DCAM classes

Dynamic Classes for Academic Mastery

Single Correct Answer Type

1. Let $[x]$ denotes the greatest integer less than or equal to x and $f(x)=[\tan^2 x]$. Then,
   a) $\lim_{x\to 0} f(x)$ does not exist
   b) $f(x)$ is continuous at $x=0$
   c) $f(x)$ is not differentiable at $x=0$
   d) $f'(0)=1$
2. The value of $f(0)$ so that $\frac{(-e^x+2^x)}{x}$ may be continuous at $x=0$ is
   a) $\log\left(\frac{1}{2}\right)$ b) 0 c) 4 d) $-1+\log 2$
3. Let $f(x)$ be an even function. Then $f'(x)$
   a) Is an even function b) Is an odd function c) May be even or odd d) None of these
4. If $f(x)=\begin{cases}[\cos\pi\ x], x<1\\ |x-2|,\ 2>x\geq 1\end{cases}$, then $f(x)$ is
   a) Discontinuous and non-differentiable at $x=-1$ and $x=1$
   b) Continuous and differentiable at $x=0$
   c) Discontinuous at $x=1/2$
   d) Continuous but not differentiable at $x=2$
5. If $f(x)=\begin{cases}\frac{|x+2|}{\tan^{-1}(x+2)}, x\neq -2\\ 2,\quad x=-2\end{cases}$, then $f(x)$ is
   a) Continuous at $x=-2$
   b) Not continuous $x=-2$
   c) Differentiable at $x=-2$
   d) Continuous but not derivable at $x=-2$
6. If $f(x)=|\log|x||$, then
   a) $f(x)$ is continuous and differentiable for all x in its domain
   b) $f(x)$ is continuous for all x in its domain but not differentiable at $x=\pm 1$
   c) $f(x)$ is neither continuous nor differentiable at $x=\pm 1$
   d) None of the above
7. If $f'(a)=2$ and $f(a)=4$, then $\lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$ equals
   a) $2a-4$ b) $4-2a$ c) $2a+4$ d) None of these
8. If $f(x)=x(\sqrt{x}+\sqrt{x+1})$, then
   a) $f(x)$ is continuous but not differentiable at $x=0$ b) $f(x)$ is differentiable at $x=0$
   c) $f(x)$ is not differentiable at $x=0$ d) None of the above
9. If $f(x)=\begin{cases}ax^2+b,\ b\neq 0, x\leq 1\\ x^2b+ax+c,\ \ x>1\end{cases}$, then, $f(x)$ is continuous and differentiable at $x=1$, if
   a) $c=0, a=2b$ b) $a=b, c\in R$ c) $a=b\,, c=0$ d) $a=b, c\neq 0$
10. For the function $f(x)=\begin{cases}|x-3|,\ x\geq 1\\ \frac{x^2}{4}-\frac{3x}{2}+\frac{13}{4},\ x<1\end{cases}$ which one of the following is incorrect?
    a) Continuous at $x=1$ b) Derivable at $x=1$ c) Continuous at $x=3$ d) Derivable at $x=3$
11. If $f: R\to R$ is defined by
    $$f(x)=\begin{cases}\frac{2\sin x-\sin 2x}{2x\cos x}, & \text{if } x\neq 0,\\ a, & \text{if } x=0\end{cases}$$
    Then the value of a so that f is continuous at 0 is
    a) 2 b) 1 c) -1 d) 0
12. $f(x)=x+|x|$ is continuous for

a) $x \in (-\infty, \infty)$ b) $x \in (-\infty, \infty) - \{0\}$ c) Only $x > 0$ d) No value of x

13. If the function

$$f(x) = \begin{cases} \{1 + |\sin x|\}^{\frac{a}{|\sin x|}}, & -\frac{\pi}{6} < x < 0 \\ b, & x = 0 \\ e^{\frac{\tan 2x}{\tan 3x}}, & 0 < x < \frac{\pi}{6} \end{cases}$$

Is continuous at $x = 0$

a) $a = \log_e b\,, b = \frac{2}{3}$ b) $b = \log_e a\,, a = \frac{2}{3}$ c) $a = \log_e b\,, b = 2$ d) None of these

14. If $f(x) = x^2 + \frac{x^2}{1+x^2} + \frac{x^2}{(1+x^2)^2} + \cdots + \frac{x^2}{(1+x^2)^n} + \cdots$, then at $x = 0, f(x)$

a) Has no limit

b) Is discontinuous

c) Is continuous but not differentiable

d) Is differentiable

15. Let $f(x) = \begin{cases} 1, & \forall \quad x < 0 \\ 1 + \sin x, & \forall \quad 0 \le x \le \pi/2 \end{cases}$, then what is the value of $f'(x)$ at $x = 0$?

a) 1 b) -1 c) ∞ d) Does not exist

16. The function $f(x) = x - |x - x^2|$ is

a) Continuous at $x = 1$ b) Discontinuous at $x = 1$

c) Not defined at $x = 1$ d) None of the above

17. If $f(x + y + z) = f(x).f(y).f(z)$ for all x, y, z and $f(2) = 4, f'(0) = 3$, then $f'(2)$ equals

a) 12 b) 9 c) 16 d) 6

18. If $f(x) = |\log_e |x||$, then $f'(x)$ equals

a) $\frac{1}{|x|}, x \neq 0$

b) $\frac{1}{x}$ for $|x| > 1$ and $\frac{-1}{x}$ for $|x| < 1$

c) $\frac{-1}{x}$ for $|x| > 1$ and $\frac{1}{x}$ for $|x| < 1$

d) $\frac{1}{x}$ for $|x| > 0$ and $-\frac{1}{x}$ for $x < 0$

19. If the function $f(x) = \begin{cases} \frac{1-\cos x}{x^2}, & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

a) 1 b) 0 c) $\frac{1}{2}$ d) -1

20. Function $f(x) = |x - 1| + |x - 2|, x \in R$ is

a) Differentiable everywhere in R

b) Except $x = 1$ and $x = 2$ differentiable everywhere in R

c) Not continuous at $x = 1$ and $x = 2$

d) Increasing in R

21. The set of points where the function $f(x) = \sqrt{1 - e^{-x^2}}$ is differentiable is

a) $(-\infty, \infty)$ b) $(-\infty, 0) \cup (0, \infty)$ c) $(-1, \infty)$ d) None of these

22. If $f(x) = x \sin\left(\frac{1}{x}\right), x \neq 0$, then the value of function at $x = 0$, so that the function is continuous at $x = 0$ is

a) 1 b) -1 c) 0 d) Indeterminate

23. The value of $f(0)$ so that the function $f(x) = \frac{2-(256-7x)^{1/8}}{(5x+32)^{1/5}-2}\,(x \neq 0)$ is continuous everywhere, is given by

a) -1 b) 1 c) 26 d) None of these

24. The derivative of $f(x) = |x|^3$ at $x = 0$ is

a) -1 b) 0 c) Does not exist d) None of these

25. If $f(x)=\begin{cases}\frac{(4^x-1)^3}{\sin\left(\frac{x}{a}\right)\log\left(1+\frac{x^2}{3}\right)}, x\neq 0\\ 9(\log 4)^3, x=0\end{cases}$ is continuous function at $x=0$, then the value of a is equal to

a) 3 b) 1 c) 2 d) 0

26. $f(x)=|[x]+x|$ in $-1<x\leq 2$ is

a) Continuous at $x=0$
b) Discontinuous at $x=1$
c) Not differentiable at $x=2,0$
d) All the above

27. Let$f(x)=[x^3-x]$, where $[x]$the greatest integer function is. Then the number of points in the interval (1, 2), where function is discontinuous is

a) 4 b) 5 c) 6 d) 7

28. If $y=\cos^{-1}\cos\,(|x|-f(x))$, where

$f(x)=\begin{cases}1, if\ x>0\\ -1, if\ x<0\\ 0, if\ x=0\end{cases}$. Then, $(dy/dx)\ x=\frac{5\pi}{4}$ is equal to

a) -1 b) 1
c) 0 d) Cannot be determined

29. Let $f(x+y)=f(x)+f(y)$ and $f(x)=x^2\,g(x)$ for all $x,y\in R$, where $g(x)$ is continuous function. Then, $f'(x)$ is equal to

a) $g'(x)$ b) $g(0)$ c) $g(0)+g'(x)$ d) 0

30. Let a function $f(x)$ be defined by $f(x)=\begin{cases}x,\ x\in Q\\ 0,\ x\in R-Q\end{cases}$ Then, $f(x)$ is

a) Everywhere continuous
b) Nowhere continuous
c) Continuous only at some points
d) Discontinuous only at some points

31. The function $f(x)=\begin{cases}1-2x+3x^2-4x^3+\cdots \text{to } \infty,\ x\neq -1\\ 1,\qquad x=-1\end{cases}$ is

a) Continuous and derivable at $x=-1$
b) Neither continuous nor derivable at $x=-1$
c) Continuous but not derivable at $x=-1$
d) None of these

32. $f(x)=\begin{cases}2a-x\ \text{ in} -a<x<a\\ 3x-2a\ \text{ in }\ a\leq x\end{cases}$. Then, which of the following is true?

a) $f(x)$ is discontinuous at $x=a$ b) $f(x)$ is not differentiable at $x=a$
c) $f(x)$ is differentiable at $x\geq a$ d) $f(x)$ is continuous at all $x<a$

33. Let $f(x+y)=f(x)f(y)$ and $f(x)=1+(\sin 2x)g(x)$ where $g(x)$ is continuous. Then, $f'(x)$ equals

a) $f(x)g(0)$ b) $2f(x)g(0)$ c) $2g(0)$ d) None of these

34. If $f(x)=[x\sin\pi\ x]$, then which of the following is incorrect?

a) $f(x)$ is continuous at $x=0$
b) $f(x)$ is continuous in $(-1,0)$
c) $f(x)$ is differentiable at $x=1$
d) $f(x)$ is differentiable in $(-1,1)$

35. If $f(x)=\begin{cases}1, x<0\\ 1+\sin x, 0\leq x\leq\frac{\pi}{2}\end{cases}$ then derivative of $f(x)$ at $x=0$

a) Is equal to 1 b) Is equal to 0 c) Is equal to -1 d) Does not exist

36. If the derivative of the function $f(x)$ is everywhere continuous and is given by

$f(x)=\begin{cases}bx^2+ax+4;\ \ x\geq -1\\ ax^2+b;\ \ x<-1\end{cases}$, then

a) $a=2,b=-3$ b) $a=3,b=2$ c) $a=-2,b=-3$ d) $a=-3,b=-2$

37. If $f(x) = \begin{cases} \frac{x \log \cos x}{\log(1+x^2)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then

a) $f(x)$ is not continuous at $x = 0$

b) $f(x)$ is not continuous and differentiable at $x = 0$

c) $f(x)$ is not continuous at $x = 0$ but not differentiable at $x = 0$

d) None of these

38. If the function $f(x) = \begin{cases} Ax - B, & x \leq 1 \\ 3x, & 1 < x < 2 \\ B\,x^2 - A, & x \geq 2 \end{cases}$ be continuous at $x = 1$ and discontinuous at $x = 2$, then

a) $A = 3 + B, B \neq 3$ b) $A = 3 + B, B = 3$ c) $A = 3 + B$ d) None of these

39. If $f(x) = \begin{cases} |x - 4|, \text{for } x \geq 1 \\ (x^3/2) - x^2 + 3x + (1/2), \text{for } x < 1 \end{cases}$, then

a) $f(x)$ is continuous at $x = 1$ and $x = 4$

b) $f(x)$ is differentiable at $x = 4$

c) $f(x)$ is continuous and differentiable at $x = 1$

d) $f(x)$ is not continuous at $x = 1$

40. The function $f(x) = a[x + 1] + b[x - 1]$, where $[x]$ is the greatest integer function, is continuous at $x = 1$, is

a) $a + b = 0$ b) $a = b$ c) $2a - b = 0$ d) None of these

41. Let $f(x) = \begin{cases} 5^{1/x}, & x < 0 \\ \lambda[x], & x \geq 0 \end{cases}$ and $\lambda \in R$, then at $x = 0$

a) f is discontinuous

b) f is continuous only, if $\lambda = 0$

c) f is continuous only, whatever λ may be

d) None of the above

42. If for a continuous function f, $f(0) = f(1) = 0, f'(1) = 2$ and $y(x) = f(e^x)e^{f(x)}$, then $y'(0)$ is equal to

a) 1 b) 2 c) 0 d) None of these

43. If $f(x) = \begin{cases} ax^2 - b, |x| < 1 \\ \frac{1}{|x|}, & |x| \geq 1 \end{cases}$ is differentiable at $x = 1$, then

a) $a = \frac{1}{2}, b = -\frac{1}{2}$ b) $a = -\frac{1}{2}, b = -\frac{3}{2}$ c) $a = b = \frac{1}{2}$ d) $a = b = -\frac{1}{2}$

44. Let $f(x) = \frac{\sin 4\,\pi\,[x]}{1+[x]^2}$, where $[x]$ is the greatest integer less than or equal to x, then

a) $f(x)$ is not differentiable at some points

b) $f'(x)$ exists but is different from zero

c) $f'(x) = 0$ for all x

d) $f'(x) = 0$ but f is not a constant function

45. The value of k which makes $f(x) = \begin{cases} \sin(1/k), & x \neq 0 \\ k, & x = 0 \end{cases}$ continuous at $x = 0$ is

a) 8 b) 1 c) -1 d) None of these

46. The function $f(x) = \max[(1 - x), (1 + x), 2], x \in (-\infty, \infty)$ is

a) Continuous at all points

b) Differentiable at all points

c) Differentiable at all points except at $x = 1$ and $x = -1$

d) None of the above

47. Let $f(x)$ be defined for all $x > 0$ and be continuous. Let $f(x)$ satisfy $f\left(\frac{x}{y}\right) = f(x) - f(y)$ for all x, y and $f(e) = 1$. Then,

a) $f(x)$ is bounded b) $f\left(\frac{1}{x}\right) \to 0$ as $x \to 0$ c) $xf(x) \to 1$ as $x \to 0$ d) $f(x) = \ln x$

48. Suppose a function $f(x)$ satisfies the following conditions for all x and y: (i) $f(x + y) = f(x)f(y)$ (ii) $f(x) = 1 + x\,g(x) \log a$, where $a > 1$ and $\lim_{x\to 0} g(x) = 1$. Then, $f'(x)$ is equal to

a) $\log a$ b) $\log a^{f(x)}$ c) $\log(f(x))^a$ d) None of these

49. Let $g(x)$ be the inverse of the function $f(x)$ and $f'(x) = \frac{1}{1+x^3}$. Then, $g'(x)$ is equal to

a) $\frac{1}{1+\left(g(x)\right)^3}$ b) $\frac{1}{1+\left(f(x)\right)^3}$ c) $1+\left(g(x)\right)^3$ d) $1+\left(f(x)\right)^3$

50. If $f(x) = |x^2 - 4x + 3|$, then
a) $f'(1) = -1$ and $f'(3) = 1$
b) $f'(1) = -1$ and $f'(3)$ does not exist
c) $f'(1) = -1$ does not exist and $f'(3) = 1$
d) Both $f'(1)$ and $f'(3)$ do not exist

51. The points of discontinuity of $\tan x$ are
a) $n\pi, n \in I$ b) $2n\pi, n \in I$ c) $(2n+1)\frac{\pi}{2}, n \in I$ d) None of these

52. Let $f(x) = ||x| - 1|$, then points where $f(x)$ is not differentiable, is/(are)
a) $0, \pm 1$ b) ± 1 c) 0 d) 1

53. $f(x) = \begin{cases} 2x, & x < 0 \\ 2x+1, & x \geq 0 \end{cases}$. Then
a) $f(x)$ is continuous at $x = 0$ b) $f(|x|)$ is continuous at $x = 0$ c) $f(x)$ is discontinuous at $x = 0$ d) None of the above

54. Let $f(x) = [x] + \sqrt{x - [x]}$, where $[x]$ denotes the greatest integer function. Then,
a) $f(x)$ is continuous on R^+
b) $f(x)$ is continuous on R
c) $f(x)$ is continuous on $R - Z$
d) None of these

55. The function $f(x) = \frac{1 - \sin x + \cos x}{1 + \sin x + \cos x}$ is not defined at $x = \pi$. The value of $f(\pi)$, so that $f(x)$ is continuous at $x = \pi$, is
a) -1/2 b) ½ c) -1 d) 1

56. Let $f(x) = \begin{cases} (x-1)\sin\frac{1}{x-1}, & \text{if } x \neq 1 \\ 0, & \text{if } x = 1 \end{cases}$. Then, which one of the following is true?
a) f is differentiable at $x = 1$ but not at $x = 0$
b) f is neither differentiable at $x = 0$ nor at $x = 1$
c) f is differentiable at $x = 0$ and at $x = 1$
d) f is differentiable at $x = 0$ but not at $x = 1$

57. If $f(x) = \begin{cases} mx + 1, & x \leq \frac{\pi}{2} \\ \sin x + n, & x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then
a) $m = 1, n = 0$ b) $m = \frac{n\pi}{2} + 1$ c) $n = \frac{m\pi}{2}$ d) $m = n = \frac{\pi}{2}$

58. Let f be differentiable for all x. If $f(1) = -2$ and $f'(x) \geq 2$ for $x \in [1, 6]$, then
a) $f(6) = 5$ b) $f(6) < 5$ c) $f(6) < 8$ d) $f(6) \geq 8$

59. If $\lim_{x \to a^+} f(x) = l = \lim_{x \to a^-} g(x)$ and $\lim_{x \to a^-} f(x) = m \lim_{x \to a^+} g(x)$, then the function $f(x)\, g(x)$
a) Is not continuous at $x = a$
b) Has a limit when $x \to a$ and it is equal to lm
c) Is continuous at $x = a$
d) Has a limit when $x \to a$ but it is not equal to lm

60. Let $f(x)$ be a function satisfying $f(x+y) = f(x)f(y)$ for all $x, y \in R$ and $f(x) = 1 + x\, g(x)$ where $\lim_{x \to 0} g(x) = 1$. Then, $f'(x)$ is equal to
a) $g'(x)$ b) $g(x)$ c) $f(x)$ d) None of these

61. The set of points where the function $f(x) = x|x|$ is differentiable is
a) $(-\infty, \infty)$ b) $(-\infty, 0) \cup (0, \infty)$ c) $(0, \infty)$ d) $[0, \infty)$

62. If $f(x+y) = f(x)f(y)$ for all real x and y, $f(6) = 3$ and $f'(0) = 10$, then $f'(6)$ is
a) 30 b) 13 c) 10 d) 0

63. If $f(x) = |x - a|\phi(x)$, where $\phi(x)$ is continuous function, then
a) $f'(a^+) = \phi(a)$ b) $f'(a^-) = \phi(a)$ c) $f'(a^+) = f'(a^-)$ d) None of these

64. If $f(x) = \begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then $f(x)$ is

a) Continuous as well as differentiable for all x
b) Continuous for all x but not differentiable at $x = 0$
c) Neither differentiable nor continuous at $x = 0$
d) Discontinuous everywhere

65. If $f(x) = \begin{cases} 3, & x < 0 \\ 2x + 1, & x \geq 0 \end{cases}$, then

a) Both $f(x)$ and $f(|x|)$ are differentiable at $x = 0$
b) $f(x)$ is differentiable but $f(|x|)$ is not differentiable at $x = 0$
c) $f(|x|)$ is differentiable but $f(x)$ is not differentiable at $x = 0$
d) Both $f(x)$ and $f(|x|)$ are not differentiable at $x = 0$

66. If $\lim_{x \to c} \frac{f(x)-f(c)}{x-c}$ exists finitely, then

a) $\lim\limits_{x \to c} f(x) = f(c)$
b) $\lim\limits_{x \to c} f'(x) = f'(c)$
c) $\lim_{x \to c} f(x)$ does not exist
d) $\lim_{x \to c} f(x)$ may or may not exist

67. The number of points at which the function $f(x) = |x - 0.5| + |x - 1| + \tan x$ does not have a derivative in the interval $(0, 2)$, is

a) 1 b) 2 c) 3 d) 4

68. If $f(x) = \begin{cases} \log_{(1-3x)}(1 + 3x), & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$ is continuous at $x = 0$, then k is equal to

a) -2 b) 2 c) 1 d) -1

69. Let $f(x)$ be a function differentiable at $x = c$. Then, $\lim_{x \to c} f(x)$ equals

a) $f'(c)$ b) $f''(c)$ c) $\frac{1}{f(c)}$ d) None of these

70. If$f(x) = ae^{|x|} + b|x|^2$; $a, b \in R$ and $f(x)$ is differentiable at $x = 0$. Then a and b are

a) $a = 0, b \in R$ b) $a = 1, b = 2$ c) $b = 0, a \in R$ d) $a = 4, b = 5$

71. Let $f(x) = (x + |x|)|x|$. The, for all x

a) f and f' are continuous
b) f is differentiable for some x
c) f' is not continuous
d) f'' is continuous

72. If $f(x) = \begin{cases} \frac{x-1}{2x^2-7x+5}, & \text{for } x \neq 1 \\ -\frac{1}{3}, & \text{for } x = 1 \end{cases}$, then $f'(1)$ is equal to

a) $-\frac{1}{9}$ b) $-\frac{2}{9}$ c) $-\frac{1}{3}$ d) $\frac{1}{3}$

73. Suppose $f(x)$ is differentiable at $x = 1$ and $\lim_{h \to 0} \frac{1}{h} f(1 + h) = 5$, then $f'(1)$ equals

a) 6 b) 5 c) 4 d) 3

74. If $f: R \to R$ is defined by

$f(x) = \begin{cases} \frac{x+2}{x^2+3x+2}, & \text{if } x \in R - \{-1, -2\} \\ -1, & \text{if } x = -2 \\ 0, & \text{if } x = -1 \end{cases}$, then f is continuous on the set

a) R b) $R - \{-2\}$ c) $R - \{-1\}$ d) $R - (-1, -2)$

75. Let $f(x) = \frac{(e^x-1)^2}{\sin\left(\frac{x}{a}\right)\log\left(1+\frac{x}{4}\right)}$ for $x \neq 0$ and$f(0) = 12$. If f is continuous at $x = 0$, then the value of a is equal to

a) 1 b) -1 c) 2 d) 3

76. If a function $f(x)$ is given by $f(x) = \frac{x}{1+x} + \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + \cdots \infty$ then at $x = 0, f(x)$

a) Has no limit
b) Is not continuous
c) Is continuous but not differentiable
d) Is differentiable

77. If $f(x)$ is continuous function and $g(x)$ be discontinuous, then
a) $f(x) + g(x)$ must be continuous
b) $f(x) + g(x)$ must be discontinuous
c) $f(x) + g(x)$ for all x
d) None of these

78. A function $f: R \to R$ satisfies the equation $f(x + y) = f(x)f(y)$ for all $x, y \in R$ and $f(x) \neq 0$ for all $x \in R$. If $f(x)$ is differentiable at $x = 0$ and $f'(0) = 2$, then $f'(x)$ equals
a) $f(x)$ b) $-f(x)$ c) $2f(x)$ d) None of these

79. Consider $f(x) = \begin{cases} \frac{x^2}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$
a) $f(x)$ is discontinuous everywhere
b) $f(x)$ is continuous everywhere
c) $f'(x)$ exists in $(-1, 1)$
d) $f'(x)$ exists in $(-2, 2)$

80. If $f(x)$ is continuous at $x = 0$ and $f(0) = 2$, then
$\lim_{x\to 0} \frac{\int_0^x f(u)du}{x}$ is
a) 0 b) 2 c) $f(2)$ d) None of these

81. Let $f(x + y) = f(x)f(y)$ for all $x, y \in R$. Suppose that $f(3) = 3$ and $f'(0) = 11$ then, $f'(3)$ is equal to
a) 22 b) 44 c) 28 d) None of these

82. If $f(x) = \begin{cases} x - 5, & \text{for } x \leq 1 \\ 4x^2 - 9, & \text{for } 1 < x < 2 \\ 3x + 4, & \text{for } x \geq 2 \end{cases}$, then $f'(2^+)$ is equal to
a) 0 b) 2 c) 3 d) 4

83. $f(x) = \sin|x|$. Then $f(x)$ is not differentiable at
a) $x = 0$ only b) All x c) Multiples of π d) Multiples of $\frac{\pi}{2}$

84. If $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} (\log_e a)^n, a > 0, a \neq 0$, then at $x = 0, f(x)$ is
a) Everywhere continuous but not differentiable
b) Everywhere differentiable
c) Nowhere continuous
d) None of these

85. The function $f(x) = [x] \cos \left[\frac{2x-1}{2}\right] \pi$ where [.] denotes the greatest integer function, is discontinuous at
a) All x b) No x
c) All integer points d) x which is not an integer

86. The function $f(x) = \begin{cases} 1, & |x| \geq 1 \\ \frac{1}{n^2}, & \frac{1}{n} < |x| < \frac{1}{n-1}, n = 2, 3, \dots \\ 0, & x = 0 \end{cases}$
a) Is discontinuous at finitely many points
b) Is continuous everywhere
c) Is discontinuous only at $x = \pm\frac{1}{n}, n \in Z - \{0\}$ and $x = 0$
d) None of these

87. Let f is a real-valued differentiable function satisfying $|f(x) - f(y)| \leq (x - y)^2$, $x, y \in R$ and $f(0) = 0$, then $f(1)$ equals
a) 1 b) 2 c) 0 d) -1

88. Let $f(x) = [2x^3 - 5]$, [] denotes the greatest integer function. Then number of points (1, 2) where the

function is discontinuous, is

a) 0 b) 13 c) 10 d) 3

89. ln$[1, 3]$ the function $[x^2 + 1]$, $[x]$ denoting the greatest integer function, is continuous

a) For all x

b) For all x except at four points

c) For all except at seven points

d) For all except at eight-points

90. If $f(x) = |\log_{10} x|$, then at $x = 1$

a) $f(x)$ is continuous and $f'(1^+) = \log_{10} e$, $f'(1^-) = -\log_{10} e$

b) $f(x)$ is continuous and $f'(1^+) = \log_{10} e$, $f'(1^-) = \log_{10} e$

c) $f(x)$ is continuous and $f'(1^-) = \log_{10} e$, $f'(1^+) = -\log_{10} e$

d) None of these

91. The function $f(x) = |\cos x|$ is

a) Everywhere continuous and differentiable

b) Everywhere continuous and but not differentiable at $(2n + 1)\pi/2, n \in Z$

c) Neither continuous nor differentiable at $(2n + 1)\pi/2, n \in Z$

d) None of these

92. Let $f(x) = \begin{cases} \frac{x-4}{|x-4|} + a, x < 4 \\ a + b, \quad x = 4 \\ \frac{x-4}{|x-4|} + b, x > 4 \end{cases}$

Then, $f(x)$ is continuous at $x = 4$ when

a) $a = 0, b = 0$ b) $a = 1, b = 1$ c) $a = -1, b = 1$ d) $a = 1, b = -1$

93. If $f(x) = \begin{cases} \frac{2^x-1}{\sqrt{1+x}-1}, -1 \le x < \infty, \ x \ne 0 \\ k, \quad x = 0 \end{cases}$ is continuous everywhere, then k is equal to

a) $\frac{1}{2}\log_e 2$ b) $\log_e 4$ c) $\log_e 8$ d) $\log_e 2$

94. The function $f(x) = \begin{cases} x^n \sin\left(\frac{1}{x}\right), x \ne 0 \\ 0, \ x = 0 \end{cases}$ is continuous and differentiable at $x = 0$, if

a) $n \in (0, 1]$ b) $n \in [1, \infty)$ c) $n \in (1, \infty)$ d) $n \in (-\infty, 0)$

95. The function $f(x) = \begin{cases} \frac{e^{1/x}-1}{e^{1/x}+1}, x \ne 0 \\ 0, \ x = 0 \end{cases}$

a) Is continuous at $x = 0$

b) Is not continuous at $x = 0$

c) Is not continuous at $x = 0$, but can be made continuous $x = 0$

d) None of these

96. A function $f(x) = \begin{cases} 1 + x, \ x \le 2 \\ 5 - x, \ x > 2 \end{cases}$ is

a) Not continuous at $x = 2$ b) Differenti8able at $x = 2$

c) Continuous but not differentiable at $= 2$ d) None of the above

97. Let $f(x + y) = f(x)f(y)$ for all $x, y \in R$. If $f'(1) = 2$ and $f(4) = 4$, then $f'(4)$ equal to

a) 4 b) 1 c) 1/2 d) 8

98. Let $f(x) = [x]$ and $g(x) = \begin{cases} 0, \ x \in Z \\ x^2, \ x \in R - Z \end{cases}$ Then, which one of the following is incorrect?

a) $\lim_{x\to1} g(x)$ exists, but $g(x)$ is not continuous at $x = 1$

b) $\lim_{x\to1} f(x)$ does not exist and $f(x)$ is not continuous at $x = 1$

c) gof is continuous for all x

d) fog is continuous for all x

99. If $f(x) = \begin{cases} x, \quad \text{for} \quad 0 < x < 1 \\ 2 - x, \ \text{for} \quad 1 \le x < 2 \\ x - (1/2)x^2, \ \text{for} \ x = 2 \end{cases}$.Then, $f'(1)$ is equal to

a) -1 b) 1 c) 0 d) None of these

100. The function $f(x) = |x| + \frac{|x|}{x}$ is

a) Discontinuous at origin because $|x|$ is discontinuous there

b) Continuous at origin

c) Discontinuous at origin because both $|x|$ and $\frac{|x|}{x}$ are discontinuous there

d) Discontinuous at the origin because $\frac{|x|}{x}$ is discontinuous there

101. $f(x) = |x-3|$ is ... at $x = 3$

a) Continuous and not differentiable b) Continuous and differentiable

c) Discontinuous and not differentiable d) Discontinuous and differentiable

102. At $x = \frac{3}{2}$ the function $f(x) = \frac{|2x-3|}{2x-3}$ is

a) Continuous b) Discontinuous c) Differentiable d) Non-zero

103. The following functions are differentiable on $(-1, 2)$

a) $\int_{x}^{2x} (\log t)^2 dt$ b) $\int_{x}^{2x} \frac{\sin t}{t} dt$ c) $\int_{x}^{2x} \frac{1-t+t^2}{1+t+t^2} dt$ d) None of these

104. Let $f(x) = \frac{1-\tan x}{4x-\pi}, x \neq \frac{\pi}{4}, \ x \in \left[0, \frac{\pi}{2}\right]$. If $f(x)$ is continuous in $\left[0, \frac{\pi}{2}\right]$, then $f\left(\frac{\pi}{4}\right)$ is

a) 1 b) 1/2 c) -1/2 d) -1

105. If $f(x) = \begin{cases} \frac{1-\cos x}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

a) 0 b) $\frac{1}{2}$ c) $\frac{1}{4}$ d) $-\frac{1}{2}$

106. Let $f(x) = |x| + |x-1|$, then

a) $f(x)$ is continuous at $x = 0$, as well as at $x = 1$

b) $f(x)$ is continuous at $x = 0$, but not at $x = 1$

c) $f(x)$ is continuous at $x = 1$, but not at $x = 0$

d) None of these

107. The function $f(x)$ is defined as $f(x) = \frac{2x - \sin^{-1} x}{2x + \tan^{-1} x}$, if $x \neq 0$. The value of f to be assigned at $x = 0$ so that the function is continuous there, is

a) $-\frac{1}{3}$ b) 1 c) $\frac{2}{3}$ d) $\frac{1}{3}$

108. Let $f(x)$ be an odd function. Then $f'(x)$

a) Is an even function b) Is an odd function c) May be even or odd d) None of these

109. If $f(x) = \begin{cases} \frac{x-1}{2x^2-7x+5}, & \text{for } x \neq 1 \\ -\frac{1}{3}, & \text{for } x = 1 \end{cases}$, then $f'(1)$ is equal to

a) $-\frac{1}{9}$ b) $-\frac{2}{9}$ c) -13 d) 1/3

110. If $f: R \to R$ given by

$$f(x) = \begin{cases} 2\cos x, \text{if } x \leq -\frac{\pi}{2} \\ a + \sin x + b, \text{if } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 1 + \cos^2 x, \text{ if } x \geq \frac{\pi}{2} \end{cases}$$ is a continuous

Function on R, then (a, b) is equal to

a) (1/2, 1/2) b) (0, -1) c) (0, 2) d) (1, 0)

111. If $f(x+y) = f(x)f(y)$ for all $x, y \in R$, $f(5) = 2$, $f'(0) = 3$. Then $f'(5)$ equals

a) 6 b) 3 c) 5 d) None of these

112. Let $f(x)$ be a function satisfying $f(x+y) = f(x) + f(y)$ and $f(x) = x\,g(x)$ for all $x, y \in R$, where $g(x)$ is continuous. Then,

a) $f'(x) = g'(x)$ b) $f'(x) = g(x)$ c) $f'(x) = g(0)$ d) None of these

113. If $f(x) = \sqrt{x + 2\sqrt{2x - 4}} + \sqrt{x - 2\sqrt{2\,x - 4}}$, then $f(x)$ is differentiable on

a) $(-\infty, \infty)$ b) $[2, \infty) - \{4\}$ c) $[2, \infty)$ d) None of these

114. If $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$, then

a) f and f' are continuous at $x = 0$

b) f is derivable at $x = 0$ and f' is continuous at $x = 0$

c) f is derivable at $x = 0$ and f' is not continuous at $x = 0$

d) f' is derivable at $x = 0$

115. If a function $f(x)$ is defined as $f(x) = \begin{cases} \frac{x}{\sqrt{x^2}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ then

a) $f(x)$ is continuous at $x = 0$ but not differentiable at $x = 0$

b) $f(x)$ is continuous as well as differentiable at $x = 0$

c) $f(x)$ is discontinuous at $x = 0$

d) None of these

116. If $f(x) = [\sqrt{2} \sin x]$, where $[x]$ represents the greatest integer function, then

a) $f(x)$ is periodic

b) Maximum value of $f(x)$ is 1 in the interval $[-2\,\pi, 2\,\pi]$

c) $f(x)$ is discontinuous at $x = \frac{n\,\pi}{2} + \frac{\pi}{4}, n \in Z$

d) $f(x)$ is differentiable at $x = n\,\pi, n \in Z$

117. $\lim_{x \to 0}[(1 + 3x)^{1/x}] = k$, then for continuity at $x = 0$, k is

a) 3 b) -3 c) e^3 d) e^{-3}

118. Let $f(x) = \begin{cases} \int_0^x \{5 + |1 - t|\}dt, & \text{if } x > 2 \\ 5x + 1, & \text{if } x \leq 2 \end{cases}$

a) $f(x)$ is continuous at $x = 2$

b) $f(x)$ is continuous but not differentiable at $x = 2$

c) $f(x)$ is everywhere differentiable

d) The right derivative of $f(x)$ at $x = 2$ does not exist

119. Let $f(x) = \begin{cases} \frac{1}{|x|} & \text{for } |x| \geq 1 \\ ax^2 + b & \text{for } |x| < 1 \end{cases}$

If $f(x)$ is continuous and differentiable at any point, then

a) $a = \frac{1}{2}, b = -\frac{3}{2}$ b) $a = -\frac{1}{2}, b = \frac{3}{2}$ c) $a = 1, b = -1$ d) None of these

120. If function $f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ 1 - x, & \text{if } x \text{ is irrational} \end{cases}$, then the number of points at which $f(x)$ is continuous, is

a) ∞ b) 1 c) 0 d) None of these

121. The function $f(x) = e^{-|x|}$ is

a) Continuous everywhere but not differentiable at $x = 0$ b) Continuous and differentiable everywhere

c) Not continuous at $x = 0$ d) None of the above

122. The value of $f(0)$, so that the function

$$f(x) = \frac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{\sqrt{a + x} - \sqrt{a - x}}$$

Becomes continuous for all x, is given by

a) $a^{3/2}$ b) $a^{1/2}$ c) $-a^{1/2}$ d) $-a^{3/2}$

123. The value of k for which the function

$f(x) = \begin{cases} \frac{1 - \cos 4x}{8x^2}, & x \neq 0 \\ k & x = 0 \end{cases}$ is continuous at $a = 0$, is

a) $k = 0$ b) $k = 1$ c) $k = -1$ d) None of these

124. The number of points at which the function $f(x) = (|x-1| + |x-2| + \cos x)$ where $x \in [0, 4]$ is not continuous, is

a) 1 b) 2 c) 3 d) 0

125. If $f(x) = \begin{cases} x \sin\frac{1}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

a) 1 b) -1 c) 0 d) 2

126. Let $f(x)$ be twice differentiable function such that $f''(x) = -f(x)$ and $f'(x) = g(x), h(x) = \{f(x)\}^2 + \{g(x)\}^2$. If $h(5) = 11$, then $h(10)$ is equal to

a) 22 b) 11 c) 0 d) None of these

127. if $f(x) = |x|^3$, then $f'(0)$ equals

a) 0 b) 1/2 c) -1 d) -1/2

128. Let function $f(x) = \sin^{-1}(\cos x)$, is

a) Discontinuous at $x = 0$

b) Continuous at $x = 0$

c) Differentiable at $x = 0$

d) None of these

129. Let $f(x) = \begin{cases} \frac{x^4-5x^2+4}{|(x-1)(x-2)|}, x \neq 1, 2 \\ 6, \; x = 10 \\ 12, \; x = 2 \end{cases}$ Then, $f(x)$ is continuous on the set

a) R b) $R - \{1\}$ c) $R - \{2\}$ d) $R - \{1, 2\}$

130. The set of points, where $f(x) = \frac{x}{1+|x|}$ is differentiable, is

a) $(-\infty, -1) \cup (-1, \infty)$ b) $(-\infty, \infty)$ c) $(0, \infty)$ d) $(-\infty, 0) \cup (0, \infty)$

131. Given $f(0) = 0$ and $f(x) = \frac{1}{(1-e^{-1/x})}$ for$x \neq 0$. Then only one of the follo0wing statements on $f(x)$ is true. That id $f(x)$, is

a) Continuous at $x = 0$

b) Not continuous at $x = 0$

c) Both continuous and differentiable at $x = 0$

d) Not defined at $x = 0$

132. Let f and g be differentiable functions satisfying $g'(a) = 2, g(a) = b$ and $fog = I$ (identify function). Then, $f'(b)$ is equal to

a) 1/2 b) 2 c) 2/3 d) None of these

133. Let $f(x) = \begin{cases} \frac{\sin \pi x}{5x}, & x \neq 0 \\ k, & x = 0 \end{cases}$, if $f(x)$ is continuous at $x = 0$, then k is equal to

a) $\frac{\pi}{5}$ b) $\frac{5}{\pi}$ c) 1 d) 0

134. The number of discontinuities of the greatest integer function $f(x) = [x], x \in \left(-\frac{7}{2}, 100\right)$ is equal to

a) 104 b) 100 c) 102 d) 103

135. For the function$f(x) = \frac{e^{1/x}-1}{e^{1/x}+1}, x = 0$, which of the following is correct?

a) $\lim_{x\to 0} f(x)$ does not exist

b) $\lim\limits_{x\to 0} f(x) = 1$

c) $\lim_{x\to 0} f(x)$ exists but $f(x)$ is not continuous at $x = 0$

d) $f(x)$ is continuous at $x = 0$

136. If $f(x) = x^4 + \frac{x^4}{1+x^4} + \frac{x^4}{(1+x^4)^2} + \cdots$ to ∞ then at $x = 0, f(x)$ is

a) Continuous but not differentiable

b) Differentiable

c) Continuous

d) None of these

137. If $f(x)=\begin{cases}1+x, & 0\le x\le 2\\ 3-x, & 2<x\le 3\end{cases}$ then the set of points of discontinuity of $g(x)=fof(x)$, is

a) $\{1, 2\}$ b) $\{0, 1, 2\}$ c) $\{0, 1\}$ d) None of these

138. Let $g(x)$ be the inverse of an invertible function $f(x)$ which is differentiable at $x=c$, then $g'(f(c))$ equals

a) $f'(c)$ b) $\frac{1}{f'(c)}$ c) $f(c)$ d) None of these

139. If $f(x)=\begin{cases}x^p\cos\left(\frac{1}{x}\right), x\neq 0\\ 0, \quad x=0\end{cases}$ is differentiable at $x=0$, then

a) $p<0$ b) $0<p<1$ c) $p=1$ d) $p>1$

140. At $x=0$, the function $f(x)=|x|$ is

a) Continuous but not differentiable b) Discontinuous and differentiable

c) Discontinuous and not differentiable d) Continuous and differentiable

141. If $f(x)=\begin{cases}(x-2)^2\sin\left(\frac{1}{x-2}\right)-|x-1|, & x\neq 2\\ -1, & x=2\end{cases}$ then the set of points where $f(x)$ is differentiable, is

a) R b) $R-\{1,2\}$ c) $R-\{1\}$ d) $R-\{2\}$

142. The value of f at $x=0$ so that function $f(x)=\frac{2^x-2^{-x}}{x}, x\neq 0$ is continuous at $x=0$, is

a) 0 b) $\log 2$ c) 4 d) $\log 4$

143. If $f(x)=|\log_e x|$, then

a) $f'(1^+)=1, f'(1^-)=-1$

b) $f'(1^-)=-1, f'(1^+)=0$

c) $f'(1)=1, f'(1^-)=0$

d) $f'(1)=-1, f'(1^+)=-1$

144. Let $f(x)$ be a function such that $f(x+y)=f(x)+f(y)$ and $f(x)=\sin x\, g(x)$ for all $x,y\in R$. If $g(x)$ is a continuous function such that $g(0)=k$, then $f'(x)$ is equal to

a) k b) kx c) $kg(x)$ d) None of these

145. The function $f(x)=|x|+|x-1|$, is

a) Continuous at $x=1$, but not differentiable

b) Both continuous and differentiable at $x=1$

c) Not continuous at $x=1$

d) None of these

146. The set of points of differentiability of the function $f(x)=\begin{cases}\frac{\sqrt{x+1}-1}{x}, \text{for } x\neq 0\\ 0, \text{for } x=0\end{cases}$ is

a) R b) $[0,\infty]$ c) $(-\infty,0)$ d) $R-\{0\}$

147. Given that $f(x)$ is a differentiable function of x and that $f(x).f(y)=f(x)+f(y)+f(xy)-2$ and that $f(2)=5$. Then, $f(3)$ is equal to

a) 10 b) 24 c) 15 d) None of these

148. If $f(x)=\frac{1}{2}x-1$, then on the interval $[0,\pi]$,

a) $\tan[f(x)]$ and $\frac{1}{f(x)}$ are both continuous

b) $\tan[f(x)]$ and $\frac{1}{f(x)}$ are both discontinuous

c) $\tan[f(x)]$ and $f^{-1}(x)$ are both continuous

d) $\tan[f(x)]$ s continuous but $\frac{1}{f(x)}$ is not

149. If $f(x)=(x+1)^{\cot x}$ be continuous at $=0$, then $f(0)$ is equal to

a) 0 b) $-e$ c) e d) None of these

150. Let $f(x)=\begin{cases}\frac{\tan x-\cot x}{x-\frac{\pi}{4}}, & x\neq\frac{\pi}{4}\\ a, & x=\frac{\pi}{4}\end{cases}$ the value of a so that $f(x)$ is continuous at $x=\frac{\pi}{4}$ is

a) 2 b) 4 c) 3 d) 1

151. If $f(x)=\int_{-1}^{x}|t|\,dt, x\geq -1$, then

a) f and f' are continuous for $x+1>0$

b) f is continuous but f' is not so for $x+1>0$

c) f and f' are continuous at $x=0$

d) f is continuous at $x=0$ but f' is not so

152. The set of points of discontinuity of the function

$f(x)=\lim_{n\to\infty}\frac{x^{-n}-x^{n}}{x^{-n}+x^{n}}, n\in Z$ is

a) $\{1\}$ b) $\{-1\}$ c) $\{-1,1\}$ d) None of these

153. The number of points of discontinuity of the function

$f(x)=\frac{1}{\log|x|}$, is

a) 4 b) 3 c) 2 d) 1

154. $f(x)=\begin{cases}\frac{\sin 3x}{\sin x}, & x\neq 0\\ k, & x=0\end{cases}$ is continuous, if k is

a) 3 b) 0 c) -3 d) -1

155. For the function $f(x)=\frac{\log_e(1+x)+\log_e(1-x)}{x}$ to be continuous at $=0$, the value of $f(0)$ is

a) -1 b) 0 c) -2 d) 2

156. Let $f(x)=\begin{cases}\frac{x-4}{|x-4|}+a, & x<4\\ a+b, & x=4\\ \frac{x-4}{|x-4|}+b, & x>4\end{cases}$

Then, $f(x)$ is continuous at $x=4$, when

a) $a=0, b=0$ b) $a=1, b=1$ c) $a=-1, b=1$ d) $a=1, b=-1$

157. If $f(x)\begin{cases}\frac{[x]-1}{x-1}, x\neq 1\\ 0, \quad x=1\end{cases}$ then at $x=1, f(x)$ is

a) Continuous and differentiable

b) Differentiable but not continuous

c) Continuous but not differentiable

d) Neither continuous nor differentiable

158. If $f(x)=\begin{cases}\frac{1-\sqrt{2}\sin x}{\pi-4x}, & \text{if } x\neq\frac{\pi}{4}\\ a, & \text{if } x=\frac{\pi}{4}\end{cases}$ is continuous at $\frac{\pi}{4}$, then a is equal to

a) 4 b) 2 c) 1 d) 1/4

159. If the function $f:R\to R$ given by $f(x)=\begin{cases}x+a, \text{if } x\leq 1\\ 3-x^2, \text{ if } x>1\end{cases}$ is continuous at $x=1$, thyen a is equal to

a) 4 b) 3 c) 2 d) 1

160. If $f:R\to R$ is defined by

$f(x)=\begin{cases}\frac{\cos 3x-\cos x}{x^2}, & \text{for } x\neq 0\\ \lambda, & \text{for } x=0\end{cases}$ and if f is continuous at $x=0$, then λ is equal to

a) -2 b) -4 c) -6 d) -8

161. For the function $f(x)=\begin{cases}\frac{x^3-a^3}{x-a}, x\neq a\\ b, \quad x=a\end{cases}$, if $f(x)$ is continuous at $x=a$, then b is equal to

a) a^2 b) $2a^2$ c) $3a^2$ d) $4a^2$

162. If $y=f(x)=\frac{1}{u^2+u-1}$ where $u=\frac{1}{x-1}$, then the function is discontinuous at $x=$

a) 1 b) 1/2 c) 2 d) -2

163. If $f(x)=\text{Min}\{\tan x, \cot x\}$, then

a) $f(x)$ is not differentiable at $x=0,\pi/4,5\pi/4$

b) $f(x)$ is continuous at $x=0,\pi/2,3\pi/2$

c) $\int_0^{\pi/2} f(x)dx = \ln\sqrt{2}$

d) $f(x)$ is periodic with period $\frac{\pi}{2}$

164. If $f(x) = \{|x| - |x-1\}^2$, then $f'(x)$ equals

a) 0 for all x

b) $2\{|x| - |x-1|\}$

c) $\begin{cases} 0 \text{ for } x < 0 \text{ and for } x > 1 \\ 4(2x-1) \text{for } 0 < x < 1 \end{cases}$

d) $\begin{cases} 0 \text{ for } x < 0 \\ 4(2x-1) \text{for } x > 0 \end{cases}$

165. If $f(x) = (x - x_0)\phi(x)$ and $\phi(x)$ is continuous at $x = x_0$, then $f'(x_0)$ is equal to

a) $\phi'(x_0)$ b) $\phi(x_0)$ c) $x_0\phi(x_0)$ d) None of these

166. The function defined by

$f(x) = \begin{cases} \left(x^2 + e^{\frac{1}{2-x}}\right)^{-1} & x \neq 2 \\ k, & x = 2 \end{cases}$ is continuous from right at the point $x = 2$, then k is equal to

a) 0 b) $\frac{1}{4}$ c) $-\frac{1}{2}$ d) None of these

167. If $f(x) = \begin{cases} \frac{1-\sin x}{(\pi-2x)^2} \cdot \frac{\log \sin x}{(\log 1+\pi^2-4\pi x+x^2)}, x \neq \frac{\pi}{2} \\ k, \ x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \pi/2$, then $k =$

a) $-\frac{1}{16}$ b) $-\frac{1}{32}$ c) $-\frac{1}{64}$ d) $-\frac{1}{28}$

168. If $f(x) = \begin{cases} \frac{\sin 5x}{x^2+2x}, & x \neq 0 \\ k + \frac{1}{2}, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

a) 1 b) -2 c) 2 d) $\frac{1}{2}$

169. Let $f(x) = \begin{cases} x^n \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. Then, $f(x)$ is continuous but not differentiable at $x = 0$, if

a) $n \in (0, 1]$ b) $n \in [1, \infty)$ c) $n \in (-\infty, 0)$ d) $n = 0$

170. The function $f(x) = \begin{cases} |x-3|, & \text{if } x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & \text{if } x < 1 \end{cases}$ is

a) Continuous and differentiable at $x = 3$

b) Continuous at $x = 3$, but not differentiable at $x = 3$

c) continuous and differentiable everywhere

d) continuous at $x = 1$, but not differentiable at $x = 1$

171. Let $f(x) = |x|$ and $g(x) = |x^3|$, then

a) $f(x)$ and $g(x)$ Both are continuous at $x = 0$

b) $f(x)$ and $g(x)$ Both are differentiable at $x = 0$

c) $f(x)$ is differentiable but $g(x)$ is not differentiable at $x = 0$

d) $f(x)$ and $g(x)$ Both are not differentiable at $x = 0$

172. If $f(x) = \begin{cases} \frac{\sin(a+1)x+\sin x}{x}, x < 0 \\ c, \ x = 0 \\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{bx\sqrt{x}}, x > 0 \end{cases}$ is continuous at $x = 0$, then

a) $a = -\frac{3}{2}, b = 0, c = \frac{1}{2}$

b) $a = -\frac{3}{2}, b = 1, c = -\frac{1}{2}$

c) $a=-\frac{3}{2}, b \in R-\{0\}, c=\frac{1}{2}$

d) None of these

173. If $f(x)=\begin{cases}\frac{36^x-9^x-4^x+1}{\sqrt{2}-\sqrt{1+\cos x}}, x \neq 0 \\ k,\ x=0\end{cases}$ is continuous at $x=0$, then k equals

a) $16\sqrt{2}\log 2\log 3$
b) $16\sqrt{2}\ln 6$
c) $16\sqrt{2}\ln 2\ln 3$
d) None of these

174. Let [ ] denotes the greatest integer function and $f(x)=[\tan^2 x]$. Then,

a) $\lim_{x\to 0} f(x)$ does not exist
b) $f(x)$ is continuous at $x=0$
c) $f(x)$ is not differentiable at $x=0$
d) $f(x)=1$

175. Let a function $f: R \to R$, where R is the set of real numbers satisfying the equation $f(x+y)=f(x)+f(y), \forall\, x, y$ if $f(x)$ is continuous at $x=0$, then

a) $f(x)$ is discontinuous, $\forall\, x \in R$
b) $f(x)$ is continuous, $\forall\, x \in R$
c) $f(x)$ is continuous for $x \in \{1, 2, 3, 4\}$
d) None of the above

176. Let $f(x)=\begin{cases}\sin x, & \text{for } x \geq 0 \\ 1-\cos x, & \text{for } x \leq 0\end{cases}$ and $g(x)=e^x$. Then, $(gof)'(0)$ is

a) 1
b) -1
c) 0
d) None of these

177. The function $f(x)\begin{cases}(x+1)^{2-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, x \neq 0 \\ 0, \quad x=0\end{cases}$ is

a) Continuous everywhere
b) Discontinuous at only one point
c) Discontinuous at exactly two points
d) None of these

178. If $f(x)=\begin{cases}\frac{\log(1+ax)-\log(1-bx)}{x}, x \neq 0 \\ k,\ x=0\end{cases}$ and $f(x)$ is continuous at $x=0$, then the value of k is

a) $a-b$
b) $a+b$
c) $\log a+\log b$
d) None of these

179. The value of $f(0)$, so that the function $f(x)=\frac{(27-2x)^{1/3}-3}{9-3(243+5\,x)^{1/5}}(x \neq 0)$ is continuous is given by

a) $\frac{2}{3}$
b) 6
c) 2
d) 4

180. The function $f: R/\{0\} \to R$ given by

$$f(x)=\frac{1}{x}-\frac{2}{e^{2x}-1}$$

Can be made continuous at $x=0$ by defining $f(0)$ as function

a) 2
b) -1
c) 0
d) 1

181. Which one of the following is not true always?

a) If $f(x)$ is not continuous at $x=a$, then it is not differentiable at $x=a$
b) If $f(x)$ is continuous at $x=a$, then it is differentiable at $x=a$
c) If $f(x)$ and $g(x)$ are differentiable at $x=a$, then $f(x)+g(x)$ is also differentiable at $x=a$
d) If a function $f(x)$ is continuous at $x=a$, then $\lim_{x\to a} f(x)$ exists

182. The value of the derivative of $|x-1|+|x-3|$ at $x=2$ is

a) 2
b) 1
c) 0
d) -2

183. On the interval $I=[-2, 2]$, the function $f(x)=\begin{cases}(x+1)\,e^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x \neq 0 \\ 0, \quad x=0\end{cases}$

a) Is continuous for all $x \in I-\{0\}$
b) Assumes all intermediate values from $f(-2)$ to $f(2)$
c) Has a maximum value equal to $3/e$
d) All the above

184. Function $f(x)=\begin{cases}x-1, x<2 \\ 2x-3, x \geq 2\end{cases}$ is a continuous function

a) For $x=2$ only
b) For all real values of x such that $x \neq 2$
c) For all real values of x
d) For all integer values of x only

185. The function $f(x) = \begin{cases} \frac{\tan x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$, is

a) Continuous but not differentiable at $x = 0$
b) Discontinuous at $x = 0$
c) Continuous and differentiable at $x = 0$
d) Not defined at $x = 0$

186. At the point $x = 1$, the function $f(x) = \begin{cases} x^3 - 1, & 1 < x < \infty \\ x - 1, & -\infty < x \leq 1 \end{cases}$

a) Continuous and differentiable
b) Continuous and not differentiable
c) Discontinuous and differentiable
d) Discontinuous and not differentiable

187. If $f(x)$ defined by $f(x) = \begin{cases} \frac{|x^2 - x|}{x^2 - x}, & x \neq 0, 1 \\ 1, & x = 0 \\ -1, & x = 1 \end{cases}$ then $f(x)$ is continuous for all

a) x
b) x except at $x = 0$
c) x except at $x = 1$
d) x except at $x = 0$ and $x = 1$

188. The value of derivative of $|x - 1| + |x - 3|$ at $x = 2$, is

a) -2
b) 0
c) 2
d) Not defined

189. If $f(x) = \begin{cases} 1 & \text{for } x < 0 \\ 1 + \sin x & \text{for } 0 \leq x \leq \pi/2 \end{cases}$, then at $x = 0$, the derivative $f'(x)$ is

a) 1
b) 0
c) Infinite
d) Does not exist

190. Let $g(x) = \frac{(x-1)^n}{\log \cos^m (x-1)}$; $0 < x < 2$, m and n are integers, $m \neq 0, n > 0$, and let p be the left hand derivative of $|x - 1|$ at $x = 1$. If $\lim_{x \to 1^+} g(x) = p$, then

a) $n = 1, m = 1$
b) $n = 1, m = -1$
c) $n = 2, m = 2$
d) $n > 2, m = n$

191. The function $f(x) = \frac{2x^2 + 7}{x^3 + 3x^2 - x - 3}$ is discontinuous for

a) $x = 1$ only
b) $x = 1$ and $x = -1$ only
c) $x = 1, x = -1, x = -3$ only
d) $x = 1, x = -1, x = -3$ and other values of x

192. If for a function $f(x), f(2) = 3, f'(2) = 4$, then $\lim_{x \to 2} [f(x)]$, where $[\cdot]$ denotes the greatest integer function, is

a) 2
b) 3
c) 4
d) Non-existent

193. A function $f(x)$ is defined as fallows for real x,

$f(x) = \begin{cases} 1 - x^2, & \text{for } x < 1 \\ 0, & \text{for } x = 1 \\ 1 + x^2, & \text{for } x > 1 \end{cases}$ Then,

a) $f(x)$, is not continuous at $x = 1$
b) $f(x)$ is continuous but not differentiable at $x = 1$
c) $f(x)$ is both continuous and differentiable at $x = 1$
d) None of the above

194. Let $f: R \to R$ be a function defined by $f(x) = \min\{x + 1, |x| + 1\}$. Then, which of the following is true?

a) $f(x) \geq 1$ for all $x \in R$
b) $f(x)$ is not differentiable at $x = 1$
c) $f(x)$ is differentiable everywhere
d) $f(x)$ is not differentiable at $x = 0$

195. If $f(x) = \begin{cases} mx + 1, & x \leq \frac{\pi}{2} \\ \sin x + n, & x > \frac{\pi}{2} \end{cases}$ is continuous t $x = \frac{\pi}{2}$, then

a) $m = 1, n = 0$
b) $m = \frac{n\pi}{2} + 1$
c) $n = m\frac{\pi}{2}$
d) $m = n = \frac{\pi}{2}$

196. If $f(x) = \frac{\log_e(1 + x^2 \tan x)}{\sin x^3}, x \neq 0$, is to be continuous at $x = 0$, then $f(0)$ must be defined as

a) 1
b) 0
c) $\frac{1}{2}$
d) -1

197. Let $f(x) = \begin{cases} x^P \sin\frac{1}{x}, x \neq 0 \\ 0, \quad x = 0 \end{cases}$ then $f(x)$ is continuous but not differentiable at $x = 0$, if

a) $0 < p \leq 1$ b) $1 \leq p < \infty$ c) $-\infty < p < 0$ d) $p = 0$

198. The function f defined by

$f(x) = \begin{cases} \frac{\sin x^2}{x}, x \neq 0 \\ 0, \; x = 0 \end{cases}$ is

a) Continuous and derivable at $x = 0$
b) Neither continuous nor derivable at $x = 0$
c) Continuous but not derivable at $x = 0$
d) None of these

199. A function f on R into itself is continuous at a point a in R, iff for each $\in > 0$, there exists, $\delta > 0$ such that

a) $|f(x) - f(a)| < \in \Rightarrow |x - a| < \delta$ b) $|f(x) - f(a)| > \in \Rightarrow |x - a| > \delta$
c) $|x - a| > \delta |f(x) - f(a)| > \in$ d) $|x - a| < \delta |f(x) - f(a)| < \in$

200. The function $f(x) = x - |x - x^2|, -1 \leq x \leq 1$ is continuous on the interval

a) $[-1, 1]$ b) $(-1, 1)$ c) $[-1, 0) \cup (0, 1]$ d) $(-1, 0) \cup (0, 1)$

201. if $f(x) = a|\sin x| + b\, e^{|x|} + c\, |x|^3$ and if $f(x)$ is differentiable at $x = 0$, then

a) $a = b = c = 0$ b) $a = 0, b = 0; c \in R$ c) $b = c = 0, a \in R$ d) $c = 0, a = 0, b \in R$

202. Let $f(x)$ be defined on R such that $f(1) = 2, f(2) = 8$ and $f(u + v) = f(u) + kuv - 2\, v^2$ for all $u, v \in R$ (k is a fixed constant). Then,

a) $f'(x) = 8x$ b) $f(x) = 8x$ c) $f'(x) = x$ d) None of these

203. If $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then $f(x)$ is differentiable on

a) $[-1, 1]$ b) $R - \{-1, 1\}$ c) $R - (-1, 1)$ d) None of these

204. Define f on R into itself by

$f(x) = \begin{cases} x \sin\frac{1}{x}, \text{ when } x \neq 0 \\ 0, \text{ when } x = 0 \end{cases}$, then

a) f is continuous at 0 but not differentiable at 0 b) f is both continuous and differentiable at 0
c) f is differentiable but not continuous at 0 d) None of the above

205. The set of points where the function $f(x) = |x - 1|e^x$ is differentiable, is

a) R b) $R - \{1\}$ c) $R - \{-1\}$ d) $R - \{0\}$

206. Let $f(x + y) = f(x)f(y)$ and $f(x) = 1 + xg(x)G(x)$, where $\lim_{x\to 0} g(x) = a$ and $\lim_{x\to 0} G(x) = b$. Then $f'(x)$ is equal to

a) $1 + ab$ b) ab c) a/b d) None of these

: ANSWER KEY :

1)	b	2)	d	3)	b	4)	c
5)	b	6)	b	7)	b	8)	c
9)	a	10)	d	11)	d	12)	a
13)	a	14)	b	15)	d	16)	a
17)	a	18)	b	19)	c	20)	b
21)	b	22)	c	23)	d	24)	b
25)	a	26)	d	27)	c	28)	b
29)	d	30)	b	31)	b	32)	b
33)	b	34)	c	35)	d	36)	c
37)	b	38)	a	39)	a	40)	a
41)	c	42)	b	43)	b	44)	c
45)	d	46)	c	47)	d	48)	b
49)	c	50)	d	51)	c	52)	a
53)	c	54)	b	55)	c	56)	d
57)	c	58)	d	59)	b	60)	c
61)	a	62)	a	63)	a	64)	b
65)	d	66)	a	67)	c	68)	d
69)	d	70)	a	71)	a	72)	b
73)	b	74)	c	75)	d	76)	b
77)	b	78)	c	79)	b	80)	b
81)	d	82)	c	83)	a	84)	b
85)	c	86)	c	87)	c	88)	b
89)	c	90)	a	91)	b	92)	d
93)	b	94)	c	95)	b	96)	c
97)	d	98)	d	99)	d	100)	d
101)	a	102)	b	103)	c	104)	c
105)	a	106)	a	107)	d	108)	a
109)	b	110)	a	111)	a	112)	c
113)	b	114)	c	115)	c	116)	c
117)	c	118)	b	119)	b	120)	c
121)	a	122)	c	123)	b	124)	d
125)	c	126)	b	127)	a	128)	b
129)	d	130)	b	131)	b	132)	a
133)	a	134)	d	135)	a	136)	d
137)	a	138)	b	139)	d	140)	a
141)	c	142)	d	143)	a	144)	a
145)	a	146)	d	147)	a	148)	b
149)	c	150)	b	151)	a	152)	c
153)	b	154)	a	155)	b	156)	d
157)	d	158)	d	159)	d	160)	b
161)	c	162)	a	163)	a	164)	c
165)	b	166)	b	167)	c	168)	c
169)	a	170)	b	171)	a	172)	c
173)	c	174)	b	175)	b	176)	c
177)	b	178)	b	179)	c	180)	d
181)	b	182)	c	183)	d	184)	c
185)	c	186)	b	187)	d	188)	b
189)	d	190)	c	191)	c	192)	c
193)	a	194)	c	195)	c	196)	a
197)	a	198)	a	199)	a	200)	a
201)	b	202)	a	203)	b	204)	a
205)	b	206)	d				

: HINTS AND SOLUTIONS :

1 **(b)**

We have,

$-\pi/4 < x < \pi/4$

$\Rightarrow -1 < \tan x < 1 \Rightarrow 0 \leq \tan^2 x < 1 \Rightarrow [\tan^2 x] = 0$

$\therefore f(x) = [\tan^2 x] = 0$ for all $x \in (-\pi/4, \pi/4)$

Thus, $f(x)$ is a constant function on $\in (-\pi/4, \pi/4)$

So, it is continuous on $\in (-\pi/4, \pi/4)$ and $f'(x) = 0$ for all $x \in (-\pi/4, \pi/4)$

2 **(d)**

Since, $f(x)$ is continuous at $x = 0$

$\therefore \quad \lim_{x\to 0} f(x) = f(0)$

$\Rightarrow \lim_{x\to 0} \frac{-e^x + 2^x}{x} = f(0)$

$\Rightarrow \lim_{x\to 0} \frac{-e^x + 2^x \log 2}{1} = f(0)$ [by L 'Hospital's rule]

$\Rightarrow f(0) = -1 + \log 2$

3 **(b)**

Since $f(x)$ is an even function

$\therefore f(-x) = f(x)$ for all x

$\Rightarrow -f'(-x) = f'(x)$ for all x

$\Rightarrow f'(-x) = -f'(x)$ for all x

$\Rightarrow f'(x)$ is an odd function

4 **(c)**

We have,

$$f(x) = \begin{cases} [\cos \pi x], x < 1 \\ |x-2|, 1 \leq x < 2 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} 2-x, & 1 \leq x < 2 \\ -1, & 1/2 < x < 1 \\ 0, & 0 < x \leq 1/2 \\ 1, & x = 0 \\ 0, & -1/2 \leq x < 0 \\ -1, & -3/2 < x < -1/2 \end{cases}$$

It is evident from the definition that $f(x)$ is discontinuous at $x = 1/2$

5 **(b)**

We have,

$$\lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h)$$

$$= \lim_{h\to 0} \frac{|-2-h+2|}{\tan^{-1}(-2-h+2)}$$

$$\Rightarrow \lim_{x\to 2^-} f(x) = \lim_{h\to 0} \frac{h}{\tan^{-1}(-h)} = \lim_{h\to 0} \frac{-h}{\tan^{-1} h} = -1$$

and,

$$\lim_{x\to -2^+} f(x) = \lim_{h\to 0} f(-2+h)$$

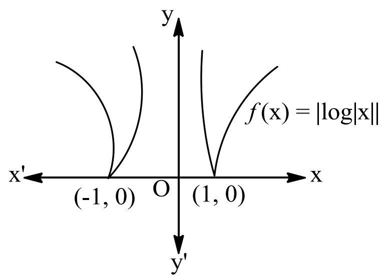
$$= \lim_{h\to 0} \frac{|-2+h+2|}{\tan^{-1}(-2+h+2)}$$

$$\Rightarrow \lim_{x\to -2^+} f(x) = \lim_{h\to 0} \frac{h}{\tan^{-1} h} = 1$$

$$\therefore \lim_{x\to -2^-} f(x) \neq \lim_{\to -2^+} f(x)$$

So, $f(x)$ is neither continuous nor differentiable at $x = -2$

6 **(b)**



From the graph of $f(x) = |\log|x||$ it is clear that $f(x)$ is everywhere continuous but not differentiable at $x = \pm 1$, due to sharp edge

7 **(b)**

We have,

$$\lim_{x\to a} \frac{xf(a) - a\,f(x)}{x-a}$$

$$= \lim_{x\to a} \frac{x\,f(a) - a\,f(a) - a\big(f(x) - f(a)\big)}{x-a}$$

$$\Rightarrow \lim_{x\to a} \frac{x\,f(a) - a\,f(x)}{x-a}$$

$$= \lim_{x\to a} \frac{f(a)(x-a)}{x-a} - a \lim_{x\to a} \frac{f(x) - f(a)}{x-a}$$

$$\Rightarrow \lim_{x\to a} \frac{x\,f(a) - a\,f(x)}{x-a} = f(a) - a\,f'(a) = 4 - 2a$$

8 **(c)**

Given, $f(x) = x\big(\sqrt{x} + \sqrt{x+1}\big)$. At $x = 0$ LHL of $\sqrt{x}$ is not defined, therefore it is not continuous at $x = 0$

Hence, it is not differentiable at $x = 0$

9 **(a)**

Here, $f'(x) = \begin{cases} 2ax, \ b \neq 0, x \leq 1 \\ 2bx + a, \ x > 1 \end{cases}$

Since, $f(X)$ is continuous at $x = 1$

$\therefore \quad \lim_{h\to 0} f(x) = \lim_{h\to 1^+} f(x)$

$\Rightarrow \quad a + b = b + a + c \Rightarrow c = 0$

Also, $f(x)$ is differentiable at $x = 1$

$\therefore$ (LHD at $x = 1$)=(RHD at $x = 1$)

$\Rightarrow \quad 2a = 2b(1) + a \quad \Rightarrow \quad a = 2b$

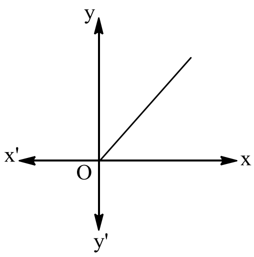
10 **(d)**

We have,

$$\lim_{x\to1^-} f(x) = \lim_{x\to1^-}\left\{\frac{x^2}{4}-\frac{3x}{4}+\frac{13}{4}\right\} = \frac{1}{4}-\frac{3}{2}+\frac{13}{4}$$
$$= 2$$
$\lim_{x\to1^+} f(x) = \lim_{x\to1} |x-3| = 2$
and, $f(1) = |1-3| = 2$
$\therefore \lim_{x\to1^-} f(x) = f(1) = \lim_{x\to1^+} f(x)$
So, $f(x)$ is continuous at $x = 1$
We have,
$\lim_{x\to3^-} f(x) = \lim_{x\to3} |x-3| = 0, \lim_{x\to3^+} f(x)$
$= \lim_{x\to3} |x-3| = 0$
and, $f(3) = 0$
$\therefore \lim_{x\to3^-} f(x) = \lim_{x\to3^+} f(x) = f(3)$
So, $f(x)$ is continuous at $x = 3$
Now,
(LHD at $x = 1$)
$$= \left\{\frac{d}{dx}\left(\frac{x^2}{4}-\frac{3x}{2}+\frac{13}{4}\right)\right\}_{x=1} = \left\{\frac{x}{2}-\frac{3}{2}\right\}_{x=1} = \frac{1}{2}-\frac{3}{2}$$
$$= -1$$
(RHD at $x = 1$) $= \left\{\frac{d}{dx}\left(-(x-3)\right)\right\}_{x=1} = -1$
$\therefore$ (LHD at $x = 1$) = (RHD at $x = 1$)
So, $f(x)$ is differentiable at $x = 1$

11 **(d)**
$$f(x) = \begin{cases} \dfrac{2\sin x - \sin 2x}{2x\cos x}, & \text{if } x \neq 0, \\ a, & \text{if } x = 0 \end{cases}$$
Now, $\lim_{x\to0} f(x) = \lim_{x\to0} \frac{2\sin x - \sin 2x}{2x\cos x}$ $\left(\frac{0}{0} \text{ form}\right)$
$$= \lim_{x\to0} \frac{2\cos x - 2\cos 2x}{2(\cos x - x\sin x)}$$
$$= \lim_{x\to0} \frac{2-2}{2(1-0)} = 0$$
Since, $f(x)$ is continuous at $x = 0$
$\therefore \quad f(0) = \lim_{x\to0} f(x)$
$\Rightarrow a = 0$

12 **(a)**
Given, $f(x) = x + |x|$
$$\therefore \quad f(x) = \begin{cases} 2x, & x \geq 0 \\ 0, & x < 0 \end{cases}$$



It is clear from the graph of $f(x)$ is continuous for every value of x
Alternate
Since, x and $|x|$ is continuous for every value of x, so their sum is also continous for every value of x

13 **(a)**
Since $f(x)$ is continuous at $x = 0$
$\therefore \lim_{x\to0^-} f(x) = f(0) = \lim_{x\to0^+} f(x)$
$\Rightarrow \lim_{x\to0}\{1+|\sin x|\}^{\frac{a}{|\sin x|}} = b = \lim_{x\to0} e^{\frac{\tan 2x}{\tan 3x}}$
$\Rightarrow e^a = b = e^{2/3} \Rightarrow a = \frac{2}{3}$ and $a = \log_e b$

14 **(b)**
We have,
$$f(x) = \begin{cases} x^2 + \dfrac{(x^2/1+x^2)}{1-(1/1+x^2)} = x^2+1, x \neq 0 \\ \qquad 0, \qquad x = 0 \end{cases}$$
Clearly, $\lim_{x\to0^-} f(x) = \lim_{x\to0^+} f(x) = 1 \neq f(0)$
So, $f(x)$ is discontinuous at $x = 0$

15 **(d)**
LHD$= \lim_{h\to0} \frac{f(0-h)-f(0)}{-h}$
$$= \lim_{h\to0} \frac{1-1}{-h} = 0$$
RHD$= \lim_{h\to0} \frac{f(0+h)-f(0)}{h}$
$$= \lim_{h\to0} \frac{1+\sin(0+h)-1}{h} = \lim_{h\to0} \frac{\sin h}{h} = 1$$
$\Rightarrow$ LHD$\neq$RHD

16 **(a)**
Given, $f(x) = x - |x - x^2|$
At $x = 1$, $f(1) = 1 - |1-1| = 1$
$\lim_{x\to1^{-1}} f(x) = \lim_{h\to0}[(1-h) - |(1-h)-(1-h)^2|]$
$= \lim_{h\to0} [(1-h) - |h-h^2|] = 1$
$\lim_{x\to1^+} f(x) = \lim_{h\to0}[(1+h) - |(1+h)-(1+h)^2|]$
$= \lim_{h\to0} [1+h-|-h^2-h|] = 1$
$\because \lim_{x\to1^{-1}} f(x) = \lim_{x\to1^+} = f(1)$

17 **(a)**
We have,
$f(x+y+z) = f(x)f(y)f(z)$ for all x, y, z ...(i)
$\Rightarrow f(0) = f(0)f(0)f(0)$ [Putting $x = y = z = 0$]
$\Rightarrow f(0)\{1-f(0)^2\} = 0$
$\Rightarrow f(0) = 1$ [$\because f(0) = 0 \Rightarrow f(x) = 0$ for all x]
Putting $z = 0$ and $y = 2$ in (i), we get
$f(x+2) = f(x)f(2)f(0)$
$\Rightarrow f(x+2) = 4f(x)$ for all x
$\Rightarrow f'(2) = 4f'(0)$ [Putting $x = 0$]
$\Rightarrow f'(2) = 4 \times 3 = 12$

18 **(b)**
For $x > 1$, we have
$$f(x) = |\log|x|| = \log x \quad \Rightarrow \quad f'(x) = \frac{1}{x}$$
For $x < -1$, we have
$$f(x) = |\log|x|| = \log(-x) \quad \Rightarrow \quad f'(x) = \frac{1}{x}$$

For $0 < x < 1$, we have

$f(x) = |\log|x|| = -\log x \quad \Rightarrow \quad f'(x) = \frac{-1}{x}$

For $-1 < x < 0$, we have

$f(x) = -\log(-x) \quad \Rightarrow \quad f'(x) = -\frac{1}{x}$

Hence, $f'(x) = \begin{cases} \frac{1}{x}, & |x| > 1 \\ -\frac{1}{x}, & |x| < 1 \end{cases}$

19 **(c)**

Since, $\lim_{x\to 0} f(x) = f(0)$

$\Rightarrow \lim_{x\to 0} \frac{1-\cos x}{x^2} = k$

$\Rightarrow \lim_{x\to 0} \frac{-(-\sin x)}{2x} = k$ [using L 'Hospital's rule]

$\Rightarrow \frac{1}{2}\lim_{x\to 0} \frac{\sin x}{x} = k \quad \Rightarrow \quad k = \frac{1}{2}$

20 **(b)**

Given, $f(X) = |x-1| + |x-2|$

$= \begin{cases} x-1+x-2, & x \ge 2 \\ x-1+2-x, & 1 \le x < 2 \\ 1-x+2-x, & x < 1 \end{cases}$

$= \begin{cases} 2x-3, & x \ge 2 \\ 1, & 1 \le x < 2 \\ 3-2x, & x < 1 \end{cases}$

$f'(x) = \begin{cases} 2, & x > 2 \\ 0, & 1 < x < 2 \\ -1, & x < 1 \end{cases}$

Hence, except $x = 1$ and $x = 2$, $f(x)$ is differentiable everywhere in R

21 **(b)**

Clearly, $f(x)$ is differentiable for all non-zero values of x. For $x \neq 0$, we have

$f'(x) = \frac{x\, e^{-x^2}}{\sqrt{1-e^{-x^2}}}$

Now,

$(\text{LHD at } x = 0) = \lim_{x\to 0^-} \frac{f(x)-f(0)}{x-0}$

$= \lim_{h\to 0} \frac{f(0-h)-f(0)}{x-0}$

$\Rightarrow (\text{LHD at } x = 0) = \lim_{h\to 0} \frac{\sqrt{1-e^{-h^2}}}{-h}$

$= \lim_{h\to 0} -\frac{\sqrt{1-e^{-h^2}}}{h}$

$\Rightarrow (\text{LHD at } x = 0) = -\lim_{h\to 0} \sqrt{\frac{e^{h^2}-1}{h^2}} \times \frac{1}{\sqrt{e^{h^2}}} = -1$

and, $(\text{RHD at } x = 0) = \lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{h\to 0} \frac{\sqrt{1-e^{-h^2}}-0}{h}$

$\Rightarrow (\text{RHD at } x = 0) = \lim_{h\to 0} \sqrt{\frac{e^{h^2}-1}{h^2}} \times \frac{1}{\sqrt{e^{h^2}}} = 1$

So, $f(x)$ is not differentiable at $x = 0$

Hence, the set of points of differentiability of $f(x)$ is $(-\infty, 0) \cup (0, \infty)$

22 **(c)**

Since $f(x)$ is continuous at $x = 0$

$\therefore f(0) = \lim_{x\to 0} x \sin\left(\frac{1}{x}\right) = 0$

23 **(d)**

For $f(x)$ to be continuous everywhere, we must have,

$f(0) = \lim_{x\to 0} f(x)$

$\Rightarrow f(0) = \lim_{x\to 0} \frac{2-(256-7x)^{1/8}}{(5x+32)^{1/5}-2} \quad \left[\text{Form}\,\frac{0}{0}\right]$

$\Rightarrow f(0) = \lim_{x\to 0} \frac{\frac{7}{8}(256-7x)^{-\frac{7}{8}}}{(5x+32)^{-4/5}} = \frac{7}{8} \times \frac{2^{-7}}{2^{-4}} = \frac{7}{64}$

24 **(b)**

We have,

$f(x) = |x|^3 = \begin{cases} x^3, & x \ge 0 \\ -x^3, & x < 0 \end{cases}$

$\therefore (\text{LHD at } x = 0) = \lim_{x\to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} -\frac{x^3}{x}$

$= 0$

and,

$\therefore (\text{RHD at } x = 0) = \lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} \frac{x^3}{x}$

$= 0$

Clearly, $(\text{LHD at } x = 0) = (\text{RHD at } x = 0)$

Hence, $f(x)$ is differentiable at $x=0$ and its derivative at $x=0$ is 0

25 **(a)**

$$\lim_{x\to 0} f(x)=\lim_{x\to 0}\left(\frac{4^x-1}{x}\right)^3\times\frac{\left(\frac{x}{a}\right)}{\sin\left(\frac{x}{a}\right)}.\frac{ax^2}{\log\left(1+\frac{1}{3}x^2\right)}$$

$$=(\log 4)^3.1.a\lim_{x\to 0}\left(\frac{x^2}{\frac{1}{3}x^2-\frac{1}{18}x^4+\ldots}\right)$$

$=3a\,(\log 4)^3$

$\because\ \lim_{x\to 0} f(x)=f(0)$

$\Rightarrow\ 3a\,(\log 4)^3=9(\log 4)^3$

$\Rightarrow\ a=3$

26 **(d)**

We have,

$f(x)=|[x]x|$ for $-1<x\le 2$

$$\Rightarrow f(x)=\begin{cases}-x, & -1<x<0\\ 0, & 0\le x<1\\ x, & 1\le x<2\\ 2x, & x=2\end{cases}$$

It is evident from the graph of this function that it is continuous but not differentiable at $x=0$. Also, it is discontinuous at $x=1$ and non-differentiable at $x=2$

27 **(c)**

Given, $f(x)=[x^3-3]$

Let $g(x)=x^3-x$ it is in increasing function

$\therefore\ g(1)=1-3=-2$

and $g(2)=8-3=5$

Here, $f(x)$ is discontinuous at six points

28 **(b)**

Given, $y=\cos^{-1}\cos(x-1),\ x>0$

$\Rightarrow\ y=x-1,\quad 0\le x-1\le\pi$

$\therefore\ y=x-1,\quad 1\le x\le\pi+1$

At $x=\frac{5\pi}{4}\in[1,\pi+1]$

$$\Rightarrow\frac{dy}{dx}=1\ \Rightarrow\ \left(\frac{dy}{dx}\right)_{x=\frac{5\pi}{4}}=1$$

29 **(d)**

We have,

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow f'(x)=\lim_{h\to 0}\frac{f(x)+f(h)-f(x)}{h}\quad[\because f(x+y)=f(x)+f(y)]$$

$$\Rightarrow f'(x)=\lim_{h\to 0}\frac{f(h)}{h}=\lim_{h\to 0}\frac{h^2g(h)}{h}$$

$$\Rightarrow f'(x)=0\times g(0)=0\quad\left[\begin{array}{l}\because g\text{ is continuous}\\ \therefore \lim_{h\to 0}g(h)=g(0)\end{array}\right]$$

30 **(b)**

Using Heine's definition of continuity, it can be shown that $f(x)$ is everywhere discontinuous

31 **(b)**

For $x\ne -1$, we have

$f(x)=1-2x+3x^2-4x^3+\cdots\infty$

$$\Rightarrow f(x)=(1+x)^{-2}=\frac{1}{(1+x)^2}$$

Thus, we have

$$f(x)=\begin{cases}\frac{1}{(1+x)^2}, & x\ne -1\\ 1, & x=-1\end{cases}$$

We have,

$\lim_{x\to -1^-} f(x)\to\infty$ and $\lim_{x\to -1^-} f(x)\to\infty$

So, $f(x)$ is not continuous at $x=-1$

Consequently, it is not differentiable there at

32 **(b)**

At $x=a$,

LHL= $\lim_{x\to a^-} f(x)=\lim_{x\to a} 2a-x=a$

And RHL= $\lim_{x\to a^+} f(x)=\lim_{x\to a} 3x-2a=a$

And $f(a)=3(a)-2a=a$

$\therefore$ LHL=RHL= $f(a)$

Hence, it is continuous at $x=a$

Again, at $x=a$

LHD= $\lim_{h\to 0}\frac{f(a-h)-f(a)}{-h}$

$$=\lim_{h\to 0}\frac{2a-(a-h)-a}{-h}=-1$$

and RHD= $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

$$=\lim_{h\to 0}\frac{3(a+h)-2a-a}{h}=3$$

$\therefore$ LHD≠RHD

Hence, it is not differentiable at $x=a$

33 **(b)**

We have,

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow f'(x)=\lim_{h\to 0}\frac{f(x)f(h)-f(x)}{h}$$

$$\Rightarrow f'(x)=f(x)\lim_{h\to 0}\frac{f(h)-1}{h}$$

$$\Rightarrow f'(x)=f(x)\lim_{h\to 0}\frac{1+(\sin 2h)g(h)-1}{h}$$

$$\Rightarrow f'(x)=f(x)\lim_{h\to 0}\frac{\sin 2h}{h}\times\lim_{h\to 0}g(h)$$

$$=2f(x)g(0)$$

34 **(c)**

If $-1 \le x \le 1$, then $0 \le x \sin \pi x \le 1/2$

$\therefore f(x) = [x \sin \pi x] = 0$, for $-1 \le x \le 1$

If $1 < x < 1 + h$, where h is a small positive real number, then

$\pi < \pi x < \pi + \pi h \Rightarrow -1 < \sin \pi x < 0 \Rightarrow -1 < x \sin \pi x < 0$

$\therefore f(x) = [x \sin \pi x] = -1$ in the right neighbourhood of $x = 1$

Thus, $f(x)$ is constant and equal to zero in $[-1, 1]$ and so $f(x)$ is differentiable and hence continuous on $(-1,1)$

At $x = 1, f(x)$ is discontinuous because

$\Rightarrow \lim_{x \to 1^-} f(x) = 0$ and $\lim_{x \to 1^+} f(x) = -1$

Hence, $f(x)$ is not differentiable at $x = 1$

35 **(d)**

We have,

$$(\text{LHD at } x = 0) = \left\{\frac{d}{dx}(1)\right\}_{x=0} = 0$$

$$(\text{RHD at } x = 0) = \left\{\frac{d}{dx}(1 + \sin x)\right\}_{x=0} = \cos 0 = 1$$

Hence, $f'(x)$ at $x = 0$ does not exist

36 **(c)**

Here, $f'(x) = \begin{cases} 2bx + a, & x \ge -1 \\ 2a, & x < -1 \end{cases}$

Given, $f'(x)$ is continuous everywhere

$\therefore \lim_{x \to -1^+} f(x) = \lim_{x \to -1^-} f(x)$

$\Rightarrow -2b + a = -2a$

$\Rightarrow 3a = 2b$

$\Rightarrow a = 2, \quad b = 3$

or $a = -2, \ b = -3$

37 **(b)**

We have,

$$\lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{\log \cos x}{\log(1 + x^2)}$$

$$\Rightarrow \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{\log(1 - 1 + \cos x)}{\log(1 + x^2)} \cdot \frac{1 - \cos x}{1 - \cos x}$$

$$\Rightarrow \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{\log\{1 - (1 - \cos x)\}}{1 - \cos x} \cdot \frac{1 - \cos x}{\log(1 + x^2)}$$

$$\Rightarrow \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = -\lim_{x \to 0} \log \frac{[1 - (1 - \cos x)]}{-(1 - \cos x)} \times \frac{2 \sin^2 \frac{x}{2}}{4\left(\frac{x}{2}\right)^2} \times \frac{x^2}{\log(1 + x^2)}$$

$$\Rightarrow \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = -\frac{1}{2}$$

Hence, $f(x)$ is differentiable and hence continuous at $x = 0$

38 **(a)**

Since $f(x)$ is continuous at $x = 1$. Therefore,

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) \Rightarrow A - B = 3 \Rightarrow A = 3 + B$ ...(i)

If $f(x)$ is continuous at $x = 2$, then

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) \Rightarrow 6 = 4B - A$ ...(ii)

Solving (i) and (ii) we get $B = 3$

As $f(x)$ is not continuous at $x = 2$. Therefore, $B \ne 3$

Hence, $A = 3 + B$ and $B \ne 3$

39 **(a)**

We have,

$$f(x) = \begin{cases} x - 4, & x \ge 4 \\ -(x - 4), & 1 \le x < 4 \\ (x^3/2) - x^2 + 3x + (1/2), & x < 1 \end{cases}$$

Clearly, $f(x)$ is continuous for all x but it is not differentiable at $x = 1$ and $x = 4$

40 **(a)**

It is given that $f(x)$ is continuous at $x = 1$

$\therefore \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

$\Rightarrow \lim_{x \to 1^-} a[x + 1] + b[x - 1] = \lim_{x \to 1^+} a[x + 1] + b[x - 1]$

$\Rightarrow a - b = 2a + 0 \times b$

$\Rightarrow a + b = 0$

41 **(c)**

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \lambda[x] = 0$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} 5^{1/x} = 0$

And $f(0) = \lambda[0] = 0$

$\therefore f$ is continuous only whatever λ may be

42 **(b)**

We have,

$y(x) = f(e^x)\, e^{f(x)}$

$\Rightarrow y'(x) = f'(e^x) \cdot e^x \cdot e^{f(x)} + f(e^x)\, e^{f(x)}\, f'(x)$

$\Rightarrow y'(0) = f'(1)e^{f(0)} + f(1)e^{f(0)}f'(0)$

$\Rightarrow y'(0) = 2 \qquad [\because f(0) = f(1) = 0, f'(1) = 2]$

43 **(b)**

Since $f(x)$ is differentiable at $x = 1$. Therefore,

$$\lim_{x\to1^-}\frac{f(x)-f(1)}{x-1}=\lim_{x\to1^+}\frac{f(x)-f(1)}{x-1}$$
$$\Rightarrow \lim_{h\to0}\frac{f(1-h)-f(1)}{-h}=\lim_{h\to0}\frac{f(1+h)-f(1)}{h}$$
$$\Rightarrow \lim_{h\to0}\frac{a(1-h)^2-b-1}{-h}=\lim_{h\to0}\frac{\frac{1}{|1+h|}-1}{h}$$
$$\Rightarrow \lim_{h\to0}\frac{(a-b-1)-2\,ah+ah^2}{-h}=\lim_{h\to0}\frac{-h}{h(1+h)}$$
$$\Rightarrow \lim_{h\to0}\frac{-(a-b-1)-2\,ah-ah^2}{h}=-1$$
$\Rightarrow -(a-b-1)=0$ and so $\lim_{h\to0}\frac{2ah-ah^2}{h}=-1$

$\Rightarrow a-b-1=0$ and $2a=-1 \Rightarrow a=-\frac{1}{2}, b=-\frac{3}{2}$

44 **(c)**

We have,

$f(x)=\frac{\sin 4\pi[x]}{1+[x]^2}=0$ for all

x $[\because 4\pi[x]$is a multiple of $\pi]$

$\Rightarrow f'(x)=0$ for all x

45 **(d)**

We have,

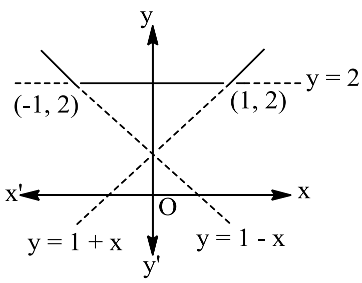
$$\lim_{x\to0}f(x)=\lim_{x\to0}\sin\frac{1}{x}$$

$\Rightarrow \lim_{x\to0} f(x)=$ An oscillating number which oscillates between -1 and 1

Hence, $\lim_{x\to0} f(x)$ does not exist

Consequently, $f(x)$ cannot be continuous at $x=0$ for any value of k

46 **(c)**



It is clear from the graph that $f(x)$ is continuous everywhere and also differentiable everywhere except $\{-1,1\}$ due to sharp edge

47 **(d)**

We have,

$\log\left(\frac{x}{y}\right)=\log x-\log y$ and $\log(e)=1$

$\therefore f(x)=\log x$

Clearly, $f(x)$ is unbounded because $f(x)\to-\infty$ as $x\to0$ and $f(x)\to+\infty$ as $x\to\infty$

We have,

$$f\left(\frac{1}{x}\right)=\log\left(\frac{1}{x}\right)=-\log x$$

As $x\to0, f\left(\frac{1}{x}\right)\to\infty$

Also,

$$\lim_{x\to0}xf(x)=\lim_{x\to0}x\log x=\lim_{x\to0}\frac{\log x}{1/x}$$
$$\Rightarrow \lim_{x\to0}xf(x)=\lim_{x\to0}\frac{1/x}{-1/x^2}=-\lim_{x\to0}x=0$$

49 **(c)**

Since $g(x)$ is the inverse of $f(x)$. Therefore,

$fog(x)=x$, for all x

$\Rightarrow \frac{d}{dx}\{fog\ (x)\}=1$, for all x

$\Rightarrow f'\big(g(x)\big)\,g'(x)=1$, for all x

$\Rightarrow \frac{1}{1+\{g(x)\}^3}\times g'(x)=1$ for all x $\quad\left[\because f'(x)=\right.$

11+x3

$\Rightarrow g'(x)=1+\{g(x)\}^3$, for all x

50 **(d)**

We have,

$f(x)=|x^2-4x+3|$

$$\Rightarrow f(x)=\begin{cases} x^2-4x+3, & \text{if } x^2-4x+3\ge0\\ -(x^2-4x+3), & \text{if } x^2-4x+3<0\end{cases}$$
$$\Rightarrow f(x)=\begin{cases} x^2-4x+3, & \text{if } x\le1 \text{ or } x\ge3\\ -x^2+4x-3, & \text{if } 1<x<3\end{cases}$$

Clearly, $f(x)$ is everywhere continuous

Now,

$$(\text{LHD at } x=1)=\left(\frac{d}{dx}(x^2-4x+3)\right)_{\text{at } x=1}$$

$\Rightarrow$ (LHD at $x=1$) $=(2x-4)_{\text{at } x=1}=-2$

and,

$$(\text{RHD at } x=1)=\left(\frac{d}{dx}(-x^2+4x-3)\right)_{\text{at } x=1}$$

$\Rightarrow$ (RHD at $x=1$) $=(-2x+4)_{\text{at } x=1}=2$

Clearly, (LHD at $x=1$) $\ne$ (RHD at $x=1$)

So, $f(x)$ is not differentiable at $x=1$

Similarly, it can be checked that $f(x)$ is not differentiable at $x=3$ also

ALITER We have,

$f(x)=|x^2-4x+3|=|x-1|\,|x-3|$

Since, $|x-1|$ and $|x-3|$ are not differentiable at 1 and 3 respectively

Therefore, $f(x)$ is not differentiable at $x=1$ and $x=3$

51 **(c)**

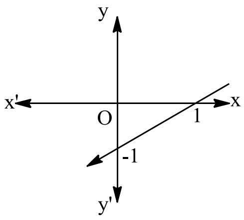
The point of discontinuity of $f(x)$ are those points where $\tan x$ is infinite.

ie, $\tan x=\tan\infty$

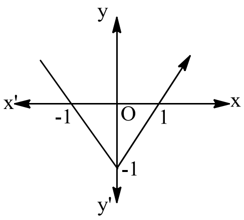
$$\Rightarrow \quad x=(2n+1)\frac{\pi}{2}, \qquad n\in I$$

52 **(a)**

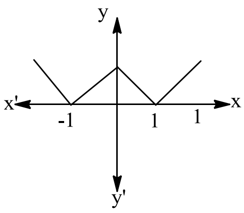
Using graphical transformation



(i) y = x - 1



(ii) y = |x| - 1



(iii) y = ||x| - 1|

As, we know the function is not differentiable at6 sharp edges and in figure (iii) $y = \left||x| - 1\right|$ we have 3 sharp edges at $x = -1, 0, 1$

$\therefore f(x)$ is not differentiable at $\{0, \pm 1\}$

53 **(c)**

$\lim_{x\to 0^-} f(x) = \lim_{h\to 0} 2(0-h) = 0$

And $\lim_{x\to 0^+} f(x) = \lim_{h\to 0} 2(0+h) + 1 = 1$

$\because \lim_{x\to 0^-} f(x) \neq \lim_{x\to 1^+} f(x)$

$\therefore \; f(x)$ is discontinuous at $x = 0$

54 **(b)**

Draw a rough sketch of $y = f(x)$ and observe its properties

55 **(c)**

$$\lim_{x\to\pi} \frac{(1+\cos x) - \sin x}{(1+\cos x) + \sin x}$$

$$= \lim_{x\to\pi} \frac{2\cos^2 x/2 - 2(\sin x/2)\cos\; x/2}{2\cos^2 x/2 + 2(\sin x/2)\cos\; x/2}$$

$$= \lim_{x\to\pi} \tan\left(\frac{\pi}{4} - \frac{\pi}{2}\right) = -1$$

Since, $f(x)$ is continuous at $x = \pi$

$\therefore \; f(\pi) = \lim_{x\to\pi} f(x) = -1$

56 **(d)**

$$f'(1^-) = \lim_{h\to 0} \frac{f(1-h) - f(1)}{-h}$$

$$= \lim_{h\to 0} \frac{(1-h-1).\sin\left(\frac{1}{1-h-1}\right) - 0}{-h}$$

$$= -\lim_{h\to 0} \sin\frac{1}{h}$$

And $f'(1^+) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$

$$= \lim_{h\to 0} \frac{(1+h-1)\sin\left(\frac{1}{1+h-1}\right) - 0}{h}$$

$$= \lim_{h\to 0} \sin\frac{1}{h}$$

$\therefore \; f'(1^-) \neq f'(1^+)$

f is not differentiable at $x = 1$

Again, now

$$f'(0^+) = \lim_{h\to 0} \frac{(0+h-1)\sin\left(\frac{1}{0+h-1}\right) - \sin 1}{h}$$

$$= \lim_{h\to 0} \frac{\left[-\left\{(h-1)\cos\left(\frac{1}{h-1}\right) \times \left(\frac{1}{(h-1)^2}\right)\right\} + \sin\left(\frac{1}{h-1}\right)\right]}{1}$$

[using L 'Hospital's rule]

$= \cos 1 - \sin 1$

And $f'(0^-) = \lim_{h\to 0} \frac{(0-h-1)\sin\left(\frac{1}{0-h-1}\right) - \sin 1}{-h}$

$$= \lim_{h\to 0} \frac{(-h-1)\cos\left(\frac{1}{-h-1}\right)\left(\frac{1}{(-h-1)^2}\right) - \sin\left(\frac{1}{-h-1}\right)}{-1}$$

[using L 'Hospital's rule]

$= \cos 1 - \sin 1$

$\Rightarrow \; f'(0^-) = f'(0^+)$

$\therefore \; f$ is differentiable at $x = 0$

57 **(c)**

As $f(x)$ is continuous at $x = \frac{\pi}{2}$

$\therefore \lim_{x\to\frac{\pi}{2}^-} f(x) = \lim_{x\to\frac{\pi}{2}^+} f(x)$

$$\Rightarrow m\frac{\pi}{2} + 1 = \sin\frac{\pi}{2} + n \Rightarrow m\frac{\pi}{2} + 1 = 1 + n \Rightarrow n = \frac{m\,\pi}{2}$$

58 **(d)**

Since, $\frac{f(6)-f(1)}{6-1} \geq 2 \quad \left[\because \; f'(x) = \frac{y_2-y_1}{x_2-x_1}\right]$

$\Rightarrow \; f(6) - f(1) \geq 10$

$\Rightarrow f(6) + 2 \geq 10$

$\Rightarrow \; f(6) \geq 8$

59 **(b)**

We have,

$$\lim_{x\to a^-} f(x)\; g(x) = \lim_{x\to a^-} f(x) . \lim_{x\to a^-} g(x) = m \times l = ml$$

and,

$$\lim_{x\to a^+} f(x)\; g(x) = \lim_{x\to a^+} f(x) \lim_{x\to a^+} g(x) = lm$$

$\therefore \lim_{x\to a^-} f(x)\; g(x) = \lim_{x\to a^+} f(x)\; g(x) = lm$

Hence, $\lim_{x\to a} f(x)\; g(x)$ exists and is equal to lm

60 **(c)**

We have,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h\to 0} \frac{f(x)f(h) - f(x)}{h}$$

$$\Rightarrow f'(x) = f(x) \lim_{h\to 0} \frac{f(h) - 1}{h} \quad [\because f(x+y) = f(x)f(y)]$$

$$\Rightarrow f'(x) = f(x) \left\{\lim_{h\to 0} \frac{1 + h\,g(h) - 1}{h}\right\} \quad [\because f(x) = 1 + x\,g(x)]$$

$$\Rightarrow f'(x) = f(x) \lim_{h\to 0} g(h) = f(x) \cdot 1 = f(x)$$

61 **(a)**

We have, $f(x) = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$

Clearly, $f(x)$ is differentiable for all $x > 0$ and for all$x < 0$. So, we check the differentiable at $x = 0$

Now, (RHD at $x = 0$)

$\left(\frac{d}{dx}(x)^2\right)_{x=0} = (2x)_{x=0} = 0$

And (LHD at $= 0$)

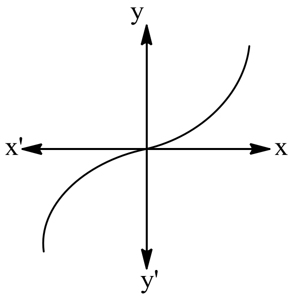
$\left(\frac{d}{dx}(-x)^2\right)_{x=0} = (-2x)_{x=0} = 0$

$\therefore$ (LHD at $x = 0$)=(RHD at $x = 0$)

So, $f(x)$ is differentiable for all x *ie*, the set of all points where $f(x)$ is differentiable is $(-\infty, \infty)$

Alternate

It is clear from the graph $f(x)$ is differentiable everywhere.



62 **(a)**

Since, $f'(0) = \lim\limits_{x\to 0} \frac{f(x)-f(0)}{x-0} = 10$

$\Rightarrow \lim\limits_{h\to 0} \dfrac{f(0+h)-f(0)}{h} = 10$

$\Rightarrow f(0)\left(\lim\limits_{h\to 0} \frac{f(h)-1}{h}\right) = 10$ ...(i)

$[\because \; f(0+h) = f(0)f(h), \text{given}]$

Now, $f(0) = f(0)f(0)$

$\Rightarrow f(0) = 1$

$\therefore$ From Eq. (i)

$\lim\limits_{h\to 0} \frac{f(h)-1}{h} = 10$ ...(ii)

Now, $f'(6) = \lim\limits_{h\to 0} \frac{f(6+h)-f(6)}{h}$

$= \lim\limits_{x\to 0} \left(\frac{f(h)-1}{h}\right) f(6)$ [from Eq. (ii)]

$= 10 \times 3 = 30$

63 **(a)**

We have,

$f'(a^+) = \lim\limits_{x\to a^+} \dfrac{f(x)-f(0)}{x-a}$

$\Rightarrow f'(a^+) = \lim\limits_{x\to a^+} \dfrac{|x-a|\phi(x)}{x-a}$

$\Rightarrow f'(a^+) = \lim\limits_{x\to a} \dfrac{(x-a)}{(x-a)}\phi(x)$ $[\because x > a \;\therefore |x-a| = x-a]$

$\Rightarrow f'(a^+) = \lim\limits_{x\to a} \phi(x)$

$\Rightarrow f'(a^+) = \phi(a)$ $[\because \phi(x)$ is continuous at $x = a]$

and,

$f'(a^-) = \lim\limits_{x\to a^-} \dfrac{f(x)-f(0)}{x-a}$

$\Rightarrow f'(a^-) = \lim\limits_{x\to a^-} \dfrac{|x-a|\phi(x)}{x-a}$

$\Rightarrow f'(a^-) = \lim\limits_{x\to a} \dfrac{(x-a)\phi(x)}{(x-a)}$ $[\because x < a \;\therefore |x-a| = -(x-a)]$

$\Rightarrow f'(a^-) = -\lim\limits_{x\to a} \phi(x)$

$\Rightarrow f'(a^-) = -\phi(a)$

$[\because \phi(x)$ is continuous at $x = a]$

64 **(b)**

LHL= $\lim_{h\to 0}(0-h)_e^{-\left(\frac{1}{|-h|}+\frac{1}{(-h)}\right)} = \lim\limits_{h\to 0}(-h) = 0$

RHL= $\lim_{h\to 0}(0+h)_e^{-\left(\frac{1}{|h|}+\frac{1}{(h)}\right)} = \lim\limits_{h\to 0} \frac{h}{e^{2/h}} = 0$

LHL=RHL= $f(0)$

Therefore, $f(x)$ is continuous for all x

Differentiability at $x = 0$

$Lf'(0) = \lim\limits_{h\to 0} \dfrac{(-h)e^{-\left(\frac{1}{h}-\frac{1}{h}\right)}}{(-h)-0} = 1$

$Rf'(0) = \lim\limits_{h\to 0} \frac{he^{-\left(\frac{1}{h}+\frac{1}{h}\right)-0}}{h-0}$

$= \lim\limits_{h\to 0} \dfrac{1}{e^{2/h}} = 0$

$\Rightarrow \; Rf'(0)Lf'(0)$

Therefore, $f(x)$ is not differentiable at $x = 0$

65 **(d)**

We have,

$f(x) = \begin{cases} 3, & x < 0 \\ 2x+1, & x \geq 0 \end{cases}$

Clearly, f is continuous but not differentiable at $x = 0$

Now,

$f(|x|) = 2|x| + 1$ for all x

Clearly, $f(|x|)$ is everywhere continuous but not differentiable at $x = 0$

67 **(c)**

We have,

$f(x) = |x-0.5| + |x-1| + \tan x, 0 < \mathrm{x} < 2$

$\Rightarrow f(x) = \begin{cases} -2x + 1.5 + \tan x, & 0 < x < 0.5 \\ 0.5 + \tan x, & 0.5 \leq x < 1 \\ 2x - 1.5 + \tan x, & 1 \leq x < 2 \end{cases}$

It is evident from the above definition that $Lf'(0.5) \neq Rf'(0.5)$ and $Lf'(1) \neq Rf'(1)$

Also, the function is not continuous at $= \pi/2$. So, it cannot be differentiable thereat

68 **(d)**

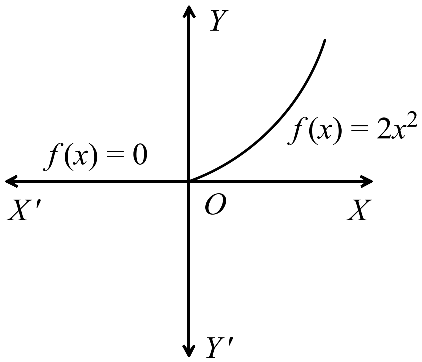
Given, $f(x) = \begin{cases} \log_{(1-3x)}(1+3x), & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$

$$\lim_{x\to0} f(x) = \lim_{x\to0}\frac{\log(1+3x)}{\log(1-3x)}$$
$$= -\lim_{x\to0}\frac{\log(1+3x)}{3x}.\frac{(-3x)}{\log(1-3x)}$$
$$= -1$$
And $f(0) = k$
$\because f(x)$ is continuous at $x = 0$
$\therefore \quad k = -1$

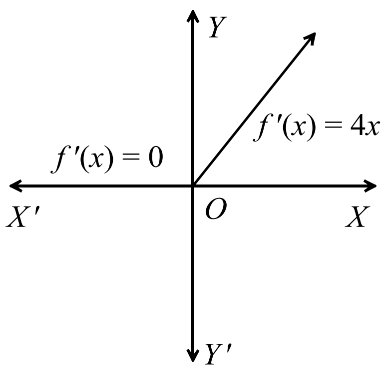
69 **(d)**
Since $f(x)$ is differentiable at $x = c$. Therefore, it is continuous at $x = c$
Hence, $\lim_{x\to c} f(x) = f(c)$

70 **(a)**
Given, $f(x) = ae^{|x|} + b\,|x|^2$
We know $e^{|x|}$ is not differentiable at $x = 0$ and $|x|^2$ is differentiable at $x = 0$
$\therefore \; f(x)$ is differentiable at $x = 0$, if $a = 0$ and $b \in R$

71 **(a)**
We have,
$$f(x) = \begin{cases}(x-x)(-x) = 0, x < 0\\ (x+x)x = 2x^2, x \ge 0\end{cases}$$



(i)



(ii)

As is evident from the graph of $f(x)$ that it is continuous and differentiable for all x
Also, we have
$$f''(x) = \begin{cases}0, x < 0\\ 4x, x \ge 0\end{cases}$$
Clearly, $f''(x)$ is continuous for all x but it is not differentiable at $x = 0$

72 **(b)**
Given, $f(x) = \begin{cases}\frac{x-1}{2x^2-7x+5}, & x \ne 1\\ -\frac{1}{3}, & x = 1\end{cases}$
$$f(x) = \begin{cases}\frac{1}{2x-5}, & x \ne 1\\ -\frac{1}{3}, & x = 1\end{cases}$$
$$f'(1) = \lim_{h\to0}\frac{f(1+h)-f(1)}{h}$$
$$= \lim_{h\to0}\frac{\frac{1}{2(1+h)-5} - \left(-\frac{1}{3}\right)}{h}$$
$$= \lim_{h\to0}\frac{\frac{1}{2h-3}+\frac{1}{3}}{h} = \lim_{h\to0}\frac{3+2h-3}{3h(2h-3)} = -\frac{2}{9}$$
$$Lf'(1) = \lim_{h\to0}\frac{f(1-h)-f(1)}{-h}$$
$$= \lim_{h\to0}\frac{\frac{1}{2(1-h)-5} - \left(-\frac{1}{3}\right)}{-h}$$
$$= \lim_{h\to0} -\frac{2}{3(2h+3)} = -\frac{2}{9}$$
$$\therefore \; f'(1) = -\frac{2}{9}$$

73 **(b)**
$$f'(1) = \lim_{h\to0}\frac{f(1+h)-f(1)}{h}$$
$$= \lim_{h\to0}\frac{f(1+h)}{h} - \lim_{h\to0}\frac{f(1)}{h}$$
Given, $\lim_{h\to0}\frac{f(1+h)}{h} = 5$
So, $\lim_{h\to0}\frac{f(1)}{h}$ must be finite as $f'(1)$ exist and $\lim_{h\to0}\frac{f(1)}{h}$ can be finite only, if $f(1) = 0$ and $\lim_{h\to0}\frac{f(1)}{h} = 0$
So, $f'(1) = \lim_{h\to0}\frac{f(1+h)}{h} = 5$

74 **(c)**
Since, $f(x)$ is continuous for every value of R except {-1, -2}. Now, we have to check that points
At $x = -2$
LHL= $\lim_{h\to0}\frac{(-2-h)+2}{(-2-h)^2+3(-2-h)+2}$
$$= \lim_{h\to0}\frac{-h}{h^2+h} = -1$$
RHL= $\lim_{h\to0}\frac{(-2+h)+2}{(-2+h)^2+3(-2+h)+2}$
$$= \lim_{h\to0}\frac{h}{h^2-h} = -1$$
$\Rightarrow$ LHL=RHL=$f(-2)$
$\therefore$ It is continuous at $x = -2$
Now, check for $x = -1$
LHL= $\lim_{h\to0}\frac{(-1-h)+2}{(-1-h)^2+3(-1-h)+2}$

$= \lim_{h\to 0} \frac{1-h}{h^2-h} = \infty$

RHL$= \lim_{h\to 0} \frac{(-1+h)+2}{(-1+h)^2+3(-1+h)+2}$

$= \lim_{h\to 0} \frac{1+h}{h^2+h} = \infty$

$\Rightarrow$ LHL=RHL$\neq f(-1)$

$\therefore$ It is not continuous at $x=-1$

The required function is continuous in $R-\{-1\}$

75 **(d)**

$f(0) = \lim_{x\to 0} \frac{(e^x-1)^2}{\sin\left(\frac{x}{a}\right)\log\left(1+\frac{x}{4}\right)}$

$\Rightarrow \quad \lim_{x\to 0} \left(\frac{e^x-1}{x}\right)^2 . \frac{\frac{x}{a}.a}{\sin\frac{x}{a}} . \frac{\frac{x}{4}.4}{\log\left(1+\frac{x}{4}\right)} = 12$

$\Rightarrow \quad 1^2.a.4 = 12$

$\Rightarrow \quad a = 3$

76 **(b)**

We have,

$f(x) = \frac{x}{1+x} + \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + \cdots \infty$

$\Rightarrow f(x) = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{x}{((r-1)x+1)\,(rx+1)}$, for $x \neq 0$

$\Rightarrow f(x) = \lim_{n\to\infty} \sum_{r=1}^{n} \left\{\frac{1}{(r-1)x+1} - \frac{1}{rx+1}\right\}$, for $x \neq 0$

$\Rightarrow f(x) = \lim_{n\to\infty} \left\{1 - \frac{2}{nx+1}\right\} = 1$, for $x \neq 0$

For $x = 0$, we have $f(x) = 0$

Thus, we have $f(x) = \begin{cases} 1, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Clearly, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) \neq f(0)$

So, $f(x)$ is not continuous at $x = 0$

77 **(b)**

If possible, let $f(x)+g(x)$ be continuous. Then, $\{f(x)+g(x)\} - f(x)$ must be continuous

$\Rightarrow g(x)$ must be continuous

This is a contradiction to the given fact that $g(x)$ is discontinuous

Hence, $f(x)+g(x)$ must be discontinuous

78 **(c)**

We have,

$f(x+y) = f(x)f(y)$ for all $x, y \in R$

$\therefore f(0) = f(0)f(0)$

$\Rightarrow f(0)\{f(0)-1\} = 0$

$\Rightarrow f(0) = 1 \qquad [\because f(0) \neq 1]$

Now,

$f'(0) = 0$

$\Rightarrow \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = 2$

$\Rightarrow \lim_{h\to 0} \frac{f(h)-1}{h} = 2 \qquad [\because f(0) = 1]$ ....(i)

$\therefore f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h\to 0} \frac{f(x)f(h)-f(x)}{h} \qquad [\because f(x+y) = f(x)f(y)]$

$\Rightarrow f'(x) = f(x)\left\{\lim_{h\to 0} \frac{f(h)-1}{h}\right\} = 2f(x)$ [Using (i)]

79 **(b)**

We have,

$f(x) = \begin{cases} \frac{x^2}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

$\Rightarrow f(x) = \begin{cases} \frac{x^2}{2} = x, & x > 0 \\ 0, & x = 0 \\ \frac{x^2}{-x} = -x, & x < 0 \end{cases}$

$\Rightarrow \lim_{x\to 0^-} f(x) = \lim_{x\to 0} -x = 0, \lim_{x\to 0^+} f(x) = \lim_{x\to 0} x = 0$ and $f(0) = 0$

So, $f(x)$ is continuous at $x = 0$. Also, $f(x)$ is continuous for all other values of x

Hence, $f(x)$ is everywhere continuous

Clearly, $Lf'(0) = -1$ and $Rf'(0) = 1$

Therefore, $f(x)$ is not differentiable at $x = 0$

80 **(b)**

Since $f(x)$ is continuous at $x = 0$

$\therefore \lim_{x\to 0} f(x) = f(0) \Rightarrow f(0) = 2$ ...(i)

Now, using L' Hospital's rule, we have

$\lim_{x\to 0} \frac{\int_0^x f(u)\,du}{x} = \lim_{x\to 0} \frac{f(x)}{1}$

$= f(0) \quad [\because f(x)$ is continuous at $x = 0]$

$\Rightarrow \lim_{x\to 0} \frac{\int_0^x f(u)\,du}{x} = 2 \qquad$ [Using (i)]

82 **(c)**

$f'(2^+) = \lim_{x\to 2^+} \left(\frac{f(x)-f(2)}{x-2}\right)$

$= \lim_{x\to 2^+} \frac{3x+4-(6+4)}{x-2} = \lim_{x\to 2} \frac{3x-6}{x-2} = 3$

83 **(a)**

Here, $f(x) = \begin{cases} \sin x, x > 0 \\ 0, \ x = 0 \\ -\sin x, \ x < 0 \end{cases}$

RHD$= \lim_{h\to 0} \frac{\sin|0+h| - \sin(0)}{h}$

$= \lim_{h\to 0} \frac{\sin h}{h} = 1$

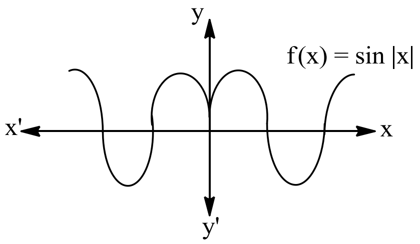
LHD= $\lim_{h\to 0} \frac{\sin|(0-h)|-\sin(0)}{-h}$

$= \frac{-\sin h}{h} = -1$

$\therefore$ LHD≠RHD at $x = 0$

$\therefore$ $f(x)$ is not derivable at $x = 0$

Alternate



It is clear from the graph that $f(x)$ is not differentiable at $x = 0$

84 **(b)**

We have,

$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} (\log_e a)^n$$

$$\Rightarrow f(x) = \sum_{n=0}^{\infty} \frac{(x \log_e a)^n}{n!} = \sum_{n=0}^{\infty} \frac{(\log_e a^x)^n}{n!}$$

$\Rightarrow f(x) = e^{\log_e a^x} = a^x$, which is everywhere continuous and differentiable

85 **(c)**

$$f(x) = [x] \cos \left[\frac{2x-1}{2}\right] \pi$$

Since, $[x]$ is always discontinuous at all integer value, hence $f(x)$ is discontinuous for all integer value

86 **(c)**

The function f is clearly continuous for $|x| > 1$

We observe that

$$\lim_{x\to -1^+} f(x) = 1, \lim_{x\to -1^-} f(x) = \frac{1}{4}$$

Also, $\lim_{x\to \frac{1}{n}+} f(x) = \frac{1}{n^2}$ and, $\lim_{x\to \frac{1}{n}-} f(x) = \frac{1}{(n+1)^2}$

Thus, f is discontinuous for $x = \pm\frac{1}{n}, n = 1, 2, 3, \ldots$

87 **(c)**

Since, $|f(x) - f(y)| \le (x-y)^2$

$\Rightarrow \lim_{x\to y} \frac{|f(x)-f(y)|}{|x-y|} \le \lim_{x\to y} |x-y|$

$\Rightarrow |f'(y)| \le 0$

$\Rightarrow f'(y) = 0$

$\Rightarrow f(y)$ =constant

$\Rightarrow f(y) = 0 \Rightarrow f(1) = 0$ [$\because f(0) = 0$, given]

88 **(b)**

Since $\phi(x) = 2x^3 - 5$ is an increasing function on $(1, 2)$ such that $\phi(1) = -3$ and $\phi(2) = 11$

Clearly, between -3 and 11 there are thirteen points where $f(x) = [2x^3 - 5]$ is discontinuous

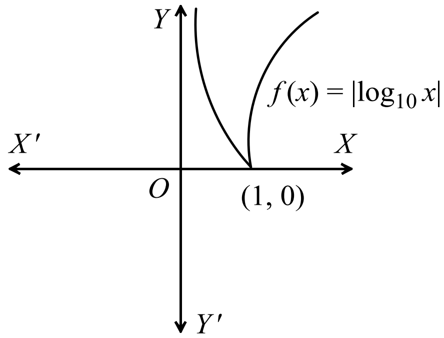
89 **(c)**

Clearly, $[x^2 + 1]$ is discontinuous at $x = \sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}$

Note that it is right continuous at $x = 1$ but not left continuous at $x = 3$

90 **(a)**

As is evident from the graph of $f(x)$ that it is continuous but not differentiable at $x = 1$



Now,

$$f''(1^+) = \lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1}$$

$$\Rightarrow f''(1^+) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$$

$$\Rightarrow f''(1^+) = \lim_{h\to 0} \frac{\log_{10}(1+h)-0}{h}$$

$$\Rightarrow f''(1^+) = \lim_{h\to 0} \frac{\log(1+h)}{h.\log_e 10} = \frac{1}{\log_e 10} = \log_{10} e$$

$$f''(1^-) = \lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1}$$

$$\Rightarrow f''(1^-) = \lim_{h\to 0} \frac{f(1-h)-f(1)}{h}$$

$$\Rightarrow f''(1^-) = \lim_{h\to 0} \frac{\log_{10}(1-h)}{h} = \lim_{h\to 0} \frac{\log_e(1-h)}{h \log_e 10}$$

$$= -\log_{10} e$$

91 **(b)**

It can be easily seen from the graph of $f(x) = |\cos x|$ that it is everywhere continuous but not differentiable at odd multiples of $\pi/2$

92 **(d)**

We have,

$$\lim_{x\to 4^-} f(x) = \lim_{h\to 0} f(4-h) = \lim_{h\to 0} \frac{4-h-4}{|4-h-4|} + a$$

$$\Rightarrow \lim_{x\to 4^-} f(x) = \lim_{h\to 0} -\frac{h}{h} + a = a - 1$$

$$\Rightarrow \lim_{x\to 4^-} f(x) = \lim_{h\to 0} f(4+h) = \lim_{h\to 0} \frac{4+h-4}{|4+h-4|} + b$$

$$= b + 1$$

and, $f(4) = a + b$

Since $f(x)$ is continuous at $x = 4$. Therefore,

$$\lim_{x\to 4^-} f(x) = f(4) = \lim_{x\to 4^+} f(x)$$

$\Rightarrow a - 1 = a + b = b + 1 \Rightarrow b = -1$ and $a = 1$

93 **(b)**

We have,

$$f(x)=\begin{cases}\dfrac{2^x-1}{\sqrt{1+x}-1}, -1\le x<\infty, & x\ne 0\\ k, & x=0\end{cases}$$

Since, $f(x)$ is continuous everywhere

$\therefore \quad \lim_{x\to 0^-} f(x)=f(0) \quad ...(i)$

Now, $\lim_{x\to 0^-} f(x)=\lim_{h\to 0}\frac{2^{(0-h)}-1}{\sqrt{1+(0-h)}-1}$

$=\lim_{h\to 0}\dfrac{2^{-h}-1}{\sqrt{1-h}-1}$

$=\lim_{h\to 0}\frac{-2^{-h}\log_e 2}{\frac{-1}{2\sqrt{1-h}}}$ [by L' Hospital's rule]

$=2\lim_{h\to 0} 2^{-h}\log_e 2\sqrt{1-h}$

$=2\log_e 2$

From Eq. (i),

$f(0)=2\log_e 2=\log_e 4$

95 **(b)**

We have,

$$\lim_{x\to 0^-} f(x)=\lim_{h\to 0} f(-h)=\lim_{h\to 0}\frac{e^{-1/h}-1}{e^{-1/h}+1}=-1$$

and,

$$\lim_{x\to 0^+} f(x)=\lim_{h\to 0} f(h)=\lim_{x\to 0}\frac{e^{1/h}-1}{e^{1/h}+1}=\lim_{h\to 0}\frac{e^{-1/h}}{e^{-1/h}}=1$$

$\therefore \lim_{x\to 0^-} f(x)\ne \lim_{x\to 0^+} f(x)$

Hence, $f(x)$ is not continuous at $x=0$

96 **(c)**

LHL= $\lim_{x\to 2^-} f(x)=\lim_{h\to 0} 1+(2-h)=3$

RHL= $\lim_{x\to 2^+} f(x)=\lim_{h\to 0} 5-(2+h)=$

3, f2=3

Hence, f is continuous at $x=2$

Now, $Rf''(2)=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$

$=\lim_{h\to 0}\dfrac{5-(2+h)-3}{h}=-1$

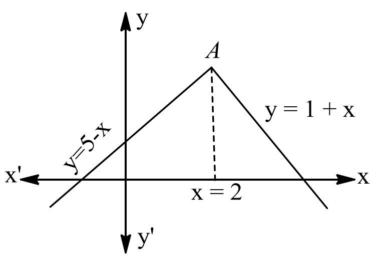
$Lf''(2)=\lim_{h\to 0}\dfrac{f(2-h)-f(2)}{-h}$

$=\lim_{h\to 0}\dfrac{1+(2-h)-3}{-h}=1$

$\therefore \quad Rf''(2)\ne Lf''(2)$

$\therefore \quad f$ is not differentiable at $x=2$

Alternate



It is clear from the graph that $f(x)$ is continuous everywhere also it is differentiable everywhere except at $x=2$

97 **(d)**

We have,

$f(x+y)=f(x)f(y)$ for all $x,y\in R$

Putting $x=1, y=0$, we get

$f(0)=f(1)f(0)\Rightarrow f(0)\big(1-f(1)\big)=0$

$\Rightarrow f(1)=1 \qquad [\because f(0)\ne 0]$

Now,

$f'(1)=2$

$\Rightarrow \lim_{h\to 0}\dfrac{f(1+h)-f(1)}{h}=2$

$\Rightarrow \lim_{h\to 0}\dfrac{f(1)f(h)-f(1)}{h}=2$

$\Rightarrow f(1)\lim_{h\to 0}\dfrac{f(h)-1}{h}=2$

$\Rightarrow \lim_{h\to 0}\frac{f(h)-1}{h}=2 \qquad$ [Using $f(1)=1$] $\qquad$ ...(i)

$\therefore f'(4)=\lim_{h\to 0}\dfrac{f(4+h)-f(4)}{h}$

$\Rightarrow f'(4)=\lim_{h\to 0}\dfrac{f(4)f(h)-f(4)}{h}$

$\Rightarrow f'(4)=\left\{\lim_{h\to 0}\dfrac{f(h)-1}{h}\right\}f(4)$

$\Rightarrow f'(4)=2\,f(4) \qquad$ [From (i)]

$\Rightarrow f'(4)=2\times 4=8$

98 **(d)**

We have,

$\lim_{x\to 1^-} g(x)=\lim_{x\to 1^+} g(x)=1$ and $g(1)=0$

So, $g(x)$ is not continuous at $x=1$ but

$\lim_{x\to 1} g(x)$ exists

We have,

$\lim_{x\to 1^-} f(x)=\lim_{h\to 0} f(1-h)=\lim_{h\to 0}[1-h]=0$

and,

$\lim_{x\to 1^+} f(x)=\lim_{h\to 0} f(1+h)=\lim_{h\to 0}[1+h]=1$

So, $\lim_{x\to 1} f(x)$ does not exist and so $f(x)$ is not continuous at $x=1$

We have, $gof(x)=g\big(f(x)\big)=g([x])=0$, for all $x\in R$

So, gof is continuous for all x

We have,

$fog(x)=f\big(g(x)\big)$

$\Rightarrow fog(x)=\begin{cases} f(0), & x\in Z\\ f(x^2), & x\in R-Z\end{cases}$

$\Rightarrow fog(x)=\begin{cases} 0, & x\in Z\\ [x^2], & x\in R-Z\end{cases}$

Which is clearly not continuous

99 **(d)**

At $x=1$,

RHD= $\lim_{h\to 0^+}\frac{f(1+h)-f(1)}{h}$

$=\lim_{h\to 0}\dfrac{2-(1+h)-(2-1)}{h}=-1$

LHD= $\lim_{h\to 0^-}\frac{f(1-h)-f(1)}{-h}$

$=\lim_{h\to 0}\frac{(1-h)-(2-1)}{-h}=1$

$\therefore$ LHD≠RHD

100 **(d)**

Given, $f(x)=|x|+\frac{|x|}{x}$

Let $f_1(x)=|x|,\ f_2(x)=\frac{|x|}{x}$

1. LHL= $\lim_{x\to 0^-} f_1(x)=\lim_{x\to 0^-}|x|=0$

And RHL $\lim_{x\to 0^+} f_1(x)=\lim_{x\to 0^+}|x|=0$

Here, LHL=RHL=$f(0)$, $f_1(x)$ is continuous

2. LHL= $\lim_{x\to 0^-}\frac{|x|}{x}=\lim_{h\to 0}\frac{|0-h|}{0-h}=-1$

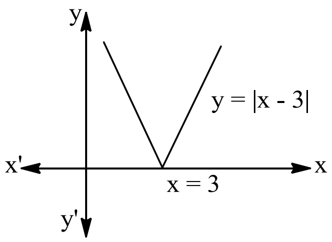
RHL= $\lim_{x\to 0^+}\frac{|x|}{x}=\lim_{h\to 0}\frac{|0+h|}{h}=1$

$\therefore$ LHL≠RHL, $f_2(x)$ is discontinuous

Hence, $f(x)$ is discontinuous at $x=0$

101 **(a)**

From the graph it is clear that $f(x)$ is continuous everywhere but not differentiable at $x=3$



102 **(b)**

Given, $f(x)=\begin{cases}\frac{2x-3}{2x-3}, & \text{if } x>\frac{3}{2}\\ \frac{-(2x-3)}{2x-3}, & \text{if } x<\frac{3}{2}\end{cases}$

$=\begin{cases}1, & \text{if } x>\frac{3}{2}\\ -1, & \text{if } x<\frac{3}{2}\end{cases}$

Now, RHL= $\lim_{x\to\frac{3}{2}^+} f(x)=\lim_{x\to\frac{3}{2}^+} 1=1$

And LHL= $\lim_{x\to\frac{3}{2}^-} f(x)=\lim_{x\to\frac{3}{2}^-}(-1)=-1$

$\because$ RHL≠LHL

$\therefore$ $f(x)$ is discontinuous at $x=\frac{3}{2}$

103 **(c)**

Since the functions $(\log t)^2$ and $\frac{\sin t}{t}$ are not defined on $(-1,2)$. Therefore, the functions in options (a) and (b) are not defined on $(-1,2)$

The function $g(t)=\frac{1-t+t^2}{1+t+t^2}$ is continuous on $(-1,2)$ and

$f(x)=\int_0^x\frac{1-t+t^2}{1+t+t^2}dt$ is the integral function of $g(t)$

Therefore, $f(x)$ is differentiable on $(-1,2)$ such that $f'(x)=g(x)$

104 **(c)**

Since, $f(x)=\frac{1-\tan x}{4x-\pi}$

Now, $\lim_{x\to\pi/4} f(x)=\lim_{x\to\pi/4}\left(\frac{1-\tan x}{4x-\pi}\right)$

$=\lim_{x\to\pi/4}\left(\frac{-\sec^2 x}{4}\right)=-\frac{1}{2}$

Since, $f(x)$ is continuous at

$x=\frac{\pi}{4}$

$\therefore$ $\lim_{x\to\pi/4} f(x)=f\left(\frac{\pi}{4}\right)=-\frac{1}{2}$

105 **(a)**

$\lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{1-\cos x}{x}=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{4\left(\frac{x}{2}\right)^2}.x=0$

Also, $f(0)=k$

For, $\lim_{x\to 0} f(x)=f(0)\quad\Rightarrow\ k=0$

106 **(a)**

We have,

$f(x)=|x|+|x-1|$

$\Rightarrow f(x)=\begin{cases}-2x+1, & x<0\\ x-x+1, & 0\le x<1\\ x+x-1, & x\ge 1\end{cases}$

$\Rightarrow f(x)=\begin{cases}-2x+1, & x<0\\ 1, & 0\le x<1\\ 2x-1, & x\ge 1\end{cases}$

Clearly, $\lim_{x\to 0^-} f(x)=1=\lim_{x\to 0^+} f(x)$ and $\lim_{x\to 1^-} f(x)=\lim_{x\to 1^+} f(x)$

So, $f(x)$ is continuous at $x=0,1$

107 **(d)**

$f(0)=\lim_{x\to 0}\frac{2x-\sin^{-1}x}{2x+\tan^{-1}x}$

$=\lim_{x\to 0}\frac{2-\frac{\sin^{-1}x}{x}}{2+\frac{\tan^{-1}x}{x}}$

$=\frac{2-1}{2+1}=\frac{1}{3}$

109 **(b)**

$f'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$

$=\lim_{h\to 0}\frac{\frac{1+h-1}{2(1+h)^2-7(1+h)+5}-\left(\frac{1}{3}\right)}{h}$

$=\lim_{h\to 0}\frac{\left(\frac{1}{2h-3}+\frac{1}{3}\right)}{h}=\lim_{h\to 0}\left(\frac{2h}{3h(2h-3)}\right)=-\frac{2}{9}$

110 **(a)**

LHL= $\lim_{h\to 0} f\left(-\frac{\pi}{2}-h\right)=\lim_{h\to 0} 2\cos\left(-\frac{\pi}{2}-h\right)=0$

RHL= $\lim_{h\to 0} f\left(-\frac{\pi}{2}+h\right) = \lim_{h\to 0} 2\,a \sin\left(-\frac{\pi}{2}+h\right)+b$
$= -a+b$
Since, function is continuous.
$\therefore$ RHL=LHL $\Rightarrow$ $a=b$
From the given options only (a) *ie*, $\left(\frac{1}{2},\frac{1}{2}\right)$ satisfies this condition

111 **(a)**
We have,
$f'(0)=3$
$\Rightarrow \lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=3$
$\Rightarrow \lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$
$= 3$ [Using: (RHD at $x=0$) $=3$]
$\Rightarrow \lim_{h\to 0}\frac{f(0)f(h)-f(0)}{h}$
$= 3$ $\begin{bmatrix}\because f(x+y)=f(x)f(y)\\ \therefore f(0+h)=f(0)f(h)\end{bmatrix}$
$\Rightarrow f(0)\left(\lim_{h\to 0}\frac{f(h)-1}{h}\right)=3$ ...(i)
Now, $f(x+y)=f(x)f(y)$ for all $x,y\in R$
$\Rightarrow f(0)=f(0)f(0)$
$\Rightarrow f(0)\{1-f(0)\}=0 \Rightarrow f(0)=1$
Putting $f(0)=1$ in (i), we get
$\lim_{h\to 0}\frac{f(h)-1}{h}=3$ ...(ii)
Now,
$f'(5)=\lim_{h\to 0}\frac{f(5+h)-f(5)}{h}$
$\Rightarrow f'(5)=\lim_{h\to 0}\frac{f(5)f(h)-f(5)}{h}$
$\Rightarrow f'(5)=\left\{\lim_{h\to 0}\frac{f(h)-1}{h}\right\}f(5)=3\times 2=6$
[Using (ii)]

112 **(c)**
We have,
$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$
$\Rightarrow f(x)'=\lim_{h\to 0}\frac{f(x)+f(h)-f(x)}{h}$
$\Rightarrow f(x)'=\lim_{h\to 0}\frac{f(h)}{h}$
$\Rightarrow f(x)'=\lim_{h\to 0}\frac{h\,g\,(h)}{h}\lim_{h\to 0}g(h)=g(0)$ [$\because g$ is conti. at $x=0$]

113 **(b)**
The domain of $f(x)$ is $[2,\infty)$
We have,
$f(x)=\sqrt{\frac{\left(\sqrt{2x-4}\right)^2}{2}+2+2\sqrt{2x-4}}$
$+\sqrt{\frac{\left(\sqrt{2x-4}\right)^2}{2}+2-2\sqrt{2x-4}}$
$\Rightarrow f(x)=\frac{1}{\sqrt{2}}\sqrt{\left(\sqrt{2x-4}\right)^2+4\sqrt{2x-4}+4}$
$+\frac{1}{\sqrt{2}}\sqrt{\left(\sqrt{2x-4}\right)^2-4\sqrt{2x-4}+4}$
$\Rightarrow f(x)=\frac{1}{\sqrt{2}}\left|\sqrt{2\,x-4}+2\right|+\frac{1}{\sqrt{2}}\left|\sqrt{2x-4}-2\right|$
$\Rightarrow f(x)=\begin{cases}\frac{1}{\sqrt{2}}\times 4, & \text{if } \sqrt{2x-4}<2\\ \sqrt{2}\cdot\sqrt{2x-4}, & \text{if } \sqrt{2x-4}\geq 2\end{cases}$
$\Rightarrow f(x)=\begin{cases}2\sqrt{2}, & \text{if } x\in[2,4)\\ 2\sqrt{x-2}, & \text{if } x\in[4,\infty)\end{cases}$
Hence, $f'(x)=\begin{cases}0 & \text{if } x\in[2,4)\\ \frac{1}{\sqrt{x-2}} & \text{if } x\in(4,\infty)\end{cases}$

114 **(c)**
We have,
$\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0}\frac{x^2\sin\left(\frac{1}{x}\right)}{x}=\lim_{x\to 0}x\sin\frac{1}{x}=0$
So, $f(x)$ is differentiable at $x=0$ such that
$f'(0)=0$
For $x\neq 0$, we have
$f'(x)=2\,x\sin\left(\frac{1}{x}\right)+x^2\cos\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)$
$\Rightarrow f'(x)=2\,x\sin\frac{1}{x}-\cos\frac{1}{x}$
$\Rightarrow \lim_{x\to 0}f'(x)=\lim_{x\to 0}2\,x\sin\frac{1}{x}-\lim_{x\to 0}\cos\left(\frac{1}{x}\right)$
$=0-\lim_{x\to 0}\cos\left(\frac{1}{x}\right)$
Since $\lim_{x\to 0}\cos\left(\frac{1}{x}\right)$ does not exist
$\therefore \lim_{x\to 0}f'(x)$ does not exist
Hence, $f'(x)$ is not continuous at $x=0$

115 **(c)**
We have,
$f(x)=\begin{cases}\frac{x}{\sqrt{x^2}}, & x\neq 0\\ 0, & x=0\end{cases}$
$\Rightarrow f(x)=\begin{cases}\frac{x}{|x|}, & x\neq 0\\ 0, & x=0\end{cases}=\begin{cases}1, & x>0\\ -1, & x<0\\ 0, & x=0\end{cases}$
Clearly, $f(x)$ is not continuous at $x=0$

117 **(c)**
Given, $\lim_{x\to 0}\left[(1+3x)^{\frac{1}{x}}\right]=k$
$\therefore$ $e^3=k$

118 **(b)**
For $x>2$, we have

$$f(x) = \int_0^x \{5 + |1-t|\}dt$$

$$\Rightarrow f(x) = \int_0^1 (5 + (1-t)dt + \int_1^x (5 - (1-t)\,dt$$

$$\Rightarrow f(x) = \int_0^1 (6-t)dt + \int_1^x (4+t)dt$$

$$\Rightarrow f(x) = \left[6t - \frac{t^2}{2}\right]_0^1 + \left[4t + \frac{t^2}{2}\right]_1^x$$

$$\Rightarrow f(x) = 1 + 4x + \frac{x^2}{2}$$

Thus, we have

$$f(x) = \begin{cases} 5x+1, & \text{if } x \le 2 \\ \frac{x^2}{2} + 4x + 1, & \text{if } x > 2 \end{cases}$$

Clearly, $f(x)$ is everywhere continuous and differentiable except possibly at $x = 2$

Now,

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2} 5x + 1 = 11$$

and,

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2}\left(\frac{x^2}{2} + 4x + 1\right) = 11$$

$$\therefore \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$$

So, $f(x)$ is continuous at $x = 2$

Also, we have (LHD at $x = 2$) $= \lim_{x\to 2^-} f'(x) = \lim_{x\to 2} 5 = 5$

119 **(b)**

The given function is clearly continuous at all points except possibly at $x = \pm 1$

For $f(x)$ to be continuous at $x = 1$, we must have

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$$

$$\Rightarrow \lim_{x\to 1} ax^2 + b = \lim_{x\to 1} \frac{1}{|x|}$$

$\Rightarrow a + b = 1 \quad$ ...(i)

Clearly, $f(x)$ is differentiable for all x, except possibly at $x = \pm 1$. As $f(x)$ is an even function, so we need to check its differentiability at $x = 1$ only

For $f(x)$ to be differentiable at $x = 1$, we must have

$$\lim_{x\to 1^-} \frac{f(x) - f(1)}{x-1} = \lim_{x\to 1^+} \frac{f(x) - f(1)}{x-1}$$

$$\Rightarrow \lim_{x\to 1} \frac{ax^2 + b - 1}{x-1} = \lim_{x\to 1} \frac{\frac{1}{|x|} - 1}{x-1}$$

$$\Rightarrow \lim_{x\to 1} \frac{ax^2 - a}{x-1} = \lim_{x\to 1} \frac{\frac{1}{x} - 1}{x-1} \quad [\because a + b = 1 \;\; \therefore b - 1 = -a]$$

$$\Rightarrow \lim_{x\to 1} a(x+1) = \lim_{x\to 1} \frac{-1}{x}$$

$\Rightarrow 2a = -1 \Rightarrow a = -1/2$

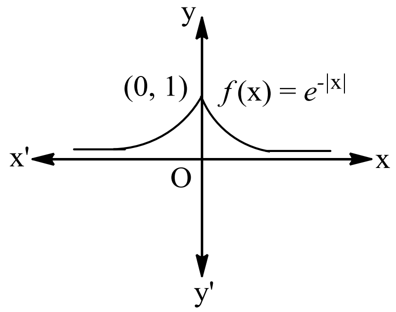
Putting $a = -1/2$ in (i), we get $b = 3/2$

120 **(c)**

At no point, function is continuous

121 **(a)**

It is clear from the figure that $f(x)$ is continuous everywhere and not differentiable at $x = 0$ due to sharp edge



122 **(c)**

$$f(x) = \frac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{\sqrt{a+x} - \sqrt{a-x}} \times \frac{\sqrt{a^2 - ax + x^2} + \sqrt{a^2 + ax + x^2}}{\sqrt{a^2 - ax + x^2} + \sqrt{a^2 + ax + x^2}} \times \frac{\sqrt{a+x} + \sqrt{a-x}}{\sqrt{a+x} + \sqrt{a-x}}$$

$$\Rightarrow \lim_{x\to 0} f(x)$$

$$= \lim_{x\to 0} \frac{-2ax(\sqrt{a+x} + \sqrt{a-x})}{2x\,(\sqrt{a^2 - ax + x^2} + \sqrt{a^2 + ax + x^2})}$$

$$= \frac{-a(2\sqrt{a})}{(a+a)} = -\sqrt{a}$$

123 **(b)**

Given, $f(x) = \begin{cases} \frac{1 - \cos 4x}{8x^2}, & x \ne 0 \\ k & x = 0 \end{cases}$

LHL$= \lim_{x\to 0^-} f(x)$

$$= \lim_{h\to 0} \frac{1 - \cos 4(0-h)}{8(0-h)^2}$$

$$= \lim_{h\to 0} \frac{1 - \sin 4h}{8h^2}$$

$= \lim_{h\to 0} \frac{4\sin 4h}{16h} = 1$ [by L 'Hospital's rule]

Since, $f(x)$ is continuous at $x = 0$

$\therefore \quad f(0) =$LHL $\Rightarrow \quad k = 1$

124 **(d)**

Given, $f(x) = |x-1| + |x-2| + \cos x$

Since, $|x-1|, |x-2|$ and $\cos x$ are continuous in $[0, 4]$

$\therefore \;\; f(x)$ being sum of continuous functions is also continuous

125 **(c)**

If function $f(x)$ is continuous at $x = 0$, then

$$f(0) = \lim_{x\to 0} f(x)$$

$\therefore \quad f(0) = k = \lim_{x \to 0} x \sin \frac{1}{x}$

$\Rightarrow k = 0 \qquad \left[\because -1 \le \sin\frac{1}{x} \le 1\right]$

126 **(b)**

We have,

$h(x) = \{f(x)\}^2 + \{g(x)\}^2$

$\Rightarrow h'(x) = 2f(x)2f'(x) + 2\,g(x)\,g'(x)$

Now,

$f'(x) = g(x)$ and $f''(x) = -f(x)$

$\Rightarrow f''(x) = g'(x)$ and $f''(x) = -f(x)$

$\Rightarrow -f(x) = g'(x)$

Thus, we have

$f'(x) = g(x)$ and $g'(x) = -f(x)$

$\therefore h'(x) = -2\,g(x)g'(x) + 2\,g\,(x)g'(x) = 0$, for all x

$\Rightarrow h(x) =$ Constant for all x

But, $h(5) = 11$. Hence, $h(x) = 11$ for all x

127 **(a)**

$$f(x) = |x|^3 = \begin{cases} 0, & x = 0 \\ x^3, & x > 0 \\ -x^3, & x < 0 \end{cases}$$

Now, $Rf'(0) = \lim_{h \to 0} \frac{h^3 - 0}{h} = 0$

And $Lf'(0) = \lim_{h \to 0} \frac{-h^3 - 0}{-h} = 0$

$\because \quad Rf'(0) = Lf'(0) = 0$

$\therefore \quad f'(0) = 0$

128 **(b)**

We have,

(LHL at $x = 0$) $= \lim_{n \to 0^-} f(x) = \lim_{h \to 0} f(0 - h)$

$\Rightarrow$ (LHL at $x = 0$) $= \lim_{n \to 0} \sin^{-1}(\cos\ (-h))$

$= \lim_{h \to 0} \sin^{-1}(\cosh h)$

$\Rightarrow$ (LHL at $x = 0$) $= \sin^{-1} 1 = \pi/2$

(RHL at $x = 0$) $= \lim_{x \to 0^+} f(x)$

$\Rightarrow$ (RHL at $x = 0$) $= \lim_{h \to 0} f(0 + h)$

$= \lim_{h \to 0} \sin^{-1}(\cos h)$

$\Rightarrow$ (RHL at $x = 0$) $= \sin^{-1}(1) = \pi/2$

and, $f(0) = \sin^{-1}(\cos 0) = \sin^{-1}(1) = \pi/2$

$\therefore$ (LHL at $x = 0$) = (RHL at $x = 0$) $= f(0)$

So, $f(x)$ is continuous at $x = 0$

Now,

$$f'(x) = \frac{-\sin x}{\sqrt{1 - \cos^2 x}} = \frac{\sin x}{|\sin x|}$$

$$= \begin{cases} \dfrac{-\sin x}{-\sin x} = 1, x < 0 \\ \dfrac{-\sin x}{\sin x} = -1, x > 0 \end{cases}$$

$\therefore$ (LHD at $x = 0$) $= 1$ and (RHD at $x = 0$) $= -1$

Hence, $f(x)$ is not differentiable at $x = 0$

129 **(d)**

For any $x \ne 1, 2$, we find that $f(x)$ is the quotient of two polynomials and a polynomial is everywhere continuous. Therefore, $f(x)$ is continuous for all $x \ne 1, 2$

Continuity at $x = 1$:

We have,

$\lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1 - h)$

$\Rightarrow \lim_{x \to 1^-} f(x)$

$$= \lim_{h \to 0} \frac{(1 - h - 2)(1 - h + 2)(1 - h + 1)(1 - h -}{|(1 - h - 1)(1 - h - 2)|}$$

$$\Rightarrow \lim_{x \to 1^-} f(x) = \lim_{h \to 0} \frac{(3 - h)(2 - h)(-1 - h)(-h)}{|(-h)(-1 - h)|}$$

$$\Rightarrow \lim_{x \to 1^-} f(x) = \lim_{h \to 0} \frac{(3 - h)(2 - h)h(h + 1)}{h(h + 1)} = 6$$

and,

$\lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1 + h)$

$\lim_{x \to 1^+} f(x)$

$$= \lim_{h \to 0} \frac{(1 + h - 2)(1 + h + 2)(1 + h + 1)(1 + h -}{|(1 + h - 1)(1 + h - 2)|}$$

$$\lim_{x \to 1^+} f(x) = \lim_{h \to 0} \frac{(h - 1)(3 + h)(2 + h)(h)}{|h(h - 1)|}$$

$$\lim_{x \to 1^+} f(x) = -\lim_{h \to 0} \frac{(h - 1)(3 + h)(2 + h)h}{h(1 - h)} = -6$$

$\therefore \lim_{x \to 1^-} f(x) \ne \lim_{x \to 1^+} f(x)$

So, $f(x)$ is not continuous at $x = 1$

Similarly, $f(x)$ is not continuous at $x = 2$

130 **(b)**

Let $f(x) = \frac{g(x)}{h(x)} = \frac{x}{1 + |x|}$

It is clear that $g(x) = x$ and $h(x) = 1 + |x|$ are differentiable on $(-\infty, \infty)$ and $(-\infty, 0) \cup (0, \infty)$ respectively

Thus, $f(x)$ is differentiable on $(-\infty, 0) \cup (0, \infty)$.Now, we have to check the differentiable at $x = 0$

$$\therefore \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{\frac{x}{1 + |x|} - 0}{x} = \lim_{x \to 0} \frac{1}{1 + |x|}$$

$= 1$

Hence, $f(x)$ is differntaible on $(-\infty, \infty)$

131 **(b)**

At $x = 0$,

LHL$= \lim_{h \to 0} \frac{1}{1 - e^{-1/(0 - h)}} = \lim_{h \to 0} \frac{1}{1 - e^{1/h}} = 0$

RHL$= \lim_{h \to 0} \frac{1}{1 - e^{-1/(0 + h)}} = \lim_{h \to 0} \frac{1}{1 - e^{-1/h}} = 1$

$\therefore$ FUnction is not continuous at $x = 0$

132 **(a)**

We have,

$fog = I$

$\Rightarrow fog(x) = x$ for all x

$\Rightarrow f'(g(x))g'(x) = 1$ for all x

$\Rightarrow f'(g(a)) = \frac{1}{g'(a)} = \frac{1}{2} \Rightarrow f'(b)$

$= \frac{1}{2} \quad [\because f(a) = b]$

133 **(a)**

Since, $\lim_{x\to 0} f(x) = f(0)$

$\Rightarrow \lim_{x\to 0} \frac{\sin \pi x}{5x} = k$

$\Rightarrow (1)\frac{\pi}{5} = k \Rightarrow k = \frac{\pi}{5} \quad \left[\because \lim_{x\to 0} \frac{\sin x}{x} = 1\right]$

134 **(d)**

Given, $f(x) = [x], x \in (-3.5, 100)$

As we know greatest integer is discontinuous on integer values.

In given interval, the integer values are $(-3, -2, -1, 0, \dots, 99)$

$\therefore$ Total numbers of integers are 103.

135 **(a)**

LHL$= \lim_{h\to 0} f(0-h)$

$= \lim_{h\to 0} \frac{e^{-1/h}-1}{e^{-1/h}+1} = -1 \quad \left[\because \lim_{h\to 0} \frac{1}{e^{1/h}} = 0\right]$

RHL$= \lim_{h\to 0} f(0+h) = \lim_{h\to 0} \frac{e^{1/h}-1}{e^{1/h}+1}$

$= \lim_{h\to 0} \frac{1-\frac{1}{e^{1/h}}}{1+\frac{1}{e^{1/h}}} = 1$

$\therefore$ LHL$\neq$RHL

So, limit does not exist at $x = 0$

136 **(d)**

We have,

$f(x) = x^4 + \frac{x^4}{1+x^4} + \frac{x^4}{(1+x^4)} + \cdots$

$\Rightarrow f(x) = \frac{x^4}{1-\frac{1}{1+x^4}} = 1 + x^4$, if $x \neq 0$

Clearly, $f(x) = 0$ at $x = 0$

Thus, we have

$f(x) = \begin{cases} 1+x^4, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Clearly, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = 1 \neq f(0)$

So, $f(x)$ is neither continuous nor differentiable at $x = 0$

137 **(a)**

We have,

$f(x) = \begin{cases} 1+x, & 0 \le x \le 2 \\ 3-x, & 2 < x \le 3 \end{cases}$

$\therefore g(x) = fof(x)$

$\Rightarrow f(x) = f(f(x))$

$\Rightarrow g(x) = \begin{cases} f(1+x), & 0 \le x \le 2 \\ f(3-x), & 2 < x \le 3 \end{cases}$

$\Rightarrow g(x) = \begin{cases} 1+(1+x), & 0 \le x \le 1 \\ 3-(1+x), & 1 < x \le 2 \\ 1+(3-x), & 2 < x \le 3 \end{cases}$

$\Rightarrow g(x) = \begin{cases} 2+x, & 0 \le x \le 1 \\ 2-x, & 1 < x \le 2 \\ 4-x, & 2 < x \le 3 \end{cases}$

Clearly, $g(x)$ is continuous in $(0,1) \cup (1,2) \cup (2,3)$ except possibly at $x = 0, 1, 2$ and 3

We observe that

$\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+} (2+x) = 2 = g(0)$

and $\lim_{x\to 3^-} g(x) = \lim_{x\to 3^-} 4 - x = 1 = g(3)$

Therefore, $g(x)$ is right continuous at $x = 0$ and left continuous at $x = 3$

At $x = 1$, we have

$\lim_{x\to 1^-} g(x) = \lim_{x\to 1^-} 2 + x = 3$

and, $\lim_{x\to 1^+} g(x) = \lim_{x\to 1^+} 2 - x = 1$

$\therefore \lim_{x\to 1^+} g(x) \neq \lim_{x\to 1^-} g(x)$

So, $g(x)$ is not continuous at $x = 1$

At $x = 2$, we have

$\lim_{x\to 2^-} g(x) = \lim_{x\to 2^-} (2-x) = 0$

and,

$\lim_{x\to 2^+} g(x) = \lim_{x\to 2^+} (4-x) = 0$

$\therefore \lim_{x\to 2^-} g(x) \neq \lim_{x\to 2^+} g(x)$

So, $g(x)$ is not continuous at $x = 2$

Hence, the set of points of discontinuity of $g(x)$ is $\{1, 2\}$

138 **(b)**

Since $g(x)$ is the inverse of function $f(x)$

$\therefore gof(x) = I(x)$, for all x

Now, $gof(x) = I(x)$, for all x

$\Rightarrow gof(x) = x$, for all x

$\Rightarrow (gof)'(x) = 1$, for all x

$\Rightarrow g'(f(x))f'(x) = 1$, for all x [Using Chain Rule]

$\Rightarrow g'(f(x)) = \frac{1}{f'(x)}$, for all x

$\Rightarrow g'(f(c)) = \frac{1}{f'(c)}$ [Putting $x = c$]

139 **(d)**

Given, $f(x) = \begin{cases} x^p \cos\left(\frac{1}{x}\right), x \neq 0 \\ 0, \ x = 0 \end{cases}$

Since, $f(x)$ is differentiable at $x = 0$, therefore it is continuous at $x = 0$

$\therefore \lim_{x\to 0} f(x) = f(0) = 0$

$\Rightarrow \lim_{x\to 0} x^p \cos\left(\frac{1}{x}\right) = 0 \Rightarrow p > 0$

As $f(x)$ is differentiable at $x = 0$

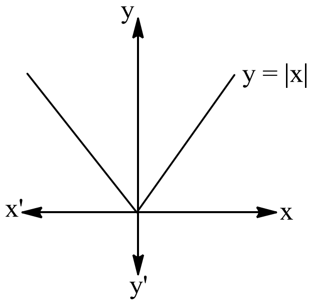
$\therefore \lim_{x\to 0} \frac{f(x)-f(0)}{x-0}$ exists finitely

$\Rightarrow \lim_{x\to 0} \frac{x^p \cos\frac{1}{x} - 0}{x}$ exists finitely

$\Rightarrow \lim_{x\to 0} x^{p-1}\cos\frac{1}{x} - 0$ exists finitely

$\Rightarrow \quad p-1>0 \quad \Rightarrow \quad p>1$

140 **(a)**



It is clear from the graph that $f(x)$ is continuous everywhere and also differentiable everywhere except at $x=0$

141 **(c)**

We know that the function

$\phi(x) = (x-a)^2 \sin\left(\frac{1}{x-a}\right)$

Is continuous and differentiable at $x=a$ whereas the function $\Psi(x) = |x-a|$ is everywhere continuous but not differentiable at $x=a$

Therefore, $f(x)$ is not differentiable at $x=1$

142 **(d)**

$\lim_{x\to 0}\frac{2^x - 2^{-x}}{x} = \lim_{x\to 0} 2^x \log 2 + 2^{-x}\log 2$

[by L' Hospital's rule]

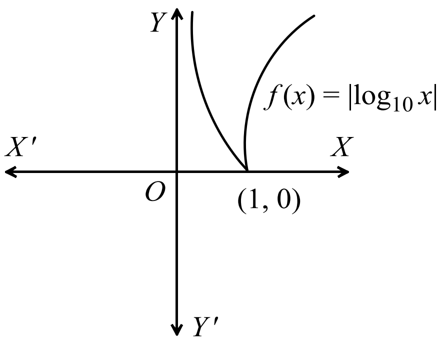
$= \log 4$

Since, the function is continuous at $x=0$

$\therefore \quad f(0) = \lim_{x\to 0} f(x) \quad \Rightarrow \quad f(0) = \log 4$

143 **(a)**

As is evident from the graph of $f(x)$ that it is continuous but not differentiable at $x=1$



Now,

$f''(1^+) = \lim_{x\to 1^+}\frac{f(x)-f(1)}{x-1}$

$\Rightarrow f''(1^+) = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$

$\Rightarrow f''(1^+) = \lim_{h\to 0}\frac{\log_{10}(1+h)-0}{h}$

$\Rightarrow f''(1^+) = \lim_{h\to 0}\frac{\log(1+h)}{h.\log_e 10} = \frac{1}{\log_e 10} = \log_{10} e$

$f''(1^-) = \lim_{x\to 1^+}\frac{f(x)-f(1)}{x-1}$

$\Rightarrow f''(1^-) = \lim_{h\to 0}\frac{f(1-h)-f(1)}{h}$

$\Rightarrow f''(1^-) = \lim_{h\to 0}\frac{\log_{10}(1-h)}{h} = \lim_{h\to 0}\frac{\log_e(1-h)}{h\log_e 10}$

$= -\log_{10} e$

144 **(a)**

We have,

$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h\to 0}\frac{f(x)+f(h)-f(x)}{h} \quad [\because f(x+y) = f(x)+f(y)]$

$\Rightarrow f'(x) = \lim_{h\to 0}\frac{f(h)}{h}$

$\Rightarrow f'(x) = \lim_{h\to 0}\frac{\sin h\ g(h)}{h} = \lim_{h\to 0}\frac{\sin h}{h}\lim_{h\to 0} g(h)$

$= g(0) = k$

145 **(a)**

We have,

$f(x) = |x| + |x-1| = \begin{cases} -2x+1, & x<0 \\ 1, & 0\le x<1 \\ 2x-1, & 1\le x \end{cases}$

Clearly,

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1} 1 = 1, \lim_{x\to 1^+} f(x) = \lim_{x\to 1}(2x-1) = 1$

and, $f(1) = 2\times 1 - 1 = 1$

$\therefore \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$

So, $f(x)$ is continuous at $x=1$

Now, $\lim_{x\to 1^-}\frac{f(x)-f(1)}{x-1} = \lim_{h\to 0}\frac{f(1-h)-f(1)}{-h} = \lim_{h\to 0}\frac{1-1}{-h} = 0$

and,

$\lim_{x\to 1^+}\frac{f(x)-f(1)}{x-1} = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$

$\Rightarrow \lim_{x\to 1^+}\frac{f(x)-f(1)}{x-1} = \lim_{h\to 0}\frac{2(1+h)-1-1}{h} = 2$

$\therefore$ (LHD at $x=1$) $\neq$ (RHD at $x=1$)

So, $f(x)$ is not differentiable at $x=1$

146 **(d)**

The given function is differentiable at all points except possibly at $x=0$

Now,

(RHD at $x=0$)

$= \lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$

$= \lim_{h\to 0}\frac{\sqrt{h+1}-1}{h^{3/2}}$

$= \lim_{h\to 0}\frac{h}{h^{3/2}(\sqrt{h+1}+1)} = \lim_{h\to 0}\frac{1}{\sqrt{h}(\sqrt{h+1}+1)}$

$\to \infty$

So, the function is not differentiable at $x = 0$

Hence, the required set is $R - \{0\}$

147 **(a)**

We have,

$f(x)\, f(y) = f(x) + f(y) + f(xy) - 2$

$\Rightarrow f(x).f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right) + f(1) - 2$

$\Rightarrow f(x).f\left(\frac{1}{x}\right)$

$= f(x)$

$+ f\left(\frac{1}{x}\right)$ $\left[\because f(1) = 2 \text{ (Putting } x = y = 1 \text{ in the given relation)}\right]$

$\Rightarrow f(x) = x^n + 1$

$\Rightarrow f(2) = 2^n + 1$

$\Rightarrow 5 = 2^n + 1$ $[\because f(2) = 5 \text{ (given)}]$

$\Rightarrow n = 2$

$\therefore f(x) = x^2 + 1 \Rightarrow f(3) = 10$

148 **(b)**

We have,

$f(x) = \frac{1}{2}x - 1$, for $0 \le x \le \pi$

$\therefore \{f(x)\} = \begin{cases} -1, \text{for } 0 \le x < 2 \\ 0, \text{for } 2 \le x \le \pi \end{cases}$

$\Rightarrow \tan[f(x)] = \begin{cases} \tan(-1) = -\tan(1), & 0 \le x < 2 \\ \tan 0 = 0, & 2 \le x \le \pi \end{cases}$

It is evident from the definition of $\tan[f(x)]$ that

$\lim_{x\to 2^-} \tan[f(x)] = -\tan 1$ and,

$\lim_{x\to 2^+} \tan[f(x)] = 0$

So, $\tan[f(x)]$ is not continuous at $x = 2$

Now,

$f(x) = \frac{1}{2}x - 1 \Rightarrow f(x) = \frac{x-2}{2} \Rightarrow \frac{1}{f(x)} = \frac{2}{x-2}$

Clearly, $f(x)$ is not continuous at $x = 2$

So, $\tan[f(x)]$ and $\tan\left[\frac{1}{f(x)}\right]$ both are discontinuous at $x = 2$

149 **(c)**

$\lim_{x\to 0}(1+x)^{\cot x} = \lim_{x\to 0}\left\{(1+x)^{\frac{1}{x}}\right\}^{x\cot x}$

$= \lim_{x\to 0} e^{x\cot x} = e$

Since $f(x)$ is continuous at $x = 0$

$\therefore \quad f(0) = \lim_{x\to 0} f(x) = e$

150 **(b)**

LHL$= \lim_{h\to 0} f\left(\frac{\pi}{4} - h\right)$

$= \lim_{h\to 0} \dfrac{\tan\left(\frac{\pi}{4} - h\right) - \cot\left(\frac{\pi}{4} - h\right)}{\frac{\pi}{4} - h - \frac{\pi}{4}}$

$= \lim_{h\to 0} \dfrac{-\sec^2\left(\frac{\pi}{4} - h\right) - \text{cosec}^2\left(\frac{\pi}{4} - h\right)}{-1} = 4$

[by L 'Hospital's rule]

Since, $f(x)$ is continuous at $x = \frac{\pi}{4}$, then

LHL$= f\left(\frac{\pi}{4}\right)$

$\therefore \quad a = 4$

151 **(a)**

If $-1 \le x < 0$, then

$f(x) = \int_{-1}^{x} |t|\, dt = \int_{-1}^{x} -t\, dt = -\frac{1}{2}(x^2 - 1)$

If $x \ge 0$, then

$f(x) = \int_{-1}^{0} -t\, dt + \int_{-1}^{x} -t\, dt = \frac{1}{2}(x^2 + 1)$

$\therefore f(x) = \begin{cases} -\frac{1}{2}(x^2 - 2), & -1 \le x < 0 \\ \frac{1}{2}(x^2 + 1), & 0 \le x \end{cases}$

It can be easily seen that $f(x)$ is continuous at $x = 0$

So, it is continuous for all $x > -1$

Also, $Rf'(0) = 0 = Lf'(0)$

So, $f(x)$ is differentiable at $x = 0$

$\therefore f'(x) = \begin{cases} -x, & -1 < x = 0 \\ 0, & x = 0 \\ x, & x > 0 \end{cases}$

Clearly, $f'(x)$ is continuous at $x = 0$

Consequently, it is continuous for all $x > -1$ i.e. for $x + 1 > 0$

Hence, f and f' are continuous for $x + 1 > 0$

152 **(c)**

We have,

$f(x) = \lim_{n\to\infty} \dfrac{x^{-n} - x^n}{x^{-n} + x^n}$

$\Rightarrow f(x) = \lim_{n\to\infty} \dfrac{1 - x^{2n}}{1 + x^{2n}}$

$\Rightarrow f(x) = \begin{cases} \frac{1-0}{1+0} = 1, & \text{if } -1 < x < 1 \\ \frac{1-1}{1+1} = 0, & \text{if } x = \pm 1 \\ \frac{0-1}{0+1} = -1, & \text{if } |x| > 1 \end{cases}$

Clearly, $f(x)$ is discontinuous at $x = \pm 1$

153 **(b)**

Clearly, $\log|x|$ is discontinuous at $x = 0$

$f(x) = \frac{1}{\log|x|}$ is not defined at $x = \pm 1$

Hence, $f(x)$ is discontinuous at $x = 0, 1, -1$

154 **(a)**

For continuity, $\lim_{x\to 0} f(x) = k$

$\Rightarrow \lim_{x\to 0} \dfrac{\sin 3x}{\sin x} = k \Rightarrow \lim_{x\to 0} \dfrac{\sin 3x}{3x}.\dfrac{3x}{\sin 3x} = k$

$\Rightarrow \quad 3 = k$

155 **(b)**

Since, the function $f(x)$ is continuous

$\therefore\ f(0)=$RHL $f(x)=$LHL$f(x)$

Now, RHL $f(X)=\lim_{h\to 0}\frac{\log(1+0+h)+\log(1-0-h)}{0+h}$

$=\lim_{h\to 0}\frac{\log(1+h)+\log(1-h)}{h}$

$=\lim_{h\to 0}\frac{\frac{1}{1+h}-\frac{1}{1-h}}{1}=0$

[by L 'Hospital's rule]

$\therefore\ f(0)=$RHL $f(x)=0$

156 **(d)**

$$f(x)=\begin{cases}\frac{x-4}{|x-4|}+a, & x<4\\ a+b, & x=4\\ \frac{x-4}{|x-4|}+b, & x>4\end{cases}=\begin{cases}-1+a, x<4\\ a+b\\ 1+b, x>4\end{cases}$$

LHL$=\lim_{x\to 4^{\mp}}f(x)=a-1$

RHL$=\lim_{x\to 4^{\mp}}f(x)=1+b$

Since, LHL=RHL$=f(4)$

$\Rightarrow\ a-1=a+b=b+1$

$a=1$ and $b=-1$

157 **(d)**

We have,

$$f(x)=\begin{cases}\frac{-1}{x-1}, & 0<x<1\\ \frac{1-1}{x-1}=0, & 1<x<2\\ 0, & x=1\end{cases}$$

Clearly, $\lim_{x\to 1^-}f(x)\to-\infty$ and $\lim_{x\to 1^+}f(x)=0$

So, $f(x)$ is not continuous at $x=1$ and hence it is not differentiable at $x=1$

158 **(d)**

$\lim_{x\to\frac{\pi}{4}}f(x)=\lim_{x\to\frac{\pi}{4}}\frac{1-\sqrt{2}\sin x}{\pi-4x}$

$=\lim_{x\to\frac{\pi}{4}}\frac{-\sqrt{2}\cos x}{4}=\frac{1}{4}$ [by L 'Hospital's rule]

Since, $f(x)$ is continuous at $x=\frac{\pi}{4}$

$\therefore\ \lim_{x\to\frac{\pi}{4}}f(x)=f\left(\frac{\pi}{4}\right)\ \Rightarrow\ \frac{1}{4}=a$

159 **(d)**

LHL$=\lim_{x\to 1^-}f(x)=\lim_{h\to 0}1-h+a=1+a$

RHL$=\lim_{x\to 1^+}f(x)=\lim_{h\to 0}3-(1+h)^2=2$

For $f(x)$ to be continuous, LHL=RHL

$\Rightarrow 1+a=2\quad\Rightarrow\quad a=1$

160 **(b)**

LHL$=\lim_{h\to 0}\frac{\cos 3(0-h)-\cos(0-h)}{(0-h)^2}$

$=\lim_{h\to 0}\frac{\cos 3h-\cos h}{h^2}$

$=\lim_{h\to 0}\frac{-3\sin 3h+\sin h}{2h}$

$=\lim_{h\to 0}\frac{-9\cos 3h+\cos h}{2}=\frac{-9+1}{2}=-4$

$\because\ \lim_{x\to 0^-}f(x)=f(0)\quad\Rightarrow\quad\lambda=-4$

161 **(c)**

LHL$=\lim_{x\to a^-}\frac{x^3-a^3}{x-a}=\lim_{h\to 0}\frac{(a-h)^3-a^3}{a-h-a}$

$=\lim_{h\to 0}\frac{(a-h-a)\{(a-h)^2+a^3+a(a-h)\}}{-h}$

$=3a^2$

Since, $f(x)$ is continuous at $x=a$

$\therefore\quad$ LHL $=f(a)$

$\Rightarrow\quad 3a^2=b$

163 **(a)**

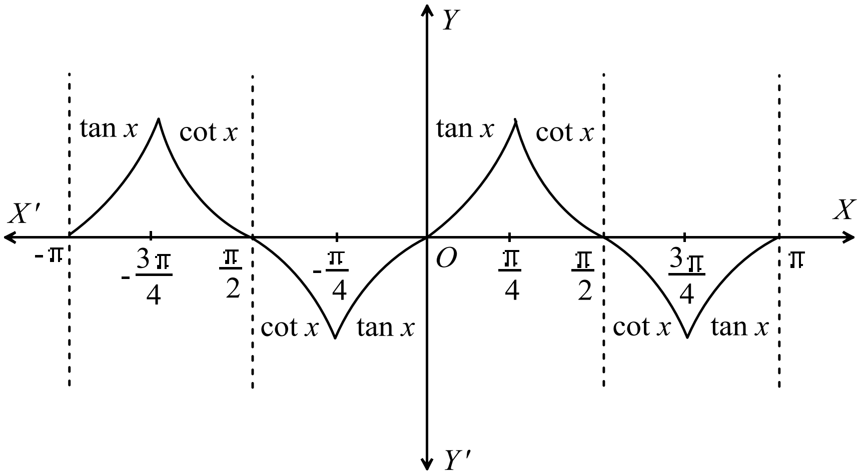
We have,

$$f(x)=\begin{cases}\tan x, & 0\le x\le\pi/4\\ \cot x, & -\pi/4\le x\le\pi/2\\ \tan x, & \pi/2<x\le 3\pi/4\\ \cot x, & 3\pi/4\le x<\pi\end{cases}$$

Since $\tan x$ and $\cot x$ are periodic functions with period π. So, $f(x)$ is also periodic with period π

It is evident from the graph that $f(x)$ is not continuous at $x=\pi/2$. Since $f(x)$ is periodic with period π. So, it is not continuous at $x=0,\pm\pi/2,\pm\pi,\neq 3\pi/2$

Also, $f(x)$ is not differentiable $x=\pi/4, 3\pi/4, 5\pi/4$ etc



164 **(c)**

We have,

$f(x) = \{|x| - |x-1\}^2$

$$\Rightarrow f(x) = \begin{cases} (-x+x-1)^2, & \text{if } x < 0 \\ (x+x-1)^2, & \text{if } 0 \le x < 1 \\ (x-x+1)^2, & \text{if } x \ge 1 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} 1, & \text{if } x < 0 \\ (2x-1)^2, & \text{if } 0 < x < 1 \\ 1, & \text{if } x \ge 1 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} 0, & \text{if } x < 0 \text{ or if } x > 1 \\ 4(2x-1), & \text{if } 0 < x < 1 \end{cases}$$

165 **(b)**

We have,

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

$$\Rightarrow f'(x_0) = \lim_{x \to x_0} \frac{(x - x_0)\phi(x) - 0}{(x - x_0)}$$

$$\Rightarrow f'(x_0) = \lim_{x \to x_0} \phi(x) = \phi(x_0) \quad [\because \phi(x) \text{ is continuous at } x = x_0]$$

166 **(b)**

Since, $\lim_{x \to 2^+} f(x) = f(2) = k$

$$\Rightarrow \quad k = \lim_{h \to 0} f(2+h)$$

$$\Rightarrow \quad k = \lim_{h \to 0} \left[(2+h)^2 + e^{\frac{1}{2-(2+h)}}\right]^{-1}$$

$$\Rightarrow \quad \lim_{h \to 0} \left[4 + h^2 + 4h + e^{-1/h}\right]^{-1} = \frac{1}{4}$$

167 **(c)**

For $f(x)$ to be continuous at $x = \pi/2$, we must have

$$\lim_{x \to \pi/2} f(x) = f(\pi/2)$$

$$\Rightarrow \lim_{x \to \pi/2} \frac{1 - \sin x}{(\pi - 2x)^2} \cdot \frac{\log \sin x}{\log(1 + \pi^2 - 4\pi x + 4x^2)} = k$$

$$\Rightarrow \lim_{h \to 0} \frac{1 - \cos h}{4h^2} \times \frac{\log \cos h}{\log(1 + 4h^2)} = k$$

$$\Rightarrow \lim_{h \to 0} \frac{1 - \cos h}{4h^2} \times \frac{\log\{1 + \cos h - 1\}}{\cos h - 1} \times \frac{4h^2}{\log(1 + 4h^2)} \times \frac{\cos h - 1}{4h^2} = k$$

$$\Rightarrow -\lim_{h \to 0} \left(\frac{1 - \cos h}{4h^2}\right)^2 \frac{\log(1 + (\cos h - 1))}{\cos h - 1} \times \frac{4\,h^2}{\log(1 + 4h^2)} = k$$

$$\Rightarrow -\lim_{h \to 0} \left(\frac{\sin^2 h/2}{2\,h^2}\right)^2 \frac{\log(1 + (\cos h - 1))}{\cos h - 1} \times \frac{4h^2}{\log(1 + 4h^2)} = k$$

$$\Rightarrow -\frac{1}{64} \lim_{h \to 0} \left(\frac{\sin h/2}{h/2}\right)^4 \frac{\log(1 + (\cos h - 1))}{\cos h - 1} \times \frac{4h^2}{\log(1 + 4h^2)} = k$$

$$\Rightarrow -\frac{1}{64} = k$$

168 **(c)**

LHL$= \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{\sin 5(0-h)}{(0-h)^2 + 2(0-h)}$

$$= -\lim_{h \to 0} \frac{\frac{\sin 5h}{5h}}{\frac{1}{5}(h-2)} = \frac{5}{2}$$

Since, it is continuous at $x = 0$, therefore

LHL$= f(0)$

$$\Rightarrow \frac{5}{2} = k + \frac{1}{2} \quad \Rightarrow \quad k = 2$$

169 **(a)**

Since $f(x)$ is continuous at $x = 0$

$$\therefore \lim_{x \to 0} f(x) = f(0) = 0$$

$$\Rightarrow \lim_{x \to 0} x^n \sin\left(\frac{1}{x}\right) = 0 \Rightarrow n > 0$$

$f(x)$ is differentiable at $x = 0$, if

$\lim_{x \to 0} \frac{f(x) - f(0)}{x - 0}$ exists finitely

$\Rightarrow \lim_{x \to 0} \frac{x^n \sin\frac{1}{x} - 0}{x}$ exists finitely

$\Rightarrow \lim_{x \to 0} x^{n-1} \sin\left(\frac{1}{x}\right)$ exists finitely

$\Rightarrow n - 1 > 0 \Rightarrow n > 1$

If $n \le 1$, then $\lim_{x \to 0} x^{n-1} \sin\left(\frac{1}{x}\right)$ does not exist and hence $f(x)$ is not differentiable at $x = 0$

Hence $f(x)$ is continuous but not differentiable at $x = 0$ for $0 < n \le 1$ i.e. $n \in (0, 1]$

170 **(b)**

Clearly, $f(x)$ is not differentiable at $x = 3$

Now, $\lim_{h\to 3^-} f(x) = \lim_{h\to 0} f(3-h)$

$= \lim_{h\to 0} |3-h-3|$

$= 0$

$\lim_{h\to 3^+} f(x) = \lim_{h\to 0} f(3+h)$

$= \lim_{h\to 0} |3+h-3| = 0$

and $f(3) = |3-3| = 0$

$\therefore$ $f(x)$ is continuous at $x = 3$

171 **(a)**

It can easily be seen from the graphs of $f(x)$ and that both are continuous at $x = 0$

Also, $f(x)$ is not differentiable at $x = 0$ whereas $g(x)$ is differentiable at $x = 0$

172 **(c)**

We have,

$$\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h)$$

$$= \lim_{h\to 0} \frac{-\sin(a+1)h - \sin h}{-h}$$

$$\Rightarrow \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h)$$

$$= \lim_{h\to 0}\left\{\frac{\sin(a+1)h}{h} + \frac{\sin h}{h}\right\}$$

$$\Rightarrow \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = (a+1)+1$$

$$= a+2$$

and, $\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h)$

$$\Rightarrow \lim_{x\to 0^+} f(x) = \lim_{h\to 0} \frac{\sqrt{h+bh^2} - \sqrt{h}}{b\,h^{3/2}}$$

$$\Rightarrow \lim_{x\to 0^+} f(x) = \lim_{h\to 0} \frac{h+bh^2-h}{bh^{3/2}(\sqrt{h+bh^2}-\sqrt{h})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{1+bh}+1} = \frac{1}{2}$$

Since, $f(x)$ is continuous at $x = 0$. Therefore,

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$$

$$\Rightarrow a+2 = \frac{1}{2} = c \Rightarrow c = \frac{1}{2}, a = -\frac{3}{2} \text{ and } b \in R - \{0\}$$

173 **(c)**

For $f(x)$ to be continuous at $x = 0$, we must have

$$\lim_{x\to 0} f(x) = f(0)$$

$$\Rightarrow \lim_{x\to 0} \frac{(9^x-1)(4^x-1)}{\sqrt{2}-\sqrt{2\cos^2 x/2}} = k$$

$$\Rightarrow \lim_{x\to 0} \frac{(9^x-1)(4^x-1)}{\sqrt{2}.\,2\sin^2 x/4} = k$$

$$\Rightarrow \lim_{x\to 0} \frac{16\times\left(\frac{9^x-1}{x}\right)\left(\frac{4^x-1}{x}\right)}{2\sqrt{2}\left(\frac{\sin x/2}{x/4}\right)^2} = k$$

$$\Rightarrow \frac{16}{2\sqrt{2}}\log 9\,.\log 4 = k = 4\sqrt{2}\log 9\,.\log 4$$

$$= 16\sqrt{2}\log 3\log 2$$

174 **(b)**

Given, $f(x) = [\tan^2 x]$

Now, $\lim_{x\to 0} f(x) = \lim_{x\to 0}[\tan^2 x] = 0$

And $f(0) = [\tan^2 0] = 0$

Hence, $f(x)$ is continuous at $x = 0$

175 **(b)**

Let, $f(x) = x$

Which is continuous at $x = 0$

Also, $f(x+y) = f(x) + f(y)$

$\Rightarrow$ $f(0+0) = f(0) + f(0)$

$= 0 + 0$

$\Rightarrow f(0) = 0$

$f(1+0) = f(1) + f(0)$

$\Rightarrow$ $f(1) = 1 + 0$

$\Rightarrow$ $f(1) = 1$

As, it satisfies it.

Hence, $f(x)$ is continous for every values of x

176 **(c)**

$$\text{Here, } gof = \begin{cases} e^{\sin x}, & x \ge 0 \\ e^{1-\cos x}, & x \le 0 \end{cases}$$

$\therefore$ LHD$= \lim_{h\to 0} \frac{gof(0-h)-gof(h)}{-h}$

$$= \lim_{h\to 0} \frac{e^{1-\cos h} - e^{1-\cos h}}{-h} = 0$$

RHD$= \lim_{h\to 0} \frac{gof(0+h)-gof(h)}{h}$

$$= \lim_{h\to 0} \frac{e^{\sin h} - e^{\sin h}}{h} = 0$$

Since, RHD=LHD=0

$\therefore$ $(gof)'(0) = 0$

177 **(b)**

We have,

$$f(x)\begin{cases} (x+1)^{2-\left(\frac{1}{x}+\frac{1}{x}\right)} = (x+1)^2, & x < 0 \\ 0, & x = 0 \\ (x+1)^{2-\left(\frac{1}{x}+\frac{1}{x}\right)} = (x+1)^{2-\frac{2}{x}}, & x > 0 \end{cases}$$

Clearly, $f(x)$ is everywhere continuous except possibly at $x = 0$

At $x = 0$, we have

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(x+1)^2 = 1$$

and, $\lim_{x\to 0^+} f(x) = \lim_{x\to 0}(x+1)^{2-\frac{2}{x}} = \lim_{x\to 0}(x+1)^{-2/x}$

$$\Rightarrow \lim_{x\to 0^+} f(x) = e^{\lim_{x\to 0} -\frac{2}{x}\log(1+x)} = e^{-2}$$

Clearly, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$

So, $f(x)$ is not continuous at $x=0$

178 **(b)**

Since $f(x)$ is continuous at $x=0$. Therefore,

$\lim_{x\to 0} f(x)=f(0)$

$\Rightarrow \lim_{x\to 0} f(x)=k$

$$\Rightarrow \lim_{x\to 0}\frac{\log(1+ax)-\log(1-bx)}{x}=k$$

$$\Rightarrow a\lim_{x\to 0}\frac{\log(1+ax)}{ax}-(-b)\lim_{x\to 0}\frac{\log(1-bx)}{-bx}=k$$

$\Rightarrow a+b=k$

179 **(c)**

Since $f(x)$ is continuous at $x=0$

$\therefore f(0)=\lim_{x\to 0} f(x)$

$$\Rightarrow f(0)=\lim_{x\to 0}\frac{(27-2x)^{1/3}-3}{9-3(243+5x)^{1/5}} \quad \left[\text{Form}\frac{0}{0}\right]$$

$$\Rightarrow f(0)=\lim_{x\to 0}\frac{\frac{1}{3}(27-2x)^{-\frac{2}{3}}(-2)}{-\frac{3}{5}(243+5x)^{-\frac{4}{5}}(5)}$$

$$=\left(-\frac{2}{3}\right)\left(-\frac{1}{3}\right)\frac{3^4}{3^2}=2$$

180 **(d)**

$\lim_{x\to 0}\frac{e^{2x}-1-2x}{x(e^{2x}-1)}$

$=\lim_{x\to 0}\frac{2e^{2x}-2}{(e^{2x}-1)+2xe^{2x}}$ [using L 'Hospital rule]

$=\lim_{x\to 0}\frac{4e^{2x}}{4e^{2x}+4xe^{2x}}=1$ [using L 'Hospital's rule]

Since, $f(x)$ is continuous at $x=0$, then

$\lim_{x\to 0} f(x)=f(0) \quad\Rightarrow\quad 1=f(0)$

181 **(b)**

If a function $f(x)$ is continuous at $x=a$, then it may or may not be differentiable at $x=a$

$\therefore$ Option (b) is correct

182 **(c)**

Let $f(x)=|x-1|+|x-3|$

$$=\begin{cases} x-1 \quad +x-3 & ,x\geq 3\\ x-1+3-x, & 1\leq x<3\\ 1-x \quad +3-x, & x\leq 1\end{cases}$$

$$=\begin{cases} 2x-4, x\geq 3\\ 2, 1\leq x<3\\ 4-2x, x\leq 1\end{cases}$$

At $x=2$, function is

$f(x)=2$

$\Rightarrow f'(x)=0$

183 **(d)**

We have,

$$f(x)=\begin{cases}(x+1)\,e^{-\left(\frac{1}{x}+\frac{1}{x}\right)}=(x+1), & x<0\\ (x+1)\,e^{-\left(\frac{1}{x}+\frac{1}{x}\right)}=(x+1)e^{-2/x}, & x>0\end{cases}$$

Clearly, $f(x)$ is continuous for all $x\neq 0$

So, we will check its continuity at $x=0$

We have,

(LHL at $x=0$) $=\lim_{x\to 0^-} f(x)=\lim_{x\to 0}(x+1)=1$

(RHL $at\ x=0$) $=\lim_{x\to 0^+} f(x)=\lim_{x\to 0}(x+1)\ e^{-2/x}$

$$=\lim_{x\to 0}\frac{x+1}{e^{2/x}}=0$$

$\therefore \lim_{x\to 0^-} f(x)\neq \lim_{x\to 0^+ f(x)}$

So, $f(x)$ is not continuous at $x=0$

Also, $f(x)$ assumes all values from $f(-2)$ to $f(2)$ and $f(2)=3/e$ is the maximum value of $f(x)$

184 **(c)**

Since, it is a polynomial function, so it is continuous for every value of x except at $x=2$

LHL= $\lim_{x\to 2^-} x-1$

$=\lim_{h\to 0} 2-h-1=1$

RHL= $\lim_{x\to 2^+} 2x-3=\lim_{h\to 0} 2(2+h)-3=1$

And $\quad f(2)=2(2)-3=1$

$\therefore$ LHL+RHL= $f(2)$

Hence, $f(x)$ is continuous for all real values of x

185 **(c)**

Continuity at $x=0$

LHL=$\lim_{x\to 0^-}\frac{\tan x}{x}=\lim_{h\to 0}\frac{-tan\ h}{-h}=1$

RHL= $\lim_{x\to 0^+}\frac{\tan x}{x}=\lim_{h\to 0}\frac{tan\ h}{h}=1$

$\therefore$ LHL=RHL= $f(0)=1$, it is continuous

Differentiability at $x=0$

LHD= $\lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}=\lim_{h\to 0}\frac{\frac{\tan(-h)}{-h}-1}{-h}$

$$=\lim_{h\to 0}\frac{+\frac{h^2}{3}+\frac{2h^4}{15}+\cdots}{-h}=0$$

RHD= $\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{\frac{\tan h}{h}-1}{h}$

$$=\lim_{h\to 0}\frac{\frac{h^2}{3}+\frac{2h^4}{15}+\cdots}{-h}=0$$

$\therefore$ LHD=RHD

Hence, it is differentiable.

186 **(b)**

We have,

$\lim_{x\to 1^-} f(x)=\lim_{x\to 1}(x-1)=0$

and,

$\lim_{x\to 1^+} f(x)=\lim_{x\to 1}(x^3-1)=0$. Also,

$f(1)=1-1=0$

So, $f(x)$ is continuous at $x=1$

Clearly, $\left(f'(1)\right)=3$ and $Rf'(1)=1$

Therefore, $f(x)$ is not differentiable at $x=1$

187 **(d)**

We have,

$$f(x)=\begin{cases}\dfrac{x^2-x}{x^2-x}=1, & \text{if } x<0 \text{ or } x>1\\ -\dfrac{(x^2-x)}{x^2-x}=-1, & \text{if } 0<x<1\\ 1, & \text{if } x=0\\ -1, & \text{if } x=1\end{cases}$$

$\Rightarrow f(x)=\begin{cases}1, \text{if } x\le 0 \text{ or } x>1\\ -1, \text{if } 0<x\le 1\end{cases}$

Now,

$\lim_{x\to 0^-} f(x)=\lim_{x\to 0} 1=1$ and, $\lim_{x\to 0^+} f(x)=\lim_{x\to 0} -1=-1$

Clearly, $\lim_{x\to 0^-} f(x)\ne \lim_{x\to 0^+} f(x)$

So, $f(x)$ is not continuous at $x=0$. It can be easily seen that it is not continuous at $x=1$

188 **(b)**

We have,

$f(x)=|x-1|+|x-3|$

$\Rightarrow f(x)=\begin{cases}-(x-1)-(x-3), & x<1\\ (x-1)-(x-3), & 1\le x<3\\ (x-1)+(x-3), & x\ge 3\end{cases}$

$\Rightarrow f(x)=\begin{cases}-2x+4, & x<1\\ 2, & 1\le x<3\\ 2x-4, & x\ge 3\end{cases}$

Since, $f(x)=2$ for $1\le x<3$. Therefore

$f'(x)=0$ for all $x\in(1,3)$

Hence, $f'(x)=0$ at $x=2$

189 **(d)**

We have,

$Lf'(0)=0$ and $Rf'(0)=0+\cos 0°=1$

$\therefore Lf'(0)\ne Rf'(0)$

Hence, $f'(x)$ does not exist at $x=0$

190 **(c)**

Given, $g(x)=\frac{(x-1)^n}{\log\cos^m(x-1)}$; $0<x<2$, $m\ne 0$, n are integers and $|x-1|=\begin{cases}x-1; & x\ge 1\\ 1-x; & x<1\end{cases}$

The left hand derivative of $|x-1|$ at $x=1$ is $p=-1$

Also, $\lim_{x\to 1^+} g(x)=p=-1$

$\Rightarrow \lim_{h\to 0}\dfrac{(1+h-1)^n}{\log\cos^m(1+h-1)}=-1$

$\Rightarrow \lim_{h\to 0}\dfrac{h^n}{m\log\cos h}=-1$

$\Rightarrow \lim_{h\to 0}\dfrac{n.h^{n-1}}{m\frac{1}{\cos h}(-\sin h)}=-1$

[using L 'Hospital's rule]

$\Rightarrow \left(\dfrac{n}{m}\right)\lim_{h\to 0}\dfrac{h^{n-2}}{\left(\frac{\tan h}{h}\right)}=1$

$\Rightarrow n=2$ and $\frac{n}{m}=1$

$\Rightarrow m=n=2$

191 **(c)**

Given, $f(x)=\frac{2x^2+7}{(x^2-1)(x+3)}$

Since, at $x=1,-1,-3$, $f(x)=\infty$

Hence, function is discontinuous

193 **(a)**

LHL$=\lim_{x\to 1^-} f(x)=\lim_{h\to 0}\ [1-(1-h)^2]=0$

RHL$=\lim_{x\to 1^+} f(x)=\lim_{h\to 0}\{1+(1+h)^2\}=2$

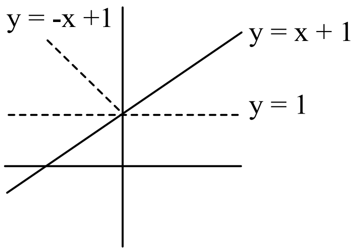
Also, $f(1)=0$

$\Rightarrow$ RHL $\ne$ LHL $=f(1)$

Hence, $f(x)$ is not continuous at $x=1$

194 **(c)**

It is clear from the graph that minimum $f(x)$ is



$f(x)=x+1, \quad \forall x\in R$

Hence, it is a straight line, so it is differentiable everywhere

195 **(c)**

Since, $f(x)$ is continuous at $x=\frac{\pi}{2}$

$\lim_{x\to\frac{\pi}{2}^{-1}}(mx+1)=\lim_{x\to\frac{\pi}{2}^{+}}(\sin x+n)$

$\Rightarrow \quad m\dfrac{\pi}{2}+1=\sin\dfrac{\pi}{2}+n$

$\Rightarrow \quad \frac{m\pi}{2}=n$

196 **(a)**

This function is continuous at $x=0$, then

$\lim_{x\to 0}\dfrac{\log_e(1+x^2\tan x)}{\sin x^3}=f(0)$

$\Rightarrow \lim_{x\to 0}\dfrac{\log_e\left\{1+x^2\left(x+\frac{x^3}{3}+\dots\right)\right\}}{x^3-\frac{x^9}{3!}+\frac{x^{15}}{5!}-\dots}=f(0)$

$\Rightarrow \lim_{x\to 0}\dfrac{\log_e(1+x^3)}{x^3-\frac{x^9}{3!}+\frac{x^{15}}{5!}-\dots}=f(0)$

[neglecting higher power of x in $x^2\tan x$]

$\Rightarrow \lim_{x\to 0}\dfrac{x^3-\frac{x^6}{2}+\frac{x^9}{3}-\cdots}{x^3+\frac{x^9}{3!}+\frac{x^{15}}{5!}-\dots}=f(0)$

$\Rightarrow \quad 1=f(0)$

197 **(a)**

Given, $f(x)$ is continuous at $x=0$

$\therefore$ Limit must exist

ie, $\lim_{x\to 0} x^p\sin\frac{1}{x}=(0)^p\sin\infty=0$, when,

$0<p<\infty$ ...(i)

Now, RHD$=\lim_{h\to 0}\frac{h^p\sin\frac{1}{h}-0}{h}=\lim_{h\to 0} h^{p-1}\sin\frac{1}{h}$

LHD= $\lim_{h\to 0} \frac{(-h)^p \sin\left(-\frac{1}{h}\right)-0}{-h}$

$= \lim_{h\to 0} (-1)^p h^{p-1} \sin\frac{1}{h}$

Since, $f(x)$ is not differentiable at $x = 0$

$\therefore \quad p \le 1$ ...(ii)

From Eqs.(i) and (iii), $0 < p \le 1$

198 **(a)**

We have,

$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\sin x^2}{x} = \lim_{x\to 0} \left(\frac{\sin x^2}{x^2}\right) x = 1 \times 0$

$= 0 = f(0)$

So, $f(x)$ is continuous at $x = 0$. $f(x)$ is also derivable at $x = 0$, because

$\lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} \frac{\sin x^2}{x} = \lim_{x\to 0} \frac{\sin x^2}{x^2} = 1$

exists finitely

199 **(a)**

A function f on R into itself is continuous at a point a in R, iff for each $\in > 0$ there exist $\delta > 0$, such that

$|f(x) - f(a)| < \in \ \Rightarrow \ |x - a| < \delta$

200 **(a)**

We have,

$f(x) = x - |x - x^2|, \qquad -1 \le x \le 1$

$\Rightarrow f(x) = \begin{cases} x + x - x^2, & -1 \le x < 0 \\ x - (x - x^2), & 0 \le x \le 1 \end{cases}$

$\Rightarrow f(x) = \begin{cases} 2x - x^2, & -1 \le x < 0 \\ x^2, & 0 \le x \le 1 \end{cases}$

Clearly, $f(x)$ is continuous at $x = 0$

Also,

$\lim_{x\to -1^+} f(x) = \lim_{x\to -1} 2x - x^2 = -2 - 1 = -3$

$= f(-1)$

and,

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x^2 = 1 = f(1)$

So, $f(x)$ is right continuous at $x = -1$ and left continuous at $x = 1$

Hence, $f(x)$ is continuous on $[-1, 1]$

201 **(b)**

Since $|\sin x|$ and $|e^{|x|}$ are not differentiable at $x = 0$. Therefore, for $f(x)$ to be differentiable at $x = 0$, we must have $a = 0, b = 0$ and c can be any real number

202 **(a)**

We have,

$f(u + v) = f(u) + kuv - 2v^2$ for all $u, v \in R$ ...(i)

Putting $u = v = 1$, we get

$f(2) = f(1) + k - 2 \Rightarrow 8 = 2 + k - 2 \Rightarrow k = 8$

Putting $u = x, v = h$ in (i), we get

$\frac{f(x + h) - f(x)}{h} = kx - 2h$

$\Rightarrow \lim_{h\to 0} \frac{f(x + h) - f(x)}{h} = kx \Rightarrow f'(x)$

$= 8x \quad [\because k = 8]$

203 **(b)**

Given, $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$\Rightarrow \ f'(x) = \frac{1}{\sqrt{1 - \left(\frac{2x}{1+x^2}\right)^2}} \times \frac{d}{dx}\left(\frac{2x}{1 + x^2}\right)$

$= \frac{1 + x^2}{\sqrt{(1 + x^2)^2}} \times \frac{2(1 - x^2)}{(1 + x^2)^2}$

$= \frac{2}{1 + x^2} \times \frac{1 - x^2}{|1 - x^2|} = \begin{cases} \frac{2}{1 + x^2}, \text{if } |x| < 1 \\ -\frac{2}{1 + x^2}, \quad \text{if } |x| > 1 \end{cases}$

$\therefore \ f'(x)$ does not exist for $|x| = i, ie, x = \pm 1$

Hence, $f(X)$ is differentiable on $R - \{-1, 1\}$

204 **(a)**

LHL= $\lim_{x\to 0^-} f(x) = \lim_{h\to 0} -h \sin\left(\frac{1}{-h}\right) = 0$

RHL= $\lim_{x\to 0^+} f(x) = \lim_{h\to 0} h \sin\left(\frac{1}{h}\right) = 0$

$\therefore$ LHL=RHL= $f(0)$, it is continuous

LHD= $\lim_{x\to 0^-} f(x) = \lim_{h\to 0} \left[\frac{f(0-h)-f(0)}{-h}\right]$

$= \lim_{h\to 0} \left[\frac{-h \sin\frac{1}{h} - 0}{-h}\right]$=does not exist

$\Rightarrow \ f(x)$ is not differentiable at $x = 0$

$\therefore \ f(x)$ is continuous at $x = 0$ but not differentiable at $x = 0$

205 **(b)**

Since, $|x - 1|$ is not differentiable at $x = 1$

So, $f(x) = |x - 1|e^x$ is not differentiable at $x = 1$

Hence, the required set is $R - \{1\}$

206 **(d)**

We have,

$f'(x) = \lim_{h\to 0} \frac{f(x + h) - f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h\to 0} \frac{f(x)f(h) - f(x)}{h} \qquad [\because f(x + y)$

$= f(x)f(y)]$

$\Rightarrow f'(x) = f(x) \lim_{h\to 0} \frac{f(h) - 1}{h}$

$\Rightarrow f'(x) = f(x) \cdot \lim_{h\to 0} \frac{1 + h\, g(h)G(h) - 1}{h}$

$\Rightarrow f'(x) = f(x) \cdot \lim_{h\to 0} g(h) \ G(h)$

$\Rightarrow f'(x) = f(x) \lim_{h\to 0} G(h) \lim_{h\to 0} g(h) = ab\, f(x)$