Single Correct Answer Type

1. A differentiable function $f(x)$ is defined for all $x > 0$ and satisfies $f(x^3) = 4x^4$ for all $x > 0$. The value of $f'(8)$ is
a) $\frac{16}{3}$ b) $\frac{32}{3}$ c) $\frac{16\sqrt{2}}{3}$ d) $\frac{32\sqrt{2}}{3}$

2. If $y = x - x^2$, then the derivatives of y^2 w.r.t. x^2 is
a) $2x^2 + 3x - 1$ b) $2x^2 - 3x + 1$ c) $2x^2 + 3x + 1$ d) $2x^2 - 3x - 1$

3. If $y = e^{\frac{1}{2}\log(1+\tan^2 x)}$, then $\frac{dy}{dx}$ is equal to
a) $\frac{1}{2}\sec^2 x$ b) $\sec^2 x$ c) $\sec x \tan x$ d) $e^{\frac{1}{2}\log(1+\tan^2 x)}$

4. For $|x| < 1$, let $y = 1 + x + x^2 + \cdots$ to ∞, then $\frac{dy}{dx} - y$ is equal to
a) $\frac{x}{y}$ b) $\frac{x^2}{y^2}$ c) $\frac{x}{y^2}$ d) xy^2

5. If $f(x) = x^n$, then the value of $f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \frac{f'''(1)}{3!} + \cdots + \frac{(-1)^n f^n(1)}{n!}$ Is
a) 2^n b) 2^{n-1} c) 0 d) 1

6. $\frac{d^2}{dx^2}(2\cos x \cos 3x)$ is equal to
a) $2^2(\cos 2x + 2^2 \cos 4x)$ b) $2^2(\cos 2x - 2^2 \cos 4x)$
c) $2^2(-\cos 2x + 2^2 \cos 4x)$ d) $-2^2(\cos 2x + 2^2 \cos 4x)$

7. If $y = x^{x^{x^{\cdots\infty}}}$, then $x(1 - y\log x)\frac{dy}{dx}$
a) x^2 b) y^2 c) xy^2 d) xy

8. If $f(x) = \frac{g(x)+g(-x)}{2} + \frac{2}{[h(x)+h(-x)]^{-1}}$ where g and h are differentiable function, then $f'(0)$ is
a) 1 b) $\frac{1}{2}$ c) $\frac{3}{2}$ d) 0

9. $\frac{d}{dx}[\sin^{-1}(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2})]$ is equal to
a) $\frac{1}{2\sqrt{x(1-x)}} - \frac{1}{\sqrt{1-x^2}}$ b) $\frac{1}{\sqrt{1-\left\{x\sqrt{1-x} - \sqrt{x(1-x^2)}\right\}^2}}$
c) $\frac{1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{x(1-x)}}$ d) $\frac{1}{\sqrt{x(1-x)(1-x)^2}}$

10. If $y = \sin^2\alpha + \cos^2(\alpha+\beta) + 2\sin\alpha\sin\beta\cos(\alpha+\beta)$, then $\frac{d^3y}{d\alpha^3}$, is
a) $\frac{\sin^3(\alpha+\beta)}{\cos\alpha}$ b) $\cos(\alpha+3\beta)$ c) 0 d) None of these

11. If $z = \log(\tan x + \tan y)$, then $(\sin 2x)\frac{\partial z}{\partial x} + (\sin 2y)\frac{\partial z}{\partial y}$ is equal to
a) 1 b) 2 c) 3 d) 4

12. Derivative of $\sec^{-1}\left(\frac{1}{1-2x^2}\right)$ w.r.t. $\sin^{-1}(3x - 4x^3)$ is
a) $\frac{1}{4}$ b) $\frac{3}{2}$ c) 1 d) $\frac{2}{3}$

13. If $(x+y)\sin u = x^2y^2$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to
a) $\sin u$ b) $\operatorname{cosec} u$ c) $2\tan u$ d) $3\tan u$

14. If $y = \left(1+\frac{1}{x}\right)\left(1+\frac{2}{x}\right)\left(1+\frac{3}{x}\right)\ldots\left(1+\frac{n}{x}\right)$ and $x \neq 0$, then $\frac{dy}{dx}$ when $x = -1$ is
a) $n!$ b) $(n-1)!$ c) $(-1)^n(n-1)!$ d) $(-1)^n n!$

15. If $y = a\sin^3\theta$ and $x = a\cos^3\theta$, then at $\theta = \frac{\pi}{3}, \frac{dy}{dx}$ is equal to

a) $\frac{1}{\sqrt{3}}$ b) $-\sqrt{3}$ c) $\frac{-1}{\sqrt{3}}$ d) $\sqrt{3}$

16. If y is a function of x and $\log(x+y) = 2xy$, then the value of $y'(0)$ is equal to

a) 1 b) -1 c) 2 d) 0

17. If $f(x)$ is a polynomial of degree $n(>2)$ and $f(x) = f(\alpha - x)$, (where α is fixed real number), then the degree of $f'(x)$ is

a) n b) $n-1$ c) $n-2$ d) None of these

18. Derivative of the function $f(x) = \log_5(\log_7 x), x > 7$ is

a) $\frac{1}{x(\log 5)(\log 7)(\log_7 x)}$ b) $\frac{1}{x(\log 5)(\log 7)}$

c) $\frac{1}{x(\log x)}$ d) None of these

19. If variables x and y are related by the equation

$x = \int_0^y \frac{1}{\sqrt{1+9u^2}} du$, then 43, $\frac{d^2y}{dx^2}$ is equal to

a) $\sqrt{1+9y^2}$ b) $\frac{1}{\sqrt{1+9y^2}}$ c) $9\,y$ d) $\frac{1}{9}y$

20. If $u = x^2 + y^2$ and $x = s + 3t, y = 2s - t$, then $\frac{d^2u}{ds^2}$ is equal to

a) 12 b) 32 c) 36 d) 10

21. If $f(x) = \sqrt{1+\cos^2(x^2)}$, then $f'\left(\frac{\sqrt{\pi}}{2}\right)$ is

a) $\frac{\sqrt{\pi}}{6}$ b) $-\sqrt{\frac{\pi}{6}}$ c) $\frac{1}{\sqrt{6}}$ d) $\frac{\pi}{\sqrt{6}}$

22. If $y = 2^{\log x}$, then $\frac{dy}{dx}$ is

a) $\frac{2^{\log x}}{\log 2}$ b) $2^{\log x}.\log 2$ c) $\frac{2^{\log x}}{x}$ d) $\frac{2^{\log x}.\log 2}{x}$

23. $\frac{d}{dx}\left[\log\left\{e^x\left(\frac{x-2}{x+2}\right)^{3/4}\right\}\right]$ is equal to

a) 1 b) $\frac{x^2+1}{x^2-4}$ c) $\frac{x^2-1}{x^2-4}$ d) $e^x.\frac{x^2-1}{x^2-4}$

24. If $f: R \to R$ is an even function which is twice differentiable on R and $f''(\pi) = 1$, then $f''(-\pi)$ is equal to

a) -1 b) 0 c) 1 d) 2

25. If $x = \int_0^y \frac{1}{\sqrt{1+4\,t^2}} dt$, then $\frac{d^2y}{dx^2}$ is

a) $2y$ b) $4y$ c) $8y$ d) $6y$

26. Let $3f(x) - 2f\left(\frac{1}{x}\right) = x$, then $f'(2)$ is equal to

a) $\frac{2}{7}$ b) $\frac{1}{2}$ c) 2 d) $\frac{7}{2}$

27. $\frac{d^n}{dx^n}(\log x)$ is equal to

a) $\frac{(n-1)!}{x^n}$ b) $\frac{n!}{x^n}$ c) $\frac{(n-2)!}{x^n}$ d) $(-1)^{n-1}\frac{(n-1)!}{x^n}$

28. Let a function $y = y(x)$ be defined parametrically by $x = 2t - |t|, y = t^2 + t|t|$. Then, $y'(x), x > 0$

a) 0 b) $4x$ c) $2x$ d) Does not exist

29. Let $f(x) = (x^3+2)^{30}$. If $f^n(x)$ is a polynomial of degree 20, where $f^n(x)$ denotes the n^{th} order derivative of $f(x)$ with respect to x, then the value of n is

a) 60 b) 40 c) 70 d) 50

30. If $y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots \text{to } \infty}}}$, then $\frac{dy}{dx}$ is equal to

a) $\frac{x}{2y-1}$ b) $\frac{2}{2y-1}$ c) $-\frac{1}{2y-1}$ d) $\frac{1}{2y-1}$

31. If $f(x) = x^3 + x^2 + f'(1) + xf''(2) + f'''(3),\ \forall x \in R$ where $f(x)$ is a polynomial of degree 3, then
a) $f(0) + f(2) = f(1)$ b) $f(0) + f(3) = 0$
c) $f(1) + f(3) = f(2)$ d) All of these

32. If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$,then $y'(0)$ is
a) 1/2 b) 0 c) 1 d) Does not exist

33. If $y = e^{\sin^{-1}x}$ and $u = \log x$, then $\frac{dy}{du}$ is
a) $\frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}$ b) $xe^{\sin^{-1}x}$ c) $\frac{xe^{\sin^{-1}x}}{\sqrt{1-x^2}}$ d) $\frac{e^{\sin^{-1}x}}{x}$

34. If $f(x) = \cos x \cos 2x \cos 4x \cos 8x \cos 16x$, then $f'\left(\frac{\pi}{4}\right)$ is
a) $\sqrt{2}$ b) $\frac{1}{\sqrt{2}}$ c) 0 d) $\frac{\sqrt{3}}{2}$

35. If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \ldots \infty}}}}$, then $\frac{dy}{dx}$ is equal to
a) $\frac{x}{2y-1}$ b) $\frac{x}{2y+1}$ c) $\frac{1}{x(2y-1)}$ d) $\frac{1}{x(1-2y)}$

36. If $y = \cos^{-1}(\cos x)$, then $\frac{dy}{dx}$ is
a) 1 in the whole plane b) −1 in the whole plane
c) 1 in the 2nd and 3rd quadrants of the plane d) −1 in the 3rd and 4th quadrants of the plane

37. If $x = a(\cos\theta + \theta \sin\theta)$ and $y = a(\sin\theta - \theta \cos\theta)$, then $\frac{dy}{dx}$ is equal to
a) $\cos\theta$ b) $\tan\theta$ c) $\sec\theta$ d) $\operatorname{cosec}\theta$

38. If $y = \log(\sin(x^2)), 0 < x < \frac{\pi}{2}$, then $\frac{dy}{dx}$ at $x = \frac{\sqrt{\pi}}{2}$ is
a) 0 b) 1 c) $\frac{\pi}{4}$ d) $\sqrt{\pi}$

39. The expression of $\frac{dy}{dx}$ of the function $y = a^{x^{a^{x\ldots\infty}}}$, is
a) $\frac{y^2}{x(1-y\log x)}$ b) $\frac{y^2 \log y}{x(1-y\log x)}$ c) $\frac{y^2 \log y}{x(1-y\log x \log y)}$ d) $\frac{y^2 \log y}{x(1+y\log x \log y)}$

40. If $f(x) = x^n + 4$, then the value of $f(1) + \frac{f'(1)}{1!} + \frac{f''(1)}{2!} + \ldots + \frac{f^n(1)}{n!}$ is
a) 2^{n-1} b) $2^n + 4$
c) $1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{2!} + \ldots + \frac{1}{n!}$ d) None of these

41. If $x = e^t \sin t, y = e^t \cos t, t$ is a parameter, then $\frac{d^2y}{dx^2}$ at (1, 1) is equal to
a) $-\frac{1}{2}$ b) $-\frac{1}{4}$ c) 0 d) $\frac{1}{2}$

42. Find the derivative of y with respect to x if $e^x + e^y = e^{x+y}$
a) $-e^{x-y}$ b) e^{x-y} c) $-e^{y-x}$ d) e^{y-x}

43. If $f'(x) = \sin(\log x)$ and $y = f\left(\frac{2x+3}{3-2x}\right)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to
a) $6 \sin \log(5)$ b) $5 \sin \log(6)$ c) $12 \sin \log(5)$ d) $5 \sin \log(12)$

44. The derivative of $f(\tan x)$ w.r.t. $g(\sec x)$ at $x = \frac{\pi}{4}$, where $f'(1) = 2$ and $g'(\sqrt{2}) = 4$, is
a) $\frac{1}{\sqrt{2}}$ b) $\sqrt{2}$ c) 1 d) None of these

45. Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then $y'(1)$ equals
a) −1

b) 1
c) log 2
d) $-\log 2$

46. If $y = x \log \left(\frac{x}{a+bx}\right)$, then $\frac{x^3 d^2y}{dx^2}$ is equal to

a) $x\frac{dy}{dx} - y$ b) $\left(x\frac{dy}{dx} - y\right)^2$ c) $y\frac{dy}{dx} - x$ d) None of these

47. The value of $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$, where y is given by $y = x^{\sin x} + \sqrt{x}$, is

a) $1 + \frac{1}{\sqrt{2\pi}}$ b) 1 c) $\frac{1}{\sqrt{2\pi}}$ d) $1 - \frac{1}{\sqrt{2\pi}}$

48. Let ϕ(x) be the inverse of the function $f(x)$ and $f'(x) = \frac{1}{1+x^5}$, then $\frac{d}{dx}\phi(x)$ is equal to

a) $\frac{1}{1+[\phi(x)]^5}$ b) $\frac{1}{1+[f(x)]^5}$ c) $1 + [\phi(x)]^5$ d) $1 + f(x)$

49. The derivative of $\left[\frac{e^x+1}{e^x}\right]$ is equal to

a) 0 b) $\frac{1}{e^x}$ c) $-\frac{1}{e^x}$ d) e^x

50. A value of x in the interval (1,2) such that $f'(x) = 0$, where $f(x) = x^3 - 3x^2 + 2x + 10$ is

a) $\frac{3+\sqrt{3}}{3}$ b) $\frac{3+\sqrt{2}}{2}$ c) $1 + \sqrt{2}$ d) $\sqrt{2}$

51. If $x^2 + y^2 = a^2$ and $k = 1/a$, then k is equal to

a) $\frac{y''}{\sqrt{1+y'}}$ b) $\frac{|y''|}{\sqrt{(1+y'^2)^3}}$ c) $\frac{2y''}{\sqrt{1+y'^2}}$ d) $\frac{y''}{2\sqrt{(1+y'^2)^3}}$

52. The value of $\frac{d}{dx}\left[\left(\frac{\tan^2 2x - \tan^2 x}{1 - \tan^2 2x \tan^2 x}\right)\cot 3x\right]$ is

a) $\tan 2x \tan x$ b) $\tan 3x \tan x$ c) $\sec^2 x$ d) $\sec x \tan x$

53. $x = \frac{1-\sqrt{y}}{1+\sqrt{y}} \Rightarrow \frac{dy}{dx}$ is equal to

a) $\frac{4}{(x+1)^2}$ b) $\frac{4(x-1)}{(1+x)^3}$ c) $\frac{x-1}{(1+x)^3}$ d) $\frac{4}{(x+1)^3}$

54. If $y = \cos^{-1}\left(\frac{2\cos x - 3\sin x}{\sqrt{13}}\right)$, then $\frac{dy}{dx}$ is

a) Zero b) Constant = 1 c) Constant ≠ 1 d) None of these

55. If $y = \sin(\log_e x)$, then $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is equal to

a) $\sin(\log_e x)$ b) $\cos(\log_e x)$ c) y^2 d) $-y$

56. If $y = \sin^{-1}\frac{x}{2} + \cos^{-1}\frac{x}{2}$, then the value of $\frac{dy}{dx}$ is

a) 1 b) −1 c) 0 d) 2

57. If $y = \cos(\sin x^2)$, then at $x = \sqrt{\frac{\pi}{2}}$, $\frac{dy}{dx}$ is equal to

a) −2 b) 2 c) $-2\sqrt{\frac{\pi}{2}}$ d) 0

58. If $f: R \to R$ is an even function having derivatives of all orders, then an odd function among the following is

a) f'' b) f''' c) $f' + f''$ d) $f'' + f'''$

59. If $z = \sec^{-1}\left(\frac{x^4+y^4-8x^2y^2}{x^2+y^2}\right)$, then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ is equal to

a) $\cot z$ b) $2\cot z$ c) $2\tan z$ d) $2\sec z$

60. If $2^x + 2^y = 2^{x+y}$, then $\frac{dy}{dx}$ is equal to

a) $\frac{2^x+2^y}{2^x-2^y}$ b) $\frac{2^x+2^y}{1+2^{x+y}}$ c) $2^{x-y}\left(\frac{2^y-1}{1-2^x}\right)$ d) $\frac{2^{x+y}-2^x}{2^y}$

61. If $f(x) = 10\cos x + (13 + 2x)\sin x$, then $f''(x) + f(x) =$

a) $\cos x$ b) $4\cos x$ c) $\sin x$ d) $4\sin x$

62. If $xy=\tan^{-1}(xy)+\cot^{-1}(xy)$, then $\frac{dy}{dx}$ is equal to

a) $\frac{y}{x}$ b) $-\frac{y}{x}$ c) $\frac{x}{y}$ d) $-\frac{x}{y}$

63. If $x^y.y^x=16$, then the value of $\frac{dy}{dx}$ at $(2,2)$ is

a) -1 b) 0 c) 1 d) None of these

64. If $x=a\cos\theta, y=b\sin\theta$, then $\frac{d^3y}{dx^3}$ is equal to

a) $-\frac{3b}{a^3}\operatorname{cosec}^4\theta\cot^4\theta$ b) $\frac{3b}{a^3}\operatorname{cosec}^4\theta\cot^4\theta$ c) $-\frac{3b}{a^3}\operatorname{cosec}^4\theta\cot\theta$ d) None of these

65. If $x=\phi(t), y=\psi(t)$, then $\frac{d^2y}{dx^2}$ is equal to

a) $\frac{\phi'\psi''-\psi'\phi''}{(\phi')^2}$ b) $\frac{\phi'\psi''-\psi'\phi''}{(\phi')^3}$ c) $\frac{\phi''}{\psi''}$ d) $\frac{\psi''}{\phi''}$

66. If $u=\sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ is equal to

a) $\sin u$ b) $\tan u$ c) $\cos u$ d) $\cot u$

67. If $y=\sec^{-1}\left(\frac{x+1}{x-1}\right)+\sin^{-1}\left(\frac{x-1}{x+1}\right)$, then $\frac{dy}{dx}$ is

a) 1 b) $\frac{x-1}{x+1}$ c) 0 d) $\frac{x+1}{x-1}$

68. If $y=x^n\log x+x(\log x)^n$, then $\frac{dy}{dx}$ is equal to

a) $x^{n-1}(1+n\log x)+(\log x)^{n-1}[n+\log x]$ b) $x^{n-2}(1+n\log x)+(\log x)^{n-1}[n+\log x]$

c) $x^{n-1}(1+n\log x)+(\log x)^{n-1}[n-\log x]$ d) None of the above

69. If $y=\sqrt{x+\sqrt{y+\sqrt{x+\sqrt{y+\cdots\infty}}}}$, then $\frac{dy}{dx}$ is equal to

a) $\frac{y+x}{y^2-2x}$ b) $\frac{y^3-x}{2y^2-2xy-1}$ c) $\frac{y^3+x}{2y^2-x}$ d) None of these

70. If $y=\int_0^x f(t)\sin\{k(x-t)\}\,dt$, then $\frac{d^2y}{dx^2}+k^2y$ equals

a) 0 b) y c) $kf(x)$ d) $k^2f(x)$

71. If $f(x)=10\cos x+(13+2x)\sin x$, then $f''(x)+f(x)$ is equal to

a) $\cos x$ b) $4\cos x$ c) $\sin x$ d) $4\sin x$

72. Observe the following statements :

I. If $f(x)=ax^{41}+bx^{-40}$, then $\frac{f''(x)}{f(x)}=1640\,x^{-2}$

II. $\frac{d}{dx}\left\{\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right\}=\frac{1}{1+x^2}$

Which of the following is correct?

a) I is true, but II is false b) Both I and II true c) Neither I nor II is true d) I is false, but II is true

73. $\frac{d}{dx}$ $[\cos x^0]$ is equal to

a) $-\frac{\pi}{180}\sin x^0$ b) $-\sin x^0$ c) $\frac{\pi}{180}\sin x^0$ d) $-\frac{\pi x}{180}\sin x$

74. If $y=e^{1+\log_e x}$, then the value of $\frac{dy}{dx}$ is equal to

a) e b) 1 c) 0 d) $\log_e x\ e^{\log_e e^x}$

75. If $y=f(x)$ and $y\cos x+x\cos y=\pi$, then the value of $f''(0)$ is

a) π b) $-\pi$ c) 0 d) 2π

76. If $x=a\sin\theta$ and $y=b\cos\theta$, then $\frac{d^2y}{dx^2}$ is equal to

a) $\frac{a}{b^2}\sec^2\theta$ b) $-\frac{b}{a}\sec^2\theta$ c) $\frac{b}{a^2}\sec^3\theta$ d) $-\frac{b}{a^2}\sec^3\theta$

77. The derivative of $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ with respect to $\cos^{-1}\sqrt{1-x^2}$ is

a) $\frac{\sqrt{1-x^2}}{1+x^2}$ b) $\frac{1}{\sqrt{1-x^2}}$ c) $\frac{2}{\sqrt{1-x^2(1+x^2)}}$ d) $\frac{2\sqrt{1-x^2}}{1+x^2}$

78. If $y=\sin^{-1}\{\sqrt{x-ax}-\sqrt{a-ax}\}$, then $\frac{dy}{dx}$ is equal to

a) $\frac{1}{\sin\sqrt{a-ax}}$ b) $\sin\sqrt{x}\sin\sqrt{a}$ c) $\frac{1}{2\sqrt{x(1-x)}}$ d) 0

79. The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is

a) -1 b) 1 c) 2 d) 4

80. Let $f(x)=e^x$, $g(x)=\sin^{-1}x$ and $h(x)=f[g(x)]$, then $\frac{h'(x)}{h(x)}$ is equal to

a) $e^{\sin^{-1}x}$ b) $\frac{1}{\sqrt{1-x^2}}$ c) $\sin^{-1}x$ d) $\frac{1}{(1-x^2)}$

81. If $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$, then $\frac{dy}{dx}$ is equal to

a) $\cot\frac{\theta}{2}$ b) $\tan\frac{\theta}{2}$ c) $\frac{1}{2}\text{cosec}^2\frac{\theta}{2}$ d) $-\frac{1}{2}\text{cosec}^2\frac{\theta}{2}$

82. A curve is given by the equations $x=a\cos\theta+\frac{1}{2}b\cos 2\theta$, $y=a\sin\theta+\frac{1}{2}b\sin 2\theta$. Then the points for which $\frac{d^2y}{dx^2}=0$ are given by

a) $\sin\theta=\frac{2a^2+b^2}{5ab}$ b) $\tan\theta=\frac{3a^2+2b^2}{4ab}$ c) $\cos\theta=-\frac{a^2+2b^2}{3ab}$ d) None of these

83. If $y=\log_a x+\log_x a+\log_x x+\log_a a$, then $\frac{dy}{dx}$ is equal to

a) $\frac{1}{x}+x\log a$ b) $\frac{\log a}{x}+\frac{x}{\log a}$ c) $\frac{1}{x\log a}+x\log a$ d) $\frac{1}{x\log a}-\frac{\log a}{x(\log x)^2}$

84. If $y=\sin[\cos^{-1}\{\sin(\cos^{-1}x)\}]$, then $\frac{dy}{dx}$ at $x=\frac{1}{2}$ is equal to

a) 0 b) -1 c) $\frac{2}{\sqrt{3}}$ d) 1

85. If $8f(x)+6f\left(\frac{1}{x}\right)=x+5$ and $y=x^2f(x)$, then $\frac{dy}{dx}$ at $x=-1$ is equal to

a) 0 b) $\frac{1}{14}$ c) $-\frac{1}{14}$ d) 1

86. If $x=\sin t$, $y=\cos pt$, then

a) $(1-x^2)y_2+xy_1+p^2y=0$ b) $(1-x^2)y_2+xy_1-p^2y=0$

c) $(1+x^2)y_2-xy_1+p^2y=0$ d) $(1-x^2)y_2-xy_1+p^2y=0$

87. The derivative of $y=(1-x)(2-x)\dots(n-x)$ at $x=1$ is equal to

a) 0 b) $(-1)(n-1)!$ c) $n!-1$ d) $(-1)^{n-1}(n-1)!$

88. If $f(x)=\sqrt{ax}+\frac{a^2}{\sqrt{ax}}$, then $f'(a)$ is equal to

a) -1 b) 1 c) 0 d) a

89. If $y=\log_2\log_2(x)$, then $\frac{dy}{dx}$ is equal to

a) $\frac{\log_2 e}{\log_e x}$ b) $\frac{\log_2 e}{x\log_x 2}$ c) $\frac{\log_2 x}{\log_e 2}$ d) $\frac{\log_2 e}{x\log_e x}$

90. The differential coefficient of $f(\sin x)$ with respect to x, where $f(x)=\log x$, is

a) $\tan x$ b) $\cot x$ c) $f(\cos x)$ d) $\frac{1}{x}$

91. If $y=e^{ax}\sin bx$, then $\frac{d^2y}{dx^2}-2a\frac{dy}{dx}+a^2y$ is equal to

a) 0 b) 1 c) $-b^2y$ d) $-by$

92. If $f(x)=(\log_{\cot x}\tan x)(\log_{\tan x}\cot x)^{-1}+\tan^{-1}\left(\frac{4x}{\sqrt{4-x^2}}\right)$, then $f'(0)$ is equal to

a) 2 b) 0 c) 1/2 d) -2

93. $f(x)$ is a polynomial of degree 2, $f(0) = 4, f'(0) = 3$ and $f''(0) = 4$, then $f(-1)$ is equal to

a) 3 b) -2 c) 2 d) -3

94. If $x = \exp\left\{\tan^{-1}\left(\frac{y-x^2}{x^2}\right)\right\}$, then $\frac{dy}{dx}$ equal

a) $2x\{1 + \tan(\log_e x)\} + x\sec^2(\log_e x)$

b) $x\{1 + \tan(\log_e x)\} + \sec^2(\log_e x)$

c) $2x\{1 + \tan(\log_e x)\} + x^2\sec^2(\log_e x)$

d) $2x\{1 + \tan(\log_e x)\} + \sec^2(\log_e x)$

95. If $y^2 = ax^2 + bx + c$ where a, b, c are constants, then $y^3\frac{d^2y}{dx^2}$, is equal to

a) A constant

b) A function of x

c) A function of y

d) A function of x and y both

96. The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ is

a) 0 b) 1 c) $\frac{1}{1-x^2}$ d) $\frac{1}{1+x^2}$

97. If $I_n = \frac{d^n}{dx^n}(x^n \log x)$, then $I_n - nI_{n-1}$ is equal to

a) n b) $n-1$ c) $n!$ d) $(n-1)!$

98. If $y = \sin(\log_e x)$, then $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is equal to

a) $\sin(\log_e x)$ b) $\cos(\log_e x)$ c) y^2 d) $-y$

99. Let $y = t^{10} + 1$ and $x = t^8 + 1$, then $\frac{d^2y}{dx^2}$ is equal to

a) $\frac{5}{2}t$ b) $20\,t^8$ c) $\frac{5}{16t^6}$ d) None of these

100. The derivative of $\sin^{-1}(2x\sqrt{1-x^2})$ with respect to $\sin^{-1}(3x - 4x^3)$ is

a) $\frac{2}{3}$ b) $\frac{3}{2}$ c) $\frac{1}{2}$ d) 1

101. If $y = (1+x)(1+x^2)(1+x^4)\ldots(1+x^{2^n})$, then the value of $\frac{dy}{dx}$ at $x = 0$ is

a) 0 b) -1 c) 1 d) None of these

102. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then $\frac{dy}{dx}$ equals

a) $\sqrt{(1-x^2)(1-y^2)}$ b) $\sqrt{\frac{1-y^2}{1-x^2}}$ c) $\sqrt{\frac{1-x^2}{1-y^2}}$ d) None of these

103. The derivative of e^{x^3} with respect to $\log x$ is

a) e^{x^3} b) $3x^2 2e^{x^3}$ c) $3x^3e^{x^3}$ d) $3x^2e^{x^3} + 3x^2$

104. The rate of change of $\sqrt{(x^2+16)}$ with respect to $\frac{x}{x-1}$ at $x = 3$ is

a) 2 b) $\frac{11}{5}$ c) $-\frac{12}{5}$ d) -3

105. If $x = \log(1+t^2)$ and $y = t - \tan^{-1} t$. Then, $\frac{dy}{dx}$ is equal to

a) $e^x - 1$ b) $t^2 - 1$ c) $\frac{\sqrt{e^x - 1}}{2}$ d) $e^x - y$

106. If $y = (1+x)(1+x^2)(1+x^4)\ldots(1+x^{2n})$, then the value of $\left(\frac{dy}{dx}\right)_{x=0}$ is

a) 0 b) -1 c) 1 d) 2

107. If $f(x) = (1-x)^n$, then the value of $f(0) + f'(0) + \frac{f''(0)}{2!} + \cdots \frac{f^n(0)}{n!}$, is

a) 2^n b) 0 c) 2^{n-1} d) None of these

108. If $f(x) = (\log_{\cot x} \tan x)(\log_{\tan x} \cot x)^{-1} + \tan^{-1}\frac{4x}{4-x^2}$, then $f'(2)$ is equal to

a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) 1 d) -1

109. If $y=\cos^2\frac{3x}{2}-\sin^2\frac{3x}{2}$, then $\frac{d^2y}{dx^2}$ is

a) $-3\sqrt{1-y^2}$ b) $9y$ c) $-9y$ d) $3\sqrt{1-y^2}$

110. If $f(x)=\log_x(\log_e x)$, then $f'(x)$ at $x=e$ is equal to

a) 1 b) 2 c) 0 d) $\frac{1}{e}$

111. If $f(x)=\cos^{-1}\left\{\frac{1+(\log_e x)^2}{1+(\log_e x)^2}\right\}$, then $f'(e)$

a) Does not exist b) Is equal to $\frac{2}{e}$ c) Is equal to $\frac{1}{e}$ d) Is equal to 1

112. If $f(x)=1+nx+\frac{n(n-1)}{2}x^2+\frac{n(n-1)(n-2)}{6}x^3+\ldots+x^n$, then $f''(1)$ is equal to

a) $n(n-1)2^{n-1}$ b) $(n-1)2^{n-1}$ c) $n(n-1)2^{n-2}$ d) $n(n-1)2^n$

113. If $2^x+2^y=2^{x+y}$, then $\frac{dy}{dx}$ is equal to

a) $\frac{(2^x+2^y)}{(2^x-2^y)}$ b) $\frac{(2^x+2^y)}{(1+2^{x+y})}$ c) $2^{x-y}\left(\frac{2^y-1}{1-2^x}\right)$ d) $\frac{2^{x+y}-2^x}{2^y}$

114. If $2x^2-3xy+y^2+x+2y-x=0$, then $\frac{dy}{dx}=$

a) $\frac{3y-4x-1}{2y-3x+2}$ b) $\frac{3y+4x+1}{2y+3x+2}$ c) $\frac{3y-4x+1}{2y-3x-2}$ d) $\frac{3y-4x+1}{2y+3x+2}$

115. If $\sin y+e^{-x\cos y}=e$, then $\frac{dy}{dx}$ at $(1,\pi)$ is

a) $\sin y$ b) $-x\cos y$ c) e d) $\sin y-x\cos y$

116. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{a-x}{1+ax}\right)\right]$is equal to

a) $-\frac{1}{1+x^2}$ b) $\frac{1}{1+a^2}-\frac{1}{1+x^2}$ c) $\frac{1}{1+\left(\frac{a-x}{1+ax}\right)^2}$ d) $\frac{-1}{\sqrt{1-\left(\frac{a-x}{1+ax}\right)^2}}$

117. If $y=\tan^{-1}\left(\frac{a\cos x-b\sin x}{b\cos x+a\sin x}\right)$, then $\frac{dy}{dx}$ is equal to

a) 2 b) -1 c) $\frac{a}{b}$ d) $\frac{b}{a}$

118. Let $y=x^{x^{x^{\cdots}}}$, then $\frac{dy}{dx}$ is equal to

a) yx^{y-1} b) $\frac{y^2}{x(1-y\log x)}$ c) $\frac{y}{x(1+y\log x)}$ d) None of these

119. If $x^xy^yz^z=c$, then $\frac{\partial z}{\partial x}$ is equal to

a) $\left(\frac{1+\log x}{1+\log z}\right)$ b) $-\left(\frac{1+\log x}{1+\log z}\right)$ c) $\left(\frac{1+\log z}{1+\log x}\right)$ d) None of these

120. If $y=\sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)+\sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$, then $\frac{dy}{dx}$ equals

a) 1 b) 0 c) $\frac{\sqrt{x}+1}{\sqrt{x}-1}$ d) $\frac{\sqrt{x}-1}{\sqrt{x}+1}$

121. If $f(x)=\sqrt{x^2-2x+1}$, then

a) $f'(x)=1$ for all x

b) $f'(x)=-1$ for all $x\le 1$

c) $f'(x)=1$ for all $x\ge 1$

d) None of these

122. If $u=\sin^{-1}\left(\frac{x}{y}\right)+\tan^{-1}\left(\frac{y}{x}\right)$, then the value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ is

a) 0 b) 1 c) 2 d) None of these

123. If $u(x,y)=y\log x+x\log y$, then $u_xu_y-u_x\log x-u_y\log y+\log x\log y$ is equal to

a) 0 b) -1 c) 1 d) 2

124. If $x = A\cos 4t + B\sin 4t$, then $\frac{d^2x}{dt^2}$ is equal to

a) $-16x$ b) $16x$ c) x d) $-x$

125. If $f(x)$ has a derivative at $x = a$, then $\lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$ is equal to

a) $f(a) - af'(a)$ b) $af(a) - f'(a)$ c) $f(a) + f'(a)$ d) $af(a) + f'(a)$

126. $\frac{d}{dx}\,[x^x + x^a + a^x + a^a] = \cdots$, a is constant

a) $x^x(1 + \log x) + a.x^{a-1}$

b) $x^x(1 + \log x) + a\,.x^{a-1} + a^x\log a$

c) $x^x(1 + \log x) + a^a\ (1 + \log a)$

d) $x^x(1 + \log x) + a^a(1 + \log a) + ax^{a-1} + a^a(1 + \log \mathrm{a})$

127. If $y = a^x.b^{2x-1}$, then $\frac{d^2y}{dx^2}$ is

a) $y^2\log ab^2$ b) $y\log ab^2$ c) y^2 d) $y(\log ab^2)^2$

128. If $f(x) = x + 2$, then the value of $f'[f(x)]$ at $x = 4$ is

a) 8 b) 1 c) 4 d) 5

129. If $ax^2 + 2hxy + by^2 = 1$, then $\frac{d^2y}{dx^2}$ equals

a) $\frac{h^2+ab}{(hx+by)^3}$ b) $\frac{h^2-ab}{(hx+by)^2}$ c) $\frac{h^2+ab}{(hx+by)^3}$ d) $\frac{h^2-ab}{(hx+by)^3}$

130. Let $f(x) = \sin x, \mathrm{g}(x) = x^2$ and $h(x) = \log_e x$. If $F(x) = (hogof)(x)$, then $F''(x)$ is equal to

a) a cosec$^3 x$ b) $2\cot x^2 - 4x^2\,\mathrm{cosec}^2\, x^2$

c) $2x\cot x^2$ d) $-2\,\mathrm{cosec}^2\, x$

131. If $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$, then $\frac{dy}{dx}$ is equal to

a) $\frac{x}{y}$ b) $-\frac{x}{y}$ c) $\frac{y}{x}$ d) $-\frac{y}{x}$

132. If $y = \tan^{-1}(\sec x - \tan x)$, then $\frac{dy}{dx}$ is

a) 2 b) -2 c) $\frac{1}{2}$ d) $-\frac{1}{2}$

133. If $x = e^t\sin t$, $y = e^t\cos t$, then $\frac{d^2y}{dx^2}$ at $x = \pi$, is

a) $2e^{\pi}$ b) $\frac{1}{2}e^{\pi}$ c) $\frac{1}{2e^{\pi}}$ d) $\frac{2}{e^{\pi}}$

134. The derivative of $f(x) = 3|2 + x|$ at the point $x_0 = -3$, is

a) 3 b) -3 c) 0 d) Does not exist

135. If variables x and y are related by the equation $x = \int_0^y \frac{1}{\sqrt{1+9u^2}}du$, then $\frac{d^2y}{dx^2}$ is equal to

a) $\sqrt{1+9y^2}$ b) $\frac{1}{1+9y^2}$ c) $9y$ d) $\frac{1}{9}y$

136. If $f(x) = x\tan^{-1}x$, then $f'(1)$ is equal to

a) $\frac{1}{2}+\frac{\pi}{4}$ b) $-\frac{1}{2}+\frac{\pi}{4}$ c) $-\frac{1}{2}-\frac{\pi}{4}$ d) $\frac{1}{2}-\frac{\pi}{4}$

137. If $f: (-1,1) \to R$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $\mathrm{g}(x) = [f(2f(x) + 2)]^2$. Then, $\mathrm{g}'(0)$ is equal to

a) 4 b) -4 c) 0 d) -2

138. The differential coefficient of the function $|x - 1| + |x - 3|$ at the point $x = 2$ is

a) -2 b) 0 c) 2 d) undefined

139. If $y = \tan^{-1}(\sec x - \tan x)$, then $\frac{dy}{dx}$ is equal to

a) 2 b) -2 c) $\frac{1}{2}$ d) $-\frac{1}{2}$

140. If $x = a\cos^4\theta, y = a\sin^4\theta$, then $\frac{dy}{dx}$ at $\theta = \frac{3\pi}{4}$ is

a) -1 b) 1 c) $-a^2$ d) a^2

141. If $\sec\left(\frac{x^2-y^2}{x^2+y^2}\right)=e^a$, then $\frac{dy}{dx}$ is equal to

a) $\frac{y^2}{x^2}$ b) $\frac{y}{x}$ c) $\frac{x}{y}$ d) $\frac{x^2-y^2}{x^2+y^2}$

142. If $y=\frac{a+bx^{3/2}}{x^{5/4}}$ and $y'=0$ at $x=5$, then the ratio $a:b$ is equal to

a) $\sqrt{5}:1$ b) 5:2 c) 3:5 d) 1:2

143. If $f(x)=be^{ax}+ae^{bx}$, then $f''(0)$ is equal to

a) 0 b) $2ab$ c) $ab(a+b)$ d) ab

144. If $x^m y^n=(x+y)^{m+n}$, then $(dy/dx)_{x=1,y=2}$ is equal to

a) 1/2 b) 2 c) $2m/n$ d) $m/2n$

145. If $f(x)$ and g (x) are two functions with g $(x)=x-\frac{1}{x}$ and $fog\,(x)=x^3-\frac{1}{x^3}$, then $f'(x)$ is

a) $3x^2+3$ b) $x^2-\frac{1}{x^2}$ c) $1+\frac{1}{x^2}$ d) $3x^2+\frac{3}{x^4}$

146. If a curve is given by $x=a\cos t+\frac{b}{2}\cos 2t$ and $y=a\sin t+\frac{b}{2}\sin 2t$, then the points for which $\frac{d^2y}{dx^2}=0$ are given by

a) $\sin t=\frac{2\,a^2+b^2}{3ab}$ b) $\cos t=-\frac{a^2+b^2}{3ab}$ c) $\tan t=a/b$ d) None of these

147. Let f be a twice differentiable function such that $f''(x)=-f(x)$ and $f'(x)=\text{g}(x)$. If $h'(x)=[f(x)^2+\text{g}(x)^2]h(1)=8$ and $h(0)=2$, then $h(2)$ is equal to

a) 1 b) 2 c) 3 d) None of these

148. If $y=\log x^x$, then the value of $\frac{dy}{dx}$ is

a) $x^x\,(1+\log x)$ b) $\log(ex)$ c) $\log\left(\frac{e}{x}\right)$ d) $\log\left(\frac{x}{e}\right)$

149. If $f''(x)=-f(x)$, where $f(x)$is a continuous double differentiable function and $\text{g}(x)=f'(x)$. If $F(x)=\left(f\left(\frac{x}{2}\right)\right)^2+\left(\text{g}\left(\frac{x}{2}\right)\right)^2$ and $F(5)=5$ then $F(10)$ is

a) 0 b) 5 c) 10 d) 25

150. If $y=\sqrt{\sin x+\sqrt{\sin x+\sqrt{\sin x+\ldots\infty}}}$, then $\frac{dy}{dx}$ is equal to

a) $\frac{\cos x}{2y-1}$ b) $\frac{-\cos x}{2y-1}$ c) $\frac{\sin x}{1-2y}$ d) $\frac{-\sin x}{1-2y}$

151. If $y=x^2e^{mx}$, where m is a constant, then $\frac{d^3y}{dx^3}$ is equal to

a) $me^{mx}(m^2x^2+6mx+6)$ b) $2m^3xe^{mx}$
c) $me^{mx}(m^2x^2+2mx+2)$ d) None of these

152. If $x^2+y^2=t-\frac{1}{t}$ and $x^4+y^4=t^2+\frac{1}{t^2}$, then $\frac{dy}{dx}$ is equal to

a) $\frac{1}{x^2y^3}$ b) $\frac{1}{xy^3}$ c) $\frac{1}{x^2y^2}$ d) $\frac{1}{x^3y}$

153. If $x=\sin^{-1}(3t-4t^3)$ and $y=\cos^{-1}(\sqrt{1-t^2})$, then $\frac{dy}{dx}$ is equal to

a) 1/2 b) 2/5 c) 3/2 d) 1/3

154. If $f(x)=\sqrt{1-\sin 2x}$, then $f'(x)$ equals

a) $-(\cos x+\sin x)$, for $x\in(\pi/4,\pi/2)$
b) $\cos x+\sin x$, for $x\in(0,\pi/4)$
c) $-(\cos x+\sin x)$, for $x\in(0,\pi/4)$
d) $\cos x-\sin x$, for $x\in(\pi/4,\pi/2)$

155. Derivative of $\sin x$ w.r.t. $\cos x$ is

a) $\cos x$ b) $\cot x$ c) $-\cot x$ d) $\tan x$

156. The derivative of $F[f\{\phi(x)\}]$is

a) $F'[f\{\phi(x)\}]$ b) $F'[f\{\phi(x)\}]f\{\phi(x)\}$

c) $F'[f\{\phi(x)\}]f'\{\phi(x)\}$ d) $F'[f\{\phi(x)\}]f'\{\phi(x)\}\phi'(x)$

157. If $x\sqrt{1+y}+y\sqrt{1+x}=0$, then $\frac{dy}{dx}$ is equal to

a) $\frac{1}{(1+x)^2}$ b) $-\frac{1}{(1+x)^2}$ c) $\frac{1}{1+x^2}$ d) $\frac{1}{1-x^2}$

158. If $\sec\left(\frac{x^2-y^2}{x^2+y^2}\right)=e^a$, then $\frac{dy}{dx}$ is equal to

a) $\frac{y^2}{x^2}$ b) $\frac{y}{x}$ c) $\frac{x}{y}$ d) $\frac{x^2-y^2}{x^2+y^2}$

159. If $f(x)=\log_a(\log_a x)$, then $f'(x)$ is

a) $\frac{\log_a e}{x\log_e x}$ b) $\frac{\log_e a}{x\log_a x}$ c) $\frac{\log_e a}{x}$ d) $\frac{x}{\log_e a}$

160. If $y=\log^n x$, where $\log^n$ means log log log...(repeated n times), then $x\log x\log^2 x\log^3 x\ldots\log^{n-1} x\log^n x\frac{dy}{dx}$ is equal to

a) $\log x$ b) $\log^n x$ c) $\frac{1}{\log x}$ d) 1

161. If $y=\sin px$ and y_n is the nth derivative of y, then $\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix}$ is equal to

a) 1 b) 0 c) -1 d) None of these

162. If $y=1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\frac{x^4}{4!}-\ldots$, then $\frac{d^2y}{dx^2}$ is equal to

a) x b) $-x$ c) $-y$ d) y

163. If $f(4)=4, f'(4)=1$, then $\lim_{x\to 4}\frac{2-\sqrt{f(x)}}{2-\sqrt{x}}$ is equal to

a) -1 b) 1 c) 2 d) -2

164. If $x=\frac{2at}{1+t^3}$ and $y=\frac{2at^2}{(1+t^3)^2}$, then $\frac{dy}{dx}$ is

a) ax b) a^2x^2 c) $\frac{x}{a}$ d) $\frac{x}{2a}$

165. $y=e^{a\sin^{-1}x}\Rightarrow(1-x^2)\,y_{n+2}-(2n+1)xy_{n+1}$ is equal to

a) $-(n^2+a^2)\,y_n$ b) $(n^2-a^2)y_n$ c) $(n^2+a^2)y_n$ d) $-(n^2-a^2)y_n$

166. If $\sec\left(\frac{x-y}{x+y}\right)=a$, then $\frac{dy}{dx}$ is

a) $\frac{y}{x}$ b) $-\frac{y}{x}$ c) $\frac{x}{y}$ d) $-\frac{x}{y}$

167. If $y=x+x^2+x^3\ldots\infty$, where $|x|<1$, then for $|y|<1$, $\frac{dx}{dy}$ is equal to

a) $y+y^2+y^3+\ldots\infty$ b) $1-y+y^2-y^3+\ldots\infty$

c) $1-2y+3y^2-\ldots\infty$ d) $1+2y+3y^2+\ldots\infty$

168. $\frac{d}{dx}\sqrt{\frac{1-\sin 2x}{1+\sin 2x}}$ is equal to

a) $\sec^2 x$ b) $-\sec^2\left(\frac{\pi}{4}-x\right)$ c) $\sec^2\left(\frac{\pi}{4}+x\right)$ d) $\sec^2\left(\frac{\pi}{4}-x\right)$

169. Derivative of $\log_{10} x$ with respect to x^2 is

a) $2x^2\log_e 10$ b) $\frac{\log_{10} e}{2x^2}$ c) $\frac{\log_e 10}{2x^2}$ d) $x^2\log_e 10$

170. If $y=\text{In}\left(\frac{x}{a+bx}\right)^x$, then $x^3\frac{d^2y}{dx^2}$ is equal to

a) $\left(\frac{dy}{dx}+x\right)^2$ b) $\left(\frac{dy}{dx}-y\right)^2$

c) $\left(x\frac{dy}{dx}+y\right)^2$ d) $\left(x\frac{dy}{dx}-y\right)^2$

171. If f be a polynomial, then the second derivative of $f(e^x)$ is

a) $f'(e^x)$
b) $f''(e^x)e^x + f'(e^x)$
c) $f''(e^x)e^{2x} + f''(e^x)$
d) $f''(e^x)e^{2x} + f'(e^x)e^x$

172. If $2x^2 - 3xy + y^2 + x + 2y - 8 = 0$, then $\frac{dy}{dx}$ is equal to

a) $\dfrac{3y - 4x - 1}{2y - 3x + 2}$
b) $\dfrac{3y + 4x + 1}{2y + 3x + 2}$
c) $\dfrac{3y - 4x + 1}{2y - 3x - 2}$
d) $\dfrac{3y - 4x + 1}{2y + 3x + 2}$

173. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \cdots \infty}}}$, then $(2y - 1)\frac{dy}{dx}$ is equal to

a) $\sin x$
b) $-\cos x$
c) $\cos x$
d) $-\sin x$

174. If $2^x + 2^y = 2^{x+y}$, then the value of $\frac{dy}{dx}$ at $x = y = 1$, is

a) 0
b) -1
c) 1
d) 2

175. If $y = \sec(\tan^{-1} x)$,then $\frac{dy}{dx}$ is equal to

a) $\dfrac{x}{\sqrt{1 + x^2}}$
b) $-\dfrac{x}{\sqrt{1 + x^2}}$
c) $\dfrac{x}{\sqrt{1 - x^2}}$
d) None of these

176. Let $f(x) = (x - 7)^2(x - 2)^7, x \in [2,7]$. The value of $\theta \in (2,7)$ such that $f'(\theta) = 0$ is equal to

a) $\dfrac{49}{4}$
b) $\dfrac{53}{9}$
c) $\dfrac{53}{7}$
d) $\dfrac{49}{9}$

177. If $x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$, then $x^3y\ \frac{dy}{dx}$ equals

a) 0
b) 1
c) -1
d) None of these

178. The value of $\frac{d}{dx}(|x - 1| + |x - 5|)$ at $x = 3$, is

a) -2
b) 0
c) 2
d) 4

179. If $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$, then $\frac{dy}{dx}$ is equal to

a) $\dfrac{x}{y}$
b) $-\dfrac{x}{y}$
c) $\dfrac{y}{x}$
d) $-\dfrac{y}{x}$

180. If $f(x) = \frac{1}{1-x}$, then the derivative of the composite function $f[f\{f(x)\}]$ is equal to

a) 0
b) $\dfrac{1}{2}$
c) 1
d) 2

181. If $x = \exp\left\{\tan^{-1}\left(\frac{y-x^2}{x^2}\right)\right\}$, then $\frac{dy}{dx}$ equals

a) $2x[1 + \tan(\log x)] + x\sec^2(\log x)$
b) $x[1 + \tan(\log x)] + \sec^2(\log x)$
c) $2x[1 + \tan(\log x)] + x^2\sec^2(\log x)$
d) $2x[1 + \tan(\log x)] + \sec^2(\log x)$

182. If $\sin y = x\sin(a + y)$, then $\frac{dy}{dx}$ is

a) $\dfrac{\sin a}{\sin^2(a + y)}$
b) $\dfrac{\sin^2(a + y)}{\sin a}$
c) $\sin a \sin^2(a + y)$
d) $\dfrac{\sin^2(a - y)}{\sin a}$

183. $f(x) = e^x \sin x$, then $f''(x)$ is equal to

a) $e^{6x}\sin 6x$
b) $2e^x\cos x$
c) $8e^x\sin x$
d) $8e^x\cos x$

184. If $f(x) = \cos x.\cos 2x.\cos 4x.\cos 8x.\cos 16x$,then the value of $f'\left(\frac{\pi}{4}\right)$ is

a) 1
b) $\sqrt{2}$
c) $\dfrac{1}{\sqrt{2}}$
d) 0

185. If $\sec^{-1}\left(\frac{1+x}{1-x}\right) = a$, then $\frac{dy}{dx}$ is

a) $\dfrac{y - 1}{x + 1}$
b) $\dfrac{y + 1}{x - 1}$
c) $\dfrac{x - 1}{y - 1}$
d) $\dfrac{x - 1}{y + 1}$

186. The derivative of $a^{\sec x}$ w.r.t. $a^{\tan x} (a > 0)$ is

a) $\sec x\ a^{\sec x - \tan x}$
b) $\sin x\ a^{\tan x - \sec x}$
c) $\sin x\ a^{\sec x - \tan x}$
d) $a^{\sec x - \tan x}$

187. If $x = a\left\{\cos\theta + \log\tan\left(\frac{\theta}{2}\right)\right\}$ and $y = a\sin\theta$, then $\frac{dy}{dx}$ is equal to

a) $\cot\theta$
b) $\tan\theta$
c) $\sin\theta$
d) $\cos\theta$

188. If $\phi(x)$ is the inverse of the function $f(x)$ and $f'(x) = \frac{1}{1+x^5}$, then $\frac{d}{dx}\phi(x)$ is

a) $\dfrac{1}{1+\{\phi(x)\}^5}$ b) $\dfrac{1}{1+\{f(x)\}^5}$ c) $1+\{\phi(x)\}^5$ d) $1+f(x)$

189. If $f(x)=3e^{x^2}$, then $f'(x)-2x\,f(x)+\frac{1}{3}f(0)-f'(0)$ is equal to

a) 0 b) 1 c) $(7/3)e^{x^2}$ d) e^{x^2}

190. If $u=\log\left(\frac{x^2+y^2}{x+y}\right)$, then the value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ is

a) -1 b) 0 c) 1 d) 2

191. If $F(x)=\frac{1}{x^2}\int_4^x(4t^2-2F'(t))\,dt$, then $F'(4)$ equals

a) $\dfrac{32}{9}$ b) $\dfrac{64}{3}$ c) $\dfrac{64}{9}$ d) $\dfrac{32}{3}$

192. If $y=\tan^{-1}\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}$, then $\frac{dy}{dx}$ is equal to

a) $\dfrac{x^2}{\sqrt{1-x^4}}$ b) $\dfrac{x^2}{\sqrt{1+x^4}}$ c) $\dfrac{x}{\sqrt{1+x^4}}$ d) $\dfrac{x}{\sqrt{1-x^4}}$

193. If $f(x)=|x^2-5x+6|$, then $f'(x)$ equals

a) $2x-5$ for $2<x<3$ b) $5-2x$ for $2<x<3$ c) $2x-5$ for $2\le x\le 3$ d) $5-2x$ for $2\le x\le 3$

194. If $\sqrt{x+y}+\sqrt{y-x}=c$, then $\frac{d^2y}{dx^2}$ equals

a) $2/c$ b) $-2/c^2$ c) $2/c^2$ d) $-2/c$

195. Derivative of x^6+6^x with respect to x is

a) $12x$ b) $x+4$ c) $6x^5+6^x\log 6$ d) $6x^5+x6^{x-1}$

196. If $f(x)=\cos^2 x+\cos^2\left(x+\frac{\pi}{3}\right)+\sin x\sin\left(x+\frac{\pi}{3}\right)$ and $g\left(\frac{5}{4}\right)=3$, then $\frac{d}{dx}(gof(x))=$

a) 1 b) 0 c) -1 d) None of these

197. If $y=\sin^{-1}\frac{x}{2}+\cos^{-1}\frac{x}{2}$, then the value of $\frac{dy}{dx}$ is

a) 1 b) -1 c) 0 d) 2

198. If $z=y+f(v)$, where $v=\left(\frac{x}{y}\right)$, then $v\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}$ is

a) -1 b) 1 c) 0 d) 2

199. If $y=\tan^{-1}\left(\frac{\sqrt{x}-x}{1+x^{3/2}}\right)$, then $y'(1)$ is equal to

a) 0 b) $\dfrac{1}{2}$ c) -1 d) $-\dfrac{1}{4}$

200. If $f(1)=1$ and $f'(1)=2$, then $\lim_{x\to 1}\frac{\sqrt{f(x)}-1}{\sqrt{x}-1}$ equals

a) 2 b) 4 c) 1 d) 1/2

201. The derivative of $\cos^3 x$ w.r.t. $\sin^3 x$ is

a) $-\cot x$ b) $\cot x$ c) $\tan x$ d) $-\tan x$

202. If $y=\log\left\{\left(\frac{1+x}{1-x}\right)^{1/4}\right\}-\frac{1}{2}\tan^{-1}x$, then $\frac{dy}{dx}=$

a) $\dfrac{x}{1-x^2}$ b) $\dfrac{x^2}{1-x^4}$ c) $\dfrac{x}{1+x^4}$ d) $\dfrac{x}{1-x^4}$

203. If $y=\left(x+\sqrt{1+x^2}\right)^n$, then $(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}$ is

a) n^2y b) $-n^2y$ c) $-y$ d) $2x^2y$

204. The value of $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\sqrt{x}(3-x)}{1-3x}\right)\right]$ is

a) $\dfrac{1}{2(1+x)\sqrt{x}}$ b) $\dfrac{3}{(1+x)\sqrt{x}}$ c) $\dfrac{2}{(1+x)\sqrt{x}}$ d) $\dfrac{3}{2(1+x)\sqrt{x}}$

205. If $y=\tan^{-1}\left[\frac{\sin x+\cos x}{\cos x-\sin x}\right]$, then $\frac{dy}{dx}$ is equal to

a) $\dfrac{1}{2}$ b) $\dfrac{\pi}{4}$ c) 0 d) 1

206. $x=\cos\theta, y=\sin 5\theta \Rightarrow (1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}$ is

a) $-5y$ b) $5y$ c) $25y$ d) $-25y$

207. If the function $f(x)$ is defined by $f(x) = a + bx$ and $f^r = fff$ ...(repeated r times), then $f^r(x)$ is equal to

a) $a + b^r x$ b) $ar + b^r x$ c) $ar + bx^r$ d) $a\left(\frac{b^r - 1}{b - 1}\right) + b^r x$

208. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is equal to

a) $(1 + \log x)^{-1}$ b) $(1 + \log x)^{-2}$ c) $\log x \cdot (1 + \log x)^{-2}$ d) None of these

209. The derivative of $\sin^2 x$ with respect to $\cos^2 x$ is

a) $\tan^2 x$ b) $\tan x$ c) $-\tan x$ d) None of these

210. If $x^p y^q = (x + y)^{p+q}$, then $\frac{dy}{dx}$ is equal to

a) $\frac{y}{x}$ b) $\frac{py}{qx}$ c) $\frac{x}{y}$ d) $\frac{qy}{px}$

211. If $y = (1 + x^2)\tan^{-1} x - x$, then $\frac{dy}{dx}$ is equal to

a) $\tan^{-1} x$ b) $2x\tan^{-1} x$ c) $2x\tan^{-1} x - 1$ d) $\frac{2x}{\tan^{-1} x}$

212. The derivative of $\sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)$ with respect to x is

a) $-\frac{1}{2\sqrt{1-x^2}}$ b) $\frac{1}{2\sqrt{1-x^2}}$ c) $\frac{2}{\sqrt{1-x^2}}$ d) $\frac{-2}{\sqrt{1-x^2}}$

213. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$ is

a) $\sqrt{1-x^2}$ b) $\frac{1}{\sqrt{1-x^2}}$ c) $\frac{1}{2\sqrt{1-x^2}}$ d) x

214. If $y = \tan^{-1} x + \cot^{-1} x + \sec^{-1} + \operatorname{cosec}^{-1} x$, then $\frac{dy}{dx}$ is equal to

a) $\frac{x^2-1}{x^2+1}$ b) π c) 0 d) 1

215. If $y = \left(\frac{ax+b}{cx+d}\right)$, then $2\frac{dy}{dx}.\frac{d^3y}{dx^3}$ is equal to

a) $\left(\frac{d^2y}{dx^2}\right)^2$ b) $3\frac{d^2y}{dx^2}$ c) $3\left(\frac{d^2y}{dx^2}\right)^2$ d) $3\frac{d^2x}{dy^2}$

216. If $y = (\log_{\cos x} \sin x)(\log_{\sin x} \cos x) + \sin^{-1}\frac{2x}{1+x^2}$, then $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$ is equal to

a) $\frac{8}{(4+\pi^2)}$ b) 0 c) $-\frac{8}{(4+\pi^2)}$ d) None of the above

217. If $y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$, then $\frac{dy}{dx}$ is equal to

a) 2 b) -1 c) $\frac{a}{b}$ d) 0

218. If $x = \cos\theta$, $y = \sin 5\theta$, then $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} =$

a) $-5y$ b) $5y$ c) $25y$ d) $-25y$

219. The differential coefficient of $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$ is

a) $\sqrt{1-x^2}$ b) $\frac{1}{\sqrt{1-x^2}}$ c) $\frac{1}{2\sqrt{1-x^2}}$ d) x

220. If $f(x) = (x-2)(x-4)(x-6)\dots(x-2n)$, then $f'(2)$ is

a) $(-1)^n 2^{n-1}(n-1)!$ b) $(-2)^{n-1}(n-1)!$ c) $(-2)^n n!$ d) $(-1)^{n-1}2^n(n-1)!$

221. Find $\frac{dy}{dx}$, if $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$.

a) $\tan\frac{3\theta}{2}$ b) $-\tan\frac{3\theta}{2}$ c) $\cot\frac{3\theta}{2}$ d) $-\cot\frac{3\theta}{2}$

222. Let $f(x) = 2^{2x-1}$ and $\phi(x) - 2^x + 2x\log 2$. If $f'(x) > \phi'(x)$, then

a) $0 < x < 1$ b) $0 \le x < 1$ c) $x > 0$ d) $x \ge 0$

223. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, then $\frac{dy}{dx} =$

a) $\frac{1}{(1+x)^2}$ b) $-\frac{1}{(1+x)^2}$ c) $\frac{1}{1+x^2}$ d) $\frac{1}{1-x^2}$

224. If $y = e^{(1/2)\log(1+\tan^2 x)}$, then $\frac{dy}{dx}$ is equal to

a) $\frac{1}{2}\sec^2 x$ b) $\sec^2 x$

c) $\sec x \tan x$ d) $e^{1/2\log(1+\tan^2 x)}$

225. If $f(x) = \frac{x-1}{4} + \frac{(x-1)^3}{12} + \frac{(x-1)^5}{20} + \frac{(x-1)^7}{28} + \cdots$, where $0 < x < 2$, then $f'(x)$ is equal to

a) $\frac{1}{4x(2-x)}$ b) $\frac{1}{4(x-2)^2}$ c) $\frac{1}{2-x}$ d) $\frac{1}{2+x}$

226. If $f(x) = \begin{vmatrix} x^3 & x^2 & 3x^2 \\ 1 & -6 & 4 \\ P & P^2 & P^3 \end{vmatrix}$, here p is a constant, then $\frac{d^3 f(x)}{dx^3}$ is

a) Proportional to x^2 b) Proportional to x

c) Proportional to x^3 d) A constant

227. If $f(x) = \arctan\left(\frac{x^x - x^{-x}}{2}\right)$, then $f'(1)$ is equal to

a) 1 b) -1 c) $\log 2$ d) None of these

228. If for all $x, y \in R$, the function f is defined by $f(x) + f(y) + f(x)f(y) = 1$ and $f(x) > 0$. Then,

a) $f'(x) = 0$ for all $x \in R$

b) $f'(0) < f'(1)$

c) $f'(x)$ does not exist

d) None of these

229. Let $f(x) = e^x$, $g(x) = \sin^{-1} x$ and $h(x) = f[g(x)]$, then $\frac{h'(x)}{h(x)}$ is equal to

a) $e^{\sin^{-1} x}$ b) $\frac{1}{\sqrt{1-x^2}}$ c) $\sin^{-1} x$ d) $\frac{1}{(1-x^2)}$

230. If $f(x,y) = \frac{\cos(x-4y)}{\cos(x+4y)}$, then $\left.\frac{\partial f}{\partial x}\right|_{y=\frac{\pi}{2}}$ is equal to

a) -1 b) 0 c) 1 d) 2

231. If $y = \sin^n x \cos nx$, then $\frac{dy}{dx}$ is

a) $n \sin^{n-1} x \sin(n+1)x$ b) $n \sin^{n-1} x \cos(n-1)x$

c) $n \sin^{n-1} x \cos nx$ d) $n \sin^{n-1} x \cos(n+1)x$

232. If $2f(x) = f'(x)$ and $f(0) = 3$, then $f(2)$ is equal to

a) $3e^4$ b) $3e^2$ c) e^4 d) None of these

233. If $y^2 = P(x)$ is a polynomial of degree 3, then $2\frac{d}{dx}\left[y^3 \frac{d^2y}{dx^2}\right]$ equals

a) $P'''(x) + P'x$ b) $P''(x).P'''(x)$ c) $P(x).P'''(x)$ d) None of these

234. If $e^{y+e^{y+\cdots\infty}}$, $x > 0$, then $\frac{dy}{dx}$ is

a) $\frac{x}{1+x}$ b) $\frac{1}{x}$ c) $\frac{1-x}{x}$ d) $\frac{1+x}{x}$

235. The 2nd derivative of $a\sin^3 t$ with respect to $a\cos^3 t$ at $t = \frac{\pi}{4}$ is

a) $\frac{4\sqrt{2}}{3a}$ b) 2 c) $\frac{1}{12a}$ d) None of these

236. The derivative of $\sin(x^3)$ w.r.t. $\cos(x^3)$ is

a) $-\tan(x^3)$ b) $\tan(x^3)$ c) $-\cot(x^3)$ d) $\cot(x^3)$

237. If $y = \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$, then $\frac{dy}{dx}$ is equal to

a) $\frac{1}{2}$ b) 2 c) -2 d) $-\frac{1}{2}$

238. If $y = \tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$ is

a) $-\frac{1}{2}$ b) $\frac{1}{2}$ c) 1 d) -1

239. If $y = \log_{\cos x} \sin x$, then $\frac{dy}{dx}$ is equal to

a) $\frac{(\cot x \log \cos x + \tan x \log \sin x)}{(\log \cos x)^2}$

b) $\frac{(\tan x \log \cos x + \cot x \log \sin x)}{(\log \cos x)^2}$

c) $\frac{(\cot x \log \cos x + \tan x \log \sin x)}{(\log \sin x)^2}$

d) None of the above

240. If $y = 2^x . 3^{2x-1}$, then $\frac{d^2y}{dx^2}$ is equal to

a) $(\log 2)(\log 3)$ b) $(\log 18)$ c) $(\log 18^2)\ y^2$ d) $(\log 18)^2\ y$

241. If $y = \sec^{-1}\frac{x+1}{x-1} + \sin^{-1}\frac{x-1}{x+1}$, then $\frac{dy}{dx}$ is

a) 1 b) 0 c) $\frac{x-1}{x+1}$ d) $\frac{x+1}{x-1}$

242. Let $g(x) = \log f(x)$, where $f(x)$ is a twice differentiable positive function on $(0, \infty)$ such that $f(x+1) = xf(x)$. Then, for $N = 1,2,3, \dots$, $g''\left(N + \frac{1}{2}\right) - g''\left(\frac{1}{2}\right)$ is equal to

a) $-4\left\{1 + \frac{1}{9} + \frac{1}{25} + \dots + \frac{1}{(2N-1)^2}\right\}$

b) $4\left\{1 + \frac{1}{9} + \frac{1}{25} + \dots + \frac{1}{(2N-1)^2}\right\}$

c) $-4\left\{1 + \frac{1}{9} + \frac{1}{25} + \dots + \frac{1}{(2N+1)^2}\right\}$

d) $4\left\{1 + \frac{1}{9} + \frac{1}{25} + \dots + \frac{1}{(2N+1)^2}\right\}$

243. If $y = \log_{x^2+4}(7x^2 - 5x + 1)$, then $\frac{dy}{dx}$ is equal to

a) $\log_e(x^2+4) \cdot \left\{\frac{14x-5}{7x^2-5x+1} - \frac{2\,xy}{x^2+4}\right\}$

b) $\frac{1}{\log_e(x^2+4)}\left\{\frac{14x-5}{7x^2-5x+1} - \frac{2xy}{x^2+4}\right\}$

c) $\log_e(7x^2-5x+1)\ \left\{\frac{2\,x}{x^2+4} - \frac{(14x-5)y}{7x^2-5x+1}\right\}$

d) $\frac{1}{\log_e(7x^2-5x+1)}\left\{\frac{2x}{x^2+4} - \frac{(14x-5)y}{7x^2-5x+1}\right\}$

244. If $\sin y = e^{-x\cos y} = e$, then $\frac{dy}{dx}$ $at\ (1, \pi)$ is equal to

a) $\sin y$ b) $-x\cos y$ c) e d) $\sin y - x\cos y$

245. If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, then $y'(0)$ is

a) $\frac{1}{2}$ b) 0 c) 1 d) -1

246. Differential coefficient of $\sqrt{\sec\sqrt{x}}$ is

a) $\frac{1}{4\sqrt{x}}\sec\sqrt{x}\sin\sqrt{x}$

b) $\frac{1}{4\sqrt{x}}\left(\sec\sqrt{x}\right)^{3/2}.\sin\sqrt{x}$

c) $\frac{1}{2}\sqrt{x}\sec\sqrt{x}\sin\sqrt{x}$

d) $\frac{1}{2}\sqrt{x}\left(\sec\sqrt{x}\right)^{3/2}.\sin\sqrt{x}$

247. If $f(x) = \sin x$ and $g(x) = sgn \sin x$, then $g'(1)$ equals

a) 0 b) $-\cos 1$ c) $\cos 1$ d) None of these

248. The derivative of $y = x^{\text{In}\, x}$ is

a) $x^{\text{In}\, x}\ \text{In}\, x$ b) $x^{\text{In}\, x - 1}\text{In}\, x$ c) $2x^{\text{In}\, x-1}\text{In}\, x$ d) $x^{\text{In}\, x - 2}$

249. If $x = e^t \sin t, y = e^t \cos t$, then $\frac{d^2y}{dx^2}$ at $x = \pi$ is

a) $2e^{\pi}$ b) $\frac{1}{2}e^{\pi}$ c) $\frac{1}{2e^{\pi}}$ d) $\frac{2}{e^{\pi}}$

250. If $f'(x) = \sin(\log x)$ and $y = f\left(\frac{2x+3}{3-2x}\right)$, then $\frac{dy}{dx}$ equals

a) $\sin(\log x) \cdot \frac{1}{x \log x}$

b) $\frac{12}{(3-2x)^2}\sin\left\{\log\left(\frac{2x+3}{3-2x}\right)\right\}$

c) $\sin\left\{\log\left(\frac{2x+3}{3-2x}\right)\right\}$

d) None of these

251. If $r = [2\phi + \cos^2(2\phi + \pi/4)]^{1/2}$, then what is the value of the derivative of $dr/d\phi$ at $\phi = \pi/4$?

a) $2\left(\frac{1}{\pi+1}\right)^{1/2}$ b) $2\left(\frac{2}{\pi+1}\right)^{2}$ c) $\left(\frac{2}{\pi+1}\right)^{1/2}$ d) $2\left(\frac{2}{\pi+1}\right)^{1/2}$

252. For $|x| < 1$, let $y = 1 + x + x^2$ ...to ∞, then $\frac{dy}{dx}$ equal to

a) $\frac{x}{y}$ b) $\frac{x^2}{y^2}$ c) $\frac{x}{y^2}$ d) $xy^2 + y$

253. If $f(x+y) = 2f(x)f(y)$, $f'(5) = 1024(\log 2)$ and $f(2) = 8$, then the value of $f'(3)$ is

a) 64 (log 2) b) 128 (log 2) c) 256 d) 256 (log 2)

254. The value of differentiation of e^{x^2} with respect to e^{2x-1} at $x = 1$ is

a) e b) 0 c) e^{-1} d) 1

255. Let $x = \log_e t, t > 0$ and $y + 1 = t^2$. Then, $\frac{d^2x}{dy^2}$ is equal to

a) $4e^{2x}$ b) $-\frac{1}{2}e^{-4x}$ c) $-\frac{3}{4}e^{5x}$ d) $4e^{x}$

256. If $y = \cot^{-1}(\cos 2x)^{1/2}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$ will be

a) $\left(\frac{2}{3}\right)^{1/2}$ b) $\left(\frac{1}{3}\right)^{1/2}$ c) $(3)^{1/2}$ d) $(6)^{1/2}$

257. If $P(x)$ is a polynomial such that $P(x^2+1) = \{P(x)\}^2 + 1$ and $P(0) = 0$, then $P'(0)$ is equal to

a) -1 b) 0 c) 1 d) None of these

258. If $y = (\cos x^2)^2$, then $\frac{dy}{dx}$ is equal to

a) $-4x \sin 2x^2$ b) $-x \sin x^2$ c) $-2x \sin 2x^2$ d) $-x \cos 2x^2$

259. Let $f(x) = 2^{2x-1}$ and $g(x) = -2^x + 2x \log 2$. Then the set of points satisfying $f'(x) > g'(x)$, is

a) $(0, 1)$ b) $[0, 1)$ c) $(0, \infty)$ d) $[0, \infty)$

260. $\frac{d}{dx}\left\{\begin{matrix}\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)\\ -\tan^{-1}\left(\frac{4x-4x^3}{1-6x^2+x^4}\right)\end{matrix}\right\}$ is equal to

a) $\frac{1}{\sqrt{1-x^2}}$ b) $-\frac{1}{\sqrt{1-x^2}}$ c) $\frac{1}{1+x^2}$ d) $-\frac{1}{1+x^2}$

261. If $y = \frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}$, then $\frac{dy}{dx}$ is equal to

a) $\frac{1}{2}\text{cosec}^2\frac{x}{2}$ b) $\frac{1}{2}\text{cosec}\frac{x}{2}$ c) $\frac{1}{2}\text{cosec}^2 x$ d) $\text{cosec}^2\frac{x}{2}$

262. The value of $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$, where y is given by $y = x^{\sin x} + \sqrt{x}$, is

a) $1 + \frac{1}{\sqrt{2\pi}}$ b) 1 c) $\frac{1}{\sqrt{2\pi}}$ d) $1 - \frac{1}{\sqrt{2\pi}}$

263. If $y = \frac{3at^2}{1+t^3}$, $x = \frac{3at}{1+t^3}$, then $\frac{dy}{dx}$ is equal to

a) $\frac{t(2-t^3)}{(1-2t^3)}$ b) $\frac{t(2+t^3)}{(1-2t^3)}$ c) $\frac{t(2-t^3)}{(1+2t^3)}$ d) $\frac{t(2+t^3)}{(1+2t^3)}$

264. If $y = \log_a x + \log_x a + \log_x x + \log_a a$, then $\frac{dy}{dx}$ is equal to

a) $\frac{1}{x} + x\log a$ b) $\frac{\log a}{x} + \frac{x}{\log a}$ c) $\frac{1}{x\log a} + x\log a$ d) None of these

265. If $8f(x) + 6f\left(\frac{1}{x}\right) = x + 5$ and $y = x^2 f(x)$, then the value of $\frac{dy}{dx}$ at $x = -1$, is

a) 0 b) $\frac{1}{14}$ c) $-\frac{1}{14}$ d) $\frac{1}{7}$

266. If $y=\sqrt{\frac{1-x}{1+x}}$, then $(1-x^2)\frac{dy}{dx}+y$ is equal to

a) 1 b) -1 c) 2 d) 0

267. If $x^y=e^{2(x-y)}$, then $\frac{dy}{dx}$ is equal to

a) $\dfrac{2(1+\log x)}{(2+\log x)^2}$ b) $\dfrac{1+\log x}{(2+\log x)^2}$ c) $\dfrac{2}{2+\log x}$ d) $\dfrac{2(1-\log x)}{(2+\log x)^2}$

268. If $x=a(1+\cos\theta), y=a(\theta+\sin\theta)$, then $\frac{d^2y}{dx^2}$ at $\theta=\frac{\pi}{2}$ is

a) $-\dfrac{1}{a}$ b) $\dfrac{1}{a}$ c) -1 d) -2

269. If $y^2=ax^2+bx+c$, where a, b, c are constants, then $y^3\frac{d^2y}{dx^2}$ is equal to

a) a constant b) a function of x

c) a function of y d) a function of x and y both

270. If $y=1+\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}+\cdots\infty$ with $|x|>1$, then $\frac{dy}{dx}$ is

a) $\dfrac{x^2}{y^2}$ b) x^2y^2 c) $\dfrac{y^2}{x^2}$ d) $-\dfrac{y^2}{x^2}$

271. If $f(x,y)=2(x-y)^2-x^4-y^4$, then $\left|(f_{xx}f_{yy}-f_{xy}^2)\right|_{0,0}$ is

a) 32 b) 16 c) 0 d) -1

272. If $y=\left[\tan^{-1}\frac{1}{1+x+x^2}+\tan^{-1}\frac{1}{x^2+3x+3}+\tan^{-1}\frac{1}{x^2+5x+7}+\ldots+\text{upto } n \text{ terms}\right]$, then $y'(0)$is equal to

a) $\dfrac{-1}{(n^2+1)}$ b) $\dfrac{-n^2}{(n^2+1)}$ c) $\dfrac{n^2}{(n^2+1)}$ d) None of these

273. Given that $\frac{d}{dx}f(x)=f'(x)$. The relationship $f'(a+b)=f'(a)+f'(b)$ is valid, if $f(x)$ is equal to

a) x b) x^2 c) x^3 d) x^4

274. nth derivative of $(x+1)^n$ is equal to

a) $(n-1)!$ b) $(n+1)!$ c) $n!$ d) $n[(n+1)]^{n-1}$

275. If $y=2^x.3^{2x-1}$, then $\frac{dy}{dx}$ is equal to

a) $(\log 2)(\log 3)$ b) $(\log 18)$ c) $(\log 18^2)y^2$ d) $(\log 18)y$

276. If $y=x^2+\cfrac{1}{x^2+\cfrac{1}{x^2+\cfrac{1}{x^2+\cdots\infty}}}$, then $\frac{dy}{dx}$ is equal to

a) $\dfrac{2xy}{2y-x^2}$ b) $\dfrac{xy}{y+x^2}$ c) $\dfrac{xy}{y-x^2}$ d) $\dfrac{2x}{2+\frac{x^2}{y}}$

277. If $y=\frac{e^x+e^{-x}}{e^x-e^{-x}}$, then $\frac{dy}{dx}$ is equal to

a) $\operatorname{sech}^2 x$ b) $\operatorname{cosech}^2 x$ c) $-\operatorname{sech}^2 x$ d) $-\operatorname{cosech}^2 x$

278. If $f(x)=|x-2|$and $g(x)=f(f(x))$, then for $x>20, g'(x)$ is equal to

a) -1 b) 0 c) 1 d) 2

279. If $5f(x)+3f\left(\frac{1}{x}\right)=x+2$ and $y=x\,f(x)$, then $\left(\frac{dy}{dx}\right)_{x=1}$ is equal to

a) 14 b) 7/8 c) 1 d) None of these

280. The differential coefficient of $f(\log x)$, where $f(x)=\log x$ is

a) $\dfrac{x}{\log x}$ b) $(x\log x)^{-1}$ c) $\dfrac{\log x}{x}$ d) None of these

281. Let $f(x)=\frac{x^2}{1-x^2}, x\neq 0, \pm 1$, then derivative of $f(x)$ with respect to x is

a) $\dfrac{2x}{(1-x^2)^2}$ b) $\dfrac{1}{(2+x^2)^3}$ c) $\dfrac{1}{(1-x^2)^2}$ d) $\dfrac{1}{(2-x^2)^2}$

282. If $f(x)=|x|^3$, then $f'(0)$ equal to

a) 0 b) 1/2 c) -1 d) $-\dfrac{1}{2}$

283. The derivative of $\log |x|$ is

a) $\frac{1}{x}, x > 0$ b) $\frac{1}{|x|}, x \neq 0$ c) $\frac{1}{x}, x \neq 0$ d) None of these

284. If $f(x) = \tan^{-1}\left\{\frac{\log\left(\frac{e}{x^2}\right)}{\log(e\,x^2)}\right\} + \tan^{-1}\left(\frac{3+2\ \log x}{1-6\ \log x}\right)$, then $\frac{d^n y}{dx^n}$ is

a) $\tan^{-1}\{(\log x)^n\}$ b) 0 c) 1/2 d) None of these

285. If $f(x) = x + 2$, then $f'(f(x))$ at $x = 4$, is

a) 8 b) 1 c) 4 d) 5

286. If $f(x) - \log_e(\log_e x)$, then $f'(x)$ at $x = e$, is

a) 0 b) 1 c) $\frac{1}{e}$ d) $\frac{e}{2}$

287. If $\sin(x+y) + \cos(x+y) = \log(x+y)$, then $\frac{d^2y}{dx^2}$ is

a) $\frac{-y}{x}$ b) 0 c) -1 d) 1

288. $10^{-x\tan x}\left[\frac{d}{dx}(10^{x\tan x})\right]$ is equal to

a) $\tan x + x\sec^2 x$ b) In 10 $(\tan x + x\sec^2 x)$

c) In 10 $\left(\tan x + \frac{x}{\cos^2 x} + \tan x \sec x\right)$ d) $x\tan x$ In 10

289. If $y = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \cdots$, then $\frac{d^2y}{dx^2}$ is equal to

a) $-x$ b) x c) y d) $-y$

290. Differential coefficient of $\sec^{-1}\frac{1}{2x^2-1}$ with respect to $\sqrt{1-x^2}$ at $x = \frac{1}{2}$ is equal to

a) 2 b) 4 c) 6 d) 1

291. If $y = \cos 2\,x \cos 3\,x$, then y_n is equal to

a) $6^n \cos\left(2\,x + \frac{n\,\pi}{2}\right)\cos\left(3\,x + \frac{n\,\pi}{2}\right)$

b) $6^n \cos\left(2\,x + \frac{n\,\pi}{2}\right)\cos\left(\frac{3\,x + n\,\pi}{2}\right)$

c) $\frac{1}{2}\left\{5^n \sin\left(5x + \frac{n\,\pi}{2}\right) + \sin\left(x + \frac{\pi}{2}\right)\right\}$

d) None of these

292. If $f(x) = log_{x^2}\,(log_e x), then\ f'(x)\ at\ x = e$ is

a) 1 b) $\frac{1}{e}$ c) $\frac{1}{2e}$ d) 0

293. Let $fx) = \sin x, g(x) = x^2$and $h(x) = \log_e x$. If $F(x) = (hogof)(x)$, then $F''(x)$ is equal to

a) $a\ \text{cosec}^3 x$ b) $2\cot x^2 - 4x^2\text{cosec}^2 x^2$ c) $2x\cot x^2$ d) $-2\ \text{cosec}^2 x$

294. $x = \cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$, $y = \sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right) \Rightarrow \frac{dy}{dx}$ is equal to

a) 0 b) $\tan t$ c) 1 d) $\sin t \cos t$

295. If $y = \tan^{-1}\left(\frac{\log(e/x^2)}{\log(ex^2)}\right) + \tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$, then $\frac{d^2y}{dx^2}$ is equal to

a) 2 b) 1 c) 0 d) -1

296. $\frac{d}{dx}\left[\sin^2\cot^{-1}\left\{\sqrt{\frac{1-x}{1+x}}\right\}\right]$ equals

a) -1 b) $\frac{1}{2}$ c) $-\frac{1}{2}$ d) 1

297. If $f(x) = \left\{\frac{\pi}{2}[x] - x^5\right\}$, $1 < x < 2$ and $[\cdot]$ denotes the greatest integer function, then $f'\left(\sqrt[5]{\frac{\pi}{2}}\right)$ is equal to

a) 0 b) $5(\pi/2)^{4/5}$ c) $-5(\pi/2)^{4/5}$ d) None of these

298. Let f be twice differentiable function such that $f''(x) = -f(x)$, and $f'(x) = g(x)$, $h(x) = \{f(x)\}^2 + \{g(x)\}^2$. If $h(5) = 11$, then $h(10)$ is equal to

a) 22 b) 11 c) 0 d) 20

299. $f(x)=\begin{vmatrix} x^3 & x^4 & 3x^2 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$, here p is a constant, then $\frac{d^4f(x)}{dx^4}$ is

a) Proportional to x^2 b) Proportional to x c) Proportional to x^3 d) A constant

300. If $y=x+e^x$, then $\frac{d^2x}{dy^2}$ is

a) e^x b) $-\frac{e^x}{(1+e^x)^3}$ c) $-\frac{e^x}{(1+e^x)^2}$ d) $\frac{1}{(1+e^x)^2}$

301. If $y=f\left(\frac{2x+3}{3-2x}\right)$ and $f(x)=\sin(\log x)$, then $\frac{dy}{dx}$ is equal to

a) $\frac{12}{9-4x^2}\cos\left\{\log\left(\frac{2x+3}{3-2x}\right)\right\}$ b) $\frac{12}{4x^2-9}\cos\left\{\log\left(\frac{2x+3}{3-2x}\right)\right\}$

c) $\frac{12}{9-4x^2}\cos\left\{\log\left(\frac{3-2x}{2x+3}\right)\right\}$ d) $\frac{12}{9-4x^2}\cos\left\{\log\left(\frac{2x+3}{2x-3}\right)\right\}$

302. If $y=\cos^{-1}\left(\frac{1-\log x}{1+\log x}\right)$, then $\frac{dy}{dx}$ at $x=e$ is

a) $-\frac{1}{e}$ b) $-\frac{1}{2e}$ c) $\frac{1}{2e}$ d) $\frac{1}{e}$

303. If $x^y=y^x$, then $x(x-y\log x)\frac{dy}{dx}$ is equal to

a) $y(y-x\log y)$ b) $y(y+x\log y)$ c) $x(x+y\log x)$ d) $x(y-x\log y)$

304. If variables x and y are related by the equation

$x=\int_0^y \frac{1}{\sqrt{1+9u^2}}du$, then $\frac{dy}{dx}$ is equal to

a) $\frac{1}{\sqrt{1+9y^2}}$ b) $\sqrt{1+9y^2}$ c) $1+9y^2$ d) $\frac{1}{1+9y^2}$

305. If $y=\sec^{-1}[\operatorname{cosec} x]+\operatorname{cosec}^{-1}[\sec x]+\sin^{-1}[\cos x]+\cos^{-1}[\sin x]$, then $\frac{dy}{dx}$ is equal

To

a) 0 b) 2 c) -2 d) -4

306. The value of $\frac{d}{dx}(|x-1|+|x-5|)$ at $x=3$ is

a) -2 b) 0 c) 2 d) 4

307. If $f(x)=\log_e\left\{\frac{u(x)}{v(x)}\right\}$, $u(1)=v(1)$ and $u'(1)=v'(1)=2$, then $f'(1)$ is equal to

a) 0 b) 1 c) -1 d) None of these

308. $\frac{d}{dx}\left\{\sin^2\left(\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)\right\}$ equals

a) -1 b) $\frac{1}{2}$ c) $-\frac{1}{2}$ d) 1

309. If $x=\frac{1-t^2}{1+t^2}$ and $y=\frac{2t}{1+t^2}$, then $\frac{dy}{dx}$ is equal to

a) $-\frac{y}{x}$ b) $\frac{y}{x}$ c) $-\frac{x}{y}$ d) $\frac{x}{y}$

310. If $x=y\sqrt{1-y^2}$, then $\frac{dy}{dx}$ is equal to

a) x b) $\frac{\sqrt{1-y^2}}{1+2y^2}$ c) 0 d) $\frac{\sqrt{1-y^2}}{1-2y^2}$

311. If $f(x)=(x+1)\tan^{-1}(e^{-2x})$, then $f'(0)$ is

a) $\frac{\pi}{2}+1$ b) $\frac{\pi}{4}-1$ c) $\frac{\pi}{6}+5$ d) $\frac{\pi}{4}+1$

312. If $y=5^x x^5$, then $\frac{dy}{dx}$ is

a) $5^x(x^5\log 5-5x^4)$ b) $x^5\log 5-5x^4$ c) $x^5\log 5+5x^4$ d) $5^x(x^5\log 5+5x^4)$

313. If $y=a\cos(\log x)+b\sin(\log x)$where a,b are parameters, then $x^2y''+xy'$is equal to

a) y b) $-y$ c) $2y$ d) $-2y$

314. If $y=\sin^{-1}\left(\frac{\sin\alpha\sin x}{1-\cos\alpha\sin x}\right)$, then $y'(0)$ is

a) 1 b) $2\tan\alpha$ c) $(1/2)\tan\alpha$ d) $\sin\alpha$

315. If $f(x)=|x-2|+|x+1|-x$, then $f'(-10)$ is equal to

a) -3 b) -2 c) -1 d) 0

316. If $f: R\to R$ is an even function which us twice differentiable on R and $f''(\pi)=1$, then $f''(-\pi)$

a) -1 b) 0 c) 1 d) 2

317. $\frac{d}{dx}[\log_e e^{\sin(x^2)}]$ is equal to

a) $2\cos(x^2)$ b) $2\cos x$ c) $2x.\cos x$ d) $2x\cos(x^2)$

318. If $f'(x)=g(x)$ and $g'(x)=-f(x)$ for all x and $f(2)=4=f'(2)$, then $f^2(4)+g^2(4)$ is

a) 8 b) 16 c) 32 d) 64

319. $z=\tan(y+ax)+\sqrt{y-ax}\Rightarrow z_{xx}-a^2z_{yy}$ is equal to

a) 0 b) 2 c) z_x+z_y d) z_xz_y

320. $\frac{d}{dx}(\log x)^4$ is equal to

a) $4\log x^3$ b) $4(\log x)^3$ c) $\frac{4(\log x)^3}{x}$ d) $\frac{(4\log x)^3}{x}$

321. If $\sin^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)=\log a$, then $\frac{dy}{dx}$ equals

a) $\frac{x}{y}$ b) $\frac{y}{x^2}$ c) $\frac{x^2-y^2}{x^2+y^2}$ d) $\frac{y}{x}$

322. The differential coefficient of $a^{\log_{10}\operatorname{cosec}^{-1}x}$, is

a) $\frac{a^{\log_{10}(\operatorname{cosec}^{-1}x)}}{\operatorname{cosec}^{-1}x}\frac{1}{x\sqrt{x^2-1}}\log_{10}a$

b) $-\frac{a^{\log_{10}(\operatorname{cosec}^{-1}x)}}{\operatorname{cosec}^{-1}x}\cdot\frac{1}{|x|\sqrt{x^2-1}}\log_{10}a$

c) $\frac{-a^{\log_{10}(\operatorname{cosec}^{-1}x)}}{\operatorname{cosec}^{-1}x}\cdot\frac{1}{|x|\sqrt{x^2-1}}\log_a 10$

d) $\frac{a^{\log_{10}\operatorname{cosec}^{-1}x}}{\operatorname{cosec}^{-1}x}\cdot\frac{1}{|x|\sqrt{x^2-1}}\log_a 10$

323. If $f(x)=\sqrt{1+\cos^2(x)^2}$, then the value of $f'\left(\frac{\sqrt{\pi}}{2}\right)$ is

a) $\frac{\sqrt{\pi}}{6}$ b) $-\sqrt{\frac{\pi}{6}}$ c) $\frac{1}{\sqrt{6}}$ d) $\frac{\pi}{\sqrt{6}}$

324. $\frac{d^n}{dx^n}(\log x)$ is equal to

a) $\frac{(n-1)!}{x^n}$ b) $\frac{n!}{x^n}$ c) $\frac{(n-2)!}{x^n}$ d) $(-1)^{n-1}\frac{(n-1)!}{x^n}$

325. $\frac{d^2x}{dy^2}$ equals

a) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ b) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$ c) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ d) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

326. The second order derivative of $a\sin^3 t$ with respect to $a\cos^3 t$ at $t=\frac{\pi}{4}$ is

a) 2 b) $\frac{1}{12a}$ c) $\frac{4\sqrt{2}}{3a}$ d) $\frac{3a}{4\sqrt{2}}$

327. If $\frac{d^2x}{dy^2}\left(\frac{dy}{dx}\right)^3+\frac{d^2y}{dx^2}=k$, then k is equal to

a) 0 b) 1 c) 2 d) None of these

328. If $e^y+xy=e$, then the value of $\frac{d^2y}{dx^2}$ for $x=0$, is

a) $1/e$ b) $1/e^2$ c) $1/e^3$ d) e

329. $\frac{d}{dx}\sin^{-1}\left(x\sqrt{1-x}+\sqrt{x}\sqrt{1-x^2}\right)$ is equal to

a) $-\frac{1}{2x\sqrt{1-x}}-\frac{1}{\sqrt{1-x^2}}$ b) $\frac{1}{2\sqrt{x}\sqrt{1-x^2}}-\frac{1}{\sqrt{1-x^2}}$
c) $\frac{1}{2\sqrt{x}\sqrt{1-x}}+\frac{1}{\sqrt{1-x^2}}$ d) $-\frac{1}{2\sqrt{x}\sqrt{1-x}}+\frac{1}{\sqrt{1-x^2}}$

330. If $f(x)=\log_x(\log_e x)$, then $f'(x)$ at $x=e$ is equal to
a) 1 b) 2 c) 0 d) $\frac{1}{e}$

331. If $y=\log_u|\cos 4x|+|\sin x|$, where $u=\sec 2x$, then $\frac{dy}{dx}$ at $x=-\frac{\pi}{6}$ is equal to
a) $\frac{-6\sqrt{3}}{\log_e 2}-\frac{\sqrt{3}}{2}$ b) $\frac{-6\sqrt{3}}{\log_e 2}+\frac{\sqrt{3}}{2}$ c) $\frac{6\sqrt{3}}{\log_e 2}+\frac{\sqrt{3}}{2}$ d) None of these

332. If $\sqrt{1-x^6}+\sqrt{1-y^6}=a(x^3-y^3)$ and $\frac{dy}{dx}=f(x,y)\sqrt{\frac{1-y^6}{1-x^6}}$, then $f(x,y)=$
a) $\frac{y}{x}$ b) $\frac{x^2}{y^2}$ c) $\frac{2y^2}{x^2}$ d) $\frac{y^2}{x^2}$

333. If $y=\log^n x$, where $\log^n$ means log log log … (repeated n times), then
$x\log x\log^2 x\log^3 x\ldots\log^{n-1} x\log^n x\frac{dy}{dx}$ is equal to
a) $\log x$ b) x c) $\frac{1}{\log x}$ d) $\log^n x$

334. If $x^2+y^2=t+\frac{1}{t}$ and $x^4+y^4=t^2+\frac{1}{t^2}$, then $\frac{dy}{dx}$ is equal to
a) $\frac{y}{x}$ b) $-\frac{y}{x}$ c) $\frac{x}{y}$ d) $-\frac{x}{y}$

335. If $y=\sqrt{x\log_e x}$,then $\frac{dy}{dx}$ at $x=e$ is
a) $\frac{1}{e}$ b) $\frac{1}{\sqrt{e}}$ c) $\sqrt{e}$ d) e^2

336. If $y=\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$, then $\frac{dy}{dx}$ equals
a) $\frac{1}{\sqrt{1-x^4}}$ b) $-\frac{1}{\sqrt{1-x^4}}$ c) $\frac{x}{\sqrt{1-x^4}}$ d) $-\frac{x}{\sqrt{1-x^4}}$

337. If $f'(x)=\sin(\log x)$ and $y=f\left(\frac{2x+3}{3-2x}\right)$, then $\frac{dy}{dx}$ at $x=1$ is equal to
a) 6 sin log (5) b) 5 sin log (6) c) 12 sin log (5) d) 5 sin log (12)

338. If $x^y=e^{x-y}$, $x>0$ then the value of $\frac{dy}{dx}$ at (1,1) is
a) 0 b) $\frac{1}{2}$ c) 1 d) 2

339. If $y=\sin^{-1}\left(\frac{5x+12\sqrt{1-x^2}}{13}\right)$, then $\frac{dy}{dx}$ is equal to
a) $-\frac{1}{\sqrt{1-x^2}}$ b) $\frac{1}{\sqrt{1-x^2}}$ c) $\frac{3}{\sqrt{1-x^2}}$ d) $\frac{x}{\sqrt{1-x^2}}$

340. If $y=e^x . e^{x^2} . e^{x^3} \ldots e^{x^n} \ldots$, for $0<x<1$, then $\frac{dy}{dx}$ at $x=\frac{1}{2}$ is
a) e b) $4e$ c) $2e$ d) $3e$

341. If $y=\sin^{-1}\left(\frac{5x+12\sqrt{1-x^2}}{13}\right)$, then $\frac{dy}{dx}$ is equal to
a) $\frac{1}{\sqrt{1-x^2}}$ b) $\frac{-1}{\sqrt{1-x^2}}$ c) $\frac{3}{\sqrt{1-x^2}}$ d) $\frac{1}{\sqrt{1+x^2}}$

342. For $0<x<2$, $\frac{d}{dx}\left(\tan^{-1}\sqrt{\frac{1+\cos\frac{x}{2}}{1-\cos\frac{x}{2}}}\right)$ is equal to
a) $-1/4$ b) $1/4$ c) $-1/2$ d) $1/2$

343. The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is

a) -1 b) 1 c) 2 d) 4

344. Let $f(x)$ be a polynomial function of the second degree. If $f(1) = f(-1)$ and a_1, a_2, a_3 are in AP, then $f'(a_1), f'(a_2), f'(a_3)$ are in

a) AP b) GP c) HP d) None of these

345. The derivative of $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to $\cot^{-1}\left(\frac{1-3x^2}{3x-x^3}\right)$ is

a) 1 b) $\frac{3}{2}$ c) $\frac{2}{3}$ d) $\frac{1}{2}$

346. If $y = \log_{\sin x} \cos x$, then $\frac{dy}{dx}$ is equal to

a) $\frac{\tan x \log(\sin x) - \cot x \log \cos x}{(\log \sin x)^2}$

b) $\frac{-\tan x \log \sin x - \cot x \log \cos x}{(\log \sin x)^2}$

c) $\frac{-\tan x \log(\sin x) + \cot x \log(\cos x)}{(\log \sin x)^2}$

d) $\frac{\tan x \log(\sin x) + \cot \log(\cos x)}{[\log(\sin x)]^2}$

347. If $y = \tan^{-1}\left(\frac{\log(e/x^2)}{\log(ex^2)}\right) + \tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$, then $\frac{d^2y}{dx^2}$ is

a) 2 b) 1 c) 0 d) -1

348. If $y = c\, e^{x/(x-a)}$, then $\frac{dy}{dx}$ equals

a) $a(x-a)^2$ b) $-\frac{ay}{(x-a)^2}$ c) $a^2(x-a)^2$ d) $a(x-a)$

349. If $y = \sin^{-1}\sqrt{1-x}$, then $\frac{\mathrm{dy}}{\mathrm{dx}}$ is equal to

a) $\frac{1}{\sqrt{1-x}}$ b) $\frac{-1}{2\sqrt{1-x}}$ c) $\frac{1}{\sqrt{x}}$ d) $\frac{-1}{2\sqrt{x}\sqrt{1-x}}$

350. If $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$, then $\frac{dy}{dx}$ is

a) $\frac{-2}{1+x^2}$ for all x b) $\frac{-2}{1+x^2}$ for all $|x| > 1$ c) $\frac{2}{1+x^2}$ for all $|x| > 1$ d) None of these

351. If $y = ae^x + be^{-x} + c$ where a, b, c are parameters, then y' is equal to

a) y b) y' c) 0 d) y''

: ANSWER KEY :

1)	b	2)	b	3)	c	4)	d	189)	b	190)	c	191)	a	192)	d
5)	c	6)	d	7)	b	8)	d	193)	b	194)	c	195)	c	196)	b
9)	c	10)	c	11)	b	12)	d	197)	c	198)	b	199)	d	200)	c
13)	d	14)	c	15)	b	16)	a	201)	a	202)	b	203)	a	204)	d
17)	b	18)	a	19)	c	20)	d	205)	d	206)	d	207)	d	208)	c
21)	b	22)	d	23)	c	24)	c	209)	d	210)	a	211)	b	212)	a
25)	b	26)	b	27)	d	28)	d	213)	c	214)	c	215)	c	216)	d
29)	c	30)	d	31)	d	32)	a	217)	b	218)	d	219)	c	220)	b
33)	c	34)	c	35)	c	36)	d	221)	a	222)	c	223)	b	224)	c
37)	b	38)	d	39)	c	40)	b	225)	a	226)	d	227)	d	228)	a
41)	a	42)	c	43)	c	44)	a	229)	b	230)	b	231)	d	232)	a
45)	a	46)	b	47)	a	48)	c	233)	c	234)	c	235)	a	236)	c
49)	c	50)	a	51)	b	52)	c	237)	d	238)	a	239)	a	240)	d
53)	b	54)	b	55)	d	56)	c	241)	b	242)	a	243)	b	244)	c
57)	d	58)	b	59)	b	60)	c	245)	a	246)	b	247)	c	248)	c
61)	b	62)	b	63)	a	64)	c	249)	d	250)	b	251)	d	252)	d
65)	b	66)	b	67)	c	68)	a	253)	a	254)	d	255)	b	256)	a
69)	d	70)	c	71)	b	72)	a	257)	c	258)	c	259)	c	260)	c
73)	a	74)	a	75)	a	76)	d	261)	a	262)	a	263)	a	264)	d
77)	d	78)	c	79)	b	80)	b	265)	c	266)	d	267)	a	268)	a
81)	a	82)	c	83)	d	84)	d	269)	a	270)	d	271)	c	272)	c
85)	c	86)	d	87)	b	88)	c	273)	b	274)	c	275)	d	276)	a
89)	d	90)	b	91)	c	92)	a	277)	d	278)	c	279)	b	280)	b
93)	a	94)	a	95)	a	96)	b	281)	a	282)	a	283)	c	284)	b
97)	d	98)	d	99)	c	100)	a	285)	b	286)	c	287)	b	288)	b
101)	c	102)	b	103)	c	104)	c	289)	c	290)	b	291)	d	292)	c
105)	c	106)	c	107)	b	108)	a	293)	d	294)	c	295)	c	296)	b
109)	c	110)	d	111)	c	112)	c	297)	c	298)	b	299)	d	300)	b
113)	c	114)	a	115)	c	116)	a	301)	a	302)	c	303)	a	304)	b
117)	b	118)	b	119)	b	120)	b	305)	d	306)	b	307)	a	308)	b
121)	d	122)	a	123)	c	124)	a	309)	c	310)	b	311)	b	312)	d
125)	a	126)	b	127)	d	128)	b	313)	b	314)	d	315)	a	316)	c
129)	d	130)	d	131)	b	132)	d	317)	d	318)	c	319)	a	320)	c
133)	d	134)	b	135)	c	136)	a	321)	d	322)	b	323)	b	324)	d
137)	b	138)	b	139)	d	140)	a	325)	d	326)	c	327)	a	328)	b
141)	b	142)	a	143)	c	144)	b	329)	c	330)	d	331)	a	332)	b
145)	a	146)	b	147)	d	148)	b	333)	d	334)	b	335)	b	336)	d
149)	b	150)	a	151)	a	152)	d	337)	c	338)	a	339)	b	340)	b
153)	d	154)	c	155)	c	156)	d	341)	a	342)	a	343)	b	344)	a
157)	b	158)	b	159)	a	160)	b	345)	c	346)	b	347)	c	348)	b
161)	b	162)	d	163)	b	164)	c	349)	d	350)	d	351)	b		
165)	c	166)	a	167)	c	168)	b								
169)	b	170)	d	171)	d	172)	a								
173)	c	174)	b	175)	a	176)	b								
177)	b	178)	b	179)	b	180)	c								
181)	a	182)	b	183)	b	184)	b								
185)	a	186)	c	187)	b	188)	c								

: HINTS AND SOLUTIONS :

1 **(b)**

$f(x^3) = 4x^4, \forall x > 0$

Let $x^3 = t \Rightarrow x = t^{1/3}$

$\therefore f(t) = 4t^{4/3}$

On differentiating w.r.t. t, we get

$f'(t) = 4.\frac{4}{3}(t)^{1/3}$

$\therefore f'(x^3) = \frac{16}{3}(x^3)^{1/3} = \frac{16}{3}x$

$\therefore f'(8) = f'(2^3) = \frac{16}{3} \times 2 = \frac{32}{3}$

2 **(b)**

$\because y = x - x^2$

$\therefore \frac{dy}{dx} = 1 - 2x$

Now,

$\frac{d(y^2)}{d(x^2)} = \frac{\frac{d(y^2)}{dx}}{\frac{d(x^2)}{dx}}$

$= \frac{2y\frac{dy}{dx}}{2x}$

$= \frac{y}{x}(1 - 2x)$

$= (1 - x)(1 - 2x)$

$= 1 - 3x + 2x^2$

$= 2x^2 - 3x + 1$

3 **(c)**

$\because y = e^{\frac{1}{2}\log(1+\tan^2 x)}$

$= (\sec^2 x)^{1/2} = \sec x$

$\therefore \frac{dy}{dx} = \sec x \tan x$

4 **(d)**

Given, $y = 1 + x + x^2 + x^3 + \ldots \infty$

Since, $|x| < 1$

$\therefore y = \frac{1}{(1-x)} = (1-x)^{-1}$

$\Rightarrow \frac{dy}{dx} = -\frac{1}{(1-x)^2}(-1) = \frac{1}{(1-x)^2}$

$\therefore \frac{dy}{dx} - y = \frac{1}{(1-x)^2} - \frac{1}{(1-x)} = \frac{x}{(1-x)^2}$

$\Rightarrow \frac{dy}{dx} - y = xy^2$

5 **(c)**

$f(x) = x^n \Rightarrow f(1) = 1$

$f'(x) = nx^{n-1} \Rightarrow f'(1) = n$

$f''(x) = n(n-1)x^{n-2} \Rightarrow f''(1) = n(n-1)$

… … … … … … …

… … … … … … …

$f^n(x) = n(n-1)(n-2) \ldots 2.1$

$\Rightarrow f^n(1) = n(n-1)(n-2) \ldots 2.1$

Now, $f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \frac{f'''(1)}{3!} + \ldots + \frac{(-1)^n f^n(1)}{n!}$

$= 1 - \frac{n}{1!} + \frac{n(n-1)}{2!}$

$- \frac{n(n-1)(n-2)}{3!} + \ldots + \frac{(-1)^n n(n-1)(n-2) \ldots}{n!}$

$= (1-1)^n = 0$

6 **(d)**

Let $y = 2\cos x \cos 3x$

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = 2\cos x(-3\sin 3x) + 2\cos 3x(-\sin x)$

$= -3(\sin 4x + \sin 2x) + (-1)[\sin 4x + \sin(-2x)]$

Again differentiating w.r.t. x, we get

$\frac{d^2y}{dx^2} = -3(4\cos 4x$

$+ 2\cos 2x)$

$- 1(4\cos 4x - 2\cos 2x)$

$= -16\cos 4x - 4\cos 2x = -4(\cos 2x + 4\cos 4x)$

$= -2^2(\cos 2x + 2^2\cos 4x)$

7 **(b)**

We have,

$y = x^{x^{x^{x^{x\ldots\infty}}}}$

$\Rightarrow y = x^y$

$\Rightarrow y = e^{y\log x}$

$\Rightarrow \frac{dy}{dx} = e^{y\log x}\frac{d}{dy}(y\log x)$

$\Rightarrow \frac{dy}{dx} = x^y\left(\frac{dy}{dx}\log x + \frac{y}{x}\right)$

$\Rightarrow \frac{dy}{dx}(1 - y\log x) = \frac{y^2}{x} \quad [\because x^y = y]$

$\Rightarrow x(1 - y\log x)\frac{dy}{dx} = y^2$

8 **(d)**

Given, $f(x) = \frac{g(x)+g(-x)}{2} + \frac{2}{[h(x)+h(-x)]^{-1}}$

$\Rightarrow (x) = \frac{g(x)+g(-x)}{2} + 2[h(x) + h(-x)]$

On differentiating w.r.t. x, we get

$f'(x) = \frac{g'(x) - g'(-x)}{2} + 2[h'(x) - h'(-x)]$

$\therefore f'(0) = \frac{g'(0) - g'(0)}{2} + 2[h'(0) - h'(0)]$

$= 0$

9 **(c)**

Let $y = \frac{d}{dx}[\sin^{-1}(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2})]$

Put $x = \sin\alpha$ and $\sqrt{x} = \sin\beta$

$$\therefore y = \frac{d}{dx}[\sin^{-1}(\sin\alpha\sqrt{1-\sin^2\beta} - \sin\beta\sqrt{1-\sin^2\alpha})]$$

$$= \frac{d}{dx}[\sin^{-1}\sin(\alpha-\beta)] = \frac{d}{dx}(\alpha-\beta)$$

$$= \frac{d}{dx}[\sin^{-1}x - \sin^{-1}\sqrt{x}]$$

$$= \frac{1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

$$= \frac{1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{x(1-x)}}$$

11 **(b)**

$z = \log(\tan x + \tan y)$

On differentiating partially w.r.t. x and y, we get

$$\frac{\partial z}{\partial x} = \frac{\sec^2 x}{\tan x + \tan y} \text{ and } \frac{\partial z}{\partial y} = \frac{\sec^2 y}{\tan x + \tan y}$$

Now, $\sin 2x\,\frac{\partial z}{\partial x} + \sin 2y\frac{\partial z}{\partial y}$

$$= \frac{\sin 2x\sec^2 x + \sin 2y\sec^2 y}{\tan x + \tan y}$$

$$= \frac{2[\tan x + \tan y]}{\tan x + \tan y} = 2$$

12 **(d)**

Let $u = \sin^{-1}\left(\frac{1}{1-2x^2}\right)$, $v = \sin^{-1}(3x - 4x^3)$

Put $x = \sin\theta$, we get

$u = \sec^{-1}(\sec 2\theta), v = \sin^{-1}(\sin 3\theta)$

$\Rightarrow \quad u = 2\theta, \quad v = 3\theta \quad \Rightarrow \quad u = 2\sin^{-1}x, \quad v = 3\sin^{-1}x$

$\Rightarrow \frac{u}{v} = \frac{2}{3} \Rightarrow u = \frac{2}{3}v$

$\Rightarrow \frac{du}{dv} = \frac{2}{3}$

13 **(d)**

$$f(x,y) = \sin u = \frac{x^2y^2}{x+y}$$

Here, $f(x)$ is a homogenous function of degree 3.

By Euler theorem,

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 3f$$

$$x\frac{\partial \sin u}{\partial x} + y\frac{\partial \sin u}{\partial y} = 3\sin u$$

$$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\tan u$$

14 **(c)**

$$y = \left(1+\frac{1}{x}\right)\left(1+\frac{2}{x}\right)\left(1+\frac{3}{x}\right)\dots\left(1+\frac{n}{x}\right)$$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \left(-\frac{1}{x^2}\right)\left(1+\frac{2}{x}\right)\left(1+\frac{3}{x}\right)\dots\left(1+\frac{n}{x}\right)$$

$$+\left(1+\frac{1}{x}\right)\left(-\frac{2}{x^2}\right)\left(1+\frac{3}{x}\right)\dots\left(1+\frac{n}{x}\right)$$

$$+\dots+\left(1+\frac{1}{x}\right)\left(1+\frac{2}{x}\right)\left(1+\frac{3}{x}\right)\dots\left(-\frac{n}{x^2}\right)$$

$$\therefore \left.\frac{dy}{dx}\right|_{x=-1} = (-1)(-1)(-2)(-3)\dots(1-n)$$

$$= (-1)^n(1)(2)(3)\dots(n-1)$$

$$= (-1)^n(n-1)!$$

15 **(b)**

Given, $y = a\sin^3\theta$ and $x = a\cos^3\theta$

On differentiating w.r.t. θ, we get

$\frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$

and $\frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta$

$$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{3a\sin^2\theta\cos\theta}{3a\cos^2\theta\sin\theta} = -\tan\theta$$

At $\theta = \frac{\pi}{3}, \frac{dy}{dx} = -\tan\frac{\pi}{3} = -\sqrt{3}$

16 **(a)**

Since, to find $\frac{dy}{dx}$ at $x = 0$

$\therefore$ At $x = 0, \log(y+0) = 0 \quad \Rightarrow \quad y = 1$

$\therefore$ To find $\frac{dy}{dx}$ at $(0,1)$

On differentiating given equation w.r.t. x, we get

$$\frac{1}{x+y}\left(1+\frac{dy}{dx}\right) = 2x\frac{dy}{dx} + 2y.1$$

$$\Rightarrow \frac{dy}{dx} = \frac{2y(x+y)-1}{1-2(x+y)x}$$

$$\therefore \left(\frac{dy}{dx}\right)_{(0,1)} = 1$$

17 **(b)**

Since $f(x)$ is a polynomial of degree n satisfying $f(x) = f(\alpha - x)$. Therefore,

$$f(x) = a_0\{x^n + (\alpha-x)^n\} + a_1\{x^{n-1} + (\alpha-x)^{n-1}\} + \dots + \dots a_{n-1}\{x + (\alpha - x)\} + a_n \text{ where } a_0 \neq 0$$

Clearly, $f'(x)$ is a polynomial of degree $(n-1)$

18 **(a)**

$f(x) = \log_5(\log_7 x)$

$$\Rightarrow f(x) = \log_5\left(\frac{\log_e x}{\log_e 7}\right)$$

$$\Rightarrow f(x) = \log_5\log_e x - \log_5\log_e 7$$

$$\Rightarrow f(x) = \frac{\log_e\log_e x}{\log_e 5} - \log_5\log_e 7$$

On differentiating w.r.t. x, we get

$$f'(x)=\frac{1}{x\log_e x\log_{e}5}-0$$
$$=\frac{1}{\frac{x\log_e x}{\log_e 7}\log_e 7.\log_e 5}$$
$$=\frac{1}{x\log_7 x\log 7.\log 5}$$

19 **(c)**

We have,

$$\frac{dy}{dx}=\sqrt{1+9y^2}$$
$$\Rightarrow\frac{d^2y}{dx^2}=\frac{9y}{\sqrt{1+9y^2}}\frac{dy}{dx}=\frac{9y}{\sqrt{1+9y^2}}\sqrt{1+9y^2}$$
$$=9y$$

20 **(d)**

Given, $u=x^2+y^2, x=s+3t, y=2s-t$

Now ,$\frac{dx}{ds}=1,\frac{dy}{ds}=2$ ... (i)

$\frac{d^2x}{ds^2}=0,\frac{d^2y}{ds^2}=0$ ... (ii)

Now, $u=x^2+y^2$

$$\frac{du}{ds}=2x\frac{dx}{ds}+2y\frac{dy}{ds}$$
$$\frac{d^2u}{ds^2}=2\left(\frac{dx}{ds}\right)^2+2x\frac{d^2x}{ds^2}+2\left(\frac{dy}{ds}\right)^2+2y\left(\frac{d^2y}{ds^2}\right)$$
$$\Rightarrow\frac{d^2u}{ds^2}=2(1)^2+2x(0)+2(2)^2+2y(0)=2+8$$
$$=10$$

21 **(b)**

Given, $f(x)=\sqrt{1+\cos^2(x^2)}$

Now, $f'(x)=\frac{-2\sin x^2\cos x^2}{\sqrt{1+\cos^2 x^2}}(x)=\frac{-\sin 2x^2}{\sqrt{1+\cos^2 x^2}}(x)$

$$\Rightarrow f'\left(\frac{\pi}{2}\right)=\frac{\sqrt{\pi}}{2}\frac{\sin 2\left(\frac{\pi}{4}\right)}{\sqrt{1+\left(\cos\frac{\pi}{4}\right)^2}}=-\sqrt{\frac{\pi}{6}}$$

22 **(d)**

Given $y=2^{\log x}$

$$\Rightarrow\frac{dy}{dx}=2^{\log x}.\log_e 2.\frac{1}{x}$$
$$\Rightarrow\frac{dy}{dx}=\frac{2^{\log x}.\log_e 2}{x}$$

23 **(c)**

Let $y=\left[\log\left\{e^x\left(\frac{x-2}{x+2}\right)^{3/4}\right\}\right]$

$$=\log e^x+\log\left(\frac{x-2}{x+2}\right)^{3/4}$$
$$\Rightarrow y=x+\frac{3}{4}[\log(x-2)-\log(x+2)]$$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{d}{dx}\left[x+\frac{3}{4}\{\log(x-2)-\log(x+2)\}\right]$$
$$=1+\frac{3}{4}\left[\frac{1}{x-2}-\frac{1}{x+2}\right]=1+\frac{3}{(x^2-4)}$$
$$\Rightarrow\frac{dy}{dx}=\frac{x^2-1}{x^2-4}$$

24 **(c)**

Let the even function be

$f(x)=\cos x$

$\Rightarrow f'(x)=-\sin x$

$\Rightarrow f''(x)=-\cos x$

At $x=\pi$, $f''(\pi)=-\cos\pi=1$

$\therefore$ Our assumption is true.

$\therefore$ At $x=-\pi$

$f''(-\pi)=-\cos(-\pi)=1$

Alternate

Since the function is twice differentiable

$\therefore f''(x)=\text{const.}\,\forall x f''(-\pi)=-f''(\pi)=1$

25 **(b)**

We have,

$$x=\int_0^y\frac{1}{\sqrt{1+4t^2}}dt$$
$$\Rightarrow\frac{dy}{dx}=\frac{1}{\frac{1}{\sqrt{1+4y^2}}}$$
$$\Rightarrow\frac{dy}{dx}=\sqrt{1+4y^2}$$
$$\Rightarrow\frac{d^2y}{dx^2}=\frac{4y}{\sqrt{1+4y^2}}\frac{dy}{dx}=\frac{4y}{\sqrt{1+4y^2}}\times\sqrt{1+4y^2}$$
$$=4y$$

26 **(b)**

Given, $3f(x)-2f\left(\frac{1}{x}\right)=x$ ... (i)

Let $\frac{1}{x}=y$, then $3f\left(\frac{1}{y}\right)-2f(y)=\frac{1}{y}$

$$\Rightarrow -2f(y)+3f\left(\frac{1}{y}\right)=\frac{1}{y}$$
$$\Rightarrow -2f(x)+3f\left(\frac{1}{x}\right)=\frac{1}{x}\text{ ... (ii)}$$

Multiply Eq. (i) by 3 and Eq. (ii) by 2 and then adding, we get

$$5f(x)=3x+\frac{2}{x}$$
$$\Rightarrow f(x)=\frac{1}{5}\left[3x+\frac{2}{x}\right]$$

On differentiating w.r.t. x, we get

$$f'(x)=\frac{1}{5}\left[3-\frac{2}{x^2}\right]$$
$$\Rightarrow f'(2)=\frac{1}{5}\left[3-\frac{2}{4}\right]=\frac{1}{2}$$

27 **(d)**

Let $y=\log x$

On differentiating w.r.t. x from 1 to n times, we get

$y_1 = \frac{1}{x}, y_2 = -\frac{1}{x^2}, y_3 = \frac{2}{x^3}, y_4 = -\frac{6}{x^4}, \ldots,$

$\therefore$ by symmctry.

$y_n = \frac{(-1)^{n-1}(n-1)!}{x^n}$

28 **(d)**

$\because x = 2t - |t| = \begin{cases} t, & t \geq 0 \\ 3t, & t < 0' \end{cases}$

$\therefore t = \begin{cases} x, & x \geq 0 \\ x/3, & x < 0 \end{cases}$

$\therefore y = t^2 + t|t| = \begin{cases} 2t^2, & x \geq 0 \\ 0, & x < 0 \end{cases}$

$= \begin{cases} 2x^2, & x \geq 0 \\ 0, & x < 0 \end{cases}$

Hence, $y'(x) = \begin{cases} 4x, & x > 0 \\ 0, & x < 0 \end{cases}$

$\therefore$ We can't find $\frac{dx}{dt}$ as the derivative does not exist at $t = 0$

29 **(c)**

We have,

$f(x) = (x^3 + 2)^{30}$

Clearly, it is a polynomial of degree 90

It is given that $f^n(x)$ is a polynomial of degree 20.

Therefore,

$\therefore 90 - \pi = 20 \Rightarrow n = 70$

30 **(d)**

$y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots \infty}}}$

$\Rightarrow y = \sqrt{x + y}$

On squaring both sides, we get

$y^2 = x + y \Rightarrow 2y\frac{dy}{dx} = 1 + \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}(2y - 1) = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{2y - 1}$

31 **(d)**

Given, $f(x)$ is a polynomial of degree 3.

$\therefore f(x) = x^3 + ax^2 + bx + c$

But $f(x) = x^3 + x^2 + f'(1) + xf''(2) + f'''(3)$

$\therefore a = f'(1), b = f''(2), c = f'''(3)$

Now, $f'(x) = 3x^2 + 2ax + b$

$f''(x) = 6x + 2a$

$f'''(x) = 6$

$\therefore a = f'(1) = 3 + 2a + b$

$\Rightarrow -a - b = 3$ ...(i)

$b = 12 + 2a$

$\Rightarrow -2a + b = 12$ ...(ii)

and $c = 6$ ...(iii)

On solving Eqs. (i), (ii) and (iii), we get

$a = -5, b = 2$ and $c = 6$

$\therefore f(x) = x^3 - 5x^2 + 2x + 6$

Thus, $f(0) = 6, f(1) = 4, f(2) = -2, f(3) = -6$

32 **(a)**

In the given equation put $x = \tan\theta$,we get

$y = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$

$= \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right) = \tan^{-1}\tan\frac{\theta}{2} = \frac{\theta}{2}$

$\Rightarrow y = \frac{1}{2}\tan^{-1}x$

$\Rightarrow y' = \frac{1}{2(1 + x^2)} \quad \Rightarrow y'(0) = \frac{1}{2}$

33 **(c)**

We have,

$y = e^{\sin^{-1}x}$ and $u = \log x$

$\therefore \frac{dy}{dx} = \frac{e^{\sin^{-1}x}}{\sqrt{1 - x^2}}, \frac{du}{dx} = \frac{1}{x} \Rightarrow \frac{dy}{du} = \frac{\frac{dy}{dx}}{\frac{du}{dx}} = \frac{x\, e^{\sin^{-1}x}}{\sqrt{1 - x^2}}$

34 **(c)**

$f\left(\frac{\pi}{4}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{2}\cos\pi\cos 2\pi\cos 4\pi = 0$ ...(i)

Taking log on given function, we get

$\log f(x) = \log\cos x + \log\cos 2x + \log\cos 4x + \log\cos 8x + \log\cos 16x$

$\Rightarrow \frac{1}{f(x)}f'(x) = -\tan x - 2\tan 2x - 4\tan 4x - 8\tan 8x - 16\tan 16x$

$\Rightarrow f'(x) = -f(x)[\tan x + 2\tan 2x + 4\tan 4x + 8\tan 8x + 16\tan 16x]$

$\Rightarrow f'\left(\frac{\pi}{4}\right) = -f\left(\frac{\pi}{4}\right)\left[\tan\frac{\pi}{4} + 2\tan\frac{\pi}{2} + 4\tan\pi + 8\tan 2\pi + 16\tan 4\pi\right]$

$= 0$ [using eq.(i)]

35 **(c)**

Given, $y = \sqrt{\log x + y} \quad \Rightarrow \quad y^2 = \log x + y$

$\Rightarrow 2y\frac{dy}{dx} = \frac{1}{x} + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{x(2y - 1)}$

36 **(d)**

Given, $y = \cos^{-1}\cos x$

For $\leq x \leq \pi, y = x$

$\Rightarrow \frac{dy}{dx} = 1$

For $\pi < x \leq 2\pi, y = \cos^{-1}\cos(2\pi - x)$

$\Rightarrow y = -x$

$\Rightarrow \ \frac{dy}{dx} = -1$

Hence, option (d) is correct.

37 **(b)**

Given, $x = a(\cos\theta + \theta\sin\theta)$

and $y = a(\sin\theta - \theta\cos\theta)$

$\therefore \ \frac{dx}{d\theta} = a(-\sin\theta + \theta\cos\theta + \sin\theta) = a\,\theta\cos\theta$

and $\frac{dy}{d\theta} = a(\cos\theta + \theta\sin\theta - \cos\theta) = a\,\theta\sin\theta$

$\therefore \ \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \tan\theta$

38 **(d)**

Given, $y = \log(\sin(x^2))$

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = \frac{1}{\sin x^2} \,.\cos x^2 .\, 2x = 2x\cot x^2$

At $x = \frac{\sqrt{\pi}}{2}, \frac{dy}{dx} = \frac{2\sqrt{\pi}}{2}\cot\left(\frac{\sqrt{\pi}}{2}\right)^2$

$= \sqrt{\pi}\cot\left(\frac{\pi}{4}\right) = \sqrt{\pi}$

39 **(c)**

We have,

$y = a^{x^{a^{x\ldots\infty}}}$

$\Rightarrow y = a^{x^y}$

$\Rightarrow \log y = x^y \log a$

$\Rightarrow \log(\log y) = y\log x + \log(\log a)$

Differentiating w.r.t. x, we get

$\frac{1}{\log y}\frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}\cdot\log x + \frac{y}{x} \Rightarrow \frac{dy}{dx}$

$= \frac{y^2\log y}{x(1 - \log x\log y)}$

40 **(b)**

$f(1) = 5, f'(x) = nx^{n-1}$ so $f'(1) = n$

$f''(1) = n(n-1), \ldots f^n(1) = 1.2. \ldots. n$

Thus, $f(1) + \frac{f'(1)}{1!} + \ldots + \frac{f^n(1)}{n!}$

$= 5 + \frac{n}{1} + \frac{n(n-1)}{2!} + \ldots + \frac{n!}{n!}$

$= (1+1)^n + 4 = 2^n + 4$

41 **(a)**

Given that, $x = e^t\sin t, y = e^t\cos t$ … (i)

At point (1,1), $1 = e^t\sin t, 1 = e^t\cos t$

$\tan t = 1 \ \Rightarrow t = \frac{\pi}{4}$

On differentiating Eq. (i) w.r.t. x, we get

$\frac{dy}{dt} = e^t(\cos t - \sin t)$

and $\frac{dx}{dt} = e^t(\sin t + \cos t)$

$\therefore \ \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t - \sin t}{\cos t + \sin t}$

Again differentiating w.r.t. x, we get

$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{\cos t - \sin t}{\cos t - \sin t}\right)\frac{dt}{dx}$

$= \left[\frac{[(\cos t + \sin t)(-\sin t - \cos t) - (\cos t - \sin t)(-\sin t + \cos t)]}{(\cos t + \sin t)^2}\right]\frac{dt}{dx}$

$= \frac{-2}{(\cos t + \sin t)^2}\cdot\frac{1}{e^t(\sin t + \cos t)}$

$= \frac{-2}{(e^t\cos t + e^t\sin t)}\cdot\frac{1}{(\cos t + \sin t)^2}$

$= \frac{-2}{x + y}\cdot\frac{1}{(\cos t + \sin t)^2}$ [from Eq. (i)]

At $t = \frac{\pi}{4}, x = 1, y = 1$

$\therefore \frac{d^2y}{dx^2} = \frac{-2}{1+1}\cdot\frac{1}{\left(\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right)^2}$

$= \frac{-1}{\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]} = -\frac{1}{2}$

42 **(c)**

$e^x + e^y = e^{x+y} = e^x e^y$

$\Rightarrow \ e^{-y} + e^{-x} = 1$

On differentiating, we get

$-e^{-y}\frac{dy}{dx} + e^{-x}(-1) = 0$

$\Rightarrow \ \frac{dy}{dx} = \frac{e^{-x}}{-e^{-y}} \Rightarrow \frac{dy}{dx} = -e^{y-x}$

43 **(c)**

$\frac{dy}{dx} = f'\left(\frac{2x+3}{3-2x}\right)\frac{d}{dx}\left(\frac{2x+3}{3-2x}\right)$

$= \sin\left[\log\ \left(\frac{2x+3}{3-2x}\right)\right]$

$\left(\frac{(3-2x)(2) - (2x+3)(-2)}{(3-2x)^2}\right)$

$= \frac{12}{(3-2x)^2}\sin\left[\ \log\left(\frac{2x+3}{3-2x}\right)\right]$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = \frac{12}{(3-2)^2}\sin\log(5)$

$= 12\sin\log 5$

44 **(a)**

Let $y = f(\tan x)$ and $u = g(\sec x)$

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = f'(\tan x)\sec^2 x$

and $\frac{du}{dx} = g'(\sec x)\sec x\tan x$

$\therefore \ \frac{dy}{du} = \frac{dy\,/dx}{du\,/dx} = \frac{f'(\tan x)\sec^2 x}{g'(\sec x)\sec x\tan x}$

$\therefore \ \left(\frac{dy}{du}\right)_{x=\frac{\pi}{4}} = \frac{f'\left(\tan\frac{\pi}{4}\right)}{g'(\sec\frac{\pi}{4})\sin\frac{\pi}{4}}$

$= \frac{f'(1).\sqrt{2}}{g'(\sqrt{2})} = \frac{2.\sqrt{2}}{4} = \frac{1}{\sqrt{2}}$

45 **(a)**

$x^{2x} - 2x^x \cot y - 1 = 0 \quad$ ...(i)

At $x = 1$,

$1 - 2\cot y - 1 = 0$

$\Rightarrow \cot y = 0 \Rightarrow y = \frac{\pi}{2}$

On differentiating Eq. (i), w.r.t.'x', we get

$2x^{2x}(1 + \log x) - 2$

$\left[x^x(-\text{cosec}^2\, y)\frac{dy}{dx} + \cot y\, x^x(1 + \log x)\right] = 0$

At $\left(1, \frac{\pi}{2}\right)$,

$2(1 + \log 1) - 2\left(1(-1)\left(\frac{dy}{dx}\right)_{\left(1,\frac{\pi}{2}\right)} + 0\right) = 0$

$\Rightarrow 2+2\left(\frac{dy}{dx}\right)_{\left(1,\frac{\pi}{2}\right)} = 0 \Rightarrow \left(\frac{dy}{dx}\right)_{\left(1,\frac{\pi}{2}\right)} = -1$

46 **(b)**

Given, $\frac{y}{x} = \log x - \log(a + bx)$

$\Rightarrow \frac{x\frac{dy}{dx} - y}{x^2} = \frac{1}{x} - \frac{1}{a + bx}b = \frac{a}{x(a + bx)}$

$\Rightarrow x\frac{dy}{dx} - y = \frac{ax}{a+bx} \quad$ ...(i)

$\Rightarrow x\frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{dy}{dx}$

$= \frac{(a + bx)a - ax.b}{(a + bx)^2}$

$\Rightarrow x\frac{d^2y}{dx^2} = \frac{a^2}{(a+bx)^2}$

$\Rightarrow x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$ [using eq(i)]

47 **(a)**

We have,

$y = x^{\sin x} + \sqrt{x}$

$\Rightarrow y = e^{\sin x \log x} + \sqrt{x}$

$\Rightarrow \frac{dy}{dx} = x^{\sin x}\left(\cos x \log x + \frac{\sin x}{x}\right) + \frac{1}{2\sqrt{x}}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} = \frac{\pi}{2}\left(0 + \frac{2}{\pi}\right) + \frac{1}{2\sqrt{\frac{\pi}{2}}} = 1 + \frac{1}{\sqrt{2\pi}}$

48 **(c)**

We have, $\phi(x) = f^{-1}(x)$

$\Rightarrow x = f[\phi(x)]$

On differentiating both sides w.r.t. x, we get

$1 = f'[\phi(x)].\phi'(x)$

$\Rightarrow \phi(x) = \frac{1}{f'[\phi(x)]}$ ...(i)

Since, $f'(x) = \frac{1}{1+x^5}$ (given)

$\Rightarrow f'[\phi(x) = \frac{1}{1 + [\phi(x)]^5}$

From Eq.(i),

$\phi'(x) = \frac{1}{f'[\phi(x)]} = 1 + [\phi(x)]^5$

49 **(c)**

$\frac{d}{dx}\left[\frac{e^x + 1}{e^x}\right] = \frac{d}{dx}[1 + e^{-x}] = -e^{-x} = -\frac{1}{e^x}$

50 **(a)**

We have, $f(x) = x^3 - 3x^2 + 2x + 10$

$f'(x) = 3x^2 - 6x + 2$

Put $f'(x) = 0, \qquad 3x^2 - 6x + 2 = 0$

$\Rightarrow x = \frac{6 \pm \sqrt{36 - 24}}{2 \times 3}$

$= \frac{3+\sqrt{3}}{3}$ $\left[\because x = \frac{3-\sqrt{3}}{3} \text{ does not lie in the given interval}\right]$

51 **(b)**

We have,

$x^2 + y^2 = a^2 \Rightarrow x + yy_1 = 0 \Rightarrow y_1 = -x/y$

Now, $yy_1 + x = 0$

$\Rightarrow y\,y_2 + y_1^2 + 1 = 0$ [Differentiating w.r.t. x]

$\Rightarrow y = -\frac{1 + y_1^2}{y^2}$ ... (i)

$\therefore k = \frac{1}{a}\left|\frac{1}{\sqrt{x^2 + y^2}}\right| = \left\{\frac{1}{y\sqrt{1 + \frac{x^2}{y^2}}}\right\}$

$\Rightarrow k = \left\{\frac{1}{y\sqrt{1 + y_1^2}}\right\}$ $[\because y_1 = -x/y]$

$\Rightarrow k = \left\{\frac{y''}{(1 + y_1^2)\sqrt{1 + y_1^2}}\right\} = \frac{|y''|}{(1 + y_1^2)^{3/2}}$

52 **(c)**

Let $y = \frac{\tan^2 2x - \tan^2 x}{1 - \tan^2 2x \tan^2 x}$

$= \frac{(\tan 2x - \tan x)}{(1 + \tan 2x \tan x)}.\frac{(\tan 2x + \tan x)}{(1 - \tan 2x \tan x)}$

$= \tan(2x - x)\tan(2x + x)$

$= \tan x \tan 3x$

On differentiating w.r.t. x, we get

$\frac{d}{dx}(y.\cot 3x) = \frac{d}{dx}(\tan x \tan 3x \cot 3x)$

$= \frac{d}{dx}(\tan x)$

$= \sec^2 x$

53 **(b)**

Given, $\frac{x}{1} = \frac{1-\sqrt{y}}{1+\sqrt{y}}$

Applying componendo and dividendo, we get

$\frac{1 + x}{1 - x} = \frac{(1 + \sqrt{y}) + (1 - \sqrt{y})}{(1 + \sqrt{y}) - (1 - \sqrt{y})}$

$\Rightarrow \frac{1 + x}{1 - x} = \frac{2}{2\sqrt{y}}$

$\Rightarrow y = \left(\frac{1 - x}{1 + x}\right)^2$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{-2(1+x)^2(1-x)-(1-x)^2.2(1+x)}{(1+x)^4}$$
$$=\frac{(1-x)(1+x)(-2-2x-2+2x)}{(1+x)^4}$$
$$=\frac{4(x-1)}{(x+1)^3}$$

54 **(b)**

We have,

$$y=\cos^{-1}\left(\frac{2\cos x-3\sin x}{\sqrt{13}}\right)$$
$$\Rightarrow y=\cos^{-1}\left\{\frac{2}{\sqrt{13}}\cos x-\sqrt{1-\frac{4}{13}}\sqrt{1-\cos^2 x}\right\}$$
$$\Rightarrow y=\cos^{-1}(\cos x)=\cos^{-1}\left(\frac{2}{\sqrt{13}}\right)$$
$$\Rightarrow y=x-\cos^{-1}\left(\frac{2}{\sqrt{13}}\right)\Rightarrow\frac{dy}{dx}=1$$

55 **(d)**

Given, $y=\sin\ (\log_e x)$ ...(i)

$\Rightarrow \frac{dy}{dx}=\cos(\log_e x).\frac{1}{x}$ ...(ii)

$$\Rightarrow \frac{d^2y}{dx^2}=\frac{-x.\sin(\log_e x).\frac{1}{x}=\cos(\log_e x).1}{x^2}$$
$$=\frac{-\sin\ (\log_e x)-\cos\ (\log_e x)}{x^2}$$

$x^2\frac{d^2y}{dx^2}=-\sin(\log_e x)-x\frac{dy}{dx}$ [using Eq. (ii)]

$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}=-y$ [using Eq. (i)]

56 **(c)**

Since, $y=\sin^{-1}\frac{x}{2}+\cos^{-1}\frac{x}{2}=\frac{\pi}{2}$

$\therefore \frac{dy}{dx}=0$

57 **(d)**

$\because\quad y=\cos\ (\sin x^2)$

$\therefore\quad \frac{dy}{dx}=-\sin(\sin x^2)\cos x^2.2x$

At $x=\sqrt{\frac{\pi}{2}}, \frac{dy}{dx}=-\sin\left(\sin\frac{\pi}{2}\right)\cos\frac{\pi}{2}.2.\sqrt{\frac{\pi}{2}}$

$=0\quad \left[\because \cos\frac{\pi}{2}=0\right]$

58 **(b)**

Let $f(x)=\cos x, f'(x)=\sin x$

$\Rightarrow\ f''(x)=-\cos x$

$\Rightarrow\ f'''(x)=\sin x$

Since, $\sin x$ is an odd function.

$\therefore\ f'''$ is an odd function.

59 **(b)**

Given, $\sec z=\frac{x^4+y^4-8x^2y^2}{x^2+y^2}$

Here, $n=2$

$$\therefore x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=2.\frac{\sec z}{\sec z.\tan z}=2\cot z$$

60 **(c)**

We have,

$2^x+2^y=2^{x+y}$

$$\Rightarrow 2^x\log 2+2^y\log 2\frac{dy}{dx}=2^{x+y}\log 2\left(1+\frac{dy}{dx}\right)$$
$$\Rightarrow (2^y-2^{x+y})\frac{dy}{dx}=2^{x+y}-2^x$$
$$\Rightarrow \frac{dy}{dx}=\frac{2^x(2^y-1)}{2^y(1-2^x)}=2^{x-y}\left(\frac{2^y-1}{1-2^x}\right)$$

61 **(b)**

We have,

$f(x)=10\cos x+(13+2x)\sin x$

$\Rightarrow f'(x)=-10\sin x+2\sin x+(13+2x)\cos x$

$\Rightarrow f''(x)=-8\cos x-(13+2x)\sin x+2\cos x$

$\therefore f'(x)+f(x)=4\cos x$

62 **(b)**

$$xy=\tan^{-1}(xy)+\cot^{-1}(xy)=\frac{\pi}{2}$$
$$\Rightarrow\ xy=\frac{\pi}{2}$$

On differentiating, we get

$$\therefore\ x\frac{dy}{dx}+y=0$$

$\Rightarrow\ \frac{dy}{dx}=\frac{-y}{x}$

63 **(a)**

Since, $x^y.y^x=16$

$\therefore \log_e x^y+\log_e y^x=\log_e 16$

$\Rightarrow y\log_e x+x\log_e y=4\log_e 2$

Now, on differentiating both sides w.r.t. x, we get

$$\frac{y}{x}+\log_e x\frac{dy}{dx}+\frac{x}{y}\frac{dy}{dx}+\log_e y.1=0$$
$$\Rightarrow \frac{dy}{dx}=-\frac{\left(\log_e y+\frac{y}{x}\right)}{\left(\log_e x+\frac{x}{y}\right)}$$
$$\therefore \left.\frac{dy}{dx}\right|_{(2,2)}=-\frac{(\log_e 2+1)}{(\log_e 2+1)}=-1$$

64 **(c)**

We have,

$y=b\sin\theta, x=a\cos\theta$

$$\Rightarrow \frac{dy}{dx}=-\frac{b}{a}\cot\theta$$
$$\Rightarrow \frac{d^2y}{dx^2}=\frac{b}{a}\ \mathrm{cosec}^2\,\theta\frac{d\theta}{dx}=-\frac{b}{a^2}\ \mathrm{cosec}^3\theta$$
$$\Rightarrow \frac{d^3y}{dx^3}=\frac{b}{a^2}3\ \mathrm{cosec}^2\theta(-\mathrm{cosec}\,\theta+\cot\theta)\frac{d\theta}{dx}$$
$$\Rightarrow \frac{d^3y}{dx^3}=\frac{3b}{a^2}\mathrm{cosec}^3\theta\cot\theta\times\frac{-1}{a\sin\theta}$$
$$=-\frac{3\ b}{a^3}\mathrm{cosec}^4\theta\cot\theta$$

65 **(b)**

Given, $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{\psi'}{\phi'}$

$$\Rightarrow \frac{d^2y}{dx^2}=\frac{d}{dt}\left(\frac{\psi'}{\phi'}\right)\frac{dt}{dx}$$

$$= \frac{\phi'\psi''-\psi'\phi''}{(\phi')^2}.\frac{1}{\phi'}$$

66 **(b)**

Since, u is not a homogeneous function. But $f(x,y)=\sin u=\frac{x^2+y^2}{x+y}$ is a homogeneous function of degree one.

Here, by Euler's theorem,

$$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=f$$

$$\Rightarrow x\frac{\partial}{\partial x}(\sin u)+y\frac{\partial}{\partial y}(\sin u)=\sin u$$

$$\Rightarrow x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\tan u$$

67 **(c)**

We have,

$$y=\sec^{-1}\left(\frac{x+1}{x-1}\right)+\sin^{-1}\left(\frac{x-1}{x+1}\right)$$

$$\Rightarrow y=\cos^{-1}\left(\left(\frac{x+1}{x-1}\right)\right)+\sin^{-1}\left(\frac{x-1}{x+1}\right)=\frac{\pi}{2}$$

$$\therefore \frac{dy}{dx}=0$$

68 **(a)**

Given, $y=x^n\log x+x(\log x)^n$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx}=nx^{n-1}\log x+x^n.\frac{1}{x}+xn(\log x)^{n-1}\left(\frac{1}{x}\right)$$
$$+1.(\log x)^n$$

$$=x^{n-1}(1+n\log x)+(\log x)^{n-1}[n+\log x]$$

69 **(d)**

We have,

$$y=\sqrt{x+\sqrt{y+\sqrt{x+\sqrt{y+\cdots\infty}}}}$$

$$\Rightarrow y^2=x+\sqrt{y+\sqrt{x+\sqrt{y+\cdots\infty}}}$$

$$\Rightarrow y^2=x+\sqrt{y+y}\Rightarrow (y^2-x)^2=2y$$

Differentiating both sides w.r.t. x, we get

$$2(y^2-x)\left(2y\frac{dy}{dx}-1\right)=2\frac{dy}{dx}\Rightarrow\frac{dy}{dx}$$
$$=\frac{y^2-x}{y^3-xy-1}$$

70 **(c)**

We have,

$$\frac{d}{dt}\left(\int_a^b f(x,t)\,dx\right)$$
$$=\int_a^b\frac{\partial f}{\partial t}dx+\frac{\partial b}{\partial t}f(b,t)$$
$$-\frac{\partial a}{\partial t}f(a,t)$$

$$\therefore y=\int_0^x f(t)\sin\{k(x-t)\}\,dt$$

$$\Rightarrow\frac{dy}{dx}=\int_0^x kf(t)\cos\{k(x-t)\}\,dt$$

Differentiating again w.r.t. x, we get

$$\frac{d^2y}{dx^2}=\int_0^x -k^2f(t)\sin\{k(x-t)\}\,dt$$
$$+kf(x)\{\cos\{k(x-x)\}-0$$

$$\Rightarrow\frac{d^2y}{dx^2}=-k^2y+kf(x)\Rightarrow\frac{d^2y}{dx^2}+k^2y=kf(x)$$

71 **(b)**

Given,

$$f(x)=10\cos x+(13+2x)\sin x$$

$$\Rightarrow f'(x)=-10\sin x+(13+2x)\cos x+2\sin x$$

$$\Rightarrow f''(x)=-10\cos x$$
$$-(13+2x)\sin x$$
$$+2\cos x+2\cos x$$

$$\Rightarrow f''(x)=-f(x)+4\cos x$$

$$\therefore f''(x)+f(x)=4\cos x$$

72 **(a)**

If $f(x)=ax^{41}+bx^{-40}$, then

$$f'(x)=41ax^{40}-40bx^{-41}$$

$$\Rightarrow f''(x)=1640\,ax^{39}+1640\,bx^{-42}$$

$$\Rightarrow f''(x)=\frac{1640}{x^2}(ax^{41}+bx^{-40})$$

$$\Rightarrow f''(x)=\frac{1640}{x^2}f(x)\Rightarrow\frac{f''(x)}{f(x)}=1640x^{-2}$$

So, statement-1 is true

We have,

$$\tan^{-1}\frac{2x}{1-x^2}=\begin{cases}\pi+2\tan^{-1}x & x<-1\\ 2\tan^{-1}x\text{, if} & -1x\le 1\\ -\pi+2\tan^{-1}x & x>1\end{cases}$$

$$\therefore\frac{d}{dx}\left(\tan^{-1}\frac{2x}{1-x^2}\right)=\frac{2}{1+x^2}\text{ for all }x$$

So, statement-II is not true

73 **(a)**

$$\frac{d}{dx}[\cos x^0]=\frac{d}{dx}\left[\cos\frac{\pi x}{180}\right]$$

$$\begin{bmatrix}\because 1^0=\frac{\pi}{180}\text{ radians}\\ \therefore x^0=\frac{\pi x}{180}\text{ radians}\end{bmatrix}$$

$$=-\sin\left(\frac{\pi x}{180}\right).\frac{\pi}{180}=-\frac{\pi}{180}\sin(x^0)$$

74 **(a)**

We have,

$y = e^{1+\log x} \Rightarrow y = e\, e^{\log_e x} = e^x \Rightarrow \frac{dy}{dx} = e$

75 **(a)**

Given, $y = \cos x + x \cos y = \pi$

On differentiating both sides w.r.t. x, we get

$\frac{dy}{dx}\cos x + y(-\sin x) + x(-\sin y)\frac{dy}{dx} + \cos y = 0$

$\Rightarrow \frac{dy}{dx} = \frac{y \sin x - \cos y}{\cos x - x \sin y}$

Again differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{\left[\begin{array}{l}(\cos x - x\sin y)\left(y\cos x + \sin x\frac{dy}{dx} + \sin y\frac{dy}{dx}\right)\\ -(y\sin x - \cos y)\left(-\sin x - \sin y - x\cos y\frac{dy}{dx}\right)\end{array}\right]}{(\cos x - x\sin y)^2}$$

At $x = 0$,

$f''(0) = \frac{1(y + \sin y) - (-1)(-\sin y)}{(1-0)^2} = y$

As $y\cos x + x\cos y = \pi$

$\therefore$ At $x = 0 \Rightarrow y = \pi$

Hence, $f''(0) = \pi$

76 **(d)**

Given that, $x = a\sin\theta$ and $y = b\cos\theta$

On differentiating w.r.t. θ, we get

$\frac{dx}{d\theta} = a\cos\theta$ and $\frac{dy}{d\theta} = -b\sin\theta$

$\Rightarrow \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-b}{a}\tan\theta$

Again differentiating w.r.t. θ, we get

$\frac{d^2y}{dx^2} = \frac{-b}{a}\sec^2\theta\frac{d\theta}{dx}$

$\Rightarrow \frac{d^2y}{dx^2} = -\frac{b}{a}\sec^2\theta\frac{1}{a\cos\theta} = -\frac{b}{a^2}\sec^3\theta$

77 **(d)**

Let $u = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ and $v = \cos^{-1}\sqrt{1-x^2}$

On differentiating w.r.t. x, respectively, we get

$\frac{du}{dx} = \frac{1}{1+\left(\frac{2x}{1-x^2}\right)^2}\cdot\left[\frac{(1-x^2)2-2x(-2x)}{(1-x^2)^2}\right]$

$= \frac{2+2x^2}{(1+x^2)^2} = \frac{2}{1+x^2}$

and $\frac{dv}{dx} = -\frac{1}{\sqrt{1-(1-x^2)}}\cdot\left[\frac{(-2x)}{2\sqrt{1-x^2}}\right]$

$= \frac{1}{\sqrt{x^2}}\left[\frac{x}{\sqrt{1-x^2}}\right] = \frac{1}{\sqrt{1-x^2}}$

$\therefore \frac{du}{dv} = \frac{\frac{2}{1+x^2}}{\frac{1}{\sqrt{1-x^2}}} = \frac{2\sqrt{1-x^2}}{1+x^2}$

Alternate

Let $u = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1}x$

$\Rightarrow \frac{du}{dx} = \frac{2}{1+x^2}$

and let $v = \cos^{-1}\sqrt{1-x^2} = \sin^{-1}x$

$\Rightarrow \frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}}$

Now, $\frac{du}{dv} = \frac{2}{1+x^2} / \frac{1}{\sqrt{1-x^2}} = \frac{2\sqrt{1-x^2}}{1+x^2}$

78 **(c)**

We have,

$y = \sin^{-1}\{\sqrt{x-ax} - \sqrt{a-ax}\}$

$\Rightarrow y = \sin^{-1}\left\{\sqrt{x}\sqrt{1-(\sqrt{a})^2} - \sqrt{a}\sqrt{1-(\sqrt{x})^2}\right\}$

$\Rightarrow y = \sin^{-1}\sqrt{x} - \sin^{-1}\sqrt{a}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}\sqrt{1-x}} = \frac{1}{2\sqrt{x(1-x)}}$

79 **(b)**

Let $p = \sin^{-1}\frac{2x}{1+x^2} = 2\tan^{-1}x$

and $q = \cos^{-1}\frac{1-x^2}{1+x^2} = 2\tan^{-1}x$

$\Rightarrow \frac{dp}{dx} = \frac{2}{1+x^2}, \frac{dq}{dx} = \frac{2}{1+x^2}$

$\therefore \frac{dp}{dq} = \frac{\frac{dp}{dx}}{\frac{dq}{dx}} = \frac{\frac{2}{(1+x^2)}}{\frac{2}{(1+x^2)}} = 1$

80 **(b)**

$f(x) = e^x$ and $g(x) = \sin^{-1}x$

and $h(x) = f[g(x)]$

$\Rightarrow h(x) = f(\sin^{-1}x) = e^{\sin^{-1}x}$

$\therefore h(x) = e^{\sin^{-1}x}$

$\Rightarrow h'(x) = e^{\sin^{-1}x}\frac{1}{\sqrt{1-x^2}}$

$\Rightarrow \frac{h'(x)}{h(x)} = \frac{1}{\sqrt{1-x^2}}$

81 **(a)**

Given, $x = a(\theta - \sin\theta)$ and $y = a(1-\cos\theta)$

$\frac{dx}{d\theta} = a(1-\cos\theta)$ and $\frac{dy}{d\theta} = a\sin\theta$

$\therefore \frac{dy}{dx} = \frac{a\sin\theta}{a(1-\cos\theta)}$

$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} = \cot\frac{\theta}{2}$

82 **(c)**

We have,

$x = a\cos\theta + \frac{1}{2}b\cos 2\theta, y = a\sin\theta + \frac{1}{2}b\sin 2\theta$

$\therefore \frac{dx}{d\theta} = -a\sin\theta - b\sin 2\theta, \frac{dy}{d\,\theta}$

$= a\cos\theta + b\sin 2\theta$

$\Rightarrow \frac{dy}{dx} = \frac{a\cos\theta + b\cos 2\theta}{-a\sin\theta - b\sin 2\theta}$

$\Rightarrow \dfrac{d^2y}{dx^2} = -\dfrac{d}{d\theta}\left(\dfrac{a\cos\theta + b\cos 2\theta}{a\sin\theta + b\sin 2\theta}\right)\dfrac{d\,\theta}{dx}$

$\therefore \dfrac{d^2y}{dx^2} = 0$

$\Rightarrow (a\sin\theta + b\sin 2\,\theta)(-a\sin\theta - b\sin 2\,\theta)$

$= (a\cos\theta + b\cos 2\,\theta)(a\cos\theta + 2\,b\cos 2\,\theta)$

$\Rightarrow -a^2 - 2\,b^2 - 3\,ab(\cos 2\,\theta\cos\theta + \sin\theta\sin 2\,\theta)$

$= 0$

$\Rightarrow a^2 + 2\,b^2 = 3\,a\,b\cos\theta \Rightarrow \cos\theta = \dfrac{a^2 + 2\,b^2}{3\,ab}$

83 **(d)**

$\because y = \log_a x + \dfrac{\log a}{\log x} + 1 + 1$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{x}\log_a e - \log a\left(\dfrac{1}{\log x}\right)^2 \dfrac{1}{x}$

$= \dfrac{1}{x\log a} - \dfrac{\log a}{x(\log x)^2}$

84 **(d)**

Given, $y = \sin[\cos^{-1}\{\sin(\cos^{-1} x)\}]$

$\Rightarrow\; y = \sin\left[cos^{-1}\left\{sin\left(\dfrac{\pi}{2} - sin^{-1} x\right)\right\}\right]$

$= \sin[\cos^{-1}(\cos\sin^{-1} x)]$

$= \sin(\sin^{-1} x)$

$\Rightarrow \quad y = x$

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = 1$

85 **(c)**

We have, $8f(x) + 6f\left(\frac{1}{x}\right) = x + 5$ for all x ...(i)

Therefore, $8f\left(\frac{1}{x}\right) + 6f(x) = \frac{1}{x} + 5$ ...(ii)

From Eqs. (i) and (ii), we have

$f(x) = \dfrac{1}{28}\left(8x - \dfrac{6}{x} + 10\right)$

Now, $y = x^2 f(x)$

$\Rightarrow \quad y = \frac{1}{28}(8x^2 - 6x + 10x^2)$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{1}{28}(24x^2 + 20x - 6)$

$\therefore \quad \left(\dfrac{dy}{dx}\right)_{x=-1} = \dfrac{1}{28}(24 - 20 - 6)$

$= -\dfrac{1}{14}$

86 **(d)**

Given, $x = \sin t$, $y = \cos pt$

$\dfrac{dx}{dt} = \cos t, \; \dfrac{dy}{dt} = -p\sin pt$

$\therefore \quad \dfrac{dy}{dx} = -\dfrac{p\sin pt}{\cos t}$

$\Rightarrow\; y_1 = \frac{-p\sqrt{1-y^2}}{\sqrt{1-x^2}}$

$\Rightarrow\; y_1\sqrt{1-x^2} = -p\sqrt{1-y^2}$

$\Rightarrow\; y_1^2(1-x^2) = p^2(1-y^2)$

$\Rightarrow\; 2y_1y_2(1-x^2) - 2xy_1^2 =$
$-2yy_1p^2$ [differentiate]

$\Rightarrow\; (1-x^2)y_2 - xy_1 + p^2y = 0$

87 **(b)**

On taking log in the given equation, we get

$\log y = \log(1-x) + \log(2-x) + \ldots + \log(n-x)$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{(1-x)}(-1) + \frac{1}{(2-x)}(-1) + \ldots + \frac{1}{(n-x)}(-1)$

$\dfrac{dy}{dx} = y$

$\left[\dfrac{\begin{matrix}(2-x)(3-x)\ldots(n-x)+\\(1-x)(3-x)\ldots(n-x)+\ldots\end{matrix}}{y}\right](-1)$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = 1.2 \ldots\ldots (n-1)(-1)$

$= (-1)(n-1)!$

88 **(c)**

$\because f(x) = \sqrt{ax} + \dfrac{a^2}{\sqrt{ax}}$

On differentiating w.r.t. x, we get

$f'(x) = \dfrac{\sqrt{a}}{2\sqrt{x}} + \dfrac{a^2}{\sqrt{a}}\left(\dfrac{-1}{2}x^{-3/2}\right)$

$\Rightarrow f'(x) = \dfrac{\sqrt{a}}{2\sqrt{x}} - \dfrac{a^2}{2\sqrt{a}}x^{-3/2}$

$\therefore f'(a) = \dfrac{\sqrt{a}}{2\sqrt{a}} - \dfrac{a^2}{2\sqrt{a}.a^{3/2}}$

$\Rightarrow f'(a) = \dfrac{1}{2} - \dfrac{a^2}{2a^2} = 0$

89 **(d)**

Given, $y = \log_2\log_2(x)$

$= \dfrac{\log_e\log_2(x)}{\log_e 2} = \dfrac{\log_e\left[\frac{\log_e x}{\log_e 2}\right]}{\log_e 2}$

$\Rightarrow y = \dfrac{\log_e\log_e x - \log_e\log_e 2}{\log_e 2}$

On differentiating w.r.t.x, we get

$\dfrac{dy}{dx} = \dfrac{1}{\log_e 2}\left[\dfrac{1}{x\log_e x} - 0\right] = \dfrac{\log_2 e}{x\log_e x}$

90 **(b)**

Let $y = f(\sin x)$

$\Rightarrow y = \log(\sin x)$

$\therefore \dfrac{dy}{dx} = \dfrac{1}{\sin x}\cos x = \cot x$

91 **(c)**

Given, $\quad y = e^{ax}\sin bx$ ...(i)

$\Rightarrow \quad \dfrac{dy}{dx} = ae^{ax}\sin bx + be^{ax}\cos bx$

$\Rightarrow \quad \frac{dy}{dx} = ay + be^{ax}\cos bx$ ...(ii)

$\Rightarrow \quad \dfrac{d^2y}{dx^2} = a\dfrac{dy}{dx} + abe^{ax}\cos bx - e^{ax}b^2\sin bx$

$\Rightarrow \frac{d^2y}{dx^2} = a\frac{dy}{dx} + a\left(\frac{dy}{dx} - ay\right) - b^2y$

[from Eqs. (i) and (ii)]

$\Rightarrow \frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + a^2y = -b^2y$

93 **(a)**

Let $f(x) = ax^2 + bx + c$ ....(i)

$\Rightarrow f'(x) = 2ax + b$

and $f''(x) = 2a$

Given, $f(0) = 4, f'(0) = 3$ and $f''(0) = 4$, we get

$c = 4, \quad b = 3, \quad a = 2$

$\therefore f(x) = 2x^2 + 3x + 4$

$\therefore f(-1) = 2(-1)^2 + 3(-1) + 4 = 3$

94 **(a)**

We have,

$x = \exp\left\{\tan^{-1}\left(\frac{y - x^2}{x^2}\right)\right\}$

$\Rightarrow \log_e x = \tan^{-1}\left(\frac{y - x^2}{x^2}\right)$

$\Rightarrow \tan(\log_e x) = \frac{y}{x^2} - 1$

$\Rightarrow y = x^2\{1 + \tan(\log_e x)\}$

$\Rightarrow \frac{dy}{dx} = 2x\{1 + \tan(\log_e x)\} + x\sec^2(\log_e x)$

95 **(a)**

We have,

$y^2 = ax^2 + bx + c$

Differentiating this with respect to x, we get

$2y\frac{dy}{dx} = 2ax + b$ ...(i)

$\Rightarrow 2\left(\frac{dy}{dx}\right)^2 + 2y\frac{d^2y}{dx^2} = 2a$

$\Rightarrow y\frac{d^2y}{dx^2} = a - \left(\frac{dy}{dx}\right)^2$

$\Rightarrow y\frac{d^2y}{dx^2} = a - \left(\frac{2ax+b}{2y}\right)^2$ ...[From (i)]

$\Rightarrow y\frac{d^2y}{dx^2} = \frac{4\,ay^2 - (2\,ax + b)^2}{4y^2}$

$\Rightarrow 4y^3\frac{d^2y}{dx^2} = 4\,a(ax^2 + bx + c) - (4a^2x^2 + 4ab\,x + b^2)$

$\Rightarrow 4y^3\frac{d^2y}{dx^2} = 4\,ac - b^2$

$\Rightarrow y^3\frac{d^2y}{dx^2} = \frac{4ac - b^2}{4} =$ a const.

96 **(b)**

Let $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}x$

$\therefore \frac{du}{dx} = \frac{2}{1 + x^2}$

and $v = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1}x$

$\therefore \frac{dv}{dx} = \frac{2}{1 + x^2}$

Hence, $\frac{du}{dv} = \frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dx}\right)} = 1$

97 **(d)**

$I_n = \frac{d^{n-1}}{dx^{n-1}}[x^{n-1} + nx^{n-1}\log x]$

$I_n = (n-1)\frac{d^{n-2}}{dx^{n-2}}x^{n-2} + n\frac{d^{n-1}}{dx^{n-1}}(x^{n-1}\log x)$

$I_n = (n-1)! + nI_{n-1}$

$\Rightarrow I_n - nI_{n-1} = (n-1)!$

98 **(d)**

We have,

$y = \sin(\log_e x)$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x}\cos(\log_e x)$

$\Rightarrow x\frac{dy}{dx} = \cos(\log_e x)$

$\Rightarrow x\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{-\sin(\log_e x)}{x} \Rightarrow x^2\frac{dy}{dx^2} + x\frac{dy}{dx} = -y$

99 **(c)**

Given, $y - 1 = t^{10},\ x - 1 = t^8$

$\Rightarrow \frac{dy}{dt} = 10t^9$ and $\frac{dx}{dt} = 8t^7$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{5}{4}t^2$

$= \frac{5}{4}\frac{t^{10}}{t^8} = \frac{5(y-1)}{4(x-1)}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{5}{4}\frac{(x-1)\frac{dy}{dx} - (y-1)1}{(x-1)^2}$

$= \frac{5}{4}\frac{1}{(x-1)}\left[\frac{dy}{dx} - \frac{(y-1)}{(x-1)}\right]$

$= \frac{5}{4}\frac{1}{(x-1)}\left[\frac{5}{4}.\frac{(y-1)}{(x-1)} - \frac{(y-1)}{(x-1)}\right]$

$= \frac{5}{4}\frac{(y-1)}{(x-1)^2}\left(\frac{5}{4} - 1\right) = \frac{5}{16t^6}$

100 **(a)**

Let $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$ and $z = \sin^{-1}(3x - 4x3)$

$\Rightarrow y = 2\sin^{-1}x$ and $z = 3\sin^{-1}x$

$\Rightarrow y = \frac{2}{3}z \quad \therefore \frac{dy}{dz} = \frac{2}{3}$

101 **(c)**

Since, $y = (1+x)(1+x^2)(1+x^4)\ldots(1+x^{2^n})$

$\Rightarrow (1-x)y = (1-x^2)(1+x^2)(1+x^4)\ldots(1+x^{2^n})$

$= (1-x^4)(1+x^4)\ldots\left(1+x^{2^n}\right)$

… … … … …

… … … … …

$= \left(1-x^{2^n}\right)\left(1+x^{2^n}\right) = 1 - x^{2^{n+1}}$

$\therefore\ y=\dfrac{1-x^{2^{n+1}}}{(1-x)}$

$\dfrac{dy}{dx}$

$=\dfrac{(1-x)(-2^{n+1}).x^{2^{n+1-1}}-\left(1-x^{2^{n+1}}\right)(-1)}{(1-x)^2}$

$\therefore \left.\dfrac{dy}{dx}\right|_{x=0}=\dfrac{(1-0)(-2^{n+1}.0)-(1-0)(-1)}{1}=1$

102 **(b)**

We have,

$\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$

Putting $x=\sin A, y=\sin B$, it reduces to

$\Rightarrow \sin^{-1}x-\sin^{-1}y=2\cot^{-1}(a)$

Differentiating w.r.t. x, we get

$\dfrac{1}{\sqrt{1-x^2}}-\dfrac{1}{\sqrt{1-y^2}}\dfrac{dy}{dx}=0\Rightarrow\dfrac{dy}{dx}=\sqrt{\dfrac{1-y^2}{1-x^2}}$

103 **(c)**

Let $y=e^{x^3}, z=\log x$

On differentiating w.r.t. x, we get

$\frac{dy}{dx}=e^{x^3}(3x^2)=3x^2e^{x^3}$and $\frac{dz}{dx}=\frac{1}{x}$

$\therefore \dfrac{dy}{dz}=\dfrac{\frac{dy}{dx}}{\frac{dz}{dx}}=\dfrac{3x^2e^{x^3}}{\left(\frac{1}{x}\right)}=3x^3e^{x^3}$

104 **(c)**

Let $y=\sqrt{x^2+16}$ and $z=\frac{x}{x-1}$

On differentiating w.r.t., x, we get

$\dfrac{dy}{dx}=\dfrac{1}{2}(x^2+16)^{-1/2}(2x)$

and $\frac{dz}{dx}=\frac{x-1-x}{(x-1)^2}=\frac{-1}{(x-1)^2}$

$\therefore \dfrac{dy}{dz}=\dfrac{dy/dx}{dz/dx}=\dfrac{-x}{\sqrt{x^2+16}}.\dfrac{1}{\frac{1}{(x-1)^2}}$

$\left(\dfrac{dy}{dz}\right)_{x=3}=\dfrac{-3(2)^2}{\sqrt{25}}=\dfrac{-12}{5}$

105 **(c)**

Given, $x=\log(1+t^2)$ and $y=t-\tan^{-1}t$

$\dfrac{dx}{dt}=\dfrac{1}{1+t^2}.2t$

and $\frac{dy}{dt}=1-\frac{1}{1+t^2}=\frac{t^2}{1+t^2}$

$\therefore \frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{t^2/(1+t^2)}{2t/(1+t^2)}=\frac{t}{2}$ ...(i)

Also, $x=\log(1+t^2)\Rightarrow t^2=e^x-1$ ...(ii)

From Eqs. (i) and (ii), we get

$\frac{dy}{dx}=\frac{\sqrt{e^x-1}}{2}$

106 **(c)**

y

$=\dfrac{(1-x)(1+x)(1+x^2)(1+x^4)\dots\dots.(1+x^{2n})}{(1-x)}$

$=\dfrac{1-x^{4n}}{1-x}$

$\Rightarrow \dfrac{dy}{dx}=\dfrac{(1-x)(-4nx^{4n-1})-(1-x^{4n})(-1)}{(1-x)^2}$

$=\dfrac{-4n(1-x)x^{4n-1}+(1-x^{4n})}{(1-x)^2}$

$\therefore \left(\dfrac{dy}{dx}\right)_{x=0}=1$

107 **(b)**

We have,

$f(x)=(1-x)^n$

$\therefore f'(x)=-n(1-x)^{n-1}, f''(x)=n(n-1)x^{n-2}$,

$f'''(x)=-n(n-1)(n-2)x^{n-3}$ and so on

$\Rightarrow f(0)=1, f'(0)=-n, f''(0)=n(n-1)$,

$f'''(0)=-n(n-1)(n-2)$ and so on

$\therefore f(0)+f'(0)+\dfrac{f''(0)}{2!}+\dots+\dfrac{f^n(0)}{n!}$

$=1-n+\dfrac{n(n-1)}{n!}+\dots+(-1)^n\dfrac{n(n-1)\dots3.2.1}{n!}$

$={}^nC_0-{}^nC_1+{}^nC_2-{}^nC_3+\dots+(-1)^n\ {}^nC_n$

$=(1-1)^n=0$

108 **(a)**

$f(x)=(\log_{\cot x}\tan x)(\log_{\tan x}\cot x)^{-1}+\tan^{-1}\dfrac{4x}{4-x^2}$

$=\dfrac{\log\tan x}{\log\cot x}.\dfrac{\log\tan x}{\log\cot x}+\tan^{-1}\left(\dfrac{4x}{4-x^2}\right)$

$=\dfrac{(\log\tan x)^2}{(-\log\tan x)^2}+\tan^{-1}\left(\dfrac{4x}{4-x^2}\right)$

$=1+\tan^{-1}\left(\dfrac{4x}{4-x^2}\right)$

$\therefore f'(x)=\dfrac{1}{1+\left(\frac{4x}{4-x^2}\right)^2}.\dfrac{(4-x^2)4-4x(-2x)}{(4-x^2)^2}$

$=\dfrac{16-4x^2+8x^2}{(4-x)^2+16x^2}=\dfrac{4(4+x^2)}{(4-x^2)^2+(4x)^2}$

Hence, $f(2)=\frac{4(4+4)}{0+(8)^2}=\frac{32}{64}=\frac{1}{2}$

109 **(c)**

Given, $y=\cos^2\frac{3x}{2}-\sin^2\frac{3x}{2}=2\cos^2\frac{3x}{2}-1$

$\Rightarrow \dfrac{dy}{dx}=2.2\cos\dfrac{3x}{2}\left(-\sin\dfrac{3x}{2}\right)\left(\dfrac{3}{2}\right)$

$\Rightarrow \dfrac{dy}{dx}=-6\cos\dfrac{3x}{2}\sin\dfrac{3x}{2}$

$\Rightarrow \dfrac{d^2y}{dx^2}=-6\left[\cos\dfrac{3x}{2}\left(\cos\dfrac{3x}{2}\right).\dfrac{3}{2}-\sin\dfrac{3x}{2}\sin\dfrac{3x}{2}.\dfrac{3}{2}\right]$

$\Rightarrow \dfrac{d^2y}{dx^2}=-9\left[\cos^2\dfrac{3x}{2}-\sin^2\dfrac{3x}{2}\right]=-9y$

Alternate

$y = \cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}$

$\Rightarrow\ y = \cos 3x$

$\Rightarrow\ \frac{dy}{dx} = -3\sin 3x$

$\Rightarrow\ \frac{d^2y}{dx^2} = -9\cos 3x = -9y$

110 **(d)**

Given,

$f(x) = \log_x(\log_e x) = \frac{\log_e \log_e x}{\log_e x}$

On differentiating w.r.t. x, we get

$f'(x) = \frac{\log_e x.\frac{1}{\log_e x}.\frac{1}{x} - \log_e \log_e x.\frac{1}{x}}{(\log_e x)^2}$

$\Rightarrow f'(x) = \frac{1-\log_e \log_e x}{x(\log_e x)^2}$

$\Rightarrow f'(e) = \frac{1-\log_e \log_e e}{e(\log_e e)^2} = \frac{1-\log_e 1}{e} = \frac{1}{e}$

111 **(c)**

We have,

$f(x) = \cos^{-1}\left\{\frac{1+(\log_e x)^2}{1+(\log_e x)^2}\right\}$

$\Rightarrow f(x) = 2\tan^{-1}(\log_e x)$ $[\because \log_e x > 0$ in the nbd of $x = e]$

$\Rightarrow f'(x) = \frac{2}{1+(\log_e x)^2} \times \frac{1}{x} \Rightarrow f'(e) = \frac{1}{e}$

112 **(c)**

Given, $f(x) = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots + x^n$

$\Rightarrow\ f(x) = (1+x)^n$

$\Rightarrow f'(x) = n(1+x)^{n-1}$

$\Rightarrow f''(x) = n(n-1)(1+x)^{n-2}$

$\Rightarrow f''(1) = n(n-1)2^{n-2}$

113 **(c)**

On differentiating w.r.t. x, we get

$2^x\log 2 + 2^y \log 2\frac{dy}{dx} = 2^{x+y}\log 2\left(1+\frac{dy}{dx}\right)$

$\Rightarrow 2^x + 2^y\frac{dy}{dx} = 2^{x+y}\left(1+\frac{dy}{dx}\right)$

$\Rightarrow\ 2^x - 2^{x+y} = \frac{dy}{dx}(2^{x+y} - 2^y)$

$\Rightarrow\ 2^{x-y}\frac{(1-2^y)}{(2^x-1)} = \frac{dy}{dx}$

114 **(a)**

We have,

$2x^2 - 3xy + y^2 + x + 2y - 8 = 0$

Differentiating w.r.t. to x, we get

$4x - 3\left(x\frac{dy}{dx} + y\right) + 2y\frac{dy}{dx} + 1 + 2\frac{dy}{dx} = 0$

$\Rightarrow 4x - 3y + 1 = \frac{dy}{dx}(3x - 2y - 2)$

$\Rightarrow \frac{dy}{dx} = \frac{3y-4x-1}{2y-3x+2}$

115 **(c)**

We have, $\sin y + e^{-x\cos y} = e$

Differentiating w.r.t. x, we get

$\cos y\frac{dy}{dx} - e^{-x\cos y}\left(\cos y - x\sin y\frac{dy}{dx}\right) = 0$

Putting $x = 1, y = \pi$, we get

$-\frac{dy}{dx} - e(-1) = 0 \Rightarrow \frac{dy}{dx} = e$

116 **(a)**

$\frac{d}{dx}\left[\tan^{-1}\left(\frac{a-x}{1+ax}\right)\right] = \frac{d}{dx}[\tan^{-1}a - \tan^{-1}x]$

$= 0 - \frac{1}{1+x^2} = -\frac{1}{1+x^2}$

117 **(b)**

$\because y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$

$= \tan^{-1}\left(\frac{\frac{a}{b} - \tan x}{1 + \frac{a}{b}\tan x}\right)$

$= \tan^{-1}\left[\tan\left(\tan^{-1}\left(\frac{a}{b}\right) - x\right)\right]$

$\Rightarrow y = \tan^{-1}\left(\frac{a}{b}\right) - x$

$\therefore \frac{dy}{dx} = 0 - 1 = -1$

118 **(b)**

Given, $y = x^y$

$\Rightarrow\ \log y = y\log x$

$\Rightarrow \frac{1}{y}.\frac{dy}{dx} = y.\frac{1}{x} + \frac{dy}{dx}.\log x$

$\Rightarrow \frac{dy}{dx}\left[\frac{1}{y} - \log x\right] = \frac{y}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{y^2}{x(1-y\log x)}$

119 **(b)**

$\because x^x y^y z^z = c$

$\Rightarrow\ x\log x + y\log y + z\log z = \log c$

On differentiating partially w.r.t. x , we get

$x.\frac{1}{x} + \log x + z.\frac{1}{z}\frac{\partial z}{\partial x} + \log z\frac{\partial z}{\partial x} = 0$

$\Rightarrow (1+\log z)\frac{\partial z}{\partial x} = -(1+\log x)$

$\Rightarrow \frac{\partial z}{\partial x} = -\left(\frac{1+\log x}{1+\log z}\right)$

121 **(d)**

We have,

$f(x) = \sqrt{(x-1)^2} = |x-1|$

$= \begin{cases} x-1, & \text{if } x \geq 1 \\ -(x-1), & \text{if } x < 1 \end{cases}$

$\therefore f'(x) = \begin{cases} 1, & \text{if } x > 1 \\ -1, & \text{if } x < 1 \end{cases}$

122 **(a)**

$\because u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$

$\therefore \frac{\partial u}{\partial x} = \frac{1}{\sqrt{\left\{1-\left(\frac{x}{y}\right)^2\right\}}}\frac{1}{y} + \frac{1}{1+\left(\frac{y}{x}\right)^2}.\left(-\frac{y}{x^2}\right)$

$\Rightarrow x\frac{\partial u}{\partial x} = \frac{x}{\sqrt{(y^2-x^2)}} - \frac{xy}{(x^2+y^2)}$ ...(i)

and $\frac{\partial u}{\partial y} = \frac{1}{\sqrt{\left\{1-\left(\frac{x}{y}\right)^2\right\}}}\left(-\frac{x}{y^2}\right) + \frac{1}{1+\left(\frac{y}{x}\right)^2}.\left(\frac{1}{x}\right)$

$\Rightarrow\ y\frac{\partial u}{\partial y} = -\frac{x}{\sqrt{(y^2-x^2)}} + \frac{xy}{(x^2+y^2)}$ ...(ii)

On adding Eqs. (i)and (ii), we get

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

123 **(c)**

On differentiating the given equation partially w.r.t. x and y respectively

$u_x = \frac{y}{x} + \log y,\quad u_y = \log x + \frac{x}{y}$

Now, $u_x u_y - u_x \log x - u_y \log y + \log x \log y$

$= \left(\frac{y}{x} + \log y\right)\left(\log x + \frac{x}{y}\right) - \left(\frac{y}{x} + \log y \log x\right)$

$-\left(\log x + \frac{x}{y}\right)\log y + \log x \log y = 1$

124 **(a)**

Here, $x = A\,cos\,4t + B\,sin\,4t$

$\Rightarrow \frac{dx}{dt} = -4A\sin 4t + 4B\cos 4t$

$\Rightarrow \frac{d^2x}{dt^2} = -16\,A\cos 4t - 16\,B\sin 4t$

$\Rightarrow \frac{d^2x}{dt^2} = -16x$

125 **(a)**

$\lim_{x\to a}\frac{xf(a) - af(x)}{x-a}$

$= \lim_{x\to a}\frac{xf(a) - af(a) - af(x) + af(a)}{x-a}$

$= \lim_{x\to a}\frac{f(a)(x-a) - a[f(x) - f(a)]}{x-a}$

$= \lim_{x\to a}\frac{f(a)(x-a)}{x-a} - a\lim_{x\to a}\frac{f(x)-f(a)}{(x-a)}$

$= f(a) - af'(a)$

126 **(b)**

$\frac{d}{dx}[x^x + x^a + a^x + a^a]$

$= x^x(1+\log x) + ax^{a-1} + a^x\log a + 0$

$= x^x(1+\log x) + ax^{a-1} + a^x\log a$

127 **(d)**

Given, $y = a^x.b^{2x-1}$

$\Rightarrow \log y = x\log a + (2x-1)\log b$

$\Rightarrow \frac{1}{y}.\frac{dy}{dx} = \log a + 2\log b$

$\Rightarrow \frac{dy}{dx} = y\log ab^2$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx}\log ab^2 = y\,(\log ab^2)^2$

129 **(d)**

Differentiating $ax^2 + 2\,h\,xy + by^2 = 1$ w.r.t. x,we get

$2\,ax + 2\,hy + 2\,hx\frac{dy}{dx} + 2\,by\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\left(\frac{ax+hy}{hx+by}\right)$

$\Rightarrow \frac{d^2y}{dx^2}$

$= \left\{\frac{(hx+by)\left(a + h\frac{dy}{dx}\right) - (ax+hy)\left(h + b\frac{dy}{dx}\right)}{(hx+by)^2}\right\}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{h^2 - ab}{(hx+by)^3}$

130 **(d)**

$[hog](x) = h(x^2) = 2\log_e x$

$\Rightarrow\ (hogof)(x) = hog(\sin x) = 2\log_e \sin x$

$\Rightarrow F(x) = 2\log_e \sin x$

$\Rightarrow F'(x) = 2\cot x$

$\Rightarrow F''(x) = -2\,\mathrm{cosec}^2\,x$

131 **(b)**

Since, $\sin^{-1}x = \frac{\pi}{2} - \sin^{-1}y$

$\Rightarrow \sin^{-1}x = \cos^{-1}y$

$\Rightarrow\ y = \sqrt{1-x^2}\quad (\because \sin^{-1}x = \cos^{-1}\sqrt{1-x^2})$

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}(-2x) = -\frac{x}{y}$

132 **(d)**

$y = \tan^{-1}(\sec x - \tan x)$

$\frac{dy}{dx} = \frac{d}{dx}\tan^{-1}\left(\frac{1-\sin x}{\cos x}\right)$

$\frac{dy}{dx} = \frac{d}{dx}\tan^{-1}\left(\frac{\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)}\right)$

$= \frac{d}{dx}\tan^{-1}\left(\frac{1-\tan x/2}{1+\tan x/2}\right)$

$= \frac{d}{dx}\tan^{-1}\left\{\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right\}$

$= \frac{d}{dx}\left(\frac{\pi}{4} - \frac{x}{2}\right) = -\frac{1}{2}$

133 **(d)**

We have,

$x = e^t\sin t$ and $y = e^t\cos t$

$\Rightarrow \frac{dx}{dt} = e^t(\sin t + \cos t)$ and $\frac{dy}{dt} = e^t(\cos t - \sin t)$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t - \sin t}{\cos t + \sin t}$

Now, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{\cos t - \sin t}{\cos t + \sin t}\right) \times \frac{1}{e^t(\sin t + \cos t)}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{-(\cos t + \sin t)^2 - (\cos t - \sin t)^2}{(\cos t + \sin t)^2}$

$\times \frac{1}{e^t(\cos t + \sin t)}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{-2}{(\cos t + \sin t)^3 e^t} \Rightarrow \left(\frac{d^2y}{dx^2}\right)_{t=\pi} = \frac{-2}{-e^\pi}$

$= \frac{2}{e^\pi}$

134 **(b)**

Given, $f(x) = 3|2 + x|$

$f(x) = \begin{cases} 3(2+x), & x \ge -2 \\ -3(2+x), & x \le -2 \end{cases}$

On differentiating w. r. t, x, we get

$f'(x) = \begin{cases} 3, & x \ge 2 \\ -3, & x \le -2 \end{cases}$

at $x = -3, f'(3) = -3$

135 **(c)**

We know that be Newton's Leibnitz formula

If $I = \int_u^v f(t)dx$,

Then $\frac{dI}{dx} = f(v)\frac{dv}{dx} - f(u)\frac{du}{dx}$

Where u and v are function of x

$\therefore \frac{dx}{dy} = \frac{1}{\sqrt{1+9y^2}}$

$\Rightarrow \frac{dy}{dx} = \sqrt{1+9y^2}$

$\therefore \frac{d^2y}{dx^2} = \frac{9y}{\sqrt{1+9y^2}} . \frac{dy}{dx}$

$= \frac{9y}{\sqrt{1+9y^2}}\sqrt{1+9y^2} = 9y$

136 **(a)**

Given, $f(x) = x\tan^{-1} x$

$\therefore f'(x) = \frac{x}{1+x^2} + \tan^{-1} x$

$\Rightarrow f'(1) = \frac{1}{1+1^2} + \tan^{-1} 1 = \frac{1}{2} + \frac{\pi}{4}$

137 **(b)**

Given , $g(x) = [f(2f(x)+2)]^2$

$\therefore g'(x) = 2. f(2f(x)+2). f'(2f(x)+2). 2f'(x)$

$= 4 f(2f(x)+2)f'(2f(x)+2)f'(x)$

$\therefore g'(0) = 4f(0)f'(0)f'(0) = -4$

138 **(b)**

Let $f(x) = |x-1| + |x-3|$

$f(x) = \begin{cases} -(x-1)-(x-3), & x < 1 \\ (x-1)-(x-3), & 1 \le x < 3 \\ (x-1)+(x-3), & x \ge 3 \end{cases}$

$f(x) = \begin{cases} 4-2x, & x < 1 \\ 2, & 1 \le x < 3 \\ 2x-4, & x \ge 3 \end{cases}$

At $x = 2$,

$f(x) = 2 \Rightarrow f'(x) = 0$

139 **(d)**

We have, $y = \tan^{-1}(\sec x - \tan x)$

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\tan^{-1}\left(\frac{1-\sin x}{\cos x}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\tan^{-1}\left(\frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}\right)$

$= \frac{d}{dx}\tan^{-1}\left[\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right]$

$= \frac{d}{dx}\left(\frac{\pi}{4} - \frac{x}{2}\right) = -\frac{1}{2}$

140 **(a)**

Given that, $x = a\cos^4\theta$ and $y = a\sin^4\theta$

On differentiating w.r.t. θ, we get

$\frac{dx}{d\theta} = 4\,a\cos^3\theta(-\sin\theta)$

and $\frac{dy}{d\theta} = 4\,a\sin^3\theta\cos\theta$

$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{4a\sin^3\theta\cos\theta}{4\,a\cos^3\theta\sin\theta}$

$= -\frac{\sin^2\theta}{\cos^2\theta} = -\tan^2\theta$

Now, $\left(\frac{dy}{dx}\right)_{\theta=\frac{3\pi}{4}} = -\tan^2\left(\frac{3\pi}{4}\right) = -1$

141 **(b)**

Given, $\left(\frac{x^2-y^2}{x^2+y^2}\right) = \sec^{-1} e^a$

$\Rightarrow \frac{\left[(x^2+y^2)\left(2x-2y\frac{dy}{dx}\right) - (x^2-y^2)\left(2x+2y\frac{dy}{dx}\right)\right]}{(x^2+y^2)^2} = 0$

$\Rightarrow (2x^3 + 2xy^2 - 2x^3 + 2xy^2) - 2y\frac{dy}{dx}$

$(x^2 + y^2 + x^2 - y^2) = 0$

$\Rightarrow 4xy^2 - 4x^2y\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$

142 **(a)**

$\because y = \frac{a + bx^{3/2}}{x^{5/4}}$

On differentiating w.r.t. x, we get

$y' = \frac{\frac{3}{2}bx^{7/4} - \frac{5}{4}(a + bx^{3/2})x^{1/4}}{(x^{5/4})^2}$

$\because y' = 0$ at $x = 5$

$\therefore \frac{3}{2}bx^{7/4} - \frac{5}{4}(a + bx^{3/2})x^{1/4} = 0$, at $x = 5$

$\Rightarrow 6bx^{3/2} - 5(a + bx^{3/2}) = 0, \text{at } x = 5$

$\Rightarrow bx^{3/2} = 5a, \text{at } x = 5 \Rightarrow b(5)^{3/2} = 5a$

$\Rightarrow \frac{a}{b} = \frac{5^{3/2}}{5} \Rightarrow a : b = \sqrt{5} : 1$

143 **(c)**

Given, $f(x) = be^{ax} + ae^{bx}$

$\Rightarrow \quad f'(x) = abe^{ax} + abe^{bx}$

$\Rightarrow \quad f''(x) = a^2be^{ax} + ab^2e^{bx}$

$\Rightarrow \quad f''(0) = a^2b + ab^2 = ab(a + b)$

144 **(b)**

Given, $x^m y^n = (x + y)^{m+n}$

$m \log x + n \log y = (m + n) \log(x + y)$

$\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = \frac{(m+n)}{(x+y)}\left[1 + \frac{dy}{dx}\right]$

$\Rightarrow \frac{dy}{dx}\left[\frac{n}{y} - \frac{(m+n)}{(x+y)}\right] = \frac{m+n}{x+y} - \frac{m}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=1, y=2} = 2$

145 **(a)**

Since, $f\left(x - \frac{1}{x}\right) = x^3 - \frac{1}{x^3}$

$f\left(x - \frac{1}{x}\right) = \left(x - \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + 1\right)$

$= \left(x - \frac{1}{x}\right)\left[\left(x - \frac{1}{x}\right)^2 + 3\right]$

$\Rightarrow f(x) = x(x^2 + 3) = x^3 + 3x$

$\Rightarrow \quad f'(x) = 3x^2 + 3$

146 **(b)**

We have,

$x = a \cos t + \frac{b}{2} \cos 2t, y = a \sin t + \frac{b}{2} \sin 2t$

$\Rightarrow \frac{dy}{dx} = -a \sin t - b \sin 2t, \frac{dy}{dt}$

$= a \cos t + b \cos 2t$

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{a \cos t + b \cos 2t}{-a \sin t - b \sin 2t}$

$\Rightarrow \frac{dy}{dx} = -\frac{(a \cos t + b \cos 2t)}{(a \sin t + b \sin 2t)}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dt}\right) \cdot \frac{dt}{dx}$

$= -\frac{d}{dt}\frac{(a \cos t + b \cos 2t)}{(a \sin t + b \sin 2t)} \cdot \frac{dt}{dx}$

$\Rightarrow \frac{d^2y}{dx^2} = -\left[\frac{a^2 + 2b^2 + 3ab \cos t}{(a \sin t + b \sin 2t)^3}\right]$

$\therefore \frac{d^2y}{dx^2} = 0 \Rightarrow a^2 + 2b^2 + 3ab \cos t = 0 \Rightarrow \cos t$

$= -\left(\frac{a^2 + 2b^2}{3ab}\right)$

147 **(d)**

$h'(x) = [f(x)^2 + g(x)^2]$

$\Rightarrow \quad h''(x) = 2f(x)f'(x) + 2g(x)g'(x)$

$\left[\begin{array}{l} \because g(x) = f'(x) \\ \Rightarrow g'(x) = f''(x) \end{array}\right]$

$\therefore \quad h''(x) = 2f(x)g(x) + 2g(x)(-f(x))$

$[\because f''(x) = -f(x)]$

$\Rightarrow h''(x) = 0$

$\Rightarrow h'(x) = C$, a constant for all $x \in R$

$\Rightarrow \quad h(x) = Cx + C_1$

$\Rightarrow h(0) = C_1$ and $h(1) = C + C_1$

$\Rightarrow 2 = C_1$ and $8 = C + C_1$

$\Rightarrow C_1 = 2$ and $C = 6$

$\therefore h(x) = 6x + 2$

$\Rightarrow h(2) = 6 \times 2 + 2 = 14$

148 **(b)**

Given, $y = x \log x$

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = \frac{x}{x} + \log x$

$\Rightarrow \quad \frac{dy}{dx} = \log e + \log x$

$\Rightarrow \quad \frac{dy}{dx} = \log(ex)$

149 **(b)**

Given, $f''(x) = -f(x)$

$\Rightarrow g'(x) = -f(x)$ and $f'(x) = g(x)$ ...(i)

Now, $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$

$\therefore F'(x) = 2\left(f\left(\frac{x}{2}\right)\right) . f'\left(\frac{x}{2}\right) . \frac{1}{2}$

$+2\left(g\left(\frac{x}{2}\right)\right) . g'\left(\frac{x}{2}\right) . \frac{1}{2} = 0$

[using Eq.(i)]

$\therefore \quad F(x)$ is a constant $\Rightarrow F(10) = F(5) = 5$

150 **(a)**

$y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots \infty}}}$

$\Rightarrow y = \sqrt{\sin x + y}$

$\Rightarrow y^2 = \sin x + y$

On differentiating w.r.t. x, we get

$2y\frac{dy}{dx} = \cos x + \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{\cos x}{2y - 1}$

151 **(a)**

Given, $y = x^2 e^{mx} \Rightarrow \frac{dy}{dx} = 2xe^{mx} + mx^2e^{mx}$

$\Rightarrow \quad \frac{d^2y}{dx^2} = 2(e^{mx} + mxe^{mx}) + m(2xe^{mx} - x^2me^{mx})$

$\Rightarrow \frac{d^2y}{dx^2} = e^{mx}(m^2x^2 + 4mx + 2)$

$$\Rightarrow \frac{d^3y}{dx^3} = e^{mx}[m^3x^2 + 4m^2x + 2m + 2m^2x + 4m]$$
$$= e^{mx}[m^3x^2 + 6m^2x + 6m]$$

152 **(d)**

Given, $x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$

$$\Rightarrow\ x^4 + y^4 + 2x^2y^2 = t^2 + \frac{1}{t^2} - 2$$
$$\Rightarrow\ x^4 + y^4 + 2x^2y^2 = x^4 + y^4 - 2$$
$$\Rightarrow\ x^2y^2 + 1 = 0 \Rightarrow y^2 = \frac{-1}{x^2}$$

On differentiating w.r.t. x, we get

$$2y\frac{dy}{dx} = \frac{2}{x^3} \Rightarrow \frac{dy}{dx} = \frac{1}{x^3y}$$

153 **(d)**

Given that, $y = \cos^{-1}\sqrt{1-t^2} = \sin^{-1} t$

and $x = \sin^{-1}(3t - 4t^3) = 3\sin^{-1} t$

On differentiating both w.r.t. t respectively, we get

$$\frac{dy}{dt} = \frac{1}{\sqrt{1-t^2}} \text{ and } \frac{dx}{dt} = \frac{3}{\sqrt{1-t^2}}$$
$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\left(\frac{1}{\sqrt{1-t^2}}\right)}{3\left(\frac{1}{\sqrt{1-t^2}}\right)} \Rightarrow \frac{dy}{dx} = \frac{1}{3}$$

154 **(c)**

We have,

$$f(x) = \sqrt{1 - \sin 2x} = \sqrt{(\cos x - \sin x)^2}$$
$$\Rightarrow f(x) = |\cos x - \sin x|$$
$$\Rightarrow f(x) = \begin{cases} \cos x - \sin x, & \text{for } 0 \le x \le \pi/4 \\ -(\cos x - \sin x), & \text{for } \pi/4 < x \le \pi/2 \end{cases}$$
$$\therefore f'(x) = \begin{cases} -(\cos x - \sin x), & \text{for } 0 < x < \pi/4 \\ (\cos x + \sin x), & \text{for } \pi/4 < x < \pi/2 \end{cases}$$

155 **(c)**

Let $u = \sin x$ and $v = \cos x$

On differentiating w.r.t. x respectively, we get

$$\frac{du}{dx} = \cos x \text{ and } \frac{dv}{dx} = -\sin x$$
$$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = -\cot x$$

156 **(d)**

Let $y = F\{f(\phi(x))\}$

On differentiating w.r.t. x, we get

$$y' = F'[f\{\phi(x)\}]\frac{d}{dx}f\{\phi(x)\}$$
$$= F'[f\{\phi(x)\}]f'\{\phi(x)\}\frac{d}{dx}\phi(x)$$
$$= F'[f\{\phi(x)\}]f'\{\phi(x)\}\phi'(x)$$

157 **(b)**

Given, $x\sqrt{1+y} = -y\sqrt{1+x}$ ...(i)

On squaring both sides, we get

$$x^2(1+y) = y^2(1+x)$$
$$\Rightarrow\ (x-y)(x+y) + xy(x-y) = 0$$
$$\Rightarrow\ (x-y)(x+y+xy) = 0$$

$x - y \neq 0$ because it does not satisfy the Eq. (i).

$$\therefore\ x + y + xy = 0\ \Rightarrow\ y = -\frac{x}{1+x}$$
$$\Rightarrow \frac{dy}{dx} = -\frac{(1+x)(1) - x(1)}{(1+x)^2} = -\frac{1}{(1+x)^2}$$

158 **(b)**

$$\because \frac{x^2 - y^2}{x^2 + y^2} = \sec^{-1}(e^a)$$

On differentiating w.r.t. x, we get

$$\frac{(x^2+y^2)\left(2x - 2y\frac{dy}{dx}\right) - (x^2-y^2)\left(2x + 2y\frac{dy}{dx}\right)}{(x^2+y^2)^2} = 0$$
$$\Rightarrow x(x^2+y^2) - y(x^2+y^2)\frac{dy}{dx}$$
$$= (x^2-y^2)x + y(x^2-y^2)\frac{dy}{dx}$$
$$\Rightarrow (x^2y - y^3 + x^2y + y^3)\frac{dy}{dx}$$
$$= (x^3 + xy^2 - x^3 + xy^2)$$
$$\Rightarrow \frac{dy}{dx} = \frac{2xy^2}{2x^2y} = \frac{y}{x}$$

159 **(a)**

We have,

$$f(x) = \log_a(\log_a x)$$
$$\Rightarrow f'(x) = \frac{1}{\log_a x \cdot \log_e a}\frac{d}{dx}(\log_a x)$$
$$\Rightarrow f'(x) = \frac{1}{\log_a x \log_e a} \times \frac{1}{x\log_e a} = \frac{\log_a e}{x\log_e x}$$

160 **(b)**

$$\because y = \log^n x$$

On differentiating w.r.t. x, we get

$$x\log x \log^2 x \log^3 x \ldots \log^{n-1} x \log^n x \frac{dy}{dx}$$
$$= \frac{x\log x \log^2 x \log^3 x \ldots \log^{n-1} x \log^n x.1}{x\log x \log^2 x \log^3 x \ldots \log^{n-1} x}$$
$$= \log^n x$$

161 **(b)**

$$\text{Let } D = \begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix}$$
$$\Rightarrow D = \begin{vmatrix} \sin px & p\cos px & -p^2\sin px \\ -p^3\cos px & p^4\sin px & p^5\cos px \\ -p^6\sin px & -p^7\cos px & p^8\sin px \end{vmatrix}$$

Taking p^3 and p^6common from R_2 and R_3row

$$= p^9\begin{vmatrix} \sin px & p\cos px & -p^2\sin px \\ -\cos px & p\sin px & p^2\cos px \\ -\sin px & -p\cos px & p^2\sin px \end{vmatrix}$$
$$= -p^9\begin{vmatrix} \sin px & p\cos px & -p^2\sin px \\ -\cos px & p\sin px & p^2\cos px \\ \sin px & p\cos px & -p^2\sin px \end{vmatrix}$$

$= 0$ ($R_1$and R_3rows are identical)

162 **(d)**

$y = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots$

$\Rightarrow y = e^{-x}$

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = -e^{-x}$

Again differentiating w.r.t. x, we get

$\frac{d^2y}{dx^2} = -e^{-x}(-1) = e^{-x} = y$

163 **(b)**

We have, $f(4) = 4$ and $f'(4) = 1$

$\therefore \lim_{x\to 4} \frac{2-\sqrt{f(x)}}{2-\sqrt{x}}$

$= \lim_{x\to 4} \frac{\frac{-f'(x)}{2\sqrt{f(x)}}}{-\frac{1}{2\sqrt{x}}}$ [Using L'Hospital's Rule]

$\Rightarrow \lim_{x\to 4} \frac{2-\sqrt{f(x)}}{2-\sqrt{x}} = \lim_{x\to 4} \frac{\sqrt{x}\, f'(x)}{\sqrt{f(x)}} = \frac{2f'(4)}{\sqrt{f(4)}} = \frac{2}{2}$

$= 1$

164 **(c)**

$\because\ x = \frac{2at}{1+t^3}$ and $y = \frac{2at^2}{(1+t^3)^2}$

$\therefore 2ay = x^2$

$\Rightarrow \frac{dy}{dx} = \frac{x}{a}$

165 **(c)**

Given, $y = e^{a\sin^{-1}x}$

On differentiating w.r.t. x, we get

$y_1 = e^{a\sin^{-1}x} a.\frac{1}{\sqrt{1-x^2}}$

$\Rightarrow\ y_1\sqrt{1-x^2} = ay$

$\Rightarrow\ (1-x^2)y_1^2 = a^2y^2$

Again, differentiating w.r.t. x, we get

$(1-x^2)2y_1y_2 - 2xy_1^2 = a^2 2yy_1$

$\Rightarrow (1-x^2)y_2 - xy_1 - a^2y = 0$

Using Leibnitz's rule,

$(1-x^2)y_{n+2} + {}^nC_1 y_{n+1}(-2x) + {}^nC_2 y_n(-2)$

$-xy_{n+1} - {}^nc_1 y_n - a^2y_n = 0$

$\Rightarrow\ (1-x^2)y_{n+2} + xy_{n+1}(-2n-1)$

$+y_n[-n(n-1) - n - a^2] = 0$

$\Rightarrow\ (1-x^2)y_{n+2} - (2n+1)xy_{n+1} = (n^2+a^2)y_n$

166 **(a)**

Since, $\frac{x-y}{x+y} = \sec^{-1} a$

$\Rightarrow\ \frac{(x+y)\left(1-\frac{dy}{dx}\right)-(x-y)\left(1+\frac{dy}{dx}\right)}{(x+y)^2} = 0$

$\Rightarrow x + y - x + y - (x+y+x-y)\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx}$

$= \frac{y}{x}$

167 **(c)**

Given, $y = x + x^2 + x^3 + \ldots \Rightarrow y = \frac{x}{1-x}$

$\Rightarrow\ x = \frac{y}{1+y} = y - y^2 + y^3 - \ldots$

On differentiating w.r.t. y, we get

$\frac{dx}{dy} = 1 - 2y + 3y^2 - \ldots$

168 **(b)**

Let $y = \sqrt{\frac{1-\sin 2x}{1+\sin 2x}} = \frac{\cos x - \sin x}{\cos x + \sin x}$

$= \frac{1-\tan x}{1+\tan x} = \tan\left(\frac{\pi}{4} - x\right)$

$\Rightarrow\ \frac{dy}{dx} = -\sec^2\left(\frac{\pi}{4} - x\right)$

169 **(b)**

Let $u = \log_{10} x$ and $v = x^2$

$\therefore \frac{du}{dx} = \frac{\log_{10} e}{x}$ and $\frac{dv}{dx} = 2x$

$\therefore\ \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{\log_{10} e}{x}/2x$

$= \frac{\log_{10} e}{2x^2}$

170 **(d)**

$\because\ y = x \operatorname{In}\left(\frac{x}{a+bx}\right) = x(\operatorname{In} x - \operatorname{In}(a+bx))$

or $\left(\frac{y}{x}\right) = \operatorname{In} x - \operatorname{In}(a+bx)$

On differentiating both sides w.r.t. x, we get

$\left(\frac{x\frac{dy}{dx} - y.1}{x^2}\right) = \frac{1}{x} - \frac{b}{a+bx} = \frac{a}{x(a+bx)}$ ... (i)

or $\left(x\frac{dy}{dx} - y\right) = \frac{ax}{a+bx}$

On taking log on both sides, we get

$\operatorname{In}\left(x\frac{dy}{dx} - y\right) = \operatorname{In}(ax) - \operatorname{In}(a+bx)$

On differentiating both sides w.r.t. x, we get

$\frac{x\frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{dy}{dx}}{\left(x\frac{dy}{dx} - y\right)} = \frac{1}{x} - \frac{b}{a+bx} = \frac{a}{x(a+bx)}$

$= \frac{\left(x\frac{dy}{dx} - y\right)}{x^2}$ [from Eq. (i)]

or $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$

172 **(a)**

On differentiating given equation w.r.t. x, we get

$4x - 3x\frac{dy}{dx} - 3y + 2y\frac{dy}{dx} + 1 + 2\frac{dy}{dx} - 0 = 0$

$\Rightarrow\ \frac{dy}{dx} = \frac{3y - 4x - 1}{2y - 3x + 2}$

173 **(c)**

Given, $y = \sqrt{\sin x + y}$ $\Rightarrow$ $y^2 = \sin x + y$

$\Rightarrow 2y\frac{dy}{dx} = \cos x + \frac{dy}{dx}$

$$\Rightarrow (2y-1)\frac{dy}{dx} = \cos x$$

174 **(b)**

We have,

$2^x + 2^y = 2^{x+y}$

Differentiating with respect to x, we get

$$2^x \log 2 + 2^y \log 2 \frac{dy}{dx} = 2^{x+y} \log 2 \left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow 2^x + 2^y \frac{dy}{dx} = 2^{x+y}\left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{2^x - 2^{x+y}}{2^{x+y} - 2y} \Rightarrow \left(\frac{dy}{dx}\right)_{(1,0)} = \frac{2-4}{4-2} = -1$$

175 **(a)**

Given, $y = \sec(\tan^{-1} x)$

$\Rightarrow\ y = \sec(\sec^{-1}\sqrt{1+x^2}) = \sqrt{1+x^2}$

On differentiating w.r.t. x,we get

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1+x^2}}(2x) = \frac{x}{\sqrt{1+x^2}}$$

176 **(b)**

Given, $f(x) = (x-7)^2(x-2)^7$

$\Rightarrow \quad f(\theta) = (\theta-7)^2(\theta-2)^7$

On differentiating w.r.t. θ, we get

$\Rightarrow \quad f'(\theta) = 2(\theta-7)(\theta-2)^7$
$\qquad + 7(\theta-2)^6(\theta-7)^2$

put $\quad f'(\theta) = 0$

$\Rightarrow \quad (\theta-7)(\theta-2)^6[2(\theta-2) + 7(\theta-7)] = 0$

$\Rightarrow \quad 9\theta = 53 \Rightarrow \theta = \frac{53}{9}$

177 **(b)**

We have,

$x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$

$$\Rightarrow (x^2+y^2)^2 = t^2 + \frac{1}{t^2} - 2$$

$\Rightarrow (x^2+y^2)^2 = x^4 + y^4 - 2$

$\Rightarrow 2x^2y^2 = -2$

$\Rightarrow x^2y^2 = -1$

$$\Rightarrow y^2 = -\frac{1}{x^2} \Rightarrow 2y\frac{dy}{dx} = \frac{2}{x^3} \Rightarrow x^3y\frac{dy}{dx} = 1$$

178 **(b)**

Let $f(x) = |x-1| + |x-5|$

$$\Rightarrow f(x) = \begin{cases} -2x+6, & x < 1 \\ 4 & 1 \le x < 5 \\ 2x-6 & x \ge 5 \end{cases}$$

$$\therefore \frac{d}{dx}(f(x)) = \begin{cases} -2, & x < 1 \\ 0 & 1 < x < 5 \\ 2 & x > 5 \end{cases}$$

Hence, $\left(\frac{d}{dx}(f(x))\right)_{x=3} = 0$

179 **(b)**

$$\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1} x = \frac{\pi}{2} - \sin^{-1} y$$

$\Rightarrow \sin^{-1} x = \cos^{-1} y$

$\Rightarrow y = \sqrt{1-x^2}$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}(-2x) = -\frac{x}{y}$$

180 **(c)**

$$f[f(f(x))] = f\left[f\left(\frac{1}{1-x}\right)\right]$$

$$= f\left(\frac{1}{1-\frac{1}{1-x}}\right)$$

$$\left[\because f(x) = \frac{1}{1-x}\right]$$

$$= f\left(\frac{1-x}{-x}\right) = \frac{1}{1+\left(\frac{1-x}{x}\right)}$$

$\Rightarrow\ f[f(f(x))] = x$

$\therefore$ The derivative of composite function is equal to 1.

181 **(a)**

Given that, $x = \exp\left\{\tan^{-1}\left(\frac{y-x^2}{x^2}\right)\right\}$

Taking log on both sides, we get

$$\log x = \tan^{-1}\left(\frac{y-x^2}{x^2}\right)$$

$$\Rightarrow \frac{y-x^2}{x^2} = \tan(\log x)$$

$\Rightarrow y = x^2 \tan(\log x) + x^2$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = 2x\tan(\log x) + x^2\frac{\sec^2(\log x)}{x} + 2x$$

$$\Rightarrow \frac{dy}{dx} = 2x\tan(\log x) + x\sec^2(\log x) + 2x$$

$$\Rightarrow \frac{dy}{dx} = 2x[1 + \tan(\log x)] + x\sec^2(\log x)$$

182 **(b)**

Given, $x = \frac{\sin y}{\sin(a+y)}$

$$\Rightarrow \frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin\cos(a+y)}{\sin^2(a+y)}$$

$$= \frac{\sin(a+y-y)}{\sin^2(a+y)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

183 **(b)**

Given, $f(x) = e^x \sin x$

$\Rightarrow \quad f'(x) = e^x\cos x + \sin x e^x$

$\Rightarrow \quad f''(x) = e^x\cos x - e^x\sin x$
$\qquad + e^x\sin x + e^x\cos x$

$= 2e^x\cos x$

184 **(b)**

We know that,

$$\cos A \cos 2A \cos 2^2 A \ldots \cos 2^{n-1} A = \frac{\sin(2^n A)}{2^n \sin A}$$

$\therefore \cos x \cos 2x \cos 4x \cos 8x \cos 16x = \dfrac{\sin 32\,x}{32 \sin x}$

$\Rightarrow f(x) = \dfrac{1}{32} . \dfrac{\sin 32x}{\sin x}$

$\therefore f'(x) = \dfrac{1}{32} \times \dfrac{\sin x(32 \cos 32x) - \sin 32x \cos x}{\sin^2 x}$

$\Rightarrow f'\left(\dfrac{\pi}{4}\right) = \dfrac{1}{\sin\frac{\pi}{4}} = \sqrt{2}$

185 **(a)**

Given, $\frac{1+x}{1-y} = \sec a$

$\Rightarrow y \sec a = \sec a - 1 - x$

$\Rightarrow \dfrac{dy}{dx} \sec a = -1$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{-1}{\sec a} = \dfrac{-1}{\left(\frac{1+x}{1-y}\right)}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{y-1}{x+1}$

186 **(c)**

Let $u = a^{\sec x}$ and $v = a^{\tan x}$

$\Rightarrow \dfrac{du}{dx} = a^{\sec x} \log_e a. \sec x \tan x$

and $\frac{dv}{dx} = a^{\tan x} \log_e a. \sec^2 x$

$\therefore \dfrac{du}{dv} = \dfrac{du/dx}{dv/dx} = \dfrac{a^{\sec x} \log_e a \sec x \tan x}{a^{\tan x} \log_e a \sec^2 x}$

$= a^{\sec x - \tan x} \sin x$

187 **(b)**

On differentiating given curves w.r.t.θ respectively, we get

$\dfrac{dx}{d\theta} = a\left(-\sin\theta + \dfrac{1}{\tan\left(\frac{\theta}{2}\right)} . \sec^2\dfrac{\theta}{2} . \dfrac{1}{2}\right)$

and $\frac{dy}{d\theta} = a \cos\theta$

$\Rightarrow \frac{dx}{d\theta} = \frac{a \cos^2\theta}{\sin\theta}$ and $\frac{dy}{d\theta} = a\cos\theta$

$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\cos\theta}{a\cos^2\theta/\sin\theta} = \tan\theta$

188 **(c)**

Since, $\phi(x) = f^{-1}(x) \Rightarrow x = f\{\phi(x)\}$

On differentiating w.r.t. x, we get

$1 = f'\{\phi(x)\}. \phi'(x)$

$\Rightarrow$

$\phi'(x) = \frac{1}{f'\{\phi(x)\}}$ ... (i)

But $f'\{\phi(x)\} = \frac{1}{1+\{\phi(x)\}^5}$ $\left(\because f'(x) = \frac{1}{1+x^5}\right)$

$\therefore$ From Eq. (i),

$\phi'(x) = \frac{1}{f'\{\phi(x)\}} = 1 + \{\phi(x)\}^5$

189 **(b)**

We have,

$f(x) = 3\,e^{x^2} \Rightarrow f'(x) = 6x\,e^{x^2}$

$\therefore f(0) = 3$ and $f'(0) = 0$

Now,

$f'(x) - 2\,x\,f(x) + \dfrac{1}{3}f(0) - f'(0)$

$= 6\,x\,e^{x^2} - 6\,x\,e^{x^2} + \dfrac{1}{3}(3) - 0$

$= 1$

190 **(c)**

On differentiating partially the given equation w.r.t. x and y

$\dfrac{\partial u}{\partial x} = \dfrac{(x+y)}{(x^2+y^2)} \times \dfrac{x^2 - y^2 + 2xy}{(x+y)^2}$

$\Rightarrow x\frac{\partial u}{\partial x} = \frac{x}{(x^2+y^2)} \times \frac{x^2-y^2+2xy}{(x+y)}$ ...(i)

and $\frac{\partial u}{\partial y} = \frac{(x+y)}{(x^2+y^2)} \times \frac{y^2-x^2+2xy}{(x+y)^2}$

$\Rightarrow y\frac{\partial u}{\partial y} = \frac{y}{(x^2+y^2)} \times \frac{(y^2-x^2+2xy)}{(x+y)}$ ...(ii)

On adding Eqs. (i) and (ii), we get

$x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \dfrac{1}{(x^2+y^2)(x+y)}$

$[x(x^2 - y^2 + 2xy) + y(y^2 - x^2 + 2xy)]$

$= \frac{1}{(x^2+y^2)(x+y)} \times (x^2+y^2)(x+y)$

$= 1$

191 **(a)**

We have,

$F(x) = \dfrac{1}{x^2}\displaystyle\int_4^x (4t^2 - 2F'(t))\,dt$

$\Rightarrow x^2F(x) = \displaystyle\int_4^x (4t^2 - 2F'(t))\,dt$

Differentiating both sides with respect to x, we get

$2x\,F(x) + x^2F'(x) = 4x^2 - 2F'(x)$

Putting $x = 4$, we get

$8F(4) + 16F'(4) = 64 - 2F'(4)$

$\Rightarrow 18F'(4) = 64 \quad [\because F(4) = 0]$

$\Rightarrow F'(4) = \dfrac{32}{9}$

192 **(d)**

Put $x^2 = \cos 2\theta$ in the given equation, we get

$y = \tan^{-1}\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}$

$= \tan^{-1}\dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$

$= \tan^{-1}\tan\left(\dfrac{\pi}{4} - \theta\right)$

$\Rightarrow y = \dfrac{\pi}{4} - \theta = \dfrac{\pi}{4} - \dfrac{1}{2}\cos^{-1}x^2$

$\Rightarrow \frac{dy}{dx} = 0 - \frac{1}{2}\left(-\frac{(2x)}{\sqrt{1-x^4}}\right) = \frac{x}{\sqrt{1-x^4}}$

193 **(b)**

We have,

$f(x) = |x^2 - 5x + 6|$

$\Rightarrow f(x) = \begin{cases} x^2 - 5x + 6, & \text{if } x \geq 3 \text{ or } x \leq 2 \\ -(x^2 - 5x + 6), & \text{if } 2x < x < 3 \end{cases}$

$\therefore f'(x) = \begin{cases} (2x - 5), & \text{if } x > 3 \text{ or } x < 2 \\ -(2x - 5), & \text{if } 2 < x < 3 \end{cases}$

195 **(c)**

Let $y = x^6 + 6^x$

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = 6x^5 + 6^x \log 6$

196 **(b)**

We have,

$f(x) = \cos^2 x + \cos^2(x + \pi/3) + \sin x \sin(x + \pi/3)$

$\Rightarrow f(x) = \frac{1}{2}\left[1 + \cos\ 2x + 1 + \cos\left(2x + \frac{2\pi}{3}\right) + \cos\frac{\pi}{3} - \cos\left(2x + \frac{\pi}{3}\right)\right]$

$\Rightarrow f(x) = \frac{1}{2}\left[\frac{5}{2} + \cos\ 2x + \cos\left(2x + \frac{2\pi}{3}\right) - \cos\left(2x + \frac{\pi}{3}\right)\right]$

$\Rightarrow f(x) = \frac{1}{2}\left[\frac{5}{2} + 2\cos\left(2x + \frac{\pi}{3}\right)\cos\frac{\pi}{3} - \cos\left(2x + \frac{\pi}{3}\right)\right]$

$\Rightarrow f(x) = \frac{5}{4}$

$\therefore gof(x) = g(5/4) = 3$ for all x

$\Rightarrow \frac{d}{dx}\big(gof(x)\big) = 0$ for all x

197 **(c)**

We have,

$y = \sin^{-1} x + \cos^{-1} y \Rightarrow y = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = 0$

198 **(b)**

Given, $z = y + f(v)$ ...(i)

Where $v = \left(\frac{x}{y}\right)$

On differentiating partially Eq. (i) w.r.t. x, we get

$\frac{\partial z}{\partial x} = f'\left(\frac{x}{y}\right).\left(\frac{1}{y}\right)$

$\Rightarrow v\frac{\partial z}{\partial x} = f'\left(\frac{x}{y}\right).\left(\frac{1}{y}\right)\left(\frac{x}{y}\right) = f'\left(\frac{x}{y}\right)\left(\frac{x}{y^2}\right)$ ...(ii)

Now, differentiating partially Eq. (i) w.r.t. y, we get

$\frac{\partial z}{\partial y} = 1 + f'\left(\frac{x}{y}\right)\left(\frac{-x}{y^2}\right)$ ...(iii)

On adding Eqs. (ii) and (iii), we get

$v\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = f'\left(\frac{x}{y}\right)\left(\frac{x}{y^2}\right) + 1 + f'\left(\frac{x}{y}\right)\left(\frac{-x}{y^2}\right) = 1$

199 **(d)**

$y = \tan^{-1}\left(\frac{\sqrt{x} - x}{1 + x^{3/2}}\right)$

$= \tan^{-1}\left(\frac{\sqrt{x} - x}{1 + \sqrt{x}.x}\right)$

$= \tan^{-1}(\sqrt{x}) - \tan^{-1}(x)$

On differentiating w.r.t. x, we get

$y' = \frac{1}{1+x}.\frac{1}{2\sqrt{x}} - \frac{1}{1+x^2}$

$\Rightarrow y'(1) = \frac{1}{2}.\frac{1}{2} - \frac{1}{2} = -\frac{1}{4}$

200 **(c)**

We have,

$f(1) = 1$ and $f'(1) = 2$

$\therefore \lim_{x\to 1}\frac{\sqrt{f(x)} - 1}{\sqrt{x} - 1}$

$= \lim_{x\to 1}\frac{\frac{f'(x)}{2\sqrt{f(x)}}}{\frac{1}{2\sqrt{x}}} = \lim_{x\to 1}\frac{\sqrt{x}\,f'(x)}{\sqrt{f(x)}} = \frac{f'(1)}{\sqrt{f(1)}} = 2$

201 **(a)**

Let $u = \cos^3 x, v = \sin^3 x$

$\frac{du}{dx} = -3\cos^2 x \sin x, \frac{dv}{dx} = 3\sin 2x \cos x$

Now, $\frac{du}{dv} = \frac{-3\cos^2 x \sin x}{3\sin^2 x \cos x} = -\cot x$

202 **(b)**

We have,

$y = \log\left(\frac{1+x}{1-x}\right)^{1/4} - \frac{1}{2}\tan^{-1} x$

$\Rightarrow y = \frac{1}{4}\log(1+x) - \frac{1}{4}\log(1-x) - \frac{1}{2}\tan^{-1} x$

$\Rightarrow \frac{dy}{dx} = \frac{1}{4(1+x)} + \frac{1}{4(1-x)} - \frac{1}{2(1+x^2)}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2(1-x^2)} - \frac{1}{2(1+x^2)} = \frac{x^2}{1-x^4}$

203 **(a)**

Given, $y = \left(x + \sqrt{1+x^2}\right)^n$

$\frac{dy}{dx} = n\left[x + \sqrt{1+x^2}\right]^{n-1}\left(1 + \frac{x}{\sqrt{x^2+1}}\right)$

$= \frac{n[x + \sqrt{1+x^2}]^n}{\sqrt{1+x^2}}$

$\Rightarrow (1+x^2)\left(\frac{dy}{dx}\right)^2 = n^2y^2$

$\Rightarrow 2\frac{dy}{dx}\frac{d^2y}{dx^2}(1+x^2) + 2x\left(\frac{dy}{dx}\right)^2 = 2n^2y\frac{dy}{dx}$

$\Rightarrow \frac{d^2y}{dx^2}(1+x^2) + x\frac{dy}{dx} = n^2y$

204 **(d)**

Let $y = \tan^{-1}\left\{\frac{3\sqrt{x} - x^{3/2}}{1-3x}\right\}$

Again let $\sqrt{x} = \tan t$

$\therefore\ y = \tan^{-1}\left\{\frac{3\tan t - \tan^3 t}{1-3\tan^2 t}\right\} = \tan^{-1}(\tan 3t)$

$\Rightarrow\ y = 3\tan^{-1}\sqrt{x}$

$\Rightarrow\ \frac{dy}{dx} = \frac{3}{1+x}.\frac{1}{2\sqrt{x}} = \frac{3}{2(1+x)\sqrt{x}}$

205 **(d)**

We have, $y = \tan^{-1}\left[\frac{\sin x + \cos x}{\cos x - \sin x}\right]$

$\Rightarrow y = \tan^{-1}\left[\frac{1+\tan x}{1-\tan x}\right]$

$\Rightarrow y = \tan^{-1}\left[\frac{\tan\left(\frac{\pi}{4}\right)+\tan x}{1-\tan\left(\frac{\pi}{4}\right)\tan x}\right]$

$\Rightarrow y = \tan^{-1}\tan\left(\frac{\pi}{4}+x\right)$

$\Rightarrow y = \left(\frac{\pi}{4}\right)+x$

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = 1$

206 **(d)**

Given, $x = \cos\theta$, $\quad y = \sin 5\theta$

$\Rightarrow\ \frac{dx}{d\theta} = -\sin\theta,\quad \frac{dy}{d\theta} = 5\cos 5\theta$

$\therefore\ \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{5\cos 5\theta}{\sin\theta}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right).\frac{d\theta}{dx}$

$= \frac{d}{d\theta}\left(\frac{-5\cos 5\theta}{\sin\theta}\right)\frac{1}{-\sin\theta}$

$= \left(\frac{\sin\theta\sin 5\theta.25 + 5\cos 5\theta\cos\theta}{\sin^2\theta}\right).\frac{1}{-\sin\theta}$

$= -\frac{25\sin 5\theta}{\sin^2\theta} - \frac{5\cos 5\theta\cos\theta}{\sin^3\theta}$

$\therefore (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx}$

$= (1-\cos^2\theta)\left(\frac{-25\sin 5\theta}{\sin^2\theta} - \frac{5\cos 5\theta\cos\theta}{\sin^3\theta}\right)$

$-\cos\theta\left(\frac{-5\cos 5\theta}{\sin\theta}\right)$

$= \sin^2\theta\left(\frac{-25\sin 5\theta}{\sin^2\theta} - \frac{5\cos\theta\cos 5\theta}{\sin^3\theta}\right)$

$+\frac{5\cos\theta\cos 5\theta}{\sin\theta}$

$= -25\sin 5\theta - \frac{5\cos\theta\cos 5\theta}{\sin\theta} + \frac{5\cos\theta\cos 5\theta}{\sin\theta}$

$= -25y$

207 **(d)**

$\because f(x) = a + bx$

$f\{f(x)\} = a + b(a+bx)$

$= ab + a + b^2x = a(1+b) + b^2x$

$f[f\{f(x)\}] = f\{a(1+b)+b^2x\}$

$= a + b\{a(1+b)+b^2x\}$

$= a(1+b+b^2) + b^3x$

$\therefore f'(x) = a(1+b+b^2+\ldots+b^{r-1}) + b^rx$

$= a\left(\frac{b^r-1}{b-1}\right) + b^rx$

208 **(c)**

We have,

$x^y = e^{x-y} \Rightarrow y\log x = (x-y) \Rightarrow y = \frac{x}{1+\log x}$

Differentiating w.r.t. x, we get $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

209 **(d)**

Let $u = \sin^2 x$ and $v = \cos^2 x$

On differentiating w.r.t. x, we get

$\frac{du}{dx} = 2\sin x\cos x = \sin 2x$

and $\frac{dv}{dx} = -2\cos x\sin x = -\sin 2x$

$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{\sin 2x}{-\sin 2x} = -1$

210 **(a)**

We have,

$x^p\, y^q = (x+y)^{p+q}$

$\Rightarrow p\log x + q\log y = (p+q)\log(x+y)$

Diff w.r.t. x, we get

$\frac{p}{x} + \frac{q}{y}\frac{dy}{dx} = \frac{p+q}{x+y}\left(1+\frac{dy}{dx}\right)$

$\Rightarrow \frac{dy}{dx}\left(\frac{q}{y} - \frac{p+q}{x+y}\right) \Rightarrow \frac{p+q}{x+y} - \frac{p}{x} \Rightarrow \frac{dy}{dx} = \frac{y}{x}$

212 **(a)**

Let $y = \sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)$

Putting $x = \cos\theta$, we get

$y = \sin^{-1}\left(\frac{1}{\sqrt{2}}\cos\frac{\theta}{2} + \frac{1}{\sqrt{2}}\sin\frac{\theta}{2}\right)$

$= \sin^{-1}\left\{\sin\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right\}$

$\Rightarrow y = \frac{\pi}{4} + \frac{1}{2}\theta = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x$

$\Rightarrow \frac{dy}{dx} = -\frac{1}{2\sqrt{1-x^2}}$

213 **(c)**

Let $y = \tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

Put $x = \cos 2\theta$

$\therefore\quad y = \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$

$= \tan^{-1}\left(\frac{\sqrt{2}\cos\theta - \sqrt{2}\sin\theta}{\sqrt{2}\cos\theta + \sqrt{2}\sin\theta}\right)$

$= \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right) = \tan^{-1}\left\{\tan\left(\frac{\pi}{4}-\theta\right)\right\}$

$= \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx}=0+\frac{1}{2}.\frac{1}{\sqrt{1-x}}=\frac{1}{2\sqrt{1-x^2}}$$

214 **(c)**

$y=\tan^{-1}x+\cot^{-1}x+\sec^{-1}x+\operatorname{cosec}^{-1}x$

$=\frac{\pi}{2}+\frac{\pi}{2}=\pi$

$\frac{dy}{dx}=0$

215 **(c)**

$\because y=\left(\frac{ax+b}{cx+d}\right)$

or $c\,xy+dy=ax+b$

On differentiating both sides w.r.t. x, we get

$c\left\{x\frac{dy}{dx}+y.1\right\}+d\frac{dy}{dx}=a$

or $x\frac{dy}{dx}+y+\left(\frac{d}{c}\right)\frac{dy}{dx}=\left(\frac{a}{c}\right)$

Again differentiating both sides w.r.t. x, we get

$x\frac{d^2y}{dx^2}+\frac{dy}{dx}+\frac{dy}{dx}+\left(\frac{d}{c}\right)\frac{d^2y}{dx^2}=0$

or $x+\frac{2\frac{dy}{dx}}{\left(\frac{d^2y}{dx^2}\right)}+\frac{d}{c}=0$

Again, on differentiating both sides w.r.t. x, we get

$$1+\frac{\left(\frac{d^2y}{dx^2}.2\frac{d^2y}{dx^2}-2\frac{dy}{dx}.\frac{d^3y}{dx^3}\right)}{\left(\frac{d^2y}{dx^2}\right)}+0=0$$

$$\Rightarrow 2\frac{dy}{dx}.\frac{d^3y}{dx^3}=3\left(\frac{d^2y}{dx^2}\right)^2$$

216 **(d)**

Given, $y=(\log_{\cos x}\sin x)(\log_{\sin x}\cos x)+$ sin−1*2x1+x2*

At $x=\frac{\pi}{2}$, $\log_{\sin x}$ cos x is not defined.

Hence, we cannot determined the derivative at $x=\frac{\pi}{2}$.

217 **(b)**

$$y=\tan^{-1}\left(\frac{a\cos x-b\sin x}{b\cos x+a\sin x}\right)$$

$$=\tan^{-1}\left(\frac{\frac{a}{b}-\tan x}{1+\frac{a}{b}\tan x}\right)$$

$=\tan^{-1}\left[\tan\left\{\tan^{-1}\left(\frac{a}{b}\right)-\right.\right.$

x

$\Rightarrow\ y=\tan^{-1}\left(\frac{a}{b}\right)-x$

$\therefore\ \frac{dy}{dx}=0-1=-1$

218 **(d)**

We have, $x\cos\theta$, $y=\sin 5\theta$

$$\therefore\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=-\frac{5\cos 5\theta}{\sin\theta}$$

$$\Rightarrow\frac{d^2y}{dx^2}=-5\frac{d}{d\theta}\left(\frac{\cos 5\theta}{\sin\theta}\right)\cdot\frac{d\theta}{dx}$$

$$\Rightarrow\frac{d^2y}{dx^2}=\frac{-25\sin\theta\sin 5\theta-5\cos\theta\cos 5\theta}{\sin^3\theta}$$

$$\therefore(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=-25\sin 5\theta=-25y$$

219 **(c)**

Let $y=\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

Put $x=\cos 2\theta\Rightarrow\theta=\frac{1}{2}\cos^{-1}x$

$$\therefore y=\tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$$

$$\Rightarrow y=\tan^{-1}\left(\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right)$$

$$\Rightarrow y=\tan^{-1}\left(\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right)$$

$$\Rightarrow y=\tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$

$$\Rightarrow y=\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\theta\right)\right)$$

$$\Rightarrow y=\frac{\pi}{4}-\theta$$

$$\Rightarrow y=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx}=-\frac{1}{2}\left(\frac{-1}{\sqrt{1-x^2}}\right)=\frac{1}{2\sqrt{1-x^2}}$$

220 **(b)**

$\because\ f(x)=(x-2)(x-4)(x-6)\dots(x-2n)$

Taking log on both sides in the given equation, we get

$\log f(x)=\log(x-2)+\log(x-4)+\ldots+\log(x-$ *2n*

on differentiating w.r.t. x, we get

$$\frac{1}{f(x)}f'(x)=\frac{1}{(x-2)}+\frac{1}{(x-4)}+\ldots+\frac{1}{(x-2n)}$$

$\Rightarrow\quad f'(x)=(x-4)(x-6)\dots(x-2n)$

$+(x-2)(x-6)\dots(x-2n)$

$+\cdots+(x-2)(x-6)\dots(x-2(n-1))$

$\therefore\ f'(2)=(-2)(-4)\dots(2-2n)$

$=(-2)^{n-1}(1.2\ldots..(n-1)=(-2)^{n-1}(n-1)!$

221 **(a)**

Given, $x=2\cos\theta-\cos 2\theta$

and $y=2\sin\theta-\sin 2\theta$

$\frac{dx}{d\theta}=-2\sin\theta+2\sin 2\theta$

and $\frac{dy}{d\theta}=2\cos\theta-2\cos 2\theta$

$$\therefore\quad\frac{dy}{dx}=\frac{2\cos\theta-2\cos 2\theta}{-2\sin\theta+2\sin 2\theta}$$

$$=\frac{\cos\theta-\cos 2\theta}{\sin 2\theta-\sin\theta}$$

$$= \frac{2\sin\left(\frac{\theta+2\theta}{2}\right)\sin\left(\frac{2\theta-\theta}{2}\right)}{2\cos\left(\frac{\theta+2\theta}{2}\right)\sin\left(\frac{2\theta-\theta}{2}\right)} = \tan\frac{3\theta}{2}$$

222 **(c)**

Since, $f'(x) > \phi'(x)$

$\Rightarrow\ 2^{2x-1} 2\log 2 > -2^x \log 2 + 2\log 2$

$\Rightarrow\ 2^{2x} > -2^x + 2$

$\Rightarrow\ 2^{2x} + 2^x - 2 > 0$

$\Rightarrow\ (2^x - 1)(2^x + 2) > 0$

$\Rightarrow\ 2^x - 1 > 0 \quad [\because 2^x + 2 > 0 \text{ for all } x]$

$\Rightarrow\ 2^x > 1$

$\therefore\ x > 0$

223 **(b)**

We have,

$x\sqrt{1+y} + y\sqrt{1+x} = 0$

$\Rightarrow x^2(1+y) = y^2(1+x)$

$\Rightarrow x^2 - y^2 = -x^2y + xy^2$

$\Rightarrow (x-y)(x+y) = -xy(x-y)$

$\Rightarrow x + y = -xy$

$$\Rightarrow y = -\frac{x}{1+x} \Rightarrow \frac{dy}{dx} = -\left\{\frac{(1+x)-x}{(1+x)^2}\right\}$$

$$= -\frac{1}{(1+x)^2}$$

224 **(c)**

$\because\ y = e^{(1/2)\log(1+\tan^2 x)}$

$\Rightarrow\ y = (\sec^2 x)^{1/2} = \sec x$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \sec x \tan x$$

225 **(a)**

Given,

$$f(x) = \frac{1}{4}\left[\frac{x-1}{1} + \frac{(x-1)^3}{3} + \frac{(x-1)^5}{5} + \frac{(x-1)^7}{7} + \cdots\right]$$

$$\Rightarrow\ f(x) = \frac{1}{4}\left[\frac{1}{2}\log\left(\frac{1+(x-1)}{1-(x-1)}\right)\right] = \frac{1}{8}\log\left(\frac{x}{2-x}\right)$$

$$\Rightarrow\ f'(x) = \frac{1}{8} \times \frac{1}{\left(\frac{x}{2-x}\right)}\left[\frac{(2-x)1 - x(-1)}{(2-x)^2}\right]$$

$$= \frac{1}{4x(2-x)}$$

226 **(d)**

$$f(x) = \begin{vmatrix} x^3 & x^2 & 3x^2 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$$

$$\Rightarrow f(x) = x^3(-6p^3 - 4p^2) - x^2(p^3 - 4p) + 3x^2(p^2 + 6p)$$

$$\Rightarrow f(x) = -6p^3x^3 - 4p^2x^3 - x^2p^3 + 4px^2 + 3p^2x^2 + 18px^2$$

On differentiating w.r.t. x, we get

$$\frac{d}{dx} f(x) = -18p^3x^2 - 12p^2x^2 - 2xp^3 + 8px + 6p^2x + 36px$$

Again differentiating w.r.t. x, we get

$$\frac{d^2}{dx^2} f(x) = -36p^3x - 24p^2x - 2p^3 + 8p + 6p^2 + 36p$$

Again differentiating w.r.t. x, we get

$$\frac{d^3}{dx^3} f(x) = -36p^3 - 24p^2 = \text{constant}$$

227 **(d)**

We have,

$$f(x) = \operatorname{arc}\tan\left(\frac{x^x - x^{-x}}{2}\right)$$

$$\Rightarrow f(x) = \tan^{-1}\left\{\frac{x^{2x} - 1}{2x^x}\right\}$$

$$\Rightarrow f(x) = -\tan^{-1}\left\{\frac{1 - x^{2x}}{2x^x}\right\}$$

$$\Rightarrow f(x) = -\cot^{-1}\left\{\frac{2x^x}{1 - x^{2x}}\right\}$$

$$\Rightarrow f(x) = \frac{-\pi}{2} + \tan^{-1}\left\{\frac{2x^x}{1 - x^{2x}}\right\}$$

$$\Rightarrow f(x) = \begin{cases} \frac{-\pi}{2} + 2\tan^{-1}(x^x), & \text{if } 0 < x < 1 \\ \frac{-\pi}{2} - \pi + 2\tan^{-1}(x^x), & \text{if } x > 1 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} \frac{-\pi}{2} + 2\tan^{-1}(x^x), & \text{if } 0 < x < 1 \\ \frac{-3\pi}{2} + 2\tan^{-1}(x^x), & \text{if } x > 1 \end{cases}$$

$\Rightarrow f'(x) = \frac{2}{1+x^{2x}} \times x^x(1 + \log_e x)$ for all

$x > 0, x \neq 1$

Clearly, $f'(1)$ does not exist

228 **(a)**

We have,

$f(x) + f(y) + f(x)f(y) = 1$ for all $x, y \in R$ ...(i)

Putting $x = y = 0$, we get

$2f(0) + \{f(0)\}^2 = 1$

$\Rightarrow \{f(0)\}^2 + 2f(0) - 1 = 0$

$$\Rightarrow f(0) = \frac{-2 \pm \sqrt{4+4}}{2}$$

$\Rightarrow f(0) = -1 \pm \sqrt{2}$

$\Rightarrow f(0) = \sqrt{2} - 1 \quad [\because f(x) > 0 \text{ for all } x]$

Putting $y = x$ in (i), we get

$\{f(x)\}^2 + 2f(x) - 1 = 0$ for all x

$\Rightarrow 2f(x)f'(x) + 2f'(x) = 0$ for all x

$\Rightarrow 2\{f(x) + 1\}f'(x) = 0$ for all x

$\Rightarrow f'(x) = 0$ for all $x\ [\because f(x) > 0 \text{ for all } x]$

229 **(b)**

Given, $f(x) = e^x$, $g(x) = \sin^{-1} x$

Since, $h(x) = f[g(x)] = e^{\sin^{-1} x}$

Now, $h'(x) = e^{\sin x}.\frac{1}{\sqrt{1-x^2}}$

$\Rightarrow h'(x) = h(x).\frac{1}{\sqrt{1-x^2}}$

$\Rightarrow \frac{h'(x)}{h(x)} = \frac{1}{\sqrt{1-x^2}}$

230 **(b)**

$\because f(x,y) = \frac{\cos(x-4y)}{\cos(x+4y)}$

$\therefore f\left(x,\frac{\pi}{2}\right) = \frac{\cos(x-2\pi)}{\cos(x+2\pi)} = \frac{\cos x}{\cos x} = 1$

$\therefore \frac{\partial f}{\partial x} = 0$

231 **(d)**

Given, $y = \sin^n x \cos nx$

$\frac{dy}{dx} = n\sin^{\text{n}-1} x \cos x \cos nx - n\sin^{\text{n}} x \sin nx$

$= n\sin^{\text{n}-1} x[\cos x \cos nx - \sin x \sin nx]$

$= n\sin^{\text{n}-1} x \cos(\text{n}+1)\, x$

232 **(a)**

Let $f(x) = 3e^{2x}$

Now, $f'(x) = 6e^{2x} = 2f(x)$

Therefore, our assumption is true.

$\therefore (2) = 3e^{2\times 2} = 3e^4$

233 **(c)**

$y^2 = P(x) \Rightarrow 2yy' = P'(x)$ ...(i)

$\Rightarrow (2y)y'' + y'(2y') = P''(x)$

$\Rightarrow 2yy'' = P''(x) - 2(y')^2$

$\Rightarrow 2y^3y'' = y^2P''(x) - 2(yy')^2$

$= y^2P''(x) - 2\frac{\{P'(x)\}^2}{4}$

[from Eq.(i)]

$\Rightarrow 2y^3.y'' = P(x)P''(x) - \frac{1}{2}\{P'(x)\}^2$

$\therefore \frac{d}{dx}(2y^3.y'')$

$= P(x)P'''(x) + P''(x)P'(x) - P'(x)P''(x)]$

$= P(x).P'''(x)$

$\Rightarrow 2\frac{d}{dx}\left(y^3\frac{d^2y}{dx^2}\right) = P(x)P'''(x)$

234 **(c)**

Given, $x = e^{y+x}$

$\Rightarrow \log x = (y+x) \Rightarrow \frac{1}{x} = \frac{dy}{dx} + 1$

$\Rightarrow \frac{dy}{dx} = \frac{1-x}{x}$

235 **(a)**

Let $y = a\sin^3 t$ and $x = a\cos^3 t$, then

On differentiating w.r.t. t, we get

$\frac{dy}{dt} = 3a\sin^2 t\cos t$

and $\frac{dx}{dt} = 3a\cos^2 t(-\sin t)$

$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{3a\sin^2 t\cos t}{3a\cos^2 t(-\sin t)} = -\tan t$

Again differentiating w.r.t. x, we get

$\frac{d^2y}{dx^2} = -\sec^2 t\frac{dt}{dx} = \frac{-\sec^2 t}{3a\cos^2 t(-\sin t)}$

$= \frac{1}{3a}\left(\frac{\sec^4 t}{\sin t}\right)$

$\therefore \left(\frac{d^2y}{dx^2}\right)_{t=\frac{\pi}{4}} = \frac{1}{3a}.\frac{4}{\frac{1}{\sqrt{2}}} = \frac{4\sqrt{2}}{3a}$

236 **(c)**

Let $u = \sin x^3$ and $v = \cos x^3$.

On differentiating w.r.t. x, we get

$\frac{du}{dx} = \cos x^3.3x^2$ and $\frac{dv}{dx} = -\sin x^3.3x^2$

$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{3x^2\cos x^3}{-3x^2\sin x^3} = -\cot x^3$

237 **(d)**

$\frac{\cos x}{1+\sin x} = \frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}$

$= \frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}} = \frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}$

$= \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$

$\therefore = \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right) = \frac{\pi}{4} - \frac{x}{2}$

$\Rightarrow \frac{dy}{dx} = -\frac{1}{2}$

238 **(a)**

Given,$= \tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}$

$= \tan^{-1}\sqrt{\frac{1-\cos\left(\frac{\pi}{2}-x\right)}{1+\cos\left(\frac{\pi}{2}-x\right)}}$

$= \tan^{-1}\left|\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right|$

$= \frac{\pi}{4} - \frac{x}{2}$ $\left[\because x = \frac{\pi}{6}\right]$

$\Rightarrow \frac{dy}{dx} = -\frac{1}{2}$

239 **(a)**

Given, $y = \frac{\log\sin x}{\log\cos x}$

$\Rightarrow \frac{dy}{dx} = \frac{\cot x\log\cos x + \tan x\log\sin x}{(\log\cos x)^2}$

240 **(d)**

Given, $y = 2^x.3^{2x-1}$

$\Rightarrow \frac{dy}{dx} = 2^x.3^{2x-1}2\log 3 + 3^{2x-1}.2^x\log 2$

$= 2^x3^{2x-1}\log 18 = y\log 18$

$\Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{dy}{dx}\log 18$

$= y(\log 18)^2$

241 **(b)**

$\because \quad y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$

$= \cos^{-1}\left(\dfrac{x-1}{x+1}\right) + \sin^{-1}\left(\dfrac{x-1}{x+1}\right)$

$\Rightarrow y = \frac{\pi}{2} \quad \Rightarrow \quad \frac{dy}{dx} = 0$

242 **(a)**

Since, $f(x) = e^{g(x)}$

$\Rightarrow e^{g(x+1)} = f(x+1) = xf(x) = xe^{g(x)}$

and $\quad g(x+1) = \log x + g(x)$

$\Rightarrow g(x+1) - g(x) = \log x$ ...(i)

Replacing x by $x - \frac{1}{2}$, we get

$g\left(x+\dfrac{1}{2}\right) - g\left(x-\dfrac{1}{2}\right) = \log(x - \dfrac{1}{2})$

$= \log(2x-1) - \log 2$

$\therefore g''\left(x+\frac{1}{2}\right) - g''\left(x-\frac{1}{2}\right) = -\frac{4}{(2x-1)^2}$ ...(ii)

On substituting, $x = 1,2,3, \ldots N$ in Eq. (ii) and adding, we get

$g''\left(N+\dfrac{1}{2}\right) - g''\left(\dfrac{1}{2}\right)$

$= -4\left\{1 + \dfrac{1}{9} + \dfrac{1}{25} + \ldots + \dfrac{1}{(2N-1)^2}\right\}$

243 **(b)**

We have,

$y = \log_{x^2+4}(7x^2 - 5x + 1) = \dfrac{\log_e(7x^2-5x+1)}{\log_e(x^2+4)}$

$= \dfrac{\log_e f(x)}{\log_e g(x)}$

$\therefore \dfrac{dy}{dx} = \dfrac{\log_e(g(x) \cdot \frac{f'(x)}{f(x)} - \log_e f(x) \cdot \frac{g'(x)}{g(x)}}{(\log_e g(x))^2}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\log_e g(x)}\left\{\dfrac{f'(x)}{f(x)} - y\dfrac{g'(x)}{g(x)}\right\}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\log_e(x^2+4)}\left\{\dfrac{14x-5}{7x^2-5x+1} - \dfrac{2xy}{x^2+4}\right\}$

245 **(a)**

We have,

$y = \tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$

$\Rightarrow y = \tan^{-1}\left(\dfrac{\sec\theta - 1}{\tan\theta}\right)$, where $x = \tan\theta$

$\Rightarrow y = \tan^{-1}\left(\dfrac{1-\cos\theta}{\sin\theta}\right) = \tan^{-1}\left(\tan\dfrac{\theta}{2}\right) = \dfrac{1}{2}\theta$

$= \dfrac{1}{2}\tan^{-1} x$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2(1+x^2)} \Rightarrow \left(\dfrac{dy}{dx}\right)_{x=0} = \dfrac{1}{2}$

246 **(b)**

Let $y = \sqrt{\sec\sqrt{x}}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2\sqrt{\sec\sqrt{x}}}.\sec\sqrt{x}.\tan\sqrt{x}.\dfrac{1}{2\sqrt{x}}$

$= \dfrac{1}{4\sqrt{x}}(\sec\sqrt{x})^{3/2}.\sin x$

247 **(c)**

We have,

$Sgn\, x = \begin{cases} 1, & \text{for } x > 0 \\ 0, & \text{for } x = 0 \\ -1, & \text{for } x < 0 \end{cases}$

$\therefore g(x) = Sgn \sin x = \begin{cases} \sin x, & \text{for } x > 0 \\ 0, & \text{for } x = 0 \\ -\sin x, & \text{for } x < 0 \end{cases}$

$\Rightarrow g'(x) = \begin{cases} \cos x, & \text{for } x > 0 \\ 0, & \text{for } x = 0 \\ -\cos x, & \text{for } x < 0 \end{cases}$

$\Rightarrow g'(1) = \cos 1$

248 **(c)**

$\because y = x^{\text{In } x}$

On taking log on both sides, we get

In $y = (\text{In } x)^2$

On differentiating w.r.t. x,we get

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{2 \text{ In } x}{x}$

$\Rightarrow \dfrac{dy}{dx} = y\dfrac{2 \text{ In } x}{x} = \dfrac{2(x^{\text{In } x})\text{In } x}{x}$

$\Rightarrow \dfrac{dy}{dx} = 2x^{\text{In } x - 1}\text{In } x$

249 **(d)**

Since, $x = e^t \sin t$ and $y = e^t \cos t$

$\Rightarrow \dfrac{dx}{dt} = e^t \cos t + \sin t\, e^t$

and $\frac{dy}{dt} = -e^t \sin t + e^t \cos t$

$\therefore \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{(\cos t - \sin t)}{(\cos t + \sin t)}$

$\frac{d^2y}{dx^2} = \frac{\begin{bmatrix}(\cos t + \sin t)(-\sin t - \cos t) \\ -(\cos t - \sin t)(-\sin t + \cos t)\end{bmatrix}}{(\cos t + \sin t)^2}\frac{dt}{dx}$

$= \dfrac{-(\sin t + \cos t)^2 - (\cos t - \sin t)^2}{(\cos t + \sin t)^2}$

$\times \dfrac{1}{e^t(\cos t + \sin t)}$

$= -\dfrac{2}{e^t(cos\, t + sin\, t)^3}$

$\Rightarrow \left(\dfrac{d^2y}{dx^2}\right)_{(x=\pi)} = \dfrac{-2}{e^\pi(\cos\pi + \sin\pi)^3} = \dfrac{2}{e^\pi}$

250 **(b)**

We have,

$f'(x) = \sin(\log x)$ and $y = f\left(\frac{2x+3}{3-2x}\right)$

$\therefore \frac{dy}{dx} = f'\left(\frac{2x+3}{3-2x}\right) \times \frac{d}{dx}\left(\frac{2x+3}{3-2x}\right)$

$\Rightarrow \frac{dy}{dx} = \sin\left\{\log\left(\frac{2x+3}{3-2x}\right)\right\} \times \frac{12}{(3-2x)^2}$ $[\because f'(x) = \sin(\log x)]$

251 **(d)**

Given, $r = \left[2\phi + \cos^2\left(2\phi + \frac{\pi}{4}\right)\right]^{1/2}$

$\frac{dr}{d\phi} = \frac{\left[2 - 2\cos\left(2\phi + \frac{\pi}{4}\right)\sin\left(2\phi + \frac{\pi}{4}\right).2\right]}{2\sqrt{2\phi + \cos^2\left(2\phi + \frac{\pi}{4}\right)}}$

$= \frac{\left[1 - \sin\left(4\phi + \frac{\pi}{2}\right)\right]}{\sqrt{2\phi + \cos^2\left(2\phi + \frac{\pi}{4}\right)}}$

$\Rightarrow \left(\frac{dr}{d\phi}\right)_{\phi=\pi/4} = \frac{\left[1 - \sin\left(\pi + \frac{\pi}{2}\right)\right]}{\sqrt{2.\frac{\pi}{4} + \cos^2\left(\frac{\pi}{2} + \frac{\pi}{4}\right)}}$

$= \frac{1+1}{\sqrt{\frac{\pi}{2} + \frac{1}{2}}} = 2\sqrt{\frac{2}{1+\pi}}$

252 **(d)**

$\because y = 1 + x + x^2 + \ldots \infty$

$\therefore y = \frac{1}{1-x} = (1-x)^{-1}$

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = -\frac{1}{(1-x)^2}(-1) = \frac{1}{(1-x)^2}$

$\therefore \frac{dy}{dx} - y = \frac{1}{(1-x)^2} - \frac{1}{(1-x)}$

$= \frac{1-1+x}{(1-x)^2} = \frac{x}{(1-x)^2}$

$\Rightarrow \frac{dy}{dx} - y = xy^2$

$\Rightarrow \frac{dy}{dx} = xy^2 + y$

253 **(a)**

$f'(5) = \lim_{h\to 0} \frac{f(5+h) - f(5)}{h}$

$= \lim_{h\to 0} \frac{2f(5)f(h) - f(5)}{h}$

$= \lim_{h\to 0} 2f(5)\left[\frac{f(h) - \frac{1}{2}}{h}\right]$

$\Rightarrow 1024\log 2 = 2f(5)f'(0)$

Again now, $f(2+3) = 2f(2)f(3)$ ... (i)

$\Rightarrow \frac{1024\log 2}{2f'(0)} = 2 \times 8 \times f(3)$

$\Rightarrow f(3) = \frac{32\log 2}{f'(0)}$ ...(ii)

$\therefore f'(3) = \lim_{h\to 0} \log \frac{f(3+h) - f(3)}{h}$

$= \lim_{h\to 0} \frac{2f(3)f(h) - f(3)}{h}$

$= 2f(3)f'(0)$

$= 2 \times \frac{32\log 2f'(0)}{f'(0)}$

$= 64\log 2$ [from Eq. (ii)]

254 **(d)**

Let $u = e^{x^2}$ and $v = e^{2x-1}$

On differentiating w.r.t. x, we get

$\frac{du}{dx} = e^{x^2}.2x$ and $\frac{dv}{dx} = e^{2x-1}(2)$

$\therefore \frac{du}{dv} = \frac{e^{x^2}.2x}{e^{2x-1}.2}$

$\Rightarrow \frac{du}{dv} = xe^{x^2-2x+1}$

$\Rightarrow \left(\frac{du}{dv}\right)_{(x=1)} = 1.e^{1-2+1} = 1$

255 **(b)**

Given, $x = log_e t$

$\Rightarrow e^x = t$ and $y + 1 = t^2 \Rightarrow y = e^{2x} - 1$

On differentiating w.r.t. y, we get

$2e^{2x}\frac{dx}{dy} = 1$

$\Rightarrow \frac{dx}{dy} = \frac{1}{2e^{2x}}$

Again, differentiating w.r.t. y, we get

$\frac{d^2x}{dy^2} = \frac{1}{2}e^{-2x}(-2)\frac{dx}{dy}$

$= -e^{-2x}.\frac{1}{2e^{2x}}$

$= -\frac{1}{2}e^{-4x}$

256 **(a)**

Given, $y = \cot^{-1}(\cos 2x)^{1/2}$

$\Rightarrow \frac{dy}{dx} = \frac{-1}{1+\cos 2x} \times \frac{1}{2\sqrt{\cos 2x}} \times -2\sin 2x$

$= \frac{2\sin x\cos x}{2\cos^2 x\sqrt{\cos 2x}}$

$\Rightarrow \frac{dy}{dx} = \frac{\tan x}{\sqrt{\cos 2x}}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{6}} = \frac{1/\sqrt{3}}{\sqrt{1/2}} = \sqrt{\frac{2}{3}}$

257 **(c)**

Polynomial $P(x)$, satisfying the given relation can be taken as x

$ie, P(x) = x$

$\therefore P'(x) = 1$

$\Rightarrow P'(0) = 1$

258 **(c)**

$y = (\cos x^2)^2$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = 2\cos x^2\,(-\sin x^2)2x = -2x\sin 2x^2$$

259 **(c)**

We have,

$f(x) = 2^{2x-1}$ and $g(x) = -2^x + 2x\log 2$

$\therefore f'(x) > g'(x)$

$\Rightarrow 2\times 2^{2x-1}\log 2 > -2^x\log 2 + 2\log 2$

$\Rightarrow 2^{2x} > -2^x + 2$

$\Rightarrow 2^{2x} + 2^x - 2 > 0$

$\Rightarrow (2^x - 1)(2^x + 2) > 0$

$\Rightarrow 2^x - 1 > 0$

$\Rightarrow 2^x > 1 \Rightarrow x > 0 \Rightarrow x \in (0, \infty)$

260 **(c)**

Let $I = \frac{d}{dx}\left\{\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) - \tan^{-1}\left(\frac{4x-4x^3}{1-6x^2+x^4}\right)\right\}$

Put $x = \tan\theta$ the given equation

$\therefore \quad I = \frac{d}{dx}\{\tan^{-1}(\tan 2\theta) + \tan^{-1}(\tan 3\theta) - \tan^{-1}(\tan 4\theta)\}$

$= \frac{d}{dx}(\theta) = \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

261 **(a)**

$$y = \frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}} \times \frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}+\sqrt{1+\sin x}}$$

$= \frac{2(1+\cos x)}{-2\sin x} = -\cot\frac{x}{2}$

$\therefore \frac{dy}{dx} = \frac{1}{2}\operatorname{cosec}^2\frac{x}{2}$

262 **(a)**

Since, $y = x^{\sin x} + \sqrt{x}$

Let $y_1 = x^{\sin x}$ and $y_2 = \sqrt{x}$

Now, $y_1 = x^{\sin x} \quad \Rightarrow \quad \log y_1 = \sin x \log x$

On differentiating w.r.t. x, we get

$\frac{1}{y_1}.\frac{dy_1}{dx} = \cos x \log x + \frac{1}{x}\sin x$

$\Rightarrow \quad \frac{dy_1}{dx} = x^{\sin x}[\cos x\log x + \frac{1}{x}\sin x]$

$\Rightarrow \left(\frac{dy_1}{dx}\right)_{x=\frac{\pi}{2}} = \left(\frac{\pi}{2}\right)^{\sin\frac{\pi}{2}}\left[\cos\frac{\pi}{2}\log\frac{\pi}{2} + \frac{2}{\pi}\sin\frac{\pi}{2}\right]$

$= \frac{\pi}{2}\times\frac{2}{\pi} = 1$

Now, $\quad y_2 = \sqrt{x} \quad \Rightarrow \quad \frac{dy_2}{dx} = \frac{1}{2\sqrt{x}}$

$\Rightarrow \left(\frac{dy_2}{dx}\right)_{x=\frac{\pi}{2}} = \frac{1}{2\sqrt{\frac{\pi}{2}}} = \sqrt{\frac{1}{2\pi}}$

Since, $y = y_1 + y_2$

$\therefore$ At $\quad x = \frac{\pi}{2}, \quad \frac{dy}{dx} = \frac{dy_1}{dx} + \frac{dy_2}{dx} \quad \Rightarrow \quad \frac{dy}{dx} = 1 + \frac{1}{\sqrt{2\pi}}$

263 **(a)**

Since, $y = \frac{3at^2}{1+t^2}$ and $x = \frac{3at}{1+t^3}$

On differentiating given curves w.r.t. t respectively

$\frac{dy}{dt} = \frac{(1+t^3)(6at) - 3at^2(3t^2)}{(1+t^3)^2} = \frac{6at - 3at^4}{(1+t^3)^2}$

and $\frac{dx}{dt} = \frac{(1+t^3)(3a) - 3at(3t^2)}{(1+t^3)^2} = \frac{3a - 6at^3}{(1+t^3)^2}$

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3at(2-t^3)}{3a(1-2t^3)} = \frac{t(2-t^3)}{(1-2t^3)}$

264 **(d)**

Given, $y = \frac{\log x}{\log a} + \frac{\log a}{\log x} + 1 + 1$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x\log a} - \frac{\log a}{x(\log x)^2}$

265 **(c)**

We have,

$8f(x) + 6f\left(\frac{1}{x}\right) = x + 5 \quad$ ...(i)

Replacing x by $\frac{1}{x}$, we get

$6f(x) + 8f\left(\frac{1}{x}\right) = \frac{1}{x} + 5 \quad$ ....(ii)

Eliminating $f\left(\frac{1}{x}\right)$ from these two equations, we get

$f(x) = \frac{1}{28}\left(8x - \frac{6}{x} + 10\right)$

$\therefore y = x^2 f(x) = \frac{1}{28}(8x^3 - 6x + 10x^2)$

$\Rightarrow \frac{dy}{dx} = \frac{1}{28}(24x^2 - 6 + 20x)$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=-1} = \frac{1}{28}(24 - 6 - 20) = -\frac{1}{14}$

266 **(d)**

Since, $y = \sqrt{\frac{1-x}{1+x}}$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{\sqrt{1+x}\times\frac{(-1)}{2\sqrt{1-x}} - \sqrt{1-x}\times\frac{1}{2\sqrt{1+x}}}{(\sqrt{1+x})^2}$$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{1}{(1+x)\sqrt{1-x^2}}\times\frac{1-x}{1-x}$

$\Rightarrow \ (1-x^2)\frac{dy}{dx} + y = 0$

267 **(a)**

$x^y = e^{2(x-y)}$

$\therefore y\ \log x = 2(x-y)$

$\Rightarrow \ y\,(\log x + 2) = 2x$

$y = \frac{2x}{\log x + 2}$

$$\frac{dy}{dx}=\frac{(\log x+2)(2)-2x.\frac{1}{x}}{(\log x+2)^2}$$
$$=\frac{2\log x+4-2}{(\log x+2)^2}=\frac{2(\log x+1)}{(\log x+2)^2}$$

268 **(a)**

$x=a(1+\cos\theta),\quad y=a(\theta+\sin\theta)$

$$\frac{dx}{d\theta}=-a\sin\theta,\ \frac{dy}{d\theta}=a\,(1+\cos\theta)$$
$$\therefore\ \frac{dy}{dx}=\frac{1+\cos\theta}{-\sin\theta}=\frac{2\cos^2\frac{\theta}{2}}{-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$
$$\Rightarrow\ \frac{dy}{dx}=-\cot\frac{\theta}{2}$$
$$\therefore\ \frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{d\theta}\left(\frac{dy}{dx}\right).\frac{d\theta}{dx}$$
$$=\frac{d}{d\theta}\left(-\cot\frac{\theta}{2}\right).\frac{1}{-a\sin\theta}$$
$$=\frac{1}{2}\operatorname{cosec}^2\frac{\theta}{2}.\frac{1}{-a\sin\theta}$$
$$\therefore\ \left(\frac{d^2y}{dx^2}\right)_{\theta=\frac{\pi}{2}}=\frac{1}{2}.2.\frac{1}{-a}=-\frac{1}{a}$$

269 **(a)**

Given, $y^2=ax^2+bx+c\Rightarrow 2y\frac{dy}{dx}=2ax+b$

$$\Rightarrow 2\left(\frac{dy}{dx}\right)^2+2y\left(\frac{d^2y}{dx^2}\right)=2a$$
$$\Rightarrow y\frac{d^2y}{dx^2}=a-\left(\frac{dy}{dx}\right)^2$$
$$\Rightarrow y\frac{d^2y}{dx^2}=a-\left(\frac{2ax+b}{2y}\right)^2$$
$$\Rightarrow\ y\frac{d^2y}{dx^2}=\frac{4ay^2-(2ax+b)^2}{4y^2}$$
$$\Rightarrow 4y^3\frac{d^2y}{dx^2}=4a(ax^2+bx+c)-(4a^2x^2+4abx+b^2)$$
$$\Rightarrow 4y^3\frac{d^2y}{dx^2}=4ac-b^2$$
$$\Rightarrow y^3\frac{d^2y}{dx^2}=\frac{4ac-b^2}{4}=\text{constant}$$

270 **(d)**

Given, $y=1+\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}+.\dots=\frac{1}{1-\frac{1}{x}}$

$\Rightarrow\ y=\frac{x}{x-1}$(GP series) ...(i)

$$\frac{dy}{dx}=\frac{1(x-1)-x.1}{(x-1)^2}=-\frac{1}{(x-1)^2}$$

$\Rightarrow\ \frac{dy}{dx}=-\frac{y^2}{x^2}$ [from Eq.(i)]

271 **(c)**

Given, $f(x,y)=2(x-y)^2-x^4-y^4$

On differentiating partially w.r.t. x, twicely

$f_x=4(x-y)-4x^3$

$\Rightarrow f_{xx}=4-12x^2$

$\Rightarrow (f_{xx})_{(0,0)}=4-0=4$

Similarly, $f_{yy}=4-12y^2$

$\Rightarrow (f_{yy})_{(0,0)}=4-0=4$

and $f_{xy}=-4+0$

$\Rightarrow (f_{xy})_{(0,0)}=-4$

$\therefore\ \left|f_{xx}\,f_{yy}-f_{xy}^2\right|_{(0,0)}=|4(4)-(-4)^2|=0$

272 **(c)**

$$y=\tan^{-1}\frac{1}{1+x+x^2}+\tan^{-1}\frac{1}{x^2+3x+3}+$$
...+upto n terms
$$=\tan^{-1}\frac{(x+1)-x}{1+x(x+1)}+\tan^{-1}\frac{(x+2)-(x+1)}{1+(x+1)(x+2)}+$$
...+ upto n terms
$$=[\tan^{-1}(x+1)-\tan^{-1}x+\tan^{-1}(x+2)-\tan^{-1}(x+1)+\cdots+\tan^{-1}(x+n)-\tan^{-1}\{x+(n-1)\}]$$
$$=\tan^{-1}(x+n)-\tan^{-1}x$$
$$\therefore\ y'(x)=\frac{1}{1+(x+n)^2}-\frac{1}{1+x^2}$$
$$\Rightarrow\ y'(0)=\frac{1}{1+n^2}-1=\frac{n^2}{1+n^2}$$

273 **(b)**

Let $f(x)=x^2$

On differentiating w.r.t. x, we get

$f'(x)=2x$

Given that, $f'(a+b)=f'(a)+f'(b)$

$\Rightarrow 2(a+b)=2a+2b$

$\Rightarrow 2a+2b=2a+2b$

274 **(c)**

Let $y=(x+1)^n$

$$\therefore\ \frac{dy}{dx}=n(x+1)^{n-1}$$
$$\frac{d^2y}{dx^2}=n(n-1)(x+1)^{n-2}$$

Similarly, $\frac{d^2y}{dx^n}=n(n-1)(n-2)\dots3.2.1=n!$

275 **(d)**

Given, $y=2^x.3^{2x-1}$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx}=2^x.3^{2x-1}\log3(2)+2^x.3^{2x-1}\log2$$
$$\Rightarrow\ \frac{dy}{dx}=2^x.3^{2x-1}[2\log3+\log2]$$
$$\Rightarrow\ \frac{dy}{dx}=y\log18$$

276 **(a)**

Given, $y=x^2+\frac{1}{y}\quad\Rightarrow\quad y^2=x^2y+1$

$\Rightarrow\ 2y\,\frac{dy}{dx}=y.2x+x^2\,\frac{dy}{dx}\ \Rightarrow\ \frac{dy}{dx}=\frac{2xy}{2y-x^2}$

277 **(d)**

$$y=\frac{\frac{(e^x+e^{-x})}{2}}{\frac{(e^x-e^{-x})}{2}}=\frac{\cosh x}{\sinh x}$$

$\Rightarrow y=\coth x \ \Rightarrow \frac{dy}{dx}=-\text{cosech}^2 x$

278 **(c)**

$f(f(x))=f(|x-2)$

$=||x-2|-2|$

$=x-4 \ \ (\because x>20)$

$\Rightarrow \ g(x)=x-4$

$\therefore g'(x)=1$

279 **(b)**

$5f(x)+3f\left(\frac{1}{x}\right)=x+2$ ... (i)

On replacing x by $\frac{1}{x}$, we get

$5f\left(\frac{1}{x}\right)+3f(x)=\frac{1}{x}+2$ ... (ii)

On multiplying Eq. (i) by 5 and Eq. (ii) by 3 and then on subtracting, we get

$\therefore 16f(x)=5x-\frac{3}{x}+4$

$\Rightarrow xf(x)=\frac{5x^2-3+4x}{16}=y$

$\therefore \frac{dy}{dx}=\frac{10x+4}{16}$

$\left.\frac{dy}{dx}\right|_{x=1}=\frac{10+4}{16}=\frac{7}{8}$

280 **(b)**

$f(\log x)=\log\log(x)$

$\Rightarrow \frac{d}{dx}\{f(\log x)\}=\frac{1}{x}.\frac{1}{\log x}=(x\log x)^{-1}$

282 **(a)**

Given, $f(x)=|x|^3=\begin{cases}0, & x=0\\ x^3, & x>0\\ -x^3, & x<0\end{cases}$

Now, $Rf'(0)=\lim_{h\to 0}\frac{f(h)-f(0)}{h}$

$=\lim_{h\to 0}\frac{h^3-0}{h}=0$

and $Lf'(0)=\lim_{h\to 0}\frac{f(-h)-f(0)}{-h}$

$=\lim_{h\to 0}\frac{-h^3-0}{-h}=0$

$\because Rf'(0)=Lf'(0)=0$

$\therefore f'(0)=0$

283 **(c)**

We have, $y=\log|x|=\begin{cases}\log x, & x>0\\ \log(-x), & x<0\end{cases}$

$\therefore \ \frac{dy}{dx}=\begin{cases}\frac{1}{x}, & x>0\\ \frac{1}{-x}(-1)=\frac{1}{x}, & x<0\end{cases}$

$\Rightarrow \ \frac{dy}{dx}=\frac{1}{x}, x\neq 0$

285 **(b)**

We have, $f(x)=x+2$

$\therefore f'(x)=1$ for all $x\Rightarrow f'(x)=1$ for all x

286 **(c)**

We have,

$f(x)-\log_x(\log_e x)$

$\Rightarrow f(x)=\frac{\log_e(\log_e x)}{\log_e x}$

$\Rightarrow f'(x)=\frac{\log_e x\times\frac{1}{x\log_e x}-\frac{1}{x}\log_e(\log_e x)}{(\log_e x)^2}$

$\Rightarrow f'(e)=\frac{1}{e}-\frac{1}{e}\times\log(1)=\frac{1}{e}$

287 **(b)**

Given, $\sin(x+y)+\cos(x+y)=\log(x+y)$

On differentiating w.r.t. x, we get

$\cos(x+y)\left(1+\frac{dy}{dx}\right)-\sin(x+y)\left(1+\frac{dy}{dx}\right)$

$=\frac{1}{(x+y)}\left(1+\frac{dy}{dx}\right)$

$\Rightarrow 1+\frac{dy}{dx}=0 \ \Rightarrow \ \frac{d^2y}{dx^2}=0$

288 **(b)**

$10^{-x\tan x}\frac{d}{dx}(10^{x\tan x})$

$=10^{-x\tan x}10^{x\tan x}\log 10(\tan x+x\sec^2 x)$

$=\log 10(\tan x+x\sec^2 x)$

289 **(c)**

Given, $y=1-x+\frac{x^2}{2!}-\frac{x^3}{3!}\dots$

$\Rightarrow \ y=e^{-x} \ \Rightarrow \ \frac{dy}{dx}=-e^{-x}$

$\therefore \ \frac{d^2y}{dx^2}=e^{-x}=y$

290 **(b)**

Let $y_1=\sec^{-1}\frac{1}{2x^2-1}$ and $y_2=\sqrt{1-x^2}$

$\Rightarrow \frac{dy_1}{dx}=\frac{-2}{\sqrt{1-x^2}}$ and $\frac{dy_2}{dx}=\frac{-x}{\sqrt{1-x^2}}$

$\Rightarrow \frac{dy_1}{dy_2}=\frac{2}{x} \ \Rightarrow \ \left(\frac{dy_1}{dy_2}\right)_{x=1/2}=4$

291 **(d)**

We have,

$y=\cos 2x\cos 3x=\frac{1}{2}[\cos 5x+\cos x]$

$\therefore y_n=\frac{1}{2}\left\{\frac{d^n}{dx^n}(\cos 5x)+\frac{d^n}{dx^n}(\cos x)\right\}$

$\Rightarrow y_n=\frac{1}{2}\left\{5^n\cos\left(\frac{n\pi}{2}+5x\right)+\cos\left(\frac{n\pi}{2}+x\right)\right\}$

292 **(c)**

Given, $f(x)=\log_{x^2}(\log_e x)=\frac{1}{2}\log_x(\log_e x)$

$\Rightarrow (x)=\frac{1}{2}\frac{\log_e\log_e x}{\log_e x}$

$\Rightarrow f'(x)=\frac{1}{2}\frac{\log_e x\left(\frac{1}{x\log_e x}\right)-\log_e\log_e x\times\frac{1}{x}}{(\log_e x)^2}$

$\Rightarrow f'(x) = \frac{1}{2}\frac{\frac{1}{x}-\frac{1}{x}\log_e \log_e x}{(\log_e x)^2}$

At $x = e, f'(e) = \frac{1}{2}\frac{\frac{1}{e}-\frac{1}{e}\log_e 1}{(1)^2}$

$\Rightarrow f'(e) = \frac{1}{2e}$

293 **(d)**

Given, $f(x) = \sin x, g(x) = x^2$

and $h(x) = \log_e x$

Also , $F(x) = (hogof)(x)$

$\therefore (hogof)(x) = (hog)(\sin x)$

$\Rightarrow \quad = h(\sin x^2)$

$\Rightarrow F(x) = 2 \log \sin x$

On differentiating, we get

$F'(x) = 2 \cot x$

Again differentiating, we get

$F''(x) = -2 \operatorname{cosec}^2 x$

294 **(c)**

Given, $x = \cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$

and

$y = \sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right)$

$\Rightarrow = \tan^{-1} t,$

and $y = \tan^{-1} t$

$\therefore y = x \Rightarrow \frac{dy}{dx} = 1$

295 **(c)**

$\because y = \tan^{-1}\left(\frac{\log\left(\frac{e}{x^2}\right)}{\log ex^2}\right) + \tan^{-1}\left(\frac{3 + 2\log x}{1 - 6\log x}\right)$

$= \tan^{-1}\left(\frac{1 - \log x^2}{1 + \log x^2}\right) + \tan^{-1}\left(\frac{3 + 2\log x}{1 - 6\log x}\right)$

$= \tan^{-1}(1) - \tan^{-1}(2\log x) + \tan^{-1}(3) + \tan^{-1}(2\log x)$

$\therefore y = \tan^{-1}(1) + \tan^{-1}(3)$

$\Rightarrow \frac{dy}{dx} = 0 \Rightarrow \frac{d^2y}{dx^2} = 0$

296 **(b)**

Let $y = \sin^2 \cot^{-1}\left\{\sqrt{\frac{1-x}{1+x}}\right\}$

Put $x = \cos\theta \Rightarrow \theta = \cos^{-1} x$

$\Rightarrow y = \sin^2 \cot^{-1}\left\{\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}\right\}$

$\Rightarrow y = \sin^2 \cot^{-1}\left\{\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}}\right\}$

$= \sin^2 \cot^{-1}\left(\tan\frac{\theta}{2}\right)$

$\Rightarrow y = \sin^3\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$

On differentiating w.r.t. θ, we get

$\frac{dy}{d\theta} = 2\sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right)\cos\left(\frac{\pi}{2} - \frac{\theta}{2}\right) - \left(-\frac{1}{2}\right)$

$\Rightarrow \frac{dy}{d\theta} = -\frac{\sin(\pi - \theta)}{2} = -\frac{\sin\theta}{2} = -\frac{1}{2}\sqrt{1 - x^2}$

$\therefore \frac{dy}{dx} = \frac{dy}{d\theta}.\frac{d\theta}{dx} = \frac{-1}{2}\sqrt{1 - x^2}\frac{d}{dx}(\cos^{-1} x)$

$= -\frac{\sqrt{1 - x^2}}{2}\left(\frac{-1}{\sqrt{1 - x^2}}\right) = \frac{1}{2}$

297 **(c)**

In the neighbourhood of $x = \sqrt[5]{\frac{\pi}{2}}$, we have

$[x] = 1$

Therefore, in the neighbourhood of $x = \sqrt[5]{\frac{\pi}{2}}$, we have

$f(x) = \sin\left\{\frac{\pi}{2}[x] - x^5\right\} = \sin\left(\frac{\pi}{2} - x^5\right) = \cos x^5$

$\Rightarrow f'(x) = -5x^4 \sin x^5$

$\Rightarrow f'\left(\sqrt[5]{\frac{\pi}{2}}\right) = -5\left(\frac{\pi}{2}\right)^{4/5} \sin\frac{\pi}{2} = -5\left(\frac{\pi}{2}\right)^{4/5}$

298 **(b)**

Since, $h'(x) = 2f(x)f'(x) + 2g(x)g'(x)$

Now, $f'(x) = g(x)$ and $f''(x) = -f(x)$

$\Rightarrow f''(x) = g'(x)$ and $f''(x) = -f(x)$

$\Rightarrow -f(x) = g'(x)$

Thus, $f'(x) = g(x)$ and $g'(x) = -f(x)$

$\therefore h'(x) = -2g(x)\, g'(x) + 2g(x)g'(x)$

$= 0, \forall x$

$\Rightarrow h(x) =$constant for all x

But $h(5) = 11$

Hence, $h(x) = 11$ for all x

299 **(d)**

$$f(x) = \begin{vmatrix} x^3 & x^4 & 3x^2 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$$

$\Rightarrow \frac{d}{dx}f(x) = \begin{vmatrix} 3x^2 & 4x^3 & 6x \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$

$\Rightarrow \frac{d^2}{dx^2}f(x) = \begin{vmatrix} 6x & 12x^2 & 6 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$

$\Rightarrow \frac{d^3}{dx^3}f(x) = \begin{vmatrix} 6 & 24x & 0 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$

$\Rightarrow \frac{d^4}{dx^4}f(x) = \begin{vmatrix} 0 & 24 & 0 \\ 1 & -6 & 4 \\ p & p^2 & p^3 \end{vmatrix}$

$= -24\begin{vmatrix} 1 & 4 \\ p & p^3 \end{vmatrix} = -24(p^3 - 4p)$

Hence, $\frac{d^4}{dx^4}f(x)$ is a constant.

301 **(a)**

Here, $y = \sin\left[\log\left(\frac{2x+3}{3-2x}\right)\right]$

$\Rightarrow \frac{dy}{dx} = \cos\left[\log\left(\frac{2x+3}{3-2x}\right)\right].\frac{3-2x}{2x+3}.\frac{12}{(3-2x)^2}$

$= \frac{12}{9-4x^2}.\cos\left[\log\left(\frac{2x+3}{3-2x}\right)\right]$

302 **(c)**

$y = \cos^{-1}\left(\frac{1-\log x}{1+\log x}\right)$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-\left(\frac{1-\log x}{1+\log x}\right)^2}}$$

$$\frac{(1+\log x)\left(\frac{-1}{x}\right)-(1-\log x)\left(\frac{1}{x}\right)}{(1+\log x)^2}$$

$$= -\frac{1}{2\sqrt{\log x}}.\frac{-1-\log x-1+\log x}{x(1+\log x)}$$

$$= \frac{1}{x\sqrt{\log x}\,(1+\log x)}$$

$\therefore \left(\frac{dy}{dx}\right)_{x=e} = \frac{1}{e(1+1)} = \frac{1}{2e}$

303 **(a)**

Since, $x^y = y^x \Rightarrow y\log x = x\log y$

On differentiating w.r.t. x, we get

$y.\frac{1}{x}+\log x\ \frac{dy}{dx} = x.\frac{1}{y}\frac{dy}{dx}+\log y$

$\Rightarrow x(x-y\log x)\frac{dy}{dx} = y(-x\log y+y)$

304 **(b)**

Since x is the integral function of $(1+9y^2)^{-1/2}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1+9y^2}} \Rightarrow \frac{dy}{dx} = \sqrt{1+9y^2}$

305 **(d)**

Given, $y = \sec^{-1}(\operatorname{cosec} x)+\operatorname{cosec}^{-1}(\sec x)+\sin^{-1}(\cos x)+\cos^{-1}(\sin x)$

$$= \sec^{-1}\left[\sec\left(\frac{\pi}{2}-x\right)\right]+\operatorname{cosec}^{-1}\left[\operatorname{cosec}\left(\frac{\pi}{2}-x\right)\right] + \sin^{-1}\left[\sin\left(\frac{\pi}{2}-x\right)\right] + \cos^{-1}\left[\cos\left(\frac{\pi}{2}-x\right)\right]$$

$= \frac{\pi}{2}-x+\frac{\pi}{2}-x+\frac{\pi}{2}-x+\frac{\pi}{2}-x$

$\Rightarrow\ y = 2\pi-4x$

On differentiating w.r.t. x, we get

$\Rightarrow \frac{dy}{dx} = -4$

306 **(b)**

Let $f(x) = |x-1|+|x-5|$

$$f(x) = \begin{cases} -(x-1)-(x-5), & x<1 \\ (x-1)-(x-5), & 1\le x\le 5 \\ x-1+x-5, & x>5 \end{cases}$$

$$f(x) = \begin{cases} 6-2x, & x<1 \\ 4, & x\le 5 \\ 2x-6, & x>5 \end{cases}$$

$\because x = 3 \in (1,5)$

$\therefore$ For $x = 3$,

$f(x) = 4 \Rightarrow f'(x) = 0$

307 **(a)**

We have,

$f(x) = \log_e\left\{\frac{u(x)}{v(x)}\right\}$

$\Rightarrow f'(x) = \frac{v(x)}{u(x)}\left\{\frac{v(x)u'(x)-u(x)v'(x)}{\{v(x)\}^2}\right\}$ for all x

$\Rightarrow f'(1) = 0$

308 **(b)**

We have,

$$y = \sin^2\left\{\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right\} = \frac{1}{\operatorname{cosec}^2\left\{\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right\}}$$

$$\Rightarrow y = \frac{1}{1+\cot^2\left\{\cot^2\sqrt{\frac{1-x}{1+x}}\right\}} = \frac{\frac{1}{1}}{1+x} = \frac{1+x}{2}$$

$\therefore \frac{dy}{dx} = \frac{1}{2}$

309 **(c)**

$\because x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$

Put $t = \tan\theta$ in both the equations, we get

$x = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$ ... (i)

and $y = \frac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta$ ... (ii)

On differentiating both the Eqs.(i) and (ii), we get

$\frac{dx}{d\theta} = -2\sin 2\theta$ and $\frac{dy}{d\theta} = 2\cos 2\theta$

Therefore, $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = -\frac{\cos 2\theta}{\sin 2\theta} = -\frac{x}{y}$

310 **(b)**

Given, $x = y\sqrt{1-y^2}$

$\Rightarrow 1 = \frac{dy}{dx}\sqrt{1-y^2}+\frac{y(-y)}{\sqrt{1-y^2}}\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}\left[\frac{1-y^2-y^2}{\sqrt{1-y^2}}\right] = 1 \Rightarrow \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{1-2y^2}$

311 **(b)**

We have,

$f(x) = (x+1)\tan^{-1}(e^{-2x})$

$f'(x) = \tan^{-1}(e^{-2x})+\frac{(x+1)(-2)}{1+e^{-4x}}e^{-2x}$

$\Rightarrow f'(0) = \tan^{-1}(1)-\frac{2}{2} = \frac{\pi}{4}-1$

312 **(d)**

Given, $y = 5^x x^5 \Rightarrow \frac{dy}{dx} = 5^x\log 5.x^5+5^x 5x^4$

$= 5^x(x^5\log 5+5x^4)$

313 **(b)**

$y = a\cos(\log x) + b\sin(\log x)$

On differentiating w.r.t. x, we get

$$y' = \frac{-a\sin(\log x)}{x} + \frac{b\cos(\log x)}{x}$$

$\Rightarrow xy' = -a\sin(\log x) + b\cos(\log x)$

Again, on differentiating w.r.t. x, we get

$$xy'' + y' = -a\cos(\log x)\frac{1}{x} - b\sin(\log x)\frac{1}{x}$$

$\Rightarrow x^2y'' + y'x = -[a\cos(\log x) + b\sin(\log x)]$

$\Rightarrow x^2y'' + xy' = -y$

314 **(d)**

We have,

$$y = \sin^{-1}\left(\frac{\sin\alpha\sin x}{1-\cos\alpha\sin x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-\frac{\sin^2\alpha\sin^2 x}{(1-\cos\alpha\sin x)^2}}}\frac{d}{dx}\left(\frac{\sin\alpha\sin x}{1-\cos\alpha\sin x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1-\cos\alpha\sin x)}{\sqrt{1+\sin^2 x - 2\cos\alpha\sin x}} \cdot \frac{\sin\alpha}{\cos\alpha}\frac{d}{dx}\left(1-\frac{1}{1-\cos\alpha\sin x}\right)$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\tan\alpha\,(1-\cos\alpha\sin x)}{\sqrt{1+\sin^2 x - 2\cos\alpha\sin x}}\frac{d}{dx}\left(1 - \frac{1}{1-\cos\alpha\sin x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\tan\alpha\,(1-\cos\alpha\sin x)}{\sqrt{1+\sin^2 x - 2\sin x\cos\alpha}}\left\{\frac{1-\cos\alpha\cos x}{(1-\cos\alpha\sin x)^2}\right\}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin\alpha\cos x}{(1-\cos\alpha\sin x)\sqrt{1+\sin^2 x - 2\cos\alpha\sin x}}$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=0} = \sin\alpha \Rightarrow y'(0) = \sin\alpha$$

315 **(a)**

For $x < -1$

$f(x) = -(x-2) - (x+1) - x = -3x + 1$

$\therefore \quad f'(x) = -3$

$\Rightarrow \quad f'(-10) = -3$

316 **(c)**

Since $f(x)$ is an even function

$\therefore f(-x) = f(x)$

$\Rightarrow -f'(-x) = f'(x)$

$\Rightarrow f''(-x) = f''(x) \Rightarrow f''(-\pi) = f''(\pi) = 1$

317 **(d)**

$$\frac{d}{dx}[\log_e e^{\sin(x^2)}] = \frac{d}{dx}[\sin(x^2)] = \cos^{(x^2)}2x$$

318 **(c)**

$$\frac{d}{dx}[f^2(x) + g^2(x)]$$

$= 2[f(x)f'(x) + g(x)g'(x)]$

$= 2[f(x)g(x) + g(x)\{-f(x)\}]$

$= 0$

$\Rightarrow f^2(x) + g^2(x) = \text{constant}$

$\therefore \; f^2(4) + g^2(4) = f^2(2) + g^2(2)$

$= (4)^2 + (4)^2$

$=32$

319 **(a)**

Given, $z = \tan(y+ax) + \sqrt{y-ax}$

$$\Rightarrow z_x = \sec^2(y+ax)a + \frac{1}{2\sqrt{y-ax}}(-a)$$

$$\Rightarrow z_{xx} = 2\sec^2(y+ax)\tan(y+ax)a^2 + \frac{1(-a^2)}{4(y-ax)^{3/2}}$$

and $z_y = \sec^2(y+ax) + \frac{1}{2\sqrt{y-ax}}$

$$\Rightarrow z_{yy} = 2\sec^2(y+ax)\tan(y+ax) - \frac{1}{4(y-ax)^{3/2}}$$

$\therefore z_{xx} - a^2 z_{yy} = 0$

320 **(c)**

$$\frac{d}{dx}(\log x)^4 = 4(\log x)^3\frac{d}{dx}(\log x) = \frac{4(\log x)^3}{x}$$

321 **(d)**

We have,

$$\sin^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = \log a$$

$$\Rightarrow \frac{x^2-y^2}{x^2+y^2} = \sin(\log a) = \lambda \text{ (say)}$$

$$\Rightarrow \frac{x^2}{y^2} = \frac{\lambda+1}{1-\lambda}$$

$$\Rightarrow \frac{d}{dx}\left(\frac{x^2}{y^2}\right) = 0$$

$$\Rightarrow \frac{2xy^2 - 2x^2y\frac{dy}{dx}}{y^4} = 0 \Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

323 **(b)**

We have, $f(x) = \sqrt{1+\cos^2(x^2)}$ ... (i)

On differentiating Eq. (i) w.r.t. x, we get

$$f'(x) = \frac{-2\sin x^2\cos x^2}{\sqrt{1+\cos^2 x^2}}(x)$$

$$\Rightarrow f'(x) = \frac{-\sin 2x^2}{\sqrt{1+\cos^2 x^2}}(x) \text{ ... (ii)}$$

Put, $x = \frac{\sqrt{\pi}}{2}$ in Eq. (ii), we get

$$f'\left(\frac{\sqrt{\pi}}{2}\right) = -\frac{\sqrt{\pi}}{2}.\frac{\sin 2\left(\frac{\pi}{4}\right)}{\sqrt{1+\frac{1}{2}}}$$

$$= -\frac{\sqrt{\pi}}{2}.\frac{\sin\frac{\pi}{2}}{\sqrt{\frac{3}{2}}} = -\sqrt{\frac{\pi}{6}}$$

324 **(d)**

Let $y = \log x$

On differentiating w.r.t. x from 1 to n times, we get

$$y_1 = \frac{1}{x}, y_2 = \frac{-1}{x^2}, y_3 = \frac{2}{x^3} \dots$$

$$y_n = \frac{(-1)^{n-1}(n-1)!}{x^n}$$

325 **(d)**

Since, $\frac{dx}{dy} = \frac{1}{dy/dx} = \left(\frac{dy}{dx}\right)^{-1}$

$$\Rightarrow \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dx}\left(\frac{dy}{dx}\right)^{-1}\frac{dx}{dy}$$

$$\Rightarrow \frac{d^2x}{dy^2} = -\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}\left(\frac{dx}{dy}\right) = -\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$$

326 **(c)**

Let $y = a\sin^3 t,\ x = a\cos^3 t$

On differentiating w.r.t. t, we get

$$\frac{dy}{dt} = 3a\sin^2 t\cos t, \frac{dx}{dt} = -3a\cos^2 t\sin t$$

$$\therefore \frac{dy}{dx} = \frac{3a\sin^2 t\cos t}{-3a\cos^2 t\sin t}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\sin t}{\cot t} = -\tan t$$

Again, differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = -\sec^2 t.\frac{dt}{dx}$$

$$= -\frac{\sec^2 t}{-3a\cos^2 t\, s\, in\, t}$$

$$= \frac{1}{3a\cos^4 t\sin t}$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{t=\frac{\pi}{4}} = \frac{1}{3a\left(\frac{1}{\sqrt{2}}\right)^4.\left(\frac{1}{\sqrt{2}}\right)}$$

$$= \frac{(\sqrt{2})^5}{3a} = \frac{4\sqrt{2}}{3a}$$

327 **(a)**

$$\frac{dy}{dx} = \left(\frac{dx}{dy}\right)^{-1}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\left(\frac{dx}{dy}\right)^{-2}\left\{\frac{d}{dx}\left(\frac{dx}{dy}\right)\right\}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\left(\frac{dx}{dy}\right)^{-2}\left\{\frac{d}{dx}\left(\frac{dx}{dy}\right)\frac{dy}{dx}\right\}$$

$$= -\left(\frac{dy}{dx}\right)^2\left\{\frac{d^2x}{dy^2}.\frac{dy}{dx}\right\} = -\left(\frac{dy}{dx}\right)^3\frac{d^2x}{dy^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3\frac{d^2x}{dy^2} = 0$$

328 **(b)**

We have,

$e^y + xy = e$

$\Rightarrow e^y\frac{dy}{dx} + y + x\frac{dy}{dx} = 0 \quad \dots$(i)

$\Rightarrow e^y\frac{d^2y}{dx^2} + e^y\left(\frac{dy}{dx}\right)^2 + 2\frac{dy}{dx} + x\frac{d^2y}{dx^2} = 0 \quad \dots$(ii)

Putting $x = 0$ in $e^y + xy = e$, we get $y = 1$

Putting $x = 0, y = 1$ in (i), we get

$$e\frac{dy}{dx} + 1 = 0 \Rightarrow \frac{dy}{dx} = -\frac{1}{e}$$

Putting $x = 0, y = 1, \frac{dy}{dx} = -\frac{1}{e}$ in (ii), we get

$$e\frac{d^2y}{dx^2} + e \times \frac{1}{e^2} - \frac{2}{e} + 0 \times \frac{d^2y}{dx^2} = 0 \Rightarrow \frac{d^2y}{dx^2} = \frac{1}{e^2}$$

329 **(c)**

Since, $\sin^{-1}\left(x\sqrt{1-x} + \sqrt{x}\sqrt{1-x^2}\right) =$ sin−1𝑥+sin−1𝑥

$$\therefore \quad \frac{d}{dx}\{\sin^{-1}(x\sqrt{1-x} + \sqrt{x}\sqrt{1-x^2})\}$$

$$= \frac{d}{dx}(\sin^{-1}x + \sin^{-1}\sqrt{x})$$

$$= \frac{1}{\sqrt{1-x^2}} + \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

330 **(d)**

$$\because \ f(x) = \log_x(\log_e x) = \frac{\log_e\log_e x}{\log_e x}$$

$$\therefore \ f(x) = \frac{\left[\log_e x\frac{1}{\log_e x}.\frac{1}{x} - \log\log_e x.\frac{1}{x}\right]}{(\log_e x)^2}$$

$$= \frac{1 - \log_e\log_e x}{x(\log_e x)^2}$$

$$\Rightarrow f'(e) = \frac{1 - \log_e\log_e e}{e(\log_e e)^2} = \frac{1 - \log_e 1}{e} = \frac{1}{e}$$

331 **(a)**

In the neighbourhood of $x = -\frac{\pi}{6}$, we observe that

$\cos 4x < 0$ and $\sin x < 0$

$\Rightarrow |\cos 4x| = -\cos 4x$ and $|\sin x| = -\sin x$

$\therefore y = \log_{\sec 2x}(-\cos\ 4x) - \sin x$

$$\Rightarrow y = \frac{\log_e(-\cos 4x)}{\log_e\sec 2x} - \sin x$$

$$\Rightarrow \frac{dy}{dx}\frac{(-4\tan 4x)\log_e\sec 2x - \log_e(-\cos 4x)\, 2\tan}{\{\log_e\sec 2x\}^2}$$

$- \cos x$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=-\pi/6} = \left(\frac{-6\sqrt{3}}{\log_e 2} - \frac{\sqrt{3}}{2}\right)$$

332 **(b)**

Let $x^3 = \sin A$ and $y^3 = \sin B$. Then

$$\sqrt{1-x^6} + \sqrt{1-y^6} = a(x^3 - y^3)$$

$$\Rightarrow \cos A + \cos B = a(\sin A - \sin B) \Rightarrow \cot\frac{A-B}{2} = a$$

$$\Rightarrow A - B = 2\cot^{-1}a \Rightarrow \sin^{-1}x^3 - \sin y^3 = 2\cot^{-1}a$$

Differentiating w.r.t. to x

$$\frac{3x^2}{\sqrt{1-x^6}} - \frac{3y^2}{\sqrt{1-y^6}} = 0 \Rightarrow \frac{dy}{dx} = \frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$$

But, $\frac{dy}{dx} = f(x,y)\sqrt{\frac{1-y^6}{1-x^6}}$

$\therefore f(x,y) = \frac{x^2}{y^2}$

333 **(d)**

We have, $y = \log^n x$

Where $\log^n$ means log log … (repeated n times).

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{x \log x \log^2 x \dots \log^{n-1} x}$$

$\therefore\ x \log x \log^2 x \dots \log^{n-1} x \log^n x \frac{dy}{dx}$

$$= \frac{x \log x \log^2 x \dots \log^{n-1} x \log^n x}{x \log x \log^2 x \dots \log^{n-1} x} = \log^n x$$

334 **(b)**

We have, $x^4 + y^4 = t^2 + \frac{1}{t^2}$

$= \left(t + \frac{1}{t}\right)^2 - 2$

$= (x^2 + y^2)^2 - 2$

$= x^4 + y^4 + 2x^2y^2 - 2$

$\Rightarrow\ x^2y^2 = 1$

On differentiating w.r.t. x, we get

$$x^2.2y\frac{dy}{dx} + y^2.2x = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$

335 **(b)**

Given, $y = \sqrt{x \log_e x}$

$$\Rightarrow\ \frac{dy}{dx} = \frac{1}{2\sqrt{x \log_e x}}\left(x \times \frac{1}{x} + 1 \times \log_e x\right)$$

$$\therefore\ \left(\frac{dy}{dx}\right)_{x=e} = \frac{1}{2\sqrt{e \times 1}}(1+1)$$

$= \frac{1}{\sqrt{e}}$

337 **(c)**

$$\frac{dy}{dx} = f'\left(\frac{2x+3}{3-2x}\right)\frac{d}{dx}\left(\frac{2x+3}{3-2x}\right)$$

$$= \sin\log\left(\frac{2x+3}{3-2x}\right)\left(\frac{(3-2x)(2)-(2x+3)(-2)}{(3-2x)^2}\right)$$

$$= \frac{12}{(3-2x)^2}\sin\left\{\log\left(\frac{2x+3}{3-2x}\right)\right\}$$

$$\therefore\ \left(\frac{dy}{dx}\right)_{x=1} = \frac{12}{(3-2)^2}.\sin\log 5 = 12\sin\log 5$$

338 **(a)**

Given, $x^y = e^{x-y}$

Taking log on both sides, we get

$y \log x = x - y$

On differentiating w.r.t. x, we get

$$\frac{y}{x} + \log x\frac{dy}{dx} = 1 - \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}(1 + \log x) = 1 - \frac{y}{x}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)}(1 + \log 1) = 1 - \frac{1}{1}$$

$$\Rightarrow\ \left(\frac{dy}{dx}\right)_{(1,1)} = 0$$

339 **(b)**

Given, $y = \sin^{-1}\left(\frac{5x+12\sqrt{1-x^2}}{13}\right)$

Put $x = \sin\theta$

$$\therefore\quad y = \sin^{-1}\left(\frac{5\sin\theta + 12\cos\theta}{13}\right)$$

$= \sin^{-1}\{\sin(\theta+\alpha)\}\left(\text{Put}\cos\alpha = \frac{5}{13}\right)$

$\Rightarrow\ y = \theta + \alpha = \sin^{-1} x + \alpha$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

340 **(b)**

Given, $y = e^x.e^{x^2}.e^{x^3}\ \dots.e^{x^n}$ ….

$\Rightarrow\quad y = e^{(x+x^2+\dots+\infty)}$

$\Rightarrow\quad y = e^{\frac{x}{1-x}}$

$$\Rightarrow\quad \frac{dy}{dx} = e^{\frac{x}{1-x}}\left[\frac{(1-x)1 - x(-1)}{(1-x^2)}\right]$$

$$= e^{\frac{x}{1-x}}.\frac{1}{(1-x)^2}$$

At $x = \frac{1}{2}$,

$$\left(\frac{dy}{dx}\right)_{x=\frac{1}{2}} = e^{\frac{1/2}{1-1/2}}.\frac{1}{\left(1-\frac{1}{2}\right)^2} = 4e$$

341 **(a)**

We have,

$$y = \sin^{-1}\left(\frac{5x + 12\sqrt{1-x^2}}{13}\right)$$

$$\Rightarrow y = \sin^{-1}\left(\frac{5}{13}x + \frac{12}{13}\sqrt{1-x^2}\right)$$

$$\Rightarrow y = \sin^{-1}\left\{x\sqrt{1-\left(\frac{12}{13}\right)^2} + \frac{12}{13}\sqrt{1-x^2}\right\}$$

$$\Rightarrow y = \sin^{-1} x + \sin^{-1}\frac{12}{13} \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

342 **(a)**

$$\frac{d}{dx} = \left(\tan^{-1}\sqrt{\frac{1+\cos\frac{x}{2}}{1-\cos\frac{x}{2}}}\right) = \frac{d}{dx}\tan^{-1}\sqrt{\frac{2\cos^2\frac{x}{4}}{2\sin^2\frac{x}{4}}}$$

$$= \frac{d}{dx}\tan^{-1}\left|\cot\left(\frac{x}{4}\right)\right|$$

$$= \frac{d}{dx}\tan^{-1}\left|\tan\left(\frac{\pi}{2} - \frac{x}{4}\right)\right|$$

$$= \frac{d}{dx}\left(\frac{\pi}{2} - \frac{x}{4}\right) = \frac{-1}{4}$$

343 **(b)**

Let $p = \sin^{-1}\frac{2x}{1+x^2} = 2\tan^{-1} x$

and $q = \cos^{-1}\frac{1-x^2}{1+x^2} = 2\tan^{-1}x$

$\therefore\ p = q \Rightarrow \frac{dp}{dq} = 1$

344 **(a)**

Let $f(x) = ax^2 + bx + c$

$\because\ f(1) = f(-1)$

$\Rightarrow a + b + c = a - b + c \Rightarrow b = 0$

$\therefore\ f(x) = ax^2 + c \Rightarrow f'(x) = 2ax$

$\therefore\ f'(a_1) = 2aa_1,\quad f'(a_2) = 2aa_2,$

$f'(a_3) = 2aa_3$

Now assume

$2f'(a_2) = f'(a_1) + f'(a_3)$

$\Rightarrow 2.2aa_2 = 2aa_1 + 2aa_3$

$\Rightarrow 2a_2 = a_1 + a_3$

$\Rightarrow a_1, a_2, a_3$ are in AP.

$\therefore f'(a_1), f'(a_2), f'(a_3)$ are in AP.

345 **(c)**

Let $y_1 = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

and $y_2 = \cot^{-1}\left(\frac{1-3x^2}{3x-x^3}\right)$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$\Rightarrow y_1 = 2\tan^{-1}x$ and $y_2 = 3\tan^{-1}x$

On differentiating w.r.t. x, we get

$$\frac{dy_1}{dx} = \frac{2}{1+x^2} \text{ and } \frac{dy_2}{dx} = \frac{3}{1+x^2}$$

$$\Rightarrow \frac{dy_1}{dy_2} = \frac{\left(\frac{dy_1}{dx}\right)}{\left(\frac{dy_2}{dx}\right)} = \frac{\left(\frac{2}{1+x^2}\right)}{\left(\frac{3}{1+x^2}\right)} = \frac{2}{3}$$

346 **(b)**

We have, $y = \log_{\sin x}\cos x$

$$= \frac{\log\cos x}{\log\sin x}$$

$$= \frac{\log\sin x\frac{1}{\cos x}(-\sin x) - \log\cos x\frac{1}{\sin x}\cos x}{(\log\sin x)^2}$$

$$= \frac{-\tan x\log\sin x - \cot x\log\cos x}{(\log\sin x)^2}$$

347 **(c)**

We have,

$$y = \tan^{-1}\left(\frac{\log e - \log x^2}{\log e + \log x^2}\right) + \tan^{-1}\left(\frac{3 + 2\log x}{1 - 6\log x}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\frac{1 - 2\log x}{1 + 2\log x}\right) + \tan^{-1}\left(\frac{3 + 2\log x}{1 - 6\log x}\right)$$

$$\Rightarrow y = \tan^{-1}1 - \tan^{-1}(2\log x) + \tan^{-1}3 + \tan^{-1}(2\log x)$$

$$\Rightarrow y = \tan^{-1}1 + \tan^{-1}3$$

$$\Rightarrow \frac{dy}{dx} = 0 \text{ and } \frac{d^2y}{dx^2} = 0$$

348 **(b)**

Differentiating $y = c\,e^{\frac{x}{x-a}}$ w.r.t. x, we get

$$\frac{dy}{dx} = c\,e^{\frac{x}{x-a}}\cdot\frac{(-a)}{(x-a)^2} = -\frac{ay}{(x-a)^2}$$

349 **(d)**

$$y = \sin^{-1}\sqrt{1-x} \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-(1-x)}}.\frac{1}{2\sqrt{1-x}}.(-1)$$

$$= \frac{-1}{2\sqrt{x}\sqrt{1-x}}$$

350 **(d)**

We have,

$$y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{\sqrt{1 - \frac{4x^2}{(1+x^2)^2}}} \times \frac{d}{dx}\left(\frac{2x}{1+x^2}\right)$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1+x^2}{\sqrt{(1-x^2)^2}} \times \frac{2(1+x^2) - 4x^2}{(1+x^2)^2}$$

$$\Rightarrow \frac{dy}{dx} = -2\frac{(1+x^2)}{|1-x^2|} \times \frac{1-x^2}{(1+x^2)^2}$$

$$\Rightarrow \frac{dy}{dx} = -2\left(\frac{1-x^2}{|1-x^2|}\right)\left(\frac{1}{1+x^2}\right)$$

$$\Rightarrow \frac{dy}{dx} = \begin{cases} \frac{2}{1+x^2}, & \text{if } |x| > 1 \\ \frac{-2}{1+x^2}, & \text{if } |x| < 1 \end{cases}$$

351 **(b)**

$y = ae^x + be^{-x} + c$

On differentiating w.r.t. x, we get

$y' = ae^x - be^{-x}$

Again differentiating w.r.t. x, we get

$y'' = ae^x + be^{-x}$

Again differentiating w.r.t. x, we get

$y''' = ae^x - be^{-x} = y'$