Single Correct Answer Type

1. For all positive integral values of $n, 3^{2n} - 2n + 1$ is divisible by
a) 2 b) 4 c) 8 d) 12

2. $3 + 13 + 29 + 51 + 79 + \cdots$ to n terms $=$
a) $2n^2 + 7n^3$ b) $n^2 + 5n^3$ c) $n^3 + 2n^2$ d) None of these

3. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all $n \geq 1$, by the principle of mathematical induction?
a) $A^n = 2^{n-1}A + (n-1)I$ b) $A^n = nA + (n-1)I$
c) $A^n = 2^{n-1}A - (n-1)I$ d) $A^n = nA - (n-1)I$

4. If $(n): 1 + 3 + 5 + \ldots + (2n - 1) = n^2$ is
a) True for all $n \in N$ b) True for $n > 1$ c) True for no n d) None of these

5. For a positive integer n, Let $a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{(2^n) - 1}$. Then
a) $a(100) \leq 100$ b) $a(100) > 100$ c) $a(200) \leq 100$ d) None of these

6. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then for $n \in N$, A^n is equal to
a) $\begin{bmatrix} \cos^n\theta & \sin^n\theta \\ -\sin^n\theta & \cos^n\theta \end{bmatrix}$ b) $\begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$ c) $\begin{bmatrix} n\cos\theta & n\sin\theta \\ -n\sin\theta & n\cos\theta \end{bmatrix}$ d) None of the above

7. The sum of the cubes of three consecutive natural numbers is divisible by
a) 7 b) 9 c) 25 d) 26

8. For all $n \in N, \sum n$
a) $< \frac{(2n+1)^2}{8}$ b) $> \frac{(2n+1)^2}{8}$ c) $= \frac{(2n+1)^2}{8}$ d) None of these

9. The product of three consecutive natural numbers is divisible by
a) 5 b) 7 c) 6 d) 4

10. For all $n \in N, 5^{2n} - 1$ is divisible by
a) 6 b) 11 c) 24 d) 26

11. $7^{2n} + 3^{n-1}.2^{3n-3}$ is divisible by
a) 24 b) 25 c) 9 d) 13

12. For $n \in N, 10^{n-2} \geq 81n$ is
a) $n > 5$ b) $n \geq 5$ c) $n < 5$ d) $n > 8$

13. For all $n \in N, \frac{n^5}{5} + \frac{n^3}{3} + \frac{7}{15n}$ is
a) An integer b) A natural number c) A positive fraction d) None of these

14. Let $S(k) = 1 + 3 + 5 \ldots . + (2k - 1) = 3 + k^2$. Then, which of the following is true?
a) $S(1)$ is correct b) $S(k) \Rightarrow S(k+1)$
c) $S(k) \nRightarrow S(k+1)$ d) Principle of mathematical induction can be used to prove the formula

15. The smallest positive integer n for which $n! < \left(\frac{n+1}{2}\right)^n$ holds, is
a) 1 b) 2 c) 3 d) 4

16. The remainder when 5^{99} is divided by 13, is
a) 6 b) 8 c) 9 d) 10

17. $10^n + 3(4^{n+2}) + 5$ is divisible by $(n \in N)$
a) 7 b) 5 c) 9 d) 17

18. For all $n \in N, n^3 + 2n$ is divisible by
a) 3 b) 8 c) 9 d) 11

19. For all $n \in N, 7^{2n} - 48n - 1$ is divisible by
a) 25 b) 26 c) 1234 d) 2304

20. $10^n + 3(4^{n+2}) + 5$ is divisible by $(n \in N)$
a) 7 b) 5 c) 9 d) 17
21. The nth term of the series $4 + 14 + 30 + 52 + 80 + 114 + \cdots$ is
a) $5n - 1$ b) $2n^2 + 2n$ c) $3n^2 + n$ d) $2n^2 + 2$
22. If $P(n)$ is a statement $(n \in N)$ such that, if $P(k)$ is true, $P(k+1)$ is true for $k \in N$, then $p(n)$ is true
a) For all n b) For all $n > 1$ c) For all $n > 2$ d) Nothing can be said
23. Using mathematical induction, then numbers $a_n's$ are defined by
a_0=1, $a_{n+1} = 3n^2 + n + a_n$, $(n \geq 0)$ Then, a_n is equal to
a) $n^3 + n^2 + 1$ b) $n^3 - n^2 + 1$ c) $n^3 - n^2$ d) $n^3 + n^2$
24. $\frac{(n+2)!}{(n-1)!}$ is divisible by
a) 6 b) 11 c) 24 d) 26
25. If $P(n) = 2 + 4 + 6 + \ldots + 2n, n \in N$, then $P(k) = k(k+1) + 2 \Rightarrow P(k+1) = (k+1)(k+2) + 2$ for all $k \in N$. So, we can conclude that $P(n) = n(n+1) + 2$ for
a) All $n \in N$ b) $n > 1$ c) $n > 2$ d) Nothing can be said
26. For all $n \in N$, $2 \cdot 4^{2n+1} + 3^{3n+1}$ is divisible by
a) 2 b) 9 c) 3 d) 11
27. For all $n \in N, n^4$ is less than
a) 10^n b) 4^n c) 5^n d) 10^{10}
28. The number $a^n - b^n (a, b$ are distinct rational numbers and $n \in N)$ is always divisible by
a) $a - b$ b) $a + b$ c) $2a - b$ d) $a - 2b$
29. If $n \in N$, then $3^{2n} + 7$ is divisible by
a) 3 b) 8 c) 9 d) 11
30. For each, $n \in N$, $10^{2n-1} + 1$ is divisible by
a) 11 b) 13 c) 9 d) None of these
31. If $10^n + 3.4^{n+2} + \lambda$ is exactly divisible by 9 for all $n \in N$, then the least positive integral value of λ is
a) 5 b) 3 c) 7 d) 1
32. For all $n \in N$, $\cos\theta \cos 2\theta \cos 4\theta \ldots \cos 2^{n-1}\theta$ equals to
a) $\frac{\sin 2^n \theta}{2^n \sin\theta}$ b) $\frac{\sin 2^n \theta}{\sin\theta}$ c) $\frac{\cos 2^n \theta}{2^n \cos 2\theta}$ d) $\frac{\cos 2^n \theta}{2^n \sin\theta}$
33. The inequality $n! > 2^{n-1}$ is true for
a) $n > 2$ b) $n \in N$ c) $n > 3$ d) None of these
34. If $P(n): 2 + 4 + 6 \ldots + (2n), n \in N$, then
$P(k) = k(k+1) + 2$implies
$P(k) = (k+1)(k+2) + 2$
is true for all $k \in N$. So, statement $P(n) = n(n+1) + 2$ is true for
a) $n \geq 1$ b) $n \geq 2$ c) $n \geq 3$ d) None of these
35. If $P(n): 3^n < n!, n \in N$, then $P(n)$ is true
a) For $n \geq 6$ b) For $n \geq 7$ c) For $n \geq 3$ d) For all n
36. Let $P(n)$ denotes the statement that $n^2 + n$ is odd. It is seen that $P(n) \Rightarrow P(n+1), P(n)$ is true for all
a) $n > 1$ b) n c) $n > 2$ d) None of these
37. The sum to n terms of the series $1^3 + 3^3 + 5^3 + \cdots$ is
a) $n^2(n^2 - 1)$ b) $n^2(2n^2 - 1)$ c) $n^2(2n^2 + 1)$ d) $n^2(n^2 + 1)$
38. If $n \in N$, then $11^{n+2} + 12^{2n+1}$ is divisible by
a) 113 b) 123 c) 133 d) None of these
39. For natural number $n, 2^n (n-1)! < n^n$, if
a) $n < 2$ b) $n > 2$ c) $n \geq 2$ d) never
40. If $n \in N$, then $n(n^2 - 1)$ is divisible by
a) 6 b) 16 c) 36 d) 24
41. $(2^{3n} - 1)$ will be divisible by $(\forall n \in N)$
a) 25 b) 8 c) 7 d) 3

42. If $n \in N$, then $x^{2n-1}y^{2n-1}$ is divisible by

a) $x+y$ b) $x-y$ c) x^2+y^2 d) None of these

43. If $x^{2n-1}+y^{2n-1}$ is divisible by $x+y$, if n is

a) A positive integer b) An even positive integer

c) An odd positive integer d) None of the above

44. If m, n are any two odd positive integer with $n < m$, then the largest positive integers which divides all the numbers of the type m^2-n^2 is

a) 4 b) 6 c) 8 d) 9

45. If x^n-1 is divisible by $x-k$, then the least positive integral value of k is

a) 1 b) 2 c) 3 d) 4

46. If n is a positive integer, then $5^{2n+2}-24n-25$ isdivisible by

a) 574 b) 575 c) 675 d) 576

47. For all $n \in N, 3^{3n}-26^n-1$ is divisible by

a) 24 b) 64 c) 17 d) 676

48. Matrix A is such that $A^2=2A-I$ where I is the identity matrix, then for $n \geq 2, A^n$ is equal to

a) $n\,A-(n-1)I$ b) $n\,A-I$ c) $2^{n-1}A-(n-1)I$ d) $2^{n-1}A-I$

49. If $a_1=1$ and $a_n=na_{n-1}$ for all positive integer $n \geq 2$, then a_5 is equal to

a) 125 b) 120 c) 100 d) 24

50. For all $n \in N, 1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\cdots+\frac{1}{\sqrt{n}}$ is

a) Equal to $\sqrt{n}$

b) Less than or equal to $\sqrt{n}$

c) Greater than or equal to $\sqrt{n}$

d) None of these

51. Let $P(n): n^2+n+1$ is an even integer. If $P(k)$ is assumed true $\Rightarrow P(k+1)$ is true. Therefore, $P(n)$ is true

a) For $n>1$ b) For all $n \in N$ c) For $n>2$ d) None of these

52. $2^{3n}-7n-1$ is divisible by

a) 64 b) 36 c) 49 d) 25

53. For all $n \in N, 3n^5+5n^3+7n$ is divisible by

a) 3 b) 5 c) 10 d) 15

54. If n is a positive integer, then n^3+2n is divisible by

a) 2 b) 6 c) 15 d) 3

55. For all $n \in N, 49^n+16n-1$ is divisible by

a) 64 b) 8 c) 16 d) 4

56. If $P(n)$ is a statement such that $P(3)$ is true. Assuming $P(k)$ is true $\Rightarrow P(k+1)$ is true for all $k \geq 3$, then $P(n)$ is true

a) For all n b) For $n \geq 3$ c) For $n>4$ d) None of these

57. If n is an odd positive integer, then a^n+b^n is divisible by

a) $a+b$ b) $a-b$ c) a^2+b^2 d) None of these

58. The nth terms of the series $3+7+13+21+\cdots$ is

a) $4n-1$ b) n^2+2n c) n^2+n+1 d) n^2+2

59. If a, b are distinct rational numbers, then for all $n \in N$ the number a^n-b^n is divisible by

a) $a-b$ b) $a+b$ c) $2a-b$ d) $a-2b$

60. $x(x^{n-1}-n\alpha^{n-1})+\alpha^n(n-1)$ is divisible by $(x-\alpha)^2$ for

a) $n>1$ b) $n>2$ c) All $n \in N$ d) None of these

61. $2^{3n}-7n-1$ is divisible by

a) 64 b) 36 c) 49 d) 25

62. The sum of n terms of the series $1+(1+a)+(1+a+a^2)+(1+a+a^2+a^3)+\cdots$, is

a) $\frac{n}{1-a}-\frac{a(1-a^n)}{(1-a)^2}$ b) $\frac{n}{1-a}+\frac{a(1-a^n)}{(1-a)^2}$ c) $\frac{n}{1-a}+\frac{a(1+a^n)}{(1-a)^2}$ d) $-\frac{n}{1-a}+\frac{a(1-a^n)}{(1-a)^2}$

63. For each $n \in N, 3^{2n}-1$ is divisible by

a) 8 b) 16 c) 32 d) None of these

64. If n is an even positive integer, then $a^n + b^n$ is divisible by

a) $a + b$ b) $a - b$ c) $a^2 - b^2$ d) None of these

65. The greatest positive integer, which divides $(n + 2)(n + 3)(n + 4)(n + 5)(n + 6)$ for all $n \in N$, is

a) 4 b) 120 c) 240 d) 24

66. If $3 + 5 + 9 + 17 + 33 + \cdots$ to n terms $= 2^{n+1} + n - 2$, then nth term of LHS is

a) $3^n - 1$ b) $2n + 1$ c) $2^n + 1$ d) $3n - 1$

67. For all $n \in N$, $10^n + 3.4^{n+2} + 5$ is divisible by

a) 23 b) 3 c) 9 d) 207

68. For all $n \in N$, $4^n - 3n - 1$ is divisible by

a) 3 b) 8 c) 9 d) 11

: ANSWER KEY :

1)	**a**	**2)**	**c**	**3)**	**d**	**4)**	**a**
5)	**a**	**6)**	**b**	**7)**	**b**	**8)**	**a**
9)	**c**	**10)**	**c**	**11)**	**b**	**12)**	**b**
13)	**b**	**14)**	**b**	**15)**	**b**	**16)**	**b**
17)	**c**	**18)**	**a**	**19)**	**d**	**20)**	**c**
21)	**c**	**22)**	**d**	**23)**	**b**	**24)**	**a**
25)	**d**	**26)**	**d**	**27)**	**a**	**28)**	**a**
29)	**b**	**30)**	**a**	**31)**	**a**	**32)**	**a**
33)	**a**	**34)**	**d**	**35)**	**b**	**36)**	**d**
37)	**b**	**38)**	**c**	**39)**	**b**	**40)**	**a**
41)	**c**	**42)**	**a**	**43)**	**a**	**44)**	**c**
45)	**a**	**46)**	**d**	**47)**	**d**	**48)**	**a**
49)	**b**	**50)**	**b**	**51)**	**d**	**52)**	**c**
53)	**d**	**54)**	**d**	**55)**	**a**	**56)**	**b**
57)	**a**	**58)**	**c**	**59)**	**a**	**60)**	**c**
61)	**c**	**62)**	**a**	**63)**	**a**	**64)**	**d**
65)	**b**	**66)**	**c**	**67)**	**c**	**68)**	**c**

: HINTS AND SOLUTIONS :

1 **(a)**

On putting $n = 2$ in $3^{2n} - 2n + 1$, we get

$3^{2\times2} - 2 \times 2 + 1 = 81 - 4 + 1 = 78$

Which is divisible by 2

2 **(c)**

Clearly, $n^3 + 2n^2$ gives the sum of the series for $n = 1, 2, 3$ etc.

3 **(d)**

$A^2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

$A^n = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ can be verified by induction. Now, taking option

(b) $\begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix} = \begin{bmatrix} n & 0 \\ n & n \end{bmatrix} + \begin{bmatrix} n-1 & 0 \\ 0 & n-1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix} \neq \begin{bmatrix} 2n-1 & 0 \\ 1 & 2n-1 \end{bmatrix}$

(d)$nA - (n-1)I = \begin{bmatrix} n & 0 \\ n & n \end{bmatrix} - \begin{bmatrix} n-1 & 0 \\ 1 & n-1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix} = A^n$

4 **(a)**

Given, $(n): 1 + 3 + 5 + \ldots + (2n-1) = n^2$

$P(1): 1 = 1$(true)

Let $P(k) = 1 + 3 + 5 + \ldots + (2k-1) = k^2$

$\therefore P(k+1) = 1 + 3 + 5 + \ldots + (2k-1) + 2k + 1$

$= k^2 + 2k + 1 = (k+1)^2$

So, it holds for all n.

5 **(a)**

It can be proved with the help of mathematical induction that $\frac{n}{2} < a(n) \leq n$.

$\therefore \dfrac{200}{2} < a(200) \Rightarrow a(200) > 100$

and $a(100) \leq 100$

6 **(b)**

Given, $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

Now, $A^2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$= \begin{bmatrix} \cos^2\theta - \sin^2\theta & 2\sin\theta\cos\theta \\ -2\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$

$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

$\therefore$ By induction, $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$

9 **(c)**

Let $P(n) \equiv n(n+1)(n+2)$

$P(1) \equiv 1 \cdot 2 \cdot 3 = 6$

$P(2) \equiv 2 \cdot 3 \cdot 4 = 24$

Hence, it is divisible by 6.

10 **(c)**

We have,

$5^{2n} - 1 = (5^2)^n - 1 = (1 + 24)^n - 1$

$\Rightarrow 5^{2n} - 1 = {}^nC_1 \times 24 + {}^nC_2 \times 24^2 + \cdots + {}^nC_n \times 24^n$

$\Rightarrow 5^{2n} - 1 = 24({}^nC_1 + {}^nC_2 \times 24 + \cdots + {}^nC_n \times 24^{n-1})$

$\Rightarrow 5^{2n} - 1$ is divisible by 24 for all $n \in N$

12 **(b)**

Let $P(n): 10^{n-2} \geq 81n$

For $n = 4, 10^2 \ngeq 81 \times 4$

For $n = 5, 10^3 \geq 81 \times 5$

Hence, by mathematical induction for $n \geq 5$, the proposition is true.

14 **(b)**

$S(k) = 1 + 3 + 5 \ldots + (2k-1) = 3 + k^2$

Put $k = 1$ in both sides, we get

LHS = 1 and RHS= $3 + 1 = 4$

$\Longrightarrow$ LHS $\neq$ RHS

Put $(k+1)$ in both sides in the place of k, we get

LHS $= 1 + 3 + 5 \ldots + (2k-1) + (2k+1)$

RHS $= 3 + (k+1)^2 = 3 + k^2 + 2k + 1$

Let LHS = RHS

Then,$1 + 3 + 5 \ldots + (2k-1) + (2k+1)$

$= 3 + k^2 + 2k + 1$

$\Longrightarrow 1 + 3 + 5 + \ldots + (2k-1) = 3 + k^2$

If $S(k)$ is true, then $S(k+1)$ is also true.

Hence, $S(k) \Longrightarrow S(k+1)$

15 **(b)**

Given, $n! < \left(\frac{n+1}{2}\right)^n$

At $n = 1$,

$1! \nless 1$

At $n = 2$,

$2! < \left(\dfrac{3}{2}\right)^2$

$\Rightarrow 2 < 2.25$ which is true.

16 **(b)**

$5^{99} = 5(5^2)^{49} = 5(25)^{49}$

$= 5(26 - 1)^{49}$

$= 5 \times 26 \times$ (Positive term) $- 5$

So, when it is divided by 13, it gives the remainder -5 or 8.

17 **(c)**

On putting $n = 2$ in $10^n + 3(4^{n+2}) + 5$, we get

$10^2 + 3 \times 4^4 + 5 = 100 + 768 + 5 = 873$

Which is divisible by 9

18 **(a)**

For $n = 1, 2, 3$, we find that $n^3 + 2n$ takes values 3, 12 and 33, which are divisible by 3

19 **(d)**

We have,

$7^{2n} - 48n - 1 = (1 + 48)^n - 48n - 1$

$\Rightarrow 7^{2n} - 48n - 1$

$= {}^nC_2 \times 48^2 + {}^nC_3 \times 48^3 + \cdots + {}^nC_n \times 48^n$

$\Rightarrow 7^{2n} - 48n - 1$ divisible by 48^2 i.e., 2304

20 **(c)**

For $n = 1, 10^n + 3 \cdot 4^{n+2} + 5$

$= 10 + 3 \cdot 4^3 + 5 = 207$ which is divisible by 9.

$\therefore$ By induction, the result is divisible by 9.

21 **(c)**

We observe that $3n^2 + n$ gives various terms of the series by putting $n = 1, 2, 3, \dots$

22 **(d)**

Unless we prove $P(1)$ is true, nothing can be said.

23 **(b)**

Given, $a_0 = 1, a_{n+1} = 3n^2 + n + a_n$

$\Longrightarrow a_1 = 3(0) + 0 + a_0 = 1$

$\Longrightarrow a_2 = 3(1)^2 + 1 + a_1 = 3 + 1 + 1 = 5$

From option (b),

Let $P(n) = n^3 - n^2 + 1$

$\therefore \quad P(0) = 0 - 0 + 1 = 1 = a_0$

$P(1) = 1^3 - 1^2 + 1 = 1 = a_1$

and $P(2) = (2)^3 - (2)^2 + 1 = 5 = a_2$

25 **(d)**

It is obvious, nothing can be said.

26 **(d)**

Let $P(n) = 2 \cdot 4^{2n+1} + 3^{3n+1}$

$P(1) \equiv 128 + 81 = 209$(divisible by 11 only)

28 **(a)**

Let $P(n) \equiv a^n - b^n$

$P(1) \equiv a - b$

$P(2) \equiv a^2 - b^2$

Hence, it is divisible by $a - b$.

29 **(b)**

$3^{2n} + 7$ is divisible by 8. This can be checked by putting $n = 1, 2, 3$ etc.

30 **(a)**

Let $P(n) = 10^{2n-1} + 1$

$P(1) = 10 + 1 = 11$

Let $P(k) \equiv 10^{2k-1} + 1 = 11I$ is true

Now, $P(k+1) = 10^{2k+1} + 1$

$= (11I - 1)100 + 1$

$= 1100I - 99 = 11I_1$

So, $P(k+1)$ is true.

33 **(a)**

Let $P(n) \equiv n! > 2^{n-1}$

$P(3) \equiv 6 > 4$

Let $P(k) \equiv k! > 2^{k-1}$ is true.

$\therefore P(k+1) = (k+1)! = (k'+1)k!$

$> (k+1)2^{k-1}$

$> 2^k \quad$ (as $k + 1 > 2$)

34 **(d)**

Here, $P(1) = 2$ and from the equation

$P(k) = k(k+1) + 2$

$\Longrightarrow \quad P(1) = 4$

So, $P(1)$ is not true

Hence, mathematical induction is not applicable.

35 **(b)**

Given that, $P(n): 3^n < n!$

Now, $P(7): 3^7 < 7!$ is true

Let $P(k): 3^k < k!$

$\Rightarrow P(k+1): 3^{k+1} = 3 \cdot 3^k < 3 \cdot k! < (k+1)! \quad (\because k + 1 > 3)$

36 **(d)**

$P(n) = n^2 + n$

It is always odd but square of any odd number is always odd and also sum of two odd number is always even. So, for no any n for which this statement is true.

37 **(b)**

$n^2(2n^2 - 1)$ gives the sum of the series for $n = 1, 2,$ etc.

38 **(c)**

On putting $n = 1$ in $11^{n+2} + 12^{2n+1}$, we get

$11^{1+2} + 12^{2\times1+1} = 11^3 + 12^3 = 3059$

Which is divisible by 133

39 **(b)**

The condition $2^n(n-1)! < n^n$ is satisfied for $n > 2$

40 **(a)**

We have,

$n(n^2 - 1) = (n-1)(n+1)$, which is product of three consecutive natural numbers and hence divisible by 6

41 **(c)**

$2^{3n} - 1 = (2^3)^n - 1$

$= 8^n - 1 = (1 + 7)^n - 1$

$= 1 + {}^nC_1 7 + {}^nC_2 7^2 + \dots + {}^nC_n 7^n - 1$

$= 7[{}^nC_1 + {}^nC_2 7 + \dots + {}^nC_n 7^{n-1}]$

$\therefore 2^{3n} - 1$ is divisible by 7

43 **(a)**

Let $P(n) \equiv x^{2n-1} + y^{2n-1} = \lambda(x + y)$

$P(1) \equiv x + y = \lambda_1(x + y)$

$P(2) \equiv x^3 + y^3 = \lambda_2(x + y)$

Hence, for $\forall\, n \in N, P(n)$ is true.

44 **(c)**

Let $m = 2k + 1, n = 2k - 1$ $(k \in N)$

$\therefore m^2 - n^2 = 4k^2 + 1 + 4k - 4k^2 + 4k - 1 = 8k$

Hence, All the numbers of the form $m^2 - n^2$ are always divisible by 8.

46 **(d)**

Let $P(n) = 5^{2n+2} - 24n - 25$

For $n = 1$

$P(1) = 5^4 - 24 - 25 = 576$

$P(2) = 5^6 - 24(2) - 25 = 15552$

$= 576 \times 27$

Here, we see that $P(n)$ is divisible by 576

47 **(d)**

We have,

$3^{3n} - 26n - 1 = 27^n - 26n - 1$

$\Rightarrow 3^{3n} - 26n - 1 = (1 + 26)^n - 26n - 1$

$\Rightarrow 3^{3n} - 26n - 1$

$= {}^nC_2 \times 26^2 + {}^nC_3 \times 26^3 + \cdots$

$+ {}^nC_n \times 26^n$

Clearly, RHS is divisible 26^2 i.e. 676

48 **(a)**

As we have $A^2 = 2A - I$

$\Longrightarrow \quad A^2A = (2A - I)A = 2A^2 - IA$

$\Longrightarrow \quad A^3 = 2(2A - I) - IA = 3A - 2I$

Similarly, $A^4 = 4A - 3I$

$A^5 = 5A - AI$

$A^n = nA - (n - 1)I$

49 **(b)**

Given, $a_n = na_{n-1}$

For $n = 2$

$a_2 = 2a_1 = 2 \quad (\because a_1 = 1\text{given})$

$a_3 = 3a_2 = 3(2) = 6$

$a_4 = 4(a_3) = 4(6) = 24$

$a_5 = 5(a_4) = 5(24) = 120$

51 **(d)**

Given, $P(n): n^2 + n + 1$

At $n = 1, P(1) : 3$, which is not an even integer.

$\therefore P(1)$ is not true (Principle of Induction is not applicable).

Also, $n(n + 1) + 1$ is always an odd integer.

52 **(c)**

Let $P(n) = 2^{3n} - 7n - 1$

$\therefore \quad P(1) = 0, P(2) = 49$

$P(1)$ and $P(2)$ are divisible by 49.

Let $P(k) \equiv 2^{3k} - 7k - 1 = 49I$

$\therefore P(k + 1) \equiv 2^{3k+3} - 7k - 8$

$= 8(49I + 7k + 1) - 7k - 8$

$= 49(8I) + 49k = 49I_1$

Alternate

$P(n) = (1 + 7)^n - 7n - 1$

$= 1 + 7n + 7^2 \dfrac{n(n-1)}{2!} + \cdots - 7n - 1$

$= 7^2 \left(\dfrac{n(n-1)}{2!} + \cdots\right)$

53 **(d)**

Putting $n = 1, 2, 3$ …, it can be checked that $3n^5 + 5n^3 + 7n$ is divisible by 15

54 **(d)**

Let $P(n) = n^3 + 2n$

$\Longrightarrow P(1) = 1 + 2 = 3$

$\Longrightarrow P(2) = 8 + 4 = 12$

$\Longrightarrow P(3) = 27 + 6 = 33$

Here, we see that all these number are divisible by 3

55 **(a)**

We observe that $49^n + 16n - 1$ takes values 64

Hence, $49^n + 16n - 1$ is divisible by 64

56 **(b)**

Since, $P(3)$ is true.

Assume $P(k)$ is true $\Rightarrow P(k + 1)$ is true means, if $P(3)$ is true $\Rightarrow P(4)$ is true $\Rightarrow P(5)$ is true and so on. So, statement is true for all $n \geq 3$.

58 **(c)**

Putting $n = 1, 2, 3$ …, we observe that $4n - 1$ is the nth term

60 **(c)**

Let $P(n) = x(x^{n-1} - n\alpha^{n-1}) + \alpha^n(n - 1) = (x - \alpha)^2 g(x)$

$P(1) \equiv 0$ is true.

Let $P(k)$ is true.

$ie, x\left(x^{k-1} - k\alpha^{k-1}\right) + \alpha^k(k - 1) = (x - \alpha)^2 g(x)$

Now, $P(k + 1) \equiv x[x^k - (k + 1)\alpha^k] + \alpha^{k+1}(k)$

$\equiv (x - \alpha)^2[xg(x) + k\alpha^{k-1}]$ (True)

So, holds for all $n \in N$.

61 **(c)**

Let $P(n) = 2^{3n} - 7n - 1$

$\therefore P(1) = 0$

$P(2) = 49$

$P(1)$ and $P(2)$ are divisible by 49.

Let $P(k) \equiv 2^{3k} - 7k - 1 = 49I$

$\therefore P(k + 1) \equiv 2^{3k+3} - 7k - 8$

$= 8(49I + 7k + 1) - 7k - 8$

$= 49(8I) + 49k = 49I_1$

Hence, by mathematical induction $2^{3n} - 7n - 1$ is divisible by 49.

63 **(a)**

Let $P(n) = 3^{2n} - 1$

At $n = 1, P(1) = 8$ which is divisible by 8.

$\therefore P(1)$ is true.

Let $P(k)$ is true, then

$P(k) \equiv 3^{2k} - 1 = 8I$

$\therefore P(k+1) \equiv 3^{2k+2} - 1 = (8I+1)9 - I$

$= 72I + 8 = 8I_1$

$\therefore P(n)$ is divisible by 8, $\forall\, n \in N$.

68 **(c)**

It can be checked that $4^n - 3n - 1$ is divisible by 9 for $n = 1, 2, 3, \ldots$

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