6.LINEAR INEQUALITIES

Single Correct Answer Type

1. If $3^x + 2^{2x} \geq 5^x$, then the solution set for x is
a) $(-\infty, 2]$ b) $[2, \infty)$ c) $[0, 2]$ d) $\{2\}$
2. $x^2 - 3|x| + 2 < 0$, then x belongs to
a) $(1, 2)$ b) $(-2, -1)$ c) $(-2, -1) \cup (1,2)$ d) $(-3, 5)$
3. Solution of $2^x + 2^{|x|} \geq 2\sqrt{2}$ is
a) $(-\infty, \log_2(\sqrt{2} + 1))$ b) $(0,8)$
c) $\left(\frac{1}{2}, \log_2(\sqrt{2} - 1)\right)$ d) $(-\infty, \log_2(\sqrt{2} - 1)) \cup \left[\frac{1}{2}, \infty\right)$
4. If $x_1, x_2, \dots, x_n$ are real numbers, then the largest value of the expression $\sin x_1 \cos x_2 + \sin x_2 \cos x_3 + \dots +$ $\sin xn \cos x1$ is
a) n b) $\frac{n}{2}$ c) $\frac{n}{4}$ d) $\sqrt{n^2 - 1}$
5. If $a < b$, then the solution $x^2 + (a + b)x + ab < 0$ is given by
a) (a, b) b) $(-\infty, a) \cup (b, \infty)$ c) $(-b, -a)$ d) $(-\infty, -b) \cup (-a, \infty)$
6. If $\log(x^3 + y^3) - \log_{10}(x^2 + y^2 - xy) \leq 2$, then the maximum value of xy, for all $x \geq 0, y \geq 0$ is
a) 2500 b) 3000 c) 1200 d) 3500
7. If $3^{x/2} + 2^x > 25$, then the solution set is
a) R b) $(2, \infty)$ c) $(4, \infty)$ d) None of these
8. If $ab = 4(a, b \in R^+)$, then
a) $a + b \leq 4$ b) $a + b = 4$ c) $a + b \geq 4$ d) None of these
9. Let $P_n(x) = 1 + 2x + 3x^2 + \dots + (n + 1)x^n$ be a polynomial such that n is even. Then, the number of real roots of $P_n(x)$, is
a) 0 b) n c) 1 d) None of these
10. $(x - 1)(x^2 - 5x + 7) < (x - 1)$,then x belongs to
a) $(1,2) \cup (3, \infty)$ b) $(2, 3)$ c) $(-\infty, 1) \cup (2,3)$ d) None of these
11. If $x = \log_{2^2} 2 + \log_{2^3} 2^2 + \log_{2^4} 2^3 + \dots + \log_{2^{n+1}} 2^n$, then
a) $x \geq \left(\frac{1}{n + 1}\right)^{1/n}$ b) $x \geq n\left(\frac{1}{n + 1}\right)^{1/n}$ c) $x \geq \left(\frac{n}{n + 1}\right)^{1/n}$ d) None of these
12. The number of real solutions (x, y, z, t) of simultaneous equations
$2y = \frac{11}{x} + x, 2z = \frac{11}{y} + y, 2t = \frac{11}{z} + z, 2x = \frac{11}{t} + t$, is
a) 0 b) 1 c) 2 d) 4
13. The solution set contained in R of the inequation $3^x + 3^{1-x} - 4 < 0$, is
a) $(1, 3)$ b) $(0, 1)$ c) $(1, 2)$ d) $(0, 2)$
14. The range of ab if $|a| \leq 1$ and $a + b = 1, (a, b \in R)$, is
a) $[0, 1/4]$ b) $[-2, 1/4]$ c) $[1/4, 2]$ d) $[0, 2]$
15. If $\sqrt{9x^2 + 6x + 1} < (2 - x)$, then
a) $x \in \left(-\frac{3}{2}, \frac{1}{4}\right)$ b) $x \in \left(-\frac{3}{2}, \frac{1}{4}\right)$ c) $x \in \left[-\frac{3}{2}, \frac{1}{4}\right]$ d) $x < \frac{1}{4}$
16. If $5^x + \left(2\sqrt{3}\right)^{2x} \geq 13^x$, then the solution set for x is
a) $[2, \infty)$ b) $\{2\}$ c) $(-\infty, 2]$ d) $[0, 2]$
17. Solution set of inequality $\log_e \frac{x-2}{x-3}$ is
a) $(2, \infty)$ b) $(-\infty, 2)$ c) $(-\infty, \infty)$ d) $(3, \infty)$
18. If $3 < 3t - 18 \leq 18$, then which one of the following is true?
a) $15 \leq 2t + 1 \leq 20$ b) $8 \leq t < 12$ c) $8 \leq t + 1 \leq 13$ d) $21 \leq 3t \leq 24$
19. Let $f(x) = ax^2 + bx + c$ and $f(-1) < 1, f(1) > -1, f(3) < -4$ and $a \neq 0$,then

a) $a > 0$ b) $a < 0$
c) Sign of a cannot be determined d) None of the above

20. The set of admissible values of x such that $\frac{2x+3}{2x-9} < 0$ is
a) $\left(-\infty, -\frac{3}{2}\right) \cup \left(\frac{9}{2}, \infty\right)$ b) $(-\infty, 0) \cup \left(\frac{9}{2}, \infty\right)$ c) $\left(-\frac{3}{2}, 0\right)$ d) $\left(-\frac{3}{2}, \frac{9}{2}\right)$

21. The number of irrational solutions of the equation $\sqrt{x^2 + \sqrt{x^2 + 11}} + \sqrt{x^2 - \sqrt{x^2 + 11}} = 4$, is
a) 0 b) 2 c) 4 d) 11

22. The number of solutions of the equation $\log_{x-3}(x^3 - 3x^2 - 4x + 8) = 3$, is
a) 1 b) 2 c) 3 d) 4

23. The number of real solutions of the equation $\log_{0.5} x = |x|$, is
a) 1 b) 0 c) 2 d) None of these

24. The number of complex roots of the equation $x^4 - 4x - 1 = 0$, is
a) 3 b) 2 c) 1 d) 0

25. If $\sin^x \alpha + \cos^x \alpha \geq 1, 0 < \alpha < \frac{\pi}{2}$, then
a) $x \in (2, \infty)$ b) $x \in (-\infty, 2]$ c) $x \in [-1,1]$ d) None of these

26. Consider the following statements:
1. $\frac{x}{1+x^2} < \tan^{-1} x < x; x > 0$
2. If $0 \leq x < \frac{\pi}{2}, \sin x + \tan x - 3x \geq 0$
Which of these is/are correct?
a) Only (1) b) Only (2) c) (1) and (2) d) None of these

27. The number of solutions of the equation $2\cos(e^x) = 3^x + 3^{-x}$, is
a) 0 b) 1 c) 2 d) None of these

28. The number of real solutions of the equation $1 - x = [\cos x]$, is
a) 1 b) 2 c) 3 d) None of these

29. Non- negative real numbers such that $a_1 + a_2 + \ldots + a_n = p$ and $q = \sum_{i<j} a_i a_j$, then
a) $q \leq \frac{1}{2}p^2$ b) $q > \frac{1}{4}p^2$ c) $q < \frac{p}{2}$ d) $q > \frac{p^2}{2}$

30. If $(\sin \alpha)^x + (\cos \alpha)^x \geq 1, 0 < \alpha < \frac{\pi}{2}$, then
a) $x \in [2, \infty)$ b) $x \in (-\infty, 2]$ c) $x \in [-1, 1]$ d) None of these

31. If $x^2 + 2ax + 10 - 3a > 0$ for all $x \in R$, then
a) $-5 < a < 2$ b) $a < -5$ c) $a > 5$ d) $2 < a < 5$

32. The least integer satisfying $49.4 - \left(\frac{27-x}{10}\right) < 47.4 - \left(\frac{27-9x}{10}\right)$, is
a) 2 b) 3 c) 4 d) None of these

33. For positive real number a, b, c which one of the following holds?
a) $a^2 + b^2 + c^2 \geq bc + ca + ab$ b) $(b + c)(c + a)(a + b) \leq 8abc$
c) $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \leq 3$ d) $a^3 + b^3 + c^3 \leq 3\, abc$

34. The least perimeter of a cyclic quadrilateral of given area A square units is
a) $\sqrt{A}$ b) $2\sqrt{A}$ c) $3\sqrt{A}$ d) $4\sqrt{A}$

35. The number of solutions of $[\sin x + \cos x] = 3 + [-\sin x] + [-\cos x]$ in the internal $[0, 2\pi]$ is (where [.] denotes the greatest integer function)
a) 0 b) 4 c) Infinite d) 1

36. The number of solutions of $3^{|x|} = |2 - |x||$ is
a) 0 b) 2 c) 4 d) Infinite

37. If C is an obtuse angle in tringle, then
a) $\tan A \tan B < 1$ b) $\tan A \tan B > 1$ c) $\tan A \tan B = 1$ d) None of these

38. If x, y, z are three real numbers such that $x + y + z = 4$ and $x^2 + y^2 + z^2 = 6$, then the exhaustive set of values of x, is

a) $[2/3, 2]$ b) $[0, 2/3]$ c) $[0, 2]$ d) $[-1/3, 2/3]$

39. The number of roots of the equation $[\sin^{-1} x] = x - [x]$, is
a) 0 b) 1 c) 2 d) None of these

40. If $3^{x/2} + 2^x > 25$, then
a) $x \in [4, \infty)$ b) $(4, \infty)$ c) $x \in (-\infty, 4]$ d) $x \in [0, 4]$

41. The number of real solutions of the equation $27^{1/x} + 12^{1/x} = 2.8^{1/x}$, is
a) 1 b) 2 c) 0 d) Infinite

42. If roots of the equation $x^4 - 8x^3 + bx^2 + cx + 16 = 0$ are positive then
a) $b = 8 = c$ b) $b = -24, c = -32$ c) $b = 24, c = -32$ d) $b = 24, c = 32$

43. If $3<|x|<6$, then x belongs to
a) $(-6, -3) \cup (3,6)$ b) $(-6, 6)$ c) $(-3, -3) \cup (3,6)$ d) None of these

44. If a, b are distinct positive real numbers, then which one of the following is true?
a) $a^4 + b^4 > a^3b + ab^3$ b) $a^4 + b^4 < a^3b + ab^3$ c) $a^3 + b^3 < a^2b + ab^2$ d) None of these

45. The solution of the inequation $4^{-x+0.5} - 7 \cdot 2^{-x} < 4, x \in R$ is
a) $(-2, \infty)$ b) $(2, \infty)$ c) $\left(2, \frac{7}{2}\right)$ d) None of tnese

46. Suppose a, b and c are real numbers such that $\frac{a}{b} > 1$ and $\frac{a}{c} < 0$. Which one of the following is true?
a) $a + b - c > 0$ b) $a > b$ c) $(a - c)(b - c) > 0$ d) $a + b + c > 0$

47. If a, b, c are positive real numbers such that $a + b + c = p$ then, which of the following is true?
a) $(p - a)(p - b)(p - c) \geq \frac{1}{27} p^3$
b) $(p - a)(p - b)(p - c) \geq 8abc$
c) $\frac{bc}{a} + \frac{ca}{b} + \frac{ab}{c} \geq p$
d) None of these

48. The number of solutions of the equation $\frac{(1+e^{x^2})\sqrt{1+x^2}}{\sqrt{1+x^4-x^2}} = 1 + \cos x$, is
a) 1 b) 2 c) 3 d) 4

49. Let n be an odd integer such that the polynomial $P_n(x) = 1 + 2x + 3x^2 + \dots \dots + (n+1)x^n$ has exactly one real root. This real root α satisfies
a) $-1 < \alpha < 0$ b) $0 < \alpha < 1$ c) $0 \leq \alpha \leq 1$ d) $-1 \leq \alpha \leq 0$

50. Let a, b be integers and $f(x)$ be a polynomial with integer coefficients such that $f(b) - f(a) = 1$. Then, the value of $b - a$, is
a) 1 b) -1 c) $1, -1$ d) None of these

51. Let $y = \sqrt{\frac{(x+1)(x-3)}{(x-2)}}$, then all real values of x for which y takes real values, are
a) $-1 \leq x < 2$ or $x \geq 3$ b) $-1 \leq x < 3$ or $x > 2$ c) $1 \leq x < 2$ *or* $x \geq 3$ d) None of these

52. If $a, b, c > 0$ and if $abc = 1$, then the value of $a + b + c + ab + bc + ca$ lies in the interval
a) $(\infty, -6)$ b) $(-6, 0)$ c) $(0, 6)$ d) $(6, \infty)$

53. The number of real roots of the equation $(\sin 2^x)(\cos 2^x) = \frac{2^x + 2^{-x}}{2}$, is
a) 1 b) 2 c) 3 d) None of these

54. The largest interval for which $x^{12} - x^9 + x^4 - x + 1 > 0$ is
a) $-4 < x \leq 0$ b) $0 < x < 1$ c) $-100 < x < 100$ d) $0 < x < \infty$

55. The number of negative real roots of $x^4 - 4x - 1 = 0$, is
a) 3 b) 2 c) 1 d) 0

56. If $0 < x < \frac{\pi}{2}$, then minimum value of $\frac{\cos^3 x}{\sin x} + \frac{\sin^3 x}{\cos x}$ is
a) $\sqrt{3}$ b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) 1

57. The number of solutions of $\sqrt{3x^2 + 6x + 7} + \sqrt{5x^2 + 10x + 14} = 4 - 2x - x^2$, is
a) 1 b) 2 c) 3 d) 4

58. The solution set of $||x|-1| < |1-x|, x \in R$ is

a) $(-1,1)$ b) $(0,\infty)$ c) $(-1,\infty)$ d) None of these

59. The minimum value of $f(x) = |3-x| + 7$ is

a) 0 b) 6 c) 7 d) 8

60. The solution set of the inequation $\frac{x+11}{x-3} > 0$ is

a) $(-\infty, 11) \cup (3,\infty)$ b) $(-\infty,-10) \cup (2,\infty)$ c) $(-100,-11) \cup (1,\infty)$ d) $(-5,0) \cup (3,7)$

61. Solution of $2x - 1 = |x+7|$ is

a) -2 b) 8 c) $-2, 8$ d) None of these

62. The number of positive real roots of $x^4 - 4x - 1 = 0$, is

a) 3 b) 2 c) 1 d) 0

63. The solution set of the inequality $\log_{\sin\left(\frac{\pi}{3}\right)}(x^2 - 3x + 2) \geq 2$ is

a) $\left(\frac{1}{2}, 2\right)$ b) $\left(1, \frac{5}{2}\right)$ c) $\left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$ d) None of these

64. If a, b, c are sides of triangle, then $\frac{(a+b+c)^2}{(ab+bc+ca)}$ always belongs to

a) [1, 2] b) [2, 3] c) [3, 4] d) [4, 5]

65. $(x-1)(x^2 - 5x + 7) < (x-1)$, then x belongs to

a) $(1,2) \cup (3,\infty)$ b) $(-\infty, 1) \cup (2,3)$ c) 2, 3 d) None of these

66. The set of values of x for which the inequalities $x^2 - 3x - 10 < 0, 10x - x^2 - 16 > 0$ hold simultaneously, is

a) (-2, 5) b) (2, 8) c) (-2, 8) d) (2, 5)

67. The solution of the inequation $\log_{1/3}(x^2 + x + 1) + 1 < 0$ is

a) $(-\infty,-2) \cup (1,\infty)$ b) $[-1,2]$ c) $(-2,1)$ d) $(-\infty,\infty)$

68. The number of values of a for which the system of equation $2^{|x|} + |x| = y + x^2 + a$ and $x^2 + y^2 = 1$ has only one solution where a, x, y are real, is

a) 1
b) 2
c) Finitely many but more than 2
d) Infinitely many

69. The solution set of the inequation $\log_{1/3}(x^2 + x + 1) + 1 > 0$, is

a) $(-\infty,-2) \cup (1,\infty)$ b) $[-1,2]$ c) $(-2,1)$ d) R

70. The set of all real numbers satisfying the inequation $x^{(\log_{10} x)^2 - 3(\log_{10} x) + 1} > 1000$, is

a) (0, 1000) b) $(1000,\infty)$ c) (0, 100) d) None of these

71. The set of all real x satisfying the inequality $\frac{3-|x|}{4-|x|} > 0$

a) $[-3,3] \cup (-\infty,-4) \cup (4,\infty)$ b) $(-\infty,-4) \cup (4,\infty)$
c) $(-\infty,-3) \cup (4,\infty)$ d) $(-\infty,-3) \cup (3,\infty)$

72. For positive real number a, b, c which of the following holds?

a) $a + b + c > 3 \Rightarrow a^2 + b^2 + c^2 > 3$ b) $a^6 + b^6 \leq 12a^2b^2 - 64$
c) $a + b + c = \alpha \Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \leq \frac{9}{\alpha}$ d) None of the above

73. If a_1, a_2, a_3 be any positive real numbers, then which of the following statements is not true

a) $3a_1, a_2, a_3 \leq a_1^3 + a_2^3 + a_3^3$
b) $\frac{a_1}{a_2} + \frac{a_2}{a_3} + \frac{a_3}{a_1} \geq 3$
c) $(a_1 + a_2 + a_3)\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right) \geq 9$
d) $(a_1 + a_2 + a_3)\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right)^3 \leq 27$

74. If $x^2+\frac{1}{x^2}=A$ and $x-\frac{1}{x}=B$, where $x \in R$ and $B>0$, then the minimum value of $\frac{A}{B}$ is

a) $\sqrt{2}$ b) $2\sqrt{2}$ c) $\sqrt{2}+2$ d) None of these

75. Let n be an odd positive integer. Then, the number of real roots of the polynomial $P_n(x)=1+2x+3x^2+\dots\dots+(n+1)x^n$, is

a) 0 b) n c) 1 d) None of these

76. The number of positive integers satisfying the inequality $n+1_{C_{n-2}}-n+1_{C_{n-1}} \leq 50$ is

a) 9 b) 8 c) 7 d) 6

77. For $\theta > \pi/3$, the value of $f(\theta)=\sec^2\theta+\cos^2\theta$ always lies in the interval

a) $(0,2)$ b) $[0,1]$ c) $(1,2)$ d) $[2,\infty)$

78. If the product of n positive numbers is n^n, then their sum is

a) A positive integer b) Divisible by n c) Equal to $n+\frac{1}{n}$ d) Never less than n^2

79. $\log_2(x^2-3x+18)<4$,then x belongs to

a) $(1,2)$ b) $(2,16)$ c) $(1,16)$ d) None of these

80. If $[x]^2=[x+6]$, where $[x]=$ the greatest integer less than or equal to x, then x must be such that

a) $x=3,-2$ b) $x \in [-2,-1)$ c) $x \in [3,4)$ d) $x \in [-2,-1) \cup [3,4)$

81. If $a,b,c>0$, the minimum value of $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}$ is

a) 1 b) $\frac{3}{2}$ c) 2 d) $\frac{5}{2}$

82. The number of real solutions of the equation $e^{|x|}-|x|=0$, is

a) 0 b) 1 c) 2 d) None of these

83. If p,q,r are any real numbers, then

a) $\max(p,q)=\max(p,q,r)$

b) $\min(p,q)=\frac{1}{2}(P+q-|p-q|)$

c) $\max(p,q)<\min(p,q,r)$

d) $\max(p,q)=\frac{1}{2}(p+q-|p-q|)$

84. The number of real solutions of $1+|e^x-1|=e^x(e^x-2)$, is

a) 1 b) 2 c) 3 d) 4

85. The solution set of the inequation $\log_{\sin 2\pi/3}(x^2-3x+2)\geq 2$, is

a) $[1/2,1)$ b) $(2,5/2]$ c) $[1/2,1)\cup(2,5/2]$ d) $[1/2,5/2]$

86. If $x^2+6x-27<0$ and $x^2-3x-4<0$, then

a) $x>3$ b) $x<4$ c) $3<x<4$ d) $x=\frac{7}{2}$

87. If $x,y \in R$, then $\frac{1}{2}(x+y+|x-y|)=x$ holds iff

a) $x>y$ b) $x<y$ c) $x=y$ d) $x \geq y$

88. The set of all x satisfying the inequality $\frac{4x-1}{3x+1}\geq 1$ is

a) $\left(-\infty,-\frac{1}{3}\right)\cup\left[\frac{1}{4},\infty\right]$ b) $\left(-\infty,-\frac{2}{3}\right)\cup\left[\frac{5}{4},\infty\right]$ c) $\left(-\infty,-\frac{1}{3}\right)\cup[2,\infty)$ d) $\left(-\infty,-\frac{2}{3}\right)[4,\infty)$

89. The number of solutions of the inequality

$E=2^{1/\sin^2\alpha_2}\cdot 3^{1/\sin^2\alpha_3}\dots n^{1/\sin^2\alpha_n}<n!$

Where $\alpha_i \in (-\pi,2\pi)$ for $i=2,3,\dots,n$ is

a) 0 b) 2^{n-1} c) 3^{n-1} d) None of these

90. The equation $e^x=x(x+1), x<0$ has

a) No real roots set

b) Exactly one real root

c) Two real roots

d) Infinitely many real roots

91. Let $F(x)$ be a function defined by $F(x)=x-[x]$, $0\neq x \in R$, where $[x]$ is the greatest integer less than or equal to x. Then, the number of solutions of $F(x)+F\left(\frac{1}{x}\right)=1$

a) 0 b) Infinite c) 1 d) 2

92. The set of all real numbers satisfying the inequation $2^x + 2^{|x|} \geq 2\sqrt{2}$, is
a) $(1/2, \infty)$
b) $(-\infty, \log_2(\sqrt{2}-1))$
c) $(-\infty, 1/2)$
d) $[1/2, \infty) \cup (-\infty, \log_2(\sqrt{2}-1))$

93. Solution of the inequality $\sin^4\left(\frac{x}{3}\right) + \cos^4\left(\frac{x}{3}\right) > \frac{1}{2}$, is given by
a) R
b) $\frac{3n\pi}{2} + \frac{3\pi}{4}$
c) $R - \left\{\left(\frac{3n\pi}{2} + \frac{3\pi}{4}\right), n \in I\right\}$
d) None of these

94. If $(\log_5 x)^2 + (\log_5 x) < 2$, then x belongs to the interval
a) $(1/25, 5)$ b) $(1/5, 1/\sqrt{5})$ c) $(1, \infty)$ d) None of these

95. The number of real roots of the equation $x^2 + x + 3 + 2\sin x = 0$ in the interval $[-\pi, \pi]$, is
a) 2 b) 4 c) 6 d) None of these

96. If $\log_{\sqrt{3}}(\sin x + 2\sqrt{2}\cos x) \geq 2, -2\pi \leq x \leq 2\pi$, then the number of solutions of x is
a) 0 b) ∞ c) 3 d) 4

97. Solution of $x^{(\log_{10} x)^2 - 3\log_{10} x + 1} > 1000$ for $x \in R$, is
a) $(10, \infty)$ b) $(100, \infty)$ c) $(1000, \infty)$ d) $(1, \infty)$

98. The largest interval for which $x^{12} - x^9 + x^4 - x + 1 > 0$ is
a) $-4 < x < 0$ b) $0 < x < 1$ c) $-100 < x < 100$ d) $-\infty < x < \infty$

99. The number of roots of the equation $\sin \pi x = |\log|x||$, is
a) 2 b) 4 c) 5 d) 6

100. The equation $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$ has
a) No solution
b) One solution
c) Two solutions
d) More than two solutions

101. If $\log_3 x - \log_x 27 < 2$, then x belongs to the interval
a) $(1/3, 27)$ b) $(1/27, 3)$ c) $(1/9, 9)$ d) None of these

102. The set of all solutions of the inequation $x^2 - 2x + 5 \leq 0$in R is
a) $R - (-\infty, -5)$ b) $R - (5, \infty)$ c) $\emptyset$ d) $R - (-\infty, -4)$

103. $2^{\sin^2 x} + 2^{\cos^2 x}$ is
a) ≤ 2 b) ≥ 2 c) ≤ 1 d) ≥ 1

104. If $a^2 + b^2 + c^2 = 1$, then $ab + bc + ca$ lies in the interval
a) $\left[-\frac{1}{2}, 1\right]$ b) $\left[0, \frac{1}{2}\right]$ c) $[0, 1]$ d) $[1, 2]$

105. The equation $\sqrt{4x+9} - \sqrt{11x+1} = \sqrt{7x+4}$ has
a) No solution
b) One solution
c) Two solutions
d) More than two solutions

106. $\left|x + \frac{2}{x}\right| < 3$, then x belongs to
a) $(-2, -1) \cup (1, 2)$
b) $(-\infty, -2) \cup (-1, 1) \cup (2, \infty)$
c) $(-2, 2)$
d) $(-3, 3)$

107. If a, b, c are the sides of a tringle, then $\frac{a}{b+c-a} + \frac{b}{c+a-b} + \frac{c}{a+b-c}$ is
a) ≤ 3 b) ≥ 3 c) ≥ 2 d) ≤ 2

108. The minimum value of the sum of the lengths of diagonals of a cyclic quadrilateral of area a^2 square units

is

a) $\sqrt{2}a$ b) $2\sqrt{2}a$ c) $2a$ d) None of these

109. $|2x-3| < |x+5|$, then x belongs to

a) $(-3,5)$ b) $(5, 9)$ c) $\left(-\frac{2}{3}, 8\right)$ d) $\left(-8, \frac{2}{3}\right)$

110. The number of real roots of the equation $1 + a_1x + a_2x^2 + \cdots + a_nx^n = 0$, where $|x| < \frac{1}{3}$ and $|a_n| < 2$, is

a) n if n is even b) 1 if n is odd c) 0 for any $n \in N$ d) None of these

111. Consider the following statements:

1. If x be real, then $-\frac{9}{2} \le \frac{3x-4}{x^2+1} \le \frac{1}{2}$

2. If x is real, then the greatest value of $\frac{x^2+14x+9}{x^2+2x+3}$ is 4

3. If $ax^2 + bx + c = 0$; $a \neq 0, a, b, c \in R$ has no real roots, then $(a+b+c)c > 0$

Which of these is/ are correct?

a) Only (1) b) Only(2) c) Only (3) d) All of these

112. If r is a real number such that $|r| < 1$ and if $a = 5(1-r)$, then

a) $0 < a < 5$ b) $-5 < a < 5$ c) $0 < a < 10$ d) $0 \le a < 10$

113. The number of integral roots of the equation $e^{x-8} + 2x - 17 = 0$, is

a) 1 b) 2 c) 4 d) 8

114. The product of real roots of the equation $x^2 + 18x + 30 = 2\sqrt{x^2 + 18x + 45}$, is

a) 720 b) 20 c) 36 d) None of these

115. The set of values of x satisfying $2 \le |x-3| < 4$ is

a) $(-1, 1] \cup [5, 7)$ b) $-4 \le x \le 2$

c) $-1 < x < 7$ or $x \ge 5$ d) $x < 7$ or $x \ge 5$

116. Let $x = \left[\frac{a+2b}{a+b}\right]$ and $y = \frac{a}{b}$, where a and b are positive integers. If $y^2 > 2$, then

a) $x^2 \le 2$ b) $x^2 < 2$ c) $x^2 > 2$ d) $x^2 \ge 2$

117. The least value of $5^{\sin x - 1} + 5^{-\sin x - 1}$ is

a) 10 b) $\frac{5}{2}$ c) $\frac{2}{5}$ d) $\frac{1}{5}$

118. If $x^2 + 2x + n > 10$ for all real numbers x, then which of the following conditions is true?

a) $n < 11$ b) $n = 10$ c) $n = 11$ d) $n > 11$

119. The minimum value of $P = bcx + cay + abz$, when $xyz = abc$, is

a) $3abc$ b) $6abc$ c) abc d) $4abc$

120. If $a_i > 0$ for $i = 1, 2, \dots, n$ and $a_1a_2 \dots a_n = 1$, then minimum value of $(1+a_1)(1+a_2) \dots (1+a_n)$ is

a) $2^{n/2}$ b) 2^n c) 2^{2n} d) 1

121. If a, b, c are positive real numbers such that $a + b + c = 2$ then, which one of the following is true?

a) $(2-a)(2-b)(2-c) \ge 8abc$

b) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \ge 2$

c) $(2-a)(2-b)(2-c) < 8abc$

d) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 2$

122. If x, y, z are positive real numbers such that $x^2 + y^2 + z^2 = 27$, then $x^3 + y^3 + z^3$ has

a) Minimum value 81 b) Maximum value 81 c) Minimum value 27 d) Maximum value 27

123. If x satisfies the inequations $2x - 7 < 11, 3x + 4 < -5$, then x lies in the interval

a) $(-\infty, 3)$ b) $(-\infty, 2)$ c) $(-\infty, -3)$ d) $(-\infty, \infty)$

124. $x^8 - x^5 - \frac{1}{x} + \frac{1}{x^4} > 0$, is satisfied for

a) Only positive values of x

b) Only negative values of x

c) All real numbers except zero

d) Only for $x > 1$

125. The solution set of the inequation $5^{(1/4)(\log_5 x)^2} \geq 5\, x^{(1/5)(\log_5 x)}$, is

a) $(0, 5^{-2\sqrt{5}}]$ b) $[5^{2\sqrt{5}}, \infty)$ c) $(0, 5^{-2\sqrt{5}}] \cup [5^{2\sqrt{5}}, \infty)$ d) $(0, \infty)$

126. If $a + b = 8$, then ab is greater when

a) $a = 4, b = 4$ b) $a = 3, b = 5$ c) $a = 6, b = 2$ d) None of these

127. The number of solutions of the equation $\cos x + |x| = 0$ is

a) 0 b) 1 c) 2 d) 3

128. If $0 < x < \frac{\pi}{2}$, then the minimum value of $\frac{\cos^3 x}{\sin x} + \frac{\sin^3 x}{\cos x}$ is

a) $\sqrt{3}$ b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) 1

129. If $x^2 + 4ax + 2 > 0$ for all values of x, then a lies in the interval

a) (-2, 4) b) (1, 2) c) $(-\sqrt{2}, \sqrt{2})$ d) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

130. If a and b are two different positive real numbers then which of the following statement is true?

a) $2\sqrt{ab} > a + b$ b) $2\sqrt{ab} < a + b$ c) $2\sqrt{ab} = a + b$ d) None of these

131. The number of negative integral solutions of $x^2 \cdot 2^{x+1} + 2^{|x-3|+2} = x^2 \cdot 2^{|x-3|+4} + 2^{x-1}$, is

a) None b) Only one c) Two d) Four

132. The number of ordered 4-tuples (x, y, z, w) where $x, y, z, w \in [0,10]$ which satisfy the inequality $2^{\sin^2 x} \times 3^{\cos^2 y} \times 4^{\sin^2 z} \times 5^{\cos^2 w} \geq 120$, is

a) 81 b) 144 c) 0 d) Infinite

133. If $a > 1, b > 1, c > 1, d > 1$, then the minimum value of $\log_b a + \log_a b + \log_d c + \log_c d$ is

a) 1 b) 2 c) 3 d) 4

134. The solution set of inequation $\log_{1/3}(2^{x+2} - 4^x) \geq -2$, is

a) $(-\infty, 2 - \sqrt{13})$ b) $(-\infty, 2 + \sqrt{13})$ c) $(-\infty, 2)$ d) None of these

135. If $\frac{2x}{2x^2+5x+2} > \frac{1}{x+1}$, then

a) $-2 > x > -1$ b) $-2 \geq x \geq -1$ c) $-2 < x < -1$ d) $-2 < x \leq -1$

136. The number of real solutions of $\log_2 x + |x| = 0$, is

a) 0 b) 1 c) 3 d) None of these

137. If $xyz = abc$, then the least value of $bcx + cay + abz$ is

a) $3abc$ b) $6abc$ c) abc d) $4abc$

138. The number of solution(s) of the inequation $\sqrt{3x^2 + 6x + 7} + \sqrt{5x^2 + 10x + 14} \leq 4 - 2x - x^2$, is

a) 1 b) 2 c) 4 d) Infinitely many

139. A stick of length 20 units is to be divided into n parts so that the product of the lengths of the parts is greater than unity. The maximum possible value of n is

a) 18 b) 19 c) 20 d) 21

140. If a, b, c are different positive real number such that $(b + c - a), (c + a - b)$ and $(a + b - c)$ are positive, then $(a + b - c)(b + c - a)(c + a - b) - abc$ is

a) Positive b) Negative c) Non-positive d) Non-negative

141. $\log_{16} x^3 + \left(\log_2 \sqrt{x}\right)^2 < 1$ iff x lies in

a) (2, 16) b) (0,1/16) c) (1/16 ,2) d) None of these

142. If $\log_{\cos x} \sin x > 2$ and $0 < x < 3\pi$, then $\sin x$ lies in the interval

a) $\left[\frac{\sqrt{5}-1}{2}, 1\right]$ b) $\left[0, \frac{\sqrt{5}-1}{2}\right]$ c) $\left[0, \frac{1}{2}\right]$ d) None of these

143. If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ such that $\min f(x) > \max g(x)$, then the relation between b and c, is

a) No real value of b and c b) $0 < c < b\sqrt{2}$ c) $|c| < |b|\sqrt{2}$ d) $|c| > |b|\sqrt{2}$

144. If the sum of the greatest integer less than or equal to x and the least integer greater than or equal to x is 5, then the solution set for x is

a) (2, 3) b) (0, 5) c) [5, 6) d) None of these

145. The total number of roots of the equation $|x - x^2 - 1| = |2x - 3 - x^2|$ is

a) 1 b) 2 c) 0 d) Infinitely many

146. For $\frac{|x-1|}{x+2} < 1, x$ lies in the interval

a) $(-\infty, -2) \cup \left(-\frac{1}{2}, \infty\right)$ b) $(-\infty, 1) \cup [2, 3]$ c) $(-\infty, -4)$ d) $\left[-\frac{1}{2}, 1\right]$

147. Number of integer solutions of $\frac{x+2}{x^2+1} > \frac{1}{2}$ is

a) 0 b) 1 c) 2 d) 3

148. Solution of the inequality $\tan\left(x + \frac{\pi}{3}\right) \geq 1$ is

a) $\left(n\pi + \frac{\pi}{12}, n\pi + \frac{\pi}{6}\right)$ b) $\left(n\pi - \frac{\pi}{12}, n\pi + \frac{\pi}{6}\right)$ c) $\left(n\pi - \frac{\pi}{6}, n\pi - \frac{\pi}{12}\right)$ d) None of these

149. If $0 < a < 1$, then the solution set of the inequation $\frac{1+(\log_a x)^2}{1+(\log_a x)} > 1$, is

a) $(1, 1/a)$ b) $(0, a)$ c) $(1, 1/a) \cup (0, a)$ d) None of these

150. Let $x = \frac{a+2b}{a+b}$ and $y = \frac{a}{b}$, wherer a and b are positive integers. If $y^2 > 2$, then

a) $x^2 \leq 2$ b) $x^2 < 2$ c) $x^2 > 2$ d) $x^2 \geq 2$

151. The minimum value of $|\sin x + \cos x + \tan x + \sec x + \operatorname{cosec} x + \cot x|$ is

a) $2\sqrt{2} - 1$ b) $2\sqrt{2} + 1$ c) $\sqrt{2} - 1$ d) $\sqrt{2} + 1$

152. If for $x \in R, \frac{1}{3} < \frac{x^2-2x+4}{x^2+2x+4} < 3$,then$\frac{9\cdot 3^{2x}-6\cdot 3^x+4}{9\cdot 3^{2x}+6\cdot 3^x+4}$ lies between

a) $\frac{1}{2}$ and 2 b) $\frac{1}{3}$ and 3 c) 0 and 2 d) None of these

153. The minimum value of $4^x + 4^{1-x}, x \in R$, is

a) 1 b) 2 c) 4 d) None of these

154. The number of real solutions of the equation $3^{-|x|} - 2^{|x|} = 0$, is

a) 0 b) 1 c) 2 d) None of these

155. The number of real roots of the equation $1 + 3^{x/2} = 2^x$, is

a) 0 b) 1 c) 2 d) None of these

156. If n is even and $n \geq 4, x_1, x_2, \dots, x_n \geq 0$ and $x_1 + x_2 + \cdots + x_n = 1$, then $P = x_1x_2 + x_2x_3 + \cdots + x_n - x_n$ cannot exceed

a) $\frac{1}{n+1}$ b) $\frac{1}{n+2}$ c) $\frac{1}{2n}$ d) None of these

157. The number of real solutions of the equation $e^{-x} = x$, is

a) 0 b) 1 c) 2 d) None of these

158. The solution set contained in R of the inequation $3^x + 3^{1-x} - 4 < 0$, is

a) $(1, 3)$ b) $(0, 1)$ c) $(1, 2)$ d) $(0, 2)$

159. The solution of the inequation $2x^2 + 3x - 9 \leq 0$ is given by

a) $\frac{3}{2} \leq x \leq 3$ b) $-3 \leq x \leq \frac{3}{2}$ c) $-3 \leq x \leq 3$ d) $\frac{3}{2} \leq x \leq 2$

160. If $0 < \theta < \pi$, then the minimum value of $\sin^5\theta + \operatorname{cosec}^5\theta$ is

a) 0 b) 1 c) 2 d) None of these

: ANSWER KEY :

1)	a	2)	c	3)	d	4)	b	85)	c	86)	c	87)	d	88)	c
5)	c	6)	a	7)	c	8)	c	89)	c	90)	b	91)	b	92)	d
9)	a	10)	c	11)	b	12)	c	93)	c	94)	a	95)	d	96)	d
13)	b	14)	b	15)	d	16)	c	97)	c	98)	d	99)	d	100)	a
17)	b	18)	c	19)	b	20)	d	101)	d	102)	c	103)	b	104)	a
21)	b	22)	a	23)	a	24)	b	105)	b	106)	a	107)	b	108)	b
25)	b	26)	c	27)	a	28)	b	109)	c	110)	c	111)	d	112)	c
29)	a	30)	b	31)	a	32)	b	113)	a	114)	b	115)	a	116)	b
33)	a	34)	d	35)	c	36)	b	117)	c	118)	d	119)	a	120)	b
37)	a	38)	a	39)	b	40)	b	121)	a	122)	a	123)	c	124)	c
41)	c	42)	c	43)	a	44)	a	125)	c	126)	a	127)	a	128)	d
45)	a	46)	c	47)	b	48)	a	129)	d	130)	b	131)	a	132)	b
49)	a	50)	c	51)	a	52)	d	133)	d	134)	c	135)	c	136)	b
53)	d	54)	d	55)	c	56)	d	137)	a	138)	a	139)	b	140)	b
57)	a	58)	d	59)	c	60)	a	141)	c	142)	b	143)	d	144)	a
61)	b	62)	c	63)	c	64)	c	145)	a	146)	a	147)	d	148)	b
65)	b	66)	d	67)	a	68)	a	149)	c	150)	b	151)	a	152)	b
69)	c	70)	d	71)	a	72)	a	153)	c	154)	b	155)	b	156)	d
73)	c	74)	a	75)	c	76)	c	157)	b	158)	b	159)	b	160)	c
77)	d	78)	d	79)	a	80)	d								
81)	b	82)	a	83)	b	84)	a								

: HINTS AND SOLUTIONS :

1 **(a)**

We have,

$3^x + 2^{2x} \geq 5^x$

$\Rightarrow \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x \geq 1$

$\Rightarrow \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x \geq \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2$

$\Rightarrow x \leq 2 \Rightarrow x \in (-\infty, 2]$ $\left[\begin{array}{c}\text{If } a^x + b^x \geq 1 \text{ and } a^2 + b^2 = 1, \\ \text{then } x \in (-\infty, 2)\end{array}\right]$

2 **(c)**

$x^2 - 3|x| + 2 < 0$

$\Rightarrow |x|^2 - 3|x| + 2 < 0$

$\Rightarrow (|x| - 1)(|x| - 2) < 0$

$\Rightarrow 1 < |x| < 2$

$\Rightarrow -2 < x < -1$ or $1 < x < 2$

$\therefore x \,\epsilon (-2, -1) \cup (1,2)$

3 **(d)**

We have, $2^x + 2^{|x|} \geq 2\sqrt{2}$

If $x \geq 0$,then $2^x + 2^x \geq 2\sqrt{2}$

$\Rightarrow 2^x \geq \sqrt{2} \Rightarrow x \geq \frac{1}{2}$

and if $x < 0$, then $2^x + 2^{-x} \geq 2\sqrt{2}$

$\Rightarrow t + \frac{1}{t} \geq 2\sqrt{2}$ (where $t = 2^x$)

$\Rightarrow t^2 - 2\sqrt{2}t + 1 \geq 0$

$\Rightarrow \left(t - (\sqrt{2} - 1)\right)\left(t - (\sqrt{2} + 1)\right) \geq 0$

$\Rightarrow t \leq \sqrt{2} - 1$ or $t \geq \sqrt{2} + 1$

But $t > 0$

$\Rightarrow 0 < 2^x \leq \sqrt{2} - 1$

Or $2^x \geq \sqrt{2} + 1$

$\Rightarrow -\infty < x \leq \log_2(\sqrt{2} - 1)$

Or $x \geq \log_2(\sqrt{2} + 1)$

Which is not possible, because $x > 0$

$\therefore x \in (-\infty, \log_2(\sqrt{2} - 1)) \cup \left[\frac{1}{2}, \infty\right)$

4 **(b)**

Using G. M. ≤ A. M., we have

$\sin x_i \cos x_{i+1} \leq \frac{\sin^2 x_i + \cos^2 x_{i+1}}{2}$ for $i = 1,2,3, \dots, n,$

where $x_{n+1} = x_1$

$\therefore \sin x_1 \cos x_2 + \sin x_2 \cos x_3 + \cdots + \sin x_n \cos x_{n+1}$

$\leq \frac{\sin^2 x_1 + \cos^2 x_2}{2} + \frac{\sin^2 x_2 + \cos^2 x_3}{2} + \cdots + \frac{\sin^2 x_n + \cos^2 x_1}{2}$

$\Rightarrow \sin x_1 \cos x_2 + \sin x_2 \cos x_3 + \cdots + \sin x_n \cos x_1 \leq \frac{n}{2}$

5 **(c)**

We have, $x^2 + (a + b)x + ab < 0$

$\Rightarrow (x + a)(x + b) < 0$

$\Rightarrow -b < x < -a$

6 **(a)**

Given, $\log_{10}(x^3 + y^3) - \log_{10}(x^2 + y^2 - xy) \leq 2$

$\Rightarrow \log_{10} \frac{(x^3 + y^3)}{x^2 + y^2 - xy} \leq 2$

$\Rightarrow \log_{10}(x + y) \leq 2 \Rightarrow x + y \leq 100$

Using AM ≥ GM

$\therefore \frac{x + y}{2} \geq \sqrt{xy}$

$\Rightarrow \sqrt{xy} \leq \frac{x + y}{2} \leq \frac{100}{2}$

$\Rightarrow xy \leq 2500$

7 **(c)**

By trial,

$3^{x/2} + 2^x \leq 25$ for $x = 1,2,3,4$

But $3^{x/2} + 2^x > 25$ for $x > 4$

Hence, solution set for $3^{x/2} + 2^x > 25$ is $(4, \infty)$

8 **(c)**
Since,

AM $\geq$ GM

$\Rightarrow \dfrac{a+b}{2} \geq \sqrt{ab}$

$\Rightarrow \frac{a+b}{2} \geq \sqrt{4}$ $(\because ab = 4, \text{given})$

$\Rightarrow a+b \geq 4$

9 **(a)**
When $x > 0, P_n(x) > 0$ and so $P_n(x) = 0$ can have no positive real roots
Now,
$P_n(x) = 1 + 2x + 3x^2 + \cdots + (n+1)x^n$
$\Rightarrow xP_n(x) = x + 2x^2 + 3x^3 + \cdots + n\,x^n + (n+1)x^{n+1}$
$\Rightarrow (1-x)P_n(x) = 1 + x + x^2 + \cdots + x^n - (n+1)x^{n+1}$
$\Rightarrow P_n(x) = \dfrac{1-(n+2)x^{n+1} + (n+1)x^{n+2}}{(1-x)^2}$
For negative values of $x, P_n(x)$ will vanish whenever
$f(x) = 1 - (n+2)x^{n+1} + (n+1)x^{n+2} = 0$
Now,
$f(-x) = 1 - (n+2)(-1)^{n+1}x^{n+1} + (n+$
1−1n+2xn+2 ...(i)
If n is even, there is no change of sign in this expression and so there is no negative real root of $f(x)$

10 **(c)**
$(x-1)(x^2 - 5x + 7) < (x-1)$

$\Rightarrow (x-1)(x^2 - 5x + 6) < 0$

$\Rightarrow (x-1)(x-2)(x-3) < 0$

$\therefore x \in (-\infty, 1) \cup (2, 3)$

11 **(b)**
Using A. M. $\geq$ G. M., we have
$\dfrac{x}{n} \geq \{\log_{2^2} 2 \times \log_{2^3} 2^2 \times \log_{2^4} 2^3 \times \ldots \times \log_{2^{n+1}} 2^n\}^{1/n}$
$\Rightarrow x \geq n(\log_{2^{n+1}} 2)^{1/n}$
$\Rightarrow x \geq n\left(\dfrac{1}{n+1}\log_2 2\right)^{1/n} \Rightarrow x \geq n\left(\dfrac{1}{n+1}\right)^{1/n}$

12 **(c)**
We have,
$\frac{1}{2}\left|a + \frac{11}{a}\right| \geq \sqrt{11}$, equality holding iff $a = \pm\sqrt{11}$
$\therefore |x| \geq \sqrt{11}, |y| \geq \sqrt{11}, |z| \geq \sqrt{11}, |t| \geq \sqrt{11}$
Let $x \geq 0$, then $x \geq \sqrt{11}, y \geq \sqrt{11}, z \geq \sqrt{11}$ and $t \geq \sqrt{11}$
Now, $y - \sqrt{11} = \dfrac{1}{2}\left(\dfrac{11}{x} + x\right) - \sqrt{11}$
$\Rightarrow y - \sqrt{11} = \dfrac{\left(x-\sqrt{11}\right)^2}{2x} = \left(\dfrac{x-\sqrt{11}}{2x}\right)\left(x - \sqrt{11}\right)$
$\Rightarrow y - \sqrt{11} = \dfrac{1}{2}\left(1 - \dfrac{\sqrt{11}}{x}\right)\left(x-\sqrt{11}\right) < \left(x - \sqrt{11}\right)$
$\Rightarrow y < x$ i. e. $x > y$
Similarly, we have
$y > z, z > t$ and $t > x \Rightarrow y > x$
$\therefore x = y = z = t = \sqrt{11}$ is the only solution for $x > 0$
We observe that (x, y, z, t) is a solution iff $(-x, -y, -z, -t)$ is a solution
Thus, $x = y = z = t = -\sqrt{11}$ is the only other solution

13 **(b)**
Given, $3 + \frac{3}{3^x} - 4 < 0 \;\Rightarrow\; 3^{2x} + 3 - 4.3^x < 0$
$\Rightarrow\; (3^x - 1)(3^x - 3) < 0$
$1 < 3^x < 3 \;\Rightarrow\; 0 < x < 1$
$\therefore$ The solution set is (0, 1)

14 **(b)**
We have, $|a| \leq 1$ and $a + b = 1$
i. e. $-1 \leq a \leq 1$ and $b = 1 - a$
$\Rightarrow -1 \leq a \leq 1$ and $0 \leq b \leq 2 \Rightarrow -2 \leq ab \leq 2$ ...(i)
Now,
$ab \leq \left(\frac{a+b}{2}\right)^2 \Rightarrow ab \leq \frac{1}{4}$ ...(ii)
From (i) and (ii), we have
$-2 \leq ab \leq \dfrac{1}{4} \Rightarrow ab \in \left[-\dfrac{2,1}{4}\right]$

15 **(d)**
Given, $\sqrt{(3x+1)^2} < (2-x)$
$\Rightarrow\; (3x+1) < 2 - x$
$\Rightarrow\; 3x + 1 < 2 - x \;\Rightarrow x < \dfrac{1}{4}$

16 **(c)**
Given, inequality can be rewritten as $\left(\frac{5}{13}\right)^x + \left(\frac{12}{13}\right)^x \geq 1$

$\therefore \cos^x \alpha + \sin^x \alpha \geq 1$

Where, $\cos\alpha = \frac{5}{13}$

If $x = 2$, the above equality holds

If $x < 2$ both $\cos\alpha$ and $\sin\alpha$ increases in positive fraction

Hence, above inequality holds for $x \in (-\infty, 2]$

17 **(b)**

Let $f(x) = \log_e \frac{x-2}{x-3}$

$f(x)$ is defined either $(x-2) > 0, (x-3) > 0$ or $(x-2) < 0$

$(x-3) < 0$ or $x \neq 2, 3$

$\Rightarrow f(x)$ is defined either $x > 3$ or $x < 2$ or $x \neq 2, 3$

$ie, x \in (-\infty, 2) \cup (3, \infty)$

18 **(c)**

$3 \leq 3t - 18 \leq 18$

$\Rightarrow \ 21 \leq 3t \leq 36$

$\Rightarrow 7 \leq t \leq 12$

$\Rightarrow 8 \leq t + 1 \leq 13$

19 **(b)**

$\because f(-1) < 1 \Rightarrow a - b + c < 1$ ...(i)

and $f(1) > -1, f(3) < -4$, then

$a + b + c > -1$ ...(ii)

$9a + 3b + c < -4$ ...(iii)

From Eq. (ii),

$-a - b - c < 1$ ....(iv)

On solving Eqs. (i), (iii) and (iv), we get

$a < -\frac{1}{8} \Rightarrow a$ is negative

20 **(d)**

Given, $\frac{2x+3}{2x-9} < 0$

$\Rightarrow 2x + 3 < 0$ and $2x - 9 > 0$

Or $2x + 3 > 0$ and $2x - 9 < 0$ and $x \neq \frac{9}{2}$

$\Rightarrow \ x < -\frac{3}{2}$ and $x > \frac{9}{2}$ or $x > -\frac{3}{2}$ and $x < \frac{9}{2}$ and $x \neq \frac{9}{2}$

$\Rightarrow x \in \left(-\frac{3}{2}, \frac{9}{2}\right)$

21 **(b)**

We have,

$$\sqrt{x^2 + \sqrt{x^2+11}} - \sqrt{x^2 - \sqrt{x^2+11}} = 4$$

Putting $x^2 + 11 = t^2$, we get

$\sqrt{t^2 + t - 11} + \sqrt{t^2 - t - 11} = 4$ ...(i)

But, $(t^2 + t - 11) - (t^2 - t - 11) = 2t$ ...(ii)

Dividing (ii) by (i), we get

$\sqrt{t^2 + t - 11} - \sqrt{t^2 - t - 11} = \frac{t}{2}$ ...(iii)

Adding (i) and (iii), we get

$2\sqrt{t^2 + t - 11} = 4 + \frac{t}{2}$

$\Rightarrow t^2 + t - 11 = 4 + t + \frac{t^2}{16}$

$\Rightarrow t^2 = 16 \Rightarrow t = 4$ $\quad \left[\because t = \sqrt{x^2+11} > 0\right]$

$\therefore x = \pm\sqrt{5}$

22 **(a)**

We have,

$\log_{x-3}(x^3 - 3x^2 - 4x + 8) = 3$ ...(i)

$\Rightarrow x^3 - 3x^2 - 4x + 8 = (x-3)^3$

$\Rightarrow 6x^2 - 31x + 35 = 0 \Rightarrow (3x-5)(2x-7) = 0$

$$\Rightarrow x = \frac{5}{3}, \frac{7}{2}$$

The equation (i) exists, if

$x - 3 > 0, x - 3 \neq 1$ and $x^3 - 3x^2 - 4x + 8 > 0$

Clearly, $x = \frac{7}{2}$ satisfies these conditions

23 **(a)**

Curves $y = \log_{0.5} x$ and $y = |x|$ intersect at one point in first quadrant. So, the equation $\log_{0.5} x = |x|$ has one real root

25 **(b)**

$\cos^x \alpha + \sin^x \alpha \geq 1$

Equality holds when $x = 2$

If $x < 2$, both $\cos\alpha$ and $\sin\alpha$ are increasing

$\therefore \cos^x \alpha + \sin^x \alpha > 1$, if $x < 2$

If $x > 2$, then $\cos^x \alpha + \sin^x \alpha < 1$

$\therefore x \in (-\infty, 2]$

27 **(a)**

We have,

$2\cos(e^x) = 3^x + 3^{-x}$

We observe that $2\cos(e^x) < 2$ and $3^x + 3^{-x} \geq 2$. So, the given equation has no solution

28 **(b)**

Graphs of $y = 1 - x$ and $y = [\cos x]$ cut each other at point $(0, 1)$ and at a point whose x-coordinate lie in $(\pi/2, \pi)$. So, the given equation has two real roots

30 **(b)**

If $a^2 + b^2 = 1$, then $a^x + b^x \geq 1$ is true for all $x \in (-\infty, 2]$

$\therefore (\sin\alpha)^x + (\cos\alpha)^x \geq 1 \Rightarrow x \in (-\infty, 2]$

31 **(a)**

If $f(x) > 0$,then $D < 0$

$4a^2 - 4(10 - 3a) < 0$

$\Rightarrow \quad (a+5)(a-2) < 0$
$\Rightarrow -5 < a < 2$

32 **(b)**

The given inequality is

$$49.4 - \left(\frac{27-x}{10}\right) < 47.4 - \left(\frac{27-9x}{10}\right)$$

$$\Rightarrow 49.4 - 47.4 < \left(\frac{27-x}{10}\right) - \left(\frac{27-9x}{10}\right)$$

$$\Rightarrow 2 < \frac{8x}{10} \Rightarrow x > \frac{5}{2}$$

$\therefore$ Least integer is 3

33 **(a)**

Since, AM $\geq$ GM

$$\therefore \frac{a^2+b^2}{2} \geq \sqrt{a^2b^2} = ab, \frac{b^2+c^2}{2} \geq bc$$

and $\frac{c^2+a^2}{2} \geq ca$

On adding, we get

$a^2 + b^2 + c^2 \geq ab + bc + ca$

$\Rightarrow (a)$ holds

Next, $\frac{b+c}{2} \geq \sqrt{bc}$, $\frac{c+a}{2} \geq \sqrt{ca}$, $\frac{a+b}{2} \geq \sqrt{ab}$

$$\Rightarrow \left(\frac{b+c}{2}\right)\left(\frac{c+a}{2}\right)\left(\frac{a+b}{2}\right) \geq \sqrt{a^2b^2c^2}$$

$\Rightarrow (b+c)(c+a)(a+b) \geq 8abc$

$\Rightarrow (b)$ does not hold

Again, $\frac{1}{3}\left(\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\right) \geq \left(\frac{a}{b}\cdot\frac{b}{c}\cdot\frac{c}{a}\right)^{1/3}$

$$\Rightarrow \frac{a}{b}+\frac{b}{c}+\frac{c}{a} \geq 3$$

$\Rightarrow (c)$does not hold

Again, $\frac{a^3+b^3+c^3}{3} \geq (a^3b^3c^3)^{1/3}$

$\Rightarrow a^3 + b^3 + c^3 \geq 3\,abc$

$\Rightarrow (d)$ does not hold

34 **(d)**

If s is the semi-perimeter of a cyclic quadrilateral of sides a, b, c and d units in length, then its area A is given by

$$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

Using A. M. $\geq$ G. M., we have

$$\frac{s-a+s-b+s-c+s-d}{4} \geq \{(s-a)(s-b)(s-c)(s-d)\}^{1/4}$$

$$\Rightarrow \frac{4s-2s}{4} \geq \sqrt{A} \Rightarrow 2s \geq 4\sqrt{A}$$

Hence, the least perimeter is $4\sqrt{A}$

35 **(c)**

Two curves

$y = [\sin x + \cos x]$

and, $y = 3 + [-\sin x] + [-\cos x]$
$= 1 + [\sin x] + [\cos x]$

intersect at infinitely many points in $[0, 2\pi]$

So, the given equation has infinitely many solutions

36 **(b)**

Two curves $y = 3^{|x|}$ and $y = \big|2 - |x|\big|$ intersect at two points only. So, the equation $3^{|x|} = \big|2 - |x|\big|$ has only two real roots

37 **(a)**

Since, angle C is obtuse, angle A and B are actute

$\therefore \tan C < 0$and $\tan A > 0, \tan B > 0$

Now, $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\ \tan B}$

$$\Rightarrow \tan(\pi - C) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\Rightarrow -\tan C = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$\Rightarrow 1 - \tan A \tan B > 0$ ($\because$ Numerator are positive)

$\Rightarrow \tan A \tan B < 1$

38 **(a)**

We have,

$x + y + z = 4$ and $x^2 + y^2 + z^2 = 6$

$\Rightarrow y + z = 4 - x$ and $y^2 + z^2 = 6 - x^2$

$$\therefore yz = \frac{1}{2}\{(y+z)^2 - (y^2+z^2)\}$$
$$= \frac{1}{2}\{(4-x)^2 - (6-x)^2\}$$

$\Rightarrow yz = x^2 - 4x + 5$

Thus, y and z are roots of the equation

$t^2 - (4-x)\,t + x^2 - 4x + 5 = 0$

As y, z are real

$\therefore (4-x)^2 - 4(x^2 - 4x + 5) \geq 0$

$\Rightarrow 3x^2 - 8x + 4 \leq 0 \Rightarrow \frac{2}{3} \leq x \leq 2$

40 **(b)**

We have,

$3^{x/2} + 2^x > 25 \Rightarrow 3^{x/2} + 4^{x/2} > 25$

Clearly, $x \in (4, \infty)$ satisfies the above inequation

41 **(c)**

We have,

$27^{1/x} + 12^{1/x} = 2 \times 8^{1/x}$

$\Rightarrow 3^{3/x} + 2^{2/x} \times 3^{1/x} = 2 \times 2^{3/x}$

$\Rightarrow \left(\frac{3}{2}\right)^{3/x} + \left(\frac{3}{2}\right)^{1/x} = 2$

$\Rightarrow y^3 + y - 2 = 0$, where $y = \left(\frac{3}{2}\right)^{1/x}$

$\Rightarrow (y-1)(y^2 + y - 2) = 0$

$\Rightarrow y = 1, y = -2 \Rightarrow \left(\frac{3}{2}\right)^{1/x} = 1 \Rightarrow \left(\frac{3}{2}\right)^{1/x} = \left(\frac{3}{2}\right)^0$

But, there is no value of x for which $\frac{1}{x}$ is zero

Hence, the given equation has no solution

42 **(c)**

Let x_1, x_2, x_3 and x_4 be four positive roots of the equation $x^4 - 8x^3 + bx^2 + cx + 16 = 0$. Then,

$x_1 + x_2 + x_3 + x_4 = 8$ and $x_1\,x_2\,x_3\,x_4 = 16$

$\Rightarrow \frac{x_1 + x_2 + x_3 + x_4}{4} = 2$ and $(x_1\,x_2\,x_3\,x_4)^{1/4} = 2$

$\Rightarrow$ A. M. and G. M. of x_1, x_2, x_3 and x_4 are equal

$\Rightarrow x_1 = x_2 = x_3 = x_4$

$\Rightarrow x_1 = x_2 = x_3 = x_4 = 2$

$\therefore x^4 - 8x^3 + bx^2 + cx + 16 = (x-2)^4$

$\Rightarrow b = {}^4C_2 \times 2^2 = 24$ and $c = -{}^4C_3 \times 2^3 = -32$

43 **(a)**

We have, $3 < |x| < 6 \Rightarrow -6 < x < -3$ or $3 < x < 6$

$\therefore x \in (-6, -3) \cup (3, 6)$

44 **(a)**

We have,

$a^4 + b^4 - a^3\,b - ab^3 = a^3(a-b) - b^3(a-b)$

$= (a^3 - b^3)(a - b)$

$\Rightarrow a^4 + b^4 - a^3 b - ab^3 > 0$ $\left[\because a^3 - b^3 \text{ and } a - b \text{ are of the same sign}\right]$

$\Rightarrow a^4 + b^4 > a^3\,b + ab^3$

45 **(a)**

The given inequation is

$4^{-x+0.5} - 7 \cdot 2^{-x} < 4, x \in R$

Let $2^{-x} = t$

$\therefore 2t^2 - 7t < 4$

$\Rightarrow 2t^2 - 7t - 4 < 0$

$\Rightarrow (2t+1)(t-4) < 0$

$\Rightarrow -\frac{1}{2} < t < 4$

$\Rightarrow 0 < t < 4 \quad (\because t = 2^{-x} > 0)$

$\Rightarrow 0 < 2^{-x} < 2^2$

As 2^x is an increasing function $-x < 2$ or $x > -2$

$\therefore x = (-2, \infty)$

46 **(c)**

Given condition are $\frac{a}{b} > 1$ and $\frac{a}{c} < 0$

1. $a > 0$ iff $c < 0$ and also $b > 0$
2. $a < 0$ iff $c > 0$ and also $b < 0$

47 **(b)**

Proceeding as in the solution of Q. no. 10, we have

$(a+b)(b+c)(c+a) \geq 8abc$

$\Rightarrow (p-a)(p-b)(p-c) \geq 8abc \quad [\because a + b + c = p]$

48 **(a)**

We have,

$\frac{(1 + e^{x^2})\sqrt{1 + x^2}}{\sqrt{1 + x^4} - x^2} = 1 + \cos x$

$\Rightarrow (1 + e^{x^2})\sqrt{1 + x^2}\left(\sqrt{1 + x^4} + x^2\right) = 1 + \cos x$

Clearly, LHS ≥ 2 and RHS ≤ 2. So, the equation exists when each side is equal to 2. This is for $x = 0$ only. Hence, it has only one solution

50 **(c)**

Let $f(x) = c_0 + c_1 x + c_2 x^2 + \cdots + c_n x^n$. Then,

$f(b) - f(a) = 1$

$\Rightarrow c_1(b-a) + c_2(b^2 - a^2) + \cdots + c_n(b^n - a^n) = 1$

$\Rightarrow (b-a)\{c_1 + c_2(b+a) + \cdots + c_n(b^{n-1} + b^{n-2}a + \cdots + a^{n-1})\} = 1$

$\Rightarrow (b-a)I = 1$, where I is an integer

$\Rightarrow b - a = \pm 1$

52 **(d)**

Using, AM > GM

$\therefore \ \dfrac{a+b+c}{3} > \sqrt[3]{abc}$

$\Rightarrow a+b+c > 3$ ...(i)

$[\because abc = 1 \text{ given}]$

Also, GM > HM

$\sqrt[3]{abc} > \dfrac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$

$\Rightarrow \ (1)^{1/3} > \dfrac{3abc}{bc+ac+ab}$

$\Rightarrow \ ab+bc+ac > 3$ ...(ii)

$\therefore$ From Eqs. (i) and (ii), we get

$a+b+c+ab+bc+ac > 6$

53 **(d)**

We have,

$\sin(2^x)\cos(2^x) = \dfrac{2^x+2^{-x}}{2}$

$\Rightarrow \sin(2^{x+1}) = 2^x + 2^{-x}$

Clearly, RHS ≥ 2 and LHS lies between -1 and 1.

So, the given equation has no solution

54 **(d)**

$x^{12} - x^9 + x^4 - x + 1 > 0$

When $0 < x < 1; x^4 > x^9$ and $1 > x$

$\therefore x^{12} + (x^4 - x^9) + (1-x) > 0$

$\Rightarrow$ Positive for all x

Again, when $x > 1$:

$x^{12} - x^9 + x^4 - x + 1 > 0$

$\therefore$ Largest interval $(0,\infty)$, also the above inequality is true for $x < 0$

56 **(d)**

$\because$ AM $\geq$ GM

$\Rightarrow \dfrac{\frac{\cos^3 x}{\sin x}+\frac{\sin^3 x}{\cos x}}{2} \geq \left(\dfrac{\cos^3 x}{\sin x}\cdot\dfrac{\sin^3 x}{\cos x}\right)^{1/2}$

$\Rightarrow \dfrac{\cos^3 x}{\sin x} + \dfrac{\sin^3 x}{\cos x} \geq 2\sin x\cos x \geq 1$

Hence, option (d) is correct

57 **(a)**

We have,

$\sqrt{3x^2+6x+7} = \sqrt{3(x+1)^2+4} \geq 2(= 2 \text{ when } x = -1)$

$\sqrt{5x^2+10x+14} = \sqrt{5(x+1)^2+9} \geq 3(= 3 \text{ when } x = -1)$

and,

$4-2x-x^2 = 5-(x+1)^2 \leq 5(= 5 \text{ when } x = -1)$

Thus, LHS ≥ 5 and RHS ≤ 5

So, the given equation is valid when each sides is equal to 5.

This happens only when $x = -1$

Hence, the given equation has only one solution

58 **(d)**

$\big||x|-1\big| < |1-x|$

Case I $x \geq 0$

$\therefore$ Inequality (i) becomes $|x-1| < x-1$ or $|1-x| < 1-x$ which is not satisfied by any x, because

$|a| \geq \forall a \in R$

Case II $-1 \leq x < 0$

$\therefore$ Inequality (i) becomes $|-1-x| < 1-x$ or $|x+1| < 1-x$

Or $x+1 < 1-x$ or $x < 0$

Thus, inequality (i) is satisfied for $-1 \leq x < 0$

Case III $x < -1$

Inequality (i) becomes $|-1-x| < 1-x \Rightarrow |1+x| < 1-x$

$\Rightarrow -(1+x) < 1-x \Rightarrow -2 < 0$, which is true

So, solution set is $(-\infty, 0)$

59 **(c)**

Minimum value of $f(x)$

Is attained at $x = 3$

$\therefore$ Minimum value of $f(x) = 7$

60 **(a)**

$\dfrac{x+11}{x-3} > 0$

$\Rightarrow \ (x-3)(x+11) > 0$

$\Rightarrow \ x < -11, x > 3$

$\Rightarrow \ x \in (-\infty,-1) \cup (3,\infty)$

61 **(b)**

$2x-1 = |x+7| = \begin{cases} x+7, \text{if } x \geq -7 \\ -(x+7), \text{if } x < -7 \end{cases}$

$\therefore$ If $x \geq -7, 2x-1 = x+7 \Rightarrow x = 8$

If $x < -7, 2x-1 = -(x+7)$

$\Rightarrow 3x = -6$

$\Rightarrow x = -2$, which is not possible

62 **(c)**

Let $f(x) = x^4 - 4x - 1$. Then, the number of changes of signs in $f(x)$ is 1. Therefore, $f(x)$ can have at most one positive real root

We have,

$f(2) > 0$ and $f(0) < 1$

Therefore, $f(x)$ has one positive real root between 1 and 2

63 **(c)**

$\log_{\sin\left(\frac{\pi}{3}\right)}(x^2 - 3x + 2) \geq 2$

$\Rightarrow x^2 - 3x + 2 \leq \frac{3}{4}$ (If $\log_a b = c \Rightarrow b = c^a$)

$\Rightarrow x^2 - 3x + \frac{5}{4} \leq 0$

$\Rightarrow 4x^2 - 12x + 5 \leq 0$

$\Rightarrow (2x - 5)(2x - 1) \leq 0$

$\Rightarrow \frac{1}{2} \leq x \leq \frac{5}{2}$ ...(i)

Also, $x^2 - 3x + 2 > 0$

$\Rightarrow (x - 1)(x - 2) > 0$

$\Rightarrow x < 1$ or $x > 2$ ...(ii)

From relation (i) and (ii), we get

$x \in \left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$

64 **(c)**

Since, $(a - b)^2 + (b - c)^2 + (c - a)^2 \geq 0$

$\Rightarrow 2(a^2 + b^2 + c^2) \geq 2(ab + bc + ca)$

$\Rightarrow \frac{a^2 + b^2 + c^2}{(ab + bc + ca)} \geq 1$

$\Rightarrow \frac{a^2 + b^2 + c^2}{ab + bc + ca} + 2 \geq 3$

Hence, option (c) is correct

65 **(b)**

Given, $(x - 1)(x^2 - 5x + 7) < (x - 1)$

$\Rightarrow (x - 1)(x^2 - 5x + 6) < 0$

$\Rightarrow (x - 1)(x - 2)(x - 3) < 0$

$\Rightarrow x \in (-\infty, 1) \cup (2, 3)$

66 **(d)**

Given inequalities are

$x^2 - 3x - 10 < 0$ and $10x - x^2 - 16 > 0$

$\Rightarrow (x + 2)(x - 5) < 0$ and $(x - 2)(x - 8) < 0$

$\Rightarrow x \in (-2, 5)$ and $x \in (2, 8)$

$\Rightarrow x \in (2, 5)$

67 **(a)**

$\log_{1/3}(x^2 + x + 1) < -1 = \log_{1/3}\left(\frac{1}{3}\right)^{-1}$

$\Rightarrow x^2 + x + 1 > \left(\frac{1}{3}\right)^{-1}$

($\because$ where $0 < a < 1$, then $\log_a x < \log_a y \Rightarrow x > y$)

$\Rightarrow x^2 + x - 2 > 0 \Rightarrow (x + 2)(x - 1) > 0$

$\Rightarrow x \in (-\infty, -2) \cup (1, \infty)$

68 **(a)**

Let (α, β) be a solution of the system for some a. Then, $(-\alpha, \beta)$ is also a solution. So, the system will have unique solution only if

$\alpha = -\alpha \Rightarrow \alpha = 0$

Putting $x = \alpha = 0$ and $y = \beta$ in $x^2 + y^2 = 1$, we get $\beta = \pm 1$

Putting $x = \alpha = 0$ and $y = \beta$ in $2^{|x|} + |x| = y + x^2 + a$, we get

$\beta + a = 1 \Rightarrow a = 1 - \beta$

$\therefore a = 0$ when $\beta = 1$ and $a = 2$ when $\beta = -1$

CASE I *When* $a = 0$

In this case, given equations become

$2^{|x|} + |x| = y + x^2$ and $x^2 + y^2 = 1$

Now, $x^2 + y^2 = 1 \Rightarrow |x| \leq 1$ and $|y| \leq 2$

$\therefore 2^{|x|} + |x| = y + x^2$ and $1 + x^2 \geq y + x^2$

$\Rightarrow 2^{|x|} + |x| \leq 1 + x^2$

$\Rightarrow 2^{|x|} + |x| \leq 1 + |x|$ [$\because x^2 \leq |x|$ when $|x| < 1$]

$\Rightarrow x = 0$

Putting $x = 0$ in $2^{|x|} + |x| = y + x^2$, we get $y = 1$

Thus, for $a = 0$, the system has unique solution $(0, 1)$

CASE II *When* $a = 2$

In this case, the system of equation is

$2^{|x|} + |x| = y + x^2 + 2$ and $x^2 + y^2 = 1$

Clearly, $(0, -1)$, $(1, 0)$ and $(-1, 0)$ satisfy these equations.

So, the system does not have unique solution

69 **(c)**

We have,

$\log_{1/3}(x^2+x+1)+1>0$

$\Rightarrow \log_{1/3}(x^2+x+1)>-1$

$\Rightarrow x^2+x+1<\left(\frac{1}{3}\right)^{-1}$

$\Rightarrow x^2+x+1<3$

$\Rightarrow x^2+x-2<0 \Rightarrow (x+2)(x-1)<0 \Rightarrow x \in (-2,1)$

70 **(d)**

We have,

$x^{(\log_{10}x)^2-3(\log_{10}x)+1}>10^3$

$\Rightarrow (\log_{10}x)^2-3(\log_{10}x)+1>\log_x 10^3$

$\Rightarrow (\log_{10}x)^2-3(\log_{10}x)+1>\frac{3}{\log_{10}x}$

$\Rightarrow \frac{(\log_{10}x)^3-3(\log_{10}x)^2+(\log_{10}x)-3}{\log_{10}x}>0$

$\Rightarrow \frac{\{(\log_{10}x)^2+1\}(\log_{10}x-3)}{\log_{10}x}>0$

$\Rightarrow \frac{(\log_{10}x-3)}{(\log_{10}x-0)}>0$

$\Rightarrow \log_{10}x<0$ or, $\log_{10}x>3$

$\Rightarrow x<1$ or, $x>10^3$

$\Rightarrow x\in(0,1)\cup(10^3,\infty)$ [$\because \log_{10}x$ is defined for $x>0$]

71 **(a)**

Given, $\frac{3-|x|}{4-|x|}\geq 0$

$\Rightarrow\ 3-|x|\leq 0$ and $4-|x|<0$

Or $3-|x|\geq 0$ and $4-|x|>0$

$\Rightarrow\ |x|\geq 3$ and $|x|>4$

Or $|x|\leq 3$ and $|x|<4$

$\Rightarrow\ |x|>4$ or $|x|\leq 3$

$\Rightarrow (-\infty,-4)\cup[-3,3]\cup(4,\infty)$

72 **(a)**

Now, $3(a^2+b^2+c^2)-(a+b+c)^2$

$=2(a^2+b^2+c^2-bc-ca-ab)$

$=(b-c)^2+(c-a)^2+(a-b)^2\geq 0$

$\Rightarrow 3(a^2+b^2+c^2)\geq(a+b+c)^2>9$

$\Rightarrow a^2+b^2+c^2>3 \Rightarrow$ (a)holds

Now, $a^6+b^6\geq 12a^2b^2-64$

If $a^6+b^6+64\geq 12a^2b^2$

ie, $a^6+b^6+2^6\geq 3\cdot 2^2\cdot a^2b^2$

ie, if $\frac{a^6+b^6+2^6}{3}\geq(2^6a^6b^6)^{1/3}$ ($\because$ AM $\geq$ GM)

$\Rightarrow (b)$does not hold

Again, since AM $\geq$ HM

$\therefore \frac{a+b+c}{3}\geq\frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$

$\Rightarrow \frac{\alpha}{3}\geq\frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$

$\Rightarrow \frac{1}{a}+\frac{1}{b}+\frac{1}{c}\geq\frac{9}{\alpha}$

$\Rightarrow (c\ )$ does not hold

73 **(c)**

Using A. M. $\geq$ G. M., we have

$\frac{a_1+a_2+a_3}{3}\geq(a_1\,a_2\,a_3)^{1/3}$ and $\frac{1}{3}(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3})$

$\geq\left(\frac{1}{a_1a_2a_3}\right)^{1/3}$

$\Rightarrow (a_1+a_2+a_3)\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}\right)\geq 9$

74 **(a)**

We have,

$x-\frac{1}{x}=B$ and $x^2+\frac{1}{x^2}=A$

$\therefore \left(x-\frac{1}{x}\right)^2=B^2$

$\Rightarrow A-2=B^2 \Rightarrow A=B^2+2 \Rightarrow \frac{A}{B}=B+\frac{2}{B}$

But, A. M. $\geq$ G. M.

$\Rightarrow B+\frac{2}{B}\geq 2\sqrt{B\times\frac{2}{B}} \Rightarrow B+\frac{2}{B}\geq 2\sqrt{2} \Rightarrow \frac{A}{B}\geq 2\sqrt{2}$

Hence, the minimum value of $\frac{A}{B}$ is $2\sqrt{2}$

75 **(c)**

As discussed in the above problem, if n is odd, there is one change of sign in (i). Therefore, $f(x)$ can have at most one negative real root. In this case, we have

$f(-1) = -2n - 2 < 0, f(0) = 1 > 0$

So, the negative real root lies between -1 and 0

76 **(c)**

Given, ${}^{n+1}C_{n-2} - {}^{n+1}C_{n-1} \leq 50$

$\Rightarrow \frac{(n-1)!}{3!(n-2)!} - \frac{(n+1)!}{2!(n-1)!} \leq 50$

$\Rightarrow \frac{(n+1)!}{3!}\left[\frac{1}{(n-2)!} - \frac{3}{(n-1)!}\right] \leq 50$

$\Rightarrow (n+1)!\left(\frac{n-1-3}{(n-1)!}\right) \leq 300$

$\Rightarrow (n+1)n(n-4) \leq 300$

For $n = 8$, it satisfy to the above inequality

But $n = 1$ it does not satisfy the above inequality

77 **(d)**

We have,

$f(\theta) = \sec^2\theta + \cos^2\theta = (\sec\theta - \cos\theta)^2 + 2 \geq 2$

$\Rightarrow f(\theta) \in [2, \infty)$

78 **(d)**

Let $a_1, a_2, \ldots, a_n$ be n positive integers such that

$a_1 a_2 \ldots a_n = n^n$

Since, AM $\geq$ GM

$\therefore \frac{a_1 + a_2 + \ldots + a_n}{n} \geq (a_1 a_2 \ldots a_n)^{1/n}$

$\Rightarrow \frac{a_1 + a_2 + \ldots + a_n}{n} \geq n$

$\Rightarrow a_1 + a_2 + \ldots + a_n \geq n^2$

79 **(a)**

$\log_2(x^2 - 3x + 18) < 4$

$\Rightarrow x^2 - 3x + 18 < 16$ (If $\log_a b < c \Rightarrow b < a^c$)

$\Rightarrow x^2 - 3x + 2 < 0$

$\Rightarrow (x-1)(x-2) < 0$

$\Rightarrow x \in (1, 2)$

80 **(d)**

We have,

$[x]^2 = [x + 6]$

$\Rightarrow [x]^2 = [x] + 6$

$\Rightarrow [x]^2 - [x] - 6 = 0$

$\Rightarrow ([x] - 3)([x] + 2) = 0$

$\Rightarrow [x] = 3, [x] = -2$

$\Rightarrow x \in [3,4)$ or $x \in [-2, -1) \Rightarrow x \in [-2, -1) \cup [3,4)$

81 **(b)**

Using AM $\geq$ GM

$\frac{\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}}{3} \geq \sqrt[3]{\frac{abc}{(a+b)(b+c)(c+a)}}$ ...(i)

Again, using AM≥GM

$\frac{a+b}{2} \geq \sqrt{ab}, \frac{b+c}{2} \geq \sqrt{bc}, \frac{c+a}{2} \geq \sqrt{ca}$

$\Rightarrow (a+b)(b+c)(c+a) \geq 8abc$

$\Rightarrow \sqrt[3]{\frac{abc}{(a+b)(b+c)(c+a)}} \leq \frac{1}{2}$

$\therefore$ From Eq. (i)

$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{3}{2}$

82 **(a)**

Two curves $y = e^{|x|}$ and $y = |x|$ does not intersect. So, the equation $e^{|x|} - |x| = 0$ has no solution

83 **(b)**

$|p - q| = \begin{cases} p - q, p \geq q \\ q - p, p < q \end{cases}$

$\therefore \min(p, q) = \begin{cases} p, p < q \\ q, q < p \end{cases}$

$\Rightarrow \text{RHS} = \frac{1}{2}(p + q - |p - q|)$, if $p > q$

$\Rightarrow \frac{1}{2}(p + q - p + q) = q$

and LHS $\min(p, q) = q$

$\therefore \min(p, q) = \frac{1}{2}(p + q - |p - q|)$

84 **(a)**

The given equation is

$1 + |e^x - 1| = e^x(e^x - 2)$

$\Rightarrow |e^x - 1| + 2 = (e^x - 1)^2$

$\Rightarrow |e^x - 1|^2 - (e^x - 1) - 2 = 0$

$\Rightarrow (|e^x - 1| - 2)(|e^x - 1| + 1) = 0$

$\Rightarrow |e^x - 1| - 2 = 0$ $[\because |e^x - 1| + 1 \neq 0]$

$\Rightarrow e^x - 1 = \pm 2 \Rightarrow e^x = 3, -1$

$\Rightarrow e^x = 3 \Rightarrow x = \log_e 3$ $[\because e^x > 0$ for all $x]$

85 **(c)**

We have,

$\log_{\sin\frac{2\pi}{3}}(x^2 - 3x + 2) \geq 2$

$\Rightarrow (x^2 - 3x + 2) \leq \left(\frac{\sqrt{3}}{2}\right)^2$ and $x^2 - 3x + 2 > 0$

$\Rightarrow 4x^2 - 12x + 5 \leq 0$ and $x^2 - 3x + 2 > 0$

$\Rightarrow (2x-1)(2x-5) \leq 0$ and $(x-1)(x-2) > 0$

$\Rightarrow \frac{1}{2} \leq x \leq \frac{5}{2}$ and $x < 1$ or $x > 2$

$\Rightarrow x \in [1/2, 1) \cup (2, 5/2]$

86 **(c)**

We have, $x^2 + 6x - 27 > 0$

$\Rightarrow (x+9)(x-3) > 0 \Rightarrow x < -9$ or $x > 3$

$\Rightarrow x \in (-\infty, -9) \cup (3, \infty)$ ....(i)

And $x^2 - 3x - 4 < 0$

$\Rightarrow (x-4)(x+1) < 0$

$\Rightarrow -1 < x < 4$ ....(i)

From relations (i) and (ii), we get

$3 < x < 4$

87 **(d)**

We have,

$\frac{1}{2}\{(x+y) + |x-y|\} = x$

$\Rightarrow \frac{1}{2}\{(x+y) + |x-y|\} = \frac{1}{2}\{(x+y) + (x-y)\}$

$\Rightarrow |x-y| = x - y \Rightarrow x \geq y$

88 **(c)**

Given, $\frac{4x-1}{3x+1} - 1 \geq 0$

$\Rightarrow \frac{x-2}{3x+1} \geq 0$

$\Rightarrow x - 2 \geq 0$ and $3x + 1 > 0$

Or $x - 2 \leq 0$ and $3x + 1 < 0$

$\Rightarrow x \geq 2$ and $x < -\frac{1}{3}$

Or $x \leq 2$ and $x > -\frac{1}{3}$

$\Rightarrow x \in \left(-\infty, -\frac{1}{3}\right) \cup [2, \infty)$

89 **(c)**

Given inequality holds only, if

$\sin^2 \alpha_i = 1$ or $\alpha_i = \pm\frac{\pi}{2}, \frac{3\pi}{2}$; $(i = 2,3, \dots, n)$

$\Rightarrow$ Number of solutions$= 3 \times 3 \times 3 \times \dots \times (n-1)$times

$= 3^{n-1}$

90 **(b)**

We have, $e^x = x(x+1), x < 0$

Consider the curves $y = e^x$ and $y = x(x+1)$ for $x < 0$. Graphs of these two curve intersect at exactly one point. So, the equation $e^x = x(x+1)$ has exactly one real root

91 **(b)**

Draw graphs of $y = 1 - x + [x]$ and $y = \frac{1}{x} - \frac{1}{[x]}$

These two curves intersect it infinitely many points

92 **(d)**

We have, $2^x + 2^{|x|} \geq 2\sqrt{2}$

Following cases arise:

<u>CASE I</u> $When\ x \geq 0$

In this case, we have

$2^x + 2^x \geq 2\sqrt{2} \Rightarrow 2^x \geq 2^{1/2} \Rightarrow x \geq \frac{1}{2} \Rightarrow x \in \left[\frac{1}{2}, \infty\right)$

<u>CASE II</u> $When\ x < 0$

In this case, we have

$2^x + 2^{-x} \geq 2\sqrt{2}$

$\Rightarrow (2^x)^2 - 2\sqrt{2} \times 2^x + 1 \geq 0$

$\Rightarrow \left(2^x - \sqrt{2}\right)^2 - 1 \geq 0$

$\Rightarrow \left(2^x - \sqrt{2} - 1\right)\left(2^x - \sqrt{2} + 1\right) \geq 0$

$\Rightarrow 2^x \leq \sqrt{2} - 1$ or, $2^x \geq \sqrt{2} + 1$

$\Rightarrow x \leq \log_2(\sqrt{2} - 1)$ or, $x \geq \log_2(\sqrt{2} + 1)$

$\Rightarrow x \leq \log_2(\sqrt{2} - 1) \Rightarrow x \in \left(-\infty, \log_2(\sqrt{2} - 1)\right)$

Hence, $x \in \left(-\infty, \log_2(\sqrt{2} - 1)\right) \cup [1/2, \infty)$

93 **(c)**

$\sin^4 \frac{x}{3} + \cos^4 \frac{x}{3} > \frac{1}{2} \Rightarrow 1 - \frac{1}{2}\sin^2 \frac{2x}{3} > \frac{1}{2}$

$\Rightarrow \sin^2 \frac{2x}{3} < 1 \Rightarrow \frac{2x}{3} \in \left(R - (2n+1)\frac{\pi}{2}\right)$

$\Rightarrow x \in R - \left(\frac{3n\pi}{2} + \frac{3\pi}{4}\right); n \in I$

94 **(a)**

We have,

$(\log_5 x)^2 + (\log_5 x) < 2$

$\Rightarrow (\log_5 x)^2 + (\log_5 x) - 2 < 0$

$\Rightarrow (\log_5 x + 2)(\log_5 x - 1) < 0$

$\Rightarrow -2 < \log_5 x < 1 \Rightarrow 5^{-1} < x < 5 \Rightarrow x \in \left(\frac{1}{25}, 5\right)$

96 **(d)**

$\because \sin x + 2\sqrt{2}\cos x \geq (\sqrt{3})^2$

$\Rightarrow \sin x + 2\sqrt{2}\cos x \geq 3$

$\Rightarrow \sin\left(a + \cos^{-1}\frac{1}{3}\right) \geq 1$

$\Rightarrow \sin\left(x + \cos^{-1}\frac{1}{3}\right) = 1$ ($\because \sin x$ cannot be greater than 1)

$\therefore x = n\pi + (-1)^n \frac{\pi}{2} - \cos^{-1}\frac{1}{3}$

For solution in the interval $[-2\pi, 2\pi], n = 0, 1, -1, -2$

97 **(c)**

$x^{(\log_{10} x)^2 - 3\log_{10} x + 1} > 1000 = 10^3$

$\Rightarrow [(\log_{10} x)^2 - 3\log_{10} x + 1]\log_{10} x > 3\log_{10} 10 = 3$

$\Rightarrow (\log_{10} x)^3 - 3(\log_{10} x)^2 + \log_{10} x > 3$

$\Rightarrow (\log_{10} x)(\log_{10} x - 3) + 1(\log_{10} x - 3) > 0$

$\Rightarrow (\log_{10} x - 3)(\log_{10} x + 1) > 0$

$\Rightarrow \log_{10} x - 3 > 0 \Rightarrow \log_{10} x > 3$

$\Rightarrow x > 10^3 = 1000$

$\Rightarrow x \in (1000, \infty)$

98 **(d)**

$x^{12} - x^9 + x^4 - x + 1 > 0$, three cases arise

Case I When $x \leq 0$

$x^{12} > 0, -x^9 > 0, x^4 > 0, -x > 0$

$\Rightarrow \; x^{12} - x^9 + x^4 + x + 1 > 0, \forall x \leq 0$ ...(i)

Case II When $0 < x \leq 1$

$x^9 < x^4, x < 1 \Rightarrow -x^9 + x^4 > 0$ and $1 - x > 0$

$\therefore \; x^{12} - x^9 + x^4 - x + 1 > 0, \forall\, 0 < x \leq 1$ ...(ii)

Case III When $x > 1$

$x^{12} > x^9, x^4 > x$

$\Rightarrow \; x^{12} - x^9 + x^4 - x + 1 > 0, \forall x > 1$ ...(iii)

$\therefore$ From Eqs. (i), (ii) and (iii) the above equation hold for $x \in R$

100 **(a)**

We have,

$\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$

$\Rightarrow x + 1 + x - 1 - 2\sqrt{x^2 - 1} = 4x - 1$

$\Rightarrow -2\sqrt{x^2 - 1} = 2x - 1$

$\Rightarrow 4(x^2 - 1) = 4x^2 - 4x + 1 \Rightarrow 4x - 5 = 0 \Rightarrow x = \frac{5}{4}$

This value of x does not satisfy the given equation. So, the equation has no solution

101 **(d)**

The LHS of the given inequality is meaningful for $x > 0$ and $x \neq 1$

Now,

$\log_3 x - \log_x 27 < 2$

$\Rightarrow \log_3 x - 3\log_x 3 < 2$

$\Rightarrow \log_3 x - \frac{3}{\log_3 x} < 2$

$\Rightarrow \frac{(\log_3 x)^2 - 3 - 2(\log_3 x)}{\log_3 x} < 0$

$\Rightarrow \frac{(\log_3 x - 3)(\log_3 x + 1)}{(\log_3 x - 0)} < 0$

$\Rightarrow \log_3 x < -1$ or, $0 < \log_3 x < 3$

$\Rightarrow x < 3^{-1}$ or, $3^0 < x < 3^3 \Rightarrow x < \frac{1}{3}$ or, $1 < x < 27$

Also, $x > 0$ and $x \neq 1$

$\therefore x \in (0, 1/3) \cup (1, 27)$

102 **(c)**

Given inequation is $x^2 - 2x + 5 \leq 0$

$\therefore$ Roots are

$x = \frac{-2 \pm \sqrt{4 - 20}}{2} = \frac{2 \pm 4i}{2}$

$\therefore$ Roots are imaginary, therefore no real solutions exist

103 **(b)**

We have,

$\frac{2^{\sin^2 x} + 2^{\cos^2 x}}{2} \geq \sqrt{2^{\sin^2 x} \times 2^{\cos^2 x}}$ [Using A. M. $\geq$ G. M.]

$\Rightarrow 2^{\sin^2 x} + 2^{\cos^2 x} \geq 2\sqrt{2} \Rightarrow 2^{\sin^2 x} + 2^{\cos^2 x} \geq 2$

104 **(a)**

Given that, $a^2 + b^2 + c^2 = 1$ ...(i)

Now, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) \geq 0$

$\Rightarrow 2(ab + bc + ca) \geq -1$ [from Eq.(i)]

$\Rightarrow ab + bc + ca \geq -\frac{1}{2}$ ...(iii)

Also, $a^2 + b^2 + c^2 - ab - bc - ca$

$= \frac{1}{2}\{(a - b)^2 + (b - c)^2 + (c - a)^2\} \geq 0$

$\Rightarrow ab + bc + ca \leq a^2 + b^2 + c^2$

$\Rightarrow ab + bc + ca \leq 1$ [from Eq.(i)] ...(iii)

From relation (ii) and (iii), we get

$-\frac{1}{2} \leq ab + bc + ca \leq 1$

105 **(b)**

We have,

$\sqrt{4x+9} - \sqrt{11x+1} = \sqrt{7x+4}$

$\Rightarrow \sqrt{4x+9} - \sqrt{7x+4} = \sqrt{11x+1}$

$\Rightarrow 4x + 9 + 7x + 4 - 2\sqrt{(4x+9)(7x+4)}$

$= 11x + 1$

$\Rightarrow -2\sqrt{(4x+9)(7x+4)} = -12$

$\Rightarrow (4x+9)(7x+4) = 36$

$\Rightarrow 28x^2 + 79x = 0 \Rightarrow x = 0, -\frac{79}{28}$

Clearly, only $x = 0$ satisfies the given equation

106 **(a)**

Since, $-3 < x + \frac{2}{x} < 3$

$\Rightarrow -3 < \frac{(x^2+2)x}{x^2} < 3$

$\Rightarrow -3x^2 < (x^2+2)x < 3x^2 \quad (x \neq 0)$

$\Rightarrow x(x^2 + 3x + 2) > 0$

And $x(x^2 - 3x + 2) < 0 \quad (x \neq 0)$

$\Rightarrow x(x+1)(x+2) > 0$

And $x(x-1)(x-2) < 0$

$\Rightarrow x \in (-2, -1) \cup (0, \infty)$ ...(i)

And $x \in (-\infty, 0) \cup (1, 2)$ ...(ii)

From relations (i) and (ii), we get

$x \in (-2, -1) \cup (1, 2)$

107 **(b)**

We have a, b, c are sides of a triangle

$\therefore b + c - a > 0, c + a - b > 0, a + b - c > 0$

Let $x = b + c - a, y = c + a - b, z = a + b - c$

$\Rightarrow y + z = 2a, z + x = 2b, x + y = 2c$

Now, $\frac{a}{b+c-a} + \frac{b}{c+a-b} + \frac{c}{a+b-c}$

$= \frac{y+z}{2x} + \frac{z+x}{2y} + \frac{x+y}{2z}$

$= \frac{1}{2}\left(\frac{x}{y} + \frac{y}{z} + \frac{z}{x} + \frac{x}{z} + \frac{y}{x} + \frac{z}{y}\right)$

$\geq \frac{6}{2}\left(\frac{y}{x} \cdot \frac{x}{y} \cdot \frac{y}{z} \cdot \frac{z}{y} \cdot \frac{z}{x} \cdot \frac{x}{z}\right) \quad (\because \text{AM} \geq \text{GM})$

$= 3$

108 **(b)**

Let d_1, d_2 be the lengths of diagonals and θ be the angle between them. Then,

Area $= \frac{1}{2} d_1 d_2 \sin\theta \Rightarrow a^2$

$= \frac{1}{2} d_1 d_2 \sin\theta \Rightarrow d_1 d_2 = \frac{2a^2}{\sin\theta}$

Using A. M. $\geq$ G. M., we have

$\frac{d_1 + d_2}{2} \geq \sqrt{d_1 d_2} \Rightarrow d_1 + d_2 \geq 2\sqrt{\frac{2a^2}{\sin\theta}} \geq 2\sqrt{2}a$

109 **(c)**

We have, $|2x - 3| < |x + 5|$

$\Rightarrow |2x - 3| - |x + 5| < 0$

$$\Rightarrow \begin{cases} 3 - 2x + x + 5 < 0, x \leq -5 \\ 3 - 2x - x - 5 < 0, -5 < x \leq \frac{3}{2} \\ 2x - 3 - x - 5 < 0, x > \frac{3}{2} \end{cases}$$

$$\Rightarrow \begin{cases} x > 8, x \leq -5 \\ x > -\frac{2}{3}, -5 < x \leq \frac{3}{2} \\ x < 8, x > \frac{3}{2} \end{cases}$$

$\Rightarrow x \in \left(-\frac{2}{3}, \frac{3}{2}\right] \cup \left(\frac{3}{2}, 8\right)$

$\Rightarrow x \in \left(-\frac{2}{3}, 8\right)$

110 **(c)**

We have,

$|a_n| < 2$ i. e. $-2 < a_n < 2$

$\therefore \max(1 + a_1 x + a_2 x^2 + \cdots a_n x^n)$

$= 1 + 2|x| + 2|x|^2 + \cdots + 2|x|^n$

$= 1 + 2|x| \left\{\frac{1 - |x|^n}{1 - |x|}\right\}$

$= 1 + 2 \cdot \frac{1}{3}\left\{\frac{1 - 1/3^n}{1 - 1/3}\right\} > 0$

and,

$\min(1 + a_1 x + a_2 x^2 + \cdots + a_n x^n)$

$= 1 - 2|x| - 2|x|^2 \ldots - 2|x|^n$

$= -2[1 + |x| + |x|^2 + \cdots + |x|^n] + 3$

$$= -2\left\{\frac{1-|x|^n}{1-|x|}\right\}+3$$

$$= -2\left\{\frac{1-1/3^n}{1-1/3}\right\}+3>0$$

Thus, the curve $y = 1 + a_1x + a_2x^2 + \cdots + a_nx^n$ does not meet x-axis for $|x| < 1/3$ and $|a_n| < 2$

Hence, the equation has no real roots

112 **(c)**

Since, $|r| < 1 \Rightarrow -1 < r < 1$

Also, $a = 5(1-r)$

$\Rightarrow 0 < a < 10 \quad \left[\begin{matrix}\because \text{ at } r=-1, & a=0\\ \text{and at } r=1, & a=0\end{matrix}\right]$

113 **(a)**

Consider the curves $y = e^{x-8}$ and $y = 17-2x$. These two curves intersect at $(8,1)$ only. Hence, the equation $e^{x-8}+2x-17=0$ has exactly one root which is equal to 8

114 **(b)**

Let $x^2+18x+30 = y$. Then,

$x^2+18x+30 = 2\sqrt{x^2+18x+45}$

$\Rightarrow y = 2\sqrt{y+15}$

$\Rightarrow y^2-4y-60 = 0 \Rightarrow (y-10)(y+6)=0 \Rightarrow y = 10$

$\therefore x^2+18x+30 = y \Rightarrow x^2+18x+20=0$

$\therefore$ Product of roots $= 20$

115 **(a)**

Since, $2 \le |x-3| < 4$

$\Rightarrow 2 \le x-3 < 4$

Or $2 \le -(x-3) < 4$

$\Rightarrow 5 \le x < 7$ or $-1 \le -x < 1$

$\Rightarrow 5 \le x < 7$ or $-1 < x \le 1$

$\therefore x \in (-1,1] \cup [5,7)$

116 **(b)**

Given that, $x = \left[\frac{a+2b}{a+b}\right]$ and $y = \frac{a}{b}$

$$\therefore x = \frac{a+2b}{a+b} = \frac{\frac{a}{b}+2}{1+\frac{a}{b}} = 1+\frac{1}{\frac{a}{b}+1}$$

$\Rightarrow x = 1+\dfrac{1}{y+1} \quad \left[\because y = \dfrac{a}{b} \text{ and } y^2 > 2 \text{ (given)}\right]$

Which shows $x^2 < 2 \quad \left[\because \frac{1}{y+1} < \text{as } y > 1\right]$

117 **(c)**

Using A.M. $\ge$ G.M., we have

$5^{\sin x-1}+5^{-\sin x-1} \ge 2\sqrt{5^{\sin x-1}\times 5^{-\sin n-1}}$

$\Rightarrow 5^{\sin x-1}+5^{-\sin x-1} \ge \dfrac{2}{5}$

118 **(d)**

As we know, if $ax+bx+c>0$, then $a>0$ and $D<0$

$\therefore (2)^2-4(n-10)<0 \Rightarrow n>11$

119 **(a)**

Since, AM $\ge$ GM

$$\Rightarrow \frac{bcx+cay+abz}{3} \ge (a^2b^2c^2\cdot xyz)^{1/3}$$

$\Rightarrow bcx+cay+abz \ge 3\,abc \quad (\because xyz = abc)$

120 **(b)**

Since, $\frac{(1+a_1)}{2} \ge \sqrt{1\cdot a_1} = \sqrt{a_1}$

$$\frac{(1+a_2)}{2} \ge \sqrt{1\cdot a_2} = \sqrt{a_2}$$

$$\vdots \qquad \vdots$$

$$\frac{(1+a_n)}{2} \ge \sqrt{1\cdot a_n} = \sqrt{a_n}$$

$$\Rightarrow \frac{1}{2^n}(1+a_1)(1+a_2)\dots(1+a_n) \ge \sqrt{a_1a_2\dots a_n} = 1$$

$\Rightarrow (1+a_1)(1+a_2)\dots(1+a_n) \ge 2^n$

121 **(a)**

Using A.M. $\ge$ G.M., we have

$a+b \ge 2\sqrt{ab}, b+c \ge 2\sqrt{bc}$ and $c+a \ge 2\sqrt{ca}$

$\Rightarrow (a+b)(b+c)(c+a) \ge 8abc$

$\Rightarrow (2-a)(2-b)(2-c) \ge 8abc \quad \left[\begin{matrix}\because a+b+c=2\\ \therefore b+c = 2-a \text{ etc}\end{matrix}\right]$

122 **(a)**

We have, $x^2+y^2+z^2 = 27$

Now,

$$\frac{(x^2)^{3/2}+(y^2)^{3/2}+(z^2)^{3/2}}{3} \ge \left(\frac{x^2+y^2+z^2}{z}\right)^{3/2}$$

$\Rightarrow x^3+y^3+z^3 \ge 81$

123 **(c)**

Given, $2x-7<11, 3x+4<-5$

$\Rightarrow x<9, x<-3$

$\Rightarrow x<-3$

$\therefore x$ lies in the interval $(-\infty,-3)$

124 **(c)**

Let

$$f(x) = x^8-x^5-\frac{1}{x}+\frac{1}{x^4} = \frac{x^{12}-x^9-x^3+1}{x^4} = \frac{(x^9-1)(x^3-1)}{x^4}$$

Clearly, $f(x) \ge 0$ for all $x<0$ and it is not defined for $x = 0$

For $0 < x < 1$, we have

$x^9 - 1 < 0$ and $x^3 - 1 < 0 \Rightarrow f(x) > 0$

For $x \geq 1$, we have $x^9 - 1 \geq 0$ and $x^3 - 1 \geq 0 \Rightarrow f(x) \geq 0$

Hence, $f(x) \geq 0$ for all $x \neq 0$

125 **(c)**

We have,

$5^{(1/4)(\log_5 x)^2} \geq 5\, x^{(1/5)(\log_5 x)}$

$\frac{1}{4}(\log_5 x)^2 \log_5 5 \geq \log_5 5 + \frac{1}{5}(\log_5 x)\log_5 x$

$\Rightarrow (\log_5 x)^2 \geq 20$

$\Rightarrow (\log_5 x)^2 - \left(2\sqrt{5}\right)^2 \geq 0$

$\Rightarrow \log_5 x \leq -2\sqrt{5}$ or, $\log_5 x \geq 2\sqrt{5}$

$\Rightarrow x \leq 5^{-2\sqrt{5}}$ or, $x \geq 5^{2\sqrt{5}}$

$\Rightarrow x \in \left(0, 5^{-2\sqrt{5}}\right] \cup \left[5^{2\sqrt{5}}, \infty\right)$ [$\log_5 x$ is defined for $x > 0$]

126 **(a)**

We know that,

AM $\geq$ GM

$\Rightarrow \frac{a+b}{2} \geq \sqrt{ab}$

$\Rightarrow 4 \geq \sqrt{ab}$ ($\because a + b = 8$ given)

$\Rightarrow ab \leq 16$

Equality holds when number are equal. So, ab is equal to 16 when $a = 4, b = 4$

127 **(a)**

Curves $y = \cos x$ and $y = -|x|$ do not intersect. So, the equation $\cos x + |x| = 0$ has no real root

128 **(d)**

Using A. M. $\geq$ G. M., we have

$\frac{\cos^3 x}{\sin x} + \frac{\sin^3 x}{\cos x} \geq 2\sqrt{\frac{\cos^3 x}{\sin x} \times \frac{\sin^3 x}{\cos x}}$ for all $x \in (0, \pi/2)$

$\Rightarrow \frac{\cos^3 x}{\sin x} + \frac{\sin^3 x}{\cos x} \geq \sin 2x$ for all $x \in (0, \pi/2)$

$\Rightarrow \frac{\cos^3 x}{\sin x} + \frac{\sin^3 x}{\cos x} \geq 1$ for all $x \in (0, \pi/2)$

129 **(d)**

$x^2 + 4ax + 20$

$\therefore\ (4a)^2 - 4 \times 2 < 0$

[$\because$ if $f(x) > 0$, then]

$\Rightarrow\ 16a^2 < 8 \Rightarrow a^2 < \frac{1}{2}$

$\Rightarrow\ -\frac{1}{\sqrt{2}} < a < \frac{1}{\sqrt{2}}$

130 **(b)**

Using A. M. $\geq$ G. M., we have

$\frac{a+b}{2} > \sqrt{ab}$ [$\because a \neq b$]

$\Rightarrow a + b > 2\sqrt{ab}$

131 **(a)**

We have,

$x^2 \cdot 2^{x+1} + 2^{|x-3|+2} = x^2 \cdot 2^{|x-3|+4} + 2^{x-1}$

Now, two cases arise

CASE I *When* $x \geq 3$:

In this case, we have $|x - 3| = x - 3$

So, the given equation reduces to

$x^2 \cdot 2^{x+1} + 2^{x-1} = x^2 \cdot 2^{x+1} + 2^{x-1}$

Which is an identity in x and hence it is true for all $x \geq 3$

CASE II *When* $x < 3$:

In this case, we have $|x - 3| = -(x - 3)$

So, the given equation reduces to

$x^2 \cdot 2^{x+1} + 2^{-x+5} = x^2 \cdot 2^{-x+7} + 2^{x-1}$

$\Rightarrow x^2 2^{x+1} - 2^{x-1} = x^2 \cdot 2^{-x+7} - 2^{-x+5}$

$\Rightarrow 2^{x-1}(4x^2 - 1) = 2^{-x+5}(4x^2 - 1)$

$\Rightarrow 2^{2x}(4x^2 - 1) = 2^6(4x^2 - 1)$

$\Rightarrow (2^{2x} - 2^6)(4x^2 - 1) = 0$

$\Rightarrow 2x = 6$ or, $4x^2 - 1 = 0$

$\Rightarrow x = 3$ or, $x = \pm\frac{1}{2}$

But, $x < 3$. Therfore, $x = \pm\frac{1}{2}$

Hence, the given equation has no negative integral root

132 **(b)**

We have,

$2^{\sin^2 x} \cdot 3^{\cos^2 y} \cdot 4^{\sin^2 z} \cdot 5^{\cos^2 \omega} \geq 120$

$\Rightarrow 2^{\sin^2 x} \cdot 3^{\cos^2 y} \cdot 4^{\sin^2 z} \cdot 5^{\cos^2 \omega} \geq 2 \times 3 \times 4 \times 5$

$\Rightarrow \sin^2 x \log 2 + \cos^2 y \log_3 + \sin^2 z \log 4 + \cos^2 \omega \log 5 \geq \log 2 + \log 3 + \log 4 + \log 5$

$\Rightarrow \cos^2 x \log 2 + \sin^2 y \log 3 + \cos^2 z \log 4 + \sin^2 \omega \log 5 \leq 0$

$\Rightarrow \cos^2 x = 0, \sin^2 y = 0, \cos^2 z = 0$ and $\sin^2 \omega = 0$

$\Rightarrow x = m\pi \pm \frac{\pi}{2}, m \in Z; y = n\pi, n \in Z$

$z = r\pi \pm \frac{\pi}{2}, r \in Z;\ \omega = t\,\pi, t \in Z$

But, $x, y, z, \omega \in [0,10]$

$\therefore x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, y = 0, \pi, 2\pi, 3\pi, z = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

and $\omega = 0, \pi, 2\pi, 3\pi$

Hence, the number of ordered 4-tuples is

$3 \times 4 \times 3 \times 4 = 144$

133 **(d)**

We have,

$\log_b a + \log_a b + \log_d c + \log_c d$

$= \left(\log_b a + \frac{1}{\log_b a}\right) + \left(\log_d c + \frac{1}{\log_d c}\right) \geq 2 + 2$

$= 4$

134 **(c)**

We have,

$\log_{1/3}(2^{x+2} - 4^x) \geq -2$

$\Rightarrow 2^{x+2} - 4^x \leq \left(\frac{1}{3}\right)^{-2}$ and $2^{x+2} - 4^x > 0$

$\Rightarrow 4(2^x) - (2^x)^2 \leq 9$ and $2^x(2^2 - 2^x) > 0$

$\Rightarrow (2^x)^2 - 4(2^x) + 9 \geq 0$ and $2^x < 2^2$

$\Rightarrow x < 2$ $[\because (2^x)^2 - 4(2^x) + 9 > 0$ for all $x \in R]$

$\Rightarrow x \in (-\infty, 2)$

135 **(c)**

Given, $\frac{2x}{(2x+1)(x+2)} - \frac{1}{(x+1)} > 0$

$\Rightarrow \frac{-3x - 2}{(x+1)(x+2)(2x+1)} > 0$

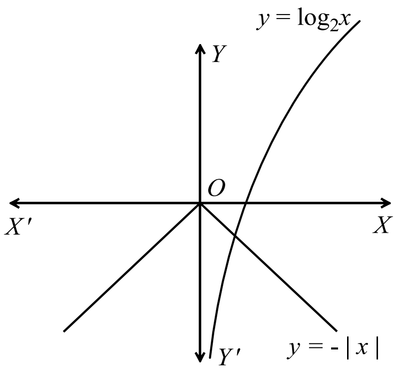
Equating each factor equal to 0, we have

$x = -2, -1, -\frac{2}{3}, -\frac{1}{2}$

It is clear $-\frac{2}{3} < x < -\frac{1}{2}$ or $-2 < x < -1$

136 **(b)**

We observe that the curves $y = \log_2 x$ and $y = -|x|$ intersect at exactly one point. So, the equation $\log_2 x + |x| = 0$ has exactly one real root



137 **(a)**

Using A. M. ≥ G. M. , we have

$\frac{bcx + cay + abz}{3} \geq (bcx \times cay \times abz)^{1/3}$

$\Rightarrow bcx + cay + abz \geq 3(a^2b^2c^2 \times xyz)^{1/3}$

$\Rightarrow bcx + cay + abz \geq 3abc$ $\quad [\because xyz = abc]$

138 **(a)**

We have,

$\sqrt{3x^2 + 6x + 7} + \sqrt{5x^2 + 10x + 14} \leq 4 - 2x - x^2$

$\Rightarrow \sqrt{3(x+1)^2 + 4} + \sqrt{5(x+1)^2 + 9} \leq (x+1)^2 + 5$

Clearly, LHS ≥ 5 and LHS ≤ 5

So, the inequation holds when each side is equal to 5

This is true when $x = -1$

Hence, the given inequation has exactly one solution

139 **(b)**

Let $a_1, a_2, a_3, \dots, a_n$ be the lengths of n parts of the stick. Then,

$a_1 + a_2 + a_3 + \dots + a_n = 20$ and $a_1a_2a_3 \dots a_n > 1$

Now, A. M. ≥ G. M.

$\Rightarrow \frac{a_1 + a_2 + \dots + a_n}{a_n} \geq (a_1a_2 \dots a_n)^{1/n}$

$\Rightarrow \frac{20}{n} > 1 \Rightarrow n < 20$

$\therefore$ Maximum possible value of n is 19

140 **(b)**

$\because$ AM $> GM$

$\frac{(a + b - c) + (b + c - a)}{2} > \sqrt{(a + b - c)(b + c - a)}$

$\Rightarrow b > \sqrt{(a + b - c)(b + c - a)}$ ...(i)

Similarly,

$\frac{(b+c-a)+(c+a-b)}{2} > \sqrt{(b + c - a)(c + a - b)}$

$\Rightarrow c > \sqrt{(b + c - a)(c + a - b)}$ ...(ii)

and $\frac{(c+a-b)+(a+b-c)}{2} > \sqrt{(c + a - b)(a + b - c)}$

$\Rightarrow a > \sqrt{(c + a - b)(a + b - c)}$ ...(iii)

On multiplying relations (i), (ii) and (iii), we get

$abc > (a + b - c)(b + c - a)(c + a - b)$

$\Rightarrow (a + b - c)(b + c - a)(c + a - b) - abc < 0$

141 **(c)**

We have,

$\log_{16} x^3 + \left(\log_2 \sqrt{x}\right)^2 < 1$

$\Rightarrow \frac{3}{4}\log_2 x + \frac{1}{4}(\log_2 x)^2 < 1$

$\Rightarrow (\log_2 x)^2 + 3\log_2 x - 4 < 0$

$\Rightarrow (\log_2 x + 4)(\log_2 x - 1) < 0$
$\Rightarrow -4 < \log_2 x < 1 \Rightarrow 2^{-4} < x < 2 \Rightarrow x \in (1/16, 2)$
Also, LHS of the given inequality is defined fro $x > 0$
Hence, $x \in (1/16, 2)$

142 **(b)**
Since, $\sin x \le \cos^2 x$, becuase $\cos x$ must be a positive proper fraction

$\sin^2 x + \sin x - 1 \le 0$

Or $\left(\sin x + \frac{1}{2}\right)^2 - \frac{5}{4} \le 0$

From the definition of logarithm

$\sin x > 0, \cos x > 0, \cos x \ne 1$

$\therefore \sin x + \frac{1}{2} \le \frac{\sqrt{5}}{2},$

$\Rightarrow 0 < \sin x \le \frac{\sqrt{5}-1}{2}$

143 **(d)**
If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$
Then, $f(x)$ is minimum and $g(x)$ is maximum at
$f(x) = \frac{-D}{4a}, \left(\because x = -\frac{-b}{a} \text{ and } f(x) = \frac{-D}{4a}\right)$
$\therefore \min\{f(x)\} = \frac{-(4b^2 - 8c^2)}{4} = (2c^2 - b^2)$
And $\max\{g(x)\} = -\frac{(4c^2+4b^2)}{4(-1)} = (b^2 + c^2)$
Since, $\min f(x) > \max g(x) \Rightarrow 2c^2 - b^2 > b^2 + c2$
$\Rightarrow c^2 > 2b^2 \Rightarrow |c| > \sqrt{2}|b|$

144 **(a)**
We have, $[x] + (x) = 5$
If $x \le 2$, then $[x] + (x) \le 2 + 2 < 5$
If $x \ge 3$, then $[x] + (x) \ge 3 + 3 > 5$
If $2 < x < 3$, then $[x] + (x) = 2 + 3 = 5$
Hence, the solution set is $(2, 3)$

145 **(a)**
We have,
$|x - x^2 - 1| = |2x - 3 - x^2|$
$\Rightarrow |x^2 - x + 1| = |x^2 - 2x + 3|$
$\Rightarrow x^2 - x + 1$
$= x^2 - 2x + 3 \quad \left[\begin{matrix} \because x^2 - x + 1 > 0 \\ \text{and } x^2 - 2x + 3 > 0 \text{ for all } x \end{matrix}\right]$
$\Rightarrow x = 2$

146 **(a)**
Given, $\frac{|x-1|}{x+2} - 1 < 0$
Case I When $x < 1, |x - 1| = 1 - x$
$\therefore \frac{1-x}{x+2} - 1 < 0 \Rightarrow \frac{-2x-1}{x+2} < 0$
$\Rightarrow \frac{2x+1}{x+2} > 0 \Rightarrow x < -2$ or $x > -\frac{1}{2}$
But $x < 1$
$\therefore x \in (-\infty, -2) \cup \left(-\frac{1}{2}, 1\right)$
Case II When $x \ge 1, |x - 1| = x - 1$
$\therefore \frac{x-1}{x+2} - 1 < 0 \Rightarrow -\frac{3}{x+2} < 0$
$\Rightarrow \frac{3}{x+2} > 0$
$\Rightarrow x > -2$
But $x \ge 1$
$\therefore x \ge 1, ie, x \in [1, \infty)$ ...(iii)
$\therefore$ From Eqs. (i) and (ii), we get
$x \in (-\infty, -2) \cup \left(-\frac{1}{2}, \infty\right)$

147 **(d)**
Given that, $\frac{x+2}{x^2+1} > \frac{1}{2}$
$\Rightarrow x^2 - 2x - 3 < 0$
$\Rightarrow (x - 3)(x + 1) < 0$
$\Rightarrow -1 < x < 3$
The integer value of x are 0, 1, 2
$\therefore$ The number of integral solutions are 3

148 **(b)**
$\because \tan\left(x + \frac{\pi}{3}\right) \ge 1 \Rightarrow \frac{\pi}{4} \le x + \frac{\pi}{3} < \frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{12} \le x < \frac{\pi}{6}$

$\Rightarrow n\pi - \frac{\pi}{12} \le x \le n\pi + \frac{\pi}{6}$

149 **(c)**
We have,
$\frac{1 + (\log_a x)^2}{1 + \log_a x} > 1$
$\Rightarrow \frac{1 + (\log_a x)^2}{1 + \log_a x} - 1 > 0$
$\Rightarrow \frac{(\log_a x)(\log_a x - 1)}{(1 + \log_a x)} > 0$
$\Rightarrow -1 < \log_a x < 0$ or, $\log_a x > 1$
$\Rightarrow a^{-1} > x > a^0$ or, $x < a \quad [\because 0 < a < 1]$
$\Rightarrow x \in (1, 1/a) \cup (0, a) \quad [\because a > 0]$

150 **(b)**
We have,
$x = \frac{y+2}{y+1}$

$\Rightarrow y = \dfrac{2-x}{x-1}$

$\Rightarrow \left(\dfrac{2-x}{x-1}\right)^2 > 2 \quad [\because y^2 > 2]$

$\Rightarrow (2-x)^2 > 2(x-1)^2 \Rightarrow x^2 < 2$

151 **(a)**

Let $x = y - \dfrac{3\pi}{4}$. Then,

$\sin x = -\left(\dfrac{\cos y + \sin y}{\sqrt{2}}\right)$ and $\cos x = -\left(\dfrac{\cos y - \sin y}{\sqrt{2}}\right)$

$\Rightarrow \sin x + \cos x = -\sqrt{2}\cos y$ and $\sin x \cos x =$ *12(2cos2y−1)*

Now,

$|\sin x + \cos x + \tan x + \sec x + \operatorname{cosec} x + \cot x|$

$= |(\sin x + \cos x) + (\tan x + \cot x) + (\sec x + \operatorname{cosec} x)|$

$= \left|(\sin x + \cos x) + \dfrac{1}{\sin x \cos x} + \dfrac{\sin x + \cos x}{\sin x \cos x}\right|$

$= \left|(\sin x + \cos x)\left(1 + \dfrac{1}{\sin x \cos x}\right) + \dfrac{1}{\sin x \cos x}\right|$

$= \left|-\sqrt{2}\cos y\left(1 + \dfrac{2}{2\cos^2 y - 1}\right) + \dfrac{2}{2\cos^2 y - 1}\right|$

$= \left|-\sqrt{2}\cos y - \dfrac{2(\sqrt{2}\cos y - 1)}{2\cos^2 y - 1}\right|$

$= \left|-\sqrt{2}\cos y - \dfrac{2}{\sqrt{2}\cos y + 1}\right| = \left|\sqrt{2}\cos y + \dfrac{2}{\sqrt{2}\cos y + 1}\right|$

$= \left|\lambda + \dfrac{2}{\lambda + 1}\right|$, where $\lambda = \sqrt{2}\cos y$

$= \left|(\lambda + 1) + \dfrac{2}{\lambda + 1} - 1\right| \geq \left|(\lambda + 1) + \dfrac{2}{(\lambda + 1)}\right| - 1$

$\geq 2\sqrt{(\lambda + 1) \times \dfrac{2}{(\lambda + 1)}} - 1 = 2\sqrt{2} - 1$ [Using AM $\geq$ GM]

152 **(b)**

$\dfrac{9 \cdot 3^{2x} - 6 \cdot 3^x + 4}{9 \cdot 3^{2x} + 6 \cdot 3^x + 4} = \dfrac{\left(3^{(x+1)}\right)^2 - 2(3^{(x+1)}) + 4}{(3^{(x+1)})^2 + 2(3^{(x+1)}) + 4}$

$= \frac{t^2 - 2t + 4}{t^2 + 2t + 4}$ (where $t = 3^{x+1}$) ...(i)

Since, $\frac{1}{3} < \frac{x^2 - 2x + 4}{x^2 + 2x + 4} < 3$

$\therefore$ From Eq.(i), the given expression lies between 1/3 and 3

153 **(c)**

Using A. M. $\geq$ G. M., we have

$4^x + 4^{1-x} \geq 2\sqrt{4^x \times 4^{1-x}} \Rightarrow 4^x + 4^{1-x} \geq 4$

154 **(b)**

We have,

$3^{-|x|} - 2^{|x|} = 0 \Rightarrow 3^{-|x|} = 2^{|x|} \Rightarrow 6^{|x|} = 1 \Rightarrow x = 0$

156 **(d)**

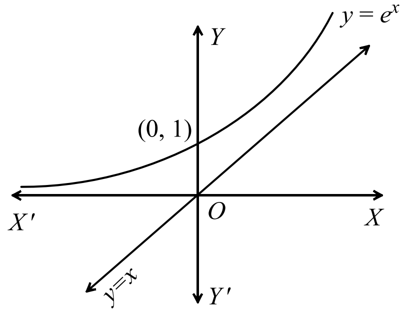
We have,

$P \leq (x_1 + x_2 + \cdots + x_{n-1})(x_2 + x_4 + x_6 + \cdots + x_n)$

$\Rightarrow P \leq \dfrac{1}{4}(x_1 + x_2 + \cdots + x_n)^2 = \dfrac{1}{4}$

157 **(b)**

We observe that $y = e^{-x}$ and $y = x$ intersect at exactly one point. So, the equation $e^{-x} = x$ has exactly one real root:



158 **(b)**

We have,

$3^x + 3^{1-x} - 4 < 0$

$\Rightarrow (3^x)^2 - 4(3^x) + 3 < 0$

$\Rightarrow (3^x - 1)(3^x - 3) < 0$

$\Rightarrow 1 < 3^x < 3 \Rightarrow 0 < x < 1 \Rightarrow x \in (0,1)$

160 **(c)**

We know that $x + \frac{1}{x} \geq 2$ for all $x > 0$

$\therefore \sin^5\theta + \operatorname{cosec}^5\theta \geq 2$ for $0 < \theta < \pi$

Hence, the minimum value of $\sin^5\theta + \operatorname{cosec}^5\theta$ is 2