16.PROBABILITY

Single Correct Answer Type

1. A fair die is tossed eight times. The probability that a third six is observed on the eight throw, is
a) $\frac{^7C_2 \times 5^5}{6^7}$ b) $\frac{^7C_2 \times 5^5}{6^8}$ c) $\frac{^7C_2 \times 5^5}{6^6}$ d) None of these
2. A natural number x is chosen at random from the first 100 natural numbers. The probability that $x + \frac{100}{x} > 50$ is
a) $\frac{1}{10}$ b) $\frac{11}{50}$ c) $\frac{11}{20}$ d) None of these
3. A coin is tossed 4 times. The probability that at least one head turns up, is
a) $\frac{1}{16}$ b) $\frac{2}{16}$ c) $\frac{14}{16}$ d) $\frac{15}{16}$
4. There are 4 white and 4 black balls in a bag and 3 balls are drawn at random. If balls of same colour are identical, the probability that none of them is black, is
a) 1/4 b) 1/14 c) 1/2 d) None of these
5. A box contains 10 mangoes out of which 4 are rotten. 2 mangoes are taken out together. If one of them is found to be good, the probability that the other is also good is
a) 1/3 b) 8/15 c) 5/13 d) 2/3
6. A five digit number is chosen at random. The probability that all the digit are distinct and digits at odd places are odd and digits at even place are even, is
a) $\frac{1}{60}$ b) $\frac{2}{75}$ c) $\frac{1}{50}$ d) $\frac{1}{75}$
7. If two dice are thrown together, then the probability that the sum of numbers appearing on them is 9, is
a) $\frac{1}{9}$ b) $\frac{1}{6}$ c) $\frac{1}{4}$ d) $\frac{1}{3}$
8. If X follows a binomial distribution with parameters $n = 6$ and p. If $4(P(X = 4)) = P(X = 2)$, then $p =$
a) 1/2 b) 1/4 c) 1/6 d) 1/3
9. A man is known to speak truth in 75% cases. If he throws an unbiased die and tells his friend that it is a six, then the probability that it is actually a six, is
a) 1/6 b) 1/8 c) 3/4 d) 3/8
10. If m rupee coins and n ten paise coins are placed in a line, then the probability that the extreme coins are ten paise coins, is
a) $^{m+n}C_m$ b) $\frac{n(n-1)}{(m+n)(m+n-1)}$ c) $^{m+n}P_m$ d) $^{m+n}P_n$
11. The number of times a die must be tossed to obtain a 6 at least once with probability exceeding 0.9 is at least
a) 13 b) 19 c) 25 d) None of these
12. Two dice are thrown n times in succession. The probability of obtaining a double six at least once is
a) $\left(\frac{1}{36}\right)^n$ b) $1 - \left(\frac{35}{36}\right)^n$ c) $\left(\frac{1}{12}\right)^n$ d) None of these
13. Seven chits are numbered 1 to 7. Four chits are drawn one by one with replacement. The probability that the least number appearing on any selected chit is 5, is
a) $\left(\frac{3}{7}\right)^4$ b) $\left(\frac{6}{7}\right)^3$ c) $\frac{5 \times 4 \times 3}{7^3}$ d) $\left(\frac{3}{4}\right)^4$
14. Probability of throwing 16 in one throw with three dice is
a) $\frac{1}{36}$ b) $\frac{1}{18}$ c) $\frac{1}{72}$ d) $\frac{1}{9}$
15. For two events A and B, if $P(A) = P\left(\frac{A}{B}\right) = \frac{1}{4}$ and $P\left(\frac{B}{A}\right) = \frac{1}{2}$, then

a) A and B are independent

b) $P\left(\frac{A'}{B}\right)=\frac{3}{4}$

c) $P\left(\frac{B'}{A'}\right)=\frac{1}{2}$

d) All of the above

16. If the probability that A and B will die with in a year are p and q respectively, then the probability that only one of them will be alive at the end of the year is

a) $p+q$ b) $p+q-2pq$ c) $p+q-pq$ d) $p+q+pq$

17. In a binomial distribution the mean is 15 and variance is 10. Then parameter n is

a) 28 b) 16 c) 45 d) 25

18. Three squares of a chess board are chosen at random, the probability that two are of one colour and one of another

a) 16/21 b) 8/21 c) 32/12 d) None of these

19. If A and B are any two events, then probability that exactly one of them occurs is

a) $P(A)+P(B)+2\,P(A\cap B)$

b) $P(A)+P(B)-P(A\cap B)$

c) $P(\bar{A})+P(\bar{B})+2\,P(\bar{A}\cap\bar{B})$

d) $P(A\cap\bar{B})+P(\bar{A}\cap B)$

20. The mean and variance of a binomial variable X are 2 and 1 respectively. The probability that X takes values greater than 1, is

a) $\frac{5}{16}$ b) $\frac{8}{16}$ c) $\frac{11}{16}$ d) $\frac{1}{16}$

21. If a fair coin is tossed 20 times and let we get head n times, then probability that n is odd, is

a) $\frac{1}{2}$ b) $\frac{1}{6}$ c) $\frac{5}{8}$ d) $\frac{7}{8}$

22. If a dice is thrown twice, the probability of occurrence of 4 at least once, is

a) 11/36 b) 7/12 c) 35/36 d) None of these

23. A box contains 9 tickets numbered 1 to 9 inclusive. If 3 tickets are drawn from the box without replacement. The probability that they are alternatively either {odd, even, odd} of {even, odd, even} is

a) $\frac{5}{17}$ b) $\frac{4}{17}$ c) $\frac{5}{16}$ d) $\frac{5}{18}$

24. If the range of a random variable X is $\{0,1,2,3,4,\dots\}$with $P(X=k)=\frac{(k+1)a}{3^k}$ for $k\geq 0$, then a is equal to

a) $\frac{2}{3}$ b) $\frac{4}{9}$ c) $\frac{8}{27}$ d) $\frac{16}{81}$

25. If A and B are two events such that $P(A)>0$ and $P(B)\neq 1$,then $P(\bar{A}|\bar{B})$ is equal to

a) $1-P(A|\bar{B})$ b) $1-P(\bar{A}|B)$ c) $\frac{1-P(A\cap B)}{P(B)}$ d) $\frac{P(\bar{A})}{P(\bar{B})}$

26. Two numbers are selected randomly from the set $S=\{1,2,3,4,5,6\}$ without replacement one by one. The probability that minimum of the two number is less than 4, is

a) $\frac{1}{15}$ b) $\frac{14}{15}$ c) $\frac{1}{5}$ d) $\frac{4}{5}$

27. A number is chosen at random among the first 120 natural numbers. The probability of the number chosen being a multiple of 5 or 15 is

a) $\frac{1}{8}$ b) $\frac{1}{5}$ c) $\frac{1}{24}$ d) $\frac{1}{6}$

28. Four numbers are chosen at random from $\{1,2,3,\dots,40\}$. The probability that they are not consecutive, is

a) $\frac{1}{2470}$ b) $\frac{4}{7969}$ c) $\frac{2469}{2470}$ d) $\frac{7965}{7969}$

29. A person draws a card from a pack of playing cards, replaces it and shuffles the pack. He continues doing this until he draws a spade. The chance that he fail the first two times is

a) $\frac{9}{64}$ b) $\frac{1}{64}$ c) $\frac{1}{16}$ d) $\frac{9}{16}$

30. Two numbers a and b are chosen at random from the set of first 30 natural numbers. The probability that $a^2 - b^2$ is divisible by 3 is

a) $\frac{9}{87}$ b) $\frac{12}{87}$ c) $\frac{15}{87}$ d) $\frac{47}{87}$

31. A coin is tossed 10 times. The probability of getting exactly six heads is

a) $\frac{512}{513}$ b) $\frac{105}{512}$ c) $\frac{100}{153}$ d) ${}^{10}C_6$

32. Let $A = \{1, 3, 5, 7, 9\}, B = \{2, 4, 6, 8\}$. If a cartesian product $A \times B$, if chosen at random, the probability of $a + b = 9$ is

a) $\frac{1}{4}$ b) $\frac{1}{5}$ c) 1 d) 0

33. One hundred identical coins, each with probability p or showing up heads are tossed once. If $0 < p < 1$ and the probability of heads showing on 50 coins is equal to that of heads showing on 51 coins, then the value of p is

a) $\frac{1}{2}$ b) $\frac{49}{101}$ c) $\frac{50}{101}$ d) $\frac{51}{101}$

34. India plays two ODI matches each with Australia and Pakistan. The probability of India getting 0, 1, 2 are 0.45, 0.05, 0.50. The probability of India getting at least 7 points in the series is

a) 0.00875 b) 0.875 c) 0.0875 d) None of these

35. For a poisson variate X, if $P(X = 2) = 3P(X = 3)$, then the mean of X is

a) 1 b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) $\frac{1}{4}$

36. If A and B are two independent events, the probability that both A and B occur is $1/8$ and the probability that neither of them occurs is $3/8$. The probability of the occurrence of A, is

a) $\frac{1}{2}, \frac{1}{4}$ b) $\frac{1}{3}, \frac{1}{4}$ c) $\frac{1}{4}, \frac{1}{6}$ d) $\frac{1}{5}, \frac{1}{2}$

37. The probability that a certain kind of component will survive a given shock test is $\frac{3}{4}$. The probability that exactly 2 of the next 4 components tested survive is

a) $\frac{9}{41}$ b) $\frac{25}{128}$ c) $\frac{1}{5}$ d) $\frac{27}{128}$

38. A bag contains 2 white and 4 black balls. A ball is drawn 5 times with replacement. The probability that at least 4 of the balls drawn are white, is

a) $\frac{8}{141}$ b) $\frac{10}{243}$ c) $\frac{11}{243}$ d) $\frac{8}{41}$

39. 6 boys and 6 girls sit in a row randomly. The probability that all 6 girls sit together, is

a) $\frac{1}{64}$ b) $\frac{1}{8}$ c) $\frac{1}{132}$ d) None of these

40. If there are 6 girls and 5 boys who sit in a row, then the probability that no two boys sit together is

a) $\frac{6!6!}{2!11!}$ b) $\frac{7!5!}{2!11!}$ c) $\frac{6!7!}{2!11!}$ d) None of these

41. In Q. 14 if $m > n$ then the probability that the mapping selected is an injective map is

a) $\frac{n!}{(n-m)!m^n}$ b) $\frac{n!}{(n-m)!n^m}$ c) $\frac{{}^nC_m}{n^m}$ d) None of these

42. If $\left(\frac{1+a}{3}\right)$ and $\left(\frac{1-a}{4}\right)$ are probability of two mutually exclusive events, then set of all values of a is

a) $-1 \le a \le 1$ b) $-7 \le a \le 5$ c) $-1 \le a \le 2$ d) $-4 \le a \le 1$

43. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours, is

a) $\frac{1}{3}$ b) $\frac{2}{7}$ c) $\frac{1}{21}$ d) $\frac{2}{23}$

44. If X is a random-variable with distribution given below:

X	0	1	2	3
$P(X = x)$	k	$3k$	$3k$	k

The value of k and its variance are

a) $1/8, 22/27$ b) $1/8, 23/27$ c) $1/8, 24/27$ d) $1/8, 3/4$

45. If the letters of the word 'MISSISSIPPI' are written down at random in a row, the probability that four S's come consecutively is

a) $\frac{8}{165}$ b) $\frac{4}{165}$ c) $\frac{161}{165}$ d) None of these

46. Probability of all 3 digit numbers having all the digits same is

a) $\frac{1}{100}$ b) $\frac{3}{100}$ c) $\frac{7}{100}$ d) None of these

47. A random variable X has the following probability distribution

$\boldsymbol{X}$	1	2	3	4
$\boldsymbol{P(X)}$	k	$2k$	$3k$	$4k$

Then, the mean of X is

a) 3 b) 1 c) 4 d) 2

48. In a test, an examines either guesses or copies or knows the answer to a multiple choice questions with four choices. The probability that he makes a guess is $\frac{1}{3}$and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct given that he copies it is $\frac{1}{8}$. The probability that his answer is correct, given that he guesses it is $\frac{1}{4}$.The probability that they knew the answer to the questions given that he correctly answered is

a) $\frac{24}{31}$ b) $\frac{31}{24}$ c) $\frac{24}{29}$ d) $\frac{29}{24}$

49. The probability that a number n chosen at random from 1 to 30, to satisfy $n + (50/n) > 27$ is

a) $\frac{7}{30}$ b) $\frac{3}{10}$ c) $\frac{3}{5}$ d) $\frac{1}{5}$

50. Let A and B are two events and $P(A') = 0.3, P(B) = 0.4, P(A \cap B') = 0.5$, then$P(A \cup B')$ is

a) 0.5 b) 0.8 c) 1 d) 0.1

51. If A and B are two events such that $P(A) \neq 0$and $P(B) \neq 1$, then $P\overline{\left(\frac{A}{B}\right)}$is equal to

a) $1 - P\left(\frac{A}{B}\right)$ b) $1 - P\left(\frac{\bar{A}}{B}\right)$ c) $\frac{1 - P(A \cup B)}{P(\bar{B})}$ d) $\frac{P(\bar{A})}{P(B)}$

52. If the letters of the word 'MISSISSIPPI' are written down at random, in a row the probability that no two $'S'$ occur together is

a) $\frac{5}{33}$ b) $\frac{7}{33}$ c) $\frac{6}{31}$ d) None of these

53. Suppose E and F are two independent events of a random experiment. If the probability of occurrence of E is$\frac{1}{5}$and the probability of occurrence of F given E is $\frac{1}{10}$,then the probability of non-occurrence of at least one of the events E and F is

a) $\frac{1}{18}$ b) $\frac{1}{2}$ c) $\frac{49}{50}$ d) $\frac{1}{50}$

54. If X is a binomial variate with the range $\{0,1,2,3,4,5,6\}$ and $P(X = 2) = 4P(X = 4)$, then the parameter p of X is

a) $\frac{1}{3}$ b) $\frac{1}{2}$ c) $\frac{2}{3}$ d) $\frac{3}{4}$

55. If A and B are arbitrary events, then

a) $P(A \cap B) \geq P(A) + P(B)$ b) $P(A \cup B) \leq P(A) + P(B)$

c) $P(A \cap B) = P(A) + P(B)$ d) None of the above

56. For two events A and B,if $P(A) = P\left(\frac{A}{B}\right) = \frac{1}{4}$and $P\left(\frac{B}{A}\right) = \frac{1}{2}$, then

a) A and B are independent events b) $P\left(\frac{A'}{B}\right) = \frac{3}{4}$

c) $P\left(\frac{B'}{A}\right) = \frac{1}{2}$ d) All of the above

57. Let $0 < P(A) < 1, 0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$. Then,
a) $P(B/A) = P(B) - P(A)$
b) $P(A^c \cup B^c) = P(A^c) + P(B^c)$
c) $P((A \cup B)^c) = P(A^c)P(B^c)$
d) $P(A/B) = P(B)$
58. A number n is chosen at random from {1, 2, 3, 4, ……….,1000}. The probability that n is a number that leaves remainder 1 when divided by 7, is
a) $\frac{71}{500}$ b) $\frac{143}{1000}$ c) $\frac{72}{500}$ d) $\frac{71}{1000}$
59. If from each of three boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, one ball is drawn at random, then the probability that 2 white and 1 black ball will be drawn, is
a) $\frac{13}{32}$ b) $\frac{1}{4}$ c) $\frac{1}{32}$ d) $\frac{3}{16}$
60. A random variable X has poisson distribution with mean 2. Then, $P(X > 1.5)$ equals
a) $\frac{3}{e^2}$ b) $1 - \frac{3}{e^2}$ c) 0 d) $\frac{2}{e^2}$
61. If $P(A \cup B) = 0.8$ and $P(A \cap B) = 0.3$, then $P(A') + P(B')$ equals to
a) 0.3 b) 0.5 c) 0.7 d) 0.9
62. The probability that a candidate secures a seat in Engineering through "EAMCET" is 1/10. 7 candidates are selected at random from a centre. The probability that exactly two will get seats is
a) $15\,(0.1)^2\,(0.9)^5$ b) $20\,(0.1)^2\,(0.9)^5$ c) $21\,(0.1)^2\,(0.9)^5$ d) $23\,(0.1)^2\,(0.9)^5$
63. A committee of five is to be chosen from a group of 9 people. The probability that a certain married couple will either serve together or not at all is
a) $\frac{1}{2}$ b) $\frac{5}{9}$ c) $\frac{4}{9}$ d) $\frac{2}{3}$
64. Three numbers are chosen from 1 to 30. The probability that they are not consecutive is
a) $\frac{144}{145}$ b) $\frac{143}{145}$ c) $\frac{142}{145}$ d) None of these
65. A bag contains 6 red and 3 white balls. Four balls are drawn one by one and not replaced. The probability that they are alternatively of different colours, is
a) $\frac{4}{42}$ b) $\frac{5}{42}$ c) $\frac{7}{42}$ d) $\frac{8}{42}$
66. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house, is
a) $\frac{7}{9}$ b) $\frac{8}{9}$ c) $\frac{1}{9}$ d) $\frac{2}{9}$
67. India plays a two ODI matches each with Australia and Pakistan. The probability of India getting points 0,1,2 are 0.45, 0.05, 0.50. The probability of India getting at least 7 points in the series is
a) 0.00875 b) 0.875 c) 0.0875 d) None of these
68. If A and B are two mutually exclusive events, then
a) $P(A) < P(\bar{B})$ b) $P(A) > P(\bar{B})$ c) $P(A) < P(B)$ d) None of these
69. If $f(x) = \lambda e^{-ax} (a > 0)$ for $0 \le x < \infty$ is a prabability density, then λ is equal to
a) a b) a^2 c) $\frac{1}{a}$ d) a^3
70. 10 different books and 2 different pens are given to 3 boys so that each gets equal number of things. The probability that the some boy does not receive both the pens is
a) 5/11 b) 7/11 c) 2/3 d) 6/11
71. From a set of 100 cards numbered 1 to 100, one card is drawn at random. The probability that the number obtained on the card is divisible by 6 or 8 but not by 24, is

a) $\frac{6}{25}$ b) $\frac{1}{4}$ c) $\frac{1}{6}$ d) $\frac{1}{5}$

72. What is the probability that when one die is thrown, the number appearing on top is even?
a) 1/6 b) 1/3 c) 1/2 d) None of these

73. If in a trial the probability of success is twice the probability of failure. In six trials the probability of at least four successes is
a) $\frac{496}{729}$ b) $\frac{400}{729}$ c) $\frac{500}{729}$ d) $\frac{600}{729}$

74. An insurance salesman sells policies to 5 men, all of identical age and in good health. The probability that a man of this particular age will be alive after 30 years is 2/3. The probability that after the lapse of 30 years all the five persons will be alive, is
a) $\frac{1}{16}$ b) $\frac{16}{81}$ c) $\frac{32}{243}$ d) None of these

75. The probability that number selected at random from the numbers 1, 2, 3, 4, 5, 6, 7, 8, ……., 100 is a prime, is
a) 0.4 b) 0.25 c) 0.45 d) 0.43

76. There are 7 seats in a row. Three persons take seats at random. The probability that the middle seat is always occupied and no two persons are consecutive is
a) 9/70 b) 9/35 c) 4/35 d) None of these

77. Given two events A and B. If odds against A are 2 : 1 and those in favour of $A \cup B$ are as 3 : 1, then
a) $\frac{1}{2} \leq P(B) \leq \frac{3}{9}$ b) $\frac{5}{12} \leq P(B) \leq \frac{3}{4}$ c) $\frac{1}{2} \leq P(B) \leq \frac{3}{5}$ d) None of these

78. The mean and variance of binomial distribution are 4 and 3 respectively. Then, the probability of getting exactly six success in this distribution is
a) ${}^{16}C_6 \left(\frac{1}{4}\right)^6 \left(\frac{3}{4}\right)^{10}$ b) ${}^{16}C_6 \left(\frac{1}{4}\right)^6 \left(\frac{3}{4}\right)^{20}$ c) ${}^{16}C_6 \left(\frac{1}{4}\right)^8 \left(\frac{3}{4}\right)^{12}$ d) ${}^{16}C_9 \left(\frac{1}{4}\right)^{16} \left(\frac{3}{4}\right)^{20}$

79. At a telephone enquiry system the number of phone calls regarding relevant enquiry follow poisson distribution with an average of 5 phone calls during 10 min time intervals. The probability that there is at the most one phone cell during a 10 min time period, is
a) $\frac{5}{6}$ b) $\frac{6}{55}$ c) $\frac{6}{e^5}$ d) $\frac{6}{5^e}$

80. The probability that the two digit number formed by digit 1, 2, 3, 4, 5, is divisible by 4 (while repetition of digit is allowed), is
a) $\frac{1}{30}$ b) $\frac{1}{20}$ c) $\frac{1}{40}$ d) None of these

81. A and B are two events. Odds against A are 2 to 1 and odds in favour of $A \cup B$ are 3 to1. If $x \leq P(B) \leq y$,then ordered pair $(x, y)$is
a) $\left(\frac{5}{12}, \frac{3}{4}\right)$ b) $\left(\frac{2}{3}, \frac{3}{4}\right)$ c) $\left(\frac{1}{3}, \frac{3}{4}\right)$ d) None of these

82. 5 boys and 5 girls are sitting in a row randomly. The probability that boys and girls sit alternatively, is
a) $\frac{5}{126}$ b) $\frac{1}{42}$ c) $\frac{4}{126}$ d) $\frac{1}{126}$

83. A problem in mathematics is given to three students E_1, E_2 and E_3 and their respective probability of solving the problem is $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. Probability that the problem is solved is
a) 3/4 b) 1/2 c) 2/3 d) 1/3

84. A number n is chosen at random from the set {11, 12, 13, ……, 30}. The probability that n is neither divisible by 3 nor divisible by 5 is
a) $\frac{7}{20}$ b) $\frac{9}{20}$ c) $\frac{11}{20}$ d) $\frac{13}{20}$

85. Events A, B, C are mutually exclusive events such that $P(A) = \frac{3x+1}{3}, P(B) = \frac{1-x}{4}$ and $P(C) = \frac{1-2x}{2}$. The set of possible values of x are in the interval

a) $\left[\frac{1}{3}, \frac{1}{2}\right]$ b) $\left[\frac{1}{3}, \frac{2}{3}\right]$ c) $\left[\frac{1}{3}, \frac{13}{3}\right]$ d) [0, 1]

86. If a dice is thrown twice, the probability of occurrence of 4 at least once is

a) $\frac{11}{36}$ b) $\frac{35}{36}$ c) $\frac{7}{12}$ d) None of these

87. The probability that at least one of the events A and B occurs is 0.7 and they occur simultaneously with probability 0.2. Then, $P(\bar{A}) + P(\bar{B}) =$

a) 1.8 b) 0.6 c) 1.1 d) 1.4

88. Two persons A and B take turns in throwing a pair of dice. The first person to through 9 from both dice will be awarded the prize. If A throws first, then the probability that B wins the game, is

a) $\frac{9}{17}$ b) $\frac{8}{17}$ c) $\frac{8}{9}$ d) $\frac{1}{9}$

89. The probability that a teacher will give an unannounced test during any class meeting is $1/5$. If a student is absent twice, the probability that he will miss atleast one test, is

a) $7/25$ b) $9/25$ c) $16/25$ d) $24/25$

90. If $P(A \cup B) = 3/4$ and $P(\bar{A}) = 2/3$, then $P(\bar{A} \cap B)$ is equal to

a) $1/12$ b) $7/12$ c) $5/12$ d) $1/2$

91. Given $P(A) = 0.5, P(B) = 0.4, P(A \cap B) = 0.3$,then $P\left(\frac{A'}{B'}\right)$is equal to

a) $\frac{1}{3}$ b) $\frac{1}{2}$ c) $\frac{2}{3}$ d) $\frac{3}{4}$

92. A natural number is selected from 1 to 1000 at random, then the probability that a particular non-zero digit appears at most once, is

a) $\frac{243}{250}$ b) $\frac{13}{15}$ c) $\frac{11}{15}$ d) $\frac{239}{250}$

93. Let A, B, C be three mutually independent events. Consider the two statements S_1 and S_2, $S_1 : A$ and $B \cup C$ are independent, $S_2 : A$ and $B \cap C$ are independent, then

a) Both S_1 and S_2 b) Only S_1 is true
c) Only S_2 is true d) Neither S_1 and S_2 is true

94. A biased die is tossed and the respective probabilities for various faces to turn up are

Face :	1	2	3	4	5	6
Probability :	0.1	0.24	0.19	0.18	0.15	0.14

If an even face has turned up, then the probability that it is face 2 or face 4, is

a) 0.25 b) 0.42 c) 0.75 d) 0.9

95. A bag X contains 2 white and 3 black bails and another bag Y contains 4 white and 2 black balls. One bag is selected at random and a ball is drawn from it. Then, the probability for the ball chosen be white, is

a) $\frac{2}{15}$ b) $\frac{7}{15}$ c) $\frac{8}{15}$ d) $\frac{14}{15}$

96. Probability $P(A) = \frac{4}{5}, P(B') = \frac{2}{5}$and $P(A \cap B) = \frac{1}{2}$,then$P(A \cap B')$ is equal to

a) $\frac{3}{10}$ b) $\frac{5}{2}$ c) $\frac{2}{5}$ d) $\frac{5}{7}$

97. $'X'$ speaks truth in 60% and $'Y'$ in 50% of the cases. The probability that they contradict each other narrating the same incident is

a) $\frac{1}{4}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$

98. Twelve tickets are numbered from 1 to 12. One ticket is drawn at random, then the probability of the number to be divisible by 2 or 3, is

a) $\frac{2}{3}$ b) $\frac{7}{12}$ c) $\frac{5}{6}$ d) $\frac{3}{4}$

99. A and B appeared for an interview for two posts. The probability of $A's$ selection is $1/3$ and that of $B's$ selection is $2/5$. The probability that only one of them is selected, is

a) $7/15$ b) $8/15$ c) $2/15$ d) $4/15$

100. If A and B are mutually exclusive events with $P(B) \neq 1$,then $P(A|\bar{B})$ is equal to (Here, $\bar{B}$is the complement of the event B)

a) $\frac{1}{P(B)}$ b) $\frac{1}{1-P(B)}$ c) $\frac{P(A)}{P(B)}$ d) $\frac{P(A)}{1-P(B)}$

101. Unbiased die is thrown, probability that outcome is greater than 4, is

a) $\frac{3}{4}$ b) $\frac{4}{5}$ c) $\frac{1}{3}$ d) $\frac{5}{6}$

102. If A and B are independent events and $P(C) = 0$, then

a) A and C are independent
b) B and C are independent
c) A, B and C are independent
d) All of these

103. A dice is thrown 100 times, getting an even number is considered a success. The variance of the number of successes is

a) 10 b) 25 c) 18 d) 10

104. A five digit number is formed by writing the digits 1, 2, 3, 4, 5, in a random order without repetitions. Then the probability that the number is divisible by 4, is

a) 3/5 b) 18/5 c) 1/5 d) 6/5

105. A coin is tossed 10 times. The probability of getting exactly six heads, is

a) $\frac{512}{513}$ b) $\frac{105}{512}$ c) $\frac{100}{153}$ d) ${}^{10}C_6$

106. Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently, equals

a) $\frac{1}{2}$ b) $\frac{7}{15}$ c) $\frac{2}{15}$ d) $\frac{1}{13}$

107. If A and B are any two events, then $P(\overline{A} \cup B)$ is equal to

a) $P(\overline{A})P(\overline{B})$ b) $1 - P(A) - P(B)$
c) $P(A) + P(B) - P(A \cap B)$ d) $P(B) - P(A \cap B)$

108. The probability of obtaining sum '8' in a single throw of two dice is

a) $\frac{1}{36}$ b) $\frac{5}{36}$ c) $\frac{4}{36}$ d) $\frac{6}{36}$

109. A pair of a dice thrown, if 5. appears on at least one of the dice, then the probability that the sum is 10 or greater, is

a) $\frac{11}{36}$ b) $\frac{2}{9}$ c) $\frac{3}{11}$ d) $\frac{1}{12}$

110. A coin is tossed n times. The probability of getting head at least once is greater than 0.8, then the least value of n is

a) 2 b) 3 c) 4 d) 5

111. Let S be the sample space of the random experiment of throwing simultaneously two unbiased dice with six faces (numbered 1 to 6) and let $E_k = \{(a, b) \in S: ab = k\}$ for $k \geq 1$.
If $p_k = P(E_k)$ for $k \geq 1$, then the correct among the following, is

a) $p_1 < p_{30} < p_4 < p_6$ b) $p_{36} < p_6 < p_2 < p_4$ c) $p_1 < p_{11} < p_4 < p_6$ d) $p_{36} < p_{11} < p_6 < p_4$

112. Suppose $n(\geq 3)$ persopns are sitting in a row. Two of them are selected at random. The probability that they are not together is

a) $1 - \frac{2}{n}$ b) $\frac{2}{n-1}$ c) $1 - \frac{1}{n}$ d) None of these

113. If $P(A \cap B) = \frac{1}{3}, P(A \cup B) = \frac{5}{6}$, and $P(A) = \frac{1}{2}$, then which one of the following is correct?

a) A and B are independent events b) A and B are mutually exclusive events
c) $P(A) = P(B)$ d) None of the above

114. A man is known to speak the truth 3 out of 4 times. He throws a die and reports that it is six. The probability that it is actually a six, is

a) $\frac{3}{8}$ b) $\frac{1}{5}$ c) $\frac{3}{4}$ d) None of these

115. A man takes a step forward with probability 0.4 and back-ward with probability 0.6. The probability that at the end of eleven steps he is one step away from the starting point is

a) ${}^{11}C_6(0.24)^5$ b) ${}^{11}C_6(0.4)^6(0.6)^5$ c) ${}^{11}C_6(0.6)^6(0.4)^5$ d) None of these

116. A coin is tossed three times. The probability of getting a head once and a tail twice is

a) $\frac{1}{8}$ b) $\frac{1}{3}$ c) $\frac{3}{8}$ d) $\frac{1}{2}$

117. The probability that a man will hit a target in shooting practice is 0.3. If he shoots 10 times, the probability that he hits the target is

a) 1 b) $1-(0.7)^{10}$ c) $(0.7)^{10}$ d) $(0.3)^{10}$

118. Seven chits are numbered 1 to 7. Four chits are drawn one by one with replacement. The probability that the least number appearing on any selected chit is 5, is

a) $\left(\frac{3}{7}\right)^4$ b) $\left(\frac{6}{7}\right)^3$ c) $\frac{5\cdot 4\cdot 3}{7^3}$ d) $\left(\frac{3}{4}\right)^3$

119. It is given the events A and B are such that $P(A)=\frac{1}{4}, P(A|B)=\frac{1}{2}$ and $P(B|A)=\frac{2}{3}$.Then, $P(B)$ is

a) $\frac{1}{2}$ b) $\frac{1}{6}$ c) $\frac{1}{3}$ d) $\frac{2}{3}$

120. A and B toss a coin alternately till one of them tosses heads and wins the game, their respective probability of winning are

a) $\frac{1}{4}$ and $\frac{3}{4}$ b) $\frac{1}{2}$ and $\frac{1}{2}$ c) $\frac{1}{3}$ and $\frac{2}{3}$ d) $\frac{1}{5}$ and $\frac{4}{5}$

121. Let E_1, E_2 be two mutually exclusive events of an experiment with $P(\text{not}E_2)=0.6=P(E_1\cup E_2)$.Then, $P(E_1)$is equal to

a) 0.1 b) 0.3 c) 0.4 d) 0.2

122. If two events A and B are such that $P(A^c)=0.3, P(B)=0.4, P(A\cap B^c)=0.5$, then $P(B/A\cup B^c)=$

a) 0.20 b) 0.25 c) 0.30 d) 0.35

123. The odds against a certain event are 5 :2 and the odds in favour of another independent event are 6 : 5. The probability that at least one of the events will happen, is

a) $\frac{25}{77}$ b) $\frac{52}{77}$ c) $\frac{12}{77}$ d) $\frac{65}{77}$

124. A and B are two independent events such that their probabilities are $\frac{3}{10}$ and $\frac{2}{5}$ respectively. The probability of exactly one of the events happening is

a) 23/50 b) 1/2 c) 31/50 d) None of these

125. A fair coin is tossed a fixed number of times. If the probability of getting 4 heads equals the probability of getting 7 heads, then the probability of getting 2 heads is

a) $\frac{55}{2048}$ b) $\frac{3}{4096}$ c) $\frac{1}{1024}$ d) None of these

126. If $P(A)=P(B)=x$ and $P(A\cap B)=P(A'\cap B')=\frac{1}{3}$, then x is equal to

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{4}$ d) $\frac{1}{6}$

127. A bag contains 7 red and 2 white balls and another bag contains 5 red and 4 white balls. Two balls are drawn, one from each bag. The probability that both the balls are white, is

a) 2/9 b) 2/3 c) 8/81 d) 35/81

128. If $P(A\cap B)=\frac{1}{3}, P(A\cup B)=\frac{5}{6}$ and $P(A)=\frac{1}{2}$, then which one of the following is correct?

a) A and B are independent events b) A and B are mutually exclusive events
c) $P(A)=P(B)$ d) $P(A)<P(B)$

129. A speaks truth 4 out of 5 times. A die is tossed. He reports that there is a six. The probability that actually there was a six, is

a) 4/9 b) 5/9 c) 3/10 d) None of these

130. There are 12 white and 12 red balls in a bag. Balls are drawn one by one with replacement from the bag. The probability that 7th drawn ball is 4th white, is

a) $\frac{1}{4}$ b) $\frac{1}{8}$ c) $\frac{1}{2}$ d) $\frac{1}{3}$

131. If birth to male child and birth to female child are equal-probable, then what is the probability that at least one of the three children born to a couple is male?

a) $\frac{4}{5}$ b) $\frac{7}{8}$ c) $\frac{8}{7}$ d) $\frac{1}{2}$

132. A letter is taken out at random from `ASSISTANT’ and another is taken out from `STATISTICS’. The probability that they are the same letters is

a) $\frac{1}{45}$ b) $\frac{13}{90}$ c) $\frac{19}{90}$ d) None of these

133. A and B toss a coin alternately on the understanding that the first to obtain head win the toss. The probability that A wins the toss

a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) $\frac{1}{4}$ d) $\frac{3}{4}$

134. If A and B are independent events such that $P(A) > 0, P(B) > 0$, then

a) A and B are mutually exclusive

b) A and $\bar{B}$ are dependent

c) $\bar{A}$ and B are dependent

d) $P(A/B) + P(\bar{A}/B) = 1$

135. The probability that in a family of 5 members, exact 2 members have birthday on Sunday, is

a) $\frac{12 \times 5^3}{7^5}$ b) $\frac{10 \times 6^2}{7^5}$ c) $\frac{2}{5}$ d) $\frac{10 \times 6^3}{7^5}$

136. A manufacture of cotter pins knows that 5% of his product is defective. He sells pins in boxes of 100 and guarantees that not more than one pin will be defective in a box. In order to find the probability that a box will fail to meet the guaranteed quality, the probability distribution one has to employ is

a) Binomial b) Poisson c) Normal d) exponential

137. The probability that a leap year will have 53 Fridays or 53 Saturdays, is

a) $\frac{2}{7}$ b) $\frac{3}{7}$ c) $\frac{4}{7}$ d) $\frac{1}{7}$

138. Five boys and three girls are seated at random in a row. The probability that no boy sits between two girls is

a) 1/56 b) 1/8 c) 3/28 d) None of these

139. A die is tossed thrice. If event of getting an even number is a success, then the probability of getting at least two successes is

a) $\frac{7}{8}$ b) $\frac{1}{4}$ c) $\frac{2}{3}$ d) $\frac{1}{2}$

140. Out of 40 consecutive integers, two are chosen at random, the probability that their sum is odd is

a) 14/29 b) 20/39 c) 1/2 d) None of these

141. A bag contains 5 blue balls and unknown numbers of red balls, two balls are drawn at random. The probability of both of them are blue is $\frac{5}{14}$, then the number of red ballsare

a) 3 b) 2 c) 4 d) 5

142. Two unbiased dice are thrown simultaneously. The probability to get a sum more than 8 is

a) $\frac{5}{36}$ b) $\frac{5}{18}$ c) $\frac{5}{12}$ d) $\frac{2}{9}$

143. If A and B are two events, then P (neither A nor B) equals

a) $1 - P(A \cup B)$ b) $P(\bar{A}) + P(\bar{B})$ c) $1 - P(A) - P(B)$ d) None of these

144. An unbiased die is tossed until a number greater than 4 appears. The probability that an even number of tosses is needed, is

a) $\frac{1}{2}$ b) $\frac{2}{5}$ c) $\frac{1}{5}$ d) $\frac{2}{3}$

145. Two coins and a die are tossed. The probability that both coins fall heads and the die shows a 3 or 6, is

a) 1/8 b) 1/12 c) 1/16 d) None of these

146. A six faced die is a biased one. It is thrice more likely to show an odd number than show an even number. It is thrown twice. The probability that the sum of the numbers in the two throws is even, is

a) 5/9 b) 5/8 c) 1/2 d) None of these

147. A man and his wife appear for an interview for two posts. The probability of the man's selection is 1/5 and that of his wife's selection is 1/7. The probability that at least one of them is selected, is

a) $\frac{9}{35}$ b) $\frac{12}{35}$ c) $\frac{2}{7}$ d) $\frac{11}{35}$

148. In Q. 90, the probability that the last digit is 2,4,6 or 8, is

a) $\frac{8^n}{5^n}$ b) $\frac{8^n - 2^n}{5^n}$ c) $\frac{4^n - 2^n}{5^n}$ d) None of these

149. A and B are two events such that $P(A) > 0, P(B) \neq 1$, then $P\left(\frac{\bar{A}}{\bar{B}}\right)$ is equal to

a) $1 - P\left(\frac{A}{B}\right)$ b) $1 - P\left(\frac{\bar{A}}{B}\right)$ c) $\frac{1 - P(A \cup B)}{P(\bar{B})}$ d) $\frac{P(\bar{A})}{P(\bar{B})}$

150. Given that X is discrete random variable which takes the values 0,1,2 and $P(X = 0) = \frac{144}{169}, P(X = 1) = \frac{1}{169}$, then the value of $P(X = 2)$ is

a) $\frac{145}{169}$ b) $\frac{24}{169}$ c) $\frac{2}{169}$ d) $\frac{143}{169}$

151. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$is

a) $\frac{2}{5}$ b) $\frac{3}{5}$ c) 0 d) 1

152. If $P(A) = P(B) = P(C) = \frac{1}{4}, P(AB) = P(CB) = 0$ and $P(AC) = \frac{1}{8}$,then $P(A + B)$is equal to

a) $\frac{5}{8}$ b) $\frac{37}{64}$ c) $\frac{3}{4}$ d) $\frac{1}{2}$

153. In a class of 125 students 70 passed in Mathematics, 55 in Statistic and 30 in both. The probability that a student selected at random from the class, has passed in only one subject is

a) 13/25 b) 3/25 c) 17/25 d) 8/25

154. When three dice are thrown the probability of getting 4 or 5 on each of the dice simultaneously, is

a) $\frac{1}{72}$ b) $\frac{1}{108}$ c) $\frac{1}{24}$ d) None of these

155. A and B are two independent events such that $P(A)\frac{1}{2}$ and $P(B) = \frac{1}{3}$, then $P(\text{neither } A \text{ nor } B)$is equal to

a) $\frac{2}{3}$ b) $\frac{1}{6}$ c) $\frac{5}{6}$ d) $\frac{1}{3}$

156. The probability that the 13th day of a randomly chosen month is a Friday, is

a) $\frac{1}{12}$ b) $\frac{1}{7}$ c) $\frac{1}{84}$ d) None of these

157. Three coins are tossed, then what is the probability that at least two heads appears on upper face?

a) $\frac{5}{8}$ b) 40 c) $\frac{8}{5}$ d) None of these

158. If the letters of the word 'REGULATION' be arranged at random, the probability that there will be exactly 4 letters between R and E is

a) 1/10 b) 1/9 c) 1/5 d) 1/2

159. A bag contains 3 black, 3 white and 2 red balls. One by one, three balls are drawn without replacement. The probability that the third ball is red, is equal to

a) $\frac{2}{56}$ b) $\frac{3}{56}$ c) $\frac{1}{56}$ d) $\frac{14}{56}$

160. If two dice are thrown simultaneously, then probability that 1 comes on first dice, is

a) 1/36 b) 5/36 c) 1/6 d) None of these

161. In a book of 500 pages, it is found that there are 250 typing errors. Assume that poisson law holds for the number of errors per page. Then, the probability that a random sample of 2 pages will contain no error, is

a) $e^{-0.3}$ b) $e^{-0.5}$ c) e^{-1} d) e^{-2}

162. If birth to a male child and birth to a female child are equal-probable, then what is the probability that at least one of the three children born to a couple is male?

a) $\frac{4}{5}$ b) $\frac{7}{8}$ c) $\frac{8}{7}$ d) $\frac{1}{2}$

163. A card is drawn from a pack of cards. The probability that the card will be a queen or a heart, is

a) $\frac{4}{3}$ b) $\frac{16}{3}$ c) $\frac{4}{13}$ d) $\frac{5}{3}$

164. A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is

a) $\frac{1}{729}$ b) $\frac{8}{9}$ c) $\frac{8}{729}$ d) $\frac{8}{243}$

165. Three letters are written to different persons and addresses to three envelops are also written. Without looking at the addresses, the probability that the letters go into right envelops is

a) 1/27 b) 1/6 c) 1/9 d) None of these

166. If $P(B)=\frac{3}{4}, P(A\cap B\cap \overline{C})=\frac{1}{3}$ and $P(\overline{A}\cap B\cap \overline{C})=\frac{1}{3}$, then $P(B\cap C)$ is

a) $\frac{1}{12}$ b) $\frac{1}{6}$ c) $\frac{1}{15}$ d) $\frac{1}{9}$

167. A person draws out two balls successively from a bag containing 6 red and 4 white balls. The probability that at least one of them will be red, is

a) $\frac{78}{90}$ b) $\frac{30}{90}$ c) $\frac{48}{90}$ d) $\frac{12}{90}$

168. A matrix is chosen at random from the set of all 2×2 matrices with elements 0 and 1 only. The probability that the value of the determinant of the matrix chosen is positive, is

a) 1/2 b) 3/16 c) 11/16 d) 13/16

169. Two dice are tossed once. The probability of getting even number at the first die or a total of 8 is

a) $\frac{1}{36}$ b) $\frac{3}{36}$ c) $\frac{11}{36}$ d) $\frac{5}{9}$

170. The probability of forming a three digit numbers with the same digits when three digit numbers are formed out of the digit 0, 2, 4, 6, 8 is

a) $\frac{1}{16}$ b) $\frac{1}{12}$ c) $\frac{1}{645}$ d) $\frac{1}{25}$

171. A bag contains a white and b black balls. Two players A and B alternately draw a ball from the bag replacing the ball each time after the draw till one of them draws a white ball and wins the game. A begins the game. If the probability of A winning the game is three times that of B, then the ratio $a:b$ is

a) 1 : 1 b) 1 : 2 c) 2 : 1 d) None of these

172. The probability that a leap year selected at random will contain either 53 Thursdays or 53 Fridays, is

a) $\frac{3}{7}$ b) $\frac{2}{7}$ c) $\frac{5}{7}$ d) $\frac{1}{7}$

173. For the three events A, B and C, P (exactly one of the events A or B occurs)$=P$(exactly one of the events B or C occurs)$-P$ (exactly one of the events C or A occurs) $=p$ and P (all the three events occur simultaneously) $=p^2$, where $0<p<\frac{1}{2}$. Then, the probability of at least one of the three events A, B and C occuring, is

a) $\frac{3p+2p^2}{2}$ b) $\frac{p+3p^2}{4}$ c) $\frac{p+3p^2}{2}$ d) $\frac{3p+2p^2}{4}$

174. A letter is taken out at random from 'ASSISTANT' and another is taken out from 'STATISTICS'. The probability that they are same letters, is

a) $\frac{1}{45}$ b) $\frac{13}{90}$ c) $\frac{19}{90}$ d) None of these

175. Four boys and three girls stand in a queue for an interview, probability that they will in alternate position,

is

a) 1/34 b) 1/35 c) 1/17 d) 1/68

176. The probability of getting at least one tail in 4 throws of a coin is

a) $\frac{15}{16}$ b) $\frac{1}{16}$ c) $\frac{1}{4}$ d) None of these

177. If a coin be tossed n times, then probability that the head comes odd times is

a) 1/2 b) $1/2^n$ c) $1/2^{n-1}$ d) None of these

178. The probability that a number selected at random from the set of numbers 1,2,3,4, … ,100 is a cube, is

a) $\frac{1}{25}$ b) $\frac{2}{25}$ c) $\frac{3}{25}$ d) $\frac{4}{25}$

179. Seven digits from the digits 1,2,3,4,5,6,7,8,9 are written in a random order. The probability that this seven digit number is divisible by 9 is

a) 2/9 b) 1/5 c) 1/3 d) 1/9

180. A bag contains 5 red, 3 white and 2 black balls. If a ball is picked at random, the probability that it is red is

a) 1/5 b) 1/2 c) 3/10 d) 9/10

181. To open a lock, a key is taken out from a collection of n keys at random. If the lock is not opened with this key, it is put back into the collection and another key is tried. The process is repeated again and again. If it is given that with only one key in the collection, the lock can be opened, then the probability that the lock will open in n trials, is

a) $\left(\frac{1}{n}\right)^n$ b) $\left(\frac{n-1}{n}\right)^n$ c) $1-\left(\frac{n-1}{n}\right)^n$ d) None of these

182. Let A and B two events such that $P(\overline{A \cup B})=\frac{1}{6}, P(A \cap B)=\frac{1}{4}$ and $P(\bar{A})=\frac{1}{4},$where $\bar{A}$ stands for complement of event A. Then, events A and B are

a) Mutually exclusive and independent b) Independent but not equally likely

c) Equally likely but not independent d) Equally likely and mutually exclusive

183. If the probability of A to fail in an examination is 0.2 and that for B is 0.3, then probability that either A or B is fail, is

a) 0.5 b) 0.44 c) 0.8 d) 0.25

184. In a binomial distribution $B\left(n, P=\frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to $\frac{9}{10}$, then n is greater than

a) $\frac{1}{\log_{10} 4-\log_{10} 3}$ b) $\frac{1}{\log_{10} 4+\log_{10} 3}$ c) $\frac{9}{\log_{10} 4-\log_{10} 3}$ d) $\frac{4}{\log_{10} 4-\log_{10} 3}$

185. A fair die is rolled. The probability that the first time 1 occurs at the even throw is

a) $\frac{1}{6}$ b) $\frac{5}{11}$ c) $\frac{6}{11}$ d) $\frac{5}{36}$

186. Two persons each make a single throw with a pair of dice. The probability that the throws are unequal is given by:

a) $\frac{1}{6^3}$ b) $\frac{73}{6^3}$ c) $\frac{51}{6^3}$ d) None of these

187. An urn contains 3 red and 5 blue balls. The probability that two balls are drawn in which 2nd ball drawn is blue without replacement, is

a) $\frac{5}{16}$ b) $\frac{5}{56}$ c) $\frac{5}{8}$ d) $\frac{20}{56}$

188. Two coins are tossed five times. The probability that an odd number of heads are obtained, is

a) $\left(\frac{1}{2}\right)^5$ b) $\frac{3}{5}$ c) $\frac{2}{5}$ d) None of these

189. Seven balls are drawn simultaneously from a bag containing 5 white and 6 green balls. The probability of drawing 3 white and 4 green balls is

a) $\frac{7}{{}^{11}C_7}$ b) $\frac{{}^5C_3+{}^6C_4}{{}^{11}C_7}$ c) $\frac{{}^5C_2\,{}^6C_2}{{}^{11}C_7}$ d) $\frac{{}^6C_3\,{}^5C_4}{{}^{11}C_7}$

190. If p is chosen at random in the interval $0 \le p \le 5$, the probability that the roots of the equation $x^2 + px + \frac{p}{4} + \frac{1}{2} = 0$ are real, is

a) 1/5 b) 2/5 c) 3/5 d) 4/5

191. One card is drawn randomly from a pack of 52 cards, then the probability that it is a king of spade, is

a) 1/26 b) 3/26 c) 4/13 d) 3/13

192. If the probability for A to fail in an examination is 0.2 and that for B is 0.3, then the probability that either A or B fails, is

a) 0.38 b) 0.44 c) 0.50 d) 0.94

193. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then, the probability of 2 successes is

a) $\frac{37}{256}$ b) $\frac{219}{256}$ c) $\frac{128}{256}$ d) $\frac{28}{256}$

194. If the mean of a binomial distribution is 25, then its standard deviation lies in the interval given below:

a) [0,5) b) (0,5] c) [0,25) d) (0,25]

195. Let $A < B$ and$C$ be the three events such that $P(A) = 0.3, P(B) = 0.4, P(C) = 0.8, P(A \cap B) = 0.08, P(A \cap C) = 0.28, P(A \cap B \cap C) = 0.09$. If $P(A \cup B \cup C) \ge 0.75$,then $P(B \cap C)$ satisfies

a) $P(B \cap C) \le 0.23$ b) $P(B \cap C) \le 0.48$

c) $0.23 \le P(B \cap C) \le 0.48$ d) $0.23 \le P(B \cap C) \ge 0.48$

196. A determinant of second order is made with the elements 0, 1. What is the probability that the determinant is positive?

a) $\frac{7}{12}$ b) $\frac{11}{12}$ c) $\frac{3}{16}$ d) $\frac{15}{16}$

197. If m and σ^2 are the mean and variance of the random variable X. whose distribution is given by

$\boldsymbol{X}$	0	1	2	3
$\boldsymbol{P(X)}$	$\frac{1}{3}$	$\frac{1}{2}$	0	$\frac{1}{6}$

then

a) $m = \sigma^2 = 2$ b) $m = 1, \sigma^2 = 2$ c) $m = \sigma^2 = 1$ d) $m = 2, \sigma^2 = 1$

198. If two coins are tossed 5 times, then the probability of getting 5 heads and 5 tails is

a) $\frac{63}{256}$ b) $\frac{1}{1024}$ c) $\frac{2}{205}$ d) $\frac{9}{64}$

199. If X and Y are independent binomial variates $B\left(5, \frac{1}{2}\right)$ and $B\left(7, \frac{1}{2}\right)$, then $P(X + Y = 3)$ is

a) $\frac{35}{47}$ b) $\frac{55}{1024}$ c) $\frac{220}{512}$ d) $\frac{11}{204}$

200. A pack of cards contains 4 aces, 4 kings, 4 queens and 4 jacks. Two cards are drawn at random from this pack without replacement. The probability that at least one of them will be an ace, is

a) $\frac{1}{5}$ b) $\frac{9}{20}$ c) $\frac{1}{6}$ d) $\frac{1}{9}$

201. A random variable has the following probability distribution.

x : 0 1 2 3 4 5 6 7

$p(x)$: 0 $2p$ $2p$ $3p$ p^2 $2p^2$ $7p^2$ $2p$

The value of p, is

a) 1/10 b) −1 c) −1/10 d) None of these

202. Let E and F be two independent events. The probability that both E and F happen is 1/12 and the probability that neither E nor F occurs is 1/2. Then,

a) $P(E) = \frac{1}{3}, P(F) = \frac{1}{4}$ b) $P(E) = \frac{1}{2}, P(F) = \frac{1}{6}$ c) $P(E) = \frac{1}{6}, P(F) = \frac{1}{2}$ d) $P(E) = \frac{1}{4}, P(F) = \frac{2}{3}$

203. Six ordinary dice are rolled. The probability that at least half of them will show at least 3 is

a) $41 \times \frac{2^4}{3^6}$ b) $\frac{2^4}{3^6}$ c) $20 \times \frac{2^4}{3^6}$ d) None of these

204. The probability that at least one of A and B occurs is 0.6. If A and B occur simultaneously with probability 0.3, then $P(A') + P(B')$ is

a) 0.9 b) 0.15 c) 1.1 d) 1.2

205. A pair of dice is rolled together till a sum of either 5 or 7 is obtained. The probability that 5 comes before 7 is

a) 2/5 b) 1/5 c) 3/5 d) None of these

206. If events are independent and $P(A) = \frac{1}{3}, P(B) = \frac{1}{3}, P(C) = \frac{1}{4}$, then $P(A' \cap B' \cap C')$ is equal to

a) $\frac{1}{4}$ b) $\frac{1}{12}$ c) $\frac{1}{3}$ d) $\frac{5}{12}$

207. Three dice are thrown. The probability that the sum of the number appearing is 15, is

a) 1/216 b) 1/72 c) 5/108 d) 1/18

208. In a poisson distribution mean is 16, then standard deviation is

a) 16 b) 256 c) 128 d) 4

209. Six faces of an unbiased die are numbered with 2, 3, 5, 7, 11 and 13. If two such dice are thrown, then the probability that the sum on the uppermost faces of the dice is an odd number, is

a) $\frac{5}{18}$ b) $\frac{5}{36}$ c) $\frac{13}{18}$ d) $\frac{25}{36}$

210. The mean and variance of a binomial distribution are 4 and 3 respectively, then the probability of getting exactly six successes in this distribution is

a) ${}^{16}C_6\left(\frac{1}{4}\right)^{10}\left(\frac{3}{4}\right)^{6}$ b) ${}^{16}C_6\left(\frac{1}{4}\right)^{6}\left(\frac{3}{4}\right)^{10}$ c) ${}^{12}C_6\left(\frac{1}{4}\right)^{10}\left(\frac{3}{4}\right)^{6}$ d) ${}^{12}C_6\left(\frac{1}{4}\right)^{6}\left(\frac{3}{4}\right)^{6}$

211. A and B are two independent events. The probability that both A and B occur is 1/6 and the probability that neither of them occurs is 1/3. Then,

a) $P(A) = 1/2, P(B) = 1/3$

b) $P(A) = 1/2, P(B) = 1/6$

c) $P(A) = 1/3, P(B) = 1/6$

d) None of these

212. If A and B are independent events of a random experiments such that

$P(A \cap B) = \frac{1}{6}$ and $P(\bar{A} \cap \bar{B}) = \frac{1}{3}$, then $P(A)$ is equal to

a) $\frac{1}{4}$ b) $\frac{1}{3}$ c) $\frac{5}{7}$ d) $\frac{2}{3}$

213. If the integers m and n are chosen at random between 1 and 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5, equals

a) $\frac{1}{4}$ b) $\frac{1}{7}$ c) $\frac{1}{8}$ d) $\frac{1}{49}$

214. Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1, r_2 and r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ is

a) $\frac{1}{18}$ b) $\frac{1}{9}$ c) $\frac{2}{9}$ d) $\frac{1}{36}$

215. A mapping is selected at random from the set of all the mappings of the set $A = \{1,2, \dots, n\}$ into itself. The probability that the mapping selected is an injection is

a) $\frac{1}{n^n}$ b) $\frac{1}{n\,!}$ c) $\frac{(n-1)\,!}{n^{n-1}}$ d) $\frac{n\,!}{n^{n-1}}$

216. An urn contains five balls. Two balls are drawn and are found to be white. The probability that the balls selected are white is

a) 3/4 b) 3/5 c) 3/10 d) 1/2

217. A single letter is selected at random from the word 'PROBABILITY'. The probability that it is a vowel is

a) 3/11 b) 4/11 c) 2/11 d) None of these

218. A die is thrown. If it shows a six, we draw a ball from a bag consisting 2 black balls and 6 white balls. If it

does not show a six, then we toss a coin. Then, the sample space of this experiment consists of

a) 13 points b) 18 points c) 10 points d) None of these

219. For a binomial variate X with $n = 6$, if $P(X = 2) = 9P(X = 4)$, then its variance is

a) $\frac{8}{9}$ b) $\frac{1}{4}$ c) $\frac{9}{8}$ d) 4

220. Out of 13 applicants for a job, there are 5 women and 8 men. It is desired to select 2 persons for the job. The probability that at least one of the selected persons will be a women is

a) 25/39 b) 14/39 c) 5/13 d) 10/13

221. A and B are two independent events such that $P(A \cup B') = 0.8$ and $P(A) = 0.3$. Then, $P(B)$ is

a) $\frac{2}{7}$ b) $\frac{2}{3}$ c) $\frac{3}{8}$ d) $\frac{1}{8}$

222. Suppose that a die (with faces marked 1 to 6) is loaded in such a manner that for $K = 1,2,3, \dots, 6$ the probability of the face marked K turning up when die is tossed is proportional to K. The probability of the event that the outcome of a toss of the die will be an even number, is equal to

a) $\frac{1}{2}$ b) $\frac{4}{7}$ c) $\frac{2}{5}$ d) $\frac{1}{21}$

223. Three are six verities of a regular hexagon are chosen at random, then the possibility that the triangle with three vertices is equilateral, is equal to

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{10}$ d) $\frac{1}{20}$

224. If a committee of 3 is to be chosen from a group of 38 people of which you are a member. What is the probability that you will be on the committee?

a) $\binom{38}{3}$ b) $\binom{37}{2}$ c) $\binom{37}{2} / \binom{38}{3}$ d) 666/8436

225. The probability that in a year of the 22nd century chosen at random there will be 53 Sundays, is

a) $\frac{3}{28}$ b) $\frac{2}{28}$ c) $\frac{7}{28}$ d) $\frac{5}{28}$

226. Two cards are drawn without replacement from a well-shuffled pack. The probability that one of them is an ace of heart, is

a) $\frac{1}{25}$ b) $\frac{1}{26}$ c) $\frac{1}{52}$ d) None of these

227. A binary operation is chosen at random from the set of all binary operations on a set A containing n elements. The probability that the binary operation is commutative, is

a) $\frac{n^n}{n^{n^2}}$ b) $\frac{n^{n/2}}{n^{n^2}}$ c) $\frac{n^{n/2}}{n^{n^2/2}}$ d) None of these

228. A lot consists of 102 good pencils, 6 with minor defects and 2 with major defects. A pencil is choosen at random. The probability that this pencil is not defective is

a) 3/5 b) 3/10 c) 4/5 d) 1/2

229. If A and B are events of the same experiments with $P(A) = 0.2, P(B) = 0.5$,then maximum value of $P(A' \cap B)$ is

a) 0.2 b) 0.5 c) 0.63 d) 0.25

230. Four tickets marked 00,01,10,11, respectively are placed in a bag. A ticket is drawn at random five times, being replaced each time. The probability that the sum of the numbers on tickets thus drawn is 23, is

a) 25/256 b) 100/256 c) 231/256 d) None of these

231. Two dice are tossed 6 times. Then the probability that 7 will show an exactly four of the tosses is

a) $\frac{225}{18442}$ b) $\frac{116}{20003}$ c) $\frac{125}{15552}$ d) None of these

232. Out of $3n$ consecutive natural numbers, 3 natural numbers are chosen at random without replacement. The probability that the sum of the chosen numbers is divisible by 3, is

a) $\frac{n(3n^2 - 3n + 2)}{2}$ b) $\frac{(3n^2 - 3n + 2)}{2(3n - 1)(3n - 2)}$ c) $\frac{(3n^2 - 3n + 2)}{(3n - 1)(3n - 2)}$ d) $\frac{n(3n - 1)(3n - 2)}{3(n - 1)}$

233. A and B are two independent witnesses (ie, there is no collusion between them) in a case. The probability

that A will speak the truth is x and the probability that B will speak the truth is y, A and B agree in a certain statement. The probability that the statement is true, is

a) $\frac{x-y}{x+y}$ b) $\frac{xy}{1+x+y+xy}$ c) $\frac{x-y}{1-x-y+2xy}$ d) $\frac{xy}{1-x-y+2xy}$

234. Five persons A, B, C, D and E are in queue of a shop. The probability that A and E always together, is

a) $\frac{1}{4}$ b) $\frac{2}{3}$ c) $\frac{2}{5}$ d) $\frac{3}{5}$

235. Three dice are thrown. The probability that the same number will appear on each of them, is

a) 1/6 b) 1/18 c) 1/36 d) None of these

236. A bag contains 8 red and 7 black balls. Two balls are drawn at random. The probability that both the balls are of the same colour, is

a) $\frac{14}{15}$ b) $\frac{11}{15}$ c) $\frac{7}{15}$ d) $\frac{4}{15}$

237. A bag contains 10 white and 3 black balls. Balls are drawn one-by-one without replacement till all the black balls are drawn. The probability that the procedure of drawing balls will come to an end at the seventh draw is

a) $\frac{105}{286}$ b) $\frac{15}{286}$ c) $\frac{181}{286}$ d) None of these

238. Two events A and B have probability 0.25 and 0.50 respectively. The probability that both A and B occur simultaneously is 0.14. Then, the probability that neither A nor B occur, is

a) 0.39 b) 0.25 c) 0.11 d) None of these

239. There are 9999 tickets bearing numbers 0001, 0002,….,9999. If one ticket is selected from these tickets at random, the probability that the number on the ticket will consists of all different digits, is

a) $\frac{5040}{9999}$ b) $\frac{5000}{9999}$ c) $\frac{5030}{9999}$ d) None of these

240. The probability of choosing randomly a number c from the set {1, 2, 3,…..,9} such that the quadratic equation $x^2 + 4x + c = 0$ has real roots is

a) $\frac{1}{9}$ b) $\frac{2}{9}$ c) $\frac{3}{9}$ d) $\frac{4}{9}$

241. If A and B are two independent events, then the probability that only one of A and B occur is

a) $P(A) + P(B) - 2\,P\,(A \cap B)$

b) $P(A) + P(B) - P(A \cap B)$

c) $P(A) + P(B)$

d) None of these

242. Let $0 < P(A) < 1, 0 < P(B) < 1$ and $P(A \cap B) = P(A) + P(B) - P(A)P(B)$, then

a) $P(B|A) = P(B) - P(A)$ b) $P(A^c \cup B^c) = P(A^c) + P(B^c)$

c) $P(A \cup B)^c = P(A^c)P(B^c)$ d) $P(A|B) = P(A) + P(B^c)$

243. The probability distribution of a random variable X is given as

X	-5	-4	-3	-2	-1	0	1	2	3	4	5
P (X)	p	$2p$	$3p$	$4p$	$5p$	$7p$	$8p$	$9p$	$10p$	$11p$	$12p$

Then, the value of P is

a) $\frac{1}{72}$

b) $\frac{3}{73}$

c) $\frac{5}{72}$

d) $\frac{1}{74}$

244. In a college 25% boys and 10% girls offer Mathematics. There are 60% girls in the college. If a Mathematics student is chosen at random, then the probability that the student is a girl, will be

a) $\frac{1}{6}$ b) $\frac{3}{8}$ c) $\frac{5}{8}$ d) $\frac{5}{6}$

245. A biased coin with probability $p, 0 < p < 1$ of heads is tossed until a head appears for the first time. If the probability that the number of tossed required is even is $\frac{2}{5}$, then p equals

a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) $\frac{2}{5}$ d) $\frac{3}{5}$

246. For any two independent events E_1 and $E_2, P\{(E_1 \cup E_2) \cap (\bar{E}_1) \cap (\bar{E}_2)\}$ is

a) $\leq 1/4$ b) $> 1/4$ c) $\geq 1/2$ d) None of these

247. A and B are the independent events. The probability that both occur simultaneously is $\frac{1}{6}$ and the probability that neither occur is $\frac{1}{3}$. The probability of occurrence of the events A and B is

a) $\frac{1}{2}, \frac{3}{2}$ b) $\frac{1}{2}, \frac{1}{3}$ c) Not possible d) None of these

248. If in a distribution each x is replaced by corresponding value of $f(x)$, then the probability of getting $f(x_i)$ when the probability of getting x_i is p_i, is

a) p_i b) $f(p_i)$ c) $f\left(\frac{1}{p_i}\right)$ d) None of these

249. The distribution of a random variable X is given below

$\boldsymbol{X}$	-2	-1	0	1	2	3
$\boldsymbol{P(X)}$	$\frac{1}{10}$	k	$\frac{1}{5}$	2k	$\frac{3}{10}$	k

The value of k is

a) $\frac{1}{10}$ b) $\frac{2}{10}$ c) $\frac{3}{10}$ d) $\frac{7}{10}$

250. The probability that a man can hit a target is 3/4. He tries 5 times. The probability that he will hit the target at least three times is

a) 291/364 b) 371/464 c) 471/502 d) 459/512

251. Two cards are drawn from a well shuffled deck of 52 cards. The probability that one is red card and the other is a queen is

a) 4/51 b) 16/221 c) 50/663 d) None of these

252. If $4P(A) = 6P(B) = 10P(A \cap B) = 1$, then$P\left(\frac{B}{A}\right)$ is equal to

a) $\frac{2}{5}$ b) $\frac{3}{5}$ c) $\frac{7}{10}$ d) $\frac{19}{60}$

253. In a binomial distribution, the mean is 4 and variance is 3. Then, its mode is

a) 5 b) 6 c) 4 d) None of these

254. If two events A and B are such that $P(A^c) = 0.3, P(B) = 0.4$ and $P(A \cap B^c) = 0.5$, then $P\left[\frac{B}{(A \cup B^c)}\right]$ is equal to

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{4}$ d) None of these

255. A and B play a game where each is asked to select a number from 1 to 25. If the two numbers match, both of them win a prize. The probability that they will not win a prize in a single trial, is

a) $\frac{1}{25}$ b) $\frac{24}{25}$ c) $\frac{2}{25}$ d) None of these

256. A box contains 100 bulbs out of which 10 are defective. A sample of 5 bulbs is drawn. The probability that none is defective, is

a) $\left(\frac{1}{10}\right)^5$ b) $\left(\frac{1}{2}\right)^5$ c) $\left(\frac{9}{10}\right)^5$ d) $\frac{9}{10}$

257. A random variable X can attain only the value 1, 2, 3, 4, 5 with respective probabilities $k, 2k, 3k, 2k, k$. If m is the mean of the probability distribution, then$(k, m)$is equal to

a) $\left(3, \frac{1}{9}\right)$ b) $\left(\frac{1}{9}, 3\right)$ c) $\left(\frac{1}{8}, 4\right)$ d) $(1, 3)$

258. A complete cycle of a traffic light takes 60 s. During each cycle the light is green for 25 s, yellow for 5 s and red for 30 s. At a randomly chosen time, the probability that the light will not be green, is

a) $\frac{1}{3}$ b) $\frac{1}{4}$ c) $\frac{4}{17}$ d) $\frac{7}{12}$

259. From a group of 8 boys and 3 girls, a committee of 5 members to be formed. Find the probability that 2 particular girls are included in the committee

a) $\frac{4}{11}$ b) $\frac{2}{11}$ c) $\frac{6}{11}$ d) $\frac{8}{11}$

260. There are n letters and n addressed envelopes, the probability that all the letters are not kept in the right envelope, is

a) $\frac{1}{n!}$ b) $1 - \frac{1}{n!}$ c) $1 - \frac{1}{n}$ d) $n!$

261. The probability that the same number appear on throwing three dice simultaneously, is

a) 1/6 b) 1/36 c) 5/36 d) None of these

262. If $P(A) = P(B) = x$ and $P(A \cap B) = P(A' \cap B') = \frac{1}{3}$,then x is equal to

a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{1}{3}$ d) $\frac{1}{6}$

263. One ticket is selected at random from 50 tickets numbered 00,01,02,….,49. Then, the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero equals

a) $\frac{1}{14}$ b) $\frac{1}{7}$ c) $\frac{5}{14}$ d) $\frac{1}{50}$

264. If n integers taken at random are multiplied together, then the probability that the last digit of the product is 1,3,7 or 9, is

a) $\frac{2^n}{5^n}$ b) $\frac{4^n - 2^n}{5^n}$ c) $\frac{4^n}{5^n}$ d) None of these

265. Among the workers in a factory only 30% receive bonus and among those receiving bonus only 20% are skilled. The probability that a randomly selected worker is skilled and is receiving bonus is

a) 0.03 b) 0.02 c) 0.06 d) 0.015

266. A box contains 10 good articles and 6 with defects, one article is chosen at random. What is the probability that it is either good or has a defect?

a) $\frac{24}{64}$ b) $\frac{40}{64}$ c) $\frac{49}{64}$ d) 1

267. A coin and six faced die. Both unbiased, are thrown simultaneously. The probability of getting a head on the coin and an odd number on the die, is

a) $\frac{1}{2}$ b) $\frac{3}{4}$ c) $\frac{1}{4}$ d) $\frac{2}{3}$

268. An anti-aircraft gun can take a maximum of four shots at any plane moving away from it. The probabilities of hitting the plane at the 1st, 2nd ,3rd and 4th shots are 0.4, 0.3, 0.2 and 0.1 respectively. What is the probability that at least one shot hits the plane?

a) 0.6976 b) 0.3024 c) 0.72 d) 0.6431

269. A coin is tossed three times. The probability of getting head and tail alternatively, is

a) $\frac{1}{8}$ b) $\frac{1}{2}$ c) $\frac{1}{4}$ d) None of these

270. A bag contains 4 tickets numbered 1,2,3,4 and another bag contains 6 tickets numbered 2,4,6,7,8,9. One bag is chosen and a ticket is drawn. The probability that the ticket bears the number 4 is

a) 1/48 b) 1/8 c) 5/24 d) None of these

271. Six coins are tossed simultaneously. The probability of getting at least 4 heads is

a) 11/64 b) 11/32 c) 15/44 d) 21/32

272. Two cards are drawn successively with replacement from a well shuffled deck of 52 cards, then the mean of the number of aces is

a) $\frac{1}{13}$ b) $\frac{3}{13}$ c) $\frac{2}{13}$ d) None of these

273. Given two mutually exclusive events A and B such that $P(A) = 0.45$ and $P(B) = 0.35, P(A \cap B)$ is equal to

a) $\frac{63}{400}$ b) 0.8 c) $\frac{63}{200}$ d) 0

274. There is an objective type question with 4 answer choices exactly one of which is correct. A student has not studied the topic on which the question has been set. The probability that the student guesses the correct answer, is

a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{1}{8}$ d) None of these

275. If E and F are two independent events such that $0 < P(E) < 1$ and $0 < P(F) < 1$, then

a) E and F^c are independent

b) E^candF^c are independent

c) $P\left(\frac{E}{F}\right) + P\left(\frac{E^c}{F^c}\right) = 1$

d) None of these

276. An integer is chosen at random from first two hundred numbers. Then, the probability that the integer chosen is divisible by 6 or 8 is

a) $\frac{1}{4}$ b) $\frac{2}{4}$ c) $\frac{3}{4}$ d) None of these

277. The mean and variance of a random variable X having a binomial distribution are 4 and 2 respectively, then $P(X = 1)$ is

a) $\frac{1}{32}$ b) $\frac{1}{16}$ c) $\frac{1}{8}$ d) $\frac{1}{4}$

278. One hundred identical coins, each with probability p of showing heads are tossed once. If $0 < p < 1$ and the probability of head showing on 50 coins is equal to that of head showing on 51 coins, the value of p is

a) $\frac{1}{2}$ b) $\frac{51}{101}$ c) $\frac{49}{101}$ d) None of these

279. The probability of choosing a number divisible by 6 or 8 from among 1 to 90 is

a) $\frac{1}{6}$ b) $\frac{1}{90}$ c) $\frac{1}{30}$ d) $\frac{23}{90}$

280. An urn contains 6 white and 4 black balls. A fair die is rolled and that number of balls are chosen from the urn. The probability that the balls selected are white is

a) 1/5 b) 1/6 c) 1/7 d) 1/8

281. In a certain population 10% of the people are rich, 5% are famous and 3% are rich and famous. The probability that a person picked at random from the population is either famous or rich but not both, is equal to

a) 0.07 b) 0.08 c) 0.09 d) 0.12

282. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane, is

a) 0.06 b) 0.14 c) 0.32 d) 0.7

283. Box A contains 2 black and 3 red balls. While box B contains 3 black and 4 red balls. Out of these two boxes one is selected at random; and the probability of choosing box A is double that of box B. If a red ball is drawn from the selected box, then the probability that it has come from box$B$. is

a) $\frac{21}{41}$ b) $\frac{10}{31}$ c) $\frac{12}{31}$ d) $\frac{13}{41}$

284. A, B, C are any three events. If $P(S)$ denotes the probability of S happening, then $P\big(A \cap (B \cup C)\big) =$

a) $P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C)$

b) $P(A) + P(B) + P(C) - P(B)P(C)$

c) $P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$

d) $P(A) + P(B) + P(C)$

285. The value of C for which $P(X = k) = C\,k^2$ can serve as the probability function of a random variable X that takes value 0, 1, 2, 3, 4 is

a) $\frac{1}{30}$ b) $\frac{1}{10}$ c) $\frac{1}{3}$ d) $\frac{1}{15}$

286. In tossing of a coin $(m+n)(m>n)$ times, the probability of coming consecutive heads at least m times is

a) $\frac{n+2}{2^{m+1}}$ b) $\frac{m-n}{2^{m+n}}$ c) $\frac{m+n}{2^{m+n}}$ d) $\frac{mn}{2^{m+n}}$

287. In $x=33^n$, n is a positive integral value, then what is the probability that x will have 3 at its units place?

a) 1/3 b) 1/4 c) 1/5 d) 1/2

288. Two numbers are selected randomly from the set $S=\{1,2,3,4,5,6\}$ without replacement one by one. The probability that minimum of the two numbers is less than, 4 is

a) 1/15 b) 14/15 c) 1/5 d) 4/5

289. A and B are two independent event such that $P(A)=\frac{1}{5}, P(A\cup B)=\frac{7}{10}$. Then, $P(\bar{B})=$

a) 3/8 b) 2/7 c) 7/9 d) None of these

290. In Q. 12 the probability that the mapping is a bijection, is

a) $\frac{1}{n^n}$ b) $\frac{1}{n\,!}$ c) $\frac{(n-1)\,!}{n^{n-1}}$ d) $\frac{n\,!}{n^{n-1}}$

291. An unbiased coin is tossed to get 2 points for turning up a head and one point for the tail. If three unbiased coins are tossed simultaneously, then the probability of getting a total of odd number of points

a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{1}{8}$ d) $\frac{3}{8}$

292. In a precision bombing attack there is a 50% chance that any one bomb will strike the target. Two direct hits are required to destroy the target completely. The minimum number of bombs which should be dropped to give a 99% chance or better of completely destroying the target is

a) 10 b) 11 c) 12 d) None of these

293. A coin is tossed n times the probability of getting head at least once is greater than 0.8. Then the least value of such n is

a) 2 b) 3 c) 4 d) 5

294. If $A_1, A_2, \dots, A_n$ are n independent events such that $P(A_i)=\frac{1}{i+1}, i=1,2,\dots,n$. The probability that none of the n events occurs, is

a) $\frac{n}{n+1}$ b) $\frac{1}{n+1}$ c) $\frac{n}{(n+1)(n+2)}$ d) None of these

295. A random variate X takes the values 0, 1, 2, 3 and its mean is 1.3. If $P(X=3)=2P(X=1)$ and $P(X=2)=0.3$,then $P(X=0)$ is equal to

a) 0.1 b) 0.2 c) 0.3 d) 0.4

296. Three dice are thrown simultaneously, then probability of throwing a total greater than 4 is

a) $\frac{1}{54}$ b) $\frac{53}{54}$ c) $\frac{5}{108}$ d) None of these

297. A bag contains 6 white and 4 black balls. Two balls are drawn at random. The probability that they are of the same colours, is

a) $\frac{1}{15}$ b) $\frac{2}{5}$ c) $\frac{4}{15}$ d) $\frac{7}{15}$

298. A box contains 3 white and 2 red balls. If we draw one ball and without replacing the first ball, the probability of drawing red ball in the second draw is

a) $\frac{8}{25}$ b) $\frac{2}{5}$ c) $\frac{3}{5}$ d) $\frac{21}{25}$

299. An unbiased coin is tossed fixed number of times. If the probability of getting 4 heads equals the probability of getting 7 heads, then the probability of getting 2 heads is

a) 55/2048 b) 1/1024 c) 3/4096 d) None of these

300. If $P(A\cap B)=\frac{1}{2}, P(\bar{A}\cap\bar{B})=\frac{1}{3}, P(A)=p, P(B)=2\,p$, then the value of p is given by

a) 1/3 b) 7/18 c) 4/9 d) 1/2

301. If the probability density function of a random variable X is $f(x)=\frac{x}{2}$ in $0\le x\le 2$, then $P\left(\frac{X>15}{X>1}\right)$ is equal

to

a) $\frac{7}{16}$ b) $\frac{3}{4}$ c) $\frac{7}{12}$ d) $\frac{21}{64}$

302. The probability that A can solve a problem is $2/3$ and B can solve it is $3/4$. If both attempt the problem, what is the probability that the problem gets solved?

a) 11/12 b) 7/12 c) 5/12 d) 9/12

303. Given $P(A \cup B) = 0.6, P(A \cap B) = 0.2$,the probability of exactly one of the event occurs is

a) 0.4 b) 0.2 c) 0.6 d) 0.8

304. Fifteen coupons are numbered 1 to 15. Seven coupons are selected at random, one at a time with replacement. The probability that the largest number appearing on a selected coupon be 9, is

a) $\left(\frac{1}{15}\right)^7$ b) $\left(\frac{8}{18}\right)^7$ c) $\left(\frac{3}{5}\right)^7$ d) None of these

305. A dice is rolled three times. The probability of getting a larger number than the previous number each time is

a) $\frac{15}{216}$ b) $\frac{5}{54}$ c) $\frac{13}{216}$ d) $\frac{1}{18}$

306. If X has binomial distribution with mean np and variance npq, then $\frac{P(X=k)}{P(X=k-1)}$ is equal to

a) $\frac{n-k}{k-1}.\frac{p}{q}$ b) $\frac{n-k+1}{k}.\frac{p}{q}$ c) $\frac{n+1}{k}.\frac{q}{p}$ d) $\frac{n-1}{k+1}.\frac{q}{p}$

307. The probability distribution of a random variable X is given by

$X = x$: 0 1 2 3 4

$P(X = x)$: 0.4 0.3 0.1 0.1 0.1

The variance of X is

a) 1.76 b) 2.45 c) 3.2 d) 4.8

308. A bag contains four tickets marked with numbers 112,121,211,222. One ticket is drawn at random from the bag. Let $E_i (i = 1,2,3)$ denote the event that ith digit on the ticket is 2. Then, which one of the following is incorrect?

a) E_1 and E_2 are independent

b) E_2 and E_3 are independent

c) E_3 and E_1 are independent

d) E_1, E_2, E_3 are independent

309. If A and B are two events, such that $P(A \cup B) = \frac{3}{4}, P(A \cap B) = \frac{1}{4}, P(\bar{A}) = \frac{2}{3}$, then $P(\bar{A} \cap B)$ is equal to

a) $\frac{5}{12}$ b) $\frac{3}{8}$ c) $\frac{5}{8}$ d) $\frac{1}{2}$

310. Let S be a set containing n elements. Two subsets A and B os S are chosen at random. The probability that $A \cup B = S$ is

a) $\frac{{}^{2n}C_n}{2^{2n}}$ b) $\left(\frac{3}{4}\right)^n$ c) $\frac{1}{{}^{2n}C_n}$ d) None of these

311. A rod of length 10 cm is broken into three parts, so that each part is having a length as an integral multiple of 1 cm,. The probability that the parts are forming a triangle, is

a) 1/4 b) 1/2 c) 3/4 d) 1/3

312. The probability that a company executive will travel by train is $\frac{2}{3}$ and that he will travel by plane is $\frac{1}{5}$. The probability of his journey by train or plane is

a) $\frac{2}{15}$ b) $\frac{13}{15}$ c) $\frac{15}{13}$ d) $\frac{15}{2}$

313. A three digit number, which is a multiple of 11, is chosen at random. Probability that the number so chosen is also a multiple of 9, is equal to

a) $\frac{1}{9}$ b) $\frac{2}{9}$ c) $\frac{1}{100}$ d) $\frac{9}{100}$

314. Four positive integers are taken at random and are multiplied together. Then the probability that the

product ends in an odd digit other than 5, is

a) 609/625 b) 16/625 c) 2/5 d) 3/5

315. A pair of fair dice is thrown independently 4 times. The probability of getting a sum of exactly 7 twice is

a) $\frac{5}{81}$ b) $\frac{25}{243}$ c) $\frac{25}{216}$ d) $\frac{125}{648}$

316. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse, is

a) $\frac{4}{5}$ b) $\frac{3}{5}$ c) $\frac{1}{5}$ d) $\frac{2}{5}$

317. A number n is chosen at random from $S = \{1, 2, 3, \dots, 50\}$.

Let $A = \left\{n \in S: n + \frac{50}{n} > 27\right\}, B = \{n \in S: n \text{ is a prime}\}$ and

$C = \{n \in S: n \text{ is a square}\}$. Then, correct order of their probabilities is

a) $P(A) < P(B) < P(C)$ b) $P(A) > P(B) > P(C)$ c) $P(B) < P(A) < P(C)$ d) $P(A) > P(C) > P(B)$

318. A bag contains 5 white and 3 black balls and 4 balls are successively drawn out and not replaced. The probability that they are alternately of different colours, is

a) $\frac{1}{196}$ b) $\frac{2}{7}$ c) $\frac{1}{7}$ d) $\frac{13}{56}$

319. Three numbers are chosen at random from 1 to 20. The probability that they are consecutive, is

a) $\frac{1}{190}$ b) $\frac{1}{120}$ c) $\frac{3}{190}$ d) $\frac{5}{190}$

320. Out of 40 consecutive natural numbers, two are chosen at random. Probability that the sum of the numbers is odd, is

a) $\frac{14}{29}$ b) $\frac{20}{39}$ c) $\frac{1}{2}$ d) None of these

321. A person puts three cards addresses to three different people in three envelopes with three different addresses without looking. What is the probability that the cards go into their respective envelopes?

a) $\frac{2}{3}$ b) $\frac{1}{6}$ c) $\frac{1}{5}$ d) $\frac{2}{5}$

322. The probability that A speaks truth is $\frac{4}{5}$ while this probability for B is $\frac{3}{4}$. The probability that they contradict each other when asked to speak on a fact, is

a) $\frac{3}{20}$ b) $\frac{1}{5}$ c) $\frac{7}{20}$ d) $\frac{4}{5}$

323. A box contains 100 tickets numbered 1,2, …,100. Two tickets are choosen at random. It is given that the maximum number on the two choosen tickets is not more than 10. The probability that the minimum number is 5 is

a) 13/15 b) 1/330 c) 1/3 d) 1/9

324. Three identical dice are rolled. The probability that the same number will appear on each of them, is

a) 1/6 b) 1/36 c) 1/18 d) 3/28

325. For $k = 1,2,3$ the box B_k contains k red balls and $(k + 1)$ white balls, Let $P(B_1) = \frac{1}{2}, P(B_2) = \frac{1}{3}$ and $P(B_3) = \frac{1}{6}$

A box is selected at random and a ball is drawn from it. If a red ball is drawn, then the probability that it has come from box B_2,is

a) $\frac{35}{78}$ b) $\frac{14}{39}$ c) $\frac{10}{13}$ d) $\frac{12}{13}$

326. Two persons each makes a single throw with a pair of dice. The probability that the throws are unequal is given by

a) $1/6^3$ b) $73/6^3$ c) $51/6^3$ d) None of these

327. Two cards are drawn at random from a pack of 52 cards. The probability of getting at least a spade and an ace is

a) 1/34 b) 8/221 c) 1/26 d) 2/51

328. A bag contain 5 black balls, 4 white balls and 3 red balls. If a ball is selected randomly, the probability that it is a black or red ball, is

a) 1/3 b) 1/4 c) 5/12 d) 2/3

329. Of a total of 600 bolts, 20% are too large and 10% are too small. The remainder are considered to be suitable. If a bolt is selected at random, the probability that it will be suitable is

a) $\frac{1}{5}$ b) $\frac{7}{10}$ c) $\frac{1}{10}$ d) $\frac{3}{10}$

330. Probability that a student will succeed in I.I.T. entrance test is 0.2 and that he will succeed in Roorkee entrance test is 0.5. If the probability that he will successful at both the places is 0.3, then the probability that he does not succeed at both the places is

a) 0.4 b) 0.3 c) 0.2 d) 0.6

331. If A and B each toss three coins. The probability that both get the same number of heads is

a) $\frac{1}{9}$ b) $\frac{3}{16}$ c) $\frac{5}{16}$ d) $\frac{3}{8}$

332. An unbiased die is tossed until a number greater than 4 appears. The probability that an even number of tosses needed, is

a) $\frac{1}{2}$ b) $\frac{2}{5}$ c) $\frac{1}{5}$ d) $\frac{2}{3}$

333. One Indian and four American men and their wifes are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife, is

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{2}{5}$ d) $\frac{1}{5}$

334. An almirah stores 5 black and 4 white socks well mixed. A boy pull out 2 socks at random. The probability that 2 are of the same colour is

a) 4/9 b) 5/8 c) 5/9 d) 7/12

335. A pack of plying cards was found to contain only 51 cards. If the first 13 cards which are examined are all red, then the probability that the missing cards is black, is

a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) $\frac{1}{2}$ d) $\frac{^{25}C_{13}}{^{51}C_{13}}$

336. In order to get at least once a head with probability ≥ 0.9, the number of times a coin needs to be tossed is

a) 3 b) 4 c) 5 d) None of these

337. Three letters are written to there different persons and addresses on the three envelopes are also written. Without looking at the addresses, the letters are kept in these envelopes. The probability that all the letters are not placed into their right envelopes is

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{6}$ d) $\frac{5}{6}$

338. Suppose $f(x) = x^3 + ax^2 + bx + c$, where a, b, c are chosen respectively by throwing a dice three times. Then, the probability that $f(x)$ is an increasing function, is

a) $\frac{4}{9}$ b) $\frac{3}{8}$ c) $\frac{2}{5}$ d) $\frac{16}{34}$

339. A sample of a 4 times is drawn at a random without replacement from a lot of 10 items containing 3 defectives. If x denotes the number of defective items in the sample, then $P(0 < x < 3)$is equal to

a) $\frac{3}{10}$ b) $\frac{4}{5}$ c) $\frac{1}{2}$ d) $\frac{1}{6}$

340. The mean and variance of a random variable X having a binomial distribution are 4 and 2 respectively. Then, $P(X > 6)$ is equal to

a) $\frac{1}{256}$ b) $\frac{3}{256}$ c) $\frac{9}{256}$ d) $\frac{7}{256}$

341. An integer is chosen at random from first two hundred digits. Then, the probability that the integer chosen is divisible by 6 or 8, is

a) $\frac{1}{4}$ b) $\frac{2}{4}$ c) $\frac{3}{4}$ d) None of these

342. For a random variable $X, E(X) = 3$ and $E(X^2) = 11$. Then, variable of X is

a) 8 b) 5 c) 2 d) 1

343. A coin is tossed n times. The probability of getting head at least once is greater than 0.8, then the least value of n is

a) 2 b) 3 c) 5 d) 4

344. If any four numbers are selected and they are multiplied, then the probability that the last digit will 1,3,5 or 7, is

a) 4/625 b) 18/625 c) 16/625 d) None of these

345. If A and B are two independent events such that $P(A \cap B') = \frac{3}{25}$ and $P(A' \cap B) = \frac{8}{25}$, then $P(A)$ is equal to

a) $\frac{1}{5}$ b) $\frac{3}{8}$ c) $\frac{2}{5}$ d) $\frac{4}{5}$

346. If, $x \in [0,5]$, then what is the probability that $x^2 - 3x + 2 \geq 0$?

a) 4/5 b) 1/5 c) 2/5 d) None of these

347. Three coins are tossed together, then the probability of getting at least one head is

a) $\frac{1}{2}$ b) $\frac{3}{4}$ c) $\frac{1}{8}$ d) $\frac{7}{8}$

348. A random variable X follows binomial distribution with mean α and variance β.Then

a) $0 < \alpha < \beta$ b) $0 < \beta < \alpha$ c) $\alpha < 0 < \beta$ d) $\beta < 0 < \alpha$

349. Probability of getting positive integral roots of the equation $x^2 - n = 0$ for the integer $n, 1 \leq n \leq 40$ is

a) $\frac{1}{5}$ b) $\frac{1}{10}$ c) $\frac{3}{20}$ d) $\frac{1}{20}$

350. The probability that the three cards drawn from a pack of 52 cards, are all black, is

a) $\frac{1}{17}$ b) $\frac{2}{17}$ c) $\frac{3}{17}$ d) $\frac{2}{19}$

351. Three six faced dice are tossed together, then the probability that exactly two of the three numbers are equal is

a) 165/216 b) 177/216 c) 51/216 d) 90/216

352. If A and B are two independent events such that $P(B) = \frac{2}{7}, P(A \cup B^c) = 0.8$, then $P(A)$is equal to

a) 0.1 b) 0.2 c) 0.3 d) 0.4

353. An experiment yields 3 mutually exclusive and exhaustive events A, B, C.If $P(A) = 2P(B) = 3P(C)$, then $P(A)$ is equal to

a) $\frac{1}{11}$ b) $\frac{2}{11}$ c) $\frac{3}{11}$ d) $\frac{6}{11}$

354. A random variable X takes values 0, 1, 2, 3,... with probability

$P(X = x) = k(x+1)\left(\frac{1}{5}\right)^x$, where k is constant, then $P(X = 0)$ is

a) $\frac{7}{25}$ b) $\frac{18}{25}$ c) $\frac{13}{25}$ d) $\frac{16}{25}$

355. If A and B are two events such that $P(A \cup B) = \frac{5}{6}$,

$P(A \cap B) = \frac{1}{3}$ and $P(\bar{B}) = \frac{1}{3}$,then the value of $P(A)$is

a) $\frac{1}{3}$ b) $\frac{1}{4}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$

356. In a bag there are three tickets numbered 1,2,3. A ticket is drawn at random and put back, and this is done four times. The probability that the sum of the numbers is even, is

a) 41/81 b) 39/81 c) 40/81 d) None of these

357. A box contains 24 identical balls of which 12 are white and 12 are black. The balls are drawn at random from the box one at a time with replacement. The probability that a white ball is drawn for the 4th time on the 7th draw, is

a) $\frac{5}{64}$ b) $\frac{27}{32}$ c) $\frac{5}{32}$ d) $\frac{1}{2}$

358. Out of 15 persons 10 can speak Hindi and 8 can speak English. If two persons are chosen at random, then the probability that one person speaks Hindi only and the other speaks both Hindi and English is

a) $\frac{3}{5}$ b) $\frac{7}{12}$ c) $\frac{1}{5}$ d) $\frac{2}{5}$

359. A purse contains 4 copper and 3 silver coins. Another purse contains 6 copper and 2 silver coins. A coin is taken out from any purse, the probability that it is a silver coin, is

a) $\frac{37}{56}$ b) $\frac{19}{56}$ c) $\frac{4}{7}$ d) $\frac{2}{3}$

360. Two dice are thrown together. If the numbers appearing on the two dice are different, then what is the probability that the sum is 6?

a) $\frac{5}{36}$ b) $\frac{1}{6}$ c) $\frac{2}{15}$ d) None of these

361. If A and B are events of a random experiment such that $P(A \cup B) = \frac{4}{5}, P(\bar{A} \cup \bar{B}) = \frac{7}{10}$ and $P(B) = \frac{2}{5}$, then $P(A)$ equals

a) $\frac{9}{10}$ b) $\frac{8}{10}$ c) $\frac{7}{10}$ d) $\frac{3}{5}$

362. Two cards are drawn one by one from a pack of cards. The probability of getting first card an ace and second a coloured one is (before drawing second card first card is not placed again in the pack)

a) 1/26 b) 5/52 c) 5/221 d) 4/13

363. 4 five-rupee coins, 3 two-rupee coins and 2 one-rupee coins are stacked together in a column at random. The probability that the coins of the same denominator are consecutive is

a) 13/9! b) 1/210 c) 1/35 d) None of these

364. If two squares are chosen at random on a chess board, the probability that they have a side in common is

a) 1/9 b) 1/18 c) 2/7 d) None of these

365. In an entrance test there are multiple choice questions. There are four possible answers to each question,of which one is correct. The probability that a student knows the answer to a question is 90%. If be gets the correct answer to a question, then the probability that he was guessing, is

a) $\frac{37}{40}$ b) $\frac{1}{37}$ c) $\frac{36}{37}$ d) $\frac{1}{9}$

366. If A and B are any two events, then $P(A \cap B')$ is equal to

a) $P(A) + P(B')$ b) $P(A)P(B)$ c) $P(B) - P(A \cap B)$ d) $P(A) - P(A \cap B)$

367. Two dice are rolled one after the other. The probability that the number on the first is smaller than the number on the second is

a) $\frac{1}{2}$ b) $\frac{3}{4}$ c) $\frac{7}{18}$ d) $\frac{5}{12}$

368. A fair coin is tossed repeatedly. If the tail appears on first four tosses, then the probability of the head appearing on the fifth toss, equals

a) $\frac{1}{2}$ b) $\frac{1}{32}$ c) $\frac{31}{32}$ d) $\frac{1}{5}$

369. A bag contains $(2n + 1)$ coins. It is known that n of these coins have a head on both sides, whereas the remaining $n + 1$ coins are fair. A coin is picked up at random from the bag and tossed. If the probability that the toss results in a head is 31/42, then n is equal to

a) 10 b) 11 c) 12 d) 13

370. The mode of the binomial distribution for which mean and standard deviation are 10 and $\sqrt{5}$ respectively, is

a) 7 b) 8 c) 9 d) 10

371. The mean and standard deviation of a binomial variate X are 4 and $\sqrt{3}$ respectively. Then, $P(X \geq 1)$ is equal to

a) $1-\left(\frac{1}{4}\right)^{16}$ b) $1-\left(\frac{3}{4}\right)^{16}$ c) $1-\left(\frac{2}{3}\right)^{16}$ d) $1-\left(\frac{1}{3}\right)^{16}$

372. A box contains 15 transistors, 5 of which are defective. An inspector takes out one transistor at random, examines it for defects and replaces it. After it has replaced another inspector does the same thing and then so does a third inspector. The probability that atleast one of the inspectors finds a defective transistor, is equal to

a) 1/27 b) 8/27 c) 19/27 d) 26/27

373. The records of a hospital show that 10% of the cases of a certain disease are fatal. If 6 patients are suffering from the disease, then the probability that only three will die, is

a) 8748×10^{-5} b) 1458×10^{-5} c) 1458×10^{-6} d) 41×10^{-6}

374. The probability that at least one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2, then $P(\bar{A})+P(\bar{B})$ is

a) 0.4 b) 0.8 c) 1.2 d) 1.4

375. The probability that in the toss of two dice, we obtain the sum 7 or 11,is

a) $\frac{1}{6}$ b) $\frac{1}{18}$ c) $\frac{2}{9}$ d) $\frac{23}{108}$

376. If $a \in [-20,0]$, then the probability that the graph of the function $y = 16\,x^2 + 8(a+5)x - 7\,a - 5$ is strictly above the x-axis is

a) 1/2 b) 1/17 c) 17/20 d) None of these

377. There are 5 duplicate and 10 original items in an automobile shop and 3 items are brought at random by a customer. The probability that none of the items is duplicate, is

a) 20/91 b) 22/91 c) 24/91 d) 89/91

378. There are four machines and it is known that exactly two of them are faulty. They are tested, one by one in a random order till both the faulty machines are identified. Then, the probability that only two tests are needed, is

a) $\frac{1}{3}$ b) $\frac{1}{6}$ c) $\frac{1}{2}$ d) $\frac{1}{4}$

379. A bag contains 50 tickets numbered 1,2,3, … ,50 of which five are drawn at random and arranged in ascending order of magnitude $(x_1 < x_2 < x_3 < x_4 < x_5)$. The probability that $x_3 = 30$, is

a) $\frac{{}^{20}C_2}{{}^{50}C_5}$ b) $\frac{{}^{29}C_2}{{}^{50}C_5}$ c) $\frac{{}^{20}C_2 \times {}^{29}C_2}{{}^{50}C_5}$ d) None of these

380. The probability of having at least one tail in 4 throws with a coin, is

a) $\frac{15}{16}$ b) $\frac{1}{16}$ c) $\frac{1}{4}$ d) 1

381. An unbiased coin is tossed n times. Let X denote the number of times head occurs. If $P(X=4)$, $P(X=5)$ and $P(X=6)$ are in AP, then the value of n can be

a) 7, 14 b) 10, 14 c) 12, 7 d) None of these

382. Three letters, to each of which corresponds an envelope, are placed in the envelopes at random. The probability that all the letters are not placed in the right envelopes, is

a) 1/6 b) 5/6 c) 1/3 d) 2/3

383. Let A and B be two finite sets having m and n elements respectively such that $m \le n$. A mapping is selected at random from the set of all mappings from A to B. The probability that the mapping selected is an injection, is

a) $\frac{n\,!}{(n-m)\,!\,m^n}$ b) $\frac{n\,!}{(n-m)\,!\,n^m}$ c) $\frac{m\,!}{(n-m)!\,n^m}$ d) $\frac{m\,!}{(n-m)\,!\,m^n}$

384. In a series of three trials the probability of exactly two successes in nine times as large as the probability of three successes. Then, the probability of success in each trial is

a) 1/2 b) 1/3 c) 1/4 d) 3/4

385. If three natural numbers from 1 to 100 are selected randomly, then probability that all are divisible by both 2 and 3, is

a) $\frac{4}{105}$ b) $\frac{4}{33}$ c) $\frac{4}{35}$ d) $\frac{4}{1155}$

386. The probability that in a family of 5 members, exactly 2 members have birthday on sunday, is

a) $\frac{12 \times 5^3}{7^5}$ b) $\frac{10 \times 6^2}{7^5}$ c) $\frac{2}{5}$ d) $\frac{10 \times 6^3}{7^5}$

387. If the mean and standard deviation of a binomial distribution are 12 and 2 respectively, then value of its parameter p is

a) 1/2 b) 1/3 c) 2/3 d) 1/4

388. In shuffling a pack of playing cards, four are accidently dropped. The probability that missing cards should be one from each suit, is

a) $\frac{1}{256}$ b) $\frac{1}{270725}$ c) $\frac{2197}{20825}$ d) None of these

389. The probability of India winning a test match against West-Indies is $\frac{1}{2}$ assuming independence from match to match the probability that in a match series India's second win occurs at the third test, is

a) $\frac{1}{8}$ b) $\frac{1}{4}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$

390. If n positive integers are taken at random and multiplied together, the probability that the last digit of the product is 2, 4, 6 or 8, is

a) $\frac{4^n + 2^n}{5^n}$ b) $\frac{4^n \times 2^n}{5^n}$ c) $\frac{4^n - 2^n}{5^n}$ d) None of these

391. A bag contains 5 white and 3 black balls and 4 balls are successively drawn out and not replaced. The probability that they are alternately of different colours, is

a) $\frac{1}{196}$ b) $\frac{2}{7}$ c) $\frac{13}{56}$ d) $\frac{1}{7}$

392. An urn contains 4 white and 3 red balls. There balls are drawn with replacement from this urn. Then, the standard deviation of the number of red balls drawn is

a) $\frac{6}{7}$ b) $\frac{36}{49}$ c) $\frac{5}{7}$ d) $\frac{25}{49}$

393. If X follows a binomial distribution with parameters $n = 100$ and $P = \frac{1}{3}$, then $P(X = r)$ is maximum when r is equal to

a) 16 b) 32 c) 33 d) None of these

394. A carton contains 20 bulbs, 5 of which are defective. The probability that, if a sample of 3 bulbs is chosen at random from the carton, 2 will be defective, is

a) 1/16 b) 3/64 c) 9/64 d) 2/3

395. In a binomial distribution, mean is 3 and standard deviation is $\frac{3}{2}$, then the probability distribution is

a) $\left(\frac{3}{4}+\frac{1}{4}\right)^{12}$ b) $\left(\frac{1}{4}+\frac{3}{4}\right)^{12}$ c) $\left(\frac{1}{4}+\frac{3}{4}\right)^{9}$ d) $\left(\frac{3}{4}+\frac{1}{4}\right)^{9}$

396. If A and B are two events than the value of the determinant choosen at random from all the determinants of order 2 with entries 0 or 1 only is positive or negative respectively. Then,

a) $P(A) > P(B)$ b) $P(A) < P(B)$ c) $P(A) = P(B) = 1/2$ d) $P(A) = P(B)$

397. If $P(A) = \frac{1}{12}$, $P(B) = \frac{5}{12}$ and $P\left(\frac{B}{A}\right) = \frac{1}{15}$, then $P(A \cup B)$ is equal to

a) $\frac{89}{180}$ b) $\frac{90}{180}$ c) $\frac{91}{180}$ d) $\frac{92}{180}$

398. A dice is thrown 100 times. Getting an even number is considered a success. The variance of the number of successes is

a) 10 b) 20 c) 25 d) 50

399. If the mean and S.D. of a binomial distribution are 20 and 4 respectively, then the number of trials is

a) 50 b) 25 c) 100 d) 80

400. Probability that in the toss of two dice we obtain an even sum or a sum less than 5, is

a) $\frac{1}{2}$ b) $\frac{1}{6}$ c) $\frac{2}{3}$ d) $\frac{5}{9}$

401. If $\frac{1+4p}{p}, \frac{1-p}{4}, \frac{1-2p}{2}$ are probabilities of three mutually exclusive events, then

a) $\frac{1}{3} \leq p \leq \frac{1}{2}$ b) $\frac{1}{2} \leq p \leq \frac{2}{3}$ c) $\frac{1}{6} \leq p \leq \frac{1}{2}$ d) None of these

402. A wire of length l is cut into three pieces. What is the probability that the three pieces from a triangle?

a) 1/2 b) 1/4 c) 2/3 d) None of these

403. A die has four blank faces and two faces marked 3. The change of getting a total of 12 in 5 throws is

a) ${}^5C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)$ b) ${}^5C_4 \left(\frac{1}{3}\right) \left(\frac{2}{3}\right)^4$ c) ${}^5C_4 \left(\frac{1}{6}\right)^5$ d) ${}^5C_4 \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)$

404. Twenty five coins are tossed simultaneously. The probability that the fifth coin will fall with head upward, is

a) $\frac{5}{25}$ b) $\frac{5}{2^{25}}$ c) $\frac{1}{2}$ d) None of these

405. Which one of the following is not correct?

a) The probability that in a family of 4 children, there will be at least one boy, is $\frac{15}{16}$

b) Two cards are drawn with replacement from a well shuffled pack. The probability of drawing both aces, is $\frac{1}{169}$

c) The probability guessing at least 8 out 10 answers in a true false examination, is $\frac{7}{64}$

d) A coin is tossed three times. The probability of getting exactly two heads, is $\frac{3}{8}$

406. A parents has two children. If one of them is boy, then the probability that other is, also a boy, is

a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{1}{3}$ d) None of these

407. The probabilities that a student will obtain grades A, B, C or D are 0.30, 0.35, 0.20 and 0.15 respectively. The probability that he will receive atleast C grade, is

a) 0.65 b) 0.85 c) 0.80 d) 0.20

408. An ordinary cube has four blank faces, one face marked 2 and another marked 3. Then the probability of obtaining 9 in 5 throws, is

a) $\frac{31}{7776}$ b) $\frac{5}{2592}$ c) $\frac{5}{1944}$ d) $\frac{5}{1296}$

409. For a party 8 guests are invited by a husband and his wife. They sit for a dinner around a round table. The probability that the husband and his wife sit together, is

a) $\frac{2}{7}$ b) $\frac{2}{9}$ c) $\frac{1}{9}$ d) $\frac{4}{9}$

410. If A and B are two independent events, then A and $\bar{B}$ are

a) Not independent b) Also independent c) Mutually exclusive d) None of these

411. Three mangoes and three apples are kept in a box. If two fruits are selected at random from the box, the probability that the selection will contain one mango and one apple, is

a) $\frac{3}{5}$ b) $\frac{5}{6}$ c) $\frac{1}{36}$ d) None of these

412. In an assembly of 4 persons the probability that at least 2 of them have the same birthday, is

a) 0.293 b) 0.24 c) 0.0001 d) 0.016

413. Four persons are chosen at random from a group of 3 men, 2 women and 4 children. The chance that exactly 2 of them are children, is

a) $\frac{10}{21}$ b) $\frac{11}{13}$ c) $\frac{13}{25}$ d) $\frac{21}{32}$

414. Out of 30 consecutive integers, 2 are chosen at random. The probability that their sum is odd, is

a) $\frac{14}{29}$ b) $\frac{16}{29}$ c) $\frac{15}{29}$ d) $\frac{10}{29}$

415. An unbiased die with faces marked 1, 2, 3, 4, 5 and 6 is rolled four times. Out of four face values obtained,

the probability that the minimum face value is not less than 2 and the maximum face value is not greater than 5, is

a) $\frac{16}{81}$ b) $\frac{1}{81}$ c) $\frac{80}{81}$ d) $\frac{65}{81}$

416. A dice is thrown 5 times, then the probability that an even number will come up exactly 3 times, is

a) $\frac{5}{16}$ b) $\frac{1}{2}$ c) $\frac{3}{16}$ d) $\frac{3}{2}$

417. Among 15 players, 8 are batsman and 7 are bowlers. The probability that a team is chosen of 6 batsman and 5 bowlers, is

a) $\frac{{}^8C_6 \times {}^7C_5}{{}^{15}C_{11}}$ b) $\frac{{}^8C_6 + {}^7C_5}{{}^{15}C_{11}}$ c) $\frac{15}{28}$ d) None of these

418. Two dice are thrown together. Then, the probability that the sum of numbers appearing on them is a prime number, is

a) $\frac{5}{12}$ b) $\frac{7}{18}$ c) $\frac{13}{36}$ d) $\frac{11}{36}$

419. If E and F are events with $P(E) \leq P(F)$ and $P(E \cup F) > 0$, then

a) Occurrence of $E \Rightarrow$ occurence of F b) Occurrence of $F \Rightarrow$ occurence of E

c) Non-occurrence of $E \Rightarrow \text{Non} - \text{ccurence of } F$ d) None of the above

420. In a college, 25% of the boys and 10% of the girls offer Mathematics. The girls constitute 60% of the total number of students. If a student is selected at random and is found to be studying Mathematics. The probability that the student is a girl is

a) 1/6 b) 3/8 c) 5/8 d) 5/6

421. Five coins whose faces are marked 2, 3 are tossed. The chance of obtaining a total of 12 is

a) 1/32 b) 1/16 c) 3/16 d) 5/16

422. A basket contains 5 apples and 7 oranges and another basket contains 4 apples and 8 oranges. One fruit is picked out from each basket. The probability that the fruits are both apples or both oranges, is

a) 24/144 b) 56/144 c) 68/144 d) 76/144

423. For n independent events $A_i's, P(A_i) = 1/(1+i), i = 1, 2, \ldots, n$. The probability that atleast one of the events occurs is

a) $1/n$ b) $1/(n+1)$ c) $n/(n+1)$ d) None of these

424. Consider two events A and B such that $P(A) = \frac{1}{4}, P\left(\frac{B}{A}\right) = \frac{1}{2}, P\left(\frac{A}{B}\right) = \frac{1}{4}$. For each of the following statements, which is true?

1. $P(A^c + B^c) = \frac{3}{4}$

2. The events A and B are mutually exclusive.

3. $P(A/B) + P(A/B^c) = 1$

a) (1) only b) (1) and (2) c) (1) and (3) d) (2) and (3)

425. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal green is

a) $\frac{3}{5}$ b) $\frac{6}{7}$ c) $\frac{20}{23}$ d) $\frac{9}{20}$

426. The probability that the roots of the equation $x^2 + nx + \frac{1}{2} + \frac{n}{2} = 0$ are real where $n \in N$ such that $n \leq 5$, is

a) 1/5 b) 2/5 c) 3/5 d) 4/5

427. The probability that in a random arrangement of the letters of the word 'UNIVERSITY', the two I's do not come together is

a) 4/5 b) 1/5 c) 1/10 d) 9/10

428. If $P(B) = \frac{3}{4}, P(A \cap B \cap \bar{C}) = \frac{1}{3}$ and $P(\bar{A} \cap B \cap \bar{C}) = \frac{1}{3}$, then $P(B \cap C)$ is

a) $\frac{1}{12}$ b) $\frac{1}{6}$ c) $\frac{1}{15}$ d) $\frac{1}{9}$

429. If $P(A) = 1/3, P(B) = 1/2$ and $P(A \cup B) = 5/6$, then events A and B are

a) Mutually exclusive
b) Independent as well as mutually exhaustive
c) Independent
d) Dependent only on A

430. If X is a poisson variate with $P(X = 0) = 0.8$, then the variance of X is

a) $\log_e 20$
b) $\log_{10} 20$
c) $\log_e \frac{5}{4}$
d) 0

431. If A and B are mutually exclusive events with $P(A) = \frac{1}{2} \times P(B)$and $A \cup B = S$,(total sample space) then $P(A)$ is equal to

a) $\frac{2}{3}$
b) $\frac{1}{3}$
c) $\frac{1}{4}$
d) $\frac{3}{4}$

432. A coin is tossed $2n$ times. The chance that the number of times one gets head is not equal to the number of times one gets tail, is

a) $\frac{(2n!)}{(n!)^2}\left(\frac{1}{2}\right)^{2n}$
b) $1 - \frac{(2n!)}{(n!)^2}$
c) $1 - \frac{(2n!)}{(n!)^2} \cdot \frac{1}{4^n}$
d) None of these

433. The probability of getting a total of at least 6 in the simultaneously throw of three dice is

a) $\frac{6}{108}$
b) $\frac{5}{27}$
c) $\frac{1}{24}$
d) $\frac{103}{108}$

434. In the above question the probability that the binary operation is non-commutative is

a) $\frac{n^{n^2} - n^{n(n+1)}}{n^{n^2}}$
b) $\frac{n^{n^2} - n^{\frac{n(n+1)}{2}}}{n^{n^2}}$
c) $\frac{n^{n^2/2} - n^{\frac{n(n+1)}{2}}}{n^{n^2/2}}$
d) None of these

435. A bag A contains 4 green and 3 red balls and bag B contains 4 red and 3 green balls. One bag is taken at random and a ball is drawn and noted to be green. The probability that it comes from bag B,is

a) $\frac{2}{7}$
b) $\frac{2}{3}$
c) $\frac{3}{7}$
d) $\frac{1}{3}$

436. From a pack of cards two are accidently dropped. Probability that they are of opposite shade is

a) $\frac{13}{51}$
b) $\frac{1}{52 \times 51}$
c) $\frac{26}{51}$
d) None of these

437. In a binomial distribution the probability of getting a success is 1/4 and standard deviation is 3, then its mean is

a) 6
b) 8
c) 12
d) 10

438. Past records reveal that during a particular was, out of 9 vessels expected to arrive at the Mumbai harbour exactly 7 reached the harbour safely. If 3 vessels were expected to arrive there on a particular data, the probability that exactly two would arrive at the harbour safely, is

a) $\frac{91}{243}$
b) $\frac{92}{243}$
c) $\frac{95}{243}$
d) None of these

439. A coin is tossed n times. The probability that head will turn up an odd number of times, is

a) $\frac{1}{2}$
b) $\frac{n+1}{2n}$
c) $\frac{n-1}{2n}$
d) $\frac{2^{n-1}-1}{2^n}$

440. If A and B are independent events such that $P(A) > 0, P(B) > 0$, then

a) A and B are mutually exclusive
b) A and $\bar{B}$ are independent
c) $P(A \cup B) = P(\bar{A})P(\bar{B})$
d) $P(A/B) = P(\bar{A}/B)$

441. The probability that the 6th day of a randomly chosen month of a year is a Sunday, is

a) $\frac{1}{12}$
b) $\frac{1}{17}$
c) $\frac{1}{84}$
d) None of these

442. If the letters of the word 'REGULATIONS' be arranged at random, the probability that there will be exactly 4 letters between R and E is

a) $\frac{6}{55}$
b) $\frac{3}{55}$
c) $\frac{49}{55}$
d) None of these

443. Five different objects A_1, A_2, A_3, A_4, A_5 are distributed randomly in 5 places marked 1,2,3,4,5. One

arrangement is picked at random. The probability that in the selected arrangement, none of the object occupies the place corresponding to its number, is

a) 119/120 b) 1/15 c) 11/30 d) None of these

444. If M and N are any two events. The probability, that exactly one of them occurs, is

a) $P(M)+P(N)-P(M\cap N)$ b) $P(M)+P(N)+P(M\cap N)$

c) $P(M)+P(N)$ d) $P(M)+P(N)-2P(M\cap N)$

445. The probability that the same number appear on throwing three dice simultaneously, is

a) $\frac{1}{36}$ b) $\frac{5}{36}$ c) $\frac{1}{6}$ d) $\frac{4}{13}$

446. One hundred cards are numbered from 1 to 100. The probability that a randomly chosen card has a digit 5 is

a) $\frac{1}{100}$ b) $\frac{9}{100}$ c) $\frac{19}{100}$ d) None of these

447. A and B stand in a ring with 10 other persons. If the arrangement of the persons is at random, then the probability that there are exactly 3 persons between A and B is

a) 2/11 b) 9/11 c) 1/11 d) None of these

448. If the random variable X takes the values x_1, x_2, x_{10} with probability $P(X=x_1)=ki$, then the value of k is equal to

a) $\frac{1}{10}$ b) $\frac{1}{4}$ c) $\frac{1}{55}$ d) $\frac{7}{12}$

449. The chances to fail in physics are 20% and the chances to fail in mathematics are 10%. What are the chances to fail in at least one object?

a) 28% b) 38% c) 72% d) 82%

450. Two friends A and B have equal number of daughters. There are three cinema tickets which are to be distributed among the daughters of A and B. The probability that all the tickets got to daughters of A is 1/20. The number of daughters each of them have is

a) 4 b) 5 c) 6 d) 3

451. Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle with these three vertices is equilateral, equals

a) $\frac{1}{2}$ b) $\frac{1}{5}$ c) $\frac{1}{10}$ d) $\frac{1}{20}$

452. If three students A, B, C can solve a problem with probabilities $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$ respectively, then the probability that the problem will be solved is

a) $\frac{3}{5}$ b) $\frac{4}{5}$ c) $\frac{2}{5}$ d) $\frac{47}{60}$

453. In the above question the probability that the number is odd is

a) 3/4 b) 1/4 c) 1/8 d) None of these

454. An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, the number of outcomes that B must have so that A and B are independent, is

a) 2,4 or 8 b) 3,6 or 9 c) 4 or 8 d) 5 or 10

455. The probability that at least one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2, then $P(\bar{A})+P(\bar{B})$, is

a) 0.4 b) 0.8 c) 1.2 d) 1.4

456. A bag contains 5 brown and 4 white socks. A man pulls out two socks. The probability that these are of the same colour, is

a) 5/108 b) 18/108 c) 30/108 d) 48/108

457. A five digits number is formed by writing the digits 1, 2, 3, 4, 5 in a random order without repetitions. Then the probability that the number is divisible by 4, is

a) $\frac{3}{5}$ b) $\frac{18}{5}$ c) $\frac{1}{5}$ d) $\frac{6}{5}$

458. In an experiment the success is twice that of failure. If the experiment is repeated 6 times, the probability

that at least 4 times favourable is

a) $\frac{64}{729}$ b) $\frac{192}{729}$ c) $\frac{240}{729}$ d) $\frac{496}{729}$

459. If $P(A) = 0.25, P(B) = 0.50$ and $P(A \cap B) = 0.14$,then $P(A \cap \overline{B})$ is equal to

a) 0.61 b) 0.39 c) 0.48 d) None of these

460. A random variable X has the probability distribution

X	1	2	3	4	5	6	7	8
$P(X)$	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events $E = \{X \text{ is a prime number}\}$ and $F = \{X < 4\}$,then $P(E \cup F)$ is

a) 0.77 b) 0.87 c) 0.35 d) 0.50

461. If X is a poisson variate such that $P(X = 1) = P(X = 2)$, then $P(X = 4)$ is equal to

a) $\frac{1}{2e^2}$ b) $\frac{1}{3e^2}$ c) $\frac{2}{3e^2}$ d) $\frac{1}{e^2}$

462. In a lottery three were 90 tickets numbered 1 to 90. Five tickets were drawn at random. The probability that two of the tickets drawn numbers 15 and 89, is

a) 2/801 b) 2/623 c) 1/267 d) 1/623

463. The probability that a man will live 10 more years is 1/4 and the probability that his wife will live 10 more years is 1/3. Then the probability that neither will be alive in 10 years, is

a) 5/12 b) 1/2 c) 7/12 d) 11/12

464. A cricket club has 15 members, of whom only 5 can bowl. If the names of 15 members are put into a box and 11 are drawn at random, then the probability of obtaining an eleven containing at least 3 bowlers is

a) 7/13 b) 6/13 c) 11/15 d) 12/13

465. A bag X contains 2 white and 3 black balls and another bag Y contains 4 white and 2 black balls. One bag is selected at random and a ball is drawn from it. Then, probability for the ball chosen be white, is

a) $\frac{2}{25}$ b) $\frac{7}{15}$ c) $\frac{8}{15}$ d) $\frac{14}{15}$

466. A bag contains 4 brown and 5 white balls. A man pulls two balls at random without replacement. The probability that the man gets both the balls of the same colour is

a) $\frac{5}{108}$ b) $\frac{1}{6}$ c) $\frac{5}{18}$ d) $\frac{4}{9}$

: ANSWER KEY :

1)	b	2)	c	3)	d	4)	a	189)	c	190)	c	191)	c	192)	b
5)	c	6)	d	7)	a	8)	d	193)	d	194)	a	195)	c	196)	c
9)	d	10)	b	11)	a	12)	b	197)	c	198)	a	199)	b	200)	b
13)	a	14)	a	15)	c	16)	b	201)	a	202)	a	203)	a	204)	c
17)	c	18)	a	19)	d	20)	c	205)	a	206)	c	207)	c	208)	d
21)	a	22)	a	23)	d	24)	b	209)	a	210)	b	211)	a	212)	b
25)	a	26)	d	27)	b	28)	c	213)	a	214)	c	215)	c	216)	d
29)	d	30)	d	31)	b	32)	b	217)	b	218)	b	219)	c	220)	a
33)	d	34)	c	35)	a	36)	a	221)	a	222)	a	223)	c	224)	c
37)	d	38)	c	39)	c	40)	c	225)	d	226)	b	227)	c	228)	a
41)	d	42)	a	43)	b	44)	d	229)	b	230)	a	231)	c	232)	c
45)	b	46)	a	47)	a	48)	c	233)	d	234)	c	235)	c	236)	c
49)	d	50)	b	51)	c	52)	b	237)	b	238)	a	239)	a	240)	d
53)	c	54)	a	55)	b	56)	d	241)	a	242)	c	243)	a	244)	b
57)	c	58)	b	59)	a	60)	b	245)	a	246)	a	247)	b	248)	a
61)	d	62)	c	63)	c	64)	a	249)	a	250)	d	251)	c	252)	a
65)	b	66)	c	67)	c	68)	a	253)	c	254)	c	255)	b	256)	c
69)	a	70)	a	71)	d	72)	c	257)	b	258)	d	259)	b	260)	b
73)	a	74)	c	75)	b	76)	c	261)	b	262)	a	263)	a	264)	a
77)	b	78)	a	79)	c	80)	d	265)	c	266)	d	267)	c	268)	a
81)	a	82)	d	83)	a	84)	c	269)	c	270)	c	271)	b	272)	c
85)	a	86)	a	87)	c	88)	b	273)	d	274)	b	275)	c	276)	a
89)	b	90)	c	91)	c	92)	a	277)	a	278)	b	279)	d	280)	a
93)	a	94)	c	95)	c	96)	a	281)	c	282)	c	283)	b	284)	c
97)	c	98)	a	99)	a	100)	d	285)	a	286)	a	287)	b	288)	d
101)	c	102)	d	103)	b	104)	c	289)	a	290)	c	291)	a	292)	a
105)	b	106)	b	107)	d	108)	b	293)	b	294)	b	295)	d	296)	b
109)	d	110)	b	111)	a	112)	a	297)	d	298)	b	299)	a	300)	b
113)	a	114)	a	115)	a	116)	c	301)	c	302)	a	303)	a	304)	c
117)	b	118)	a	119)	c	120)	c	305)	b	306)	b	307)	a	308)	d
121)	d	122)	b	123)	b	124)	a	309)	a	310)	b	311)	a	312)	b
125)	a	126)	a	127)	c	128)	a	313)	a	314)	b	315)	c	316)	d
129)	a	130)	c	131)	b	132)	c	317)	b	318)	c	319)	c	320)	b
133)	b	134)	d	135)	d	136)	b	321)	b	322)	c	323)	d	324)	b
137)	b	138)	c	139)	d	140)	b	325)	b	326)	d	327)	c	328)	d
141)	a	142)	b	143)	a	144)	b	329)	b	330)	d	331)	c	332)	b
145)	b	146)	b	147)	d	148)	c	333)	c	334)	a	335)	b	336)	b
149)	c	150)	b	151)	d	152)	d	337)	b	338)	a	339)	b	340)	c
153)	a	154)	d	155)	d	156)	c	341)	a	342)	c	343)	b	344)	c
157)	d	158)	b	159)	d	160)	c	345)	a	346)	a	347)	d	348)	b
161)	c	162)	b	163)	c	164)	d	349)	c	350)	b	351)	d	352)	c
165)	b	166)	a	167)	a	168)	b	353)	d	354)	d	355)	c	356)	c
169)	d	170)	d	171)	c	172)	a	357)	c	358)	c	359)	b	360)	c
173)	a	174)	c	175)	b	176)	a	361)	c	362)	c	363)	b	364)	b
177)	a	178)	a	179)	d	180)	b	365)	b	366)	d	367)	d	368)	a
181)	c	182)	b	183)	b	184)	a	369)	a	370)	d	371)	b	372)	c
185)	b	186)	c	187)	c	188)	d	373)	b	374)	c	375)	c	376)	a

377)	c	378)	b	379)	c	380)	a
381)	a	382)	b	383)	b	384)	c
385)	d	386)	d	387)	c	388)	c
389)	b	390)	c	391)	d	392)	a
393)	c	394)	c	395)	a	396)	d
397)	a	398)	c	399)	b	400)	d
401)	d	402)	b	403)	a	404)	c
405)	c	406)	c	407)	d	408)	d
409)	b	410)	b	411)	a	412)	d
413)	a	414)	c	415)	a	416)	a
417)	a	418)	a	419)	d	420)	b
421)	d	422)	d	423)	c	424)	a

425)	c	426)	d	427)	a	428)	a
429)	a	430)	c	431)	b	432)	c
433)	d	434)	b	435)	c	436)	c
437)	c	438)	d	439)	a	440)	b
441)	c	442)	a	443)	c	444)	d
445)	a	446)	c	447)	a	448)	c
449)	a	450)	d	451)	c	452)	a
453)	a	454)	d	455)	c	456)	d
457)	c	458)	d	459)	d	460)	a
461)	c	462)	a	463)	b	464)	d
465)	c	466)	d				

: HINTS AND SOLUTIONS :

1 **(b)**

Required probability= ${}^7C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^5 \times \frac{1}{6}$

$= \frac{{}^7C_2 \times 5^5}{6^8}$

2 **(c)**

$x + \frac{100}{x} > 50$

$\Rightarrow x^2 + 100 > 50\,x \quad (\because x \in N)$

$\Rightarrow x^2 - 50x + 100 > 0$

$\Rightarrow (x-25)^2 > 525$

$\Rightarrow x - 25 < -\sqrt{525}$ or $x - 25 > \sqrt{525}$

$\Rightarrow x < 25 - 22.9$ or $x > 25 + 22.9$

$\Rightarrow x \le 2$ or $x \ge 48$

Hence, the number of favourable cases = 2+ 53 = 55

Thus, required probability $= \frac{55}{100} = \frac{11}{20}$

3 **(d)**

The probability of getting head and tail in one toss is $\frac{1}{2}$.

P (atleast one H) $= 1 - P$ (no head in four toss)

$= 1 - P$ (four tails)

$= 1 - \left(\frac{1}{2}\right)^4 = 1 - \frac{1}{16} = \frac{15}{16}$

4 **(a)**

Three balls can be selected in the following ways:

White : 3 2 1 0

Black : 0 1 2 3

∴ Total number of ways = 4,

Clearly, there is only one favourable ways in which three balls are white

Hence, required probability $= \frac{1}{4}$

NOTE It should be noted that the number of ways of selecting 3 white balls from 4 white balls is not equal to ${}^4C_3$, because all white balls are identical and all black balls are also identical

5 **(c)**

Let A be the event that the first mango is good and B denotes the event that the second is good. Then,

Required probability $= P(B/A) = \frac{P(A\cap B)}{P(A)}$

Now,

$P(A \cap B)$ = Probability that both mangoes are good

$\Rightarrow P(A \cap B) = \frac{{}^6C_2}{{}^{15}C_2}$

$P(A)$ = Probability that first mango is good

$\Rightarrow P(A) = \frac{{}^6C_2 + {}^6C_1 \times {}^4C_1}{{}^{15}C_2}$

∴ Required probability $= P(B/A) = \frac{{}^6C_2}{{}^6C_2 + {}^6C_1 \times {}^4C_1}$

$= \frac{15}{15+24} = \frac{5}{13}$

6 **(d)**

Total number of 5 digit number $= 9 \times 10 \times 10 \times 10 \times 10 = 90000$

Number of favourable numbers $= 5 \times 5 \times 4 \times 4 \times 3 = 12000$

Thus, required probability $= \frac{1200}{90000} = \frac{1}{75}$

7 **(a)**

Total number of cases $= 6^2 = 36$

Let E =Event of getting sum is 9

$= \{(3,6), (6,3), (4,5), (5,4)\}$

$\therefore\ n(E) = 4$

∴ Required probability= $\frac{4}{36} = \frac{1}{9}$

8 **(d)**

We have,

$4\,P(X=4)P(X=2)$

$\Rightarrow 4\ {}^6C_4\,p^4q^2 = {}^6C_2\,p^2q^4$

$\Rightarrow 4\,p^2 = q^2$

$\Rightarrow 4\,p^2 = (1-p)^2 \Rightarrow 3\,p^2 + 2\,p - 1 = 0 \Rightarrow p = \frac{1}{3}$

9 **(d)**

Let E denote the event that a six occurs and A the event that the man reports it is a six. We have

$P(E) = 1/6, P(E') = 5/6, P(A/E) = \frac{3}{4}$ and $P(A/E') = 1/4$

By Bay's theorem

$P(E/A) = \frac{P(E)\cdot P(A/E)}{P(E)\cdot P(A/E) + P(E')\cdot P(A/E')}$

$= \frac{\left(\frac{1}{6}\right)\left(\frac{3}{4}\right)}{\left(\frac{1}{6}\right)\left(\frac{3}{4}\right) + \left(\frac{5}{6}\right)\left(\frac{1}{4}\right)} = \frac{3}{8}$

10 **(b)**

m one rupee coins and n ten paise coins can be placed in a line in $\frac{(m+n)!}{m!\,n!}$ ways

∴ Total number of elementary events $= \frac{(m+n)!}{m!\,n!}$

If the extreme coins are ten paise coins, then the remaining $(n-2)$ ten paise coins and m one rupee coins can be arranged in a line a $\frac{(m+n-2)!}{m!(n-2)!}$

ways

$\therefore$ Favourable number of elementary events

$= \frac{(m+n-2)!}{m!(n-2)!}$

$\therefore$ Required probability $= \frac{\frac{(m+n-2)!}{m!(n-2)!}}{\frac{(m+n)!}{m!n!}} = \frac{n(n-1)}{(m+n)(m+n-1)}$

11 **(a)**

Let the die be tossed n times and let X denote the number of sixes obtained in n tosses

Clearly, $p =$ Probability of getting a six in a toss $= \frac{1}{6}$

Now,

$P(X \geq 1) \geq 0.9$

$\Rightarrow 1 - P(X = 0) \geq 0.9$

$\Rightarrow 1 - \left(\frac{5}{6}\right)^n \geq 0.9$

$\Rightarrow \left(\frac{5}{6}\right)^n \leq 0.1$

$\Rightarrow n(\log_{10} 5 - \log_{10} 6) \leq -1$

$\Rightarrow n \geq \frac{1}{\log_{10} 6 - \log_{10} 5} = 12.6$

$\Rightarrow n = 13, 14, 15, \ldots$

The least value of n is 13

12 **(b)**

Here, p

$=$ Probability of getting double six in two dice

$= \frac{1}{6^2} = \frac{1}{36}$ and $q = \frac{35}{36}$

$\therefore$ Required probability

$= 1 -$ (Probability of not getting double six)^n

$= 1 - \left(\frac{35}{36}\right)^n$

13 **(a)**

The probability that the number appearing on the selected chit is greater than or equal to 5 is 3/7. Therefore, the probability that in each of the four draws, the chits bear 5 or 6 or 7 is $\left(\frac{3}{7}\right)^4$

14 **(a)**

The total number of possible ways

$n(S) = 6 \times 6 \times 6 = 216$

Now, we find out how many ways are favourable to the total of 16 points in one throw. Let A, B, C be three dice, then favourable ways happen only as follows

(6,6,4),(6,4,6),(6,5,5),(5,5,6),(5,6,5),(4,6,6).

Hence, the total number of favourable ways

$n(A) = 6$

$\therefore$ Required probability$= \frac{n(A)}{n(S)} = \frac{6}{216} = \frac{1}{36}$

15 **(c)**

Given that, $P\left(\frac{B}{A}\right) = \frac{1}{2} \Rightarrow \frac{P(B \cap A)}{P(A)} = \frac{1}{2}$

$\Rightarrow P(B \cap A) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

$P\left(\frac{A}{B}\right) = \frac{1}{4} \Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{1}{4}$,

$\Rightarrow P(B) = 4P(A \cap B) \Rightarrow P(B) = \frac{1}{2}$

$\therefore P(A \cap B) = \frac{1}{8} = \frac{1}{2} \cdot \frac{1}{4} = P(A) \cdot P(B)$

$\because$ Events A and B are independent.

Now, $P\left(\frac{A'}{B}\right) = \frac{P(A' \cap B')}{P(B)}$

$= \frac{P(A')P(B')}{P(B)} = \frac{3}{4}$

and $P\left(\frac{B'}{A'}\right) = \frac{P(B' \cap A')}{P(A')}$

$= \frac{P(B')P(A')}{P(A')} = \frac{1}{2}$

16 **(b)**

Given, $P(A) = p, \ P(B) = q$

$\Rightarrow P(\bar{A}) = 1 - P, \qquad P(\bar{B}) = 1 - q$

Probability that one person is alive there are two cases arise,

(i) A dies and B lives

(ii) B dies and A lives

$\therefore$ Required probability$= p(1-q) + q(1-p)$

$= p + q - 2pq$

17 **(c)**

Given mean, $np = 15$

and variance $np(1-P) = 10$

$\therefore \ 1 - p = \frac{10}{15} = \frac{2}{3} \Rightarrow p = \frac{1}{3}$

$\therefore \ n = 15 \times 3 = 45$

18 **(a)**

Three squares can be chosen out of 64 squares in $^{64}C_3$ ways. Two squares of one colour and one another colour can be the following chosen in two mutually exclusive ways:

(i) two white and one black

and (ii) two black and one white

$\therefore$ Favourable number of ways $= \ ^{32}C_2 \times \ ^{32}C_1 + \ ^{32}C_1 \times \ ^{32}C_2$

Hence, required probability $= \frac{2 \times \ ^{32}C_1 \times \ ^{32}C_2}{^{64}C_3} = \frac{16}{21}$

20 **(c)**

Given, mean $np = 2$ ...(i)

And variance $npq = 1$ ...(ii)

From Eqs.(i) and (ii), we get

$q=\frac{1}{2}$

$\therefore\ p=1-q=\frac{1}{2}$

From Eq.(i), $n\times\frac{1}{2}=2$

$\therefore\ n=4$

The binomial distribution is $\left(\frac{1}{2}+\frac{1}{2}\right)^4$

Now, $P(X>1)=P(X=2)+P(X=3)+P(X=4)$

$={}^4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2+{}^4C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1+{}^4C_4\left(\frac{1}{2}\right)^4$

$=\frac{6+4+1}{16}$

$=\frac{11}{16}$

21 **(a)**

Probability of getting head in one trial, $P=\frac{1}{2}$ and probability of not getting head, $q=\frac{1}{2}$

Probability of getting head odd times

$={}^{26}C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{19}$

$+{}^{20}C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{17}+\ldots+{}^{20}C_{19}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{19}$

$=\frac{1}{2^{20}}[{}^{20}C_1+{}^{20}C_3+\ldots+{}^{20}C_{19}]$

$=\frac{1}{2^{20}}\times 2^{20-1}$

$=\frac{2^{19}}{2^{20}}=\frac{1}{2}$

22 **(a)**

Probability of occurrence of $4=\frac{1}{6}$

Probability of non-occurrence of $4=\frac{5}{6}$

Required probability

$={}^2C_1\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)+{}^2C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^0$

$=2\cdot\frac{5}{36}+1\cdot\frac{1}{36}=\frac{10}{36}+\frac{1}{36}=\frac{11}{36}$

23 **(d)**

In out of 9 tickets, 5 tickets are odd number and 4 tickets are even numbers.

∴Required probability

$=\left\{\frac{{}^5C_1}{{}^9C_1}\times\frac{{}^4C_1}{{}^8C_1}\times\frac{{}^4C_1}{{}^7C_1}+\frac{{}^4C_1}{{}^9C_1}\times\frac{{}^5C_1}{{}^8C_1}\times\frac{{}^3C_1}{{}^7C_1}\right\}$

$=\frac{5}{9}\times\frac{4}{8}\times\frac{4}{7}+\frac{4}{9}\times\frac{5}{8}\times\frac{3}{7}$

$=\frac{140}{504}=\frac{5}{18}$

24 **(b)**

Given, $P(X=k)=\frac{(k+1)a}{3^k}$, for $x\in\{0,1,2,\ldots\infty\}$

As we know that

$P(0)+P(1)+P(2)+\ldots\infty=1$

$\Rightarrow\ a+\frac{2a}{3}+\frac{3a}{3^2}+\ldots\infty=1$ ...(i)

Let $S=a\left(1+\frac{2}{3}+\frac{3}{3^2}+\frac{4}{3^3}+\ldots\infty\right)$

$\Rightarrow\ \frac{\frac{1}{3}S=a\left(\frac{1}{3}+\frac{2}{3^2}+\frac{3}{3^3}+\ldots\infty\right)}{S-\frac{1}{3}S=a\left(1+\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\infty\right)}$

$\Rightarrow\frac{2}{3}S=a\left(\frac{1}{1-\frac{1}{3}}\right)\ \Rightarrow\ S=\frac{9a}{4}$

∴ From Eq.(i)

$\frac{9a}{4}=1\ \Rightarrow\ a=\frac{4}{9}$

25 **(a)**

We know that, $P(A|\bar{B})+P(\bar{A}|\bar{B})=1$

$\Rightarrow P(\bar{A}|\bar{B})=1-P(A|\bar{B})$

26 **(d)**

Total ways $=6\times5=30$

Favourable events = The minimum of the two numbers is less than 4.

$n(E)=6\times4=24$

[We can select one from {1, 2, 3, 4} and other from (1 ,2, 3, 4, 5, 6)]

∴ Required probability $=\frac{24}{30}=\frac{4}{5}$

27 **(b)**

In first 120 natural number total number of multiple of 5, $n(A)=24$ and total number of multiple of 15, $n(B)=8$ and $n(A\cap B)=8$

$\therefore\ n(A\cup B)=n(A)+n(B)-n(A\cap B)$

$=24+8-8=24$

∴ Required probability$=\frac{24}{120}=\frac{1}{5}$

28 **(c)**
Probability that four of the numbers are consecutive
$= \frac{^{37}C_1}{^{40}C_4}$
Now, probability that four of the numbers are not consecutive
$= 1 - \frac{^{37}C_1}{^{40}C_4} = 1 - \frac{37}{91390} = \frac{2469}{2470}$

29 **(d)**
Total number of cards =52
Probability of getting spade $= \frac{13}{52} = \frac{1}{4}$
Probability of not getting spade $= 1 - \frac{1}{4} = \frac{3}{4}$
$\therefore$ Required probability $= \frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$

30 **(d)**
The total number of ways of choosing two numbers out of 1,2,3, … ,30 is $^{30}C_2 = 435$.
Now, $a^2 - b^2$ will be divisible by 3 iff either a and b both are divisible by 3 or none of a and b is divisible by 3
$\therefore$ Favourable number of ways $= {^{10}C_2} + {^{20}C_2} = 235$
Hence, required probability $= \frac{235}{435} = \frac{47}{87}$

31 **(b)**
Let $p = P$(getting a head)$= \frac{1}{2}, q = P$(getting no head)$= \frac{1}{2}$
By using binomial distribution,
Required probability P(six heads)
$= {^{10}C_6} \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^4 = \frac{10!}{6!\,4!} \times \frac{1}{2^{10}}$
$= \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2} \times \frac{1}{2^{10}} = \frac{105}{512}$

32 **(b)**
Total number of cases=20
Favorable cases=$\{(1,8), (3,6), (5,4), (7,2)\} = 4$
$\therefore$ Required probability $= \frac{4}{20} = \frac{1}{5}$

33 **(d)**
Let X be the number of coins showing heads. Let X be a binomial variate with parameters $n = 100$ and p probability of success. According to question
$P(X = 50) = P(X = 51)$
$\Rightarrow {^{100}C_{50}}\, p^{50}(1-p)^{50} = {^{100}C_{51}}(p)^{51}(1-p)^{49}$
$\Rightarrow \frac{(100)!}{(50!)(50!)} \cdot \frac{(51!) \times (49!)}{100!} = \frac{p}{1-p}$
$\Rightarrow \frac{p}{1-p} = \frac{51}{50} \Rightarrow p = \frac{51}{101}$

34 **(c)**
Matches played by India = 4
Maximum points in any match = 2
$\therefore$ Maximum points in four matches can be 8 only.
Therefore, atleast 7 points means 7 or 8 points.
$\therefore$ Required probability $= P(7) + P(8)$
$= {^4C_1}(0.05)(0.5)^3 + (0.5)^4$
$= 0.0250 + 0.0625 = 0.0875$

35 **(a)**
Given, $P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$
Since, $P(X = 2) = 3P(X = 3)$
$\Rightarrow \frac{\lambda^2 e^{-\lambda}}{2!} = 3.\frac{\lambda^3 e^{-\lambda}}{3!} \Rightarrow \lambda = 1$
$\therefore$ Mean of poisson distribution is 1

36 **(a)**
Since A and B are independent events
$\therefore P(A \cap B) = \frac{1}{8}$ and $P(\bar{A} \cap \bar{B}) = \frac{3}{8}$
$\Rightarrow P(A)P(B) = \frac{1}{8}$ and $P(\bar{A})P(\bar{B}) = \frac{3}{8}$
Now,
$P(\bar{A} \cap \bar{B}) = \frac{3}{8}$
$\Rightarrow 1 - P(A \cup B) = \frac{3}{8}$
$\Rightarrow 1 - \{P(A) + P(B) - P(A \cap B)\} = \frac{3}{8}$
$\Rightarrow 1 - \{P(A) + P(B)\} + \frac{1}{8} = \frac{3}{8}$
$\Rightarrow P(A) + P(B) = \frac{3}{4}$
The quadratic equation whose roots are $P(A)$ and $P(B)$ is
$x^2 - x\{P(A) + P(B)\} + P(A)P(B) = 0$
$\Rightarrow x^2 - \frac{3}{4}x + \frac{1}{8} = 0 \Rightarrow 8x^2 - 6x + 1 = 0 \Rightarrow x = \frac{1}{2}, \frac{1}{4}$
$\therefore P(A) = \frac{1}{2}, \frac{1}{4}$

37 **(d)**
Required probability $= {^4C_2} \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^2$
$= \frac{4!}{2!\,2!} \times \frac{9}{16} \times \frac{1}{16} = \frac{27}{128}$

38 **(c)**
Probability for selecting a white ball $= \frac{2}{6} = \frac{1}{3}$
Probability for selecting a black ball $= \frac{4}{6} = \frac{2}{3}$

$\therefore$ Required probability $= {}^5C_5\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^0 +$
${}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)$
$=\left(\frac{1}{3}\right)^5 + 5\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)$
$=\left(\frac{1}{3}\right)^4\left[\frac{1}{3}+5\cdot\frac{2}{3}\right]$
$=\frac{11}{3^5}=\frac{11}{243}$

39 **(c)**
Assuming all six girls as one unit.
$\therefore$ Required probability $=\frac{7!\,6!}{12!}=\frac{1}{132}$

40 **(c)**
Six girls and 5 boys can sit in a row in 11 ! ways
$\therefore$ Total number of elementary events = 11 !
Six girl can sit in a row in 6 ! and in each such arrangement there are 7 places between them in which 5 boys can be seated in ${}^7C_5 \times 5\,!$ ways.
Therefore, the total number of ways in which no two boys sit together $= 6\,! \times {}^7C_5 \times 5\,!$
Hence, required probability $=\frac{6\,!\cdot {}^7C_5\times 5\,!}{11\,!}=\frac{6\,!7\,!}{2\,!11\,!}$

41 **(d)**
If $m > n$, then there is no injective map from A to B
$\therefore$ Required probability = 0

42 **(a)**
Since, $0 \le \frac{1+a}{3} \le 1 \Rightarrow -1 \le a \le 2$ ...(i)
and $0 \le \frac{1-a}{4} \le 1 \Rightarrow -3 \le a \le 1$ ...(ii)
Also, as $\frac{1+a}{3}$ and $\frac{1-a}{4}$ are the probabilities of two mutually exclusive events.
$\therefore 0 \le \frac{1+a}{3}+\frac{1-a}{4} \le 1 \Rightarrow -7 \le a \le 5$ ...(iii)
From relations (i), (ii) and (iii), we get
$-1 \le a \le 1$

43 **(b)**
Total number of cases $= {}^9C_3 = 84$
Number of favourable cases $= {}^3C_1.{}^4C_1.{}^2C_1 = 24$
$\therefore P=\frac{24}{84}=\frac{2}{7}$

44 **(d)**
The given distribution will be a probability distribution, if
$P(X=0)+P(X=1)+P(X=2)+P(X=3)=1$
$\Rightarrow k+3k+3k+k=1 \Rightarrow k=\frac{1}{8}$

Computation of Variance:

X	$p(x)$	$xp(x)$	$x^2p(x)$
0	$\frac{1}{8}$	0	0
1	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$
2	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{12}{8}$
3	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{9}{8}$
Total		$\sum xp(x) = \frac{12}{8}$	$\sum x^2p(x) = \frac{24}{8}$

$\therefore$ Variance $=\sum x^2p(x) - \{\sum xp(x)\}^2$
$\Rightarrow$ Variance $=\frac{24}{8}-\left(\frac{12}{8}\right)^2 = 3-\frac{9}{4}=\frac{3}{4}$

45 **(b)**
Total number of permutations of the 11 letters of the word 'MISSISSIPPI', in which 4 are of one kind (viz, S), 4 of other kind (viz, I), 2 of third kind (viz, P) and one of fourth kind (viz; M), is
$\frac{11\,!}{4\,!4\,!2\,!1\,!}$
The number of ways in which for S come together
$=\frac{8!}{4\,!2\,!}$
Hence, required probability $=\frac{8\,!}{4\,!2\,!1\,!}+\frac{11\,!}{4\,!4\,!2\,!1\,!}=\frac{4}{165}$

46 **(a)**
There are 9 favourable cases in which all three digits are same.
$\therefore$ Required probability $=\frac{9}{900}=\frac{1}{100}$

47 **(a)**
We know, sum of probability distribution is 1

$\therefore k+2k+3k+4k=1 \Rightarrow k=\frac{1}{10}$

Now, mean

$\bar{X}=k\times 1+2k\times 2+3k\times 3+4k\times 4$

$=k+4k+9k+16k=30k$

$\Rightarrow \bar{X}=30\times\frac{1}{10}=3$

48 **(c)**
Let E_1, E_2 and E_3 are the examines guesses, copies and knows the answer

and E =Event that he answers correctly

Then, $P(E_1)=\frac{1}{3}, P(E_2)=\frac{1}{6}$

and $P(E_3) = 1 - \left(\frac{1}{3} + \frac{1}{6}\right) = \frac{1}{2}$

∴ Required probability= $P\left(\frac{E_3}{E}\right)$

$$= \frac{P\left(\frac{E}{E_3}\right).P(E_3)}{P\left(\frac{E}{E_1}\right).P(E_1) + P\left(\frac{E}{E_2}\right).P(E_2) + P\left(\frac{E}{E_3}\right).P(E_3)}$$

$$= \frac{1 \times \frac{1}{2}}{\left(\frac{1}{4} \times \frac{1}{3}\right) + \left(\frac{1}{8} \times \frac{1}{6}\right) + \left(1 \times \frac{1}{2}\right)} = \frac{24}{29}$$

49 **(d)**
Total outcomes=30
Now, $n + \left(\frac{50}{n}\right) > 27$
$\Rightarrow n^2 - 27n + 50 > 0$
$\Rightarrow (n-2)(n-25) > 0$
Favourable outcomes are 1,26,27,28,29,30
Number of favourable outcomes=6
∴ Required probability= $\frac{6}{30} = \frac{1}{5}$

50 **(b)**
Here, $P(B') = 1 - 0.4 = 0.6$

and $P(A) = 1 - 0.3 = 0.7$

∴ $P(A \cup B') = P(A) + P(B') - P(A \cap B')$

$= 0.7 + 0.6 - 0.5 = 0.8$

51 **(c)**

$$P\overline{\left(\frac{A}{B}\right)} = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})} = \frac{P(\overline{A \cup B})}{P(\bar{B})}$$

$$= \frac{1 - P(A \cup B)}{P(\bar{B})}$$

52 **(b)**
We have,
Total number of arrangements $= \frac{11!}{4!2!2!1!} = 34650$
The number of ways in which 4 I's, 2 P's and 1 M can be arranged in a row $= \frac{7!}{2!4!} = 105$
After arranging this 7 letters (viz, IIIIPPM) there are 8 places in which $4S'$s can be arranged in $^8C_4$ ways
∴ Number of ways in which no 2 S's are together $= \frac{7!}{2!4!} \times {}^8C_4$
∴ Required probability $= \frac{7!}{2!4!} \times {}^8C_4 + \frac{11!}{4!2!2!1!} = \frac{7}{33}$

53 **(c)**
Probability of both occurrence,

$P(E \cap F) = P(E)P(F)$

$= \frac{1}{5}.\frac{1}{10} = \frac{1}{50}$

Required probability $= 1 - P(E \cap F)$

$= 1 - \frac{1}{50} = \frac{49}{50}$

54 **(a)**
Here, $n = 6$

According to the question

${}^6C_2p^2q^4 = 4.\ {}^6C_4p^4q^2$

$\Rightarrow q^2 = 4p^2$

$\Rightarrow (1-p)^2 = 4p^2$

$\Rightarrow 3p^2 + 2P - 1 = 0$

$\Rightarrow (p+1)(3p-1) = 0$

$\Rightarrow p = \frac{1}{3}$ ($\because p$ cannot be negative)

55 **(b)**
We know, $P(A \cup B) = P(A) + P(B) - P(A \cap B) \leq P(A) + P(B)$ $[\because P(A \cap B) \geq 0]$

56 **(d)**
Given, $P\left(\frac{B}{A}\right) = \frac{1}{2} \Rightarrow P(B \cap A) = \frac{1}{8}$

and $P\left(\frac{A}{B}\right) = \frac{1}{4} \Rightarrow P(B) = \frac{1}{2}$

∴ $P(A \cap B) = \frac{1}{8} = P(A).P(B)$

∴ Events are independent

Now, $P\left(\frac{A'}{B}\right) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)} = \frac{3}{4}$

and $P\left(\frac{B'}{A}\right) = \frac{P(A \cap B')}{P(A)} = \frac{P(A) - P(A \cap B)}{P(A)} = \frac{1}{2}$

57 **(c)**
We know that
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
But, it is given that $P(A \cup B) = P(A) + P(B) - P(A)P(B)$
∴ $P(A \cap B) = P(A)P(B)$
$\Rightarrow A$ and B are independent events
$\Rightarrow P((A \cup B)^c) = P(A^c \cap B^c) = P(A^c)P(B^c)$

and, $P(A/B) = P(A)$

58 **(b)**

Let A be the set of all numbers from $\{1,2,3,...,1000\}$ that leave remainder 1 when divided by 7

$A = \{1,8,15,22, ...,995\}$

$\Rightarrow\ n(A) = \frac{995-1}{7} + 1 = 143$

$\therefore\ P(A) = \frac{143}{1000}$

59 **(a)**

$P(2 \text{ white and } 1 \text{ black})$

$= P(W_1W_2B_3 \text{ or } W_1B_2W_3 \text{ or } B_1W_2W_3)$

$= P(W_1)P(W_2)P(B_3) + P(W_1)P(B_2)P(W_3) + P(B_1)P(W_2)P(W_3)$

$= \left(\frac{3}{4}\right)\left(\frac{2}{4}\right)\left(\frac{3}{4}\right) + \left(\frac{3}{4}\right)\left(\frac{2}{4}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)\left(\frac{2}{4}\right)\left(\frac{1}{4}\right)$

$= \frac{1}{32}(9+3+1) = \frac{13}{32}$

60 **(b)**

$\therefore\ P(X > 1.5) = P(2) + P(3) + \ldots\infty$

$= 1 - [P(O) + P(1)]$

$= 1 - \left(e^{-2} + \frac{e^{-2}\times 2}{1}\right) = 1 - \frac{3}{e^2}$

61 **(d)**

Now, $P(A' \cap B') = P(A \cup B)'$

$= 1 - P(A \cup B) = 1 - 0.8 = 0.2$

and $P(A' \cup B') = 1 - P(A \cap B) = 1 - 0.3 = 0.7$

But $P(A' \cup B') = P(A') + P(B') - P(A' \cap B')$

$\Rightarrow 0.7 = P(A') + P(B') - 0.2$

$\Rightarrow\ P(A') + P(B') = 0.9.$

62 **(c)**

We have,

Required probability = ${}^7C_2\left(\frac{1}{10}\right)^2\left(\frac{9}{10}\right)^5 = 21(0.1)^2(0.9)^5$

63 **(c)**

The couple server the committee in ${}^7C_3 \times {}^2C_2$ ways. The couple does not serve the committee in ${}^7C_5$ ways

$\therefore$Required probability= $\frac{{}^7C_3\times{}^2C_2+{}^7C_5}{{}^9C_5}$

$= \frac{56}{126} = \frac{4}{9}$

64 **(a)**

The total number of ways in which 3 numbers can be chosen out of 30 numbers = ${}^{30}C_3 = 4060$

The number of ways of choosing 3 consecutive numbers is 28. Therefore, the number of ways in which the three numbers chosen are not consecutive is $4060 - 28 = 4032$

Hence, required probability = $\frac{4032}{4060} = \frac{144}{145}$

65 **(b)**

Four balls can be drawn alternatively in the following two ways $RWRW$ or $WRWR$

If red ball is drawn first, then probability of drawing the balls alternatively

$= \frac{6}{9}\times\frac{3}{8}\times\frac{5}{7}\times\frac{2}{6} = \frac{5}{84}$

If white ball is drawn first, then probability of drawing the balls alternatively

$= \frac{3}{9}\times\frac{6}{8}\times\frac{2}{7}\times\frac{5}{6} = \frac{5}{84}$

Since, these two are mutually exclusive ways.

$\therefore$ Required probability $= \frac{5}{84} + \frac{5}{84} = \frac{10}{84} = \frac{5}{42}$

66 **(c)**

Since, all three persons have three options to apply.

$\therefore$ Total cases= 3^3

Favorable cases=3

$\therefore$ Required probability = $\frac{3}{3^3} = \frac{1}{9}$

67 **(c)**

Maximum points in four matches can be 8 only. Therefore, at least 7 points means 7 or 8 points

$\therefore$Required probability= $P(7) + P(8)$

$=\ {}^4C_1(0.05)(0.5)^3 + (0.5)^4$

$= 0.0250 + 0.0625$

$= 0.0875$

68 **(a)**

Since, A and B are mutually exclusive events, therefore

$A \cap B = \phi \Rightarrow A \subseteq \bar{B}$ and $B \subseteq \bar{A}$

$\Rightarrow\ P(A) \le P(\bar{B})$ and $P(B) \le P(\bar{A})$

69 **(a)**

Given, $f(x) = \lambda e^{-ax}$, for $0 \le x < \infty$ and $a > 0$

$\therefore \int_0^\infty \lambda e^{-ax} = 1$ [$\because$ sum of total distribution is one]

$\Rightarrow\ \lambda\left[\frac{e^{-ax}}{-a}\right]_0^\infty = 1$

$\Rightarrow\ \lambda\left[0 + \frac{1}{a}\right] = 1 \Rightarrow \lambda = a$

70 **(a)**

Ten different books and 2 different pens can be distributed equally among 3 boys in

${}^{12}C_4 \times {}^8C_4 \times {}^4C_4$ ways

Required probability

= 1 − Probability that 2 pens are received by the same boy

$= 1 - \left(\frac{{}^{10}C_2 \times {}^{8}C_4 \times {}^{4}C_4}{{}^{12}C_4 \times {}^{8}C_4 \times {}^{4}C_4}\right) \times 3! = 1 - \frac{6}{1} = \frac{5}{11}$

71 **(d)**

E =Set of number divisible by 6

$= \{6,12,18,24,30, \ldots\ldots,96\}$

F =Set of number divisible by 8

$= \{8,16,24, \ldots.,96\}$

$E \cap F$ =Set of number divisible by 24

$= \{24,48,72,96\}$

$\therefore n(E) = 16, n(F) = 12. n(E \cap F) = 4$

Let $n(A) = n(E) + n(F) - 2n(E \cap F)$

$= 16 + 12 - 8 = 20$

and $n(S) = 100$

$\therefore$ Required probability

$= \frac{n(E \cup F)}{n(S)} = \frac{20}{100} = \frac{1}{5}$

72 **(c)**

Number of favourable cases = 3

Required probability = $\frac{3}{6} = \frac{1}{2}$

73 **(a)**

Let the probability of success and failure are p and q respectively

$\therefore\ p = 2q$ and $p + q = 1$

$\Rightarrow\ 3q = 1 \Rightarrow\ q = \frac{1}{3}$ and $p = \frac{2}{3}$

$\therefore$ Required probability

$= {}^{6}C_4 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^2 + {}^{6}C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right) + {}^{6}C_6 \left(\frac{2}{3}\right)^6$

$= \frac{240}{729} + \frac{192}{729} + \frac{64}{729} = \frac{496}{729}$

74 **(c)**

Probability that one person is alive = $\frac{2}{3}$

Probability that all five are alive = $\left(\frac{2}{3}\right)^5 = \frac{32}{243}$

75 **(b)**

Let E =Event of getting prime number from 1 to 100

$= \{2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,$

$\therefore n(E) = 25$

$\therefore$Required probability= $\frac{25}{100} = 0.25$

76 **(c)**

Seven persons can sit in a row in 7! Ways.

No two persons will sit consecutively if one person sits at any one of the two marked seats on the left and other sits at any one of the marked seats on the right of the middle seat. Number of seating arrangements in which no two persons sit consecutively = ${}^{2}C_1 \times {}^{2}C_1 \times {}^{1}C_1 \times 3! = 24$

Hence, required probability = $\frac{24}{7!} = \frac{4}{35}$

78 **(a)**

Given, $np = 4,\ npq = 3$

$\Rightarrow\ P = \frac{1}{4}, q = \frac{3}{4}$

$\therefore\ P(X = 6) = {}^{16}C_6 \left(\frac{1}{4}\right)^6 . \left(\frac{3}{4}\right)^{10}$

79 **(c)**

Required probability = $P(X = 0) + P(X = 1)$

$= \frac{e^{-5}}{0!} . 5^0 + \frac{e^{-5}}{1!} . 5^1$

$= e^{-5} + 5e^{-5} = \frac{6}{e_5}$

80 **(d)**

Total number of numbers = $(5)^2$

Number of numbers which are divisible by 4 = 5

$\therefore$ Required probability = $\frac{5}{25} = \frac{1}{5}$

81 **(a)**

Given A and B are two events odds against A are 2 to 1, odds in favour of $A \cup B$ are 3 to 1 also $x \le P(B) \le y$

Given $P(A) = \frac{1}{3}, P(A \cup B) = \frac{3}{4}$

$\therefore\ P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow\ \frac{1}{3} + P(B) - P(A \cap B) = \frac{3}{4}$

$\Rightarrow\ P(A \cap B) = P(B) - \frac{5}{12}$

$\Rightarrow\ P(B) \ge \frac{5}{12}$

Again, $P(B) = \frac{5}{12} + P(A \cap B)$

$\Rightarrow\ P(B) \le \frac{5}{12} + P(A)\ \ [\because P(A \cap B) \le P(A)]$

$\Rightarrow\ P(B) \le \frac{5}{12} + \frac{1}{3} \ \Rightarrow\ P(B) \le \frac{3}{4}$

$\therefore\ \frac{5}{12} \le P(B) \le \frac{3}{4}$

But$x \le P(B) \le y$

82 **(d)**

Total number of ways in which 5 boys and 5 girls are sitting in a row alternatively $= 2.5!\,5!$

$\therefore$ Required probability$= \frac{2.5!5!}{10!} = \frac{1}{126}$

83 **(a)**

Given, $P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{3}$ and $P(E_3) = \frac{1}{4}$

$\therefore\ P(E_1 \cup E_2 \cup E_3) = 1 - P(\bar{E}_1)P(\bar{E}_2)P(\bar{E}_3)$

$= 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)$

$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{3}{4}$

84 **(c)**

Number which are divisible by either 3 or 5 are 12,15,18,20,21,24,25,27,30

Total number=9

P(number either divisible by 3 or 5)$= \frac{9}{20}$

P(number neither divisible by 3 nor 5)

$= 1 - P$(either divisible by 3 or5)

$= 1 - \frac{9}{20} = \frac{11}{20}$

85 **(a)**

Since, $0 \le P(A) \le 1, 0 \le P(B) \le 1, 0 \le P(C) \le 1$

and $0 \le P(A) + P(B) + P(C) \le 1$

$\therefore\ 0 \le \frac{3x+1}{3} \le 1$

$\Rightarrow\ -\frac{1}{3} \le x \le \frac{2}{3}$ ...(i)

$0 \le \frac{1-x}{4} \le 1\ \Rightarrow\ -3 \le x \le 1$ ...(ii)

$0 \le \frac{1-2x}{2} \le 1\ \Rightarrow\ -\frac{1}{2} \le x \le \frac{1}{2}$ ...(iii)

and $0 \le \frac{3x+1}{3} + \frac{1-x}{4} + \frac{1-2x}{2} \le 1$

$\Rightarrow\ \frac{1}{3} \le x \le \frac{13}{3}$ ...(iv)

From Eqs. (i), (ii), (iii) and (iv), we get

$\frac{1}{3} \le x \le \frac{1}{2}$

86 **(a)**

Probability of getting $4 = \frac{1}{6}$

Probability of not getting $4 = 1 - \frac{1}{6} = \frac{5}{6}$

$\therefore$ Probability of getting 4 at least once in two throw of dice

$= {}^2C_1\left(\frac{1}{6}\right)\left(\frac{5}{6}\right) + {}^2C_2\left(\frac{1}{6}\right)^2$

$= 2.\frac{5}{36} + 1.\frac{1}{36} = \frac{11}{36}$

87 **(c)**

We have,

$P(A \cup B) = 0.7$ and $P(A \cap B) = 0.2$

Now,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A) + P(B) = 0.9$

$\Rightarrow 1 - P(\bar{A}) + 1 - P(\bar{B}) = 0.9$

$\Rightarrow P(\bar{A}) + P(\bar{B}) = 1.1$

88 **(b)**

The probability of throwing 9 with two dice $= \frac{4}{36} = \frac{1}{9}$

$\therefore$ The probability of not throwing 9 with two dice $= 1 - \frac{1}{9} = \frac{8}{9}$

If A is to win, he should throw 9, in Ist or 3rd or 5th and so on.

If B is to win, he should throw, 9 in 2nd, 4th and so on.

B can be get the chance only when A should not get it.

$\therefore$ B's chances $= \left(\frac{8}{9}\right) \cdot \frac{1}{9} + \left(\frac{8}{9}\right)^3 \cdot \frac{1}{9} + \ldots$

$= \frac{\frac{8}{9} \times \frac{1}{9}}{1 - \left(\frac{8}{9}\right)^2} = \frac{8}{17}$

89 **(b)**

Probability of student (if he miss no test)

$= \left(1 - \frac{1}{5}\right) \times \left(1 - \frac{1}{5}\right)$

$= \frac{16}{25}$

Hence, probability that he will miss atleast one test

$= 1 - \frac{16}{25} = \frac{9}{25}$

90 **(c)**

Since $\bar{A} \cap B$ and A are mutually exclusive events such that

$A \cup B = (\bar{A} \cap B) \cup A$

$\therefore P(A \cup B) = P(\bar{A} \cap B) + P(A)$

$\Rightarrow \frac{3}{4} = P(\bar{A} \cap B) + 1 - \frac{2}{3} \Rightarrow P(\bar{A} \cap B) = \frac{5}{12}$

91 **(c)**

$P(A'|B') = \frac{P(A' \cap B')}{P(B')}$

$= \frac{P(A \cup B')}{P(B')}$

$= \frac{1 - P(A \cup B)}{1 - P(B)}$

$= \frac{1 - P(A) + P(B) - P(A \cap B)}{1 - P(B)}$

$= \frac{1 - [-0.5 + 0.4 - 0.3]}{1 - 0.4} = \frac{0.4}{0.6} = \frac{2}{3}$

92 **(a)**

There is only one two digit number not satisfying the given property. Again there are 27 three digit numbers not satisfying the given property. Hence, required probability

$= 1 - \frac{27 + 1}{1000} = \frac{243}{250}$

94 **(c)**

Let A be the even that an even face turns up and B be the event that it is 2 or 4. Then,

$P(A) = 0.24 + 0.18 + 0.14 = 0.56$ and,

$P(B) = 0.24 + 0.18 = 0.42$

$\therefore$ Required probability $= P(B/A)$

$\Rightarrow$ Required probability $= \frac{P(A \cap B)}{P(A)} = \frac{P(B)}{P(A)}$ $[\because B \subset A]$

$\Rightarrow$ Required probability $= \frac{0.42}{0.56} = \frac{3}{4} = 0.75$

95 **(c)**

Probability of selecting a white ball from X bag= $\frac{2}{5}$

Probability of selecting a white ball from Y bag

$= \frac{4}{6} = \frac{2}{3}$

Probability of selecting a white ball from X or Y bags

$= \frac{2}{5} + \frac{2}{3} = \frac{16}{15}$

Probability of selecting the white ball from one of the bags

$= \frac{1}{2} . \frac{16}{15} = \frac{8}{15}$

96 **(a)**

Here, $P(B) = 1 - P(B') = 1 - \frac{2}{5} = \frac{3}{5}$

$\therefore\ P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{4}{5} + \frac{3}{5} - \frac{1}{2} = \frac{9}{10}$

$\therefore\ P(A \cap B') = P(A \cup B) - P(B)$

$= \frac{9}{10} - \frac{3}{5} = \frac{3}{10}$

97 **(c)**

Given, $P(X) = 60\% = \frac{3}{5}$,

$P(Y) = 50\% = \frac{1}{2}$

$\Rightarrow\ P(X') = 1 - \frac{3}{5} = \frac{2}{5}$,

$P(Y') = 1 - \frac{1}{2} = \frac{1}{2}$

[$\because$ If both speak truth or both speak false narrating the same incident, then thry will not contradict each other].

$\therefore$ Required probability that they contradict each other narrating the same incident= $1 - [P(X) \times P(Y') + P(X') \times P(Y)]$

$= 1 - \left[\frac{3}{5} \times \frac{1}{2} + \frac{2}{5} \times \frac{1}{2}\right]$

$= 1 - \left[\frac{3}{10} + \frac{2}{10}\right] = \frac{1}{2}$

98 **(a)**

Let E be the event of numbers to be divisible by 2 or 3

$\therefore\ E = \{2, 3, 4, 6, 8, 9, 10,12\}$

$\Rightarrow\ n(E) = 8$ and $n(S) = 12$

Hence, required probability $= \dfrac{n(E)}{n(S)}$

$= \dfrac{8}{12} = \dfrac{2}{3}$

100 **(d)**

Given A and B are mutually exclusive events

$\therefore P(A \cap B) = \phi$

Now, $P(A|\bar{B}) = \frac{P(A \cap \bar{B})}{P(B)}$

$= \dfrac{P(A) - P(A \cap B)}{P(\bar{B})}$

$= \dfrac{P(A)}{1 - P(B)}$

101 **(c)**

$S = \{1,2,3,4,5,6\}$ and $E = \{5,6\}$

$\therefore\ n(S) = 6$ and $n(E) = 2$

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

102 **(d)**

Clearly, $A \cap C, B \cap C$ and $A \cap B \cap C$ are subsets of C

$\therefore P(A \cap C) \leq P(C), P(B \cap C)$
$\leq P(C), P(A \cap B \cap C) \leq P(C)$

$\Rightarrow P(A \cap C) \leq 0, P(B \cap C) \leq 0, P(A \cap B \cap C) \leq 0$

But, $P(A \cap C) \geq 0, P(B \cap C) \geq 0, P(A \cap B \cap C) \geq 0$

$\therefore P(A \cap C) = 0 = P(B \cap C) = P(A \cap B \cap C)$

$\Rightarrow P(A \cap C) = 0 = P\ (A)\ P\ (C)$

$P(B \cap C) = 0 = P(B)\ P(C)$

and, $P(A \cap B \cap C) = 0 = P\ (A)\ P\ (B)\ P\ (C)$

Hence, A and C; B and C and A, B and C are independent events

103 **(b)**

Let E =Event of getting an even numbers

$= \{2,4,6\}$

$n(E) = 3$

$\therefore$ Probability of success, $P = \frac{3}{6} = \frac{1}{2}$

and probability of failure, $q = \frac{1}{2}$

$\therefore$ Variance$= npq = 100 \times \frac{1}{2} \times \frac{1}{2} = 25$

104 **(c)**

The number of divisible by 4, if last two digits are 12, 24, 32 and 52.

Remaining three place can be filled by 3! ways.

$\therefore$ Favourable cases $= 3! \times 4$

Required probability $= \frac{3! \times 4}{5!}$

$= \dfrac{3! \times 4}{5 \times 4 \times 3!} = \dfrac{1}{5}$

105 **(b)**

$\therefore P = P(\text{getting a head}) = \frac{1}{2}$

$\therefore\ q = \dfrac{1}{2}$

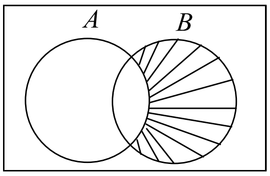
$\therefore$ Required probability

$= {}^{10}C_6 \left(\dfrac{1}{2}\right)^6 \left(\dfrac{1}{2}\right)^4$

$= \dfrac{10!}{6!\,4!} \times \dfrac{1}{2^{10}} = \dfrac{105}{512}$

107 **(d)**

$P(\overline{A} \cap B) = P(B) - P(A \cap B)$



$P(\bar{A} \cap B)$

108 **(b)**

Favourable ways are (2, 6), (3, 5), (4, 4), (5, 3) and (6, 2)

$\therefore$ Required probability $= \frac{5}{36}$

109 **(d)**

Favourable cases of getting 10 or greater than 10, if 5 appears on atleast one of dice.

$= \{(5,6), (6,5), (5,5)\}$

Number of favourable cases =3

Total number of cases = 36

$\therefore$ Required probability $= \frac{3}{36} = \frac{1}{12}$

110 **(b)**

Let X be the number of heads getting in n tossed.

X follows binomial distribution with parameters $n, p = \frac{1}{2}, q = \frac{1}{2}$.

Given that, $P(X \geq 1) \geq 0.8$

$\Rightarrow 1 - P(X = 0) \geq 0.8$

$\Rightarrow P(X = 0) \leq 0.2$

$\Rightarrow\ {}^{n}C_0 \left(\dfrac{1}{2}\right)^n \left(\dfrac{1}{2}\right)^0 \leq 0.2$

$\Rightarrow\ \dfrac{1}{2^n} \leq \dfrac{1}{5}$

$\Rightarrow\ 2^n \geq 5$

$\therefore$ The least value of n is 3.

111 **(a)**

Given, $E_k = \{(a,b) \in S: ab = k\}$for $k \geq 1$ and $p_k = P(E_k)$

Now, $E_1 = \{1,1\} \Rightarrow p_1 = P(E_1)$

$\Rightarrow P_1 = \frac{1}{36}$

$E_2 = \{(1,2),(2,1)\} \Rightarrow p_2 = P(E_2)$

$\Rightarrow P_2 = \frac{2}{36}$

$E_4 = \{(1,4),(4,1),(2,2)\}$

$\Rightarrow P_4 = P(E_4)$

$\Rightarrow P_4 = \frac{3}{36}$

$E_6 = \{(1,6),(6,1),(2,3),(3,2)\}$

$\Rightarrow P_6 = P(E_6) \Rightarrow P_6 = \frac{4}{36}$

and $E_{30} = \{(5,6),(6,5)\} \Rightarrow p_{30} = p(E_{30})$

$\Rightarrow p_{30} = \frac{2}{36}$

∴ From the above results, we get

$p_1 < p_{30} < p_4 < p_6$

112 **(a)**

The total number of ways of choosing 2 persons out of n is ${}^nC_2$

After selecting two persons when the remaining $(n-2)$ persons sit in row $(n-1)$ places are created between them in which 2 persons can be arranged in ${}^{n-1}C_2 \times 2\,!$ ways

So, required probability $= \frac{{}^{n-1}C_2 \times 2\,!}{{}^nC_2} = \frac{n-2}{n} = 1 - \frac{2}{n}$

113 **(a)**

Given that, $P(A \cap B) = \frac{1}{3}, P(A \cup B) = \frac{5}{6}$, and $P(A) = \frac{1}{2}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{5}{6} = \frac{1}{2} + P(B) - \frac{1}{3}$

$\Rightarrow P(B) = \frac{5}{6} + \frac{1}{3} - \frac{1}{2} = \frac{4}{6} = \frac{2}{3}$

$\therefore P(A) \times P(B) = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3} = P(A \cap B)$

This shows that A and B are independent events.

114 **(a)**

Let E denote the event that a six occurs and A the event that the man reports that it is a '6'. We have

$P(E) = \frac{1}{6}, P'(E) = \frac{5}{6}$

$P\left(\frac{A}{E}\right) = \frac{3}{4}$ and $P\left(\frac{A}{E'}\right) = \frac{1}{4}$

From Baye's theorem,

$$P\left(\frac{E}{A}\right) = \frac{P(E).P\left(\frac{A}{E}\right)}{P(E).P\left(\frac{A}{E}\right) + P(E').P\left(\frac{A}{E'}\right)}$$

$$= \frac{\frac{1}{6} \times \frac{3}{4}}{\frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4}} = \frac{3}{8}$$

115 **(a)**

The man will be one step away from the starting point if (i) either he is one step ahead or (ii) one step behind the starting point

∴ Required probability $= P(i) + P(ii)$

The man will be one step ahead at the end of eleven steps if he moves six steps forward and five steps backward.

The probability of this even is ${}^{11}C_6 (0.4)^6\,(0.6)^5$

The man will be one step behind at the end of eleven steps if he moves six steps backward and five steps forward

The probability of this event is ${}^{11}C_6 (0.6)^6\,(0.4)^5$

Hence, required probability

$= {}^{11}C_6 (0.4)^6 (0.6)^5 + {}^{11}C_6 (0.6)^6 (0.4)^5$

$= {}^{11}C_6 (0.4)^5 (0.6)^5 (0.4 + 0.6) = {}^{11}C_6 (0.24)^5$

116 **(c)**

Total number of outcomes=8

Favorable cases are HTT, THT, TTH

∴ Number of favourable outcomes =3

∴ Required probability= $\frac{3}{8}$

117 **(b)**

We have,

Required probability

= 1 − Probability that he does not hit the target in any trial

$= 1 - (0.7)^{10}$

118 **(a)**

The probability that the number appearing on the selected chit is greater than or equal to 5 is 3/7

∴ Required probability= $\left(\frac{3}{7}\right)^4$

119 **(c)**

We know, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

and $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$

$$\therefore P(B) = \frac{P\left(\frac{B}{A}\right).P(A)}{P\left(\frac{A}{B}\right)}$$

$$= \frac{\left(\frac{2}{3}\right)\left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)} = \frac{1}{3}$$

120 **(c)**

A and B toss a coin alternately till one of them tosses heads and win the game, their respectively probabilities of winning are $\frac{1}{3}$ and $\frac{2}{3}$ respectively.

121 **(d)**

Since, E_1, E_2 are mutually exclusive events, then

$P(E_1 \cap E_2) = 0$

Now, $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$\Rightarrow 0.6 = P(E_1) + 1 - P(\overline{E_2}) - 0$

$\Rightarrow 0.6 = P(E_1) + 0.4$

$\Rightarrow P(E_1) = 0.2$

122 **(b)**

We have,

$P(A \cap B^c) = P(A) - P(A \cap B)$

$\Rightarrow P(A \cap B) = P(A) - P(A \cap B^c) = 0.7 - 0.5$

$= 0.2$

Now, $P(A \cup B^c) = P(A) + P(B^c) - P(A \cap B^c)$

$\Rightarrow P(A \cup B^c) = 0.7 + 0.6 - 0.5 = 0.8$

$\therefore P\{B \cap (A \cup B^c)\} = P[(B \cap A) \cup (B \cap B^c)]$

$= P(A \cap B)$

$\Rightarrow P(A \cup B^c)P(B/A \cup B^c) = P(A \cap B)$

$\Rightarrow 0.8\, P(B/A \cup B^c) = 0.2 \Rightarrow P(B/A \cup B^c) = \frac{0.2}{0.8}$

$= 0.25$

123 **(b)**

Let A and B are two events

$\therefore\ P(A) = \frac{2}{7}, P(B) = \frac{6}{11}$

$\therefore$Required probability$= 1 - P(\bar{A})P(\bar{B})$

$= 1 - \left(1 - \frac{2}{7}\right)\left(1 - \frac{6}{11}\right)$

$= \frac{52}{77}$

124 **(a)**

We have, $P(A) = \frac{3}{10}$ and $P(B) = \frac{2}{5}$

Required probability $= P(A) + P(B) - 2P(A \cap B)$

$\Rightarrow$ Required probability $= P(A) + P(B) -$
$2P\ (A)\ P(B)$

[$\because$ A and B are independent events]

$\Rightarrow$ Required probability $= \frac{3}{10} + \frac{2}{5} - 2 \times \frac{3}{10} \times \frac{2}{5} = \frac{23}{50}$

125 **(a)**

Let the coin be tossed n times and let X denote the number of heads in n tosses. Then,

$P(X = r) = {}^nC_r \left(\frac{1}{2}\right)^n$

Now, $P(X = 4) = P(X = 7)$

$\Rightarrow {}^nC_4 \left(\frac{1}{2}\right)^n = {}^nC_7 \left(\frac{1}{2}\right)^n$

$\Rightarrow {}^nC_4 = {}^nC_7$

$\Rightarrow n = 11 \qquad [\because {}^nC_x = {}^nC_y \Rightarrow x + y = n]$

$\therefore P(X = 2) = {}^{11}C_2 \left(\frac{1}{2}\right)^{11} = \frac{55}{2048}$

126 **(a)**

$P(A' \cap B') = 1 - P(A \cup B) = \frac{1}{3}$ [given]

$\Rightarrow\ P(A \cup B) = \frac{2}{3}$

$\therefore\ P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{2}{3} = x + x - \frac{1}{3} \Rightarrow\ x = \frac{1}{2}$

127 **(c)**

Required probability $= \frac{{}^2C_1}{{}^9C_1} \times \frac{{}^4C_1}{{}^9C_1} = \frac{8}{81}$

128 **(a)**

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{5}{6} = \frac{1}{2} + P(B) - \frac{1}{3}$

$\Rightarrow\ P(B) = \frac{2}{3}$

Also, $P(A).P(B) = \frac{1}{2}.\frac{2}{3} = \frac{1}{3}$

$\therefore\ P(A \cap B) = P(A).P(B)$

Hence, A and B are independent events

130 **(c)**

Since, balls are replaced, then the probability that 7th drawn ball is 4th white$= \frac{1}{2}$

131 **(b)**

Let

$S = \{BBB, BBG, BGB, GBB, GGB, GBG, BGG, GGG\}$

and $E = \{BBB, BBG, BGB, GBB, GGB, GBG, BGG\}$

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{7}{8}$

132 **(c)**

ASSISTANT : A A I N SSS TT

STATISTICS : A C II SSS TTT

Same letters can be A, I, S, T

Probability of choosing, $A = \frac{{}^2C_1}{{}^9C_1} \times \frac{{}^1C_1}{{}^{10}C_1} = \frac{1}{45}$

Probability choosing, $I = \frac{^1C_1}{^9C_1} \times \frac{^2C_1}{^{10}C_1} = \frac{1}{45}$

Probability of choosing, $S = \frac{^3C_1}{^9C_1} \times \frac{^3C_1}{^{10}C_1} = \frac{1}{10}$

Probability of choosing, $T = \frac{^2C_1}{^9C_1} \times \frac{^3C_1}{^{10}C_1} = \frac{1}{15}$

So, total probability $= \frac{19}{90}$

133 **(b)**

$P(H) = P(T) = \frac{1}{2}$

$\therefore$ Required probability $= P(A) + P(\bar{A} \cap \bar{B} \cap A + \ldots$

$= \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \ldots = \frac{\frac{1}{2}}{1 - \frac{1}{4}} = \frac{2}{3}$

134 **(d)**

Since A and B are independent events

$\therefore P(A \cap B) = P(A)P(B), P(A/B) = P(A)$

and, $P(\bar{A}/B) = P(\bar{A})$

Now,

$P(A \cap B) = P(A)P(B) \neq 0$

$\Rightarrow A$ and B are not mutually exclusive

Since $A \cap B$ and $A \cap \bar{B}$ are mutually exclusive events such that

$(A \cap B) \cup (A \cap \bar{B}) = A$

$\therefore P(A \cap B) + P(A \cap \bar{B}) = P(A)$

$\Rightarrow P(A)P(B) + P(A \cap \bar{B}) = P(A)$

$\Rightarrow P(A \cap \bar{B}) = P(A) - P(A)P(B)$

$\Rightarrow P(A \cap \bar{B}) = P(A)(1 - P(B))$

$\Rightarrow P(A \cap \bar{B}) = P(A)P(\bar{B})$

So, A and $\bar{B}$ are independent events

Similarly, $\bar{A}$ and B are also independent events

Now,

$P(A/B) = P(A)$ and $P(\bar{A}/B) = P(\bar{A})$

$\Rightarrow P(A/B) + P(\bar{A}/B) = P(A) + P(\bar{A}) = 1$

136 **(b)**

Required probability distribution is poisson distribution.

137 **(b)**

In a leap year there are 366 days in which 52 weeks and two days. The combination of 2 days may be: Sun-Mon, Mon-Tue, Tue-Wed, Wed-Thu, Thu-Fri, Fri-Sat, Sat-Sun.

$P(53\text{Fri}) = \frac{2}{7}; P(53 \text{ Sat}) = \frac{2}{7}$ and

$P(53 \text{ Fri and } 53 \text{ Sat}) = \frac{1}{7}$

$\therefore P(53 \text{ Fri or Sat})$

$= P(53 \text{ Fri}) + P(53 \text{ Sat}) - P(53 \text{ Fri and Sat})$

$= \frac{2}{7} + \frac{2}{7} - \frac{1}{7} = \frac{3}{7}$

138 **(c)**

Five boys and 3 girls can sit in a row in 8 ! ways. Considering three girls as one individual. There are 6 persons, who can sit in a row in 6 ! ways. But, 3 girls can sit together in 3! Ways

$\therefore$ Number of ways in which 5 boys and 3 girls can sit in a row when 3 girls sit together $= 6! \times 3!$

Hence, required probability $= \frac{6! \times 3}{8!} = \frac{3}{28}$

139 **(d)**

Let $E = \{2,4,6\}$ [$\because A$ and B are independent events]

$\therefore\ n(E) = 3$ and $n(S) = 6$

$\therefore\ P(E) = \frac{1}{2}$

Probability of failure $= 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore$ Probability of at least two success

$= P(X = 2) + P(X = 3)$

$= {}^3C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^1 + {}^3C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^0$

$= \frac{3}{8} + \frac{1}{8} = \frac{1}{2}$

140 **(b)**

The total number of ways in which 2 integers can be chosen from the given 40 integers is

${}^{40}C_2 = 780$

The sum of the selected numbers is odd if exactly one of them is odd and other is even

$\therefore$ Favourable number of ways $= {}^{20}C_1 \times {}^{20}C_1 = 400$

Hence, required probability $= \frac{400}{780} = \frac{20}{39}$

141 **(a)**

Let the number of red balls be x.

According to the given condition,

$\frac{^5C_2}{^{5+x}C_2} = \frac{5}{14}$

$\Rightarrow\ \frac{20(3 + x)!}{(5 + x)!} = \frac{5}{14}$

$\Rightarrow\ 56 = (5 + x)(4 + x)\ \Rightarrow\ x = 3$

142 **(b)**

Total number of outcomes=36

Favourable outcomes are

(3,6),(4,5),(5,5),(6,5),(6,4),(4,6),(6,3),(5,4),(6,6),(5,6)

Number of favourable outcomes =10

$\therefore$ Required probability= $\frac{10}{36} = \frac{5}{18}$

143 **(a)**

Required probability = $P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$

144 **(b)**

p = Probability of success $(s) = \frac{2}{6} = \frac{1}{3}$

q = Probability of failure $(f) = 1 - \frac{1}{3} = \frac{2}{3}$

Probability that success occurs in even number of tosses

$= P(fs) + P(f\,f\,fs) + P(f\,f\,f\,f\,fs) + \ldots$

$= qp + q^3p + q^5p + \ldots = \frac{qp}{1-q^2}$

$= \frac{\frac{2}{3} \times \frac{1}{3}}{1 - \left(\frac{2}{3}\right)^5} = \frac{\frac{2}{9}}{1 - \frac{4}{9}} = \frac{2}{9} \times \frac{9}{5} = \frac{2}{5}$

145 **(b)**

Consider the following events:

$A \rightarrow$ Getting head on first coin

$B \rightarrow$ Getting head on second coin

$C \rightarrow$ Getting 3 or 6 on die

These three events are independent with respective probabilities given by

$P(A) = \frac{1}{2}, P(B) = \frac{1}{2}, P(C) = \frac{2}{6} = \frac{1}{3}$

$\therefore$ Required probability = $P(A \cap B \cap C)$

$= P\,(A)\,P\,(B)\,P\,(C)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{12}$

146 **(b)**

Let p be the probability of getting an even number. Then by hypothesis, the probability of getting an odd number in $3p$. Since the events of getting an even number and an odd number are mutually exclusive and exhaustive

$\therefore p + 3\,p = 1 \Rightarrow p = 1/4$

Thus, the probability of getting an odd number in a single throw is $3/4$ and that of an even number is $1/4$

If the die is thrown twice, then the sum of the numbers in two throws is even if both the numbers are even or both are odd

$\therefore$ Required probability = $\frac{3}{4} \times \frac{3}{4} + \frac{1}{4} \times \frac{1}{4} = \frac{10}{16} = \frac{5}{8}$

147 **(d)**

$P(A \cup B) = P(A) + P(B) - P(AB)$

$P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$

(A and B are independents events)

$= \frac{1}{5} + \frac{1}{7} - \frac{1}{35} = \frac{11}{35}$

148 **(c)**

If the last digit is 2,4,6 or 8, none of the numbers can end in 0 or 5 and one of the last digits must be even. Now 8n is the number of ways in which 0 and 5 can be excluded and of these we have further to include 4n cases in which the last digit can be selected solely from 1,3,7 or 9

$\therefore$ Favourable number of ways = $8^n - 4^n$

Hence, required probability = $\frac{8^n - 4^n}{10^n} = \frac{4^n - 2^n}{5^n}$

150 **(b)**

Given, $P(X=0) = \frac{144}{169}$ and $P(X=1) = \frac{1}{169}$

$P(X=2) = 1 - P(X=1) - P(X=0)$

$= 1 - \frac{1}{169} - \frac{144}{169} = \frac{169 - 145}{169} = \frac{24}{169}$

151 **(d)**

Since, $A = \{4,5,6\}$and $B = \{1,2,3,4\}$

$\therefore A \cap B = \{4\}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = \frac{3}{6} + \frac{4}{6} - \frac{1}{6} = 1$

152 **(d)**

Given, $P(A) = P(B) = P(C) = \frac{1}{4}$

$P(AB) = P(CB) = 0$

and $P(AC) = \frac{1}{8}$

Now, $P(A+B) = P(A) + P(B) - P(AB)$

$= \frac{1}{4} + \frac{1}{4} - 0 = \frac{1}{2}$

153 **(a)**

Consider the following events:

A = A student is passed in Mathematics,

B = A student is passed in Statistics

Clearly,

$P(A) = \frac{70}{125}, P(B) = \frac{55}{125}, P(A \cap B) = \frac{30}{125}$

$\therefore$ Required probability = $P(A \cap \bar{B}) + P(\bar{A} \cap B)$

$\Rightarrow$ Required probability = $P(A) - P(A \cap B) + P(B) - P(A \cap B)$

$\Rightarrow$ Required probability = $\frac{70}{125} + \frac{55}{125} - \frac{60}{125} = \frac{65}{125} = \frac{13}{25}$

154 **(d)**

Total number of cases= $6^3 = 216$

Favorable cases are

(4,4,5),(4,5,4),(5,4,4),(4,5,5),(5,4,5),(5,5,4),(5,5,5),(4,4,4,).

$\therefore$Total number of cases=8

∴Required probability= $\frac{8}{216}=\frac{1}{27}$

155 **(d)**

Since, A and B are independent events.

$$\therefore\ P(A\cap B)=P(A).P(B)=\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$$

Now, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$=\frac{1}{2}+\frac{1}{3}-\frac{1}{6}=\frac{2}{3}$$

$$\therefore\ P(\bar{A}\cap\bar{B})=1-P(A\cup B)$$

$$=1-\frac{2}{3}=\frac{1}{3}$$

156 **(c)**

Consider the following events :

$A=$ Selecting a month of the year

$B=13^{th}$ day of a month is Friday

We have,

$P(A)=\frac{1}{12}$ and $P(B/A)=\frac{1}{7}$

∴ Required probability $=P(A\cap B)$

$$=P(A)P(B/A)=\frac{1}{12}\times\frac{1}{7}=\frac{1}{84}$$

157 **(d)**

Here, $n(S)=2^3=8$

and $E=\{HTH,HHT,THH,HHH\}$

and $n(E)=4$

∴ Required probability $=\frac{n(E)}{n(S)}=\frac{4}{8}=\frac{1}{2}$

158 **(b)**

There are 10 letters in the word “REGULATION”.

These 10 letters can be arranged in 10 ! ways

Exactly 4 letters can be put between R and E in $^8C_4\times 4\,!\times 2\,!\times 5\,!$ ways

Hence, required probability $=\frac{^8C_4\times 4\,!\times 2\,!\times 5\,!}{10\,!}=\frac{1}{9}$

159 **(d)**

Three possible cases are

Ist draw	IIst draw	IIIst draw
Red	Non Red	Red
Non Red	Red	Red
Non Red	Non Red	Red

∴ Required probability

$$=\frac{2}{8}\times\frac{6}{7}\times\frac{1}{6}+\frac{6}{8}\times\frac{2}{7}\times\frac{1}{6}+\frac{6}{8}\times\frac{5}{7}\times\frac{2}{6}$$

$$=\frac{14}{56}$$

161 **(c)**

Here, number of errors per page

$$p=\frac{150}{500}=\frac{1}{2}$$

and $n=2$

$$\therefore\ \lambda=np=2\times\frac{1}{2}=1$$

and probability of no error

$$P(X=0)=\frac{e^{-1}\times(1)^0}{0!}=e^{-1}$$

162 **(b)**

$S=\{BBB,BBG,BGB,GBB,GGB,GBG,BGG,GGG\}$

and $E=\{BBB,BBG,BGB,GBB,GGB,GBG,BGG\}$

$n(E)=7$ and $n(S)=8$

$$\therefore P(E)=\frac{n(E)}{n(S)}=\frac{7}{8}$$

163 **(c)**

P (getting a queen)= $\frac{4}{52}=\frac{1}{13}$

P (getting a heart) $=\frac{13}{52}=\frac{1}{4}$

P (getting a heart queen) $=\frac{1}{52}$

P (queen or heart) $=\frac{4}{52}+\frac{13}{52}-\frac{1}{52}=\frac{4}{13}$

164 **(d)**

Probability of getting score 9 in a single throw

$$=\frac{4}{36}=\frac{1}{9}$$

Probability of getting score 9 exactly in double throw

$$=\ ^3C_2\times\left(\frac{1}{9}\right)^2\times\frac{8}{9}=\frac{8}{243}$$

165 **(b)**

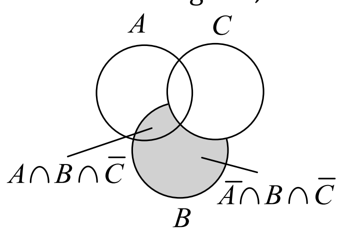
Three letters can be placed in three envelops in 3 ! ways.

There is only one way of putting all the letters in the correct envelops

Hence, required probability = 1/6

166 **(a)**

From Venn diagram, we can see that



$$P(B\cap C)=P(B)-P\left(A\cap B\cap\bar{C}\right)-P(\bar{A}\cap B\cap\bar{C})$$

$$=\frac{3}{4}-\frac{1}{3}-\frac{1}{3}=\frac{1}{12}$$

167 **(a)**

Required probability$=P(WR)+P(RW)+P(RR)$

$=\frac{6}{10}\times\frac{4}{9}+\frac{4}{10}\times\frac{6}{9}+\frac{6}{10}\times\frac{5}{9}$

$=\frac{24+24+30}{90}=\frac{78}{90}$

168 **(b)**

Since each entry (element) of a 2×2 matrix, with elements 0 and 1 only, can be filled in 2 ways.

Therefore, the total number of 2×2 matrices

$=2^4=16$

There are three 2×2 matrices whose determinants are positive viz. $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, \begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}$ and $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$

Hence, the required probability $=\frac{3}{16}$

169 **(d)**

Let $A=$ Event of getting even number on first die

$=\{(2,1)\ldots\ldots\ldots(2,6)$

$(4,1)\ldots\ldots\ldots(4,6)$

$(6,1)\ldots\ldots\ldots(6,6)\}$

and $B=$ Events of getting a sum of 8

$=\{(2,6),(3,5),(4,4),(5,3),(6,2)\}$

$A\cap B=\{(2,6),(4,4),(6,2)\}$

$\therefore\ n(A)=18, n(B)=5, n(A\cap B)=3$

$\Rightarrow\ n(A\cup B)=18+5-3=20$

$\therefore\ P(A\cup B)=\frac{n(A\cup B)}{n(S)}=\frac{20}{36}=\frac{5}{9}$

170 **(d)**

Total number of cases$=4\times 5\times 5=100$

Favourable cases are 222, 444, 666, 888

Number of favourable cases= 4

$\therefore$ Required probability$=\frac{4}{100}=\frac{1}{25}$

171 **(c)**

Let W denotes the events of drawing a white ball at any draw and B that for a black ball.

Then $P(W)=\frac{a}{a+b}, P(B)=\frac{b}{a+b}$

$P(A$ wins the game$)=P(W$ or $BBBBW$ or $\ldots.)$

$=P(W)+P(B)P(B)P(W)$

$+P(B)P(B)P(B)P(B)P(W)+.\ldots$

$=\frac{P(W)}{1-P(B)^2}=\frac{\frac{a}{a+b}}{1-\frac{b^2}{(a+b)^2}}\frac{a(a+b)}{a^2+2ab}=\frac{(a+b)}{a+2b}$

Also $P(B$ wins the game$)=1-\frac{a+b}{a+2b}=\frac{b}{a+2b}$

According to the given condition,

$\frac{a+b}{a+2b}=3\cdot\frac{b}{a+2b}\ \Rightarrow a=2b\ \Rightarrow a:b=2:1$

172 **(a)**

A leap year consists of 366 days *ie*, 52 full weeks and two extra days. These two extra days can be any one of the following possible outcomes; (i) Monday and Tuesday, (ii) Tuesday and Wednesday, (iii) Wednesday and Thursday, (iv) Thursday and Friday, (v) Friday and Saturday, (vi) Saturday and Sunday, (vii) Sunday and Monday.

Let A and B be the events that a leap year contains 53 Thursdays and 53 Fridays respectively. Then,

$P(A)=\frac{2}{7}, P(B)=\frac{2}{7}$ and $P(A\cap B)=\frac{1}{7}$

$\therefore$ Required probability is given by

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=\frac{2}{7}+\frac{2}{7}-\frac{1}{7}=\frac{3}{7}$

173 **(a)**

We know that, P (exactly one of A or B occurs)

$=P(A)+P(B)-2P(A\cap B)$

Therefore, $P(A)+P(B)-2P(A\cap B)=p$ ...(i)

Similarly, $P(B)+P(C)-2P(B\cap C)=p$ ...(ii)

and $P(C)+P(A)-2P(C\cap A)=p$ ....(iii)

Adding Eqs. (i), (ii) and (iii), we get

$2[P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)$

$-P(C\cap A)]=3P$

$\Rightarrow P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)=\frac{3P}{2}$ ....(iv)

We are also given that

$P(A\cap B\cap C)=p^2$ ...(v)

Now, P (at least one of A, B and C)

$=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)$

[from Eqs. (iv) and (v)]

$=\frac{3p}{2}+p^2$

$=\frac{3p+p^2}{2}$

174 **(c)**

Given, ASSISTANT→AA I N SSS TT→

STATISTICS→AC II SSS TTT

Hence, N and C are not common

Same letters can be A, I, S, T

Probability of choosing 'A'

$\frac{^2C_1}{^9C_1}\times\frac{^1C_1}{^{10}C_1}=\frac{2}{9}\times\frac{1}{10}=\frac{1}{45}$

Probability of choosing '*I*'

$= \frac{1}{^9C_1} \times \frac{^2C_1}{^{10}C_1} = \frac{1}{9} \times \frac{2}{10} = \frac{1}{45}$

Probability of choosing 'S'

$= \frac{^3C_1}{^9C_1} \times \frac{^3C_1}{^{10}C_1} = \frac{3}{9} \times \frac{3}{10} = \frac{1}{10}$

Probability of choosing 'T'

$= \frac{^2C_1}{^9C_1} \times \frac{^3C_1}{^{10}C_1} = \frac{2}{9} \times \frac{3}{10} = \frac{1}{15}$

Hence, required probability

$= \frac{1}{45} + \frac{1}{45} + \frac{1}{10} + \frac{1}{15} = \frac{19}{90}$

175 **(b)**

First we fix the position of a girl between one place. Girls can be arranged in 3! ways.

There are four alternate places to be left.

Therefore four boys occupy the place in 4! ways.

$\therefore$ The favourable cases $= 4! \times 3!$

Hence, the required probability

$= \frac{4! \times 3!}{7!} = \frac{6}{7 \times 6 \times 5} = \frac{1}{35}$

176 **(a)**

Required probability$= 1 - \left(\frac{1}{2}\right)^4 = \frac{15}{16}$

177 **(a)**

A coin is tossed n times.

$\therefore$ Total number of ways $= 2^n$

If head comes odd times, then favourable ways $= 2^{n-1}$.

$\therefore$ Required probability of getting odd times head

$= \frac{2^{n-1}}{2^n} = \frac{1}{2}$

178 **(a)**

Out of numbers 1,2,3, … ,100 one number can be chosen in $^{100}C_1$ ways.

Clearly, 1,8,27,64 are cubes between 1 and 100

$\therefore$ Required probability $= \frac{4}{100} = \frac{1}{25}$

179 **(d)**

We have, $1 + 2 + 3 + \cdots + 9 = 45$.

Therefore, a seven digit number formed will be divisible by 9, if the two digits which are not used are

1,8 or 2,7 or 3,6 or 4,5

$\therefore$ Favourable number of ways $= 4$

Since two digits out of 9 can be left in $^9C_2$ ways

$\therefore$ Total number of ways $= {}^9C_2$

Thus, the probability of the required event

$= \frac{4}{^9C_2} = \frac{1}{9}$

180 **(b)**

Required probability $= \frac{^5C_1}{^{10}C_1} = \frac{5}{10} = \frac{1}{2}$

181 **(c)**

Required probability

$= 1 - P(\text{not opened the lock in } n \text{ trials})$

$= 1 - \left(\frac{n-1}{n}\right)^n$

182 **(b)**

Given, $P(\overline{A \cup B}) = \frac{1}{6}$

$\Rightarrow 1 - P(A \cup B) = \frac{1}{6}$

$\Rightarrow 1 - P(A) - P(B) + P(A \cap B = \frac{1}{6}$

$\Rightarrow P(\bar{A}) - P(B) + P(A \cap B) = \frac{1}{6}$

$\Rightarrow \frac{1}{4} - P(B) + \frac{1}{4} = \frac{1}{6}$

$\Rightarrow P(B) = \frac{1}{3}$

and $P(A) = 1 - P(\bar{A}) = 1 - \frac{1}{4} = \frac{3}{4}$

Since, $P(A \cap B) = P(A).P(B)$ so the events A and B are independent events but not equally likely

183 **(b)**

Here, A and B are independent events

$\therefore\ P(A \cap B) = P(A).P(B) = 0.06$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.2 + 0.3 - 0.06 = 0.44$

184 **(a)**

According to the condition $1 - \left(\frac{3}{4}\right)^n \geq \frac{9}{10}$

$\Rightarrow \left(\frac{3}{4}\right)^n \leq 1 - \frac{9}{10} = \frac{1}{10}$

$\Rightarrow \left(\frac{4}{3}\right)^n \geq 10$

$\Rightarrow\ n[\log_{10} 4 - \log_{10} 3] \geq \log_{10} 10 = 1$

$\Rightarrow\ n \geq \frac{1}{\log_{10} 4 - \log_{10} 3}$

185 **(b)**

Let E be the event of getting 1 on a die

$\Rightarrow P(E) = \frac{1}{6}$ and $P(\bar{E}) = \frac{5}{6}$

$\therefore P(\text{first time 1 occurs at the even throw})$

$= t_2$ or t_4 or t_6 or t_8 … and so on.

$= \{P(\bar{E}_1).P(E_2)\}$
$+ \{P(\bar{E}_1)P(\bar{E}_2)P(\overline{E_3})P(E_4)\} + \ldots\infty$
$= \left(\frac{5}{6}.\frac{1}{6}\right) + \left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right) + \ldots\infty$
$= \frac{\frac{5}{36}}{1 - \frac{25}{36}} = \frac{5}{11}$

187 **(c)**
Required probability
$= P(I$st is red and IInd is blue)
$= P$(Ist is blue and IInd is also blue)
$= \frac{3}{8} \times \frac{5}{7} + \frac{5}{8} \times \frac{4}{7} = \frac{5}{8}$

189 **(c)**
Favourable ways,
$= {}^5C_3 \times^6 C_4 = {}^5C_2 \times {}^6C_2$
and total number of ways= ${}^{11}C_7$
∴Required probability= $\frac{{}^5C_2 \times {}^6C_2}{{}^{11}C_7}$

190 **(c)**
The roots of the equation $x^2 + px + \frac{p}{4} + \frac{1}{2} = 0$ are real, if
$p^2 - 4\left(\frac{p}{4} + \frac{1}{2}\right) \geq 0$
$\Rightarrow p^2 - p - 2 \geq 0 \Rightarrow p \geq 2$ or $p \leq -1 \Rightarrow 2 \leq p \leq 5$ [$\because 0 \leq p \leq 5$]
Hence, required probability $= \frac{\int_2^5}{\int_0^5 dp} = \frac{5-2}{5-0} = \frac{3}{5}$

191 **(c)**
Probability of getting a king $= \frac{4}{52} = \frac{1}{13}$
Probability of getting a spade $= \frac{13}{52} = \frac{1}{4}$
Probability of getting king and a spade $= \frac{1}{52}$
∴ P(king or spade)= P(king)+P(spade)−P(king and spade)
$= \frac{1}{13} + \frac{1}{4} - \frac{1}{52} = \frac{4 + 13 - 1}{52} = \frac{16}{52} = \frac{4}{13}$

192 **(b)**
$P(\overline{A}) = 0.2, P(\overline{B}) = 0.3$
$P(A) = 0.8, P(B) = 0.7$
∴ Required probability
$= P(\overline{A})P(B) + P(A)P(\overline{B}) + P(\overline{A})P(\overline{B})$
$= 0.2 \times 0.7 + 0.8 \times 0.3 + 0.2 \times 0.3$
$= 0.44$

193 **(d)**
Given, $np = 4, npq = 2$

$\Rightarrow p = q = \frac{1}{2}$

Also, $n = 8$

∴ Probability of 2 successes,

$P(X = 2) = \frac{8!}{2! \times 6!} \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^6 = \frac{28}{256}$

194 **(a)**
We have, $np = 25$
Now,
$0 \leq p < 1$ and $0 \leq q \leq 1$
$\Rightarrow 0 \leq n\,pq \leq np$
$\Rightarrow 0 \leq \sqrt{npq} \leq \sqrt{np}$
$\Rightarrow 0 \leq \text{S.D.} \leq 5$
But, $p \neq 0$, therefore $0 \leq \text{S.D.} < 5 \Rightarrow S.D. \in [0, 5)$

195 **(c)**
Given,$P(A) = 0.3, P(B) = 0.4, P(C) = 0.8,$

$P(A \cap B) = 0.08, P(A \cap C) = 0.28,$

$P(A \cap B \cap C) = 0.09$

Since, $P(A \cup B \cup C) \geq 0.75$

$\Rightarrow P(A) + P(B) + P(C) - P(A \cap C) - P(A \cap C)$

$-P(B \cap C) + P(A \cap B \cap C) \geq 0.75$

$\Rightarrow 0.3 + 0.4 + 0.8 - 0.08 - 0.28 - P(B \cap C)$
$+ 0.09 \geq 0.75$

$\Rightarrow P(B \cap C) \leq 0.48$

Also, $P(A \cup B \cup C) \leq 1$

$\Rightarrow 1.23 - P(B \cap C) \leq 1$

$\Rightarrow P(B \cap C) \geq 0.23$

$\therefore 0.23 \leq P(B \cap C) \leq 0.48$

196 **(c)**
Total number of ways =16
The favourable ways are
$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$
∴ Required probability $= \frac{3}{16}$

197 **(c)**
Given, distribution is

X	0	1	2	3
$\boldsymbol{P(X)}$	$\frac{1}{3}$	$\frac{1}{2}$	0	$\frac{1}{6}$

∴ Mean, $m = \sum_{i=1}^{4} p_i x_i$

$= 0 \times \frac{1}{3} + 1 \times \frac{1}{2} + 2 \times 0 + 3 \times \frac{1}{6}$

$= 0 + \frac{1}{2} + 0 + \frac{1}{2} = 1$

Variance, $\sigma^2 = \sum_{i=1}^{4} p_i(x_1 - m)^2$

$= \frac{1}{3}(0-1)^2 + \frac{1}{2}(1-1)^2 + 0(2-1)^1 + \frac{1}{6}(3-1)^2$

$= \frac{1}{3} + 0 + 0 + \frac{2}{3} = 1$

$\therefore\ m = \sigma^2 = 1$

198 **(a)**

Probability of getting a head $= \frac{1}{2}$

$ie, p = \frac{1}{2} \Rightarrow q = \frac{1}{2}$

$\therefore$ Required probability $= {}^{10}C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^5$

$= \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2} \left(\frac{1}{2}\right)^{10} = \frac{63}{256}$

199 **(b)**

For X binomial variate $B\left(5, \frac{1}{2}\right)$

$\Rightarrow p = \frac{1}{2}, n = 5, q = \frac{1}{2}$

For Y binomial variate $B\left(7, \frac{1}{2}\right)$

$\Rightarrow p = \frac{1}{2}, n = 7, q = \frac{1}{2}$

Now, $X + Y = 3$

(i) When $X = 0, Y = 3$, possible cases

$= {}^5C_0 \left(\frac{1}{2}\right)^5 . {}^7C_3 \left(\frac{1}{2}\right)^7$

$= 35 \left(\frac{1}{2}\right)^{12}$

(ii) When $X = 1, Y = 2$, possible cases

$= {}^5C_1 \left(\frac{1}{2}\right)^5 . {}^7C_2 \left(\frac{1}{2}\right)^7$

$= 105 \left(\frac{1}{2}\right)^{12}$

(iii) When $X = 2, Y = 1$, possible cases

$= {}^5C_2 \left(\frac{1}{2}\right)^5 . {}^7C_1 \left(\frac{1}{2}\right)^7$

$= 70 \left(\frac{1}{2}\right)^{12}$

(iv) When $X = 3, Y = 0$, possible cases

$= {}^5C_3 \left(\frac{1}{2}\right)^3 . {}^7C_0 \left(\frac{1}{2}\right)^7$

$= 10 \left(\frac{1}{2}\right)^{12}$

$\therefore$ Total cases$= \left(\frac{1}{2}\right)^{12} [35 + 105 + 70 + 10]$

$= \frac{220}{2^{12}} = \frac{55}{1024}$

201 **(a)**

Since the given distribution is a probability distribution

$\therefore 0 + 2p + 2p + 3p + p^2 + 2p^2 + 7p^2 + 2p = 1$

$\Rightarrow 10p^2 + 9p - 1 = 0 \Rightarrow (10p - 1)(p + 1) = 0$

$\Rightarrow p = 1/10$

202 **(a)**

We have,

$P(E \cap F) = \frac{1}{12}$ and $P(\bar{E} \cap \bar{F}) = \frac{1}{2}$

$\Rightarrow P(E)P(F) = \frac{1}{12}$ and $P(\bar{E})P(\bar{F}) = \frac{1}{2}$

[$\because E$ and F are independent events]

$\Rightarrow P(E)P(F) = \frac{1}{12}$ and $\{1 - P(E)\}\{1 - P(F)\} = \frac{1}{2}$

$\Rightarrow P(E)P(F) = \frac{1}{12}$ and $1 - \{P(E) + P(F)\} + P(E)P(F) = \frac{1}{2}$

$\Rightarrow P(E)P(F) = \frac{1}{12}$ and $1 - \{P(E) + P(F)\} + \frac{1}{12} = \frac{1}{2}$

$\Rightarrow P(E)P(F) = \frac{1}{12}$ and $P(E) + P(F) = \frac{7}{12}$

The quadratic equation having $P(E)$ and $P(F)$ as its roots is

$x^2 - \{P(E) + P(F)\}x + P(E)P(F) = 0$

$\Rightarrow x^2 - \frac{7}{12}x + \frac{1}{12} = 0 \Rightarrow x = \frac{1}{3}, \frac{1}{4}$

$\therefore P(E) = \frac{1}{3}$ and $P(F) = \frac{1}{4}$ or, $P(E) = \frac{1}{4}$ and $P(F) = \frac{1}{3}$

203 **(a)**

We have,

$p =$ Probability of getting at least 3 in a throw

$= \frac{4}{6} = \frac{2}{3}$

$\therefore q = 1 - p = \frac{1}{3}$

Required probability

$= {}^6C_3\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^3 + {}^6C_4\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2 + {}^6C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right) + {}^6C_6\left(\frac{2}{3}\right)^6$

$= 41 \times \frac{2^4}{3^6}$

204 **(c)**

Given $P(A \cup B) = 0.6, P(A \cap B) = 0.3$

$\therefore P(A') + P(B')$

$= 1 - P(A) + 1 - P(B) = 2 - \{P(A) + P(B)\}$

$= 2 - \{P(A \cup B) + P(A \cap B)\}$

$= 2 - \{0.6 + 0.3\} = 2 - 0.9 = 1.1$

206 **(c)**

Given, $P(A) = \frac{1}{3}, P(B) = \frac{1}{3}$ and $P(C) = \frac{1}{4}$

$\therefore P(A') = \frac{2}{3}, P(B') = \frac{2}{3}$ and $P(C') = \frac{3}{4}$

Now, $P(A' \cap B' \cap C') = P(A')P(B')P(C')$

[$\because$ A, B and C are independent events]

$= \frac{2}{3} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{3}$

207 **(c)**

Total number of elementary events $= 6^3 = 216$

Favourable number of elementary events

$=$ Coeff. of x^{15} in $(x^1 + x^2 + x^3 + \cdots + x^6)^3$

$=$ Coeff. of x^{15} in $x^3\left(\frac{1-x^6}{1-x}\right)^3$

$=$ Coeff. of x^{12} in $(1 - 3x^6 + 3x^{12} - x^{18})(1-x)^{-3}$

$=$ Coeff. of x^{12} in $(1-x)^{-3} - 3$ Coeff. of x^6 in $(-x)^{-3}$ $+ 3$ Coeff. of x^0 in $(1-x)^{-3}$

$= {}^{12+3-1}C_{3-1} - 3 \times {}^{6+3-1}C_{3-1} + 3 = {}^{14}C_2 - 3$

$= {}^{14}C_2 - 3 \times {}^8C_2 + 3 = 91 - 84 + 3 = 10$

So, required probability $= \frac{10}{216} = \frac{5}{108}$

208 **(d)**

For a Poisson distribution, mean = variance

$\Rightarrow$ Variance = 16

$\therefore$ Standard deviation= $\sqrt{\text{Variance}}$

$= \sqrt{16} = 4$

209 **(a)**

The sum of two numbered on a dice is odd only, whence once is odd and second is even.

$\therefore$ Required probability

$= 2\ \times$probability of odd number $\times$probability of even number

[$\because$Here, we multiply by 2 because either the even number is on first or second dice.]

$= 2 \times \left(\frac{5}{6}\right) \times \left(\frac{1}{6}\right) = \frac{5}{18}$

210 **(b)**

In binomial distribution, variance $= npq$ and mean $= np$.

From the given condition

$npq = 3$ and $np = 4$

$\therefore \frac{npq}{np} = \frac{3}{4}$

$\Rightarrow q = \frac{3}{4}, p = \frac{1}{4}$ and $n = 16$

Probability of exactly six success

$= {}^{16}C_6\left(\frac{1}{4}\right)^6\left(\frac{3}{4}\right)^{10}$

211 **(a)**

Since A and B are independent events

$\therefore P(A \cap B) = \frac{1}{6}$ and $P(\bar{A} \cap \bar{B}) = \frac{1}{3}$

$\Rightarrow P(A)P(B) = \frac{1}{6}$ and $P(\bar{A})P(\bar{B}) = \frac{1}{3}$

$\Rightarrow P(A)P(B) = \frac{1}{6}$ and $\{(1 - P(A)\}\{(1 - P(B)\} = \frac{1}{3}$

$\Rightarrow 1 - [P(A) + P(B)] + \frac{1}{6} = \frac{1}{3}$

$\Rightarrow P(A) + P(B) = \frac{5}{6}$

Solving $P(A)\ P(B) = \frac{1}{6}$ and $P(A) + P(B) = \frac{5}{6}$, we get

$P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$ or, $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{2}$

Hence, option (a) is correct

212 **(b)**

Since, A and B are independent events

$\therefore P(A)P(B) = \frac{1}{6}$ and $P(\bar{A})P(\bar{B}) = \frac{1}{3}$

$\Rightarrow [1 - P(A)][1 - P(B)] = \frac{1}{3}$

$\Rightarrow 1 - [P(A) + P(B)] + P(A)P(B) = \frac{1}{3}$

$\Rightarrow 1 + \frac{1}{6} - \frac{1}{3} = P(A) + P(B)$

$\Rightarrow P(A) + P(B) = \frac{5}{6}$

$\Rightarrow P(A) = \frac{1}{2}, P(B) = \frac{1}{3},$

or $P(A) = \frac{1}{3}, P(B) = \frac{1}{2}$

213 **(a)**

$7^1 = 7, 7^2 = 49, 7^3 = 343, 7^4 = 2401, ...$

Therefore, for $7^r, r \in N$ the no. ends at unit place 7, 9, 3, 1, 7,..... .

$\therefore 7^m + 7^n$ will be divisible by 5, if it end at 5 or 0.

But it cannot end at 5.

Also it cannot end at 0.

For this m and n should be as follows :

	m	n
1	$4r$	$4r+2$
2	$4r+1$	$4r+3$
3	$4r+2$	$4r$
4	$4r+3$	$4r+1$

For any given value of m, there will be 25 values of n. Hence, the probability of the required event is $\frac{100\times25}{100\times100} = \frac{1}{4}$.

214 **(c)**

A dice is thrown thrice, $n(S) = 6 \times 6 \times 6$

Favorable events of $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$

ie, (r_1, r_2, r_3) are ordered triplets which can take values,

$\left.\begin{matrix}(1,2,3),(1,5,3),(4,2,3),(4,5,3)\\(1,2,6),(1,5,6),(4,2,6),(4,5,6)\end{matrix}\right\}$

ie, 8 ordered triplets and each can be arranged in 3! ways = 6

$\therefore n(E) = 8 \times 6$

$\Rightarrow P(E) = \frac{8\times6}{6\times6\times6}$

$= \frac{2}{9}$

215 **(c)**

We have,

Total number of functions from A to itself $= n^n$

Out of these functions, $n!$ Function are injections

So, required probability $= \frac{n!}{n^n} = \frac{(n-1)!}{n^{n-1}}$

216 **(d)**

Let $A_i (i = 1,2,3,4)$ be the event that the urn contains 2,3,4 or 5 white balls and E the event that two white balls are drawn. Since the four events A_1, A_2, A_3, A_4 are equally likely. Therefore, $P(A_i) = \frac{1}{4}$, i = 1,2,3,4

We have,

$P(E/A_1)$ = Prob. that the urn contains 2 white balls and both have been drawn

$\Rightarrow P(E/A_1) = \frac{{}^2C_2}{{}^5C_2} = \frac{1}{10}$

Similarly, we have

$P(E/A_2) = \frac{{}^3C_2}{{}^5C_2} = \frac{3}{10}, P(E/A_3) = \frac{{}^4C_2}{{}^5C_2}$

$= \frac{3}{5}, P(E/A_4) = \frac{{}^5C_2}{{}^5C_2} = 1$

Required probability $= P(A_4/E) = \frac{P(A_4)P(E/A_4)}{\sum_{i=1}^{4} P(A_i)P(E/A_i)}$

$= \frac{\frac{1}{4}\times1}{\frac{1}{4}\left(\frac{1}{10}+\frac{3}{10}+\frac{3}{5}+1\right)} = \frac{1}{2}$

217 **(b)**

There are 11 letters in word 'PROBABILITY' out of which 1 can be selected in ${}^{11}C_1$ ways.

There are four vowels viz. A, I, O. Therefore,

Number of ways of selecting a vowel $= {}^4C_1 = 4$

Hence, required probability $= \frac{4}{11}$

218 **(b)**

If the show a six, then number of outcomes =8

If die not show a six. Then number of outcomes=2

$\therefore$ Sample space $= 1 \times 8 + 2 \times 5 = 18$ points

219 **(c)**

Given, $n = 6$ and

$P(X = 2) = 9P(X = 4)$

$\Rightarrow {}^6C_2 p^2 q^4 = 9.{}^6C_4 p^4 q^2$

$\Rightarrow 9p^2 = q^2$

$\Rightarrow P = \frac{1}{3}q$

$\therefore$ We know that $p + q = 1$

$\Rightarrow \frac{q}{3} + q = 1$

$\Rightarrow q = \frac{3}{4}$ and $p = \frac{1}{4}$

$\therefore$ Variance $= npq$

$= 6.\frac{1}{4}.\frac{3}{4} = \frac{9}{8}$

220 **(a)**

Required probibility $= \frac{{}^5C_1 \times {}^8C_1}{{}^{13}C_2} + \frac{{}^5C_2}{{}^{13}C_2} = \frac{25}{39}$

221 **(a)**

$P(A \cup B') = P(A) + P(B') - P(A)P(B')$

$\therefore 0.8 = 0.3 + P(B') - 0.3P(B')$

$\Rightarrow 0.5 = P(B')(0.7)$

$\Rightarrow P(B') = \frac{5}{7}$

$\therefore P(B) = 1 - \frac{5}{7} = \frac{2}{7}$

222 **(a)**

Required probability $= \frac{3}{6} = \frac{1}{2}$

223 **(c)**

There are two equilateral triangles in a regular hexagon

$\therefore$ Required probability$= \frac{2}{20} = \frac{1}{10}$

224 **(c)**

From the given condition it is clear that a particular person is always in a committee of 3 persons. It means we have to select 2 person out of 37 persons.

$\therefore$ Required probability $= \frac{{}^{37}C_2}{{}^{38}C_3}$

225 **(d)**

We know a leap year is fallen within 4 yr, so its probability $= \frac{25}{100} = \frac{1}{4}$.

In a century the probability of 53rd Sunday in a leap year $= \frac{1}{4} \times \frac{2}{7} = \frac{2}{28}$

Non-leap year in century = 75

Probability of selecting is non-leap year $= \frac{75}{100} = \frac{3}{4}$

53rd Sunday in non-leap year $= \frac{1}{7}$

Similarly, in a century probabilities of 53rd Sunday in a non-leap year

$= \frac{3}{4} \times \frac{1}{7} = \frac{3}{28}$

$\therefore$ Required probability $= \frac{2}{28} + \frac{3}{28} = \frac{5}{28}$

226 **(b)**

There are two conditions arise.

(i) When first is an ace of heart and second one is non-ace of heart, the probability $= \frac{1}{52} \times \frac{51}{51} = \frac{1}{52}$

(ii) When first is non-ace of heart and second one is an ace of heart, the probability $= \frac{51}{52} \times \frac{1}{51} = \frac{1}{52}$

$\therefore$ Required probability $= \frac{1}{52} + \frac{1}{52} = \frac{1}{26}$

227 **(c)**

We have,

Total number of binary operations on $A = n^{n^2}$

Total number of commutative binary operations on A

$= n^{\frac{n(n+1)}{2}}$

$\therefore$ Required probability $= \frac{n^{\frac{n(n+1)}{2}}}{n^{n^2}} = \frac{n^{n/2}}{n^{n^2/2}}$

228 **(a)**

Required probability $= \frac{{}^{12}C_1}{{}^{20}C_1} = \frac{3}{5}$

229 **(b)**

$P(A') = 1 - P(A) = 0.8, P(A' \cap B)$ will maximum, if $B \subseteq A'$ in which case $A' \cap B = B$. So,

$P(A' \cap B) = P(B) = 0.5$

230 **(a)**

The total number of ways in which 4 tickets can be drawn 5 times $= 4^5 = 1024$

The number of ways of getting a sum of 23

$= \text{Coeff. of } x^{23} \text{ in } (x^{00} + x^{01} + x^{10} + x^{11})^5$

$= \text{Coeff. of } x^{23} \text{ in } [(1 + x)(1 + x^{10})]^5$

$= \text{Coeff. of } x^{23} \text{ in } (1 + x)^5(1 + x^{10})^5$

$= \text{Coeff. of } x^{23} \text{ in } \{(1 + 5\,x + 10\,x^2 + 10\,x^3 + 5\,x^4 + x^5)\,(1 + 5\,x^{10} + 10\,x^{20} + 10\,x^{30} + \cdots)\}$

$= 100$

Hence, required probability $= \frac{100}{1024} = \frac{25}{256}$

231 **(c)**

Required probability $= {}^6C_4 \left(\frac{1}{4}\right)^4 \left(\frac{5}{6}\right)^2 = \frac{125}{15552}$

232 **(c)**

In $3n$ conseecutive natural numbers, either

(i) n numbers are of from $3P$

(ii) n numbers are of from $3P + 1$

$(iii) n$ numbers are of from $3P + 2$

Here favourable number of cases= Either we can select three numbers from any of the set or we can select one from each set

$= {}^nC_3 + {}^nC_3 + {}^nC_3 + ({}^nC_1 \times {}^nC_1 \times {}^nC_1)$

$= 3\left(\frac{n(n-1)(n-2)}{6}\right) + n^3$

$= \frac{n(n-1(n-2)}{2} + n^3$

Total number of selections $= {}^{3n}C_3$

$\therefore$Required probability

$= \frac{\frac{n(n-1)(n-2)}{2} + n^3}{\frac{3n(3n-1)(3n-2)}{6}}$

$= \frac{3n^2 - 3n + 2}{(3n-1)(3n-2)}$

233 **(d)**

A and B will agree in a certain statement if both speak truth or both tell a lie. We define following events

$E_1 = A$ and B both speak truth $\Rightarrow P(E_1) = xy$

$E_2 = A$ and B both tell a lie $\Rightarrow P(E_2) = (1 - x)(1 - y)$

$E = A$ and B agree in a certain statement

Clearly, $P(E|E_1) = 1$ and $P(E|E_2) = 1$

The required probability is $P(E_1|E)$

Using Bayes' theorem

$$P(E_1|E) = \frac{P(E_1)P(E|E_1)}{P(E_1)P(E|E_1) + P(E_2)P(E|E_2)}$$

$$= \frac{xy.1}{xy.1 + (1-x)(1-y).1} = \frac{xy}{1-x-y+2xy}$$

234 **(c)**

$\because$ Total number of ways $= 5!$

and favourable number of ways $= 2 \cdot 4!$

Hence, required probability $= \frac{2 \cdot 4!}{5!} = \frac{2}{5}$

235 **(c)**

Three dice can be thrown in $6^3 = 216$ ways.

The same number can appear on three dice in the following ways :

$(1,1,1), (2,2,2), (3,3,3), (4,4,4), (5,5,5), (6,6,6)$

$\therefore$ Favourable number of elementary events $= 6$

Hence, required probability $= \frac{6}{216} = \frac{1}{36}$

236 **(c)**

Probability that both are of red colours$= \frac{^8C_2}{^{15}C_2} = \frac{4}{15}$

And probability that both are of black colours

$= \frac{^7C_2}{^{13}C_2} = \frac{3}{15}$

$\therefore$Probability that they are of same colour

$= \frac{4}{15} + \frac{3}{15} = \frac{7}{15}$

237 **(b)**

Consider the following events :

$A =$ Getting 2 black balls and 4 white in first 6 draws

$B =$ Getting a black ball in 7th draw

Required probability $= P(A \cap B) = P(A)P(B/A)$

$\Rightarrow$ Required probability $= \frac{^3C_2 \times {^{10}C_4}}{^{13}C_6} \times \frac{1}{7} = \frac{15}{286}$

238 **(a)**

$P(A) = 0.25, P(B) = 0.50, P(A \cap B) = 0.14$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.25 + 0.50 - 0.14 = 0.61$

$\therefore P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B})$=1$-$ $P(A \cup B)$

$= 1 - 0.61 = 0.39$

239 **(a)**

Total number of cases $= 9999$

Favourable cases $= 10 \times 9 \times 8 \times 7 = 5040$

$\therefore$ Probability $= \frac{5040}{9999}$

240 **(d)**

Given, $x^2 + 4x + c = 0$

For real roots, $D = b^2 - 4ac \geq 0$

$= 16 - 4c \geq 0$

$\Rightarrow c = 1, 2, 3, 4$ will satisfy the above inequality

$\therefore$ Required probability $= \frac{4}{9}$

241 **(a)**

The required probability is given by

$P\{(A \cap \bar{B}) \cup (\bar{A} \cap B)\}$

$= P(A \cap \bar{B}) + P(\bar{A} \cap B)$

$\begin{bmatrix}\text{By add. Theorem for mutually} \\ \text{exclusive events}\end{bmatrix}$

$= P(A)P(\bar{B}) + P(\bar{A})P(B)$

[$\because$ A, B are independent events]

$= P(A)(1 - P(B) + (1 - (A))P(B)$

$= P(A) + P(B) - 2\,P(A)P(B)$

$= P(A) + P(B) - 2\,P(A \cap B)$

242 **(c)**

Since, $P(A \cap B) = P(A)P(B)$

$\Rightarrow$ A and B are independent events

$\Rightarrow$ $A^c$and B^cwill also independent events

Hence, $P(A \cup B)^c = P(A^c \cap B^c)$

$= P(A^c)P(B^c)$

243 **(a)**

Sum of Probabilities=1

$\Rightarrow\; p + 2p + 3p + 4p + 5p + 7p + 8p + 9p$

$+10p + 11p + 12p = 1$

$\Rightarrow\; 72p = 1\; \Rightarrow p = \frac{1}{72}$

244 **(b)**

Let the total number of students be 100, then in which 60 girls and 40 boys

As 25% of boys offer Mathematics $= \frac{25}{100} \times 40$

$= 10$ boys

and 10% of girls offer Mathematics $= \frac{10}{100} \times 60$

$= 6$ girls

$\therefore$ Total number of students, whose offers Mathematics is 16

$\therefore$ Required probability $= \frac{6}{16} = \frac{3}{8}$

245 **(a)**

Let $q = 1 - p$. Since, head appears first time in an even throw 2 or 4 or 6

$\therefore\; \frac{2}{5} = qp + q^3p + q^5p + \ldots$

$\therefore\; \frac{2}{5} = \frac{qp}{1-q^2}$

$$\Rightarrow \frac{2}{5} = \frac{(1-p)p}{1-(1-p)^2}$$
$$\Rightarrow \frac{2}{5} = \frac{1-p}{2-p}$$
$$\Rightarrow 4 - 2P = 5 - 5P \Rightarrow p = \frac{1}{3}$$

246 **(a)**

We have,

$P[(E_1 \cup E_2) \cap (\bar{E}_1) \cap (\bar{E}_2)]$

$= P[(E_1 \cup E_2) \cap (\bar{E}_1 \cap \bar{E}_2)]$

$= P[(E_1 \cup E_2) \cap (\overline{E_1 \cup E_2})] = P(\phi) = 0 \leq 1/4$

247 **(b)**

Given, $P(A \cap B) = \frac{1}{6}$

$\Rightarrow P(A)P(B) = \frac{1}{6}$ ...(i)

and $P(\bar{A} \cap \bar{B}) = \frac{1}{3}$

$$\Rightarrow P(\bar{A})P(\bar{B}) = \frac{1}{3}$$

$$\Rightarrow \{1 - P(A)\}\{1 - P(B)\} = \frac{1}{3}$$

$$\Rightarrow 1 - \frac{1}{3} + P(A)P(B) = P(A) + P(B)$$

$\Rightarrow \frac{2}{3} + \frac{1}{6} = P(A) + P(B)$ [from Eq.(i)]

$\Rightarrow P(A) + P(B) = \frac{5}{6}$ ...(ii)

On solving Eqs. (i) and (ii), we get

$$P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$$

or $P(A) = \frac{1}{3}, P(B) = \frac{1}{2}$

249 **(a)**

We know, total probability distribution is 1.

$$\therefore \frac{1}{10} + k + \frac{1}{5} + 2k + \frac{3}{10} + k = 1$$

$$\Rightarrow \frac{6}{10} + 4k = 1$$

$$\Rightarrow k = \frac{1}{10}$$

250 **(d)**

We have, $p = 3/4$ and $n = 5$

$\therefore$ Required probability

$$= {}^5C_3\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2 + {}^5C_4\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right) + {}^5C_5\left(\frac{3}{4}\right)^5$$
$$= \frac{459}{512}$$

251 **(c)**

One red card one queen can be drawn in the following mutually exclusive ways:

(I) By drawing one red card out of 24 red cards (excluding 2 red queens) and one red queen out of 2 red queens. Let this event be A

(II) By drawing one red card out of 26 red cards (including 2 red queens) and one queen out of 2 black queens. Let B

$\therefore$ Required probability $= P(A \cup B) = P(A) + P(B)$

$$= \frac{{}^{24}C_1 \times {}^2C_1}{{}^{52}C_2} + \frac{{}^{26}C_1 \times {}^2C_1}{{}^{52}C_2} = \frac{50}{663}$$

252 **(a)**

Given, $4P(A) = 6P(B) = 10P(A \cap B) = 1$

$$\therefore P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{10}}{\frac{1}{4}} = \frac{2}{5}$$

253 **(c)**

Given, $np = 4, npq = 3V \Rightarrow p = \frac{1}{4}, q = \frac{3}{4}$

Mode is an integer x such that

$$\Rightarrow 4 + \frac{1}{4} > x > 4 - \frac{3}{4}$$

$\Rightarrow 3.25 < x < 4.25$

$\therefore x = 4$

254 **(c)**

$$P\left(\frac{B}{A \cup B^C}\right) = \frac{P(B \cap (A \cup B^c))}{P(A \cup B^c)}$$

$$= \frac{P(A \cap B)}{P(A) + P(B^c) - P(A \cap B^c)}$$

$$= \frac{P(A) - P(A \cap B^c)}{P(A) + P(B^c) - P(A \cap B^c)}$$

$$= \frac{0.7 - 0.5}{0.8} = \frac{1}{4}$$

255 **(b)**

The number of ways in which either player can choose a number from 1 to 25 is 25, so the total number of ways a choosing numbers is $25 \times 25 = 625$. So, the probability that they will not win a prize in a single trial

$= 1 - \frac{1}{25} = \frac{24}{25}$

256 **(c)**

Let X be the number of defective bulbs in a sample of 5 bulbs.

Probability that a bulb is defective $= p = \frac{10}{100} = \frac{1}{10}$

Then, $P(X = r) = {}^5C_r \left(\frac{1}{10}\right)^r \left(\frac{9}{10}\right)^{5-r}$

$\therefore$ Required probability $= P(X = 0) =$

${}^5C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^5 = \left(\frac{9}{10}\right)^5$

257 **(b)**

We know sum of probability distribution is 1

$\therefore\ k + 2k + 3k + 2k + k = 1$

$\Rightarrow\ k = \frac{1}{9}$

$\therefore$ Mean, $m = \sum_{i=1}^{5} P_i x_i$

$= k(1) + 2k(2) + 3k(3) + 2k(4) + k(5)$

$= k(1 + 4 + 9 + 8 + 5) = \frac{1}{9} \times 27 = 3$

$\therefore\ (k, m) = \left(\frac{1}{9}, 3\right)$

258 **(d)**

$\therefore$ Required probability $= \frac{30 + 5}{60} = \frac{7}{12}$

259 **(b)**

Total number of ways$=\ {}^{11}C_5 = 462$

Number of ways in which 2 particular girls are included

${}^9C_3 = 84$

$\therefore$Required probability$= \frac{84}{462} = \frac{2}{11}$

260 **(b)**

Required probability$= 1 - P$(all letters in right envelope)

$= 1 - \frac{1}{n!}$

261 **(b)**

Total number of ways $= 6 \times 6 \times 6$

Favourable number of ways $= 6$

$\therefore$ Required probability $= \frac{6}{6\times6\times6} = \frac{1}{36}$

262 **(a)**

Given, $P(A) = P(B) = x$

and $P(A \cap B) = P(A' \cap B') = \frac{1}{3}$

$\therefore\ P(A' \cap B') = 1 - P(A \cup B)$

$\Rightarrow\ P(A \cup B) = 1 - \frac{1}{3} = \frac{2}{3}$ ...(i)

Also, $P(A \cup B) = P(A) + P(B) - P(A) \cap B)$

$\Rightarrow \frac{2}{3} = 2x - \frac{1}{3}$ [from Eq (i)]

$\Rightarrow\ x = \frac{1}{2}$

263 **(a)**

$S = \{00, 01, 02, \ldots, 49\}$

Let A be the event that sum of the digits on the selected ticket is 8, then

$A = \{08, 17, 26, 35, 44\}$

Let Bbe the event that the product of the digits is zero

B ={00, 01, 02, 03,….,09, 10, 20, 30, 40}

$\therefore A \cap B = \{8\}$

$\therefore$Required probability=$P\left(\frac{A}{B}\right)$

$= \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{50}}{\frac{14}{50}} = \frac{1}{14}$

264 **(a)**

If any number the last digits can be 0,1,2,3,4,5,6,7,8,9. Therefore, last digit of each number can be chosen in 10 ways.

$\therefore$ The last digit of all numbers can be chosen in 10^n ways. If the last digit is to be 1,3,7, or 9, then none of the numbers can be even or end in 0 or 5. Thus, we have a choice of 4 digits viz. 1,3,7, or 9 with which each of n numbers should end.

So, favourable number of ways $= 4^n$

Hence, required probability $= \frac{4^n}{10^n} = \left(\frac{2}{5}\right)^n$

265 **(c)**

Consider the following events:

$A = A$ worker receives bonus, $B = A$ worker is skilled.

We have,

$P(A) = \frac{30}{100}$ and $P(B/A) = \frac{20}{100}$

$\therefore$ Required probability $= P(A \cap B) = P(A)P(B/A)$

$\Rightarrow$ Required probability $= \frac{30}{100} \times \frac{20}{100} = 0.06$

266 **(d)**

$\therefore$ Required probability$=\frac{^{10}C_1+^{6}C_1}{^{16}C_1}$

$=\frac{16}{16}=1$

267 **(c)**

Let E =Event of getting a head from a coin

F =Event of getting an odd number {1, 3,5}, from a die

$P(E)=\frac{1}{2}, P(F)=\frac{3}{6}=\frac{1}{2}$

Since, E and F are independent events

$\therefore\ P(E\cap F)=P(E)\cap P(F)=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$

268 **(a)**

Probability that at least one shot hits the plane

$=1-P$(none of the shot hits the plane)

$=1-0.6\times 0.7\times 0.8\times 0.9$

$=1-0.3024=0.6976$

269 **(c)**

Number of favourable cases $(HTH, HTH)=2$

Number of total cases $=2^3=8$

$\therefore$ Required probability $=\frac{2}{8}=\frac{1}{4}$

270 **(c)**

Consider the following events:

E_1 = Selecting first bag

E_2 = Selecting second bag

A = Getting a ticket bearing number 4

$\therefore$ Required probability $=P\big((E_1\cap A)\cup(E_2\cap A)\big)$

$=P(E_1\cap A)+P(E_2\cap A)$

$=P(E_1)P(A/E_1)+P(E_2)P(A/E_2)$

$=\frac{1}{2}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{6}=\frac{5}{24}$

271 **(b)**

We have,

Required probability $=\ ^6C_4\left(\frac{1}{2}\right)^6+\ ^6C_5\left(\frac{1}{2}\right)^6+$

$^6C_6\left(\frac{1}{2}\right)^6=\frac{11}{32}$

272 **(c)**

Let X denote the number of aces.

Probability of selecting aces,

$P=\frac{4}{52}=\frac{1}{13}$

Probability of not selecting aces,

$q=1-\frac{1}{13}=\frac{12}{13}$

$P(X=1)=2\times\left(\frac{1}{13}\right)\times\left(\frac{12}{13}\right)=\frac{24}{169}$

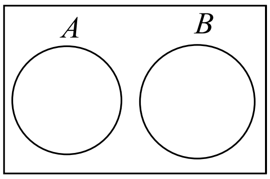
$P(X=2)=2\left(\frac{1}{13}\right)^2.\left(\frac{12}{13}\right)^0=\frac{2}{169}$

Mean $=\Sigma P_1X_i=\frac{24}{169}+\frac{2}{169}=\frac{2}{13}$

273 **(d)**

$P(A)=0.45,$

$P(B)=0.35$ (events are mutually exclusive)



$P(A\cap B)=0$

274 **(b)**

Total cases = 4

Correct option = 1

So, probability of correct answer $=\frac{1}{4}$

275 **(c)**

$P(E\cap F)=P(E).P(F)$

Now, $P(E\cap F)=P(E)-P(E\cap F)=P(E)[1-P(F)]$

$=P(E).P(F^c)$

and $P(E^c\cap F^c)=1-P(E\cup F)$

$=1-[P(E)+P(F)-P(E\cap F)$

$=[1-P(E)][1-P(F)]=P(E^c)P(F^c)$

Also $P(E/F)=P(E)$ and $P(E^c/F^c)=P(E^c)$

$\Rightarrow\ P(E/F)+P(E^c/F^c)=1$

276 **(a)**

One integer can be chosen out of 200 integers in $^{200}C_1$ways. Let A be the event that an integer selected is divisible by 6 and B that it is divisible by 8

Then, $P(A)=\frac{33}{200}, P(B)=\frac{25}{200}$

and $P(A\cap B)=\frac{8}{200}$

$\therefore P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$= \frac{33}{200} + \frac{25}{200} - \frac{8}{200} = \frac{1}{4}$$

277 **(a)**

Given, $np = 4, npq = 2$

$$\Rightarrow p = q = \frac{1}{2}, n = 8$$

We know, $P(X = r) = {}^nC_r p^r q^{n-r}$

$$\therefore P(X = 1) = {}^8C_1 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^1 = 8 \times \frac{1}{2^8} = \frac{1}{32}$$

278 **(b)**

Let X be binomial variate with parameter $n = 100$ and P

Since, $P(X = 50) = P(X = 51)$ [given]

$$\Rightarrow {}^{100}C_{50} p^{50}(1-p)^{50} = {}^{100}C_{51} p^{51}(1-p)^{49}$$

$$\Rightarrow \frac{100!}{50!\,50!} \times \frac{51!\,49!}{100!} = \frac{p}{1-p}$$

$$\Rightarrow \frac{51}{50} = \frac{p}{1-p}$$

$$\Rightarrow p = \frac{51}{101}$$

279 **(d)**

Total number=90

Number divisible by 6 are {6,12,18,24,30,36,42,48,54,60,66,72,78,84,90}

Numbers divisible by 8 are {8,16,24,32,40,48,56,64,72,80,88}

Numbers divisible by 6 and 8 are {24,48,72}

Total number of numbers divisible by 6 or 8 $= 15 + 11 - 3 = 23$

$\therefore$ Required probability$= \frac{23}{90}$

280 **(a)**

Let A_i denote the event that the number i appears on the die, and let E denote the event that only white balls are drawn. Then,

$P(A_i) = \frac{1}{6}$ and, $P(E/A_i) = \frac{{}^6C_i}{{}^{10}C_i}, i = 1,2,\dots,6$

Required probability $= P(E)$

$$= P\left(\bigcup_{i=1}^{6}(E \cap A_i)\right)$$

$$= \sum_{i=1}^{6} P(E \cap A_i)$$

$$= \sum_{i=1}^{6} P(A_i)P\,(E/A_i)$$

$$= \frac{1}{6}\left\{\frac{6}{10} + \frac{15}{45} + \frac{20}{120} + \frac{15}{210} + \frac{6}{252} + \frac{1}{210}\right\} = \frac{1}{5}$$

281 **(c)**

Let $P(R) = 10\% = \frac{1}{10}$

$$P(F) = 5\% = \frac{1}{20}$$

$$P(R \cap F) = 3\% = \frac{3}{100}$$

Probability of getting either rich or famous but not both

$$= P(R \cap F') + P(R' \cap F)$$

$$= P(R) - P(R \cap F) + P(F) - P(R \cap F)$$

$$= P(R) + P(F) - 2P(R \cap F)$$

$$= \frac{1}{10} + \frac{1}{20} - \frac{6}{100} = \frac{10 + 5 - 6}{100} = 0.09$$

282 **(c)**

Let A and B are the Ist and IInd aeroplane hit the target respectively and their corresponding probabilities are

$P(A) = 0.3$ and $P(B) = 0.2$

$\Rightarrow P(\bar{A}) = 0.7$ and $P(\bar{B}) = 0.8$

$\therefore$ Required probability

$= P(\bar{A})P(B) + P(\bar{A})P(\bar{B})P(\bar{A})P(B) + \dots$

$= (0.7)(0.2) + (0.7)(0.8)(0.7)(0.2) + (0.7)(0.8)(0.7)(0.8)(0.7)(0.2) + \dots$

$= 0.14[1 + (0.56) + (0.56)^2 + \dots]$

$$= 0.14\left(\frac{1}{1 - 0.56}\right) = 0.32$$

283 **(b)**

Let probability of box B, $P(B) = P$

According to given condition

$$P(A) = 2P(B) = 2P$$

Now, $P\left(\frac{R}{A}\right) = \frac{{}^3C_1}{{}^5C_1} = \frac{3}{5}$

and $P\left(\frac{R}{B}\right)=\frac{{}^4C_1}{{}^7C_1}=\frac{4}{7}$

$$\therefore\ P\left(\frac{B}{R}\right)=\frac{P(B).P\left(\frac{R}{B}\right)}{P(A).P\left(\frac{R}{A}\right)+P(B).P\left(\frac{R}{B}\right)}$$

$$=\frac{p.\frac{4}{7}}{2p.\frac{3}{5}+p.\frac{4}{7}}=\frac{10}{31}$$

284 **(c)**

Clearly,

$P[A\cap(B\cup C)]=P[(A\cap B)\cup(A\cap C)]$

$=P(A\cap B)+P(A\cap C)-P[(A\cap B)\cap(A\cap C)]$

$=P(A\cap B)+P(B\cap C)-P(A\cap B\cap C)$

285 **(a)**

We have,

$$\sum_{k=0}^{4}P(X=k)=1$$

$$\Rightarrow\sum_{k=0}^{4}C\,k^2=1\Rightarrow C(1^2+2^2+3^2+4^2)=1\Rightarrow C=\frac{1}{30}$$

286 **(a)**

Let $'H'$denote the head and any of them come is $'A'$.Suppose the sequence of m consecutive heads start with first throw, then

$P[(HH\ ....m\text{ times})(AA\ ...n\text{ times})]$

$$=\left(\frac{1}{2}.\frac{1}{2}....m\text{ times}\right)(1..1.....n\text{ times})$$

$$=\left(\frac{1}{2}\right)^m$$

Now, suppose the sequence of m consecutive heads start with second throw, the first must be a tail,

$\therefore\ P[T(H.H\ ...m\text{ times})(A.A\ ...n-1\text{ times})]$

$$=\frac{1}{2}.\frac{1}{2^m}\times(1)^{n-1}=\frac{1}{2^{m+1}}$$

Similarly, as above

∴Required probability

$$=\frac{1}{2^m}+\left(\frac{1}{2^{m+1}}+\frac{1}{2^{m+1}}+\cdots n\text{ times}\right)$$

$$=\frac{1}{2^m}+\frac{n}{2^{m+1}}=\frac{n+2}{2^{m+1}}$$

287 **(b)**

Given that, $x=33^n$

Where, n is a positive integral value.

Here, only four digits may be at the unit place ie, 1, 3, 7, 9.

$\therefore n(S)=4$

Let E be the event of getting 3 at its units place.

$n(E)=1$

$$\therefore P(E)=\frac{n(E)}{n(S)}=\frac{1}{4}$$

288 **(d)**

Here, two numbers are selected from {1,2,3,4,5,6,}

$\Rightarrow n(S)=6\times 5$ {as one by one without replacement}

Favourable cases,

First number	Possible value for second number
1	2, 3, 4, 5, 6
2	3, 4, 5, 6
3	4, 5, 6

There are 12 ways but the numbers may be interchanged

$\therefore\ n(E)=2\times 12=24$

$\therefore$ Required probability$=\frac{n(E)}{n(S)}=\frac{24}{30}=\frac{4}{5}$

289 **(a)**

We have, $P(A)=\frac{1}{5}$ and $P(A\cup B)=\frac{7}{10}$

Now,

$$P(A\cup B)=\frac{7}{10}$$

$\Rightarrow 1-P(\bar{A})P(\bar{B})=\frac{7}{10}$ [$\because$ A and B are independent events]

$$\Rightarrow 1-\frac{4}{5}P(\bar{B})=\frac{7}{10}\Rightarrow\frac{4}{5}P(\bar{B})=\frac{3}{10}\Rightarrow P(\bar{B})=\frac{3}{8}$$

290 **(c)**

Since A is a finite set, therefore every injective map from A to itself is bijective also

∴Required probability $=\frac{n\,!}{n^n}=\frac{(n-1)\,!}{n^{n-1}}$

291 **(a)**

We are getting a odd number of points, if it will comes (two heads, one tail and three tails)

$$\because\ P(H)=P(T)=\frac{1}{2}$$

∴Required probability=Probability of getting two heads and one tail + Probability of all three tails

$$=\ {}^3C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1+\left(\frac{1}{2}\right)^3$$

$$=3\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^3$$

$$=\frac{3}{8}+\frac{1}{8}=\frac{1}{2}$$

292 **(a)**

We have,

p = Probability that the bomb strikes the target $= 1/2$

Let n be the number of bombs which should be dropped to ensure 99% chance or better of completely destroying the target. Then, the probability that out of n bombs, at least two strike the target, is greater than 0.99

Let X denote the number of bombs striking the target. Then,

$$P(X = r) = {}^nC_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{n-r} = {}^nC_r \left(\frac{1}{2}\right)^n, r = 0, 1, 2, \dots, n$$

Now,

$P(X \geq 2) \geq 0.99$

$\Rightarrow \{1 - P(X < 2)\} \geq 0.99$

$\Rightarrow 1 - \{P(X = 0) + P(X = 1)\} \geq 0.99$

$\Rightarrow 1 - \left\{(1 + n)\frac{1}{2^n}\right\} \geq 0.99$

$\Rightarrow 0.01 \geq \frac{1 + n}{2^n}$

$\Rightarrow 2^n > 100 + 100\,n \;\Rightarrow n \geq 11$

Thus, the minimum number of bombs is 11

293 **(b)**

The probability of getting head at least once in n times

$= 1 - P$(None of the trial getting head)

$= 1 - \left(\frac{1}{2}\right)^n$

Given $1 - \left(\frac{1}{2}\right)^n > 0.8 \Rightarrow \left(\frac{1}{2}\right)^n < 0.2$

$\Rightarrow 2^n > \frac{1}{0.2} \Rightarrow 2^n > 5$

Hence, least value of n is 3

294 **(b)**

Required probability $= P(\bar{A}_1 \cap \bar{A}_2 \cap \dots \cap \bar{A}_n)$

$= P(\bar{A}_1)P(\bar{A}_2) \dots P(\bar{A}_n) = \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \dots \cdot \frac{n}{n+1}$

$= \frac{1}{n+1}$

295 **(d)**

Given, mean$= \Sigma X_k P(X = k) = 1.3$

$\Rightarrow X_0 P(X = 0) + X_1 (X = 1) + X_2 P(X = 2)$

$+ X_3 P(X = 3) = 1.3$

$\Rightarrow 0.P(X = 0) + 1.P(X = 1) + 2.P(X = 2)$

$+3.P(X = 3) = 1.3$

$\Rightarrow P(X = 1) + 2(0.3) + 3.2P(X = 1) = 1.3$

$\Rightarrow 7P(X = 1) = 0.7$

$\Rightarrow P(X = 1) = 0.1$

Since, $P(X = 3) = 2P(X = 1) = 2(0.1) = 0.2$

Also, $P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = 1$

$\Rightarrow P(X = 0) + 0.1 + 0.3 + 0.2 = 1$

$\Rightarrow P(X = 0) = 1 - 0.6 = 0.4$

296 **(b)**

P(getting a sum greater than 4)

$= 1 - P$(getting a sum less than 5) ...(i)

For sum 3, Number of cases=1

For sum 4, Number of cases=3

∴ Total number of cases=4

From Eq.(i),

$P = 1 - \frac{4}{216} = 1 - \frac{1}{54} = \frac{53}{54}$

297 **(d)**

The required probability$= \frac{{}^6C_2 + {}^4C_2}{{}^{10}C_2} = \frac{7}{15}$

298 **(b)**

There are two cases arise.

Case I If Ist ball is white, then

$P = \frac{{}^3C_1}{{}^5C_1} \times \frac{{}^2C_1}{{}^4C_1} = \frac{6}{20} = \frac{3}{10}$

Case II If Ist ball is red, then

$P = \frac{{}^2C_1}{{}^5C_1} \times \frac{{}^1C_1}{{}^4C_1} = \frac{2}{20} = \frac{1}{10}$

∴ Required probability $= \frac{3}{10} + \frac{1}{10} = \frac{2}{5}$

299 **(a)**

Let the coin be tossed n times and let X denote the number of heads obtained. Then,

$P(X = r) = {}^nC_r \left(\frac{1}{2}\right)^n$

We have,

$P(X = 4) = P(X = 7) \Rightarrow {}^nC_4 = {}^nC_7 \Rightarrow n = 11$

$\therefore P(X = 2) = {}^{11}C_2 \left(\frac{1}{2}\right)^{11} = \frac{55}{2048}$

300 **(b)**

We have,

$P(\bar{A} \cap \bar{B}) = \frac{1}{3}$

$\Rightarrow P(\overline{A \cup B}) = \frac{1}{3} \Rightarrow 1 - P(A \cup B) = \frac{1}{3} \Rightarrow P(A \cup B) = \frac{2}{3}$

$$\Rightarrow P(A)+P(B)-P(A\cap B)=\frac{2}{3}\Rightarrow p+2p-\frac{1}{2}$$
$$=\frac{2}{3}\Rightarrow p=\frac{7}{18}$$

301 **(c)**

Given, $f(x)=\frac{x}{2}\quad[0\le x\le 2]$

$$\therefore P(X>1.5)=\int_{1.5}^{2}\frac{x}{2}dx=\left[\frac{x^2}{4}\right]_{1.5}^{2}$$

$=0.4375$

and $P(X>1)=\int_1^2\frac{x}{2}dx=\left[\frac{x^2}{4}\right]_1^2=0.75$

$$\therefore P\left(\frac{X>1.5}{X>1}\right)=\frac{P(X>1.5)}{P(X>1)}$$

$$=\frac{0.4375}{0.75}=\frac{7}{12}$$

302 **(a)**

Required probability $=1-\left(1-\frac{2}{3}\right)\left(1-\frac{3}{4}\right)=\frac{11}{12}$

303 **(a)**

Given, $P(A\cup B)=0.6, P(A\cap B)=0.2$

Probability of exactly one of the event occurs is

$P(\bar{A}\cap B)+P(A\cap\bar{B})$

$=P(B)-P(A\cap B)+P(A)-P(A\cap B)$

$=P(A\cup B)+P(A\cap B)-2P(A\cap B)$

$[\because P(A\cup B)=P(A)+P(B)-P(A\cap B)]$

$=P(A\cup B)-P(A\cap B)$

$=0.6-0.2=0.4$

304 **(c)**

Probability of each case $=\frac{9}{15}=\frac{3}{5}$

Required probability (with replacement)$=\left(\frac{3}{5}\right)^7$

305 **(b)**

The total number of ways $=6^3=216$

If the second number is $i(i>1)$, then the total number of favourable ways

$$=\sum_{i=1}^{5}(i-1)(6-i)=20$$

$\therefore$ Required probability $=\frac{20}{216}=\frac{5}{54}$

306 **(b)**

$$\frac{P(X=K)}{P(X=k-1)}=\frac{{}^nC_kp^kq^{n-k}}{{}^nC_{k-1}p^{k-1}q^{n-k+1}}$$

$$=\left(\frac{n-k+1}{k}\right).\frac{p}{q}$$

308 **(d)**

We have, $P(E_i)=\frac{1}{2}$ for $i=1,2,3$

For $i\ne j$, we have,

$P(E_i\cap E_j)=\frac{1}{4}=P(E_i)P(E_j)$

$\Rightarrow E_i$ and E_j are independent events for $i\ne j$

Also, $P(E_1\cap E_2\cap E_3)=\frac{1}{4}\ne P(E_1)\,P(E_2)\,P(E_3)$

$\Rightarrow E_1,E_2,E_3$ are not independent

Hence option (d) is not correct

309 **(a)**

$P(A)=1-P(\bar{A})=1-\frac{2}{3}=\frac{1}{3}$

Using, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow\frac{3}{4}=\frac{1}{3}+P(B)-\frac{1}{4}\Rightarrow P(B)=\frac{2}{3}$

Now, $P(\bar{A}\cap B)=(B)-P(A\cap B)=\frac{2}{3}-\frac{1}{4}=\frac{5}{12}$

311 **(a)**

Let x,y and z be the parts and $x\le y\le z$. Then, $(x,y,z)\in$ $(1,1,8),(1,2,7),(1,3,6),(1,4,5),(2,2,6),(2,3,5),$ .Only the cases when (x,y,z) formed a triangle are $\{(3,3,4),(2,4,4)\}$,

Required probability $=\frac{2}{8}=\frac{1}{4}$.

312 **(b)**

Let E_1 denote the event of travelling by train and E_2 denote the event travelling by plane.

$P(E_1)=\frac{2}{3}, P(E_2)=\frac{1}{5}$

$P(E_1\cup E_2)=P(E_1)+P(E_2)$

$=\frac{2}{3}+\frac{1}{5}=\frac{13}{15}$

313 **(a)**

Three digit numbers multiple of 11 are 110, 121,…,990 (81 numbers). Now number also divisible by 9 are divisible by 99. So, numbers are 198, 297,…,990(9 numbers).

So, required probability $=\frac{9}{81}=\frac{1}{9}$.

314 **(b)**

If any number the last digit can be 0,1,2,3,4,5,6,7,8,9. We want that the last digit in the product is an odd digit other than 5 i.e. it is any one of the digits 1,3,7,9. This means that the product is not divisible by 2 or 5. The probability that a number is divisible by 2 or 5 is $\frac{6}{10}$, and in the case the last digit can be one of 0,2,4,5,6 or 8. The probability that the number is not divisible by 2 or 5, is $1-\frac{6}{10}=\frac{2}{5}$

In order that the product is not divisible by 2 or 5, none of the constituent numbers should be divisible by 2 or 5 and its probability is $\left(\frac{2}{5}\right)^4=\frac{16}{125}$

315 **(c)**

Let E =Event of getting sum of 7 in two dice

$=\{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

Now, $P(E)=\frac{6}{36}=\frac{1}{6}$ (say)

$\Rightarrow\ p=\frac{1}{6}$

$\therefore\ q=1-p=\frac{5}{6}$

Required probability= ${}^4C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2$

$=6\times\frac{5^2}{6^4}=\frac{25}{216}$

316 **(d)**

The probability that Mr. A selected the loosing horse

$=\frac{4}{5}\times\frac{3}{4}=\frac{3}{5}$

The probability that Mr. A selected the winning horse

$=1-\frac{3}{5}=\frac{2}{5}$

317 **(b)**

Given, $S=\{1,2,3\ldots,50\}$

$A=\left\{n\in S:n+\frac{50}{n}>27\right\}$

$=\{n\in S:n^2-27n+50>0\}$

$=\{n\in S:n<2 \text{ or } n>25\}$

$=\{1,26,27,\ldots,50\}$

$\Rightarrow\ n(A)=26$

$B=\{n\in S:n \text{ is a prime}\}$

$=\{2,3,5,7,11,13,17,19,23,29,31,37,41,43,47\}$

$\Rightarrow\ n(B)=15$

$\therefore\ C=\{n\in S:n \text{ is a square}\}$

$=\{1,4,9,16,25,36,49\}$

$\Rightarrow\ n(C)=7$

$\therefore\ P(A)=\frac{n(A)}{n(S)}=\frac{26}{50},P(B)=\frac{15}{50},P(C)=\frac{7}{50}$

$\Rightarrow\ P(A)>P(B)>P(C)$

318 **(c)**

$\therefore$ Required probability

$=P(WBWB)+(BWBW)$

$=\frac{{}^5C_1\times{}^3C_1\times{}^4C_1\times{}^2C_1}{{}^8C_1\times{}^7C_1\times{}^6C_1\times{}^5C_1}+\frac{{}^3C_1\times{}^5C_1\times{}^2C_1\times{}^4C_1}{{}^8C_1\times{}^7C_1\times{}^6C_1\times{}^5C_1}$

$=2\left(\frac{5\times3\times4\times2}{8\times7\times6\times5}\right)=\frac{1}{7}$

319 **(c)**

The total number of favourable cases, $n(E)=18$

The total number of cases, $n(S)=\ {}^{20}C_3$

$=\frac{20\times19\times18}{3\times2\times1}=1140$

$\therefore$ Required probability $=\frac{18}{1140}=\frac{3}{190}$

320 **(b)**

The sum of two numbers is odd only when one is odd and other is even.

$\therefore$ Required probability= $\frac{{}^{20}C_1.{}^{20}C_1}{{}^{40}C_2}$

$=\frac{20\times20}{\frac{40\times39}{2\times1}}=\frac{20\times20}{20\times39}$

$=\frac{20}{39}$

321 **(b)**

Total number of ways placing 3 letters in three envelopes

$=3!=3\times2\times1=6$

Out of these ways only one way is correct

$\therefore$ The required probability $=\frac{1}{6}$

322 **(c)**

Given probability of speaking truth are

$P(A)=\frac{4}{5}$ and $P(B)=\frac{3}{4}$

And their corresponds probabilities of not speaking truth are

$P(\bar{A})=\frac{1}{5}$ and $P(\bar{B})=\frac{1}{4}$

The probability that they contradict each other

$=P(A)\times P(\bar{B})+P(\bar{A})\times P(B)$

$=\frac{4}{5}\times\frac{1}{4}+\frac{1}{5}\times\frac{3}{4}$

$=\frac{1}{5}+\frac{3}{20}$

$=\frac{7}{20}$

323 **(d)**

Consider the following events:

A = Numbers on two tickets are not more than 10

B = Lowest number on two tickets is 5

$\therefore$ Required probability $=P(B/A)$

$\Rightarrow$ Required Probability

$= \frac{P(A \cap B)}{P(A)} = \frac{{}^5C_1 \times {}^1C_1 / {}^{100}C_2}{{}^{10}C_2 / {}^{100}C_2} = \frac{1}{9}$

324 **(b)**

Let A_i denote an event of getting number $i(i = 1,2, \dots ,6)$ on each die. Then, A_i, i = 1,2, … ,6 are mutually exclusive events

∴ Required probability $= P(A_1) + P(A_2) + \dots + P(A_6)$ … (i)

Now,

$P(A_i) =$ Probability of getting number i on each die

$\Rightarrow P(A_i) = \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$

So, required probability $= \frac{6}{216} = \frac{1}{36}$ [From (i)]

ALITER We have,

Total number of elementary events $= 6 \times 6 \times 6 = 216$

Same number can be obtained on each die in one of the following ways:

(1,1,1), (2,2,2), (3,3,3), (4,4,4), (5,5,5), (6,6,6)

Favourable number of elementary events = 6

Hence, required probability $= \frac{6}{216} = \frac{1}{36}$

325 **(b)**

$P\left(\frac{B_2}{R}\right)$

$$= \frac{P(B_2)P\left(\frac{R}{B_2}\right)}{P(B_1)P\left(\frac{R}{B_1}\right) + P(B_2)P\left(\frac{R}{B_2}\right) + P(B_3)P\left(\frac{R}{B_3}\right)}$$

$$= \frac{\frac{1}{3} \times \frac{2}{5}}{\frac{1}{2} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{5} + \frac{1}{6} \times \frac{3}{7}}$$

$$= \frac{\frac{2}{15}}{\frac{1}{6} + \frac{2}{15} + \frac{1}{14}} = \frac{14}{39}$$

326 **(d)**

Total number of elementary events associated to the random experiment is $36 \times 36 = 36^2$

Throws of the two persons are equal means the sum of the numbers are same. The sum of the numbers can be 2,3, … ,12.

∴ Favourable number of elementary events

$= 2(1 \times 1 + 2 \times 2 + \dots + 3 \times 3 + 5 \times 5) + 6 \times 6$
$= 146$

Hence, required probability $= -1 \frac{146}{36^2} = \frac{572}{648}$

327 **(c)**

At least one spade and one ace can be drawn in two mutually exclusive ways:

(i) Drawing one spade and one ace from 3 aces other than ace of spade

(ii) Drawing ace of spade and one other spade.

∴ Required probability

$$= \frac{{}^{13}C_1 \times {}^3C_1 + {}^{12}C_1 \times {}^1C_1}{{}^{52}C_2} = \frac{51}{{}^{52}C_2} = \frac{1}{26}$$

328 **(d)**

P (selecting a black ball) $= \frac{{}^5C_1}{{}^{12}C_1}$

P (selecting a red ball)$= \frac{{}^3C_1}{{}^{12}C_1}$

P (black ball or red ball) $= \frac{{}^5C_1 + {}^3C_1}{{}^{12}C_1} = \frac{2}{3}$

329 **(b)**

Given total number of bolts=600

Numbers of large bolts=20% of 600

$= \frac{20}{100} \times 600 = 120$

Number of small bolts=10% of 600

$= \frac{10}{100} \times 600 = 60$

∴ Number of suitable bolts

$= 600 - 120 - 60 = 420$

∴ Probability of selecting suitable bolt

$= \frac{420}{600} = \frac{7}{10}$

330 **(d)**

Let A denote the event that the student is selected in I.I.T. entrance test and B denotes the event that he is selected in Roorkee entrance test. Then,

$P(A) = 0.2, P(B) = 0.5$ and $P(A \cap B) = 0.3$

∴Required probability $= P(\bar{A} \cap \bar{B})$

$\Rightarrow$ Required probability $= 1 - P(A \cup B)$

$\Rightarrow$ Required probability $= 1 - \{P(A) + P(B) - P(A \cap B)\}$

$\Rightarrow$ Required probability $= 1 - (0.2 + 0.5 - 0.3) = 0.6$

331 **(c)**

The probability that A get r heads in the three tosses of a coin is

$P(X=r)= {}^3C_r\left(\frac{1}{2}\right)^3$. The probability that A and B both get r heads in three tosses of a coin is

$${}^3C_r\left(\frac{1}{2}\right)^3 . {}^3C_r\left(\frac{1}{2}\right)^3$$

$$=({}^3C_r)^2\left(\frac{1}{2}\right)^6$$

$\therefore$ Required probability $=\sum_{r=0}^{3}({}^3C_r)^2\left(\frac{1}{2}\right)^6$

$$=\left(\frac{1}{2}\right)^6(1+9+9+1)=\frac{20}{64}=\frac{5}{16}$$

332 **(b)**

We have, $P(S)=P\{5,6\}=\frac{2}{6}=\frac{1}{3}$

Let us denote the occurrence of a number greater than 4 in a single throw of the die and F denote its failure.

$\Rightarrow P(F)=\frac{2}{3}$

P (an even number of tosses is needed)

$=P(FS \text{ or } FFFS \text{ or } FFFFFS \text{ or } \dots)$

$=P(F)P(S)+P(F)^3P(S)+P(F)^5P(S)+\dots$

$$=\frac{P(F)P(S)}{1-P(F)^2}=\frac{\frac{2}{9}}{1-\frac{4}{9}}=\frac{2}{5}$$

333 **(c)**

Let E =event when each American man is seated adjacent to his wife

and A =event when Indian man is seated adjacent to his wife.

Now, $n(A\cap E)=(4!)\times(2!)^5$

Even when each American man is seated adjacent to his wife.

Again, $n(E)=(5!)\times(2!)^4$

$$\therefore P\left(\frac{A}{E}\right)=\frac{n(A\cap E)}{n(E)}$$

$$=\frac{(4!)\times(2!)^3}{(5!)\times(2!)^4}=\frac{2}{5}$$

334 **(a)**

Required probability $=\frac{{}^5C_2+{}^4C_2}{{}^9C_2}=\frac{4}{9}$

335 **(b)**

Let A_1 be the event that the black card is lost, A_2 be the event that red card is lost and let E be the event that first 13 cards examined are red. Then,

Required probability $=P(A_1/E)$

We have,

$P(A_1)=P(A_2)=1/2$, as black and red cards were initially equal in number

Also, $P(E/A_1)=\frac{{}^{26}C_{13}}{{}^{51}C_{13}}$ and $P(E/A_2)=\frac{{}^{25}C_{13}}{{}^{51}C_{13}}$

$\therefore$Required probability $=P(A_1/E)$

$$=\frac{P(E/A_1)P(A_1)}{P(E/A_1)P(A_1)+P(E/A_2)P(A_2)}$$

$$=\frac{\frac{1}{2}\times\frac{{}^{26}C_{13}}{{}^{51}C_{13}}}{\frac{1}{2}\times\frac{{}^{26}C_{13}}{{}^{51}C_{13}}+\frac{1}{2}\times\frac{{}^{25}C_{13}}{{}^{51}C_{13}}}=\frac{2}{3}$$

336 **(b)**

The probability of getting at least one head in n tosses of a coin $=1-\left(\frac{1}{2}\right)^n$

$$\therefore 1-\left(\frac{1}{2}\right)^n\geq 0.9\Rightarrow\left(\frac{1}{2}\right)^n\leq 0.1\Rightarrow 2^n\geq 10\Rightarrow n\geq 4$$

Hence, the least value of n is 4

337 **(b)**

Favourable ways $=3!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right)=2$

Total ways $=3!$

$\therefore$ Probability $=\frac{2}{3!}=\frac{1}{3}$

338 **(a)**

$\because f(x)=x^3+ax^2+bx+c$

$\therefore f'(x)=3x^2+2ax+b$

$y=f(x)$ is increasing.

$\Rightarrow f'(x)\geq 0,\forall x$

And for $f'(x)=0$ should not form an interval.

$\Rightarrow 4a^2-4\times 3\times b\leq 0$

$\Rightarrow a^2-3b\leq 0$

This is true for exactly 16 ordered pairs $(a,b), 1\leq a,b\leq 6$ namely (1, 1), (1, 2), (1, 3), (1, 4); (1, 5), (1, 6), (2, 2),(2, 3), (2, 4), (2, 5), (2, 6); (3, 3),(3, 4),(3, 5),(3, 6) and (4, 6).

Thus, required probability $=\frac{16}{36}=\frac{4}{9}$

339 **(b)**

$P(0<x<3)=P(x=1)+P(x=2)$

$$=\frac{{}^3C_1\times{}^7C_3}{{}^{10}C_4}+\frac{{}^3C_2\times{}^7C_2}{{}^{10}C_4}=\frac{3\times 35}{210}+\frac{3\times 21}{210}=\frac{4}{5}$$

340 **(c)**

Given, $np=4, npq=2$

$$\Rightarrow p=q=\frac{1}{2}$$

$\therefore\ n = 4 \times 2 = 8$

$\therefore\ P(X > 6) = {}^8C_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right) + {}^8C_8\left(\frac{1}{2}\right)^8$

$= \frac{8}{256} + \frac{1}{256} = \frac{9}{256}$

341 **(a)**

Given, one integer is chosen at random from the first 200 positive integers and integer chosen is divisible by 6 or 8.

$\therefore$ One integer can be chosen out of 200 integers in ${}^{200}C_1$ ways.

Let A be the event that an integer selected is divisible by 6 and B that it is divisible by 8.

Then, $P(A) = \frac{33}{200}, P(B) = \frac{25}{200}$

and $P(A \cap B) = \frac{8}{200}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{33}{200} + \frac{25}{200} - \frac{8}{200} = \frac{1}{4}$

342 **(c)**

Given that, $E(X) = 3$ and $E(X^2) = 11$

Variance of $X = E(X^2) - [E(X)]^2$

$= 11 - (3)^2 = 11 - 9 = 2$

343 **(b)**

Let X be the number of heads getting in n tossed. Therefore, X follows binomial distribution with parameters

$n, P = \frac{1}{2}, q = \frac{1}{2}$

Since, $P(X \geq 1) \geq 0.8$ [given]

$\therefore 1 - P(X = 0) \geq 0.8$

$\Rightarrow\ P(X = 0) \leq 0.2$

$\Rightarrow\ {}^nC_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^n \leq 0.2$

$\Rightarrow\ \frac{1}{2^n} \leq \frac{1}{5}\ \Rightarrow\ 2^n \geq 5$

Hence, least value of n is 3

344 **(c)**

Total number of digits in any number at the unit place is 10.

$\therefore\ n(S) = 10$

To get the last digit in product is 1, 3, 5 or 7, it is necessary the last digit in each number must be 1, 3, 5 or 7.

$n(A) = 4,$

$\therefore\ P(A) = \frac{4}{10} = \frac{2}{5}$

Hence, required probability $= \left(\frac{2}{5}\right)^4 = \frac{16}{625}$

345 **(a)**

Since, events are independent, so

$P(A \cap B') = P(A) \times P(B') = \frac{3}{25}$

$\Rightarrow P(A) \times [(1 - P(B)] = \frac{3}{25}$ ...(i)

Similarly, $P(B) \times [1 - P(A)] = \frac{8}{25}$ ...(ii)

$\therefore$ From Eqs.(i) and (ii), $P(A) = \frac{1}{5}$

346 **(a)**

We have,

$x^2 - 3\,x + 2 \geq 0 \Rightarrow (x-1)(x-2) \geq 0 \Rightarrow x < 1$ or $x > 2$

$\therefore$ Required probability $= \frac{\int_0^1 dx + \int_2^5 dx}{\int_0^5 dx} = \frac{4}{5}$

347 **(d)**

P(At least on head)$= 1 - P$(zero head)

$= 1 - P$(all three tails)

$= 1 - \frac{1}{8} = \frac{7}{8}$

348 **(b)**

For binomial distribution

0<variance<mean

$\Rightarrow\ 0 < \beta < \alpha$

349 **(c)**

Given, $x^2 - n = 0$

$\Rightarrow\ x = \pm\sqrt{n}$

$\therefore\ n = 1, 4, 9, 16, 25, 36$

$\therefore$Required probability $= \frac{6}{40} = \frac{3}{20}$

350 **(b)**

In a pack of 52 cards, there are 26 black cards.

$\therefore$ Required probability $= \frac{{}^{26}C_3}{{}^{52}C_3}$

$= \frac{26 \times 25 \times 24}{3 \times 2 \times 1} \times \frac{3 \times 2 \times 1}{52 \times 51 \times 50}$

$= \frac{2}{17}$

351 **(d)**

We have,

Total number of elementary events $= 6^3 = 216$

Exactly two of three dice will show the same number in

$^6C_1 \times {}^5C_1 \times \frac{3\,!}{2\,!}$ ways

∴ Favourable number of elementary events

$= {}^6C_1 \times {}^5C_1 \times \frac{3\,!}{2\,!} = 90$

Hence, required probability $= \frac{90}{216}$

352 **(c)**

Since, $P(B) = \frac{2}{7}$ and $P(A \cup B^c) = 0.8$

$P(B^c) = 1 - \frac{2}{7} = \frac{5}{7}$

Using, $P(A \cup B^c) = P(A) + P(B^c) - P(A).P(B^c)$

$\Rightarrow 0.8 = P(A) + \frac{5}{7} - \frac{5}{7}P(A)$

$\Rightarrow 0.8 = \frac{5}{7} + \frac{2}{7}P(A)$

$\Rightarrow P(A) = 0.3$

353 **(d)**

Clearly, $P(A \cup B \cup C) = 1$

$\Rightarrow P(A) + P(B) + P(C) = 1$

$\Rightarrow P(A) + \frac{1}{2}P(A) + \frac{1}{3}P(A) = 1$

$\Rightarrow \frac{11}{6}P(A) = 1$

$\Rightarrow P(A) = \frac{6}{11}$

354 **(d)**

$P(X = 0) = k, P(X = 1)2k\left(\frac{1}{5}\right)^1$

$P(X = 2) = 3k\left(\frac{1}{5}\right)^2, \ldots$

Since, $P(X = 0) + P(X = 1) + P(X = 2) + \ldots = 1$

$\therefore k + 2k\left(\frac{1}{5}\right) + 3k\left(\frac{1}{5}\right)^2 + \ldots = 1$

and $+\frac{k}{5} + 2k\left(\frac{1}{5}\right)^2 + \ldots = \frac{1}{5}$

$- \quad - \quad - \quad - \quad -$

$k + k\left(\frac{1}{5}\right) + k\left(\frac{1}{5}\right)^2 + \ldots = \frac{4}{5}$

$\Rightarrow \frac{k}{1 - \frac{1}{5}} = \frac{4}{5}$

$\Rightarrow k = \frac{16}{25}$

$\therefore P(X = 0) = \frac{16}{25}(0 + 1)\left(\frac{1}{5}\right)^0 = \frac{16}{25}$

355 **(c)**

Given, $P(\bar{B}) = \frac{1}{3} \Rightarrow P(B) = \frac{2}{3}$,

$P(A \cup B) = \frac{5}{6}, P(A \cap B) = \frac{1}{3}$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{5}{6} = P(A) + \frac{2}{3} - \frac{1}{3}$

$\Rightarrow P(A) = \frac{5}{6} - \frac{2}{3} + \frac{1}{3} = \frac{1}{2}$

356 **(c)**

We have,

Total number of ways of selecting 4 tickets

$= 3^4 = 81$

Favourable number of ways

= Sum of the coefficients of $x^2, x^4, \ldots$ in $(x + x^2 +$ $x34$

= Sum of the coefficients of $x^2, x^4, \ldots$ in $x^4(1 + x + x^2)^4$

Let $(1 + x + x^2)^4 = 1 + a_1\,x + a_2\,x^2 + \cdots + a_8\,x^8$

Putting $x = 1$ and $x = -1$ respectively, we get

$3^4 = 1 + a_1 + a_2 + a_3 + \cdots + a_8$
and, $1 = 1 - a_1 + a_2 - a_3 + \cdots + a_8$
$\therefore 3^4 + 1 = 2(1 + a_2 + a_4 + a_6 + a_8)$
$\Rightarrow a_2 + a_4 + a_6 + a_8 = 40$
Thus, the sum of the coefficients of $x^2, x^4, \ldots = 40$
Hence, required probability $= \frac{40}{81}$

357 **(c)**
Probability of getting a white ball at any draw is, $p = \frac{12}{24} = \frac{1}{2}$.
The probability of getting a white ball 4th in the 7th draw
$= P$ (getting 3 white balls in 6 draws)$\times$ P (white ball at the 7th draw)
$= {}^6C_3 \left(\frac{1}{2}\right)^6 \cdot \frac{1}{2} = \frac{20}{2^7} = \frac{5}{32}$

358 **(c)**
Total number of persons $= 15$
and number of persons who can speak Hindi and English both
$= 10 + 8 - 15 = 3$
$\therefore$ Required probability$= \frac{{}^7C_1 \times {}^3C_1}{{}^{15}C_2} = \frac{7\times 3}{\frac{15\times 14}{2}} = \frac{1}{5}$

359 **(b)**
Required probability$= \frac{1}{2}\left(\frac{{}^3C_1}{{}^7C_1} + \frac{{}^2C_1}{{}^8C_1}\right)$
$= \frac{1}{2}\left(\frac{3}{7} + \frac{2}{8}\right) = \frac{19}{56}$

360 **(c)**
Favourable cases will be (5,1),(4,2),(2,4),(1,5)
Hence, required probability $= \frac{4}{6.5} = \frac{2}{15}$

361 **(c)**
Given, $P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = \frac{7}{10}$

Since,$P(A \cap B) + P(\overline{A \cap B})$=1

$\Rightarrow P(A \cap B) = 1 - \frac{7}{10} = \frac{3}{10}$

Also, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{4}{5} = P(A) + \frac{2}{5} - \frac{3}{10}$

$\Rightarrow P(A) = \frac{4}{5} - \frac{2}{5} + \frac{3}{10}$

$= \frac{2}{3} + \frac{3}{10} = \frac{7}{10}$

362 **(c)**
Probability of getting an ace
$P(E_1) = \frac{4}{52} = \frac{1}{13}, P\left(\frac{E_2}{E_1}\right) = \frac{15}{51} = \frac{5}{17}$

$P(E_1 \cap E_2) = P(E_1) \cdot P\left(\frac{E_2}{E_1}\right)$
$= \frac{1}{13} \cdot \frac{5}{17} = \frac{5}{221}$

363 **(b)**
4 five-rupee, 3 two-rupee and 2 one-rupee coins can be stacked together in a column in $\frac{9!}{4!3!2!}$ ways
The number of ways in which coins of the same denomination the consecutive is same as the number of ways of arranging 3 distinct items i.e. 3! Ways
Hence, required probability $= \frac{3!}{\frac{9!}{4!\,3!\,2!}} = \frac{1}{210}$

364 **(b)**
The total number of ways of selecting two squares is 64×63
For each of the four corner squares, the favourable number of way is 2
For each of the 24 non-corner squares on either side of chess board, the favourable number of cases is 3
For each of the 36 remaining squares, the favourable, number of ways is 4
Thus, the total number of favourable ways
$= 4 \times 2 + 24 \times 3 + 36 \times 4 = 224$
Hence, required probability $= \frac{224}{64\times 63} = \frac{1}{18}$

365 **(b)**
We define the following events:

A_1: He knows the answer;

A_2:He does not know the answer;

E:He gets the correct answer

Then, $P(A_1) = \frac{9}{10}, P(A_2) = 1 = \frac{9}{10} = \frac{1}{10}$,

$P(E|A_1) = 1, P(E|A_2) = \frac{1}{4}$

$\therefore$ Required probability $= P(A_2|E)$

$= \frac{P(A_2)P(E/A_2)}{P(A_1)P(E|A_1) + P(A_2)P(E|A_2)} = \frac{\frac{1}{10}.\frac{1}{4}}{\frac{9}{10}.1 + \frac{1}{10}.\frac{1}{4}}$
$= \frac{1}{37}$

366 **(d)**
$P(A \cap B') = P(A) - P(A \cap B)$

367 **(d)**
Let A =Event of getting i on first dice

and B =Event of getting more than i on second dice

$\therefore$ Required probability $= \sum_{i=1}^{5} P(A_i \cap B_1)$

$$= \frac{1}{6}[P(B_1) + P(B_2) + P(B_3) + P(B_4) + P(B_5)]$$

$$= \frac{1}{6}\left[\frac{5}{6} + \frac{4}{6} + \frac{3}{6} + \frac{2}{6} + \frac{1}{6}\right]$$

$$= \frac{15}{36} = \frac{5}{12}$$

368 **(a)**

The event that the fifth toss results in a head is independent of the event that the first four tosses result in tails.

$\therefore$ Probability of the required event $= \frac{1}{2}$

369 **(a)**

Let A_1 denote the event that a coin having heads on both sides is chosen, and A_2 denote the event that a fair coin is chosen. Let E denote the event that head occurs. Then

$$P(A_1) = \frac{n}{2n+1}, P(A_2) = \frac{n+1}{2n+1}, P(E/A_1) = 1, P(E/A_2) = \frac{1}{2}$$

Now, $P(E) = P(A_1 \cap E) + P(A_2 \cap E)$

$\Rightarrow P(E) = P(A_1)P(E/A_1) + P(A_2)P(E/A_2)$

$\Rightarrow \frac{31}{42} = \frac{n}{2n+1} \times 1 + \frac{n+1}{2n+1} \times \frac{1}{2}$

$\Rightarrow \frac{31}{42} = \frac{3n+1}{2(2n+1)}$

$\Rightarrow 124n + 62 = 126n + 42$

$\Rightarrow 2n = 20 \Rightarrow n = 10$

370 **(d)**

In binomial distribution, mean$= np = 10$, variance$= npq = 5$

$\therefore p = q = \frac{1}{2}$

Let x be the mode, then

$np + p < x > np - q$

$\therefore 10 + \frac{1}{2} > x > 10 - \frac{1}{2}$

$\Rightarrow \frac{21}{2} > x > \frac{19}{2} \Rightarrow 9.5 < x < 10.5$

$\therefore x = 10$

371 **(b)**

Mean$= np = 4$, variance$= nqp = 3$

On solving, we get $q = \frac{3}{4}, n = 16, p = \frac{1}{4}$

Now, $P(X \geq 1) = 1 - P(X = 0) = 1 - {}^nC_0 p^0 q^{n-0}$

$$= 1 - \left(\frac{3}{4}\right)^{16}$$

372 **(c)**

Probability of defective transistor $= \frac{5}{15} = \frac{1}{3}$ and probability of non-defective transistor

$= 1 - \frac{1}{3} = \frac{2}{3}$

Probability that the inspectors finds non-defective transistors

$= \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$

Hence, probability that atleast one of the inspectors finds a defective transistor

$= 1 - \frac{8}{27} = \frac{19}{27}$

373 **(b)**

The probability of suffering of a disease is 10%

$$p = \frac{10}{100} = \frac{1}{10} \text{ and } q = \frac{9}{10}$$

Total number of patients, $n = 6$

$\therefore$ Required probability

$$= {}^6C_3 \left(\frac{1}{10}\right)^3 \left(\frac{9}{10}\right)^3$$

$$= \frac{6.5.4}{3.2.1} \times \frac{1}{1000} \times \frac{9 \times 9 \times 9}{1000}$$

$$= \frac{2}{10^5} \times 729 = 1458 \times 10^{-5}$$

374 **(c)**

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore 0.6 = P(A) + P(B) - 0.2$

$\Rightarrow P(A) + P(B) = 0.8$

$\Rightarrow P(\bar{A}) + P(\bar{B}) = 12 \; [\because P(A) = 1 - P(\bar{A})]$

375 **(c)**

$n(S) = 36$

Let E =Event of getting sum 7

$= \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$

$\therefore\ n(E) = 6$

E =Event of getting sum 11

$= \{(6,5), (5,6)\}$

$\therefore\ n(F) = 2$

Also $n(E \cap F) = 0$

$\therefore\ n(E \cup F) = n(E) + n(F) - n(E \cap F)$

$= 6 + 2 = 8$

$\therefore$ Required probability$= \frac{8}{36} = \frac{2}{9}$

376 **(a)**

Since the graph of $y = 16\,x^2 + 8(a+5)x - 7\,a - 5$ is strictly above x-axis. Therefore,
$y > 0$ for all x
$\Rightarrow 16\,x^2 + 8(a+5)x - 7\,a - 5 > 0$ for all x
$\Rightarrow 64(a+5)^2 + 64(7\,a+5) < 0\quad [\because \text{Disc} < 0]$
$\Rightarrow a^2 + 17\,a + 30 < 0$
$\Rightarrow -15 < a < -2$

$$\therefore \text{Required probability} = \frac{\int_{-15}^{-2} dx}{\int_{-20}^{0} dx} = \frac{13}{20}$$

377 **(c)**

Duplicate =5, original = 10
Taking 3 times.
The probability that none of the items is duplicate *ie*, all the three are original

$$= \frac{{}^{10}C_3}{{}^{15}C_3} = \frac{24}{91}$$

378 **(b)**

The probability that only two tests are needed = (probability that the first tested machine is faulty) × (probability that the second tested machine is faulty given the first machine tested is faulty)$= \frac{2}{4} \times \frac{1}{3} = \frac{1}{6}$.

379 **(c)**

Five tickets out of 50 can be drawn in ${}^{50}C_5$ ways.
Since $x_1 < x_2 < x_3 < x_4 < x_5$ and $x_3 = 30$.
Therefore, $x_1, x_2 < 30$ i.e. x_1 and x_2 should come from tickets numbered 1 to 29 and this may happen in ${}^{29}C_2$ ways. Remaining two i.e. $x_4, x_5 < 30$, should come from 20 tickets numbered 31 to 50 in ${}^{29}C_2$ ways.
So, favourable number of elementary events
$= {}^{29}C_2 \times {}^{20}C_2$

Hence, required probability $= \frac{{}^{29}C_2 \times {}^{20}C_2}{{}^{50}C_5}$

380 **(a)**

Probability of no tail in four throws $= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$

Probability of atleast one tail $= 1 - \frac{1}{16} = \frac{15}{16}$

381 **(a)**

Clearly, X is a binomial variate with $p = 1/2$

$$\therefore P(X = r) = {}^nC_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{n-r} = {}^nC_r \left(\frac{1}{2}\right)^n$$

It is given that $P(X = 4), P(X = 5)$ and $P(X = 6)$ are in AP
$\therefore 2\,P(X = 5) = P(X = 4) + P(X = 6)$
$\Rightarrow 2\ {}^nC_5 = {}^nC_4 + {}^nC_6$

$$\Rightarrow 2 = \frac{{}^nC_4}{{}^nC_5} + \frac{{}^nC_6}{{}^nC_5}$$

$$\Rightarrow 2 = \frac{5}{n-4} + \frac{n-5}{6}$$

$\Rightarrow n^2 - 21\,n + 98 = 0 \ \Rightarrow n = 7, 14$

382 **(b)**

Three letters can be placed in 3 envelopes in 3! ways, whereas there is only one way of placing them in their right envelopes. So. Probability that all the letters are placed in the right envelopes$= \frac{1}{3!}$

Hence, required probability$= 1 - \frac{1}{3!} = \frac{5}{6}$

383 **(b)**

We have,
Total number of mappings from A to $B = n^m$
Number of injective mappings from A to $B = {}^nC_m \times m\,!$

Hence, required probability $= \frac{{}^nC_m \times m\,!}{n^m} = \frac{n\,!}{(n-m)\,!\,n^m}$

384 **(c)**

Let x be the probability of success in each trial, then $(1-x)$ will be the probability of failure in each trial.
Thus, probability of exactly successes in a series of three trials
$= P(\overline{E}_1\,E_2\,E_3 + E_1\overline{E}_2\,E_3 + E_1\,E_2\overline{E}_3)$
$= (1-x)x \cdot x + x(1-x)x + x \cdot x(1-x)$
$= 3x^2(1-x)$
and the probability of three success
$P(E_1E_2E_3) = x \cdot x \cdot x = x^3$
According to question,
$9x^3 - 3x^2(1-x)$
$\Rightarrow 3x = 1 - x$
$\Rightarrow 4x = 1$

$$\Rightarrow x = \frac{1}{4}$$

Hence, the probability of success in each trial is $\frac{1}{4}$.

385 **(d)**

Let $E = E$ =Events of numbers divisible by 2 and 3 [*ie*, divisible by 6]

$= (6, 12, \dots, 96)$

$n(E) = 16$

$\therefore$Required probability $= \frac{^{16}C_3}{^{100}C_3}$

$= \frac{\frac{16\times15\times14}{3\times2\times1}}{\frac{100\times99\times98}{3\times2\times1}} = \frac{4}{1155}$

386 **(d)**

Probability of getting a Sunday in a week,

$p = \frac{1}{7}, q = \frac{6}{7}$

Required probability= $^5C_2\left(\frac{1}{7}\right)^2\left(\frac{6}{7}\right)^3 = \frac{10\times6^3}{7^5}$

387 **(c)**

Given that, $np = 12$ ...(i)

and $\sqrt{npq} = 2 \Rightarrow npq = 4$ ...(ii)

From Eqs. (i) and (ii), we get

$12 \times q = 4 \Rightarrow q = \frac{1}{3}$

and we know that,

$p + q = 1 \Rightarrow p = 1 - \frac{1}{3} = \frac{2}{3}$

388 **(c)**

Total cases = $^{52}C_4$

Favourable cases = $(^{13}C_1)^4$

So, probability = $\frac{(^{13}C_1)^4}{^{52}C_4}$

$= \frac{13\times13\times13\times13\times1\times2\times3\times4}{52\times51\times50\times49}$

$= \frac{2197}{20825}$

389 **(b)**

Required probability

$P(A_1 \cap A_2' \cap A_3) + P(A_1{}' \cap A_2 \cap A_3)$

$= P(A_1)P(A_2')P(A_3) + P(A_1')P(A_2)P(A_3)$

$= \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3$

$= \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$

390 **(c)**

The last digit of the product will be 1, 2, 3, 4, 5, 6, 7, 8 or 9 if and only if each of the n positive integers ends in any of these digits. Now the probability of an integer ending

in 1, 2, 3, 4, 5, 6, 7, 8 or 9 is $\frac{8}{10}$. Therefore the probal product of n integer in

1, 2, 3, 4, 5, 6, 7, 8 or 9 is $\left(\frac{4}{5}\right)^n$.The probability for an integer to

end in 1, 3, 7 or 9 is $\frac{4}{10} = \frac{2}{5}$

Therefore the probability for the product of n positive integers to end in 1, 3, 7 or 9 is $\left(\frac{2}{5}\right)^n$

Hence the required probability= $\left(\frac{4}{5}\right)^n - \left(\frac{2}{5}\right)^n = \frac{4^n - 2^n}{5^n}$

391 **(d)**

Required probability $= P(WBWB) + P(BWBW)$

$= \left(\frac{^5C_1 \times {}^3C_1 \times {}^4C_1 \times {}^2C_1}{^8C_1 \times {}^7C_1 \times {}^6C_1 \times {}^5C_1}\right) + \left(\frac{^5C_1 \times {}^3C_1 \times {}^4C_1 \times {}^2C_1}{^8C_1 \times {}^7C_1 \times {}^6C_1 \times {}^5C_1}\right)$

$= \frac{1}{14} + \frac{1}{14} = \frac{2}{14} = \frac{1}{7}$

392 **(a)**

Let X denotes the number of red balls. Here probability of getting red balls, $p = \frac{3}{7}$ and probability of getting red bills, $q = \frac{4}{7}$

1. $P_1(X = 0) = {}^3C_0 \left(\frac{3}{7}\right)^0 \left(\frac{4}{7}\right)^3 = \frac{64}{(7)^3}$

2. $P_2(X = 1) = {}^3C_1 \left(\frac{3}{7}\right)^1 \left(\frac{4}{7}\right)^2 = \frac{144}{(7)^3}$

3. $P_3\ (X = 2) = {}^3C_2 \left(\frac{3}{7}\right)^2 \left(\frac{4}{7}\right)^1 \frac{108}{(7)^3}$

4. $P_4\ (X = 3) = {}^3C_3 \left(\frac{3}{7}\right)^3 = \frac{27}{(7)^2}$

$\therefore$ Variance$= \sum_{i=0}^{3} P_i x_i^2 - \left(\sum_{i=0}^{3} P_i x_i\right)^2$

$$= \left[\frac{64}{(7)^3} \times 0 + \frac{144}{(7)^3} \times (1)^2 + \frac{108}{(7)^3} \times (2)^2 + \frac{27}{(7)^3} \times (3)^2\right]$$

$$- \left[\frac{64}{(7)^3} \times 0 + \frac{144}{(7)^3} \times 1 + \frac{108}{(7)^3} \times 2 + \frac{27}{(7)^3} \times 3\right]^2$$

$$= \left[0 + \frac{144}{343} + \frac{432}{343} + \frac{243}{343}\right] - \left[0 + \frac{144}{343} + \frac{216}{343} + \frac{81}{343}\right]^2$$

$$= \frac{819}{343} - \left(\frac{441}{343}\right)^2$$

$$= \frac{280917 - 194481}{(343)^2} = \frac{36}{49}$$

Now, standard deviation $= \sqrt{\text{variance}}$

$$= \sqrt{\frac{36}{49}} = \frac{6}{7}$$

393 **(c)**

Since, $n(n+1)P = \frac{101}{3}$ is not an integer

Therefore, $P(X = r)$is maximum when $r = \left[\frac{101}{3}\right] = 33$

394 **(c)**

We have,

$p =$ Probability that a bulb is defective $= \frac{5}{20} = \frac{1}{4}$

$\therefore q = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$

Let X denote the number of defective bulbs in a sample of 3 bulbs. Then, X is a binomial variate with parameter $n = 3$ and $p = \frac{1}{4}$ such that

$$P(X = r) = {}^3C_r \left(\frac{1}{4}\right)^r \left(\frac{3}{4}\right)^{3-r}$$

$$\Rightarrow P(X = 2) = {}^3C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{2}\right) = \frac{9}{64}$$

395 **(a)**

Given, $np = 3 = \sqrt{npq} = \frac{3}{2}$

$$\Rightarrow q = \frac{npq}{np} = \frac{9}{4 \times 3} = \frac{3}{4}$$

$$\Rightarrow p = 1 - \frac{3}{4} = \frac{1}{4}$$

Also, $np = 3 \Rightarrow n = 12$

Hence, binomial distribution is

$$(q + p)^n = \left(\frac{3}{4} + \frac{1}{4}\right)^{12}$$

396 **(d)**

Since each element of a determinant of order 2 can be 0 or 1. Therefore, the total number of determinants with entries 0 or 1 is $2^4 = 16$. Out of these 16 determinants, there are 3 positive and 3 negative

$$\therefore P(A) = P(B) = \frac{3}{16} \neq \frac{1}{2}$$

397 **(a)**

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$\therefore \frac{1}{15} = \frac{P(A \cap B)}{\frac{1}{12}}$$

$$\Rightarrow P(A \cap B) = \frac{1}{180}$$

Also, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow P(A \cup B) = \frac{1}{12} + \frac{5}{12} - \frac{1}{180} = \frac{89}{180}$$

399 **(b)**

We have,

$np = 20$ and $npq = 4 \Rightarrow q = \frac{1}{5} \Rightarrow p = \frac{4}{5}$

Now, $np = 20 \Rightarrow n = 25$

400 **(d)**

Let Abe the event of obtaining an even sum and B be the event of obtaining a sum less five.

Then, we have to find $P(A \cup B)$.Since, A, B are not mutually exclusive, we have

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{18}{36} + \frac{6}{36} - \frac{4}{36}$$

$$= \frac{5}{9}$$

[Since, there are 18 ways to get an even sum and 6 ways to get a sum less than 5 *ie*. (1,3), (3,1), (2,2), (1,2), (2,1),(1,1)

and 4 ways to get an even sum less than 5, *ie*, (1,3), (3,1), (2,2), (1,1).]

401 **(d)**

Since $\frac{1+4p}{p}, \frac{1-p}{2}$ and $\frac{1-2p}{2}$ are probabilities of three mutually exclusive events

$$\therefore 0 \le \frac{1+4p}{p} 1, 0 \le \frac{1-p}{2} \le 1, 0 \le \frac{1-2p}{2} \le 1$$

and,

$$0 \le \frac{1+4p}{p} + \frac{1-p}{2} + \frac{1-2p}{2} \le 1$$

$$\Rightarrow -\frac{1}{4} \le p \le \frac{3}{4}, -1 \le p \le 1, -\frac{1}{2} \le p \le \frac{1}{2} \text{ and } \frac{1}{2} \le p \le \frac{5}{2}$$

$$\Rightarrow \max\left\{-\frac{1}{4}, -1, -\frac{1}{2}, \frac{1}{2}\right\} \le p \le \min\left\{\frac{3}{4}, 1, \frac{1}{2}, \frac{5}{2}\right\}$$

$$\Rightarrow \frac{1}{2} \le p \le \frac{1}{2} \Rightarrow p = \frac{1}{2}$$

402 **(b)**

Let the lengths of three parts of the rod be x, y and $a - (x + y)$. Then,

$x > 0, y > 0$ and $a - (x + y) > 0$, i. e. $x + y < a$ or $y < a - x$

Since in a triangle, the sum of any two sides is greater than the third. Therefore,

$$\left.\begin{array}{l} x + y > a - (x + y) \Rightarrow y > \frac{a}{2} - x \\ x + a - (x + y) > y \Rightarrow y < \frac{a}{2} \\ y + a - (x + y) > x \Rightarrow y < \frac{a}{2} \end{array}\right\} \Rightarrow \begin{array}{c} \frac{a}{2} - x < y < \\ \text{and} \\ 0 < x < a/2 \end{array}$$

$$\therefore \text{Required probability} = \frac{\int_0^{1/2} \int_{1/2-3}^{1/2} dy\, dx}{\int_0^a \int_0^{a-x} dy\, dx}$$

$$= \frac{\int_0^{a/2} [a/2 - (a/2 - x)] dx}{\int_0^a (a - x)\, dx} = \frac{\int_0^{a/2} x\, dx}{\int_0^a (a - x) dx}$$

$$= \left[\frac{x^2}{2}\right]_0^{a/2} \quad [ax - x^2/2]_0^a = \frac{a^2/8}{a^2/2} = \frac{1}{4}$$

403 **(a)**

Probability of success, $P = \frac{2}{6} = \frac{1}{3}$ and probability of failure, $q = \frac{2}{3}$

$\therefore$ Required probability= ${}^5C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)$

404 **(c)**

There is no condition on other coins only 5th coin will fall head upwards. So, probability $= \frac{1}{2}$

405 **(c)**

(a) P (no boy in family of 4) = P (all girls in it)= $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$

Hence, the probability of having at least one boy

$$= 1 - \frac{1}{16} = \frac{15}{16}$$

(b)P (first card is an ace)= $\frac{1}{13}$

and P (second card is an ace) $= \frac{1}{13}$

Therefore, P (both cards are aces) $= \frac{1}{13} \times \frac{1}{13} = \frac{1}{169}$

(c) Let guessing correctly one answer as a success. Then, we have

$$p = \frac{1}{2}, q = \frac{1}{2}, n = 10$$

$$\therefore P(8) + P(9) + P(10)$$

$$= {}^{10}C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^2 + {}^{10}C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right) + {}^{10}C_{10} \left(\frac{1}{2}\right)^{10}$$

$$= \frac{45 + 10 + 1}{1024} = \frac{7}{128}$$

(d) we have, $n = 3, p = \frac{1}{2}, q = \frac{1}{2}$

Where obtaining a head has been reckoned a success.

Now, $P(2) = {}^3C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1 = \frac{3}{8}$

Hence, it is clear that option (c) is not correct.

406 **(c)**

Total cases are $BB.BG, GB, GG = 4$

Favourable cases are $BB, BG, GB = 3$

Let $P(A)$ =Probability of a boy in two children

$= \frac{3}{4}$

Let $P(B)$ =The probability that the second child is also a boy

$= \frac{1}{4}$

Here, $P(A \cap B) = \frac{1}{4}$

$\therefore P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$

407 **(d)**

Grade order is $A > B > C > D$

For atleast C grade, he will get grade A or grade B or grade C

$\therefore$ Required probability $= 0.30 + 0.35 + 0.20 = 0.85$

408 **(d)**

A total of 9 can be obtained in the following mutually exclusive ways:

(I) 2 occurs in 3 throws out of 5 and 3 occurs in one out of the remaining 2 throws. The number of such ways is ${}^5C_3 \cdot {}^2C_1$

(II) 3 occurs three times out of 5 throws. The number of such ways is ${}^5C_3$

So, required probability

$= P(I) + P(II) = \frac{{}^5C_3 \times {}^2C_1}{6^5} + \frac{{}^5C_3}{6^5} = \frac{5}{1296}$

409 **(b)**

Total number of ways of sitting= 9!

and number of favourable ways of sitting= $2 \times 8!$

$\therefore$ Required probability= $\frac{2\times 8!}{9!} = \frac{2}{9}$

410 **(b)**

We have, $P(A \cap B) = P(A)P(B)$

Now,

$P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$\Rightarrow P(A \cap \bar{B}) = P(A) - P(A)P(B) = P(A)P(\bar{B})$

$\therefore A$ and $\bar{B}$ are independent events

412 **(d)**

Required probability

$= P$(when two persons have some birthday)

$+P$(when three persons have same birthday)

$+P$(when four persons have same birthday)

$$= {}^4C_2 \times \frac{1}{365} \times \left(\frac{364}{365}\right)^2 + {}^4C_3 \frac{1}{(365)^2} \times \frac{364}{365} + {}^4C_4\left(\frac{1}{365}\right)^3$$

$$= \frac{6(364)^2 + 4 \times 364 + 1}{(365)^3}$$

$$= \frac{794976 + 1456 + 1}{(365)^3}$$

$$= \frac{796433}{48627125} = 0.016$$

413 **(a)**

Let E =Number of ways of choosing 2 children out of 4 and 2 persons out (3+2)persons

$\therefore\ n(E) = ({}^4C_2 \times {}^5C_2)$

$\therefore$Required probability= $\frac{{}^4C_2 \times {}^5C_2}{{}^9C_4} = \frac{60}{126} = \frac{10}{21}$

414 **(c)**

The sum of the selected numbers is odd, if exactly one of them is even and one is odd.

$\therefore$ Favourable number of cases $= {}^{15}C_1 \cdot {}^{15}C_1$

$\therefore$ Required probability $= \frac{{}^{15}C_1 \cdot {}^{15}C_1}{{}^{30}C_2} = \frac{15}{29}$

415 **(a)**

Let A = getting not less than 2 and not greater than 5.

$\Rightarrow A = \{2, 3, 4, 5\}$

$\Rightarrow P(A) = \frac{4}{6}$

But dice is rolled four times, therefore the probability in getting four throws.

$= \left(\frac{4}{6}\right)\left(\frac{4}{6}\right)\left(\frac{4}{6}\right)\left(\frac{4}{6}\right) = \frac{16}{81}$

416 **(a)**

Required probability= ${}^5C_3\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^2 = \frac{5}{16}$

417 **(a)**

Total number of ways of selecting 11 players $={}^{15}C_{11}$

Favourable cases = ${}^8C_6 \times {}^7C_5$

$\therefore$ Required probability $= \frac{{}^8C_6 \times {}^7C_5}{{}^{15}C_{11}}$

418 **(a)**

The prime numbers between 2 to12 are 2, 3, 5, 7, 11

Case I When sum is 2, total cases are (1, 1) *ie*, 1

Case II When sum is 3, total cases are (1, 2), (2, 1)

$ie, 2$

Case III When sum is 5, total cases are (1, 4), (2, 3), (4, 1), (3, 2) $ie, 4$

Case IV When sum is 7, total cases are (1, 6), (2, 5), (3, 4), (6, 1), (5, 2), (4, 3) $ie, 6$

Case V When sum is 11, total cases are (5, 6), (6, 5) $ie, 2$

$\therefore$ Required probability $= \frac{15}{36} = \frac{5}{12}$

420 **(b)**

Let total number of students be 100 in which 60% girls and 40% boys.

Number of boys =40,number of girls =60

25% of boys offer Mathematics $= \frac{25}{100} \times 40 = 10$ boys

10% girls offer Mathematics $= \frac{10}{100} \times 60 = 6$ girls

It means, 16 students offer Mathematics.

$\therefore$ Required probability $= \frac{6}{16} = \frac{3}{8}$

421 **(d)**

The probability of getting a number either 2 or 3 in one toss is $\frac{1}{2}$

Condition for getting the sum 12 in five tossed is (2, 2, 2, 3, 3).

$\therefore$ Required probability $= {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$

$= \frac{5 \times 4}{2 \times 1} \left(\frac{1}{2}\right)^5$

$= 10 \cdot \frac{1}{2^5} = \frac{5}{16}$

422 **(d)**

P (selecting an apple from both baskets)

$= P$(apple from first basket) $\cdot$ P(apple from second basket)

$= \frac{{}^5C_1}{{}^{12}C_1} \cdot \frac{{}^4C_1}{{}^{12}C_1}$

P (selecting a orange from both baskets)

$= P$(orange from first basket) $\cdot$ P(orange from second basket)

$= \frac{{}^7C_1}{{}^{12}C_1} \cdot \frac{{}^8C_1}{{}^{12}C_1}$

Required probability $= \frac{{}^5C_1\,{}^4C_1}{{}^{12}C_1\,{}^{12}C_1} + \frac{{}^7C_1\,{}^8C_1}{{}^{12}C_1\,{}^{12}C_1}$

$= \frac{20 + 56}{144} = \frac{76}{144}$

423 **(c)**

The probability $= P(\overline{A}_1 \cap \overline{A}_2 \cap \overline{A}_3 \cap \ldots .\cap \overline{A}_n)$ (none of the events occur)

$= P(\overline{A}_1)P(\overline{A}_2)P(\overline{A}_3) \ldots P(\overline{A}_n)$

$= \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \ldots \frac{n}{n+1}$

$= \frac{1}{(n+1)}$

$\therefore$ Probability that atleast one of the events occurs

$= 1 - \frac{1}{n+1} = \frac{n}{n+1}$

424 **(a)**

$\therefore P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$\Rightarrow \frac{1}{2} = \frac{P(A \cap B)}{\frac{1}{4}}$

$\Rightarrow P(A \cap B) = \frac{1}{8}$

Hence, event A and B are not mutually exclusive.

$\therefore$ Statement 2 is incorrect.

$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow P(B) = \frac{P(A \cap B)}{P\left(\frac{A}{B}\right)} = \frac{\frac{1}{8}}{\frac{1}{4}}$

$\Rightarrow P(B) = \frac{1}{2}$

$\therefore$ Event A and B are independent events.

$P\left(\frac{A^c}{B^c}\right) = \frac{P(A^c \cap B^c)}{P(B^c)} = \frac{P(A^c)P(B^c)}{P(B^c)}$

$= \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{2}{1} = \frac{3}{4}$

Hence, statement 1 is correct.

Again, $P\left(\frac{A}{B}\right) + P\left(\frac{A}{B^c}\right) = \frac{1}{4} + \frac{P(A \cap B^c)}{P(B^c)}$

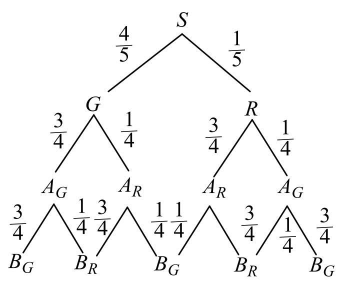
$= \frac{1}{4} + \frac{P(A) - P(A \cap B)}{P(B^c)}$

$= \frac{1}{4} + \frac{\frac{1}{4} - \frac{1}{8}}{\frac{1}{2}} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

Hence, statement 3 is correct.

425 **(c)**

From the three diagram it follows that



$P(B_G) = \frac{46}{80}$

$P(B_G \middle| G) = \frac{10}{16} = \frac{5}{8}$

$$P(B_G \cap G) = \frac{5}{8} \times \frac{4}{5} = \frac{1}{2}$$

$$P(G|B_G) = \frac{P(B_G \cap G)}{P(B_G)} = \frac{1}{2} \times \frac{80}{46} = \frac{20}{23}$$

426 **(d)**

Clearly n taken values 1,2,3,4,5. Therefore.

Total number of ways = 5

The equation $x^2 + nx + \frac{1}{2}n + \frac{1}{2} = 0$ will have real roots, if

$$n^2 - 4\left(\frac{n}{2} + \frac{1}{2}\right) \geq 0 \Rightarrow n^2 - 2n - 2 \geq 0 \Rightarrow \text{n} = 2,3,4,5.$$

So, favourable number of ways = 4

Hence, required probability = 4/5

427 **(a)**

The total number of arrangements of the letters of the word 'UNIVERSITY' is $\frac{10!}{2!}$ as there are two I's

Considering 2 I's as one letter, number of ways of arrangements in which both I's are together = 9 !. Therefore,

Number of ways in which 2 I's are not together $= \frac{10!}{2!} - 9!$

Hence, required probability $= \frac{\frac{10!}{2!} - 9!}{\frac{10!}{2!}} = 1 - \frac{2!9!}{10!} =$

45

428 **(a)**

$$P(\bar{A} \cap (B \cap \bar{C})\} = P(B \cap \bar{C}) - P(A \cap B \cap \bar{C})$$
$$= P(B) - P(B \cap C) - P(A \cap B \cap \bar{C})$$
$$\Rightarrow -P(\bar{A} \cap B \cap \bar{C}) - P(A \cap B \cap \bar{C}) + P(B) = P(B \cap C)$$
$$\Rightarrow P(B \cap C) = \frac{3}{4} - \frac{1}{3} - \frac{1}{3} = \frac{1}{12}$$

429 **(a)**

We know, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\frac{5}{6} = \frac{1}{3} + \frac{1}{2} - P(A \cap B)$$
$$\Rightarrow \frac{5}{6} = \frac{5}{6} - P(A \cap B)$$
$$\Rightarrow P(A \cap B) = 0$$

$\therefore$ Events A and B are mutually exclusive.

430 **(c)**

Given, $P(X) = \frac{e^{-m}m^x}{x!}$

$$\therefore P(X = 0) = \frac{e^{-m}1}{1}$$
$$\Rightarrow 0.8 = e^{-m} \Rightarrow -m \log_e 0.8$$
$$\Rightarrow m = \log_e \frac{10}{8} = \log_e \frac{5}{4}$$

$\therefore$ Variance $= m = \log_e \frac{5}{4}$

431 **(b)**

Given, $A \cup B = S$

$$\therefore P(A \cup B) = P(S) = 1$$
$$\Rightarrow P(A) + P(B) = 1 \quad [\because P(A \cap B) = 0]$$
$$\Rightarrow P(A) + 2P(A) = 1$$
$$\Rightarrow P(A) = \frac{1}{3} \quad [\because P(B) = 2P(A), \text{given}]$$

432 **(c)**

The required probability = 1 − probability of equal number of heads and tails.

Out of $2n$ tossed n times heads and n times tails.

$$= 1 - {}^{2n}C_n \left(\frac{1}{2}\right)^n \left(\frac{1}{2}\right)^{2n-n}$$
$$= 1 - \frac{(2n)!}{n!\,n!}\left(\frac{1}{2}\right)^{2n} = 1 - \frac{(2n)!}{(n!)^2} \cdot \frac{1}{4^n}$$

433 **(d)**

Total number of cases= $6^3 = 216$

Let A denote the event of getting at least 6, $\bar{A}$ will denote the event of getting less than 6 for three dice.

$\therefore$

$\bar{A}$={(1,1,1),(1,12),(1,1,3,),(1,2,1),(1,3,1),(2,1,2),(1,2,2),(2,2,1),(2,1,1),(3,1,1)}

$\Rightarrow n(\bar{A})$=10

Probability of getting less than 6 $= \frac{10}{216} = \frac{5}{108}$

Now, $P(A) = 1 - P(\bar{A}) = 1 - \frac{5}{108} = \frac{103}{108}$

434 **(b)**

Required probability $= 1 - \frac{n^{\frac{n(n+1)}{2}}}{n^{n^2}}$

435 **(c)**

$$P\left(\frac{G}{A}\right) = \frac{{}^4C_1}{{}^7C_1} = \frac{4}{7} \text{ and } P\left(\frac{G}{B}\right) = \frac{{}^3C_1}{{}^7C_1} = \frac{3}{7}$$

Now, $P\left(\frac{B}{G}\right)=\frac{P(G).P\left(\frac{G}{B}\right)}{P(A)P\left(\frac{G}{A}\right)+P(B)P\left(\frac{G}{B}\right)}$

$$=\frac{\frac{1}{2}\cdot\frac{3}{7}}{\frac{1}{2}\cdot\frac{4}{7}+\frac{1}{2}\cdot\frac{3}{7}}=\frac{3}{7}$$

436 **(c)**

P(getting 2 different coloured cards)

$$=\frac{{}^{26}C_1\times{}^{26}C_1}{{}^{52}C_2}$$

$$=\frac{26\times 26\times 2}{52\times 51}=\frac{26}{51}$$

437 **(c)**

Given that, probability of success $p=\frac{1}{4}$ and probability of unsuccess $q=\frac{3}{4}$

$\therefore$ Mean $=np$

and standard deviation $=\sqrt{\text{variance}}$

$\Rightarrow 3=\sqrt{\text{Variance}}$

$\Rightarrow$ variance $=9$

$\Rightarrow npq=9$

$\Rightarrow n\cdot\frac{1}{4}\cdot\frac{3}{4}=9$

$\Rightarrow n=\frac{9\cdot 4\cdot 4}{3}\ \Rightarrow n=48$

$\therefore$ Mean $=np=\frac{1}{4}\times 48=12$

438 **(d)**

On the basis of past records, the probability of safe arrival of a vessel on Mumbai harbor is 7/9. Since, arrival of all the vessels is independent and there are only two possibilities on every namely safe arrival and unsafe arrival. We can use the binomial distribution. Here

$n=3, p=\frac{7}{9}, q=\frac{2}{9}$ and $r=2$

$\therefore P(2)={}^3C_2\left(\frac{7}{9}\right)^2\left(\frac{2}{9}\right)=\frac{98}{243}$

439 **(a)**

Let X denote the number of heads in n trials. Then,

$$P(X=r)={}^nC_r\left(\frac{1}{2}\right)^r\left(\frac{1}{2}\right)^{n-r}={}^nC_r\left(\frac{1}{2}\right)^n$$

$\therefore$ Required probability

$=P(X=1)+P(X=3)+P(X=5)+\cdots$

$={}^nC_1\left(\frac{1}{2}\right)^n+{}^nC_3\left(\frac{1}{2}\right)^n+{}^nC_5\left(\frac{1}{2}\right)^n+\cdots$

$=\left(\frac{1}{2}\right)^n\{{}^nC_1+{}^nC_3+{}^nC_5+\cdots\}=\frac{1}{2^n}(2^{n-1})=\frac{1}{2}$

441 **(c)**

Any month out of 12 months can be chosen with probability $\frac{1}{12}$

There are 7possiblle ways, in which the month can start and it will be a Sunday on 6th day, if the first day of the month is Tuesday, whose probability is $\frac{1}{7}$

$\therefore$ Required probability is $\frac{1}{7}$

$\therefore$ Required probability$=\frac{1}{12}\times\frac{1}{7}=\frac{1}{84}$

442 **(a)**

There are 11 letters in the word "REGULATIONS" which can be arranged in 111 ways

Other than R and E there are 9 letters out of which 4 can be chosen in ${}^9C_4$ ways. These four letters can be arranged between R and E in 4! Ways. Also, R and E can interchange their positions in 2! Ways.

$\therefore$ Number of ways in which there are exactly four letters between R and $E={}^9C_4\times 4!\times 2!$

Considering this group of 6 letters as one letters and the remaining 5 letters can be arranged in 6 ! ways

$\therefore$ Number of arrangements of the letters of the word "REGULATIONS" in which there are exactly four letters between R and $E={}^9C_4\times 4!\times 2!\times 6!$

Hence, required probability $=\frac{{}^9C_4\times 4!\times 2!\times 6!}{11!}=\frac{6}{55}$

443 **(c)**

We have,

Total number of arrangements of 5 objects $=5!$

We know that the total number of de-rangements of n objects in which none of the object occupies its original position is given by

$$n!\left\{1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\ldots+\frac{(-1)^n}{n!}\right\}$$

Therefore, the total number of de-rangements in which none of the 5 object occupies the place corresponding to it

$$=5!\left\{1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right\}=44$$

Hence, required probability $=\frac{44}{120}=\frac{11}{30}$

444 **(d)**

$\because$ Probability $=P(M\cap\overline{N})+P(\overline{M}\cap N)$

$\Rightarrow\ P=P(M)-P(M\cap N)+P(N)-P(M\cap N)$

$\Rightarrow\ P=P(M)+P(N)-2P(M\cap N)$

445 **(a)**

Total number of favorable cases =6

Total number of cases =216

$\therefore$ Required probability $=\frac{6}{216}=\frac{1}{36}$

446 **(c)**

There are 10 numbers from 50 to 59 such that each has a digit 5 and there are 9 other numbers, 5,15,25,35,45,65,75,85,95 each containing a digit 5

So, the favourable number of elementary events = 19

Total number of elementary events is 100

Hence, required probability $= \frac{19}{100}$

447 **(a)**

The total number of ways in which 12 persons can stand in a ring $= 11\,!$. Three persons between A and B can be selected in ${}^{10}C_3$ ways. A and B can interchange their positions in $2\,!$ ways. Also, 3 persons between A and B can stand in $3\,!$ ways and the other in 7 in $7\,!$ ways

$\therefore$ Favourable number of ways $= 2\,!\,3\,!\,7\,! \cdot {}^{10}C_3 = 2!\,10!$

Hence, required probability $= \frac{2\,!10\,!}{11\,!} = \frac{2}{11}$

448 **(c)**

As we know, the sum of probability density function is one.

$$\therefore P(X = x_1) + P(X = x_2) + \ldots + P(X = x_{10}) = 1$$

$$\Rightarrow 1k + 2k + 3k + \ldots + 10k = 1$$

$$\Rightarrow \frac{10(10+1)}{2}k = 1$$

$$\Rightarrow k = \frac{1}{55}$$

449 **(a)**

Let $P(A) = \frac{20}{100} = \frac{1}{5}, P(B) = \frac{10}{100} = \frac{1}{10}$

Since, events are independent and we have to find

$$P(A \cup B) = P(A) + P(B) - P(A).P(B)$$

$$= \frac{1}{5} + \frac{1}{10} - \frac{1}{5}.\frac{1}{10}$$

$$= \frac{14}{50} \times 100 = 28\%$$

450 **(d)**

Let each of the friends has x daughters. Then, Probability that all the tickets go to the daughters of $A = \frac{{}^{x}C_3}{{}^{2x}C_3}$

$$\therefore \frac{{}^{x}C_3}{{}^{2x}C_3} = \frac{1}{20} \Rightarrow \frac{x-2}{4(2x-1)} = \frac{1}{20} \Rightarrow 5x - 10 = 2x - 1 \Rightarrow x = 3$$

451 **(c)**

We can choose three vertices out of 6 in ${}^{6}C_3 = 20$ ways. Chosen vertices can form an equilateral triangle in just two ways

viz, A_1, A_3, A_5 and A_2, A_4, A_6.

$\therefore$ Required probability $= \frac{2}{20} = \frac{1}{10}$.

452 **(a)**

Given, the probability of solving the problem are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$ respectively and corresponding probabilities of not solving the problem are $\frac{2}{3}, \frac{3}{4}$ and $\frac{4}{2}$ respectively

$\therefore$ Required probability

$$= 1 - P(\text{not solving the problem})$$

$$= 1 - \left(\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}\right)$$

$$= 1 - \frac{2}{5} = \frac{3}{5}$$

453 **(a)**

The number formed is odd if it has 1,3 or 5 at units place. Therefore, units place can be filled in 3 ways and the remaining 3 places can be filled with other digits in $3\,!$ ways.

Hence, the number of ways in which odd numbers can be formed is $3(3\,!) = 18$

Hence, required probability $= \frac{18}{24} = \frac{3}{4}$

454 **(d)**

$$P(A) = \frac{2}{5}$$

For independent events,

$$P(A \cap B) = P(A)P(B)$$

$$\Rightarrow P(A \cap B) \leq \frac{2}{5}$$

$$\Rightarrow P(A \cap B) = \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}$$

[Maximum 4 outcomes may be in $P(A \cap B)$]

1. When $P(A \cap B) = \frac{1}{10}$

$$\Rightarrow P(A).P(B) = \frac{1}{10}$$

$\Rightarrow P(B) = \frac{1}{10} \times \frac{5}{2} = \frac{1}{4}$, not possible

2. When $P(A \cap B) = \frac{2}{10} \Rightarrow \frac{2}{5} \times P(B) = \frac{2}{10}$

$\Rightarrow P(B) = \frac{5}{10}$, outcomes of $B = 5$

3. When $P(A \cap B) = \frac{3}{10}$

$\Rightarrow P(A)P(B) = \dfrac{3}{10}$

$\Rightarrow \dfrac{2}{5} \times P(B) = \dfrac{3}{10}$

$P(B) = \frac{3}{4}$,not possible

4. When $P(A \cap B) = \frac{4}{10}$

$\Rightarrow P(A).P(A) = \dfrac{4}{10}$

$\Rightarrow P(B) = 1$, outcomes of $B = 10$

455 **(c)**

We have, $P(A \cup B) = 0.6$ and $P(A \cap B) = 0.2$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow 0.6 = P(A) + P(B) - 0.2$

$\Rightarrow P(A) + P(B) = 0.8$

$\Rightarrow 1 - P(\bar{A}) + 1 - P(\bar{B}) = 0.8 \Rightarrow P(\bar{A}) + P(\bar{B}) = 1.2$

456 **(d)**

Total number of socks = 5+4 = 9

The number of ways to select 2 socks out of 9 $= {}^9C_2$

Number of ways to select both brown socks $= {}^5C_2$

And number of ways to select both white socks $= {}^4C_2$

$\therefore P(\text{Either both brown or white}) = \frac{{}^5C_2 + {}^4C_2}{{}^9C_2}$

$= \dfrac{\frac{5!}{3!\cdot 2!} + \frac{4!}{2!\cdot 2!}}{\frac{9!}{7!\cdot 2!}} = \dfrac{10+6}{36}$

$= \dfrac{16}{36} \times \dfrac{3}{3} = \dfrac{48}{108}$

457 **(c)**

Total number of ways to from the numbers of five digits with 1, 2, 3, 4, 5 are $= 5! = n\,(S)$

Total number of numbers which are divisible by 4 $n(E) = 3! \times 4 = 4!$

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{4!}{5!} = \frac{1}{5}$

458 **(d)**

Let p be the probability of success in a trial. Then,

$p = 2(1-p) \Rightarrow p = \dfrac{2}{3}$

$\therefore$ Required probability

$= {}^6C_4 \left(\dfrac{2}{3}\right)^4 \left(\dfrac{1}{3}\right)^2 + {}^6C_5 \left(\dfrac{2}{3}\right)^5 \left(\dfrac{1}{3}\right) + {}^6C_6 \left(\dfrac{2}{3}\right)^6$

$= \dfrac{496}{729}$

460 **(a)**

$P(E) = P(X=2) + P(X=3) + P(X=5) + P(X=7)$

$= 0.23 + 0.12 + 0.20 + 0.07 = 0.62$

$P(F) = P(X=1) + P(X=2) + P(X=3)$

$= 0.15 + 0.23 + 0.12 = 0.5$

$P(E \cap F) = P(X=2) + P(X=3)$

$= 0.23 + 0.12 = 0.35$

$\therefore P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= 0.62 + 0.5 - 0.35 = 0.77$

461 **(c)**

Given $P(X=1) = P(X=2)$

$\therefore \dfrac{e^{-\lambda}\lambda^1}{1!} = \dfrac{e^{-\lambda}\lambda^2}{2!} \Rightarrow \lambda = 2$

$\therefore P(X=4) = \dfrac{e^{-2}(2)^4}{4!} = \dfrac{2}{3e^2}$

462 **(a)**

Out of 90 tickets, two tickets already considered, instead of selecting 5 tickets we have to select only 3 tickets out of 88 tickets.

$\therefore$ Required probability $= \frac{{}^{88}C_3}{{}^{90}C_5}$

$= \dfrac{\frac{88\times87\times86}{3\times2\times1}}{\frac{90\times89\times88\times87\times86}{5\times4\times3\times2\times1}}$

$= \dfrac{5\times4}{90\times89} = \dfrac{2}{801}$

463 **(b)**

Required probability $= \left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{3}\right) = \frac{1}{2}$

464 **(d)**

The total number of ways of choosing 11 players out of 15 is ${}^{15}C_{11}$

A team of 11 players containing at least 3 bowlers can be chosen in the following mutually exclusive ways:

(I) Three bowlers out of 5 bowlers and 8 other players out of the remaining 10 players

(II) Four bowlers out of 5 bowlers and 7 other players out of the remaining 10 players

(III) Five bowlers out of 5 bowlers and 6 other players out of the remaining 10 players

$\therefore$ required probability $= P(\text{I}) + P(\text{II}) + P(\text{III})$

$$= \frac{^5C_3 \times {}^{10}C_8}{^{15}C_{11}} + \frac{^5C_4 \times {}^{10}C_7}{^{15}C_{11}} + \frac{^5C_5 \times {}^{10}C_6}{^{15}C_{11}}$$

$$= \frac{1260}{1365} = \frac{12}{13}$$

465 **(c)**

Let A be the event of selecting bag X, B be the event of selecting bag Y and E be the event of drawing a white ball, then probability of selecting a bag

$P(A) = \frac{1}{2}, P(B) = \frac{1}{2}$,

$P\left(\frac{E}{A}\right) = \frac{2}{5}, P\left(\frac{E}{B}\right) = \frac{4}{6} = \frac{2}{3}$.

Required probability,

$$P(E) = P(A)\, P\left(\frac{E}{A}\right) + P(B)\, P\left(\frac{E}{B}\right)$$

$$= \frac{1}{2} \cdot \frac{2}{5} + \frac{1}{2} \cdot \frac{2}{3}$$

$$= \frac{8}{15}$$

466 **(d)**

$\therefore$ Required probability$= \frac{^4C_2 + {}^5C_2}{^9C_2}$

$$= \frac{6+10}{36} = \frac{16}{36}$$

$$= \frac{4}{9}$$