DCAM classes

Dynamic Classes for Academic Mastery

13.LIMITS AND DERIVATIVES

Single Correct Answer Type

1. The value of $\lim_{x\to 0} \frac{\sqrt{1-\cos x^2}}{1-\cos x}$, is

a) 1/2 b) 2 c) $\sqrt{2}$ d) None of these

2. If $l(x)$ is the least integer not less than x and $g(x)$ is the greatest integer not greater than x, then $\lim_{x\to e+\pi} \{l(x)+g(x)\}$ is equal to

a) 9 b) 13 c) 1 d) None of these

3. If $f(x)$ is differentiable and strictly increasing function, then the value of $\lim_{x\to 0} \frac{f(x^2)-f(x)}{f(x)-f(0)}$ is

a) 1 b) 0 c) -1 d) 2

4. The value of $\lim_{x\to\infty} \left(\frac{\pi}{2} - \tan^{-1} x\right)^{1/x}$, is

a) 0 b) 1 c) -1 d) e

5. If $f(1) = g(1) = 2$, then $\lim_{x\to 1} \frac{f(1)g(x)-f(x)g(1)-f(1)+g(1)}{f(x)-g(x)}$ is equal to

a) 0 b) 1 c) 2 d) -2

6. The value of $\lim_{x\to\infty}\{\log_{(n-1)} n \,.\, \log_n(n+1) \,.\, \log_{(n+1)}(n+2) \ldots \{\ldots \log_{(n^k-1)}(n^k)\}$, is

a) ∞ b) n c) k d) None of these

7. $\lim_{n\to\infty} \frac{1^2+2^2+3^2+\ldots+n^2}{n^3}$ is equal to

a) $\frac{1}{2}$ b) $\frac{2}{3}$ c) $\frac{1}{3}$ d) $\frac{1}{6}$

8. The value of $\lim_{x\to 0}(\cos x)^{\cot^2 x}$ is

a) e^{-1} b) $e^{-1/2}$ c) 1 d) Not existing

9. The value of $\lim_{x\to 0} \frac{e^{ax}-e^{bx}}{x}$ is equal to

a) $a+b$ b) $a-b$ c) e^{ab} d) 1

10. $\lim_{h\to 0} \frac{(a+h)^2 \sin(a+h) - a^2 \sin a}{h}$ is equal to

a) $2a \sin a$ b) $a^2 \cos a$ c) $a^2 \cos a + 2a \sin a$ d) None of these

11. $\lim_{x\to 0} x^2 \sin\frac{\pi}{x}$ is equal to

a) 1 b) 0 c) Does not exist d) ∞

12. The value of $\lim_{x\to\pi/2}(\sin x)^{\tan x}$, is

a) 0 b) 1 c) -1 d) ∞

13. Let $< a_n >$ be a sequence such that $a_1 = 1$ and $a_{n+1} = \cos a_n, n \geq 1$. If $a = \lim_{n\to\infty} a_n$, then a belongs to the interval

a) $(0, \pi/6)$ b) $(\pi/6, \pi/3)$ c) $(\pi/3, \pi/2)$ d) $(\pi/2, \pi/3)$

14. $\lim_{x\to\infty} \frac{\int_0^{2x} x e^{x^2} dx}{e^{4x^2}}$ equals

a) 0 b) ∞ c) 2 d) $\frac{1}{2}$

15. $\lim_{x\to\infty} x \sin\left(\frac{2}{x}\right)$ is equal to

a) ∞ b) 0 c) 2 d) $\frac{1}{2}$

16. $\lim_{x\to 1} \frac{\tan(x^2-1)}{x-1}$ is equal to

a) 2 b) $\frac{1}{2}$ c) -2 d) $-\frac{1}{2}$

17. Let $f(x) = \begin{cases} \sin x, & x \neq n\pi \\ 2, & x = n\pi \end{cases}$, where $n \in I$ and $g(x) = \begin{cases} x^2+1, & x \neq 2 \\ 3, & x = 2 \end{cases}$, then $\lim_{x\to 0} g[f(x)]$ is

a) 1 b) 0 c) 3 d) Does not exist

18. The value of $\lim_{x\to 0}\frac{1+\sin x-\cos x+\log(1-x)}{x^3}$ is

a) -1 b) $\frac{1}{2}$ c) $-\frac{1}{2}$ d) 1

19. $\lim_{x\to\frac{\pi}{4}}\frac{\tan x-1}{x-\frac{\pi}{4}}$ is equal to

a) 1 b) 1/2 c) 2 d) 0

20. Given $f(x)=\frac{ax+b}{x+1}$, $\lim_{x\to\infty}f(x)=1$ and $\lim_{x\to 0}f(x)=2$, then $f(-2)$ is

a) 0 b) 1 c) 2 d) 3

21. The value of the constant α and β such that $\lim_{x\to\infty}\left(\frac{x^2+1}{x+1}-\alpha x-\beta\right)=0$ are respectively

a) (1, 1) b) (-1, 1) c) (1, -1) d) (0, 1)

22. If $0<p<q$, then $\lim_{n\to\infty}(q^n+p^n)^{1/n}$ is equal to

a) e b) p c) q d) 0

23. If $f(a)=2, f'(a)=1, g(a)=-1, g'(a)=2$, then the value of $\lim_{x\to a}\frac{g(x)f(a)-g(a)f(x)}{x-a}$, is

a) -5 b) $1/5$ c) 5 d) $-1/5$

24. If $f(5)=7$ and $f'(5)=7$, then $\lim_{x\to 5}\frac{xf(5)-5f(x)}{x-5}$ is given by

a) 35 b) -35 c) 28 d) -28

25. If $f(x)=\begin{cases}\sin x, x\neq n\pi, n\in Z\\ 0, \text{ otherwise}\end{cases}$ and $g(x)=\begin{cases}x^2+1, x\neq 0,2\\ 4, x=0\\ 5, x=2\end{cases}$, then $\lim_{x\to 0}g(f(x))$ is

a) 1 b) 5 c) 6 d) 7

26. If $z_r=\cos\frac{r\alpha}{n^2}+i\sin\frac{r\alpha}{n^2}$, where $r=1,2,3,\ldots,n$, then $\lim_{n\to\infty}z_1z_2\ldots z_n$ is equal to

a) $\cos\alpha+i\sin\alpha$ b) $\cos\left(\frac{\alpha}{2}\right)-i\sin\left(\frac{\alpha}{2}\right)$ c) $e^{i\alpha/2}$ d) $\sqrt[3]{e^{i\alpha}}$

27. The value of $\lim_{x\to 2}\frac{x(5^x-1)}{1-\cos x}$, is

a) $5\log 2$ b) $2\log 5$ c) $\frac{1}{2}\log 5$ d) $\frac{1}{5}\log 2$

28. $\lim_{x\to 0}\left[\frac{\sqrt{a+x}-\sqrt{a-x}}{4x}\right]$ is equal to

a) $\frac{1}{2\sqrt{a}}$ b) $\frac{1}{4\sqrt{a}}$ c) $\frac{1}{3\sqrt{a}}$ d) $\frac{2}{\sqrt{a}}$

29. $\lim_{x\to -1}\frac{\sqrt{\pi}-\sqrt{\cos^{-1}x}}{\sqrt{x+1}}$ is given by

a) $\frac{1}{\sqrt{\pi}}$ b) $\frac{1}{\sqrt{2\pi}}$ c) 1 d) 0

30. $\lim_{x\to 0}\frac{\tan x-\sin x}{x^3}$ is equal to

a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) 0 d) 1

31. The value of $\lim_{x\to 2}\frac{e^{3x-6}-1}{\sin(2-x)}$ is

a) $\frac{3}{2}$ b) 3 c) -3 d) -1

32. The value of

$$\lim_{x\to\infty}\frac{1.\sum_{r=1}^{n}r+2.\sum_{r=1}^{n-1}r+3.\sum_{r=1}^{n-2}r+\cdots n.1}{n^4}$$ is

a) $\frac{1}{24}$ b) $\frac{1}{12}$ c) $\frac{1}{6}$ d) None of these

33. The value of

$\lim_{n\to\infty}\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\ldots+\frac{1}{(2n-1)(2n+1)}$ is

a) 1/2 b) 1/3 c) 1/4 d) None of these

34. The value of $\lim_{n\to\infty}\left\{\frac{1}{3}+\frac{2}{21}+\frac{3}{91}+\cdots+\frac{n}{n^4+n^2+1}\right\}$, is

a) 1 b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) None of these

35. $\lim_{x\to\infty}[\sqrt{x^2+2x-1}-x]$ is equal to

a) ∞ b) $\frac{1}{2}$ c) 4 d) 1

36. $\lim_{n\to\infty}\left(1+\sin\frac{a}{n}\right)^n$ equals

a) e^a b) e c) e^{2a} d) 0

37. $\lim_{x\to\frac{\pi}{2}}\frac{a^{\cot x}-a^{\cos x}}{\cot x-\cos x}$, $a>0$ is equal to

a) $\log_e\frac{\pi}{2}$ b) $\log_e 2$ c) $\log_e a$ d) al

38. $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}e^{r/n}$ is

a) e b) $e-1$ c) $1-e$ d) $e+1$

39. The value of $\lim_{x\to 0}\frac{\int_0^x t\,dt}{x\tan(x+\pi)}$ is equal to

a) 0 b) 2 c) 1/2 d) 1

40. $\lim_{x\to 0}\frac{x}{\tan^{-1}2x}$ is equal to

a) 0 b) 1/2 c) 1 d) ∞

41. The value of $\lim_{x\to\infty}\left(\frac{1+3x}{2+3x}\right)^{\frac{1-\sqrt{x}}{1+x}}$, is

a) 0 b) -1 c) e d) 1

42. The value of $\lim_{x\to 1}(1-x)\tan\left(\frac{\pi x}{2}\right)$, is

a) $\frac{\pi}{2}$ b) $\pi+2$ c) $\frac{2}{\pi}$ d) None of these

43. $\lim_{x\to\infty}\frac{x^n}{e^x}=0$, ($n$ integer), for

a) No values of n
b) All values of n
c) Only negative values of n
d) Only positive values of n

44. The value of $\lim_{x\to 3}\frac{3^x-x^2}{x^x-3^2}$, is

a) $\frac{\log 3-1}{\log 3+1}$ b) $\frac{\log 3+1}{\log 3-1}$ c) 1 d) None of these

45. $\lim_{x\to\frac{\pi}{2}}\frac{\left[1-\tan\left(\frac{x}{2}\right)\right](1-\sin x)}{\left[1+\tan\left(\frac{x}{2}\right)\right](\pi-2x)^3}$ is

a) 1/8 b) 0 c) 1/32 d) ∞

46. The value of $\lim_{x\to\infty}\sqrt{a^2x^2+ax+1}-\sqrt{a^2x^2+1}$, is

a) $\frac{1}{2}$ b) 1 c) 2 d) None of these

47. $\lim_{x\to 0}\frac{1-\cos^3 x}{x\sin x\cos x}$ is equal to

a) $\frac{2}{5}$ b) $\frac{3}{5}$ c) $\frac{3}{2}$ d) $\frac{3}{4}$

48. $\lim_{x\to 1}(1-x)\tan\left(\frac{\pi x}{2}\right)$ is equal to

a) $\frac{\pi}{2}$ b) π c) $\frac{2}{\pi}$ d) 0

49. $\lim_{\theta\to 0}\frac{4\theta(\tan\theta-2\theta\tan\theta)}{(1-\cos 2\theta)}$ is

a) $1/\sqrt{2}$ b) 1/2 c) 1 d) 2

50. $\lim_{x\to 0^-} \frac{\sin x}{\sqrt{x}}$ is equal to

a) 0 b) 1 c) $-\frac{1}{2}$ d) None of these

51. $\lim_{\theta\to\frac{\pi}{2}} \frac{\frac{\pi}{2}-\theta}{\cot\theta}$ is equal to

a) 0 b) -1 c) 1 d) ∞

52. $\lim_{x\to 0} \frac{a^x-b^x}{e^x-1}$ is equal to

a) $\log_e\left(\frac{a}{b}\right)$ b) $\log_e\left(\frac{b}{a}\right)$ c) $\log_e(ab)$ d) $\log_e(a+b)$

53. $\lim_{x\to -\infty} \frac{2x-1}{\sqrt{x^2+2x+1}}$ is equal to

a) 2 b) -2 c) 1 d) -1

54. $\lim_{x\to 2} = \frac{\sqrt{1+\sqrt{2+x}}-\sqrt{3}}{x-2}$ is equal to

a) $\frac{1}{8\sqrt{3}}$ b) $\frac{1}{\sqrt{3}}$ c) $8\sqrt{3}$ d) $\sqrt{3}$

55. $\lim_{x\to 0} \left\{\frac{1}{x\sqrt[3]{8+x}} - \frac{1}{2x}\right\}$ is equal to

a) $\frac{1}{12}$ b) $\frac{-4}{3}$ c) $\frac{-16}{3}$ d) $\frac{-1}{48}$

56. The value of $\lim_{x\to 0} \frac{a^x-b^x}{x}$, is

a) $\log\left(\frac{a}{b}\right)$ b) $\log\left(\frac{b}{a}\right)$ c) $\log(ab)$ d) $-\log(ab)$

57. $\lim_{x\to 1} \frac{\sum_{r=1}^{n} x^r - n}{x-1}$ is equal to

a) $\frac{n}{x}$ b) $\frac{n(n+1)}{2}$ c) 1 d) 0

58. The value of $\lim_{x\to 1} (\log_5 5x)^{\log_x 5}$ is

a) 1 b) e c) -1 d) None of these

59. $\lim_{x\to 0} \frac{e^{\tan x - e^x}}{\tan x - x} =$

a) 1 b) e c) $e-1$ d) 0

60. The value of $\lim_{x\to\infty} \left(\frac{x^2-2x+1}{x^2-4x+2}\right)^x$, is

a) e^2 b) e^{-2} c) e^6 d) None of these

61. If $\lim_{x\to 0} \frac{\log(x+a)-\log a}{x} + k \lim_{x\to e} \frac{\log x - 1}{x-e} = 1$, then the value of k is

a) $1-\frac{1}{a}$ b) $e(1-a)$ c) $e\left(1-\frac{1}{a}\right)$ d) $e(1+a)$

62. The value of $\lim_{x\to\infty} \frac{\sin x}{x}$, is

a) 1 b) 0 c) -1 d) None of these

63. $\lim_{x\to 0} x \log \sin x$ is equal to

a) 0 b) ∞

c) 1 d) Cannot be determined

64. $\lim_{x\to 0} \frac{d}{dx} \int \frac{1-\cos x}{x^2} dx$ is equal to

a) 1 b) 0 c) 1/2 d) None of these

65. $\lim_{x\to 0} \frac{1}{x}\left\{\int_y^a e^{\sin^2 t} dt - \int_{x+y}^a e^{\sin^2 t}\, dt\right\}$ is equal to (where a is a constant)

a) $e^{\sin^2 y}$ b) $\sin 2y\, e^{\sin^2 y}$ c) 0 d) None of these

66. Let $f''(x)$ be continuous at $x=0$ and $f''(0)=4$. Then $\lim_{x\to 0} \frac{2f(x)-3f(2x)+f(4x)}{x^2}$ is equal to

a) 11 b) 2 c) 12 d) None of these

67. If $\lim_{x\to 0} \frac{[(a-n)nx - \tan x] \sin nx}{x^2} = 0$, where n is non-zero real number, then a is equal to

a) 0 b) $\frac{n+1}{n}$ c) n d) $n+\frac{1}{n}$

68. The values of a and b such that $\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}=0$, are

a) $\frac{5}{2},\frac{3}{2}$ b) $\frac{5}{2},-\frac{3}{2}$ c) $-\frac{5}{2},-\frac{3}{2}$ d) None of these

69. The value of $\lim_{x\to\infty}\left(\frac{x^2-2x+1}{x^2-4x+2}\right)^x$ is

a) e^2 b) e^{-2} c) e^6 d) None of these

70. The value of $\lim_{x\to 0}\frac{(1-\cos 2x)}{x^2}$ is

a) Does not exist b) Infinite c) 0 d) 2

71. The value of $\lim_{x\to 0}\frac{1+\sin x-\cos x+\log(1-x)}{x^3}$, is

a) 1/2 b) $-1/2$ c) 0 d) 1

72. $\lim_{x\to 0}\left\{\frac{1+\tan x}{1+\sin x}\right\}^{\operatorname{cosec} x}$ is equal to

a) $\frac{1}{e}$ b) 1 c) e d) e^2

73. $\lim_{x\to\frac{\pi}{6}}\frac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1}$ is equal to

a) 3 b) -3 c) 6 d) 0

74. The value of $\lim_{x\to 0}\left(\frac{1+5x^2}{1+3x^2}\right)^{1/x^2}$ is

a) e^2 b) e c) $\frac{1}{e}$ d) $\frac{1}{e^2}$

75. If $f(x)=\begin{cases} x, x<0 \\ 1, x=0 \\ x^2, x>0 \end{cases}$, then $\lim_{x\to 0} f(x)$ is

a) 0 b) 1 c) 2 d) Does not exist

76. If x is a real number in [0, 1], then the value of $\lim_{m\to\infty}\lim_{n\to\infty}[1+\cos^{2m}(n!\,\pi\,x)]$ is given by

a) 2 or 1 according as x is rational or irrational

b) 1 or 2 according as x is rational or irrational

c) 1 for all x

d) 2 or 1 for all x

77. $\lim_{x\to 1}(1+\cos\pi x)\cot^2\pi$ is equal to

a) 1 b) -1 c) 1/2 d) $-1/2$

78. If $\lim_{x\to 0}\frac{(e^{kx}-1)\sin kx}{x^2}=4$, then k is equal to

a) 2 b) -2 c) ± 2 d) ± 4

79. $\lim_{x\to 0}\frac{\log(1+x^3)}{\sin^3 x}$ is equal to

a) 0 b) 1 c) 3 d) None of these

80. If $l_1=\lim_{x\to 2^+}(x+[x])$, $l_2=\lim_{x\to 2^-}(2x-[x])$ and $l_3=\lim_{x\to\pi/2}\frac{\cos x}{(x-\pi/2)}$, then

a) $l_1<l_2<l_3$ b) $l_2<l_3<l_1$ c) $l_3<l_2<l_1$ d) $l_1<l_3<l_2$

81. If $f(x)=\begin{cases} \frac{\sin(1+[x])}{[x]}, & for\ [x]\neq 0 \\ 0, & for\ [x]=0 \end{cases}$

Where $[x]$ denotes the greatest integer not exceeding x, then $\lim_{x\to 0^-} f(x)$ is equal to

a) -1 b) 0 c) 1 d) 2

82. $\lim_{x\to 0}\frac{(1-cos\ 2x)\sin 5x}{x^2\sin 3x}$ equals

a) 10/3 b) 3/10 c) 6/5 d) 5/6

83. If $f(x)=\frac{\sin(e^{x-2}-1)}{\log(x-1)}$, then $\lim_{x\to 2} f(x)$ is given by

a) -2 b) -1 c) 0 d) 1

84. If $S_n = \sum_{k=1}^{n} a_k$ and $\lim_{n\to\infty} a_n = a$, then $\lim_{n\to\infty} \frac{S_{n+1}-S_n}{\sqrt{\sum_{k=1}^{n} k}}$ is equal to

a) 0 b) a c) $\sqrt{2}\, a$ d) $2a$

85. $\lim_{h\to 0} \frac{\sin\sqrt{x+h}-\sin\sqrt{x}}{h}$ is equal to

a) $\cos\sqrt{x}$ b) $1/(2\sin\sqrt{x})$ c) $(\cos\sqrt{x})/2\sqrt{x}$ d) $\sin\sqrt{x}$

86. The value of $\lim_{x\to\frac{\pi}{2}} (\sin x)^{\tan x}$ is

a) 1 b) 0 c) e d) None of these

87. The value of

$\lim_{n\to\infty} \cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{4}\right)\cos\left(\frac{x}{8}\right)\ldots\cos\left(\frac{x}{2^n}\right)$ is

a) $\frac{x}{\sin x}$ b) $\frac{x}{\cos x}$ c) $\frac{(\sin x)}{x}$ d) $\frac{(\cos x)}{x}$

88. The value of the limit $\lim_{x\to 1} \frac{\sin(e^{x-1}-1)}{\log x}$ is

a) 0 b) e c) $\frac{1}{e}$ d) 1

89. If $l = \lim_{x\to -2} \frac{\tan \pi x}{x+2} + \lim_{x\to\infty}\left(1+\frac{1}{x^2}\right)^x$, then which one of the following is not correct?

a) $l > 3$
b) $l > 4$
c) $l < 4$
d) l is a transcendental number

90. $\lim_{x\to 1} \frac{\sqrt{1-\cos 2(x-1)}}{x-1}$

a) Exists and is equals $\sqrt{2}$
b) Exists and is equals $-\sqrt{2}$
c) Does not exist because $x-1\to 0$
d) Does not exist because left hand limit is not equal to right hand limit

91. If $f'(2) = 2, f''(2) = 1$, then $\lim_{x\to 2} \frac{2x^2-4f'(x)}{x-2}$, is

a) 4 b) 0 c) 2 d) ∞

92. $\lim_{x\to\pi/4} \frac{\int_2^{\sec^2 x} f(t)dt}{x^2-\pi^2/16}$ equals

a) $\frac{8}{\pi}f(2)$ b) $\frac{2}{\pi}f(2)$ c) $\frac{2}{\pi}f\left(\frac{1}{2}\right)$ d) $4f(2)$

93. If $f(a) = 2,\ f'(a) = 1, g(a) = 3, g'(a) = -1$, then $\lim_{x\to a} \frac{f(a)g(x)-f(x)g(a)}{x-a}$ is equal to

a) 6 b) 1 c) -1 d) -5

94. If $f(x) = \left(\frac{x^2+5x+3}{x^2+x+2}\right)^x$ then $\lim_{x\to\infty} f(x)$ is equal to

a) e^4 b) e^3 c) e^2 d) 24

95. $\lim_{x\to 0} \frac{\sin^{-1}x-x}{x^3\cos x}$ is equal to

a) 1/2 b) 1/3 c) 1/6 d) 1/12

96. For $x > 0$, $\lim_{x\to 0}\left((\sin x)^{1/x} + \left(\frac{1}{x}\right)^{\sin x}\right)$ is

a) 0 b) -1 c) 1 d) 2

97. The value of $\lim_{x\to\infty}\left(\frac{3x-4}{3x+2}\right)^{\frac{x+1}{3}}$ is equal to

a) $e^{-1/3}$ b) $e^{-2/3}$ c) e^{-1} d) e^{-2}

98. $\lim_{x\to 0} \frac{1}{x}\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is equal to

a) -2 b) 0 c) 2 d) ∞

99. $\lim_{x\to 0}\frac{2\sin^2 3x}{x^2}$ is equal to

a) 0 b) 1 c) 18 d) 36

100. $\lim_{x\to 0}\frac{a^x+a^{-x}-2}{x^2}$ is equal to

a) $(\log a)^2$ b) $\log a$ c) 0 d) None of these

101. Let $f(x)=\begin{cases}1, \text{when } x \text{ is rational}\\ 0, \text{when } x \text{ irrational}\end{cases}$, then $\lim_{x\to 0} f(x)$ is

a) 0 b) 1 c) $\frac{1}{2}$ d) None of these

102. For $x\in R$ $\lim_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x$ is equal to

a) e b) e^{-1} c) e^{-5} d) e^5

103. The value of $\lim_{x\to\infty} x\cos\left(\frac{\pi}{4x}\right)\sin\left(\frac{\pi}{4x}\right)$, is

a) $\frac{\pi}{2}$ b) $\frac{\pi}{4}$ c) 1 d) None of these

104. The derivative of function $f(x)$ is$\tan^4 x$. If $f(x)=0$, then $\lim_{x\to 0}\frac{f(x)}{x}$is equal to

a) 1 b) 0 c) -1 d) None of these

105. Let $f(x)=\begin{cases}(1/2)\{g(x)+(x)\}\sin(x), x\geq 1\\ \sin x/x,\ \ x<1\end{cases}$

Where $g(x)=\begin{cases}1, & \text{if } x>0\\ -1, & \text{if } x<0\\ 0, & \text{if } x=0\end{cases}$Then, $\lim_{x\to 1} f(x)$ is equal to

a) 0 b) 2 c) $\sin 1$ d) None of these

106. If $\lim_{x\to\infty}\left[\frac{x^3+1}{x^2+1}-(ax+b)\right]=2$, then

a) $a=1$ and $b=1$ b) $a=1$ and $b=-1$ c) $a=1$ and $b=-2$ d) $a=1$ and $b=2$

107. If $f: R\to R$ is defined by

$$f(x)=\begin{cases}\dfrac{x-2}{x^2-3x+2}, & \text{if } x\in R-\{1,2\}\\ 2, & \text{if } x=1\\ 1, & \text{if } x=2\end{cases}$$

Then $\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}$ is equal to

a) 0 b) -1 c) 1 d) $-\frac{1}{2}$

108. Let $f: R\to R$ be a differentiable function such that $f(3)=3, f'(3)=\frac{1}{2}$, Then, the value of $\lim_{x\to 3}\frac{\int_3^{f(x)} 2t^3\,dt}{x-3}$ is

a) 25 b) 26 c) 27 d) None of these

109. Let $f(a)=g(a)=k$ and their nth derivatives $f^n(a), g^n(a)$ exist and are not equal for some$n$. Further if $\lim_{x\to a}\frac{f(a)g(x)-f(a)-g(a)f(x)+g(a)}{g(x)-f(x)}=4$, then the value of k is equal to

a) 4 b) 2 c) 1 d) 0

110. The value of $\lim_{x\to 0}\frac{\sin x}{\sqrt{x^2}}$, is

a) 1 b) -1 c) 0 d) None of these

111. The value of $\lim_{h\to 0}\frac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$, is

a) $2a\sin a+a^2\cos a$ b) $2a\sin a-a^2\cos a$ c) $2a\cos a+a^2\sin a$ d) None of these

112. If $f(x)$ is differentiable function and $f''(0)=a$, then $\lim_{x\to 0}\frac{2f(x)-3f(2x)+f(4x)}{x^2}$ is equal to

a) $3a$ b) $2a$ c) $5a$ d) $4a$

113. The value of $\lim_{x\to 0}\frac{e^x+\log(1+x)-(1-x)^{-2}}{x^2}$ is equal to

a) 0 b) -3 c) -1 d) Infinity

114. If for some real number k

$\lim_{x\to 0} kx \text{ cosec }(x) = \lim_{x\to 0} x \text{ cosec }(kx)$, then the possible values of k are

a) 1, -1 b) 0, 1 c) 1, 2 d) 0, π

115. The value of $\lim_{x\to 0} \frac{|x|}{x}$ is

a) 1 b) -1 c) 0 d) None of these

116. $\lim_{x\to 0} x^2 \sin\frac{\pi}{x}$, us

a) 1 b) 0 c) Non-existent d) ∞

117. The value of $\lim_{x\to\infty} \left(\frac{x^2+bx+4}{x^2+ax+5}\right)$ is

a) $\frac{b}{a}$ b) 0 c) 1 d) $\frac{4}{5}$

118. If $f(x)$ is the integral function of the function $\frac{2\sin x-\sin 2x}{x^3}$, $x \neq 0$, then $\lim_{\to} f'(x)$ is equal to

a) 0 b) 1 c) -1 d) None of these

119. The value of $\lim_{x\to\infty} \left(\frac{3x-4}{3x+4}\right)^{\left(\frac{x+1}{3}\right)}$, is

a) $e^{-2/3}$ b) $e^{-1/3}$ c) e^{-2} d) e^{-1}

120. $\lim_{x\to 0} \frac{(1+x)^8-1}{(1+x)^2-1}$ is equal to

a) 8 b) 6 c) 4 d) 2

121. The value of $\lim_{x\to\pi/4} \frac{2\sqrt{2}-(\cos x+\sin x)^3}{1-\sin 2x}$, is

a) $\frac{3}{\sqrt{2}}$ b) $\frac{\sqrt{2}}{3}$ c) $\frac{1}{\sqrt{2}}$ d) $\sqrt{2}$

122. $\lim_{x\to-\infty}\left(3x+\sqrt{9x^2-x}\right)$ equals

a) 1/3 b) 1/6 c) $-1/6$ d) $-1/3$

123. $\lim_{x\to 0} \frac{e^{5x}-e^{4x}}{x}$ is equal to

a) 1 b) 2 c) 4 d) 5

124. Let $L = \lim_{x\to 0} \frac{a-\sqrt{a^2-x^2}-x^2/4}{x^4}$, $a > 0$. If L is finite, then

a) $a = 2, L = \frac{1}{64}$ b) $a = 1, L = \frac{1}{64}$ c) $a = 3, L = \frac{1}{32}$ d) $a = 1, L = \frac{1}{32}$

125. $\lim_{x\to 0} x \log_e(\sin x)$ is equal to

a) -1 b) $\log_e 1$ c) 1 d) None of these

126. The value of $\lim_{x\to 0^+} x^m (\log x)^n, m, n. N$ is

a) 0 b) m/n c) mn d) n/m

127. The value of $\lim_{x\to\infty} \frac{\sqrt{1+x^4}-(1+x^2)}{x^2}$ is equal to

a) 0 b) -1 c) 2 d) None of these

128. $\lim_{x\to 0}(\text{cosec } x)^{1/\log x}$ is equal to

a) 0 b) 1 c) $1/e$ d) None of these

129. If $f(1) = 2$ and $f'(1) = 1$, then value of $\lim_{x\to 1} \frac{2x-f(x)}{x-1}$ is

a) -1 b) 0 c) 1 d) 2

130. The value of $\lim_{x\to 0} \frac{1-\cos(1-\cos x)}{x^4}$ is

a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{1}{6}$ d) $\frac{1}{8}$

131. The value of $\lim_{x\to\infty} x^{3/2}\left(\sqrt{x^3+1}-\sqrt{x^3-1}\right)$, is

a) 1 b) -1 c) 0 d) None of these

132. If $\lim_{x\to a} \frac{a^x-x^a}{x^x-a^a} = -1$, then a equal to

a) 1 b) 0 c) e d) $(1/e)$

133. If $\lim_{x\to 0}\left\{\frac{x^3+1}{x^2+1}-(ax+b)\right\} = 2$, then

a) $a=1, b=1$ b) $a=1, b=2$ c) $a=1, b=-2$ d) None of these

134. $\lim_{x\to 0}\frac{e^{x^2}-\cos x}{x^2}$ is equal to

a) 0 b) $\frac{1}{2}$ c) 1 d) $\frac{3}{2}$

135. $\lim_{n\to\infty}\left(\frac{1^2}{1-n^3}+\frac{2^2}{1-n^3}+\ldots+\frac{n^2}{1-n^3}\right)$ is equal to

a) $\frac{1}{3}$ b) $-\frac{1}{3}$ c) $\frac{1}{6}$ d) $-\frac{1}{6}$

136. $\lim_{x\to\frac{\pi}{6}}\frac{\sin 2x}{\sin x}$ is equal to

a) $\sqrt{3}$ b) $\frac{1}{\sqrt{3}}$ c) 2 d) $\frac{1}{2}$

137. The value of $\lim_{x\to 0}\frac{1-\cos(1-\cos x))}{x^4}$, is

a) $\frac{1}{8}$ b) $\frac{1}{2}$ c) $\frac{1}{4}$ d) None of these

138. Let α and β be the roots of the equation $ax^2+bx+c=0$, where $1<\alpha<\beta$. If $\lim_{x\to m}\frac{|ax^2+bx+c|}{ax^2+bx+c}=1$, then

a) $a<0$ and $\alpha<m<\beta$ b) $a>0$ and $m>1$ c) $a>0$ and $m<1$ d) All the above

139. $\lim_{x\to 1}\frac{x^m-1}{x^n-1}$ is equal to

a) $\frac{n}{m}$ b) $\frac{m}{n}$ c) $\frac{2m}{n}$ d) $\frac{2n}{m}$

140. Let α and β be the distinct roots of $ax^2+bx+c=0$, then $\lim_{x\to\alpha}\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to

a) $\frac{1}{2}(\alpha-\beta)^2$ b) $-\frac{a^2}{2}(\alpha-\beta)^2$ c) 0 d) $\frac{a^2}{2}(\alpha-\beta)^2$

141. $\lim_{x\to 0}\left[\frac{2^x-1}{\sqrt{1+x}-1}\right]$ is equal to

a) $\log_e 2$ b) $\log_e\sqrt{2}$ c) $\log_e 4$ d) 2

142. $\lim_{x\to 0}\left(\frac{x}{\sqrt{1+x}-\sqrt{1-x}}\right)$is equal to

a) 0 b) 1 c) 2 d) -1

143. If $\lim_{x\to 0}\frac{\log(3+x)-\log(3-x)}{x}=k$, the value of k is

a) 0 b) -1/3 c) 2/3 d) -2/3

144. If a,b,c,d are positive, then $\lim_{x\to\infty}\left(1+\frac{1}{a+bx}\right)^{c+dx}=$

a) $e^{d/b}$ b) $e^{c/a}$ c) $e^{(c+d)/a+b}$ d) e

145. The value of $\lim_{x\to 0}\left(\frac{\int_0^{x^2}\sec^2 t\,dt}{x\sin x}\right)$ is

a) 3 b) 2 c) 1 d) 0

146. $\lim_{x\to\infty}\frac{\sin x}{x}$ is equal to

a) ∞ b) 1 c) 0 d) Does not exist

147. $\lim_{x\to 0}\frac{x\cos x-\log(1+x)}{x^2}$ equals

a) 1/2 b) 0 c) 1 d) $-1/2$

148. Given that $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{\log(r+n)-\log n}{n}=2\left(\log 2-\frac{1}{2}\right)$, $\lim_{n\to\infty}\frac{1}{n^k}[(n+1)^k(n+2)^k\ldots(n+n)^k]^{1/n}$, is

a) $\frac{4k}{e}$ b) $\sqrt[k]{\frac{4}{e}}$ c) $\left(\frac{4}{e}\right)^k$ d) $\left(\frac{e}{4}\right)^k$

149. $\lim_{x\to 0}\frac{\sin|x|}{x}$ is equal to

a) 1 b) 0 c) positive infinity d) does not exist

150. The value of $\lim_{x\to\infty}\left\{\frac{x^2\sin\left(\frac{1}{x}\right)-x}{1-|x|}\right\}$, is

a) 0 b) 1 c) -1 d) None of these

151. If $l_1=\lim_{x\to-2}(x+|x|)$, $l_2=\lim_{x\to-2}(2x+|x|)$ and $l_3=\lim_{x\to\pi/2}\frac{\cos x}{x-\pi/2}$, then

a) $l_1<l_2<l_3$ b) $l_2<l_3<l_1$ c) $l_3<l_2<l_1$ d) $l_1<l_3<l_2$

152. $\lim_{x\to\infty}\left(\frac{x+2}{x+1}\right)^{x+3}$ is equal to

a) 1 b) e c) e^2 d) e^3

153. $\lim_{x\to0}\frac{1}{x^{12}}\left\{1-\cos\frac{x^2}{2}-\cos\frac{x^4}{4}+\cos\frac{x^2}{2}\cos\frac{x^4}{4}\right\}$ is equal to

a) $\frac{1}{32}$ b) $\frac{1}{256}$ c) $\frac{1}{16}$ d) $-\frac{1}{256}$

154. If $f(x)=\sqrt{\frac{x-\sin x}{x+\cos^2 x}}$, then $\lim_{x\to\infty}f(x)$ is

a) 0 b) ∞ c) 1 d) None of these

155. The value of $\lim_{x\to1}(2-x)^{\tan\frac{\pi x}{2}}$ is equal to

a) $e^{-2/\pi}$ b) $e^{1/\pi}$ c) $e^{2/\pi}$ d) $e^{-1/\pi}$

156. $\lim_{n\to\infty}r^n=0$, then r is equal to

a) $\frac{4}{5}$ b) $\frac{5}{4}$ c) 2 d) 1

157. The value of $\lim_{x\to0}\frac{5^x-5^{-x}}{2x}$ is

a) $\log 5$ b) 0 c) 1 d) $2\log 5$

158. $\lim_{x\to0}\frac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$ is

a) λ b) -1 c) 0 d) Does not exist

159. The value of $\lim_{x\to\infty}a^x\sin\left(\frac{b}{a^x}\right)$ is $(a>1)$

a) $b\log a$ b) $a\log b$ c) b d) None of these

160. $\lim_{x\to\infty}\left(\frac{x^3}{3x^2-4}-\frac{x^2}{3x+2}\right)$ is equal to

a) $-\frac{1}{4}$ b) $-\frac{1}{2}$ c) 0 d) $\frac{2}{9}$

161. $\lim_{x\to0}\frac{e^{x^2}-\cos x}{x^2}$ is equal to

a) 3/2 b) 1/2 c) 2/3 d) None of these

162. The value of $\lim_{x\to\infty}\left\{\frac{a_1^{1/x}+a_2^{1/x}+\cdots+a_n^{1/x}}{n}\right\}^1$, is

a) $a_1+a_2+\cdots+a_n$ b) $e^{a_1+a_2+\cdots+a_n}$ c) $\frac{a_1+a_2+\cdots+a_n}{n}$ d) $a_1\,a_2\,\ldots\ldots a_n$

163. The value of $\lim_{x\to0}\frac{\sin^2 x+\cos x-1}{x^2}$ is

a) 1 b) $\frac{1}{2}$ c) $-\frac{1}{2}$ d) 0

164. $\lim_{x\to0}\frac{\sin 4x}{1-\sqrt{1-x}}$, is

a) 4 b) 8 c) 10 d) 2

165. $\lim_{x\to1}\frac{x^8-2x+1}{x^4-2x+1}$ equals

a) 3 b) 0 c) -3 d) 1

166. The value of $\lim_{x\to\infty}\sqrt{a^2x^2+ax+1}-\sqrt{a^2x^2+1}$ is

a) $\frac{1}{2}$ b) 1 c) 2 d) None of these

167. $\lim_{x\to0}(-1)^{[x]}$, where [.] denotes the greatest integer function is equal to

a) 0 b) 1 c) -1 d) Does not exist

168. $\lim_{x\to 0}\frac{(1-e^x)\sin x}{x^2+x^3}$ is equal to

a) -1 b) 0 c) 1 d) 2

169. The value of $\lim_{x\to\infty}\left(\frac{x+3}{x+1}\right)^{x+2}$ is

a) 0 b) 1 c) e^2 d) e^4

170. If α is a repeated root of $ax^2+bx+c=0$, then $\lim_{x\to\alpha}\frac{\sin(ax^2+bx+c)}{(x-\alpha)^2}$ is

a) 0 b) a c) b d) c

171. The value of $\lim_{x\to 0}\frac{e^x-(1+x)}{x^2}$, is

a) 0 b) 1/2 c) 2 d) e

172. $\lim_{x\to\infty}\frac{(2x+1)^{40}(4x-1)^5}{(2x+3)^{45}}$ is equal to

a) 16 b) 24 c) 32 d) 8

173. If $f: R\to R$ is defined by $f(x)=[x-3]+[x-4]$ for $x\in R$, then $\lim_{x\to 3^-}f(x)$ is equal to

a) -2 b) -1 c) 0 d) 1

174. The value of $\lim_{x\to 2^-}\{x+(x-[x]^2\}$, is

a) 0 b) 1 c) 2 d) 3

175. The value of $\lim_{x\to\infty}\left(\frac{x^2+2x+3}{2x^2+x+5}\right)^{\frac{3x-2}{3x+2}}$, is

a) $e^{1/2}$ b) $e^{3/2}$ c) e^3 d) None of these

176. $\lim_{x\to 0}\left[\frac{e^x-e^{\sin x}}{x-\sin x}\right]$ is equal to

a) -1 b) 0 c) 1 d) None of these

177. The value of $\lim_{x\to 1}(\log_2 2x)^{\log_x 5}$ is

a) 5/2 b) $e^{\log_2 5}$ c) $\log 5/\log 2$ d) $e^{\log_5 2}$

178. Let $f: R\to R$ be appositive increasing function withlim$_{x\to\infty}\frac{f(3x)}{f(x)}=1$. Then, $\lim_{x\to\infty}\frac{f(2x)}{f(x)}$ is equal to

a) 1 b) $\frac{2}{3}$ c) $\frac{3}{2}$ d) 3

179. If $f(x)=\begin{cases}x^2-3, & 2<x<3\\ 2x+5, & 3<x<4\end{cases}$, the equation whose roots are $\lim_{x\to 3^-}f(x)$ and $\lim_{x\to 3^+}f(x)$, is

a) $x^2-7x+3=0$ b) $x^2-20x+66=0$ c) $x^2-17x+66=0$ d) $x^2-18x+60=0$

180. If $f(x)=\begin{cases}x\sin\left(\frac{1}{x}\right), x\neq 0\\ 0, x=0\end{cases}$ Then, $\lim_{x\to 0}f(x)$

a) Is equal to 1 b) Is equal to -1 c) Is equal to 0 d) Does not exist

181. The value of $\lim_{x\to-\pi}\frac{|x+\pi|}{\sin x}$

a) Is equal to -1 b) Is equal to 1 c) Is equal to π d) Does not exist

182. Let $f(x)=\frac{1}{\sqrt{18-x^2}}$. The value of $\lim_{x\to 3}\frac{f(x)-f(3)}{x-3}$, is

a) 0 b) $-\frac{1}{9}$ c) $-\frac{1}{3}$ d) $\frac{1}{9}$

183. The value of $\lim_{x\to\infty}\frac{5^{x+1}-7^{x+1}}{5^x-7^x}$, is

a) 5 b) -5 c) 7 d) -7

184. If $A_i=\frac{x-a_i}{|x-a_i|}, i=1,2,\dots,n$ and if $a_1<a_2<a_3<\dots<a_n$. Then, $\lim_{x\to a_m}(A_1A_2\dots A_n)$, $1\le m\le n$

a) Is equal to $(-1)^m$ b) Is equal to $(-1)^{m+1}$ c) Is equal to $(-1)^{m-1}$ d) Does not exist

185. $\lim_{x\to -1}\frac{(1+x)(1-x^2)(1+x^3)(1-x^4)\dots(1-x^{4n})}{[(1+x)(1-x^2)(1+x^3)(1-x^4)\dots(1-x^{2n})]^2}$ is equal to

a) ${}^{4n}C_{2n}$ b) ${}^{2n}C_n$ c) $2.\ {}^{4n}C_{2n}$ d) $2.\ {}^{4n}C_n$

186. The value of $\lim_{x\to\infty}\frac{\log x}{x^n}, n>0$ is

a) 0 b) 1 c) $\frac{1}{n}$ d) $\frac{1}{n!}$

187. If f be a function such that $f(9)=9$ and $f'(9)=3$, then $\lim_{x\to 9}\frac{\sqrt{f(x)}-3}{\sqrt{x}-3}$ is equal to

a) 9 b) 3 c) 1 d) None of these

188. Let $f(x)=\lim_{x\to\infty}\frac{x^{2n}-1}{x^{2n}+1}$, then

a) $f(x)=\begin{cases}1, |x|>1\\ -1, |x|<1\end{cases}$

b) $f(x)=\begin{cases}1, |x|<1\\ -1, |x|>1\end{cases}$

c) $f(x)$ is not defined for any value of x

d) $f(x)=1$ for $|x|=1$

189. $\lim_{x\to 1}\frac{e^{-x}-e^{-1}}{x-1}$ is equal to

a) $\frac{1}{e}$ b) $-\frac{1}{e}$ c) 1 d) None of these

190. The value of $\lim_{n\to\infty}\frac{x^n}{x^n+1}$, where $x<-1$ is

a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) 1 d) None of these

191. If $a=\min\{x^2+4x+5: x\in R\}$ and $b=\lim_{\theta\to 0}\frac{1-\cos 2\theta}{\theta^2}$, then the value of $\sum_{r=0}^{n}\ {}^nC_r\ a^r\ b^{n-r}$, is

a) $2n$ b) 3^n c) 2^{n+1} d) 2^{n-1}

192. If $0<x<y$, then $\lim_{n\to\infty}(y^n+x^n)^{1/n}$ is equal to

a) e b) x c) y d) None of these

193. $\lim_{x\to 1}(\log ex)^{1/\log x}$ is equal to

a) e^{-1} b) e c) e^2 d) 0

194. If $\lim_{x\to\infty}\left\{ax-\frac{x^2+1}{x+1}\right\}=b$, a finite number, then

a) $a=1, b=1$ b) $a=0, b=1$ c) $a=-1, b=1$ d) $b=-1, a=-1$

195. The value of $\lim_{x-0}\frac{1}{x^3}\int_0^x\frac{t\log(1+t)}{t^4+4}dt$ is

a) 0 b) $\frac{1}{12}$ c) $\frac{1}{24}$ d) $\frac{1}{64}$

196. For the function

$f(x)=\lim_{n\to\infty}\frac{\log(2+x)+x^{2n}\sin x}{1+x^{2n}}$ Which of the flowing is true?

a) $\lim_{x\to 1^-}f(x)$ does not exist

b) $\lim_{x\to 1^+}f(x)$ does not exist

c) Both limits exist and $\lim_{x\to 1^-}f(x)=\lim_{x\to 1^+}f(x)$

d) Both limits exist and $\lim_{x\to 1^-}f(x)\neq\lim_{x\to 1^+}f(x)$

197. $\lim_{x\to\infty}\frac{\int_0^{2x}x\,e^{x^2}\,dx}{e^{4x^2}}$ equals

a) 0 b) ∞ c) 2 d) 1/2

198. The value of

$\lim_{x\to\pi/2}\tan^2 x\ \left(\sqrt{2\sin^2 x+3\sin x+4}-\sqrt{\sin^2 x+6\sin x+2}\right)$ is equal to

a) $\frac{1}{10}$ b) $\frac{1}{11}$ c) $\frac{1}{12}$ d) $\frac{1}{8}$

199. $\lim_{x\to\infty}\left(\frac{x+2}{x+1}\right)^{x+3}$ is

a) 1 b) e c) e^2 d) e^3

200. If $f(x)=\cot^{-1}[(3x-x^3)/(1-3x^2)]$ and $g(x)=\cos^{-1}[(1-x^2)/(1+x^2)]$, then $\lim_{x\to a}\frac{f(x)-f(a)}{g(x)-g(a)}\Big(0<$ *a<12* is

a) $-\frac{3}{2}$ b) $\frac{1}{2}$ c) $\frac{3}{2}$ d) None of these

201. Let $f(x)=\begin{cases} x^2, & x\in Z \\ \frac{k(x^2-4)}{2-x}, & x\notin Z \end{cases}$ Then, $\lim_{x\to 2} f(x)$

a) Exists only when $k=1$

b) Exists for every real k

c) Exists for every real k except $k=1$

d) Does not exist

202. The value of $\lim_{x\to 0}\frac{1-\cos^3 x}{x\sin x\cos x}$, is

a) 2/5 b) 3/5 c) 3/2 d) 3/4

203. $\lim_{x\to\infty}\sqrt{\frac{x+\sin x}{x-\cos x}}=$

a) 0 b) 1 c) -1 d) None of these

204. It is given that $f'(a)$ exists, then $\lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$ is equal to

a) $f(a)-a\,f'(a)$ b) $f'(a)$ c) $-f'(a)$ d) $f(a)+a\,f'(a)$

205. $\lim_{h\to 0}\frac{\sin(a+3h)-3\sin(a+2h)+3\sin(a+h)-\sin a}{h^3}$ is equal to

a) $\sin a$ b) $-\sin a$ c) $\cos a$ d) $-\cos a$

206. $\lim_{x\to 0}\left\{\frac{1^x+2^x+3^x+\cdots+n^x}{n}\right\}^{1/x}$ is equal to

a) $(n!)^n$ b) $(n!)^{1/n}$ c) $n!$ d) $\ln n!$

207. The value of $\lim_{x\to 0}\frac{\int_0^{x^2}\cos t^2\,dt}{x\sin x}$ is

a) 3/2 b) 1 c) -1 d) None of these

208. If $f(x)=\frac{2}{x-3}$, $g(x)=\frac{x-3}{x+4}$ and $h(x)=-\frac{2(2x+1)}{x^2+x-12}$, then $\lim_{x\to 3}\{f(x)+g(x)+h(x)\}$, is

a) -2 b) -1 c) $-2/7$ d) 0

209. $\lim_{x\to 0}\left[\frac{8\sin x+x\cos x}{3\tan x+x^2}\right]$ is equal to

a) 3 b) 2 c) -1 d) 4

210. If $\lim_{x\to 0}\frac{\log(3+x)-\log(3-x)}{x}=k$, the value of k is

a) $-2/3$ b) 0 c) $-1/3$ d) 2/3

211. The value of $\lim_{x\to\infty}\left(\frac{x^2+6}{x^2-6}\right)^x$ is given by

a) 0 b) 1 c) -1 d) None of these

212. $\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+2}\right)^x$ is equal to

a) e^4 b) e^2 c) e^3 d) e

213. If $G(x)=\sqrt{25-x^2}$, then $\lim_{x\to 1}\frac{G(x)-G(1)}{x-1}$ has the value

a) $\frac{1}{\sqrt{24}}$ b) $\frac{1}{5}$ c) $-\sqrt{24}$ d) $-1/5$

214. $\lim_{x\to 1}\cos^{-1}\left(\frac{1-\sqrt{x}}{1-x}\right)$ is equal to

a) $\frac{\pi}{3}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$

215. $\lim_{x\to 1}\frac{\sin(e^x-1)}{\log x}$ is equal to

a) 1 b) 0 c) e d) e^{-1}

216. $\lim_{x\to\frac{\pi}{6}}\left[\frac{3\sin x-\sqrt{3}\cos x}{6x-\pi}\right]$

a) $\sqrt{3}$ b) $\frac{1}{\sqrt{3}}$ c) $-\frac{1}{\sqrt{3}}$ d) $-\frac{1}{3}$

217. Let $f: R\to R$ be a differentiable function such that $f(2)=2$. Then, the value of

$\lim_{x\to 2}\int_2^{f(x)}\frac{4\,t^3}{x-2}dt$, is

a) $6f'(2)$ b) $12f'(2)$ c) $32f'(2)$ d) None of these

218. $\lim_{x\to\frac{\pi}{4}}\frac{\int_2^{\sec^2 x}f(t)dt}{x^2-\frac{\pi^2}{16}}$ equals

a) $\frac{8}{\pi}f(2)$ b) $\frac{2}{\pi}f(2)$ c) $\frac{2}{\pi}f\left(\frac{1}{2}\right)$ d) $4f(2)$

219. If $f(x)=\sqrt{\frac{x-\sin x}{x+\cos^2 x}}$, then $\lim_{x\to\infty}f(x)$ is

a) 0 b) ∞ c) 1 d) None of these

220. Let $f(x)$ be twice differentiable function such that $f''(0)=2$. Then, $\lim_{x\to 0}\frac{2f(x)-3f(2x)+f(4x)}{x^2}$ is

a) 6 b) 3 c) 12 d) None of these

221. The value of $\lim_{x\to\pi/2}\frac{\cot x-\cos x}{(\pi-2x)^3}$

a) 1 b) $\frac{1}{16}$ c) 16 d) None of these

222. The value of $\lim_{x\to\infty}\left(\frac{x+3}{x-1}\right)^{x+3}$, is

a) e b) e^2 c) e^4 d) $1/e$

223. The value of $\lim_{x\to 7}\frac{2-\sqrt{x-3}}{x^2-49}$ is

a) 2/9 b) -2/49 c) 1/64 d) -1/56

224. $\lim_{x\to\infty}\frac{\sqrt{x^2+1}-\sqrt[3]{x^3+1}}{\sqrt[4]{x^4+1}-\sqrt[5]{x^4+1}}$ equals

a) 1 b) 0 c) -1 d) None of these

225. Let $f: R\to R$ be a differentiable function having $f(2)=6, f'(2)=\left(\frac{1}{48}\right)$. Then, $\lim_{x\to 2}\frac{\int_6^{f(x)}4\,t^3\,dt}{x-2}$ is equals

a) 18 b) 12 c) 36 d) 24

226. If $g(x)$ is a polynomial satisfying $g(x)g(y)=g(x)+g(y)+g(xy)-2$ for all real x and y and $g(2)=5$, then $\lim_{x\to 3}g(x)$ is

a) 9 b) 10 c) 25 d) 20

227. $\lim_{x\to\pi/4}\frac{1-\cot^3 x}{2-\cot x-\cot^3 x}$, is

a) $\frac{11}{4}$ b) $\frac{3}{4}$ c) $\frac{1}{2}$ d) None of these

228. If $[x]$ denotes the greatest integer less than or equal to x, then the value of $\lim_{x\to 1}\{1-x+[x-1]+[1-x]\}$ is

a) 0 b) 1 c) -1 d) None of these

229. If $\lim_{x\to 1}\frac{ax^2+bx+c}{(x-1)^2}=2$, then (a, b, c) is

a) (2, -4, 2) b) (2, 4, 2) c) (2, 4, -2) d) (2, -4, -2)

230. $\lim_{x\to 1}\frac{2x^2+x-3}{3x^3-3x^2+2x-2}$ is equal to

a) 1 b) 2 c) -1 d) -2

231. $\lim_{x\to\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$ is equal to

a) ½ b) 0 c) 1 d) None of these

232. The value of $\lim_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4}$, is

a) e b) e^2 c) e^4 d) e^5

233. Let α and β be the roots of $a\,x^2+b\,x+c=0$, then $\lim_{x\to\alpha}\frac{1-\cos(a\,x^2+bx+c)}{(x-\alpha)^2}$ is equal to

a) 0 b) $\frac{1}{2}(\alpha-\beta)^2$ c) $\frac{\alpha^2}{2}(\alpha-\beta)^2$ d) $-\frac{\alpha^2}{2}(\alpha-\beta)^2$

234. $\lim_{x\to 0}(1-ax)^{1/x}$ is equal to

a) e^{-a} b) e c) e^{a} d) 1

235. If $\lim_{n\to\infty} \frac{1-(10)^n}{1+(10)^{n+1}} = \frac{-\alpha}{10}$, then the value of α is

a) 0 b) -1 c) 1 d) 2

236. The value of $\lim_{n\to\infty} \frac{\sqrt{n^2+1}+\sqrt{n}}{\sqrt[4]{n^3+n}-\sqrt[4]{n}}$, is

a) 0 b) 1 c) -1 d) None of these

237. The value of $\lim_{x\to 2} \frac{5}{\sqrt{2}-\sqrt{x}}$ is

a) $10\sqrt{2}$ b) $+\infty$ c) $-\infty$ d) Does not exist

238. $\lim_{n\to\infty} \frac{2^{-n}(n^2+5n+6)}{(n+4)(n+5)}$ is equal to

a) 0 b) 1 c) ∞ d) $-\infty$

239. If $f(x) = \begin{cases} x \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$, then $\lim_{x\to 0} f(x)$ is equal to

a) 1 b) 0 c) -1 d) None of these

240. $\lim_{x\to a} \frac{\log(x-a)}{\log(e^x-e^a)}$ is equal to

a) 0 b) 1 c) a d) Does not exist

241. The value of $\lim_{x\to\infty} \left(\frac{x-1}{x+1}\right)^x$, is

a) 0 b) e^{-1} c) e^{-2} d) e^{-3}

242. $\lim_{x\to 0} \left[\frac{3^x+3^{-x}-2}{x^2}\right]$ is equal to

a) $2\log 3$ b) $-(\log 3)^2$ c) $-2\log 3$ d) $(\log 3)^2$

243. $\lim_{x\to 0} \left(\frac{e^x+e^{-x}-2}{x^2}\right)^{1/x^2}$ is equal to

a) $e^{1/2}$ b) $e^{1/4}$ c) $e^{1/b}$ d) $e^{1/12}$

244. The value of $\lim_{x\to\infty} \left(\frac{\pi}{2} - \tan^{-1} x\right)^{1/x}$ is

a) 0 b) 1 c) -1 d) e

245. The value of $\lim_{x\to 1} \left\{\frac{x^3+2x^2+x+1}{x^2+2x+3}\right\}^{\frac{1-\cos(x-1)}{(x-1)^2}}$, is

a) e b) $e^{1/2}$ c) 1 d) None of these

246. $\lim_{x\to -3} \frac{3x^2+ax+a-7}{x^2+2x-3}$ exists, then a is equal to

a) 10 b) 15 c) -15 d) -10

247. $\lim_{x\to\infty} \left(1 - \frac{4}{x-1}\right)^{3x-1}$ is equal to

a) e^{12} b) e^{-12} c) e^{4} d) e^{3}

248. $\lim_{n\to\infty} \left\{\frac{1}{1-n^2} + \frac{2}{1-n^2} + \frac{3}{1-n^2} + \cdots + \frac{n}{1-n^2}\right\}$ is equal to

a) 0 b) $-1/2$ c) $1/2$ d) 1

249. $\lim_{x\to 0} \frac{x \tan 2x - 2x \tan x}{(1-\cos 2x)^2}$, is

a) 2 b) -2 c) $1/2$ d) $-1/2$

250. $\lim_{n\to\infty} \frac{3.2^{n+1}-4.5^{n+1}}{5.2^n+7.5^n}$ is equal to

a) $\frac{3}{5}$ b) $-\frac{4}{7}$ c) $-\frac{20}{7}$ d) 0

251. The value of $\lim_{x\to 2} \left\{\left[\frac{x^3-4x}{x^3-8}\right]^{-1} - \left(\frac{x+\sqrt{2x}}{x-2} - \frac{\sqrt{2}}{\sqrt{x}-\sqrt{2}}\right)^{-1}\right\}$ is

a) 1/2 b) 2 c) 1 d) None of these

252. If $\lim_{x\to\infty}\{\sqrt{x^2-x+1} - ax - b\} = 0$, then

a) $a = 1, b = \frac{1}{2}$ b) $a = 1, b = -\frac{1}{2}$ c) $a = -1, b = \frac{1}{2}$ d) None of these

253. $\lim_{x\to\infty}\frac{(2x-3)(3x-4)}{(4x-5)(5x-6)}$ is equal to

a) $\frac{1}{10}$ b) 0 c) $\frac{1}{5}$ d) $\frac{3}{10}$

254. The value of $\lim_{h\to 0}\frac{\ln(1+2h)-2\ln(1+h)}{h^2}$ is

a) 1 b) -1 c) 0 d) None of these

255. The value of $\lim_{x\to\infty} x^{1/x}$ equals

a) 0 b) 1 c) e d) e^{-1}

256. The value of $\lim_{x\to a}\frac{\log(x-a)}{\log(e^x-e^a)}$, is

a) 1 b) -1 c) 0 d) 2

257. If $a_1=1$ and $a_{n+1}=\frac{4+3a_n}{3+2a_n}, n\geq 1$ and if $\lim_{n\to\infty} a_n=a$, then the value of a is

a) $\sqrt{2}$ b) $-\sqrt{2}$ c) 2 d) None of these

258. If $\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x}=1$, then a, b are

a) $\frac{1}{2}, -\frac{3}{2}$ b) $\frac{5}{2}, \frac{3}{2}$ c) $-\frac{5}{2}, -\frac{3}{2}$ d) None of these

259. The value of

$\lim_{\alpha\to 0}\frac{[\operatorname{cosec}^{-1}(\sec\alpha)+\cot^{-1}(\tan\alpha)+\cot^{-1}\cos(\sin^{-1}\alpha)]}{\alpha}$ is

a) 0 b) -1 c) -2 d) 1

260. If $\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x}=e^2$, then the values of a and b are

a) $a\in R, b\in R$ b) $a=1, b\in R$ c) $a\in R, b=2$ d) $a=1, b=2$

261. The value of $\lim_{x\to 0}\left\{\frac{\sin x-x+\frac{x^3}{6}}{x^5}\right\}$ is

a) 0 b) 1 c) 1/60 d) 1/120

262. Let $f(2)=4$ and$f'(2)=4$. Then, $\lim_{x\to 2}\frac{xf(2)-2f(x)}{x-2}$ is given by

a) 2 b) -2 c) -4 d) 3

263. Let $f: R\to R$ be a differentiable function and$f(1)=4$. Then, the value of $\lim_{x\to 1}\frac{\int_4^{f(x)} 2t}{x-1}dt$, if $f'(1)=2$ is

a) 16 b) 8 c) 4 d) 2

264. If $0<a<b$, then $\lim_{n\to\infty}\frac{a^n+b^n}{a^n-b^n}$

a) Equals 0 b) Equals -1 c) Equals 1 d) Does not exist

265. The value $\lim_{x\to a}\left(\frac{\sin x}{\sin a}\right)^{\frac{1}{x-a}}$, is

a) $e^{\sin a}$ b) $e^{\tan a}$ c) $e^{\cot a}$ d) 1

266. If $x_1=3$ and $x_{n+1}=\sqrt{2+x_n}, n\geq 1$, then $\lim_{n\to\infty} x_n$ is equal to

a) -1 b) 2 c) $\sqrt{5}$ d) 3

267. $\lim_{h\to 0}\frac{f(2h+2+h^2)-f(2)}{f(h-h^2+1)-f(1)}$, given that $f'(2)=$ and $f'(1)=4$

a) does not exist b) is equal to -3/2 c) is equal to 3/2 d) is equal to 3

268. The value of $\lim_{x\to 0}\left(\frac{1+5x^2}{1+3x^2}\right)^{1/x^2}$, is

a) e^2 b) e c) e^{-1} d) None of these

269. The value of $\lim_{x\to\infty} x\left\{\tan^{-1}\left(\frac{x+1}{x+2}\right)-\tan^{-1}\left(\frac{x}{x+2}\right)\right\}$ is

a) 1 b) -1 c) 1/2 d) $-1/2$

270. $\lim_{x\to\infty}\left(\frac{3x^2+2x+1}{x^2+x+2}\right)^{\frac{6x+1}{3x+2}}$ is equal to

a) 3 b) 6 c) 9 d) None of these

271. The value of $\lim_{n\to\infty} \cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{4}\right)\cos\left(\frac{x}{8}\right)\dots\cos\left(\frac{x}{2^n}\right)$, is

a) 1 b) $\frac{\sin x}{x}$ c) $\frac{x}{\sin x}$ d) None of these

272. The value of $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{\sin x}{x-\sin x}}$, is

a) e^{-1} b) e c) 1 d) None of these

273. The value of $\lim_{x\to 0}\frac{x^2\sin\left(\frac{1}{x}\right)}{\sin x}$, is

a) 1 b) 0 c) 1/2 d) None of these

274. If $f(1)=1, f'(1)=2$, then $\lim_{x\to 1}\frac{\sqrt{f(x)}-1}{\sqrt{x}-1}$ is

a) 2 b) 4 c) 1 d) 1/2

275. $\lim_{x\to 0}\frac{3x+|x|}{7x-|x|}$, is

a) 2 b) 1/6 c) 0 d) None of these

276. Let $f(x)=\frac{\sqrt{x+3}}{x+1}$, then the value of $\lim_{x\to -3-0} f(x)$ is

a) 0 b) Does not exist c) $\frac{1}{2}$ d) $-\frac{1}{2}$

277. The value of $\lim_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4}$ is equal to

a) 1/5 b) 1/6 c) 1/4 d) 1/2

278. $\lim_{x\to 0}\frac{e^{1/x}}{e^{1/x+1}}$ is equal to

a) 0 b) 1 c) Does not exist d) None of these

279. The value of $\lim_{x\to 0}\frac{\sqrt[3]{1+\sin x}-\sqrt[3]{1-\sin x}}{x}$, is

a) $\frac{2}{3}$ b) $\frac{-2}{3}$ c) $\frac{3}{2}$ d) $\frac{-3}{2}$

280. The value of $\lim_{n\to\infty}\left(\cos\frac{x}{n}\right)^n$, is

a) e b) e^{-1} c) 1 d) None of these

281. If $f(9)=9, f'(9)=4$, then $\lim_{x\to 9}\frac{\sqrt{f(x)}-3}{\sqrt{x}-3}$ equals

a) 4 b) 0 c) c d) 9

282. $\lim_{x\to 0}\frac{\sin x^n}{(\sin x)^m}$, $(m<n)$ is equal to

a) 1 b) 0 c) n/m d) None of these

: ANSWER KEY :

1)	c	2)	d	3)	c	4)	a	145)	c	146)	c	147)	a	148)	c
5)	d	6)	c	7)	c	8)	b	149)	d	150)	a	151)	b	152)	b
9)	b	10)	c	11)	b	12)	b	153)	b	154)	c	155)	c	156)	a
13)	b	14)	d	15)	c	16)	a	157)	a	158)	d	159)	c	160)	d
17)	a	18)	c	19)	c	20)	a	161)	a	162)	d	163)	b	164)	b
21)	c	22)	c	23)	c	24)	d	165)	a	166)	a	167)	d	168)	a
25)	a	26)	c	27)	b	28)	b	169)	c	170)	b	171)	b	172)	c
29)	b	30)	a	31)	c	32)	a	173)	c	174)	d	175)	d	176)	c
33)	a	34)	b	35)	d	36)	a	177)	b	178)	a	179)	c	180)	c
37)	c	38)	b	39)	c	40)	b	181)	d	182)	d	183)	c	184)	d
41)	d	42)	c	43)	b	44)	d	185)	a	186)	a	187)	b	188)	a
45)	c	46)	a	47)	c	48)	c	189)	b	190)	c	191)	b	192)	c
49)	d	50)	d	51)	c	52)	a	193)	b	194)	a	195)	b	196)	d
53)	b	54)	a	55)	d	56)	a	197)	d	198)	c	199)	b	200)	a
57)	b	58)	b	59)	a	60)	a	201)	b	202)	c	203)	b	204)	a
61)	c	62)	b	63)	a	64)	c	205)	d	206)	b	207)	b	208)	c
65)	a	66)	c	67)	d	68)	c	209)	a	210)	d	211)	b	212)	a
69)	a	70)	d	71)	b	72)	b	213)	a	214)	a	215)	a	216)	b
73)	b	74)	a	75)	a	76)	a	217)	c	218)	a	219)	c	220)	a
77)	c	78)	c	79)	b	80)	c	221)	b	222)	c	223)	d	224)	b
81)	b	82)	a	83)	d	84)	a	225)	a	226)	b	227)	b	228)	c
85)	c	86)	a	87)	c	88)	d	229)	a	230)	a	231)	a	232)	d
89)	c	90)	d	91)	a	92)	a	233)	c	234)	a	235)	c	236)	d
93)	d	94)	a	95)	c	96)	c	237)	d	238)	a	239)	b	240)	b
97)	b	98)	c	99)	c	100)	a	241)	c	242)	d	243)	d	244)	b
101)	b	102)	c	103)	b	104)	b	245)	d	246)	a	247)	b	248)	b
105)	c	106)	c	107)	b	108)	c	249)	c	250)	c	251)	a	252)	b
109)	a	110)	d	111)	a	112)	a	253)	d	254)	b	255)	b	256)	a
113)	b	114)	a	115)	d	116)	b	257)	a	258)	d	259)	c	260)	b
117)	c	118)	b	119)	a	120)	c	261)	d	262)	c	263)	a	264)	b
121)	a	122)	b	123)	a	124)	a	265)	c	266)	b	267)	d	268)	a
125)	b	126)	a	127)	a	128)	c	269)	c	270)	c	271)	b	272)	a
129)	c	130)	d	131)	a	132)	a	273)	b	274)	a	275)	d	276)	b
133)	c	134)	d	135)	d	136)	a	277)	b	278)	d	279)	a	280)	c
137)	a	138)	d	139)	b	140)	d	281)	a	282)	b				
141)	c	142)	b	143)	c	144)	a								

: HINTS AND SOLUTIONS :

1 **(c)**

We have,

$$\lim_{x\to 0}\frac{\sqrt{1-\cos x^2}}{1-\cos x}=\lim_{x\to 0}\frac{\sqrt{2\sin^2(x^2/2)}}{2\sin^2 x/2}$$

$$=\frac{1}{\sqrt{2}}\lim_{x\to 0}\frac{\left(\frac{\sin x^2/2}{x^2/2}\right)}{\left(\frac{\sin x/2}{x/2}\right)^2}\times\left\{\frac{x^2/2}{x^2/4}\right\}=\sqrt{2}$$

2 **(d)**

$$\lim_{x\to e+\pi}\{l(x)+g(x)\}=\lim_{x\to 5.81}\{l(x)+g(x)\}$$

$=(5.81)+(5.81)$

$=6+5=11$

3 **(c)**

Here, $\lim_{x\to 0}\frac{f(x^2)-f(x)}{f(x)-f(0)}=\lim_{x\to 0}\frac{f'(x^2).2x-f'(x)}{f'(x)}$

$$=\frac{-f'(0)}{f'(0)}=-1$$

4 **(a)**

Let $y=\lim_{x\to\infty}\left(\frac{\pi}{2}-\tan^{-1}x\right)^{\frac{1}{x}}$

$\therefore \log y=\lim_{x\to\infty}\frac{1}{x}\log\left(\frac{\pi}{2}-\tan^{-1}x\right)$

$\Rightarrow \log y=\lim_{x\to\infty}\frac{\log(\pi/2-\tan^{-1}x)}{x}$ $\quad\left[\frac{\infty}{\infty}\text{ form}\right]$

$\Rightarrow \log y=\lim_{x\to\infty}\frac{\left(-\frac{1}{1+x^2}\right)}{\frac{\pi}{2}-\tan^{-1}x}$ [Using L' Hospital's Rule]

$\Rightarrow \log y=\lim_{x\to\infty}\frac{\frac{2x}{(1+x^2)^2}}{\left(-\frac{1}{1+x^2}\right)}$ [Using L' Hospital's Rule]

$$\Rightarrow \log y=\lim_{x\to\infty}\frac{-2x}{1+x^2}=0\Rightarrow y=e^0=1$$

5 **(d)**

We have, $f(1)=g(1)=2$

$$\therefore \lim_{x\to 1}\frac{f(1)g(x)-f(x)g(1)-f(1)+g(1)}{f(x)-g(x)}$$

$$=\lim_{x\to 1}\frac{2g(x)-2f(x)}{f(x)-g(x)}=\lim_{x\to 1}-2=-2$$

6 **(c)**

We have,

$\log_b a\times\log_c b=\log_c a$

$$\therefore \lim_{x\to\infty}\{\log_{(n-1)}n\,.\log_n(n+1)\,.\log_{(n+1)}(n+2)\ldots\log_{(n^k-1)}(n^k)\}$$

$=\lim_{n\to\infty}\{\log_{(n-1)}n^k\}$

$$=\lim_{n\to\infty}\frac{\log_e n^k}{\log_e(n-1)}$$

$$=k\lim_{n\to\infty}\frac{\log_e n}{\log_e(n-1)}$$

$=k\lim_{n\to\infty}\frac{1/n}{1/n-1}$ [Using L' Hospital's Rule]

$$=k\lim_{n\to\infty}\frac{n-1}{n}=k$$

7 **(c)**

$$\lim_{n\to\infty}\frac{1^2+2^2+3^2+\cdots+n^2}{n^3}$$

$$=\lim_{n\to\infty}\frac{\sum n^2}{n^3}$$

$$=\lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6n^3}$$

$$=\lim_{n\to\infty}\frac{1}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)$$

$$=\frac{1}{3}$$

8 **(b)**

$$\lim_{x\to 0}(\cos x)^{\cot^2 x}=\lim_{x\to 0}\left(1-2\sin^2\frac{x}{2}\right)^{\cot^2 x}$$

$$=e^{-\lim_{x\to 0}2\sin^2\frac{x}{2}\cot^2 x}$$

$$=e^{-\lim_{x\to 0}2\sin^2\frac{x}{2}\times\frac{\cos^2 x}{4\sin^2\frac{x}{2}\cos^2\frac{x}{2}}}$$

$$=e^{-\lim_{x\to 0}\frac{\cos^2 x}{2\cos^2\frac{x}{2}}}$$

$$=e^{-\frac{1}{2}}$$

9 **(b)**

$$\lim_{x\to 0}\frac{e^{ax}-e^{bx}}{x}$$

$$=\lim_{x\to 0}\frac{\left(1+\frac{ax}{1!}+\frac{(ax)^2}{2!}+\ldots\right)-\left(1+\frac{bx}{1!}+\frac{(bx)^2}{2!}+\ldots\right)}{x}$$

$=a-b$

Alternate

$$\lim_{x\to 0}\frac{e^{ax}-e^{bx}}{x}=\lim_{x\to 0}\frac{ae^{ax}-be^{bx}}{1}=a-b$$

10 **(c)**

Here, $\lim_{h\to 0}\frac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$

$$=\lim_{h\to 0}\left[\frac{a^2\{\sin(a+h)-\sin a\}}{h}+\frac{h\{2a\sin(a+h)+h\sin(a+h)\}}{h}\right]$$

$$=\lim_{h\to 0}\frac{a^2.\,2\cos\left[a+\frac{h}{2}\right].\sin\frac{h}{2}}{2.\frac{h}{2}}+\lim_{h\to 0}(2a+h)\sin(a+h)$$

$=a^2\cos a+2a\sin a$

11 **(b)**

$$\lim_{x\to 0} x^2 \sin\left(\frac{\pi}{x}\right) = \lim_{x\to 0} \pi x . \frac{\sin\frac{\pi}{x}}{\frac{\pi}{x}} = 0(1) = 0$$

12 **(b)**

We have,

$$\lim_{x\to\pi/2} (\sin x)^{\tan x}$$

$$= \lim_{x\to\pi/2} \{1 + (\sin x - 1)\}^{\tan x}$$

$$= e^{\lim_{x\to\pi/2} (\sin x - 1)\tan x}$$

$$= e^{\lim_{x\to\pi/2} \left(\frac{\sin x - 1}{\cos x}\right)\sin x}$$

$$= e^{\lim_{x\to\pi/2} \frac{\sin^2 x - \sin x}{\cos x}} = e^{\lim_{x\to\pi/2} \frac{\sin 2x - \cos x}{-\sin x}} = e^{\frac{0}{-1}} = 1$$

14 **(d)**

$$\lim_{x\to\infty} \frac{\int_0^{2x} x e^{x^2} dx}{e^{4x^2}} = \lim_{x\to\infty} \frac{\int_0^{2x} e^{x^2} d(x)^2}{2e^{4x^2}}$$

$$= \lim_{x\to\infty} \frac{\left[e^{x^2}\right]_0^{2x}}{2e^{4x^2}}$$

$$= \lim_{x\to\infty} \frac{e^{4x^2} - 1}{2e^{4x^2}}$$

$$= \lim_{x\to\infty} \left(\frac{1}{2} - \frac{1}{e^{4x^2}}\right) = \frac{1}{2}$$

15 **(c)**

$$\lim_{x\to\infty} x \sin\left(\frac{2}{x}\right) = \lim_{x\to\infty} \frac{\sin\left(\frac{2}{x}\right)}{\frac{1}{2}\left(\frac{2}{x}\right)} = 2$$

16 **(a)**

We have, $\lim_{x\to 1} \frac{\tan(x^2-1)}{x-1}$ $\left[\frac{0}{0} \text{ from}\right]$

$$= \lim_{x\to 1} \frac{\sec^2(x^2-1).2x}{1}$$

[using L'Hospital's rule]

$= 2.\sec^2(0) = 2$

17 **(a)**

$$g[f(x)] = \begin{cases} [f(x)]^2 + 1, & f(x) \neq 2 \\ 3, & f(x) = 2 \end{cases}$$

$$\Rightarrow \quad g[f(x)] = \begin{cases} \sin^2 x + 1, & x \neq n\pi \\ 3, & x = n\pi \end{cases}$$

RHL$= \lim_{h\to 0} g[f(0+h)]$

$= \lim_{h\to 0} (\sin^2 h + 1) = 1$

And LHL$= \lim_{h\to 0} g[f(0-h)]$

$= \lim_{h\to 0} (\sin^2 h + 1) = 1$

$\therefore \quad \lim_{x\to 0} g[f(x)] = 1$

18 **(c)**

$$\lim_{x\to 0} \frac{1 + \sin x - \cos x + \log(1-x)}{x^3}$$

$$= \lim_{x\to 0} \frac{1 + \left(x - \frac{x^3}{3!} + \cdots\right) - \left(1 - \frac{x^2}{2!} + \cdots\right) - \left(x - \frac{x^2}{2} + \frac{x^3}{3} + \ldots\right)}{x^3}$$

$$= \lim_{x\to 0} \frac{-\frac{x^3}{2} + \text{higher power of } x}{x^3}$$

$$= -\frac{1}{2} + 0 = -\frac{1}{2}$$

19 **(c)**

$$\lim_{x\to\frac{\pi}{4}} \frac{\tan x - 1}{x - \frac{\pi}{4}} \qquad \left[\frac{0}{0} \text{ from}\right]$$

Applying L'Hospital's rule,

$$= \lim_{x\to\frac{\pi}{4}} = \frac{\sec^2 x}{1} = 2$$

20 **(a)**

Given, $\lim_{x\to\infty} f(x) = 1$

$$\Rightarrow \quad \lim_{x\to\infty} = \frac{ax + b}{x + 1} = 1$$

$$\Rightarrow \quad \lim_{x\to\infty} \frac{a + \frac{b}{x}}{1 + \frac{1}{x}} = 1$$

$\Rightarrow \quad a = 1$

Also, $\lim_{x\to 0} f(x) = 2$

$$\Rightarrow \quad \lim_{x\to 0} \frac{ax + b}{x + 1} = 2$$

$\Rightarrow \quad b = 2$

Now, $f(-2) = \frac{a(-2)+b}{(-2)+1}$

$$= \frac{-2 + 2}{-2 + 1} = 0$$

21 **(c)**

Given, $\lim_{x\to\infty} \left(\frac{x^2+1}{x+1} - \alpha x - \beta\right) = 0$

$\Rightarrow \quad \lim_{x\to\infty} \left(\frac{x^2+1-\alpha(x^2+x)-\beta(x+1)}{x+1}\right) = 0$

$$\Rightarrow \quad \lim_{x\to\infty} \left(\frac{2x - \alpha(2x+1) - \beta(1)}{1}\right) = 0$$

[by L' Hospital's rule]

If this limit is zero, then the function

$2x - \alpha(2x+1) - \beta = 0$

or $x(2 - 2\alpha) - (\alpha + \beta) = 0$

Equating the coefficient of x and constant terms, we get

$2 - 2\alpha = 0$ and $\alpha + \beta = 0$

$\Rightarrow \quad \alpha = 1, \quad \beta = -1$

22 **(c)**

$$\lim_{n\to\infty} (q^n + p^n)^{1/n} = q \lim_{n\to\infty} \left[1 + \left(\frac{p}{q}\right)^n\right]^{1/n} = q$$

23 **(c)**

We have,

$$\lim_{x\to a} \frac{g(x)f(a) - g(a)f(x)}{x - a}$$

$= \lim_{x\to a} \frac{g'(x)f(a) - g(a)f'(x)}{1}$ [By L' Hospital's Rule]

$= g'(a)f(a) - g(a)f'(a) = (2 \times 2) - (-1 \times 1)$

$= 5$

24 **(d)**

$$\lim_{x\to5}\frac{xf(5)-5f(x)}{x-5}$$
$$=\lim_{x\to5}\frac{f(5)-5f'(x)}{1-0}=f(5)-5f'(5)$$
$$=7-5.7=7-35=-28$$

25 **(a)**

We have,

$$g(f(x))=\begin{vmatrix}\{f(x)\}^2+1,\text{if } f(x)\neq0,2\\ 4,\text{if } f(x)=0\\ 5,\text{if } f(x)=2\end{vmatrix}$$
$$g(f(x))=\begin{vmatrix}\sin^2x+1,\text{if } (x)\neq n\pi,n\in Z\\ 5,\text{if } f(x)=n\pi,n\in Z\end{vmatrix}$$
$$\Rightarrow\lim_{x\to0}g(f(x))=\lim_{x\to0}\sin^2x+1=1$$

26 **(c)**

$$\lim_{n\to\infty}z_1z_2\dots z_n=\lim_{n\to\infty}\left(\cos\frac{\alpha}{n^2}+i\sin\frac{\alpha}{n^2}\right)\times\left(\cos\frac{2\alpha}{n^2}+i\sin\frac{2\alpha}{n^2}\right)\dots\left(\cos\frac{n\alpha}{n^2}+i\sin\frac{n\alpha}{n^2}\right)$$
$$=\lim_{n\to\infty}\left[\text{cols}\left\{\frac{\alpha}{n^2}(1+2+3+\dots+n)\right\}+i\sin\left\{\frac{\alpha}{n^2}(1+2+3+\dots+n)\right\}\right]$$
$$=\lim_{n\to\infty}\left[\cos\frac{\alpha n(n+1)}{2n^2}+i\sin\frac{\alpha n(n+1)}{2n^2}\right]$$
$$=\cos\frac{\alpha}{2}+i\sin\frac{\alpha}{2}=e^{\frac{i\alpha}{2}}$$

27 **(b)**

We have,

$$\lim_{x\to2}\frac{x(5^x-1)}{1-\cos x}=\lim_{x\to0}\frac{\left(\frac{5^x-1}{x}\right)}{\frac{1-\cos x}{x^2}}=2\log5$$

28 **(b)**

$$\lim_{x\to0}\left[\frac{\sqrt{a+x}-\sqrt{a-x}}{4x}\right]$$
$$=\lim_{x\to0}\left[\frac{a+x-a+x}{4x(\sqrt{a+x}+\sqrt{a-x})}\right]$$
$$=\lim_{x\to0}\left[\frac{2x}{4x(\sqrt{a+x}+\sqrt{a-x})}\right]$$
$$=\frac{1}{4\sqrt{a}}$$

29 **(b)**

Let $\cos^{-1}x=y$. Then, $x\to-1^+\Rightarrow y\to\pi^-$

$$\therefore\lim_{x\to-1^+}\frac{\sqrt{\pi}-\sqrt{\cos^{-1}x}}{\sqrt{x+1}}$$
$$=\lim_{y\to\pi^-}\frac{\sqrt{\pi}-\sqrt{y}}{\sqrt{1+\cos y}}$$
$$=\lim_{y\to\pi^-}\frac{\sqrt{\pi}-\sqrt{y}}{\sqrt{2}\cos y/2}$$
$$=\lim_{y\to\pi^-}\frac{\sqrt{\pi}-\sqrt{y}}{\sqrt{2}\sin\left(\frac{\pi}{2}-\frac{y}{2}\right)}\times\frac{\left(\frac{\pi}{2}-\frac{y}{2}\right)}{\left(\frac{\pi}{2}-\frac{y}{2}\right)}$$
$$\lim_{y\to\pi^-}\frac{1}{\frac{\sqrt{2}}{2}(\sqrt{\pi}+\sqrt{y})}\cdot\frac{1}{\left\{\frac{\sin\left(\frac{\pi}{2}-\frac{y}{2}\right)}{\frac{\pi}{2}-\frac{y}{2}}\right\}}=\frac{1}{\sqrt{2\pi}}$$

30 **(a)**

$$\lim_{x\to0}\frac{\tan x-\sin x}{x^3}=\lim_{x\to0}\frac{\sec^2x-\cos x}{3x^2}$$

[using L'Hospital's rule]

$$=\lim_{x\to0}\frac{2\sec^2x\tan x+\sin x}{6x}$$

[using L'Hospital's rule]

$$=\lim_{x\to0}\frac{[2(\sec^2x\sec^2x+2\sec x\times\sec x\tan x\tan x)}{6}$$

[by L'Hospital's rule]

$$=\frac{2[1.1+2(0)+1]}{6}=\frac{1}{2}$$

31 **(c)**

$$\lim_{x\to2}\frac{e^{3x-6}-1}{\sin(2-x)}=\lim_{x\to2}\frac{e^{3x-6}(3)}{-\cos(2-x)}$$

[using L' Hospital's rule]

$$=-\frac{3e^0}{\cos0}=-3$$

32 **(a)**

We have,

$$1\cdot\sum_{r=1}^{n}r+2\cdot\sum_{r=1}^{n-1}r+3\cdot\sum_{r=1}^{n-2}+\dots+n.1$$
$$=\sum_{k=1}^{n}\left\{k\sum_{r=1}^{n-k+1}r\right\}$$
$$=\sum_{k=1}^{n}\left\{k\frac{(n-k+1)(n-k+2)}{2}\right\}$$
$$=\frac{1}{2}\sum_{k=1}^{n}k\{(n+1)-k\}\{(n+2)-k\}$$
$$=\frac{1}{2}\sum_{k=1}^{n}\{(n+1)(n+2)k-(2n+3)k^2+k^3\}$$
$$=\frac{1}{2}\left[(n+1)(n+2)\frac{n(n+1)}{2}-(2n+3)\frac{n(n+1)(2n+1)}{6}+\left\{\frac{n(n+1)}{2}\right\}^2\right]$$

$$= \frac{1}{2}\left[\frac{n(n+1)^2(n+2)}{2} - \frac{n(n+1)(2n+1)(2n+3)}{6} + \frac{n^2(n+1)^2}{4}\right]$$

$$= \frac{n(n+1)}{24}[6(n+1)(n+2) - 2(2n+1)(2n+3) + 3n(n+1)]$$

$$= \frac{n(n+1)}{24}[6n^2 + 18n + 12 - 8n^2 - 16n - 6 + 3n^2 + 3n]$$

$$= \frac{n(n+1)}{24}(n^2 + 5n + 6)$$

$$= \frac{n(n+1)(n+2)(n+3)}{24}$$

∴ Required limit

$$= \lim_{n\to\infty} \frac{n(n+1)(n+2)(n+3)}{24n^4}$$

$$= \frac{1}{24}\lim_{n\to\infty}\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right) = \frac{1}{24}$$

33 **(a)**

Let $s_n = \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots + \frac{1}{(2n-1)(2n+1)}$

$$= \frac{1}{2}\left[\left(1-\frac{1}{3}\right) + \left(\frac{1}{3}-\frac{1}{5}\right) + \ldots + \left(\frac{1}{2n-1} - \frac{1}{2n+1}\right)\right]$$

$$= \frac{1}{2}\left[1 - \frac{1}{2n+1}\right]$$

$$\therefore \quad \lim_{n\to\infty} s_n = \lim_{n\to\infty} \frac{1}{2}\left[1 - \frac{1}{2n+1}\right] = \frac{1}{2}$$

34 **(b)**

We have,

$$\left\{\frac{1}{3} + \frac{2}{21} + \frac{3}{91} + \cdots + \frac{n}{n^4+n^2+1}\right\}$$

$$= \sum_{r=1}^{n} \frac{r}{r^4+r^2+1}$$

$$= \sum_{r=1}^{n} \frac{r}{(r^2+1)^2 - r^2}$$

$$= \frac{1}{2}\sum_{r=1}^{n} \frac{2r}{(r^2+r+1)(r^2-r+1)}$$

$$= \frac{1}{2}\sum_{r=1}^{n}\left\{\frac{1}{r^2-r+1} - \frac{1}{r^2+r+1}\right\}$$

$$= \frac{1}{2}\left\{\left(\frac{1}{1}-\frac{1}{3}\right) + \left(\frac{1}{3}-\frac{1}{7}\right) + \left(\frac{1}{7}-\frac{1}{13}\right) + \cdots + \left(\frac{1}{n^2-n+1} - \frac{1}{n^2+n+1}\right)\right\}$$

$$= \frac{1}{2}\left(1 - \frac{1}{n^2+n+1}\right) = \frac{n^2+n}{2(n^2+n+1)}$$

∴ Requires limit $= \lim_{n\to\infty} \frac{n^2+n}{2(n^2+n+1)} = \frac{1}{2}$

35 **(d)**

$\lim_{x\to\infty}[\sqrt{x^2+2x-1} - x] = \lim_{x\to\infty}\left[\frac{x^2+2x-1-x^2}{\sqrt{x^2+2x-1}+x}\right]$

$$= \lim_{x\to\infty}\left[\frac{2-\frac{1}{x}}{\sqrt{1+\frac{2}{x}-\frac{1}{x^2}}+1}\right] = 1$$

36 **(a)**

$$\lim_{n\to\infty}\left(1+\sin\frac{a}{n}\right)^n = e^{\lim_{n\to\infty}\frac{\sin a/n}{a/n.\frac{1}{a}}} = e^a$$

37 **(c)**

$$\lim_{x\to\frac{\pi}{2}} \frac{a^{\cot x} - a^{\cos x}}{\cot x - \cos x}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\left[\begin{array}{l} 1 + \cot x \log_e a + \frac{\cot^2 x}{2!}(\log_e a)^2 + \cdots \\ -1 - \cos x \log_e a - \frac{\cos^2 x}{2!}(\log_e a)^2 - \ldots \end{array}\right]}{\cot x - \cos x}$$

$$= \lim_{x\to\frac{\pi}{2}}\left\{\log_e a + \frac{\cot x + \cos x}{2!}(\log_e a)^2 + \ldots\right\}$$

$= \log_e a$

38 **(b)**

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}e^{\frac{r}{n}} = \int_0^1 e^x dx = [e^x]_0^1 = e - 1$$

39 **(c)**

We have,

$$\lim_{x\to 0}\frac{\int_0^x t\,dt}{x\tan(x+\pi)} = \lim_{x\to 0}\frac{x^2}{2x\tan x} = \lim_{x\to 0}\frac{x}{2\tan x} = \frac{1}{2}$$

40 **(b)**

We have,

$$\lim_{x\to 0}\frac{x}{\tan^{-1}2x} = \frac{1}{2}\lim_{x\to 0}\frac{1}{\left(\frac{\tan^{-1}2x}{2x}\right)} = \frac{1}{2}$$

41 **(d)**

We have,

$$\lim_{x\to\infty}\left(\frac{1+3x}{2+3x}\right)^{\frac{1-\sqrt{x}}{1+x}}$$

$$= 1^0 = 1 \quad \left[\because \lim_{x\to\infty}\frac{1+3x}{2+3x} = 1 \;\&\; \lim_{x\to\infty}\frac{1-\sqrt{x}}{1+x} = 0\right]$$

42 **(c)**

We have,

$$\lim_{x\to 1}(1-x)\tan\left(\frac{\pi x}{2}\right) = \lim_{x\to 1}(1-x)\tan\left(\frac{\pi}{2} - \frac{\pi}{2}x\right)$$

$$= \frac{\pi}{2}\lim_{x\to 1}\frac{\frac{\pi}{2}(1-x)}{\tan\frac{\pi}{2}(1-x)} = \frac{2}{\pi}\times 1 = \frac{2}{\pi}$$

43 **(b)**

If n is a negative integer, then $n = -m$, where $m \in N$

$$\therefore \lim_{x\to\infty}\frac{x^n}{e^x}\lim_{x\to\infty}\frac{x^{-m}}{e^x}\lim_{x\to\infty}\frac{1}{x^m e^x} = 0$$

If $n = 0$, then

$$\lim_{x\to\infty}\frac{x^n}{e^x} = \lim_{x\to\infty}\frac{1}{e^x} = 0$$

If $n \in N$, then,

$\lim_{x\to\infty}\frac{x^n}{e^x} = \lim_{x\to\infty}\frac{n!}{e^x} = 0$ [By L' Hospital' Rule]

Hence, $\lim_{x\to\infty}\frac{x^n}{e^x} = 0$ for all values of n

44 **(d)**

We have,

$$\lim_{x\to 3}\frac{3^x - x^2}{x^x - 3^2}$$

$= \lim_{x\to 3}\frac{3^x \log_e 3 - 2x}{x^x(1+\log_e x) - 0}$ [Using L' Hospital's Rule]

$$= \frac{3^3\log_e 3 - 6}{3^3(1+\log_e 3)} = \frac{9\log_e 3 - 2}{9\,(\log_e 3 + 1)}$$

45 **(c)**

$$\lim_{x\to\frac{\pi}{2}}\frac{\left(1-\tan\frac{x}{2}\right)(1-\sin x)}{\left(1+\tan\frac{x}{2}\right)(\pi-2x)^3}$$

$$= \lim_{h\to 0}\frac{1-\tan\left(\frac{\pi}{4}-\frac{h}{2}\right)}{1+\tan\left(\frac{\pi}{4}-\frac{h}{2}\right)}.\frac{(1-\cos h)}{(2h)^3}$$

$\left[let\ x = \frac{\pi}{2} - h \text{ as } x\to\frac{\pi}{2}, h\to 0\right]$

$= \lim_{h\to 0}\tan\frac{h}{2}.\frac{2\sin^2\frac{h}{2}}{8h^3}$ $\left[\because \tan\left(\frac{\pi}{4}-x\right) = \frac{1-\tan x}{1+\tan x}\right]$

$$= \lim_{h\to 0}\frac{1}{4}.\frac{\tan\frac{h}{2}}{\frac{h}{2}\times 2}\times\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)^2\times\frac{1}{4} = \frac{1}{32}$$

46 **(a)**

We have,

$$\lim_{x\to\infty}\sqrt{a^2x^2+ax+1} - \sqrt{a^2x^2+1}$$

$$= \lim_{x\to\infty}\frac{ax}{\sqrt{a^2x^2+ax+1}+\sqrt{a^2x^2+1}}$$

$$= \lim_{x\to\infty}\frac{a}{\sqrt{a^2+\frac{a}{x}+\frac{1}{x^2}}+\sqrt{a^2+\frac{1}{x^2}}} = \frac{a}{2\,a} = \frac{1}{2}$$

47 **(c)**

$$\lim_{x\to 0}\frac{1-\cos^3 x}{x\sin x\cos x}$$

$$= \lim_{x\to 0}\frac{(1-\cos x)(1+\cos^2 x+\cos x)}{x^2\cos x\,.\frac{\sin x}{x}}$$

$$= 3\lim_{x\to 0}\frac{1-\cos x}{x^2}$$

$$= 3\times\frac{1}{2}$$

$$= \frac{3}{2}$$

48 **(c)**

Put, $1 - x = y$ as $x\to 1, y\to 0$

$$\therefore \lim_{y\to 0} y\tan\frac{\pi(1-y)}{2} = \lim_{y\to 0}\frac{2}{\pi}\frac{\left(\frac{\pi y}{2}\right)}{\tan\left(\frac{\pi y}{2}\right)} = \frac{2}{\pi}$$

49 **(d)**

$$\lim_{\theta\to 0}\frac{4(\tan\theta - 2\theta^2\tan\theta)}{(1-\cos 2\theta)}$$

$$= \frac{4(\theta\sec^2\theta + \tan\theta - 4\theta\tan\theta - 2\theta^2\sec^2\theta)}{2\sin 2\theta}$$

[using L' Hospital's rule]

$$= \lim_{\theta\to 0}\frac{\left[4\begin{pmatrix}\sec^2\theta + 2\theta\sec^2\theta\tan\theta + \sec^2\theta - 4\tan\theta - \\ 4\theta\sec^2\theta - 4\theta\sec^2\theta - 4\theta^2\sec^2\theta\tan\theta\end{pmatrix}\right]}{4\cos 2\theta}$$

[using L' Hospital's rule]

$$= \frac{4(1+0+1)}{4} = 2$$

50 **(d)**

We have,

$$\lim_{x\to 0^-}\frac{\sin x}{\sqrt{x}} = \lim_{h\to 0}\frac{\sin(-h)}{\sqrt{-h}} = -\lim_{h\to 0}\frac{\sin h}{\sqrt{-h}}$$

Clearly, $\sqrt{-h}$ is not defined

$\therefore \lim_{x\to 0^-}\frac{\sin x}{\sqrt{x}}$ does not exist in R

51 **(c)**

We have, $\lim_{\theta\to\frac{\pi}{2}}\frac{\frac{\pi}{2}-\theta}{\cot\theta} = \lim_{\theta\to\frac{\pi}{2}}\frac{-1}{-\text{cosec}^2\theta}$

$$= \lim_{\theta\to\frac{\pi}{2}}\sin^2\theta = 1$$

52 **(a)**

$$\lim_{x\to 0}\frac{a^x - b^x}{e^x - 1} = \lim_{x\to 0}\frac{a^x - b^x}{x}.\frac{x}{a^x - 1}$$

$$= \left[\lim_{x\to 0}\left(\frac{a^x-1}{x}\right) - \lim_{x\to 0}\left(\frac{b^x-1}{x}\right)\right].\lim_{x\to 0}\frac{x}{e^x-1}$$

$$= (\log_e a - \log_e b).\lim_{x\to 0}\frac{1}{\frac{e^x-1}{x}}$$

$$= \log_e\left(\frac{a}{b}\right)$$

53 **(b)**

$$\lim_{x\to-\infty}\frac{2x-1}{\sqrt{x^2+2x+1}} = \lim_{y\to\infty}\frac{-2x-\frac{1}{y}}{\sqrt{1-\frac{2}{y}+\frac{1}{y^2}}}$$

[put $x = -y$ $\therefore$ $x\to-\infty$ $ie, y\to\infty$]

$$= -\frac{2}{1} = -2$$

54 **(a)**

$$\lim_{x\to 2} = \frac{\sqrt{1+\sqrt{2+x}} - \sqrt{3}}{x-2}$$

$$= \lim_{x\to 2}\frac{(1+\sqrt{2+x}-3)}{(x-2)(\sqrt{1+\sqrt{2+x}}+\sqrt{3})}$$

$$= \lim_{x\to 2} \frac{x-2}{(x-2)\left(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right)(\sqrt{2+x}+2)}$$

$$= \lim_{x\to 2} \frac{1}{(\sqrt{1+\sqrt{2+x}}+\sqrt{3})(\sqrt{2+x}+2)}$$

$$= \frac{1}{(\sqrt{1+2}+\sqrt{3})(\sqrt{2+2}+2)} = \frac{1}{8\sqrt{3}}$$

55 **(d)**

We have,

$$\lim_{x\to 0}\left\{\frac{1}{x\sqrt[3]{8+x}} - \frac{1}{2x}\right\} \quad [\infty-\infty \text{ form}]$$

$$= \lim_{x\to 0}\frac{1}{2x}\left\{\left(1+\frac{x}{8}\right)^{-1/3} - 1\right\}$$

$$= \frac{1}{16}\lim_{x\to 0}\frac{\left(1+\frac{x}{8}\right)^{-1/3} - 1^{-1/3}}{\left(1+\frac{x}{8}\right)-1}$$

$$= \frac{1}{16}\lim_{y\to 1}\frac{y^{-1/3}-1^{-1/3}}{y-10}, \text{where } y = 1+\frac{x}{8}$$

$$= \frac{1}{16}\times\frac{-1}{3}(1)^{-1/3-1} = -\frac{1}{48}$$

56 **(a)**

We have,

$$\lim_{x\to 0}\frac{a^x-b^x}{x}$$

$$= \lim_{x\to 0}\left\{\left(\frac{a^x-1}{x}\right)-\left(\frac{b^x-1}{x}\right)\right\} = \log(a)-\log(b)$$

$$= \log\left(\frac{a}{b}\right)$$

57 **(b)**

We have,

$$\lim_{x\to 1}\frac{\sum_{r=1}^{n} x^r - n}{x-1}$$

$$= \lim_{x\to 1}\frac{x-1}{x-1}+\frac{x^2-1^2}{x-1}+\frac{x^3-1^3}{x-1} + \cdots + \frac{x^n-1^n}{x-1}$$

$$= 1+2+3+\cdots+n = \frac{n(n+1)}{2}$$

58 **(b)**

We have,

$$\lim_{x\to 1}(\log_4 5x)^{\log_x 5} = \lim_{x\to 1}(\log_5 5+\log_5 x)^{\log_x 5}$$

$$= \lim_{x\to 1}(1+\log_5 x)^{\frac{1}{\log_5 x}} = e^{\lim_{x\to 1}\log_5 x \cdot \frac{1}{\log_x 5}} = e^1 = e$$

59 **(a)**

We have,

$$\lim_{x\to 0}\frac{e^{\tan x}-e^x}{\tan x - x} = \lim_{x\to 0}\frac{e^x\{e^{\tan x - x}-1\}}{\tan x - x}$$

$$= \lim_{x\to 0} e^x \times \lim_{x\to 0}\frac{e^{\tan x - x}-1}{\tan x - x} e^0 \times 1 = 1$$

60 **(a)**

We have,

$$\lim_{x\to\infty}\left(\frac{x^2-2x+1}{x^2-4x+2}\right)^x$$

$$= \lim_{x\to\infty}\left(1+\frac{2x-1}{x^2-4x+2}\right)^x = e^{\lim_{x\to\infty}\frac{(2x-1)x}{x^2-4x+2}} = e^2$$

61 **(c)**

$$\lim_{x\to 0}\frac{\log(x+a)-\log a}{x} + k\lim_{x\to e}\frac{\log x - 1}{x-e} = 1$$

Using L' Hospital's rule

$$\lim_{x\to 0}\frac{\frac{1}{x+a}}{1} + k\lim_{x\to 0}\frac{\frac{1}{x}}{1} = 1$$

$$\Rightarrow \quad \frac{1}{a}+\frac{k}{e} = 1$$

$$\Rightarrow \quad k = e\left(1-\frac{1}{a}\right)$$

62 **(b)**

We have,

$$\lim_{x\to\infty}\frac{\sin x}{x} = \lim_{y\to 0} y\sin\left(\frac{1}{y}\right) = 0$$

63 **(a)**

$$\lim_{x\to 0} x\log\sin x$$

$$= \lim_{x\to 0}\frac{\log\sin x}{1/x} \qquad \left(\frac{0}{0}\text{ form}\right)$$

$$= \lim_{x\to 0}\frac{\frac{1}{\sin x}\cos x}{\frac{-1}{x^2}} = \lim_{x\to 0} -\frac{x^2}{\tan x}$$ [by L'Hospital's rule]

$$= \lim_{x\to 0}\frac{-2x}{\sec^2 x}$$ [by L'Hospital's rule]

$= 0$

64 **(c)**

$$\lim_{x\to 0}\frac{d}{dx}\int\left(\frac{1-\cos x}{x^2}\right)dx = \lim_{x\to 0}\frac{1-\cos x}{x^2}$$

$$= \lim_{x\to 0}\frac{2\sin^2 x/2}{4.x^2/4}$$

$$= \frac{1}{2}\lim_{x\to 0}\left(\frac{\sin x/2}{x/2}\right)^2 = \frac{1}{2}$$

65 **(a)**

We have,

$$\lim_{x\to 0}\frac{\left(\int_y^a e^{\sin^2 t}dt - \int_{x+y}^a e^{\sin^2 t}\,dt\right)}{x}$$

$$= \lim_{x\to 0}\frac{\left(\int_y^a e^{\sin^2 t}dt + \int_a^{x+y} e^{\sin^2 t}\,dt\right)}{x}$$

$$= \lim_{x\to 0}\frac{\int_y^{x+y} e^{\sin^2 t\,dt}}{x}$$

$$= \lim_{x\to 0}\frac{\frac{d}{dx}(x+y)e^{\sin^2(x+y)}-0}{1}$$ [Using L' Hospital's Rule]

$$= \lim_{x\to 0} 1.e^{\sin^2(x+y)} = e^{\sin^2 y}$$

66 **(c)**

We have,

$\lim_{x\to 0}\frac{2f(x)-3f(2x)+f(4x)}{x^2}$ $\left[\text{Form}\frac{0}{0}\right]$

$=\lim_{x\to 0}\frac{2f'(x)-6f'(2x)+4f'(4x)}{2x}$ [By L' Hospital's Rule]

$=\lim_{x\to 0}\frac{2f'(x)-3f'(2x)+2f'(4x)}{x}$ $\left[\text{Form}\frac{0}{0}\right]$

$=\lim_{x\to 0}\frac{2f''(x)-6f''(2x)+8f''(4x)}{1}$ [By L' Hospital's Rule]

$=f''(0)-6f''(0)+8f''(0)=3f''(0)=3\times 4$
$=12$

67 **(d)**

Given, $\lim_{x\to 0}\frac{\{(a-n)nx-\tan x\}\sin nx}{x^2}=0$

$\Rightarrow \lim_{x\to 0}\left((a-n)n-\frac{\tan x}{x}\right).\frac{\sin nx}{x}=0$

$\Rightarrow [\{a-n\}n-1]n=0$

$\Rightarrow a=n+\frac{1}{n}$

68 **(c)**

We have,

$\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}=1$

$\Rightarrow \lim_{x\to 0}\frac{x\left\{1+a\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\cdots\right)\right\}-b\left\{x-\frac{x^3}{3!}\right.}{x^3}$
$=1$

$\Rightarrow \lim_{x\to 0}\frac{(1+a-b)+x^2\left(\frac{b}{3!}-\frac{a}{2!}\right)+x^4\left(\frac{a}{4!}-\frac{b}{5!}\right)+\cdots}{x^2}=1$ ...(i)

If $1+a-b\neq 0$, then LHS $\to\infty$ as $x\to 0$ which RHS $=1$

$\therefore 1+a-b=0$

From (i), we have

$\lim_{x\to 0}\frac{x^2\left(\frac{b}{3!}-\frac{a}{2!}\right)+x^4\left(\frac{a}{4!}-\frac{b}{5!}\right)+\cdots}{x^2}=1$

$\therefore \frac{b}{3!}-\frac{a}{2!}=1\Rightarrow b-3a=6$

Solving $1+a-b=0$ and $b-3a=6$, we get $a=-\frac{5}{2}, b=-\frac{3}{2}$

69 **(a)**

$\lim_{x\to\infty}\left(\frac{x^2-2x+1}{x^2-4x+2}\right)^x=\lim_{x\to\infty}\left(1+\frac{2x-1}{x^2-4x+2}\right)^x$

$=e^{\lim_{x\to\infty}\left(\frac{x(2x-1)}{x^2-4x+2}\right)}=e^2$

70 **(d)**

$\lim_{x\to 0}\frac{(1-\cos 2x)}{x^2}$

$=\lim_{x\to 0}\frac{2\sin^2 x}{x^2}$

$=2\lim_{x\to 0}\left(\frac{\sin x}{x^2}\right)^2=2$

71 **(b)**

We have,

$\lim_{x\to 0}\frac{1+\sin x-\cos x+\log(1-x)}{x^3}$

$=\lim_{x\to 0}\frac{1+\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots\right)-\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}\ldots\right.}{x^3}$

$=\lim_{x\to 0}\left(-\frac{1}{3!}-\frac{1}{3}\right)+x^2\left(\frac{1}{5!}-\frac{1}{5}\right)+\cdots=-\frac{1}{6}-\frac{1}{3}$
$=-\frac{1}{2}$

72 **(b)**

$\lim_{x\to 0}\left\{\frac{1+\tan x}{1+\sin x}\right\}^{\operatorname{cosec} x}$

$=\lim_{x\to 0}\frac{\left[\left(1+\frac{\sin x}{\cos x}\right)^{\frac{\cos x}{\sin x}}\right]^{1/\cos x}}{(1+\sin x)^{1/\sin x}}$

$=\frac{e^{\lim_{x\to 0}\frac{1}{\cos x}}}{e}$

$=\frac{e}{e}=1$

73 **(b)**

$\lim_{x\to\frac{\pi}{6}}\frac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1}$

$=\lim_{x\to\frac{\pi}{6}}\frac{4\sin x\cos x+\cos x}{4\sin x\cos x-3\cos x}$ [by L'Hospital's rule]

$=\lim_{x\to\frac{\pi}{6}}\frac{\cos x\,(4\sin x+1)}{\cos x\,(4\sin x-3)}$

$=\frac{4\sin\frac{\pi}{6}+1}{4\sin\frac{\pi}{6}-3}=-3$

74 **(a)**

$\lim_{x\to 0}\left(\frac{1+5x^2}{1+3x^2}\right)^{1/x^2}=\lim_{x\to 0}\left(1+\frac{2x^2}{1+3x^2}\right)^{1/x^2}$

$=e^{\lim_{x\to 0}\frac{1}{x^2}\left(\frac{2x^2}{1+3x^2}\right)}=e^2$

75 **(a)**

We have,

$\lim_{x\to 0^-}f(x)=\lim_{x\to 0}x=0$ and, $\lim_{x\to 0^+}f(x)=\lim_{x\to 0}x^2=0$

Hence $\lim_{x\to 0}f(x)=0$

76 **(a)**

If $x \in Q$, then $n!\,\pi\, x$ will be an integral multiple of π for large values of n. Therefore, $\cos(n!\,\pi x)$ will be either 1 or -1 and so $\cos^{2\,m}(n!\,\pi x) = 1$

$\therefore \lim_{m\to\infty}\lim_{n\to\infty}[1+\cos^{2m}(n!\ \pi\, x)] = 1 + 1 = 2$

If $x \notin Q$, $n!\,\pi\, x$ will not be an integral multiple of π and so $\cos(n!\,\pi x)$ will lie between -1 and 1

Thus, $\lim_{m\to\infty}\cos^{2m}(n!\,\pi x) = 0$

$\Rightarrow \lim_{m\to\infty}\lim_{n\to\infty}[1+\cos^{2m}(n!\,\pi x)] = 1 + 0 = 1$

77 **(c)**

We have,

$\lim_{x\to 1}(1+\cos\pi\, x)\cot^2\pi\, x$

$= \lim_{x\to 1}\dfrac{(1+\cos\pi x)(\cos^2\pi x)}{(1-\cos^2\pi x)} = \lim_{x\to 1}\dfrac{\cos^2\pi x}{1-\cos\pi x} = \dfrac{1}{2}$

78 **(c)**

$\lim_{x\to 0}\dfrac{\left(e^{kx}-1\right)\sin kx}{x^2} = 4$

$\Rightarrow \quad \lim_{x\to 0}\dfrac{e^{kx}-1}{kx}\times k\times\dfrac{\sin kx}{kx}\times k = 4$

$\Rightarrow \quad k^2 = 4$

$\Rightarrow \quad k = \pm 2$

79 **(b)**

$$\lim_{x\to 0}\frac{\log(1+x^3)}{\sin^3 x} = \lim_{x\to 0}\frac{\left(\frac{x^3}{1}-\frac{(x^3)^2}{2}+\frac{(x^3)^3}{3}-\ldots\infty\right)}{\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots\infty\right)^3}$$

$$= \lim_{x\to 0}\frac{\left(1-\frac{x^3}{2}+\frac{(x^3)^2}{3}-\ldots\infty\right)}{\left(1-\frac{x^2}{3!}+\frac{x^4}{5!}-\ldots\infty\right)^3} = 1$$

80 **(c)**

$l_1 = \lim_{x\to 2^+}(x+[x])$

$= \lim_{h\to 0}2+h+[2+h] = 4$

$l_2 = \lim_{x\to 2^-}(2x-[x])$

$= \lim_{h\to 0}\{2(2-h)-[2-h]\}$

$= \lim_{h\to 0}\{2(2-h)-1\} = 3$

$l_3 = \lim_{x\to\frac{\pi}{2}}\dfrac{\cos x}{x-\frac{\pi}{2}} = \lim_{x\to\frac{\pi}{2}} -\sin x = -1$

[by L'Hospital's rule]

Thus, $\quad l_3 < l_2 < l_1$

81 **(b)**

$\lim_{x\to 0^-}f(x) = \lim_{x\to 0^-}\dfrac{\sin(1+[x])}{[x]}$

$= \dfrac{\sin(1-1)}{-1} = 0$

82 **(a)**

$\lim_{x\to 0}\dfrac{(1-cos\ 2x)\sin 5x}{x^2\sin 3x}$

$= \lim_{x\to 0}\dfrac{1-cos\ 2x}{x^2}\lim_{x\to 0}\dfrac{\sin 5x}{\sin 3x}$

$= \lim_{x\to 0}\dfrac{2\sin^2 x}{x^2}\lim_{x\to 0}\dfrac{\sin 5x}{5x}\cdot\dfrac{3x}{\sin 3x}\cdot\dfrac{5x}{3x}$

$= \lim_{x\to 0}2\left(\dfrac{\sin x}{x}\right)^2\cdot\dfrac{5}{3} = \dfrac{10}{3}$

83 **(d)**

We have,

$\Rightarrow \lim_{x\to 2}f(x) = \lim_{x\to 2}\dfrac{\sin(e^{x-2}-1)}{\log(x-1)}$

$\Rightarrow \lim_{x\to 2}f(x)$

$= \lim_{x\to 2}\left\{\dfrac{\sin(e^{x-2}-1)}{e^{x-2}-1}\cdot\dfrac{e^{x-2}-1}{x-2}\cdot\dfrac{x-2}{\log\left(1+(x-2)\right)}\right\}$

$\Rightarrow \lim_{x\to 2}f(x) = 1\times 1\times 1 = 1$

84 **(a)**

We have,

$\lim_{n\to\infty}\dfrac{S_{n+1}-S_n}{\sqrt{\sum_{k=1}^{n}k}} = \lim_{n\to\infty}\dfrac{a_{n+1}}{\sqrt{\frac{n(n+1)}{2}}} = 0$

85 **(c)**

$\lim_{h\to 0}\dfrac{\sin\sqrt{x+h}-\sin\sqrt{x}}{h}$

Applying L'Hospital's rule,

$= \lim_{h\to 0}\dfrac{\frac{\cos\sqrt{x+h}}{2\sqrt{x+h}}}{1} = \dfrac{\cos\sqrt{x}}{2\sqrt{x}}$

86 **(a)**

Let $y = \lim_{x\to\frac{\pi}{2}}(\sin x)^{\tan x}$

$\Rightarrow \quad \log y = \lim_{x\to\frac{\pi}{2}}\tan x\log\sin x$

$= \lim_{x\to\frac{\pi}{2}}\dfrac{\log\sin x}{\cot x}$

$= \lim_{x\to\frac{\pi}{2}}\dfrac{\frac{1}{\sin x}.\cos x}{-\mathrm{cosec}^2 x}$ [by L' Hospital's rule]

$= 0$

$\Rightarrow \quad y = e^0 = 1$

87 **(c)**

We know that

$\cos A\cos 2A\cos 4A\ldots\cos 2^{n-1}A = \dfrac{\sin 2^n A}{2^n\sin A}$

$\lim_{n\to\infty}\cos\left(\dfrac{x}{2}\right)\cos\left(\dfrac{x}{4}\right)\ldots\cos\left(\dfrac{x}{2^{n-1}}\right)\cos\left(\dfrac{x}{2^n}\right)$

$= \lim_{n\to\infty}\dfrac{\sin x}{2^n\sin(x/2^n)}$ $\left[\text{put } A = \dfrac{x}{2^n}\right]$

$= \lim_{n\to\infty}\dfrac{\sin x}{x}\cdot\dfrac{(x/2^n)}{\sin(x/2^n)}$

$= \frac{\sin x}{x}$

88 **(d)**

$\lim_{x \to 1} \frac{\sin(e^{x-1} - 1)}{\log x}$

$= \lim_{h \to 0} \frac{\sin(e^h - 1)}{\log(1+h)}$

$= \lim_{h \to 0} \frac{\sin(e^h - 1)}{(e^h - 1)} \times \frac{(e^h - 1)}{\log(1+h)}$

$= 1 \times \lim_{h \to 0} \frac{\left(h + \frac{h^2}{2!} + \ldots\right)}{\left(h - \frac{h^2}{2!} + \ldots \infty\right)} = 1 \times 1 = 1$

89 **(c)**

We have,

$l = \lim_{x \to -2} \frac{\tan \pi x}{x+2} + \lim_{x \to \infty} \left(1 + \frac{1}{x^2}\right)^x$

$\Rightarrow l = \lim_{x \to -2} \frac{\tan(2\pi + \pi x)}{x+2} + \lim_{x \to \infty} \left(1 + \frac{1}{x^2}\right)^x$

$\Rightarrow l = \pi \lim_{x \to -2} \frac{\tan \pi (x+2)}{\pi (x+2)} + e^{\lim_{x \to \infty} \frac{x}{x^2}} = \pi + e^0$

$= \pi + 1$

90 **(d)**

RHL$= \lim_{h \to 0} f(1+h)$

$= \lim_{h \to 0} \frac{\sqrt{1 - \cos 2h}}{h}$

$= \lim_{h \to 0} \frac{\sqrt{2} \sin h}{h} = \sqrt{2}$

LHL$= \lim_{h \to 0} f(1-h) \lim_{h \to 0} \frac{\sqrt{1 - \cos(-2h)}}{h}$

$= \lim_{h \to 0} \sqrt{2} \frac{\sin h}{-h} = -\sqrt{2}$

Here, LHL≠RHL

So, limit does not exist.

91 **(a)**

We have,

$\lim_{x \to 2} \frac{2x^2 - 4f'(x)}{x-2} = \lim_{x \to 0} \frac{4x - 4f''(x)}{1}$ [Using L' Hospital's Rule]

$\Rightarrow \lim_{x \to 2} \frac{2x^2 - 4f'(x)}{x-2} = 8 - 4f''(2) = 8 - 4 = 4$

92 **(a)**

$\lim_{x \to \pi/4} \frac{\int_2^{\sec^2 x} f(t)dt}{x^2 - \pi^2/16}$

$= \lim_{x \to \pi/4} \frac{2 \sec^2 x \tan x \, f(\sec^2 x)}{2x}$ [Using Leibniz and L' Hospital's rules]

$= \frac{\sec^2 \frac{\pi}{2} f\left(\sec^2 \frac{\pi}{4}\right) \tan \frac{\pi}{4}}{\pi/4} = \frac{8}{\pi} f(2)$

93 **(d)**

$\lim_{x \to a} \frac{f(a)g(x) - f(x)g(a)}{x-a}$ $\left[\frac{0}{0} \text{ from}\right]$

$= \lim_{x \to a} \frac{f(a)g'(x) - f'(x)g(a)}{1-0}$ [by L' Hospital's rule]

$= f(a)g'(a) - f'(a)g(a)$

$= 2(-1) - 1(3) = -2 - 3 = -5$

94 **(a)**

We have,

$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \left(\frac{x^2 + 5x + 3}{x^2 + x + 2}\right)^x$

$\Rightarrow \lim_{x \to \infty} f(x) = \lim_{x \to \infty} \left(1 + \frac{4x+1}{x^2 + x + 2}\right)^x$

$= e^{\lim_{x \to \infty} \frac{x(4x+1)}{x^2+x+2}} = e^4$

95 **(c)**

$\lim_{x \to 0} \frac{\sin^{-1} x - x}{x^3 \cos x} = \lim_{x \to 0} \frac{\frac{1}{\sqrt{1-x^2}} - 1}{x^3(-\sin x) + 3x^2 \cos x}$

[using L'Hospital's rule]

$= \lim_{x \to 0} \frac{1 - \sqrt{1-x^2}}{\sqrt{1-x^2} . x^2(-x \sin x + 3 \cos x)} \times \frac{1 + \sqrt{1-x^2}}{1 + \sqrt{1-x^2}}$

$= \lim_{x \to 0} \frac{1}{\left[\begin{array}{c}\sqrt{1-x^2}\left(1+\sqrt{1-x^2}\right) \\ (-x \sin x + 3 \cos x)\end{array}\right]}$

$= \frac{1}{1(1+1)(3)} = \frac{1}{6}$

96 **(c)**

Here, $\lim_{x \to 0} (\sin x)^{1/x} + \lim_{x \to 0} \left(\frac{1}{x}\right)^{\sin x} = 0 +$ $\lim_{x \to 0} e^{\log\left(\frac{1}{x}\right)^{\sin x}}$

$\left[\begin{array}{l}\lim_{x \to 0} (\sin x)^{\frac{1}{x}} \to 0 \\ \text{as,} \quad 0 < \sin x < 1\end{array}\right]$

$= e^{\lim_{x \to 0} \frac{\log(1/x)}{\operatorname{cosec} x}} = e^{\lim_{x \to 0} \frac{x\left(-\frac{1}{x^2}\right)}{-\operatorname{cosec} x \cot x}}$

[by L'Hospital's rule]

$= e^{\lim_{x \to 0} \frac{\sin x}{x} \tan x} = e^0 = 1$

97 **(b)**

$\lim_{x \to \infty} \left(\frac{3x-4}{3x+2}\right)^{\frac{x+1}{3}}$

$= \lim_{x \to \infty} \left[1 + \frac{-6}{3x+2}\right]^{\frac{x+1}{3}}$

$= \left[\lim_{x \to \infty} \left\{1 + \frac{-6}{3x+2}\right\}^{\frac{3x+2}{-6}}\right]^{\frac{-6}{3x+2} \times \frac{x+1}{3}}$

$= [e]^{\lim_{x \to \infty} \frac{-6}{3x+2} \times \frac{x+1}{3}}$ $\left[\because \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e\right]$

$= e^{\lim_{x \to \infty} \frac{-2x-2}{3x+2}} = e^{-2/3}$

98 **(c)**

Put $x = \tan \theta \Rightarrow \theta = \tan^{-1} x$

As $x \to 0 \Rightarrow \theta \to 0$

$\therefore \lim_{\theta \to 0} \frac{1}{\tan \theta} \sin^{-1}\left(\frac{2 \tan \theta}{1 + \tan^2 \theta}\right)$

$= \lim_{\theta \to 0} \frac{1}{\tan\theta} \sin^{-1}(\sin 2\theta)$

$= \lim_{\theta \to 0} \frac{2\theta}{\tan\theta} = 2$

99 **(c)**

$\lim_{x\to 0} \frac{2\sin^2 3x}{x^2} = \lim_{x\to 0} 2\left(\frac{\sin 3x}{3x}\right)^2 \times \frac{9}{1} = 18$

100 **(a)**

$\lim_{x\to 0} \frac{a^x + a^{-x} - 2}{x^2} = \lim_{x\to 0} \frac{a^x \log a - a^{-x}\log a}{2x}$ [by L' Hospital's rule]

$= \lim_{x\to 0} \frac{a^x (\log a)^2 + a^{-x} (\log a)^2}{2}$

$= (\log a)^2$ [by L' Hospital's rule]

101 **(b)**

LHL= $\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = 1$ [$\because$ $(0-h)$ is rational]

RHL= $\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = 1$ [$\because$ $(0+h)$ is rational]

Hence, LHL=RHL=1

102 **(c)**

$\lim_{x\to\infty} \left(\frac{x-3}{x+2}\right)^x = \lim_{x\to\infty} \left[\frac{1-\frac{3}{x}}{1+\frac{2}{x}}\right]^x$

$= \frac{e^{-3}}{e^2} = e^{-5}$

103 **(b)**

We have,

$\lim_{x\to\infty} \frac{x}{2}\sin\left(\frac{\pi}{2x}\right)$

$= \lim_{x\to\infty} \frac{\sin\left(\frac{\pi}{2x}\right)}{\frac{\pi}{2x}} \cdot \frac{\pi}{4} = \frac{\pi}{4}\lim_{y\to 0} \frac{\sin y}{y} = \frac{\pi}{4}$, where $y = \frac{\pi}{2x}$

104 **(b)**

$\lim_{x\to 0} \frac{f(x)}{x} = \lim_{x\to 0} \frac{f'(x)}{1}$

$= \lim_{x\to 0} \frac{\tan^4 x}{1} = 0$

105 **(c)**

RHL= $\lim_{x\to 1^+} \frac{1}{2}\{g(x) + (x)\}\sin x$

$= \lim_{x\to 1^+} \frac{1}{2}\{1 + x\}\sin x$

$= \frac{1}{2}.(1+1)\sin 1 = \sin 1$

and LHL= $\lim_{x\to 1^-} \frac{\sin x}{x} = \sin 1$

Since, RHL=LHL=sin 1

$\therefore \lim_{x\to 1} f(x) = \sin 1$

106 **(c)**

Given, $\lim_{x\to\infty} \left[\frac{x^3+1}{x^2+1} - (ax+b)\right] = 2$

$\Rightarrow \lim_{x\to\infty} \left[\frac{x(1-a) - b - \frac{a}{x} + \frac{(1-b)}{x^2}}{1 + \frac{1}{x^2}}\right] = 2$

This limit will exist, if

$1 - a = 0$

and $b = -2$

$\Rightarrow a = 1$

and $b = -2$

107 **(b)**

$\lim_{x\to 2} \frac{f(x) - f(2)}{x-2} = \lim_{x\to 2} \frac{\frac{x-2}{x^2-3x+2} - 1}{x-2}$

$= \lim_{x\to 2} \frac{x - 2 - (x^2 - 3x + 2)}{(x-2)(x^2-3x+2)}$

$= \lim_{x\to 2} \frac{-(x-2)^2}{(x-2)(x-2)(x-1)}$

$= -\lim_{x\to 2} \frac{1}{x-1}$

$= -1$

108 **(c)**

$\lim_{x\to 3} \frac{\int_3^{f(x)} 2t^3\,dt}{x-3} = \lim_{x\to 3} \frac{2[f(x)]^3.f'(x)}{1}$

$= 2[f(3)]^3.f'(3) = 2\times 3^3 \times \frac{1}{2}$

$= 27$

109 **(a)**

Given, $\lim_{x\to a} \frac{f(a)g(x) - f(a) - g(a)f(x) + g(a)}{g(x) - f(x)} = 4$

Applying L' Hospital's rule,

$\Rightarrow \lim_{x\to a} \frac{f(a)g'(x) - g(a)f'(x)}{g'(x) - f'(x)} = 4$

$\Rightarrow \lim_{x\to a} \frac{kg'^{(x)} - kf'(x)}{g'(x) - f'(x)} = 4$

$\Rightarrow k = 4$

110 **(d)**

We have,

$\lim_{x\to 0} \frac{\sin x}{\sqrt{x^2}} = \lim_{x\to 0} \frac{\sin x}{|x|}$

Now, $\lim_{x\to 0^-} \frac{\sin x}{|x|} = \lim_{x\to 0} \frac{\sin x}{x|} = -1$

and, $\lim_{x\to 0^+} \frac{\sin x}{|x|} = \lim_{x\to 0} \frac{\sin x}{x} = 1$

Hence, $\lim_{x\to 0} \frac{\sin x}{|x|}$ does not exist

111 **(a)**

We have,

$\lim_{h\to 0} \frac{(a+h)^2 \sin(a+h) - a^2 \sin a}{h}$

$= \left\{\frac{d}{dx}(x^2 \sin x)\right\}_{\text{at } x=a} = 2a\sin a + a^2 \cos a$

112 **(a)**

$$\lim_{x\to 0}\frac{2f(x)-3f(2x)+f(4x)}{x^2}$$
$$=\lim_{x\to 0}\frac{2f'(x)-6f'(2x)+4f'(4x)}{2x}$$
$$=\lim_{x\to 0}\frac{2f''(x)-12f''(2x)+16f''(4x)}{2}$$
$$=\frac{2f''(0)-12f''(0)+16f''(0)}{2}=\frac{6a}{2}=3a$$

113 **(b)**

$\lim_{x\to 0}\frac{e^x+\log(1+x)-(1-x)^{-2}}{x^2}$ $\left[\frac{0}{0}\text{ form}\right]$

$$=\lim_{x\to 0}\frac{e^x+(1+x)^{-1}-2(1-x)^{-3}}{2x}$$ [by L' Hospital's rule]

$$=\lim_{x\to 0}\frac{e^x-(1+x)^{-2}-6(1-x)^{-4}}{2}$$ [by L' Hospital's rule]

$$=\frac{e^0-1-6}{2}=-3$$

114 **(a)**

Given,

$\lim_{x\to 0} kx\ \text{cosec}\ (x)=\lim_{x\to 0}\ x\ \text{cosec}\ (kx)$

$$\Rightarrow\quad k\lim_{x\to 0}\frac{x}{\sin x}=\lim_{x\to 0}\frac{x}{\sin kx}\times\frac{k}{k}$$
$$\Rightarrow\qquad k=\frac{1}{k}$$
$$\Rightarrow\qquad k=\pm 1$$

115 **(d)**

We have,

$$\lim_{x\to 0^+}\frac{|x|}{x}=\lim_{x\to 0^+}\frac{x}{x}=1$$

and, $\lim_{x\to 0^-}\frac{|x|}{x}=\lim_{x\to 0^+}\frac{|-x|}{-x}=\lim_{x\to 0^+}\frac{x}{-x}=-1$

Hence, $\lim_{x\to 0}\frac{|x|}{x}$ does not exist

116 **(b)**

We have,

$$\lim_{x\to 0}x^2\sin\frac{\pi}{x}$$

$=0\times$ (A finite oscillating number) $=0$

117 **(c)**

$$\lim_{x\to\infty}\left(\frac{x^2+bx+4}{x^2+ax+5}\right)=\lim_{x\to\infty}\frac{\left(1+\frac{b}{x}+\frac{4}{x^2}\right)x^2}{\left(1+\frac{a}{x}+\frac{5}{x^2}\right)x^2}=1$$

118 **(b)**

Since, $f(x)$ is the integral function of $\frac{2\sin x-\sin 2x}{x^3}$, therefore by definition

$$f'(x)=\frac{2\sin x-\sin 2x}{x^3}$$
$$\Rightarrow\quad\lim_{x\to 0}f'(x)=\lim_{x\to 0}\frac{2\sin x-\sin 2x}{x^3}$$
$$=\lim_{x\to 0}=\frac{2\sin x}{x}.\frac{1-\cos x}{x^2}=1$$

119 **(a)**

We have,

$$\lim_{x\to\infty}\left(\frac{3x-4}{3x+4}\right)^{\left(\frac{x+1}{3}\right)}=\lim_{x\to\infty}\left(1+\frac{-6}{3x+2}\right)^{\frac{x+1}{2}}$$
$$=e^{\lim_{x\to\infty}\frac{-6}{3x+2}\times\frac{x+1}{3}}=e^{\lim_{x\to\infty}\frac{-2x-2}{3x+2}}=e^{-2/3}$$

120 **(c)**

$$\lim_{x\to 0}\frac{(1+x)^8-1}{(1+x)^2-1}$$
$$=\lim_{x\to 0}\frac{[(1+x)^4+1][(1+x)^2+1][(1+x)^2-1]}{(1+x)^2-1}$$
$$=2\times 2=4$$

Alternate

$\lim_{x\to 0}\frac{(1+x)^8-1}{(1+x)^2-1}$ $\left(\frac{0}{0}\text{form}\right)$

$\lim_{x\to 0}\frac{8(1+x)^7}{2(1+x)}$ (by L' Hospital's rule)

$=4$

121 **(a)**

We have,

$$\lim_{x\to\pi/4}\frac{2\sqrt{2}-(\cos x+\sin x)^3}{1-\sin 2x}$$
$$=\lim_{x\to\pi/4}\frac{2^{3/2}-\{(\cos x+\sin x)^2\}^{3/2}}{2-(1+\sin 2x)}$$
$$=\lim_{x\to\pi/4}\frac{2^{3/2}-(1+\sin 2x)^{3/2}}{2-(1+\sin 2x)}$$

$=\lim_{y\to 2}\frac{y^{3/2}-2^{3/2}}{y-2}$, where $y=1+\sin 2x$

$$=\frac{3}{2}(2)^{3/2-1}=\frac{3}{2}\times\sqrt{2}=\frac{3}{\sqrt{2}}$$

122 **(b)**

We have,

$$\lim_{x\to-\infty}\left(3x+\sqrt{9x^2-x}\right)$$

$=\lim_{y\to\infty}\left(-3y+\sqrt{9y^2+y}\right)$, where $y=-x$

$$=\lim_{y\to\infty}\frac{-9y^2+9y^2+y}{(3y+\sqrt{9y^2}+y)}=\lim_{y\to\infty}\frac{y}{3y+\sqrt{9y^2}+y}$$
$$=\frac{1}{3+3}=\frac{1}{6}$$

123 **(a)**

$$\lim_{x\to 0}\frac{e^{5x}-e^{4x}}{x}$$
$$=\lim_{x\to 0}\frac{\left[\left(1+\frac{5x}{1!}+\frac{(5x)^2}{2!}+\cdots\right)-\left(1+\frac{4x}{1!}+\frac{(4x)^2}{2!}+\ldots\right)\right]}{x}$$
$$=\lim_{x\to 0}\frac{x\left[\left(\frac{5}{1!}+\frac{25x}{2!}+\cdots\right)-\left(\frac{4}{1!}+\frac{16}{2!}+\ldots\right)\right]}{x}$$

$= 1$

124 **(a)**

We have,

$$L = \lim_{x\to 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}$$

$$\Rightarrow L = \lim_{x\to 0} \frac{a - a\left(1 - \frac{x^2}{a^2}\right)^{1/2} - \frac{x^2}{4}}{x^4}$$

$\Rightarrow L$

$$= \lim_{x\to 0} \frac{a - a\left\{1 - \frac{1}{2}\cdot\frac{x^2}{a^2} - \frac{1}{8}\cdot\frac{x^4}{a^4} + \frac{1}{16}\cdot\frac{x^6}{a^6}\dots\right\} - \frac{x^2}{4}}{x^4}$$

$$\Rightarrow L = \lim_{x\to 0} \frac{\left(\frac{1}{2}\cdot\frac{x^2}{a} + \frac{1}{8}\cdot\frac{x^4}{a^3} - \frac{1}{16}\cdot\frac{x^6}{a^5}\dots\right) - \frac{x^2}{4}}{x^4}$$

$$\Rightarrow L = \lim_{x\to 0} \frac{x^2}{2}\left(\frac{1}{a} - \frac{1}{2}\right) + \frac{1}{8a^3} - \frac{1}{16}\cdot\frac{x^2}{a^5} + \cdots$$

$\Rightarrow \frac{1}{a} - \frac{1}{2} = 0$ and in that case $L = \frac{1}{8a^3}$ [$\because L$ is finite]

$\Rightarrow a = 2$ and $L = \frac{1}{64}$

125 **(b)**

$$\lim_{x\to 0} \log_e(\sin x)^x = \log_e\left[\lim_{x\to 0}(\sin x)^x\right]$$

$$= \log_e\left[\lim_{x\to 0}(1 + \sin x - 1)^{\frac{x(\sin x - 1)}{(\sin x - 1)}}\right]$$

$$= \log_e\left[e^{\lim_{x\to 0} x(\sin x - 1)}\right]$$

$= \log_e 1$

126 **(a)**

We have,

$$\lim_{x\to 0^+} x^m (\log x)^n = \lim_{x\to 0^+} \frac{(\log x)^n}{x^{-m}}$$

$\Rightarrow \lim_{x\to 0^+} x^m (\log x)^n = \lim_{x\to 0^+} \frac{n(\log x)^{n-1}\frac{1}{x}}{-m\, x^{-m-1}}$ [By L' Hospital's Rule]

$$\Rightarrow \lim_{x\to 0^+} x^m (\log x)^n = \lim_{x\to 0^+} \frac{n(\log x)^{n-1}}{-m\, x^{-m}}$$

$\Rightarrow \lim_{x\to 0^+} x^m (\log x)^n = \lim_{x\to 0^+} \frac{n(n-1)(\log x)^{n-2}\frac{1}{x}}{(-m)^2\, x^{-m-1}}$

[By L' Hospital's Rule]

$$\Rightarrow \lim_{x\to 0^+} x^m (\log x)^n = \lim_{x\to 0^+} \frac{n(n-1)(\log x)^{n-2}}{m^2 x^{-m}}$$

= ....

$= \lim_{x\to 0^+} \frac{n!}{(-m)^n x^{-m}}$ $\left[\begin{matrix}\text{Diff. numerator and} \\ \text{denominator } n \text{ times}\end{matrix}\right]$

$= 0$

127 **(a)**

$$\lim_{x\to\infty} \frac{\sqrt{1 + x^4} - (1 + x^2)}{x^2}$$

$$= \lim_{x\to\infty} \frac{\sqrt{1 + \frac{1}{x^4}} - \left(1 + \frac{1}{x^2}\right)}{1}$$

$$= \frac{1 - 1}{1} = 0$$

128 **(c)**

Let $y = \lim_{x\to 0}(\operatorname{cosec} x)^{1/\log x}$

$$\Rightarrow \quad \log y = \lim_{x\to 0} \frac{\log \operatorname{cosec} x}{\log x}$$

$= \lim_{x\to 0} \frac{-\cot x}{1/x}$ [by L'Hospital's rule]

$$= -\lim_{x\to 0} \frac{x}{\tan x} = -1$$

$\Rightarrow \quad \log y = -1$

$$\Rightarrow \quad y = \frac{1}{e}$$

129 **(c)**

$$\lim_{x\to 1} \frac{2x - f(x)}{x - 1}$$

$= \lim_{x\to 1} \frac{2 - f\prime(x)}{1}$ (by L' Hospital's rule)

$= 2 - f'(1)$

$= 2 - (1) = 1$

130 **(d)**

$$f(0) = \lim_{x\to 0} \frac{1 - \cos(1 - \cos x)}{x^4}$$

$$= \lim_{x\to 0} \frac{\sin(1 - \cos x).\sin x}{4x^3}$$

$$= \frac{1}{4}.\lim_{x\to 0} \frac{\sin(1 - \cos x)}{1 - \cos x}.\frac{1 - \cos x}{x^2}.\frac{\sin x}{x}$$

$$= \frac{1}{4}.1.\frac{1}{2}.1 = \frac{1}{8}$$

131 **(a)**

We have,

$$\lim_{x\to\infty} x^{3/2}\left(\sqrt{x^3 + 1} - \sqrt{x^3 - 1}\right)$$

$$= \lim_{x\to\infty} \frac{2x^{3/2}}{\sqrt{x^3 + 1} + \sqrt{x^3 - 1}}$$

$$= \lim_{x\to\infty} \frac{2x^{3/2}}{\sqrt{1 + \frac{1}{x^3}} + \sqrt{1 - \frac{1}{x^3}}} = \frac{2}{1 + 1} = 1$$

132 **(a)**

Given, $\lim_{x\to a} \frac{a^x - x^a}{x^x - a^a} = -1$

$\Rightarrow \quad \lim_{x\to a} \frac{a^x \log_e a - a x^{a-1}}{x^x(1 + \log_e x) - 0} = -1$

[by L' Hospital's rule]

$$\Rightarrow \quad \frac{a^a \log_e a - a^a}{a^a(1 + \log_e a)} = -1$$

$\Rightarrow \quad 2\log_e a = 0 \quad \Rightarrow \quad a = 1$

133 **(c)**

We have,

$$\lim_{x\to\infty} \left\{\frac{x^3 + 1}{x^2 + 1} - (ax + b)\right\} = 2$$

$$\Rightarrow \lim_{x\to\infty} \frac{x^3(1 - a) - bx^2 - ax + (1 - b)}{x^2 + 1} = 2$$

$\Rightarrow 1 - a = 0$ and $-b = 2 \Rightarrow a = 1, b = -2$

134 **(d)**

$\lim_{x\to 0}\frac{e^{x^2}-\cos x}{x^2}$ $\left(\frac{0}{0}\text{ from}\right)$

$=\lim_{x\to 0}\frac{2xe^{x^2}-\sin x}{2x}$ $\left(\frac{0}{0}\text{ from}\right)$

$$=\lim_{x\to 0}\frac{2e^{x^2}+4x^2e^{x^2}+\cos x}{2}$$

$$=\frac{2+0+1}{2}=\frac{3}{2}$$

135 **(d)**

$$\lim_{n\to\infty}\frac{1}{1-n^3}\sum_{r=1}^{n} r^2$$

$$=\lim_{n\to\infty}\frac{1}{n^3\left(\frac{1}{n^3}-1\right)}\frac{n(n+1)(2n+1)}{6}$$

$$=\lim_{n\to\infty}\frac{n^3\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{n^3\left(\frac{1}{n^3}-1\right)6}=-\frac{1}{3}$$

136 **(a)**

$$\lim_{x\to\frac{\pi}{6}}\frac{\sin 2x}{\sin x}=\lim_{x\to\frac{\pi}{6}}\frac{2\sin x\cos x}{\sin x}$$

$$=2\lim_{x\to\frac{\pi}{6}}\cos x=\sqrt{3}$$

137 **(a)**

We have,

$$\lim_{x\to 0}\frac{1-\cos(1-\cos x)}{x^4}=\lim_{x\to 0}\frac{1-\cos(2\sin^2 x/2)}{x^4}$$

$$=\lim_{x\to 0}\frac{2\sin^2(\sin^2 x/2)}{x^4}=2\lim_{x\to 0}\left\{\frac{\sin(\sin^2 x/2)}{x^2}\right\}^2$$

$$=2\lim_{x\to 0}\left\{\frac{\sin(\sin^2 x/2)}{\sin^2 x/2}\times\frac{\sin^2 x/2}{x^2/4}\times\frac{1}{4}\right\}^2=2\left(\frac{1}{2}\right)^2$$

$$=\frac{1}{8}$$

138 **(d)**

Let $f(x)=ax^2+bx+c$

We have,

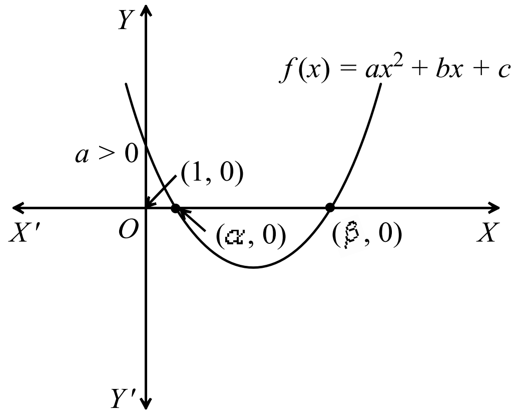
$$\lim_{x\to m}\frac{|ax^2+bx+c|}{ax^2+bx+c}=1$$

$\Rightarrow ax^2+bm+c>0$

$\Rightarrow f(m)>0$

$\Rightarrow$ Point $(m, f(m))$ must be on darkened part of the curve $y=f(x)$

Thus, options (a), (b) and (c) are true



139 **(b)**

$\lim_{x\to 1}\frac{x^m-1}{x^n-1}=\lim_{x\to 1}\frac{mx^{m-1}}{nx^{n-1}}$ [by L' Hospital's rule]

$$=\frac{m}{n}$$

140 **(d)**

$$\lim_{x\to\alpha}\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$$

$$=\lim_{x\to\alpha}\frac{2\sin^2\left(\frac{ax^2+bx+c}{2}\right)}{(x-\alpha)^2}$$

[Since α and β are the roots of $ax^2+bx+c=0$, so it can be written as $a(x-\alpha)(x-\beta)=0$]

$$=\lim_{x\to\alpha}\frac{2\sin^2\left(\frac{a(x-\alpha)(x-\beta)}{2}\right)}{(x-\alpha)^2}$$

$$=\lim_{x\to\alpha}\frac{2\sin^2\left(\frac{a}{2}(x-\alpha)(x-\beta)\right)\left(\frac{a}{2}\right)^2(x-\beta)^2}{\left[\left(\frac{a}{2}\right)(x-\alpha)(x-\beta)\right]^2}$$

$$=\lim_{x\to\alpha}2\left(\frac{a}{2}\right)^2(x-\beta)^2=\frac{a^2}{2}(\alpha-\beta)^2$$

141 **(c)**

$\lim_{x\to 0}\left[\frac{2^x-1}{\sqrt{1+x}-1}\right]=\lim_{x\to 0}\frac{2^x\log_e 2}{\frac{1}{2\sqrt{1+x}}}$ [by L' Hospital's rule]

$=2\log_e 2=\log_e 4$

142 **(b)**

$$\lim_{x\to 0}\left(\frac{x}{\sqrt{1+x}-\sqrt{1-x}}\right)$$

$$=\lim_{x\to 0}\left(\frac{x(\sqrt{1+x}+\sqrt{1-x})}{2x}\right)$$

$=1$

143 **(c)**

Given, $\lim_{x\to 0}\frac{\log(3+x)-\log(3-x)}{x}=k$

using L' Hospital's rule

$$\Rightarrow \lim_{x\to 0}\frac{\left(\frac{1}{3+x}+\frac{1}{3-x}\right)}{1}=k$$

$$\Rightarrow \frac{1}{3}+\frac{1}{3}=k \quad\Rightarrow\quad k=\frac{2}{3}$$

144 **(a)**

We have,

$$\lim_{x\to\infty}\left(1+\frac{1}{a+bx}\right)^{c+dx}=e^{\lim_{x\to\infty}\frac{c+dx}{a+bx}}=e^{d/b}$$

145 **(c)**

$$\lim_{x\to 0}\left(\frac{\int_0^{x^2}\sec^2 t\,dt}{x\sin x}\right)=\lim_{x\to 0}\frac{\sec^2 x^2.2x}{\sin x+x\cos x}$$

$$=\lim_{x\to 0}\frac{2x.\sec^2 x^2}{x\left(\frac{\sin x}{x}+\cos x\right)}$$

$=\frac{2\times 1}{1+1}=1$ $\left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$

146 **(c)**

It is fundamental co0ncept of indeterminate

ie, $\lim_{x\to\infty}\frac{\sin x}{x}=\frac{\sin\infty}{\infty}$

$=0\times$ finite term $=0$

147 **(a)**

Using expressions of $\cos x$ and $\log(1+x)$, the given limit is equal to

$$\lim_{x\to 0}\frac{x\left\{1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}\dots\right\}-\left\{x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}\dots\right\}}{x^2}$$

$$=\lim_{x\to 0}\left(\frac{1}{2}-\frac{x}{2!}-\frac{x}{3}+\cdots\right)=\frac{1}{2}$$

148 **(c)**

Let $A=\lim_{n\to\infty}\frac{1}{n^k}\{(n+1)^k(n+2)^k\dots(n+$ *nk…n+nk1/n*

Then,

$$A=\lim_{n\to\infty}\left\{\left(1+\frac{1}{n}\right)^k\left(1+\frac{2}{n}\right)^k\dots\left(1+\frac{1}{n}\right)^k\right\}^{1/n}$$

$$\Rightarrow \log A=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1^n}\log\left(1+\frac{r}{n}\right)^k$$

$$=k\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\log\left(1+\frac{r}{n}\right)$$

$$\Rightarrow \log A=2k(\log 2-1/2)=\log 4^k-k$$

$$\Rightarrow A=\left(\frac{4}{e}\right)^k$$

149 **(d)**

LHL$=\lim_{x\to 0}\frac{-\sin x}{x}$

$=-1$

RHL$=\lim_{x\to 0}\frac{\sin x}{x}$

$=1$

$\Rightarrow$ LHL $\neq$ RHL

$\Rightarrow$ $\lim_{x\to 0}\frac{\sin|x|}{x}$ Does not exist.

150 **(a)**

We have,

$$\lim_{x\to\infty}\left\{\frac{x^2\sin\left(\frac{1}{x}\right)-x}{1-|x|}\right\}$$

$$=\lim_{x\to\infty}\frac{x^2\sin(x^{-1})-x}{1-x}$$

$$=\lim_{x\to\infty}\frac{\left(\frac{\sin(x^{-1})}{x^{-1}}\right)-1}{x^{-1}-1}=\frac{1-1}{0-1}=0$$

151 **(b)**

We have,

$l_1=\lim_{x\to -2}(x+|x|)=-2+2=0$

$l_2=\lim_{x\to -2}(2x+|x|)=-4+2=-2$

and $l_3=\lim_{x\to\pi/2}\frac{\cos x}{x-\pi/2}=\lim_{x\to\pi/2}\frac{\sin(\pi/2)}{-(\pi/2-x)}=-1$

$\therefore l_2<l_3<l_1$

152 **(b)**

We have,

$$\lim_{x\to\infty}\left(\frac{x+2}{x+1}\right)^{x+3}=\lim_{x\to\infty}\left(1+\frac{1}{x+1}\right)^{x+3}=e^{\lim_{x\to\infty}\frac{x+3}{x+1}}$$

$$=e$$

153 **(b)**

We have,

$$\lim_{x\to 0}\frac{1}{x^{12}}\left\{1-\cos\frac{x^2}{2}-\cos\frac{x^4}{4}+\cos\frac{x^2}{2}\cos\frac{x^4}{4}\right\}$$

$$=\lim_{x\to 0}\frac{\left(1-\cos\frac{x^2}{2}\right)\left(1-\cos\frac{x^4}{4}\right)}{x^8}$$

$$=\frac{1}{64}\lim_{x\to 0}\frac{1-\cos\frac{x^2}{2}}{\left(\frac{x^2}{2}\right)^2}\times\frac{1-\cos\frac{x^4}{4}}{\left(\frac{x^4}{4}\right)^2}$$

$=\frac{1}{64}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{256}$ $\left[\because \lim_{\theta\to 0}\frac{1-\cos\theta}{\theta^2}=\frac{1}{2}\right]$

154 **(c)**

$$\lim_{x\to\infty}f(x)=\lim_{x\to\infty}\sqrt{\frac{x-\sin x}{x+\cos^2 x}}$$

$$=\lim_{x\to\infty}\sqrt{\frac{x-\frac{\sin x}{x}}{1+\frac{\cos^2 x}{x}}}$$

$$=\sqrt{\frac{1-0}{1+0}}$$

$\left[\because \frac{\sin x}{x}\to 0,\frac{\cos^2 x}{x}\to 0 \text{ as } x\to\infty\right]$

$=1$

155 **(c)**

We have,

$$\lim_{x\to 1}(2-x)^{\tan\frac{\pi x}{2}}=\lim_{x\to 1}\{1+(1-x)\}^{\tan\frac{\pi x}{2}}$$

$$=e^{\lim_{x\to 1}(1-x)\tan\frac{\pi x}{2}}$$

$$=e^{\lim_{h\to 0}-h\tan\left(\frac{\pi}{2}+\frac{\pi h}{2}\right)}=e^{\lim_{h\to 0}h\cot\frac{\pi h}{2}}=e^{\lim_{h\to 0}\frac{h}{\tan\left(\frac{\pi h}{2}\right)}}$$

$$=e^{2/\pi}$$

156 **(a)**

We know that, if $r<1$, then

$\lim_{n\to\infty} r^n = 0$

And if $r > 1$, then

$\lim_{n\to\infty} r^n = \infty$

Here, $\lim_{n\to\infty} r^n = 0$

$\therefore \quad r < 1 \quad ie, \quad r = \frac{4}{5}$

157 **(a)**

$$\lim_{x\to 0}\frac{5^x-5^{-x}}{2x} = \lim_{x\to 0}\frac{5^x\log 5 - 5^{-x}\log 5}{2}$$

[by L' Hospital's rule]

$= \frac{\log 5 + \log 5}{2}$

$= \log 5$

158 **(d)**

$\lim_{x\to 0}\frac{\sqrt{1-\cos 2x}}{\sqrt{2}x} = \lim_{x\to 0}\frac{\sqrt{2\sin^2 x}}{\sqrt{2}x}$

$= \lim_{x\to 0}\frac{\sin|x|}{x} = f(x)$ [say]

Now, $f(0+0) = \lim_{h\to 0}\frac{|\sin(0+h)|}{0+h} = 1$

$f(0-0) = \lim_{h\to 0}\frac{|\sin(0-h)|}{-h} = -1$

$\because \quad f(0+0) \neq f(0-0)$

$\therefore$ The limit of function does not exist.

159 **(c)**

We have,

$$\lim_{x\to\infty} a^x \sin\left(\frac{b}{a^x}\right) = \lim_{x\to\infty}\frac{\sin\left(\frac{b}{a^x}\right)}{\left(\frac{b}{a^x}\right)}.b = 1\cdot b = b$$

160 **(d)**

$\lim_{x\to\infty}\left(\frac{x^3}{3x^2-4} - \frac{x^2}{3x+2}\right)$

$= \lim_{x\to\infty}\frac{x^3(3x+2) - x^2(3x^2-4)}{(3x^2-4)(3x+2)}$

$= \lim_{x\to\infty}\frac{2x^3+4x^2}{9x^3+6x^2-12x-8}$

$= \lim_{x\to\infty}\frac{2+4/x}{9+6/x-12/x^2-8/x^3} = \frac{2}{9}$

161 **(a)**

We have,

$\lim_{x\to 0}\frac{e^{x^2}-\cos x}{x^2} = \lim_{x\to 0}\left\{\frac{(e^{x^2}-1)}{x^2} + \frac{(1-\cos x)}{x^2}\right\}$

$= \lim_{x\to 0}\frac{e^{x^2}-1}{x^2} + \lim_{x\to 0}\frac{1-\cos x}{x^2} = 1 + \frac{1}{2} = \frac{3}{2}$

162 **(d)**

Let $x = \frac{1}{y}$. Then,

$\lim_{x\to\infty}\left\{\frac{a_1^{1/x}+a_2^{1/x}+\cdots+a_n^{1/x}}{n}\right\}^{nx}$

$= \lim_{y\to 0}\left\{\frac{a_1^y+a_2^y+\cdots+a_n^y}{n}\right\}^{n/y}$

$= \lim_{y\to 0}\left\{\frac{1+a_1^y+a_2^y+\cdots+a_n^y-n}{n}\right\}^{n/y}$

$= e^{\lim_{y\to 0}\left\{\frac{a_1^y-1}{y}+\frac{a_2^y-1}{y}+\cdots+a_n^y-1\right\}^{n/y}}$

$= e^{\log a_1+\log a_2+\cdots+\log a_n} = e^{\log(a_1 a_2 \ldots\ldots a_n)}$

$= a_1 a_2 a_3 \ldots a_n$

163 **(b)**

$\lim_{x\to 0}\frac{\sin^2 x + \cos x - 1}{x^2}$

$= \lim_{x\to 0}\frac{\cos x + \cos^2 x}{x^2}$

$= \lim_{x\to 0}\cos x\,.\frac{1-\cos x}{x^2}$

$= 1.\frac{1}{2} = \frac{1}{2}$

164 **(b)**

We have,

$\lim_{x\to 0}\frac{\sin 4x}{1-\sqrt{1-x}} = \lim_{x\to 0}\frac{\sin 4x}{x}\left(1+\sqrt{1-x}\right)$

$= \lim_{x\to 0} 4\left(\frac{\sin 4x}{4x}\right)\left(1+\sqrt{1-x}\right) = 8$

165 **(a)**

$\lim_{x\to 1}\frac{x^8-2x+1}{x^4-2x+1} = \lim_{x\to 1}\frac{8x^7-2}{4x^3-2}$

$= \frac{8-2}{4-2} = 3$ [using L' Hospital's rule]

166 **(a)**

$\lim_{x\to\infty}\sqrt{a^2x^2+ax+1} - \sqrt{a^2x^2+1}$

$= \lim_{x\to\infty}\frac{a^2x^2+ax+1-a^2x^2-1}{\sqrt{a^2x^2+ax+1}+\sqrt{a^2x^2+1}}$

$= \lim_{x\to\infty}\frac{ax}{x\left[\sqrt{a^2+\frac{a}{x}+\frac{1}{x^2}}+\sqrt{a^2+\frac{1}{x^2}}\right]}$

$= \frac{a}{\sqrt{a^2}+\sqrt{a^2}} = \frac{1}{2}$

167 **(d)**

LHL$= \lim_{x\to 0^-}(-1)^{[x]}$

$= \lim_{h\to 0}(-1)^{[0-h]} = (-1)^{-1} = -1$

RHL$= \lim_{x\to 0^+}(-1)^{[x]} = \lim_{h\to 0}(-1)^{[0+h]}$

$= (-1)^0 = 1$

$\because$ LHL$\neq$RHL

$\therefore$ Limit does not exist.

168 **(a)**

$\lim_{x\to 0}\frac{(1-e^x)\sin x}{(x+x^2)x}$

$= \lim_{x\to 0}\frac{\left(-x-\frac{x^2}{2!}-\frac{x^3}{3!}-\ldots\right)}{x(1+x)} \times \lim_{x\to 0}\frac{\sin x}{x}$

$= -1 \times 1 = -1$

169 **(c)**

$$\lim_{x\to\infty}\left(\frac{x+3}{x+1}\right)^{x+2} = \lim_{x\to\infty}\left[\left(1+\frac{2}{x+1}\right)^{\left(\frac{x+2}{2}\right)}\right]^{\frac{2}{x+2}\times(x+2)}$$

$$= \lim_{x\to\infty}\left(1+\frac{2}{x+1}\right)^{\left(\frac{x+2}{2}\right)\left[\frac{2(x+2)}{x+1}\right]}$$

$$= e^{\lim_{x\to\infty}\frac{(2+4/x)}{(1+1/x)}}$$

$$= e^2$$

170 **(b)**

Since, α is a repeated root.

$\therefore \quad ax^2+bx+c = a(x-\alpha)^2$

Now, $\quad \lim_{x\to\alpha}\frac{\sin(ax^2+bx+c)}{(x-\alpha)^2}$

$$= \lim_{x\to\alpha}\frac{\sin a(x-\alpha)^2}{a(x-\alpha)^2}\times a$$

$$= \lim_{x\to\alpha} a(1) = a$$

171 **(b)**

We have,

$\lim_{x\to 0}\frac{e^x-1-x}{x^2} = \lim_{x\to 0}\frac{e^x-1}{2x} = \frac{1}{2}$ [Using L' Hospital's Rule]

172 **(c)**

$$\lim_{x\to\infty}\frac{(2x+1)^{40}(4x-1)^5}{(2x+3)^{45}}$$

$$= \lim_{x\to\infty}\frac{x^{40}\left(2+\frac{1}{x}\right)^{40}x^5\left(4-\frac{1}{x}\right)^5}{x^{45}\left(2+\frac{3}{x}\right)^{45}}$$

$$= \frac{(2+0)^{40}(4-0)^5}{(2+0)^{45}} = \frac{2^{50}}{2^{45}} = 32$$

173 **(c)**

$$\therefore \quad \lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}([x-3]+|x-4|)$$

$$= \lim_{h\to 0}([3-h-3]+|3-h-4|)$$

$$= \lim_{h\to 0}([-h]+1+h)$$

$$= -1+1+0 = 0$$

174 **(d)**

We have,

$$\lim_{x\to 2^-}\{x+(x-[x]^2\} = \lim_{x\to 2^-} x + \lim_{x+2^-}(x-[x])^2$$

175 **(d)**

We have,

$$\lim_{x\to\infty}\left(\frac{x^2+2x+3}{2x^2+x+5}\right)^{\frac{3x-2}{3x+2}} = \lim_{x\to\infty}\left(\frac{1+\frac{2}{x}+\frac{3}{x^2}}{2+\frac{1}{x}+\frac{5}{x^2}}\right)^{\frac{1-2/3x}{1+2/3x}}$$

$$= \left(\frac{1}{2}\right)^1 = \frac{1}{2}$$

176 **(c)**

$$\lim_{x\to 0}\left[\frac{e^{\sin x}-e^x}{\sin x - x}\right] = \lim_{x\to 0}\left[\frac{e^x(e^{\sin x - x}-1)}{\sin x - x}\right]$$

$$= \lim_{x\to 0} e^x \lim_{x\to 0}\left[\frac{e^{\sin x - x}-1}{\sin x - x}\right]$$

$$= e^0 \times 1 = 1$$

177 **(b)**

We have,

$$\lim_{x\to 1}(\log_2 2x)^{\log_x 5}$$

$$= \lim_{x\to 1}(\log_2 2 + \log_2 x)^{\log_x 5}$$

$$= \lim_{x\to 1}(1+\log_2 x)^{1/\log_5 x} = e^{\lim_{x\to 1}\log_2 x \cdot \frac{1}{\log_5 x}}$$

$$= e^{\lim_{x\to 1}\log_2 5} = e^{\log_2 5}$$

178 **(a)**

$f(x)$ is a positive increasing function

$\Rightarrow \quad 0 < f(x) < f(2x) < f(3x)$

$$\Rightarrow \quad 0 < 1 < \frac{f(2x)}{f(x)} < \frac{f(3x)}{f(x)}$$

$$\Rightarrow \quad \lim_{x\to\infty} 1 < \lim_{x\to\infty}\frac{f(2x)}{f(x)} < \lim_{x\to\infty}\frac{f(3x)}{f(x)}$$

By Sandwich theorem

$$\lim_{x\to\infty}\frac{f(2x)}{f(x)} = 1$$

179 **(c)**

LHL$= \lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} x^2 - 3$

$= 9 - 3 = 6$

And RHL$= \lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} 2x+5$

$= 2\times 3 + 5 = 11$

$\therefore$ 6 and 11 are the roots of equation

$\therefore$ Required equation is

$x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$

$\Rightarrow \quad x^2 - (11+6)x + (11\times 6) = 0$

$\Rightarrow \quad x^2 - 17x + 66 = 0$

180 **(c)**

We have,

$$f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right), & x\neq 0 \\ 0, & x = 0 \end{cases}$$

$$\therefore \lim_{x\to 0^-} f(x) = \lim_{x\to 0} f(0-h) = \lim_{x\to 0} -h\sin\left(-\frac{1}{h}\right)$$

$$\Rightarrow \lim_{x\to 0^-} f(x) = \lim_{x\to 0} h\sin\left(\frac{1}{h}\right) = 0$$

Similarly, we have $\lim_{x\to 0^+} f(x) = 0$

Hence, $\lim_{x\to 0} f(x) = 0$

181 **(d)**

We have, $f(x)\frac{|x+\pi|}{\sin x}$

$$\therefore \lim_{x\to -\pi^-} f(x) = \lim_{h\to 0} f(-\pi - h)$$

$$= \lim_{h\to 0}\frac{|-\pi-h+\pi|}{\sin(-\pi-h)}$$

$$\Rightarrow \lim_{x\to -\pi^-} f(x) = -\lim_{h\to 0}\frac{h}{\sin(\pi+h)} = \lim_{h\to 0}\frac{h}{\sin h} = 1$$

and,

$$\lim_{x\to -\pi^+} f(x) = \lim_{h\to 0} f(-\pi+h) = \lim_{h\to 0}\frac{|-\pi+h+\pi|}{\sin(-\pi+h)}$$

$\Rightarrow \lim_{x\to-\pi^+} f(x) = -\lim_{h\to 0}\frac{h}{\sin h} = -1$

Hence, $\lim_{x\to-\pi} f(x)$ does not exist

182 **(d)**

We have,

$$\lim_{x\to 3}\frac{f(x)-f(3)}{x-3} = f'(3)$$

Now, $f'(x) = \frac{x}{(18-x^2)^{3/2}} \Rightarrow f'(3) = \frac{3}{(9)^{3/2}} = \frac{1}{9}$

183 **(c)**

We have,

$$\lim_{x\to\infty}\frac{5^{x+1}-7^{x+1}}{5^x-7^x} = \lim_{x\to\infty}\frac{5.\left(\frac{5}{7}\right)^x-7}{\left(\frac{5}{7}\right)^x-1} = \frac{5\times 0-7}{0-1}$$

$$= 7$$

184 **(d)**

We have,

$A_i = \frac{x-a_i}{|x-a_i|}, i = 1,2,\dots,n$ and $a_1 < a_2 < \cdots < a_{n-1} < a_n$

Let x be in the left neighbourhood of a_m. Then,

$x - a_i < 0$ for $i = m, m+1, \dots, n$

and,

$x - a_i > 0$ for $i = 1, 2, \dots, m-1$

$$A_i = \begin{cases} = \frac{(x-a_i)}{-(x-a_i)} = -1 \text{ for } i = m, m+1, \dots, n \\ = \frac{x-a_i}{x-a_i} = 1 \text{ for } i = 1,2,\dots,m-1 \end{cases}$$

Similarly, if x is in the right neighbourhood of a_m. Then,

$x - a_i < 0$ for $i = m+1, \dots, n$ and $x - a_i > 0$ for $i = 1, 2, \dots., m$

$$\therefore A_i = \begin{cases} A_i = \frac{x-a_i}{-(x-a_i)} = -1 \text{ for } i = m+1,\dots,n \\ A_i = \frac{x-a_i}{x-a_i} = 1 \text{ for } i = 1,2,\dots,m \end{cases}$$

Thus, we have

$$\lim_{x\to a_m^-}(A_1 A_2 \dots A_n) = (-1)^{n-m+1}$$

and,

$$\lim_{x\to a_m^+}(A_1 A_2 \dots A_n) = (-1)^{n-m}$$

Hence, $\lim_{x\to a_m}(A_1 A_2 \dots A_n)$ does not exist

185 **(a)**

We have,

$$\lim_{x\to-1}\frac{(1+x)(1-x^2)(1+x^3)(1-x^4)\dots(1+x^{4n-}}{[(1+x)(1-x^2)(1+x^3)(1-x^4)\dots(1-}$$

$$= \lim_{x\to-1}\frac{(1+x^{2n+1})(1-x^{2n+2})\dots(1+x^{4n-1})(1-}{(1+x)(1-x^2)(1+x^3)(1-x^4)\dots(1-x}$$

$$= \lim_{x\to-1}\left\{\frac{1+x^{2n+1}}{1+x}\times\frac{1-x^{2n+2}}{1-x^2}\times\frac{1+x^{2n+3}}{1+x^3}\times\dots \times\frac{1-x^{4n}}{1-x^{2n}}\right\}$$

$$= \lim_{x\to-1}\left\{\frac{x^{2n+1}+1}{x+1}\times\frac{x^{2n+2}-1}{x^2-1}\times\frac{x^{2n+3}+1}{x^3+1}\times\dots \times\frac{x^{4n}-1}{x^{2n}-1}\right\}$$

$$= \lim_{x\to-1}\left\{\frac{x^{2n+1}-(-1)^{2n+1}}{x-(-1)}\times\frac{x^{2n+2}-(-1)^{2n+2}}{x^2-(-1)^2}\right\}$$

$$\times\left\{\frac{x^{2n+3}-(-1)^{2n+3}}{x^3-(-1)^3}\times\dots\times\frac{x^{4n}-(-1)^{4n}}{x^{2n}-(-1)^{2n}}\right\}$$

$$= \frac{2n+1}{1}\times\frac{2n+2}{2}\times\frac{2n+3}{3}\times\dots\times\frac{4n}{2n}$$

$$= \frac{4n!}{\{(2n)!\}^2} = {}^{4n}C_{2n}$$

186 **(a)**

We have,

$\lim_{x\to\infty}\frac{\log x}{x^n} = \lim_{x\to\infty}\frac{1}{n\,x^n} = 0$ [By L' Hospital's Rule]

187 **(b)**

$$\lim_{x\to 9}\frac{\sqrt{f(x)}-3}{\sqrt{x}-3} = \lim_{x\to 9}\frac{f(x)-9}{x-9}\times\frac{\sqrt{x}+3}{\sqrt{f(x)}+3}$$

$$= f'(9)\times\left(\frac{\sqrt{9}+3}{\sqrt{f(9)}+3}\right)$$

$$= f'(9)\times\frac{3+3}{3+3} = 3$$

188 **(a)**

We have,

$$\lim_{n\to\infty} x^{2n} = \begin{cases} 0, \text{if } |x| < 1 \\ 1, \text{if } |x| = 1 \\ \infty, \text{if } |x| > 1 \end{cases}$$

$$\therefore f(x) = \lim_{n\to\infty}\frac{1-\frac{1}{x^{2n}}}{1+\frac{1}{x^{2n}}} = \lim_{n\to\infty}\begin{cases} -1, |x| < 1 \\ 0, |x| = 1 \\ 1, |x| > 1 \end{cases}$$

189 **(b)**

$$\lim_{x\to 1}\frac{e^{-x}-e^{-1}}{x-1} = \frac{1}{e}\lim_{x\to 1}\frac{e^{-(x-1)}-1}{x-1} = -\frac{1}{e}$$

190 **(c)**

$$\lim_{n\to\infty}\frac{x^n}{x^n+1} = \lim_{n\to\infty}\frac{x^n}{(1+1/x^n)x^n}$$

$$= \lim_{n\to\infty}\frac{1}{1+\frac{1}{x^n}} = 1$$

191 **(b)**

We have,

$x^2 + 4x + 5 = (x+2)^2 + 1 \geq 1$ for all x

$\therefore a = 1$

$$b = \lim_{\theta\to 0}\frac{1-\cos^2\theta}{\theta^2} = 1$$

$$\therefore \sum_{r=0}^{n} {}^nC_r\, a^r\, b^{n-r} = (a+b)^n = 2^n$$

192 **(c)**

We have,

$$\lim_{n\to\infty}\left\{1+\left(\frac{x}{y}\right)^n\right\}^{1/n}=\lim_{n\to\infty} y\,(1+0)^{1/n}=y\times 1^0$$
$$=y$$

193 **(b)**

$$\lim_{x\to 1}(\log ex)^{1/\log x}=\lim_{x\to 1}[\log e+\log x]^{1/\log x}$$
$$=\lim_{x\to 1}[1+\log x]^{1/\log x}$$
$$=e^{\lim_{x\to 1}\frac{\log x}{\log x}}=e$$

194 **(a)**

We have,

$$\lim_{x\to\infty}\left\{ax-\frac{x^2+1}{x+1}\right\}=b\Rightarrow \lim_{x\to\infty}\frac{(a-1)x^2+ax-1}{x+1}$$
$$=b$$

Since b is a finite number. Therefore, degree of numerator must be less than or equal to that of the denominator

$\therefore a-1=0\Rightarrow a=1$

Now,

$$\lim_{x\to\infty}\frac{(a-1)x^2+ax-1}{x+1}=b\Rightarrow\lim_{x\to\infty}\frac{ax-1}{x+1}=b$$
$$\Rightarrow a=b$$

Hence, $a=b=1$

195 **(b)**

Given limit

$$=\lim_{x\to 0}\frac{\int_0^x\frac{t\log(1+t)}{t^4+4}dt}{x^3}$$

Using L' Hospital's rule,

$$=\lim_{x\to 0}\frac{\frac{x\log(1+x)}{x^4+4}}{3x^2}$$
$$=\lim_{x\to 0}\frac{\log(1+x)}{3x}.\frac{1}{x^4+4}$$
$$=\frac{1}{3}.\frac{1}{4}=\frac{1}{12}$$

196 **(d)**

We know, $\lim_{n\to\infty}x^{2n}=\begin{cases}\infty, & \text{if } x^2>1\\ 1, & \text{if } x^2=1\\ 0, & \text{if } x^2<1\end{cases}$

Given, $f(x)=\lim_{n\to\infty}\frac{\log(2+x)-x^{2n}\sin x}{1+x^{2n}}$

For $x^2=1$, $f(x)=\lim_{n\to\infty}\frac{\log 3-\sin 1}{2}$

$$=\frac{1}{2}(\log 3-\sin 1)$$

For $x^2<1$,

$f(x)=\log(2+x)$

Foe $x^2>1$,

$$f(x)=\lim_{n\to\infty}\frac{\left(\frac{1}{x^{2n}}\right)\log(2+x)-\sin x}{\left(1+\frac{1}{x^{2n}}\right)}$$
$$=-\sin x$$
$$\therefore\quad f(x)=\begin{cases}\log(2+x), & x^2<1\\ \frac{1}{2}(\log 3-\sin 1), & x=1\\ -\sin x, & x^2>1\end{cases}$$
$$\therefore\lim_{x\to 1^-}f(x)=\lim_{h\to 0}\log(2+1-h)$$
$=\log 3$
$$\lim_{x\to 1^+}f(x)=\lim_{h\to 0}[-\sin(1+h)]$$
$$=-\sin 1$$

It is clear that both limits exist and $\lim_{x\to 1^-}f(x)\neq\lim_{x\to 1^+}f(x)$

197 **(d)**

We have,

$$\lim_{x\to\infty}\frac{\int_0^{2x}x\,e^{x^2}dx}{e^{4x^2}}=\lim_{x\to\infty}\frac{\int_0^{2x}e^{x^2}d\,(x^2)}{2e^{4x^2}}$$
$$=\lim_{x\to\infty}\frac{\left[e^{x^2}\right]^{2x}}{2e^{4x^2}}$$
$$\Rightarrow\lim_{x\to\infty}\frac{\int_0^{2x}x\,e^{x^2}dx}{e^{4x^2}}=\lim_{x\to\infty}\frac{e^{4x^2}-1}{2\,e^{4x^2}}$$
$$=\lim_{x\to\infty}\left(\frac{1}{2}-\frac{1}{e^{4x^2}}\right)=\frac{1}{2}$$

198 **(c)**

We have,

$$\lim_{x\to\pi/2}\tan^2 x\left(\sqrt{2\sin^2 x+3\sin x+4}-\sqrt{\sin^2 x+6\sin x+2}\right)$$
$$=\lim_{x\to\pi/2}\tan^2 x\frac{(2\sin^2 x+3\sin x+4-\sin^2 x-6\sin x-2)}{\sqrt{2\sin^2 x+3\sin x+4}+\sqrt{\sin^2 x+6\sin x+2}}$$

$$= \lim_{x\to\pi/2} \frac{\tan^2 x(\sin^2 x - 3\sin x + 2)}{\sqrt{2\sin^2 x + 3\sin x + 4} + \sqrt{\sin^2 x + 6\sin x + 2}}$$

$$= \lim_{x\to\pi/2} \frac{\sin^2 x(\sin x - 1)(\sin x - 2)}{(1-\sin^2 x)(\sqrt{2\sin^2 x + 3\sin x + 4} + \sqrt{\sin^2 x + 6\sin x + 2})}$$

$$= \lim_{x\to\pi/2} \frac{-\sin^2 x(\sin x - 2)}{(1+\sin x)(\sqrt{2\sin^2 x + 3\sin x + 4} + \sqrt{\sin^2 x + 6\sin x + 2})}$$

$$= \frac{1}{2(\sqrt{9}+\sqrt{9})} = \frac{1}{12}$$

199 **(b)**

$$\lim_{x\to\infty}\left(1 + \frac{1}{x+1}\right)^{x+3}$$

$$= e^{\lim_{x\to\infty}\frac{x+3}{x+1}} = e$$

200 **(a)**

$$f(x) = \cot^{-1}\left(\frac{3x - x^3}{1-3x^2}\right) = \frac{\pi}{2} - 3\tan^{-1} x$$

and $g(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\tan^{-1} x$

$$\therefore \quad \lim_{x\to a}\frac{f(x)-f(a)}{g(x)-g(a)}$$

$$= \lim_{x\to a}\frac{\frac{\pi}{2} - 3\tan^{-1}x - \frac{\pi}{2} + 3\tan^{-1}a}{2\tan^{-1}x - 2\tan^{-1}a}$$

$$= -\frac{3}{2}\lim_{x\to a}\frac{\tan^{-1}x - \tan^{-1}a}{\tan^{-1}x - \tan^{-1}a} = -\frac{3}{2}$$

201 **(b)**

We have,

$$\lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h)$$

$$\Rightarrow \lim_{x\to 2^-} f(x) = \lim_{h\to 0}\frac{k((2-h)^2 - 4)}{2-(2-h)}$$

$$= k\lim_{h\to 0}\frac{h(h-4)}{h} = -4k$$

$$\lim_{x\to 2^+} f(x) = \lim_{h\to 0} f(2+h) = \lim_{h\to 0}\frac{k((2+h)^2-4)}{2-(2+h)}$$

$$\Rightarrow \lim_{x\to 2^+} f(x) = k\lim_{h\to 0}\frac{h(h+4)}{-h} = -4k$$

$\therefore \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$ for all $k \in R$

202 **(c)**

We have,

$$\lim_{x\to 0}\frac{1-\cos^3 x}{x\sin x\cos x}$$

$$= \lim_{x\to 0}\frac{(1-\cos x)(1+\cos x+\cos^2 x)}{x\sin x\cos x}$$

$$= \lim_{x\to 0}\frac{2\sin^2_{x/2}}{x.2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)} \times \frac{(1-\cos x + \cos^2 x)}{\cos x}$$

$$= \lim_{x\to 0}\frac{\sin\left(\frac{x}{2}\right)}{2(x/2)} \times \frac{1+\cos x+\cos^2 x}{\cos\left(\frac{x}{2}\right)\cos x} = \frac{1}{2}\times 3 = \frac{3}{2}$$

203 **(b)**

We have,

$$\lim_{x\to\infty}\sqrt{\frac{x+\sin x}{x-\cos x}} = \lim_{x\to\infty}\sqrt{\frac{1+\frac{\sin x}{x}}{1-\frac{\cos x}{x}}} = \sqrt{\frac{1+0}{1-0}} = 1$$

204 **(a)**

Since, $f'(a)$ exists.

$$\therefore \quad f'(a) = \lim_{h\to 0}\frac{f(x)-f(a)}{x-a}$$

Now, $\quad \lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$

$$= \lim_{x\to a}\frac{xf(a)-af(a)+af(a)-af(x)}{x-a}$$

$$= \lim_{x\to a}\frac{f(a)-(x-a)}{(x-a)} - a\lim_{x\to a}\frac{f(x)-f(a)}{(x-a)}$$

$$= f(a) - a\,f'(a)$$

205 **(d)**

We have,

$$\lim_{h\to 0}\frac{\sin(a+3h) - 3\sin(a+2h) + 3\sin(a+h) - s}{h^3}$$

$$= \lim_{h\to 0}\frac{\{\sin(a+3h)-\sin a\} - 3\{\sin(a+2h)-\sin(}{h^3}$$

$$= \lim_{h\to 0}\frac{2\sin\frac{3h}{2}\cos\left(a+\frac{3h}{2}\right) - 6\cos\left(a+\frac{3h}{2}\right)\sin\frac{h}{2}}{h^3}$$

$$= \lim_{h\to 0}\frac{2\cos\left(a+\frac{3h}{2}\right)\left(\sin\frac{3h}{2} - 3\sin\frac{h}{2}\right)}{h^3}$$

$$= -8\lim_{h\to 0}\cos\left(a+\frac{3h}{2}\right)\frac{\sin^3\frac{h}{2}}{h^3}$$

$$= -\lim_{h\to 0}\cos\left(a+\frac{3h}{2}\right)\left\{\frac{\sin\frac{h}{2}}{h/2}\right\}^3 = -\cos a$$

206 **(b)**

We have,

$$\lim_{x\to 0}\left\{\frac{1^x + 2^x + \cdots + n^x}{n}\right\}^{1/x}$$

$$= \lim_{x\to 0}\left\{1 + \frac{1^x - 1}{n} + \frac{2^x-1}{n} + \cdots + \frac{n^x-1}{n}\right\}^{1/x}$$

$$= e^{\lim_{x\to 0}\frac{1}{n}\left\{\frac{1^x-1}{n} + \frac{2^x-1}{n} + \cdots + \frac{n^x-1}{n}\right\}}$$

$$= e^{\frac{1}{n}[\log 1 + \log 2 + \cdots + \log n]}$$

$$= e^{\frac{1}{n}(\log n!)} = e^{\log(n!)^{\frac{1}{n}}} = (n!)^{\frac{1}{n}}$$

207 **(b)**

We have,

$$\lim_{x\to 0}\frac{\int_0^{x^2}\cos t^2 dt}{x\sin x}$$

$= \lim_{x\to\infty}\frac{2x\cos x^4}{x\cos x+\sin x}$ [Using L' Hospital's Rule]

$$= \lim_{x\to\infty}\frac{2\cos x^4 - 8x^4\sin x^4}{2\cos x - x\sin x} = \frac{2-0}{2-0} = 1$$

208 **(c)**

We have,

$$f(x)+g(x)+h(x) = \frac{x^2-4x+17-4x-2}{x^2+x-12}$$

$$= \frac{(x-3)(x-5)}{(x-3)(x+4)}$$

$$\therefore \lim_{x\to 3}[f(x)+g(x)+h(x)]$$

$$= \lim_{x\to 3}\frac{(x-3)(x-5)}{(x-3)(x+4)} = -\frac{2}{7}$$

209 **(a)**

$$\lim_{x\to 0}\left[\frac{8\sin x + x\cos x}{3\tan x + x^2}\right] = \lim_{x\to 0}\left[\frac{\frac{8\sin x}{x}+\cos x}{\frac{3\tan x}{x}+x}\right]$$

$$= \frac{\lim_{x\to 0}\left[\frac{8\sin x}{x}+\cos x\right]}{\lim_{x\to 0}\left[\frac{3\tan x}{x}+x\right]}$$

$$= \frac{9}{3} = 3$$

210 **(d)**

We have,

$$\lim_{x\to 0}\frac{\log_e(3+x)-\log_e(3-x)}{x} = k$$

$$\Rightarrow \lim_{x\to 0}\frac{\log_e\left(1+\frac{x}{3}\right)-\log_e\left(1-\frac{x}{3}\right)}{x} = k$$

$$\Rightarrow \lim_{x\to 0}\frac{1}{3}\times\frac{\log_e\left(1+\frac{x}{3}\right)}{\frac{x}{3}} + \frac{1}{3} + \lim_{x\to 0}\frac{\log\left(1-\frac{x}{3}\right)}{\left(-\frac{x}{3}\right)} = k$$

$$\Rightarrow \frac{1}{3}+\frac{1}{3} = k \Rightarrow k = \frac{2}{3}$$

211 **(b)**

We have,

$$\lim_{x\to\infty}\left(\frac{x^2+6}{x^2-6}\right)^x = \lim_{x\to\infty}\left(1+\frac{12}{x^2-6}\right)^x = e^{\lim_{x\to\infty}\frac{12x}{x^2-6}}$$

$$= e^0 = 1$$

212 **(a)**

$$\lim_{x\to\infty}\left[1+\frac{4x+1}{x^2+x+2}\right]^x = \lim_{x\to\infty} e^{\lim_{x\to\infty}\frac{4x^2+x}{x^2+x+2}} = e^4$$

213 **(a)**

We have,

$\lim_{x\to 1}\frac{G(x)-G(1)}{x-1} = G'(1)$ [By def. of derivative]

Now,

$$G(x) = -\sqrt{25-x^2} \Rightarrow G'(x) = \frac{x}{\sqrt{25-x^2}} \Rightarrow G'(1)$$

$$= \frac{1}{\sqrt{24}}$$

214 **(a)**

$$\lim_{x\to 1}\cos^{-1}\left(\frac{1-\sqrt{x}}{1-x}\right)$$

$$= \lim_{x\to 1}\cos^{-1}\left(\frac{1-\sqrt{x}}{(1-\sqrt{x})(1+\sqrt{x})}\right)$$

$$= \lim_{x\to 1}\cos^{-1}\left(\frac{1}{1+\sqrt{x}}\right)$$

$$= \cos^{-1}\left(\frac{1}{2}\right)$$

$$= \frac{\pi}{3}$$

215 **(a)**

We have,

$$\lim_{x\to 1}\frac{\sin(e^{x-1}-1)}{\log x} = \lim_{h\to 0}\frac{\sin(e^h-1)}{\log(1+h)}$$

$$= \lim_{h\to 0}\frac{\sin(e^h-1)}{(e^h-1)}\times\frac{(e^{-h}-1)}{h}\times\frac{h}{\log(1+h)} = 1$$

216 **(b)**

$$\lim_{x\to\frac{\pi}{6}}\frac{3\sin x-\sqrt{3}\cos x}{6x-\pi} = \lim_{x\to\frac{\pi}{6}}\frac{3\cos x+\sqrt{3}\cos x}{6}$$

$$= \frac{3\cos\frac{\pi}{6}+\sqrt{3}\sin\frac{\pi}{6}}{6}$$

$$= \frac{3.\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}}{6}$$

$$= \frac{1}{\sqrt{3}}$$

217 **(c)**

We have,

$$\lim_{x\to 2}\int_2^{f(x)}\frac{4t^3}{x-2}dt = \lim_{x\to 2}\frac{[t^4]_2^{f(x)}}{x-2}$$

$$= \lim_{x\to 2}\frac{\{f(x)\}^4-16}{x-2}$$

$= \lim_{x\to 2}\frac{4\{f(x)\}^3 f\prime(x)}{1}$ [Applying L' Hospital's Rule]

$$= 4\big(f(2)\big)^3 f'(2) = 32f'(2)$$

218 **(a)**

$$\lim_{x\to\frac{\pi}{4}}\frac{\int_2^{\sec^2 x}f(t)dt}{x^2-\frac{\pi^2}{16}} \qquad \left[\frac{0}{0}\text{ from}\right]$$

$$= \lim_{x\to\frac{\pi}{4}}\frac{f(\sec^2 x)2\sec x\sec x\tan x}{2x}$$

$$\therefore \quad L = \frac{2f(2)}{\pi/4} = \frac{8f(2)}{\pi}$$

219 **(c)**

We have,

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty}\sqrt{\frac{x-\sin x}{x+\cos^2 x}} = \lim_{x\to\infty}\frac{\sqrt{1-\frac{\sin x}{x}}}{1+\frac{\cos^2 x}{x}}$$

$$\Rightarrow \lim_{x\to\infty} f(x) = \sqrt{\frac{1-0}{1+0}}$$

$$= 1 \left[\because \frac{\sin x}{x} \to 0, \frac{\cos^2 x}{x} \to 0 \text{ as } x \to \infty\right]$$

220 **(a)**

We have,

$$\lim_{x\to 0}\frac{2f(x)-3f(2x)+f(4x)}{x^2} \quad \left[\frac{0}{0}\text{ form}\right]$$

$= \lim_{x\to 0}\frac{2f'(x)-6f'(2x)+4f'(4x)}{2}$ [Using L' Hospital's Rule]

$= \lim_{x\to 0}\frac{2f''(x)12f''(2x)+16f''(4x)}{2}$ [Using L' Hospital's Rule]

$$= \frac{2f''(0)-12f''(0)+16f''(0)}{2}$$

$= 3f''(0) = 3\times 2 = 6$

221 **(b)**

$$\lim_{x\to\frac{\pi}{2}}\frac{\cot x-\cos x}{(\pi-2x)^3}$$

$$= \lim_{h\to 0}\frac{\cot\left(\frac{\pi}{2}+h\right)-\cos\left(\frac{\pi}{2}+h\right)}{(-2h)^3}$$

$$= \lim_{h\to 0}\frac{-\tan h+\sin h}{(-2h)^3}$$

$$= \lim_{h\to 0}\frac{\sin h(1-\cos h)}{\cos h\times 8h^3}$$

$$= \frac{1}{8}\lim_{h\to 0}\frac{\sin h}{h}\times\frac{2\sin^2 h/2}{4(h/2)^2}\times\frac{1}{\cos h} = \frac{1}{16}$$

222 **(c)**

We have,

$$\lim_{x\to\infty}\left(\frac{x+3}{x-1}\right)^{x+3} = \lim_{x\to\infty}\left(1+\frac{4}{x-1}\right)^{x+3}$$

$$= e^{\lim_{x\to\infty}\frac{4(x+3)}{(x+1)}} = e^4$$

223 **(d)**

$$\lim_{x\to 7}\frac{2-\sqrt{x-3}}{x^2-49} = \lim_{x\to 7}\frac{-\frac{1}{2\sqrt{x-3}}}{2x}$$

$$= -\frac{1}{4.7.2} = -\frac{1}{56}$$

224 **(b)**

$$\lim_{x\to\infty}\frac{\sqrt{x^2+1}-\sqrt[3]{x^3+1}}{\sqrt[4]{x^4+1}-\sqrt[5]{x^4+1}}$$

$$= \lim_{x\to\infty}\frac{\sqrt{1+\frac{1}{x^2}}-\sqrt[3]{1+\frac{1}{x^3}}}{\sqrt[4]{1+\frac{1}{x^4}}-\sqrt[5]{\frac{1}{x}+\frac{1}{x^5}}} = \frac{1-1}{1-0} = 0$$

225 **(a)**

$$\lim_{x\to 2}\frac{\int_6^{f(x)} 4\,t^3\,dt}{x-2}\lim_{x\to 2}\frac{4\{f(x)\}^3}{1}.f'(x)$$

$= 4\{f(2)\}^3.f'(2)$

$$= 4\times(6)^3.\frac{1}{48}$$

$= 18$

226 **(b)**

Since, $g(x)g(y) = g(x)+g(y)+g(xy)-2$ ...(i)

Now, at $x=0$, $y=2$, we get

$g(0)g(2) = g(0)+g(2)+g(0)-2$

$\Rightarrow \quad 5g(0) = 5+2g(0)-2 \qquad [\because \; g(2)=5]$

$\Rightarrow \quad g(0) = 1$

$g(x)$ is given in a polynomial and by the relation given $g(x)$ cannot be linear.

Let $g(x) = x^2+k$

$\Rightarrow \quad g(x) = x^2+1 \qquad [\because \; g(0)=1]$

$\therefore \quad g(x)$ is satisfied in Eq. (i)

$\therefore \quad \lim_{x\to 3} g(x) = g(3) = 3^2+1 = 10$

227 **(b)**

We have,

$$\lim_{x\to\pi/4}\frac{1-\cot^3 x}{2-\cot x-\cot^3 x}$$

$= \lim_{y\to 1}\frac{1-y^3}{2-y-y^3}$, where $y = \cot x$

$$= \lim_{y\to 1}\frac{y^3-1}{y^3+y-2}$$

$$= \lim_{y\to 1}\frac{(y-1)(y^2+y+1)}{(y-1)(y^2+y+2)} = \lim_{y\to 1}\frac{y^2+y+1}{y^2+y+2} = \frac{3}{4}$$

228 **(c)**

We have,

$\lim_{x\to 1^-}\{1-x+[x-1]+[1-x]\}$

$= \lim_{h\to 0}\{1-(1-h)+[1-h-1]+[1-(1-h)]\}$

$= \lim_{h\to 0}\{h+[-h]+[h]\} = \lim_{h\to 0}(h-1+0) = -1$

and,

$\lim_{x\to 1^+}\{1-x+[x-1]+[1-x]\}$

$= \lim_{h\to 0}\{1-(1+h)+[1+h-1]+[1-(1+h)]\}$

$= \lim_{h\to 0}\{-h+[h]+[-h]\} = \lim_{h\to 0}(-h+0-1) = -1$

Hence, $\lim_{x\to 1} f(x) = -1$

229 **(a)**

Given, $\lim_{x\to 1}\frac{ax^2+bx+c}{(x-1)^2}=2$

This limit will exist, if

$ax^2+bx+c=2(x-1)^2$

$\Rightarrow \quad ax^2+bx+c=2x^2-4x+2$

$\Rightarrow \quad a=2, \quad b=-4, \quad c=2$

230 **(a)**

$$\lim_{x\to 1}\frac{2x^2+x-3}{3x^3-3x^2+2x-2}=\lim_{x\to 1}\frac{(2x+3)(x-1)}{(3x^2+2)(x-1)}$$

$$=\lim_{x\to 1}\frac{2x+3}{3x^2+2}=1$$

231 **(a)**

$$\left(\lim_{x\to\infty}\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right.$$

$$\left.\times\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}\right)$$

$$=\lim_{x\to\infty}\left(\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}\right)$$

$$=\lim_{y\to 0}\frac{(\sqrt{1+\sqrt{y}})/\sqrt{y}}{\sqrt{\frac{1}{y}+\frac{\sqrt{1+\sqrt{y}}}{\sqrt{y}}}+\sqrt{\frac{1}{y}}} \qquad \left[\text{put } x=\frac{1}{y}\right]$$

$$=\lim_{y\to 0}\frac{\sqrt{1+\sqrt{y}}}{\sqrt{1+\sqrt{y(1+\sqrt{y})}+1}}=\frac{1}{2}$$

232 **(d)**

We have,

$$\lim_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4}=\lim_{x\to\infty}\left(1+\frac{5}{x+1}\right)^{x+4}$$

$$=e^{\lim_{x\to\infty}\frac{5(x+4)}{x+1}}=e^5$$

233 **(c)**

We have,

$$\lim_{x\to a}\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$$

$$=2\lim_{x\to\alpha}\frac{\sin^2\left\{\frac{(ax^2+bx+c)}{2}\right\}}{(x-\alpha)^2}$$

$$=2\lim_{x\to\alpha}\frac{\sin^2\left\{\frac{a(x-\alpha)(x-\beta)}{2}\right\}}{(x-\alpha)^2} \quad \left[\begin{array}{l}\because \alpha,\beta \text{ are roots of}\\ ax^2+bx+c=0\\ \therefore ax^2+bx+c\\ =a(x-\alpha)(x-\beta)\end{array}\right]$$

$$=2\lim_{x\to\alpha}\left[\frac{\sin\left\{\frac{a(x-\alpha)(x-\beta)}{2}\right\}}{\frac{a(x-\alpha)(x-\beta)}{2}}\right]^2\times\frac{a^2}{4}(x-\beta)^2$$

$$=2(1)^2\times\frac{a^2}{4}(\alpha-\beta)^2=\frac{a^2}{2}(\alpha-\beta)^2$$

234 **(a)**

$$\lim_{x\to 0}(1-ax)^{1/x}=\lim_{x\to 0}\left[(1-ax)^{-\frac{1}{ax}}\right]^{-a}=e^{-a}$$

235 **(c)**

$$\lim_{n\to\infty}\frac{1-(10)^n}{1+(10)^{n+1}}$$

$$=\lim_{n\to\infty}\frac{10^n}{10^{n+1}}\left(\frac{\frac{1}{10^n}-1}{\frac{1}{10^{n+1}}+1}\right)$$

$\Rightarrow \quad -\frac{\alpha}{10}=\frac{1}{10}\left(\frac{0-1}{0+1}\right)=-\frac{1}{10}$

[given]

$\Rightarrow \quad \alpha=1$

236 **(d)**

We have,

$$\lim_{n\to\infty}\frac{\sqrt{n^2+1}+\sqrt{n}}{\sqrt[4]{n^3+n}-\sqrt[4]{n}}$$

$$=\lim_{n\to\infty}\frac{n\left\{\sqrt{1+\frac{1}{n^2}}+\frac{1}{\sqrt{n}}\right\}}{n^{3/4}\left\{\sqrt[4]{1+\frac{1}{n^2}}-\frac{1}{\sqrt{n}}\right\}}$$

$$=\lim_{n\to\infty}n^{1/4}\frac{\left\{\sqrt{1+\frac{1}{n^2}}+\frac{1}{\sqrt{n}}\right\}}{\left\{\sqrt[4]{1+\frac{1}{n^2}}-\frac{1}{\sqrt{n}}\right\}}=\infty\times 2=\infty$$

237 **(d)**

LHL= $\lim_{x\to 2^-}\frac{5}{\sqrt{2}-\sqrt{x}}$

$$=\lim_{h\to 0}\frac{5}{(\sqrt{2}-\sqrt{2-h})}\times\frac{(\sqrt{2}+\sqrt{2-h})}{(\sqrt{2}+\sqrt{2-h})}$$

$$=\lim_{h\to 0}\frac{5(\sqrt{2}+\sqrt{2-h})}{2-2+h}=\infty$$

RHL= $\lim_{x\to 2^+}\frac{5}{\sqrt{2}-\sqrt{x}}$

$$=\lim_{h\to 0}\frac{5}{(\sqrt{2}-\sqrt{2+h})}\times\frac{(\sqrt{2}+\sqrt{2+h})}{(\sqrt{2}+\sqrt{2+h})}$$

$= \lim_{h \to 0} \frac{5(\sqrt{2} + \sqrt{2+h})}{2-2-h} = -\infty$

$\therefore$ LHL≠RHL

Hence, limit does not exist.

238 **(a)**

$\lim_{n \to \infty} \frac{2^{-n}(n^2+5n+6)}{(n+4)(n+5)}$

$= \lim_{n \to \infty} \frac{n^2(1+\frac{5}{n}+\frac{6}{n^2})}{2^n . n^2 \left(1+\frac{4}{n}\right)(1+\frac{5}{n})}$

$= 0$

239 **(b)**

$\lim_{x \to 0} f(x) = \lim_{x \to 0} x . \sin\frac{1}{x}$

$= 0 \times \text{finite term} = 0$

240 **(b)**

$\lim_{x \to a} \frac{\log(x-a)}{\log(e^x - e^a)} = \lim_{x \to a} \frac{\frac{1}{x-a}}{\frac{e^x}{e^x - e^a}}$

[by L' Hospital's rule]

$= \lim_{x \to a} \frac{e^x - e^a}{e^x(x-a)}$

$= \lim_{x \to a} \frac{e^x}{e^x(x-a)+e^x}$

[by L' Hospital's rule]

$= \frac{e^a}{0+e^a} = 1$

241 **(c)**

We have,

$\lim_{x \to \infty} \left(1 - \frac{2}{x+1}\right)^x = e^{\lim_{x \to \infty} \frac{-2x}{x+1}} = e^{-2}$

242 **(d)**

$\lim_{x \to 0} \left[\frac{3^x + 3^{-x} - 2}{x^2}\right]$

$= \lim_{x \to 0} \frac{\begin{bmatrix} 1 + x\log 3 + \frac{x^2}{2!}(\log 3)^2 + \cdots + \\ 1 - x\log 3 + \frac{x^2}{2!}(\log 3)^2 - \ldots - 2 \end{bmatrix}}{x^2} = (\log 3)^2$

243 **(d)**

We have,

$\lim_{x \to 0} \left(\frac{e^x + e^{-x} - 2}{x^2}\right)^{\frac{1}{x^2}}$

$= \lim_{x \to 0} \left\{\frac{2\left(\frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots\right)}{x^2}\right\}^{\frac{1}{x^2}}$

$= \lim_{x \to 0} \left(1 + 2\frac{x^2}{4!} + 2\frac{x^4}{6!} + \cdots\right)^{\frac{1}{x^2}}$

$= e^{\lim_{x \to 0}\left(\frac{x^2}{4!} + \frac{2x^4}{6!} + \cdots\right) \times \frac{1}{x^2}} = e^{1/12}$

244 **(b)**

Let $y = \lim_{x \to \infty} \left(\frac{\pi}{2} - \tan^{-1} x\right)^{1/x}$

$\Rightarrow \quad \log y = \lim_{x \to \infty} \frac{1}{x} \log\left(\frac{\pi}{2} - \tan^{-1} x\right)$

$\left[\frac{\infty}{\infty} \text{ form}\right]$

$= \lim_{x \to \infty} \frac{\left(-\frac{1}{1+x^2}\right)}{\left(\frac{\pi}{2} - \tan^{-1} x\right)}$ [using L'Hospital's rule]

$= \lim_{x \to \infty} \frac{\frac{2x}{(1+x^2)^2}}{-\left(\frac{1}{1+x^2}\right)} = \lim_{x \to \infty} \frac{-2x}{1+x^2}$

[using L'Hospital's rule]

$\Rightarrow \quad \log y = 0 \quad \Rightarrow \quad y = 1$

245 **(d)**

We have,

$\lim_{x \to 1} \left\{\frac{x^3 + 2x^2 + x + 1}{x^2 + 2x + 3}\right\}^{\frac{1 - \cos(x-1)}{(x-1)^2}}$

$= \lim_{x \to 1} \left\{\frac{x^3 + 2x^2 + x + 1}{x^2 + 2x + 3}\right\}^{\frac{2\sin^2(x-1)/2}{(x-1)^2}}$

$= \left(\frac{5}{6}\right)^{\frac{2}{4}\left\{\frac{\sin\left(\frac{x-1}{2}\right)}{\frac{x-1}{2}}\right\}^2} = \sqrt{\frac{5}{6}}$

246 **(a)**

Here, $\lim_{x \to -3} x^2 + 2x - 3 = 0$

$\therefore \lim_{x \to -3} 3x^2 + ax + a - 7$ must be zero, in order to limit exist.

$\Rightarrow \quad 3(-3)^2 + a(-3) + a - 7 = 0$

$\Rightarrow \quad 27 - 2a - 7 = 0$

$\Rightarrow \quad 2a = 20$

$\Rightarrow \quad a = 10$

247 **(b)**

$\lim_{x \to \infty} \left(1 - \frac{4}{x-1}\right)^{3x-1} =$

$\lim_{x \to \infty} \left[\left(1 - \frac{4}{x-1}\right)^{\frac{-(x-1)}{4}}\right]^{-4\left(\frac{3x-1}{x-1}\right)}$

$= e^{-4 \lim_{x \to \infty} (3 - 1/x)(1 - 1/x)} = e^{-12}$

248 **(b)**

We have,

$\lim_{n \to \infty} \left\{\frac{1}{1-n^2} + \frac{2}{1-n^2} + \frac{3}{1-n^2} + \cdots + \frac{n}{1-n^2}\right\}$

$= \lim_{n \to \infty} \frac{1+2+3+\cdots+n}{1-n^2} = \lim_{n \to \infty} \frac{n}{2(1-n)} = -\frac{1}{2}$

250 **(c)**

$\lim_{n \to \infty} \frac{3.2^{n+1} - 4.5^{n+1}}{5.2^n + 7.5^n}$

$= \lim_{n \to \infty} \frac{5^n \left(6.\left(\frac{2}{5}\right)^n - 20\right)}{5^n \left(5.\left(\frac{2}{5}\right)^n + 7\right)} = -\frac{20}{7}$

251 **(a)**

We have,

$$\lim_{x\to 2}\left\{\left(\frac{x^3-4x}{x^3-8}\right)^{-1}-\left(\frac{x+\sqrt{2x}}{x-2}-\frac{\sqrt{2}}{\sqrt{x}-\sqrt{2}}\right)^{-1}\right\}$$

$$=\lim_{x\to 2}\left\{\frac{x^2+2x+4}{x(x+2)}-\left(\frac{\sqrt{x}(x-2)-\sqrt{2}(x-2)}{(x-2)(\sqrt{x}-\sqrt{2})}\right)^{-1}\right\}$$

$$=\lim_{x\to 2}\left\{\frac{x^2+2x+4}{x(x+2)}-\left(\frac{(x-2)(\sqrt{x}-\sqrt{2})}{(x-2)(\sqrt{x}-\sqrt{2})}\right)^{-1}\right\}$$

$$=\lim_{x\to 2}\left\{\frac{x^2+2x+4}{x(x+2)}\right\}=\frac{12}{8}-1=\frac{1}{2}$$

252 **(b)**

We have,

$$\lim_{x\to\infty}\{\sqrt{x^2-x+1}-(ax+b)\}=0$$

$$\Rightarrow \lim_{x\to\infty}\frac{x^2-x+1-(ax+b)^2}{\sqrt{x^2-x+1}+(ax+b)}=0$$

$$\Rightarrow \lim_{x\to\infty}\frac{x^2(1-a^2)-x(1+2ab)+1-b^2}{\sqrt{x^2}-x+1+ax+b}=0$$

$\Rightarrow 1-a^2=0$ and $1+2ab=0$

$\Rightarrow a=\pm 1$ and $b=\mp 1/2$

For $a=-1$ and $b=1/2$, we observe that

$$\lim_{x\to\infty}\left(\sqrt{x^2-x+1}-ax-b\right)$$

$$=\lim_{x\to\infty}\left(\sqrt{x^2-x+1}+x-\frac{1}{2}\right)$$

$$\to\infty$$

Hence, $a=1$ and $b=-\frac{1}{2}$

253 **(d)**

$$\lim_{x\to\infty}\frac{(2x-3)(3x-4)}{(4x-5)(5x-6)}=\lim_{x\to\infty}\frac{6x^2-17x+12}{20x^2-49x+30}$$

$=\lim_{x\to\infty}\frac{12x-17}{40x-49}$ [using L' Hospital's rule]

$=\lim_{x\to\infty}\frac{12}{40}=\frac{3}{10}$ [using L' Hospital's rule]

254 **(b)**

We have,

$$\lim_{h\to 0}\frac{\log(1+2h)-2\log(1+h)}{h^2}$$

$$=\lim_{h\to 0}\frac{\log\left\{\frac{1+2h}{(1+h)^2}\right\}}{h^2}=-\lim_{h\to 0}\frac{\log\left\{\frac{(1+h)^2}{1+2h}\right\}}{h^2}$$

$$=-\lim_{h\to 0}\frac{\log\left\{1+\frac{h^2}{1+2h}\right\}}{\left\{\frac{h^2}{1+2h}\right\}(1+2h)}=-1=-1$$

255 **(b)**

Let $A=\lim_{x\to\infty}x^{1/x}$. Then,

$\log A=\lim_{x\to\infty}\frac{1}{x}\log x=0$ $\left[\because \lim_{x\to\infty}\frac{\log x}{x^m}=0,\ \text{for all } m>0\right]$

$\Rightarrow A=e^0=1$

256 **(a)**

We have,

$$\lim_{x\to a}\frac{\log(x-a)}{\log(e^x-e^a)}$$

$$=\lim_{x\to a}\frac{e^x-e^a}{(x-a)e^x}=\lim_{x\to a}\frac{e^x}{(x-a)e^x+e^x}=\frac{e^a}{e^a}=1$$

257 **(a)**

We have,

$$a_{n+1}=\frac{4+3a_n}{3+2a_n}$$

$$\Rightarrow \lim_{n\to\infty}a_{n+1}=\lim_{n\to\infty}\frac{4+3a_n}{3+2a_n}$$

$\Rightarrow a=\frac{4+3a}{3+2a}$, where $a=\lim_{x\to\infty}a_n$

$\Rightarrow 2a^2=4\Rightarrow a=\pm\sqrt{2}$

But, $a\neq-\sqrt{2}$ because each $a_n>0$

Hence $a=\sqrt{2}$

258 **(d)**

We have,

$$\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x}=1$$

$$\Rightarrow \lim_{x\to 0}\left\{(1+a\cos x)-b\frac{\sin x}{x}\right\}=1$$

$\Rightarrow 1+a-b=1\Rightarrow a-b=0$

Clearly, none of the options satisfy this relation

259 **(c)**

$$\lim_{\alpha\to 0}\frac{\begin{bmatrix}\operatorname{cosec}^{-1}(\sec\alpha)+\cot^{-1}(\tan\alpha)+\\ \cot^{-1}\cos(\sin^{-1}\alpha)\end{bmatrix}}{\alpha}$$

$$=\lim_{\alpha\to 0}\frac{\begin{bmatrix}\operatorname{cosec}^{-1}\left(\operatorname{cosec}\left(\frac{\pi}{2}-\alpha\right)\right)+\cot^{-1}\left(\cot\left(\frac{\pi}{2}-\alpha\right)\right)\\ +\cot^{-1}\cos[\cos^{-1}\sqrt{(1-\alpha^2)}]\end{bmatrix}}{\alpha}$$

$$=\lim_{\alpha\to 0}\frac{\pi-2\alpha+\cot^{-1}\sqrt{1-\alpha^2}}{\alpha}$$

$$=\lim_{\alpha\to 0}\frac{-2-\frac{1}{1+1-\alpha^2}\left(\frac{1}{2\sqrt{1-\alpha^2}}(-2\alpha)\right)}{1}$$

$=-2$ [by L'Hospital's rule]

260 **(b)**

$$\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x}$$

$$=\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x\left(\frac{a/x+b/x^2}{a/x+b/x^2}\right)}$$

$$=e^{\lim_{x\to\infty}2x(a/x+b/x^2)}$$

$\left[\because \lim_{x\to\infty}(1+x)^{\frac{1}{x}}=e\right]$

$\Rightarrow\ e^2=e^{\lim_{x\to\infty}2(a+b/x)}=e^{2a}$ [given]

$\Rightarrow\ a=1$

and $b\in R$

261 **(d)**

We have,

$$\lim_{x\to 0}\frac{\sin x - x + \frac{x^3}{6}}{x^5}$$

$$= \lim_{x\to 0}\frac{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots - x + \frac{x^3}{6}}{x^5}$$

$$= \lim_{x\to 0}\frac{1}{5!} - \frac{x^2}{7!} + \cdots = \frac{1}{120}$$

262 **(c)**

$$\lim_{x\to 2}\frac{xf(2) - 2f(x)}{x-2}$$

$$= \lim_{x\to 2}\frac{xf(2) - 2f(2) + 2f(2) - 2f(x)}{x-2}$$

$$= f(2) - 2f'(2)$$

$$= 4 - 2\times 4 = -4$$

263 **(a)**

$$\lim_{x\to 1}\frac{\int_4^{f(x)} 2t\,dt}{x-1} = \lim_{x\to 1}\frac{2f(x).f'(x)}{1}$$

$$= 2f(1).f'(1) = 16$$

264 **(b)**

$$\lim_{n\to\infty}\frac{a^n + b^n}{a^n - b^n} = \lim_{n\to\infty}\frac{\left(\frac{a}{b}\right)^n + 1}{\left(\frac{a}{b}\right)^n - 1}$$

$$= -1$$

$$\left[\text{since}, 0 < \frac{a}{b} < 1 \text{ implies} \left(\frac{a}{b}\right)^n \to 0 \text{ as } n\to\infty\right]$$

265 **(c)**

We have,

$$\lim_{x\to a}\left(\frac{\sin x}{\sin a}\right)^{\frac{1}{x-a}}$$

$$= \lim_{x\to a}\left\{1 + \frac{\sin x - \sin a}{\sin a}\right\}^{\frac{1}{x-a}}$$

$$= e^{\lim_{x\to a}\frac{\sin x - \sin a}{x-a}\times\frac{1}{\sin a}} = e^{\frac{\cos a}{\sin a}} = e^{\cot a}$$

266 **(b)**

We have,

$x_1 = 3, x_{n+1} = \sqrt{2 + x_n}$

$\therefore x_2 = \sqrt{2 + x_1} = \sqrt{2+3} = \sqrt{5}, x_3 = \sqrt{2 + x_2}$

$= \sqrt{2 + \sqrt{5}}$

$\therefore x_1 > x_2 > x_3$

It can be easily shown by mathematical induction that the sequence $x_1, x_2, \ldots x_n, \ldots$ is a monotonically decreasing sequence bounded below by 2. So, it is convergent. Let $\lim x_n = x$. Then,

$x_{n+1} = \sqrt{2 + x_n}$

$\Rightarrow \lim x_{n+1} = \sqrt{2 + \lim\ x_n}$

$\Rightarrow x = \sqrt{2 + x}$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x-2)(x+1) = 0$

$\Rightarrow x = 2 \qquad [\because x_n > 0 \text{ for all } n \in N\ \therefore x > 0]$

267 **(d)**

$$\lim_{h\to 0}\frac{f(2h + 2 + h^2) - f(2)}{f(h - h^2 + 1) - f(1)}$$

$$= \lim_{h\to 0}\frac{\{f'(2h + 2 + h^2)\}.(2 + 2h) - 0}{\{f'(h - h^2 + 1)\}.(1 - 2h) - 0}$$

[using L' Hospital's rule]

$$= \frac{f'(2).2}{f'(1).1} = \frac{6.2}{4.1} = 3$$

268 **(a)**

We have,

$$\lim_{x\to 0}\left(\frac{1 + 5x^2}{1 + 3x^2}\right)^{\frac{1}{x^2}} = e^{\lim_{x\to 0}\frac{2x^2}{1+3x^2}\cdot\frac{1}{x^2}} = e^{\lim_{x\to 0}\frac{2}{1+3x^2}} = e^2$$

269 **(c)**

We have,

$$\lim_{x\to\infty} x\left\{\tan^{-1}\frac{x+1}{x+2} - \tan^{-1}\frac{x}{x+2}\right\}$$

$$= \lim_{x\to\infty} x\tan^{-1}\left\{\frac{\frac{x+1}{x+2} - \frac{x}{x+2}}{1 + \frac{x+1}{x+2}.\frac{x}{x+2}}\right\}$$

$$= \lim_{x\to\infty} x\tan^{-1}\left(\frac{x+2}{2x^2 + 5x + 4}\right)$$

$$= \lim_{x\to\infty}\left\{\frac{\tan^{-1}\left(\frac{x+2}{2x^2+5x+4}\right)}{\frac{x+2}{2x^2+5x+4}}\right\}\times\frac{x(x+2)}{2x^2 + 5x + 4}$$

$$= 1\times\frac{1}{2} = \frac{1}{2}$$

270 **(c)**

We have,

$$\lim_{x\to\infty}\left(\frac{3x^2 + 2x + 1}{x^2 + x + 2}\right) = 3, \text{and} \lim_{x\to\infty}\frac{6x+1}{3x+2} = 2$$

$$\therefore \lim_{x\to\infty}\left(\frac{3x^2 + 2x + 1}{x^2 + x + 2}\right)^{\frac{6x+1}{3x+2}} = 3^2 = 9$$

271 **(b)**

We know that

$$\cos A\cos 2A\cos 4A \ldots \cos 2^{n-1}A = \frac{\sin 2^n A}{2^n \sin A}$$

Taking $A = \frac{x}{2^n}$, we get

$$\cos\left(\frac{x}{2^n}\right)\cos\left(\frac{x}{2^{n-1}}\right)\ldots\cos\left(\frac{x}{4}\right)\cos\left(\frac{x}{2}\right)$$

$$= \frac{\sin x}{2^n \sin\left(\frac{x}{2^n}\right)}$$

$$\therefore \lim_{n\to\infty}\cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{4}\right)\ldots\cos\left(\frac{x}{2^{n-1}}\right)\cos\left(\frac{x}{2^n}\right)$$

$$= \lim_{n\to\infty}\frac{\sin x}{2^n\sin\left(\frac{x}{2^n}\right)} = \lim_{n\to\infty}\frac{\sin x}{x}\frac{(x/2^n)}{\sin(x/2^n)} = \frac{\sin x}{x}$$

272 **(a)**

We have,

$$\lim_{x\to0}\left(\frac{\sin x}{x}\right)^{\frac{\sin x}{x-\sin x}}$$

$$=\lim_{x\to0}\left(1+\frac{\sin x-x}{x}\right)^{\frac{\sin x}{x-\sin x}}$$

$$=e^{\lim_{x\to0}\frac{\sin x-x}{x}\times\frac{\sin x}{x-\sin x}}=e^{-\lim_{x\to0}\frac{\sin x}{x}}=e^{-1}$$

273 **(b)**

$$\lim_{x\to0}\frac{x^2\sin\left(\frac{1}{x}\right)}{\sin x}=\lim_{x\to0}\frac{x}{\sin x}\times x\sin\left(\frac{1}{x}\right)$$

$= 1\times 0\times$ (An oscillating number) $= 0$

274 **(a)**

$\lim_{x\to1}\frac{\sqrt{f(x)}-1}{\sqrt{x}-1}=\lim_{x\to1}\frac{\sqrt{x}-f'(x)}{\sqrt{f(x)}}$ [by L' Hospital's rule]

$$=\frac{1.f(1)}{f(1)}=\frac{2}{1}=2$$

275 **(d)**

We have,

$$\lim_{x\to0^-}\frac{3x+|x|}{7x-|x|}=\lim_{x\to0}\frac{3x-x}{7x+5x}=\frac{1}{6}$$

and, $\lim_{x\to0^+}\frac{3x+|x|}{7x-5|x|}=\lim_{x\to0}\frac{3x+x}{7x-5x}=2$

So, $\lim_{x\to0}\frac{3x+|x|}{7x-5|x|}$ does not exist

276 **(b)**

$$\lim_{x\to-3-0}=\frac{\sqrt{x+3}}{x+1}$$

But $\sqrt{x+3}$ is not defined on left hand limit of -3.

Hence, function is not defined.

277 **(b)**

We have,

$$\lim_{x\to0}\frac{\cos(\sin x)-\cos x}{x^4}$$

$$=\lim_{x\to0}\frac{2\sin\left(\frac{x+\sin x}{2}\right)\sin\left(\frac{x-\sin x}{2}\right)}{x^4}$$

$$=2\lim_{x\to0}\frac{\sin\left(\frac{x+\sin x}{2}\right)}{\left(\frac{x+\sin x}{2}\right)}$$

$$\times\frac{\sin\left(\frac{x-\sin x}{2}\right)}{\left(\frac{x-\sin x}{2}\right)}\left(\frac{x+\sin x}{2x}\right)\left(\frac{x-\sin x}{2\,x^3}\right)$$

$$=2\lim_{x\to0}\frac{\sin\left(\frac{x+\sin x}{2}\right)}{\frac{x+\sin x}{2}}\times\frac{\sin\left(\frac{x-\sin x}{2}\right)}{\frac{x-\sin x}{2}}\left(\frac{1}{2}\right.$$

$$\left.+\frac{\sin x}{2x}\right)\left(\frac{x-\sin x}{2\,x^3}\right)$$

$$=2\times1\times1\times\left(\frac{1}{2}+\frac{1}{2}\right)\lim_{x\to0}\frac{x-\sin x}{2x^3}$$

$$=2\lim_{x\to0}\frac{x-\sin x}{2x^3}=\lim_{x\to0}\frac{x-\sin x}{x^3}$$

$$=\lim_{x\to0}\frac{x-\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}\right)}{x^3}=\lim_{x\to0}\left(\frac{1}{3}+\frac{x^2}{5!}\dots\right)=\frac{1}{3!}$$

$$=\frac{1}{6}$$

278 **(d)**

$$\lim_{x\to0}\frac{e^{1/x}}{e^{1/x+1}}=\lim_{x\to0}\frac{e^{1/x}}{e^{1/x}.e}=\frac{1}{e}$$

279 **(a)**

We have,

$$\lim_{x\to0}\frac{\sqrt[3]{1+\sin x}-\sqrt[3]{1-\sin x}}{x}$$

$$=\lim_{x\to0}\frac{(1+\sin x)^{1/3}-(1-\sin x)^{1/3}}{x}$$

$$=\lim_{x\to0}\frac{2\left\{\frac{1}{3}\sin x+\frac{\frac{1}{3}\left(-\frac{2}{3}\right)}{2!}(\sin x)^2+\cdots\right\}}{x}$$

$$=\frac{2}{3}\lim_{x\to0}\frac{\sin x}{x}+0=\frac{2}{3}$$

280 **(c)**

We have,

$$\lim_{n\to\infty}\left(\cos\frac{x}{n}\right)^n$$

$$=\lim_{n\to\infty}\left\{1+\left(\cos\frac{x}{n}-1\right)\right\}^n=\lim_{n\to\infty}\left\{1-2\sin^2\frac{x}{2n}\right\}^n$$

$$=e^{\lim_{n\to\infty}-2n\sin^2\frac{x}{2n}}$$

$$=e^{-\lim_{n\to\infty}\left(\frac{\sin\frac{x}{2n}}{x/2n}\right)^2\times\frac{1}{2n}}=e^{-1\times0}=e^0=1$$

281 **(a)**

We have,

$\lim_{x\to9}\frac{\sqrt{f(x)}-3}{\sqrt{x}-3}=\lim_{x\to9}\frac{\frac{f'(x)}{2\sqrt{f(x)}}}{\frac{1}{2\sqrt{x}}}$ [Using L' Hospital Rule]

$$=\lim_{x\to9}\frac{\sqrt{x}f'(x)}{\sqrt{f(x)}}=\frac{3\times4}{3}=4$$

282 **(b)**

We have,

$$\lim_{x\to0}\frac{\sin x^n}{(\sin x)^m}$$

$$=\lim_{x\to0}\left(\frac{\sin x^n}{x^n}\right)\left(\frac{x^n}{x^m}\right)\left(\frac{x}{\sin x}\right)^m$$

$$=\lim_{x\to0}x^{n-m}=0\qquad[\because m<n]$$