DCAM classes

Dynamic Classes for Academic Mastery

9.DIFFERENTIAL EQUATIONS

Single Correct Answer Type

1. Let F denotes the family of ellipses whose centre is at the origin and major axis is the y-axis. Then, equation of the family F is

a) $\frac{d^2y}{dx^2}+\frac{dy}{dx}\left(x\,\frac{dy}{dx}-y\right)=0$ b) $xy\,\frac{d^2y}{dx^2}-\frac{dy}{dx}\left(x\,\frac{dy}{dx}-y\right)=0$

c) $xy\,\frac{d^2y}{dx^2}+\frac{dy}{dx}\left(x\,\frac{dy}{dx}-y\right)=0$ d) $\frac{d^2y}{dx^2}-\frac{dy}{dx}\left(x\,\frac{dy}{dx}-y\right)=0$

2. The order and degree of the differential equation $\rho=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ are respectively

a) 2, 2 b) 2, 3 c) 2, 1 d) None of these

3. The differential equation of the family of curves $y=e^{2x}(a\cos x+b\sin x)$, where a and b are arbitrary constants, is given by

a) $y_2-4y_1+5y=0$ b) $2y_2-y_1+5y=0$ c) $y_2+4y_1-5y=0$ d) $y_2-2y_1+5y=0$

4. The solution of $\frac{dy}{dx}=\frac{ax+g}{by+f}$ represents a circle when

a) $a=b$ b) $a=-b$ c) $a=-2b$ d) $a=2b$

5. Solution of the differential equation $\frac{dy}{dx}+\frac{y}{x}=x^2$ is

a) $y=\frac{x^2}{4}+cx^{-2}$ b) $y=x^{-1}+cx^{-3}$ c) $y=\frac{x^3}{4}+cx^{-1}$ d) $xy=x^2+c$

6. The differential equation satisfied by the family of curves $y=ax\cos\left(\frac{1}{x}+b\right)$, where a and b are parameters, is

a) $x^2y_2+y=0$ b) $x^4y_2+y=0$ c) $xy_2-y=0$ d) $x^4y_2-y=0$

7. The solution of the differential equation $y\,dx+(x+x^2y)dy=0$ is

a) $-\frac{1}{xy}=c$ b) $-\frac{1}{xy}+\log y=c$ c) $\frac{1}{xy}+\log y=c$ d) $\log y=cx$

8. The degree of the differential equation $2\left(\frac{d^2y}{dx^2}\right)+3\left(\frac{dy}{dx}\right)^2+4y^{3\cdot}=x$, is

a) 0 b) 1 c) 2 d) 3

9. The differential equation cot $y\,dx=x\,dy$ has a solution of the form

a) $y=\cos x$ b) $x=c\sec y$ c) $x=\sin y$ d) $y=\sin x$

10. The solution of the differential equation $(3xy+y^2)dx+(x^2+xy)dy=0$ is

a) $x^2(2xy+y^2)=c^2$ b) $x^2(2xy-y^2)=c^2$ c) $x^2(y^2-2xy)=c^2$ d) None of these

11. The order and degree of the differential equation $\sqrt{\frac{dy}{dx}}-4\frac{dy}{dx}-7x=0$ are

a) 1 and 1/2 b) 2 and 1 c) 1 and 1 d) 1 and 2

12. Solution of the differential equation $\cos x\;dy=y(\sin x-y)dx, 0<x<\frac{\pi}{2}$, is

a) $\sec x=(\tan x+c)y$ b) $y\sec x=\tan x+c$

c) $y\tan x=\sec x+c$ d) $\tan x=(\sec x+x)y$

13. If $x=A\cos 4t+B\sin 4t$, then $\frac{d^2x}{dt^2}$ is equal to

a) $-16\,x$ b) $16\,x$ c) x d) $-x$

14. If $\frac{dy}{dx}+y=2e^{2x}$, then y is equal to

a) $ce^x+\frac{2}{3}e^{2x}$ b) $(1+x)e^{-x}+\frac{2}{3}\,e^{2x}+c$

c) $ce^{-x}+\frac{2}{3}\,e^{2x}$ d) $e^{-x}+\frac{2}{3}\,e^{2x}+c$

15. The solution of differential equation $\frac{dt}{dx} = \frac{t\left(\frac{d}{dx}(\mathrm{g}(x))\right)-t^2}{\mathrm{g}(x)}$ is

a) $t = \frac{\mathrm{g}(x)+c}{x}$ b) $t = \frac{\mathrm{g}(x)}{x} + c$ c) $t = \frac{\mathrm{g}(x)}{x+c}$ d) $t = \mathrm{g}(x) + x + c$

16. The differential equation of all circles of radius a is of order

a) 2 b) 3 c) 4 d) None of these

17. The solution of the differential equation $xy^2dy - (x^3 + y^3)dx = 0$ is

a) $y^3 = 3x^3 + c$ b) $y^3 = 3x^3 \log(cx)$ c) $y^3 = 3x^3 + \log(cx)$ d) $y^3 + 3x^3 = \log(cx)$

18. The solution of the differential equation $x\frac{dy}{dx} + y = x\cos x + \sin x$, given that $y = 1$ when $x = \frac{\pi}{2}$, is

a) $y = \sin x - \cos x$ b) $y = \cos x$ c) $y = \sin x$ d) $y = \sin x + \cos x$

19. The differential equation obtained by eliminating the arbitrary constants a and b from $xy = ae^x + be^{-x}$ is

a) $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - xy = 0$ b) $\frac{d^2y}{dx^2} + 2y\frac{dy}{dx} - xy = 0$

c) $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + xy = 0$ d) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - xy = 0$

20. The general solution of the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 2e^{3x}$ is given by

a) $y = (c_1 + c_2x)\,e^x + \frac{e^{3x}}{8}$ b) $y = (c_1 + c_2x)e^{-x} + \frac{e^{-3x}}{8}$

c) $y = (c_1 + c_2x)e^{-x} + \frac{e^{3x}}{8}$ d) $y = (c_1 + c_2x)e^x + \frac{e^{-3x}}{8}$

21. The solution of the differential equation $(2y-1)dx - (2x+3)dy = 0$, is

a) $\frac{2x-1}{2y+3} = C$ b) $\frac{2x+3}{2y-1} = C$ c) $\frac{2x-1}{2y-1} = C$ d) $\frac{2y+1}{2x-3} = C$

22. A curve passes through the point $(0, 1)$ and the gradient at (x, y) on it is $y(xy-1)$. The equation of the curve is

a) $y(x-1) = 1$ b) $y(x+1) = 1$ c) $x(y+1) = 1$ d) $x(y-1) = 1$

23. Solution of the differential equation $xdy - ydx = 0$ represents a

a) Parabola b) Circle c) Hyperbola d) Straight line

24. If $\frac{dy}{dx} = 1 + x + y + xy$ and $y(-1) = 0$, then the function y is

a) $e^{(1-x)^2/2}$ b) $e^{(1+x)^2/2} - 1$ c) $\log_e(1+x) - 1$ d) $(1+x)$

25. An integrating factor of the differential equation $x\frac{dy}{dx} + y\log x = xe^x x^{-\frac{1}{2}\log x}, (x > 0)$, is

a) $x^{\log x}$ b) $\left(\sqrt{x}\right)^{\log x}$ c) $\left(\sqrt{e}\right)^{(\log x)^2}$ d) e^{x^2}

26. The differential equation $y\frac{dy}{dx} + x = c$ represents

a) A family of hyperbolas

b) A family of circles whose centres are on the y-axis

c) A family of parabolas

d) A family of circles whose centres are on the x-axis

27. If $y' = \frac{x-y}{x+y}$, then its solution is

a) $y^2 + 2xy - x^2 = c$ b) $y^2 + 2xy + x^2 = c$ c) $y^2 - 2xy - x^2 = c$ d) $y^2 - 2xy + x^2 = c$

28. The order of differential equation whose general solution is given by $y = c_1e^{2x+c_2} + c_3e^x + c_4\sin(x + c_5)$ is

a) 5 b) 4 c) 3 d) 2

29. The solution of the differential equation $\frac{x}{x^2+y^2}dy = \left(\frac{y}{x^2+y^2} - 1\right)dx$, is

a) $y = x\cot(C - x)$ b) $\cos^{-1}\frac{y}{x} = (-x + C)$ c) $y = x\tan(C - x)$ d) $\frac{y^2}{x^2} = x\tan(C - x)$

30. If $dx + dy = (x + y)(dx - dy)$, then $\log(x + y)$ is equal to

a) $x + y + c$ b) $x + 2y + c$ c) $x - y + c$ d) $2x + y + c$

31. The solution of the equation $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$ is

a) $y = c(x + a)(1 - ay)$ b) $y = c(x + a)(1 + ay)$

c) $y = c(x - a)(1 + ay)$ d) None of these

32. The order and degree of the differential equation $\left(1 + 3\frac{dy}{dx}\right)^{2/3} = 4\frac{d^3y}{dx^3}$ are

a) $\left(1, \frac{2}{3}\right)$ b) $(3,1)$ c) $(3,3)$ d) $(1,2)$

33. The equation of one of the curves whose slope at any point is equal to $y + 2x$ is

a) $y = 2(e^x + x - 1)$ b) $y = 2(e^x - x - 1)$ c) $y = 2(e^x - x + 1)$ d) $y = 2(e^x + x + 1)$

34. The degree of the differential equation of all curves having normal of constant length c, is

a) 1 b) 3 c) 4 d) None of these

35. The solution of $\frac{dy}{dx} + 1 = \text{cosec}\,(x + y)$ is

a) $\cos(x + y) + x = c$ b) $\cos(c + y) = c$

c) $\sin(x + y) + x = c$ d) $\sin(x + y) + \sin(x + y) = c$

36. The solution of the differential equation $\frac{dy}{dx} = e^{3x-2y} + x^2e^{-2y}$, is

a) $e^{2y} = e^{3x} + x^3 + c$ b) $\frac{1}{2}e^{2y} = \frac{1}{3}(e^{3x} + x^3) + c$

c) $\frac{1}{2}\,e^{2y} = \frac{1}{3}(e^{3x} + x^3) + c$ d) $e^{2y} = e^{3x} + x^3 + c$

37. The differential equation whose solution is $(x - h)^2 + (y - k)^2 = a^2$ is (a is a constant)

a) $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = a^2\frac{d^2y}{dx^2}$

b) $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$

c) $\left[1 + \left(\frac{dy}{dx}\right)\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$

d) None of these

38. The degree and order of the differential equation whose solution is a parabola whose axis is x-axis, are

a) 1,1 b) 1,2 c) 1,0 d) 2,1

39. General solution of the differential equation $\frac{dy}{dx} = \frac{x+y+1}{x+y-1}$ is given by

a) $x + y = \log|x + y| + c$ b) $x - y = \log|x + y| + c$

c) $y = x + \log|x + y| + c$ d) $y = x\log|x + y| + c$

40. Integral curve satisfying $y' = \frac{x^2+y^2}{x^2-y^2}, y(1) = 2$ has the slope at the point $(1, 0)$ of the curve is equal to

a) $-5/3$ b) -1 c) 1 d) $5/3$

41. The slope at any point of a curve $y = f(x)$ is given by $\frac{dy}{dx} = 3x^2$ and it passes through (-1,1). The equation of the curve is

a) $y = x^3 + 2$ b) $y = -x^3 - 2$ c) $y = 3x^3 + 4$ d) $y = -x^3 + 2$

42. The solution of the differential equation $(x + y)^2\frac{dy}{dx} = a^2$ is

a) $(x + y)^2 = \frac{a^2x}{2} + c$ b) $(x + y)^2 = a^2x + c$

c) $(x + y)^2 = 2a^2x + c$ d) None of these

43. $y = cx - c^2$, is the general solution of the differential equation

a) $(y')^2 - xy' + y = 0$ b) $y'' = 0$

c) $y' = c$ d) $(y')^2 + xy' + y = 0$

44. The solution of $\frac{dv}{dt} + \frac{k}{m}v = -g$ is

a) $v = c\, e^{-\frac{k}{m}t} - \frac{mg}{k}$ b) $v = c - \frac{mg}{k} e^{-\frac{k}{m}t}$ c) $ve^{-\frac{k}{m}t} = c - \frac{mg}{k}$ d) $v\, e^{\frac{k}{m}t} = c - \frac{mg}{k}$

45. The solution of $\frac{dy}{dx} + y\tan x = \sec x$ is

a) $y\sec x = \tan x + c$ b) $y\tan x = \sec x + c$ c) $\tan x = y\tan x + c$ d) $x\sec x = \tan y + c$

46. If $\frac{dy}{dx} = \frac{y + x\tan\frac{y}{x}}{x}$, then $\sin\frac{y}{x}$ is equal to

a) cx^2 b) cx c) cx^3 d) cx^4

47. The differential equation of the family of curve $y^2 = 4a(x+1)$, is

a) $y^2 = 4\frac{dy}{dx}\left(x + \frac{dy}{dx}\right)$

b) $2y = \frac{dy}{dx} + 4a$

c) $y^2\left(\frac{dy}{dx}\right)^2 + 2xy\,\frac{dy}{dx} - y^2 = 0$

d) $y^2\frac{dy}{dx} + 4y = 0$

48. The solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is

a) $e^x + e^y = c$ b) $e^x - e^y = c$ c) $e^x + e^{-y} = c$ d) $e^x - e^{-y} = c$

49. The solution of differential equation $t = 1 + (ty)\frac{dy}{dt} + \frac{(ty)^2}{2!}\left(\frac{dy}{dx}\right)^2 + \ldots\infty$ is

a) $y = \pm\sqrt{(\log t)^2 + c}$ b) $ty = t^y + c$ c) $y = \log t + c$ d) $y = (\log t)^2 + c$

50. The solution of $x\,dy - y\,dx + x^2e^x\,dx = 0$ is

a) $\frac{y}{x} + e^x = c$ b) $\frac{x}{y} + e^x = c$ c) $x + e^y = c$ d) $y + e^x = c$

51. The solution of $\frac{dy}{dx} + 2y\tan x = \sin x$, is

a) $y\sec^3 x = \sec^2 x + C$ b) $y\sec^2 x = \sec x + C$ c) $y\sin x = \tan x + C$ d) None of these

52. The equation of the curve passing through the origin and satisfying the differential equation $(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$ is

a) $(1+x^2)y = x^3$ b) $2(1+x^2)y = 3\,x^3$ c) $3(1+x^2)y = 4\,x^3$ d) None of these

53. The solution of the differential equation $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ is

a) $y = e^{x-y} - x^2e^{-y} + c$ b) $e^y - e^x = \frac{1}{3}x^3 + c$ c) $e^x + e^y = \frac{1}{3}x^3 + c$ d) $e^x - e^y = \frac{1}{3}x^3 + c$

54. The solution of the equation $x^2\frac{d^2y}{dx^2} = \log x$ when $x = 1, y = 0$ and $\frac{dy}{dx} = -1$ is

a) $\frac{1}{2}(\log x)^2 + \log x$ b) $\frac{1}{2}(\log x)^2 - \log x$ c) $-\frac{1}{2}(\log x)^2 + \log x$ d) $-\frac{1}{2}(\log x)^2 - \log x$

55. Observe the following statements

A:Interating factor of $\frac{dy}{dx} + y = x^2$ is e^x

R: Integrating factor of $\frac{dy}{dx} + P(x)y = Q(x)$ is $e^{\int P(x)dx}$

Then, the true statement among the following is

a) A is true, R is false b) A is false, R is true

c) A is true, R is true, R ⇒ A d) A is false, R is false

56. The differential equation of the family of ellipse having major and minor axes respectively along the x and y-axes and the minor axis is equal to half of the major axis, is

a) $xy' - 4y = 0$ b) $4xy' + y = 0$ c) $4yy' + x = 0$ d) $yy' + 4x = 0$

57. The differential equation of system of concentric circles with centre (1,2) is

a) $(x-2) + (y-1)\frac{dy}{dx} = 0$ b) $(x-1) + (y-2)\frac{dy}{dx} = 0$

c) $(x+1)\frac{dy}{dx} + (y-2) = 0$ d) $(x+2)\frac{dy}{dx} + (y-1) = 0$

58. The equation of family of a curve is $y^2 = 4a(x+a)$ then differential equation of the family is

a) $y = y' + x$ b) $y = y'' + x$ c) $y = 2y'x + yy'^2$ d) $y'' + y' + y^2 = 0$

59. $y = Ae^x + Be^{2x} + Ce^{3x}$ satisfies the differential equation

a) $y''' - 6y' + 11y' - 6y = 0$ b) $y''' + 6y'' + 11y' + 6y = 0$

c) $y''' + 6y'' - 11y' + 6y = 0$ d) $y''' - 6y'' - 11y' + 6y = 0$

60. The differential equation of all parabolas whose axes are parallel to y-axis, is

a) $\frac{d^3y}{dx^3} = 0$ b) $\frac{d^2x}{dy^2} = c$ c) $\frac{d^3y}{dx^3} + \frac{d^2x}{dy^2} = 0$ d) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = c$

61. The differential equation of all circles which passes through the origin and whose centre lies on y-axis, is

a) $(x^2 - y^2)\frac{dy}{dx} - 2xy = 0$ b) $(x^2 - y^2)\frac{dy}{dx} + 2xy = 0$

c) $(x^2 - y^2)\frac{dy}{dx} - xy = 0$ d) $(x^2 - y^2)\frac{dy}{dx} + xy = 0$

62. The equation of the curve whose tangent at any point (x, y) makes an angle $\tan^{-1}(2x + 3y)$ with x-axis and which passes through (1,2) is

a) $6x + 9y + 2 = 26e^{3(x-1)}$ b) $6x - 9y + 2 = 26e^{3(x-1)}$

c) $6x + 9y - 2 = 26e^{3(x-1)}$ d) $6x - 9y - 2 = 26e^{3(x-1)}$

63. The solution of the differential equation $\frac{dy}{dx} - \frac{\tan y}{x} = \frac{\tan y \sin y}{x^2}$ is

a) $\frac{x}{\sin y} + \log x = c$ b) $\frac{y}{\sin x} + \log x = c$ c) $\log x + x = c$ d) $\log x + y = c$

64. The order and degree of the differential equation

$5\left(\frac{d^2y}{dx^2}\right)^5 + 4\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{dy}{dx}\right)^3 + 2y + x^3 = 0$ are respectively

a) (2,5) b) (3,2) c) (1,3) d) (2,3)

65. The order and degree of the differential equation $y = \frac{dy}{dx} + \sqrt{a^2\left(\frac{dy}{dx}\right)^2 + b^2}$ is

a) 3,1 b) 1,2 c) 2,1 d) 1,3

66. The differential equation of all 'Simple Harmonic Motions' of given period $\frac{2\pi}{n}$, is

a) $\frac{d^2x}{dt^2} + nx = 0$ b) $\frac{d^2x}{dt^2} + n^2x = 0$ c) $\frac{d^2x}{dt^2} - n^2x = 0$ d) $\frac{d^2x}{dt^2} + \frac{1}{n^2}x = 0$

67. Solution of the differential equation $\frac{dy}{x} + \frac{dy}{y} = 0$, is

a) $\log x = \log y$ b) $\frac{1}{x} + \frac{1}{y} = c$ c) $x + y = c$ d) $xy = c$

68. The differential equation of all non-vertical lines in a plane is

a) $\frac{d^2y}{dx^2} = 0$ b) $\frac{d^2x}{dy^2} = 0$ c) $\frac{dy}{dx} = 0$ d) $\frac{dx}{dy} = 0$

69. The solution of $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$ is

a) $e^{y/x} = kx$ b) $e^{y/x} = ky$ c) $e^{x/y} = kx$ d) $e^{-y/x} = ky$

70. The order and degree of the differential equation $\left(1 + 4\frac{dy}{dx}\right)^{2/3} = 4\frac{d^2y}{dx^2}$ are respectively

a) $1, \frac{2}{3}$ b) 3,2 c) 2,3 d) $2, \frac{2}{3}$

71. The solution of the differential equation $y' = 1 + x + y^2 + xy^2, y(0) = 0$ is

a) $y^2 = \exp\left(x + \frac{x^2}{2}\right) - 1$

b) $y^2 = 1 + C\exp\left(x + \frac{x^2}{2}\right)$

c) $y = \tan(C + x + x^2)$

d) $y = \tan\left(x + \frac{x^2}{2}\right)$

72. Solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = \sin x$ is

a) $x(y + \cos x) = \sin x + c$ b) $x(y - \cos x) = \sin x + c$

c) $x(y\cos x) = \sin x + c$ d) $x(y - \cos x) = \cos x + c$

73. The integrating factor of the differential equation $(y\log y)dx = (\log y - x)dy$ *is*

a) $\frac{1}{\log y}$ b) $\log(\log y)$ c) $1 + \log y$ d) $\log y$

74. The family of curves $y = e^{a\sin x}$, where a is an arbitrary constant, is represented by the differential equation

a) $\log y = \tan x \frac{dy}{dx}$ b) $y\log y = \tan x \frac{dy}{dx}$ c) $y\log y = \sin x \frac{dy}{dx}$ d) $\log y = \cos x \frac{dy}{dx}$

75. The order of differential equation of all parabolas having directrix parallel to x-axis is

a) 3 b) 1 c) 4 d) 2

76. The solution of the differential equation $x\,dy - y\,dx = \sqrt{x^3 + y^2}\,dx$, is

a) $x + \sqrt{x^2 + y^2} = Cx^2$ b) $y - \sqrt{x^2 + y^2} = Cx$ c) $x - \sqrt{x^2 + y^2} = Cx$ d) $y + \sqrt{x^2 + y^2} = Cx^2$

77. The integral factor of equation $(x^2 + 1)\frac{dy}{dx} + 2xy = x^2 - 1$ is

a) $x^2 + 1$ b) $\frac{2x}{x^2 + 1}$ c) $\frac{x^2 - 1}{x^2 + 1}$ d) None of these

78. The difference equation of the family of circles with fixed radius r and with centre on y-axis is

a) $y^2(1 + y_1^2) = r^2 y_1^2$ b) $y^2 = r^2 y_1 + y_1^2$ c) $x^2(1 + y_1^2) = r^2 y_1^2$ d) $x^2 = r^2 y_1 + y_1^2$

79. The differential equation of the family $y = ae^x + bx\,e^x + cx^2\,e^x$ of curves, where a, b, c are arbitrary constants, is

a) $y''' + 3y'' + 3y' + y = 0$ b) $y''' + 3y'' - 3y' - y = 0$

c) $y''' - 3y'' - 3y' + y = 0$ d) $y''' - 3y' + 3y' - y = 0$

80. $y = ae^{mx} + be^{-mx}$ satisfies which of the following differential equations

a) $\frac{dy}{dx} - my = 0$ b) $\frac{dy}{dx} + my = 0$ c) $\frac{d^2y}{dx^2} - m^2y = 0$ d) None of these

81. The order and degree of the differential equation $\sqrt{\sin x}\ (dx + dy) = \sqrt{\cos x}\ (dx - dy)$ are

a) (1,2) b) (2,2) c) (1,1) d) (2,1)

82. The order of the differential equation of all circles of radius r, having centre on y –axis and passing through the origin, is

a) 1 b) 2 c) 3 d) 4

83. An integrating factor of the differential equation $(1 + y + x^2y)dx + (x + x^3)dy = 0$ is

a) $\log x$ b) x c) e^x d) $\frac{1}{x}$

84. The solution of $\frac{dy}{dx} = \left(\frac{y}{x}\right)^{1/3}$ is

a) $x^{2/3} + y^{2/3} = c$ b) $x^{1/3} + y^{1/3} = c$ c) $y^{2/3} - x^{2/3} = c$ d) $y^{1/3} - x^{1/3} = c$

85. The differential equation of the curve for which the initial ordinate of any tangent is equal to the corresponding subnormal, is

a) Non-linear b) Homogeneous

c) In variable separable form d) None of the above

86. The solution of differential equation $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$ is

a) $(x+a)(x+ay) = cy$
b) $(x+a)(1-ay) = cy$
c) $(x+a)(1-ay) = -cy$
d) None of these

87. The equation of family of curves for which the length of the normal is equal to the radius vector, is

a) $y^2 \mp x^2 = k^2$
b) $y \pm x = k$
c) $y^2 = kx$
d) None of these

88. The equation of the curve in which the portion of y-axis cut off between the origin and the tangent varies as the cube of the abscissa of the point of contact is

a) $y = \frac{k\,x^3}{3} + c\,x$
b) $y = -\frac{k\,x^2}{2} + c$
c) $y = -\frac{k\,x^3}{2} + c\,x$
d) $y = \frac{k\,x^3}{3} + \frac{c\,x^2}{2}$

(k is constant of proportionality)
(where c is arbitrary constant)

89. The solution of differential equation $(\sin x + \cos x)dy + (\cos x - \sin x)dx = 0$ is

a) $e^x(\sin x + \cos x) + c = 0$
b) $e^y(\sin x + \cos x) = c$
c) $e^y(\cos x - \sin x) = c$
d) $e^x(\sin x - \cos x) = c$

90. A particles moves in a straight line with a velocity given by $\frac{dx}{dt} = x + 1$ (x is the distance described). The time taken by a particle to traverse a distance of 99 metres is

a) $\log_{10} e$
b) $2\log_e 10$
c) $2\log_{10} e$
d) $\frac{1}{2}\log_{10} e$

91. The differential equation of all parabolas having their axis of symmetry coinciding with the axis of X, is

a) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$
b) $x\frac{d^2x}{dy^2} + \left(\frac{dx}{dy}\right)^2 = 0$
c) $y\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$
d) None of these

92. The solution of the differential equation $\frac{dy}{dx} = \frac{xy+y}{xy+x}$ is

a) $x + y = \log\left(\frac{cy}{x}\right)$
b) $x + y = \log(cxy)$
c) $x - y = \log\left(\frac{cx}{y}\right)$
d) $y - x = \log\left(\frac{cx}{y}\right)$

93. Observe the following statements.

I. IF $dy + 2xy\,dx = 2e^{-x^2}\,dx$, then $ye^{x^2} = 2x + c$

II.IF $ye^{x^2} - 2x = c$, then $dx = \left(2e^{-x^2} - 2xy\right)dy$

Which is/are correct statements?

a) Both I and II are true
b) Neither I nor II is true
c) I is true, II is false
d) I is false, II is true

94. The degree of the differential equation corresponding to the family of curves $y = a(x+a)^2$, where a is an arbitrary constant is

a) 1
b) 2
c) 3
d) None of these

95. The order and degree of the differential equation $\frac{d^2y}{dx^2} = \sqrt[3]{1 - \left(\frac{dy}{dx}\right)^4}$ are respectively

a) 2,3
b) 3,2
c) 2,4
d) 2,2

96. Solution of differential equation $\sec x\ dy - \operatorname{cosec} y\ dx = 0$ is

a) $\cos x + \sin y = c$
b) $\sin x + \cos y = c$
c) $\sin y - \cos x = c$
d) $\cos y - \sin x = c$

97. The solution of the differential equation $\frac{dy}{dx} = \frac{x-2y+1}{2x-4y}$ is

a) $(x-2y)^2 + 2x = c$
b) $(x-2y)^2 + x = c$
c) $(x-2y) + 2x^2 = c$
d) $(x-2y) + x^2 = c$

98. The differential equation of all circles in the first quadrant which touch the coordinate axes is of order

a) 1
b) 2
c) 3
d) None of these

99. The solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x}$ satisfying the condition $y(1) = 1$ is

a) $y = x\log x + x$
b) $y = \log x + x$
c) $y = x\log x + x^2$
d) $y = xe^{(x-1)}$

100. The slope of the tangent at (x, y) to a curve passing through a point $(2, 1)$ is $\frac{x^2+y^2}{2xy}$, then the equation of the curve is

a) $2(x^2 - y^2) = 3x$ b) $2(x^2 - y^2) = 6y$ c) $x(x^2 - y^2) = 6$ d) $x(x^2 + y^2) = 10$

101. The order and degree of the differential equation $\sqrt{y + \frac{d^2y}{dx^2}} = x + \left(\frac{dy}{dx}\right)^{3/2}$ are

a) 2,2 b) 2,1 c) 1,2 d) 2,3

102. The solution of $\frac{dy}{dx} + y = e^x$ is

a) $2y = e^{2x} + c$ b) $2ye^x = e^2 + c$ c) $2ye^x = e^{2x} + c$ d) $2ye^{2x} = 2e^x + c$

103. If $\phi(x) = \phi'(x)$ and $\phi(1) = 2$, then $\phi(3)$ equals

a) e^2 b) $2\,e^2$ c) $3\,e^2$ d) $2\,e^3$

104. The general solution of the differential equation $\frac{dy}{dx} + \sin\left(\frac{x+y}{2}\right) = \sin\left(\frac{x-y}{2}\right)$ is

a) $\log\tan\left(\frac{y}{2}\right) = c - 2\sin x$ b) $\log\tan\left(\frac{y}{4}\right) = c - 2\sin\left(\frac{x}{2}\right)$

c) $\log\tan\left(\frac{y}{2} + \frac{\pi}{4}\right) = c - 2\sin x$ d) $\log\tan\left(\frac{y}{4} + \frac{\pi}{4}\right) = c - 2\sin\left(\frac{x}{2}\right)$

105. The differential equation of family of curves $x^2 + y^2 - 2ax = 0$, is

a) $x^2 - y^2 - 2xy\,y' = 0$ b) $y^2 - x^2 = 2xy\,y'$ c) $x^2 + y^2 + 2y'' = 0$ d) None of these

106. The order of the differential equation whose general solution is given by $y = (c_1 + c_2)\cos(x + c_3) - c_4e^{x+c_5}$ where c_1, c_2, c_3, c_4, c_5 are arbitrary constants, is

a) 4 b) 3 c) 2 d) 5

107. The degree of the equation $e^x + \sin\left(\frac{dy}{dx}\right) = 3$ is

a) 2 b) 0

c) Degree is not defined d) 1

108. If $x = \sin t$, $y = \cos pt$, then

a) $(1 - x^2)y_2 + xy_1 + p^2y = 0$ b) $(1 - x^2)y_2 + xy_1 - p^2y = 0$

c) $(1 + x^2)y_2 - xy_1 + p^2y = 0$ d) $(1 - x^2)y_2 - xy_1 + p^2y = 0$

109. The differential equation representing the family of curves $y = xe^{cx}$ (c is a constant) is

a) $\frac{dy}{dx} = \frac{y}{x}\left(1 - \log\frac{y}{x}\right)$ b) $\frac{dy}{dx} = \frac{y}{x}\log\left(\frac{y}{x}\right) + 1$ c) $\frac{dy}{dx} = \frac{y}{x}\left(1 + \log\frac{y}{x}\right)$ d) $\frac{dy}{dx} + 1 = \frac{y}{x}\log\left(\frac{y}{x}\right)$

110. The degree and order of the differential equation $y = px + \sqrt[3]{a^2\,p^2 + b^2}$, where $p = \frac{dy}{dx}$, are respectively

a) 3,1 b) 1,3 c) 1,1 d) 3,3

111. The degree of the differential equation $y_3^{2/3} + 2 + 3y_2 + y_1 = 0$, is

a) 1 b) 2 c) 3 d) None of these

112. If $x^2 + y^2 = 1$, then $\left(y' = \frac{dy}{dx}, y'' = \frac{d^2y}{dx^2}\right)$

a) $yy'' - (2y')^2 + 1 = 0$ b) $yy'' + (y')^2 + 1 = 0$ c) $y'' - (y')^2 - 1 = 0$ d) $y'' + 2(y')^2 + 1 = 0$

113. The solution of the differential equation $\frac{dy}{dx} = \frac{x\log x^2 + x}{\sin y + y\cos y}$, is

a) $y\sin y = x^2\log x + C$

b) $y\sin y = x^2 + C$

c) $y\sin y = x^2 + \log x + C$

d) $y\sin y = x\log x + C$

114. To reduce the differential equation $\frac{dy}{dx} + P(x).y = Q(x).y^n$ to the linear form, the substitution is

a) $v = \frac{1}{y^n}$ b) $v = \frac{1}{y^{n-1}}$ c) $v = y^n$ d) $v = y^{n-1}$

115. The equation of the curve whose subnormal is equal to a constant a is

a) $y = ax + b$ b) $y^2 = 2ax + 2b$ c) $ay^2 - x^3 = a$ d) None of these

116. A particle starts at the origin and moves along the x-axis in such a way that its velocity at the point $(x, 0)$ is given by the formula $\frac{dx}{dt} = \cos^2 \pi x$. Then, the particle never reaches the point on

a) $x = \frac{1}{4}$ b) $x = \frac{3}{4}$ c) $x = \frac{1}{2}$ d) $x = 1$

117. The solution of the equation $\frac{dy}{dx} = \frac{x+y}{x-y}$ is

a) $c(x^2 + y^2)^{1/2} + e^{\tan^{-1}(y/x)} = 0$ b) $c(x^2 + y^2)^{1/2} = e^{\tan^{-1}(y/x)}$

c) $c(x^2 - y^2) = e^{\tan^{-1}(y/x)}$ d) None of the above

118. The solution of the equation $\frac{d^2y}{dx^2} = e^{-2x}$ is

a) $\frac{e^{-2x}}{4}$ b) $\frac{e^{-2x}}{4} + cx + d$ c) $\frac{1}{4} e^{-2x} + cx^2 + d$ d) $\frac{1}{4} e^{-2x} + c + d$

119. If $x^2 + y^2 = 1$, then

a) $yy'' - (2y')^2 + 1 = 0$ b) $yy'' + (y')^2 + 1 = 0$

c) $yy'' - (y')^2 - 1 = 0$ d) $yy'' + 2(y')^2 + 1 = 0$

120. The equation of the curve whose slope is $\frac{y-1}{x^2+x}$ and which passes through the point $(1, 0)$ is

a) $xy + x + y - 1 = 0$ b) $xy - x - y - 1 = 0$ c) $(y - 1)(x + 1) = 2x$ d) $y(x + 1) - x + 1 = 0$

121. The solution of the differential equation $x \frac{dy}{dx} = 2y + x^3 e^x$, where $y = 0$ when $x = 1$, is

a) $y = x^3(e^x - e)$ b) $y = x^3(e - e^x)$ c) $y = x^2(e^x - e)$ d) $y = x^2(e - e^x)$

122. The solution of $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ is

a) $3x(1 + y^2) = 4y^3 + c$ b) $3y(1 + x^2) = 4x^3 + c$

c) $3x(1 - y^2) = 4y^3 + c$ d) $3y(1 + y^2) = 4x^3 + c$

123. A normal is drawn at a $P(x, y)$ of a curve. It meets the x-axis at Q. if PQ is of constant length k, then the differential equation describing such a curve is

a) $y \frac{dy}{dx} = \pm\sqrt{k^2 - y^2}$ b) $x \frac{dy}{dx} = \pm\sqrt{k^2 - x^2}$ c) $y \frac{dy}{dx} = \pm\sqrt{y^2 - k^2}$ d) $x \frac{dy}{dx} = \pm\sqrt{x^2 - k^2}$

124. The solution of the differential equation $y_1 y_3 = 3y_2^2$ is

a) $x = A_1 y^2 + A_2 y + A_3$ b) $x = A_1 y + A_2$ c) $x = A_1 y^2 + A_2 y$ d) None of these

125. If $x = A \cos 4t + B \sin 4t$, then $\frac{d^2x}{dt^2}$ is equal to

a) $-16x$ b) $16x$ c) x d) $-x$

126. The order of the differential equation associated with the primitive $y = c_1 + c_2 e^x + c_3 e^{-2x+c_4}$, where c_1, c_2, c_3, c_4 are arbitrary constants, is

a) 3 b) 4 c) 2 d) None of these

127. The differential equation of all parabolas whose axes are parallel to axis of x, is

a) $\frac{d^3y}{dx^3} = 0$ b) $\frac{d^3x}{dy^3} = 0$ c) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ d) $\frac{d^2x}{dy^2} = 0$

128. The solution of the differential equation $(x^2 - yx^2) \frac{dy}{dx} + y^2 + xy^2 = 0$ is

a) $\log\left(\frac{x}{y}\right) = \frac{1}{x} + \frac{1}{y} + c$ b) $\log\left(\frac{y}{x}\right) = \frac{1}{x} + \frac{1}{y} + c$

c) $\log(xy) = \frac{1}{x} + \frac{1}{y} + c$ d) $\log(xy) + \frac{1}{x} + \frac{1}{y} = c$

129. The solution of the differential equation $x\, dy - y\, dx - \sqrt{x^2 - y^2}\, dx = 0$ is

a) $y - \sqrt{x^2 + y^2} = cx^2$ b) $y + \sqrt{x^2 + y^2} = cx^2$

c) $y + \sqrt{x^2 + y^2} = cy^2$ d) $x - \sqrt{x^2 + y^2} = cy^2$

130. Solution of $\frac{dy}{dx} = \frac{x \log x^2 + x}{\sin y + y \cos y}$ is

a) $y \sin y = x^2 \log x + c$ b) $y \sin y = x^2 + c$

c) $y \sin y = x^2 + \log x$ d) $y \sin y = x \log x + c$

131. If integrating factor of $x(1-x^2)dy+(2x^2y-y-ax^3)dx=0$ is $e^{\int P dx}$, then P is equal to

a) $\frac{2x^2-ax^3}{x(1-x^2)}$ b) $2x^2-1$ c) $\frac{2x^2-1}{ax^3}$ d) $\frac{2x^2-1}{x(1-x^2)}$

132. The solution of the differential equation $\frac{dy}{dx}+\frac{y}{x}=x^2$, is

a) $y=\frac{x^2}{4}+C\,x^{-2}$ b) $y=x^{-1}+C\,x^{-3}$ c) $y=\frac{x^3}{4}+C\,x^{-1}$ d) $xy=x^2+C$

133. The differential equation of all circles passing through the origin and having their centres on the x-axis is

a) $x^2=y^2+xy\frac{dy}{dx}$ b) $x^2=y^2+3xy\frac{dy}{dx}$ c) $y^2=x^2+2xy\frac{dy}{dx}$ d) $y^2=x^2-2xy\frac{dy}{dx}$

134. If $y''-3y'+2y=0$ where $y(0)=1, y'(0)=0$, then the value of y at $x=\log 2$ is

a) 1 b) -1 c) 2 d) 0

135. The differential equation of all straight lines touching the circle $x^2+y^2=a^2$ is

a) $\left(y-\frac{dy}{dx}\right)^2=a^2\left[1+\left(\frac{dy}{dx}\right)^2\right]$ b) $\left(y-x\frac{dy}{dx}\right)^2=a^2\left[1+\left(\frac{dy}{dx}\right)^2\right]$

c) $\left(y-x\frac{dy}{dx}\right)=a^2\left[1+\frac{dy}{dx}\right]$ d) $\left(y-\frac{dy}{dx}\right)=a^2\left[1-\frac{dy}{dx}\right]$

136. The solution of the differential equation $(x^2-yx^2)\frac{dy}{dx}+y^2+xy^2=0$ is

a) $\log\left(\frac{x}{y}\right)=\frac{1}{x}+\frac{1}{y}+c$ b) $\log\left(\frac{y}{x}\right)=\frac{1}{x}+\frac{1}{y}+c$ c) $\log(xy)=\frac{1}{x}+\frac{1}{y}+c$ d) $\log(xy)+\frac{1}{x}+\frac{1}{y}=c$

137. The equation of the curve satisfying the equation $(xy-x^2)\frac{dy}{dx}=y^2$ and passing through the point $(-1,1)$ is

a) $y=(\log y-1)x$ b) $y=(\log y+1)x$ c) $x=(\log x-1)y$ d) $x=(\log x+1)y$

138. $y=2e^{2x}-e^{-x}$ is a solution of the differential equation

a) $y_2+y_1+2y=0$ b) $y_2-y_1+2y=0$ c) $y_2+y_1=0$ d) $y_2-y_1-2y=0$

139. The solution of $y'-y=1, y(0)=-1$ is given by $y(x)$, which is equal to

a) $-\exp(x)$ b) $-\exp(-x)$ c) -1 d) $\exp(x)-2$

140. The differential equation of the family of circles with fixed radius 5 unit and centre on the line $y=2$, is

a) $(x-2)^2y'^2=25-(y-2)^2$ b) $(x-2)y'^2=25-(y-2)^2$

c) $(y-2)y'^2=25-(y-2)^2$ d) $(y-2)^2y'^2=25-(y-2)^2$

141. Solution of the differential equation $\frac{dy}{dx}+y\sec^2 x=\tan x\sec^2 x$ is

a) $y=\tan x-1+ce^{-\tan x}$ b) $y^2=\tan x-1+ce^{\tan x}$

c) $ye^{\tan x}=\tan x-1+c$ d) $ye^{-\tan x}=\tan x-1+c$

142. The differential equation $y\frac{dy}{dx}+x=a$ (a is any constant) represents

a) A set of circles having centre on the y –axis

b) A set of circles on the x –axis

c) A set of ellipses

d) None of these

143. The equation of the curve for which the square of the ordinate is twice the rectangle contained by the abscissa and the intercept of the normal on x-axis and passing through (2, 1) is

a) $x^2+y^2-x=0$ b) $4x^2+2y^2-9y=0$ c) $2x^2+4y^2-9x=0$ d) $4x^2+2y^2-9x=0$

144. The general solution of $ydx-xdy-3x^2y^2e^{x^3}dx=0$, is equal to

a) $\frac{x}{y}=e^{x^3}+C$ b) $\frac{y}{x}=e^{x^3}+C$ c) $xy=e^{x^3}+C$ d) $xy=e^x+C$

145. The solution of $\frac{dy}{dx}=\frac{ax+h}{by+k}$ represents a parabola, when

a) $a=0, b=0$ b) $a=1, b=2$ c) $a=0, b\neq 0$ d) $a=2,\ b=1$

146. The differential equation of all ellipses centred at the origin is

a) $y_2 + x\, y_1^2 - y\, y_1 = 0$
b) $xy\, y_2 + x\, y_1^2 - y\, y_1 = 0$
c) $y\, y_2 + x\, y_1^2 - x\, y_1 = 0$
d) None of these

147. If $y = ax^{n+1}$, then $x^2\ \frac{d^2y}{dx^2}$ is equal to
a) $n(n-1)$ b) $n(n+1)y$ c) ny d) n^2y

148. The differential equation of the family of curves $y = a\cos(x+b)$ is
a) $\frac{d^2y}{dx^2} - y = 0$ b) $\frac{d^2y}{dx^2} + y = 0$ c) $\frac{d^2y}{dx^2} + 2y = 0$ d) None of these

149. If $y(t)$ is a solution of $(1+t)\frac{dy}{dt} - ty = 1$ and $y(0) = -1$, then $y(1)$ is equal to
a) $-\frac{1}{2}$ b) $e + \left(\frac{1}{2}\right)$ c) $e - \frac{1}{2}$ d) $\frac{1}{2}$

150. The integrating factor of the differential equation $\frac{dy}{dx} + \frac{y}{(1-x)\sqrt{x}} = 1 - \sqrt{x}$ is
a) $\frac{1-\sqrt{x}}{1+\sqrt{x}}$ b) $\frac{1+\sqrt{x}}{1-\sqrt{x}}$ c) $\frac{1-x}{1+x}$ d) $\frac{\sqrt{x}}{1-\sqrt{x}}$

151. The solution of the differential equation $(x^2+y^2)dx = 2xy\, dy$ is (here c is an arbitrary constant)
a) $x^2 + y^2 = cy$ b) $c(x^2 - y^2) = x$ c) $x^2 - y^2 = cy$ d) $x^2 + y^2 = cx$

152. The real value of n for which the substitution $y = u^n$ will transform the differential equation $2x^4y\ \frac{dy}{dx} + y^4 = 4x^6$ into a homogenous equation is
a) $1/2$ b) 1 c) $3/2$ d) 2

153. The differential equation satisfied by the family of curves $y = ax\cos\left(\frac{1}{x} + b\right)$ where a, b are parameters is
a) $x^2y_2 + y = 0$ b) $x^4y_2 + y = 0$ c) $xy_2 - y = 0$ d) $x^4y_2 - y = 0$

154. The solution of the differential equation $\frac{dy}{dx} = x\log x$ is
a) $y = x^2\log x - \frac{x^2}{2} + c$ b) $y = \frac{x^2}{2}\log x - \frac{x^2}{4} + c$
c) $y = \frac{x^2}{2} + \frac{x^2}{2}\log x + c$ d) None of these

155. Differential equation of $y = \sec(\tan^{-1}x)$ is
a) $(1+x^2)\frac{dy}{dx} = y + x$ b) $(1+x^2)\frac{dy}{dx} = y - x$ c) $(1+x^2)\frac{dy}{dx} = xy$ d) $(1+x^2)\frac{dy}{dx} = \frac{x}{y}$

156. Solution of the differential equation $\frac{dy}{dx}\tan y = \sin(x+y) + \sin(x-y)$ is
a) $\sec y + 2\cos x = c$ b) $\sec y - 2\cos x = c$ c) $\cos y - 2\sin x = c$ d) $\tan y - 2\sec x = c$

157. The differential equation of the family of parabolas with focus at the origin and the x-axis as axis, is
a) $y\left(\frac{dy}{dx}\right)^2 + 4x\frac{dy}{dx} = 4y$ b) $-y\left(\frac{dy}{dx}\right)^2 = 2x\frac{dy}{dx} - y$
c) $y\left(\frac{dy}{dx}\right)^2 + y = 2xy\frac{dy}{dx}$ d) $y\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx} + y = 0$

158. The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$, is
a) $\frac{x}{e^x}$ b) $\frac{e^x}{x}$ c) $x\, e^x$ d) e^x

159. The differential equation of all coaxial parabola $y^2 = 4a(x-b)$, where a and b are arbitrary constants, is
a) $y\ \frac{d^2y}{dx^2} + \frac{dy}{dx} = 1$ b) $y\ \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 1$ c) $y\ \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ d) $y\ \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

160. If $\frac{d^2y}{dx^2}\sin x = 0$, then the solution of differential equation is
a) $y = \sin x + cx + d$ b) $y = \cos x + cx^2 + d$ c) $y = \tan x + c$ d) $y = \log\sin x + cx$

161. The solution of $\frac{dy}{dx}+y=e^{-x}, y(0)=0$ is

a) $y=e^{-x}(x-1)$ b) $y=xe^{x}$ c) $y=xe^{-x}+1$ d) $y=xe^{-x}$

162. A curve having the condition that the slope of tangent at some point is two times the slope of the straight line joining the same point to the origin of coordinates, is a/an

a) Circle b) Ellipse c) Parabola d) Hyperbola

163. The solution of the differential equation $x\frac{dy}{dx}+2y=x^2$ is

a) $y=\frac{x^2+c}{4x^2}$ b) $y=\frac{x^2}{4}+c$ c) $y=\frac{x^4+c}{x^2}$ d) $y=\frac{x^4+c}{4x^2}$

164. The solution of the differential equation $y\frac{dy}{dx}=x-1$ satisfying $y(1)=1$ is

a) $y^2=x^2-2x+2$ b) $y^2=2x^2-x-1$ c) $y=x^2-2x+2$ d) None of these

165. The differential equation of the family of lines whose slope is equal to y-intercept, is

a) $(x+1)\frac{dy}{dx}-y=0$ b) $(x+1)\frac{dy}{dx}+y=0$

c) $\frac{dy}{dx}=\frac{x-1}{y-1}$ d) $\frac{dy}{dx}=\frac{x+1}{y+1}$

166. Solution of the equation $x^2y-x^3\frac{dy}{dx}=y^4\cos x$, when $y(0)=1$ is

a) $y^3=3x^3\sin x$ b) $x^3=3y^3\sin x$ c) $x^3=y^3\sin x$ d) $x^3=y^3\cos x$

167. A curve $y=f(x)$ passes through the point $P(1,1)$. The normal to the curve at point P is $a(y-1)+(x-1)=0$. If the slope of the tangent at any point on the curve is proportional to the ordinate at that point, then the equation of the curve is

a) $y=e^{ax}-1$ b) $y=e^{ax}+1$ c) $y=e^{ax}-a$ d) $y=e^{a(x-1)}$

168. The solution of $\frac{dy}{dx}+1=e^{x+y}$ is

a) $e^{-(x+y)}+x+c=0$ b) $e^{-(x+y)}-x+c=0$ c) $e^{x+y}+x+c=0$ d) $e^{x+y}-x+c=0$

169. The solution of the differential equation $\left\{\frac{1}{x}-\frac{y^2}{(x-y)^2}\right\}dx+\left\{\frac{x^2}{(x-y)^2}-\frac{1}{y}\right\}dy=0$ is

a) $\text{In}\left|\frac{x}{y}\right|+\frac{xy}{(x-y)}=c$ b) $\text{In}\,|xy|+\frac{xy}{(x-y)}=c$ c) $\frac{xy}{(x-y)}=ce^{x/y}$ d) $\frac{xy}{(x-y)}=ce^{xy}$ (where c is arbitrary constant)

170. Degree of differential equation $e^{dy/dx}=x$ is

a) 1 b) 2 c) 3 d) None of these

171. The order and degree of the differential equation $\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/4}=\left(\frac{d^2y}{dx^2}\right)^{1/3}$ is

a) (2,4) b) (2,3) c) (6,4) d) (6,9)

172. $\tan^{-1}x+\tan^{-1}y=C$ is the general solution of the differential equation

a) $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$

b) $\frac{dy}{dx}=\frac{1+x^2}{1+y^2}$

c) $(1+x^2)dy+(1+y^2)dx=0$

d) $\frac{dy}{dx}=\frac{1-y^2}{1-x^2}$

173. The solution of $y'=1+x+y^2+xy^2, y(0)=0$ is

a) $y^2=\exp\left(x+\frac{x^2}{2}\right)-1$ b) $y^2=1+c\exp\left(x+\frac{x^2}{2}\right)$

c) $y=\tan(c+x+x^2)$ d) $y=\tan\left(x+\frac{x^2}{2}\right)$

174. If $\frac{dy}{dx}=e^{-2y}$ and $y=0$ when $x=5$, the value of x and $y=3$ is

a) e^5 b) $e^6 + 1$ c) $\frac{e^6 + 9}{2}$ d) $\log_e 6$

175. The solution of differential equation $(\sin x + \cos x)dy + (\cos x - \sin x)dx = 0$ is
   a) $e^x(\sin x + \cos x) + c = 0$ b) $e^y(\sin x + \cos x) = c$
   c) $e^y(\cos x - \sin x) = c$ d) $e^x(\sin x - \cos x + x) = c$
176. If $x\ dy = y(dx + y\ dy), y(1) = 1$ and $y(x) > 0$, then $y(-3)$ is equal to
   a) 3 b) 2 c) 1 d) 0
177. The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$ is
   a) $\phi\left(\frac{y}{x}\right) = kx$ b) $x\phi\left(\frac{y}{x}\right) = k$ c) $\phi\left(\frac{y}{x}\right) = ky$ d) $y\phi\left(\frac{y}{x}\right) = k$
178. The solution of $(x + y + 1)\frac{dy}{dx} = 1$ is
   a) $y = (x + 2) + ce^x$ b) $y = -(x + 2) + ce^x$
   c) $x = -(y + 2) + ce^y$ d) $x = (y + 2)^2 + ce^y$
179. The differential equation of the family of the curves $x^2 + y^2 - 2ax = 0$ is
   a) $x^2 - y^2 - 2xy'' = 0$ b) $y^2 - x^2 = 2xyy'$
   c) $x^2 + y^2 + 2y'' = 0$ d) None of these
180. If c_1, c_2, c_3, c_4, c_5 and c_6 are constants, then the order of the differential equation whose general solution is given by
   $y = c_1 \cos(x + c_2) + c_3 \sin(x + c_4) + c_5 e^x + c_6$ is
   a) 6 b) 5 c) 4 d) 3
181. The solution of the differential equation $\frac{x + y\frac{dy}{dx}}{y - x\frac{dy}{dx}} = x^2 + 2y^2 + \frac{y^4}{x^2}$ is
   a) $\frac{y}{4} + \frac{1}{x^2 + y^2} = c$ b) $\frac{y}{x} - \frac{1}{x^2 + y^2} = c$ c) $\frac{x}{y} - \frac{1}{x^2 + y^2} = c$ d) None of these
182. The solution of differential equation $(1 + x)y\ dx + (1 - y)x\ dy = 0$ is
   a) $\log_e(xy) + x - y = c$ b) $\log_e\left(\frac{x}{y}\right) + x + y = c$
   c) $\log_e\left(\frac{x}{y}\right) - x + y = c$ d) $\log_e(xy) - x + y = c$
183. The differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where $c > 0$ is a parameter is of order and degree as follows
   a) Order 2, degree 2 b) Order 1, degree 3 c) Order 1, degree 1 d) Order 1, degree 2
184. The solution of the differential equation $\frac{dy}{dx} = \frac{1}{x^2 + y^2}$ is
   a) $y = -x^2 - 2x - 2 + ce^x$ b) $y = x^2 + 2x + 2 - ce^x$
   c) $x = -y^2 - 2y + 2 - ce^y$ d) $x = -y^2 - 2y - 2 + ce^y$
185. Integrating factor of $(x + 2y^3)\frac{dy}{dx} = y^2$ is
   a) $e^{\left(\frac{1}{y}\right)}$ b) $e^{-\left(\frac{1}{y}\right)}$ c) y d) $\frac{-1}{y}$
186. The curve in which the slope of the tangent at any point equals the ratio of the abscissa to the ordinate of the point is
   a) An ellipse
   b) A parabola
   c) A rectangular hyperbola
   d) A circle
187. The solution of the differential equation $(1 + y^2) + \left(x - e^{\tan^{-1} y}\right)\frac{dy}{dx} = 0$ is
   a) $2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + c$ b) $xe^{\tan^{-1} y} = {}^{\tan^{-1} y} + c$

c) $xe^{2\tan^{-1}y} = e^{\tan^{-1}y} + c$ d) $(x-2) = ce^{-\tan^{-1}y}$

188. The differential equation $(e^x+1)y\,dy = (y+1)e^x\,dx$, has the solution

a) $(y-1)(e^x-1) = ce^y$ b) $(y-1)(e^x+1) = ce^y$
c) $(y+1)(e^x-1) = ce^y$ d) $(y+1)(e^x+1) = ce^y$

189. The differential equation of all straight lines passing through origin is

a) $y = \sqrt{x}\frac{dy}{dx}$ b) $\frac{dy}{dx} = y + x$ c) $\frac{dy}{dx} = y - x$ d) None of these

190. The solution of the differential equation $\frac{dy}{dx} = \sin\,(x+y)\tan(x+y) - 1$ is

a) $\operatorname{cosec}(x+y) + \tan(x+y) = x + c$ b) $x + \operatorname{cosec}(x+y) = c$
c) $x + \tan(x+y) = c$ d) $x + \sec(x+y) = c$

191. The differential equation for which $\sin^{-1}x + \sin^{-1}y = c$ is given by

a) $\sqrt{1-x^2}dy + \sqrt{1-y^2}dx = 0$ b) $\sqrt{1-x^2}\,dx + \sqrt{1-y^2}\,dy = 0$
c) $\sqrt{1-x^2}\,dx - \sqrt{1-y^2}\,dy = 0$ d) $\sqrt{1-x^2}\,dy - \sqrt{1-y^2}\,dx = 0$

192. The integrating factor of $x\frac{dy}{dx} + (1+x)y = x$ is

a) x b) $2x$ c) $e^{x\log x}$ d) xe^x

193. The solution of the differential equation $(x+2y^3)\frac{dy}{dx} = y$, is

a) $x = y^2 + C$ b) $y = x^2 + C$ c) $x = y(y^2 + C)$ d) $y = x(x^2 + C)$

194. The order of the differential equation $\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^3}$, is

a) 2 b) 1 c) 3 d) 4

195. The number of solutions of $y' = \frac{y+1}{x-1}$, $y(1) = 2$ is

a) Zero b) One c) Two d) Infinite

196. The solution of the differential equation $x(x-y)\frac{dy}{dx} = y(x+y)$, is

a) $\frac{x}{y} + \log(xy) = c$ b) $\frac{y}{x} + \log(xy) = c$ c) $\frac{x}{y} + y\log x = c$ d) $\frac{x}{y} + x\log y = c$

197. The general solution of differential equation $\frac{dy}{dx} = \frac{x^2}{y^2}$, is

a) $x^3 - y^3 = C$ b) $x^3 + y^3 = C$ c) $x^2 + y^2 = C$ d) $x^2 - y^2 = C$

198. The solution of the differential equation $\frac{d^2y}{dx^2} = e^{-2x}$ is $y = c_1e^{-2x} + c_2x + x_3$, where c_1 is

a) 1 b) $\frac{1}{4}$ c) $\frac{1}{2}$ d) 2

199. Solution of the equation $x\left(\frac{dy}{dx}\right)^2 + 2\sqrt{xy}\frac{dy}{dx} + y = 0$ is

a) $x + y = a$ b) $\sqrt{x} - \sqrt{y} = \sqrt{a}$ c) $x^2 + y^2 = a^2$ d) $\sqrt{x} + \sqrt{y} = c$

200. Form of the differential equation of all family of lines $y = mx + \frac{4}{m}$ by eliminating the arbitrary constant m is

a) $\frac{d^2y}{dx^2} = 0$ b) $x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} + 4 = 0$
c) $x\left(\frac{dy}{dx}\right)^2 + y\frac{dy}{dx} + 4 = 0$ d) $\frac{dy}{dx} = 0$

201. The general solution $e^x\cos y\;dx - e^x\sin y\;dy = 0$, is

a) $e^x(\sin y + \cos y) = C$ b) $e^x\sin y = C$ c) $e^x = C\cos y$ d) $e^x\cos y = C$

202. $y = ae^{mx} + be^{-mx}$ satisfies which of the following differential equation?

a) $\frac{dy}{dx} - my = 0$ b) $\frac{dy}{dx} + my = 0$ c) $\frac{d^2y}{dx^2} + m^2y = 0$ d) $\frac{d^2y}{dx^2} - m^2y = 0$

203. The solution of $\frac{dy}{dx} + y = e^{-x}$, $y(0) = 0$, is

a) $y = e^{-x}(x-1)$ b) $y = xe^{-x}$ c) $y = xe^{-x} + 10$ d) $y = (x+1)e^{-x}$

204. The general solution of the differential equation $(1+y^2)dx+(1+x^2)dy=0$ is

a) $x-y=c(1-xy)$ b) $x-y=c(1+xy)$ c) $x+y=c(1-xy)$ d) $x+y=c(1+xy)$

205. If the integrating factor of the differential equation $\frac{dy}{dx}+P(x)y=Q(x)$ is x, then $P(x)$ is

a) x b) $x^2/2$ c) $1/x$ d) $1/x^2$

206. The order of the differential equation $\frac{d^2y}{dx^2}=\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ is

a) 3 b) 2 c) 1 d) 4

207. The solution of $\frac{dy}{dx}+\sqrt{\left(\frac{1-y^2}{1-x^2}\right)}=0$ is

a) $\tan^{-1}x+\cot^{-1}x=c$ b) $\sin^{-1}x+\sin^{-1}y=c$
c) $\sec^{-1}x+\mathrm{cosec}^{-1}x=c$ d) None of these

208. Solution of the differential equation $x\,dy-y\,dx=0$ represents

a) A parabola whose vertex is at the origin
b) A circle whose centre is at the origin
c) A rectangular hyperbola
d) Straight lines passing through the origin

209. The differential equation of the family of circles passing through the fixed points $(a,0)$ and $(-a,0)$ is

a) $y_1(y^2-x^2)+2xy+a^2=0$ b) $y_1y^2+xy+a^2x^2=0$
c) $y_1(y^2-x^2+a^2)+2xy=0$ d) $y_1(y^2+x^2)-2xy+a^2=0$

210. The solution of differential equation $(x+y)(dx-dy)=dx+dy$ is

a) $x-y=ke^{x-y}$ b) $x+y=ke^{x+y}$ c) $x+y=ke^{x-y}$ d) $(x-y)=ke^{x+y}$

211. The general solution of $y^2dx+(x^2-xy+y^2)dy=0$ is

a) $\tan^{-1}\left(\frac{x}{y}\right)+\log y+c=0$ b) $2\tan^{-1}\left(\frac{x}{y}\right)+\log x+c=0$
c) $\log\left(y+\sqrt{x^2+y^2}\right)+\log y+c=0$ d) $\sin h^{-1}\left(\frac{x}{y}\right)+\log y+c=0$

212. The order and degree of the following differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{5/2}=\frac{d^3y}{dx^3}$ are respectively

a) 3,2 b) 3,10 c) 2,3 d) 3,5

213. The solution of $x\,dy-y\,dx+x^2e^xdx=0$ is

a) $\frac{y}{x}+e^x=c$ b) $\frac{x}{y}+e^x=c$ c) $x+e^y=c$ d) $y+e^x=c$

214. The solution of the differential equation $\frac{dy}{dx}=\frac{x-y+3}{2(x-y)+5}$ is

a) $2(x-y)+\log(x-y)=x+c$ b) $2(x-y)-\log(x-y+2)=x+c$
c) $2(x-y)+\log(x-y+2)=x+c$ d) None of the above

215. The differential equation whose solution is $Ax^2+By^2=1$, where A and B are arbitrary constants, is of

a) First order and second degree b) First order and first degree
c) Second order and first degree d) Second order and second degree

216. If $y=f(x)$ is the equation of the curve an its differential equation is given by $\frac{dy}{dx}=\frac{x+2}{y+3}$, then the equation of the curve, if it passes through (2, 2), is

a) $x^2-y^2+4x-6y+4=0$ b) $x^2-y^2+4x+6y=0$
c) $x^2-y^2-4x-6y=0$ d) $x^2-y^2-4x-6y-4=0$

217. The differential equation of the family of curves $y^2=4\,a\,(x+a)$, is

a) $y^2=4\frac{dy}{dx}\left(x+\frac{dy}{dx}\right)$

b) $y^2\left(\frac{dy}{dx}\right)^2+2xy\,\frac{dy}{dx}-y^2=0$

c) $2y\,\frac{dy}{dx}=4a$

d) $y^2\frac{dy}{d}+4y=0$

218. The integrating factor of the differential equation $x\log x\,\frac{dy}{dx}+y=2\log x$ is given by

a) e^x b) $\log x$ c) $\log(\log x)$ d) x

219. The differential equation which represents the family of plane curves $y=\exp(cx)$ is

a) $y'=cy$ b) $xy'-\log y=0$ c) $x\log y=yy'$ d) $y\log y=xy'$

220. The solution of $\frac{dy}{dx}+y\tan x=\sec x$ is

a) $y\sec x=\tan x+c$ b) $y\tan x=\sec x+c$ c) $\tan x=y\tan x+c$ d) $x\sec x=y\tan y+c$

221. The function $f(\theta)=\frac{d}{d\theta}\int_0^{\theta}\frac{dx}{1-\cos\theta\cos x}$ satisfies the differential equation

a) $\frac{df}{d\theta}+2f(\theta)=0$ b) $\frac{df}{d\theta}-2f(\theta)=0$ c) $\frac{df}{d\theta}-2f(\theta)=\tan\theta$ d) $\frac{df}{d\theta}+2f(\theta)\cot\theta=0$

222. The solution of $\frac{dy}{dx}=\left(\frac{y}{x}\right)^{1/3}$, is

a) $x^{2/3}+y^{2/3}=C$ b) $x^{1/3}+y^{1/3}=C$ c) $y^{2/3}-x^{2/3}=C$ d) $y^{1/3}-x^{1/3}=C$

223. If $x\sin\left(\frac{y}{x}\right)dy=\left[y\sin\left(\frac{y}{x}\right)-y\right]dx$ and $y(1)=\frac{\pi}{2}$, then the value of $\cos\left(\frac{y}{x}\right)$ is equal to

a) x b) $\frac{1}{x}$ c) $\log x$ d) e^x

224. The solution of the differential equation $\frac{dy}{dx}=\frac{y}{x}+\frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$ is

a) $x\,\phi\left(\frac{y}{x}\right)=k$ b) $\phi\left(\frac{y}{x}\right)=kx$ c) $y\,\phi\left(\frac{y}{x}\right)=k$ d) $\phi\left(\frac{y}{x}\right)=ky$

225. If $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$, $y(1)=1$, then one of the values of x_0 satisfying $y(x_0)=e$ is given by

a) $e\sqrt{2}$ b) $e\sqrt{3}$ c) $e\sqrt{5}$ d) $e/\sqrt{2}$

226. Solution of $\frac{dy}{dx}=3^{x+y}$ is

a) $3^{x+y}=c$ b) $3^x+3^y=c$ c) $3^{x-y}=c$ d) $3^x+3^{-y}=c$

227. Order of the differential equation of the family of all concentric circles centred at (h,k) is

a) 1 b) 2 c) 3 d) 4

228. The solution of $\frac{dy}{dx}=\cos(x+y)+\sin(x+y)$ is

a) $\log\left[1+\tan\left(\frac{x+y}{2}\right)\right]+c=0$ b) $\log\left[1+\tan\left(\frac{x+y}{2}\right)\right]=x+c$

c) $\log\left[1-\tan\left(\frac{x+y}{2}\right)\right]=x+c$ d) None of these

229. The general solution of the differential equation $\frac{dy}{dx}=\frac{(1+y^2)}{xy(1+x^2)}$ is

a) $(1+x^2)(1+y^2)=c$ b) $(1+x^2)(1+y^2)=cx^2$

c) $(1-x^2)(1-y^2)=c$ d) $(1+x^2)(1+y^2)=cy^2$

230. The general solution of $\frac{dy}{dx}=\frac{2x-y}{x+2y}$ is

a) $x^2-xy+y^2=c$ b) $x^2-xy-y^2=c$ c) $x^2+xy-y^2=c$ d) $x^2+xy^2=c$

231. The differential equation representing the family of curves $y^2=2c(x+c^{2/3})$, where c is a positive parameter, is of

a) Order 3, degree 3 b) Order 2, degree 4 c) Order 1, degree 5 d) Order 5, degree 1

232. The solution of the differential equation $\frac{dx}{x}+\frac{dy}{y}=0$ is

a) $xy=c$ b) $x+y=c$ c) $\log x\log y=c$ d) $x^2+y^2=c$

233. If $y=\left(x+\sqrt{1+x}\right)^n$, then $(1+x^2)\frac{d^2y}{dx^2}+x\ \frac{dy}{dx}$ is

a) n^2y b) $-n^2y$ c) $-y$ d) $2x^2y$

234. The order of the differential equation whose general solution is given by $y=(c_1+c_2)\cos(x+c_3)-c4ex+c5$ where $c1,c2,c3,c4$ and $c5$ are arbitrary constants is

a) 5 b) 6 c) 3 d) 2

235. The differential equation obtained by eliminating arbitrary constants from $y=ae^{bx}$ is

a) $y\ \frac{d^2y}{dx^2}+\frac{dy}{dx}=0$ b) $y\ \frac{d^2y}{dx^2}-\frac{dy}{dx}=0$ c) $y\ \frac{d^2y}{dx^2}-\left(\frac{dy}{dx}\right)^2=0$ d) $y\ \frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$

236. The differential equation of all non-horizontal lines in a plane is

a) $\frac{d^2y}{dx^2}=0$ b) $\frac{dx}{dy}=0$ c) $\frac{dy}{dx}=0$ d) $\frac{d^2x}{dy^2}=0$

237. The degree of the differential equation satisfying $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$ is

a) 1 b) 2 c) 3 d) None of these

238. The solution of the differential equation $\frac{dy}{dx}=e^{y+x}+e^{y-x}$ is

a) $e^{-y}=e^x-e^{-x}+c$ b) $e^{-y}=e^{-x}-e^x+c$ c) $e^{-y}=e^x+e^{-x}+c$ d) $e^{-y}+e^x+e^{-x}=c$

239. The integrating factor of the differential equation $\frac{dy}{dx}+\frac{1}{x}.y=3x$ is

a) x b) $\ln x$ c) 0 d) ∞

240. The solution of the differential equation $\sec^2 x\tan y\ dx+\sec^2 y\tan x\ dy=0$ is

a) $\tan y\tan x=c$ b) $\frac{\tan y}{\tan x}=c$ c) $\frac{\tan^2 x}{\tan y}=c$ d) None of these

241. The equation of the curve in which subnormal varies as the square of the ordinate is (λ is constant of proportionality)

a) $y=C\ e^{2\lambda x}$ b) $y=C\ e^{\lambda x}$ c) $\frac{y^2}{2}+\lambda x=C$ d) $y^2+\lambda x^2=C$

242. The general solution of the differential equation $\frac{dy}{dx}+\frac{1+\cos 2y}{1-\cos 2x}=0$ is given by

a) $\tan y+\cot x=c$ b) $\tan y-\cot x=c$ c) $\tan x-\cot y=c$ d) $\tan x+\cot y=c$

243. The solution of the differential equation $\left(e^{-2\sqrt{x}}-\frac{y}{\sqrt{x}}\right)\frac{dy}{dx}=1$ is given by

a) $ye^{2\sqrt{x}}=x+c$ b) $ye^{-2\sqrt{x}}=\sqrt{x}+c$ c) $y=\sqrt{x}$ d) $y=3\sqrt{x}$

244. The solution of the differential equation $e^{-x}(y+1)dy+(\cos^2 x-\sin 2x)y\ dx=0$ subjected to the condition that $y=1$ when $x=0$ is

a) $y+\log y+e^x\cos^2 x=2$ b) $\log(y+1)+e^x\cos^2 x=1$

c) $y+\log y=e^x\cos^2 x$ d) $(y+1)+e^x\cos^2 x=2$

245. The solution of the differential equation $\frac{dy}{dx}=(4x+y+1)^2$, is

a) $(4x+y+1)=\tan(2x+c)$ b) $(4x+y+1)^2=2\tan(2x+c)$

c) $(4x+y+1)^3=3\tan(2x+c)$ d) $(4x+y+1)=2\tan(2x+c)$

246. An integrating factor of the differential equation

$x+\frac{dy}{dx}+y\log x=xe^x x^{-\frac{1}{2}\log x}$, $(x,0)$ is

a) $x^{\log x}$ b) $\left(\sqrt{x}\right)^{\log x}$ c) $\left(\sqrt{e}\right)^{(\log x)^2}$ d) e^{x^2}

247. The order of differential equation of all parabola with it's axis parallel to y-axis and touch x-axis is

a) 2 b) 3 c) 1 d) None of these

248. The differential equation obtained on eliminating A and B from the equation $y=A\cos\omega t+B\sin\omega t$ is

a) $y_2=-\omega^2y$ b) $y_1+y=0$ c) $y_2+y_1=0$ d) $y_1-\omega^2y=0$

249. The solution of the differential equation $\frac{dy}{dx}\tan y=\sin(x+y)+\sin(x-y)$ is

a) $\sec y+2\cos x=c$ b) $\sec y-2\cos x=c$ c) $\cos y-2\sin x=c$ d) $\tan y-2\sec y=c$

250. The degree of the differential equation satisfying the relation $\sqrt{1+x^2}+\sqrt{1+y^2}=\lambda\left(x\sqrt{1+y^2}-\right.$ $y1+x2$, is

a) 1 b) 2 c) 3 d) None of these

251. The solution of the differential equation $\frac{dy}{dx}-y\tan x=e^x\sec x$ is

a) $y=e^x\cos x+c$ b) $y\cos x=e^x+c$ c) $y=e^x\sin x+c$ d) $y\sin x=e^x+c$

252. The degree of the differential equations $x=1+\left(\frac{dy}{dx}\right)+\frac{1}{2!}\left(\frac{dy}{dx}\right)^2+\frac{1}{3!}\left(\frac{dy}{dx}\right)^3+\cdots$

a) 3 b) 2 c) 1 d) Not defined

253. If $y=y(x)$ and $\frac{2+\sin x}{y+1}\left(\frac{dy}{dx}\right)=-\cos x, y(0)=1$, then $y\left(\frac{\pi}{2}\right)$ equals

a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) $-\frac{1}{3}$ d) 1

254. The solution of $\cos y\frac{dy}{dx}=e^{x+\sin y}+x^2e^{\sin y}$ is

a) $e^x-e^{\sin y}+\frac{x^3}{3}=c$ b) $e^{-x}-e^{-\sin y}+\frac{x^3}{3}=c$

c) $e^x+e^{-\sin y}+\frac{x^3}{3}=c$ d) $e^x-e^{\sin y}-\frac{x^3}{3}=c$

255. The solution of $y\,dx-x\,dy+3x^2y^2e^{x^3}dx=0$ is

a) $\frac{x}{y}+e^{x^3}=C$ b) $\frac{x}{y}-e^{x^3}=0$ c) $-\frac{x}{y}+e^{x^3}=C$ d) None of these

256. The general solution of $\frac{dy}{dx}=\frac{2x-y}{x+2y}$ is

a) $x^2-xy+y^2=c$ b) $x^2-xy-y^2=c$ c) $x^2+xy-y^2=c$ d) $x^2+xy^2=c$

257. $y+x^2=\frac{dy}{dx}$ has the solution

a) $y+x^2+2x+2=ce^x$ b) $y+x+x^2+2=ce^{2x}$

c) $y+x+2x^2+2=ce^x$ d) $y^2+x+x^2+2=ce^x$

258. The equation of curve passing through the point $\left(1,\frac{\pi}{4}\right)$ and having slope of tangent at any point (x,y) as $\frac{y}{x}-\cos^2\left(\frac{y}{x}\right)$, is

a) $x=e^{1+\tan\left(\frac{y}{x}\right)}$ b) $x=e^{1-\tan\left(\frac{y}{x}\right)}$ c) $x=e^{1+\tan\left(\frac{x}{y}\right)}$ d) $x=e^{1-\tan\left(\frac{x}{y}\right)}$

259. The solution of $\frac{dy}{dx}=1+y+y^2+x+xy+xy^2$ is

a) $\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right)=x+x^2+c$ b) $4\tan^{-1}\left(\frac{4y+1}{\sqrt{3}}\right)=\sqrt{3}(2x+x^2)+c$

c) $\sqrt{3}\tan^{-1}\left(\frac{3y+1}{3}\right)=4(1+x+x^2)+c$ d) $4\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right)=\sqrt{3}(2x+x^2)+c$

260. The solution of $\frac{dy}{dx}=2^{y-x}$ is

a) $2^x+2^y=c$ b) $2^x-2^y=c$ c) $\frac{1}{2^x}-\frac{1}{2^y}=c$ d) $\frac{1}{2^x}+\frac{1}{2^y}=c$

261. A function $y=f(x)$ has a second order derivative $f''=6(x-1)$. If its graph passes through the point (2,1) and at point the tangent to the graph is $y=3x-5$ then the function is

a) $(x-1)^2$ b) $(x-1)^3$ c) $(x+1)^3$ d) $(x+1)^2$

262. The solution of $\log\left(\frac{dy}{dx}\right)=ax+by$ is

a) $\frac{e^{by}}{b}=\frac{e^{ax}}{a}+c$ b) $\frac{e^{-by}}{-b}=\frac{e^{ax}}{a}+c$ c) $\frac{e^{-by}}{a}=\frac{e^{ax}}{b}+c$ d) None of these

263. For solving $\frac{dy}{dx}=4x+y+1$, suitable substitution is

a) $y=vx$ b) $y=4x+v$ c) $y=4x$ d) $y+4x+1=v$

264. The differential equation $\frac{dy}{dx}=\frac{x(1+y^2)}{y(1+x^2)}$ represents a family of

a) Parabola b) Hyperbola c) Circle d) Ellipse

265. The differential equation of the system of all circles of radius r in the xy-plane, is

a) $\left[1+\left(\frac{dy}{dx}\right)^3\right]^2 = r^2\left(\frac{d^2y}{dx^2}\right)^2$
b) $\left[1+\left(\frac{dy}{dx}\right)^3\right]^2 = r^2\left(\frac{d^2y}{dx^2}\right)^3$
c) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = r^2\left(\frac{d^2y}{dx^2}\right)^2$
d) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = r^2\left(\frac{d^2y}{dx^2}\right)^3$

266. The differential equation of the family of parabola with focus as the origin and the axis as x-axis, is

a) $y\left(\frac{dy}{dx}\right)^2 + 4x\frac{dy}{dx} = 4y$
b) $-y\left(\frac{dy}{dx}\right)^2 = 2x\frac{dy}{dx} - y$
c) $y\left(\frac{dy}{dx}\right)^2 + y = 2xy\frac{dy}{dx}$
d) $y\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx} + y = 0$

267. The equation of curve through point (1,0) which satisfies the differential equation $(1+y^2)dx - xy\,dy = 0$ is

a) $x^2+y^2=4$
b) $x^2-y^2=1$
c) $2x^2+y^2=2$
d) None of these

268. The equation of the curve through the point (3, 2) and whose slope is $\frac{x^2}{y+1}$, is

a) $\frac{y^2}{2}+y=\frac{x^3}{3}+5$
b) $y+y^2-x^3-21$
c) $y^2+2y=\frac{2x^3}{3}-10$
d) $\frac{y^2}{2}+y=\frac{x^3}{3}-5$

269. The equation of the curve through the point (1,0) and whose slope is $\frac{y-1}{x^2+x}$, is

a) $2x+(y-1)(x+1)=0$
b) $2x-(y-1)(x+1)=0$
c) $2x+(y-1)(x-1)=0$
d) None of these

270. If $y(t)$ is a solution of $(1+t)\frac{dy}{dt} - ty = 1$ and $y(0)=-1$, then $y(1)$ is equal to

a) $-\frac{1}{2}$
b) $e+\frac{1}{2}$
c) $e-\frac{1}{2}$
d) $\frac{1}{2}$

271. The order of the differential equation of all tangent lines to the parabola $y=x^2$ is

a) 1
b) 2
c) 3
d) 4

272. The differential equation for the family of curves $x^2+y^2-2ay=0$, where a is an arbitrary constant, is

a) $2(x^2-y^2)y'=xy$
b) $2(x^2+y^2)y'=xy$
c) $(x^2-y^2)y'=2xy$
d) $(x^2+y^2)y'=2xy$

273. The solution of $\frac{dy}{dx}+1=\text{cosec}\,(x+y)$ is

a) $\cos(x+y)+x=c$
b) $\cos(x+y=c$
c) $\sin(x+y)+x=c$
d) $\sin(x+y)+\sin(x+y)=c$

274. The solution of the differential equation $9y\frac{dy}{dx}+4x=0$ is

a) $\frac{y^2}{9}+\frac{x^2}{4}=c$
b) $\frac{y^2}{4}+\frac{x^2}{9}=c$
c) $\frac{y^2}{9}-\frac{x^2}{4}=c$
d) $y^2-\frac{x^2}{9}=c$

275. The differential equation of the rectangular hyperbola whose axes are the asymptotes of the hyperbola, is

a) $y\frac{dy}{dx}=x$
b) $x\frac{dy}{dx}=-y$
c) $x\frac{dy}{dx}=y$
d) $x\,dy+y\,dx=c$

276. A particular solution of $\log\left(\frac{dy}{dx}\right)=3x+4y,\ y(0)=0$ is

a) $e^{3x}+3e^{-4y}=4$
b) $4e^{3x}-3e^{-4y}=3$
c) $3e^{3x}+4e^{-4y}=7$
d) $4e^{3x}+3e^{-4y}=7$

277. The differential equation $\frac{d^2y}{dx^2}=2$ represents

a) A parabola whose axis is parallel to x-axis
b) A parabola whose axis is parallel to y-axis
c) A circle
d) None of the above

278. If $x\frac{dy}{dx}=y(\log y-\log x+1)$, then the solution of the equation is

a) $\log\left(\frac{x}{y}\right)=cy$
b) $\log\left(\frac{y}{x}\right)=cx$
c) $x\log\left(\frac{y}{x}\right)=cy$
d) $y\log\left(\frac{x}{y}\right)=cx$

279. The general solution of $y^2dx+(x^2-xy+y^2)dy=0$ is

a) $\tan^{-1}\left(\frac{y}{x}\right)=\log y+c$
b) $2\tan^{-1}\left(\frac{x}{y}\right)+\log x+c=0$

c) $\log(y+\sqrt{x^2+y^2})+\log y+c=0$ d) $\sinh^{-1}\left(\frac{x}{y}\right)+\log y+c=0$

280. The equation of the curve satisfying the differential equation $y_2(x^2+1)=2xy_1$ passing through the point (0,1) and having slope of tangent at $x=0$ as 3 is
a) $y=x^3+3x+1$ b) $y=x^3-3x+1$ c) $y=x^2+3x+1$ d) $y=x^2-3x+1$

281. The solution of differential equation $(1+y^2)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$ is
a) $2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$ b) $2xe^{\tan^{-1}y}=e^{\tan^{-1}y}+k$
c) $xe^{\tan^{-1}y}=e^{\tan^{-1}y}+k$ d) $xe^{\tan^{-1}y}=e^{\tan^{-1}y}+k$

282. The solution of $e^{dy/dx}=(x+1), y(0)=3$ is
a) $y=x\ \log x-x+2$ b) $y=(x+1)\log|x+1|-x+3$
c) $y=(x+1)\log|x+1|+x+3$ d) $y=x\log x+x+3$

283. The solution of the equation $x^2\frac{d^2y}{dx^2}=\log x$ when $x=1, y=0$ and $\frac{dy}{dx}=-1$ is
a) $y=\frac{1}{2}(\log x)^2+\log x$ b) $y=\frac{1}{2}(\log x)^2-\log x$
c) $y=-\frac{1}{2}(\log x)^2+\log x$ d) $y=-\frac{1}{2}(\log x)^2-\log x$

284. The order of the differential equation whose solution is $y=a\cos x+b\sin x+ce^{-x}$, is
a) 3 b) 1 c) 2 d) 4

285. The differential equation for which $\sin^{-1}x+\sin^{-1}y=c$, is given by
a) $\sqrt{1-x^2}dx+\sqrt{1-y^2}dy=0$ b) $\sqrt{1-x^2}dy+\sqrt{1-y^2}dx=0$
c) $\sqrt{1-x^2}dy-\sqrt{1-y^2}dx=0$ d) $\sqrt{1-x^2}dx-\sqrt{1-y^2}dy=0$

286. A continuously differential function $\phi(x)$ in $(0,\pi)$ satisfying $y'=1+y^2, y(0)=0=y(\pi)$, is
a) $\tan x$ b) $x(x-\pi)$ c) $(x-\pi)(1-e^x)$ d) Not possible

287. A solution of the differential equation $\left(\frac{dy}{dx}\right)^2-x\frac{dy}{dx}+y=0$ is
a) $y=2$ b) $y=2x$ c) $y=2x-4$ d) $y=2x^2-4$

288. If $y=a\sin(5x+c)$, then
a) $\frac{dy}{dx}=5y$ b) $\frac{dy}{dx}=-5y$ c) $\frac{d^2y}{dx^2}=-25y$ d) $\frac{d^2y}{dx^2}=25y$

289. An integrating factor of the differential equation $(1-x^2)\frac{dy}{dx}-xy=1$ is
a) $-x$ b) $-\frac{x}{(1-x^2)}$ c) $\sqrt{(1-x^2)}$ d) $\frac{1}{2}\log(1-x^2)$

290. The slope of a curve at any point is the reciprocal of twice the ordinate at the point and it passes through the point (4, 3). The equation of the curve is
a) $x^2=y+5$ b) $y^2=x-5$ c) $y^2=x+5$ d) $x^2=y-5$

291. The integrating factor of the differential equation $\cos x\left(\frac{dy}{dx}\right)+y\sin x=1$ is
a) $\sec x$ b) $\tan x$ c) $\sin x$ d) $\cot x$

292. The solution of the differential equation $\frac{dy}{dx}+\frac{2x}{1+x^2}.y=\frac{1}{(1+x^2)^2}$ is
a) $y(1-x^2)=\tan^{-1}x+c$ b) $y(1+x^2)=\tan^{-1}x+c$
c) $y(1+x^2)^2=\tan^{-1}x+c$ d) $y(1-x^2)^2=\tan^{-1}x+c$

293. The second order differential equation is
a) $y'^2+x=y^2$ b) $y'y''+y=\sin x$ c) $y'''+y''+y=0$ d) $y'=y$

294. The differential equation $\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{y}$ determines a family of circles with
a) Variable radii and a fixed centre t (0,1)
b) Variable radii and a fixed centre at (0,-1)
c) Fixed radius 1 and variable centres along the x-axis

d) Fixed radius 1 and variable centres along the y-axis

295. If $\frac{dy}{dx} + y = 2e^{2x}$, then y is equal to

a) $ce^{x} + \frac{2}{3}e^{2x}$
b) $(1-x)e^{-x} + \frac{2}{3}e^{2x} + c$
c) $ce^{-x} + \frac{2}{3}e^{2x}$
d) $e^{-x} + \frac{2}{3}e^{2x} + c$

296. If the function $y = \sin^{-1} x$, then $(1-x^2)\frac{d^2y}{dx^2}$ is equal to

a) $-x\frac{dy}{dx}$
b) 0
c) $x\frac{dy}{dx}$
d) $x\left(\frac{dy}{dx}\right)^2$

297. The solution of $dy = \cos x\,(2 - y \operatorname{cosec} x)dx$, where $y = \sqrt{2}$, when $x = \pi/4$ is

a) $y = \sin x + \frac{1}{2}\operatorname{cosec} x$
b) $y = \tan(x/2) + \cot(x/2)$
c) $y = \left(1/\sqrt{2}\right)\sec(x/2) + \sqrt{2}\cos(x/2)$
d) None of the above

298. The solution of the differential equation $(1+y^2)\tan^{-1} x\ dx + y(1+x^2)dy = 0$ is

a) $\log\left(\frac{\tan^{-1} x}{x}\right) + y(1+x^2) = c$
b) $\log(1+y^2) + (\tan^{-1} x)^2 = c$
c) $\log(1+x^2) + \log(\tan^{-1} y) + c$
d) $(\tan^{-1} x)(1+y^2) + c = 0$

299. The solution of the differential equation $\frac{dy}{dx} = y\tan x - 2\sin x$, is

a) $y\sin x = c + \sin 2x$
b) $y\ \cos x = c + \frac{1}{2}\sin 2x$
c) $y\cos x = c - \sin 2x$
d) $y\cos x = c + \frac{1}{2}\cos 2x$

300. If $y(t)$ is a solution of $(1+t)\frac{dy}{dt} - ty = 1$ and $y(0) = -1$ then $y(1)$ is equal to

a) $-\frac{1}{2}$
b) $e + \frac{1}{2}$
c) $e - \frac{1}{2}$
d) $\frac{1}{2}$

301. The differential equation of all straight lines passing through origin is

a) $y = \sqrt{x}\frac{dy}{dx}$
b) $\frac{dy}{dx} = y + x$
c) $\frac{dy}{dx} = y - x$
d) None of these

302. The solution of $\frac{dy}{dx} = \frac{x\log x^2 + x}{\sin y + y\cos y}$ is

a) $y\sin y = x^2\log x + c$
b) $y\sin y = x^2 + c$
c) $y\sin y = x^2 + \log x + c$
d) $y\sin y = x\log x + c$

303. If c is an arbitrary constant, then the general solution of the differential equation $y\,dx - x\,dy = xy\,dx$ is given by

a) $y = cxe^{-x}$
b) $y = cye^{-x}$
c) $y + e^{x} = cx$
d) $ye^{x} = cx$

304. Solution of $x\frac{dy}{dx} + y = x\,e^{x}$, is

a) $xy = e^{x}(x+1) + C$
b) $xy = e^{x}(x-1) + C$
c) $xy = e^{x}(1-x) + C$
d) $xy = e^{y}(y-1) + C$

305. The general solution of the differential equation $100\frac{d^2y}{dx^2} - 20\frac{dy}{dx} + y = 0$ is

a) $y = (c_1 + c_2x)e^{x}$
b) $y = (c_1 + c_2x)e^{-x}$
c) $y = (c_1 + c_2x)e^{\frac{x}{10}}$
d) $y = c_1e^{x} + c_2e^{-x}$

306. The equation of the curve whose subnormal is twice the abscissa, is

a) A circle
b) A parabola
c) An ellipse
d) A hyperbola

307. The solution of $2(y+3) - xy\frac{dy}{dx} = 0$ with $y = -2$, where $x = 1$, is

a) $y + 3 = x^2$
b) $x^2(y+3) = 1$
c) $x^4(y+3) = 1$
d) $x^2(y+3)^3 = e^{y+2}$

308. The solution of $\frac{dy}{dx} - y = 1, y(0) = 1$ is given by $y(x) =$

a) $-\exp(x)$
b) $-\exp(-x)$
c) -1
d) $2\exp\ (x) - 1$

309. The solution of the differential equation $2x\frac{dy}{dx} - y = 3$ represents

a) Straight lines
b) Circles
c) Parabola
d) Ellipse

: ANSWER KEY :

1)	c	2)	a	3)	a	4)	b	189)	d	190)	b	191)	a	192)	d
5)	c	6)	b	7)	b	8)	b	193)	c	194)	a	195)	a	196)	a
9)	b	10)	a	11)	d	12)	a	197)	a	198)	b	199)	d	200)	b
13)	a	14)	c	15)	c	16)	a	201)	d	202)	d	203)	b	204)	c
17)	b	18)	c	19)	a	20)	c	205)	c	206)	b	207)	b	208)	d
21)	b	22)	b	23)	d	24)	b	209)	c	210)	c	211)	a	212)	a
25)	c	26)	d	27)	a	28)	b	213)	a	214)	c	215)	c	216)	a
29)	c	30)	c	31)	a	32)	c	217)	b	218)	b	219)	d	220)	a
33)	b	34)	d	35)	a	36)	b	221)	d	222)	c	223)	c	224)	b
37)	b	38)	b	39)	c	40)	c	225)	b	226)	d	227)	a	228)	b
41)	a	42)	d	43)	a	44)	a	229)	b	230)	b	231)	c	232)	a
45)	a	46)	b	47)	c	48)	c	233)	a	234)	c	235)	c	236)	d
49)	a	50)	a	51)	b	52)	c	237)	a	238)	b	239)	a	240)	a
53)	b	54)	d	55)	c	56)	c	241)	b	242)	b	243)	a	244)	a
57)	b	58)	c	59)	a	60)	a	245)	d	246)	c	247)	a	248)	a
61)	a	62)	a	63)	a	64)	b	249)	a	250)	a	251)	b	252)	c
65)	b	66)	b	67)	a	68)	a	253)	a	254)	c	255)	a	256)	a
69)	b	70)	c	71)	d	72)	a	257)	a	258)	b	259)	d	260)	c
73)	d	74)	b	75)	a	76)	d	261)	b	262)	b	263)	d	264)	b
77)	a	78)	c	79)	d	80)	c	265)	c	266)	b	267)	b	268)	d
81)	c	82)	a	83)	b	84)	c	269)	a	270)	a	271)	a	272)	c
85)	b	86)	b	87)	a	88)	c	273)	a	274)	b	275)	b	276)	c
89)	b	90)	b	91)	a	92)	d	277)	b	278)	b	279)	a	280)	a
93)	c	94)	c	95)	a	96)	b	281)	a	282)	b	283)	d	284)	a
97)	a	98)	a	99)	a	100)	a	285)	b	286)	d	287)	c	288)	c
101)	a	102)	c	103)	b	104)	b	289)	c	290)	c	291)	a	292)	b
105)	a	106)	b	107)	c	108)	d	293)	b	294)	c	295)	c	296)	c
109)	c	110)	a	111)	b	112)	b	297)	a	298)	b	299)	d	300)	a
113)	a	114)	b	115)	b	116)	c	301)	d	302)	a	303)	d	304)	b
117)	b	118)	b	119)	b	120)	a	305)	c	306)	d	307)	d	308)	d
121)	c	122)	b	123)	a	124)	a	309)	c						
125)	a	126)	a	127)	b	128)	a								
129)	b	130)	a	131)	d	132)	c								
133)	c	134)	d	135)	b	136)	a								
137)	a	138)	d	139)	c	140)	d								
141)	a	142)	b	143)	d	144)	a								
145)	c	146)	b	147)	b	148)	b								
149)	a	150)	b	151)	b	152)	c								
153)	b	154)	b	155)	c	156)	a								
157)	b	158)	b	159)	c	160)	a								
161)	d	162)	c	163)	d	164)	a								
165)	a	166)	b	167)	d	168)	a								
169)	a	170)	a	171)	a	172)	c								
173)	d	174)	c	175)	b	176)	a								
177)	a	178)	c	179)	b	180)	c								
181)	b	182)	a	183)	b	184)	d								
185)	a	186)	c	187)	a	188)	d								

: HINTS AND SOLUTIONS :

1 **(c)**

Equation of family of ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\Rightarrow \quad \frac{2x}{a^2}+\frac{2y}{b^2}\cdot\frac{dy}{dx}=0$

$\Rightarrow \quad \frac{x}{a^2}+\frac{y}{b^2}\cdot\frac{dy}{dx}=0$

...(i)

$\Rightarrow \quad \frac{1}{a^2}+\frac{y}{b^2}\cdot\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\frac{1}{b^2}=0$

$\Rightarrow \quad \frac{b^2}{a^2}+y\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)^2=0$

$\Rightarrow \quad -\frac{y}{x}\cdot\frac{dy}{dx}+y\,\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$

[from Eq. (i), $\frac{b^2}{a^2}=-\frac{y}{x}\cdot\frac{dy}{dx}$]

$\Rightarrow \quad xy\,\frac{d^2y}{dx^2}+\frac{dy}{dx}\left(x\,\frac{dy}{dx}-y\right)=0$

2 **(a)**

The given equation can be rewritten as

$\rho\cdot\frac{d^2y}{dx^2}=\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}$

On squaring both sides, we get

$\left(\rho\cdot\frac{d^2y}{dx^2}\right)=\left[1+\left(\frac{dy}{dx}\right)^2\right]^3$

$\Rightarrow$ order = 2, degree = 2.

3 **(a)**

Since,

$y=e^{2x}(a\cos x+b\sin x)$

...(i)

$\Rightarrow \quad y_1=e^{2x}(-a\sin x+b\cos x)+(a\cos x+b\sin x)2e^{2x}$

$\Rightarrow \quad y_1=e^{2x}(-a\sin x+b\cos x)+2y$

...(ii)

$\Rightarrow \quad y_2=e^{2x}(-a\cos x-b\sin x)+(-a\sin x+b\cos x)e^{2x}2+2y_1$

$=-y+2e^{2x}(-a\sin x+b\cos x)+2y_1$ (using eq.(ii))

$\Rightarrow \quad y_2=-y+2(y_1-2y)+2y_1$

(using eq.(ii))

$\Rightarrow \quad y_2=-y+4y_1-4y$

$\Rightarrow \quad y_2-4y_1+5y=0$

4 **(b)**

We have, $\frac{dy}{dx}=\frac{ax+g}{by+f}$

$\Rightarrow \quad (by+f)dy=(ax+g)dx$

On integrating, we get

$\frac{by^2}{2}+fy=\frac{ax^2}{2}+gx+c$

$\Rightarrow \quad ax^2-by^2+2gx-2fy+c=0$

This represents a circle, if $a=-b$

5 **(c)**

Given, $\frac{dy}{dx}+\frac{y}{x}=x^2$

$\therefore \quad \text{IF}=e^{\int\frac{1}{x}dx}=e^{\log x}=x$

$\therefore$ Complete solution is

$y.x=\int x.x^2\,dx+c$

$\Rightarrow \quad y.x=\frac{1}{4}x^4+c$

$\Rightarrow \quad y=\frac{1}{4}x^3+cx^{-1}$

6 **(b)**

Given, $y=ax\cos\left(\frac{1}{x}+b\right)$

$\Rightarrow \quad y_1=-ax\sin\left(\frac{1}{x}+b\right)\times\left(-\frac{1}{x^2}\right)+a\cos(\frac{1}{x}+b)$

$\Rightarrow \quad y_1=\frac{a}{x}\sin\left(\frac{1}{x}+b\right)+a\cos\left(\frac{1}{x}+b\right)$

$\Rightarrow \quad xy_1=a\sin\left(\frac{1}{x}+b\right)+y$

$\Rightarrow \quad y_1+xy_2=a\cos\left(\frac{1}{x}+b\right)\left(-\frac{1}{x^2}\right)+y_1$

$\Rightarrow \quad x^3y_2=-a\cos\left(\frac{1}{x}+b\right)$

$\Rightarrow \quad x^4y_2+y=0$

7 **(b)**

Given, $\frac{y\,dx+x\,dy}{x^2y^2}=-\frac{1}{y}\,dy$

$\Rightarrow \quad d\left(-\frac{1}{xy}\right)=-\frac{1}{y}\,dy$

$\Rightarrow \quad -\frac{1}{xy}=-\log y+c$

[integrating]

$\Rightarrow \quad -\frac{1}{xy}+\log y=c$

8 **(b)**

The given differential equation is

$2\left(\frac{d^2y}{dx^2}\right)+3\left(\frac{dy}{dx}\right)^2+4y^3=x$

Here, highest order is 2 and degree is 1.

9 **(b)**

Given, $\cot y\,dx=x\,dy$

$\Rightarrow\frac{dx}{x}=\frac{dy}{\cot y}\Rightarrow\frac{dx}{x}=\tan y\,dy$

On integrating both sides, we get

$\int \frac{1}{x} dx = \int \tan\cdot y \, dy$

$\Rightarrow \log x = \log \sec y + \log c$

$\Rightarrow \log x = \log c \sec y$

$\Rightarrow x = c \sec y$

10 **(a)**

Given, $\frac{dy}{dx} = -\frac{3xy+y^2}{x^2+xy}$

Put $y = vx$

$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$

$\Rightarrow v + x \frac{dv}{dx} = -\frac{3v+v^2}{1+v}$

$\Rightarrow x \frac{dv}{dx} = \frac{-2v(v+2)}{v+1}$

$\Rightarrow \frac{1}{x} dx = -\frac{(v+1)}{2v(v+2)} dv$

$\Rightarrow -\frac{2}{x} dx = -\left[\frac{1}{2(v+2)} + \frac{1}{2v}\right] dv$

$\Rightarrow -2 \log_e x = \frac{1}{2}\log(v+2) + \frac{1}{2}\log v - \log c$

$\Rightarrow v(v+2)x^4 = c^2$

$\Rightarrow \frac{y}{x}\left(\frac{y}{x} + 2\right) x^4 = c^2$

$\Rightarrow (y^2 + 2xy)x^2 = c^3$

11 **(d)**

Given equation is $\sqrt{\frac{dy}{dx}} - 4\frac{dy}{dx} - 7x = 0$

$\Rightarrow \frac{dy}{dx} = 16\left(\frac{dy}{dx}\right)^2 + 49x^2 + 56x\frac{dy}{dx}$

Obviously, it is first order and second degree differential equation.

12 **(a)**

Since, $\cos x \; dy = y \sin x \; dx - y^2 \, dx$

$\Rightarrow \frac{1}{y^2} \frac{dy}{dx} - \frac{1}{y}\tan x = -\sec x$

Put , $-\frac{1}{y} = z \Rightarrow \frac{1}{y^2} \frac{dy}{dx} = \frac{dz}{dx}$

$\Rightarrow \frac{dz}{dx} + (\tan x)z = -\sec x$

This is a linear differential equation.

Therefore,

$\text{IF} = e^{\int \tan x \; dx} = e^{\int \log \sec x} = \sec x$

Hence, the solution is

$z.(\sec x) = \int -\sec x . \sec x \; dx + c_1$

$\Rightarrow -\frac{1}{y} \sec x = -\tan x + c_1$

$\Rightarrow \sec x = y(\tan x + c)$

13 **(a)**

Given, $x = A \cos 4t + B \sin 4t$

...(i)

$\Rightarrow \frac{dx}{dt} = -4A \sin 4t + 4B \cos 4t$

$\Rightarrow \frac{d^2x}{dt^2} - 16A \cos 4t - 16\,B \sin 4t$

$\Rightarrow \frac{d^2x}{dt^2} = -16x$

[from Eq. (i)]

14 **(c)**

Given, $\frac{dy}{dx} + y = 2e^{2x}$

$\therefore \quad IF = e^{\int 1\,dx} = e^x$

$\therefore$ Required solution is

$ye^x = 2\int e^{2x} \; e^x \, dx = \frac{2}{3}\, e^{3x} + c$

$\Rightarrow y = \frac{2}{3}\, e^{2x} + ce^{-x}$

15 **(c)**

$\frac{dt}{dx} - t\frac{g'(x)}{g(x)} = -\frac{t^2}{g(x)}$

$\Rightarrow -\frac{1}{t^2}\frac{dt}{dx} + \frac{1}{t}\frac{g'(x)}{g(x)} = \frac{1}{g(x)}$ ....(i)

Let $z = \frac{1}{t} \Rightarrow -\frac{1}{t^2}\frac{dt}{dx} = \frac{dz}{dx}$

$\therefore$ From Eq. (i)

$\frac{dz}{dx} + \frac{g'(x)}{g(x)} z = \frac{1}{g(x)}$

On comparing with $\frac{dz}{dx} + Pz = Q$, we get

$P = \frac{g'(x)}{g(x)}, Q = \frac{1}{g(x)}$

$\therefore IF = e^{\int \frac{g'(x)}{g(x)} dx}$

$= e^{\log[g(x)]} = g(x)$

Thus, complete solution is

$z \cdot g(x) = \int g(x) \cdot \frac{1}{g(x)} dx + c$

$\Rightarrow \frac{1}{t}\, g(x) = x + c \Rightarrow \frac{g(x)}{x+c} = t$

16 **(a)**

The equation of the family of circles of radius a is $(x-h)^2 + (y-k^2) = a^2$, which is a two parameter family of curves. So, its differential equation is of order two

17 **(b)**

Given, $\frac{dy}{dx} = \frac{x^3+y^3}{xy^2}$

Put $y = vx$

$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$

$\Rightarrow x\frac{dv}{dx} + v = \frac{1+v^3}{v^2}$

$\Rightarrow v^2 dv = \frac{dx}{x}$

$\Rightarrow \frac{v^3}{3} = \log x + \log c$

$\Rightarrow \frac{1}{3}\left(\frac{y}{x}\right)^3 = \log x + \log c$

$\Rightarrow y^3 = 3x^2 \log cx$

18 **(c)**

Given, $\frac{dy}{dx}+\frac{y}{x}=\cos x+\frac{\sin x}{x}$

$\therefore$ IF $= e^{\int \frac{1}{x}dx} = e^{\log x} = x$

$\therefore$ Complete solution is

$$xy=\int(x\cos x+\sin x)dx$$

$\Rightarrow \quad xy = x\sin x + c$

At $y=1, x=\frac{\pi}{2}, c=0$

$\therefore \quad y=\sin x$

19 **(a)**

Given, $xy = ae^x + be^{-x}$

$\Rightarrow \quad x\frac{dy}{dx}+y = ae^x - be^{-x}$

$\Rightarrow \quad x\frac{d^2y}{dx^2}+\frac{dy}{dx}+\frac{dy}{dx} = ae^x + be^{-x}$

$\Rightarrow \quad x\frac{d^2y}{dx^2}+2\frac{dy}{dx}-xy=0$ [from eq. (I)]

20 **(c)**

The given equation can be written as

$(D^2+2D+1)y=2e^{3x}$, where $\frac{d}{dx}=D$

Here, $F(D)=D^2+2D+1$ and $Q=2e^{3x}$

The auxiliary equation is

$$m^2+2m+1=0$$

$\Rightarrow \quad (m+1)^2=0$

$\Rightarrow \quad m=-1,-1$

$\therefore$ The CF $=(c_1+c_2x)e^{-1}$

and $PI=\frac{1}{F(D)}2e^{3x}=2\frac{1}{D^2+2D+1}e^{3x}$

$= 2\frac{e^{3x}}{9+6+1}=\frac{e^{3x}}{8}$

$\therefore$ The complete solution is

$$y=(c_1+c_2x)e^{-x}+\frac{e^{3x}}{8}$$

21 **(b)**

We have,

$(2y-1)dx=(2x+3)dy$

$\Rightarrow \frac{1}{2x+3}dx=\frac{1}{2y-1}dy$

$\Rightarrow \int\frac{2}{2x+3}dx=\int\frac{2}{2y-1}dy$

$\Rightarrow \log(2x+3)=\log(2y-1)+\log C \Rightarrow \frac{2x+3}{2y-1} = C$

22 **(b)**

We have,

$\frac{dy}{dx}=y(xy-1)$

$\Rightarrow dy = xy^2dx - y\,dx$

$\Rightarrow y\,dx+dy=xy^2\,dx \Rightarrow \frac{y\,dx+dy}{y^2}=x\,dx$

$\Rightarrow \frac{y\,e^{-x}dx+e^{-x}dy}{y^2}=x\,e^{-x}dx \Rightarrow -d\left(\frac{e^{-x}}{y}\right) = xe^{-x}dx$

On integrating, we get

$-\frac{e^{-x}}{y}=-xe^{-x}-e^{-x}+C \Rightarrow \frac{1}{y}=x+1+Ce^{-x}$ ...(i)

It passes through (0, 1). Therefore, $1=1+C \Rightarrow C=0$

Putting $C=0$ in (i), we get $\frac{1}{y}=x+1 \Rightarrow y(x+1)=1$

ALTER We have,

$\frac{dy}{dx}=y(xy-1)$

$\Rightarrow \frac{dy}{dx}+y=xy^2$

$\Rightarrow \frac{1}{y^2}\frac{dy}{dx}+\frac{1}{y}=x$

$\Rightarrow -\frac{dv}{dx}+v=x$, where $v=\frac{1}{y}$

$\Rightarrow \frac{dv}{dx}-v=-x$ ....(ii)

$\text{I.F.}=e^{-\int 1.dx}=e^{-x}$

Multiplying (i) by e^{-x} and integrating, we get

$v\,e^{-x}=xe^{-x}+e^{-x}+C \Rightarrow \frac{1}{y}=x+1+C\,e^{x}$

It passes through (0, 1)

Hence, the equation of the curve is

$\frac{1}{y}=(x+1)\Rightarrow(x+1)y=1$

23 **(d)**

Given differential equation is

$$x\,dy=y\,dx$$

$\Rightarrow \quad \frac{dy}{y}=\frac{dx}{x}\Rightarrow\int\frac{dy}{y}=\int\frac{dx}{x}$

$\Rightarrow \quad \log_e y=\log_e x+\log_e c$

$\Rightarrow \quad y=cx$

Which is a straight line.

24 **(b)**

Given, $\frac{dy}{dx}=(1+x)(1+y)$

$\Rightarrow \quad \frac{1}{1+y}dy=(1+x)dx$

$\Rightarrow \quad \log(1+y)=x+\frac{x^2}{2}+c$ [integrating]

At $y(-1)=0$

$\Rightarrow \quad c=\frac{1}{2}$

$\therefore \quad \log(1+y)=\frac{x^2+2x+1}{2}$

$\Rightarrow \quad y=e^{\frac{(1+x)^2}{2}}-1$

25 **(c)**

Given, $\frac{dy}{dx} + y.\frac{1}{x}\log x = e^x\, x^{-(1/2)\log x}$

$\therefore \quad IF = e^{\int \frac{1}{x}\log x\, dx} = e^{\int \frac{(\log x)^2}{2}} = (\sqrt{e})^{(\log x)^2}$

26 **(d)**

Given differential equation is

$$ydy = (c-x)dx$$

$$\Rightarrow \quad \frac{y^2}{2} = cx - \frac{x^2}{2} + d$$

$$\Rightarrow \quad y^2 + x^2 - 2cx - 2d = 0$$

Hence, it represents a family of circles whose centres are on the x-axis.

27 **(a)**

Given, $\frac{dy}{dx} = \frac{x-y}{x+y}$

This is a homogeneous equation

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Given equation becomes

$$v + x\frac{dv}{dx} = \frac{x - vx}{x + vx}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1-v}{1+v} - v$$

$$\Rightarrow \frac{1+v}{2-(1+v)^2}dv = \frac{dx}{x}$$

On integrating both sides, we get

$$\int \frac{1+v}{2-(1+v)^2}dv = \int \frac{dx}{x}$$

Put $(1+v)^2 = t \Rightarrow 2(1+v)dv = dt$

$$\Rightarrow \frac{1}{2}\int \frac{dt}{2-t} = \int \frac{dx}{x}$$

$$\Rightarrow -\frac{1}{2}\log(2-t) = \log x + \log c$$

$$\Rightarrow -\frac{1}{2}\log[2-(1+v)^2] = \log xc$$

$$\Rightarrow -\frac{1}{2}\log[-v^2 - 2v + 1] = \log xc$$

$$\Rightarrow \log\frac{1}{\sqrt{1-2v-v^2}} = \log xc$$

$$\Rightarrow x^2c^2(1-2v-v^2) = 1$$

$$\Rightarrow x^2c^2\left(1 - \frac{2y}{x} - \frac{y^2}{x^2}\right) = 1 \quad \left(\because v = \frac{y}{x}\right)$$

$$\Rightarrow \frac{x^2c^2(x^2 - 2yx - y^2)}{x^2} = 1$$

$$\Rightarrow y^2 + 2xy - x^2 = c$$

28 **(b)**

Given equation is

$y = c_1e^{2x+c_2} + c_3e^x + c_4\sin(x+c_5)$

$= c_1e^{c_2}e^{2x} + c_3e^x + c_4(\sin x\cos c_5 + \cos x\sin c_5)$

$= Ae^{2x} + c_3e^x + B\sin x + D\cos x$

Here, $A = c_1e^{c_2}, B = c_4\cos c_5, D = c_4\sin c_5$

29 **(c)**

We have,

$$\frac{x}{x^2+y^2}dy = \left(\frac{y}{x^2+y^2} - 1\right)dx$$

$$\Rightarrow \frac{x\,dy - y\,dx}{x^2+y^2} = -dx$$

$$\Rightarrow d\left\{\tan^{-1}\left(\frac{y}{x}\right)\right\} = -dx$$

$$\Rightarrow \tan^{-1}\frac{y}{x} = -x + C \Rightarrow y = x\tan(C - x)$$

30 **(c)**

Given differential equation can be rewritten as

$$\frac{dy}{dx} = \frac{x+y-1}{x+y+1}$$

Put $\quad x + y = t$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dt}{dx} - 1$$

$$\therefore \quad \frac{dt}{dx} - 1 = \frac{t-1}{t+1}$$

$$\Rightarrow \quad \frac{1}{2}(t + \log t) = x + \frac{c}{2}$$

$$\Rightarrow \quad \frac{1}{2}(t + \log t) = x - y + c$$

$$\Rightarrow \quad \log(x+y) = x - y + c$$

31 **(a)**

Given, $\quad y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$

$$\Rightarrow \quad \frac{dy}{dx}(a+x) = y - ay^2$$

$$\Rightarrow \quad \int\left(\frac{1}{y} + \frac{a}{1-ay}\right)dy = \int\frac{dx}{a+x}$$

$\Rightarrow \quad \log y - \log(1-ay) = \log(a+x) + \log c$

$$\Rightarrow \quad \log y = \log(1-ay)(a+x)c$$

$$\Rightarrow \quad y = c(1-ay)(a+x)$$

32 **(c)**

Given differential equation can be rewritten as

$$\left(1 + 3\frac{dy}{dx}\right)^2 = 64\left(\frac{d^3y}{dx^3}\right)^3$$

33 **(b)**

$$\frac{dy}{dx} = y + 2x$$

$$\Rightarrow \quad \frac{dy}{dx} - y = 2x$$

$$\text{IF} = e^{\int -1\,dx} = e^{-x}$$

$\therefore$ Solution of the differential equation is

$$y.e^{-x} = 2\int xe^{-x}\,dx$$

$$= 2(-xe^{-x} - e^{-x}) + c$$

$$\Rightarrow \quad y = 2e^x(-xe^{-x} - e^{-x}) + ce^x$$

$$\Rightarrow \quad y = -2x - 2 + ce^x$$

For $c = 2$

We get $\quad y = 2(e^x - x - 1)$

34 **(d)**

We have,

$$y\sqrt{1+\left(\frac{dy}{dx}\right)^2}=c$$

$$\Rightarrow y^2\left\{1+\left(\frac{dy}{dx}\right)^2\right\}=c^2 \Rightarrow y^2\left(\frac{dy}{dx}\right)^2+y^2=c^2$$

Clearly, it is a differential equation of degree 2

35 **(a)**

Given, $\frac{dy}{dx}+1=\text{cosec}\,(x+y)$

Put $x+y=t$

$\Rightarrow \quad 1+\frac{dy}{dx}=\frac{dt}{dx}$

$\therefore \quad \frac{dt}{\text{cosec}\,t}=dx$

$\Rightarrow \quad \int \sin t\,dt=\int dx$

$\Rightarrow \quad -\cos t=x-c$

$\Rightarrow \quad \cos(x+y)+x=c$

36 **(b)**

Given differential equation can be rewritten as

$$e^{2y}dy=(e^{3x}+x^2)dx$$

On integrating, we get

$\Rightarrow \quad \frac{e^{2y}}{2}=\frac{e^{3x}}{3}+\frac{x^3}{3}+c$

37 **(b)**

We have,

$(x-h)^2+(y-k)^2=a^2$ ...(i)

Differentiating w.r.t. x, we get

$$2(x-h)+2(y-k)\frac{dy}{dx}=0$$

$\Rightarrow (x-h)+(y-k)\frac{dy}{dx}=0$ ...(ii)

Differentiating w.r.t. x, we get

$1+\left(\frac{dy}{dx}\right)^2+(y-k)\frac{d^2y}{dx^2}=0$ ...(iii)

From (iii), we get

$y-k=-\frac{1+p^2}{q}$, where $p=\frac{dy}{dx}, q=\frac{d^2y}{dx^2}$

Putting the value of $y-k$ in (ii), we get

$$x-h=\frac{(1+p^2)p}{q}$$

Substituting the values of $x-h$ and $y-k$ in (i), we get

$$\left(\frac{1+p^2}{q}\right)^2(1+p^2)=a^2$$

$$\Rightarrow \left\{1+\left(\frac{dy}{dx}\right)^2\right\}^3=a^2\left(\frac{d^2y}{dx^2}\right)^2,$$

which is the required differential equation

38 **(b)**

The general equation of parabola whose axis is x-axis,is

$$y^2=4a(x-h)$$

$\Rightarrow \quad 2y\frac{dy}{dx}=4a$

$\Rightarrow \quad y\frac{dy}{dx}=2a$

$\Rightarrow \quad \left(\frac{dy}{dx}\right)^2+y\frac{d^2y}{dx^2}=0$

$\therefore$ Degree=1, order=2

39 **(c)**

$\frac{dy}{dx}=\frac{x+y+1}{x+y-1}$

Put $x+y=t$

$\Rightarrow \quad 1+\frac{dy}{dx}=\frac{dt}{dx}$

$\Rightarrow \quad \frac{dy}{dx}=\frac{dt}{dx}-1$

$\therefore \quad \frac{dt}{dx}-1=\frac{t+1}{t-1}$

$\Rightarrow \quad \frac{dt}{dx}=\frac{t+1+t-1}{t-1}$

$\Rightarrow \quad \frac{dt}{dx}=\frac{2t}{t-1}$

$\Rightarrow \quad \left(\frac{t-1}{2t}\right)dt=dx$

$\Rightarrow \quad \left(\frac{1}{2}-\frac{1}{2t}\right)dt=dx$

On integrating, we get

$$\frac{1}{2}t-\frac{1}{2}\log t=x+c_1$$

$\Rightarrow \quad t-\log t=2x+2c_1$

$\Rightarrow \quad x+y-\log(x+y)=2x+2c_1$

$\Rightarrow \quad y=x+\log(x+y)+c$

40 **(c)**

$\frac{dy}{dx}=\frac{x^2+y^2}{x^2-y^2}$, where, $\frac{dy}{dx}$ is the slope of the curve.

$$\therefore \left(\frac{dy}{dx}\right)_{(1,0)}=\frac{1+0}{1-0}=1$$

41 **(a)**

Given, $\frac{dy}{dx}=3x^3$

$\Rightarrow \quad dy=3x^3\ dx$

On integrating, we get

$$y=\frac{3x^3}{3}+c$$

$\Rightarrow \qquad y = x^3 + c$

It passes through (-1, 1).

$\therefore \qquad 1 = (-1)^3 + c$

$\Rightarrow \qquad c = 2$

$\therefore \qquad y = x^3 + 2$

42 **(d)**

Given, $(x+y)^2 \frac{dy}{dx} = a^2$

Put $x + y = v$

$\Rightarrow \qquad \frac{dy}{dx} = \frac{dv}{dx} - 1$

$\therefore \qquad v^2\left(\frac{dv}{dx} - 1\right) = a^2$

$\Rightarrow \qquad \frac{dv}{dx} = \frac{a^2+v^2}{v^2}$

$\Rightarrow \qquad \left(1 - \frac{a^2}{a^2+v^2}\right) dv = dx$

$\Rightarrow \qquad v - a\tan^{-1}\left(\frac{v}{a}\right) = x + c$

[integrating]

$\Rightarrow \qquad (x+y) - a\tan^{-1}\left(\frac{x+y}{a}\right) = x + c$

$\Rightarrow \qquad y = a\tan^{-1}\left(\frac{x+y}{a}\right) + c$

43 **(a)**

We have, $y = cx - c^2$

On differentiating w.r.t $'x'$, we get

$y' = c$

On putting this value in Eq. (i), we get

$y = x(y') - (y')^2$

$\Rightarrow \qquad (y')^2 - xy' + y = 0$

44 **(a)**

We have,

$\frac{dv}{dt} + \frac{k}{m}v = -g$

$\Rightarrow \frac{dv}{dt} = -\frac{k}{m}\left(v + \frac{mg}{k}\right)$

$\Rightarrow \frac{dv}{v + mg/k} = -\frac{k}{m}dt$

$\Rightarrow \log\left(v + \frac{mg}{k}\right) = -\frac{k}{m}t + \log C$

$\Rightarrow v + \frac{mg}{k} = Ce^{-k/m\,t} \Rightarrow v = Ce^{-k/m\,t} - \frac{mg}{k}$

45 **(a)**

Given equation is $\frac{dy}{dx} + y\tan x = \sec x$

Here, $P = \tan x$ and $Q = \sec x$

$\therefore \text{IF} = e^{\int p\,dx} = e^{\int \tan x\,dx}$

$= e^{\log\sec x} = \sec x$

Hence, required solution is

$y\sec x = \int \sec^2 x\,dx + c$

$\Rightarrow y\sec x = \tan x + c$

46 **(b)**

Given, $\frac{dy}{dx} = \frac{y + x\tan\frac{y}{x}}{x}$

Put $y = vx$

$\Rightarrow \qquad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore \qquad v + x\frac{dv}{dx} = \frac{vx + x\tan\left(\frac{vx}{x}\right)}{x}$

$\Rightarrow \qquad x\frac{dv}{dx} = v + \tan v - v$

$\Rightarrow \qquad \int \cot v\,dv = \int \frac{dx}{x}$

$\Rightarrow \qquad \log\sin v = \log x + \log c \Rightarrow \sin\frac{y}{x} = xc$

47 **(c)**

We have,

$y^2 = 4a(x+a)$ ...(i)

$\Rightarrow 2y\frac{dy}{dx} = 4a \Rightarrow a = \frac{1}{2}y\frac{dy}{dx}$

Substituting the value of a in (i), we get

$y^2 = 2y\frac{dy}{dx}\left(x + \frac{1}{2}y\frac{dy}{dx}\right) \Rightarrow y^2$

$= y\frac{dy}{dx}\left(2x + y\frac{dy}{dx}\right)$

$\Rightarrow y^2\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx} - y^2 = 0$

48 **(c)**

Given, $\frac{dy}{dx} = e^x e^y$

$\Rightarrow \qquad \int e^{-y}\,dy = \int e^x\,dx$

$\Rightarrow \qquad -e^{-y} = e^x - c$

$\Rightarrow \qquad e^x + e^{-y} = c$

49 **(a)**

The given equation is

$t = 1 + (ty)\left(\frac{dy}{dt}\right) + \frac{(ty)^2}{2!}\left(\frac{dy}{dt}\right)^2 + \ldots\infty$

$\Rightarrow t = e^{ty\left(\frac{dy}{dt}\right)}$

$\Rightarrow \log t = ty\frac{dy}{dt}$

$\Rightarrow y\,dy = \frac{\log t}{t}dt$

On integrating both sides, we get

$\frac{y^2}{2} = \frac{(\log t)^2}{2} + k$

$\Rightarrow y = \pm\sqrt{(\log t)^2 + 2k}$

$\Rightarrow y = \pm\sqrt{(\log t)^2 + c}$

50 **(a)**

Given, $\frac{x\,dy - y\,dx}{x^2} + e^x dx = 0$

$\Rightarrow \qquad d\left(\frac{y}{x}\right) + d(e^x) = 0$

$\Rightarrow \quad \frac{y}{x} + e^x = c$

[integrating]

51 **(b)**

We have,

$\frac{dy}{dx} + 2y \tan x = \sin x \quad \ldots$ (i)

It is a linear differential equation with integrating factor

$\text{I.F.} = e^{\int 2\tan x\, dx} = e^{2\log\sec x} = \sec^2 x$

Multiplying (i) by $\sec^2 x$ and integrating, we get

$y\sec^2 x = \int \sin x \sec^2 x \; dx$

$\Rightarrow y\sec^2 x = \int \sec x \tan x \; dx \Rightarrow y\sec^2 x$

$= \sec x + C$

53 **(b)**

Given, $\frac{dy}{dx} = e^{-y}(e^x + x^2)$

$\Rightarrow \quad \int e^y\, dy = \int e^x\, dx + \int x^2\, dx$

$\Rightarrow \quad e^y = e^x + \frac{x^3}{3} + c$

$\Rightarrow \quad e^y - e^x = \frac{x^3}{3} + c$

54 **(d)**

Given, $\frac{d^2y}{dx^2} = \frac{\log x}{x^2}$

$\Rightarrow \quad \frac{dy}{dx} = \frac{-(\log x + 1)}{x} + c$

[integrating]

Since, $\left(\frac{dy}{dx}\right)_{(1,0)} = -1$

$\Rightarrow \quad \frac{-1}{1} + c = -1 \Rightarrow c = 0$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{(\log x + 1)}{x} + 0$

$\Rightarrow \quad y = -\frac{1}{2}(\log x)^2 - \log x + c_1$

[integrating]

At $x = 1, y = 0 \Rightarrow c_1 = 0$

$\therefore \quad y = -\frac{1}{2}(\log x)^2 - \log x$

55 **(c)**

Given, $\frac{dy}{dx} + y = x^2$

$\therefore \quad \text{IF} = e^{\int 1\; dx} = e^x$

$\frac{dy}{dx} + P(x)y = Q(x)$

$\therefore \quad \text{IF} = e^{\int P(x)dx}$

$\therefore$ Both statements A and B are true and R $\Rightarrow$ A

56 **(c)**

The equation of the given family of ellipses is

$\frac{x^2}{4a^2} + \frac{y^2}{a^2} = 1$ or, $x^2 + 4y^2 = 4a^2 \quad \ldots$(i)

Differentiating with respect to x, we get

$2x + 8y\,\frac{dy}{dx} = 0 \Rightarrow x + 4yy' = 0$

This is the required differential equation

57 **(b)**

Given that centre of circle is (1, 2).

Let radius of circle is a.

$\therefore \quad (x-1)^2 + (y-2)^2 = a^2$

$\Rightarrow \quad 2(x-1) + 2(y-2)\frac{dy}{dx} = 0$

$\Rightarrow \quad (x-1) + (y-2)\frac{dy}{dx} = 0$

58 **(c)**

Given, $\quad y^2 = 4ax + 4a^2 \quad \ldots$(i)

$\Rightarrow \quad 2yy' = 4a$

On putting the value of $4a$ in eq(i), we get

$y^2 = 2yy'x + 4.\frac{y^2y'^2}{4}$

$\Rightarrow \quad y = 2y'x + yy'^2$

59 **(a)**

Given, $\quad y = Ae^x + Be^{2x} + Ce^{3x}$

...(i)

$\Rightarrow \quad y'Ae^x + 2Be^{2x} + 3Ce^{3x}$

From Eq. (i),

$Ae^x = y - Be^{2x} - Ce^{3x}$

$\Rightarrow \quad y' = y + Be^{2x} - Ce^{3x}$

$\therefore \quad y'' = y' + Be^{2x} + 6Ce^{3x}$

...(ii)

From Eq. (ii),

$Be^{2x} = y' - y - 2Ce^{3x}$

$\therefore \quad y'' = y' + 2y' - 2y - 4Ce^{3x} + 6Ce^{3x}$

$\Rightarrow \quad y'' = 3y' - 2y + 2Ce^{3x}$

...(iii)

Again, differentiating w. r. t. x, we get

$y''' = 3y'' - 2y' + 6Ce^{3x}$

From Eq. (iii),

$2Ce^{3x} = y'' - 3y' + 2y$

$\therefore \quad y''' = 3y'' - 2y' + 3(y'' - 3y' + 2y)$

$\Rightarrow \quad y''' - 6y'' + 11y' - 6y = 0$

60 **(a)**

The equation of a member of the family of parabolas having axis parallel to y-axis is

$y = Ax^2 + Bx + C$

$\Rightarrow \quad \frac{dy}{dx} = 2Ax + B$

$\Rightarrow \quad \frac{d^2y}{dx^2} = 2A \Rightarrow \frac{d^3y}{dx^3} = 0$

61 **(a)**

Let $\quad x^2 + y^2 - 2ky = 0$

$\Rightarrow \quad 2x + 2y\,\frac{dy}{dx} - 2k\,\frac{dy}{dx} = 0$

$\Rightarrow \quad k = \frac{k}{\left(\frac{dy}{dx}\right)} + y$

From Eq. (i),

$x^2 + y^2 - 2\left(\frac{x}{(dy/dx)} + y\right)y = 0$

$\Rightarrow \quad (x^2 - y^2)\frac{dy}{dx} - 2xy = 0$

62 **(a)**

Given, $\frac{dy}{dx} = \tan\theta = 2x + 3y$

Put $2x + 3y = z \Rightarrow 2 + 3 \frac{dy}{dx} = \frac{dz}{dx}$

$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{dz}{dx} - 2\right)\frac{1}{3}$

$\therefore \quad \frac{dz}{dx} - 2 = 3z \Rightarrow \frac{dz}{3z+2} = dx$

On integrating, we get

$\frac{\log(3z+2)}{3} = x + C$

$\Rightarrow \quad \frac{\log(6x+9y+2)}{3} = x + C$

Since, it passes through (1,2).

$\therefore \quad \frac{\log(6+18+2)}{3} = 1 + C$

$\Rightarrow \quad C = \frac{\log 26}{3} - 1$

$\therefore \quad \frac{\log(6x+9y+2)}{3} = x + \frac{\log 26}{3} - 1$

$\Rightarrow \quad \log\left(\frac{6x+9y+2}{26}\right) = 3(x-1)$

$\Rightarrow \quad 6x + 9y + 2 = 26e^{3(x-1)}$

63 **(a)**

Given, $\frac{dy}{dx} - \frac{\tan y}{x} = \frac{\tan y \ \sin y}{x^2}$

$\Rightarrow \quad \cot y \ cosec\, y \frac{dy}{dx} - \frac{\text{cosec}\, y}{x} = \frac{1}{x^2}$

Put $\quad -\text{cosec}\, y = t$

$\Rightarrow \quad \cot y \ \text{cosec}\, y \frac{dy}{dx} = \frac{dt}{dx}$

$\therefore \quad \frac{dt}{dx} + \frac{t}{x} = \frac{1}{x^2}$

$\therefore \quad \text{IF} = e^{\int P\, dx} = e^{\int \frac{1}{x} dx} = x$

$\therefore$ Solution is $tx = \int x . \frac{1}{x^2} dx - c$

$\Rightarrow \quad -\text{cosec}\, y . x = \log x - c$

$\Rightarrow \quad \frac{x}{\sin y} + \log x = c$

64 **(b)**

Given differential equation is

$$5\left(\frac{d^2y}{dx^2}\right)^5 + 4\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{dy}{dx}\right)^3 + 2y + x^3 = 0$$

Here, highest order derivative is 3 whose degree is 2.

65 **(b)**

Given differential equation can be rewritten as

$$y - x\frac{dy}{dx} = \sqrt{a^2\left(\frac{dy}{dx}\right)^2 + b^2}$$

$\Rightarrow \quad y^2 + \left(x\frac{dy}{dx}\right)^2 - 2yx\frac{dy}{dx} = a^2\left(\frac{dy}{dx}\right)^2 + b^2$

Here, order is 1 and degree is 2.

66 **(b)**

The displacement x for all SHM is given by

$x = a\cos(nt + b)$

$\Rightarrow \frac{dx}{dt} = -na\sin(nt + b)$

$\Rightarrow \frac{d^2x}{dt^2} = -n^2a\cos(nt + b)$

$\Rightarrow \frac{d^2x}{dt^2} = -n^2x \Rightarrow \frac{d^2x}{dt^2} = +n^2x = 0$

67 **(a)**

We have,

$\frac{dx}{x} + \frac{dy}{y} = 0 \Rightarrow \log x + \log y = \log c \Rightarrow xy = c$

68 **(a)**

The general equation of all non-vertical lines in a plane is $ax + by = 1$, where $b \neq 0$

$\therefore \quad a + b\frac{dy}{dx} = 0$

$\Rightarrow \quad b\frac{d^2y}{dx^2} = 0$

$\Rightarrow \quad \frac{d^2y}{dx^2} = 0$

69 **(b)**

Given, $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$

Put $\quad y = vx$

$\Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore \quad v + x\frac{dv}{dx} = \frac{v^2x^2}{vx^2 - x^2}$

$\Rightarrow \quad x\frac{dv}{dx} = \frac{v}{v-1}$

$\Rightarrow \quad \left(1 - \frac{1}{v}\right)dv = \frac{dx}{x}$

$\Rightarrow \quad v - \log v = \log x + \log k$

[integrating]

$\Rightarrow \quad \frac{y}{x} = \log x \ k \ \frac{y}{x}$

$\Rightarrow \quad \frac{y}{x} = \log ky$

$\Rightarrow \quad ky = e^{v/x}$

70 **(c)**

Given differentiating equation is

$$\left(1 + 4\,\frac{dy}{dx}\right)^{2/3} = 4\,\frac{d^2y}{dx^2}$$

$$\Rightarrow \quad \left(1 + 4\,\frac{dy}{dx}\right)^{2} = 4^3\left(\frac{d^2y}{dx^2}\right)^3$$

Here, highest order is 2 and degree is 3.

71 **(d)**

We have,

$\frac{dy}{dx} = 1 + x + y^2 + xy^2$

$\Rightarrow \frac{dy}{dx} = (1+x)(1+y^2)$

$\Rightarrow \frac{1}{1+y^2}dy = (1+x)dx$

$\Rightarrow \tan^{-1} y = \left(x + \frac{x^2}{2}\right) + C$ ... (i)

It is given that $y(0) = 0$ *i.e.* $y = 0$ when $x = 0$

$\therefore \tan^{-1} 0 = 0 + C \Rightarrow c = 0$

Hence, $\tan^{-1} y = x + \frac{x^2}{2} \Rightarrow y = \tan\left(x + \frac{x^2}{2}\right)$

72 **(a)**

Given, $\frac{dy}{dx} + \frac{y}{x} = \sin x$

$\therefore$ IF $= e^{\int \frac{1}{x}dx} = x$

$\therefore$ Solution is $y.x = \int x \sin x \; dx + c$

$\Rightarrow xy = -x\cos x + \sin x + c$

$\Rightarrow x(y + \cos x) = \sin x + c$

73 **(d)**

Given differential equation can be rewritten as

$$\frac{dx}{dy} = \frac{(\log y - x)}{y \log y}$$

$$\Rightarrow \quad \frac{dx}{dy} + \frac{x}{y\log y} = \frac{1}{y}$$

$$\therefore \quad \text{IF} = e^{\int \frac{1}{y\log y}dy}$$

$$= e^{\log\log y} = \log y$$

74 **(b)**

Curve is $y = e^{a\sin x} \Rightarrow \sin x = \frac{\log y}{a}$

$\therefore \quad \frac{dy}{dx} = e^{a\sin x}\, a\cos x$

$\Rightarrow \quad \frac{dy}{dx} = y\cos x . \frac{\log y}{\sin x}$

$\Rightarrow \quad y\log y = \tan x\, \frac{dy}{dx}$

75 **(a)**

Let the equation of parabola having the directrix parallel to x-axis is

$x^2 = 4a(y+k)$ .....(i)

and equation of directrix is $y = p$ ....(ii)

Here, 3 unknowns are in Eqs. (i) and (ii).

$\therefore$ Order of DE such parabolas having directrix parallel to x-axis is 3.

76 **(d)**

We have,

$x\,dy - y\,dx = \sqrt{x^2+y^2}dx \Rightarrow \frac{dy}{dx} - \frac{y}{x} = \frac{\sqrt{x^2+y^2}}{x}$

Putting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, we get

$v + x\frac{dv}{dx} - v = \sqrt{1+v^2} \Rightarrow \frac{1}{\sqrt{1+v^2}}dv = \frac{dx}{x}$

Integrating, we get

$\log\left|v + \sqrt{v^2+1}\right| = \log x + \log C$

$\Rightarrow v + \sqrt{v^2+1} = Cx \Rightarrow y + \sqrt{x^2+y^2} = Cx^2$

77 **(a)**

Given, $(x^2+1)\frac{dy}{dx} + 2xy = x^2 - 1$

or $\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{x^2-1}{x^2+1}$

It is a linear differential equation.

On comparing with the standard equation $\frac{dy}{dx} + Py = Q$, we get

$P = \frac{2x}{1+x^2}, Q = \frac{x^2-1}{x^2+1}$

$\therefore$ IF $= e^{\int P\,dx} = e^{\int \frac{2x}{1+x^2}dx}$

$= e^{\log(1+x^2)} = 1 + x^2$

78 **(c)**

The equation of the family of circles is

$x^2 + (y-k)^2 = r^2$ ...(i)

Where k is a parameter

Differentiating w.r.t. x, we get

$2x + 2(y-k)y_1 = 0 \Rightarrow y - k = -\frac{x}{y_1}$ ...(ii)

Eliminating k from (i) and (ii), we obtain

$x^2 + \frac{x^2}{y_1^2} = r^2 \Rightarrow x^2 = \frac{r^2y_1^2}{1+y_1^2} \Rightarrow x^2(y_1^2+1)$

$= r^2y_1^2$

79 **(d)**

Given, $y = ae^x + bx\,e^x + cx^2e^x$ ...(i)

On differentiating $\text{w.r.t.}\,x$, we get

$y' = ae^x + b(xe^x + e^x) + c(x^2e^x + 2xe^x)$

$\Rightarrow \quad y' = ae^x + bxe^x + cx^2e^x + be^x + 2cxe^x$

$\Rightarrow \quad y' = y + be^x + 2cxe^x$ ...(ii)

Again, differentiating $\text{w.r.t.}\,x$, we get

$y'' = y' + be^x + 2c(xe^x + e^x)$

$\Rightarrow \quad y'' = y' + be^x + 2cxe^x + 2ce^x$

$\Rightarrow \quad y'' = 2y' - y + 2ce^x$ ...(iii)

[from

Eq. (ii)]

Again, differentiating w.r.t. x, we get

$$y''' = 2y'' - y' + 2ce^x$$

$\Rightarrow$ $y''' = 2y'' - y' + (y'' - 2y' + y)$

[from eq. (iii)]

$\Rightarrow$ $y''' - 3y'' + 3y' - y = 0$

80 **(c)**

Given, $y = ae^{mx} + be^{-mx}$

$\Rightarrow$ $\frac{dy}{dx} = mae^{mx} - m\,be^{-mx}$

$\Rightarrow$ $\frac{d^2y}{dx^2} = m^2ae^{mx} + m^2be^{-mx} = m^2y$

$\Rightarrow$ $\frac{d^2y}{dx^2} - m^2y = 0$

81 **(c)**

Given, $\sqrt{\sin x}\left(1 + \frac{dy}{dx}\right) = \sqrt{\cos x}\left(1 - \frac{dy}{dx}\right)$

$\Rightarrow$ $\frac{dy}{dx} = \frac{\sqrt{\cos x} - \sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}$

$\therefore$ Order=1, degree=1

82 **(a)**

The equation of a family of circles of radius r passing through the origin and having centre on y-axis is

$(x - 0)^2 + (y - r)^2 = r^2 \Rightarrow x^2 + y^2 - 2ry = 0$

This is one parameter family of circles so its differential equation is of order one

83 **(b)**

Given, $\frac{dy}{dx} = -\frac{1 + y + x^2y}{x + x^3}$

$\Rightarrow$ $\frac{dy}{dx} + \frac{y}{x} = -\frac{1}{x(1 + x^2)}$

$\therefore$ $IF = e^{\int \frac{1}{x} dx} = x$

84 **(c)**

Given, $\frac{dy}{dx} = \left(\frac{y}{x}\right)^{1/3}$

$\Rightarrow \frac{dy}{dx} = \frac{y^{1/3}}{x^{1/3}} \Rightarrow \frac{dy}{y^{1/3}} = \frac{dx}{x^{1/3}}$

On integrating both sides, we get

$$\int \frac{dy}{y^{1/3}} = \int \frac{dy}{x^{1/3}}$$

$\Rightarrow \frac{y^{2/3}}{\frac{2}{3}} = \frac{x^{2/3}}{\frac{2}{3}} + c_1$

$\Rightarrow \frac{3}{2}y^{2/3} = \frac{3}{2}x^{2/3} + c_1$

$\Rightarrow$ $y^{2/3} - x^{2/3} = c$ (where $c = \frac{2}{3}c_1$)

85 **(b)**

Let $y = f(x)$ be the curve. The equation of tangent at (x, y) to this curve is

$$Y - y = f'(x)(X - x)$$

....(i)

Put $X = 0$ in Eq. (i), we get

$$Y = y - x\ f'(x)$$

This ordinate is called the initial ordinate of the tangent. It is given that,

Initial ordinate of the tangent = Subnormal

$\Rightarrow$ $y - x\,f'(x) = y\,\frac{dy}{dx}$

$\Rightarrow$ $\frac{dy}{dx} = \frac{y}{x + y}$ [put $f'(x) = \frac{dy}{dx}$]

Hence, it is a homogenous differential equation.

86 **(b)**

$$y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$$

$\Rightarrow y - ay^2 = a\,\frac{dy}{dx} + x\,\frac{dy}{dx}$

$\Rightarrow y(1 - ay) = (a + x)\frac{dy}{dx}$

$\Rightarrow \frac{dx}{(a + x)} = \frac{dy}{y(1 - ay)}$

On integrating both sides, we get

$$\int \frac{dx}{(a + x)} = \int \frac{dy}{y(1 - ay)}$$

$\Rightarrow \log(a + x) = \int \left[\frac{1}{y} + \frac{a}{(1 - ay)}\right] dx$

$\Rightarrow \log(a + x) = \log y + \frac{a\log(1 - ay)}{-a} + \log c$

$\Rightarrow \log(a + x) = \log y - \log(1 - ay) + \log c$

$\Rightarrow \log(x + a)(1 - ay) = \log cy$

$\Rightarrow (x + a)(1 - ay) = cy$

87 **(a)**

We have,

Length of the normal $= y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

It is given that

$y\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{x^2 + y^2}$ [$\because$ Radius vector $= R = \sqrt{x^2 + y^2}$]

$\Rightarrow y^2 + y^2\left(\frac{dy}{dx}\right)^2 = x^2 + y^2$

$\Rightarrow y^2\left(\frac{dy}{dx}\right)^2 = x^2 \Rightarrow y\,dy \pm x\,dx = 0 \Rightarrow y^2 \pm x^2 = k^2$

88 **(c)**

Equation of tangent at (x, y) is

$$Y - y = \frac{dy}{dx}(X - x)$$

For y-axis $X = 0$.

Then, $Y = y - x\frac{dy}{dx}$

Given, $\left(y - x\frac{dy}{dx}\right) \propto x^3$

$\Rightarrow y - x\frac{dy}{dx} = kx^3$

$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = -kx^2$

$\text{IF} = e^{\int 1/x\,dx} = e^{-\text{In}\,x} = e^{\text{In}(1/x)} = \frac{1}{x}$

Then, solution is

$y\left(\frac{1}{x}\right) = \int \frac{-kx^2}{x}dx$

$\Rightarrow \frac{y}{x} = -\frac{kx^2}{2} + c$

or $y = -\frac{kx^3}{2} + cx$

89 **(b)**

Given, $dy = -\left(\frac{\cos x - \sin x}{\sin x + \cos x}\right)dx$

$\Rightarrow \quad y = -\log(\sin x + \cos x) + \log c$

[integrating]

$\Rightarrow \quad y = \log\left(\frac{c}{\sin x + \cos x}\right)$

$\Rightarrow \quad e^y(\sin x + \cos x) = c$

90 **(b)**

We have,

$\frac{dx}{dt} = x + 1$

$\Rightarrow \frac{1}{x+1}dx = dt \Rightarrow \log(x+1) = t + C$

Putting $t = 0, x = 0$, we get

$\log 1 = C \Rightarrow C = 0$

$\therefore t = \log(x+1)$

Putting $x = 99$, we get

$t = \log_e 100 = 2\log_e 10$

91 **(a)**

The equation to all given parabolas is

$y^2 = 4\,a(x - b)$

$\Rightarrow 2\,y\frac{dy}{dx} = 4\,a \Rightarrow y\frac{dy}{dx} = 2\,a \Rightarrow y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

92 **(d)**

Given differential equation is

$\Rightarrow \quad \frac{(1+y)}{y}dy = \frac{(1+x)}{x}dx$

$\Rightarrow \quad \int\left(\frac{1}{y} + 1\right)dy = \int\left(\frac{1}{x} + 1\right)dx$

$\Rightarrow \quad \log y + y = \log x + x + \log c$

$\Rightarrow \quad y - x = \log\left(\frac{cx}{y}\right)$

93 **(c)**

I. $\frac{dy}{dx} + 2xy = 2e^{-x^2}$

$\therefore \quad \text{IF} = e^{\int 2x\,dx} = e^{x^2}$

$\therefore$ Complete solution is

$ye^{x^2} = 2\int e^{-x^2}e^{x^2}dx + c$

$\Rightarrow \quad ye^{x^2} = 2x + c$

II. $ye^{x^2} - 2x = c$

$\Rightarrow \quad ye^{x^2}.2x + e^{x^2}.\frac{dy}{dx} - 2 = 0$

$\Rightarrow \quad e^{x^2}.\frac{dy}{dx} = 2 - 2xy\,e^{x^2}$

$\Rightarrow \quad \frac{dy}{dx} = 2e^{-x^2} - 2xy$

$\therefore$ I is true and II is false.

94 **(c)**

We have,

$y = a(x+a)^2$ ...(i)

$\Rightarrow \frac{dy}{dx} = 2a(x+a)$ ...(ii)

Dividing (i) by (ii), we get

$\frac{y}{\frac{dy}{dx}} = \frac{x+a}{2} \Rightarrow x + a = \frac{2y}{y_1}$, where $y_1 = \frac{dy}{dx}$

Substituting $a = \frac{2y}{y_1} - x$ in (i), we get

$y = \left(\frac{2y}{y_1} - x\right)\left(\frac{2y}{y_1}\right)^2 \Rightarrow y_1^3 y = 4(2y - x\,y_1)y^2$

Clearly, it is a differential equation of degree 3

95 **(a)**

Given differential equation is

$$\frac{d^2y}{dx^2} = \sqrt[3]{1 - \left(\frac{dy}{dx}\right)^4}$$

$\Rightarrow \quad \left(\frac{d^2y}{dx^2}\right)^3 = 1 - \left(\frac{dy}{dx}\right)^4$

$\therefore$ Order=2, degree=3

96 **(b)**

Given, $\sin y\;dy = \cos x\;dx$

$\Rightarrow \quad -\cos y + c = \sin x$

[integrating]

$\Rightarrow \quad \sin x + \cos y = c$

97 **(a)**

Given, $\frac{dy}{dx} = \frac{x - 2y + 1}{2x - 4y}$

Put $x - 2y = z \Rightarrow 1 - 2\,\frac{dy}{dx} = \frac{dz}{dx}$

$\therefore \quad \frac{1}{2}\left[-\frac{dz}{dx} + 1\right] = \frac{z+1}{2z}$

$\Rightarrow \quad zdz = -dx$

$\Rightarrow \quad \frac{z^2}{2} = -x + c_1$ [integrating]

$\Rightarrow \quad (x - 2y)^2 + 2x = c$

98 **(a)**

The equation of the family of circles which touch both the axes is

$(x-a)^2 + (y-a)^2 = a^2$, where a is a parameter

This is one parameter family of curves

So its differential equation is of order one

99 **(a)**

Given equation can be rewritten as

$$\frac{dy}{dx}-\frac{1}{x}.y=1$$

$\therefore \quad IF=e^{-\int\frac{1}{x}dx}=e^{-\log x}=\frac{1}{x}$

$\therefore$ Required solution is

$$y\left(\frac{1}{x}\right)=\int\frac{1}{x}\,dx=\log x+c$$

Since, $\quad y(1)=1$

$\Rightarrow \quad c=1$

$\therefore \quad y=x\log x+x$

100 **(a)**

Given, $\frac{dy}{dx}=\frac{x^2+y^2}{2xy}$

Put $y=vx$ and $\frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore v+x\dfrac{dv}{dx}=\dfrac{x^2+v^2x^2}{2vx^2}$

$\Rightarrow x\dfrac{dv}{dx}=\left(\dfrac{1+v^2}{2v}-v\right)$

$\Rightarrow \dfrac{2v}{1-v^2}dv=\dfrac{dx}{x}$

On integrating both sides, we get

$\int\dfrac{2v}{1-v^2}dv=\int\dfrac{1}{x}dx$

$\Rightarrow -\log(1-v^2)=\log x+\log c$

$\Rightarrow -\log\left(1-\frac{y^2}{x^2}\right)=\log x+\log c \quad$ ....(i)

This curve passes through (2, 1).

$\therefore -\log\left(1-\dfrac{1}{4}\right)=\log 2+\log c$

$\Rightarrow -\log\left(\dfrac{3}{4}\right)=\log 2c$

$\Rightarrow \log\left(\dfrac{4}{3}\right)=\log 2c$

$\Rightarrow c=\dfrac{2}{3}$

On putting $c=\frac{2}{3}$ in Eq. (i), we get

$\log\left(\dfrac{x^2}{x^2-y^2}\right)=\log\dfrac{2}{3}x$

$\Rightarrow 2(x^2-y^2)=3x$

101 **(a)**

The given differential equation can be rewritten as

$$y+\frac{d^2y}{dx^2}=\left[a+\left(\frac{dy}{dx}\right)^{3/2}\right]^2$$

$$\Rightarrow \quad y+\frac{d^2y}{dx^2}=x^2+\left(\frac{dy}{dx}\right)^3+2x\left(\frac{dy}{dx}\right)^{3/2}$$

$$\Rightarrow \quad \left[y+\frac{d^2y}{dx^2}-x^2-\left(\frac{dy}{dx}\right)^3\right]^2=\left[2x\left(\frac{dy}{dx}\right)^{3/2}\right]^2$$

$\therefore$ Order and degree of the given differential equation is 2 and 2 respectively.

102 **(c)**

Given differential equation is $\frac{dy}{dx}+y=e^x$

$\therefore \quad \text{IF}=e^{\int P\,dx}=e^{\int 1\,dx}=e^x$

Now, solution is

$$ye^x=\int e^{2x}\,dx$$

$\Rightarrow \quad ye^x=\frac{e^{2x}}{2}+\frac{c}{2}$

$\Rightarrow \quad 2ye^x=e^{2x}+c$

103 **(b)**

We have,

$\phi(x)=\phi'(x)$

$\Rightarrow \dfrac{\phi'(x)}{\phi(x)}=1$

$\Rightarrow \log\phi(x)=x+\log C\Rightarrow \phi(x)=C\,e^x$

Putting $x=1,\phi(1)=2$, we get $C=\frac{2}{e}$

$\therefore \phi(x)=2e^{x-1}\Rightarrow \phi(3)=2\,e^2$

104 **(b)**

Given equation

$\dfrac{dy}{dx}+\sin\left(\dfrac{x+y}{2}\right)=\sin\left(\dfrac{x-y}{2}\right)$

$\Rightarrow \dfrac{dy}{dx}=\sin\left(\dfrac{x-y}{2}\right)-\sin\left(\dfrac{x+y}{2}\right)$

$\Rightarrow \dfrac{dy}{dx}=-2\sin\left(\dfrac{y}{2}\right)\cos\left(\dfrac{x}{2}\right)$

$\Rightarrow \operatorname{cosec}\left(\dfrac{y}{2}\right)dy=-2\cos\left(\dfrac{x}{2}\right)dx$

On integrating both sides, we get

$\int\operatorname{cosec}\left(\dfrac{y}{2}\right)dy=-\int 2\cos\left(\dfrac{x}{2}\right)dx+c$

$\Rightarrow \dfrac{\log(\tan\frac{y}{4})}{\frac{1}{2}}=-\dfrac{2\sin\left(\frac{x}{2}\right)}{\frac{1}{2}}+c$

$\Rightarrow \log(\tan\dfrac{y}{4})=c-2\sin\left(\dfrac{x}{2}\right)$

105 **(a)**

The family of curves is

$x^2+y^2-2ax=0 \quad$ ...(i)

Differentiating w.r.t. to x, we get

$2x+2y\dfrac{dy}{dx}-2a=0\Rightarrow a=x+y\dfrac{dy}{dx}$

Substituting the value of a in (i), we obtain

$x^2+y^2-2x\left(x+y\frac{dy}{dx}\right)=0$ or, $y^2-x^2-2xy\frac{dy}{dx}=0$

106 **(b)**

Given, $y = (c_1 + c_2)\cos(x + c_3) - c_4 e^{x+c_5}$

$\Rightarrow \quad y = (c_1 \cos c_3 + c_2 \cos c_3)\cos x$

$-(c_1 \sin c_3 + c_2 \sin c_3)\sin x - c_4 e^{c_5} e^x$

$\Rightarrow \quad y = A\cos x - B\sin x + Ce^x$

Where, $\quad A = c_1 \cos c_3 + c_2 \cos c_3$

$\quad B = c_1 \sin c_3 + c_2 \sin c_3$

And $\quad C = -c_4 e^{c_5}$

Which is an equation containing three arbitrary constant. Hence, the order of the differential equation is 3.

107 **(c)**

Given equation is $e^x + \sin\left(\frac{dy}{dx}\right) = 3$

Since, the given differential equation cannot be written as a polynomial in all the differential coefficients, the degree of the equation is not defined.

108 **(d)**

Given, $\quad x = \sin t,\ y = \cos pt$

$\frac{dx}{dt} = \cos t,\ \frac{dy}{dt} = -p\sin pt$

$\therefore \quad \frac{dy}{dx} = -\frac{p\sin pt}{\cos t}$

$\Rightarrow \quad y_1 = \frac{-p\sqrt{1-y^2}}{\sqrt{1-x^2}}$

$\Rightarrow \quad y_1\sqrt{1-x^2} = -p\sqrt{1-y^2}$

$\Rightarrow \quad y_1^2(1-x^2) = p^2(1-y^2)$

$\Rightarrow \quad 2y_1y_2(1-x^2) - 2xy_1^2 = -2yy_1p^2$

[differentiating]

$\Rightarrow \quad (1-x^2)y_2 - xy_1 + p^2y = 0$

109 **(c)**

Given, $\quad y = xe^{cx}$

...(i)

$\Rightarrow \quad \frac{dy}{dx} = e^{cx} + xe^{cx}.c = \frac{y}{x} + y.c$

...(ii)

From Eq. (ii),

Log $y = \log x + cx$

$\Rightarrow c\frac{1}{x}\log\frac{y}{x}$

$\frac{dy}{dx} = \frac{y}{x} + \frac{y}{x}\log\frac{y}{x}$

$= \frac{y}{x}\left(1 + \log\frac{y}{x}\right)$

110 **(a)**

Given differential equation is

$$y = x\frac{dy}{dx} + \left(a^2\left(\frac{dy}{dx}\right)^2 + b^2\right)^{\frac{1}{3}}$$

$$\Rightarrow \quad \left(y - x\frac{dy}{dx}\right)^3 = a^2\left(\frac{dy}{dx}\right)^2 + b^2$$

$\therefore$ Order and degree of the above differential equations are 1 and 3 respectively.

111 **(b)**

We have,

$y_3^{2/3} + 2 + 3y_2 + y_1 = 0$

$\Rightarrow y_3^{2/3} = -(3y_2 + y_1 + 2)$

$\Rightarrow y_2^3 = -(3y_2 + y_1 + 2)^3$

Clearly, it is differential equation of third order and second degree

112 **(b)**

Given, $x^2 + y^2 = 1 \quad$ ....(i)

On differentiating w. r. t. x, we get

$2x + 2yy' = 0 \ \Rightarrow x + yy' = 0$

Again, on differentiating w. r. t. x, we get

$1 + (y')^2 + yy'' = 0$

113 **(a)**

We have,

$$\frac{dy}{dx} = \frac{x\log x^2 + x}{\sin y + y\cos y}$$

$$\Rightarrow \int(\sin y + y\cos y)dy = 2\int x\log x\ dx + \int x\,dx$$

$\Rightarrow y\sin y = x^2\log x + C$

114 **(b)**

The given differential equation is

$\frac{dy}{dx} + P(x)y = Q(x).y^n$

$\Rightarrow \quad \frac{1}{y^n}.\frac{dy}{dx} + y^{-n+1}P(x) = Q(x)$

Put $\quad \frac{1}{y^{n-1}} = v$

$\Rightarrow \quad (-n+1)y^{-n}\frac{dy}{dx} = \frac{dv}{dx}$

$\therefore \quad \frac{1}{(-n+1)}.\frac{dv}{dx} + P(x).v = Q(x)$

$\Rightarrow \quad \frac{dv}{dx} + (1-n)P(x)v = (1-n)Q(x)$

Hence, required substitution is $v = \frac{1}{y^{n-1}}$

115 **(b)**

Since, length of subnormal $= a$

$\Rightarrow \quad y\frac{dy}{dx} = a \ \Rightarrow y\,dy = a\,dx$

On integrating both sides, we get

$\frac{y^2}{2} = ax + b$

Where b is a constant of integration

$\Rightarrow \quad y^2 = 2ax + 2b$

116 **(c)**

Given, $\frac{dx}{dt} = \cos^2 \pi x$

On differentiating w.r.t. x, we get

$\frac{d^2x}{dt^2} = -2\pi \sin 2\pi x = \text{negative}$

The particle never reaches the point, it means

$\frac{d^2x}{dt^2} = 0 \Rightarrow -2\pi \sin 2\pi x = 0$

$\Rightarrow \sin 2\pi x = \sin \pi$

$\Rightarrow 2\pi x = \pi \Rightarrow x = \frac{1}{2}$

The particle never reaches at $x = \frac{1}{2}$

117 **(b)**

Given, $\frac{dy}{dx} = \frac{x+y}{x-y}$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore \quad v + x\frac{dv}{dx} = \frac{1+v}{1-v}$

$\Rightarrow \quad x\frac{dv}{dx} = \frac{1+v^2}{1-v}$

$\Rightarrow \quad \frac{1}{x}dx = \left(\frac{1}{1+v^2} - \frac{v}{1+v^2}\right)dv$

$\Rightarrow \quad \log_e x = \tan^{-1} v - \frac{1}{2}\log_e(1+v^2) - \log_e c$ [integrating]

$\Rightarrow \quad \log_e x = \tan^{-1}\left(\frac{y}{x}\right) - \frac{1}{2}\log_e\Big[1 +$

$yx2 - \log ec$

$\Rightarrow \quad c(x^2+y^2)^{1/2} = e^{\tan^{-1}(y/x)}$

118 **(b)**

Given, $\frac{d^2y}{dx^2} = e^{-2x}$

$\Rightarrow \quad \frac{dy}{dx} = \frac{e^{-2x}}{-2} + c$

[integrating]

$\Rightarrow \quad y = \frac{e^{-2x}}{4} + cx + d$

[integrating]

119 **(b)**

Given, $x^2 + y^2 = 1$

On differentiating w.r.t. x, we get

$$2x + 2yy' = 0$$

$\Rightarrow \quad x + yy' = 0$

Again , differentiating, we get

$$1 + yy'' + (y')^2 = 0$$

120 **(a)**

We have,

$\frac{dy}{dx} = \frac{y-1}{x^2+x}$

$\Rightarrow \frac{1}{x^2+x}dx = \frac{1}{y-1}dy$

$\Rightarrow \int \frac{1}{x(x+1)}dx = \int \frac{1}{y-1}dy$

$\Rightarrow \int \frac{1}{x(x+1)}dx = \int \frac{1}{y-1}dy$

$\Rightarrow \int \left(\frac{1}{x} - \frac{1}{x+1}\right)dx = \int \frac{1}{y-1}dy$

$\Rightarrow \log x - \log(x+1) = \log(y-1) + \log C$

$\Rightarrow \frac{x}{x+1} = C(y-1) \quad$ ...(i)

This passes through (1, 0)

$\therefore \frac{1}{2} = -C$

Substituting the value of C in (i), we get

$\frac{x}{x+1} = -\frac{1}{2}(y-1)$

$\Rightarrow (x+1)(y-1) = -2x \Rightarrow xy + x + y - 1 = 0$

This is the required curve

121 **(c)**

Given, $\frac{dy}{dx} - \frac{2}{x}y = x^2e^x$

$\therefore \quad \text{IF} = e^{-\int \frac{2}{x}dx} = e^{-\log x^2} = \frac{1}{x^2}$

$\therefore$ Complete solution is $\frac{y}{x^2} = \int \frac{x^2e^x}{x^2}dx + c$

$\Rightarrow \quad \frac{y}{x^2} = e^x + c$

$\Rightarrow \quad y = x^2(e^x + c)$

When $y = 0, x = 1$, then $c = -e$

$\therefore \quad y = x^2(e^x - e)$

122 **(b)**

Given, $\frac{dy}{dx} + \frac{2x}{1+x^2}.y = \frac{4x^2}{1+x^2}$

$\therefore \quad \text{IF} = e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2) = (1+x^2)}$

$\therefore$ Complete solution is

$$y.(1+x^2) = \int (1+x^2).\frac{4x^2}{1+x^2}dx$$

$\Rightarrow \quad y(1+x^2) = \frac{4x^3}{3} + c_1$

$\Rightarrow \quad 3y(1+x^2) = 4x^3 + c$

123 **(a)**

Given, $k = PQ =$ length of normal

$\Rightarrow \quad k = y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

$\Rightarrow \quad \frac{k^2}{y^2} = 1 + \left(\frac{dy}{dx}\right)^2$

$\therefore \quad y\frac{dy}{dx} = \pm\sqrt{k^2 - y^2}$

124 **(a)**

We have,

$y_1y_3 = 3y_2^2 \Rightarrow \frac{y_3}{y_2} = 3\frac{y_2}{y_1}$

Integrating both sides, we get

$\log y_2 = 3\log y_1 + \log c_1$

$\Rightarrow y_2 = c_1 y_1^3 \Rightarrow \frac{y_2}{y_1^3} = c_1 \Rightarrow \frac{d\,y_1}{y_1^3} = c_1$

Integrating both sides w.r.t. x, we get

$-\frac{1}{2y_1^2} = c_1 x + c_2$

$\Rightarrow y_1^2 = \frac{1}{(-2\,c_1)x + (-2\,c_2)}$

$\Rightarrow y_1^2 = \frac{1}{ax+b}$, where $a = -2c_1, b = -2\,c_2$

$\Rightarrow y_1 = \frac{1}{\sqrt{ax+b}}$

Integrating both sides w.r.t. x, we get

$y = \frac{2}{a}\sqrt{ax+b} + c_3$

$\Rightarrow \frac{ay - c_3}{2} = \sqrt{ax+b}$

$\Rightarrow ax + b = \left(\frac{ax - c_3}{2}\right)^2$

$\Rightarrow x = \frac{a}{4}y^2 - \frac{c^3}{2}y + \frac{1}{a}\left(\frac{c_3^2}{4} - b\right) \Rightarrow x = A_1 y^2 + A_2 y + A_3$,

where $= A_1 = \frac{a}{4}, A_2 = -\frac{c_3}{2}$ and $A_3 = \frac{1}{a}\left(\frac{c_3^2}{4} - b\right)$

125 **(a)**

Here, $x = A\cos 4t + B\sin 4t$

On differentiating w. r. t. t, we get

$\frac{dx}{dt} = -4A\sin 4t + 4B\cos 4t$

Again, on differentiating w. r. t. t, we get

$\frac{d^2x}{dt^2} = -16A\cos 4t - 16B\sin 4t$

$= -16(A\cos 4t + B\sin 4t)$

$\Rightarrow \frac{d^2x}{dt^2} = -16x$

126 **(a)**

We have,

$y = c_1 + c_2 e^x + c_3 e^{-2x+c_4}$

$\Rightarrow y = c_1 + c_2 e^x + c_3 e^{-2x} \cdot e^{c_4}$

$\Rightarrow y = c_1 + c_2 e^x + c_3{}' e^{-2x}$, where $c_3' = c_3 e^{c_4}$

It is an equation containing three arbitrary constants. So, the associated differential equation is of order 3

127 **(b)**

Equation of parabolas family can be taken as

$x = ay^2 + by + c$

Differentiating $w.r.t.,y$ we get

$\frac{dx}{dy} = 2ay + b$

$\Rightarrow \frac{d^2x}{dy^2} = 2a \Rightarrow \frac{d^3x}{dy^3} = 0$

128 **(a)**

Given $\frac{1-y}{y^2}\,dy + \frac{1+x}{x^2}\,dx = 0$

$\Rightarrow \int\left(\frac{1}{y^2} - \frac{1}{y}\right)dy + \int\left(\frac{1}{x^2} + \frac{1}{x}\right)\,dx = 0$

$\Rightarrow \log\left(\frac{x}{y}\right) = \frac{1}{x} + \frac{1}{y} + c$

129 **(b)**

Given, $\frac{dy}{dx} = \frac{\sqrt{x^2+y^2}+y}{x}$

Put $y = vx$

$\Rightarrow \frac{dy}{dx} = v + x\,\frac{dv}{dx}$

$\therefore v + x\frac{dv}{dx} = \frac{\sqrt{x^2+v^2x^2}+vx}{x}$

$\Rightarrow \frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$

$\Rightarrow \log\left(v + \sqrt{1+v^2}\right) = \log x + \log c$

$\Rightarrow \log\left(\frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}}\right) = \log cx$

$\Rightarrow y + \sqrt{x^2+y^2} = cx^2$

130 **(a)**

Given, $\frac{dy}{dx} = \frac{x\log x^2 + x}{\sin y + y\cos y}$

$\Rightarrow (\sin y + y\cos y)dy = (x\log x^2 + xdx$

$\left(\frac{d}{dy}y\sin y\right)dy = \left(\frac{d}{dx}x^2\log x\right)dx$

$\Rightarrow y\sin y = x^2\log x + c$

131 **(d)**

Given, $x(1-x^2)dy + (2x^2y - y - ax^3)dx = 0$

$\Rightarrow \frac{dy}{dx} + \frac{(2x^2-1)}{x(1-x)}y = \frac{ax^3}{(1-x^2)}$

Here, $P = \frac{2x^2-1}{x(1-x^2)}$

132 **(c)**

We have,

$\frac{dy}{dx} + \frac{y}{x} = x^2 \Rightarrow x\frac{dy}{dx} + y = x^3 \Rightarrow \frac{d}{dx}(xy) = x^3$

Integrating, we get

$xy = \frac{x^4}{4} + C \Rightarrow y = \frac{x^3}{4} + C\,x^{-1}$

133 **(c)**

Let $x^2 + y^2 - 2gx = 0$ ....(i)

$\Rightarrow 2x + 2y\frac{dy}{dx} - 2g = 0$

$\Rightarrow 2g = \left(2x + 2y\frac{dy}{dx}\right)$

On putting the value of 2g in eq. (i), we get

$x^2 + y^2 - \left(2x + 2y\,\frac{dy}{dx}\right)x = 0$

$\Rightarrow y^2 = x^2 + 2xy\,\frac{dy}{dx}$

134 **(d)**

Given differential equation can be written as

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$$

$\Rightarrow \quad (m^2 - 3m + 2)y = 0$

$\Rightarrow \quad (m-1)(m-2)y = 0$

$\Rightarrow \quad m = 1,2$

$\therefore$ Solution is $y = c_1e^x + c_2e^{2x}$

$y' = c_1e^x + 2c_2e^{2x}$

From given condition

$y(0) = 1$

$\Rightarrow c_1 + c_2 = 1$.....(i)

And y'(0)=0

$\Rightarrow c_1 + c_2 = 1 \ldots\ldots(ii)$

On solving Eqs. (i) and (ii)we get

$-c_2 = 1$

$\Rightarrow \quad c_2 = -1$

And $c_1 = 2$

$\therefore$y=$2e^x - e^{2x}$

$\therefore \quad at\ x = \log_e 2$

$y = 2e^{log2} - e^{2\log 2}$

$= 2 \times 2 - 2^2 = 0$

135 **(b)**

The equation of straight line touching the given circle is

$$x\cos\theta + y\sin\theta = a$$

...(i)

On differentiating w. r. t. x, regarding θ as a constant

$\Rightarrow \quad \cos\theta + \frac{dy}{dx}\sin\theta = 0$

...(ii)

From eqs. (i) and (ii), we get

$$\cos\theta = \frac{a\frac{dy}{dx}}{x\frac{dy}{dx}-y} \text{ and } \sin\theta = -\frac{a}{x\frac{dy}{dx}-y}$$

$\therefore \quad \cos^2\theta + \sin^2\theta = 1$

$$\therefore \quad \frac{a^2\left(\frac{dy}{dx}\right)^2 + a^2}{\left(x\frac{dy}{dx} - y\right)^2} = 1$$

$$\Rightarrow \quad \left(y - x\frac{dy}{dx}\right)^2 = a^2\left[1 + \left(\frac{dy}{dx}\right)^2\right]$$

136 **(a)**

The given differential equation can be rewritten as

$$\Rightarrow \quad \left(\frac{1}{y^2} - \frac{1}{y}\right)dy = -\left(\frac{1}{x^2} + \frac{1}{x}\right)dx$$

$$\Rightarrow \quad -\frac{1}{y} - \log y = -\left(-\frac{1}{x} + \log x\right) + c$$

[integrating]

$$\Rightarrow \quad \log\left(\frac{x}{y}\right) = \frac{1}{x} + \frac{1}{y} + c$$

137 **(a)**

We have, $(xy - x^2) = y^2$

$$\Rightarrow \quad y^2\frac{dx}{dy} = xy - x^2$$

$$\Rightarrow \quad \frac{1}{x^2}\frac{dx}{dy} - \frac{1}{x}.\frac{1}{y} = -\frac{1}{y^2}$$

Put $\frac{1}{x} = v \Rightarrow -\frac{1}{x^2}\frac{dx}{dy} = \frac{dv}{dy}$

$\therefore \quad \frac{dv}{dy} + \frac{v}{y} = \frac{1}{y^2}$, which is linear

$$\therefore \quad IF = e^{\int\frac{1}{y}dy} = e^{\log y} = y$$

$\therefore$ The solution is $vy = \int\frac{1}{y^2}\,y\,dy + c$

$$\Rightarrow \quad \frac{y}{x} = \log y + c$$

$$\Rightarrow \quad y = x(\log y + c)$$

This passes through the point (-1,1)

$\therefore \quad 1 = -1(\log 1 + c)$

ie., $\quad c = -1$

thus, the equation of the curve is

$$y = x(\log y - 1)$$

138 **(d)**

Given, $y = 2e^{2x} - e^{-x}$

$\Rightarrow \quad y_1 = 4e^{2x} + e^{-x}$

$\Rightarrow \quad y_2 = 8e^{2x} - e^{-x}$

$\Rightarrow \quad y_2 = 4e^{2x} + e^{-x} + 4e^{2x} - 2e^{-x}$

$\Rightarrow \quad y_2 = y_1 + 2(2e^{2x} - e^{-x})$

$\Rightarrow \quad y_2 = y_1 + 2y$

$\Rightarrow \quad y_2 = y_1 + 2y$

$\Rightarrow \quad y_2 - y_1 - 2y = 0$

139 **(c)**

Given equation is $\frac{dy}{dx} - y = 1 \Rightarrow \frac{dy}{1+y} = dx$

On integrating both sides, we get

$$\int\frac{1}{1+y}dy = \int dx$$

$\Rightarrow \log(1+y) = x + c$

$\Rightarrow 1 + y = e^x \cdot e^c \quad$ ...(i)

At $x = 0, y = -1$

Then $1 - 1 = e^0 \cdot e^c \Rightarrow e^c = 0$

On putting the value of e^c in Eq. (i).

Therefore, solution becomes

$1 + y = e^x \times 0 \Rightarrow y(x) = -1$

140 **(d)**

Let family of circles be

$$(x-\alpha)^2 + (y-2)^2 = 5^2$$

$\Rightarrow \quad x^2 + \alpha^2 - 2\alpha x + y^2 - 21 - 4y = 0$ ...(i)

$\Rightarrow \quad 2x - 2\alpha + 2y\frac{dy}{dx} - 4\frac{dy}{dx} = 0$

$\Rightarrow \quad \alpha = x + \frac{dy}{dx}(y-2)$

On putting the value of α in Eq. (i), we get

$$\left(x - x - \frac{dy}{dx}(y-2)\right)^2 + (y-2)^2 = 5^2$$

$\Rightarrow \quad \left(\frac{dy}{dx}\right)^2 (y-2)^2 = 25 - (y-2)^2$

141 **(a)**

It is a linear differential equation of the form of $\frac{dy}{dx} + Py = Q$.

$\Rightarrow P = \sec^2 x, Q = \tan x \sec^2 x$

$\therefore$ IF $= e^{\int P\,dx} = e^{\int \sec^2 x\,dx} = e^{\tan x}$

Solution is $ye^{\tan x} = \int \tan x e^{\tan x} \sec^2 x\,dx + c$

$\Rightarrow ye^{\tan x} = \tan x\, e^{\tan x} - e^{\tan x} + c$

$\Rightarrow y = \tan x - 1 + ce^{-\tan x}$

142 **(b)**

We have,

$y\frac{dy}{dx} + x = a \Rightarrow y\,dy + x\,dx = a\,dx$

Integrating, we get

$\frac{y^2}{2} + \frac{x^2}{2} = ax + C \Rightarrow x^2 + y^2 - 2\,ax + 2\,C = 0$,

which represents a set of circles having centre on x-axis

143 **(d)**

$\because$ Equation of normal at (x, y) is

$Y - y = \frac{dx}{dy}(X - x)$

Put, $y = 0$

Then, $X = x + y\frac{dy}{dx}$

Given, $y^2 = 2x\,X$

$\Rightarrow y^2 = 2x\left(x + y\frac{dy}{dx}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{y^2 - 2x^2}{2xy} = \frac{\left(\frac{y}{x}\right)^2 - 2}{2\left(\frac{y}{x}\right)}$

Put $y = vx$, we get

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

Then, $v + x\frac{dv}{dx} = \frac{v^2-2}{2v}$

$\Rightarrow x\frac{dv}{dx} = -\frac{(2+v^2)}{2v}$

$\Rightarrow \frac{2v\,dv}{(2+v^2)} + \frac{dv}{x} = 0$

On integrating both sides, we get

In $(2 + v^2)$ + In$|x|$ = In c

$\Rightarrow$ In $\left(|x|(2+v^2)\right)$ = In c

$\Rightarrow |x|\left(2 + \frac{y^2}{x^2}\right) = c$

$\because$ It passes through (2, 1), then

$2\left(2 + \frac{1}{4}\right) = c$

$\Rightarrow c = \frac{9}{2}$

Then, $|x|\left(2 + \frac{y^2}{x^2}\right) = \frac{9}{2}$

$\Rightarrow 2x^2 + y^2 = \frac{9}{2}|x|$

$\Rightarrow 4x^2 + 2y^2 = 9|x|$

144 **(a)**

We have,

$y\,dx - x\,dy - 3x^2y^2\,e^{x^3}dx = 0$

$\Rightarrow y\,dx - x\,dy = 3x^2y^2e^{x^3}dx$

$\Rightarrow \frac{y\,dx - x\,dy}{y^2} = 3x^2e^{x^3}dx$

$\Rightarrow d\left(\frac{x}{y}\right) = d\left(e^{x^3}\right) \Rightarrow \frac{x}{y} = e^{x^3} + C$

145 **(c)**

Given, $\frac{dy}{dx} = \frac{ax+h}{by+k}$

$\Rightarrow \quad \int(by+k)\,dy = -\int(ax+h)\,dx$

$\Rightarrow \quad \frac{by^2}{2} + ky = \frac{ax^2}{2} + hx + c$

Thus, above equation represents a parabola, if

$a = 0$ and $b \neq o$

Or $\quad b = 0$ and $a \neq 0$

146 **(b)**

The equations of the ellipses centred at the origin are given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a, b are arbitrary constants

Differentiating both sides w.r.t. to x, we get

$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$

$\Rightarrow \frac{x}{a^2} + \frac{y\,y_1}{b^2} = 0$ ...(i)

Differentiating (i) w.r.t. x, we get

$\frac{1}{a^2} + \frac{y_1^2}{b^2} + \frac{y\,y_2}{b^2} = 0$ ...(ii)

Multiplying (ii) by x and subtracting it from (i), we get

$\frac{1}{b^2}\{y\,y_1 - x\,y_1^2 - x\,y\,y_2\} = 0 \Rightarrow xy\,y_2 + xy_1^2 - y\,y_1 = 0$

147 **(b)**

Given equation is $y = ax^{n+1} + bx^{-n}$

On differentiating with respect to x, we get

$$\frac{dy}{dx} = a(n+1)x^n - bnx^{-n-1}$$

Again, on differentiating , we get

$$\frac{d^2y}{dx^2} = an(n+1)x^{n-1} + bn(n+1)x^{-n-2}$$

$$\Rightarrow x^2\frac{d^2y}{dx^2} = an(n+1)x^{n+1} + bn(n+1)x^{-n}$$

$$\Rightarrow x^2\frac{d^2y}{dx^2} = n(n+1)(ax^{n+1} + bx^{-n})$$

$$\Rightarrow \frac{x^2 d^2y}{dx^2} = n(n+1)y$$

148 **(b)**

Given, $y = a\cos(x+b)$

$$\Rightarrow \frac{dy}{dx} = -a\sin(x+b)$$

$$\Rightarrow \frac{d^2y}{dx^2} = -a\cos(x+b) = -y$$

$$\Rightarrow \frac{d^2y}{dx^2} + y = 0$$

149 **(a)**

Here, $\frac{dy}{dt} - \left(\frac{1}{1+t}\right)y = \frac{1}{(1+t)}$ and $y(0) = -1$

Which represents linear differential equation of first order.

$$\text{IF} = e^{\int -\left(\frac{t}{1+t}\right)dt} = e^{-t+\log|1+t|} = e^{-t}(1+t)$$

$\therefore$ Required solution is

$$y(\text{IF}) = \left[\int Q(\text{IF})dt\right] + c$$

$$\Rightarrow ye^{-t}(1+t) = \int \frac{1}{1+t}\cdot e^{-t}(1+t)dt + c$$

$$= \int e^{-t}dt + c$$

$$\Rightarrow ye^{-t}(1+t) = -e^{-t} + c$$

Since, $y(0) = -1 \Rightarrow -1\cdot e^0(1+0) = -e^0 + c$

$\Rightarrow c = 0$

$\therefore y = -\frac{1}{(1+t)}$ and $y(1) = -\frac{1}{2}$

150 **(b)**

Given, $\frac{dy}{dx} + \frac{y}{(1-x)\sqrt{x}} = 1 - \sqrt{x}$

$\therefore$ $\text{IF} = e^{\int \frac{1}{(1-x)\sqrt{x}}dx}$

Put $\sqrt{x} = t$

$\Rightarrow \frac{1}{2\sqrt{x}}dx = dt$

$\therefore$ $\text{IF} = e^{\int \frac{2}{1-t^2}dt}$

$$= e^{\frac{2}{2}\log\left(\frac{1+t}{1-t}\right)} = \frac{1+t}{1-t} = \frac{1+\sqrt{x}}{1-\sqrt{x}}$$

151 **(b)**

Given, $\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$

Put $y = vx$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ $v + x\frac{dv}{dx} = \frac{x^2+v^2x^2}{2xvx}$

$\Rightarrow x\frac{dv}{dx} = \frac{1-v^2}{2v}$

$\Rightarrow \frac{2v}{1-v^2}dv = \frac{dx}{x}$

$\Rightarrow -\log(1-v^2) = \log x + \log c$

$\Rightarrow \log(1-v^2)^{-1} = \log xc$

$\Rightarrow \left(\frac{x^2-y^2}{x^2}\right)^{-1} = xc$

$\Rightarrow \frac{x^2}{x^2-y^2} = xc$

$\Rightarrow x = c(x^2-y^2)$

152 **(c)**

$\because y = u^n$

$$\therefore \frac{dy}{dx} = nu^{n-1}\frac{du}{dx}$$

On substituting the values of y and $\frac{dy}{dx}$ in the given equation, then

$$2x^4\cdot u^n\cdot nu^{n-1}\frac{du}{dx} + u^{4n} = 4x^6$$

$$\Rightarrow \frac{du}{dx} = \frac{4x^6 - u^{4n}}{2nx^4u^{2n-1}}$$

Since, it is homogeneous. Then, the degree of $4x^6 - u^{4n}$ and $2nx^4u^{2n-1}$ must be same.

$\therefore 4n = 6$ and $4 + 2n - 1 = 6$

Then, we get $n = \frac{3}{2}$

153 **(b)**

Given equation is $y = ax\cos\left(\frac{1}{x}+b\right)$ .....(i)

On differentiating Eq. (i), we get

$$y_1 = a\left[\cos\left(\frac{1}{x}+b\right) - x\sin\left(\frac{1}{x}+b\right)\left(\frac{-1}{x^2}\right)\right]$$

$$\Rightarrow y_1 = a\left[\cos\left(\frac{1}{x}+b\right) + \frac{1}{x}\sin\left(\frac{1}{x}+b\right)\right] \quad .....(ii)$$

Again, on differentiating Eq. (ii), we get

$$y_2 = a\left[-\sin\left(\frac{1}{x}+b\right)\left(-\frac{1}{x^2}\right) + \frac{1}{x}\cos\left(\frac{1}{x}+b\right)\left(-\frac{1}{x^2}\right) - \frac{1}{x^2}\sin\left(\frac{1}{x}+b\right)\right]$$

$$\Rightarrow y_2 = \frac{-a}{x^3}\cos\left(\frac{1}{x}+b\right) = \frac{-ax}{x^4}\cos\left(\frac{1}{x}+b\right) = \frac{-y}{x^4}$$

$\Rightarrow x^4y_2 + y = 0$

154 **(b)**

Given, $dy = x\log x\ dx$

$\Rightarrow y = \frac{x^2}{2}\log x - \int \frac{x}{2}dx$

[integrating]

$\Rightarrow y = \frac{x^2}{2}\log x - \frac{x^2}{4} + c$

155 **(c)**

Given equation is $y = \sec(\tan^{-1}x)$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \sec(\tan^{-1} x)\tan(\tan^{-1} x)\cdot\frac{1}{1+x^2}$$

$$= \frac{xy}{1+x^2} \quad [\because \tan(\tan^{-1} x) = x]$$

$$\Rightarrow (1+x^2)\frac{dy}{dx} = xy$$

156 **(a)**

$$\frac{dy}{dx}\tan y = \sin(x+y) + \sin(x-y)$$

$$\Rightarrow \frac{dy}{dx}\tan y = 2\sin x\cos y$$

$$\Rightarrow \int \tan y \sec y\, dy = 2\int \sin x\, dx$$

$$\Rightarrow \sec y + 2\cos x = c$$

157 **(b)**

Equation of family of parabolas with focus at (0, 0) and x-axis as axis is

$y^2 = 4a(x+a)$ ....(i)

On differentiating Eq. (i), we get

$2yy_1 = 4a$, putting the value of a in Eq. (i)

$$\Rightarrow y^2 = 2yy_1\left(x + \frac{yy_1}{2}\right)$$

$$\Rightarrow y = 2xy_1 + yy_1^2$$

$$\Rightarrow y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} = y$$

158 **(b)**

We have,

$$\frac{dy}{dx} + y = \frac{1+y}{x} \Rightarrow \frac{dy}{dx} + \left(1 - \frac{1}{x}\right)y = \frac{1}{x}$$

$$\therefore \text{I.F.} = e^{\int\left(1-\frac{1}{x}\right)dx} = e^{x-\log x} = \frac{1}{x}e^x$$

159 **(c)**

Given, $y^2 = 4a(x-b)$

$$\Rightarrow \quad 2y\frac{dy}{dx} = 4a$$

$$\Rightarrow \quad 2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 = 0$$

$$\Rightarrow \quad y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

160 **(a)**

The given equation can be rewritten as,

$$\frac{d^2y}{dx^2} = -\sin x.$$

On integrating the given equation

$$\int \frac{d^2y}{dx^2}dx = \int -\sin x\, dx + c$$

$$\Rightarrow \frac{dy}{dx} = -(-\cos x) + c = \cos x + c$$

Again, on integrating, we get

$$\int \frac{dy}{dx}dx = \int \cos x\, dx + \int c\, dx + d$$

$$y = \sin x + cx + d$$

161 **(d)**

Given that, $\frac{dy}{dx} + y = e^{-x}$

It is a linear differential equation, comparing with the standard equation

$$\frac{dy}{dx} + Py = Q$$

$$\Rightarrow P = 1, Q = e^{-x}$$

$$\therefore \text{IF} = e^{\int P\,dx} = e^x$$

$\therefore$ Required solution is

$$ye^x = \int e^{-x}e^x\, dx + c = \int 1\, dx + c$$

$$\Rightarrow ye^x = x + c$$

At $x = 0, y = 0 \ \therefore c = 0$

Hence, the required solution is

$$ye^x = x \Rightarrow y = xe^{-x}$$

162 **(c)**

Given, $\frac{dy}{dx} = 2\frac{y}{x}$ $\qquad (\because y = mx)$

$$\Rightarrow \quad \int\frac{dy}{y} = 2\int\frac{dx}{x}$$

$$\Rightarrow \quad \log y = 2\log x + \log c$$

$$\Rightarrow \quad y = cx^2$$

Which represent a parabola of the form

$$x^2 = 4ay$$

163 **(d)**

Given, $\frac{dy}{dx} + \frac{2}{x}y = x$

$\therefore$ Integrating factor $= e^{\int\frac{2}{x}dx} = x^2$

$\therefore$ Required solution is

$$y.x^2 = \int x^3\, dx = \frac{x^4}{4} + \frac{c}{4}$$

$$\therefore \quad y = \frac{x^4+c}{4x^2}$$

164 **(a)**

We have,

$$y\frac{dy}{dx} = x - 1 \Rightarrow y\,dy = (x-1)dx \Rightarrow \frac{y^2}{2}$$

$$= \frac{x^2}{2} - x + C$$

For $x = 1$, we have $y = 1$

$$\therefore \frac{1}{2} = \frac{1}{2} - 1 + C \Rightarrow C = 1$$

Hence, $\frac{y^2}{2} = \frac{x^2}{2} - x + 1 \Rightarrow y^2 = x^2 - 2x + 2$

165 **(a)**

Equation of line whose slope is equal to y intercept, is

$$y = cx + c = c(x+1)$$

$$\Rightarrow \quad \frac{dy}{dx} = c$$

$$\therefore \quad \frac{dy}{dx} = \frac{y}{x+1}$$

$$\Rightarrow \quad (x+1)\frac{dy}{dx} - y = 0$$

166 **(b)**

Given that, $x^2y - x^3\ \frac{dy}{dx} = y^4 \cos x$

i.e., $\qquad x^3\frac{dy}{dx} - x^2y = -y^4\cos x$

on dividing by $-y^4x^3$, we get

$$-\frac{1}{y^4}\ \frac{dy}{dx} + \frac{1}{y^3}.\frac{1}{x} = \frac{1}{x^3}\cos x$$

Put $\ \frac{1}{y^3} = V$

$\Rightarrow \qquad -\frac{1}{y^4}\ \frac{dy}{dx} = \frac{1}{3}\frac{dV}{dx}$

$\therefore \qquad \frac{1}{3}\ \frac{dV}{dx} + \frac{1}{x}V = \frac{1}{x^3}\cos x$

$\Rightarrow \qquad \frac{dV}{dx} + \frac{3}{x}\ V = \frac{3}{x^3}\cos x$

Which is linear in V.

$\therefore \qquad IF = e^{\int\frac{3}{x}dx} = e^{3\log x} = x^3$

So, the solution is

$$x^3V = \int x^3.\frac{3}{x^3}\cos x\ \ dx + c$$

$$= 3\sin x + c$$

$\Rightarrow \qquad \frac{x^3}{y^3} = 3\sin x + c$

Putting $x = 0, y = 1$, we get $c = 0$

Hence, the solution is $x^3\ = 3y^3\sin x$

167 **(d)**

$\because$ Equation of normal at $P(1,1)$ is

$ay + x = a + 1$ $\qquad$ (given)

$\because$ Slope of normal at $(1,1) = -\frac{1}{a}$

$\therefore$ Slope of tangent at $(1,1) = a$ $\quad$ ...(i)

Also, given $\frac{dy}{dx} \propto y$

$\Rightarrow \frac{dy}{dx} = ky$

$\left.\frac{dy}{dx}\right|_{(1,1)} = k = a$ [from Eq. (i)]

Then, $\frac{dy}{dx} = ay$

$\Rightarrow \frac{dy}{y} = a\ dx$

$\Rightarrow \text{In}|y| = ax + c$

$\because$ It is passing through (1,1), then $c = -a$

$\Rightarrow \text{In}|y| = a(x - 1)$

$\Rightarrow |y| = e^{a(x-1)}$

168 **(a)**

Given, $\ \frac{dy}{dx} + 1 = e^{x+y}$

Put $\qquad x + y = z$

$\Rightarrow \qquad 1 + \frac{dy}{dx} = \frac{dz}{dx}$

$\therefore \qquad \frac{dz}{dx} = e^z$

$\Rightarrow \qquad \int e^{-z}\,dz = \int dx$

$\Rightarrow \qquad -e^{-z} = x + c$

$\Rightarrow \qquad x + e^{-(x+y)} + c = 0$

169 **(a)**

The given equation can be written as

$$\left(\frac{dx}{x} - \frac{dy}{y}\right) + \frac{(x^2dy - y^2dx)}{(x-y)^2} = 0$$

$$\Rightarrow \left(\frac{dx}{x} - \frac{dy}{y}\right) + \frac{\left(\frac{dy}{y^2} - \frac{dx}{x^2}\right)}{\left(\frac{1}{y} - \frac{1}{x}\right)^2} = 0$$

$$\Rightarrow \left(\frac{dx}{x} - \frac{dy}{y}\right) + \frac{\frac{dy}{y^2} - \frac{dx}{x^2}}{\left(\frac{1}{x} - \frac{1}{y}\right)^2} = 0$$

On integrating both sides, we get

$\text{In}\,|x| - \text{In}\,|y| - \frac{1}{\left(\frac{1}{x}-\frac{1}{y}\right)} = c$

$\Rightarrow \text{In}\left|\frac{x}{y}\right| - \frac{xy}{(y-x)} = c$

$\Rightarrow \text{In}\left|\frac{x}{y}\right| + \frac{xy}{(x-y)} = c$

170 **(a)**

We have, $e^{dy/dx} = x$

$\Rightarrow \frac{dy}{dx} = \log x$

$\therefore$ Degree is 1.

171 **(a)**

Given differential equation can be rewritten as

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{4}\times 12} = \left(\frac{d^2y}{dx^2}\right)^{\frac{1}{3}\times 12}$$

$$\Rightarrow \qquad \left[1 + \left(\frac{dy}{dx}\right)^2\right]^9 = \left(\frac{d^2y}{dx^2}\right)^4$$

Here, we see that order of highest derivative is 2 and degree is 4.

172 **(c)**

We have,

$\tan^{-1}x + \tan^{-1}y = C$

Differentiating w.r.t. to x, we get

$$\frac{1}{1+x^2} + \frac{1}{1+y^2}\frac{dy}{dx} = 0$$

$\Rightarrow (1+x^2)dy + (1+y^2)dx = 0,$
which is the required differential equation

173 **(d)**

Given that, $\frac{dy}{dx} = 1 + x + y^2 + xy^2$

This can be rewritten as, we get

$\frac{dy}{1+y^2} = (1+x)dx$

On integrating both sides, we get

$\int \frac{dy}{1+y^2} = \int (1+x)dx$

$\Rightarrow \tan^{-1} y = x + \frac{x^2}{2} + c$

At $x = 0, y = 0$

$\Rightarrow 0 = 0 + 0 + c \ \Rightarrow c = 0$

$\therefore \tan^{-1} y = x + \frac{x^2}{2} \Rightarrow y = \tan\left(x + \frac{x^2}{2}\right)$

175 **(b)**

$\frac{dy}{dx} = -\left(\frac{\cos x - \sin x}{\sin x + \cos x}\right)$

$\Rightarrow dy = -\left(\frac{\cos x - \sin x}{\sin x + \cos x}\right) dx$

On integrating both sides, we get

$y = -\log(\sin x + \cos x) + \log c$

$\Rightarrow y = \log\left(\frac{c}{\sin x + \cos x}\right)$

$\Rightarrow e^y = \frac{c}{\sin x + \cos x}$

$\Rightarrow e^y(\sin x + \cos x) = c$

176 **(a)**

Given, $\frac{x\ dy - y\ dx}{y^2} = dy$

$\Rightarrow \quad d\left(\frac{x}{y}\right) = -dy$

$\Rightarrow \quad \frac{x}{y} = -y + c$

[integrating]

As $\quad y(1) = 1 \ \Rightarrow c = 2$

$\therefore \quad \frac{x}{y} + y = 2$

Again, for $x = -3$

$-3 + y^2 = 2y$

$\Rightarrow \quad (y+1)(y-3) = 0$

Also, $\quad y > 0$

$\Rightarrow \quad y = 3$

[neglecting $y = -1$]

177 **(a)**

Given equation is, $\frac{dy}{dx} = \frac{y}{x} + \frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$ ...(i)

Put $y = vx \ \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Now, Eq. (i) becomes

$v + x\frac{dv}{dx} = v + \frac{\phi(v)}{\phi'(v)}$

$\Rightarrow \frac{\phi'(v)}{\phi(v)} dv = \frac{dx}{x}$

On integrating both sides, we get

$\int \frac{\phi'(v)}{\phi(v)} dv = \int \frac{1}{x} dx$

$\Rightarrow \log \phi(v) = \log x + \log k$

$\Rightarrow \log \phi(v) = \log xk$

$\Rightarrow \phi(v) = kx \ \Rightarrow \phi\left(\frac{y}{x}\right) = kx \quad \left(\because v = \frac{x}{y}\right)$

178 **(c)**

Given, $\frac{dx}{dy} = x + y + 1 \Rightarrow \frac{dx}{dy} - x = y + 1$

$\therefore \quad \text{IF} = e^{\int -1\, dy} = e^{-y}$

$\therefore$ Solution is $x.e^{-y} = \int (y+1)\, e^{-y} dy$

$\Rightarrow \quad xe^{-y} = -(y+1)e^{-y} + \int e^{-y}\ dy$

$\Rightarrow \quad xe^{-y} = -(y+1)e^{-y} - e^{-y} + c$

$\Rightarrow \quad x = -(y+2) + ce^y$

179 **(b)**

Given, $\quad x^2 + y^2 - 2ax = 0$ ...(i)

$\Rightarrow \quad 2x + 2yy' - 2a = 0$

$\Rightarrow \quad a = x + yy'$

On putting the value of a in Eq. (i), we get

$x^2 + y^2 - 2x(x + yy') = 0$

$\Rightarrow \quad y^2 - x^2 = 2xyy'$

180 **(c)**

Given, $\quad y = c_1 \cos(x + c_2) + c_3 \sin(x + c_4) + c_5 e^x + c_6$

$y = c_1[\cos x \cos c_2 - \sin x \sin c_2] + c_3[\sin x \cos c_4 + \cos x \sin c_4] + c_5 e^x + c_6$

$= \cos x\,(c_1 \cos c_2 + c_3 \sin c_4) + \sin x\,(-c_1 \sin c_2 + c_3 \cos c_4) + c_5 e^x + c_6$

$= A \cos x + B \sin x + Ce^x + D$

Where $\quad A = c_1 \cos c_2 + c_3 \sin c_4$

$B = -c_1 \sin c_2 + c_3 \cos c_4, \ C = c_5, D = c_6$

Hence, order is 4.

182 **(a)**

Given, $(1+x)y\ dx + (1-y)x\ dy = 0$

$\Rightarrow \quad \frac{(1-y)}{y}\,dy + \frac{(1+x)}{x}\,dx = 0$

$\Rightarrow \quad \int\left(\frac{1}{y}-1\right)dy + \int\left(\frac{1}{x}+1\right)dx = 0$

$\Rightarrow \quad \log_e y - y + \log_e x + x = c$

$\Rightarrow \quad \log_e(xy) + x - y = c$

183 **(b)**

Given, $\quad y^2 = 2c(x+\sqrt{c})$

$\Rightarrow \quad 2yy_1 = 2c$

$\Rightarrow \quad c = yy_1$

$\therefore \quad y^2 = 2yy_1(x+\sqrt{yy_1})$

$\Rightarrow \quad y^2 - 2yy_1x = \sqrt{yy_1}.\,2yy_1$

$\Rightarrow \quad (y^2-2yy_1x)^2 = 4(yy_1)^3$

$\therefore$ The degree of above equation is 3 and order is 1.

184 **(d)**

Given differential equation is

$\frac{dy}{dx} = \frac{1}{x+y^2}$

$\Rightarrow \quad \frac{dx}{dy} - x = y^2$

Here, $\; P = -1,\; Q = y^2$

$\text{IF} = e^{\int -1\,dy} = e^{-y}$

$\therefore$ Solution is

$xe^{-y} = \int e^{-y}\; y^2\;\; dy$

$= -e^{-y}\,y^2 + \int 2e^{-y}\; y\,dy$

$= -e^{-y}\,y^2 + 2[-e^{-y}\,y +$

$e{-}y\,dy{+}c$

$= -e^{-y}\,y^2 + 2[-e^{-y}\,y - e^{-y}] +$

c

$\Rightarrow \quad xe^{-y} = e^{-y}(-y^2-2y-2)+c$

$\Rightarrow \quad x = -y^2-2y-2+ce^{y}$

185 **(a)**

Given differential equation can be rewritten as

$\Rightarrow \quad \frac{dx}{dy} - \frac{x}{y^2} = 2y$

$\therefore \quad \text{IF} = e^{-\int\frac{1}{y^2}dy} = e^{1/y}$

186 **(c)**

It is given that $\frac{dy}{dx} = \frac{x}{y}$

On integration, we get $y^2 - x^2 = C$, which is a rectangular hyperbola

187 **(a)**

Given differential equation is

$(1+y^2) + \left(x - e^{\tan^{-1}y}\right)\frac{dy}{dx} = 0$

$\Rightarrow \quad (1+y^2)\frac{dx}{dy} = -x + e^{\tan^{-1}y}$

$\Rightarrow \quad \frac{dx}{dy} + \frac{x}{1+y^2} = \frac{e^{\tan^{-1}y}}{1+y^2}$

Which is a linear differential equation,

Here, $\quad P = \frac{1}{1+y^2},\; Q = \frac{e^{\tan^{-1}y}}{1+y^2}$

$\text{IF} = e^{\int P\,dy} = e^{\int\frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$

$\therefore$ Solution is

$x.\text{IF} = \int Q.\text{IF}\;dy + c$

$xe^{\tan^{-1}y} = \int\frac{e^{\tan^{-1}y}}{1+y^2}\,.e^{\tan^{-1}y} + \frac{c}{2}$

$\Rightarrow \quad xe^{\tan^{-1}y} = \frac{e^{2\tan^{-1}y}}{2} + \frac{c}{2}$

$\therefore \quad 2xe^{\tan^{-1}y} = e^{2\tan^{-1}y} + c$

188 **(d)**

Given differential equation can be rewritten as

$\frac{y\,dy}{y+1} = \frac{e^{x}dx}{e^x+1}$

$\Rightarrow \quad \left(1-\frac{1}{y+1}\right)dy = \frac{e^x}{e^x+1}dx$

$\Rightarrow \quad y - \log(y+1) = \log(e^x+1) - \log c$

[integrating]

$\Rightarrow \quad y = \log\frac{(e^x+1)(y+1)}{c}$

$\Rightarrow \quad (e^x+1)(y+1) = ce^{y}$

189 **(d)**

The equation of all the straight lines passing through origin is

$y = mx$

$\Rightarrow \quad \frac{dy}{dx} = m$

...(i)

$\therefore$ From Eq. (i), $\quad y = \frac{dy}{dx}\,x$

190 **(b)**

Given, $\frac{dy}{dx} = \sin(x+y)\tan(x+y) - 1$

Put $\quad x+y = z \Rightarrow 1 + \frac{dy}{dx} = \frac{dz}{dx}$

$\therefore \quad \frac{dz}{dx} - 1 = \sin z\tan z - 1$

$\Rightarrow \quad \int\frac{\cos z}{\sin^2 z}dz = \int dx$

Put $\quad \sin z = t$

$\therefore \quad \int\frac{1}{t^2}\,dt = x - c \Rightarrow -\frac{1}{t} = x - c$

$\Rightarrow \quad -\text{cosec}\, z = x - c$

$\Rightarrow \quad x + \text{cosec}\,(x+y) = c$

191 **(a)**

Given, $\quad \sin^{-1}x + \sin^{-1}y = c$

$\Rightarrow \quad \frac{dx}{\sqrt{1-x^2}} + \frac{dy}{\sqrt{1-y^2}} = 0$

$\Rightarrow \quad \sqrt{1-y^2}\,dx+\sqrt{1-x^2}\,dy=0$

192 **(d)**

$x\,\frac{dy}{dx}+(1+x)y=x$

$\Rightarrow \quad \frac{dy}{dx}+\frac{1+x}{x}y=1$

$\text{IF}=e^{\int\frac{1+x}{x}dx}$

$=e^{\int\frac{dx}{x}+\int dx}$

$=e^{\log x+x}$

$=xe^x$

193 **(c)**

We have,

$(x+2y^3)\frac{dy}{dx}=y$

$\Rightarrow y\frac{dy}{dx}=x+2y^3 \Rightarrow \frac{dx}{dy}-\frac{x}{y}=2y^2 \quad$ ...(i)

This is linear differential equation with

$\text{I.F.}=e^{\int-\frac{1}{y}dy}=e^{-\log y}=\frac{1}{y}$

Multiplying (i) by I.F. and integrating, we get

$\frac{x}{y}=\int 2y\,dy \Rightarrow \frac{x}{y}=y^2+C \Rightarrow x=y(y^2+C)$

194 **(a)**

We have,

$\frac{d^2y}{dx^2}=\sqrt{1+\left(\frac{dy}{dx}\right)^3} \Rightarrow \left(\frac{d^2y}{dx^2}\right)^2=1+\left(\frac{dy}{dx}\right)^3$

Clearly, it is a second order second degree differential equation

195 **(a)**

Given equation is $\frac{dy}{dx}=\frac{y+1}{x-1} \Rightarrow \frac{dy}{y+1}=\frac{dx}{x-1}$

On integrating both sides

$\int\frac{dy}{y+1}=\int\frac{dx}{x-1}$

$\Rightarrow \log(y+1)=\log(x-1)+\log c$

$\Rightarrow \log(y+1)=\log(x-1)c$

$\Rightarrow y+1=(x-1)c$

At $x=1 \Rightarrow y=-1$

Whereas $y(1)=2$.

Hence, the above solution is not possible.

196 **(a)**

Given, $\quad \frac{dy}{dx}=\frac{y(x+y)}{x(x-y)}$

Put $\quad y=vx$

$\Rightarrow \quad \frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore \quad v+x\,\frac{dv}{dx}=\frac{vx(x+vx)}{x(x-vx)}$

$x\,\frac{dv}{dx}=\frac{2v^2}{1-v}$

$\Rightarrow \quad \frac{1}{2}\left[\frac{1}{v^2}-\frac{1}{v}\right]dv=\frac{dx}{x}$

$\Rightarrow \quad \frac{1}{2}\left[-\frac{1}{v}-\log v\right]=\log x+c_1$

$\Rightarrow \quad \frac{x}{v}+\log\left(\frac{y}{x}\right)+2\log x=-2c$

$\Rightarrow \quad \frac{x}{y}+\log(xy)=c$

[let $c=-2c_1$]

197 **(a)**

We have,

$y^2dy=x^2dx$

Integrating we get $y^3-x^3=C$

198 **(b)**

Given, $\quad \frac{d^2y}{dx^2}=e^{-2x}$

$\Rightarrow \quad \frac{dy}{dx}=-\frac{e^{-2x}}{2}+c_2$

[integrating]

$\Rightarrow \quad y=\frac{e^{-2x}}{4}+c_2x+c_3$

[integrating]

But $\quad y=c_1e^{-2x}+c_2x+c_3$

[given]

$\therefore \quad c_1=\frac{1}{4}$

199 **(d)**

Given, $\quad x\left(\frac{dy}{dx}\right)^2+2\sqrt{xy}\frac{dy}{dx}+y=0$

$\Rightarrow \quad \left(\sqrt{x}\,\frac{dy}{dx}+\sqrt{y}\right)^2=0$

$\Rightarrow \quad \frac{1}{\sqrt{y}}dy+\frac{1}{\sqrt{x}}dx=0$

$\Rightarrow \quad 2\sqrt{y}+2\sqrt{x}=c_1$

$\Rightarrow \quad \sqrt{x}+\sqrt{y}=c$

200 **(b)**

$y=mx+\frac{4}{m} \quad$ ...(i)

$\therefore \quad \frac{dy}{dx}=m$

From Eq. (i), we get

$y=x\left(\frac{dy}{dx}\right)+\frac{4}{\left(\frac{dy}{dx}\right)}$

$\Rightarrow \quad y\left(\frac{dy}{dx}\right)=x\left(\frac{dy}{dx}\right)^2+4$

$\Rightarrow \quad x\left(\frac{dy}{dx}\right)^2-y\,\frac{dy}{dx}+4=0$

Which is required differential equations.

201 **(d)**

We have,
$e^x \cos y\ dx - e^x \sin y\ dy = 0$
$\Rightarrow \cos y\, d(e^x) + e^x d(\cos y) = 0$
$\Rightarrow d(e^x \cos y) = 0 \Rightarrow e^x \cos y = C$ [On integrating]

202 **(d)**
$y = ae^{mx} + be^{-mx}$
On differentiating w. r. t. x, we get
$\frac{dy}{dx} = mae^{mx} - mbe^{-mx}$
Again, on differentiating, we get
$\frac{d^2y}{dx^2} = m^2ae^{mx} + m^2be^{-mx}$
$= m^2(ae^{mx} + be^{-mx}) = m^2y$
$\Rightarrow \frac{d^2y}{dx^2} - m^2y = 0$

203 **(b)**
We have,
$\frac{dy}{dx} + y = e^{-x}$ ...(i)
This is a linear differential equation with
$\text{I. F.} = e^{\int 1\cdot dx} = e^x$
Multiplying both sides of (i) by I.F. $= e^x$ and integrating, we get
$y\, e^x = \int e^x\, e^{-x} dx + C \Rightarrow y\, e^x = x + C$
It is given that $y = 0$ when $x = 0$
$\therefore 0 = 0 + C \Rightarrow C = 0$
Hence, $y\, e^x = x \Rightarrow y = xe^{-x}$

204 **(c)**
Given, $\frac{dx}{1+x^2} + \frac{dy}{1+y^2} = 0$
$\Rightarrow \quad \tan^{-1} x + \tan^{-1} y = \tan^{-1} c$
[integrating]
$\Rightarrow \quad \frac{x+y}{1-xy} = c$
$\Rightarrow \quad x + y = c(1 - xy)$

205 **(c)**
Given, IF $= x$
$\therefore \quad e^{\int P\, dx} = x$
$\Rightarrow \quad \int P\, dx = \log x$
$\Rightarrow \quad P = \frac{d}{dx} \log x = \frac{1}{x}$

206 **(b)**
Given differential equation is

$$\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$\Rightarrow \quad \left(\frac{d^2y}{dx^2}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$

Hence, order is 2.

207 **(b)**
Given, $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$
$\Rightarrow \quad \int \frac{dy}{\sqrt{1-y^2}} + \int \frac{dx}{\sqrt{1-x^2}} = 0$
$\Rightarrow \quad \sin^{-1} x + \sin^{-1} y = c$

208 **(d)**
We have,
$x\, dy - y\, dx = 0$
$\Rightarrow \frac{dy}{y} - \frac{dx}{x} = 0$
$\Rightarrow \log y - \log x = \log C$ [On integrating]
$\Rightarrow \frac{y}{x} = C \Rightarrow y = C\, x$
Clearly, it represents a family of straight lines passing through the origin

209 **(c)**
Let the equation of circle passing through given points is
$x^2 + y^2 - 2fy = a^2$
$\Rightarrow \quad 2x + 2yy_1 - 2fy_1 = 0$
...(i)
$\Rightarrow \quad x = y_1(f - y)$
$\Rightarrow \quad x = y_1\left(\frac{x^2+y^2-a^2}{2y} - y\right)$
[from Eq. (i)]
$\Rightarrow \quad y_1(y^2 - x^2 + a^2) + 2xy = 0$

211 **(a)**
Given, $\frac{dy}{dx} + \left(\frac{x}{y}\right)^2 - \left(\frac{x}{y}\right) + 1 = 0$
Put $v = \frac{x}{y} \Rightarrow x = vy$
$\Rightarrow \quad \frac{dx}{dy} = v + y\frac{dv}{dy}$
$\therefore \quad v + y\frac{dv}{dy} + v^2 - v + 1 = 0$
$\Rightarrow \quad \frac{dv}{v^2+1} + \frac{dy}{y} = 0$
$\Rightarrow \quad \tan^{-1} v + \log y + c = 0$
[integrating]
$\Rightarrow \quad \tan^{-1}\frac{x}{y} + \log y + c = 0$

212 **(a)**
Given, $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{5/2} = \frac{d^3y}{dx^3}$
$\Rightarrow \quad \left(\frac{d^3y}{dx^3}\right)^2 = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^5$
Here, order=3, degree=2

213 **(a)**
Given equation is

$x\,dy - y\,dx + x^2 e^x dx = 0$

$\Rightarrow \dfrac{x\,dy - y\,dx}{x^2} + e^x dx = 0$

$\Rightarrow d\left(\dfrac{y}{x}\right) + d(e^x) = 0$

$\Rightarrow \dfrac{y}{x} + e^x = c$

214 **(c)**

Given differential equation is

$\frac{dy}{dx} = \frac{x-y+3}{2(x-y)+5}$

Put $\quad x - y = v \Rightarrow \frac{dy}{dx} = 1 - \frac{dv}{dx}$

$\therefore \quad 1 - \frac{dv}{dx} = \frac{v+3}{2v+5} \Rightarrow \frac{dv}{dx} = \frac{v+2}{2v+5}$

$\Rightarrow \quad \int \left(2 + \frac{1}{v+2}\right) dv = \int dx$

$\Rightarrow \quad 2v + \log(v+2) = x + c$

$\Rightarrow \quad 2(x-y) + \log(x - y + 2) = x + c$

215 **(c)**

The given equation is $Ax^2 + By^2 = 1$

$\Rightarrow \quad 2Ax + 2By\,\frac{dy}{dx} = 0$

...(i)

$\Rightarrow \quad 2A + 2B\left\{\left(\frac{dy}{dx}\right)^2 + y\,\frac{d^2y}{dx^2}\right\} = 0$

...(ii)

Eliminating A and B from Eqs. (i) and (ii), we get

$$y\,\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 - \frac{y}{x}.\frac{dy}{dx} = 0$$

Here, order $=2$, degree $=1$

216 **(a)**

The given equation is

$(y+3)dy = (x+2)dx$

$\Rightarrow \dfrac{y^2}{2} + 3y = \dfrac{x^2}{2} + 2x + c$

Since, it passes through (2, 2).

$\therefore 2 + 6 = 2 + 4 + c \ \Rightarrow c = 2$

$\therefore \dfrac{y^2}{2} + 3y = \dfrac{x^2}{2} + 2x + 2$

$\Rightarrow y^2 + 6y = x^2 + 4x + 4$

$\Rightarrow x^2 + 4x - y^2 - 6y + 4 = 0$

217 **(b)**

We have,

$y^2 = 4a(x+a) \qquad$ ...(i)

Clearly, it is a one parameter family of parabolas

Differentiating (i) w.r.t. to x, we get

$2y\dfrac{dy}{dx} = 4a \Rightarrow a = \dfrac{1}{2}y\dfrac{dy}{dx}$

Substituting this value of a in (i), we get

$y^2 = 2y\dfrac{dy}{dx}\left(x + \dfrac{1}{2}y\dfrac{dy}{dx}\right)$

$\Rightarrow y^2\left(\dfrac{dy}{dx}\right) + 2xy\dfrac{dy}{dx} - y^2 = 0$

218 **(b)**

Given differential equation can be rewritten as

$\frac{dy}{dx} + \frac{1}{x\log x}.y = \frac{2}{x}$

$\therefore \quad \text{IF} = e^{\int \frac{1}{x\log x}dx} = e^{\int \frac{1/x}{\log x}dx} = e^{\log\log x} = \log x$

220 **(a)**

Given, $\quad \frac{dy}{dx} + y\tan x = \sec x$

$\therefore \quad IF e^{\int P\,dx} = e^{\int \tan x\,dx} = \sec x$

$\therefore$ Solution is $\ y\sec x = \int \sec^2 x\ dx + c$

$\Rightarrow \quad y\sec x = \tan x + c$

222 **(c)**

We have,

$\dfrac{dy}{dx} = \left(\dfrac{y}{x}\right)^{1/3}$

$\Rightarrow y^{-1/3}dy = x^{-1/3}dx$

$\Rightarrow \displaystyle\int y^{-1/3}dy = \int x^{-1/3}dx$

$\Rightarrow \dfrac{3}{2}y^{2/3} = \dfrac{3}{2}x^{2/3} + C$

$\Rightarrow y^{2/3} = x^{2/3} + C'$, where $C' = 2C$

$\Rightarrow y^{2/3} - x^{2/3} + C'$

223 **(c)**

Given, $\frac{dy}{dx} = \frac{y\sin\left(\frac{y}{x}\right) - x}{x\sin\left(\frac{y}{x}\right)} = \frac{\frac{y}{x}\sin\left(\frac{y}{x}\right) - 1}{\sin\left(\frac{y}{x}\right)}$

Put $\quad \frac{y}{x} = u$

$\Rightarrow \quad \frac{dy}{dx} = x\,\frac{du}{dx} + u$

$\therefore \quad x\,\frac{du}{dx} + u = \frac{u\sin u - 1}{\sin u}$

$\Rightarrow \quad -\sin u\ du = \frac{1}{x}\ dx$

$\Rightarrow \quad \cos u = \log x + c$

[integrating]

$\Rightarrow \quad \cos\left(\frac{y}{x}\right) = \log x + c$

$\therefore \quad y(1) = \frac{\pi}{2}$

$\therefore \quad \cos\frac{\pi}{2} = \log 1 + c$

$\Rightarrow \quad c = 0$

Thus, $\quad \cos\left(\frac{y}{x}\right) = \log x$

224 **(b)**

Given, $\quad \frac{dy}{dx} - \frac{y}{x} = \frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$

$\Rightarrow \quad \frac{\phi'\left(\frac{y}{x}\right)\left(\frac{x\,dy - y\,dx}{x^2}\right)}{\phi\left(\frac{y}{x}\right)} = \frac{1}{x}\,dx$

$\Rightarrow \quad \int \frac{\phi'\left(\frac{y}{x}\right) d\left(\frac{y}{x}\right)}{\phi\left(\frac{y}{x}\right)} = \int \frac{1}{x}\,dx + \log k$

$\Rightarrow \quad \log \phi\left(\frac{y}{x}\right) = \log x + \log k$

$\Rightarrow \quad \phi\left(\frac{y}{x}\right) = kx$

225 **(b)**

Given, $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$

Put $y = vx$

$\Rightarrow \quad \frac{dy}{dx} = v + x\,\frac{dv}{dx}$

$\therefore \quad v + x\frac{dv}{dx} = \frac{x^2 v}{x^2(1+v^2)}$

$\Rightarrow \quad \int \frac{1+v^2}{v^3}\,dv = -\int \frac{dx}{x}$

$\Rightarrow \quad -\frac{1}{2v^2} + \log v = -\log x + \log c$

$\Rightarrow \quad -\frac{1}{2}.\frac{x^2}{y^2} + \log|y| = \log c$

$\therefore \quad y(1) = 1,\ -\frac{1}{2} = \log c$

$\therefore \quad -\frac{1}{2}.\frac{x^2}{y^2} + \log|y| = -\frac{1}{2}$

$\Rightarrow \quad \log_e |y| + \frac{1}{2} = \frac{x^2}{2y^2}$

Again, when $x = x_0, y = e$

$1 + \frac{1}{2} = \frac{x_0^2}{2e^2} \Rightarrow x_0 = \sqrt{3}e$

226 **(d)**

Given, $3^{-y}dy = 3^x dx$

$\Rightarrow \quad \int 3^{-y}\,dy = \int 3^x\,dx$

$\Rightarrow \quad \frac{-3^{-y}}{\log 3} = \frac{3^x}{\log 3} + k$

$\Rightarrow \quad 3^x + 3^{-y} = c$, where $c = -k\log 3$

227 **(a)**

$(x-h)^2 + (y-k)^2 = r^2$, here only one arbitrary constant r. So, order of differential equation = 1.

229 **(b)**

Given differential equation can be rewritten as

$$\frac{y}{(1+y^2)}dy = \frac{dx}{x(1+x^2)}$$

$\Rightarrow \quad \frac{1}{2}\int \frac{2y}{(1+y^2)}dy = \frac{1}{2}\int \frac{2x}{x^2(1+x^2)}dx$

$\Rightarrow \quad \frac{1}{2}\int \frac{2y}{(1+y^2)}dy = \frac{1}{2}\int \frac{dt}{t(1+t)}$

[put $x^2 = t$ in RHS integral]

$\Rightarrow \quad \frac{1}{2}\int \frac{2y\,dy}{1+y^2} = \frac{1}{2}\int \left(\frac{1}{t} - \frac{1}{1+t}\right)dt$

$\Rightarrow \quad \frac{1}{2}\log(1+y^2) = \frac{1}{2}[\log t - \log(1+$
$t+12$]$\log c$

$\Rightarrow \quad \log(1+y^2) = \log x^2 - \log(1+x^2) + \log c$

$\Rightarrow \quad \log(1+y^2)(1+x^2) = \log cx^2$

$\Rightarrow \quad (1+y^2)(1+x^2) = cx^2$

230 **(b)**

Given, $\frac{dy}{dx} = \frac{2x-y}{x+2y}$

Put $y = vx$

$\Rightarrow \quad \frac{dy}{dx} = v + \frac{xdv}{dx}$

$\therefore \quad v + x\,\frac{dv}{dx} = \frac{2-v}{1+2v}$

$\Rightarrow \quad \frac{xdv}{dx} = \frac{2-v-v(1+2v)}{1+2v}$

$\Rightarrow \quad \int \frac{1+2v}{2(1-v-v^2)}\,dv = \int \frac{1}{x}\,dx$

$\Rightarrow \quad \log k - \frac{1}{2}\log(1-v-v^2) = \log x$

$\Rightarrow \quad \log c = \log[x^2(1-v-v^2)]$

[put $k^2 = c$]

$\Rightarrow \quad x^2 - xy - y^2 = c$

[put $v = \frac{y}{x}$]

231 **(c)**

$y^2 = 2c(x + c^{2/3})$

$\Rightarrow \quad 2y\,\frac{dy}{dx} = 2c \Rightarrow c = y\,\frac{dy}{dx}$

$\therefore \quad y^2 = 2y\,\frac{dy}{dx}\left(x + \left(y\,\frac{dy}{dx}\right)^{2/3}\right)$

$\Rightarrow \quad \left(\frac{y}{2\frac{dy}{dx}} - x\right) = \left(y\,\frac{dy}{dx}\right)^{2/3}$

$\Rightarrow \quad \left(y - 2x\,\frac{dy}{dx}\right)^3 = \left(2\,\frac{dy}{dx}\right)^3\left(y\,\frac{dy}{dx}\right)^2$

$\Rightarrow \quad \left(y - 2x\,\frac{dy}{dx}\right)^3 = 8y^2\left(\frac{dy}{dx}\right)^5$

Here, order=1, degree=5

232 **(a)**

Given equation is $\frac{dx}{x} + \frac{dy}{y} = 0$

On integrating, we get

$$\int \frac{dx}{x} + \int \frac{dy}{y} = 0$$

$\Rightarrow \log x + \log y = \log c$

$\Rightarrow \log(xy) + \log c \Rightarrow xy = c$

233 **(a)**

Given, $y = \left(x + \sqrt{1+x^2}\right)^n$

$\Rightarrow \quad \frac{dy}{dx} = n\left[x + \sqrt{1+x^2}\right]^{n-1}\left(1 + \frac{x}{\sqrt{x^2+1}}\right)$

$= \frac{n\left[x+\sqrt{1+x^2}\right]^n}{\sqrt{1+x^2}}$

$\Rightarrow \quad \left(\frac{dy}{dx}\right)^2(1+x^2) = n^2y^2$

Again, differentiating, we get

$$2\frac{dy}{dx}.\frac{d^2y}{dx^2}(1+x^2)+2x\left(\frac{dy}{dx}\right)^2 =$$
$2n^2y\frac{dy}{dx}$

$\Rightarrow \quad \frac{d^2y}{dx^2}(1+x^2)+x\frac{dy}{dx}=n^2y$

[divide by $2\frac{dy}{dx}$]

234 **(c)**

$y=(c_1+c_2)\cos(x+c_3)-c_4e^{x+c_5}$
$y_1=-(c_1+c_2)\sin(x+c_3)-c_4\,e^{x+c_5}$
$y_2=-(c_1+c_2)\cos(x+c_3)-c_4\,e^{x+c_5}$
$\quad =-y-2c_4e^{x+c_5}$
$y_3=-y_1-2c_4e^{x+c_5}$
$y_3=-y_1+y_2-y$

∴ Differential equation is

$y_3-y_2+y_1-y=0$

Which is order 3

235 **(c)**

The given equation is

$$y=ae^{bx}$$

$\Rightarrow \quad \frac{dy}{dx}=abe^{bx}$ …(i)

$\Rightarrow \quad \frac{d^2y}{dx^2}=ab^2e^{bx}$ …(ii)

$\Rightarrow \quad ae^{bx}\ \frac{d^2y}{dx^2}=a^2b^2e^{2bx}$

$\Rightarrow \quad y\ \frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$

[from eq. (ii)]

236 **(d)**

Let $ax+by=1$, where $a\neq 0$

$\Rightarrow \quad a\frac{dx}{dy}+b=0$

$\Rightarrow \quad a\frac{d^2x}{dy^2}=0$

$\Rightarrow \quad \frac{d^2x}{dy^2}=0$

237 **(a)**

We have, $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$

Putting $x=\sin A$, $y=\sin B$, we get

$\cos A+\cos B=a(\sin A-\sin B)$

$\Rightarrow \quad \cot\frac{A-B}{2}=a$

$\Rightarrow \quad A-B=2\cot^{-1}a$

$\Rightarrow \quad \sin^{-1}x-\sin^{-1}y=2\cot^{-1}a$

On differentiating w.r.t. x, we get

$$\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=0$$

$\Rightarrow \quad \frac{dy}{dx}=\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

Clearly, it is differential equation of the first order and first degree.

238 **(b)**

Given differential equation is

$$\frac{dy}{dx}=e^{y+x}+e^{y-x}$$

$\Rightarrow \quad \int e^{-y}\,dy=\int(e^x-e^{-x})\,dx$

$\Rightarrow \quad -e^{-y}=e^x-e^{-x}-c$

$\Rightarrow \quad e^{-y}=e^{-x}-e^{-x}+c$

239 **(a)**

Given, $\frac{dy}{dx}+\frac{1}{x}.y=3x$

∴ $\quad \text{IF}=e^{\int\frac{1}{x}dx}=e^{\log x}=x$

240 **(a)**

Given, $\frac{\sec^2 x}{\tan x}dx=-\frac{\sec^2 y}{\tan y}dy$

$\Rightarrow \quad \int\frac{\sec^2 x}{\tan x}dx=-\int\frac{\sec^2 y}{\tan y}dy$

Put $\quad \tan x=u$

$\Rightarrow \quad \sec^2 x\ dx=du$

And $\quad \tan y=v$

$\Rightarrow \quad \sec^2 y\ dy=dv$

∴ $\quad \int\frac{du}{u}=-\int\frac{dv}{v}$

$\Rightarrow \quad \log u=-\log v+\log c\Rightarrow uv=c$

∴ $\quad \tan x.\tan y=c$

241 **(b)**

We have,

$y\frac{dy}{dx}=\lambda\,y^2\Rightarrow\frac{dy}{dx}=\lambda\,y$

$\Rightarrow\frac{1}{y}dy=\lambda\,dx\Rightarrow\log y=\lambda\,x+\log C\Rightarrow y=Ce^{\lambda\,x}$

242 **(b)**

We have,

$$\frac{dy}{dx}+\frac{1+\cos 2y}{1-\cos 2x}=0$$

Given, $\frac{dy}{dx}=-\frac{1+\cos 2y}{1-\cos 2x}=-\frac{2\cos^2 y}{2\sin^2 x}$

$\Rightarrow \quad \int\sec^2 y\ dy=-\int\text{cosec}^2x\ dx$

$\Rightarrow \tan y=\cot x+c.$

243 **(a)**

Given differential equation can be rewritten as

$$\frac{dy}{dx}+\frac{y}{\sqrt{x}}=e^{-2\sqrt{x}}$$

Here, $P=\frac{1}{\sqrt{x}}, Q=e^{-2\sqrt{x}}$

∴ $\quad \text{IF}=e^{\int\frac{1}{\sqrt{x}}dx}=e^{2\sqrt{x}}$

∴ Solution is

$$ye^{2x} = \int e^{2\sqrt{x}} e^{-2\sqrt{x}} dx = \int 1 \ dx$$

$\Rightarrow \quad ye^{2\sqrt{x}} = x + c$

244 **(a)**

Given, $\left(1+\frac{1}{y}\right) dy = -e^x (\cos^2 x - \sin 2x)dx$

On integrating both sides, we get

$y + \log y = -e^x \cos^2 x + \int e^x \sin 2x \ dx - \int e^x \sin 2x \ dx + c$

$\Rightarrow \quad y + \log y = -e^x \cos^2 x + c$

At x=0, y=1

$1 + 0 = -e^0 \cos 0 + c \Rightarrow c = 2$

$\therefore$ Required solution is

$y + \log y = -e^x \cos^2 x + 2$

245 **(d)**

Given, $\frac{dy}{dx} = (4x + y + 1)^2$

Put $\quad 4x + y + 1 = v$

$\Rightarrow \quad \frac{dy}{dx} = \frac{dv}{dx} - 4$

$\therefore \quad \frac{dv}{dx} - 4 = v^2$

$\Rightarrow \quad \frac{dv}{v^2+4} = dx$

$\Rightarrow \quad \frac{1}{2}\tan^{-1}\left(\frac{v}{2}\right) = x + c$

[integrating]

$\Rightarrow \quad \tan^{-1}\left(\frac{4x+y+1}{2}\right) = 2x + c$

$\Rightarrow \quad 4x + y + 1 = 2\tan(2x + c)$

246 **(c)**

Given differential equation is

$x\frac{dy}{dx} + y\log x = xe^x x^{-\frac{1}{2}\log x}$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x}\log x = e^x x^{-\frac{1}{2}\log x}$

Here, $P = \frac{1}{x}\log x$ and $Q = e^x x^{-\frac{1}{2}\log x}$

$\therefore$ IF $= e^{\int \frac{\log x}{x}dx} = e^{\frac{(\log x)^2}{2}} = \left(\sqrt{e}\right)^{(\log x)^2}$

247 **(a)**

Let us assume the equation of parabola whose axis is parallel to y-axis and touch x-axis.

$y = ax^2 + bx + c$ ...(i)

and $b^2 = 4ac$ ($\because$ curve touches x-axis)

$\because$ There are two arbitrary constant.

$\therefore$ Order of this equation is 2.

248 **(a)**

Here, $y = A\cos\omega t + B\sin\omega t$ ....(i)

On differentiating w. r. t. t, we get

$\frac{dy}{dt} = -\omega A\sin\omega t + \omega B\cos\omega t$

Again, on differentiating w. r. t. t, we get

$\frac{d^2y}{dt^2} = -\omega^2 A\cos\omega t - \omega^2 B\sin\omega t$

$\Rightarrow \frac{d^2y}{dt^2} = -\omega^2(A\cos\omega t - B\sin\omega t)$

$\therefore y_2 = -\omega^2 y$ [from Eq. (i)]

249 **(a)**

Given, $\frac{dy}{dx}\tan y = \sin(x + y) + \sin(x - y)$

$\Rightarrow \quad \frac{dy}{dx}\tan y = 2\sin x\cos y$

$\Rightarrow \quad \tan y\sec y \ dy = 2\sin x \ dx$

$\Rightarrow \quad \sec y = -2\cos x + c$

[integrating]

$\Rightarrow \quad \sec y + 2\cos x = c$

250 **(a)**

Putting $x = \tan A$, and $y = \tan B$ in the given relation, we get

$\cos A + \cos B = \lambda(\sin A - \sin B)$

$\Rightarrow \tan\left(\frac{A-B}{2}\right) = \frac{1}{\lambda}$

$\Rightarrow \tan^{-1} x - \tan^{-1} y = 2\tan^{-1}\left(\frac{1}{\lambda}\right)$

Differentiating w.r.t. to x, we get

$\frac{1}{1+x^2} - \frac{1}{1+y^2}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

Clearly, it is a differential equation of degree 1

251 **(b)**

Given, $\frac{dy}{dx} - y\tan x = e^x \sec x$

$\therefore \quad$ IF $= e^{-\int \tan x \, dx} = e^{-\log\sec x} = \frac{1}{\sec x}$

$\therefore$ Complete solution is

$\Rightarrow \quad y.\frac{1}{\sec x} = \int e^x \sec x . \frac{1}{\sec x} \ dx$

$\Rightarrow \quad \frac{y}{\sec x} = e^x + c$

$\Rightarrow \quad y\cos x = e^x + c$

252 **(c)**

$x = 1 + \frac{dy}{dx} + \frac{1}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{1}{3!}\left(\frac{dy}{dx}\right)^3 + \cdots.$

$\Rightarrow \quad x = e^{\frac{dy}{dx}} \Rightarrow \frac{dy}{dx} = \log_e x$

$\Rightarrow$ Degree of differential equation is 1.

253 **(a)**

Given, $\frac{dy}{y+1} = \frac{-\cos x}{2+\sin x} \ dx$

$\Rightarrow \quad \int\frac{dy}{y+1} = -\frac{\int \cos x}{2+\sin x}dx$

$\Rightarrow \quad \log(y + 1) = -\log(2 + \sin x) + \log c$

When $\quad x = 0, y = 1$

$\Rightarrow \quad c = 4$

$\therefore \quad y + 1 = \frac{4}{2+\sin x}$

At $x=\frac{\pi}{2}$, $\quad y+1=\frac{4}{2+1}$

$\Rightarrow \quad y=\frac{1}{3}$

254 **(c)**

Given, $\cos y\frac{dy}{dx}=e^{x+\sin y}+x^2e^{\sin y}$

$\Rightarrow \quad \cos y\ \frac{dy}{dx}=e^{\sin y}(e^x+x^2)dx$

$\Rightarrow \quad \int\frac{\cos y}{e^{\sin y}}\ dy=\int(e^x+x^2)\ dx$

Put $\sin y=t$ in LHS$\Rightarrow \cos y\ dy=dt$

$\therefore \quad \int\frac{dt}{e^t}=\int(e^x+x^2)\ dx$

$\Rightarrow \quad -e^{-t}=e^x+\frac{x^3}{3}-c$

$\Rightarrow \quad e^x+e^{\sin y}+\frac{x^3}{3}=c$

255 **(a)**

The given differential equation can be written as

$$\frac{y\ dx-x\ dy}{y^2}+3x^2e^{x^3}dx=0\Rightarrow d\left(\frac{x}{y}\right)+d\left(e^{x^3}\right)$$

$$=0\Rightarrow\frac{x}{y}+e^{x^3}=C$$

256 **(a)**

Given that, $\frac{dy}{dx}=\frac{2x-y}{x+2y}$ ....(i)

Let $y=vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore v+x\frac{dv}{dx}=\frac{2-v}{1+2v}$

$\Rightarrow x\frac{dv}{dx}=\frac{2-v-v(1+2v)}{1+2v}$

$\Rightarrow \int\frac{1+2v}{2(1-v-v^2)}dv=\int\frac{1}{x}dx$

$\Rightarrow \log k-\frac{1}{2}\log(1-v-v^2)=\log x$

$\Rightarrow 2\log k-\log(1-v-v^2)=2\log x$

$\Rightarrow \log c=\log[x^2(1-v-v^2)]$

$\Rightarrow c=x^2\left(1-\frac{y}{x}-\frac{y^2}{x^2}\right)$

$\Rightarrow x^2-xy-y^2=c$

257 **(a)**

Given $y+x^2=\frac{dy}{dx} \Rightarrow \frac{dy}{dx}-y=x^2$

This is the linear differential equation of the form

$\frac{dy}{dx}+Py=Q$

$\Rightarrow P=-1, Q=x^2$

$\therefore$ IF $=e^{\int P\,dx}=e^{\int -1\,dx}=e^{-x}$

Hence, required solution is

$ye^{-x}=\int x^2e^{-x}\ dx$

$y\cdot e^{-x}=-x^2e^{-x}-2xe^{-x}-2e^{-x}+c$

$\Rightarrow y+x^2+2x+2=ce^x$

258 **(b)**

Given, $\quad \frac{x\ dy-y\ dx}{x}=-\left(\cos^2\frac{y}{x}\right)dx$

$\Rightarrow \quad \sec^2\left(\frac{y}{x}\right)\left(\frac{x\ dy-y\ dx}{x^2}\right)=-\frac{dx}{x}$

$\Rightarrow \quad \sec^2\left(\frac{y}{x}\right)d\left(\frac{y}{x}\right)=-\frac{dx}{x}$

$\Rightarrow \quad \tan\frac{y}{x}=-\log x+c$

[integrating]

When $x=1$, $y=\frac{\pi}{4} \quad \Rightarrow c=1$

$\therefore \ \tan\left(\frac{y}{x}\right)=1-\log x \quad \Rightarrow x=e^{1-\tan\left(\frac{y}{x}\right)}$

259 **(d)**

Given, $\quad \frac{dy}{1+y+y^2}=(1+x)dx$

$\Rightarrow \quad \int\frac{dy}{\left(y+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}=\int(1+x)\ dx$

$\Rightarrow \quad \frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\left(\frac{y+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)=x+\frac{x^2}{2}+\frac{c}{2}$

$\Rightarrow \quad 4\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right)=\sqrt{3}(2x+x^2)+c$

260 **(c)**

Given equation is

$\frac{dy}{dx}=2^y\cdot 2^{-x} \ \Rightarrow 2^{-y}dy=2^{-x}dx$

On integrating both sides, we get

$\frac{2^{-y}}{\log 2}(-1)=\frac{2^{-x}}{\log 2}(-1)+c_1$

$\Rightarrow -\frac{2^{-y}}{\log 2}=-\frac{2^{-x}}{\log 2}+c_1$

$\Rightarrow -2^{-y}=-2^{-x}+c_1\log 2$

$\therefore \frac{1}{2^x}-\frac{1}{2^y}=c_1\log 2=c$

261 **(b)**

Since, $\quad f''(x)=6(x-1)$

$\Rightarrow \quad f'(x)=3(x-1)^2+c$

[integrating] ...(i)

Also, at the point (2,1) the tangent to graph is

$y=3x-5$

Slope of tangent=3

$\Rightarrow \quad f'(2)=3$

$\quad 3(2-1)^2+c=3$

[from eq. (i)]

$\Rightarrow \quad 3+c=3$

$\Rightarrow \quad c=0$

From Eq. (i),

$\quad f'(x)=3(x-1)^2$

$\Rightarrow \quad f(x)=(x-1)^2+k$

[integrating] ...(ii)

Since, it passes through (2,1)

$\therefore \quad 1=(2-1)^2+k \quad \Rightarrow k=0$

Hence, equation of function is

$$f(x) = (x-1)^2$$

262 **(b)**

$\because \ \log\left(\frac{dy}{dx}\right) = ax + by$

$\Rightarrow \frac{dy}{dx} = e^{ax+by} = e^{ax}e^{by}$

$\Rightarrow e^{-by}dy = e^{ax}dx$

On integrating both sides, we get

$\int e^{-by}dy = \int e^{ax}dx$

$\Rightarrow \frac{e^{-by}}{-b} = \frac{e^{ax}}{a} + c$

263 **(d)**

$y + 4x + 1 = V$ is the suitable substitution

$\because \frac{dy}{dx} = f(ax + by + c)$ is

Solvable for substituting

$ax + by + c = V$

264 **(b)**

Given, $\frac{dy}{dx} = \frac{(1+y^2)x}{y(1+x^2)}$

$\Rightarrow \quad \int \frac{2y}{1+y^2}dy = \int \frac{2x}{1+x^2}dx$

$\Rightarrow \quad \log(1+y^2) = \log(1+x^2) + \log k$

$\Rightarrow \quad (1+y^2) = (1+x^2)k$

This equation represents a family of hyperbola.

265 **(c)**

The equation of the family of circles of radius r is

$$(x-a)^2 + (y-b)^2 = r^2 \text{.....(i)}$$

Where a and b are arbitrary constants

$\Rightarrow \quad 2(x-a) + 2(y-b)\frac{dy}{dx} = 0$

$\Rightarrow \quad (x-a) + (y-b)\frac{dy}{dx} = 0 \quad \text{...(ii)}$

$\Rightarrow \quad 1 + (y-b)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

$\Rightarrow \quad (y-b) = -\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}$

...(iii)

From eq. (ii),

$$(x-a) = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]\frac{dy}{dx}}{\frac{d^2y}{dx^2}}$$

...(iv)

On putting the value of $(y-b)$ and $(x-a)$, in eq. (i), we get

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]\left(\frac{dy}{dx}\right)^2}{\left(\frac{d^2y}{dx^2}\right)^2} + \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^2}{\left(\frac{d^2y}{dx^2}\right)^2} = r^2$$

$\Rightarrow \quad \left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = r^2\left[\frac{d^2y}{dx^2}\right]^2$

266 **(b)**

Given,

Focus $S = (0,0)$ let $P(x,y)$ be any point on the parabola,

Since, $SP^2 = PM^2$

$\Rightarrow \quad (x-0)^2 + (y-0)^2 = (x+a)^2$

$\Rightarrow \quad y^2 = 2ax + a^2$

...(i)

$\Rightarrow \quad 2y\frac{dy}{dx} = 2a$

...(ii)

From Eqs. (i) and (ii), we get

$$y^2 = 2y\frac{dy}{dx}.x + \left(y\frac{dy}{dx}\right)^2$$

$\Rightarrow \quad y^2\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx} = y^2$

$\Rightarrow \quad -y\left(\frac{dy}{dx}\right)^2 = 2x\frac{dy}{dx} - y$

267 **(b)**

Given, $\frac{dx}{x} = \frac{ydy}{1+y^2}$

$\Rightarrow \quad \log x = \frac{1}{2}\log(1+y^2) + \log c$

$\Rightarrow \quad x = c\sqrt{1+y^2}$

But it passes through (1,0), so we get $c = 1$

$\therefore$ Solution is $x^2 - y^2 = 1$

268 **(d)**

Given that, $\frac{dy}{dx} = \frac{x^2}{y+1}$

$\Rightarrow (y+1)dy = x^2dx$

$\Rightarrow \frac{y^2}{2} + y = \frac{x^2}{3} + c$

This curve passes through the point (3, 2).

$2 + 2 = 9 + c$

$\Rightarrow c = -5$

$\therefore$ Required curve is $\frac{y^2}{2} + y = \frac{x^3}{3} - 5$

269 **(a)**

Given, $\frac{dy}{dx} = \frac{y-1}{x^2+x}$

$\Rightarrow \quad \int\left(\frac{1}{x} - \frac{1}{x+1}\right)dx = \int\frac{1}{y-1}\,dy$

$\Rightarrow \quad \log x - \log(x+1) = \log(y-1) + \log c$

$\Rightarrow \quad \frac{x}{x+1} = (y-1)c$

...(i)

Since, this curve passes through $(1,0)c = -\frac{1}{2}$

$\therefore$ From Eq. (i) $2x + (y-1)(x+!) = 0$

270 **(a)**

Given, $\frac{dy}{dt} - \left(\frac{1}{1+t}\right) y = \frac{1}{(1+t)}$ and $y(0) = -1$

$\therefore \quad IF = e^{\int -\left(\frac{t}{1+t}\right)dt} = e^{-\int\left(1-\frac{1}{1+t}\right)dt}$

$e^{-t+\log(1+t)} = e^{-t}(1+t)$

$\therefore$ Required solution is,

$$ye^{-t}(1+t) = \int \frac{1}{1+t} e^{-t}(1+t)dt + c$$

$$= \int e^{-t}\, dt + c$$

$\Rightarrow ye^{-1}(1+t) = -e^{-1} + c$

Since, $y(0) = -1$

$\Rightarrow \quad c = 0$

$\therefore \quad y = -\frac{1}{(1+t)}$

$\Rightarrow \quad y(1) = -\frac{1}{2}$

271 **(a)**

Given curve is $y = x^2$

For this curve there is only one tangent line ie,

x-axis $(y = 0)$

$\therefore \quad \frac{dy}{dx} = 0$

Hence, order is 1.

272 **(c)**

Given, $x^2 + y^2 - 2ay = 0$

....(i)

$\Rightarrow \quad 2x + 2yy' - 2ay' = 0$

$\Rightarrow \quad \frac{2x+2yy'}{y'} = 2a$

...(ii)

$\therefore$ From Eq. (i)

$$2a = \frac{x^2+y^2}{y}$$

$\Rightarrow \quad \frac{2x+2yy'}{y'} = \frac{x^2+y^2}{y}$

[from Eq. (ii)]

$\Rightarrow \quad (x^2 - y^2)y' = 2xy$

273 **(a)**

$\because \frac{dy}{dx} + 1 = \operatorname{cosec}(x+y)$

Let $x + y = t$

and $1 + \frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{dt}{\operatorname{cosec} t} = dx$

$\therefore \int \sin t\, dt = \int dx$

$\Rightarrow -\cos t = x - c$

$\Rightarrow \cos(x+y) + x = c$

274 **(b)**

Given, $\frac{y}{4}\, dy = -\frac{x}{9} dx$

$\Rightarrow \quad \frac{y^2}{4.2} = -\frac{x^2}{9.2} + \frac{c}{2}$

$\Rightarrow \quad \frac{y^2}{4} + \frac{x^2}{9} = c$

275 **(b)**

The differential equation of the rectangular hyperbola $xy = c^2$ is

$y + x\frac{dy}{dx} = 0 \Rightarrow x\frac{dy}{dx} = -y$

276 **(c)**

Given, $\log\left(\frac{dy}{dx}\right) = 3x + 4y$

$\Rightarrow \quad \frac{dy}{dx} = e^{3x} e^{4y}$

$\Rightarrow \quad e^{-4y}\, dy = e^{3x}\, dx$

On integrating both sides, we get

$$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + c$$

At $\quad x = 0, y = 0$

$$-\frac{1}{4} = \frac{1}{3} + c$$

$\Rightarrow \quad c = -\frac{7}{12}$

$\therefore$ Solution is

$$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12}$$

$\Rightarrow \quad 4e^{3x} + 3e^{-4y} = 7$

277 **(b)**

Given differential equation is

$\frac{d^2y}{dx^2} = 2 \Rightarrow \frac{dy}{dx} = 2x + a$

$\Rightarrow y = x^2 + ax + b$

$\therefore$ It represents a parabola whose axis is parallel to y-axis.

278 **(b)**

Given, $\frac{dy}{dx} = \left(\frac{y}{x}\right)\left[\log\left(\frac{y}{x}\right) + 1\right]$

Put $\quad \frac{y}{x} = t$

$\Rightarrow \quad y = xt$

$\Rightarrow \quad \frac{dy}{dx} = t + x\frac{dt}{dx}$

$\therefore \quad t + x\frac{dt}{dx} = t(\log t + 1)$

$\Rightarrow \quad \frac{1}{t\log t} dt = \frac{dx}{x}$

$\Rightarrow \quad \log(\log t) = \log x + \log c$

[integrating]

$\Rightarrow \quad \log\left(\frac{y}{x}\right) = cx$

279 **(a)**

Given, $\frac{dy}{dx} = \frac{-y^2}{x^2-xy+y^2}$

Put $y = vx$

$\Rightarrow \quad \frac{dy}{dx} = v + x\ \frac{dv}{dx}$

$\therefore \quad v + x\ \frac{dv}{dx} = \frac{-v^2}{v^2-v+1}$

$x\ \frac{dv}{dx} = \frac{-v^3-v}{v^2-v+1}$

$\Rightarrow \quad \frac{(v^2-v+1)}{-v^3-v}\ dv = \frac{1}{x}\ dx$

$\Rightarrow \quad \frac{-(v^2+1)+v}{v(v^2+1)}\ dv = \frac{1}{x}\ dx$

$\Rightarrow \quad \int -\frac{1}{v} dv + \int \frac{1}{v^2+1} dv = \int \frac{1}{x}\ dx$

$\Rightarrow \quad -\log v + \tan^{-1} v = \log x + c$

$\Rightarrow \quad \tan^{-1} v = \log xv + c$

$\Rightarrow \quad \tan^{-1}\left(\frac{y}{x}\right) = \log y + c$

280 **(a)**

Given, $\frac{d^2y}{dx^2}(x^2+1) = 2x\ \frac{dy}{dx}$

$\Rightarrow \quad \frac{\frac{d^2y}{dx^2}}{\frac{dy}{dx}} = \frac{2x}{x^2+1}$

On integrating both sides, we get

$\log \frac{dy}{dx} = \log(x^2+1) + \log c$

$\Rightarrow \quad \frac{dy}{dx} = c(x^2+1) \quad$ ...(i)

As at $x = 0,\ \frac{dy}{dx} = 3$

$\therefore \quad 3 = c(0+1)$

$\Rightarrow \quad c = 3$

$\therefore$ From Eq. (i),

$\frac{dy}{dx} = 3(x^2+1)$

$\Rightarrow \quad dy = 3(x^2+1)dx$

Again, integrating both sides, we get

$y = 3\left(\frac{x^3}{3} + x\right) + c_1$

At point (0,1)

$1 = 3(0+0) + c_1 \Rightarrow c_1 = 1$

$\therefore \quad y = 3\left(\frac{x^3}{3} + x\right) + 1$

$\Rightarrow \quad y = x^3 + 3x + 1$

281 **(a)**

Given equation can be rewritten as

$\frac{dx}{dy} + \frac{1}{(1+y^2)}x = \frac{e^{\tan^{-1} y}}{(1+y^2)}$

$\therefore \quad IF = e^{\int \frac{1}{1+y^2} dy} = e^{\tan^1 y}$

$\therefore$ Required solution is

$xe^{\tan^1 y} = \int \frac{e^{\tan^{-1} y} e^{\tan^{-1} y}}{1+y^2}\ dy$

Put $\quad e^{\tan^{-1} y} = t \Rightarrow e^{\tan^{-1} y}\ \frac{1}{1+y^2} dy = dt$

$\therefore \quad xe^{\tan^{-1} y} = \int t\ dt = \frac{t^2}{2} + c$

$\Rightarrow \quad 2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + k$

282 **(b)**

Given, $\frac{dy}{dx} = \log(x+1)$

$\Rightarrow \quad dy = \log(x+1)\,dx$

$\Rightarrow \quad \int dy = \int \log(x+1)\,dx$

$\Rightarrow \quad y = (x+1)\log|x+1| - x + c$

$\therefore \quad x = 0,\ y = 3$

$\therefore \quad c = 3$

$\therefore \quad y = (x+1)\log|x+1| - x + 3$

283 **(d)**

Given equation is

$\frac{d^2y}{dx^2} = \frac{\log x}{x^2}$

On integrating, both sides we get

$\int \frac{d^2y}{dx^2} dx = \int \frac{\log x}{x^2} dx$

$\Rightarrow \frac{dy}{dx} = -\frac{\log x}{x} + \int \frac{1}{x^2} dx + c$

$\Rightarrow \frac{dy}{dx} = -\frac{\log x}{x} - \frac{1}{x} + c$

At $x = 1, y = 0$ and $\frac{dy}{dx} = -1 \Rightarrow c = 0$

$\therefore \frac{dy}{dx} = -\frac{(\log x + 1)}{x}$

Again on integrating, both sides we get

$\int \frac{dy}{dx} dx = -\int \frac{\log x + 1}{x} dx + c_1$

$y = -\frac{1}{2}(\log x)^2 - \log x + c_1$

At $x = 1, y = 0$

$\Rightarrow c_1 = 0$

$\therefore y = -\frac{1}{2}(\log x)^2 - \log x$

285 **(b)**

Given equation is

$\sin^{-1} x + \sin^{-1} y = c \quad$ ....(i)

On differentiating Eq. (i) w.r.t. x, we get

$\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

$\Rightarrow \sqrt{1-x^2}\ dy + \sqrt{1-y^2} dx = 0$

This is the required differential equation.

286 **(d)**

Given that, $\frac{dy}{dx} = 1 + y^2$

$\Rightarrow \frac{dy}{1+y^2} = dx$

On integrating both sides, we get

$\int \frac{dy}{1+y^2} = \int dx$

$\Rightarrow \tan^{-1} y = x + c$
At $x = 0, y = 0$, then $c = 0$
At $x = \pi, y = 0$, then $\tan^{-1} 0 = \pi + c \Rightarrow c = -\pi$
$\therefore \tan^{-1} y = x \Rightarrow y = \tan x = \phi(x)$
Therefore, solution becomes $y = \tan x$
But $\tan x$ is not continuous function in $(0, \pi)$
So, $\phi(x)$ is not possible in $(0, \pi)$.

287 **(c)**
Let $p = \frac{dy}{dx}$
$\therefore$ Given differential equation reduces to
$p^2 - xp + y = 0$
Differentiating both sides w.r.t. x, we get
$$2p\frac{dp}{dx} - x\frac{dp}{dx} - p + p = 0$$
$$\Rightarrow \frac{dp}{dx}(2p - x) = 0$$
$\Rightarrow$ Either $\frac{d^2y}{dx^2} = 0$ or $\frac{dy}{dx} = \frac{x}{2}$
$\Rightarrow y = 2x - 4$ will satisfy.

288 **(c)**
Given, $\quad y = a\sin(5x + c)$
$$\Rightarrow \quad \frac{dy}{dx} = 5\,a\cos(5x + c)$$
$$\Rightarrow \quad \frac{d^2y}{dx^2} = -25a\sin(5x + c) = -25y$$

289 **(c)**
Given, $(1 - x^2)\frac{dy}{dx} - xy = 1$
$$\Rightarrow \frac{dy}{dx} - \frac{x}{1 - x^2}y = \frac{1}{1 - x^2}$$
This is a linear equation, comparing with the equation
$$\frac{dy}{dx} + Py = Q$$
$$\Rightarrow P = -\frac{x}{1 - x^2}, Q = \frac{1}{1 - x^2}$$
$\therefore IF = e^{\int P\,dx} = e^{\int \frac{-x}{1-x^2}dx}$
$\Rightarrow \text{IF} = e^{\frac{1}{2}\log(1-x^2)} = \sqrt{1 - x^2}$

290 **(c)**
We have,
Slope $= \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{2y} \Rightarrow 2\,y\,dy = dx$
Integrating both sides, we get $y^2 = x + C$
This passes through (4, 3)
$\therefore 9 = 4 + C \Rightarrow C = 5$
So, the equation of the curve is $y^2 = x + 5$

291 **(a)**
The given differential equation is
$$\frac{dy}{dx} + y\frac{\sin x}{\cos x} = \frac{1}{\cos x}$$
$\therefore IF = e^{\int \frac{\sin x}{\cos x}dx} = e^{\log\sec x} = \sec x$

292 **(b)**
Given, $\frac{dy}{dx} + \frac{2x}{1+x^2}.y = \frac{1}{(1+x^2)^2}$
$\therefore \quad \text{IF} = e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = 1 + x^2$
The complete solution is
$$y(1 + x^2) = \int (1 + x^2).\frac{1}{(1+x^2)^2}\,dx + c$$
$$\Rightarrow \quad y(1 + x^2) = \tan^{-1} x + c$$

293 **(b)**
$\because$ The order of the differential equation is the order of highest derivative in the differential equation.
$\therefore$ The second order differential equation is in option (b) *ie*,
$y'y'' + y = \sin x$

294 **(c)**
Given, $\quad \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$
$$\Rightarrow \quad \int \frac{y}{\sqrt{1-y^2}}\,dy = \int dx$$
$$\Rightarrow \quad -\sqrt{1 - y^2} = x + c$$
$$\Rightarrow \quad (x + c)^2 + y^2 = 1$$
$\therefore$ Centre $(-c, 0)$, radius=1

295 **(c)**
Given, $\quad \frac{dy}{dx} + y = 2e^{2x}$
$\therefore \quad \text{IF} = e^{\int 1\,dx} = e^x$
$\therefore$ Required solution is
$$ye^x = 2\int e^{2x}e^x dx = \frac{2}{3}e^{3x} + c$$
$$\Rightarrow \quad y = \frac{2}{3}e^{2x} + ce^{-x}$$

296 **(c)**
Given, $\quad y = \sin^{-1} x$
$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \quad \text{...(i)}$$
$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{0 - \frac{1}{2}.\frac{(-2x)}{\sqrt{1-x^2}}}{\left(\sqrt{1-x^2}\right)^2}$$
$$\Rightarrow \quad (1 - x^2)\,\frac{d^2y}{dx^2} = x\,\frac{dy}{dx} \text{ ....[from Eq.(i)]}$$

297 **(a)**
Given, $\quad \frac{dy}{dx} = 2\cos x - y\cos x\,\operatorname{cosec} x$
$$\Rightarrow \quad \frac{dy}{dx} + y\cot x = 2\cos x$$
$\therefore \quad \text{IF} = e^{\int \cot x\,dx} = e^{\log(\sin x)} = \sin x$
$\therefore$ Solution is $\quad y\sin x = \int 2\cos x\sin x\,dx + c$
$$\Rightarrow \quad y\sin x = \int \sin 2x\,dx + c$$
$$\Rightarrow \quad y\sin x = \frac{-\cos 2x}{2} + c$$
At $\quad x = \frac{\pi}{4}, y = \sqrt{2}$

$\therefore \quad \sqrt{2}\sin\frac{\pi}{4} = \frac{-\cos 2(\pi/4)}{2} + c$

$\Rightarrow \quad c = 1$

$\therefore \quad y\sin x = -\frac{1}{2}\cos 2x + 1$

$\Rightarrow \quad y = -\frac{1}{2}.\frac{\cos 2x}{\sin x} + \text{cosec}\, x$

$\Rightarrow \quad y = -\frac{1}{2\sin x}(1 - 2\sin^2 x) + \text{cosec}\, x$

$\Rightarrow \quad y = \frac{1}{2}\ \text{cosec}\, x + \sin x$

298 **(b)**

Given, $\frac{\tan^{-1} x}{1+x^2}dx + \frac{y}{1+y^2}dy = 0$

$\Rightarrow \quad \frac{(\tan^{-1} y)^2}{2} + \frac{1}{2}\log(1+y^2) = \frac{c}{2}$

[integrating]

$\Rightarrow \quad (\tan^{-1} x)^2 + \log(1+y^2) = c$

299 **(d)**

Given, $\frac{dy}{dx} - y\tan x = -2\sin x$

$\therefore \quad \text{IF} = e^{-\int \tan x\, dx} = \cos x$

$\therefore$ Solution is

$y(\cos x) = \int -2\sin x\cos x\ dx + c = -\int \sin 2x\ dx + c$

$\Rightarrow \quad y\cos x = \frac{\cos 2x}{2} + c$

300 **(a)**

Given, $\frac{dy}{dt} - \left(\frac{1}{1+t}\right)y = \frac{1}{(1+t)}$ and $y(0) = -1$

$\therefore \quad \text{IF} = e^{\int -\left(\frac{t}{1+t}\right)dt} = e^{-\int\left(1-\frac{1}{1+t}\right)dt}$

$= e^{-t+\log(1+t)} = e^{-t}(1+t)$

$\therefore$ Required solution is

$ye^{-t}(1+t) = \int \frac{1}{1+t}e^{-t}(1+t)dt + c$

$= \int e^{-t}\ dt + c$

$\Rightarrow \quad ye^{-t}(1+t) = -e^{-t} + c$

Since, $\quad y(0) = -1$

$\Rightarrow \quad c = 0$

$\therefore \quad y = -\frac{1}{(1+t)}$

$\Rightarrow \quad y(1) = -\frac{1}{2}$

301 **(d)**

The equation of all the straight lines passing through origin (0, 0) is

$y = mx$ ....(i)

Hence, required differential equation of all such lines is

$y = \left(\frac{dy}{dx}\right)x \quad \left(\because m = \frac{dy}{dx}\right)$

302 **(a)**

Given equation is $\frac{dy}{dx} = \frac{x\log x^2 + x}{\sin y + y\cos y}$

$\Rightarrow (\sin y + y\cos y)dy = (x\log x^2 + x)dx$

On integrating both sides, we get

$\int(\sin y + y\cos y)dy = \int(x\log x^2 + x)dx$

$\Rightarrow -\cos y + y\sin y + \cos y = \frac{x^2}{2}\log x^2 - \int\frac{x^2}{2}\frac{1}{x^2}2x\ dx + \int x\ dx + c$

$\Rightarrow y\sin y = \frac{x^2}{2}2\log x - \int x\ dx + \int x\ dx + c$

$\Rightarrow y\sin y = x^2\log x + c$

303 **(d)**

Given, $\quad y(1-x)dx = xdy$

$\Rightarrow \quad \left(\frac{1}{x} - 1\right)dx = \frac{1}{y}dy$

$\Rightarrow \quad \log x - x = \log y - \log c$

[integrating]

$\Rightarrow \quad x = \log\frac{xc}{y}$

$\Rightarrow \quad ye^x = xc$

304 **(b)**

We have,

$x\frac{dy}{dx} + y = x\, e^x$

$\Rightarrow x\, dy + y\, dx = x\, e^x dx$

$\Rightarrow d(xy) = x\, e^x dx$

$\Rightarrow \int 1 \cdot d(xy) = \int x\, e^x\ dx \Rightarrow xy = e^x(x-1) + C$

305 **(c)**

Differential equation is

$$100\,\frac{d^2y}{dx^2} - 20\,\frac{dy}{dx} + y = 0$$

Here Auxiliary equation is

$$(100m^2 - 20m + 1)y = 0$$

$\Rightarrow \quad (10m-1)^2 y = 0$

$\Rightarrow \quad m = \frac{1}{10}, \frac{1}{10}$

Hence the required solution is

$$y = (c_1 + c_2 x)e^{\frac{1}{10}}$$

306 **(d)**

We have,

$y\frac{dy}{dx} = 2x \Rightarrow y\, dy = 2x\, dx$

On integrating, we obtain

$\frac{y^2}{2} = x^2 + C \Rightarrow y^2 - 2x^2 = 2C$

Clearly, it represents a hyperbola

307 **(d)**

$\because \ 2(y+3) - xy\,\frac{dy}{dx} = 0$

$\Rightarrow 2(y+3) = xy\,\dfrac{dy}{dx}$

$\Rightarrow \int \frac{2}{x}\,dx = \int \frac{y}{y+3}\,dx$

$\Rightarrow 2\log x = y - 3\log(y+0) + c$

Put $x = 1$ and $y = -2$

$\Rightarrow 2 = c$

$\therefore x^2(y+3)^3 = e^{y+2}$

308 **(d)**

We have,

$\dfrac{dy}{dx} - y = 1$

$\Rightarrow \dfrac{dy}{dx} = y + 1$

$\Rightarrow \dfrac{1}{y+1}dy = dx$

$\Rightarrow \displaystyle\int \frac{1}{y+1}dy = \int dx$

$\Rightarrow \log(y+1) = x + C$ ... (i)

It is given that $y(0) = 1$ i.e. $y = 1$ when $x = 0$

$\therefore \log 2 = C$

Substituting the value of C in (i), we get

$\log(y+1) = x + \log 2$

$\Rightarrow y + 1 = 2e^x \Rightarrow y = 2\,e^x - 1$

309 **(c)**

Given differential equation is

$2x\,\dfrac{dy}{dx} - y = 3$

$\Rightarrow 2x\,\dfrac{dy}{dx} = 3 + y$

$\Rightarrow \displaystyle\int \frac{dy}{3+y} = \int \frac{dx}{2x}$

$\Rightarrow \log(3+y) = \dfrac{1}{2}\log x + \log c$

$\Rightarrow \log(3+y) = \log c \cdot \sqrt{x}$

$\Rightarrow 3 + y = c \cdot \sqrt{x}$

$\Rightarrow (3+y)^2 = c^2 x$

Which is an equation of a parabola.