

7. INTEGRALS

Single Correct Answer Type

1. The value of $\int_0^{\pi/2} \frac{1}{9 \cos x + 12 \sin x} dx$, is
 a) $\frac{1}{15} \log_{10} 6$ b) $\frac{1}{15} \log_e 6$ c) $\log\left(\frac{6}{15}\right)$ d) $\log\left(\frac{15}{6}\right)$
2. The value of $\int_{-2}^0 [x^3 + 3x^2 + 3x + 3 + (x + 1) \cos(x + 1)] dx$ is
 a) 0 b) 3 c) 4 d) 1
3. $\int \frac{e^{2x} - 2e^x}{e^{2x} + 1} dx$ is equal to
 a) $\log(e^{2x} + 1) - \tan^{-1}(e^x) + C$
 b) $\frac{1}{2} \log(e^{2x} + 1) - \tan^{-1}(e^x) + C$
 c) $\frac{1}{2} \log(e^{2x} + 1) - 2 \tan^{-1}(e^x) + C$
 d) None of these
4. The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$ is
 a) 1 b) 0 c) -1 d) None of these
5. $\int \sqrt{1 + \sin\left(\frac{x}{4}\right)} dx$ is equal to
 a) $8\left(\sin\frac{x}{8} + \cos\frac{x}{8}\right) + c$ b) $8\left(\sin\frac{x}{8} - \cos\frac{x}{8}\right) + c$
 c) $8\left(\cos\frac{x}{8} - \sin\frac{x}{8}\right) + c$ d) $\frac{1}{8}\left(\sin\frac{x}{8} - \cos\frac{x}{8}\right) + c$
6. The value of the integral $\int_a^b \frac{\sqrt{x} dx}{\sqrt{x} + \sqrt{a+b-x}}$ is
 a) π b) $\frac{1}{2}(b-a)$ c) $\frac{\pi}{2}$ d) $b-a$
7. The value of the integral $\int_0^3 \frac{dx}{\sqrt{x+1} + \sqrt{5x+1}}$, is
 a) $\frac{11}{15}$ b) $\frac{14}{15}$ c) $\frac{2}{5}$ d) None of these
8. The value of $\int_0^{12a} \frac{f(x)}{f(x) + f(12a-x)} dx$ is
 a) a b) $2a$ c) $3a$ d) $6a$
9. The value of $\int_{-2}^2 \left[p \log\left(\frac{1+x}{1-x}\right) + q \log\left(\frac{1-x}{1+x}\right)^{-2} + r \right] dx$ depends on
 a) The value of p b) The value of q
 c) The value of r d) The value of p and q
10. The antiderivative of $\frac{3^x}{\sqrt{1-9^x}}$ with respect to x is
 a) $(\log_3 e) \sin^{-1}(3^x) + C$
 b) $\sin^{-1}(3^x) + C$
 c) $(\log_3 e) \cos^{-1}(3^x)$
 d) None of these
11. If $\int \frac{dx}{x \log x} = f(x) + \text{constant}$, then $f(x)$ is equal to
 a) $1/\log x$ b) $\log x$ c) $\log \log x$ d) $x/\log x$
12. The solution of the equation $\int_{\log 2}^x \frac{1}{e^x - 1} dx = \log \frac{3}{2}$ is given by $x =$
 a) e^2 b) $1/e$ c) $\log 4$ d) None of these

13. If $f(x)$ is continuous for all real values of x , then $\sum_{r=1}^{10} \int_0^1 (r-1+x) dx$ is equal to
 a) $\int_0^{10} f(x) dx$ b) $\int_0^1 f(x) dx$ c) $10 \int_0^1 f(x) dx$ d) $9 \int_0^1 f(x) dx$
14. The value of the integral $\int_{-1}^1 [x^2 + \{x\}] dx$, where $[\cdot]$ and $\{\cdot\}$ denote respectively the greatest integer function and fractional part function, is equal to
 a) $\frac{5 + \sqrt{5}}{2}$ b) $\frac{5 - \sqrt{5}}{2}$ c) $-\frac{5 + \sqrt{5}}{2}$ d) None of these
15. $g(x) = \underbrace{(f \text{ of } o \dots \text{ of})}_{n \text{ times}}(x)$.
 Let $f(x) = \frac{x}{(1+nx^{1/n})}$ for $n \geq 2$ and $g(x) = (f \text{ of } o \dots \text{ of})(x)$. Then, $\int x^{n-2} g(x) dx$ equals
 a) $\frac{1}{n(n-1)} (1+nx^n)^{1-\frac{1}{n}} + c$
 b) $\frac{1}{n-1} (1+nx^n)^{1-\frac{1}{n}} + c$
 c) $\frac{1}{n(n+1)} (1+nx^n)^{1+\frac{1}{n}} + c$
 d) $\frac{1}{n+1} (1+nx^n)^{1+\frac{1}{n}} + c$
16. Let $I_n = \int_0^{\pi/2} \sin^n x dx, n \in N$. Then
 a) $I_n \cdot I_{n-2} = n : (n-1)$ b) $I_n > I_{n-2}$ c) $n(I_{n-2} - I_n) = I_{n-2}$ d) None of these
17. The value of $\int_0^3 x \sqrt{1+x} dx$, is
 a) $\frac{9}{2}$ b) $\frac{27}{4}$ c) $\frac{126}{15}$ d) None of these
18. $\int_0^1 \log \left\{ \sin \left(\frac{\pi x}{2} \right) \right\} dx$ is equal to
 a) $-\frac{\pi}{2} \log 2$ b) $-\log 2$ c) $-\frac{2}{\pi} \log 2$ d) $\frac{\pi}{2} \log 2$
19. $\int \frac{dx}{x(x^7+1)}$ is equal to
 a) $\log \left(\frac{x^7}{x^7+1} \right) + c$ b) $\frac{1}{7} \log \left(\frac{x^7}{x^7+1} \right) + c$ c) $\log \left(\frac{x^7+1}{x^7} \right) + c$ d) $\frac{1}{7} \log \left(\frac{x^7+1}{x^7} \right) + c$
20. $\int \cos \left\{ 2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right\} dx$ is equal to
 a) $\frac{1}{8} (x^2 - 1) + c$ b) $\frac{x^2}{4} + c$ c) $\frac{x}{2} + c$ d) $\frac{x^2}{2} + c$
21. The value of $\int \frac{x^2+1}{x^2-1} dx$ is
 a) $\log \left(\frac{x-1}{x+1} \right) + c$ b) $\log \left(\frac{x+1}{x-1} \right) + c$
 c) $x + \log \left(\frac{x-1}{x+1} \right) + c$ d) $\log (x^2 - 1) + c$
22. $\int e^{\tan^{-1} x} \left| \frac{1+x+x^2}{1+x^2} \right| dx$ is equal to
 a) $x e^{\tan^{-1} x} + c$ b) $x^2 e^{\tan^{-1} x} + c$ c) $\frac{1}{x} e^{\tan^{-1} x} + c$ d) None of these
23. $\int \frac{dx}{x+\sqrt{x}}$ is equal to
 a) $\frac{1}{2} \log(1 + \sqrt{x}) + c$ b) $2 \log(1 + \sqrt{x}) + c$ c) $\frac{1}{4} \log(1 + \sqrt{x}) + c$ d) $3 \log(1 + \sqrt{x}) + c$
24. $\int \frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}} dx$ is equal to
 a) $2 \operatorname{cosec} \alpha \sqrt{\cos \alpha + \sin \alpha \tan x} + C$

- b) $-2 \operatorname{cosec} \alpha \sqrt{\cos \alpha + \sin \alpha \cot x} + C$
 c) $\operatorname{cosec} \alpha \sqrt{\cos \alpha + \sin \alpha \cot x} + C$
 d) None of these
25. $\int_0^{\pi/2} \log \sin x \, dx$ is equal to
 a) $-\left(\frac{\pi}{2}\right) \log 2$ b) $\pi \log \frac{1}{2}$ c) $-\pi \log \frac{1}{2}$ d) $\frac{\pi}{2} \log 2$
26. $\int_0^1 \frac{1}{x+\sqrt{x}} \, dx$ is equal to
 a) $\log 3$ b) $\log 1$ c) $\log 4$ d) $\log 2$
27. The tangent of the curve $y = f(x)$ at the point with abscissa $x = 1$ form an angle of $\pi/6$ and at the point $x = 2$ an angle of $\pi/3$ and at the point $x = 3$ an angle of $\pi/4$. If $f''(x)$ is continuous, then the value of $\int_1^3 f''(x) f'(x) dx + \int_2^3 f''(x) dx$, is
 a) $\frac{4\sqrt{3}-1}{3\sqrt{3}}$ b) $\frac{3\sqrt{3}-1}{2}$ c) $\frac{4-3\sqrt{3}}{3}$ d) None of these
28. If $g(x) = \frac{f(x)-f(-x)}{2}$ defined over $[-3,3]$ and $f(x) = 2x^2 - 4x + 1$, then $\int_{-3}^3 g(x) dx$ is equal to
 a) 0 b) 4 c) -4 d) 8
29. $\int \frac{x+\sin x}{1+\cos x} dx$ is equal to
 a) $x \tan \frac{x}{2} + C$ b) $\cot \frac{x}{2} + C$ c) $\log(1 + \cos x) + C$ d) $\log(x + \sin x) + C$
30. The value of the integral $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x+\sqrt{x}}} dx$ is
 a) $\frac{3}{2}$ b) 2 c) 1 d) $\frac{1}{2}$
31. $\int \frac{dx}{(a^2+x^2)^{3/2}}$ is equal to
 a) $\frac{x}{(a^2+x^2)^{1/2}} + c$ b) $\frac{x}{a^2(a^2+x^2)^{1/2}} + c$ c) $\frac{1}{a^2(a^2+x^2)^{1/2}} + c$ d) None of these
32. The value of the integral $\int_0^{\pi/2} (\sin^{100} x - \cos^{100} x) dx$ is
 a) $\frac{1}{100}$ b) $\frac{100!}{(100)^{100}}$ c) $\frac{\pi}{100}$ d) 0
33. $\int e^x [f(x) + f'(x)] dx$ is equal to
 a) $e^x f(x) + c$ b) $e^x + c$ c) $e^x f'(x) + c$ d) None of these
34. $\int \frac{dx}{1-\cos x - \sin x}$ is equal to
 a) $\log \left| 1 + \cot \frac{x}{2} \right| + c$ b) $\log \left| 1 - \tan \frac{x}{2} \right| + c$ c) $\log \left| 1 - \cot \frac{x}{2} \right| + c$ d) $\log \left| 1 + \tan \frac{x}{2} \right| + c$
35. The value of $\alpha \in [0, 2\pi]$ which does not satisfy the equation $\int_{\pi/2}^{\alpha} \sin x \, dx = \sin 2\alpha$, is
 a) π b) $\frac{3\pi}{2}$ c) $\frac{7\pi}{6}$ d) $\frac{11\pi}{6}$
36. If $\int_{-1}^4 f(x) \, dx = 4$ and $\int_2^4 (3 - f(x)) dx = 7$, then the value of $\int_2^{-1} f(x) dx$, is
 a) 2 b) -3 c) -5 d) None of these
37. If $f(x) = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$ and $g(x) = e^{\sin^{-1} x}$, then $\int f(x)g(x) dx$ is equal to
 a) $e^{\sin^{-1} x} (\sin^{-1} x - 1) + c$ b) $e^{\sin^{-1} x} + c$
 c) $e^{(\sin^{-1} x)^2} + c$ d) $e^{2\sin^{-1} x} + c$
38. The value of $\int_{-1}^3 (|x-2| + [x]) \, dx$ is ($[x]$ stands for greatest integer less than or equal to x)
 a) 7 b) 5 c) 4 d) 3
39. Let $g(x) = \int_0^x f(t) dt$, where $\frac{1}{2} \leq f(t) \leq 1, t \in [0, 1]$ and $0 \leq f(t) \leq \frac{1}{2}$ for $t \in (1, 2]$, then

- a) $-\frac{3}{2} \leq g(2) < \frac{1}{2}$ b) $0 \leq g(2) < 2$ c) $\frac{3}{2} < g(2) \leq \frac{5}{2}$ d) $2 < g(2) < 4$
40. Let $\int \frac{x^2}{\sqrt{1-x}} dx = p\sqrt{(1-x)(3x^2+4x+8)}$, then value of p is
a) $\frac{-2}{15}$ b) $\frac{2}{15}$ c) $\frac{4}{15}$ d) None of these
41. $\int_0^1 \{x\} d\{x\}$ is equal to (where $\{ \cdot \}$ denotes fractional part of x)
a) 0 b) 1 c) $\frac{1}{2}$ d) None of these
42. If $I = \int \frac{x^5}{\sqrt{1+x^3}} dx$, then I is equal to
a) $\frac{2}{9}(1+x^3)^{\frac{5}{2}} + \frac{2}{3}(1+x^3)^{\frac{3}{2}} + c$ b) $\log|\sqrt{x} + \sqrt{1+x^3}| + c$
c) $\log|\sqrt{x} - \sqrt{1+x^3}| + c$ d) $\frac{2}{9}(1+x^3)^{\frac{3}{2}} - \frac{2}{3}(1+x^3)^{\frac{1}{2}} + c$
43. The value of the integral $\int_0^{100\pi} \sqrt{1-\cos 2x} dx$, is
a) $100\sqrt{2}$ b) $200\sqrt{2}$ c) 0 d) $400\sqrt{2}$
44. The value of $I = \int_{-\pi/2}^{\pi/2} |\sin x| dx$ is
a) 0 b) 2 c) -2 d) $-2 < I < 2$
45. The value of $\int_{-2}^3 |1-x^2| dx$ is
a) $\frac{28}{3}$ b) $\frac{14}{3}$ c) $\frac{7}{3}$ d) $\frac{1}{3}$
46. $\int_{-2}^2 (x - |x|) dx$ is equal to
a) 0 b) 2 c) 4 d) -4
47. The value of $\int_0^1 \log[x] dx$ is
a) 0 b) 1 c) $\log 1$ d) None of these
48. $\int_0^1 \frac{dx}{x+\sqrt{1-x^2}}$ is equal to
a) $\pi/3$ b) $\pi/2$ c) $1/2$ d) $\pi/4$
49. $\int \frac{(\sin\theta+\cos\theta)}{\sqrt{\sin 2\theta}} d\theta$ is equal to
a) $\log|\cos\theta - \sin\theta + \sqrt{\sin 2\theta}| + c$ b) $\log|\sin\theta - \cos\theta + \sqrt{\sin 2\theta}| + c$
c) $\sin^{-1}(\sin\theta - \cos\theta) + c$ d) $\sin^{-1}(\sin\theta + \cos\theta) + c$
50. $\int \frac{\cos x + x \sin x}{x(x+\cos x)} dx$ is equal to
a) $\log(x(x+\cos x)) + C$ b) $\log\left(\frac{x}{x+\cos x}\right) + C$ c) $\log\left(\frac{x+\cos x}{x}\right)$ d) None of these
51. If $\int \frac{dx}{x^2+2x+2} = f(x) + c$, then $f(x)$ is equal to
a) $\tan^{-1}(x+1)$ b) $2\tan^{-1}(x+1)$ c) $-\tan^{-1}(x+1)$ d) $3\tan^{-1}(x+1)$
52. $\int_0^{\pi/2} \frac{\cos x - \sin x}{1+\cos x \sin x} dx$ is equal to
a) 0 b) $\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$
53. $\int \operatorname{cosec}^4 x dx$ is equal to
a) $\cot x + \frac{\cot^3 x}{3} + c$ b) $\tan x + \frac{\tan^3 x}{3} + c$ c) $-\cot x - \frac{\cot^3 x}{3} + c$ d) $-\tan x - \frac{\tan^3 x}{3} + c$
54. $\int_0^{\pi/2} \frac{\sin^{2/3} x}{\sin^{2/3} x + \cos^{2/3} x} dx$ is equal to
a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\frac{3\pi}{4}$ d) π

- c) $g'(x)h[g(x)] - f'(x)h[f(x)]$ d) None of the above
95. If $\int_0^\infty e^{-ax} dx = \frac{1}{a}$, then $\int_0^\infty x^n e^{-ax} dx$, is
 a) $\frac{(-1)^n n!}{a^{n+1}}$ b) $\frac{(-1)^n (n-1)!}{a^n}$ c) $\frac{n!}{a^{n+1}}$ d) None of these
96. The integral $\int_{-1}^1 \frac{|x+2|}{x+2} dx$ is equal to
 a) 1 b) 2 c) 0 d) -1
97. The value of the integral $\int \frac{dx}{x(1+\log x)^2}$ is equal to
 a) $\frac{-1}{1+x} + c$ b) $\frac{-1}{1+\log x} + c$ c) $\frac{1}{1+\log x} + c$ d) $\frac{1}{1+x} + c$
98. The value of the integral $\int_\alpha^\beta \sqrt{(x-\alpha)(\beta-x)} dx$, is
 a) $\frac{\pi}{4}(\beta-\alpha)^2$ b) $\frac{\pi}{2}(\beta-\alpha)^2$ c) $\frac{\pi}{8}(\beta-\alpha)^2$ d) None of these
99. $\int_0^{\pi/2} x \sin x dx$ is equal to
 a) 0 b) 1 c) -1 d) 2
100. The value of $\int \frac{d(\sin x)}{\sqrt{1-\sin^2 x}}$ is equal to
 a) $x+c$ b) $3x+c$ c) x^2+c d) None of these
101. $\int \sin^{-1} x dx$ is equal to
 a) $\cos^{-1} x + c$ b) $x \sin^{-1} x + \sqrt{1-x^2} + c$
 c) $\frac{1}{\sqrt{1-x^2}} + c$ d) $x \sin^{-1} x - \sqrt{1-x^2} + c$
102. Let m be any integer. Then, the integral $\int_0^\pi \frac{\sin 2m x}{\sin x} dx$ equals
 a) 0 b) π c) 1 d) None of these
103. $I_1 = \int_0^{\pi/2} \sin x dx$, $I_2 = \int_0^{\pi/2} \sin^3 x dx$ then
 a) $I_1 > I_2$ b) $I_1 < I_2$ c) $I_1 = I_2$ d) $I_2 = 0$
104. $f(x) = \min\{x+2, 1, 2-x\}$, then $\int_{-2}^2 f(x) dx$ is equal to
 a) 1 b) 2 c) 3 d) 0
105. $\int_0^3 |x^3 + x^2 + 3x| dx$ is equal to
 a) $\frac{171}{2}$ b) $\frac{171}{4}$ c) $\frac{170}{4}$ d) $\frac{170}{3}$
106. $\int_{-1/7}^{20/7} e^{5\{x\}} dx$ is equal to (where $\{ \cdot \}$ denotes fractional part of x)
 a) $\frac{3}{5}(e^5)$ b) $\frac{3}{5}(e^5 - 1)$ c) $\frac{3}{5}$ d) None of these
107. $\int_{\pi/3}^{\pi/2} \operatorname{cosec}^2 x dx$ is equal to
 a) -1 b) 1 c) 0 d) $\frac{1}{2}$
108. If $\frac{C_0}{1} + \frac{C_1}{2} + \frac{C_2}{3} = 0$, where C_0, C_1, C_2 are all real, then the quadratic equation $C_2 x^2 + C_1 x + C_0 = 0$ has
 a) At least one root in $(0, 1)$
 b) One root in $(1, 2)$ and the other in $(3, 4)$
 c) One root in $(-1, 1)$ and the other in $(-5, -2)$
 d) Both roots imaginary
109. $\int \frac{dx}{2\sqrt{x}(1+x)}$ is equal to
 a) $\frac{1}{2} \tan^{-1}(\sqrt{x}) + c$ b) $\tan^{-1}(\sqrt{x}) + c$ c) $2 \tan^{-1}(\sqrt{x}) + c$ d) None of these
110. If $f(x)$ is an odd function defined on $[-T/2, T/2]$ and has period T , then $\phi(x) = \int_a^x f(t) dt$ is
 a) A periodic function with period $T/2$

- b) A periodic function with period T
 c) Not a periodic function
 d) A periodic function with period $T/4$
111. $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\cot x}}$ is equal to
 a) $\frac{\pi}{3}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{12}$ d) $\frac{\pi}{2}$
112. $\int_{-1/7}^{20/7} \sin(x - [x])dx$ (where $[\cdot]$ denotes greatest integer) is
 a) 0 b) $3(\cos 1 - 1)$ c) $3(1 - \cos 1)$ d) None of these
113. If $f(x)$ is a function satisfying $f\left(\frac{1}{x}\right) + x^2 f(x) = 0$ for all non-zero x , then $\int_{\sin \theta}^{\operatorname{cosec} \theta} f(x)dx$ is equal to
 a) 0 b) 1 c) 2 d) 3
114. $\int_{-\pi/2}^{\pi/2} \sin^4 x \cos^6 x dx$ is equal to
 a) $\frac{3\pi}{64}$ b) $\frac{3\pi}{572}$ c) $\frac{3\pi}{256}$ d) $\frac{3\pi}{128}$
115. The value of the integral $\int_0^1 \cot^{-1}(1 - x + x^2)dx$, is
 a) $\pi - \log 2$ b) $\frac{\pi}{2} - \log 2$ c) $\pi + \log 2$ d) $\frac{\pi}{2} + \log 2$
116. $\int_{-100}^{100} f(x) dx$ is equal to
 a) $\int_{-100}^{100} f(x^2) dx$ b) $\int_{-100}^{100} f(-x^2) dx$ c) $\int_{-100}^{100} f\left(\frac{1}{x}\right) dx$ d) $\int_{-100}^{100} f(-x) dx$
117. $\int \frac{\cos x - 1}{\sin x + 1} e^x dx$ is equal to
 a) $\frac{e^x \cos x}{1 + \sin x} + c$ b) $c - \frac{e^x \sin x}{1 + \sin x}$ c) $c - \frac{e^x}{1 + \sin x}$ d) $c - \frac{e^x \cos x}{1 + \sin x}$
118. If $\int (\log x)^2 dx = x[f(x)]^2 + Ax[f(x) - 1] + c$, then
 a) $f(x) = \log x, A = 2$ b) $f(x) = \log x, A = -2$
 c) $f(x) = -\log x, A = 2$ d) $f(x) = -\log x, A = -2$
119. Let $f(x)$ be an integrable function defined on $[a, b], b > a > 0$. If $I_1 = \int_{\pi/6}^{\pi/3} f(\tan \theta + \cot \theta) \sec^2 \theta d\theta$ and, $I_2 = \int_{\pi/6}^{\pi/3} f(\tan \theta + \cot \theta) \operatorname{cosec}^2 \theta d\theta$, then $\frac{I_1}{I_2} =$
 a) A positive integer b) A negative integer c) An irrational number d) None of these
120. The value of the integral $\int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x} dx$ is
 a) 0 b) $2/3$ c) $4/3$ d) None of these
121. The value of the integral $\int \frac{1+x^2}{1+x^4} dx$ is equal to
 a) $\tan^{-1} x^2 + C$
 b) $\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{2}x} \right)$
 c) $\frac{1}{2\sqrt{2}} \log \left(\frac{x^2 + \sqrt{2}x + 1}{x^2 - \sqrt{2}x + 1} \right) + C$
 d) None of these
122. If $(-1, 2)$ and $(2, 4)$ are two points on the curve $y = f(x)$ and if $g(x)$ is the gradient of the curve at point (x, y) , then the value of the integral $\int_{-1}^2 g(x) dx$, is
 a) 2 b) -2 c) 0 d) 1
123. $\int_0^{\pi/3} [\sqrt{3} \tan x] dx =$

- a) $\frac{5\pi}{6}$ b) $\frac{5\pi}{6} - \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ c) $\frac{\pi}{2} - \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ d) None of these
124. If $I = \int \frac{1}{x^4\sqrt{a^2+x^2}} dx$, then I equals
a) $\frac{1}{a^4} \left\{ \frac{1}{x} \sqrt{a^2+x^2} - \frac{1}{3x^3} \sqrt{a^2+x^2} \right\} + C$
b) $\frac{1}{a^4} \left\{ \frac{1}{x} \sqrt{a^2+x^2} - \frac{1}{3x^3} (a^2+x^2)^{3/2} \right\} + C$
c) $\frac{1}{a^4} \left\{ \frac{1}{x} \sqrt{a^2+x^2} - \frac{1}{2\sqrt{x}} (a^2+x^2)^{3/2} \right\} + C$
d) None of these
125. The value of the integral $\int_0^{\pi/2} \log |\tan x| dx$ is
a) $\pi \log 2$ b) 0 c) $-\pi \log 2$ d) None of these
126. The value of the integral $\int_{-1}^1 \sin^{11} x dx$, is
a) $\frac{10}{11}, \frac{8}{9}, \frac{6}{7}, \frac{4}{5}, \frac{2}{3}$ b) $\frac{10}{11}, \frac{8}{9}, \frac{6}{7}, \frac{4}{5}, \frac{2}{3}, \frac{\pi}{2}$ c) 1 d) 0
127. If $\int \cos^4 x dx = Ax + B \sin 2x + C \sin 4x + D$, then $\{A, B, C\}$ equals
a) $\left\{ \frac{3}{8}, \frac{1}{32}, \frac{1}{4} \right\}$ b) $\left\{ \frac{3}{8}, \frac{1}{4}, \frac{1}{32} \right\}$ c) $\left\{ \frac{1}{32}, \frac{1}{4}, \frac{3}{8} \right\}$ d) $\left\{ \frac{1}{4}, \frac{3}{8}, \frac{1}{32} \right\}$
128. If $\int e^x \left(\frac{1-\sin x}{1-\cos x} \right) dx = f(x) + \text{constant}$, then $f(x)$ is equal to
a) $e^x \cot\left(\frac{x}{2}\right) + c$ b) $e^{-x} \cot\left(\frac{x}{2}\right) + c$ c) $-e^x \cot\left(\frac{x}{2}\right) + c$ d) $-e^{-x} \cot\left(\frac{x}{2}\right) + c$
129. The value of $\int_1^a [x] f'(x) dx$, $a > 1$, where $[x]$ denotes the greatest integer not exceeding x , is
a) $[a]f(a) - \{f(1) + f(2) + \dots + f([a])\}$ b) $[a]f([a]) - \{f(1) + f(2) + \dots + f(a)\}$
c) $af([a]) - \{f(1) + f(2) + \dots + f(a)\}$ d) $af(a) - \{f(1) + f(2) + \dots + f([a])\}$
130. If $f(x) = f(a+b-x)$ for all $x \in [a, b]$ and $\int_a^b x f(x) dx = k \int_a^b f(x) dx$, then the value of k is
a) $\frac{a+b}{2}$ b) $\frac{a-b}{2}$ c) $\frac{a^2+b^2}{2}$ d) $\frac{a^2-b^2}{2}$
131. $\int |x| dx$ is equal to
a) $\frac{1}{2}x^2 + c$ b) $-\frac{x^2}{2} + c$ c) $x|x| + c$ d) $\frac{1}{2}x|x| + c$
132. The value of $\int_{-1}^1 \sin^{-1} \left[x^2 + \frac{1}{2} \right] dx + \int_{-1}^1 \cos^{-1} \left[x^2 - \frac{1}{2} \right] dx$, where $[\cdot]$ denotes the greatest integer function, is
a) π b) 2π c) 4π d) 0
133. $\int \frac{x^2-2}{x^3\sqrt{x^2-1}} dx$ is equal to
a) $\frac{x^2}{\sqrt{x^2-1}} + C$ b) $-\frac{x^2}{\sqrt{x^2-1}} + C$ c) $\frac{\sqrt{x^2-1}}{x^2} + C$ d) $-\frac{\sqrt{x^2-1}}{x^2} + C$
134. $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \sin \sqrt{t} dt}{x^3}$ is equal to
a) $1/3$ b) 1 c) $2/3$ d) None of these
135. $\int e^x \frac{x^2+1}{(x+1)^2} dx$ is equal to
a) $\frac{-e^x}{x+1} + c$ b) $\frac{e^x}{x+1} + c$ c) $e^x \frac{x-1}{x+1} + c$ d) $\frac{xe^x}{x+1} + c$
136. If $\int x \log(1+1/x) dx = f(x) \cdot \log(x+1) + g(x) \cdot x^2 + Ax + C$, then
a) $f(x) = \frac{1}{2}x^2$ b) $g(x) = \log x$ c) $A = 1$ d) None of these
137. Suppose that $f(x) \geq 0$ for all $x \in [0,1]$ and f is continuous in $[0,1]$ and $\int_0^1 f(x) dx = 0$, then $\forall x \in [0,1]$, f is
a) Entirely increasing b) Entirely decreasing c) Constant d) None of these

138. If $f\left(\frac{3x-4}{3x+4}\right) = x + 2$, then $\int f(x) dx$ is

- a) $e^{x+2} \log\left|\frac{3x-4}{3x+4}\right| + c$ b) $-\frac{8}{3} \log|1-x| + \frac{2}{3}x + c$
 c) $\frac{8}{3} \log|1-x| + \frac{x}{3} + c$ d) $e^{[(3x-4)/(3x+4)]} - \frac{x^2}{2} - 2x + c$

139. The value of $\int e^x \frac{1+nx^{n-1}-x^{2n}}{(1-x^n)\sqrt{1-x^{2n}}} dx$ is equal to

- a) $e^x (\sqrt{1-x^2}) + c$ b) $e^x \frac{\sqrt{1+x^{2n}}}{1+x^{2n}} + c$ c) $\frac{e^x \sqrt{1-x^n}}{1-x^{2n}} + c$ d) $\frac{e^x \sqrt{1-x^{2n}}}{1-x^n} + c$

140. The anti-derivative of $f(x) = \log(\log x) + (\log x)^{-2}$ whose graph passes through (e, e) , is

- a) $x[\log(\log x) + (\log x)^{-1}]$ b) $x[-\log(\log x) + (\log x)^{-1}] + e$
 c) $x[\log(\log x) - (\log x)^{-1}] + 2e$ d) None of the above

141. If $\int_a^b \frac{x^n}{x^n + (16-x)^n} dx = 6$, then

- a) $a = 4, b = 12, n \in R$ b) $a = 2, b = 14, n \in R$ c) $a = -4, b = 20, n \in R$ d) $a = 2, b = 8, n \in R$

142. $\int \tan^{-1} x dx = \dots + c$

- a) $\frac{1}{1+x^2}$ b) $x \tan^{-1} x + \frac{1}{2} \log|1+x^2|$
 c) $x \tan^{-1} x + \frac{1}{2} \cdot \frac{\tan^{-1} x}{1+x^2}$ d) $x \tan^{-1} x - \frac{1}{2} \log|1+x^2|$

143. $\int x^{51} (\tan^{-1} x + \cot^{-1} x) dx$

- a) $\frac{x^{52}}{52} (\tan^{-1} x + \cot^{-1} x) + c$ b) $\frac{x^{52}}{52} (\tan^{-1} x - \cot^{-1} x) + c$
 c) $\frac{\pi x^{52}}{104} + \frac{\pi}{2} + c$ d) $\frac{x^{52}}{52} + \frac{\pi}{2} + c$

144. If $f(x) = |x-1|$, then $\int_0^2 f(x) dx$ is

- a) 1 b) 0 c) 2 d) -2

145. $\int \frac{3-x^2}{1-2x+x^2} \cdot e^x dx = e^x f(x) + c \Rightarrow f(x)$

- a) $\frac{1+x}{1-x}$ b) $\frac{1-x}{1+x}$ c) $\frac{1+x}{x-1}$ d) $\frac{x-1}{1+x}$

146. $\int_0^1 \tan^{-1}\left(\frac{1}{x^2-x+1}\right) dx$ is equal to

- a) $\log 2$ b) $-\log 2$ c) $\frac{\pi}{2} + \log 2$ d) $\frac{\pi}{2} - \log 2$

147. Let $f: R \rightarrow R$ be a differentiable function and $f(1) = 4$. Then, the value of $\lim_{x \rightarrow 1} \int_4^{f(x)} \frac{2t}{x-1} dt$ is

- a) $8f'(1)$ b) $4f'(1)$ c) $2f'(1)$ d) $f'(1)$

148. $\int e^x (\log \sin x + \cot x) dx$ is equal to

- a) $e^x \cot x + c$ b) $e^x \log \sin x + c$
 c) $e^x \log \sin x + \tan x + c$ d) $e^x + \sin x + c$

149. The value of $\int_{-\pi}^{\pi} (1-x^2) \sin x \cos^2 x dx$, is

- a) 0 b) $\pi - \frac{\pi^3}{3}$ c) $2\pi - \pi^3$ d) $\frac{\pi}{2} - 2\pi^3$

150. $\int \frac{x}{(x^2-a^2)(x^2-b^2)} dx$ is equal to

- a) $\frac{1}{a^2-b^2} \log\left|\frac{x^2-a^2}{x^2-b^2}\right| + c$ b) $\frac{1}{a^2-b^2} \log\left|\frac{x^2-b^2}{x^2-a^2}\right| + c$
 c) $\frac{1}{2(a^2-b^2)} \log\left|\frac{x^2-a^2}{x^2-b^2}\right| + c$ d) $\frac{1}{2(a^2-b^2)} \log\left|\frac{x^2-b^2}{x^2-a^2}\right| + c$

151. If $\int_0^{\pi} f(\cos^2 x) dx = k \int_0^{\pi} f(\cos x) dx$, then the value of k , is

167. $\int_0^{\pi/4} (\cos x - \sin x) dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x) dx + \int_{2\pi}^{\pi/4} (\cos x - \sin x) dx$ is equal to
 a) $\sqrt{2} - 2$ b) $2\sqrt{2} - 2$ c) $3\sqrt{2} - 2$ d) $4\sqrt{2} - 2$
168. Let f be a real-valued function defined on the interval $(-1,1)$ such that $e^{-x}f(x) = 2 + \int_0^x \sqrt{t^4 + 1} dt$, for all $x \in (-1,1)$ and let f^{-1} be the inverse function of f . Then, $(f^{-1})'(2)$ is equal to
 a) 1 b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{1}{e}$
169. $\int \frac{f'(x)}{\sqrt{f(x)}} dx = \dots + c; f(x) \neq 0$
 a) $\frac{1}{2} \sqrt{f(x)}$ b) $2 \sqrt{f(x)}$ c) $\frac{1}{2} f(x)$ d) $2 f(x)$
170. $3a \int_0^1 \left(\frac{ax-1}{a-1}\right)^2 dx$ is equal to
 a) $a - 1 + (a - 1)^{-2}$ b) $a + a^{-2}$ c) $a - a^2$ d) $a^2 + \frac{1}{a^2}$
171. The value of $\int_1^e 10^{\log_e x} dx$ is equal to
 a) $10 \log_e(10 e)$ b) $\frac{10 e - 1}{\log_e 10 e}$ c) $\frac{10 e}{\log_e 10 e}$ d) $(10 e) \log_e(10 e)$
172. The value of $\sqrt{2} \int \frac{\sin x dx}{\sin(x - \frac{\pi}{4})}$ is
 a) $x - \log \left| \cos \left(x - \frac{\pi}{4} \right) \right| + c$ b) $x + \log \left| \cos \left(x - \frac{\pi}{4} \right) \right| + c$
 c) $x - \log \left| \sin \left(x - \frac{\pi}{4} \right) \right| + c$ d) $x + \log \left| \sin \left(x - \frac{\pi}{4} \right) \right| + c$
173. $\int \frac{1}{x(x^4-1)} dx$ is equal to
 a) $\frac{1}{4} \log \left| \frac{x^4}{x^4 - 1} \right| + C$ b) $\frac{1}{4} \log \left| \frac{x^4 - 1}{x^4} \right| + C$ c) $\log \left| \frac{x^4 - 1}{x^4} \right| + C$ d) $\log \left| \frac{x^4}{x^4 - 1} \right| + C$
174. If $g(x) = \int_0^x \cos^4 t dt$, then $g(x + \pi)$ is equal to
 a) $g(x) + g(\pi)$ b) $g(x) - g(\pi)$ c) $g(x) \cdot g(\pi)$ d) $\frac{g(x)}{g(\pi)}$
175. The integral $\int_0^{\pi/2} f(\sin 2x) \sin x dx$ is equal to
 a) $\int_0^{\pi/2} f(\cos 2x) \sin x dx = \sqrt{2} \int_0^{\pi/2} f(\cos 2x) \sin x dx$
 b) $\int_0^{\pi/2} f(\sin 2x) \cos x dx = \sqrt{2} \int_0^{\pi/4} f(\cos 2x) \cos x dx$
 c) $\int_0^{\pi/2} f(\cos 2x) \cos x dx = \sqrt{2} \int_0^{\pi/2} f(\cos 2x) \cos x dx$
 d) $\int_0^{\pi/2} f(\sin 2x) \cos x dx = \sqrt{2} \int_0^{\pi/2} f(\cos 2x) \cos x dx$
176. Let $f(x) = \int_1^x \sqrt{2-t^2} dt$. Then, real roots of the equation $x^2 - f'(x) = 0$ are
 a) ± 1 b) $\pm \frac{1}{\sqrt{2}}$ c) $\pm \frac{1}{2}$ d) 0 and 1
177. Evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x + \sqrt{\cos x}}} dx$ is
 a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) zero d) 1

178. $\int_0^3 \frac{3x+1}{x^2+9} dx$ is equal to
 a) $\log(2\sqrt{2}) + \frac{\pi}{12}$ b) $\log(2\sqrt{2}) + \frac{\pi}{2}$ c) $\log(2\sqrt{2}) + \frac{\pi}{6}$ d) $\log(2\sqrt{2}) + \frac{\pi}{3}$
179. If $\phi(x) = \cos x - \int_0^x (x-t)\phi(t)dt$, then $\phi''(x) + \phi(x)$ is equal to
 a) $-\cos x$ b) 0 c) $\int_0^x (x-t)\phi(t)dt$ d) $-\int_0^{-x} (x-t)\phi(t)dt$
180. If $I_n = \int_0^\infty e^{-x} x^{n-1} dx$, then $\int_0^\infty e^{-\lambda x} x^{n-1} dx$ is equal to
 a) λI_n b) $\frac{1}{\lambda} I_n$ c) $\frac{I_n}{\lambda^n}$ d) $\lambda^n I_n$
181. $\int \frac{x^{49} \tan^{-1}(x^{50})}{(1+x^{100})} dx = k[\tan^{-1}(x^{50})]^2 + c$, then k is equal to
 a) $\frac{1}{50}$ b) $-\frac{1}{50}$ c) $\frac{1}{100}$ d) $-\frac{1}{100}$
182. If $I = \int_{-\pi}^{\pi} \frac{\sin^2 x}{1+a^x} dx$, $a > 0$, then I equals
 a) π b) $\frac{\pi}{2}$ c) $a\pi$ d) $\frac{a\pi}{2}$
183. $\int \frac{x \tan^{-1} x}{\sqrt{(1+x^2)^{3/2}}} dx$ is equal to
 a) $\frac{x - \tan^{-1} x}{1 - x^2} + c$ b) $\frac{x + \tan^{-1} x}{\sqrt{1 - x^2}} + c$ c) $\frac{x - \tan^{-1} x}{\sqrt{1 + x^2}} + c$ d) $\frac{x + \sqrt{1 - x^2}}{\sqrt{1 + x^2}} + c$
184. If $f(x)$ is a quadratic polynomial in x such that $6 \int_0^1 f(x)dx - \left\{ f(0) + 4f\left(\frac{1}{2}\right) \right\} = kf(1)$, then $k =$
 a) -1 b) 0 c) 1 d) 2
185. If $f(x)$ and $g(x)$ are continuous functions satisfying $f(x) = f(a-x)$ and $g(x) + g(a-x) = 2$, then $\int_0^a f(x) g(x)dx$ is equal to
 a) $\int_0^a g(x)dx$
 b) $\int_0^a f(x)dx$
 c) 0
 d) None of these
186. $\int_{3\pi/2}^{5\pi/3} [2 \cos x] dx =$
 a) $\frac{5\pi}{3}$ b) $\frac{4\pi}{3}$ c) $\frac{2\pi}{3}$ d) None of these
187. $\int_0^{\pi/8} \cos^3 4\theta d\theta$ is equal to
 a) $5/3$ b) $5/4$ c) $1/3$ d) $1/6$
188. If $\int_{[x]}^{[x]+1} f(t)dt = [x]$, then the value of $\int_{-2}^4 f(x)dx$ is equal to
 a) 1 b) 2 c) -2 d) 3
189. $\int |x| \log |x| dx$ equals ($x \neq 0$)
 a) $\frac{x^2}{2} \log |x| - \frac{x^2}{4} + c$ b) $\frac{1}{2} x|x| \log x + \frac{1}{4} x|x| + c$
 c) $-\frac{x^2}{2} \log |x| + \frac{x^2}{4} + c$ d) $\frac{1}{2} x|x| \log |x| - \frac{1}{4} x|x| + c$
190. Let $f(x) = \frac{x}{(1+x^n)^{1/n}}$ for $n \geq 2$ and $g(x) = f \circ f \circ f \circ \dots \circ f$ (n times). Then, $\int x^{n-2} g(x) dx$ equals
 a) $\frac{1}{n(n-1)} (1 + nx^n)^{1-\frac{1}{n}} + k$

- b) $\frac{1}{n-1}(1+nx^n)^{1-\frac{1}{n}}+k$
 c) $\frac{1}{n(n-1)}(1+nx^n)^{1+\frac{1}{n}}+k$
 d) $\frac{1}{n-1}(1+nx^n)^{1+\frac{1}{n}}+k$
191. The value of the integral $\int_0^{2\pi} \frac{\sin 2\theta}{a-b \cos \theta} d\theta$ when $a > b > 0$, is
 a) 1 b) π c) $\pi/2$ d) 0
192. Suppose f is such that $f(-x) = -f(x)$ for every real x and $\int_0^1 f(x)dx = 5$, then $\int_{-1}^0 f(t)dt$ is equal to
 a) 10 b) 5 c) 0 d) -5
193. If $f(x) = f(a-x)$ and $g(x) + g(a-x) = 2$, then the value of $\int_0^a f(x)g(x)dx$ is
 a) $\int_0^a f(x)dx$ b) $\int_0^a g(x)dx$ c) $\int_0^a [g(x) - f(x)]dx$ d) $\int_0^a [g(x) + f(x)]dx$
194. $\int_{-10}^{10} \log\left(\frac{a+x}{a-x}\right) dx$ is equal to
 a) 0 b) $-2 \log(a+10)$ c) $2 \log\left(\frac{a+10}{a-10}\right)$ d) $2 \log(a+10)$
195. The value of $\int_0^\pi |\sin^3 \theta| d\theta$ is
 a) 0 b) $3/8$ c) $4/3$ d) π
196. $\int_0^{\pi/6} \frac{\sin x}{\cos^3 x} dx$ is equal to
 a) $\frac{2}{3}$ b) $\frac{1}{6}$ c) 2 d) $\frac{1}{3}$
197. $\int_0^2 \sqrt{\frac{2+x}{2-x}} dx$ is equal to
 a) $\pi + 2$ b) $\pi + \frac{3}{2}$ c) $\pi + 1$ d) π
198. If $\int f(x)dx = f(x) + c$, then $\int (f(x))^2 dx$ is
 a) $(f(x))^2 + k$ b) $\frac{1}{3}(f(x))^3 + k$ c) $f(x) + k$ d) $\frac{1}{2}(f(x))^2 + k$
199. $\int \frac{\sqrt{x-1}}{\sqrt{x+1}} dx$ is equal to
 a) $2\sqrt{x^2+1} + \sin^{-1} x + c$ b) $\sqrt{x^2-1} - \sin^{-1} x + c$
 c) $2\sqrt{x^2-1} + \sin^{-1} x + c$ d) $\sqrt{\frac{x^2-1}{2}} + \sin^{-1} x + c$
200. $\int \sin x d(\cos x)$ is equal to
 a) $\frac{1}{4} \sin 2x + \frac{x}{2} + c$ b) $\frac{1}{4} \sin 2x - \frac{x}{2} + c$ c) $2 \sin 2x + c$ d) $\sin x + \cos x + c$
201. Value of the integral $\int_{-\pi/2}^{\pi/2} \cos x dx$ is
 a) 4 b) 2 c) 0 d) 1
202. If $f(x) = f(a+x)$ and $\int_0^a f(x)dx = k$, then $\int_0^{na} f(x)dx$ is equal to
 a) nk b) $(n-1)k$ c) $(n+1)k$ d) 0
203. The value of $\int_0^\pi \frac{1}{5+3 \cos x} dx$, is
 a) π b) $2\pi/3$ c) $\pi/4$ d) 2
204. If $I_1 = \int_0^x e^{zx} e^{-z^2} dz$ and $I_2 = \int_0^x e^{-z^2/4} dz$, then
 a) $I_1 = e^x I_2$ b) $I_1 = e^{x^2} I_2$ c) $I_1 = e^{x^2/2} I_2$ d) None of these
205. If $f(x)$ is an integrable function over every interval on the real line such that $f(t+x) = f(x)$ for every x

and every x real t , then $\int_a^{a+t} f(x) dx$ is equal to

- a) $\int_0^a f(x) dx$ b) $\int_0^t f(x) dx$ c) $\int_a^t f(x) dx$ d) None of these

206. If $f(\pi) = 2$ and $\int_0^\pi [f(x) + f''(x)] \sin x dx = 5$, then $f(0)$ is equal to, (it is given that $f(x)$ is continuous in $[0, \pi]$)

- a) 7 b) 3 c) 5 d) 1

207. If $\int \frac{x+2}{2x^2+6x+5} dx = P \int \frac{4x+6}{2x^2+6x+5} dx + \frac{1}{2} \int \frac{dx}{2x^2+6x+5}$, then the value of P is

- a) $\frac{1}{3}$ b) $\frac{1}{2}$ c) $\frac{1}{4}$ d) 2

208. $\int (1+x-x^{-1})e^{x+x^{-1}} dx$ is equal to

- a) $(x+1)e^{x+x^{-1}} + c$ b) $(x-1)e^{x+x^{-1}} + c$ c) $-xe^{x+x^{-1}} + c$ d) $xe^{x+x^{-1}} + c$

209. If $I_1 = \int_0^{3\pi} f(\cos^2 x) dx$ and $I_2 = \int_0^\pi f(\cos^2 x) dx$, then

- a) $I_1 = I_2$ b) $3I_1 = I_2$ c) $I_1 = 3I_2$ d) $I_1 = 5I_2$

210. $\int_0^{\pi/2} (x\sqrt{\tan x} + \sqrt{\cot x}) dx$ equals

- a) $\frac{\pi}{2\sqrt{2}}$ b) $\frac{\pi^2}{2}$ c) $\frac{\pi^2}{2\sqrt{2}}$ d) $\frac{\pi^2}{2\sqrt{3}}$

211. $\int_0^\pi \frac{x dx}{a^2 \cos^2 x + b^2 \sin^2 x}$ is equal to

- a) $\frac{\pi}{2ab}$ b) $\frac{\pi}{ab}$ c) $\frac{\pi^2}{2ab}$ d) $\frac{\pi^2}{ab}$

212. If $f(x) = ae^{2x} + cx$, satisfies the conditions $f(0) = -1$, $f'(\log 2) = 31$, $\int_0^{\log 4} (f(x) - cx) dx = \frac{39}{2}$, then

- a) $a = 5, b = 6, c = 3$ b) $a = 5, b = -6, c = 3$ c) $a = -5, b = 6, c = 3$ d) None of these

213. The value of the integral $\int_0^1 \frac{1}{(x^2+1)^{3/2}} dx$, is

- a) $1/2$ b) $\sqrt{2}/2$ c) 1 d) $\sqrt{2}$

214. If $\int x f(x) dx = \frac{f(x)}{2}$, then $f(x)$ is equal to

- a) e^x b) e^{-x} c) $\log x$ d) $\frac{e^{x^2}}{2}$

215. $\int_0^\pi \frac{1}{1+\sin x} dx$ is equal to

- a) 1 b) 2 c) -1 d) -2

216. Let $f(x) = x - [x]$, for every real number x , where $[x]$ is the integral part of x . Then, $\int_{-1}^1 f(x) dx$ is equal to

- a) 1 b) 2 c) 0 d) $-\frac{1}{2}$

217. The value of $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ is

- a) $-\frac{(x^4+1)^{1/4}}{x} + c$ b) $\frac{(x^4+1)^{1/4}}{x} + c$ c) Zero d) None of these

218. If $I_1 = \int_{1-k}^k x \sin\{x(1-x)\} dx$ and $I_2 = \int_{1-k}^k \sin\{x(1-x)\} dx$, then

- a) $I_1 = 2I_2$ b) $2I_1 = I_2$ c) $I_1 = I_2$ d) None of these

219. If the primitive of $\frac{1}{f(x)}$ is equal to $\log\{f(x)\}^2 + c$, then $f(x)$ is

- a) $x + d$ b) $\frac{x}{2} + d$ c) $\frac{x^2}{2} + d$ d) $x^2 + d$

220. $\int \frac{x^4+x^2+1}{x^2-x+1} dx$ is equal to

- a) $\frac{x^3}{3} - \frac{x^2}{2} + x + c$ b) $\frac{x^3}{3} + \frac{x^2}{2} + x + c$ c) $\frac{x^3}{3} - \frac{x^2}{2} - x + c$ d) $\frac{x^3}{3} + \frac{x^2}{2} - x + c$

- b) $\frac{n}{m+1} I(m+1, n-1)$
 c) $\frac{2^n}{m+1} + \frac{n}{m+1} I(m+1, n-1)$
 d) $\frac{m}{m+1} I(m+1, n-1)$
236. $\int \frac{dx}{x(\log x)(\log \log x) \dots (\log \log \dots x)}$ is equal to
 a) $\frac{(\log \log \dots x)}{8 \text{ times}} + c$ b) $\frac{(\log \log \dots x)}{7 \text{ times}} + c$ c) $\frac{(\log \log \dots x)}{9 \text{ times}} + c$ d) None of these
237. If $I = \left| \int_2^5 \frac{\sin x dx}{(1+x^2)} \right|$, then
 a) $I \geq \frac{1}{4}$ b) I lies in the interval $\left(\frac{1}{4}, \frac{1}{5}\right)$
 c) I lies in the interval $\left(\frac{1}{5}, \frac{1}{6}\right)$ d) $I \leq \frac{3}{10}$
238. $\lim_{n \rightarrow \infty} \left(\frac{1}{n} + \frac{1}{n+1} + \dots + \frac{1}{3n}\right)$ is equal to
 a) $\log 2$ b) $\log 3$ c) $\log 5$ d) 0
239. The difference between the greatest and least values of the function $\phi(x) = \int_0^x (t+1) dt$ on $[2, 3]$, is
 a) 3 b) 2 c) $7/2$ d) $11/2$
240. The value of the integral $\int_0^\infty \frac{x \log x}{(1+x^2)^2} dx$ is
 a) 1 b) 0 c) 2 d) None of these
241. The value of $\int_0^1 \frac{2^{2x+1} - 5^{2x-1}}{10^x} dx$, is
 a) $\frac{3}{5} \left\{ \frac{2}{\log_e \left(\frac{2}{5}\right)} + \frac{1}{2 \log_e \left(\frac{5}{2}\right)} \right\}$
 b) $-\frac{3}{5} \left\{ \frac{2}{\log_e \left(\frac{2}{5}\right)} + \frac{1}{2 \log_e \left(\frac{5}{2}\right)} \right\}$
 c) $\frac{3}{5} \left\{ \frac{2}{\log_e \left(\frac{2}{5}\right)} - \frac{1}{2 \log_e \left(\frac{5}{2}\right)} \right\}$
 d) None of these
242. The integral $\int_0^r \pi \sin^{2n} x dx$ is equal to
 a) $r \int_0^\pi \sin^{2n} x dx$ b) $2r \int_0^\pi \sin^{2n} x dx$ c) $r \int_0^{\pi/2} \sin^{2n} x dx$ d) None of these
243. $\int \{f(x)g''(x) - f''(x)g(x)\} dx$ is equal to
 a) $\frac{f(x)}{g'(x)} + c$ b) $f'(x)g(x) - f(x)g'(x) + c$
 c) $f(x)g'(x) - f'(x)g(x) + c$ d) $f(x)g'(x) + f'(x)g(x) + c$
244. $\int_0^\lambda \frac{y dy}{\sqrt{y+\lambda}}$ is equal to
 a) $\frac{2}{3} (2 - \sqrt{2}) \lambda \sqrt{\lambda}$ b) $\frac{2}{3} (2 + \sqrt{2}) \lambda \sqrt{\lambda}$ c) $\frac{1}{3} (2 - \sqrt{2}) \lambda \sqrt{\lambda}$ d) $\frac{1}{3} (2 + \sqrt{2}) \lambda \sqrt{\lambda}$
245. Let f be a non-negative function defined on the interval $[0, 1]$. If $\int_0^x \sqrt{1 - (f'(t))^2} dt = \int_0^x f(t) dt$, $0 \leq x \leq 1$ and $f(0) = 0$, then
 a) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

- b) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$
 c) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$
 d) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$
246. $\int_1^2 e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$ is equal to
 a) $e - \frac{e^2}{2}$ b) $\frac{e^2}{2} - e$ c) $\frac{e^2}{2} + e$ d) $\frac{e^2}{2} - 2$
247. If a be a positive integer, the number of values of a satisfying $\int_0^{\pi/2} \left\{ a^2 \left(\frac{\cos 3x}{4} + \frac{3}{4} \cos x \right) + a \sin x - 20 \cos x dx \leq -a23 \right\}$, is
 a) Only one b) Two c) Three d) Four
248. $\int (x+1)^{3/2} [\sin^{-1}(\log x) + \cos^{-1}(\log x)] dx$ is equal to
 a) $\frac{2\pi}{2} (x+1)^{5/2} + c$ b) $\frac{\pi}{5} (x+1)^{5/2} + c$ c) Does not exist d) None of these
249. If $\int_0^{\pi/3} \frac{\cos x}{3+4 \sin x} dx = k \log\left(\frac{3+2\sqrt{3}}{3}\right)$, then k is
 a) $1/2$ b) $1/3$ c) $1/4$ d) $1/8$
250. If $F(x) = \int_{x^2}^{x^3} \log t dt$ ($x > 0$), then $F'(x)$ equals
 a) $(9x^2 - 4x) \log x$ b) $(4x - 9x^2) \log x$ c) $(9x^2 + 4x) \log x$ d) None of these
251. $\int \frac{\sin x \cos x}{\sqrt{1-\sin^4 x}} dx$ is equal to
 a) $\frac{1}{2} \sin^{-1}(\sin^2 x) + c$ b) $\frac{1}{2} \cos^{-1}(\sin^2 x) + c$ c) $\tan^{-1}(\sin^2 x) + c$ d) $\tan^{-1}(2 \sin^2 x) + c$
252. $\int \frac{1+x}{1+\sqrt[3]{x}} dx$ is equal to
 a) $\frac{3}{5} x^{5/3} - \frac{3}{4} x^{4/3} + x + C$
 b) $\frac{3}{5} x^{5/3} + \frac{3}{4} x^{4/3} + x + C$
 c) $\frac{3}{5} x^{5/3} - \frac{3}{4} x^{4/3} + C$
 d) None of these
253. The value of the integral $\int_0^1 x(1-x)^n dx$, is
 a) $\frac{1}{n+1} + \frac{1}{n+2}$ b) $\frac{1}{(n+1)(n+2)}$ c) $\frac{1}{n+2} - \frac{1}{n+1}$ d) $2\left(\frac{1}{n+1} - \frac{1}{n+2}\right)$
254. $\int_0^1 \frac{x dx}{[x+\sqrt{1-x^2}]\sqrt{1-x^2}}$ is equal to
 a) 0 b) 1 c) $\pi/4$ d) $\pi^2/2$
255. $\int_{-1}^1 |1-x| dx$ is equal to
 a) -2 b) 0 c) 2 d) 4
256. The value of $\int \frac{\sec x dx}{\sqrt{\sin(2x+\theta)+\sin\theta}}$ is
 a) $\sqrt{(\tan x + \tan \theta) \sec \theta} + c$ b) $\sqrt{2(\tan x + \tan \theta) \sec \theta} + c$
 c) $\sqrt{2(\sin x + \tan \theta) \sec \theta} + c$ d) None of the above
257. If an anti-derivate of $f(x)$ is e^x and that of $g(x)$ is $\cos x$, then $\int f(x) \cos x dx + \int g(x) e^x dx$ is equal to
 a) $f(x)g(x) + c$ b) $f(x) + g(x) + c$ c) $e^x \cos x + c$ d) $f(x) - g(x) + c$
258. If $P = \int_0^{3\pi} f(\cos^2 x) dx$ and $Q = \int_0^{\pi} f(\cos^2 x) dx$, then
 a) $P - Q = 0$ b) $P - 2Q = 0$ c) $P - 3Q = 0$ d) $P - 5Q = 0$
259. If $f(x) = \int_{x^2}^{x^4} \sin \sqrt{t} dt$, then $f'(x)$ equals
 a) $\sin x^2 - \sin x$ b) $4x^3 \sin x^2 - 2x \sin x$ c) $x^4 \sin x^2 - x \sin x$ d) None of these

260. $\int_0^{50\pi} |\cos x| dx =$
a) 100 b) 50 c) 0 d) None of these
261. If $\int_{-1}^4 f(x) dx = 4$ and $\int_2^4 \{3 - f(x)\} dx = 7$, then the value of $\int_{-1}^2 f(x) dx$ is
a) -2 b) 3 c) 4 d) 5
262. $\int \frac{(x+1)^2}{x(x^2+1)} dx$ is equal to
a) $\log_e x + c$ b) $\log_e x + 2 \tan^{-1} x + c$ c) $\log_e \frac{1}{x^2+1} + c$ d) $\log_e \{x(x^2+1)\} + c$
263. $\lim_{x \rightarrow \infty} \frac{\pi}{n} \left\{ \sin \frac{\pi}{n} + \sin \frac{2\pi}{n} + \dots + \sin \frac{(n-1)\pi}{n} \right\}$ equals
a) 0 b) π c) 2 d) None of these
264. $f(x)$ is a continuous function for all real values of x and satisfies $\int_0^x f(t) dt = \int_x^1 t^2 f(t) dt + \frac{x^{16}}{8} + \frac{x^6}{3} + a$, then the value of 'a' is equal to
a) $-\frac{1}{24}$ b) $\frac{17}{168}$ c) $\frac{1}{7}$ d) None of these
265. If $f: R \rightarrow R$ and $g: R \rightarrow R$ are one to one, real valued functions, then the value of the integral $\int_{-\pi}^{\pi} [f(x) + f(-x)][g(x) - g(-x)] dx$ is
a) 0 b) π c) 1 d) None of these
266. If $\int_0^{\pi} x f(\sin x) dx = A \int_0^{\pi/2} f(\sin x) dx$, then A is equal to
a) 0 b) π c) $\frac{\pi}{4}$ d) 2π
267. The number of solutions of $I(x) = \int_0^x [t]^2 dt = 2(x-1), \forall x > 0$, where $[\cdot]$ denotes the greatest integer function, is
a) 2 b) 3 c) 4 d) 5
268. If n is an odd natural number, then $\int_{-\pi/6}^{\pi/6} \frac{\pi + 4x^n}{1 - \sin(pq + \frac{\pi}{6})} dx =$
a) 4π b) $2\pi + \frac{1}{\sqrt{3}}$ c) $2\pi - \sqrt{3}$ d) $4\pi + \sqrt{3} - \frac{1}{\sqrt{3}}$
269. If $I_1 = \int_0^{\pi/4} \sin^2 x dx$ and $I_2 = \int_0^{\pi/4} \cos^2 x dx$, then
a) $I_1 = I_2$ b) $I_1 < I_2$ c) $I_1 > I_2$ d) $I_2 = I_1 + \frac{\pi}{4}$
270. The set of values of $\alpha (\alpha > 0)$ for which the inequality $\int_{-\alpha}^{\alpha} e^x dx > \frac{3}{2}$ holds true, is
a) $(0, \infty)$ b) $(2, \infty)$ c) $(\log 2, \infty)$ d) None of these
271. If $\frac{d}{dx} \left[a \tan^{-1} x + b \log \left(\frac{x-1}{x+1} \right) \right] = \frac{1}{x^4-1}$, then $a - 2b$ is equal to
a) 1 b) -1 c) 0 d) 2
272. $\int_{\alpha}^{\beta} \frac{\sqrt{x-\alpha}}{\sqrt{\beta-x}} dx$ is equal to
a) $\frac{\pi}{2} (\alpha - \beta)$ b) $\frac{\pi}{2} (\beta - \alpha)$ c) $\pi (\alpha - \beta)$ d) $\pi (\beta - \alpha)$
273. $\int \frac{x dx}{x^2+4x+5}$ is equal to
a) $\frac{1}{2} \log(x^2 + 4x + 5) + 2 \tan^{-1}(x) + c$ b) $\frac{1}{2} \log(x^2 + 4x + 5) - \tan^{-1}(x + 2) + c$
c) $\frac{1}{2} \log(x^2 + 4x + 5) + \tan^{-1}(x + 2) + c$ d) $\frac{1}{2} \log(x^2 + 4x + 5) - 2 \tan^{-1}(x + 2) + c$

274. $\int \cos \left[2 \cot^{-1} \sqrt{\frac{1-x}{1+x}} \right] dx$ is equal to

a) $\frac{1}{2}x^2 + c$

b) $\frac{1}{2} \sin \left[2 \cot^{-1} \sqrt{\frac{1-x}{1+x}} \right] + c$

c) $-\frac{1}{2}x^2 + c$

d) $\frac{1}{2}x + c$

275. The value of $\int_0^a \sqrt{\frac{a-x}{x}} dx$ is

a) $\frac{a}{2}$

b) $\frac{a}{4}$

c) $\frac{\pi a}{2}$

d) $\frac{\pi a}{4}$

276. $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos^2 x (\sin x + \cos x) dx =$

a) 2/15

b) 4/15

c) 2/5

d) 8/15

277. The value of $\int_{-1}^1 \log \left(\frac{x-1}{x+1} \right) dx$ is

a) 1

b) 2

c) 0

d) 4

278. If for every integer $n, \int_n^{n+1} f(x) dx = n^2$, then the value of $\int_{-2}^4 f(x) dx$ is

a) 16

b) 14

c) 19

d) None of these

279. The value of $\int_{-1}^1 x|x| dx$, is

a) 2

b) 1

c) 0

d) None of these

280. If $f(x) = \begin{vmatrix} \sin x + \sin 2x + \sin 3x & \sin 2x & \sin 3x \\ 3 + 4 \sin x & 3 & 4 \sin x \\ 1 + \sin x & \sin x & 1 \end{vmatrix}$, then the value of $\int_0^{\pi/2} f(x) dx$ is

a) 3

b) 2/3

c) 1/3

d) 0

281. The value of the integral $\int_{\alpha}^{\beta} \frac{1}{\sqrt{(x-\alpha)(\beta-x)}} dx$ for $\beta > \alpha$, is

a) $\sin^{-1}(\alpha/\beta)$

b) $\pi/2$

c) $\sin^{-1}(\beta/2\alpha)$

d) π

282. $\int_{\pi/2}^{3\pi/2} [2 \cos x] dx$ is equal to

a) $\frac{5\pi}{3}$

b) $-\frac{5\pi}{3}$

c) $-\pi$

d) -2π

283. $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$ is equal to

a) $\log(\sin^{-1} x) + c$

b) $\frac{1}{2} (\sin^{-1} x)^2 + c$

c) $\log(\sqrt{1-x^2}) + c$

d) $\sin(\cos^{-1} x) + c$

284. $\int_{-\pi/2}^{\pi/2} \frac{dx}{1+\cos x}$ is equal to

a) 0

b) 1

c) 2

d) 3

285. If f is a positive function and

$$I_1 = \int_{1-k}^k x f[x(1-x)] dx,$$

$$I_2 = \int_{1-k}^k f[x(1-x)] dx,$$

Where $2k - 1 > 0$, then $\frac{I_1}{I_2}$ is

a) 2

b) k

c) $\frac{1}{2}$

d) 1

286. $\int \frac{1}{(a^2+x^2)^{3/2}} dx$ is equal to

a) $\frac{x}{a^2 \sqrt{a^2+x^2}} + C$

b) $\frac{x}{(a^2+x^2)^{3/2}} + C$

c) $\frac{1}{a^2 \sqrt{a^2+x^2}} + C$

d) None of these

287. $\int \frac{\cos x - \sin x}{1 + 2 \sin x \cos x} dx$ is equal to
 a) $-\frac{1}{\cos x - \sin x} + c$ b) $\frac{\cos x + \sin x}{\cos x - \sin x} + c$ c) $-\frac{1}{\sin x + \cos x} + c$ d) $\frac{x}{\sin x + \cos x} + c$
288. The value of the integral $\int_0^{\pi/2} |\sin x - \cos x| dx$, is
 a) 0 b) $2(\sqrt{2} - 1)$ c) $2\sqrt{2}$ d) $2(\sqrt{2} + 1)$
289. $\int_0^{\pi/4} (\cos x - \sin x) + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x) dx + \int_{2\pi}^{\pi/4} (\cos x - \sin x) dx$ is equal to
 a) $\sqrt{2} - 2$ b) $2\sqrt{2} - 2$ c) $3\sqrt{2} - 2$ d) $4\sqrt{2} - 2$
290. The value of $\int_1^4 e^{\sqrt{x}} dx$, is
 a) e^2 b) $2e^2$ c) $4e^2$ d) $3e^2$
291. Let $I_n = \int_0^{\pi/2} \cos^n x \cos n x dx$. Then, $I_n : I_{n+1}$ is equal to
 a) 3 : 1 b) 2 : 3 c) 2 : 1 d) 3 : 4
292. If $\int \frac{2x^2 + 3}{(x^2 - 1)(x^2 + 4)} dx = a \log \left(\frac{x-1}{x+1} \right) + b \tan^{-1} \left(\frac{x}{2} \right) + c$, then the value of a and b are
 a) (1, -1) b) (-1, 1) c) $\left(\frac{1}{2}, -\frac{1}{2} \right)$ d) $\left(\frac{1}{2}, \frac{1}{2} \right)$
293. $\int_0^{\pi/2} \frac{\cos x - \sin x}{1 + \cos x \sin x} dx$ is equal to
 a) 0 b) $\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$
294. $\int_0^{\pi/2} \log \sin x dx$ is equal to
 a) $-\pi \log 2$ b) $\pi \log 2$ c) $-\frac{\pi}{2} \log 2$ d) $\frac{\pi}{2} \log 2$
295. Let $\int e^x (f(x) - f'(x)) dx = \phi(x)$
 Then, $\int e^x f(x) dx$ is equal to
 a) $\phi(x) + e^x f(x)$ b) $\phi(x) - e^x f(x)$ c) $\frac{1}{2} [\phi(x) + e^x f(x)]$ d) $\frac{1}{2} [\phi(x) + e^x f'(x)]$
296. If $f(x)$ is an odd function and has a period T , then $\phi(x) = \int_0^x f(t) dt$ is
 a) A periodic function with period $T/2$ b) A periodic function with period T
 c) Not a periodic function d) A periodic function with period $T/4$
297. If x is any arbitrary constant, then $\int 2^{2^{2^x}} \cdot 2^{2^x} \cdot 2^x dx$ is equal to
 a) $\frac{\int 2^{2^x} dx}{(\log 2)^3} + c$ b) $\frac{\int 2^{2^{2^x}} dx}{(\log 2)^3} + c$
 c) $\int 2^{2^{2^x}} (\log 2)^3 dx + c$ d) None of these
298. The value of $\int \frac{2 dx}{\sqrt{1-4x^2}}$ is
 a) $\tan^{-1}(2x) + c$ b) $\cot^{-1}(2x) + c$ c) $\cos^{-1}(2x) + c$ d) $\sin^{-1}(2x) + c$
299. $\int \frac{d(\cos \theta)}{\sqrt{1-\cos^2 \theta}}$ is equal to
 a) $\cos^{-1} \theta + c$ b) $\theta + c$ c) $\sin^{-1} \theta + c$ d) $\sin^{-1}(\cos \theta) + c$
300. The value(s) of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2} dx$ is (are)
 a) $\frac{22}{7} - \pi$ b) $\frac{2}{105}$ c) 0 d) $\frac{71}{15} - \frac{3\pi}{2}$
301. Let $f(x) = \int_0^x |x - 2| dx, x \geq 0$. Then, $f'(x)$ is
 a) Continuous and non differentiable at $x = 2$
 b) Discontinuous at $x = 4$
 c) Neither continuous nor differentiable at $x = 2$
 d) Non-differentiable at $x = 4$

317. The value of the integral $\int \frac{\log(x+1) - \log x}{x(x+1)} dx$ is
- $\frac{1}{2} [\log(x+1)]^2 + \frac{1}{2} (\log x)^2 + \log(x+1) \log x + C$
 - $-\{[\log(x+1)]^2 - (\log x)^2\} + \log(x+1) \cdot \log x + C$
 - $\frac{1}{2} [\log(1 + 1/x)]^2 + C$
 - None of these
318. The value of $\int_0^{2\pi} [2 \sin x] dx$, where $[\cdot]$ represents the greatest integral functions, is
- $-\frac{5\pi}{3}$
 - $-\pi$
 - $\frac{5\pi}{3}$
 - -2π
319. $\int_0^1 \frac{d}{dx} \left[\sin^{-1} \left(\frac{2x}{1+x^2} \right) \right] dx$ is equal to
- 0
 - π
 - $\frac{\pi}{2}$
 - $\frac{\pi}{4}$
320. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $g(x)$ be a function that satisfies $f(x) + g(x) = x^2$. Then, the value of the integral $\int_0^1 f(x) g(x) dx$, is
- $e - \frac{e^2}{2} - \frac{5}{2}$
 - $e + \frac{e^2}{2} - \frac{3}{2}$
 - $e - \frac{e^2}{2} - \frac{3}{2}$
 - $e + \frac{e^2}{2} + \frac{5}{2}$
321. The value of the integral $\int_0^\pi \log(1 + \cos x) dx$ is
- $\frac{\pi}{2} \log 2$
 - $-\pi \log 2$
 - $\pi \log 2$
 - None of these
322. $\int e^{\tan^{-1} x} \left(\frac{1+x+x^2}{1+x^2} \right) dx$ is equal to
- $x e^{\tan^{-1} x} + C$
 - $x^2 e^{\tan^{-1} x} + C$
 - $\frac{1}{x} e^{\tan^{-1} x} + C$
 - None of these
323. Let $I_1 = \int_a^{\pi-a} x f(\sin x) dx$, $I_2 = \int_a^{\pi-a} f(\sin x) dx$, then I_2 is equal to
- $\frac{\pi}{2} I_1$
 - πI_1
 - $\frac{2}{\pi} I_1$
 - $2I_1$
324. $\int \cos^{-1} \left(\frac{1}{x} \right) dx$ equals
- $x \sec^{-1} x + \cosh^{-1} x + c$
 - $x \sec^{-1} x - \cosh^{-1} x + c$
 - $x \sec^{-1} x - \sin^{-1} x + c$
 - None of these
325. The value of $\int_0^1 \frac{dx}{x + \sqrt{1-x^2}}$ is
- $\frac{\pi}{3}$
 - $\frac{\pi}{2}$
 - $\frac{1}{2}$
 - $\frac{\pi}{4}$
326. For any natural number n , the value of the integral $\int_0^{\sqrt{n}} [x^2] dx$, is
- $n\sqrt{n} + \sum_{r=1}^n \sqrt{r}$
 - $n\sqrt{n} - \sum_{r=1}^n \sqrt{r}$
 - $\sum_{r=1}^n \sqrt{r} - n\sqrt{n}$
 - None of these
327. $\int \frac{1}{x} (\log_{ex} e) dx$ is equal to
- $\log_e (1 - \log_e x) + c$
 - $\log_e (\log_e ex - 1) + c$
 - $\log_e (\log_e x - 1) + c$
 - $\log_e (1 + \log_e x) + c$
328. Let f be integrable over $[0, a]$ for any real a . If we define
- $$I_1 = \int_0^{\pi/2} \cos \theta f(\sin \theta + \cos^2 \theta) d\theta$$
- and $I_2 = \int_0^{\pi/2} \sin 2\theta f(\sin \theta + \cos^2 \theta) d\theta$, then
- $I_1 = I_2$
 - $I_1 = -I_2$
 - $I_1 = 2I_2$
 - $I_1 = -2I_2$
329. Consider the following statements:
- $\int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x} dx = \frac{3}{4}$

$$2. \int_0^4 (|x-1| + |x-3|) dx = 10$$

Which of these is/are correct?

- a) Only (1) b) Only (2) c) Both of these d) None of these
330. The value of the integral $\int_{-\pi/4}^{\pi/4} \sin^{-4} x dx$ is
 a) $-\frac{8}{3}$ b) $\frac{3}{2}$ c) $\frac{8}{3}$ d) None of these
331. The value of $\lim_{x \rightarrow \infty} \frac{(\int_0^x e^x dx)^2}{\int_0^x e^{2x^3} dx}$, is
 a) 1 b) 2 c) 3 d) 0
332. If $\int_{\sin x}^1 t^2 f(t) dt = 1 - \sin x, \forall x \in [0, \pi/2]$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is
 a) 3 b) $\sqrt{3}$ c) $\frac{1}{3}$ d) None of these
333. If a is fixed real number such that $f(a-x) + f(a+x) = 0$, then $\int_0^{2a} f(x) dx =$
 a) $\frac{a}{2}$ b) 0 c) $-\frac{a}{2}$ d) $2a$
334. $\int \frac{dx}{\sin(x-a)\sin(x-b)}$ is
 a) $\frac{1}{\sin(a-b)} \log \left| \frac{\sin(x-a)}{\sin(x-b)} \right| + c$
 b) $\frac{-1}{\sin(a-b)} \log \left| \frac{\sin(x-a)}{\sin(x-b)} \right| + c$
 c) $\log \sin(x-a) \sin(x-b) + c$
 d) $\log \left| \frac{\sin(x-a)}{\sin(x-b)} \right| + c$
335. Let $\int \sqrt{\frac{5-x}{2+x}} dx$ equal
 a) $\sqrt{x+2} \sqrt{5-x} + 3 \sin^{-1} \sqrt{\frac{x+2}{3}} + C$
 b) $\sqrt{x+2} \sqrt{5-x} + 7 \sin^{-1} \sqrt{\frac{x+2}{7}} + C$
 c) $\sqrt{x+2} \sqrt{5-x} + 5 \sin^{-1} \sqrt{\frac{x+2}{5}} + C$
 d) None of these
336. $\int \frac{2x^2+3}{(x^2-1)(x^2+4)} dx = a \log \left(\frac{x+1}{x-1} \right) + b \tan^{-1} \frac{x}{2}$, then (a, b) is
 a) $(-1/2, 1/2)$ b) $(1/2, 1/2)$ c) $(-1, 1)$ d) $(1, -1)$
337. $\int e^x \frac{(x-1)}{x^2} dx$ is equal to
 a) $\frac{e^x}{x^2} + c$ b) $\frac{-e^x}{x^2} + c$ c) $\frac{e^x}{x} + c$ d) $\frac{-e^x}{x} + c$
338. The value of the integral $\int_0^{3\alpha} \operatorname{cosec}(x-\alpha) \operatorname{cosec}(x-2\alpha) dx$, is
 a) $2 \sec \alpha \log \left(\frac{1}{2} \operatorname{cosec} \alpha \right)$ b) $2 \sec \alpha \log \left(\frac{1}{2} \sec \alpha \right)$ c) $2 \operatorname{cosec} \alpha \log(\sec \alpha)$ d) $2 \operatorname{cosec} \alpha \log \left(\frac{1}{2} \sec \alpha \right)$
339. $\int_0^3 \frac{3x+1}{x^2+9} dx$ is equal to
 a) $\log(2\sqrt{2}) + \frac{\pi}{12}$ b) $\log(2\sqrt{2}) + \frac{\pi}{2}$ c) $\log(2\sqrt{2}) + \frac{\pi}{6}$ d) $\log(2\sqrt{2}) + \frac{\pi}{3}$

340. $\int \frac{dx}{\sqrt{(1-x)(x-2)}}$ is equal to
 a) $\sin^{-1}(2x - 3) + c$ b) $\sin^{-1}(2x + 5) + c$ c) $\sin^{-1}(3 - 2x) + c$ d) $\sin^{-1}(5 - 2x) + c$
341. If $f(x) = \lim_{n \rightarrow \infty} [2x + 4x^3 + \dots + 2nx^{2n-1}]$ ($0 < x < 1$), then $\int f(x) dx$ is equal to
 a) $-\sqrt{1-x^2}$ b) $\frac{1}{\sqrt{1-x^2}}$ c) $\frac{1}{x^2-1}$ d) $\frac{1}{1-x^2}$
342. $\int \frac{dx}{\sin x - \cos x + \sqrt{2}}$ equals
 a) $-\frac{1}{\sqrt{2}} \tan\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$ b) $\frac{1}{\sqrt{2}} \tan\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$ c) $\frac{1}{\sqrt{2}} \cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$ d) $-\frac{1}{\sqrt{2}} \cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$
343. $\int_0^{\pi/2} \frac{\cos x}{1+\sin x} dx$ is equal to
 a) $\log 2$ b) $2 \log 2$ c) $(\log 2)^2$ d) $\frac{1}{2} \log 2$
344. The integral $\int_0^1 \frac{2 \sin^{-1} x}{x} dx$ equals
 a) $\int_0^{\pi/6} \frac{x}{\tan x} dx$ b) $\int_0^{\pi/6} \frac{2x}{\tan x} dx$ c) $\int_0^{\pi/2} \frac{2x}{\tan x} dx$ d) $\int_0^{\pi/6} \frac{x}{\sin x} dx$
345. If $\int_2^e \left(\frac{1}{\log x} - \frac{1}{(\log x)^2}\right) dx = a + \frac{b}{\log 2}$, then
 a) $a = e, b = -2$ b) $a = e, b = 2$ c) $a = -e, b = 2$ d) None of these
346. The value of $\int_0^8 |x - 5| dx$ is
 a) 17 b) 12 c) 9 d) 18
347. $\int_0^1 \frac{x dx}{[x + \sqrt{1-x^2}] \sqrt{1-x^2}}$ is equal to
 a) 0 b) 1 c) $\frac{\pi}{4}$ d) $\frac{\pi^2}{2}$
348. $\int_0^1 \cot^{-1}(1 - x + x^2) dx$ is equal to
 a) $\pi - \log 2$ b) $\pi + \log 2$ c) $\frac{\pi}{2} + \log 2$ d) $\frac{\pi}{2} - \log 2$
349. $\int_8^{15} \frac{dx}{(x-3)\sqrt{x+1}}$ is equal to
 a) $\frac{1}{2} \log \frac{5}{3}$ b) $\frac{1}{3} \log \frac{5}{3}$ c) $\frac{1}{2} \log \frac{3}{5}$ d) $\frac{1}{5} \log \frac{3}{5}$
350. $\int \{1 + 2 \tan x (\tan x + \sec x)\}^{1/2} dx$ is equal to
 a) $\log \sec x (\sec x - \tan x) + C$
 b) $\log \operatorname{cosec} (\sec x + \tan x) + C$
 c) $\log \sec x (\sec x + \tan x + C)$
 d) $\log(\sec x + \tan x) + C$
351. If $I_1 = \int_0^\infty \frac{1}{1+x^4} dx$ and $I_2 = \int_0^\infty \frac{x^2}{1+x^4} dx$. Then $\frac{I_1}{I_2} =$
 a) 1 b) 2 c) 1/2 d) None of these
352. $\int \frac{\sin x dx}{3+4 \cos^2 x}$ is equal to
 a) $\log(3 + 4 \cos^2 x) + c$ b) $\frac{1}{2\sqrt{3}} \tan^{-1}\left(\frac{\cos x}{\sqrt{3}}\right) + c$
 c) $-\frac{1}{2\sqrt{3}} \tan^{-1}\left(\frac{2 \cos x}{\sqrt{3}}\right) + c$ d) $\frac{1}{2\sqrt{3}} \tan^{-1}\left(\frac{2 \cos x}{\sqrt{3}}\right) + c$
353. For any integer n , the integral $\int_0^\pi e^{\cos^2 x} \cos^3(2n+1)x dx$ has the value
 a) π b) 1 c) 0 d) None of these
354. If $\frac{d}{dx} \{f(x) = \frac{1}{1+x^2}\}$, then $\frac{d}{dx} \{f(x^3)\}$ is

- a) $\frac{3x}{1+x^3}$ b) $\frac{3x^2}{1+x^6}$ c) $\frac{-6x^5}{(1+x^6)^2}$ d) $\frac{-6x^5}{1+x^6}$
355. $\int_0^\pi [\cot x] dx$, [.] denotes the greatest integer function, is equal to
a) $\frac{\pi}{2}$ b) 1 c) -1 d) $-\frac{\pi}{2}$
356. $\int_{-3}^2 \{|x+1| + |x+2| + |x-1|\} dx$ is equal to
a) $\frac{31}{2}$ b) $\frac{35}{2}$ c) $\frac{47}{2}$ d) $\frac{39}{2}$
357. $\int_0^3 |x^3 + x^2 + 3x| dx$ is equal to
a) $\frac{171}{2}$ b) $\frac{171}{4}$ c) $\frac{170}{4}$ d) $\frac{170}{3}$
358. $\int \frac{dx}{\sin x \cos x}$ is equal to
a) $\log |\sin x| + c$ b) $\log |\tan x| + c$ c) $\log |\sec x| + c$ d) None of these
359. $\int_0^{2n\pi} \left\{ |\sin x| - \left| \frac{1}{2} \sin x \right| \right\} dx$ equals
a) n b) $2n$ c) $-2n$ d) None of these
360. If $\int_a^b x^3 dx = 0$ and if $\int_a^b x^2 dx = \frac{2}{3}$, then the values of a and b are respectively
a) 1, 1 b) -1, -1 c) 1, -1 d) -1, 1
361. The value of $\int_0^{\pi/2} \operatorname{cosec}(x - \pi/3) \operatorname{cosec}(x - \pi/6) dx$, is
a) $2 \log 3$ b) $-2 \log 3$ c) $\log 3$ d) None of these
362. The primitive function of the function $f(x) = \frac{\sqrt{(a^2-x^2)}}{x^4}$ is
a) $c + \frac{\sqrt{a^2-x^2}}{3a^2x^3}$ b) $c - \frac{(a^2-x^2)^{3/2}}{2a^2x^2}$ c) $c - \frac{(a^2-x^2)^{3/2}}{3a^2x^3}$ d) None of these
363. If $f(x) = \begin{cases} x, & \text{for } x < 1 \\ x-1, & \text{for } x \geq 1 \end{cases}$, then $\int_0^2 x^2 f(x) dx$ is equal to
a) 1 b) $\frac{4}{3}$ c) $\frac{5}{3}$ d) $\frac{5}{2}$
364. $\int \frac{x^2 + x - 6}{(x-2)(x-1)} dx$
a) $x + 2 \log(x-1) + c$ b) $2x + 2 \log(x-1) + c$ c) $x + 4 \log(1-x) + c$ d) $x + 4 \log(x-1) + c$
365. $\int \frac{x^3}{(1+x^2)^{1/3}} dx$ is equal to
a) $\frac{20}{3} (1+x^2)^{2/3} (2x^2-3) + C$
b) $\frac{3}{20} (1+x^2)^{2/3} (2x^2-3) + C$
c) $\frac{3}{20} (1+x^2)^{2/3} (2x^2+3) + C$
d) None of these
366. The equation $\int_{-\pi/4}^{\pi/4} \left\{ a|\sin x| + \frac{b \sin x}{1+\cos x} + c \right\} dx = 0$, where a, b, c are constants, gives a relation between
a) a, b and c b) a and c c) a and b d) b and c
367. The value of $\int_2^4 \{|x-2| + |x-3|\} dx$ is
a) 1 b) 2 c) 3 d) 5
368. If $g(x) = \int_0^x \cos^4 t dt$, then $g(x+\pi)$ equals
a) $g(x) + g(\pi)$ b) $g(x) - g(\pi)$ c) $g(x)g(\pi)$ d) $\frac{g(x)}{g(\pi)}$
369. $\int \cos^3 x e^{\log(\sin x)} dx$ is equal to

- a) $-\frac{\sin^4 x}{4} + C$ b) $-\frac{\cos^4 x}{4} + C$ c) $\frac{e^{\sin x}}{4} + C$ d) None of these
370. $\int_8^{15} \frac{dx}{(x-3)\sqrt{x+1}}$ is equal to
a) $\frac{1}{2} \log \frac{5}{3}$ b) $\frac{1}{3} \log \frac{5}{3}$ c) $\frac{1}{5} \log \frac{3}{5}$ d) $\frac{1}{2} \log \frac{3}{5}$
371. The value of $\int \frac{ax^2-b}{x\sqrt{c^2x^2-(ax^2+b)^2}} dx$, is
a) $\sin^{-1}\left(\frac{ax + \frac{b}{x}}{c}\right) + k$ b) $\sin^{-1}\left(\frac{ax^2 + \frac{b}{x^2}}{c}\right) + k$ c) $\cos^{-1}\left(\frac{ax + b/x}{c}\right) + k$ d) $\cos^{-1}\left(\frac{ax^2 + \frac{b}{x^2}}{c}\right) + k$
372. If $f(x) = \int_{-1}^x |t| dt$, then for any $x \geq 0$, $f(x)$ equals
a) $\frac{1}{2}(1-x^2)$ b) $\frac{1}{2}x^2$ c) $\frac{1}{2}(1+x^2)$ d) None of these
373. Let $I_1 = \int_1^2 \frac{1}{\sqrt{1+x^2}} dx$ and $I_2 = \int_1^2 \frac{1}{x} dx$. Then
a) $I_1 > I_2$ b) $I_2 > I_1$ c) $I_1 = I_2$ d) $I_1 > 2I_2$
374. The value of $\int_{-\pi}^{\pi} (1-x^2) \sin x \cos^2 x dx$ is
a) 0 b) $\pi - \frac{\pi^3}{3}$ c) $2\pi - \pi^3$ d) $\frac{7}{2} - 2\pi^3$
375. If $I_n = \int_0^{\pi/2} x^n \sin x dx$, then $I_4 + 12I_2$ is equal to
a) 4π b) $3\left(\frac{\pi}{2}\right)^3$ c) $\left(\frac{\pi}{2}\right)^2$ d) $4\left(\frac{\pi}{2}\right)^3$
376. The value of the integral $\int_0^2 x[x] dx$, is
a) $\frac{7}{2}$ b) $\frac{3}{2}$ c) $\frac{5}{2}$ d) None of these
377. $\int_{-1}^0 \frac{dx}{x^2+2x+2}$ is equal to
a) 0 b) $\pi/4$ c) $\pi/2$ d) $-\pi/4$
378. The value of $\int_{-\pi/4}^{\pi/4} x^3 \sin^4 x dx$ is
a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{8}$ d) 0
379. Let f be a positive function. Let $I_1 = \int_{1-k}^k xf\{x(1-x)\}$, $I_2 = \int_{1-k}^k f\{x(1-x)\} dx$ where $2k-1 > 0$. Then, $\frac{I_1}{I_2}$ is
a) 2 b) k c) $\frac{1}{2}$ d) 1
380. If $I_n = \int_0^{\pi/4} \tan^n x dx$, then $\lim_{n \rightarrow \infty} n(I_{n+1} + I_{n-1})$ equals
a) 1 b) 2 c) $\pi/4$ d) π
381. $\int_1^e \frac{1}{x} dx$ is equal to
a) ∞ b) 0 c) 1 d) $\log(1+e)$
382. $\lim_{n \rightarrow \infty} \frac{1^{99} + 2^{99} + \dots + n^{99}}{n^{100}} =$
a) $\frac{99}{100}$ b) $\frac{1}{100}$ c) $\frac{1}{99}$ d) $\frac{1}{101}$
383. $\int \frac{\cos 2x}{\cos x} dx$ is equal to
a) $2 \sin x + \log(\sec x - \tan x) + C$
b) $2 \sin x - \log(\sec x - \tan x) + C$
c) $2 \sin x + \log(\sec x + \tan x) + C$
d) None of these

384. $\int f'(ax + b) \{f(ax + b)\}^n dx$ is equal to
 a) $\frac{1}{n+1} \{f(ax + b)\}^{n+1} + C$, for all n except $n = -1$
 b) $\frac{1}{n+1} \{f(ax + b)\}^{n+1} + C$, for all n
 c) $\frac{1}{a(n+1)} \{f(ax + b)\}^{n+1} + C$, for all n except $n = -1$
 d) $\frac{1}{a(n+1)} \{f(ax + b)\}^{n+1} + C$, for all n
385. If $I_1 = \int_0^1 2x^2 dx$, $I_2 = \int_0^1 2x^3 dx$, $I_3 = \int_1^2 2x^2 dx$ and $I_4 = \int_1^2 2x^3 dx$, then
 a) $I_3 > I_4$ b) $I_3 = I_4$ c) $I_1 > I_2$ d) $I_2 > I_1$
386. $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx = \dots + c$; $x \neq \frac{k\pi}{2}$ and $\tan x > 0$
 a) $\frac{1}{2\sqrt{\tan x}}$ b) $\sqrt{2 \tan x}$ c) $2\sqrt{\tan x}$ d) $\sqrt{\tan x}$
387. $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2+r^2}}$ equals
 a) $1 + \sqrt{5}$ b) $-1 + \sqrt{5}$ c) $-1 + \sqrt{2}$ d) $1 + \sqrt{2}$
388. $\int_0^{\infty} \frac{x \log x dx}{(1+x^2)^2}$ is equal to
 a) 0 b) 1 c) ∞ d) None of these
389. The value of $\int_{-1}^1 [x[1 + \sin \pi x] dx$ is ($[\cdot]$ denotes the greatest integer)
 a) 2 b) 0 c) 1 d) None of these
390. If $\int_0^{\infty} e^{-x^2} dx = \sqrt{\frac{\pi}{2}}$ then $\int_0^{\infty} e^{-ax^2} dx$, $a > 0$ is
 a) $\frac{\sqrt{\pi}}{2}$ b) $\frac{\sqrt{\pi}}{2a}$ c) $2 \frac{\sqrt{\pi}}{a}$ d) $\frac{1}{2} \frac{\sqrt{\pi}}{a}$
391. If $\int \frac{2^x}{\sqrt{1-4^x}} dx = K \sin^{-1}(2^x) + C$, then K is equal to
 a) $\log 2$ b) $\frac{1}{2} \log 2$ c) $\frac{1}{2}$ d) $\frac{1}{\log 2}$
392. The value of the integral $\int_{-\pi}^{\pi} (\cos ax - \sin bx)^2 dx$, where (a and b integers), is
 a) $-\pi$ b) 0 c) π d) 2π
393. $\int \frac{mx^{m+2n-1} - nx^{n-1}}{x^{2m+2n+2} + 2x^{m+n+1}} dx$ is equal to
 a) $\frac{x^m}{x^{m+n} + 1} + c$ b) $\frac{x^n}{x^{m+n} + 1} + c$ c) $\frac{x^{m+n} - 1}{x^{m+n} + 1} + c$ d) $-\frac{x^n}{x^{m+n} + 1} + c$
394. The value of $\int_0^{16\pi/3} |\sin x| dx$, is
 a) 21 b) $21/2$ c) 10 d) 11
395. If $f(x)$ and $g(x)$, $x \in \mathbb{R}$ are continuous functions, then value of integral
 $\int_{-\pi/2}^{\pi/2} [f(x) + f(-x)][g(x) - g(-x)] dx$ is
 a) π b) $\frac{\pi}{2}$ c) 1 d) 0
396. $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$ is equal to
 a) $2\sqrt{\tan x} + C$ b) $2\sqrt{\cot x} + C$ c) $\frac{\sqrt{\tan x}}{2} + C$ d) None of these
397. $\int_0^1 \sin \left\{ 2 \tan^{-1} \sqrt{\frac{1+x}{1-x}} \right\} dx =$
 a) $\pi/6$ b) $\pi/4$ c) $\pi/2$ d) π
398. If $\int_{-\pi/3}^{\pi/3} \left(\frac{a}{3} |\tan x| + \frac{b \tan x}{1 + \sec x} \right) dx = 0$ where a, b, c are constants, then $c =$

- a) $a \ln 2$ b) $\frac{a}{\pi} \ln 2$ c) $-\frac{a}{\pi} \ln 2$ d) $\frac{2a}{\pi} \ln 2$

399. If the tangent to the graph function $y = f(x)$ makes angles $\frac{\pi}{4}$ and $\frac{\pi}{3}$ with the x -axis is at the point $x = 2$ and $x = 4$ respectively, the value of $\int_2^4 f'(x)f''(x) dx$

- a) $f(4)f(2)$ b) $f(4)$ c) $f(2)$ d) 1

400. $\int \frac{dx}{\cos^3 \sqrt{2} \sin 2x}$ is equal to

- a) $\sqrt{\tan x} + \frac{\tan^{5/2} x}{5} + c$ b) $\sqrt{\tan x} + \frac{2}{5} \tan^{5/2} x + c$
 c) $2\sqrt{\tan x} + \frac{2}{5} \tan^{5/2} x + c$ d) None of these

401. Let $f(x) = \int \frac{x^2 dx}{(1+x^2)(1+\sqrt{1+x^2})}$ and $f(0) = 0$. Then, $f(1)$ is

- a) $\log(1 + \sqrt{2})$ b) $\log(1 + \sqrt{2}) - \frac{\pi}{4}$ c) $\log(1 + \sqrt{2}) + \frac{\pi}{4}$ d) None of these

402. The value of the integral $\int_0^1 \frac{1}{(1+x^2)^{3/2}} dx$, is

- a) $1/2$ b) $1/\sqrt{2}$ c) 1 d) $\sqrt{2}$

403. If $I(m, n) = \int_0^1 t^m (1+t)^n dt$, then the expression for $I(m, n)$ in terms of $I(m+1, n-1)$ is

- a) $\frac{2^n}{m+1} - \frac{n}{m+1} I(m+1, n-1)$ b) $\frac{n}{m+1} I(m+1, n-1)$
 c) $\frac{2^n}{m+1} + \frac{n}{m+1} I(m+1, n-1)$ d) $\frac{m}{n+1} I(m+1, n-1)$

404. The value of $\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$ is

- a) $\frac{\pi}{4}$ b) $\frac{\pi^2}{32}$ c) 1 d) None of these

405. The value of $\int_3^5 \frac{x^2}{x^2-4} dx$ is

- a) $2 - \log_e \left(\frac{15}{7}\right)$ b) $2 + \log_e \left(\frac{15}{7}\right)$
 c) $2 + 4 \log_e 3 - 4 \log_e 7 + 4 \log_e 5$ d) $2 - \tan^{-1} \left(\frac{15}{7}\right)$

406. $\int \sqrt{x^2 + a^2} dx$ equals

- a) $\frac{x}{2} \sqrt{x^2 + a^2} - \frac{a^2}{2} \log \{x + \sqrt{x^2 + a^2}\} + c$ b) $\frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log \{x + \sqrt{x^2 + a^2}\} + c$
 c) $\frac{x}{2} \sqrt{x^2 + a^2} - \frac{a^2}{2} \log \{x - \sqrt{x^2 + a^2}\} + c$ d) $\frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log \{x - \sqrt{x^2 + a^2}\} + c$

407. $I_n = \int_0^{\pi/4} \tan^n x dx$, then $\lim_{n \rightarrow \infty} n[I_n + I_{n+2}]$ is equal to

- a) $\frac{1}{2}$ b) 1 c) ∞ d) zero

408. $\int (x^2 + 1)\sqrt{x+1} dx$ is equal to

- a) $\frac{(x+1)^{7/2}}{7} - 2 \frac{(x+1)^{5/2}}{5} + 2 \frac{(x+1)^{3/2}}{3} + c$ b) $2 \left[\frac{(x+1)^{7/2}}{7} - 2 \frac{(x+1)^{5/2}}{5} + 2 \frac{(x+1)^{3/2}}{3} \right] + c$
 c) $\frac{(x+1)^{7/2}}{7} - 2 \frac{(x+1)^{5/2}}{5} + 5$ d) $\frac{(x+1)^{7/2}}{7} - 3 \frac{(x+1)^{5/2}}{5} + 11(x+1)^{1/2} + c$

409. $\int \frac{f'(x)}{f(x) \log[f(x)]} dx$ is equal to

- a) $\frac{f(x)}{\log f(x)} + c$ b) $f(x) \log f(x) + c$ c) $\log[\log f(x)] + c$ d) $\frac{1}{\log[\log f(x)]} + c$

410. Consider the integrals

$I_1 = \int_0^1 e^{-x} \cos^2 x \, dx$, $I_2 = \int_0^1 e^{-x^2} \cos^2 x \, dx$, $I_3 = \int_0^1 e^{-x^2} \, dx$ and $I_4 = \int_0^1 e^{-(1/2)x^2} \, dx$. The greatest of these integral is

- a) I_1 b) I_2 c) I_3 d) I_4
411. $\int \frac{1}{(1+x^2)\sqrt{1-x^2}} dx$ is equal to
a) $\frac{1}{2} \tan^{-1} \left(\frac{\sqrt{2x}}{\sqrt{1-x^2}} \right)$ b) $\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{\sqrt{2x}}{\sqrt{1+x^2}} \right)$ c) $\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{\sqrt{2x}}{\sqrt{1-x^2}} \right)$ d) None of these
412. The value of the integral $\int_{1/n}^{(an-1)/n} \frac{\sqrt{x}}{\sqrt{a-x+\sqrt{x}}} dx$ is
a) $\frac{a}{2}$ b) $\frac{na+2}{2n}$ c) $\frac{na-2}{2n}$ d) None of these
413. The value of $\int e^x (x^5 + 5x^4 + 1) dx$ is
a) $e^x \cdot x^5 + c$ b) $e^x \cdot x^5 + e^x + c$ c) $e^{x+1} \cdot x^5 + c$ d) $5x^4 \cdot e^x + c$
414. If $\int_{-1/2}^{1/2} \cos x \log \left(\frac{1+x}{1-x} \right) dx = k \cdot \log 2$, then k equals
a) 0 b) -1 c) -2 d) $\frac{1}{2}$
415. $\int \sin \sqrt{x} \, dx$ is equal to
a) $\sin \sqrt{x} - \sqrt{x} \cos \sqrt{x}$ b) $2(\sin \sqrt{x} - \sqrt{x} \cos \sqrt{x}) + c$
c) $\cos \sqrt{x} - \sqrt{x} \sin \sqrt{x} + c$ d) $2(\cos \sqrt{x} - \sqrt{x} \sin \sqrt{x}) + c$
416. If $f(x) = \int_x^{x^2} (t-1) dt$, $1 \leq x \leq 2$, then global maximum value of $f(x)$ is
a) 1 b) 2 c) 3 d) 4
417. $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \frac{r}{\sqrt{n^2+r^2}}$ is equal to
a) $1 - \sqrt{2}$ b) $\sqrt{2} - 1$ c) $\sqrt{2}$ d) $-\sqrt{2}$
418. The value of $\int_0^{2\pi} \cos^{99} x \, dx$, is
a) 1 b) -1 c) 99 d) 0
419. The value of $\int_{-2}^2 (ax^3 + bx + c) dx$ depends on the
a) Value of b b) Value of c c) Value of a d) Values of a and b
420. The value of integral $\int_0^\pi x f(\sin x) \, dx$ is
a) 0 b) $\pi \int_0^{\pi/2} f(\sin x) \, dx$ c) $\frac{\pi}{4} \int_0^\pi f(\sin x) \, dx$ d) None of these
421. The value of the integral $\int \frac{\log(x+1) - \log x}{x(x+1)} dx$ is
a) $-\frac{1}{2} [\log(x+1)]^2 - \frac{1}{2} (\log x)^2 + \log(x+1) \log x + c$
b) $-[\{\log(x+1)\}^2 - (\log x)^2] + \log(x+1) \log x + c$
c) $\left[\log \left(1 + \frac{1}{x} \right) \right]^2 + c$
d) None of the above
422. The value of $I = \int_0^1 x \left| x - \frac{1}{2} \right| dx$ is
a) $\frac{1}{3}$ b) $\frac{1}{4}$ c) $\frac{1}{8}$ d) None of these
423. $\int \frac{a^{x/2}}{\sqrt{a^{-x} - a^x}} dx =$
a) $\frac{1}{\log a} \sin^{-1}(a^x)$ b) $\frac{1}{\log a} \tan^{-1}(a^x)$ c) $2\sqrt{a^{-x} - a^x}$ d) $\log(a^x - 1)$
424. If $\int_{\log 2}^x \frac{du}{(e^u - 1)^{1/2}} = \frac{\pi}{6}$, then e^x is equal to

- a) 1 b) 2 c) 4 d) -1
425. $\int \{f(x)g''(x) - f''(x)g(x)\}dx$ is equal to
 a) $\frac{f(x)}{g'(x)}$
 b) $f'(x)g(x) - f(x)g'(x)$
 c) $f(x)g'(x) - f'(x)g(x)$
 d) $f(x)g'(x) + f'(x)g(x)$
426. The value of $\int_{-\pi/4}^{\pi/4} x^3 \sin^4 x \, dx$, is equal to
 a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{8}$ d) 0
427. The value of $\int_0^{\pi/2} \sin^8 x \, dx$ is
 a) $\frac{105\pi}{32(4!)}$ b) $\frac{105\pi}{14(4!)}$ c) $\frac{105}{16\pi(4!)}$ d) None of these
428. $\int_{-\pi/2}^{\pi/2} \frac{|x|}{8 \cos^2 2x+1} dx$ has the value
 a) $\frac{\pi^2}{6}$ b) $\frac{\pi^2}{12}$ c) $\frac{\pi^2}{24}$ d) None of these
429. If $\int f(x) \cos x \, dx = \frac{1}{2}\{f(x)\}^2 + c$, then $f(x)$ is
 a) $x + c$ b) $\sin x + c$ c) $\cos x + c$ d) c
430. If $f(x) = \frac{x+2}{2x+3}$. Then, $\int \left(\frac{f(x)}{x^2}\right)^{1/2} dx$ is equal to $\frac{1}{\sqrt{2}}g\left(\frac{1+\sqrt{2f(x)}}{1-\sqrt{2f(x)}}\right) - \sqrt{\frac{2}{3}}h\left(\frac{\sqrt{3f(x)+\sqrt{2}}}{\sqrt{3f(x)-\sqrt{2}}}\right) + c$ where
 a) $g(x) = \tan^{-1} x, h(x) = \log|x|$
 b) $g(x) = \log|x|, h(x) = \tan^{-1} x$
 c) $g(x) = h(x) = \tan^{-1} x$
 d) $g(x) = \log|x|, h(x) = \log|x|$
431. $\int 5^{5^{5^x}} \cdot 5^{5^x} \cdot 5^x \, dx$ is equal to
 a) $\frac{5^{5^x}}{(\log 5)^3} + C$ b) $5^{5^{5^x}} (\log 5)^3 + C$ c) $\frac{5^{5^{5^x}}}{(\log 5)^3} + C$ d) None of these
432. If $f(t) = \int_{-t}^t \frac{e^{-|x|}}{2} dx$, then $\lim_{t \rightarrow \infty} f(t)$ is equal to
 a) 1 b) $\frac{1}{2}$ c) 0 d) -1
433. $\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$ is equal to
 a) $\frac{-1}{\sin x + \cos x} + C$
 b) $\log(\sin x + \cos x) + C$
 c) $\log(\sin x - \cos x) + C$
 d) $\log(\sin x + \cos x)^2 + C$
434. If the primitive of $\sin^{-3/2} x \sin^{-1/2}(x + \theta)$ is $-2 \operatorname{cosec} \theta \sqrt{f(x)} + c$, then
 a) $f(x) = \frac{\sin x}{\sin(x + \theta)}$ b) $f(x) = \tan(x + \theta)$ c) $f(x) = \frac{\sin(x + \theta)}{\sin x}$ d) $f(x) = \frac{\tan(x + \theta)}{\sin x}$
435. $\int \frac{x^4-1}{x^2 \sqrt{x^4+x^2+1}} dx$ is equal to
 a) $\frac{x}{\sqrt{x^4+x^2+1}} + C$ b) $\frac{\sqrt{x^4+x^2+1}}{x} + C$ c) $\frac{2x}{\sqrt{x^4+x^2+1}} + C$ d) $\frac{\sqrt{x^4+x^2+1}}{2x} + C$
436. $\int 32x^3(\log x)^2 \, dx$ is equal to
 a) $8x^4(\log x)^2 + c$ b) $x^4\{8(\log x)^2 - 4 \log x + 1\} + c$
 c) $x^4\{8(\log x)^2 - 4 \log x\} + c$ d) $x^3\{(\log x)^2 + 2 \log x\} + c$
437. $\int \frac{e^x(1+\sin x)}{1+\cos x} dx$ is equal to
 a) $e^x \tan\left(\frac{x}{2}\right) + c$ b) $e^x \tan x + c$ c) $e^x \left(\frac{1+\sin x}{1-\cos x}\right) + c$ d) $c - e^x \cot\left(\frac{x}{2}\right)$

438. If $\int \log(\sqrt{1-x} + \sqrt{1+x}) dx = x f(x) + Ax + B \sin^{-1} x + C$, then
- $f(x) = \log(\sqrt{1-x} + \sqrt{1+x})$
 - $A = 1/3$
 - $B = 2/3$
 - $B = -1/2$
439. $\int_0^{\pi/4} \log(1 + \tan x) dx$ is equal to
- $\frac{\pi}{8} \log_e 2$
 - $\frac{\pi}{4} \log_2 e$
 - $\frac{\pi}{4} \log_e 2$
 - $\frac{\pi}{8} \log_e \left(\frac{1}{2}\right)$
440. $\int_{-1}^2 \sin^3 x |x| dx$ is equal to
- 0
 - 1
 - 4
 - None of these
441. $\int_0^{1000} e^{x-[x]} dx$ is
- $e^{1000} - 1$
 - $\frac{e^{1000} - 1}{e - 1}$
 - $1000(e - 1)$
 - $\frac{e - 1}{1000}$
442. $\int \frac{(\tan^{-1} x)^3}{(1+x^2)} dx$ is equal to
- $3(\tan^{-1} x)^2 + c$
 - $\frac{(\tan^{-1} x)^4}{4} + c$
 - $(\tan^{-1} x)^4 + c$
 - None of these
443. The value of $\int_0^{\pi} |\sin^3 \theta| d\theta$ is
- 0
 - π
 - $\frac{4}{3}$
 - $\frac{3}{8}$
444. If $d[f(x)] = e^{\tan x} \sec^2 x dx$, then $f(x)$ is equal to
- $e^{\tan x} + c$
 - $e^{\sec^2 x} + c$
 - $e^{\sin x} + c$
 - None of these
445. The value of $\int_1^2 \{f(g(x))\}^{-1} f'(g(x)) g'(x) dx$, where $g(1) = g(2)$, is equal to
- 1
 - 2
 - 0
 - None of these
446. Let f be a function such that $f(1) = 4$ and $f'(x) \geq 2$ for $1 \leq x < 4$. How small can $f(4)$ possibly be?
- 8
 - 12
 - 16
 - 10
447. $\int \frac{f'(x)}{f(x) \log[f(x)]} dx$ is equal to
- $\frac{f(x)}{\log f(x)} + c$
 - $f(x) \cdot \log f(x) + c$
 - $\log[\log f(x)] + c$
 - $\frac{1}{\log[\log f(x)]} + c$
448. $\int_0^{\pi/2} \frac{2\sqrt{\cos \theta}}{3(\sqrt{\sin \theta} + \sqrt{\cos \theta})} d\theta$ is equal to
- $\frac{\pi}{6}$
 - $\frac{\pi}{4}$
 - $\frac{\pi}{2}$
 - None of these
449. The value of $\lim_{n \rightarrow \infty} \left[\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \dots + \frac{n}{n^2+n^2} \right]$
- $\frac{\pi}{4}$
 - $\log 2$
 - 0
 - 1
450. The value of $\int_{-\pi}^{\pi} \sin x f(\cos x) dx$ is
- π
 - 2π
 - $2f(1)$
 - None of these
451. If $\int \frac{\sin x}{\cos x(1+\cos x)} dx = f(x) + c$, then $f(x)$ is equal to
- $\log \left| \frac{1 + \cos x}{\cos x} \right|$
 - $\log \left| \frac{\cos x}{1 + \cos x} \right|$
 - $\log \left| \frac{\sin x}{1 + \sin x} \right|$
 - $\log \left| \frac{1 + \sin x}{\sin x} \right|$
452. If $f(x)$ satisfies the requirements of Rolle's Theorem in $[1, 2]$ and $f'(x)$ is continuous in $[1, 2]$, then $\int_1^2 f'(x) dx$ is equal to
- 0
 - 1
 - 3
 - 1

453. The value of $\int e^{2x}(2 \sin 3x + 3 \cos 3x)dx$ is
 a) $e^{2x} \sin 3x + c$ b) $e^{2x} \cos 3x + c$ c) $e^{2x} + c$ d) $e^{2x} (2 \sin 3x) + c$
454. $\int_{-2}^2 |x| dx$ is equal to
 a) 1 b) 2 c) 3 d) 4
455. If $\int_0^{\pi/2} \cos^n x \sin^n x dx = \lambda \int_0^{\pi/2} \sin^n x dx$, then $\lambda =$
 a) $\frac{1}{2^{n-1}}$ b) $\frac{1}{2^{n+1}}$ c) $\frac{1}{2^n}$ d) $\frac{1}{2}$
456. If x satisfies the equation

$$x^2 \left(\int_0^1 \frac{dt}{t^2 + 2t \cos \alpha + 1} \right) - x \left(\int_{-3}^3 \frac{t^2 \sin 2t}{t^2 + 1} dt \right) - 2 = 0$$
 ($0 < \alpha < \pi$), then the value of x is
 a) $\pm 2 \sqrt{\frac{\sin \alpha}{\alpha}}$ b) $\pm \sqrt{\frac{\sin \alpha}{\alpha}}$ c) $\pm 4 \sqrt{\frac{\sin \alpha}{\alpha}}$ d) None of these
457. $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$ is equal to
 a) $\frac{\pi}{12}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{4}$
458. $\int_0^2 [x^2] dx$ is
 a) $2 - \sqrt{2}$ b) $2 + \sqrt{2}$ c) $\sqrt{2} - 1$ d) $-\sqrt{2} - \sqrt{3} + 5$
459. If $f(x) \sin x \cos x dx = \frac{1}{2(b^2 - a^2)} \log f(x) + c$, the $f(x)$ is equal to
 a) $\frac{1}{a^2 \sin^2 x + b^2 \cos^2 x}$ b) $\frac{1}{a^2 \sin^2 x - b^2 \cos^2 x}$ c) $\frac{1}{a^2 \cos^2 x + b^2 \sin^2 x}$ d) $\frac{1}{a^2 \cos^2 x - b^2 \sin^2 x}$
460. If $f(x) = f(a - x)$, then $\int_0^a f(x) dx$ is equal to
 a) $\int_0^a f(x) dx$ b) $\frac{a^2}{2} \int_0^a f(x) dx$ c) $\frac{a}{2} \int_0^a f(x) dx$ d) $-\frac{a}{2} \int_0^a f(x) dx$
461. The value of the integral $\int_{1/e}^e |\log x| dx$, is
 a) $2 \left(\frac{e-1}{e} \right)$ b) $2 \left(\frac{1-e}{e} \right)$ c) $2 - \frac{1}{e}$ d) None of these
462. $\int_0^{\pi/8} \cos^3 4\theta d\theta$ is equal to
 a) $\frac{5}{3}$ b) $\frac{5}{4}$ c) $\frac{1}{3}$ d) $\frac{1}{6}$
463. $\int_{-1}^1 \frac{\cosh x}{1+e^{2x}} dx$ is equal to
 a) 0 b) 1 c) $\frac{e^2 - 1}{2e}$ d) $\frac{e^2 + 2}{2e}$
464. If $u_{10} = \int_0^{\pi/2} x^{10} \sin x dx$, then the value of $u_{10} + 90 u_8$, is
 a) $9 \left(\frac{\pi}{2} \right)^8$ b) $\left(\frac{\pi}{2} \right)^9$ c) $10 \left(\frac{\pi}{2} \right)^9$ d) $9 \left(\frac{\pi}{2} \right)^9$
465. $\int \frac{x^3 \sin[\tan^{-1}(x^4)]}{1+x^8} dx$ is equal to
 a) $\frac{1}{4} \cos[\tan^{-1}(x^4)] + c$ b) $\frac{1}{4} \sin[\tan^{-1}(x^4)] + c$
 c) $-\frac{1}{4} \cos[\tan^{-1}(x^4)] + c$ d) $\frac{1}{4} \sec^{-1}[\tan^{-1}(x^4)] + c$
466. $\left[\sum_{n=1}^{10} \int_{-2n-1}^{-2n} \sin^{27} x dx \right] + \left[\sum_{n=1}^{10} \int_{2n}^{2n+1} \sin^{27} x dx \right]$ equals
 a) 27^2 b) -54 c) 54 d) 0

467. $\int \frac{1}{\sin^2 x \cos^2 x} dx$ is equal to
 a) $\sin x - \cos x + c$ b) $\tan x + \cot x + c$ c) $\cos x + \sin x + c$ d) $\tan x - \cot x + c$
468. The value of the integral $\int_{-1}^1 (x - [2x]) dx$, is
 a) 1 b) 0 c) 2 d) 4
469. The function $F(x) = \int_0^x \log\left(\frac{1-x}{1+x}\right) dx$, is
 a) An even function b) An odd function c) A periodic function d) None of these
470. If $\int \frac{1}{x\sqrt{1-x^3}} dx = a \log\left|\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1}\right| + b$, then a is equal to
 a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) $-\frac{1}{3}$ d) $-\frac{2}{3}$
471. $\frac{d}{dx} \left(\int_{f(x)}^{g(x)} \phi(t) dt \right)$ is equal to
 a) $\phi(g(x)) - \phi(f(x))$
 b) $\frac{1}{2} [\phi(g(x))]^2 - \frac{1}{2} [\phi(f(x))]^2$
 c) $g'(x)\phi(g(x)) - f'(x)\phi(f(x))$
 d) $\phi'(g(x))g'(x) - \phi'(f(x))f'(x)$
472. $\int_1^3 (x-1)(x-2)(x-3) dx$ is equal to
 a) 3 b) 2 c) 1 d) 0
473. If $f(x) = \begin{vmatrix} \sin x + \sin 2x + \sin 3x & \sin 2x & \sin 3x \\ 3 + 4 \sin x & 3 & 4 \sin x \\ 1 + \sin x & \sin x & 1 \end{vmatrix}$, then the value of $\int_0^{\pi/2} f(x) dx$ is
 a) 3 b) $\frac{2}{3}$ c) $\frac{1}{3}$ d) 0
474. If $\int g(x) dx = g(x)$, then $\int g(x)\{f(x) + f'(x)\} dx$ is equal to
 a) $g(x)f(x) - g(x)f'(x) + C$
 b) $g(x)f'(x) + C$
 c) $g(x)f(x) + C$
 d) $g(x)f^2(x) + C$
475. $\int \left(\frac{x+2}{x+4}\right)^2 e^x dx$ is equal to
 a) $e^x \left(\frac{x}{x+4}\right) + c$ b) $e^x \left(\frac{x+2}{x+4}\right) + c$ c) $e^x \left(\frac{x-2}{x+4}\right) + c$ d) $\left(\frac{2xe^x}{x+4}\right) + c$
476. If $P = \int_0^{3\pi} f(\cos^2 x) dx$ and $Q = \int_0^{\pi} f(\cos^2 x) dx$, then
 a) $P - Q = 0$ b) $P - 2Q = 0$ c) $P - 3Q = 0$ d) $P - 5Q = 0$
477. If $I_1 = \int_0^{3\pi} f(\cos^2 x) dx$ and $I_2 = \int_0^{\pi} f(\cos^2 x) dx$ then
 a) $I_1 = I_2$ b) $I_1 = 2I_2$ c) $I_1 = 5I_2$ d) None of these
478. For any $n \in \mathbb{N}$ and $x \in \mathbb{R}^+$, the value of the integral $\int_0^{n[x]} (x - [x]) dx$, is
 a) $n[x]$ b) $[x]$ c) $\frac{n}{2}[x]$ d) None of these
479. $\int_0^1 \frac{x^3 dx}{(x^2+1)^{3/2}}$ is equal to
 a) $(\sqrt{2} - 1)^2$ b) $\frac{(\sqrt{2} - 1)^2}{2}$ c) $\frac{\sqrt{2} - 1}{2}$ d) None of these
480. The value of $\int_0^{2\pi} |\cos x - \sin x| dx$, is
 a) $\frac{4}{\sqrt{2}}$ b) $2\sqrt{2}$ c) $\frac{2}{\sqrt{2}}$ d) $4\sqrt{2}$
481. The value of $\int_{-\pi/2}^{\pi/2} (x^3 + x \cos x + \tan^5 x + 1) dx$ is equal to

496. $\int \sqrt[3]{x} \sqrt{1 + \sqrt[3]{x^4}} dx$ is equal to
 a) $\frac{21}{32} \{1 + \sqrt[3]{x^4}\}^{8/7} + C$ b) $\frac{32}{21} \{1 + \sqrt[3]{x^4}\}^{8/7} + C$ c) $\frac{7}{32} \{1 + \sqrt[3]{x^4}\}^{8/7} + C$ d) None of these
497. If $\int \sin^{-1} \left(\frac{2x}{1+x^2} \right) dx = f(x) - \log(1+x^2) + c$, then $f(x)$ is equal to
 a) $2x \tan^{-1} x$ b) $-2x \tan^{-1} x$ c) $x \tan^{-1} x$ d) $-x \tan^{-1} x$
498. If $I_{10} = \int_0^{\pi/2} x^{10} \sin x dx$. Then, the value of $I_{10} + 90I_8$ is
 a) $10 \left(\frac{\pi}{2}\right)^3$ b) $10 \left(\frac{\pi}{2}\right)^9$ c) $\frac{\pi}{2}$ d) 0
499. $\int_0^{\pi} \frac{x dx}{a^2 \cos^2 x + b^2 \sin^2 x}$ is equal to
 a) $\frac{\pi}{ab}$ b) $\frac{\pi}{2ab}$ c) $\frac{\pi^2}{ab}$ d) $\frac{\pi^2}{2ab}$
500. Let $f(x) = x - [x]$, for every real x , where $[x]$ is the greatest integer less than or equal to x . Then,
 $\int_{-1}^1 f(x) dx$ is
 a) 1 b) 2 c) 3 d) 0
501. $\int_{-\pi}^{\pi} [\cos px - \sin qx]^2 dx$, where p, q are integers is equal to
 a) $-\pi$ b) 0 c) π d) 2π
502. $\lim_{n \rightarrow \infty} \sum_{r=1}^n \left| \frac{r^3}{r^4 + n^4} \right|$ equals
 a) $\log 2$ b) $\frac{1}{2} \log 2$ c) $\frac{1}{3} \log 2$ d) $\frac{1}{4} \log 2$
503. If $\int_{\pi/2}^x \sqrt{3 - 2 \sin^2 u} du + \int_0^y \cos t dt = 0$ then, $\frac{dy}{dx}$ is equal to
 a) $\frac{\sqrt{4 - 3 \sin^2 x}}{\cos y}$ b) $-\frac{\sqrt{3 - 2 \sin^2 x}}{\cos y}$ c) $\sqrt{3 - 2 \sin^2 x} + \cos y$ d) None of these
504. If $I_n = \int \tan^n x dx$ for $n \geq 2$, then $I_n + I_{n-2}$ is equal to
 a) $\tan^n x + c$ b) $\frac{(\tan^{n-1} x)}{n-1} + c$ c) $\frac{\tan^n x}{n} + c$ d) $n \tan^n x + c$
505. The value of $\int_0^{\pi} \frac{\sin\left(n + \frac{1}{2}\right)x}{\sin\left(\frac{x}{2}\right)} dx$, is
 a) $\frac{\pi}{2}$ b) 0 c) π d) 2π
506. $\int (\sin^6 x + \cos^6 x + 3 \sin^2 x \cos^2 x) dx$ is equal to
 a) $x + c$ b) $\frac{3}{2} \sin 2x + c$
 c) $-\frac{3}{2} \cos 2x + c$ d) $\frac{1}{3} \sin 3x - \cos 3x + c$
507. $\int_a^b \frac{|x|}{x} dx$, $a < 0 < b$, is equal to
 a) $|b| - |a|$ b) $|b| + |a|$ c) $|a - b|$ d) None of these
508. $\int_0^{10\pi} |\sin x| dx$ is
 a) 20 b) 8 c) 10 d) 18
509. If $\int_0^1 f(x) dx = M$, $\int_0^1 g(x) dx = N$, then which of the following is correct?
 a) $\int_0^1 (f(x) + g(x)) dx = M + N$
 b) $\int_0^1 f(x)g(x) dx = MN$

$$c) \int_0^1 \frac{1}{f(x)} dx = \frac{1}{M}$$

$$d) \int_0^1 \frac{f(x)}{g(x)} dx = \frac{M}{N}$$

510. $\int \frac{(\sec^2 x - 7)}{\sin^7 x} dx$ is equal to

- a) $\frac{\tan x}{\sin^7 x} + c$ b) $\frac{\cos x}{\sin^7 x} + c$ c) $\frac{\sin x}{\cos^7 x} + c$ d) $\frac{\sin x}{\tan^7 x} + c$

511. The value of $\int_{-1/2}^{1/2} \left| x \cos\left(\frac{\pi x}{2}\right) \right| dx$, is

- a) $\frac{\pi\sqrt{2} - 4\sqrt{2} - 8}{\pi^2}$ b) $\frac{\sqrt{2} + 4\pi\sqrt{2} - 8}{\pi^2}$ c) $\frac{\pi\sqrt{2} + 4\sqrt{2} + 8}{\pi^2}$ d) None of these

512. $\int \frac{\log_e(\tan x)}{\sin x \cos x} dx$ is equal to

- a) $[\log_e(\tan x)]^2 + c$ b) $\frac{1}{2}(\log_e \tan x)^2 + c$ c) $\log_e(\log_e \tan x) + c$ d) $\log_e \tan x + c$

513. $\int \left(\frac{x+2}{x+4}\right)^2 e^x dx$ is equal to

- a) $e^x \left(\frac{x}{x+4}\right) + c$ b) $e^x \left(\frac{x+2}{x+4}\right) + c$ c) $e^x \left(\frac{x-2}{x+4}\right) + c$ d) $\left(\frac{2xe^x}{x+4}\right) + c$

514. $\int \frac{1}{\sin(x-a) \cos(x-b)} dx$ is equal to

- a) $\frac{1}{\sin(a-b)} \log \left| \frac{\sin(x-a)}{\cos(x-b)} \right| + C$
 b) $\frac{1}{\cos(a-b)} \log \left| \frac{\sin(x-a)}{\cos(x-b)} \right| + C$
 c) $\frac{1}{\sin(a+b)} \log \left| \frac{\sin(x-a)}{\cos(x-b)} \right| + C$
 d) $\frac{1}{\cos(a+b)} \log \left| \frac{\sin(x-a)}{\cos(x-b)} \right| + C$

515. $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x} dx =$

- a) 1 b) 0 c) -1 d) None of these

516. The value of $\int_{-1}^3 \left[\tan^{-1}\left(\frac{x}{x^2+1}\right) + \tan^{-1}\left(\frac{x^2+1}{x}\right) \right] dx$ is

- a) 2π b) π c) $\pi/2$ d) $\pi/4$

517. If for every integer n $\int_n^{n+1} f(x) dx = n^2$, then the value of $\int_{-2}^4 f(x) dx$, is

- a) 16 b) 14 c) 19 d) None of these

518. $\int_0^{2\pi} (\sin x + |\sin x|) dx$ is equal to

- a) 4 b) 0 c) 1 d) 8

519. If $f(x) = \lim_{y \rightarrow x} \frac{\sin^2 y - \sin^2 x}{y^2 - x^2}$, then $\int 4x f(x) dx$ is equal to

- a) $\cos 2x + c$ b) $2 \cos 2x + c$ c) $-\cos 2x + c$ d) $-2 \cos 2x + c$

520. $\int \frac{x^4+1}{(1-x^4)^{3/2}} dx$ is equal to

- a) $\frac{x}{\sqrt{1-x^4}} + C$ b) $\frac{-x}{\sqrt{1-x^4}} + C$ c) $\frac{2x}{\sqrt{1-x^4}} + C$ d) $\frac{-2x}{\sqrt{1-x^4}} + C$

521. $\int \frac{x e^x}{(1+x)^2} dx$ is equal to

- a) $\frac{e^x}{x+1} + C$ b) $e^x(x+1) + C$ c) $-\frac{e^x}{(x+1)^2} + C$ d) $\frac{e^x}{1+x^2} + C$
522. $\int_{-\pi/2}^{\pi/2} \sin|x| dx$ is equal to
a) 0 b) 1 c) 2 d) π
523. $\int \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x}$ is equal to
a) $\frac{1}{ab} \tan^{-1}\left(\frac{a \tan x}{b}\right) + c$ b) $\frac{a}{b} \tan^{-1}\left(\frac{a \tan x}{b}\right) + c$
c) $\frac{b}{a} \tan^{-1}\left(\frac{b \tan x}{a}\right) + c$ d) None of these
524. If $\int \frac{\sin^8 x - \cos^8 x}{1 - 2 \sin^2 x \cos^2 x} dx = A \sin 2x + B$, then
a) $A = -\frac{1}{2}$ b) $A = \frac{1}{2}$ c) $A = -1$ d) $A = 1$
525. $\int \frac{4^{x+1} - 7^{x-1}}{28^x} dx$ is equal to
a) $\frac{1}{7 \log_e 4} 4^{-x} - \frac{4}{\log_e 7} 7^{-x} + c$ b) $\frac{1}{7 \log_e 4} 4^{-x} + \frac{4}{\log_e 7} 7^{-x} + c$
c) $\frac{4^{-x}}{\log_e 7} - \frac{7^{-x}}{\log_e 4} + c$ d) $\frac{4^{-x}}{\log_e 4} - \frac{7^{-x}}{\log_e 7} + c$
526. If $\int \frac{dx}{5+4 \cos x} = A \tan^{-1}(B \tan x/2) + C$, then
a) $A = 1, B = 1/3$ b) $A = 2/3, B = 1/3$ c) $A = -1, B = 1/3$ d) $A = 1/3, B = 2/3$
527. Let $f(x)$ be a function such that, $f(0) = f'(0) = 0, f''(x) = \sec^4 x + 4$, then the function is
a) $\log|(\sin x)| + \frac{1}{3} \tan^3 x + x$ b) $\frac{2}{3} \log|(\sec x)| + \frac{1}{6} \tan^2 x + 2x^2$
c) $\log|(\cos x)| + \frac{1}{6} \cos^2 x + \frac{x^2}{5}$ d) None of the above
528. $\int 32x^3(\log x)^2 dx$ is equal to
a) $8x^4(\log x)^2 + c$ b) $x^4\{8(\log x)^2 - 4 \log x + 1\} + c$
c) $x^4\{8(\log x)^2 - 4 \log x\} + c$ d) $x^3\{(\log x)^2 - 2 \log x\} + c$
529. $\int_1^{\frac{4\sqrt{3}-1}{5}} \frac{x+2}{\sqrt{x^2+2x-3}} dx$ equals to
a) $\frac{2\sqrt{3}}{3} - \frac{1}{2} \log 3$ b) $\frac{2\sqrt{3}}{3} + \frac{1}{2} \log 3$ c) $\frac{2\sqrt{3}}{3} - \frac{1}{2} \log(\sqrt{3} + 2)$ d) $\frac{2\sqrt{3}}{3} + \frac{1}{2} \log(\sqrt{3} + 2)$
530. $\int_{-2}^2 |x| dx$ is equal to
a) 1 b) 2 c) 3 d) 4
531. Let $F(x) = f(x) + f\left(\frac{1}{x}\right)$, where
 $f(x) = \int_1^x \frac{\log t}{1+t} dt$. Then, $F(e)$ equals
a) $1/2$ b) 0 c) 1 d) 2
532. $\int_{\pi/6}^{\pi/3} \frac{1}{1 + \tan^3 x} dx$ is
a) $\frac{\pi}{12}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{6}$
533. If $\int f(x) dx = g(x) + c$, then $\int f^{-1}(x) dx$ is equal to
a) $xf^{-1}(x) + c$ b) $f\{g^{-1}(x)\} + c$
c) $xf^{-1}(x) - g\{f^{-1}(x)\} + c$ d) $g^{-1}(x) + c$
534. $\int x \log x dx$ is equal to

- a) $\frac{1}{(x+4)^2} + c$ b) $\frac{e^x}{(x+4)^2} + c$ c) $\frac{e^x}{x+4} + c$ d) $\frac{e^x}{x+3} + c$
550. $\int_0^{\pi/2} x \sin^2 x \cos^2 x \, dx$ is equal to
a) $\frac{\pi^2}{32}$ b) $\frac{\pi^2}{16}$ c) $\frac{\pi}{32}$ d) None of these
551. If $\int uv'' \, dx = uv' - vu' + a$, then a is equal to
a) $\int u'' v \, dx$ b) $\int u' v \, dx$ c) $\int uv' \, dx$ d) $\int u'' \, dx$
552. $\int (\sqrt[3]{x}) (\sqrt[5]{1 + \sqrt[3]{x^4}}) \, dx$ is equal to
a) $(1 + x^{\frac{4}{3}})^{\frac{6}{5}} + c$ b) $(1 + x^{\frac{4}{3}})^{\frac{6}{5}} + c$ c) $\frac{5}{8} (1 + x^{\frac{4}{3}})^{\frac{6}{5}} + c$ d) $\frac{1}{6} (1 + x^{\frac{4}{3}})^{\frac{6}{5}} + c$
553. $\int \frac{1}{\cos x - \sin x} \, dx$ is equal to
a) $\frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} + \frac{3\pi}{8} \right) \right| + C$
b) $\frac{1}{\sqrt{2}} \log \left| \cot \frac{x}{2} \right| + C$
c) $\frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} - \frac{3\pi}{8} \right) \right| + C$
d) $\frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} - \frac{\pi}{8} \right) \right| + C$
554. The value of $\int_0^1 (1 + e^{-x^2}) \, dx$, is
a) -1 b) 2 c) $1 + e^{-1}$ d) None of these
555. The value of $\int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{1 + \sin 2x}} \, dx$ is
a) 0 b) 1 c) 2 d) 3
556. $\int \sqrt{x} e^{\sqrt{x}} \, dx$ is equal to
a) $2\sqrt{x} - e^{\sqrt{x}} - 4\sqrt{x e^{\sqrt{x}}} + c$ b) $(2x - 4\sqrt{x} + 4)e^{\sqrt{x}} + c$
c) $(2x + 4\sqrt{x} + 4)e^{\sqrt{x}} + c$ d) $(1 - 4\sqrt{x})e^{\sqrt{x}} + c$
557. The value of $\lim_{n \rightarrow \infty} \left\{ \left(1 + \frac{1}{n^2}\right) \left(1 + \frac{2^2}{n^2}\right) \dots \left(1 + \frac{n^2}{n^2}\right) \right\}^{1/n}$, is
a) $\frac{e^{\pi/2}}{2e^2}$ b) $2e^2 e^{\pi/2}$ c) $\frac{2}{e^2} e^{\pi/2}$ d) None of these
558. The solution of the equation $\int^x \sqrt{2} \frac{1}{x\sqrt{x^2-1}} \, dx = \frac{\pi}{12}$, is
a) $x = 3$ b) $x = 4$ c) $x = 1$ d) None of these
559. If $f(a+x) = f(x)$, then $\int_0^{na} f(x) \, dx$ is equal to ($n \in N$)
a) $(n-1) \int_0^a f(x) \, dx$ b) $n \int_0^a f(x) \, dx$ c) $\int_0^{(n-1)a} f(x) \, dx$ d) None of these
560. The value of the integral $\int_{-\pi/3}^{\pi/3} \frac{x \sin x}{\cos^2 x} \, dx$, is
a) $(\pi/3 - \log \tan 3\pi/2)$
b) $2(2\pi/3 - \log \tan 5\pi/12)$
c) $3(\pi/2 - \log \sin \pi/12)$
d) None of these
561. $\int \frac{a^{x/2}}{\sqrt{a-x-a^x}} \, dx$ is equal to
a) $\frac{1}{\log a} \sin^{-1}(a^x) + c$ b) $\frac{1}{\log a} \tan^{-1}(a^x) + c$ c) $2\sqrt{a-x-a^x} + c$ d) $\log(a^x - 1) + c$

562. $\int \frac{x+\sin x}{1+\cos x} dx$ is equal to
 a) $x \tan \frac{x}{2} + c$ b) $\log(1 + \cos x) + c$ c) $\cot \frac{x}{2} + c$ d) $\log(x + \sin x) + c$
563. $\int_{-1}^1 \sin^3 x \cos^2 x dx$ is equal to
 a) -1 b) 1 c) 0 d) None of these
564. $\int x\{f(x^2)g''(x^2) - f''(x^2)g(x^2)\}dx$ is equal to
 a) $f(x^2)g'(x^2) - g(x^2)f'(x^2) + c$ b) $\frac{1}{2}\{f(x^2)g(x^2)f'(x^2)\} + c$
 c) $\frac{1}{2}\{f(x^2)g'(x^2) - g(x^2)f'(x^2)\} + c$ d) None of the above
565. If $I = \int_{-2}^2 |1 - x^4| dx$, then I equals
 a) 6 b) 8 c) 12 d) 21
566. $\int_{-1}^1 [x + [x + [x]]] dx$, where $[\cdot]$ denotes greatest integer function, is equal to
 a) -2 b) -1 c) -3 d) -4
567. The value of $\int_0^\pi (\sum_{r=0}^3 a_r \cos^{3-r} x \sin^r x) dx$ depends upon
 a) a_1 and a_2 b) a_0 and a_3 c) a_2 and a_3 d) a_1 and a_3
568. The value of the integral $\int \frac{x \sin x^2 e^{\sec x^2}}{\cos^2 x^2} dx$, is
 a) $\frac{1}{2} e^{\sec x^2} + C$ b) $\frac{1}{2} e^{\sin x^2} + C$ c) $\frac{1}{2} \sin x^2 e^{\cos^2 x^2} + C$ d) None of these
569. $\int \frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}} dx =$
 a) $\frac{1}{2} \sqrt{1+x} + C$ b) $\frac{2}{3} (1+x)^{3/2} + C$ c) $\sqrt{1+x} + C$ d) $2(1+x)^{3/2} + C$
570. $\int_0^\infty \frac{dx}{[x + \sqrt{x^2 + 1}]^3} =$
 a) $\frac{3}{8}$ b) $\frac{1}{8}$ c) $-\frac{3}{8}$ d) None of these
571. If $f(y) = e^y, g(y) = y; y > 0$ and $F(t) = \int_0^t f(t-y)g(y) dy$, then
 a) $F(t) = 1 - e^{-t}(1+t)$ b) $F(t) = e^t - (1+t)$
 c) $F(t) = te^t$ d) $F(t) = te^{-t}$
572. $\int_0^\pi [2 \sin x] dx =$
 a) $2\pi/3$ b) $-5\pi/3$ c) $-\pi$ d) -2π
573. The value of $\int_1^{e^2} \frac{dx}{x(1+\log x)^2}$ is
 a) $\frac{2}{3}$ b) $\frac{1}{3}$ c) $\frac{3}{2}$ d) $\ln 2$
574. $\int \frac{dx}{(x+1)\sqrt{4x+3}}$ is equal to
 a) $\tan^{-1} \sqrt{4x+3} + c$ b) $3 \tan^{-1} \sqrt{4x+3} + c$
 c) $2 \tan^{-1} \sqrt{4x+3} + c$ d) $4 \tan^{-1} \sqrt{4x+3} + c$
575. If $f(a+b-x) = f(x)$, then $\int_a^b x f(x) dx$ is equal to
 a) $\frac{a+b}{2} \int_a^b f(b-x) dx$ b) $\frac{a+b}{2} \int_a^b f(x) dx$ c) $\frac{b-a}{2} \int_a^b f(x) dx$ d) $\frac{a+b}{2} \int_a^b f(a+b+x) dx$

576. If $f(x)$ is differentiable and $\int_0^{t^2} x f(x) dx = \frac{2}{5} t^5$, then $f\left(\frac{4}{25}\right)$ equals
 a) $\frac{2}{5}$ b) $-\frac{5}{2}$ c) 1 d) $\frac{5}{2}$
577. Let $f: (0, \infty) \rightarrow R$ and $F(x) = \int_0^x f(x) dt$. If $F(x^2) = x^2(1+x)$, then $f(4)$ equals
 a) $5/4$ b) 7 c) 4 d) 2
578. $\int_0^{\sqrt{2}} [x^2] dx$, is
 a) $2 - \sqrt{2}$ b) $2 + \sqrt{2}$ c) $\sqrt{2} - 1$ d) $\sqrt{2} - 2$
579. $\int \frac{e^x}{(2+e^x)(e^x+1)} dx$ is equal to
 a) $\log\left(\frac{e^x+1}{e^x+2}\right) + c$ b) $\log\left(\frac{e^x+2}{e^x+1}\right) + c$ c) $\left(\frac{e^x+1}{e^x+2}\right) + c$ d) $\left(\frac{e^x+2}{e^x+1}\right) + c$
580. The value of $\int_0^{\pi/2} \frac{\sin^3 x \cos x}{\sin^4 x + \cos^4 x} dx$, is
 a) $\pi/8$ b) $\pi/4$ c) $\pi/2$ d) π
581. If $\int_0^1 e^{x^2} (x - \alpha) dx = 0$, then
 a) $1 < \alpha < 2$ b) $\alpha < 0$ c) $0 < \alpha < 1$ d) $\alpha = 0$
582. $\int_0^{\pi/4} \sin(x - [x]) dx$ is equal to
 a) $\frac{1}{2}$ b) $1 - \frac{1}{\sqrt{2}}$ c) 1 d) None of these
583. The value of $\int_{-1}^2 \frac{|x|}{x} dx$, is
 a) 0 b) 1 c) 3 d) None of these
584. $\int \frac{1+x^2}{x\sqrt{1+x^4}} dx$ is equal to
 a) $-\log\left|x - \frac{1}{x} + \sqrt{\left(x - \frac{1}{x}\right)^2 - 2}\right| + C$
 b) $\log\left|x - \frac{1}{x} + \sqrt{\left(x - \frac{1}{x}\right)^2 + 2}\right| + C$
 c) $\log\left|x - \frac{1}{x} + \sqrt{\left(x - \frac{1}{x}\right)^2 - 2}\right| + C$
 d) None of these
585. $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \tan \sqrt{t} dt}{x^3}$ is equal to
 a) $\frac{2}{3}$ b) $\frac{3}{2}$ c) 1 d) None of these
586. If $f(t)$ is an odd function, then $\int_0^x f(t) dt$ is
 a) An odd function b) An even function
 c) Neither even nor odd d) 0
587. The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$, is
 a) 1 b) 0 c) -1 d) 2
588. The value of $\int_{\pi/4}^{3\pi/4} \frac{x}{1+\sin x} dx$ is equal to
 a) $(\sqrt{2}-1)\pi$ b) $(\sqrt{2}+1)\pi$ c) π d) None of these
589. Let $f(x)$ be a differentiable function such that $f(1) = 2$. If $\lim_{x \rightarrow 1} \int_2^{f(x)} \frac{2t}{x-1} dt = 4$, then the value of $f'(1)$ is
 a) 1 b) 2 c) 4 d) None of these
590. Let $f(x) = \frac{\sin^2 \pi x}{1+\pi^x}$. Then, $\int [f(x) + f(-x)] dx$ is equal to

- a) 2 b) -3 c) -1 d) 1
605. Let $f : R \rightarrow R$ be a continuous function such that $f(x)$ is not identically equal to zero. If $\int_2^5 f(x) dx = \int_{-2}^5 f(x) dx$, then $f(x)$ is
a) An even function b) An odd function c) A periodic function d) None of these
606. For $y = f(x) = \int_0^x 2|t| dt$, the tangent lines parallel to the bisector of the first quadrant angle are
a) $y = x \pm \frac{1}{4}$ b) $y = x \pm \frac{3}{2}$ c) $y = x \pm \frac{1}{2}$ d) None of these
607. $\int \cos \left[2 \cot^{-1} \sqrt{\frac{1-x}{1+x}} \right] dx$ is equal to
a) $\frac{1}{2}x^2 + c$ b) $\frac{1}{2} \sin \left[2 \cot^{-1} \sqrt{\frac{1-x}{1+x}} \right] + c$
c) $-\frac{1}{2}x^2 + c$ d) $\frac{1}{2}x + c$
608. The value of $\int_2^3 \frac{x+1}{x^2(x-1)} dx$ is
a) $\log \frac{16}{9} + \frac{1}{6}$ b) $\log \frac{16}{9} - \frac{1}{6}$ c) $2 \log 2 - \frac{1}{6}$ d) $\log \frac{4}{3} - \frac{1}{6}$
609. The value of $\int x(x^x)^x (2 \log x + 1) dx$ is
a) $(x^x)^x + c$ b) $x^x + c$ c) $x^{\log x} + c$ d) None of these
610. $\int \frac{dx}{(\sin x - 2 \cos x)(2 \sin x + \cos x)}$ is equal to
a) $\log_e \sqrt[5]{\frac{\tan x - 2}{2 \tan x + 1}} + c$ b) $\frac{1}{4} \log \left(\frac{\tan x - 2}{\tan x + 2} \right) + c$
c) $-\frac{1}{4} \log \left(\frac{2 \sin x + \cos x}{\sin x + 2 \cos x} \right) + c$ d) None of the above
611. The value of $\int_0^\infty \frac{dx}{(a^2 + x^2)}$ is equal to
a) $\frac{\pi}{2}$ b) $\frac{\pi}{2a}$ c) $\frac{\pi}{a}$ d) $\frac{1}{2a}$
612. The value of the integral $\int_1^e (\log x)^3 dx$ is
a) $6 + 2e$ b) $6 - 2e$ c) $2e - 6$ d) None of these
613. The value of $\int_1^{\sqrt[7]{2}} \frac{1}{x(2x^7+1)} dx$, is
a) $\log(6/5)$ b) $6 \log(6/5)$ c) $(1/7) \log(6/5)$ d) $(1/12) \log(6/5)$
614. If $\int f(x) dx = F(x)$, then $\int x^3 f(x^2) dx$ is equal to
a) $\frac{1}{2} \{x^2 F(x^2) - \int F(x^2) dx\}$ b) $\frac{1}{2} \{x^2 F(x^2) - \int F(x^2) dx\}$
c) $\frac{1}{2} \left(x^2 F(x) - \frac{1}{2} \int F(x^2) dx \right)$ d) None of the above
615. If $\int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx$, then
a) $f(2a - x) = -f(x)$
b) $f(2a - x) = f(x)$
c) $f(x)$ is an odd function
d) $f(x)$ is an even function
616. The value of the integral $\int_0^\pi x \log \sin x dx$, is
a) $\frac{\pi}{2} \log 2$ b) $\frac{\pi^2}{2} \log 2$ c) $-\frac{\pi^2}{2} \log 2$ d) None of these
617. The value of $\int_1^2 [f\{g(x)\}]^{-1} f'\{g(x)\} g'(x) dx$, where $g(1) = g(2)$ is equal to

- a) 1 b) 2 c) 0 d) None of these
618. If $f(t)$ is a continuous function defined on $[a, b]$ such that $f(t)$ is an odd function, then the function $\phi(x) = \int_a^x f(t) dt$
- a) Is an odd function
b) Is an even function
c) Is an increasing function on $[a, b]$
d) None of these
619. If $\int \frac{x^5}{\sqrt{1+x^3}} dx$ is equal to
- a) $\frac{2}{9}(1+x^3)^{5/2} + \frac{2}{3}(1+x^3)^{3/2} + C$
b) $\frac{2}{9}(1+x^3)^{3/2} - \frac{2}{3}(1+x^3)^{1/2} + C$
c) $\log|\sqrt{x} + \sqrt{1+x^3}| + C$
d) $x^2 \log(1+x^3) + C$
620. The value of the integral $\int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x} dx$ is
- a) 0 b) $\frac{4}{3}$ c) $\frac{2}{3}$ d) $\frac{1}{5}$
621. The value of the integral $\int_0^\pi \frac{x dx}{1+\cos \alpha \sin x}$ ($0 < \alpha < \pi$) is
- a) $\frac{\pi \alpha}{\sin \alpha}$ b) $\frac{\pi \alpha}{\cos \alpha}$ c) $\frac{\pi \alpha}{1+\sin \alpha}$ d) $\frac{\pi \alpha}{1+\cos \alpha}$
622. The value of $\int_{-3}^3 (ax^5 + bx^3 + cx + k) dx$, where a, b, c, k are constant, depends only on
- a) a and k b) a and b c) a, b and c d) k
623. $\lim_{n \rightarrow \infty} \frac{1}{n^2} \sum_{r=1}^n r e^{r/n} =$
- a) 0 b) 1 c) e d) $2e$
624. If $I_n = \int x^n \cdot e^{cx} dx$ for $n \geq 1$, then $c \cdot I_n + n \cdot I_{n-1}$ is equal to
- a) $x^n e^{cx}$ b) x^n c) e^{cx} d) $x^n + e^{cx}$
625. The value of $\int_0^{\pi/2} \frac{1+2 \cos x}{(2+\cos x)^2} dx$ is
- a) $-\frac{1}{2}$ b) 2 c) $\frac{1}{2}$ d) None of these
626. The value of $\int_0^{\pi/4} (\pi x - 4x^2) \log(1 + \tan x) dx$ is
- a) $\frac{\pi^3}{192} \log_e 2$ b) $\frac{\pi^3}{192} \log \sqrt{2}$ c) $\frac{\pi^3}{36} \log 2$ d) $\frac{\pi^3}{48} \log \sqrt{2}$
627. $\int_2^{\pi/2} \frac{\cos x}{(1+\sin x)(2+\sin x)} dx$ is equal to
- a) $\log \frac{4}{3}$ b) $\log \frac{1}{3}$ c) $\log \frac{3}{4}$ d) None of these
628. $\int \frac{x^2}{(a+bx^2)^{5/2}} dx$ is equal to
- a) $-\frac{1}{3a} \left(\frac{x^2}{a+bx^2} \right)^{3/2} + C$
b) $\frac{1}{3a} \left(\frac{x^2}{a+bx^2} \right)^{3/2} + C$
c) $\frac{1}{2a} \left(\frac{x^2}{a+bx^2} \right)^{2/3} + C$
d) None of these

629. If $f(x)$ and $g(x)$ are two integrable functions defined on $[a, b]$ then $\left| \int_a^b f(x)g(x)dx \right|$, is

- a) Less than $\sqrt{\int_a^b f(x)dx \int_a^b g(x)dx}$
 b) Less than or equal to $\sqrt{\int_a^b f^2(x)dx \int_a^b g^2(x)dx}$
 c) Less than or equal to $\sqrt{\left\{ \int_a^b f^2(x)dx \right\} \left\{ \int_a^b g^2(x)dx \right\}}$
 d) None of these

630. $\int \frac{3^x}{\sqrt{9^x-1}} dx$ is equal to

- a) $\frac{1}{\log 3} \log|3^x + \sqrt{9^x - 1}| + c$ b) $\frac{1}{\log 3} \log|9^x + \sqrt{9^x - 1}| + c$
 c) $\frac{1}{\log 9} \log|3^x + \sqrt{9^x - 1}| + c$ d) $\frac{1}{\log 9} \log|3^x - \sqrt{9^x - 1}| + c$

631. $\int \{1 + 2 \tan x(\tan x + \sec x)\}^{1/2} dx$ is equal to

- a) $\log(\sec x + \tan x) + c$ b) $\log(\sec x + \tan x)^{1/2} + c$
 c) $\log \sec x(\sec x + \tan x) + c$ d) None of the above

632. If $f(x) = \int_{-1}^x |t|dt$, then for any $x \geq 0$, $f(x)$ is equal to

- a) $1 - x^2$ b) $\frac{1}{2}(1 + x^2)$ c) $1 + x^2$ d) $\frac{1}{2}(1 - x^2)$

633. The value of the integral $\int_a^b \frac{|x|}{x} dx$, $a < b$ is

- a) $|a| - |b|$ b) $|b| - |a|$ c) $|a| - b$ d) $|b| - a$

634. $\int \frac{1+\tan^2 x}{1-\tan^2 x} dx$ equals

- a) $\log\left(\frac{1-\tan x}{1+\tan x}\right) + c$ b) $\log\left(\frac{1+\tan x}{1-\tan x}\right) + c$ c) $\frac{1}{2}\log\left(\frac{1-\tan x}{1+\tan x}\right) + c$ d) $\frac{1}{2}\log\left(\frac{1+\tan x}{1-\tan x}\right) + c$

635. If $\int_0^a f(x)dx = \lambda$ and $\int_0^a f(2a-x)dx = \mu$, then $\int_0^{2a} f(x)dx =$

- a) $\lambda + \mu$ b) $\lambda - \mu$ c) $2\lambda + \mu$ d) $\lambda + 2\mu$

636. $\int_2^3 \{x\}dx$ is equal to (where $\{ \cdot \}$ denotes fractional part of x)

- a) $\frac{17}{2}$ b) $\frac{7}{2}$ c) $\frac{5}{2}$ d) $\frac{1}{2}$

637. The value of $\int_0^{\pi/2} \frac{1}{1+\tan^3 x} dx$, is

- a) 0 b) 1 c) $\pi/2$ d) $\pi/4$

638. $\int_{-\pi/4}^{\pi/4} e^{-x} \sin x dx$ is

- a) $-\frac{\sqrt{2}}{2} e^{-\pi/4}$ b) $\frac{\sqrt{2}}{2} e^{-\pi/4}$ c) $-\sqrt{2}(e^{-\pi/4} - e^{\pi/4})$ d) Zero

639. The greatest value of $f(x) = \int_{-1/2}^x |t|dt$ on the interval $[-1/2, 1/2]$ is

- a) $\frac{3}{8}$ b) $\frac{1}{4}$ c) $-\frac{3}{8}$ d) $-\frac{1}{2}$

640. Let a, b, c be non-zero real numbers such that $\int_0^1 (1 + \cos^8 x)(ax^2 + bx + c)dx$

$$= \int_0^2 (1 + \cos^8 x)(ax^2 + bx + c)dx$$

Then, the quadratic equation $ax^2 + bx + c = 0$ has

- a) No root in $(0, 2)$ b) At least one root in $(1, 2)$
 c) A double root in $(0, 2)$ d) Two imaginary roots

641. If $f(x) = A \sin\left(\frac{\pi x}{2}\right) + B$, $f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x)dx = \frac{2A}{\pi}$, then constant A and B are

- a) $\frac{\pi}{2}$ and $\frac{\pi}{2}$ b) $\frac{2}{\pi}$ and $\frac{3}{\pi}$ c) 0 and $-\frac{4}{\pi}$ d) $\frac{4}{\pi}$ and 0

642. If $I = \int_0^1 \frac{dx}{\sqrt{1+x^4}}$, then
 a) $I > 0.78$ b) $I < 0.78$ c) $I > 1$ d) None of these
643. If $I_k = \int_{-2k\pi}^{2k\pi} |\sin x| [\sin x] dx, \forall k \in N$, where $[\cdot]$ denotes the greatest integer function, then $\sum_{k=1}^{10} I_k$ is equal to
 a) -110 b) -440 c) -330 d) -220
644. $\int_0^{10} |x - 5| dx$ is equal to
 a) 17 b) 9 c) 12 d) 18
645. $\int \frac{e^x}{(2 + e^x)(e^x + 1)} dx =$
 a) $\log\left(\frac{e^x + 1}{e^x + 2}\right) + C$ b) $\log\left(\frac{e^x + 2}{e^x + 1}\right) + C$ c) $\frac{e^x + 1}{e^x + 2} + C$ d) $\frac{e^x + 2}{e^x + 1} + C$
646. $\int_{-1}^2 |x|^3 dx$ is equal to
 a) $\frac{5}{4}$ b) $\frac{17}{4}$ c) $\frac{15}{4}$ d) $\frac{4}{5}$
647. The value of $\int_{-1}^1 x|x| dx$, is
 a) 2 b) 1 c) 0 d) None of these
648. $\int \frac{(x+1)}{x(1+xe^x)^2} dx$ is
 a) $\log\left(\frac{xe^x}{xe^x + 1}\right) + \frac{1}{1 + xe^x} + c$ b) $\log\left(\frac{xe^x}{xe^x + 1}\right) - \frac{1}{1 + xe^x} + c$
 c) $\log\left(\frac{xe^x + 1}{xe^x}\right) + \frac{1}{1 + xe^x} + c$ d) None of these
649. The greater value of $F(x) = \int_1^x |t| dt$ on the interval $[-1/2, 1/2]$, is
 a) $\frac{3}{8}$ b) $\frac{1}{2}$ c) $-\frac{3}{8}$ d) $-\frac{1}{2}$
650. The value of the integral $\int_0^{\pi/2} \frac{f(x)}{f(x)+f(\frac{\pi}{2}-x)} dx$, is
 a) $\pi/4$ b) $\pi/2$ c) π d) 0
651. The value of $\int \frac{(x^2-1)dx}{x^3\sqrt{2x^4-2x^2+1}}$ is
 a) $2\sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}} + c$
 b) $2\sqrt{2 + \frac{2}{x^2} + \frac{1}{x^4}} + c$
 c) $\frac{1}{2}\sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}} + c$
 d) None of the above
652. The value of the integral $I = \int_0^1 x(1-x)^n dx$ is
 a) $\frac{1}{n+1}$ b) $\frac{1}{n+2}$ c) $\frac{1}{n+1} - \frac{1}{n+2}$ d) $\frac{1}{n+1} + \frac{1}{n+2}$
653. $\int x^2(ax+b)^{-2} dx$ is equal to
 a) $\frac{2}{a^2}\left(x - \frac{b}{a}\log(ax+b)\right) + c$ b) $\frac{2}{a^2}\left(x - \frac{b}{a}\log(ax+b)\right) - \frac{x^2}{a(ax+b)} + c$
 c) $\frac{2}{a^2}\left(x + \frac{b}{a}\log(ax+b)\right) + \frac{x^2}{a(ax+b)} + c$ d) $\frac{2}{a^2}\left(x + \frac{b}{a}\log(ax+b)\right) - \frac{x^2}{a(ax+b)} + c$
654. If $0 < a < 1$, then $\int_{-1}^1 \frac{1}{\sqrt{1-2ax+a^2}} dx$ is equal to

667. $\int \frac{(x^4-x)^{1/4}}{x^5} dx$ is equal to
 a) $\frac{4}{15} \left(1 - \frac{1}{x^3}\right)^{5/4} + C$ b) $\frac{4}{5} \left(1 - \frac{1}{x^3}\right)^{5/4} + C$ c) $\frac{4}{15} \left(1 + \frac{1}{x^3}\right)^{5/4} + C$ d) None of these
668. If $f(x) = \begin{vmatrix} \sin x + \sin 2x + \sin 3x & \sin 2x & \sin 3x \\ 3 + 4 \sin x & 3 & 4 \sin x \\ 1 + \sin x & \sin x & 1 \end{vmatrix}$, then the value of $\int_0^{\pi/2} f(x) dx$ is
 a) 3 b) $\frac{2}{3}$ c) $\frac{1}{3}$ d) 0
669. $\int \frac{(x^3+3x^2+3x+1)}{(x+1)^5} dx$ is equal to
 a) $-\frac{1}{(x+1)} + c$ b) $\frac{1}{5} \log(x+1) + c$ c) $\log(x+1) + c$ d) $\tan^{-1} x + c$
670. If $\int \sqrt{\frac{x}{a^3-x^3}} dx = g(x) + c$, then $g(x)$ is equal to
 a) $\frac{2}{3} \cos^{-1} x$ b) $\frac{2}{3} \sin^{-1} \left(\frac{x^3}{a^3}\right)$ c) $\frac{2}{3} \sin^{-1} \left(\sqrt{\frac{x^3}{a^3}}\right)$ d) $\frac{2}{3} \cos^{-1} \left(\frac{x}{a}\right)$
671. If $I_n = \int \sin^n x dx$, then $nI_n - (n-1)I_{n-2}$ equals
 a) $\sin^{n-1} x \cos x$ b) $\cos^{n-1} x \sin x$ c) $-\sin^{n-1} x \cos x$ d) $-\cos^{n-1} x \sin x$
672. $\int_{-2}^2 e^{\sin^2 x} \cos^3 x dx$ is equal to
 a) -1 b) 0 c) 1 d) π
673. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\int_{\pi/2}^x (2^{-\cos t} - 1) dt}{\int_{\pi^2/4}^{x^2} (\sqrt{t} - \frac{\pi}{2}) dt}$ is equal to
 a) $\frac{\log_e 2}{2\pi}$ b) $\frac{\log_e 2}{\pi}$ c) $\frac{4 \log_e 2}{\pi}$ d) None of these
674. $\int [f(x)g''(x) - f''(x)g(x)] dx$ is equal to
 a) $\frac{f(x)}{g'(x)}$ b) $f'(x)g(x) - f(x)g'(x)$
 c) $f(x)g'(x) - f'(x)g(x)$ d) $f(x)g'(x) + f'(x)g(x)$
675. If $f(x) = A \sin\left(\frac{\pi x}{2}\right) + B$, $f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x) dx = \frac{2A}{\pi}$, then the constants A and B are respectively
 a) $\frac{\pi}{2}$ and $\frac{\pi}{2}$ b) $\frac{2}{\pi}$ and $\frac{3}{\pi}$ c) 0 and $-\frac{4}{\pi}$ d) $\frac{4}{\pi}$ and 0
676. $\int \frac{dx}{\cos x - \sin x}$ is equal to
 a) $\frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} - \frac{\pi}{8}\right) \right| + c$ b) $\frac{1}{\sqrt{2}} \log \left| \cot \left(\frac{x}{2}\right) \right| + c$
 c) $\frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} - \frac{3\pi}{8}\right) \right| + c$ d) $\frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} + \frac{3\pi}{8}\right) \right| + c$
677. The value of $\int \frac{(x+1)^2}{x(x^2+1)} dx$ is equal to
 a) $\log_e x + c$ b) $\log_e x + 2 \tan^{-1} x + c$
 c) $\log_e \left(\frac{1}{x^2+1}\right) + c$ d) None of these
678. If $\int \sqrt{2} \sqrt{1 + \sin x} dx = -4 \cos(ax + b) + c$, then the value of a, b are respectively
 a) $\frac{1}{2}, \frac{\pi}{4}$ b) $1, \frac{\pi}{2}$ c) 1, 1 d) None of these
679. If $f(x) = x(x-1)$, $0 \leq x \leq 1$ and $f(x+1) = f(x)$, $\forall x \in R$, then $\left| \int_2^4 f(x) dx \right|$ is
 a) 1 sq unit b) $\frac{1}{3}$ sq unit c) $\frac{2}{3}$ sq unit d) $\frac{1}{6}$ sq unit

- a) $x^2 + c$ b) $\frac{x^2}{2} + c$ c) $\frac{x^3}{3} + c$ d) $\frac{x}{2} + c$
693. The value of the integral $\int_0^{2a} \frac{f(x)}{f(x)+f(2a-x)} dx$ is equal to
a) 0 b) $2a$ c) a d) None of these
694. If $I_1 = \int_0^1 \frac{1}{|x|} dx$ and $I_2 = \int_0^1 \frac{1}{\sqrt{1+x^2}} dx$, then
a) $I_1 = I_2$ b) $I_1 < I_2$ c) $I_1 > I_2$ d) $\frac{I_1}{I_2} = 2$
695. The value of $\int_0^{\frac{3\pi}{2}} \frac{|\tan^{-1} \tan x| - |\sin^{-1} \sin x|}{|\tan^{-1} \tan x| + |\sin^{-1} \sin x|} dx$ is equal to
a) $\frac{\pi}{2}$ b) π c) $\frac{3\pi}{2}$ d) None of these
696. $\int \sqrt{\frac{\cos x - \cos^3 x}{1 - \cos^3 x}} dx$ is equal to
a) $\frac{2}{3} \sin^{-1}(\cos^{3/2} x) + C$ b) $\frac{3}{2} \sin^{-1}(\cos^{3/2} x) + C$ c) $\frac{2}{3} \cos^{-1}(\cos^{3/2} x) + C$ d) None of these
697. Let $x^2 \neq n\pi - 1, n \in N$. The indefinite integral of

$$x \sqrt{\frac{2 \sin(x^2 + 1) - \sin 2(x^2 + 1)}{2 \sin(x^2 + 1) + \sin 2(x^2 + 1)}}$$
is but for an arbitrary constant
a) $\log \left| \frac{1}{2} \sec(x^2 + 1) \right|$ b) $\log \left| \frac{1}{2} \sec \left(\frac{x^2 + 1}{2} \right) \right|$ c) $\frac{1}{2} \log |\sec(x^2 + 1)|$ d) None of these
698. The value of the integral $\int_0^\pi \frac{x dx}{1 + \cos \alpha \sin x}, 0 < \alpha < \pi$, is
a) $\frac{\pi \alpha}{\sin \alpha}$ b) $\frac{\pi \alpha}{1 + \sin \alpha}$ c) $\frac{\pi \alpha}{\cos \alpha}$ d) $\frac{\pi \alpha}{1 + \cos \alpha}$
699. $\int \tan(\sin^{-1} x) dx$ is equal to
a) $\frac{1}{\sqrt{1-x^2}} + c$ b) $\sqrt{1-x^2} + c$ c) $\frac{-x}{\sqrt{1-x^2}} + c$ d) $-\sqrt{1-x^2} + c$
700. The value of integral $\int_0^1 \sqrt{\frac{1-x}{1+x}} dx$ is
a) $\frac{\pi}{2} + 1$ b) $\frac{\pi}{2} - 1$ c) -1 d) 1
701. The value of $\int_\pi^{2\pi} [2 \sin x] dx$ is
a) $\frac{\pi}{3}$ b) $-\frac{5\pi}{3}$ c) $\frac{4\pi}{3}$ d) $-\frac{\pi}{3}$
702. If $\int \frac{1}{(\sin x + 4)(\sin x - 1)} dx = A \frac{1}{\tan \frac{x}{2} - 1} + B \tan^{-1}(f(x)) + C$, then
a) $A = \frac{1}{5}, B = \frac{-2}{5\sqrt{15}}, f(x) = \frac{4 \tan x + 3}{\sqrt{15}}$
b) $A = -\frac{1}{5}, B = \frac{1}{\sqrt{15}}, f(x) = \frac{4 \tan \left(\frac{x}{2}\right) + 1}{\sqrt{15}}$
c) $A = \frac{2}{5}, B = \frac{-2}{5}, f(x) = \frac{4 \tan x + 1}{5}$
d) $A = \frac{2}{5}, B = \frac{-2}{5\sqrt{15}}, f(x) = \frac{4 \tan x/2 + 1}{\sqrt{15}}$
703. If $\int e^x (1 + x) \cdot \sec^2(xe^x) dx = f(x) + \text{constant}$, then $f(x)$ is equal to
a) $\cos(xe^x)$ b) $\sin(xe^x)$ c) $2 \tan^{-1}(x)$ d) $\tan(xe^x)$
704. The value of $\int_0^{1000} e^{x-[x]} dx$, is

705. $\int \cos^{-3/7} x \sin^{-11/7} x dx$ is equal to
- a) $\frac{e^{1000} - 1}{1000}$ b) $\frac{e^{1000} - 1}{e - 1}$ c) $1000(e - 1)$ d) $\frac{e - 1}{1000}$
- a) $\log |\sin^{4/7} x| + c$ b) $\frac{4}{7} \tan^{4/7} x + c$ c) $-\frac{7}{4} \tan^{-4/7} x + c$ d) $\log |\cos^{3/7} x| + c$
706. If $\int \frac{2e^x + 3e^{-x}}{3e^x + 4e^{-x}} dx = Ax + B \log(3e^{2x} + 4) + c$, then
- a) $A = -\frac{3}{4}, B = \frac{1}{24}$ b) $A = \frac{3}{4}, B = -\frac{1}{24}$ c) $A = \frac{1}{4}, B = \frac{1}{24}$ d) $A = -\frac{3}{4}, B = \frac{1}{4}$
707. $\int \frac{x^2}{1+x^6} dx$ is equal to
- a) $x^3 + c$ b) $\frac{1}{3} \tan^{-1}(x^3) + c$ c) $\log(1 + x^3) + c$ d) None of these
708. $\int_0^{\pi} x \sin x \cos^4 x dx =$
- a) $\frac{\pi}{10}$ b) $\frac{\pi}{5}$ c) $-\frac{\pi}{5}$ d) None of these
709. $\int \frac{(2x^{12} + 5x^9)}{(x^5 + x^3 + 1)^3} dx$ is equal to
- a) $\frac{x^2 + 2x}{(x^5 + x^3 + 1)^2} + c$ b) $\frac{x^{10}}{2(x^5 + x^3 + 1)^2} + c$
- c) $\log |x^5 + x^3 + 1| + \sqrt{(2x^7 + 5x^4)} + c$ d) None of the above
710. The value of $\int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}} dx$ is
- a) 2 b) π c) $\frac{\pi}{4}$ d) 2π
711. $\int_0^{\pi} \sqrt{\frac{\cos 2x + 1}{2}} dx$ is equal to
- a) 0 b) 2 c) -2 d) None of these
712. $\int e^{x \log a} e^x dx$ is equal to
- a) $\frac{a^x}{\log ae} + c$ b) $\frac{e^x}{1 + \log_e a} + c$ c) $(ae)^x + c$ d) $\frac{(ae)^x}{\log_e ae} + c$
713. $\int_{-1}^1 \log(x + \sqrt{x^2 + 1}) dx$ is equal to
- a) 0 b) $\log 2$ c) $\log \frac{1}{2}$ d) None of these
714. If $I_n = \int (\log x)^n dx$, then $I_n + nI_{n-1}$ is equal to
- a) $(x \log x)^n$ b) $x(\log x)^n$ c) $n(\log x)^n$ d) $(\log x)^{n-1}$
715. If $I = \int_0^1 \frac{dx}{1+x^{\pi/2}}$, then
- a) $\log_e 2 < I < \pi/4$ b) $\log_e 2 > I$ c) $I = \pi/4$ d) $I = \log_e 2$
716. The value of $\int_a^b \frac{x}{|x|} dx, a < b < 0$ is
- a) $-(|a| + |b|)$ b) $|b| - |a|$ c) $|a| - |b|$ d) $|a| + |b|$
717. Given $f(x) = \begin{vmatrix} 0 & x^2 - \sin x & \cos x - 2 \\ \sin x - x^2 & 0 & 1 - 2x \\ 2 - \cos x & 2x - 1 & 0 \end{vmatrix}$, $\int f(x) dx$ is equal to
- a) $\frac{x^3}{3} - x^2 \sin x + \sin 2x + C$
- b) $\frac{x^3}{3} - x^2 \sin x - \cos 2x + C$
- c) $\frac{x^3}{3} - x^2 \cos x - \cos 2x + C$
- d) None of these

718. If $f(x) = \lim_{n \rightarrow \infty} n^2(x^{1/n} - x^{1/(n+1)})$, $x > 0$, then $\int x f(x) dx$ is equal to
 a) $x^2/2$ b) 0 c) $x^2 \log x - \frac{1}{2}x^2 + c$ d) None of these
719. $\int \frac{1}{x^{1/2}(1+x^2)^{5/4}} dx$ is equal to
 a) $\frac{-2\sqrt{x}}{\sqrt[4]{1+x^2}} + C$ b) $\frac{2\sqrt{x}}{\sqrt[4]{1+x^2}} + C$ c) $\frac{-\sqrt{x}}{\sqrt[4]{1+x^2}} + C$ d) $\frac{\sqrt{x}}{\sqrt[4]{1+x^2}} + C$
720. $\int_0^1 (\{x\}[x+1]) dx$ (where $\{\cdot\}$ denotes fractional part of x and $[\cdot]$ denotes greatest integer of x) is equal to
 a) 1 b) 0 c) $\frac{1}{2}$ d) None of these
721. $\int_0^2 (x - \log_2 a) dx = 2 \log_2 \left(\frac{2}{a}\right)$, if
 a) $a = 2$ b) $a > 2$ c) $a = 4$ d) $a = 8$
722. $\int_{-2}^{10} \text{sgn}(x - [x]) dx$ Equals, where $[\cdot]$ denotes greatest integer function
 a) -12 b) 10 c) 8 d) 12
723. The value of the integral $\int_0^{\infty} \frac{1}{1+x^4} dx$, is
 a) $\frac{\pi}{2}$ b) $\frac{\pi}{\sqrt{2}}$ c) $\frac{\pi}{2\sqrt{2}}$ d) None of these
724. $\int \frac{x^2}{x^2+4} dx$ is equal to
 a) $x - 2 \tan^{-1} \left(\frac{x}{2}\right) + c$ b) $x + 2 \tan^{-1} \left(\frac{x}{2}\right) + c$ c) $x - 4 \tan^{-1} \left(\frac{x}{2}\right) + c$ d) $x + 4 \tan^{-1} \left(\frac{x}{2}\right) + c$
725. $\int \frac{1+x}{x+e^{-x}} dx$ is equal to
 a) $\log|(x - e^{-x})| + c$ b) $\log|(x + e^{-x})| + c$ c) $\log|(1 + xe^x)| + c$ d) $(1 + xe^x)^2 + c$
726. $\int \frac{dx}{\sqrt[4]{(x+1)^5(x+2)^3}}$ is equal to
 a) $4 \left(\frac{x+1}{x+2}\right)^{1/4} + c$ b) $-4 \left(\frac{x+1}{x+2}\right)^{-1/4} + c$ c) $-4 \left(\frac{x+2}{x+1}\right)^{1/4} + c$ d) None of these
727. $\int_0^{\infty} \left[\frac{x+1}{e^x}\right] dx$ (where $[\cdot]$ denotes greatest integer function) is equal to
 a) 0 b) 1 c) ∞ d) None of these
728. $\int \frac{(\tan^{-1} x)^3}{1+x^2} dx$ is equal to
 a) $3 (\tan^{-1} x)^2 + c$ b) $\frac{(\tan^{-1} x)^4}{4} + c$ c) $(\tan^{-1} x)^4 + c$ d) None of these
729. $\int x^x (1 + \log|x|) dx$ is equal to
 a) $x^x \log|x| + c$ b) $e^{x^x} + c$ c) $x^x + c$ d) None of these
730. $\int \left\{ \frac{\log x - 1}{1 + (\log x)^2} \right\}^2 dx$ is equal to
 a) $\frac{x}{(\log x)^2 + 1} + c$ b) $\frac{xe^x}{1 + x^2} + c$ c) $\frac{x}{x^2 + 1} + c$ d) $\frac{\log x}{(\log x)^2 + 1} + c$
731. $\lim_{n \rightarrow \infty} \frac{1+2^4+3^4+\dots+n^4}{n^5} - \lim_{n \rightarrow \infty} \frac{1+2^3+3^3+\dots+n^3}{n^5}$ is
 a) $\frac{1}{30}$ b) 0 c) $\frac{1}{4}$ d) $\frac{1}{5}$
732. $\int \frac{dx}{\sqrt{1-e^{2x}}}$ is equal to
 a) $\log \left| e^{-x} + \sqrt{e^{-2x} - 1} \right| + c$ b) $\log \left| e^x + \sqrt{e^{2x} - 1} \right| + c$
 c) $-\log \left| e^{-x} + \sqrt{e^{-2x} - 1} \right| + c$ d) $-\log \left| e^{-2x} + \sqrt{e^{-2x} - 1} \right| + c$
733. If $I = \int_0^{\pi/2} \cos(\sin x) dx$,

$J = \int_0^{\pi/2} \sin(\cos x) dx$ and $K = \int_0^{\pi/2} \cos x dx$. Then,

- a) $K > I > J$ b) $J > I > K$ c) $I > J > K$ d) $I > K > J$
734. $\int_{-1}^1 \frac{17x^5 - x^4 + 29x^3 - 31x + 1}{x^2 + 1} dx$ is
- a) $\frac{4}{5}$ b) $\frac{5}{4}$ c) $\frac{4}{3}$ d) $\frac{3}{4}$
735. $\int_0^1 \sin\left(2 \tan^{-1} \sqrt{\frac{1+x}{1-x}}\right) dx$ is equal to
- a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) π
736. $\int e^{2x}(2 \sin 3x + 3 \cos 3x) dx$ is equal to
- a) $e^{2x} \sin 2x + c$ b) $e^{2x} \cos 3x + c$ c) e^{2x+c} d) $e^{2x}(2 \sin 3x) + c$
737. The value of the integral $\int_0^2 |x^2 - 1| dx$ is
- a) 0 b) 2 c) $-\frac{1}{3}$ d) -2
738. If
- $$I_1 = \int_0^1 2^{x^2} dx, I_2 = \int_0^1 2^{x^3} dx, I_3 = \int_1^2 2^{x^2} dx \text{ and } I_4 = \int_1^2 2^{x^3} dx$$
- Then
- a) $I_1 > I_2$ and $I_4 > I_3$ b) $I_2 > I_1$ and $I_3 > I_4$ c) $I_1 > I_2$ and $I_3 > I_4$ d) None of these
739. $\int (\sin 2x - \cos 2x) dx = \frac{1}{\sqrt{2}} \sin(2x - a) + b$, then
- a) $a = \frac{5\pi}{4}, b \in R$ b) $a = -\frac{5\pi}{4}, b \in R$ c) $a = \frac{\pi}{4}, b \in R$ d) None of these
740. $\int (1 + x - x^{-1}) e^{x+x^{-1}} dx$ is equal to
- a) $(1 + x) e^{x+x^{-1}} + c$ b) $(x - 1) e^{x+x^{-1}} + c$ c) $-x e^{x+x^{-1}} + c$ d) $x e^{x+x^{-1}} + c$
741. The value of the integral $\sum_{k=1}^n \int_0^1 f(k-1+x) dx$, is
- a) $\int_0^1 f(x) dx$ b) $\int_0^2 f(x) dx$ c) $\int_0^n f(x) dx$ d) $n \int_0^1 f(x) dx$
742. $\int_0^\pi \frac{1}{1+3\cos x} dx$ is equal to
- a) π b) 0 c) $\pi/2$ d) None of these
743. The value of the integral $\int_0^1 \frac{1}{x^2+2x \cos \alpha+1} dx$ is equal to
- a) $\sin \alpha$ b) $\alpha \sin \alpha$ c) $\frac{\alpha}{2 \sin \alpha}$ d) $\frac{\alpha}{2} \sin \alpha$
744. The value of the integral $\int_0^{\pi/4} \frac{\sin \theta + \cos \theta}{9+16 \sin 2\theta} d\theta$, is
- a) $\log 3$ b) $\log 2$ c) $\frac{1}{20} \log 3$ d) $\frac{1}{20} \log 2$
745. $\int_0^\infty \frac{dx}{(x+\sqrt{x^2+1})^3}$ is equal to
- a) $\frac{3}{8}$ b) $\frac{1}{8}$ c) $-\frac{3}{8}$ d) None of these
746. $\int \frac{dx}{x^2+4x+13}$ is equal to
- a) $\log(x^2 + 4x + 13) + c$ b) $\frac{1}{3} \tan^{-1} \left(\frac{x+2}{3}\right) + c$ c) $\log(2x + 4) + c$ d) $\frac{2x+4}{(x^2+4x+13)^2} + c$

747. Let $I = \int \frac{e^x}{e^{4x} + e^{2x} + 1} dx, J = \int \frac{e^{-x}}{e^{-4x} + e^{-2x} + 1} dx$. Then, for an arbitrary constant c , the value of $J - I$ equals
- $\frac{1}{2} \log \left| \frac{e^{4x} - e^{2x} + 1}{e^{4x} + e^{2x} + 1} \right| + c$
 - $\frac{1}{2} \log \left| \frac{e^{2x} + e^x + 1}{e^{2x} - e^x + 1} \right| + c$
 - $\frac{1}{2} \log \left| \frac{e^{2x} - e^x + 1}{e^{2x} + e^x + 1} \right| + c$
 - $\frac{1}{2} \log \left| \frac{e^{4x} + e^{2x} + 1}{e^{4x} - e^{2x} + 1} \right| + c$
748. If $b > a$, then $\int_a^b \frac{dx}{\sqrt{(x-a)(b-x)}}$ is equal to
- $\frac{\pi}{2}$
 - π
 - $\frac{\pi}{2}(b-a)$
 - $\frac{\pi}{4}(b-a)$
749. If $\int_0^{\pi/2} \sin^6 x \, dx = \frac{5\pi}{32}$, then the value of $\int_{-\pi}^{\pi} (\sin^6 x + \cos^6 x) \, dx$ is
- $5\pi/8$
 - $5\pi/16$
 - $5\pi/2$
 - $5\pi/4$
750. If $I_m = \int_1^x (\log x)^m \, dx$ satisfies the relation $I_m = k - II_{m-1}$, then
- $k = e$
 - $l = m$
 - $k = \frac{1}{e}$
 - None of these
751. $\int x^{-2/3} (1 + x^{1/2})^{-5/3} \, dx$ is equal to
- $3(1 + x^{-1/2})^{-1/3} + C$
 - $3(1 + x^{-1/2})^{-2/3} + C$
 - $3(1 + x^{1/2})^{-2/3} + C$
 - None of these
752. $\int (1 - \cos x) \operatorname{cosec}^2 x \, dx$ is equal to
- $\tan \frac{x}{2} + c$
 - $-\cot \frac{x}{2} + c$
 - $2 \tan \frac{x}{2} + c$
 - $-2 \cot \frac{x}{2} + c$
753. If $\int_0^a f(2a-x) \, dx = m$ and $\int_0^a f(x) \, dx = n$, then $\int_0^{2a} f(x) \, dx$ is equal to
- $2m + n$
 - $m + 2n$
 - $m - n$
 - $m + n$
754. If $I_n = \int (\log x)^n \, dx$, then $I_n + nI_{n-1}$ is equal to
- $x(\log x)^n$
 - $(x \log x)^n$
 - $(\log x)^{n-1}$
 - $n(\log x)^n$
755. The points of extremum of $\int_0^{x^2} \frac{t^2 - 5t + 4}{2 + e^t} \, dt$ are
- $x = 0, \pm 1, \pm 1$
 - $x = \pm 1, \pm 2, \pm 3$
 - $x = 0, 1, 2, 3$
 - None of these
756. The value of $\int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}} \, dx$ is
- 2
 - π
 - $\pi/4$
 - 2π
757. $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} e^{r/n}$ is
- e
 - $e - 1$
 - $1 - e$
 - $e + 1$
758. Let $f : R \rightarrow R, g : R \rightarrow R$ be continuous functions. Then the value of the integral $\int_{-\pi/2}^{\pi/2} [f(x) + f(-x)][g(x) - g(-x)] \, dx$ is
- π
 - 1
 - 1
 - 0
759. The integral $\int \frac{2x-3}{(x^2+x+1)^2} \, dx$ is equal to
- $-\frac{8x+7}{x^2+x+1} - \frac{16}{3\sqrt{3}} \tan^{-1} \left(\frac{2x+1}{3} \right) + C$
 - $-\frac{1}{x^2+x+1} - \frac{4}{3} \tan^{-1}(4x+3) + C$
 - $\frac{1}{2(x^2+x+1)} - \frac{(2x+1)^2}{(x^2+x+1)^2} + C$

- d) $\frac{1}{4(x^2 + x + 1)} + \frac{2}{3}\tan^{-1}(2x + 1) + C$
760. $\int \frac{\sqrt{x}}{x+1} dx$ is equal to
 a) $2(\sqrt{x} + \tan^{-1} \sqrt{x}) + c$ b) $2(\sqrt{x} + \cot^{-1} \sqrt{x}) + c$
 c) $2(\sqrt{x} - \cot^{-1} \sqrt{x}) + c$ d) $2(\sqrt{x} - \tan^{-1} \sqrt{x}) + c$
761. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ is equal to
 a) $\pi^2/4$ b) π^2 c) 0 d) $\pi/2$
762. The value of $\lim_{n \rightarrow \infty} \left\{ \left(1 + \frac{1}{n}\right) \left(1 + \frac{2}{n}\right) \left(1 + \frac{3}{n}\right) \dots (2) \right\}^{1/n}$, is
 a) $4/e$ b) $e/4$ c) $4e$ d) None of these
763. $\int_{-2}^2 |x| dx$ is equal to
 a) 0 b) 1 c) 2 d) 4
764. The value of the definite integral $\int_{a+2\pi}^{a+5\pi/2} (\sin^{-1}(\cos x) + \cos^{-1}(\sin x)) dx$ is equal to
 a) $\frac{\pi^2}{8}$ b) $\frac{\pi^2}{4}$ c) $\frac{\pi^2}{2}$ d) π^2
765. Let $T \neq 0$ be a fixed number suppose f is a continuous function such that for all $x \in R, f(x + T) = f(x)$. If $I = \int_0^T f(x) dx$, then the value of $\int_3^{3+3T} f(2x) dx$ is
 a) $\frac{3}{2}I$ b) $2I$ c) $3I$ d) $6I$
766. $\int \frac{\log(x+\sqrt{1+x^2})}{\sqrt{1+x^2}} dx$ is equal to
 a) $[\log(x + \sqrt{1+x^2})]^2 + c$ b) $x \log(x + \sqrt{1+x^2}) + c$
 c) $\frac{1}{2} \log(x + \sqrt{1+x^2}) + c$ d) $\frac{1}{2} [\log(x + \sqrt{1+x^2})]^2 + c$
767. If $\int_{\log 2}^x \frac{1}{\sqrt{e^x-1}} dx = \frac{\pi}{6}$, then x is equal to
 a) e^2 b) $1/e$ c) $\log 4$ d) None of these
768. $\int \frac{\cos x + x \sin x}{x^2 + x \cos x} dx$ is equal to
 a) $\log \left| \frac{\sin x}{1 + \cos x} \right| + c$ b) $\log \left| \frac{\sin x}{x + \cos x} \right| + c$ c) $\log \left| \frac{2 \sin x}{x + \cos x} \right| + c$ d) $\log \left| \frac{x}{x + \cos x} \right| + c$
769. $\int \frac{dx}{x\sqrt{x^6-16}}$ is equal to
 a) $\frac{1}{3} \sec^{-1} \left(\frac{x^3}{4} \right) + c$ b) $\cos^{-1} \left(\frac{x^3}{4} \right) + c$ c) $\frac{1}{12} \sec^{-1} \left(\frac{x^3}{4} \right) + c$ d) $\sec^{-1} \left(\frac{x^3}{4} \right) + c$
770. If $F(x) = \int_{x^2}^{x^3} \log t dt, (x > 0)$, then $F'(x)$ is equal to
 a) $(9x^2 - 4x) \log x$ b) $(4x - 9x^2) \log x$ c) $(9x^2 + 4x) \log x$ d) None of these
771. If $\int_0^{\infty} \frac{\log(1+x^2)}{1+x^2} dx = k \int_0^1 \frac{\log(1+x)}{1+x^2} dx$, then $k =$
 a) 4 b) 8 c) π d) 2π
772. The value of integral $\int_0^4 |x - 1| dx$ is
 a) 4 b) 5 c) 7 d) 9
773. If f is a continuous function, then
 a) $\int_{-2}^2 f(x) dx = \int_0^2 [f(x) - f(-x)] dx$ b) $\int_{-3}^5 2f(x) dx = \int_{-6}^{10} f(x - 1) dx$
 c) $\int_{-3}^5 f(x) dx = \int_{-4}^4 f(x - 1) dx$ d) $\int_{-3}^5 f(x) dx = \int_{-2}^6 f(x - 1) dx$
774. $\int [\sin(\log x) + \cos(\log x)] dx$ is equal to

- a) $x \cos(\log x) + c$ b) $\cos(\log x) + c$ c) $x \sin(\log x) + c$ d) $\sin(\log x) + c$
775. Let $f(x)$ be a continuous function such that $f(a - x) + f(x) = 0$ for all $x \in [0, a]$. Then, the value of the integral $\int_0^a \frac{1}{1+e^{f(x)}} dx$ is equal to
- a) a b) $\frac{a}{2}$ c) $f(a)$ d) $\frac{1}{2}f(a)$
776. $\int_{-1}^1 |1 - x| dx$ is equal to
- a) -2 b) 0 c) 2 d) 4
777. The integral $\int_{-1}^3 \left[\tan^{-1} \frac{x}{x^2+1} + \tan^{-1} \frac{x^2+1}{x} \right] dx$ is equal to
- a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) π d) 2π
778. The value of $\int \frac{x^2}{1+x^6} dx$ is
- a) $x^3 + c$ b) $\frac{1}{3} \tan^{-1}(x^3) + c$ c) $\log(1 + x^3)$ d) None of these
779. $\int \frac{\cos 2x-1}{\cos 2x+1} dx$ is equal to
- a) $\tan x - x + c$
b) $x + \tan x + c$
c) $x - \tan x + c$
d) $-x - \cot x + c$
780. $\int \frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}} dx$ is equal to
- a) $\frac{1}{2} \sqrt{1+x} + c$ b) $\frac{2}{3} (1+x)^{3/2} + c$ c) $\sqrt{1+x} + c$ d) $2(1+x)^{3/2} + c$
781. The primitive of the function $f(x) = (2x + 1)|\sin x|$, when $\pi < x < 2\pi$ is
- a) $-(2x + 1) \cos x + 2 \sin x + C$
b) $(2x + 1) \cos x - 2 \sin x + C$
c) $(x^2 + x) \cos x + C$
d) None of these
782. If $k \int_0^1 x \cdot f(3x) dx = \int_0^3 t \cdot f(t) dt$, then the value of k is
- a) 9 b) 3 c) $\frac{1}{9}$ d) $\frac{1}{3}$
783. $\int \frac{x^{e-1}+e^{x-1}}{x^e+e^x} dx$ is equal to
- a) $\log(x^e + e^x) + c$ b) $e \log(x^e + e^x) + c$
c) $\frac{1}{e} \log(x^e + e^x) + c$ d) None of these
784. $\int \operatorname{cosec}(x - a) \operatorname{cosec} x dx$ is equal to
- a) $\frac{-1}{\sin a} \log|\sin x \operatorname{cosec}(x - a)| + c$ b) $\frac{-1}{\sin a} \log[\sin(x - a) \sin x] + c$
c) $\frac{1}{\sin a} \log[\sin(x - a) \operatorname{cosec} x] + c$ d) $\frac{1}{\sin a} \log[\sin(x - a) \sin x] + c$
785. The value of $\int_0^{\pi/2} \frac{\cos \theta}{\sqrt{4-\sin^2 \theta}} d\theta$ is
- a) $\frac{\pi}{2}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{5}$
786. The value of the integral $\int_0^\pi \frac{1}{a^2 - 2a \cos x + 1} dx (a < 1)$ is
- a) $\frac{\pi}{1-a^2}$ b) $\frac{\pi}{a^2-1}$ c) $\frac{2\pi}{a^2-1}$ d) $\frac{3\pi}{4}$
787. The value of integral $\int_{-1}^1 \frac{|x+2|}{x+2} dx$ is

788. The value of $\int_1^5 (\sqrt{x+2\sqrt{x-1}} + \sqrt{x-2\sqrt{x-1}}) dx$ is
- a) 1 b) 2 c) 0 d) -1
- a) $\frac{8}{3}$ b) $\frac{16}{3}$ c) $\frac{32}{3}$ d) $\frac{64}{3}$
789. If $\int \frac{1}{(\sin x+4)(\sin x-1)} dx = A \frac{1}{\tan \frac{x}{2}-1} + B \tan^{-1}(f(x)) + C_1$. Then,
- a) $A = \frac{1}{5}, B = \frac{-2}{5\sqrt{15}}, f(x) = \frac{4 \tan x + 3}{\sqrt{15}}$ b) $A = -\frac{1}{5}, B = \frac{1}{\sqrt{15}}, f(x) = \frac{4 \tan \left(\frac{x}{2}\right) + 1}{\sqrt{15}}$
- c) $A = \frac{2}{5}, B = \frac{-2}{5}, f(x) = \frac{4 \tan x + 1}{5}$ d) $A = \frac{2}{5}, B = \frac{-2}{5\sqrt{15}}, f(x) = \frac{4 \tan \frac{x}{2} + 1}{\sqrt{15}}$
790. $\int (\sqrt{\tan x} + \sqrt{\cot x}) dx$ is equal to
- a) $\sqrt{2} \sin^{-1}(\sin x - \cos x) + c$ b) $\sqrt{2} \sin^{-1}(\sin x + \cos x) + c$
- c) $\sqrt{2} \tan^{-1}(\sin x - \cos x) + c$ d) None of the above
791. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ is
- a) $\frac{\pi^2}{4}$ b) π^2 c) zero d) $\frac{\pi}{2}$
792. If $\int \frac{\sin x}{\sin(x-\alpha)} dx = Ax + B \log_e \sin(x-\alpha) + C$, then
- a) $A = \sin \alpha, B = \cos \alpha$
- b) $A = \cos \alpha, B = -\sin \alpha$
- c) $A = \cos \alpha, B = \sin \alpha$
- d) None of these
793. The value of the integral $\int_0^{100} \sin(x - [x]) \pi dx$, is
- a) $100/\pi$ b) $200/\pi$ c) 100π d) 200π
794. If $f(x) = \cos x - \cos^2 x + \cos^3 x - \dots \infty$, then $\int f(x) dx$ is equal to
- a) $\tan \frac{x}{2} + c$ b) $x - \tan \frac{x}{2} + c$ c) $x - \frac{1}{2} \tan \frac{x}{2} + c$ d) $\frac{x - \tan \frac{x}{2}}{2} + c$
795. If $\int_0^a f(2a-x) dx = \mu$ and $\int_0^a f(x) dx = \lambda$, then $\int_0^{2a} f(x) dx$ equals
- a) $2\lambda - \mu$ b) $\lambda + \mu$ c) $\mu - \lambda$ d) $\lambda - 2\mu$
796. $\int \frac{f'(x)}{f(x) \log\{f(x)\}} dx =$
- a) $\frac{f(x)}{\log\{f(x)\}} + C$ b) $f(x) \log f(x) + C$ c) $\log\{\log f(x)\} + C$ d) $\frac{1}{\log\{\log f(x)\}} + C$
797. The value of $\int_1^4 |x-3| dx$ is equal to
- a) 2 b) $\frac{5}{2}$ c) $\frac{1}{2}$ d) $\frac{3}{2}$
798. $\int e^x \left(\log x + \frac{1}{x}\right) dx$ is equal to
- a) $e^x \log x + c$ b) $\frac{e^x}{\log x} + c$ c) $\frac{\log x}{x} + c$ d) $\frac{e^x}{x} + c$
799. $\int_{-3\pi/2}^{-\pi/2} [(x+\pi)^3 + \cos^2(x+3\pi)] dx$ is equal to
- a) $\left(\frac{\pi^4}{32}\right) + \left(\frac{\pi}{2}\right)$ b) $\left(\frac{\pi}{2}\right)$ c) $\left(\frac{\pi}{4}\right) - 1$ d) $\frac{\pi^4}{32}$
800. $\int_1^x \frac{\log(x^2)}{x} dx$ is equal to
- a) $(\log x)^2$ b) $\frac{1}{2}(\log x)^2$ c) $\frac{\log x^2}{2}$ d) None of these

801. If $f'(x) = x + \frac{1}{x}$, then the value of $f(x)$ is
 a) $x^2 + \log x + c$ b) $\frac{x^2}{2} + \log x + c$ c) $\frac{x}{2} + \log x + c$ d) None of these
802. $\int e^{-\log x} dx$ is equal to
 a) $e^{-\log x} + c$ b) $-xe^{-\log x} + c$ c) $e^{\log x} + c$ d) $\log|x| + c$
803. $\int \frac{dx}{x(x+1)}$ equals
 Where c is an arbitrary constant
 a) $\log\left|\frac{x+1}{x}\right| + c$ b) $\log\left|\frac{x}{x+1}\right| + c$ c) $\log\left|\frac{x-1}{x}\right| + c$ d) $\log\left|\frac{x-1}{x+1}\right| + c$
804. $\int_0^{\pi^2/4} \sin\sqrt{x} dx$ is equal to
 a) 0 b) 1 c) 2 d) 4
805. Limit of $\int_0^x \left[\frac{1}{\sqrt{1+t^2}} - \frac{1}{1+t}\right] dt$ as $x \rightarrow \infty$ is
 a) $\log_2 e$ b) $\log_e 2$ c) $\log_2\left(\frac{1}{e}\right)$ d) $\log_{1/e} 2$
806. If $I_1 = \int \sin^{-1} x dx$ and $I_2 = \int \sin^{-1} \sqrt{1-x^2} dx$, then
 a) $I_1 = I_2$ b) $I_2 = \frac{\pi}{2} I_1$ c) $I_1 + I_2 = \frac{\pi}{2} x$ d) $I_1 + I_2 = \frac{\pi}{2}$
807. $\int \cos^3 x \cdot e^{\log \sin x} dx$ is equal to
 a) $-\frac{\sin^4 x}{4} + c$ b) $-\frac{\cos^4 x}{4} + c$ c) $\frac{e^{\sin x}}{4} + c$ d) None of these
808. The value of the integral $\int_0^\pi \sqrt{\frac{1+\cos 2x}{2}} dx$ is
 a) -2 b) 2 c) 0 d) -3
809. $\int_0^{\pi/2} \sin 2x \log \tan x dx$ is equal to
 a) π b) $\frac{\pi}{2}$ c) 0 d) 2π
810. The value of $\int_{-\pi}^\pi (1-x^2) \sin x \cos^2 x dx$ is
 a) 0 b) $\pi - \frac{\pi^2}{3}$ c) $2\pi - \pi^3$ d) $\frac{7}{2} - 2\pi^3$
811. $\lim_{n \rightarrow \infty} \left[\frac{1}{n^2} \sec^2 \frac{1}{n^2} + \frac{2}{n^2} \sec^2 \frac{2}{n^2} + \dots + \frac{n}{n^2} \sec^2 1 \right]$ equals
 a) $\frac{1}{2} \tan 1$ b) $\tan 1$ c) $\frac{1}{2} \operatorname{cosec} 1$ d) $\frac{1}{2} \sec 1$
812. $\int \frac{1}{1+\cos ax} dx$ is equal to
 a) $\cot \frac{ax}{2} + c$ b) $\frac{1}{a} \tan \frac{ax}{2} + c$
 c) $\frac{1}{a} (\operatorname{cosec} ax - \cot ax) + c$ d) $\frac{1}{a} (\operatorname{cosec} ax + \cot ax) + c$
813. $\int \sqrt{1+\cos x} dx$ is equal to
 a) $2\sqrt{2} \cos \frac{x}{2} + c$ b) $2\sqrt{2} \sin \frac{x}{2} + c$ c) $\sqrt{2} \cos \frac{x}{2} + c$ d) $\sqrt{2} \sin \frac{x}{2} + c$
814. If $f(x) = \begin{cases} e^{\cos x} \sin x, & |x| \leq 2 \\ 2, & \text{otherwise} \end{cases}$, then $\int_{-2}^3 f(x) dx$ is equal to
 a) 0 b) 1 c) 2 d) 3
815. If $\int \frac{x e^x}{\sqrt{1+e^x}} dx = f(x)\sqrt{1+e^x} - 2 \log g(x) + C$, then
 a) $f(x) = x - 1$ b) $g(x) = \frac{\sqrt{1+e^x} - 1}{\sqrt{1+e^x} - 1}$ c) $g(x) = \frac{\sqrt{1+e^x} + 1}{\sqrt{1+e^x} - 1}$ d) $f(x) = 2(x+2)$

816. If a particle is moving with velocity $v(t) = \cos \pi t$ along a straight line such that at $t = 0, s = 4$ its position function is given by
- a) $\frac{1}{\pi} \cos \pi t + 2$ b) $-\frac{1}{\pi} \sin \pi t + 4$ c) $\frac{1}{\pi} \sin \pi t + 4$ d) None of these
817. $\int \cos(\log_e x) dx$ is equal to
- a) $\frac{1}{2} x [\cos(\log_e x) + \sin(\log_e x)]$ b) $x [\cos(\log_e x) + \sin(\log_e x)]$
c) $\frac{1}{2} x [\cos(\log_e x) - \sin(\log_e x)]$ d) $x [\cos(\log_e x) - \sin(\log_e x)]$
818. $\int \frac{(e^x - e^{-x}) dx}{(e^x + e^{-x}) \log(\cosh x)}$ is equal to
- a) $\log(\tanh x) + c$ b) $2 \log(e^x + e^{-x}) + c$
c) $2 \log(e^x - e^{-x}) + c$ d) $\log[\log(\cosh x)] + c$
819. If $u = -f''(\theta) \sin \theta + f'(\theta) \cos \theta$ and $v = f''(\theta) \cos \theta + f'(\theta) \sin \theta$, then
- $f \left[\left(\frac{du}{d\theta} \right)^2 + \left(\frac{dv}{d\theta} \right)^2 \right]^{\frac{1}{2}} d\theta$ is equal to
- a) $f(\theta) - f'(\theta) + c$ b) $f(\theta) + f''(\theta) + c$ c) $f'(\theta) + f''(\theta) + c$ d) $f'(\theta) - f''(\theta) + c$
820. The value of the integral $\int_0^{\pi} \frac{\sin kx}{\sin x} dx$ (k is an even integer) is equal to
- a) π b) $\frac{\pi}{2}$ c) $\frac{k\pi}{2}$ d) 0
821. $\int \frac{\sin x}{\sin(x-\alpha)} dx$ is
- a) $x \sin \alpha + \cos \alpha \log \sin(x + \alpha) + c$ b) $x \sin \alpha + \cos \alpha \log \sin(x - \alpha) + c$
c) $x \cos \alpha + \sin \alpha \log \cos(x + \alpha) + c$ d) $x \cos \alpha + \sin \alpha \log \sin(x - \alpha) + c$
822. To find the numerical value of $\int_{-2}^2 (px^3 + qx + s) dx$ it is necessary to know the values of the constants:
- a) p b) q c) s d) p and s
823. $\int x^3 \log x dx$ is equal to
- a) $\frac{x^4 \log x}{4} + c$ b) $\frac{1}{16} [4x^4 \log x - x^4] + c$
c) $\frac{1}{8} [x^4 \log x - 4x^2] + c$ d) $\frac{1}{16} [4x^4 \log x + x^4] + c$
824. $\int \frac{\sin^3 x}{(1 + \cos^2 x) \sqrt{1 + \cos^2 x + \cos^4 x}} dx$ is equal to
- a) $\sec^{-1}(\sec x + \cos x) + C$
b) $\sec^{-1}(\sec x - \cos x) + C$
c) $\sec^{-1}(\sec x - \tan x) + C$
d) None of these
825. If $x = \int_2^{\sin t} \sin^{-1} \theta d\theta$ and $y = \int_n^{\sqrt{t}} \frac{\sin \theta^2}{\theta} d\theta$ then $\frac{dy}{dx}$ is equal to
- a) $\frac{\tan t}{2t^2}$ b) $\frac{2t^2}{\tan t}$ c) $\frac{\tan t}{t^2}$ d) None of these
826. $\int_0^{\pi/2} \frac{200 \sin x + 100 \cos x}{\sin x + \cos x} dx$ is equal to
- a) 50π b) 25π c) 75π d) 150π
827. If $\frac{d}{dx} f(x) = g(x)$ for $a \leq x \leq b$ then, $\int_a^b f(x)g(x) dx$ equals
- a) $f(b) - f(a)$ b) $g(b) - g(a)$ c) $\frac{[f(b)]^2 - [f(a)]^2}{2}$ d) $\frac{[g(b)]^2 - [g(a)]^2}{2}$
828. If $f(x) = \int_0^x (\sin^4 t + \cos^4 t) dt$, then $f(x + \pi)$ will be equal to
- a) $f(x) + f\left(\frac{\pi}{2}\right)$ b) $f(x) + f(\pi)$ or $f(x) + 2f\left(\frac{\pi}{2}\right)$
c) $f(x) - f(\pi)$ d) $f(x) - 2f\left(\frac{\pi}{2}\right)$

829. $\int_0^1 \frac{x^7}{\sqrt{1-x^4}} dx$ is equal to

a) 1

b) $\frac{1}{3}$

c) $\frac{2}{3}$

d) $\frac{\pi}{3}$

830. The value of $\int \frac{1}{[(x-1)^3(x+2)^5]^{1/4}} dx$ is

a) $\frac{4}{3} \left(\frac{x-1}{x+2}\right)^{1/4} + c$

b) $\frac{4}{3} \left(\frac{x+1}{x+2}\right)^{1/4} + c$

c) $\frac{4}{3} \left(\frac{x+1}{x-2}\right)^{1/4} + c$

d) $\frac{4}{3} \left(\frac{x-1}{x-2}\right)^{1/4} + c$

831. The value of $\int \frac{dx}{x+\sqrt{x-1}}$ is

a) $\log(x + \sqrt{x-1}) + \sin^{-1} \left(\sqrt{\frac{x-1}{x}}\right) + c$

b) $\log(x + \sqrt{x-1}) + c$

c) $\log(x + \sqrt{x-1}) - \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right) + c$

d) None of the above

832. If $2f(x) - 3f(1/x) = x$, then $\int_1^2 f(x) dx$ is equal to

a) $(3/5) \log 2$

b) $(-3/5)(1 + \log 2)$

c) $(-3/5) \log 2$

d) None of these

833. $\int e^x \left(\frac{2}{x} - \frac{2}{x^2}\right) dx$ is equal to

a) $\frac{e^x}{x} + c$

b) $\frac{e^x}{2x^2} + c$

c) $\frac{2e^x}{x} + c$

d) $\frac{2e^x}{x^2} + c$

834. The value of the integral $\int_2^4 \frac{\sqrt{x^2-4}}{x^4} dx$, is

a) $\sqrt{\frac{3}{32}}$

b) $\frac{\sqrt{3}}{32}$

c) $\frac{32}{\sqrt{3}}$

d) $-\frac{\sqrt{3}}{32}$

835. $\int e^{3 \log x} (x^4 + 1)^{-1} dx$ is equal to

a) $\log(x^4 + 1) + C$

b) $\frac{1}{4} \log(x^4 + 1) + C$

c) $-\log(x^4 + 1)$

d) None of these

836. If $\int_0^p \frac{dx}{1+4x^2} = \frac{\pi}{8}$, then the value of p is

a) $\frac{1}{4}$

b) $-\frac{1}{2}$

c) $\frac{3}{2}$

d) $\frac{1}{2}$

837. $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx$ is equal to

a) $\frac{\pi}{2}$

b) $\sqrt{2} \log(\sqrt{2} + 1)$

c) $\frac{1}{\sqrt{2}} \log(\sqrt{2} + 1)$

d) None of these

838. $\int \frac{\sqrt{x^2+1} [\log(x^2+1) - 2 \log x]}{x^4} dx$ is equal to

a) $\frac{1}{3} \left(1 + \frac{1}{x^2}\right)^{1/2} \left[\log \left(1 + \frac{1}{x^2}\right) + \frac{2}{3}\right] + C$

b) $-\frac{1}{3} \left(1 + \frac{1}{x^2}\right)^{3/2} \left[\log \left(1 + \frac{1}{x^2}\right) - \frac{2}{3}\right] + C$

c) $\frac{2}{3} \left(1 + \frac{1}{x^2}\right)^{3/2} \left[\log \left(1 + \frac{1}{x^2}\right) + \frac{2}{3}\right] + C$

d) None of these

839. The value of $\int_3^5 \frac{x^2}{x^2-4} dx$, is

a) $2 - \log_e \left(\frac{15}{7}\right)$

b) $2 + \log_e \left(\frac{15}{7}\right)$

c) $2 + 4 \log_e 3 - 4 \log_e 7 + 4 \log_e 5$

d) $2 - \tan^{-1} \left(\frac{15}{7}\right)$

840. $\int_{\pi}^{10\pi} |\sin x| dx$ is equal to
 a) 20 b) 8 c) 10 d) 18
841. Let $\Delta(y) = \begin{vmatrix} y+a & y+b & y+a-c \\ y+b & y+c & y-1 \\ y+c & y+d & y-b+d \end{vmatrix}$ and, $\int_0^2 \Delta(y) dy = -16$, where a, b, c, d are in A.P., then the common difference of the A.P. is equal to
 a) ± 1 b) ± 2 c) ± 3 d) None of these
842. $\int \operatorname{cosec}^4 x dx$ is equal to
 a) $\cot x + \frac{\cot^3 x}{3} + c$ b) $\tan x + \frac{\tan^3 x}{3} + c$
 c) $-\cot x - \frac{\cot^3 x}{3} + c$ d) $-\tan x - \frac{\tan^3 x}{3} + c$
843. The value of $\int_0^1 (1 + e^{-x^2}) dx$ is
 a) -1 b) 2 c) $1 + e^{-1}$ d) None of these
844. The value of $\int e^{\sec x} \cdot \sec^3 x (\sin^2 x + \cos x + \sin x + \sin x \cos x) dx$ is
 a) $e^{\sec x} (\sec^2 x + \sec x \tan x) + C$
 b) $e^{\sec x} + C$
 c) $e^{\sec x} (\sec x + \tan x) + C$
 d) None of these
845. $\int \frac{\log \sqrt{x}}{3x} dx$ is equal to
 a) $\frac{1}{3} (\log \sqrt{x})^2 + c$ b) $\frac{2}{3} (\log \sqrt{x})^2 + c$ c) $\frac{2}{3} (\log x)^2 + c$ d) $\frac{1}{3} (\log x)^2 + c$
846. If $I = \int_0^{2\pi} \sin^2 x dx$, then
 a) $I = 4 \int_0^{\pi/4} \sin^2 x = 4 \int_0^{\pi/2} \sin^2 x dx$
 b) $I = \int_0^{2\pi} \cos^2 x dx$
 c) $I = 8 \int_0^{\pi/4} \sin^2 x dx$
 d) None of these
847. The smallest interval $[a, b]$ such that $\int_0^1 \frac{1}{\sqrt{1+x^4}} dx \in [a, b]$ is
 a) $\left[\frac{1}{\sqrt{2}}, 1\right]$ b) $[0, 1]$ c) $\left[\frac{1}{2}, 1\right]$ d) $\left[\frac{3}{4}, 1\right]$
848. The value of $\int \frac{3x+2}{(x-2)^2(x-3)} dx$ is
 a) $11 \log \frac{x-3}{x-2} - \frac{8}{x-2} + c$ b) $11 \log \frac{x+3}{x+2} - \frac{8}{x-2} + c$
 c) $11 \log \frac{x-3}{x-2} + \frac{8}{x-2} + c$ d) $11 \log \frac{x+3}{x+2} + \frac{8}{x-2} + c$
849. $\int \frac{\cos x - 1}{\sin x + 1} \cdot e^x dx$ is equal to
 a) $\frac{e^x \cos x}{1 + \sin x} + c$ b) $c - \frac{e^x \sin x}{1 + \sin x}$ c) $c - \frac{e^x}{1 + \sin x}$ d) $c - \frac{e^x \cos x}{1 + \sin x}$
850. $\int_0^{2\pi} \sin^6 x \cos^5 x dx$ is equal to
 a) 2π b) $\pi/2$ c) 0 d) $-\pi$
851. $\int \cos^{-3/7} x \sin^{-11/7} x dx$ is equal to

- a) $\log|\sin^{4/7} x| + c$ b) $\frac{4}{7}\tan^{4/7} x + c$ c) $-\frac{7}{4}\tan^{-4/7} x + c$ d) $\log|\cos^{3/7} x| + c$
852. $\lim_{n \rightarrow \infty} \left\{ \frac{1}{na} + \frac{1}{na+1} + \frac{1}{na+2} + \dots + \frac{1}{nb} \right\}$ is equal to
a) $\log\left(\frac{b}{a}\right)$ b) $\log\left(\frac{a}{b}\right)$ c) $\log a$ d) $\log b$
853. The value of the integral $\int_{-a}^a \frac{xe^{x^2}}{1+x^2} dx$ is
a) e^{a^2} b) 0 c) e^{-a^2} d) a
854. $\int \frac{(\sin\theta + \cos\theta)}{\sqrt{\sin 2\theta}} d\theta$ is equal to
a) $\log|\cos\theta - \sin\theta + \sqrt{\sin 2\theta}| + c$ b) $\log|\sin\theta - \cos\theta + \sqrt{\sin 2\theta}| + c$
c) $\sin^{-1}(\sin\theta - \cos\theta) + c$ d) $\sin^{-1}(\sin\theta + \cos\theta) + c$
855. If $\int \frac{e^x - 1}{e^x + 1} dx = f(x) + c$, then $f(x)$ is equal to
a) $2\log(e^x + 1)$ b) $\log(e^{2x} - 1)$ c) $2\log(e^x + 1) - x$ d) $\log(e^{2x} + 1)$
856. $\int e^{3\log x} (x^4 + 1)^{-1} dx$ equals
a) $\log(x^4 + 1) + c$ b) $\frac{1}{4}\log(x^4 + 1) + c$ c) $-\log(x^4 + 1) + c$ d) None of these
857. The anti-derivative of $f(x) = \frac{1}{3+5\sin x+3\cos x}$, whose graph passes through the point (0, 0) is
a) $\frac{1}{5} \left(\log \left| 1 - \frac{5}{3} \tan(x/2) \right| \right)$ b) $\frac{1}{5} \left(\log \left| 1 + \frac{5}{3} \tan(x/2) \right| \right)$
c) $\frac{1}{5} \left(\log \left| 1 + \frac{5}{3} \cot(x/2) \right| \right)$ d) None of these
858. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $g(x)$ be a function that satisfies $f(x) + g(x) = x^2$. Then, the value of the integral $\int_0^1 f(x) g(x) dx$, is
a) $e + \frac{e^2}{2} + \frac{5}{2}$ b) $e - \frac{e^2}{2} - \frac{5}{2}$ c) $e + \frac{e^2}{2} - \frac{3}{2}$ d) $e - \frac{e^2}{2} - \frac{3}{2}$
859. If $f(x) = \int_0^{x^2} \sqrt{1+t^2} dt$, then $f'(x)$ equals
a) $\sqrt{1+t^2}$ b) $\sqrt{1+x^4}$ c) $2x\sqrt{1+x^4}$ d) None of these
860. If $I = \int_3^4 \frac{1}{\sqrt[3]{\log x}} dx$, then
a) $0.92 < I < 1$ b) $I > 1$ c) $I < 0.92$ d) None of these
861. If $f(y) = e^y, g(y) = y; y > 0$ and $F(t) = \int_0^t f(t-y) g(y) dy$, then
a) $F(t) = t e^{-t}$
b) $F(t) = 1 - e^{-t}(t+1)$
c) $F(t) = e^t - (1+t)$
d) $F(t) = t e^t$
862. The value of the integral $I = \int_1^\infty \frac{x^2-2}{x^3\sqrt{x^2-1}} dx$, is
a) 0 b) $\frac{2}{3}$ c) $\frac{4}{3}$ d) None of these
863. $\int (\sin 2x - \cos 2x) dx = \frac{1}{\sqrt{2}} \sin(2x - a) + b$, then
a) $a = \frac{5\pi}{4}, b \in R$ b) $a = -\frac{5\pi}{4}, b \in R$ c) $a = \frac{\pi}{4}, b \in R$ d) None of these
864. $\int \frac{\sec x}{\sec x + \tan x} dx$ is equal to
a) $\tan x - \sec x + c$ b) $\log(1 + \sec x) + c$
c) $\sec x + \tan x + c$ d) $\log \sin x + \log \cos x + c$
865. Primitive of $\cos^{-1} x$ w. r. t x is
a) $x \cos^{-1} x - \frac{1}{2} \sqrt{1-x^2} + c$ b) $x \cos^{-1} x - \sqrt{1-x^2} + c$

- c) $x \cos^{-1} x + \sqrt{1-x^2} + c$ d) $x \cos^{-1} x + \frac{1}{2}\sqrt{1-x^2} + c$
866. If $\int_0^{36} \frac{1}{2x+9} dx = \log k$, then k is equal to
 a) 3 b) $9/2$ c) 9 d) 81
867. $\int \log 2x dx$ is equal to
 a) $x \log 2x - \frac{x^2}{2} + c$ b) $x \log 2x - \frac{x}{2} + c$ c) $x^2 \log 2x - \frac{x}{2} + c$ d) $x \log 2x - x + c$
868. If $f(0) = f'(0) = 0$ and $f''(x) = \tan^2 x$, then $f(x)$ is
 a) $\log \sec x - \frac{x^2}{2}$ b) $\log \cos x + \frac{x^2}{2}$ c) $\log \sec x + \frac{x^2}{2}$ d) None of these
869. $\int \frac{dx}{x(x^5+1)}$ is equal to
 a) $\frac{1}{5} \log x^5(x^5+1) + c$ b) $\frac{1}{5} \log \left(\frac{x^5+1}{x^5} \right) + c$ c) $\frac{1}{5} \log \left(\frac{x^5}{x^5+1} \right) + c$ d) None of these
870. If $\int \frac{1}{1+\sin x} dx = \tan \left(\frac{x}{2} + a \right) + b$, then
 a) $a = -\frac{\pi}{4}, b \in R$ b) $a = \frac{\pi}{4}, b \in R$ c) $a = \frac{5\pi}{4}, b \in R$ d) None of these
871. $\int \sqrt{e^x-1} dx$ is equal to
 a) $2[\sqrt{e^x-1} - \tan^{-1} \sqrt{e^x-1}] + c$ b) $\sqrt{e^x-1} - \tan^{-1} \sqrt{e^x-1} + c$
 c) $\sqrt{e^x-1} + \tan^{-1} \sqrt{e^x-1} + c$ d) $2[\sqrt{e^x-1} + \tan^{-1} \sqrt{e^x-1}] + c$
872. If $\int \frac{dx}{x^4+x^3} = \frac{A}{x^2} + \frac{B}{x} + \log \left| \frac{x}{x+1} \right| + C$, then
 a) $A = \frac{1}{2}, B = 1$ b) $A = 1, B = -\frac{1}{2}$ c) $A = -\frac{1}{2}, B = 1$ d) $A = 1, B = 1$
873. The value of $\int_{\alpha}^{\beta} x |x| dx$, where $\alpha < 0 < \beta$, is
 a) $\frac{1}{2}(\alpha^2 + \beta^2)$ b) $\frac{1}{3}(\beta^2 - \alpha^2)$ c) $\frac{1}{3}(\alpha^3 + \beta^3)$ d) None of these
874. The value of $\alpha \in (-\pi, 0)$ satisfying $\sin \alpha + \int_{\alpha}^{2\alpha} \cos 2x dx = 0$, is
 a) $-\pi/2$ b) $-\pi$ c) $-\pi/3$ d) 0
875. If $I_1 = \int_0^{\pi/2} x \sin x dx$ and $I_2 = \int_0^{\pi/2} x \cos x dx$, then which one of the following is true?
 a) $I_1 + I_2 = \frac{\pi}{2}$ b) $I_2 - I_1 = \frac{\pi}{2}$ c) $I_1 + I_2 = 0$ d) $I_1 = I_2$
876. $\int_{1/2}^2 |\log_{10} x| dx$ is equals to
 a) $\log_{10}(8/e)$ b) $\frac{1}{2} \log_{10}(8/e)$ c) $\log_{10}(2/e)$ d) None of these
877. $\int_0^1 |\sin 2\pi x| dx$ is equal to
 a) 0 b) $-\frac{1}{\pi}$ c) $\frac{1}{\pi}$ d) $\frac{2}{\pi}$
878. If $I = \int_{-\pi}^{\pi} \frac{e^{\sin x}}{e^{\sin x} + e^{-\sin x}} dx$, then I equals
 a) $\frac{\pi}{2}$ b) 2π c) π d) $\frac{\pi}{4}$
879. $\int \frac{1}{1+\cos x+\sin x} dx$ is equal to
 a) $\log \left| 1 + \tan \frac{x}{2} \right| + c$ b) $\frac{1}{2} \log \left| 1 + \tan \frac{x}{2} \right| + c$
 c) $2 \log \left| 1 + \tan \frac{x}{2} \right| + c$ d) $\frac{1}{2} \log \left| 1 - \tan \frac{x}{2} \right| + c$
880. $\int \frac{x^4-1}{x^2(x^4+x^2+1)^{1/2}} dx$ is equal to

$$a) \sqrt{\frac{x^4 + x^2 + 1}{c}} + c$$

$$b) \frac{x^2}{\sqrt{x^4 + x^2 + 1}} + c$$

$$c) x(x^4 + x^2 + 1)^{3/2} + c$$

$$d) \frac{\sqrt{x^4 + x^2 + 1}}{x} + c$$

881. The value of $\int_0^1 \frac{x^3}{1+x^8} dx$, is

$$a) \frac{\pi}{4}$$

$$b) \frac{\pi}{8}$$

$$c) \frac{\pi}{16}$$

d) None of these

882. $\int \frac{\sin x + 8 \cos x}{4 \sin x + 6 \cos x} dx$ is equal to

$$a) x + \frac{1}{2} \log(4 \sin x + 6 \cos x) + c$$

$$b) 2x + \log(2 \sin x + 3 \cos x) + c$$

$$c) x + 2 \log(2 \sin x + 3 \cos x) + c$$

$$d) \frac{1}{2} \log(4 \sin x + 6 \cos x) + c$$

883. $\int x^2(ax + b)^{-2} dx$ is equal to

$$a) \frac{2}{a^2} \left(x - \frac{b}{a} \log(ax + b) \right) + c$$

$$b) \frac{2}{a^2} \left(x - \frac{b}{a} \log(ax + b) \right) - \frac{x^2}{a(ax + b)} + c$$

$$c) \frac{2}{a^2} \left(x + \frac{b}{a} \log(ax + b) \right) + \frac{x^2}{a(ax + b)} + c$$

$$d) \frac{2}{a^2} \left(x + \frac{b}{a} \log(ax + b) \right) - \frac{x^2}{a(ax + b)} + c$$

884. The value of $\int_1^4 e^{\sqrt{x}} dx$ is

$$a) e^2$$

$$b) 2e^2$$

$$c) 4e^2$$

$$d) 3e^2$$

885. If $\int \frac{1}{x+x^5} dx = f(x) + c$, then the value of $\int \frac{x^4}{x+x^5} dx$ equals

$$a) \log x - f(x) + c$$

$$b) f(x) + \log x + c$$

$$c) f(x) - \log x + c$$

d) None of these

886. $\int e^x(1 - \cot x + \cot^2 x) dx$ equals

$$a) e^x \cot x + c$$

$$b) e^x \operatorname{cosec} x + c$$

$$c) -e^x \cot x + c$$

$$d) -e^x \operatorname{cosec} x + c$$

887. If $f(x)$ is defined $[-2, 2]$ by $f(x) = 4x^2 - 3x + 1$ and $g(x) = \frac{f(-x) - f(x)}{x^2 + 3}$,

then $\int_{-2}^2 g(x) dx$ is equal to

$$a) 64$$

$$b) -48$$

$$c) 0$$

$$d) 24$$

888. $\int \sqrt{1 + \sin \frac{x}{4}} dx$ is equal to

$$a) 8 \left(\sin \frac{x}{8} - \cos \frac{x}{8} \right) + c$$

$$b) \left(\sin \frac{x}{8} + \cos \frac{x}{8} \right) + c$$

$$c) \frac{1}{8} \left(\sin \frac{x}{8} - \cos \frac{x}{8} \right) + c$$

$$d) 8 \left(\cos \frac{x}{8} - \sin \frac{x}{8} \right) + c$$

889. If $h(a) = h(b)$, the value of the integral $\int_a^b [f(g(h(x)))]^{-1} f'(g(h(x))) g'(h(x)) h'(x) dx$ is equal to

$$a) 0$$

$$b) f(a) - f(b)$$

$$c) f(g(a)) - f(g(b))$$

d) None of these

890. $\int_0^{\pi/2} \frac{\sin^2 x}{\sin^2 x + \cos^2 x} dx$ is equal to

$$a) \frac{\pi}{4}$$

$$b) 2\pi$$

$$c) \pi^2$$

$$d) \frac{1}{2} \pi^2$$

891. The value of the integral $\int_0^1 \log \sin \left(\frac{\pi x}{2} \right) dx$, is

$$a) \log 2$$

$$b) -\log 2$$

$$c) \frac{\pi}{2} \log 2$$

$$d) -\frac{\pi}{2} \log 2$$

892. Let $\frac{d}{dx} F(x) = \left(\frac{e^{\sin x}}{x} \right)$, $x > 0$, If $\int_1^4 \frac{3}{x} e^{\sin x^3} dx = F(k) - F(1)$, then one of the possible values of k ,

is

$$a) 15$$

$$b) 16$$

$$c) 63$$

$$d) 64$$

893. $\int_0^{10} |x(x-1)(x-2)| dx$ is equal to

$$a) 160.05$$

$$b) 1600.5$$

$$c) 16.005$$

d) None of these

894. $\int \frac{dx}{\sin x - \cos x + \sqrt{2}}$ equals
- a) $-\frac{1}{\sqrt{2}} \tan\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$ b) $\frac{1}{2} \tan\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$
c) $\frac{1}{\sqrt{2}} \cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$ d) $-\frac{1}{\sqrt{2}} \cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$
895. The value of the integral $\int_{-\pi/4}^{\pi/4} \sin^{-4} x \, dx$, is
- a) $-\frac{8}{3}$ b) $\frac{3}{2}$ c) $\frac{8}{3}$ d) None of these
896. If $I_n = \int_0^{\pi/4} \tan^n \theta \, d\theta$, then $I_8 + I_6$ is equal to
- a) $\frac{1}{7}$ b) $\frac{1}{4}$ c) $\frac{1}{5}$ d) $\frac{1}{6}$
897. $\int \frac{a^{\sqrt{x}}}{\sqrt{x}} \, dx$ is equal to
- a) $\frac{a^{\sqrt{x}}}{\log a} + C$ b) $\frac{2a^{\sqrt{x}}}{\log a} + C$ c) $2a^{\sqrt{x}} \cdot \log a + C$ d) None of these
898. $\int_a^b \sqrt{(x-a)(b-x)} \, dx$, ($b > a$) is equal to
- a) $\frac{\pi(b-a)^2}{8}$ b) $\frac{\pi(b+a)^2}{8}$ c) $(b-a)^2$ d) $(b+a)^2$
899. If $\int_0^{x^2} f(t) \, dt = x \cos \pi x$, then the value of $f(4)$ is
- a) 1 b) $\frac{1}{4}$ c) -1 d) $-\frac{1}{4}$
900. If $f'(x) = \frac{dx}{(1+x^2)^{3/2}}$ and $f(0) = 0$, then $f(1)$ is equal to
- a) $\sqrt{2}$ b) $-\frac{1}{\sqrt{2}}$ c) $\frac{1}{\sqrt{2}}$ d) None of these
901. The value of $\int_0^{\pi/2} \frac{x + \sin x}{1 + \cos x} \, dx$, is
- a) π b) 2π c) $\pi/2$ d) $3\pi/2$
902. If $\int f(x) \sin x \cos x \, dx = \frac{1}{2(b^2-a^2)} \log\{f(x)\} + C$, then $f(x)$ is equal to
- a) $\frac{1}{a^2 \sin^2 x + b^2 \cos^2 x}$ b) $\frac{1}{a^2 \sin^2 x - b^2 \cos^2 x}$ c) $\frac{1}{a^2 \cos^2 x + b^2 \sin^2 x}$ d) $\frac{1}{a^2 \cos^2 x - b^2 \sin^2 x}$
903. $\int \sqrt{\frac{1+x}{1-x}} \, dx$ is equal to
- a) $-\sin^{-1} x - \sqrt{1-x^2} + c$ b) $\sin^{-1} x + \sqrt{1-x^2} + c$
c) $\sin^{-1} x - \sqrt{1-x^2} + c$ d) $-\sin^{-1} x - \sqrt{x^2-1} + c$
904. The value of $\int_1^2 \frac{dx}{x(1+x^4)}$ is
- a) $\frac{1}{4} \log \frac{17}{32}$ b) $\frac{1}{4} \log \frac{32}{17}$ c) $\log \frac{17}{2}$ d) $\frac{1}{4} \log \frac{17}{2}$
905. $\int \frac{1}{(x+1)^2 \sqrt{x^2+2x+2}} \, dx$ is equal to
- a) $\frac{\sqrt{x^2+2x+2}}{x+1} + C$ b) $\frac{\sqrt{x^2+2x+2}}{(x+1)^2} + C$ c) $\frac{-\sqrt{x^2+2x+2}}{x+1} + C$ d) None of these
906. If $I = \int_{-1}^1 \left\{ [x^2] + \log\left(\frac{2+x}{2-x}\right) \right\} dx$ where $[x]$ denotes the greatest integer less than or equal to x , the I equals
- a) -2 b) -1 c) 0 d) 1
907. $\int e^x \left(\frac{1-\sin x}{1-\cos x} \right) dx$ is equal to
- a) $-e^x \tan\left(\frac{x}{2}\right) + c$ b) $-e^x \cot\left(\frac{x}{2}\right) + c$ c) $-\frac{1}{2} e^x \tan\left(\frac{x}{2}\right) + c$ d) $\frac{1}{2} e^x \cot\left(\frac{x}{2}\right) + c$

908. $\int \frac{x^2-1}{(x^4+3x^2+1)\tan^{-1}\left(x+\frac{1}{x}\right)} dx$ is equal to
- a) $\tan^{-1}\left(x+\frac{1}{x}\right)+c$ b) $\cot^{-1}\left(x+\frac{1}{x}\right)+c$
c) $\log\left(x+\frac{1}{x}\right)+c$ d) $\log\left[\tan^{-1}\left(x+\frac{1}{x}\right)\right]+c$
909. $\int_0^1 \frac{x}{(1-x)^{3/4}} dx =$
- a) $\frac{12}{5}$ b) $\frac{16}{5}$ c) $-\frac{16}{5}$ d) None of these
910. Least value of the function $f(x) = \int_0^x (1 - \cos t) dt$ on the interval $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ is
- a) $\frac{3\pi}{2} + 1$ b) $\frac{\pi}{2} - 1$ c) $\frac{\pi}{2} + 1$ d) None of these
911. $\int \frac{x^2-2}{x^3\sqrt{x^2-1}} dx$ is equal to
- a) $\frac{x^2}{\sqrt{x^2-1}} + c$ b) $-\frac{x^2}{\sqrt{x^2-1}} + c$ c) $\frac{\sqrt{x^2-1}}{x^2} + c$ d) $-\frac{\sqrt{x^2-1}}{x^2} + c$
912. The value of $\int \frac{(x-x^3)^{1/3}}{x^4} dx$ is
- a) $\frac{3}{8}\left(\frac{1}{x^2}-1\right)^{4/3} + C$ b) $-\frac{3}{8}\left(\frac{1}{x^2}-1\right)^{4/3} + C$ c) $\frac{1}{8}\left(1-\frac{1}{x^2}\right)^{4/3} + 1$ d) None of these
913. $\int_0^\infty \log\left(x+\frac{1}{x}\right) \frac{dx}{1+x^2}$ is equal to
- a) $\pi \log 2$ b) $-\pi \log 2$ c) $\left(\frac{\pi}{2}\right) \log 2$ d) $-\left(\frac{\pi}{2}\right) \log 2$
914. If $\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right] dx = x[f(x) - g(x)] + c$, then
- a) $f(x) = \log(\log x); g(x) = \frac{1}{\log x}$ b) $f(x) = \log x; g(x) = \frac{1}{\log x}$
c) $f(x) = \frac{1}{\log x}; g(x) = \log(\log x)$ d) $f(x) = \frac{1}{x \log x}; g(x) = \frac{1}{\log x}$
915. Integration of $\frac{1}{\sqrt{x^2+9}}$ with respect to (x^2+1) is equal to
- a) $\sqrt{x^2+9} + C$ b) $\frac{1}{\sqrt{x^2+9}} + C$ c) $2\sqrt{x^2+9} + C$ d) None of these
916. $\int \frac{\sqrt{x}}{1+\sqrt[4]{x^3}} dx$ is equal to
- a) $\frac{4}{3}[1+x^{3/4} + \log_e(1+x^{3/4})] + C$
b) $\frac{4}{3}[1+x^{3/4} - \log_e(1+x^{3/4})] + C$
c) $\frac{4}{3}[1-x^{3/4} + \log_e(1+x^{3/4})] + C$
d) None of these
917. $\int e^{x \log a} \cdot e^x dx$ is equal to
- a) $(ae)^x$ b) $\frac{(ae)^x}{\log(ae)}$ c) $\frac{e^x}{1+\log a}$ d) None of these
918. If $\int f(x) dx = g(x)$, then $f(x)g(x)$ is equal to
- a) $\frac{1}{2}f^2(x)$ b) $\frac{1}{2}g^2(x)$ c) $\frac{1}{2}[g'(x)]^2$ d) $f'(x)g(x)$
919. If $\int \frac{\cos 4x+1}{\cos x - \tan x} dx = A \cos 4x + B$, then
- a) $A = -\frac{1}{2}$ b) $A = -\frac{1}{8}$ c) $A = -\frac{1}{4}$ d) None of these

920. $\int_0^\pi k(\pi x - x^2)^{100} \sin 2x \, dx$ is equal to
 a) π^{100} b) $\frac{1}{2}(\pi^{100} - \pi^{97})$ c) $\frac{1}{2}(\pi^{100} + \pi^{97})$ d) 0
921. The value of $\int_\pi^{2\pi} [2 \sin x] \, dx$, where $[]$ represents the greatest integer function, is
 a) $-\frac{5\pi}{3}$ b) $-\pi$ c) $\frac{5\pi}{3}$ d) -2π
922. $\int \frac{1}{\sqrt{\sin^3 x \cos x}} \, dx$ is equal to
 a) $\frac{-2}{\sqrt{\tan x}} + C$ b) $2\sqrt{\tan x} + c$ c) $\frac{2}{\sqrt{\tan x}} + C$ d) $-2\sqrt{\tan x} + c$
923. $\int_0^{1.5} [x^2] \, dx$ is
 a) $4 + 2\sqrt{2}$ b) $2 + \sqrt{2}$ c) $2 - \sqrt{2}$ d) None of these
924. $\int_0^\pi |\cos x| \, dx$ is equal to
 a) $\frac{1}{2}$ b) -2 c) 1 d) 2
925. $\int \frac{x + \sqrt[3]{x^2} + \sqrt[6]{x}}{x(1 + \sqrt[3]{x})} \, dx$
 a) $\frac{3}{2}x^{2/3} + 6 \tan^{-1} x^{1/6} + C$
 b) $\frac{3}{2}x^{2/3} - 6 \tan^{-1} x^{1/6} + C$
 c) $-\frac{3}{2}x^{2/3} + 6 \tan^{-1} x^{1/6} + C$
 d) None of these
926. For any integer n , the integral $\int_0^\pi e^{\cos^2 x} \cos^3(2n - 1)x \, dx$ has the value
 a) π b) 1 c) 0 d) None of these
927. The value of $\int_0^{\pi/2} \frac{\cos 3x + 1}{2 \cos x - 1} \, dx$ is
 a) 2 b) 1 c) $\frac{1}{2}$ d) 0
928. $\int_1^3 \frac{\cos(\log x)}{x} \, dx$ is equal to
 a) 1 b) $\cos(\log 3)$ c) $\sin(\log 3)$ d) $\pi/4$
929. $\int (e^x + e^{-x}) \cdot (e^x - e^{-x}) \, dx$ is equal to
 a) $e^x + c$ b) $\frac{1}{2} (e^x - e^{-x})^2 + c$ c) $\frac{1}{2} (e^x + e^{-x})^2 + c$ d) $\frac{1}{3} (e^x + e^{-x})^3 + c$
930. $\int \frac{dx}{\sin x \cos x}$ is equal to
 a) $\log |\sin x| + c$ b) $\log |\tan x| + c$ c) $\log |\cos x| + c$ d) None of these
931. The value of the integral $\int_0^{\pi/2} \sin^5 x \, dx$
 a) $\frac{4}{15}$ b) $\frac{8}{5}$ c) $\frac{8}{15}$ d) $\frac{4}{5}$
932. $\int_{-\pi/2}^{\pi/2} \log_e \left\{ \frac{(ax^2 + bx + c)}{(ax^2 - bx + c)} (a + b) |\sin x| \right\} \, dx$ is equal to
 a) $\pi \log_e (a + b)$ b) $\pi \log_e \left(\frac{a + b}{2} \right)$ c) $\frac{\pi}{2} \log_e (a + b)$ d) None of these
933. The value of $\int_{1/e}^e \frac{|\log x|}{x^2} \, dx$, is
 a) 2 b) $\frac{2}{e}$ c) $2 \left(1 - \frac{1}{e} \right)$ d) 0
934. The value of $\int_{-1}^1 x|x| \, dx$ is

- a) 2 b) 1 c) 0 d) None of these
935. If $\int \frac{1}{x^3+x^4} dx = \frac{A}{x^2} + \frac{B}{x} + \log \left| \frac{x}{x+1} \right| + C$, then
a) $A = \frac{1}{2}, B = 1$ b) $A = 1, B = -\frac{1}{2}$ c) $A = -\frac{1}{2}, B = 1$ d) None of these
936. If $\int \frac{1}{(x^2+1)(x^2+4)} dx = A \tan^{-1} x + B \tan^{-1} \frac{x}{2} + C$, then
a) $A = 1/3, B = -2/3$ b) $A = -1/3, B = 2/3$ c) $A = -1/3, B = \frac{1}{3}$ d) $A = \frac{1}{3}, B = -\frac{1}{6}$
937. The value of the integral $\int_{-1/2}^{1/2} \left\{ \left(\frac{x+1}{x-1} \right)^2 + \left(\frac{x-1}{x+1} \right)^2 - 2 \right\}^{1/2} dx$, is
a) $\log \left(\frac{4}{3} \right)$ b) $4 \log \left(\frac{3}{4} \right)$ c) $4 \log \left(\frac{4}{3} \right)$ d) $\log \left(\frac{3}{4} \right)$
938. If $\int \operatorname{cosec} x dx = f(x) + \text{constant}$, then $f(x)$ is equal to
a) $\tan x/2$ b) $\log |\tan(x/2)|$ c) $\log |\sin x|$ d) $\log |\cos x|$
939. $\int \frac{dx}{x(x^n+1)}$ is equal to
a) $\frac{1}{n} \log \left(\frac{x^n}{x^n+1} \right) + c$ b) $\frac{1}{n} \log \left(\frac{x^n+1}{x^n} \right) + c$ c) $\log \left(\frac{x^n}{x^n+1} \right) + c$ d) None of the above
940. $\int \frac{ax^3+bx^2+c}{x^4} dx$ equals
a) $a \log x + \frac{b}{x^2} + \frac{c}{3x^3} + k$ b) $a \log x + \frac{b}{x} - \frac{c}{3x^3} + k$
c) $a \log x - \frac{b}{x} - \frac{c}{3x^3} + k$ d) None of these
941. $\int \frac{x+2}{(x^2+3x+3)\sqrt{x+1}} dx$ is equal to
a) $\frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{x}{\sqrt{3(x+1)}} \right)$ b) $\frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{x}{\sqrt{3(x+1)}} \right)$ c) $\frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{x}{\sqrt{x+1}} \right)$ d) None of these
942. The value of $\int_0^{\pi/2} \frac{dx}{1+\cot x}$ is
a) π b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{4}$
943. $\int \frac{\sin 2x}{1+\cos^2 x} dx$ is equal to
a) $-\frac{1}{2} \log(1+\cos^2 x) + c$ b) $2 \log(1+\cos^2 x) + c$
c) $\frac{1}{2} \log(1+\cos 2x) + c$ d) $c - \log(1+\cos^2 x)$
944. For any integer n , the integral $\int_0^\pi e^{\sin^2 x} \cos^3(2n+1)x dx$ has the value
a) π b) 1 c) 0 d) None of these
945. If $\int \frac{\log(x+\sqrt{1+x^2})}{\sqrt{1+x^2}} dx = gof(x) + \text{constant}$, then
a) $f(x) = \log(x+\sqrt{x^2+1})$ b) $f(x) = \log(x+\sqrt{x^2+1})$ and $g(x) = x^2$
c) $f(x) = \log(x+\sqrt{x^2+1})$ and $g(x) = \frac{x^2}{2}$ d) $f(x) = \frac{x^2}{2}$ and $g(x) = \log(x+\sqrt{x^2+1})$
946. $\int \frac{1+\tan^2 x}{1-\tan^2 x} dx$ is equal to
a) $\log \left(\frac{1-\tan x}{1+\tan x} \right) + c$ b) $\log \left(\frac{1+\tan x}{1-\tan x} \right) + c$ c) $\frac{1}{2} \log \left(\frac{1-\tan x}{1+\tan x} \right) + c$ d) $\frac{1}{2} \log \left(\frac{1+\tan x}{1-\tan x} \right) + c$
947. $\int (x+1)(x+2)^7(x+3) dx$ is equal to
a) $\frac{(x+2)^{10}}{10} - \frac{(x+2)^8}{8} + c$ b) $\frac{(x+1)^2}{2} - \frac{(x+2)^8}{8} - \frac{(x+3)^2}{2} + c$
c) $\frac{(x+2)^{10}}{10} + c$ d) $\frac{(x+1)^2}{2} + \frac{(x+2)^8}{8} + \frac{(x+3)^2}{2} + c$

: ANSWER KEY :

1)	b	2)	c	3)	c	4)	b	189)	d	190)	a	191)	d	192)	d
5)	b	6)	b	7)	d	8)	d	193)	a	194)	a	195)	c	196)	b
9)	c	10)	a	11)	c	12)	c	197)	a	198)	a	199)	b	200)	b
13)	a	14)	b	15)	a	16)	c	201)	b	202)	a	203)	c	204)	d
17)	d	18)	b	19)	b	20)	d	205)	b	206)	b	207)	c	208)	d
21)	c	22)	a	23)	b	24)	b	209)	c	210)	c	211)	c	212)	b
25)	a	26)	c	27)	c	28)	a	213)	b	214)	d	215)	b	216)	a
29)	a	30)	a	31)	b	32)	d	217)	a	218)	b	219)	b	220)	b
33)	a	34)	c	35)	a	36)	c	221)	b	222)	d	223)	a	224)	c
37)	a	38)	a	39)	b	40)	a	225)	c	226)	a	227)	a	228)	c
41)	c	42)	d	43)	b	44)	b	229)	d	230)	b	231)	a	232)	c
45)	a	46)	d	47)	d	48)	d	233)	d	234)	b	235)	a	236)	c
49)	c	50)	b	51)	a	52)	a	237)	d	238)	b	239)	c	240)	b
53)	c	54)	a	55)	d	56)	c	241)	b	242)	c	243)	c	244)	a
57)	b	58)	c	59)	d	60)	b	245)	c	246)	b	247)	d	248)	c
61)	b	62)	d	63)	b	64)	b	249)	c	250)	a	251)	a	252)	a
65)	c	66)	d	67)	a	68)	a	253)	b	254)	c	255)	c	256)	b
69)	c	70)	c	71)	d	72)	b	257)	c	258)	c	259)	b	260)	a
73)	c	74)	c	75)	d	76)	c	261)	d	262)	b	263)	c	264)	d
77)	c	78)	b	79)	a	80)	b	265)	a	266)	b	267)	a	268)	a
81)	d	82)	a	83)	c	84)	d	269)	b	270)	c	271)	b	272)	b
85)	b	86)	a	87)	a	88)	c	273)	d	274)	c	275)	c	276)	b
89)	b	90)	d	91)	c	92)	d	277)	c	278)	c	279)	c	280)	c
93)	c	94)	c	95)	c	96)	b	281)	d	282)	b	283)	b	284)	c
97)	b	98)	c	99)	b	100)	a	285)	c	286)	a	287)	c	288)	b
101)	b	102)	a	103)	a	104)	c	289)	d	290)	b	291)	c	292)	d
105)	b	106)	b	107)	b	108)	a	293)	a	294)	c	295)	c	296)	b
109)	b	110)	b	111)	c	112)	c	297)	d	298)	d	299)	d	300)	a
113)	a	114)	c	115)	b	116)	d	301)	a	302)	b	303)	c	304)	d
117)	a	118)	b	119)	a	120)	c	305)	d	306)	c	307)	c	308)	c
121)	b	122)	a	123)	c	124)	b	309)	a	310)	d	311)	b	312)	c
125)	b	126)	d	127)	b	128)	c	313)	b	314)	c	315)	a	316)	a
129)	a	130)	a	131)	d	132)	b	317)	a	318)	a	319)	c	320)	c
133)	d	134)	c	135)	c	136)	d	321)	b	322)	a	323)	c	324)	b
137)	c	138)	b	139)	d	140)	c	325)	d	326)	b	327)	d	328)	a
141)	b	142)	d	143)	a	144)	a	329)	b	330)	a	331)	d	332)	a
145)	a	146)	d	147)	a	148)	b	333)	b	334)	a	335)	b	336)	a
149)	a	150)	c	151)	b	152)	c	337)	c	338)	d	339)	a	340)	a
153)	d	154)	b	155)	d	156)	c	341)	d	342)	c	343)	a	344)	b
157)	c	158)	b	159)	d	160)	b	345)	a	346)	a	347)	c	348)	d
161)	a	162)	c	163)	b	164)	c	349)	a	350)	c	351)	a	352)	c
165)	c	166)	a	167)	d	168)	b	353)	c	354)	b	355)	d	356)	c
169)	b	170)	a	171)	b	172)	d	357)	b	358)	b	359)	b	360)	d
173)	b	174)	a	175)	b	176)	a	361)	b	362)	c	363)	c	364)	d
177)	a	178)	a	179)	a	180)	c	365)	b	366)	b	367)	c	368)	a
181)	c	182)	b	183)	c	184)	c	369)	b	370)	a	371)	a	372)	c
185)	b	186)	a	187)	d	188)	d	373)	b	374)	a	375)	c	376)	b

377) b	378) d	379) c	380) a	581) c	582) b	583) b	584) b
381) c	382) b	383) a	384) c	585) d	586) b	587) b	588) a
385) c	386) c	387) b	388) a	589) a	590) d	591) b	592) b
389) a	390) d	391) d	392) d	593) d	594) d	595) c	596) d
393) d	394) b	395) d	396) a	597) d	598) c	599) c	600) b
397) b	398) c	399) d	400) a	601) b	602) b	603) a	604) a
401) b	402) b	403) a	404) b	605) d	606) a	607) c	608) b
405) b	406) b	407) b	408) b	609) a	610) a	611) b	612) b
409) c	410) d	411) c	412) c	613) c	614) a	615) b	616) c
413) b	414) a	415) b	416) d	617) c	618) b	619) b	620) b
417) b	418) d	419) b	420) b	621) a	622) d	623) b	624) a
421) a	422) c	423) a	424) c	625) c	626) a	627) a	628) b
425) c	426) d	427) a	428) b	629) c	630) a	631) c	632) b
429) b	430) d	431) c	432) a	633) b	634) d	635) b	636) d
433) b	434) c	435) b	436) b	637) d	638) a	639) b	640) b
437) a	438) a	439) a	440) a	641) d	642) a	643) d	644) a
441) c	442) b	443) c	444) a	645) b	646) b	647) c	648) a
445) c	446) d	447) c	448) a	649) c	650) a	651) c	652) c
449) a	450) d	451) a	452) a	653) b	654) d	655) a	656) a
453) a	454) d	455) c	456) a	657) d	658) b	659) a	660) b
457) a	458) d	459) a	460) c	661) b	662) d	663) b	664) c
461) a	462) d	463) c	464) c	665) a	666) b	667) a	668) c
465) c	466) d	467) d	468) a	669) a	670) c	671) c	672) b
469) a	470) a	471) c	472) d	673) b	674) c	675) d	676) d
473) c	474) c	475) a	476) c	677) b	678) a	679) b	680) a
477) d	478) c	479) d	480) d	681) a	682) a	683) a	684) d
481) c	482) a	483) c	484) b	685) c	686) d	687) a	688) c
485) c	486) c	487) d	488) c	689) c	690) a	691) a	692) c
489) d	490) c	491) b	492) a	693) c	694) c	695) d	696) c
493) b	494) a	495) c	496) a	697) b	698) a	699) d	700) b
497) a	498) b	499) d	500) a	701) b	702) d	703) d	704) c
501) d	502) d	503) b	504) b	705) c	706) b	707) b	708) b
505) c	506) a	507) a	508) a	709) b	710) c	711) b	712) d
509) a	510) a	511) a	512) b	713) a	714) b	715) a	716) b
513) a	514) b	515) a	516) a	717) d	718) d	719) b	720) c
517) c	518) a	519) c	520) a	721) a	722) c	723) c	724) a
521) a	522) c	523) a	524) a	725) c	726) c	727) a	728) b
525) a	526) b	527) b	528) b	729) c	730) a	731) d	732) c
529) b	530) d	531) a	532) a	733) d	734) c	735) b	736) a
533) c	534) a	535) a	536) c	737) b	738) a	739) b	740) d
537) b	538) b	539) b	540) a	741) c	742) c	743) c	744) c
541) a	542) d	543) d	544) c	745) a	746) b	747) c	748) b
545) b	546) a	547) d	548) c	749) d	750) b	751) b	752) a
549) c	550) d	551) a	552) c	753) d	754) a	755) a	756) c
553) a	554) d	555) c	556) b	757) b	758) d	759) a	760) d
557) c	558) d	559) b	560) b	761) b	762) a	763) d	764) b
561) a	562) a	563) c	564) c	765) c	766) d	767) c	768) d
565) c	566) c	567) d	568) a	769) c	770) a	771) b	772) b
569) b	570) a	571) b	572) a	773) b	774) c	775) b	776) c
573) a	574) c	575) b	576) a	777) d	778) b	779) c	780) b
577) c	578) c	579) a	580) a	781) b	782) a	783) c	784) a

785) b	786) a	787) b	788) c	873) c	874) c	875) a	876) b
789) d	790) a	791) b	792) c	877) d	878) c	879) a	880) d
793) b	794) b	795) b	796) c	881) c	882) a	883) b	884) b
797) b	798) a	799) b	800) a	885) a	886) c	887) c	888) a
801) b	802) d	803) b	804) c	889) a	890) a	891) b	892) d
805) b	806) c	807) b	808) c	893) b	894) d	895) a	896) a
809) c	810) a	811) a	812) b	897) b	898) a	899) b	900) c
813) b	814) c	815) d	816) c	901) c	902) a	903) c	904) b
817) a	818) d	819) b	820) d	905) c	906) c	907) b	908) d
821) d	822) c	823) b	824) a	909) b	910) b	911) d	912) b
825) a	826) c	827) c	828) b	913) a	914) a	915) c	916) b
829) b	830) a	831) c	832) b	917) b	918) b	919) b	920) d
833) c	834) b	835) b	836) d	921) a	922) a	923) c	924) d
837) c	838) b	839) b	840) d	925) a	926) c	927) b	928) c
841) b	842) c	843) d	844) c	929) d	930) b	931) c	932) b
845) a	846) b	847) a	848) c	933) c	934) c	935) c	936) a
849) a	850) c	851) c	852) a	937) c	938) b	939) a	940) c
853) b	854) c	855) c	856) b	941) b	942) d	943) d	944) c
857) b	858) d	859) c	860) a	945) c	946) d	947) a	948) b
861) d	862) a	863) b	864) a	949) a	950) d	951) b	952) d
865) b	866) a	867) d	868) d	953) d	954) c	955) b	
869) c	870) a	871) a	872) c				

: HINTS AND SOLUTIONS :

1 (b)

We have,

$$I = \int_0^{\pi/2} \frac{1}{9 \cos x + 12 \sin x} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{1 + \tan^2 x/2}{9 - 9 \tan^2 x/2 + 24 \tan x/2} dx$$

$$\Rightarrow I = \int_0^1 \frac{2dt}{9 - 9t^2 + 24t}, \text{ where } t = \tan \frac{x}{2}$$

$$\Rightarrow I = -\frac{2}{9} \int_0^1 \frac{dt}{t^2 - \frac{8}{3}t - 1}$$

$$\Rightarrow I = -\frac{2}{9} \int_0^1 \frac{1}{\left(t - \frac{4}{3}\right)^2 - \left(\frac{5}{3}\right)^2} dt$$

$$\Rightarrow I = \frac{2}{9} \int_0^1 \frac{1}{\left(\frac{5}{3}\right)^2 - \left(t - \frac{4}{3}\right)^2} dt$$

$$\Rightarrow I = \frac{2}{9} \times \frac{3}{10} \left[\log \left(\frac{t - \frac{4}{3} + \frac{5}{3}}{\frac{5}{3} - t + \frac{4}{3}} \right) \right]_0^1$$

$$\Rightarrow I = \frac{1}{15} \left[\log \left(\frac{3t + 1}{9 - 3t} \right) \right]_0^1 = \frac{1}{15} \left(\log \frac{4}{6} - \log \frac{1}{9} \right)$$

$$= \frac{1}{15} \log_e 6$$

2 (c)

$$I = \int_{-2}^0 [x^3 + 3x^2 + 3x + 3 + (x + 1) \cos(x + 1)] dx$$

$$= \int_{-2}^0 [(x + 1)^3 + 2 + (x + 1) \cos(x + 1)] dx$$

Put, $x + 1 = t \Rightarrow dx = dt$

$$\therefore I = \int_{-1}^1 t^3 dt + 2 \int_{-1}^1 dt + \int_{-1}^1 t \cos t dt$$

$$= 0 + 2[1 - (-1)] + 0$$

$$\Rightarrow I = 4 \left[\begin{array}{l} \because t^3 \text{ and } t \cos t \text{ are odd functions.} \\ \therefore \int_{-1}^1 t^3 dt = \int_{-1}^1 t \cos t dt = 0 \end{array} \right]$$

3 (c)

We have,

$$I = \int \frac{e^{2x} - 2e^x}{e^{2x} + 1} dx$$

$$\Rightarrow I = \int \frac{e^{2x}}{e^{2x} + 1} dx - \int \frac{2e^x}{e^{2x} + 1} dx$$

$$\Rightarrow I = \frac{1}{2} \int \frac{1}{e^{2x} + 1} d(e^{2x} + 1)$$

$$- 2 \int \frac{1}{(e^x)^2 + 1^2} d(e^x)$$

$$\Rightarrow I = \frac{1}{2} \log(e^{2x} + 1) - 2 \tan^{-1}(e^x) + C$$

4 (b)

$$\text{Let } I = \int_0^1 \tan^{-1} \left(\frac{2x-1}{1+x-x^2} \right) dx$$

$$= \int_0^1 \tan^{-1} x dx + \int_0^1 \tan^{-1}(x-1) dx$$

$$= \int_0^1 \tan^{-1} x dx + \int_0^1 \tan^{-1}(1-x-1) dx$$

$$= \int_0^1 \tan^{-1} x dx - \int_0^1 \tan^{-1} x dx = 0$$

5 (b)

$$\int \sqrt{1 + \sin \left(\frac{x}{4} \right)} dx = \int \sqrt{1 + 2 \sin \frac{x}{8} \cos \frac{x}{8}} dx$$

$$= \int \sqrt{\sin^2 \frac{x}{8} + \cos^2 \frac{x}{8} + 2 \sin \frac{x}{8} \cos \frac{x}{8}} dx$$

$$= \int \sqrt{\left(\sin \frac{x}{8} + \cos \frac{x}{8} \right)^2} dx$$

$$= \int \left(\sin \frac{x}{8} + \cos \frac{x}{8} \right) dx$$

$$= \frac{-\cos \frac{x}{8}}{1/8} + \frac{\sin \frac{x}{8}}{1/8} + c$$

$$= 8 \left(\sin \frac{x}{8} - \cos \frac{x}{8} \right) + c$$

6 (b)

$$\text{Let } I = \int_a^b \frac{\sqrt{x} dx}{\sqrt{x} + \sqrt{a+b-x}} \dots (i)$$

$$\therefore I = \int_a^b \frac{\sqrt{a+b-x}}{\sqrt{a+b-x} + \sqrt{x}} \dots (ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_a^b dx = [x]_a^b = b - a$$

$$\Rightarrow I = \frac{b - a}{2}$$

7 (d)

We have,

$$I = \int_0^3 \frac{1}{\sqrt{x+1} + \sqrt{5x+1}} dx$$

$$= \int_0^3 \frac{\sqrt{x+1} - \sqrt{5x+1}}{-4x} dx$$

$$\Rightarrow I = -\frac{1}{4} \int_0^3 \frac{\sqrt{x+1}}{x} dx + \frac{1}{4} \int_0^3 \frac{\sqrt{5x+1}}{x} dx$$

$$\Rightarrow I = I_1 + I_2, \text{ where}$$

$$I_1 = -\frac{1}{4} \int_0^3 \frac{\sqrt{x+1}}{x} dx \text{ and } I_2 = \frac{1}{4} \int_0^3 \frac{\sqrt{5x+1}}{x} dx$$

Putting $x+1 = t^2$ in I_1 , we get

$$I_1 = -\frac{1}{2} \int_1^2 \frac{t^2}{t^2-1} dt$$

$$\Rightarrow I_1 = -\frac{1}{2} \int_1^2 \frac{t^2-1+1}{t^2-1} dt$$

$$\Rightarrow I_1 = -\frac{1}{2} \int_1^2 \left(1 + \frac{1}{t^2-1}\right) dt$$

$$\Rightarrow I_1 = -\frac{1}{2} \left[t + \frac{1}{2} \log \left| \frac{t-1}{t+1} \right| \right]_1^2$$

$$\Rightarrow I_1 = -\frac{1}{2} \left[1 - \frac{1}{2} \lim_{t \rightarrow 1} \log \left| \frac{t-1}{t+1} \right| \right]$$

Now, putting $5x+1 = t^2$ in I_2 we get

$$I_2 = \frac{1}{4} \int_1^4 \frac{2t^2}{t^2-1} = \frac{1}{2} \int_1^4 \frac{t^2}{t^2-1} dt$$

$$= \frac{1}{2} \int_1^4 \frac{t^2-1+1}{t^2-1} dt$$

$$= \frac{1}{2} \left[t + \frac{1}{2} \log \left| \frac{t-1}{t+1} \right| \right]_1^4$$

$$= \frac{1}{2} \left[3 + \frac{1}{2} \log \frac{3}{5} - \frac{1}{2} \lim_{t \rightarrow 1} \left| \frac{t-1}{t+1} \right| \right]$$

$$\therefore I = I_1 + I_2 = 1 + \frac{1}{4} \log \frac{3}{5}$$

8 (d)

$$\text{Let } I = \int_0^{12a} \frac{f(x)}{f(x)+f(12a-x)} dx \dots (i)$$

$$\Rightarrow I = \int_0^{12a} \frac{f(12a-x)}{f(12a-x)+f(x)} dx \dots (ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{12a} 1 dx = [x]_0^{12a}$$

$$\Rightarrow I = 6a$$

9 (c)

We have,

$$I = \int_{-2}^2 \left[p \log \left(\frac{1+x}{1-x} \right) + q \log \left(\frac{1-x}{1+x} \right)^{-2} + r \right] dx$$

Since, $\log \left(\frac{1+x}{1-x} \right)$ is an odd function

$$\therefore \int_{-2}^2 \log \left(\frac{1+x}{1-x} \right) dx = 0$$

$$\Rightarrow I = \int_{-2}^2 r dx = r(2+2) = 4r$$

10 (a)

We have,

$$I = \int \frac{3^x}{\sqrt{1-9^x}} dx = \frac{1}{\log_e 3} \int \frac{1}{\sqrt{1^2 - (3^x)^2}} d(3^x)$$

$$= (\log_3 e) \sin^{-1}(3^x) + C$$

11 (c)

$$\text{Put } \log x = t \Rightarrow \frac{1}{x} dx = dt$$

$$\therefore \int \frac{dx}{x \log x} = \int \frac{dt}{t} = \log t + \text{constant}$$

$$= \log(\log x) + \text{constant} = f(x) + \text{constant}$$

$$\therefore f(x) = \log(\log x)$$

12 (c)

We have,

$$\int_{\log 2}^x \frac{1}{e^x - 1} dx = \log \left(\frac{3}{2} \right)$$

$$\Rightarrow \int_{\log 2}^x \frac{e^{-x}}{1 - e^{-x}} dx = \log \left(\frac{3}{2} \right)$$

$$\Rightarrow [\log(1 - e^{-x})]_{\log 2}^x = \log \frac{3}{2}$$

$$\Rightarrow \log(1 - e^{-x}) - \log(1 - e^{-\log 2}) = \log \frac{3}{2}$$

$$\Rightarrow \log(1 - e^{-x}) - \log \left(1 - \frac{1}{2} \right) = \log \frac{3}{2}$$

$$\Rightarrow \log(1 - e^{-x}) = \log \frac{3}{2} + \log \frac{1}{2}$$

$$\Rightarrow \log(1 - e^{-x}) = \log \frac{3}{4}$$

$$\Rightarrow 1 - e^{-x} = \frac{3}{4} \Rightarrow e^{-x} = \frac{1}{4} \Rightarrow e^x = 4 \Rightarrow x = \log 4$$

13 (a)

$$I = \sum_{r=1}^{10} \int_0^1 f(r-1+x) dx$$

$$I_t = \int_0^1 f(t-1+x) dx$$

$$\text{Put } t-1+x = y \Rightarrow dx = dy$$

$$I_t = \int_{t-1}^t f(y) dy \Rightarrow I_t = \int_{t-1}^t f(x) dx$$

$$\begin{aligned} \therefore I_1 &= \int_0^1 f(x) dx, I_2 = \int_1^2 f(x) dx, \\ I_3 &= \int_2^3 f(x) dx, \dots, I_{10} = \int_9^{10} f(x) dx \\ \text{So, } I &= I_1 + I_2 + \dots + I_{10} = \int_0^{10} f(x) dx \end{aligned}$$

14 (b)

We have,

$$\begin{aligned} I &= \int_{-1}^1 [x^2 + \{x\}] dx \\ \Rightarrow I &= \int_{-1}^0 [x^2 + \{x\}] dx + \int_0^1 [x^2 + \{x\}] dx \\ \Rightarrow I &= \int_{-1}^0 [x^2 + x + 1] dx + \int_0^1 [x^2 + x] dx \\ \Rightarrow I &= \int_{-1}^0 ([x^2 + x] + 1) dx + \int_0^1 [x^2 + x] dx \\ \Rightarrow I &= \int_{-1}^0 [x^2 + x] dx + \int_{-1}^0 dx + \int_0^1 [x^2 + x] dx \\ \Rightarrow I &= \int_{-1}^0 0 dx + \int_{-1}^0 dx + \int_0^1 [x^2 + x] dx \\ &\quad + \int_{\frac{\sqrt{5}-1}{2}}^1 [x^2 + x] dx \\ \Rightarrow I &= 1 + \int_0^{\frac{\sqrt{5}-1}{2}} 0 dx + \int_{\frac{\sqrt{5}-1}{2}}^1 1 dx = 1 + 1 - \frac{\sqrt{5}-1}{2} \\ &= \frac{\sqrt{5}-5}{2} \end{aligned}$$

15 (a)

$$\begin{aligned} \text{Here, } f f(x) &= \frac{f(x)}{[1+f(x)^n]^{1/n}} = \frac{x}{(1+2x^n)^{1/n}} \\ \text{and } f f f(x) &= \frac{x}{(1+3x^n)^{1/n}} \\ g(x) &= \underbrace{(f \text{ of } o \dots \text{ of } f)}_{n \text{ times}}(x). \\ \therefore &= \frac{x}{(1+nx^n)^{1/n}} \end{aligned}$$

$$\begin{aligned} \text{Let } I &= \int x^{n-2} g(x) dx = \int \frac{x^{n-1} dx}{(1+nx^n)^{1/n}} \\ &= \frac{1}{n^2} \int \frac{n^2 x^{n-1} dx}{(1+nx^n)^{1/n}} \\ &= \frac{1}{n^2} \int \frac{\frac{d}{dx}(1+nx^n)}{(1+nx^n)^{1/n}} dx \\ \therefore I &= \frac{1}{n(n-1)} (1+nx^n)^{1-\frac{1}{n}} + c \end{aligned}$$

16 (c)

We have,

$$\begin{aligned} I_n &= \int_0^{\pi/2} \sin^n x dx \\ \Rightarrow I_n &= \int_0^{\pi/2} \sin^{n-1} x \sin x dx \\ &\quad \begin{matrix} I & II \end{matrix} \\ \Rightarrow I_n &= [-\sin^{n-1} x \cos x]_0^{\pi/2} \\ &\quad + (n-1) \int_0^{\pi/2} \sin^{n-2} x \cos^2 x dx \\ \Rightarrow I_n &= (n-1) \int_0^{\pi/2} \sin^{n-2} x (1 - \sin^2 x) dx \\ \Rightarrow I_n &= (n-1) \int_0^{\pi/2} \sin^{n-2} x dx - (n-1) \int_0^{\pi/2} \sin^n x dx \\ \Rightarrow I_n &= (n-1)I_{n-2} - (n-1)I_n \\ \Rightarrow nI_n &= (n-1)I_{n-2} \\ \Rightarrow I_{n-2} &= n(I_{n-2} - I_n) \end{aligned}$$

17 (d)

We have,

$$\begin{aligned} I &= \int_0^3 x \sqrt{1+x} dx \\ \Rightarrow I &= \left[\frac{2x}{3} (1+x)^{3/2} \right]_0^3 - \frac{2}{3} \int_0^3 (1+x)^{3/2} x \\ \Rightarrow I &= \left[\frac{2x}{3} (1+x)^{3/2} \right]_0^3 - \frac{4}{15} [(1+x)^{5/2}]_0^3 = \frac{116}{15} \end{aligned}$$

18 (b)

$$\begin{aligned} \text{Let } I &= \int_0^1 \log \left\{ \sin \left(\frac{\pi x}{2} \right) \right\} dx \\ \text{Put } \frac{\pi x}{2} &= \theta \Rightarrow dx = \frac{2}{\pi} d\theta \\ \therefore I &= \frac{2}{\pi} \int_0^{\pi/2} \log \sin \theta d\theta \\ &= \frac{2}{\pi} \left(-\frac{\pi}{2} \log 2 \right) = -\log 2 \end{aligned}$$

19 (b)

$$\begin{aligned} \text{Let } I &= \int \frac{dx}{x(x^7+1)} \\ \text{Put } x^7 &= t \Rightarrow dx = \frac{1}{7x^6} dt \\ \therefore I &= \frac{1}{7} \int \frac{dt}{t(t+1)} = \frac{1}{7} \int \left(\frac{1}{t} - \frac{1}{t+1} \right) dt \\ &= \frac{1}{7} [\log t - \log(t+1)] + c \end{aligned}$$

$$= \frac{1}{7} \log \left(\frac{t}{t+1} \right) + c$$

$$= \frac{1}{7} \log \left(\frac{x^7}{x^7+1} \right) + c$$

20 (d)

$$\text{Let } I = \int \cos \left\{ 2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right\} dx$$

Put $x = \cos \theta$, then

$$I = \int \cos \left\{ 2 \tan^{-1} \sqrt{\frac{1-\cos \theta}{1+\cos \theta}} \right\} dx$$

$$= \int \cos \left\{ 2 \tan^{-1} \left(\tan \frac{\theta}{2} \right) \right\} dx$$

$$= \int \cos \theta dx = \int x dx = \frac{x^2}{2} + c$$

21 (c)

$$\text{Let } I = \int \frac{x^2+1}{x^2-1} dx$$

$$\Rightarrow I = \int \frac{x^2+1-1+1}{x^2-1} dx$$

$$\Rightarrow I = \int \frac{x^2-1}{x^2-1} dx + \int \frac{2}{x^2-1} dx$$

$$\Rightarrow I = \int 1 dx + 2 \int \frac{1}{x^2-1} dx$$

$$\Rightarrow I = x + 2 \cdot \frac{1}{2} \log \left(\frac{x-1}{x+1} \right) + c$$

$$\Rightarrow I = x + \log \left(\frac{x-1}{x+1} \right) + c$$

23 (b)

$$\text{Let } I = \int \frac{dx}{x+\sqrt{x}} = \int \frac{1}{\sqrt{x}(1+\sqrt{x})} dx$$

$$\text{Put } 1+\sqrt{x} = t \Rightarrow \frac{1}{\sqrt{x}} dx = 2dt$$

$$\therefore I = 2 \int \frac{1}{t} dt = 2 \log t + c$$

$$= 2 \log(1+\sqrt{x}) + c$$

24 (b)

We have,

$$I = \int \frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}} dx$$

$$\Rightarrow I = \int \frac{1}{\sqrt{\sin^4 x (\cos \alpha + \cot x \sin \alpha)}} dx$$

$$\Rightarrow I = \int \frac{1}{\sqrt{\cos \alpha + \cot x \sin \alpha}} \operatorname{cosec}^2 x dx$$

$$\Rightarrow I = -\frac{1}{\sin \alpha} \int \frac{1}{\sqrt{\cos \alpha + \cot x \sin \alpha}} d(\cos \alpha + \cot x \sin \alpha)$$

$$\Rightarrow I = -\frac{2}{\sin \alpha} \sqrt{\cos \alpha + \cot x \sin \alpha} + C$$

$$\Rightarrow I = -2 \operatorname{cosec} \alpha \sqrt{\cos \alpha + \cot x \sin \alpha} + C$$

25 (a)

$$\text{Let } I = \int_0^{\pi/2} \log \sin x dx \quad \dots(i)$$

$$I = \int_0^{\pi/2} \log \sin \left(\frac{\pi}{2} - x \right) dx = \int_0^{\pi/2} \log \cos x dx$$

...(ii)

On adding Eqs.(i) and (ii), we get

$$2I = \int_0^{\pi/2} \log \sin x \cos x dx$$

$$= \int_0^{\pi/2} \log \sin 2x dx - \int_0^{\pi/2} \log 2 dx$$

$$= \frac{1}{2} \int_0^{\pi} \log \sin t dt - \frac{\pi}{2} \log 2$$

$$= \frac{1}{2} \cdot 2 \int_0^{\pi/2} \log \sin t dt - \frac{\pi}{2} \log 2$$

$$\Rightarrow 2I = I - \frac{\pi}{2} \log 2 \Rightarrow I = -\frac{\pi}{2} \log 2$$

26 (c)

$$\text{Let } I = \int_0^1 \frac{dx}{x+\sqrt{x}} = \int_0^1 \frac{dx}{\sqrt{x}(\sqrt{x}+1)}$$

$$\text{Put } \sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt$$

$$\Rightarrow \frac{1}{\sqrt{x}} dx = 2dt$$

$$I = \int_0^1 \frac{2dt}{t+1} = 2[\log(t+1)]_0^1$$

$$= 2(\log 2 - \log 0) = \log 4$$

27 (c)

It is given that

$$f'(3) = \tan \frac{\pi}{4}, f'(2) = \tan \frac{\pi}{3}, f'(1) = \tan \frac{\pi}{6}$$

Now,

$$\int_1^3 f''(x) f'(x) dx + \int_2^3 f''(x) dx$$

$$= \left[\frac{\{f'(x)\}^2}{2} \right]_1^3 + [f'(x)]_2^3$$

$$= \frac{1}{2} [\{f'(3)\}^2 - \{f'(1)\}^2] + [f'(3) - f'(2)]$$

$$= \frac{1}{2} \left[1 - \frac{1}{3} \right] + \left[1 - \frac{1}{\sqrt{3}} \right] = \frac{4}{3} - \frac{1}{\sqrt{3}}$$

28 (a)

$$\text{Given, } f(x) = 2x^2 - 4x + 1$$

$$\therefore g(x) = \frac{2x^2 - 4x + 1 - 2x^2 - 4x - 1}{2} = -4x$$

$$\therefore \int_{-3}^3 g(x) dx = -4 \int_{-3}^3 x dx$$

$$= -4 \left[\frac{x^2}{2} \right]_{-3}^3 = -2[9 - 9] = 0$$

29 (a)

$$I = \int \frac{x + \sin x}{1 + \cos x} dx = \int \frac{x}{2} \sec^2 \frac{x}{2} + \tan \frac{x}{2} dx$$

$$\Rightarrow I = \int x \frac{1}{2} \sec^2 \frac{x}{2} dx + \int \tan \frac{x}{2} dx$$

$$\Rightarrow I = x \tan \frac{x}{2} - \int \tan \frac{x}{2} dx + \int \tan \frac{x}{2} dx + C$$

$$= x \tan \frac{x}{2} + C$$

30 (a)

$$\text{Let } I = \int_3^6 \frac{\sqrt{x}}{\sqrt{9-x} + \sqrt{x}} dx \dots \text{(i)}$$

$$= \int_3^6 \frac{\sqrt{9-x}}{\sqrt{9-9+x} + \sqrt{9-x}} dx$$

$$\Rightarrow I = \int_3^6 \frac{\sqrt{9-x}}{\sqrt{x} + \sqrt{9-x}} dx \dots \text{(ii)}$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_3^6 1 dx = [x]_3^6 = 6 - 3 \Rightarrow I = \frac{3}{2}$$

31 (b)

$$\text{Let } I = \int \frac{dx}{(a^2 + x^2)^{3/2}}$$

$$\text{Put } x = a \tan \theta \Rightarrow dx = a \sec^2 \theta d\theta$$

$$\therefore I = \int \frac{a \sec^2 \theta}{(a^2 + a^2 \tan^2 \theta)^{3/2}} d\theta$$

$$= \int \frac{a \sec^2 \theta}{a^3 (\sec^2 \theta)^{3/2}} d\theta$$

$$\Rightarrow I = \frac{1}{a^2} \int \frac{d\theta}{\sec \theta} = \frac{1}{a^2} \int \cos \theta d\theta$$

$$= \frac{1}{a^2} \sec \theta + c$$

$$\Rightarrow \frac{x}{a^2(x^2 + a^2)^{1/2}} + c$$

32 (d)

$$\text{Let } I = \int_0^{\pi/2} \sin^{100} x dx - \int_0^{\pi/2} \cos^{100} x dx$$

$$= 0 - 0 = 0$$

$$= 0$$

33 (a)

$$\int [e^x f(x) + f'(x) e^x] dx$$

$$= e^x f(x) - \int e^x f'(x) dx + \int e^x f'(x) dx$$

$$= e^x f(x) + c$$

34 (c)

$$\text{Let } I = \int \frac{dx}{1 - \cos x - \sin x}$$

$$\text{Put } \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \text{ and } \sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$\therefore I = \int \frac{dx}{1 - \frac{(1 - \tan^2 \frac{x}{2})}{(1 + \tan^2 \frac{x}{2})} - \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}}$$

$$= \int \frac{\sec^2 \frac{x}{2} dx}{\left[1 + \tan^2 \frac{x}{2} - 1 + \tan^2 \frac{x}{2} - 2 \tan \frac{x}{2}\right]}$$

$$= \int \frac{\sec^2 \frac{x}{2} dx}{2 \tan^2 \frac{x}{2} - 2 \tan \frac{x}{2}} = \int \frac{\frac{1}{2} \sec^2 \frac{x}{2} dx}{\tan^2 \frac{x}{2} - \tan \frac{x}{2}}$$

$$\text{Put } \tan \frac{x}{2} = t \Rightarrow \frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

$$\therefore I = \int \frac{dt}{t^2 - t} = \int \frac{dt}{(t-1)}$$

$$= \int \left[\frac{1}{t-1} - \frac{1}{t} \right] dt = \int \frac{dt}{t-1} - \int \frac{dt}{t}$$

$$= \log(t-1) - \log t + c = \log \left| \frac{t-1}{t} \right| + c$$

$$= \log \left| \frac{\tan \frac{x}{2} - 1}{\tan \frac{x}{2}} \right| + c = \log \left| 1 - \cot \frac{x}{2} \right| + c$$

35 (a)

We have,

$$\int_{\pi/2}^{\alpha} \sin x dx = \sin 2\alpha$$

$$\Rightarrow -\cos \alpha = \sin 2\alpha$$

$$\Rightarrow \cos \alpha (2 \sin \alpha + 1) = 0$$

$$\Rightarrow \cos \alpha = 0 \text{ or } \sin \alpha = -\frac{1}{2} \Rightarrow \alpha = \frac{\pi}{2}, \frac{3\pi}{2} \Rightarrow \alpha = \frac{7\pi}{6}, \frac{11\pi}{6}$$

36 (c)

We have,

$$\int_2^4 \{3 - f(x)\} dx = 7 \Rightarrow 6 - \int_2^4 f(x) dx = 7$$

$$\Rightarrow \int_2^4 f(x) dx = -1$$

$$\therefore I = \int_2^{-1} f(x) dx$$

$$\Rightarrow I = - \int_{-1}^2 f(x) dx$$

$$\Rightarrow I = - \left\{ \int_{-1}^4 f(x) dx + \int_4^2 f(x) dx \right\}$$

$$\Rightarrow I = - \left\{ \int_{-1}^4 f(x) dx - \int_2^4 f(x) dx \right\} = -(4 + 1)$$

$$= -5$$

37 (a)

$$\int f(x)g(x)dx = \int \frac{\sin^{-1} x}{\sqrt{1-x^2}} e^{\sin^{-1} x} dx$$

$$\text{Put } \sin^{-1} x = t \Rightarrow \frac{1}{\sqrt{1-x^2}} dx = dt$$

$$\begin{aligned} \therefore \int f(x)g(x)dx &= \int te^t dt = te^t - e^t + c \\ &= e^{\sin^{-1}x}(\sin^{-1}x - 1) + c \end{aligned}$$

38 (a)

We have,

$$I = \int_{-1}^3 \{|x-2| + [x]\} dx = \int_{-1}^3 |x-2| dx + \int_{-1}^3 [x] dx$$

Now,

$$\int_{-1}^3 |x-2| dx = \int_{-1}^2 |x-2| dx + \int_2^3 |x-2| dx$$

$$= \int_{-1}^2 (2-x) dx + \int_2^3 (x-2) dx$$

$$\begin{aligned} &= \left[2x - \frac{x^2}{2} \right]_{-1}^2 + \left[\frac{x^2}{2} - 2x \right]_2^3 \\ &= \left(2 + \frac{5}{2} \right) + \left(\frac{-3}{2} + 2 \right) = 5 \end{aligned}$$

and,

$$\int_{-1}^3 [x] dx = \int_{-1}^0 -1 dx + \int_0^1 0 dx + \int_1^2 1 dx + \int_2^3 2 dx$$

$$\Rightarrow \int_{-1}^3 [x] dx = -1 + 1 + 2 = 2$$

$$\therefore I = \int_{-1}^3 (|x-2| + [x]) dx = 5 + 2 = 7$$

39 (b)

$$\text{Here, } g(2) = \int_0^2 f(t) dt$$

$$= \int_0^1 f(t) dt + \int_1^2 f(t) dt$$

$$\text{As } \frac{1}{2} \leq f(t) \leq 1 \text{ for } 0 \leq t \leq 1$$

$$\Rightarrow \int_0^1 \frac{1}{2} dt \leq \int_0^1 f(t) dt \leq \int_0^1 dt$$

$$\Rightarrow \frac{1}{2} \leq \int_0^1 f(t) dt \leq 1 \quad \dots(i)$$

$$\text{As } 0 \leq f(t) \leq \frac{1}{2} \text{ for } 1 < t \leq 2$$

$$\Rightarrow \int_1^2 0 dt \leq \int_1^2 f(t) dt \leq \int_1^2 \frac{1}{2} dt$$

$$\therefore 0 \leq \int_1^2 f(t) dt \leq \frac{1}{2} \quad \dots(ii)$$

On adding Eqs.(i) and (ii), we get

$$\frac{1}{2} \leq g(2) \leq \frac{3}{2}$$

$\therefore g(2)$ satisfies the inequality $0 \leq g(2) < 2$

40 (a)

$$\int \frac{x^2}{\sqrt{1-x}} dx = p\sqrt{(1-x)}(3x^2 + 4x + 8)$$

Put $1-x = t^2$ in LHS, we get

$$\begin{aligned} I &= - \int \frac{2t}{t} (1-t^2)^2 dt \\ &= -2 \int (1+t^4 - 2t^2) dt \\ &= -2 \left[t + \frac{t^5}{5} - \frac{2t^3}{3} \right] \\ &= -2\sqrt{1-x} \left[1 + \frac{(1-x)^2}{5} - \frac{2}{3}(1-x) \right] \\ &= -2\sqrt{1-x} \left[\frac{15 + 3(1+x^2 - 2x) - 10(1-x)}{15} \right] \\ &= \frac{-2}{15} \sqrt{1-x} (3x^2 + 4x + 8) \end{aligned}$$

$$\text{But } I = p\sqrt{1-x}(3x^2 + 4x + 8)$$

$$\therefore p = \frac{-2}{15}$$

41 (c)

$$\{x\} = x, 0 \leq x < 1$$

$$I = \int_0^1 x dx = \left[\frac{x^2}{2} \right]_0^1 = \frac{1}{2}$$

42 (d)

$$\text{Given, } I = \int \frac{x^5}{\sqrt{1+x^3}} dx$$

$$\text{Let } 1+x^3 = t \Rightarrow 3x^2 dx = dt$$

$$\therefore I = \int \frac{(t-1) dt}{\sqrt{t}} \cdot \frac{1}{3} = \frac{1}{3} \int (\sqrt{t} - t^{-1/2}) dt$$

$$= \frac{1}{3} \left[\frac{2t^{3/2}}{3} - 2t^{1/2} \right] + c$$

$$= \frac{2}{9} (1+x^3)^{3/2} - \frac{2}{3} (1+x^3)^{1/2} + c$$

44 (b)

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\sin x| dx$$

$$= 2 \int_0^{\frac{\pi}{2}} \sin x dx = -2[\cos x]_0^{\frac{\pi}{2}}$$

$$= -2 \left(\cos \frac{\pi}{2} - \cos 0 \right) = 2$$

45 (a)

$$\int_{-2}^3 |1-x^2| dx = \int_{-2}^{-1} (x^2-1) dx$$

$$+ \int_{-1}^1 (1-x^2) dx + \int_1^3 (x^2-1) dx$$

$$= \left[\frac{x^3}{3} - x \right]_{-2}^{-1} + \left[x + \frac{x^3}{3} \right]_{-1}^1 + \left[\frac{x^3}{3} - x \right]_1^3$$

$$= \frac{4}{3} + \frac{4}{3} + \frac{20}{3} = \frac{28}{3}$$

46 (d)

We know that, $|x| = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$

$$\begin{aligned} \therefore \int_{-2}^2 (x - |x|) dx &= \int_{-2}^0 \{x - (-x)\} dx \\ &+ \int_0^2 (x - x) dx \end{aligned}$$

$$= \int_{-2}^0 2x dx + 0$$

$$= 2 \left[\frac{x^2}{2} \right]_{-2}^0$$

$$= 2 \left[0 - \frac{(-2)^2}{2} \right] = -4$$

47 (d)

$$\text{Given, } \int_0^1 \log[x] dx = \int_0^1 \log 0 dx$$

Since, $\log 0$ does not exist, so we cannot find the value of given integral.

48 (d)

$$\text{Let } I = \int_0^1 \frac{dx}{x + \sqrt{1-x^2}}$$

$$\text{Put } x = \cos \theta \Rightarrow dx = -\sin \theta d\theta$$

$$\therefore I = \int_{\pi/2}^0 \frac{-\sin \theta d\theta}{\cos \theta + \sqrt{1 - \cos^2 \theta}}$$

$$= \int_0^{\pi/2} \frac{\sin \theta}{\sin \theta + \cos \theta} d\theta \dots (i)$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\sin(\frac{\pi}{2} - \theta)}{\sin(\frac{\pi}{2} - \theta) + \cos(\frac{\pi}{2} - \theta)} d\theta$$

$$= \int_0^{\pi/2} \frac{\cos \theta}{\sin \theta + \cos \theta} d\theta \dots (ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2} d\theta = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

49 (c)

$$\text{Let } I = \int \frac{\sin \theta + \cos \theta}{\sqrt{1 + \sin 2\theta - 1}} d\theta$$

$$= \int \frac{\sin \theta + \cos \theta}{\sqrt{1 - (\sin \theta - \cos \theta)^2}} d\theta$$

$$\text{Put } \sin \theta - \cos \theta = t$$

$$\Rightarrow (\cos \theta + \sin \theta) d\theta = dt$$

$$\therefore I = \int \frac{1}{\sqrt{1 - t^2}} dt$$

$$= \sin^{-1} t + c$$

$$= \sin^{-1}(\sin \theta - \cos \theta) + c$$

50 (b)

We have,

$$\frac{\cos x + x \sin x}{x(x + \cos x)} = \frac{(x + \cos x) - x + x \sin x}{x(x + \cos x)}$$

$$= \frac{1}{x} - \frac{(1 - \sin x)}{x + \cos x}$$

$$\therefore I = \int \frac{\cos x + x \sin x}{x(x + \cos x)} dx = \int \frac{1}{x} - \frac{1 - \sin x}{x + \cos x} dx$$

$$\Rightarrow I = \log x - \log(x + \cos x) + C$$

$$= \log \left(\frac{x}{x + \cos x} \right) + C$$

51 (a)

$$\text{Let } I = \int \frac{dx}{x^2 + 2x + 2} = \int \frac{dx}{1 + (x+1)^2}$$

$$= \tan^{-1}(x + 1) + c = f(x) + c \text{ [given]}$$

$$\therefore f(x) = \tan^{-1}(x + 1)$$

52 (a)

$$\text{Let } I = \int_0^{\pi/2} \frac{\cos x - \sin x}{1 + \cos x \sin x} \dots (i)$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\cos(\frac{\pi}{2} - x) - \sin(\frac{\pi}{2} - x)}{1 + \cos(\frac{\pi}{2} - x) \sin(\frac{\pi}{2} - x)} dx$$

$$\Rightarrow I = - \int_0^{\pi/2} \frac{(\cos x - \sin x)}{(1 + \cos x \sin x)} dx \dots (ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2} 0 dx = 0 \Rightarrow I = 0$$

53 (c)

$$\int \operatorname{cosec}^4 x dx = \int \operatorname{cosec}^2 x \cdot \operatorname{cosec}^2 x dx$$

$$= \int \operatorname{cosec}^2 x (1 + \cot^2 x) dx$$

$$= \int \operatorname{cosec}^2 x dx + \int \cot^2 x \cdot \operatorname{cosec}^2 x dx$$

$$= -\cot x - \frac{\cot^3 x}{3} + c$$

54 (a)

$$\text{Let } I = \int_0^{\pi/2} \frac{\sin^{2/3} x}{\sin^{2/3} x + \cos^{2/3} x} dx \dots (i)$$

$$\text{and } I = \int_0^{\pi/2} \frac{\sin^{2/3}(\frac{\pi}{2} - x)}{\sin^{2/3}(\frac{\pi}{2} - x) + \cos^{2/3}(\frac{\pi}{2} - x)} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\cos^{2/3} x}{\cos^{2/3} x + \sin^{2/3} x} dx \dots (ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{(\sin^{2/3} x + \cos^{2/3} x)}{(\sin^{2/3} x + \cos^{2/3} x)} dx$$

$$\Rightarrow 2I = \int_0^{\pi/2} 1 dx \Rightarrow I = \frac{1}{2} [x]_0^{\pi/2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

55 (d)

$$\text{Let } I = \int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx \dots (i)$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx \dots (ii)$$

$$\left[\text{put } x = \left(\frac{\pi}{2} - x \right) \right]$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2} 1 \, dx = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

56 (c)

$$\text{Let } I = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx, a > 0 \dots(i)$$

$$I = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^{-x}} dx \dots(ii)$$

On adding Eqs. (i) and (ii), we get

$$\begin{aligned} 2I &= \int_{-\pi}^{\pi} \cos^2 x \, dx \\ &= \int_{-\pi}^{\pi} \frac{(\cos 2x + 1)}{2} dx \end{aligned}$$

$$\Rightarrow 2I = \frac{1}{2} \left[\left(\frac{\sin 2x}{2} + x \right) \right]_{-\pi}^{\pi} = \frac{1}{2} (\pi + \pi)$$

$$\Rightarrow I = \frac{\pi}{2}$$

57 (b)

$$\text{Let } I = \int_0^{\pi} x \sin^3 x \, dx \dots(i)$$

$$\text{Also, } I = \int_0^{\pi} (\pi - x) \sin^3 x \, dx \dots(ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \pi \int_0^{\pi} \sin^3 x \, dx$$

$$= \frac{\pi}{4} \int_0^{\pi} (3 \sin x - \sin 3x) dx$$

$$= \frac{\pi}{4} \left[-3 \cos x + \frac{\cos 3x}{3} \right]_0^{\pi} = \frac{4\pi}{3}$$

$$\text{Hence, } I = \frac{2\pi}{3}$$

58 (c)

$$\text{Let } I = \int_{-2}^2 (px^3 + qx + s) dx = 0 + 0 + -22s dx$$

[$\because px^3$ and qx are odd functions]

$$= 2s[x]_0^2 = 4s$$

Hence, it is necessary to know the value of s

59 (d)

$$\text{Let } I = \int e^{\tan^{-1} x} dx + \int e^{\tan^{-1} x} \cdot \frac{x}{(1+x^2)} dx$$

$$= \int \frac{d}{dx} (x e^{\tan^{-1} x}) dx + c$$

$$= x e^{\tan^{-1} x} + c$$

60 (b)

$$\int_0^{\pi/2} \frac{\cos \theta}{\sqrt{4 - \sin^2 \theta}} d\theta = \left[\sin^{-1} \left(\frac{\sin \theta}{2} \right) \right]_0^{\pi/2}$$

$$= \sin^{-1} \left(\frac{1}{2} \right) = \frac{\pi}{6}$$

61 (b)

Let

$$A = \lim_{n \rightarrow \infty} \left\{ \left(1 + \frac{1}{n} \right) \left(1 + \frac{2}{n} \right) \left(1 + \frac{3}{n} \right) \dots \left(1 + \frac{n}{n} \right) \right\}^{1/n}$$

$$\Rightarrow \log A = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \log \left(1 + \frac{r}{n} \right)$$

$$\Rightarrow \log A = \int_0^1 \log(1+x) dx$$

$$\begin{aligned} \Rightarrow \log A &= [x \log(1+x) - x + \log(x+1)]_0^1 \\ &= \log \left(\frac{4}{e} \right) \end{aligned}$$

$$\Rightarrow A = \frac{4}{e}$$

62 (d)

$$\text{Let } I = \int_2^k (2x+1) dx$$

$$= \left[\frac{2x^2}{2} + x \right]_2^k$$

$$= k^2 + k - 4 - 2$$

$$= k^2 + k - 6$$

But given, $I = 6$

$$\Rightarrow k^2 + k - 6 = 6$$

$$\Rightarrow (k+4)(k-3) = 0$$

$$\Rightarrow k = 3 \quad [\because k \neq -4]$$

63 (b)

We have,

$$I = \int_0^1 \cos \left\{ 2 \cot^{-1} \sqrt{\frac{1-x}{1+x}} \right\} dx$$

Putting $x = \cos \theta$ and $dx = -\sin \theta d\theta$, we get

$$I = \int_{\pi/2}^0 \cos \left\{ 2 \cot^{-1} \left(\tan \frac{\theta}{2} \right) \right\} (-\sin \theta) d\theta$$

$$\Rightarrow I = \int_{\pi/2}^0 \cos \left\{ 2 \cot^{-1} \left(\cot \left(\frac{\pi}{2} - \frac{\theta}{2} \right) \right) \right\} (-\sin \theta) d\theta$$

$$\Rightarrow I = \int_0^{\pi/2} \cos \left\{ 2 \left(\frac{\pi}{2} - \frac{\theta}{2} \right) \right\} \sin \theta d\theta$$

$$\Rightarrow I = - \int_0^{\pi/2} \sin \theta \cos \theta d\theta$$

$$\Rightarrow I = - \frac{1}{2} \int_0^{\pi/2} \sin 2\theta d\theta = \frac{1}{4} [\cos 2\theta]_0^{\pi/2}$$

$$= \frac{1}{4} (-1 - 1) = -\frac{1}{2}$$

64 (b)

$$\text{Let } I = \int (\sin x - \cos x)^4 (\sin x + \cos x) dx$$

Put $\sin x - \cos x = t \Rightarrow (\cos x + \sin x)dx = dt$

$$\therefore I = \int t^4 dt = \frac{t^5}{5} + c = \frac{(\sin x - \cos x)^5}{5} + c$$

65 (c)

$$\text{Let } I = \int \frac{1}{\sqrt{x}(4+\sqrt{x})} dx$$

Put $4 + \sqrt{x} = t$

$$\Rightarrow \frac{1}{2\sqrt{x}} dx = dt$$

$$\Rightarrow \frac{1}{\sqrt{x}} dx = 2dt$$

$$\therefore I = 2 \int \frac{1}{t} dt = 2 \log |4 + \sqrt{x}| + c$$

66 (d)

$$\text{Let } I = \int_0^{\pi/2} \frac{1}{1+\tan^3 x} dx = \int_0^{\pi/2} \frac{1}{1+\frac{\sin^3 x}{\cos^3 x}} dx$$

$$= \int_0^{\pi/2} \frac{\cos^3 x}{\cos^3 x + \sin^3 x} dx \quad \dots(i)$$

$$= \int_0^{\pi/2} \frac{\cos^3 \left(\frac{\pi}{2} - x\right)}{\cos^3 \left(\frac{\pi}{2} - x\right) + \sin^3 \left(\frac{\pi}{2} - x\right)} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} dx \quad \dots(ii)$$

On adding Eqs.(i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{\sin^3 x + \cos^3 x}{\sin^3 x + \cos^3 x} dx$$

$$\Rightarrow 2I = \int_0^{\pi/2} 1 dx$$

$$\Rightarrow 2I = [x]_0^{\pi/2} = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

67 (a)

$$\int \frac{dx}{\cos x + \sqrt{3} \sin x} = \int \frac{dx}{2 \left(\frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x \right)}$$

$$= \frac{1}{2} \int \sec \left(x - \frac{\pi}{3} \right) dx$$

$$= \frac{1}{2} \log \tan \left(\frac{x}{2} - \frac{\pi}{6} + \frac{\pi}{4} \right) + c$$

$$= \frac{1}{2} \log \tan \left(\frac{x}{2} + \frac{\pi}{12} \right) + c$$

68 (a)

The integrand is an odd function. So, the value of given integral is zero

69 (c)

Since $\sqrt{3+x^3}$ is strictly increasing function on (1, 3)

$$\therefore \sqrt{3+1} < \sqrt{3+x^3} < \sqrt{3+(3^3)} \text{ for all } x \in (1, 3)$$

$$\Rightarrow 2 < \sqrt{3+x^3} < \sqrt{30} \text{ for all } x \in (1, 3)$$

$$\Rightarrow 2 \int_1^3 dx < \int_1^3 \sqrt{3+x^3} dx < \int_1^3 \sqrt{30} dx$$

$$\Rightarrow 4 < \int_1^3 \sqrt{3+x^3} dx < 2\sqrt{30}$$

$$\Rightarrow I = \int_1^3 \sqrt{3+x^3} dx \in (4, 2\sqrt{30})$$

70 (c)

Since $x^3, x \cos x$ and $\tan^5 x$ are odd functions

$$\therefore \int_{-\pi/2}^{\pi/2} x^3 dx = 0, \int_{-\pi/2}^{\pi/2} x \cos x dx$$

$$= 0, \int_{-\pi/2}^{\pi/2} \tan^5 x dx = 0$$

$$\Rightarrow \int_{-\pi/2}^{\pi/2} (x^3 + x \cos x + \tan^5 x + 1) dx = \int_{-\pi/2}^{\pi/2} dx$$

$$= \pi$$

71 (d)

$$\int_{-1}^1 [x \sin \pi x] dx = 2 \int_0^1 [x \sin \pi x] dx \quad \dots(i)$$

(as $[x \sin \pi x]$ is a even function)

Now, $0 \leq x \leq 1$

$$\Rightarrow 0 \leq \pi x \leq \pi$$

$$\Rightarrow 0 \leq \sin \pi x \leq 1 \quad \dots(ii)$$

$$\Rightarrow 0 \leq x \sin \pi x < 1$$

$$\Rightarrow [x \sin \pi x] = 0$$

From Eqs. (i) and (ii), we get

$$\int_0^1 [x \sin \pi x] dx = 0$$

72 (b)

$$\text{Let } I = \int_{e^{-1}}^{e^2} \left| \frac{\log_e x}{x} \right| dx$$

$$= \int_{e^{-1}}^1 \left| \frac{\log_e x}{x} \right| dx + \int_1^{e^2} \left| \frac{\log_e x}{x} \right| dx$$

$$= \int_{e^{-1}}^1 \left(-\frac{\log_e x}{x} \right) dx + \int_1^{e^2} \frac{\log_e x}{x} dx$$

$$\text{Put } \log_e x = z \Rightarrow \frac{1}{x} dx = dz$$

$$\therefore I = \int_{-1}^0 (-z) dz + \int_0^2 z dz$$

$$= \left[-\frac{z^2}{2} \right]_{-1}^0 + \left[\frac{z^2}{2} \right]_0^2$$

$$= -\frac{1}{2} [0 - 1] + \frac{1}{2} [4 - 0]$$

$$= \frac{1}{2} + 2 = \frac{5}{2}$$

73 (c)

We have, $f(x) = -\cos^5 x - \sin^2 x$

$$\therefore I = \int_0^{\pi/2} f(x) dx = \int_0^{\pi/2} (-\cos^5 x - \sin^2 x) dx$$

$$\Rightarrow I = -\int_0^1 (1-t^2)^2 dt - \int_0^{\pi/2} \frac{1-\cos 2x}{2} dx, \text{ where}$$

$$t = \sin x$$

$$\Rightarrow I = -\frac{\pi}{4} - \frac{8}{15}$$

74 (c)

Putting $\sin x = t$ in the given integral, we get

$$I = \int \frac{1-t^2+(1-t^2)^2}{t^2+t^4} dt$$

$$= \int \frac{(1-t^2)(2-t^2)}{t^2+t^4} dt$$

$$\Rightarrow I = \left(1 + \frac{2}{t^2} - \frac{6}{1+t^2}\right) dt$$

$$= t - \frac{2}{t} - 6 \tan^{-1}(t) + C$$

$$\Rightarrow I = \sin x - 2(\sin x)^{-1} - 6 \tan^{-1}(\sin x) + C$$

75 (d)

$$\text{Let } I = \int_0^1 x^{3/2} \sqrt{1-x} dx$$

$$\text{Put } x = \sin^2 \theta \Rightarrow dx = 2 \sin \theta \cos \theta d\theta$$

$$\therefore I = \int_0^{\pi/2} \sin^3 \theta \cdot \sqrt{1-\sin^2 \theta} \cdot 2 \sin \theta \cos \theta d\theta$$

$$= 2 \int_0^{\pi/2} \sin^4 \theta \cos^2 \theta d\theta$$

$$= 2 \left[\frac{3.1.1}{6.4.2} \cdot \frac{\pi}{2} \right] \quad [\text{using Walli's formula}]$$

$$= \frac{\pi}{16}$$

76 (c)

$$\text{Let } I = \int_0^1 x \left| x - \frac{1}{2} \right| dx$$

$$= -\int_0^{1/2} x \left(x - \frac{1}{2} \right) dx + \int_{1/2}^1 x \left(x - \frac{1}{2} \right) dx$$

$$= \int_0^{1/2} \left(\frac{1}{2}x - x^2 \right) dx + \int_{1/2}^1 \left(x^2 - \frac{1}{2}x \right) dx$$

$$= \left[\frac{x^2}{4} - \frac{x^3}{3} \right]_0^{1/2} + \left[\frac{x^3}{3} - \frac{x^2}{4} \right]_{1/2}^1$$

$$= \left(\frac{1}{16} - \frac{1}{24} \right) + \left(\frac{1}{3} - \frac{1}{4} - \frac{1}{24} + \frac{1}{16} \right)$$

$$= \left(\frac{6-4}{96} \right) + \left(\frac{32-24-4+6}{96} \right) = \frac{12}{96} = \frac{1}{8}$$

77 (c)

Since $\phi(x) = \sin\{\log(x + \sqrt{x^2 + 1})\}$ is the composition of $f(x) = \log(x + \sqrt{x^2 + 1})$ and $g(x) = \sin x$ i.e. $\phi(x) = g \circ f(x)$ and $f(x)$ and $g(x)$ both are odd functions. Therefore, $\phi(x)$ is an odd function

$$\text{Hence, } \int_{-\pi/2}^{\pi/2} \phi(x) dx = 0$$

78 (b)

We have,

$$\frac{x^3 - 1}{x^3 + x} = 1 - \frac{1}{x} + \frac{x}{x^2 + 1} - \frac{1}{x^2 + 1}$$

$$\therefore \int \frac{x^3 - 1}{x^3 + x} dx = x - \log x + \frac{1}{2} \log(1 + x^2) - \tan^{-1} x + C$$

79 (a)

We have,

$$\int \sqrt{\frac{1 + \sin \theta - \sin^2 \theta - \sin^3 \theta}{2 \sin \theta - 1}} d\theta$$

$$= \int \sqrt{\frac{(1 + \sin \theta)(1 - \sin^2 \theta)}{2 \sin \theta - 1}} d\theta$$

$$= \int \sqrt{\frac{1 + \sin \theta}{2 \sin \theta - 1}} \cos \theta d\theta$$

$$= \int \sqrt{\frac{x + 1}{2x - 1}} dx, \text{ where } x = \sin \theta$$

$$= \int \frac{x + 1}{\sqrt{2x^2 + x - 1}} dx$$

$$= \frac{1}{4} \int \frac{4x + 1}{\sqrt{2x^2 + x - 1}} dx$$

$$+ \frac{3}{4\sqrt{2}} \int \frac{1}{\sqrt{\left(x + \frac{1}{4}\right)^2 - \frac{9}{16}}} dx$$

$$= \frac{1}{2} \sqrt{2x^2 + x - 1}$$

$$+ \frac{3}{4\sqrt{2}} \left\{ \log \left| x + \frac{1}{4} \right| + \sqrt{\left(x + \frac{1}{4}\right)^2 - \frac{9}{16}} \right\} + C$$

$$= \frac{1}{2} \sqrt{2x^2 + x - 1}$$

$$+ \frac{3}{4\sqrt{2}} \left\{ \log |4x + 1| + 2\sqrt{2} \sqrt{2x^2 + x - 1} \right\} + C$$

$$= \frac{1}{2} \sqrt{2 \sin^2 \theta + \sin \theta - 1}$$

$$+ \frac{3}{4\sqrt{2}} \left\{ (4 \sin \theta + 1) + 2\sqrt{2} \sqrt{2 \sin^2 \theta + \sin \theta - 1} \right\} + C$$

$$= \frac{1}{2} \sqrt{\sin \theta - \cos 2\theta}$$

$$+ \frac{3}{4\sqrt{2}} \log |(4 \sin \theta + 1)|$$

$$+ 2\sqrt{2} \sqrt{\sin \theta - \cos 2\theta} + C$$

80 (b)

$$\begin{aligned}
& \int_{-1}^1 \max(\{x\}, \{-x\}) dx \\
&= \int_{-1}^{-0.5} \{-x\} dx \\
&\quad + \int_{-0.5}^0 \{x\} dx + \int_0^{0.5} \{-x\} dx \\
&\quad + \int_{0.5}^1 \{x\} dx \\
&= \int_{-1}^{-0.5} \{-x\} dx + \int_{-0.5}^0 \{x - [x]\} dx \\
&\quad + \int_0^{0.5} (-x - [-x]) dx \\
&\quad + \int_{0.5}^1 (x - [x]) dx \\
&= \left[-\frac{x^2}{2}\right]_{-1}^{-0.5} + \left[\frac{x^2}{2} + x\right]_{-0.5}^0 + \left[x - \frac{x^2}{2}\right]_0^{0.5} \\
&\quad + \left[\frac{x^2}{2}\right]_{0.5}^1 \\
&= -\frac{0.25}{2} + \frac{1}{2} + \left(-\frac{0.25}{2} + 0.5\right) + 0.5 - \frac{0.25}{2} + \frac{1}{2} \\
&\quad - \frac{0.25}{2} \\
&= 2 - \frac{1}{2} = \frac{3}{2}
\end{aligned}$$

81 (d)

$$\begin{aligned}
I &= \int_{-1}^1 \frac{x^3 + |x| + 1}{x^2 + 2|x| + 1} dx \\
\Rightarrow I &= \int_{-1}^1 \frac{x^3}{x^2 + 2|x| + 1} dx + \int_{-1}^1 \frac{|x| + 1}{x^2 + 2|x| + 1} dx \\
\Rightarrow I &= 0 + 2 \int_0^1 \frac{|x| + 1}{(|x| + 1)^2} dx \\
&\left[\because \frac{x^3}{x^2 + 2|x| + 1} \text{ is an odd function} \right] \\
\Rightarrow I &= 2 \int_0^1 \frac{x + 1}{(x + 1)^2} dx = 2[\log_e(x + 1)]_0^1 \\
&= 2 \log_e 2 = 2 \ln 2
\end{aligned}$$

82 (a)

$$\begin{aligned}
\text{Let } I &= \int_0^2 \frac{2x-2}{2x-x^2} dx \\
\text{Put } 2x - x^2 &= t \Rightarrow (2 - 2x) dx = dt \\
I &= - \int_0^0 \frac{dt}{t} = 0
\end{aligned}$$

83 (c)

$$\text{Let } I = \int_0^\pi \frac{x dx}{1 + \sin x} \dots (i)$$

$$\Rightarrow I = \int_0^\pi \frac{(\pi - x) dx}{1 + \sin x} \dots (ii)$$

On adding Eqs. (i) and (ii), we get

$$\begin{aligned}
2I &= \int_0^\pi \frac{\pi dx}{1 + \sin x} \\
&= \pi \int_0^\pi \frac{1 - \sin x}{1 - \sin^2 x} dx \\
\Rightarrow 2I &= \pi \int_0^\pi (\sec^2 x - \sec x \tan x) dx \\
\Rightarrow 2I &= \pi [\tan x - \sec x]_0^\pi = 2\pi \\
\Rightarrow I &= \pi
\end{aligned}$$

84 (d)

Let $I_n = \int_0^{\pi/2} \frac{\sin^2 nx}{\sin^2 x} dx$. Then,

$$\begin{aligned}
I_{n+1} - 2I_n + I_{n-1} &= \int_0^{\pi/2} \frac{\sin^2(n+1)x - 2\sin^2 nx + \sin^2(n-1)x}{\sin^2 x} dx \\
&= \int_0^{\pi/2} \frac{\sin(2n+1)x \sin x - \sin(2n-1)x \sin x}{\sin^2 x} dx \\
&= \int_0^{\pi/2} \frac{\sin(2n+1) - \sin(2n-1)x}{\sin x} dx \\
&= \int_0^{\pi/2} \frac{2 \sin x \cos 2n x}{\sin x} dx \\
&= 2 \int_0^{\pi/2} \cos 2n x dx = \frac{1}{n} [\sin 2n x]_0^{\pi/2} \\
&= \frac{1}{n} [\sin n\pi - 0] = 0
\end{aligned}$$

$\therefore I_{n+1} + I_{n-1} = 2I_n$ for all $n \geq 1$

$\Rightarrow I_1, I_2, I_3, \dots$ are in A.P.

So, option (c) is true

Clearly, $I_1 = \int_0^{\pi/2} 1 \cdot dx = \pi/2$

and, $I_2 = \int_0^{\pi/2} \frac{\sin^2 2x}{\sin^2 x} dx = 4 \int_0^{\pi/2} \cos^2 x dx = 4 \times \pi/4 = \pi$

$\therefore I_2 - I_1 = \pi - \pi/2 = \pi/2$

Thus, I_1, I_2, I_3, \dots are in AP with first term $\pi/2$ and common difference $\pi/2$

$\therefore I_n = \pi/2 + (n-1)\pi/2 = n\pi/2$

So, option (a) is true

Now, $\sin(I_{15}) = \sin(15\pi/2) = 1$

Hence, option (d) is not true

86 (a)

We have,

$$\begin{aligned}
f(x) &= a e^{2x} + b e^x + c x, f(0) = -1, f'(\log 2) \\
&= 31
\end{aligned}$$

$$\int_0^{\log 4} \{f(x) - cx\} dx = \frac{39}{2}$$

$$\text{Now, } f(0) = -1$$

$$\Rightarrow a + b = -1 \quad \dots(i)$$

$$\text{and, } f'(x) = 2ae^{2x} + be^x + c$$

$$\Rightarrow f'(\log 2) = 2a e^{2 \log 2} + b e^{\log 2} + c \\ = 8a + 2b + c$$

$$\therefore f'(\log 2) = 31$$

$$\Rightarrow 8a + 2b + c = 31 \quad \dots(ii)$$

Now,

$$\int_0^{\log 4} \{f(x) - cx\} dx = \frac{39}{2}$$

$$\Rightarrow \int_0^{\log 4} (ae^{2x} + be^x) dx = \frac{39}{2}$$

$$\Rightarrow \left[\frac{a}{2} e^{2x} + b e^x \right]_0^{\log 4} = \frac{39}{2}$$

$$\Rightarrow \left(\frac{a}{2} e^{2 \log 4} + b e^{\log 4} \right) - \left(\frac{a}{2} + b \right) = \frac{39}{2}$$

$$\Rightarrow \frac{a}{2} (16) + b(4) - \frac{a}{2} - b = \frac{39}{2}$$

$$\Rightarrow \frac{15}{2} a + 3b = \frac{39}{2}$$

$$\Rightarrow 15a + 6b = 39$$

$$\Rightarrow 5a + 2b = 13 \quad \dots(iii)$$

Solving (i), (ii) and (iii), we get

$$a = 5, b = -6, c = -7$$

87 (a)

$$\text{Let } I = \int_0^{\infty} \frac{dx}{(x^2+4)(x^2+9)}$$

$$= \frac{1}{5} \left(\int_0^{\infty} \frac{1}{(x^2+4)} dx - \int_0^{\infty} \frac{1}{(x^2+9)} dx \right)$$

$$= \frac{1}{5} \left[\left[\frac{1}{2} \tan^{-1} \frac{x}{2} \right]_0^{\infty} - \left[\frac{1}{3} \tan^{-1} \frac{x}{3} \right]_0^{\infty} \right]$$

$$= \frac{1}{5} \left[\frac{1}{2} \cdot \frac{\pi}{2} - 0 - \left(\frac{1}{3} \cdot \frac{\pi}{2} - 0 \right) \right]$$

$$= \frac{1}{5} \left(\frac{\pi}{4} - \frac{\pi}{6} \right) = \frac{\pi}{60}$$

88 (c)

We have,

$$I_{m,n} = \int_0^1 x^m (\log x)^n dx$$

$$\Rightarrow I_{m,n} = \left[(\log x)^n \frac{x^{m+1}}{m+1} \right]_0^1 \\ - \int_0^1 n (\log x)^{n-1} \cdot \frac{1}{x} \cdot \frac{x^{m+1}}{m+1} dx$$

$$\Rightarrow I_{m,n} = 0 - \frac{n}{m+1} \int_0^1 x^m (\log x)^{n-1} dx \\ = -\frac{n}{m+1} I_{m,n-1}$$

89 (b)

$$\int e^{3 \log x} (x^4 + 1)^{-1} dx$$

$$= \int \frac{x^3}{1+x^4} dx = \frac{1}{4} \log(x^4 + 1) + c$$

90 (d)

$$\text{Put } x = \tan \theta \Rightarrow dx = \sec^2 \theta d\theta$$

$$\therefore \int_0^{\infty} \frac{x dx}{(1+x)(1+x^2)} \\ = \int_0^{\pi/2} \frac{\tan \theta \sec^2 \theta d\theta}{(1+\tan \theta)(1+\tan^2 \theta)}$$

$$= \int_0^{\pi/2} \frac{\tan \theta}{1+\tan \theta} d\theta$$

$$= \int_0^{\pi/2} \frac{\sin \theta}{\cos \theta + \sin \theta} d\theta$$

$$\text{Let } I = \int_0^{\pi/2} \frac{\sin \theta}{\cos \theta + \sin \theta} d\theta \quad \dots(i)$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\sin \left(\frac{\pi}{2} - \theta \right)}{\cos \left(\frac{\pi}{2} - \theta \right) + \sin \left(\frac{\pi}{2} - \theta \right)} d\theta$$

$$= \int_0^{\pi/2} \frac{\cos \theta}{\sin \theta + \cos \theta} d\theta \quad \dots(ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2} 1 d\theta = [\theta]_0^{\pi/2} = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

$$\therefore \int_0^{\infty} \frac{x dx}{(1+x)(1+x^2)} = \frac{\pi}{4}$$

91 (c)

$$f(x) = \tan x - \tan^3 x + \tan^5 x - \dots \infty$$

$$f(x) = \frac{\tan x}{1+\tan^2 x} = \frac{\tan x}{\sec^2 x} = \frac{\sin 2x}{2}$$

$$\therefore \int_0^{\pi/4} f(x) dx = \int_0^{\pi/4} \frac{\sin 2x}{2} dx$$

$$= \left[-\frac{\cos 2x}{4} \right]_0^{\pi/4} = \frac{1}{4}$$

92 (d)

$$\text{Let } I = \int_0^1 \frac{x^4+1}{x^2+1} dx$$

$$= \int_0^1 \frac{x^4+1-1+1}{x^2+1} dx$$

$$= \int_0^1 \left[\frac{x^4-1}{x^2+1} + \frac{2}{x^2+1} \right] dx$$

$$\begin{aligned}
&= \int_0^1 \left(x^2 - 1 + \frac{2}{x^2 + 1} \right) dx \\
&= \left[\frac{x^3}{3} - x + 2 \tan^{-1} x \right]_0^1 \\
&= \left[\frac{1}{3} - 1 + 2 \tan^{-1}(1) - 0 \right] \\
&= -\frac{2}{3} + 2 \cdot \frac{\pi}{4} = \frac{3\pi - 4}{6}
\end{aligned}$$

93 (c)

We have,

$$I = \int_0^{\pi/2} \log \tan x \, dx \quad \dots(i)$$

$$\Rightarrow I = \int_0^{\pi/2} \log \tan \left(\frac{\pi}{2} - x \right) dx = \int_0^{\pi/2} \log \cot x \, dx \quad \dots(ii)$$

Adding (i) and (ii), we get

$$2I = 0 \Rightarrow I = 0$$

94 (c)

$$\frac{d}{dx} \int_{f(x)}^{g(x)} h(t) dt = h[g(x)]g'(x) - h[f(x)]f'(x)$$

95 (c)

Differentiate both sides n times with respect to a

96 (b)

We have,

$$I = \int_{-1}^1 \frac{|x+2|}{x+2} dx$$

$$= \int_{-1}^1 \frac{x+2}{x+2} dx \left[\begin{array}{l} \because x+2 \geq 0 \text{ for } -1 \leq x \leq 1 \\ \therefore |x+2| = x+2 \end{array} \right]$$

$$\Rightarrow I = \int_{-1}^1 1 \cdot dx = 2$$

97 (b)

$$\text{Put } 1 + \log x = t \Rightarrow \frac{1}{x} dx = dt$$

$$\begin{aligned} \int \frac{dx}{x(1 + \log x)^2} &= \int t^{-2} dt = \frac{t^{-1}}{-1} + c \\ &= \frac{-1}{(1 + \log x)} + c \end{aligned}$$

99 (b)

$$\begin{aligned} \int_0^{\pi/2} x \sin x \, dx &= [x(-\cos x)]_0^{\pi/2} \\ &\quad - \int_0^{\pi/2} 1(-\cos x) dx \\ &= 0 + [\sin x]_0^{\pi/2} = 1 \end{aligned}$$

100 (a)

$$\text{Let } \sin x = z \Rightarrow d(\sin x) = dz$$

$$\therefore \int \frac{dz}{\sqrt{1-z^2}} = \sin^{-1} z + c$$

$$= \sin^{-1}(\sin x) + c = x + c$$

101 (b)

$$\begin{aligned} \int 1 \cdot \sin^{-1} x \, dx &= (\sin^{-1} x)x - \int \frac{1}{\sqrt{1-x^2}} \cdot x \, dx \\ &= x \sin^{-1} x - \frac{t \, dt}{t} \quad (\text{put } 1-x^2 = t \Rightarrow \\ &\quad -2x \, dx = 2t \, dt) \\ &= x \sin^{-1} x + \sqrt{1-x^2} + c \end{aligned}$$

102 (a)

$$\text{Let } I_m = \int_0^{\pi} \frac{\sin 2m x}{\sin x} dx. \text{ Then,}$$

$$I_m - I_{m-1} = \int_0^{\pi} \frac{\sin 2m x - \sin 2(m-1)x}{\sin x} dx$$

$$\Rightarrow I_m - I_{m-1} = \int_0^{\pi} 2 \cos(2m-1)x \, dx$$

$$\Rightarrow I_m - I_{m-1} = \frac{2}{2m-1} [\sin(2m-1)x]_0^{\pi} = 0$$

$$\Rightarrow I_m - I_{m-1} \text{ for all } m \in \mathbb{N}$$

$$\Rightarrow I_m - I_{m-1} = I_{m-2} = \dots = I_1$$

$$\text{But, } I_1 = \int_0^{\pi} \frac{\sin 2x}{\sin x} dx = 2 \int_0^{\pi} \cos x \, dx = 0$$

$$\therefore I_m = 0 \text{ for all } m \in \mathbb{N}$$

103 (a)

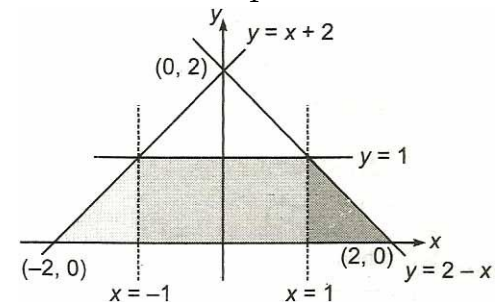
$$\sin x > \sin^3 x$$

$$\Rightarrow \int_0^{\pi/2} \sin x \, dx > \int_0^{\pi/2} \sin^3 x \, dx$$

$$\Rightarrow I_1 > I_2$$

104 (c)

$$\begin{aligned} \int_{-2}^2 f(x) dx &= \int_{-2}^{-1} (x+2) dx + \int_{-1}^1 dx \\ &\quad + \int_1^2 (2-x) dx \end{aligned}$$



$$= \frac{1}{2} + 2 + \frac{1}{2} = 3$$

105 (b)

Let $f(x) = x^3 + x^2 + 3x = (x(x^2 + x + 3))$

Since $x^2 + x + 3 > 0$ for all x . Therefore,

$f(x) = x(x^2 + x + 3) > 0$ for all $x > 0$

$$\therefore I = \int_0^3 |x^3 + x^2 + 3x| dx = \int_0^3 (x^3 + x^2 + 3x) dx$$

$$\Rightarrow I = \left[\frac{x^4}{4} + \frac{x^3}{3} + \frac{3x^2}{2} \right]_0^3 = \frac{81}{4} + 9 + \frac{27}{2} = \frac{171}{4}$$

106 (b)

$$\int_{-1/7}^{20/7} e^{5\{x\}} dx = \int_{-1/7}^{3-\frac{1}{7}} e^{5\{x\}} dx$$

$$= 3 \int_0^1 e^{5\{x\}} dx = 3 \int_0^1 e^{5x} dx \quad (\{x\} = x \text{ as } 0 \leq x < 1)$$

$$= 3 \left[\frac{e^{5x}}{5} \right]_0^1 = 3 \left(\frac{e^5 - 1}{5} \right)$$

107 (b)

$$\int_{\pi/4}^{\pi/2} \operatorname{cosec}^2 x dx = [-\cot x]_{\pi/4}^{\pi/2}$$

$$= \left(-\cot \frac{\pi}{2} + \cot \frac{\pi}{4} \right) = 1$$

108 (a)

Let $f(x) = C_2 \frac{x^3}{3} + C_1 \frac{x^2}{2} + C_0 x$

Clearly, $f(x)$, being a polynomial, is continuous on $[0, 1]$ and differentiable on $(0, 1)$

Also, $f(0) = 0$ and $f(1) = \frac{C_2}{3} + \frac{C_1}{2} + C_0 = 0$

[Given]

$\Rightarrow f(0) = f(1)$

Hence, $f(x)$ satisfies conditions of Rolle's theorem. Consequently there exists $\alpha \in (0, 1)$ such that

$f'(\alpha) = 0$

$\Rightarrow C_2 \alpha^2 + C_1 \alpha + C_0 = 0$

$\Rightarrow C_2 \alpha^2 + C_1 \alpha + C_0 = 0 \Rightarrow \alpha$ is a root of

$C_2 x^2 + C_1 x + C_0 = 0$

109 (b)

Let $I = \int \frac{dx}{2\sqrt{x}(1+x)}$

Put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$$\therefore I = \int \frac{dt}{1+t^2} = \tan^{-1} t + c = \tan^{-1} \sqrt{x} + c$$

111 (c)

Let $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\cot x}}$

$$\Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad \dots(i)$$

$$\Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)} + \sqrt{\cos\left(\frac{\pi}{2}-x\right)}} dx$$

$$\left[\because \int_a^b f(x) dx = \int_a^b f(a+b-x) dx \right]$$

$$\Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \quad \dots(ii)$$

On adding Eqs.(i) and (ii), we get

$$2I = \int_{\pi/6}^{\pi/3} 1 dx = [x]_{\pi/6}^{\pi/3}$$

$$\Rightarrow I = \frac{1}{2} \left(\frac{\pi}{3} - \frac{\pi}{6} \right) = \frac{\pi}{12}$$

112 (c)

$$I = \int_{-1/7}^{20/7} \sin(x - [x]) dx = \int_{-1/7}^{3-\frac{1}{7}} \sin(\{x\}) dx$$

$$= 3 \int_0^1 \sin x dx = 3[-\cos x]_0^1 = 3(1 - \cos 1)$$

113 (a)

$$f\left(\frac{1}{x}\right) + x^2 f(x) = 0$$

Let $I = \int_{\sin \theta}^{\operatorname{cosec} \theta} f(x) dx$

Put $x = \frac{1}{t} \Rightarrow dx = -\frac{1}{t^2} dt$

$$\therefore I = - \int_{\operatorname{cosec} \theta}^{\sin \theta} f\left(\frac{1}{t}\right) \cdot \frac{1}{t^2} dt$$

$$= \int_{\sin \theta}^{\operatorname{cosec} \theta} \frac{1}{x^2} f\left(\frac{1}{x}\right) dx$$

$$\Rightarrow I = - \int_{\sin \theta}^{\operatorname{cosec} \theta} f(x) dx \Rightarrow I = 0$$

114 (c)

$$I = \int_{-\pi/2}^{\pi/2} \sin^4 x \cos^6 x dx$$

This is an even function.

$$\therefore I = 2 \int_0^{\pi/2} \sin^4 x \cos^6 x dx$$

Applying Gamma function

$$\Rightarrow I = \frac{\sqrt{\frac{4+1}{2}} - \sqrt{\frac{6+1}{2}}}{2|6} = \frac{2\sqrt{\frac{5}{2}} - \sqrt{\frac{7}{2}}}{2|6}$$

$$= \frac{\frac{3}{2} \cdot \frac{1}{2} \cdot \sqrt{\pi} \cdot \frac{5}{2} \cdot \frac{3}{2} \cdot \frac{1}{2} \cdot \sqrt{\pi}}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{3\pi}{2^8} = \frac{3\pi}{256}$$

115 (b)

We have,

$$I = \int_0^1 \cot^{-1}(1-x+x^2) dx$$

$$\begin{aligned} \Rightarrow I &= \int_0^1 \tan^{-1} \left\{ \frac{1}{1-x(1-x)} \right\} dx \\ \Rightarrow I &= \int_0^1 \tan^{-1} \left\{ \frac{x+(1-x)}{1-x(1-x)} \right\} \\ \Rightarrow I &= \int_0^1 [\tan^{-1} x - \tan^{-1}(1-x)] dx \\ \Rightarrow I &= \int_0^1 \tan^{-1} x dx - \int_0^1 \tan^{-1}(1-x) dx \\ \Rightarrow I &= \int_0^1 \tan^{-1} x dx - \int_0^1 \tan^{-1}(1-(1-x)) dx \\ \Rightarrow I &= 2 \int_0^1 \tan^{-1} x dx = \frac{\pi}{2} - \log 2 \end{aligned}$$

116 (d)

$$\begin{aligned} \text{Put } x &= -t \\ \Rightarrow dx &= -dt \\ \therefore \int_{-100}^{100} f(x) dx &= - \int_{100}^{-100} f(-t) dt \\ &= \int_{-100}^{100} f(-t) dt \\ &= \int_{-100}^{100} f(-x) dx \end{aligned}$$

117 (a)

$$\begin{aligned} &\int \frac{\cos x - 1}{\sin x + 1} e^x dx \\ &= \int \frac{\cos x}{1 + \sin x} e^x dx - \int \frac{1}{1 + \sin x} e^x dx \\ &= \frac{(\cos x) e^x}{1 + \sin x} + \int \frac{-(1 + \sin x) \sin x - \cos^2 x}{(1 + \sin x)^2} e^x dx \\ &\quad - \int \frac{e^x}{\sin x + 1} dx + c \\ &= \frac{e^x \cos x}{1 + \sin x} + \int \frac{1}{1 + \sin x} e^x dx \\ &\quad - \int \frac{e^x}{1 + \sin x} dx + c \\ &= \frac{e^x \cos x}{1 + \sin x} + c \end{aligned}$$

118 (b)

$$\begin{aligned} \text{LHS} &= \int (\log x)^2 dx \\ &= x (\log x)^2 - \int x \cdot 2 \log x \frac{1}{x} dx \\ &= x (\log x)^2 - 2 \left[x \log x - \int x \cdot \frac{1}{x} dx \right] + c \\ &= x (\log x)^2 - 2[x \log x - x] \\ &= x (\log x)^2 - 2x[\log x - 1] + c \end{aligned}$$

But RHS is given by
 $x[f(x)]^2 + Ax[f(x) - 1] + c$
 $\therefore f(x) = \log x$ and $A = -2$

119 (a)

We have,

$$\begin{aligned} I_1 - I_2 &= \int_{\pi/6}^{\pi/3} f(\tan \theta + \cot \theta) (\sec^2 \theta \\ &\quad - \operatorname{cosec}^2 \theta) d\theta \\ \Rightarrow I_1 - I_2 &= \int_{\pi/6}^{\pi/3} f(\tan \theta + \cot \theta) d(\tan \theta + \cot \theta) \\ \Rightarrow I_1 - I_2 &= \int_{4/\sqrt{3}}^{4/\sqrt{3}} f(t) dt, \text{ where } t = \tan \theta + \cot \theta \\ \Rightarrow I_1 - I_2 &= 0 \Rightarrow I_1 = I_2 \Rightarrow \frac{I_1}{I_2} = 1 \end{aligned}$$

120 (c)

We have,

$$\begin{aligned} I &= \int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x} dx \\ \Rightarrow I &= \int_{-\pi/2}^{\pi/2} \sqrt{\cos x} |\sin x| dx \\ \Rightarrow I &= 2 \int_0^{\pi/2} \sqrt{\cos x} |\sin x| dx \\ \Rightarrow I &= 2 \int_0^{\pi/2} \sqrt{\cos x} \sin x dx = -2 \int_1^0 \sqrt{t} dt, \text{ where } t \\ &= \cos x \\ \Rightarrow I &= -\frac{4}{3} (0 - 1) = \frac{4}{3} \end{aligned}$$

121 (b)

We have,

$$I = \int \frac{1+x^2}{1+x^4} dx = \int \frac{(1+1/x^2)}{x^2+(1/x^2)} dx$$

$$\Rightarrow I = \int \frac{1 + (1/x^2)}{\left(x - \frac{1}{x^2}\right)^2 + C} dx$$

$$= \int \frac{1}{\left(x - \frac{1}{x}\right)^2 + (\sqrt{2})^2} d\left(x - \frac{1}{x}\right)$$

$$\Rightarrow I = \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{2}x} \right)$$

122 (a)

We have,

$$g(x) = f'(x), f(-1) = 2 \text{ and } f(2) = 4$$

$$\therefore \int_{-1}^2 g(x) dx = \int_{-1}^2 f'(x) dx = f(2) - f(-1)$$

$$= 4 - 2 = 2$$

124 (b)

We have,

$$I = \int \frac{1}{x^4 \sqrt{a^2 + x^2}} dx$$

$$\Rightarrow I = \int \frac{1}{x^5 \sqrt{\left(\frac{a}{x}\right)^2 + 1}} dx = \int \frac{\frac{1}{x^2}}{x^3 \sqrt{\left(\frac{a}{x}\right)^2 + 1}} dx$$

$$\Rightarrow I = -\frac{1}{2a^4} \int \frac{\left(\frac{a}{x}\right)^2}{\sqrt{\left(\frac{a}{x}\right)^2 + 1}} \times \frac{-2a^2}{x^3} dx$$

$$\Rightarrow I = -\frac{1}{2a^4} \int \frac{t}{\sqrt{t+1}} dt, \text{ where } t = \frac{a^2}{x^2}$$

$$\Rightarrow I = -\frac{1}{2a^4} \int \frac{(t+1) - 1}{\sqrt{t+1}} dt$$

$$\Rightarrow I = -\frac{1}{2a^4} \int \left(\sqrt{t+1} - \frac{1}{\sqrt{t+1}} \right) dt$$

$$\Rightarrow I = -\frac{1}{2a^4} \left\{ \frac{2}{3} (t+1)^{3/2} - 2(t+1)^{1/2} \right\} + C$$

$$\Rightarrow I = \frac{1}{a^4} \left\{ \frac{\sqrt{a^2 + x^2}}{x} - \frac{(a^2 + x^2)^{3/2}}{3x^3} \right\} + C$$

125 (b)

$$\text{Let } I = \int_0^{\pi/2} \log |\tan x| dx \quad \dots(i)$$

$$\Rightarrow I = \int_0^{\pi/2} \log \left| \tan \left(\frac{\pi}{2} - x \right) \right| \dots(ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2} \log |\tan x \cot x| dx$$

$$= \int_0^{\pi/2} \log 1 dx = 0$$

$$\Rightarrow I = 0$$

126 (d)

We have,

$$[\sin(-x)]^{11} = (-\sin x)^{11} = -\sin^{11} x$$

$$\therefore \sin^{11} x \text{ is an odd function } x$$

$$\text{Hence, } \int_{-1}^1 \sin^{11} x dx = 0$$

127 (b)

$$\int \cos^4 x dx = \frac{1}{4} (2 \cos^2 x)^2 dx$$

$$= \frac{1}{4} \int (1 + \cos 2x)^2 dx$$

$$= \frac{1}{4} \int (1 + 2 \cos 2x + \cos^2 2x) dx$$

$$= \frac{1}{8} \int \{2 + 4 \cos 2x + (1 + \cos 4x)\} dx$$

$$= \frac{1}{8} \left\{ 3x + 2 \sin 2x + \frac{1}{4} \sin 4x \right\} + D$$

$$= \frac{3}{8} x + \frac{1}{4} \sin 2x + \frac{1}{32} \sin 4x + D$$

On comparing, we get

$$A = \frac{3}{8}, B = \frac{1}{4}, C = \frac{1}{32}$$

128 (c)

$$\int e^x \left(\frac{1 - \sin x}{1 - \cos x} \right) dx$$

$$= \int e^x \left(\frac{1 - 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \sin^2 \frac{x}{2}} \right) dx$$

$$= \frac{1}{2} \int e^x \left(\operatorname{cosec}^2 \frac{x}{2} \right) dx - \int e^x \cot \frac{x}{2} dx$$

$$= \frac{1}{2} \left[-e^x \cot \frac{x}{2} \cdot 2 + \int e^x \cot \frac{x}{2} \cdot 2 dx \right]$$

$$- \int e^x \cot \frac{x}{2} dx + c$$

$$= -e^x \cot \frac{x}{2} + c$$

129 (a)

$$\text{Since, } \int_1^a [x] f'(x) dx = \int_1^2 f'(x) dx$$

$$+ \int_2^3 2f'(x) dx + \dots + \int_{[a]}^a [a] f'(x) dx$$

$$= [f(x)]_1^2 + 2[f(x)]_2^3 + \dots + [a][f(x)]_{[a]}^a$$

$$= f(2) - f(1) + 2f(3) - 2f(2) + \dots$$

$$+ [a]f(a) - [a]f([a])$$

$$= [a]f(a) - \{f(1) + f(2) + \dots + f([a])\}$$

130 (a)

We have,

$$\int_a^b x f(x) dx = k \int_a^b f(x) dx \quad \dots(i)$$

$$\Rightarrow \int_a^b (a + b - x)(a + b - x) dx = k \int_a^b f(x) dx$$

$$\begin{aligned} &\Rightarrow \int_a^b (a+b-x)f(x)dx \\ &= k \int_a^b f(x) dx [\because f(a+b-x) \\ &= f(x)] \\ &\Rightarrow (a+b) \int_a^b f(x) dx - \int_a^b xf(x) dx = k \int_a^b f(x) dx \\ &\Rightarrow (a+b) \int_a^b f(x) dx - k \int_a^b f(x) dx \\ &= k \int_a^b f(x) dx \quad [\text{Using (i)}] \\ &\Rightarrow (a+b) \int_a^b f(x) dx = 2k \int_a^b f(x) dx \Rightarrow k = \frac{a+b}{2} \end{aligned}$$

131 (d)

$$\begin{aligned} \text{Let } I &= \int |x| \cdot 1 dx \\ &= |x|x - \int \frac{|x|}{x} x dx = x|x| - \int |x| dx \\ &\Rightarrow 2I = x|x| \\ &\Rightarrow I = \frac{x|x|}{2} + c \end{aligned}$$

132 (b)

$$\begin{aligned} \text{Let} \\ I_1 &= \int_{-1}^1 \sin^{-1} \left[x^2 + \frac{1}{2} \right] dx \\ I_1 &= 2 \int_0^1 \sin^{-1} \left[x^2 + \frac{1}{2} \right] dx \\ I_1 &= 2 \left[\int_0^{1/\sqrt{2}} \sin^{-1} \left[x^2 + \frac{1}{2} \right] dx \right. \\ &\quad \left. + \int_{1/\sqrt{2}}^1 \sin^{-1} \left[x^2 + \frac{1}{2} \right] dx \right] \\ I_1 &= 2 \left[\int_0^{1/\sqrt{2}} \sin^{-1} 0 dx + \int_{1/\sqrt{2}}^1 \sin^{-1} 1 dx \right] \\ I_1 &= 2 \left[\frac{\pi}{2} \int_{1/\sqrt{2}}^1 dx \right] = \pi \left(1 - \frac{1}{\sqrt{2}} \right) \end{aligned}$$

and,

$$I_2 = \int_{-1}^1 \cos^{-1} \left[x^2 - \frac{1}{2} \right] dx$$

$$\begin{aligned} &\Rightarrow I_2 = 2 \int_0^1 \cos^{-1} \left[x^2 - \frac{1}{2} \right] dx \\ &\Rightarrow I_2 = 2 \left[\int_0^{1/\sqrt{2}} \cos^{-1} \left[x^2 - \frac{1}{2} \right] dx \right. \\ &\quad \left. + \int_{1/\sqrt{2}}^1 \cos^{-1} \left[x^2 - \frac{1}{2} \right] dx \right] \\ &\Rightarrow I_2 = 2 \left[\int_0^{1/\sqrt{2}} \cos^{-1}(-1) dx + \int_{1/\sqrt{2}}^1 \cos^{-1} 0 dx \right] \\ &\Rightarrow I_2 = 2 \left[\int_0^{1/\sqrt{2}} \pi dx + \int_{1/\sqrt{2}}^1 \frac{\pi}{2} dx \right] \\ &\Rightarrow I_2 = 2 \left[\pi \left(\frac{1}{\sqrt{2}} - 0 \right) + \frac{\pi}{2} \left(1 - \frac{1}{\sqrt{2}} \right) \right] \\ &\Rightarrow I_2 = 2 \left[\pi \left(\frac{1}{2\sqrt{2}} + \frac{1}{2} \right) \right] = \pi \left(\frac{1}{\sqrt{2}} + 1 \right) \\ &\therefore \int_{-1}^1 \sin^{-1} \left[x^2 + \frac{1}{2} \right] dx + \int_{-1}^1 \cos^{-1} \left[x^2 - \frac{1}{2} \right] dx \\ &= I_1 + I_2 = \pi \left(1 - \frac{1}{\sqrt{2}} \right) + \pi \left(\frac{1}{\sqrt{2}} + 1 \right) = 2\pi \end{aligned}$$

133 (d)

We have,

$$\begin{aligned} I &= \int \frac{x^2 - 2}{x^3 \sqrt{x^2 - 1}} dx \\ &\Rightarrow I = \int \frac{x^2}{x^3 \sqrt{x^2 - 1}} dx - 2 \int \frac{1}{x^3 \sqrt{x^2 - 1}} dx \\ &\Rightarrow I \\ &= \int \frac{1}{x \sqrt{x^2 - 1}} dx \\ &\quad - 2 \int \frac{1}{\sec^3 \theta \tan \theta} \sec \theta \tan \theta d\theta, \text{ (where } x \\ &= \sec \theta) \\ &\Rightarrow I = \sec^{-1} x - \int (1 + \cos 2\theta) d\theta \\ &\Rightarrow I = \sec^{-1} x - \left(\theta + \frac{\sin 2\theta}{2} \right) + C \\ &\Rightarrow I = \sec^{-1} x - \sec^{-1} x - \frac{\sqrt{x^2 - 1}}{x^2} + C \\ &= -\frac{\sqrt{x^2 - 1}}{x^2} + C \end{aligned}$$

134 (c)

Using L' Hospital's rule, we have

$$\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \sin \sqrt{t} dt}{x^3} = \lim_{x \rightarrow 0} \frac{2x \sin x}{3x^2} = \frac{2}{3} \lim_{x \rightarrow 0} \frac{\sin x}{x} = \frac{2}{3}$$

135 (c)

$$\begin{aligned}
\text{Let } I &= \int e^x \frac{x^2+1}{(x+1)^2} dx \\
&= \int e^x \frac{(x-1)}{(x+1)} dx + 2 \int \frac{e^x}{(x+1)^2} dx \\
&= \int e^x dx - 2 \int \frac{e^x}{(x+1)} dx \\
&\quad + 2 \left[-\frac{e^x}{(x+1)} - \int -\frac{e^x}{(x+1)} dx \right] \\
&= e^x - \frac{2e^x}{x+1} + c = e^x \left(\frac{x-1}{x+1} \right) + c
\end{aligned}$$

136 (d)

We have,

$$\begin{aligned}
I &= \int x \log \left(1 + \frac{1}{x} \right) dx \\
\Rightarrow I &= \int x \log(x+1) dx - \int x \log x dx \\
\Rightarrow I &= \frac{x^2}{2} \log(x+1) - \frac{1}{2} \int \frac{x^2}{x+1} dx - \frac{x^2}{2} \log x \\
&\quad + \frac{1}{2} \int \frac{x^2}{2} dx \\
\Rightarrow I &= \frac{x^2}{2} \log(x+1) - \frac{1}{2} \int \left(x - 1 + \frac{1}{x} + 1 \right) dx \\
&\quad - \frac{x^2}{2} \log x + \frac{1}{4} x^2 \\
\Rightarrow I &= \frac{x^2}{2} \log(x+1) - \frac{x^2}{2} \log x - \frac{1}{2} \left(\frac{x^2}{2} - x \right) \\
&\quad - \frac{1}{2} \log(x+1) + \frac{1}{4} x^2 + C \\
\Rightarrow I &= \frac{x^2}{2} \log(x+1) - \frac{x^2}{2} \log x - \frac{1}{2} \log(x+1) \\
&\quad + \frac{1}{2} x + C
\end{aligned}$$

Hence, $f(x) = \frac{x^2}{2} - \frac{1}{2}$, $g(x) = -\frac{1}{2} \log x$ and $A = \frac{1}{2}$

137 (c)

Let $I = \int_0^1 f(x) dx = 0$ and $f(x) \geq 0 \Rightarrow f(x) = 0$

138 (b)

Given, $f\left(\frac{3x-4}{3x+4}\right) = x + 2$

Let $\frac{3x-4}{3x+4} = X$

$$\Rightarrow \frac{3x-4+3x+4}{3x-4-(3x+4)} = \frac{X+1}{X-1}$$

$$\Rightarrow \frac{6x}{-8} = \frac{X+1}{X-1}$$

$$\Rightarrow x = \frac{4(X+1)}{3(X-1)}$$

$$\therefore f(x) = \frac{4(X+1)}{3(X-1)} + 2 = \frac{10-2X}{3(1-X)}$$

$$\begin{aligned}
&= \frac{2}{3} \left[\int dx + \int \frac{4}{1-x} dx \right] \\
&= \frac{2}{3} [x - 4 \log(1-x)] + c \\
&= \frac{2}{3} x - \frac{8}{3} \log(1-x) + c
\end{aligned}$$

139 (d)

Let $I = \int \frac{e^x 1 + nx^{n-1} - x^{2n}}{(1-x^n)\sqrt{1-x^{2n}}} dx$

$$= \int e^x \left\{ \frac{\sqrt{1-x^{2n}}}{1-x^n} + \frac{nx^{n-1}}{(1-x^n)\sqrt{1-x^{2n}}} \right\} dx$$

Let $f(x) = \frac{\sqrt{1-x^{2n}}}{(1-x^n)}$

$$\Rightarrow f'(x) = \frac{\frac{(1-x^n)(-2nx^{2n-1})}{2\sqrt{1-x^{2n}}} + \sqrt{1-x^{2n}}(nx^{n-1})}{(1-x^n)^2}$$

$$\Rightarrow f'(x) = \frac{(1-x^{2n})(nx^{n-1}) - n(1-x^n)x^{2n-1}}{(1-x^n)^2\sqrt{1-x^{2n}}}$$

$$= \frac{n(1-x^n)x^{n-1}\{(1+x^n) - x^n\}}{(1-x^n)^2\sqrt{1-x^{2n}}}$$

$$= \frac{nx^{n-1}}{(1-x^n)\sqrt{1-x^{2n}}}$$

$$\therefore I = \frac{e^x \sqrt{1-x^{2n}}}{(1-x^n)} + c$$

140 (c)

An anti-derivative of $f(x)$

$= F(x)$

$$= \int [\log(\log x) + (\log x)^{-2}] dx + c$$

$$= x \log(\log x) - \int \frac{x}{x \log x} dx + \int (\log x)^{-2} dx + c$$

(integrating by parts the first integral)

$$= x \log(\log x) - [x(\log x)^{-1}$$

$$+ \int (\log x)^{-2} dx] + \int (\log x)^{-2} dx + c$$

[again integrating by parts $(\log x)^{-1}$]

$$= x \log(\log x) - x(\log x)^{-1} + c$$

On putting $x = e$, we have, $e = 0 - e + c$ so,

$$c = 2e$$

Thus, $F(x) = x[\log(\log x) - (\log x)^{-1}] + 2e$

141 (b)

We have,

$$\int_a^b \frac{x^n}{x^n + (16-x)^n} dx = 6$$

$$\Rightarrow \frac{b-a}{2} = 6 \text{ and } a+b = 16 \Rightarrow a = 2, b = 14$$

142 (d)

Let $I = \int \tan^{-1} \frac{x}{1-x} dx$

$$= \tan^{-1} x \cdot x - \int \frac{1}{1+x^2} \cdot x dx$$

$$= x \tan^{-1} x - I_1 \quad \dots(i)$$

$$\text{Where } I_1 = \int \frac{x}{1+x^2} dx$$

$$\text{Put } 1+x^2 = t \Rightarrow x dx = \frac{1}{2} dt$$

$$\therefore I_1 = \frac{1}{2} \int \frac{1}{t} dt = \frac{1}{2} \log t$$

$$= \frac{1}{2} \log |1+x^2|$$

\therefore From eq. (i), we get

$$I = x \tan^{-1} x - \frac{1}{2} \log |1+x^2| + c$$

143 (a)

$$\int x^{51} (\tan^{-1} x + \cot^{-1} x) dx = \int x^{51} \cdot \frac{\pi}{2} dx$$

$$= \frac{\pi x^{52}}{104} + c$$

$$= \frac{x^{52}}{52} (\tan^{-1} x + \cot^{-1} x) + c$$

144 (a)

We have,

$$\int_0^2 |x-1| dx = \int_0^1 -(x-1) dx$$

$$+ \int_1^2 (x-1) dx$$

$$= -\left[\frac{x^2}{2} - x\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^2$$

$$= \left[-\left(\frac{1}{2} - 1\right) - (0 - 0)\right]$$

$$+ \left[\left(\frac{4}{2} - 2\right) - \left(\frac{1}{2} - 1\right)\right] = 1$$

145 (a)

$$\text{Let } I = \int \frac{3-x^2}{(1-x)^2} \cdot e^x dx = \int \left(\frac{2}{(1-x)^2} + \frac{1+x}{1-x}\right) e^x dx$$

$$\Rightarrow I = e^x \left(\frac{1+x}{1-x}\right) + c$$

But $I = e^x f(x) + c$ [given]

$$\therefore f(x) = \frac{1+x}{1-x}$$

146 (d)

$$\int_0^1 \tan^{-1} \left(\frac{1}{x^2-x+1}\right) dx$$

$$= \int_0^1 \tan^{-1} \left(\frac{x-(x-1)}{1+x(x-1)}\right)$$

$$= \int_0^1 \tan^{-1} x dx - \int_0^1 \tan^{-1}(x-1) dx$$

$$= \int_0^1 \tan^{-1} x dx + \int_0^1 \tan^{-1}(1-1+x) dx$$

$$= \int_0^1 \tan^{-1} x dx + \int_0^1 \tan^{-1} x dx$$

$$= 2 \int_0^1 \tan^{-1} x dx$$

$$= 2 \left[x \tan^{-1} x - \int \frac{x}{1+x^2} dx \right]_0^1$$

$$= 2 \left[x \tan^{-1} x - \frac{1}{2} \log(1+x^2) \right]_0^1$$

$$= 2 \left[\left\{ 1 \tan^{-1} 1 - \frac{1}{2} \log(2) \right\} - \left\{ 0 - \frac{1}{2} \log 1 \right\} \right]$$

$$= 2 \left[\frac{\pi}{4} - \frac{1}{2} \log 2 - 0 \right] = \frac{\pi}{2} - \log 2$$

147 (a)

$$\lim_{x \rightarrow 1} \int_4^{f(x)} \frac{2t}{x-1} dt = \lim_{x \rightarrow 1} \frac{\int_4^{f(x)} 2t dt}{x-1} \quad (\text{using L'}$$

Hospital's rule)

$$= \lim_{x \rightarrow 1} \frac{2f(x) \cdot f'(1)}{1}$$

$$= 2f(1) \cdot f'(1), \text{ where } f(1) = 4$$

$$= 8f'(1)$$

148 (b)

We know, $\int e^x [f(x) + f'(x)] dx = e^x f(x) + c$

\therefore If $f(x) = \log \sin x$

$$\Rightarrow f'(x) = \frac{1}{\sin x} \cdot \cos x = \cot x$$

$$\therefore \int e^x (\log \sin x + \cot x) dx = e^x \log \sin x + c$$

149 (a)

Let $f(x) = (1-x^2) \sin x \cos^2 x$. Then,

$$f(-x) = \{1-(x^2)\} \sin(-x) \cos^2(-x)$$

$$\Rightarrow f(-x) = -(1-x^2) \sin x \cos^2 x$$

$$\Rightarrow f(-x) = -f(x)$$

$$\text{Hence, } \int_{-\pi}^{\pi} (1-x^2) \sin x \cos^2 x dx = 0$$

150 (c)

$$\text{Let } I = \int \frac{xdx}{(x^2-a^2)(x^2-b^2)}$$

$$\text{Put } x^2 = t \Rightarrow xdx = \frac{1}{2} dt$$

$$\therefore I = \frac{1}{2} \int \frac{dt}{(t-a^2)(t-b^2)}$$

$$= \frac{1}{2} \int \left\{ \frac{dt}{(a^2-b^2)(t-a^2)} - \frac{dt}{(a^2-b^2)(t-b^2)} \right\}$$

$$= \frac{1}{2(a^2-b^2)} \log \left| \frac{t-a^2}{t-b^2} \right| + c$$

$$\Rightarrow I = \frac{1}{2(a^2-b^2)} \log \left| \frac{x^2-a^2}{x^2-b^2} \right| + c$$

151 (b)

Since $\cos^2 x$ is a periodic function with period π

Therefore, so is $f(\cos^2 x)$. Hence,

$$\int_0^{n\pi} f(\cos^2 x) dx = n \int_0^{\pi} f(\cos^2 x) dx \Rightarrow k = n$$

152 (c)

$$\text{Let } I = \int (1 - \cos^2 x) \cos^2 x \cdot \sin x \, dx$$

$$\text{Put } \cos x = t$$

$$\Rightarrow -\sin x \, dx = dt$$

$$\therefore I = - \int (1 - t^2) t^2 dt = \int (t^4 - t^2) dt$$

$$= \frac{t^5}{5} - \frac{t^3}{3} + c$$

$$= \frac{(\cos x)^5}{5} - \frac{(\cos x)^3}{3} + c$$

153 (d)

$$\int_{\pi/2}^{\alpha} \sin x \, dx = \sin 2\alpha, \alpha \in (0, 2\pi)$$

$$\Rightarrow -[\cos x]_{\pi/2}^{\alpha} = \sin 2\alpha$$

$$\Rightarrow -\cos \alpha = \sin 2\alpha$$

$$\Rightarrow \sin\left(\alpha - \frac{\pi}{2}\right) = \sin 2\alpha$$

$$\text{Hence, } \alpha = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}$$

154 (b)

$$\text{Let } \int_{-\pi/4}^{\pi/4} \frac{\tan^2 x}{1+a^x} dx$$

$$\text{Using } \int_{-a}^a f(x) dx = \int_0^a \{f(x) + f(-x)\} dx, \text{ we have}$$

$$I = \int_0^{\pi/4} \left(\frac{\tan^2 x}{1+a^x} + \frac{\tan^2 x}{1+a^{-x}} \right) dx$$

$$\Rightarrow I = \int_0^{\pi/4} \tan^2 x \, dx = \int_0^{\pi/4} (\sec^2 x - 1) dx$$

$$= [\tan x - x]_0^{\pi/4} = \frac{4 - \pi}{4}$$

155 (d)

We have,

$$I = \int e^x \frac{1 + n x^{n-1} - x^{2n}}{(1-x^n)\sqrt{1-x^{2n}}} dx$$

$$\Rightarrow I = \int e^x \frac{(1-x^{2n}) + n x^{n-1}}{(1-x^n)\sqrt{1-x^{2n}}} dx$$

$$\Rightarrow I = \int e^x \left\{ \sqrt{\frac{1+x^n}{1-x^n}} + n x^{n-1} \times \frac{1}{(1-x^n)^2} \times \sqrt{\frac{1-x^n}{1+x^n}} \right\} dx$$

$$\Rightarrow I = e^x \sqrt{\frac{1+x^n}{1-x^n}} + C = e^x \sqrt{\frac{1-x^{2n}}{1-x^n}} + C$$

156 (c)

$$(A) \int_0^1 e^x dx = [e^x]_0^1 = e - 1$$

$$(B) \int_0^1 2^x dx = \left[\frac{2^x}{\log_e 2} \right]_0^1 = \frac{1}{\log 2} \cdot (2 - 2^0) = \frac{1}{\log 2}$$

$$(C) \int_0^1 \sqrt{x} dx = \left[\frac{x^{3/2}}{3/2} \right]_0^1 = \frac{2}{3}$$

$$(D) \int_0^1 x dx = \left[\frac{x^2}{2} \right]_0^1 = \frac{1}{2}$$

158 (b)

We have,

$$\int_1^a (a-4x) dx \geq 6-5a$$

$$\Rightarrow [ax - 2x^2]_1^a \geq 6-5a$$

$$\Rightarrow -a^2 - a + 2 \geq 6-5a$$

$$\Rightarrow -a^2 + 4a - 4 \geq 0$$

$$\Rightarrow a^2 - 4a + 4 \leq 0 \Rightarrow (a-2)^2 \leq 0 \Rightarrow a = 2$$

159 (d)

$$\int_0^2 (3x) dx = \int_0^2 (3x - [3x]) dx$$

$$= 3 \cdot \int_0^2 x dx - \int_0^2 [3x] dx$$

$$= 3 \left[\frac{x^2}{2} \right]_0^2 - \frac{1}{3} \int_0^6 [x] dx \text{ (by property)}$$

$$= 6 - \frac{1}{3} \sum_{r=0}^5 \int_r^{r+1} [x] dx$$

$$= 6 - \frac{1}{3} \sum_{r=0}^5 r$$

$$= 6 - \frac{1}{3} (0 + 1 + 2 + \dots + 5)$$

$$= 6 - \frac{1}{3} \cdot 15 = 1$$

160 (b)

We have,

$$I_n = \int_0^{\pi/4} \tan^n x \, dx \Rightarrow I_{n-2} = \int_0^{\pi/4} \tan^{n-2} x \, dx$$

$$\therefore I_2 + I_{n-2} = \int_0^{\pi/4} (\tan^n x + \tan^{n-2} x) dx$$

$$\Rightarrow I_2 + I_{n-2} = \int_0^1 \tan^{n-2} x \, d(\tan x)$$

$$\Rightarrow I_2 + I_{n-2} = \left[\frac{\tan^{n-1} x}{n-1} \right]_0^{\pi/4} = \frac{1}{n-1}$$

161 (a)

$$\text{Let, } I = \int [e^{a \log x} + e^{x \log a}] dx$$

$$= \int [e^{\log_e x^a} + e^{\log_e a^x}] dx$$

$$= \int [x^a + a^x] dx = \frac{x^{a+1}}{a+1} + \frac{a^x}{\log a} + c$$

162 (c)

We have,

$$\begin{aligned}
I &= \left| \int_{10}^{19} \frac{\sin x}{1+x^8} dx \right| \\
\Rightarrow I &\leq \int_{10}^{19} \left| \frac{\sin x}{1+x^8} \right| dx = \int_{10}^{19} \frac{|\sin x|}{1+x^8} dx \\
\Rightarrow I &\leq \int_{10}^{19} \frac{1}{1+x^8} dx \\
\Rightarrow I &< \int_{10}^{19} \frac{1}{x^8} dx = \frac{1}{7}(10^{-7} - 19^{-7}) < \frac{10^{-7}}{7} \\
&< 10^{-7}
\end{aligned}$$

163 (b)

We have,

$$7 \leq 10 + 3 \cos x \leq 13$$

$$\Rightarrow \frac{1}{13} \leq \frac{1}{10 + 3 \cos x} \leq \frac{1}{7} \Rightarrow \frac{2\pi}{13}$$

$$\leq \int_0^{2\pi} \frac{1}{10 + 3 \cos x} dx \leq \frac{2\pi}{7}$$

164 (c)

Let $I = \int_0^{\pi/2} \log \left(\frac{4+3 \sin x}{4+3 \cos x} \right) dx$. Then,

$$\Rightarrow I = \int_0^{\pi/2} \log \left(\frac{4+3 \cos x}{4+3 \sin x} \right) dx$$

$$\Rightarrow I = - \int_0^{\pi/2} \log \left(\frac{4+3 \sin x}{4+3 \cos x} \right) dx \Rightarrow I = -I \Rightarrow 2I = 0 \Rightarrow I = 0$$

165 (c)

We have,

$$I = \int \frac{x \tan^{-1} x}{\sqrt{1+x^2}} dx$$

$$\Rightarrow I = \frac{1}{2} \int \tan^{-1} x \frac{2x}{\sqrt{1+x^2}} dx$$

$$\Rightarrow I = \frac{1}{2} \left[\tan^{-1} x \cdot 2\sqrt{1+x^2} - \int \frac{1}{1+x^2} \cdot 2\sqrt{1+x^2} dx \right]$$

$$\Rightarrow I = \frac{1}{2} \left[2\sqrt{1+x^2} \tan^{-1} x - 2 \int \frac{1}{\sqrt{1+x^2}} dx \right]$$

$$\Rightarrow I = \sqrt{1+x^2} \tan^{-1} x - \log \left(x + \sqrt{1+x^2} \right) + C$$

Hence, $f(x) = \tan^{-1} x$ and $A = 1$

166 (a)

We have,

$$I = \int_{-2}^4 x[x] dx$$

$$\begin{aligned}
\Rightarrow I &= \int_{-2}^{-1} -2x dx + \int_{-1}^0 -x dx + \int_0^1 0 \cdot x dx \\
&\quad + \int_1^2 x dx + \int_2^3 2x dx + \int_3^4 3x dx \\
\Rightarrow I &= -(1-4) + \left(0 + \frac{1}{2}\right) + 0 + \left(2 - \frac{1}{2}\right) \\
&\quad + (9-4) + \frac{3}{2}(16-9) \\
\Rightarrow I &= 3 + \frac{1}{2} + \frac{3}{2} + 5 + \frac{21}{2} = \frac{6+1+3+10+21}{2} \\
&= \frac{41}{2}
\end{aligned}$$

167 (d)

$$\begin{aligned}
&\int_0^{\pi/4} (\cos x - \sin x) dx \\
&\quad + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x) dx \\
&\quad + \int_{2\pi}^{\pi/4} (\cos x - \sin x) dx \\
&= [\sin x + \cos x]_0^{\pi/4} - [\sin x + \cos x]_{\pi/4}^{5\pi/4} \\
&\quad + [\sin x + \cos x]_{2\pi}^{\pi/4} \\
&= \left[\sin \frac{\pi}{4} + \cos \frac{\pi}{4} - (\sin 0 + \cos 0) \right] \\
&\quad - \left[\sin \frac{5\pi}{4} + \cos \frac{5\pi}{4} - \left(\sin \frac{\pi}{4} + \cos \frac{\pi}{4} \right) \right] \\
&\quad + \left[\sin \frac{\pi}{4} + \cos \frac{\pi}{4} - (\sin 2\pi + \cos 2\pi) \right] \\
&= \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 \right] - \left[-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) \right] \\
&\quad + \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 \right] \\
&= [\sqrt{2} - 1] - [-\sqrt{2} - \sqrt{2}] + [\sqrt{2} - 1] \\
&= [\sqrt{2} - 1 + 2\sqrt{2} + \sqrt{2} - 1] = 4\sqrt{2} - 2
\end{aligned}$$

168 (b)

We have,

$$e^{-x} f(x) = 2 + \int_0^x \sqrt{t^4 + 1} dt, x \in (-1, 1)$$

On differentiating w.r.t x , we get

$$e^{-x} (f'(x) - f(x)) = \sqrt{x^4 + 1}$$

$$\Rightarrow f'(x) = f(x) + \sqrt{x^4 + 1} e^x$$

$\therefore f^{-1}$ is the inverse of f

$$\therefore f^{-1}(f(x)) = x$$

$$\Rightarrow f^{-1}'(f(x)) f'(x) = 1$$

$$\Rightarrow f^{-1}'(f(x)) = \frac{1}{f'(x)}$$

$$\Rightarrow f^{-1}'(f(x)) = \frac{1}{f(x) + \sqrt{x^4 + 1} e^x}$$

As $x = 0$, $f(x) = 2$

and $f^{-1}(2) = \frac{1}{2+1} = \frac{1}{3}$

169 (b)

Let $I = \int \frac{f'(x)}{\sqrt{f(x)}} dx$

Put $f(x) = t \Rightarrow f'(x)dx = dt$

$$\therefore I = \int \frac{1}{\sqrt{t}} dt = 2\sqrt{t} + c = 2\sqrt{f(x)} + c$$

170 (a)

$$3a \int_0^1 \left(\frac{ax-1}{a-1}\right)^2 dx = \frac{3a}{(a-1)^2} \left[\frac{(ax-1)^3}{3} \times \frac{1}{a}\right]_0^1$$

$$= \frac{1}{(a-1)^2} [(a-1)^3 + 1]$$

$$= (a-1) + (a-1)^{-2}$$

171 (b)

Let $I = \int_1^e 10^{\log_e x} dx$

Again, let $I_1 = \int 10^{\log_e x} dx$

$$\Rightarrow I_1 = x \cdot 10^{\log_e x} - \int x \cdot 10^{\log_e x} \cdot \frac{\log_e 10}{x} dx$$

$$\Rightarrow I_1 = x \cdot 10^{\log_e x} - \int 10^{\log_e x} \log_e 10 dx$$

$$\Rightarrow (1 + \log_e 10)I_1 = x \cdot 10^{\log_e x}$$

$$\Rightarrow I_1 = \frac{x \cdot 10^{\log_e x}}{1 + \log_e 10}$$

$$\therefore I = \left[\frac{x \cdot 10^{\log_e x}}{1 + \log_e 10} \right]_1^e$$

$$= \left[\frac{10e - 1}{1 + \log_e 10} \right] = \frac{10e - 1}{\log_e 10e}$$

172 (d)

Let $I = \sqrt{2} \int \frac{\sin x}{\sin(x - \frac{\pi}{4})} dx$

Put $x - \frac{\pi}{4} = t \Rightarrow dx = dt$

$$\therefore I = \sqrt{2} \int \frac{\sin(\frac{\pi}{4} + t)}{\sin t} dt$$

$$= \sqrt{2} \int \frac{\sin \frac{\pi}{4} \cos t + \cos \frac{\pi}{4} \sin t}{\sin t} dt$$

$$= \sqrt{2} \int \left(\frac{1}{\sqrt{2}} \frac{\cos t}{\sin t} + \frac{1}{\sqrt{2}} \right) dt$$

$$= \int (\cot t + 1) dt$$

$$= \log |\sin t| + t + c_1$$

$$= x + \log \left| \sin \left(x - \frac{\pi}{4} \right) \right| + c, \left(\because c_1 - \frac{\pi}{4} = c \right)$$

173 (b)

We have,

$$I = \int \frac{1}{x(x^4 - 1)} dx = \int \frac{x^3}{x^4(x^4 - 1)} dx$$

$$\Rightarrow I = \frac{1}{4} \int \frac{1}{x^4(x^4 - 1)} d(x^4)$$

$$= \frac{1}{4} \int \left(\frac{1}{x^4 - 1} - \frac{1}{x^4} \right) d(x^4)$$

$$\Rightarrow I = \frac{1}{4} \{ \log_e(x^4 - 1) - \log_e x^4 \} + C$$

$$= \frac{1}{4} \log_e \left(\frac{x^4 - 1}{x^4} \right) + C$$

174 (a)

$$g(x + \pi) = \int_0^{\pi+x} \cos^4 t dt$$

$$= \int_0^{\pi} \cos^4 t dt + \int_{\pi}^{\pi+x} \cos^4 t dt$$

$$I_1 = I_2$$

where, $I_1 = g(\pi)$ and $I_2 = \int_{\pi}^{\pi+x} \cos^4 t dt$

$$\Rightarrow I_2 = \int_0^x \cos^4(y + \pi) dy$$

$$= \int_0^x \cos^4 y dy = g(x)$$

$$\therefore g(\pi + x) = g(x) + g(\pi)$$

175 (b)

Let $I = \int_0^{\pi/2} f(\sin 2x) dx$. Then, ... (i)

$$I = \int_0^{\pi/2} f(\sin 2x) \cos x dx, \left[\because \int_0^a f(x) dx = \int_a^0 f(a-x) dx \right] \dots (ii)$$

$$\therefore 2I = \int_0^{\pi/2} f(\sin 2x)(\sin x + \cos x) dx$$

$$\Rightarrow 2I = \sqrt{2} \int_0^{\pi/2} f(\sin 2x) \cos \left(x - \frac{\pi}{4} \right) dx$$

$$\Rightarrow 2I = \sqrt{2} \int_{-\pi/4}^{\pi/4} f \left\{ \sin \left(\frac{\pi}{2} + 2t \right) \right\} \cos t dt, \text{ where } x - \frac{\pi}{4} = t,$$

$$\Rightarrow 2I = \sqrt{2} \int_{-\pi/4}^{\pi/4} f(\cos 2t) \cos t dt$$

$$\Rightarrow 2I = \sqrt{2} \int_0^{\pi/4} f(\cos 2t) \cos t dt \quad \left[\because \text{Integrand is an even function} \right]$$

$$\Rightarrow I = \sqrt{2} \int_0^{\pi/4} f(\cos 2x) \cos x dx$$

176 (a)

Given, $f(x) = \int_1^x \sqrt{2-t^2} dt$

$\Rightarrow f'(x) = \sqrt{2-x^2}$

Now, $x^2 - f'(x) = 0$

$\Rightarrow x^2 - \sqrt{2-x^2} = 0$

$\Rightarrow x^4 + x^2 - 2 = 0$

$\Rightarrow (x^2 + 2)(x^2 - 1) = 0$

$\Rightarrow x^2 - 1 = 0 \Rightarrow x = \pm 1$

177 (a)

Let $I = \int_0^{\pi/2} \frac{\sqrt{\sin x} dx}{\sqrt{\cos x + \sqrt{\sin x}}} \dots$ (i)

$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x + \sqrt{\cos x}}} dx \dots$ (ii)

[put $x = \frac{\pi}{2} - x$]

On adding Eqs. (i) and (ii), we get

$2I = \int_0^{\pi/2} dx \Rightarrow I = \frac{\pi}{4}$

178 (a)

$\int_0^3 \frac{3x+1}{x^2+9} dx = \frac{3}{2} \int_0^3 \frac{2x}{x^2+9} dx + \int_0^3 \frac{dx}{x^2+9}$

$= \left[\frac{3}{2} \log(x^2+9) + \frac{1}{3} \tan^{-1} \left(\frac{x}{3} \right) \right]_0^3$

$= \frac{3}{2} [(\log 3^2 + 9)$

$-\log(0+9)$

$+ \frac{1}{3} [\tan^{-1} \left(\frac{3}{3} \right) - \tan^{-1} 0]$

$= \frac{3}{2} (\log 18 - \log 9) + \frac{1}{3} \left(\frac{\pi}{4} \right)$

$= \frac{3}{2} \log 2 + \frac{\pi}{12} = \log(2\sqrt{2}) + \frac{\pi}{12}$

179 (a)

$\phi(x) = \cos x - \int_0^x (x-t)\phi(t) dt$

$= \cos x - x \int_0^x \phi(t) dt + \int_0^x t \phi(t) dt$

$\Rightarrow \phi'(x) = -\sin x - \int_0^x \phi(t) dt - x\phi(x) + x\phi(x)$

$\Rightarrow \phi''(x) = -\cos x - \phi(x)$

$\Rightarrow \phi(x) + \phi''(x) = -\cos x$

180 (c)

Putting $\lambda x = t, \lambda dx = dt$, we get

$\int_0^\infty e^{-\lambda x} x^{n-1} dx = \frac{1}{\lambda^n} \int_0^\infty e^{-t} t^{n-1} dt$

$= \frac{1}{\lambda^n} \int_0^\infty e^{-x} x^{n-1} dx = \frac{I_n}{\lambda^n}$

181 (c)

Let $I = \int x^{49} \frac{\tan^{-1}(x^{50})}{(1+x^{50})^2} dx$

Put $x^{50} = t \Rightarrow 50x^{49} dx = dt$

$\therefore I = \frac{1}{50} \int \frac{\tan^{-1} t}{1+t^2} dt$

Again, put $\tan^{-1} t = u \Rightarrow \frac{1}{1+t^2} dt = du$

$\therefore I = \frac{1}{50} \int u du = \frac{u^2}{100} + c$

$= \frac{(\tan^{-1} x^{50})^2}{100} + c$

But $I = k(\tan^{-1} x^{50})^2 + c$ [given]

$\therefore k = \frac{1}{100}$

182 (b)

We have,

$I = \int_{-\pi}^{\pi} \frac{\sin^2 x}{1+a^x} dx$

$\Rightarrow I = \int_0^{\pi} \left\{ \frac{\sin^2 x}{1+a^x} + \frac{\sin^2(-x)}{1+a^{-x}} \right\} dx$

{Using : $\int_{-a}^a f(x) dx = \int_0^a \{f(x) + f(-x)\} dx$ }

$\Rightarrow I = \int_0^{\pi/2} \sin^2 x dx$

$\Rightarrow I = 2 \int_0^{\pi/2} \sin^2 x dx$

$\left[\because \int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx, \text{ if } f(2a-x) = f(x) \right]$

$\Rightarrow I = 2 \times \frac{\pi}{4} = \frac{\pi}{2}$

183 (c)

Let $I = \int \frac{x \tan^{-1} x}{(1+x^2)^{3/2}} dx$

Put $x = \tan \theta \Rightarrow dx = \sec^2 \theta d\theta$

$\therefore I = \int \frac{\tan \theta \tan^{-1}(\tan \theta)}{(1+\tan^2 \theta)^{3/2}} \sec^2 \theta d\theta$

$\Rightarrow I = \int \frac{\theta \tan \theta \sec^2 \theta}{\sec^3 \theta} d\theta = \int \theta \sin \theta d\theta$

$\Rightarrow I = \theta(-\cos \theta) + \int \cos \theta d\theta$

$= -\theta \cos \theta + \sin \theta + c$

$\Rightarrow I = -\tan^{-1} x \cdot \frac{1}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} + c$

$\Rightarrow I = \frac{(x - \tan^{-1} x)}{\sqrt{1+x^2}} + c$

184 (c)

Let $f(x) = ax^2 + bx + c$. Then,

$f(0) = c, f(1) = a + b + c$ and $f\left(\frac{1}{2}\right) = \frac{a}{4} + \frac{b}{2} + c$

$$\begin{aligned} &\therefore 6 \int_0^1 f(x) dx - \left\{ f(0) + 4f\left(\frac{1}{2}\right) \right\} \\ &= 6 \left[\frac{ax^3}{3} + \frac{bx^2}{2} + cx \right]_0^1 - \{c + a + 2b + 4c\} \\ &= 2a + 3b + 6c - (a + 2b + 5c) = a + b + c \\ &= f(1) \end{aligned}$$

$$\therefore k = 1$$

185 (b)

We have,

$$I = \int_0^a f(x) g(x) dx$$

$$\Rightarrow I = \int_0^a f(a-x) g(a-x) dx$$

$$\Rightarrow I$$

$$= \int_0^a f(x) \{2 - g(x)\} dx \quad \left[\begin{array}{l} \because f(a-x) = f(x), \\ g(a-x) = 2 - g(x) \end{array} \right]$$

$$\Rightarrow I = 2 \int_0^a f(x) dx - \int_0^a f(x) g(x) dx$$

$$\Rightarrow I = 2 \int_0^a f(x) dx - I$$

$$\therefore 2I = 2 \int_0^a f(x) dx \Rightarrow I = \int_0^a f(x) dx$$

187 (d)

$$\text{Let } I = \int_0^{\pi/8} \cos^3 4\theta d\theta$$

$$= \int_0^{\pi/8} \cos^2 4\theta \cdot \cos 4\theta d\theta$$

$$= \int_0^{\pi/8} \left(\frac{1 + \cos 8\theta}{2} \right) \cos 4\theta d\theta$$

$$= \frac{1}{2} \int_0^{\pi/8} \cos 4\theta d\theta + \frac{1}{2} \int_0^{\pi/8} \cos 8\theta \cos 4\theta d\theta$$

$$= \frac{1}{2} \left[\frac{\sin 4\theta}{4} \right]_0^{\pi/8} + I_1$$

$$I = \frac{1}{8} + I_1$$

$$\text{Where, } I_1 = \frac{1}{2} \int_0^{\pi/8} \cos 8\theta \cos 4\theta d\theta$$

$$= \frac{1}{4} \int_0^{\pi/8} 2 \cos 8\theta \cos 4\theta d\theta$$

$$= \frac{1}{4} \left[\int_0^{\pi/8} \cos 12\theta d\theta + \int_0^{\pi/8} \cos 4\theta d\theta \right]$$

$$= \frac{1}{4} \left[\frac{\sin 12\theta}{12} + \frac{\sin 4\theta}{4} \right]_0^{\pi/8}$$

$$= \frac{1}{4} \left[-\frac{1}{21} + \frac{1}{4} \right] = \frac{1}{4} \left[\frac{2}{12} \right] 0$$

$$I_1 = \frac{1}{24} \quad \dots \text{(iii)}$$

From Eqs.(i) and (ii), we get

$$I = \frac{1}{8} + \frac{1}{24} - \frac{3+1}{24} = \frac{1}{6}$$

188 (d)

$$\int_{[x]}^{[x]+1} f(t) dt = [x]$$

$$\int_{-2}^{-2+1} f(x) dx =$$

$$-2, \int_{-1}^0 f(x) dx = -1, \dots, \int_3^4 f(x) dx = 3$$

$$\therefore \int_{-2}^4 f(x) dx = -2 - 1 + 1 + 2 + 3 = 3$$

189 (d)

Case I If $x > 0$, then $|x| = x$

$$\therefore \int |x| \log|x| dx = \int x \log x dx$$

$$= \log x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2}{2} \cdot \log x - \frac{x^2}{4} + c$$

$$= -\frac{x^2}{2} \log|x| - \frac{x^2}{4} + c$$

Case II If $x < 0$, then $|x| = -x$

$$\therefore \int |x| \log|x| dx = - \int x \log(-x) dx$$

$$= - \left\{ \log(-x) \cdot \frac{x^2}{2} - \frac{x^2}{4} \right\} + c$$

$$= -\frac{x^2}{2} \log|x| + \frac{x^2}{2} + c$$

On combining both cases, we get

$$\int |x| \log|x| dx = \frac{1}{2} x|x| \log|x| - \frac{1}{4} x|x| + c$$

190 (a)

We have,

$$\begin{aligned} f \circ f(x) &= f(f(x)) = \frac{f(x)}{[1 + \{f(x)\}^n]^{1/n}} \\ &= \frac{x}{(1 + 2x^n)^{1/n}} \end{aligned}$$

Similarly,

$$\begin{aligned} f \circ f \circ f(x) &= \frac{x}{(1 + 3x^n)^{1/n}}, (f \circ f \circ f \circ \dots \circ f)(x) \\ &= \frac{x}{(1 + nx^n)^{1/n}} \end{aligned}$$

n - times

$$\therefore I = \int x^{n-2} g(x) = \int \frac{x^{n-1}}{(1+nx^n)^{1/n}} dx$$

$$\Rightarrow I = \frac{1}{n^2} \int (1+nx^n)^{-1/n} d(1+nx^n)$$

$$\Rightarrow I = \frac{1}{n^2} \frac{(1+nx^n)^{-\frac{1}{n}+1}}{\left(-\frac{1}{n}+1\right)} + k$$

$$= \frac{1}{n(n-1)} (1+nx^n)^{1-\frac{1}{n}} + k$$

191 (d)

Let $I = \int_0^{2\pi} \frac{\sin 2\theta}{a-b \cos \theta} d\theta$. Then,

$$I = \int_0^{2\pi} \frac{\sin 2(2\pi - \theta)}{a-b \cos(2\pi - \theta)} d\theta,$$

$$\Rightarrow I = \int_0^{2\pi} \frac{-\sin 2\theta}{a-b \cos \theta} d\theta \Rightarrow I = -I \Rightarrow 2I = 0 \Rightarrow I = 0$$

192 (d)

Given, $f(-x) = -f(x)$, \forall values of real x

We know that,

$$\int_{-a}^a f(x) dx = 0 = \int_{-a}^0 f(x) dx + \int_0^a f(x) dx \quad [$$

$$\therefore f(-x) = -f(x)]$$

$$\Rightarrow \int_{-1}^0 f(x) dx + \int_0^1 f(x) dx = 0$$

$$\Rightarrow \int_{-1}^0 f(x) dx = -5 \quad [\therefore \int_0^1 f(x) dx = 5]$$

$$\Rightarrow \int_{-1}^0 f(t) dt = -5$$

193 (a)

Let $I = \int_0^a f(x)g(x) dx$

$$= \int_0^a f(a-x)g(a-x) dx$$

$$\Rightarrow I = \int_0^a f(x)[2-g(x)] dx$$

$$[\therefore f(x) = f(a-x), g(x) + g(a-x) = 2]$$

$$= \int_0^a 2f(x) dx - \int_0^a f(x)g(x) dx$$

$$\Rightarrow I = \int_0^a 2f(x) dx - I$$

$$\Rightarrow I = \int_0^a f(x) dx$$

194 (a)

Let $f(x) = \log\left(\frac{a+x}{a-x}\right)$

$$\Rightarrow f(-x) = \log\left(\frac{a-x}{a+x}\right)$$

$$= -\log\left(\frac{a+x}{a-x}\right)$$

$$= -f(x)$$

$\Rightarrow f(x)$ is an odd function.

$$\therefore \int_{-10}^{10} f(x) dx = 0$$

195 (c)

Let $I = \int_0^\pi |\sin^3 \theta| d\theta$

Since, $\sin \theta$ is positive in interval $(0, \pi)$

$$\therefore I = \int_0^\pi \sin^3 \theta d\theta = \int_0^\pi \sin \theta \cdot \sin^2 \theta d\theta$$

$$= \int_0^\pi \sin \theta (1 - \cos^2 \theta) d\theta$$

$$= \int_0^\pi \sin \theta d\theta + \int_0^\pi (-\sin \theta) \cos^2 \theta d\theta$$

$$= [-\cos \theta]_0^\pi + \left[\frac{\cos^3 \theta}{3}\right]_0^\pi$$

$$= -[\cos \pi - \cos 0] + \frac{1}{3}[\cos^3 \pi - \cos^3 0]$$

$$= -[-1 - 1] + \frac{1}{3}[-1 - 1] = 2 - \frac{2}{3} = \frac{4}{3}$$

196 (b)

Let $I = \int_0^{\pi/6} \frac{\sin x}{\cos^3 x} dx$

Put $\cos x = t \Rightarrow -\sin x dx = dt$

$$\therefore I = - \int_1^{\sqrt{3}/2} \frac{1}{t^3} = \left[\frac{1}{2t^2}\right]_1^{\sqrt{3}/2} = \frac{1}{6}$$

197 (a)

Let $I = \int_0^2 \sqrt{\frac{2+x}{2-x}} dx = \int_0^2 \frac{2+x}{\sqrt{4-x^2}} dx$

$$= \int_0^2 \frac{2}{\sqrt{4-x^2}} dx + \int_0^2 \frac{x}{\sqrt{4-x^2}} dx$$

Put $x = 2\sin \theta$ in the first integral and

$4-x^2 = t$ in the second integral

$$I = 2 \int_0^{\pi/2} \frac{2 \cos \theta d\theta}{2 \cos \theta} + \int_4^0 -\frac{dt}{2\sqrt{t}}$$

$$= 2 \int_0^{\pi/2} d\theta + \frac{1}{2} \int_0^4 t^{-1/2} dt$$

$$= 2[\theta]_0^{\pi/2} + \frac{1}{2} \left[\frac{t^{1/2}}{1/2}\right]_0^4$$

$$= \pi + 2$$

198 (a)

Given, $\int f(x) dx = f(x) + c$

$$\Rightarrow f(x) = f'(x)$$

$$\therefore \int (f(x)^2) dx = \int f(x) \cdot f'(x) dx$$

$$= \frac{1}{2} (f(x))^2 + c$$

199 (b)

$$\begin{aligned} \text{Let } I &= \int \sqrt{\frac{x-1}{x+1}} dx = \int \frac{x-1}{\sqrt{x^2-1}} dx \\ &= \int \frac{x}{\sqrt{x^2-1}} dx - \int \frac{1}{\sqrt{x^2-1}} dx \\ &= \sqrt{x^2-1} - \sin^{-1} x + c \end{aligned}$$

200 (b)

$$\begin{aligned} \text{Let } I &= \int \sin x d(\cos x) \\ \text{Put } \cos x &= t \Rightarrow d(\cos x) = dt \\ \therefore I &= \int \sqrt{1-t^2} dt \\ &= \frac{t}{2} \sqrt{1-t^2} + \frac{1}{2} \sin^{-1}(t) + c \\ &= \frac{\cos x}{2} \cdot \sin x + \frac{1}{2} \sin^{-1}(\cos x) + c \\ &= \frac{1}{4} \sin 2x + \frac{1}{2} \sin^{-1} \left[\sin \left(\frac{\pi}{2} - x \right) \right] + c \\ &= \frac{1}{4} \sin 2x - \frac{x}{2} + c \end{aligned}$$

201 (b)

$$\begin{aligned} \text{Let } I &= \int_{-\pi/2}^{\pi/2} \cos x dx \\ \text{Since, } \cos(-x) &= \cos x \\ \therefore I &= 2 \int_0^{\pi/2} \cos x dx = 2[\sin x]_0^{\pi/2} = 2 \end{aligned}$$

202 (a)

$$\begin{aligned} \text{Given, } f(x) &= f(a+x) \text{ and } \int_0^a f(x) dx = k \\ \int_a^{na} f(x) dx &= n \int_0^a f(x) = nk \\ [\because f(x) &\text{ is periodic function}] \end{aligned}$$

203 (c)

$$\begin{aligned} \text{Let } I &= \int_0^{\pi} \frac{1}{5+3 \cos x} dx \quad \dots(i) \\ \text{Then,} \\ I &= \int_0^{\pi} \frac{1}{5-3 \cos x} dx \quad \dots(ii) \quad [\because \int_0^a f(x) dx = \int_a^0 f(a-x) dx] \\ \text{Adding (i) and (ii), we get} \\ 2I &= 10 \int_0^{\pi} \frac{1}{25-9 \cos^2 x} dx \\ &\Rightarrow I = 5 \times 2 \int_0^{\pi/2} \frac{1}{25-9 \cos^2 x} dx \\ &\Rightarrow I = 10 \int_0^{\pi/2} \frac{\sec^2 x}{25+16 \tan^2 x} dx \\ &\Rightarrow I = 10 \times \frac{1}{20} \left[\tan^{-1} \left(\frac{4 \tan x}{5} \right) \right]_0^{\pi/2} = \frac{\pi}{4} \end{aligned}$$

204 (d)

We have,

$$I_1 = \int_0^x e^{zx} e^{-z^2} dz \text{ and } I_2 = \int_0^x e^{-\frac{z^2}{4}} dz$$

Putting $z = \frac{x+t}{2}$ and $dt = 2 dz$ in I_1 , we get

$$\begin{aligned} I_1 &= \frac{1}{2} \int_{-x}^x e^{\left(\frac{x+t}{2}\right)x} \cdot e^{-\left(\frac{x+t}{2}\right)^2} dt \\ &\Rightarrow I_1 = \frac{1}{2} \int_{-x}^x e^{\frac{1}{2}\{x^2+tx-\frac{1}{2}(x+t)^2\}} dt \\ &\Rightarrow I_1 = \frac{1}{2} \int_{-x}^x e^{\frac{1}{2}\left\{\frac{1}{2}x^2-\frac{1}{2}t^2\right\}} dt \\ &\Rightarrow I_1 = \frac{1}{2} \int_{-x}^x e^{\frac{1}{4}x^2} e^{-\frac{1}{4}t^2} dt \\ &\Rightarrow I_1 = \frac{1}{2} e^{\frac{1}{4}x^2} \int_{-x}^x e^{-\frac{1}{4}t^2} dt \\ &\Rightarrow I_1 = \frac{1}{2} e^{\frac{1}{4}x^2} \times 2 \int_0^x e^{-\frac{1}{4}t^2} dt \quad \left[\because e^{-\frac{1}{4}t^2} \text{ is an even function} \right] \\ &\Rightarrow I_1 = e^{\frac{1}{4}x^2} \int_0^x e^{-\frac{1}{4}t^2} dt = e^{\frac{1}{4}x^2} I_2 \end{aligned}$$

206 (b)

$$\begin{aligned} \int_0^{\pi} f(x) \sin x dx + \int_0^{\pi} f''(x) \sin x dx &= 5 \\ \Rightarrow [f(x)(-\cos x)]_0^{\pi} &+ \int_0^{\pi} f'(x) \cos x dx \\ &+ \int_0^{\pi} f''(x) \sin x dx = 5 \\ \Rightarrow [-f(x) \cos x]_0^{\pi} + [f'(x) \sin x]_0^{\pi} &- \int_0^{\pi} f''(x) \sin x dx \\ &+ \int_0^{\pi} f''(x) \sin x dx = 5 \\ \Rightarrow f(\pi) + f(0) = 5 \Rightarrow f(0) = 5 - f(\pi) = 3 \end{aligned}$$

207 (c)

$$\begin{aligned} \int \frac{x+2}{2x^2+6x+5} dx &= \frac{1}{4} \int \frac{4x+6+2}{2x^2+6x+5} dx \\ &= \frac{1}{4} \int \frac{4x+6}{2x^2+6x+5} dx + \frac{1}{2} \int \frac{dx}{2x^2+6x+5} \\ &\Rightarrow P = \frac{1}{4} \end{aligned}$$

208 (d)

$$\int (1+x-x^{-1})e^{x+x^{-1}} dx$$

$$\int \left[x e^{x+x^{-1}} \left(1 - \frac{1}{x^2} \right) + e^{x+x^{-1}} \right] dx$$

$$= \frac{x^{x+x^{-1}}}{e} - \int e^{x+x^{-1}} dx + \int e^{x+x^{-1}} dx$$

$$= x e^{x+x^{-1}} + c$$

209 (c)

$$\because \cos^2(\pi + x) = \cos^2 x$$

$$\Rightarrow I_1 = \int_0^{3\pi} f(\cos^2 x) dx = 3 \int_0^{\pi} f(\cos^2 x) dx$$

$$= 3I_2$$

210 (c)

$$\text{Let } I = \int_0^{\pi/2} (x\sqrt{\tan x} + \sqrt{\cot x}) dx \quad \dots(i)$$

$$\text{Then, } I = \int_0^{\pi/2} \left(\frac{\pi}{2} - x \right) (\sqrt{\cot x} + \sqrt{\tan x}) dx$$

$$\Rightarrow I = \frac{\pi}{2} \int_0^{\pi/2} (\sqrt{\cot x} + \sqrt{\tan x}) dx - I$$

$$\Rightarrow 2I = \frac{\pi}{2} \int_0^{\pi/2} \frac{\cos x + \sin x}{\sqrt{(\sin x - \cos x)}} dx$$

$$\Rightarrow 2I = \frac{\pi}{\sqrt{2}} \int_0^{\pi/2} \frac{\cos x + \sin x}{\sqrt{1 - (\sin x - \cos x)^2}} dx$$

$$\Rightarrow 2I = \frac{\pi}{\sqrt{2}} \sin^{-1}[(\sin x - \cos x)]_0^{\pi/2}$$

$$\Rightarrow 2I = \frac{\pi}{\sqrt{2}} \{ \sin^{-1}(1) - \sin^{-1}(-1) \}$$

$$\Rightarrow 2I = \frac{\pi}{\sqrt{2}} \times \pi \Rightarrow I = \frac{\pi^2}{2\sqrt{2}}$$

211 (c)

$$\text{Let } I = \int_0^{\pi} \frac{x dx}{a^2 \cos^2 x + b^2 \sin^2 x} \dots (i)$$

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{a^2 \cos^2 x + b^2 \sin^2 x} dx \dots (ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = 2\pi \int_0^{\pi/2} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

$$\Rightarrow 2I = 2\pi \int_0^{\pi/2} \frac{\sec^2 x}{a^2 + b^2 \tan^2 x} dx$$

Now, put $\tan x = t \Rightarrow dx = \frac{dt}{\sec^2 x}$

$$\therefore 2I = 2\pi \int_0^{\infty} \frac{dt}{a^2 + b^2 t^2}$$

$$= \frac{1}{b^2} \times 2\pi \int_0^{\infty} \frac{dt}{\frac{a^2}{b^2} + t^2}$$

$$= \left[\frac{2\pi}{b^2} \cdot \frac{b}{a} \tan^{-1} \frac{bt}{a} \right]_0^{\infty}$$

$$= \frac{2\pi}{ab} [\tan^{-1} \infty - \tan^{-1} 0] = \frac{2\pi}{ab} \times \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi^2}{2ab}$$

212 (b)

We have,

$$f(x) = ae^{2x} + be^x + cx \Rightarrow f'(x) = 2ae^{2x} + be^x + c$$

Now,

$$f(0) = -1 \Rightarrow a + b = -1 \quad \dots(i)$$

$$f'(\log 2) = 31 \Rightarrow 8a + 2b + c = 31 \quad \dots(ii)$$

$$\text{and } \int_0^{\log 4} (f(x) - cx) dx = \frac{39}{2}$$

$$\int_0^{\log 4} (ae^{2x} + be^x) dx = \frac{39}{2}$$

$$\Rightarrow \left[\frac{1}{2} ae^{2x} + be^x \right]_0^{\log 4} = \frac{39}{2} \Rightarrow \frac{15}{2} a + 3b = \frac{39}{2}$$

...(iii)

Solving (i), (ii) and (iii), we get $a = 5, b = -6, c = 3$

213 (b)

Putting $x = \tan \theta$ and $dx = \sec^2 \theta d\theta$, we get

$$\int_0^1 \frac{1}{(x^2 + 1)^{3/2}} dx = \int_0^{\pi/4} \frac{\sec^2 \theta d\theta}{\sec^3 \theta} = \int_0^{\pi/4} \cos \theta d\theta$$

$$= \frac{1}{\sqrt{2}}$$

214 (d)

Let us assume $f(x) = \frac{e^{-x^2}}{2}$

$$\therefore \int x \cdot \frac{e^{-x^2}}{2} dx = \int 2x \cdot \frac{e^{-x^2}}{2} dx$$

Put $x^2 = t \Rightarrow 2x dx = dt$

$$\therefore \int \frac{e^t}{4} dt = \frac{e^t}{4} + c = \frac{1}{2} \left(\frac{e^{-x^2}}{2} \right) + c$$

Hence, $\int x f(x) dx = \frac{1}{2} f(x) + c$, when $f(x) = \frac{e^{-x^2}}{2}$

215 (b)

$$\text{Let } I = \int_0^{\pi} \frac{1}{1 + \sin x} dx$$

$$= \int_0^{\pi} \frac{1}{1 + \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}} dx$$

$$= \int_0^{\pi} \frac{\sec^2 \frac{x}{2}}{\left(1 + \tan \frac{x}{2} \right)^2} dx$$

Put $\tan \frac{x}{2} = t \Rightarrow \frac{1}{2} \sec^2 \frac{x}{2} dx = dt$

$$\therefore I = \int_0^{\infty} \frac{2dt}{(1+t)^2} = \left[-\frac{2}{1+t} \right]_0^{\infty} = 2$$

216 (a)

$$\int_{-1}^1 f(x) dx = \int_{-1}^1 (x - [x]) dx$$

$$\begin{aligned}
&= \int_{-1}^1 x \, dx - \int_{-1}^1 [x] \, dx \\
&= 0 - \int_{-1}^1 [x] \, dx \quad (\because x \text{ is an odd function}) \\
\text{But } [x] &= \begin{cases} -1, & \text{if } -1 \leq x < 0 \\ 0, & \text{if } 0 \leq x < 1 \\ 1, & \text{if } x = 1 \end{cases} \\
\therefore \int_{-1}^1 [x] \, dx &= \int_{-1}^0 [x] \, dx + \int_0^1 [x] \, dx \\
&= \int_{-1}^0 [-1] \, dx + \int_0^1 0 \, dx = -[x]_{-1}^0 + 0 = 1 \\
\text{Thus, } \int_{-1}^1 f(x) \, dx &= 1
\end{aligned}$$

217 (a)

$$\begin{aligned}
\text{Let } I &= \int \frac{dx}{x^2(x^4+1)^{3/4}} = \int \frac{dx}{x^2 \cdot x^3(1+\frac{1}{x^4})^{3/4}} \\
\text{Put } 1+x^{-4} &= t \Rightarrow \frac{-4}{x^5} dx = dt \\
\therefore I &= -\frac{1}{4} \int \frac{dt}{t^{3/4}} = -\frac{1}{4} \cdot \frac{t^{1/4}}{1/4} + c \\
&= -\left(1 + \frac{1}{x^4}\right)^{1/4} + c \\
&= \frac{-(x^4+1)^{1/4}}{x} + c
\end{aligned}$$

218 (b)

$$\begin{aligned}
\text{We have,} \\
I_1 &= \int_{1-k}^k x \sin\{x(1-x)\} \, dx \text{ and} \\
I_2 &= \int_{1-k}^k \sin\{x(1-x)\} \, dx \\
\Rightarrow I_1 &= \int_{1-k}^k (1-x) \sin\{x(1-x)\} \, dx \\
&\quad \int_{1-k}^k x \sin\{x(1-x)\} \, dx \\
\Rightarrow I_1 &= \int_{1-k}^k x \sin\{x(1-x)\} \, dx \\
&\quad - \int_{1-k}^k x \sin\{x(1-x)\} \, dx \\
\Rightarrow I_1 &= I_2 - I_1 \Rightarrow 2I_1 = I_2
\end{aligned}$$

219 (b)

$$\begin{aligned}
\text{Given that, } \int \frac{1}{f(x)} dx &= \log\{f(x)\}^2 + c \\
\Rightarrow \frac{1}{f(x)} &= \frac{1}{\{f(x)\}^2} 2f(x)f'(x) \\
\Rightarrow f'(x) &= \frac{1}{2} \\
\Rightarrow f(x) &= \frac{x}{2} + d
\end{aligned}$$

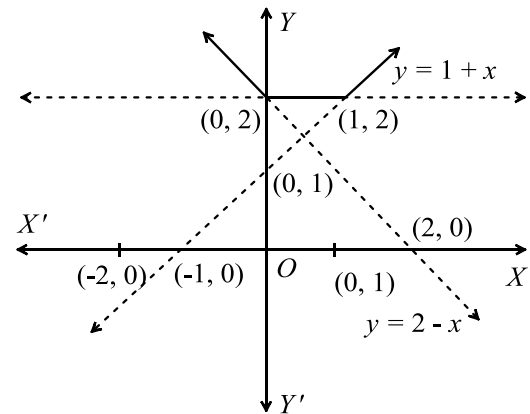
(where d is a integration constant)

220 (b)

$$\begin{aligned}
\int \frac{x^4 + x^2 + 1}{x^2 - x + 1} dx &= \int (x^2 + x + 1) dx \\
&= \frac{x^3}{3} + \frac{x^2}{2} + x + c
\end{aligned}$$

221 (b)

Let $f(x) = \max.\{2-x, 2, 1+x\}$
The graph of $f(x)$ is shown by continuous lines in Fig.



Clearly,

$$\begin{aligned}
f(x) &= \begin{cases} 2-x, & x \leq 0 \\ 2, & 0 \leq x \leq 1 \\ 1+x, & x \geq 1 \end{cases} \\
\therefore \int_{-1}^1 f(x) \, dx &= \int_{-1}^0 (2-x) \, dx + \int_0^1 2 \, dx \\
\Rightarrow \int_{-1}^1 f(x) \, dx &= \left[2x - \frac{x^2}{2}\right]_{-1}^0 + [2x]_0^1 = \frac{9}{2}
\end{aligned}$$

222 (d)

$$\begin{aligned}
\int_0^1 \frac{x^2}{1+x^2} dx &= \int_0^1 \frac{1+x^2}{1+x^2} dx - \int_0^1 \frac{1}{1+x^2} dx \\
&= \int_0^1 1 \, dx - \int_0^1 \frac{1}{1+x^2} dx \\
&= [x]_0^1 - [\tan^{-1} x]_0^1 = 1 - \frac{\pi}{4}
\end{aligned}$$

223 (a)

$$\begin{aligned}
\text{Let } I &= \int \frac{e^x}{(2+e^x)(e^x+1)} dx \\
\text{Put } e^x &= t \Rightarrow e^x dx = dt \\
\therefore I &= \int \frac{1}{(2+t)(1+t)} dt \\
&= \int \left[\left(\frac{1}{1+t}\right) - \left(\frac{1}{2+t}\right) \right] dt \\
&= \log(1+t) - \log(2+t) + c \\
&= \log\left(\frac{1+e^x}{2+e^x}\right) + c
\end{aligned}$$

224 (c)

We have,

$$\phi(x) \begin{cases} \int_0^x (1-t) dt, & 0 \leq x < 1 \\ \int_0^1 (1-t) dt + \int_1^x 0 dt, & 1 \leq x < 2 \\ \int_0^1 (1-t) dt + \int_1^2 0 dt + \int_2^x (2-t)^2 dt, & 2 \leq x \end{cases}$$

$$\Rightarrow \phi(x) \begin{cases} \left(x - \frac{x^2}{2}\right), & 0 \leq x \leq 1 \\ \frac{1}{2}, & 1 \leq x \leq 2 \\ \frac{1}{2} + \frac{(x-2)^3}{3}, & 2 \leq x \leq 3 \end{cases}$$

226 (a)

We know that $\int_a^{a+T} f(x) dx$ is independent of a , if $f(x)$ is a periodic function with period T .
Since $\sin^4 x + \cos^4 x$ is a periodic function with period $\pi/2$

$\therefore \int_a^{a+\pi/2} (\sin^4 x + \cos^4 x) dx$ is independent of a

227 (a)

$$\text{Let } I = \int_{-1/2}^{1/2} \cos x \log \left(\frac{1-x}{1+x} \right) dx \quad \dots(i)$$

On replacing x by $-x$, we get

$$I = \int_{-1/2}^{1/2} \cos(-x) \log \left(\frac{1+x}{1-x} \right) dx$$

$$\Rightarrow I = - \int_{-1/2}^{1/2} \cos x \log \left(\frac{1-x}{1+x} \right) dx \quad \dots(ii)$$

On adding Eqs.(i) and (ii), we get

$$2I = \int_{-1/2}^{1/2} \cos x \log \left(\frac{1-x}{1+x} \right) dx - \int_{-1/2}^{1/2} \cos x \log \left(\frac{1-x}{1+x} \right) dx$$

$$\Rightarrow 2I = 0$$

$$\Rightarrow I = 0$$

228 (c)

$$\int \frac{x^2 - 1}{x^4 + x^2 + 1} dx$$

$$\Rightarrow I = \int \frac{1 - \frac{1}{x^2}}{x^2 + \frac{1}{x^2} + 1} dx$$

$$= \int \frac{1 - \frac{1}{x^2}}{\left(x - \frac{1}{x}\right)^2 - 1} dx$$

$$\text{Put } x + \frac{1}{x} = t \Rightarrow \left(1 - \frac{1}{x^2}\right) dx = dt$$

$$\therefore I = \int \frac{dt}{t^2 - 1^2} = \frac{1}{2 \times 1} \log \frac{t-1}{t+1} + c$$

$$= \frac{1}{2} \log \frac{x^2 + 1 - x}{x^2 + 1 + x} + c$$

229 (d)

$$\text{Let } I = \int \frac{\sqrt{x}}{x+1} dx = \int \frac{\sqrt{x} dx}{(\sqrt{x})^2 + 1}$$

$$\text{Put } \sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt \Rightarrow dx = 2\sqrt{x} dt$$

$$\therefore I = \int \frac{2t^2 dt}{t^2 + 1} = 2 \left[\int \frac{t^2 + 1}{t^2 + 1} dt - \int \frac{1}{t^2 + 1} dt \right]$$

$$= 2t - 2 \tan^{-1}(t^2 + 1) + c$$

$$= 2\sqrt{x} - 2 \tan^{-1}(x + 1) + c$$

$$\text{But } I = A\sqrt{x} + B \tan^{-1}(x + 1) + c$$

$$\Rightarrow A = 2, B = -2$$

230 (b)

$$\text{Let } I = \int x e^{x^2} dx$$

$$\text{Put } x^2 = t \Rightarrow 2x dx = dt \Rightarrow x dx = \frac{dt}{2}$$

$$\therefore I = \frac{1}{2} \int e^t dt = \frac{e^t}{2} + c = \frac{e^{x^2}}{2} + c$$

231 (a)

$$\text{Let } f(\theta) = \log \left(\frac{2 - \sin \theta}{2 + \sin \theta} \right)$$

$$\text{Now, } f(-\theta) = \log \left(\frac{2 + \sin \theta}{2 - \sin \theta} \right)$$

$$= - \log \left(\frac{2 - \sin \theta}{2 + \sin \theta} \right) = -f(\theta)$$

$\therefore f(\theta)$ is an odd function.

$$\text{So, } \int_{-\pi/2}^{\pi/2} \log \left(\frac{2 - \sin \theta}{2 + \sin \theta} \right) d\theta = 0$$

232 (c)

$$\text{Let } I = \int \frac{dx}{\left[(\sqrt{x+99})^2 + 1 \right] \sqrt{x+99}}$$

$$\text{Put } \sqrt{x+99} = t \Rightarrow \frac{1}{\sqrt{x+99}} dx = 2dt$$

$$\therefore I = \int \frac{2dt}{t^2 + 1} = 2 \tan^{-1} t + c$$

$$= 2 \tan^{-1} \sqrt{x+99} + c$$

$$\text{But } 2 \tan^{-1} \sqrt{x+99} + c = f(x) + c \quad [\text{given}]$$

$$\therefore f(x) = 2 \tan^{-1} \sqrt{x+99}$$

233 (d)

For $n \leq 1$ and $-1 \leq x \leq 1$, we have

$$\Rightarrow 1 \leq \frac{1}{\sqrt{1-x^{2n}}} \leq \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \int_0^{1/2} 1 \cdot dx \leq \int_0^{1/2} \frac{1}{\sqrt{1-x^{2n}}} dx \leq \int_0^{1/2} \frac{1}{\sqrt{1-x^2}} dx$$

$$\Rightarrow \frac{1}{2} \leq \int_0^{1/2} \frac{1}{\sqrt{1-x^{2n}}} dx \leq [\sin^{-1} x]_0^{1/2}$$

$$\Rightarrow \frac{1}{2} \leq I \leq \frac{\pi}{6}$$

234 (b)

$$\text{Let } I = \int \frac{\operatorname{cosec} x}{\cos^2(1 + \log \tan \frac{x}{2})} dx$$

$$\text{Put } 1 + \log \tan \frac{x}{2} = t$$

$$\Rightarrow \frac{1}{\tan \frac{x}{2}} \cdot \sec^2 \frac{x}{2} \cdot \frac{1}{2} dx = dt$$

$$\Rightarrow \operatorname{cosec} x dx = dt$$

$$\therefore I = \int \frac{dt}{\cos^2 t} = \int \sec^2 t dx = \tan t + c$$

$$= \tan \left(1 + \log \tan \frac{x}{2} \right) + c$$

235 (a)

$$\text{Here, } I(m, n) = \int_0^1 t^m (1+t)^n dt$$

$$\Rightarrow I(m, n) = \left\{ (1+t)^n \cdot \frac{t^{m+1}}{m+1} \right\}_0^1 - \int_0^1 n(1+t)^{n-1} \cdot \frac{t^{m+1}}{m+1} dt$$

$$= \frac{2^n}{m+1} - \frac{n}{m+1} \int_0^1 (1+t)^{n-1} \cdot t^{m+1} dt$$

$$\therefore I(m, n) = \frac{2^n}{m+1} - \frac{n}{m+1} \cdot I(m+1, n-1)$$

236 (c)

$$I = \int \frac{dx}{x(\log x)(\log \log x) \dots \underbrace{(\log \log \dots x)}_{8 \text{ times}}}$$

$$\text{Put } \underbrace{(\log \log \dots x)}_{8 \text{ times}} = t$$

$$\Rightarrow I = \frac{1}{\underbrace{(\log \log \dots x)}_{7 \text{ times}} \dots (\log \log x)(\log x)x} dx = dt$$

$$\therefore I = \int \frac{dt}{t} = \log t + c = \underbrace{(\log \log \log \dots x)}_{9 \text{ times}} + c$$

237 (d)

$$\text{Given, } I = \left| \int_2^5 \frac{\sin x dx}{(1+x^2)} \right|$$

$$\leq \int_2^5 \left| \frac{\sin x}{1+x^2} \right| dx$$

$$\leq \int_2^5 \frac{1}{1+x^2} dx \leq \int_2^5 \frac{1}{x^2} dx$$

$$= \left[\frac{x^{-1}}{-1} \right]_2^5 = - \left[\frac{1}{5} - \frac{1}{2} \right]$$

$$\Rightarrow I \leq \frac{3}{10}$$

238 (b)

$$\lim_{n \rightarrow \infty} \left(\frac{1}{n} + \frac{1}{n+1} + \dots + \frac{1}{n+2n} \right)$$

$$= \sum_{r=0}^{2n} \frac{1}{n+r} = \frac{1}{n} \sum_{r=0}^{2n} \frac{1}{1+\frac{r}{n}}$$

$$= \int_0^2 \frac{1}{1+x} dx = [\log(1+x)]_0^2$$

$$= \log 3 - \log 1 = \log 3$$

239 (c)

We have,

$$\phi(x) = \int_0^x (t+1) dt \Rightarrow \phi'(x) = x+1$$

Clearly, $\phi'(x) = x+1 > 0$ for $x \in [2, 3]$

Therefore, $\phi(x)$ is an increasing function on $[2, 3]$

So, its greatest and least values are $\phi(3)$ and $\phi(2)$

respectively. Therefore, required difference is

given by

$$\phi(3) - \phi(2) = \int_0^3 (t+1) dt - \int_0^2 (t+1) dt = \frac{7}{2}$$

240 (b)

Putting $x = \tan \theta$, we have

$$I = \int_0^{\infty} \frac{x \log x}{(1+x^2)^2} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\tan \theta \log \tan \theta \sec^2 \theta d\theta}{(1+\tan^2 \theta)^2}$$

$$\Rightarrow I = \int_0^{\pi/2} \sin \theta \cos \theta \log \tan \theta d\theta \quad \dots(i)$$

$$\Rightarrow I = \int_0^{\pi/2} \sin \left(\frac{\pi}{2} - \theta \right) \cos \left(\frac{\pi}{2} - \theta \right) \log \tan \left(\frac{\pi}{2} - \theta \right) d\theta$$

$$\Rightarrow I = \int_0^{\pi/2} \cos \theta \sin \theta \log \cot \theta d\theta \quad \dots(ii)$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2} \sin \theta \cos \theta (\log \tan \theta + \log \cot \theta) d\theta$$

$$\Rightarrow 2I = \int_0^{\pi/2} \sin \theta \cos \theta \cdot \log 1 d\theta = 0 \Rightarrow I = 0$$

241 (b)

Let

$$I = \int_0^1 \frac{2^{2x+1} - 5^{2x-1}}{10^x} dx = \int_0^1 \frac{2^{2x+1} - 5^{2x-1}}{2^x \cdot 5^x} dx$$

$$\Rightarrow I = 2 \int_0^1 \left(\frac{2}{5} \right)^x dx - \frac{1}{2} \int_0^1 \left(\frac{5}{2} \right)^{x-1} dx,$$

$$\Rightarrow I = 2 \left[\frac{(2/5)^x}{\log_e(2/5)} \right]_0^1 - \frac{1}{2} \left[\frac{(5/2)^{x-1}}{\log_e 5/2} \right]_0^1$$

$$\Rightarrow I = 2 \left[\frac{2/5 - 1}{\log_e(2/5)} \right] - \frac{1}{2} \left[\frac{1 - 2/5}{\log_e 5/2} \right]$$

$$\Rightarrow I = -\frac{6}{5 \log_e(2/5)} - \frac{3}{10 \log_e(5/2)}$$

$$\Rightarrow I = -\frac{3}{5} \left\{ \frac{2}{\log_e(2/5)} + \frac{1}{2 \log_e(5/2)} \right\}$$

242 (c)

Let $I = \int_0^{\pi r} \sin^{2n} x \, dx$

Since $\sin^{2n} x$ is a periodic function with period $\pi/2$. Therefore,

$$\therefore I = \int_0^{\pi r} \sin^{2n} x \, dx = \int_0^{2r(\pi/2)} \sin^{2n} x \, dx$$

$$\Rightarrow I = 2r \int_0^{\pi/2} \sin^{2n} x \, dx$$

$$\Rightarrow I$$

$$= r \int_0^{\pi} \sin^{2n} x \, dx \quad \left[\begin{array}{l} \because \sin^{2n}(\pi - x) = \sin^{2n} x \\ \therefore \int_0^{\pi} \sin^{2n} x \, dx = 2 \int_0^{\pi/2} \sin^{2n} x \, dx \end{array} \right]$$

244 (a)

Let $I = \int_0^{\lambda} \frac{y \, dy}{\sqrt{y+\lambda}}$

$$= \int_0^{\lambda} \left[\sqrt{y+\lambda} - \frac{\lambda}{\sqrt{y+\lambda}} \right] dy$$

$$= \left[\frac{(y+\lambda)^{3/2}}{3/2} \right]_0^{\lambda} - \left[\frac{\lambda \sqrt{y+\lambda}}{1/2} \right]_0^{\lambda}$$

$$= \frac{2}{3} [(2\lambda)^{3/2} - \lambda^{3/2}] - 2\lambda [(2\lambda)^{1/2} - (\lambda)^{1/2}]$$

$$= 2\lambda\sqrt{\lambda} \left[\frac{2\sqrt{2}-1}{3} - (\sqrt{2}-1) \right]$$

$$= \frac{2}{3} \lambda\sqrt{\lambda}(2-\sqrt{2})$$

245 (c)

Given, $\int_0^x \sqrt{1 - (f'(t))^2} \, dt = \int_0^x f(t) \, dt, 0 \leq x \leq 1$

1

Applying Leibnitz theorem, we get

$$\sqrt{1 - (f'(x))^2} = f(x)$$

$$\Rightarrow 1 - (f'(x))^2 = f^2(x)$$

$$\Rightarrow (f'(x))^2 = 1 - f^2(x)$$

$$\Rightarrow f'(x) = \pm \sqrt{1 - f^2(x)}$$

$$\Rightarrow \frac{dy}{dx} = \pm \sqrt{1 - y^2}, \text{ where } y = f(x)$$

$$\Rightarrow \frac{dy}{\sqrt{1 - y^2}} = \pm dx$$

On integrating both sides, we get

$$\sin^{-1}(y) = \pm x + C$$

$$\because f(0) = 0 \Rightarrow C = 0$$

$$\therefore y = \pm \sin x$$

$$y = \sin x = f(x) \text{ given } f(x) \geq 0 \text{ for } x \in [0, 1]$$

It is known that $\sin x < x, \forall x \in R^+$

$$\therefore \sin\left(\frac{1}{2}\right) < \frac{1}{2} \Rightarrow f\left(\frac{1}{2}\right) < \frac{1}{2} \text{ and } \sin\left(\frac{1}{3}\right) < \frac{1}{3}$$

$$\Rightarrow f\left(\frac{1}{3}\right) < \frac{1}{3}$$

246 (b)

$$\int_1^2 e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx$$

$$= \left[\frac{1}{x} e^x \right]_1^2$$

$$+ \int_1^2 \frac{e^x}{x^2} dx - \int_1^2 \frac{e^x}{x^2} dx$$

$$= \frac{e^2}{2} - e$$

248 (c)

$\because \sin^{-1} x$ and $\cos^{-1} x$ are defined for $[-1, 1]$.

For $x > e, \log x > 1$

$\Rightarrow \sin^{-1}(\log x) + \cos^{-1}(\log x)$ is not defined.

Hence, the given integral does not exist.

249 (c)

We have,

$$I = \int_0^{\pi/3} \frac{\cos x}{3 + 4 \sin x} dx$$

$$\Rightarrow I = \frac{1}{4} \int_0^{\pi/2} \frac{1}{3 + 4 \sin x} d(3 + 4 \sin x)$$

$$\Rightarrow I = \frac{1}{4} [\log(3 + 4 \sin x)]_0^{\pi/4}$$

$$\Rightarrow I = \frac{1}{4} \log \left(3 + 4 \frac{\sqrt{3}}{2} \right) - \frac{1}{4} \log 3$$

$$= \frac{1}{4} \log \left(\frac{3 + 2\sqrt{3}}{3} \right)$$

$$\therefore \int_0^{\pi/3} \frac{\cos x}{3 + 4 \sin x} dx = k \log \left(\frac{3 + 2\sqrt{3}}{3} \right)$$

$$\Rightarrow \frac{1}{4} \log \left(\frac{3 + 2\sqrt{3}}{3} \right) = k \log \left(\frac{3 + 2\sqrt{3}}{3} \right)$$

$$\Rightarrow k = \frac{1}{4}$$

250 (a)

We have,

$$F(x) = \int_{x^2}^{x^3} \log t \, dt, (x > 0)$$

$$\Rightarrow F'(x) = 3x^2(\log x^3) - 2x \log x^2$$

$$\Rightarrow F'(x) = 9x^2 \log x - 4x \log x = (9x^2 - 4x) \log x$$

251 (a)

Let $I = \int \frac{\sin x \cos x}{\sqrt{1 - \sin^4 x}} dx$

Put $\sin^2 x = t \Rightarrow 2 \sin x \cos x \, dx = dt$

$$\begin{aligned} \therefore I &= \int \frac{dt}{2\sqrt{1-t^2}} = \frac{1}{2} \sin^{-1} t + c \\ &= \frac{1}{2} \sin^{-1}(\sin^2 x) + c \end{aligned}$$

252 (a)

We have,

$$\begin{aligned} I &= \int \frac{1+x}{1+x^{1/3}} dx = \int \frac{1^3 + (x^{1/3})^3}{1+x^{1/3}} dx \\ \Rightarrow I &= \int (1-x^{1/3} + x^{2/3}) dx \\ &= x - \frac{3}{4} x^{4/3} + \frac{3}{5} x^{5/3} + C \end{aligned}$$

253 (b)

We have,

$$\begin{aligned} I &= \int_0^1 x(1-x)^n dx \\ I &= \int_0^1 (1-x)(1-(1-x))^n dx \left[\because \int_0^a f(x) dx \right. \\ &\quad \left. = \int_0^a f(a-x) dx \right] \\ \Rightarrow I &= \int_0^1 x^n (1-x) dx \\ \Rightarrow I &= \int_0^1 x^n - x^{n+1} dx \Rightarrow I = \frac{1}{n+1} - \frac{1}{n+2} \\ &= \frac{1}{(n+1)(n+2)} \end{aligned}$$

254 (c)

$$\begin{aligned} \text{Let } I &= \int_0^1 \frac{x dx}{[x+\sqrt{1-x^2}]\sqrt{1-x^2}} \\ \text{Put } x &= \sin \theta \Rightarrow dx = \cos \theta d\theta \\ \therefore I &= \int_0^{\pi/2} \frac{\sin \theta \cdot \cos \theta d\theta}{(\sin \theta + \cos \theta) \cdot \cos \theta} \\ I &= \int_0^{\pi/2} \frac{\sin \theta}{\sin \theta + \cos \theta} d\theta \quad \dots(i) \\ \text{Now, } I &= \int_0^{\pi/2} \frac{\sin(\frac{\pi}{2}-\theta)}{\sin(\frac{\pi}{2}-\theta) + \cos(\frac{\pi}{2}-\theta)} d\theta \\ \Rightarrow I &= \int_0^{\pi/2} \frac{\cos \theta}{\sin \theta + \cos \theta} d\theta \quad \dots(ii) \\ \text{On adding Eqs. (i) and (ii), we get} \\ 2I &= \int_0^{\pi/2} \left(\frac{\sin \theta + \cos \theta}{\sin \theta + \cos \theta} \right) d\theta \\ &= \int_0^{\pi/2} 1 d\theta = \frac{\pi}{2} \\ \Rightarrow I &= \frac{\pi}{4} \end{aligned}$$

255 (c)

$$I = \int_{-1}^1 |1-x| dx$$

Here, $-1 \leq x \leq 1 \Rightarrow 1-x \geq 0$

$$\begin{aligned} \therefore I &= \int_{-1}^1 (1-x) dx = \left[x - \frac{x^2}{2} \right]_{-1}^1 \\ &= 1 - \frac{1}{2} + \frac{1}{2} + 1 = 2 \end{aligned}$$

256 (b)

$$\begin{aligned} &\int \frac{\sec x dx}{\sqrt{\sin(2x+\theta) + \sin \theta}} \\ &= \int \frac{\sec x dx}{\sqrt{2 \sin(x+\theta) \cos x}} \\ &= \frac{1}{\sqrt{2}} \int \frac{\sec^{3/2} x dx}{\sqrt{\sin x \cos \theta + \sin \theta \cos x}} \\ &= \frac{1}{\sqrt{2}} \int \frac{\sec^2 x dx}{\sqrt{\tan x \cos \theta + \sin \theta}} \\ \text{Put } \tan x &= t \Rightarrow \sec^2 x dx = dt \\ \therefore I &= \frac{1}{\sqrt{2}} \int \frac{dt}{\sqrt{t \cos \theta + \sin \theta}} \\ &= \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{t \cos \theta + \sin \theta}}{\cos \theta} + c \\ &= \sqrt{2}(\tan x \sec \theta + \tan \theta \sec \theta) + c \end{aligned}$$

257 (c)

$$\begin{aligned} \text{Given, } \int f(x) dx &= e^x \text{ and } \int g(x) dx = \cos x \\ \Rightarrow f(x) &= e^x \text{ and } g(x) = -\sin x \\ \therefore \int f(x) \cos x dx &+ \int g(x) e^x dx \\ &= \int e^x \cos x dx - \int e^x \sin x dx \\ &= e^x \cos x + \int \sin x e^x dx - \int e^x \sin x dx \\ &= e^x \cos x + c \end{aligned}$$

258 (c)

$$\begin{aligned} \text{Given, } P &= \int_0^{3\pi} f(\cos^2 x) dx \quad \dots(i) \\ \text{and } Q &= \int_0^{\pi} f(\cos^2 x) dx \quad \dots(ii) \\ \text{From Eq. (i),} \\ P &= 3 \int_0^{\pi} f(\cos^2 x) dx = 3Q \\ \therefore P - 3Q &= 0 \end{aligned}$$

259 (b)

We have,

$$\begin{aligned} f(x) &= \int_{x^2}^{x^4} \sin \sqrt{t} dt \\ \Rightarrow f'(x) &= \frac{d}{dx}(x^4) (\sin \sqrt{x^4}) - \frac{d}{dx}(x^2) (\sin \sqrt{x^2}) \\ \Rightarrow f'(x) &= 4x^3 \sin x^2 - 2x \sin x \end{aligned}$$

261 (d)

$$\text{Since, } \int_2^4 \{3 - f(x)\} dx = 7$$

$$\Rightarrow 3[x]_2^4 - \int_2^4 f(x) dx = 7$$

$$\Rightarrow \int_2^4 f(x) dx = 6 - 7 = -1$$

$$\begin{aligned} \therefore \int_{-1}^2 f(x) dx &= \int_{-1}^4 f(x) dx - \int_2^4 f(x) dx \\ &= 4 - (-1) = 5 \end{aligned}$$

262 (b)

$$\begin{aligned} \int \frac{(x+1)^2}{x(x^2+1)} dx &= \int \frac{x^2+1+2x}{x(x^2+1)} dx \\ &= \int \frac{x^2+1}{x(x^2+1)} dx + 2 \int \frac{x}{x(x^2+1)} dx \\ &= \int \frac{dx}{x} + 2 \int \frac{dx}{x^2+1} \\ &= \log_e x + 2 \tan^{-1} x + c \end{aligned}$$

263 (c)

We have,

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{\pi}{n} \left\{ \sin \frac{\pi}{n} + \sin \frac{2\pi}{n} + \dots + \sin \frac{(n-1)\pi}{n} \right\} \\ = \lim_{n \rightarrow \infty} \frac{\pi}{n} \sum_{r=1}^{(n-1)} \sin \left(\frac{r\pi}{n} \right) = \pi \int_0^1 \sin x \pi dx \\ = \int_0^{\pi} \sin t dt = 2 \end{aligned}$$

264 (d)

$$\int_0^x f(t) dt = \int_x^1 t^2 f(t) dt + \frac{x^{16}}{8} + \frac{x^6}{3} + a \quad \dots(i)$$

For $x = 1$

$$\int_0^1 f(t) dt = \frac{1}{8} + \frac{1}{3} + a \quad \dots(ii)$$

On differentiating Eq.(i) w.r.t. x , we get

$$f(x) = -x^2 f(x) + 2x^{15} + 2x^5$$

$$\Rightarrow f(x) = \frac{2(x^{15} + x^5)}{1 + x^2}$$

Now, from Eq. (ii)

$$2 \int_0^1 \left(\frac{x^{15} + x^5}{1 + x^2} \right) dx = \frac{1}{8} + \frac{1}{3} + a \Rightarrow a = -\frac{167}{840}$$

265 (a)

Let $\phi(x) = [f(x) + f(-x)][g(x) - g(-x)]$,
then $\phi(-x) = [f(-x) + f(x)][g(-x) - g(x)]$
 $\phi(-x) = -\phi(x)$

\Rightarrow It is an odd function.

$$\therefore \int_{-\pi}^{\pi} \phi(x) dx = 0$$

$$\Rightarrow \int_{-\pi}^{\pi} [f(x) + f(-x)][g(x) - g(-x)] dx = 0$$

266 (b)

$$\text{Let } I = \int_0^{\pi} x f(\sin x) dx \quad \dots(i)$$

$$= \int_0^{\pi} (\pi - x) f[\sin(\pi - x)] dx$$

$$\Rightarrow I = \int_0^{\pi} (\pi - x) f(\sin x) dx \quad \dots(ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi} \pi f(\sin x) dx = 2\pi \int_0^{\pi/2} f(\sin x) dx$$

$$\Rightarrow I = \pi \int_0^{\pi/2} f(\sin x) dx = A \int_0^{\pi/2} f(\sin x) dx$$

$$\therefore A = \pi$$

267 (a)

$$\because x = [x] + \{x\} = n + f \quad (\text{say})$$

Where $[x] = n, \{x\} = f (0 \leq f < 1)$

$$\text{Now, } \int_0^x [t^2] dt = 2(x-1)$$

$$\Rightarrow \int_0^{n+f} [t^2] dt = 2(n+f-1)$$

$$\Rightarrow \sum_{r=0}^{n-1} \int_r^{r+1} [t^2] dt + \int_n^{n+f} [t^2] dt = 2(n+f-1)$$

$$\Rightarrow \sum_{r=0}^{n-1} r^2 + n^2 f = 2(n+f-1)$$

$$\Rightarrow \frac{(n-1)n(2n-1)}{6} - 2(n-1) = (2-n^2)f$$

$$\text{For } n = 1, 0 - 0 = f \Rightarrow f = 0$$

$$\text{Then, } x = n + f = 1 + 0 = 1$$

$$\text{For } n = 2, 1 - 2 = -2f \Rightarrow f = \frac{1}{2}$$

$$\text{Then, } x = 2 + \frac{1}{2} = \frac{5}{2}$$

For $n > 2, f < 0$ impossible

$$\text{Hence, } x = 1, \frac{5}{2}$$

268 (a)

We have,

$$I = \int_{-\pi/6}^{\pi/6} \frac{\pi + 4x^n}{1 - \sin\left(|x| + \frac{\pi}{6}\right)} dx$$

$$\begin{aligned} \Rightarrow I &= \pi \int_{-\pi/6}^{\pi/6} \frac{1}{1 - \sin\left(|x| + \frac{\pi}{6}\right)} \\ &\quad + 4 \int_{-\pi/6}^{\pi/6} \frac{x^n}{1 - \sin\left(|x| + \frac{\pi}{6}\right)} dx \end{aligned}$$

$$\Rightarrow I = 2\pi \int_0^{\pi/6} \frac{1}{1 - \sin\left(|x| + \frac{\pi}{6}\right)} dx + 4 \times 0$$

$$\Rightarrow I = 2\pi \int_0^{\pi/6} \frac{1}{1 + \cos\left(\frac{2\pi}{3} + x\right)} dx$$

$$\Rightarrow I = \frac{2\pi}{2} \int_0^{\pi/6} \sec^2\left(\frac{\pi}{3} + \frac{x}{2}\right) dx$$

$$\Rightarrow I = 2\pi \left[\tan\left(\frac{\pi}{3} + \frac{x}{2}\right) \right]_0^{\pi/6}$$

$$\Rightarrow I = 2\pi \left(\tan\frac{5\pi}{12} - \tan\frac{\pi}{3} \right) = \frac{2\pi \times \sin\left(\frac{5\pi}{12} - \frac{\pi}{3}\right)}{\cos\frac{5\pi}{12} \cos\frac{\pi}{3}}$$

$$\Rightarrow I = \frac{4\pi \sin 15^\circ}{\cos 75^\circ} = 4\pi$$

269 (b)

For $x \in \left(0, \frac{\pi}{4}\right)$ $\cos^2 x > \sin^2 x$

$$\Rightarrow \int_0^{\pi/4} \cos^2 x \, dx > \int_0^{\pi/4} \sin^2 x \, dx$$

$$\Rightarrow I_1 < I_2$$

270 (c)

Since, $2e^{2\alpha} - 3e^\alpha - 2 > 0$

$$\Rightarrow e^\alpha < -\frac{1}{2} \text{ or } e^\alpha > 2$$

But $e^\alpha > 0, \forall \alpha \in R \Rightarrow e^\alpha > 2$

$$\therefore \alpha \in (\log 2, \infty)$$

271 (b)

$$\text{Given, } \frac{d}{dx} \left[a \tan^{-1} x + b \log\left(\frac{x-1}{x+1}\right) \right] = \frac{1}{x^2-1}$$

On integrating both sides, we get

$$a \tan^{-1} x + b \log\left(\frac{x-1}{x+1}\right) = \frac{1}{2} \int \left[\frac{1}{x^2-1} - \frac{1}{x^2+1} \right] dx$$

$$\Rightarrow a \tan^{-1} x + b \log\left(\frac{x-1}{x+1}\right) = \frac{1}{4} \int \log\left(\frac{x-1}{x+1}\right) - \frac{1}{2} \tan^{-1} x$$

$$\Rightarrow a = -\frac{1}{2}, \quad b = \frac{1}{4}$$

$$\therefore a - 2b = -\frac{1}{2} - 2\left(\frac{1}{4}\right) = -1$$

272 (b)

$$\text{Let } I = \int_\alpha^\beta \sqrt{\frac{x-\alpha}{\beta-x}} dx$$

Put $x = \alpha \cos^2 \theta + \beta \sin^2 \theta$,

$$\therefore x - \alpha = (\beta - \alpha) \sin^2 \theta, \beta - x = (\beta - \alpha) \cos^2 \theta$$

and $dx = 2(\beta - \alpha) \sin \theta \cos \theta d\theta$

$$\therefore I = \int_0^{\pi/2} \sqrt{\frac{(\beta - \alpha) \sin^2 \theta}{(\beta - \alpha) \cos^2 \theta}} \times 2(\beta - \alpha) \sin \theta \cos \theta d\theta$$

$$= (\beta - \alpha) \int_0^{\pi/2} (1 - \cos 2\theta) d\theta$$

$$= (\beta - \alpha) \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{\pi/2} = (\beta - \alpha) \cdot \frac{\pi}{2}$$

273 (d)

$$\int \frac{x \, dx}{x^2 + 4x + 5} = \int \frac{x}{x^2 + 4x + 4 + 1} dx$$

$$= \int \frac{x + 2 - 2}{(x + 2)^2 + 1} dx$$

$$= \frac{1}{2} \int \frac{2(x + 2) dx}{(x + 2)^2 + 1} - 2 \int \frac{dx}{1 + (x + 2)^2}$$

$$= \frac{1}{2} \log[(x + 2)^2 + 1] - 2 \tan^{-1}(x + 2) + c$$

$$= \frac{1}{2} \log(x^2 + 4x + 5) - 2 \tan^{-1}(x + 2) + c$$

274 (c)

$$\int \cos \left[2 \cot^{-1} \sqrt{\frac{1-x}{1+x}} \right] dx$$

$$= \int \cos[\cos^{-1}(-x)] dx$$

$$= \int (-x) dx = -\frac{x^2}{2} + c$$

275 (c)

$$\text{Let } I = \int_0^a \sqrt{\frac{a-x}{x}} dx$$

$$\text{Put } x = a \sin^2 \theta \Rightarrow dx = 2a \sin \theta \cos \theta d\theta$$

$$\therefore I = \int_0^{\pi/2} \sqrt{\frac{\cos^2 \theta}{\sin^2 \theta}} \cdot 2a \sin \theta \cos \theta d\theta$$

$$= 2a \int_0^{\pi/2} \cos^2 \theta d\theta$$

$$= 2a \times \frac{1}{2} \times \frac{\pi}{2} = \frac{\pi a}{2}$$

276 (b)

We have,

$$I = \int_{-\pi/2}^{\pi/2} \sin^2 x \cos^2 x (\sin x + \cos x) dx$$

$$\Rightarrow I =$$

$$\int_{-\pi/2}^{\pi/2} \sin^3 x \cos^2 x \, dx - \int_{-\pi/2}^{\pi/2} \sin^2 x \cos^3 x \, dx$$

...(i)

Since $\sin^3 x \cos^2 x$ is an odd function and $\sin^2 x \cos^3 x$ is an even function

$$\therefore \int_{-\pi/2}^{\pi/2} \sin^3 x \cos^2 x \, dx = 0$$

and,

$$\int_{-\pi/2}^{\pi/2} \sin^2 x \cos^3 x \, dx = 2 \int_0^{\pi/2} \sin^2 x \cos^3 x \, dx$$

$$\begin{aligned} \therefore I &= 2 \int_0^{\pi/2} \sin^2 x \cos^3 x \, dx \\ \Rightarrow I &= 2 \int_0^1 t^2(1-t^2) dt, \text{ where } t = \sin x \\ \Rightarrow I &= 2 \int_0^1 (t^2 - t^4) dt = 2 \left[\frac{1}{3} - \frac{1}{5} \right] = \frac{4}{15} \end{aligned}$$

277 (c)

Let $f(x) = \log\left(\frac{x-1}{x+1}\right)$
 $f(-x) = \log\left(\frac{-x-1}{-x+1}\right) = -\log\left(\frac{x-1}{x+1}\right)$
 $= -f(x)$
 $\Rightarrow f(x)$ is an odd function
 $\therefore \int_{-1}^1 \log\left(\frac{x-1}{x+1}\right) dx = 0$

278 (c)

Given, $\int_n^{n+1} f(x) dx = n^2$
 On putting $n = -2, -1, 0, 1, 2, 3$, we get
 $\int_{-2}^{-1} f(x) dx = 4, \int_{-1}^0 f(x) dx = 1,$
 $\int_0^1 f(x) dx = 0, \int_1^2 f(x) dx = 1,$
 $\int_2^3 f(x) dx = 4, \int_3^4 f(x) dx = 9,$
 $\therefore \int_{-2}^4 f(x) dx = 4 + 1 + 0 + 1 + 4 + 9 = 19$

279 (c)

The function $f(x) = x|x|$ is an odd function
 $\therefore \int_{-1}^1 x|x| dx = 0$

280 (c)

Using $C_1 \rightarrow C_1 - C_2 - C_3$, we get

$$f(x) = \begin{vmatrix} \sin x & \sin 2x & \sin 3x \\ 0 & 3 & 4 \sin x \\ 0 & \sin x & 1 \end{vmatrix}$$

$$= 3 \sin x - 4 \sin^3 x = \sin 3x$$

$$\therefore \int_0^{\pi/2} f(x) dx = \int_0^{\pi/2} \sin 3x dx = \left[-\frac{\cos 3x}{3} \right]_0^{\pi/2} = \frac{1}{3}$$

281 (d)

We have,

$$I = \int_a^\beta \frac{1}{\sqrt{(x-a)(\beta-x)}} dx$$

$$\begin{aligned} \Rightarrow I &= \int_a^\beta \frac{1}{\sqrt{\left(\frac{\beta-a}{2}\right)^2 - \left(x - \frac{\alpha+\beta}{2}\right)^2}} dx \\ \Rightarrow I &= \left[\sin^{-1} \left(\frac{x - \frac{\alpha+\beta}{2}}{\frac{\beta-a}{2}} \right) \right]_a^\beta = \sin^{-1} 1 - \sin^{-1}(-1) \\ &= \pi \end{aligned}$$

283 (b)

Let $I = \int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$
 Put $\sin^{-1} x = t$
 $\Rightarrow \frac{1}{\sqrt{1-x^2}} dx = dt$
 $\therefore I = \int t dt = \frac{t^2}{2} + c$
 $= \frac{(\sin^{-1} x)^2}{2} + c$

284 (c)

Let $I = \int_{-\pi/2}^{\pi/2} \frac{dx}{2 \cos^2 \frac{x}{2}}$
 $= \int_0^{\pi/2} \sec^2 \frac{x}{2} dx = \left[2 \tan \frac{x}{2} \right]_0^{\pi/2} = 2$

285 (c)

$I_1 = \int_{1-k}^k (1-x)f[x(1-x)] dx$
 $\Rightarrow 2I_1 = I_2 \Rightarrow \frac{I_1}{I_2} = \frac{1}{2}$

286 (a)

Putting $x = a \tan \theta$ and $dx = a \sec^2 \theta d\theta$, we get

$$\int \frac{1}{(a^2 + x^2)^{3/2}} dx = \int \frac{a \sec^2 \theta}{a^3 \sec^3 \theta} d\theta$$

$$\Rightarrow I = \frac{1}{a^2} \int \cos \theta d\theta = \frac{1}{a^2} \sin \theta + C$$

$$= \frac{1}{a^2} \frac{x}{\sqrt{a^2 + x^2}} + C$$

287 (c)

Let $I = \int \frac{\cos x - \sin x}{1 + 2 \sin x \cos x} dx$
 Put $\cos x + \sin x = t$
 $\Rightarrow (\cos x - \sin x) dx = dt$
 $\therefore I = \int \frac{1}{t^2} dt$
 $= -\frac{1}{t} + c$
 $= -\frac{1}{\sin x + \cos x} + c$

288 (b)

We have,

$$\begin{aligned}
I &= \int_0^{\pi/2} |\sin x - \cos x| dx \\
\Rightarrow I &= \int_0^{\pi/4} -(\sin x - \cos x) dx \\
&\quad + \int_{\pi/4}^{\pi/2} (\sin x - \cos x) dx \\
\Rightarrow I &= [\cos x + \sin x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{\pi/2} \\
\Rightarrow I &= \left(\frac{2}{\sqrt{2}} - 1\right) + \left(-1 + \frac{2}{\sqrt{2}}\right) = 2\left(\frac{2}{\sqrt{2}} - 1\right) \\
&= 2(\sqrt{2} - 1)
\end{aligned}$$

289 (d)

$$\begin{aligned}
\text{Let } I &= \int_0^{\pi/4} (\cos x - \sin x) dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x) dx + 2\pi \cdot 4(\cos x - \sin x) dx \\
&= [\sin x + \cos x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{5\pi/4} \\
&\quad + [\cos x - \sin x]_{2\pi}^{\pi/4} \\
&= \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right] \left[-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right] \\
&\quad + \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1\right] \\
&= \left[\frac{2}{\sqrt{2}} - 1\right] - \left[-\frac{4}{\sqrt{2}}\right] + \left[\frac{2}{\sqrt{2}} - 1\right] \\
&= (\sqrt{2} - 1) + 2\sqrt{2} + (\sqrt{2} - 1) = 4\sqrt{2} - 2
\end{aligned}$$

290 (b)

$$\begin{aligned}
\text{We have,} \\
I &= \int_1^4 e^{\sqrt{x}} dx = 2 \int_1^2 t e^t dt, \text{ where } x = t^2 \\
\Rightarrow I &= 2[t e^t - e^t]_1^2 = 2e^2
\end{aligned}$$

291 (c)

$$\begin{aligned}
\text{We have,} \\
I_n &= \int_0^{\pi/2} \cos^n x \cos nx dx \\
\therefore I_{n+1} - I_n &= \int_0^{\pi/2} (\cos^{n+1} x \cos(n+1)x \\
&\quad - \cos^n x \cos nx) dx \\
\Rightarrow I_{n+1} - I_n &= \int_0^{\pi/2} \cos^n x \{\cos(n+1)x \cos x \\
&\quad - \cos nx\} dx \\
\Rightarrow I_{n+1} - I_n &= \int_0^{\pi/2} \cos^n x \{\cos(n+1)x \cos x \\
&\quad - \cos\{(n+1)x - x\}\} dx
\end{aligned}$$

$$\begin{aligned}
\Rightarrow I_{n+1} - I_n &= \int_0^{\pi/2} \cos^n x \{-\sin(n+1)x \sin x\} dx \\
\Rightarrow I_{n+1} - I_n &= \int_0^{\pi/2} \sin(n+1)x \cos^n x (-\sin x) dx \\
&\quad I \qquad \qquad \qquad II \\
\Rightarrow I_{n+1} - I_n &= \left[\sin(n+1)x \frac{\cos^{n+1} x}{n+1} \right]_0^{\pi/2} \\
&\quad - \int_0^{\pi/2} \cos(n+1)x \cos^{n+1} x dx \\
\Rightarrow I_{n+1} - I_n &= -I_{n+1} \\
\therefore 2I_{n+1} &= I_n
\end{aligned}$$

292 (d)

$$\begin{aligned}
\text{Let } I &= \int \frac{2x^2+3}{(x^2-1)(x^2+4)} dx = \int \frac{dx}{x^2-1} + \int \frac{dx}{x^2+4} \\
\left[\because \frac{2x^2+3}{(x^2-1)(x^2+4)} = \frac{1}{x^2-1} + \frac{1}{x^2+4} \right] \\
\Rightarrow I &= \frac{1}{2} \log \left(\frac{x-1}{x+1} \right) + \frac{1}{2} \tan^{-1} \frac{x}{2} + c \\
\text{But } I &= a \log \left(\frac{x-1}{x+1} \right) + b \tan^{-1} \left(\frac{x}{2} \right) + c \\
\therefore a &= \frac{1}{2}, \quad b = \frac{1}{2}
\end{aligned}$$

293 (a)

$$\begin{aligned}
\text{Let } I &= \int_0^{\pi/2} \frac{\cos x - \sin x}{1 + \cos x \sin x} dx \quad \dots(i) \\
\text{On putting } x &= \left(\frac{\pi}{2} - x\right) \text{ in Eq.(i), we get} \\
I &= \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2} - x\right) - \sin\left(\frac{\pi}{2} - x\right)}{1 + \cos\left(\frac{\pi}{2} - x\right) \sin\left(\frac{\pi}{2} - x\right)} dx \\
&= \int_0^{\pi/2} -\left(\frac{\cos x - \sin x}{1 + \cos x \sin x}\right) dx \quad \dots(ii) \\
\text{On adding Eqs.(i) and (ii), we get} \\
2I &= \int_0^{\pi/2} 0 dx = 0 \Rightarrow I = 0
\end{aligned}$$

294 (c)

$$\begin{aligned}
\text{Let } I &= \int_0^{\pi/2} \log \sin x dx \quad \dots(i) \\
I &= \int_0^{\pi/2} \log \sin\left(\frac{\pi}{2} - x\right) dx \\
&= \int_0^{\pi/2} \log \cos x dx \quad \dots(ii) \\
\text{On adding Eqs.(i) and (ii), we get} \\
2I &= \int_0^{\pi/2} (\log \sin x + \log \cos x) dx \\
&= \int_0^{\pi/2} \log \sin 2x + dx - \log 2 \int_0^{\pi/2} dx \\
\text{Put } 2x &= t \Rightarrow dx = \frac{dt}{2}
\end{aligned}$$

$$\begin{aligned} \therefore 2I &= \int_0^{\pi} \frac{\log \sin t}{2} dt - \log 2 [x]_0^{\pi/2} \\ &= \frac{1}{2} \times 2 \int_0^{\pi/2} \log \sin t dt - \frac{\pi}{2} \log 2 \\ &= \int_0^{\pi/2} \log \sin x dx - \frac{\pi}{2} \log 2 \\ &= I - \frac{\pi}{2} \log 2 \\ \Rightarrow I &= -\frac{\pi}{2} \log 2 \end{aligned}$$

295 (c)

$$\text{Given, } \int e^x (f(x) - f'(x)) dx = \phi(x)$$

$$\Rightarrow e^x (f(x) - f'(x)) = \phi'(x)$$

$$\text{Let } I = \int e^x f(x) dx$$

$$= f(x)e^x - \int f'(x)e^x dx$$

$$= f(x)e^x - \left[\int e^x f(x) dx - \int \phi'(x) dx \right]$$

$$\Rightarrow 2I = f(x)e^x + \phi(x)$$

$$\therefore I = \frac{1}{2} [e^x f(x) + \phi(x)]$$

296 (b)

$$\because \phi'(x) = f(x)$$

$\therefore \phi(x)$ is a periodic function with period T .

Because $f(x)$ has the period T

297 (d)

$$\text{Let } I = \int 2^{2^{2^x}} \cdot 2^{2^x} \cdot 2^x dx$$

$$\text{Put } 2^{2^{2^x}} = t \Rightarrow 2^{2^{2^x}} \cdot 2^{2^x} \cdot 2^x (\log 2)^3 dx = dt$$

$$\therefore I = \int \frac{dt}{(\log 2)^3} = \frac{t}{(\log 2)^3} = \frac{2^{2^{2^x}}}{(\log 2)^3} + c$$

298 (d)

$$\text{Let } I = \int \frac{2dx}{\sqrt{1-4x^2}}$$

$$\text{Put } 2x = \sin \theta \Rightarrow 2dx = \cos \theta d\theta$$

$$\therefore I = \int \frac{\cos \theta d\theta}{1 - \sin^2 \theta}$$

$$= \int \frac{\cos \theta}{\cos \theta} d\theta = \int 1 d\theta$$

$$\Rightarrow I = \theta + c$$

$$\therefore I = \sin^{-1}(2x) + c$$

299 (d)

$$\int \frac{d(\cos \theta)}{\sqrt{1 - \cos^2 \theta}} = \sin^{-1}(\cos \theta) + c$$

300 (a)

$$\text{Let } I = \int_0^1 \frac{x^4(1-x)^4}{1+x^2} dx$$

$$= \int_0^1 \frac{(x^4 - 1)(1-x)^4 + (1-x)^4}{(1+x^2)} dx$$

$$= \int_0^1 (x^2 - 1)(1-x)^4 + \int_0^1 \frac{(1+x^2 - 2x)^2}{(1+x^2)} dx$$

$$= \int_0^1 \left\{ (x^2 - 1)(1-x)^4 + (1+x^2) - 4x + \frac{4x^2}{(1+x^2)} \right\} dx$$

$$= \int_0^1 \left((x^2 - 1)(1-x)^4 + (1+x^2) - 4x + 4 - \frac{4}{1+x^2} \right) dx$$

$$= \int_0^1 \left(x^6 - 4x^5 + 5x^4 - 4x^2 + 4 - \frac{4}{1+x^2} \right) dx$$

$$= \left[\frac{x^7}{7} - \frac{4x^6}{6} + \frac{5x^5}{5} - \frac{4x^3}{3} + 4x - 4 \tan^{-1} x \right]_0^1$$

$$= \frac{1}{7} - \frac{4}{6} + \frac{5}{5} - \frac{4}{3} + 4 - 4 \left(\frac{\pi}{4} - 0 \right)$$

$$= \frac{22}{7} - \pi$$

301 (a)

We have,

$$f(x) = \int_0^x |x-2| dx \Rightarrow f'(x) = |x-2|$$

Clearly, $f'(x)$ is everywhere continuous and differentiable except at $x = 2$

302 (b)

If $f(t)$ is an odd function, then $\int_0^x f(t) dt$ is an even function

303 (c)

$$\int \frac{\sec x \operatorname{cosec} x}{2 \cot x - \sec x \operatorname{cosec} x} dx$$

$$= \int \frac{\frac{1}{\cos x \sin x}}{\frac{2 \cos x}{\sin x} - \frac{1}{\sin x \cos x}} dx$$

$$= \int \frac{dx}{2 \cos^2 x - 1}$$

$$= \int \frac{dx}{\cos 2x} = \int \sec 2x dx$$

$$= \frac{1}{2} \log |\sec 2x + \tan 2x| + c$$

304 (d)

$$\text{Let } I = \int_0^{\pi} \frac{\theta \sin \theta}{1 + \cos^2 \theta} d\theta \dots (i)$$

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi - \theta) \sin \theta}{1 + \cos^2 \theta} d\theta \dots (ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^\pi \frac{\pi \sin \theta}{1 + \cos^2 \theta} d\theta$$

Put $\cos \theta = t \Rightarrow -\sin \theta d\theta = dt$

$$\therefore 2I = -\pi \int_1^{-1} \frac{1}{1+t^2} dt = 2\pi \int_0^1 \frac{1}{1+t^2} dt$$

$$= 2\pi [\tan^{-1} t]_0^1 = 2\pi \cdot \frac{\pi}{4}$$

$$\Rightarrow I = \frac{\pi^2}{4}$$

305 (d)

$$\text{Let } f(\theta) = \left[\int_0^{\sin^2 \theta} \sin^{-1} \sqrt{\phi} d\phi + \int_0^{\cos^2 \theta} \cos^{-1} \sqrt{\phi} d\phi \right]$$

$$f'(\theta) = \left(\frac{d}{d\theta} \sin^2 \theta \right) \left[\sin^{-1} \sqrt{\sin^2 \theta} \right] + \left(\frac{d}{d\theta} \cos^2 \theta \right) \left[\cos^{-1} \sqrt{\cos^2 \theta} \right]$$

$$(2 \sin \theta \cos \theta) \theta - (2 \sin \theta \cos \theta) \theta = 0$$

$$\therefore f(\theta) = \text{constant} = a \quad [\text{say}]$$

$$\therefore f\left(\frac{\pi}{4}\right) = a$$

$$\Rightarrow \int_0^{1/2} (\sin^{-1} \sqrt{\phi} + \cos^{-1} \sqrt{\phi}) d\phi = a$$

$$\Rightarrow \frac{\pi}{2} [\phi]_0^{1/2} = a \Rightarrow \frac{\pi}{4} = a$$

306 (c)

$$\text{Let } I = \int_0^{\pi/2} \frac{dx}{1 + \tan^3 x} =$$

$$0\pi/2 \cos 3x \sin 3x + \cos 3x dx \dots (i)$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\cos^3 \left(\frac{\pi}{2} - x\right)}{\sin^3 \left(\frac{\pi}{2} - x\right) + \cos^3 \left(\frac{\pi}{2} - x\right)} dx$$

$$I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} dx \dots (ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{\sin^3 x + \cos^3 x}{\sin^3 x + \cos^3 x} dx$$

$$= \int_0^{\pi/2} 1 dx = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

307 (c)

$$\text{Since, } I = \int_0^1 \frac{\sin x}{\sqrt{x}} dx < \int_0^1 \frac{x}{\sqrt{x}} dx$$

Because in $x \in (0,1), x > \sin x$

$$I < \int_0^1 \sqrt{x} dx = \frac{2}{3} [x^{3/2}]_0^1 \Rightarrow I < \frac{2}{3}$$

$$\text{For } x \in (0,1), \frac{\cos x}{\sqrt{x}} < \frac{1}{\sqrt{x}}$$

$$\text{And } J = \int_0^1 \frac{\cos x}{\sqrt{x}} dx < \int_0^1 x^{-1/2} dx = 2 \Rightarrow J < 2$$

308 (c)

$$\int_{-1}^1 |x| dx + \int_1^3 |x-2| dx$$

$$= 2 \int_0^1 x dx + \int_1^2 (2-x) dx + \int_2^3 (x-2) dx$$

$$= 2 \times \frac{1}{2} + \left[2x - \frac{x^2}{2} \right]_1^2 + \left[\frac{x^2}{2} - 2x \right]_2^3$$

$$= 1 + \frac{1}{2} + \frac{1}{2} = 2$$

309 (a)

Putting $x^n + 1 = t$ and $n x^{n-1} dx = dt$, we get

$$I = \int \frac{1}{x(x^n + 1)} dx$$

$$= \frac{1}{n} \int \frac{1}{t(t-1)} dt$$

$$= \frac{1}{n} \int \left(\frac{1}{t-1} - \frac{1}{t} \right) dt$$

$$\Rightarrow I = \frac{1}{n} \log \left(\frac{t-1}{t} \right) + C = \frac{1}{n} \log \left(\frac{x^n}{x^n + 1} \right) + C$$

310 (d)

We have,

$$I = \int_0^{1/2} |\sin \pi x| dx = \frac{1}{\pi} \int_0^{\pi/2} |\sin t| dt, \text{ where } t = \pi x$$

$$\Rightarrow I = \frac{1}{\pi} \int_0^{\pi/2} \sin t dt = \frac{1}{\pi}$$

312 (c)

We know that,

$$\left| \int_a^b f(x)g(x) dx \right| \leq \sqrt{\int_a^b f^2(x) dx} \cdot \sqrt{\int_a^b g^2(x) dx}$$

Putting $f(x) = 1$ and $g(x) = \sqrt{1+x^3}$, we get

$$I \int_0^1 \sqrt{1+x^3} dx \leq \sqrt{\int_0^1 1 dx} \cdot \sqrt{\int_0^1 1+x^3 dx} = \sqrt{1 + \frac{1}{4}} = \frac{\sqrt{5}}{2}$$

$$\Rightarrow I < 2 \text{ and } I < \frac{\sqrt{7}}{2}$$

313 (b)

We have,

$$I = \frac{1}{c} \int_{ac}^{bc} f\left(\frac{x}{c}\right) dx$$

Putting $\frac{x}{c} = t$ and $dx = c dt$, we get

$$I = \frac{1}{c} \int_a^b f(t) c dt = \int_a^b f(t) dt = \int_a^b f(x) dx$$

314 (c)

$$\begin{aligned} \int e^x \left(\frac{1-x}{1+x^2} \right)^2 dx &= \int e^x \frac{(1+x^2-2x)}{(1+x^2)^2} dx \\ &= \int e^x \left(\frac{1}{1+x^2} - \frac{2x}{(1+x^2)^2} \right) dx \\ &= e^x \cdot \frac{1}{1+x^2} + \int \frac{2xe^x}{(1+x^2)^2} dx - \int \frac{2xe^x}{(1+x^2)^2} dx \\ &= \frac{e^x}{1+x^2} + C \end{aligned}$$

315 (a)

We have,

$$\begin{aligned} \frac{d}{dx}(f(x)) &= \frac{e^{\sin x}}{x} \Rightarrow \int \frac{e^{\sin x}}{x} dx = F(x) \\ \therefore \int_1^4 \frac{3}{x} e^{\sin x^3} dx &= \int_1^4 \frac{e^{\sin x^3}}{x^3} \cdot 3x^2 dx \\ &= \int_1^{64} \frac{e^{\sin x^3}}{x^3} d(x^3) \end{aligned}$$

$$= F(64) - F(1)$$

Hence, $k = 64$

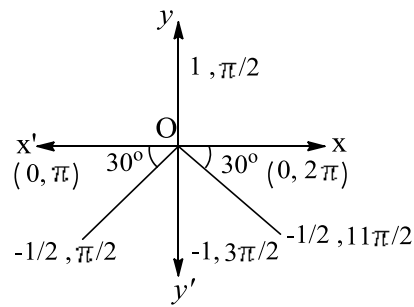
317 (a)

We have,

$$\begin{aligned} I &= \int \frac{\log(x+1) - \log x}{x(x+1)} dx = \int \frac{\log(1+1/x)}{(x^2+x)} dx \\ \Rightarrow I &= \int \frac{\log(1+1/x)}{(x^2(1+1/x))} dx \\ &= - \int \log\left(1 + \frac{1}{x}\right) d\left\{\log\left(1 + \frac{1}{x}\right)\right\} \\ \Rightarrow I &= -\frac{1}{2} \left\{\log\left(1 + \frac{1}{x}\right)\right\}^2 + C \\ \Rightarrow I &= -\frac{1}{2} [\log(x+1) - \log x]^2 + C \\ \Rightarrow I &= -\frac{1}{2} [\{\log(x+1)\}^2 + (\log x)^2 - 2\log(x+1) \\ &\quad \cdot \log x] + C \\ \Rightarrow I &= -\frac{1}{2} \{\log(x+1)\}^2 - \frac{1}{2} (\log x)^2 + \log(x+1) \\ &\quad \cdot \log x + C \end{aligned}$$

318 (a)

It is a question of greatest integer function. We have, subdivide the interval π to 2π as under keeping in view that we have to evaluate $[2 \sin x]$



We know that, $\sin \frac{\pi}{6} = \frac{1}{2}$

$$\sin\left(\pi + \frac{\pi}{6}\right) = \sin \frac{7\pi}{6} = -\frac{1}{2}$$

$$\sin \frac{11\pi}{6} = \sin\left(2\pi - \frac{\pi}{6}\right) = -\sin \frac{\pi}{6} = -\frac{1}{2}$$

$$\sin \frac{9\pi}{6} = \sin \frac{3\pi}{6} = -1$$

Hence, we divide the interval

π to 2π as $\left(\pi, \frac{7\pi}{6}\right), \left(\frac{7\pi}{6}, \frac{11\pi}{6}\right), \left(\frac{11\pi}{6}, 2\pi\right)$

$$\sin x = \left(0, -\frac{1}{2}\right), \left(-1, -\frac{1}{2}\right), \left(0, -\frac{1}{2}\right)$$

$$2 \sin x = (0, -1), (-2, -1), (0, -1)$$

$$[2 \sin x] = -1$$

$$\therefore I = I_1 + I_2 + I_3$$

$$= \int -1 dx + \int -2 dx + \int -1 dx$$

Between proper limits

$$= -\frac{\pi}{6} - 2\left(\frac{4\pi}{6}\right) - \frac{\pi}{6} = -\frac{10\pi}{6} = -\frac{5\pi}{3}$$

319 (c)

$$\begin{aligned} \int_0^1 \frac{d}{dx} \left[\sin^{-1} \left(\frac{2x'}{1+x^2} \right) \right] dx \\ = \left[\sin^{-1} \left(\frac{2x}{1+x^2} \right) \right]_0^1 = \sin^{-1}(1) - \sin^{-1}(0) = \frac{\pi}{2} \end{aligned}$$

320 (c)

Let $F(x) = e^x$

Also, $f(x) + g(x) = x^2 \Rightarrow g(x) = x^2 - e^x$

Now, $\int_0^1 f(x)g(x) dx = \int_0^1 e^x(x^2 - e^x) dx$

$$= \int_0^1 x^2 e^x dx - \int_0^1 e^{2x} dx$$

$$= [x^2 e^x - \int 2x e^x dx]_0^1 - \frac{1}{2} [e^{2x}]_0^1$$

$$= [x^2 e^x - 2x e^x + 2e^x]_0^1 - \frac{1}{2} (e^2 - 1)$$

$$= [(1 - 2 + 2)e^1 - (0 - 0 + 2)e^0] - \frac{1}{2} e^2 + \frac{1}{2}$$

$$= e - \frac{e^2}{2} - \frac{3}{2}$$

321 (b)

We have,

$$\begin{aligned}
I &= \int_0^{\pi} \log(1 + \cos x) dx \\
\Rightarrow I &= \int_0^{\pi} \log\left(2 \cos^2 \frac{x}{2}\right) dx \\
\Rightarrow I &= \int_0^{\pi} \left(\log 2 + 2 \log \cos \frac{x}{2}\right) dx \\
\Rightarrow I &= \pi \log 2 + 2 \int_0^{\pi} \log \cos \frac{x}{2} dx \\
\Rightarrow I &= \pi \log 2 + 2 \times 2 \int_0^{\pi/2} \log \cos t dt, \text{ where } t \\
&= \frac{x}{2} \text{ and } dx = 2dt \\
\Rightarrow I &= \pi \log 2 + 4 \times -\frac{\pi}{2} \log 2 = -\pi \log 2
\end{aligned}$$

322 (a)

Putting $\tan^{-1} x = t$, we have

$$\begin{aligned}
I &= \int e^{\tan^{-1} x} \left(\frac{1+x+x^2}{1+x^2}\right) dx \\
\Rightarrow I &= \int e^t (\tan t + \sec^2 t) dt = e^t \tan t + C \\
&= x e^{\tan^{-1} x} + C
\end{aligned}$$

323 (c)

Given,

$$\begin{aligned}
I_1 &= \int_a^{\pi-a} x f(\sin x) dx \\
\text{and } I_2 &= \int_a^{\pi-a} f(\sin x) dx \\
\text{Now, } I_1 &= \int_a^{\pi-a} x f(\sin x) dx \\
&= \int_a^{\pi-a} (\pi - x) f[\sin(\pi - x)] dx \\
&= \int_a^{\pi-a} (\pi - x) f(\sin x) dx \\
&= \int_a^{\pi-a} \pi f(\sin x) dx - I_1 \\
\Rightarrow 2I_1 &= \pi I_2 \Rightarrow I_2 = \frac{2}{\pi} I_1
\end{aligned}$$

324 (b)

$$\begin{aligned}
\int \cos^{-1}\left(\frac{1}{x}\right) dx &= \int \sec^{-1} x \cdot 1 dx \\
&= \sec^{-1} x \int dx - \int \left[\frac{d}{dx} \sec^{-1} x \int dx\right] dx \\
&= x \sec^{-1} x - \int \frac{1}{x\sqrt{x^2-1}} x dx \\
&= x \sec^{-1} x - \int \frac{1}{\sqrt{x^2-1}} dx \\
&= x \sec^{-1} x - \cosh^{-1} x + c
\end{aligned}$$

325 (d)

$$\begin{aligned}
\text{Let } I &= \int_0^1 \frac{dx}{x+\sqrt{1-x^2}} \\
\text{Put } x &= \sin \theta \Rightarrow dx = \cos \theta d\theta \\
I &= \int_0^{\pi/2} \frac{\cos \theta d\theta}{\sin \theta + \cos \theta} \dots (i) \\
\Rightarrow I &= \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2} - \theta\right) d\theta}{\sin\left(\frac{\pi}{2} - \theta\right) + \cos\left(\frac{\pi}{2} - \theta\right)} \\
&= \int_0^{\pi/2} \frac{\sin \theta}{\cos \theta + \sin \theta} d\theta \dots (ii) \\
\text{On adding Eqs. (i) and (ii), we get} \\
2I &= \int_0^{\pi/2} 1 d\theta \Rightarrow I = \frac{\pi}{4}
\end{aligned}$$

326 (b)

Let

$$\begin{aligned}
I &= \int_0^{\sqrt{n}} [x^2] dx \\
\Rightarrow I &= \sum_{r=1}^n \int_{\sqrt{r-1}}^{\sqrt{r}} [x^2] dx \\
\Rightarrow I &= \sum_{r=1}^n \int_{\sqrt{r-1}}^{\sqrt{r}} (r-1) dx \\
\Rightarrow I &= \sum_{r=1}^n (r-1)(\sqrt{r} - \sqrt{r-1}) \\
\Rightarrow I &= (\sqrt{2} - \sqrt{1}) + 2(\sqrt{3} - \sqrt{2}) + 3(\sqrt{4} - \sqrt{3}) \\
&+ \dots + (n-1)(\sqrt{n} - \sqrt{n-1}) \\
\Rightarrow I &= n\sqrt{n} - (\sqrt{1} + \sqrt{2} + \dots + \sqrt{n}) \\
&= n\sqrt{n} - \sum_{r=1}^n \sqrt{r}
\end{aligned}$$

327 (d)

$$\begin{aligned}
\text{Let } I &= \int \frac{1}{x} (\log_e x) dx = \int \frac{1}{x(1+\log_e x)} dx \\
\text{Put } \log_e x &= t \Rightarrow \frac{1}{x} dx = dt \\
\therefore I &= \int \frac{dt}{(1+t)} = \log_e(1+t) + c \\
&= \log_e(1 + \log_e x) + c
\end{aligned}$$

328 (a)

$$\begin{aligned}
I_1 - I_2 &= \int_0^{\pi/2} (\cos \theta - \sin 2\theta) f(\sin \theta + \cos^2 \theta) d\theta \\
\text{Put } \sin \theta + \cos^2 \theta &= t \\
\Rightarrow (\cos \theta - \sin 2\theta) d\theta &= dt \\
\text{Then, } I_1 - I_2 &= \int_1^1 f(t) dt = 0 \\
\therefore I_1 &= I_2
\end{aligned}$$

329 (b)

$$(1) I = \int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x} dx$$

$$= \int_{-\pi/2}^{\pi/2} \sqrt{\cos x} \sin x \, dx$$

$$= 2 \int_0^{\pi/2} \sqrt{\cos x} \sin x \, dx$$

$$= -\frac{4}{3} [\cos^{3/2} x]_0^{\pi/2} = \frac{4}{3}$$

$$(2) I = \int_0^1 |x-1| dx + \int_1^4 |x-1| dx + \int_0^3 |x-3| dx + \int_3^4 |x-3| dx$$

$$= \int_0^1 -(x-1) dx$$

$$+ \int_1^4 (x-1) dx + \int_0^3 -(x-3) dx$$

$$+ \int_3^4 (x-3) dx$$

$$= 10$$

330 (a)

$$\text{Let } I = \int_{-\pi/4}^{\pi/4} \sin^{-4} x \, dx = \int_{-\pi/4}^{\pi/4} \operatorname{cosec}^{-4} x \, dx$$

$$= \int_{-\pi/4}^{\pi/4} (1 + \cot^2 x) \operatorname{cosec}^2 x \, dx$$

$$\text{Put } \cot x = t$$

$$\Rightarrow -\operatorname{cosec}^2 x \, dx = dt$$

$$\therefore I = - \int_{-1}^1 (1+t^2) dt = -2 \int_0^1 (1+t^2) dt$$

$$= -2 \left[t + \frac{t^3}{3} \right]_0^1 = -2 \left[1 + \frac{1}{3} \right] = -\frac{8}{3}$$

331 (d)

We have,

$$\lim_{x \rightarrow \infty} \frac{(\int_0^x e^x \, dx)^2}{\int_0^x e^{2x^2} \, dx} = \lim_{x \rightarrow \infty} \frac{(e^x - 1)^2}{\int_0^x e^{2x^2} \, dx}$$

$$= \lim_{x \rightarrow \infty} \frac{2(e^x - 1)e^x}{\int_0^x e^{2x^2} \, dx} \quad [\text{Using L'Hospital's rule}]$$

$$= 2 \lim_{x \rightarrow \infty} \frac{e^x - 1}{e^{2x^2 - x}}$$

$$= 2 \lim_{x \rightarrow \infty} \frac{e^x}{e^{2x^2 - x(4x-1)}} \quad [\text{Using L'Hospital's rule}]$$

$$= 2 \lim_{x \rightarrow \infty} \frac{1}{e^{2x^2 - 2x(4x-1)}} = 0$$

332 (a)

$$\text{Given, } \int_{\sin x}^1 t^2 f(t) dt = 1 - \sin x$$

$$\text{Now, } \frac{d}{dx} \int_{\sin x}^1 t^2 f(t) dt = \frac{d}{dx} (1 - \sin x)$$

$$\Rightarrow [1^2 f(1)] \cdot (0) - (\sin^2 x) \cdot f(\sin x) \cdot \cos x = -\cos x$$

[by Leibnitz formula]

$$\Rightarrow \text{Put } \sin x = 1/\sqrt{3}$$

$$\therefore f\left(\frac{1}{\sqrt{3}}\right) = (\sqrt{3})^2 = 3$$

333 (b)

We have,

$$f(a-x) + f(a+x) = 0$$

$$\Rightarrow f(2a-x) + f(x) = 0 \quad [\text{On replacing } x \text{ by } x-a]$$

$$\Rightarrow f(2a-x) = -f(x)$$

$$\therefore \int_0^{2a} f(x) dx = \int_0^a \{f(x) + f(2a-x)\} dx$$

$$\therefore \int_0^{2a} f(x) dx = \int_0^a \{f(x) - f(x)\} dx = 0$$

334 (a)

$$\int \frac{dx}{\sin(x-a) \sin(x-b)}$$

$$= \frac{1}{\sin(a-b)} \int \frac{\sin\{(x-b) - (x-a)\}}{\sin(x-a) \sin(x-b)} dx$$

$$= \frac{1}{\sin(a-b)}$$

$$\int \frac{\sin(x-b) \cos(x-a) - \cos(x-b) \sin(x-a)}{\sin(x-a) \sin(x-b)} dx$$

$$= \frac{1}{\sin(a-b)} \left[\int \cot(x-a) dx - \int \cot(x-b) dx \right]$$

$$= \frac{1}{\sin(a-b)} [\log \sin(x-a) - \log \sin(x-b)] + c$$

$$= \frac{1}{\sin(a-b)} \log \left| \frac{\sin(x-a)}{\sin(x-b)} \right| + c$$

335 (b)

Putting $2+x = t^2$, we get

$$I = \int \sqrt{\frac{5-x}{2+x}} dx = 2 \int \sqrt{7-t^2} dt$$

$$\Rightarrow I = t \sqrt{7-t^2} + 7 \sin^{-1} \frac{t}{\sqrt{7}} + C$$

$$\Rightarrow I = \sqrt{x+2} \sqrt{5-x} + 7 \sin^{-1} \frac{\sqrt{x+2}}{7} + C$$

336 (a)

We have,

$$I = \int \frac{2x^2 + 3}{(x^2 - 1)(x^2 + 4)} dx$$

$$= \int \frac{1}{x^2 - 1} dx + \int \frac{1}{x^2 + 4} dx$$

$$\Rightarrow I = \frac{1}{2} \log \left(\frac{x-1}{x+1} \right) + \frac{1}{2} \tan^{-1} \left(\frac{x}{2} \right) + C$$

$$\Rightarrow I = -\frac{1}{2} \log \left(\frac{x+1}{x-1} \right) + \frac{1}{2} \tan^{-1} x + C$$

$$\therefore a = -1/2 \text{ and } b = 1/2$$

337 (c)

$$\int e^x \frac{(x-1)}{x^2} dx$$

$$= \int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx$$

$$= \frac{e^x}{x} + c$$

338 (d)

We have,

$$I = \int_0^{3\alpha} \operatorname{cosec}(x - \alpha) \operatorname{cosec}(x - 2\alpha) dx$$

$$\Rightarrow I = \frac{1}{\sin \alpha} \int_0^{3\alpha} \frac{\sin \alpha}{\sin(x - \alpha) \sin(x - 2\alpha)} dx$$

$$\Rightarrow I = \frac{1}{\sin \alpha} \int_0^{3\alpha} \frac{\sin\{(x - \alpha) - (x - 2\alpha)\}}{\sin(x - \alpha) \sin(x - 2\alpha)} dx$$

$$\Rightarrow I = \frac{1}{\sin \alpha} \int_0^{3\alpha} \cot(x - 2\alpha) - \cot(x - \alpha) dx$$

$$\Rightarrow I = \frac{1}{\sin \alpha} \left[\log \frac{\sin(x - 2\alpha)}{\sin(x - \alpha)} \right]_0^{3\alpha}$$

$$\Rightarrow I = \frac{1}{\sin \alpha} \left[\log \frac{\sin \alpha}{\sin 2\alpha} - \log \frac{\sin 2\alpha}{\sin \alpha} \right]$$

$$\Rightarrow I = \frac{1}{\sin \alpha} \left[\log \left(\frac{\sin \alpha}{2 \sin \alpha \cos \alpha} \right) \right]$$

$$= \frac{2}{\sin \alpha} \log \left(\frac{1}{2} \sec \alpha \right)$$

$$\Rightarrow I = 2 \operatorname{cosec} \alpha \log \left(\frac{1}{2} \sec \alpha \right)$$

339 (a)

$$\int_0^3 \frac{3x + 1}{x^2 + 9} dx = \frac{3}{2} \int_0^3 \frac{2x}{x^2 + 9} dx$$

$$+ \int_0^3 \frac{1}{x^2 + 9} dx$$

$$= \frac{3}{2} [\log(x^2 + 9)]_0^3 + \frac{1}{3} \left[\tan^{-1} \frac{x}{3} \right]_0^3$$

$$= \frac{3}{2} [\log 18 - \log 9] + \frac{1}{3} [\tan^{-1}(1) - \tan^{-1}(0)]$$

$$= \frac{3}{2} [\log 2] + \frac{\pi}{12}$$

$$= \log(2\sqrt{2}) + \frac{\pi}{12}$$

340 (a)

$$\text{Let } I = \int \frac{dx}{\sqrt{(1-x)(x-2)}}$$

$$= \int \frac{dx}{\sqrt{-x^2 + 3x - 2}} = \int \frac{dx}{\sqrt{\frac{1}{4} - \left(x - \frac{3}{2}\right)^2}}$$

$$= \sin^{-1} \left(\frac{\left(x - \frac{3}{2}\right)}{\frac{1}{2}} \right) + c = \sin^{-1}(2x - 3) + c$$

341 (d)

$$\text{Let } g_n(x) = 1 + x^2 + x^4 + \dots + x^{2n} = \frac{x^{2n+2} - 1}{x^2 - 1}$$

$$\therefore h_n(x) = g'_n(x) = \frac{2x(nx^{2n+2} - (n+1)x^{2n} + 1)}{(x^2 - 1)^2}$$

$$\text{Now, } f(x) = \lim_{n \rightarrow \infty} h_n(x) = \frac{2x}{(x^2 - 1)^2}$$

As $0 < x < 1$

$$\text{Thus, } \int f(x) dx = \int \frac{2x}{(x^2 - 1)^2} dx$$

$$= -\frac{1}{x^2 - 1} = \frac{1}{1 - x^2}$$

342 (c)

$$\int \frac{dx}{\sin x - \cos x + \sqrt{2}}$$

$$= \int \frac{dx}{\sin x \frac{\sqrt{2}}{\sqrt{2}} - \cos x \frac{\sqrt{2}}{\sqrt{2}} + \sqrt{2}}$$

$$= \int \frac{dx}{\sqrt{2}(\sin x \sin \frac{\pi}{4} - \cos x \cos \frac{\pi}{4} + 1)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{1 - \cos \left(x + \frac{\pi}{4}\right)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{1 - \cos 2 \left(\frac{x}{2} + \frac{\pi}{8}\right)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{2 \sin^2 \left(\frac{x}{2} + \frac{\pi}{8}\right)}$$

$$= \frac{1}{2\sqrt{2}} \int \operatorname{cosec}^2 \left(\frac{x}{2} + \frac{\pi}{8}\right) dx$$

$$= -\frac{1}{2\sqrt{2}} \cdot \frac{-\cot \left(\frac{x}{2} + \frac{\pi}{8}\right)}{\frac{1}{2}} + c = \frac{1}{\sqrt{2}} \cot \left(\frac{x}{2} + \frac{\pi}{8}\right) + c$$

343 (a)

$$\text{Let } I = \int_0^{\pi/2} \frac{\cos x}{1 + \sin x} dx$$

$$= [\log(1 + \sin x)]_0^{\pi/2}$$

$$= \log 2 - \log 1 = \log 2$$

344 (b)

$$\text{Let } I = \int_0^1 \frac{2 \sin^{-1} \frac{x}{2}}{x} dx$$

$$\text{Put } \sin^{-1} \frac{x}{2} = t \Rightarrow \frac{x}{2} = \sin t$$

$$\Rightarrow x = 2 \sin t$$

$$\Rightarrow dx = 2 \cos t dt$$

$$\therefore I = \int_0^{\pi/6} \frac{2t}{2 \sin t} \cdot 2 \cos t dt$$

$$= \int_0^{\pi/6} \frac{2t}{\tan t} dt$$

$$= \int_0^{\pi/6} \frac{2x}{\tan x} dx$$

345 (a)

$$\text{Given, } \int_2^e \left(\frac{1}{\log x} - \frac{1}{(\log x)^2} \right) dx = a + \frac{b}{\log 2}$$

$$\text{Put } \log x = z \Rightarrow x = e^z \Rightarrow dx = e^z dz$$

$$\begin{aligned} \therefore \int_2^e \left(\frac{1}{\log x} - \frac{1}{(\log x)^2} \right) dx \\ = \int_{\log 2}^1 \left(\frac{1}{z} - \frac{1}{z^2} \right) e^z dz \\ = \int_{\log 2}^1 e^z \left(\frac{1}{z} + d \left(\frac{1}{z} \right) \right) dz \\ = \left[e^z \cdot \frac{1}{z} \right]_{\log 2}^1 = e - \frac{2}{\log 2} \\ \therefore a = e \text{ and } b = -2 \end{aligned}$$

347 (c)

$$\begin{aligned} \text{Let } I &= \int_0^1 \frac{x dx}{[x+\sqrt{1-x^2}]\sqrt{1-x^2}} \\ \text{Put } x &= \sin \theta \Rightarrow dx = \cos \theta d\theta \\ \therefore I &= \int_0^{\pi/2} \frac{\sin \theta \cos \theta d\theta}{(\sin \theta + \cos \theta) \cdot \cos \theta} \\ \Rightarrow I &= \int_0^{\pi/2} \frac{\sin \theta}{\sin \theta + \cos \theta} d\theta \dots (i) \\ \Rightarrow I &= \int_0^{\pi/2} \frac{\sin \left(\frac{\pi}{2} - \theta \right)}{\sin \left(\frac{\pi}{2} - \theta \right) + \cos \left(\frac{\pi}{2} - \theta \right)} d\theta \\ \Rightarrow I &= \int_0^{\pi/2} \frac{\cos \theta}{\sin \theta + \cos \theta} d\theta \dots (ii) \end{aligned}$$

On adding Eqs. (i) and (ii), we get

$$\begin{aligned} 2I &= \int_0^{\pi/2} \left(\frac{\sin \theta + \cos \theta}{\sin \theta + \cos \theta} \right) d\theta \\ &= \int_0^{\pi/2} 1 d\theta = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4} \end{aligned}$$

348 (d)

$$\begin{aligned} \text{Let } I &= \int_0^1 \cot^{-1}(1-x+x^2) dx \\ &= \int_0^1 \tan^{-1} \left(\frac{1}{x^2-x+1} \right) dx \\ &= \int_0^1 \tan^{-1} \left(\frac{x-(x-1)}{1+x(x-1)} \right) dx \\ &= \int_0^1 \tan^{-1} x dx - \int_0^1 \tan^{-1}(x-1) dx \\ &= \int_0^1 \tan^{-1} x dx - \int_0^1 \tan^{-1}(1-x-1) dx \\ &= 2 \int_0^1 \tan^{-1} x dx \\ &= 2 \left[x \tan^{-1} x - \int \frac{x}{1+x^2} dx \right]_0^1 \\ &= 2 \left[x \tan^{-1} x - \frac{1}{2} \log(1+x^2) \right]_0^1 \\ &= 2 \left[\left(1 \tan^{-1} 1 - \frac{1}{2} \log 2 \right) - \left(0 - \frac{1}{2} \log 1 \right) \right] \end{aligned}$$

$$= \frac{\pi}{2} - \log 2$$

349 (a)

$$\begin{aligned} \text{Let } I &= \int_8^{15} \frac{dx}{(x-3)\sqrt{x+1}} \\ \text{Put } x &= \tan^2 \theta \Rightarrow \theta = \tan^{-1} \sqrt{x} \\ dx &= 2 \tan \theta \sec^2 \theta d\theta \\ \therefore I &= \int_{\tan^{-1} \sqrt{8}}^{\tan^{-1} \sqrt{15}} \frac{2 \tan \theta \sec^2 \theta}{(\tan^2 \theta - 3)\sqrt{\tan^2 \theta + 1}} d\theta \\ &= \int_{\tan^{-1} \sqrt{8}}^{\tan^{-1} \sqrt{15}} \frac{2 \tan \theta \sec^2 \theta}{(\sec^2 \theta - 1 - 3) \sec \theta} d\theta \\ &= \int_{\tan^{-1} \sqrt{8}}^{\tan^{-1} \sqrt{15}} \frac{2 \tan \theta \sec^2 \theta}{(\sec^2 \theta - 4) \sec \theta} d\theta \\ &= \int_{\tan^{-1} \sqrt{8}}^{\tan^{-1} \sqrt{15}} \frac{2 \tan \theta \sec \theta}{(\sec^2 \theta - 4)} d\theta \\ &= \left[\frac{1}{2} \log \left(\frac{\sec \theta - 2}{\sec \theta + 2} \right) \right]_{\tan^{-1} \sqrt{8}}^{\tan^{-1} \sqrt{15}} \left[\because \int \frac{1}{x^2 - a^2} dx \right. \\ &\quad \left. = \frac{1}{2a} \log \left(\frac{x-a}{x+a} \right) \right] \\ &= \frac{1}{2} \left[\log \left(\frac{\sec \tan^{-1} \sqrt{15} - 2}{\sec \tan^{-1} \sqrt{15} + 2} \right) \right. \\ &\quad \left. - \log \left(\frac{\sec \tan^{-1} \sqrt{8} - 2}{\sec \tan^{-1} \sqrt{8} + 2} \right) \right] \\ &= \frac{1}{2} \left[\log \left(\frac{\sec \sec^{-1} 4 - 2}{\sec \sec^{-1} 4 + 2} \right) \right. \\ &\quad \left. - \log \left(\frac{\sec \sec^{-1} 3 - 2}{\sec \sec^{-1} 3 + 2} \right) \right] \\ &= \frac{1}{2} \left[\log \frac{2}{6} - \log \frac{1}{5} = \frac{1}{2} \log \frac{5}{3} \right] \end{aligned}$$

350 (c)

We have,

$$\begin{aligned} \Rightarrow I &= \int \{1 + 2 \tan x (\tan x + \sec x)\}^{1/2} dx \\ \Rightarrow I &= \int \{1 + \tan^2 x + \tan^2 x + 2 \tan x \sec x\}^{1/2} dx \\ \Rightarrow I &= \int (\sec^2 x + \tan^2 x + 2 \tan x \sec x)^{1/2} dx \\ \Rightarrow I &= \int (\tan x + \sec x) dx \\ \Rightarrow I &= \log \sec x + \log(\sec x + \tan x) + C \\ \Rightarrow I &= \log \sec x (\sec x + \tan x) + C \end{aligned}$$

351 (a)

We have,

$$I_1 = \int_0^{\infty} \frac{1}{1+x^4} dx \text{ and } I_2 = \int_0^{\infty} \frac{x^2}{1+x^4} dx$$

Putting $x = \frac{1}{t}$ in I_1 , we get

$$I_1 = \int_{-\infty}^0 \frac{t^4}{1+t^4} \times -\frac{1}{t^2} dt = \int_0^{\infty} \frac{t^2}{t+t^4} dt = I_2 \Rightarrow \frac{I_1}{I_2} = 1$$

352 (c)

$$\text{Let } I = \int \frac{\sin x}{3+4\cos^2 x} dx$$

$$\text{Put } \cos x = t \Rightarrow -\sin x dx = dt$$

$$\therefore I = \int \frac{-dt}{3+4t^2} = \frac{-1}{4} \int \frac{dt}{t^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow I = -\frac{1}{4 \cdot \frac{\sqrt{3}}{2}} \cdot \tan^{-1} \frac{t}{\left(\frac{\sqrt{3}}{2}\right)} + c$$

$$= -\frac{1}{2\sqrt{3}} \tan^{-1} \frac{2t}{\sqrt{3}} + c$$

$$\Rightarrow I = \frac{1}{2\sqrt{3}} \tan^{-1} \left(\frac{2\cos x}{\sqrt{3}} \right) + c$$

353 (c)

$$\text{Let } f(x) = e^{\cos^2 x} \cdot \cos^3(2n+1)x$$

$$\text{Then, } f(\pi-x) = e^{\cos^2(\pi-x)} \cdot \cos^3[(2n+1)\pi - 2n+1x]$$

$$= -e^{\cos^2 x} \cdot \cos^3(2n+1)x$$

$$\Rightarrow f(\pi-x) = -f(x)$$

$\therefore f(x)$ is an odd function.

$$\text{Hence, } \int_0^{\pi} e^{\cos^2 x} \cdot \cos^3(2n+1)x dx = 0$$

354 (b)

$$\text{Given, } \frac{d}{dx} \{f(x)\} = \frac{1}{1+x^2}$$

On integrating both sides, we get

$$f(x) = \tan^{-1} x$$

$$\therefore \frac{d}{dx} f(x^3) = \frac{d}{dx} (\tan^{-1} x^3)$$

$$= \frac{1}{1+(x^3)^2} \cdot 3x^2$$

$$= \frac{3x^2}{1+x^6}$$

355 (d)

$$\text{Let } I = \int_0^{\pi} [\cot x] dx \dots (i)$$

$$\Rightarrow I = \int_0^{\pi} [\cot(\pi-x)] dx$$

$$= \int_0^{\pi} [-\cot x] dx \dots (ii)$$

On adding Eqs. (i) and (ii),

$$2I = \int_0^{\pi} [\cot x] dx + \int_0^{\pi} [-\cot x] dx$$

$$= \int_0^{\pi} (-1) dx$$

$$[\because [x] + [-x] = -, \text{ if } x \notin \mathbb{Z} \text{ and } [x] + [-x] =$$

$$0, \text{ if } x \in \mathbb{Z}]$$

$$= [-x]_0^{\pi} = -\pi$$

$$\therefore I = -\frac{\pi}{2}$$

356 (c)

$$\text{Let } I = \int_{-3}^2 (|x+1| + |x+2| + |x-1|) dx$$

$$\text{Again, let } f(x) = |x+1| + |x+2| + |x-1|$$

$$= \begin{cases} -(x+1) - (x+2) - (x-1), & -3 < x \leq -2 \\ -(x+1) + x+2 - (x-1), & -2 < x \leq -1 \\ 1+x+x+2 - (x-1) & -1 < x \leq 0 \\ 1+x+x+2 - (x-1) & 0 \leq x < 1 \\ 1+x+x+2+x-1 & 1 \leq x < 2 \end{cases}$$

$$= \begin{cases} -3x-2, & -3 < x \leq -2 \\ -x+2, & -2 < x \leq -1 \\ x+4, & -1 \leq x < 1 \\ 3x+2, & 1 \leq x < 2 \end{cases}$$

$$\therefore I = \int_{-3}^{-2} (-3-2) dx + \int_{-2}^{-1} (-x+2) dx$$

$$+ \int_{-1}^1 (x+4) dx + \int_1^2 (3x+2) dx$$

$$= \left[-\frac{3x^2}{2} - 2x \right]_{-3}^{-2} + \left[-\frac{x^2}{2} + 2x \right]_{-2}^{-1}$$

$$+ \left[\frac{x^2}{2} + 4x \right]_{-1}^1 + \left[\frac{3x^2}{2} + 2x \right]_1^2$$

$$= \left[-6 + 4 - \left(-\frac{27}{2} + 6 \right) \right] + \left[-\frac{1}{2} - 2 - (-2 - 4) \right]$$

$$+ \left[\frac{1}{2} + 4 - \left(\frac{1}{2} - 4 \right) \right] + \left[6 + 4 - \left(\frac{3}{2} + 2 \right) \right]$$

$$= \frac{11}{2} + \frac{7}{2} + 8 + \frac{13}{2}$$

$$= \frac{31}{2} + 8 = \frac{47}{2}$$

Alternate

$$\text{Let } I = \int_{-3}^2 \{|x+1| + |x+2| + |x-1|\} dx$$

$$= \int_{-3}^{-1} |x+1| dx$$

$$+ \int_{-1}^2 |x+1| dx + \int_{-3}^{-2} |x+2| dx$$

$$+ \int_{-2}^2 |x+2| dx + \int_{-3}^1 |x-1| dx$$

$$+ \int_1^2 |x-1| dx$$

$$\begin{aligned}
&= - \int_{-3}^{-1} (x+1) dx \\
&\quad + \int_{-1}^2 (x+1) dx \\
&\quad - \int_{-3}^{-2} (x+2) dx \\
&+ \int_{-2}^2 (x+2) dx \\
&\quad - \int_{-3}^1 (x-1) dx + \int_1^2 (x-1) dx \\
&= - \left(\frac{x^2}{2} + x \right)_{-3}^{-1} + \left(\frac{x^2}{2} + x \right)_{-1}^2 - \left(\frac{x^2}{2} + 2x \right)_{-3}^{-2} \\
&+ \left(\frac{x^2}{2} + 2x \right)_{-2}^2 - \left(\frac{x^2}{2} - x \right)_{-3}^1 - \left(\frac{x^2}{2} - x \right)_{1}^2 \\
&= \frac{47}{2}
\end{aligned}$$

357 (b)

$$\begin{aligned}
\int_0^3 |x^3 + x^2 + 3x| dx &= \int_0^3 (x^3 + x^2 + 3x) dx \\
&= \left[\frac{x^4}{4} + \frac{x^3}{3} + \frac{3x^2}{2} \right]_0^3 \\
&= \frac{81}{4} + \frac{27}{3} + \frac{27}{2} = \frac{171}{4}
\end{aligned}$$

358 (b)

$$\begin{aligned}
\int \frac{dx}{\sin x \cos x} &= \int \frac{\cos x}{\sin x \cos^2 x} dx \\
&= \int \frac{1}{\tan x} \sec^2 x dx = \log |\tan x| + c
\end{aligned}$$

359 (b)

$$\begin{aligned}
\text{Let } I &= \int_0^{2n\pi} \left\{ |\sin x| - \frac{1}{2} |\sin x| \right\} dx \\
&= \int_0^{2n\pi} \frac{1}{2} |\sin x| dx \\
&= \frac{1}{2} \left[\int_0^{2\pi} |\sin x| dx + \int_{2\pi}^{4\pi} |\sin x| dx + \dots \right. \\
&\quad \left. + \int_{2(n-1)\pi}^{2n\pi} |\sin x| dx \right]
\end{aligned}$$

$$\begin{aligned}
\text{Now, } I_1 &= \int_0^{2\pi} |\sin x| dx \\
&= \int_0^{\pi} \sin x dx - \int_{\pi}^{2\pi} \sin x dx \\
&= [-\cos x]_0^{\pi} + [\cos x]_{\pi}^{2\pi} = 4 \\
\therefore I &= \frac{1}{2} [4 + 4 + 4 + \dots + n \text{ times}] \\
&= \frac{1}{2} (4n) = 2n
\end{aligned}$$

$$I = \int_0^{2n\pi} \frac{1}{2} |\sin x| dx$$

Since $|\sin x|$ is periodic with period π .

$$\therefore I = 2n \int_0^{\pi} \frac{|\sin x| dx}{2}$$

$$I = n \int_0^{\pi} \sin x dx$$

$$= [-\cos x]_0^{\pi}$$

$$\Rightarrow I = 2n$$

Alternate

$$I = \int_0^{2n\pi} \frac{1}{2} |\sin x| dx$$

Since, $|\sin x|$ is periodic with period π .

$$\therefore I = 2n \int_0^{\pi} \frac{|\sin x|}{2} dx$$

$$= n \int_0^{\pi} \sin x dx = n[-\cos x]_0^{\pi}$$

$$\Rightarrow I = 2n$$

360 (d)

$$\text{Given, } \int_a^b x^3 dx = 0 \text{ and } \int_a^b x^2 dx = \frac{2}{3}$$

If we take $a = -1$ and $b = 1$, then it will satisfy the given integration.

361 (b)

Let

$$I = \int_0^{\pi/2} \operatorname{cosec}(x - \pi/3) \operatorname{cosec}(x - \pi/6) dx$$

$$\Rightarrow I = 2 \int_0^{\pi/2} \frac{\sin[(x - \pi/6) - (x - \pi/3)]}{\sin(x - \pi/6) \sin(x - \pi/3)} dx$$

$$\Rightarrow I = 2 \int_0^{\pi/2} [\cot(x - \pi/3) - \cot(x - \pi/6)] dx$$

$$\Rightarrow I = 2 \left[\log \sin \left(x - \frac{\pi}{3} \right) - \log \sin \left(x - \frac{\pi}{6} \right) \right]_0^{\pi/2}$$

$$\Rightarrow I = 2 \left[\log \left(\frac{\sin(x - \pi/3)}{\sin(x - \pi/6)} \right) \right]_0^{\pi/2}$$

$$\Rightarrow I = 2 \left[\log \left(\frac{1/2}{\sqrt{3}/2} \right) - \log \left(\frac{\sqrt{3}/2}{1/2} \right) \right]$$

$$\Rightarrow I = 2[-\log \sqrt{3} - \log \sqrt{3}] = -4 \log \sqrt{3} = -2 \log 3$$

362 (c)

Primitive function means indefinite integral.

\therefore Primitive function of $f(x)$

$$I = \int \frac{\sqrt{a^2 - x^2}}{x^4} dx \quad (\text{say})$$

$$= \int \frac{x \sqrt{\frac{a^2}{x^2} - 1}}{x \cdot x^3} dx$$

$$= \int \frac{1}{x^3} \cdot \sqrt{\left(\frac{a^2}{x^2} - 1\right)} dx$$

Put $\frac{a^2}{x^2} - 1 = t^2$

$$\Rightarrow -\frac{2a^2}{x^3} dx = 2t dt$$

$$\Rightarrow \frac{1}{x^3} dx = -\frac{1}{a^2} t dt$$

Then, $I = -\frac{1}{a^2} \int t^2 dt$

$$= -\frac{1}{3a^2} t^3 + c$$

$$= -\frac{1}{3a^2} \left(\frac{a^2}{x^2} - 1\right)^{3/2} + c$$

$$= -\frac{(a^2 - x^2)^{3/2}}{3a^2 x^3} + c$$

363 (c)

We have,

$$f(x) = \begin{cases} x, & \text{for } x < 1 \\ x - 1 & \text{for } x \geq 1 \end{cases}$$

$$\Rightarrow x^2 f(x) = \begin{cases} x^3, & \text{for } x < 1 \\ x^3 - x^2 & \text{for } x \geq 1 \end{cases}$$

$$\Rightarrow \int_0^2 x^2 f(x) dx = \int_0^1 x^3 dx + \int_1^2 (x^3 - x^2) dx = \frac{5}{3}$$

364 (d)

$$\int \frac{x^2 + x - 6}{(x-2)(x-1)} dx = \int \frac{x+3}{x-1} dx$$

$$= \int 1 dx + \int \frac{4}{(x-1)} dx$$

$$= x + 4 \log(x-1) + c$$

365 (b)

We have,

$$I = \int \frac{x^3}{(1+x^2)^{1/3}} dx = \frac{1}{2} \int \frac{x^2}{(1+x^2)^{1/3}} \cdot 2x dx$$

$$\Rightarrow I = \frac{1}{2} \int \frac{(1+x^2) - 1}{(1+x^2)^{1/3}} d(1+x^2)$$

$$\Rightarrow I = \frac{1}{2} \int \{(1+x^2)^{2/3} - (1+x^2)^{-1/3}\} d(1+x^2)$$

$$\Rightarrow I = \frac{1}{2} \left\{ \frac{3}{5} (1+x^2)^{5/3} - \frac{3}{2} (1+x^2)^{2/3} \right\} + C$$

$$\Rightarrow I = \frac{1}{2} (1+x^2)^{2/3} \left\{ \frac{3}{5} (1+x^2) - \frac{2}{3} \right\} + C$$

$$\Rightarrow I = \frac{1}{20} (1+x^2)^{2/3} (6x^2 - 9) + C$$

$$= \frac{3}{20} (1+x^2)^{2/3} (2x^2 - 3) + C$$

366 (b)

We have,

$$\int_{-\pi/4}^{\pi/4} \left\{ a|\sin x| + \frac{b \sin x}{1 + \cos x} + c \right\} dx = 0$$

$$\Rightarrow a \int_{-\pi/4}^{\pi/4} |\sin x| dx + b \int_{-\pi/4}^{\pi/4} \frac{\sin x}{1 + \cos x} dx$$

$$+ c \int_{-\pi/4}^{\pi/4} dx = 0$$

$$\Rightarrow 2a \int_0^{\pi/4} |\sin x| dx + b \times 0 + 2c \int_0^{\pi/4} dx = 0$$

$$\Rightarrow 2a \int_0^{\pi/4} \sin x dx + 2c \times \frac{\pi}{4} = 0$$

$$\Rightarrow -2a \left(\cos \frac{\pi}{4} - \cos 0 \right) + \frac{\pi c}{2} = 0$$

$$\Rightarrow -2a \left(\frac{1}{\sqrt{2}} - 1 \right) + \frac{\pi c}{2} = 0 \Rightarrow a(2 - \sqrt{2}) + \frac{\pi c}{2} = 0$$

367 (c)

Let $I = \int_2^4 \{|x-2| + |x-3|\} dx$

$$= \int_2^3 \{(x-2) + (3-x)\} dx$$

$$+ \int_3^4 \{(x-2) + (x-3)\} dx$$

$$= \int_2^3 dx + \int_3^4 (2x-5) dx$$

$$= [x]_2^3 + [x^2 - 5x]_3^4$$

$$= 3 - 2 + [16 - 20 - (9 - 15)]$$

$$= 1 + 2 = 3$$

368 (a)

$g(x) = \int_0^x \cos^4 t dt$ (given)

$$g(x + \pi) = \int_0^{\pi+x} \cos^4 t dt$$

$$= \int_0^{\pi} \cos^4 t dt + \int_{\pi}^{\pi+x} \cos^4 t dt = I_1 + I_2$$

$$\Rightarrow I_1 = g(\pi)$$

$$I_2 = \int_{\pi}^{\pi+x} \cos^4 t dt$$

Put $t = \pi + y \Rightarrow dt = dy$

$$I_2 = \int_0^x \cos^4(y + \pi) dy = \int_0^x [\cos(\pi + y)]^4 dy$$

$$= \int_0^x (-\cos y)^4 dy = \int_0^x \cos^4 y dy = g(x)$$

$$\therefore g(x + \pi) = g(x) + g(\pi)$$

369 (b)

We have,

$$\Rightarrow I = \int \cos^3 x e^{\log(\sin x)} dx = \int \cos^3 x \sin x dx$$

$$\Rightarrow I = - \int \cos^3 x d(\cos x) = -\frac{\cos^4 x}{4} + C$$

370 (a)

$$\text{Put } x + 1 = t^2 \Rightarrow dx = 2t dt$$

$$\text{At } x = 8, t = 3 \text{ and } x = 15, t = 4$$

$$\begin{aligned} \therefore I &= \int_3^4 \frac{2t dt}{(t^2 - 1 - 3)t} \\ &= \int_3^4 \frac{2dt}{(t^2 - 4)} = 2 \cdot \frac{1}{4} \left[\log \frac{t-2}{t+2} \right]_3^4 \\ &= \frac{1}{2} \left[\log \frac{1}{3} - \log \frac{1}{5} \right] = \frac{1}{2} \log \frac{5}{3} \end{aligned}$$

371 (a)

$$\text{Using } \sin^2 x = \frac{1 - \cos 2x}{2} \text{ and } \cos^2 x = \frac{1 + \cos 2x}{2},$$

We get

$$\begin{aligned} I &= \int \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx = \int \frac{(1 - \cos 2x)^2}{2(1 + \cos^2 2x)} dx \\ \Rightarrow I &= \frac{1}{2} \int 1 - \frac{2 \cos 2x}{2 - \sin^2 2x} dx \\ \Rightarrow I &= \frac{1}{2} \left\{ x - \frac{1}{2\sqrt{2}} \log \left| \frac{\sqrt{2} + \sin 2x}{\sqrt{2} - \sin 2x} \right| \right\} + C \end{aligned}$$

372 (c)

We have,

$$f(x) = \begin{cases} \int_{-1}^x -t dt, & -1 \leq x \leq 0 \\ \int_{-1}^0 -t dt + \int_0^x t dt, & x \geq 0 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} \frac{1}{2}(1 - x^2), & -1 \leq x \leq 0 \\ \frac{1}{2}(1 + x^2), & x \geq 0 \end{cases}$$

373 (b)

Since $\sqrt{1+x^2} > x^2$ for all $x \in [1, 2]$. Therefore,

$$\begin{aligned} \frac{1}{\sqrt{1+x^2}} &< \frac{1}{x} \text{ for all } x \in [1, 2] \\ \Rightarrow \int_1^2 \frac{1}{\sqrt{1+x^2}} dx &< \int_1^2 \frac{1}{x} dx \Rightarrow I_1 < I_2 \end{aligned}$$

374 (a)

$$\begin{aligned} \text{Let } f(x) &= (1 - x^2) \sin x \cos^2 x \\ f(-x) &= [1 - (-x)^2][\sin(-x)] \cos^2(-x) \\ &= -(1 - x^2) \sin x \cos^2 x = -f(x) \\ \Rightarrow f(x) &\text{ is an odd function.} \\ \therefore \int_{-\pi}^{\pi} (1 - x^2) \sin x \cos^2 x dx &= 0 \end{aligned}$$

375 (c)

We have,

$$I_n = \int_0^{\pi/2} x^n \sin x dx$$

I II

$$\Rightarrow I_n = [-x^n \cos x]_0^{\pi/2} + n \int_0^{\pi/2} x^{n-1} \cos x dx$$

I II

$$\Rightarrow I_n = n[x^{n-1} \sin x]_0^{\pi/2} - n(n-1) \int_0^{\pi/2} x^{n-2} \sin x dx$$

$$\Rightarrow I_n = n \left(\frac{\pi}{2}\right)^{n-1} - n(n-1)I_{n-2}$$

Putting $n = 4$, we get

$$I_4 + 12I_2 = 4 \left(\frac{\pi}{2}\right)^3$$

376 (b)

We have,

$$\int_0^2 x[x] dx = \int_0^1 x \times 0 dx + \int_1^2 x dx = \frac{3}{2}$$

377 (b)

$$\begin{aligned} \int_{-1}^0 \frac{dx}{x^2 + 2x + 2} &= \int_{-1}^0 \frac{dx}{(x+1)^2 + 1} \\ &= [\tan^{-1}(x+1)]_{-1}^0 \\ &= [\tan^{-1} 1 - \tan^{-1} 0] = \frac{\pi}{4} - 0 = \frac{\pi}{4} \end{aligned}$$

378 (d)

$$\text{Let } I = \int_{-\pi/4}^{\pi/4} x^3 \sin^4 x dx$$

Since, $x^3 \sin^4 x$ is an odd function

$$\therefore I = 0$$

379 (c)

$$\begin{aligned} I_1 &= \int_{1-k}^k xf\{x(1-x)\}dx \\ &= \int_{1-k}^k (1-x)f\{(1-x)\{1-(1-x)\}\}dx \text{ (put } x \\ &\quad = 1-x) \\ &= \int_{1-k}^k (1-x)f\{x(1-x)\}dx \\ &= \int_{1-k}^k f\{x(1-x)\}dx - \int_{1-k}^k xf\{x(1-x)\}dx \\ &= I_2 - I_1 \\ \therefore 2I_1 &= I_2 \\ \Rightarrow \frac{I_1}{I_2} &= \frac{1}{2} \end{aligned}$$

380 (a)

We have,

$$I_n = \int_0^{\pi/4} \tan^n x \, dx$$

$$\therefore I_{n+1} + I_{n-1} = \int_0^{\pi/4} (\tan^{n+1} x + \tan^{n-1} x) \, dx$$

$$\Rightarrow I_{n+1} + I_{n-1} = \int_0^{\pi/4} (\tan^{n-1} x \sec^2 x) \, dx$$

$$\Rightarrow I_{n+1} + I_{n-1} = \left[\frac{\tan^n x}{n} \right]_0^{\pi/4} = \frac{1}{n}$$

$$\Rightarrow n(I_{n+1} + I_{n-1}) = 1$$

$$\Rightarrow \lim_{n \rightarrow \infty} (I_{n+1} + I_{n-1}) = 1$$

381 (c)

$$\int_1^e \frac{1}{x} \, dx = [\log x]_1^e = \log e - \log 1 = 1$$

382 (b)

We have,

$$\lim_{n \rightarrow \infty} \frac{1^{99} + 2^{99} + \dots + n^{99}}{n^{100}} = \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{r^{99}}{n^{100}}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \left(\frac{r}{n} \right)^{99}$$

$$= \int_0^1 x^{99} \, dx = \left[\frac{x^{100}}{100} \right]_0^1 = \frac{1}{100} - 0 = \frac{1}{100}$$

383 (a)

We have,

$$I = \int \frac{\cos 2x}{\cos x} \, dx = \int 2 \cos x - \sec x \, dx$$

$$\Rightarrow I = 2 \sin x - \log(\sec x + \tan x) + C$$

$$\Rightarrow I = 2 \sin x + \log(\sec x - \tan x) + C$$

384 (c)

We have,

$$I = \int f'(ax+b) \{f(ax+b)\}^n \, dx$$

$$\Rightarrow I = \frac{1}{a} \int \{f(ax+b)\}^n d\{f(ax+b)\}$$

$$= \frac{1}{a} \times \frac{\{f(ax+b)\}^{n+1}}{n+1} + C$$

385 (c)

$$\therefore 2^{x^3} < 2^{x^2}, 0 < x < 1 \text{ and } 2^{x^3} > 2^{x^2},$$

$$x > 1$$

$$\therefore I_4 > I_3 \text{ and } I_2 < I_1$$

386 (c)

$$\text{Let } I = \int \frac{\sqrt{\tan x}}{\sin x \cos x} \, dx$$

$$= \int \frac{\sqrt{\sin x}}{\sqrt{\cos x} \sin x \cos x} \, dx$$

$$= \int \frac{1}{\sqrt{\sin x} \cos^{3/2} x} \, dx$$

$$= \int \frac{1}{\sqrt{\tan x} \cos^2 x} \, dx$$

$$= \int \frac{\sec^2 x}{\sqrt{\tan x}} \, dx$$

$$\text{Put } \tan x = t \Rightarrow \sec^2 x \, dx = dt$$

$$\therefore I = \int \frac{1}{\sqrt{t}} \, dt = 2\sqrt{t} = 2\sqrt{\tan x}$$

387 (b)

$$\text{Let } I = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2+r^2}}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r}{n\sqrt{1+(r/n)^2}}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r/n}{\sqrt{1+(r/n)^2}}$$

$$\text{Put } \frac{r}{n} = x, \frac{1}{n} \, dx, \lim_{n \rightarrow \infty} \sum_{r=1}^{2n} = \int_0^2$$

$$\therefore I = \int_0^2 \frac{x}{\sqrt{1+x^2}} \, dx = [\sqrt{1+x^2}]_0^2 = \sqrt{5} - 1$$

388 (a)

$$\therefore I = \int_0^{\infty} \frac{x \log x}{(1+x^2)^2} \, dx$$

$$\text{Put } x = \tan \theta \Rightarrow dx = \sec^2 \theta \, d\theta$$

$$\therefore I = \int_0^{\pi/2} \frac{\tan \theta \log(\tan \theta)}{\sec^4 \theta} \sec^2 \theta \, d\theta$$

$$= \int_0^{\pi/2} \frac{\sin \theta \log(\tan \theta)}{\cos \theta \cdot \frac{1}{\cos^2 \theta}} \, d\theta$$

$$= \int_0^{\pi/2} \sin \theta \cos \theta \log(\tan \theta) \, d\theta$$

$$= \frac{1}{2} \int_0^{\pi/2} \sin 2\theta \log(\tan \theta) \, d\theta$$

$$= 0 \left(\because \int_0^{\pi/2} \sin 2\theta \log \tan \theta \, d\theta = 0 \right)$$

389 (a)

$$\int_{-1}^1 [x[1 + \sin \pi x] + 1] \, dx$$

$$= \int_{-1}^0 [x[1 + \sin \pi x] + 1] \, dx$$

$$+ \int_0^1 [x[1 + \sin \pi x] + 1] \, dx$$

$$\text{Now, } -1 < x < 0 \Rightarrow [1 + \sin \pi x] = 0$$

$$0 < x < 1 \Rightarrow [1 + \sin \pi x] = 1$$

$$\Rightarrow [x[1 + \sin \pi x] + 1] = 1$$

$$\therefore \int_{-1}^1 [x[1 + \sin \pi x] + 1] dx = 2$$

390 (d)

We have,

$$I = \int_0^{\infty} e^{-ax^2} dx$$

$$\Rightarrow I = \frac{1}{\sqrt{a}} \int_0^{\infty} e^{-(\sqrt{a}x)^2} d(\sqrt{a}x)$$

$$\Rightarrow I = \frac{1}{\sqrt{a}} \times \frac{\sqrt{\pi}}{2} \Rightarrow I = \frac{1}{2} \frac{\sqrt{\pi}}{a} \left[\because \int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2} \right]$$

391 (d)

Let $I = \int \frac{2x}{\sqrt{1-4x}} dx$. Then,

$$I = \frac{1}{\log 2} \int \frac{1}{1-(2^x)^2} d(2^x) = \frac{1}{\log 2} \sin^{-1}(2^x) + C$$

$$\therefore K = \frac{1}{\log 2}$$

392 (d)

We have,

$$I = \int_{-\pi}^{\pi} (\cos ax - \sin bx)^2 dx$$

$$\Rightarrow I = \int_{-\pi}^{\pi} (\cos^2 ax - \sin^2 bx - 2 \cos ax \sin bx) dx$$

$$\Rightarrow I = \int_{-\pi}^{\pi} (\cos^2 ax - \sin^2 bx) dx$$

$$- 2 \int_{-\pi}^{\pi} \cos ax \sin bx dx$$

$$\Rightarrow I = 2 \int_{-\pi}^{\pi} (\cos^2 ax + \sin^2 bx) dx - 0$$

$$\Rightarrow I = 2 \int_0^{\pi} \frac{1 + \cos 2ax}{2} + \frac{1 - \cos 2bx}{2} dx$$

$$\Rightarrow I = \int_0^{\pi} 2 + \cos 2ax - \cos 2bx dx = 2\pi$$

393 (d)

$$\text{Let } I = \int \frac{mx^{m+2n-1} - nx^{n-1}}{x^{2m+2n} + 2x^{m+n} + 1} dx$$

$$= \int \frac{mx^{m+2n-1} - nx^{n-1}}{(1+x^{m+n})^2} dx$$

$$\therefore I = \int \frac{mx^{m-1} - \frac{n}{x^{n+1}}}{\left(x^m + \frac{1}{x^n}\right)^2} dx$$

$$\text{Let } x^m + \frac{1}{x^n} = t$$

$$\Rightarrow mx^{m-1} - \frac{n}{x^{n+1}} dx = dt$$

$$\therefore I = \int \frac{1}{t^2} dt = -\frac{1}{t} + c$$

$$= \frac{-1}{\left(x^m + \frac{1}{x^n}\right)} + c = \frac{-x^n}{x^{m+n} + 1} + c$$

394 (b)

We have,

$$I = \int_0^{16\pi/3} |\sin x| dx = \int_0^{5\pi} |\sin x| dx + \int_{5\pi}^{16\pi/3} |\sin x| dx$$

$$\Rightarrow I$$

$$= 5 \int_0^{\pi} |\sin x| dx$$

$$+ \int_0^{\pi/3} |\sin x| dx$$

$$[\because |\sin x| \text{ is periodic with period } \pi]$$

$$\Rightarrow I = 5 \int_0^{\pi} \sin x dx + \int_0^{\pi/3} \sin x dx$$

$$= 5 \times 2 + \left(-\frac{1}{2} + 1\right) = \frac{21}{2}$$

395 (d)

$$\text{Let } I = \int_{-\pi/2}^{\pi/2} \{f(x) + f(-x)\} \{g(x) - g(-x)\} dx$$

Again, let

$$h(x) = \{f(x) + f(-x)\} \{g(x) - g(-x)\}$$

$$\Rightarrow h(-x) = \{f(-x) + f(x)\} \{g(-x) - g(x)\}$$

$$\Rightarrow h(-x) = -h(x)$$

Hence, $h(x)$ is an odd function

$$\therefore I = 0$$

396 (a)

We have,

$$I = \int \frac{\sqrt{\tan x}}{\sin x \cos x} dx = \int \frac{\sqrt{\tan x}}{\tan x} \sec^2 x dx$$

$$\Rightarrow I = \int (\tan x)^{-1/2} d(\tan x) = 2\sqrt{\tan x} + C$$

398 (c)

We have,

$$\int_{-\pi/3}^{\pi/3} \left\{ \frac{a}{3} |\tan x| + \frac{6 \tan x}{1 + \sec x} + c \right\} dx = 0$$

$$\Rightarrow \frac{a}{3} \int_{-\pi/3}^{\pi/3} |\tan x| dx + b \int_{-\pi/3}^{\pi/3} \frac{\tan x}{1 + \sec x} dx + c \int_{-\pi/3}^{\pi/3} dx = 0$$

$$\Rightarrow 2a \int_0^{\pi/3} \tan x dx + b \times 0 + c \left(\frac{2\pi}{3}\right) = 0$$

$$\Rightarrow \frac{2a}{3} [\log_e |\sec x|]_0^{\pi/3} + \frac{2\pi}{3} c = 0$$

$$\Rightarrow \frac{2a}{3} \ln 2 + \frac{2\pi}{3} c = 0 \Rightarrow c = -\frac{a}{\pi} \ln 2$$

399 (d)

$$\text{Given, } f'(2) = \tan \frac{\pi}{4} = 1, f'(4) = \tan \frac{\pi}{3} = \sqrt{3}$$

$$\therefore \int_2^4 f'(x) f''(x) dx = \left[\frac{1}{2} [f'(x)]^2 \right]_2^4$$

$$= \frac{1}{2} [f'(4)]^2 - \frac{1}{2} [f'(2)]^2$$

$$= \frac{3}{2} - \frac{1}{2} = 1$$

400 (a)

$$\begin{aligned} \text{Let } I &= \int \frac{dx}{\cos^3 x \sqrt{2} \sin 2x} \\ &= \int \frac{dx}{\cos^3 x \sqrt{4} \sin x \cos x} = \frac{1}{2} \int \frac{dx}{\cos^{7/2} x \sin^{1/2} x} \\ &= \frac{1}{2} \int \frac{\sec^4 x}{\sqrt{\tan x}} dx = \frac{1}{2} \int \frac{(1 + \tan^2 x) \sec^2 x}{\sqrt{\tan x}} dx \end{aligned}$$

$$\text{Put } \tan x = t \Rightarrow \sec^2 x dx = dt$$

$$\therefore I = \frac{1}{2} \int \frac{1+t^2}{\sqrt{t}} dt$$

$$= \frac{1}{2} \int t^{-1/2} dt + \frac{1}{2} \int t^{3/2} dt = t^{1/2} + \frac{t^{5/2}}{5} + c$$

$$= \sqrt{\tan x} + \frac{1}{5} \tan^{5/2} x + c$$

401 (b)

$$\text{On putting, } x = \tan \theta \Rightarrow dx = \sec^2 \theta d\theta, \text{ we get}$$

$$f(x) = \int \frac{\tan^2 \theta \cdot \sec^2 \theta}{\sec^2 \theta (1 + \sec \theta)} d\theta$$

$$= \int \frac{\sec^2 \theta - 1}{1 + \sec \theta} d\theta = \int (\sec \theta - 1) d\theta$$

$$= \log(\sec \theta + \tan \theta) - \theta + c$$

$$\Rightarrow f(x) = \log(\sqrt{1+x^2} + x) - \tan^{-1} x + c$$

$$\text{At } x = 0, f(0) = \log(1+0) - 0 + c \Rightarrow c = 0$$

$$\therefore f(x) = \log(\sqrt{1+x^2} + x) - \tan^{-1} x$$

$$\text{At } x = 1, f(1) = \log(1 + \sqrt{2}) - \frac{\pi}{4}$$

402 (b)

We have,

$$I = \int_0^1 \frac{1}{(1+x^2)^{3/2}} dx = \int_0^{\pi/4} \cos \theta, \text{ where } x = \tan \theta$$

$$\Rightarrow I = \frac{1}{\sqrt{2}}$$

403 (a)

$$\text{We have, } I(m, n) = I = \int_0^1 t^m (1+t)^n dt$$

$$\Rightarrow I(m, n) = \left[(1+t)^n \cdot \frac{t^{m+1}}{m+1} \right]_0^1$$

$$- \frac{n}{m+1} \int_0^1 (1+t)^{n-1} t^{m+1} dt$$

$$\Rightarrow I(m, n) = \frac{2^n}{m+1} - \frac{n}{m+1} I(m+1, n-1)$$

404 (b)

$$\text{Let } I = \int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$$

$$\text{Put } \tan^{-1} x = t \Rightarrow \frac{1}{1+x^2} dx = dt$$

$$\therefore I = \int_0^{\pi/4} t dt = \left[\frac{t^2}{2} \right]_0^{\pi/4}$$

$$= \frac{1}{2} \left[\left(\frac{\pi}{4}\right)^2 - 0^2 \right] = \frac{\pi^2}{32}$$

405 (b)

$$\text{Let } I = \int_3^5 \frac{x^2}{x^2-4} dx = \int_3^5 \left(\frac{x^2-4}{x^2-4} + \frac{4}{x^2-4} \right) dx$$

$$= \int_3^5 \left(1 + \frac{4}{x^2-4} \right) dx$$

$$= \left[x + \frac{4}{2 \times 2} \log_e \left(\frac{x-2}{x+2} \right) \right]_3^5$$

$$= \left[5 + \log_e \left(\frac{5-2}{5+2} \right) - 3 - \log_e \left(\frac{3-2}{3+2} \right) \right]$$

$$= 2 + \log_e \left(\frac{3}{7} \right) - \log_e \left(\frac{1}{5} \right)$$

$$= 2 + \log_e \left(\frac{3}{7} \times \frac{5}{1} \right) = 2 + \log_e \left(\frac{15}{7} \right)$$

406 (b)

$$\int \sqrt{x^2 + a^2} dx = \int \sqrt{x^2 + a^2} \cdot 1 dx$$

$$= \sqrt{x^2 + a^2} \int 1 dx$$

$$- \int \left[\frac{d}{dx} (\sqrt{x^2 + a^2}) \int 1 dx \right] dx$$

$$= x\sqrt{x^2 + a^2} - \int \left[\frac{2x}{2\sqrt{x^2 + a^2}} x \right] dx$$

$$= x\sqrt{x^2 + a^2} - \int \left[\frac{x^2 + a^2 - a^2}{\sqrt{x^2 + a^2}} \right] dx$$

$$= x\sqrt{x^2 + a^2} - \int \left[\sqrt{x^2 + a^2} - \frac{a^2}{\sqrt{x^2 + a^2}} \right] dx$$

$$= x\sqrt{x^2 + a^2} - \int \sqrt{x^2 + a^2} dx + a^2 \int \frac{dx}{\sqrt{x^2 + a^2}}$$

$$= x\sqrt{x^2 + a^2} - I + a^2 \log[x + \sqrt{x^2 + a^2}] + c$$

$$\Rightarrow 2I = x\sqrt{x^2 + a^2} + a^2 \log[x + \sqrt{x^2 + a^2}] + c$$

$$\Rightarrow I = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log [x \sqrt{x^2 + a^2}] + c$$

407 (b)

$$\begin{aligned} I_n + I_{n+2} &= \int_0^{\pi/4} \tan^n x (1 \\ &\quad + \tan^2 x) dx \\ &= \int_0^{\pi/4} \tan^n x \sec^2 x dx \end{aligned}$$

$$\text{Put } \tan x = t \Rightarrow \sec^2 x dx = dt$$

$$\therefore I_n + I_{n+2} = \int_0^1 t^n dt = \frac{1}{n+1}$$

$$\begin{aligned} \Rightarrow \lim_{n \rightarrow \infty} n[I_n + I_{n+2}] \\ &= \lim_{n \rightarrow \infty} \frac{n}{n+1} \\ &= \lim_{n \rightarrow \infty} \frac{1}{\left(1 + \frac{1}{n}\right)} = 1 \end{aligned}$$

408 (b)

$$\text{Put } x + 1 = t^2$$

$$\Rightarrow dx = 2t dt$$

$$\Rightarrow x^2 + 1 = (t^2 - 1)^2 + 1$$

$$= t^4 - 2t^2 + 2$$

\therefore Given integral

$$= \int (t^4 - 2t^2 + 2)t \cdot 2t dt$$

$$= 2 \int (t^6 - 2t^4 + 2t^2) dt$$

$$= 2 \left[\frac{t^7}{7} - 2 \frac{t^5}{5} + 2 \frac{t^3}{3} \right] + c$$

$$= 2 \left[\frac{(x+1)^{\frac{7}{2}}}{7} - 2 \frac{(x+1)^{\frac{5}{2}}}{5} + 2 \frac{(x+1)^{\frac{3}{2}}}{3} \right] + c$$

409 (c)

$$\int \frac{f'(x)}{f(x) \log[f(x)]} dx = \log[\log f(x)] + c$$

410 (d)

We have, $0 < x < 1$

$$\therefore \frac{1}{2} x^2 < x^2 < x$$

$$\Rightarrow -x^2 > -x$$

$$\Rightarrow e^{-x^2} > e^{-x}$$

$$\Rightarrow e^{-x^2} \cos^2 x > e^{-x} \cos^2 x$$

$$\Rightarrow \int_0^1 e^{-x^2} \cos^2 x dx > \int_0^1 e^{-x} \cos^2 x dx$$

$$\Rightarrow I_2 > I_1 \quad \dots(i)$$

Also, $\cos^2 x \leq 1$

$$\Rightarrow e^{-x^2} \cos^2 x < e^{-x^2}$$

$$\begin{aligned} < e^{-(1/2)x^2} \quad \left[\because -\frac{1}{2}x^2 > -x^2 \right. \\ &\quad \left. > -x \right] \end{aligned}$$

$$\Rightarrow \int_0^1 e^{-x^2} \cos^2 x dx < \int_0^1 e^{-x^2} dx < \int_0^1 e^{-(1/2)x^2} dx$$

$$\Rightarrow I_2 < I_3 < I_4 \quad \dots(ii)$$

From (i) and (ii), we get $I_1 < I_2 < I_3 < I_4$

Hence, I_4 is the greatest integral

411 (c)

We have,

$$I = \int \frac{1}{(1+x^2)\sqrt{1-x^2}} dx$$

$$\Rightarrow I = \int \frac{-t dt}{(t^2+1)\sqrt{t^2-1}}, \text{ where } x = \frac{1}{t}, dx = -\frac{1}{t^2} dt$$

$$\Rightarrow I = \int \frac{-u du}{(u^2+2)\sqrt{u^2}}, \text{ where } t^2 - 1 = u^2$$

$$\Rightarrow I = - \int \frac{1}{u^2 + (\sqrt{2})^2} du = -\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{u}{\sqrt{2}} \right) + K$$

$$\Rightarrow I = -\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{\sqrt{1-x^2}}{\sqrt{2}x} \right) + K$$

$$\begin{aligned} \Rightarrow I &= -\frac{1}{\sqrt{2}} \left\{ \frac{\pi}{2} - \cot^{-1} \left(\frac{\sqrt{1-x^2}}{\sqrt{2}x} \right) \right\} \\ &\quad + K \quad [\because \tan^{-1} x + \cot^{-1} x = \pi/2] \end{aligned}$$

$$\Rightarrow I = -\frac{1}{\sqrt{2}} \left(\frac{\sqrt{1-x^2}}{\sqrt{2}x} \right) + \left(K - \frac{\pi}{2\sqrt{2}} \right)$$

$$\Rightarrow I = -\frac{1}{\sqrt{2}} \left(\frac{\sqrt{2}x}{\sqrt{1-x^2}} \right) + C, \text{ where } C = K - \frac{\pi}{2\sqrt{2}}$$

412 (c)

$$I = \int_{1/n}^{(an-1)/n} \frac{\sqrt{x}}{\sqrt{a-x} + \sqrt{x}} dx$$

$$= \int_{1/n}^{a-(1/n)} \frac{\sqrt{x}}{\sqrt{a-x} + \sqrt{x}} dx \quad \dots(i)$$

$$= \int_{1/n}^{a-(1/n)} \frac{\sqrt{\frac{1}{n} + a - \frac{1}{n} - x}}{\sqrt{a - \left(\frac{1}{n} + a - \frac{1}{n} - x\right)} + \sqrt{\left(\frac{1}{n} + a - \frac{1}{n} - x\right)}} dx$$

$$\Rightarrow I = \int_{\frac{1}{n}}^{a-\frac{1}{n}} \frac{\sqrt{a-x}}{\sqrt{x} + \sqrt{a-x}} dx \quad \dots(ii)$$

On adding Eqs.(i) and (ii), we get

$$2I = \int_{1/n}^{a-(1/n)} 1 dx = [x]_{\frac{1}{n}}^{a-\frac{1}{n}}$$

$$\Rightarrow 2I = a - \frac{1}{n} - \frac{1}{n} = \frac{na-2}{n}$$

$$\Rightarrow I = \frac{na - 2}{2n}$$

413 (b)

$$\begin{aligned} \text{Let } I &= \int e^x(x^5 + 5x^4 + 1)dx \\ &= \int e^x x^5 dx + 5 \int e^x x^4 dx + \int e^x dx \\ &= x^5 e^x - \int 5x^4 e^x dx + 5 \int e^x x^4 dx + e^x \\ &= x^5 e^x + e^x + c \end{aligned}$$

414 (a)

$$\begin{aligned} \cos x \log\left(\frac{1+x}{1-x}\right) \text{ is an odd function} \\ \therefore \int_{-1/2}^{1/2} \cos x \log\left(\frac{1+x}{1-x}\right) dx = 0 \\ \therefore k = 0 \end{aligned}$$

415 (b)

$$\begin{aligned} \text{Let } I &= \int \sin \sqrt{x} dx \\ \text{Put } \sqrt{x} &= t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt \\ \therefore I &= \int 2t \sin t dt \\ &= 2[-t \cos t + \int \cos t dt] \\ &= 2[-t \cos t + \sin t] + c \\ &= 2[-\sqrt{x} \cos \sqrt{x} + \sin \sqrt{x}] + c \end{aligned}$$

416 (d)

$$\begin{aligned} f'(x) &= (x^2 - 1)(2x) - (x - 1) \\ &= (x - 1)(2x^2 + 2x - 1) \end{aligned}$$

Which is positive for $x > 1$. Hence, f increases in $[1, 2]$

Hence, global maximum of f is $f(2) = 4$

417 (b)

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \frac{\binom{n}{r}}{\sqrt{1 + \left(\frac{r}{n}\right)^2}} &= \frac{1}{2} \int_0^1 \frac{2x}{\sqrt{1+x^2}} dx \\ &= \frac{1}{2} \left[\frac{\sqrt{1+x^2}}{1/2} \right]_0^1 = \sqrt{2} - 1 \end{aligned}$$

418 (d)

We have,

$$\begin{aligned} I &= \int_0^{2\pi} \cos^{99} x dx, \\ \Rightarrow I &= 2 \int_0^{\pi} \cos^{99} x dx \quad [\because \cos^{99}(2\pi - x) \\ &= \cos^{99} x] \end{aligned}$$

$$\text{But, } \int_0^{\pi} \cos^{99} x dx = 0 \quad [\because \cos^{99}(\pi - x) = -\cos^{99} x]$$

$$\therefore I = 2 \times 0 = 0$$

419 (b)

$$\text{Let } I = \int_{-2}^2 (ax^3 + bx + c) dx$$

In the given integral, ax^3 and bx are odd functions. Hence, it depends only on the value of c .

420 (b)

$$\begin{aligned} \text{Let } I &= \int_0^{\pi} x f(\sin x) dx \\ \Rightarrow I &= \int_0^{\pi} (\pi - x) f\{\sin(\pi - x)\} dx \\ &= \pi \int_0^{\pi} f(\sin x) dx - x \int_0^{\pi} f(\sin x) dx \\ \Rightarrow I &= \pi \int_0^{\pi} f(\sin x) dx - I \\ \Rightarrow 2I &= 2\pi \int_0^{\pi/2} f(\sin x) dx \\ \Rightarrow I &= \pi \int_0^{\pi/2} f(\sin x) dx \end{aligned}$$

421 (a)

$$\begin{aligned} \text{Let } I &= \int \frac{\log(x+1) - \log x}{x(x+1)} dx = \int \frac{\log\left(1 + \frac{1}{x}\right)}{x(x+1)} dx \\ &= \int \frac{\log\left(1 + \frac{1}{x}\right)}{x^2 \left(1 + \frac{1}{x}\right)} dx \\ \text{Put } 1 + \frac{1}{x} &= t \Rightarrow -\frac{1}{x^2} dx = dt \\ \therefore I &= - \int \frac{\log t}{t} dt = -\frac{1}{2} (\log t)^2 + c \\ &= -\frac{1}{2} \left[\log\left(1 + \frac{1}{x}\right) \right]^2 + c \\ &= -\frac{1}{2} [\log(x+1) - \log x]^2 + c \\ &= -\frac{1}{2} \{\log(x+1)\}^2 - \frac{1}{2} (\log x)^2 \\ &\quad + \log(x+1) \cdot \log x + c \end{aligned}$$

422 (c)

$$\begin{aligned} \text{Let } I &= \int_0^1 x \left| x - \frac{1}{2} \right| dx \\ &= - \int_0^{1/2} x \left(x - \frac{1}{2} \right) dx + \int_{1/2}^1 x \left(x - \frac{1}{2} \right) dx \\ &= \left[\frac{x^2}{4} - \frac{x^3}{3} \right]_0^{1/2} + \left[\frac{x^3}{3} - \frac{x^2}{4} \right]_{1/2}^1 \\ &= \left(\frac{1}{16} - \frac{1}{24} \right) + \left(\frac{1}{3} - \frac{1}{4} - \frac{1}{24} + \frac{1}{16} \right) = \frac{1}{8} \end{aligned}$$

423 (a)

Let

$$I = \int \frac{a^{x/2}}{\sqrt{a^{-x} - a^x}} dx = \int \frac{a^x}{\sqrt{1 - (a^x)^2}} dx$$

$$\Rightarrow I = \frac{1}{\log a} \int \frac{1}{\sqrt{1^2 - (a^x)^2}} d(a^x)$$

$$= \frac{1}{\log a} \sin^{-1}(a^x) + C$$

424 (c)

$$\text{Let } I = \int_{\log 2}^x \frac{e^u}{e^u(e^u-1)^{1/2}} du$$

$$\text{Put } e^u - 1 = t^2 \Rightarrow e^u du = 2t dt$$

$$\therefore I = \int_1^{\sqrt{e^x-1}} \frac{2t}{(t^2+1)t} dt$$

$$= 2 \int_1^{\sqrt{e^x-1}} \frac{dt}{(1+t^2)}$$

$$= 2 [\tan^{-1}t]_1^{\sqrt{e^x-1}}$$

$$= 2 \left[\tan^{-1} \sqrt{e^x-1} - \frac{\pi}{4} \right] = \frac{\pi}{6} \quad [\text{given}]$$

$$\Rightarrow \tan^{-1} \sqrt{e^x-1} = \frac{\pi}{12} + \frac{\pi}{4} = \frac{\pi}{3}$$

$$\Rightarrow \sqrt{e^x-1} = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$

$$\Rightarrow e^x = 3 + 1 = 4$$

425 (c)

We have,

$$I = \int \{f(x)g''(x) - f''(x)g(x)\} dx$$

$$\Rightarrow I = \int f(x)g''(x) dx - \int f''(x)g(x) dx$$

$$\Rightarrow I = \left\{ f(x)g'(x) - \int f'(x)g'(x) dx \right\}$$

$$- \left\{ g(x)f'(x) - \int g'(x)f'(x) dx \right\}$$

$$\Rightarrow I = f(x)g'(x) - f'(x)g(x)$$

426 (d)

$$\text{Let } I = \int_{-\pi/4}^{\pi/4} x^3 \sin^4 x dx$$

Since, $x^3 \sin^4 x$ is an odd function.

$$\therefore I = 0$$

427 (a)

$$\text{Let } I = \int_0^{\pi/2} \sin^8 x dx = \frac{7.5.3.1}{8.6.4.2} \cdot \frac{\pi}{2} \quad [\text{by Walli's}$$

formula]

$$= \frac{105\pi}{32(4.3.2.1)}$$

$$= \frac{105\pi}{32.4!}$$

428 (b)

We have,

$$I = \int_{-\pi/2}^{\pi/2} \frac{|x|}{8 \cos^2 2x + 1} dx$$

$$= 2 \int_0^{\pi/2} \frac{x}{8 \cos^2 2x + 1} dx$$

$$= 2I_1, \text{ where } I_1 = \int_0^{\pi/2} \frac{x}{8 \cos^2 2x + 1} dx$$

Now,

$$I_1 = \int_0^{\pi/2} \frac{x}{8 \cos^2 2x + 1} dx \quad \dots(i)$$

$$\Rightarrow I_1 = \int_0^{\pi/2} \frac{\frac{\pi}{2} - x}{8 \cos^2(\pi - 2x) + 1} dx$$

$$\Rightarrow I_1 = \int_0^{\pi/2} \frac{\frac{\pi}{2} - x}{8 \cos^2 2x + 1} dx \quad \dots(ii)$$

Adding (i) and (ii), we get

$$2I_1 = \frac{\pi}{2} \int_0^{\pi/2} \frac{x}{8 \cos^2 2x + 1} dx$$

$$\Rightarrow I_1 = \frac{\pi}{4} \int_0^{\pi/2} \frac{\sec^2 2x}{9 + \tan^2 2x} dx$$

$$= 2 \times \frac{\pi}{4} \int_0^{\pi/2} \frac{\sec^2 2x}{9 + \tan^2 2x} dx$$

$$\Rightarrow I_1 = \frac{\pi}{4} \int_0^{\pi/4} \frac{2 \sec^2 2x}{9 + \tan^2 2x} dx$$

$$\Rightarrow I_1 = \frac{\pi}{4} \int_0^{\pi/4} \frac{1}{3^2 + \tan^2 2x} d(\tan 2x)$$

$$\Rightarrow I_1 = \frac{\pi}{4} \times \frac{1}{3} \left[\tan^{-1} \left(\frac{\tan 2x}{3} \right) \right]_0^{\pi/4} = \frac{\pi}{12} \times \frac{\pi}{2} = \frac{\pi^2}{24}$$

$$\text{Hence, } I = 2I_1 = \frac{\pi^2}{12}$$

429 (b)

$$\therefore f(x) \cos x = \frac{1}{2} \cdot 2f(x)f'(x)$$

Then, $f'(x) = \cos x$

$$\therefore f(x) = \sin x + c$$

430 (d)

On putting $y^2 = f(x) = \frac{x+2}{2x+3}$, we have

$$x = \frac{3y^2-2}{1-2y^2} \text{ and } dx = -\frac{2y}{(1-2y^2)^2} dy$$

$$\therefore \int (f(x))^{1/2} \frac{dx}{x} = - \int y \frac{2y}{(1-2y^2)^2} \cdot \frac{1-2y^2}{3y^2-2} dy$$

$$= 2 \int \frac{y^2}{(2y^2-1)(3y^2-2)} dy$$

$$= -2 \int \left[\frac{1}{2y^2-1} - \frac{2}{3y^2-2} \right] dy$$

$$= \frac{1}{\sqrt{2}} \log \left| \frac{1+\sqrt{2y}}{1-\sqrt{2y}} \right| - \sqrt{\frac{2}{3}} \log \left| \frac{\sqrt{3y}+\sqrt{2}}{\sqrt{3y}-\sqrt{2}} \right| + c$$

Thus, $g(x) = \log|x|$ and $h(x) = \log|x|$

431 (c)

Let $5^{5^{5^x}} = t$. Then, $5^{5^{5^x}} \cdot 5^{5^x} \cdot 5^x (\log 5)^3 dx = dt$,

$$\begin{aligned} \therefore I &= \int 5^{5^{5^x}} \cdot 5^{5^x} \cdot 5^x dx = \frac{1}{(\log 5)^3} \int 1 \cdot dt \\ &= \frac{5^{5^{5^x}}}{(\log 5)^3} + C \end{aligned}$$

432 (a)

$$\begin{aligned} f(t) &= \int_{-t}^t \frac{e^{-|x|}}{2} dx = 2 \int_0^t \frac{e^{-x}}{2} dx \\ &= -[e^{-x}]_0^t = -e^{-t} + 1 \end{aligned}$$

Now, $\lim_{t \rightarrow \infty} f(t) = -\lim_{t \rightarrow \infty} e^{-t} + 1 = 1$

433 (b)

We have,

$$\begin{aligned} I &= \int \frac{\cos 2x}{(\sin x + \cos x)^2} dx = \int \frac{\cos x - \sin x}{(\sin x + \cos x)^2} dx \\ \Rightarrow I &= \log(\sin x + \cos x) + C \end{aligned}$$

434 (c)

The primitive of the given function is

$$\begin{aligned} &\int \frac{dx}{\sqrt{\sin^3 x \sin(x + \theta)}} \\ &= \int \frac{dx}{\sqrt{\sin^3 x (\sin x \cos \theta + \cos x \sin \theta)}} \\ &= \int \frac{\operatorname{cosec}^2 x dx}{\sqrt{\cos \theta + \cot x \sin \theta}} \\ \text{Put } \cot x &= t \Rightarrow -\operatorname{cosec}^2 x dx = dt \\ &= -\frac{1}{\sqrt{\sin \theta}} \int \frac{dt}{\sqrt{\cot \theta + 1}} \\ &= -\frac{2}{\sqrt{\sin \theta}} (\cot \theta + t)^{1/2} + c \\ &= -\frac{2}{\sin \theta} (\cos \theta + t \sin \theta)^{1/2} + c \\ &= \frac{-2 \operatorname{cosec} \theta (\sin x \cos \theta + \sin \theta \cos x)^{1/2}}{\sqrt{\sin x}} + c \\ &= -2 \operatorname{cosec} \theta \left(\frac{\sin(\theta + x)}{\sin x} \right)^{1/2} + c \end{aligned}$$

435 (b)

We have,

$$\begin{aligned} I &= \int \frac{x^4 - 1}{x^2 \sqrt{x^4 + x^2 + 1}} dx \\ &= \int \frac{(x^2 - 1)(x^2 + 1)}{x^2 \sqrt{x^4 + x^2 + 1}} dx \\ \Rightarrow I &= \int \frac{\left(x + \frac{1}{x}\right) \left(x - \frac{1}{x}\right)}{\sqrt{x^2 + \frac{1}{x^2} + 1}} dx \\ &= \int \frac{\left(x + \frac{1}{x}\right)}{\sqrt{\left(x + \frac{1}{x}\right)^2 - 1^2}} d\left(x + \frac{1}{x}\right) \end{aligned}$$

$$\Rightarrow I = \frac{1}{2} \int \frac{2\left(x + \frac{1}{x}\right)}{\sqrt{\left(x + \frac{1}{x}\right)^2 - 1^2}} d\left(x + \frac{1}{x}\right)$$

$$\Rightarrow I = \frac{1}{2} \int \frac{1}{\sqrt{\left(x + \frac{1}{x}\right)^2 - 1^2}} d\left\{\left(x + \frac{1}{x}\right)^2 - 1^2\right\}$$

$$\Rightarrow I = \sqrt{\left(x + \frac{1}{x}\right)^2 - 1^2} + C = \sqrt{x^2 + \frac{1}{x^2} + 1} + C$$

$$\Rightarrow I = \sqrt{\frac{x^4 + x^2 + 1}{x}} + C$$

436 (b)

$$\begin{aligned} &\int 32x^3 (\log x)^2 dx \\ &= 32 \left\{ (\log x)^2 \frac{x^4}{4} - \int 2 \log x \cdot \frac{1}{x} \cdot \frac{x^4}{4} dx \right\} \\ &= 8x^4 (\log x)^2 - 16 \int x^3 \log x dx \\ &= 8x^4 (\log x)^2 - 16 \left\{ \log x \cdot \frac{x^4}{4} - \int \frac{1}{x} \cdot \frac{x^4}{4} dx \right\} \\ &= 8x^4 (\log x)^2 - 4x^4 \log x + 4 \int x^3 dx \\ &= 8x^4 (\log x)^2 - 4x^4 + \log x + x^4 + c \\ &= x^4 [8(\log x)^2 - 4 \log x + 1] + c \end{aligned}$$

437 (a)

$$\begin{aligned} \text{Let } I &= \int \frac{e^x(1+\sin x)}{1+\cos x} dx \\ &= \int \frac{1}{2} e^x \sec^2 \frac{x}{2} dx + \int e^x \tan \frac{x}{2} dx \\ &= \frac{1}{2} \left[2e^x \tan \frac{x}{2} - \int 2e^x \tan \frac{x}{2} dx \right] + \int e^x \tan \frac{x}{2} dx \\ &= e^x \tan \frac{x}{2} + c \end{aligned}$$

438 (a)

We have,

$$\begin{aligned} I &= \int \log(\sqrt{1-x} + \sqrt{1+x}) \cdot 1 dx \\ &\quad \text{I} \qquad \qquad \qquad \text{II} \\ \Rightarrow I &= x \log(\sqrt{1-x} + \sqrt{1+x}) \\ &\quad - \int \frac{1}{2} \frac{1}{\sqrt{1-x^2}} \left(\frac{\sqrt{1-x^2}}{x} - \frac{1}{x} \right) \\ &\quad \cdot x dx \\ \Rightarrow I &= x \log(\sqrt{1-x} + \sqrt{1+x}) \\ &\quad - \frac{1}{2} \int \left(1 - \frac{1}{\sqrt{1-x^2}} \right) dx \\ \Rightarrow I &= x \log(\sqrt{1-x} + \sqrt{1+x}) - \frac{x}{2} + \frac{1}{2} \sin^{-1} x + c \end{aligned}$$

Hence, $f(x) = \log(\sqrt{1-x} + \sqrt{1+x})$, $A = -\frac{1}{2}$ and

$B = \frac{1}{2}$

439 (a)

$$\begin{aligned} \text{Let } I &= \int_0^{\pi/4} \log(1 + \tan x) dx \dots (i) \\ \Rightarrow I &= \int_0^{\pi/4} \log \left[1 + \tan \left(\frac{\pi}{4} - x \right) \right] dx \\ &= \int_0^{\pi/4} \log \left[1 + \frac{1 - \tan x}{1 + \tan x} \right] dx \\ &= \int_0^{\pi/4} \log 2 dx - \int_0^{\pi/4} \log(1 + \tan x) dx \\ \Rightarrow I &= \log 2 [x]_0^{\pi/4} - I \\ \Rightarrow 2I &= \frac{\pi}{4} \log_e 2 \Rightarrow I = \frac{\pi}{8} \log_e 2 \end{aligned}$$

440 (a)

$f(x) = |x| \sin^3 x$ is an odd function

$$\therefore \int_{-2}^2 |x| \sin^3 x dx = 0$$

441 (c)

Since, $e^{x-[x]}$ is a periodic function with period 1.

$$\begin{aligned} \therefore \int_0^{1000} e^{x-[x]} dx &= 1000 \int_0^1 e^x dx = 1000 [e^x]_0^1 \\ &= 1000(e - 1) \end{aligned}$$

442 (b)

$$\begin{aligned} \text{Put } \tan^{-1} x &= t \Rightarrow \frac{1}{(1+x^2)} dx = dt \\ \therefore \int \frac{(\tan^{-1} x)^3}{(1+x^2)} dx &= \int t^3 dt = \frac{t^4}{4} + c \\ &= \frac{(\tan^{-1} x)^4}{4} + c \end{aligned}$$

443 (c)

$$\begin{aligned} \text{Let } I &= \int_0^{\pi} \sin^3 \theta d\theta \\ \left[\because \sin \theta > 0 \right. \\ &\left. \text{for } 0 < \theta < \pi \right] \\ &= \int_0^{\pi} \sin \theta (1 - \cos^2 \theta) d\theta \\ \text{Put } \cos \theta &= t \Rightarrow -\sin \theta d\theta = dt \\ \therefore I &= \int_{-1}^1 (1 - t^2) dt = \left[t - \frac{t^3}{3} \right]_{-1}^1 \\ &= 2 - \frac{2}{3} \\ &= \frac{4}{3} \end{aligned}$$

444 (a)

We have, $d[f(x)] = e^{\tan x} \sec^2 x dx$

On integrating w. r. t x , we get

$$f(x) = e^{\tan x} + c$$

445 (c)

We have,

$$\begin{aligned} I &= \int_1^2 \{f(g(x))\}^{-1} f'(g(x)) g'(x) dx \\ \Rightarrow I &= \int_1^2 \frac{1}{f \circ g(x)} f'(g(x)) g'(x) dx \\ \Rightarrow I &= \int_1^2 \frac{1}{f \circ g(x)} d(f \circ g(x)) = [\log(f \circ g(x))]_1^2 \\ \Rightarrow I &= \log\{f(g(2))\} - \log\{f(g(1))\} \\ &= 0 \quad [\because g(1) = g(2)] \end{aligned}$$

446 (d)

Given, $f'(x) \geq 2$

On integrating both sides w. r. t. x , we get

$$f(x) \geq 2x + c$$

$$f(1) \geq 2 \times 1 + c$$

$$\Rightarrow c \leq 4 - 2 \Rightarrow c \leq 2$$

$$\Rightarrow f(x) \geq 2x + 2$$

$$\therefore f(4) \geq 2 \times 4 + 2 = 10$$

447 (c)

$$\begin{aligned} \text{Let } I &= \int \frac{f'(x)}{f(x) \log f(x)} dx \\ \text{Put } \log f(x) &= t \Rightarrow \frac{f'(x)}{f(x)} dx = dt \\ \therefore I &= \int \frac{1}{t} dt = \log t + c \\ &= \log \log f(x) + c \end{aligned}$$

448 (a)

$$\begin{aligned} \text{Let } I &= \frac{2}{3} \int_0^{\pi/2} \frac{\sqrt{\cos \theta}}{(\sqrt{\sin \theta} + \sqrt{\cos \theta})} d\theta \dots (i) \\ \text{And } I &= \frac{2}{3} \int_0^{\pi/2} \frac{\sqrt{\cos(\pi/2 - \theta)}}{\sqrt{\sin(\pi/2 - \theta)} + \sqrt{\cos(\pi/2 - \theta)}} d\theta \\ \Rightarrow I &= \frac{2}{3} \int_0^{\pi/2} \frac{\sqrt{\sin \theta}}{\sqrt{\sin \theta} + \sqrt{\cos \theta}} d\theta \dots (ii) \\ \text{On adding Eqs. (i) and (ii), we get} \\ 2I &= \frac{2}{3} \int_0^{\pi/2} 1 d\theta = \frac{2}{3} [\theta]_0^{\pi/2} \Rightarrow I = \frac{1}{3} \cdot \frac{\pi}{2} = \frac{\pi}{6} \end{aligned}$$

449 (a)

$$\begin{aligned} \lim_{n \rightarrow \infty} \left[\frac{n}{n^2 + 1^2} + \frac{n}{n^2 + 2^2} + \dots + \frac{n}{n^2 + n^2} \right] \\ &= \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{n}{n^2 + r^2} \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \frac{1}{1 + \left(\frac{r}{n}\right)^2} \\ &= \int_0^1 \frac{dx}{1 + x^2} = [\tan^{-1} x]_0^1 = \frac{\pi}{4} \end{aligned}$$

450 (d)

Integrand is an odd function. So, the value of the integral is zero

451 (a)

$$\text{Let } I = \int \frac{\sin x}{\cos x(1+\cos x)} dx$$

$$\text{Put } \cos x = t \Rightarrow -\sin x dx = dt$$

$$\therefore I = \int \frac{-dt}{t(1+t)} = -\int \left[\frac{1}{t} - \frac{1}{(1+t)} \right] dt$$

$$= \log\left(\frac{t+1}{t}\right) + c = f(x) + c \quad [\text{given}]$$

$$\therefore f(x) = \log\left(\frac{t+1}{t}\right) = \log\left(\frac{1+\cos x}{\cos x}\right)$$

452 (a)

Since $f(x)$ satisfies conditions of Rolle's theorem on $[1, 2]$. Therefore, $f(1) = f(2)$

$$\text{Hence, } \int_1^2 f'(x) dx = f(2) - f(1) = 0$$

453 (a)

$$\int e^{2x}(2 \sin 3x + 3 \cos 3x) dx$$

$$= 2 \int e^{2x} \sin 3x + dx + 3 \int e^{2x} \cos 3x dx$$

$$= e^{2x} \sin 3x - 3 \int e^{2x} \cos 3x dx$$

$$+ 3 \int e^{2x} \cos 3x dx$$

$$= e^{2x} \sin 3x + c$$

454 (d)

$$\int_{-2}^2 |[x]| dx$$

$$= \int_{-2}^{-1} |[x]| dx + \int_{-1}^0 |[x]| dx + \int_0^1 |[x]| dx$$

$$+ \int_1^2 |[x]| dx$$

$$= \int_{-2}^{-1} 2 dx + \int_{-1}^0 1 dx + \int_0^1 0 dx + \int_1^2 1 dx$$

$$= 2[x]_{-2}^{-1} + [x]_{-1}^0 + 0 + [x]_1^2$$

$$= 2(-1+2) + (0+1) + (2-1) = 2+1+1 = 4$$

455 (c)

We have,

$$\int_0^{\pi/2} \cos^n x \sin^n x dx = \lambda \int_0^{\pi/2} \sin^n x dx$$

$$\Rightarrow \int_0^{\pi/2} (\sin 2x)^n dx = 2^n \lambda \int_0^{\pi/2} \sin^n x dx$$

$$\Rightarrow \int_0^{\pi} (\sin t)^n dt = 2^{n+1} \lambda \int_0^{\pi/2} \sin^n x dx$$

$$\Rightarrow 2 \int_0^{\pi/2} \sin^n t dt = 2^{n+1} \lambda \int_0^{\pi/2} \sin^n x dx$$

$$\left[\because \int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx \text{ if } f(2a-x) = f(x) \right]$$

$$\Rightarrow 2 = 2^{n+1} \lambda \Rightarrow \lambda = \frac{1}{2^n}$$

456 (a)

Clearly, $\frac{t^2 \sin 2t}{t^2+1}$ is an odd function

$$\therefore \int_{-3}^3 \frac{t^2 \sin 2t}{t^2+1} dt = 0$$

Also,

$$\int_0^1 \frac{1}{t^2+2t \cos \alpha + 1} dt = \frac{1}{\sin \alpha} \left[\tan^{-1} \left(\frac{t+\cos \alpha}{\sin \alpha} \right) \right]_0^1$$

$$= \frac{\alpha}{2 \sin \alpha}$$

Thus the given equation reduces to

$$x^2 \frac{\alpha}{2 \sin \alpha} - 2 = 0 \Rightarrow x = \pm 2 \sqrt{\frac{\sin \alpha}{\alpha}}$$

457 (a)

$$\text{Let } I = \int_{\pi/3}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$$

$$\int_{\pi/6}^{\pi/3} \left(\frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} \right) dx \dots (i)$$

$$\Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} \dots (ii)$$

$$\left(\because \int_a^b f(x) dx = \int_a^b f(a+b-x) dx \right)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_{\pi/6}^{\pi/3} 1 dx = [x]_{\pi/6}^{\pi/3} = \frac{\pi}{6}$$

$$\Rightarrow I = \frac{\pi}{12}$$

458 (d)

$$\int_0^2 [x^2] dx$$

$$= \int_0^1 [x^2] dx + \int_1^{\sqrt{2}} [x^2] dx$$

$$= \int_{\sqrt{2}}^{\sqrt{3}} [x^2] dx + \int_{\sqrt{3}}^2 [x^2] dx$$

$$= \int_0^1 0 dx + \int_1^{\sqrt{2}} 1 dx + \int_{\sqrt{2}}^{\sqrt{3}} 2 dx + \int_{\sqrt{3}}^2 3 dx$$

$$= [x]_1^{\sqrt{2}} + [2x]_{\sqrt{2}}^{\sqrt{3}} + [3x]_{\sqrt{3}}^2$$

$$= 5 - \sqrt{3} - \sqrt{2}$$

459 (a)

$$\text{Given, } \int f(x) \sin x \cos x \, dx = \frac{1}{2(b^2 - a^2)} \log\{f(x)\} + c$$

On differentiating both sides, we get

$$f(x) \sin x \cos x = \frac{1}{2(b^2 - a^2)} \cdot \frac{1}{f(x)} f'(x)$$

$$\Rightarrow 2(b^2 - a^2) \sin x \cos x = \frac{f'(x)}{[f(x)]^2}$$

$$\begin{aligned} \Rightarrow \int (2b^2 \sin x \cos x - 2a^2 \sin x \cos x) \, dx \\ = \int \frac{f'(x)}{[f(x)]^2} \, dx \end{aligned}$$

$$I_1 - I_2 = \int \frac{f'(x)}{[f(x)]^2} \, dx \quad \dots(i)$$

$$\text{Where } I_1 = \int 2b^2 \sin x \cos x \, dx \text{ and } I_2 = \int 2a^2 \sin x \cos x \, dx$$

$$\text{Now, } I_1 = 2b^2 \int \sin x \cos x \, dx$$

$$\text{Put } \cos x = t \Rightarrow -\sin x \, dx = dt$$

$$\therefore I_1 = -\int 2b^2 t \, dt = -b^2 t^2 = -b^2 \cos^2 x$$

$$\text{and } I_2 = \int 2a^2 \sin x \cos x \, dx$$

$$\text{Put } \sin x = t \Rightarrow \cos x \, dx = dt$$

$$\therefore I_2 = \int 2a^2 t \, dt = a^2 t^2 = a^2 \sin^2 x$$

\(\therefore\) From Eq. (i),

$$-b^2 \cos^2 x - a^2 \sin^2 x = -\frac{1}{f(x)}$$

$$\Rightarrow f(x) = \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x}$$

460 (c)

$$\text{Let } I = \int_0^a x f(x) \, dx \quad \dots(i)$$

$$\Rightarrow I = \int_0^a (a - x) f(a - x) \, dx$$

$$I = \int_0^a (a - x) f(x) \, dx$$

$$[\because f(x) = f(a - x) \text{ given}]$$

$$\Rightarrow I = a \int_0^a f(x) \, dx - \int_0^a x f(x) \, dx \quad \dots(ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = a \int_0^a f(x) \, dx$$

$$\Rightarrow I = \frac{a}{2} \int_0^a f(x) \, dx$$

461 (a)

We have,

$$I = \int_{1/e}^e |\log x| \, dx$$

$$\Rightarrow I = \int_{1/e}^e -\log x \, dx + \int_1^e \log x \, dx$$

$$\Rightarrow I = [x - x \log x]_{1/e}^1 + [x \log x - x]_1^e = 2 \left(\frac{e-1}{e} \right)$$

462 (d)

$$\text{Let } I = \int_0^{\pi/8} \cos^3 4\theta \, d\theta$$

$$= \int_0^{\pi/8} \left(\frac{3 \cos 4\theta + \cos 12\theta}{4} \right) d\theta$$

$$= \frac{1}{4} \left[\frac{3 \sin 4\theta}{4} + \frac{\sin 12\theta}{12} \right]_0^{\pi/8}$$

$$= \frac{1}{4} \left[\frac{3}{4} \left(\sin \frac{\pi}{2} - 0 \right) + \frac{1}{12} \left(\sin \frac{3\pi}{2} - 0 \right) \right]$$

$$= \frac{1}{4} \left[\frac{3}{4} + \frac{1}{12} (-1) \right] = \frac{1}{4} \left(\frac{8}{12} \right) = \frac{1}{6}$$

463 (c)

$$I = \int_{-1}^1 \frac{\cosh x}{1 + e^{2x}} \, dx$$

$$= \int_{-1}^1 \frac{e^x + e^{-x}}{2(1 + e^{2x})} \, dx \quad \left[\because \cosh x = \frac{e^x + e^{-x}}{2} \right]$$

$$= \frac{1}{2} \int_{-1}^1 \frac{1 + e^{2x}}{(1 + e^{2x})e^x} \, dx = \frac{1}{2} \int_{-1}^1 e^{-x} \, dx$$

$$= -\frac{1}{2} [e^{-x}]_{-1}^1 = -\frac{1}{2} (e^{-1} - e^1) = \frac{e^2 - 1}{2e}$$

464 (c)

We have,

$$u_{10} = \int_0^{\pi/2} x^{10} \sin x \, dx$$

$$\Rightarrow u_{10} = [-x^{10} \cos x]_0^{\pi/2} + 10 \int_0^{\pi/2} x^9 \cos x \, dx$$

$$\Rightarrow u_{10} = 10 [x^9 \sin x]_0^{\pi/2} - 90 \int_0^{\pi/2} x^8 \sin x \, dx$$

$$= 10 \left(\frac{\pi}{2} \right)^9 - 90 u_8$$

$$\Rightarrow u_{10} + 90 u_8 = 10 \left(\frac{\pi}{2} \right)^9$$

465 (c)

$$\text{Let } I = \int \frac{x^3 \sin[\tan^{-1}(x^4)] \, dx}{1+x^8}$$

$$\text{Put } x^4 = t \Rightarrow 4x^3 \, dx = dt$$

$$\therefore I = \int \frac{1}{4} \cdot \frac{\sin[\tan^{-1}(t)] \, dt}{1+t^2}$$

$$\text{Again, put } \tan^{-1} t = u \Rightarrow \frac{1}{1+t^2} \, dt = du$$

$$\begin{aligned}\therefore I &= \int \frac{1}{4} \sin u \, du = -\frac{1}{4} \cos u + c \\ &= -\frac{1}{4} \cos[\tan^{-1}(x^4)] + c\end{aligned}$$

466 (d)

We have,

$$\begin{aligned}&\sum_{n=1}^{10} \int_{-2n-1}^{-2n} \sin^{27} x \, dx + \sum_{n=1}^{10} \int_{2n}^{2n+1} \sin^{27} x \, dx \\ &= -\sum_{n=1}^{10} \int_{2n}^{2n+1} \sin^{27} t \, dt \\ &\quad + \sum_{n=1}^{10} \int_{2n}^{2n+1} \sin^{27} x \, dx, \text{ where } x \\ &= -t \\ &= -\sum_{n=1}^{10} \int_{2n}^{2n+1} \sin^{27} x \, dx + \sum_{n=1}^{10} \int_{2n}^{2n+1} \sin^{27} x \, dx = 0\end{aligned}$$

467 (d)

$$\begin{aligned}\int \frac{1}{\sin^2 x \cdot \cos^2 x} \, dx &= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} \, dx \\ &= \int \left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} \right) dx \\ &= \int (\sec^2 x + \operatorname{cosec}^2 x) \, dx \\ &= \tan x - \cot x + c\end{aligned}$$

468 (a)

We have,

$$\begin{aligned}I &= \int_{-1}^1 (x - [2x]) \, dx = \int_{-1}^1 x \, dx - \int_{-1}^1 [2x] \, dx \\ \Rightarrow I &= 0 - \left[\int_{-1}^{-1/2} [2x] \, dx + \int_{-1/2}^0 [2x] \, dx + \int_0^{1/2} [2x] \, dx \right. \\ &\quad \left. + \int_{1/2}^1 [2x] \, dx \right] \\ \Rightarrow I &= - \left[\int_{-1}^{-1/2} -2 \, dx + \int_{-1/2}^0 -1 \, dx + \int_0^{1/2} 0 \, dx \right. \\ &\quad \left. + \int_{1/2}^1 1 \, dx \right] \\ \Rightarrow I &= - \left[-2 \left(-\frac{1}{2} + 1 \right) - 1 \left(0 + \frac{1}{2} \right) + 0 \right. \\ &\quad \left. + \left(1 - \frac{1}{2} \right) \right] \\ \Rightarrow I &= - \left[-1 - \frac{1}{2} + \frac{1}{2} \right] = 1\end{aligned}$$

469 (a)

We know that if $f(t)$ is an odd function. Then,

$\int_0^x f(t) \, dt$ is an even function.

Since the function $f(x) = \log \frac{1-x}{1+x}$ is an odd function. Therefore, $F(x)$ is an even function

471 (c)

We have

$$\begin{aligned}\frac{d}{dx} \left\{ \int_{f(x)}^{g(x)} \phi(t) \, dt \right\} \\ = g'(x) \cdot \phi(g(x)) - f'(x) \cdot \phi(f(x))\end{aligned}$$

472 (d)

$$\begin{aligned}&\int_1^3 (x-1)(x-2)(x-3) \, dx \\ &= \int_1^3 (x-1)(x^2-5x+6) \, dx \\ &= \int_1^3 (x^3-6x^2+11x-6) \, dx \\ &= \left[\frac{x^4}{4} - \frac{6x^3}{3} + \frac{11x^2}{2} - 6x \right]_1^3 \\ &= \left[\frac{81}{4} - \frac{162}{3} + \frac{99}{2} - 18 - \left(\frac{1}{4} - \frac{6}{3} + \frac{11}{2} - 6 \right) \right] \\ &= \left[-\frac{27}{12} + \frac{27}{12} \right] = 0\end{aligned}$$

473 (c)

Given,

$$f(x) = \begin{vmatrix} \sin x + \sin 2x + \sin 3x & \sin 2x & \sin 3x \\ 3 + 4 \sin x & 3 & 4 \sin x \\ 1 + \sin x & \sin x & 1 \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 - (C_2 + C_3)$, we get

$$\begin{aligned}&= \begin{vmatrix} \sin x & \sin 2x & \sin 3x \\ 0 & 3 & 4 \sin x \\ 0 & \sin x & 1 \end{vmatrix} \\ &= \sin x (3 - 4 \sin^2 x) = 3 \sin x - 4 \sin^3 x \\ f(x) &= \sin 3x\end{aligned}$$

$$\begin{aligned}\therefore \int_0^{\pi/2} f(x) \, dx &= \int_0^{\pi/2} \sin 3x \, dx \\ &= \left[-\frac{\cos 3x}{3} \right]_0^{\pi/2} = \left(-\frac{1}{3} \right) [\cos 3x]_0^{\pi/2} \\ &= \left(-\frac{1}{3} \right) \left[\cos \left(3 \times \frac{\pi}{2} \right) - \cos 0 \right] \\ &= \left(-\frac{1}{3} \right) (0 - 1) = \frac{-1}{3} (-1) = \frac{1}{3}\end{aligned}$$

474 (c)

We have,

$$\begin{aligned}I &= \int g(x) \{f(x) + f'(x)\} \, dx \\ \Rightarrow I &= \int g(x) f(x) \, dx + \int g(x) f'(x) \, dx \\ \Rightarrow I &= f(x) \left(\int g(x) \, dx \right) - \int \left(f'(x) \int g(x) \, dx \right) \, dx \\ &\quad + \int g(x) f'(x) \, dx\end{aligned}$$

$$\begin{aligned} \Rightarrow I &= f(x)g(x) - \int g(x)g'(x)dx \\ &\quad + \int g(x)g'(x)dx + C \\ \Rightarrow I &= f(x)g(x) + C \quad \left[\because \int g(x)dx = g(x) \right] \end{aligned}$$

475 (a)

$$\begin{aligned} \int \left(\frac{x+2}{x+4} \right)^2 e^x dx &= \int e^x \left[\frac{x^2 + 4x + 4}{(x+4)^2} \right] dx \\ &= \int e^x \left[\frac{x(x+4)}{(x+4)^2} + \frac{4}{(x+4)^2} \right] dx \\ &= \int e^x \left[\frac{x}{x+4} + \frac{4}{(x+4)^2} \right] dx \\ &= \int \frac{e^x x}{x+4} dx + \int \frac{4e^x}{(x+4)^2} dx \\ &= e^x \left(\frac{x}{x+4} \right) - \int \frac{4e^x}{(x+4)^2} dx + \int \frac{4e^x}{(x+4)^2} dx \\ &= \frac{xe^x}{x+4} + c \end{aligned}$$

476 (c)

$$\text{Given, } P = \int_0^{3\pi} f(\cos^2 x) dx \quad \dots(i)$$

$$\text{and } Q = \int_0^\pi f(\cos^2 x) dx \quad \dots(ii)$$

From Eq. (i)

$$P = 3 \int_0^\pi f(\cos^2 x) dx$$

$$\Rightarrow P = 3Q \Rightarrow P - 3Q = 0$$

477 (d)

Let $g(x) = f(\cos^2 x)$. Then,

$$g(x + n\pi) = f\{\cos^2(x + n\pi)\} = f(\cos^2 x) = g(x)$$

$$\therefore \int_0^{n\pi} g(x) dx = n \int_0^\pi g(x) dx$$

$$\Rightarrow \int_0^{3\pi} g(x) dx = 3 \int_0^\pi g(x) dx \quad [\text{Putting } n = 3]$$

$$\Rightarrow \int_0^{3\pi} f(\cos^2 x) dx = 3 \int_0^\pi f(\cos^2 x) dx$$

$$\Rightarrow I_1 = 3I_2$$

478 (c)

We know that $x - [x]$ is a periodic function with period 1 unit. Therefore,

$$\begin{aligned} \int_0^{n[x]} (x - [x]) dx &= n[x] \int_0^1 (x - [x]) dx \\ &= n[x] \int_0^1 x dx = \frac{n}{2}[x] \end{aligned}$$

479 (d)

Put $t = x^2 + 1 \Rightarrow dt = 2x dx$

$$\int_0^2 \frac{x^3}{(x^2 + 1)^{3/2}} dx = \frac{1}{2} \int_1^5 \frac{(t-1)}{t^{3/2}} dt$$

$$\begin{aligned} &= \frac{1}{2} \int_1^5 [t^{-1/2} - t^{-3/2}] dt \\ &= \frac{1}{2} \left[2\sqrt{t} + 2\frac{1}{\sqrt{t}} \right]_1^5 \\ &= \frac{1}{2} \left[2\sqrt{5} + \frac{2}{\sqrt{5}} - 2 - 2 \right] \\ &= \left[\sqrt{5} + \frac{1}{\sqrt{5}} - 2 \right] = \frac{6 - 2\sqrt{5}}{\sqrt{5}} \end{aligned}$$

480 (d)

We have,

$$I = \int_0^{2\pi} |\cos x - \sin x| dx$$

$$\Rightarrow I = \sqrt{2} \int_0^{2\pi} \left| \cos \left(x + \frac{\pi}{4} \right) \right| dx$$

$$\Rightarrow I = \sqrt{2} \int_{\pi/4}^{9\pi/4} |\cos t| dt, \text{ where } t = x + \frac{\pi}{4}$$

$$\Rightarrow I = \sqrt{2} \left\{ \int_{\pi/4}^{\pi/2} \cos t dt + \int_{\pi/2}^{3\pi/2} (-\cos t) dt + \int_{3\pi/2}^{9\pi/4} \cos t dt \right\}$$

$$\begin{aligned} \Rightarrow I &= \sqrt{2} \left[\left(1 - \frac{1}{\sqrt{2}} \right) - (-1 - 1) + \left(\frac{1}{\sqrt{2}} + 1 \right) \right] \\ &= 4\sqrt{2} \end{aligned}$$

481 (c)

$$\text{Let } I = \int_{-\pi/2}^{\pi/2} (x^3 + x \cos x + \tan^5 x + 1) dx$$

Since, x^3 , $x \cos x$ and $\tan^5 x$ are odd functions, therefore

$$I = \int_{-\pi/2}^{\pi/2} 1 dx = [x]_{-\pi/2}^{\pi/2} = \pi$$

482 (a)

$$\because f'(x) = g(x)$$

$$\Rightarrow \int_a^b f(x)g(x) dx = \int_a^b f(x)f'(x) dx$$

$$= \left[\frac{(f(x))^2}{2} \right]_a^b$$

$$= \frac{1}{2} [(f(b))^2 - (f(a))^2]$$

483 (c)

We have,

$$I = \int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}} dx = \int \frac{4e^{2x} + 6}{9e^{2x} - 4} dx$$

$$\begin{aligned} \Rightarrow I &= \int \frac{4e^{2x}}{9e^{2x}-4} dx + 6 \int \frac{1}{9e^{2x}-4} dx \\ \Rightarrow I &= \frac{2}{9} \int \frac{18e^{2x}}{9e^{2x}-4} dx + 6 \int \frac{e^{-2x}}{9-4e^{-2x}} dx \\ \Rightarrow I &= \frac{2}{9} \log(9e^{2x}-4) + \frac{6}{8} \log(9-4e^{-2x}) + C \\ \Rightarrow I &= \frac{2}{9} \log(9e^{2x}-4) + \frac{3}{4} \log(9e^{2x}-4) \\ &\quad - \frac{3}{4} \log e^{2x} + C \\ \Rightarrow I &= \frac{-3}{2}x + \frac{35}{36} \log(9e^{2x}-4) + C \end{aligned}$$

Hence, $A = -\frac{3}{2}$, $b = \frac{35}{36}$ and $C \in R$

484 (b)

$$\begin{aligned} \text{Let } I &= \int \frac{\cos 4x+1}{\cot x-\tan x} dx \\ &= \int \frac{2 \cos^2 2x}{\frac{\cos^2 x - \sin^2 x}{\sin x \cos x}} dx \\ &= \int \frac{2 \cos^2 2x}{\cos 2x} \cdot \sin x \cos x dx \\ &= \int \cos 2x \cdot \sin 2x dx \\ &= \frac{1}{2} \int 2 \cos 2x \sin 2x dx \\ &= \frac{1}{2} \int \sin 4x dx = \frac{1}{2} \frac{(-\cos 4x)}{4} + c \\ \Rightarrow I &= \frac{-\cos 4x}{8} + c = k \cos 4x + c \\ \therefore k &= -\frac{1}{8} \end{aligned}$$

485 (c)

$$\begin{aligned} \text{Let } I &= \int_{-2}^4 |x+1| dx \\ &= \int_{-2}^{-1} -(x+1) dx + \int_{-1}^4 (1+x) dx \\ &= -\left[\frac{x^2}{2} + x\right]_{-2}^{-1} + \left[x + \frac{x^2}{2}\right]_{-1}^4 \\ &= -\left[\frac{1}{2} - 1 - (2 - 2)\right] + \left[4 + 8 - \left(1 + \frac{1}{2}\right)\right] \\ &= \frac{1}{2} + \left(\frac{25}{2}\right) = 13 \end{aligned}$$

486 (c)

$$\begin{aligned} \int_2^3 \frac{dx}{x^2-x} \\ &= \int_2^3 \frac{dx}{x(x-1)} = \int_2^3 \left[\frac{1}{x-1} - \frac{1}{x}\right] dx \\ &= [\log(x-1)]_2^3 - [\log x]_2^3 \\ &= [\log 2 - \log 1] - [\log 3 - \log 2] \\ &= 2 \log 2 - \log 3 = \log \frac{4}{3} \end{aligned}$$

487 (d)

$$\begin{aligned} \text{Let } I &= \int \frac{x^2+1}{x^4-x^2+1} dx = \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}-1} dx \\ &= \frac{\left(1+\frac{1}{x^2}\right)}{\left(x-\frac{1}{x}\right)^2+1} dx \\ \text{Put } x-\frac{1}{x} &= t \Rightarrow \left(1+\frac{1}{x^2}\right) dx = dt \\ \therefore I &= \int \frac{dt}{t^2+1} = \tan^{-1} t + c \\ &= \tan^{-1} \left(x-\frac{1}{x}\right) + c \\ &= \tan^{-1} \left(\frac{x^2-1}{x}\right) + c \end{aligned}$$

488 (c)

$$\begin{aligned} \text{Let } I &= \int e^x \sec x dx + \int e^x \sec x \tan x dx \\ &= e^x \sec x - \int e^x \sec x \tan x dx \\ &\quad + \int e^x \sec x \tan x dx + c \\ &= e^x \sec x + c \end{aligned}$$

489 (d)

$$\begin{aligned} f'(x) &= 2xe^{-(x^2+1)^2} - 2xe^{-x^4} \\ &= 2x \left(e^{-(x^2+1)^2} - e^{-x^4} \right) \quad \left[\text{here } e^{-x^4} \right. \\ &\quad \left. > e^{-(x^2+1)^2} \right] \end{aligned}$$

So, $f'(x) > 0$, when $x < 0$ [as $x^4 < (x^2+1)^2$]

491 (b)

$$\begin{aligned} \text{Given, } \int_0^a x dx &\leq a+4 \\ \Rightarrow \left[\frac{x^2}{2}\right]_0^a &\leq a+4 \Rightarrow \frac{a^2}{2} \leq a+4 \\ \Rightarrow a^2 &\leq 2a+8 \Rightarrow a^2-2a-8 \leq 0 \\ \Rightarrow (a-4)(a+2) &\leq 0 \Rightarrow -2 \leq a \leq 4 \end{aligned}$$

492 (a)

$$\begin{aligned} \text{Given, } u_n &= \int_0^{\pi/4} \tan^n x dx \\ \text{or } u_n &= \int_0^{\pi/4} \tan^{n-2} x (\sec^2 x - 1) dx \\ &= \int_0^{\pi/4} \tan^{n-2} x \sec^2 x dx - \int_0^{\pi/4} \tan^{n-2} x dx \\ \text{On putting } \tan x &= t, \sec^2 x dx = dt \text{ in 1st integral,} \\ \text{we get} \end{aligned}$$

$$\begin{aligned} u_n &= \int_0^1 t^{n-2} dt - u_{n-2} \\ \Rightarrow u_n + u_{n-2} &= \left[\frac{t^{n-1}}{n-1}\right]_0^1 = \left[\frac{1^{n-1}}{n-1} - 0\right] = \frac{1}{n-1} \end{aligned}$$

493 (b)

$$\text{Let } I = \int_0^\pi x \sin^4 x dx \dots (i)$$

$$I = \int_0^\pi (\pi - x) \sin^4 x \, dx \dots \text{(ii)}$$

On adding Eqs. (i) and (ii), we get

$$\begin{aligned} 2I &= \pi \int_0^\pi \sin^4 x \, dx \\ &= 2\pi \int_0^{\pi/2} \sin^4 x \, dx \\ &= 2\pi \cdot \frac{4-1}{4} \cdot \frac{4-3}{4-2} \cdot \frac{\pi}{2} \\ &= 2\pi \cdot \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2} = \frac{3\pi^2}{8} \\ \Rightarrow I &= \frac{3\pi^2}{16} \end{aligned}$$

494 (a)

We have,

$$\begin{aligned} &\int \frac{\sin x - \cos x}{\sqrt{1 - \sin 2x}} e^{\sin x} \cos x \, dx \\ \Rightarrow I &= \int \frac{\sin x - \cos x}{\sin x - \cos x} e^{\sin x} \cos x \, dx \\ &= \int e^{\sin x} \cos x \, dx = e^{\sin x} + C \end{aligned}$$

495 (c)

$$\begin{aligned} \text{Let } I &= \int \frac{dx}{\sqrt{x} + \sqrt[3]{x}} \\ \text{Now, let } x &= t^6 \Rightarrow dx = 6t^5 dt \\ \Rightarrow I &= \int \frac{6t^5}{t^3 + t^2} dt = 6 \int \frac{t^3}{t+1} dt \\ &= 6 \int \left(t^2 - t + 1 - \frac{1}{t+1} \right) dt \\ &= 2t^3 - 3t^2 + 6t - 6 \log(t+1) + C \\ &= 2\sqrt[6]{x} - 3(\sqrt[3]{x}) + 6(\sqrt[6]{x}) - 6 \log(\sqrt[6]{x} + 1) + C \end{aligned}$$

496 (a)

We have,

$$\begin{aligned} I &= \int \sqrt[3]{x} \sqrt{1 + \sqrt[3]{x^4}} dx \\ &= \frac{3}{4} \int \sqrt{1 + x^{4/3}} \times \frac{4}{3} x^{1/3} dx \\ \Rightarrow I &= \frac{3}{4} \int (1 + x^4)^{1/7} d(1 + x^{4/3}) \\ \Rightarrow I &= \frac{3}{4} \times \frac{7}{8} \times (1 + x^{4/3})^{8/7} + C \\ &= \frac{21}{32} (1 + x^{4/3})^{8/7} + C \end{aligned}$$

497 (a)

$$\begin{aligned} \text{Let } I &= \int \sin^{-1} \left(\frac{2x}{1+x^2} \right) dx \\ \text{Put } x &= \tan \theta \Rightarrow dx = \sec^2 \theta \, d\theta \\ \therefore I &= \int \sin^{-1}(\sin 2\theta) \cdot \sec^2 \theta \, d\theta \end{aligned}$$

$$\begin{aligned} &= 2 \int \theta \sec^2 \theta \, d\theta = 2[\theta \tan \theta - \int \tan \theta \, d\theta] \\ &= 2[\theta \tan \theta + \log \cos \theta] + c \\ &= 2 \left[x \tan^{-1} x + \log \frac{1}{\sqrt{1+x^2}} \right] + c \\ &= 2x \tan^{-1} x - \log(1+x^2) + c \\ &= f(x) - \log(1+x^2) + c \quad [\text{given}] \\ \therefore f(x) &= 2x \tan^{-1} x \end{aligned}$$

498 (b)

$$\begin{aligned} \text{Given } I_{10} &= \int_0^{\pi/2} x^{10} \sin x \, dx \\ &= [-x^{10} \cos x]_0^{\pi/2} + \int_0^{\pi/2} 10x^9 \cdot \cos x \, dx \\ &= 0 + [10x^9 \sin x]_0^{\pi/2} - \int_0^{\pi/2} 90x^8 \sin x \, dx \\ &= 10 \left(\frac{\pi}{2} \right)^9 - 90I_8 \\ \Rightarrow I_{10} + 90I_8 &= 10 \left(\frac{\pi}{2} \right)^9 \end{aligned}$$

499 (d)

$$\begin{aligned} \text{Let } I &= \int_0^\pi \frac{x \, dx}{a^2 \cos^2 x + b^2 \sin^2 x} \dots \text{(i)} \\ \text{and } I &= \int_0^\pi \frac{(\pi-x) \, dx}{a^2 \cos^2(\pi-x) + b^2 \sin^2(\pi-x)} \\ \Rightarrow I &= \int_0^\pi \frac{(\pi-x) \, dx}{a^2 \cos^2 x + b^2 \sin^2 x} \dots \text{(ii)} \\ \text{On adding Eqs. (i) and (ii), we get} \\ 2I &= \int_0^\pi \frac{(x + \pi - x) \, dx}{a^2 \cos^2 x + b^2 \sin^2 x} \\ \Rightarrow 2I &= \pi \int_0^\pi \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x} \\ \Rightarrow I &= \frac{\pi}{2} \int_0^\pi \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x} \\ \Rightarrow I &= 2 \cdot \frac{\pi}{2} \int_0^{\pi/2} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x} \end{aligned}$$

On dividing numerator and denominator by $\cos^2 x$, we get

$$\begin{aligned} I &= \pi \int_0^{\pi/2} \frac{\sec^2 x \, dx}{a^2 + b^2 \tan^2 x} \\ \text{Put } b \tan x &= t \Rightarrow b \sec^2 x \, dx = dt \\ \therefore I &= \frac{\pi}{b} \int_0^\infty \frac{dt}{a^2 + t^2} = \frac{\pi}{b} \cdot \frac{1}{a} \left[\tan^{-1} \frac{t}{a} \right]_0^\infty \\ &= \frac{\pi}{ab} \left[\frac{\pi}{2} - 0 \right] = \frac{\pi^2}{2ab} \end{aligned}$$

500 (a)

$$\begin{aligned} &\int_{-1}^1 (x - [x]) \, dx \\ &= \int_{-1}^0 (x + 1) \, dx + \int_0^1 (x - 0) \, dx \end{aligned}$$

$$= \left[\frac{x^2}{2} + x \right]_{-1}^0 + \left[\frac{x^2}{2} \right]_0^1 = \frac{1}{2} + \frac{1}{2} = 1$$

501 (d)

We have,

$$I = \int_{-\pi}^{\pi} (\cos px - \sin qx)^2 dx$$

$$\Rightarrow I = \int_{-\pi}^{\pi} (\cos^2 px + \sin^2 qx - 2 \cos px \sin qx) dx$$

Since $\cos^2 px$ and $\sin^2 qx$ are even functions of x and $\cos px \sin qx$ an odd function. Therefore,

$$\int_{-\pi}^{\pi} \cos^2 px dx = 2 \int_0^{\pi} \cos^2 px dx, \quad \int_{-\pi}^{\pi} \sin^2 qx dx = 2 \int_0^{\pi} \sin^2 qx dx$$

$$\text{and, } \int_{-\pi}^{\pi} \cos px \sin qx dx = 0$$

$$\therefore I = 2 \int_0^{\pi} \cos^2 px dx + 2 \int_0^{\pi} \sin^2 qx dx$$

$$\Rightarrow I = 2 \int_0^{\pi} \left(\frac{1 + \cos 2px}{2} \right) dx + 2 \int_0^{\pi} \left(\frac{1 - \cos 2qx}{2} \right) dx$$

$$\Rightarrow I = \int_0^{\pi} (1 + \cos 2px) dx + \int_0^{\pi} (1 - \cos 2qx) dx$$

$$\Rightarrow I = \left[x + \frac{\sin 2px}{2p} \right]_0^{\pi} + \left[x - \frac{\sin 2qx}{2q} \right]_0^{\pi} = 2\pi$$

502 (d)

We have,

$$\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{r^3}{r^4 + n^4} = \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{(r/n)^3}{1 + (r/n)^4} \cdot \frac{1}{n}$$

$$= \int_0^1 \frac{x^3}{1 + x^4} dx = \frac{1}{4} [\log(1 + x^4)]_0^1 = \frac{1}{4} \log 2$$

503 (b)

We have,

$$\int_{\pi/2}^x \sqrt{3 - 2 \sin^2 u} du + \int_0^y \cos t dt = 0$$

Differentiating w.r.t. x , we get

$$\sqrt{3 - 2 \sin^2 x} + \frac{dy}{dx} \cos y = 0$$

$$\Rightarrow \frac{dy}{dx} = - \frac{\sqrt{3 - 2 \sin^2 x}}{\cos y}$$

504 (b)

$$I_n = \int \tan^n x dx = \int \tan^{n-2} x (\sec^2 x - 1) dx$$

$$= \int \tan^{n-2} x \cdot \sec^2 x dx - I_{n-2}$$

$$\Rightarrow I_n + I_{n-2} = \int \tan^{n-1} x \cdot \sec^2 x dx$$

$$\text{Put } \tan x = t \Rightarrow \sec^2 x dx = dt$$

$$\therefore I_n + I_{n-2} = \int t^{n-2} dt = \frac{t^{n-1}}{n-1} + c$$

$$\Rightarrow I_n + I_{n-2} = \frac{(\tan^{n-1} x)}{n-1} + c$$

505 (c)

We have,

$$I_{n+1} - I_n = 2 \int_0^{\pi} \cos(n+1)x dx = 0$$

$$\therefore I_{n+1} = I_n$$

$$\Rightarrow I_{n+1} = I_n = \dots = I_0 \Rightarrow I_n = \pi \text{ for all } n \geq 0$$

506 (a)

Let

$$I = \int (\sin^6 x + \cos^6 x + 3 \sin^2 x \cos^2 x) dx$$

$$= \int \{ (\sin^2 x)^3 + (\cos^2 x)^3 + 3 \sin^2 x \cos^2 x \} dx$$

$$= \int [(\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x) - \sin^2 x \cos^2 x + 3 \sin^2 x \cos^2 x] dx$$

$$= \int [(\sin^2 x + \cos^2 x) - 3 \sin^2 x \cos^2 x + 3 \sin^2 x \cos^2 x] dx$$

$$= \int 1 dx = x + c$$

507 (a)

$$\int_a^b \frac{|x|}{x} dx = \int_a^0 \frac{|x|}{x} dx + \int_0^b \frac{|x|}{x} dx \quad [\because a < 0 < b]$$

$$= \int_a^0 -1 dx + \int_0^b 1 dx$$

$$= a + b = b - (-a)|b| - |a|$$

508 (a)

$$\int_0^{10\pi} |\sin x| dx$$

$$= 10 \left[\int_0^{\pi/2} \sin x dx \right.$$

$$\left. + \int_{\pi/2}^{\pi} \sin x dx \right]$$

$$[\because |\sin x| \text{ is periodic with period } \pi]$$

$$= 10 [[-\cos x]_0^{\pi/2} + [-\cos x]_{\pi/2}^{\pi}]$$

$$= 10 [1 + 1] = 20$$

510 (a)

$$\begin{aligned} \text{Let } I &= \int \frac{(\sec^2 x - 7)}{\sin^7 x} dx \\ &= \int \frac{\sin^7 x \sec^2 x - 7 \sin^7 x}{(\sin^7 x)^2} dx \\ &= \int \frac{d}{dx} \left(\frac{\tan x}{\sin^7 x} \right) dx \\ &= \frac{\tan x}{\sin^7 x} + c \end{aligned}$$

511 (a)

We have,

$$\begin{aligned} I &= \int_{-1/2}^{1/2} \left| x \cos \frac{\pi}{2} x \right| dx \\ \Rightarrow I &= - \int_{-1/2}^0 x \cos \frac{\pi x}{2} dx + \int_0^{1/2} x \cos \frac{\pi x}{2} dx \\ &= \frac{\pi\sqrt{2} + 4\sqrt{2} - 8}{\pi^2} \end{aligned}$$

512 (b)

$$\text{Let } I = \int \frac{\log_e(\tan x)}{\sin x \cos x} dx = \int \frac{\log_e(\tan x)}{\tan x} \sec^2 x dx$$

$$\text{Put } \tan x = t \Rightarrow \sec^2 x dx = dt$$

$$\therefore I = \int \frac{\log_e t}{t} dt$$

$$\text{Again, put } \log_e t = u \Rightarrow \frac{1}{t} dt = du$$

$$\therefore I = \int u du = \frac{u^2}{2} + c$$

$$= \frac{(\log_e t)^2}{2} + c$$

$$= \frac{1}{2} [\log_e \tan x]^2 + c$$

513 (a)

$$\text{Let } I = \int \frac{(x+2)^2}{(x+4)^2} e^x dx = \int e^x \left[\frac{x^2+4+4x}{(x+4)^2} \right] dx$$

$$\Rightarrow I = \int e^x \left[\frac{x(x+4)}{(x+4)^2} + \frac{4}{(x+4)^2} \right] dx$$

$$\Rightarrow I = \int \frac{e^x x}{x+4} dx + \int \frac{4e^x}{(x+4)^2} dx$$

$$\Rightarrow I = e^x \left(\frac{x}{x+4} \right) - \int \frac{4e^x}{(x+4)^2} dx$$

$$+ \int \frac{4e^x}{(x+4)^2} dx$$

$$\Rightarrow I = \frac{xe^x}{(x+4)} + c$$

514 (b)

We have,

$$I = \int \frac{1}{\sin(x-a) \cos(x-b)} dx$$

$$\Rightarrow I = \frac{1}{\cos(a-b)} \int \frac{\cos\{(x-a) - (x-b)\}}{\sin(x-a) \cos(x-b)} dx$$

$$\Rightarrow I = \frac{1}{\cos(a-b)} \int \{ \cot(x-a) + \tan(x-b) \} dx$$

$$\Rightarrow I = \frac{1}{\cos(a-b)} \{ \log \sin(x-a) - \log \cos(x-b) \} + C$$

$$\Rightarrow I = \frac{1}{\cos(a-b)} \log \left| \frac{\sin(x-a)}{\cos(x-b)} \right| + C$$

515 (a)

We have,

$$I = \int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x} dx$$

$$\Rightarrow I = \int_{-\pi/2}^0 \frac{\cos x}{1+e^x} dx + \int_0^{\pi/2} \frac{\cos x}{1+e^x} dx \quad \dots(i)$$

Putting $x = -t$ in $\int_{-\pi/2}^0 \frac{\cos x}{1+e^x} dx$, we get

$$\int_{-\pi/2}^0 \frac{\cos x}{1+e^x} dx = \int_0^{\pi/2} \frac{e^x \cos x}{1+e^x} dx$$

$$\therefore I = \int_0^{\pi/2} \frac{e^x \cos x}{1+e^x} dx + \int_0^{\pi/2} \frac{\cos x}{1+e^x} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{(1+e^x) \cos x}{(1+e^x)} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \cos x dx = [\sin x]_0^{\pi/2} = 1$$

516 (a)

$$\int_{-1}^3 \left[\tan^{-1} \left(\frac{x}{x^2+1} \right) + \tan^{-1} \left(\frac{x^2+1}{x} \right) \right] dx$$

$$= \int_{-1}^3 \left[\tan^{-1} \left(\frac{x}{x^2+1} \right) \right. \\ \left. + \cot^{-1} \left(\frac{x}{x^2+1} \right) \right] dx \quad \left(\because \tan^{-1} = \cot^{-1} \frac{1}{x} \right)$$

$$= \int_{-1}^3 \left(\frac{\pi}{2} \right) dx = \left[\frac{\pi x}{2} \right]_{-1}^3 = \frac{\pi}{2} [3+1]$$

$$= 2\pi \quad \left(\because \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2} \right)$$

517 (c)

$$\text{We have, } \int_n^{n+1} f(x) dx = n^2$$

Putting $n = 2, -1, 0, 1, 2, 3$, we get

$$\int_{-2}^{-1} f(x) dx = 4, \int_{-1}^0 f(x) dx = 1, \int_0^1 f(x) dx$$

$$= 0, \int_1^2 f(x) dx = 1,$$

$$\int_2^3 f(x) dx = 4 \text{ and } \int_3^4 f(x) dx = 9$$

$$\therefore \int_{-2}^4 f(x) dx = 4 + 1 + 0 + 1 + 4 + 9 = 19$$

518 (a)

We have, $\int_0^{2\pi} (\sin x + |\sin x|) dx$

$$= \int_0^{\pi} (\sin x + \sin x) dx + \int_{\pi}^{2\pi} (\sin x - \sin x) dx$$

$$= \int_0^{\pi} 2 \sin x dx + 0 = 2[-\cos x]_0^{\pi}$$

$$= -2(\cos \pi - \cos 0) = 4$$

519 (c)

$$f(x) = \lim_{y \rightarrow x} \frac{\sin^2 y - \sin^2 x}{y^2 - x^2} \left[\frac{0}{0} \text{ form} \right]$$

$$= \lim_{y \rightarrow x} \frac{2 \sin y \cos y - 0}{2y - 0}$$

$$= \frac{\sin 2x}{2x}$$

$$\therefore \int 4x f(x) dx = \int 4x \left(\frac{\sin 2x}{2x} \right) dx$$

$$= 2 \int \sin 2x dx$$

$$= -\cos 2x + c$$

520 (a)

We have,

$$I = \int \frac{x^4 + 1}{(1 - x^4)^{3/2}} dx = \int \frac{x^2 \left(\frac{1}{x^2} + x^2 \right)}{x^3 \left(\frac{1}{x^2} - x^2 \right)^{3/2}} dx$$

$$\Rightarrow I = \int \frac{x + \frac{1}{x^3}}{\left(\frac{1}{x^2} - x^2 \right)^{3/2}} dx = -\frac{1}{2} \int \frac{-2x - \frac{2}{x^3}}{\left(\frac{1}{x^2} - x^2 \right)^{3/2}} dx$$

$$\Rightarrow I = -\frac{1}{2} \int \left(\frac{1}{x^2} - x^2 \right)^{-3/2} d \left(-x^2 + \frac{1}{x} \right)$$

$$\Rightarrow I = -\frac{1}{2} \left\{ \frac{\left(\frac{1}{x^2} - x^2 \right)^{-1/2}}{-1/2} \right\} + C$$

$$\Rightarrow I = \frac{1}{\sqrt{\frac{1}{x^2} - x^2}} + C = \frac{x}{\sqrt{1 - x^4}} + C$$

521 (a)

We have,

$$I = e^x \frac{x}{(x+1)^2} dx = \int e^x \frac{x+1-1}{(x+1)^2} dx$$

$$\Rightarrow I = \int e^x \left\{ \frac{1}{x+1} + \frac{-1}{(x+1)^2} \right\} dx = e^x \frac{1}{x+1} + C$$

522 (c)

$$\text{Let } I = \int_{-\pi/2}^{\pi/2} \sin|x| dx = 2 \int_0^{\pi/2} \sin x dx$$

$$= 2[-\cos x]_0^{\pi/2} = 2$$

523 (a)

$$\int \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x}$$

$$= \int \frac{\sec^2 x dx}{a^2 \tan^2 x + b^2}$$

$$= \frac{1}{a^2} \int \frac{dt}{t^2 + \left(\frac{b}{a}\right)^2}$$

Where $t = \tan x$

$$= \frac{1}{a^2} \cdot \frac{a}{b} \tan^{-1} \frac{t}{b/a} + c$$

$$= \frac{1}{ab} \tan^{-1} \frac{a(\tan x)}{b} + c$$

524 (a)

$$\int \frac{\sin^8 x - \cos^8 x}{1 - 2 \sin^2 x \cos^2 x} dx$$

$$= \int \frac{(\sin^4 x - \cos^4 x)(\sin^4 x + \cos^4 x)}{(1 - 2 \sin^2 x \cos^2 x)} dx$$

$$= \int \frac{(\sin^4 x - \cos^4 x)(\sin^4 x + \cos^4 x)}{(1 - 2 \sin^2 x \cos^2 x) \times [(\sin^2 x \cos^2 x)^2 - 2 \sin^2 x \cos^2 x]} dx$$

$$= - \int \frac{\cos 2x (1 - 2 \sin^2 x \cos^2 x)}{(1 - 2 \sin^2 x \cos^2 x)} dx$$

$$= - \int \cos 2x dx$$

$$\Rightarrow A \sin 2x + B = -\frac{1}{2} \sin 2x + B \quad (\text{given})$$

$$\therefore A = -\frac{1}{2}$$

525 (a)

$$\text{Let } I = \int \frac{4^{x+1} - 7^{x-1}}{28^x} dx$$

$$= 4 \int \frac{1}{7^x} dx - \frac{1}{7} \int \frac{1}{4^x} dx$$

$$= -\frac{4 \cdot 7^{-x}}{\log_e 7} + \frac{1}{7 \log_e 4} 4^{-x} + c$$

526 (b)

We have,

$$I = \int \frac{1}{5 + 4 \cos x} dx$$

$$\Rightarrow I = \int \frac{1 + \tan^2 x/2}{5(1 + \tan^2 x/2) + 4(1 - \tan^2 x/2)} dx$$

$$\Rightarrow I = 2 \int \frac{1}{3^2 + \left(\tan \frac{x}{2}\right)^2} d \left(\tan \frac{x}{2} \right)$$

$$= \frac{2}{3} \tan^{-1} \left(\frac{\tan x/2}{3} \right) + C$$

Hence, $A = 2/3$ and $B = 1/3$

527 (b)

Since, $f''(x) = \sec^4 x + 4$

$$f''(x) = (1 + \tan^2 x) \sec^2 x + 4$$

$$\Rightarrow f'(x) = \tan x + \frac{\tan^3 x}{3} + 4x + c$$

$$\Rightarrow f'(0) = 0$$

$$\Rightarrow 0 = c$$

$$\text{Then, } f'(x) = \tan x + \frac{\tan^3 x}{3} + 4x$$

$$= \tan x + \frac{1}{3} \tan x (\sec^2 x - 1) + 4x$$

$$\Rightarrow f'(x) = \frac{2}{3} \tan x + \frac{1}{3} \tan x \sec^2 x + 4x$$

$$\therefore f(x) = \frac{2}{3} \log |\sec x| + \frac{\tan^2 x}{6} 2x^2 + d$$

$$\text{But } f(0) = 0$$

$$\Rightarrow d = 0$$

$$\text{Then, } f(x) = \frac{2}{3} \log |\sec x| + \frac{1}{6} \tan^2 x + 2x^2$$

528 (b)

$$\text{Let } I = \int 32x^3 (\log x)^2 dx$$

$$= 32 \left\{ (\log x)^2 \frac{x^4}{4} - \int 2 \log x \cdot \frac{1}{x} \cdot \frac{x^4}{4} dx \right\}$$

$$= 8x^4 (\log x)^2 - 16 \left\{ \log x \cdot \frac{x^4}{4} - \int \frac{1}{x} \cdot \frac{x^4}{4} dx \right\}$$

$$= 8x^4 (\log x)^2 - 4x^4 \log x + x^4 + c$$

$$= x^4 [8 (\log x)^2 - 4 \log x + 1] + c$$

529 (b)

We have,

$$I = \int_1^{\frac{4\sqrt{3}-1}{5}} \frac{x+2}{\sqrt{x^2+2x-3}} dx$$

$$\Rightarrow I = \frac{1}{2} \int_1^{\frac{4\sqrt{3}-1}{5}} \frac{(2x+2)}{\sqrt{x^2+2x-3}} dx$$

$$+ \int_1^{\frac{4\sqrt{3}-1}{5}} \frac{1}{\sqrt{(x+1)^2 - 2^2}} dx$$

$$\Rightarrow I = \left[\sqrt{x^2+2x-3} \right]_1^{\frac{4\sqrt{3}-1}{5}} + \left[\log(x+1) \right]_1^{\frac{4\sqrt{3}-1}{5}}$$

$$+ \left[\log(x+1) \right]_1^{\frac{4\sqrt{3}-1}{5}}$$

$$+ \left[\sqrt{x^2+2x-3} \right]_1^{\frac{4\sqrt{3}-1}{5}}$$

$$\Rightarrow I = \frac{2\sqrt{3}}{3} + \frac{1}{2} \log 3$$

530 (d)

$$\text{Let } I = \int_{-2}^2 |[x]| dx$$

$$\begin{aligned} &= \int_{-2}^{-1} |[x]| dx + \int_{-1}^0 |[x]| dx \\ &\quad + \int_0^1 |[x]| dx + \int_1^2 |[x]| dx \\ &= \int_{-2}^{-1} 2 dx + \int_{-1}^0 1 dx + \int_0^1 0 dx + \int_1^2 1 dx \\ &= 2[x]_{-2}^{-1} + [x]_{-1}^0 + 0 + [x]_1^2 = 4 \end{aligned}$$

531 (a)

$$F = (e) = f(e) + f\left(\frac{1}{e}\right)$$

$$\Rightarrow F(e) = \int_1^e \frac{\log t}{1+t} dt + \int_1^{1/e} \frac{\log t}{1+t} dt$$

On putting $t = \frac{1}{t}$ in second integration, we get

$$F(e) = \int_1^e \frac{\log t}{1+t} dt + \int_1^e \frac{\log t}{t(1+t)} dt$$

$$= \int_1^e \frac{\log t}{t} dt = \left[\frac{(\log t)^2}{2} \right]_1^e$$

$$= \frac{1}{2} [(\log e)^2 - (\log 1)^2] = \frac{1}{2}$$

532 (a)

$$\text{Let } I = \int_{\pi/6}^{\pi/3} \frac{1}{1+\tan^3 x} dx$$

$$= \int_{\pi/6}^{\pi/3} \frac{\cos^3 x}{\sin^3 x + \cos^3 x} dx \dots (i)$$

$$\Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} dx \dots (ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_{\pi/6}^{\pi/3} 1 dx = [x]_{\pi/6}^{\pi/3} = \frac{\pi}{6}$$

$$\Rightarrow I = \frac{\pi}{12}$$

533 (c)

$$\text{Let } f^{-1}(x) = u \Rightarrow x = f(u)$$

$$\Rightarrow dx = f'(u) du$$

$$\therefore I = \int f^{-1}(x) dx = \int u f'(u) du$$

$$\Rightarrow I = uf(u) - \int f(u) du$$

$$= uf(u) - g(u) + c \quad [\because \int f(x) dx = g(x) + c, \text{ given}]$$

$$\therefore \text{On putting } u = f^{-1}(x) \text{ and } f(u) = x$$

$$\text{We get, } I = xf^{-1}(x) - g\{f^{-1}(x)\} + c$$

534 (a)

$$\int \frac{x \log x}{x^2} dx = \log x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2}{2} \log x - \frac{1}{2} \frac{x^2}{2} + c$$

$$= \frac{x^2}{4} (2 \log x - 1) + c$$

535 (a)

We have,

$$I = \int_{1/3}^1 \frac{(x - x^3)^{1/3}}{x^4} dx$$

$$\Rightarrow I = \int_{1/3}^1 \left(\frac{1}{x^2} - 1\right)^{1/3} \frac{1}{x^3} dx$$

Putting $\frac{1}{x^2} - 1 = t$ and $-\frac{2}{x^3} dx = dt$, we get

$$I = \int_0^8 t^{1/3} \times -\frac{1}{2} dt = \frac{1}{2} \int_0^8 t^{1/3} dt = \frac{1}{2} \times \frac{3}{4} [t^{4/3}]_0^8$$

$$\Rightarrow I = \frac{3}{8} \times 2^4 = 6$$

536 (c)

Let $I = \int_{1/3}^3 \frac{1}{x} \sin\left(\frac{1}{x} - x\right) dx$. Then,

$$\Rightarrow I = \int_3^1 t \sin\left(t - \frac{1}{t}\right) \times -\frac{1}{t^2} dt, \text{ where } x = \frac{1}{t}$$

$$\Rightarrow I = \int_{1/3}^3 \frac{1}{t} \sin\left(t - \frac{1}{t}\right) dt$$

$$\Rightarrow I = - \int_{1/3}^3 \frac{1}{t} \sin\left(\frac{1}{t} - t\right) dt$$

$$\Rightarrow I = -I$$

$$\Rightarrow 2I = 0 \Rightarrow I = 0$$

537 (b)

$$\text{Let } I_1 = \int_0^1 \frac{x e^{x^2}}{x^2 + 1} dx$$

Putting $x^2 = t$, we get

$$I_1 = \frac{1}{2} \int_0^1 \frac{e^t}{t + 1} dt = \frac{1}{2} I$$

538 (b)

$$\int (\sin^4 x - \cos^4 x) dx$$

$$= \int (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) dx$$

$$= \int (\sin^2 x - \cos^2 x) dx = - \int (\cos^2 x - \sin^2 x) dx$$

$$= - \int \cos 2x dx = \frac{-\sin 2x}{2} + c$$

539 (b)

We know that

$$\int_0^{\pi/2} \sin^m x \cos^n x dx = \frac{\Gamma \frac{m+1}{2} \Gamma \frac{n+1}{2}}{2 \Gamma \frac{m+n+2}{2}}$$

$$\int_0^{\pi/2} \sin^6 x dx = \frac{\Gamma \frac{7}{2} \Gamma \frac{1}{2}}{2 \sqrt{4}} = \frac{\frac{5}{2} \cdot \frac{3}{2} \cdot \frac{1}{2} \Gamma \frac{1}{2} \Gamma \frac{1}{2}}{2(3!)} = \frac{5\pi}{32}$$

540 (a)

We know that

$$\int_0^{2a} f(x) dx = \int_0^a \{f(x) + f(2a - x)\} dx$$

$$\Rightarrow \int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a - x) dx$$

$$= \lambda + \mu$$

541 (a)

$$\text{Let } I = \int \frac{dx}{7 + 5 \cos x} = \int \frac{dx}{7 + 5 \frac{1 - \tan^2(\frac{x}{2})}{1 + \tan^2(\frac{x}{2})}}$$

$$= \int \frac{\sec^2(\frac{x}{2}) dx}{7 + 7 \tan^2(\frac{x}{2}) + 5 - 5 \tan^2(\frac{x}{2})}$$

$$= \int \frac{\sec^2(\frac{x}{2}) dx}{12 + 2 \tan^2(\frac{x}{2})} = \int \frac{\frac{1}{2} \sec^2(\frac{x}{2}) dx}{6 + \tan^2(\frac{x}{2})}$$

$$\text{Put } \tan \frac{x}{2} = t \Rightarrow \frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

$$\therefore I = \int \frac{dt}{t^2 + (\sqrt{6})^2} = \frac{1}{\sqrt{6}} \tan^{-1}\left(\frac{t}{\sqrt{6}}\right) + c$$

$$= \frac{1}{\sqrt{6}} \tan^{-1} \left| \frac{\tan(\frac{x}{2})}{\sqrt{6}} \right| + c$$

542 (d)

$$\text{Let } I = \int_0^{\pi/3} \frac{\cos x + \sin x}{\sqrt{1 + 2 \sin x \cos x}} dx$$

$$= \int_0^{\pi/3} \frac{\cos x + \sin x}{\sqrt{\cos^2 x + \sin^2 x + 2 \sin x \cos x}} dx$$

$$= \int_0^{\pi/3} \frac{\cos x + \sin x}{\sqrt{(\cos x + \sin x)^2}} dx$$

$$= \int_0^{\pi/3} \frac{\cos x + \sin x}{\cos x + \sin x} dx$$

$$= \int_0^{\pi/3} 1 dx = \frac{\pi}{3}$$

543 (d)

$$\therefore \int_0^n [x] dx = \int_0^n (x - \{x\}) dx$$

$$= \int_0^n x dx - \int_0^n \{x\} dx$$

$$= \int_0^n x dx - n \int_0^1 \{x\} dx$$

$$= \int_0^n x dx - n \int_0^1 x dx = \left(\frac{n^2}{2} - \frac{n}{2} \right)$$

$$\therefore \frac{\int_0^n [x] dx}{\int_0^n \{x\} dx} = \frac{\frac{n^2}{2} - \frac{n}{2}}{\frac{n}{2}} = n - 1$$

544 (c)

$$\text{Let } I = \int_{\pi/4}^{\pi/2} e^x (\log \sin x + \cot x) dx$$

$$\Rightarrow I = \int_{\pi/4}^{\pi/2} e^x \log \sin x dx + \int_{\pi/4}^{\pi/2} e^x \cot x dx$$

$$= \int_{\pi/4}^{\pi/2} e^x \log \sin x dx + [e^x \log \sin x]_{\pi/4}^{\pi/2}$$

$$- \int_{\pi/4}^{\pi/2} e^x \log \sin x dx$$

$$= e^{\pi/2} \log \sin \frac{\pi}{2} - e^{\pi/4} \log \sin \frac{\pi}{4}$$

$$= -e^{\pi/4} \log \left(\frac{1}{\sqrt{2}} \right) = \frac{1}{2} e^{\pi/4} \log 2$$

545 (b)

$$\text{Let}$$

$$I = \int_0^1 \cot^{-1}(1 - x + x^2) dx$$

$$I = \int_0^1 \tan^{-1} \left\{ \frac{1}{1 - x(1 - x)} \right\} dx$$

$$= \int_0^1 \tan^{-1} \left\{ \frac{x + (1 - x)}{1 - x(1 - x)} \right\} dx$$

$$I = \int_0^1 \{ \tan^{-1} x + \tan^{-1}(1 - x) \} dx$$

$$I = \int_0^1 \tan^{-1} x dx + \int_0^1 \tan^{-1}(1 - x) dx$$

$$I = \int_0^1 \tan^{-1} x dx + \int_0^1 \tan^{-1}(1 - (1 - x)) dx$$

$$= 2 \int_0^1 \tan^{-1} x dx$$

$$\therefore k = 2$$

546 (a)

$$\text{Let } I = \int_0^{\pi/2} \frac{(\sin^2 x - \cos^2 x) dx}{(\sin x + \cos x)(\sin^2 + \cos^2 x - \sin x \cos x)}$$

$$I = \int_0^{\pi/2} \frac{(\sin x - \cos x) dx}{(1 - \sin x \cos x)}$$

$$I = \int_0^{\pi/2} \frac{\cos x - \sin x}{1 - \sin x \cos x} dx \quad \left(\because \int_0^a f(x) \right.$$

$$\left. = \int_0^a f(a - x) \right)$$

$$\Rightarrow I = 0$$

548 (c)

$$\int \sqrt{1 + \sin \frac{x}{2}} dx$$

$$= \int \sqrt{\sin^2 \frac{x}{4} + \cos^2 \frac{x}{4} + 2 \sin \frac{x}{4} \cos \frac{x}{4}} dx$$

$$= \int \left[\sin \frac{x}{4} + \cos \frac{x}{4} \right] dx$$

$$= 4 \left(\sin \frac{x}{4} - \cos \frac{x}{4} \right) + c$$

549 (c)

$$\int \frac{(x+3)e^x}{(x+4)^2} dx = \int \frac{(x+4-1)e^x dx}{(x+4)^2}$$

$$= \int \left(\frac{x+4}{(x+4)^2} - \frac{1}{(x+4)^2} \right) e^x dx$$

$$= \int e^x \frac{1}{x+4} dx - \int e^x \frac{1}{(x+4)^2} dx$$

$$= \frac{e^x}{x+4} + \int e^x \frac{1}{(x+4)^2} dx - \int \frac{e^x}{(x+4)^2} dx + c$$

$$= \frac{e^x}{x+4} + c$$

550 (d)

$$\text{Let } I = \int_0^{\pi/2} x \sin^2 x \cos^2 x dx \dots (i)$$

$$\Rightarrow I = \int_0^{\pi/2} \left(\frac{\pi}{2} - x \right) \sin^2 x \cos^2 x dx \dots (ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \frac{\pi}{2} \int_0^{\pi/2} \sin^2 x \cos^2 x dx$$

$$= \frac{\pi}{8} \int_0^{\pi/2} \sin^2 2x dx$$

$$= \frac{\pi}{8} \int_0^{\pi/2} \left(\frac{1 - \cos 4x}{2} \right) dx$$

$$\Rightarrow 2I = \frac{\pi}{16} \left[x - \frac{\sin 4x}{4} \right]_0^{\pi/2}$$

$$\Rightarrow 2I = \frac{\pi}{16} \left[\frac{\pi}{2} \right] \Rightarrow I = \frac{\pi^2}{64}$$

551 (a)

$$\begin{aligned} \text{Now, } \int uv'' dx &= uv' - \int u'v' dx \\ &= uv' - \left[vu' - \int vu'' dx \right] \\ &= uv' - vu' + \int u''v dx \end{aligned}$$

$$\therefore a = \int u''v dx$$

552 (c)

$$\text{Let } I = \int (\sqrt[3]{x}) \left(\sqrt[5]{1 + \sqrt[3]{x^4}} \right) dx$$

$$\text{Put } \sqrt[3]{x^4} = t$$

$$\Rightarrow \frac{4}{3} \cdot \sqrt[3]{x} dx = dt$$

$$\therefore I = \frac{3}{4} \int (\sqrt[5]{1+t}) dt$$

$$I = \frac{3}{4} \left[\frac{(1+t)^{\frac{1}{5}+1}}{\frac{1}{5}+1} \right] + c$$

$$= \frac{5}{8} \left[(1 + \sqrt[3]{x^4})^{6/5} \right] + c$$

553 (a)

Let

$$I = \int \frac{1}{\cos x - \sin x} dx$$

$$\begin{aligned} \Rightarrow I &= \frac{1}{\sqrt{2}} \frac{1}{\left(\frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x \right)} dx \\ &= \frac{1}{\sqrt{2}} \int \frac{1}{\cos \left(x + \frac{\pi}{4} \right)} dx \end{aligned}$$

$$\begin{aligned} \Rightarrow I &= \frac{1}{\sqrt{2}} \int \sec \left(x + \frac{\pi}{4} \right) dx \\ &= \frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} + \frac{\pi}{8} + \frac{\pi}{4} \right) \right| + C \end{aligned}$$

$$\Rightarrow I = \frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} + \frac{3\pi}{8} \right) \right| + C$$

554 (d)

If $f(x)$ is a continuous function defined on $[a, b]$ then

$$m(b-a) \leq \int_a^b f(x) dx \leq M(b-a)$$

Where m and M are respectively minimum and maximum values of $f(x)$ in $[a, b]$

Clearly, $f(x) = 1 + e^{-x^2}$ is continuous on $[0, 1]$

Also,

$$0 < x < 1 \Rightarrow x^2 < x \Rightarrow e^{x^2} < e^x \Rightarrow e^{-x^2} > e^{-x}$$

and,

$$0 < x < 1 \Rightarrow x^2 > 0 \Rightarrow e^{x^2} > e^0 \Rightarrow e^{-x^2} < 1$$

$$\therefore e^{-x} < e^{-x^2} < 1 \text{ for all } x \in [0, 1]$$

$$\Rightarrow 1 + e^{-x} < 1 + e^{-x^2} < 2 \text{ for all } x \in [0, 1]$$

$$\Rightarrow \int_0^1 1 + e^{-x} dx < \int_0^1 1 + e^{-x^2} dx < \int_0^1 2 dx$$

$$\Rightarrow 2 - \frac{1}{e} < \int_0^1 1 + e^{-x^2} dx < 2$$

555 (c)

$$\text{Let } I = \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{\sin^2 x + \cos^2 x + 2 \sin x \cos x}} dx$$

$$= \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{(\sin x + \cos x)^2}} dx$$

$$= \int_0^{\pi/2} (\sin x + \cos x) dx$$

$$= [-\cos x + \sin x]_0^{\pi/2} = -0 + 1 + 1 - 0 = 2$$

556 (b)

$$\text{Let } I = \int \sqrt{x} e^{\sqrt{x}} dx$$

$$\text{Put } x = t^2$$

$$\Rightarrow dx = 2t dt$$

$$\therefore I = 2 \int t^2 e^t dt$$

$$= 2[t^2 e^t - (2t)e^t + 2e^t] + c$$

$$= (2x - 4\sqrt{x} + 4)e^{\sqrt{x}} + c$$

557 (c)

Let

$$A = \lim_{n \rightarrow \infty} \left\{ \left(1 + \frac{1}{n^2} \right) \left(1 + \frac{2^2}{n^2} \right) \left(1 + \frac{3^2}{n^2} \right) \dots \left(1 + \frac{n^2}{n^2} \right) \right\}^{1/n}$$

$$\Rightarrow \log A = \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} \log \left(1 + \frac{r^2}{n^2} \right)$$

$$\Rightarrow \log A = \int_0^1 \log(1 + x^2) dx$$

$$= [x \log(1 + x^2)]_0^1 - 2 \int_0^1 \frac{x^2}{1 + x^2} dx$$

$$\Rightarrow \log A = \log 2 - 2[x - \tan^{-1} x]_0^1$$

$$\Rightarrow \log A = \log 2 - 2 \left[1 - \frac{\pi}{4} \right] = \log 2 + \frac{\pi}{2} - 2$$

$$\Rightarrow A = e^{\log 2 + \frac{\pi}{2} - 2} = \frac{2}{e^2} e^{\pi/2}$$

558 (d)

We have,

$$\int_{\sqrt{2}}^x \frac{1}{x \sqrt{x^2 - 1}} dx = \frac{\pi}{12}$$

$$\Rightarrow [\sec^{-1} x]_{\sqrt{2}}^x = \frac{\pi}{12}$$

$$\Rightarrow \sec^{-1} x - \sec^{-1} \sqrt{2} = \frac{\pi}{12}$$

$$\Rightarrow \sec^{-1} x - \frac{\pi}{4} = \frac{\pi}{12} \Rightarrow \sec^{-1} x = \frac{\pi}{3} \Rightarrow x = \sec \frac{\pi}{3} = 2$$

559 (b)

We have,

$$I = \int_0^{na} f(x) dx$$

$$\Rightarrow I = \sum_{r=1}^n \int_{(r-1)a}^{ra} f(x) dx$$

$$\Rightarrow I = \sum_{r=1}^n \int_0^a f(y + (r-1)a) dy, \text{ where } x = y + (r-1)a$$

$$\Rightarrow I = \sum_{r=1}^n \int_0^a f(y) dy \quad [\because f(y+a) = f(y)]$$

$$\therefore f\{y + (r-1)a\} = f(y)$$

$$\Rightarrow I = n \int_0^a f(x) dx$$

560 (b)

We have,

$$I = \int_{-\pi/3}^{\pi/3} \frac{x \sin x}{\cos^2 x} dx$$

$$\Rightarrow I = 2 \int_0^{\pi/3} \frac{x \sin x}{\cos^2 x} dx \quad [\because \text{integrand is an even function}]$$

$$\Rightarrow I = 2 \int_0^{\pi/3} x \tan x \sec x dx$$

I II

$$\Rightarrow I = 2[x \sec x - \log(\sec x + \tan x)]_0^{\pi/3}$$

$$\Rightarrow I = 2[2\pi/3 - \log(2 + \sqrt{3})]$$

$$\Rightarrow I = 4\pi/3 - 2 \log \tan 5\pi/12 = 2(2\pi/3 - \log \tan 5\pi/12)$$

561 (a)

$$\text{Let } I = \int \frac{a^x}{\sqrt{a^x - a^{-x}}} dx = \int \frac{a^x}{\sqrt{1 - a^{-2x}}} dx$$

$$\text{Put } a^x = t \Rightarrow a^x \log a dx = dt$$

$$\therefore I = \frac{1}{\log a} \int \frac{1}{\sqrt{1 - t^2}} dt = \frac{1}{\log a} \sin^{-1} t + c$$

$$= \frac{1}{\log a} \sin^{-1}(a^x) + c$$

562 (a)

$$\text{Let } I = \int \frac{x + \sin x}{1 + \cos x} dx = \int \left(\frac{x}{2} \sec^2 \frac{x}{2} + \tan \frac{x}{2} \right) dx$$

$$= x \tan \frac{x}{2} - \int \tan \frac{x}{2} dx + \int \tan \frac{x}{2} dx$$

$$= x \tan \frac{x}{2} + c$$

563 (c)

$$\text{Let } I = \int_{-1}^1 \sin^3 x \cos^2 x dx$$

$$\text{Again, let } f(x) = \sin^3 x \cos^2 x$$

$$f(-x) = \sin^3(-x) \cos^2(-x)$$

$$= -\sin^3 x \cos^2 x$$

$\therefore f(x)$ is an odd function.

$$\text{Hence, } \int_{-1}^1 \sin^3 x \cos^2 x dx = 0$$

564 (c)

$$\text{Let } I = \int x \{f(x^2)g''(x^2) - f''(x^2)g(x^2)\} dx$$

$$\text{Put } x^2 = t \Rightarrow 2x dx = dt$$

$$\therefore I = \frac{1}{2} [\{f(t)g''(t)\}dt - \int f''(t^2)g(t^2)dt]$$

$$= \frac{1}{2} [f(t)g'(t) - \int f'(t)g(t)dt - g(t)f'(t) + \int g'(t)f'(t)dt] + c$$

$$= \frac{1}{2} [f(t)g'(t) - g(t)f'(t)] + c$$

$$= \frac{1}{2} [f(x^2)g'(x^2) - g(x^2)f'(x^2)] + c$$

565 (c)

We have,

$$I = \int_{-2}^2 |1 - x^4| dx$$

$$\Rightarrow I = 2 \int_0^2 |1 - x^4| dx \quad [\because f(x) = 1 - x^4 \text{ is an even function}]$$

$$\Rightarrow I = 2 \left\{ \int_0^1 (1 - x^4) dx + \int_1^2 (x^4 - 1) dx \right\}$$

$$\Rightarrow I = 2 \left\{ \left[x - \frac{x^5}{5} \right]_0^1 + \left[\frac{x^5}{5} - x \right]_1^2 \right\} = 12$$

566 (c)

$$I = \int_{-1}^0 [x + [x + [x]]] dx + \int_0^1 [x + [x + [x]]] dx$$

$$\Rightarrow I = I_1 + I_2$$

$$\text{For } I_1 \rightarrow [x] = -1 \text{ and } -1 < x < 0$$

$$-2 < x + [x] < -1 \Rightarrow [x + [x]] = -2$$

$$\text{and now, } -3 < x + [x + [x]] < -2$$

$$\Rightarrow [x + [x + [x]]] = -3$$

$$I_1 = \int_{-1}^0 (-3) dx + -3[x]_{-1}^0 = -3$$

$$\text{For } I_2 \rightarrow [x] = 0 \text{ and } 0 < x < 1$$

$$\Rightarrow 0 < x + [x] < 1 \Rightarrow [x + [x]] = 0$$

$$\text{Again, } 0 \leq x + [x + [x]] < 1$$

$$\Rightarrow [x + [x + [x]]] = 0$$

$$\text{So, } I_2 = 0$$

$$\therefore I_1 + I_2 = -3$$

567 (d)

$$\begin{aligned} \text{Let } I &= \int_0^\pi \sum_{r=0}^3 a_r \cos^{3-r} x \sin^r x \, dx \\ &= \int_0^\pi a_0 \cos^3 x \, dx + \int_0^\pi a_1 \cos^2 x \sin x \, dx \\ &+ \int_0^\pi a_2 \cos x \sin^2 x \, dx + \int_0^\pi a_3 \sin^3 x \, dx \end{aligned}$$

$$\begin{aligned} \text{Since, } \int_0^{2a} f(x) \, dx &= \begin{cases} 2 \int_0^a f(x) \, dx & , \text{ if } f(2a-x) = f(x) \\ 0 & , \text{ if } f(2a-x) = -f(x) \end{cases} \end{aligned}$$

∴ Integral Ist and IIIrd become zero.

∴ The given integral is depend upon a_1 and a_3 .

568 (a)

Let

$$\begin{aligned} \int \frac{x \sin x^2 e^{\sec x^2}}{\cos^2 x^2} \, dx &= \frac{1}{2} \int e^{\sec x^2} \tan x^2 \sec x^2 \, 2x \, dx \\ \Rightarrow I &= \frac{1}{2} \int e^{\sec x^2} d(\sec x^2) = \frac{1}{2} e^{\sec x^2} + C \end{aligned}$$

569 (b)

We have,

$$\begin{aligned} \int \frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}} \, dx &= \int \sqrt{1+x} \, dx \\ &= \frac{2}{3} (1+x)^{3/2} + C \end{aligned}$$

570 (a)

Putting $x = \tan \theta$, we get

$$\begin{aligned} I &= \int_0^\infty \frac{dx}{[x+\sqrt{x^2+1}]^3} = \int_0^{\pi/2} \frac{\sec^2 \theta \, d\theta}{(\tan \theta + \sec \theta)^3} \\ &= \int_0^{\pi/2} \frac{\cos \theta}{(1+\sec \theta)^3} \, d\theta = \left[-\frac{1}{2(1+\sin \theta)^2} \right]_0^{\pi/2} \\ &= -\frac{1}{8} + \frac{1}{2} = \frac{3}{8} \end{aligned}$$

571 (b)

$$\begin{aligned} F(t) &= \int_0^1 f(t-y)g(y) \, dy \\ &= \int_0^t e^{t-y} \cdot y \, dy = e^t \int_0^t e^{-y} y \, dy \\ &= e^t [(-y)e^{-y}]_0^t - (e^{-y})_0^t \\ &= e^t [-te^{-t} - e^{-t} + 1] = e^t - (1+t) \end{aligned}$$

573 (a)

$$\begin{aligned} \text{Let } I &= \int_1^{e^2} \frac{dx}{x(1+\log x)^2} \\ \text{Put } 1 + \log x &= t \Rightarrow \frac{1}{x} dx = dt \\ \therefore I &= \int_1^3 \frac{1}{t^2} dt = \left[-\frac{1}{t} \right]_1^3 = \frac{2}{3} \end{aligned}$$

574 (c)

$$\begin{aligned} \text{Let } I &= \int \frac{dx}{(x+1)\sqrt{4x+3}} \\ \text{Put } 4x+3 &= t^2 \Rightarrow 4 \, dx = 2t \, dt \\ \therefore I &= \frac{1}{2} \int \frac{t \, dt}{\left(\frac{t^2-3}{4}+1\right)t} = 2 \int \frac{dt}{1+t^2} \\ &= 2 \tan^{-1} t + c = 2 \tan^{-1} \sqrt{4x+3} + c \end{aligned}$$

575 (b)

$$\begin{aligned} \text{Let } I &= \int_a^b x f(x) \, dx \quad \dots (i) \\ \Rightarrow I &= \int_a^b (a+b-x)f(x) \, dx \\ [\text{since, } f(a+b-x) &= f(x), \text{ given}] \\ \Rightarrow I &= (a+b) \int_a^b f(x) \, dx - I \text{ [from Eq. (i)]} \\ \Rightarrow 2I &= (a+b) \int_a^b f(x) \, dx \\ \Rightarrow I &= \left(\frac{a+b}{2}\right) \int_a^b f(x) \, dx \end{aligned}$$

576 (a)

$$\begin{aligned} \text{Here, } \int_0^{t^2} \{x f(x)\} \, dx &= \frac{2}{5} t^5 \\ (\text{Using Newton Leibnitz formula):} & \text{ differentiating both sides, we get} \\ t^2 \{f(t^2)\} \cdot \left\{ \frac{d}{dt} (t^2) \right\} - 0 \cdot f(0) \left\{ \frac{d}{dt} (0) \right\} &= 2t^4 \\ \Rightarrow t^2 f(t^2) \cdot 2t &= 2t^4 \\ \Rightarrow f(t^2) &= t \\ \therefore f\left(\frac{4}{25}\right) &= \pm \frac{2}{5} \quad \left[\text{putting } t = \pm \frac{2}{5} \right] \\ \Rightarrow f\left(\frac{4}{25}\right) &= \frac{2}{5} \quad \left[\text{neglecting negative} \right] \end{aligned}$$

577 (c)

$$\begin{aligned} \text{Given, } F(x^2) &= x^2(1+x) \\ \Rightarrow \int_0^{x^2} f(t) \, dt &= x^2(1+x) \\ \text{On differentiating both sides w.r.t., we get} \\ 2xf(x^2) &= 2x + 3x^2 \\ \Rightarrow f(x^2) &= 1 + \frac{3x}{2} \\ \Rightarrow f(2^2) &= 1 + \frac{3}{2}(2) = 4 \Rightarrow f(4) = 4 \end{aligned}$$

578 (c)

$$\begin{aligned} \text{We have,} \\ \int_0^{\sqrt{2}} [x^2] \, dx &= \int_0^1 [x^2] \, dx + \int_1^{\sqrt{2}} [x^2] \, dx \\ &= \int_0^1 0 \, dx + \int_1^{\sqrt{2}} 1 \, dx = \sqrt{2} - 1 \end{aligned}$$

579 (a)

$$\text{Let } I = \int \frac{e^x}{(2+e^x)(e^x+1)} dx$$

$$\text{Put } e^x = t \Rightarrow e^x dx = dt$$

$$\begin{aligned} \therefore I &= \int \frac{dt}{(2+t)(t+1)} \\ &= \int \left[\frac{1}{(1+t)} - \frac{1}{(2+t)} \right] dt \\ &= \log(1+t) - \log(2+t) + c \\ &= \log\left(\frac{1+t}{2+t}\right) + c \\ &= \log\left(\frac{1+e^x}{2+e^x}\right) + c \end{aligned}$$

580 (a)

$$\text{Let } I = \int_0^{\pi/2} \frac{\sin^3 x \cos x}{\sin^4 x + \cos^4 x} dx \quad \dots(i)$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\cos^3 x \sin x}{\cos^4 x + \sin^4 x} dx \quad \dots(ii)$$

Adding (i) and (ii), we get

$$\begin{aligned} 2I &= \int_0^{\pi/2} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx \\ \Rightarrow 2I &= \int_0^{\pi/2} \frac{\sin x \cos x}{\left(\frac{1-\cos 2x}{2}\right)^2 + \left(\frac{1+\cos 2x}{2}\right)^2} dx \\ \Rightarrow 2I &= \int_0^{\pi/2} \frac{2 \sin x \cos x}{1 + \cos^2 2x} dx \\ \Rightarrow 2I &= -\frac{1}{2} \int_1^{-1} \frac{1}{1+t^2} dt, \text{ where } t = \cos 2x \\ \Rightarrow 2I &= -\frac{1}{2} [\tan^{-1} t]_1^{-1} = -\frac{1}{2} \left(-\frac{\pi}{4} - \frac{\pi}{4}\right) \\ \Rightarrow I &= \frac{\pi}{8} \end{aligned}$$

581 (c)

We have,

$$\int_0^1 e^{x^2} (x - \alpha) dx = 0 \Rightarrow \int_0^1 e^{x^2} x dx = \int_0^1 e^{x^2} \alpha dx$$

$$\Rightarrow \frac{1}{2} \int_0^1 e^t dt = \alpha \int_0^1 e^{x^2}, \text{ where } t = x^2$$

$$\Rightarrow \frac{1}{2}(e-1) = \alpha \int_0^1 e^{x^2} dx \quad \dots(i)$$

Since e^{x^2} is an increasing function for $0 \leq x \leq 1$

$$\therefore 1 \leq e^{x^2} \leq e \text{ when } 0 \leq x \leq 1$$

$$\Rightarrow 1(1-0) \leq \int_0^1 e^{x^2} dx \leq e(1-0) \Rightarrow 1 \leq$$

$$\int_0^1 e^{x^2} dx \leq e \quad \dots(ii)$$

From (i) and (ii) we find that LHS of (i) is positive and $\int_0^1 e^{x^2} dx$ lies between 1 and e . Therefore, α is a positive real number. Also, from (i), we have

$$\therefore \alpha = \frac{\frac{1}{2}(e-1)}{\int_0^1 e^{x^2} dx} \quad \dots(iii)$$

The denominator of (iii) is greater than unity and the numerator lies between 0 and 1. Therefore, $0 < \alpha < 1$

582 (b)

For $x \in [0, \pi/4]$, we have $x - [x] = x$

$$\begin{aligned} \therefore \int_0^{\pi/4} \sin x d(x - [x]) &= \int_0^{\pi/4} \sin x dx = [-\cos x]_0^{\pi/4} \\ &= 1 - \frac{1}{\sqrt{2}} \end{aligned}$$

583 (b)

We know that

$$\int_a^b \frac{|x|}{x} dx = |b| - |a|$$

$$\therefore \int_{-1}^2 \frac{|x|}{x} dx = |2| - |-1| = 1$$

584 (b)

We have,

$$I = \int \frac{1+x^2}{x\sqrt{1+x^4}} dx = \int \frac{1+\frac{1}{x^2}}{\sqrt{x^2+\frac{1}{x^2}}} dx$$

$$\Rightarrow I = \int \frac{1}{\sqrt{\left(x-\frac{1}{x}\right)^2 + 2}} d\left(x-\frac{1}{x}\right)$$

$$\Rightarrow I = \log \left| \left(x-\frac{1}{x}\right) + \sqrt{\left(x-\frac{1}{x}\right)^2 + 2} \right| + C$$

585 (d)

$$\lim_{x \rightarrow 0} \frac{2x \tan |x|}{3x^2} = \frac{2}{3} \lim_{x \rightarrow 0} \frac{\tan |x|}{x}$$

$$\text{RHL is } \frac{2}{3} \text{ and LHL is } -\frac{2}{3}$$

So, limit does not exist

586 (b)

If $f(t)$ is an odd function, then $\int_0^x f(t) dt$ is an even function.

587 (b)

We have,

$$I = \int_0^1 \tan^{-1} \left(\frac{2x-1}{1+x-x^2} \right) dx$$

$$\Rightarrow I = \int_0^1 \tan^{-1} \left\{ \frac{x+x-1}{1-x(x-1)} \right\} dx$$

$$\Rightarrow I = \int_0^1 \tan^{-1} x + \tan^{-1}(x-1) dx$$

$$\begin{aligned} \Rightarrow I &= \int_0^1 \tan^{-1} x \, dx + \int_0^1 \tan^{-1}(x-1) \, dx \\ \Rightarrow I &= \int_0^1 \tan^{-1} x \, dx + \int_0^1 \tan^{-1}(1-x-1) \, dx \\ \Rightarrow I &= \int_0^1 \tan^{-1} x \, dx + \int_0^1 \tan^{-1}(-x) \, dx \\ \Rightarrow I &= \int_0^1 \tan^{-1} x \, dx - \int_0^1 \tan^{-1} x \, dx = 0 \end{aligned}$$

588 (a)

$$\text{Let } I = \int_{\pi/4}^{3\pi/4} \frac{x}{1+\sin x} \, dx \quad \dots(i)$$

Then,

$$I = \int_{\pi/4}^{3\pi/4} \frac{x - \pi}{1 + \sin(\pi - x)} \, dx$$

$$I = \int_{\pi/4}^{3\pi/4} \frac{\pi - x}{1 + \sin x} \, dx \quad \dots(ii)$$

Adding (i) and (ii), we get

$$2I = \int_{\pi/4}^{3\pi/4} \frac{\pi}{1 + \sin x} \, dx$$

$$\Rightarrow 2I = \pi \int_{\pi/4}^{3\pi/4} \frac{1 - \sin x}{\cos^2 x} \, dx$$

$$\Rightarrow 2I = \pi \int_{\pi/4}^{3\pi/4} (\sec^2 x - \tan x \sec x) \, dx$$

$$\Rightarrow 2I = \pi [\tan x - \sec x]_{\pi/4}^{3\pi/4}$$

$$\Rightarrow 2I = \pi [(\tan 3\pi/4 - \sec 3\pi/4) - (\tan \pi/4 - \sec \pi/4)]$$

$$\Rightarrow 2I = \pi [(-1 + \sqrt{2}) - (1 - \sqrt{2})]$$

$$\Rightarrow 2I = 2\pi(\sqrt{2} - 1)$$

$$\Rightarrow I = \pi(\sqrt{2} - 1)$$

589 (a)

We have,

$$\lim_{x \rightarrow 1} \int_2^{f(x)} \frac{2t \, dt}{x-1} = 4$$

$$\Rightarrow \lim_{x \rightarrow 1} \frac{\int_2^{f(x)} 2t \, dt}{x-1} = 4$$

$$\Rightarrow \lim_{x \rightarrow 1} 2f'(x)f(x) = 4 \quad [\text{Using De' L'}$$

Hospital's rule]

$$\Rightarrow 2f'(1)f(1) = 4 \Rightarrow f'(1) = 1$$

590 (d)

$$f(x) + f(-x) = \frac{\sin^2 \pi x}{1 + \pi^x} + \frac{\sin^2(-\pi x)}{1 + \pi^{-x}}$$

$$\begin{aligned} &= \frac{\sin^2 \pi x}{1 + \pi^x} + \frac{(\sin^2 \pi x)\pi^x}{\pi^x + 1} \\ &= \sin^2 \pi x = \frac{1 - \cos 2\pi x}{2} \end{aligned}$$

$$\therefore \int [f(x) + f(-x)] \, dx = \int \left[\frac{1 - \cos 2\pi x}{2} \right] \, dx$$

$$= \frac{1}{2}x - \frac{\sin 2\pi x}{4\pi} + c$$

591 (b)

$$\int_0^{2\pi} (\sin x + |\sin x|) \, dx$$

$$= \int_0^{\pi} (\sin x + \sin x) \, dx + \int_{\pi}^{2\pi} (\sin x - \sin x) \, dx$$

$$= \int_0^{\pi} 2 \sin x \, dx + \int_{\pi}^{2\pi} 0 \, dx = 2[-\cos x]_0^{\pi} + 0$$

$$= -2(\cos \pi - \cos 0) = -2(-1 - 1) = 4$$

592 (b)

We have,

$$f(x + 2\pi) = \int_0^{x+2\pi} \sin^4 t \, dt$$

$$= \int_0^x \sin^4 t \, dt + \int_x^{x+2\pi} \sin^4 t \, dt$$

$$\Rightarrow f(x + 2\pi) = f(x) + \int_0^{2\pi} \sin^4 t \, dt$$

$$[\because \sin^4 t \text{ is periodic with period } \frac{\pi}{2}]$$

$$\Rightarrow f(x + 2\pi) = f(x) + f(2\pi)$$

593 (d)

$$\frac{d}{dx} \int_2^{x^2} (t-1) \, dt = \frac{d}{dx} \left\{ \frac{x^4}{2} - x^2 \right\} = 2x(x^2 - 1)$$

594 (d)

We have,

$$\int_0^2 \left| \cos \frac{\pi}{2} x \right| \, dx$$

$$= \int_0^1 \cos \frac{\pi}{2} x \, dx + \int_1^2 -\cos \frac{\pi}{2} x \, dx$$

$$\begin{aligned} &= \frac{2}{\pi} \left[\sin \frac{\pi x}{2} \right]_0^1 - \frac{2}{\pi} \left[\sin \frac{\pi x}{2} \right]_1^2 = \left(\frac{2}{\pi} - 0 \right) - \left(0 - \frac{2}{\pi} \right) \\ &= \frac{4}{\pi} \end{aligned}$$

595 (c)

$$\text{We have, } I = \int_{-1}^1 |1-x| \, dx$$

$$\text{Here, } -1 \leq x \leq 1 \Rightarrow 1-x \geq 0$$

$$\therefore I = \int_{-1}^1 (1-x) \, dx$$

$$= \left[x - \frac{x^2}{2} \right]_{-1}^1 = 1 - \frac{1}{2} + 1 + \frac{1}{2} = 2$$

596 (d)

$$\text{Let } I = 2 \int_0^{\pi} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$$

$$= 4 \int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} \dots (i)$$

$$\Rightarrow I = 4 \int_0^{\pi/2} \frac{\cos^4 x}{\sin^4 x + \cos^4 x} dx \dots (ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = 4 \int_0^{\pi/2} 1. dx = 2\pi$$

$$\Rightarrow I = \pi$$

597 (d)

$$\text{Let } f(x) = \sin^3 x \cos^2 x$$

$$f(-x) = -\sin^3 x \cos^2 x = -f(x)$$

$\Rightarrow f(x)$ is an odd function

$$\therefore \int_{-\pi}^{\pi} \sin^3 x \cos^2 x dx = 0$$

598 (c)

$$\text{Let } I = \int \frac{1 - \frac{1}{x^2}}{x^2 + \frac{1}{x^2} + 1} dx = \int \frac{1 - \frac{1}{x^2}}{\left(x + \frac{1}{x}\right)^2 - 1} dx$$

$$\text{Put } x + \frac{1}{x} = t \Rightarrow \left(1 - \frac{1}{x^2}\right) dx = dt$$

$$\therefore I = \int \frac{dt}{t^2 - 1^2} = \frac{1}{2 \times 1} \log \frac{t-1}{t+1} + c$$

$$= \frac{1}{2} \log \frac{x^2 + 1 - x}{x^2 + 1 + x} + c$$

599 (c)

$$\int \frac{\cos 2x + 2 \sin^2 x}{\cos^2 x} dx$$

$$= \int \frac{2(\sin^2 x + \cos^2 x) - 1}{\cos^2 x} dx$$

$$= \int \sec^2 x dx = \tan x + c$$

600 (b)

$$\text{Let } \int_0^{\pi/4} \frac{\sin x + \cos x}{3 + \sin 2x} dx$$

Putting $\sin x - \cos x = t$ and $(\sin x + \cos x) dx = dt$, we get

$$I = \int_{-1}^0 \frac{dt}{3 + 1 - t^2}$$

$$I = \int_{-1}^0 \frac{dt}{2^2 - t^2} = \frac{1}{4} \left[\log \left(\frac{2+t}{2-t} \right) \right]_{-1}^0 = -\frac{1}{4} \log \frac{1}{3}$$

$$= \frac{1}{4} \log 3$$

601 (b)

$$\int_0^1 \sqrt{\frac{1-x}{1+x}} dx = \int_0^1 \frac{1-x}{\sqrt{1-x^2}} dx$$

$$= \int_0^1 \frac{1}{\sqrt{1-x^2}} dx - \int_0^1 \frac{x}{\sqrt{1-x^2}} dx$$

$$= [\sin^{-1} x]_0^1 + \int_1^0 \frac{t}{t} dt \quad (\text{where, } t^2 = 1 - x^2)$$

$$\Rightarrow t dt = -x dx$$

$$= (\sin^{-1} 1 - \sin^{-1} 0) + [t]_1^0$$

$$= \frac{\pi}{2} - 1$$

602 (b)

We have,

$$p'(x) = p'(1-x), \forall x \in [0,1], p(0) = 1, p(1) = 41$$

$$\Rightarrow p(x) = -p(1-x) + C$$

$$\text{At } x = 0, p(0) = -p(1) + C$$

$$\Rightarrow 1 = -41 + C$$

$$\Rightarrow C = 42$$

$$\therefore p(x) + p(1-x) = 42$$

$$\text{Now, } I = \int_0^1 p(x) dx = \int_0^1 p(1-x) dx$$

$$\Rightarrow 2I = \int_0^1 (p(x) + p(1-x)) dx = \int_0^1 42 dx$$

$$= 42$$

$$\Rightarrow I = 21$$

603 (a)

We have,

$$I = \int \frac{x^2 - 1}{x \sqrt{(x^2 + \alpha x + 1)(x^2 + \beta x + 1)}} dx$$

$$\Rightarrow I \int \frac{1 - \frac{1}{x^2}}{\sqrt{\left(x + \frac{1}{x} + \alpha\right) \left(x + \frac{1}{x} + \beta\right)}} dx$$

$$\Rightarrow I = \int \frac{1}{\sqrt{\left(x + \frac{1}{x}\right)^2 + (\alpha + \beta) \left(x + \frac{1}{x}\right) + \alpha\beta + \frac{1}{x}}} d \left(x + \frac{1}{x} \right)$$

$$\Rightarrow I = \int \frac{1}{\sqrt{t^2 + (\alpha + \beta)t + \alpha\beta}} dt, \text{ where } t = x + \frac{1}{x}$$

$$= x + \frac{1}{x}$$

$$\Rightarrow I = \int \frac{1}{\sqrt{\left(t + \frac{\alpha + \beta}{2}\right)^2 - \left(\frac{\alpha - \beta}{2}\right)^2}} dt$$

$$\Rightarrow I = \log \left| t + \frac{\alpha + \beta}{2} \right|$$

$$+ \sqrt{\left(t + \frac{\alpha + \beta}{2}\right)^2 - \left(\frac{\alpha - \beta}{2}\right)^2} \Big| + C$$

$$\Rightarrow I = \log \left| t + \frac{\alpha + \beta}{2} + \sqrt{t^2 + (\alpha + \beta)t + \alpha\beta} \right| + C$$

$$\begin{aligned} \Rightarrow I &= \log|(t + \alpha) + (t + \beta) + 2\sqrt{(t + \alpha)(t + \beta)}| \\ &\quad + C - \log 2 \\ \Rightarrow I &= \log\{\sqrt{t + \alpha} + \sqrt{t + \beta}\}^2 + C_1 \\ \Rightarrow I &= 2 \log\{\sqrt{t + \alpha} + \sqrt{t + \beta}\} + C_1 \\ \Rightarrow I &= 2 \log\left\{\frac{\sqrt{x^2 + \alpha x + 1} + \sqrt{x^2 + \beta x + 1}}{\sqrt{x}}\right\} + C_1 \end{aligned}$$

604 (a)

$$f(a) = \frac{e^a}{1+e^a} \dots(i)$$

$$\text{and } f(-a) = \frac{e^{-a}}{1+e^{-a}} = \frac{1}{1+e^a} \dots(ii)$$

On adding Eqs.(i) and (ii), we get

$$f(a) + f(-a) = 1$$

$$\Rightarrow f(a) = 1 - f(-a)$$

Let $f(-a) = t$, then $f(a) = 1 - t$

$$\text{Now, } I_1 = \int_t^{1-t} xg[x(1-x)]dx \dots(i)$$

$$\Rightarrow I_1 = \int_t^{1-t} (1-x)g[x(1-x)]dx \dots(ii)$$

On adding Eqs. (i) and (ii). We get

$$2I_1 = \int_t^{1-t} g[x(1-x)]dx = I_2$$

$$\Rightarrow \frac{I_2}{I_1} = 2$$

605 (d)

We have,

$$\int_2^5 f(x) dx = \int_{-2}^5 f(x) dx$$

$$\Rightarrow \int_2^5 f(x) dx - \int_{-2}^5 f(x) dx = 0$$

$$\Rightarrow \int_2^5 f(x) dx + \int_5^{-2} f(x) dx = 0$$

$$\Rightarrow \int_2^{-2} f(x) dx = 0$$

$$\Rightarrow \int_{-2}^2 f(x) dx = 0 \Rightarrow \int_0^2 \{f(x) + f(-x)\}dx = 0$$

Thus, $f(x)$ may be an odd function

In general, nothing can be said about $f(x)$ defined on R

606 (a)

We have,

$$f(x) = \int_0^x 2|t| dt \Rightarrow f'(x) = 2|x|$$

Now,

$$f'(x) = 1 \Rightarrow 2|x| = 1 \Rightarrow x = \pm \frac{1}{2}$$

$$\text{For } x = \frac{1}{2}, \text{ we have } y = f(x) = \int_0^{1/2} 2t dt = \frac{1}{4}$$

$$\text{and, for } x = -\frac{1}{2}, \text{ we have } y = \int_0^{-1/2} -2t dt = -\frac{1}{4}$$

The equations of the tangents at $(1/2, 1/4)$ and $(-1/2, -1/4)$ are $y = x - \frac{1}{4}$ and $y = x + \frac{1}{4}$

respectively

607 (c)

$$\text{Let } I = \int \cos \left[2 \cot^{-1} \sqrt{\frac{1-x}{1+x}} \right] dx$$

$$= \int \cos \left[2 \tan^{-1} \sqrt{\frac{1+x}{1-x}} \right] dx$$

$$= \int \cos[\cos^{-1}(-x)] dx \left[\because 2 \tan^{-1} x \right. \\ \left. = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) \right]$$

$$= -\frac{x^2}{2} + c$$

608 (b)

$$\text{Let } I = \int_2^3 \frac{x+1}{x^2(x-1)} dx$$

$$= \int_2^3 \left(\frac{-2}{x} - \frac{1}{x^2} + \frac{2}{x-1} \right) dx$$

$$= \left[-2 \log x + \frac{1}{x} + 2 \log(x-1) \right]_2^3$$

$$= \left[2 \log \left(\frac{x-1}{x} \right) + \frac{1}{x} \right]_2^3 \\ = \left[2 \left(\log \frac{2}{3} - \log \frac{1}{2} \right) + \frac{1}{3} - \frac{1}{2} \right]$$

$$= 2 \log \frac{4}{3} - \frac{1}{6} = \log \frac{16}{9} - \frac{1}{6}$$

609 (a)

$$\text{Let } I = \int x (x^x)^x (2 \log x + 1) dx$$

$$\text{Let } (x^x)^x = t, x^2 \log x = \log t$$

$$\Rightarrow \left(x^2 \cdot \frac{1}{x} + 2x \log x \right) dx = \frac{1}{t} dt$$

$$\Rightarrow x(1 + 2 \log x) dx = \frac{1}{t} dt$$

$$\therefore I = \int t \cdot \frac{1}{t} dt = t + c = (x^x)^x + c$$

610 (a)

$$\text{Let } I = \int \frac{dx}{(\sin x - 2 \cos x)(2 \sin x + \cos x)}$$

$$= \int \frac{\sec^2 x \, dx}{(\tan x - 2)(2 \tan x + 1)}$$

Put $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

$$\therefore I = \int \frac{dt}{(t-2)(2t+1)}$$

$$= \frac{1}{5} \int \frac{t}{t-2} dt - \frac{2}{5} \int \frac{1}{2t+1} dt$$

$$= \frac{1}{5} \log(t-2) - \frac{1}{5} \log(2t+1) + c$$

$$= \log \sqrt[5]{\frac{\tan x - 2}{2 \tan x + 1}} + c$$

611 (b)

$$\int_0^\infty \frac{dx}{(a^2 + x^2)} = \left[\frac{1}{a} \tan^{-1} \frac{x}{a} \right]_0^\infty = \frac{1}{a} \left(\frac{\pi}{2} - 0 \right)$$

$$= \frac{\pi}{2a}$$

612 (b)

We have,

$$\int_1^e (\log x)^3 dx = [x(\log x)^3]_1^e - 3 \int_1^e (\log x)^2 dx$$

$$\Rightarrow \int_1^e (\log x)^3 dx = e - 3 \left[\{x(\log x)^2\}_1^e - 2 \int_1^e (\log x) dx \right]$$

$$\Rightarrow \int_1^e (\log x)^3 dx = e - 3[e - 2\{x \log x - x\}_1^e]$$

$$= 6 - 2e$$

613 (c)

We have,

$$I = \int_1^{\sqrt[7]{2}} \frac{1}{x(2x^7 + 1)} dx$$

$$\Rightarrow I = \frac{1}{7} \int_1^2 \frac{1}{y(2y + 1)} dy, \text{ where } y = x^7$$

$$\Rightarrow I = \frac{1}{7} \left[\log \left(\frac{y}{2y + 1} \right) \right]_1^2 = \frac{1}{7} \left[\log \frac{2}{5} - \log \frac{1}{3} \right]$$

$$= \frac{1}{7} \log \left(\frac{6}{5} \right)$$

614 (a)

We have, $\int f(x) dx = F(x) \dots(i)$

Let $I = \int x^3 f(x^2) dx$

Put $x^2 = t \Rightarrow 2x \, dx = dt$

$$\therefore I = \frac{1}{2} \int t f(t) dt$$

$$= \frac{1}{2} \left\{ t \int f(t) dt - \int \left(\int f(t) dt \right) dt \right\}$$

$$= \frac{1}{2} \{ tF(t) - \int F(t) dt \} \quad [\text{From Eq. (i)}]$$

$$= \frac{1}{2} \{ x^2 F(x^2) - \int F(x^2) dx^2 \}$$

615 (b)

We have,

$$\int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx$$

$$\therefore \int_0^a f(x) dx + \int_a^{2a} f(x) dx = 2 \int_0^a f(x) dx$$

$$\Rightarrow \int_a^{2a} f(x) dx = \int_0^a f(x) dx \dots(i)$$

Putting $2a - x = t$ in the integral on LHS, we get

$$\int_a^{2a} f(x) dx = \int_a^0 f(2a - t) (-dt) = \int_0^a f(2a - t) dt$$

$$\Rightarrow \int_a^{2a} f(x) dx = \int_0^a f(2a - x) dx$$

$$\therefore \int_0^a f(2a - x) dx = \int_0^a f(x) dx \Rightarrow f(2a - x) = f(x)$$

616 (c)

Let $I = \int_0^\pi x \log \sin x \, dx$. Then,

$$I = \int_0^\pi (\pi - x) \log \sin(\pi - x) dx$$

$$\Rightarrow I = \int_0^\pi (\pi - x) \log \sin x \, dx$$

$$\Rightarrow I = \pi \int_0^\pi \log \sin x \, dx - \int_0^\pi x \log \sin x \, dx$$

$$\Rightarrow I = 2\pi \int_0^{\pi/2} \log \sin x \, dx - I$$

$$\Rightarrow I = \pi \int_0^{\pi/2} \log \sin x \, dx = \pi \left(\frac{-\pi}{2} \log 2 \right)$$

$$= -\frac{\pi^2}{2} \log 2$$

617 (c)

Let $I = \int_1^2 [f\{g(x)\}]^{-1} f'\{g(x)\} g'(x) dx$

Put $f\{g(x)\} = t \Rightarrow f'\{g(x)\} g'(x) dx = dt$

$$\therefore I = \int_1^2 t^{-1} dt = [\log f\{g(x)\}]_1^2$$

$$= \log f\{g(2)\} - \log f\{g(1)\} = 0$$

[$\because g(2) = g(1)$]

618 (b)

We have,

$$\phi(x) = \int_a^x f(t) dt,$$

$$\Rightarrow \phi(-x) = \int_a^{-x} f(t) dt$$

$$\Rightarrow \phi(-x) = \int_a^x f(-u) (-du), \text{ where } t = -u$$

$$\Rightarrow \phi(-x) = \int_a^x f(u) du \quad [\because f(-u) = -f(u)]$$

$$\begin{aligned} \Rightarrow \phi(-x) &= \int_a^x f(u) du + \int_a^x f(u) du \\ &= 0 + \int_a^x f(u) du = \phi(x) \end{aligned}$$

Hence, $\phi(x)$ is an odd function

619 (b)

We have,

$$I = \int \frac{x^5}{\sqrt{1+x^3}} dx = \frac{1}{3} \int \frac{x^3}{\sqrt{1+x^3}} 3x^2 dx$$

$$\Rightarrow I = \frac{1}{3} \int \frac{t}{\sqrt{1+t}} dt, \text{ where } t = x^3$$

$$\Rightarrow I = \frac{1}{3} \int \frac{(u^2-1)2u du}{\sqrt{u^2}}, \text{ where } 1+t = u^2$$

$$\Rightarrow I = \frac{2}{3} \int (u^2-1) du = \frac{2}{3} \left(\frac{u^3}{3} - u \right) + C$$

$$\Rightarrow I = \frac{2}{9} (1+x^3)^{3/2} - \frac{2}{3} (1+x^3)^{1/2} + C$$

620 (b)

$$\text{Let } I = \int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x} dx$$

$$= \int_{-\pi/2}^{\pi/2} \sqrt{\cos x} |\sin x| dx$$

$$= 2 \int_0^{\pi/2} \sqrt{\cos x} \sin x dx$$

$$\text{Put } \cos x = t \Rightarrow -\sin x dx = dt$$

$$\therefore I = -2 \int_1^0 \sqrt{t} dt = -\frac{4}{3} [t^{3/2}]_1^0 = \frac{4}{3}$$

622 (d)

$$\int_{-3}^3 (ax^5 + bx^3 + cx + k) dx$$

$$= \left[\frac{ax^6}{6} + \frac{bx^4}{4} + \frac{cx^2}{2} + kx \right]_{-3}^3$$

$$\begin{aligned} &= \frac{a \cdot 3^6}{6} + \frac{b \cdot 3^4}{4} + \frac{c \cdot 3^2}{2} + k(3) - \frac{a \cdot 3^6}{6} - \frac{b \cdot 3^4}{4} \\ &\quad - \frac{c \cdot 3^2}{2} + k(3) \end{aligned}$$

$$= 6k$$

ie, Integral depends upon k

623 (b)

We have,

$$\lim_{n \rightarrow \infty} \frac{1}{n^2} \sum_{r=1}^n r e^{r/n} = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \left(\frac{r}{n} \right) e^{r/n}$$

$$= \int_0^1 x e^x dx = [e^x(x-1)]_0^1 = 1$$

624 (a)

$$\text{Given, } I_n = \int x^n \cdot e^{cx} dx$$

$$= \frac{e^{cx}}{c} \cdot x^n - \int \frac{e^{cx}}{c} \cdot nx^{n-1} dx$$

$$\Rightarrow I_n = \frac{e^{cx} \cdot x^n}{c} - \frac{n}{c} I_{n-1}$$

$$\Rightarrow cI_n + nI_{n-1} = e^{cx} \cdot x^n$$

625 (c)

$$\int_0^{\pi/2} \frac{1+2\cos x}{(2+\cos x)^2} dx$$

$$= \int_0^{\pi/2} \frac{d}{dx} \left(\frac{\sin x}{2+\cos x} \right) dx$$

$$= \left[\frac{\sin x}{2+\cos x} \right]_0^{\pi/2} = \frac{1}{2}$$

626 (a)

$$\text{Let } \int_0^{\pi/4} (\pi x - 4x^2) \log(1 + \tan x) dx \quad \dots(i)$$

Then,

$$I = \int_0^{\pi/4} \left\{ \pi \left(\frac{\pi}{4} - x \right) - 4 \left(\frac{\pi}{4} - x \right)^2 \right\} \log \left(1 + \tan \left(\frac{\pi}{4} - x \right) \right) dx$$

$$\Rightarrow I = \int_0^{\pi/4} (\pi x - 4x^2) \log \frac{1}{1+\tan x} dx \quad \dots(ii)$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/4} (\pi x - 4x^2) \log 2 dx$$

$$\Rightarrow I = \frac{1}{2} \log_e 2 \left[\frac{\pi x^2}{2} - \frac{4x^3}{3} \right]_0^{\pi/4}$$

$$\Rightarrow I = \frac{\pi^3}{192} \log_e 2$$

627 (a)

$$\text{Let } I = \int_0^{\pi/2} \frac{\cos x}{(1+\sin x)(2+\sin x)} dx$$

$$\text{Put } \sin x = t \Rightarrow \cos x dx = dt$$

$$\therefore I = \int_0^1 \frac{dt}{(1+t)(2+t)}$$

$$= \int_0^1 \frac{1}{(1+t)} dt - \int_0^1 \frac{1}{(2+t)} dt$$

$$\begin{aligned}
&= [\log(1+t) - \log(2+t)]_0^1 \\
&= (\log 2 - \log 3) - (0 - \log 2) \\
&= \log \frac{4}{3}
\end{aligned}$$

628 (b)

We have,

$$\begin{aligned}
I &= \int \frac{x^2}{(a+bx^2)^{5/2}} dx \\
\Rightarrow I &= \int \frac{1}{x^3 \left(\frac{a}{x^2} + b\right)^{5/2}} dx \\
\Rightarrow I &= -\frac{1}{2a} \int \left(\frac{a}{x^2} + b\right)^{-5/2} \times \frac{-2a}{x^3} dx \\
\Rightarrow I &= -\frac{1}{2a} \int \left(\frac{a}{x^2} + b\right)^{-5/2} d\left(\frac{a}{x^2} + b\right) \\
\Rightarrow I &= -\frac{1}{2a} \times -\frac{2}{3} \times \left(\frac{a}{x^2} + b\right)^{-3/2} + C \\
&= \frac{1}{3a} \left(\frac{x^2}{a+bx^2}\right)^{3/2} + C
\end{aligned}$$

629 (c)

This is a standard result and its proof is beyond the scope of this book

630 (a)

$$\begin{aligned}
\text{Let } I &= \int \frac{3^x}{\sqrt{9^x-1}} dx = \frac{3^x dx}{\sqrt{(3^x)^2-1}} \\
\text{Put } 3^x &= t \Rightarrow 3^x \log 3 dx = dt \\
\Rightarrow 3^x dx &= \frac{dt}{\log 3} \\
\therefore I &= \frac{1}{\log 3} \int \frac{dt}{\sqrt{t^2-1}} \\
&= \frac{1}{\log 3} \log [t + \sqrt{t^2-1}] + c \\
&= \frac{1}{\log 3} \log [3^x + \sqrt{9^x-1}] + c
\end{aligned}$$

631 (c)

$$\begin{aligned}
\text{Let } I &= \int \{1 + 2 \tan x (\tan x + \sec x)\}^{1/2} dx \\
&= \int \{1 + 2 \tan^2 x + 2 \tan x \sec x\}^{1/2} dx \\
&= \int \{\sec^2 x + \tan^2 x + 2 \tan x \sec x\}^{1/2} dx \\
&= \int (\sec x + \tan x) dx \\
&= \log(\sec x + \tan x) + \log \sec x + c \\
&= \log \sec x (\sec x + \tan x) + c
\end{aligned}$$

632 (b)

$$\begin{aligned}
\text{Given, } f(x) &= \int_{-1}^x |t| dt \\
&= \int_{-1}^0 |t| dt + \int_0^x |t| dt \\
&= \int_{-1}^0 -t dt + \int_0^x t dt \\
&= -\left[\frac{t^2}{2}\right]_{-1}^0 + \left[\frac{t^2}{2}\right]_0^x \\
&= -\left[0 - \frac{1}{2}\right] + \left[\frac{x^2}{2} - 0\right] = \frac{1}{2}(1+x^2)
\end{aligned}$$

633 (b)

If $0 \leq a < b$, then $|x| = x$

$$\therefore \int_a^b \frac{|x|}{x} dx = \int_a^b 1 dx = b - a = |b| - |a|$$

If $a < b < 0$, then $|x| = -x$

$$\therefore \int_a^b \frac{|x|}{x} dx = -\int_a^b 1 dx = a - b = |b| - |a|$$

If $a < 0 < b$, then

$$\begin{aligned}
\int_a^b \frac{|x|}{x} dx &= \int_a^0 \frac{|x|}{x} dx + \int_0^b \frac{|x|}{x} dx \\
\Rightarrow \int_a^b \frac{|x|}{x} dx &= -\int_a^0 1 dx + \int_0^b 1 dx = b + a = |b| - |a|
\end{aligned}$$

Hence, $\int_a^b \frac{|x|}{x} dx = |b| - |a|$

634 (d)

$$\begin{aligned}
\text{Let } I &= \int \frac{1+\tan^2 x}{1-\tan^2 x} dx = \int \frac{\sec^2 x}{1-\tan^2 x} dx \\
\text{Put } \tan x &= t \Rightarrow \sec^2 x dx = dt \\
\therefore I &= \int \frac{dt}{1-t^2} \\
&= \frac{1}{2 \times 1} \log \left[\frac{1+t}{1-t}\right] + c = \frac{1}{2} \log \left(\frac{1+\tan x}{1-\tan x}\right) + c
\end{aligned}$$

636 (d)

$$\begin{aligned}
\int_2^3 \{x\} dx &= \int_2^3 (x - [x]) dx = \int_2^3 (x - 2) dx \\
&= \left[\frac{x^2}{2} - 2x\right]_2^3 = \frac{1}{2}
\end{aligned}$$

637 (d)

We have,

$$I = \int_0^{\pi/2} \frac{\cos^3 x}{\cos^3 x + \sin^3 x} dx,$$

$$\begin{aligned} \Rightarrow I &= \int_0^{\pi/2} \frac{\cos^3\left(\frac{\pi}{2}-x\right)}{\cos^3\left(\frac{\pi}{2}-x\right) + \sin^3\left(\frac{\pi}{2}-x\right)} dx \\ &= \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} dx \\ \therefore 2I &= \int_0^{\pi/2} \frac{\cos^3 x + \sin^3 x}{\sin^3 x + \cos^3 x} dx = \frac{\pi}{2} - 0 \Rightarrow \frac{\pi}{4} \end{aligned}$$

639 (b)

We have,

$$f(x) = \int_{-1/2}^x |t| dt$$

$$\Rightarrow f'(x) = |x| > 0 \text{ for all } x \in [-1/2, 1/2]$$

$$\Rightarrow f(x) \text{ is increasing on } [-1/2, 1/2]$$

$$\Rightarrow f\left(\frac{1}{2}\right) \text{ is the greatest value of } f(x)$$

$$\text{Now, } f\left(\frac{1}{2}\right) = \int_{-1/2}^{1/2} |t| dt = 2 \int_0^{1/2} t dt = \frac{1}{4}$$

640 (b)

$$\text{Consider } f(x) = \int_0^x (1 + \cos^8 x)(ax^2 + bx + c) dx$$

Obviously, $f(x)$ is continuous and differentiable on $[1, 2]$ and $(1, 2)$ respectively.

$$\text{Also, } f(1) = f(2) \quad (\text{given})$$

\therefore By Rolle's theorem there exist at least one point $k \in (1, 2)$ such that $f'(k) = 0$

$$\text{Now, } f'(x) = (1 + \cos^8 x)(ax^2 + bx + c)$$

$$f'(k) = 0$$

$$\Rightarrow (1 + \cos^8 k)(ak^2 + bk + c) = 0$$

$$\Rightarrow ak^2 + bk + c = 0 \quad [\text{as } (1 + \cos^8 k) \neq 0]$$

$$\therefore x = k \text{ is root of } ax^2 + bx + c = 0,$$

$$\text{Where } k \in (1, 2)$$

Hence, at least one root $\in (1, 2)$

641 (d)

$$\text{We have, } f(x) = A \sin\left(\frac{\pi x}{2}\right) + B, f'\left(\frac{1}{2}\right) = \sqrt{2}$$

$$\text{and } \int_0^1 f(x) dx = \frac{2A}{\pi}$$

$$f'(x) = \frac{A\pi}{2} \cos \frac{\pi x}{2}$$

$$\Rightarrow f'\left(\frac{1}{2}\right) = \frac{A\pi}{2} \cos \frac{\pi}{4} = \frac{A\pi}{2\sqrt{2}}$$

$$\text{But } f'\left(\frac{1}{2}\right) = \sqrt{2}$$

$$\text{Therefore, } \frac{A\pi}{2\sqrt{2}} = \sqrt{2} \Rightarrow A = \frac{4}{\pi}$$

$$\text{Now, } \int_0^1 f(x) dx = \frac{2A}{\pi}$$

$$\Rightarrow \int_0^1 \left\{ A \sin\left(\frac{\pi x}{2}\right) + B \right\} dx = \frac{2A}{\pi}$$

$$\Rightarrow \left[-\frac{2A}{\pi} \cos \frac{\pi x}{2} + Bx \right]_0^1 = \frac{2A}{\pi}$$

$$\Rightarrow B + \frac{2A}{\pi} = \frac{2A}{\pi} \Rightarrow B = 0$$

642 (a)

We have,

$$1 + x^4 < (1 + x^2)^2$$

$$\Rightarrow \sqrt{1 + x^4} < 1 + x^2$$

$$\Rightarrow \frac{1}{\sqrt{1 + x^4}} > \frac{1}{1 + x^2}$$

$$\Rightarrow \int_0^1 \frac{1}{\sqrt{1 + x^4}} dx > \int_0^1 \frac{1}{1 + x^2} dx$$

$$\Rightarrow \int_0^1 \frac{1}{\sqrt{1 + x^4}} dx > [\tan^{-1} x]_0^1$$

$$\Rightarrow \int_0^1 \frac{1}{\sqrt{1 + x^4}} dx > \frac{\pi}{4} > 0.78$$

$$\text{Also, } \frac{1}{\sqrt{1 + x^4}} \leq 1$$

$$\Rightarrow \int_0^1 \frac{1}{\sqrt{1 + x^4}} \leq \int_0^1 1 \cdot dx = 1$$

Hence, $I > 0.78$ and $I \leq 1$

643 (d)

$$\therefore I_k = \int_{-2k\pi}^0 |\sin x| |\sin x| dx$$

$$+ \int_0^{2k\pi} |\sin x| |\sin x| dx$$

$$= - \int_{2k\pi}^0 |\sin x| [-\sin x] dx$$

$$+ \int_0^{2k\pi} |\sin x| [\sin x] dx$$

$$= \int_0^{2k\pi} |\sin x| ([\sin x] + [-\sin x]) dx$$

$$\begin{aligned}
&= 2k \int_0^\pi |\sin x| ([\sin x] + [-\sin x]) dx \\
&= 2k \int_0^\pi \sin x(0 - 1) dx \\
&= -2k[-\cos x]_0^\pi \quad \left(\begin{array}{l} \because \sin x > 0, x \in (0, \pi) \\ \therefore -\sin x < 0, x \in (0, \pi) \end{array} \right) \\
&= -4k \\
\therefore \sum_{k=1}^{10} I_k &= -4 \sum_{k=1}^{10} k = -4 \cdot \frac{10 \cdot 11}{2} = -220
\end{aligned}$$

644 (a)

$$\begin{aligned}
\text{Let } I &= \int_0^8 |x - 5| dx \\
&= \int_0^5 -(x - 5) dx + \int_5^8 (x - 5) dx \\
&= \left[-\frac{x^2}{2} + 5x \right]_0^5 + \left[\frac{x^2}{2} - 5x \right]_5^8 \\
&= \left(\frac{25}{2} \right) + \left(-\frac{16}{2} + \frac{25}{2} \right) = 17
\end{aligned}$$

645 (b)

$$\begin{aligned}
\text{Let } I &= \int \frac{e^x}{(2 + e^x)(e^x + 1)} dx \\
\Rightarrow I &= \int \frac{1}{(t + 2)(t + 1)} dt, \text{ where } t = e^x \\
\Rightarrow I &= \int \left(\frac{1}{t + 1} - \frac{1}{t + 2} \right) dt = \log \left(\frac{t + 1}{t + 2} \right) + C \\
&= \log \left(\frac{e^x + 1}{e^x + 2} \right) + C
\end{aligned}$$

646 (b)

$$\begin{aligned}
\int_{-1}^2 |x|^3 dx &= \int_{-1}^0 -x^3 dx + \int_0^2 x^3 dx \\
&= -\left[\frac{x^4}{4} \right]_{-1}^0 + \left[\frac{x^4}{4} \right]_0^2 \\
&= -\frac{1}{4}[0 - 1] + \frac{1}{4}[2^4] \\
&= \frac{1}{4} + 4 = \frac{17}{4}
\end{aligned}$$

647 (c)

$$\begin{aligned}
\text{Since } f(x) = x|x| \text{ is an odd function} \\
\therefore \int_{-1}^1 f|x| dx = 0
\end{aligned}$$

648 (a)

$$\begin{aligned}
\int \frac{(x + 1)}{x(1 + xe^x)^2} dx &= \int \frac{e^x + xe^x}{xe^x(1 + xe^x)^2} dx \\
\text{Put } 1 + xe^x &= t \Rightarrow e^x + xe^x dx = dt \\
\therefore \int \frac{dt}{(t - 1)t^2} &= \int \left[\frac{1}{t - 1} - \frac{(t + 1)}{t^2} \right] dt
\end{aligned}$$

$$\begin{aligned}
&= \log(t - 1) - \log t + \frac{1}{t} + c \\
&= \log \left(\frac{t - 1}{t} \right) + \frac{1}{t} + c \\
&= \log \left(\frac{xe^x}{xe^x + 1} \right) + \frac{1}{1 + xe^x} + c
\end{aligned}$$

649 (c)

We have,

$$\begin{aligned}
F(x) &= \int_1^x |t| dt \\
\Rightarrow F'(x) &= |x| \text{ for all } x \in [-1/2, 1/2] \\
\Rightarrow F'(x) &> 0 \text{ for all } x \in [-1/2, 1/2] \\
\Rightarrow F(x) &\text{ is increasing on } [-1/2, 1/2] \\
\Rightarrow F(-1/2) &\text{ and } F(1/2) \text{ are minimum and} \\
&\text{ maximum values of } F(x) \\
\therefore \text{Required value} &= F\left(\frac{1}{2}\right) \\
&= \int_1^{1/2} |t| dt = \left(\frac{t^2}{2}\right)_1^{1/2} = \frac{1}{2} \left(\frac{1}{4} - 1\right) = -\frac{3}{8}
\end{aligned}$$

650 (a)

We have,

$$\begin{aligned}
I &= \int_0^{\pi/2} \frac{f(x)}{f(x) + f(\frac{\pi}{2} - x)} dx \quad \dots(i) \\
\Rightarrow I &= \int_0^{\pi/2} \frac{f(\frac{\pi}{2} - x)}{f(\frac{\pi}{2} - x) + f(x)} dx \quad \dots(ii)
\end{aligned}$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2} 1 \cdot dx = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

651 (c)

$$\text{Let } I = \int \frac{(x^2 - 1)dx}{x^3 \sqrt{2x^4 - 2x^2 + 1}}$$

On dividing Nr and Dr by x^5 , we get

$$I = \int \frac{\left(\frac{1}{x^3} - \frac{1}{x^5}\right) dx}{\sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}}}$$

$$\text{Put } 2 - \frac{2}{x^2} + \frac{1}{x^4} = t \Rightarrow \left(\frac{4}{x^3} - \frac{4}{x^5}\right) dx = dt$$

$$\therefore I = \frac{1}{4} \int \frac{dt}{\sqrt{t}} = \frac{1}{2} \sqrt{t} + c = \frac{1}{2} \sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}} + c$$

652 (c)

$$I = \int_0^1 x(1 - x)^n dx$$

$$\text{Put } 1 - x = z \Rightarrow -dx = dz$$

$$\therefore I = \int_1^0 (1 - z)z^n (-dz)$$

$$= \int_0^1 (z^n - z^{n+1}) dz$$

$$= \left[\frac{z^{n+1}}{n+1} - \frac{z^{n+2}}{n+2} \right]_0^1 = \frac{1}{n+1} - \frac{1}{n+2}$$

Alternate Given integral can be rewritten as

$$I = \int_0^1 x^{2-1} (1-x)^{(n+1)-1} dx$$

It is the form of beta function

$$\begin{aligned} \therefore I &= B(2, n+1) = \frac{\sqrt{2}\sqrt{n+1}}{\sqrt{(n+3)}} \\ &= \frac{1}{(n+1)(n+2)} = \frac{1}{n+1} - \frac{1}{n+2} \end{aligned}$$

653 (b)

$$\text{Let } I = \int \frac{x^2}{(ax+b)} dx$$

$$\text{Put } ax+b = t \Rightarrow dx = \frac{1}{a} dt \text{ and } x = \left(\frac{t-b}{a}\right)$$

$$\therefore I = \frac{1}{a^3} \int \frac{(t-b)^2}{t^2} dt = \frac{1}{a^3} \int \left(1 + \frac{b^2}{t^2} - \frac{2b}{t}\right) dt$$

$$= \frac{1}{a^3} \left(t - \frac{b^2}{t} - 2b \log t \right) + c$$

$$= \frac{1}{a^3} \left[ax + b - \frac{b^2}{ax+b} - 2b \log(ax+b) \right] + c$$

$$= \frac{1}{a^3} \left[\frac{a^2x^2 + b^2 + 2axb - b^2}{(ax+b)} - 2b \log(ax+b) \right] + c$$

$$= \frac{1}{a^3} \left[\frac{2ax(ax+b) - a^2x^2}{ax+b} - 2b \log(ax+b) \right] + c$$

$$= \frac{1}{a^3} \left[2ax - \frac{a^2x^2}{ax+b} - 2b \log(ax+b) \right] + c$$

$$= \frac{2}{a^2} \left[x - \frac{b}{a} \log(ax+b) \right] - \frac{x^2}{a(ax+b)} + c$$

654 (d)

$$I = \int_{-1}^1 \frac{1}{\sqrt{1-2ax+a^2}} dx$$

$$\Rightarrow I = \int_{-1}^1 (1-2ax+a^2)^{1/2} dx$$

$$\Rightarrow \left[-\frac{1}{a} (1-2ax+a^2)^{1/2} \right]_{-1}^1$$

$$\Rightarrow I = -\frac{1}{a} \sqrt{(1-a)^2} + \frac{1}{a} \sqrt{(1+a)^2}$$

$$\Rightarrow I = -\frac{1}{a} |1-a| + \frac{1}{a} (1+a)$$

$$\Rightarrow I = 2$$

656 (a)

$$\text{Let } I = \int \frac{\sin x + \cos x}{3 + \sin 2x} dx = \int \frac{\sin x + \cos x}{4 + \sin 2x - 1} dx$$

$$\Rightarrow I = \int \frac{\sin x + \cos x}{4 - (\sin x - \cos x)^2} dx$$

$$\text{Put } \sin x - \cos x = t \Rightarrow (\cos x + \sin x) dx = dt$$

$$\therefore I = \int \frac{dt}{4-t^2} = \frac{1}{4} \log \frac{2-t}{2+t} + c$$

$$= \frac{1}{4} \log \left(\frac{2 - \sin x + \cos x}{2 + \sin x - \cos x} \right) + c$$

657 (d)

$$\text{Let } I = \int_{e^{-1}}^e \frac{dt}{t(t+1)}$$

$$= \int_{e^{-1}}^e \frac{1}{t} dt - \int_{e^{-1}}^e \frac{1}{t+1} dt$$

$$= [\log t - \log(t+1)]_{e^{-1}}^e$$

$$= [\log e - \log(e+1)] - [\log e^{-1} - \log(e^{-1}+1)]$$

$$= \log \left(\frac{e}{e+1} \right) - [-\log e - \log(e^{-1}+1)]$$

$$= \log \left(\frac{e}{e+1} \right) + \left[\log e \left(\frac{1}{e} + 1 \right) \right]$$

$$= \log_e e = 1$$

658 (b)

Let

$$I = \int_0^{\pi/2} \frac{\cos 3x + 1}{2 \cos x - 1} dx = \int_0^{\pi/2} \frac{\cos 3x - \cos 3\pi/3}{2(\cos x - \cos \pi/3)} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{(4 \cos^3 x - 3 \cos x) - (4 \cos^3 \pi/3 - 3 \cos \pi/3)}{2(\cos x - \cos \pi/3)} dx$$

$$\Rightarrow I = 2 \int_0^{\pi/2} \left(\frac{\cos^3 x - \cos^3 \pi/3}{\cos x - \cos \pi/3} \right) dx$$

$$- \frac{3}{2} \int_0^{\pi/2} \left(\frac{\cos x - \cos \pi/3}{\cos x - \cos \pi/3} \right) dx$$

$$\Rightarrow I = 2 \int_0^{\pi/2} (\cos^2 x + \cos^2 \pi/3 + \cos \pi/3) dx$$

$$- \frac{3}{2} \int_0^{\pi/2} 1 \cdot dx$$

$$\Rightarrow I = \int_0^{\pi/2} \left(1 + \cos 2x + \frac{1}{2} + \cos x \right) dx - \frac{3\pi}{4}$$

$$= \frac{3\pi}{4} + 1 - \frac{3\pi}{4} = 1$$

659 (a)

Putting $x = \frac{1}{1+y}$, $dx = -\frac{1}{(1+y)^2} dy$, we have

$$I(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1} dx$$

$$\Rightarrow I(m, n) = \int_0^1 \frac{1}{(1+y)^{m-1}} \left(1 - \frac{1}{1+y}\right)^{n-1} dy$$

$$= \int_0^1 \frac{(-1)^{n-1}}{(1+y)^{m+n-1}} dy$$

$$\Rightarrow I(m, n) = \int_0^1 \frac{y^{n-1}}{(1+y)^{m+n}} dy = \int_0^1 \frac{x^{n-1}}{(1+x)^{m+n}} dx$$

But $I(m, n) = I(n, m)$

$$\therefore I(m, n) = \int_0^1 \frac{x^{m-1}}{(1+x)^{m+n}} dx = \int_0^1 \frac{x^{n-1}}{(1+x)^{m+n}} dx$$

660 (b)

$\int_{-1}^{15} \operatorname{sgn}(\{x\}) dx = \int_0^{16} \operatorname{sgn}(\{x-1\}) dx$ (by property)

$$= \int_0^{16} \operatorname{sgn}(\{x\}) dx$$

$$= 16 \int_0^1 \operatorname{sgn}(\{x\}) dx = 16 \int_0^1 \operatorname{sgn}(x) dx$$

$$= 16 \int_0^1 1 \cdot dx = 16$$

661 (b)

Let $I = \int_5^{10} \frac{1}{(x-1)(x-2)} dx$

$$\int_5^{10} \left[\frac{-1}{x-1} + \frac{1}{x-2} \right] dx$$

$$= [-\log(x-1) + \log(x-2)]_5^{10}$$

$$= -\log 9 + \log 8 + \log 4 - \log 3$$

$$= -2 \log 3 + 3 \log 2 + 2 \log 2 - \log 3$$

$$= -3 \log 3 + 5 \log 2$$

$$= -\log 27 + \log 32$$

$$= \log \frac{32}{27}$$

662 (d)

Let $f(x) = (e^{x^3} + e^{-x^3})(e^x - e^{-x})$

$$\Rightarrow f(-x) = (e^{-x^3} + e^{x^3})(e^{-x} - e^x)$$

$$= -(e^{x^3} + e^{-x^3})(e^x - e^{-x})$$

$$= -f(x)$$

$\Rightarrow f(x)$ is an odd function.

$$\therefore \int_{-1}^1 f(x) dx = 0$$

663 (b)

$$\int_{b-1}^b \frac{e^{-t}}{t-b-1} dt = \int_{-1}^0 \frac{e^{-(t+b)}}{(t+b)-b-1} dt$$

$$= e^{-b} \int_{-1}^0 \frac{e^{-t}}{t-1} dt$$

$$= e^{-b} \int_0^1 \frac{e^{-(t-1)}}{t-2} dt$$

Put $t-1 = -s$

$$\Rightarrow dt = -ds$$

$$= -e^{-b} \int_1^0 \frac{e^s}{-(s+1)} ds$$

$$= e^{-b} \int_1^0 \frac{e^s}{s+1} ds$$

$$= -e^{-b} \int_0^1 \frac{e^t}{t+1} dt$$

$$= -ae^{-b}$$

664 (c)

Let $x = \alpha \cos^2 \theta + \beta \sin^2 \theta$. Then,
 $x - \alpha = (\beta - \alpha) \sin^2 \theta$, $\beta - x = (\beta - \alpha) \cos^2 \theta$
 and, $dx = 2(\beta - \alpha) \sin \theta \cos \theta d\theta$
 Also, $x = \alpha \Rightarrow \theta = 0$ and, $x = \beta \Rightarrow \theta = \pi/2$

$$\therefore \int_{\alpha}^{\beta} \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}}$$

$$= \int_0^{\pi/2} \frac{2(\beta-\alpha) \sin \theta \cos \theta d\theta}{(\beta-\alpha) \sin \theta \cos \theta}$$

$$= 2 \int_0^{\pi/2} d\theta = \pi$$

665 (a)

We have,

$$I = \int \frac{1}{\{(x-1)^3(x+2)^5\}^{1/4}} dx$$

$$= \int \frac{1}{\left(\frac{x-1}{x+2}\right)^{3/4} (x+2)^2} dx$$

$$\Rightarrow I = \frac{1}{3} \int \left(\frac{x-1}{x+2}\right)^{-3/4} d\left(\frac{x-1}{x+2}\right)$$

$$= \frac{4}{3} \left(\frac{x-1}{x+2}\right)^{1/4} + C$$

666 (b)

Let

$$I = \int_{-\pi/3}^{\pi/3} x \sec x \tan x dx \quad [x \sec x \tan x \text{ is an even}]$$

$$= 2 \int_0^{\pi/3} x \sec x \tan x dx$$

$$= 2 \left[x \sec x - \log \left\{ \tan \left(\frac{\pi}{4} + \frac{x}{2} \right) \right\} \right]_0^{\pi/3}$$

$$= \frac{4\pi}{3} - 2 \log \tan \frac{5\pi}{12}$$

667 (a)

We have,

$$I = \int \frac{(x^4 - x)^{1/4}}{x^5} dx = \int \frac{1}{x^4} \left(1 - \frac{1}{x^3}\right)^{1/4} dx$$

$$\Rightarrow I = \frac{1}{3} \int \left(1 - \frac{1}{x^3}\right)^{1/4} d\left(1 - \frac{1}{x^3}\right)$$

$$= \frac{4}{15} \left(1 - \frac{1}{x^3}\right)^{5/4} + C$$

668 (c)

$f(x)$

$$= \begin{vmatrix} \sin x + \sin 2x + \sin 3x & \sin 2x & \sin 3x \\ 3 + 4 \sin x & 3 & 4 \sin x \\ 1 + \sin x & \sin x & 1 \end{vmatrix}$$

[Applying $C_1 \rightarrow C_1 - (C_2 + C_3)$]

$$= \begin{vmatrix} \sin x & \sin 2x & \sin 3x \\ 0 & 3 & 4 \sin x \\ 0 & \sin x & 1 \end{vmatrix}$$

$$= \sin x (3 - 4 \sin^2 x)$$

$$= \sin 3x$$

$$\text{Then, } \int_0^{\pi/2} f(x) dx = \int_0^{\pi/2} \sin 3x dx$$

$$= - \left[\frac{\cos 3x}{3} \right]_0^{\pi/2}$$

$$= \left(-\frac{1}{3}\right) \left[\cos \left(3 \times \frac{\pi}{2}\right) - \cos 0 \right]$$

$$= \left(-\frac{1}{3}\right) (0 - 1)$$

$$= \frac{1}{3}$$

669 (a)

$$\text{Let } I = \int \frac{(x^3 + 3x^2 + 3x + 1)}{(x+1)^5} dx$$

$$= \int \frac{(x+1)^3}{(x+1)^5} dx = \int \frac{1}{(x+1)^2} dx$$

$$= -\frac{1}{(x+1)} + c$$

670 (c)

$$\text{Put } x = a (\sin \theta)^{2/3}$$

$$\Rightarrow dx = \frac{2}{3} a (\sin \theta)^{-1/3} \cos \theta d\theta$$

$$\therefore \int \sqrt{\frac{x}{a^3 - x^3}} dx$$

$$= \int \frac{a^{1/2} (\sin \theta)^{1/3} \cdot \frac{2}{3} a (\sin \theta)^{-1/3} \cos \theta}{\sqrt{a^3 - a^3 \sin^2 \theta}} d\theta$$

$$= \frac{2}{3} a^{3/2} \int \frac{\cos \theta}{a^{3/2} \sqrt{1 - \sin^2 \theta}} d\theta$$

$$= \frac{2}{3} \sin^{-1} \left(\frac{x}{a} \right)^{3/2} + c = g(x) + c$$

$$\therefore g(x) = \frac{2}{3} \sin^{-1} \sqrt{\frac{x^3}{a^3}}$$

671 (c)

We know that, if

$$I_n = \int \sin^n x dx, \text{ then}$$

$$I_n = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n+1}{n} I_{n-2}$$

where n is a positive integer

$$\Rightarrow n I_n - (n-1) I_{n+2} = -\sin^{n-1} x \cos x$$

672 (b)

$$\text{Let } I = \int_0^{\pi} e^{\sin^2 x} \cos^3 x dx \dots (i)$$

$$\Rightarrow I = \int_0^{\pi} e^{\sin^2(\pi-x)} \cos^3(\pi-x) dx$$

$$\Rightarrow I = -\int_0^{\pi} e^{\sin^2 x} \cos^3 x dx \dots (ii)$$

On adding Eqs.(i) and (ii), we get

$$2I = 0 \Rightarrow I = 0$$

673 (b)

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\int_{\pi/2}^x (2^{-\cos t} - 1) dt}{\int_{\pi^2/4}^{x^2} (\sqrt{t} - \frac{\pi}{2}) dt} = \lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{2^{-\cos x} - 1}{\left(x - \frac{\pi}{2}\right) \cdot 2x} \right)$$

Applying L' Hospital's rule

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{2^{-\cos x} (\log_e 2) (\sin x)}{2x + 2 \left(x - \frac{\pi}{2}\right)} = \frac{\log_e 2}{\pi}$$

674 (c)

$$\int f(x) g''(x) dx - \int f''(x) g(x) dx$$

$$= [f(x) g'(x) - \int f'(x) g'(x) dx] - [g(x) f'(x)$$

$$- \int g'(x) f'(x) dx]$$

$$= f(x) g'(x) - f'(x) g(x)$$

675 (d)

We have,

$$f(x) = A \sin \left(\frac{\pi x}{2} \right) + B, f' \left(\frac{1}{2} \right)$$

$$= \sqrt{2} \text{ and, } \int_0^1 f(x) dx = \frac{2A}{\pi}$$

$$\therefore f'(x) = \frac{A\pi}{2} \cos \left(\frac{\pi x}{2} \right)$$

$$\Rightarrow f' \left(\frac{1}{2} \right) = \frac{A\pi}{2} \cos \frac{\pi}{4} = \frac{A\pi}{2\sqrt{2}} \Rightarrow \frac{A\pi}{2\sqrt{2}} = \sqrt{2} \Rightarrow A = \frac{4}{\pi}$$

Now,

$$\int_0^1 f(x) dx = \frac{2A}{\pi}$$

$$\Rightarrow \int_0^1 \left\{ A \sin\left(\frac{\pi x}{2}\right) + B \right\} dx = \frac{2A}{\pi}$$

$$\Rightarrow \left[-\frac{2A}{\pi} \cos\left(\frac{\pi x}{2}\right) + Bx \right]_0^1 = \frac{2A}{\pi} \Rightarrow B + \frac{2A}{\pi} = \frac{2A}{\pi}$$

$$\Rightarrow B = 0$$

676 (d)

$$I = \int \frac{dx}{\cos x - \sin x}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\left(\frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x\right)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\cos\left(x + \frac{\pi}{4}\right)}$$

$$= \frac{1}{\sqrt{2}} \int \sec\left(x + \frac{\pi}{4}\right) dx$$

$$= \frac{1}{\sqrt{2}} \log \left| \tan\left(\frac{\pi}{4} + \frac{x}{2} + \frac{\pi}{8}\right) \right| + c$$

$$= \frac{1}{\sqrt{2}} \log \left| \tan\left(\frac{x}{2} + \frac{3\pi}{8}\right) \right| + c$$

677 (b)

$$\text{Let } I = \int \frac{(x+1)^2}{x(x^2+1)} dx = \int \frac{x^2+1+2x}{x(x^2+1)} dx$$

$$= \int \frac{1}{x} dx + \int \frac{2}{x^2+1} dx$$

$$\Rightarrow I = \log_e x + 2 \tan^{-1} x + c$$

678 (a)

$$\text{Let } I = \int \sqrt{2} \sqrt{1 + \sin x} dx = \sqrt{2} \int \left(\sin \frac{x}{2} + \cos x \right) dx$$

$$= 2 \int \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) dx = -4 \cos\left(\frac{x}{2} + \frac{\pi}{4}\right) + c$$

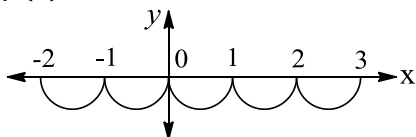
But $I = -4 \cos(ax + b) + c$ (given)

On comparing, we get

$$a = \frac{1}{2}, b = \frac{\pi}{4}$$

679 (b)

Graph of $y = f(x)$ will be of the following form. From the graph we can observe that period of $f(x)$ is 1 unit.



$$\therefore \left| \int_2^4 f(x) dx \right| = 2 \left| \int_0^1 x(x-1) dx \right| = \frac{1}{3} \text{ sq unit}$$

680 (a)

$$\text{Let } I = \int_0^{\pi/2} \frac{\sin 8x \log \cot x}{\cos 2x} dx \quad \dots(i)$$

Then,

$$I = \int_0^{\pi/2} \frac{\sin 8\left(\frac{\pi}{2} - x\right) \log \cot\left(\frac{\pi}{2} - x\right)}{\cos 2\left(\frac{\pi}{2} - x\right)} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\sin 8x \log \tan x}{\cos 2x} dx \quad \dots(ii)$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{\sin 8x}{\cos 2x} \log 1 dx = 0$$

$$\Rightarrow I = 0$$

681 (a)

We have,

$$f(x) = \begin{cases} 3[x] - 5, & x > 0 \\ 2, & x = 0 \\ 3[x] + 5, & x < 0 \end{cases}$$

$$\therefore I = \int_{-3/2}^2 f(x) dx$$

$$\Rightarrow I = \int_{-3/2}^{-1} (3[x] + 5) dx + \int_{-1}^0 (3[x] + 5) dx$$

$$+ \int_0^1 (3[x] - 5) dx + \int_1^2 (3[x] - 5) dx$$

$$\Rightarrow I = \int_{-3/2}^{-1} (-6 + 5) dx + \int_{-1}^0 (-3 + 5) dx$$

$$+ \int_0^1 (-5) dx + \int_1^2 (3 - 5) dx$$

$$\Rightarrow I = -\left(-1 + \frac{3}{2}\right) + 2(0 + 1) - 5(1 - 0) + (-2)(2 - 1)$$

$$\Rightarrow I = -\frac{1}{2} + 2 - 5 - 2 = \frac{-11}{2}$$

682 (a)

$$\text{Let } I = \int \frac{(1+x^4)}{(1-x^4)^{3/2}} dx$$

$$= \int \frac{x^2 \left(\frac{1}{x^2} + x^2\right)}{x^3 \left(\frac{1}{x^2} - x^2\right)} dx = \int \frac{\left(\frac{1}{x^3} + x\right)}{\left(\frac{1}{x^2} - x^2\right)} dx$$

$$\text{Put } \frac{1}{x^2} - x^2 = z$$

$$\Rightarrow \left(\frac{1}{x^3} + x\right) dx = -\frac{dz}{z}$$

$$\therefore I = -\frac{1}{2} \int \frac{dz}{z^{3/2}}$$

$$= z^{-1/2} + c = \frac{x}{\sqrt{1-x^4}} + c$$

683 (a)

We have,

$$\begin{aligned}
I &= \int \frac{\sin \theta - \cos \theta}{(\sin \theta + \cos \theta) \sqrt{\sin \theta \cos \theta + \sin^2 \theta \cos^2 \theta}} d\theta \\
&\Rightarrow I \\
&= \int \frac{\sin^2 \theta - \cos^2 \theta}{(\sin \theta + \cos \theta)^2 \sqrt{\left(\sin \theta \cos \theta + \frac{1}{2}\right)^2 - \frac{1}{4}}} d\theta \\
&\Rightarrow I = - \int \frac{2 \cos 2\theta}{(1 + \sin 2\theta) \sqrt{(1 + \sin 2\theta)^2 - 1}} d\theta \\
&\Rightarrow I = - \int \frac{1}{(1 + \sin 2\theta) \sqrt{(1 + \sin 2\theta)^2 - 1 + \sin 2\theta}} d(1 + \sin 2\theta) \\
&\Rightarrow I = \operatorname{cosec}^{-1}(1 + \sin 2\theta) \\
&\text{Hence, } f(\theta) = 1 + \sin 2\theta
\end{aligned}$$

684 (d)

$$\begin{aligned}
&\because f(x) = \cos x - \cos^2 x + \cos^3 x - \dots \infty \\
&= \frac{\cos x}{1 + \cos x} \\
&\therefore \int f(x) dx = \int \frac{1 + \cos x}{1 + \cos x} dx \\
&\quad - \int \frac{1}{1 + \cos x} dx \\
&= \int 1 dx - \frac{1}{2} \int \sec^2 \frac{x}{2} dx \\
&= x - \frac{1}{2} \tan \frac{x}{2} + c \\
&= x - \tan \frac{x}{2} + c
\end{aligned}$$

685 (c)

We have,

$$\begin{aligned}
F(x) &= \frac{1}{x^2} \int_4^x [4t^2 - 2F'(t)] dt \\
&\Rightarrow F'(x) = \frac{1}{x^2} [4x^2 - 2F'(x)] \\
&\quad - \frac{2}{x^3} \int_4^x \{4t^2 - 2F'(t)\} dt \\
&\Rightarrow F'(4) = \frac{1}{16} [64 - 2F'(4)] - 0 \\
&\Rightarrow 18F'(4) = 64 \Rightarrow F'(4) = \frac{32}{9}
\end{aligned}$$

686 (d)

We have, $f'(x) = f(x)$

$$\begin{aligned}
&\therefore \frac{f'(x)}{f(x)} = 1 \\
&\Rightarrow \log f(x) = x \log C
\end{aligned}$$

$$\Rightarrow f(x) = C e^x$$

$$\Rightarrow 1 = C e^0 \Rightarrow C = 1 \quad [\because f(0) = 1]$$

$$\therefore f(x) = e^x \text{ and } g(x) = x^2 - e^x$$

$$\begin{aligned}
\therefore \int_0^1 f(x) g(x) dx &= \int_0^1 e^x (x^2 - e^x) dx \\
&= e - \frac{1}{2} e^2 - \frac{3}{2}
\end{aligned}$$

687 (a)

We have,

$$\int_0^\pi \frac{1}{a + b \cos x} dx = \frac{\pi}{\sqrt{a^2 - b^2}}$$

$$\Rightarrow \frac{d}{da} \int_0^\pi \frac{1}{a + b \cos x} dx = - \frac{\pi}{2(a^2 - b^2)^{3/2}} \times 2a$$

$$\Rightarrow \int_0^\pi - \frac{1}{(a + b \cos x)^2} dx = - \frac{\pi a}{(a^2 - b^2)^{3/2}}$$

$$\Rightarrow \int_0^\pi \frac{1}{(a + b \cos x)^2} dx = \frac{\pi a}{(a^2 - b^2)^{3/2}}$$

688 (c)

$$\begin{aligned}
&\int \frac{e^x(1 + \sin x)}{(1 + \cos x)} dx \\
&= \int e^x \left[\frac{1}{2} \sec^2 \frac{x}{2} + \tan \frac{x}{2} \right] dx \\
&= \frac{1}{2} \int e^x \sec^2 \frac{x}{2} dx + \int e^x \tan \frac{x}{2} dx \\
&= e^x \tan \frac{x}{2} - \int e^x \tan \frac{x}{2} dx + \int e^x \tan \frac{x}{2} dx \\
&= e^x \tan \frac{x}{2} + c \\
&\text{But } I = e^x f(x) + c \text{ (given)} \\
&\therefore f(x) = \tan \frac{x}{2}
\end{aligned}$$

689 (c)

$$\begin{aligned}
\text{Let } I &= \int \frac{dx}{(e^x + e^{-x})^2} = \int \frac{e^{2x} dx}{(e^{2x} + 1)^2} \\
\text{Put } e^{2x} + 1 &= z \\
&\Rightarrow 2e^{2x} dx = dz \\
&\therefore I = \frac{1}{2} \int \frac{dz}{z^2} \\
&= -\frac{1}{2z} + c \\
&= -\frac{1}{2(e^{2x} + 1)} + c \\
&= -\frac{1}{2} (e^{2x} + 1)^{-1} + c
\end{aligned}$$

690 (a)

We have,

$$I = \int \frac{x \cos x + 1}{\sqrt{2x^3 e^{\sin x} + x^2}} dx$$

$$\begin{aligned} \Rightarrow I &= \int \frac{1}{2x e^{\sin x}} \times \frac{(2x \cos x + 2) e^{\sin x}}{\sqrt{2x e^{\sin x} + 1}} dx \\ \Rightarrow I &= 2 \int \frac{1}{t^2 - 1} dt, \text{ where } 2x e^{\sin x} + 1 = t^2 \\ \Rightarrow I &= \log \left| \frac{t-1}{t+1} \right| + C \\ &= \log \left| \frac{\sqrt{2x e^{\sin x} + 1} - 1}{\sqrt{2x e^{\sin x} + 1} + 1} \right| + C \end{aligned}$$

691 (a)

We have,

$$\begin{aligned} I &= \int_1^3 \frac{3 \log_e |\sin x^3|}{x} dx \\ \Rightarrow I &= \int_1^{27} \frac{3x^2 \log_e |\sin x^3|}{x^3} dx \\ \Rightarrow I &= \int_1^3 \frac{\log_e |\sin x^3|}{x^3} d(x^3), \text{ where } t = x^3 \\ \Rightarrow I &= \int_1^{27} \frac{\log_e |\sin t|}{t} dt \\ \Rightarrow I &= \int_1^{27} \phi'(t) dt = [\phi(t)]_1^{27} = \phi(27) - \phi(1) \end{aligned}$$

Hence, $k = 27$

692 (c)

$$\begin{aligned} \int \frac{e^{5 \log_e x} - e^{4 \log_e x}}{e^{3 \log_e x} - e^{2 \log_e x}} dx &= \int \frac{e^{\log_e x^5} - e^{\log_e x^4}}{e^{\log_e x^3} - e^{\log_e x^2}} dx \\ &= \int \frac{x^5 - x^4}{x^3 - x^2} dx = \int \frac{x^4(x-1)}{x^2(x-1)} dx \\ &= \int x^2 dx = \frac{x^3}{3} + c \end{aligned}$$

693 (c)

$$\begin{aligned} \text{Let } I &= \int_0^{2a} \frac{f(x)}{f(x)+f(2a-x)} dx. \text{ Then,} \\ I &= \int_0^{2a} \frac{f(2a-x)}{f(2a-x)+f(x)} dx \\ \therefore 2I &= \int_0^{2a} \frac{f(x)+f(2a-x)}{f(x)+f(2a-x)} dx = \int_0^{2a} 1 \cdot dx = 2a \\ \Rightarrow I &= a \end{aligned}$$

694 (c)

$$\begin{aligned} |x| < \sqrt{1+x^2} &\Rightarrow \frac{1}{|x|} > \frac{1}{\sqrt{1+x^2}} \\ \therefore \int_0^1 \frac{1}{|x|} dx &> \int_0^1 \frac{1}{\sqrt{1+x^2}} dx \\ \Rightarrow I_1 &> I_2 \end{aligned}$$

695 (d)

\therefore Integrand is discontinuous at $\frac{\pi}{2}$, then

$$\int_0^{\pi/2} 0 \cdot dx + \int_{\pi/2}^{3\pi/2} 0 \cdot dx = 0$$

$\therefore 0 < x < \frac{\pi}{2}, |\tan^{-1} \tan x| = |\sin^{-1} \sin x|$

and $\frac{\pi}{2} < x < \frac{3\pi}{2}, |\tan^{-1} \tan x| = |\sin^{-1} \sin x|$

696 (c)

We have,

$$\begin{aligned} I &= \int \sqrt{\frac{\cos x - \cos^3 x}{1 - \cos^3 x}} dx \\ &= \int \sqrt{\frac{\cos x}{1 - \cos^3 x}} \sin x dx \\ \Rightarrow I &= -\frac{2}{3} \int \frac{1}{1^2 - (\cos^{3/2} x)^2} d(\cos^{3/2} x) \\ \Rightarrow I &= \frac{2}{3} \cos^{-1}(\cos^{3/2} x) + C \end{aligned}$$

697 (b)

We have,

$$\begin{aligned} \int x \sqrt{\frac{2 \sin(x^2+1) - \sin 2(x^2+1)}{2 \sin(x^2+1) + \sin 2(x^2+1)}} dx \\ = \int x \sqrt{\frac{2 \sin(x^2+1) - 2 \sin(x^2+1) \cos(x^2+1)}{2 \sin(x^2+1) + 2 \sin(x^2+1) \cos(x^2+1)}} dx \\ = \int x \sqrt{\frac{1 - \cos(x^2+1)}{1 + \cos(x^2+1)}} dx \\ = \int x \tan\left(\frac{x^2+1}{2}\right) dx \\ = \int \tan\left(\frac{x^2+1}{2}\right) d\left(\frac{x^2+1}{2}\right) \\ = \log \left| \sec\left(\frac{x^2+1}{2}\right) \right| + C \end{aligned}$$

698 (a)

Let $I = \int_0^{\pi} \frac{x dx}{1 + \cos \alpha \sin x}$. Then,

$$\begin{aligned} I &= \int_0^{\pi} \frac{\pi - x}{1 + \cos \alpha \sin(\pi - x)} dx \\ \Rightarrow I &= \int_0^{\pi} \frac{\pi}{1 + \cos \alpha \sin x} dx - I \\ \Rightarrow 2I &= \pi \int_0^{\pi} \frac{\sec^2 x/2}{1 + \tan^2(x/2) + 2 \tan(x/2) \cos \alpha} dx \\ \Rightarrow 2I &= 2\pi \int_0^{\infty} \frac{dt}{t^2 + 2t \cos \alpha + 1}, \text{ where } t = \tan \frac{x}{2} \\ \Rightarrow 2I &= 2\pi \int_0^{\infty} \frac{dt}{(t + \cos \alpha)^2 + \sin^2 \alpha} \end{aligned}$$

$$\begin{aligned} \Rightarrow I &= \frac{\pi}{\sin \alpha} \left[\tan^{-1} \left(\frac{t + \cos \alpha}{\sin \alpha} \right) \right]_0^\infty \\ \Rightarrow I &= \frac{\pi}{\sin \alpha} \left[\frac{\pi}{2} - \tan^{-1}(\cot \alpha) \right] \\ \Rightarrow I &= \frac{\pi}{\sin \alpha} \left[\frac{\pi}{2} - \tan^{-1} \tan \left(\frac{\pi}{2} - \alpha \right) \right] \Rightarrow I = \frac{\pi \alpha}{\sin \alpha} \end{aligned}$$

699 (d)

$$\begin{aligned} \text{Let, } I &= \int \tan(\sin^{-1} x) dx \\ &= \int \tan \left(\tan^{-1} \frac{x}{\sqrt{1-x^2}} \right) dx = \int \frac{x}{\sqrt{1-x^2}} dx \\ \text{Put, } 1-x^2 &= t^2 \\ \Rightarrow -2x dx &= 2t dt \\ \therefore I &= - \int \frac{t dt}{t} = -t + c = -\sqrt{1-x^2} + c \end{aligned}$$

700 (b)

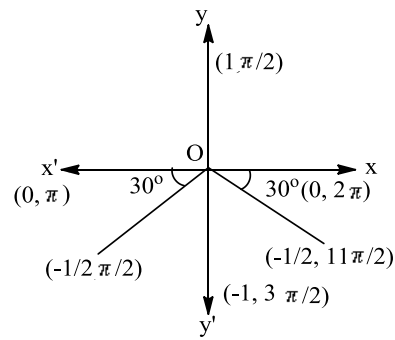
$$\begin{aligned} \text{Let } I &= \int_0^1 \sqrt{\frac{1-x}{1+x}} dx = \int_0^1 \frac{1-x}{\sqrt{1-x^2}} dx \\ &= \int_0^1 \frac{1}{\sqrt{1-x^2}} dx - \int_0^1 \frac{x}{\sqrt{1-x^2}} dx \\ &= [\sin^{-1} x]_0^1 - \int_0^1 \frac{x}{\sqrt{1-x^2}} dt \\ \text{Put } t^2 &= 1-x^2 \Rightarrow t dt = -x dx \\ \therefore I &= [\sin^{-1} 1 - \sin^{-1} 0] + \int_1^0 \frac{t}{t} dt \\ &= \frac{\pi}{2} + [t]_1^0 = \frac{\pi}{2} - 1 \end{aligned}$$

Alternate

$$\begin{aligned} \text{Put } x &= \cos 2\theta \Rightarrow dx = -2 \sin 2\theta d\theta \\ \therefore I &= \int_{\pi/4}^0 - \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}} \cdot 2 \sin 2\theta d\theta \\ &= 2 \int_0^{\pi/4} \frac{\sin \theta}{\cos \theta} \cdot 2 \sin \theta \cos \theta d\theta \\ &= 2 \int_0^{\pi/4} (1 + \cos 2\theta) d\theta = 2 \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{\pi/4} \\ &= \frac{\pi}{2} - 1 \end{aligned}$$

701 (b)

$$\begin{aligned} \text{We know that } \sin \frac{\pi}{6} &= \frac{1}{2} \\ \sin \left(\pi + \frac{\pi}{6} \right) &= \sin \frac{7\pi}{6} = -\frac{1}{2} \end{aligned}$$



$$\sin \frac{11\pi}{6} = \sin \left(2\pi - \frac{\pi}{6} \right)$$

$$= -\sin \frac{\pi}{6} = -\frac{1}{2}$$

$$\sin \frac{9\pi}{6} = \sin \frac{3\pi}{6} = -1$$

Hence, we divide the interval π to 2π as

$$\left(\pi, \frac{7\pi}{6} \right), \left(\frac{7\pi}{6}, \frac{11\pi}{6} \right), \left(\frac{11\pi}{6}, 2\pi \right)$$

$$\sin x = \left(0, -\frac{1}{2} \right), \left(-\frac{1}{2}, -1 \right), \left(-\frac{1}{2}, 0 \right)$$

$$2\sin x = (0, -1), (-1, -2), (-1, 0)$$

$$\begin{aligned} \therefore \int_{\pi}^{2\pi} [2 \sin x] dx &= \int_{\pi}^{7\pi/6} -1 dx + \int_{7\pi/6}^{11\pi/6} -2 dx \\ &+ \int_{11\pi/6}^{2\pi} -1 dx \\ &= -\frac{5\pi}{3} \end{aligned}$$

702 (d)

We have,

$$\begin{aligned} I &= \int \frac{1}{(\sin x + 4)(\sin x - 1)} dx \\ \Rightarrow I &= \frac{1}{5} \int \frac{(\sin x + 4) - (\sin x - 1)}{(\sin x + 4)(\sin x - 1)} dx \\ \Rightarrow I &= \frac{1}{5} \int \frac{1}{\sin x - 1} dx - \frac{1}{5} \int \frac{1}{\sin x + 4} dx \\ \Rightarrow I &= \frac{1}{5} \int \frac{2dt}{2t - 1 - t^2} \\ &\quad - \frac{1}{5} \int \frac{2dt}{2t + 4(1+t^2)}, \text{ where } t \\ &= \tan \frac{x}{2} \\ \Rightarrow I &= -\frac{2}{5} \int \frac{dt}{t^2 - 2t + 1} - \frac{1}{10} \int \frac{dt}{t^2 + \frac{1}{2}t + 1} \\ \Rightarrow I &= -\frac{2}{5} \int \frac{1}{(1-t)^2} dt \\ &\quad - \frac{1}{10} \int \frac{1}{\left(t + \frac{1}{4}\right)^2 + \left(\frac{\sqrt{15}}{4}\right)^2} dt \end{aligned}$$

$$\Rightarrow I = \frac{2}{5} \frac{1}{(t-1)} - \frac{2}{5\sqrt{15}} \tan^{-1} \left(\frac{4t+1}{\sqrt{15}} \right) + C$$

$$\Rightarrow I = \frac{2}{5} \cdot \frac{1}{\tan \frac{x}{2} - 1} - \frac{2}{5\sqrt{15}} \tan^{-1} \left(\frac{4 \tan \frac{x}{2} + 1}{\sqrt{15}} \right) + C$$

Hence, $A = \frac{2}{5}, B = \frac{-2}{5\sqrt{15}}, f(x) = \frac{4 \tan \frac{x}{2} + 1}{\sqrt{15}}$

703 (d)

Given, $\int e^x (1+x) \cdot \sec^2(xe^x) dx = f(x) + \text{constant}$

Put $xe^x = t \Rightarrow (e^x + xe^x) dx = dt$ in LHS, we get

$$\text{LHS} = \int \sec^2 t dt$$

$$= \tan t + \text{constant}$$

$$\Rightarrow \tan(xe^x) + \text{constant} = f(x) + \text{constant}$$

$$\Rightarrow f(x) = \tan(xe^x)$$

704 (c)

Since $e^{x-[x]}$ is a periodic function with period 1

$$\therefore \int_0^{1000} e^{x-[x]} dx = 1000 \int_0^1 e^{x-[x]} dx$$

$$= 1000 \int_0^1 e^x dx = 1000(e-1)$$

706 (b)

$$\int \frac{2e^x + 3e^{-x}}{3e^x + 4e^{-x}} dx = \int \frac{2e^{2x} + 3}{3e^{2x} + 4} dx$$

$$\text{Let } 2e^{2x} + 3 = A(3e^{2x} + 4) + B(6e^{2x})$$

$$\Rightarrow 2e^{2x} + 3 = (3A + 6B)e^{2x} + 4A$$

On comparing both sides, we get

$$2 = 3A + 6B \quad \dots(i)$$

$$\text{and } 3 = 4A \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$2 = \frac{9}{4} + 6B$$

$$\Rightarrow 6B = 2 - \frac{9}{4} = -\frac{1}{4}$$

$$\Rightarrow B = -\frac{1}{24}$$

$$\therefore \int \frac{2e^{2x} + 3}{3e^{2x} + 4} dx$$

$$= \int \frac{3(3e^{2x} + 4)}{4(3e^{2x} + 4)} dx$$

$$- \frac{1}{24} \int \frac{6e^{2x}}{3e^{2x} + 4} dx$$

$$= \frac{3}{4}x - \frac{1}{24} \log(3e^{2x} + 4) + c$$

707 (b)

$$\text{Let } I = \int \frac{x^2}{1+(x^3)^2} dx$$

$$\text{Put } x^3 = t \Rightarrow 3x^2 dx = dt$$

$$\therefore I = \frac{1}{3} \int \frac{dt}{1+t^2}$$

$$= \frac{1}{3} \tan^{-1} t + c = \frac{1}{3} \tan^{-1}(x^3) + c$$

708 (b)

We have,

$$I = \int_0^{\pi} x \sin x \cos^4 x dx$$

$$\Rightarrow I = \int_0^{\pi} (\pi - x) \sin(\pi - x) \cos^4(\pi - x) dx$$

$$\Rightarrow I = \int_0^{\pi} (\pi - x) \sin x \cos^4 x dx$$

$$\Rightarrow I = \pi \int_0^{\pi} \sin x \cos^4 x dx - I$$

$$\Rightarrow 2I = \pi \left[\frac{-\cos^5 x}{5} \right]_0^{\pi} = \frac{\pi}{5} (1+1) = \frac{2\pi}{5}$$

$$\Rightarrow I = \frac{\pi}{5}$$

709 (b)

$$\text{Let } I = \int \frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3} dx$$

$$= \int \frac{\left(\frac{2}{x^3} + \frac{5}{x^6}\right)}{\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^3} dx$$

$$\text{Put } 1 + \frac{1}{x^2} + \frac{1}{x^5} = t$$

$$\Rightarrow \left(-\frac{2}{x^3} - \frac{5}{x^6}\right) dx = dt$$

$$\text{Then, } I = -\int \frac{dt}{t^3} = \frac{1}{2t^2} + c$$

$$= \frac{1}{2\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^2} + c$$

$$= \frac{x^{10}}{2(x^5 + x^3 + 1)^2} + c$$

710 (c)

$$\text{Let } I = \int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}} dx \quad \dots(i)$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{2^{\cos x}}{2^{\cos x} + 2^{\sin x}} dx \quad \dots(ii)$$

$$\left[\text{put } x = \left(\frac{\pi}{2} - x\right) \right]$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2} 1 dx = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

711 (b)

$$\int_0^\pi \sqrt{\frac{\cos 2x + 1}{2}} dx$$

$$= \int_0^\pi \sqrt{\cos^2 x} dx = \int_0^\pi |\cos x| dx$$

$$= \int_0^{\pi/2} \cos x dx - \int_{\pi/2}^\pi \cos x dx$$

$$= 2$$

712 (d)

Let $I = \int e^{x \log a} \cdot e^x dx = \int a^x \cdot e^x dx$

$$= \int (ae)^x dx = \frac{(ae)^x}{\log_e ae} + c$$

713 (a)

Let $f(x) = \log(x + \sqrt{1 + x^2})$ and Replacing x by $-x$, we get

$$f(-x) = \log(\sqrt{1 + x^2} - x)$$

$$= \log(\sqrt{1 + x^2} - x) \frac{(\sqrt{1 + x^2} + x)}{(\sqrt{1 + x^2} + x)}$$

$$= \log \frac{[(1 + x^2) - x^2]}{(\sqrt{1 + x^2} + x)}$$

$$= \log 1 - \log(\sqrt{1 + x^2} + x)$$

$$= -\log(\sqrt{1 + x^2} + x)$$

$$\Rightarrow f(-x) = -f(x)$$

Hence, $f(x)$ is an odd function.

$$\therefore \int_{-1}^1 \log(x + \sqrt{1 + x^2}) dx = 0$$

714 (b)

$$I_n = \int (\log x)^n dx$$

$$= (\log x)^n \cdot x - \int x \cdot n(\log x)^{n-1} \frac{1}{x} dx$$

$$\Rightarrow I_n = x(\log x)^n - nI_{n-1}$$

$$\Rightarrow I_n + nI_{n-1} = x(\log x)^n$$

715 (a)

$$\because x^2 < x^{\frac{\pi}{2}} < x$$

$$\Rightarrow 1 + x^2 < 1 + x^{\frac{\pi}{2}} < 1 + x$$

$$\Rightarrow \frac{1}{1 + x} < \frac{1}{1 + x^{\frac{\pi}{2}}} < \frac{1}{1 + x^2}$$

$$\Rightarrow \int_0^1 \frac{dx}{1 + x} < \int_0^1 \frac{1}{1 + x^{\frac{\pi}{2}}} dx < \int_0^1 \frac{dx}{1 + x^2}$$

$$\Rightarrow \{\log(1 + x)\}_0^1 < I < (\tan^{-1} x)_0^1$$

$$\Rightarrow \log 2 < I < \frac{\pi}{4}$$

716 (b)

Let $I = \int_a^b \frac{x}{|x|} dx = \int_a^b -1 dx$

$$\Rightarrow I = -[x]_a^b = [-b + a]$$

$$= |b| - |a| \quad [\because a < b < 0]$$

717 (d)

We have,

$$f(x) = \begin{vmatrix} 0 & x^2 - \sin x & \cos x - 2 \\ \sin x - x^2 & 0 & 1 - 2x \\ 2 - \cos x & 2x - 1 & 0 \end{vmatrix}$$

$$\Rightarrow f(x) = \begin{vmatrix} 0 & \sin x - x^2 & 2 - \cos x \\ x^2 - \sin x & 0 & 2x - 1 \\ \cos x - 2 & 1 - 2x & 0 \end{vmatrix}$$

[Interchanging rows and columns]

$$\Rightarrow f(x)$$

$$= (-1)^3 \begin{vmatrix} 0 & x^2 - \sin x & \cos x - 2 \\ \sin x - x^2 & 0 & 2x - 1 \\ 2 - \cos x & 2x - 1 & 0 \end{vmatrix}$$

[Taking (-1) common from each column]

$$\Rightarrow f(x) = -f(x) \Rightarrow f(x) = 0 \Rightarrow \int f(x) dx = 0$$

718 (d)

$$f(x) = \lim_{n \rightarrow \infty} n^2 x^{1/(n+1)} \left[x^{\frac{1}{n} - \frac{1}{n+1}} - 1 \right]$$

$$= \lim_{n \rightarrow \infty} \frac{x^{1/(n+1)} \left(\frac{1}{x^{n(n+1)}} - 1 \right)}{\frac{1}{n(n+1)} \times \frac{n(n+1)}{n^2}} \log x = \log x$$

Hence, $\int x f(x) dx = \int x \log x dx$

$$= \frac{x^2}{2} \log x - \frac{1}{4} x^2 + c$$

719 (b)

We have,

$$I = \int \frac{1}{x^{1/2}(1 + x^2)^{5/4}} dx = \int \frac{1}{x^3(1 + x^2)^{5/4}} dx$$

$$\Rightarrow I = -\frac{1}{2} \int \left(1 + \frac{1}{x^2}\right)^{-5/4} d\left(1 + \frac{1}{x^2}\right)$$

$$\Rightarrow I = -\frac{1}{2} \left\{ \frac{\left(1 + \frac{1}{x^2}\right)^{-1/4}}{-\frac{1}{4}} \right\} + C = \frac{2\sqrt{x}}{(x^2 + 1)^{1/4}} + C$$

720 (c)

$\{x\} = x$ when $0 < x < 1$ and $[x + 1] = 1$

$$\therefore \int_0^1 (\{x\}[x + 1]) dx = \int_0^1 x dx = \frac{1}{2}$$

721 (a)

We have,

$$\int_0^2 (x - \log_2 a) dx = 2 \log_2 \left(\frac{2}{a}\right)$$

$$\Rightarrow 2 - 2 \log_2 a = 2 \log_2 \left(\frac{2}{a}\right)$$

$$\Rightarrow 1 = \log_2 a + \log_a 2 - \log_a a$$

$$\Rightarrow 2 = \log_2 a + \log_a 2$$

$$\Rightarrow 2 = \log_2 a + \frac{1}{\log_2 a} \Rightarrow \log_2 a = 1 \Rightarrow a = 2$$

722 (c)

By definition $\text{sgn}(x - [x]) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$

$$\begin{aligned} \therefore \int_{-2}^{10} \text{sgn}(x - [x]) dx &= - \int_{-2}^0 1 dx + \int_0^{10} 1 dx \\ &= -[x]_{-2}^0 + [x]_0^{10} = 8 \end{aligned}$$

723 (c)

We have,

$$I = \int_0^{\infty} \frac{1}{1+x^4} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\cos^2 \theta}{\cos^4 \theta + \sin^4 \theta} d\theta, \text{ where } x = \tan \theta \quad \dots(i)$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\sin^2 \theta}{\cos^4 \theta + \sin^4 \theta} d\theta \quad \dots(ii)$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{1}{\cos^4 \theta + \sin^4 \theta} d\theta$$

$$\Rightarrow 2I = \int_0^{\pi/2} \frac{\sec^4 \theta}{1 + \tan^4 \theta} d\theta$$

$$\Rightarrow 2I = \int_0^{\infty} \frac{t^2 + 1}{t^4 + 1} dt$$

$$= \int_0^{\infty} \frac{1}{\left(t - \frac{1}{t}\right)^2 + (\sqrt{2})^2} d\left(t - \frac{1}{t}\right)$$

$$\Rightarrow 2I = \frac{1}{\sqrt{2}} \left[\tan^{-1} \left(\frac{t - 1/t}{\sqrt{2}} \right) \right]_0^{\infty} = \frac{1}{\sqrt{2}} \left(\frac{\pi}{2} + \frac{\pi}{2} \right) = \frac{\pi}{\sqrt{2}}$$

$$\Rightarrow I = \frac{\pi}{2\sqrt{2}}$$

724 (a)

$$\int \frac{x^2}{x^2 + 4} dx = \int \frac{x^2 + 4 - 4}{x^2 + 4} dx$$

$$= \int \left(\frac{x^2 + 4}{x^2 + 4} - \frac{4}{x^2 + 4} \right) dx$$

$$= \int \left[1 - \frac{4}{x^2 + 4} \right] dx = \int dx - \int \frac{4}{x^2 + 4} dx$$

$$= x - 4 \int \frac{dx}{x^2 + (2)^2} = x - \frac{4}{2} \tan^{-1} \frac{x}{2} + c$$

$$= x - 2 \tan^{-1} \frac{x}{2} + c$$

725 (c)

$$\text{Let } I = \int \frac{1+x}{x+e^{-x}} dx = \int \frac{e^x(x+1)}{xe^{x+1}} dx$$

$$\text{Put } xe^x + 1 = t$$

$$\Rightarrow (xe^x + e^x) dx = dt$$

$$\Rightarrow (x+1)e^x dx = dt$$

$$\therefore I = \int \frac{dt}{t} = \log t + c$$

$$= \log |(xe^x + 1)| + c$$

726 (c)

$$\text{Let } I = \int \frac{dx}{\sqrt[4]{(x+1)^5(x+2)^3}}$$

$$\text{Put } \frac{x+1}{x+2} = t \Rightarrow \frac{1}{(x+2)^2} dx = dt$$

$$\therefore I = \int \frac{dt}{t^{5/4}} \Rightarrow I = \frac{t^{-1/4}}{\left(-\frac{1}{4}\right)} + c$$

$$\Rightarrow I = -4 \left(\frac{x+2}{x+1} \right)^{1/4} + c$$

727 (a)

$$\frac{x+1}{e^x} < 1 \quad \dots(i)$$

$$\Rightarrow \frac{[x+1]}{e^x} < 1 \quad \dots(ii)$$

$$(\text{as } [x+1] < x+1)$$

$$\therefore \left[\frac{[x+1]}{e^x} \right] = 0 \quad [\text{from Eqs.(i) and (ii)}]$$

728 (b)

$$\text{Let } \tan^{-1} x = t$$

$$\Rightarrow \frac{1}{1+x^2} dx = dt$$

$$\therefore \int \frac{(\tan^{-1} x)^3}{1+x^2} dx = \int t^3 dt$$

$$= \frac{t^4}{4} + c = \frac{(\tan^{-1} x)^4}{4} + c$$

729 (c)

$$\text{Put } x^x = t$$

$$\Rightarrow x^x (1 + \log_e |x|) dx = dt$$

$$\therefore \int x^x (1 + \log_e |x|) dx = t + c = x^x + c$$

730 (a)

$$\int \left\{ \frac{(\log x - 1)}{1 + (\log x)^2} \right\}^2 dx$$

$$= \int \frac{(\log x)^2 + 1 - 2 \log x}{[(\log x)^2 + 1]^2} dx$$

$$= \int \frac{(\log x)^2 + 1 - 2x \left(\log x \cdot \frac{1}{x} \right)}{[(\log x)^2 + 1]^2} dx$$

$$= \int \frac{d}{dx} \left(\frac{x}{(\log x)^2 + 1} \right) dx = \frac{x}{(\log x)^2 + 1} + c$$

731 (d)

$$\lim_{n \rightarrow \infty} \frac{1 + 2^4 + 3^4 + \dots + n^4}{n^5}$$

$$- \lim_{n \rightarrow \infty} \frac{1 + 2^3 + 3^3 + \dots + n^3}{n^5}$$

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \left(\frac{r}{n} \right)^4 - \lim_{n \rightarrow \infty} \frac{1}{n} \times \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \left(\frac{r}{n} \right)^3$$

$$\begin{aligned}
&= \int_0^1 x^4 dx - \lim_{n \rightarrow \infty} \frac{1}{n} \times \int_0^1 x^3 dx \\
&= \left[\frac{x^5}{5} \right]_0^1 - 0 = \frac{1}{5}
\end{aligned}$$

732 (c)

$$\begin{aligned}
\text{Let } I &= \int \frac{dx}{\sqrt{1-e^{2x}}} = \int \frac{e^{-x}}{\sqrt{e^{-2x}-1}} dx \\
\text{Put } e^{-x} &= t \Rightarrow e^{-x} dx = -dt \\
\therefore I &= - \int \frac{dt}{\sqrt{t^2-1}} = -\log |t + \sqrt{t^2-1}| + c \\
&= -\log |e^{-x} + \sqrt{e^{-2x}-1}| + c
\end{aligned}$$

733 (d)

Since, $\cos(\sin x) > \cos x > \sin(\cos x)$, for $x \in 0, \pi/2$

$$\begin{aligned}
&\Rightarrow \int_0^{\pi/2} \cos(\sin x) dx > \int_0^{\pi/2} \cos x dx \\
&> \int_0^{\pi/2} \sin(\cos x) dx \\
&\Rightarrow I > K > J
\end{aligned}$$

734 (c)

$$\begin{aligned}
\text{Let } I &= \int_{-1}^1 \frac{17x^5 + 29x^3 - 31x}{x^2 + 1} dx - \int_{-1}^1 \frac{x^4 - 1}{x^2 + 1} dx \\
&= 0 - 2 \int_0^1 \frac{(x^2 - 1)(x^2 + 1)}{(x^2 + 1)} dx \\
&= -2 \left[\left(\frac{x^3}{3} - x \right) \right]_0^1 = \frac{4}{3}
\end{aligned}$$

735 (b)

$$\begin{aligned}
\text{Let } I &= \int_0^1 \sin \left(2 \tan^{-1} \sqrt{\frac{1+x}{1-x}} \right) dx \\
\text{Put } x &= \cos 2\theta, \\
\text{then } \sin \left[2 \tan^{-1} \sqrt{\frac{1+\cos 2\theta}{1-\cos 2\theta}} \right] \\
&= \sin [2 \tan^{-1}(\cot \theta)] \\
&= \sin \left[2 \tan^{-1} \left(\tan \left(\frac{\pi}{2} - \theta \right) \right) \right] \\
&= \sin \left[2 \left(\frac{\pi}{2} - \theta \right) \right] = \sin(\pi - 2\theta) = \sin 2\theta \\
&= \sqrt{1 - \cos^2 2\theta} = \sqrt{1 - x^2} \\
\text{Now, } \int_0^1 \sin \left(2 \tan^{-1} \sqrt{\frac{1+x}{1-x}} \right) dx &= \int_0^1 \sqrt{1 - x^2} dx \\
&= \left[\frac{1}{2} x \cdot \sqrt{1 - x^2} \right]_0^1 + \frac{1}{2} [\sin^{-1} x]_0^1 \\
&= \frac{1}{2} [1\sqrt{1-1} - 0] + \frac{1}{2} [\sin^{-1}(1) - 0] \\
&= \frac{1}{2} [0] + \frac{1}{2} \left(\frac{\pi}{2} \right) = \frac{\pi}{4}
\end{aligned}$$

736 (a)

$$\begin{aligned}
\text{Let } I &= 2 \int e^{2x} \sin 3x dx + 3 \int e^{2x} \cos 3x dx \\
&= 2 \left[\frac{e^{2x}}{2} \sin 3x - \int \frac{e^{2x}}{2} 3 \cos 3x dx \right] \\
&\quad + 3 \int e^{2x} \cos 3x dx \\
&= e^{2x} \sin 3x + c
\end{aligned}$$

737 (b)

$$\begin{aligned}
\int_0^2 |x^2 - 1| dx &= \int_0^1 -(x^2 - 1) dx \\
&\quad + \int_1^2 (x^2 - 1) dx \\
&= \left[-\frac{x^3}{3} + x \right]_0^1 + \left[\frac{x^3}{3} - x \right]_1^2 \\
&= -\frac{1}{3} + 1 + \frac{8}{3} - 2 - \frac{1}{3} + 1 \\
&= 2
\end{aligned}$$

738 (a)

For $0 < x < 1$, we get

$$x^2 > x^3$$

For, $1 < x < 2$, we have $x^3 + x^2$

$\therefore 2x^2 > 2x^3$ for $0 < x < 1$ and $2x^2 < 2x^3$ for

$$1 < x < 2$$

$$\Rightarrow \int_0^1 2x^2 dx > \int_0^1 2x^3 dx \text{ and } \int_1^2 2x^2 dx < \int_1^2 2x^3 dx$$

$$\Rightarrow I_1 > I_2 \text{ and } I_3 > I_4$$

739 (b)

$$\begin{aligned}
\int (\sin 2x - \cos 2x) dx &= \frac{1}{\sqrt{2}} \sin(2x - a) + b \\
&\Rightarrow \sqrt{2} \int \left(\frac{1}{\sqrt{2}} \sin 2x - \frac{1}{\sqrt{2}} \cos 2x \right) dx \\
&= \frac{1}{\sqrt{2}} \sin(2x - a) + b \\
&\Rightarrow -\sqrt{2} \int \left(\frac{1}{2} \cos 2x - \frac{1}{\sqrt{2}} \sin 2x \right) dx \\
&= \frac{1}{\sqrt{2}} \sin(2x - a) + b \\
&\Rightarrow -\sqrt{2} \int \left(2x + \frac{\pi}{4} \right) dx = \frac{1}{\sqrt{2}} \sin(2x - a) + b \\
&\Rightarrow -\frac{\sqrt{2}}{2} \sin \left(2x + \frac{\pi}{4} \right) + c = \frac{1}{\sqrt{2}} \sin(2x - a) + b \\
&\Rightarrow \frac{1}{\sqrt{2}} \sin \left(2x + \frac{5\pi}{4} \right) + c = \frac{1}{\sqrt{2}} \sin(2x - a) + b \\
\therefore a &= -\frac{5\pi}{4}, b \in R
\end{aligned}$$

740 (d)

$$\text{Let } I = \int e^{x+x^{-1}} \cdot 1 dx + \int \left(x - \frac{1}{x} \right) e^{x+x^{-1}} dx$$

$$= xe^{x+x^{-1}} - \int \left(1 - \frac{1}{x^2}\right) xe^{x+x^{-1}} dx$$

$$+ \int \left(x - \frac{1}{x}\right) e^{x+x^{-1}} dx$$

$$= xe^{x+x^{-1}} + c$$

741 (c)

We have,

$$I = \int_0^1 f(k-1+x) dx$$

$$\Rightarrow I = \int_{k-1}^k f(t) dt, \text{ where } t = k-1+x$$

$$\Rightarrow I = \int_{k-1}^k f(x) dx$$

$$\therefore I = \sum_{k=1}^n \int_{k-1}^k f(x) dx$$

$$\Rightarrow I = \int_0^1 f(x) dx + \int_1^2 f(x) dx + \dots + \int_{n-1}^n f(x) dx$$

$$= \int_0^n f(x) dx$$

742 (c)

$$\text{Let } I = \int_0^\pi \frac{1}{1+3\cos x} dx \quad \dots(i)$$

Then,

$$I = \int_0^\pi \frac{1}{1+3\cos(\pi-x)} dx$$

$$\Rightarrow I = \int_0^\pi \frac{1}{3+e^{-\cos x}} dx$$

$$\Rightarrow I = \int_0^\pi \frac{3\cos x}{3\cos x+1} dx \quad \dots(ii)$$

Adding (i) and (ii), we get

$$2I = \int_0^\pi 1 \cdot dx = \pi \Rightarrow I = \frac{\pi}{2}$$

ALITER We have,

$$I = \int_0^\pi \frac{1}{1+3\cos x} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \left\{ \frac{1}{1+3\cos x} + \frac{1}{3\cos(\pi-x)} \right\} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \left\{ \frac{1}{1+3\cos x} + \frac{1}{1+3^{-\cos x}} \right\} dx$$

$$\Rightarrow I = \int_0^{\pi/2} dx = \frac{\pi}{2}$$

743 (c)

We have,

$$I = \int_0^1 \frac{1}{x^2 + 2x \cos \alpha + 1} dx$$

$$\Rightarrow I = \int_0^1 \frac{1}{(x + \cos \alpha)^2 \sin^2 \alpha} dx$$

$$\Rightarrow I = \frac{1}{\sin \alpha} \left\{ \tan^{-1} \left(\frac{x + \cos \alpha}{\sin \alpha} \right) \right\}_0^1$$

$$\Rightarrow I = \frac{1}{\sin \alpha} \left[\tan^{-1} \frac{1 + \cos \alpha}{\sin \alpha} - \tan^{-1} \frac{\cos \alpha}{\sin \alpha} \right]$$

$$\Rightarrow I = \frac{1}{\sin \alpha} \left[\tan^{-1} \left(\cot \frac{\alpha}{2} \right) - \tan^{-1} (\cot \alpha) \right]$$

$$\Rightarrow I = \frac{1}{\sin \alpha} \left[\left(\frac{\pi}{2} - \frac{\alpha}{2} \right) - \left(\frac{\pi}{2} - \alpha \right) \right] = \frac{\alpha}{2 \sin \alpha}$$

744 (c)

$$\text{Let } I = \int_0^{\pi/4} \frac{\sin \theta + \cos \theta}{9 + 16 \sin 2\theta} d\theta$$

Putting $\sin \theta - \cos \theta = t$ and $(\sin \theta + \cos \theta) d\theta = dt$, we get

$$I = \int_{-1}^0 \frac{1}{9 + 16(1-t^2)} dt = \int_{-1}^0 \frac{1}{25 - 16t^2} dt$$

$$\Rightarrow I = \frac{1}{40} \left[\log \left(\frac{5+4t}{5-4t} \right) \right]_{-1}^0 = -\frac{1}{40} \log \left(\frac{1}{9} \right)$$

$$= \frac{1}{20} \log 3$$

745 (a)

$$\text{Let } I = \int_0^\infty \frac{dx}{(x + \sqrt{x^2+1})^3}$$

Put $x = \tan \theta \Rightarrow dx = \sec^2 \theta d\theta$

$$\therefore I = \int_0^{\pi/2} \frac{\sec^2 \theta d\theta}{(\tan \theta + \sqrt{\tan^2 \theta + 1})^3}$$

$$= \int_0^{\pi/2} \frac{\sec^2 \theta d\theta}{(\tan \theta + \sec \theta)^3}$$

$$= \int_0^{\pi/2} \frac{1/\cos^2 \theta}{\left(\frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta} \right)^3} d\theta$$

$$= \int_0^{\pi/2} \frac{\cos \theta}{(\sin \theta + 1)^3} d\theta$$

$$= \left[-\frac{1}{2(\sin \theta + 1)^2} \right]_0^{\pi/2} = \frac{-1}{8} + \frac{1}{2} = \frac{3}{8}$$

746 (b)

$$\int \frac{dx}{x^2 + 4x + 13} = \int \frac{dx}{x^2 + 4x + 4 + 9}$$

$$= \int \frac{dx}{(x+2)^2 + 3^2} = \frac{1}{3} \tan^{-1} \left(\frac{x+2}{3} \right) + c$$

747 (c)

$$\text{Since, } J = \int \frac{e^{3x}}{1+e^{2x}+e^{4x}} dx$$

$$\begin{aligned} \therefore J - I &= \int \frac{(e^{3x} - e^x)}{1 + e^{2x} + e^{4x}} dx \\ &= \int \frac{(u^2 - 1)}{1 + u^2 + u^4} du \quad [u = e^x] \\ &= \int \frac{\left(1 - \frac{1}{u^2}\right)}{1 + \frac{1}{u^2} + u^2} du = \int \frac{\left(1 - \frac{1}{u^2}\right)}{\left(u + \frac{1}{u}\right)^2 - 1} du \\ &= \int \frac{dt}{t^2 - 1} \quad \left[\text{put } u + \frac{1}{u} = t \Rightarrow \left(1 - \frac{1}{u^2}\right) du = dt\right] \\ &= \frac{1}{2} \log \left| \frac{t-1}{t+1} \right| + c = \frac{1}{2} \log \left| \frac{u^2 - u + 1}{u^2 + u + 1} \right| + c \\ &= \frac{1}{2} \log \left| \frac{e^{2x} - e^x + 1}{e^{2x} + e^x + 1} \right| + c \end{aligned}$$

748 (b)

$$\begin{aligned} \int_a^b \frac{dx}{\sqrt{(x-a)(b-x)}} &= \int_a^b \frac{1}{\sqrt{-x^2 + (a+b)x - ab}} dx \\ &= \int_a^b \frac{1}{\sqrt{\left(\frac{b-a}{2}\right)^2 - \left(x - \frac{a+b}{2}\right)^2}} dx \\ &= \left[\sin^{-1} \left(\frac{x - \frac{a+b}{2}}{\frac{b-a}{2}} \right) \right]_a^b \\ &= \sin^{-1} 1 - \sin^{-1}(-1) \\ &= \frac{\pi}{2} + \frac{\pi}{2} = \pi \end{aligned}$$

749 (d)

$$\begin{aligned} \text{Let } I &= \int_{-\pi}^{\pi} (\sin^6 x + \cos^6 x) dx = \\ &= 2 \int_0^{\pi} (\sin^6 x + \cos^6 x) dx \\ &= 4 \int_0^{\pi/2} \sin^6 x dx + 4 \int_0^{\pi/2} \cos^6 \left(\frac{\pi}{2} - x\right) dx \\ &= 8 \int_0^{\pi/2} \sin^6 x dx = 8 \times \frac{5.3.1}{6.4.2} \times \frac{\pi}{2} = \frac{5\pi}{4} \end{aligned}$$

750 (b)

We have,

$$\begin{aligned} I_m &= \int_1^x (\log x)^m dx \\ \Rightarrow I_m &= [(\log x)^m x]_1^x - \int_1^x m(\log x)^{m-1} \times \frac{1}{x} \times x dx \\ \Rightarrow I_m &= (\log x)^m x - m I_{m-1} \\ \Rightarrow k - l I_{m-1} &= x(\log x)^m \\ &\quad - m I_{m-1} \quad [\because I_m = k - l I_{m-1}] \\ \Rightarrow k &= x(\log x)^m, l = m \end{aligned}$$

751 (b)

We have,

$$\begin{aligned} I &= \int x^{-2/3} (1 + x^{1/2})^{-5/3} dx \\ &= \int x^{3/2} (1 + x^{-1/2})^{-5/3} dx \end{aligned}$$

$$\begin{aligned} \Rightarrow I &= -2 \int (1 + x^{-1/2})^{-5/3} d(1 + x^{-1/2}) \\ \Rightarrow I &= -2 \times \frac{(1 + x^{-1/2})^{-2/3}}{-2/3} + C \\ &= 3(1 + x^{-1/2})^{-2/3} + C \end{aligned}$$

752 (a)

$$\begin{aligned} \int (1 - \cos x) \operatorname{cosec}^2 x dx &= \int \frac{1 - \cos x}{1 - \cos^2 x} dx \\ &= \int \frac{1}{1 + \cos x} dx \\ &= \frac{1}{2} \int \sec^2 \frac{x}{2} dx = \tan \frac{x}{2} + c \end{aligned}$$

753 (d)

$$\begin{aligned} \therefore \int_0^{2a} f(x) dx &= \int_0^a \{f(2a - x) + f(x)\} dx \\ &= \int_0^a f(2a - x) dx + \int_0^a f(x) dx \\ &= m + n \end{aligned}$$

754 (a)

$$\begin{aligned} I_n &= \int (\log x)^n dx \quad \dots(i) \\ \therefore I_{n-1} &= \int (\log x)^{n-1} dx \quad \dots(ii) \\ \text{Now, } I_n &= \int (\log x)^n \cdot 1 dx \\ &= (\log x)^n x - n \int (\log x)^{n-1} \frac{1}{x} dx \\ &= x(\log x)^n - n \int (\log x)^{n-1} dx \\ \Rightarrow I_n &= x(\log x)^n - n I_{n-1} \\ \therefore I_n + n I_{n-1} &= x(\log x)^n \end{aligned}$$

755 (a)

$$\begin{aligned} \text{Let } f(x) &= \int_0^{x^2} \frac{t^2 - 5t + 4}{2 + e^t} dt. \text{ Then,} \\ f'(x) &= 2x \frac{(x^4 - 5x^2 + 4)}{(2 + e^{x^2})} \end{aligned}$$

For points of extremum, we must have

$$f'(x) = 0 \Rightarrow x = 0, x = \pm 1, \pm 2$$

756 (c)

$$\begin{aligned} \text{Let } I &= \int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}} dx \quad \dots(i) \\ \Rightarrow I &= \int_0^{\pi/2} \frac{2^{\sin x(\pi/2 - x)}}{2^{\sin(\pi/2 - x)} + 2^{\cos x(\pi/2 - x)}} dx \\ \Rightarrow I &= \int_0^{\pi/2} \frac{2^{\cos x}}{2^{\cos x} + 2^{\sin x}} dx \quad \dots(ii) \\ \text{On adding Eqs.(i) and (ii), we get} \\ 2I &= \int_0^{\pi/2} \frac{2^{\sin x} + 2^{\cos x}}{2^{\sin x} + 2^{\cos x}} dx = \frac{\pi}{2} \\ \Rightarrow I &= \frac{\pi}{4} \end{aligned}$$

757 (b)

$$\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} e^{r/n} = \int_0^1 e^x dx = [e^x]_0^1$$

$$= e - 1$$

758 (d)

The integrand is an odd function. So the value of the integral is zero

759 (a)

We have,

$$I = \int \frac{2x - 3}{(x^2 + x + 1)^2} dx = \int \frac{(2x + 1) - 4}{(x^2 + x + 1)^2} dx$$

$$\Rightarrow I = \int \frac{2x + 1}{(x^2 + x + 1)^2} dx - 4 \int \frac{1}{[(x + 1/2)^2 + (\sqrt{3}/2)^2]^2} dx$$

$$\Rightarrow I = -\frac{1}{x^2 + x + 1} - 4I_1, \text{ where } I_1 = \int \frac{1}{[(x + 1/2)^2 + (\sqrt{3}/2)^2]^2} dx$$

Putting $x + 1/2 = (\sqrt{3}/2) \tan \theta$ in I_1 , we get

$$I_1 = \int \frac{\sqrt{3}/2 \sec^2 \theta d\theta}{[(\sqrt{3}/2 \tan \theta)^2 + (\sqrt{3}/2)^2]^2}$$

$$\Rightarrow I_1 = \frac{8}{3\sqrt{3}} \int \cos^2 \theta d\theta = \frac{8}{3\sqrt{3}} \int \frac{1 + \cos 2\theta}{2} d\theta$$

$$\Rightarrow I_1 = \frac{4}{3\sqrt{3}} \{\theta + \sin 2\theta\} + C$$

$$\Rightarrow I_1 = \frac{4}{3\sqrt{3}} \left\{ \tan^{-1} \frac{2x + 1}{\sqrt{3}} + \frac{\sqrt{3}}{4} \left(\frac{2x + 1}{x^2 + x + 1} \right) \right\} + C$$

$$\therefore I = -\frac{1}{x^2 + x + 1} - \frac{16}{3\sqrt{3}} \tan^{-1} \frac{2x + 1}{\sqrt{3}} - \frac{4}{3} \left(\frac{2x + 1}{x^2 + x + 1} \right) + C$$

760 (d)

$$\text{Let } I = \int \frac{\sqrt{x}\sqrt{x}}{\sqrt{x}(x+1)} dx$$

$$= \int \frac{x + 1}{\sqrt{x}(x + 1)} dx - \int \frac{1}{\sqrt{x}(x + 1)} dx$$

$$= \int \frac{1}{\sqrt{x}} dx - \int \frac{1}{\sqrt{x}(x + 1)} dx$$

$$= 2\sqrt{x} - 2 \tan^{-1} \sqrt{x} + c$$

$$= 2(\sqrt{x} - \tan^{-1} \sqrt{x}) + c$$

761 (b)

We have,

$$I = \int_{-\pi}^{\pi} \frac{2x(1 + \sin x)}{1 + \cos^2 x} dx$$

$$\Rightarrow I = \int_{-\pi}^{\pi} \frac{2x}{1 + \cos^2 x} dx + 2 \int_{-\pi}^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$$

$$\Rightarrow I = 0 + 2 \times 2 \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$$

$$\Rightarrow I = 4 \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$$

$$\Rightarrow I = 4 \int_0^{\pi} \frac{(\pi - x) \sin(\pi - x)}{1 + \cos^2(\pi - x)} dx$$

$$\Rightarrow I = 4 \int_0^{\pi} \frac{\pi \sin x}{1 + \cos^2 x} dx - 4 \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$$

$$\Rightarrow I = -4\pi [\tan^{-1}(\cos x)]_0^{\pi} - I$$

$$\Rightarrow 2I = -4\pi[-\pi/4 - \pi/4] \Rightarrow I = \pi^2$$

762 (a)

Let

$$A = \lim_{n \rightarrow \infty} \left\{ \left(1 + \frac{1}{n}\right) \left(1 + \frac{2}{n}\right) \left(1 + \frac{3}{n}\right) \dots \left(1 + \frac{n}{n}\right) \right\}^{1/n}$$

$$\Rightarrow \log A = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \log \left(1 + \frac{r}{n}\right)$$

$$\Rightarrow \log A = \lim_{n \rightarrow \infty} \sum_{r=1}^n \log \left(1 + \frac{r}{n}\right)$$

$$\Rightarrow \log A = \int_0^1 \log(1 + x) dx$$

$$\Rightarrow \log A = [n \log(1 + x) - x + \log(x + 1)]_0^1$$

$$\Rightarrow \log A = 2 \log 2 - 1 \Rightarrow A = \frac{4}{e}$$

763 (d)

$$\int_{-2}^2 |x| dx = -\int_{-2}^0 x dx + \int_0^2 x dx$$

$$= -\left[\frac{x^2}{2}\right]_{-2}^0 + \left[\frac{x^2}{2}\right]_0^2$$

$$= -(0 - 2) + (2 - 0) = 4$$

764 (b)

$\therefore \sin^{-1}(\cos x) + \cos^{-1}(\sin x)$ is periodic with period 2π , then

$$\int_{a+2\pi}^{a+5\pi/2} (\sin^{-1}(\cos x) + \cos^{-1}(\sin x)) dx$$

$$= \int_0^{\pi/2} \left(\sin^{-1} \left\{ \cos \left(\frac{\pi}{2} - x \right) \right\} \right.$$

$$\left. + \cos^{-1} \left\{ \sin \left(\frac{\pi}{2} - x \right) \right\} \right) dx$$

$$= \int_0^{\pi/2} (\sin^{-1} \sin x + \cos^{-1} \cos x) dx$$

$$= 2 \int_0^{\pi/2} x dx = 2 \left\{ \frac{x^2}{2} \right\}_0^{\pi/2} = \frac{\pi^2}{4}$$

766 (d)

$$\text{Put } x + \sqrt{1 + x^2} = t$$

$$\Rightarrow \left(1 + \frac{1}{2\sqrt{1+x^2}} \cdot 2x\right) dx = dt$$

$$\Rightarrow \frac{x + \sqrt{1+x^2}}{\sqrt{1+x^2}} dx = dt$$

$$\Rightarrow \frac{dx}{\sqrt{1+x^2}} = \frac{dt}{t}$$

$$\therefore \int \frac{\log(x + \sqrt{1+x^2})}{\sqrt{1+x^2}} dx = \int \frac{\log t}{t} dt$$

$$= \frac{(\log t)^2}{2} + c$$

$$= \frac{1}{2} [\log(x + \sqrt{1+x^2})]^2 + c$$

767 (c)

Putting $e^x - 1 = t^2$ and $e^x dx = 2t dt$, we have

$$I = \int_{\log 2}^x \frac{dx}{\sqrt{e^x - 1}}$$

$$\Rightarrow I = \int_1^t \frac{dt}{t^2 + 1} = 2[\tan^{-1} t]_1^t$$

$$\Rightarrow I = 2[\tan^{-1} t - \tan^{-1} 1] = 2 \tan^{-1} t = -\frac{\pi}{2}$$

$$\therefore \int_{\log 2}^x \frac{dx}{\sqrt{e^x - 1}} = \frac{\pi}{6}$$

$$\Rightarrow 2 \tan^{-1} t - \frac{\pi}{2} = \frac{\pi}{6}$$

$$\Rightarrow \tan^{-1} t = \frac{\pi}{3}$$

$$\Rightarrow t = \sqrt{3} = t^2 = 3 \Rightarrow e^x - 1 = 3 \Rightarrow e^x = 4 \Rightarrow x = \log_e 4$$

768 (d)

$$\int \frac{\cos x + x \sin x}{x^2 + x \cos x} dx = \int \frac{x + \cos x - x + x \sin x}{x^2 + x \cos x} dx$$

$$= \int \frac{dx}{x} - \int \frac{x(1 - \sin x)}{x(x + \cos x)} dx$$

$$= \log x - \log(x + \cos x) + c$$

$$= \log \left| \frac{x}{x + \cos x} \right| + c$$

769 (c)

$$\text{Let } I = \int \frac{dx}{x\sqrt{x^6-16}}$$

$$= \frac{1}{3} \int \frac{3x^2}{x^3\sqrt{(x^3)^2-4^2}} dx$$

$$\text{Put } x^3 = t \Rightarrow 3x^2 dx = dt$$

$$\therefore I = \frac{1}{3} \int \frac{dt}{t\sqrt{t^2-4^2}}$$

$$= \frac{1}{3 \times 4} \sec^{-1} \left(\frac{t}{4} \right) + c$$

$$= \frac{1}{12} \sec^{-1} \left(\frac{x^3}{4} \right) + c$$

770 (a)

$$F(x) = \int_{x^2}^{x^3} \log t dt$$

Applying Leibnitz theorem, we get

$$F'(x) = \log x^3 \cdot \frac{d}{dx} x^3 - \log x^2 \cdot \frac{d}{dx} x^2$$

$$= 3 \log x \cdot 3x^2 - 2 \log x \cdot 2x$$

$$= (9x^2 - 4x) \log x$$

771 (b)

We have,

$$\int_0^{\infty} \frac{\log(1+x^2)}{1+x^2} dx = k \int_0^1 \frac{\log(1+x)}{1+x^2} dx$$

$$\Rightarrow \int_0^{\pi/2} \log \sec^2 \theta d\theta$$

$$= k \int_0^{\pi/4} \log(1 + \tan \theta) d\theta, \text{ where } x = \tan \theta$$

$$= \tan \theta$$

$$\Rightarrow -2 \int_0^{\pi/2} \log \cos \theta d\theta = k \int_0^{\pi/4} \log(1 + \tan \theta) d\theta$$

$$\Rightarrow -2 \times -\frac{\pi}{2} \log_e 2 = k \left(\frac{\pi}{8} \log 2 \right)$$

$$\Rightarrow k = 8$$

772 (b)

$$\int_0^4 |x-1| dx = \int_0^1 (1-x) dx + \int_1^4 (x-1) dx$$

$$= \left[x - \frac{x^2}{2} \right]_0^1 + \left[\frac{x^2}{2} - x \right]_1^4$$

$$= 1 - \frac{1}{2} + \left[8 - 4 - \left(\frac{1}{2} - 1 \right) \right]$$

$$= \frac{1}{2} + 4 + \frac{1}{2} = 5$$

773 (b)

Given, $f(x)$ is a continuous function.

Let us consider $f(x) = x$

$$\therefore \int_{-3}^5 2x dx = [x^2]_{-3}^5 = 16$$

$$\text{and } \int_{-6}^{10} (x-1) dx = \left[\frac{x^2}{2} - x \right]_{-6}^{10} = 16$$

$$\therefore \int_{-3}^5 2f(x) dx = \int_{-6}^{10} f(x-1) dx$$

774 (c)

$$\int [\sin(\log x) + \cos(\log x)] dx$$

$$= \int \frac{d}{dx} [x \sin(\log x)] dx$$

$$= x \sin(\log x) + c$$

775 (b)

$$\text{Let } I = \int_0^a \frac{1}{1+e^{f(x)}} dx \quad \dots(i)$$

Then,

$$\Rightarrow I = \int_0^a \frac{1}{1+e^{f(a-x)}} dx$$

$$\Rightarrow I = \int_0^a \frac{1}{1+e^{-f(x)}} dx \quad [\because f(a-x) + f(x) = 0]$$

$$\Rightarrow I = \int_0^a \frac{e^{f(x)}}{e^{f(x)}+1} dx \quad \dots(ii)$$

Adding (i) and (ii), we get

$$2I = \int_0^a \frac{e^{f(x)} + 1}{e^{f(x)} + 1} dx = \int_0^a 1 dx = a \Rightarrow I = \frac{a}{2}$$

776 (c)

$$\text{Let } I = \int_{-1}^1 |1-x| dx = \int_{-1}^1 (1-x) dx$$

$$= \left[x - \frac{x^2}{2} \right]_{-1}^1 = 2$$

777 (d)

$$\text{Let } I = \int_{-1}^3 \left[\tan^{-1} \frac{x}{x^2+1} + \tan^{-1} \frac{x^2+1}{x} \right] dx$$

$$\text{Now, } \tan^{-1} \frac{x}{x^2+1} + \tan^{-1} \frac{x^2+1}{x}$$

$$= \tan^{-1} \left[\frac{\frac{x}{x^2+1} + \frac{x^2+1}{x}}{1 - \frac{x}{x^2+1} \cdot \frac{x^2+1}{x}} \right]$$

$$= \tan^{-1} \left[\frac{x^2 + (x^2+1)^2}{x(x^2+1) \cdot 0} \right] = \tan^{-1}(\infty) = \frac{\pi}{2}$$

$$\text{Hence, } I = \int_{-1}^3 \frac{\pi}{2} dx = 2\pi$$

778 (b)

$$\text{Let } I = \int \frac{x^2}{1+x^6} dx$$

$$\text{Put } x^3 = t \Rightarrow x^2 dx = \frac{1}{3} dt$$

$$\therefore I = \frac{1}{3} \int \frac{1}{1+t^2} dt = \frac{1}{3} \tan^{-1}(x^3) + c$$

780 (b)

$$\int \frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}} dx$$

$$= \int \frac{\sqrt{(1+x)^2} + \sqrt{x}\sqrt{x+1}}{\sqrt{x}+\sqrt{1+x}} dx$$

$$= \int \sqrt{1+x} dx = \frac{2}{3} (1+x)^{3/2} + c$$

781 (b)

We have,

$$f(x) = (2x+1)|\sin x|, \pi < x < 2\pi$$

$$\Rightarrow f(x) = -(2x+1) \sin x$$

Hence, required primitive is given by

$$-\int (2x+1) \sin x dx$$

$$= -[-(2x+1) \cos x + 2 \sin x] + C$$

$$I \quad II$$

$$= (2x+1) \cos x - 2 \sin x + C$$

782 (a)

$$\text{Let } I = k \int_0^1 x f(3x) dx$$

$$\text{Let } 3x = t \Rightarrow dx = \frac{dt}{3}$$

$$\therefore I = k \int_0^3 \frac{t}{3} \cdot f(t) \cdot \frac{dt}{3} = \frac{k}{9} \int_0^3 t \cdot f(t) dt$$

$$\text{Now, } \frac{k}{9} \int_0^3 t \cdot f(t) dt = \int_0^3 t \cdot f(t) dt \quad [\text{given}]$$

$$\Rightarrow \frac{k}{9} = 1 \Rightarrow k = 9$$

783 (c)

$$\text{Put } x^e + e^x = t \Rightarrow e(x^{e-1} + e^{x-1}) dx = dt$$

$$\therefore \int \frac{x^{e-1} + e^{x-1}}{x^e + e^x} dx = \frac{1}{e} \int \frac{dt}{t} = \frac{1}{e} \log t + c$$

$$= \frac{1}{e} \log(x^e + e^x) + c$$

784 (a)

$$\text{Let } I = \int \operatorname{cosec}(x-a) \operatorname{cosec} x dx$$

$$I = \int \frac{\sin a}{\sin a \sin(x-a) \sin x} dx$$

$$= -\frac{1}{\sin a} \int \left[\frac{\sin(x-a) \cos x - \cos(x-a) \sin x}{\sin(x-a) \sin x} \right] dx$$

$$= -\frac{1}{\sin a} \int [\cot x - \cot(x-a)] dx$$

$$= -\frac{1}{\sin a} [\log |\sin x| - \log |\sin(x-a)|] + c$$

$$= -\frac{1}{\sin a} [\log |\sin x \operatorname{cosec}(x-a)|] + c$$

785 (b)

$$\text{Let } I = \int_0^{\pi/2} \frac{\cos \theta}{\sqrt{4-\sin^2 \theta}} d\theta$$

$$\text{Put } \sin \theta = t \Rightarrow \cos \theta d\theta = dt$$

$$\therefore I = \int_0^1 \frac{dt}{\sqrt{4-t^2}}$$

$$= \left[\sin^{-1} \frac{t}{2} \right]_0^1$$

$$= \sin^{-1} \frac{1}{2} = \frac{\pi}{6}$$

786 (a)

We have,

$$I = \int_0^{\pi} \frac{1}{a^2 - 2a \cos x + 1} dx$$

$$\Rightarrow I = \int_0^{\pi} \frac{1 + \tan^2 x/2}{(a^2 + 1)(1 + \tan^2 x/2) - 2a(1 - \tan^2 x/2)} dx$$

$$\Rightarrow I = \int_0^{\infty} \frac{2 dt}{(a^2 + 1)(1 + t^2) - 2a(1 - t^2)}, \text{ where } \tan \frac{x}{2} = t,$$

$$\Rightarrow I = 2 \int_0^{\infty} \frac{dt}{t^2(a+1)^2 + (a-1)^2}$$

$$\Rightarrow I = \frac{2}{(a+1)^2} \int_0^{\infty} \frac{dt}{t^2 + \left(\frac{1-a}{1+a}\right)^2}$$

$$\Rightarrow I = \frac{2}{(a+1)^2} \left(\frac{1+a}{1-a}\right) \left[\tan^{-1} \frac{t(1+a)}{1-a} \right]_0^{\infty}$$

$$\Rightarrow I = \frac{2}{1-a^2} (\tan^{-1} \infty - \tan^{-1} 0) = \frac{\pi}{1-a^2}$$

787 (b)

$$\text{Let } I = \int_{-1}^1 \frac{|x+2|}{x+2} dx$$

For $-1 \leq x \leq 1$, $|x+2| = 2+x$

$$\therefore I = \int_{-1}^1 \frac{x+2}{x+2} dx = \int_{-1}^1 1 dx$$

$$= [x]_{-1}^1 = 1 - (-1)$$

$$= 2$$

788 (c)

$$\therefore \sqrt{x + 2\sqrt{(x-1)}} = \sqrt{(\sqrt{(x-1)})^2 + 1^2 + 2\sqrt{(x-1)}}$$

$$= \sqrt{(x-1)} + 1$$

and

$$\sqrt{x - 2\sqrt{(x-1)}} = \sqrt{(\sqrt{(x-1)})^2 + 1^2 - 2\sqrt{(x-1)}}$$

$$= |\sqrt{(x-1)} - 1|$$

Then, $\int_1^5 (\sqrt{x + 2\sqrt{(x-1)}} + \sqrt{x - 2\sqrt{(x-1)}}) dx$

$$= \int_1^5 (\sqrt{(x-1)} + 1) dx + \int_1^5 |\sqrt{(x-1)} - 1| dx$$

$$= \int_1^5 (\sqrt{(x-1)} + 1) dx + \int_1^2 (1 - \sqrt{(x-1)}) dx + \int_2^5 (\sqrt{(x-1)} - 1) dx$$

$$= \int_0^4 (\sqrt{x} + 1) dx + \int_0^1 (1 - \sqrt{x}) dx + \int_1^4 (\sqrt{x} - 1) dx$$

$$= \left[\frac{2}{3} (x^{3/2}) + x \right]_0^4 + \left[x - \frac{2}{3} x^{3/2} \right]_0^1 + \left[\frac{2}{3} x^{3/2} - x \right]_1^4$$

$$= \left(\frac{16}{3} + 4 \right) + \left(1 - \frac{2}{3} \right) + \left(\frac{16}{3} - 4 \right) + \left(\frac{2}{3} - 1 \right)$$

$$= \frac{32}{3}$$

789 (d)

$$\text{Let } I = \int \frac{1}{(\sin x + 4)(\sin x - 1)} dx$$

$$= \frac{1}{5} \int \frac{(\sin x + 4) - (\sin x - 1)}{(\sin x + 4)(\sin x - 1)} dx$$

$$= \frac{1}{5} \int \frac{1}{(\sin x - 1)} dx - \frac{1}{5} \int \frac{1}{(\sin x + 4)} dx$$

$$= \frac{1}{5} \int \frac{\sec^2 x/2}{2 \tan x/2 - 1 - \tan^2 x/2} dx - \frac{1}{5} \int \frac{\sec^2 x/2}{2 \tan x/2 + 4 + 4 \tan^2 x/2} dx$$

$$\text{Put } \tan \frac{x}{2} = t$$

$$\Rightarrow \sec^2 \frac{x}{2} dx = 2t dt$$

$$\therefore I = \frac{1}{5} \int \frac{2t dt}{2t - 1 - t^2} - \frac{1}{5} \int \frac{2t dt}{[2t + 4(1 + t^2)]}$$

$$\therefore I = -\frac{2}{5} \int \frac{dt}{2t - 1 - t^2} - \frac{1}{10} \int \frac{dt}{t^2 - \frac{1}{2}t + 1}$$

$$= \frac{-2}{5} \int \frac{1}{(t-1)^2} dt - \frac{1}{10} \int \frac{1}{\left(t + \frac{1}{4}\right)^2 \left(\frac{\sqrt{15}}{4}\right)^2}$$

$$= \frac{2}{5} \frac{1}{(t-1)} - \frac{2}{5\sqrt{15}} \tan^{-1} \left(\frac{4t+1}{\sqrt{15}} \right) + c$$

$$= \frac{2}{5} \frac{1}{(\tan \frac{x}{2} - 1)} - \frac{2}{5\sqrt{15}} \tan^{-1} \left(\frac{4 \tan \frac{x}{2} + 1}{\sqrt{15}} \right) + c$$

$$\therefore A = \frac{2}{5}, B = \frac{-2}{5\sqrt{15}}, f(x) = \frac{4 \tan \frac{x}{2} + 1}{\sqrt{15}}$$

790 (a)

$$\int (\sqrt{\tan x} + \sqrt{\cot x}) dx$$

$$= \int \left(\frac{\sqrt{\sin x}}{\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}} \right) dx$$

$$= \int \frac{(\sin x + \cos x)}{\sqrt{\sin x \cos x}} dx$$

$$= \int \frac{\sqrt{2}(\sin x + \cos x)}{\sqrt{2 \sin x \cos x}} dx$$

$$= \sqrt{2} \int \frac{\sin x + \cos x}{\sqrt{1 - (\sin x - \cos x)^2}} dx$$

Put $\sin x - \cos x = t$

$$\Rightarrow (\cos x + \sin x) dx = dt$$

$$= \sqrt{2} \int \frac{dt}{\sqrt{1 - t^2}}$$

$$= \sqrt{2} \sin^{-1}(\sin x - \cos x) + c$$

791 (b)

$$\text{Let } I = \int_{-\pi}^{\pi} \frac{2x}{1 + \cos^2 x} dx + 2 \int_{-\pi}^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$$

$$\Rightarrow I = 0 + 4 \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx \dots (i)$$

$$\left[\because \int_{-\pi}^{\pi} \frac{2x}{1 + \cos^2 x} dx \text{ is an odd function} \right]$$

$$\Rightarrow I = 4 \int_0^{\pi} \frac{(\pi - x) \sin x}{1 + \cos^2 x} dx \dots (ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = 4\pi \int_0^{\pi} \frac{\sin x}{1 + \cos^2 x} dx$$

Put $\cos x = t \Rightarrow \sin x dx = -dt$

$$\begin{aligned} \therefore I &= 2\pi \int_1^{-1} \frac{-dt}{1 + t^2} \\ &= 2\pi [\tan^{-1} t]_{-1}^1 = 2\pi \left[\frac{\pi}{4} + \frac{\pi}{4} \right] \\ &= \pi^2 \end{aligned}$$

792 (c)

We have,

$$I = \int \frac{\sin x}{\sin(x - \alpha)} dx = \int \frac{\sin\{(x - \alpha) + \alpha\}}{\sin(x - \alpha)} dx$$

$$\Rightarrow I = \int \{\cos \alpha + \sin \alpha \cot(x - \alpha)\} dx$$

$$\Rightarrow I = x \cos \alpha + \sin \alpha \log_e \sin(x - \alpha) + C$$

$$\therefore A = \cos \alpha, B = \sin \alpha$$

793 (b)

Since $x - [x]$ is a periodic function with period 1.

Therefore, $\sin(x - [x]) \pi$ is also a periodic function with period 1

$$\begin{aligned} \therefore I &= \int_0^{100} \sin(x - [x]) \pi dx \\ &= 100 \int_0^1 \sin(x - [x]) \pi dx \\ \Rightarrow I &= 100 \int_0^1 \sin \pi x dx = -\frac{100}{\pi} [\cos \pi x]_0^1 = \frac{200}{\pi} \end{aligned}$$

794 (b)

$$\because f(x) = \cos x - \cos^2 x + \cos^3 x - \dots \infty$$

$$= \frac{\cos x}{1 + \cos x}$$

$$\therefore \int f(x) dx = \int \frac{1 + \cos x}{1 + \cos x} dx - \int \frac{1}{1 + \cos x} dx$$

$$= x - \frac{1}{2} \int \sec^2 \frac{x}{2} dx$$

$$= x - \frac{1}{2} \tan \frac{x}{2} \cdot 2 + c = x - \tan \frac{x}{2} + c$$

795 (b)

$$\text{We have, } \int_0^a f(2a - x) dx = \mu$$

$$\text{and } \int_0^a f(x) dx = \lambda$$

Now, using properties of definite integral

$$\int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a - x) dx$$

$$\Rightarrow \int_0^{2a} f(x) dx = \lambda + \mu$$

796 (c)

Let

$$I = \int \frac{f'(x)}{f(x) \log\{f(x)\}} dx$$

$$\begin{aligned} \Rightarrow I &= \int \frac{1}{\log\{f(x)\}} d[\log\{f(x)\}] \\ &= \log[\log\{f(x)\}] + C \end{aligned}$$

797 (b)

$$\begin{aligned} \int_1^4 |x - 3| dx &= \int_1^3 (3 - x) dx + \int_3^4 (x - 3) dx \\ &= \left[3x - \frac{x^2}{2} \right]_1^3 + \left[\frac{x^2}{2} - 3x \right]_3^4 \\ &= \left(9 - \frac{9}{2} \right) - \left(3 - \frac{1}{2} \right) + \left(\frac{16}{2} - 12 \right) - \left(\frac{9}{2} - 9 \right) \\ &= \frac{5}{2} \end{aligned}$$

798 (a)

$$\begin{aligned} \int e^x \left(\log x + \frac{1}{x} \right) dx &= \int e^x \log x dx + \int e^x \cdot \frac{1}{x} dx \\ &= [e^x \log x] - \int e^x \cdot \frac{1}{x} dx + \int e^x \cdot \frac{1}{x} dx \\ &= e^x \log x + c \end{aligned}$$

799 (b)

$$\text{Let } I = \int_{-3\pi/2}^{-\pi/2} [(x + \pi)^3 + \cos^2 x] dx \dots (i)$$

$$\Rightarrow I = \int_{-3\pi/2}^{-\pi/2} \left[\left(-\frac{\pi}{2} - \frac{3\pi}{2} - x + \pi \right)^3 \right]$$

$$= \cos^2 \left(-\frac{\pi}{2} - \frac{3\pi}{2} - x \right) dx$$

$$\Rightarrow I = \int_{-3\pi/2}^{-\pi/2} [-(x + \pi)^3 + \cos^2 x] dx \dots (ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_{-3\pi/2}^{-\pi/2} 2 \cos^2 x dx$$

$$= \int_{-3\pi/2}^{-\pi/2} (1 + \cos 2x) dx$$

$$\begin{aligned}
&= \left[x + \frac{\sin 2x}{2} \right]_{-3\pi/2}^{-\pi/2} \\
&= \left[-\frac{\pi}{2} + \frac{\sin(-\pi)}{2} - \left(-\frac{3\pi}{2} + \frac{\sin(-3\pi)}{2} \right) \right] = \pi \\
&\Rightarrow I = \frac{\pi}{2}
\end{aligned}$$

800 (a)

$$\text{Let } I = \int_1^x \frac{\log(x^2)}{x} dx = 2 \int_1^x \log x \cdot \frac{1}{x} dx$$

$$\text{Put } \log x = t \Rightarrow \frac{1}{x} dx = dt$$

$$\therefore I = 2 \int_0^{\log x} t dt = 2 \left[\frac{t^2}{2} \right]_0^{\log x} = (\log x)^2$$

801 (b)

$$\text{Given, } f'(x) = x + \frac{1}{x}$$

On integrating both sides, we get

$$f(x) = \frac{x^2}{2} + \log x + c$$

802 (d)

$$\text{Let } I = \int e^{-\log x} dx = \int \frac{1}{x} dx = \log|x| + c$$

803 (b)

$$\int \frac{dx}{x(x+1)} = \int \frac{1}{x} dx - \int \frac{1}{x+1} dx$$

$$= \log x - \log(x+1) + c$$

$$= \log \left| \frac{x}{x+1} \right| + c$$

804 (c)

We have,

$$I = \int_0^{\pi^2/4} \sin \sqrt{x} dx = 2 \int_0^{\pi/2} \sin t dt, \text{ where } t^2 = x$$

$$\Rightarrow I = 2[-t \cos t + \sin t]_0^{\pi/2} = 2$$

805 (b)

$$\text{Let } f(x) = \int_0^x \left[\frac{1}{\sqrt{1+t^2}} - \frac{1}{1+t} \right] dt$$

$$= \int_0^x \frac{1}{\sqrt{1+t^2}} dt - \int_0^x \frac{1}{1+t} dt$$

$$\text{Let } I_1 = \int_0^x \frac{1}{\sqrt{1+t^2}} dt$$

$$\text{Put } t = \tan \theta \Rightarrow dt = \sec^2 \theta d\theta$$

$$\therefore I_1 = \int_0^{\tan^{-1} x} \frac{\sec^{-1} \theta}{\sec \theta} d\theta$$

$$= [\log(\sec \theta + \tan \theta)]_0^{\tan^{-1} x}$$

$$= \log(\sec \tan^{-1} x + x)$$

$$\therefore f(x) = \log(\sec \tan^{-1} x + x) - \log(1+x)$$

$$\therefore \lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \log \left(\frac{\sec \tan^{-1} x + x}{1+x} \right)$$

$$\begin{aligned}
&= \lim_{x \rightarrow \infty} \log \left(\frac{\sqrt{1+x^2} + x}{1+x} \right) \\
&= \log \lim_{x \rightarrow \infty} \left(\frac{\sqrt{\frac{1}{x^2} + 1} + 1}{\frac{1}{x} + 1} \right)
\end{aligned}$$

$$= \log_e 2$$

806 (c)

$$\text{Given, } I_1 = \int \sin^{-1} x dx$$

$$\text{and } I_2 = \int \sin^{-1} \sqrt{1-x^2} dx$$

$$\Rightarrow I_2 = \int \cos^{-1} x dx$$

$$\text{Now, } I_1 + I_2 = \int (\sin^{-1} x + \cos^{-1} x) dx$$

$$= \int \frac{\pi}{2} dx = \frac{\pi}{2} x$$

$$\therefore I_1 + I_2 = \frac{\pi}{2} x$$

807 (b)

$$\text{Let } I = \int \cos^3 x \cdot e^{\log \sin x} dx = \int \cos^3 x \sin x dx$$

$$\text{Put } \cos x = t \Rightarrow -\sin x dx = dt$$

$$\therefore I = -\int t^3 dt = -\frac{t^4}{4} + c = -\frac{\cos^4 x}{4} + c$$

808 (c)

We have,

$$\sqrt{\frac{1+\cos 2x}{2}} = |\cos x| = \begin{cases} \cos x, & 0 \leq x \leq \pi/2 \\ -\cos x, & \pi/2 \leq x \leq \pi \end{cases}$$

$$\therefore I = \int_0^{\pi/2} \sqrt{\frac{1+\cos 2x}{2}} dx = \int_0^{\pi/2} |\cos x| dx$$

$$\Rightarrow I = \int_0^{\pi/2} \cos x dx + \int_{\pi/2}^{\pi} -\cos x dx$$

$$= [\sin x]_0^{\pi/2} - [\sin x]_{\pi/2}^{\pi} = 2$$

809 (c)

$$\text{Let } \int_0^{\pi/2} \sin 2x \log \tan x dx \dots (i)$$

$$I = \int_0^{\pi/2} \sin 2 \left(\frac{\pi}{2} \right.$$

$$\left. - x \right) \log \tan \left(\frac{\pi}{2} \right.$$

$$\left. - x \right) dx \left[\because \int_0^a f(x) dx \int_0^a f(a \right.$$

$$\left. - x \right) dx \right]$$

$$I = \int_0^{\pi/2} \sin 2x \log \cot x dx \dots (ii)$$

On adding Eqs.(i) and (ii), we get

$$2I = \int_0^{\pi/2} \sin 2x \log(\tan x \cot x) dx$$

$$= \int_0^{\pi/2} \sin 2x \log 1 \, dx = \int_0^{\pi/2} 0 \, dx$$

$$\Rightarrow I = 0$$

810 (a)

Since, the given function is an odd function.

$$\therefore I = \int_{-\pi}^{\pi} (1 - x^2) \sin x \cos^2 x \, dx = 0$$

811 (a)

$$\text{Let } A = \lim_{n \rightarrow \infty} \frac{1}{n} \left[\frac{1}{n} \sec^2 \left(\frac{1}{n} \right)^2 + \frac{2}{n} \sec^2 \left(\frac{2}{n} \right)^2 + \dots + \frac{n}{n} \sec^2 \left(\frac{n}{n} \right)^2 \right]$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \left(\frac{r}{n} \right) \sec^2 \left(\frac{r}{n} \right)^2$$

$$\Rightarrow A = \int_0^1 x \sec^2(x)^2 \, dx$$

$$\text{Put } x^2 = t \Rightarrow x \, dx = \frac{dt}{2}$$

$$\therefore A = \frac{1}{2} \int_0^1 \sec^2 t \, dt = \frac{1}{2} [\tan t]_0^1 = \tan 1$$

812 (b)

$$\begin{aligned} \int \frac{1}{1 + \cos ax} \, dx &= \int \frac{1}{2 \cos^2(ax/2)} \, dx \\ &= \frac{1}{2} \int \sec^2 \frac{ax}{2} \, dx = \frac{1}{2} \cdot \frac{\tan \, ax/2}{a/2} \\ &= \frac{1}{a} \tan \frac{ax}{2} + c \end{aligned}$$

813 (b)

$$\begin{aligned} \int \sqrt{1 + \cos x} \, dx &= \int \sqrt{2 \cos^2 \frac{x}{2}} \, dx \\ &= \sqrt{2} \int \cos \frac{x}{2} \, dx = 2\sqrt{2} \cdot \sin \frac{x}{2} + c \end{aligned}$$

814 (c)

$$\begin{aligned} \int_{-2}^3 f(x) \, dx &= \int_{-2}^2 f(x) \, dx + \int_2^3 f(x) \, dx \\ &= \int_{-2}^2 e^{\cos x} \sin x \, dx + \int_2^3 2 \, dx \end{aligned}$$

Function of 1st integral is an odd function, therefore the value of 1st integral is zero.

$$\therefore \int_{-2}^3 f(x) \, dx = 0 + 2[x]_2^3$$

$$= 0 + 2(3 - 2) = 2$$

815 (d)

$$\text{Let } I = \int x \frac{e^x}{\sqrt{1+e^x}} \, dx$$

Putting $1 + e^x = t^2$ i.e. $x = \log(t^2 - 1)$, we get

$$I = 2 \int \log(t^2 - 1) \, dt$$

$$\Rightarrow I = 2 \left\{ t \log(t^2 - 1) - 2 \int \frac{t^2}{t^2 - 1} \, dt \right\}$$

$$\Rightarrow I = 2 \left\{ t \log(t^2 - 1) - 2 \int 1 + \left(\frac{1}{t^2 - 1} \right) \, dt \right\}$$

$$\Rightarrow I = 2 \left\{ t \log(t^2 - 1) - 2t - \log \left(\frac{t-1}{t+1} \right) \right\} + C$$

$$\begin{aligned} \Rightarrow I &= 2x \sqrt{1+e^x} - 4 \sqrt{1+e^x} \\ &\quad - 2 \log \left\{ \frac{\sqrt{1+e^x} - 1}{\sqrt{1+e^x} + 1} \right\} + C \end{aligned}$$

Hence, $f(x) = 2x - 4 = 2(x - 2)$ and

$$g(x) = \frac{\sqrt{1+e^x} - 1}{\sqrt{1+e^x} + 1}$$

816 (c)

$$\therefore v(t) = \cos \pi t,$$

$$\Rightarrow \frac{ds}{dt} = \cos \pi t$$

$$\Rightarrow s = \frac{\sin \pi t}{\pi} + c, \text{ at } t = 0, s = 4, c = 4$$

$$\Rightarrow s = \frac{\sin \pi t}{\pi} + 4$$

817 (a)

$$\text{Let } I = \int \cos(\log_e x) \, dx = \int \cos(\log_e x) \cdot 1 \, dx$$

$$= \cos(\log_e x) x - \int \frac{-\sin \log_e x}{x} \cdot x \, dx$$

$$= x \cos(\log_e x) + \int \sin(\log_e x) \, dx$$

$$= x \cos(\log_e x) + \int \sin(\log_e x) \cdot 1 \, dx$$

$$\begin{aligned} &= x \cos(\log_e x) - \sin(\log_e x) x \\ &\quad - \int \frac{\cos(\log_e x)}{x} x \, dx \end{aligned}$$

$$= x \cos(\log_e x) + x \sin(\log_e x) - I$$

$$\Rightarrow 2I = x[\cos(\log_e x) + \sin(\log_e x)]$$

$$\Rightarrow I = \frac{x}{2} [\cos(\log_e x) + \sin(\log_e x)]$$

818 (d)

$$\text{Let } I = \int \frac{(e^x - e^{-x})}{(e^x + e^{-x}) \log(\cosh x)} \, dx$$

$$\text{Put } \log(\cosh x) = t \Rightarrow \frac{1}{\cosh x} \cdot \sinh x \, dx = dt$$

$$\Rightarrow \frac{e^x - e^{-x}}{e^x + e^{-x}} \, dx = dt$$

$$[\because \sinh x = \frac{e^x - e^{-x}}{2}]$$

$$\text{and } \cosh x = \frac{e^x + e^{-x}}{2}]$$

$$\therefore I = \int \frac{1}{t} \, dt = \log t + c$$

$$= \log(\log \cosh x) + c$$

819 (b)

$$\text{Given, } u = -f''(\theta) \sin \theta + f'(\theta) \cos \theta$$

$$\text{and } v = f''(\theta) \cos \theta + f'(\theta) \sin \theta$$

On differentiating w. r. t θ respectively, we get

$$\begin{aligned} \frac{du}{d\theta} &= -f'''(\theta) \sin \theta \\ &\quad - f''(\theta) \cos \theta \\ &\quad + f''(\theta) \cos \theta - f'(\theta) \sin \theta \end{aligned}$$

$$= -f'''(\theta) \sin \theta - f'(\theta) \sin \theta$$

$$\text{and } \frac{dv}{d\theta} = f'''(\theta) \cos \theta - f''(\theta) \sin \theta + f''\theta \sin \theta + f'\theta \cos \theta$$

$$= -f'''(\theta) \cos \theta + f'(\theta) \cos \theta$$

$$\begin{aligned} \therefore \left(\frac{du}{d\theta}\right)^2 + \left(\frac{dv}{d\theta}\right)^2 &= [f'''(\theta)]^2 + [f'(\theta)]^2 \\ &\quad + 2f'(\theta)f'''(\theta) \end{aligned}$$

$$= [f'''(\theta) + f'(\theta)]^2$$

$$\begin{aligned} \text{Now, } \int \left[\left(\frac{du}{d\theta}\right)^2 + \left(\frac{dv}{d\theta}\right)^2 \right]^{1/2} d\theta &= \int [f'''(\theta) + f'(\theta)] d\theta \\ &= f''(\theta) + f(\theta) + c \end{aligned}$$

820 (d)

We have,

$$I = \int_0^{\pi} \frac{\sin kx}{\sin x} dx = \int_0^{\pi} \frac{\sin k(\pi - x)}{\sin(\pi - x)} dx$$

$$\Rightarrow I = - \int_0^{\pi} \frac{\sin kx}{\sin x} dx = -I \Rightarrow 2I = 0 \Rightarrow I = 0$$

821 (d)

$$\text{Let } I = \int \frac{\sin x}{\sin(x-\alpha)} dx$$

$$\text{Put } x - \alpha = t \Rightarrow dx = dt$$

$$\therefore I = \frac{\sin(\alpha + t)}{\sin t} dt$$

$$= \int \frac{\sin \alpha \cot t + \cos \alpha \sin t}{\sin t} dt$$

$$= \sin \alpha \int \cot t dt + \cos \alpha \int dt$$

$$= \sin \alpha \log \sin t + \cos \alpha \cdot t + c_1$$

$$= \sin \alpha \log \sin(x - \alpha) + \cos \alpha \cdot (x - \alpha) + c_1$$

$$= \sin \alpha \log \sin(x - \alpha) + x \cos \alpha + c$$

$$[\text{let } c = -\alpha \cos \alpha + c_1]$$

823 (b)

$$\int x^3 \log x dx$$

$$= \frac{x^4}{4} \log x - \int \frac{x^4}{4} \cdot \frac{1}{x} dx$$

$$= \frac{x^4}{4} \log x - \int \frac{x^3}{4} dx = \frac{x^4}{4} \log x - \frac{x^4}{16} + c$$

$$= \frac{1}{16} [4x^4 \log x - x^4] + c$$

824 (a)

We have,

$$I = \int \frac{\sin^3 x}{(1 + \cos^2 x)\sqrt{1 + \cos^2 x + \cos^4 x}} dx$$

$$\Rightarrow I$$

$$= - \int \frac{(1 - \cos^2 x)}{(1 + \cos^2 x)\sqrt{1 + \cos^2 x + \cos^4 x}} d(\cos x)$$

$$\Rightarrow I = - \int \frac{1-t^2}{(1+t^2)\sqrt{1+t^2+t^4}} dt, \text{ where } t = \cos x$$

$$\Rightarrow I = \int \frac{\left(1 - \frac{1}{t^2}\right)}{\left(t + \frac{1}{t}\right)\sqrt{\left(t + \frac{1}{t}\right)^2 - 1}} dt$$

$$\Rightarrow I = \int \frac{1}{\left(1 + \frac{1}{t}\right)\sqrt{\left(t + \frac{1}{t}\right)^2 - 1^2}} d\left(t + \frac{1}{t}\right)$$

$$\Rightarrow I = \sec^{-1}\left(t + \frac{1}{t}\right) + C$$

$$= \sec^{-1}(\cos x + \sec x) + C$$

825 (a)

We have,

$$x = \int_2^{\sin t} \sin^{-1} \theta d\theta \text{ and } y = \int_n^{\sqrt{t} \frac{\sin \theta^2}{\theta}} d\theta$$

$$\Rightarrow \frac{dx}{dt} = \cos t \sin^{-1}(\sin t) \text{ and } \frac{dy}{dt} = \frac{1}{2\sqrt{t}} \times \frac{\sin t}{\sqrt{t}}$$

$$\Rightarrow \frac{dx}{dt} = t \cos t \text{ and } \frac{dy}{dt} = \frac{\sin t}{2t}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin t}{2t} \times \frac{1}{t \cos t} = \frac{\tan t}{2t^2}$$

826 (c)

$$\text{Let } I = 100 \int_0^{\pi/2} \frac{(\sin x + \cos x) + \sin x}{\sin x + \cos x} dx$$

$$= 100 \left[\int_0^{\pi/2} 1 dx + \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx \right]$$

$$\text{Let } I_1 = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx \dots (i)$$

$$\Rightarrow I_1 = \int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x} dx \dots (ii)$$

$$\left[\text{put } x = \frac{\pi}{2} - x \right]$$

On adding Eqs. (i) and (ii), we get

$$2I_1 = \int_0^{\pi/2} 1 dx = \frac{\pi}{2}$$

$$\Rightarrow I_1 = \frac{\pi}{4}$$

$$\therefore I = 100 \left[\frac{\pi}{2} + \frac{\pi}{4} \right] = 100 \times \frac{3\pi}{4} = 75\pi$$

827 (c)

$$\text{We have, } \frac{d}{dx}(f(x)) = g(x) \Rightarrow df(x) = g(x)dx$$

Now,

$$I = \int_a^b f(x)d(f(x))$$

$$\Rightarrow I = \frac{1}{2} \{[f(x)]^2\}_a^b \Rightarrow I = \frac{1}{2} \{[f(b)]^2 - [f(a)]^2\}$$

828 (b)

$\because \sin^4 t + \cos^4 t$ is periodic with period $\frac{\pi}{2}$

Now, $f(x + \pi) = \int_0^{\pi+x} (\sin^4 t + \cos^4 t) dt$

$$= \int_0^x (\sin^4 t + \cos^4 t) dt + \int_0^{\pi} (\sin^4 t + \cos^4 t) dt$$

$$= f(x) + \int_0^{\pi} (\cos^4 t + \sin^4 t) dt = f(x) + f(\pi)$$

or

$$= f(x) + 2 \int_0^{\pi/2} (\cos^4 t + \sin^4 t) dt$$

$$= f(x) + 2f\left(\frac{\pi}{2}\right)$$

829 (b)

$$\text{Let } I = \int_0^1 \frac{x^7}{\sqrt{1-x^4}} dx = \int_0^1 \frac{x^6 x dx}{\sqrt{1-x^4}}$$

$$\text{Put } x^2 = \sin \theta \Rightarrow 2x dx = \cos \theta d\theta$$

$$\therefore I = \frac{1}{2} \int_0^{\pi/2} \frac{\sin^3 \theta \cos \theta d\theta}{\sqrt{1-\sin^2 \theta}}$$

$$\Rightarrow I = \frac{1}{2} \int_0^{\pi/2} \frac{\sin^3 \theta \cos \theta d\theta}{\cos \theta} = \frac{1}{2} \int_0^{\pi/2} \sin^3 \theta d\theta$$

$$\Rightarrow I = \frac{1}{2} \frac{\sqrt{2} \sqrt{\left(\frac{1}{2}\right)}}{\sqrt{2} \cdot \sqrt{\left(\frac{5}{2}\right)}} = \frac{\sqrt{\left(\frac{1}{2}\right)}}{4 \cdot \frac{3}{2} \cdot \frac{1}{2} \cdot \sqrt{\left(\frac{1}{2}\right)}} = \frac{1}{3}$$

830 (a)

$$\text{Let } \frac{x-1}{x+2} = t \Rightarrow \frac{1}{(x+2)^2} dx = \frac{1}{3} dt$$

$$\therefore \int \frac{1}{[(x-1)^3(x+2)^5]^{1/4}} dx = \frac{1}{3} \int t^{-3/4} dt$$

$$= \frac{4}{3} t^{1/4} + c$$

$$= \frac{4}{3} \left(\frac{x-1}{x+2}\right)^{1/4} + c$$

831 (c)

$$\text{Let } I = \int \frac{dx}{x+\sqrt{x-1}}$$

$$\text{Put } x = t^2 + 1 \Rightarrow dx = 2t dt$$

$$\therefore I = \int \frac{2t}{t^2 + t + 1} dt$$

$$= \int \frac{2t+1}{t^2+t+1} dt - \int \frac{1}{t^2+t+1} dt$$

$$= \log(t^2+t+1) - \int \frac{1}{\left(t+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dt$$

$$= \log(t^2+t+1) - \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{2t+1}{\sqrt{3}}\right) + c$$

$$= \log(x+\sqrt{x-1}) - \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right) + c$$

832 (b)

$$\text{Given, } 2f(x) - 3f\left(\frac{1}{x}\right) = x \quad \dots(i)$$

Put $x = \frac{1}{x}$ in Eq.(i), we get

$$2f\left(\frac{1}{x}\right) - 3f(x) = \frac{1}{x} \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$f(x) = -\frac{3+2x^2}{5x}$$

$$\text{Now, } \int_1^2 f(x) dx = -\int_1^2 \frac{3+2x^2}{5x} dx$$

$$= -\int_1^2 \left(\frac{3}{5x} + \frac{2x}{5}\right) dx$$

$$= -\frac{1}{5} \left[3 \log x + \frac{2x^2}{2}\right]_1^2$$

$$= -\frac{1}{5} (3 \log 2 + 4 - 3 \log 1 - 1)$$

$$= -\frac{1}{5} (3 \log 2 + 4 - 0 - 1)$$

$$= -\frac{3}{5} (1 + \log 2)$$

833 (c)

$$\int e^x \left(\frac{2}{x} - \frac{2}{x^2}\right) dx$$

$$= \frac{2e^x}{x} + c \quad [\because f(x) = \frac{2}{x} \text{ and } f'(x) = -\frac{2}{x^2}]$$

834 (b)

We have,

$$I = \int_2^4 \frac{\sqrt{x^2-4}}{x^4} dx = \int_2^4 \frac{1}{x^3} \sqrt{1-\frac{4}{x^2}} dx$$

$$\text{Let, } 1 - \frac{4}{x^2} = t \Rightarrow \frac{8}{x^3} dx = dt$$

$$\therefore I = \frac{1}{8} \int_0^{\sqrt{3}} \sqrt{t} dt = \frac{1}{8} \times \frac{2}{3} [t^{3/2}]_0^{\sqrt{3}} = \frac{1}{12} \times \frac{3}{4} \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{32}$$

835 (b)

We have,

$$\int e^{3 \log x} (x^4 + 1)^{-1} dx = \int e^{\log x^3} \frac{1}{x^4 + 1} dx$$

$$\Rightarrow I = \int \frac{x^3}{x^4 + 1} dx = \frac{1}{4} \log(x^4 + 1) + C$$

836 (d)

$$\text{Given, } \int_0^p \frac{dx}{1+4x^2} = \frac{\pi}{8}$$

$$\text{LHS} = \int_0^p \frac{dx}{1+4x^2} = \frac{1}{4} \int_0^p \frac{dx}{\frac{1}{4}+x^2}$$

$$= \frac{1}{4} \times \frac{1}{2} \left[\tan^{-1} \left(\frac{x}{1/2}\right) \right]_0^p$$

$$\Rightarrow \frac{1}{2} [\tan^{-1} 2p] = \frac{\pi}{8}$$

$$\Rightarrow \tan^{-1} 2p = \frac{\pi}{4}$$

$$\Rightarrow 2p = 1$$

$$\Rightarrow p = \frac{1}{2}$$

837 (c)

$$\text{Let, } I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx \quad \dots(i)$$

Then,

$$I = \int_0^{\pi/2} \frac{\sin^2\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\cos^2 x}{\sin x + \cos x} dx \quad \dots(ii)$$

Adding (i) and (ii), we get

$$\Rightarrow 2I = \int_0^{\pi/2} \frac{\sin^2 x + \cos^2 x}{\sin x + \cos x} dx$$

$$\Rightarrow 2I = \int_0^{\pi/2} \frac{1}{\sin x + \cos x} dx$$

$$\Rightarrow 2I = \frac{1}{\sqrt{2}} \int_0^{\pi/2} \frac{1}{\cos\left(x - \frac{\pi}{4}\right)} dx$$

$$\Rightarrow 2I = \frac{1}{\sqrt{2}} \int_0^{\pi/2} \sec\left(x - \frac{\pi}{4}\right) dx$$

$$\Rightarrow 2I = \frac{1}{\sqrt{2}} \left[\log\left(\sec\left(x - \frac{\pi}{4}\right) + \tan\left(x - \frac{\pi}{4}\right)\right) \right]_0^{\pi/2}$$

$$\Rightarrow 2I = \frac{1}{\sqrt{2}} \left[\log(\sqrt{2} + 1) - \log(\sqrt{2} - 1) \right]$$

$$\Rightarrow 2I = \frac{1}{\sqrt{2}} \log\left(\frac{\sqrt{2} + 1}{\sqrt{2} - 1}\right) = \frac{1}{\sqrt{2}} \log(\sqrt{2} + 1)^2 = \sqrt{2} \log(\sqrt{2} + 1)$$

$$\therefore I = \frac{1}{\sqrt{2}} \log(\sqrt{2} + 1)$$

838 (b)

$$\text{Let } I = \int \frac{\sqrt{x^2+1} [\log(x^2+1) - 2\log x]}{x^4} dx \cdot \text{Then,}$$

$$I = \int \sqrt{1 + \frac{1}{x^2}} \log\left(1 + \frac{1}{x^2}\right) \cdot \frac{1}{x^3} dx$$

Putting $1 + \frac{1}{x^2} = t$ and $-\frac{2}{x^3} dx = dt$, we get

$$I = -\frac{1}{2} \int \sqrt{t} \log t dt$$

$$\Rightarrow I = -\frac{1}{2} \left\{ \log t \left(\frac{t^{3/2}}{3/2}\right) - \frac{2}{3} \int \frac{1}{t} t^{3/2} dt \right\}$$

$$\Rightarrow I = -\frac{1}{2} \left\{ \frac{2}{3} (\log t) t^{3/2} - \frac{2}{3} \left(\frac{2}{3} t^{3/2}\right) \right\} + C$$

$$\Rightarrow I = -\frac{1}{3} t^{3/2} \log t + \frac{2}{9} t^{3/2} + C$$

$$\Rightarrow I = -\frac{1}{3} \left(1 + \frac{1}{x^2}\right)^{3/2} \left\{ \log\left(1 + \frac{1}{x^2}\right) - \frac{2}{3} \right\} + C$$

839 (b)

We have,

$$I = \int_3^5 \frac{x^2}{x^2 - 4} dx = \int_3^5 \left(\frac{4}{x^2 - 4}\right) dx$$

$$\Rightarrow I = \left[x + \frac{4}{2 \times 2} \log\left(\frac{x-2}{x+2}\right) \right]_3^5 = 2 + \left(\log\frac{3}{7} - \log\frac{1}{5}\right) = 2 + \log\frac{15}{7}$$

840 (d)

Since $|\sin x|$ is a periodic function with period π

$$\therefore \int_{\pi}^{10\pi} |\sin x| dx = 9 \int_0^{\pi} |\sin x| dx = 9 \int_0^{\pi} \sin x dx = 18$$

841 (b)

We have,

$$\Delta(y) = \begin{vmatrix} a-c & b-d & a-c+b-d \\ b-c & c-d & b-d-1 \\ y+c & y+d & y-b+d \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 - R_3$ and $R_2 \rightarrow R_2 - R_3$

$$\begin{aligned} \therefore \int_0^2 \Delta(y) dy &= \int_0^2 \begin{vmatrix} a-c & b-d & a-c+b-d \\ b-c & c-d & b-d-1 \\ y+c & y+d & y-b+d \end{vmatrix} dy \\ &= \int_0^2 \begin{vmatrix} a-c & b-d & a-c+b-d \\ b-c & c-d & b-d-1 \\ 2+2c & 2+2d & 2-2(b-d) \end{vmatrix} dy \\ \Rightarrow \int_0^2 \Delta(y) dy &= \begin{vmatrix} a-c & b-d & a-c+b-d \\ b-c & c-d & b-d-1 \\ 2+2c & 2+2d & 2-2(b-d) \end{vmatrix} \end{aligned}$$

Let x be the common difference of the A.P.

a, b, c, d . The,

$$\int_0^2 \Delta(y) dy = \begin{vmatrix} -2x & -2x & -4x \\ -x & -x & -2x-1 \\ 2x+2c & 2+2d & 2+2x \end{vmatrix}$$

$$\Rightarrow \int_0^2 \Delta(y) dy = \begin{vmatrix} 0 & 0 & 2 \\ -x & -x & -2x-1 \\ 2+2c & 2+2d & 2+2x \end{vmatrix}$$

$$\text{Applying } R_1 \rightarrow R_1 - 2R_2$$

$$\Rightarrow \int_0^2 \Delta(y) dy = 2 \begin{vmatrix} -x & -x \\ 2+2c & 2+2d \end{vmatrix}$$

$$\Rightarrow \int_0^2 \Delta(y) dy = -4x \begin{vmatrix} 1 & 1 \\ 1+c & 1+d \end{vmatrix} = -4x(d-c) = -4x^2$$

$$\therefore \int_0^2 \Delta(y) dy = -16 \Rightarrow -4x^2 = -16 \Rightarrow x = \pm 2$$

842 (c)

$$\begin{aligned} \int \operatorname{cosec}^4 x dx &= \int \operatorname{cosec}^2 x \cdot \operatorname{cosec}^2 x dx \\ &= \int \operatorname{cosec}^2 x (1 + \cot^2 x) dx \\ &= \int \operatorname{cosec}^2 x dx + \int \cot^2 x \cdot \operatorname{cosec}^2 x dx \\ &= -\cot x - \frac{\cot^3 x}{3} + c \end{aligned}$$

843 (d)

If $f(x)$ is a continuous function defined on $[a, b]$, then

$$m(b-a) \leq \int_a^b f(x) dx \leq M(b-a)$$

Where M and m are maximum and minimum values respectively of $f(x)$ in $[a, b]$

Here, $f(x) = 1 + e^{-x^2}$ is continuous on $[0, 1]$.

$$\text{Now, } 0 < x < 1 \Rightarrow x^2 < x$$

$$\Rightarrow e^{x^2} < e^x \Rightarrow e^{-x^2} > e^{-x}$$

$$\text{Again, } 0 < x < 1 \Rightarrow x^2 > 0$$

$$\Rightarrow e^{x^2} < e^0 \Rightarrow e^{-x^2} > 1$$

$$\therefore e^{-x} < e^{-x^2} < 1 \text{ for all } x \in [0, 1]$$

$$\Rightarrow 1 + e^{-x} < 1 + e^{-x^2} < 2 \text{ for all } x \in [0, 1]$$

$$\Rightarrow \int_0^1 (1 + e^{-x}) dx < \int_0^1 (1 + e^{-x^2}) dx < \int_0^1 2 dx$$

$$\Rightarrow 2 - \frac{1}{e} < \int_0^1 (1 + e^{-x^2}) dx < 2$$

844 (c)

We have,

$$I = \int e^{\sec x} (\tan^2 x + \sec x + \sec^2 x + \tan x \sec^2 x$$

$$+ \tan x \sec x) dx$$

$$\Rightarrow I = \int e^{\sec x} [\sec x \tan x (\sec x + \tan x)$$

$$+ (\sec x \tan x + \sec^2 x)] dx$$

$$\Rightarrow I = \int e^{\sec x} \sec x \tan x \cdot (\sec x + \tan x) dx$$

$$+ \int e^{\sec x}$$

$$\cdot (\sec x \tan x + \sec^2 x) dx$$

$$\Rightarrow I = (\sec x + \tan x) e^{\sec x}$$

$$- \int (\sec x + \tan x$$

$$+ \sec^2 x) e^{\sec x} dx$$

$$+ \int e^{\sec x} (\sec x \tan x + \sec^2 x) dx$$

$$\Rightarrow I = e^{\sec x} (\sec x + \tan x) + C$$

845 (a)

$$\text{Let } I = \int \frac{\log \sqrt{x}}{3x} dx$$

$$\text{Put } \sqrt{x} = t$$

$$\frac{1}{\sqrt{x}} dx = 2 dt$$

$$\therefore I = \int \frac{2 \log t}{3t} dt$$

$$= \frac{2}{3} \int \frac{\log t}{t} dt$$

$$= \frac{2}{3} \cdot \frac{(\log t)^2}{2} + c$$

$$= \frac{1}{3} (\log \sqrt{x})^2 + c$$

846 (b)

We have,

$$I = \int_0^{2\pi} \sin^2 x dx$$

$$\Rightarrow I = 2 \int_0^{\pi} \sin^2 x dx \quad [\because \sin^2(2\pi - x) = \sin^2 x]$$

$$\Rightarrow I = 4 \int_0^{\pi/2} \sin^2 x dx \quad [\because \sin^2(\pi - x) = \sin^2 x]$$

847 (a)

We have,

$$0 \leq x \leq 1$$

$$\Rightarrow 1 \leq 1 + x^4 \leq 2$$

$$\Rightarrow 1 \leq \sqrt{1 + x^4} \leq \sqrt{2}$$

$$\Rightarrow \frac{1}{\sqrt{2}} \leq \frac{1}{\sqrt{1 + x^4}} \leq 1$$

$$\Rightarrow \frac{1}{\sqrt{2}} (1 - 0) \leq \int_0^1 \frac{1}{\sqrt{1 + x^4}} dx \leq 1(1 - 0)$$

$$\Rightarrow \frac{1}{\sqrt{2}} \leq \int_0^1 \frac{1}{\sqrt{1 + x^4}} dx \leq 1$$

$$\Rightarrow \int_0^1 \frac{1}{\sqrt{1 + x^4}} dx \in \left[\frac{1}{\sqrt{2}}, 1 \right]$$

848 (c)

$$\text{Let } I = \int \frac{3x+2}{(x-2)^2(x-3)} dx \quad \dots(i)$$

Again, let

$$\frac{3x+2}{(x-2)^2(x-3)} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2} + \frac{C}{(x-3)} \quad \dots(ii)$$

$$\Rightarrow 3x + 2 = A(x-2)(x-3) + B(x-3) + C(x-2)^2 \dots(iii)$$

On putting the values of $x = 2, 3$ respectively, we get

$$3 \times 2 + 2 = B(2 - 3) \Rightarrow B = -8$$

$$\text{and } 3 \times 3 + 2 = C(3 - 2)^2 \Rightarrow C = 11$$

On equation the coefficient of x^2 in Eq. (iii), we get
 $0 = A + C \Rightarrow A = -11$

On putting the values of A, B and C in Eq. (ii), we get

$$\frac{3x + 2}{(x - 2)^2(x - 3)}$$

$$= -\frac{11}{(x - 2)} - \frac{8}{(x - 2)^2} + \frac{11}{(x - 3)}$$

$$\therefore I = \int \left[-\frac{11}{(x - 2)} - \frac{8}{(x - 2)^2} + \frac{11}{(x - 3)} \right] dx$$

$$= -11 \log(x - 2) + \frac{8}{(x - 2)} + 11 \log(x - 3) + c$$

$$= 11 \log \left(\frac{x - 3}{x - 2} \right) + \frac{8}{(x - 2)} + c$$

849 (a)

$$\text{Let } I = \int \frac{\cos x}{1 + \sin x} \cdot e^x dx - \int \frac{1}{\sin x + 1} \cdot e^x dx$$

$$= \frac{(\cos x)e^x}{1 + \sin x}$$

$$- \int \left[\frac{-(1 + \sin x) \sin x - \cos^2 x}{(1 + \sin x)^2} \right] \cdot e^x dx$$

$$- \int \frac{e^x}{\sin x + 1} dx$$

$$= \frac{e^x \cos x}{1 + \sin x} + \int \frac{1}{1 + \sin x} \cdot e^x dx - \int \frac{e^x dx}{1 + \sin x}$$

$$= \frac{e^x \cos x}{1 + \sin x} + c$$

850 (c)

$$\text{Let } I = \int_0^{2\pi} \sin^6 x \cos^5 x dx$$

$$= 2 \int_0^\pi \sin^6 x \cos^5 x dx \quad [\because f(2\pi - x) = f(x)]$$

$$\text{Let } f(x) = \sin^6 x \cos^5 x$$

$$f(\pi - x) = \sin^6(\pi - x) \cos^5(\pi - x)$$

$$= -\sin^6 x \cdot \cos^5 x = -f(x)$$

$$\therefore I = 0$$

851 (c)

$$\int \cos^{-3/7} x \sin^{-11/7} x dx$$

$$= \int \frac{\sin^{-11/7} x}{\cos^{-11/7} x} \cdot \sec^2 x dx$$

$$= \int \tan^{-11/7} x \sec^2 x dx$$

$$\text{Put } \tan x = t \Rightarrow \sec^2 x dx = dt$$

$$\therefore I = \int t^{-11/7} dt = -\frac{7}{4} \tan^{-4/7} x + c$$

852 (a)

We have,

$$\lim_{n \rightarrow \infty} \left\{ \frac{1}{na} + \frac{1}{na + 1} + \dots + \frac{1}{na + n(b - a)} \right\}$$

$$= \lim_{n \rightarrow \infty} \sum_{r=1}^{n(b-a)} \frac{1}{na + r}$$

$$= \lim_{n \rightarrow \infty} \sum_{r=1}^{n(b-a)} \frac{1}{a + (r/n)} \times \frac{1}{n} = \int_0^{b-a} \frac{1}{a + x} dx$$

$$= \log \left(\frac{b}{a} \right)$$

853 (b)

Since, $\frac{xe^{x^2}}{1+x^2}$ is an odd function

$$\therefore \int_{-a}^a \frac{xe^{x^2}}{1+x^2} dx = 0$$

854 (c)

$$\text{Let } I = \int \frac{\sin \theta + \cos \theta}{\sqrt{1 + \sin 2\theta - 1}} d\theta$$

$$= \int \frac{\sin \theta + \cos \theta}{\sqrt{1 - (\sin \theta - \cos \theta)^2}} d\theta$$

$$\text{Put } \sin \theta - \cos \theta = t \Rightarrow (\cos \theta + \sin \theta) d\theta = dt$$

$$\therefore I = \int \frac{1}{\sqrt{1 - t^2}} dt = \sin^{-1} t + c$$

$$= \sin^{-1}(\sin \theta - \cos \theta) + c$$

855 (c)

$$\int \frac{e^x - 1}{e^x + 1} dx = \int \left(\frac{2e^x}{e^x + 1} - 1 \right) dx$$

$$= 2 \log(e^x + 1) - x + c = f(x) + c \quad [\text{given}]$$

$$\therefore f(x) = 2 \log(e^x + 1) - x$$

856 (b)

$$\text{Let } I = \int e^{3 \log x} (x^4 + 1)^{-1} dx$$

$$= \int e^{\log x^3} (x^4 + 1)^{-1} dx$$

$$= \int x^3 (x^4 + 1)^{-1} dx = \frac{1}{4} \int \frac{4x^3}{(x^4 + 1)} dx$$

$$\text{Let } x^4 + 1 = t \Rightarrow 4x^3 dx = dt$$

$$\therefore I = \frac{1}{4} \int \frac{1}{t} dt = \frac{1}{4} \log t + c$$

$$\Rightarrow I = \frac{1}{4} \log(x^4 + 1) + c$$

857 (b)

$$y = \int \frac{dx}{(3 + 5 \sin x + 3 \cos x)}$$

$$= \int \frac{\sec^2(x/2)}{10 \tan(x/2) + 6} dx$$

$$= \frac{1}{5} \log |5 \tan(x/2) + 3| + c$$

$$= \frac{1}{5} \log \left| \frac{5}{3} \tan(x/2) + 1 \right| + c$$

This curve passes through (0, 0).

$$\therefore c = 0$$

$$\text{Then, } y = \frac{1}{5} \log \left| 1 + \frac{5}{3} \tan \left(\frac{x}{2} \right) \right|$$

858 (d)

We have,

$$f'(x) = f(x)$$

$$\Rightarrow \frac{f'(x)}{f(x)} = 1$$

$$\Rightarrow \log f(x) = x + \log C$$

$$\Rightarrow f(x) = C e^x \Rightarrow f(0) = C e^0 \Rightarrow 1 = C$$

$$\therefore f(x) = e^x$$

Now,

$$\int_0^1 f(x) g(x) dx$$

$$= \int_0^1 e^x (x^2 - e^x) dx \quad [\because f(x) + g(x) = x^2]$$

$$\Rightarrow \int_0^1 f(x) g(x) dx = \int_0^1 x^2 e^x dx - \int_0^1 e^{2x} dx$$

$$\Rightarrow \int_0^1 f(x) g(x) dx$$

$$= [(x^2 - 2x + 2)e^x]_0^1 - \frac{1}{2} [e^{2x}]_0^1$$

$$= e - \frac{e^2}{2} - \frac{3}{2}$$

859 (c)

We have,

$$f(x) = \int_0^{x^2} \sqrt{1+t^2} dt,$$

$$\Rightarrow f'(x) = \frac{d}{dx} \left\{ \int_0^{x^2} \sqrt{1+t^2} dt \right\}$$

$$\Rightarrow f'(x) = \int_0^{x^2} 0 dt + 2x \sqrt{1+x^4} - 0$$

$$= 2x \sqrt{1+x^4}$$

860 (a)

For $x > e$, we know that

$$1 < \log x < \frac{x}{e}$$

$$\Rightarrow 1 > \frac{1}{\sqrt[2]{\log x}} > \left(\frac{e}{x} \right)^{1/3}$$

$$\Rightarrow \int_3^4 dx > I > \int_3^4 e^{1/3} x^{-1/3} dx$$

$$\Rightarrow 1 > I > \frac{3}{2} e^{1/3} (4^{2/3} - 3^{2/3})$$

$$\Rightarrow 1 > I > \frac{3}{2} e^{1/3} (16^{1/3} - 9^{1/3})$$

$$\Rightarrow 1 > I > 0.92$$

861 (d)

We have,

$$f(y) = e^y \text{ and } g(y) = y$$

$$\therefore F(t) = \int_0^t f(t-y) g(y) dy$$

$$\Rightarrow F(t) = \int_0^t e^{t-y} \cdot y dy$$

$$\Rightarrow F(t) = [-y e^{t-y} - e^{t-y}]_0^t$$

$$\Rightarrow F(t) = (-t - 1) - (0 - e^t) = e^t - t - 1$$

862 (a)

Putting $x^2 - 1 = t^2$ and $x dx = t dt$, we get

$$I = \int_0^\infty \frac{t^2 - 1}{(t^2 + 1)^2} dt$$

$$\Rightarrow I = \int_0^1 \frac{t^2 - 1}{(t^2 + 1)^2} dt + \int_1^\infty \frac{t^2 - 1}{(t^2 + 1)^2} dt$$

$$\Rightarrow I = I_1 + I_2,$$

$$\text{Where } I_1 = \int_0^1 \frac{t^2 - 1}{(t^2 + 1)^2} dt \text{ and } I_2 = \int_1^\infty \frac{t^2 - 1}{(t^2 + 1)^2} dt$$

Putting $t = \frac{1}{u}$ in I_2 , we get,

$$I_2 = - \int_0^1 \frac{u^2 - 1}{(u^2 + 1)^2} du = -I_1$$

$$\text{Hence, } I = I_1 + I_2 = I_1 - I_1 = 0$$

863 (b)

We have,

$$I = \int (\sin 2x - \cos 2x) dx$$

$$\Rightarrow I = \sqrt{2} \int \left(\frac{1}{\sqrt{2}} \sin 2x - \frac{1}{\sqrt{2}} \cos 2x \right) dx$$

$$\Rightarrow I = \sqrt{2} \int \cos \left(2x + \frac{\pi}{4} \right) + \text{constant}$$

$$I = \frac{1}{\sqrt{2}} \sin \left(\pi + 2x + \frac{\pi}{4} \right) + \text{constant}$$

$$I = \frac{1}{\sqrt{2}} \sin \left(\frac{5\pi}{4} + 2x \right) + \text{constant}$$

$$\text{But, } \int (\sin 2x - \cos 2x) dx = \frac{1}{\sqrt{2}} \sin(2x - a) + b$$

$$\therefore a = -\frac{5\pi}{4} \text{ and } b = \text{any constant}$$

864 (a)

$$\text{Let } I = \int \frac{\sec x}{\sec x + \tan x} dx$$

$$= \int \frac{\sec x (\sec x - \tan x)}{\sec^2 x - \tan^2 x} dx$$

$$= \int (\sec^2 x - \sec x \tan x) dx$$

$$= \tan x - \sec x + c$$

865 (b)

$$\begin{aligned} \int 1 \cos^{-1} x \, dx &= x \cos^{-1} x + \int x \cdot \frac{1}{\sqrt{1-x^2}} \, dx \\ &= x \cos^{-1} x - \frac{1}{2} \int \frac{(-2x)}{\sqrt{1-x^2}} \, dx \\ &= x \cos^{-1} x - \sqrt{1-x^2} + c \end{aligned}$$

866 (a)

We have,

$$\int_0^{36} \frac{1}{2x+9} \, dx = \log k$$

$$\begin{aligned} \Rightarrow \frac{1}{2} [\log(2x+9)]_0^{36} &= \log k \Rightarrow \log\left(\frac{81}{9}\right) = \log k^2 \\ &\Rightarrow k = 3 \end{aligned}$$

867 (d)

$$\text{Let } I = \int 1 \cdot \log 2x \, dx$$

$$\Rightarrow I = x \log 2x - \int \frac{1}{2x} \cdot 2 \cdot x \, dx$$

$$\Rightarrow I = x \log 2x - x + c$$

868 (d)

$$\text{Given, } f''(x) = \tan^2 x = (1 - \sec^2 x)$$

On integrating both sides, we get

$$f'(x) = x - \tan x + c_1$$

$$f'(0) = 0 - 0 + c_1 \Rightarrow c_1 = 0$$

$$\Rightarrow f'(x) = x - \tan x$$

Again integrating both sides, we get

$$\Rightarrow f(x) = \frac{x^2}{2} - \log \sec x + c_2$$

$$\Rightarrow f(0) = 0 - \log 1 + c_2 \Rightarrow c_2 = 0$$

$$\therefore f(x) = \frac{x^2}{2} - \log \sec x$$

869 (c)

$$\text{Let } I = \int \frac{dx}{x(x^5+1)}$$

$$= \int \frac{x^4 dx}{x^5(x^5+1)}$$

$$\text{Put } x^5 = t \text{ and } x^4 dx = \frac{1}{5} dt$$

$$\therefore I = \frac{1}{5} \int \frac{dt}{t(t+1)}$$

$$= \frac{1}{5} \int \left[\frac{1}{t} - \frac{1}{t+1} \right] dt$$

$$= \frac{1}{5} [\log t - \log(t+1)] + c$$

$$= \frac{1}{5} \log \frac{t}{t+1} + c$$

$$= \frac{1}{5} \log \frac{x^5}{x^5+1} + c$$

870 (a)

We have,

$$I = \int \frac{1}{1+\sin x} \, dx = \int \frac{1}{1+\cos\left(\frac{\pi}{2}-x\right)} \, dx$$

$$\Rightarrow I = \frac{1}{2} \int \sec^2 \left(\frac{\pi}{4} - \frac{x}{2} \right) \, dx = -\tan \left(\frac{\pi}{4} - \frac{x}{2} \right) + b$$

$$\Rightarrow I = \tan \left(-\frac{\pi}{4} + \frac{x}{2} \right) + b, \text{ where } b = \text{const}$$

$$\text{Hence, } a = -\frac{\pi}{4} \text{ and } b \in \mathbb{R}$$

871 (a)

$$\text{Let } I = \int \sqrt{e^x-1} \, dx = \int \frac{(\sqrt{e^x-1})e^x}{1+(\sqrt{e^x-1})} \, dx$$

$$\text{Put } e^x - 1 = t^2 \Rightarrow e^x dx = 2t \, dt$$

$$\therefore I = 2 \int \frac{t^2 dt}{1+t^2}$$

$$= 2 \int \left(\frac{1+t^2}{1+t^2} \right) dt - 2 \int \frac{1}{1+t^2} dt$$

$$= 2[t - \tan^{-1} t] + c$$

$$= 2[\sqrt{e^x-1} - \tan^{-1} \sqrt{e^x-1}] + c$$

872 (c)

$$\int \frac{dx}{x^4+x^3} = \int \frac{(x+1)-x}{x^3(x+1)} \, dx$$

$$= \int \left(\frac{1}{x^3} - \frac{1}{x^2(x+1)} \right) dx$$

$$= \int \left(\frac{1}{x^3} - \frac{1}{x^2} + \frac{1}{x(x+1)} \right) dx$$

$$= \int \left(\frac{1}{x^3} - \frac{1}{x^2} + \frac{1}{x} - \frac{1}{x+1} \right) dx$$

$$= -\frac{1}{2x^2} + \frac{1}{x} + \log|x| - \log|x+1| + c$$

$$= -\frac{1}{2x^2} + \frac{1}{x} + \log \left| \frac{x}{x+1} \right| + c$$

$$\therefore A = -\frac{1}{2}$$

$$\text{and } B = 1$$

873 (c)

We have,

$$I = \int_{\alpha}^{\beta} x|x| \, dx$$

$$\Rightarrow I = \int_{\alpha}^0 x|x| \, dx + \int_0^{\beta} x|x| \, dx$$

$$\Rightarrow I = -\int_{\alpha}^0 x^2 \, dx + \int_0^{\beta} x^2 \, dx$$

$$\begin{aligned} \Rightarrow I &= -\left[\frac{x^3}{3} \right]_{\alpha}^0 + \left[\frac{x^3}{3} \right]_0^{\beta} = -\left[0 - \frac{\alpha^3}{3} \right] + \left[\frac{\beta^3}{3} - 0 \right] \\ &= \frac{\alpha^3 + \beta^3}{3} \end{aligned}$$

874 (c)

We have,

$$\begin{aligned} \sin \alpha + \int_{\alpha}^{2\alpha} \cos 2x \, dx &= 0 \\ \Rightarrow \sin \alpha + \frac{1}{2} [\sin 2x]_{\alpha}^{2\alpha} &= 0 \\ \Rightarrow \sin \alpha + \frac{1}{2} (\sin 4\alpha - \sin 2\alpha) &= 0 \\ \Rightarrow \sin \alpha + \sin \alpha \cos 3\alpha &= 0 \\ \Rightarrow \sin \alpha = 0 \text{ or } (1 + \cos 3\alpha) &= 0 \\ \Rightarrow \alpha = 0, \alpha = -\pi, \alpha = -\pi/3 &\Rightarrow \alpha \\ &= -\frac{\pi}{3} \quad [\because \alpha \in (-\pi, 0)] \end{aligned}$$

875 (a)

$$\begin{aligned} I_1 &= \int_0^{\pi/2} x \sin x \, dx \\ &= -[x \cos x]_0^{\pi/2} - \int_0^{\pi/2} (-\cos x) \, dx \\ &= [0 - 0] + [\sin x]_0^{\pi/2} = 1 \quad \dots(i) \\ \text{And } I_2 &= \int_0^{\pi/2} x \cos x \, dx = [x \sin x]_0^{\pi/2} - \\ &\int_0^{\pi/2} \sin x \, dx \\ &= \left[\frac{\pi}{2} - 0 \right] + [\cos x]_0^{\pi/2} \\ &= \frac{\pi}{2} - 1 \quad \dots(ii) \end{aligned}$$

From Eqs. (i) and (ii), we get

$$I_1 + I_2 = \frac{\pi}{2}$$

876 (b)

We have,

$$\begin{aligned} I &= \int_{1/2}^2 |\log_{10} x| \, dx \\ \Rightarrow I &= \int_{1/2}^2 |\log_e x \cdot \log_{10} e| \, dx \\ \Rightarrow I &= |\log_{10} e| \left[\int_{1/2}^1 -\log_e x \, dx + \int_1^2 \log_e x \, dx \right] \\ \Rightarrow I &= \log_{10} e \left[-\{x \log x - x\}_{1/2}^1 + \{x \log x - x\}_1^2 \right] \\ \Rightarrow I &= \log_{10} e \left[1 + \left(-\frac{1}{2} \log 2 - \frac{1}{2} \right) + 2 \log 2 - 2 \right. \\ &\quad \left. + 1 \right] \\ \Rightarrow I &= \log_{10} e \left[-\frac{1}{2} + \frac{3}{2} \log 2 \right] \\ \Rightarrow I &= \frac{1}{2} (\log_{10} e) (\log 8 - 1) \\ \Rightarrow I &= \frac{1}{2} (\log_{10} e) \log_e \left(\frac{8}{e} \right) = \frac{1}{2} \log_{10} \left(\frac{8}{e} \right) \end{aligned}$$

877 (d)

We have,

$$\begin{aligned} I &= \int_0^1 |\sin 2\pi x| \, dx \\ \Rightarrow I &= \int_0^{1/2} \sin 2\pi x \, dx + \int_{1/2}^1 -(\sin 2\pi x) \, dx \\ \Rightarrow I &= -\frac{1}{2\pi} [\cos 2\pi x]_0^{1/2} + \frac{1}{2\pi} [\cos 2\pi x]_{1/2}^1 = \frac{2}{\pi} \end{aligned}$$

878 (c)

$$\begin{aligned} \text{Given, } I &= \int_{-\pi}^{\pi} \frac{e^{\sin x}}{e^{\sin x} + e^{-\sin x}} \, dx \quad \dots(i) \\ \Rightarrow I &= \int_{-\pi}^{\pi} \frac{e^{\sin(2\pi-x)}}{e^{\sin(2\pi-x)} + e^{-\sin(2\pi-x)}} \, dx \\ \Rightarrow I &= \int_{-\pi}^{\pi} \frac{e^{-\sin x}}{e^{-\sin x} + e^{\sin x}} \, dx \quad \dots(ii) \\ \text{On adding Eqs.(i) and (ii), we get} \\ 2I &= \int_{-\pi}^{\pi} \frac{e^{\sin x} + e^{-\sin x}}{e^{\sin x} + e^{-\sin x}} \, dx \\ &= \int_{-\pi}^{\pi} 1 \, dx = [x]_{-\pi}^{\pi} \\ I &= \pi \end{aligned}$$

879 (a)

$$\begin{aligned} \text{Let } I &= \int \frac{1}{1 + \cos x + \sin x} \, dx \\ &= \int \frac{1}{1 + \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} + \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}} \, dx \\ &= \int \frac{\sec^2 \frac{x}{2}}{2 \left(1 + \tan \frac{x}{2} \right)} \, dx \\ \text{Put } \tan \frac{x}{2} &= t \\ \Rightarrow \frac{1}{2} \sec^2 \frac{x}{2} \, dx &= dt \\ \therefore I &= \int \frac{dt}{1+t} = \log(1+t) + c \\ &= \log \left| 1 + \tan \frac{x}{2} \right| + c \end{aligned}$$

880 (d)

$$\begin{aligned} & \int \frac{x^4 - 1}{x^2(x^4 + x^2 + 1)^{1/2}} dx \\ &= \int \frac{2x^4 + x^2 - (x^4 + x^2 + 1)}{x^2\sqrt{x^4 + x^2 + 1}} dx \\ &= \int \frac{\frac{x(4x^3 + 2x)}{2\sqrt{x^4 + x^2 + 1}} - \sqrt{x^4 + x^2 + 1}}{x^2} dx \\ &= \int \frac{d}{dx} \left(\frac{\sqrt{x^4 + x^2 + 1}}{x} \right) = \frac{\sqrt{x^4 + x^2 + 1}}{x} + c \end{aligned}$$

881 (c)

Let $I = \int_0^1 \frac{x^3}{1+x^8} dx$
 Putting $x^4 = t$, $4x^3 dx = dt$, we get

$$I = \frac{1}{4} \int_0^1 \frac{dt}{1+t^2} = \frac{1}{4} [\tan^{-1} t]_0^1 = \frac{\pi}{16}$$

882 (a)

Let $I = \int \frac{\sin x + 8 \cos x}{4 \sin x + 6 \cos x} dx$
 We can write
 $\sin x + 8 \cos x = A(4 \sin x + 6 \cos x) + B \frac{d}{dx} (4 \sin x + 6 \cos x)$
 $\Rightarrow \sin x + 8 \cos x = A(4 \sin x + 6 \cos x) + B(4 \cos x - 6 \sin x)$

On equating the coefficients of $\sin x$ and $\cos x$, we get

$$1 = 4A - 6B, \quad 8 = 6A + 4B$$

$$\Rightarrow A = 1, \quad B = \frac{1}{2}$$

$\therefore I$

$$\begin{aligned} &= \int \frac{(4 \sin x + 6 \cos x) + \frac{1}{2}(4 \cos x - 6 \sin x)}{4 \sin x + 6 \cos x} dx \\ &= \int \left(1 + \frac{1}{2} \cdot \frac{4 \cos x - 6 \sin x}{4 \sin x + 6 \cos x} \right) dx \\ &= x + \frac{1}{2} \log(4 \sin x + 6 \cos x) + c \end{aligned}$$

883 (b)

$$\begin{aligned} I &= \int \frac{x^2}{(ax + b)^2} dx \\ \text{Put } ax + b &= t \Rightarrow dx = \frac{1}{a} dt \\ \text{and } x &= \left(\frac{t-b}{a} \right) \\ \therefore I &= \frac{1}{a^3} \int \frac{(t-b)^2}{t^2} dt = \frac{1}{a^3} \int \left(1 + \frac{b^2}{t^2} - \frac{2b}{t} \right) dt \\ &= \frac{1}{a^3} \left(t - \frac{b^2}{t} - 2b \log t \right) + c \end{aligned}$$

$$\begin{aligned} &= \frac{1}{a^3} \left(ax + b - \frac{b^2}{ax + b} - 2b \log(ax + b) \right) + c \\ &= \frac{2}{a^2} \left(x - \frac{b}{a} \log(ax + b) \right) - \frac{x^2}{a(ax + b)} + c \end{aligned}$$

884 (b)

Let $I = \int_1^4 e^{\sqrt{x}} dx$
 Put $x = t^2 \Rightarrow dx = 2t dt$
 $\therefore I = \int_1^2 2e^t t dt = 2[te^t - e^t]_1^2$
 $= 2[(2e^2 - e) - e^2 + e] = 2e^2$

885 (a)

Let $I = \int \frac{x^4}{x+x^5} dx = \int \frac{x^{4+1-1}}{x+x^5} dx$
 $\Rightarrow I = \int \frac{1}{x} dx - \int \frac{1}{x^5 + x} dx$
 $\Rightarrow I = \log x - [f(x) + c]$ [given]
 $\Rightarrow I = \log x - f(x) + c_1$ [where $c_1 = -c$]

886 (c)

$$\begin{aligned} & \int e^x(1 - \cot x + \cot^2 x) dx \\ &= \int e^x(-\cot x + \operatorname{cosec}^2 x) dx \\ &= -\int e^x \cot x dx + \int e^x \operatorname{cosec}^2 x dx \\ &= -e^x \cot x - \int e^x \operatorname{cosec}^2 x dx \int e^x \operatorname{cosec}^2 x dx \\ &= e^x(-\cot x) + c = -e^x \cot x + c \end{aligned}$$

887 (c)

Given, $f(x) = 4x^2 - 3x + 1$, $g(x) = \frac{f(-x) - f(x)}{x^2 + 3}$
 $\therefore g(x) = \frac{(4x^2 + 3x + 1) - (4x^2 - 3x + 1)}{x^2 + 3}$
 $= \frac{6x}{x^2 + 3}$

Now, $g(-x) = -\frac{6x}{x^2 + 3} = -g(x)$

Which is an odd function

$$\therefore \int_{-2}^2 g(x) dx = 0$$

888 (a)

$$\begin{aligned} & \int \sqrt{1 + \sin\left(\frac{x}{4}\right)} dx \\ &= \int \sqrt{\left(\sin^2 \frac{x}{8} + \cos^2 \frac{x}{8}\right) + \left(2 \sin \frac{x}{8} \cos \frac{x}{8}\right)} dx \\ &= \int \sqrt{\left(\sin \frac{x}{8} + \cos \frac{x}{8}\right)^2} dx = \int \left(\sin \frac{x}{8} + \cos \frac{x}{8}\right) dx \\ &= \frac{-\cos \frac{x}{8}}{\left(\frac{1}{8}\right)} + \frac{\sin \frac{x}{8}}{\left(\frac{1}{8}\right)} + c = \left(8 \sin \frac{x}{8} - \cos \frac{x}{8}\right) + c \end{aligned}$$

889 (a)

If we put $h(x) = t$, then the integral reduces to

$$\int_{h(a)}^{h(b)} [f(g(h(t)))]^{-1} f'(g(t))g'(t)dt$$

$$= 0 \quad [\because h(a) = h(b)]$$

890 (a)

Let $I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin^2 x \cos^2 x} dx \dots$ (i)

$$\Rightarrow I = \int_0^{\pi/2} \frac{\cos^2 x}{\sin^2 x + \cos^2 x} dx \dots$$
 (ii)

On adding Eqs. (i) and(ii), we get

$$2I = \int_0^{\pi/2} 1 dx = [x]_0^{\pi/2} = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

891 (b)

Putting $\frac{\pi x}{2} = t$ and $dx = \frac{2}{\pi} dt$, we get

$$\int_0^1 \log \sin \left(\frac{\pi x}{2}\right) dx = \frac{2}{\pi} \int_0^{\pi/2} \log \sin t dt$$

$$= \frac{2}{\pi} \left(-\frac{\pi}{2} \log 2\right) = -\log 2$$

892 (d)

Given, $\frac{d}{dx} F(x) = \frac{e^{\sin x}}{x}, x > 0$

On integrating both sides, we get

$$F(x) = \int \frac{e^{\sin x}}{x} dx \dots$$
 (i)

Also $\int_1^4 \frac{3}{x} e^{\sin x^3} dx = \int_1^4 \frac{3x^2}{x^3} \cdot e^{\sin x^3} dx = Fk - F(1)$

Let $x^3 = z \Rightarrow 3x^2 dx = dz$

$$\therefore \int_1^{64} \frac{e^{\sin z}}{z} dz = F(k) - F(1)$$

$$\Rightarrow [F(z)]_1^{64} = F(k) - F(1) \quad [\text{from Eq.(i)}]$$

$$\Rightarrow F(64) - F(1) = F(k) - F(1)$$

$$\Rightarrow k = 64$$

893 (b)

$$\int_0^{10} |x(x-1)(x-2)| dx$$

$$= \int_0^1 (x^3 - 3x^2 + 2x) dx$$

$$+ \int_1^2 (-x^3 + 3x^2 - 2x) dx$$

$$= \int_2^{10} (x^3 - 3x^2 + 2x) dx$$

$$= \left[\frac{x^4}{4} - x^3 + x^2 \right]_0^1 + \left[-\frac{x^4}{4} + x^3 - x^2 \right]_1^2$$

$$+ \left[\frac{x^4}{4} - x^3 + x^2 \right]_2^{10}$$

$$= 1600.5$$

894 (d)

Let $I = \int \frac{dx}{\sin x - \cos x + \sqrt{2}}$

$$= \int \frac{dx}{\sin x \frac{\sqrt{2}}{\sqrt{2}} - \cos x \frac{\sqrt{2}}{\sqrt{2}} + \sqrt{2}}$$

$$= \int \frac{dx}{\sqrt{2} \left(\sin x \sin \frac{\pi}{4} - \cos x \cos \frac{\pi}{4} + 1 \right)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{1 - \cos \left(x + \frac{\pi}{4} \right)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{1 - \cos 2 \left(\frac{x}{2} + \frac{\pi}{8} \right)}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{2 \sin^2 \left(\frac{x}{2} + \frac{\pi}{8} \right)}$$

$$= \frac{1}{2\sqrt{2}} \int \operatorname{cosec}^2 \left(\frac{x}{2} + \frac{\pi}{8} \right) dx$$

$$= \frac{1}{2\sqrt{2}} \cdot \frac{-\cot \left(\frac{x}{2} + \frac{\pi}{8} \right)}{\frac{1}{2}} + c$$

$$= -\frac{1}{\sqrt{2}} \cot \left(\frac{x}{2} + \frac{\pi}{8} \right) + c$$

895 (a)

We have,

$$I = \int_{-\pi/4}^{\pi/4} \sin^{-4} x dx$$

$$\Rightarrow I = \int_{-\pi/4}^{\pi/4} \frac{1}{\sin^4 x} dx$$

$$\Rightarrow I = \int_{-\pi/4}^{\pi/4} \operatorname{cosec}^4 x dx$$

$$\Rightarrow I = \int_{-1}^1 (1+t^2) dt, \text{ where } \cot x = t$$

$$\Rightarrow I = -2 \int_0^1 (1+t^2) dt = -2 \left[t + \frac{t^3}{3} \right]_0^1 = -\frac{8}{3}$$

896 (a)

$$I_8 + I_6 = \int_0^{\pi/4} \tan^8 \theta d\theta + \int_0^{\pi/4} \tan^6 \theta d\theta$$

$$= \int_0^{\pi/4} \sec^2 \theta \tan^6 \theta d\theta$$

$$= \left[\frac{\tan^7 \theta}{7} \right]_0^{\pi/4} = \frac{1}{7}$$

897 (b)

We have,

$$\int \frac{a^{\sqrt{x}}}{\sqrt{x}} dx = 2 \int \frac{a^{\sqrt{x}}}{2\sqrt{x}} dx = 2 \int a^{\sqrt{x}} d(\sqrt{x})$$

$$= \frac{2a^{\sqrt{x}}}{\log a} + C$$

898 (a)

$$\text{Let } I = \int_a^b \sqrt{(x-a)(b-x)} dx$$

$$\text{Put } x = a \cos^2 \theta + b \sin^2 \theta$$

$$\Rightarrow dx = -2a \cos \theta \sin \theta + 2b \sin \theta \cos \theta$$

$$\Rightarrow dx = \sin 2\theta (b-a)$$

$$\therefore I = \int_0^{\pi/2} \sqrt{(b-a) \sin^2 \theta (b-a) \cos^2 \theta} (b-a) \sin 2\theta d\theta$$

$$= \int_0^{\pi/2} \frac{(b-a)^2}{2} \sin^2 2\theta d\theta$$

$$= \frac{(b-a)^2}{2} \int_0^{\pi/2} \left[\frac{1 - \cos 4\theta}{2} \right] d\theta$$

$$= \frac{(b-a)^2}{4} \left[\theta - \frac{\sin 4\theta}{4} \right]_0^{\pi/2}$$

$$= \frac{(b-a)^2}{4} \left[\frac{\pi}{2} - 0 \right]$$

$$= \frac{\pi(b-a)^2}{8}$$

899 (b)

$$\int_0^{x^2} f(t) dt = x \cos \pi x$$

On differentiating both sides, we get

$$2x f(x^2) = \frac{-x \sin \pi x}{\pi} + \cos \pi x$$

$$\Rightarrow f(x^2) = -\frac{x \sin \pi x}{2\pi x} + \frac{\cos \pi x}{2x}$$

$$\therefore f(4) = f(2^2) = \frac{1}{4}$$

900 (c)

$$f'(x) = \frac{dx}{(1+x^2)^{3/2}}$$

On integrating both sides,

$$f(x) = \int \frac{dx}{(1+x^2)^{3/2}} + c$$

$$\text{Put } x = \tan \theta \Rightarrow dx = \sec^2 \theta d\theta$$

$$\therefore f(x) = \int \frac{\sec^2 \theta}{\sec^3 \theta} d\theta + c = \int \cos \theta d\theta + c$$

$$\Rightarrow f(x) = \sin \theta + c$$

$$\Rightarrow f(x) = \frac{x}{\sqrt{1+x^2}} + c$$

$$\Rightarrow f(0) = 0 + c \Rightarrow c = 0$$

$$\therefore f(x) = \frac{x}{\sqrt{1+x^2}} \Rightarrow f(1) = \frac{1}{\sqrt{2}}$$

901 (c)

We have,

$$I = \int_0^{\pi/2} \frac{x + \sin x}{1 + \cos x} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{1}{2} x \sec^2 \frac{x}{2} + \int_0^{\pi/2} \tan \frac{x}{2} dx$$

$$\Rightarrow I = \left[x \tan \frac{x}{2} \right]_0^{\pi/2} - \int_0^{\pi/2} \tan \frac{x}{2} dx + \int_0^{\pi/2} \tan \frac{x}{2} dx$$

$$= \pi/2$$

902 (a)

We have,

$$\int f(x) \sin x \cos x dx = \frac{1}{2(b^2 - a^2)} \log\{f(x)\} + C$$

$$\Rightarrow f(x) \sin x \cos x = \frac{1}{2(b^2 - a^2)} \cdot \frac{1}{f(x)} f'(x)$$

$$\Rightarrow 2(b^2 - a^2) \sin x \cos x = \frac{f'(x)}{\{f(x)\}^2}$$

$$\Rightarrow \int (2b^2 \sin x \cos x - 2a^2 \sin x \cos x) dx$$

$$= \int \frac{f'(x)}{\{f(x)\}^2} dx$$

$$\Rightarrow -b^2 \cos^2 x - a^2 \sin^2 x = -\frac{1}{f(x)}$$

$$\Rightarrow f(x) = \frac{1}{(a^2 \sin^2 x + b^2 \cos^2 x)}$$

903 (c)

$$\int \sqrt{\frac{1+x}{1-x}} dx = \int \frac{1+x}{\sqrt{1-x^2}} dx$$

$$= \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x}{\sqrt{1-x^2}} dx$$

$$= \sin^{-1} x - \sqrt{1-x^2} + c$$

904 (b)

$$\text{Let } I = \int_1^2 \frac{dx}{x(1+x^4)} = \int_1^2 \frac{x^3}{x^4(1+x^4)} dx$$

$$\text{Put } x^4 = t \Rightarrow 4x^3 dx = dt$$

$$\therefore I = \frac{1}{4} \int_1^{16} \frac{dt}{t(1+t)} = \frac{1}{4} \int_1^{16} \left(\frac{1}{t} - \frac{1}{1+t} \right) dt$$

$$= \frac{1}{4} \left(\log \frac{t}{1+t} \right)_1^{16} = \frac{1}{4} \left(\log \frac{16}{17} - \log \frac{1}{2} \right)$$

$$= \frac{1}{4} \log \frac{32}{17}$$

905 (c)

We have,

$$I = \int \frac{1}{(x+1)^2 \sqrt{x^2+2x+2}} dx$$

$$= \int \frac{1}{(x+1)^2 \sqrt{(x+1)^2+1}} dx$$

$$\Rightarrow I = \int \frac{\sec^2 \theta d\theta}{\tan^2 \theta \sec \theta}, \text{ where } x+1 = \tan \theta$$

$$\Rightarrow I = \int \frac{\cos \theta}{\sin^2 \theta} d\theta = \int (\sin \theta)^{-2} d(\sin \theta)$$

$$\Rightarrow I = -\frac{1}{\sin \theta} + C = -\frac{\sqrt{x^2+2x+2}}{x+1} + C$$

906 (c)

We have,

$$I = \int_{-1}^1 \left\{ [x^2] + \log \left(\frac{2+x}{2-x} \right) \right\} dx$$

$$= \int_{-1}^1 [x^2] dx + \int_{-1}^1 \log \left(\frac{2+x}{2-x} \right) dx$$

$\Rightarrow I$

$$= \int_{-1}^1 0 dx$$

$$+ 0 \quad \left[\because \log \left(\frac{2+x}{2-x} \right) \text{ is an odd function} \right]$$

$\Rightarrow I = 0$

907 (b)

$$\int e^x \left(\frac{1-\sin x}{1-\cos x} \right) dx = \int e^x \left(\frac{1-\sin x}{2 \sin^2 \frac{x}{2}} \right) dx$$

$$= \int e^x \left(\frac{1}{2} \operatorname{cosec}^2 \frac{x}{2} - \cot \frac{x}{2} \right) dx$$

$$\frac{1}{2} \int e^x \operatorname{cosec}^2 \frac{x}{2} dx - \int e^x \cot \frac{x}{2} dx$$

$$= \frac{1}{2} \int e^x \operatorname{cosec}^2 \frac{x}{2} dx - e^x \cot \frac{x}{2} \int e^x \operatorname{cosec}^2 \frac{x}{2} dx$$

$$= e^x \left(-\cot \frac{x}{2} \right) + c$$

$$= -e^x \cot \frac{x}{2} + c$$

908 (d)

$$\text{Let } I = \int \frac{x^2-1}{(x^4+3x^2+1) \tan^{-1} \left(x+\frac{1}{x} \right)} dx$$

$$= \int \frac{\left(1-\frac{1}{x^2} \right)}{\left(x^2+\frac{1}{x^2}+3 \right) \tan^{-1} \left(x+\frac{1}{x} \right)} dx$$

$$\text{Put } x+\frac{1}{x} = t$$

$$\Rightarrow \left(1-\frac{1}{x^2} \right) dx = dt \text{ and } \left(x+\frac{1}{x} \right)^2 = t^2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = t^2 - 2$$

$$\therefore I = \int \frac{1}{(t^2+1) \tan^{-1} t} dt$$

$$= \log(\tan^{-1} t) + c$$

$$= \log \left[\tan^{-1} \left(x+\frac{1}{x} \right) \right] + c$$

909 (b)

We have,

$$\int_0^1 x(1-x)^{-3/4} dx$$

$$= \left[-4x + (1-x)^{1/4} \right]_0^1$$

$$+ 4 \int_0^1 (1-x)^{1/4} = \frac{16}{5}$$

910 (b)

$f'(x) = 1 - \cos x$ always positive, so $f(x)$ is an increasing function so least value of $f(x)$ on

$$\left[\frac{\pi}{2}, \frac{3\pi}{2} \right] \text{ is } f\left(\frac{\pi}{2}\right)$$

$$\therefore f\left(\frac{\pi}{2}\right) = \int_0^{\pi/2} (1 - \cos t) dt$$

$$= [t - \sin t]_0^{\pi/2} = \left(\frac{\pi}{2} - 1\right)$$

911 (d)

$$\int \frac{x^2-2}{x^3-\sqrt{x^2-1}} dx$$

$$= \int \frac{dx}{x\sqrt{x^2-1}} - 2 \int \frac{dx}{x^3\sqrt{x^2-1}}$$

$$= \sec^{-1} x - 2 \int \frac{\sec \theta \tan \theta}{\sec^3 \theta \tan \theta} d\theta$$

[Putting $x = \sec \theta \Rightarrow dx = \sec \theta \tan \theta d\theta$]

$$= \sec^{-1} x - 2 \int \cos^2 \theta d\theta$$

$$= \sec^{-1} x - \int (1 + \cos 2\theta) d\theta$$

$$= \sec^{-1} x - \left(\theta + \frac{\sin 2\theta}{2} \right) + c$$

$$= \sec^{-1} x - \sec^{-1} x - \frac{\sqrt{x^2-1}}{x^2} + c$$

$$= -\frac{\sqrt{x^2-1}}{x^2} + c$$

912 (b)

We have,

$$I = \int \frac{(x-x^3)^{1/3}}{x^4} dx = \int \frac{\left(\frac{1}{x^2}-1\right)^{1/3}}{x^3} dx$$

$$\Rightarrow -\frac{1}{2} \int \left(\frac{1}{x^2}-1\right)^{1/3} d\left(\frac{1}{x^2}-1\right)$$

$$= -\frac{3}{8} \left(\frac{1}{x^2}-1\right)^{4/3} + c$$

913 (a)

$$\text{Let } I = \int_0^\infty \log\left(x + \frac{1}{x}\right) \frac{1}{1+x^2} dx$$

$$\text{Put } x = \tan \theta \Rightarrow dx = \sec^2 \theta d\theta$$

$$\therefore I = \int_0^{\pi/2} \log\left(\tan \theta + \frac{1}{\tan \theta}\right) \frac{\sec^2 \theta}{\sec^2 \theta} d\theta$$

$$\Rightarrow I = \int_0^{\pi/2} \log \frac{(1 + \tan^2 \theta)}{\tan \theta} d\theta$$

$$\Rightarrow I = 2 \int_0^{\pi/2} \log \sec \theta d\theta - \int_0^{\pi/2} \log \tan \theta d\theta$$

$$\Rightarrow I = 2 \int_0^{\pi/2} \log \sec \theta d\theta \left(\because \int_0^{\pi/2} \log \tan \theta d\theta = 0 \right)$$

$$\begin{aligned} \Rightarrow I &= -2 \int_0^{\pi/2} \log \cos \theta d\theta \\ &= -2 \\ &\times \frac{-\pi}{2} \log 2 \left(\because \int_0^{\pi/2} \log \cos \theta d\theta = -\frac{\pi}{2} \log 2 \right) \end{aligned}$$

$$\therefore I = \pi \log 2$$

914 (a)

$$\text{Given, } \int \left[\log(\log x) + \frac{1}{(\log x)^2} \right] dx =$$

$$xfx - gx + c$$

$$\text{LHS} = \int \left[\frac{1}{x} \cdot \log(\log x) dx + \int \frac{1}{(\log x)^2} dx \right]$$

$$= x \log(\log x) - \int \frac{1}{\log x} dx + \int \frac{1}{(\log x)^2} dx$$

$$= x \log(\log x) - \frac{x}{\log x} - \int \frac{1}{(\log x)^2} dx + \int \frac{1}{(\log x)^2} dx + c$$

$$= x \left[\log(\log x) - \frac{1}{\log x} \right] + c$$

$$\therefore f(x) = \log(\log x), g(x) = \frac{1}{\log x}$$

915 (c)

Required integral I is given by

$$I = \int \frac{1}{\sqrt{x^2 + 9}} d(x^2 + 1)$$

$$\Rightarrow I = \int \frac{1}{\sqrt{x^2 + 9}} d(x^2 + 9) \quad [\because d(x^2 + 1) = d(x^2 + 9)]$$

$$\Rightarrow I = 2\sqrt{x^2 + 9} + C$$

916 (b)

We have,

$$I = \int \frac{\sqrt{x}}{1 + x^{3/4}} dx = \int \frac{t^2 \times 4t^3}{1 + t^3} dt, \text{ where } x = t^4$$

$$\Rightarrow I = \frac{4}{3} \int \frac{t^3}{1 + t^3} \times 3t^2 dt = \frac{4}{3} \int \frac{u - 1}{u} du, \text{ where } u = 1 + t^3$$

$$\Rightarrow I = \frac{4}{3} |u - \log u| + C$$

$$= \frac{4}{3} [1 + x^{3/4} - \log(1 + x^{3/4})] + C$$

917 (b)

We have,

$$\int e^{x \log a} e^x dx = \int e^{\log a^x} \cdot e^x dx = \int a^x e^x dx$$

$$\Rightarrow \int e^{x \log a} e^x dx = \int (ae)^x dx = \frac{(ae)^x}{\log(ae)} + C$$

918 (b)

Given, $\int f(x) dx = g(x)$

$$\therefore \int f(x)g(x) dx$$

$$= g(x) \int f(x) dx$$

$$- \int [g'(x) \int f(x) dx] dx$$

$$= g(x)g(x) - \int g'(x)g(x)dx - [g(x)]^2$$

$$- \int f(x)g(x)dx$$

$$\Rightarrow \int f(x)g(x)dx = \frac{1}{2} [g(x)]^2$$

919 (b)

$$\text{Given, } \int \frac{\cos 4x + 1}{\cos x - \tan x} dx = A \cos 4x + B$$

$$\text{Let } I = \int \frac{\cos 4x + 1}{\cos x - \tan x} dx$$

$$= \int \frac{2 \cos^2 2x}{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}} dx = \int \frac{2 \cos^2 x}{\sin x \cos x} dx$$

$$= \int \sin 2x \cos 2x dx$$

$$= \frac{1}{2} \int 2 \sin 2x \cos 2x dx$$

$$= \frac{1}{2} \int \sin 4x dx = \frac{1}{2} \left[-\frac{\cos 4x}{4} \right] + B$$

$$= -\frac{1}{8} \cos 4x + B$$

$$\text{But } I = A \cos 4x + B \quad (\text{given})$$

$$\therefore A = -\frac{1}{8}$$

920 (d)

$$\text{Let } I = \int_0^\pi (\pi x - x^2)^{100} \sin 2x dx. \text{ Then,}$$

$$\Rightarrow I = \int_0^\pi \{ \pi(\pi - x) - (\pi - x)^2 \} \sin 2(\pi - x) dx$$

$$\Rightarrow I = - \int_0^\pi (\pi x - x^2) \sin 2x dx$$

$$\Rightarrow I = -I \Rightarrow 2I = 0 \Rightarrow I = 0$$

921 (a)

We have,

$$I = \int_{\pi}^{2\pi} [2 \sin x] dx$$

$$\Rightarrow I = \int_{\pi}^{2\pi/6} [2 \sin x] dx + \int_{7\pi/6}^{11\pi/6} [2 \sin x] dx$$

$$+ \int_{11\pi/6}^{2\pi} [2 \sin x] dx$$

$$\Rightarrow I = \int_{\pi}^{7\pi/6} -1 dx + \int_{7\pi/6}^{11\pi/6} -2 dx + \int_{11\pi/6}^{2\pi} -1 dx$$

$$= -\frac{5\pi}{3}$$

922 (a)

We have,

$$I = \int \frac{1}{\sqrt{\sin^3 x \cos x}} dx = \int \frac{\sec^2 x}{\sqrt{\tan^3 x}} dx$$

$$\Rightarrow I = \int (\tan x)^{-3/2} d(\tan x) = \frac{-2}{\sqrt{\tan x}} + C$$

923 (c)

$$\text{Let } \int_0^{1.5} [x^2] dx = \int_0^1 [x^2] dx + \int_1^{\sqrt{2}} [x^2] dx + 21.5x2dx$$

$$= \int_0^1 0 dx + \int_1^{\sqrt{2}} 1 dx + \int_{\sqrt{2}}^{1.5} 2 dx = 2 - \sqrt{2}$$

924 (d)

$$\int_0^{\pi} |\cos x| dx = \int_0^{\pi/2} \cos x dx - \int_{\pi/2}^{\pi} \cos x dx$$

$$= [\sin x]_0^{\pi/2} - [\sin x]_{\pi/2}^{\pi}$$

$$= \sin \frac{\pi}{2} - \sin 0 - \sin \pi + \sin \frac{\pi}{2}$$

$$= 2 \sin \frac{\pi}{2} = 2$$

925 (a)

We have,

$$\int \frac{x + \sqrt[3]{x^2} + \sqrt[6]{x}}{x(1 + \sqrt[3]{x})} dx$$

$$\Rightarrow I = \int \frac{(t^6 + t^4 + t)}{t^6(1 + t^2)} 6t^2 dt, \text{ where } x = t^6$$

$$\Rightarrow I = 6 \int \frac{t^5 + t^3 + 1}{t^2 + 1} dt = 6 \int \frac{t^3(t^2 + 1) + 1}{t^2 + 1} dt$$

$$\Rightarrow I = 6 \int \left(t^3 + \frac{1}{t^2 + 1} \right) dt$$

$$\Rightarrow I = 6 \left\{ \frac{t^4}{4} + \tan^{-1} t \right\} + C$$

$$= \frac{3}{2} x^{2/3} + 6 \tan^{-1} x^{1/6} + C$$

926 (c)

$$\text{Given, } \int_0^{\pi} e^{\cos^2 x} \cdot \cos^3(2n+1)x dx$$

$$\text{Let } f(x) = e^{\cos^2 x} \cdot \cos^3(2n+1)x$$

$$\text{Then, } f(\pi - x) = e^{\cos^2(\pi-x)} \cdot \cos^3[(2n+1)\pi - 2n+1x]$$

$$= -e^{\cos^2 x} \cdot \cos^3(2n+1)x$$

$$\Rightarrow f(\pi - x) = -f(x)$$

Then, $f(x)$ is an odd function

$$\therefore \int_0^{\pi} e^{\cos^2 x} \cdot \cos^3(2n+1)x dx = 0$$

927 (b)

$$\text{Let } I = \int_0^{\pi/2} \frac{\cos 3x + 1}{2 \cos x - 1} dx$$

$$= \int_0^{\pi/2} \frac{\cos 3x - \cos \frac{3\pi}{3}}{2(\cos x - \cos \frac{\pi}{3})} dx$$

$$= \int_0^{\pi/2} \frac{(4 \cos^3 x - 3 \cos x) - (4 \cos^3 \frac{\pi}{3} - 3 \cos \frac{\pi}{3})}{2(\cos x - \cos \frac{\pi}{3})} dx$$

$$= 2 \int_0^{\pi/2} \left(\frac{\cos^3 x - \cos^3 \frac{\pi}{3}}{\cos x - \cos \frac{\pi}{3}} \right) dx - \frac{3}{2} \int_0^{\pi/2} \left(\frac{\cos x - \cos \frac{\pi}{3}}{\cos x - \cos \frac{\pi}{3}} \right) dx$$

$$= 2 \int_0^{\pi/2} \left(\cos^2 x + \cos^2 \frac{\pi}{3} + \cos x \cos \frac{\pi}{3} \right) dx - \frac{3}{2} \int_0^{\pi/2} 1 dx$$

$$= \int_0^{\pi/2} \left(1 + \cos 2x + \frac{1}{2} + \cos x \right) dx - \frac{3\pi}{4}$$

$$= \frac{3\pi}{4} + 1 - \frac{3\pi}{4} = 1$$

928 (c)

$$\int_1^3 \frac{\cos(\log x)}{x} dx = [\sin(\log x)]_1^3$$

$$= \sin(\log 3) - \sin(\log 1) = \sin(\log 3)$$

929 (d)

$$\text{Let } I = \int (e^x + e^{-x})^2 \cdot (e^x - e^{-x}) dx$$

$$\text{Put } e^x + e^{-x} = t \Rightarrow (e^x - e^{-x}) dx = dt$$

$$\therefore I = \int t^2 dt = \frac{t^3}{3} + c$$

$$= \frac{(e^x + e^{-x})^3}{3} + c$$

930 (b)

$$\text{Let } I = \int \frac{dx}{\sin x \cos x} = 2 \int \frac{dx}{\sin 2x}$$

$$= 2 \int \operatorname{cosec} 2x dx$$

$$= 2 \log \left| \tan \left(\frac{2x}{2} \right) \right| \times \frac{1}{2} + c$$

$$= \log |\tan x| + c$$

931 (c)

Using Walli's formula, we get

$$\begin{aligned} & \int_0^{\pi/2} \sin^n x \, dx \\ &= \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \frac{n-5}{n-4} \cdots \frac{2}{3} \text{ if } n \text{ is odd} \\ \Rightarrow \int_0^{\pi/2} \sin^5 x \, dx &= \frac{5-1}{5} \cdot \frac{5-3}{5-2} \\ &= \frac{4}{5} \cdot \frac{2}{3} \\ &= \frac{8}{15} \end{aligned}$$

932 (b)

Let

$$\begin{aligned} I &= \int_{-\pi/2}^{\pi/2} \log_e \left\{ \left(\frac{ax^2 + bx + c}{ax^2 - bx + c} \right) (a+b) |\sin x| \right\} dx \\ \Rightarrow I &= \int_{-\pi/2}^{\pi/2} \log_e \left(\frac{ax^2 + bx + c}{ax^2 - bx + c} \right) dx + \\ & \int_{-\pi/2}^{\pi/2} \log_e (a+b) dx + \int_{-\pi/2}^{\pi/2} \log_e |\sin x| dx \\ \Rightarrow I &= 0 + \pi \log_e (a+b) + 2 \int_0^{\pi/2} \log_e \sin x \, dx \\ \Rightarrow I &= \pi \log_e (a+b) + 2 \times -\frac{\pi}{2} \log 2 \\ &= \pi \log_e \left(\frac{a+b}{2} \right) \end{aligned}$$

933 (c)

Let $I = \int_{1/e}^e \frac{|\log x|}{x^2} dx$. Then,

$$\begin{aligned} I &= - \int_{1/e}^e |\log t| dt, \text{ where } x = \frac{1}{t} \\ \Rightarrow I &= \int_{1/e}^e |\log t| dt \\ \Rightarrow I &= \left\{ \int_{1/e}^1 -\log t \, dt + \int_1^e \log t \, dt \right\} \\ \Rightarrow I &= \left\{ -[(t \log t - t)]_{1/e}^1 + [t \log t - t]_1^e \right\} \\ \Rightarrow I &= \left(1 - \frac{2}{e} + 1 \right) = 2 \left(1 - \frac{1}{e} \right) \end{aligned}$$

934 (c)

Let $f(x) = x|x|$

$f(-x) = -x|x|$

$= -f(x)$

$\therefore f(x)$ is an odd function

Hence, $I = 0$

935 (c)

We have,

$$\begin{aligned} I &= \int \frac{1}{x^3 + x^4} dx = \int \frac{1}{x^3(x+1)} dx \\ &= \int \frac{1}{x^2\{x(x+1)\}} dx \\ \Rightarrow I &= \int \frac{1}{x^2} \left(\frac{1}{x} - \frac{1}{x+1} \right) dx \\ &= \int \frac{1}{x} \left\{ \frac{1}{x^2} - \frac{1}{x(x+1)} \right\} dx \\ \Rightarrow I &= \int \frac{1}{x} \left\{ \frac{1}{x^2} - \left(\frac{1}{x} - \frac{1}{x+1} \right) \right\} dx \\ &= \int \frac{1}{x^3} - \frac{1}{x^2} + \frac{1}{x(x+1)} dx \\ \Rightarrow I &= \int \left(\frac{1}{x^3} - \frac{1}{x^2} + \frac{1}{x} - \frac{1}{x+1} \right) dx \\ &= -\frac{1}{2x^2} + \frac{1}{x} + \log \left| \frac{x}{x+1} \right| + C \\ \therefore A &= -\frac{1}{2} \text{ and } B = 1 \end{aligned}$$

936 (a)

We have,

$$\begin{aligned} I &= \int \frac{1}{(x^2+1)(x^2+4)} dx \\ &= \frac{1}{3} \left(\frac{1}{x^2+1} - \frac{1}{x^2+4} \right) dx \\ \Rightarrow I &= \frac{1}{3} \tan^{-1} x - \frac{1}{6} \tan^{-1} \frac{x}{2} + C \\ \text{Hence, } A &= \frac{1}{3} \text{ and } B = -\frac{1}{6} \end{aligned}$$

937 (c)

We have,

$$\begin{aligned} I &= \int_{-1/2}^{1/2} \left\{ \left(\frac{x+1}{x-1} \right)^2 + \left(\frac{x-1}{x+1} \right)^2 - 2 \right\}^{1/2} dx \\ \Rightarrow I &= \int_{-1/2}^{1/2} \left\{ \left(\frac{x+1}{x-1} - \frac{x-1}{x+1} \right)^2 \right\}^{1/2} dx \\ \Rightarrow I &= \int_{-1/2}^{1/2} \left| \frac{4x}{x^2-1} \right| dx \\ \Rightarrow I &= \int_{-1/2}^0 \left| \frac{4x}{1-x^2} \right| dx + \int_0^{1/2} \left| \frac{4x}{1-x^2} \right| dx \\ \Rightarrow I &= -4 \int_{-1/2}^0 \frac{x}{1-x^2} dx + 4 \int_0^{1/2} \frac{x}{1-x^2} dx \\ \Rightarrow I &= 2 [\log(1-x^2)]_{-1/2}^0 - 2 [\log(1-x^2)]_0^{1/2} \\ \Rightarrow I &= -2 \log \left(1 - \frac{1}{4} \right) - 2 \log \left(1 - \frac{1}{4} \right) \end{aligned}$$

$$\Rightarrow I = -4 \log \frac{3}{4} = 4 \log \frac{4}{3}$$

938 (b)

$$\begin{aligned} \int \operatorname{cosec} x \, dx &= \log |\operatorname{cosec} x - \cot x| + c \\ &= \log \left| \frac{(1 - \cos x)}{\sin x} \right| + c \\ &= \log \left| \frac{2 \sin^2 \frac{x}{2}}{2 \sin \frac{x}{2} \cdot \cos \frac{x}{2}} \right| + c \\ &= \log \left| \tan \frac{x}{2} \right| + c = f(x) + \text{constant} \\ \therefore f(x) &= \log \left| \tan \frac{x}{2} \right| \end{aligned}$$

939 (a)

$$\begin{aligned} \text{Let } I &= \int \frac{dx}{x(x^n+1)} \\ \text{Put } x^n + 1 &= t \Rightarrow nx^{n-1} dx = dt \\ \therefore I &= \frac{1}{n} \int \frac{dt}{t(t-1)} = \frac{1}{n} \int \left(\frac{1}{t-1} - \frac{1}{t} \right) dt \\ &= \frac{1}{n} [\log(t-1) - \log t] + c \\ &= \frac{1}{n} \log \left(\frac{t-1}{t} \right) + c = \frac{1}{n} \log \left(\frac{x^n}{x^n+1} \right) + c \end{aligned}$$

940 (c)

$$\begin{aligned} \int \frac{ax^3 + bx^2 + c}{x^4} \, dx &= \int \left[\frac{a}{x} + \frac{b}{x^2} + \frac{c}{x^4} \right] dx \\ &= a \int \frac{1}{x} dx + b \int x^{-2} dx + c \int x^{-4} dx \\ &= a \log x - bx^{-1} - \frac{c}{3} x^{-3} + k \\ &= a \log x - \frac{b}{x} - \frac{c}{3x^3} + k \end{aligned}$$

941 (b)

$$\begin{aligned} \text{Let } I &= \int \frac{x+2}{(x^2+3x+3)\sqrt{x+1}} dx \\ \text{Putting } x+1 &= t^2 \text{ and } dx = 2t \, dt, \text{ we get} \\ I &= 2 \int \frac{t^2+1}{t^4+t^2+1} dt \\ \Rightarrow I &= 2 \int \frac{1+(1/t)^2}{(t-\frac{1}{t})^2+3} dt \\ &= 2 \int \frac{1}{(t-\frac{1}{t})^2+(\sqrt{3})^2} d\left(t-\frac{1}{t}\right) \\ \Rightarrow t &= \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{t-\frac{1}{t}}{\sqrt{3}} \right) + C \\ &= \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{x}{\sqrt{3(x+1)}} \right) + C \end{aligned}$$

942 (d)

$$\text{Let } I = \int_0^{\pi/2} \frac{dx}{1+\cot x} = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x} dx \dots (i)$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2}-x\right)}{\cos\left(\frac{\pi}{2}-x\right) + \sin\left(\frac{\pi}{2}-x\right)} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\sin x}{\cos x + \sin x} dx \dots (ii)$$

On adding Eqs.(i) and (ii), we get

$$2I = \int_0^{\pi/2} 1 \, dx = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

943 (d)

$$\text{Let } I = \int \frac{2 \sin x \cos x}{1+\cos^2 x} dx$$

$$\text{Put } 1 + \cos^2 x = t \Rightarrow -2 \cos x \sin x \, dx = dt$$

$$\begin{aligned} \therefore I &= - \int \frac{dt}{t} = -\log t + c \\ &= -\log(1 + \cos^2 x) + c \end{aligned}$$

944 (c)

We have,

$$I = \int_0^{\pi} e^{\sin^2 x} \cos^3(2n+1)x \, dx,$$

$$\Rightarrow I = \int_0^{\pi} e^{\sin^2(\pi-x)} \cos^3\{(2n+1)(\pi-x)\} dx$$

$$\Rightarrow I = \int_0^{\pi} e^{\sin^2 x} \cos^3\{(2n+1)\pi - (2n+1)x\} dx$$

$$\Rightarrow I = - \int_0^{\pi} e^{\sin^2 x} \cos^3(2n+1)x \, dx = -I$$

$$\Rightarrow 2I = 0 \Rightarrow I = 0$$

945 (c)

$$\text{On putting } \log(x + \sqrt{1+x^2}) = t$$

$$\Rightarrow \frac{1}{x + \sqrt{1+x^2}} \times \left(1 + \frac{x}{\sqrt{1+x^2}} \right) dx = dt$$

$$\Rightarrow \frac{dx}{\sqrt{1+x^2}} = dt$$

$$\therefore \int \frac{\log(x + \sqrt{1+x^2})}{\sqrt{1+x^2}} dx = \int t \, dt = \frac{1}{2} t^2 + c$$

$$= \frac{1}{2} \log(x + \sqrt{x^2+1})^2 + c$$

$$\text{Thus, } f(x) = \log(x + \sqrt{x^2+1})$$

$$\text{and } g(x) = \frac{x^2}{2}$$

946 (d)

$$\text{Let } I = \int \frac{1+\tan^2 x}{1-\tan^2 x} dx = \int \frac{\sec^2 x}{1-\tan^2 x} dx$$

$$\text{Put } \tan x = t \Rightarrow \sec^2 x \, dx = dt$$

$$\therefore I = \int \frac{dt}{1-t^2} = \frac{1}{2 \times 1} \log \left(\frac{1+t}{1-t} \right) + c$$

$$= \frac{1}{2} \log \left(\frac{1 + \tan x}{1 - \tan x} \right) + c$$

947 (a)

$$(x+1)(x+3) = (x+2-1)(x+2+1) \\ = (x+2)^2 - 1$$

$$\therefore \int (x+1)(x+2)^7(x+3) dx$$

$$= \int \{(x+2)^9 - (x+2)^7\} dx$$

$$= \frac{(x+2)^{10}}{10} - \frac{(x+2)^8}{8} + c$$

948 (b)

$$\text{Let } I = \int_1^2 \frac{dx}{(x+1)\sqrt{x^2-1}}$$

$$\text{Put } x = \sec \theta \Rightarrow dx = \sec \theta \tan \theta d\theta \quad \dots (i)$$

$$\therefore I = \int_0^{\pi/3} \frac{\sec \theta \tan \theta}{(\sec \theta + 1) \tan \theta} d\theta$$

$$= \int_0^{\pi/3} \frac{1}{1 + \cos \theta} d\theta = \int_0^{\pi/3} \frac{\sec^2 \frac{\theta}{2}}{2} d\theta$$

$$= \frac{1}{2} \left[\frac{\tan \frac{\theta}{2}}{\frac{1}{2}} \right]_0^{\pi/3} = \frac{1}{\sqrt{3}}$$

949 (a)

$$\text{Let } I = \int_0^\pi \cos^3 x dx \quad \dots (i)$$

$$\Rightarrow I = \int_0^\pi \cos^3(\pi - x) dx = - \int_0^\pi \cos^3 x dx$$

... (ii)

On adding Eqs. (i) and (ii), we get

$$2I = 0 \Rightarrow I = 0$$

950 (d)

$$\int_0^a \sqrt{a^2 - x^2} dx =$$

$$\left[\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) \right]_0^a$$

$$= \left[0 + \frac{a^2}{2} \sin^{-1}(1) - 0 - \frac{a^2}{2} \sin^{-1}(0) \right]$$

$$= \frac{a^2}{2} \cdot \frac{\pi}{2} - 0 = \frac{a^2 \pi}{4}$$

951 (b)

We have,

$$I = \int \frac{x-1}{(x+1)\sqrt{x^3+x^2+x}} dx$$

$$\Rightarrow I = \int \frac{x^2-1}{(x+1)^2 \sqrt{x^3+x^2+x}} dx$$

$$\Rightarrow I = \int \frac{1 - \frac{1}{x^2}}{(x + \frac{1}{x} + 2) \sqrt{x + \frac{1}{x} + 1}} dx$$

$$\Rightarrow I = \int \frac{1}{(x + \frac{1}{x} + 2) \sqrt{x + \frac{1}{x} + 1}} d \left(x + \frac{1}{x} + 1 \right)$$

$$\Rightarrow I = 2 \int \frac{1}{t^2 + 1} dt, \text{ where } x + \frac{1}{x} + 1 = t^2$$

$$\Rightarrow I = 2 \tan^{-1} \left(x + \frac{1}{x} + 1 \right) + C$$

$$= 2 \tan^{-1} \left(\frac{x^2 + x + 1}{x} \right) + C$$

952 (d)

$$\text{Let } A = \lim_{n \rightarrow \infty} \left\{ \frac{n!}{(kn)^n} \right\}^{1/n}$$

$$\Rightarrow A = \lim_{n \rightarrow \infty} \left\{ \frac{1}{kn} \cdot \frac{2}{kn} \dots \frac{n}{kn} \right\}^{1/n}$$

$$\Rightarrow \log A = \lim_{n \rightarrow \infty} \frac{1}{n} \left\{ \log \frac{1}{kn} + \log \frac{2}{kn} + \dots + \log \frac{n}{kn} \right\}$$

$$\Rightarrow \log A = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \log \left(\frac{r}{kn} \right)$$

$$\Rightarrow \log A = \int_0^1 \log \left(\frac{x}{k} \right) dx$$

$$\Rightarrow \log A = \left[x \log \frac{x}{k} - x \right]_0^1$$

$$\Rightarrow \log A = \log \left(\frac{1}{k} \right) - \log e \Rightarrow A = \frac{1}{ke}$$

953 (d)

$$\text{Put } 10^x + x^{10} = t$$

$$\therefore (10^x \log_e 10 + 10x^9) dx = dt$$

$$\therefore \int \frac{10x^9 + 10x \log_e 10}{10^x + x^{10}} dx = \int \frac{dt}{t}$$

$$= \log_e t + c$$

$$= \log_e (10^x + x^{10}) + c$$

954 (c)

$$\text{Let } I = \int e^{\sin \theta} (\log \sin \theta) \cos \theta d\theta +$$

$$\int e^{\sin \theta} \operatorname{cosec}^2 \theta \cos \theta d\theta$$

$$\text{Put } \sin \theta = t \Rightarrow \cos \theta d\theta = dt$$

$$\therefore I = \int e^t \log t dt + \int e^t t^{-2} dt$$

$$= \log t e^t - \int \frac{e^t}{t} dt + \frac{e^t t^{-1}}{-1} - \int \frac{e^t t^{-1}}{-1} dt$$

$$= e^t \left(\log t - \frac{1}{t} \right) + c$$

$$= e^{\sin \theta} (\log \sin \theta - \operatorname{cosec} \theta) + c$$

955 (b)

We have,

$$f(x) = \int \frac{1}{(1+x^2)^{3/2}} dx$$

$$\Rightarrow f(x) = \int \cos \theta d\theta, \text{ where } x = \tan \theta$$

$$\Rightarrow f(x) = \sin \theta + C$$

$$\Rightarrow f(x) = \frac{x}{\sqrt{x^2 + 1}} + C$$

Now, $f(0) = 0 \Rightarrow C = 0$

$$\therefore f(x) = \frac{x}{\sqrt{x^2 + 1}} \Rightarrow f(1) = \frac{1}{\sqrt{2}}$$