow (4a^2 - 12ab + 9b^2) + (9b^2 - 24bc + 16c^2) + (16c^2 - 16ca + 4a^2) = 0$$

$$\Rightarrow (2a-3b)^2 + (3b-4c)^2 + (4c-2a)^2 = 0$$

$$\Rightarrow 2a = 3b, 3b = 4c, 4c = 2a$$

$$\Rightarrow 2a = 3b = 4c = k \quad \text{[say]}$$

$$\Rightarrow a = \frac{k}{2}, b = \frac{k}{3}, c = \frac{k}{4}$$

$\Rightarrow a, b, c$ are in HP.

86 **(c)**

We have,

$$\sum n = \left(\frac{1}{5}\right)\sum n^2$$

$$\Rightarrow n\frac{(n+1)}{2} = \frac{1}{5}\left\{\frac{n(n+1)(2\,n+1)}{6}\right\}$$

$$\Rightarrow 2\,n + 1 = 15 \;\Rightarrow n = 7$$

87 **(c)**

Given, $\frac{\frac{p}{2}[2a_1+(p-1)d]}{\frac{q}{2}[2a_1+(q-1)d]} = \frac{p^2}{q^2}$

$$\Rightarrow \frac{(2a_1-d)+pd}{(2a_1-d)+qd} = \frac{p}{q}$$

$$\Rightarrow (2a_1-d)(p-q) = 0$$

$$\Rightarrow a_1 = \frac{d}{2}$$

Now, $\frac{a_6}{a_{21}} = \frac{a_1+5d}{a_2+20d}$

$$= \frac{\frac{d}{2}+5d}{\frac{d}{2}+20d} = \frac{11}{41}$$

91 **(b)**

Let r be the common ration of the G.P. Then,

$b = ar, c = ar^2$

Since $a-b, c-a$ and $b-c$ are in H.P.

$$\therefore \frac{2}{c-a} = \frac{1}{a-b} + \frac{1}{b-c}$$

$$\Rightarrow \frac{2}{ar^2-a} = \frac{1}{a-ar} + \frac{1}{ar-ar^2}$$

$$\Rightarrow \frac{2}{r^2-1} = \frac{1}{1-r} + \frac{1}{r-r^2}$$

$$\Rightarrow \frac{2}{r^2-1} = \frac{r+1}{r(1-r)}$$

$$\Rightarrow 2\,r(1-r) = (r^2-1)(r+1)$$

$$\Rightarrow 2\,r = -(r+1)^2 \qquad [\because r \neq 1]$$

$$\Rightarrow r^2 + 4\,r + 1 = 0 \;\Rightarrow ar^2 + 4\,ar + a = 0$$

$$\Rightarrow c + 4\,b + a = 0$$

92 **(a)**

Let t_n denotes nth term of an AP whose first term and common difference are a and d respectively.

$\therefore \; t_n = a + (n-1)d$

Now, $\frac{t_1+t_3+t_5+\ldots+t_{2n+1}}{t_2+t_4+t_6+\ldots+t_{2n}}$

$$= \frac{a+(a+2d)+(a+4d)+\ldots+(a+2nd)}{(a+d)+(a+3d)+(a+5d)+\ldots+a+(2n-1)d}$$

$$= \frac{\frac{n+1}{n}[a+(a+2nd)]}{\frac{n}{2}[(a+d)+\{a+(2n-1)d\}]}$$

$$= \frac{n+1}{n}$$

93 **(a)**

Let α and β are the roots of the equation

$x^2 - 2ax + a^2 = 0$

$\therefore \quad \alpha + \beta = 2a$ and $\alpha\beta = a^2$ ...(i)

Since, $A = \frac{\alpha+\beta}{2}$ and $G = \sqrt{\alpha\beta}$

$\Rightarrow A = a$ and $G^2 = a^2$ [from Eq. (i)]

$\Rightarrow G^2 = A^2 \;\Rightarrow G = A$

95 **(a)**

We have,

$1^3 - 2^3 + 3^3 - 4^3 + \cdots + 9^3$

$= (1^3 + 2^3 + \cdots + 9^3) - 2(2^3 + 4^3 + 6^3 + 8^3)$

$$= \left\{\frac{9(9+1)}{2}\right\}^2 - 2 \times 2^3(1^3+2^3+3^3+4^3)$$
$$= (9 \times 5)^2 - 16\left\{\frac{4(4+1)}{2}\right\}^2$$
$$= (9 \times 5)^2 - 16 \times (10)^2 = 425$$

96 **(c)**

We have,

$\log_2 \log_2 \log_4 256 + 2\log_{\sqrt{2}} 2$

$= \log_2(\log_2(\log_4 4^4) + 2\log_{2^{1/2}} 2$

$= \log_2(\log_2 4) + 4\log_2 2$

$= \log_2 2 + 4\log_2 2 = 1 + 4 = 5$

97 **(c)**

Given, $1 + \sin x + \sin^2 x + \ldots \infty = 4 + 2\sqrt{3}$

$\Rightarrow \dfrac{1}{1-\sin x} = 4 + 2\sqrt{3}$

$\Rightarrow 1 - \sin x = \dfrac{1}{4+2\sqrt{3}} \times \dfrac{4-2\sqrt{3}}{4-2\sqrt{3}}$

$\Rightarrow 1 - \sin x = \dfrac{4-2\sqrt{3}}{4}$

$\Rightarrow \sin x = \dfrac{2\sqrt{3}}{4} = \dfrac{\sqrt{3}}{2}$

$\Rightarrow x = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$

98 **(c)**

Let the sides of the right triangle be $a-d, a, a+d$, then hypotenuse being the greatest side $ie, a+d$.

So, $(a+d)^2 = a^2 + (a-d)^2$

$\Rightarrow a^2 + d^2 + 2ad = a^2 - 2ad + d^2$

$\Rightarrow a = 4d$

Therefore, ratio in the sides $= a - d : a : a + d = (4d - d) : 4d : (4d + d) = 3 : 4 : 5$

99 **(a)**

We have,

$3\log\dfrac{81}{80} + 5\log\dfrac{25}{24} + 7\log\dfrac{16}{15}$

$= 3\log\left(\dfrac{3^4}{2^4 \times 5}\right) + 5\log\left(\dfrac{5^2}{2^3 \times 3}\right) + 7\log\left(\dfrac{2^4}{3 \times 5}\right)$

$= \log\left\{\left(\dfrac{3^4}{2^4 \times 5}\right)^3 \times \left(\dfrac{5^2}{2^3 \times 3}\right)^5 \times \left(\dfrac{2^4}{3 \times 5}\right)^7\right\}$

$= \log\left\{\dfrac{3^{12}}{2^{12} \times 5^3} \times \dfrac{5^{10}}{2^{15} \times 3^5} \times \dfrac{28}{3^7 \times 5^7}\right\} = \log 2$

101 **(d)**

Given, $\frac{1}{e^{3x}}(e^x + e^{5x}) = a_0 + a_1 x + a_2 x^2 + \ldots$

$\Rightarrow (e^{-2x} + e^{2x}) = a_0 + a_1 x + a_2 x^2 + \ldots$

$\Rightarrow 2\left[1 + \dfrac{(2x)^2}{2!} + \dfrac{(2x)^4}{4!} + \ldots\right]$

$= a_0 + a_1 x + a_2 x^2 + \ldots$

$\Rightarrow a_1 = a_3 = a_5 = \ldots = 0$

$\therefore 2a_1 + 2^3 a_3 + 2^5 a_5 + \ldots = 0$

102 **(b)**

We have,

$a^{\frac{\log_b(\log_b x)}{\log_b a}} = a^{\log_b(\log_b x)\cdot\log_a b} = a^{\log_a(\log_b x)}$

$= \log_b x$

103 **(b)**

Now, $\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \ldots + \frac{1}{a_{4000} a_{4001}}$

$= \dfrac{1}{d}\left(\dfrac{a_2 - a_1}{a_1 a_2} + \dfrac{a_3 - a_2}{a_2 a_3} + \ldots + \dfrac{a_{4001} - a_{4000}}{a_{4000} a_{4001}}\right)$

$= \dfrac{1}{d}\left(\dfrac{1}{a_1} - \dfrac{1}{a_2} + \dfrac{1}{a_2} - \dfrac{1}{a_3} + \ldots + \dfrac{1}{a_{4000}} - \dfrac{1}{a_{4001}}\right)$

$= \frac{1}{d}\left(\frac{1}{a_1} - \frac{1}{a_{4001}}\right) = \frac{400}{a_1 a_{4001}} = 10$ (given)

$\Rightarrow a_1 a_{4001} = 400$ ...(i)

$a_1 + a_{4001} = a_2 + a_{4000} = 50$ ...(ii)

$\therefore (a_1 - a_{4001})^2 = (a_1 + a_{4000})^2 - 4a_1 a_{4001}$

$= (50)^2 - 1600$

$\Rightarrow |a_1 - a_{4001}| = 30$

104 **(b)**

It is given that

p, q, r are in AP $\Rightarrow 2q = p + r$

Let a be the first term and R be the common ration of the GP.

Then,

$T_p = p$th term $= aR^{p-1}, T_q = q$th term $= aR^{q-1}$, $T_r = r$th term $= aR^{r-1}$

Now,

$T_q^2 = (aR^{q-1})^2 = a^2 R^{2q-2}$

$\Rightarrow T_q^2 = a^2 R^{p+r-2}$ $[2q = p = r]$

$\Rightarrow T_q^2 = (a R^{p-1})(a R^{r-1}) = T_p T_r$

$\Rightarrow T_p, T_q, T_r$ are in GP.

105 **(c)**

$S_n = 1(1!) + 2(2!) + 3(3!) + \ldots + n(n!)$

$= (2 - 1(1!) + (3-1)(2!) + (4-1)(3!) + \ldots + [(n+1) - 1](n!)$

$= (2 \cdot 1! - 1!) + (3 \cdot 2! - 2!) + (4 \cdot 3! - 3!) + \ldots] + [(n+1)(n!) - (n!)$

$= (n+1)! - 1!$

106 **(c)**

Let a be the first term and x be the common ratio of the A.P. Then,

$a + 5x = 2 \Rightarrow a = 2 - 5x$

Let $P = a_1\, a_4\, a_5 = a(a+3\,x)(a+4\,x) = (2-5\,x)(2-2\,x)(2-x)$

$\Rightarrow P = 2(-5\,x^3 + 17\,x^2 - 16\,x + 4)$

$\Rightarrow \frac{dP}{dx} = 0 \Rightarrow x = 8/5, 2/3$

Clearly, $\frac{d^2 P}{dx^2} > 0$ for $x = \frac{2}{3}$

Hence, P is least for $x = 2/3$

107 **(b)**

$\frac{n}{2}[2\times 2 + (n-1)\times 2] = 240$

$\Rightarrow\ n(2+n-1) = 240$

$\Rightarrow\ n(n+1) = 15\times 16$

$\Rightarrow n = 15$

108 **(c)**

Let the three numbers in AP are $a-d, a, a+d$

Since, $a-d+a+a+d = 15 \Rightarrow a = 5$

Since, $(a-d+1), (a+4), (a+d+19)$ are in GP.

$\therefore (a+4)^2 = (a-d+1)(a+d+19)$

$\Rightarrow\ 9^2 = (6-d)(24+d)\quad [\because a = 5]$

$\Rightarrow\ d^2 + 18d - 63 = 0$

$\Rightarrow\ d = 3, -21$

$\therefore$ Required series are 2, 5, 8 and 26, 5, -16

109 **(d)**

We have,

$1 + \cos\alpha + \cos^2\alpha + \ldots$ to $\infty = 2-\sqrt{2}$

$\Rightarrow\ \frac{1}{1-\cos\alpha} = 2-\sqrt{2}$

$\Rightarrow\ 1-\cos\alpha = \frac{1}{2-\sqrt{2}} = 1 + \frac{1}{\sqrt{2}}$

$\Rightarrow\ \cos\alpha = -\frac{1}{\sqrt{2}} \Rightarrow\ \alpha = \frac{3\pi}{4}$

110 **(c)**

$\because (\alpha+\beta), (\alpha^2+\beta^2), (\alpha^3+\beta^3)$ are in GP.

$\Rightarrow\ (\alpha^2+\beta^2)^2 = (\alpha+\beta)(\alpha^3+\beta^3)$

$\Rightarrow\ \alpha^4+\beta^4+2\alpha^2\beta^2 = \alpha^4+\beta^4+\alpha\beta^3+\beta\alpha^3$

$\Rightarrow\ \alpha\beta(\alpha^2+\beta^2-2\alpha\beta) = 0$

$\Rightarrow\ \alpha\beta(\alpha-\beta)^2 = 0$

$\Rightarrow\ \alpha\beta = 0$ or $\alpha = \beta$

$\Rightarrow\ \frac{c}{a} = 0$ or $\Delta = 0$

$\Rightarrow c\Delta = 0$

111 **(c)**

In n geometric means $G_1, G_2, \ldots, G_n$ are to be inserted between two positive real numbers a and b, then $a, G_1, G_2, \ldots, G_n, b$ are in GP, then

$G_1 = ar, G_2 = ar^2, \ldots, G_n = ar^n$

So, $b = ar^{n+1} \Rightarrow r = \left(\frac{b}{a}\right)^{\frac{1}{(n+1)}}$

Now, nth geometric mean

$(G_n) = ar^n = a\left(\frac{a}{b}\right)^{\frac{n}{n+1}}$

112 **(a)**

We have,

$$(x+y)(x-y) + \frac{1}{2\,!}(x+y)(x-y)(x^2+y^2) + \frac{1}{3\,!}(x+y)(x-y)(x^4+y^4+x^2y^2) + \cdots\infty$$

$$= (x^2-y^2) + \frac{1}{2\,!}(x^4-y^4) + \frac{1}{3\,!}(x^6-y^6) + \cdots\infty$$

$$= \left\{x^2 + \frac{(x^2)^2}{2\,!} + \frac{(x^2)^3}{3\,!} + \cdots\right\} - \left\{y^2 + \frac{(y^2)^2}{2\,!} + \frac{(y^2)^3}{3\,!} + \cdots\right\}$$

$$= \left(e^{x^2}-1\right) - \left(e^{y^2}-1\right) = e^{x^2} - e^{y^2}$$

113 **(b)**

We have,

$2.\overline{357} = 2 + 0.357357357357\ \ldots.$

$\Rightarrow 2.\overline{357} = 2 + 0.357 + 0.000357 + 0.000000357 + \cdots$

$\Rightarrow 2.\overline{357} = 2 + \frac{357}{10^3} + \frac{357}{10^6} + \frac{357}{10^9} + \cdots$

$\Rightarrow 2.\overline{357} = 2 + \frac{\frac{357}{10^3}}{1-\frac{1}{10^3}} = 2 + \frac{357}{999} = \frac{2355}{999}$

114 **(c)**

We have,

$\left.\begin{array}{l} x, 1, z \text{ are in AP} \Rightarrow 2 = x+z \\ x, 2, z \text{ are in GP} \Rightarrow 4 = xz \end{array}\right\}\quad \ldots(i)$

Since (i) does not satisfy $8 = x+z$ and $16 = xz$. But, it satisfies the relation $4 = \frac{2\,xz}{x+2}$. Hence, $x, 4, z$ are in HP

115 **(c)**

We have,

$$\frac{1^2.2}{1\,!} + \frac{2^2.3}{2\,!} + \frac{3^2.4}{3\,!} + \frac{4^2.5}{4\,!} + \cdots \text{ to } \infty$$

$$= \sum_{n=1}^{\infty}\frac{n^2(n+1)}{n\,!} = \sum_{n=1}^{\infty}\frac{n^3}{n\,!} + \frac{n^2}{n\,!}$$

$$= \sum_{n=1}^{\infty}\frac{n^3}{n\,!} + \sum_{n=1}^{\infty}\frac{n^2}{n\,!} = 5e + 2e = 7e$$

116 **(b)**

We have,

$\log_{10} 343 = 2.5353$

$\Rightarrow \log_{10} 7^3 = 2.5353$

$\Rightarrow 3\log_{10} 7 = 2.5353 \Rightarrow \log_{10} 7 = \frac{2.5353}{3}$

Now, $7^n > 10^5$

$\Rightarrow \log_{10} 7^n > \log_{10} 10^5$

$$\Rightarrow n\log_{10}7>5\Rightarrow n\frac{(2.5353)}{3}>5\Rightarrow n>\frac{15}{2.5353}$$
$$\Rightarrow n>5.916$$

Hence, the least value of n is 6

117 **(a)**

Since, a, b, c are in HP, then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP, let $\frac{1}{a}=p-q, \frac{1}{b}=p$ and $\frac{1}{c}=p+q$,

Where $p, q>0$ and $p>q$

Now, $\frac{3a+2b}{2a-b}+\frac{3c+2b}{2c-b}=\frac{\frac{3}{p-q}+\frac{2}{p}}{\frac{2}{p-q}-\frac{1}{p}}\frac{\frac{3}{p+q}+\frac{2}{p}}{\frac{2}{p+q}-\frac{1}{p}}$

$$=\frac{\frac{5p-2q}{p(p-q)}}{\frac{p+q}{p(p-q)}}+\frac{\frac{5p+2q}{p(p+q)}}{\frac{p-q}{p(p+q)}}$$
$$=\frac{5p-2q}{p+q}+\frac{5p+2q}{p-q}$$
$$=\frac{(5p-2q)(p-q)+(5p+2q)(p+q)}{(p^2+q^2)}$$
$$=10+\frac{14q^2}{p^2-q^2}$$

Which is obviously greater than 10 (as $p>q>0$).

118 **(b)**

We have,

$$\frac{\log x}{a-b}=\frac{\log y}{b-c}=\frac{\log z}{c-a}=\lambda(\text{say})$$
$$\Rightarrow \log x=\lambda(a-b), \log y=\lambda(b-c), \log z=\lambda(c-a)$$
$$\Rightarrow \log x+\log y+\log z=0$$
$$\Rightarrow \log xyz=0\Rightarrow xyz=1$$

119 **(a)**

We have,

$$a=\sum_{n=0}^{\infty}\frac{x^{3n}}{(3n)!}, b=\sum_{n=1}^{\infty}\frac{x^{3n-2}}{(3n-2)!} \text{ and } c=\sum_{n=1}^{\infty}\frac{x^{3n-1}}{(3n-1)!}$$
$$\Rightarrow a+b+c=\sum_{n=0}^{\infty}\frac{x^{3n}}{(3n)!}+\sum_{n=1}^{\infty}\frac{x^{3n-2}}{(3n-2)}+\sum_{n=1}^{\infty}\frac{x^{3n-1}}{(2n-1)!}$$
$$\Rightarrow a+b+c=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots=e^x$$

Also,

$$a+b\,\omega+c\,\omega^2=1+\omega\,x+\frac{\omega^2x^2}{2!}+\frac{\omega^3x^3}{3!}+\cdots=e^{\omega\,x}$$

and,

$a+b\,\omega^2+c\omega=e^{\omega^2x}$, where ω is an imaginary cube root of unity

Now,

$$a^3+b^3+c^3-3\,abc$$
$$=(a+b+c)(a+b\omega+c\omega^2)(a+b\,\omega^2+c\,\omega)$$
$$=e^x\,e^{\omega x}\,e^{\omega^2x}=e^{x(1+\omega+\omega^2)}=e^{x.0}=e^0=1$$

120 **(a)**

We have,

$$a_n=\log\left(\frac{5^{n+1}}{3^{n-1}}\right)$$
$$\Rightarrow a_n=\log\left\{\left(\frac{5}{3}\right)^{n+1}\times 9\right\}$$
$$\Rightarrow a_n=\log 9+(n+1)\log\frac{5}{3}$$
$$\Rightarrow a_n=2\log 3+(n+1)\log\left(\frac{5}{3}\right)$$
$$=n\log\left(\frac{5}{3}\right)+\log 15$$

Clearly, it is a linear expression in n. So, the sequence $<a_n>$ is an A.P. with common difference $\log\left(\frac{5}{3}\right)$

Clearly, the sequence $<b_n>$ forms a G.P. with common ratio equal to $\log\left(\frac{5}{3}\right)$

121 **(d)**

Let a_r be the r^{th} term of the given series. Then,

$$a_r=\frac{1}{r(r+1)(r+2)(r+3)}$$
$$\Rightarrow a_r=\frac{1}{6r}-\frac{1}{2(r+1)}+\frac{1}{2(r+2)}-\frac{1}{6(r+3)} \quad \text{[Using partial fractions]}$$
$$\Rightarrow a_r=\frac{1}{6}\left(\frac{1}{r}-\frac{1}{r+1}\right)-\frac{1}{3}\left(\frac{1}{r+1}-\frac{1}{r+2}\right)+\frac{1}{6}\left(\frac{1}{r+2}-\frac{1}{r+3}\right)$$
$$\Rightarrow \sum_{r=1}^{n}a_r=\frac{1}{6}\left(1-\frac{1}{n+1}\right)-\frac{1}{3}\left(\frac{1}{2}-\frac{1}{n+2}\right)+\frac{1}{6}\left(\frac{1}{3}-\frac{1}{n+3}\right)$$
$$\Rightarrow \sum_{r=1}^{n}a_r=\frac{n^3+6n^2+11n}{18(n+1)(n+2)(n+3)}$$

122 **(d)**

Let the GP series be $a, ar, ar^2, \ldots$

According to the given condition

$$a+ar+ar^2+\ldots=a^2+a^2r^2+a^2r^4+\ldots=3$$
$$\Rightarrow \frac{a}{1-r}=\frac{a^2}{1-r^2}=3$$
$$\Rightarrow a=3(1-r) \quad \ldots\text{(i)}$$

and $a^2=3(1-r^2)$ ...(ii)

Eliminating a from Eqs. (i) and (ii), we get

$$[3(1-r)]^2=3(1-r^2)$$

$\Rightarrow\ 3(1-r)=(1+r) \quad (\because r\neq 1)$

$\Rightarrow 4r=2\ \Rightarrow r=\frac{1}{2}$

$\therefore\ a=3(1-r)=3\left(1-\frac{1}{2}\right)=\frac{3}{2}$

124 **(c)**

We have,

$\log 2, \log(2^x-1), \log(2^x+3)$ are in A.P.

$\Rightarrow 2, 2^x-1, 2^x+3$ are in G.P.

125 **(a)**

We have,

$\log_2 a+\log_4 b+\log_4 c=2$

$\log_9 a+\log_3 b+\log_9 c=2$

$\log_{16} a+\log_{16} b+\log_4 c=2$

$\Rightarrow \log_2 a+\frac{1}{2}\log_2 b+\frac{1}{2}\log_2 x=2$

$\Rightarrow \frac{1}{2}\log_3 a+\log_3 b+\frac{1}{2}\log_3 c=2$

$\Rightarrow \frac{1}{2}\log_4 a+\frac{1}{2}\log_4 b+\log_4 c=2$

$\Rightarrow \log_2(a^2bc)=4, \log_3(ab^2c)=4, \log_4(abc^2)=4$

$\Rightarrow a^2bc=2^4, ab^2c=3^4$ and $abc^2=4^4$

$\Rightarrow (a^2bc)(ab^2c)(abc^2)=2^4\times 3^4\times 4^4$

$\Rightarrow (abc)^4=(2\times 3\times 4)^4 \Rightarrow abc=24$

Now, $a^2bc=2^4$ and $abc=24 \Rightarrow a=\frac{16}{24}=\frac{2}{3}$

$ab^2c=3^4$ and $abc=24 \Rightarrow b=\frac{81}{21}=\frac{27}{8}$

$abc^2=4^4$ and $abc=24 \Rightarrow c=\frac{256}{24}=\frac{32}{3}$

126 **(b)**

Given that $T_p=a+(p-1)d=q$ ...(i)

And $T_q=a+(q-1)d=p$ ...(ii)

From Eqs. (i) and (ii), we get

$d=-\frac{(p-q)}{p-q)}=-1$ ...(iii)

On putting the value of d in Eq.(i),we get

$a+(p-1)(-1)=q$

$\Rightarrow a=p+q-1$ ...(iv)

Now, rth term is given by AP

$T_r=a+(r-1)d=(p+q-1)+(r-1)(-1)$

[from Eqs. (iii) and (iv)]

$=\ p+q-r$

127 **(b)**

We know that,

$e+e^{-1}=2+\frac{2}{2!}+\frac{2}{4!}+\ldots\infty$

$\Rightarrow \frac{e^2+1-2e}{e}=2\left(\frac{1}{2!}+\frac{1}{4!}+\ldots\infty\right)$

$\Rightarrow \frac{(e-1)^2}{2e}=\frac{1}{2!}+\frac{1}{4!}+\ldots\infty$

128 **(b)**

We have, $|a|<1, |b|<1$

$\therefore |ab|=|a|\,|b|<1$

Now,

$a(a+b)+a^2(a^2+b^2)+a^3(a^3+b^3)+\cdots$ upto ∞

$=[(a^2+a^4+a^6+\cdots \text{ to } \infty)]+[(ab+(ab)^2+(ab)^3+\cdots \text{ to } \infty)]$

$=\frac{a^2}{1-a^2}+\frac{ab}{1-ab}$

129 **(a)**

$x(y^3-1)=1 \Rightarrow x=\frac{1}{y^3-1}=\frac{1}{k}$ [say]

$\Rightarrow k=\frac{1}{x}$

Then, $\frac{2}{x}+\frac{2}{3x^3}+\frac{2}{5x^5}+\ldots=2k+\frac{2}{3}k^3+\frac{2}{5}k^5+\ldots$

$=\log\frac{1+k}{1-k}=\log\left\{\frac{1+y^3-1}{1-y^3+1}\right\}$

$=\log\frac{y^3}{2-y^3}$

130 **(c)**

$a_1a_2\ldots a_n=b_n\left(\frac{a_1a_2\ldots a_n}{b_n}\right)$

$=a_nb_n\left(\frac{a_1a_2\ldots a_{n-1}}{b_n}\right)$

$=\left[(x)^{\frac{1}{2^{n-1}}}-(y)^{\frac{1}{2^{n-1}}}\right]\left(\frac{a_1a_2\ldots a_{n-1}}{b_n}\right)$

$=b_{n-1}\cdot a_{n-1}\left(\frac{a_1a_2\ldots a_{n-2}}{b_n}\right)=\ldots=\frac{x-y}{b_n}$

131 **(d)**

$\because T_n=\frac{5+(n-1)4}{[3+(n-1)4]^2[7+(n-1)4]^2}$

$=\frac{1}{8}\left\{\frac{1}{(4n-1)^2}-\frac{1}{(4n+3)^2}\right\}$

$\therefore S_n=T_1+T_2+\ldots+T_n$

$=\frac{1}{8}\left\{\frac{1}{3^2}-\frac{1}{7^2}+\frac{1}{7^2}-\frac{1}{11^2}+\frac{1}{11^2}-\ldots+\frac{1}{(4n-1)^2}-\frac{1}{(4n+3)^2}\right\}$

$=\frac{1}{8}\left\{\frac{1}{3^2}-\frac{1}{(4n+3)^2}\right\}$

$\Rightarrow S_\infty=\frac{1}{8}\left\{\frac{1}{9}-0\right\}=\frac{1}{72}$

132 **(d)**

Given, $a=7, ar^{n-1}=448$ and $S_n=\frac{a(r^n-1)}{r-1}=889$

$\Rightarrow \frac{ar^{n-1}r-a}{r-1}=889 \Rightarrow \frac{448r-7}{r-1}=889$

$\Rightarrow\ 448r - 7 = 889(r-1)$
$\Rightarrow r = 2$

133 **(b)**
Let a the first term and r be the common ratio.
Then,
$\frac{a}{1-r} = 20$ and $\frac{a^2}{1-r^2} = 100$ [Given]
$\Rightarrow \frac{\{20(1-r)\}^2}{1-r^2} = 100$
$\Rightarrow 400\left(\frac{1-r}{1+r}\right) = 100 \Rightarrow 4 - 4r = 1 + r \Rightarrow r = 3/5$

134 **(b)**
Since, a, b, c are in HP.
$\therefore \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP.
$\Rightarrow \frac{abc}{a}, \frac{abc}{b}, \frac{abc}{c}$ are in AP
$\Rightarrow bc, ca, ab$ are in AP
$\Rightarrow \frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in HP.

135 **(c)**
$i^2 + i^4 + i^6 + \cdots$upto $(2k+1)$ terms
$= \frac{i^2[1-(i^2)^{2k+1}]}{1-i^2} = \frac{-1[1-(-1)^{2k+1}]}{1+1}$
$= \frac{-[1-(-1)]}{2} = -1$

136 **(b)**
We have,
$a^x = b, b^y = c$ and $c^z = a$
$\Rightarrow b = (c^z)^x$ [$\because b = a^x$ and $a = c^z$]
$\Rightarrow b = c^{zx}$
$\Rightarrow b = (b^y)^{zx}$ [$\because c = b^y$]
$\Rightarrow b = b^{xyz} \Rightarrow xyz = 1$

137 **(b)**
Since, $a_1, a_2, \dots, a_n$ are in AP.
Then, $a_2 - a_1 = a_3 - a_2 = \ldots = a_n - a_{n-1} = d$
Where d is common difference
Now, $\frac{1}{\sqrt{a_2}+\sqrt{a_1}} + \frac{1}{\sqrt{a_3}+\sqrt{a_2}} + \cdots + \frac{1}{\sqrt{a_n}+\sqrt{a_{n-1}}}$
$= \frac{\sqrt{a_2}-\sqrt{a_1}}{d} + \frac{\sqrt{a_3}-\sqrt{a_2}}{d} + \cdots + \frac{\sqrt{a_n}-\sqrt{a_{n-1}}}{d}$
$= \frac{1}{d}(\sqrt{a_n}-\sqrt{a_1}) \times \frac{\sqrt{a_n}+\sqrt{a_1}}{\sqrt{a_n}+\sqrt{a_1}}$
$= \frac{1}{d}\left(\frac{a_n - a_1}{\sqrt{a_n}+\sqrt{a_1}}\right) = \frac{n-1}{\sqrt{a_n}+\sqrt{a_1}}$

138 **(c)**
Since, $n^m + 1$ divides $1 + n + n^2 + \ldots + n^{127}$
Therefore, $\frac{1+n+n^2+\ldots+n^{127}}{n^m+1}$is an integer.
$\Rightarrow \frac{1-n^{128}}{1-n} \times \frac{1}{n^m+1}$ is an integer.
$\Rightarrow \frac{(1-n^{64})(1+n^{64})}{(1-n)(n^m+1)}$ is an integer when largest
$m = 64$

139 **(b)**
Since, $2\log_3(3^{1-x}+2)^{1/2} = 1 + \log_3(4.3^x - 1)$
$\Rightarrow\ \log_3(3^{1-x}+2) = \log_3 3 + \log_3(4.3^x - 1)$
$\Rightarrow 3^{1-x} + 2 = 3(4.3^x - 1)$
$\Rightarrow 3.3^{-x} + 2 = 12.3^x - 3$
Let $3^x = t$
$\therefore \frac{3}{t} + 2 = 12t - 3$
$\Rightarrow 12t^2 - 5t - 3 = 0$
$\Rightarrow\ t = -\frac{1}{3}, \frac{3}{4}$
$\therefore\ 3^x = \frac{3}{4}$ [$\because 3^x$ cannot be negative]
$\Rightarrow\ x = \log_3\left(\frac{3}{4}\right) \Rightarrow x = 1 - \log_3 4$

140 **(c)**
We have,
$\log_5\left(1+\frac{1}{5}\right) + \log_5\left(1+\frac{1}{6}\right) + \log_5\left(1+\frac{1}{7}\right) + \cdots + \log_5\left(1+\frac{1}{624}\right)$
$= \log_5\left(\frac{6}{5}\times\frac{7}{6}\times\frac{8}{7}\times\ldots\times\frac{625}{624}\right)$
$= \log_5\left(\frac{625}{5}\right) = \log_5 5^3 = 3$

141 **(a)**
$2\log_e x - \log_e\left\{\left(1+\frac{1}{x}\right)x\right\} - \log_e\left\{\left(1-\frac{1}{x}\right)x\right\}$
$= 2\log_e x - \left\{\log_e\left(1+\frac{1}{x}\right) + \log_e x\right\} - \left\{\log_e\left(1-\frac{1}{x}\right) + \log_e x\right\}$
$= -\left\{\log_e\left(1+\frac{1}{x}\right) + \log_e\left(1-\frac{1}{x}\right)\right\} = 2\left[\frac{1}{2x^2} + \frac{1}{4x^4} + \ldots\right\}$
$\therefore$ The coefficients of $x^{-4} = 2 \times \frac{1}{4} = \frac{1}{2}$

142 **(b)**
If a, b, c are in H.P., then
$a^n + c^n > 2b^n \Rightarrow a^5 + c^5 > 2b^5$

143 **(c)**
Since, $\frac{a+b}{2} = \frac{a^n+b^n}{a^{n-1}+b^{n-1}}$
$\Rightarrow (a+b)(a^{n-1}+b^{n-1}) = 2(a^n+b^n)$

$$\Rightarrow \frac{ab^n}{b}+\frac{ba^n}{a}=a^n+b^n$$

$$\Rightarrow\ a^n\left(\frac{a-b}{a}\right)=-b^n\left(\frac{b-a}{b}\right)$$

$$\Rightarrow \frac{a^n}{b^n}=\frac{a}{b}\Rightarrow n=1$$

144 **(b)**

Taking x as the product of variates $x_1, x_2, \dots, x_r$ corresponding to r set of observations *ie*, $x=x_1x_2\dots\dots, x_r$, we have

$$\log x=\log x_1+\log x_2+\dots+\log x_r$$

$$\Rightarrow \sum \log x=\sum \log x_1+\sum \log x_2+\cdots+\sum \log x_r$$

$$\Rightarrow \frac{1}{n}\sum \log x=\frac{1}{n}\sum \log x_1+\frac{1}{n}\sum \log x_2+\cdots+\frac{1}{n}\sum \log x_r$$

$$\Rightarrow\ \log G=\log G_1+\log G_2+\dots+\log G_r$$

$$\Rightarrow\ G=G_1G_2\dots G_r$$

145 **(a)**

Two digit odd numbers are 11,13,15, ... ,99.

Clearly, these numbers form an A.P. consisting of 45 terms.

$\therefore$ Required sum $=\frac{45}{2}(11+99)=2475$

146 **(a)**

Let $S=\frac{1}{1.2.3}+\frac{1}{3.4.5}+\frac{1}{5.6.7}+\dots\infty$

$$\therefore\ T_n=\frac{1}{(2n-1)(2n)(2n+1)}$$

$$=\frac{1}{2(2n-1)}-\frac{1}{2n}+\frac{1}{2(2n+1)}$$

$$=\frac{1}{2}\left[\frac{1}{2n-1}-\frac{1}{2n}\right]-\frac{1}{2}\left[\frac{1}{2n}-\frac{1}{2n+1}\right]$$

On putting $n=1,2,3,\dots$

$$T_1=\frac{1}{2}\left[\frac{1}{1}-\frac{1}{2}\right]-\frac{1}{2}\left[\frac{1}{2}-\frac{1}{3}\right]$$

$$T_2=\frac{1}{2}\left[\frac{1}{3}-\frac{1}{4}\right]-\frac{1}{2}\left[\frac{1}{4}-\frac{1}{5}\right]$$

… … … … … … … … … … … … … … … … …

… … … … … … … … … … … … … … … … …

$$\therefore\ S=T_1+T_2+T_3+\dots+T_n+\dots$$

$$=\frac{1}{2}\left[1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\frac{1}{7}-\dots\right]-\frac{1}{2}\left[\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}+\frac{1}{6}-\frac{1}{7}+\dots\right]$$

$$=\frac{1}{2}\log_e(1+1)+\frac{1}{2}\left[-1+\left\{1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots\right\}\right]$$

$$=\frac{1}{2}\log_e 2-\frac{1}{2}+\frac{1}{2}\log_e(1+1)$$

$$=\log_e 2-\frac{1}{2}$$

147 **(b)**

We have,

$$\log_a ab=x$$

$$\Rightarrow \log_a a+\log_a b=x$$

$$\Rightarrow 1+\log_a b=x\Rightarrow \log_a b=x-1$$

$$\Rightarrow \log_b a=\frac{1}{x-1}$$

Now,

$$\log_b ab=\log_b a+\log_b b=\frac{1}{x-1}+1=\frac{x}{x-1}$$

148 **(c)**

We have $1.1!+2.2!+3.3!+\cdots.+n.n!$

$$=\sum_{r=1}^{n} r.(r!)=\sum_{r=1}^{n}[(r+1)r!-r!]$$

$$=\sum_{r=1}^{n}[(r+1)!-r\,!]$$

$$=(2!-1!)+(3!-2!)+\dots+[(n+1)!-n!]$$

$$=(n+1)!-1!=(n+1)!-1$$

149 **(b)**

We have,

$$\log_2[\log_2\{\log_3(\log_3 27^3)\}]$$

$$=\log_2[\log_2\{\log_3(\log_3 3^9)\}]$$

$$=\log_2[\log_2(\log_3 9)]=\log_2[\log_2 2]=\log_2 1=0$$

150 **(b)**

$$2^2+4^2+6^2+\dots+(2n)^2$$

$$=2^2(1^2+2^2+\cdots+n^2)$$

$$=\frac{4n(n+1)(2n+1)}{6}$$

$$=\frac{2n(n+1)(2n+1)}{3}$$

151 **(a)**

Let the two numbers be a and b, then

$A=\frac{a+b}{2}, G=\sqrt{ab}$, H= $\frac{2ab}{a+b}$

$\Rightarrow H=\frac{ab}{A} \Rightarrow H=\frac{G^2}{A}$

152 **(c)**

Let $S=1+\frac{1}{2!}+\frac{1.3}{4!}+\frac{1.3.5}{6!}+\ldots\infty$

$\therefore T_n=\frac{1.3.5.\ldots.(2n-1)}{(2n)!}\times\frac{2.4.\ldots.2n}{2.4.\ldots.2n}$

$=\frac{(2n)!}{(2n)!\,2^n(n)!}=\frac{1}{2^n(n)!}$

$\therefore S=1+\sum T_n=1+\frac{1}{2(1)!}+\frac{1}{2^2(2)!}+\ldots\infty$

$=e^{1/2}=\sqrt{e}$

153 **(b)**

Since, $a_1,a_2,a_3,\ldots,a_n$ are in AP.

$\Rightarrow\ d=a_2-a_1=a_3-a_2=\ldots=a_n-a_{n-1}$

$\therefore \sin d$ (cosec $a_1$cosec $a_2+\ldots+$cosec a_{n-1} cosec a_r

$=\frac{\sin(a_2-a_1)}{\sin a_1\sin a_2}+\ldots+\frac{\sin(a_n-a_{n-1})}{\sin a_{n-1}\sin a_n}$

$=\frac{(\sin a_2\cos a_1-\cos a_2\sin a_1)}{\sin a_1\sin a_2}+\ldots+\frac{(\sin a_n\cos}{\text{si}}$

$=(\cot a_1-\cot a_2)$

$+(\cot a_2-\cot a_3)+\ldots+(\cot a_{n-1}$

$-\cot a_n)$

$=\cot a_1-\cot a_n$

154 **(a)**

Given, $S_n=\frac{lr-a}{r-1}=364$

$\Rightarrow\ \frac{3\times243-a}{2}=364\ \Rightarrow a=1$

Since, $l=ar^{n-1}\Rightarrow 243=1(3)^{n-1}$

$\Rightarrow\ 3^5=3^{n-1}\Rightarrow n=6$

155 **(b)**

It is given that

a,b,c are pth,qth,rth terms of a H.P.

$\Rightarrow\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are pth,qth,rth terms of the corresponding A.P.

Let A be its first term and D be the common difference. Then,

$\frac{1}{a}=A+(p-1)D$ ... (i)

$\frac{1}{b}=A+(q-1)D$ ... (ii)

$\frac{1}{c}=A+(r-1)D$ ... (iii)

Subtracting (iii) from (ii), we get

$\frac{1}{b}-\frac{1}{c}=(q-r)D\Rightarrow bc(q-r)=\frac{c-b}{D}$

Similarly, we have

$ca(r-p)=\frac{(a-c)}{D}$ and $ab(p-q)=\frac{(b-a)}{D}$

$\therefore bc(q-r)+ca(r-p)+ab(p-q)$

$=\frac{1}{D}(c-b+a-c+b-a)=0$

$\Rightarrow\begin{vmatrix}bc & ca & ab\\ p & q & r\\ 1 & 1 & 1\end{vmatrix}=0$

157 **(a)**

We have,

$\log(x+z)+\log(x-2y+z)=2\log(x-z)$

$\Rightarrow(x+z)(x-2y+z)=(x-z)^2$

$\Rightarrow(x+z)^2-2\,xy-2\,yz=(x-z)^2$

$\Rightarrow xy+yz=2\,xz$

$\Rightarrow\frac{1}{z}+\frac{1}{x}=\frac{2}{y}\Rightarrow x,y,z$ are in H.P.

158 **(b)**

Since x,y,z are three consecutive positive integers

$\therefore 2\,y=x+z$

$\Rightarrow 4\,y^2=(x+z)^2$

$\Rightarrow 4\,y^2=(x-z)^2+4\,xz$

$\Rightarrow 4\,y^2=(-2)^2+4\,x\,z$ $\quad[\because z-x=-2]$

$\Rightarrow y^2=1+xz$

Now,

$\frac{1}{2}\log_e x+\frac{1}{2}\log_e z+\frac{1}{1+2\,x\,z}+\frac{1}{3}\left(\frac{1}{1+2\,x\,z}\right)^3$

$+\cdots$

$=\frac{1}{2}\left[\log_e x+\log_e z+2\left\{\left(\frac{1}{1+2\,x\,z}\right)\right.\right.$

$\left.\left.+\frac{1}{3}\left(\frac{1}{1+2\,x\,z}\right)^3+\cdots\right\}\right]$

$=\frac{1}{2}\left\{\log_e x\,x+\log_e\left(\frac{1+\frac{1}{1+2\,x\,z}}{1-\frac{1}{1+2\,x\,z}}\right)\right\}$

$=\frac{1}{2}\left\{\log_e xz+\log_e\left(\frac{1+xz}{xz}\right)\right\}$

$=\frac{1}{2}\log_e(1+xz)=\frac{1}{2}\log_e y^2$ $\quad$ [From (i)]

$=\log_e y$

160 **(c)**

$1+\frac{(a+bx)=}{1!}+\frac{(a+bx)^2}{2!}+\frac{(a+bx)^3}{3!}$

$=\cdots\infty=e^{a+bx}$

$\therefore$ Coefficient of x^nin $e^ae^{bx}=e^a.\frac{(b)^n}{n!}$

162 **(d)**

Since, $\frac{2}{3},k$ and $\frac{5}{8}$ are in AP.

$\therefore 2k=\frac{2}{3}+\frac{5}{8}\ \Rightarrow k=\frac{31}{48}$

163 **(c)**

Since, $\log(1+x)-\log(1-x)$

$$= 2\left[x+\frac{x^3}{3}+\frac{x^5}{5}+\ldots\infty\right]$$

Put $x=\frac{1}{2}$ on both sides, we get

$$\log\left(\frac{3}{2}\right)-\log\left(\frac{1}{2}\right)$$

$$= 2\left(\frac{1}{2}+\frac{1}{3}.\frac{1}{2^3}+\frac{1}{5}.\frac{1}{2^5}+\ldots\infty\right)$$

$$\Rightarrow \log 3 = 1+\frac{1}{3}.\frac{1}{4}+\frac{1}{5}.\frac{1}{4^2}+\ldots$$

164 **(a)**

Let $f(x)=Ax^2+Bx+C$

$\therefore f(1)=A+B+C$

and $f(-1)=A-B+C$

$\because f(1)=f(-1)$

$\Rightarrow A+B+C=A-B+C$

$\Rightarrow B=0$

$\therefore f(x)=Ax^2+C$

$\Rightarrow f'(x)=2Ax$

$\Rightarrow f'(a)=2Aa, f'(b)=2Ab$ and $f'(c)=2Ac$

Also, a,b,c are in AP

$\therefore 2Aa, 2Ab, 2Ac$ are in AP

$\Rightarrow f'(a), f'(b), f'(c)$ are in AP

165 **(c)**

Here, $T_n=n(n+1)(n+2)$

$=n^3+3n^2+2n$

$\therefore S_n=\sum T_n$

$$=\left[\frac{n(n+1)}{2}\right]^3+\frac{3n(n+1)(2n+1)}{6}+\frac{2n(n+1)}{2}$$

$$=\frac{1}{4}n(n+1)(n+2)(n+3)$$

166 **(b)**

Since, $\frac{x+y}{2\sqrt{xy}}=\frac{p}{q}\Rightarrow\frac{(x+y)^2}{4xy}=\frac{p^2}{q^2}$ ...(i)

On subtracting both sides by1, we get

$$\frac{(x-y)^2}{4xy}=\frac{p^2-q^2}{q^2}\ \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\left(\frac{x+y}{x-y}\right)^2=\frac{p^2}{p^2-q^2}\Longrightarrow\frac{x+y}{x-y}=\frac{p}{\sqrt{p^2-q^2}}$$

$$\Rightarrow\frac{2x}{2y}=\frac{p+\sqrt{p^2-q^2}}{p-\sqrt{p^2-q^2}}$$

[by componendo-dividendo rule]

$\therefore x:y=(p+\sqrt{p^2-q^2}):(p-\sqrt{p^2-q^2})$

167 **(b)**

We know that

$$AM>GM\Rightarrow\frac{a+b}{2}>\sqrt{ab}\Rightarrow a+b>2\sqrt{ab}$$

168 **(b)**

The general term is

$$T_n=\frac{\frac{n}{2}.\frac{n+1}{2}}{1^3+2^3+3^3+\ldots+n^3}$$

$$=\frac{1}{n(n+1)}=\frac{1}{n}-\frac{1}{n+1}$$

$$\therefore S_n=1-\frac{1}{n+1}=\frac{n}{n+1}$$

169 **(b)**

We have,

$(666\ldots6)=6+6.10+6.10^2+\cdots+6.10^{n-1}$

$$=\frac{2}{3}(10^n-1)$$

$n-$digits

$$\therefore (666\ldots6)^2=\frac{4}{9}(10^n-1)^2$$

Similarly, we have

$$(888\ldots8)=\frac{8}{9}(10^n-1)$$

$\therefore (66\ldots6)^2+(888\ldots8)$

$$=\frac{4}{9}(10^n-1)^2+\frac{8}{9}(10^n-1)$$

$n-$digits $\quad n-$digits

$$\Rightarrow (66\ldots6)^2+(888\ldots8)=\frac{4}{9}(10^{2n}-1)$$

$n-$digits $\quad n-$digits

170 **(b)**

We have,

$$S_n=(2-1)+\left(2-\frac{1}{2}\right)+\left(2-\frac{1}{3}\right)+\cdots+\left(2-\frac{1}{n}\right)$$

$\Rightarrow S_n=2n-H_n$

172 **(b)**

Here, $a=1,\ r=\frac{1}{5},\ d=3$

$$\because S_n=\frac{a}{1-r}+\frac{dr(1-r^{n-1})}{(1-r)^2}-\frac{\{a+(n-1)d\}r^n}{1-r}$$

$$\therefore S_n=\frac{1}{1-\frac{1}{5}}+\frac{3.\frac{1}{5}\left[1-\left(\frac{1}{5}\right)^{n-1}\right]}{\left(1-\frac{1}{5}\right)^2}-\frac{\{1+(n-1)3\}}{1-\frac{1}{5}}\left(\frac{1}{5}\right)^n$$

$$= \frac{5}{4} + \frac{15}{16}\left(1 - \frac{1}{5^{n-1}}\right) - \frac{1}{4}.\frac{(3n-2)}{5^{n-1}}$$

But, $S_n = l + \frac{15}{16}\left(1 - \frac{1}{5^{n-1}}\right) - \frac{(3n-2)}{4.5^{n-1}}$

$\therefore\ l = \frac{5}{4}$

173 **(b)**

$0.5737373 = 0.\overline{573}$

$0.\underbrace{X}_{\text{1 term}}\ \underbrace{Y}_{\text{2 term}}$

$\therefore\ S = \frac{XY - X}{\underbrace{9}_{\text{2 digits}}\ \underbrace{0}_{\text{1 digits}}}$

It is of the from 0 X Y

$\therefore\quad S = \frac{XY-X}{9\quad 0} = \frac{573-5}{990} = \frac{568}{990} = \frac{284}{495}$

174 **(b)**

Since, $y^2 = xz$ ...(i)

Now, $a^x = b^y = c^z = m$ [say]

$\Rightarrow x\log_e a = y\log_e b = z\log_e c = \log_e m$

$\Rightarrow x\log_e a = \log_e m, y\log_e b$

$= \log_e m, z\log_e c = \log_e m$

$\Rightarrow x = \log_a m, y = \log_b m, z = \log_e m$

From Eq. (i),

$(\log_b m)^2 = \log_a m \log_c m$

$\Rightarrow \frac{\log_b m}{\log_a m} = \frac{\log_c m}{\log_b m}$

$\Rightarrow \log_b a = \log_c b$

175 **(d)**

Since, $ar^{n-1} = ar^n + ar^{n+1}$

$\Rightarrow \frac{1}{r} = 1 + r \ \Rightarrow r^2 + r - 1 = 0$

$\Rightarrow r = \frac{\sqrt{5}-1}{2} \quad \left[\because\ r \neq \frac{-\sqrt{5}-1}{2}\right]$

176 **(c)**

Consider three numbers $\log_2 3, \log_2 6$ and $\log_2 12$.

We have,

$\log_2 6 = \log_2(3 \times 2)$

$= \log_2 3 + \log_2 2 = 1 + \log_2 3$

And, $\log_2 12 = \log_2(2^2 \times 3) = \log_2 3 +$
2log22=2+log23

$\because \log_2 3, 1 + \log_2 3$ and $2 + \log_2 3$ are in A.P.

$\Rightarrow \log_1 3, \log_2 6, \log_2 12$ are in A.P.

$\Rightarrow \log_3 2, \log_6 2, \log_{12} 2$ are in H.P.

177 **(a)**

Since, p, q, r are positive and are in AP.

$\therefore\ q = \frac{p+r}{2}$ ......(i)

$\because$ The roots of the equation $px^2 + qx + r = 0$ are real.

$\Rightarrow q^2 \geq 4pr \Rightarrow \left[\frac{p+r}{2}\right]^2 \geq 4pr$ [from Eq. (i)]

$\Rightarrow p^2 + r^2 - 14pr \geq 0$

$\Rightarrow \left(\frac{r}{p}\right)^2 - 14\left(\frac{r}{p}\right) + 1 \geq 0 \quad (\because p > 0)$

$\Rightarrow \left(\frac{r}{p} - 7\right)^2 - 48 \geq 0$

$\Rightarrow \left(\frac{r}{p} - 7\right)^2 - (4\sqrt{3})^2 \geq 0$

$\Rightarrow \left|\frac{r}{p} - 7\right| \geq 4\sqrt{3}$

178 **(c)**

Let $S = \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots$ Then,

$S = \frac{1/3}{1 - 1/3} = \frac{1}{2}$

$\therefore (0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\cdots\infty\right)}$

$= \{(0.4)^2\}^{\log_{\frac{1}{0.4}}\left(\frac{1}{2}\right)} = \{(0.4)^2\}^{\log_{(0.4)^{-1}} 2^{-1}}$

$= \{(0.4)^2\}\log_{0.4}^2$

$= \{(0.4)^2\}^{\log_{0.4}^2} = (0.4)^{\log_{0.4} 4} = 4$

179 **(c)**

We have,

$\frac{\log_a(\log_b a)}{\log_b(\log_a b)} = \frac{\log(\log_b a)}{\log a} \times \frac{\log b}{\log(\log_a b)}$

$= \frac{\log\left(\frac{\log a}{\log b}\right)}{\log a} \times \frac{\log b}{\log\left(\frac{\log b}{\log a}\right)}$

$= \frac{\log(\log a) - \log(\log b)}{\log a} \times \frac{\log b}{\log(\log b) - \log(\log a)}$

$= -\frac{\log b}{\log a} = -\log_a b$

180 **(b)**

Let $S = \frac{1}{1.3} + \frac{1}{2.5} + \frac{1}{3.7} + \frac{1}{4.9} + \ldots$

$\therefore T_n = \frac{1}{n(2n+1)} = \frac{1}{n} - \frac{2}{(2n+1)}$

$\Rightarrow S = \sum_{n=1}^{\infty} T_n = \frac{1}{1} - \frac{2}{3} + \frac{1}{2} - \frac{2}{5} + \frac{1}{3} - \frac{2}{7} + \frac{1}{4} - \frac{2}{9}$

$+ \frac{1}{5} - \ldots$

$= 1 - \left(-\frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots\right)$

$= 1 - (-1 + \log_e 2)$

$= 2 - \log_e 2$

181 **(a)**

We have,

$S_2 = \frac{n_1}{2}\{2a + (n_1 - 1)d\}$,

$S_2 = \frac{n_2}{2}\{2a + (n_2 - 1)d\}$ and $S_3 = \frac{n_3}{2}\{2a +$
$n3-1\,d$

$\therefore \frac{2\,S_1}{n_1}(n_2-n_3)+\frac{2\,S_2}{n_2}(n_3-n_1)+\frac{2\,S_3}{n_3}(n_1-n_2)$
$=\{2\,a+(n-1)\,d\}(n_2-n_3)$
$+\{2\,a+(n_2-1)\,d\}\,(n_3-n_1)$
$+\{2\,a+(n_3-1)\,d\}\,(n_1-n_2)$
$=0$

182 **(a)**
Here, $a=0.9=\frac{9}{10}$ and $r=\frac{1}{10}=0.1$
$\therefore\ S_{100}=a\left(\frac{1-r^{100}}{1-r}\right)$
$=\frac{9}{10}\left(\frac{1-\frac{1}{10^{100}}}{1-\frac{1}{10}}\right)=1-\left(\frac{1}{10^{100}}\right)$

183 **(c)**
Here, $T_n=\frac{1+2+3+...+n}{1^3+2^3+3^3+...+n^3}=\frac{\Sigma n}{\Sigma n^3}$
$=\frac{n(n+1)/2}{\{n(n+1)/2\}^2}=\frac{2}{n(n+1)}$
$=2\left(\frac{1}{n}-\frac{1}{n+1}\right)$
$\therefore S_n=\Sigma T_n$
$=2\left(\frac{1}{1}-\frac{1}{2}\right)+2\left(\frac{1}{2}-\frac{1}{3}\right)+\ldots+2\left(\frac{1}{n}-\frac{1}{n+1}\right)$
$=2\left(1-\frac{1}{n+1}\right)=2-\frac{2}{n+1}$
$\leq 2\quad\left[\because \frac{2}{n+1}\leq 1\right]$

184 **(d)**
We have,
$\log_4(3x^2+11x)>1$
$\Rightarrow 3x^2+11x>4$
$\Rightarrow 3x^2+11x-4>0\Rightarrow (x+4)(3x-1)>0\Rightarrow$
$x<-4$ or $x>\frac{1}{3}$
But, $\log_4(3x^2+11x)$ is defined for $3x^2+11x>0$
i.e. for $x>0$ or $x<-\frac{11}{3}$
Hence, $x\in(-\infty,-4)\cup(1/3,\infty)$

185 **(b)**
Let a be the first term and R be the common ratio of the given G.P. Then,
$x=a\,R^{p-1},y=a\,R^{q-1},z=a\,R^{r-1}$
$\therefore x^{q-r}y^{r-p}z^{p-q}$
$=a^{q-r+r-p+p-q}R^{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)}$
$=a^0R^0=1$

186 **(a)**
We have,
$5^{3x^2\log_{10}2}=2^{\left(x+\frac{1}{2}\right)\log_{10}25}$
$\Rightarrow 5^{3x^2\log_{10}2}=2^{2\left(x+\frac{1}{2}\right)\log_{10}5}$
$\Rightarrow 5^{\log_{10}2^{3x^2}}=2^{\log_{10}5^{2x+1}}$
$\Rightarrow \left(2^{3x^2}\right)^{\log_{10}5}=2^{\log_{10}5^{2x+1}}\quad[\because x^{\log_a y}=y^{\log_a x}]$
$\Rightarrow 3x^2.\log_{10}5=\log_{10}5^{2x+1}$
$\Rightarrow 5^{3x^2}=5^{2x+1}$
$\Rightarrow 3x^2=2x+1\Rightarrow 3x^2-2x-1=0\Rightarrow x=1,-\frac{1}{3}$

187 **(c)**
We have,
$e^{(x-1)-\frac{1}{2}(x-1)^2+\frac{1}{3}(x-1)^3-\frac{1}{4}(x-1)^4+\cdots}$
$=e^{\log\{1+(x-1)\}}=e^{\log x}=x$

188 **(b)**
$e^{e^x}=1+\frac{e^x}{1!}+\frac{e^{2x}}{2!}+\ldots$
$=1+\frac{1}{1!}\left(1+x+\frac{x^2}{2!}+\ldots\right)$
$+\frac{1}{2!}\left(1+\frac{2x}{1!}+\frac{(2x)^2}{2!}+\ldots\right)+\ldots$
$\therefore e^{e^x}=\left(1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\ldots\right)$
$+x\left(\frac{1}{1!}+\frac{2}{2!\,1!}+\ldots\right)+\ldots$
$\therefore\ a_0=e$

189 **(b)**
We have,
$\log\left(\frac{1}{1+x+x^2+x^3}\right)$
$=\log\left(\frac{1-x}{1-x^4}\right)=\log(1-x)-\log(1-x^4)$
$=-\sum_{r=1}^{\infty}\frac{x^r}{r}+\sum_{r=1}^{\infty}\frac{x^{4r}}{r}$
When n is odd, there is no term in the second series containing x^n. Therefore, the coefficient of x^n is zero in the second series, and in the first series the coefficient of x^n is $-1/n$
Hence, when n is odd, we have
Coefficient of $x^n=-\frac{1}{n}+0=-\frac{1}{n}$

190 **(a)**
Let two numbers be p and Q.
∴Given, $\frac{2PQ}{P+Q}=14\frac{2}{5}=\frac{72}{5}$ ....(i)
And $\sqrt{PQ}=24\Rightarrow PQ=576\quad ...(ii)$
From Eq.(i),
$P+Q=\frac{10PQ}{72}$
$\Rightarrow P+Q=\frac{10\times 576}{72}$
$=80\ [from\ Eq.(ii)]\ ...(iii)$

Now, $(P-Q)^2=(P=Q)^2-4PQ$
$=80^2-4\times 576=4096$
[From Eqs. (ii)and (iii)]
$\Rightarrow \quad P-Q=64$ ....(iv)
On solving Eqs. (iii) and (iv), we get
$P=72, Q=8$
Hence, greater number is 72.

191 **(b)**
We have,
$\frac{n}{2}[2\times 2+(n-1)3]=60100$
$\Rightarrow n(3\,n+1)=120200$
$\Rightarrow 3\,n^2+n-120200=0$
$\Rightarrow 3\,n^2-600\,n+601\,n-120200=0$
$\Rightarrow (n-200)(3\,n+601)=0 \Rightarrow n=200$

192 **(c)**
The given series is clearly an AG, the corresponding AP is $1+4+7+10+\ldots$ having nth term $=3n-2$.
and corresponding GP is $1+\frac{1}{5}+\frac{1}{5^2}+\ldots$ having nth term $=\frac{1}{5^{n-1}}$
Hence, required nth term of the series is $\frac{3n-2}{5^{n-1}}$.

193 **(c)**
$$y=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots=\log(1+x)$$
$$\Rightarrow 1+x=e^y=1+y+\frac{y^2}{2!}+\ldots$$
$$\Rightarrow x=y+\frac{y^2}{2!}+\frac{y^3}{3!}+\ldots$$

194 **(d)**
$$\frac{a+(a+1)+(a+2)+\ldots+(a+6)}{7}=m$$
$\Rightarrow 7a+21=7m$
$\Rightarrow a+3=m$
$$\therefore \quad \frac{(a+2)+(a+3)+(a+4)+\cdots+(a+1}{11}$$
$$=\frac{11a+77}{11}$$
$=a+7$
$=m-3+7$
$=m+4$

195 **(d)**
We have,
$$\log 2+2\left(\frac{1}{5}+\frac{1}{3}\cdot\frac{1}{5^3}+\frac{1}{5}\cdot\frac{1}{5^3}+\cdots\infty\right)$$
$$=\log 2+\log\left(\frac{1+\frac{1}{5}}{1-\frac{1}{5}}\right)\ \left[\because \log\left(\frac{1+x}{1-x}\right)=2\left(x+\frac{x^3}{3}+\frac{x^5}{5}-\cdots\right)\right]$$
$$=\log 2+\log\left(\frac{3}{2}\right)=\log 3$$

196 **(d)**
Given that,
$a_1+a_5+a_{10}+a_{15}+a_{20}+a_{24}=225$
$\Rightarrow (a_1+a_{24}+(a_5+a_{20})+(a_{10}+a_{15})=225$
$\Rightarrow 3(a_1+a_{24})=225$
$\Rightarrow a_1+a_{24}=75$ ...(i)
($\because$ In an AP the sum of the terms equidistant from the beginning and the end is same and is equal to the sum of the first and last term)
$\therefore \; a_1+a_2+\ldots+a_{24}=\frac{n}{2}[a+l)$
$=\frac{24}{2}(a_1+a_{24})$
$=12\times 75=900$ [from Eq. (i)]

197 **(b)**
Since, x,y,z are in GP.
$\therefore \; y^2=xz$
$\Rightarrow 2\log y=\log x+\log z$
$\Rightarrow 2(\log y+1)=(1+\log x)+(1+\log z)$
$\Rightarrow 1+\log x, 1+\log y, 1+\log z$ are in AP
$\Rightarrow \frac{1}{1+\log x},\frac{1}{1+\log y},\frac{1}{1+\log z}$ are in HP.

198 **(a)**
We have,
$\log\{(1+x)^{1+x}\cdot(1-x)^{1-x}\}$
$=(1+x)\log(1+x)+(1-x)\log(1-x)$
$=\log(1+x)+\log(1-x)$
$\quad +x\{\log(1+x)-\log(1-x)\}$
$$=\log(1-x^2)+x\log\left(\frac{1+x}{1-x}\right)$$
$$=-\left(x^2+\frac{x^4}{2}+\frac{x^6}{3}+\cdots\right)$$
$$+2x\left(x+\frac{x^3}{3}+\frac{x^5}{5}+\cdots\right)$$
$$=2\left\{\left(x^2-\frac{x^2}{2}\right)+\left(\frac{x^4}{3}-\frac{x^4}{4}\right)+\left(\frac{x^6}{5}-\frac{x^6}{6}\right)+\cdots\right\}$$
$$=2\left\{\frac{x^2}{1.2}+\frac{x^4}{3.4}+\frac{x^6}{5.6}+\cdots\right\}$$

199 **(b)**
Let a be the first term and r be the common ratio of the G.P. $a_1, a_2, a_3, \ldots$
We have, $a_5=5\Rightarrow ar^4=2$
Now,
$a_1a_2a_3\ldots a_9=a\;ar\;ar^2\ldots ar^8$

$\Rightarrow a_1 a_2 a_3 \dots a_9 = a^9\, r^{1+2+\cdots+8}$

$\Rightarrow a_1 a_2 a_3 \dots a_9 = a^9\, r^{36} = (ar^4)^9 = 2^9 = 512$

200 **(a)**

We have,

$\log_3 x \times \log_x 2x \times \log_{2x} y = \log_x x^2$

$\Rightarrow \log_3 y = 2\log_x x \Rightarrow y = 3^2 = 9$

201 **(c)**

It is given that a, b, c are in H.P.

$\therefore \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

$\Rightarrow \frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c}$ are in A.P.

$\Rightarrow 1+\frac{b+c}{a}, 1+\frac{a+c}{b}, 1+\frac{a+b}{c}$ are in A.P.

$\Rightarrow \frac{b+c}{a}, \frac{a+c}{b}, \frac{a+b}{c}$ are in A.P.

$\Rightarrow \frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in H.P.

202 **(d)**

mth mean between $a, 2b$ is $a+\frac{m(2b-a)}{n+1}$ and mth mean between $2a, b$ is $2a+\frac{m(b-2a)}{n+1}$.

According to given condition

$$a+\frac{m(2b-a)}{n+1} = 2a+\frac{m(b-2a)}{n+1}$$

$\Rightarrow\ m(2b-a) = a(n+1)+m(b-2a)$

$\Rightarrow\ a(n-m+1) = bm$

$\Rightarrow \frac{a}{b} = \frac{m}{n-m+1}$

203 **(c)**

We have,

$$\lim_{n\to\infty} S_n = \sum_{n=1}^{\infty} \frac{n^2(n+1)}{n\,!}$$

$$\Rightarrow \lim_{n\to\infty} S_n = \sum_{n=1}^{\infty}\frac{n^3}{n\,!}+\sum_{n=1}^{\infty}\frac{n^2}{n\,!} = 5e+2e = 7e$$

204 **(a)**

Let the numbers are a and b

$\therefore \sqrt{ab} = 10$

$\Rightarrow\ ab = 100$ ...(i)

and $\frac{2ab}{a+b} = 8$

$\Rightarrow \frac{200}{a+b} = 8$

$\Rightarrow\ a+b = 25$ ...(ii)

On solving Eqs. (i) and (ii), we get

$a = 5, b = 20$

205 **(c)**

Let nth term of GP is 2187.

$\therefore\ 3(\sqrt{3})^{n-1} = 2187 \ \Rightarrow 3^{(n/2-1/2+1)} = 3^7$

$\Rightarrow \frac{n}{2}+\frac{1}{2} = 7 \Rightarrow n = 13$

206 **(b)**

Required two digit numbers are 12, 19, ...,96 which leave a remainder 5 when they are divided by 7.

Here, $a = 12, d = 7, l = 96$

$\therefore\quad l = a+(n-1)d$

$\Rightarrow\quad 96 = 12+7(n-1)$

$\Rightarrow\quad n = 13$

$\therefore\quad S_{13} = \frac{13}{2}(12+96) = \frac{13\times108}{2} = 702$

207 **(a)**

Let T_n denote the n^{th} term of the given series.

We have,

$$T_n = \frac{n^2(n+1)^2}{n\,!} = \frac{n^4}{n\,!}+2\cdot\frac{n^3}{n\,!}+\frac{n^2}{n\,!}$$

$\therefore$ Sum of the series

$$= \sum_{n=1}^{\infty}\frac{n^4}{n\,!}+2\sum_{n=1}^{\infty}\frac{n^3}{n\,!}+\sum_{n=1}^{\infty}\frac{n^2}{n\,!} = 15e+2(5e)+2e$$

$$= 27\,e$$

208 **(c)**

Sum of the integers which are divided by both 3 and 5

$= 15+30+45+\ldots+90$

$= \frac{6}{2}(15+90) = 315$

209 **(a)**

Since H is the harmonic mean between a and b

$\therefore H = \frac{2\,ab}{a+b}$

$\Rightarrow \frac{H}{a} = \frac{2\,b}{a+b}$ and $\frac{H}{b} = \frac{2\,a}{a+b}$

$\Rightarrow \frac{H}{a}+\frac{H}{b} = \frac{2\,b}{a+b}+\frac{2\,a}{a+b} = \frac{2(a+b)}{a+b} = 2$

210 **(b)**

Given that, $a_1 = a_2 = 2$ and $a_n = a_{n-1}-1$

$\therefore\ a_3 = a_2-1 = 2-1 = 1$

$a_4 = a_3-1 = 1-1 = 0$

$a_5 = a_4-1 = 0-1 = -1$

211 **(a)**

$0.2\overline{34} = 0.2343434\ \dots$

$= 0.2+0.034+0.00034+\cdots$

$= \frac{2}{10}+34\left[\frac{1}{10^3}+\frac{1}{10^5}+\ldots\right]$

$= \frac{2}{10}+34\times\frac{1}{1000}\times\frac{100}{99}$

$= \frac{2}{10}+\frac{34}{990} = \frac{232}{990}$

212 **(a)**

We have,

$\sqrt{\log_{0.5}^2 4} = \sqrt{(\log_{0.5} 4)^2}$

$= \log_{2^{-1}} 4 = \log_{2^{-1}} 2^2 = \frac{2}{-1}\log_2 2 = -2$

214 **(b)**

We have, 4, 14, 30, 52, 80, 114, … as the given sequence. The differences of the successive terms form an A.P.

So, let its n^{th} term be

$a_n = an^2 + bn + c$

Putting $n = 1,2,3$, we get

$a + b + c = 4, 4a + 2b + c = 14$ and,

$9a + 3b + c = 30$

Solving these equation, we get

$a = 3, b = 1$ and $c = 0$

$\therefore a_n = 3n^2 + n$

215 **(d)**

Given, $y = 3x + 6x^2 + 10x^3 + \ldots$

$\therefore 1 + y = 1 + 3x + 6x^2 + 10x^3 + \ldots$

$\Rightarrow 1 + y = (1 - x)^{-3}$

$\Rightarrow 1 - x = (1 + y)^{-1/3}$

$\Rightarrow x = 1 - (1 + y)^{-1/3}$

216 **(b)**

We have,

$\log_a(1 + x) = \log_e(1 + x) \cdot \log_a e$

$\log_a(1 + x) = \log_a e \left\{\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n\,!}\right\}$

$\therefore$ Coefficient of x^n in $\log_a(1 + x) = \frac{(-1)^{n-1}}{n\,!}\log_a e$

217 **(a)**

We have,

$$a_n = 1 + \left(\frac{1}{2} + \frac{1}{3}\right) + \left(\frac{1}{4} + \cdots + \frac{1}{7}\right) + \left(\frac{1}{8} + \cdots + \frac{1}{15}\right) + \cdots + \left(\frac{1}{2^{n-1}} + \cdots + \frac{1}{2^n - 1}\right)$$

$\Rightarrow a_n < 1 + \frac{2}{2} + \frac{4}{4} + \frac{8}{8} + \cdots + \frac{2^{n-1}}{2^{n-1}}$

$\Rightarrow a_n < n$

$a_{100} < 100$

218 **(c)**

Now, $e^2 = 1 + \frac{2}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \ldots$

and $e^{-2} = 1 - \frac{2}{1!} + \frac{2^2}{2!} - \frac{2^3}{3!} + \ldots$

$\Rightarrow e^2 + e^{-2} = 2\left[1 + \frac{2^2}{2!} + \frac{2^4}{4!} + \ldots\right]$

$\Rightarrow \frac{e^2 + e^{-2}}{2} - 1 = \left[\frac{2^2}{2!} + \frac{2^4}{4!} + \ldots\right]$

$\Rightarrow \frac{e^4 + 1 - 2e^2}{2e^2} = \left[\frac{2^2}{2!} + \frac{2^4}{4!} + \ldots\right]$

$\Rightarrow \frac{(e^2 - 1)^2}{2e^2} = \frac{2^2}{2!} + \frac{2^4}{4!} + \ldots$

219 **(a)**

Let the numbers in GP be $\frac{a}{r}, a, ar$

Given, $\frac{a}{r} + a + ar = 38$ …(i)

and $\frac{a}{r}. a. ar = 1728$

$\Rightarrow a^3 = 1728 \Rightarrow a = 12$ …(ii)

$\therefore$ From Eq. (i)

$a\left(\frac{1}{r} + 1 + r\right) = 38$

$\Rightarrow \frac{1}{r} + 1 + r = \frac{38}{12}$ [from Eq. (ii)]

$\Rightarrow 6r^2 - 13r + 6 = 0$

$\Rightarrow (3r - 2)(2r - 3) = 0 \Rightarrow r = \frac{2}{3}$ or $\frac{3}{2}$

Hence, required GP is 18, 12, 8 or 8, 12, 18

$\therefore$ Greatest number is 18.

220 **(c)**

We have,

$\frac{\log_2 a}{3} = \frac{\log_2 b}{4} = \frac{\log_2 c}{5\lambda} = k(\text{say})$

$\Rightarrow a = 2^{3k}, b = 2^{4k}$ and $c = 2^{5\lambda k}$

$\therefore a^{-3}b^{-4}c = 1$

$\Rightarrow 2^{-9k} \times 2^{-16k} \times 2^{5\lambda k} = 1$

$\Rightarrow 2^{5\lambda k - 25k} = 2^0 \Rightarrow 5\lambda k - 25k = 0 \Rightarrow \lambda = 5$

221 **(c)**

The nth term of 1,3,6,10, … is $\frac{n(n+1)}{2}$ and so

$(n - 1)^{\text{th}}$ term of 1,3,6,10, … is $\frac{n(n-1)}{2}$

Since terms in the nth group are

$$\left\{\frac{n(n-1)}{2} + 1\right\}^3, \left\{\frac{n(n-1)}{2} + 2\right\}^3, \ldots, \left\{\frac{n(n+1)}{2}\right\}^3$$

Required sum

= Sum of the cubes of first $\frac{n(n+1)}{2}$ natural numbers

− Sum of the cube of first $\frac{n(n-1)}{2}$ natural numbers

$$= \frac{\left\{\frac{n(n+1)}{2}\right\}^2 \left\{\frac{n(n+1)}{2} + 1\right\}^2}{4} - \frac{\left\{\frac{n(n-1)}{2}\right\}^2 \left\{\frac{n(n-1)}{2} + 1\right\}^2}{4}$$

$= \frac{n^3}{8}(n^2 + 1)(n^2 + 3)$

222 **(b)**

Let T_n be the n^{th} term of the given series

We have,

$T_n = \frac{1^2 + 2^2 + \cdots + n^2}{n\,!}$

$\Rightarrow T_n = \frac{n(n + 1)(2n + 1)}{6. n\,!}$

$\Rightarrow T_n = \frac{1}{6}\left\{\frac{2n^3 + 3n^2 + n}{n\,!}\right\}$

$$\Rightarrow T_n = \frac{1}{6}\left\{2\cdot\frac{n^2}{n\,!}+3\cdot\frac{n^2}{n\,!}+\frac{n}{n\,!}\right\}$$

$\therefore$ Sum of the series

$$= \frac{1}{6}\left\{2\sum_{n=1}^{\infty}\frac{n^3}{n\,!}+3\sum_{n=1}^{\infty}\frac{n^2}{n\,!}+\sum_{n=1}^{\infty}\frac{n}{n\,!}\right\}$$

Sum of the series $= \frac{1}{6}\{2\times 5e+3\times 2e+e\}$

$$= \frac{17\,e}{6}$$

223 **(d)**

$$\sum_{k=1}^{\infty}\frac{1}{k!}\left(\sum_{n=1}^{k}2^{n-1}\right)=\sum_{k=1}^{\infty}\frac{2^k-1}{k!}$$

$$\sum_{k=1}^{\infty}\frac{2^k}{k!}-\sum_{n=1}^{\infty}\frac{1}{k!}$$

$$= e^2-1-(e-1)=e^2-e$$

224 **(c)**

We have,

$a_n = S_n - S_{n-1}$ for all $n \geq 2$

$$\Rightarrow a_n = \frac{1}{6}[2\{n^3-(n-1)^3\}+9\{n^2-(n-1)^2\} + 13\{n-(n-1)\}]$$

$\Rightarrow a_n = (n+1)^2$

Also, $a_1 = S_1 = 4 = (1+1)^2$

$\therefore a_r = (r+1)^2$ for $r = 1,2,\ldots$

$$\Rightarrow \sum_{r=1}^{n}\sqrt{a_r}=\sum_{r=1}^{n}(r+1)=\frac{n}{2}(n+3)$$

225 **(c)**

Given $x = 1+a+a^2+\ldots\infty = \frac{1}{1-a}$

$y = 1+b+b^2+\ldots\infty = \frac{1}{1-b}$

and $z = 1+c+c^2+\ldots\infty = \frac{1}{1-c}$

Now, a, b, c are in AP.

$\Rightarrow 1-a, 1-b, 1-c$ are in AP.

$\Rightarrow \frac{1}{1-a},\frac{1}{1-b},\frac{1}{1-c}$ are in HP.

$\Rightarrow x, y, z$ are in HP

226 **(c)**

Clearly, it is a G.P. with first term $a = \frac{10}{9}$ and common ration $r = \sqrt{\frac{3}{5}}$

$$\therefore T_5 = ar^4 = \frac{10}{9}\times\left(\sqrt{\frac{3}{5}}\right)^4 = \frac{10}{9}\times\frac{9}{25}=\frac{2}{5}$$

227 **(b)**

Since $x, |x+1|$ and $|x-1|$ are in A.P. Therefore

$2|x+1| = x+|x-1|$

Now, three cases arise

<u>CASE I</u> $When\ x \geq 1$

In this case, we have

$|x+1| = x+1$ and $|x-1| = x-1$

$\therefore 2|x+1| = x+|x-1|$

$\Rightarrow 2(x+1) = x+(x-1) \Rightarrow 2 = -1$, which is absurd

<u>CASE II</u> $When\ x < -1$

In this case, we have

$|x+1| = -(x+1)$ and $|x-1| = -(x-1)$

$\therefore 2|x+1| = x+|x-1|$

$\Rightarrow -2(x+1) = x-(x-1) \Rightarrow -2x-2 = 1 \Rightarrow x = -\frac{3}{2}$

Thus, the three terms of the A.P. are $-\frac{3}{2},\frac{1}{2},\frac{5}{2}$

Clearly, common difference of the A.P. is 2 and first term is $-3/2$

$$\therefore \text{Sum of 20 terms} = \frac{20}{2}\left\{2\times-\frac{3}{2}+(20-1)+2\right\}$$

$= 10[-3+38] = 350$

CASE III $When -1 \leq x < 1$

In this case, we have

$|x+1| = x+1$ and $|x-1| = -(x-1)$

$\therefore 2|x+1| = x+|x-1|$

$\Rightarrow 2(x+1) = x-(x-1) \Rightarrow 2x+2 = 1 \Rightarrow x = -\frac{1}{2}$

So, first three terms of the A.P. are $-\frac{1}{2},\frac{1}{2}$ and $\frac{3}{2}$

In this case, we have

First term $= -\frac{1}{2}$ and, Common difference $= 1$

$$\therefore \text{Sum of 20 terms} = 10\left\{2\times-\frac{1}{2}+(20-1)\times 1\right\}$$

$= 180$

228 **(c)**

Let the numbers be a, ar the ar^2. Then the number obtained by adding the middle number are

$a+ar, 2ar, ar+ar^2$

Clearly, these numbers are neither in A.P. nor in G.P.

Now,

$$\frac{1}{2ar}-\frac{1}{a+ar}=\frac{a-ar}{2ar(a+ar)}=\frac{1-r}{2\,ar(1+r)}$$

and, $\frac{1}{ar+ar^2}-\frac{1}{2ar}=\frac{ar-ar^2}{2ar(ar+ar^2)}$

$$=\frac{1-r}{2\,ar(1+r)}$$

We have,

$$\frac{1}{2ar}-\frac{1}{a+ar}=\frac{1}{ar+ar^2}-\frac{1}{2ar}$$

So, $a+ar, 2ar, ar+ar^2$ are in H.P.

231 **(b)**

We have,

$\log(x+y) = \log 2 + \frac{1}{2}\log x + \frac{1}{2}\log y$

$\Rightarrow \log(x+y)^2 = \log(4\,xy)$

$\Rightarrow (x+y)^2 = 4xy \Rightarrow (x-y)^2 = 0 \Rightarrow x = y$

232 **(b)**

We have,

$x^{2\log_{10} x} = 1000\,x$

$\Rightarrow 2\log_{10} x = \log_x 1000\,x$

$\Rightarrow 2\log_{10} x = \log_x 10^3 + \log_x x$

$\Rightarrow 2\log_{10} x = 3\log_x 10 + 1$

$\Rightarrow 2y^2 - y - 3 = 0$ where $y = \log_{10} x$

$\Rightarrow (2y-3)(y+1) = 0$

$\Rightarrow y = \frac{3}{2}, -1$

$\Rightarrow \log_{10} x = \frac{3}{2}, -1 \Rightarrow x = 10^{3/2}, 10^{-1} \Rightarrow x$

$= 10\sqrt{10}, 10^{-1}$

233 **(b)**

We have,

$\log_b a \times \log_c b \times \log_a c$

$= (\log_b a \times \log_c b)\log_a c$

$= \log_c a \times \log_a c = \log_a a = 1$

234 **(b)**

Suppose that the added number be x, then $x+2, x+14, x+62$ are in GP.

$\therefore\ (x+14)^2 = (x+2)(x+62)$

$\Rightarrow\ x^2 + 196 + 28x = x^2 + 64x + 124$

$\Rightarrow\ 36x = 72\ \Rightarrow x = 2$

235 **(c)**

According to the given condition,

$S_{2n} = S_n$

$\Rightarrow \frac{2n}{2}[2 \times 2 + (2n-1) \times 3$

$= \frac{n}{2}[2 \times 57 + (n-1) \times 2]$

$\Rightarrow (4 + 6n - 3) = \frac{1}{2}(114 + 2n - 2)$

$\Rightarrow 6n + 1 = 57 + n - 1$

$\Rightarrow 5n = 55$

$\Rightarrow n = 11$

236 **(d)**

Let $S = \frac{1}{3\times7} + \frac{1}{7\times11} + \frac{1}{11\times15} + \ldots\infty$

$= \frac{1}{4}\left[\left\{\frac{1}{3} - \frac{1}{7}\right\} + \left\{\frac{1}{7} - \frac{1}{11}\right\} + \ldots\right]$

$= \frac{1}{4}\left[\frac{1}{3} + 0\right] = \frac{1}{12}$

237 **(c)**

Let the first term and the common ratio be a and r respectively. Then,

$a_n = \frac{a_{n+1} + a_{n+2}}{2}$

$\Rightarrow ar^{n-1} = \frac{ar^n + ar^{n+1}}{2}$

$\Rightarrow 2 = r + r^2 \Rightarrow r^2 + r - 2 = 0$

$\Rightarrow (r+2)(r-1) = 0 \Rightarrow r = -2\ \ [\because r \neq 1]$

238 **(a)**

From symmetry, we observe that S_{50} has 50 terms. First term of $S_1, S_2, S_3, S_4, \ldots, S_{50}$ are 1, 2, 4, 7, ... , 50.

Let T_n be the first term of nth set. Then

$S = T_1 + T_2 + T_3 + \ldots + T_n$

$\Rightarrow\ S = 1 + 2 + 4 + 7 + 11 + \ldots + T_{n-1} + T_n$

or $S = 1 + 2 + 4 + 7 + \ldots\quad + T_{n-1} + T_n$

therefore, on subtracting

$0 = 1 + [1 + 2 + 3 + 4 + \ldots + (T_n - T_{n-1})] - T_n$

or $0 = 1 + \frac{n(n-1)}{2} - T_n$

$\Rightarrow\ T_n = 1 + \frac{n(n-1)}{2}$

$\Rightarrow\ T_{50} =$ First term in $S_{50} = 1226$

Therefore, sum of the terms in S_{50}

$= \frac{50}{2}[2 \times 1226 + (50-1) \times 1]$

$= 25(2452 + 49) = 25(2501) = 62525$

239 **(a)**

Since, a, b, c are in H.P.

$\therefore b = \frac{2\,ac}{a+c}$

$\Rightarrow \frac{a-b}{b-c} = \frac{a - \frac{2\,ac}{a+c}}{\frac{2\,ac}{a+c} - c} = \frac{a^2 - ac}{ac - c^2} = \frac{a}{c}$

240 **(b)**

Since, a, b and c are in AP.

Let d be the common difference.

$\therefore\ \ a = b - d, b = d, c = b + d$

Also, $abc = 4$

$\Rightarrow\ (b-d)d(b+d) = 4$

$\Rightarrow (b^2 - d^2)b = 4$

$\Rightarrow b^3 = 4 + d^2 b$

$\Rightarrow\ b^3 \geq 4$

$\Rightarrow b \geq (2)^{2/3}$

241 **(a)**

Let α and β be the roots of equation $x^2 - 18x + 9 = 0$.

$\therefore$ GM of α and $\beta = \sqrt{\alpha\beta} = \sqrt{9} = 3$

243 **(a)**

We have,

$\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots = \frac{1/4}{1 - 1/2} = \frac{1}{2}$

let $y = 0.2^{\log_{\sqrt{5}}\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots\right)}$. Then,

$$y = 0.2^{\log_{\sqrt{5}}\left(\frac{1}{2}\right)} = \left(\frac{1}{5}\right)^{\log_{\sqrt{5}} 2^{-1}} = 5^{-1\times -2\log_5 2}$$
$$= 5^{\log_5 4} = 4$$

244 **(c)**

$(32)(32)^{1/6}(32)^{1/36} \ldots = 32^{1+\frac{1}{6}+\frac{1}{36}+\ldots} = 32^{\frac{1}{1-1/6}}$

$= (2^5)^{6/5} = 64$

245 **(c)**

We have,

$\log_4 2 + \log_4 4 + \log_4 16 + \log_4 x = 6$

$\Rightarrow \log_4(2\times 4\times 16\times x) = 6$

$\Rightarrow \log_4 128x = 6 \Rightarrow 128x = 4^6 \Rightarrow x = \frac{4^6}{128} = 32$

246 **(c)**

$x, 1, z$ are in AP, then $2 = x + z$ ...(i)

And $x, 2, z$ are in GP, then $4 = xz$ ...(ii)

Divide Eq.(ii) by Eq.(i), we get

$\frac{xz}{x+z} = \frac{4}{2} \Rightarrow \frac{2xz}{x+z} = 4$

Hence, $x, 4, z$ will be in HP.

249 **(c)**

It is given that

a, b, c are in A.P. $\Rightarrow 2b = a + c$ ... (i)

a, x, b are in G.P. $\Rightarrow x^2 = ab$ ... (ii)

b, y, c are in G.P. $\Rightarrow y^2 = cb$ ... (iii)

From (i) and (iii), we have

$y^2 = (2b - a)b$

$\Rightarrow y^2 = 2b^2 - ab$

$\Rightarrow y^2 = 2b^2 - x^2$ [Using (ii)]

$\Rightarrow x^2 + y^2 = 2b^2 \Rightarrow x^2, b^2, y^2$ are in A.P.

250 **(c)**

We have,

$$S = \sum_{n=0}^{\infty} \frac{(\log x)^{2n}}{(2\,n)\,!} = \left(\frac{e^{\log x} + e^{-\log x}}{2}\right) = \frac{x + x^{-1}}{2}$$

251 **(c)**

We have,

$$\sum_{r=1}^{n} \log\left(\frac{a^r}{b^{r-1}}\right)$$
$$= \frac{n}{2}\left\{\log a + \log\left(\frac{a^n}{b^{n-1}}\right)\right\} = \frac{n}{2}\log\left(\frac{a^{n+1}}{b^{n-1}}\right)$$

253 **(c)**

$\log_e \frac{1+3x}{1-2x} = \log_e(1+3x) - \log_e(1-2x)$

$$= \left[3x - \frac{(3x)^2}{2} + \frac{(3x)^3}{3} - \ldots\right] + \left[2x + \frac{(2x)^2}{2} + \frac{(2x)^3}{3} + \ldots\right]$$
$$= 5x - \frac{5x^2}{2} + \frac{35x^3}{3} - \ldots$$

254 **(d)**

We have,

$\log_7\{\log_4(\sqrt{x+5} + \sqrt{x})\} = 0$

$\Rightarrow \log_4(\sqrt{x+4} + \sqrt{x}) = 1$

$\Rightarrow \sqrt{x+4} + \sqrt{x} = 4^1$

$\Rightarrow \sqrt{x+4} = 4 - \sqrt{x}$

$\Rightarrow x + 4 = 16 - 8\sqrt{x} + x \Rightarrow 2\sqrt{x} = 3 \Rightarrow x = \frac{9}{4}$

255 **(c)**

We have,

$2^x \times 9^{2x+3} = 7^{x+5}$

$\Rightarrow 2^x \times 9^{2x} \times 9^3 = 7^x \times 7^5$

$\Rightarrow \log(2^x \times 3^{4x} \times 3^6) = \log(7^x \times 7^5)$

$\Rightarrow x\log 2 + 4x\log 3 + 6\log 3 = x\log 7 + 5\log 7$

$\Rightarrow x(\log 2 + 4\log 3 - \log 7) = 5\log 7 - 6\log 3$

$$\Rightarrow x = \frac{5\log 7 - 6\log 3}{\log 2 + 4\log 3 - \log 7} = \frac{5\log 7 - 6\log 3}{\log 162 - \log 7}$$

256 **(b)**

Since, $ar^4 = 2$

$$\therefore\ a \times ar \times ar^2 \times ar^3 \times ar^4 \times ar^5 \times ar^6 \times ar^7 \times ar^8$$
$$= a^9 r^{36} = (ar^4)^9 = 2^9 = 512$$

257 **(b)**

Given, $b = \frac{a^1}{1} + \frac{a^2}{2} + \frac{a^3}{3} + \ldots\infty$

$\Rightarrow\ b = -\log(1-a)$ $[\because |a| < 1]$

$\Rightarrow\ e^{-b} = (1-a)$

$$\Rightarrow\ a = \frac{b}{1!} - \frac{b^2}{2!} + \frac{b^3}{3!} - \ldots\infty$$
$$= \sum_{k=1}^{\infty} \frac{(-1)^{k-1} b^k}{k!}$$

258 **(b)**

Let t_n be the n^{th} term of the series

$4 + 11 + 22 + 37 + 56 + ..$

Since the differences of the successive terms in this series are in A.P. So, let

$t_n = an^2 + bn + c$

Putting $n = 1,2,3$, we get

$a + b + c = 4, 4a + 2b + c = 11$ and $9a + 3b + c = 22$

Solving these equations, we obtain

$a = 2, b = 1$ and $c = 1$

$\therefore t_n = 2n^2 + n + 1, n = 1,2, \ldots$

$\therefore$ Sum of the series

$$= \sum_{n=1}^{\infty} \frac{2n^2 + n + 1}{n\,!}$$

$$= 2\sum_{n=1}^{\infty}\frac{n^2}{n\,!}+\sum_{n=1}^{\infty}\frac{n}{n\,!}+\sum_{n=1}^{\infty}\frac{1}{n\,!}$$
$$= 2(2e)+e+(e-1)=6e-1$$

259 **(c)**

We have,

$\frac{a+b}{1-ab}, b, \frac{b+c}{1-bc}$ are in A. P.

$$\Rightarrow b-\frac{a+b}{1-ab}=\frac{b+c}{1-bc}-b$$
$$\Rightarrow -\frac{a(b^2+1)}{1-ab}=\frac{c(b^2+1)}{1-bc}$$
$$\Rightarrow -\left(\frac{1-ab}{a}\right)=\frac{1-bc}{c}$$
$$\Rightarrow -\frac{1}{a}+b=\frac{1}{c}-b$$
$\Rightarrow 2\,b=\frac{1}{a}+\frac{1}{c}\Rightarrow a,\frac{1}{b},c$ are in HP.

260 **(b)**

Since, $\log 2, \log(2^n-1)$ and $\log(2^n+3)$ are in AP.

$\therefore\ 2\log(2^n-1)=\log 2+\log(2^n+3)$

$\Rightarrow\ (2^n-1)^2=2(2^n+3)$

$\Rightarrow\ (2^n-5)(2^n+1)=0$

As 2^n cannot be negative hence, $2^n-5=0$

$\Rightarrow\ 2^n=5\Rightarrow n=\log_2 5$

261 **(a)**

We have,

$$1+\frac{1+a}{2\,!}+\frac{1+a+a^2}{3\,!}+\frac{1+a+a^2+a^3}{4\,!}+\cdots\infty$$
$$=\sum_{n=1}^{\infty}\frac{1+a+a^2+\cdots+a^{n-1}}{n\,!}$$
$$=\sum_{n=1}^{\infty}\frac{1}{n\,!}\left(\frac{1-a^n}{1-a}\right)$$
$$=\frac{1}{(1-a)}\left\{\sum_{n=1}^{\infty}\frac{1}{n\,!}-\sum_{n=1}^{\infty}\frac{a^n}{n\,!}\right\}$$
$$=\frac{1}{1-a}\{(e-1)-(e^a-1)\}=\frac{e-e^a}{1-a}=\frac{e^a-e}{a-1}$$

262 **(b)**

Let $S_1=5+9+13+\ldots+n$ terms

$$\Rightarrow\ S_1=\frac{n}{2}[2\times5+(n-1)4]=n(3+2n)$$

and $S_2=7+9+11+\ldots+12$ terms

$$=\frac{12}{1}[2\times7+(12-1)2]=6(36)$$
$$=216$$

Since, $\frac{S_1}{S_2}=\frac{5}{12}$ (given)

$$\Rightarrow\frac{n(3+2n)}{216}=\frac{5}{12}$$
$$\Rightarrow 2n^2+3n-90=0$$
$$\Rightarrow(2n+15)(n-6)=0$$

$\Rightarrow n=6$ ($\because n$ cannot be negative)

263 **(a)**

Let $S=1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+\ldots$

$\Rightarrow S-1=\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}=\ldots$ ...(i)

$\Rightarrow\frac{S-1}{3}=\frac{2}{3^2}+\frac{6}{3^3}+\frac{10}{3^4}+\frac{14}{3^5}+\cdots$ ...(ii)

On subtracting Eq.(ii) from Eq. (i), we get

$$\frac{2}{3}(S-1)=\frac{2}{3}+\frac{4}{3^2}+\frac{4}{3^3}+\frac{4}{3^4}+\ldots$$
$$\Rightarrow S-1=1+\frac{2}{3}+\frac{2}{3^2}+\frac{2}{3^3}+\ldots$$
$$\Rightarrow\ S=2+\frac{3}{1-\frac{1}{3}}$$
$$=2+1=3$$

264 **(b)**

The nth term of the sequence is

$\ldots n(n+1)n\ldots321$

$$=1\times10^{2n}+2\times10^{2n-1}+3\times10^{2n-2}+\cdots+n\times10^{n+1}+(n+1)\times10^n\times10^{n-1}+\ldots+3\times10^2+2\times10+1$$

Let $S=1\times10^{2n}+2\times10^{2n-1}+\ldots+n\times10^n+1$ (AG series)

$$\frac{1}{10}S_1=1\times10^{2n-1}+\ldots+(n-1)\times10^{n-1}+n\times10^n$$

On solving, we get

$$\frac{9}{10}S_1=1\times10^{2n}+\ldots+1\times10^{n+1}-n\times10^n$$
$$S_1=\frac{10}{9}[10^{2n}+10^{2n-1}+\cdots+10^{n+1}-n\times10^n]$$
$$S_1=\frac{10}{9}\left[\frac{10^{2n}\left(1-\frac{1}{10^n}\right)}{1-\frac{1}{10}}-n\times10^n\right]$$
$$=\left(\frac{10}{9}\right)^2\left[10^{2n}\left(1-\frac{1}{10^n}\right)\right]-\left[\left(\frac{10}{9}\right)n\times10^n\right]$$

Again, let

$$S_2=(n+1)\times10^n+n\times10^{n-1}+\ldots+2\times10+1$$
$$\frac{1}{10}S_2=(n+1)\times10^{n-1}+\ldots+3\times10+2+\frac{1}{10}$$
$$\frac{9}{10}S_2=(n+1)\times10^n-\left[10^{n-1}+10^{n-2}+\ldots+1+\right.$$
$$=(n+1)\times10^n-\frac{10^{n-1}\left(1-\frac{1}{10^{n-1}}\right)}{1-\frac{1}{10}}$$
$$\Rightarrow\ S_2=\frac{10}{9}(n+1)\times10^n-\left(\frac{10}{9}\right)^2 10^{n-1}\left(1-\frac{1}{10^{n-1}}\right)$$

$\therefore$ The nth term $=S_1+S_2$

$= \left(\frac{10}{9}\right)^2 \left[10^{2n}\left(1-\frac{1}{10^{n-1}}\right) - 10^{n-1}\left(1-\frac{1}{10^{n-1}}\right)\right]$
$+\frac{10}{9}[-n \times 10^n + (n+1) \times 10^n]$
$= \left(\frac{10}{9}\right)^2 \left[10^{2n} - 2.10^{n-1} + \frac{1}{10^2}\right]$
$= \frac{10}{9}\left[10^n - \frac{1}{10}\right]^2$
$= \left[\frac{10^{n-1}-1}{9}\right]^2$
$= \left[\frac{999 \ldots (n+1)\text{times}}{9}\right]^2$
$= [111 \ldots (n+1)\text{times}]^2 = (\text{odd number})^2$

266 **(a)**

Let common difference $d_1 = -3$ and first term be a.

$\therefore$ Series become

$a, a-3, a-6, \ldots, a-27$

$\therefore S = 10a + (-3-6-\ldots-27)$

$\Rightarrow -30 = 10a - 3(1+2+\ldots+9)$

$\Rightarrow -30 = 10a - 3\left[\frac{9(9+1)}{2}\right]$

$\Rightarrow -30 = 10a - 135$

$\Rightarrow 10a = 105 \quad \Rightarrow a = \frac{105}{10}$

Now, correct common difference $d_2 = 3$

$\therefore$ Required sum $= \frac{10}{2}\left[2 \times \frac{105}{10} + (10-1)3\right]$

$= 5\left(\frac{105}{5} + 27\right) = 5 \times 48 = 240$

267 **(b)**

Let sum of $2n$ terms of the AP 2, 5, 8, 11, … is S_{2n}

$\therefore S_{2n} = \frac{2n}{2}[2 \times 2 + (2n-1)3]$
$= n(4 + 6n - 3)$
$= n(6n+1)$

And sum of n terms of the AP 57, 59, 61, 63,… is S_n

$\therefore \quad S_n = \frac{n}{2}[2 \times 57 + (n-1)2]$
$= \frac{n}{2}(2n + 112)$

According to question, $S_{2n} = S_n$

$\Rightarrow n(6n+1) = \frac{n}{2}(2n+112)$
$\Rightarrow 12n + 2 = 2n + 112$
$\Rightarrow 10n = 110$
$\Rightarrow n = 11$

268 **(a)**

$\frac{x^{n+1}+y^{n+1}}{x^n+y^n} = \sqrt{xy}$
$\Rightarrow x^{n+1} + y^{n+1} = (xy)^{\frac{1}{2}}(x^n + y^n)$
$\Rightarrow x^n.x + y^n.y = x^n.x^{\frac{1}{2}}y^{\frac{1}{2}} + y^n.x^{\frac{1}{2}}y^{\frac{1}{2}}$
$\Rightarrow x^n(x - \sqrt{xy}) + y^n(y - \sqrt{xy}) = 0$
$\Rightarrow x^n.\sqrt{x}(\sqrt{x} - \sqrt{y}) + \sqrt{y}.y^n(\sqrt{y} - \sqrt{x}) = 0$
$\Rightarrow (\sqrt{x} - \sqrt{y})(x^n.\sqrt{x} - y^n.\sqrt{y}) = 0$

For $x \neq y$

$x^n\sqrt{x} = y^n\sqrt{y}$
$\Rightarrow \left(\frac{x}{y}\right)^{n+\frac{1}{2}} = 1$
$\Rightarrow n + \frac{1}{2} = 0$
$\Rightarrow n = -\frac{1}{2}$

270 **(d)**

We have,

$2\log_8 a = x, \log_2 2a = y$ and $y - x = 4$
$\Rightarrow \frac{2}{3}\log_2 a = x$ and $\log_2 2 + \log_2 a = y$ and $y - x = 4$
$\Rightarrow 1 + \frac{3}{2}x = y$ and $y - x = 4 \Rightarrow x = 6$

271 **(d)**

Given that, $x_1x_2x_3 \ldots x_n = 1 \quad \ldots$(i)

We know that, AM $\geq$ GM

$\therefore \quad \left(\frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}\right) \geq (x_1x_2x_3 \ldots x_n)^{1/n}$
$= (1)^{1/n} = 1 \quad$ [from Eq.(i)]

$\Rightarrow\ x_1+x_2+x_3+\ldots+x_n \geq n$

$\therefore\ x_1+x_2+x_3+\ldots+x_n$

Can never be less than n

272 **(c)**

$9^{1/3}\times 9^{1/9}\times 9^{1/27}\times \ldots\infty=(9)^{\left[\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots\infty\right]}$

$=(9)^{\frac{1}{3}\left(\frac{1}{1-1/3}\right)}=9^{1/2}$

$=3$

273 **(a)**

Here, $T_n=n(2n+1)^2=4n^3+4n^2+n$

$\therefore S_n=\Sigma T_n=\Sigma(4n^3+4n^2+n)$

$=4\Sigma n^3+4\Sigma n^2+\Sigma n$

$=4\left\{\frac{n}{2}(n+1)\right\}^2+\frac{4}{6}n(n+1)(2n+1)+\frac{n}{2}(n+1)$

$=n(n+1)\left[n^2+n+\frac{4}{6}(2n+1)+\frac{1}{2}\right]$

$=\frac{n}{6}(n+1)(6n^2+14n+7)$

274 **(d)**

$\because\quad \text{AM}=\dfrac{\cos^2 x+\sec^2 x}{2}$

and GM$=\sqrt{\cos^2 x.\sec^2 x=1}$

We know that, AM$\geq$ GM

$\therefore\ \dfrac{\cos^2 x+\sec^2 x}{2}\geq 1$

$\Rightarrow \cos^2 x+\sec^2 x\geq 2\Rightarrow f(x)\geq 2$

275 **(b)**

We have,

$\log_7\{\log_5(\sqrt{x+5}+\sqrt{x})\}=0$

$\Rightarrow \log_5(\sqrt{x+5}+\sqrt{x})=7^0$

$\Rightarrow \sqrt{x+5}+\sqrt{x}=5$

$\Rightarrow \sqrt{x+5}=5-\sqrt{x}$

$\Rightarrow x+5=25+x-2\times 5\times\sqrt{x}$

$\Rightarrow 0=20-10\sqrt{x}\Rightarrow x=4$

276 **(b)**

We have,

a,b,c are in A.P. $\Rightarrow 2\,b=a+c$ ...(i)

b,c,d are in G.P. $\Rightarrow c^2=bd$ ...(ii)

c,d,e are in H.P. $\Rightarrow d=\frac{2\,ce}{c+e}$ ...(iii)

From (i), (ii) and (iii), we have

$c^2=\dfrac{a+c}{2}\times\dfrac{2\,ce}{c+e}$

$\Rightarrow c(c+e)=(a+c)e$

$\Rightarrow c^2+ce=ae+ce\Rightarrow c^2=ae\Rightarrow a,c,e$ are in G.P.

277 **(d)**

$\log_4 2-\log_8 2+\log_{16} 2-\log_{32} 2+\ldots$

$=\log_{2^2} 2-\log_{2^3} 2+\log_{2^4} 2-\log_{2^5} 2+\ldots$

$=\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}+\ldots+1-1$

$=1-\left(1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\ldots\right)=1-\log_e 2$

278 **(a)**

Since, $y=1+x+x^2+x^3+\cdots=\frac{1}{1-x}$

$\therefore y-yx=1\ \Rightarrow\ x=\dfrac{y-1}{y}$

279 **(a)**

We have,

$(1-x)(1-2x)(1-2^2\,x)(1-2^3\,x)\ldots(1-2^{15}\,x)$

$=2\,.\,2^2.\,2^3\ldots 2^{15}(x-1)\left(x-\frac{1}{2}\right)\left(x-\frac{1}{2^2}\right)\ldots\left(x-\frac{1}{2^{15}}\right)$

$=2^{120}\left\{x^{16}-x^{15}\left(1+\frac{1}{2}+\cdots+\frac{1}{2^{15}}\right)+\cdots\right\}$

$\therefore$ Coefficient of x^{15}

$=-2^{120}(1+2^{-1}+2^{-2}+\cdots+2^{-15})$

$=-2^{120}\left\{\dfrac{1-\left(\frac{1}{2}\right)^{16}}{1-\frac{1}{2}}\right\}=-2^{121}\left(1-\frac{1}{2^{16}}\right)$

$=-2^{121}+2^{105}$

280 **(c)**

We have,

$\log_{10} 5=x$

$\Rightarrow \log_{10}\left(\frac{10}{2}\right)=x$

$\Rightarrow \log_{10} 10-\log_{10} 2=x$

$\Rightarrow 1-\log_{10} 2=x\Rightarrow \log_{10} 2=1-x$

Now,

$\log_5 1250=\log_5 5^4\times 2$

$=4\log_5 5+\log_5 2=4+\log_5 2$

$=4+\dfrac{1}{\log_2 5}=4+\dfrac{1}{\log_2\frac{10}{2}}=4+\dfrac{1}{\log_2 10-\log_2 2}$

$=4+\dfrac{1}{\frac{1}{1-x}-1}=4+\dfrac{1-x}{x}=\dfrac{1+3x}{x}=3+\dfrac{1}{x}$

281 **(a)**

Let $a^{1/x}=b^{1/y}=c^{1/z}=k$

$\Rightarrow a=k^x, b=k^y, c=k^z$

Now, a,b,c are in GP.

$\Rightarrow\ b^2=ac\ \Rightarrow k^{2y}=k^x\cdot k^z=k^{x+z}$

$\Rightarrow\ 2y=x+z$

$\therefore x,y,z$ are in AP.

282 **(b)**

We have, $\sum_{n=1}^{\infty}\frac{2n}{(2n+1)!}=\sum_{n=1}^{\infty}\frac{2n+1-1}{(2n+1)!}$

$$= \sum_{n=1}^{\infty} \left(\frac{1}{(2n)!} - \frac{1}{(2n+1)!} \right)$$

$$\sum_{n=1}^{\infty} \frac{1}{(2n)!} - \sum_{n=1}^{\infty} \frac{1}{(2n+1)!}$$

$$= \left[\frac{e + e^{-1}}{2} - 1 \right] - \left[\frac{e - e^{-1}}{2} - 1 \right] = e^{-1}$$

283 **(b)**

We have,

$$e^x = y + \sqrt{1 + y^2}$$

$$\Rightarrow (e^x - y)^2 = y^2 + 1$$

$$\Rightarrow e^{2x} - 2y e^x = 1 \Rightarrow y = \frac{e^{2x} - 1}{2e^x}$$

$$= \frac{1}{2}(e^x - e^{-x})$$

284 **(b)**

Let AM = 15, GM=12

We know, $(\text{GM})^2 = \text{AM} \times \text{HM}$

$$\Rightarrow \text{HM} = \frac{144}{15}$$

286 **(b)**

Here, $T_n = \frac{2n}{(2n+1)!} = \frac{1}{2n!} - \frac{1}{(2n+1)!}$

$$\sum T_n = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \dots$$

$$= 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \dots$$

$$= e^{-1}$$

287 **(c)**

We have,

$$y + \frac{y^3}{3} + \frac{y^5}{5} + \dots = \infty\, 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \dots \infty \right)$$

$$\Rightarrow \frac{1}{2} \log\left(\frac{1+y}{1-y} \right) = \log\left(\frac{1+x}{1-x} \right)$$

$$\Rightarrow \log\left(\frac{1+y}{1-y} \right) = \log\left(\frac{1+x}{1-x} \right)^2$$

$$\Rightarrow \frac{1+y}{1-y} = \frac{(1+x)^2}{(1-x)^2}$$

$$\Rightarrow \frac{2}{2y} = \frac{(1+x)^2 + (1-x)^2}{(1+x)^2 - (1-x)^2}$$

$$\Rightarrow \frac{1}{y} = \frac{2(1+x^2)}{4x}$$

$$\Rightarrow y = \frac{2x}{1+x^2} \Rightarrow x^2 y = 2x - y$$

288 **(c)**

Let the first three terms of an AP are $a - d, a$ and $a + d$

Since, $(a - d) + (a + d) = 12$

$\Rightarrow a = 6$

and $a(a - d) = 24$

$\Rightarrow 6 - d = 4$

$\Rightarrow d = 2$

$\therefore$ First term is $a - d = 4$

289 **(a)**

Given equation is $ax^2 + bx + c = 0$ and let the roots are α, β.

So $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$.

Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$$= \frac{b^2}{a^2} - \frac{2c}{a}$$

Now, $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2} = \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c^2}{a^2}} = \frac{b^2 - 2ac}{c^2}$

According to given condition,

$$\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

$$\Rightarrow -\frac{b}{a} = \frac{b^2 - 2ac}{c^2}$$

$$\Rightarrow -bc^2 = ab^2 - 2a^2c$$

Hence, $2a^2c = ab^2 + bc^2$

$\Rightarrow ab^2, ca^2, bc^2$ or bc^2, ca^2 be in AP.

291 **(c)**

Let a, ar, ar^2 are in GP $a, ar, ar^2 - 64$ are in AP, we get

$a(r^2 - 2r + 1) = 64$ ...(i)

Again, $a, ar - 8, ar^2 - 64$ are in GP.

$\therefore (ar - 8)^2 = a(ar^2 - 64)$

$\Rightarrow a(16r - 64) = 64$ ...(ii)

On solving Eqs. (i) and (ii), we get $r = 5, a = 4$

Thus, required numbers are 4, 20, 100

293 **(c)**

We have,

$$y = 2^{\frac{1}{\log_x 8}} \Rightarrow y = 2^{\log_8 x}$$

$$\Rightarrow y = 2^{\log_{2^3} x} = 2^{(1/3)\log_2 x} = 2^{\log_2 x^{1/3}} = x^{1/3}$$

$$\Rightarrow y^3 = x$$

294 **(a)**

It is given that a, b, c are in A.P. and $(b - a), (c - b, a$ are in G.P.

$\therefore 2b = a + c$

And,

$(c - b)^2 = (b - a)a$

$\Rightarrow (b - a)^2 = (b - a)a$ $[2b = a + c \Rightarrow b - a = c - b]$

$\Rightarrow b = 2a$

$\Rightarrow c = 3a$ [Using : $2b = a + c$]

$\Rightarrow a : b : c = 1 : 2 : 3$

295 **(b)**

Let the two numbers be a and b

Since, $\frac{2ab}{a+b}=4,\ \ A=\frac{a+b}{2}$

and $G=\sqrt{ab}$

$\therefore\ \frac{2ab}{2A}=4\ \Rightarrow A=\frac{ab}{4}$

and $G^2=4A$

Given, $2A+G^2=27$

$\therefore\ 2A+4A=27$

$\Rightarrow\ A=\frac{9}{2}$

296 **(c)**

Given, $S_n=1^3+2^3+\ldots+n^3=\Sigma n^3$

and $T_n=1+2+\ldots+n=\Sigma n$

$\because S_n=\Sigma n^3=\left[\frac{n(n+1)}{2}\right]\Rightarrow S_n=\{\Sigma n\}^2=T_n^2$

297 **(d)**

$\because a_1,a_2,\ldots,a_{n+1}$ are in AP and common difference $=d$

Let $S=\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+\ldots+\frac{1}{a_na_{n+1}}$

$=\frac{1}{d}\left\{\frac{a_2-a_1}{a_1a_2}+\frac{a_3-a_2}{a_2a_3}+\ldots+\frac{a_{n+1}-a_n}{a_na_{n+1}}\right\}$

$=\frac{1}{d}\left\{\frac{1}{a_1}-\frac{1}{a_2}+\frac{1}{a_2}-\frac{1}{a_3}+\ldots+\frac{1}{a_n}-\frac{1}{a_{n+1}}\right\}$

$=\frac{1}{d}\left\{\frac{a_{n+1}-a_1}{a_1a_{n+1}}\right\}$

$=\frac{nd}{d(a_1a_{n+1})}=\frac{n}{a_1a_{n+1}}$

298 **(a)**

We have,

d,e,f are in G.P. $\Rightarrow e^2=df$ ... (i)

Now,

$dx^2+2ex+f=0$

$\Rightarrow dx^2+2\sqrt{df}x+f=0$ [Using (i)]

$\Rightarrow\left(\sqrt{d}\,x+\sqrt{f}\right)^2=0\Rightarrow x=-\frac{\sqrt{f}}{\sqrt{d}}$

Putting $x=-\frac{\sqrt{f}}{\sqrt{d}}$ in $ax^2+2bx+c=0$, we get

$a\frac{f}{d}+c=2b\sqrt{\frac{f}{d}}$ ... (ii)

$\Rightarrow\frac{a}{b}+\frac{c}{f}=\frac{2\,b}{\sqrt{fd}}$

$\Rightarrow\frac{a}{d}+\frac{c}{f}=\frac{2\,b}{e}$ [Using (i)]

$\Rightarrow\frac{d}{a},\frac{e}{b},\frac{f}{c}$ are in H.P.

Again from (ii), we have

$af+cd=2\,b\sqrt{fd}$

$\Rightarrow aef+ced=2\,be\sqrt{fd}\Rightarrow aef+cde$

$=2\,b\,df$ [Using (i)]

299 **(c)**

Since, a,b,c be in HP.

Then, $b=\frac{2ac}{a+c}$

GM between a and $c=\sqrt{ac}$

We know, GM > HM

$\Rightarrow\sqrt{ac}>b$ or $ac>b^2$ ...(i)

Similarly, $\sqrt{bd}>c$ or $bd>c^2$ ...(ii)

On adding relations (i) and (ii), we get

$ac+bd>b^2+c^2$

300 **(b)**

In an AP, the distance of one of them from the beginning is same as that of the other from the end is equal to the sum of first and last terms.

301 **(c)**

Since, $S_\infty=\frac{x}{1-r}=5\quad\Rightarrow r=\frac{5-x}{x}$ [thus $|r|<1$]

$\Rightarrow -1<\frac{5-x}{5}<1\ \Rightarrow\ 0<x<10$

302 **(a)**

For $n\geq 1$, we have

$a_n=\sum_{r=1}^{n}a_r-\sum_{r=1}^{n-1}a_r$

$=\frac{1}{6}n(n+1)(n+2)$

$-\frac{1}{6}(n-1)(n)(n+1)$

$\Rightarrow a_n=\frac{n(n+1)}{2}$

$\therefore\sum_{r=1}^{n}\frac{1}{a_r}=2\sum_{r=1}^{n}\frac{1}{r(r+1)}=2\sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{r+1}\right)$

$\Rightarrow\sum_{r=1}^{n}\frac{1}{a_r}=2\left(1-\frac{1}{n+1}\right)$

$\Rightarrow\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{a_r}=\lim_{n\to\infty}2\left(1+\frac{1}{n+1}\right)=2$

303 **(c)**

We have,

$S_n=\frac{1}{1^3}+\frac{1+2}{1^3+2^3}+\frac{1+2+3}{1^3+2^3+3^3}+\cdots$

$+\frac{1+2+3+\cdots+n}{1^3+2^3+3^3+\cdots+n^3}$

or, $S_n=a_1+a_2+a_3+\cdots+a_n$, where

$$a_n = \frac{1+2+\cdots+n}{1^3+2^3+\cdots+n^3} = \frac{\frac{n(n+1)}{2}}{\left\{\frac{n(n+1)}{2}\right\}^2}$$

$$\Rightarrow a_n = \frac{2}{n(n+1)} = \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$\therefore S_n = 2\left(\frac{1}{1}-\frac{1}{2}\right) + 2\left(\frac{1}{2}-\frac{1}{3}\right) + 2\left(\frac{1}{3}-\frac{1}{4}\right) + \cdots + 2\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$= 2\left(1 - \frac{1}{n+1}\right) = 2 - \frac{2}{n+1} < 2$$

304 **(d)**

We have,

$$\log_4 2 - \log_8 2 + \log_{16} 2 - \log_{32} 2 + \ldots$$

$$= \frac{1}{\log_2 4} - \frac{1}{\log_2 8} + \frac{1}{\log_2 16} - \frac{1}{\log_2 32} + \cdots$$

$$= \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \cdots$$

$$= 1 - \left(\frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots\right) = 1 - \log_e 2$$

305 **(a)**

Since, the first 11 terms are in AP, $d = 2$

$\therefore\ a_{11} = a + 10d = a + 20$

The middle term of AP is

$T_6 = a + 5d = a + 10$

For the next 11 terms in GP,

$r = 2$

$\therefore$ The middle term of GP is $b(2)^5$ where b is the first term of a GP which is the last term of AP.

$\therefore\ b(2)^5 = (a+20)32$

According to the given condition,

$a + 10 = (a+20)32$

$\Rightarrow 31a = 10 - 640$

$$\Rightarrow a = -\frac{630}{31}$$

$\therefore$ Middle term of entire sequence is 11th term.

$$\therefore\ T_{11} = -\frac{630}{31} + 10 \times d$$

$$= -\frac{630}{31} + 10 \times 2 = -\frac{10}{31}$$

306 **(b)**

We have,

$\log_{12} 27 = a$

$\Rightarrow \log_{12} 3^3 = a$

$\Rightarrow 3\log_{12} 3 = a$

$$\Rightarrow \frac{3}{a} = \log_3 12$$

$$\Rightarrow \frac{3}{a} = \log_3(2^2 \times 3) = 2\log_3 2 + \log_3 3$$

$$\Rightarrow \frac{3}{a} = 2\log_3 2 + 1$$

$$\Rightarrow \frac{3-a}{a} = 2\log_3 2 \Rightarrow \log_2 3 = \frac{2a}{3-a} \quad \ldots \text{(i)}$$

Now,

$$\log_6 16 = \log_6 2^4 = 4\log_6 2 = \frac{4}{\log_2 6}$$

$$\Rightarrow \log_6 16 = \frac{4}{\log_2 3 + \log_2 2}$$

$$= \frac{4}{\frac{2a}{3-a} + 1} \quad \text{[Using (i)]}$$

$$\Rightarrow \log_6 16 = 4\left(\frac{3-a}{3+a}\right)$$

307 **(c)**

We have,

$x = \log_3 5$ and $y = \log_{17} 25$

$\Rightarrow x = \log_3 5$ and $y = 2\log_{17} 5$

$\Rightarrow \frac{1}{x} = \log_5 3$ and $\frac{1}{y} = \frac{1}{2}\log_5 17$

$\Rightarrow \frac{1}{x} = \frac{1}{2}\log_5 9$ and $\frac{1}{2}y = \frac{1}{2}\log_5 17$

$\Rightarrow \frac{1}{y} > \frac{1}{x} \Rightarrow x > y$

308 **(b)**

Let $a, ar, ar^2, \ldots, ar^{m-1}$ be the given G.P. Then, the required sum S is given by

$$S = \frac{1}{2}[(a + ar^2 + \cdots + ar^{m-1})^2 - (a^2 + a^2r^2 + a^2r^4 + \cdots + a^2r^{2m-2})]$$

$$\Rightarrow S = \frac{1}{2}\left\{a^2\left(\frac{1-r^m}{1-r}\right)^2 - a^2\left(\frac{1-r^{2m}}{1-r^2}\right)\right\}$$

$$\Rightarrow S = \frac{1}{2}a^2\left(\frac{1-r^m}{1-r}\right)\left\{\frac{1-r^m}{1-r} - \frac{1+r^m}{1+r}\right\}$$

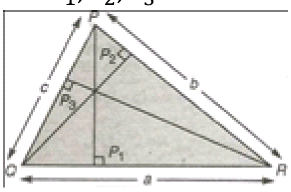
$$\Rightarrow S = \frac{1}{2}a^2\left(\frac{1-r^m}{1-r}\right)2r\left(\frac{1-r^{m-1}}{1-r}\right)\left(\frac{1}{1+r}\right)$$

$$\Rightarrow S = a\left(\frac{1-r^m}{1-r}\right)\cdot a\left(\frac{1-r^{m-1}}{1-r}\right)\cdot\frac{r}{1+r}$$

$$= \frac{r}{r+1}S_m S_{m-1}$$

309 **(b)**

Let P_1, P_2, P_3 be altitudes from P, Q and R



$\therefore P_1 = c\sin Q = \lambda bc, P_2 = a\sin R = \lambda ca, P_3 = b\sin P = \lambda ab$

$$\left[\because \frac{\sin P}{a} = \frac{\sin Q}{b} = \frac{\sin R}{c} = \lambda\right]$$

Since, P_1, P_2, P_3 are in AP.
$\Rightarrow \lambda bc, \lambda ca, \lambda ab$ are in AP.
$\Rightarrow bc, ca\ ab$ are in AP .
$\Rightarrow \frac{abc}{a}, \frac{abc}{b}, \frac{abc}{c}$ are in AP.
$\therefore\ a, b, c$ are in HP.
ie, sides of the triangle are in HP.

310 **(a)**
Since, $H = \frac{2PQ}{P+Q}$
$$\Rightarrow \frac{H}{P} = \frac{2Q}{P+Q}$$
and $\frac{H}{Q} = \frac{2P}{P+Q}$
$$\therefore\ \frac{H}{P} + \frac{H}{Q} = \frac{2Q}{P+Q} + \frac{2P}{P+Q} = 2$$

311 **(b)**
We have,
$x = \log_a bc, y = \log_b ca, z = \log_c ab$
$\Rightarrow 1 + x = \log_a a + \log_a bc, 1 + y$
$= \log_b b + \log_b ca, 1 + z$
$= \log_c c + \log_c ab$
$\Rightarrow 1 + x = \log_a abc, 1 + y = \log_b abc, 1 + z$
$= \log_c abc$
$$\Rightarrow \frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z}$$
$= \log_{abc} a + \log_{abc} b + \log_{abc} c$
$\Rightarrow = \log_{abc} abc = 1$

312 **(a)**
If a, b, c are in GP, then $b^2 = ac$
Taking log on both sides, we get
$2 \log_e b = \log_e a + \log_e c$
$\Rightarrow 2n \log_e b = n \log_e a + n \log_e c$
$\Rightarrow 2 \log_e b^n = \log_e a^n + \log_e c^n$
$\Rightarrow \log_e a^n, \log_e b^n, \log_e c^n$ be in AP.

314 **(c)**
Since, a, b, c are in AP.
$\therefore\ 2b = a + c$
$\Rightarrow\ 2bx = (a + c)x$ for all x
$\Rightarrow\ 2bx = (a + c)x + 20$ for all x
$\Rightarrow\ 2(bx + 10) = (ax + 10) + (cx + 10)$
$\therefore\ 10^{2(bx+10)} = 10^{(ax+10)+(cx+10)}$
$\Rightarrow\ \left(10^{bx+10}\right)^2 = 10^{ax+10}.10^{cx+10}$
$\Rightarrow\ 10^{ax+10}, 10^{bx+10}, 10^{cx+10}$
Are in GP for all x

315 **(b)**
We have,
$$\frac{5^{1+x} + 5^{1-x}}{2} = 13$$
$$\Rightarrow 5\left(5^x + \frac{1}{5^x}\right) = 26$$
$\Rightarrow 5y^2 - 26y + 5 = 0$
$\Rightarrow (5y - 1)(y - 5) = 0$
$$\Rightarrow y = 5, \frac{1}{5} \Rightarrow 5^x = 5, 5^{-1} \Rightarrow x = 1, -1$$

317 **(d)**
Given series is $2^3 + 4^3 + 6^3 + \ldots$
$\therefore T_n = (2n)^3 = 8n^3$
$$\therefore \Sigma T_n = 8\Sigma n^3 = \frac{8n^2(n+1)^2}{4}$$
$\Rightarrow 3528 = 2n^2(n + 1)^2 \Rightarrow n = 6$

318 **(c)**
Since, $\frac{2}{a+b}, \frac{1}{b}, \frac{2}{b+c}$ are in AP.
$$\Rightarrow \frac{2}{b} = 2\left(\frac{1}{a+b} + \frac{1}{b+c}\right)$$
$$\Rightarrow \frac{1}{b} = \left(\frac{2b + a + c}{ab + ac + b^2 + bc}\right)$$
$\Rightarrow ab + ac + b^2 + bc = 2b^2 + ab + bc$
$\Rightarrow b^2 = ac$
$\Rightarrow a, b, c$ are in GP.

319 **(b)**
Let the numbers be x and y
Since, $\frac{x+y}{2} = 2\sqrt{xy}$
$\Rightarrow x + y = 4\sqrt{xy}$
$\Rightarrow (x + y)^2 = 16xy$ ...(i)
Also $(x - y)^2 = (x + y)^2 - 4xy$
$\therefore\ (x - y)^2 = 16xy - 4xy = 12xy$
$\Rightarrow\ x - y = 2\sqrt{3xy}$ ...(ii)
On solving Eqs. (i) and (ii), we get
$x = (2 + \sqrt{3})\sqrt{xy}$
and $y = (2 - \sqrt{3})\sqrt{xy}$
$\therefore$ Required ratio $= \frac{x}{y} = \frac{(2+\sqrt{3})\sqrt{xy}}{(2-\sqrt{3})\sqrt{xy}}$
$= \left(2 + \sqrt{3}\right)^2 = 7 + 4\sqrt{3}$

320 **(d)**
Let the two numbers be a and b
$\therefore\ a - b = 48$ and $\frac{a+b}{2} - \sqrt{ab} = 18$
$\Rightarrow \left(\sqrt{a} - \sqrt{b}\right)\left(\sqrt{a} + \sqrt{b}\right) = 48$
and $\left(\sqrt{a} - \sqrt{b}\right) = 6$
$\Rightarrow \sqrt{a} + \sqrt{b} = 8$ and $\left(\sqrt{a} - \sqrt{b}\right) = 6$
$\Rightarrow \sqrt{a} = 7$ and $\sqrt{b} = 1$
$\Rightarrow a = 49$ and $b = 1$
Hence, the greater number is 49.

321 **(b)**
We have
$a_n = \frac{2(-1)^{n-1}}{n}$, if n is a multiple of 3
$\therefore a_3 + a_6 + a_9 + \cdots$

$= 2\left(\frac{1}{3} - \frac{1}{6} + \frac{1}{9} - \frac{1}{12} + \cdots\right)$

$= \frac{2}{3}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots\right) = \frac{2}{3}\log 2$

ALITER We have

$\log(1 - x + x^2)$

$= x(a_1 + a_4x^3 + a_7x^6 + \cdots)$

$+ x^2(a_2 + a_5 x^3 + \cdots)$

$+(a_3x^3 + a_6x^6 + \cdots)$

Put $x = 1, \omega, \omega^2$ respectively and add to get the value of

$a_3 + a_6 + a_9 + \cdots$

323 **(a)**

Given series is $1 \cdot 3^2 + 2 \cdot 5^2 + 3 \cdot 7^2 + \ldots \infty$

This is an arithmetico-geometric series whose nth term is equal to

$T_n = n(2n + 1)^2 = 4n^3 + 4n^2 + n$

$\therefore S_n = \sum_1^n T_n = \sum_1^n (4n^3 + 4n^2 + n)$

$= 4\sum_1^n n^3 + 4\sum_1^n n^2 + \sum_1^n n$

$= 4\left(\frac{n}{2}(n + 1)\right)^2 + \frac{4}{6}n(n + 1)(2n + 1) + \frac{n}{2}(n + 1)$

$= n(n + 1)\left[n^2 + n + \frac{4}{6}(2n + 1) + \frac{1}{2}\right]$

$= \frac{n}{6}(n + 1)(6n^2 + 14n + 7)$

324 **(d)**

Since, it is an infinite GP whose common ratio is 0.24.

$\therefore \ S_\infty = \frac{a}{1 - r} = \frac{5.05}{1 - 0.24}$

$= 6.64474$

325 **(b)**

$2^{1/4}.4^{1/8}.8^{1/16} \ldots \infty = 2^{\frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \ldots \infty}$

$= (2)^{\frac{1}{4}\left\{1 + \frac{2}{2} + \frac{3}{2^2} + \frac{4}{3^2} + \ldots\right\}}$

$= (2)^{\frac{1}{4}\left\{\frac{1}{1 - 1/2} + \frac{1.1/2}{(1 - 1/2)^2}\right\}}$

$= 2^{\frac{1}{4}(2+2)} = 2$

326 **(d)**

We have,

$\left(1 + \frac{a^2x^2}{2\,!} + \frac{a^4x^4}{4\,!} + \cdots\right)^2 - \left(ax + \frac{a^3x^3}{3\,!} + \frac{a^5x^5}{5\,!} + \cdots\right)^2$

$= \left(\frac{e^{ax} + e^{-ax}}{2}\right)^2 - \left(\frac{e^{ax} - e^{-ax}}{2}\right)^2$

$= \frac{1}{4}(4\, e^{ax} \cdot e^{-ax}) = 1$

327 **(a)**

Here, $T_n = \frac{n^3}{n!} = \frac{n^2 - 1}{(n-1)!} + \frac{1}{(n-1)!}$

$= \frac{n + 1}{(n - 2)!} + \frac{1}{(n - 1)!}$

$= \frac{1}{(n - 3)!} + \frac{3}{(n - 2)!} + \frac{1}{(n - 1)!}$

$\Rightarrow \sum T_n = \sum\left(\frac{1}{(n - 3)!} + \frac{3}{(n - 2)!} + \frac{1}{(n - 1)!}\right)$

$= e + 3e + e = 5e$

328 **(c)**

$1 + \frac{(\log_e n)^2}{2!} + \frac{(\log_e n)^4}{4!} + \ldots$

$= \frac{e^{\log n} + e^{-\log n}}{2}$

$= \frac{e^{\log n} + e^{\log n^{-1}}}{2} = \frac{n + n^{-1}}{2}$

329 **(a)**

We have,

$\left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \left(x^3 + \frac{1}{x^3}\right)^3 + \cdots + \left(x^{10} + \frac{1}{x^{10}}\right)^2$

$= (x^2 + x^4 + x^6 + \cdots + x^{20}) + \left(\frac{1}{x^2} + \frac{1}{x^4} + \frac{1}{x^6} + \cdots + \frac{1}{x^{20}}\right) + 20$

$x^2\frac{(x^{20} - 1)}{(x^2 - 1)} + \frac{1}{x^2}\frac{\left(1 - \frac{1}{x^{20}}\right)}{\left(1 - \frac{1}{x^2}\right)} + 20$

$= \left(\frac{x^{20} - 1}{x^2 - 1}\right)\left(\frac{x^{22} + 1}{x^{20}}\right) + 20$

330 **(c)**

We have,

$x = \sum_{n=0}^{\infty} a^n, y = \sum_{n=0}^{\infty} b^n, \ z = \sum_{n=0}^{\infty} c^n$

$\Rightarrow x = \frac{1}{1 - a}, y = \frac{1}{1 - b}, z = \frac{1}{1 - c}$

$\Rightarrow a = 1 - \frac{1}{x}, b = 1 - \frac{1}{y}, c = 1 - \frac{1}{z}$

Now,

a, b, c are in A.P.
$\Rightarrow a-1, b-1-1$ are in A.P.
$\Rightarrow \frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P. $\Rightarrow x, y, z$ are in H.P.

331 **(c)**
Let the number of days be n. Hence, a worker can do $\left(\frac{1}{150n}\right)$th part of the work in day.
According to given condition
$(150+146+142+\ldots+\text{upto } \frac{(n+8)}{\text{terms}}) \times \frac{1}{150n} = 1$
$\Rightarrow \frac{n+8}{2}[300+(n+8-1)(-4)] = 1$
$\Rightarrow (n+8)(272-4n) = 300n$
$\Rightarrow 4n^2+60n-2176 = 0$
$\Rightarrow n^2+15n-544 = 0$
$\Rightarrow n = 17, -32$
We do not take negative value
$\therefore n = 17$
Therefore, number of total days
$= 17+8 = 25$

332 **(a)**
Using AM>GM [$\therefore a, b$ *and* c *are* distinct]
$\therefore \frac{a^2+b^2+c^2}{3} > (a^2b^2c^2)^{1/3}$
$\Rightarrow 3(a^2b^2c^2)^{\frac{1}{3}}$ [$\therefore a^2+b^2+c^2 = 1$ *given*]

333 **(b)**
$\because a^{-1}, b^{-1}, c^{-1}$ are in AP
$\therefore a, b, c$ are in HP.
Now, for numbers $a^{101}, b^{101}, c^{101}$
AM > GM
$\Rightarrow \frac{a^{101}+c^{101}}{2} > \left(\sqrt{ac}\right)^{101} > b^{101}$ $(\because \sqrt{ac} > b)$
$\Rightarrow 2b^{101}-a^{101}-c^{101} < 0$ ...(i)
Now, product of roots of given equation
$= \frac{2b^{101}-a^{101}-c^{101}}{1} < 0$ [from relation (i)]

334 **(b)**
We have,
$\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b} = \lambda(\text{say})$
$\Rightarrow a = 10^{\lambda(b-c)}, b = 10^{\lambda(c-a)}, c = 10^{\lambda(a-b)}$
$\therefore a^a b^b c^c = 10^{\lambda\, a(b-c)}.10^{\lambda\, b(c-a)}.10^{\lambda\, c(a-b)}$
$\Rightarrow a^a b^b c^c = 10^{\lambda\{a(b-c)+b(c-a)+c(a-b)\}} = 10^{\lambda\times 0}$
$= 10^0 = 1$

335 **(d)**
$d = T_2 - T_1 = (S_2 - S_1) - S_1$
$= S_2 - 2S_1$
$= 2P+Q-2P$
$= Q \left[\because S_n = \frac{n}{2}\{2P+(n-1)Q\}\right]$

336 **(b)**
Given, $a_0 = p$ and $a_n - a_{n-1} = ra_{n-1}$
$\Rightarrow a_n = a_{n-1}(r+1)$
For $n = 1, a_1 = a_0(r+1) = p(r+1)$
$n = 2,\ a_2 = a_1(r+1) = p(r+1)^2$
$n = 3,\ a_3 = a_2(r+1) = p(r+1)^3$
This shows that the sequence is a geometric progression.

337 **(c)**
Since a, b, c, d are in H.P. Therefore,
$\frac{1}{b}-\frac{1}{a} = \frac{1}{c}-\frac{1}{b} = \frac{1}{d}-\frac{1}{c} = \lambda(\text{say})$ and $\frac{1}{d}-\frac{1}{a} = 3\lambda$
$\Rightarrow a-b = \lambda(ab), b-c = \lambda(bc)$ and $c-d = \lambda(cd)$
$\Rightarrow a-b+b-c+c-d = \lambda(ab+bc+cd)$
$\Rightarrow a-d = \lambda(ab+bc+cd)$
$\Rightarrow 3\lambda ad = \lambda(ab+bc+cd)$
$\Rightarrow ab+bc+cd = 3ad$

338 **(b)**
We have,
$1+2x+4x^2+8x^3+16x^4+32x^5 = \frac{1-p^6}{1-p}$
$\Rightarrow \frac{1-(2x)^6}{1-2x} = \frac{1-p^6}{1-p} \Rightarrow p = 2x \Rightarrow \frac{p}{x} = 2$

339 **(d)**
Let $b = a+d$ and $c = a+2d$, where d is common ratio.
Now, $2^{bx+1} = 2^{(a+d)x+1} = 2^{ax+1}.2^{dx}$
and $2^{cx+1} = 2^{(a+2d)x+1} = 2^{ax+1}.(2^{dx})^2$
$\therefore$ These numbers are in GP, for all values of x

340 **(b)**
$\log_a(1+x) = \log_e(1+x)\log_a e =$
$(\log_a e)\left[\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}\right]$
So, the coefficient of x^n in $\log_a(1+x)$ is $\frac{(-1)^{n-1}}{n}\log_a e$

342 **(b)**
Since, $a_1, a_2, \ldots, a_n$ are n AM's between a and b, then
$a_i = a + i\,\frac{(b-a)}{n+1}$
$\therefore 2\sum_{i=1}^{n} a_i = 2\left[\sum a + \frac{(b-a)}{n+1}\sum i\right]$

$$= 2\left[na + \frac{(b-a)}{n+1}.n\ \frac{(n+1)}{2}\right]$$
$$= n(a+b)$$

343 **(b)**

Since, m is a root of the given equation

$\therefore\ (1-ab)m^2 - (a^2+b^2)m - (1+ab) = 0$

$\Rightarrow\ m(a^2+b^2) + (m^2+1)ab = m^2 - 1$ ...(i)

Now, H_1 = first HM between a and b

$$= \frac{(m+1)ab}{a+mb}$$

And $H_m = \frac{(m+1)ab}{b+ma}$

$$\therefore H_m - H_1 = (m+1)ab\left[\frac{1}{b+ma} - \frac{1}{a+mb}\right]$$
$$= (m+1)ab\frac{[(m-1)(b-a)]}{(b+ma)(a+mb)}$$
$$= \frac{(m^2-1)ab((b-a)}{m(a^2+b^2)+(m^2+1)ab}$$
$$= \frac{(m^2-1)ab(b-a)}{m^2-1} \quad \text{[from Eq.(i)]}$$
$$= ab(b-a)$$

344 **(b)**

Since, a, b, c are in AP.

$\Rightarrow 2b + a + c$

$\therefore\ 3^{2b} = 3^{a+c} \Rightarrow (3^b)^2 = 3^a \cdot 3^c$

ie, $3^a, 3^b, 3^c$ are in GP.

345 **(c)**

Since, $\frac{1}{b-a} + \frac{1}{b-c} = \frac{1}{a} + \frac{1}{c}$

$$\Rightarrow \frac{b-a+b-c}{b^2-(a+c)b+ac} = \frac{a+c}{ac}$$
$$\Rightarrow 2abc - (a+c)ac = b^2(a+c) - b(a+c)^2 + ac(a+c)$$
$$\Rightarrow 2ac(b-a-c) = b(a+c)(b-a-c)$$
$$\Rightarrow \frac{2ac}{a+c} = b$$

$\Rightarrow a, b, c$ are in HP.

346 **(d)**

We have,

$$S = \frac{1}{1-\frac{1}{2}} = 2$$

And,

$$S_n = \frac{1-1/2^n}{1-1/2} = 2\left(1-\frac{1}{2^n}\right) = 2 - \frac{1}{2^{n-1}}$$
$$\therefore S - S_n < \frac{1}{1000}$$
$$\Rightarrow \frac{1}{2^{n-1}} < \frac{1}{1000}$$
$$\Rightarrow 2^{n-1} > 1000$$
$$\Rightarrow \frac{1}{2^{n-1}} < \frac{1}{1000} \Rightarrow n-1 \geq 10 \Rightarrow n \geq 11$$

Hence, the least value of n is 11

347 **(b)**

Let the three numbers $(a-d), a$ and $(a+d)$ are in AP. Then,

$(a-d) + a + (a+d) = 18 \quad \Rightarrow a = 6$ ...(i)

and $(a-d)^2 + a^2 + (a+d)^2 = 158$

$\Rightarrow 3a^2 + 2d^2 = 158$

$\Rightarrow 2d^2 = 158 - 3 \times 36$ [from Eq. (i)]

$\Rightarrow\ d = \pm 5$

Hence, the required numbers are 1, 6, 11 or 11, 6, 1

Thus, greatest number is 11.

348 **(a)**

We have,

$$\frac{1}{1.2.3.4} + \frac{4}{3.4.5.6} + \frac{9}{4.6.7.8} + \frac{16}{7.8.9.10} + \cdots \text{to } \infty$$
$$= \sum_{n=1}^{\infty} \frac{n^2}{(2n-1)(2n)(2n+1)(2n+2)}$$
$$= \frac{1}{2}\sum_{n=1}^{\infty} \frac{n}{(2n-1)(2n+1)(2n+2)}$$
$$= \frac{1}{2}\sum_{n=1}^{\infty}\left\{\frac{1}{12(2n-1)} + \frac{1}{4(2n+1)} - \frac{1}{3(2n+2)}\right\}$$
$$= \frac{1}{2}\sum_{n=1}^{\infty}\left\{\frac{1}{12}\left(\frac{1}{2n-1} - \frac{1}{2n}\right) + \frac{1}{12}\left(\frac{1}{2n} - \frac{1}{2n+1}\right) + \frac{1}{3}\left(\frac{1}{2n+1} - \frac{1}{2n+2}\right)\right\}$$
$$= \frac{1}{2}\left\{\frac{1}{12}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5}\dots\right) + \frac{1}{12}\left(\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6}\dots\right) + \frac{1}{3}\left(\frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6}\dots\right)\right\}$$
$$= \frac{1}{2}\left\{\frac{1}{12}\log_e 2 - \frac{1}{12}(\log_e 2 - 1) + \frac{1}{3}\left(\log_e 2 - \frac{1}{2}\right)\right\}$$
$$= \frac{1}{6}\log_e 2 - \frac{1}{24}$$

349 **(c)**

We have,

$a = 1 + \log_x yz = \log_x x + \log_x yz = \log_x xyz$

$b = 1 + \log_y zx = \log_y y + \log_y zx = \log_y xyz$

and, $c = 1 + \log_z xy = \log_z z + \log_z xy = \log_z xyz$

$$\therefore \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \log_{xyz} x + \log_{xyz} y + \log_{xyz} z = \log_{xyz} xyz = 1$$

$\Rightarrow ab + bc + ca = abc$

350 **(a)**

$\text{GM} = (7.7^2.7^3 \dots 7^n)^{1/n}$

$= (7^{1+2+3+\cdots+n})^{1/n}$

$= \left(7^{\frac{n(n+1)}{2}}\right)^{1/n} = 7^{\left(\frac{n+1}{2}\right)}$

351 **(a)**

We have the following cases :

<u>CASE I</u> *When n is even*

Let $n = 2m$. Then,

$1 - 3 + 5 - 7 + 9 - 11$ ... to $2m$ terms

$= [1 + 5 + 9 + \cdots$ to m terms$] - [3 + 7 + 11 + \cdots$ to m terms$]$

$= \frac{m}{2}\{2 \times 1 + (m-1) \times 4\} - \frac{m}{2}\{2 \times 3 + (m-1) \times 4\}$

$-2m = -n$

<u>CASE II</u> *When n is odd*

Let $n = 2m + 1$. Then,

$1 - 3 + 5 - 7 + 9 - 11 + \cdots$ to n terms

$= \{1 + 5 + 9 + \cdots$ to $(m+1)$ terms$\} - \{3 + 7 + 11 + \cdots$ to m terms$\}$

$= \frac{m+1}{2}\{2 \times 1 + (m + 1 - 1) \times 4\} - \frac{m}{2}\{2 \times 3 + (m-1) \times 4\}$

$= 2m + 1 = n$

352 **(b)**

The sum of integers from 1 to 100 that are divisible by 2 or 5 = sum of series divisible by 2+ sum of series divisible by 5− sum of series divisible by 2 ad 5.

$= (2 + 4 + 6 + \ldots + 100) + (5 + 10 + 15 + \ldots + 100) - (10 + 20 + 30 + \ldots + 100)$

$= \frac{50}{2}\{2 \times 2 + (50-1)2\} + \frac{20}{2}\{2 \times 5 + (20-1)5\} - \frac{10}{2}\{10 \times 2 + (10-1)10\}$

$= 25(102) + 10(105) - 5(110)$

$= 2550 + 1050 - 550 = 3050$

353 **(c)**

Given, $y = \frac{3^x}{3} + \frac{1}{3.3^x} = \frac{1}{3}(3^x + 3^{-x})$

Since, $\frac{3^x + 3^{-x}}{2} \geq \sqrt{3^x . 3^{-x}}$ $[\because AM \geq GM]$

$\Rightarrow 3^x + 3^{-x} \geq 2$

$\Rightarrow y \geq \frac{2}{3}$

Therefore, least value of y is $\frac{2}{3}$

354 **(c)**

Let $S_n = 1 + 3 + 5 + \ldots + (2n-1)$

$= \frac{n}{2}[1 + (2n-1)] = n^2$

$\therefore$ Arithmetic mean $= \frac{n^2}{n} = n$

355 **(c)**

Let there be $2n$ terms in the given GP with first term a and the common ratio r

According to the given condition

$a\frac{(r^{2n}-1)}{(r-1)} = 5a\frac{(r^{2n}-1)}{(r^2-1)}$

$\Rightarrow r + 1 = 5 \Rightarrow r = 4$

356 **(d)**

Since, $1 + r + r^2 + \ldots \infty = S$

$\therefore \frac{1}{1-r} = S \Rightarrow r = \frac{S-1}{S}$ ...(i)

Now, $1 + r^2 + r^4 + \ldots \infty = \frac{1}{1-r^2} = \frac{1}{1-\left(\frac{S-1}{S}\right)^2}$

[from Eq. (i)]

$= \frac{S^2}{S^2 - (S-1)^2}$

$= \frac{S^2}{(2S-1)}$

357 **(d)**

Since, $T_n = \frac{1}{4}(n+2)(n+3)$

$\frac{1}{T_n} = \frac{4}{(n+2)(n+3)} = 4\left[\frac{1}{n+2} - \frac{1}{n+3}\right]$

$\therefore \frac{1}{T_1} + \frac{1}{T_2} + \ldots + \frac{1}{T_{2003}} = 4\left(\frac{1}{3} - \frac{1}{4}\right) + 4\left(\frac{1}{4} - \frac{1}{5}\right) + \ldots + 4\left(\frac{1}{2005} - \frac{1}{2006}\right)$

$= 4\left(\frac{1}{3} - \frac{1}{2006}\right) = \frac{4 \times 2003}{3 \times 2006} = \frac{4006}{3009}$

358 **(d)**

Let$S_n = \frac{1}{\sqrt{1}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \ldots$

$= \frac{1}{2}[(\sqrt{3}-1) + (\sqrt{5}-\sqrt{3}) + (\sqrt{7}-\sqrt{5}) + \ldots + (\sqrt{2n+1} - \sqrt{2n-1})]$

$= \frac{1}{2}(\sqrt{2n+1} - 1)$

359 **(a)**

We know that,

$\left\{\frac{n}{2}(n+1)^2\right\} = (1 + 2 + \ldots + n)^2$

$= \sum_{i=1}^{n} x_i^2 + 2\sum_{i<j} x_1 x_j$

$$\Rightarrow \sum_{i<j} x_i x_j = \frac{1}{2}\left\{\frac{n^2(n+1)^2}{4} - \frac{n(n+1)(2n+1)}{6}\right\}$$

$$= \frac{n}{24}(n-1)(n+1)(3n+2)$$

360 **(d)**

We have,

$$1 + \frac{x}{2\,!} + \frac{2\,x^2}{3\,!} + \frac{3\,x^3}{4\,!} + \cdots \infty$$

$$= 1 + \sum_{n=1}^{\infty} \frac{(n-1)x^{n-1}}{n\,!}$$

$$= 1 + \sum_{n=1}^{\infty} \frac{x^{n-1}}{(n-1)\,!} - \sum_{n=1}^{\infty} \frac{x^{n-1}}{n\,!}$$

$$= 1 + \sum_{n=1}^{\infty} \frac{x^{n-1}}{(n-1)\,!} - \frac{1}{x}\sum_{n=1}^{\infty} \frac{x^{n}}{n\,!}$$

$$= 1 + e^x - \frac{1}{x}(e^x - 1) = \frac{e^x(x-1)+x+1}{x}$$

361 **(c)**

Let $S_n = cn^2$, then

$S_{n-1} = c(n-1)^2 = cn^2 + c - 2cn$

$\therefore\ T_n = 2\,cn - c\quad (\because T_n = S_n - S_{n-1})$

$T_n^2 = (2cn - c)^2 = 4c^2n^2 + c^2 - 4c^2n$

$$\therefore \text{Sum} = \sum T_n^2 = \frac{4c^2.n(n+1)(2n+1)}{6} + nc^2 - 2c^2n(n+1)$$

$$= \frac{2c^2n(n+1)(2n+1) + 3nc^2 - 6c^2n(n+1)}{3}$$

$$= \frac{nc^2(4n^2+6n+2+3-6n-6)}{3}$$

$$= \frac{nc^2(4n^2-1)}{3}$$

362 **(c)**

Since, the given equation is cubic, therefore we take three roots.

Let the roots be $a-b, a, a+d$.

Sum of three numbers in AP $= 3a = 12$

$\Rightarrow\ a = 4$ is a root.

$\therefore$ The given equation $x^3 - 12x^2 + 39x - 28 = 0$ can be rewritten as

$(x-4)(x^2 - 8x + 7) = 0$

$\therefore\ x = 1, 4, 7$ or $7, 4, 1$

$\therefore\ d = \pm 3$

363 **(c)**

Let $a^x = b^y = c^z = d^u = \lambda$. Then,

$a = \lambda^{\frac{1}{x}}, b = \lambda^{\frac{1}{y}}, c = \lambda^{\frac{1}{z}}, d = \lambda^{\frac{1}{u}}$

Now,

a, b, c, d are in G.P.

$\Rightarrow \frac{b}{a} = \frac{c}{d} = \frac{d}{c}$

$\Rightarrow \lambda^{\frac{1}{y}-\frac{1}{x}} = \lambda^{\frac{1}{z}-\frac{1}{y}} = \lambda^{\frac{1}{u}-\frac{1}{z}}$

$\Rightarrow \frac{1}{y} - \frac{1}{x} = \frac{1}{z} - \frac{1}{y} = \frac{1}{u} - \frac{1}{z}$

$\Rightarrow \frac{1}{x}, \frac{1}{y}, \frac{1}{z}, \frac{1}{u}$ are in A.P.

$\Rightarrow x, y, z, u$ are in H.P.

364 **(a)**

We have,

$$e^x = 1 + x + \frac{x^2}{2\,!} + \frac{x^3}{3\,!} + \frac{x^4}{4\,!} + \cdots$$

Putting $x = \frac{1}{2}$, we get

$$\sqrt{e} = 1 + \frac{1}{2} + \frac{1}{2\,!}\left(\frac{1}{2}\right)^2 + \frac{1}{3\,!}\left(\frac{1}{2}\right)^3 + \frac{1}{4\,!}\left(\frac{1}{2}\right)^4 + \frac{1}{5\,!}\left(\frac{1}{2}\right)^5 + \cdots$$

$\Rightarrow \sqrt{e} = 1 + 0.5 + 0.12500 + 0.02083 + 0.00260 + 0.00026$

$\Rightarrow \sqrt{e} = 1.64869$

$\Rightarrow \sqrt{e} = 1.648$ (Rounded off to three places of decimals)

365 **(b)**

Let the first term of an AP be a and common difference be d.

Since, $a + 3d = \frac{5}{3}$ ...(i)

and $a + 7d = 3$ ...(ii)

On solving Eqs.(i) and (ii), we get

$a = \frac{2}{3},\ \ d = \frac{1}{3}$

$\therefore T_6 = a + 5d = \frac{2}{3} + \frac{5}{3} = \frac{7}{3}$

$\Rightarrow$ 6th term of HP is $\frac{3}{7}$.

366 **(b)**

We have,

$(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) \leq 0$ ...(i)

LHS $= (a^2p^2 - 2abp + b^2) + (b^2p^2 - 2bcp +$ *c2+(c2p2–2cdp+d2)* ...(ii)

$= (ap - b^2) + (bp - c^2) + (cp - d^2) \geq 0$

Since, the sum of square of real number is non-negative

From Eqs.(i) and (ii), we get

$(ap - b)^2 + (bp - c)^2 + (cp - d)^2 = 0$

$\Rightarrow\ ap - b = 0 = bp - c = cp - d$

$\Rightarrow \frac{b}{a} = \frac{c}{d} = \frac{d}{c} = p$

$\therefore\ a, b, c, d$ are in GP.

367 **(c)**

The corresponding terms of HP in terms of AP is 10 and 25

$\therefore T_7 = a + 6d = 10$ ... (i)

and $T_{12} = a + 11d = 25$ ...(ii)

On solving Eqs. (i) and (ii), we get

$a = -8, d = 3$

$\therefore\ T_{20} = -8 + (20-1)3 = 49$

Then, 20th term of HP is $\frac{1}{49}$.

368 **(c)**

Given, $\log(1 + 3x + 2x^2)$

$= \log(1+x) - \log(1+2x)$

$$= \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots\right) + \left(2x - \frac{(2x)^2}{2} + \frac{(2x)^3}{3} - \dots\right)$$

$$= 3x - \frac{5}{2}x^2 + \frac{9}{3}x^3 - \frac{17}{4}x^4 + \dots$$

369 **(c)**

Here, $r = \frac{1}{3}\sqrt{\frac{20}{3} \times \frac{9}{10}} = \sqrt{\frac{3}{5}}$

$$\therefore\ T_5 = ar^4 = \frac{10}{9}\left(\frac{3}{5}\right)^2 = \frac{2}{5}$$

371 **(a)**

We have,

$\log_2 \sin x - \log_2 \cos x - \log_2(1 - \tan^2 x) = -1$

$$\Rightarrow \log_2\left\{\frac{\sin x}{\cos x(1 - \tan^2 x)}\right\} = -1$$

$$\Rightarrow \frac{\tan x}{1 - \tan^2 x} = 2^{-1}$$

$$\Rightarrow \frac{2\tan x}{1 - \tan^2 x} = 1$$

$$\Rightarrow \tan 2x = \tan\frac{\pi}{4} \Rightarrow 2x = n\pi + \frac{\pi}{4}, n \in Z$$

$$\Rightarrow x = \frac{n\pi}{2} + \frac{\pi}{8}, n \in Z$$

372 **(a)**

We have,

$$\frac{x-1}{x+1} + \frac{1}{2}\frac{x^2-1}{(x+1)^2} + \frac{1}{3}\frac{x^3-1}{(x+1)^3} + \cdots$$

$$= \left\{\frac{x}{x+1} + \frac{1}{2}\left(\frac{x}{x+1}\right)^2 + \frac{1}{3}\left(\frac{x}{x+1}\right)^3 + \cdots\right\}$$

$$- \left\{\frac{1}{x+1} + \frac{1}{2}\cdot\frac{1}{(x+1)^2} + \frac{1}{3}\cdot\frac{1}{(x+1)^3} + \cdots\right\}$$

$$= -\log_e\left(1 - \frac{x}{x+1}\right) + \log_e\left(1 - \frac{1}{x+1}\right)$$

$$= -\log_e\left(\frac{1}{x+1}\right) + \log_e\left(\frac{x}{x+1}\right) = \log_e x$$

373 **(c)**

Let $S = \sum_{n=1}^{\infty}\frac{2n^2+n+1}{n!}$

$$= \sum_{n=1}^{\infty}\left(\frac{2n}{(n-1)!} + \frac{1}{(n-1)!} + \frac{1}{n!}\right)$$

$$= \sum_{n=1}^{\infty}\left(\frac{2}{(n-2)!} + \frac{3}{(n-1)!} + \frac{1}{n!}\right)$$

$$= 2\left(1 + \frac{1}{1!} + \frac{1}{2!} + \dots\infty\right) + 3\left(1 + \frac{1}{1!} + \dots\infty\right) + \left(\frac{1}{1!} + \frac{1}{2!} + \dots\right)$$

$= 2e + 3e + e - 1$

$= 6e - 1$

374 **(d)**

Let the four numbers be $\frac{a}{r}, a, ar, 2ar - a$ ...(i)

Where first three numbers are in GP and last three in AP.

Given that the common difference of AP is 6, so ...(ii)

$ar - a = 6$

And also given $\frac{a}{r} = 2ar - a$

$\Rightarrow \frac{a}{r} = 2(ar - a) + a$

$\Rightarrow \frac{a}{r} = 2(6) + a$ [from Eq.(ii)]

$\Rightarrow \left(\frac{a}{r}\right) - a = 12$

$\Rightarrow a(1 - r) = 12r$

$\Rightarrow -6 = 12r$ [from Eq.(ii)]

$\Rightarrow r = -\frac{1}{2}$

From Eq. (ii), we get

$a\left[\left(-\frac{1}{2}\right) - 1\right] = 6$

$\Rightarrow a = -4$

On putting the value of a and r in Eq. (i), the required numbers are $8, -4, 2, 8$.

376 **(b)**

Given, $\frac{\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\dots}{1+\frac{1}{3!}+\frac{1}{5!}+\dots} = \frac{\frac{e+e^{-1}}{2}-1}{\frac{e-e^{-1}}{2}}$

$$= \frac{e^2 + 1 - 2e}{e^2 - 1} = \frac{e-1}{e+1}$$

377 **(d)**

Since, $\frac{a}{1-r} = 4 \quad \Rightarrow a = 4(1-r)$ ...(i)

and $ar = \frac{3}{4}$

$\Rightarrow 4(1-r)r = \frac{3}{4}$ [from Eq. (i)]

$\Rightarrow 16r^2 - 16r + 3 = 0 \Rightarrow (4r-1)(4r-3) = 0$

$\Rightarrow r = \frac{1}{4}, \frac{3}{4}$

If $r = \frac{1}{4}$, then $a = 3$

378 **(a)**

$$\sum_{k=1}^{5} \frac{1^3 + 2^3 + \ldots + k^3}{1 + 3 + 5 + \ldots + (2k-1)} = \sum_{k=1}^{5} \frac{\left(\frac{k(k+1)}{2}\right)^2}{k^2}$$

$$= \sum_{k=1}^{5} \frac{(k+1)^2}{4}$$

$$= \frac{2^2 + 3^2 + 4^2 + 5^2 + 6^2}{4}$$

$$= \frac{4 + 9 + 16 + 25 + 36}{4}$$

$$= \frac{90}{4} = 22.5$$

379 **(c)**

We have,

$$\left(1 + x + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots\right)$$

$$= \left(\frac{e^x + e^{-x}}{2}\right)^2$$

$$= \frac{1}{4}\{e^{2x} + e^{-2x} + 2\}$$

$$= \frac{1}{4}\left[\left(1 + 2x + \frac{2^2 x^2}{2!} + \frac{2^3 x^3}{3!} + \cdots\right) + \left(1 - 2x + \frac{2^2 x^2}{2!} - \frac{2^3 x^3}{3!} + \cdots\right) + 2\right]$$

$$= \frac{1}{2}\left\{1 + \frac{2^2 x^2}{2!} + \frac{2^4 x^4}{4!} + \cdots + 1\right\}$$

$$= 1 + \frac{2x^2}{2!} + \frac{2^3 x^4}{4!} + \cdots$$

380 **(a)**

We have, $\tan n\theta = \tan m\theta$

$\Rightarrow n\theta = N\pi + (m\theta)$

$\Rightarrow \theta = \frac{N\pi}{n-m}$, putting $N = 1, 2, 3, \ldots$ we get $\frac{\pi}{n-m}, \frac{2\pi}{n-m}, \frac{3\pi}{n-m}$ ... which are obviously in AP.

381 **(a)**

We have,

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$$

$$= \left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \frac{1}{7^2} + \cdots\right) - \left(\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \cdots\right)$$

$$\frac{\pi^2}{6} - \frac{1}{4}\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right) = \frac{\pi^2}{6} - \frac{1}{4}\left(\frac{\pi^2}{6}\right) = \frac{\pi^2}{8}$$

382 **(c)**

It is given that a, b, c are in G.P.

$\therefore b^2 = ac$

$\Rightarrow 2 \log_\lambda b = \log_\lambda a + \log_\lambda c$

$$\Rightarrow \frac{2}{\log_b \lambda} = \frac{1}{\log_a \lambda} + \frac{1}{\log_c \lambda}$$

$\Rightarrow \log_a \lambda, \log_b \lambda, \log_c \lambda$ are in H.P.

384 **(b)**

The resulting progression will have $n + 2$ terms with 2 as the first term and 38 as the last term. So,

The sum of the progression $= \frac{n+2}{2}(2 + 38) = 20(n+2)$

Also,

Sum = 200

$\Rightarrow 20(n+2) = 200 \Rightarrow n = 8$

385 **(b)**

We have,

$fn(x) = n \log_{n!} x = \log_{n!} x^n$

$\Rightarrow (n!)^{fn(x)} = x^n$

386 **(c)**

$\because T_6 = 2 \Rightarrow a + 5d = 2$

Now, let $P = T_1 T_4 T_5$

$= a(a+3d)(a+4d)$

$= (2-5d)(2-2d)(2-d)$

$= 2\{4 - 16d + 17d^2 - 5d^3\}$

Now, $\frac{dp}{dd} = 2\{-16 + 34d - 15d^2\}$

Put $\frac{dp}{dd} = 0$ for maximum or minimum

$-16 + 34d - 15d^2 = 0$

$\Rightarrow d = \frac{2}{3}$ and $\frac{8}{5}$

Also, $\frac{d^2P}{dd^2} = 2\{34 - 30d\}$

$$\left(\frac{d^2P}{dd^2}\right)_{d=2/3} > 0$$

Thus, P is least.

Thus, the value of $d = 2/3$.

387 **(d)**

Let $S_n = 12 + 16 + 24 + \ldots + T_n$

$S_n = 12 + 16 + \ldots + T_n$

On subtraction

$0 = 12 + 4 + 8 + 16 + \ldots - T_n$

$$\Rightarrow T_n = 12 + \frac{4(2^{n-1} - 1)}{2 - 1}$$

$= 2^{n+1} + 8$

$$S_n = \sum T_n = 2^2 + 2^4 + \ldots + 8n$$

$$= \frac{2^2(2^n - 1)}{2 - 1} + 8n$$

$= 4(2^n - 1) + 8n$

388 **(b)**

The sequence is a G.P. with common ratio 1/3

$$\therefore \sum_{r=1}^{20} a_r = 2\left\{\frac{1-\left(\frac{1}{3}\right)^{20}}{1-\frac{1}{3}}\right\} = 3\left(1-\frac{1}{3^{20}}\right)$$

389 **(c)**

We have,

$a_1\, a_2\, a_3 \dots a_n$

$$= \frac{b_n\, a_1\, a_2\, a_3 \dots a_n}{b_n} = \frac{(a_1\, a_2 \dots a_{n-1})(a_n b_n)}{b_n}$$

$$= \frac{(a_1\, a_2 \dots\dots a_{n-1})\, b_{n-1}}{b_n} \quad \text{[Using def. of } b_n]$$

$$= \frac{(a_1\, a_2 \dots a_{n-2})(a_{n-1} b_{n-1})}{b_n}$$

$$= \frac{(a_1\, a_2 \dots a_{n-2})\, b_{n-2}}{b_n}$$

$$= \frac{a_1 b_1}{b_n} = \frac{(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})}{b_n} = \frac{x-y}{b_n}$$

390 **(d)**

We have,

$$\frac{e^{7x}+e^x}{e^{3x}} = e^{4x} + e^{-2x}$$

$$= \sum_{n=0}^{\infty} \frac{(4x)^n}{n!} + \sum_{n=0}^{\infty} \frac{(-2x)^n}{n!}$$

$\therefore$ Coefficient of x^n in $\left(\frac{e^{7x}+e^x}{e^{3x}}\right) = \frac{4^n}{n!} + \frac{(-1)^n 2^n}{n!} = \frac{4^n+(-2)^n}{n!}$

391 **(d)**

Given series, is a arithmetic geometric series.

Here, $a_1 = 1,\ d = 1,\ r = a$

$$\therefore S_\infty = \frac{a_1}{1-r} + \frac{d.r}{(1-r)^2}$$

$$= \frac{1}{1-a} + \frac{1.a}{(1-a)^2} = \frac{1}{(1-a)^2}$$

392 **(b)**

Let $b = ar, c = ar^2$ and $d = ar^3$. Then,

$$\frac{1}{a^3+b^3} = \frac{1}{a^3(1+r^3)}, \frac{1}{b^3+c^3} = \frac{1}{a^3\, r^3(1+r^3)}$$

and, $\frac{1}{c^3+d^3} = \frac{1}{a^3\, r^3(1+r^3)}$

Clearly, $(a^3+b^3)^{-1}, (b^3+c^3)^{-1}$ and $(c^3+d^3)^{-1}$ are in G.P. with common ratio $1/r^3$

393 **(b)**

Here, $T_{12} = a + 11d$

and $T_{22} = a + 21d$

Since, $100 = T_{12} + T_{22}$

$\therefore 100 = a + 11d + a + 21d$

$\Rightarrow a + 16d = 50$ ...(i)

Now, $S_{33} = \frac{33}{2}[2a + (33-1)d]$

$= 33(a + 16d)$

$= 33 \times 50 = 1650$ [from Eq. (i)]

395 **(a)**

We have,

$$10^{x-14} + 10^{-x-1} = \frac{1}{3}$$

$$\Rightarrow 10^x + 10^{-x} = \frac{10}{3}$$

$\Rightarrow 3 \times (10^x)^2 - 10(10^x) + 3 = 0$

$\Rightarrow (10^x - 3)(310^x - 1) = 0$

$\Rightarrow 10^x = 3$ or $10^x = \frac{1}{3} \Rightarrow x = \log_{10} 3$ or

$x = -\log_{10} 3$

396 **(c)**

Let

$x^a = x^{b/2}\, z^{b/2} = z^c = \lambda$

$\Rightarrow x = \lambda^{\frac{1}{a}}, z = \lambda^{\frac{1}{c}}, xz = \lambda^{\frac{2}{b}}$

$\Rightarrow \lambda^{\frac{1}{a}+\frac{1}{c}} = \lambda^{\frac{2}{b}}$ $[\because x \times z = xz]$

$\Rightarrow \frac{1}{a} + \frac{1}{c} = \frac{2}{b} \Rightarrow a, b, c$ are in HP

397 **(b)**

The nth term T_n of the given series is given by

$$T_n = \frac{1 \cdot 3 \cdot 5 \cdot 7 \dots (2n-1)}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \dots (2n)}$$

$\Rightarrow T_n$

$$= \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \dots (2n-2)(2n-1)(2n)}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \dots (2n-1)(2n)}$$

$$\times \frac{1}{2 \cdot 4 \cdot 6 \dots (2n-2)(2n)}$$

$$\Rightarrow T_n = \frac{1}{2^n\, n!}$$

$$\therefore \sum_{n=1}^{\infty} T_n = \sum_{n=1}^{\infty} \frac{\left(\frac{1}{2}\right)^n}{n!} = \left(\sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}\right)^n}{n!}\right) - 1 = e^{1/2} - 1$$

398 **(c)**

Let d be the common difference of the A.P. Now,

$$\sum_{r=1}^{n-1} \frac{1}{(a_{r+1})^{2/3} + (a_{r+1})^{1/3}(a_r)^{1/3} + (a_r)^{2/3}}$$

$$= \sum_{r=1}^{n-1} \frac{(a_{r+1})^{1/3} - (a_r)^{1/3}}{a_{r+1} - a_r}$$

$$= \frac{1}{d} \sum_{r=1}^{n-1} \{(a_{r+1})^{1/3} - (a_r)^{1/3}\} \quad [\because a_{r+1} - a_r = d]$$

$$= \frac{1}{d}\left[(a_{n+1})^{1/3} - (a_1)^{1/3}\right]$$

$= \frac{1}{d} \times \frac{a_n - a_1}{(a_n)^{2/3} + (a_n a_1)^{1/3} + (a_1)^{2/3}}$

$= \frac{n-1}{a_n^{2/3} + a_n^{1/3} a_1^{1/3} + a_1^{2/3}}$ $[\because a_n - a_1 = (n-1)\, d]$

399 **(a)**

Let the two numbers be x and y, then

$A = \frac{1}{2}(x+y), \sqrt{xy} = G$ or $G^2 = xy$

And $\frac{2xy}{(x+y)} = 4 \Rightarrow G^2 = 4A$

We have, $2A + G^2 = 27 \Rightarrow 2A + 4A = 27$

$\Rightarrow A = \frac{9}{2}$

$\Rightarrow x + y = 9$ ...(i)

So, $xy = 18$ ...(ii)

Solving Eqs.(i) and (ii), we get

$x = 6, y = 3$

400 **(a)**

We have,

$3^{2x+1}.4^{x-1} = 36$

$\Rightarrow 3^{2x+1} \times 2^{2x-2} = 36$

$\Rightarrow 3 \times 3^{2x} \times \frac{2^{2x}}{4} = 36$

$\Rightarrow (3 \times 2)^{2x} = 48 \Rightarrow 6^{2x} = 48 \Rightarrow 36^x = 48 \Rightarrow x = \log_{36} 48$

401 **(b)**

If p, q, r, s are in A.P., then in an A.P. or a G.P. or an H.P. $a_1, a_2, a_3, \ldots$, the terms a_p, a_q, a_r are in A.P., G.P. or H.P. respectively

402 **(c)**

$T_n = \frac{\frac{n(n+1)}{2.2}}{1^3 + 2^3 + 3^3 + \ldots + n^3}$

$= \frac{\frac{n(n+1)}{4}}{\left(\frac{n(n+1)}{2}\right)^2} = \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$

$\therefore T_n = \Sigma\left(\frac{1}{n} - \frac{1}{n+1}\right)$

$= \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \ldots + \left(\frac{1}{n} - \frac{1}{n+1}\right)$

$= 1 - \frac{1}{n+1} = \frac{n}{n+1}$

403 **(a)**

Since, $a_1, a_2, a_3, \ldots, a_n$ are in AP.

Then, $a_2 - a_1 = a_3 - a_2 = \ldots = a_n - a_{n-1} = d$

Where d is the common difference of the give AP

Also, $a_n = a_1 + (n-1)d$

Then, by rationalizing each term

$\frac{1}{\sqrt{a_2} + \sqrt{a_1}} + \frac{1}{\sqrt{a_3} + \sqrt{a_2}} + \ldots + \frac{1}{\sqrt{a_n} + \sqrt{a_{n-1}}}$

$= \frac{\sqrt{a_2} - \sqrt{a_1}}{a_2 - a_1} + \frac{\sqrt{a_3} - \sqrt{a_2}}{a_3 - a_2} + \ldots + \frac{\sqrt{a_n} - \sqrt{a_{n-1}}}{a_n - a_{n-1}}$

$= \frac{1}{d}\,(\sqrt{a_2} - \sqrt{a_1} + \sqrt{a_3} - \sqrt{a_2} + \ldots + \sqrt{a_n} - \sqrt{a_{n-1}}$

$= \frac{1}{d}\,(\sqrt{a_n} - \sqrt{a_1}) \times \frac{\sqrt{a_n} + \sqrt{a_1}}{\sqrt{a_n} + \sqrt{a_1}}$

$= \frac{1}{d}\left(\frac{a_n - a_1}{\sqrt{a_n} + \sqrt{a_1}}\right) = \frac{1}{d}\left(\frac{(n-1)d}{\sqrt{a_n} + \sqrt{a_1}}\right)$

$= \frac{n-1}{\sqrt{a_n} + \sqrt{a_1}}$

404 **(a)**

We have,

$\frac{1}{x^2} + \frac{1}{2x^4} + \frac{1}{3x^6} + \cdots$ ad. inf.

$= -\log_e\left(1 - \frac{1}{x^2}\right)$

$= -\log_e\left(1 - \frac{2}{y+1}\right)$ $\left[\because y = 2x^2 - 1 \therefore x^2 = \frac{y+1}{2}\right]$

$= -\log_e\left(\frac{y-1}{y+1}\right) = \log_e\left(\frac{y+1}{y-1}\right)$

405 **(c)**

Let the number of sides of the polygon be n.Then, the sum of interior angles of the polygon

$= (2n-4)\frac{\pi}{4} = (n-2)\pi$

Since, the angles are in AP and $a = 120°, d = 5$

Therefore, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \frac{n}{2}[2 \times 120 + (n-1)5] = (n-2)180$

$\Rightarrow n^2 - 25n + 144 = 0$

$\Rightarrow (n-9)(n-16) = 0$

$\Rightarrow n = 9, 16$

Take $n = 16$

$T_{16} = a + 15d = 120° + 15(5°) = 195°$, which is impossible, an interior angle cannot be greater than 180°.

Hence, $n = 9$

406 **(d)**

We have,

$\log_x\left(4.x^{\log_5 x} + 5\right) = 2\log_5 x$

$\Rightarrow \log_x\left(4.x^{\log_5 x} + 5\right) = \log_5 x^2$

$\Rightarrow 4.x^{\log_5 x} + 5 = x^{\log_5 x^2}$

$\Rightarrow 4.x^{\log_5 x} + 5 = x^{2\log_5 x}$

$\Rightarrow 4y + 5 = y^2$, where $y = x^{\log_5 x}$

$\Rightarrow y^2 - 4y - 5 = 0$

$\Rightarrow y = 5, -1$

$\Rightarrow x^{\log_5 x} = 5 \qquad [\because y \neq -1]$

$\Rightarrow \log_5 x = \log_x 5$

$\Rightarrow (\log_5 x)^2 = 1 \Rightarrow \log_5 x = \pm 1 \Rightarrow x = 5, 5^{-1}$

407 **(d)**

Since, $x = 1 + a + a^2 + \ldots \infty$

$$\Rightarrow x = \frac{1}{1-a} \quad \Rightarrow \quad a = \frac{x-1}{x}$$

Similarly, $b = \frac{y-1}{y}$ and $c = \frac{z-1}{z}$

Since, a, b, c are in AP.

$$\therefore b = \frac{a+c}{2}$$

$$\Rightarrow \frac{y-1}{y} = \frac{\frac{x-1}{x} + \frac{z-1}{z}}{2}$$

$\Rightarrow 2xz(y-1) = y[z(x-1) + x(z-1)]$

$\Rightarrow 2xz = xy + yz$

408 **(a)**

We have,

$x^{(3/2)(\log_2 x - 3)} = 2^{-3}$

$\Rightarrow \frac{3}{2}(\log_2 x - 3) = \log_x 2^{-3}$

$\Rightarrow \frac{3}{2}(\log_2 x - 3) = -3 \log_x 2$

$\Rightarrow \frac{1}{2}(\log_2 x - 3) = -\frac{1}{\log_2 x}$

$\Rightarrow (\log_2 x)^2 - 3(\log_2 x) + 2 = 0$

$\Rightarrow (\log_2 x - 1)(\log_2 x - 2) = 0$

$\Rightarrow \log_2 x = 1, 2 \Rightarrow x = 2, 2^2$

411 **(b)**

Since, a, b, c are in AP.

$\Rightarrow 2b = a + c$, then straight line $ax + by + c = 0$ will pass through$(1,-2)$ because it satisfies condition $a - 2b + c = 0$ or $2b = a + c$.

412 **(a)**

We have,

$$\frac{e^x}{1-x} = B_0 + B_1 x + B_2 x^2 + \cdots + B_n x^n + \cdots$$

$$\Rightarrow \sum_{r=0}^{\infty} \frac{x^r}{r!} = (B_0 + B_1 x + B_2 x^2 + \cdots + B_n x^n + \cdots)(1-x)$$

On equating the coefficients of x^n on both sides, we get

$$\frac{1}{n!} = B_n - B_{n-1}$$

413 **(b)**

We have,

$$\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$$

Applying componendo and dividendo rule, we get

$$\frac{2a}{2bx} = \frac{2b}{2cx} = \frac{2c}{2dx}$$

$$\Rightarrow \frac{a}{b} = \frac{b}{c} = \frac{c}{d}$$

$\Rightarrow b^2 = ac$ and $c^2 = bd$

$\Rightarrow a, b, c$ and b, c, d are in GP, therefore a, b, c, d are in GP.

414 **(c)**

We have,

$$\sum_{r=1}^{n} \frac{1}{(2r-1)^2} = \frac{\pi^2}{8}$$

$$\Rightarrow \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$$

Let $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{5^2} + -\text{to } \infty = x$

$$\Rightarrow \left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right) + \frac{1}{2^2}\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right) = x$$

$$\Rightarrow \frac{\pi^2}{8} + \frac{x}{4} = x \Rightarrow x = \frac{\pi^2}{6}$$

415 **(c)**

It will take 10yr for Jairam to pay off Rs 10000 in 10 yearly installments.

$\therefore$ He pays 10% annual interest on remaining amount.

$\therefore$ Money given in the first year

$$= 1000 + \frac{10000 \times 10}{100} = 1000 + 1000$$

$=$ Rs 2000

Money given in second year

$= 1000 +$ interest of $(10000 - 1000)$

$= 1000 + \frac{9000 \times 10}{100} = 100 + 900 =$ Rs 1900

Similarly, money paid in third year $=$ Rs 1800 etc.

So, money given by Jairam in 10 yr will be Rs 2000, Rs 1900, Rs 1800, Rs 1700 …

Which is in arithmetic progression, whose first term

$a = 2000$ and $d = -100$

Total money given in 10 yr

$= \frac{10}{2}[2(2000) + (10-1)(-100)] =$ Rs 15500

Therefore, total money given by Jairam

$= 5000 + 15500 =$ Rs 20500

416 **(a)**

We have,

a, b, c are in A.P. …(i)

$\Rightarrow \frac{a}{abc}, \frac{b}{abc}, \frac{c}{abc}$ are in A.P. $\Rightarrow \frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in A.P. …(ii)

From (i) and (ii), we obtain

$$a + \frac{1}{bc}, b + \frac{1}{ca}, c + \frac{1}{ab} \text{ are in A. P.}$$

417 **(d)**

We observe that the successive differences of the terms of the sequence 12,28,50,78, ... are in A.P.

So, let its n^{th} term be

$t_n = an^2 + bn + c$,

Putting $n = 1,2,3$, we get

$t_1 = a + b + c \Rightarrow a + b + c = 12$

$t_2 = 4a + 2b + c \Rightarrow 4a + 2b + c = 28$

$t_3 = 9a + 3b + c \Rightarrow 9a + 3b + c = 50$

Solving these equations, we get

$a = 3, b = 7, c = 2$

$\therefore t_n = 3n^2 + 7n + 2$

Hence,

$$\frac{12}{2!} + \frac{28}{3!} + \frac{50}{4!} + \frac{78}{5!} + \cdots$$

$$= \sum_{n=1}^{\infty} \frac{3n^2 + 7n + 2}{(n+1)!}$$

$$= \sum_{n=2}^{\infty} \frac{3(n-1)^2 + 7(n-1) + 2}{n!}$$

$$= \sum_{n=2}^{\infty} \frac{3n^2 + n - 2}{n!}$$

$$= 3\sum_{n=2}^{\infty} \frac{n^2}{n!} + \sum_{n=2}^{\infty} \frac{n}{n!} - 2\sum_{n=2}^{\infty} \frac{1}{n!}$$

$$= 2(2e-1) + (e-1) - 2(e-2) = 5e$$

418 **(b)**

We have,

$$\frac{x^2}{2} + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \frac{4}{5}x^5 + \cdots$$

$$= \sum_{n=1}^{\infty} \frac{n}{n+1}\ x^{n+1}$$

$$= \sum_{n=1}^{\infty} \frac{n+1-1}{n+1}\ x^{n+1}$$

$$= \sum_{n=1}^{\infty} \left(1 - \frac{1}{n+1}\right) x^{n+1}$$

$$= \sum_{n=1}^{\infty} x^{n+1} - (n = 1 \frac{\infty}{} \frac{x^{n+1}}{n+1}$$

$$= \frac{x^2}{1-x} + x - \sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1}$$

$$= \frac{x^2}{1-x} + x + \log(1-x) = \frac{x}{1-x} + \log(1-x)$$

419 **(c)**

$$\left(\frac{1}{3}\right)^2 + \frac{1}{3}\left(\frac{1}{3}\right)^4 + \frac{1}{5}\left(\frac{1}{3}\right)^6 + \ldots$$

$$= \frac{1}{3}\left[\left(\frac{1}{3}\right) + \frac{1}{3}\left(\frac{1}{3}\right)^3 + \frac{1}{5}\left(\frac{1}{3}\right)^5 + \ldots\right]$$

$$= \frac{1}{3}\cdot\frac{1}{2}\log\left(\frac{1+\frac{1}{3}}{1-\frac{1}{3}}\right) \quad \left[\because \frac{1}{2}\log\left(\frac{1+x}{1-x}\right) = x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\right]$$

$$= \frac{1}{6}\log_e 2$$

420 **(a)**

Since, $T_m = \frac{1}{n} \Rightarrow a + (m-1)d$ ...(i)

and $T_n = \frac{1}{m} = a + (n-1)d$ ...(ii)

On solving Eqs. (i) and (ii), we get

$a = \frac{1}{mn}$ and $d = \frac{1}{mn}$

$$\therefore\ a - d = \frac{1}{mn} - \frac{1}{mn} = 0$$

421 **(c)**

Given, sum

$$= (x+2)^{n-1}\left\{1 + \left(\frac{x+1}{x+2}\right) + \left(\frac{x+1}{x+2}\right)^2 + \ldots + \left(\frac{x+1}{x+2}\right)^{n-1}\right\}$$

$$= (x+2)^{n-1}\left\{\frac{1 - \left(\frac{x+1}{x+2}\right)^n}{1 - \left(\frac{x+1}{x+2}\right)}\right\}$$

$$= \frac{(x+2)^{n-1}\{(x+2)^n - (x+1)^n\}\cdot(x+2)}{(x+2)^n}$$

$$= (x+2)^n - (x+1)^n$$

422 **(a)**

Given $b^2 = ac$, $x = \frac{a+b}{2}$ and $y = \frac{b+c}{2}$

$$\therefore\ \frac{1}{x} + \frac{1}{y} = \frac{2}{a+b} + \frac{2}{b+c}$$

$$= \frac{2(2b+a+c)}{ab + b^2 + bc + ac}$$

$$= \frac{2(2b+a+c)}{ab + 2b^2 + bc}$$

$$= \frac{2(2b+a+c)}{b(2b+a+c)}$$

$$= \frac{2}{b}$$

423 **(c)**

$\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + \ldots$

$1 \times \sqrt{2} + 2\sqrt{2} + 3\sqrt{2} + 4\sqrt{2} + \ldots$

$= \sqrt{2}(1 + 2 + 3 + 4 + \ldots$ upto 24 terms)

$= \sqrt{2} \times \dfrac{24 \times 25}{2}$

$= 300\sqrt{2} \quad \left[\because \Sigma n = \dfrac{n(n=1)}{2}\right]$

424 **(c)**

Let $S = 1^3 + 2^3 + 3^3 + \ldots + 15^3$

$= \sum_{n=1}^{15} n^3 = \left(\dfrac{15(15+1)}{2}\right)^2$

$= 14400$

426 **(c)**

Since, $\Sigma n = \left(\frac{1}{5}\right) \Sigma n^2$

$\Rightarrow \dfrac{n(n+1)}{2} = \dfrac{1}{5} \dfrac{n(n+1)(2n+1)}{6}$

$\Rightarrow 2n + 1 = 15 \Rightarrow n = 7$

427 **(a)**

We know that, $\frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots$

Put $x = \frac{1}{2}$, we get

$\dfrac{e^{1/2} + e^{-1/2}}{2} = 1 + \left(\dfrac{1}{2}\right)^2 \dfrac{1}{2!} + \left(\dfrac{1}{2}\right)^4 \dfrac{1}{4!} + \ldots$

$\Rightarrow \dfrac{e+1}{2\sqrt{e}} = 1 + \dfrac{1}{4.2!} + \dfrac{1}{16.4!} + \ldots \infty$

428 **(b)**

Let the three distinct real numbers in G.P. be a, ar, ar^2, where $r \neq \pm 1$

It is given that

$a^2 + a^2 r^2 + a^2 r^4 = S^2$

And,

$a + ar + ar^2 = \alpha S$

$\therefore \dfrac{a^2(1 + r + r^2)^2}{a^2(1 + r^2 + r^4)} = \dfrac{\alpha^2 S^2}{S^2}$

$\Rightarrow \dfrac{(1 + r + r^2)^2}{(r^2 + r + 1)(r^2 - r + 1)} = \alpha^2$

$\Rightarrow \dfrac{r^2 + r + 1}{r^2 - r + 1} = \alpha^2$

$\Rightarrow r^2(\alpha^2 - 1) - r(\alpha^2 + 1) + (\alpha^2 - 1) = 0$

$\Rightarrow (\alpha^2 + 1)^2 - 4(\alpha^2 - 1)^2 \geq 0 \quad [\because r \text{ is real}]$

$\Rightarrow (3\alpha^2 - 1)(\alpha^2 - 3) \leq 0 \Rightarrow \dfrac{1}{3} \leq \alpha^2 \leq 3$

But, $\alpha^2 = 3$ for $r = 1$ and $\alpha^2 = \frac{1}{3}$ for $r = -1$

$\therefore \dfrac{1}{3} < \alpha^2 < 3 \ i.e. \alpha^2 \in (1/3, 3)$

429 **(c)**

Using, $S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

Here, $a = 1, \ r = \frac{1}{5}, \ d = 3$

$\therefore S_\infty = \frac{1}{1-\frac{1}{5}} + \frac{3 \times \frac{1}{5}}{\left(1-\frac{1}{5}\right)^2}$

$= \dfrac{5}{4} + \dfrac{3}{5 \times \frac{16}{25}}$

$= \dfrac{5}{4} + \dfrac{15}{16} = \dfrac{35}{16}$

430 **(b)**

Since a, b, c are in HP.

$\therefore b = \dfrac{2\,ac}{a + c}$

$\Rightarrow \dfrac{b}{a} = \dfrac{2\,c}{a + c}$

$\Rightarrow \frac{b+a}{b-a} = \frac{3\,c+a}{c-a}$ [Applying componendo dividendo]

Again,

$b = \dfrac{2\,ac}{a + c}$

$\Rightarrow \dfrac{b}{c} = \dfrac{2\,a}{a + c}$

$\Rightarrow \dfrac{b + c}{b - c} = \dfrac{3\,a + c}{a - c}$

$\therefore \dfrac{b + a}{b - a} + \dfrac{b + c}{b - c} = \dfrac{3\,c + a}{c - a} + \dfrac{3\,a + c}{a - c} = 2$

431 **(a)**

Here, $T_n = 1 + 2 + 2^2 + 2^3 + \ldots + 2^n$

$= \dfrac{1(2^{n+1} - 1)}{2 - 1} = 2^{n+1} - 1$

$\therefore \Sigma T_n = \Sigma(2^{n+1} - 1) = \Sigma 2^{n+1} - \Sigma 1$

$= (2^2 + 2^3 + \ldots + 2^{n+1}) - n$

$= 2(2 + 2^2 + \ldots + 2^n) - n$

$= \dfrac{4(2^n - 1)}{2 - 1} - n = 2^{n+2} - n - 4$

432 **(b)**

$\dfrac{3}{4.8} - \dfrac{3.5}{4.8.12} + \dfrac{3.5.7}{4.8.12.16} - \ldots + \dfrac{3}{4} - \dfrac{3}{4}$

$= 1 - \dfrac{1}{4} + \dfrac{1.3}{2.4.4} - \dfrac{1.3.5}{4.4.2.4.3} + \ldots - \dfrac{3}{4}$

$= \left[1 + \dfrac{1}{1!}\left(-\dfrac{1}{4}\right) + \dfrac{1(1+2)}{2!}\left(-\dfrac{1}{4}\right)^2 + \dfrac{1(1+2)(1+4)}{3!}\left(-\dfrac{1}{4}\right)^3 + \ldots\right] - \dfrac{3}{4}$

$= \left(1 - \dfrac{1}{4}\right)^{-1/2} - \dfrac{3}{4} = \sqrt{\dfrac{2}{3}} - \dfrac{3}{4}$

433 **(c)**

$$\therefore 1-\log 2+\frac{(\log 2)^2}{2!}-\frac{(\log 2)^3}{3!}+\cdots$$
$$=e^{-\log 2}$$
$$=e^{\log 2^{-1}}=\frac{1}{2}$$

434 **(c)**

Let S_n and S' be the sums of n terms of two AP's and T_{11} and $T_{11}^{'}$ be the respective 11th term, then

$\frac{S_n}{S_n'}=\frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}(2a'+(n-1)d')}=\frac{7n+1}{4n+27}$ (given)

$$\Rightarrow \frac{a+\frac{(n-1)}{2}d}{a'+\frac{(n-1)}{2}d'}=\frac{7n+1}{4n+27}$$

Now put, $n=21$, we get

$$\frac{a+10d}{a'+10d'}=\frac{T_{11}}{T_{11}'}=\frac{148}{111}$$
$$=\frac{4}{3}$$

435 **(a)**

$2.\overline{357}=2+0.357+0.000357+\ldots$

$$\Rightarrow 2.\overline{357}=2+\frac{357}{10^3}+\frac{357}{10^6}+\ldots$$

$$\Rightarrow 2.\overline{357}=2+\frac{\frac{357}{10^3}}{1-\frac{1}{10^3}}=2+\frac{357}{999}=\frac{2355}{999}$$

436 **(c)**

Here, $T_n=\frac{1+2+3+\ldots+n}{n!}=\frac{n(n+1)}{2(n)!}\ \left[\because \sum n=\right.$ *n(n+1)2*

$$=\frac{(n+1)}{2(n-1)!}=\frac{1}{2(n-2)!}+\frac{1}{(n-1)!}$$

$$T_1=0+\frac{1}{1}$$
$$T_2=\frac{1}{2}.\frac{1}{1}+\frac{1}{1!}$$
$$T_3=\frac{1}{2}.\frac{1}{1}+\frac{1}{2!}$$
$$T_4=\frac{1}{2}.\frac{1}{2}+\frac{1}{3!}$$
$$\vdots \quad \vdots$$
$$\vdots \quad \vdots$$

$\therefore S=\Sigma T_n$

$$=\frac{1}{2}\left(1+\frac{1}{1!}+\frac{1}{2!}+\ldots\right)+\left(1+\frac{1}{1!}+\frac{1}{2!}+\ldots\right)$$
$$=\frac{1}{2}e+e=\frac{3}{2}e$$

437 **(b)**

Let $S=3+7+13+21+\ldots+T_n$

$\Rightarrow T_n=n^2+n+1$

Let $T_r=\cot^{-1}(r^2+r+1)$

$=\tan^{-1}(r+1)-\tan^{-1}r$

Put $r=1,2,\ldots,n$

$T_1=\tan^{-1}2-\tan^{-1}1$

$T_2=\tan^{-1}3-\tan^{-1}2$

$\cdots\ \cdots\ \cdots$

$\cdots\ \cdots\ \cdots$

$T_n=\tan^{-1}(n+1)-\tan^{-1}n$

On adding all these, we get

$T_1+T_2+\ldots+T_n$

$$=\tan^{-1}(n+1)-\tan^{-1}1$$
$$=\tan^{-1}\left(\frac{n}{n+2}\right)=\cot^{-1}\left(\frac{n+2}{n}\right)$$

439 **(a)**

Let $x^{18}=y^{21}=z^{28}=k$

Then,

$18\log x=21\log y=28\log z=\log k$

$$\Rightarrow \log_y x=\frac{21}{18},\log_z y=\frac{28}{21},\log_x z=\frac{18}{28}$$

$$\Rightarrow 3\log_y x=\frac{7}{2},3\log_z y=4,7\log_x z=\frac{9}{2}$$

$\Rightarrow 3,3\log_y z,3\log_z y,7\log_x z$ are in A.P.

440 **(b)**

For $0<x<\pi/2$, we have $0<\sin^2 x<1$

$\therefore y=\exp[(\sin^2 x+\sin^4 x+\sin^6 x+\cdots\infty)\log_e 2]$

$$\Rightarrow y=\exp\left[\left(\frac{\sin^2 x}{1-\sin^2 x}\right)\log_e 2\right]$$
$$=\exp[\tan^2 x\log_2 2]$$

$$\Rightarrow y=e^{\log_e 2^{\tan^2 x}}=2^{\tan^2 x}$$

Since y satisfies the equation $x^2-9x+8=0$.

Therefore,

$y^2-9y+8=0 \Rightarrow (y-1)(y-8)=0 \Rightarrow y=1$ or, $y=8$

Now,

$y=1\Rightarrow 2^{\tan^2 x}=1\Rightarrow 2^{\tan^2 x}=2^0\Rightarrow \tan x=0$

$\Rightarrow x=0$

But, $0<x<\pi/2$. Therefore, $y\neq 1$.

Consequently, we have

$y=8\Rightarrow 2^{\tan^2 x}=2^3\Rightarrow \tan^2 x=3\Rightarrow \tan x=\sqrt{3}$

$\Rightarrow x=\pi/3$

$$\therefore \frac{\sin x+\cos x}{\sin x-\cos x}=\frac{\tan x+1}{\tan x-1}=\frac{\sqrt{3}+1}{\sqrt{3}-1}=2+\sqrt{3}$$

441 **(c)**

$$\left(a+\frac{1}{p}-\frac{1}{q}\right)^2+\left(a+\frac{1}{q}-\frac{1}{r}\right)^2+\left(a+\frac{1}{r}-\frac{1}{s}\right)^2\leq 0$$

$$\Rightarrow \frac{1}{p}-\frac{1}{q}=\frac{1}{q}-\frac{1}{r}=\frac{1}{r}-\frac{1}{s}$$

$\Rightarrow p,q,r,s$ are in HP.

442 **(d)**

$$\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}-\dots$$
$$=\left(1-\frac{1}{2}\right)-\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)-\dots$$
$$=2\left(1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots\right)-1=\log_e\frac{4}{e}$$

443 **(b)**
We have,
$x^{\log_x(x^2-4x+5)}=x-1$
$\Rightarrow x^2-4x+5=x-1\Rightarrow x^2-5x+6=0\Rightarrow x=2,3$

444 **(b)**
It is given that
$y-x, 2(y-a), (y-z)$ are in H.P.
$\Rightarrow \frac{1}{y-x}, \frac{1}{2(y-a)}, \frac{1}{y-z}$ are in A. P.
$\Rightarrow \frac{1}{2(y-a)}-\frac{1}{y-x}=\frac{1}{y-z}-\frac{1}{2(y-a)}$
$\Rightarrow \frac{2a-y-x}{y-x}=\frac{y+z-2a}{y-z}$
$\Rightarrow \frac{(x-a)+(y-a)}{(x-a)-(y-a)}=\frac{(y-a)+(z-a)}{(y-a)-(z-a)}$
$\Rightarrow \frac{x-a}{y-a}=\frac{y-a}{z-a}$
$x-a,\ y-a,\ z-a$ are in G.P.

445 **(b)**
The some of n terms of given series$=\frac{n(n+1)^2}{2}$ if n is even. Let n is odd $ie, n=2m+1$
Then, $S_{2m+1}=S_{2m}+(2m+1)$th term
$=\frac{(n-1)n^2}{2}+n$th term
$=\frac{(n-1)n^2}{2}+n^2$ $[\because n \text{ is odd }=2m+1]$
$=n^2\left[\frac{n-1+2}{2}\right]=\frac{(n+1)n^2}{2}$

446 **(c)**
LHS $=\frac{1(1-\lambda^{n+1})}{1-\lambda}=\frac{1-\lambda^{n+1}}{1-\lambda}$
And RHS $=(1+\lambda)(1+\lambda^2)(1+\lambda^4)(1+\lambda^8)(1+\lambda^{16})$
$$=\frac{(1-\lambda)(1+\lambda)(1+\lambda^2)(1+\lambda^4)(1+\lambda^8)(1+\lambda^{16})}{(1-\lambda)}$$
$$=\frac{(1-\lambda^2)(1+\lambda^2)(1+\lambda^4)(1+\lambda^8)(1+\lambda^{16})}{1-\lambda}$$
$$=\frac{(1-\lambda^{32})}{1-\lambda}$$
$\therefore \frac{1-\lambda^{n+1}}{1-\lambda}=\frac{1-\lambda^{32}}{1-\lambda}$
$\Rightarrow 1-\lambda^{n+1}=1-\lambda^{32}$
$\therefore n+1=32 \Rightarrow n=31$

447 **(a)**
$\because (x+1)+(x+4)+(x+7)+\dots+(x+28)=155$
Let n be the number of terms in the AP on LHS.
$\therefore x+28=(x+1)+(n-1)3$
$\Rightarrow n=10$
$\therefore \frac{10}{2}[(x+1)+(x+28)]=155$
$\Rightarrow x=1$

448 **(a)**
Let $'r'$ be the common ratio,
$\therefore \frac{\sum_{n=1}^{100}a_{2n}}{\sum_{n=1}^{100}a_{2n-1}}=\frac{a_2+a_4+a_6+\dots+a_{200}}{a_1+a_3+a_5+\dots+a_{199}}$
$=\frac{a_1(r+r^3+r^5+\dots+r^{199})}{a_1(1+r^2+r^4+\dots+r^{198})}=r$
$\Rightarrow \frac{\alpha}{\beta}=r$

449 **(a)**
Let $S=2+7+14+23+34+\dots+T_n$ ...(i)
and $S=2+7+14+23+34+\dots T_{n-1}+T_n$ ...(ii)
On subtracting Eqs. (i) from (ii),we get
$\therefore S-S=2+[5+7+9+11+\dots+T_n-T_{n-1}]-T_n$
$\Rightarrow T_n=2+\left[\frac{n-1}{2}\{2\times5+(n-2)2\}\right]$
$\Rightarrow T_n=2+(n-1)(n+3)$
$\therefore T_{99}=2+98\times102=9998$

450 **(b)**
Here, $(a^2+b^2+c^2)p^2-2(ab+bc+cd)p+(b^2+c^2+d^2)\le 0$
$\Rightarrow (a^2p^2-2abp+b^2)+(b^2p^2-2bcp+c^2)+(c^2p^2-2cdp+d^2)\le 0$
$\Rightarrow (ap-b)^2+(bp-c)^2+(cp-d)^2\le 0$
(Since sum of squares is never less than zero).
$\Rightarrow$ Each of the square is zero.
$\therefore (ap-b)^2=(bp-c)^2=(cp-d)^2=0$
$\Rightarrow p=\frac{b}{a}=\frac{c}{b}=\frac{d}{c}$
$\therefore a,b,c,d$ are in GP.

451 **(c)**
Since, $\frac{\frac{2p}{3}+\frac{2p}{3}+\frac{2p}{3}+\underbrace{\frac{3q}{5}+\dots+\frac{3q}{5}}_{5\text{times}}+\underbrace{\frac{4r}{7}+\dots+\frac{4r}{7}}_{7\text{times}}}{15}$
$\ge 15\sqrt{\left(\frac{2p}{3}\right)^3\left(\frac{3q}{5}\right)^5\left(\frac{4r}{7}\right)^7}$ $(\because \text{AM}\ge\text{GM})$

$\Rightarrow\ p^3q^5r^7\ \dfrac{2^3 3^5 4^7}{3^3 5^5 7^7} \le 1$

$\Rightarrow\ p^3q^5r^7 \le \frac{5^5\ 7^7}{2^3\ 3^2\ 4^7}$

452 **(c)**

Let $S = 1 + 10 + 10^2 + \ldots + 10^{90}$

$= \dfrac{1 \cdot (10^{91}-1)}{10-1} = \dfrac{(10^{13})^7 - 1}{10^{13}-1} \times \dfrac{10^{13}-1}{10-1}$

$= [(10^{13})^6 + (10^{13})^5 + (10^{13})^4 + \ldots + 1] \times (10^{12} + 10^{11} + \ldots +)$

$\therefore$ It is the product of two integers and hence not prime.

453 **(d)**

Let $S = 1 + 2x + 3x^2 + 4x^3 + \ldots\infty$ ...(i)

$xS = x + 2x^2 + 3x^3 + \ldots\infty$ ...(ii)

Subtracting Eq. (ii) from Eq. (i), we get

$(1-x)S = 1 + x + x^2 + x^3 + \ldots\infty$

$\Rightarrow\ S = \dfrac{1}{(1-x)}\left(\dfrac{1}{1-x}\right) = \dfrac{1}{(1-x)^2}$

454 **(c)**

We have,

$5^{\sqrt{\log_5 7}} - 7^{\sqrt{\log_7 5}}$

$= 5^x - 7^{\frac{1}{x}}$, where $x = \sqrt{\log_5 7}$

$= 5^x - \left(5^{x^2}\right)^{\frac{1}{x}}$ $\left[\because x = \sqrt{\log_5 7} \Rightarrow x^2 = \log_5 7 \Rightarrow 7 = 5^{x^2}\right]$

$= 5^x - 5^x = 0$

455 **(b)**

We have, $\frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3}, \ldots, \frac{1}{x_n}$ are in AP.

$\therefore \frac{1}{x_2} - \frac{1}{x_1} = \frac{1}{x_3} - \frac{1}{x_2} = \ldots = \frac{1}{x_n} - \frac{1}{x_{n-1}} = d$ (say)

$\therefore\ \dfrac{x_1 - x_2}{x_1x_2} = \dfrac{x_2 - x_3}{x_2x_3} = \ldots = \dfrac{x_{m-1} - x_n}{x_{n-1}x_n} = d$

Now, $x_1x_2 + x_2x_3 + \ldots + x_{n+1}x_n$

$= \dfrac{1}{d}[x_1 - x_2 + x_2 - x_3 + \cdots + x_{n-1} - x_n]$

$= \dfrac{x_1 - x_n}{d}$

But $\frac{1}{x_n} = \frac{1}{x_1} + (n-1)d$

$\therefore\ \dfrac{x_1 - x_n}{x_1x_n} = (n-1)d$

or $\frac{x_1 - x_n}{d} = (n-1)x_1x_n$

$\therefore x_1x_2 + x_2x_3 + \ldots + x_{n-1}x_n = (n-1)x_1x_n$

456 **(d)**

Given a, b, c are in GP and $4a, 5b, 4c$ are in AP.

$\therefore\ b^2 = ac$ and $5b = \frac{4a+4c}{2}$

$\Rightarrow b^2 = ac$ and $5b = 2a + 2c$

Now, $a + b + c = 70$ (given)

$\Rightarrow 2a + 2c + 2b = 140$

$\Rightarrow\ 5b + 2b = 140$

$\Rightarrow\ b = 20$

457 **(d)**

Since, p, q and r in HP.

$\Rightarrow q = \frac{2pr}{p+r} \Rightarrow \frac{q}{2} = \frac{pr}{p+r} = K$ (say)

$\Rightarrow q = 2K, pr = (p+r)K$

Also, p^2, q^2, r^2 are in AP.

$\therefore\ 2q^2 = p^2 + r^2 = (p+r)^2 - 2pr$

$\Rightarrow 8K^2 = (p+r)^2 - 2(p+r)K$

$\Rightarrow (p+r)^2 - 2(p+r)K - 8K^2 = 0$

$\Rightarrow p + r = 4K, -2K$

When $p + r = 4K$, then $pr = 4K^2$

$\therefore (p-r)^2 = (p+r)^2 - 4pr = 16K^2 - 16K^2 = 0$

$\Rightarrow\ p = r$

But this is not possible ($\because p \neq r$)

$\therefore p += -2K\ \Rightarrow pr = -2K \cdot K = -2K^2$

Now, $(p-r)^2 = (p+r)^2 - 4pr$

$= 4K^2 - 4(-2K^2) = 12\ K^2$

$\Rightarrow p - r = \pm 2\sqrt{3}K$

$\Rightarrow p = \left(-1 \pm \sqrt{3}\right)K$

And $2r = -2K \mp \sqrt{3}K$

$\Rightarrow\ r = \left(-1 \mp \sqrt{3}\right)K$

$\therefore p : q : r = \left(-1 \mp \sqrt{3}\right)K : 2K : \left(-1 \mp \sqrt{3}\right)K$

$= -1 \mp \sqrt{3} : 2 : -1 \mp \sqrt{3}$

$= (-1 \mp \sqrt{3} : (-2) : (-1 \mp \sqrt{3})$

458 **(a)**

Given, $x = 1 + \frac{2}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \ldots$

$\Rightarrow x = e^2$

$\Rightarrow x^{-1} = e^{-2}$

459 **(a)**

Since, $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots\infty = \frac{\pi^4}{90}$

$\Rightarrow \dfrac{\pi}{90} = \left(\dfrac{1}{1^4} + \dfrac{1}{3^4} + \ldots\infty\right) + \left(\dfrac{1}{2^4} + \dfrac{1}{4^4} + \dfrac{1}{6^4} + \ldots\infty\right)$

$\Rightarrow \dfrac{\pi^4}{90} = \left(\dfrac{1}{1^4} + \dfrac{1}{3^4} + \ldots\infty\right) + \dfrac{1}{2^4}\left(\dfrac{1}{1^4} + \dfrac{1}{2^4} + \dfrac{1}{3^4} + \ldots\infty\right)$

$\Rightarrow \dfrac{\pi^4}{90} = \left(\dfrac{1}{1^4} + \dfrac{1}{3^4} + \ldots\infty\right) + \dfrac{1}{16}\left(\dfrac{\pi^4}{90}\right)$

$\Rightarrow \dfrac{\pi^4}{96} = \dfrac{1}{1^4} + \dfrac{1}{3^4} + \ldots\infty$

460 **(a)**

We have,

$(x+y) + (x^2 + xy + y^2) + (x^3 + x^2y + x\,y^2 + y^3) + \cdots\infty$

$$=\frac{x^2-y^2}{x-y}+\frac{x^3-y^3}{x-y}+\frac{x^4-y^4}{x-y}+\cdots \text{ to } \infty$$

$$=\frac{1}{x-y}\{(x^2+x^3+x^4+\cdots)$$

$$-(y^2+y^3+y^4+\cdots)\}$$

$$=\frac{1}{x-y}\left\{\frac{x^2}{1-x}-\frac{y^2}{1-y}\right\}=\frac{x+y-xy}{1-x-y+xy}$$

461 **(d)**

Let a, H_1, H_2, b are in HP.

$\therefore\ H_1=\frac{3ab}{a+2b},\ \ H_2=\frac{3ab}{2a+b}$

Now, $\frac{H_1+H_2}{H_1H_2}=\frac{1}{H_1}+\frac{1}{H_2}$

$=\frac{2a+b}{3ab}+\frac{a+2b}{3ab}=\frac{a+b}{ab}$ ...(i)

Also, $2A=a+b$ ...(ii)

and $ab=G^2$ ...(iii)

From Eqs. (i), (ii) and (iii), we get

$$\frac{H_1+H_2}{H_1H_2}=\frac{2A}{G^2}$$

462 **(a)**

We have,

$$704\left\{1+\frac{1}{2}+\frac{1}{2^2}+\cdots+\frac{1}{2^{n-1}}\right\}$$

$$=1984\left\{1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\cdots+\left(-\frac{1}{2}\right)^{n-1}\right\}$$

$$\Rightarrow 704\left\{\frac{1-\frac{1}{2^n}}{1-\frac{1}{2}}\right\}=1984\left\{\frac{1-\left(-\frac{1}{2}\right)^n}{1+\frac{1}{2}}\right\}$$

$$\Rightarrow 128=\frac{2112}{2^n}-\frac{1984(-1)^n}{2^n}$$

If n is odd, we get $2^n=32 \Rightarrow n=5$

If n is even, we get $128=\frac{128}{2^n}\Rightarrow n=0$

463 **(a)**

Let $S=1+\frac{3}{2}+\frac{7}{4}+\frac{15}{8}+\frac{31}{16+\ldots}$

$$=1+\frac{(4-1)}{2}+\frac{(8-1)}{4}+\frac{(16-1)}{8}+\frac{(32-1)}{16}+\ldots$$

$$=1+2-\frac{1}{2}+2-\frac{1}{4}+2-\frac{1}{8}+2-\frac{1}{16}+\ldots$$

$$=1+2(n-1)-\left[\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+(n-1)\right]$$

$$1+2(n-1)-\left[\frac{\frac{1}{2}\left(1-\frac{1}{2^{n-1}}\right)}{1-\frac{1}{2}}\right]$$

$$=1+2(n-1)-1+\frac{1}{2^{n-1}}=2(n-1)+\frac{1}{2^{n-1}}$$

464 **(b)**

Since, $y=\frac{2xz}{x+z}$

Now, $x-2y+z=x+z-2\left(\frac{2xz}{x+z}\right)$

$$=x+z-\frac{4xz}{x+z}=\frac{(x-z)^2}{x+z}$$

$\Rightarrow \log(x-2y+z)=\log(x-z)^2-\log(x+z)$

$\Rightarrow \log(x-2y+z)+\log(x+z)$

$=2\log(x-z)$

465 **(d)**

Since, $\frac{a+ar+ar^2}{a+ar+ar^2+ar^3+ar^4+ar^5}=\frac{125}{162}$

$$\Rightarrow \frac{1+r+r^2}{(1+r+r^2)(1+r^3)}=\frac{125}{162}$$

$$\Rightarrow 1+r^3=\frac{152}{125}$$

$$\Rightarrow r^3=\frac{27}{125}=\left(\frac{3}{5}\right)^3$$

$$\Rightarrow r=\frac{3}{5}$$

466 **(c)**

Let T_r be the rth term of the given series. Then,

$$T_r=1+x+x^2+\cdots+x^{r-1}=\frac{1-x^r}{1-x}$$

$\therefore$ Required sum

$$=\sum_{r=1}^{n}T_r$$

$$=\sum_{r=1}^{n}\frac{1-x^r}{1-x}=\frac{1}{1-x}\sum_{r=1}^{n}(1-x^r)$$

$$\Rightarrow \text{Required sum}=\frac{1}{1-x}\left(\sum_{r=1}^{n}1-\sum_{r=1}^{n}x^r\right)$$

$$\Rightarrow \text{Required sum}=\frac{1}{1-x}\left\{n-x\left(\frac{1-x^n}{1-x}\right)\right\}$$

$$\Rightarrow \text{Required sum}=\frac{n(1-x)-x(1-x)^n}{(1-x)^2}$$

467 **(b)**

Since, a, b, c are in GP.

$\Rightarrow\ b^2=ac$

And $\log a-\log 2b, \log 2b-\log 3c$ and $\log 3c-\log a$ are in AP.

$\Rightarrow 2(\log 2b-\log 3c)$

$=\log a-\log 2b+\log 3c-\log a$

$\therefore\ b^2=ac$ and $2b=3c$

$\Rightarrow\ b=\frac{2a}{3}$ and $c=\frac{4a}{9}$

Since, $a+b=\frac{5a}{3}>c, b+c=\frac{10a}{9}>a,$

$c+a=\frac{13a}{9}>b$

It implies that a, b, c form a triangle with a as the greatest side.

Now, let us find the greatest angle A of $\Delta\, ABC$ by using the cosine formula.

$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{\frac{4a^2}{9}+\frac{16a^2}{81}-a^2}{\frac{4a}{3}\cdot\frac{4a}{9}}$$

$$=-\frac{29}{48}<0$$

$\therefore$ The angle A is obtuse.

469 **(b)**

Required sum

$$=\sum_{n=0}^{\infty}\frac{(\log_e x)^n}{n\,!}=e^{\log_e x}=x$$

470 **(b)**

Sum of an infinite GP $=\frac{a}{1-r}=S$

$$\Rightarrow\ a=S(1-r)\ \Rightarrow\ r=\frac{S-a}{S}$$

471 **(b)**

We have,

$$\sum_{n=1}^{\infty}\frac{2\,n}{(2n+1)\,!}=\sum_{n=1}^{\infty}\frac{2n+1-1}{(2n+1)\,!}$$

$$\Rightarrow\sum_{n=1}^{\infty}\frac{2\,n}{(2n+1)\,!}=\sum_{n=1}^{\infty}\left\{\frac{1}{(2n)\,!}-\frac{1}{(2n+1)\,!}\right\}$$

$$\Rightarrow\sum_{n=1}^{\infty}\frac{2\,n}{(2n+1)}=\frac{1}{2\,!}-\frac{1}{3\,!}+\frac{1}{4\,!}-\frac{1}{5\,!}+\frac{1}{6\,!}-\frac{1}{7\,!}+\cdots=e^{-1}$$

472 **(d)**

Let $S=1+3x+6x^2+10x^3+\ldots\infty$ ...(i)

$xS=x+3x^2+6x^3+\ldots\infty$ ...(ii)

On subtracting Eq. (ii) from Eq. (i), we get

$S(1-x)=1+2x+3x^2+4x^3+\ldots\infty$ ...(iii)

$\Rightarrow\ x(1-x)S=x+2x^2+3x^3+\ldots\infty$ ...(iv)

Again, subtracting Eq. (iv) from Eq. (iii), we get

$$S[(1-x)-x(1-x)]=(1+x+x^2+x^3+\ldots\infty)$$

$$\Rightarrow S[(1-x)(1-x)]=\frac{1}{1-x}$$

$$\Rightarrow S=\frac{1}{(1-x)^3}$$

473 **(a)**

We have, $\frac{1}{n!}+\frac{1}{2!(n-2)!}+\frac{1}{4!(n-4)!}+\ldots\infty$

$$\frac{1}{n!}({}^nC_o+{}^nC_2+{}^nC_4+\ldots\infty)=\frac{2^{n-1}}{n!}$$

474 **(b)**

Let the first term and common difference of the A.P. be a and d respectively. Then,

Middle term $=30\Rightarrow 6th$ term $=30\Rightarrow a+5d=30$

Now,

$$S_{11}=\frac{11}{2}\{2a+10d\}=11\times(a+5d)=11\times 30=330$$

475 **(c)**

We have,

$(2.3)^x=(0.23)^y=1000$

$\Rightarrow 2.3=10^{3/x}$ and $0.23=10^{3/y}$

$\Rightarrow 2.3=10^{3/x}$ and $2.3=10^{3/y+1}$

$\Rightarrow \log_{10}2.3=\frac{3}{x}$ and $\log_{10}2.3=\frac{3}{y}+1$

$$\Rightarrow\frac{3}{x}-\frac{3}{y}=1\Rightarrow\frac{1}{x}-\frac{1}{y}=\frac{1}{3}$$

477 **(b)**

We have,

$\log_6(x+3)-\log_6 x=2$

$$\Rightarrow\log_6\left(\frac{x+3}{x}\right)=2\Rightarrow\frac{x+3}{x}=6^2\Rightarrow x+3=36x$$

$$\Rightarrow x=\frac{3}{35}$$

478 **(d)**

Let $S=1+3+7+15+\ldots+T_n$

$$\Rightarrow\frac{S=1+3+7+\ldots+T_{n-1}+T_n}{0=1+2+4+8+\ldots-T_n}$$

$\Rightarrow T_n=1+2+4+\ldots n$ terms

$$=\frac{(2^n-1)}{2-1}=2^n-1$$

$$\therefore\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\ldots=\sum\frac{T_n}{2^n}=\sum\frac{2^n-1}{2^n}$$

$$=\sum(1-2^{-n})$$

$$=n-\frac{\frac{1}{2}\left(1-\frac{1}{2^n}\right)}{1/2}=2^{-n}+n-1$$

479 **(a)**

Since, $T_7=a+6d=40$ ...(i)

and $S_{13}=\frac{13}{2}[2a+12d]$

$=13[a+6d]$

$=13\times 40=520$ [from Eq. (i)]

481 **(a)**

Since $\log_x a, a^{x/2}$ and $\log_b x$ are in G.P. Therefore,

$$\left(a^{x/2}\right)^2=\log_x a\cdot\log_b x\Rightarrow a^x=\log_b a\Rightarrow x=\log_a(\log_b a)$$

482 **(a)**

Let

$S = i - 2 - 3i + 4 + 5i \ldots. + 100\, i^{100}$

$\Rightarrow S = i + 2i^2 + 3i^3 + 4i^4 + 5i^5 \ldots. + 100\, i^{100}$

$\Rightarrow iS = i^2 + 2i^3 + 3i^4 \ldots. + 99\, i^{100} + 100\, i^{101}$

$\therefore S - iS = i + \{i^2 + i^3 + i^4 + \cdots + i^{100}\}$
$- 100\, i^{101}$

$$S(1-i) = i + i^2\left\{\frac{(1-i^{99})}{(1-i)}\right\} - 100\, i^{101}$$

$$\Rightarrow S(1-i) = i - \frac{(1+i)}{(1-i)} - 100\, i$$

$= i + 1 - 1 - i - 100\, i = -100\, i$

$$\Rightarrow S = \frac{-100\, i}{1-i} = -50\, i(1+i) = -50(i-1)$$

$= 50(1-i)$

483 **(a)**

We have,

$$\log_2 x + \log_4 x + \log_{16} x = \frac{21}{4}$$

$$\Rightarrow \log_2 x + \frac{1}{2}\log_2 x + \frac{1}{4}\log_2 x = \frac{21}{4}$$

$$\Rightarrow \frac{7}{4}\log_2 x = \frac{21}{4} \Rightarrow \log_2 x = 3 \Rightarrow x = 2^3 = 8$$

484 **(c)**

The given numbers are 13, 17, … , 97. This is an AP with the first term 13 and common difference 4. Let the number of term be n. Then

$97 = 13 + (n-1)4 \Rightarrow 4n = 88 \Rightarrow n = 22$

Therefore, the sum of the numbers

$$S = \frac{n}{2}(a + l)$$

$$= \frac{22}{2}[13 + 97] = 11(110) = 1210$$

485 **(c)**

Let $S_n = \frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \ldots + \frac{1}{(3n-1)(3n+2)}$

$$= \frac{1}{3}\left[\frac{1}{2} - \frac{1}{5} + \frac{1}{5} - \frac{1}{8} + \ldots\ldots + \frac{1}{3n-1} - \frac{1}{3n+2}\right]$$

$$= \frac{1}{3}\left[\frac{1}{2} - \frac{1}{3n+2}\right] = \frac{n}{6n+4}$$

486 **(a)**

We have, $4a^2 + 9b^2 + 16c^2 - 6ab - 12bc - 8ac = 0$

$\Rightarrow 8a^2 + 18b^2 + 32c^2 - 12ab - 24bc - 16ac = 0$

$\Rightarrow 4a^2 + 9b^2 - 12ab + 9b^2 + 16c^2 - 24bc$
$+ 16c^2 + 4a^2 - 16ac = 0$

$\Rightarrow (2a - 3b)^2 + (3b - 4c)^2 + (4c - 2a)^2 = 0$

$\Rightarrow 2a = 3b = 4c = k$

$$\Rightarrow a = \frac{k}{2}, b = \frac{k}{3}, c = \frac{k}{4}$$

$\Rightarrow a, b, c$ are in HP

GM $\geq$ HM

$\therefore \sqrt{ac} \geq b$

487 **(c)**

Let A_j, H_j where $j = 1, 2, 3, \ldots, 9$ denote the 9 AM's and HM's between 2 and 3.

Then $2, A_1, A_2, A_3, \ldots, A_9, 3$ are in AP, let d be the common difference of this AP, then

$$3 = 2 + 10d \Rightarrow d = \frac{1}{10}$$

If A denotes the jth arithmetic mean, then

$A = 2 + jd = 2 + \left(\frac{j}{10}\right) \quad \left(\because d = \frac{1}{10}\right)$ …(i)

Again, $2, H_1, H_2, \ldots, H_9, 3$ will be in HP.

$\Rightarrow \frac{1}{2}, \frac{1}{H_1}, \frac{1}{H_2}, \ldots \frac{1}{H_9}, \frac{1}{3}$ will be in AP

Let d be the common difference of this AP, then

$$\frac{1}{3} = \frac{1}{2} + 10d \Rightarrow d = \frac{-1}{60}$$

If H be the jth harmonic mean, then

$\frac{1}{H} = \frac{1}{2} + jd = \frac{1}{2} - \frac{j}{60}$ …(ii)

$\therefore A + \frac{6}{H} = 2 + \frac{j}{10} + 6\left(\frac{1}{2} - \frac{j}{60}\right)$ [from Eqs.(i) and (ii)]

$$= 5 + \frac{j}{10} - \frac{j}{10} = 5$$

488 **(c)**

Since a, b, c are in G.P.

$\therefore b^2 = ac$

$\Rightarrow 2\log_x b = \log_x a + \log_x c$

$$\Rightarrow \frac{2}{\log_b x} = \frac{1}{\log_a x} + \frac{1}{\log_c x}$$

$\Rightarrow \log_a x, \log_b x, \log_c x$ are in H.P.

489 **(b)**

$\because 13, a_1, a_2, \ldots, a_{20}, 67$ are in AP

$$\therefore a_1 + a_2 + a_3 + \ldots + a_{20} = 20\left(\frac{13 + 67}{2}\right) = 800$$

Also, AM > GM

$$\Rightarrow \frac{a_1 + a_2 + a_3 + \ldots + a_{20}}{20} \geq (a_1\, a_2\, a_3 \ldots a_{20})^{1/20}$$

$\Rightarrow 40 \geq (a_1 \cdot a_2 \cdot a_3 \ldots, a_{20})^{1/20}$

Hence, maximum value of $a_1 \cdot a_2 \cdot a_3 \ldots, a_{20}$ is $(40)^{20}$

490 **(a)**

We have,

$$\log_{10}\left(\frac{n}{n-1}\right) = \log_e\left(\frac{n}{n-1}\right) \cdot \log_{10} e$$

$$\Rightarrow \log_{10}\left(\frac{n}{n-1}\right) = -\log_e\left(\frac{n-1}{n}\right) \cdot \log_{10} e$$

$$\Rightarrow \log_{10}\left(\frac{n}{n-1}\right) = -\log_{10} e \cdot \log_e\left(1 - \frac{1}{n}\right)$$

$\Rightarrow \log_{10}\left(\frac{n}{n-1}\right)$

$$= \log_{10} e \left\{\sum_{r=1}^{\infty} \frac{1}{r}\left(\frac{1}{n}\right)^r\right\}$$

$$= \sum_{r=1}^{\infty}\left\{\frac{1}{r}\log_{10} e\right\} n^{-r}$$

$\therefore$ Coefficient of $n^{-r} = \frac{1}{r}\log_{10} e = \frac{1}{r\log_e 10}$

491 **(c)**

We have,

$$\frac{1}{1-x} - \frac{1}{1+\sqrt{x}} = \frac{\sqrt{x}}{1-x}$$

and, $\frac{1}{1-\sqrt{x}} - \frac{1}{1-x} = \frac{\sqrt{x}}{1-x}$

hence, the terms are in A.P.

492 **(d)**

$$\left[(0.16)^{\log_{0.25}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\cdots\right)}\right]^{1/2}$$

$$= \left[(0.16)^{\log_{0.25}\left(\frac{\frac{1}{3}}{1-\frac{1}{3}}\right)}\right]^{1/2}$$

$$= \left[(0.16)^{\log_{(0.5)^2} 0.5}\right]^{1/2}$$

$$= \left[(0.16)^{1/2}\right]^{1/2} = (0.4)^{1/2}$$

$$= \frac{2}{\sqrt{10}}$$

493 **(b)**

Number of notes that the person counts in 10 min

$$= 10 \times 150 = 1500$$

Since $a_{10}, a_{11}, a_{12}, \ldots$ are in AP with common difference -2

Let n be the time taken to count remaining 3000 notes, then

$$\frac{n}{2}[2 \times 148 + (n-1) \times -2] = 3000$$

$\Rightarrow \quad n^2 - 149n + 3000 = 0$

$\Rightarrow \quad (n-24)(n-125) = 0$

$\Rightarrow \quad n = 24, 125$

Then, the total time taken by the person to count all notes.

$$= 10 + 24 = 34 \text{ min}$$

494 **(d)**

The given series is an A.P. with first term $a = 20$ and common diference $d = 2\frac{2}{3} = 8/3$

Let S_n denote the sum of n terms. Then,

$S_n > 1568$

$\Rightarrow \frac{n}{2}\left[40 + (n-1)\frac{8}{3}\right] > 1568$

$\Rightarrow n^2 + 14n - 1176 > 0$

$\Rightarrow (n+42)(n-28) > 0$

$\Rightarrow n > 28$

$\Rightarrow$ The least value of n is 29

495 **(b)**

Let a_p, a_q, a_r, a_s be $p^{\text{th}}, q^{\text{th}}, r^{\text{th}}$ and s^{th} terms of the A.P. such that they are in G.P. with common ratio R.

$\therefore a_q = a_p R, a_r = a_p R^2$ and $a_s = a_p R^3$

$\Rightarrow a_q - a_p = a_p(R-1), a_r - a_q$

$= a_p R(R-1), a_s - a_r$

$= a_p R^2(R-1)$

$\Rightarrow a_q - a_p, a_r - a_q, a_s - a_r$ are in G.P.

$\Rightarrow a_p - a_q, a_q - a_r, a_r - a_s$ are in G.P.

$\Rightarrow (p-q)d, (q-r)d, (r-s)d$ are in G.P., where d is the common difference of the A.P.

$\Rightarrow p-q, q-r, r-s$ are in G.P.

496 **(a)**

Since, $2\tan^{-1} q = \tan^{-1} p + \tan^{-1} r$

$\Rightarrow \tan^{-1}\frac{2q}{1-q^2} = \tan^{-1}\frac{p+r}{1-p^r}$

$\Rightarrow \quad 2q = p + r \qquad [\therefore q^2 = pr]$

$\Rightarrow \quad p, q, r$ *are in* AP.

But p, q, r are in GP.

$\Rightarrow p = q = r$

498 **(c)**

It is given that $H_1, H_2, H_3, \ldots, H_n$ are n harmonic means between a and b. So,

$a, H_1, H_2, H_3, \ldots, H_n, b$ are in HP

$\Rightarrow \frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \ldots, \frac{1}{H_n}, \frac{1}{b}$ are in A.P. with common difference $D = \frac{a-b}{(n+1)ab}$

$\therefore \frac{1}{H_1} = \frac{1}{a} + D$ and $\frac{1}{H_n} = \frac{1}{a} + n\,D$

$\Rightarrow \frac{1}{H_1} = \frac{1}{a} + \frac{a-b}{(n+1)ab}$ and $\frac{1}{H_n} = \frac{1}{a} + \frac{n(a-b)}{(n+1)\,ab}$

$\Rightarrow \frac{1}{H_1} = \frac{nb+a}{(n+1)\,ab}$ and $\frac{1}{H_n} = \frac{n\,a+b}{(n+1)\,ab}$

$\Rightarrow \frac{H_1}{a} = \frac{nb+b}{nb+a}$ and $\frac{H_n}{b} = \frac{na+a}{na+b}$

$\Rightarrow \frac{H_1+a}{H_1-a} = \frac{2nb+(a+b)}{b-a}$ and $\frac{H_n+b}{H_n-b}$

$$= \frac{2na+a+b}{a-b}$$

$\Rightarrow \frac{H_1+a}{H_1-a}+\frac{H_n+b}{H_n-b}=2n$

499 **(a)**

We have,

$S_n=\frac{1}{2}\left\{\left(\sum_{r=1}^{n} r\right)^2-\sum_{r=1}^{n} r^2\right\}$

$\Rightarrow S_n=\frac{1}{2}\left[\left\{\frac{n(n+1)}{2}\right\}^2-\frac{n(n+1)(2n+1)}{6}\right]$

$\Rightarrow S_n=\frac{n(n^2-1)(3n+2)}{24}$

$\Rightarrow \frac{S_n}{(n+1)\,!}=\frac{1}{24}\left\{\frac{n(n^2-1)(3n+2)}{(n+1)\,!}\right\}$

$=\frac{1}{24}\left\{\frac{3n+2}{(n-2)\,!}\right\}$

$\Rightarrow \frac{S_n}{(n+1)\,!}=\frac{1}{24}\left\{\frac{3(n-2)+8}{(n-2)\,!}\right\}$

$=\frac{1}{8}\frac{1}{(n-3)\,!}+\frac{1}{3}\cdot\frac{1}{(n-2)\,!}$

$\therefore \sum_{n=0}^{\infty}\frac{S_n}{(n+1)\,!}=\frac{1}{8}\sum_{n=0}^{\infty}\frac{1}{(n-3)\,!}+\frac{1}{2}\sum_{n=0}^{\infty}\frac{1}{(n-2)\,!}$

$\Rightarrow \sum_{n=0}^{\infty}\frac{S_n}{(n+1)\,!}=\frac{e}{8}+\frac{e}{3}=\frac{11e}{24}$

501 **(b)**

If x, y, z are in HP, then

$y=\frac{2xz}{x+z}$ ...(i)

Now, $\log(x+z)+\log(x-2y+z)$

$=\log[(x+z)(x-2y+z)]$

$=\log\left[(x+z)\left(x+z-\frac{4xz}{x+z}\right)\right]$ [from Eq.(i)]

$=\log[(x+z)^2-4xz]$

$=\log(x-z)^2$

$=2\log(x-z)$

502 **(c)**

Let a be the first term and d be the common difference of the A.P. Then,

$a_m=a+(m-1)d$ ... (i)

$a_n=a+(n-1)d$ ... (ii)

$a_p=a+(p-1)d$ ... (iii)

Multiplying (i), (ii) and (iii) respectively by $(n-p),(p-m)$ and $(m-n)$ and adding, we get

$a_m(n-p)+a_n(p-m)+a_p(m-n)=0$ ...(iv)

Expanding along first row, we have

$\Delta=a_m(n-p)+a_n(p-m)+a_p(m-n)$

$\Rightarrow \Delta=0$ [Using (iv)]

503 **(c)**

Let $S=\frac{1}{n!}\left[\frac{n!}{1!(n-1)!}+\frac{n!}{3!(n-3)!}+\frac{n!}{5!(n-5)!}+\dots\right]$

$=\frac{1}{n!}[{}^nC_1+{}^nC_3+{}^nC_5+\dots]$

$=\frac{1}{n!}2^{n-1}$ for all value of n only

504 **(b)**

The given equation is meaningful if $x-1>0$ and $x-3>0$ i.e. $x>3$

Now,

$\log_4(x-1)=\log_2(x-3)$

$\Rightarrow \frac{1}{2}\log_2(x-1)=\log_2(x-3)$

$\Rightarrow \log_2(x-1)=2\log_2(x-3)$

$\Rightarrow \log_2(x-1)=\log_2(x-3)^2$

$\Rightarrow x-1=(x-3)^2$

$\Rightarrow x^2-7x+10=0 \Rightarrow x=5,2 \Rightarrow x=5$ $[\because x>3]$

Hence, the given equation has just one solution

505 **(a)**

Let T_r be the rth term of the given series. Then,

$T_r=1+2+2^2+\dots+2^r=2^{r+1}-1$

$\therefore$ Required sum $=\sum_{r=1}^{n}T_r=\sum_{r=1}^{n}(2^{r+1}-1)$

$\Rightarrow$ Required sum $=2^2\left(\frac{2^n-1}{2-1}\right)-n=2^{n+2}-n-4$

506 **(c)**

We have,

$1+\frac{(\log_e n)^2}{2\,!}+\frac{(\log_e n)^4}{4\,!}+\dots$ to ∞

$=\frac{e^{\log_e n}+e^{-\log_e n}}{2}=\frac{1}{2}(n+n^{-1})$

507 **(a)**

$\log_e 3-\frac{\log_e 9}{2^2}+\frac{\log_e 27}{3^2}-\frac{\log_e 81}{4^2}+\dots$

$=(\log_e 3)\left(1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}\dots\right)$

$=(\log_e 3)\log_e 2$

509 **(c)**

Given that, $T_m=a+(m-1)d=\frac{1}{n}$ ...(i)

And $T_n=a+(n-1)d=\frac{1}{m}$ ...(ii)

Where a and b are the first term and common difference respectively.

On solving Eqs. (i) and (ii), we get

$a=\frac{1}{mn}$ and $d=\frac{1}{mn}$

$\therefore\ T_{mn}=a+(mn-1)d$

$=\frac{1}{m}+(mn-1)\frac{1}{mn}=1$

510 **(a)**

Since, $l=A+(n-1)d$

$\therefore c=a+(n-1)(b-a)$

$\Rightarrow (n-1) = \dfrac{c-a}{b-a}$

$\Rightarrow n = \dfrac{b+c-2a}{b-a}$

511 **(c)**

Given, $x_n = x_{n+1}\sqrt{2}$

$\therefore\ x_1 = x_2\sqrt{2}, x_2 = x_3\sqrt{2}, \dots, x_n = x_{n+1}\sqrt{2}$

On multiplying $x_1 = x_{n+1}(\sqrt{2})^n$

$\Rightarrow x_{n+1} = x/(\sqrt{2})^n$

Hence, $x_n = \frac{x_1}{(\sqrt{2})^{n-1}}$

Area of $S_n = x_n^2 = \frac{x_1^2}{2^{n-1}} < 1 \Rightarrow 2^{n-1} > x_1^2$ $(\because x_1 = 10)$

$\therefore\ 2^{n-1} > 100$

But $2^7 > 100, 2^8 > 100$ etc.

$\therefore\ n-1 = 7, 8, 9, \dots \Rightarrow n = 8, 9, 10, \dots$

512 **(a)**

We have,

$x = \dfrac{1}{1-a}, y = \dfrac{1}{1-b}$

$\Rightarrow a = 1 - \dfrac{1}{x}, b = 1 - \dfrac{1}{y}$

$\Rightarrow \mathrm{a} = \dfrac{x-1}{x}, b = \dfrac{y-1}{y}$

$\therefore 1 + ab + a^2b^2 + \dots = \dfrac{1}{1-ab} = \dfrac{xy}{x+y-1}$

513 **(c)**

Given, $d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \dots = a_n - a_{n-1}$

$\therefore\ (\sin d)[\sec a_1 \sec a_2 + \sec a_2 \sec a_3 + \dots + \sec a_{n-1} \sec a_n]$

$= \dfrac{\sin d}{\cos a_1 \cos a_2} + \dfrac{\sin d}{\cos a_2 \cos a_3} + \dots + \dfrac{\sin d}{\cos a_{n-1} \cos a_n}$

$= \frac{\sin(a_2 - a_1)}{\cos a_1 \cos a_2} + \frac{\sin(a_3 - a_2)}{\cos a_2 \cos a_3} + \dots + \frac{\sin(a_n - a_{n-1})}{\cos a_{n-1} \cos a_n}$

$= \tan a_2 - \tan a_1 + \tan a_3 - \tan a_2 + \dots + \tan a_n - \tan a_{n-1}$

$= \tan a_n - \tan a_1$

515 **(c)**

Since, $a_1, a_2, a_3, \dots, a_n$ are in HP.

$\therefore \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \dots, \frac{1}{a_n}$ are in AP.

Let d be common difference of AP, then

$\frac{1}{a_2} - \frac{1}{a_1} = d$

$\Rightarrow\ a_1 - a_2 = a_1 a_2 d$

Similarly, $a_2 - a_3 = a_2 a_3 d$

$\dots\ \dots\dots\dots$

$\dots\ \dots\dots\dots$

$a_{n-1} - a_n = a_{n-1} a_n d$

On adding all of these, we get

$a_1 - a_n = d(a_1 a_2 + a_2 a_3 + \dots + a_{n-1} a_n)$ ...(i)

Also, $\frac{1}{a_n} = \frac{1}{a_1} + (n-1)d$

$\Rightarrow d = \dfrac{a_1 - a_2}{a_1 a_n (n-1)}$

On putting the value of d in Eq. (i), we get

$a_1 - a_n = \dfrac{a_1 - a_n}{a_1 a_n (n-1)}(a_1 a_2 + a_2 a_3 + \dots + a_{n-1} a_n)$

$\Rightarrow\ a_1 a_2 + a_2 a_3 + \dots + a_{n-1} a_n = a_1 a_n (n-1)$

517 **(b)**

We have,

$2^{\log_{10} 3\sqrt{3}} = 3^{k \log_{10} 2}$

$\Rightarrow (3\sqrt{3})^{\log_{10} 2} = 3^{k \log_{10} 2}$ $[\because x^{\log_a y} = y^{\log_a x}]$

$\Rightarrow 3^{\frac{3}{2}\log_{10} 2} = 3^{k \log_{10} 2} \Rightarrow k = \dfrac{3}{2}$

518 **(b)**

Since, $\beta\beta\gamma\delta = 1$ ...(i)

As we know, AM≥GM

$\Rightarrow \frac{1+\alpha}{2} \geq \sqrt{\alpha} \Rightarrow 1 + \alpha \geq 2\sqrt{\alpha}$ .....(ii)

Similarly, $1 + \beta \geq 2\sqrt{\beta}$ ... (iii)

$1 + \gamma \geq 2\sqrt{\gamma}$ ...(iv)

And $1 + \delta \geq 2\sqrt{\delta}$ ... (v)

On multiplying Eqs. (ii), (iii), (iv) and (v), we get

$(1+\alpha)(1+\beta)(1+\gamma)(1+\delta) \geq 16\sqrt{\alpha\beta\gamma\delta}$

Least value of $(1+\alpha)(1+\beta)(1+\gamma)(1+\delta) \geq 16$

519 **(b)**

Let $S = 6 + 66 + 666 + \dots n$ terms

$= \frac{6}{9}(9 + 99 + 999 + \dots n$ terms)

$= \frac{2}{3}[(10-1) + (100-1) + (1000-1) + \dots n$ terms]

$= \dfrac{2}{3}\left[10 \cdot \dfrac{(10^n - 1)}{9} - n\right] = \dfrac{2}{27}[10^{n+1} - 10 - 9n]$

520 **(b)**

$\frac{4}{3} + \frac{10}{9} + \frac{28}{27} + \dots$upto n terms

$= \left(1 + \frac{1}{3}\right) + \left(1 + \frac{1}{9}\right) + \left(1 + \frac{1}{27}\right) + \dots$upto n terms

$= (1 + 1 + 1 + \dots n$ terms$) + \left(\dfrac{1}{3} + \dfrac{1}{3^2} + \dfrac{1}{3^3} + \dots n \text{ terms}\right)$

$$= n + \frac{1}{3}\left(\frac{1-\frac{1}{3^n}}{1-\frac{1}{3}}\right) = \frac{3^n(2n+1)-1}{2(3^n)}$$

521 **(c)**

Since, a, b, c, d, e, f are in AP.

So, $b-a = c-b = d-c = e-d = f-e = k$

Where k is the common difference

Now, $d-c = e-d \Rightarrow e+c = 2d$

$\Rightarrow e-c+2c = 2d \Rightarrow e-c = 2(d-c)$

522 **(b)**

Let A and R be the first term and common ratio of the GP, then

$a = AR^{p-1}, b = AR^{q-1}$ and $= AR^{r-1}$ ...(i)

Again, if x and d be the first term and common difference of an AP corresponding to the given HP, then

$\frac{1}{a} = x+(p-1)d,\ \frac{1}{b} = x+(q-1)d,\ \frac{1}{c} = x+(r-1)d$ ...(ii)

From Eq.(i), $\frac{a}{b} = R^{p-q}$

$\Rightarrow \left(\frac{a}{b}\right)^{1/c} = (R^{p-q})^{1/c} = R^k,$

Where $k = \frac{p-q}{c} = (p-q)\{x+(r-1)d\}$ [from Eq. (ii)]

$= (p-q)x + (p-q)(r-1)d$

$= (p-q)x - (p-q)d + (rp-rq)d$ ...(iii)

Similarly, $\left(\frac{b}{c}\right)^{1/a} = (R^{q-r})^{1/a} = R^n,$

Where $n = \frac{(q-r)}{a} = (q-r)\times\{x+(p-1)d\}$ [from Eq.(ii)]

$\Rightarrow n = (q-r)x - (q-r)d + (pq-pr)d$ ...(iv)

And $\left(\frac{c}{a}\right)^{1/b} = (R^{r-p})^{1/b} = R^m$

Where $m = \frac{r-p}{b} = (r-p)\{x+(q-1)d\}$ [from Eq.(ii)]

$= (r-p)x(r-p)d + (rq-qp)d$ ...(v)

$\therefore \left(\frac{a}{b}\right)^{1/c}\left(\frac{b}{c}\right)^{1/a}\left(\frac{c}{a}\right)^{1/b} = R^kR^mR^n = R^{m+n+k} = R^0 = 1$

[Since, $k+m+n = 0$, adding Eqs. (iii), (iv) and (v)]

Taking log on both sides, we get

$$\frac{1}{c}(\log a - \log b) + \frac{1}{a}(\log b - \log c) + \frac{1}{b}(\log c - \log a) = 1\log(1)$$

$$\Rightarrow \left(\frac{1}{c}-\frac{1}{b}\right)\log a + \left(\frac{1}{a}-\frac{1}{c}\right)\log b + \left(\frac{1}{b}-\frac{1}{a}\right)\log c = 0$$

$$\Rightarrow \left(\frac{b-c}{bc}\right)\log a + \left(\frac{c-a}{ac}\right)\log b + \left(\frac{a-b}{ab}\right)\log c = 0$$

$$\Rightarrow a(b-c)\log a + b(c-a)\log b + c(a-b)\log c = 0$$

523 **(a)**

Here, $T_n = \sum_{n=1}^{\infty} \frac{1}{(n+a)(n+1+a)}$

$$= \sum_{n=1}^{\infty} \left(\frac{1}{n+a} - \frac{1}{n+1+a}\right)$$

$$\therefore S_n = \Sigma T_n = \left(\frac{1}{1+a} - \frac{1}{2+a}\right) + \left(\frac{1}{2+a} - \frac{1}{3+a}\right)$$

$$+ \ldots + \left(\frac{1}{n+a} - \frac{1}{n+1+a}\right)$$

$$\Rightarrow S_n = \frac{1}{1+a} - \frac{1}{n+1+a}$$

$$\Rightarrow \lim_{n\to\infty} S_n = \frac{1}{1+a}$$

524 **(b)**

The two sides of the equation are meaningful, if $-x > 0$ and $x + 1 > 0$ i.e. if $x \in (-1,0)$

Now,

$\log(-x) = 2\log(x+1)$

$\Rightarrow -x = (x+1)^2$

$$\Rightarrow x^2 + 3x + 1 = 0 \Rightarrow x = \frac{-3+\sqrt{5}}{2} \quad [\because x \in (-1,0)]$$

525 **(b)**

Let $S = 0.4\dot{2}\dot{3}$. Then,

$S = 0.42323232323 \ldots$

$\Rightarrow S = 0.4 + 0.023 + 0.00023 + \cdots$

$\Rightarrow S = 0.4 + 23 \times 10^{-3} + 23 \times 10^{-5} + \cdots$

$$\Rightarrow S = 0.4 + \frac{23 \times 10^{-3}}{1 - 10^{-2}} = 0.4 + \frac{23}{990} = \frac{419}{990}$$

526 **(b)**

$$\sum_{r=1}^{n}\sum_{s=1}^{n} S_{rs} 2^r 3^s = 2 \cdot 3 + 2^2 \cdot 3^2 + 2^3 \cdot 3^3 + \ldots + 2^n \cdot 3^n$$

(as $S_{rs} = 0$, if $r \neq s$ and $S_{rs} = 1$, if $r = s$)

$$= \frac{6(6^n - 1)}{6 - 1} = \frac{6}{5}(6^n - 1)$$

527 **(b)**

We have,

$$2b = a + c, d = \frac{2ce}{c+e} \text{ and } c^2 = bd$$

On eliminating b and d, we obtain

$c^2 = ae \Rightarrow a, c, e$ are in G.P.

528 **(a)**

$$2\left[\frac{1}{7} + \frac{1}{3}.\frac{1}{7^3} + \frac{1}{5}.\frac{1}{7^5} + \ldots\right] = \log_e\left[\frac{1 + 1/7}{1 - 1/7}\right]$$

$$= \log_e \frac{4}{3}$$

529 **(d)**

$t_{11} + t_{12} + t_{13} = 141$

And $t_{21} + t_{22} + t_{23} = 261$

$\therefore 3a + 33d = 141$

$\Rightarrow a + 11d = 47$ ...(i)

And $3a + 63d = 261$

$\Rightarrow a + 21d = 87$ ...(ii)

On solving Eqs. (i) and (ii), we get

$a = 3, \quad d = 4$

530 **(c)**

We have,

2^{n+10}

$= 2 \times 2^2 + 3 \times 2^3 + 4 \times 2^4 + \cdots + (n-1) \times 2^{n-1} + n \times 2^n$ ...(i)

$\Rightarrow 2 \times 2^{n+10}$

$= 2 \times 2^3 + 3 \times 2^4 + \cdots + (n-1)2^n + n \times 2^{n+1}$ ...(ii)

Subtracting (ii) from (i), we get

$-2^{n+10} = 2 \times 2^2 + (2^3 + 2^4 + \cdots + 2^n) - n \times 2^{n+1}$

$\Rightarrow -2^{n+10} = 8 + 8(2^{n-2} - 1) - n \times 2^{n+1}$

$-2^{n+10} = 2^{n+1} - n \times 2^{n+1}$

$\Rightarrow -2^{10} = 2 - 2n \Rightarrow n = 513$

531 **(c)**

As we know, sum infinite terms of GP,

$$S_\infty = \begin{cases} \frac{a}{1-r}, & |r| < 1 \\ \infty, & |r| \geq 1 \end{cases}$$

$\therefore \; S_\infty = \frac{x}{1-r} = 5 \qquad \{\text{thus } |r| < 1\}$

$$\Rightarrow 1 - r = \frac{x}{5}$$

$\Rightarrow r = \frac{5-x}{5}$ exists only when $|r| < 1$

$$\Rightarrow -1 < \frac{5-x}{5} < 1$$

$\therefore \; -10 < -x < 0 \Rightarrow 0 < x < 10$

532 **(c)**

For $x = -2$, we have

$$\log_4\left(\frac{x^2}{4}\right) - 2\log_4(4x^4)$$

$$= \log_4 1 - 2\log_{2^2}(2^6) = 0 - 2 \times \frac{6}{2}\log_2 2 = -6$$

533 **(c)**

We have,

$$\frac{\log 3}{x - y} = \frac{\log 5}{y - z} = \frac{\log 7}{z - x} = \lambda(\text{say})$$

$\Rightarrow \log 3 = \lambda(x-y), \log 5 = \lambda(y-z), \log 7$
$= \lambda(z-x)$
$\Rightarrow 3 = 10^{\lambda(x-y)}, 5 = 10^{\lambda(y-z)}, 7 = 10^{\lambda(z-x)}$
$\Rightarrow 3^{x+y}.5^{z+x}.7^{z+x}$
$= 10^{\lambda(x^2-y^2)}.10^{\lambda(y^2-z^2)}.10^{\lambda(z^2-x^2)}$
$\Rightarrow 3^{x+y}.5^{y+z}.7^{z+x} = 10^{\lambda(x^2-y^2+y^2-z^2+z^2-x^2)}$
$= 10^0 = 1$

534 **(d)**

$$\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j}1 = \sum_{i=1}^{n}\sum_{j=1}^{i}j$$

$$= \sum_{i=1}^{n}\frac{i(i+1)}{2} = \frac{1}{2}\sum_{i=1}^{n}(i^2+i)$$

$$= \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$$

$$= \frac{n(n+1)}{12}(2n+4)$$

$$= \frac{n(n+1)(n+2)}{6}$$

535 **(b)**

We have,

$\frac{2}{3!} + \frac{4}{5!} + \frac{6}{7!} + \cdots$ to ∞

$$= \sum_{n=1}^{\infty}\frac{2n}{(2n+1)} = \sum_{n=1}^{\infty}\frac{(2n+1)}{(2n+1)!}$$

$$= \sum_{n=1}^{\infty}\left\{\frac{1}{2n!} - \frac{1}{(2n+1)!}\right\}$$

$= \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!} - \frac{1}{7!} + \cdots$ to ∞

$= e^{-1}$

536 **(b)**

Let $S_n = \frac{1}{5}\left(\frac{1}{6} - \frac{1}{11} + \frac{1}{11} - \frac{1}{16} + \ldots + \frac{1}{5n+1} - \frac{1}{5n+6}\right)$

$$= \frac{1}{5}\left(\frac{1}{6} - \frac{1}{11} + \frac{1}{11} - \frac{1}{16} + \ldots + \frac{1}{5n+1} - \frac{1}{5n+6}\right)$$

$$= \frac{1}{5}\left(\frac{1}{6} - \frac{1}{5n+6}\right) = \frac{n}{6(5n+6)}$$

$$\Rightarrow 6S_n = \frac{n}{5n+6}$$

537 **(c)**

We have,

$\log(x-y) - \log 5 - \frac{1}{2}\log x - \frac{1}{2}\log y = 0$

$\Rightarrow 2\log(x-y) - 2\log 5 - \log x - \log y = 0$

$$\Rightarrow \frac{(x-y)^2}{25xy} = 1 \Rightarrow \left(\frac{x-y}{\sqrt{xy}}\right)^2 = 25$$

$$\Rightarrow \left(\sqrt{\frac{x}{y}} - \sqrt{\frac{y}{x}}\right)^2 = 25 \Rightarrow \frac{x}{y} + \frac{y}{x} - 2 = 25 \Rightarrow \frac{x}{y} + \frac{y}{x} = 27$$

538 **(b)**

It is given that

$\log_a x, \log_b x, \log_c x$ are in A.P.

$\Rightarrow 2\log_b x = \log_a x + \log_c x$

$$\Rightarrow \frac{2\log x}{\log b} = \frac{\log x}{\log a} + \frac{\log x}{\log c}$$

$$\Rightarrow \frac{2\log a \log c}{\log b} = \log a + \log c$$

$$\Rightarrow \frac{2\log a \log c}{\log b} = \log(ac)$$

$\Rightarrow 2\log c \log_b a = \log ac$

$\Rightarrow \log c^{2\log_b a} = \log ac$

$\Rightarrow c^{2\log_b a} = ac \Rightarrow (c^2) = (ac)^{\log_a b}$

539 **(a)**

Let $a^{1/x} = b^{1/y} = c^{1/z} = k$ [say]

$\Rightarrow \log a = x\log k, \log b = y\log k$

and $\log c = z\log k$

Since, $b^2 = ac$

$\Rightarrow 2\log b = \log a + \log c$

$\Rightarrow 2(y\log k) = x\log k + z\log k$

$\Rightarrow 2y = x + z$

$\Rightarrow x, y, z$ are in AP.

540 **(a)**

$(1-2x-x^2)(e^x)$

$$= (1-2x-x^2)\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^k}{k!} + \ldots\infty\right)$$

$$= \left(1 + x + \frac{x^2}{2!} + \ldots + \frac{x^k}{k!} + \ldots\infty\right)$$

$$-2\left(x + x^2 + \frac{x^3}{2!} + \ldots + \frac{x^k}{(k-1)!} + \frac{x^{k+1}}{k!} + \ldots\infty\right)$$

$$-\left(x^2+x^3+\frac{x^4}{2!}+\cdots\frac{x^k}{(k-2)!}+\frac{x^{k+1}}{(k-1)!}+\frac{x^{k+2}}{k!}+\ldots\infty\right)$$

∴Cofficient of x^kin $\left(\frac{1-2x-x^2}{e^{-x}}\right)=\frac{1}{k!}-\frac{2}{(k-1)!}-\frac{1}{(k-2)!}$

$$=\frac{1}{k!}-\frac{2k}{k!}-\frac{k(k-1)}{k!}$$

$$=\frac{1-k-k^2}{k!}$$

541 **(a)**

Here, T_n of the AP 1,2,3 ... $= n$

and T_n of the AP 3,5,7 ... $= 2n+1$

∴ $T_n$of given series$= n(2n+1)^2 = 4n^3 + 4n^2 + n$

Hence, $S=\sum_{n=1}^{20} T_n = 4\sum_{n=1}^{20} n^3 +$

4n=120n2+n=120n

$= 4\frac{1}{4}20^2.21^2+4\frac{1}{6}20.21.41+\frac{1}{2}20.21$

$= 188090$

542 **(c)**

$$\frac{b}{c+a}-\frac{a}{b+c}=\frac{c}{a+b}-\frac{b}{c+a}\quad [T_2-T_1 = T_3-T_2]$$

$$\Rightarrow \frac{b^2+bc-ac-a^2}{(c+a)(b+c)}=\frac{c^2+ac-ab-b^2}{(a+b)(c+a)}$$

$$\Rightarrow \{b^2-a^2-c(a-b)\}(a+b) = \{c^2-b^2-a(b-c)\}(b+c)$$

$$\Rightarrow (b^2-a^2)(b+a+c) = (c^2-b^2)(a+b+c)$$

$$\Rightarrow 2b^2=a^2+c^2$$

$\Rightarrow a^2,b^2,c^2$are in AP

543 **(a)**

Since, $\frac{(S_n)_1}{(S_n)_2}=\frac{2n+3}{6n+5}$ ...(i)

$$\Rightarrow \frac{\frac{n}{2}[2a_1+(n-1)d_1]}{\frac{n}{2}[2a_2+(n-1)d_2]}=\frac{2n+3}{6n+5}$$

$$\Rightarrow \frac{a_1+\frac{(n-1)}{2}d_1}{a_2+\frac{(n-1)}{2}d_2}=\frac{2n+3}{6n+5}$$

Put $\frac{n-1}{2}=12 \Rightarrow n=25$

$$\therefore \frac{a_1+12d_1}{a_2+12d_2}=\frac{53}{155}$$

$$\Rightarrow \frac{(T_{13})_1}{(T_{13})_2}=\frac{53}{155}$$

544 **(c)**

It is given that

$\frac{x+y}{1-xy}, y, \frac{y+z}{1-yz}$ are in A. P.

$$\Rightarrow y-\frac{x+y}{1-xy}=\frac{y+z}{1-yz}-y$$

$$\Rightarrow \frac{y-xy^2-x-y}{1-xy}=\frac{y+z-y+y^2z}{1-yz}$$

$$\Rightarrow -\frac{x}{1-xy}=\frac{z}{1-yz}$$

$$\Rightarrow -x+xyz=z-xyz$$

$$\Rightarrow 2\,xyz=x+z$$

$\Rightarrow y=\frac{x+z}{2\,xz} \Rightarrow \frac{1}{y}=\frac{2\,xz}{x+z} \Rightarrow x,\frac{1}{y},z$ are in H. P.

545 **(a)**

We have, $x+y+z=15$, if $9,x,y,z,a$ are in AP.

$\therefore$ Sum $=9+15+a=\frac{5}{2}(9+a)$

$$\Rightarrow 24+a=\frac{5}{2}(9+a)$$

$$\Rightarrow 48+2a=45+5a$$

$\Rightarrow 3a=3 \Rightarrow a=1$ ...(i)

And $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+\frac{5}{3}$, if $9,x,y,z,a$ are in HP.

Sum $=\frac{1}{9}+\frac{5}{3}+\frac{1}{a}=\frac{5}{2}\left[\frac{1}{9}+\frac{1}{a}\right] \Rightarrow a=1$

546 **(d)**

Since, a,b,c are in AP.

(A)$\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in HP $\Rightarrow \frac{k}{a},\frac{k}{b},\frac{k}{c}$ are in HP.

(B)$a+k, b+k, c+k$ are in AP.

(C)ka, kb, kc are in AP.

(D)a^2,b^2,c^2 are in AP.

Then, $b^2-a^2=c^2-b^2$

$$\therefore (b-a)(b+a)=(c-b)(c+b) = (b-a)(c+b)$$

$\left[\begin{array}{l}\because a,b,c \text{ are in AP}\\ \therefore b-a=c-b\end{array}\right]$

$$\Rightarrow b+a=c+b$$

$\Rightarrow a=c$, which is not true

547 **(d)**

We have,

$$\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\cdots\infty=\frac{\frac{1}{3}}{1-\frac{1}{3}}=\frac{1}{2}$$

$$\therefore \log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\cdots\infty\right)$$

$$=\log_{2.5}^{\left(\frac{1}{2}\right)}=\log_{\left(\frac{2}{5}\right)}-1\,(2)^{-1}$$

$$=\log_{(2/5)}2=\log_{0.4}2$$

$$\therefore (0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\cdots\infty\right)}$$

$$=(0.16)^{\log_{0.4}2}$$

$= \{(0.4)^2\}^{\log_{0.4} 2} = (0.4)^{2\log_{0.4} 2} = (0.4)^{\log_{0.4} 2^2}$
$= 2^2 = 4$

548 **(a)**
We have,
$\frac{3+5+7+\dots+n \text{ (terms)}}{5+8+11+\dots+10 \text{ terms}} = 7$
$\Rightarrow \frac{\frac{n}{2}\{6+(n-1)2\}}{\frac{10}{2}\{10+(10-1)3\}} = 7$
$\Rightarrow \frac{n(n+2)}{5\times 37} = 7$
$\Rightarrow n^2+2n = 35\times 37 \Rightarrow (n+37)(n-35) = 0$
$\Rightarrow n = 35$

550 **(c)**
Let $a_2 = ra_1, a_3 = r^2a_1, \dots$,so on
$\therefore \frac{a_1 - a_3 + a_5 - \dots + a_{49}}{a_2 - a_4 + a_6 - \dots + a_{50}}$
$= \frac{a_1 - r^2a_1 + r^4 a_1 - \dots + r^{48}a_1}{a_1r - r^3a_1 + r^5a_1 - \dots + r^{49}a_1}$
$= \frac{a_1(1 - r^2 + r^4 - \dots + r^{48})}{a_1r(1 - r^2 + r^4 - \dots + r^{48})}$
$= \frac{1}{r} = \frac{a_1}{a_2}$

551 **(c)**
As a, b, c, d are in HP. So, b is HM between a and c.
Also, GM between a and $c = \sqrt{ac}$.
Now, GM > HM
$\Rightarrow \sqrt{ac} > b$
$\Rightarrow ac > b^2$ ...(i)
Again, a, b, c, d are in HP. So c is HM between b and d.
Therefore, $bd > c^2$ ...(ii)
On multiplying relations (i) and (ii), we get
$abcd > b^2c^2 \Rightarrow ad > bc$
Hence, option (b) is true.
Now, AM between a and $c = \frac{1}{2}(a+c)$
Now, as AM > HM
So, here $a + c > 2b$ ...(iii)
And c is HM between b and d
$\Rightarrow b + d > 2c$ ...(iv)
On adding relations (iii) and (iv), we get
$(a+c)+(b+d) > 2(b+c)$
$\Rightarrow a + d > b + c$
So, both (a) and (b) are correct.

552 **(d)**
The sum to infinity of the given G.P. exists, iff.
$\left|\frac{3}{x}\right| < 1 \Leftrightarrow |x| > 3$

554 **(d)**
We have,
$9a^2 + 4b^2 = 18ab$
$\Rightarrow 9a^2 + 12ab + 4b^2 = 30ab$
$\Rightarrow (3a+2b)^2 = 30ab$
$\Rightarrow 2\log(3a+2b) = \log(5a \times 3b \times 2)$
$\Rightarrow \log(3a+2b) = \frac{1}{2}\{\log 5a + \log 3b + \log 2\}$

555 **(d)**
We have,
$S = \sum_{n=1}^{\infty}\left\{\frac{{}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n}{{}^nP_n}\right\} = \sum_{n=1}^{\infty}\frac{2^n}{n!}$
$= e^2 - 1$

556 **(a)**
The number of divisors of $3 \times 7^3 = (1+1)(3+1) = 8$
The number of divisors of $7 \times 11^2 = (1+1)(2+1) = 6$
And the number of divisors of $2 \times 61 = (1+1)(1+1) = 4$
$\Rightarrow$ 8, 6, 4 are in AP with common difference-2

557 **(b)**
We have,
$\frac{1}{a} + \frac{1}{c} + \frac{1}{a-b} + \frac{1}{c-b} = 0$
$\Rightarrow \frac{1}{a} + \frac{1}{c-b} = \frac{1}{b-a} - \frac{1}{c}$
$\Rightarrow \frac{a+c-b}{a(c-b)} = \frac{c-b+a}{c(b-a)}$
$\Rightarrow a(c-b) = c(b-a)$
$\Rightarrow ac - ab = bc - ac$
$\Rightarrow 2ac = ab + bc \Rightarrow \frac{2ac}{a+c} = b \Rightarrow a, b, c$ are in H.P.

558 **(b)**
Let four numbers are $a-3d, a-d, a+d, a+3d$.
$\therefore (a-3d)+(a+3d) = 8$
$\Rightarrow (a-d)(a+d) = 15$
and $(a-d)(a+d) = 15$
$\Rightarrow a^2 - d^2 = 15$
$\Rightarrow d = 1$
Thus, required numbers are 1, 3, 5, 7 .
Hence, greatest number is 7.

559 **(d)**
We have,
$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots + \frac{1}{\sqrt{n^2-1}+\sqrt{n^2}}$
$= (\sqrt{2}-\sqrt{1}) + (\sqrt{3}-\sqrt{2}) + (\sqrt{4}-\sqrt{3}) + \cdots + (\sqrt{n^2}-\sqrt{n^2-1})$
$= \sqrt{n^2} - 1 = n - 1$

560 **(b)**

Using A. M. $>G.M.$, we have

$$\left.\begin{array}{l}x+y>2\sqrt{xy}\\ y+z>2\sqrt{yz}\\ y+x>2\sqrt{xz}\end{array}\right\}\Rightarrow(x+y)(y+z)(z+x)>8\,xyz$$

561 **(c)**

Put $X=\frac{x-y}{x}$ in $\log_e(1-X)=-\left[\frac{X}{1}+\frac{X^2}{2}+\right.$
X33+...

We get

$\log_e\left(1-\frac{x-y}{x}\right)=-\left[\frac{x-y}{x}+\frac{1}{2}\left(\frac{x-y}{x}\right)^2+\right.$
13x−yx3+...

$$\Rightarrow\frac{x-y}{x}+\frac{1}{2}\left(\frac{x-y}{x}\right)^2+\frac{1}{3}\left(\frac{x-y}{x}\right)^3+\ldots$$
$$=-\log_e\left(\frac{y}{x}\right)=\log_e\frac{x}{y}$$

562 **(b)**

Let $x=n, y=n+1$ and $z=n+2$, where n is a positive integer.

$\therefore\ \log_e\sqrt{x}+\log_e\sqrt{z}+\left(\frac{1}{2xz+1}\right)+\frac{1}{3}\left(\frac{1}{2xz+1}\right)^3+\frac{1}{5}\left(\frac{1}{2xz+1}\right)^5+\ldots$

$$=\log_e\sqrt{xz}+\frac{1}{2}\log_e\left[\frac{1+\frac{1}{2xz+1}}{1-\frac{1}{2xz+1}}\right]$$
$$=\log_e\sqrt{xz}+\frac{1}{2}\log_e\left(\frac{2xz+2}{2xz}\right)$$
$$=\log_e\sqrt{n(n+2)}+\log_e\sqrt{\frac{n(n+2)+1}{n(n+2)}}$$
$$=\log_e\sqrt{(n+1)^2}=\log_e(n+1)$$
$$=\log_e y$$

564 **(b)**

Let a and b be the same first and last terms of the three progressions, each having $(2n+1)$ terms. Then,

The middle term of the A.P. $=\frac{a+b}{2}$

The middle term of the G.P. $=\sqrt{ab}$

The middle term of the H.P. $=\frac{2\,ab}{a+b}$

Obviously, these terms are in G.P.

565 **(b)**

Let $S=\frac{1}{1.2}+\frac{1.3}{1.2.3.4}+\frac{1.3.5}{1.2.3.4.5.6}+\ldots$

$$\therefore T_n=\frac{1.3.5.\ldots.(2n-1)}{1.2.3.\ldots.(2n-1)2n}\times\frac{(2.4.8.\ldots.2n)}{(2.4.8.\ldots.2n)}$$
$$=\frac{(2n)!}{(2n)!.\,2^n(n!)}=\frac{1}{2^n(n!)}$$
$$\therefore S=\sum T_n=\frac{1}{2.1!}+\frac{1}{2^2.2!}+\frac{1}{2^3.3!}+\ldots$$
$$=e^{1/2}-1$$

566 **(c)**

We know that, arithmetic mean of a and $b=\frac{a+b}{2}$.

But given that $\frac{a+b}{2}=\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$

$$\Rightarrow\ a^n+b^n+\frac{ab^n}{b}+\frac{ba^n}{a}=2(a^n+b^n)$$
$$\Rightarrow\ \frac{a}{b}b^n+\frac{b}{a}a^n=a^n+b^n$$
$$\Rightarrow a^n\left(\frac{a-b}{a}\right)=-b^n\left(\frac{b-a}{b}\right)$$
$$\Rightarrow\ \left(\frac{a}{b}\right)^n=\left(\frac{a}{b}\right)$$
$$\therefore\ n=1$$

567 **(d)**

0.14189189189 ...
$$=0.14+0.00189+0.00000189+\cdots$$
$$=\frac{14}{100}+189\left[\frac{1}{10^5}+\frac{1}{10^8}+\ldots\infty\right]$$
$$=\frac{7}{50}+\frac{189}{999\times100}$$
$$=\frac{7}{50}+\frac{7}{3700}=\frac{21}{148}$$

568 **(c)**

The series is $\log_{3^2}3+\log_{3^3}3-\log_{3^4}3+\log_{3^5}3-\ldots$
$$=\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\ldots+1-1$$
$$=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\ldots$$
$$=\log_e(1+1)=\log_e 2$$

569 **(a)**

Since, $\quad 2b=a+c$

Now, $(a+2b-c)(2b+c-a)(a+2b+c)$
$$=(a+a+c-c)(a+c+c-a)(2b+2b)$$
$$=2a.2c.4b=16abc$$

570 **(a)**

$1\cdot n+2(n-1)+3(n-2)+\ldots+n\cdot 1$
$$=\sum_{r=1}^{n}(n+1)r-\sum_{r=1}^{n}r^2$$
$$=(n+1)\sum n-\sum n^2$$
$$=\frac{(n+1)n(n+1)}{2}-\frac{n(n+1)(2n+1)}{6}$$
$$=\frac{n(n+1)}{6}\{3n+3-2n-1\}$$
$$=\frac{n(n+1)(n+2)}{6}$$

571 **(b)**

We have,

$a^x = b^y = c^z = d^\omega$

$\Rightarrow a^x = b^y, a^x = c^z$ and $a^x = d^\omega$

$\Rightarrow x\log a = y\log b, x\log a = z\log c$ and $x\log a = \omega\log d$

$\Rightarrow \frac{x}{y} = \log_a b, \frac{x}{z} = \log_a c$ and $\frac{x}{\omega} = \log_a d$

$\Rightarrow \frac{x}{y} + \frac{x}{z} + \frac{x}{\omega} = \log_a b + \log_a c + \log_a d$

$\Rightarrow x\left(\frac{1}{y} + \frac{1}{z} + \frac{1}{\omega}\right) = \log_a bcd$

572 **(c)**

We have,

$2^{\frac{3}{\log_3 x}} = \frac{1}{64} \Rightarrow 2^{\frac{3}{\log_3 x}} = 2^{-6} \Rightarrow \frac{3}{\log_3 x} = -6$

$\Rightarrow \log_3 x = -\frac{1}{2} \Rightarrow x = 3^{-1/2} = \frac{1}{\sqrt{3}}$

573 **(d)**

Let $S = 1 + 2 + 3 + \ldots + 100$

$= \frac{100}{2}(1 + 100) = 50(101) = 5050$

Let $S_1 = 3 + 6 + 9 + 12 + \ldots + 99$

$= 3(1 + 2 + 3 + 4 + \ldots + 33)$

$= 3.\frac{33}{2}(1 + 33) = 99 \times 17 = 1683$

Let $S_2 = 5 + 10 + 15 + \ldots + 100$

$= 5(1 + 2 + 3 + \ldots + 20)$

$= 5.\frac{20}{2}(1 + 20) = 50 \times 21 = 1050$

Let $S_3 = 15 + 30 + 45 + \ldots + 90$

$= 15(1 + 2 + 3 + \ldots + 6)$

$= 15.\frac{6}{2}(1 + 6) = 45 \times 7 = 315$

$\therefore$ Required sum $= S - S_1 - S_2 + S_3$

$= 5050 - 1683 - 1050 + 315 = 2632$

574 **(d)**

Given that, $T_3 = ar^2 = P$

Let first five terms of GP series be

a, ar, ar^2, ar^3, ar^4

Now, $a.ar.ar^2.ar^3.ar^4 = a^5r^{10} = (ar^2)^5 = P^5$

575 **(c)**

Let A be the first term and D be the common difference of the AP. Then,

$S_n = an(n-1)$

$\Rightarrow \frac{n}{2}\{2A + (n-1)D\} = an(n-1)$

$\Rightarrow 2A + (n-1)D = 2an - 2a$

$\Rightarrow 2A - D = -2a$ and $D = 2a$

$\Rightarrow A = 0, D = 2a$

The sum of the squares of the n terms of the sequence is

$S = A^2 + (A+D)^2 + (A+2D)^2 + \cdots + \{A + (n-1)D\}^2$

$\Rightarrow S = D^2\{1^2 + 2^2 + 3^2 + \cdots + (n-1)^2\}$

$\Rightarrow S = 4a^2\frac{n(n-1)(2n-1)}{6}$

$= \frac{2a^2}{3}n(n-1)(2n-1)$

576 **(a)**

We have,

$\log_{30} 8 = \log_{30} 2^3 = 3\log_{30} 2 = 3\log_{30}\left(\frac{30}{15}\right)$

$= 3(\log_{30} 30 - \log_{30} 15)$

$= 3(1 - \log_{10} 3 - \log_{30} 5) = 3(1 - x - y)$

577 **(c)**

We have,

$3^{\frac{4}{\log_4 9}} + 27^{\frac{1}{\log_{36} 9}} + 81^{\frac{1}{\log_5 3}}$

$= 3^{\frac{4}{\log_2 3}} + 27^{\frac{1}{\log_6 3}} + 81^{\frac{1}{\log_5 3}}$

$= 3^{4\log_3 2} + 27^{\log_3 6} + 81^{\log_3 5}$

$= 3^{\log_3 2^4} + (3^3)^{\log_3 6} + (3^4)^{\log_3 5}$

$= 3^{\log_3 16} + (3^3)^{\log_3 6} + (3^4)^{\log_3 5}$

$= 3^{\log_3 16} + 3^{\log_3 6^3} + 3^{\log_3 5^4}$

$= 16 + 6^3 + 5^4 = 16 + 216 + 625 = 857$

578 **(a)**

Let $a_2 - a_1 = a_3 - a_2 = \ldots = a_n - a_{n-1} = d$

$\therefore \frac{1}{d}\left(\frac{d}{a_1a_2} + \frac{d}{a_2a_3} + \ldots + \frac{d}{a_{n-1}a_n}\right)$

$= \frac{1}{d}\left[\frac{a_2 - a_1}{a_1a_2} + \frac{a_3 - a_2}{a_2a_3} + \ldots + \frac{a_n - a_{n-1}}{a_{n-1}a_n}\right]$

$= \frac{1}{d}\left[\frac{1}{a_1} - \frac{1}{a_2} + \frac{1}{a_2} - \frac{1}{a_3} + \ldots + \frac{1}{a_{n-1}} - \frac{1}{a_n}\right]$

$= \frac{1}{d}\left[\frac{1}{a_1} - \frac{1}{a_n}\right] = \frac{1}{d}\left[\frac{a_n - a_1}{a_1a_n}\right]$

$= \frac{n-1}{a_1a_n}$

579 **(d)**

We have,

$S = \sum_{n=2}^{\infty} \frac{^nC_2}{(n+1)!}$

$\Rightarrow S = \sum_{n=2}^{\infty} \frac{n!}{(n-2)!(n+1)!2!}$

$\Rightarrow S = \frac{1}{2}\sum_{n=2}^{\infty} \frac{n(n-1)}{(n+1)!}$

$\Rightarrow S = \frac{1}{2}\sum_{n=2}^{\infty}\left(\frac{n^2 - 1 - n - 1 + 2}{(n+1)!}\right)$

$\Rightarrow S = \frac{1}{2}\sum_{n=2}^{\infty}\left(\frac{n-1}{n!} - \frac{1}{n!} + \frac{2}{(n+1)!}\right)$

$$\Rightarrow S=\frac{1}{2}\sum_{n=2}^{\infty}\left(\frac{1}{(n-1)\,!}-\frac{2}{n\,!}+\frac{2}{(n+1)\,!}\right)$$

$$\Rightarrow S=\frac{1}{2}\left\{(e-1)-2(e-2)+2\left(e-\frac{5}{2}\right)\right\}=\frac{e}{2}-1$$

580 **(b)**

Since a, b, c are in H.P. Therefore,

$$b=\frac{2ac}{a+c}$$

$$\therefore \frac{1}{\frac{2ac}{a+c}-a}+\frac{1}{\frac{2ac}{a+c}-c}$$

$$=\frac{a+c}{ac-a^2}+\frac{a+c}{ac-c^2}$$

$$=\frac{a+c}{a(c-a)}+\frac{a+c}{c(a-c)}=\frac{(a+c)(c-a)}{ac(c-a)}=\frac{1}{a}+\frac{1}{c}$$

581 **(a)**

Given, $S_\infty=\frac{4}{3}$ and $a=\frac{3}{4}$

Let r be the common ratio.

$$\therefore \frac{a}{1-r}=\frac{4}{3}$$

$$\Rightarrow \frac{4}{3}-\frac{4}{3}r=\frac{3}{4}$$

$$\Rightarrow \frac{16-9}{12}=\frac{4}{3}r$$

$$\Rightarrow \frac{7}{12}=\frac{4}{3}r$$

$$\Rightarrow r=\frac{7}{16}$$

582 **(d)**

We have,

$$\log_{10}\left\{98+\sqrt{(x-6)^2}\right\}=2$$

$$\Rightarrow 98+|x-6|=10^2$$

$$\Rightarrow |x-6|=2 \Rightarrow x-6=\pm 2 \Rightarrow x=8,4$$

583 **(a)**

We have,

$$a=\sum_{n=1}^{\infty}\frac{2\,n}{(2\,n-1)\,!}$$

$$\Rightarrow a=\sum_{n=1}^{\infty}\frac{2n-1+1}{(2n-1)\,!}$$

$$\Rightarrow a=\sum_{n=1}^{\infty}\left\{\frac{1}{(2n-2)\,!}+\frac{1}{(2n-1)\,!}\right\}$$

$$\Rightarrow a=\left(1+\frac{1}{1\,!}\right)+\left(\frac{1}{3\,!}+\frac{1}{2\,!}\right)+\left(\frac{1}{5\,!}+\frac{1}{4\,!}+\cdots\right)$$
$$=e$$

and,

$$b=\sum_{n=1}^{\infty}\frac{2\,n}{(2n+1)\,!}$$

$$\Rightarrow b=\sum_{n=1}^{\infty}\frac{2\,n+1-1}{(2n+1)\,!}$$

$$\Rightarrow b=\sum_{n=1}^{\infty}\left\{\frac{1}{(2\,n)\,!}-\frac{1}{(2n+1)\,!}\right\}$$

$$\Rightarrow b=\left(\frac{1}{2\,!}-\frac{1}{3\,!}\right)+\left(\frac{1}{4\,!}-\frac{1}{5\,!}\right)+\left(\frac{1}{6\,!}-\frac{1}{7\,!}\right)+\cdots$$
$$=e^{-1}$$

$$\therefore ab=e\cdot e^{-1}=1$$

584 **(a)**

Let

$$S_n=1+\frac{3}{2}+\frac{7}{4}+\frac{15}{8}+\frac{31}{16}+\cdots \text{to } n \text{ terms}$$

$$\Rightarrow S_n=1+\frac{2^2-1}{2}+\frac{2^3-1}{2^2}+\frac{2^4-1}{2^3}+\frac{2^5-1}{2^4}+\cdots+\frac{2^n-1}{2^{n-1}}$$

$$\Rightarrow S_n=(2-1)+\left(2-\frac{1}{2}\right)+\left(2-\frac{1}{2^2}\right)+\left(2-\frac{1}{2^3}\right)+\left(2-\frac{1}{2^4}\right)+\cdots+\left(2-\frac{1}{2^{n-1}}\right)$$

$$\Rightarrow S_n=2n-\left(1+\frac{1}{2}+\frac{1}{2^2}+\cdots\frac{1}{2^{n-1}}\right)$$

$$\Rightarrow S_n=2n-\left(\frac{1-\frac{1}{2^n}}{1-\frac{1}{2}}\right)=2(n-1)+\frac{1}{2^{n-1}}$$

585 **(b)**

Since, $x=\sum_{n=0}^{\infty}\cos^{2n}\phi$

$=1+\cos^2\phi+\cos^4\phi+\ldots$

$=\frac{1}{1-\cos^2\phi}=\frac{1}{\sin^2\phi}$ $[\because |\cos x|<1]$

Similarly, $y=\frac{1}{1-\sin^2\phi}=\frac{1}{\cos^2\phi}$

and $z=\frac{1}{1-\sin^2\phi\cos^2\phi}$

$$=\frac{1}{1-\frac{1}{x}.\frac{1}{y}}=\frac{xy}{xy-1}$$

$$\Rightarrow xyz=xy+z$$

586 **(d)**

Since, $a+23d=100$ ...(i)

$$\therefore S_{47}=\frac{47}{2}[2a+46d]=47[a+23d]$$

$=47\times 100=4700$ [from Eq. (i)]

587 **(b)**

Since, a, b, c, d, e, f are six A.M.'s. between 2 and 12

$$\therefore a+b+c+d+e+f=\frac{6}{2}(a+f)=\frac{6}{2}(2+12)$$
$$=42$$

588 **(a)**

We have,

$x=\log_2 3$ and $y=\log_{1/2} 5$

$\Rightarrow x = \log_2 3$ and $y = \log_2^{-1} 5$
$\Rightarrow x = \log_2 3$ and $y = -\log_2 5$
$\Rightarrow x > 0$ and $y < 0 \Rightarrow x > y$

590 **(b)**
If $2^2 < x < 2^3$, then $2 < \log_2 x < 3$
$\therefore 2 < \log_2 7 < 3 \quad [\because 2^2 < 7 < 2^3]$
Let $\log_2 7$ be a rational number equal to $\frac{m}{n}$, where $m, n \in Z, n \neq 0$. Then,
$7 = 2^{m/n} \Rightarrow 7^n = 2^m$
This is not possible as LHS is an odd natural number and RHS is an even natural number

591 **(d)**
We have,
$4^{\log_9 3} + 9^{\log_2 4} = 10^{\log_x 83}$
$\Rightarrow 4^{\log_{3^2} 2} + 9^{\log_2 2^2} = 10^{\log_x 83}$
$\Rightarrow 4^{1/2} + 9^2 = 10^{\log_x 83}$
$\Rightarrow 83 = 10^{\log_x 83} \Rightarrow \log_{10} 83 = \log_x 83 \Rightarrow x = 10$

592 **(c)**
Since, the given series $\log_a x, \log_b x, \log_c x$ be in HP.
$\Rightarrow \frac{\log x}{\log a}, \frac{\log x}{\log b}, \frac{\log x}{\log c}$ are in HP.
$\Rightarrow \frac{\log a}{\log x}, \frac{\log b}{\log x}, \frac{\log c}{\log x}$ are in AP.
$\Rightarrow \log_x a, \log_x b, \log_x c$ are in AP.
$\therefore a, b, c$ are in GP.

594 **(c)**
Since, $a + ar = a(1 + r) = 12$ ...(i)
and $ar^2 + ar^3 = ar^2(1 + r) = 48$ ...(ii)
From Eqs. (i) and (ii),
$r^2 = 4$
$\Rightarrow r = -2$
(Since, the series is alternately sign, so we take negative values).
On putting the value of r in Eq. (i), we get
$a = -12$

595 **(a)**
We have,
$2\left\{\frac{m-n}{m+n} + \frac{1}{3}\left(\frac{m-n}{m+n}\right)^3 + \frac{1}{5}\left(\frac{m-n}{m+n}\right)^5 + \cdots\right\}$
$= \log\left(\frac{1 + \frac{m-n}{m+n}}{1 - \frac{m-n}{m+n}}\right) = \log\left(\frac{m}{n}\right)$

596 **(c)**
$\sum_{k=1}^{n}(k^2 + 2k) = \sum_{k=1}^{n} k^2 + 2\sum_{k=1}^{n} k$
$= \frac{n(n+1)(2n+1)}{6} + \frac{2n(n+1)}{2}$
$= \frac{n(n+1)(2n+7)}{6}$

597 **(a)**
Let a be the first term and d be the common difference of the A.P. Then, it is given that
$x = \frac{p}{2}[2a + (p-1)d]$,
$y = \frac{q}{2}[2a + (q-1)d]$,
$z = \frac{r}{2}[2a + (r-1)d]$
$\therefore \frac{2x}{p} = 2a + (p-1)d$ ... (i)
$\frac{2y}{q} = 2a + (q-1)d$ ... (ii)
$\frac{2z}{r} = 2a + (r-1)d$ ... (iii)
Multiplying (i), (ii), and (iii) by $(q-r), (r-p)$ and $(p-q)$ respectively and adding, we get
$\frac{2x}{p}(q-r) + \frac{2y}{q}(r-p) + \frac{2z}{r}(p-q) = 0$
$\Rightarrow \frac{x}{p}(q-r) + \frac{y}{q}(r-p) + \frac{z}{r}(p-q) = 0$

598 **(b)**
It is given that
$1, \log_9(3^{1-x} + 2), \log_3(4.3^x - 1)$ are in A.P.
$\Rightarrow (3^{1-x} + 2)^{1/2}, 3, (4 \times 3^x - 1)$ are in G.P
$\Rightarrow 3^{1-x} + 2 = 3(4.3^x - 1)$
$\Rightarrow 3 + 2.3^x = 12(3x)^2 - 3(3^x)$
$\Rightarrow 12(3^x)^2 - 5(3^x) - 3 = 0$
$\Rightarrow (4.3^x - 3)(3.3^x + 1) = 0$
$\Rightarrow 3^x = \frac{3}{4} \quad [\because 3.3^x + 1 \neq 0]$
$\Rightarrow x = \log_3\left(\frac{3}{4}\right) \Rightarrow x$
$= \log_3 3 - \log_3 4 = 1 - \log_3 4$

599 **(c)**
Given that, AM =8, GM =5, if α, β are the roots of quadratic equation, then the required quadratic equation is
$x^2 - x(\alpha + \beta) + \alpha\beta = 0$ ...(i)
Here, AM $= \frac{\alpha+\beta}{2} = 8 \Rightarrow \alpha + \beta = 16$
And GM $= \sqrt{\alpha\beta} = 5 \Rightarrow \alpha\beta = 25$
From Eq. (I)
$x^2 - 16x + 25 = 0$

600 **(a)**

Let $S_n = \frac{1}{\sqrt{2}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{8}} + \ldots + \frac{1}{\sqrt{3n-1}+\sqrt{3n+2}}$

$$\frac{\sqrt{2}-\sqrt{5}}{-3} + \frac{\sqrt{5}-\sqrt{8}}{-3} + \ldots + \frac{\sqrt{3n-1}-\sqrt{3n+2}}{-3}$$

$$= -\frac{1}{3}(\sqrt{2}-\sqrt{3n+2}) = \frac{1}{3}(\sqrt{3n+2}-\sqrt{2})$$

601 **(a)**

We have,

$$1 + \log_e 2 + \frac{(\log_e 2)^2}{2!} + \frac{(\log_e 2)^3}{3!} + \cdots$$

$$= e^{\log_e 2} = 2$$

602 **(c)**

We have,

$x^{\frac{1}{2}} x^{\frac{1}{4}} x^{\frac{1}{8}} x^{\frac{1}{16}} \ldots$ to ∞

$$= x^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots \text{to}\infty} = x^{\frac{1/2}{1-1/2}} = x$$

603 **(b)**

It is given that

$\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

$$\Rightarrow \frac{2}{c+a} = \frac{1}{b+c} + \frac{1}{a+b}$$

$\Rightarrow 2\, b^2 = a^2 + c^2 \Rightarrow a^2, b^2, c^2$ are in A.P.

604 **(d)**

It is given that

a, b, c are in G.P. $\Rightarrow b^2 = ac$

Also, a, b, c are in A.P. $\Rightarrow 2b = a + c$

Now, $b^2 = ac$ and $2b = a + c$

$$\Rightarrow \left(\frac{a+c}{2}\right)^2 = ac \qquad \text{[Eliminating } b\text{]}$$

$\Rightarrow (a+c)^2 - 4ac = 0 \Rightarrow (a-c)^2 = 0 \Rightarrow a = c$

Putting $a = c$ in $2b = a + c$, we get

$2b = 2a \Rightarrow b = a$

Hence, $a = b = c$

605 **(c)**

We have,

$$\left(1 + \frac{x^2}{2\,!} + \frac{x^4}{4\,!} + \cdots\right)^2$$

$$= \left(\frac{e^x + e^{-x}}{2}\right)^2$$

$$= \frac{1}{4}(e^{2x} + e^{-2x} + 2)$$

$$= \frac{1}{2}\left(\frac{e^{2x}+e^{-2x}}{2}\right) + \frac{1}{2}$$

$$= \frac{1}{2}\left\{1 + \frac{(2x)^2}{2\,!} + \frac{(2x)^4}{4\,!} + \cdots\right\} + \frac{1}{2}$$

$\therefore$ Coefficient of x^n (when n is odd) $= 0$

606 **(a)**

Let the two quantities be a and b. Then,

a, A_1, A_2, b are in AP.

$\therefore A_1 - a = b - A_2 \Rightarrow A_1 + A_2 = a + b \quad \ldots$(i)

Again a, G_1, G_2, b are in GP.

$$\therefore \frac{G_1}{a} = \frac{b}{G_2}$$

$\Rightarrow G_1 G_2 = ab \quad \ldots$(ii)

Also, a, H_1, H_2, b are in HP.

$$\therefore \frac{1}{H_1} - \frac{1}{a} = \frac{1}{b} - \frac{1}{H_2}$$

$$\Rightarrow \frac{1}{H_1} + \frac{1}{H_2} = \frac{1}{b} + \frac{1}{a}$$

$$\Rightarrow \frac{H_1 + H_2}{H_1 H_2} = \frac{a+b}{ab}$$

$\Rightarrow \frac{H_1+H_2}{H_1H_2} = \frac{A_1+A_2}{G_1G_2}$[using Eqs, (i) and (ii)]

$$\Rightarrow \frac{G_1 G_2}{H_1 H_2} = \frac{A_1 + A_2}{H_1 + H_2}$$

607 **(a)**

Here, $T_n = \frac{n}{(2n+1)!} = \frac{1}{2}\left[\frac{2n+1-1}{(2n+1)!}\right]$

$$= \frac{1}{2}\left[\frac{1}{(2n)!} - \frac{1}{(2n+1)!}\right]$$

$$\therefore T_1 = \frac{1}{2}\left(\frac{1}{2!} - \frac{1}{3!}\right)$$

$$T_2 = \frac{1}{2}\left(\frac{1}{4!} - \frac{1}{5!}\right)$$

$\cdots \quad \cdots \quad \cdots$

$\cdots \quad \cdots \quad \cdots$

$\cdots \quad \cdots \quad \cdots$

$$\therefore S = \sum_{n=1}^{\infty} T_n = \frac{1}{2}\left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \ldots \infty + 1 - 1\right] = \frac{e^{-1}}{2}$$

608 **(a)**

The required numbers are

3,9,15,...,999

Here, $\quad l = a + (n-1)d$

$\therefore 999 = 3 + (n-1)6$

$\Rightarrow 6n = 1002 \Rightarrow n = 167$

$$\therefore S = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{167}{2}(6 + 166 \times 6)$$

$$= \frac{167}{2}(1002)$$

$= 83667$

609 **(b)**

Given series is

$\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \ldots + n$ terms

Let T_n be the nth term of the series

$$\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+\dots$$

Then, $T_n=\frac{n}{1+n^2+n^4}=\frac{n}{(1+n^2)^2-n^2}$

$=\frac{n}{(n^2+n+1)(n^2-n+1)}$

$=\frac{1}{2}\left[\frac{1}{n^2-n+1}-\frac{1}{n^2+n+1}\right]$

$=\frac{1}{2}\left[\frac{1}{1+(n-1)n}-\frac{1}{1+(n+1)}\right]$

$\therefore T_1=\frac{1}{2}\left[\frac{1}{1}-\frac{1}{1+1\cdot 2}\right]$

$T_2=\frac{1}{2}\left[\frac{1}{1+1\cdot 2}-\frac{1}{1+2\cdot 3}\right]$

$T_3=\frac{1}{2}\left[\frac{1}{1+2\cdot 3}-\frac{1}{1+3\cdot 4}\right]$

$\dots\ \dots\ \dots$

$\dots\ \dots\ \dots$

$T_n=\frac{1}{2}\left[\frac{1}{1+(n-1)n}-\frac{1}{1+(n+1)}\right]$

Adding all these equations, we get

$\sum_{r=1}^{n}T_r=\frac{1}{2}\left[1-\frac{1}{1+(n+1)}\right]=\frac{n(n+1)}{2(n^2+n+1)}$

610 **(b)**

$\because$ Common terms are 5, 11, 17, …

$T_n=5+(n-1)6$

$=6n-1$

100th term of the first sequence

$=2+(100-1)3=299$

and 100th term of the second sequence

$=3+(100-1)2=201$

Now, $201>6-1$

$\Rightarrow n\le 33\frac{2}{3}$

$\Rightarrow n=33 \quad (\because n\in N)$

611 **(c)**

Here, $T_n=\frac{n^2}{(n+2)!}=\frac{n^2-4+4}{(n+2)!}$

$=\frac{(n-2)}{(n+1)!}+\frac{4}{(n+2)!}$

$\Rightarrow T_n=\frac{1}{n!}-\frac{3}{(n+1)!}+\frac{4}{(n+2)!}$

$\therefore S=\Sigma T_n=\sum_{n=1}^{\infty}\frac{1}{n!}$

$-3\sum_{n=1}^{\infty}\frac{1}{(n+1)!}$

$+4\sum_{n=1}^{\infty}\frac{1}{(n+2)!}$

$=(e-1)-3(e-2)+4\left(e-2-\frac{1}{2}\right)$

$=2e-5$

612 **(a)**

We have,

$\log_\pi(\log_2(\log_7 x))=0$

$\Rightarrow \log_2(\log_7 x)=\pi^0\Rightarrow \log_7 x=2^1\Rightarrow x=7^2$

613 **(b)**

$\underset{n\text{ digits}}{(666\dots 6)}=6+6\times 10+6\times 10^2+\dots+6\times 10^{n-1}$

$=6(1+10+10^2+\dots+10^{n-1})$

$=\frac{6}{9}(10^n-1)=\frac{2}{3}(10^n-1)$

Similarly, $\underset{n\text{ digits}}{(888\dots 8)}=\frac{8}{9}(10^n-1)$

Hence, required sum

$=\frac{4}{9}(10^n-1)^2+\frac{8}{9}(10^n-1)$

$=\frac{4}{9}(10^{2n}-2.10^n+1+2.10^n-2)$

$=\frac{4}{9}(10^{2n}-1)$

614 **(a)**

Given series is $1.3^2+2.5^2+3.7^2+\dots\infty$

This is an arithmetic-geometric series whose nth term is equal to

$T_n=n(2n+1)^2=4n^3+4n^2+n$

$\therefore S_n=\sum_1^n T_n=\sum_1^n(4n^3+4n^2+n)$

$=4\sum_1^n n^3+4\sum_1^n n^2+\sum_1^n n$

$=4\left(\frac{n}{2}(n+1)\right)^2+\frac{4}{6}n(n+1)(2n+1)+\frac{n}{2}(n+1)$

$=n(n+1)\left[n^2+n+\frac{4}{6}(2n+1)+\frac{1}{2}\right]$

$=\frac{n}{6}(n+1)(6n^2+14n+7)$

615 **(b)**

We have,

$S=\sum_{n=2}^{\infty}{}^nC_2\frac{3^{n-2}}{n!}$

$\Rightarrow S=\sum_{n=2}^{\infty}\frac{n!}{(n-2)!\,2!}\cdot\frac{3^{n-2}}{n!}=\frac{1}{2}\sum_{n=2}^{\infty}\frac{3^{n-2}}{(n-2)!}$

$=\frac{1}{2}e^3$

616 **(c)**

Let there be $2n$ terms in the given G.P. with first term a and the common ratio r. Then,

Sum of all terms $=5$ (Sum of odd terms)

$\Rightarrow a_1 + a_2 + \cdots + a_{2n} = 5(a_1 + a_3 + \cdots + a_{2n-1})$
$\Rightarrow a + ar + ar^2 + \cdots + ar^{2n-1}$
$= 5(a + ar^2 + \cdots + ar^{2n-2})$
$\Rightarrow a\dfrac{(r^{2n}-1)}{(r-1)} = 5a\dfrac{(r^{2n}-1)}{(r^2-1)}$
$\Rightarrow r + 1 = 5 \;\Rightarrow r = 4$

617 **(a)**
Let $S = a + ar + ar^2 + \ldots + ar^{n-1}$
$= \dfrac{a(1-r^n)}{1-r} \quad [\because r < 1]$
$P = a.ar.ar^2 \ldots .ar^{n-1} = a^n r^{1+2+\ldots+(n-1)}$
$= a^n r^{\frac{n(n-1)}{2}}$
and $R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \cdots + \frac{1}{ar^{n-1}}$
$= \dfrac{\frac{1}{a}\left(\frac{1}{r^n} - 1\right)}{\frac{1}{r} - 1} = \dfrac{\frac{1}{a}(1-r^n)}{r^{n-1}(1-r)} \quad \left[\because \frac{1}{r} > 1\right]$
Now, $\left(\frac{S}{R}\right)^n = \left\{\dfrac{\frac{a(1-r^n)}{(1-r)}}{\frac{\frac{1}{a}(1-r^n)}{r^{n-1}(1-r)}}\right\}^n = a^{2n} r^{n(n-1)}$
$= \left\{a^n r^{\frac{n(n-1)}{2}}\right\}^2 = P^2$

618 **(c)**
Let $A = m\lambda, H = n\lambda$
$\therefore\; G^2 = AH = mn\lambda^2$
Also, a and b be the roots of
$x^2 - (a+b)x + ab = 0$
$\Rightarrow\; x^2 - 2m\lambda x + mn\lambda^2 = 0$
$\Rightarrow\; x = \lambda\sqrt{m}\{\sqrt{m} \pm \sqrt{m-n}\}$
$\therefore\; a:b = (\sqrt{m} + \sqrt{m-n}):(\sqrt{m} - \sqrt{m-n})$
or $(\sqrt{m} - \sqrt{m-n}) : (\sqrt{m} + \sqrt{m-n})$

619 **(d)**
Let $S = 1 + 2x + 3x^2 + 4x^3 + \ldots\infty$ ...(i)
$S = x + 2x^2 + 3x^3 + \ldots\infty$ ...(ii)
On subtracting Eq. (ii) from Eq. (i), we get
$(1-x)S = (1 + x + x^2 + \ldots\infty)$
$\Rightarrow S = \dfrac{1}{(1-x)}\left(\dfrac{1}{1-x}\right) = \dfrac{1}{(1-x)^2}$
Alternate Here, $a = 1, d = 1, r = x$
$\therefore S_\infty = \dfrac{a}{1-r} + \dfrac{d.r}{(1-r)^2}$
$= \dfrac{1}{1-x} + \dfrac{1.x}{(1-x)^2} = \dfrac{1}{(1-x)^2}$

620 **(d)**
Let $4, G_1, G_2, G_3, \frac{1}{4}$ are in GP.
$\therefore\; G_1 = ar = 4r$
$G_2 = 4r^2, G_3 = 4r^3$
$G_4 = 4 \times r^4 = \dfrac{1}{4}$
$\Rightarrow\; r = \dfrac{1}{2}$
$\therefore$ Product of GM $= G_1 \cdot G_2 \cdot G_3$
$= ar \cdot ar^2 \cdot ar^3$
$= a^3 r^6$
$= 4^3 \times \left(\dfrac{1}{2}\right)^6 = \dfrac{4^3}{4^3} = 1$

621 **(a)**
Let the two quantities be a and b. Then a, A_1, A_2, b are in AP.
$\therefore\; A_1 - a = b - A_2 \;\Rightarrow A_1 + A_2 = a + b$ ...(i)
Again, a, G_1, G_2, b are in GP.
$\therefore\; \frac{G_1}{a} = \frac{b}{G_2} \;\Rightarrow G_1G_2 = ab$ ...(ii)
Also, a, H_1, H_2, b are in HP.
$\therefore\; \dfrac{1}{H_1} - \dfrac{1}{a} = \dfrac{1}{b} - \dfrac{1}{H_2}$
$\Rightarrow\; \dfrac{1}{H_1} + \dfrac{1}{H_2} = \dfrac{1}{a} + \dfrac{1}{b}$
$\Rightarrow\; \frac{H_1+H_2}{H_1H_2} = \frac{a+b}{ab} = \frac{A_1+A_2}{G_1G_2}$ [from Eqs.(i) and (ii)]
$\Rightarrow\; \dfrac{G_1G_2}{H_1H_2} = \dfrac{A_1+A_2}{H_1+H_2}$

622 **(a)**
$\dfrac{1}{2!} + \dfrac{2}{3!} + \ldots + \dfrac{999}{1000!}$
$= \dfrac{2-1}{2!} + \dfrac{3-1}{3!} + \ldots + \dfrac{1000-1}{1000!}$
$= \dfrac{1}{1!} - \dfrac{1}{2!} + \dfrac{1}{2!} - \dfrac{1}{3!} + \ldots + \dfrac{1}{999!} - \dfrac{1}{1000!}$
$1 - \dfrac{1}{1000!} = \dfrac{1000! - 1}{1000!}$

623 **(a)**
Let the sum of the series
$\dfrac{1}{1^4} + \dfrac{1}{3^4} + \dfrac{1}{5^4} + \cdots$ to ∞ be x
it is given that
$\dfrac{1}{1^4} + \dfrac{1}{2^4} + \dfrac{1}{3^4} + \dfrac{1}{4^4} + \dfrac{1}{5^4} + \cdots = \dfrac{\pi^4}{90}$
$\Rightarrow \left(\dfrac{1}{1^4} + \dfrac{1}{3^4} + \dfrac{1}{5^4} + \cdots\right) + \dfrac{1}{2^4}\left(\dfrac{1}{1^4} + \dfrac{1}{2^4} + \dfrac{1}{3^4} + \cdots\right)$
$= \dfrac{\pi^4}{90}$
$\Rightarrow x + \dfrac{1}{2^4} \times \dfrac{\pi^4}{90} = \dfrac{\pi^4}{90} \Rightarrow x = \dfrac{\pi^4}{96}$

624 **(b)**
We have,
$\dfrac{\log a}{3} = \dfrac{\log b}{4} = \dfrac{\log c}{5} = \lambda(\text{say})$
$\Rightarrow a = 10^{3\lambda}, b = 10^{4\lambda}$ and $c = 10^{5\lambda} \Rightarrow b^2 = ac$

625 **(a)**

Given $2b = a + c$
and $(c-b)^2 = (b-a)a$
$\therefore (b-a)^2 = (b-a)a$
$\Rightarrow b = 2a$
$\Rightarrow c = 3a$
$\Rightarrow a:b:c = 1:2:3$

626 **(a)**
Since, $T_{p+q} = m = a(r)^{p+q-1}$ ...(i)
And $T_{p-q} = n = a(r)^{p-q-1}$ ...(ii)
On multiplying Eqs. (i) and (ii), we get
$mn = a^2(r)^{2p-2}$
$\Rightarrow a(r)^{p-1} = (mn)^{1/2}$
$\therefore T_p = (mn)^{1/2}$

627 **(b)**
Now, we assume $(b-c)^2, (c-a)^2, (a-b)^2$ are in AP, then we have
$(c-a)^2 - (b-c)^2 = (a-b)^2 - (c-a)^2$
$\Rightarrow (b-a)(2c-a-b) = (c-b)(2a-b-c)$ ...(i)
Also, if $\frac{1}{b-c}, \frac{1}{c-a}, \frac{1}{a-b}$ are in AP, then
$$\frac{1}{c-a} - \frac{1}{b-c} = \frac{1}{a-b} - \frac{1}{c-a}$$
$$\Rightarrow \frac{b+a-2c}{(c-a)(b-c)} = \frac{c+b-2a}{(a-b)(c-a)}$$
$\Rightarrow (a-b)(b+a-2c) = (b-c)(c+b-2a)$
$\Rightarrow (b-a)(2c-a-b) = (c-b)(2a-b-c)$
Which is equal to Eq. (i), so, our hypothesis is true.

628 **(b)**
We have,
$2\log x - \log(x+1) - \log(x-1)$
$$= \log\left(\frac{x^2}{x^2-1}\right)$$
$$= -\log\left(\frac{x^2-1}{x^2}\right)$$
$$= -\log\left(1 - \frac{1}{x^2}\right) = \frac{1}{x^2} + \frac{1}{2x^4} + \frac{1}{3x^6} + \cdots$$

629 **(c)**
We have,
$$\log_3\left\{\log_6\left(\frac{x^2+x}{x-1}\right)\right\} = 0$$
$$\Rightarrow \log_6\left(\frac{x^2+x}{x-1}\right) = 3^0$$
$$\Rightarrow \frac{x^2+x}{x-1} = 6 \Rightarrow x^2 - 5x + 6 = 0 \Rightarrow x = 2,3$$

630 **(c)**
Clearly, it is an AGP with $a = 1, d = 3$ and $r = 1/5$
$$\therefore 1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \dots\dots\dots\dots$$
$$= \frac{1}{1-\frac{1}{5}} + \frac{3/5}{\left(1-\frac{1}{5}\right)^5} \quad \left[\text{Using : } S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}\right]$$
$$= \frac{5}{4} + \frac{15}{16} = \frac{35}{16}$$

631 **(d)**
Since, $a_1, a_2, a_3, \dots, a_{10}$ be in AP.
$\therefore a_{10} = a_1 + 9d$
$\Rightarrow 3 = a_1 + 9d$
$\Rightarrow 3 = 2 + 9d$
$\Rightarrow d = \frac{1}{9}$
Now, $a_4 = a_1 + 3d$
$\Rightarrow a_4 = 2 + 3\left(\frac{1}{9}\right) = 2 + \frac{1}{3} = \frac{7}{3}$
Again, $h_1, h_1, h_3, \dots, h_{10}$ be in HP.
$\Rightarrow \frac{1}{h_1}, \frac{1}{h_2}, \frac{1}{h_3}, \dots, \frac{1}{h_{10}}$ be in AP
Since, $h_1 = 2, h_{10} = 3$ (given)
$\therefore \frac{1}{h_{10}} = \frac{1}{h_1} + 9d_1$
$\Rightarrow \frac{1}{3} = \frac{1}{2} + 9d_1$
$\Rightarrow \frac{1}{3} - \frac{1}{2} = 9d_1$
$\Rightarrow -\frac{1}{6} = 9d_1$
$\Rightarrow d_1 = -\frac{1}{54}$
Now, $\frac{1}{h_7} = \frac{1}{2} + \frac{6\times1}{-54} \Rightarrow \frac{1}{h_7} = \frac{1}{2} - \frac{1}{9}$
$\Rightarrow \frac{1}{h_7} = \frac{9-2}{18} \Rightarrow h_7 = \frac{18}{7}$
$\therefore a_4h_7 = \frac{7}{3} \times \frac{18}{7} = 6$

632 **(d)**
We have,
$$\frac{e^{5x}+e^x}{e^{3x}} = e^{2x} + e^{-2x} = 2\left\{1 + \frac{2^2x^2}{2!} + \frac{2^4x^4}{4!}\right\}$$
This expansion does not contain any odd power of x
$\therefore$ Coefficient of $x^n = 0$

633 **(b)**
We have,
$\log_{10} x = y$
$\therefore \log_{10^3} x^2 = \frac{2}{3}\log_{10} x = \frac{2}{3}y$

634 **(d)**
By the properties of AP and GP
$a_1 + a_{2n} = a_2 + a_{2n-1} = \ldots = a_n + a_{n+1} = a + b$

and $\quad g_1 g_{2n} = g_2 g_{2n-1} = \ldots = g_n g_{n+1} = ab$

$\therefore \dfrac{a_1 + a_{2n}}{g_1 g_{2n}} + \dfrac{a_2 + a_{2n-1}}{g_2 g_{2n-1}} + \ldots + \dfrac{a_n + a_{n+1}}{g_n g_{n+1}}$

$= \dfrac{a+b}{ab} + \dfrac{a+b}{ab} + \ldots + \dfrac{a+b}{ab}$

$= \dfrac{n(a+b)}{(ab)} = \dfrac{2n}{h}$

635 **(d)**

As, $S_{25} = \left\{\frac{26}{25}, \frac{51}{25}, \frac{76}{25}, \ldots \text{upto 25 terms}\right\}$

Here, $a = \frac{26}{25},\ n = 25, d = 1$

$\therefore S_{25} = \dfrac{25}{2}\left(\dfrac{52}{25} + 24\right) = 326$

636 **(c)**

Let a be the first term and d be the common difference. Then,

$S_1 = \dfrac{n}{2}\{2\,a + (n-1)\,d\}$,

$S_2 = \dfrac{2\,n}{2}\{2a + (2n-1)\,d\}$,

$S_3 = \dfrac{3\,n}{2}\{2\,a + (3\,n-1)\,d\}$

$\Rightarrow S_2 - S_1 = \dfrac{n}{2}\{2\,a + (3n-1)\,d\}$

$\Rightarrow S_2 - S_1 = \dfrac{1}{3}\left\{\dfrac{3n}{2}\{2a + (n-1)\,d\}\right\} = \dfrac{1}{3}S_3$

$\Rightarrow S_3 = 3(S_2 - S_1)$

637 **(a)**

Here,$\alpha \in \left(0, \frac{\pi}{2}\right) \Rightarrow \tan\alpha\ is\ (+ve)$

[as, we know if

$a, b > 0 \Rightarrow \frac{a+b}{2} \geq \sqrt{ab}\ ie, AM \geq GM$]

$\dfrac{\sqrt{x^2+x} + \frac{\tan^2\alpha}{\sqrt{x^2+x}}}{2}$

$\geq \sqrt{\sqrt{x^2+x} . \dfrac{\tan^2\alpha}{\sqrt{x^2+x}}}\,[using\ AM\ \geq GM]$

$\Rightarrow \sqrt{x^2+x} + \dfrac{\tan^2\alpha}{\sqrt{x^2+x}} \geq 2\tan\alpha$

638 **(b)**

We have,

$\log_3 e - \log_9 e + \log_{27} e - \log_{81} e + \cdots \infty$

$= \log_3 e - \dfrac{1}{2}\log_3 e + \dfrac{1}{3}\log_3 e - \dfrac{1}{4}\log_3 e + \cdots \infty$

$= (\log_3 e)\left(1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} + \cdots\right)$

$= (\log_3 e)(\log_e 2) = \log_3 2$

641 **(a)**

We have,

Coefficient of $x^6 = 2\left(\dfrac{1}{5.6}\right) = \dfrac{1}{15}$

642 **(c)**

Let r be the common ratio of give GP.

$\therefore \dfrac{x}{1-r} = 5$

$\Rightarrow r = \left(1 - \dfrac{x}{5}\right)$

$\because$ For infinite GP,

$|r| < 1$

$\Rightarrow -1 < 1 - \dfrac{x}{5} < 1$

$\Rightarrow 10 > x > 0$

$\Rightarrow 0 < x < 10$

643 **(a)**

Let three numbers in GP are $\frac{a}{r}, a, ar$.

From the given condition

$\frac{a}{r} + a + ar = 14 \Rightarrow a\left(\frac{1}{r} + 1 + r\right) = 14$ ...(i)

and $\frac{a}{r} + 1, a + 1$ and $ar - 1 = \frac{a}{r}(1 + r^2)$ ...(ii)

From Eqs. (i) and (ii), we get

$a = 4$ and $r = 2$

So, required numbers are 2, 4, 8

Hence, greatest number is 8

644 **(a)**

We have,

$2\,b = a + c$ and $b^2 = \dfrac{2\,a^2c^2}{a^2+c^2}$

$\therefore \left(\dfrac{a+c}{2}\right)^2 = \dfrac{2\,a^2c^2}{a^2+c^2}$

$\Rightarrow (a+c)^2(a^2+c^2) = 8\,a^2c^2$

$\Rightarrow (a^2+c^2)^2 + 2\,ac(a^2+c^2) - 8a^2c^2 = 0$

$\Rightarrow (a^2-c^2)^2 + 2\,ac(a-c)^2 = 0$

$\Rightarrow (a-c)^2[(a+c)^2 + 2\,ac] = 0$

$\Rightarrow a = c$

Hence, $a = b = c$

645 **(d)**

Given that, $S_n = nA + n^2B$

Putting $n = 1, 2, 3, \ldots$, we get

$S_1 = A + B, S_2 = 2\,A + 4B, S_3 = 3A + 9B$

..........,

Therefore,

$T_1 = S_1 = A + B, T_2 = S_2 - S_1 = A + 3B$,

$T_3 = S_3 - S_2 = A + 5B$

Hence, the sequence is $(A+B), (A+3B), (A+5B, \ldots$

$\therefore$ Common difference, $d = A + 3B - (A + B) = 2B$

646 **(d)**

The Coefficient of x^3 in the expansion of 3^x

$= \dfrac{(\log 3)^3}{3!} = \dfrac{(\log 3)^3}{6}$

647 **(c)**

$\sin^2 B - \sin^2 A = \sin^2 C - \sin^2 B$ [$\because$ a^2, b^2, c^2 are in AP]

$\Rightarrow \sin(B+A)\sin(B-A) = \sin(C+B)\sin(C-B)$

$\Rightarrow \sin C(\sin B\cos A - \cos B\sin A) = \sin A(\sin C\cos B - \cos C\sin B)$

$\Rightarrow 2\cot B = \cot A + \cot C$ [divide by $\sin A\sin B\sin C$]

$\Rightarrow \cot A, \cot B, \cot C$ are in AP.

648 **(a)**

We have,

$\frac{1}{\log_3 \pi} + \frac{1}{\log_4 \pi} > x$

$\Rightarrow \log_\pi 3 + \log_\pi 4 > x$

$\Rightarrow \log_\pi 12 > x$

$\Rightarrow \pi^x < 12$

Hence, the greatest integral value of x is 2

649 **(c)**

Let $a_n = \frac{1}{(n+1)(n+2)\dots(n+k)}$

$\Rightarrow a_n = \frac{1}{(k-1)}\left(\frac{(n+k)-(n+1)}{(n+1)(n+2)\dots(n+k)}\right)$

$\Rightarrow \frac{1}{(k-1)}\left(\frac{1}{(n+1)(n+2)\dots(n+k-1)}\right) - \left(\frac{1}{(n+2)(n+3)\dots(n+k)}\right)$

$\therefore S_n = a_1 + a_2 + \dots + a_n$

$= \frac{1}{(k-1)}\left(\frac{1}{2\cdot 3\cdot 4\dots k} - \frac{1}{(n+2)(n+3)\dots(n+k)}\right)$

$\Rightarrow \lim_{n\to\infty} S_n = \frac{1}{(k-1)k!}$

650 **(c)**

We have,

$1 + \frac{a+bx}{1!} + \frac{(a+bx)}{2!} + \frac{(a+bx)^3}{3!} + \cdots$

$= e^{a+bx} = e^a e^{bx} = e^a \sum_{n=0}^{\infty} \frac{(bx)^n}{n!} = e^a \sum_{n=0}^{\infty} \frac{b^n x^n}{n!}$

So, Coefficient of $x^n = \frac{e^a b^n}{n!}$

651 **(b)**

Suppose that $\angle A = x$, then $\angle B = x + 10°$, $\angle C = x + 20°$ and $\angle D = x + 30°$

So, we know that $\angle A + \angle B + \angle C + \angle D = 360°$

On putting these values, we get

$(x) + (x + 10°) + (x + 20°) + (x + 30°) = 360°$

$\Rightarrow x = 75°$

Hence, the angles of the quadrilateral are $75°, 85°, 95°, 105°$.

652 **(a)**

We have,

$(n^2 - 1^2) + 2(n^2 - 2^2) + 3(n^2 - 3^2) + \cdots + (n-1)\{n^2 - (n-1)^2\} + n(n^2 - n^2)$

$= \sum_{r=1}^{n} r(n^2 - r^2) = n^2 \sum_{r=1}^{n} r - \sum_{r=1}^{n} r^3$

$= n^2 \frac{n(n+1)}{2} - \left\{\frac{n(n+1)}{2}\right\}^2 = \frac{n^2}{4}(n^2 - 1)$

653 **(a)**

We have,

$\frac{3 + \log 343}{2 + \frac{1}{2}\log\frac{49}{4} + \frac{1}{3}\log\left(\frac{1}{125}\right)}$

$= \frac{3 + \log 7^3}{2 + \frac{1}{2}(\log 7^2 - \log 2^2) + \frac{1}{3}\log 5^{-3}}$

$= \frac{3 + 3\log 7}{2 + (\log 7 - \log 2) - \log 5}$

$= \frac{3(1 + \log 7)}{2 + \log 7 - (\log 2 + \log 5)}$

$= \frac{3(1+\log 7)}{2 + \log 7 - \log 10} = \frac{3(1+\log 7)}{1 + \log 7} = 3$

654 **(c)**

Given series is $9 - 3 + 1 - \frac{1}{3} + \dots\infty$

This is an infinite GP series.

$\therefore S_\infty = \frac{a}{1-r} = \frac{9}{1-\left(-\frac{1}{3}\right)} = \frac{27}{4}$

655 **(a)**

We have,

$\log_e x + \log_e(1+x) = 0$

$\Rightarrow \log_e x(1+x) = 0 \Rightarrow x(1+x) = e^0$

$\Rightarrow x^2 + x - 1 = 0$

656 **(c)**

We have,

$\log_{\sqrt{2}} \sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}$

$= \log_{\sqrt{2}}\left(2^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}}\right) = \log_{\sqrt{2}} 2^{\frac{15}{16}} = \frac{\frac{15}{16}}{\frac{1}{2}}\log_2 2 = \frac{15}{8}$

657 **(d)**

We have,

$A = \frac{a+b}{2}$ and, $S = \frac{n}{2}(a+b)$

$\Rightarrow S = nA \Rightarrow \frac{S}{A} = n$

658 **(b)**

When n is odd, the last term will be n^2. Therefore, the required sum is given by

$1^2 + 2.2^2 + 3^2 + 2.4^2 + \cdots + 2 \cdot (n-1)^2 + n^2$

$= \{1^2 + 2.2^2 + 3^2 + 2.4^2 + \cdots + 2 \cdot (n-1)^2\} + n^2$

$= \frac{(n-1)n^2}{2} + n^2 \left[\text{Replacing } n \text{ by } (n - 1) \text{ in } \frac{n(n+1)^2}{2}\right]$

$= \frac{n^2(n+1)}{2}$

660 **(c)**

Let $S_n = 2n + 3n^2$

$\therefore\ S_{n-1} = 2(n-1) + 3(n-1)^2$

$= 2n - 2 + 3n^2 + 3 - 6n$

$= 3n^2 - 4n + 1$

$\therefore\ T_n = S_n - S_{n-1}$

$= 2n + 3n^2 - (3n^2 - 4n + 1)$

$= 6n - 1$

$\therefore\ T_n = 6r - 1$

661 **(d)**

Given, $\left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots\right)^2 = \left(\frac{e^x + e^{-x}}{2}\right)^2$

$= \frac{1}{4}(e^{2x} + e^{-2x} + 2)$

$= \frac{1}{4}\left[4 + 2\left\{\frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} + \ldots\right\}\right]$

$= 1 + \frac{(2x)^2}{2.2!} + \frac{(2x)^4}{2.4!} + \ldots$

662 **(c)**

Let a be the first term and d be the common difference of the A.P. Then,

$x = a + (p-1)d, y = a + (q-1)d, z = a + (r-1)d$

Let A be the first term and R be the common ratio of the GP

Then,

$x = AR^{p-1}, y = AR^{q-1}, z = AR^{r-1}$

$\therefore x^{y-z}\ y^{z-x}\ z^{x-y}$

$= (AR^{p-1})^{(q-r)d}(AR^{q-1})^{(r-p)d}(AR^{r-1})^{(p-q)d}$

$= A^0R^0 = 1$

663 **(a)**

We have,

$3 + \log_5 x = 2\log_{25} y$

$\Rightarrow 3\log_5 5 + \log_5 x = \frac{2}{2}\log_5 y$

$\Rightarrow \log_5(x \times 5^3) = \log_5 y \Rightarrow 125\ x = y \Rightarrow x = \frac{y}{125}$

664 **(c)**

Given, AP is $3, a_1, a_2, a_3, a_4, a_5, a_6, 31$

$\therefore\ 31 = 3 + 7d$

$\Rightarrow d = 4$

$\therefore\ a_1 = 3 + 4 = 7$

$a_5 = a + 5d = 3 + 20 = 23$

and $a_6 = a + 6d = 3 + 24 = 27$

$\therefore\ a_6 - a_5 = 27 - 23 = 4$

and $a_1 + a_6 = 7 + 27 = 34$

665 **(a)**

a, A_1, A_2, b are in AP

$\therefore\ A_1 - a = b - A_2$

$\Rightarrow\ A_1 + A_2 = a + b$

And a, G_1, G_2, b are in GP.

$\therefore\ \frac{G_1}{a} = \frac{b}{G_2}$

$\Rightarrow G_1G_2 = ab$

$\therefore\ \frac{A_1 + A_2}{G_1G_2} = \frac{a+b}{ab}$

666 **(a)**

We have,

$\log_5(\log_5(\log_2 x)) = 0$

$\Rightarrow \log_5(\log_2 x) = 5^0 \Rightarrow \log_2 x = 5 \Rightarrow x = 2^5$

667 **(d)**

We have,

(a) $0.5 + 0.55 + 0.555 + \ldots = \frac{5}{9}[0.9 + 0.99 +$ *0.999+...*

$= \frac{5}{9}[(1 - 0.1) + (1 - 0.01) + (1 - 0.001) + \ldots$ to n terms]

$= \frac{5}{9}[(1 + 1 + \ldots$ to n terms$) - \left(\frac{1}{10} + \frac{1}{10^2} +\right.$ *1103+...+*to n term

$$= \frac{5}{9}\left[n - \frac{\frac{1}{10}\left\{1 - \frac{1}{10^n}\right\}}{1 - \frac{1}{10}}\right]$$

$$= \frac{5n}{9} - \frac{5}{81}(1 - 10^{-n})$$

(b) $8 + 88 + 888 + \ldots$ to n terms

$= \frac{8}{9}[9 + 99 + 999 + \ldots$ to n terms]

$= \frac{8}{9}[(10 - 1) + (10^2 - 1) + (10^3 - 1) + \ldots$ to n terms]

$= \frac{8}{9}[(10 + 10^2 + 10^3 + \cdots +$ to n terms$) - (1 + 1 + 1 + \ldots +$to n terms$)]$

$$= \frac{8}{9}\left[\frac{10(10^n - 1)}{10 - 1} - n\right]$$

$$= \frac{80}{81}(10^n - 1) - \frac{8n}{9}$$

(c) the nth terms in the sequence is

$$x_n = 1^2 + 2^2 + 3^2 + \ldots + n^2$$

$$= \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$$

$\therefore$ The required sum $= \sum x_n$

$$= \frac{1}{3}\sum n^3 + \frac{1}{2}\sum n^2 + \frac{1}{6}\sum n$$

$$= \frac{1}{3}\left[\frac{n(n+1)}{2}\right]^2 + \frac{1}{2}.\frac{n(n+1)(2n+1)}{6} + \frac{1}{6}.\frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{12}[(n+1) + (2n+1) + 1$$

$$= \frac{n(n+1)}{2}[n^2 + 3n + 2]$$

$$= \frac{n(n+1)}{2}(n+1)(n+2)$$

$$= \frac{n(n+1)^2(n+2)}{12}$$

668 **(b)**

We have,

$\log(1 - x + x^2)$

$= \log\{(1 + \omega x)(1 + \omega^2 x)\}$

$= \log(1 + \omega x) + \log(1 + \omega^2 x)$

$$= \sum_{n=1}^{\infty}(-1)^{n-1}\frac{(\omega x)^x}{n} + \sum_{n=1}^{\infty}(-1)^{n-1}\frac{(\omega^2 x)^n}{n}$$

$$= \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}(\omega^n + \omega^{2n})x^n$$

$\therefore$ Coeff. of x^n in $\log(1 - x + x^2)$

$$= \frac{(-1)^{n-1}}{n}(\omega^n + \omega^{2n})$$

$$= \begin{cases} \frac{(-1)^n}{n}, & \text{if } n \text{ is not a multiple of 3} \\ \frac{2(-1)^{n-1}}{n}, & \text{if a multiple of 3} \end{cases}$$