Single Correct Answer Type

1. The circle $x^2 + y^2 + 4x - 7y + 12 = 0$ cuts an intercept on y-axis of length
   a) 3 b) 4 c) 7 d) 1
2. If the eccentricities of the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ and the hyperbola $\frac{x^2}{64} - \frac{y^2}{b^2} = 1$ are reciprocals of each other, then b^2 is equal to
   a) 192 b) 64 c) 16 d) 32
3. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which is turn in inscribed in another ellipse that passes through the point (4, 0). Then, the equation of the ellipse is
   a) $x^2 + 12y^2 = 16$ b) $4x^2 + 48y^2 = 48$ c) $4x^2 + 64y^2 = 48$ d) $x^2 + 16y^2 = 16$
4. The Cartesian equation of the directrix of the parabola whose parametric equations are $x = 2t + 1, y = t^2 + 2$, is
   a) $y = 2$ b) $y = 1$ c) $y = -1$ d) $y = -2$
5. The line $x - 1 = 0$ is the directrix of the parabola $y^2 - kx + 8 = 0$. Then one of the value of k is
   a) $\frac{1}{8}$ b) 8 c) 4 d) $\frac{1}{4}$
6. The equation of the axes of the ellipse $3x^2 + 4y^2 + 6x - 8y - 5 = 0$, are
   a) $x + 3, y = 5$ b) $x + 3 = 0, y - 5 = 0$ c) $x - 1 = 0, y = 0$ d) $x + 1 = 0, y - 1 = 0$
7. Locus of the mid points of the chord of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, so that chord is always touching the circle $x^2 + y^2 = c^2, (c < a, c < b)$ is
   a) $(b^2x^2 + a^2y^2)^2 = c^2(b^4x^2 + a^4y^2)$ b) $(a^2x^2 + b^2y^2)^2 = c^2(a^4x^2 + b^4y^2)$
   c) $(b^2x^2 + a^2y^2)^2 = c^2(b^2x^4 + a^2y^4)$ d) None of the above
8. The length intercepted by the curve $y^2 = 4x$ on the line satisfying $dy/dx = 1$ and passing through point (0, 1), is given by
   a) 1 b) 2 c) 0 d) None of these
9. Two vertices of an equilateral triangle are $(-1,0)$ and $(1,0)$ and its third vertex lies above the x-axis. The equation of its circumcircle, is
   a) $x^2 + y^2 - \frac{1}{\sqrt{3}}y - 1 = 0$
   b) $x^2 + y^2 + \frac{2}{\sqrt{3}}y - 1 = 0$
   c) $x^2 + y^2 - \frac{2}{\sqrt{3}}y - 1 = 0$
   d) None of these
10. The tangents to $x^2 + y^2 = a^2$ having inclinations α and β intersect at P. If $\cot\alpha + \cot\beta = 0$, then the locus of P is
    a) $x + y = 0$ b) $x - y = 0$ c) $xy = 0$ d) None of these
11. The parametric representation $(2 + t^2, 2t + 1)$ represents
    a) A parabola with focus at (2,1)
    b) A parabola with vertex at (2,1)
    c) An ellipse with centre at (2,1)
    d) None of these
12. Product of the perpendicular from the foci upon any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a < b)$ is equal to
    a) $2a$ b) a^2 c) b^2 d) ab^2
13. The equations of the sides AB, BC, CA of a ΔABC are $x + y = 1, 4x - y + 4 = 0$ and$2x + 3y = 6$. Circles are drawn on AB, BC, CA as diameter. The point of concurrence of the common chord is
    a) Centroid of the triangle b) Orthocenter

c) Circumcentre d) Incentre

14. The sum of the distances of a point $(2, -3)$ from the foci of an ellipse $16\,(x-2)^2 + 25\,(y+3)^2 = 400$ is
a) 8 b) 6 c) 50 d) 32

15. If the equation of a given circle is $x^2 + y^2 = 36$, then the length of the chord which lies along the line $3x + 4y - 15 = 0$ is
a) $3\sqrt{6}$ b) $2\sqrt{3}$ c) $6\sqrt{3}$ d) None of these

16. The normal chord of a parabola $y^2 = 4ax$ at (x_1, x_1) subtends a right angle at the
a) Focus
b) Vertex
c) End of the latusrectum
d) None of these

17. The equation of the circle which has a tangent $2x - y - 1 = 0$ at $(3,5)$ on it and with the centre on $x + y = 5$, is
a) $x^2 + y^2 + 6x - 16y + 28 = 0$
b) $x^2 + y^2 - 6x + 16y - 28 = 0$
c) $x^2 + y^2 + 6x + 6y - 28 = 0$
d) $x^2 + y^2 - 6x - 6y - 28 = 0$

18. The equation of the tangent to the parabola $y^2 = 9x$ which goes through the point $(4, 10)$, is
a) $x + 4y + 1 = 0$ b) $9x + 4y + 4 = 0$ c) $x + 4y + 36 = 0$ d) $9x - 4y + 4 = 0$

19. The length of the chord of the circle $x^2 + y^2 + 4x - 7y + 2 = 0$ along the y-axis, is
a) 1 b) 2 c) 1/2 d) None of these

20. What is the slope of the tangent drawn to the hyperbola $xy = a, (a \neq 0)$ at the point $(a, 1)$?
a) $\frac{1}{a}$ b) $-\frac{1}{a}$ c) a d) $-a$

21. The equation $ax^2 + 2\,hxy + by^2 + 2\,gx + 2\,fy + c = 0$ represents a rectangular hyperbola if
a) $\Delta \neq 0, h^2 > ab, a + b = 0$
b) $\Delta \neq 0, h^2 < ab, a + b = 0$
c) $\Delta \neq 0, h^2 = ab, a + b = 0$
d) None of these

22. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$ meets its auxiliary circle at the point M. Then, the area of the triangle with vertices at A, M and the origin O is
a) $\frac{31}{10}$ b) $\frac{29}{10}$ c) $\frac{21}{10}$ d) $\frac{27}{10}$

23. From the point $(-1, -6)$ two tangents are drawn to the parabola $y^2 = 4x$. Then, the angle between the two tangents is
a) 30° b) 45° c) 60° d) 90°

24. The centre of the ellipse $4x^2 + 9y^2 + 16x - 18y - 11 = 0$ is
a) $(-2, -1)$ b) $(-2,1)$ c) $(2, -1)$ d) None of these

25. The circle whose equation are $x^2 + y^2 + c^2 = 2ax$ and $x^2 + y^2 + c^2 - 2by = 0$ will touch one another externally if
a) $\frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a^2}$ b) $\frac{1}{c^2} + \frac{1}{a^2} = \frac{1}{b^2}$ c) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$ d) None of these

26. In an ellipse the distance between the foci is 8 and the distance between the directrices is 25. The length of major axis is
a) $10\sqrt{2}$ b) $20\sqrt{2}$ c) $30\sqrt{2}$ d) None of these

27. If $lx + my + n = 0$ represents a chord of the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ whose eccentric angles differ by 90°, then
a) $a^2l^2 + b^2m^2 = n^2$ b) $\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2 - b^2)^2}{n^2}$
c) $a^2l^2 + b^2m^2 = 2n^2$ d) None of these

28. If the latusrectum of a hyperbola forms an equilateral triangle with the vertex at the centre of the hyperbola, then the eccentricity of the hyperbola is

a) $\frac{\sqrt{5}+1}{2}$ b) $\frac{\sqrt{11}+1}{2}$ c) $\frac{\sqrt{13}+1}{2\sqrt{3}}$ d) $\frac{\sqrt{13}-1}{2\sqrt{3}}$

29. The eccentricity of the conic $4x^2 + 16y^2 - 24x - 32y = 1$ is

a) $\frac{1}{2}$ b) $\sqrt{3}$ c) $\frac{\sqrt{3}}{2}$ d) $\frac{\sqrt{3}}{4}$

30. If the chords of contact of tangents from two points (x_1, y_1) and (x_2, y_2) to the hyperbola $4x^2 - 9y^2 - 36 = 0$ are at right angles, then $\frac{x_1 x_2}{y_1 y_2}$ is equal to

a) $\frac{9}{4}$ b) $-\frac{9}{4}$ c) $\frac{81}{16}$ d) $-\frac{81}{16}$

31. The equation of a circle which cuts the three circles
$x^2 + y^2 - 2x - 6y + 14 = 0$
$x^2 + y^2 - x - 4y + 8 = 0$
$x^2 + y^2 + 2x - 6y + 9 = 0$
orthogonally, is

a) $x^2 + y^2 - 2x - 4y + 1 = 0$
b) $x^2 + y^2 + 2x + 4y + 1 = 0$
c) $x^2 + y^2 - 2x + 4y + 1 = 0$
d) $x^2 + y^2 - 2x - 4y - 1 = 0$

32. The length of the common chord of the ellipse $\frac{(x-1)^2}{9} + \frac{(y-2)^2}{4} = 1$ and the circle $(x-1)^2 + (y-2)^2 = 1$ is

a) 2 b) $\sqrt{3}$ c) 4 d) None of these

33. The mirror image of the directrix of the parabola $y^2 = 4(x+1)$ in the line mirror $x + 2y = 3$, is

a) $x = -2$ b) $4y - 3x = 16$ c) $x - 3y = 0$ d) $x + y = 0$

34. The line $x = at^2$ meets the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the real points, if

a) $|t| < 2$ b) $|t| \leq 1$ c) $|t| > 1$ d) None of these

35. The length of the latusrectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$, is

a) $\frac{2a^2}{b}$ b) $\frac{2b^2}{a}$ c) $\frac{b^2}{a}$ d) $\frac{a^2}{b}$

36. The condition that the chord $x \cos\alpha = 0 + y \sin\alpha - p = 0$ of $x^2 + y^2 - a^2 = 0$ may subtend a right angle at the centre of the circle is

a) $a^2 = 2p^2$ b) $p^2 = 2a^2$ c) $a = 2p$ d) $p = 2a$

37. Given that circle $x^2 + y^2 - 2x + 6y + 6 = 0$ and $x^2 + y^2 - 5x + 6y + 15 = 0$ touch, the equation to their common tangent is

a) $x = 3$ b) $y = 6$ c) $7x - 12y - 21 = 0$ d) $7x + 12y + 21 = 0$

38. The number of common tangents of the circles $x^2 + y^2 - 2x - 1 = 0$ and $x^2 + y^2 - 2y - 7 = 0$ is

a) 1 b) 2 c) 3 d) 4

39. A ray of light incident at the point $(-2, -1)$ gets reflected from the tangent at $(0, -1)$ to the circle $x^2 + y^2 = 1$. The reflected ray touches the circle. The equation of the line along which the incident ray moved is

a) $4x - 3y + 11 = 0$ b) $4x + 3y + 11 = 0$ c) $3x + 4y + 11 = 0$ d) None of these

40. If the points $A(2,5)$ and B are symmetrical about the tangent to the circle $x^2 + y^2 - 4x + 4y = 0$ at the origin, then the coordinates of B are

a) $(5, -2)$ b) $(1,5)$ c) $(5,2)$ d) None of these

41. A rectangular hyperbola whose centre is C is cut by any circle of radius r in four points P, Q, R and S. Then, $CP^2 + CQ^2 + CR^2 + CS^2 =$

a) r^2 b) $2r^2$ c) $3r^2$ d) $4r^2$

42. If PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that OPQ is an equilateral triangle, O being the centre of the hyperbola. Then, the eccentricity e of the hyperbola satisfies

a) $1 < e < \frac{2}{\sqrt{3}}$ b) $e = \frac{2}{\sqrt{3}}$ c) $e = \frac{\sqrt{3}}{2}$ d) $e > \frac{2}{\sqrt{3}}$

43. If e and e_1, are the eccentricities of the hyperbolas $xy = c^2$ and $x^2 - y^2 = c^2$, then $e^2 + e_1^2$ is equal to
a) 1 b) 4 c) 6 d) 5

44. If e and e_1 are the eccentricities of hyperbolas $xy = c^2$ and $x^2 - y^2 = c^2$, then $e^2 + e_1^2$ is
a) 1 b) 4 c) 6 d) 8

45. The eccentricity of the hyperbola in the standard form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, passing through (3, 0) and (3,$\sqrt{2}$, 2) is
a) $\frac{13}{3}$ b) $\sqrt{13}$ c) $\sqrt{3}$ d) $\frac{\sqrt{13}}{3}$

46. Which of the following is a point on the common chord of the circles $x^2 + y^2 + 2x - 3y + 6 = 0$ and$x^2 + y^2 + x - 8y - 13 = 0$?
a) $(1, -2)$ b) $(1, 4)$ c) $(1, 2)$ d) $(1, -4)$

47. If the chord of contact of tangents drawn from a point P to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ subtends a right-angle at its centre, then P lies on
a) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{a^2} + \frac{1}{b^2}$ b) $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \left(\frac{1}{a} + \frac{1}{b}\right)^2$ c) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{a^4} + \frac{1}{b^4}$ d) $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{a^2} + \frac{1}{b^2}$

48. The locus of a point which moves such that the difference of its distances from two fixed points is always a constant, is
a) a circle b) a straight line c) a hyperbola d) an ellipse

49. Eccentricity of the ellipse $x^2 + 2y^2 - 2x + 3y + 2 = 0$ is
a) $\frac{1}{\sqrt{2}}$ b) $\frac{1}{2}$ c) $\frac{1}{2\sqrt{2}}$ d) $\frac{1}{\sqrt{3}}$

50. If e is the eccentricity of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and θ be the angle between the asymptotes, then $\sec\frac{\theta}{2}$ equals
a) e^2 b) $\frac{1}{e}$ c) $2e$ d) e

51. If $P(-3,2)$ is one end of the focal chord PQ of the parabola $y^2 + 4x + 4y = 0$, then the slope of the normal at Q is
a) $-1/2$ b) 2 c) $1/2$ d) -2

52. The equation of the circumcircle of the triangle formed by the lines $y + \sqrt{3}\,x = 6, y - \sqrt{3}\,x = 6$ and $y = 0$ is
a) $x^2 + y^2 - 4\,y = 0$
b) $x^2 + y^2 + 4\,x = 0$
c) $x^2 + y^2 - 4\,y - 12 = 0$
d) $x^2 + y^2 + 4\,x = 12$

53. The centre of the circle $r^2 - 4r(\cos\theta + \sin\theta) - 4 = 0$ in Cartesian coordinates is
a) $(1, 1)$ b) $(-1, -1)$ c) $(2, 2)$ d) $(-2, -2)$

54. The locus of the middle of chords of length 4 of the circle $x^2 + y^2 = 16$ is
a) A straight line b) A circle of radius 2 c) A circle of radius $2\sqrt{3}$ d) An ellipse

55. The normal at P to a hyperbola of eccentricity e, intersects its transverse and conjugate axes at L and M respectively. If locus of the mid point of LM is hyperbola, then eccentricity of the hyperbola is
a) $\left(\frac{e+1}{e-1}\right)$ b) $\frac{e}{\sqrt{(e^2-1)}}$ c) e d) None of these

56. If the chords of the rectangular hyperbola $x^2 - y^2 = a^2$ touch the parabola $y^2 = 4ax$, then the locus of their mid-points is
a) $x^2(y-a) = y^3$ b) $y^2(x-a) = x^3$ c) $x(y^2-a) = y$ d) $y(x^2-a) = x$

57. If the tangent at point P on the circle $x^2 + y^2 + 6x + 6y - 2 = 0$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then length PQ
a) 4 b) $2\sqrt{5}$ c) 5 d) $3\sqrt{5}$

58. An ellipse is described by using an endless string which is passed over two pins. If the axes are 6 cm and 4

cm, the necessary length of the string and the distance between the pins respectively in cms. are

a) $6, 2\sqrt{5}$ b) $6, \sqrt{5}$ c) $4, 2\sqrt{5}$ d) None of these

59. The slope of tangents drawn form a point $(4, 10)$ to the parabola $y^2 = 9x$ are

a) $\frac{1}{4}, \frac{3}{4}$ b) $\frac{1}{4}, \frac{9}{4}$ c) $\frac{1}{4}, \frac{1}{3}$ d) None of these

60. The area of the triangle formed by the tangents from the point (4,3) to the circle $x^2 + y^2 = 9$ and the line joining their points of contact, is

a) $\frac{25}{192}$ sq. units b) $\frac{192}{25}$ sq. units c) $\frac{384}{25}$ sq. units d) None of these

61. The value of m, for which the line $y = mx + 2$ becomes a tangent to the conic $4x^2 - 9y^2 = 36$ are

a) $\pm\frac{2}{3}$ b) $\pm\frac{2\sqrt{2}}{3}$ c) $\pm\frac{8}{9}$ d) $\pm\frac{4\sqrt{2}}{3}$

62. If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line. $5x - 2y + 6 = 0$ at a point Q on the y-axis, then the length of PQ is

a) 4 b) $2\sqrt{5}$ c) 5 d) $3\sqrt{5}$

63. Consider a family of circles, which are passing through the point $(-1, 1)$ and are tangent to x-axis. If (h, k) are the coordinates of the centre of the circles, then the set of values of k is given by the interval

a) $0 < k < \frac{1}{2}$ b) $k \geq \frac{1}{2}$ c) $-\frac{1}{2} \leq k \leq \frac{1}{2}$ d) $k \leq \frac{1}{2}$

64. The equation of the circle passing through the point (1, 1) and through the points of intersection of the circles $x^2 + y^2 = 6$ amnd $x^2 + y^2 - 6y + 8 = 0$ is

a) $x^2 + y^2 + 3y - 13 = 0$ b) $x^2 + y^2 - 3y + 1 = 0$
c) $x^2 + y^2 - 3x + 1 = 0$ d) $5x^2 + 5y^2 + 6y + 16 = 0$

65. The number of distinct normal that can be drawn from (11/4,1/4) to the parabola $y^2 = 4x$, is

a) 3 b) 2 c) 1 d) 4

66. For the hyperbola $\frac{x^2}{\cos^2 \alpha} - \frac{y^2}{\sin^2 \alpha} = 1$, which of the following remains constant when α varies?

a) Eccentricity b) Directrix c) Abscissae of vertices d) Abscissae of foci

67. The equation of the circumcircle of the triangle formed by the lines $x = 0, y = 0, 2x + 3y = 5$, is

a) $6(x^2 + y^2) + 5(3x - 2y) = 0$
b) $x^2 + y^2 - 2x - 3y + 5 = 0$
c) $x^2 + y^2 + 2x - 3y - 5 = 0$
d) $6(x^2 + y^2) - 5(3x + 2y) = 0$

68. If t_1 and t_2 be the parameters of the end points of a focal chord for the parabola $y^2 = 4ax$, then which one is true?

a) $t_1t_2 = 1$ b) $\frac{t_1}{t_2} = 1$ c) $t_1t_2 = -1$ d) $t_1 + t_2 = -1$

69. The two circles
$x^2 + y^2 - 2x + 22y + 5 = 0$ and
$x^2 + y^2 + 14x + 6y + k = 0$ intersect orthogonally provided k is equal to

a) 47 b) -47 c) 49 d) -49

70. The ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the straight line $y = mx + c$ intersect in real points only if

a) $a^2m^2 < c^2 - b^2$ b) $a^2m^2 > c^2 - b^2$ c) $a^2m^2 \geq c^2 - b^2$ d) $c \geq b$

71. If four points to be taken on a rectangular hyperbola such that the chord joining any two is perpendicular to the chord joining the other two and if $\alpha, \beta, \gamma, \delta$ be the inclination to either asymptote of the straight line joining these points to the centre. Then, $\tan\alpha \tan\beta \tan\gamma \tan\delta$ is equal to

a) 1 b) 0 c) 2 d) 3

72. If the distance between the foci and the distance between the directrices of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are in the ratio 3 : 2, then a : b is

a) $\sqrt{2} : 1$ b) $\sqrt{3} : \sqrt{2}$ c) $1 : 2$ d) $2 : 1$

73. If m_1 and m_2 are the slopes of tangents to the circle $x^2+y^2=4$ from the point (3, 2), then m_1-m_2 is equal to

a) $\frac{5}{12}$ b) $\frac{12}{5}$ c) $\frac{3}{2}$ d) 0

74. The length of the axes of the conic $9x^2+4y^2-6x+4y+1=0$, are

a) $\frac{1}{2},9$ b) $3,\frac{2}{5}$ c) $1,\frac{2}{3}$ d) 3,2

75. For different values of α, the locus of the point of intersection of the two straight lines $\sqrt{3}x-y-4\sqrt{3}\alpha=0$ and $\sqrt{3}\alpha x+\alpha y-4\sqrt{3}=0$ is

a) a hyperbola with eccentricity 2

b) an ellipse with eccentricity $\sqrt{\frac{2}{3}}$

c) an hyperbola with eccentricity $\sqrt{\frac{19}{16}}$

d) an ellipse with eccentricity $\frac{3}{4}$

76. If the area of the circle $4x^2+4y^2-8x+16y+k=0$ is 9π sq unit, then the4 value of k is

a) 4 b) 16 c) -16 d) ±16

77. $ABCD$ is a square whose side is a. The equation of the circle circumscribing the square, taking AB and AD as axes of reference, is

a) $x^2+y^2+ax+ay=0$

b) $x^2+y^2+ax-ay=0$

c) $x^2+y^2-ax-ay=0$

d) $x^2+y^2-ax+ay=0$

78. If the circle $x^2+y^2+2gx+2fy+c=0$ bisects the circumference of the circle $x^2+y^2+2g'x+2f'y+c'=0$, then

a) $2\,g(g-g')+2\,f(f-f')=c-c'$

b) $2\,g'(g-g')+2\,f'(f-f')=c'-c$

c) $2\,g'(g-g')+2\,f'(f-f')=c-c'$

d) $2\,g(g-g')+2\,f(f-f')=c'-c$

79. If the parabolas $y^2=4x$ and $x^2=32y$ intersect at (16,8) at an angle θ, then θ is equal to

a) $\tan^{-1}(3/5)$ b) $\tan^{-1}(4/5)$ c) π d) $\pi/2$

80. The equation of the circle, which cuts orthogonally each of three circles

$x^2+y^2-2x+3y-7=0$,

$x^2+y^2+5x-5y+9=0$

and $x^2+y^2+7x-9y+29=0$

a) $x^2+y^2-16x-18y-4=0$

b) $x^2+y^2=a^2$

c) $x^2+y^2-16x=0$

d) $y^2-x^2+2x=0$

81. The angle between the tangents drawn from the origin to the parabola $y^2=4\,a(x-a)$, is

a) 90° b) 30° c) $\tan^{-1}(1/2)$ d) 45°

82. If for the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, y-axis is the minor axis and the length of the latusrectum is one half of the length of its minor axis, then its eccentricity is

a) $\frac{1}{\sqrt{2}}$ b) $\frac{1}{2}$ c) $\frac{\sqrt{3}}{2}$ d) $\frac{3}{4}$

83. The coordinates of the centre of the circle which intersects circles $x^2+y^2+4x+7=0$, $2x^2+2y^2+3x+5y+9=0$ and $x^2+y^2+y=0$ orthogonally are

a) $(-2,1)$ b) $(-2,-1)$ c) $(2,-1)$ d) $(2,1)$

84. Equation $ax^2+2hxy+by^2+2gx+2fy+c=0$

$(abc+2fgh-af^2-bg^2-ch^2\neq0)$ represents a parabola, if

a) $h^2=ab$ b) $h^2>ab$ c) $h^2<ab$ d) None of these

85. The ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and the hyperbola $\frac{x^2}{25}-\frac{y^2}{16}=1$ have in common

a) centre only

b) Centre, foci and directrices

c) Centre, foci and vertices | d) Centre and vertices only

86. The eccentricity of the hyperbola $\frac{x^2}{16} - \frac{y^2}{25} = 1$ is

a) $\frac{3}{4}$ b) $\frac{3}{5}$ c) $\frac{\sqrt{41}}{4}$ d) $\frac{\sqrt{41}}{5}$

87. One equation of common tangent to ellipse $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2$ is

a) $2y = \sqrt{3}bx + ab$ b) $y = 2\sqrt{3}\frac{b}{a}x + 2b$

c) No common tangent d) $ay = \sqrt{3}bx + 2ab$

88. If $lx + my + n = 0$ is a tangent to the rectangular hyperbola $xy = c^2$, then

a) $l < m < 0$ b) $l > 0, m < 0$ c) $l < 0, m > 0$ d) None of these

89. The normals at three points P, Q, R of the parabola $y^2 = 4\,ax$ meet in (h, k). The centroid of triangle PQR lies on

a) $x = 0$ b) $y = 0$ c) $x = -a$ d) $y = a$

90. If the point $P(4, -2)$ is the one end of the focal chord PQ of the parabola $y^2 = x$, then the slope of the tangent at Q is

a) $-1/4$ b) $1/4$ c) 4 d) -4

91. Equation of normal to the parabola $y^2 = 4x$ which passes through (3,0) is

a) $x + y = 3$ b) $x + y + 3 = 0$ c) $x - 2y = 3$ d) None of these

92. Let C be the centre of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. If the tangent at any point on the ellipse cuts the coordinate axes in P and Q respectively, then $\frac{a^2}{CP^2} + \frac{b^2}{CQ^2} =$

a) 1 b) 2 c) 3 d) 4

93. The equation of the circle having $x - y - 2 = 0$ and $x - y + 2 = 0$ as two tangents and $x - y = 0$ as a diameter is

a) $x^2 + y^2 + 2x - 2y + 1 = 0$ b) $x^2 + y^2 - 2x + 2y - 1 = 0$

c) $x^2 + y^2 = 2$ d) $x^2 + y^2 = 1$

94. If $(-3, 2)$ lies on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ which is concentric with the circle $x^2 + y^2 + 6x + 8y - 5 = 0$, then c is equal to

a) 11 b) -11 c) 24 d) 100

95. The equation of the circumcircle of the triangle formed by the lines $x = 0, y = 0, 2x + 3y = 5$ is

a) $6(x^2 + y^2) + 5(3x - 2y) = 0$ b) $x^2 + y^2 - 2x - 3y + 5 = 0$

c) $x^2 + y^2 + 2x - 3y - 5 = 0$ d) $6(x^2 + y^2) - 5(3x + 2y) = 0$

96. Circles are drawn through the point (2,0) to cut intercepts of length 5 units on the x-axis. If their centres lie in the first quadrant, then their equation is

a) $x^2 + y^2 - 9\,x + 2\,ky + 14 = 0$

b) $3\,x^2 + 3\,y^2 + 27\,x - 2\,ky + 42 = 0$

c) $x^2 + y^2 - 9\,x - 2\,ky + 14 = 0$

d) $x^2 + y^2 - 2\,kx - 9\,y + 14 = 0$

97. The number of points with integral coordinates with lie in the interior of the region common to the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 4x$ is

a) 8 b) 10 c) 16 d) None of these

98. If the chords of contact of the tangents from a point on the circle $x^2 + y^2 = a^2$ to the circle $x^2 + y^2 = b^2$ touch the circle $x^2 + y^2 = c^2$, then the roots of the equation $ax^2 + 2bx + c = 0$, are

a) Imaginary b) Real and equal c) Real and unequal d) Rational

99. If the vertex and focus of a parabola are (3,3) and (−3,3) respectively, then its equation is

a) $x^2 + 6\,x - 24\,y + 63 = 0$

b) $x^2 - 6\,x + 24\,y - 63 = 0$

c) $y^2 - 6\,y + 24\,x - 63 = 0$

d) $y^2 + 6\,y - 24\,x + 63 = 0$

100. If the length of the major axis of an ellipse is three times the length of its minor axis, its eccentricity, is

a) $\frac{1}{3}$ b) $\frac{1}{\sqrt{3}}$ c) $\frac{1}{\sqrt{2}}$ d) $\frac{2\sqrt{2}}{3}$

101. The number of integral values of $'a'$ for which the radius of the circle $x^2+y^2+ax+(1-a)y+5=0$ cannot exceed 5, is

a) 14 b) 18 c) 16 d) None of these

102. The number of common tangents to the circles $x^2+y^2-2x-4y+1=0$ and $x^2+y^2-12x-16y+91=0$, is

a) 1 b) 2 c) 3 d) 4

103. If two tangents drawn from a point P to the parabola $y^2=4x$ are at right angles, then the locus of P is

a) $x=1$ b) $2x+1=0$ c) $x=-1$ d) $2x-1=0$

104. A point P moves in such a way that the ratio of its distance from two coplanar points is always a fixed number($\neq 1$). Then, its locus is a

a) Parabola b) Circle c) Hyperbola d) Pair of straight lines

105. Two circles, each of radius 5, have a common tangent at (1,1) whose equation is $3x+4y-7=0$. Then their centres are

a) $(4,-5),(-2,3)$ b) $(4,-3),(-2,5)$ c) $(4,5),(-2,-3)$ d) None of these

106. The tangent at (1, 7) to the curve $x^2=y-6$ touches the circle $x^2+y^2+16x+12y+c=0$ at

a) (6, 7) b) (−6, 7) c) (6, −7) d) (−6, −7)

107. If the latusrectum subtends a right angle at the centre of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, then its eccentricity is

a) $\frac{\sqrt{13}}{2}$ b) $\frac{\sqrt{5}-1}{2}$ c) $\frac{\sqrt{5}+1}{2}$ d) $\frac{\sqrt{3}+1}{2}$

108. If e_1 is the eccentricity of the ellipse $\frac{x^2}{16}+\frac{y^2}{7}=1$ and e_2 is the eccentricity of the hyperbola $\frac{x^2}{9}-\frac{y^2}{7}=1$, then e_1+e_2 is equal to

a) $\frac{16}{7}$ b) $\frac{25}{4}$ c) $\frac{25}{12}$ d) $\frac{16}{9}$

109. If $y=mx-\frac{(a^2-b^2)m}{\sqrt{a^2+b^2m^2}}$ is normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ for all values of m belonging to

a) (0, 1) b) $(0,\infty)$ c) R d) None of these

110. The area of the quadrilateral formed by the tangents at the end points of latus rectum to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$is

a) 27/4 sq units b) 9 sq units c) 27/2 sq units d) 27 sq units

111. If the tangent at any point P on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ meets the lines $bx-ay=0$ and $bx+ay=0$ in the points Q and R, then $CQ\cdot CR=$

a) a^2b^2 b) a^2-b^2 c) a^2+b^2 d) None of these

112. From a point T a tangent is drawn at the point $P(16,16)$ of the parabola $y^2=16\,x$. If S be the focus of the parabola, then $\angle TPS$ can be equal to

a) $\tan^{-1}(3/4)$ b) $\frac{1}{2}\tan^{-1}(1/2)$ c) $\tan^{-1}(1/2)$ d) $\pi/4$

113. The number of common tangents to two circles $x^2+y^2=4$ and $x^2+y^2-8x+12=0$ is

a) 1 b) 2 c) 5 d) 3

114. The circle $x^2+y^2+2gx+2fy+c=0$ cuts the parabola $x^2=4ay$ at points $(x_i,y_i), i=1,2,3,4$, then

a) $\sum y_i=0$ b) $\sum y_i=-4(f+2a)$ c) $\sum x_i=-4(g+2a)$ d) $\sum x_i=-2(g+2a)$

115. A straight rod of length 9 units with its ends A,B always on x and y axes respectively. then, the locus of the centroid of $\Delta\,OAB$, is

a) $x^2+y^2=3$ b) $x^2+y^2=9$ c) $x^2+y^2=1$ d) $x^2+y^2=81$

116. If a focal chord of the parabola $y^2=ax$ is $2x-y-8=0$, then the equation of the directrix is

a) $x+4=0$ b) $x-4=0$ c) $y-4=0$ d) $y+4=0$

117. The locus of the point of intersection of the tangents to the circle $x=r\cos\theta, y=r\sin\theta$ at points whose parametric angles differ by a right angle is

a) $x^2+y^2=\frac{r^2}{2}$ b) $x^2+y^2=2\,r^2$ c) $x^2+y^2=4\,r^2$ d) None of these

118. If $P(1,3)$ and $Q(1,1)$ are two points on the parabola $y^2=4x$ such that a point dividing PQ internally in the ratio $1:\lambda$ is an interior point of the parabola, then λ lies in the interval

a) $(0,1)$ b) $(-3/5,1)$ c) $(1/2,3/5)$ d) None of these

119. The value of c, for which the line $y=2x+c$ is a tangent to the circle $x^2+y^2=16$, is

a) $-16\sqrt{5}$ b) $4\sqrt{5}$ c) $16\sqrt{5}$ d) 20

120. How many common tangents can be drawn to the following circles $x^2+y^2=6x$ and $x^2+y^2+6x+2y+1=0$?

a) 4 b) 3 c) 2 d) 1

121. The equation of the unit circle concentric with $x^2+y^2.8x+4y-8=0$ is

a) $x^2+y^2-8\,x+4\,y-8=0$

b) $x^2+y^2-8\,x+4\,y+8=0$

c) $x^2+y^2-8\,x+4\,y-28=0$

d) $x^2+y^2-8\,x+4\,y+19=0$

122. If $(9a,6a)$ is a point bounded in region formed by parabola $y^2=16x$ and $x=9$, then

a) $a\in(0,1)$ b) $a<\frac{1}{4}$ c) $a<1$ d) $0<a<4$

123. If the coordinates of the vertices of an ellipse are $(-6,1)$ and $(4,1)$ and the equation of a focal chord passing through the focus on the right side of the centre is $2x-y-5=0$. The equation of the ellipse is

a) $\frac{(x+1)^2}{25}+\frac{(y+1)^2}{16}=1$

b) $\frac{(x+1)^2}{25}+\frac{(y-1)^2}{16}=1$

c) $\frac{(x-1)^2}{25}+\frac{(y+1)^2}{16}=1$

d) None of these

124. The radius of the circle $r=\sqrt{3}\sin\theta+\cos\theta$ is

a) 1 b) 2 c) 3 d) 4

125. If the latusrectum of the hyperbola $\frac{x^2}{16}-\frac{y^2}{b^2}=1$ is $\frac{9}{2}$, then its eccentricity is

a) 4/5 b) 5/4 c) 3/4 d) 4/3

126. S and T are the foci of an ellipse and B is end point of the minor axis . If STB is an equilateral triangle, the eccentricity of the ellipse is

a) $\frac{1}{4}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$

127. The eccentricity of the hyperbola can never be equal to

a) $\sqrt{\frac{9}{5}}$ b) $2\sqrt{\frac{1}{9}}$ c) $3\sqrt{\frac{1}{8}}$ d) 2

128. If the tangent at (α,β) to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ cuts the auxiliary circle at points whose ordinates are y_1 and y_2, then $\frac{1}{y_1}+\frac{1}{y_2}=$

a) $\frac{1}{\alpha}$ b) $\frac{2}{\alpha}$ c) $\frac{1}{\beta}$ d) $\frac{2}{\beta}$

129. The eccentricity of the hyperbola $\frac{\sqrt{1999}}{3}(x^2-y^2)=1$, is

a) $\sqrt{2}$ b) 2 c) $2\sqrt{2}$ d) $\sqrt{3}$

130. If the line $3x-4y-k=0, (k>0)$ touches the circle $x^2+y^2-4x-8y-5=0$ at (a,b), then $k+a+b$

is equal to

a) 20 b) 22 c) -30 d) -28

131. The length of the latusrectum of the parabola whose focus is $(3,3)$ and directrix is $3x-4y-2=0$, is

a) 2 b) 1 c) 4 d) None of these

132. The equation of the tangent from the point $(0, 1)$ to the circle $x^2+y^2-2x-6y+6=0$, is

a) $y-1=0$ b) $4x+3y+3=0$ c) $4x-3y-3=0$ d) $y+1=0$

133. The circles $x^2+y^2+6x+6y=0$ and $x^2+y^2-12x-12y=0$

a) Cut orthogonally b) Touch each other internally c) Intersect two points d) Touch each other externally

134. If tangents at A and B on the parabola $y^2=4ax$ intersect at point C, then ordinates of A, C and B are

a) Always in AP b) Always in GP c) Always in HP d) None of these

135. The equations of the asymptotes of the hyperbola $2x^2+5xy+2y^2-11x-7y-4=0$ are

a) $2x^2+5xy+2y^2-11x-7y-5=0$ b) $2x^2+4xy+2y^2-7x-11y+5=0$ c) $2x^2+5xy+2y^2-11x-7y+5=0$ d) None of the above

136. The circle $x^2+y^2+2g_1x-a^2=0$ and $x^2+y^2+2g_2x-a^2=0$ cut each other orthogonally. If p_1, p_2 are perpendicular from $(0, a)$ and $(0, -a)$ on a common tangent of these circles, then p_1p_2 is equal to

a) $\frac{a^2}{2}$ b) a^2 c) $2a^2$ d) a^2+2

137. If $(a\cos\alpha, b\sin\alpha)$, $(a\cos\beta, b\sin\beta)$ are the end points of a focal chord of an ellipse $b^2x^2+a^2y^2=a^2b^2$, then which of the following is correct?

a) $e=\frac{\sin\alpha-\sin\beta}{\sin(\alpha-\beta)}$ b) $e=\frac{\cos\left(\frac{\alpha-\beta}{2}\right)}{\cos\left(\frac{\alpha+\beta}{2}\right)}$

c) $\frac{e-1}{e+1}=\tan\frac{\alpha}{2}\tan\frac{\beta}{2}$ d) None of these

138. A line meets the coordinates axes in A and B. A circle is circumscribed about the ΔOAB. The distances from the points A and B of the side AB to the tangent at O are equal to m and n respectively. Then, the diameter of the circle is

a) $m(m+n)$ b) $n(m+n)$ c) $m-n$ d) None of these

139. A line L passing through the focus of the parabola $(y-2)^2=4(x+1)$ intersects the parabola in two distinct points. If m be the slope of the line L, then

a) $m\in(-1,1)$

b) $m\in(-\infty,-1)\cup(1,\infty)$

c) $m\in(-\infty,0)\cup(0,\infty)$

d) None of these

140. If $a>2b>0$, then the positive value of m fro which $y=mx-b\sqrt{1+m^2}$ is a common tangent to $x^2+y^2=b^2$ and $(x-a)^2+y^2=b^2$, is

a) $\frac{2b}{\sqrt{a^2-4b^2}}$ b) $\frac{\sqrt{a^2-4b^2}}{2b}$ c) $\frac{2b}{a-2b}$ d) $\frac{b}{a-2b}$

141. For an equilateral triangle the centre is the origin and the length of altitude is a. Then, the equation of the circumcircle is

a) $x^2+y^2=a^2$ b) $3x^2+3y^2=2a^2$ c) $x^2+y^2=4a^2$ d) $9x^2+9y^2=4a^2$

142. the tangents drawn from the ends of latusrectum of $y^2=12x$ meets at

a) Directrix b) Vertex c) Focus d) None of these

143. If B and B' are the ends of minor axis and S and S' are the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$, then area of the rhombus $SBS'B'$ will be

a) 12. sq. units b) 48 sq. units c) 24 sq. units d) 36 sq. units

144. A point P moves so that sum of its distances from $(-ae, 0)$ and $(ae, 0)$ is $2a$. Then, the locus of P is

a) $\frac{x^2}{a^2}-\frac{x^2}{a^2(1-e^2)}=1$ b) $\frac{x^2}{a^2}+\frac{y^2}{a^2(1-e^2)}=1$ c) $\frac{x^2}{a^2}+\frac{y^2}{a^2(1+e^2)}=1$ d) $\frac{x^2}{a^2}-\frac{y^2}{a^2(1+e^2)}=1$

145. Tangents are drawn from the point on the line $x-y-5=0$ to $x^2+4y^2=4$, then all the chords of contact pass through a fixed point, whose coordinates are

a) $\left(\frac{1}{5},\frac{4}{5}\right)$ b) $\left(\frac{4}{5},\frac{1}{5}\right)$ c) $\left(-\frac{4}{5},-\frac{1}{5}\right)$ d) $\left(\frac{4}{5},-\frac{1}{5}\right)$

146. If the chord $y=mx+c$ subtends a right angle at the vertex of the parabola $y^2=4\,ax$, then the value of c is

a) -4am b) 4am c) -2am d) 2am

147. If the chord of contact of tangents drawn from a point on the circle $x^2+y^2=a^2$ to the circle $x^2+y^2=b^2$ touches the circle $x^2+y^2=c^2$, then a,b,c are in

a) AP b) GP c) HP d) None of these

148. The length of the subnormal to the parabola $y^2=4\,ax$ at any point is equal to

a) $a\sqrt{2}$ b) $2\sqrt{2}\,a$ c) $a/\sqrt{2}$ d) 2a

149. If P is a point such that the ratio of the tangents from P to the circles $x^2+y^2+2x-4y-20=0$ and $x^2+y^2-4x+2y-44=0$ is $2:3$, then the locus of P is a circle with centre

a) $(7,-8)$ b) $(-7,8)$ c) $(7,8)$ d) $(-7,-8)$

150. The intercepts on the line $y=x$ by the circle $x^2+y^2-2x=0$isAB. Equation of the circle on AB as a diameter is

a) $x^2+y^2-x-y=0$ b) $x^2+y^2-x+y=0$
c) $x^2+y^2+x+y=0$ d) $x^2+y^2+x-y=0$

151. The equation of the normal at the point $(a\sec\theta, b\tan\theta)$ of the curve $b^2x^2-a^2y^2=a^2b^2$ is

a) $\frac{ax}{\cos\theta}+\frac{by}{\sin\theta}=a^2+b^2$ b) $\frac{ax}{\tan\theta}+\frac{by}{\sec\theta}=a^2+b^2$
c) $\frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2+b^2$ d) $\frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2-b^2$

152. The equation of normal to the circle $2x^2+2y^2-2x-5y+3=0$ at $(1, 1)$ is

a) $2x+y=3$ b) $x-2y=3$ c) $x+2y=3$ d) None of these

153. The product of perpendicular distances from any point on the hyperbola $9x^2-16y^2=144$ to its asymptotes is

a) $\frac{25}{12}$ b) $\frac{144}{25}$ c) $\frac{144}{7}$ d) $\frac{25}{144}$

154. The two parabolas $y^2=4x$ and $x^2=4\,y$ intersect at a point P, whose abscissae is not zero, such that

a) They both touch each other at P
b) They cut at right angles at P
c) The tangents to each curve at P make complementary angles with the x-axis
d) None of these

155. If the four points of the intersection of the lines $2x-y+11=0$ and $x-2y+3=0$ with the axes lie on a circle, then the coordinates of the centre of the circle are

a) $(7/5, 5/2)$ b) $(7/4, 5/4)$ c) $(-7/4, 5/4)$ d) $(7/4,-5/4)$

156. The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and having its centre $(0, 3)$ is

a) 4 b) $\frac{3}{7}$ c) $\sqrt{12}$ d) $\frac{7}{2}$

157. The curve with parametric equations $x=\alpha+5\cos\theta,\ y=\beta+4\sin\theta$ (where θ is parameter) is

a) A parabola b) An ellipse c) A hyperbola d) None of these

158. If p and q are the segments of a focal chord of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then

a) $a^2(p+q)=2bpq$ b) $b^2(p+q)=2apq$ c) $a(p+q)=2b^2pq$ d) $b(p+q)=2a^2pq$

159. The curve with parametric equation $x=e^t+e^{-t}\ y=e^t-e^{-t}$ and is

a) A circle b) An ellipse c) A hyperbola d) A parabola

160. The equation of the circle which passes through the points of intersection of the circles $x^2+y^2-6x=0$

and $x^2+y^2-6y=0$ and has its centre at $\left(\frac{3}{2},\frac{3}{2}\right)$, is

a) $x^2+y^2+3x+3y+9=0$
b) $x^2+y^2+3x+3y=0$
c) $x^2+y^2-3x-3y=0$
d) $x^2+y^2-3x-3y+9=0$

161. If $(1,a),(b,2)$ are conjugate points with respect to the circle $x^2+y^2=25$, then $4a+2b$ is equal to

a) 25
b) 50
c) 100
d) 150

162. The equation $(10x-5)^2+(10y-4)^2=(3x+4y-1)^2$ represents

a) A circle
b) A pair of straight lines
c) An ellipse
d) A parabola

163. The difference in focal distances of any point on the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ is

a) 8
b) 9
c) 0
d) 6

164. The chord of contact of tangents drawn from any point on $x-1=0$ to $y^2-6y+4x+9=0$ passes through the point

a) $(-1,3)$
b) $(1,-3)$
c) $(3,-1)$
d) $(3,1)$

165. The equation of the circle passing through (4, 5) and having the centre (2, 2), is

a) $x^2+y^2+4x+4y-5=0$
b) $x^2+y^2-4x-4y-5=0$
c) $x^2+y^2-4x=13$
d) $x^2+y^2-4x-4y+5=0$

166. The product of lengths of perpendicular from any point on the hyperbola $x^2-y^2=8$ to its asymptotes is

a) 8
b) 6
c) 2
d) 4

167. The foci of an ellipse are $(0,\pm4)$ and the equations for the directrices are $y=\pm9$. The equation for the ellipse is

a) $5x^2+9y^2=4$
b) $2x^2-6y^2=28$
c) $6x^2+3y^2=45$
d) $9x^2+5y^2=180$

168. Tangents at any points on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ cut the axes at A and B respectively. If the rectangle $OAPB$, where O is the origin is completed, then locus of point P is given by

a) $\frac{a^2}{x^2}-\frac{b^2}{y^2}=1$
b) $\frac{a^2}{x^2}+\frac{b^2}{y^2}=1$
c) $\frac{a^2}{y^2}-\frac{b^2}{x^2}=1$
d) None of these

169. Let P be the point (1,0) and Q a point on the locus of $y^2=8x$, The locus of mid point of PQ is

a) $x^2-4y+2=0$
b) $x^2+4y+2=0$
c) $y^2+4x+2=0$
d) $y^2-4x+2=0$

170. The equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents an ellipse if

a) $\Delta=0, h^2<ab$
b) $\Delta\neq0, h^2<ab$
c) $\Delta\neq0, h^2>ab$
d) $\Delta\neq0, h^2=ab$

171. If the lengths of major and semi-minor axes of an ellipse are 4 and $\sqrt{3}$ and their corresponding equations are $y-5=0$ and $x+3=0$, then the equation of the ellipse is

a) $3x^2+4y^2+18x-40y+115=0$
b) $4x^2-3y^2-24x+30y+99=0$
c) $3x^2-4y^2-18x+40y+115=0$
d) $4x^2+3y^2+24x-30y+99=0$

172. The pole of the straight line $9x+y-28=0$ with respect to the circle $2x^2+2y^2-3x+5y-7=0$ is

a) (3,1)
b) (1,3)
c) $(3,-1)$
d) $(-3,1)$

173. The locus of middle points of chords of hyperbola $3x^2-2y^2+4x-6y=0$ parallel to $y=2x$ is

a) $3x-4y=4$
b) $3y-4x+4=0$
c) $4x-3y=3$
d) $3x-4y=2$

174. If the circle $x^2+y^2-10x-14y+24=0$ cuts an intercepts on y-axis of length

a) 5
b) 10
c) 1
d) None of these

175. The locus of a point $P(\alpha,\beta)$ moving under the condition that the line $y=\alpha x+\beta$ is a tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, is

a) A hyperbola
b) A parabola
c) A circle
d) An ellipse

176. If y_1, y_2 and y_3 are the ordinates of the vertices of a triangle inscribed in the parabola $y^2=4ax$, then its area is

a) $\frac{1}{2a}(y_1-y_2)(y_2-y_3)(y_3-y_1)$
b) $\frac{1}{4a}(y_1-y_2)(y_2-y_3)(y_3-y_1)$

c) $\frac{1}{8a}(y_1-y_2)(y_2-y_3)(y_3-y_1)$ d) None of the above

177. A variable tangent to the parabola $y^2=4ax$ meets the parabola $y^2=-4ax$ at P and Q. The locus of the mid-point of PQ is

a) $y^2=-2ax$ b) $y^2=-ax$ c) $y^2-\frac{4}{3}ax$ d) $y^2=-4ax$

178. P is a point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, N is the foot of the perpendicular from P on the transverse axis. The tangent to the hyperbola at P meets the transverse axis at T. If O is the centre of the hyperbola, then $OT\cdot ON$ is equal to

a) e^2 b) a^2 c) b^2 d) b^2/a^2

179. If the eccentricity of the hyperbola $x^2-y^2\sec^2\theta=4$ *is* $\sqrt{3}$ times the eccentricity of the ellipse $x^2\sec^2\theta+y^2=16$, then the value of θ equals

a) $\frac{\pi}{6}$ b) $\frac{3\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

180. If two circles of the same radius r and centres at $(2, 3)$ and $(5, 6)$ respectively cut orthogonally, then the value of r is

a) 3 b) 2 c) 1 d) 5

181. If the circle$x^2+y^2+4x+22y+c=0$ bisects the circumference of the circle$x^2+y^2-2x+8y-d=0$, then $c+d$ is equal to

a) 30 b) 50 c) 40 d) 56

182. If C is the centre of the ellipse $9x^2+16y^2=144$ and S is one focus. The ratio of CS to major axis, is

a) $\sqrt{7}:16$ b) $\sqrt{7}:4$ c) $\sqrt{5}:\sqrt{7}$ d) None of these

183. The angle between the normal to the parabola $y^2=24x$ at points $(6,12)$ and $(6,-12)$, is

a) 30° b) 45° c) 60° d) 90°

184. If the circle $C_1: x^2+y^2=16$ intersects another circle C_2 of radius 5 in such a manner that the common chord is of maximum length and has a slope equal to $3/4$, the coordinates of the centre of C_2 are

a) $(-9/5,12/5),(9/5,-12/5)$

b) $(-9/5,-12/5),(9/5,12/5)$

c) $(12/5,-9/5),(-12/5,9/5)$

d) None of these

185. The normal at the point $(bt_1^2, 2bt_1)$ on a parabola $y^2=4bv$ meets the parabola again int he point $(bt_2^2, 2bt_2)$, then

a) $t_2=-t_1-\frac{2}{t_1}$ b) $t_2=-t_1-\frac{2}{t_1}$ c) $t_2=t_1-\frac{2}{t_1}$ d) $t_2=t_1+\frac{2}{t_1}$

186. Equation of tangents to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$, which are perpendicular to the line $3x+4y=7$, are

a) $4x-3y=\pm6\sqrt{5}$ b) $4x-3y=\pm\sqrt{12}$ c) $4x-3y=\pm\sqrt{2}$ d) $4x-3y=\pm1$

187. Any point on the hyperbola $\frac{(x+1)^2}{16}-\frac{(y-2)^2}{4}=1$ is of the form

a) $(4\sec\theta, 2\tan\theta)$ b) $(4\sec\theta+1, 2\tan\theta-2)$

c) $(4\sec\theta-1, 2\tan\theta-2)$ d) $(4\sec\theta-1, 2\tan\theta+2)$

188. The distances from the foci of point $P(x_1,y_1)$ on the ellipse $\frac{x^2}{9}+\frac{y^2}{25}=1$ are

a) $4\pm\frac{5}{4}y_1$ b) $5\pm\frac{4}{5}x_1$ c) $5\pm\frac{4}{5}y_1$ d) None of these

189. The locus of the points of trisection of the double ordinates of the parabola $y^2=4ax$ is

a) $y^2=ax$ b) $9y^2=4ax$ c) $9y^2=ax$ d) $y^2=9ax$

190. The tangent drawn at any point P to the parabola $y^2=4ax$ meets the directrix at the point K, then the angle which KP subtends at its focus is

a) 30° b) 45° c) 60° d) 90°

191. The set of values of 'c' so that the equation $y=|x|+c$ and $x^2+y^2-8|x|-9=0$ have no solution, is

a) $(-\infty,-3)\cup(3,\infty)$ b) $(-3,3)$

c) $\left(-\infty, 5\sqrt{2}\right) \cup (5\sqrt{2}, \infty)$ d) $(5\sqrt{2}-4, \infty)$

192. The radius of the circle passing through the point (6, 2) and two of whose diameters are $x+y=6$ and $x+2y=4$, is

a) 4 b) 6 c) 20 d) $\sqrt{20}$

193. The coordinates of the point on the circle $x^2+y^2-12x-4y+30=0$, which is farthest from the origin are

a) (9,3) b) (8,5) c) (12,4) d) None of these

194. The points of contact of tangents to the circle $x^2+y^2=25$ which are inclined at an angle of 30° to the x-axis are

a) $(\pm 5/2, \pm 1/2)$ b) $(\pm 1/2, \pm 5/2)$ c) $(\mp 5/2, \mp 1/2)$ d) None of these

195. How many real tangents can be drawn to the ellipse $5x^2+9y^2=32$ form the point $(2,3)$?

a) 2 b) 1 c) 0 d) 3

196. If the line $2x+\sqrt{6}y=2$ touches the hyperbola $x^2-2y^2=4$, then the point of contact is

a) $\left(-2, \sqrt{6}\right)$ b) $(-5, 2\sqrt{6})$ c) $\left(\frac{1}{2}, \frac{1}{\sqrt{6}}\right)$ d) $(4, -\sqrt{6})$

197. The locus of the mid-points of focal chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, is

a) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{x}{a^2}$ b) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{ex}{a^2}$ c) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{x^2}{a^4}$ d) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{ex}{a}$

198. The curve described parametrically by $x=t^2+2t-1, y=3t+5$ represents

a) An ellipse b) A hyperbola c) A parabola d) A circle

199. The number of points on the circle $2(x^2+y^2)=3x$ which are a distance 2 from the point $(-2,1)$ is

a) 2 b) 0 c) 1 d) None of these

200. The number of normal drawn to the parabola $y^2=4x$ from the point (1,0) is

a) 0 b) 1 c) 2 d) 3

201. If tangents at extremities of a focal chord AB of the parabola $y^2=4ax$ intersect at a point C, then $< ACB$ is equal to

a) $\frac{\pi}{4}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{6}$

202. A point moves in a plane so that its distances PA and PB from two fixed points A and B in the plane satisfy the relation $PA-PB=k(k \neq 0)$, then the locus of P is

a) A parabola
b) An ellipse
c) A hyperbola
d) A branch of a hyperbola

203. If OAB is an equilateral triangle inscribed in the parabola $y^2=4ax$ with O as the vertex, then the length of the side of the ΔOAB is

a) $8a\sqrt{3}$ b) $4a\sqrt{3}$ c) $2a\sqrt{3}$ d) $a\sqrt{3}$

204. The equation of the parabola whose vertex is at $(2,-1)$ and focus at $(2,-3)$ is

a) $x^2+4x-8y-12=0$
b) $x^2-4x+8y+12=0$
c) $x^2+8y=12$
d) $x^2-4x+12=0$

205. The locus of the point of intersection of the tangents at the end points of the focal chord of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, (b<a)$ is

a) $x=\pm\frac{a^2}{\sqrt{a^2-b^2}}$ b) $y=\pm\frac{b^2}{\sqrt{a^2-b^2}}$ c) $x=\pm\frac{ab}{\sqrt{a^2-b^2}}$ d) None of these

206. Length of the straight line $x-3y=1$intercepted by the hyperbola $x^2-4y^2=1$ is

a) $\frac{3}{5}\sqrt{10}$ b) $\frac{6}{5}\sqrt{10}$ c) $\frac{5}{3}\sqrt{10}$ d) $\frac{5}{6}\sqrt{10}$

207. The length of latusrectum of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ equals

a) $\frac{a}{e^2-1}$ b) $2a(e^2-1)$ c) $2a^2(e^2-1)$ d) $\frac{e^2-1}{2a}$

208. The number of common tangents to circles $x^2+y^2+2x+8y-23=0$ and $x^2+y^2-4x-10y+9=0$, is

a) 1 b) 3 c) 2 d) None of these

209. The inverse point of $(1,-1)$ with respect to $x^2+y^2=4$ is

a) $(-1,1)$ b) $(-2,2)$ c) $(1,-1)$ d) $(2,-2)$

210. If the area of the circle $4x^2+4y^2-8x+16y+k=0$ is 9π sq unit, then the value of k is

a) 4 units b) 16 units c) -16 units d) ± 16 units

211. $(-6,0),(0,6)$ And $(-7,7)$ are the vertices of $\Delta\, ABC$. The incircle of the triangle has the equation

a) $x^2+y^2-9x-9y+36=0$ b) $x^2+y^2+9x-9y+36=0$

c) $x^2+y^2+9x+9y-36=0$ d) $x^2+y^2+18x-18y+36=0$

212. Minimum distance between the curves $y^2=x-1$ and $x^2=y-1$ is equal to

a) $\frac{3\sqrt{2}}{4}$ b) $\frac{5\sqrt{2}}{4}$ c) $\frac{7\sqrt{2}}{4}$ d) $\frac{\sqrt{2}}{4}$

213. If $x^2+6x+20y-51=0$, then axis of parabola is

a) $x+3=0$ b) $x-3=0$ c) $x=1$ d) $x+1=0$

214. Eccentricity of hyperbola $\frac{x^2}{k}+\frac{y^2}{k^2}=1 (k<0)$ is

a) $\sqrt{1+k}$ b) $\sqrt{1-k}$ c) $\sqrt{1+\frac{1}{k}}$ d) $\sqrt{1-\frac{1}{k}}$

215. The focus of the parabola $y^2-x-2y+2=0$ is

a) $\left(\frac{1}{4},0\right)$ b) $(1,2)$ c) $\left(\frac{5}{4},1\right)$ d) $\left(\frac{3}{4},\frac{5}{2}\right)$

216. The locus of the middle points of the focal chord of the parabola $y^2=4\,ax$ is

a) $y^2=a(x-a)$ b) $y^2 2\,a(x-a)$ c) $y^2=4\,a(x-a)$ d) None of these

217. The conditions that $ax+by+c=0$ is tangent to the parabola $y^2=4ax$, is

a) $a^2=b^2=c^2$ b) $a=b$ c) $b^2=c$ d) $b^2=a$

218. The circle drawn on the line segment joining the foci of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ as diameter cuts the asymptotes at

a) (a,a) b) (b,a) c) $(\pm b,\pm a)$ d) $(\pm a,\pm b)$

219. The coordinates of the focus of the parabola described parametrically by $x=5t^2+2, y+\ 10t\ +\ 4$ are

a) $(7,4)$ b) $(3,4)$ c) $(3,-4)$ d) $(-7,4)$

220. The equation of the common tangent touching the circle $(x-3)^2+y^2=9$ and parabola $y^2=4x$ above the x-axis is

a) $\sqrt{3}y=3x+1$ b) $\sqrt{3}y=-(x+3)$ c) $\sqrt{3}y=x+3$ d) $\sqrt{3}y=-(3x+1)$

221. The equation of the circle having radius 5 and touching the circle $x^2+y^2-2x-4y-20=0$ at (5,5) is

a) $(x^2+y^2)+18\,x+16\,y+120=0$

b) $(x^2+y^2)+18\,x-16\,y+120=0$

c) $(x^2+y^2)-18\,x+16\,y+120=0$

d) $(x^2+y^2)-18\,x-16\,y+120=0$

222. The ends of a line segment are $P(1,3)$ and $Q(1,1)$. R is a point on the line segment PQ such that $PR:QR=1:\lambda$. If R is an interior point of the parabola $y^2=4x$, then

a) $\lambda\in(0,1)$ b) $\lambda\in\left(-\frac{3}{5},1\right)$ c) $\lambda\in\left(\frac{1}{2},\frac{3}{5}\right)$ d) None of these

223. The chord AB of the parabola $y^2=4ax$ cuts the axis of the parabola at C. If $A=(at_1^2,2at_1)$, $B=(at_2^2,2at_2)$ and $AC:AB=1:3$, then

a) $t_2=2t_1$ b) $t_2+2t_1=0$ c) $t_1+2t_2=0$ d) None of these

224. The eccentricity of an ellipse whose pair of a conjugate diameter are $y = x$ and $3\,y = -2\,x$ is

a) $2/3$ b) $1/3$ c) $1/\sqrt{3}$ d) None of these

225. The eccentricity of the conic $x^2 - 4\,x + 4y^2 = 12$ is

a) $\frac{\sqrt{3}}{2}$ b) $\frac{2}{\sqrt{3}}$ c) $\sqrt{3}$ d) None of these

226. The equation of the directrix of the parabola $x^2 + 8y - 2x = 7$ is

a) $y = 3$ b) $y = -3$ c) $y = 2$ d) $y = 0$

227. The locus of the centre of the circles which touch both the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = 4ax$ externally has the equation

a) $12(x-a)^2 - 4\,y^2 = 3\,a^2$
b) $9(x-a)^2 - 5\,y^2 = 2\,a^2$
c) $8\,x^2 - 3(y-a)^2 = 9\,a^2$
d) None of these

228. If P_1, P_2, P_3 are the perimeter of the three circles $x^2 + y^2 + 8x - 6y = 0, 4x^2 + 4y^2 - 4x - 12y - 186 = 0$ and $x^2 + y^2 - 6x + 6y - 9 = 0$ respectively, then

a) $P_1 < P_2 < P_3$ b) $P_1 < P_3 < P_2$ c) $P_3 < P_2 < P_1$ d) $P_2 < P_3 < P_1$

229. The angle between the tangents drawn from the point (3,4) to the parabola $y^2 - 2y + 4x = 0$ is

a) $\tan^{-1}(8\sqrt{5}/7)$ b) $\tan^{-1}(12/\sqrt{5})$ c) $\tan^{-1}(\sqrt{5}/7)$ d) None of these

230. If S and S' are two foci of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a < b)$ and $P(x_1, y_1)$ a point on it, then $SP + S'P$ is equal to

a) $2a$ b) $2b$ c) $a + e\,x_1$ d) $b + ey_1$

231. The locus of the mid-points of the chords of the circle $x^2 + y^2 = 16$ which are tangents to the hyperbola $9x^2 - 16y^2 = 144$ is

a) $(x^2 + y^2)^2 = 16x^2 - 9y^2$
b) $(x^2 + y^2)^2 = 9x^2 - 16y^2$
c) $(x^2 - y^2)^2 = 16x^2 - 9y^2$
d) None of these

232. Equation of the circle which of the mirror image of the circle $x^2 + y^2 - 2x = 0$ in the line $x + y = 2$ is

a) $x^2 + y^2 - 2x + 4y + 3 = 0$ b) $2(x^2 + y^2) + x + y + 1 = 0$
c) $x^2 + y^2 - 4x - 2y + 4 = 0$ d) None of the above

233. If the area of the quadrilateral by the tangent from the origin to the circle $x^2 + y^2 + 6x - 10y + c = 0$ and the pair of radii at the points of contract of these tangents to the circle is 8 sq unit, then c is a root of the equation

a) $c^2 - 32c + 64 = 0$ b) $c^2 - 34c + 64 = 0$ c) $c^2 + 2c - 64 = 0$ d) $c^2 + 34c - 64 = 0$

234. Circle $x^2 + y^2 - 2x - \lambda x - 1 = 0$ passes through two fixed points coordinates of the points are

a) $(0, \pm 1)$ b) $(\pm 1, 0)$ c) $(0, 1)$ and $(0, 2)$ d) $(0, -1)$ and $(0, -2)$

235. In an ellipse, the distances between its foci is 6 and minor axis is 8. Then, its eccentricity is

a) $\frac{1}{2}$ b) $\frac{4}{5}$ c) $\frac{1}{\sqrt{5}}$ d) $\frac{3}{5}$

236. The line segment joining the points $(4, 7)$ and $(-2, -1)$ is a dismeter of a circle. If the circle intersects the x-axis at A and B, then AB is equal to

a) 4 b) 5 c) 6 d) 8

237. $ABCD$Is a square whose side is$a$. If AB and AD are axes of coordinates, the equation of the circle circumscribing the square will be

a) $x^2 + y^2 = a^2$ b) $x^2 + y^2 = a(x+y)$ c) $x^2 + y^2 = 2a(x+y)$ d) $x^2 + y^2 = \frac{a^2}{4}$

238. The locus of a point which moves such that the sum of its distance from two fixed points is always a constant, is

a) A straight line b) A circle c) An ellipse d) A hyperbola

239. If $(-4,3)$ and $(8,3)$ are the vertices of an ellipse whose eccentricity is $5/6$ then the equation of the ellipse is

a) $\dfrac{(x-2)^2}{11}+\dfrac{(y-1)^2}{36}=1$

b) $\dfrac{(x-2)^2}{36}+\dfrac{(y-3)^2}{11}=1$

c) $\dfrac{(x-3)^2}{11}+\dfrac{(y-2)^2}{11}=1$

d) None of these

240. One of the limit point of the coaxial system of circles containing $x^2+y^2-6x-6y+4=0$, $x^2+y^2-2x-4y+3=0$, is

a) $(-1,1)$ b) $(-1,2)$ c) $(-2,1)$ d) $(-2,2)$

241. The locus of the middle point of chords of the circle $x^2+y^2=a^2$ which pass through the fixed point (h,k) is

a) $x^2+y^2-hx-ky=0$

b) $x^2+y^2+hx+ky=0$

c) $x^2+y^2-2hx-2ky=0$

d) $x^2+y^2+2hx+2ky=0$

242. If $x\cos\alpha+y\sin\alpha=p$ is a tangent to the ellipse, then

a) $a^2\sin^2\alpha+b^2\cos^2\alpha=p^2$ b) $a^2+b^2\sin^2\alpha=\mathrm{p}^2\mathrm{cosec}^2\alpha$

c) $a^2\cos^2\alpha+b^2\sin^2\alpha=p^2$ d) None of the above

243. Equation of chord of the parabola $y^2=16x$ whose mid point is (1, 1), is

a) $x+y=2$ b) $x-y=0$ c) $8x+y=9$ d) $8x-y=7$

244. A parabola has the origin as its focus and the line $x=2$ as the directrix. Then, the vertex of the parabola is at

a) (2, 0) b) (0, 2) c) (1, 0) d) (0,1)

245. The equation of the common tangent to the hyperbola $3x^2-y^2=3$ and to the parabola $y^2=8x$ is

a) $2x-y-1=0$ b) $x-2y+1=0$ c) $2x+y-1=0$ d) $2x+y+1=0$

246. For any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, tangents are drawn to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=2$, then area cut off by the chord of contact on the region between the asymptotes is equal to

a) ab b) $2ab$ c) $3ab$ d) $4ab$

247. The locus of the foot of the perpendicular from the centre of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ on any tangent is given by $(x^2+y^2)^2=lx^2+my^2$ where

a) $l=a^2, m=b^2$ b) $l=b^2, m=a^2$ c) $l=m=a$ d) $l=m=b$

248. If $y=mx+c$ is a tangent to the ellipse $x^2+2y^2=6$, then $c^2=$

a) $36/m^2$ b) $6m^2-3$ c) $3m^2+6$ d) $6m^2+3$

249. The angle between the tangents drawn from a point $(-a,2a)$ to $y^2=4ax$, is

a) $\pi/4$ b) $\pi/2$ c) $\pi/3$ d) $\pi/6$

250. A tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ cuts the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in P and Q. The locus of the mid-point of PQ is

a) $\left(\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}\right)^2=\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}$

b) $\left(\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}\right)^2=\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}$

c) $\left(\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}\right)^2=\dfrac{2x^2y^2}{a^2b^2}$

d) None of these

251. The equation of the ellipse whose focus is $S(1,-1)$, directrix the line $x-y-3=0$ and eccentricity 1/2, is

a) $7x^2+2xy+7y^2-10x+10y+7=0$

b) $7x^2+2xy+7y^2+7=0$

c) $7x^2 + 2xy + 7y^2 + 10x - 10y - 7 = 0$
d) None of these

252. If the slope of the focal chord of $y^2 = 16x$ is 2, then the length of the chord is
a) 22 b) 24 c) 20 d) 18

253. The radius of any circle touching the lines $3x - 4y + 5 = 0$ and $6x - 8y - 9 = 0$ is
a) 1.9 b) 0.95 c) 2.9 d) 1.45

254. If the tangents are drawn to the ellipse $x^2 + 2y^2 = 2$, then the locus of the mid point of the intercept made by the tangents between the coordinate axes is
a) $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$ b) $\frac{1}{4x^2} + \frac{1}{2y^2} = 1$ c) $\frac{x^2}{2} + \frac{y^2}{4} = 1$ d) $\frac{x^2}{4} + \frac{y^2}{2} = 1$

255. The sum of the focal distances from any point on the ellipse $9x^2 + 16y^2 = 144$ is
a) 3 b) 6 c) 8 d) 4

256. Let P be a variable point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, with foci S_1 and S_2, then coordinates of P such that area of $\Delta S_1 PS_2$ is maximum, are
a) $(0, b)$ b) $\left(\frac{a}{2}, \frac{\sqrt{3}}{2}b\right)$ c) $\left(\frac{\sqrt{3}}{2}a, \frac{b}{2}\right)$ d) None of these

257. The values of α in $[0, 2\pi]$ so that $x^2 + y^2 + 2\sqrt{\sin\alpha}\ x + (\cos\alpha - 1) = 0$ having intercept on x-axis always greater than 2 is/are
a) $(\pi/4, 3\pi/2)$ b) $(\pi/4, \pi)$ c) $(\pi/4, 5\pi/4)$ d) $[0, \pi]$

258. The locus of the mid point of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with the directrix
a) $x = -a$ b) $x = -\frac{a}{2}$ c) $x = 0$ d) $x = \frac{a}{2}$

259. If $\left(m_i, \frac{1}{m_i}\right), i = 1, 2, 3, 4$ are concyclic points, then the value of $m_1 m_2 m_3 m_4$ is
a) 1 b) -1 c) 0 d) None of these

260. The locus of the foot of the perpendicular from the focus upon a tangent to the parabola $y^2 = 4ax$ is
a) The directrix b) Tangent at the vertex c) $x = a$ d) None of these

261. The focal distance of a point P on the parabola $y^2 = 12x$, if the ordinate of P is 6, is
a) 12 b) 6 c) 3 d) 9

262. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord to the circle with centre (2, 1), then the radius of the circle is
a) $\sqrt{3}$ b) $\sqrt{2}$ c) 3 d) 2

263. Locus of the point of intersection of perpendicular tangents to the circle $x^2 + y^2 = 16$ is
a) $x^2 + y^2 = 8$ b) $x^2 + y^2 = 32$ c) $x^2 + y^2 = 64$ d) $x^2 + y^2 = 16$

264. If the coordinates of the centre, a focus and adjacent vertex are $(2, -3)$, $(3, -3)$ and $(4, -3)$ respectively, then the equation of the ellipse is
a) $\frac{(x-2)^2}{4} + \frac{(y+3)^2}{3} = 1$
b) $\frac{(x-3)^2}{4} + \frac{(y-2)^2}{3} = 1$
c) $\frac{(x-2)^2}{8} + \frac{(y+3)^2}{6} = 1$
d) $\frac{(x+2)^2}{4} + \frac{(y-3)^2}{3} = 1$

265. If the circles $x^2 + y^2 = 9$ and $x^2 + y^2 + 2\alpha x + 2y + 1 = 0$ tuoch each other internally, then α is equal to
a) $\pm\frac{4}{3}$ b) 1 c) $\frac{4}{3}$ d) $-\frac{4}{3}$

266. The locus of a point represented by
$x = \frac{a}{2}\left(\frac{t+1}{t}\right), y = \frac{a}{2}\left(\frac{t-1}{t}\right)$ is

a) An ellipse b) A circle c) A pair of lines d) None of these

267. The locus of the mid point of the line joining the focus and any point on the parabola $y^2 = 4ax$ is a parabola with the equation of directrix as

a) $x + a = 0$ b) $2x + a = 0$ c) $x = 0$ d) $x = \frac{a}{2}$

268. Tangents drawn from the point (c, d) to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ make angles α and β with the x-axis. If $\tan\alpha \tan\beta = 1$, then $c^2 - d^2 =$

a) $a^2 - b^2$ b) $b^2 - a^2$ c) $a^2 + b^2$ d) None of these

269. If $P(at^2, 2at)$ be one end of a focal chord of the parabola $y^2 = 4ax$, then the length of the chord is

a) $a\left(t - \frac{1}{t}\right)^2$ b) $a\left(t - \frac{1}{t}\right)$ c) $a\left(t + \frac{1}{t}\right)$ d) $a\left(t + \frac{1}{t}\right)^2$

270. Focus of hyperbola is $(\pm 3, 0)$ and equation of tangent is $2x + y - 4 = 0$, find the equation of hyperbola

a) $4x^2 - 5y^2 = 20$ b) $5x^2 - 4y^2 = 20$ c) $4x^2 - 5y^2 = 1$ d) $5x^2 - 4y^2 = 1$

271. The length of the common chord of the circles $x^2 + y^2 + 4x + 1 = 0$ and $x^2 + y^2 + 4y - 1 = 0$

a) $\sqrt{\frac{15}{2}}$ b) $\sqrt{15}$ c) $2\sqrt{15}$ d) None of these

272. The line $x = at^2$ meets the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, in the real points, iff

a) $|t| < 2$ b) $|t| \leq 1$ c) $|t| > 1$ d) None of these

273. If $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is an ellipse, then length of its latusrectum is

a) $\frac{2b^2}{a}$ b) $\frac{2a^2}{b}$

c) Depends on whether $a > b$ or $b > a$ d) None of the above

274. C_1 Is a circle of radius 2 touching the x-axis and the y-axis. C_2 Is another circle of radius >2 and touching the axes as well as the circle C_1. Then, the radius of C_2 is

a) $6 - 4\sqrt{2}$ b) $6 + 4\sqrt{2}$ c) $6 - 4\sqrt{3}$ d) $6 + 4\sqrt{3}$

275. The intersection point of the normals drawn at the end points of latusrectum of the parabola $x^2 = -2y$ is

a) $\left(-\frac{1}{2}, -\frac{3}{2}\right)$ b) $\left(\frac{1}{2}, -\frac{3}{2}\right)$ c) $(0,1)$ d) $\left(0, -\frac{3}{2}\right)$

276. The equation of the circle whose one diameter is PQ, where the ordinates of P, Q are the roots of the equation $x^2 + 2x - 3 = 0$ and the abscissae are the roots of the equation $y^2 + 4y - 12 = 0$, is

a) $x^2 + y^2 + 2x + 4y - 15 = 0$

b) $x^2 + y^2 - 4x - 2y - 15 = 0$

c) $x^2 + y^2 + 4x + 2y - 15 = 0$

d) None of these

277. The locus of the point of intersection of tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ which meet at right angle is

a) A circle b) A parabola c) An ellipse d) A hyperbola

278. A circle passes through the origin and has its centre on $y = x$. If it cuts $x^2 + y^2 - 4x6y + 10 = 0$ orthogonally, then the equation of the circle is

a) $x^2 + y^2 - x - y = 0$

b) $x^2 + y^2 - 6x - 4y = 0$

c) $x^2 + y^2 - 2x - 2y = 0$

d) $x^2 + y^2 + 2x + 2y = 0$

279. Which of the following is a point on the common chord of the circle $x^2 + y^2 + 2x - 3y + 6 = 0andx^2 + y^2 + x - 8y - 13 = 0$?

a) $(1, -2)$ b) $(1, 4)$ c) $(1, 2)$ d) $(1, -4)$

280. Area of the equilateral triangle inscribed in the circle $x^2 + y^2 - 7x + 9y + 5 = 0$ is

a) $\frac{155}{8}\sqrt{3}$sq units b) $\frac{165}{8}\sqrt{3}$sq units c) $\frac{175}{8}\sqrt{3}$sq units d) $\frac{185}{8}\sqrt{3}$sq units

281. The equation $y^2 - 8y - x + 19 = 0$ represents
a) A parabola whose focus is $\left(\frac{1}{4}, 0\right)$ and directrix is $x = \frac{-1}{4}$
b) A parabola whose vertex is (3,4) and directrix is $x = \frac{11}{4}$
c) A parabola whose focus is $\left(\frac{13}{4}, 4\right)$ and vertex is (0,0)
d) A curve which is not a parabola

282. The two circles
$x^2 + y^2 - 5 = 0$ and
$x^2 + y^2 - 2x - 4y - 15 = 0$
a) Touch each other externally
b) Touch each other internally
c) Cut each other orthogonally
d) Do not intersect

283. The length of the common chord of the parabolas $y^2 = x$ and $x^2 = y$ and is
a) $2\sqrt{2}$
b) 1
c) $\sqrt{2}$
d) $\frac{1}{\sqrt{2}}$

284. The range of values of 'a' such that the angle θ between the pair of tangents drawn from $(a, 0)$ to the circle $x^2 + y^2 = 1$ satisfies $\frac{\pi}{2} < \theta < \pi$, is
a) $(1, 2)$
b) $(1, \sqrt{2})$
c) $(-\sqrt{2}, -1)$
d) $(-\sqrt{2}, -1) \cup (1, \sqrt{2})$

285. The eccentricity of the hyperbola which passes through (3, 0) and $(3\sqrt{2}, 2)$, is
a) $\sqrt{(13)}$
b) $\frac{\sqrt{13}}{3}$
c) $\sqrt{\frac{13}{4}}$
d) None of these

286. If two circles, each of radius 5 unit, touch each other at (1, 2) and the equation of their common tangent is $4x + 3y = 10$, then equation of the circle a portion of which lies in all the quadrants, is
a) $x^2 + y^2 - 10x - 10y + 25 = 0$
b) $x^2 + y^2 + 6x + 2y - 15 = 0$
c) $x^2 + y^2 + 2x + 6y - 15 = 0$
d) $x^2 + y^2 + 10x + 10y + 25 = 0$

287. If $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ are two variable points on the curve $y^2 = 4\,ax$ and PQ subtends a right angle at the vertex, then $t_1 t_2$ is equal to
a) -1
b) -2
c) -3
d) -4

288. The number of common tangents to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x - 8y + 24 = 0$ is
a) 3
b) 4
c) 2
d) 1

289. If a variable tangent of the circle $x^2 + y^2 = 1$ intersects the ellipse $x^2 + 2y^2 = 4$ at points P and Q, then the locus of the point of intersection of tangent at P and Q is
a) A circle of radius 2 unit
b) A parabola with focus as (2, 3)
c) An ellipse with eccentricity $\frac{\sqrt{3}}{2}$
d) None of the above

290. Consider the following statements :
I. Circle $x^2 + y^2 - x - y - 1 = 0$ is completely inside the circle $x^2 + y^2 - 2x + 2y - 7 = 0$
II. Number of common tangents of the circles $x^2 + y^2 + 14x + 12y + 21 = 0$ and $x^2 + y^2 + 2x - 4y - 4 = 0$ is 4

Which of these is/are correct?
a) Only (1)
b) Only (2)
c) Both of these
d) None of these

291. If P is any point on the ellipse $9x^2 + 36y^2 = 324$ whose foci are S and S'. Then, $SP + S'P$ equals
a) 3
b) 12
c) 36
d) 324

292. If the polar with respect to $y^2 = 4ax$ touches the ellipse $\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1$, the locus of its pole is
a) $\frac{x^2}{\alpha^2} - \frac{y^2}{(4a^2\alpha^2/\beta^2)} = 1$
b) $\frac{x^2}{\alpha^2} + \frac{\beta^2 y^2}{4\,a^2} = 1$
c) $\alpha^2 x^2 + \beta^2 y^2 = 1$
d) None of these

293. The equation of the chord of contact of tangents from $(1, 2)$ to the hyperbola $3x^2 - 4y^2 = 3$ is

a) $3x - 16y = 3$ b) $3x - 8y - 3 = 0$ c) $\frac{x}{3} - \frac{y}{4} = 1$ d) $\frac{x}{4} - \frac{y}{3} = 1$

294. Tangents PT_1 and PT_2 are drawn from a point P to the circle $x^2 + y^2 = a^2$. If the point P lies on the line $px + qy - r = 0$, then the locus of the circumcircle of the triangle $P\,T_1T_2$

a) $px + qy = \frac{r}{2}$

b) $2px + 2py + r = 0$

c) $px + qy = r$

d) $(x - p)^2 + (y - q)^2 = r^2$

295. The locus of the centre of a circle of radius 2 which rolls on the outside of the circle, is $x^2 + y^2 + 3x - 6y - 9 = 0$ is

a) $x^2 + y^2 + 3x - 6y + 5 = 0$ b) $x^2 + y^2 + 3x - 6y - 31 = 0$

c) $x^2 + y^2 + 3x - 6y + \frac{29}{4} = 0$ d) None of the above

296. A Basic Terms of Conics is defined by the equations $x = -1 + \sec t, y = 2 + 3\tan t$. The coordinates of the foci are

a) $(-1 - \sqrt{10}, 2)$ and $(-1 + \sqrt{10}, 2)$ b) $(-1 - \sqrt{8}, 2)$ and $(-1 + \sqrt{8}, 2)$

c) $(-1, 2 - \sqrt{8})$ and $(-1, 2 + \sqrt{8})$ d) $(-1, 2 - \sqrt{10})$ and $(-1, 2 + \sqrt{10})$

297. If A and B are two fixed points and P is a variable point such that $PA + PB = 4$, the locus of P is

a) A parabola b) An ellipse c) A hyperbola d) None of these

298. If the vertices of an ellipse are $(-12,4)$ and $(14,4)$ and eccentricity $12/13$, then the equation of the ellipse is

a) $\frac{(x-4)^2}{25} + \frac{(y-1)^2}{169} = 1$

b) $\frac{(x-4)^2}{169} + \frac{(y-1)^2}{25} = 1$

c) $\frac{(x-1)^2}{169} + \frac{(y-4)^2}{25} = 1$

d) $\frac{(x+1)^2}{169} + \frac{(y+4)^2}{25} = 1$

299. If C is the centre of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the normal at an end of a latusrectum cuts the major axis in G, then $CG =$

a) ae b) a^2e^2 c) ae^2 d) a^2e^3

300. The locus of the point of intersection of perpendicular tangents to the hyperbola $\frac{x^2}{3} - \frac{y^2}{1} = 1$ is

a) $x^2 + y^2 = 2$ b) $x^2 + y^2 = 3$ c) $x^2 - y^2 = 3$ d) $x^2 + y^2 = 4$

301. The eccentricity of the ellipse represented by the equation $25\,x^2 + 16\,y^2 - 150\,x - 175 = 0$ is

a) $2/5$ b) $3/5$ c) $4/5$ d) None of these

302. Axis of a parabola is $y = x$ and vertex and focus are at a distance $\sqrt{2}$ and $2\sqrt{2}$ Respectively from the origin. Then, equation of the parabola is

a) $(x - y)^2 = 8(x + y - 2)$ b) $(x + y)^2 = 2(x + y - 2)$

c) $(x - y)^2 = 4(x + y - 2)$ d) $(x + y)^2 = 2(x - y + 2)$

303. Let PQ and PS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect at a point x on the circumference of the circle, then $2r$ equals

a) $\sqrt{PQ.RS}$ b) $\frac{PQ + RS}{2}$ c) $\frac{2PQ.RS}{PQ + RS}$ d) $\sqrt{\frac{PQ^2 + RS^2}{2}}$

304. If the vertex of a parabola is the point $(-3,0)$ and the directrix is the line $x + 5 = 0$, then its equation is

a) $y^2 = 8(x + 3)$ b) $x^2 = 8(y + 3)$ c) $y^2 = -8(x + 3)$ d) $y^2 = 8(x + 5)$

305. The length of the latusrectum of the parabola $169\{(x - 1)^2 + (y - 3)^2\} = (5x - 12y + 17)^2$ is

a) 14/13 b) 12/13 c) 28/13 d) None of these

306. If P is a point on the parabola $y^2 = 4\,ax$ such that the subtangent and subnormal at P are equal, then the coordinates of P are
a) $(a, 2a)$ or $(a, -2a)$
b) $(2a, 2\sqrt{2}\,a)$ or $(2a, -2\sqrt{2}\,a)$
c) $(4a, -4a)$ or $(4a, 4a)$
d) None of these

307. In the normal at the end of latusrectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with eccentricity e, passes through one end of the minor axis, then
a) $e^2(1 + e^2) = 0$ b) $e^2(1 + e^2) = 1$ c) $e^2(1 + e^2) = -1$ d) $e^2(1 + e^2) = 2$

308. The pole of a straight line with respect to the circle $x^2 + y^2 = a^2$ lies on the circle $x^2 + y^2 = 9a^2$. If the straight line touches the circle $x^2 + y^2 = r^2$, then
a) $9a^2 = r^2$ b) $9r^2 = a^2$ c) $r^2 = a^2$ d) None of these

309. The equation of the latusrectum of the parabola $x^2 + 4x + 2y = 0$, is equal to
a) $2y + 3 = 0$ b) $3y = 2$ c) $2y = 3$ d) $3y + 2 = 0$

310. If the normal at any point P on the ellipse cuts the major and minor axes in G and g respectively and C be the centre of the ellipse, then
a) $a^2(CG)^2 + b^2(Cg)^2 = (a^2 - b^2)^2$
b) $a^2(CG)^2 - b^2(Cg)^2 = (a^2 - b^2)^2$
c) $a^2(CG)^2 - b^2(Cg)^2 = (a^2 + b^2)^2$
d) None of the above

311. The value of m, for which the line $y = mx + \frac{25\sqrt{3}}{3}$ is a normal to the conic $\frac{x^2}{16} - \frac{y^2}{9} = 1$, is
a) $\pm\frac{2}{\sqrt{3}}$ b) $\pm\sqrt{3}$ c) $\pm\frac{\sqrt{3}}{2}$ d) None of these

312. Equation of the latusrectum of the ellipse $9x^2 + 4y^2 - 18x - 8y - 23 = 0$ are
a) $y = \pm\sqrt{5}$ b) $y = \pm\sqrt{5}$ c) $y = 1 \pm \sqrt{5}$ d) $y = -1 \pm \sqrt{5}$

313. The number of circles belonging to the system of circles $2(x^2 + y^2) + \lambda\,x - (1 + \lambda^2)y - 10 = 0$ and orthogonal to $x^2 + y^2 + 4x + 6y + 3 = 0$, is
a) 2 b) 1 c) 0 d) None of these

314. The length of the semi-transverse axis of the rectangular hyperbola $xy = 32$ is
a) 32 b) 16 c) 64 d) 8

315. If $y = 2x + 3$ is a tangent to the parabola, $y^2 = 24\,x$, then its distance from the parallel normal is
a) $5\sqrt{5}$ b) $10\sqrt{5}$ c) $15\sqrt{5}$ d) $3\sqrt{5}$

316. If $P(\alpha, \beta)$ is a point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with foci S and S' and eccentricity e, then area of $\Delta SPS'$ is
a) $ae\sqrt{a^2 - \alpha^2}$ b) $be\sqrt{b^2 - \alpha^2}$ c) $ae\sqrt{b^2 - \alpha^2}$ d) $be\sqrt{a^2 - \alpha^2}$

317. If a circle passes through the point (1, 2) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the equation of the locus of its centre is
a) $x^2 + y^2 - 3x - 8y + 1 = 0$
b) $x^2 + y^2 - 2x - 6y - 7 = 0$
c) $2x + 4y - 9 = 0$
d) $2x + 4y - 1 = 0$

318. Three sides of a triangle have the equations $L_r \equiv y - m_r x - c_r = 0, r = 1,2,3$. If λ, μ, v are non-zero real numbers such that $\lambda\,L_2L_3 + \mu L_3L_1 + v\,L_1L_2 = 0$ represents the circumcircle of the triangle, then
a) $\lambda(m_2 + m_3) + \mu(m_3 + m_1) + v(m_1 + m_2) = 0$
b) $\lambda(m_2m_3 - 1) + \mu(m_3m_1 - 1) + v(m_1m_2 - 1) = 0$
c) Both (a) and (b) hold together
d) None of these

319. The distinct points $A(0, 0), B(0, 1), C(1, 0)$ and $D(2a, 3a)$ are concyclic, then
a) 'a' can attain only rational values
b) 'a' is irrational
c) Cannot be concyclic for any 'd'
d) None of the above

320. For the given circles $x^2 + y^2 - 6x - 2y + 1 = 0andx^2 + y^2 + 2x - 8y + 13 = 0$, which of the following is true?

a) One circles lies inside the other
b) One circle lies completely outside the other
c) Two circles intersect in two points
d) They touch each other externally

321. The eccentricity of the ellipse $9x^2 + 5y^2 - 18x - 20y - 16 = 0$ is
a) $\frac{1}{2}$
b) $\frac{2}{3}$
c) $\frac{3}{2}$
d) 2

322. The normal to the parabola $y^2 = 8x$ at the point $(2, 4)$meets the parabola again at the point
a) $(-18, -12)$
b) $(-18, 12)$
c) $(18, 12)$
d) $(18, -12)$

323. To which of the following circles, the line $y - x + 3 = 0$ is normal at the point $\left(3 + \frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$?
a) $\left(x - 3 - \frac{3}{\sqrt{2}}\right)^2 + \left(y - \frac{3}{\sqrt{2}}\right)^2 = 9$
b) $\left(x - \frac{3}{\sqrt{2}}\right)^2 + \left(y - \frac{3}{\sqrt{2}}\right)^2 = 9$
c) $x^2 + (y - 3)^2 = 9$
d) $(x - 3)^2 + y^2 = 9$

324. The point on the curve $3x^2 - 4y^2 = 72$, which is nearest to the line $3x + 2y - 1 = 0$, is
a) $(6, 3)$
b) $(6, -3)$
c) $(6, 6)$
d) $(6, 5)$

325. The equation of the mirror that can reflect all incident rays from origin parallel to y-axis is
a) $x^2 = 4a(y + a)$
b) $y^2 = 4a(x + a)$
c) $y^2 = -4a(x - a)$
d) None of these

326. For the two circles $x^2 + y^2 = 16$ and $x^2 + y^2 - 2y = 0$ there is/are
a) One pair of common tangents
b) Only one common tangent
c) Three common tangents
d) No common tangent

327. The point on the curve $y^2 = ax$ the tangent at which makes an angle of 45° with x-axis will be given by
a) $(a/2, a/4)$
b) $(-a/2, a/4)$
c) $(a/4, a/2)$
d) $(-a/4, a/2)$

328. The equation of hyperbola whose foci are $(2, 4)$ and $(-2,4)$ and eccentricity is $\frac{4}{3}$, is
a) $x^2 - (y - 4)^2 = 5$
b) $\frac{x^2}{9} - \frac{(y - 4)^2}{7} = \frac{1}{4}$
c) $\frac{x^2}{9} - \frac{y^2}{7} = \frac{1}{4}$
d) None of these

329. If the line $lx + my + n = 0$ intersects the curve $ax^2 + 2hxy + by^2 = 1$ at P and Q such that the circle with PQ as a diameter passes through the origin, then $l^2 + m^2 =$
a) $n^2(a + b)$
b) $n^2(a + b)^2$
c) $n^2(a^2 - b^2)$
d) $n^2(a^2 + b^2)$

330. One of the diameter of the circle $x^2 + y^2 - 12x + 4y + 6 = 0$ is given by
a) $x + y = 0$
b) $x + 3y = 0$
c) $x = y$
d) $3x + 2y = 0$

331. The length of the latusrectum of the parabola $x^2 - 4x - 8y + 12 = 0$ is
a) 4
b) 6
c) 8
d) 10

332. The coordinates of a point on the parabola $y^2 = 8\,x$ whose focal distance is 4, are
a) $(1/2, \pm 2)$
b) $(1, \pm 2\sqrt{2})$
c) $(2, \pm 4)$
d) None of these

333. The eccentricity of the conic $36x^2 + 144y^2 - 36x - 96y - 119 = 0$ is
a) $\frac{\sqrt{3}}{2}$
b) $\frac{1}{2}$
c) $\frac{\sqrt{3}}{4}$
d) $\frac{1}{\sqrt{3}}$

334. If the tangent and normal at any point P of a parabola meet the axes in $T$and G respectively, then
a) $ST \neq SG = SP$
b) $ST - SG \neq SP$
c) $ST = SG = SP$
d) $ST = SG \cdot SP$

335. The tangents to the hyperbola $x^2 - y^2 = 3$ are parallel to the straight line $2x + y + 8 = 0$ at the following points:
a) $(2, 1), (1, 2)$
b) $(2, -1), (-2,1)$
c) $(-2, -1), (1, 2)$
d) $(-2, -1), (-1, -2)$

336. The equation of the tangents to the ellipse $4x^2 + 3y^2 = 5$, which are parallel to the line $y = 3x + 7$ are
a) $y = 3x \pm \sqrt{\frac{155}{3}}$
b) $y = 3x \pm \sqrt{\frac{155}{12}}$
c) $y = 3x \pm \sqrt{\frac{95}{12}}$
d) None of these

337. Tangent to the ellipse $\frac{x^2}{32}+\frac{y^2}{18}=1$ having slope $-\frac{3}{4}$ meet the coordinate axes in A and B. Find the area of the ΔAOB, where O is the origin

a) 12 sq unit b) 8 sq unit c) 24 sq unit d) 32 sq unit

338. If the straight line $lx+my+n=0$ touches the parabola, $y^2=4ax$, then

a) $nm=al^2$ b) $nl=am^2$ c) $nl=am$ d) $ml=an^2$

339. If $3x+y+k=0$ is a tangent to the circle $x^2+y^2=10$, the values of k are

a) ± 7 b) ± 5 c) ± 10 d) ± 9

340. The area of the triangle formed by the lines $x+y=0, x-y=0$ and any tangent to the hyperbola $x^2-y^2=a^2$ is

a) $4a^2$ sq. units b) $3a^2$ sq. units c) $2a^2$ sq. units d) a^2 sq. units

341. The values of λ so that the line $3x-4y=\lambda$ touches $x^2+y^2-4x-8y-5=0$ are

a) $-35, 15$ b) $3, -5$ c) $35, -15$ d) $-3, 5$

342. The common chord of $x^2+y^2-4x-4y=0$ and $x^2+y^2=16$ subtends at the origin an angle equal to

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

343. The distance between the foci of an ellipse is 16 and eccentricity is $1/2$. Length of the major axis of the ellipse is

a) 8 b) 64 c) 16 d) 32

344. The centres of three circles $x^2+y^2=1, x^2+y^2+6x-2y=1, x^2+y^2-12x+4y=1$ are

a) Collinear b) Non-collinear c) Nothing to be said d) None of these

345. If the normal at the end of latusrectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through $(0,-b)$, then e^4+e^2(where e is eccentricity) equals

a) 1 b) $\sqrt{2}$ c) $\frac{\sqrt{5}-1}{2}$ d) $\frac{\sqrt{5}+1}{2}$

346. The equation of the pair of asymptotes of the hyperbola $xy-4x+3y=0$ is

a) $xy-4x+3y-1=0$

b) $xy-4x+3y-10=0$

c) $xy-4x+3y-12=0$

d) None of these

347. On the ellipse $4x^2+9y^2=1$ the point at which the tangent are parallel to $8x=9y$ are

a) $\left(\frac{2}{5},\frac{1}{5}\right)$ or $\left(-\frac{2}{5},\frac{1}{5}\right)$ b) $\left(-\frac{2}{5},\frac{1}{5}\right)$ or $\left(\frac{2}{5},-\frac{1}{5}\right)$ c) $\left(-\frac{2}{5},-\frac{1}{5}\right)$ d) $\left(-\frac{3}{5},-\frac{2}{5}\right)$ or $\left(\frac{3}{5},\frac{2}{5}\right)$

348. If roots of quadratic equation $ax^2+2bx+c=0$ are not real, then $ax^2+2bxy+cy^2+dx+ey+f=0$ represents a/an

a) Ellipse b) Circle c) Parabola d) Hyperbola

349. The equation of the hyperbola whose vertices are at (5,0) and $(-5,0)$ and one of the directrices is $x=\frac{25}{7}$, is

a) $\frac{x^2}{25}-\frac{y^2}{24}=1$ b) $\frac{x^2}{24}-\frac{y^2}{25}=1$ c) $\frac{x^2}{16}-\frac{y^2}{25}=1$ d) $\frac{x^2}{25}-\frac{y^2}{16}=1$

350. The radius of the circle passing through the foci of the ellipse $\frac{x^2}{4}+\frac{4}{7}y^2=1$ and having its centre at $\left(\frac{1}{2},2\right)$, is

a) $\sqrt{5}$ b) 3 c) $\sqrt{12}$ d) $\frac{7}{2}$

351. The directrix of the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ is

a) $y=\frac{6}{\sqrt{13}}$ b) $x=\frac{6}{\sqrt{13}}$ c) $y=\frac{9}{\sqrt{13}}$ d) $x=\frac{9}{\sqrt{13}}$

352. If the line $y=7x-25$ meets the circle $x^2+y^2=25$ in the points A, B, then the distance between A and B is

a) $\sqrt{10}$ b) 10 c) $5\sqrt{2}$ d) 5

353. Sides of an equilateral $\triangle ABC$ touch the parabola $y^2 = 4x$, then the points A, B and C lie on

a) $y^2 = (x+a)^2 + 4ax$ b) $y^2 = 3(x+a)^2 + ax$

c) $y^2 = 3(x+a)^2 + 4ax$ d) $y^2 = (x+a)^2 + ax$

354. A tangent to the ellipse $x^2 + 4y^2 = 4$ meets the ellipse $x^2 + 2y^2 = 6$ at P and Q. The angle between the tangent at P and Q of the ellipse $x^2 + 2y^2 = 6$ is

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$

355. The equation of the tangents to the circle $x^2 + y^2 = 4$, which are parallel to $x + 2y + 3 = 0$, are

a) $x - 2y = 2$ b) $x + 2y = \pm 2\sqrt{3}$ c) $x + 2y = \pm 2\sqrt{5}$ d) $x - 2y = \pm 2\sqrt{5}$

356. Tangents are drawn at the ends of any focal chord of the parabola $y^2 = 16x$. Then which of the following statements about the point of intersection of tangents is true

a) Its abscissae is independent of the extremities of the focal chord

b) Its ordinate is independent of the extremities of the focal chord

c) It is at a distance of 8 units from the vertex of the parabola

d) It is at a distance of 16 units from the focus of the parabola

357. The locus of points whose polars with respect to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are at a distance d from the centre of the ellipse, is

a) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{d^2}$ b) $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{d^2}$ c) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{d^4}$ d) None of these

358. If $x = my + c$ is a normal to the parabola $x^2 = 4\,ay$, then the value of c is

a) $-2\,am - am^3$ b) $2\,am + am^3$ c) $-\frac{2\,a}{m} - \frac{a}{m^2}$ d) $\frac{2\,a}{m} + \frac{a}{m^3}$

359. If the line $y = 2x + \lambda$ be a tangent to the hyperbola $36x^2 - 25y^2 = 3600$, then λ is equal to

a) 16 b) -16 c) ± 16 d) None of these

360. The number of common tangents to the circles $x^2 + y^2 - x = 0, x^2 + y^2 + x = 0$ is

a) 2 b) 1 c) 4 d) 3

361. Equation of the circle through the origin and making intercepts of 3 and 4 on the positive sides of the axes is

a) $x^2 + y^2 + 3\,x + 4\,y = 0$

b) $x^2 + y^2 + 3\,x - 4\,y = 0$

c) $x^2 + y^2 + 3\,x - 4\,y = 0$

d) $x^2 + y^2 - 3\,x + 4\,y = 0$

362. The straight line $y = mx + c$ cuts the circle $x^2 + y^2 = a^2$ in real points if

a) $\sqrt{a^2(1+m^2)} < c$ b) $\sqrt{a^2(1-m^2)} < c$ c) $\sqrt{a^2(1+m^2)} > c$ d) $\sqrt{a^2(1-m^2)} > c$

363. If the chords of contact of tangents from two points (x_1, y_1) amd (x_2, y_2) to the hyperbola $4x^2 - 9y^2 - 36 = 0$ are at right angles, then $\frac{x_1 x_2}{y_1 y_2}$ is equal to

a) $\frac{9}{4}$ b) $-\frac{9}{4}$ c) $\frac{81}{16}$ d) $-\frac{81}{16}$

364. The condition for the coaxial system $x^2 + y^2 + 2\lambda x + c = 0$, where λ is a parameter and c is a constant to have distinct limiting points, is

a) $c = 0$ b) $c < 0$ c) $c = -1$ d) $c > 0$

365. On the ellipse $2x^2 + 3y^2 = 1$ the points at which the tangent is parallel to $4x = 3y + 4$, are

a) $\left(\frac{2}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right)$ or $\left(-\frac{2}{\sqrt{11}}, -\frac{1}{\sqrt{11}}\right)$ b) $\left(-\frac{2}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right)$ or $\left(\frac{2}{\sqrt{11}}, -\frac{1}{\sqrt{11}}\right)$

c) $\left(-\frac{2}{5}, -\frac{1}{5}\right)$ d) $\left(-\frac{3}{5}, -\frac{2}{5}\right)$ or $\left(\frac{3}{5}, \frac{2}{5}\right)$

366. The locus of the middle points of the chords of the parabola $y^2 = 4ax$, which passes through the origin is

a) $y^2 = ax$ b) $y^2 = 2ax$ c) $y^2 = 4ax$ d) $x^2 = 4ay$

367. The eccentricity of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose latusrectum is half of its major axis is

a) $\frac{1}{\sqrt{2}}$ b) $\sqrt{\frac{2}{3}}$ c) $\frac{\sqrt{3}}{2}$ d) None of these

368. On the parabola $y = x^2$, the point at least distance from the straight line $y = 2x - 4$ is
a) $(1, 1)$ b) $(1, 0)$ c) $(1, -1)$ d) $(0, 0)$

369. For the curve $7x^2 - 2y^2 + 12xy - 2x + 14y - 22 = 0$ which of the following is true?
a) It is an hyperbola with eccentricity $\sqrt{3}$
b) It is an hyperbola with directrix $2x + y - 1 = 0$
c) It is an hyperbola with focus (1,2)
d) All of the above

370. If e_1 is the eccentricity of the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$ and e_2 is the eccentricity of the hyperbola passing through the foci of the ellipse and $e_1 e_2 = 1$, then equation of the hyperbola is
a) $\frac{x^2}{9} - \frac{y^2}{16} = 1$ b) $\frac{x^2}{16} - \frac{y^2}{9} = -1$ c) $\frac{x^2}{9} - \frac{y^2}{25} = 1$ d) None of these

371. The tangent to the parabola $y^2 = 16x$, which is perpendicular to a line $y - 3x - 1 = 0$, is
a) $3y + x + 36 = 0$ b) $3y - x - 36 = 0$ c) $x + y - 36 = 0$ d) $x - y + 36 = 0$

372. The locus of appoint which moves so that the ratio of the length of the tangents to the circles $x^2 + y^2 + 4x + 3 = 0$ and $x^2 + y^2 - 6x + 5 = 0$ is $2: 3$, is
a) $5x^2 + 5y^2 - 60x + 7 = 0$ b) $5x^2 + 5y^2 + 60x - 7 = 0$
c) $5x^2 + 5y^2 - 60x - 7 = 0$ d) $5x^2 + 5y^2 + 60x + 7 = 0$

373. The equation of normal of $x^2 + y^2 - 2x + 4y - 5 = 0$ at (2, 1) is
a) $y = 3x - 5$ b) $2y = 3x - 4$ c) $y = 3x + 4$ d) $y = x + 1$

374. If the abscissa and ordinates of two points P and Q are roots of the equations $x^2 + 2ax - b^2 = 0$ and $y^2 + 2py - q^2 = 0$ respectivaly, then the equation of the circle with PQ as diameter, is
a) $x^2 + y^2 + 2ax + 2py - b^2 - q^2 = 0$
b) $x^2 + y^2 - 2ax + 2py + b^2 + q^2 = 0$
c) $x^2 + y^2 - 2ax - 2py - b^2 - q^2 = 0$
d) $x^2 + y^2 + 2ax + 2py + b^2 + q^2 = 0$

375. The angle between the pair of tangents drawn from the point (1,1/2) to the circle $x^2 + y^2 + 4x + 2y - 4 = 0$, is
a) $\cos^{-1}\frac{4}{5}$ b) $\sin^{-1}\frac{4}{5}$ c) $\sin^{-1}\frac{3}{5}$ d) None of these

376. Suppose S and S' are foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$. If P is a variable point on the ellipse and if Δ is area of the triangles PSS', then the maximum value Δ is
a) 8 b) 12 c) 16 d) 20

377. The length of the latusrectum of the ellipse $3x^2 + y^2 = 12$ is
a) 4 b) 3 c) 8 d) $4/\sqrt{3}$

378. Centre of circle whose normal's are $x^2 - 2xy - 3x + 6y = 0$, is
a) $\left(3, \frac{3}{2}\right)$ b) $\left(3, -\frac{3}{2}\right)$ c) $\left(\frac{3}{2}, 3\right)$ d) None of these

379. The focus of the parabola $y = 2x^2 + x$ is
a) (0,0) b) $\left(\frac{1}{2}, \frac{1}{4}\right)$ c) $\left(-\frac{1}{4}, 0\right)$ d) $\left(-\frac{1}{4}, \frac{1}{8}\right)$

380. The centre of the circle, which cuts orthogonally each of the three circles $x^2 + y^2 + 2x + 17y + 4 = 0$ and $x^2 + y^2 + 7x + 6y + 11 = 0$, $x^2 + y^2 - x + 22y + 3 = 0$, is
a) (3, 2) b) (1, 2) c) (2, 3) d) (0, 2)

381. If I denotes the semi-latusrectum of the parabola $y^2 = 4ax$ and SP and SQ denote the segments of any focal chord PQ, S being the focus, then SP, l and SQ are in the relation
a) AP b) GP c) HP d) $l^2 = SP^2 + SQ^2$

382. The number of the tangents that can be drawn from (1,2) to $x^2 + y^2 = 5$, is

a) 1 b) 2 c) 3 d) 0

383. The difference in focal distance of any point on the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ is

a) 8 b) 9 c) 0 d) 6

384. The equation of the circle of radius 3 that lies in the fourth quadrant and touching the lines $x=0$ and $y=0$ is

a) $x^2+y^2-6x+6y+9=0$
b) $x^2+y^2-6x-6y+9=0$
c) $x^2+y^2+6x-6y+9=0$
d) $x^2+y^2+6x+6y+9=0$

385. If t is the parameter for one end of a focal chord of the parabola $y^2=4\,ax$, then its length is

a) $a\left(t+\frac{1}{t}\right)^2$ b) $b\left(t-\frac{1}{t}\right)^2$ c) $a\left(t+\frac{1}{t}\right)$ d) $a\left(t-\frac{1}{t}\right)$

386. The number of points with integral coordinates $(2a, a-1)$ that fall in the interior of the larger segment of the circle $x^2+y^2=25$ cut off by the parabola $x^2+4y=0$, is

a) 1 b) 2 c) 3 d) None of these

387. The radius of the larger circle lying in the first quadrant and touching the line $4x+3y-12=0$ and the coordinate axes, is

a) 5 b) 6 c) 7 d) 8

388. The equation of a directrix of the ellipse $\frac{x^2}{16}+\frac{y^2}{25}=1$ is

a) $3y=\pm 5$ b) $y=\pm 5$ c) $3y=\pm 25$ d) $y=\pm 3$

389. If a circle passes through the point (a, b) and cuts the circle $x^2+y^2=p^2$ orthogonally, then the equation of the locus of its centre is

a) $2\,ax+2\,by-(a^2+b^2+p^2)=0$
b) $2\,ax+2\,by-(a^2-b^2+p^2)=0$
c) $x^2+y^2-3\,ax-4\,by+(a^2+b^2-p^2)=0$
d) $x^2+y^2-2\,ax-3\,by+(a^2-b^2-p^2)=0$

390. The parametric representation of a point of the ellipse whose foci are (3, 0) and (−1,0) and eccentricity 2/3 is

a) $(1+3\cos\theta, \sqrt{3}\sin\theta)$
b) $(1+3\cos\theta, 5\sin\theta)$
c) $(1+3\cos\theta, 1+\sqrt{5}\sin\theta)$
d) $(1+3\cos\theta, \sqrt{5}\sin\theta)$

391. Which of the following equations gives circle?

a) $r=2\sin\theta$
b) $r^2\cos\,2\theta=1$
c) $r(4\cos\theta+5\sin\theta)=3$
d) $5=r(1+\sqrt{2}\cos\theta)$

392. The equation of the chord of the circle $x^2+y^2=81$, which is bisected at the point $(-2,3)$, is

a) $3x-y=13$ b) $3x-4y=13$ c) $2x-3y=13$ d) $2x-3y=-13$

393. The circles $x^2+y^2+x+y=0$ and x^2+y^2+x-y intersect at an angle of

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

394. If $\frac{x}{a}+\frac{y}{b}=\sqrt{2}$ touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ then the eccentric angle of the point of contact is equal to

a) 0° b) 90° c) 45° d) 60°

395. Area of the circle in which a chord of length $\sqrt{2}$ makes an angle $\pi/2$ at the centre is

a) $\pi/2$ b) $2\,\pi$ c) π d) $\pi/4$

396. If the chord of contact of tangents from a point P to the parabola $y^2=4ax$ touches the parabola $x^2=4by$, then the locus of P is a/an

a) Circle b) Parabola c) Ellipse d) Hyperbola

397. If $2x+y+k=0$ is a normal to the parabola $y^2=-8x$, then the values of k is

a) −16 b) −8 c) −24 d) 24

398. The equation of circle touches the line $x=y$ at origin and passes through the point (2, 1) is$x^2+y^2+px+qy=0$. Then p, q are

a) −5, −5 b) −5, 5 c) 5, −5 d) None of these

399. If the line $x+y-1=0$, is a tangent to the parabola $y^2-y+x=0$, then the point of contact is

a) $(0,1)$ b) $(1,0)$ c) $(0,-1)$ d) $(-1,0)$

400. A variable chord is drawn through the origin to the circle $x^2+y^2-2ax=0$. The locus of the centre of the circle drawn on this chord as diameter is

a) $x^2+y^2+ax=0$ b) $x^2+y^2-ax=0$ c) $x^2+y^2+ay=0$ d) $x^2+y^2-ay=0$

401. The centre of the ellipse $\frac{(x+y-2)^2}{9}+\frac{(x-y)^2}{16}=1$ is

a) (0,0) b) (1,1) c) (1,0) d) (0,1)

402. The equation of the directrix of $(x-1)^2=2(y-2)$ is

a) $2y+3=0$ b) $2x+1=0$ c) $2x-1=0$ d) $2y-3=0$

403. The point of contact of the line $x-2y-1=0$ with the parabola $y^2=2(x-3)$, is

a) (5,2) b) $(5,-2)$ c) (2,5) d) (5,3)

404. The equation of the hyperbola whose directrix $x+2y=1$, focus (2,1) and eccentricity 2 is

a) $x^2+16xy-11y^2-12x+6y+21=0$

b) $x^2-16xy-11y^2-12x+6y+21=0$

c) $x^2-4xy-y^2-12x+6y+21=0$

d) None of these

405. Locus of mid point of any focal chord of $y^2=4ax$ is

a) $y^2=a(x-2a)$ b) $y^2=2a(x-2a)$ c) $y^2=2a(x-a)$ d) None of these

406. The angle between the pair of tangents drawn from the point (1, 2) to the ellipse $3x^2+2y^2=5$, is

a) $\tan^{-1}(12/5)$ b) $\tan^{-1}(6/\sqrt{5})$ c) $\tan^{-1}(12/\sqrt{5})$ d) $\tan^{-1}(6/5)$

407. If the foci of an ellipse are $(\pm\sqrt{5},0)$ and its eccentricity is $\sqrt{5}/3$, then the equation of the ellipse is

a) $9x^2+4y^2=36$ b) $4x^2+9y^2=36$ c) $36x^2+9y^2=4$ d) $9x^2+36y^2=4$

408. The locus of the mid-points of chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ that touch the circle $x^2+y^2=b^2$, is

a) $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^2=\frac{x^2}{a^4}+\frac{y^2}{b^4}$

b) $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^2=b^2\left(\frac{x^2}{a^4}+\frac{y^2}{b^4}\right)$

c) $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^2=a^2\left(\frac{x^2}{a^4}+\frac{y^2}{b^4}\right)$

d) None of these

409. The equations of the normal at the ends of the latus rectum of the parabola $y^2=4ax$ are given by

a) $x^2-y^2-6ax+9a^2=0$

b) $x^2-y^2-6ax-6ay-6ay+9a^2=0$

c) $x^2-y^2-6ay+9a^2=0$

d) None of these

410. The value of k, if (1, 2), $(k,-1)$ are conjugate points with respect to the ellipse $2x^2+3y^2=6$, is

a) 2 b) 4 c) 6 d) 8

411. The angle between the asymptotes of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is equal to

a) $2\tan^{-1}\left(\frac{b}{a}\right)$ b) $\tan^{-1}\left(\frac{a}{b}\right)$ c) $2\tan^{-1}\left(\frac{a}{b}\right)$ d) $\tan^{-1}\left(\frac{b}{a}\right)$

412. The eccentricity of the conic $\frac{(x+2)^2}{7}+(y-1)^2=14$ is

a) $\sqrt{\frac{7}{8}}$ b) $\sqrt{\frac{6}{17}}$ c) $\frac{\sqrt{3}}{2}$ d) $\sqrt{\frac{6}{7}}$

413. The circle $x^2+y^2=4$ cuts the circle $x^2+y^2+2x+3y-5=0$ in A and B. Then the equation of the circle on AB as diameter is

a) $13(x^2+y^2)-4x-6y-50=0$

b) $9(x^2+y^2)+8x-4y+25=0$

c) $x^2 + y^2 - 5\,x + 2\,y + 72 = 0$
d) None of these

414. If θis the angle between the tangents from $(-1, 0)$ to the circle $x^2 + y^2 - 5x + 4y - 2 = 0$, then θ is equal to
a) $2\tan^{-1}\left(\frac{7}{4}\right)$ b) $\tan^{-1}\left(\frac{7}{4}\right)$ c) $2\cot^{-1}\left(\frac{7}{4}\right)$ d) $\cot^{-1}\left(\frac{7}{4}\right)$

415. The equation to the circle having $y = mx$ as a diameter where $y = mx$ is a chord of the circle, through the origin, of radius a and having the x-axis as diameter is
a) $(1 + m^2)(x^2 + y^2) - 2a(x + my) = 0$
b) $(1 - m^2)(x^2 + y^2) - 2a(x + my) = 0$
c) $(1 + m^2)(x^2 + y^2) + 2a(x + my) = 0$
d) None of these

416. A triangle ABC of area Δ is inscribed in the parabola $y^2 = 4ax$ such that A is the vertex and BC is a focal chord of the parabola. The difference of the ordinates of B and C is
a) $\frac{2\,\Delta}{a}$ b) $\frac{\Delta}{a}$ c) $\frac{2\,a^3}{\Delta}$ d) $\frac{2\,\Delta^2}{a^3}$

417. Let S, S' be the foci and BB' be the minor axis of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. If $\angle BSS' = \theta$, then the eccentricity e of the ellipse is equal to
a) $\sin\theta$ b) $\cos\theta$ c) $\tan\theta$ d) $\cot\theta$

418. An isosceles right angles triangle is inscribed in the circle$x^2 + y^2 = r^2$. If the coordinates of an end of the hypotenuse are (a, b), the coordinates of the vertex are
a) $(-a, -b)$ b) $(b, -a)$ c) (b, a) d) $(-b, -a)$

419. If $(-3, 2)$ lies on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ which is concentric with the circle $x^2 + y^2 + 6x + 8y - 5 = 0$, then c is equal to
a) 11 b) -11 c) 24 d) 100

420. Let S and S' be two foci of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. If the circle described on SS' as diameter touches the ellipse in real points, then the eccentricity of the ellipse is
a) $\frac{2}{\sqrt{3}}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{1}{\sqrt{2}}$ d) $\frac{1}{\sqrt{3}}$

421. The distance between the chords of contact of the tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ from the origin and the point (g, f), is
a) $g^2 + f^2$ b) $\frac{1}{2}(g^2 + f^2 + c)$ c) $\frac{g^2 + f^2 + c}{2\sqrt{g^2 + f^2}}$ d) $\frac{g^2 + f^2 - c}{2\sqrt{g^2 + f^2}}$

422. If $P(-1, -3)$ is a centre of similitude for the circles $x^2 + y^2 = 1$ and $x^2 + y^2 - 2x - 6y + 6 = 0$, then the length of the common tangent through P to the circles is
a) 2 b) 3 c) 4 d) 5

423. The equation $x^2 + y^2 - 2x - 2\,\lambda\,y - 8 = 0$ represents a system of circles, λ being a parameter, passing through two fixed points P and Q. The circle on PQ as a diameter, is
a) $x^2 + y^2 - 2y = 0$
b) $x^2 + y^2 - 2x - 8 = 0$
c) $x^2 + y^2 - 2y = 8$
d) $x^2 + y^2 - 2x - 2y = 8$

424. The radius of the circle $x^2 + y^2 + 4x + 6y + 13 = 0$ is
a) $\sqrt{26}$ b) $\sqrt{13}$ c) $\sqrt{23}$ d) 0

425. If the vertex of a parabola is (0,2) and the extremities of latusrectum are $(-6,4)$ and (6,4), then, its equation is
a) $x^2 - 4y + 8 = 0$ b) $x^2 + 4y - 8 = 0$ c) $x^2 - 8y + 16 = 0$ d) $x^2 + 8y - 16 = 0$

426. One of the diameters of the circle $x^2 + y^2 - 12\,x + 4y + 6 = 0$ is given by
a) $x + y = 0$ b) $x + 3\,y = 0$ c) $x = y$ d) $3\,x + 2\,y = 0$

427. The equation of the director circle of the hyperbola $9x^2 - 16y^2 = 144$ is
a) $x^2 + y^2 = 7$ b) $x^2 + y^2 = 9$ c) $x^2 + y^2 = 16$ d) $x^2 + y^2 = 25$

428. A line is drawn through a fixed point $P(\alpha, \beta)$ to cut the circle $x^2 + y^2 = r^2$ at A and B. Then, $PA \cdot PB$ is equal to
a) $(\alpha + \beta)^2 - r^2$ b) $\alpha^2 + \beta^2 - r^2$ c) $(\alpha - \beta)^2 + r^2$ d) None of these

429. If (α, β) is a point on the chord PQ of the circle $x^2 + y^2 = 25$, where the coordinates of P and Q are $(3, -4)$ and $(4,3)$ respectively, then
a) $3 \leq \alpha \leq 4$ and $-4 \leq \beta \leq 3$
b) $-4 \leq \alpha \leq 3$ and $3 \leq \beta \leq 4$
c) $\alpha = 3$ and $-4 \leq \beta \leq 4$
d) None of these

430. The parabola $y^2 = 8x$ and the circle $x^2 + y^2 = 2$
a) Have only two common tangents which are mutually perpendicular
b) Have only two common tangents which are parallel to each other
c) Have infinitely many common tangents
d) Does not have any common tangent

431. A parabola is drawn with focus at $(3, 4)$ and vertex at the focus of the parabola $y^2 - 12x - 4y + 4 = 0$. The equation of the parabola is
a) $x^2 - 6x - 8y + 25 = 0$ b) $y^2 - 8x - 6y + 25 = 0$
c) $x^2 - 6x + 8y - 25 = 0$ d) $x^2 + 6x - 8y - 25 = 0$

432. The foci of the ellipse, $25(x + 1)^2 + 9(y + 2)^2 = 225$ are at,
a) $(-1,2)$ and $(-1, -6)$
b) $(-2,1)$ and $(-2,6)$
c) $(-1, -2)$ and $(-2, -1)$
d) $(-1, -2)$ and $(-1, -6)$

433. Radius of circle in which a chord length $\sqrt{2}$ makes an angle $\frac{\pi}{2}$ at the centre, is
a) 1 b) $\sqrt{3}$ c) $\frac{\sqrt{3}}{2}$ d) None of these

434. If the focus and vertex of a parabola are the points $(0,2)$ and $(0,4)$ respectively, then its equation is
a) $y^2 = 8x + 32$ b) $y^2 = -88x + 32$ c) $x^2 + 8y = 32$ d) $x^2 - 8y = 32$

435. The foci of an ellipse are $(0, \pm 6)$ and the equations of the directrices are $y = \pm 9$. The equation of the ellipse is
a) $5x^2 + 9y^2 = 4$ b) $2x^2 - 6y^2 = 28$ c) $6x^2 + 3y^2 = 45$ d) $9x^2 + 5y^2 = 180$

436. The point of intersection of tangents at the ends of the latusrectum of the parabola $y^2 = 4x$, is equal to
a) $(1,0)$ b) $(-1,0)$ c) $(0, 1)$ d) $(0, -1)$

437. If a variable circle $x^2 + y^2 - 2ax + 4ay = 0$ intersects the hyperbola $xy = 4$ at the points $(x_i, y_i) =$ 1, 2, 3, 4, then locus of the point $\left(\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}\right)$ is
a) $y + 2x = 0$ b) $y - 2x + 5 = 0$ c) $y - 2x = 0$ d) $y + 4x - 7 = 0$

438. The equation $13[(x - 1)^2 + (y - 2)^2] = 3(2x + 3y - 2)^2$ represents
a) Parabola b) Ellipse c) Hyperbola d) None of these

439. The equation of the circle of radius 5 in the first quadrant which touches x-axis and the line $4y = 3x$ is
a) $x^2 + y^2 - 24x - y - 25 = 0$ b) $x^2 + y^2 - 30x - 10y + 225 = 0$
c) $x^2 + y^2 - 16x - 18y + 64 = 0$ d) $x^2 + y^2 - 20x - 12y + 144 = 0$

440. AB, AC are tangents to a parabola $y^2 = 4ax$; p_1, p_2, p_3 are the lengths of the perpendiculars from A, B, C on any tangent to the curve, then p_2, p_1, p_3 are in
a) AP b) GP c) HP d) None of these

441. A circle cuts rectangular hyperbola $xy = 1$ in the points $(x_r, y_r), r = 1,2,3,4$, then
a) $y_1y_2y_3y_4 = 1$ b) $x_1x_2x_3x_4 = 1$
c) $x_1x_2x_3x_4 = y_1y_2y_3y_4 = -1$ d) $y_1y_2y_3y_4 = 0$

442. If the line $y = 3x + \lambda$ touches the hyperbola $9x^2 - 5y^2 = 45$, then the value of λ is

a) 36 b) 45 c) 6 d) 15

443. The equation of the circle touching $x = 0, y = 0$ and $x = 4$ is

a) $x^2 + y^2 - 4x - 4y + 16 = 0$
b) $x^2 + y^2 - 8x - 8y + 16 = 0$
c) $x^2 + y^2 + 4x + 4y - 4 = 0$
d) $x^2 + y^2 - 4x - 4y + 4 = 0$

444. A, B, C and D are the points of intersection with the coordinate axes of the lines $ax + by = ab$ and $bx + ay = ab$, then

a) A, B, C, D are concylic
b) A, B, C, D form a parallelogram
c) A, B, C, D from a rhombus
d) None of the above

445. The condition for a line $y = 2x + c$ to touch the circle $x^2 + y^2 = 16$ is

a) $c = 10$ b) $c^2 = 80$ c) $c = 12$ d) $c^2 = 64$

446. If $(\sqrt{3})bx + ay = 2ab$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then eccentric angle ϕ is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

447. The equation of the ellipse whose foci are at $(\pm 2, 0)$ and eccentricity is $\frac{1}{2}$, is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Then,

a) $a^2 = 16, b^2 = 12$ b) $a^2 = 12, b^2 = 16$ c) $a^2 = 16, b^2 = 4$ d) $a^2 = 4, b^2 = 16$

448. In the standard form of an ellipse sum of the focal distances of a point is

a) 1 b) $-2a$ c) $2a$ d) None of these

449. The number of common tangents to the circles $x^2 + y^2 - y = 0$ and $x^2 + y^2 + y = 0$ is

a) 2 b) 3 c) 0 d) 1

450. If asymptotes of a hyperbola are at 90°, then

a) Eccentricity is $\sqrt{2}$
b) Eccentricity is 2
c) Eccentricity depends on equation of asymptotes
d) None of the above

451. The centre of a circle is $(2, -3)$ and the circumference is 10π. Then, the equation of the circle is

a) $x^2 + y^2 + 4x + 6y + 12 = 0$
b) $x^2 + y^2 - 4x + 6y + 12 = 0$
c) $x^2 + y^2 - 4x + 6y - 12 = 0$
d) $x^2 + y^2 - 4x - 6y - 12 = 0$

452. The equation of the pair of straight lines parallel to x-axis and touching the circle $x^2 + y^2 - 6x - 4y - 12 = 0$, is

a) $y^2 - 4y - 21 = 0$ b) $y^2 + 4y - 21 = 0$ c) $y^2 - 4y + 21 = 0$ d) $y^2 + 4y + 21 = 0$

453. The equation $y^2 - 8y - x + 19 = 0$ represents

a) a parabola whose focus is $\left(\frac{1}{4}, 0\right)$ and directrix is $x = \frac{-1}{4}$
b) a parabola whose vertex is $(3, 4)$ and directrix is $x = \frac{11}{4}$
c) a parabola whose focus is $\left(\frac{13}{44}, 4\right)$ and vertex is (0, 0)
d) a curve which is not a parabola

454. Centre of circle whose normals are $x^2 - 2xy + 3x + 6y = 0$, is

a) $\left(3, \frac{3}{2}\right)$ b) $\left(3, -\frac{3}{2}\right)$ c) $\left(\frac{3}{2}, 3\right)$ d) None of these

455. If the normal at (1,2) on the parabola $y^2 = 4x$ meets the parabola again at the point $(t^2, 2t)$, then the value of t is

a) 1 b) 3 c) -3 d) 1

456. The locus of the point of intersection of the straight lines $\frac{x}{a} + \frac{y}{b} = \lambda$ and $\frac{x}{a} - \frac{y}{b} = \frac{1}{\lambda}$ (λ is a variable) is

a) A circle b) A parabola c) An ellipse d) A hyperbola

457. The equation of the hyperbola whose foci are $(6, 5), (-4, 5)$ and eccentricity 5/4, is

a) $\frac{(x-1)^2}{16} - \frac{(y-5)^2}{9} = 1$

b) $\frac{x^2}{16}-\frac{y^2}{9}=1$
c) $\frac{(x-1)^2}{16}-\frac{(y-5)^2}{9}=-1$
d) None of these
458. The directrix of the parabola $x^2-4x-8y+12=0$ is
a) $y=0$ b) $x=1$ c) $y=-1$ d) $x=-1$
459. The locus of the vertices of the family of parabolas $y=\frac{a^3x^2}{3}+\frac{a^2x}{2}-2a$ is
a) $xy=\frac{3}{4}$ b) $xy=\frac{35}{16}$ c) $xy=\frac{64}{105}$ d) $xy=\frac{105}{64}$
460. The limiting points of coaxial-system determined by the circles $x^2+y^2+5x+y+4=0$ and $x^2+y^2+10x-4y-1=0$ are
a) $(0,3)$ and $(2,1)$ b) $(0,-3)$ and $(-2,-1)$
c) $(0,3)$ and $(1,2)$ d) $(0,-3)$ and $(2,1)$
461. If the tangent at any point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the tangents at the ends B and B' of minor axis at L and L' respectively, then $BL\cdot B'L'=$
a) a^2 b) b^2 c) a^2+b^2 d) a^2-b^2
462. The equation of the ellipse (referred to its axes as the axes of x and y respectively) which passes through the point $(-3,1)$ and has eccentricity $\sqrt{\frac{2}{5}}$, is
a) $3x^2+6y^2=33$ b) $5x^2+3y^2=48$ c) $3x^2+5y^2=32$ d) None of these
463. The locus of the point which moves such that the ratio of its distance from two fixed point in the plane is always a constant $k(<1)$ is
a) hyperbola b) ellipse c) straight line d) circle
464. The equation $\frac{x^2}{12-k}+\frac{y^2}{8-k}=1$ represents
a) A hyperbola if $k<8$
b) An ellipse if $k>8$
c) A hyperbola if $8<k<12$
d) None of these
465. The straight lines joining the origin to the points of intersection of the line $4x+3y=24$ with the curve $(x-3)^2+(y-4)^2=25$
a) Are coincident
b) Are perpendicular
c) Make equal angles with x-axis
d) None of these
466. The value of k so that $x^2+y^2+kx+4y+2=0$ and $2(x^2+y^2)-4x-3y+k=0$ cut orthogonally, is
a) $\frac{10}{3}$ b) $-\frac{8}{3}$ c) $-\frac{10}{3}$ d) $\frac{8}{3}$
467. The diameter of $16x^2-9y^2=144$ which is conjugate to $x=2y$ is
a) $y=\frac{16x}{9}$ b) $y=\frac{32x}{9}$ c) $x=\frac{16y}{9}$ d) $x=\frac{32y}{9}$
468. The radius of the circle passing through the point $P(6,2)$ and two of whose diameter are $x+y=6$ and $x+2y=4$, is
a) 4 b) 6 c) 20 d) $\sqrt{20}$
469. The total number of tangents through the points $(3,5)$ that can be drawn to the ellipses $3x^2+5y^2=32$ and $25x^2+9y^2=450$ is
a) 0 b) 2 c) 3 d) 4
470. If a point $(x,y)=(\tan\theta+\sin\theta,\tan\theta-\sin\theta)$, then locus of (x,y) is
a) $(x^2y)^{2/3}+(xy^2)^{2/3}=1$ b) $x^2-y^2=4xy$

c) $(x^2-y^2)^2=16xy$ d) $x^2-y^2=6xy$

471. The latusrectum of the ellipse $9x^2+15y^2=144$ is

a) 4 b) $\frac{11}{4}$ c) $\frac{7}{2}$ d) $\frac{9}{2}$

472. If the tangents are drawn to the circle $x^2+y^2=12$ at the point where it meets the circle $x^2+y^2-5x+3y-2=0$, then the point of intersection of these tangents is

a) $(6,-6)$ b) $(6,18/5)$ c) $(6,-18/5)$ d) None of these

473. The difference of the focal distances of any point on the hyperbola is equal to its

a) Latusrectum b) Eccentricity c) Transverse axis d) Conjugate axis

474. Equation of the hyperbola whose vertices are $(\pm 3,0)$ and foci at $(\pm 5,0)$, is

a) $16x^2-9y^2=144$ b) $9x^2-16y^2=144$ c) $25x^2-9y^2=225$ d) $9x^2-25y^2=81$

475. The tangent at point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ cuts the minor axis in Q and PR is drawn perpendicular to the minor axis. If C is the centre of the ellipse, then $CQ\cdot CR=$

a) b^2 b) $2b^2$ c) a^2 d) $2a^2$

476. If equation $(10x-5)^2+(10y-4)^2=\lambda^2(3x+4y-1)^2$ represents a hyperbola, then

a) $-2<\lambda<2$ b) $\lambda>2$ c) $\lambda<-2$ or $\lambda>2$ d) $0<\lambda<2$

477. The tangents at the points $(at_1^2,2at_1),(at_2^2,2at_2)$ on the parabola $y^2=4ax$ are at right angles if

a) $t_1t_2=-1$ b) $t_1t_2=1$ c) $t_1t_2=2$ d) $t_1t_2=-2$

478. The angle between lines joining the origin to the point of intersection of the line $\sqrt{3x+y=2}$ and the curve $y^2-x^2=4$ is

a) $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ b) $\frac{\pi}{6}$ c) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ d) $\frac{\pi}{2}$

479. In an ellipse length of minor axis is 8 and eccentricity is $\frac{\sqrt{5}}{3}$. The length of major axis is

a) 6 b) 12 c) 10 d) 16

480. Three normals to the parabola $y^2=x$ through point $(a,0)$. Then,

a) $a=\frac{1}{2}$ b) $a=\frac{1}{4}$ c) $a>\frac{1}{2}$ d) $a<\frac{1}{2}$

481. If a bar of given length moves with its extremities on two fixed straight lines at right angles, then the locus of any point on bar marked on the bar describes a/an

a) Circle b) Parabola c) Ellipse d) Hyperbola

482. If $3x+y=0$ is a tangent to the circle with centre at the point $(2,-1)$, then the equation of the other tangent to the circle from the origin is

a) $x-3y=0$ b) $x+3y=0$ c) $3x-y=0$ d) $2x+y=0$

483. The combined equation of the asymptotes of the hyperbola $2x^2+5xy+2y^2+4x+5y=0$ is

a) $2x^2+5xy+2y^2=0$ b) $2x^2+5xy+2y^2-4x+5y+2=0$

c) $2x^2+5xy+2y^2+4x+5y-2=0$ d) $2x^2+5xy+2y^2+4x+5y+2=0$

484. Tangents drawn from the point $P(1,8)$ to the circle $x^2+y^2-6x-4y-11=0$ touch the circle at the points $A$and$B$. The equation of the circumcircle of the triangle PAB is

a) $x^2+y^2+4x-6y+19=0$ b) $x^2+y^2-4x-10y+19=0$

c) $x^2+y^2-2x+6y-29=0$ d) $x^2+y^2-6x-4y+19=0$

485. The straight line $x+y=c$ will be tangent to the ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$, c is equal to

a) 8 b) ± 5 c) ± 10 d) ± 6

486. If the vertex of the parabola $y=x^2-8x+c$ lies on x-axis, then the value of c is

a) -16 b) -4 c) 4 d) 16

487. The equation of tangent drawn from the origin to the circle $x^2+y^2-2rx+2hy+h^2=0$ are

a) $x=0,y=0$ b) $x=1,y=0$

c) $(h^2-r^2)x-2rhy=0,y=0$ d) $(h^2-r^2)x-2rhy=0,x=0$

488. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is

a) $2x-\sqrt{5}y-20=0$ b) $2x-\sqrt{5}y+4=0$ c) $3x-4y+8=0$ d) $4x-3y+4=0$

489. Let E be the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and C be the circle $x^2+y^2=9$. Let P and Q be the points (1,2) and (2,1) respectively. Then

a) Q lies inside C but outside E

b) Q lies outside both C and E

c) P lies inside both C and E

d) P lies inside C but outside E

490. All ellipse has its centre at $(1,-1)$ and semi-major axis $=8$ and it passes through the point (1,3). The equation of the ellipse is

a) $\frac{(x+1)^2}{64}+\frac{(y+1)^2}{16}=1$

b) $\frac{(x-1)^2}{64}+\frac{(y+1)^2}{16}=1$

c) $\frac{(x-1)^2}{16}+\frac{(y+1)^2}{64}=1$

d) $\frac{(x+1)^2}{64}+\frac{(y-1)^2}{16}=1$

491. The equation the tangent parallel to $y-x+5=0$, drawn to $\frac{x^2}{3}-\frac{y^2}{2}=1$ is

a) $x-y-1=0$ b) $x-y+2=0$ c) $x+y-1=0$ d) $x+y+2=0$

492. The equation of normal at $\left(at,\frac{a}{t}\right)$ to the hyperbola $xy=a^2$ is

a) $xt^3-yt+at^4-a=0$ b) $xt^3-yt-at^4+a=0$

c) $xt^3+yt+at^4-a=0$ d) None of these

493. S and T are the foci of an ellipse and B is an end of the minor axis If ΔSTB is equilateral , then e is

a) $\frac{1}{4}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) None of these

494. The locus of the point of intersection of the perpendicular tangents to ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ is

a) $x^2+y^2=4$ b) $x^2+y^2=9$ c) $x^2+y^2=5$ d) $x^2+y^2=13$

495. The equations to the common tangents to the two hyperbolas $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$ are

a) $y=\pm x\pm\sqrt{b^2-a^2}$ b) $y=\pm x\pm\sqrt{a^2-b^2}$ c) $y=\pm x\pm(a^2-b^2)$ d) $y=\pm x\pm\sqrt{a^2+b^2}$

496. The equation of the image of the circle $(x-3)^2+(y-2)=1$ in the mirror $x+y=19$, is

a) $(x-14)^2+(y-13)^2=1$

b) $(x-15)^2+(y-14)^2=1$

c) $(x-16)^2+(y-15)^2=1$

d) $(x-17)^2+(y-16^2)=1$

497. The equation of the chord of the ellipse $2x^2+5y^2=20$ which is bisected at the point (2,1) is

a) $4x+5y+13=0$ b) $4x+5y=13$ c) $5x+4y+13=0$ d) None of these

498. The locus of a point which moves so that the ratio of the length of the tangents to the circles $x^2+y^2+4x+3=0$ and $x^2+y^2-6x+5=0$ is 2: 3, is

a) $5x^2+5y^2+60x-7=0$ b) $5x^2+5y^2-60x-7=0$

c) $5x^2+5y^2+60x+7=0$ d) $5x^2+5y^2+60x+12=0$

499. The equation of circle is$x^2+y^2-2x=0$. The point $P(-1,0)$ lies

a) On the circle b) Inside the circle

c) Outside the circle d) On the centre of the circle

500. The area of the circle whose centre is at (2, 3) and passing through (4, 6), is

a) 5π sq units b) 10π sq units c) 13π sq units d) None of these

501. A line through (0, 0) cuts the circle $x^2+y^2-2ax=0$ at A and B, then locus of the centre of the circle drewn AB as diameter is

a) $x^2+y^2-2ay=0$ b) $x^2+y^2+ay=0$ c) $x^2+y^2+ax=0$ d) $x^2+y^2-ax=0$

502. If distance between directrices of a rectangular hyperbola is 10, then distance between its foci will be

a) $10\sqrt{2}$ b) 5 c) $5\sqrt{2}$ d) 20

503. The length of latusrectum of the ellipse $9x^2 + 16y^2 = 144$ is
a) 4 b) $\frac{11}{4}$ c) $\frac{7}{2}$ d) $\frac{9}{2}$

504. If the circle $x^2 + y^2 = a^2$ intersects the hyperbola $xy = c^2$ in four points $P(x_1, y_1), Q(x_2, y_2), R(x_3, y_3)$ and $S(x_4, y_4)$, then
a) $x_1 + x_2 + x_3 + x_4 = 0$ b) $y_1 + y_2 + y_3 + y_4 = 0$
c) $x_1x_2x_3x_4 = c^4$ d) All of these

505. AB is a chord of the parabola $y^2 = 4ax$ with vertex at $A.BC$ is drawn perpendicular to AB meeting the axes at C. The projection of BC on the axis of the parabola is
a) 2 b) $2a$ c) $4a$ d) $8a$

506. The number of common tangents that can be drawn to the circles $x^2 + y^2 - 4x - 6y - 3 = 0$ and $x^2 + y^2 + 2x + 2y + 1 = 0$ is
a) 1 b) 2 c) 3 d) 4

507. If (x_1, y_1) and (x_2, y_x) are the ends of a focal chord of $y^2 4ax$, then $x_1x_2 + y_1y_2$ is equal to
a) $-3a^2$ b) $3a^2$ c) $-4a^2$ d) $4a^2$

508. If two distinct chords drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$ (where $pq \neq 0$) are bisected by the x-axis, then
a) $p^2 = q^2$ b) $p^2 = 8q^2$ c) $p^2 < 8q^2$ d) $p^2 > 8q^2$

509. An equilateral triangle is inscribed in the parabola $y^2 = 4ax$, whose vertex is at the vertex of the parabola. The length of its side is
a) $2a\sqrt{3}$ b) $4a\sqrt{3}$ c) $6a\sqrt{3}$ d) $8a\sqrt{3}$

510. The two circles $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2 + y^2 - 4x - 6y - 8 = 0$ are such that
a) They touch each other
b) They intersect each other
c) One lies inside the other
d) Each lies outside the other

511. One of the directrices of the ellipse $8x^2 + 6y^2 - 16x + 12y + 13 = 0$ is
a) $3y - 3 = \sqrt{6}$ b) $3y + 3 = \sqrt{6}$ c) $y + 1 = \sqrt{3}$ d) $y - 1 = -\sqrt{3}$

512. The line $3x - 2y = k$ meets the circle $x^2 + y^2 = 4r^2$ at only one point if k^2 is equal to
a) $52r^2$ b) $20r^2$ c) $\frac{20}{9}r^2$ d) $\frac{52}{9}r^2$

513. The equation of the parabola with vertex $(-1,1)$ and focus $(2,1)$ is
a) $y^2 - 2y - 12x - 11 = 0$ b) $x^2 + 2x - 12y + 13 = 0$
c) $y^2 - 2y + 12x + 11 = 0$ d) $y^2 - 2y - 12x + 13 = 0$

514. If any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ makes intercepts p and q on the coordinate axes, then $\frac{a^2}{p^2} + \frac{b^2}{q^2} =$
a) 1 b) 2 c) 3 d) 4

515. Equation of the ellipse whose foci are $(2,2)$ and $(4,2)$ and the major axis is of length 10 is
a) $\frac{(x+3)^2}{24} + \frac{(y+2)^2}{25} = 1$ b) $\frac{(x-3)^2}{24} + \frac{(y-2)^2}{25} = 1$
c) $\frac{(x+3)^2}{25} + \frac{(y+2)^2}{24} = 1$ d) $\frac{(x-3)^2}{25} + \frac{(y-2)^2}{24} = 1$

516. If $5x - 12y + 10 = 0$ and $12y - 5x + 16 = 0$ are two tangents to a circle, the radius of the circle is
a) 1 b) 2 c) 4 d) 6

517. If P is a point on the ellipse $\frac{x^2}{16} + \frac{y^2}{20} = 1$ whose foci are S and S'. Then $PS + PS'$ is
a) 8 b) $4\sqrt{5}$ c) 10 d) 4

518. The arithmetic mean of the ordinates of the feet of the normals from $(3, 5)$ to the parabola $y^2 = 8x$ is
a) 4 b) 0 c) 8 d) None of these

519. The points of intersection of the curves whose parametric equations are $x = t^2 + 1, y = 2t$ and $x =$

$2s, y = 2/s$ is given by

a) $(1, -3)$ b) $(2, 2)$ c) $(-2, 4)$ d) $(1, 2)$

520. The eccentricity of a rectangular hyperbola is

a) 2 b) $\sqrt{2}$ c) 0 d) None of these

521. The equation of the circle which cuts orthogonally the circle $x^2 + y^2 - 6x + 4y - 3 = 0$, passes through $(3, 0)$ and touches the axis of y is

a) $x^2 + y^2 + 6x - 6y + 9 = 0$ b) $x^2 + y^2 - 6x + 6y - 9 = 0$
c) $x^2 + y^2 - 6x - 6y + 9 = 0$ d) None of the above

522. The number of rational point (s) (a point (a, b) is rational, if a and b both are rational numbers) on the circumference of a circle having centre (π, e) is

a) At most one b) At least two c) Exactly two d) Infinite

523. The equation of the hyperbola in the standard form (with transverse axis along the x-axis) having the length of the latusrectum= 9 unit and eccentricity $= \frac{5}{4}$, is

a) $\frac{x^2}{16} - \frac{y^2}{18} = 1$ b) $\frac{x^2}{36} - \frac{y^2}{27} = 1$ c) $\frac{x^2}{64} - \frac{y^2}{36} = 1$ d) $\frac{x^2}{36} - \frac{y^2}{64} = 1$

524. The equation of the hyperbola whose foci are $(6, 4)$ and $(-4, 4)$ and eccentricity 2, is

a) $\frac{4(x-1)^2}{25} + \frac{4(y-4)^2}{25} = 1$ b) $\frac{4(x+1)^2}{25} + \frac{4(y+4)^2}{75} = 1$
c) $\frac{4(x-1)^2}{75} - \frac{4(y-4)^2}{25} = 1$ d) $\frac{4(x-1)^2}{25} - \frac{4(y-4)^2}{75} = 1$

525. The subtangent, ordinate and subnormal to the parabola $y^2 = 4ax$ at a point (different from the origin) are in

a) AP b) GP c) HP d) None of these

526. If a line $21x + 5y = 116$ is a tangent to the hyperbola $7x^2 - 5y^2 = 232$, then point of contact is

a) $(-6, 3)$ b) $(6, -2)$ c) $(8, 2)$ d) None of these

527. The equation of the ellipse passing through (2,1) having $e = 1/2$ is

a) $3x^2 + 4y^2 = 16$ b) $3x^2 + 5y^2 = 17$ c) $5x^2 + 3y^2 = 23$ d) None of these

528. For the ellipse $25x^2 + 9y^2 - 150x - 90y + 225 = 0$, the eccentricity e is equal to

a) $\frac{2}{5}$ b) $\frac{3}{5}$ c) $\frac{4}{5}$ d) $\frac{1}{5}$

529. The circles $x^2 + y^2 - 10x + 16 = 0$ and $x^2 + y^2 = r^2$ intersect each other at two distinct points, if

a) $r < 2$ b) $r > 8$ c) $2 < r < 8$ d) $2 \le r \le 8$

530. Three distinct normals to the parabola $y^2 = x$ are drawn through a point $(c, 0)$, then

a) $c = \frac{1}{4}$ b) $c = \frac{1}{2}$ c) $c > \frac{1}{2}$ d) None of these

531. If the length of the major axis of an ellipse is $\frac{17}{8}$ times the length of the minor axis, then the eccentricity of the ellipse is

a) $\frac{8}{17}$ b) $\frac{15}{17}$ c) $\frac{9}{17}$ d) $\frac{2\sqrt{2}}{17}$

532. The eccentricity of the hyperbola conjugate to $x^2 - 3y^2 = 2x + 8$ is

a) $\frac{2}{\sqrt{3}}$ b) $\sqrt{3}$ c) 2 d) None of these

533. The locus of the poles of the focal chords of a parabola is …. of the parabola

a) The axis
b) A focal chord
c) The directrix
d) The tangent at the vertex

534. The eccentricity of the conic $4x^2 + 16y^2 - 24x - 32y = 1$ is

a) $1/2$ b) $\sqrt{3}$ c) $\sqrt{3}/2$ d) $\sqrt{3}/4$

535. If $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ $(abc + 2fgh - af^2 - bg^2 - ch^2 \neq 0)$ represents an ellipse, if

a) $h^2 = ab$ b) $h^2 > ab$ c) $h^2 < ab$ d) None of these

536. The abscissae of two points A and B are the roots of the equation $x^2 - 2ax - b^2 = 0$, and their ordinates are the roots of the equation $x^2 + 2px - q^2 = 0$. The radius of the circle with AB as diameter is

a) $\sqrt{a^2 + p^2}$ b) $\sqrt{b^2 + q^2}$ c) $\sqrt{a^2 + b^2}$ d) $\sqrt{a^2 + b^2 + p^2 + q^2}$

537. The range of values of a for which the point $(a, 4)$ is outside the circles $x^2 + y^2 + 10x = 0$ and $x^2 + y^2 - 12x + 20 = 0$, is

a) $(-\infty, -8) \cup (-2,6) \cup (6, \infty)$

b) $(-8, -2)$

c) $(-\infty, -8) \cup (-2, \infty)$

d) None of these

538. The equation of an ellipse whose eccentricity is $\frac{1}{2}$ and the vertices are, $(4,0)$ and $(10, 0)$ is

a) $3x^2 + 4y^2 - 42x + 120 = 0$ b) $3x^2 + 4y^2 + 42x + 120 = 0$

c) $3x^2 + 4y^2 + 42x - 120 = 0$ d) $3x^2 + 4y^2 - 42x - 120 = 0$

539. If the length of the tangent from any point on the circle $(x - 3)^2 + (y - 2)^2 = 5r^2$ to the circle $(x - 3)^2 + (y + 2)^2 = r^2$ is 16 units, then the area between the two circles in sq units is

a) 32π b) 4π c) 8π d) 256π

540. A tangent is drawn to the circle $2(x^2 + y^2) - 3x + 4y = 0$ and it touches the circle at point A. If the tangent passes through the point $P(2,1)$ then $PA =$

a) 4 b) 2 c) $2\sqrt{2}$ d) None of these

541. Suppose a circle passes through (2, 2) and (9, 9) and touches the x-axis at$P$. If O is the origin, then OP is equal to

a) 4 b) 5 c) 6 d) 9

542. Two perpendicular tangents to $y^2 = 4ax$ always intersect on the line, if

a) $x = a$ b) $x + a = 0$ c) $x + 2a = 0$ d) $x + 4a = 0$

543. The area of the triangle inscribed in the parabola $y^2 = 4x$ the ordinates of whose vertices are 1,2 and 4 is

a) 7/2 sq. units b) 5/2 sq. units c) 3/2 sq. unity d) 3/4 sq. units

544. P is a point on the circle $x^2 + y^2 = c^2$. The locus of the mid-points of chords of contact of P with respect to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, is

a) $c^2\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right) = x^2 + y^2$

b) $c^2\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^2 = x^2 + y^2$

c) $c^2\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right) = (x^2 + y^2)^2$

d) None of these

545. The condition that the chord $x\cos\alpha + y\sin\alpha - p = 0$ of $x^2 + y^2 - a^2 = 0$ may subtend a right angle at the centre of circle, is

a) $a^2 = 2p^2$ b) $p^2 = 2a^2$ c) $a = 2p$ d) $p = 2a$

546. The locus of the equation $x^2 - y^2 = 0$, is

a) a circle b) a hyperbola

c) a pair of lines d) a pair of lines at right angles

547. The eccentricity of the hyperbola $x^2 - y^2 = 2004$ is

a) $\sqrt{3}$ b) 2 c) $2\sqrt{2}$ d) $\sqrt{2}$

548. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points, if

a) $-85 < m < -35$ b) $-35 < m < 15$ c) $15 < m < 65$ d) $35 < m < 85$

549. The length of the tangent drawn to the circle $x^2 + y^2 - 2x + 4y - 11 = 0$ from the point (1, 3) is

a) 1 b) 2 c) 3 d) 4

550. The product of the lengths of perpendiculars drawn from any point on the hyperbola $x^2 - 2y^2 - 2 = 0$ to

its asymptotes is

a) 1/2 b) 2/3 c) 3/2 d) 20

551. If tangent at any point P on the ellipse $7x^2 + 26y^2 = 12$ cuts the tangent at the end points of the major axis at the points A and B, then the circle with AB as diameter passes through a fixed point whose coordinates are

a) $\left(\pm\sqrt{a^2 - b^2}, 0\right)$ b) $\left(\pm\sqrt{a^2 + b^2}, 0\right)$ c) $\left(0, \pm\sqrt{a^2 - b^2}\right)$ d) $\left(0, \pm\sqrt{a^2 + b^2}\right)$

552. The circles $x^2 + y^2 - 8x + 4y + 4 = 0$ touches

a) x-axis b) y-axis

c) Both axes d) Neither x-axis not y-axis

553. If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct points P and Q, then the line $5x + by - a = 0$ passes through P and Q for

a) Exactly two values of a b) Infinitely many values of a

c) No value of a d) Exactly one value of a

554. The two ends of latusrectum of a parabola are the points (3,6) and (−5,6). The focus is

a) (1,6) b) (−1,6) c) (1,−6) d) (−1,−6)

555. Equation of the chord of the hyperbola $25x^2 - 16y^2 = 400$ which is bisected at the point (6, 2), is

a) $16x - 75y = 418$ b) $75x - 16y = 418$ c) $25x - 4y = 400$ d) None of these

556. Equation of the circle passing through the intersection of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, is

a) $x^2 + y^2 = a^2$ b) $x^2 + y^2 = b^2$ c) $x^2 + y^2 = \frac{a^2b^2}{a^2 + b^2}$ d) $x^2 + y^2 = \frac{2a^2b^2}{a^2 + b^2}$

557. Let a focal chord of parabola $y^2 = 16x$ cuts it at points (f, g) and (h, k) Then, $f.h$ is equal to

a) 12 b) 16 c) 14 d) None of these

558. If $y = 3x$ is a tangent to a circle with centre (1, 1), then the other tangent drawn through (0, 0) to the circle is

a) $3y = x$ b) $y = -3x$ c) $y = 2x$ d) $y = -2x$

559. If the curve $xy = R^2 - 16$ represents a rectangular hyperbola whose branches lies only in the quadrant in which abscissa and ordinate are opposite in sigh but not equal in magnitude, then

a) $|R| < 4$ b) $|R| \geq 4$ c) $|R| = 4$ d) None of these

560. Locus of the middle points of all chords of the parabola $y^2 = 4x$ which are drawn through the vertex is

a) $y^2 = 8x$ b) $y^2 = 2x$ c) $x^2 + 4y^2 = 16$ d) $x^2 = 2y$

561. The circles on focal radii of a parabola as diameter touch

a) The tangent at the vertex

b) The axis

c) The directrix

d) None of these

562. Let O be the origin and A be a point on the curve $y^2 = 4x$. Then, the locus of the mid point of OA, is

a) $x^2 = 4y$ b) $x^2 = 2y$ c) $x^2 = 16y$ d) $y^2 = 2x$

563. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is

a) $2ax + 2by + (a^2 + b^2 + 4) = 0$ b) $2ax + 2by - (a^2 + b^2 + 4) = 0$

c) $2ax - 2by + (a^2 + b^2 + 4) = 0$ d) $2ax - 2by - (a^2 + b^2 + 4) = 0$

564. The length of the normal of the parabola $y^2 = 4x$ which subtends a right angle at the vertex is

a) $6\sqrt{3}$ b) $3\sqrt{3}$ c) 2 d) 1

565. The eccentricity of the hyperbola $3x^2 - 4y^2 = -12$ is

a) $\sqrt{\frac{7}{3}}$ b) $\frac{\sqrt{7}}{2}$ c) $-\sqrt{\frac{7}{3}}$ d) $-\frac{\sqrt{7}}{2}$

566. A variable circle passes through the fixed point (2,0) and touches y-axis. Then, the locus of its centre is

a) A parabola b) A circle c) An ellipse d) A hyperbola

567. If the lines $2x-3y=5$ and $3x-4y=7$ are two diameters of a circle of radius 7, then the equation of the circle is

a) $x^2+y^2+2x-4y-47=0$
b) $x^2+y^2=49$
c) $x^2+y^2-2x+2y-47=0$
d) $x^2+y^2=17$

568. If a point P moves such that its distances from the point $A(1,1)$ and the line $x+y+2=0$ are equal, then the locus of P is

a) A straight line
b) A pair of straight lines
c) A parabola
d) An ellipse

569. Consider the two curves $C_1: y^2=4x$
$C_2: x^2+y^2-6x+1=0$, then

a) C_1 and C_2 touch each other only at one point
b) C_1 and C_2 touch each other exactly at two points
c) C_1 and C_2 intersect (but do not touch) at exactly two point
d) C_1 and C_2 neither intersect nor touch each other

570. If $g^2+f^2=c$, then the equation $x^2+y^2+2gx+2fy+c=0$ will represent

a) A circle of radius g
b) A circle of radius f
c) A circle of diameter $\sqrt{c}$
d) A circle of radius zero

571. The two lines through (2,3) from which the circle $x^2+y^2=25$ intercepts chords of length 8 units have equations

a) $2x+3y=13, x+5y=17$
b) $y=3, 12x+5y=39$
c) $x=2, 9x-11y=51$
d) None of these

572. The shortest distance between the parabola $y^2=4x$ and the circle $x^2+y^2+6x-12y+20=0$ is

a) $4\sqrt{2}-5$
b) 0
c) $3\sqrt{2}+5$
d) 1

573. The equation of normal at the point $(0,3)$ of the ellipse $9x^2+5y^2=45$, is

a) x-axis
b) y-axis
c) $y+3=0$
d) $y-3=0$

574. If the line $x\cos\alpha+y\sin a=p$ be normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then

a) $p^2(a^2\cos^2\alpha+b^2\sin^2\alpha)=a^2-b^2$
b) $p^2(a^2\cos^2\alpha+b^2\sin^2\alpha)=(a^2-b^2)^2$
c) $p^2(a^2\sec^2\alpha+b^2\operatorname{cosec}^2\alpha)=a^2-b^2$
d) $p^2(a^2\sec^2\alpha+b^2 cosec^2\alpha)=(a^2-b^2)^2$

575. If the chord of contact of tangents drawn from P to the parabola $y^2=4ax$ touches the rectangular hyperbola $x^2-y^2=a^2$, then P lies on

a) $4x^2-y^2=a^2$
b) $y^2-4x^2=4a^2$
c) $4x^2+y^2=4a^2$
d) $4y^2-x^2=4a^2$

576. Two perpendicular tangents drawn to the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ intersect on the curve

a) $x=\frac{a}{e}$
b) $x^2+y^2=41$
c) $x^2+y^2=9$
d) $x^2-y^2=41$

577. A line through $P(1,4)$ intersect a circle $x^2+y^2=16$ at A and B, then $PA.PB$ is equal to

a) 1
b) 2
c) 3
d) 4

578. If the circle $x^2+y^2+4x+22y+c=0$ bisects the circumference of the circle $x^2+y^2-2x+8y-d=0$, then $c+d$ is equal to

a) 60
b) 50
c) 40
d) 30

579. Equation of tangent to the parabola $y^2=16x$ at $P(3,6)$ is

a) $4x-3y+12=0$
b) $3y-4x-12=0$
c) $4x-3y-24=0$
d) $3y-x-24=0$

580. Let (α,β) be a point from which two perpendicular tangents can be drawn to the ellipse $4x^2+5y^2=20$. If $F=4\alpha+3\beta$, then

a) $-15\le F\le 15$
b) $F\ge 0$
c) $-5\le F\le 20$
d) $F\le -5\sqrt{5}$ or $F\ge 5\sqrt{5}$

581. If $\tan\theta_1, \tan\theta_2=\frac{a^2}{b^2}$, then the chord joining two points θ_1 and θ_2 on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ will subtend a

right angle at

a) Focus b) Centre

c) End of the major axis d) End of the minor axis

582. The equation of the circle which passes through the intersection of $x^2+y^2+13x-3y=0$ and $2x^2+2y^2+4x-7y-25=0$ and whose centre lies on $13x+30y=0$, is

a) $x^2+y^2+30x-13y-25=0$ b) $4x^2+4y^2+30x-13y-25=0$

c) $2x^2+2y^2+30x-13y-25=0$ d) $x^2+y^2+30x-13y+25=0$

583. If P and Q are the points of intersection of the circles

$x^2+y^2+3x+7y+2p-5=0$

$x^2+y^2+2x+2y-p^2=0$, then there is a circle passing through P, Q and $(1, 1)$ and

a) All values of p b) All except one value of p

c) All except two values of p d) Exactly one value of p

584. The number of distinct normals that can be drawn to parabola $y^2=16x$ from the point $(2, 0)$, is

a) 1 b) 2 c) 3 d) 0

585. P is any point on the ellipse $81x^2+144y^2=1944$, whose foci are S and S'. Then, $SP+S'P$ equals

a) 3 b) $4\sqrt{6}$ c) 36 d) 324

586. If the tangent at P and Q on the parabola meet in T, then SP, ST and SQ are in

a) AP b) GP c) HP d) None of these

587. Tangent is drawn to the ellipse $\frac{x^2}{27}+y^2=1$ at $(3\sqrt{3}\cos\theta, \sin\theta)$ $\left[\text{where } \theta \in \left(0, \frac{\pi}{2}\right)\right]$. Then, the value of θ such that sum of intercepts on axes made by this tangent is minimum, is

a) $\pi/3$ b) $\pi/6$ c) $\pi/8$ d) $\pi/4$

588. The length of the common chord of the circles $x^2+y^2+2x+3y+1=0$ and $x^2+y^2+4x+3y+2=0$ is

a) $\frac{9}{2}$ b) $2\sqrt{2}$ c) $3\sqrt{2}$ d) $\frac{3}{2}$

589. The circles $x^2+y^2-10x+16=0$ and $x^2+y^2=r^2$ intersect each other at two distinct points, if

a) $r<2$ b) $r>8$ c) $2<r<8$ d) $2\leq r\leq 8$

590. The number of circles that touch all the straight lines $x+y-4=0, x-y+2=0$ and $y=2$, is

a) 1 b) 2 c) 3 d) 4

591. The equation of a diameter conjugate to a diameter $y=\frac{b}{a}x$ of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

a) $y=-\frac{b}{a}x$ b) $y=-\frac{a}{b}x$ c) $y=\frac{a}{b}x$ d) None of these

592. The set of points on the axis of the parabola $y^2-2y-4x+5=0$ from which all the three normals to the parabola are real, is

a) $\{(x, 1): x\geq 3\}$ b) $\{(x, -1): x\geq 1\}$ c) $\{(x, 3): x\geq 1\}$ d) $\{(x, -3): x\geq 3\}$

593. A variable circle passes through the fixed point (2,0) and touches the y-axis. Then, the locus of its centre, is

a) A parabola b) A circle c) An ellipse d) A hyperbola

594. Equation of the circle passing through the point (3, 4) and concentric with the circle $x^2+y^2-2x-4y+1=0$ is

a) $x^2+y^2-2x-4y=0$ b) $x^2+y^2-2x-4y+3=0$

c) $x^2+y^2-2x-4y-3=0$ d) None of the above

595. The point of the straight line $y=2x+11$ which is nearest to the circle $16(x^2+y^2)+32x-8y-50=0$, is

a) $9/2, 2$ b) $(-9/2, 2)$ c) $(9/2, -2)$ d) None of these

596. The distance of the centre of ellipse $x^2+2y^2-2=0$ to those tangents of the ellipse which are equally inclined from both the axes, is

a) $\frac{3}{\sqrt{2}}$ b) $\sqrt{\frac{3}{2}}$ c) $\frac{\sqrt{2}}{3}$ d) $\frac{\sqrt{3}}{2}$

597. The equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a hyperbola, if

a) $\Delta \neq 0, h^2 < ab$ b) $\Delta \neq 0, h^2 > ab$ c) $\Delta \neq 0, h^2 = ab$ d) $\Delta \neq 0, a + b = 0$

598. Two circles $x^2 + y^2 - 2x - 3 = 0$ and $x^2 + y^2 - 4x - 6y - 8 = 0$ are such that
a) They touch internally b) They touch externally
c) They intersect at two points d) They are non-intersecting

599. If the eccentricity of a hyperbola is $\sqrt{3}$, then the eccentricity of its conjugate hyperbola is
a) $\sqrt{2}$ b) $\sqrt{3}$ c) $\sqrt{\frac{3}{2}}$ d) $2\sqrt{3}$

600. If the normal at $(ap^2, 2ap)$ on the parabola $y^2 = 4ax$, meets the parabola again at $(aq^2, 2aq)$, then
a) $p^2 + pq + 2 = 0$ b) $p^2 - pq + 2 = 0$ c) $q^2 + pq + 2 = 0$ d) $p^2 + pq + 1 = 0$

601. The equation of the conic with focus at $(1, -1)$, directrix along $x - y + 1 = 0$ and with eccentricity $\sqrt{2}$ is
a) $x^2 - y^2 = 1$
b) $xy = 1$
c) $2xy - 4x + 4y + 1 = 0$
d) $2xy - 4x + 4y - 1 = 0$

602. PQ is a chord of the circle $x^2 + y^2 - 2x - 8 = 0$ whose midpoint is (2,2). The circle passing through P, Q and (1,2) is
a) $x^2 + y^2 - 7x + 10y + 28 = 0$
b) $x^2 + y^2 - 7x - 10y + 22 = 0$
c) $x^2 + y^2 + 7x + 10y - 22 = 0$
d) $x^2 + y^2 + 7x + 10y - 22 = 0$

603. The circle $x^2 + y^2 - 3x - 4y + 2 = 0$ cuts x-axis at
a) $(2, 0), (-3, 0)$ b) $(3, 0), (4, 0)$ c) $(1, 0), (-1, 0)$ d) $(1, 0), (2, 0)$

604. The normal at $(ap^2, 2\,ap)$ on $y^2 = 4\,ax$, meets the curve again at $(aq^2, 2\,aq)$ then
a) $p^2 + pq + 2 = 0$ b) $p^2 - pq + 2 = 0$ c) $q^2 + pq + 2 = 0$ d) $p^2 + pq + 1 = 0$

605. Let L_1 be a straight line passing through the origin and L_2 be the straight line $x + y = 1$. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 and L_2 are equal, then L_1 can be represented by
a) $x + y = 0$ b) $x - y = 0$ c) $7x + y = 0$ d) $x - 7y = 0$

606. The angle between the asymptotes of the hyperbola $2x^2 - 2y^2 = 9$ is
a) $\pi/4$ b) $\pi/3$ c) $\pi/6$ d) $\pi/2$

607. A focus of an ellipse is at the origin. The directrix is the line $x = 4$ and the eccentricity is $\frac{1}{2,}$ then length of semi major axis is
a) 5/3 b) 8/3 c) 2/3 d) 4/3

608. If $3x + y = 0$ is tangent to the circle having its centre at $(2, -1)$, then the equation of other tangent to the circle from the origin is
a) $x - 3y = 0$ b) $x + 3y = 0$ c) $3x - y = 0$ d) $2x + y = 0$

609. The equation of the image of the circle $x^2 + y^2 + 16x - 24y + 183 = 0$ *by the line mirror* $4x + 7y + 13 = 0$ *is*
a) $x^2 + y^2 + 32\,x - 4\,y + 235 = 0$
b) $x^2 + y^2 + 32\,x + 4\,y - 235 = 0$
c) $x^2 + y^2 + 32\,x - 4\,y - 235 = 0$
d) $x^2 + y^2 + 32\,x + 4\,y + 235 = 0$

610. The equation $x^2 + y^2 + 4\,x + 6\,y + 13 = 0$ represents
a) A circle
b) A pair of two straight line
c) A pair of containing straight lines
d) A point

611. The radical centre of the circles
$x^2 + y^2 - 16x + 60 = 0,$
$x^2 + y^2 - 12x + 27 = 0$

and $x^2 + y^2 - 12x + 8 = 0$ is

a) $\left(13, \frac{33}{4}\right)$
b) $\left(\frac{33}{4}, -13\right)$
c) $\left(\frac{33}{4}, 13\right)$
d) None of these

612. The equation of the tangent to the conic $x^2 - y^2 - 8x + 2y + 11 = 0$ at $(2, 1)$ is

a) $x + 2 = 0$
b) $2x + 1 = 0$
c) $x + y + 1 = 0$
d) $x - 2 = 0$

613. An ellipse has OBas semi minor axis, F and F it's foci and the angle FBF' is a right angle. Then, the eccentricity of the ellipse is

a) $\frac{1}{\sqrt{3}}$
b) $\frac{1}{4}$
c) $\frac{1}{2}$
d) $\frac{1}{\sqrt{2}}$

614. The eccentric angle of the point of contact of the line $\frac{x}{a} + \frac{y}{b} = \sqrt{2}$ with the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, is

a) 0
b) $\pi/3$
c) $\pi/4$
d) $\pi/2$

615. The length of the latusrectum of the ellipse $5x^2 + 9y^2 = 45$ is

a) $\frac{5}{3}$
b) $\frac{10}{3}$
c) $\frac{2\sqrt{5}}{5}$
d) $\frac{\sqrt{5}}{3}$

616. If the lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are two diameters of a circle of area 49π sq unit, the equation of the circle is

a) $x^2 + y^2 + 2x - 2y - 62 = 0$
b) $x^2 + y^2 - 2x + 2y - 62 = 0$
c) $x^2 + y^2 - 2x + 2y - 47 = 0$
d) $x^2 + y^2 + 2x - 2y - 47 = 0$

617. The angle made by a double ordinate of length $8a$ at the vertex of the parabola $y^2 = 4ax$ is

a) $\frac{\pi}{3}$
b) $\frac{\pi}{2}$
c) $\frac{\pi}{4}$
d) $\frac{\pi}{6}$

618. Number of common tangents to the parabola $y^2 = 4ax$ and $x^2 = 4by$ is

a) 4
b) 3
c) 2
d) 1

619. The number of normals to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ from an external point is

a) 6
b) 5
c) 4
d) 2

620. The tangent drawn from (α, β) to an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ touches the circle $x^2 + y^2 = c^2$, then the locus of (α, β) is

a) An ellipse
b) A circle
c) A parabola
d) None of these

621. The eccentricity of the hyperbola whose asymptotes are $3x + 4y = 2$ and $4x - 3y + 5 = 0$, is

a) 1
b) 2
c) $\sqrt{2}$
d) None of these

622. The normal at $(a, 2a)$ on $y^2 = 4\,ax$, meets the curve again at $(at^2, 2at)$, then the value of t is

a) 1
b) 3
c) -1
d) -3

623. The curve represented by $x = a(\cosh\theta + \sinh\theta)$, $y = b(\cosh\theta - \sinh\theta)$, is

a) A hyperbola
b) An ellipse
c) A parabola
d) A circle

624. If $x - 2y - a = 0$ is a chord of $y^2 = 4ax$, then its length is

a) $4a\sqrt{5}$
b) $40\,a$
c) $20\,a$
d) $15\,a$

625. The equation of the normal at the point of contact of a tangent $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$, is

a) $y = mx - 2\,am - am^3$
b) $m^3y = m^2x - 2\,am^2 - a$
c) $m^3y = 2\,am^2 - m^2x + a$
d) None of these

626. The point, at shortest distance from the line $x + y = 7$ and lying on an ellipse $x^2 + 2y^2 = 6$, has coordinates

a) $(\sqrt{2}, \sqrt{2})$
b) $(0, \sqrt{3})$
c) $(2, 1)$
d) $\left(\sqrt{5}, \frac{1}{\sqrt{2}}\right)$

627. The equation of any tangent to the circle $x^2 + y^2 - 2x + 4y - 4 = 0$, is

a) $y = m(x-1)^2 + 3\sqrt{1+m^2} - 2$
b) $y = mx + 3\sqrt{1+m^2}$

c) $y = mx + 3\sqrt{1+m^2} - 2$
d) None of these

628. Origin is a limiting point of a coaxial system of which $x^2 + y^2 - 6x - 8y + 1 = 0$ is a member. The other limiting point is

a) $(-2, -4)$ b) $(3/25, 4/25)$ c) $(-3/25, -4/25)$ d) $4/25, 3/25$

629. The line $5x + 12y = 9$ touches the hyperbola $x^2 - 9y^2 = 9$ at the point

a) $(-5, 4/3)$ b) $(5, -4/3)$ c) $(3, -1/2)$ d) None of these

630. Two perpendicular tangents to the circle $x^2 + y^2 = a^2$ meet at P. Then, the locus of P has the equation

a) $x^2 + y^2 = 2a^2$ b) $x^2 + y^2 = 3a^2$ c) $x^2 + y^2 = 4a^2$ d) None of these

631. Two points P and Q are taken on the line joining the points $A(0,0)$ and $B(3a, 0)$ such that $AP = PQ = QB$. Circles are drawn on AP, PQ and QB as diameters
The locus of the points, the sum of the squares of the tangents from which to the three circles is equal to b^2, is

a) $x^2 + y^2 - 3ax + 2a^2 - b^2 = 0$
b) $3(x^2 + y^2) - 9ax + 8a^2 - b^2 = 0$
c) $x^2 + y^2 - 5ax + 6a^2 - b^2 = 0$
d) $x^2 + y^2 - ax - b^2 = 0$

632. The value of c, for which the line $y = 2x + c$, is tangent to the parabola $y^2 = 4a(x + a)$, is

a) a b) $\frac{3a}{2}$ c) $2a$ d) $\frac{5a}{2}$

633. The equation of a parabola which passes through the intersection of a straight line $x + y = 0$ and the circle $x^2 + y^2 + 4y = 0$ is

a) $y^2 = 4x$ b) $y^2 = x$ c) $y^2 = 2x$ d) None of these

634. If the line $lx + my = 1$ is a normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $\frac{a^2}{l^2} - \frac{b^2}{m^2}$ is equal to

a) $a^2 - b^2$ b) $a^2 + b^2$ c) $(a^2 + b^2)^2$ d) $(a^2 - b^2)^2$

635. The circle $x^2 + y^2 - 4x - 4y + 4 = 0$ is inscribed in a triangle which has two of its sides along the coordinate axes. If the locus of the circumcentre of the triangle is $x + y - xy + k\sqrt{x^2 - y^2} = 0$, then the value of k is equal to

a) 2 b) 1 c) -2 d) 3

636. Locus of the point which divides double ordinate of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the ratio 1:2 internally, is

a) $\frac{x^2}{a^2} - \frac{9y^2}{b^2} = \frac{1}{9}$ b) $\frac{x^2}{a^2} + \frac{9y^2}{b^2} = 1$ c) $\frac{9x^2}{a^2} + \frac{9y^2}{b^2} = 1$ d) None of these

637. Equation of the parabola with its vertex at (1,1) and focus (3,1) is

a) $(x-1)^2 = 8(y-1)$ b) $(y-1)^2 = 8(x-3)$ c) $(y-1)^2 = 8(x-1)$ d) $(x-3)^2 = 8(y-1)$

638. A circle of radius 5 touches another circle
$x^2 + y^2 - 2x - 4y - 20 = 0$ at $(5, 5)$, then its equation is

a) $x^2 + y^2 + 18x + 16y + 120 = 0$ b) $x^2 + y^2 - 18x - 16y + 120 = 0$
c) $x^2 + y^2 - 18x + 16y + 120 = 0$ d) None of the above

639. If the chord of contact of tangents from a point $P(x_1, y_1)$ to the circle $x^2 + y^2 = a^2$ touches the circle $(x - a)^2 + y^2 = a^2$, then the locus of (x_1, y_1), is

a) A circle b) A parabola c) An ellipse d) A hyperbola

640. A line is at a constant distance c from the origin and meets the coordinate axes in A and B. The locus of the centre of the circle passing through O, A, B is

a) $x^{-2} + y^{-2} = c^{-2}$ b) $x^{-2} + y^{-2} = 2c^{-2}$ c) $x^{-2} + y^{-2} = 3c^{-2}$ d) $x^{-2} + y^{-2} = 4c^{-2}$

641. An equilateral triangle SAB is inscribed in the parabola $y^2 = 4ax$ having it's focus at S. If chord AB lies towards the left S, then length of the side of this triangle is

a) $3a(2 - \sqrt{3})$ b) $4a(2 - \sqrt{3})$ c) $2a(2 - \sqrt{3})$ d) $8a(2 - \sqrt{3})$

642. If the foci and vertices of an ellipse be $(\pm 1, 0)$ and $(\pm 2, 0)$ then the minor axis of the ellipse is

a) $2\sqrt{5}$ b) 2 c) 4 d) $2\sqrt{3}$

643. If the circle $x^2+y^2+2x+3y+1=0$ cuts $x^2+y^2+4x+3y+2=0$ in A and B, then the equation of the circle on AB as diameter is
a) $x^2+y^2+x+3y+3=0$
b) $2x^2+2y^2+2x+6y+1=0$
c) $x^2+y^2+x+6y+1=0$
d) None of these

644. Two tangents to the circle $x^2+y^2=4$ at the points A and B meet at $p(-4,0)$. The area of the quadrilateral $PAOB$, where O is the origin, is
a) 4 sq units b) $6\sqrt{2}$ sq units c) $4\sqrt{3}$ sq units d) None of these

645. Tangent at a point of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is drawn which cuts the coordinate axes at A and B. The minimum area of the ΔOAB is (O being the origin)
a) ab b) $\frac{a^3+ab+b^3}{3}$ c) a^2+b^2 d) $\frac{(a^2+b^2)}{4}$

646. The vertex of the parabola $x^2+2y=8x-7$ is
a) $\left(\frac{9}{2},0\right)$ b) $\left(4,\frac{9}{2}\right)$ c) $\left(2,\frac{9}{2}\right)$ d) $\left(4,\frac{7}{2}\right)$

647. Let C be the circle with centre (0, 0) and radius 3 unit. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $\frac{2\pi}{3}$ at its centre, is
a) $x^2+y^2=1$ b) $x^2+y^2=\frac{27}{4}$ c) $x^2+y^2=\frac{9}{4}$ d) $x^2+y^2=\frac{3}{2}$

648. The equation of the circle passing through (1, 1) and the points of intersection of $x^2+y^2+13x-3y=0$ and $2x^2+2y^2+4x-7y-25=0$ is
a) $4x^2+4y^2-30x-10y=25$
b) $4x^2+4y^2+30x-13y-25=0$
c) $4x^2+4y^2-17x-10y+25=0$
d) None of the above

649. If the line $x\cos\alpha+y\sin\alpha=p$ be normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then
a) $p^2(a^2\cos^2\alpha+b^2\sin^2\alpha)=a^2-b^2$
b) $p^2(a^2\cos^2\alpha+b^2\sin^2\alpha)=(a^2-b^2)^2$
c) $p^2(a^2\sec^2\alpha+b^2\text{cosec}^2\alpha)=a^2-b^2$
d) $p^2(a^2\sec^2\alpha+b^2\text{cosec}^2\alpha)=(a^2-b^2)^2$

650. Asymptotes of a hyperbola $\frac{x^2}{25}-\frac{y^2}{16}=1$ are
a) $x=\pm\frac{25}{16}y$ b) $x=\pm\frac{4}{5}y$ c) $y=\pm\frac{5}{4}x$ d) $y=\pm\frac{4}{5}x$

651. The line among the following which touches the parabola $y^2=4ax$, is
a) $x+my+am^3=0$ b) $x-my+am^2=0$ c) $x+my-am^2=0$ d) $y+mx+am^2=0$

652. The tangents from a point $(2\sqrt{2},1)$ to the hyperbola $16x^2-25y^2=400$ include an angle equal to
a) $\pi/2$ b) $\pi/4$ c) π d) $\pi/3$

653. The limiting points of the coaxial system of circles $x^2+y^2+2\lambda y+4=0$ are
a) $(0,\pm4)$ b) $(\pm2,0)$ c) $(0,\pm1)$ d) $(0,\pm2)$

654. The equation of the circle which passes through the origin and cuts orthogonally each of the circles $x^2+y^2-6x+8=0$ and $x^2+y^2-2x-2y-7=0$ is
a) $3x^2+3y^2-8x-13y=0$
b) $3x^2+3y^2-8x+29y=0$
c) $3x^2+3y^2+8x+29y=0$
d) $3x^2+3y^2-8x-29y=0$

655. If a normal of slope m to the parabola $y^2=4ax$ touches the hyperbola $x^2-y^2=a^2$, then
a) $m^6-4m^4-3m^2+1=0$
b) $m^6-4m^4+3m^2-1=0$
c) $m^6+4m^4-3m^2+1=0$
d) $m^6+4m^4+3m^2+1=0$

656. If two tangents drawn from a point P to the parabola $y^2 = 4x$ be such that the slope of one tangent is double of the other, then P lies on the curve

a) $9y = 2x^2$ b) $9x = 2y^2$ c) $2x = 9y^2$ d) None of these

657. The other end of the diameter through the point $(-1, 1)$ on the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ is

a) $(-7, 5)$ b) $(-7, -5)$ c) $(7, -5)$ d) $(7, 5)$

658. Angle between tangents drawn from the point $(5, 4)$ to the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, is

a) $60°$ b) $90°$ c) $120°$ d) $45°$

659. A common tangent to circle $x^2 + y^2 = 16$ and an ellipse is $\frac{x^2}{49} + \frac{y^2}{4} = 1$ is

a) $y = x + 4\sqrt{5}$ b) $y = x + \sqrt{53}$ c) $y = \frac{2}{\sqrt{11}}x + \frac{4\sqrt{4}}{\sqrt{11}}$ d) None of these

660. $y = 4x^2$ and $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$ intersect, if

a) $|a| \leq \frac{1}{\sqrt{2}}$ b) $a < -\frac{1}{\sqrt{2}}$ c) $a > -\frac{1}{\sqrt{2}}$ d) None of these

661. Let AB be the intercept of the line $y = x$ the circle$x^2 + y^2 - 2x = 0$. Then, the equation of the circle with AB as its diameter is

a) $x^2 + y^2 - x - y = 0$ b) $x^2 + y^2 + x + y = 0$

c) $x^2 + y^2 + 2(x - y) = 0$ d) $x^2 + y^2 - 2x + y = 0$

662. The ends of the latusrectum of the conic $x^2 + 10x - 16y + 25 = 0$ are

a) $(3, -4), (13, 4)$ b) $(-3, -4), (13, -4)$ c) $(3, 4), (-13, 4)$ d) $(5, -8), (-5, 8)$

663. Equation of a circle passing through the origin and making intercept by the line $4x + 3y = 12$ with coordinate axes, is

a) $x^2 + y^2 + 3x + 4y = 0$ b) $x^2 + y^2 + 3x - 4y = 0$

c) $x^2 + y^2 - 3x + 4y = 0$ d) $x^2 + y^2 - 3x - 4y = 0$

664. The area of square inscribed in a circle $x^2 + y^2 - 6x - 8y = 0$ is

a) 100 sq unit b) 50 sq unit c) 25 sq unit d) None of these

665. The length of the transverse axis of the rectangular hyperbola $xy = 18$ is

a) 6 b) 12 c) 18 d) 9

666. Equation of the circle which touches $3x + 4y = 7$ and passes through $(1, -2)$ and $(4, -3)$ is

a) $x^2 + y^2 - 94x + 18y + 55 = 0$

b) $15x^2 + 15y^2 - 94x + 18y + 55 = 0$

c) $15x^2 + 15y^2 + 94x + 18y + 55 = 0$

d) $x^2 + y^2 - 94x - 18y + 55 = 0$

667. The line $3x + 5y = 15\sqrt{2}$ is a tangent to the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$, at a point whose eccentric angle is

a) $\pi/6$ b) $\pi/4$ c) $\pi/3$ d) $2\pi/3$

668. The coordinates of the centre of the smallest circle passing through the origin and having $y = x + 1$ as a diameter are

a) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ b) $\left(\frac{1}{2}, \frac{1}{3}\right)$ c) $(-1, 0)$ d) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

669. If the tangent at the point $(a \sec \alpha, b \tan \alpha)$ to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meets the transverse axis at T, then the distance of T from a focus of the hyperbola is

a) $b(e - \cos \alpha)$ b) $b(e + \cos \alpha)$ c) $a(e + \cos \alpha)$ d) $\sqrt{a^2e^2 + b^2 \cot^2 \alpha}$

670. For an ellipse with eccentricity $1/2$ the centre is at the origin. If one directrix is $x = 4$, then the equation of the ellipse is

a) $3x^2 + 4y^2 = 1$ b) $3x^2 + 4y^2 = 12$ c) $4x^2 + 3y^2 = 1$ d) $4x^2 + 3y^2 = 12$

671. If the focal distance of an end of the minor axis of an ellipse (referred to its axes as the axes of x and y respectively) is k and the distance between its foci is $2h$, then its equation is

a) $\frac{x^2}{k^2}+\frac{y^2}{h^2}=1$ b) $\frac{x^2}{k^2}+\frac{y^2}{k^2-h^2}=1$ c) $\frac{x^2}{k^2}+\frac{y^2}{h^2-k^2}=1$ d) $\frac{x^2}{k^2}+\frac{y^2}{k^2+h^2}=1$

672. The straight line $x+y-1=0$ meets the circle $x^2+y^2-6x-8y=0$ at $A$and B. Then, the equation of the circle of which AB is a diameter is

a) $x^2+y^2-2y-6=0$ b) $x^2+y^2+2y-6=0$

c) $2(x^2+y^2)+2y-6=0$ d) $3(x^2+y^2)+2y-6=0$

673. The curve represented by $x=3\,(\cos t+\sin t),\ y=4(\cos t-\sin t)$ is

a) Ellipse b) Parabola c) Hyperbola d) Circle

674. C_1 And C_2 are circles of unit radius with centres at $(0,0)$ and $(1,0)$ respectively. C_3 Is a circle of unit radius, passes through the centres of the circles C_1 and C_2 and have its centre above x-axis. Equation of the common tangent to C_1 and C_2 which does not pass through C_2, is

a) $x-\sqrt{3}y+2=0$ b) $\sqrt{3}x-y+2=0$ c) $\sqrt{3}x-y-2=0$ d) $x+\sqrt{3}y+2=0$

675. The normal at a point P on the ellipse $x^2+4y^2=16$ meets the x-axis at Q. If M is the mid point of the line segment PQ, then the locus of M intersects the latusrectum of the given ellipse at the points

a) $\left(\pm\frac{3\sqrt{5}}{2},\pm\frac{2}{7}\right)$ b) $\left(\pm\frac{3\sqrt{5}}{2},\pm\frac{\sqrt{19}}{4}\right)$ c) $\left(\pm2\sqrt{3},\pm\frac{1}{7}\right)$ d) $\left(\pm2\sqrt{3},\pm\frac{4\sqrt{3}}{7}\right)$

676. If (0,6)and (0,3) are respectively the vertex and focus of a parabola, then its equation is

a) $x^2+12y=72$ b) $x^2-12y=72$ c) $y^2-12x=72$ d) $y^2+12x=72$

677. Set of values of m for which a chord of slope m of the circle $x^2+y^2=4$ touches the parabola $y^2=4x$, is

a) $\left(-\infty,-\sqrt{\frac{\sqrt{2}-1}{2}}\right)\cup\left(\sqrt{\frac{\sqrt{2}-1}{2}},\infty\right)$

b) $(-\infty,-1)\cup(1,\infty)$

c) $(-1,1)$

d) R

678. The parabola $y^2=4ax$ passes through the point $(2,-6)$, then the length of its latusrectum is

a) 18 b) 9 c) 6 d) 16

679. If the circle $x^2+y^2+2gx+2fy+c=0$ is touched by $y=x$ at P such that $OP=6\sqrt{2}$, then the value of c is

a) 36 b) 144 c) 72 d) None of these

680. From a point on the circle$x^2+y^2=a^2$, two tangents are drawn to the circle$x^2+y^2=a^2\sin^2\alpha$. The angle between them is

a) α b) $\frac{\alpha}{2}$ c) 2α d) None of these

681. The sum of the squares of the perpendiculars on any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ from two points on the minor axis each at a distance $\sqrt{a^2-b^2}$ from the centre is

a) $2a^2$ b) $2b^2$ c) a^2+b^2 d) a^2-b^2

682. If two chords having lengths a^2-1 and $3(a+1)$, where a is a constant of a circle bisect each other, then the radius of the circle is

a) 6 b) $\frac{15}{2}$ c) 8 d) $\frac{19}{2}$

683. The eccentric angle of a point on the ellipse $\frac{x^2}{6}+\frac{y^2}{2}=1$, whose distance from the centre of the ellipse is 2, is

a) $\frac{\pi}{4}$ b) $\frac{3\pi}{2}$ c) $\frac{5\pi}{3}$ d) $\frac{7\pi}{6}$

684. The equation of the parabola whose focus is the point $(0,0)$ and the tangent at the vertex is $x-y+1=0$, is

a) $x^2+y^2-2xy-4x+4y-4=0$ b) $x^2+y^2-2xy+4x-4y-4=0$

c) $x^2+y^2+2xy-4x+4y-4=0$ d) $x^2+y^2+2xy-4x-4y+4=0$

685. If $a \neq 0$ and the line $2bx + 3cy + 4d = 0$, passes through the points of intersection of the parabolas$y^2 = 4ax$ $x^2 = 4ay$, then

a) $d^2 + (2b + 3c)^2 = 0$ b) $d^2 + (3b + 2c)^2 = 0$ c) $d^2 + (2b - 3c)^2 = 0$ d) $d^2 + (3b - 2c)^2 = 0$

686. The foci of the Basic Terms of Conics $25x^2 + 16y^2 - 150x = 175$ are

a) $(0, \pm 3)$ b) $(0, \pm 2)$ c) $(3, \pm 3)$ d) $(0, \pm 1)$

687. The inverse of the point (1, 2) with respect to the circle $x^2 + y^2 - 4x - 6y + 9 = 0$, is

a) $\left(1, \frac{1}{2}\right)$ b) (2, 1) c) (0, 1) d) (1, 0)

688. the equation $ax^2 + 2\,hxy + by^2 + 2\,gx + 2\,fy + c = 0$ represents a circle, the condition will be

a) $a = b$ and $c = 0$ b) $f = g$ and $h = 0$ c) $a = b$ and $h = 0$ d) $f = g$ and $c = 0$

689. The equation of a circle with origin as a centre and passing through an equilateral triangle whose median is of length $3a$, is

a) $x^2 + y^2 = 9a^2$ b) $x^2 + y^2 = 16a^2$ c) $x^2 + y^2 = 4a^2$ d) $x^2 + y^2 = a^2$

690. A hyperbola, having the transverse axis of length $2\sin\theta$, is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then, its equation is

a) $x^2\text{cosec}^2\theta - y^2\sec^2\theta = 1$ b) $x^2\sec^2\theta - y^2\text{cosec}^2\theta = 1$

c) $x^2\sin^2\theta - y^2\cos^2\theta = 1$ d) $x^2\cos^2\theta - y^2\sin^2\theta = 1$

691. The lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to the same circle. Then, its radius is

a) 1/4 b) 1/2 c) 3/4 d) None of these

692. The values of θ in $[0, 2\pi]$ so that circles $x^2 + y^2 + 2(\sin\alpha)x + 2(\cos\alpha)y + \sin^2\theta = 0$ always lies inside the square of unit side length, is /are

a) $(\pi/3, 2\pi/3)$ b) $[4\pi, 5\pi/3]$ c) $(\pi/4, 2\pi/3)$ d) None of these

693. The locus of the point intersection of tangents to the parabola $y^2 = 4(x + 1)$ and $y^2 = 8(x + 2)$ which are perpendicular to each other is

a) $x + 7 = 0$ b) $x - y = 4$ c) $x + 3 = 0$ d) $y - x = 12$

694. The equation of the parabola whose focus is $(3, -4)$ and directrix $6x - 7y + 5 = 0$, is

a) $(7x + 6y)^2 - 570x + 750y + 2100 = 0$ b) $(7x + 6y)^2 + 570x - 750y + 2100 = 0$

c) $(7x - 6y)^2 - 570x + 750y + 2100 = 0$ d) $(7x - 6y)^2 + 570x - 750y + 2100 = 0$

695. If one end of a diameter of the ellipse $4x^2 + y^2 = 16$ is $(\sqrt{3}, 2)$, then the other end is

a) $(-\sqrt{3}, 2)$ b) $(\sqrt{3}, -2)$ c) $(-\sqrt{3}, -\sqrt{2})$ d) (0,0)

696. The angle between the asymptotes of the hyperbola $27x^2 - 9y^2 = 24$ is

a) 30° b) 120° c) 45° d) 240°

697. The point $(\sin\theta, \cos\theta)$, θ being any real number, lie inside the circle $x^2 + y^2 - 2x - 2y + \lambda = 0$, if

a) $\lambda < 1 + 2\sqrt{2}$ b) $\lambda > 2\sqrt{2} - 1$ c) $\lambda < -1 - 2\sqrt{2}$ d) $\lambda > 1 + 2\sqrt{2}$

698. The angle between the pair of tangents drawn to the ellipse $3x^2 + 2y^2 = 5$ from the point (1, 2) is

a) $\tan^{-1}\left(\frac{12}{5}\right)$ b) $\tan^{-1}(6\sqrt{5})$ c) $\tan^{-1}\left(\frac{12}{\sqrt{5}}\right)$ d) $\tan^{-1}(12\sqrt{5})$

699. The maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with vertex at one at one end of major axis is

a) $\sqrt{3}\,ab$ b) $\frac{3\sqrt{3}}{4}ab$ c) $\frac{5\sqrt{3}}{4}ab$ d) None of these

700. If the minor axis of an ellipse subtends an angle of 60° at each focus of the ellipse, then its eccentricity is

a) $\frac{\sqrt{3}}{2}$ b) $\frac{1}{\sqrt{2}}$ c) $\frac{2}{\sqrt{3}}$ d) None of these

701. A man running round a race course notes that the sum of the distances of two flag posts from him is always 10m and the distance between the flag posts is 8m. The area of the path he encloses (in square metre) is

a) 15π b) 12π c) 18π d) 8π

702. The middle point of the chord $x + 3y = 2$ of the conic $x^2 + xy - y^2 = 1$ is

a) $(5,-1)$ b) $(1,1)$ c) $(2,0)$ d) $(-1,1)$

703. The circles $x^2+y^2-4x-6y-12=0$ and $x^2+y^2+4x+6y+4=0$

a) Touch externally b) Do not intersect
c) Intersect at two points d) Are concentric

704. The equation of the director circle of the hyperbola $\frac{x^2}{16}-\frac{y^2}{4}=1$, is given by

a) $x^2+y^2=16$ b) $x^2+y^2=4$ c) $x^2+y^2=20$ d) $x^2+y^2=12$

705. If the lines $2x+3y+1=0$ and $3x-y-4=0$ lie along diameters of acircle of circumference 10π, then the equation of the circle is

a) $x^2+y^2-2x+2y-23=0$ b) $x^2+y^2-2x-2y-23=0$
c) $x^2+y^2+2x+2y-23=0$ d) $x^2+y^2+2x-2y-23=0$

706. The value of λ, for which the circle $x^2+y^2+2\lambda x+6y+1=0$ intersects the circle $x^2+y^2+4x+2y=0$ orthogonally, is

a) $\frac{11}{8}$ b) -1 c) $\frac{-5}{4}$ d) $\frac{5}{2}$

707. The area of the triangle formed by any tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ with its asymptotes is

a) $4a^2b^2$ b) a^2b^2 c) $4ab$ d) ab

708. If the chords of contact of tangents drawn from P to the hyperbola $x^2-y^2=a^2$ and its auxiliary circle are at right angle, then P lies on

a) $x^2-y^2=3a^2$ b) $x^2-y^2=2a^2$ c) $x^2-y^2=0$ d) $x^2-y^2=1$

709. Let P be a point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, of eccentricity e. If A, A' are the vertices and S, S' are the foci of the ellipse, then Area $\Delta\, PSS'$: Area $\Delta\, APA'$ =

a) $e^3:1$ b) $e^2:1$ c) $e:1$ d) $\frac{1}{e}:1$

710. The centre of the ellipse $9x^2+25y^2-18x-100y-166=0$, is

a) (1,1) b) (−1,2) c) (−1,1) d) (1,2)

711. Length of major axis of ellipse $9x^2+7y^2=63$ is

a) 3 b) 9 c) 6 d) $2\sqrt{7}$

712. Equation of the normal to the ellipse $4(x-1)^2+9(y-2)^2=36$, which is parallel to the line $3x-y=1$, is

a) $3x-y=\sqrt{5}$ b) $3x-y=\sqrt{5}-3$
c) $3x-y=\sqrt{5}+2$ d) $3x-y=\sqrt{5}(\sqrt{5}+1)$

713. If e and e' be the eccentricities of a hyperbola and its conjugate, then $\frac{1}{e^2}+\frac{1}{(e)^2}$ is equal to

a) 0 b) 1 c) 2 d) 3

714. In the parabola $y^2=4ax$, the length of the chord passing through the vertex inclined to the axis at $\frac{\pi}{4}$ is

a) $4a\sqrt{2}$ b) $2a\sqrt{2}$ c) $a\sqrt{2}$ d) a

715. If e is eccentricity of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b)$ and e' is eccentricity of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a<b)$, then

a) $e=e'$ b) $ee'=1$ c) $\frac{1}{e^2}+\frac{1}{(e')^2}=1$ d) None of these

716. The area (in square unit) of the circle which touches the lines $4x+3y=15$ and $4x+3y=5$ is

a) 4π b) 3π c) 2π d) π

717. The sum of the coefficients in the expansion of $(\alpha^2x^2-2\,\alpha\, x+1)^{51}$, as a polynomial in x, vanishes. Position of the point $(\alpha, 2\,\alpha^2)$ with respect to the circle $x^2+y^2=4$, is

a) Outside b) Inside c) On side d) Cannot be decided

718. The locus of the poles of tangents to the auxiliary circle with respect to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, is

a) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{a^2}$ b) $\frac{x^2}{a^4}+\frac{y^2}{b^4}=\frac{1}{b^2}$ c) $\frac{x^2}{a^4}+\frac{y^2}{b^4}=\frac{1}{a^2}$ d) None of these

719. The points $(5,-7)$ lies outside the circle

a) $x^2+y^2-8x=0$
b) $x^2+y^2-5x+7y=0$
c) $x^2+y^2-5x+7y-1=0$
d) $x^2+y^2-8x+7y-2=0$

720. If the lines joining the foci of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$, where $a>b$ and an extremity of its minor axis are inclined at an angle 60°, then the eccentricity of the ellipse is
a) $-\frac{\sqrt{3}}{2}$
b) $\frac{1}{2}$
c) $\frac{\sqrt{5}}{2}$
d) $\frac{\sqrt{7}}{3}$

721. The two parabolas $x^2=4y$ and $y^2=4x$ meet in two distinct points. One of these is the origin and the other is
a) (2,2)
b) $(4,-4)$
c) (4,4)
d) $(-2,2)$

722. If the circles $x^2+y^2+2gx+2fy=0$ and $x^2+y^2+2g'x+2f'y=0$ touch each other, then
a) $ff'=gg'$
b) $fg=f'g'$
c) $(fg)^2=(f'g')^2$
d) $fg'=f'g$

723. If the normals at two points P and Q of a parabola $y^2=4ax$ intersect at a third point R on the curve, then the product of ordinates of P and Q is
a) $4a^2$
b) $2a^2$
c) $-4a^2$
d) $8a^2$

724. The equation of the line which is tangent to both the circle $x^2+y^2=5$ and the parabola $y^2=40x$ is
a) $2x-y\pm 5=0$
b) $2x-y+5=0$
c) $2x-y-5=0$
d) $2x+y+5=0$

725. The length of the chord of the parabola $y^2=4ax$, which passes through the vertex and makes an angle α with the axis of the parabola is
a) $4a\cos\alpha\,\text{cosec}^2\alpha$
b) $4a\cos\alpha\,\text{cosec}^2\alpha$
c) $a\cos\alpha\,\text{cosec}^2\alpha$
d) $a\cos^2\alpha\, a\,\text{cosec}\,\alpha$

726. $x=1$ is the radical axis of the two orthogonally intersecting circles. If $x^2+y^2=4$ is one of the circles, then the circles, then the other circle is
a) $x^2+y^2-4x+4=0$
b) $x^2+y^2-8x+4=0$
c) $x^2+y^2-8x-4=0$
d) None of these

727. The equation $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$ represents a circle whose centre is
a) $\left(\frac{x_1-x_2}{2},\frac{y_1-y_2}{2}\right)$
b) $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$
c) (x_1,y_1)
d) (x_2,y_2)

728. $x=4(1+\cos\theta)$ and $y=3(1+\sin\theta)$ are the parametric equations of
a) $\frac{(x-3)^2}{9}+\frac{(y-4)^2}{16}=1$
b) $\frac{(x+4)^2}{16}+\frac{(y+3)^2}{9}=1$
c) $\frac{(x-4)^2}{16}-\frac{(y-3)^2}{9}=1$
d) $\frac{(x-4)^2}{16}+\frac{(y-3)^2}{9}=1$

729. If the chord of contact of tangents from a point on the circle $x^2+y^2=r_1^2$ to the circle $x^2+y^2=r_2^2$ touches the circle $x^2+y^2=r_3^2$, then r,r_2,r_3 are in
a) AP
b) HP
c) GP
d) AGP

730. The locus of the poles of tangents to the director circle of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with respect to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, is
a) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{a^2+b^2}$
b) $\frac{x^2}{a^4}+\frac{y^2}{a^4}=\frac{1}{a^2+b^2}$
c) $\frac{x^2}{a^6}+\frac{y^2}{b^6}=\frac{1}{a^2+b^2}$
d) None of these

731. If chords of the hyperbola $x^2-y^2=a^2$ touch the parabola $y^2=4ax$. Then, the locus of the middle points of these chord is
a) $y^2=(x-a)x^3$
b) $y^2(x-a)=x^3$
c) $x^2(x-a)=x^3$
d) None of these

732. If the parabola $y^2=4ax$ passes through the point $(1,-2)$, then tangent at this point is
a) $x-y-1=0$
b) $x+y+1=0$
c) $x-y+1=0$
d) None of these

733. If the circles $x^2+y^2+2x+2ky+11=6$ and $x^2+y^2+2ky+K=0$ intersect orthogonally, then k is
a) 2 or $-\frac{3}{2}$
b) -2 or $\frac{3}{2}$
c) 2 or $\frac{3}{2}$
d) $-2\ \frac{3}{2}$

734. The foci of the hyperbola $2x^2-3y^2=5$ are
a) $\left(\pm\frac{5}{\sqrt{6}},0\right)$
b) $\left(\pm\frac{5}{6},0\right)$
c) $\left(\pm\frac{\sqrt{5}}{6},0\right)$
d) None of these

735. Consider the circles $x^2 + (y-1)^2 = 9, (x-1)^2 + y^2 = 25$. They are such that
a) These circles touch each other
b) One of these circles lies entirely inside the other
c) Each of these circle lies outside the other
d) They intersect in two point

736. The circle $x^2 + y^2 + 8y - 4 = 0$, cuts the real circle $x^2 + y^2 + gx + 4 = 0$ orthogonally, if
a) For any real value of g
b) For no real value of g
c) $g = 0$
d) $g < -2, g > 2$

737. If P is any point on the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$ and S and S' are the foci, then $PS + PS'$ is equal to
a) 4
b) 8
c) 10
d) 12

738. If A, A' are the vertices, S, S' are the foci and Z, Z' are the feet of the directrices of an ellipse with centre C, then CS, CA, CZ are in
a) A.P.
b) G.P.
c) H.P.
d) None of these

739. The condition that the parabola $y^2 = 4c(x-d)$ and $y^2 = 4ax$ have a common normal other than x-axis $(a > 0, c > 0)$, if
a) $2a < 2c + d$
b) $2c < 2a + d$
c) $2d < 2a + c$
d) $2d < 2c + a$

740. The equation of the circle whose diameter is the common chord of the circles
$x^2 + y^2 + 2x + 3y + 2 = 0$
and $x^2 + y^2 + 2x - 3y - 4 = 0$ is
a) $x^2 + y^2 + 2x + 2y + 2 = 0$
b) $x^2 + y^2 + 2x + 2y - 1 = 0$
c) $x^2 + y^2 + 2x + 2y + 1 = 0$
d) $x^2 + y^2 + 2x + 2y + 3 = 0$

741. The coaxial system of circles given by $x^2 + y^2 + 2gx + c = 0$ for $c < 0$ respects
a) Intersecting circles
b) Non-intersecting circles
c) Touching circles
d) Touching or non-intersecting circles

742. The value of λ, for which the line $2x - \frac{8}{3}\lambda y = -3$ is a normal to the conic $x^2 + \frac{y^2}{4} = 1$ is
a) $-\frac{\sqrt{3}}{2}$
b) $\frac{1}{2}$
c) -3
d) $\pm\frac{\sqrt{3}}{2}$

743. If the length of the latusrectum of the ellipse $x^2 \tan^2\theta + y^2 \sec^2\theta = 1$ is $1/2$, then $\theta =$
a) $\pi/12, 5\pi/12$
b) $\pi/6, 5\pi/6$
c) $7\pi/12$
d) None of these

744. If $2y = x$ and $3y + 4x = 0$ are the equations of a pair of conjugate diameters of an ellipse, then the eccentricity of the ellipse is
a) $\sqrt{\frac{2}{3}}$
b) $\sqrt{\frac{2}{5}}$
c) $\sqrt{\frac{1}{3}}$
d) $\sqrt{\frac{1}{2}}$

745. Equation $\frac{1}{r} = \frac{1}{8} + \frac{3}{8}\cos\theta$ represents
a) A rectangular hyperbola
b) A hyperbola
c) An ellipse
d) A parabola

746. The length of the latusrectum of an ellipse is one third of its major axis. Its eccentricity would be
a) $\frac{2}{3}$
b) $\sqrt{\frac{2}{3}}$
c) $\frac{1}{\sqrt{3}}$
d) $\frac{1}{\sqrt{2}}$

747. The locus of centre of a circle which passes through the origin and cuts off a length of 4 unit from the line $x = 3$ is
a) $y^2 + 6x = 0$
b) $y^2 + 6x = 13$
c) $y^2 + 6x = 10$
d) $x^2 + 6y = 13$

748. The equation of latusrectum of a parabola is $x + y = 8$ and the equation of the tangent at the vertex is $x + y = 12$, then length of the latusrectum is
a) $4\sqrt{2}$
b) $2\sqrt{2}$
c) 8
d) $8\sqrt{2}$

749. The equation of the hyperbola referred to the axes of coordinate and whose distance between the foci is 16 and eccentricity is $\sqrt{2}$, is

a) $x^2 - y^2 = 16$ b) $x^2 - y^2 = 32$ c) $x = 2y^2 = 16$ d) $y^2 - x^2 = 16$

750. A circle touches y-axis at (0,2) and has an intercept of 4 units on the positive side of x-axis. The equation of the circle is

a) $x^2 + y^2 - 4(\sqrt{2}\,x + y) + 4 = 0$

b) $x^2 + y^2 - 4(x + \sqrt{2}y) + 4 = 0$

c) $x^2 + y^2 - 2(\sqrt{2}\,x + y) + 4 = 0$

d) None of these

751. If t is a parameter, then $x = a\left(t + \frac{1}{t}\right), y = b\left(t - \frac{1}{t}\right)$ represents

a) An ellipse b) A circle

c) A pair of straight lines d) A hyperbola

752. A common tangent to $9x^2 - 16y^2 = 144$ and $x^2 + y^2 = 9$, is

a) $y = \frac{3}{\sqrt{7}}x + \frac{15}{\sqrt{7}}$ b) $y = 3\sqrt{\frac{2}{7}}x + \frac{15}{\sqrt{7}}$ c) $y = 2\sqrt{\frac{3}{7}}x + 15\sqrt{7}$ d) None of these

753. Equation of the circle with centre on the y-axis and passing through the origin and (2,3) is

a) $x^2 + y^2 + 13y = 0$

b) $3\,x^2 + 3\,y^2 - 13\,y = 0$

c) $x^2 + y^2 + 13\,x + 3 = 0$

d) $6\,x^2 + 6\,y^2 - 13\,x = 0$

754. The angle between the asymptotes of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$, is

a) $\pi - 2\tan^{-1}\frac{3}{4}$ b) $\pi - 2\tan^{-1}\frac{3}{2}$ c) $\tan^{-1}\frac{3}{4}$ d) $\pi - 2\tan^{-1}\frac{4}{3}$

755. If the point $\left(-5 + \frac{\lambda}{\sqrt{2}}, -3 + \frac{\lambda}{\sqrt{2}}\right)$ is an interior point of the larger segment of the circle $x^2 + y^2 = 16$ cut off by the line $x + y = 2$, then

a) $\lambda \in (-\infty, 5\sqrt{2})$

b) $\lambda \in (4\sqrt{2} - \sqrt{14}, 5\sqrt{2})$

c) $\lambda \in (4\sqrt{2} - \sqrt{14}, 4\sqrt{2} + \sqrt{14})$

d) None of these

756. If the circle $x^2 + y^2 + 6x - 2y + k = 0$ bisects the circumference of the circle $x^2 + y^2 + 2x - 6y - 15 = 0$, then k is equal to

a) 21 b) −21 c) 23 d) −23

757. If $x = 9$ is the chord of contact of the hyperbola $x^2 - y^2 = 9$, then the equation of the corresponding pair of tangents is

a) $9x^2 - 8y^2 + 18x - 9 = 0$ b) $9x^2 - 8y^2 - 18x + 9 = 0$

c) $9x^2 - 8y^2 - 18x - 9 = 0$ d) $9x^2 - 8y^2 + 18x + 9 = 0$

758. If the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ cut the coordinate axes in concyclic points, then

a) $|a_1a_2| = |b_1b_2|$ b) $|a_1b_1| = |a_2b_2|$ c) $|a_1b_2| = |a_2b_1|$ d) None of these

759. One of the diameters of the circle circumscribing the rectangle $ABCD$ is $4y = x + 7$. If A and B are the points (−3,4) and (5,4) respectively, then the area of the rectangle is

a) 16 sq. units b) 24 sq. units c) 32 sq. units d) None of these

760. The intercept on the line $y = x$ by the circle $x^2 + y^2 - 2x = 0$ is AB. The equation of the circle with AB as diameter is

a) $x^2 + y^2 + x + y = 0$ b) $x^2 + y^2 = x + y$ c) $x^2 + y^2 - 3x + y = 0$ d) None of these

761. Let LL' be the latusrectum and S be a focus of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. If $\Delta\, SLL'$ is equilateral, then the eccentricity of the ellipse is

a) $1/\sqrt{5}$ b) $1/\sqrt{3}$ c) $1/\sqrt{2}$ d) $\sqrt{2}/3$

762. The radius of the circle $r^2 - 2\sqrt{2}\,r(\cos\theta + \sin\theta) - 5 = 0$, is

a) 9 b) 5 c) 3 d) 2

763. If $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b)$ and $x^2-y^2=c^2$ cut at right angles, then

a) $a^2+b^2=2c^2$ b) $b^2-a^2=2c^2$ c) $a^2-b^2=2c^2$ d) $a^2b^2=2c^2$

764. If the chords of constant of tangents from two points (x_1, y_1) and (x_2, y_2) to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are at right angles, then $\frac{x_1x_2}{y_1y_2}$ is equal to

a) $\frac{a^2}{b^2}$ b) $-\frac{b^2}{a^2}$ c) $-\frac{a^4}{b^4}$ d) $-\frac{b^4}{a^4}$

765. The equation of the hyperbola of given transverse axis 2a with its vertex mid-way between the centre and the corresponding focus is

a) $3x^2-y^2=a^2$ b) $3x^2-y^2=3a^2$ c) $x^2-3y^2=a^2$ d) $x^2-3y^2=a^2$

766. The equation of the circle concentric to the circle $2x^2+2y^2-3x+6y+2=0$ and having double the area of this circle, is

a) $8x^2+8y^2-24x+48y-13=0$
b) $16x^2+16y^2+24x-48y-13=0$
c) $16x^2+16y^2-24x+48y-13=0$
d) $8x^2+8y^2+24x-48y-13=0$

767. If a tangent, having slope $-\frac{4}{3}$, to the ellipse $\frac{x^2}{18}+\frac{y^2}{32}=1$ intersects the major and minor axes in points A and B respectively, then the area of $\Delta\, OAB$ is equal to

a) 12 sq. units b) 48 sq. units c) 64 sq. units d) 24 sq. units

768. The angle of intersection between the curves $x^2=4(y+1)$ and $x^2=-4(y+1)$ is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) 0 d) $\frac{\pi}{2}$

769. The equation $|\sqrt{x^2+(y-1)^2}-\sqrt{x^2+(y+1)^2}|=k$ will represent a hyperbola for

a) $k\in(0,2)$ b) $k\in(0,1)$ c) $k\in(1,\infty)$ d) $k\in R^+$

770. Angle between tangent drawn to circle $x^2+y^2=20$, from the point (6, 2) is

a) $\frac{\pi}{2}$ b) π c) $\frac{\pi}{4}$ d) 2π

771. The foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide. Then, the value of b^2 is

a) 1 b) 5 c) 7 d) 9

772. If $\frac{x^2}{36}-\frac{y^2}{k^2}=1$, is a hyperbola, then which of the following statements can by true?

a) (−3,1) lies on the hyperbola b) (3,1) lies on the hyperbola
c) (10,4) lies on the hyperbola d) (5,2) lies on the hyperbola

773. The locus of the point of intersection of perpendicular tangents to the parabola $x^2=4ay$ is

a) $y=a$ b) $y=-a$ c) $x=a$ d) $x=-a$

774. If a chord which is normal to the parabola $y^2=4ax$ at one end subtends a right angle at the vertex, then its slope is

a) 1 b) $\sqrt{3}$ c) $\sqrt{2}$ d) 2

775. The equation of the circle on the common chord of the circles $(x-a)^2+y^2=a^2$ and $x^2+(y+b)^2=b^2$ as diameter, is

a) $x^2+y^2=2ab(bx+ay)$ b) $x^2+y^2=bx+ay$
c) $(a^2+b^2)(x^2+y^2)=2ab(bx-ay)$ d) $(a^2+b^2)(x^2+y^2)=2(bx+ay)$

776. If in a $\Delta\, ABC$ (whose circumcentre is at the origin), $a\le\sin A$, then for any point (x,y) inside the circumcircle of $\Delta\, ABC$, we have

a) $|xy|<\frac{1}{8}$ b) $|xy|>\frac{1}{8}$ c) $\frac{1}{8}<xy<\frac{1}{2}$ d) None of these

777. The locus of the mid points of the chords of the circle $x^2+y^2=4$ which subtend a right angle at the origin is

a) $x^2+y^2=1$ b) $x^2+y^2=2$ c) $x+y=1$ d) $x+y=2$

778. The directrix of the parabola $y^2+4x+3=0$ is

a) $x-\frac{4}{3}=0$ b) $x+\frac{1}{4}=0$ c) $x-\frac{3}{4}=0$ d) $x-\frac{1}{4}=0$

779. The point $P(9/2,6)$ lies on the parabola $y^2=4ax$, then parameter of the point P is

a) $\frac{3a}{2}$ b) $\frac{2}{3a}$ c) $\frac{2}{3}$ d) $\frac{3}{2}$

780. The length of the latusrectum of the parabola whose focus is $(3,3)$ and directix is $3x-4y=0$, is

a) 2 b) 1 c) 4 d) None of these

781. If $5x^2+\lambda y^2=20$ represents a rectangular hyperbola, then λ equals

a) 5 b) 4 c) -5 d) None of these

782. Tangents at $P(t_1)$ and $Q(t_2)$ on the curve $y^2=4ax$ are meeting at a point R on the axis of the parabola. the area of ΔPQR is

a) $-8a^2t_1^3$ b) $2a^2t_1^2t_2$ c) $4a^2t_1t_2^2$ d) None of these

783. A variable circle passes through the fixed point $A(p,q)$ and touches x-axis. The locus of the other end of the diameter through A is

a) $(x-p)^2=4qy$ b) $(x-q)^2=4py$ c) $(y-p)^2=4qx$ d) $(y-q)^2=4px$

784. The equation of the directrix of the parabola $x^2-4x-3y+10=0$, is

a) $y=-\frac{5}{4}$ b) $y=\frac{5}{4}$ c) $y=-\frac{3}{4}$ d) $x=\frac{5}{4}$

785. The angle between the two asymptotes of the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ is

a) $\pi-2\tan^{-1}\left(\frac{3}{4}\right)$ b) $\pi-2\tan^{-1}\left(\frac{3}{2}\right)$ c) $2\tan^{-1}\left(\frac{3}{4}\right)$ d) $\pi-2\tan^{-1}\left(\frac{4}{3}\right)$

786. The equation of the ellipse whose one focus is at $(4,0)$ and whose eccentricity is $4/5$ is

a) $\frac{x^2}{3^2}+\frac{y^2}{5^2}=1$ b) $\frac{x^2}{5^2}+\frac{y^2}{3^2}=1$ c) $\frac{x^2}{5^2}+\frac{y^2}{4^2}=1$ d) $\frac{x^2}{4^2}+\frac{y^2}{5^2}=1$

787. The locus of centres of family of circle passing through the origin and cutting the circle $x^2+y^2+4x-6y-13=0$ orthogonally, is

a) $4x+6y+13=0$ b) $4x-6y+13=0$ c) $4x+6y-13=0$ d) $4x-6y-13=0$

788. The angle of intersection of the circles $x^2+y^2=4$ and $x^2+y^2=2x+2y$, is

a) $\pi/2$ b) $\pi/3$ c) $\pi/6$ d) $\pi/4$

789. Let $P(a\sec\theta, b\tan\theta)$ and $Q(a\sec\phi, b\tan\phi)$, where $\theta+\phi=\frac{\pi}{2}$ be two points on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If (h,k) is the point of intersection of normals at P and Q, then k is equal to

a) $\frac{a^2+b^2}{a}$ b) $-\left[\frac{a^2+b^2}{a}\right]$ c) $\frac{a^2+b^2}{b}$ d) $-\left[\frac{a^2+b^2}{b}\right]$

790. The circle $x^2+y^2+2\lambda x=0, \lambda\in R$, touches the parabola $y^2=4x$ externally. Then

a) $\lambda>0$ b) $\lambda<0$ c) $\lambda>1$ d) None of these

791. If the circles $x^2+y^2-2x-2y-7=0$ and $x^2+y^2+4x+2y+k=0$ cut orthogonally, then the length of the common chord of the circle is

a) $\frac{12}{\sqrt{13}}$ b) 2 c) 5 d) 8

792. The radical axis of the coaxial system of circles with limiting points $(1,2)$ and $(-2,1)$is

a) $x+3y=0$ b) $3x+y=0$ c) $2x+3y=0$ d) $3x+2y=0$

793. If $P(1,1/2)$ is a centre of similitude for the circles $x^2+y^2+4x+2y-4=0$ and $x^2+y^2-4x-2y+4=0$, then the length of the common tangent through P to the circles is

a) 4 b) 3 c) 2 d) 1

794. Number of tangents from $(7,6)$ to ellipse $\frac{x^2}{16}+\frac{y^2}{25}=1$ is

a) 0 b) 1 c) 2 d) None of these

795. The equation of the common tangent of the two touching circles, $y^2+x^2-6x-12y+37=0$ and $x^2+y^2-6y+7=0$ is

a) $x+y-5=0$ b) $x-y+5=0$ c) $x-y-5=0$ d) $x+y+5=0$

796. Let E be the ellipse $\frac{x^2}{8}+\frac{y^2}{4}=1$ and C be the circle $x^2+y^2=9$. Let P and Q be the points $(1,2)$ and $(2,1)$ respectively. Then

a) Q lies inside C but outside E
b) Q lies outside both C and E
c) P lies inside both C and E
d) P lies inside C but outside E

797. An isosceles triangle is inscribed in the circle $x^2+y^2-6x-8y=0$ with vertex at the origin and one of the equal sides along the axis of$x$. Equation of the other side through the origin is

a) $7x-24y=0$
b) $24x-7y=0$
c) $7x+24y=0$
d) $24x+7y=0$

798. The locus of the middle point of the chords of the circle $x^2+y^2=a^2$ such that the chords pass through a given point (x_1,y_1), is

a) $x^2+y^2-xx_1-yy_1=0$
b) $x^2+y^2=x_1^2+y_1^2$
c) $x+y=x_1+y_1$
d) $x+y=x_1^2+y_1^2$

799. If p is the length of the perpendicular from a facus upon the tangent at any point p of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and r is the distance of p from the focus, then $\frac{2a}{r}-\frac{b^2}{p^2}$ is equal to

a) -1
b) 0
c) 1
d) 2

800. Equation of the ellipse with eccentricity $\frac{1}{2}$ and foci at $(\pm 1,0)$ is

a) $\frac{x^2}{3}+\frac{y^2}{4}=1$
b) $\frac{x^2}{4}+\frac{y^2}{3}=1$
c) $\frac{x^2}{4}+\frac{y^2}{3}=\frac{4}{3}$
d) None of these

801. If the tangent at a point P on $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ cuts one of its directrices at Q, then the angle made by PQ at the corresponding focus, is

a) 45°
b) 30°
c) 60°
d) 90°

802. The common tangent to the parabola $y^2=4\,ax$ and $x^2=4\,ay$, is

a) $x+y+a=0$
b) $x+y-a=0$
c) $x-y+a=0$
d) $x-y-a=0$

803. The equation of the tangent at the vertex of the parabola $x^2+4x+2\,y=0$ is

a) $x=-2$
b) $x=2$
c) $y=2$
d) $y=-2$

804. If the normal at the $P(\theta)$ to the ellipse $\frac{x^2}{14}+\frac{y^2}{5}=1$ intersect it again at the point $Q(2\theta)$, then $\cos\theta$ is equal to

a) $\frac{2}{3}$
b) $-\frac{2}{3}$
c) $\frac{1}{3}$
d) $-\frac{1}{3}$

805. The vertex of the parabola $y^2+6x-2y+13=0$, is

a) $(1,-1)$
b) $(-2,1)$
c) $(3/2,1)$
d) $(-7/2,1)$

806. The equation to the hyperbola having its eccentricity 2 and the distance between its foci is 8, is

a) $\frac{x^2}{12}-\frac{y^2}{4}=1$
b) $\frac{x^2}{4}-\frac{y^2}{12}=1$
c) $\frac{x^2}{8}-\frac{y^2}{2}=1$
d) $\frac{x^2}{16}-\frac{y^2}{9}=1$

807. If P is a point such that the ratio of the square of the lengths of the tangents from P to the circles $x^2+y^2+2x-4y-20=0$ and $x^2+y^2-4x+2y-44=0$ is $2:3$, then the locus of p is a circle with centre

a) $(7,-8)$
b) $(-7,8)$
c) $(7,8)$
d) $(-7,-8)$

808. The equation of a circle C is $x^2+y^2-6c-8y-11=0$. The number of real points at which the circle drawn with points (1,8) and (0,0) as the ends of a diameter cuts the circle, C is

a) 0
b) 1
c) 2
d) None of these

809. The length of transverse axis of the hyperbola $3x^2-4y^2=32$ is

a) $\frac{8\sqrt{2}}{\sqrt{3}}$
b) $\frac{16\sqrt{2}}{\sqrt{3}}$
c) $\frac{3}{32}$
d) $\frac{64}{3}$

810. Vertex of the parabola $9x^2-6x+36y+9=0$ is

a) $(1/3,-2/9)$
b) $(-1/3,-1/2)$
c) $(-1/3,1/2)$
d) $(1/3,1/2)$

811. If the latusrectum of a hyperbola through one focus subtends 60° angle at the other focus, then its eccentricity e is

a) $\sqrt{2}$
b) $\sqrt{3}$
c) $\sqrt{5}$
d) $\sqrt{6}$

812. If a circle passes through the point of intersection of the coordinates axes with the lines $\lambda x - y + 1 = 0$ and $x - 2y + 3 = 0$, then the value of λ is

a) 2 b) 4 c) 6 d) 3

813. Let AB be a chord of the circle $x^2 + y^2 = r^2$ subtending a right angle at the4 centre. Then, the locus of the centroid of the $\Delta\, PAB$ as P moves on the circle is

a) A parabola b) A circle
c) An ellipse d) A pair of straight lines

814. The locus of the centre of a circle which cuts orthogonally the circle $x^2 + y^2 - 20x + +4 = 0$ and which touches $x = 2$, is

a) $y^2 = 16x + 4$ b) $x^2 = 16y$ c) $x^2 = 16y + 4$ d) $y^2 = 16x$

815. If from any point P on the circle $x^2 + y^2 + 2\,gx + 2\,fy + c = 0$, tangents are drawn to the circle
$x^2 + y^2 + 2\,gx + 2\,fy + c\sin^2\alpha + (g^2 + f^2)\cos^2\alpha = 0$,
then the angle between the tangents is

a) α b) $2\,\alpha$ c) $\alpha/2$ d) None of these

816. The equation of the chord joining two points (x_1, y_1) and (x_2, y_2) on the rectangular hyperbola $xy = c^2$ is

a) $\frac{x}{x_1 + x_2} + \frac{y}{y_1 + y_2} = 1$ b) $\frac{x}{x_1 - x_2} + \frac{y}{y_1 - y_2} = 1$
c) $\frac{x}{y_1 + y_2} + \frac{y}{x_1 + x_2} = 1$ d) $\frac{x}{y_1 + y_2} + \frac{y}{x_1 + x_2} = 1$

817. If the line $y\cos\alpha = x\sin\alpha + a\cos\alpha$ be a tangent to the circle $x^2 + y^2 = a^2$, then

a) $\sin^2\alpha = 1$ b) $\cos^2\alpha = 1$ c) $\sin^2\alpha = a^2$ d) $\cos^2\alpha = a^2$

818. The tangent at P, any point on the circle $x^2 + y^2 = 4$, meets the coordinate axes in A and B, then

a) Length of AB is constant
b) PA and PB are always equal
c) The locus of the mid-point of AB is $x^2 + y^2 = x^2y^2$
d) None of these

819. The equation of the circle which touches both the axes and the straight line $4\,x + 3\,y = 6$ in the first quadrant and lies below it is

a) $4\,x^2 + 4\,y^2 - 4\,x - 4\,y + 1 = 0$
b) $x^2 + y^2 - 6\,x - 6\,y + 9 = 0$
c) $x^2 + y^2 - 6\,x - y + 9 = 0$
d) $4(x^2 + y^2 - x - 6\,y) + 1 = 0$

820. The equation of circle which touches the x-axis and y-axis at the points (1, 0) and (0, 1) respectively, is

a) $x^2 + y^2 - 4y + 3 = 0$ b) $x^2 + y^2 - 2y - 2 = 0$
c) $x^2 + y^2 - 2x - 2y + 2 = 0$ d) $x^2 + y^2 - 2x - 2y + 1 = 0$

821. The shortest distance between the parabolas $y^2 = 4x$ and $y^2 = 2x - 6$ is

a) 2 b) $\sqrt{5}$ c) 3 d) None of these

822. The eccentricity of the ellipse $9x^2 + 5y^2 - 30y = 0$ is

a) 1/3 b) 2/3 c) 3/4 d) 4/5

823. The equation of the circle concentric to the circle $2x^2 + 2y^2 - 3x + 6y + 2 = 0$ and having area double the area of this circle, is

a) $8x^2 + 8y^2 - 24x + 48y - 13 = 0$ b) $16x^2 + 16y^2 + 24x - 48y - 13 = 0$
c) $16x^2 + 16y^2 - 24x + 48y - 13 = 0$ d) $8x^2 + 8y^2 + 24x - 48y - 13 = 0$

824. Equation of the normal to the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$ perpendicular to the line $2x + y = 1$ is

a) $\sqrt{21}(x - 2y) = 41$ b) $x - 2y = 1$ c) $\sqrt{41}(x - 2y) = 41$ d) $\sqrt{21}(x - 2y) = 21$

825. The point of the parabola $y^2 = 18x$, for which the ordinate is three times the abscissa is

a) (6,2) b) $(-2 - 6)$ c) (3,18) d) (2,6)

826. The equation of tangent to the hyperbola $4x^2 - 9y^2 = 1$, which is parallel to the line $4y = 5x + 7$, are

a) $y = 30x \pm 161$ b) $24y = 30x \pm \sqrt{161}$ c) $24y = x \pm 161$ d) None of these

827. Two circles with centres (2, 3) and (5, 6) cut orthogonally. If radius of both circles are equal, then radius is

equal to

a) 1 b) 2 c) 3 d) 4

828. If the straight line $x - 2y + 1 = 0$ intersects the circle $x^2 + y^2 = 25$ in points P and Q, then the coordinates of the point of intersection of tangents drawn at P and Q to the circle $x^2 + y^2 = 25$ are

a) (25,50) b) (−25, −50) c) (−25,50) d) (25, −50)

829. The angle between the tangents drawn from the point (1, 4) to the parabola $y^2 = 4x$ is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

830. Coordinates of foci of hyperbola are (-5,3) and (7, 3) and eccentricity is 3/2. Then ,length of its latusrectum is

a) 20 b) 10 c) 40 d) None of these

831. Three points A, B, C are taken on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with eccentric angles $\theta, \theta + \alpha$ and $\theta + 2\alpha$, then

a) The area ΔABC is independent of θ
b) The area ΔABC is independent of α
c) The maximum value of area is $\frac{\sqrt{3}}{4} ab$
d) The maximum value of area is $\frac{3\sqrt{3}}{4} ab$

832. Equation of the parabola with its vertex at (1,1) and focus (3,1) is

a) $(x - 1)^2 = 8(y - 1)$ b) $(y - 1)^2 = 8(x - 3)$ c) $(y - 1)^2 = 8(x - 1)$ d) $(x - 3)^2 = 8(y - 1)$

833. If the tangent at the point $\left(4 \cos \phi, \frac{16}{\sqrt{11}} \sin \phi\right)$ to the ellipse $16x^2 + 11y^2 = 256$ is also a tangent to the circle $x^2 + y^2 - 2x = 15$, then the value of ϕ is

a) $\pm \pi/2$ b) $\pm \pi/4$ c) $\pm \pi/3$ d) $\pm \pi/6$

834. If the circles $x^2 + y^2 + 2ky + 2x + 6 = 0$ and $x^2 + y^2 + 2ky + k = 0$ intersects orthogonally. Then, the value of k is

a) $\frac{3}{2}$ b) -2 c) $-\frac{3}{2}$ d) $\frac{1}{2}$

835. The equation of the asymptotes of the hyperbola $3x^2 + 4y^2 + 8xy - 8x - 4y - 6 = 0$ is

a) $3x^2 + 4y^2 + 8xy - 8x - 4y - 3 = 0$
b) $3x^2 + 4y^2 + 8xy - 8x - 4y + 3 = 0$
c) $3x^2 + 4y^2 + 8xy - 8x - 4y + 6 = 0$
d) $4x^2 + 3y^2 + 2xy - x + y + 3 = 0$

836. The equation of the chord of the circle $x^2 + y^2 - 4x = 0$, whose mid point is (1, 0) is

a) $y = 2$ b) $y = 1$ c) $x = 2$ d) $x = 1$

837. The equation of a circle which passes through $(2a, 0)$ and whose radical axis in relation to the circle $x^2 + y^2 = a^2$ is $x = a/2$, is

a) $x^2 + y^2 - ax = 0$ b) $x^2 + y^2 + 2\,ax = 0$ c) $x^2 + y^2 - 2\,ax = 0$ d) $x^2 + y^2 + ax = 0$

838. The range of values of $\theta \in [0,2\,\pi]$ for which $(1 + \cos\theta, \sin\theta)$ is on interior point of the circle $x^2 + y^2 = 1$, is

a) $(\pi/6, 5\,\pi/6)$ b) $(2\,\pi/3, 5\,\pi/3)$ c) $(\pi/6, 7\,\pi/6)$ d) $2\,\pi/3, 4\,\pi/3$

839. The equation $(x - 2)^2 + (y - 3)^2 = \left(\frac{3x+4y-2}{5}\right)^2$ represents

a) A parabola
b) A pair of straight lines
c) An ellipse
d) A hyperbola

840. Two diameters of the circle $3x^2 + 3y^2 - 6x - 18y - 7 = 0$ are along the lines $3x + y = c_1$and$x - 3y = c_2$. Then, the value of c_1c_2 is

a) −48 b) 80 c) −72 d) 54

841. If e_1 and e_2 are respectively the eccentricities of the ellipse $\frac{x^2}{18} + \frac{y^2}{4} = 1$ and the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$, then the relation between e_1 and e_2 is

a) $3\,e_1^2 + e_2^2 = 2$ b) $e_1^2 + 2\,e_2^2 = 3$ c) $2\,e_1^2 + e_2^2 = 3$ d) $e_1^2 + 3\,e_2^2 = 2$

842. $x^2 + y^2 - 6x - 6y + 4 = 0$, $x^2 + y^2 - 2x - 4y + 3 = 0$, $x^2 + y^2 + 2kx + 2y + 1 = 0$. If the radical centre of the above three circles exists, then which of the following cannot be the value of k

a) 2 b) 1 c) 5 d) 4

843. The equations of tangents to the ellipse $3x^2 + 4y^2 = 5$, which are inclined at 30° to the x-axis, are

a) $y=\sqrt{3}x\pm\frac{5}{2}$ b) $y=\frac{1}{\sqrt{3}}x\pm\frac{5}{2}$ c) $y=\frac{1}{\sqrt{3}}x\pm 1$ d) None of these

844. The slope of the tangent at the point (h,h) to the circle $x^2+y^2=a^2$ is
a) 0 b) 1 c) -1 d) Will depend on h

845. Coordinates of the foci of the ellipse $5x^2+9y^2+10x-36y-4=0$, are
a) (1,2) and $(-3,2)$ b) (2,1) and $(-3,2)$ c) (1,2) and (3,2) d) None of these

846. The parametric coordinates of any point on the parabola $y^2=4ax$ can be
a) $(a-at^2,-2at)$ b) $(a-at^2,2at)$ c) $(a\sin^2 t,-2a\sin t)$ d) $(a\sin\,t,-2a\cos t)$

847. The latusrectum of the parabola $y^2=4ax$ whose focal chord is PSQ such that $SP=3$ and $SQ=2$, is given by
a) $\frac{24}{5}$ b) $\frac{12}{5}$ c) $\frac{6}{5}$ d) $\frac{1}{5}$

848. The one which does not represent a hyperbola is
a) $xy=1$ b) $x^2-y^2=5$ c) $(x-1)(y-3)=0$ d) $x^2-y^2=0$

849. Equation of the directrix of parabola $2x^2=14y$ is equal to
a) $y=-\frac{7}{4}$ b) $x=-\frac{7}{4}$ c) $y=\frac{7}{4}$ d) $y=\frac{7}{4}$

850. The angle between the asymptotes of the hyperbola $3x^2-y^2=3$ is
a) $\frac{\pi}{3}$ b) $\frac{\pi}{5}$ c) $\frac{2\pi}{3}$ d) $\frac{2\pi}{5}$

851. The equation of the ellipse whose foci are $(\pm 2,0)$ and eccentricity $\frac{1}{2}$ is
a) $\frac{x^2}{12}+\frac{y^2}{16}=1$ b) $\frac{x^2}{16}+\frac{y^2}{12}=1$ c) $\frac{x^2}{16}+\frac{y^2}{8}=1$ d) None of these

852. The area of the triangle formed by the tangent at (3, 4) to the circle $x^2+y^2=25$ and the coordinate axes is
a) $\frac{24}{25}$ b) 0 c) $\frac{325}{24}$ d) $-\left(\frac{24}{25}\right)$

853. If $2x+3y-6=0$ and $9x+6y-18=0$ cuts the axes in concyclic points, then the centre of the circle, is
a) (2,3) b) (3,2) c) (5,5) d) $(5/2,5/2)$

854. The point of intersection of two tangents to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, the product of whose slope is c^2, lies on the curve
a) $y^2-b^2=c^2(x^2+a^2)$ b) $y^2+a^2=c^2(x^2-b^2)$
c) $y^2+a^2=c^2(x^2-a^2)$ d) $y^2-a^2=c^2(x^2+b^2)$

855. A parabola is drawn with its focus at (3, 4) and vertex at the focus of the parabola $y^2-12x-4y+4=0$. The equation of the parabola is
a) $y^2-8x+6y+25=0$ b) $y^2-6x+8y-25=0$
c) $x^2-6x-8y+25=0$ d) $x^2+6x-8y-25=0$

856. The common tangent of the parabolas $y^2=4x$ and $x^2=-8y$ is
a) $y=x+2$ b) $y=x-2$ c) $y=2x+3$ d) None of these

857. The circle $x^2+y^2+4x-4y+4=0$ touches
a) x-axis b) y-axis c) x-axis and y-axis d) None of these

858. The line $2x-3y=5$ and $3x-4y=7$ are the diameter of a circle of ares 154 sq unit. The equation of this circle is $(\pi=22/7)$
a) $x^2+y^2+2x-2y=62$ b) $x^2+y^2+2x-2y=47$
c) $x^2+y^2-2x+2y=47$ d) $x^2+y^2-2x+2y=62$

859. If $P=(x,y),F_1=(3,0),F_2=(-3,0)$ and $16x^2+25y^2=400$, then PF_1+PF_2 equals
a) 8 b) 6 c) 10 d) 12

860. The distance between the directrices of the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ is
a) $\frac{9}{\sqrt{5}}$ b) $\frac{24}{\sqrt{5}}$ c) $\frac{18}{\sqrt{5}}$ d) None of these

861. A tangent at any point to the ellipse $4x^2 + 9y^2 = 36$ is cut by the tangent at the extremities of the major axis at T and T'. The circle on TT' as diameter passes through the point

a) $(0, \sqrt{5})$ b) $(\sqrt{5}, 0)$ c) $(2, 1)$ d) $(0, -\sqrt{5})$

862. If θ and ϕ are eccentric angle of the ends of a pair of conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then $\theta - \phi$ is equal to

a) $\pm\frac{\pi}{2}$ b) $\pm\pi$ c) 0 d) None of these

863. The radius of any circle touching the lines $3x - 4y + 5 = 0$ and $6x - 8y - 9 = 0$ is

a) 1.9 b) 0.95 c) 2.9 d) 1.45

864. If the tangent at the point $(2\sec\theta, 3\tan\theta)$ to the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$ is parallel to $3x - y + 4 = 0$, then the value of θ, is

a) 45° b) 60° c) 30° d) 75°

865. A circle passes through $(0, 0), (a, 0)$ and $(0, b)$ the coordinates of its centre are

a) $\left(\frac{b}{2}, \frac{a}{2}\right)$ b) $\left(\frac{a}{2}, \frac{b}{2}\right)$ c) (b, a) d) (a, b)

866. The Polar equation of the circle with centre $\left(2, \frac{\pi}{2}\right)$ and radius 3 units is

a) $r^2 + 4r\cos\theta = 5$ b) $r^2 + 4r\sin\theta = 5$ c) $r^2 - 4r\sin\theta = 5$ d) $r^2 - 4r\cos\theta = 5$

867. The locus of the centre of circle which cuts the circles $x^2 + y^2 + 4x - 6y + 9 = 0$ and $x^2 + y^2 - 4x + 6y + 4 = 0$ orthogonally, is

a) $12x + 8y + 5 = 0$ b) $8x + 12y + 5 = 0$ c) $8x - 12y + 5 = 0$ d) None of these

868. If $2x - 4y = 9$ and $6x - 12y + 7 = 0$ are common tangents to the circle, then radius of the circle is

a) $\frac{\sqrt{3}}{5}$ b) $\frac{17}{6\sqrt{5}}$ c) $\frac{\sqrt{2}}{3}$ d) $\frac{17}{3\sqrt{5}}$

869. Let $p(x_1, y_1)$ and $Q(x_2, y_2)$ are two points such that their abscissa x_1 and x_2 are the roots of the equation $x^2 + 2x - 3 = 0$ while the ordinates y_1 and y_2 are the roots of theequation$y^2 + 4y - 12 = 0$. The centre of the circle with PQ as diameter is

a) $(-1, -2)$ b) $(1, 2)$ c) $(1, -2)$ d) $(-1, 2)$

870. Angle between the tangents drawn to $y^2 = 4x$ at the points where it is intersected by the line $y = x - 1$is equal to

a) $\frac{\pi}{4}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{2}$

871. The coordinates of the focus of the parabola $x^2 - 4x - 8y - 4 = 0$ are

a) (0,2) b) (2,1) c) (1,2) d) $(-2, -1)$

872. A line touches the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$, then equation of tangent is

a) $y = x + 3$ b) $y = x + 2$ c) $y = x + 4$ d) $y = x + 1$

873. The locus of middle points of chords of hyperbola $3x^2 - 2y^2 + 4x - 6y = 0$ parallel to $y = 2x$ is

a) $3x - 4y = 4$ b) $3y - 4x + 4 = 0$ c) $4x - 3y = 3$ d) $3x - 4y = 2$

874. The line $ax + by + c = 0$ is normal to the circle $x^2 + y^2 + 2gx + 2fy + d = 0$, if

a) $ag + bf + c = 0$ b) $ag + bf - c = 0$ c) $ag - bf + c = 0$ d) $ag - bf - c = 0$

875. The equation of the ellipse whose vertices are $(-4,1), (6,1)$ and one of the focal chord is $x - 2y - 2 = 0$, is

a) $\frac{(x-1)^2}{25} + \frac{(y-1)^2}{9} = 1$

b) $\frac{(x+1)^2}{25} + \frac{(y+1)^2}{9} = 1$

c) $\frac{(x-1)^2}{16} + \frac{(y-1)^2}{25} = 1$

d) $\frac{(x+1)^2}{16} + \frac{(y+1)^2}{25} = 1$

876. For the ellipse $24x^2 + 9y^2 - 120x - 90y + 225 = 0$, the eccentricity is equal to

a) $\frac{2}{5}$ b) $\frac{3}{5}$ c) $\sqrt{\frac{15}{24}}$ d) $\frac{1}{5}$

877. If $x + y = k$ is normal to the parabola $y^2 = 12x$, then k is
a) 3 b) 9 c) -9 d) -3

878. If the circles $x^2 + y^2 + 4x + 8y = 0$ and $x^2 + y^2 + 8x + 2ky = 0$ touch each other, then k is equal to
a) 12 b) 8 c) -8 d) 4

879. If the circles $(x - a)^2 + (y - b)^2 = c^2$ and $(x - b)^2 + (y - a)^2 = c^2$ touch each other, then
a) $a = b \pm 2\,c$ b) $a = b \pm \sqrt{2}\,c$ c) $a = b \pm c$ d) None of these

880. The sum of the focal distances of any point on the conic $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is
a) 10 b) 9 c) 41 d) 18

881. The line $y = mx + 1$ is a tangent to the parabola $y^2 = 4x$, if
a) $m = 1$ b) m $= 2$ c) $m = 4$ d) $m = 3$

882. If in a hyperbola, the distance between the foci is 10 and the transverse axis has length 8, then the length of its latusrectum is
a) 9 b) $\frac{9}{2}$ c) $\frac{32}{3}$ d) $\frac{64}{3}$

883. Extremities of a diagonal of a rectangle are (0,0) and (4,3). The equations of the tangents to the circumcircle of the rectangle which are parallel to the diagonal are
a) $16x + 8y \pm 25 = 0$ b) $6x - 8y \pm 25 = 0$ c) $8x + 6y \pm 25 = 0$ d) None of these

884. The number of values of c such that the line $y = 4x + c$ touches the curve $\frac{x^2}{4} + y^2 = 1$ is
a) 1 b) 2 c) ∞ d) 0

885. Two tangents are drawn from the point $(-2, -1)$ to the parabola $y^2 = 4x$. If α is the angle between these tangents, then tan α is equal to
a) 3 b) 1/3 c) 2 d) 1/2

886. The locus of the point of intersection of the perpendicular tangents to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is
a) $x^2 + y^2 = 9$ b) $x^2 + y^2 = 4$ c) $x^2 + y^2 = 13$ d) $x^2 + y^2 = 5$

887. The locus of centre of a circle $x^2 + y^2 - 2x - 2y + 1 = 0$, which rolls outside the circle $x^2 + y^2 - 6x + 8y = 0$, is
a) $x^2 + y^2 - 2x - 2y - 34 = 0$
b) $x^2 + y^2 - 6x - 8y + 11 = 0$
c) $x^2 + y^2 - 6x + 8y - 11 = 0$
d) None of the above

888. The length of the chord of the circle $x^2 + y^2 = 25$ passing through (5,0) and perpendicular to the line $x + y = 0$ is
a) $5\sqrt{2}$ b) $5/\sqrt{2}$ c) $2\sqrt{5}$ d) None of these

889. The equation of a tangent parallel to $y = x$ drawn to $\frac{x^2}{3} - \frac{y^2}{2} = 1$, is
a) $x - y + 1 = 0$ b) $x - y + 2 = 0$ c) $x + y - 1 = 0$ d) $x - y + 2 = 0$

890. If $y = 2x + k$ is a tangent to the curve $x^2 = 4y$, then k is equal to
a) 4 b) 1/2 c) -4 d) $-1/2$

891. If the line $\frac{x}{a} + \frac{y}{b} = 1$ moves such that $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$, where c is a constant, then the locus of the foot of the perpendicular from the origin to the line is
a) Straight line b) Circle c) Parabola d) Ellipse

892. If the tangent at any point P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the tangents at the vertices A and A' in L and L' respectively, then $AL \cdot A'L' =$
a) $a + b$ b) $a^2 + b^2$ c) a^2 d) b^2

893. The slope of the normal at the point $(at^2, 2\,at)$ of the parabola, $y^2 = 4ax$, is
a) $\frac{1}{t}$ b) t c) $-t$ d) $-\frac{1}{t}$

894. Two rods of lengths a and b slide along the x-axis and y-axis respectively in such a manner that their ends

are concylic. The locus of the centre of the circle passing through the end points is

a) $4(x^2+y^2)=a^2+b^2$ b) $x^2+y^2=a^2+b^2$

c) $4(x^2-y^2)=a^2-b^2$ d) $x^2-y^2=a^2-b^2$

895. The point $(3,-4)$ lies on both the circles $x^2+y^2-2x+8y+13=0$and$x^2+y^2-4x+6y+11=0$. Then, the angle between the circles is

a) $60°$ b) $\tan^{-1}\left(\frac{1}{2}\right)$ c) $\tan^{-1}\left(\frac{3}{5}\right)$ d) $45°$

896. The equation of the normal at the point $P(2,3)$ on the ellipse $9x^2+16y^2=180$ is

a) $3y=8x-10$ b) $3y-8x+7=0$ c) $8y+3x+7=0$ d) $3x+2y+7=0$

897. AB is a chord of the parabola $y^2=4ax$ with vertex A, BC is drawn perpendicular to AB meeting the axis at C. The projection of BC on the axis of the parabola is

a) a b) $2a$ c) $4a$ d) $8a$

898. The distance of the point $'\theta'$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ from a focus is

a) $a(e+\cos\theta)$ b) $a(e-\cos\theta)$ c) $a(1+e\cos\theta)$ d) $a(1+2e\cos\theta)$

899. The number of real tangents that can be drawn to the curve $y^2+2xy+x^2+2x+3y+1=0$ from the point $(1,-2)$ is

a) One b) Two c) Zero d) None of these

900. A general point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

a) $(a\sin\theta, b\cos\theta)$ (where θ is parameter) b) $(a\tan\theta, b\sec\theta)$ (where θ is parameter)

c) $\left(a\frac{e^t+e^{-t}}{2}, b\frac{e^t-e^{-t}}{2}\right)$ (where t is parameter) d) None of the above

901. If one end of the diameter is $(1, 1)$ and the other end lies on the line $x+y=3$, then locus of centre of circle is

a) $x+y=1$ b) $2(x-y)=5$ c) $2x+2y=5$ d) None of these

902. The equation of the smallest circle passing through the points $(2, 2)$ and $(3, 3)$ is

a) $x^2+y^2+5x+5y+12=0$ b) $x^2+y^2-5x-5y+12=0$

c) $x^2+y^2+5x-5y+12=0$ d) $x^2+y^2-5x+5y-12=0$

903. The equation of the directrix of parabola $y^2+4y+4x+2=0$ is

a) $x=-1$ b) $x=1$ c) $x=-\frac{3}{2}$ d) $x=\frac{3}{2}$

904. If a point $P(x,y)$ moves along the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and if C is the centre of the ellipse, then the sum of maximum and minimum values of CP is

a) 25 b) 9 c) 4 d) 5

905. If the tangent to the parabola $y^2=ax$ makes an angle of 45° with x-axis, then the point of contact is

a) $\left(\frac{a}{2},\frac{a}{2}\right)$ b) $\left(\frac{a}{4},\frac{a}{4}\right)$ c) $\left(\frac{a}{2},\frac{a}{4}\right)$ d) $\left(\frac{a}{4},\frac{a}{2}\right)$

906. The locus of the point (l,m) so that $lx+my=1$ touches the circle $x^2+y^2=a^2$, is

a) $x^2+y^2-ax=0$ b) $x^2+y^2=\frac{1}{a^2}$

c) $y^2=4ax$ d) $x^2+y^2-ax-ay+a^2=0$

907. The eccentricity of the hyperbola $9x^2-16y^2-18x-64y-199=0$ is

a) $\frac{16}{9}$ b) $\frac{5}{4}$ c) $\frac{25}{16}$ d) Zero

908. If b and C are the lengths of the segments of any focal chord of a parabola $y^2=4ax$, then the length of the semilatusrectum is

a) $\frac{bc}{b+c}$ b) $\sqrt{bc}$ c) $\frac{b+c}{2}$ d) $\frac{2bc}{b+c}$

909. If the area of the auxiliary circle of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b)$ is twice the area of the ellipse, then the eccentricity of the ellipse is

a) $\frac{1}{\sqrt{2}}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{1}{\sqrt{3}}$ d) $\frac{1}{2}$

910. The distance of the mid point of line joining two points $(4, 0)$ and $(0, 4)$ from the centre of the circle $x^2 + y^2 = 16$ is

a) $\sqrt{2}$ b) $2\sqrt{2}$ c) $3\sqrt{2}$ d) $2\sqrt{3}$

911. The equation of the chord of the circle, $x^2 + y^2 = a^2$ having (x_1, y_1) as its mid point, is

a) $xy_1 + yx_1 = a^2$ b) $x_1 + y_1 = a$ c) $xx_1 + yy_1 = x_1^2 + y_1^2$ d) $xx_1 + yy_1 = a^2$

912. The angle between the pair of tangents drawn from (1,3) to the parabola $y^2 = 8x$, is

a) $\tan^{-1} 2$ b) $\tan^{-1} \frac{1}{2}$ c) $\tan^{-1} \frac{1}{3}$ d) $\tan^{-1} 3$

913. The equations of the tangents to circle $x^2 + y^2 - 6x + 4y - 12 = 0$, which are parallel to line $4x + 3y + 15$are

a) $4x + 3y + 11 = 0$ and $4x + 3y + 8 = 0$
b) $4x + 3y - 9 = 0$ and $4x + 3y + 7 = 0$
c) $4x + 3y + 19 = 0$ and $4x + 3y - 31 = 0$
d) $4x + 3y - 10 = 0$ and $4x + 3y + 12 = 0$

914. Eccentricity of hyperbola whose asymptotes are $3x - 4y = 7$ and $4x + 3y = 8$, is

a) $\sqrt{2}$
b) 2
c) Not sufficient information
d) None of the above

915. Length of the tangents from the point (1, 2) to the circles $x^2 + y^2 + x + y - 4 = 0$ and $3x^2 + 3y^2 - x - y - k = 0$ are in the ration 4:3, then k is equal to

a) 37/2 b) 4/37 c) 12 d) 39/4

916. If $P(\theta)$ and $Q(\pi/2 + \theta)$ are two points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Locus of the mid-point of PQ is

a) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{2}$ b) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 4$ c) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$ d) None of these

917. The circle $ax^2 + ay^2 + 2g_1x + 2f_1y + c_1 = 0$
and $bx^2 + by^2 + 2g_2x + 2f_2y + c_2 = 0$
($a \neq 0$ and $b \neq 0$) cut orthogonally, if

a) $g_1g_2 + f_1f_2 = ac_1 + bc_2$
b) $2(g_1g_2 + f_1f_2) = bc_1 + bc_2$
c) $bg_1g_2 + af_1f_2 = ac_1 + bc_2$
d) $g_1g_2 + f_1f_2 = c_1 + c_2$

918. The curve represented by the equation $4x^2 + 16y^2 - 24x - 32y - 12 = 0$ is

a) A parabola
b) A pair of straight lines
c) An ellipse with eccentricity 1/2
d) An ellipse with eccentricity $\sqrt{3}/2$

919. If the chord of contact of tangents drawn from the point (h, k) to the circle $x^2 + y^2 = a^2$ subtends a right angle at the centre, then

a) $h^2 + k^2 = a^2$ b) $2(h^2 + k^2) = a^2$ c) $h^2 - k^2 = a^2$ d) $h^2 + k^2 = 2a^2$

920. The tangent to $x^2 + y^2 = 9$ which is parallel to y-axis and does not lie in the third quadrant touches the circle at the point

a) (3,0) b) (−3,0) c) (0,3) d) (0, −3)

921. The equation of the circle of radius 5 and touching the coordinate axes in third quadrant is

a) $(x - 5)^2 + (y + 5)^2 = 25$
b) $(x + 4)^2 + (y + 4)^2 = 25$
c) $(x + 6)^2 + (y + 6)^2 = 25$
d) $(x + 5)^2 + (y + 5)^2 = 25$

922. The straight line $x + y = \sqrt{2}p$ will touch the hyperbola $4x^2 - 9y^2 = 36$, if

a) $p^2 = 2$ b) $p^2 = 5$ c) $5p^2 = 2$ d) $2p^2 = 5$

923. If P is a point such that the ratio of the squares of the lengths of the tangents from P to the circles $x^2 + y^2 + 2x - 4y - 20 = 0$ and $x^2 + y^2 - 4x + 2y - 44 = 0$ is 2: 3, then the locus of P is a circle with centre

a) (7, −8) b) (−7, 8) c) (7, 8) d) (−7, −8)

924. The length of the latusrectum of the ellipse $\frac{x^2}{36} + \frac{y^2}{49} = 1$ is

a) 98/6 b) 72/7 c) 72/14 d) 98/12

925. The equation $\frac{x^2}{2-\lambda} - \frac{y^2}{\lambda-5} - 1 = 0$, represent an ellipse, if

a) $\lambda > 5$ b) $\lambda < 2$ c) $2 < \lambda < 5$ d) $2 > \lambda > 5$

926. If the normals from any point to the parabola $x^2 = 4y$ cuts the line $y = 2$ in points whose abscissae are in A.P., then the slopes of the tangents at the three conormal points are in

a) A.P. b) G.P. c) H.P. d) None of these

927. The area of the quadrilateral formed by the tangents at the end points of latusrectum to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ is

a) 27/4 sq unit b) 9 sq unit c) 27/2 sq unit d) 27 sq unit

928. The mid-point of the chord $2x + y - 4 = 0$ of the parabola $y^2 = 4x$ is

a) $(5/2, -1)$ b) $(-1, 5/2)$ c) $(3/2, -1)$ d) None of these

929. The equation of the circle with centre (2, 1) and touching the line $3x + 4y = 5$ is

a) $x^2 + y^2 - 4x - 2y + 5 = 0$ b) $x^2 + y^2 - 4x - 2y - 5 = 0$

c) $x^2 + y^2 - 4x - 2y + 4 = 0$ d) $x^2 + y^2 - 4x - 2y - 4 = 0$

930. If $(-1, -2\sqrt{2})$ is one of extremity of a focal chord of the parabola $y^2 = -8x$, then the other extremity is

a) $(-1, -\sqrt{2})$ b) $(2\sqrt{2}, -1)$ c) $(-4, 4\sqrt{2})$ d) $(4, 4\sqrt{2})$

931. The length of the chord joining the points $(4\cos\theta, 4\sin\theta)$ and $(4\cos(\theta + 60°), 4\sin(\theta + 60°))$ of the circle $x^2 + y^2 = 16$ is

a) 4 b) 8 c) 16 d) 2

932. The distance between the foci of the hyperbola $x^2 - 3y^2 - 4x - 6y - 11 = 0$ is

a) 4 b) 6 c) 8 d) 10

933. The locus of the centre of the circle which cuts the circles $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $x^2 + y^2 + 2g_2 + 2f_2y + c_2 = 0$ orthogonally, is

a) An ellipse

b) The radical axis of the given circles

c) A conic

d) Another circle

934. The radius of the circle with the polar equation $r^2 - 8r(\sqrt{3}\cos\theta + \sin\theta) + 15 = 0$ is

a) 8 b) 7 c) 6 d) 5

935. The centre of the circle passing through $(0,0)(a, 0)$and $(0, b)$ is

a) (a, b) b) $a/2, b/2$ c) $(-a/2, -b/2)$ d) $-a, -b$

936. The mid-point of the line joining the common points of the line $2x - 3y + 8 = 0$ and $y^2 = 8x$, is

a) (3,2) b) (5,6) c) $(4, -1)$ d) $(2, -3)$

937. The equation of the tangent to the parabola $y^2 = 8x$ which is perpendicular to the line $x - 3y + 8 = 0$ is

a) $9x + 3y + 2 = 0$ b) $3x + y + 2 = 0$ c) $3x - y - 1 = 0$ d) $9x - 3y + 2 = 0$

938. The length of the chord of the parabola $y^2 = 4ax$ passing through the vertex and making an angle θ with the axis is

a) $4a\,\text{cosec}^2\,\theta$ b) $4a\cos\theta\,\text{cosec}^2\,\theta$ c) $4a\cot\theta\,\text{cosec}^2\,\theta$ d) $2a\,\text{cosec}^2\,\theta$

939. The distance between the foci of the ellipse $5x^2 + 9y^2 = 45$, is

a) $2\sqrt{2}$ b) 4 c) $4\sqrt{2}$ d) 2

940. The equation of the parabola with focus (0,0) and directrix $x + y = 4$, is

a) $x^2 + y^2 - 2xy + 8x + 8y - 16 = 0$ b) $x^2 + y^2 - 2xy + 8x + 8y = 0$

c) $x^2 + y^2 + 8x + 8y - 16 = 0$ d) $x^2 - y^2 + 8x + 8y - 16 = 0$

941. The length of the chord cut off by $y = 2x + 1$ from the circle $x^2 + y^2 = 2$, is

a) $\frac{5}{6}$ b) $\frac{6}{5}$ c) $\frac{6}{\sqrt{5}}$ d) $\frac{\sqrt{5}}{6}$

942. The two circles $x^2 + y^2 - 2x + 6y + 6 = 0$ and $x^2 + y^2 - 5x + 6y + 15 = 0$ touch each other

a) Externally b) Internally c) Coincide d) None of these

943. The equation of the normal to the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ at $(-4,0)$ is

a) $2x-3y=1$ b) $x=0$ c) $x=1$ d) $y=0$

944. The locus of the centre of the circle for which one end of a diameter is $(1, 1)$ while the other end is on the line $x+y=3$, is

a) $x+y=1$ b) $2(x-y)=5$ c) $2x+2y=5$ d) None of these

945. The mid-point of the chord intercepted by the hyperbola $9x^2-16y^2=144$ on the line $9x-8y-10=0$, is

a) $(1,2)$ b) $(-1,2)$ c) $(-2,1)$ d) $(2,1)$

946. The radius of the circle, which is touched by the line $y=x$ and has its centre on the positive direction of x-axis and also cuts-off a chord of length 2 unit along the line $\sqrt{3}y-x=0$, is

a) $\sqrt{5}$ b) $\sqrt{3}$ c) $\sqrt{2}$ d) 1

947. If a focal chord of the parabola $y^2=ax$ is $2x-y-8=0$, then the equation of the directrix is

a) $x+4=0$ b) $x-4=0$ c) $y-4=0$ d) $y+4=0$

948. The focus of the parabola $x^2+2y+6x=0$ is

a) $(-3,4)$ b) $(3,4)$ c) $(3,-4)$ d) $(-3,-4)$

949. The normal drawn at a point $(at_1^2, 2\,at_1)$ of the parabola $y^2=4ax$ meets it again on the point $(at_2^2, 2\,at_2)$, then

a) $t_1=2\,t_2$ b) $t_1^2=2\,t_2$ c) $t_1t_2=-1$ d) None of these

950. If tangent and normal to a rectangular hyperbola $xy=c^2$ cut off intercepts a_1 and a_2 on one axis and b_1, b_2 on the other, then

a) $a_1=b_1$ b) $a_2=b_2$ c) $\frac{a_1}{a_2}=\frac{b_1}{b_2}$ d) $a_1a_2+b_1b_2=0$

951. The range of α, which the point (α,α) lies inside the region bounded by the curves $y=\sqrt{1-x^2}$ and $x+y=1$ is

a) $\frac{1}{2}<\alpha<\frac{1}{\sqrt{2}}$ b) $\frac{1}{2}<\alpha<\frac{1}{3}$ c) $\frac{1}{3}<\alpha<\frac{1}{\sqrt{3}}$ d) $\frac{1}{4}<\alpha<\frac{1}{2}$

952. The normal at the point (3,4) on a circle at the point $(-1,-2)$. The equation of the circle, is

a) $x^2+y^2+2x-2y-13=0$

b) $x^2+y^2-2x-2y-11=0$

c) $x^2+y^2-2x+2y+12=0$

d) $x^2+y^2-2x-2y+14=0$

953. The focal chord to $y^2=16x$ is tangent to $(1-6)^2+y^2=2$, then the possible values of the slope of this chord are

a) $\{-1,1\}$ b) $\{-2,2\}$ c) $\left\{-2,\frac{1}{2}\right\}$ d) $\left\{2,-\frac{1}{2}\right\}$

954. The equation of the parabola with its vertex at the origin, axis on the y-axis and passing through the point $(6,-3)$ is

a) $y^2=12x+6$ b) $x^2=12y$ c) $x^2=-12y$ d) $y^2=-12x+6$

955. The sum of focal distance of any point on the ellipse with major and minor axes as $2a$ and $2b$ respectively, is equal to

a) $2a$ b) $2\frac{a}{b}$ c) $2\frac{b}{a}$ d) $\frac{b^2}{a}$

956. The maximum number of points with rational coordinates on a circle whose centre is $(\sqrt{3},0)$ is

a) One b) Two c) Four d) Infinite

957. The length of major and minor axis of an ellipse are 10 and 8 respectively and its major axis along y-axis the equation of the ellipse referred to its centre as origin is

a) $\frac{x^2}{25}+\frac{y^2}{16}=1$ b) $\frac{x^2}{16}+\frac{y^2}{25}=1$ c) $\frac{x^2}{100}+\frac{y^2}{64}=1$ d) $\frac{x^2}{64}+\frac{y^2}{100}=1$

958. The equation of the circle which touches the axes of the coordinates and the line $\frac{x}{3}+\frac{y}{4}=1$ and whose

centre lies in the first quadrant is $x^2+y^2-2cx-2cy+c^2=0$, where c is

a) 1,6 b) 2,1 c) 3,6 d) 6,4

959. The area of the triangle formed by three points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose eccentric angles are α,β and γ is

a) $2\,ab\sin\frac{\alpha-\beta}{2}\cos\frac{\beta-\gamma}{2}\cos\frac{\gamma-\alpha}{2}$

b) $2ab\sin\frac{\alpha-\beta}{2}\sin\frac{\beta-\gamma}{2}\cos\frac{\gamma-\alpha}{2}$

c) $2ab\sin\frac{\alpha-\beta}{2}\sin\frac{\beta-\gamma}{2}\sin\frac{\gamma-\alpha}{2}$

d) $2\,ab\cos\frac{\alpha-\beta}{2}\cos\frac{\beta-\gamma}{2}\cos\frac{\gamma-\alpha}{2}$

960. The eccentricity of the conic $4x^2+16y^2-24x-32y=1$ is

a) $\frac{1}{2}$ b) $\sqrt{3}$ c) $\frac{\sqrt{3}}{2}$ d) $\frac{\sqrt{3}}{4}$

961. The equation of the ellipse having vertices at $(\pm 5,0)$ and $(\pm 4,0)$ is

a) $\frac{x^2}{25}+\frac{y^2}{16}=1$ b) $9x^2+25y^2=225$ c) $\frac{x^2}{9}+\frac{y^2}{25}=1$ d) $4x^2+5y^2=20$

962. The circle S_1 with centre $C_1(a_1,b_1)$ and radius r_1 touches externally the circle S_2 with centre $C_2(a_2,b_2)$ and radius r_2. If the tangent at their common point passes through the origin, then

a) $(a_1^2+a_2^2)+(b_1^2+b_2^2)=r_1^2+r_2^2$

b) $(a_1^2-a_2^2)+(b_1^2-b_2^2)=r_1^2-r_2^2$

c) $(a_1^2-b_2^2)+(a_2^2+b_2^2)=r_1^2+r_2^2$

d) $(a_1^2-b_1^2)+(a_2^2+b_2^2)=r_1^2+r_2^2$

963. If eccentricity of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is e and e' is the eccentricity of its conjugate hyperbola, then

a) $e=e'$ b) $ee'=1$ c) $\frac{1}{e^2}+\frac{1}{(e')^2}$ d) None of these

964. If the equation of tangent to the circle $x^2+y^2-2x+6y-6=0$ parallel to $3x-4y+7=0$ is $3x-4y+k=0$, then the value of k are

a) $5,-35$ b) $-5,35$ c) $7,-32$ d) $-7,32$

965. If circle $x^2+y^2+2gx+2fy+k=0$ intersect hyperbola $xy=c^2$ at four points $(x_i,y_1), i=1,2,3,4$ then

a) $x_1+x_2+x_3+x_4=-g$ b) $x_1+x_2+x_3+x_4=-2g$

c) $x_1+x_2+x_3+x_4=-4g$ d) $x_1+x_2+x_3+x_4=2g$

966. If tangents are drawn to the ellipse $x^2+2y^2=2$, then the locus of the mid point of the intercept made by the tangents between the coordinate axes is

a) $\frac{1}{2x^2}+\frac{1}{4y^2}=1$ b) $\frac{1}{4x^2}+\frac{1}{2y^2}=1$ c) $\frac{x^2}{2}+\frac{y^2}{4}=1$ d) $\frac{x^2}{4}+\frac{y^2}{2}=1$

967. The curve represented by the equation $4x^2+16y^2-24x-32y-12=0$ is

a) A parabola b) A pair of straight lines

c) An ellipse with eccentricity $\frac{1}{2}$ d) An ellipse with eccentricity $\frac{\sqrt{3}}{2}$

968. If the eccentricity of the two ellipse $\frac{x^2}{169}+\frac{y^2}{25}=1$ and $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are equal, then the value of $\frac{a}{b}$ is

a) $\frac{5}{13}$ b) $\frac{6}{13}$ c) $\frac{13}{5}$ d) $\frac{13}{6}$

969. The distance between the directrices of the hyperbola $x=8\sec\theta, y=8\tan\theta$ is

a) $8\sqrt{2}$ b) $16\sqrt{2}$ c) $4\sqrt{2}$ d) $6\sqrt{2}$

970. The equation to the line touching both the parabolas $y^2=4x$ and $x^2=-32y$, is

a) $x+2y+4=0$ b) $2x+y-4=0$ c) $x-2y-4=0$ d) $x-2y+4=0$

971. The locus of the mid point of the chord of the circle $x^2+y^2-2x-2y-2=0$ which makes an angle of $120°$ at the centre, is

a) $x^2+y^2-2x-2y-1=0$ b) $x^2+y^2+x+y-1=0$
c) $x^2+y^2-2x-2y+1=0$ d) None of the above

972. If $\frac{x}{\alpha}+\frac{y}{\beta}=1$ touches the circle $x^2+y^2=a^2$, the point $(1/\alpha, 1/\beta)$ lies on a/an
a) Straight line b) Circle c) Parabola d) Ellipse

973. The graph represented by the equations $x=\sin^2 t, y=2\cos t$, is
a) A portion of a parabola
b) A parabola
c) A part of a sine graph
d) A part of a hyperbola

974. The equation of tangent to the ellipse $x^2+4y^2=5$ at $(-1,1)$, is
a) $x+4y+5=0$ b) $x-4y-5=0$ c) $x+4y-5=0$ d) $x-4y+5=0$

975. Two circles, each of radius 5, have a common tangent at (1,1) whose equation is $3x+4y-7=0$. Then, their centres are
a) $(4,-5)(-2,3)$ b) $(4,-3)(-2,5)$ c) $(4,5)(-2,-3)$ d) None of these

976. The equation of the circle having its centre on the line $x+2y-3=0$ and passing through the point of intersection of the circles
$x^2+y^2-4y+1=0$ and $x^2+y^2-4x-2y+4=0$ is
a) $x^2+y^2-6x+7=0$
b) $x^2+y^2-3x+4=0$
c) $x^2+y^2-2x-2y+1=0$
d) $x^2+y^2+2x-4y+4=0$

977. If two different tangents of $y^2=4x$ are the normals to $x^2=4by$, then
a) $|b|>\frac{1}{2\sqrt{2}}$ b) $|b|<\frac{1}{2\sqrt{2}}$ c) $|b|>\frac{1}{\sqrt{2}}$ d) $|b|<\frac{1}{\sqrt{2}}$

978. The line $3x-2y=k$ meets the circle $x^2+y^2=4r^2$ at only one point, if k^2 is
a) $20r^2$ b) $52r^2$ c) $\frac{52}{9}r^2$ d) $\frac{20}{9}r^2$

979. The distance between the foci of the conic $7x^2-9y^2=63$ is equal to
a) 8 b) 4 c) 3 d) 7

980. If the circle $x^2+y^2=a^2$ intersects the hyperbola $xy=c^2$ in four points (x_i, y_i), for $i=1,2,3$ and 4, then $y_1+y_2+y_3+y_4$ equals
a) 0 b) c c) a d) c^4

981. Consider the following statements :
1. The equation of the parabola whose focus is at the origin is $y^2=4a(x+a)$
2. The line $lx+my+n=0$ will touch the parabola $y^2=4ax$, if $ln=am^2$
Which of these is/are correct
a) Only (1) b) Only (2) c) Both of these d) None of these

982. If M_1 and M_2 are the feet of the perpendiculars from the foci S_1 and S_2 of the ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$ on the tangent at any point P on the ellipse, then $(S_1M_1)(S_2M_2)$ is equal to
a) 16 b) 9 c) 4 d) 3

983. The equation of two circles which touch the y-axis at (0, 3) and make an intercept of 8 unit on x-axis, are
a) $x^2+y^2\pm 10x-6y+9=0$ b) $x^2+y^2\pm 6x-10y+9=0$
c) $x^2+y^2-8x\pm 10y+9=0$ d) $x^2+y^2\pm 10x\pm 6y+9=0$

984. If $ax^2+by^2+2gx+2fy+c=0$ represents an ellipse, then
a) It's major axis is parallel to x-axis
b) It's major axis is parallel to y-axis
c) It's axes (ie, major axis and minor axis) are neither parallel to x-axis nor parallel to y-axis
d) It's axes are parallel to coordinates axes

985. Which of the following is a point on the common chord of the circles $x^2+y^2+2x-3y+6=0$ and x^2+

$y^2 + x - 8y - 13 = 0$

a) $(1, 4)$ b) $(1, -2)$ c) $(1, -4)$ d) $(1, 2)$

986. AB is a diameter of $x^2 + 9y^2 = 25$. The eccentric angle of A is $\pi/6$. Then, the eccentric angle of B is

a) $5\pi/6$ b) $-5\pi/6$ c) $-2\pi/3$ d) None of these

987. The mirror image of the parabola $y^2 = 4x$ in the tangent to the parabola at the point $(1, 2)$ is

a) $(x-1)^2 = 4(y+1)$ b) $(x+1)^2 = 4(y+1)$ c) $(x+1)^2 = 4(y-1)$ d) $(x-1)^2 = 4(y-1)$

988. The equation of the tangent to circle $5x^2 + 5y^2 = 1$, parallel to line $3x + 4y = 1$ are

a) $3x + 4y = \pm 2\sqrt{5}$ b) $6x + 8y = \pm\sqrt{5}$ c) $3x + 4y = \pm\sqrt{5}$ d) None of these

989. The diameter of a circle are along $2x + y - 7 = 0$and$x + 3y - 11 = 0$. Then, the equation of this circle, which also passes through $(5, 7)$, is

a) $x^2 + y^2 - 4x - 6y - 16 = 0$ b) $x^2 + y^2 - 4x - 6y - 20 = 0$

c) $x^2 + y^2 - 4x - 6y - 12 = 0$ d) $x^2 + y^2 + 4x + 6y - 12 = 0$

990. If the tangent at a point $(a\cos\theta, b\sin\theta)$ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the auxiliary circle in two points, the chord joining them subtends a right angle at the centre, then the eccentricity of the ellipse is given by

a) $(1+\cos^2\theta)^{-1/2}$ b) $(1+\sin^2\theta)$ c) $(1+\sin^2\theta)^{-1/2}$ d) $(1+\cos^2\theta)$

991. The value of m for which $y = mx + 6$ is a tangent to the hyperbola $\frac{x^2}{100} - \frac{y^2}{49} = 1$, is

a) $\sqrt{\frac{17}{20}}$ b) $\sqrt{\frac{20}{17}}$ c) $\sqrt{\frac{3}{20}}$ d) $\sqrt{\frac{20}{3}}$

992. The parametric equation of a parabola is $x = t^2 + 1, y = 2t + 1$. The cartesian equation of its directrix is

a) $x = 0$ b) $x + 1 = 0$ c) $y = 0$ d) None of these

993. The length of the subtangent to the parabola $y^2 = 16x$ at the point whose abscissa is 4, is

a) 2 b) 4 c) 8 d) None of these

994. The angle between the asymptotes of the hyperbola $x^2 + 2xy - 3y^2 + x + 7y + 9 = 0$ is

a) $\tan^{-1}(\pm 2)$ b) $\tan^{-1}(\pm\sqrt{3})$ c) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ d) $\tan^{-1}\left(\frac{1}{2}\right)$

995. If the points $(2, 0)$, $(0, 1)$, $(4, 5)$ and $(0, c)$ are concyclic, then the value of c is

a) 1 b) $\frac{14}{3}$ c) 5 d) None of these

996. If the line $x + y - 1 = 0$ is a tangent to the parabola $y^2 - y + x = 0$, then the point of contact is

a) $(0, 1)$ b) $(1, 0)$ c) $(0, -1)$ d) $(-1, 0)$

997. If $4x - 3y + k = 0$ touches the ellipse $5x^2 + 9y = 45$, then k is equal to

a) $\pm 3\sqrt{21}$ b) $3\sqrt{21}$ c) $-3\sqrt{21}$ d) $2\sqrt{21}$

998. The point of the parabola $y^2 = 18x$, for which the ordinate is three times the abscissa, is

a) $(6, 2)$ b) $(-2, -6)$ c) $(3, 18)$ d) $(2, 6)$

999. Tangent at the vertex divides the distance between directrix and latusrectum in the ratio

a) 1: 1 b) 1: 2

c) Depends on directrix and focus d) None of the above

1000. The point $(4, -3)$ with respect to the ellipse $4x^2 + 5y^2 = 1$ is

a) Lies on the curve b) Is inside the curve c) Is outside the curve d) Is focus of the curve

1001. If $x - y + 1 = 0$ meets the circle $x^2 + y^2 + y - 1 = 0$ at A and B, then the equation of the circle with AB as diameter is

a) $2(x^2 + y^2) + 3x - y + 1 = 0$ b) $2(x^2 + y^2) + 3x - y + 2 = 0$

c) $2(x^2 + y^2) + 3x - y + 3 = 0$ d) $x^2 + y^2 + 3x - y + 1 = 0$

1002. The equation of circle with centre $(1, 2)$ and tangent $x + y - 5 = 0$ is

a) $x^2 + y^2 + 2x - 4y + 6 = 0$ b) $x^2 + y^2 - 2x - 4y + 3 = 0$

c) $x^2 + y^2 - 2x + 4y + 8 = 0$ d) $x^2 + y^2 - 2x - 4y + 8 = 0$

1003. Equation of chord of an ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$, whose mid point is $(1,1)$, is

a) $25x+9y=36$ b) $9x+25y=34$ c) $9x-25y=34$ d) None of these

1004. The latusrectum of the parabola $y^2=4ax$, whose focal chord is PSQ, such that $SP=3$ and $SQ=2$ is given by

a) $\frac{24}{5}$ b) $\frac{12}{5}$ c) $\frac{6}{5}$ d) $\frac{1}{5}$

1005. Length of normal chord $y=x+c$ to the parabola $y^2=8x$ is

a) $6\sqrt{2}$ unit b) $12\sqrt{2}$ unit c) $16\sqrt{2}$ unit d) None of these

1006. If the circle $x^2+y^2+6x-2y+k=0$ bisects the circumference of the circle $x^2+y^2+2x-6y-15=0$, then k is equal to

a) 21 b) -21 c) 23 d) -23

1007. The equation of the normal at the point $(2,3)$ on the ellipse $9x^2+16y^2=180$ is

a) $3y=8x-10$ b) $3y-8x+7=0$ c) $8y+3x+7=0$ d) $3x+2y+7=0$

1008. If $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$ are two hyperbola, then

a) Their asymptotes are same
b) Their eccentricity are same
c) Their transverse axes are same
d) Asymptotes of Ist are angle bisectors of asymptotes of IInd hyperbola

1009. If the chord joining points $P(\alpha)$ and $Q(\beta)$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ subtends a right angle at the vertex $A(a,0)$, then $\tan\alpha/2\tan\beta/2=$

a) $\frac{a^2}{b^2}$ b) $-\frac{a^2}{b^2}$ c) $\frac{b^2}{a^2}$ d) $-\frac{b^2}{a^2}$

1010. Equation of asymptotes of $xy=7x+5y$ are

a) $x=7, y=5$ b) $x=5, y=7$ c) $xy=35$ d) None of these

1011. The point diametrically opposite to the point $P(1,0)$ on the circle $x^2+y^2+2x+4y-3=0$ is

a) $(3,4)$ b) $(3,-4)$ c) $(-3,4)$ d) $(-3,-4)$

1012. The equation of the circle passing through (0,0) and belonging to the system of circles of which (3,1) and $(-1,5)$ are limiting points, is

a) $x^2+y^2-x+3y=0$
b) $x^2+y^2-11x+3y=0$
c) $x^2+y^2=1$
d) None of these

1013. The angle between the tangent drawn from the point $(1,4)$ to the parabola $y^2=4x$ is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

1014. The equations of the circle which pass through the origin and makes intercepts of lengths 4 and 8 on the x and y-axes respectively are

a) $x^2+y^2\pm 4x\pm 8y=0$ b) $x^2+y^2\pm 2x\pm 4y=0$
c) $x^2+y^2\pm 8x\pm 16y=0$ d) $x^2+y^2\pm x\pm y=0$

1015. The equation of the tangent to the hyperbola $4y^2=x^2-1$ at the point (1,0), is

a) $x=1$ b) $y=1$ c) $y=4$ d) $x=4$

1016. The parametric representation of a point on the ellipse whose foci are $(-1,0)$ and $(7,0)$ and eccentricity

6. 1/2 is

a) $(3+8\cos\theta, 4\sqrt{3}\sin\theta)$ b) $(8\cos\theta, 4\sqrt{3}\sin\theta)$
c) $(3+4\sqrt{3}\cos\theta, 8\sin\theta)$ d) None of these

1017. If a focal chord of the parabola $y^2 = ax$ is $2x - y - 8 = 0$, then the equation of the directrix is

a) $x + 4 = 0$ b) $x - 4 = 0$ c) $y - 4 = 0$ d) $y + 4 = 0$

1018. The line $x + y = 6$ is a normal to the parabola $y^2 = 8x$ at the point

a) $(18, -12)$ b) $(4, 2)$ c) $(2, 4)$ d) $(8, 8)$

1019. The focal chord to $y^2 = 16x$ is tangent to $(x-6)^2 + y^2 = 2$, then the possible values of the slope of this chord, are

a) $\{-1, 1\}$ b) $\{-2, 2\}$ c) $\{-2, 1/2\}$ d) $\{2, -1/2\}$

1020. If from the origin a chord is drawn to the circle $x^2 + y^2 - 2x = 0$, then the locus of the mid point of the chord has equation

a) $x^2 + y^2 + x + y = 0$
b) $x^2 + y^2 + 2x + y = 0$
c) $x^2 + y^2 - x = 0$
d) $x^2 + y^2 - 2x + y = 0$

1021. Four distinct points $(2k, 3k)$, $(1,0)$, $(0,1)$ and $(0,0)$ lie on a circle for

a) All integral values of k b) $0 < k < 1$ c) $k < 0$ d) For two values of k

1022. The equation of parabola with focus $(0, 0)$ and directrix $x + y = 4$, is

a) $x^2 + y^2 - 2xy + 8x + 8y - 16 = 0$ b) $x^2 + y^2 - 2xy + 8x + 8y = 0$
c) $x^2 + y^2 + 8x + 8y - 16 = 0$ d) $x^2 - y^2 + 8x + 8y - 16 = 0$

1023. One of the points on the parabola $y^2 = 12x$ with focal distance 12, is

a) $(3, 6)$ b) $(9, 6\sqrt{3})$ c) $(7, 2\sqrt{21})$ d) $(8, 4\sqrt{6})$

1024. The equation of family of circles with centre at (h, k) touching the x-axis is given by

a) $x^2 + y^2 - 2hx + h^2 = 0$ b) $x^2 + y^2 - 2hx - 2ky + h^2 = 0$
c) $x^2 + y^2 - 2hx - 2ky - h^2 = 0$ d) $x^2 + y^2 - 2hx - 2ky = 0$

1025. The parabola with directrix $x + 2y - 1 = 0$ and focus $(1,0)$ is

a) $4x^2 - 4xy + y^2 - 8x + 4y + 4 = 0$ b) $4x^2 + 4xy + y^2 - 8x + 4y + 4 = 0$
c) $4x^2 + 5xy + y^2 + 8x - 4y + 4 = 0$ d) $4x^2 - 4xy + y^2 - 8x - 4y + 4 = 0$

1026. The square of the length of the tangent from $(3, -4)$ to the circle $x^2 + y^2 - 4x - 6y + 3 = 0$, is

a) 20 b) 30 c) 40 d) 50

1027. Let P, Q, R be three points on parabola $y^2 = 4x$ and normal at P and R meet at Q, then the locus of the mid-point of the chord PR is a parabola whose vertex is at

a) $(2,0)$ b) $(0, -2)$ c) $(-2,0)$ d) None of these

1028. The equations to the directrices of the ellipse $4(x-3)^2 + 9(y+2)^2 = 144$ are

a) $5x - 15 \pm 18\sqrt{5} = 0$
b) $5x + 15 \pm 2\sqrt{5} = 0$
c) $15x + 5 \pm 2\sqrt{5} = 0$
d) $15x - 5 \pm 18\sqrt{5} = 0$

102 Let P be a variable point on the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ with foci at S and S'. If A be the area of triangle PSS',

9. then the maximum value of A is

a) 24 sq. units b) 12 sq. units c) 36 sq. units d) None of these

1030. The focal distance of a point on the parabola $y^2 + 16x$ whose ordinate is twice the abscissa, is

a) 6 b) 8 c) 10 d) 12

1031. If θ is a parameter, then $x = a(\sin\theta + \cos\theta), y = b(\sin\theta - \cos\theta)$ represents

a) An ellipse
b) A circle
c) A pair of straight lines
d) A hyperbola

1032. The circles $x^2 + y^2 + x + y = 0$ and $x^2 + y^2 + x - y = 0$ intersect at an angle

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

1033. In the two circles $(x+7)^2 + (y-3)^2 = 36$ and $(x-5)^2 + (y+2)^2 = 49$ touch each other externally, then the point of contact is

a) $\left(\frac{-19}{13}, \frac{19}{13}\right)$ b) $\left(\frac{-19}{13}, \frac{9}{13}\right)$ c) $\left(\frac{17}{13}, \frac{9}{13}\right)$ d) $\left(\frac{-17}{13}, \frac{9}{13}\right)$

1034. If y_1, y_2 are the ordinates of two points P and Q on the parabola and y_3 is the ordinate of the point of intersection of tangents at P and Q, then

a) y_1, y_2, y_3 are in AP b) y_1, y_3, y_2 are in AP c) y_1, y_2, y_3 are in GP d) y_1, y_3, y_2 are in GP

1035. One of the diameter of the circle $x^2 + y^2 - 2x + 4y - 4 = 0$ is

a) $x - y - 3 = 0$ b) $x + y - 3 = 0$ c) $-x + y - 3 = 0$ d) $x + y + 3 = 0$

1036. If $5x - 12y + 10 = 0$ and $12y - 5x + 16 = 0$ are two tangents to a circle, then the radius of the circle is

a) 1 b) 2 c) 4 d) 6

1037. The image of the centre of the circle $x^2 + y^2 = a^2$ with respect to the mirror $x + y = 1$ is

a) $\left(\frac{1}{\sqrt{2}}, \sqrt{2}\right)$ b) $(\sqrt{2}, \sqrt{2})$ c) $(\sqrt{2}, 2\sqrt{2})$ d) None of these

1038. The eccentricity of the ellipse $25x^2 + 16y^2 - 150x - 175 = 0$ is

a) $\frac{2}{5}$ b) $\frac{2}{3}$ c) $\frac{4}{5}$ d) $\frac{3}{5}$

1039. If the vertex of the parabola $y = x^2 - 16x + k$ lies on x-axis, then the value of k is

a) 16 b) 8 c) 64 d) -64

1040. The latusractum of the hyperbola $9x^2 - 16y^2 + 72x - 32 - 16 = 0$ is

a) $\frac{9}{2}$ b) $-\frac{9}{2}$ c) $\frac{32}{3}$ d) $-\frac{32}{3}$

1041. Equation of hyperbola passing through origin and whose asymptotes are $3x + 4y = 5$ and $4x + 3y = 5$ is

a) $x^2 - y^2 = 1$
b) $12x^2 + 12y^2 + 35xy - 15x - 15y = 0$
c) $12x^2 + 12y^2 + 25xy - 35x - 35y = 0$
d) $12x^2 + 12y^2 + 25xy - 25x - 25y = 0$

1042. If $g^2 + f^2 = c$, then the equations $x^2 + y^2 + 2gx + 2fy + c = 0$ will represent

a) A circle of radius g
b) A circle of radius f
c) A circle of diameter $\sqrt{c}$
d) A circle of radius 0

1043. The equation of parabola whose focus is $(5,3)$ and directrix is $3x-4y+1=0$, is

a) $(4x+3y)^2-256x-142y+849=0$
b) $(4x-3y)^2-256x-142y+849=0$
c) $(3x+4y)^2-142x-256y+849=0$
d) $(3x-4y)^2-256x-142y+849=0$

1044. If the radical axis of the circles $x^2+y^2+2gx+2fy+c=0$ and $2x^2+2y^2+3x+8y+2c=0$, touches the circle $x^2+y^2+2x+2y+1=0$, then

a) $g=\frac{3}{4}$ and $f\neq 2$
b) $g\neq\frac{3}{4}$ and $f=2$
c) $g=\frac{3}{4}$ or $f=2$
d) None of these

1045. If the normal at point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the axes in R and S respectively, then $PR:RS$ is equal to

a) $a:b$
b) $a^2:b^2$
c) $b^2:a^2$
d) $b:a$

1046. The mid point of the chord $4x-3y=5$ of the hyperbola $2x^2-3y^2=12$ is

a) $\left(0,-\frac{5}{3}\right)$
b) $(2,1)$
c) $\left(\frac{5}{4},0\right)$
d) $\left(\frac{11}{4},2\right)$

1047. The circle on focal radii of a parabola as diameter touches the

a) Axis
b) Directrix
c) Tangent at the vertex
d) None of these

1048. A set of points is such that each point is three times as far away from the y-axis as it is from the point $(4,0)$. Then, the locus of the points is

a) Hyperbola
b) Parabola
c) Ellipse
d) Circle

1049. The number of common tangents to the two circles $x^2+y^2-8x+2y=0$ and $x^2+y^2-2x-16y+25=0$ is

a) 1
b) 2
c) 3
d) 4

1050. If transverse and conjugate axes of hyperbola are equal then it's eccentricity is

a) $\sqrt{3}$
b) $\sqrt{2}$
c) $\frac{1}{\sqrt{2}}$
d) 2

1051. Distance between foci is 8 and distance between directrices is 6 of hyperbola, then length of latusrectum is

a) $4\sqrt{3}$
b) $\frac{4}{\sqrt{3}}$
c) $\sqrt{\frac{3}{4}}$
d) None of these

1052. The eccentricity of the hyperbola $5x^2-4y^2+20x+8y=4$ is

a) $\sqrt{2}$
b) $\frac{3}{2}$
c) 2
d) 3

1053. A line is drawn through the point $P(3,11)$ to cut the circle$x^2+y^2=9$ at A and B. Then, $PA.PB$ is equal to

a) 9
b) 121
c) 205
d) 139

1054. Locus of the point of intersection of straight lines $\frac{x}{a}-\frac{y}{b}=m$ and $\frac{x}{a}+\frac{y}{b}=\frac{1}{m}$ is

a) An ellipse
b) A circle
c) A hyperbola
d) A parabola

1055. Consider the set of hyperbola $xy=k, k\in R$. Let e_1 be the eccentricity when $k=4$ and e_2 be the eccentricity when $k=9$, then e_1-e_2 is equal to

a) -1
b) 0
c) 2
d) -3

1056. The product of the perpendicular from two foci on any tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, is

a) a^2
b) b^2
c) $-a^2$
d) $-b^2$

105 The equation of the tangents to the circle $x^2+y^2=13$ at the points whose abscissa is 2, are

7.

a) $2x + 3y = 13, 2x - 3y = 13$ b) $3x + 2y = 13, 2x - 3y = 13$
c) $2x + 3y = 13, 3x - 2y = 13$ d) None of the above

1058. The equation of the tangent to the circle $x^2 + y^2 = 4$, which are parallel to $x + 2y + 3 = 0$, are

a) $x - 2y = 2$ b) $x + 2y = \pm 2\sqrt{3}$ c) $x + 2y = \pm 2\sqrt{5}$ d) $x - 2y = \pm 2\sqrt{5}$

1059. The equations of the tangents to the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ which are parallel to the line $4x + 3y + 5 = 0$, are

a) $4x + 3y + 11 = 0$ and $4x + 3y + 8 = 0$ b) $4x + 3y - 9 = 0$ and $4x + 3y + 7 = 0$
c) $4x + 3y + 19 = 0$ and $4x + 3y - 31 = 0$ d) $4x + 3y - 10 = 0$ and $4x + 3y + 12 = 0$

1060. The tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ at

a) (6,7) b) (−6, 7) c) (6, −7) d) (−6, −7)

1061. The equation of the tangent to the circle $x^2 + y^2 + 4x - 4y + 4 = 0$ which makes equal intercepts on the positive coordinate axes, is

a) $x + y = 2$ b) $x + y = 2\sqrt{2}$ c) $x + y = 4$ d) $x + y = 8$

1062. From the point $P(16, 7)$ tangents PQ and PR are drawn to the circle$x^2 + y^2 - 2x - 4y - 20 = 0$. If C be the centre of the circle, then area of quadrilateral $PQCR$ is

a) 450 sq units b) 15 sq units c) 50 sq units d) 75 sq units

1063. Any point on the hyperbola $\frac{(x+1)^2}{16} - \frac{(y-2)^2}{4} = 1$ is of the form

a) $(4 \sec \theta, 2 \tan \theta)$ b) $(4 \sec \theta - 1, 2 \tan \theta + 2)$
c) $(4 \sec \theta - 1, 2 \tan \theta - 2)$ d) $(4 \sec \theta - 4, 2 \tan \theta - 2)$

1064. The centre of the circ le $x = 2 + 3\cos\theta, y = 3\sin\theta - 1$ is

a) (3, 3) b) (2, −1) c) (−2, 1) d) (1, −2)

1065. The asymptotes of the hyperbola $xy = hx + ky$ are

a) $x = k, y = h$ b) $x = h, y = k$ c) $x = h, y = h$ d) $x = k, y = k$

1066. If the equation $\lambda x^2 + (2\lambda - 3)y^2 - 4x - 1 = 0$ represents a circle, then its radius is

a) $\frac{\sqrt{11}}{3}$ b) $\frac{\sqrt{13}}{3}$ c) $\frac{\sqrt{7}}{3}$ d) $\frac{1}{3}$

1067. If $\frac{x^2}{f(4a)} + \frac{y^2}{f(a^2-5)}$ represents an ellipse with major axis as y-axis and f is a decreasing function, then

a) $a \in (-\infty, 1)$ b) $a \in (5, \infty)$ c) $a \in (1, 4)$ d) $a \in (-1, 5)$

1068. If the two circles $(x - 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then

a) $2 < r < 8$ b) $r < 2$ c) $r = 2$ d) $r > 2$

1069. The angle between the tangents drawn at the points (5, 12) and (12, −5) to the circle $x^2 + y^2 = 169$ is

a) 45° b) 60° c) 30° d) 90°

1070. If the point $(\lambda, \lambda + 1)$ lies in the interior of the region bounded by $y = \sqrt{25 - x^2}$ and x-axis, then λ lies in the interval

a) (−4,3) b) $(-\infty, -1) \cup (3, \infty)$ c) (−1,3) d) None of these

1071. The equation of the common tangent to the curves $y^2 = 8x$ and $xy = -1$ is

a) $3y = 9x + 2$ b) $y = 2x + 1$ c) $2y = y + 8$ d) $y = x + 2$

1072. If $(a\cos\theta_i, a\sin\theta_i), i = 1, 2, 3$ represents the vertices of an equilibrium triangle inscribed in a circle, then

a) $\cos\theta_1 + \cos\theta_2 + \cos\theta_3 = 0$
b) $\sec\theta_1 + \sec\theta_2 + \sec\theta_3 = 0$
c) $\tan\theta_1 + \tan\theta_2 + \tan\theta_3 = 0$
d) $\cot\theta_1 + \cot\theta_2 + \cot\theta_3 = 0$

1073. The circle $x^2 + y^2 - 8x + 4y + 4 = 0$ touches
a) x-axis
b) y-axis
c) Both axis
d) Neither x-axis nor y-axis

1074. The number of maximum normals which can be drawn from a point to ellipse is
a) 4
b) 2
c) 1
d) 3

1075. The equation of the parabola with vertex at the origin and directrix $y = 2$ is
a) $y^2 = 8x$
b) $y^2 = -8x$
c) $y^2 = \sqrt{8}x$
d) $x^2 = -8y$

1076. The equation to the chord of the circle $x^2 + y^2 = 9$ whose middle point is $(1, -2)$ is
a) $x - 2y = 9$
b) $x - 2y - 4 = 0$
c) $x - 2y - 5 = 0$
d) $x - 2y + 5 = 0$

1077. The equation of the circle radius $2\sqrt{2}$ whose centre lies on the line $x - y = 0$ and which touches the line $x + y = 4$, and whose centre is coordinate satisfy $x + y > 4$, is
a) $x^2 + y^2 - 8x - 8y + 24 = 0$
b) $x^2 + y^2 = 8$
c) $x^2 + y^2 - 8x + 8y - 24 = 0$
d) None of these

1078. The greatest distance of the point $P(10, 7)$ from the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ is
a) 10
b) 15
c) 5
d) None of these

1079. The two circles $x^2 + y^2 - 2\,x - 2\,y - 7 = 0$ and $3(x^2 + y^2) - 8x + 29y = 0$
a) Touch externally
b) Touch internally
c) Cut each other orthogonally
d) Do not cut each other

1080. The equation of the circle described on the common chord of the circles $x^2 + y^2 + 2\,x = 0$ and $x^2 + y^2 + 2\,y = 0$ as diameter is
a) $x^2 + y^2 + x - y = 0$
b) $x^2 + y^2 - x - y = 0$
c) $x^2 + y^2 - x + y = 0$
d) $x^2 + y^2 + x + y = 0$

1081. The product of perpendiculars drawn from any point of a hyperbola to its asymptotes is
a) $\dfrac{a^2b^2}{a^2 + b^2}$
b) $\dfrac{a^2 + b^2}{a^2b^2}$
c) $\dfrac{ab}{\sqrt{a} + \sqrt{b}}$
d) $\dfrac{ab}{a^2 + b^2}$

1082. Number of points from where perpendicular tangents to the curve $\frac{x^2}{16} - \frac{y^2}{25} = 1$ can be drawn, is
a) 1
b) 2
c) 0
d) None of these

1083. Suppose S and S' are foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{6} = 1$. If P is variable point on the ellipse and if Δ is area of the triangle PSS', then the maximum value of Δ is
a) 8
b) 12
c) 16
d) 20

1084. For the ellipse $3x^2 + 4y^2 + 6x - 8y - 5 = 0$ the eccentricity is
a) 1/3
b) 1/2
c) 1/4
d) 1/5

1085. The locus of the poles of normal chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, is
a) $\dfrac{x^2}{a^4} + \dfrac{y^2}{b^4} = a^2 + b^2$

b) $\frac{x^2}{a^4}+\frac{y^2}{b^2}=a^2-b^2$
c) $\frac{a^6}{x^2}+\frac{b^6}{y^2}=(a^2-b^2)^2$
d) $\frac{a^2}{x^2}+\frac{b^4}{y^2}=(a^2-b^2)^2$

1086. The product of the lengths of perpendicular drawn from any point on the hyperbola $x^2-2y^2-2=0$ to its asymptotes, is
a) 1/2 b) 2/3 c) 3/2 d) 2

1087. If e_1 and e_2 are the eccentricities of a hyperbola $3x^2-3y^2=25$ and its conjugate, then
a) $e_1^2+e_2^2=2$ b) $e_1^2+e_2^2=4$ c) $e_1+e_2=4$ d) $e_1+e_2=\sqrt{2}$

1088. The straight line $x+y=k$ touches the parabola $y=x-x^2$, if k =
a) 0 b) −1 c) 1 d) None of these

1089. A hyperbola has the asymptotes $x+2y=3$ and $x-y=0$ and passes through (2, 1). Its centre is
a) (1, 2) b) (2, 2) c) (1, 1) d) (2, 1)

1090. The angular between the tangent drawn from the origin to the circle $(x-7)^2+(y+1)^2=25$ is
a) $\frac{\pi}{3}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{8}$

1091. The length of the latusrectum of the hyperbola $xy-3x-3y+7=0$ is
a) 2 b) 4 c) $2\sqrt{2}$ d) None of these

1092. The circle $x^2+y^2=4$ cuts the circle $x^2+y^2-2x-4=0$ at the points A and B. If the circle $x^2+y^2-4x-k=0$ passes through A and B then the value of k is
a) −4 b) 0 c) −8 d) 4

1093. The equation of the ellipse whose distance between foci is equal to 8 and distance between the directrix is 18, is
a) $5x^2-9y^2=180$ b) $9x^2+5y^2=180$ c) $x^2+9y^2=180$ d) $5x^2+9y^2=180$

1094. AB is a diameter of a circle and C is any point on the circumference of the circle. Then,
a) The area of $\Delta\, ABC$ is maximum when it is isosceles
b) The area of $\Delta\, ABC$ is minimum when it is isosceles
c) The perimeter $\Delta\, ABC$ is maximum when it is isosceles
d) None of these

1095. The area of the circle centred at (1, 2) and passing through (4, 6), is
a) 5π sq units b) 10π sq units c) 25π sq units d) None of these

1096. If (3, −2) is the centre of a circle and $4x+3y+19=0$ is a tangent to the circle, then the equation of the circle is
a) $x^2+y^2-6x+4y+25=0$ b) $x^2+y^2-6x+4y+12=0$
c) $x^2+y^2-6x+4y-12=0$ d) $x^2+y^2-6x+4y+13=0$

: ANSWER KEY :

1)	d	2)	a	3)	a	4)	b	189)	b	190)	d	191)	d	192)	d
5)	c	6)	d	7)	a	8)	c	193)	a	194)	d	195)	a	196)	d
9)	c	10)	c	11)	b	12)	b	197)	b	198)	c	199)	b	200)	b
13)	b	14)	b	15)	c	16)	a	201)	c	202)	c	203)	a	204)	b
17)	a	18)	d	19)	a	20)	b	205)	a	206)	b	207)	b	208)	c
21)	c	22)	d	23)	d	24)	b	209)	d	210)	c	211)	b	212)	a
25)	c	26)	a	27)	c	28)	c	213)	a	214)	d	215)	c	216)	b
29)	c	30)	d	31)	a	32)	d	217)	c	218)	d	219)	a	220)	c
33)	b	34)	b	35)	a	36)	a	221)	d	222)	a	223)	b	224)	c
37)	a	38)	a	39)	b	40)	c	225)	a	226)	a	227)	a	228)	b
41)	d	42)	b	43)	b	44)	b	229)	a	230)	b	231)	a	232)	c
45)	d	46)	d	47)	d	48)	c	233)	b	234)	a	235)	d	236)	d
49)	a	50)	d	51)	a	52)	c	237)	b	238)	c	239)	b	240)	a
53)	c	54)	c	55)	b	56)	b	241)	a	242)	c	243)	d	244)	c
57)	c	58)	d	59)	b	60)	b	245)	d	246)	d	247)	a	248)	d
61)	b	62)	c	63)	b	64)	b	249)	b	250)	a	251)	a	252)	c
65)	a	66)	d	67)	d	68)	c	253)	b	254)	a	255)	c	256)	a
69)	a	70)	c	71)	a	72)	a	257)	b	258)	c	259)	a	260)	b
73)	b	74)	c	75)	a	76)	c	261)	b	262)	c	263)	b	264)	a
77)	c	78)	c	79)	a	80)	a	265)	a	266)	d	267)	c	268)	c
81)	a	82)	c	83)	b	84)	a	269)	d	270)	a	271)	a	272)	b
85)	d	86)	c	87)	d	88)	a	273)	c	274)	b	275)	d	276)	a
89)	b	90)	c	91)	a	92)	a	277)	a	278)	c	279)	d	280)	b
93)	c	94)	b	95)	d	96)	c	281)	b	282)	b	283)	c	284)	d
97)	a	98)	b	99)	c	100)	d	285)	b	286)	b	287)	d	288)	b
101)	c	102)	d	103)	c	104)	b	289)	c	290)	a	291)	b	292)	a
105)	c	106)	d	107)	c	108)	c	293)	b	294)	a	295)	b	296)	a
109)	c	110)	d	111)	c	112)	c	297)	b	298)	c	299)	c	300)	a
113)	d	114)	b	115)	b	116)	a	301)	b	302)	a	303)	a	304)	a
117)	b	118)	a	119)	b	120)	a	305)	c	306)	a	307)	b	308)	b
121)	d	122)	a	123)	b	124)	a	309)	c	310)	a	311)	a	312)	c
125)	b	126)	c	127)	b	128)	d	313)	a	314)	d	315)	c	316)	d
129)	a	130)	a	131)	a	132)	a	317)	c	318)	c	319)	a	320)	d
133)	d	134)	a	135)	c	136)	b	321)	b	322)	d	323)	d	324)	b
137)	b	138)	d	139)	c	140)	a	325)	a	326)	d	327)	c	328)	b
141)	d	142)	a	143)	c	144)	d	329)	a	330)	b	331)	c	332)	c
145)	d	146)	a	147)	b	148)	d	333)	a	334)	c	335)	b	336)	b
149)	b	150)	a	151)	c	152)	c	337)	c	338)	b	339)	c	340)	d
153)	b	154)	c	155)	c	156)	a	341)	a	342)	d	343)	d	344)	a
157)	b	158)	b	159)	c	160)	c	345)	a	346)	c	347)	b	348)	a
161)	b	162)	c	163)	a	164)	a	349)	a	350)	a	351)	d	352)	c
165)	b	166)	d	167)	d	168)	a	353)	c	354)	a	355)	c	356)	a
169)	d	170)	b	171)	a	172)	c	357)	b	358)	a	359)	c	360)	d
173)	a	174)	b	175)	a	176)	c	361)	b	362)	c	363)	d	364)	d
177)	c	178)	b	179)	b	180)	a	365)	b	366)	b	367)	a	368)	a
181)	b	182)	d	183)	d	184)	a	369)	d	370)	b	371)	a	372)	d
185)	a	186)	a	187)	d	188)	c	373)	a	374)	a	375)	b	376)	b

377)	d	378)	a	379)	c	380)	a
381)	c	382)	a	383)	a	384)	a
385)	a	386)	b	387)	b	388)	c
389)	a	390)	d	391)	a	392)	d
393)	d	394)	c	395)	c	396)	d
397)	d	398)	b	399)	a	400)	b
401)	b	402)	d	403)	a	404)	b
405)	c	406)	c	407)	b	408)	b
409)	a	410)	c	411)	a	412)	d
413)	a	414)	a	415)	a	416)	a
417)	b	418)	b	419)	b	420)	c
421)	d	422)	b	423)	b	424)	d
425)	c	426)	b	427)	a	428)	b
429)	a	430)	a	431)	a	432)	a
433)	a	434)	c	435)	d	436)	b
437)	d	438)	c	439)	b	440)	b
441)	a	442)	c	443)	d	444)	a
445)	b	446)	a	447)	a	448)	c
449)	b	450)	a	451)	c	452)	a
453)	b	454)	a	455)	c	456)	d
457)	a	458)	c	459)	d	460)	b
461)	a	462)	c	463)	b	464)	c
465)	b	466)	c	467)	b	468)	d
469)	c	470)	c	471)	d	472)	b
473)	c	474)	a	475)	a	476)	c
477)	a	478)	c	479)	b	480)	c
481)	c	482)	a	483)	d	484)	b
485)	b	486)	d	487)	d	488)	b
489)	d	490)	b	491)	a	492)	b
493)	c	494)	d	495)	b	496)	d
497)	b	498)	c	499)	c	500)	c
501)	d	502)	d	503)	d	504)	d
505)	c	506)	c	507)	a	508)	d
509)	d	510)	b	511)	b	512)	a
513)	a	514)	a	515)	d	516)	a
517)	b	518)	b	519)	b	520)	b
521)	c	522)	a	523)	c	524)	d
525)	b	526)	b	527)	a	528)	c
529)	c	530)	c	531)	b	532)	c
533)	c	534)	c	535)	c	536)	d
537)	a	538)	a	539)	d	540)	b
541)	c	542)	b	543)	d	544)	a
545)	a	546)	b	547)	d	548)	b
549)	c	550)	b	551)	a	552)	b
553)	c	554)	b	555)	b	556)	d
557)	b	558)	a	559)	a	560)	b
561)	a	562)	d	563)	b	564)	a
565)	a	566)	a	567)	c	568)	c
569)	b	570)	d	571)	b	572)	a
573)	b	574)	d	575)	c	576)	b
577)	a	578)	b	579)	b	580)	a

581)	b	582)	b	583)	b	584)	a
585)	b	586)	b	587)	b	588)	b
589)	c	590)	d	591)	a	592)	a
593)	a	594)	c	595)	b	596)	d
597)	b	598)	c	599)	c	600)	a
601)	c	602)	b	603)	d	604)	a
605)	b	606)	d	607)	b	608)	a
609)	d	610)	d	611)	d	612)	d
613)	d	614)	c	615)	b	616)	c
617)	b	618)	d	619)	c	620)	a
621)	c	622)	d	623)	a	624)	c
625)	c	626)	c	627)	a	628)	b
629)	b	630)	a	631)	b	632)	d
633)	c	634)	c	635)	b	636)	b
637)	c	638)	b	639)	b	640)	d
641)	b	642)	d	643)	b	644)	a
645)	a	646)	b	647)	c	648)	b
649)	d	650)	d	651)	b	652)	a
653)	d	654)	b	655)	d	656)	b
657)	c	658)	b	659)	d	660)	a
661)	a	662)	c	663)	d	664)	b
665)	b	666)	b	667)	b	668)	d
669)	c	670)	b	671)	b	672)	a
673)	a	674)	b	675)	c	676)	a
677)	a	678)	a	679)	c	680)	c
681)	a	682)	b	683)	a	684)	c
685)	a	686)	c	687)	c	688)	c
689)	c	690)	a	691)	c	692)	d
693)	c	694)	a	695)	c	696)	b
697)	c	698)	c	699)	b	700)	a
701)	a	702)	d	703)	c	704)	d
705)	a	706)	c	707)	d	708)	c
709)	c	710)	d	711)	c	712)	d
713)	b	714)	a	715)	c	716)	d
717)	a	718)	c	719)	a	720)	b
721)	c	722)	d	723)	d	724)	a
725)	b	726)	b	727)	b	728)	d
729)	c	730)	b	731)	b	732)	b
733)	a	734)	a	735)	b	736)	a
737)	d	738)	b	739)	c	740)	c
741)	a	742)	d	743)	a	744)	c
745)	b	746)	b	747)	b	748)	d
749)	b	750)	a	751)	d	752)	b
753)	b	754)	a	755)	b	756)	d
757)	b	758)	a	759)	c	760)	b
761)	b	762)	c	763)	c	764)	c
765)	b	766)	c	767)	d	768)	c
769)	a	770)	a	771)	c	772)	c
773)	b	774)	c	775)	c	776)	a
777)	b	778)	d	779)	d	780)	a
781)	c	782)	c	783)	a	784)	b

785)	c	786)	b	787)	d	788)	d
789)	d	790)	a	791)	a	792)	b
793)	c	794)	c	795)	c	796)	d
797)	d	798)	a	799)	c	800)	b
801)	d	802)	a	803)	c	804)	b
805)	b	806)	b	807)	b	808)	c
809)	a	810)	a	811)	b	812)	a
813)	b	814)	d	815)	b	816)	a
817)	b	818)	c	819)	a	820)	d
821)	b	822)	b	823)	c	824)	a
825)	d	826)	b	827)	c	828)	c
829)	c	830)	b	831)	d	832)	c
833)	c	834)	c	835)	a	836)	d
837)	c	838)	d	839)	a	840)	a
841)	c	842)	c	843)	d	844)	c
845)	a	846)	c	847)	a	848)	d
849)	a	850)	c	851)	b	852)	c
853)	d	854)	c	855)	c	856)	d
857)	c	858)	c	859)	c	860)	c
861)	b	862)	a	863)	b	864)	c
865)	b	866)	c	867)	c	868)	b
869)	a	870)	d	871)	b	872)	b
873)	a	874)	b	875)	a	876)	c
877)	b	878)	b	879)	b	880)	a
881)	a	882)	b	883)	b	884)	b
885)	a	886)	c	887)	c	888)	a
889)	a	890)	c	891)	b	892)	d
893)	c	894)	c	895)	d	896)	b
897)	c	898)	c	899)	c	900)	c
901)	c	902)	b	903)	d	904)	b
905)	d	906)	b	907)	b	908)	d
909)	b	910)	b	911)	c	912)	c
913)	c	914)	a	915)	d	916)	a
917)	b	918)	d	919)	d	920)	a
921)	d	922)	d	923)	b	924)	b
925)	b	926)	b	927)	d	928)	a
929)	c	930)	c	931)	a	932)	c
933)	b	934)	b	935)	b	936)	b
937)	a	938)	b	939)	b	940)	a
941)	c	942)	b	943)	d	944)	c

945)	d	946)	c	947)	a	948)	a
949)	d	950)	d	951)	a	952)	b
953)	a	954)	c	955)	a	956)	b
957)	b	958)	a	959)	c	960)	c
961)	b	962)	b	963)	c	964)	a
965)	b	966)	a	967)	d	968)	c
969)	a	970)	d	971)	c	972)	b
973)	b	974)	d	975)	c	976)	a
977)	b	978)	b	979)	a	980)	a
981)	c	982)	a	983)	a	984)	d
985)	c	986)	b	987)	c	988)	c
989)	c	990)	b	991)	a	992)	a
993)	c	994)	a	995)	b	996)	a
997)	a	998)	d	999)	a	1000)	c
1001)	a	1002)	b	1003)	b	1004)	a
1005)	c	1006)	d	1007)	c	1008)	a
1009)	d	1010)	b	1011)	d	1012)	b
1013)	c	1014)	a	1015)	a	1016)	a
1017)	a	1018)	c	1019)	a	1020)	c
1021)	d	1022)	a	1023)	b	1024)	b
1025)	a	1026)	c	1027)	c	1028)	a
1029)	b	1030)	b	1031)	a	1032)	d
1033)	b	1034)	b	1035)	a	1036)	a
1037)	d	1038)	d	1039)	c	1040)	a
1041)	c	1042)	d	1043)	a	1044)	c
1045)	c	1046)	b	1047)	c	1048)	c
1049)	b	1050)	b	1051)	b	1052)	b
1053)	b	1054)	c	1055)	b	1056)	b
1057)	a	1058)	c	1059)	c	1060)	d
1061)	b	1062)	d	1063)	b	1064)	b
1065)	a	1066)	c	1067)	d	1068)	a
1069)	d	1070)	c	1071)	d	1072)	a
1073)	b	1074)	a	1075)	d	1076)	c
1077)	a	1078)	b	1079)	c	1080)	d
1081)	a	1082)	c	1083)	b	1084)	b
1085)	c	1086)	b	1087)	b	1088)	c
1089)	c	1090)	c	1091)	b	1092)	d
1093)	d	1094)	a	1095)	c	1096)	c

: HINTS AND SOLUTIONS :

1 **(d)**

The circle $x^2+y^2+2\,gx+2\,fy+c=0$ cuts an intercept of length $2\sqrt{f^2-c}$ on y-axis.

For the circle $x^2+y^2+4\,x-7\,y+12=0$, we have

$g=2, f=-7/2$ and $c=12$

$\therefore y-\text{intercept}=2\sqrt{f^2-c}=2\sqrt{\frac{49}{4}-12}=1$

2 **(a)**

$\because$ Eccentricity of ellipse$=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{3}{4}}=\frac{1}{2}$

$\because$ Eccentricity of hyperbola$=2$

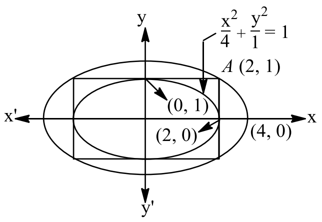
$\Rightarrow \sqrt{1+\frac{b^2}{64}} \Rightarrow\ 2$

$\Rightarrow\ 4=1+\frac{b^2}{64}\ \Rightarrow 192=b^2$

3 **(a)**

Let the equation of the required ellipse be $\frac{x^2}{16}+\frac{y^2}{b^2}=1$

But the ellipse passes through the point $(2,1)$



$\Rightarrow\ \frac{1}{4}+\frac{1}{b^2}=1$

$\Rightarrow\ \frac{1}{b^2}=\frac{3}{4}\ \Rightarrow\ b^2=\frac{4}{3}$

Hence, equation is

$\frac{x^2}{16}+\frac{3y^2}{4}=1$

$\Rightarrow\ x^2+12y^2=16$

4 **(b)**

We have,

$x=2t+1, y=t^2+2$

$\Rightarrow y=\left(\frac{x-1}{2}\right)^2+2$

$\Rightarrow (x-1)^2=4(y-2)$

The equation of the directrix of this parabola is $y-2=-1$ or, $y=1$ [Using $y=-a$]

5 **(c)**

Given equation can be rewritten as

$y^2=\frac{4k}{4}\left(x-\frac{8}{k}\right)$

The standard equation of parabola is

$Y^2=4AX$, where $A=\frac{k}{4}$

$\therefore$ Equation of directrix is $X+\frac{k}{4}=0$

$\Rightarrow x-\frac{8}{k}+\frac{k}{4}=0$

But the given equation of directrix is $x-1=0$

Since, both equations are same

$\therefore\ \frac{8}{k}-\frac{k}{4}=1$

$\Rightarrow 32-k^2=4k\ \Rightarrow k=-8,4$

6 **(d)**

The equation of the ellipse is

$3(x+1)^2+4(y-1)^2=12$ or, $\frac{(x+1)^2}{2^2}+\frac{(y-1)^2}{(\sqrt{3})^2}=1$

The equations of its major and minor axes are $y-1=0$ and $x+1=0$ respectively

7 **(a)**

Let mid point of the chord be (h,k), then equation of the chords be

$\frac{hx^2}{a^2}+\frac{ky^2}{b^2}-1=\frac{h^2}{a^2}+\frac{k^2}{b^2}-1$

$\Rightarrow y=-\frac{b^2}{a^2}\cdot\frac{h}{k}x+\left(\frac{h^2}{a^2}+\frac{k^2}{b^2}\right)\frac{b^2}{k}$ ...(i)

Since, line (i) is touching the circle $x^2+y^2=c^2$

$\therefore \left(\frac{h^2}{a^2}+\frac{k^2}{b^2}\right)\frac{b^4}{k^2}=c^2\left(1+\frac{b^4h^2}{a^4k^2}\right)$

Hence, locus is $(b^2x^2+a^2y^2)^2=c^2(b^4x^2+a^4y^2)$

8 **(c)**

Given curve is $y^2=4x$ ...(i)

Let the equation of line be $y=mx+c$

Since, $\frac{dy}{dx}=m=1$ and above line is passing through the point $(0,1)$

$1=1(0)+c\ \Rightarrow\ c=1$

$y = x + 1$ ...(ii)

On solving Eqs. (i) and (ii), we get

$x = 1$ and $y = 2$

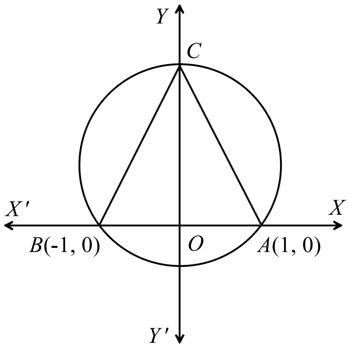
This shows that line touch the curve at one point. So, length of intercept is zero.

9 **(c)**

We have, $AB = 2$

Since $\Delta\, ABC$ is equilateral. Therefore,

$AC = BC = 2$ and $OC = \frac{\sqrt{3}}{2}(\text{Side}) = \sqrt{3}$



Thus, the coordinates of C are $(0, \sqrt{3})$

Let the circumcircle of $\Delta\, ABC$ be

$x^2 + y^2 + 2gx + 2fy + c = 0$

It passes through $(1,0)$, $(-1,0)$ and $(0, \sqrt{3})$

$\therefore 1 + 2g + c = 0, 1 - 2g + c = 0$ and $3 + 2\sqrt{3}\, f + c = 0$

Solving these three equations, we get

$g = 0, c = -1$ and $f = -\frac{1}{\sqrt{3}}$

Thus, the equation of the circumcircle is

$$x^2 + y^2 - \frac{2}{\sqrt{3}}y - 1 = 0$$

10 **(c)**

The coordinates of P be (h, k)

Let the equation of a tangent from $P(h, k)$ to the circle

$x^2 + y^2 = a^2$ be $y = mx + a\sqrt{1 + m^2}$

Since $P(h, k)$ lies on $y = mx + a\sqrt{1 + m^2}$

$\therefore k = mh + a\sqrt{1 + m^2}$

$\Rightarrow (k = mh)^2 = a(1 + m^2)$

$\Rightarrow m^2(h^2 - a^2) - 2\, mkh + k^2 - a^2 = 0$

This is a quadric in m. Let the two roots be m_1 and m_2. Then,

$$m_1 + m_2 = \frac{2\, hk}{h^2 - a^2}$$

But, $\tan\alpha = m_1, \tan\beta = m_2$ and it is given that $\cot\alpha + \cot\beta = 0$

$$\Rightarrow \frac{1}{m_1} + \frac{1}{m_2} = 0 \Rightarrow m_1 + m_2 = 0 \Rightarrow \frac{2\, hk}{k^2 - a^2} = 0$$

$$\Rightarrow hk = 0$$

Hence, the locus of (h, k) is $xy = 0$

11 **(b)**

We have,

$x = 2 + t^2, y = 2\, t + 1$

$\Rightarrow x - 2 = t^2$ and $y - 1 = 2t$

$\Rightarrow (y - 1)^2 = 4t^2$ and $x - 2 = t^2$

$\Rightarrow (y - 1)^2 = 4(x - 2),$

Which is a parabola with vertex at (2,1)

12 **(b)**

Given equation of ellipse is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a < b)$

It is a vertical ellipse with foci $(0, \pm be)$

Equation of any tangent line to the above ellipse is

$y = mx + \sqrt{a^2m^2 + b^2}$

$\therefore$ Required product

$= \left|\frac{-be + \sqrt{a^2m^2 + b^2}}{\sqrt{m^2 + 1}}\right| \left|\frac{be + \sqrt{a^2m^2 + b^2}}{\sqrt{m^2 + 1}}\right|$

$= \left|\frac{a^2m^2 + b^2 - b^2e^2}{m^2 + 1}\right|$

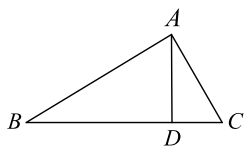
$= \left|\frac{a^2m^2 + b^2(1 - e^2)}{m^2 + 1}\right|$

$= \left|\frac{a^2m^2 + a^2}{m^2 + 1}\right|$ $[\because a^2 = b^2(1 - e^2)]$

$= a^2$

13 **(b)**

Since, $\angle\, ADB = \angle ADC = 90°$, circle on AB and AC as dismeters pass through D and therefore the altitude AD is the common chord. Similarly, the other two common chords are the other two altitudes and hence they concur at the orthocenter



14 **(b)**

Given equation of ellipse can be rewritten as

$$\frac{(x - 2)^2}{25} + \frac{(y + 3)^2}{16} = 1 \Rightarrow \frac{X^2}{25} + \frac{Y^2}{16} = 1$$

Where $X = x - 2, Y = y + 3$

Here, $a > b$

$$\therefore\ e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$$

$\therefore$ Focus $(\pm ae, 0) = (\pm 3,0)$

$\Rightarrow\ x-2=\pm 3, y+3=0$

$\Rightarrow\ x=5, =-1, y=-3$

$\therefore$ Foci are $(-1,-3)$and $(-1,-3)$

Distance between (2,-3) and (-1,-3)

$= \sqrt{(2+1)^2+(-3+3)^2}=3$

and distance between $(2,-3)$ and $(5,-3)$

$= \sqrt{(2-5)^2+(-3+3)^2}=3$

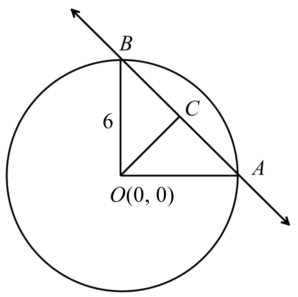
Hence, sum of the distance of point $(2,-3)$from the foci

$= 3+3=6$

15 **(c)**
We have,
$OC =$ Length of the perpendicular from (0,0) on the line $3\,x+4\,y-15=0$

$\Rightarrow OC = \dfrac{15}{\sqrt{3^2+4^2}} = 3$

$\therefore AB = 2\,AC = 2\sqrt{OA^2-OC^2} = 2\sqrt{36-9} = 6\sqrt{3}$



16 **(a)**
We know that the normal at $(at_1^2, 2at_1)$ meets the parabola at $(at_2^2, 2\,at_2)$, if $t_2=-t_1-\frac{2}{t_1}$
Here, the normal is drawn at (x_1, x_1)

$\therefore at_1^2 = 2\,at_1 \Rightarrow t_1 = 2 \Rightarrow t_2 = -2-\dfrac{2}{2} = -3$

The coordinates of the end points of the normal chord are $P(4a, 4a)$ and $Q(9a, -6a)$
Clearly, PQ makes a right angle at the focus $(a, 0)$

17 **(a)**
The equation of the family of circles touching $2x-y-1=0$ at (3,5) is
$(x-3)^2+(y-5)^2+\lambda(2x-y-1)=0$ ...(i)
It has its centre $\left(-\lambda+3, \frac{\lambda+10}{2}\right)$ on the line $x+y=5$

$\therefore -\lambda+3+\dfrac{\lambda+10}{2} = 5 \Rightarrow \lambda = 6$

Putting $\lambda=6$ in (i), we get
$x^2+y^2+6x-16y+28=0$
As the equation of the required circle

18 **(d)**
Given that equation of parabola is $y^2=9x$
On comparing with $y^2=4ax$, we get $a=\frac{9}{4}$
Now, equation of tangent to the parabola $y^2=9x$ is
$y = mx + \frac{9/4}{m}$ ...(i)
If this tangent passing through the point (4, 10), then

$10 = 4m + \dfrac{9}{4m}$

$\Rightarrow 16m^2-40m+9=0$
$\Rightarrow (4m-9)(4m-1)=0$

$\Rightarrow m = \dfrac{1}{4}, \dfrac{9}{4}$

On putting the values of m in Eq. (i)
$4y=x+36$ and $4y=9x+4$
$\Rightarrow x-4y+36=0$ and $9x-4y+4=0$

19 **(a)**
Required length $=$ y-intercept $= 2\sqrt{\frac{9}{4}-2} = 1$

20 **(b)**
Given equation is $xy=a$
On differentiating, we get

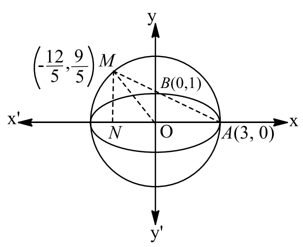
$x\dfrac{dy}{dx}+y=0$

$\Rightarrow \dfrac{dy}{dx} = -\dfrac{y}{x}$

$\Rightarrow\ \left(\dfrac{dy}{dx}\right)_{(a,1)} = -\dfrac{1}{a}$

22 **(d)**
Equation of auxiliary circle is

$x^2+y^2=9$ ... (i)



Equation of AM is $\frac{x}{3}+\frac{y}{1}=1$ ... (ii)

On solving Eqs. (i) and (ii) , we get $M\left(-\frac{12}{5}, \frac{9}{5}\right)$

Now, area of $\Delta AOM = \frac{1}{2}.OA \times MN$

$= \dfrac{27}{10}$ sq unit

23 **(d)**

Equation of tangent to $y^2 = 4x$ is $y = mx + \frac{1}{m}$

Since, tangent passes through $(-1, -6)$

$\therefore -6 = -m + \dfrac{1}{m} \Rightarrow m^2 - 6m - 1 = 0$

Here, $m_1 m_2 = -1$

$\therefore$ Angle between them is 90°

24 **(b)**

The equation of the ellipse is

$4(x^2 + 4x + 4) + 9(y^2 - 2y + 1) = 36$

$$\Rightarrow \frac{(x+2)^2}{3^2} + \frac{(y-1)^2}{2^2} = 1$$

So, the coordinates of the centre are $(-2, 1)$

25 **(c)**

The two circles are

$x^2 + y^2 - 2\,ax + c^2 = 0$ and $x^2 + y^2 - 2\,by + c^2 = 0$

Centres and radii of these two circles are :

Centres : $C_1(a, 0)$ $\quad C_2(0, b)$

Radii : $r_1 = \sqrt{a^2 - c^2}$ $\quad r_2 = \sqrt{b^2 - c^2}$

Since the two circles touch each other externally.

$\therefore C_1C_2 = r_1 + r_2$

$\Rightarrow \sqrt{a^2 + b^2} = \sqrt{a^2 - c^2} + \sqrt{b^2 - c^2}$

$\Rightarrow a^2 + b^2 = a^2 - c^2 + b^2 - c^2 + 2\sqrt{a^2 - c^2}\sqrt{b^2 - c^2}$

$\Rightarrow c^4 = a^2b^2 - c^2(a^2 + b^2) + c^4$

$\Rightarrow a^2b^2 = c^2(a^2 + b^2) \Rightarrow \dfrac{1}{a^2} + \dfrac{1}{b^2} = \dfrac{1}{c^2}$

26 **(a)**

It is given that $2ae = 8$ and $\frac{2a}{e} = 25$

$\Rightarrow 2ae \times \dfrac{2a}{e} = 8 \times 25 \Rightarrow 4a^2 = 200 \Rightarrow a = 5\sqrt{2}$

$\Rightarrow 2a = 10\sqrt{2}$

27 **(c)**

Equation of chord joining points

$P(a\cos\alpha, b\sin\alpha)$ and $Q(a\cos\beta, b\sin\beta)$ is

$$\frac{x}{a}\cos\left(\frac{\alpha+\beta}{2}\right) + \frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\alpha-\beta}{2}\right)$$

Now, $\beta = \alpha + 90°$

$$\frac{x}{a}\cos\left(\frac{2\alpha+90°}{2}\right) + \frac{y}{b}\sin\left(\frac{2\alpha+90°}{2}\right) = \frac{1}{\sqrt{2}}$$

now, compare it with $lx + my = -n$, we get

$$\frac{\cos\left(\frac{2\alpha+90°}{2}\right)}{al} = \frac{\sin\left(\frac{2\alpha+90°}{2}\right)}{bm} = -\frac{1}{\sqrt{2}n}$$

$\because \cos^2\theta + \sin^2\theta = 1$

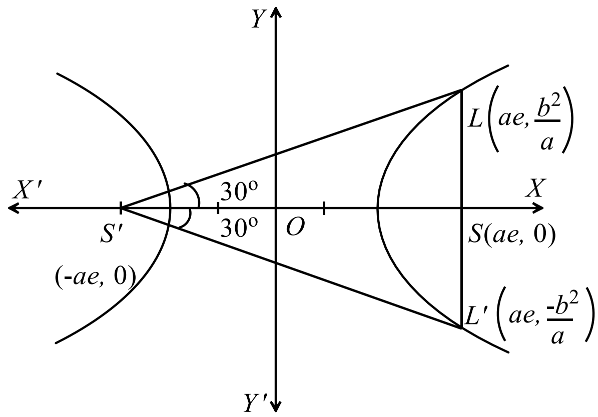
$\Rightarrow a^2l^2 + b^2m^2 = 2n^2$

28 **(c)**

Let LSL'' be a latusrectum and C be the centre of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. It is given that CLL'' is equilateral triangle. Therefore, $\angle LCS = 30°$

In ΔCSL, we have

$\tan 30° = \dfrac{SL}{CS}$



$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{b^2/a}{ae}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{b^2}{a^2e}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{e^2 - 1}{e} \Rightarrow \sqrt{3}\,e^2 - e - \sqrt{3} = 0 \Rightarrow e = \dfrac{1 + \sqrt{13}}{2\sqrt{3}}$

29 **(c)**

Given equation can be rewritten as

$\Rightarrow 4(x^2 - 6x + 9) + 16(y^2 - 2y + 1) - 36 - 6 = 1$

$$\Rightarrow \frac{(x-3)^2}{\frac{53}{4}} + \frac{(y-1)^2}{\frac{53}{16}} = 1$$

Here, $a^2 = \frac{53}{4}$ and $b^2 = \frac{53}{16}$

$\therefore$ Eccentricity of ellipse is $e = \frac{\sqrt{a^2 - b^2}}{a^2}$

$$\Rightarrow e = \frac{\sqrt{\frac{53}{4} - \frac{53}{16}}}{\frac{53}{4}}$$

$\Rightarrow e = \dfrac{\sqrt{3}}{2}$

30 **(d)**

The equation of hyperbola is

$4x^2 - 9y^2 = 36$

$\Rightarrow \frac{x^2}{9} - \frac{y^2}{4} = 1$ ...(i)

The equation of the chords of contact of tangents from (x_1, y_1) and (x_2, y_2) to the given hyperbola are

$\frac{x\,x_1}{9} - \frac{y\,y_1}{4} = 1$ ...(ii)

and $\frac{x\,x_2}{9} - \frac{y\,y_2}{4} = 1$ ...(iii)

Lines (ii) and (iii) are at right angles.

$\therefore \frac{9}{4} \cdot \frac{x_1}{y_1} \times \frac{4}{9} \cdot \frac{x_2}{y_2} = -1$

$\Rightarrow \frac{x_1 x_2}{y_1 y_2} = -\left(\frac{9}{4}\right)^2 = -\frac{81}{16}$

31 **(a)**

The circle having centre at the radical centre of three given circles and radius equal to the length of the tangent from it to any one of three circles cuts all the three circles orthogonally. The given circles are

$x^2 + y^2 - 3\,x - 6\,y + 14 = 0$ ...(i)

$x^2 + y^2 - x - 4\,y + 8 = 0$ ...(ii)

$x^2 + y^2 + 2\,c - 6\,y + 9 = 0$ ...(iii)

The radical axes of (i), (ii) and (ii), (iii) are respectively

$x + y - 3 = 0$ ...(iv)

and, $3\,x - 2y + 1 = 0$ ...(v)

Solving (iv) and (v), we get $x = 1, y = 2$

Thus, the coordinates of the radical centre are (1,2)

The length of the tangent from (1,2) to circle (i) is given by

$r = \sqrt{1 + 4 - 3 - 12 + 14} = 2$

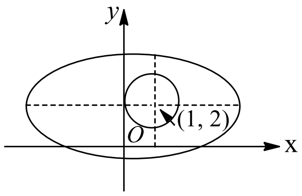
Hence, the required circle is

$(x-1)^2 + (y-2)^2 = 2^2$

$\Rightarrow x^2 + y^2 - 2x - 4\,y + 1 = 0$

32 **(d)**

It is clear from the figure that the two curves do not intersect each other



33 **(b)**

Directrix of $y^2 = 4(x+1)$ is $x = -2$. Any point on $x = -2$ is $(-2, k)$

Now mirror image (x, y) of $(-2, k)$ in the line $x + 2y = 3$ is given by

$\frac{x+2}{1} = \frac{y-k}{2} = -2\left(\frac{-2+2k-3}{5}\right)$

$\Rightarrow x = \frac{10-4k}{5} - 2$

$\Rightarrow x = -\frac{4k}{5}$ ...(i)

And $y = \frac{20-8k}{5} + k$

$\Rightarrow y = \frac{20-3k}{5}$ ...(ii)

From Eqs. (i) and (ii), we get

$y = 4 + \frac{3}{5}\left(\frac{5x}{4}\right)$

$\Rightarrow y = 4 + \frac{3x}{4}$

$\Rightarrow 4y = 16 + 3x$ is the equation of the mirror image of the directrix

34 **(b)**

Putting $x = at^2$ in $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,

We get, $t^4 + \frac{y^2}{b^2} = 1$

$ie, y^2 = b^2(1 - t^4) = b^2(1 + t^2)(1 - t^2)$

y is real, if $1 - t^2 \geq 0$

$ie, |t| \leq 1$

36 **(a)**

The combined equation of the lines joining the origin to the points of intersection of $x \cos\alpha + y \sin\alpha = p$ and $x^2 + y^2 - a^2 = 0$ is a homogeneous equation of second degree given by

$x^2 + y^2 - a^2\left(\frac{x\cos\alpha + y\sin\alpha}{p}\right)^2 = 0$

$\Rightarrow x^2(p^2 - a^2\cos^2\alpha) + y^2(p^2 - a^2\sin^2\alpha) - (\alpha^2 \sin 2\alpha)xy = 0$

The lines given by this equation are at right angle

Coeff. of x^2 + Coeff. of $y^2 = 0$

$\Rightarrow p^2 - a^2\cos^2\alpha + p^2 - a^2\sin^2\alpha = 0 \Rightarrow 2\,p^2 = a^2$

37 **(a)**

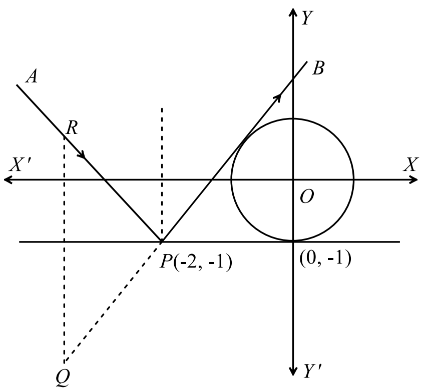
Using $S_1 - S_2 = 0$, we obtain $3\,x - 9 = 0$ or, $x = 3$ as the equation of the required common tangent

38 **(a)**

Since the difference of the radii of two circles is equal to the distance between their centres. Therefore, two circles touch each other internally and so only one common tangent can be drawn to given two circles

39 **(b)**

Clearly, the incidence ray passes through the point $P(-2, -1)$ and the image of any point Q on BP is $y = -1$



Let us find the equation of PB. Let its equation be $y+1=m(x+2)$

It touches the circle $x^2+y^2=1$

$\therefore \left|\frac{2m-1}{\sqrt{m^2+1}}\right|=1 \Rightarrow m=0,\frac{4}{3}$

So, the equation of PB is

$y+1=\frac{4}{3}(x+2)$ or, $4x-3y+5=0$

Let $Q(-5,5)$ be a point on PB. The image of Q in $y=-1$ is $R(-5,3)$. So, the equation of RP is

$y-3=\frac{3+1}{-5+2}(x+5)$ or, $4x+3y+11=0$

40 **(c)**

The equation of the tangent to the given circle at the origin is $y=x$. Image of the point $A(2,5)$ in $y=x$ is $(5,2)$.

Thus, the coordinates of B are $(5,2)$

42 **(b)**

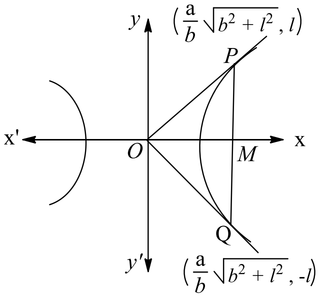
$\because PQ$ Is the double ordinate. Let $MP=MQ=l$.

Given that ΔOPQ is an equilateral, then $OP=OQ=PQ$

$\Rightarrow (OP)^2=(OQ)^2=(PQ)^2$

$\Rightarrow \frac{a^2}{b^2}(b^2+l^2)+l^2=\frac{a^2}{b^2}(b^2+l^2)+l^2=4l^2$

$\Rightarrow \frac{a^2}{b^2}(b^2+l^2)+3l^2$



$\Rightarrow a^2=l^2\left(3-\frac{a^2}{b^2}\right)$

$\Rightarrow \; l^2=\frac{a^2b^2}{(3b^2-a^2)}>0$

$\therefore \; 3b^2-a^2>0$

$\Rightarrow \; 3b^2>a^2$

$\Rightarrow 3a^2(e^2-1)>a^2$

$\Rightarrow e^2>4/3$

$\therefore \; e>\frac{2}{\sqrt{3}}$

43 **(b)**

Clearly, $x^2-y^2=c^2$ and $xy=c^2$ are rectangular hyperbolas each of eccentricity $\sqrt{2}$

$\therefore e=e_1=\sqrt{2} \Rightarrow e^2+e_1^2=4$

44 **(b)**

Since, both the given hyperbolas are rectangular hyperbolas

$\therefore \; e=\sqrt{2}, e_1=\sqrt{2}$

Hence, $e^2+e_1^2=2+2=4$

45 **(d)**

Since, $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, passes through $(3,0)$ and $(3\sqrt{2},2)$

$\therefore \; \frac{9}{a^2}=1$

$\Rightarrow \quad a^2=9$

and $\frac{9\times 2}{9}-\frac{4}{b^2}=1 \Rightarrow b^2=4$

$\therefore \; e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{1+\frac{4}{9}}=\frac{\sqrt{13}}{3}$

46 **(d)**

Let the equation of circles are

$S_1 \equiv x^2+y^2+2x-3y+6=0$ ...(i)

$S_2 \equiv x^2+y^2+x-8y-13=0$ ...(ii)

$\therefore$ Equation of common chord is

$S_1-S_2=0$

$\Rightarrow (x^2+y^2+2x-3y+6)$
$\qquad -(x^2+y^2+x-8y-13)=0$

$\Rightarrow \; x+5y+19=0$ ...(iii)

In the given option only the point $(1,-4)$ satisfied the Eq. (iii)

47 **(d)**

Let $P(h,k)$ be the given point. Then, the chord of contact of tangents drawn from P to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

$\frac{hx}{a^2}+\frac{ky}{b^2}=1$ ... (i)

This subtends a right angle at the centre $C(0,0)$ of the ellipse. The combined equation of the pair of straight lines joining C to the points of intersection of (i) and the ellipse is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\left(\frac{hx}{a^2}+\frac{ky}{b^2}\right)^2$$

This equation represents a pair of perpendicular straight lines.

$$\therefore \frac{1}{a^2}-\frac{h^2}{a^4}+\frac{1}{b^2}-\frac{k^2}{b^4}=0 \Rightarrow \frac{h^2}{a^4}+\frac{k^2}{b^4}=\frac{1}{a^2}+\frac{1}{b^2}$$

Hence, the locus of (h,k) is $\frac{x^2}{a^4}+\frac{y^2}{b^4}=\frac{1}{a^2}+\frac{1}{b^2}$

48 **(c)**

The locus is a hyperbola.

49 **(a)**

Given equation of ellipse can be rewritten as

$$\frac{(x-1)^2}{1/8}+\frac{\left(y+\frac{3}{4}\right)^2}{1/16}=1$$

$$\therefore \text{Eccentricity}=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{8}{16}}=\frac{1}{\sqrt{2}}$$

51 **(a)**

The equation of the tangent at $(-3,2)$ to the parabola $y^2+4x+4y=0$ is

$2\,y+2(x-3)+2(y+2)=0$

$\Rightarrow 2\,x+4\,y-2=0$

$\Rightarrow x+2\,y-1=0$

Since the tangent at one end of the focal chord is parallel to the normal at the other end. Therefore, the slope of the normal at the other end of the focal chord is $-1/2$

52 **(c)**

Solving the equations of lines in pairs, we obtain that the vertices of the $\Delta\, ABC$ are

$A(0,6), B(-2\sqrt{3},0)$ and $C(2\sqrt{3},0)$

Clearly, $AB=BC=CA$

So, ΔABC is an equilateral triangle. Therefore, centroid of the triangle ABC coincides with the circumcentre. Co-ordinates of the circumcentre are $O'(0,2)$ and the radius $=O'A=4$.

Hence, the equation of the circumcircle is

$(x-0)^2+(y-2)^2=4^2$ or, $x^2+y^2-4\,y=12$

53 **(c)**

Given, $r^2-4r(\cos\theta+\sin\theta)-4=0$ ...(i)

Put $x=r\cos\theta$, $y=r\sin\theta$, then $r^2=x^2+y^2$

$\therefore$ From Eq. (i)

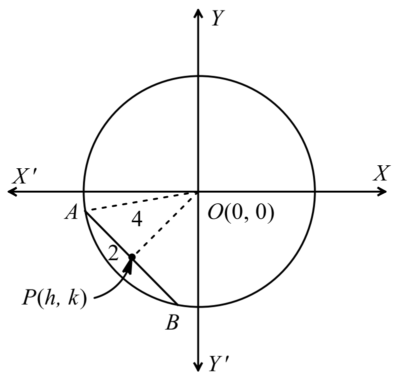
$x^2+y^2-4(x+y)-4=0$

$\Rightarrow\; x^2+y^2-4x-4y-4=0$

$\therefore$ Centre of circle is (2, 2)

54 **(c)**

Let $P(h,k)$ be the mid-point of a chord AB of length 4 units



In $\Delta\, OPA$, we have

$OA^2=OP^2+AP^2 \Rightarrow 4^2=h^2+k^2+2^2$

$\Rightarrow h^2+k^2=12$

Hence, the locus of $P(h,k)$ is $x^2+y^2=12$, which is a circle of radius $2\sqrt{3}$

55 **(b)**

Equation of normal at $P\ (a\sec\phi, b\tan\phi)$ is

$a\,x\cos\phi+by\cot\phi=a^2+b^2$

Then, coordinates of L and M are

$\left(\frac{a^2+b^2}{a}.\sec\phi,0\right)$ and $\left(0,\frac{a^2+b^2}{b}\tan\phi\right)$ respectively.

Let mid point of ML is $Q\ (h,k)$,

Then $h=\frac{(a^2+b^2)}{2a}\sec\phi$

$$\therefore \sec\phi=\frac{2ah}{(a^2+b^2)} \quad \text{... (i)}$$

and $k=\frac{(a^2+b^2)}{2b}\tan\phi$

$\therefore\ \tan\phi=\frac{2bk}{(a^2+b^2)}$ ...(ii)

From Eqs.(i) and (ii), we get

$$\sec^2\phi-\tan^2\phi=\frac{4a^2h^2}{(a^2+b^2)^2}-\frac{4b^2k^2}{(a^2+b^2)^2}$$

Hence, required locus is

$$\frac{x^2}{\left(\frac{a^2+b^2}{2a}\right)}-\frac{y^2}{\left(\frac{a^2+b^2}{2b}\right)^2}=1$$

Let eccentricity of this curve is e_1.

$$\Rightarrow \left(\frac{a^2+b^2}{2a}\right)^2=\left(\frac{a^2+b^2}{2a}\right)^2(e_1^2-1)$$

$\Rightarrow a^2=b^2(e_1^2-1)$

$\Rightarrow\ a^2=a^2(e^2-1)(e_1^2-1)$ $[\because b^2=a^2(e^2-1)]$

$\Rightarrow e^2e_1^2-e^2-e_1^2+1=1$

$\Rightarrow\ e_1^2(e^2-1)=e^2$

$$\Rightarrow\ e_1=\frac{e}{\sqrt{e^2-1}}$$

56 **(b)**

Let (h,k) be the mid-point of a chord of the hyperbola $x^2-y^2=a^2$. Then, the equation of the chord is

$hx-ky=h^2-k^2$ [Using : $T=S'$]

$$\Rightarrow y=\frac{h}{k}x+\frac{k^2-h^2}{k}$$

This touches the parabola $y^2 = 4ax$

$\therefore \dfrac{k^2 - h^2}{k} = \dfrac{a}{h/k}$ [Using: $c = a/m$]

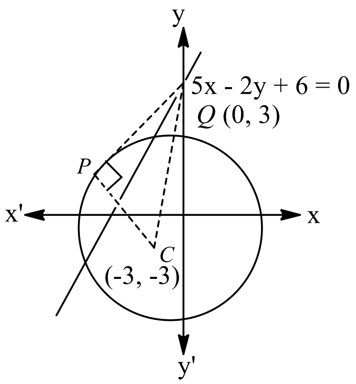
$\Rightarrow h(k^2 - h^2) = ak^2$

Hence, the locus of (h, k) is $x(y^2 - x^2) = ay^2$ or, $y^2(x - a) = x^3$

57 **(c)**

$x^2 + y^2 + 6x + 6y - 2 = 0$

Centre $(-3, -3)$, radius $= \sqrt{9 + 9 + 2} = \sqrt{20}$



Now, $QC = \sqrt{(-3)^2 + 6^2} = \sqrt{45}$

In right ΔCPQ

$PQ = \sqrt{45 - 20} = 5$

58 **(d)**

We have, $2a = 6, 2b = 4$

$\therefore e = \sqrt{1 - \dfrac{b^2}{a^2}} \Rightarrow e = \sqrt{\dfrac{5}{3}}$

So, distance between foci $= 2ae = 6\sqrt{\frac{5}{3}} = 2\sqrt{5}$

and, length of the string $= 2a + 2ae = 6 + 2\sqrt{5}$

59 **(b)**

The equation of a tangent to the given parabola is

$y = mx + \dfrac{9}{4m}$

If it passes through (4,10), then

$10 = 4m + \dfrac{9}{4m}$

$\Rightarrow 16m^2 - 40m + 9 = 0$

$\Rightarrow (4m - 1)(4m - 9) = 0 \Rightarrow m = \dfrac{1}{4}, \dfrac{9}{4}$

60 **(b)**

We know that the area Δ of the triangle formed by the tangent drawn from (x_1, y_1) to the circle $x^2 + y^2 = a^2$ and their chord of contact is given by

$\Delta = \dfrac{a(x_1^2 + y_1^2 - a^2)^{3/2}}{x_1^2 + y_1^2}$

Here, the point is $P(4,3)$ and the circle is $x^2 + y^2 = 9$

$\therefore$ Required area $= \dfrac{3(4^2 + 3^2 - 9)^{3/2}}{4^2 + 3^2}$ sq. units

$= \dfrac{192}{25}$ sq. units

61 **(b)**

Given, $y = mx + 2$

and $\frac{x^2}{9} - \frac{y^2}{4} = 1$

Condition of tangency, $c = \pm\sqrt{a^2m^2 - b^2}$

$2 = \pm\sqrt{9m^2 - 4} \Rightarrow m = \pm\dfrac{2\sqrt{2}}{3}$

62 **(c)**

Let any point $P(x_1, y_1)$ outside the circle. Then, equation of tangent to the circle $x^2 + y^2 + 6x + 6y = 2$ at the point P is

$xx_1 + yy_1 3(x + x_1) + 3(y + y_1) - 2 = 0$ ...(i)

The Eq. (i) and the line $5x - 2y + 6 = 0$ intersect at a point Q on y-axis $ie, x = 0$

$\Rightarrow 5(0) - 2y + 6 = 0 \Rightarrow y = 3$

$\therefore$ Coordinates of Q are (0, 3)

Point Q satisfies Eq. (i)

$\therefore 3x_1 + 6y_1 + 7 = 0$ ...(ii)

Distance between P and Q is given by

$PQ^2 = x_1^2 + (y_1 - 3)^2$

$= x_1^2 + y_1^2 - 6y_1 + 9$

$= 11 - 6x_1 - 12y_1$ $(\because x_1^2 + y_1^2 + 6x_1 + 6y_1 - 2 = 0)$

$= 11 - 2(3x_1 - 6y_1)$

$= 11 - 2(-7) = 25$ [from Eq. (ii)]

$\therefore PQ = 5$

63 **(b)**

Equation of circle which touches x-axis and coordinates of centre are (h, k) is

$(x - h)^2 + (y - k)^2 = k^2$

Since, it is passing through $(-1, 1)$, then

$(-1 - h)^2 + (1 - k)^2 = k^2$

$\Rightarrow h^2 + 2h - 2k + 2 = 0$

For real circles, $D \geq 0$,

$\Rightarrow (2)^2 - 4(-2k + 2) \geq 0 \Rightarrow k \geq \dfrac{1}{2}$

64 **(b)**

The required equation of circle is

$(x^2 + y^2 - 6) + \lambda(x^2 + y^2 - 6y + 8) = 0$ ...(i)

It passes through (1, 1)

$\therefore (1 + 1 - 6) + \lambda(1 + 1 - 6 + 8) = 0$

$\Rightarrow -4 + 4\lambda = 0$

$\Rightarrow \lambda = 1$

∴ required equation of circle is

$x^2 + y^2 - 6 + x^2 + y^2 - 6y + 8 = 0$

$\Rightarrow\ 2x^2 + 2y^2 - 6y + 2 = 0$

$\Rightarrow\ x^2 + y^2 - 3y + 1$

65 **(a)**

The equation of a normal to $y^2 = 4x$ is $y = mx - 2m - m^3$.

If it passes through (11/4,1/4), then

$\frac{1}{4} = \frac{11\,m}{4} - 2m - m^3$

$\Rightarrow 1 = 11\,m - 8\,m - 4\,m^3$

$\Rightarrow 4\,m^3 - 3m + 1 = 0 \Rightarrow m = \frac{1}{2}, \frac{-1 \pm \sqrt{3}}{2}$

Hence, three normals can be drawn from (11/4,1/4) to $y^2 = 4x$

66 **(d)**

Here, $a^2 = \cos^2\alpha$ and $b^2 = \sin^2\alpha$

Now, $e = \sqrt{1 + \frac{b^2}{a^2}}\ \Rightarrow\ e = \sqrt{1 + \frac{\sin^2\alpha}{\cos^2\alpha}}$

$\Rightarrow\ \ e = \sqrt{1 + \tan^2\alpha}\ \Rightarrow e = sec\alpha$

Coordinates of foci are $(\pm ae, 0) ie, (\pm 1, 0)$

Hence, abscissae of foci remain constant when α varies.

68 **(c)**

It is a known result

$t_1 t_2 = -1$

69 **(a)**

Here, $g_1 = -1, f_1 = 11, c_1 = 5$

and $g_2 = 7, f_2 = 3, c_2 = k$

$\Rightarrow\ 2(-1.7 + 11.3) = 5 + k\ \Rightarrow k = 47$

70 **(c)**

If the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the straight line $y = ma + c$ intersect in real points, then the quadratic equation $\frac{x^2}{a^2} + \frac{(mx+c)^2}{b^2} = 1$ must have real roots.

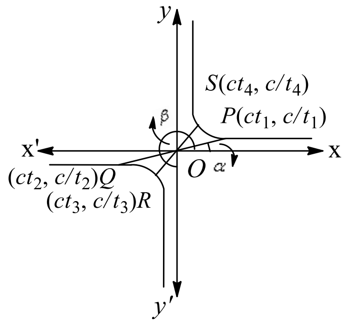
∴Discriminant $\geq 0 \Rightarrow c^2 \leq a^2 m^2 + b^2$

71 **(a)**

Let the equation of rectangular hyperbola is $xy = c^2$.

Take any four points on the hyperbola $P\left(ct_1, \frac{c}{t_1}\right), Q\left(ct_2, \frac{c}{t_2}\right), R\left(ct_3, \frac{c}{t_3}\right)$ and $S\left(ct_4, \frac{c}{t_4}\right)$

Such that PQ is perpendicular toRS.



Since, OP makes angle α withOX.

Therefore, $\tan\alpha = \frac{\frac{c}{t_1}}{ct_1} = \frac{1}{t_1^2}$

Similarly, $\tan\beta = \frac{1}{t_2^2}, \tan\gamma = \frac{1}{t_3^2}$ and $\tan\delta = \frac{1}{t_4^2}$

∴ $\tan\alpha\tan\beta\tan\gamma\tan\delta = \frac{1}{t_1^2 t_2^2 t_3^2 t_4^2}$ ...(i)

Now, PQ is perpendicular to RS.

∴ $\frac{\frac{c}{t_2} - \frac{c}{t_1}}{ct_2 - c_1} \times \frac{\frac{c}{t_4} - \frac{c}{t_3}}{ct_4 - ct_3} = -1$

$\Rightarrow -\frac{1}{t_1 t_2} \times \left(-\frac{1}{t_3 t_4}\right) = -1$

$\Rightarrow \frac{1}{t_1 t_2 t_3 t_4} = -1$

$\Rightarrow t_1 t_2 t_3 t_4 = -1$

From Eq.(i),

$\tan\alpha\tan\beta\tan\gamma\tan\delta = 1$

72 **(a)**

Equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Distance between foci of hyperbola= $2ae$

and its distance between directrices = $\frac{2a}{e}$

According to the question,

$\frac{2ae}{2a/e} = \frac{3}{2}$

$\Rightarrow\ e^2 = \frac{3}{2}$

Using , $b^2\ = a^2(e^2\ - 1)\ \Rightarrow\ \frac{b^2}{a^2} = \frac{3}{2} - 1$

$\Rightarrow\ \frac{a}{b} = \frac{\sqrt{2}}{1}$

73 **(b)**

Equation of pair of tangents is

$SS_1 = T^2$

$\Rightarrow\ (x^2 + y^2 - 4)(9 + 4 - 4) = (3x + 2y - 4)^2$

$\Rightarrow\ 5y^2 + 16y - 12xy + 24x - 50 = 0$

$\therefore\ m_1 + m_2 = -\frac{2h}{b} = \frac{12}{5}$

and $m_1 m_2 = 0$

Now, $m_1 - m_2 = \sqrt{(m_1+m_2)^2 - 4m_1m_2}$

$= \sqrt{\left(\frac{12}{5}\right)^2 - 0} = \frac{12}{5}$

74 **(c)**

Given equation is $9x^2 + 4y^2 - 6x + 4y + 1 = 0$

$\Rightarrow 9\left(x^2 - \frac{2}{3}x + \frac{1}{3^2}\right) + 4\left(y^2 + y + \frac{1}{4}\right) + 1 - 1 - 1 = 0$

$\Rightarrow \frac{\left(x-\frac{1}{3}\right)^2}{\left(\frac{1}{3}\right)^2} + \frac{\left(y+\frac{1}{2}\right)^2}{\left(\frac{1}{2}\right)^2} = 1$ (here, $a < b$)

Length of major axis $= 2b = 2\left(\frac{1}{2}\right) = 1$

Length of minor axis $= 2a = 2\left(\frac{1}{3}\right) = \frac{2}{3}$

75 **(a)**

Equation of two straight lines are

$\sqrt{3}\,x - y = 4\sqrt{3}\alpha$

and $\sqrt{3}\,x + y = \frac{4\sqrt{3}}{\alpha}$

Solving above equations, we get

$3x^2 - y^2 = 48 \Rightarrow \frac{x^2}{16} - \frac{y^2}{48} = 1$

Which is a hyperbola

Whose eccentricity

$e = \sqrt{\frac{48+16}{16}} = \sqrt{4} = 2$

76 **(c)**

Given equation of circle can be rewritten as

$x^2 + y^2 - 2x + 4y + \frac{k}{4} = 0$

$\therefore$ Radius of circle $= \sqrt{1 + 4 - \frac{k}{4}} = \sqrt{5 - \frac{k}{4}}$

Area of circle $= 9\pi$ (given)

$\Rightarrow\ \pi\left(5 - \frac{k}{4}\right) = 9\pi$

$\Rightarrow\ 5 - 9 = \frac{k}{4} \Rightarrow k = -16$

77 **(c)**

The circle passes through $(0,0)$, $(a,0)$, $(0,a)$ and (a,a)

Hence, the required equation is $x^2 + y^2 - ax - ay = 0$

78 **(c)**

It is given that the circle $x^2 + y^2 + 2\,gx + 2\,fy + c = 0$ bisects the circumference of the circle $x^2 + y^2 + 2\,g'x + 2\,f'y + c' = 0$. Therefore, the common chord of these two circles passes through the centre $(-g', -f')$ of $x^2 + y^2 + 2\,g'x + 2\,f'y + c' = 0$

The equation of the common chord of the two given circles is

$2\,x(g - g') + 2\,y(f - f') + c - c' = 0$

This passes through $(-g', -f')$

$\therefore -2\,g'(g - g') - 2f'(f - f') + c - c' = 0$

$\Rightarrow 2g'(g - g') + 2f'(f - f') = c - c'$

79 **(a)**

The slope of the tangent to $y^2 = 4x$ at (16,8) is given by

$m_1 = \left(\frac{dy}{dx}\right)_{(16,8)} = \left(\frac{4}{2\,y}\right)_{(16,8)} = \frac{2}{8} = \frac{1}{4}$

The slope of the tangent to $x^2 = 32\,y$ at (16,8) is given by

$m_2 = \left(\frac{dy}{dx}\right)_{(16,8)} = \left(\frac{2\,x}{32}\right)_{(16,8)} = 1$

$\therefore \tan\theta = \frac{1 - \frac{1}{4}}{1 + \frac{1}{4}} = \frac{3}{5} \Rightarrow \theta = \tan^{-1}\left(\frac{3}{5}\right)$

80 **(a)**

Let the equation of circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

Given equation of circles are

$x^2 + y^2 - 2x + 3y - 7 = 0$ ...(ii)

$x^2 + y^2 + 5x - 5y + 9 = 0$ ...(iii)

and $x^2 + y^2 + 7x - 9y + 29 = 0$ ...(iv)

Since, the circle (i) cut all three circles orthogonally,

$\therefore\ 2g(-1) + 2f(3/2) = c - 7 \Rightarrow -2g + 3f - c = -7$ ...(v)

$2g(5/2) + 2f(-5/2) = c + 29 \Rightarrow 5g - 5f - c = 9$ ...(vi)

$2g\left(\frac{7}{2}\right) + 2f\left(-\frac{9}{2}\right) = c + 29 \Rightarrow 7g - 9f - c = 29$ ...(vii)

On solving Eqs. (v), (vi) and (vii), we get

$g = -8,\ f = -9$ and $c = -4$

On putting the values of g, f and c in Eq. (i), we get

$x^2 + y^2 - 16x - 18y - 4 = 0$

81 **(a)**

Using $SS' = T^2$, the combined equation of the

tangents drawn from (0,0) to $y^2 = 4\,a(x-a)$ is
$(y^2 - 4\,ax + 4\,a^2)(0 - 0 + 4\,a^2)$
$= [y \cdot 0 - 2\,a(x + 0 - 2a)]^2$
$\Rightarrow (y^2 - 4\,ax + 4\,a^2)(4a^2) = 4\,a^2(x-2a)^2$
$\Rightarrow y^2 - 4\,ax + 4\,a^2 = (x-2a)^2$
$\Rightarrow x^2 - y^2 = 0$
Clearly, Coeff. of x^2 + Coeff. of $y^2 = 0$. Therefore, the required angle is a right angle
ALITER The point (0,0) lies on the directrix $x = 0$ of the parabola $y^2 = 4\,a(x-a)$, therefore the tangents are at right angle

82 **(c)**
We know that length of latusrectum of an ellipse= $\frac{2b^2}{a}$ and length of its minor axis = $2b$

Then, $\frac{2b^2}{a} = b \;\Rightarrow\; 2b = a$

$$\therefore\; e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{b^2}{4b^2}} = \frac{\sqrt{3}}{2}$$

83 **(b)**
The required point is the radical centre of the given circles

84 **(a)**
Equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a parabola, if $h^2 = ab$

85 **(d)**
Let e and e' be the eccentricities of the ellipse and hyperbola

$$\therefore e = \sqrt{\frac{a^2 - b^2}{a^2}} = \sqrt{\frac{25-16}{25}} = \frac{3}{5}$$

and $e' = \sqrt{\frac{a^2+b^2}{a^2}} = \sqrt{\frac{25+16}{25}} = \frac{\sqrt{41}}{5}$

1. Centre of ellipse is (0, 0) and centre of hyperbola is (0, 0)

2. Foci of ellipse are $(\pm ae, 0)$ or $(\pm 3, 0)$ foci of hyperbola are $(\pm ae', 0)$ or $\left(\pm\sqrt{41}, 0\right)$

3. Directrices of ellipse are $x = \pm\frac{a}{e} \Rightarrow x = \pm\frac{25}{3}$

   Directrices of hyperbola are $x = \pm\frac{a}{e}$

   $\Rightarrow\; x = \pm\frac{25}{\sqrt{41}}$

4. Vertices of ellipse are $(\pm a, 0)$ or $(\pm 5, 0)$

   Vertices of hyperbola are $(\pm a, 0)$ or $(\pm 5, 0)$

From the above discussions, their are common in centre and vertices.

86 **(c)**
Given equation is $\frac{x^2}{16} - \frac{y^2}{25} = 1$

$$\therefore\; e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{25}{16}} = \frac{\sqrt{41}}{4}$$

87 **(d)**
Equation of tangent to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is
$y = mx + \sqrt{a^2m^2 + b^2}$
And equation of tangent to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2$ is
$y = mx + \sqrt{2a^2m^2 + 2b^2}$
For common tangent,
$a^2m^2 + b^2 = 2a^2m^2 - 2b^2$
$\Rightarrow a^2m^2 = 3b^2 \;\Rightarrow m = \pm\frac{\sqrt{3}b}{a}$
$\therefore$ Equation of common tangent is $y = \frac{\sqrt{3}b}{a}x + 2b$.

88 **(a)**
The equation of a tangent to $xy = c^2$ is
$\frac{x}{t} + yt = 2c$ (i)
If $lx + my + n = 0$ is a tangent to $xy = c^2$, then it should be of the form of equation (i).
$\therefore \frac{l}{1/t} = \frac{m}{t} = \frac{-n}{2c}$
$\Rightarrow lt = \frac{m}{t} = -\frac{n}{2c}$
$\Rightarrow lt = -\frac{n}{2c}$ and $\frac{m}{t} = -\frac{n}{2c}$
$\Rightarrow lm = \frac{n^2}{4c^2}$
$\Rightarrow lm > 0 \Rightarrow l$ and m are of the same sign

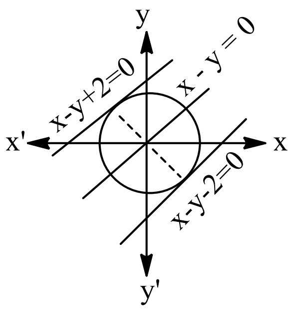
90 **(c)**
The equation of the tangent at $(4, -2)$ to $y^2 = x$ is
$-2\,y = \frac{1}{2}(x+4) \Rightarrow x + 4\,y + 4 = 0$
Its slope is $-1/4$. Therefore, the slope of the perpendicular line is 4. Since the tangents at the end points of a focan chord of a parabola are at right angles. Therefore, the slope of the tangent at Q is 4

91 **(a)**
The equation of a normal to $y^2 = 4x$ is

$y + tx = 2t + t^3$ ...(i)
If it passes through (3,0), then
$3t = 2t + t^3 \Rightarrow t = 0, \pm 1$
Putting the values of t in (i), we get
$y = 0, y + x = 3$ and $y - x = -3$
As the equation of the normals

92 **(a)**
Let $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ be tangent at
$P(a\cos\theta, b\sin\theta)$.
Its cuts the coordinates axes at $P(a\sec\theta, 0)$ and $Q(0, b\ \text{cosec}\ \theta)$
$\therefore CP = a\sec\theta$ and $CQ = b\ \text{cosec}\ \theta$
$\Rightarrow \frac{a^2}{CP^2} + \frac{b^2}{CQ^2} = 1$

93 **(c)**
Since, the equation of tangents $x - y - 2 = 0$ and $x - y + 2 = 0$ are parallel.
$\therefore$ Distance between them=Diameter of the circle$= \frac{2-(-2)}{\sqrt{1^2+1^2}}$
$\left(\because \frac{c_2 - c_1}{\sqrt{a^2 + b^2}}\right)$
$= \frac{4}{\sqrt{2}} = 2\sqrt{2}$

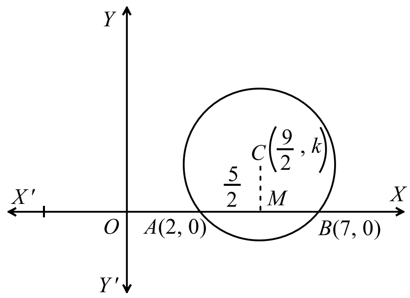


$\therefore$ Radius $= \frac{1}{2}(2\sqrt{2}) = \sqrt{2}$
It is clear from the figure that centre lies on the origin.
$\therefore$Equation of circle is
$(x - 0)^2 + (y - 0)^2 = (\sqrt{2})^2$
$\Rightarrow x^2 + y^2 = 2$

94 **(b)**
Equation of family of concentric circles to the circle $x^2 + y^2 + 6x + 8y - 5 = 0$ is $x^2 + y^2 + 6x + 8y + \lambda = 0$ which is similar to$x^2 + y^2 + 2gx + 2fy + c = 0$. Since, it is equation of concentric circle to the circle$x^2 + y^2 + 6x + 8y - 5 = 0$. Thus, the point $(-3, 2)$ lies on the circle
$x^2 + y^2 + 6x + 8y + c = 0$
$\Rightarrow (-3)^2 + (2)^2 + 6(-3) + 8(2) + c = 0$
$\Rightarrow 9 + 4 - 18 + 16 + c = 0$
$\Rightarrow c = -11$

95 **(d)**
On solving the given equations, we get$(0, 0), B(0,5/3), C(5/2, 0)$.
Let equation of circle be
$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)
Eq. (i) passes through $A(0, 0)$, we get $c = 0$
Similarly, Eq. (i) passes through $B(0,5/3)$ and $C(5/2, 0)$, we get
$2f = -5/3$ and $2g = -5/2$
$\therefore$ Required equation of circle is
$x^2 + y^2 - \frac{5}{2}x - \frac{5}{3}y = 0$
$\Rightarrow 6x^2 + 6y^2 - 15x - 10y = 0$

96 **(c)**
We have,
$OM = OA + AM = 2 + 5/2 = 9/2$
So, the x-coordinate of the centre is 9/2
$\therefore$ Radius $= CA = \sqrt{(9/2 - 2)^2 + (k - 0)^2}$
Hence, the equation of the circle is
$(x - 9/2)^2 + (y - k)^2 = \sqrt{(9/2 - 2)^2 + k^2}$
$\Rightarrow x^2 + y^2 - 9x - 2ky + 14 = 0$



98 **(b)**
Let $P(x_1, y_1)$ be a point on $x^2 + y^2 = a^2$. Then,
$x_1^2 + y_1^2 = a^2$ ...(i)
Let QR be the chord of contact of tangents drawn from $P(x_1, y_1)$ to the circle $x^2 + y^2 = b^2$. Then, the equation QR is
$xx_1 + yy_1 = b^2$ ...(ii)
This touches the circle $x^2 + y^2 = c^2$
$\therefore \left|\frac{0x_1 + 0y_1 - b^2}{\sqrt{x_1^2 + y_1^2}}\right| = c \Rightarrow b^2 = ac$ [Usin : (i)]
Let D be the discriminant of $ax^2 + 2bx + c = 0$. Then,
$D = 4(b^2 - ac) = 0$ $[\because b^2 = ac]$
Hence, the roots of the given equal are real and equal

99 **(c)**
The equation of the line joining (3,3) and $(-3,3)$ i.e. axis of the parabola is $y - 3 = 0$.
Since the directrix is a line perpendicular to the axis. Therefore, its equation is $x + \lambda = 0$.
The directrix intersects with the axis at $(-\lambda, 3)$

and the vertex is the mid point of the line segment joining the focus and the point of intersection of the directrix and axis

$\therefore \frac{-\lambda-3}{2} = 3 \Rightarrow \lambda = -9$

So, the equation of the directrix is $x - 9 = 0$

Let $P(x, y)$ be any point on the parabola. Then, by definition, we have

$(x+3)^2 + (y-3)^2 = (x-9)^2$

$\Rightarrow y^2 - 6\,y + 24\,x - 63 = 0$

100 **(d)**

Let the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

It is given that,

$2\,a = 3(2\,b) \Rightarrow a^2 = 9\,b^2 = a^2 = 9\,a^2(1-e^2)$

$$\Rightarrow e = \frac{2\sqrt{2}}{3}$$

101 **(c)**

We have,

$x^2 + y^2 + ax + (1-a)y + 5 = 0$

It is given that the radius of this circle is less than or equal to 5

$\therefore \frac{a^2}{4} + \frac{(1-a)^2}{4} - 5 \le 25$

$\Rightarrow 2a^2 - 2a - 119 \le 0 \Rightarrow -7.2 \le a \le 8.2 \Rightarrow a \in [-7, 8]$

But, a is an integer

$\therefore a = -7, -6, -5, -4, -3, -2, -1, 1, 0, 1, 2, 3, 4, 5, 6, 7, 8$

Hence, these are 16 integral values of a

102 **(d)**

Given equation of circles are $x^2 + y^2 - 2x - 4y + 1 = 0$ and $x^2 + y^2 - 12x - 16y + 91 = 0$ whose centre and radius are $C_1(1, 2), r_1 = 2$ and $C_2(6, 8), r_2 = 3$

$\therefore C_1C_2 = \sqrt{(1-6)^2 + (2-8)^2}$

$= \sqrt{25 + 36} = \sqrt{61}$

And $r_1 + r_2 = 2 + 3 = 5$

$\because C_1C_2 > r_1 + r_2$

$\therefore$ Number of common tangents $=4$

103 **(c)**

We know that the locus of point P from which two perpendicular tangents are drawn to the parabola, is the directrix of the parabola.

Hence, the required locus is $x = 1$

104 **(b)**

Let two coplanar points be $(0, 0)$ and $(a, 0)$

$\therefore \quad \frac{\sqrt{x^2 + y^2}}{\sqrt{(x-a)^2 + y^2}} = \lambda \quad [\lambda \ne 1]$

[where λ is any number]

$\Rightarrow \; x^2 + y^2 + \left(\frac{\lambda^2}{\lambda^2 - 1}\right)(a^2 - 2ax) = 0$

Which is the equation of circle

105 **(c)**

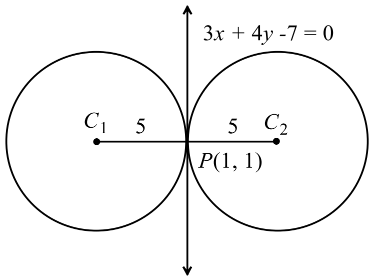
The equation of line C_1C_2 is

$\frac{x-1}{3/5} = \frac{y-1}{4/5}$

So, the coordinates of C_1 and C_2 are given by

$\frac{x-1}{3/5} = \frac{y-1}{4/5} = \pm 5 \Rightarrow x = 1 \pm 3, y = 1 \pm 4$

Thus, the coordinates of the centres are $(4, 5), (-2, -3)$



106 **(d)**

The tangent at $(1, 7)$ to the parabola $x^2 = y - 6$ is

$x = \frac{1}{2}(y + 7) - 6$

$\Rightarrow 2x = y + 7 - 12$

$\Rightarrow y = 2x + 5$

Which is also tangent to the circle

$x^2 + y^2 + 16x + 12y + c = 0$

$\therefore x^2 + (2x+5)^2 + 16x + 12(2x+5) + c = 0$

Or $\; 5x^2 + 60x + 85 + c = 0$

Must have equal roots

Let α and β are the roots of the equation

$\Rightarrow \; \alpha + \beta = -12 \;\Rightarrow\; \alpha = -6 \quad (\because\; \alpha = \beta)$

$\therefore\; x = -6$ and $y = 2x + 5 = -7$

$\Rightarrow$ point of contact is $(-6, -7)$

107 **(c)**

Let $C(0,0)$ be the centre and $L(ae, b^2/a)$ and $L'(-ae, b^2/a)$ be the vertices of latusrectum LL'. Then,

$m_1 =$ Slope of $CL = \frac{b^2/a - 0}{ae - 0} = \frac{b^2}{a^2e}$

$m_2 =$ Slope of $CL' = \frac{b^2/a - 0}{-ae - 0} = \frac{-b^2}{a^2e}$

It is given that $\angle LCL' = \pi/2$

$\therefore m_1m_2 = -1$

$\Rightarrow \frac{b^2}{a^2e} \times \frac{-b^2}{a^2e} = -1$

$\Rightarrow (e^2 - 1)^2 = e^2$

$\Rightarrow e^2 - 1 = e \Rightarrow e^2 - e - 1 = 0 \Rightarrow e = \frac{1+\sqrt{5}}{2}$

108 **(c)**

Given, ellipse $\frac{x^2}{16}+\frac{y^2}{7}=1$

$\therefore\ e_1=\sqrt{1-\frac{7}{16}}=\frac{3}{4}$

and hyperbola $\frac{x^2}{9}-\frac{y^2}{7}=1$

$\therefore\ e_2=\sqrt{1+\frac{7}{9}}=\frac{4}{3}$

Now, $e_1+e_2=\frac{3}{4}+\frac{4}{3}=\frac{25}{12}$

109 **(c)**

The equation of normal to the given ellipse at $P(a\cos\theta, b\sin\theta)$ is

$ax\sec\theta-by\operatorname{cosec}\theta-a^2=b^2$

$\Rightarrow y=\left(\frac{a}{b}\tan\theta\right)x-\frac{a^2-b^2}{b}\sin\theta$ ...(i)

Let $\frac{a}{b}\tan\theta=m$, then $\sin\theta=\frac{bm}{\sqrt{a^2+b^2m^2}}$

$\therefore$ From Eq. (i), we get

$y=mx-\frac{(a^2-b^2)m}{\sqrt{a^2+b^2m^2}}$

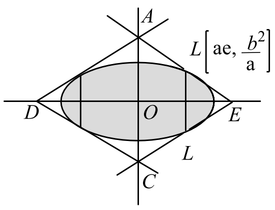
$\because \frac{a}{b}\tan\theta\in R\Rightarrow m\in R$

110 **(d)**

Given, $\frac{x^2}{9}+\frac{y^2}{5}=1$

Latusrectum of an ellipse be

$ae=\sqrt{a^2-b^2}=\sqrt{4}=2$



By symmetry the quadrilateral is rhombus

$\Rightarrow$ Equation of tangent at $\left(ae,\frac{b^2}{a}\right)=\left(2,\frac{5}{3}\right)$

$ie, \frac{2}{9}x+\frac{5}{3}.\frac{y}{5}=1$

$\Rightarrow \frac{x}{9/2}+\frac{y}{3}=1$

$\therefore$ Area of quadrilateral $ABCD=4$(area of $\Delta\, AOB$)

$=4.\left\{\frac{1}{2}.\frac{9}{2}.3\right\}$

$=27$ sq units

111 **(c)**

The equation of the tangent at $P(a\sec\theta, b\tan\theta)$ to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

$\frac{x}{a}\sec\theta-\frac{y}{b}\tan\theta=1$

This cuts the line $\frac{x}{a}-\frac{y}{b}=0$ and $\frac{x}{a}+\frac{y}{b}=0$ at Q and R

The coordinates of Q and $R$are

$Q\left(\frac{a}{\sec\theta-\tan\theta},\frac{b}{\sec\theta-\tan\theta}\right), R\left(\frac{a}{\sec\theta+\tan\theta},\frac{-}{s}\right.$

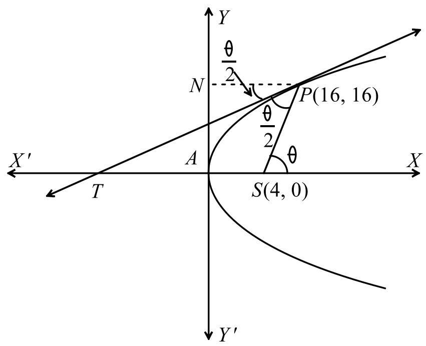
$\therefore CQ\cdot CR=\frac{\sqrt{a^2+b^2}}{(\sec\theta-\tan\theta)}\times\frac{\sqrt{a^2+b^2}}{(\sec\theta+\tan\theta)}$

$=a^2+b^2$

112 **(c)**

We know that PT bisects $\angle NPS$

Let $\angle NPT=\angle TPS=\frac{\theta}{2}$. Then,



$\angle PSX=\theta$

$\Rightarrow \tan\theta=\frac{16-0}{16-4}$

$\Rightarrow \tan\theta=\frac{4}{3}$

$\Rightarrow \frac{2\tan\theta/2}{1-\tan^2\theta/2}=\frac{4}{3}$

$\Rightarrow 3\tan\frac{\theta}{2}=2-2\tan^2\frac{\theta}{2}$

$\Rightarrow 2\tan^2\frac{\theta}{2}+3\tan\frac{\theta}{2}-2=0$

$\Rightarrow \left(2\tan\frac{\theta}{2}-1\right)\left(\tan\frac{\theta}{2}+2\right)=0$

$\Rightarrow \tan\frac{\theta}{2}=\frac{1}{2}$ $\left[\because \frac{\theta}{2} \text{ is acute}\right]$

$\Rightarrow \frac{\theta}{2}=\tan^{-1}\left(\frac{1}{2}\right)\Rightarrow \angle TPS=\tan^{-1}\left(\frac{1}{2}\right)$

113 **(d)**

The centres and radii of gives circles are

$C_1(0,0), C_2(4,0)$ and $r_1=2,\ r_2=2$

Now, $C_1C_2=\sqrt{(4-0)^2+0}=4$

and $r_1+r_2=2+2=4$

$\therefore\ C_1C_2=r_1+r_2$

Hence, three common tangents are possible

114 **(b)**

Given, circle cuts the parabola

$\therefore x^2 + \left(\frac{x^2}{4a}\right)^2 + 2gx + 2f\left(\frac{x^2}{4a}\right) + c = 0$

$\Rightarrow x^4 + 16a^2x^2 + 8afx^2 + 32gxa^2 + 16a^2c = 0$

$\sum x_i = 0$ ...(i)

$\sum x_1x_2 = 16a^2 + 8af$ ...(ii)

Now, $\sum y_i = \frac{1}{4a}\sum x_i^2$

$= \frac{1}{4a}[(x_1 + x_2 + x_3 + x_4)^2 - 2\sum x_1x_2]$

$= -\frac{1}{2a}(16a^2 + 8af) = -4(f + 2a)$

115 **(b)**

Let the coordinates of A and B be $(a, 0)$ and $(0, b)$ respectively. then,

$a^2 + b^2 = 9^2$ ...(i)

Let $P(h, k)$ be the centroid of $\Delta\, OAB$. Then,

$h = \frac{a}{3}$ and $k = \frac{b}{3} \Rightarrow a - 3h$ and $b = 3k$

Substituting the values of a and b in (i), we get

$9h^2 + 9k^2 = 9^2 \Rightarrow h^2 + k^2 = 9$

Hence, the locus of (h, k) is $x^2 + y^2 = 9$

116 **(a)**

Given focal chord of parabola $y^2 = ax$ is $2x - y - 8 = 0$

Since, this chord passes through focus $\left(\frac{a}{4}, 0\right)$

$\therefore\ 2.\frac{a}{4} - 0 - 8 = 0 \ \Rightarrow\ a = 16$

Hence, directrix is $x = -4 \ \Rightarrow\ x + 4 = 0$

117 **(b)**

Let one of the points be $P(r\cos\theta, r\sin\theta)$. Then, the other point is $Q(r\cos(\pi/2 + \theta)), (r\sin(\pi/2 + \theta))$ i.e. $Q(-r\sin\theta, r\cos\theta)$. The equations of tangents at P and Q are

$x\cos\theta + y\sin\theta = r$ and $-x\sin\theta + y\cos\theta = r$

The locus of the point of intersection of these two is obtained by eliminating θ from these two equations. Squaring and adding the two equations, we get

$(x\cos\theta + y\sin\theta)^2 + (-x\sin\theta + y\cos\theta)^2$

$= r^2 + r^2$

or, $x^2 + y^2 = 2\,r^2$, which is the required locus

118 **(a)**

The coordinates of a point dividing PQ internally in the ratio $1 : \lambda$ are

$\left(\frac{1+\lambda}{\lambda+1}, \frac{1+3\,\lambda}{\lambda+1}\right)$

This point is an interior point of the parabola

$y^2 = 4x$

$\therefore \left(\frac{1+3\,\lambda}{\lambda+1}\right)^2 - 4\left(\frac{1+\lambda}{\lambda+1}\right) < 0$

$\Rightarrow (3\,\lambda + 1)^2 - 4(\lambda + 1)^2 < 0$

$\Rightarrow 5\,\lambda^2 - 2\,\lambda - 3 < 0$

$\Rightarrow (5\,\lambda + 3)(\lambda - 1) < 0$

$\Rightarrow \lambda - 1 < 0$ $[\because \lambda > 0]$

$\Rightarrow 0 < \lambda < 1 \Rightarrow \lambda \in (0,1)$

119 **(b)**

Given that, $y = 2x + c$ ...(i)

And $x^2 + y^2 = 16$ ...(ii)

We know that, if $y = mx + c$ is tangent to the circle

$x^2 + y^2 = a^2$, then $c = \pm a\sqrt{1 + m^2}$, here, $m = 2, a = 4$

$\therefore\ c = \pm 4\sqrt{1 + 2^2} = \pm 4\sqrt{5}$

120 **(a)**

Given, $x^2 + y^2 = 6x$ ...(i)

and $x^2 + y^2 + 6x + 2y + 1 = 0$ ...(ii)

From Eq. (i), $x^2 - 6x + y^2 = 0$

$\Rightarrow\ (x - 3)^2 + y^2 = 3^2$

$\therefore$ Centre $(3, 0), r = 3$

From Eq. (ii),

$x^2 + 6x + y^2 + 2y + 1 + 3^2 = 3^2$

$\Rightarrow\ (x + 3)^2 + (y + 1)^2 = 3^2$

$\therefore$ Centre $(-3, -1)$, radius=3

Now, distance between centres

$= \sqrt{(3 + 3)^2 + 1}$

$= \sqrt{37} > r_1 + r_2 = 6$

$\therefore$ Circles do not cut each other

$\Rightarrow$ 4 tangents (two direct and two transversal) are possible

121 **(d)**

Centre of the given circle is $(4, -2)$. Therefore, the equation of the unit circle concentric with the given circle is $(x - 4)^2 + (y + 2)^2 = 1 \Rightarrow x^2 + y^2 - 8\,x + 4\,y + 19 = 0$

122 **(a)**

Since, the point $(9a, 6a)$ is bounded in the region formed by the parabola $y^2 = 16x$ and $x = 9$, then

$y^2 - 16x < 0, x - 9 < 0$

$\Rightarrow 36a^2 - 16 \cdot 9a < 0, 9a - 9 < 0$

$\Rightarrow 36a(a - 4) < 0, a < 1$

$0 < a < 4, a < 1\ \Rightarrow 0 < a < 1$

123 **(b)**

It is given that the coordinates of the vertices are $A'(-6,1)$ and $A(4,1)$. So, centre of the ellipse is at $C(-1,1)$ and length of major axis is $2a = 10$

Let e be the eccentricity of the ellipse. Then, coordinates its focus on the right side of centre

ar$(ae, 1)$ or $(5e, 1)$
It is given that $2x - y - 5 = 0$ is a focal chord of the ellipse.
So, it passes through $(5e, 1)$
$\therefore 10e - 1 - 5 = 0 \Rightarrow e = \frac{3}{5}$
So, $b^2 = a^2(1 - e^2) = 25\left(1 - \frac{9}{25}\right) = 16$
Hence, the equation of the ellipse is
$$\frac{(x+1)^2}{25} + \frac{(y-1)^2}{16} = 1$$

124 **(a)**
Given, $r = \sqrt{3}\sin\theta + \cos\theta$
Put $x = r\cos\theta, y = r\sin\theta$
$\therefore \quad r = \sqrt{3}\frac{y}{r} + \frac{x}{r}$
$\Rightarrow \quad r^2 = \sqrt{3}y + x$
$\Rightarrow x^2 + y^2 - \sqrt{3}y - x = 0$
$\therefore$Radius$= \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = 1$

125 **(b)**
We have,
$2\left(\frac{b^2}{4}\right) = \frac{9}{2} \Rightarrow b^2 = 9 \Rightarrow 16(e^2 - 1) = 9$
$\Rightarrow 16\,e^2 = 25 \Rightarrow e = \frac{5}{4}$

126 **(c)**
Form right $\Delta\, OSB$

$\tan 0° = \frac{b}{ae}$

$\Rightarrow \quad \sqrt{3} = \frac{b}{ae}$

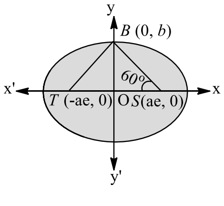
$\Rightarrow \quad b = \sqrt{3}\; ae$

Also, $\quad b^2 = a^2(1 - e^2)$

$\Rightarrow 3a^2e^2 = a^2(1 - e^2)$

$\Rightarrow 3e^2 = 1 - e^2 \Rightarrow \quad 4e^2 = 1$

$\Rightarrow \quad e = \frac{1}{2}$



127 **(b)**
The eccentricity of a hyperbola is never less than or equal to 1. So option (b) is correct

128 **(d)**
The equation of the tangent at (α, β) to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{\alpha\, x}{a^2} - \frac{\beta\, y}{b^2} = 1$
The ordinates of the points of intersection of this tangent and the auxiliary circle $x^2 + y^2 = a^2$ are the roots of the equation
$$\left\{\frac{a^2}{\alpha}\left(1 + \frac{\beta\, y}{b^2}\right)\right\}^2 + y^2 = a^2$$
$$\Rightarrow \frac{a^4}{\alpha^2}\left(1 + \frac{\beta^2 y^2}{b^4} + \frac{2\,\beta\, y}{b^2}\right) + y^2 = a^2$$
$$\Rightarrow y^2\left(\frac{\alpha^2}{a^4} + \frac{\beta^2}{b^4}\right) + \frac{2\,\beta}{b^2}y - \frac{\alpha^2}{a^2} + 1 = 0$$
Clearly, y_1 and y_2 are the roots of this equation
$$\therefore y_1 + y_2 = -\frac{2\beta/b^2}{\frac{\alpha^2}{a^4} + \frac{\beta^2}{b^4}} \text{ and, } y_1 y_2 = \frac{1 - \frac{\alpha^2}{a^2}}{\frac{\alpha^2}{a^4} + \frac{\beta^2}{b^4}}$$
$$\Rightarrow \frac{1}{y_1} + \frac{1}{y_2} = \frac{-2\,\beta/b^2}{1 - \frac{\alpha^2}{a^2}}$$
$$= \frac{-2\,\beta/b^2}{-\frac{\beta^2}{b^2}} \quad \left[\because \frac{\alpha^2}{a^2} - \frac{\beta^2}{b^2} = 1\right]$$
$$\Rightarrow \frac{1}{y_1} + \frac{1}{y_2} = \frac{2}{\beta}$$

129 **(a)**
Given hyperbola is a rectangular hyperbola whose eccentricity is $\sqrt{2}$

130 **(a)**
Since, the given line touches the given circle, the length of the perpendicular from the centre $(2, 4)$ of the circle to the line $3x - 4y - k = 0$ is equal to the radius $\sqrt{4 + 16 + 5} = 5$ of the circle
$\therefore \quad \frac{3 \times 2 - 4 \times 4 - k}{\sqrt{9 + 16}} = \pm 5$
$\Rightarrow k = 15 \quad (\because k > 0)$
Now, equation of the tangent at (a, b) to the given circle is
$xa + yb - 2(x + a) - 4(y + b) - 5 = 0$
$\Rightarrow (a - 2)x + (b - 4)y - (2a + 4b + 5) = 0$
If it represents the given line $3x - 4y - k = 0$
Then, $\frac{a-2}{3} = \frac{b-4}{-4} = \frac{2a+4b+5}{k} = l \quad$ (say)
$\Rightarrow a = 3l + 2, b = 4 - 4l$
and $\;2a + 4b + 5 = kl$
$\Rightarrow 2(3l + 2) + 4(4 - 4l) + 5 = 15l \quad (\because k = 15)$
$\Rightarrow \quad l = 1 \quad \Rightarrow a = 5, b = 0$
$\therefore \; k + a + b = 15 + 5 + 0 = 20$

131 **(a)**
Since, the distance between the focus and directrix of the parabola is half of the length of the latusrectum. Therefore length of latusrectum $=2$

(length of the perpendicular from (3, 3) to $3x - 4y - 2 = 0$)

$= 2\left|\frac{9-12-2}{\sqrt{9+16}}\right| = 2 \cdot \frac{5}{5} = 2$

132 **(a)**

Given equation of circle is

$x^2 + y^2 - 2x - 6y + 6 = 0$ ...(i)

Its centre is (1, 3) and radius $= \sqrt{1+9-6} = 2$

Equation of any line through (0, 1) is

$y - 1 = m(x - 0)$

$\Rightarrow mx - y + 1 = 0$ ...(ii)

If it touches the circle (i), then the length of perpendicular from centre (1, 3) to the circle is equal to radius 2

$\therefore \frac{m-3+1}{\sqrt{m^2+1}} = \pm 2$

$\Rightarrow (m-2)^2 = 4(m^2+1)$

$\therefore m = 0, -\frac{4}{3}$

On substituting these values of m in Eq. (ii), the required tangent are $y - 1 = 0$ and $4x + 3y - 3 = 0$

133 **(d)**

The centres of given circles are $C_1(-3,-3)$ and $C_2(6,6)$respectively and radii are $r_1 = \sqrt{9+9+0} = 3\sqrt{2}$ and $r_2 = \sqrt{36+36+0} = 6\sqrt{2}$ respectively

Now, $C_1C_2 = \sqrt{(6+3)^2 + (6+3)^2} = 9\sqrt{2}$

and $r_1 + r_2 = 3\sqrt{2} + 6\sqrt{2} = 9\sqrt{2}$

$\Rightarrow C_1C_2 = r_1 + r_2$

$\therefore$ Both circles touch each other externally

134 **(a)**

Let $A \equiv (at_1^2, 2at_1), B \equiv (at_2^2, 2at_2)$

Tangents, at A and B will intersect at the point C, whose coordinate is given by $\{at_1t_2, a(t_1+t_2)\}$.

Clearly, ordinates of A, C and B are always in AP

135 **(c)**

The pair of asymptotes and second degree curve differ by a constant.

$\therefore$ Pair of asymptotes is

$2x^2 + 5xy + 2y^2 - 11x - 7y + \lambda = 0$ ...(i)

Hence, Eq. (i) represents a pair of straight lines.

$\therefore \Delta = 0$

$$\Rightarrow 2 \times 2 \times \lambda + 2 \times -\frac{7}{2} \times -\frac{11}{2} \times \frac{5}{2} - 2 \times \left(-\frac{7}{2}\right)^2 - 2 \times \left(-\frac{11}{2}\right)^2 - \lambda \times \left(\frac{5}{2}\right)^2 = 0$$

$\Rightarrow \lambda = 5$

From Eq.(i), pair of asymptotes is

$2x^2 + 5xy + 2y^2 - 11x - 7y + 5 = 0$

136 **(b)**

Since, the given circles cut each other orthogonally

$\therefore g_1g_2 + a^2 = 0$ ...(i)

If $lx + my = 1$ is a common tangent of these circles, then

$\frac{-lg_1 - 1}{\sqrt{l^2+m^2}} = \pm\sqrt{g_1^2 + a^2}$

$\Rightarrow (lg_1 + 1)^2 = (l^2+m^2)(g_1^2 + a^2)$

$\Rightarrow m^2g_1^2 - 2lg_1 + a^2(l^2+m^2) - 1 = 0$

Similarly, $m^2g_2^2 - 2lg_2 + a^2(l^2+m^2) - 1 = 0$

So, that g_1, g_2 are the roots of the equation

$m^2g^2 - 2lg + a^2(l^2+m^2) - 1 = 0$

$\Rightarrow g_1g_2 = \frac{a^2(l^2+m^2)-1}{m^2} = -a^2$ [from Eq. (i)]

$\Rightarrow a^2(l^2+m^2) = 1 - a^2m^2$ ...(ii)

Now, $p_1p_2 = \frac{|ma-1|}{\sqrt{l^2+m^2}} \cdot \frac{|-ma-1|}{\sqrt{l^2+m^2}}$

$= \frac{|1-m^2a^2|}{l^2+m^2} = a^2$ [from Eq. (ii)]

137 **(b)**

If $(a\cos\alpha, b\sin\alpha)$and $(a\cos\beta, b\sin\beta)$ are the end points of chord, then equation of chord is

$\frac{x}{a}\cos\left(\frac{\alpha+\beta}{2}\right) + \frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\alpha-\beta}{2}\right)$

If it is a focal chord, it passes through $(ae, 0)$, so

$e\cos\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\alpha-\beta}{2}\right)$

$\Rightarrow e = \frac{\cos\left(\frac{\alpha-\beta}{2}\right)}{\cos\left(\frac{\alpha+\beta}{2}\right)}$

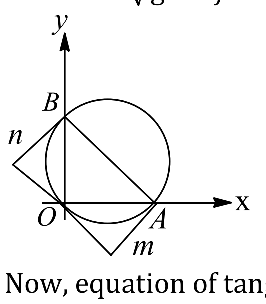
138 **(d)**

Let the equation of circle be

$x^2 + y^2 + 2gx + 2fgy = 0$

(passing through origin)

Radius $= \sqrt{g^2 + f^2}$



Now, equation of tangents at $O(0,0)$ is

$x(0) + y(0) + g(x) + f(y) = 0$

$\Rightarrow gx + fy = 0$

Distance from $A(2g, 0) = \frac{2g^2}{\sqrt{g^2+f^2}} = m$

and distance from $B(0, 2f) = \frac{2f^2}{\sqrt{g^2+f^2}} = n$

$\Rightarrow \frac{2r^2}{r} = m + n \Rightarrow 2r = m + n$

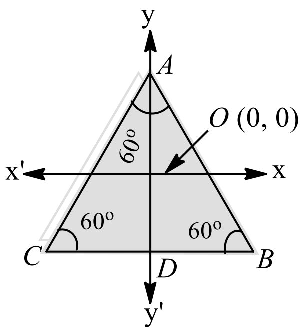
139 **(c)**

We know that every line passing through the focus of a parabola intersects the parabola in two distinct points except lines parallel to the axis.
The equation $(y-2)^2 = 4(x+1)$ represents a parabola with vertex $(-1,2)$ and axis parallel to x-axis. So, the line of slope m will cut the parabola in two distinct points if $m \neq 0$ i.e.
$m \in (-\infty, 0) \cup (0, \infty)$

140 **(a)**
Given that, any tangent to the circle $x^2 + y^2 = b^2$ is$y = mx - b\sqrt{1+m^2}$. It touches the circle $(x-a)^2 + y^2 = b^2$, then

$$\frac{ma - b\sqrt{1+m^2}}{\sqrt{m^2+1}} = b$$
$$\Rightarrow ma = 2b\sqrt{1+m^2}$$
$$\Rightarrow m^2a^2 = 4b^2 + 4b^2m^2$$
$$\therefore\ m = \pm\frac{2b}{\sqrt{a^2-4b^2}}$$

141 **(d)**
Centre of triangle is (0, 0)
Since, triangle is an equilateral, the centre of circumcircle is also (0, 0)
$AD = a$ (given)



$$\therefore\ AC = BC = AB$$
$$= \frac{a}{\sin 60°} = \frac{2a}{\sqrt{3}}$$
$$\therefore \text{Circumradius} = \frac{AC}{2\sin B}$$
$$= \frac{2a}{2\sqrt{3}} \times \frac{2}{\sqrt{3}} = \frac{2a}{3} \quad [\because B = 60°]$$
$\therefore$ required equation of circumcircle is
$$x^2 + y^2 = \frac{4a^2}{9}$$
$$\Rightarrow\ 9x^2 + 9y^2 = 4a^2$$

142 **(a)**
The coordinates of end point of latusrectum are $(a, 2a)$ and $(a, -2a)$ *ie*, $(3, 6)$ and $(3, -6)$
The equation of directrix is $x = -3$
The equation of tangents from the above points are $6y = 6(x+3)$ and $-6y = 6(x+3)$
$\Rightarrow x - y + 3 = 0$ and $x + y + 3 = 0$
The intersection point is $(-3, 0)$
The equation of directrix of the parabola $y^2 = 12x$ is $x = -3$
$\Rightarrow$ Intersection point $(-3, 0)$ lies on the directrix

143 **(c)**
We have, $\frac{x^2}{25} + \frac{y^2}{9} = 1$
The eccentricity of this ellipse is $\frac{4}{5}$. So, the coordinates of foci S and S' are (4,0) and (−4,0)
$\therefore$Area of rhombus $= \frac{1}{2} \times$ Product of diagonals
$\Rightarrow$ Area of rhombus $= \frac{1}{2}(BB' \times SS')$
$\Rightarrow$ Area of rhombus $= \frac{1}{2} \times 6 \times 8$ sq. units $=$ 24 sq. units

144 **(d)**
Let the equation of ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
$$\because\ b^2 = a^2(1-e^2)$$
$$\therefore\ \frac{x^2}{a^2} + \frac{y^2}{a^2(1-e^2)} = 1$$

145 **(d)**
Any point on the line $x - y - 5 = 0$ will be of the form $(t, t-5)$ Chord of contact of this point with respect to curve $x^2 + 4y^2 = 4$ is
$tx + 4(t-5)y - 4 = 0$
$\Rightarrow (-20y - 4) + t(x + 4y) = 0$
Which is a family of straight lines, each member of this family pass through point of intersection of straight lines $-20y - 4 = 0$ and $x + 4y = 0$ which is $\left(\frac{4}{5}, -\frac{1}{5}\right)$

146 **(a)**
The combined equation of the lines joining the origin (vertex) to the points of intersection of $y^2 = 4\,ax$ and $y = mx + c$ is
$$y^2 = 4\,ax\left(\frac{y-mx}{c}\right) \Rightarrow cy^2 - 4\,axy + 4\,am\,x^2 = 0$$
This represents a pair of perpendicular lines
$\therefore c + 4\,am = 0 \Rightarrow c = -4\,am$

147 **(b)**
Let the point on $x^2 + y^2 = a^2$ is $(a\cos\theta, a\sin\theta)$
Equation of chord of contact is
$ax\cos\theta + ay\sin\theta = b^2$
It touches circle $x^2 + y^2 = c^2$
$$\therefore\ \left|\frac{-b^2}{\sqrt{a^2\cos^2\theta + a^2\sin^2\theta}}\right| = c$$
$$\Rightarrow b^2 = ac$$

$\therefore a, b, c$ are in GP

148 **(d)**

We have,

$y^2 = 4ax \Rightarrow \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = \frac{2a}{y_1}$

$\therefore$Length of the sub-normal at $P(x_1, y_1)$

$= y_1 \left(\frac{dy}{dx}\right)_P = y_1 \times \frac{2a}{y_1} = 2a$

149 **(b)**

Let $P(h, k)$ be the point such that the ratio of the squares of the lengths of the tangents from P to the circles $x^2 + y^2 + 2x - 4y - 20 = 0$ and $x^2 + y^2 - 4x + 2y - 44 = 0$ is $2 : 3$.

Then,

$\frac{h^2 + k^2 + 2h - 4k - 20}{h^2 + k^2 + 4h + 2k - 44} = \frac{2}{3}$

$\Rightarrow h^2 + k^2 + 14h - 16k + 22 = 0$

So, the locus of $P(h, k)$ is $x^2 + y^2 + 14x - 16y + 22 = 0$

Clearly, it represents a circle having its centre at $(-7,8)$

150 **(a)**

The intersection of given line and circle is

$x^2 + y^2 - 2x = 0$

$\Rightarrow\ \ 2x(x - 1) = 0$

$\Rightarrow x = 0, x = 1$

And $\ y = 0, 1$

Let coordinates of A are (0, 0) and coordinates of B are (1, 1).

$\therefore$ Equition of circle (AB as a diameter) is

$(x - x_1)(x - x_2) + (y\_y_1)(y - y_2) = 0$

$\Rightarrow\ \ (x - 0)(x - 1) + (y - 0)(y - 1) = 0$

$\Rightarrow x^2 + y^2 - x - y = 0$

151 **(c)**

Equation of normal to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at

$(a \sec \theta, b \tan \theta)$ is $\frac{ax}{\sec \theta} + \frac{by}{\tan \theta} = a^2 + b^2$

152 **(c)**

The equation of tangent to the given circle $2x^2 + 2y^2 - 2x - 5y + 3 = 0$ at point (1, 1) is

$2x + 2y - (x + 1) - \frac{5}{2}(y + 1) + 3 = 0$

$\Rightarrow x - \frac{1}{2}y - \frac{1}{2} = 0$

$\Rightarrow 2x - y - 1 = 0$

$\Rightarrow y = 2x - 1$

Slope of tangent=2, therefore slope of normal = $-\frac{1}{2}$

Hence, equation of normal at point (1, 1) and having slope $\left(-\frac{1}{2}\right)$ is

$y - 1 = -\frac{1}{2}(x - 1)$

$\Rightarrow 2y - 2 = -x + 1$

$\Rightarrow x + 2y = 3$

153 **(b)**

The product of perpendicular distance from any point on $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to its asymptotes is $\frac{a^2b^2}{a^2+b^2}$

(See illustration 3 on page 26.12)

$\therefore$ Required product $= \frac{16\times9}{16+9} = \frac{144}{25}$

154 **(c)**

$x^2 = 4y$ and $y^2 = 4x$ intersect at $O(0,0)$ and (4,4). Therefore, the coordinates of P are (4,4).

The equations of the tangents to the two parabolas at (4,4) are :

$2x - y - 4 = 0$ ...(i)

and, $x - 2y + 4 = 0$ ...(ii)

Now, $m_1 =$ Slope of (i) $= 2, m_2 =$ Slope of (ii) $= 1/2$

Clearly, $m_1 m_2 = 1$

$\Rightarrow \tan \theta_1 \tan \theta_2 = 1$

$\Rightarrow \tan \theta_1 = \cot \theta_2$

$\Rightarrow \theta_1$ and θ_2 are such that $\theta_1 + \theta_2 = \pi/2$

155 **(c)**

The equation of a second degree curve passing through the points of intersection of the lines $2x - y + 11 = 0$ and $x - 2y + 3 = 0$ with the coordinate axes is

$(2x - y + 11)(x - 2y + 3) + \lambda xy = 0$ ...(i)

This equation will represent a circle, if

Coeff. of $x^2 =$ Coeff. of y^2 and Coeff. of $xy = 0$

$\Rightarrow \lambda - 5 = 0 \Rightarrow \lambda = 5$

Putting the value of λ in (i), we obtain that the equation of the circle is

$(2x - y + 11)(x - 2y + 3) + 5xy = 0$

$\Rightarrow 2x^2 + 2y^2 + 7x - 5y + 3 = 0$

The coordinates of its centre are $(-7/2, 5/2)$

156 **(a)**

Given , $\frac{x^2}{16} + \frac{y^2}{9} = 1$

$\therefore e = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$

$\therefore$ Coodinates of foci are $(\pm\sqrt{7}, 0)$

Since, centre of circle is (0, 3) and passing through foci $(\pm 7, 0)$

∴ Radius of circle= $\sqrt{\left(0 \pm \sqrt{7}\right)^2 + (3-0)^2}$

$= \sqrt{7+9} = 4$

157 **(b)**

Given equation of curve is $x = \alpha + 5\cos\theta,\ y = \beta + 4\sin\theta$

Or $\cos\theta = \frac{x-\alpha}{5}, \sin\theta = \frac{y-\beta}{4}$

$\because \cos^2\theta + \sin^2\theta = 1$

$\Rightarrow \left(\frac{x-\alpha}{5}\right)^2 + \left(\frac{y-\beta}{4}\right)^2 = 1$

This represents the equation of an ellipse.

158 **(b)**

Let PQ be a focal chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ having focus S. Then,

$\frac{2\,SP \cdot SQ}{SP + SQ} = \frac{b^2}{a} \Rightarrow \frac{2pq}{p+q} = \frac{b^2}{a} \Rightarrow b^2(p+q) = 2apq$

159 **(c)**

Given, parametric equations are $x = e^t + e^{-t}$ and $y = e^t - e^{-t}$

Now, on squaring and then on subtracting, we get

$x^2 - y^2 = 4$

160 **(c)**

Intersection points of given circles are (0, 0) and (3, 3) let equation of required circle whose centre $\left(\frac{3}{2}, \frac{3}{2}\right)$, is

$x^2 + y^2 - 3x - 3y + c = 0$

Since, this circle passes through (0, 0), thus equation of circle becomes,

$x^2 + y^2 - 3x - 3y = 0$

161 **(b)**

Equation of circle is

$x^2 + y^2 = 25$ ..(i)

Polar equation of a circle with respect to the point $(1, a)$ and $(b, 2)$ is

$x + ay = 25$ ...(ii)

and $bx + 2y = 25$ ...(iii)

since, $(1, a)$ and $(b, 2)$ are the conjugate point of a circle, therefore point $(1, a)$ satisfy the Eq. (iii), we get

$b + 2a = 25 \Rightarrow 2b + 4a = 50$

163 **(a)**

Given, $\frac{x^2}{16} - \frac{y^2}{9} = 1$

We know that the difference of focal distances of any point of the hyperbola is equal to major axis

∴ Required distance= $2a = 2 \times 4 = 8$

164 **(a)**

We have,

$y^2 - 6y + 4x + 9 = 0 \Rightarrow (y-3)^2 = -4(x-0)$

The coordinate of the focus of this parabola are $(-1,3)$ and the equation of the directrix is $x - 1 = 0$

We know that the chord of contact of tangents drawn from any point on the directrix always passes through the focus.

Hence, the required point is $(-1,3)$

<u>ALITER</u> Let $P(1, \lambda)$ be an arbitrary point on $x - 1 = 0$. The chord of contact of tangents drawn from $P(1, \lambda)$ to the parabola $y^2 - 6y + 4x + 9 = 0$ is

$\lambda y - 3(y + \lambda) + 2(x+1) + 9 = 0$

$\Rightarrow (2x - 3y + 11) + \lambda(y - 3) = 0$

Clearly, it represents a family of lines passing through the intersection of the lines

$2x - 3y + 11 = 0$ and $y - 3 = 0$ i.e. $(-1,3)$

165 **(b)**

Equation of circle whose centre is at (2, 2) and radius r is

$(x-2)^2 + (y-2)^2 = r^2$ ...(i)

This circle passes through (4, 5), then

$(4-2)^2 + (5-2)^2 = r^2$

$\Rightarrow \quad r^2 = 13$

On putting this values in Eq. (i), we get

$(x-2)^2 + (y-2)^2 - 13 = 0$

$\Rightarrow x^2 + y^2 - 4x - 4y - 5 = 0$

166 **(d)**

The equations of asymptotes of $x^2 - y^2 = 8$ are given by

$x^2 - y^2 = 0$ or, $x + y = 0$ and $x - y = 0$

Let (x_1, y_1) be a point on the hyperbola $x^2 - y^2 = 8$. Then, product of perpendicular from (x_1, y_1) on the asymptotes

$= \left|\frac{x_1 - y_1}{\sqrt{2}}\right| \left|\frac{x_1 + y_1}{\sqrt{2}}\right|$

$= \left|\frac{x_1^2 - y_1^2}{2}\right| = \left|\frac{8}{2}\right| = 4 \quad [\because x_1^2 - y_1^2 = 8]$

167 **(d)**

Given foci of ellipse are $(0, -4)$and $(0, 4)$

∴ Focal distance is $2be = 8$

$be = 4$ ...(i)

Also, since equation of directrices are $\frac{b}{e} = \pm 9$ ...(ii)

From, Eqs. (i) and (ii), we get

$b^2 = 36 \Rightarrow b = 6$ and $e = \frac{2}{3}$

$\because a^2 = b^2(1 - e^2) = 36\left(1 - \frac{4}{9}\right) = 20$

$\therefore \frac{x^2}{20} + \frac{y^2}{36} = 1$

$\Rightarrow 9x^2 + 5y^2 = 180$

168 **(a)**

The equation of tangent is

$\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$

$\therefore$ Coordinates of A and B are $(a\cos\theta, 0)$ and $(0, -b\cot\theta)$respectively.

Let coordinates of P are(h, k).

$\therefore h = a\cos\theta, k = -b\cot\theta$

$\Rightarrow \frac{k}{h} = -\frac{b}{a\sin\theta}$

$\Rightarrow \sin\theta = -\frac{bh}{ak}$

$\Rightarrow \frac{b^2h^2}{a^2k^2} = \sin^2\theta$

$\Rightarrow \frac{b^2h^2}{a^2k^2} + \frac{h^2}{a^2} = 1$

$\Rightarrow \frac{b^2}{k^2} + 1 = \frac{a^2}{h^2}$

$\Rightarrow \frac{a^2}{h^2} - \frac{b^2}{k^2} = 1$

Hence, the locus of P is $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$

169 **(d)**

The coordinates of P are $(1, 0)$. A gerneral point Q on $y^2 = 8x$ is $(2t^2, 4t)$. Let mid point of PQ is (h, k)

$\therefore 2h = 2t^2 + 1$ and $2k = 4t \Rightarrow t = \frac{k}{2}$

$\therefore 2h = \frac{2k^2}{4} + 1 \Rightarrow 4h = k^2 + 2$

Hence, the locus of (h, k) is $y^2 - 4x + 2 = 0$

171 **(a)**

The equation of the ellipse is

$\frac{(x+3)^2}{2^2} + \frac{(y-5)^2}{(\sqrt{3})^2} = 1$

$\Rightarrow 3x^2 + 4y^2 + 18x - 40y + 115 = 0$

172 **(c)**

Let (h, k) be the pole of the line $9x + y - 28 = 0$ with respect to the circle $x^2 + y^2 - \frac{3}{2}x + \frac{5}{2}y - \frac{7}{2} = 0$. Then, the equation of the polar is

$hx + ky - \frac{3}{4}(x + h) + \frac{5}{4}(y + k) - \frac{7}{2} = 0$

$\Rightarrow x\left(h - \frac{3}{4}\right) + y\left(k + \frac{5}{4}\right) - \frac{3}{4}h + \frac{5}{4}k - \frac{7}{2} = 0$

$\Rightarrow x(4h - 3) + y(4k + 5) - 3h + 5k - 14 = 0$

This equation and $9x + y - 28 = 0$ represent the same line.

$\therefore \frac{4h - 3}{9} = \frac{4k + 5}{1} = \frac{-3h + 5k - 14}{-28} = \lambda$ (say)

$\Rightarrow h = \frac{3 + 9\lambda}{4}, k = \frac{\lambda - 5}{4}, -3h + 5k - 14 = -28\lambda$

$\Rightarrow -3\left(\frac{3 + 9\lambda}{4}\right) + 5\left(\frac{\lambda - 5}{4}\right) - 14 = -28\lambda$

$\Rightarrow -9 - 27\lambda + 5\lambda - 25 - 56 = -112\lambda$

$\Rightarrow -22\lambda - 90 = -112\lambda$

$\Rightarrow 90\lambda = 90 \Rightarrow \lambda = 1$

Hence, the pole of the given line is $(3, -1)$

173 **(a)**

Let (h, k) is mid point of chord.

Then, its equation is $T = S_1$

$\therefore 3hx - 2ky + 2(x + h) - 3(y + k) = 3h^2 - 2k^2 + 4h - 6k$

$x(3h + 2) + y(-2k - 3) = 3h^2 - 2k^2 + 2h - 3k$

Since, this line is parallel to $y = 2x$

$\frac{3h + 2}{2k + 3} = 2$

$\Rightarrow 3h - 4k = 4$

Thus, locus of point is $3x - 4y = 4$

174 **(b)**

If circle $x^2 + y^2 - 10x - 14y + 24 = 0$ cuts an intercept on y-axis, then

Length of intercept$= 2\sqrt{f^2 - c} = 2\sqrt{49 - 24} = 10$

175 **(a)**

Given line $y = ax + \beta$ is a tangent to given hyperbola, if $\beta^2 = a^2\alpha^2 - b^2$

Hence, locus of (α, β) is $y^2 = a^2x^2 - b^2$, which represents a hyperbola

176 **(c)**

Let the points are $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3)

$\therefore y_1^2 = 4ax_1, y_2^2 = 4ax_2, y_3^2 = 4ax_3$

$\therefore$ Area of triangle whose vertices are

$(x_1, y_1), (x_2, y_2)$ and (x_3, y_3)

$$= \frac{1}{2}\left\|\begin{matrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{matrix}\right\| = \frac{1}{2}\left\|\begin{matrix} \frac{y_1^2}{4a} & y_1 & 1 \\ \frac{y_2^2}{4a} & y_2 & 1 \\ \frac{y_3^2}{4a} & y_3 & 1 \end{matrix}\right\|$$

$$= \frac{1}{8a}\left\|\begin{matrix} y_1^2 & y_1 & 1 \\ y_2^2 & y_2 & 1 \\ y_3^2 & y_3 & 1 \end{matrix}\right\|$$

$\Rightarrow$ Area of triangle

$$= \frac{1}{8a}(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)$$

177 **(c)**

Let $y = mx + \frac{a}{m}$ be a tangent to $y^2 = 4ax$ cutting $y^2 = -4ax$ at P and Q. Let (h, k) be mid-point of PQ. Then, equation of PQ is

$ky + 2a(x + h) = k^2 + 4ah$ [Using : $T = S'$]

or, $ky = -2ax + k^2 + 2ah$

But, equation of PQ is

$$y = mx + \frac{a}{m}$$

$$\therefore m = -\frac{2a}{k} \text{ and } \frac{k^2 + 2ah}{k} = \frac{a}{m}$$

$$\Rightarrow -\frac{2a}{k}(k^2 + 2ah) = ak$$

$\Rightarrow -2(k^2 + 2ah) = k^2 \Rightarrow 3k^2 + 4ah = 0$

Hence, the locus of (h, k) is $3y^2 + 4ax = 0$ or, $y^2 = -\frac{4a}{3}x$

178 **(b)**

Let $P(x_1, y_1)$ be a point on the hyperbola. Then the coordinates of N are $(x_1, 0)$

The equation of the tangent at (x_1, y_1) is $\frac{x\,x_1}{a^2} - \frac{y\,y_1}{b^2} = 1$

This meets x-axis at $T\left(\frac{a^2}{x_1}, 0\right)$

$$\therefore OT.ON = \frac{a^2}{x_1} \times x_1 = a^2$$

180 **(a)**

The equation of circles whose radius is r and centres (2, 3) and (5, 6) is

$(x - 2)^2 + (y - 3)^2 = r^2$

And $(x - 5)^2 + (y - 6)^2 = r^2$

$\Rightarrow x^2 + y^2 - 4x - 6y + (-r^2 + 13) = 0$

And $x^2 + y^2 - 10x - 12y + (-r^2 + 61) = 0$

Since, circles cut orthogonally, then

$2g_1g_2 + 2f_1f_2 = c_1 + c_2$

$\Rightarrow 2(2)(5) + 2(3)(6) = 13 - r^2 + 61 - r^2$

$\Rightarrow 2r^2 = 18 \Rightarrow r = 3$

181 **(b)**

Given that, $S_1 \equiv x^2 + y^2 + 4x + 22y + c = 0$, bisects the circumference of the circle

$S_2 \equiv x^2 + y^2 - 2x + 8y - d = 0$

The common chord of the given circle is

$S_1 - S_2 = 0$

ie, $6x + 14y + c + d = 0$ ...(i)

So, Eq. (i) passes through the centre of the second circle, *ie*, $(1, -4)$

$\therefore 6 + 56 + c + d = 0$

$\Rightarrow c + d = 50$

182 **(d)**

We have, $a^2 = 16, b^2 = 9$

$$\therefore e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{\sqrt{7}}{4}$$

Coordinates of S are $(\sqrt{7}, 0)$. Therefore, $CS = \sqrt{7}$

$\therefore CS$: Major axis $= \sqrt{7} : 2a = \sqrt{7} : 8$

183 **(d)**

The given points are the ends of the latusrectum where the normals are always at right angle

184 **(a)**

Let (h, k) be the coordinates of the centre of circle C_2. Then its equation is

$(x - h)^2 + (y - k)^2 = 5^2$

The equation of C_1 is $x^2 - y^2 = 4^2$ and so the equation of the common chord of C_1 and C_2 is

$2\,hx + 2\,ky = h^2 + k^2 - 9$ ...(i)

Let p be the length of the perpendicular from the centre (0,0) of C_1 to (i). Then,

$$p = \left|\frac{h^2 + k^2 - 9}{\sqrt{4\,h^2 + 4\,k^2}}\right|$$

The length of the common chord is $2\sqrt{4^2 - p^2}$ which will be of maximum length, if

$p = 0 \Rightarrow h^2 + k^2 - 9 = 0$ ...(ii)

Now, Slope of common chord $= \frac{3}{4}$

$$\therefore -\frac{h}{k} = \frac{3}{4} \Rightarrow k = -\frac{4\,h}{3} \quad \text{... (iii)}$$

Putting the value of k in (ii), we get

$$h = \pm\frac{9}{5} \Rightarrow k = \mp\frac{12}{5} \quad \text{[From (iii)]}$$

Hence, the centres of circle C_2 are $(9/5, -12/5)$ and $(-9/5, 12/5)$

185 **(a)**

Equation of the normal at point $(bt_1^2, 2bt_1)$ on parabola is

$y = -t_1x + 2bt_1 + bt_1^3$

It is also passes through $(bt_2^2, 2bt_2)$, then

$2bt_2 = t_1.bt_2^2 + 2bt_1 + bt_1^3$

$\Rightarrow\ 2t_2 - 2t_1 = t_1(t_1^2 - t_1^2)$

$\Rightarrow\ \ 2 = -t_1(t_2 + t_1)$

$\Rightarrow\ t_2 = -t_1 - \dfrac{2}{t_1}$

186 **(a)**

Let the equation of tangent which is perpendicular to the line $3x + 4y = 7$, is $4x - 3y = \lambda \Rightarrow\ y = \frac{4}{3}x - \frac{\lambda}{3}$

Since, it is a tangent to the ellipse

$\therefore \left(\dfrac{\lambda}{3}\right)^2 = 9 \times \left(\dfrac{4}{3}\right)^2 + 4\ \ [\because a^2 = 9, b^2 = 4]$

$\Rightarrow\ \lambda^2 = 180\ \Rightarrow\ \lambda = \pm 6\sqrt{5}$

$\therefore$ Equation is $4x - 3y = \pm 6\sqrt{5}$

187 **(d)**

Any point on the hyperbola

$\frac{(x+1)^2}{16} - \frac{(y-2)^2}{4} = 1$, is of the form

$(4\sec\theta - 1, 2\tan\theta + 2)$

188 **(c)**

In the given equation we observe that the denominator of y^2 is greater than that of x^2. So, the two foci lie on y-axis and their coordinates are $(0, \pm be)$, where

$b = 5$ and $e = \sqrt{1 - \dfrac{a^2}{b^2}} = \sqrt{1 - \dfrac{9}{25}} = \dfrac{4}{5}$

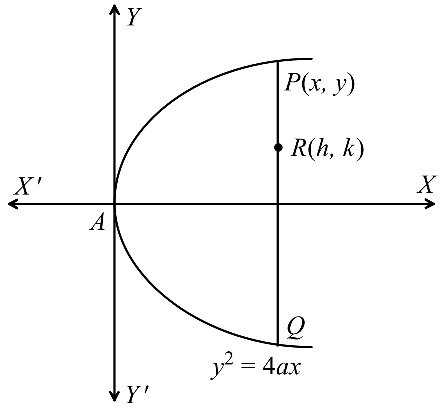
The focal distances of a point $P(x_1, y_1)$ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $b^2 > a^2$ are given by $b \pm ey_1$

Hence, required distances $= b \pm ey_1 = 5 \pm \frac{4}{5}y_1$

189 **(b)**

Let PQ be a double ordinate of $y^2 = 4\,ax$, and let $R(h, k)$ be a point of trisection. Let the coordinates of P be (x, y). Then,

$x = h$ and $y = 3\,k$



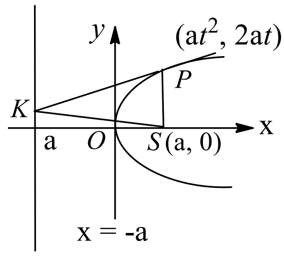
But, $(x - y)$ lies on $y^2 = 4\,ax$

$\therefore 9\,k^2 = 4\,ah$

Hence, the locus of (h, k) is $9y^2 = 4\,ax$

190 **(d)**

Let $P(at^2, 2at)$ any point on the parabola and focus is $(a, 0)$



The equation of tangent at P is $yt = x + at^2$

Since, it meets the directrix $x = -a$ at K

Then, the coordinate of K is $\left(-a, \frac{at^2 - a}{t}\right)$

Slope of $SP = m_1 = \frac{2at}{a(t^2-1)}$

Slope of $SK = m_2 = \frac{a(t^2-1)}{-2at}$

$\therefore m_1 m_2 = \dfrac{2at}{a(t^2 - 1)} \cdot \dfrac{a(t^2 - 1)}{(-2at)} = -1$

$\therefore \angle PSK = 90^\circ$

191 **(d)**

Since, $y = |x| + c$ and $x^2 + y^2 - 8|x| - 9 = 0$ both are symmetrical about y-axis for$x > 0, y = x + c$. Equation of tangent to circle $x^2 + y^2 - 8x - 9 = 0$ which is parallel to $y = x + c$ is $y = (x - 4) + 5\sqrt{1 + 1}$

$\Rightarrow y = x + (5\sqrt{2} - 4)$

For no solution $c > 5\sqrt{2} - 4$,

$\therefore c \in \left(5\sqrt{2} - 4, \infty\right)$

192 **(d)**

Centre is the point of intersection of two diameter, *ie*, the point of intersection of two diameters is $C(8, -2)$, therefore the distance from the centre to the point $P(6, 2)$ is

$r = CP = \sqrt{4 + 16} = \sqrt{20}$

193 **(a)**

Only the point (9,3) lies on the given circle

194 **(d)**

The equation of a tangent of slope m to the circle $x^2+y^2=a^2$ is $y=mx\pm a\sqrt{1+m^2}$ and the coordinates of point of contact are

$$\left(\mp\frac{am}{\sqrt{1+m^2}},\pm\frac{a}{\sqrt{1+m^2}}\right)$$

Here, $a=5$ and $m=\tan 30=1/\sqrt{3}$

So, the coordinates of the points of contact are

$$\left(\mp\frac{5}{2},\pm\frac{5\sqrt{3}}{2}\right)$$

195 **(a)**

Given, $\frac{x^2}{32/5}+\frac{y^2}{32/9}=1$

Let the equation of tangent be $y=mx+c$

$$y=mx\pm\sqrt{\frac{32}{5}m^2+\frac{32}{9}} \quad \dots(i)$$

$[\therefore c^2=a^2m^2+b^2 \text{for } a>b]$

Since, (2,3) lies on Eq. (i)

$$\Rightarrow\ 3=m.2\pm\sqrt{\frac{32}{5}m^2+\frac{32}{9}}$$

$$45(3-2m)^2=288m^2+160$$

$$\Rightarrow 108m^2+540m-245=0$$

$$\therefore D=(540)^2+4.180.245>0\ \Rightarrow\ D>0$$

⇒ Two values of m will exist

⇒ Two tangents will exist

Alternate

Let $S\equiv 5x^2+9y^2-32$

Now, $S(2,3)\equiv 20+81-32>0$

∴ Point (2,3) lies outside ellipse

Thus, two tangents can be drawn

196 **(d)**

As we know equation of tangent to the given hyperbola at (x_1,y_1) is $xx_1-2yy_1=4$ which is same as $2x+\sqrt{6}y=2$

$$\Rightarrow x_1=4 \text{ and } y_1=\sqrt{6}$$

Thus, the point of contact is $(4,-\sqrt{6})$

197 **(b)**

Let (h,k) be the mid-point of a focal chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Then, its equation is

$$\frac{hx}{a^2}+\frac{ky}{b^2}=\frac{h^2}{a^2}+\frac{k^2}{b^2}$$

It passes through the focus $S(ae,0)$

$$\therefore \frac{he}{a}=\frac{h^2}{a^2}+\frac{k^2}{b^2}$$

Hence, the locus of (h,k) is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{xe}{a}$

198 **(c)**

Given, $x=t^2+2t-1$ ...(i)

and $y=3t+5\Rightarrow t=\frac{y-5}{3}$ ...(ii)

On putting the value of t in Eq. (i), we get

$$x=\left(\frac{y-5}{3}\right)^2+2\left(\frac{y-5}{3}\right)-1$$

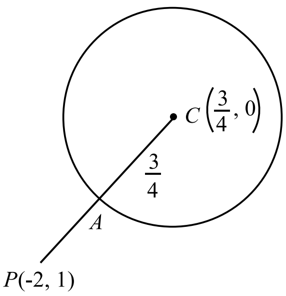
$$\Rightarrow\ x=\frac{1}{9}\{y^2-4y-14\}$$

$$\Rightarrow\ (y-2)^2=9(x+2)$$

This is an equation of a parabola

199 **(b)**

We observe that the minimum distance between point P and the given circle is



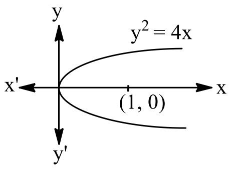
$$PA=CP-CA=\frac{\sqrt{137}}{4}-\frac{3}{4}=\frac{\sqrt{137}-3}{4}>2$$

So, there is no point on the circle whose distance from P is 2 units

200 **(b)**

Given curve is $y^2=4x$

Also, point (1, 0) is the focus of the parabola. It is clear from the graph that only normal is possible



201 **(c)**

Let the extremities of focal chords be $A(at_1^2, 2at_1)$ and $B(at_2^2, 2at_2)$

The equation of tangents at A and B are

$t_{1y} = x + at_2^2$ and $t_2 y = x + at_2^2$

which meets the points C

Slopes of these lines are $m_1 = \frac{1}{t_1}, m_2 = \frac{1}{t_2}$

Now, $m_1 m_2 = \frac{1}{t_1} \times \frac{1}{t_2}$

$= \frac{1}{-1}$ $(\because t_1 t_2 = -1)$

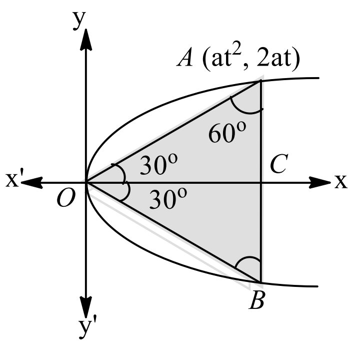
Hence, $\angle ACB = 90° = \frac{\pi}{2}$

202 **(c)**

We know that the difference of the focal distances of any point on a hyperbola is constant equal to its transverse axis. Therefore, the locus of P is a hyperbola

203 **(a)**

In ΔOCA, $\tan 30° = \frac{AC}{OC}$



$\Rightarrow \frac{1}{\sqrt{3}} = \frac{2at}{at^2}, t = 2\sqrt{3}$

Again in ΔOCA,

$OA = \sqrt{OC^2 + AC^2} = \sqrt{(at^2)^2 + (2at)^2}$

$= \sqrt{\left[\left(2\sqrt{3}\right)^2\right]^2 a^2 + 4a^2\left(2\sqrt{3}\right)^2} = \sqrt{192a^2}$

$= 8a\sqrt{3}$

204 **(b)**

Let (x_1, y_1) be the point of intersection of the axis of the parabola with the directrix.

Since vertex is the mid-point of the segment joining the focus and the point of intersection of axis and directrix.

$\therefore \frac{x_1 + 2}{2} = 2$ and $\frac{y_1 - 3}{2} = -1$

$\Rightarrow x_1 = 2$ and $y_1 = 1$

Since directrix is perpendicular to the axis and passes through $(2,1)$. Clearly, axis is parallel to y-axis. So, directrix is parallel to x-axis and passes through $(2,1)$. So, its equation is $y = 1$.

Thus, the focus and directrix of the parabola are $(2, -3)$ and $y = 1$ respectively.

Hence, the equation of the parabola is

$\sqrt{(x-2)^2 + (y+3)^2} = \left|\frac{y-1}{\sqrt{0+1}}\right|$

$\Rightarrow (x-2)^2 + (y+3)^2 = (y-1)^2$

$\Rightarrow x^2 - 4x + 8y + 12 = 0$

205 **(a)**

Since, locus of the point of intersection of the tangents at the end points of a focal chord is directrix

$\therefore$ Required locus is $x = \pm\frac{a}{e} = \pm\frac{a^2}{\sqrt{a^2 - b^2}}$

206 **(b)**

The intersection points of given curves are $(1, 0)$ and $\left(-\frac{13}{5}, -\frac{6}{5}\right)$

$\therefore$ The distance between these two points

$= \sqrt{\left(1 + \frac{13}{5}\right)^2 + \left(0 + \frac{6}{5}\right)^2}$

$= \frac{1}{5}\sqrt{324 + 36}$

$= \frac{6}{5}\sqrt{10}$

207 **(b)**

The length of latusrectum of a hyperbola

$= \frac{2b^2}{a} = \frac{2a^2(e^2 - 1)}{a} = 2a(e^2 - 1)$

208 **(c)**

The centres and radii of given circles are

$C_1(-1, -4), C_2(2, 5)$

and $r_1 = \sqrt{1 + 16 + 23} = \sqrt{40}$,

$r_2 = \sqrt{4 + 25 - 9} = \sqrt{20}$

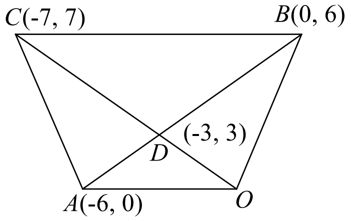
Now, $C_1 C_2 = \sqrt{(2+1)^2 + (5+4)^2} = \sqrt{90}$

And $r_1 + r_2 = \sqrt{40} + \sqrt{20}$
Here, $C_1C_2 < r_1 + r_2$
$\therefore$ Two common tangents can be drawn

209 **(d)**
We know that two point are inverse point with respect to a circle if each lies on the polar of the other.
The polar of $(1, -1)$ with respect to $x^2 + y^2 = 4$ is $x - y = 4$
Clearly, $(2, -2)$ lies on it. Hence, the inverse point of $(1, -1)$ with respect of $x^2 + y^2 = 4$ is $(1, -1)$

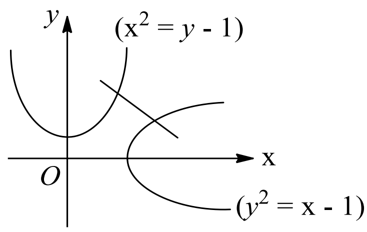
210 **(c)**
Given, $x^2 + y^2 - 2x + 4y + \frac{k}{4} = 0$
$\therefore$ Radius of circle $= \sqrt{1 + 4 - \frac{k}{4}}$
Area of circle $= 9\pi$ [given]
$\Rightarrow \pi\left(5 - \frac{k}{4}\right) = 9\pi \Rightarrow k = -16$

211 **(b)**
The triangle is isosceles and therefore the median through C is the bisector of $\angle C$. The equation of the angle bisector can be taken as $y = -x$and $l = (-a, a)$, where a is positive



Equation of AC is $y - 0 = -7(x + 6)$ or $7x + y + 42 = 0$ and equation of AB is $x - y + 6 = 0$
The length of the perpendicular from l to AB and AC are equal
$\therefore \left|\frac{-7a + a + 42}{\sqrt{50}}\right| = \left|\frac{-a - a + 6}{\sqrt{2}}\right|$
Giving the positive value $a = \frac{9}{2}$
$\therefore$ Centre is $\left(-\frac{9}{2}, \frac{9}{2}\right)$ and radius $= \frac{3}{\sqrt{2}}$
The equation of the circle is
$\left(x + \frac{9}{2}\right)^2 \left(y - \frac{9}{2}\right)^2 = \frac{9}{2}$
$\Rightarrow x^2 + y^2 + 9x - 9y + 36 = 0$

212 **(a)**
General point on the curve $y^2 = x - 1$ is $(t^2 + 1, t)$ and the general point on the curve $x^2 = y - 1$ is $(t, t^2 + 1)$. Since, both curves are symmetrical about line $y = x$. For nearest point on curve $y^2 = x - 1$ from the line $y = x$



Let $D = \frac{t^2+1-t}{\sqrt{2}}$
$\Rightarrow \frac{dD}{dt} = \frac{1}{\sqrt{2}}(2t - 1)$
Put $\frac{dD}{dt} = 0 \Rightarrow t = \frac{1}{2}$. Then point is $\left(\frac{5}{4}, \frac{1}{2}\right)$
Similarly, point on the other curve is $\left(\frac{1}{2}, \frac{5}{4}\right)$
Distance between them
$= \sqrt{\left(\frac{5}{4} - \frac{1}{2}\right)^2 + \left(\frac{1}{2} - \frac{5}{4}\right)^2}$
$= \sqrt{\frac{18}{16}} = \frac{3\sqrt{2}}{4}$

213 **(a)**
Given, $(x + 3)^2 = -20(y - 3)$
This is of the form $X^2 = -4\,aY$
$\therefore$ Axis of such parabola is given by
$X = 0$
$\Rightarrow (x + 3) = 0$

214 **(d)**
Given equation can be rewritten as $\frac{y^2}{k^2} - \frac{x^2}{(-k)} = 1 (-k > 0)$
$e^2 = 1 + \frac{(-k)}{k^2} = 1 - \frac{k}{k^2}$
$\Rightarrow e = \sqrt{1 - \frac{1}{k}}$

215 **(c)**
Given, $(y - 1)^2 = x = 1$
$\Rightarrow Y^2 = X$, where $Y = y - 1, X = x - 1$
Here, $a = \frac{1}{4}$
$\therefore$ Focus is $(a, 0) ie, \left(\frac{1}{4}, 0\right)$
$\Rightarrow X = \frac{1}{4}, Y = 0$
$\Rightarrow x - 1 = \frac{1}{4}, y - 1 = 0 \Rightarrow x = \frac{5}{4}, y = 1$

$\therefore$ Required focus is $\left(\frac{5}{4}, 1\right)$

216 **(b)**

Let $P(h, k)$ be the mid-point of a focal chord of the parabola $y^2 = 4ax$. Then, its equation is

$ky - 2a(x + h) = k^2 = 4ah$ [Using : $T = S'$]

It passes through the focus $(a, 0)$

$\therefore -2a(a + h) = k^2 - 4ah$

$\Rightarrow k^2 = 2a(h - a)$

Hence, the locus of (h, k) is $y^2 = 2\,a(x - a)$

217 **(c)**

Since, the line $y = \frac{1}{b}x - \frac{c}{b}$ is tangent to the parabola $y^2 = 4ax$, then

$$-\frac{c}{b} = \frac{a}{-\frac{a}{b}} \Rightarrow c = b^2$$

218 **(d)**

The circle drawn on foci $(ae, 0)$ and $(-ae, 0)$ as diameter is

$(x - ae)(x + ae) + (y - 0)^2 = 0$

or, $x^2 + y^2 = a^2e^2$ or, $x^2 + y^2 = a^2 + b^2$ ... (i)

The equations of asymptotes are $y = \pm\frac{b}{a}x$

These two intersect at $(\pm\,a, \pm\,b)$

219 **(a)**

Given parametric curves are

$x = 5t^2 + 2,\ y = 10\,t + 4$

or $\frac{x-2}{5} = t^2, \frac{y-4}{10} = t$

$$\Rightarrow \frac{x-2}{5} = \left(\frac{y-4}{10}\right)^2$$

$\Rightarrow (y - 4)^2 = 20(x - 2)$

$\Rightarrow Y^2 = 20X$, where $Y = y - 4, X = x - 2$

$\therefore$ Coodinates of focus are (5, 0)

ie, $x - 2 = 5, y - 4 = 0$

$\Rightarrow x = 7, y = 4$

Hence, required coordinates are (7, 4)

221 **(d)**

The centre of the given circle is (1,2) and its radius is 5. Since the radii of the two circles are equal. Therefore, the two circles are equal. Therefore, the two circles will touch externally and the point of contact will lie mid-way between the two centres. Let the coordinates of the centre of the required circle be (h, k). Then,

$$\frac{h+1}{2} = 5 \text{ and } \frac{k+2}{2} = 5 \Rightarrow h = 9 \text{ and } k = 8$$

Thus, the centre of the required circle is (9,8). Its equation is $(x - 9)^2 + (y - 8)^2 = 5^2 \Rightarrow x^2 + y^2 - 18x - 16y + 120 = 0$

222 **(a)**

Let any point on the line segment PQ is $R(\alpha, \beta)$, then

$\alpha = \frac{\lambda(1)+1}{\lambda+1} = 1,$

And $\beta = \frac{3\lambda+1}{\lambda+1}$ ($\because \lambda > 0$ as R is on segment AB)

A point is inside parabola $y^2 = 4x$, if

$y^2 - 4x < 0$

$$\Rightarrow \left(\frac{3\lambda+1}{\lambda+1}\right)^2 - 4(1) < 0$$

$$\Rightarrow \left(\frac{3\lambda+1}{\lambda+1} + 2\right)\left(\frac{3\lambda+1}{\lambda+1} - 2\right) < 0$$

$\Rightarrow (5\lambda + 3)(\lambda - 1) < 0$

$$\Rightarrow -\frac{3}{5} < \lambda < 1$$

So, $0 < \lambda < 1$ (but $\lambda > 0$)

223 **(b)**

$\because A(at_1^2, 2at_1)$ and $B(at_2^2, 2at_2)$ are such that $AC: AB = 1: 3$

$\therefore$ Coordinates of C are $\left(\frac{2at_1^2+at_2^2}{3}, \frac{4at_1+2at_2}{3}\right)$

Point C lies on x-axis, then

$$\frac{4at_1 + 2at_2}{3} = 0$$

$\Rightarrow t_2 + 2t_1 = 0$

224 **(c)**

Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Since $y = x$ and $3\,y = -2\,x$ is a pair of conjugate diameters.

$$\therefore m_1m_2 = -\frac{b^2}{a^2}$$

$$\Rightarrow 1 \times \left(-\frac{2}{3}\right) = -\frac{b^2}{a^2}$$

$$\Rightarrow 2\,a^2 = 3\,b^2 \Rightarrow 2a^2 = 3\,a^2(1 - e^2) \Rightarrow e^2 = \frac{1}{3}$$

$$\Rightarrow e = \frac{1}{\sqrt{3}}$$

225 **(a)**

We have,

$x^2 - 4\,x + 4\,y^2 = 12$

$\Rightarrow (x - 2)^2 + 4(y - 0)^2 = 8$

$$\Rightarrow \frac{(x-2)^2}{\left(2\sqrt{2}\right)^2} + \frac{(y-0)^2}{\left(\sqrt{2}\right)^2} = 1$$

This is an ellipse whose major and minor axes are $2\,a = 2\sqrt{2}$ and $2\,b = \sqrt{2}$ respectively. Therefore, its eccentricity e is given by

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{2}{8}} = \frac{\sqrt{3}}{2}$$

226 **(a)**

Given equation can be rewritten as

$$(x-1)^2 = -4 \times (2)(y-1)$$

$\Rightarrow \quad X^2 = -4aY$, where $X = x - 1, Y = y - 1$

So, equation of directrix is

$$Y = a$$

$$\Rightarrow \; y - 1 = 2 \Rightarrow \; y = 3$$

227 **(a)**

If the circle $(x-h)^2 + (y-k)^2 = r^2$ touches both the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 - 4\,ax = 0$ externally. Then,

$\sqrt{h^2 + k^2} = r + a$ and $\sqrt{(h - 2\,a)^2 + k^2} = r + 2\,a$

$\therefore \sqrt{(h-2\,a)^2 + k^2} - \sqrt{h^2 + k^2} = a$

$\Rightarrow \sqrt{(h-2\,a)^2 + k^2} = a + \sqrt{h^2 + k^2}$

$\Rightarrow (h - 2\,a)^2 + k^2 = a^2 + h^2 + k^2 + 2\,a\sqrt{h^2 + k^2}$

$\Rightarrow -4\,ah + 3\,a^2 = 2\,a\sqrt{h^2 + k^2}$

$\Rightarrow (3\,a - 4\,h)^2 = 4(h^2 + k^2)$

$\Rightarrow 12(h - a)^2 - 4\,k^2 = 3\,a^2$

Hence, the locus of (h, k) is $12(x - a)^2 - 4\,y^2 = 3\,a^2$

228 **(b)**

Let r_1, r_2 and r_3 be the radii of the respective circles, then

$$r_1 = \sqrt{(-4)^2 + (-3)^2 + 0} = \sqrt{25} = 5$$

$$r_2 = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{3}{2}\right)^2 + \left(\frac{186}{4}\right)} = \sqrt{49} = 7$$

$$r_3 = \sqrt{(3)^2 + (3)^2 + 9} = \sqrt{27} = 3\sqrt{3}$$

$\therefore \; P_1 = 2\pi r_1 = 10\pi, P_2 = 2\pi r_2 = 14\pi, P_3 = 2\pi r_3 = 6\sqrt{3}\pi$

$\therefore \; P_1 < P_3 < P_2$

229 **(a)**

The equation of the parabola is

$$(y-1)^2 = -4\left(x - \frac{1}{4}\right)$$

The equation of any tangent to this parabola is

$$y - 1 = m\left(x - \frac{1}{4}\right) - \frac{1}{m}$$

If it passes through (3,4), then

$$3 = \frac{11m}{4} - \frac{1}{m}$$

$\Rightarrow 12m = 11m^2 - 4 \Rightarrow 11m^2 - 12m - 4 = 0$

Let m_1, m_2 be the roots of this equation. Then,

$m_1 + m_2 = \frac{12}{11}$ and $m_1 m_2 = -\frac{4}{11}$

Let θ be the angle between the tangents. Then,

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\Rightarrow \tan\theta = \frac{\sqrt{(m_1 + m_2)^2 - 4m_1 m_2}}{1 + m_1 m_2}$$

$$\Rightarrow \tan\theta = \frac{\sqrt{\frac{144}{121} + \frac{16}{11}}}{1 - \frac{4}{11}} = \frac{\sqrt{144 + 176}}{7} = \frac{\sqrt{320}}{7}$$

$$= \frac{8\sqrt{5}}{7}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{8\sqrt{5}}{7}\right)$$

ALITER The combined equation of the pair of tangents drawn from (3,4) to the parabola $y^2 - 2y + 4x = 0$ is

$$(y^2 - 2y + 4x)(16 - 8 + 12) = \{4y - (y + 4) + 2(x + 3)\}^2$$

$\Rightarrow 4x^2 + 12xy - 11y^2 - 72x + 12y + 4 = 0$

Let θ be the angle between the lines given by this equation.

Then,

$$\tan\theta = \left|\frac{2\sqrt{36 + 44}}{4 - 11}\right|$$

$$= \frac{8\sqrt{5}}{7} \quad \left[\because \tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b}\right]$$

230 **(b)**

The equations of the directrices of the given ellipse are $y = \pm b/e$

Let PM and PM' be perpendiculars from $P(x_1, y_1)$ on these two directrices. Then, by definition

$SP = e(PM)$ and $S'P = e(PM')$

$\Rightarrow SP + S'P = e(PM + PM')$

$$= e\left(y_1 + \frac{b}{e} + \frac{b}{e} - y_1\right) = 2b$$

ALITER The sum of the focal distances of a point is the major axis of the ellipse

231 **(a)**

Let $P(h, k)$ be the mid-point of a chord of the circle $x^2 + y^2 = 16$. Then, the equation of the chord is

$hx + ky - 16 = h^2 + k^2 - 16$ or, $y = \left(-\frac{h}{k}\right)x + \left(\frac{h^2 + k^2}{k}\right)$

It touches the hyperbola $9x^2 - 16y^2 = 144$

$$\therefore \left(\frac{h^2 + k^2}{k}\right)^2 = 16\left(-\frac{h}{k}\right)^2 - 9 \quad [\text{Using } c^2 = a^2 m^2 - b^2]$$

$\Rightarrow (h^2+k^2)^2 = 16h^2 - 9k^2$

Hence, the locus of (h, k) is $(x^2+y^2)^2 = 16x^2 - 9y^2$

232 **(c)**

Centre and radius of the given circle are (1, 0)and 1.

Let the centre of the image circle be (x_1,y_1)

Hence, (x_1, y_1) be the image of the point (1, 0) w.r.t. the line $x+y=2$, then

$$\frac{x_1-1}{1} = \frac{y_1-0}{1} = \frac{-2[1(1)+1(0)-2]}{(1)^2+(1)^2}$$

$$\Rightarrow \quad \frac{x_1-1}{1} = \frac{y_1}{1} = 1$$

$\Rightarrow \quad x_1 = 2, y_1 = 1$

$\therefore$ Equation of the imaged circle is

$(x-2)^2 + (y-1)^2 = 1^2$

$\Rightarrow x^2+y^2-4x-2y+4=0$

233 **(b)**

Let OA, OB be the tangents from the origin to the given circle with centre $C(-3, 5)$ and radius $\sqrt{9+25-c} = \sqrt{34-c}$

Then, area of the quadrilateral $OACB = 2 \times$ area of the ΔOAC

$= 2 \times \left(\frac{1}{2}\right) \times OA \times AC$

Now, $OA =$ length of the tangent from the origin to the given circle $= \sqrt{c}$

And $AC =$ radius of the circle$= \sqrt{34-c}$

So, that $\sqrt{c}\sqrt{34-c} = 8$ (given)

$\Rightarrow \quad c(34-c) = 34$

$\Rightarrow c^2 - 34c + 64 = 0$

234 **(a)**

$(x^2+y^2-2x-1)+\lambda x = 0$, they pass through intersection points of line $x=0$ and circle $x^2+y^2-2x-1=0$

$\Rightarrow y = \pm 1$

$\therefore$ Required points are $(0, \pm 1)$

235 **(d)**

Given, $2ae = 6$ and $2b = 8$

$\Rightarrow \quad ae = 3$ and $b = 4$

$$\Rightarrow \frac{ae}{b} = \frac{3}{4}, \frac{b^2}{a^2} = \frac{16e^2}{9}$$

$$\frac{b^2}{a^2} = 1-e^2 \Rightarrow \frac{16e^2}{9} = 1-e^2$$

$$\Rightarrow \left(\frac{16+9}{9}\right)e^2 = 1 \Rightarrow e = \frac{3}{5}$$

236 **(d)**

Equation of circle is

$(x-4)(x+2)+(y-7)(y+1) = 0$

$\Rightarrow x^2-2x-8+y^2+y-7y-7=0$

$\Rightarrow x^2+y^2-2x-6y-15=0$

Here, $g=-1, c=-15$

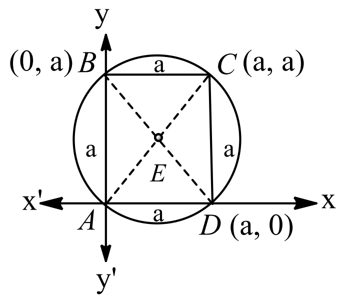
$\therefore \quad AB = 2\sqrt{g^2-c}$

$= 2\sqrt{1+15}$

$= 8$

237 **(b)**

Since, E is the mid point of AC, therefore, the coordinates of D are $\left(\frac{a}{2}, \frac{a}{2}\right)$



Now, $AC = \sqrt{a^2+a^2} = \sqrt{2}a$

$$\therefore \quad AE = \frac{1}{2}AC = \frac{a}{\sqrt{2}}$$

$\therefore$ The equation of circle whose centre is $\left(\frac{a}{2}, \frac{a}{2}\right)$ and radius $\frac{a}{\sqrt{2}}$, is

$$\left(x-\frac{a}{2}\right)^2 + \left(y-\frac{a}{2}\right)^2 = \left(\frac{a}{\sqrt{2}}\right)^2$$

$\Rightarrow \quad x^2+y^2 = a(x+y)$

239 **(b)**

The centre of the ellipse is at (2,3) and its axes are parallel to the coordinate axes. So, let its equation be

$$\frac{(x-2)^2}{a^2} + \frac{(y-3)^2}{b^2} = 1$$

We have,

$2a =$ Disatnce between vertices =12$\Rightarrow a = 6$

Also, $e = 5/6$

$\therefore b^2 = a^2(1-e^2) \Rightarrow 36-25 = 11$

Hence, the equation of the ellipse is $\frac{(x-2)^2}{36} + \frac{(y-3)^2}{11} = 1$

240 **(a)**

The equation of the family of coaxial system of circles having $x^2+y^2-6x-6y+4=0$ and $x^2+y^2-2x-4y+3=0$ as two members is

$x^2+y^2-6x-6y+4+\lambda(-4x-2y+1)=0$

[Using : $S_1 - S_2 = 0$]

$\Rightarrow x^2+y^2-2x(3+2\lambda)-2y(3+\lambda)+4+\lambda = 0$ ...(i)

Coordinates of centre of circle (i) are $(3+2\lambda, 3+$

$\lambda)$

Radius $= \sqrt{(3+2\lambda)^2+(3+\lambda)^2-(4+\lambda)}$

For limiting points, we must have

Radius $= 0 \Rightarrow 5\lambda^2+17\lambda+14=0 \Rightarrow \lambda$

$= -2, -7/5$

Hence, limiting points are $(-1,1)$ and $(1/5, 8/5)$

241 **(a)**

Let (α, β) be the mid point of a chord of the circle $x^2+y^2=a^2$. Then its equation is

$\alpha x + \beta y = \alpha^2+\beta^2$ [Using $S' = T$]

This passes through (h, k)

$\therefore \alpha h + \beta k = \alpha^2 + \beta^2$

Hence, the locus of (α, β) is

$x^2+y^2 = hx+ky \Rightarrow x^2+y^2-hx-ky=0$

242 **(c)**

We have, $x\cos\alpha + y\sin\alpha = p$

$\Rightarrow y = -x\cot\alpha + p\operatorname{cosec}\alpha$

Since, above line is tangent to the ellipse

$\therefore c^2 = a^2m^2+b^2$

$\Rightarrow p^2\text{coses}^2\alpha = a^2\cot^2\alpha + b^2$

$\Rightarrow a^2\cos^2\alpha + b^2\sin^2\alpha = p^2$

243 **(d)**

Given equation of parabola is $y^2 = 16x$

If $(1, 1)$ is the mid point of the chord, then its equation of chord is

$T = S_1$

$\therefore y(1) - 8(x+1) = 1-16$

$\Rightarrow y - 8x - 8 = -15$

$\Rightarrow 8x - y = 7$

244 **(c)**

The vertex in a mid point of focus and directrix.

Hence, coordinate of vertex is $(1, 0)$

245 **(d)**

The equation of a tangent to $y^2 = 8x$ is

$y = mx + \dfrac{2}{m}$ ... (i)

This will touch the hyperbola $\frac{x^2}{1} - \frac{y^2}{3} = 1$, if

$\dfrac{4}{m^2} = m^2 - 3$ [Using : $c^2 = a^2m^2 - b^2$]

$\Rightarrow m^4 - 3m^2 - 4 = 0 \Rightarrow (m^2-4)(m^2+1) = 0$

$\Rightarrow m = \pm 2$

So, equations of common tangents are

$y = (\pm 2)x \pm 1$ or, $2x - y + 1 = 0$ and $2x + y + 1 = 0$

247 **(a)**

Equation of any tangent to the given ellipse is

$y = mx \pm \sqrt{a^2m^2+b^2}$

$\Rightarrow y - mx = \pm\sqrt{a^2m^2+b^2}$ ...(i)

Equation of perpendicular line is

$my + x = \lambda$

It passes through the centre (0, 0)

$\therefore \lambda = 0$

$\therefore my + x = 0$ ...(ii)

On squaring and adding Eqs. (i) and (ii)

$y^2 + m^2x^2 + m^2y^2x^2 = a^2m^2 + b^2$

$(1+m^2)(x^2+y^2) = a^2m^2+b^2$

$\Rightarrow \left(1 + \dfrac{x^2}{y^2}\right)(x^2+y^2) = \dfrac{a^2x^2}{y^2} + b^2$

$\Rightarrow (x^2+y^2)^2 = a^2x^2 + b^2y^2$

But $(x^2+y^2)^2 = lx^2 + my^2$

$\therefore l = a^2, m = b^2$

248 **(d)**

If $y = mx + c$ is a tangent to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then $c^2 = a^2m^2 + b^2$. We have, $a^2 = 6, b^2 = 3$

$\therefore c^2 = 6m^2 + 3$

249 **(b)**

The given point $(-a, 2a)$ lies on the directrix $x = -a$ of the parabola $y^2 = 4ax$. Thus, the tangents are at right angle

251 **(a)**

Let $P(x, y)$ be any point on the ellipse. Then, by definition, we have

$SP = e\,PM$, where PM is the length of perpendicular from P on the directrix

$\Rightarrow \sqrt{(x-1)^2+(y+1)^2} = \dfrac{1}{2}\left|\dfrac{x-y-3}{\sqrt{2}}\right|$

$\Rightarrow (x-1)^2 + (y+1)^2 = \dfrac{1}{8}(x-y-3)^2$

$\Rightarrow 7x^2 + 2xy + 7y^2 - 10x + 10y + 7 = 0$

Hence, the equation of the ellipse is

$7x^2 + 2xy + 7y^2 - 10x + 10y + 7 = 0$

252 **(c)**

Let $P(4t_2^2 . 8t_2)$ be the end-points of a focal chord of the parabola $y^2 = 16x$. Then,

$PQ = 4(t_2 - t_1)^2$

Now, Slope of $PQ = 2$

$\Rightarrow \dfrac{8t_2 - 8t_1}{4t_2^2 - 4t_1^2} = 2 \Rightarrow t_2 + t_1 = 1$

$\therefore PQ = 4(t_2 - t_1)^2 = 4\{(t_2 + t_1)^2 - 4t_1t_2\}$

$\Rightarrow PQ = 4\{(t_2 + t_1)^2 + 4\} = 4(1 + 4) = 20$

ALITER We know that the length of a focal chord of the parabola $y^2 = 4ax$ making an angle θ with the axis of the parabola is $4a \operatorname{cosec}^2 \theta$

Here, we have

$a = 4$ and $\tan\theta = 2$

$\therefore$ Length of the focal chord $= 16\left(1 + \frac{1}{4}\right) = 20$

253 **(b)**

Since, given lines are parallel to each other, so the line segment joining the points of contact is diameter of the circle. Distance between the lines $3x - 4y + 5 = 0$ and $3x - 4y - \frac{9}{2} = 0$ is

$$\left|\frac{5 + \frac{9}{2}}{\sqrt{3^2 + 4^2}}\right| = \left|\frac{19}{10}\right| = 1.9$$

Length of diameter of the circle is 1.9

$\therefore$ Radius of circle $= \frac{1.9}{2} = 0.95$

254 **(a)**

Let the point be $P(\sqrt{2}\cos\theta, \sin\theta)$ on $\frac{x^2}{2} + \frac{y^2}{1} = 1$

$\Rightarrow$ Equation of tangent is

$$\frac{x\sqrt{2}}{2}\cos\theta + y\sin\theta = 1$$

Whose intercept on coordinate axes are

$A(\sqrt{2}\sec\theta, 0)$ and $B(0, \operatorname{cosec}\theta)$

$\therefore$ Mid point of its intercept between axes is

$$\left(\frac{\sqrt{2}}{2}\sec\theta, \frac{1}{2}\operatorname{cosec}\theta\right) = (h, k)$$

$$\cos\theta = \frac{1}{\sqrt{2}h} \text{ and } \sin\theta = \frac{1}{2k}$$

Thus, locus of mid point M is

$$\cos^2\theta + \sin^2\theta = \frac{1}{2h^2} + \frac{1}{4k^2}$$

$$\Rightarrow \frac{1}{2x^2} + \frac{1}{4y^2} = 1$$

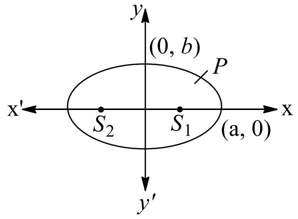
255 **(c)**

Given, $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Sum of the focal distance $= 2a = 2 \times 4 = 8$

256 **(a)**

To maximise the area of ΔS_1PS_2 altitude should be maximum as base S_1, S_2 is fixed. So, P should be $(0, b)$



257 **(b)**

For real circle, we must have

$\sin\alpha \geq 0 \Rightarrow \alpha \in [0, \pi]$

Now, $x -$ intercept > 2

$\Rightarrow \sqrt{\sin\alpha - \cos\alpha + 1} > 2$

$\Rightarrow \sin\alpha - \cos\alpha + 1 > 1$

$\Rightarrow \sin\alpha - \cos\alpha > 0$

$\Rightarrow \sin\left(\alpha - \frac{\pi}{4}\right) > 0 \Rightarrow 0 < \alpha - \frac{\pi}{4} < \pi \Rightarrow \frac{\pi}{4} < \alpha < \frac{5\pi}{4}$

But, $\alpha \in [0, \pi]$ $\quad [\therefore \frac{\pi}{4} < \alpha \leq \pi$ i. e. $\alpha \in (\pi/4, \pi)]$

258 **(c)**

Let any point on the parabola be $(at^2, 2at)$

If the equation of parabola is $y^2 = 4ax$, then focus is $(a, 0)$

Let the focus of a point be (α, β) if it is a mid point

$\therefore \alpha = \frac{at^2 + a}{2}, \beta = \frac{2at + 0}{2}$

$\Rightarrow 2\alpha = at^2 + a, \qquad \beta = at$

$\therefore 2\alpha = a\left(\frac{\beta}{a}\right)^2 + a$

$\Rightarrow 2a\alpha = \beta^2 + a^2$

$\Rightarrow \beta^2 = -a^2 + 2a\alpha$

$\Rightarrow \beta^2 = \frac{4a}{2}\left(\alpha - \frac{a}{2}\right)$

$\therefore$ The locus is $y^2 = \frac{4a}{2}\left(x - \frac{a}{2}\right)$

The directrix is $X = -\frac{a}{2}$

$\Rightarrow x - \frac{a}{2} = -\frac{a}{2}$

$\Rightarrow x = 0$

259 **(a)**

Let the equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$

Since, $\left(m, \frac{1}{m}\right)$ lies on this circle

$\therefore \; m^2 + \frac{1}{m^2} + 2gm + \frac{2f}{m} + c = 0$

$\Rightarrow m^4 + 2gm^3 + cm^2 + 2fm + 1 = 0$

$\Rightarrow m_1m_2m_3m_4 = 1$

260 **(b)**

Any line touching the parabola $y^2 = 4\,ax$ can be written as

$y = mx + \frac{a}{m}$ ... (i)

Equation of a line passing through the focus $(a, 0)$ and perpendicular to (i) is

$y = -\frac{1}{m}(x - a)$ ... (ii)

Let $P(h, k)$ be the point of intersection of (i) and (ii). Then,

$k = mh + \frac{a}{m}$ and $k = -\frac{1}{m}(h - a)$

$\Rightarrow mh + \frac{a}{m} = -\frac{1}{m}(h - a)$

$\Rightarrow mh = -\frac{h}{m} \Rightarrow (m^2 + 1)h = 0 \Rightarrow h = 0$

Hence, the locus of $P(h, k)$ is $x = 0$,

Which is a line tangent to the parabola $y^2 = 4\,ax$ at the vertex

261 **(b)**

Given parabola is $y^2 = 12\,x$

Here, $a = 3$

For point $P(x, y), y = 6$

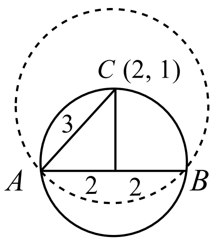
This point lie on the parabola

$\therefore \quad (6)^2 = 12x \Rightarrow x = 3$

Thus , focal distance of point P is 6

262 **(c)**

The centre of given circle is (1, 3) and radius is 2. So, AB is a diameter of the given circle has its mid point as (1, 3). The radius of the required circle is 3



263 **(b)**

We know that, if two perpendicular tangents to the circle $x^2 + y^2 = a^2$ meet at P, then the point P lies on a director circle

$\therefore$ Required locus is $x^2 + y^2 = 32$

264 **(a)**

It is given that the coordinates of the centre, focus and adjacent vertex of an ellipse are $(2, -3), (3, -3)$ and $(4, -3)$ respectively. So, equation of the ellipse is

$\frac{(x-2)^2}{a^2} + \frac{(y+3)^2}{b^2} = 1$

Clearly,

ae = Distance between centre $(2, -3)$ and focus $(3, -3)$

and, a = Distance between centre $(2, -3)$ and vertex $(4, -3)$

$\Rightarrow ae = 1$ and $a = 2 \Rightarrow a = 2, e = \frac{1}{2}$

$\therefore b^2 = a^2(1 - e^2) \Rightarrow b^2 = 3$

So, the equation of the ellipse is $\frac{(x-2)^2}{4} + \frac{(y+3)^2}{3} = 1$

265 **(a)**

Centres and radii of the given circles are

$C_1(0, 0), r_1 = 3$

And $C_2(-\alpha, -1)$ and $r_2 = \sqrt{\alpha^2 + 1 - 1} = |\alpha|$

Since, two circles touch internally,

$\therefore \quad C_1C_2 = r_1 - r_2$

$\Rightarrow \sqrt{\alpha^2 + 1^2} = 3 - |\alpha|$

$\Rightarrow \quad \alpha^2 + 1 = 9 + \alpha^2 - 6|\alpha|$

$\Rightarrow \quad 6|\alpha| = 8$

$\Rightarrow \quad |\alpha| = \frac{4}{3}$

$\Rightarrow \quad \alpha \pm \frac{4}{3}$

266 **(d)**

We have,

$x = \frac{a}{2}\left(\frac{t+1}{t}\right), y = \frac{a}{2}\left(\frac{t-1}{t}\right)$

$\Rightarrow \frac{2\,x}{a} = 1 + \frac{1}{t}, \frac{2\,y}{a} = 1 - \frac{1}{t}$

$\Rightarrow \frac{2\,x}{a} - 1 = \frac{1}{t}, 1 - \frac{2\,y}{a} = \frac{1}{t}$

$\Rightarrow \frac{2\,x}{a} - 1 = 1 - \frac{2\,y}{a} \Rightarrow x + y = a$, which is a straight line

267 **(c)**

Let the coordinates of focus be $S\ (a, 0)$

Let any point on the parabola be $P(at^2, 2at)$. Let the coordinates of mid point of P and S be (x_1, y_1)

$\therefore \quad x_1 = \frac{a + at^2}{2}, y_1 = \frac{0 + 2at}{2}$

$\Rightarrow at^2 = 2x_1 - a, \qquad y_1 = at$

$\Rightarrow a\left(\frac{y_1}{a}\right)^2 = 2x_1 - a$

$\Rightarrow y_1^2 = 2x_1a - a^2$ =Hence , the locus of the mid point is

$y^2 = 2a\left(x - \frac{a}{2}\right)$

$\therefore$ Equation of directrix is $x - \frac{a}{2} = -\frac{a}{2} \Rightarrow x = 0$

268 **(c)**

The equation of any tangent to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

$y = mx + \sqrt{a^2m^2 - b^2}$

If it passes through (c, d), then

$d = mc + \sqrt{a^2m^2 - b^2}$

$\Rightarrow m^2(c^2 - a^2) - 2mcd + d^2 + b^2 = 0$

This equation gives two values of m i.e. slopes of tangents passing through (c, d). This means that $\tan\alpha$ and $\tan\beta$ are its roots.

$\therefore \tan\alpha \tan\beta = \dfrac{d^2 + b^2}{c^2 - a^2}$

$\Rightarrow 1 = \dfrac{d^2 + b^2}{c^2 - a^2} \Rightarrow c^2 - d^2 = a^2 + b^2$

269 **(d)**

If $P(at^2, 2at)$ be one end of a focal chord of the parabola $y^2 = 4ax$, then another end of chord will be $Q\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$

$\therefore$ Length of focal chord$= PQ$

$$= \sqrt{\left(\frac{a}{t^2} - at^2\right)^2 + \left(-\frac{2a}{t} - 2at\right)^2}$$

$$= a\left(\frac{1}{t} + t\right)\sqrt{\left(\frac{1}{t} - t\right)^2 + 4}$$

$$= a\left(\frac{1}{t} + t\right)^2$$

270 **(a)**

Given, $(\pm ae, 0) = (\pm 3, 0)$

$\Rightarrow ae = 3$

$\Rightarrow a^2e^2 = 9$

$\Rightarrow b^2 + a^2 = 9$ ...(i)

$\because 2x + y - 4 = 0$

$\Rightarrow y = -2x + 4$

is the tangent to the hyperbola

$\therefore (4)^2 = a^2(-2)^2 - b^2$

$\Rightarrow 4a^2 - b^2 = 16$ ...(ii)

On solving Eqs. (i) and (ii), we get

$a^2 = 5, b^2 = 4$

$\therefore$ Equation of hyperbola is $\frac{x^2}{5} - \frac{y^2}{4} = 1$

$\Rightarrow 4x^2 - 5y^2 = 20$

271 **(a)**

Given circles intersect orthogonally. So, the length of their common chord is

$l = \dfrac{2\, r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$

where r_1 and r_2 are the radii of the given circles

Here $r_1 = \sqrt{5}$ and $r_2 = \sqrt{3}$

$\therefore l = \dfrac{2\sqrt{15}}{\sqrt{5+3}} = \sqrt{\dfrac{15}{2}}$

272 **(b)**

Put $x = at^2$ in the given equation, we get

$\dfrac{a^2t^4}{a^2} + \dfrac{y^2}{b^2} = 1$

$y^2 = b^2(1 - t^2)(1 + t^2)$

This will give real values of y, if

$1 - t^2 \geq 0 \Rightarrow |t| \leq 1$

274 **(b)**

Since a radius of circle C_1 is 2 and this circle touches both the axes

So, centre of circle $C_1 = (2, 2)$ and let radius of another circle is r and this circle also touches both the axes so centre of circle $C_2 = (r, r)$

Since, both circles touches each other

$\sqrt{(r-2)^2 + (r-2)^2} = 2 + r$

$\Rightarrow 2(r-2)^2 = (r+2)^2$

$\Rightarrow r^2 - 12r + 4 = 0$

$\Rightarrow r = \dfrac{12 \pm \sqrt{128}}{2} = 6 \pm 4\sqrt{2}$

$\Rightarrow r = 6 + 4\sqrt{2}$ $[\because r > 2]$

275 **(d)**

Given parabola is $x^2 = -2y$

Coordinates of end points of a latusrectum are $A\left(1, -\frac{1}{2}\right)$ and $B\left(-1, -\frac{1}{2}\right)$

Now, $2x = -2\frac{dy}{dx}$

$\Rightarrow \dfrac{dy}{dx} = -x$

And slope of normal is $-\frac{dx}{dy} = \frac{1}{x}$

The equations of normals at points A and B are

$y + \dfrac{1}{2} = \dfrac{1}{1}(x - 1)$

$\Rightarrow 2y - 2x = -3$ ...(i)

And $y+\frac{1}{2}=-\frac{1}{1}(x+1)$
$\Rightarrow 2y+2x=-3$ ...(ii)
On solving Eqs. (i) and (ii), we get
$x=0, y=-\frac{3}{2}$

276 **(a)**
Let $P(x_1,y_1)$ and $Q(x_2,y_2)$ be end points of diameter PQ. Then,
$x_1+x_2=-2, x_1x_2=-3, y_1+y_2=-4$ and $y_1y_2=-12$
The equation of the circle having PQ as a diameter is
$x^2+y^2-x(x_1+x_2)-y(y_1+y_2)+x_1x_2+y_1y_2=0$
$\Rightarrow x^2+y^2+2x+4y-3-12=0$
$\Rightarrow x^2+y^2+2x+4y-15=0$

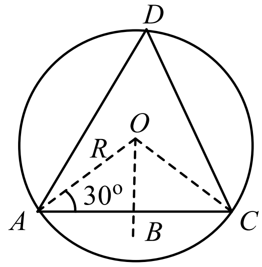
277 **(a)**
The locus of the point of intersection of perpendicular tangents to an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is its director circle $x^2+y^2=a^2+b^2$

278 **(c)**
Let the required circle be
$x^2+y^2+2\,gx+2\,fy+x=0$ ...(i)
This passes through (0,0). Therefore, $c=0$
The centre $(-g,-f)$ of circle (i) lies on $y=x$. Therefore, $g=f$
Since (i) cuts the circle $x^2+y^2-4\,x-6\,y+10=0$ orthogonally
$\therefore 2(-2\,g-3\,f)=c+10$
$\Rightarrow -10\,g=10$ [$\because g=f$ and $c=0$]
$\Rightarrow g=f=-1$
Hence, the required circle is $x^2+y^2-2x-2y=0$

279 **(d)**
$\therefore$ equation of common chord is
$(x^2+y^2+2x-3y+6)-(x^2+y^2+x-8y-13)=0$
[$\because S_1-S_2=0$]
$\Rightarrow x+5y+19=0$
In the given option, only the point $(1,-4)$ satisfies this equation

280 **(b)**
Given, $x^2+y^2-7x+9y+5=0$
$\therefore R=\sqrt{\left(\frac{-7}{2}\right)^2+\left(\frac{9}{2}\right)^2-5}$
$=\sqrt{\frac{49}{4}+\frac{81}{4}-5}=\frac{\sqrt{110}}{2}$

In ΔOAB, $\cos 30°=\frac{AB}{R} \Rightarrow \frac{\sqrt{3}}{2}=\frac{AB}{\frac{\sqrt{110}}{2}}$



$\Rightarrow \frac{\sqrt{330}}{4}=AB$
$\therefore$ Length $AC=\frac{\sqrt{330}}{2}$
$\therefore$ Area of equilateral $\Delta=\frac{\sqrt{3}}{4}(a)^2$
$=\frac{\sqrt{3}}{4}\times\frac{330}{4}=\frac{165\sqrt{3}}{4}$ sq units

281 **(b)**
Given equation is
$y^2-8y-x+19=0$
$\Rightarrow (y-4)^2=x-3$
$\Rightarrow Y^2=4AX$, where $Y-y-4, A=\frac{1}{4}$ and $X=x-3$
$\therefore$ Focus is $(A,0)=\left(\frac{1}{4},0\right)=\left(\frac{13}{4},4\right)$
Vertex is $(0,0)=(3,4)$
Directrix is $x=-A \Rightarrow x-3=-\frac{1}{4}$
$\Rightarrow x=\frac{11}{4}$

282 **(b)**
Centre and radii of two circles are
$C_1(0,0), C_2(1,2)$ and $r_1=\sqrt{5}, r_2=2\sqrt{5}$
Since, $C_1C_2=\sqrt{5}=r_2-r_1$, therefore, the two circles touch each other internally

283 **(c)**
The point of intersection of given curves are (0, 0) and (1, 1)

$\therefore$ Length of common chord$=\sqrt{1^2+1^2}=\sqrt{2}$

284 **(d)**
Equation of pair of tangents is
$(a^2-1)y^2-x^2+2ax-a^2=0$
If θ be the angle between the tangents, then
$\tan\theta=\frac{2\sqrt{(h^2-ab)}}{a+b}$
$=\frac{2\sqrt{-(a^2-1)(-1)}}{a^2-2}$
$=\frac{2\sqrt{a^2-1}}{a^2-1}$

$\because$ θ lies in II quadrant, the $\tan\theta < 0$

$\therefore \dfrac{2\sqrt{a^2-1}}{a^2-2} < 0$

$\Rightarrow\ a^2 - 1 > 0$ and $a^2 - 2 < 0$

$\Rightarrow\ 1 < a^2 < 2$

$\Rightarrow\ a \in (-\sqrt{2}, -1) \cup (1, \sqrt{2})$

285 **(b)**

Let the equation of hyperbola is

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ ...(i)

It passes through the point (3, 0) and $(3\sqrt{2}, 2)$.

$\therefore \dfrac{9}{a^2} = 1 \Rightarrow a^2 = 9$

And $\frac{18}{a^2} - \frac{4}{b^2} = 1 \Rightarrow \frac{4}{b^2} = \frac{18}{9} - 1$

$\Rightarrow \dfrac{4}{b^2} = 1 \Rightarrow b^2 = 4$

$\therefore$ Eccentricity of hyperbola,

$$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{4}{9}} = \frac{\sqrt{13}}{3}$$

286 **(b)**

The centres of the two circles will lie on the line through $P(1, 2)$ perpendicular to the common tangent $4x + 3y = 10$. If C_1 and C_2 are the centres of these circles, then $PC_1 = 5 = r_1, PC_2 = -5 = r_2$. Also, C_1, C_2 lie on the line $\frac{x-1}{\cos\theta} = \frac{y-2}{\sin\theta} = r$, where $\tan\theta = \frac{3}{4}$. When $r = r_1$ the coordinates of C_1 are $(5\cos\theta + 1, 5\sin\theta + 2)$ or (5, 5) as $\cos\theta = \frac{4}{5}, \sin\theta = \frac{3}{5}$

When $r = r_2$, the coordinates of C_2 are $(-3, -1)$

The circle with centre $C_1(5, 5)$ and radius 5 touches both the coordinates axes and hence lies completely in the first quadrant

Therefore, the required circle is with centre $(-3, -1)$ and radius 5, so its equation is

$(x + 3)^2 + (y + 1)^2 = 5^2$

or $x^2 + y^2 + 6x + 2y - 15 = 0$

Since, the origin lies inside the circle, a portion of the circle lies in all the quadrants

287 **(d)**

The slopes of AP and AQ (A is the vertex) are given by

$$m_1 = \frac{2\,at_1 - 0}{at_1^2 - 0} = \frac{2}{t_1} \text{ and } m_2 = \frac{2\,at_2 - 0}{at_2^2 - 0} = \frac{2}{t_2}$$

Now, $AP \perp AQ \Rightarrow m_1 m_2 = -1 \Rightarrow \dfrac{2}{t_1}\cdot\dfrac{2}{t_2} = -1$

$$\Rightarrow t_1 t_2 = -4$$

288 **(b)**

The centre and radii of circles are

$C_1(0, 0), C_2(3, 4)$ and

$r_1 = 2, r_2 = \sqrt{9 + 16 - 24} = 1$

Now, $C_1C_2 = \sqrt{(3-0)^2 + (4-0)^2} = 5$

$r_1 + r_2 = 2 + 1 = 3$

Since, $C_1C_2 > r_1 + r_2$

$\therefore$ Number of common tangents=4

289 **(c)**

Let point of intersection be (h, k). Then, equation of the line passing through P and Q is $hx + 2ky = 4$ (chord of contact)

Since, $hx + 2ky = 4$ touches $x^2 + y^2 = 1, \frac{16}{4k^2} = 1 + \frac{h^2}{4k^2}$

$ie, 4k^2 + h^2 = 16$. So, required locus is $4y^2 + x^2 = 16$, which is an ellipse of eccentricity $\frac{\sqrt{3}}{2}$ and length of latusrectum is 2 unit

290 **(a)**

1. The centre and radius of circle

$x^2 + y^2 - x - y - 1 = 0$

are $\left(\frac{1}{2}, \frac{1}{2}\right)$ and $\sqrt{\frac{3}{2}}$ respectively and the centre and radius of circle

$x^2 + y^2 - 2x + 2y - 7 = 0$

are $(1, -1)$ are 3 respectively

Distance between the centres is $\sqrt{\frac{5}{2}} < 3 - \sqrt{\frac{3}{2}}$

$(\because C_1C_2 < r_1 - r_2)$

$\therefore$ First circle is completely inside the second circle

2. The centre and radius of circle

$x^2 + y^2 + 14x + 12y + 21 = 0$

are $(-7, 6)$ and 8 respectively and the centre and radius of circle

$x^2 + y^2 + 2x - 4y - 4 = 0$

are $(-1, 2)$ and 1 respectively

Distance between the centres is $2\sqrt{13} > 8 + 1$ $(\because C_1C_2 > r_1 + r_2)$

These two circles intersect each other, therefore the number of common tangents is 2. Hence, only first statements is correct

291 **(b)**

We know that the sum of the focal distance of a point on an ellipse is equal to the length of the major axis of the ellipse

$\therefore SP + S'P = 12$

292 **(a)**

Let (h,k) be the pole. Then, the equation of the polar is

$ky = 2a(x+h) \Rightarrow y = \left(\frac{2a}{k}\right)x + \frac{2ah}{k}$

This touches the ellipse $\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1$

$\therefore \left(\frac{2ah}{k}\right)^2 = \alpha^2\left(\frac{2a}{k}\right)^2 + \beta^2 \Rightarrow 4a^2 h^2$
$= 4a^2\alpha^2 + \beta^2k^2$

Hence, the locus of (h,k) is $4a^2 x^2 = 4a^2\alpha^2 + \beta^2y^2$

293 **(b)**

The equation of the chord of contact of tangents drawn from (1,2) to $3x^2 - 4y^2 = 3$ is $3x - 8y = 3$

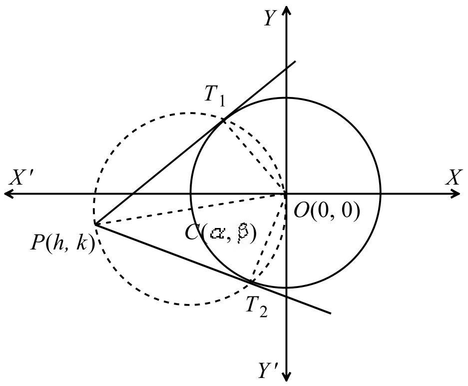
294 **(a)**

Let $C(\alpha,\beta)$ be the circumcentre of $\Delta\, PT_1T_2$. Then,

$\alpha = \frac{h}{2}$ and $\beta = \frac{k}{2} \Rightarrow h = 2\,\alpha$ and $k = 2\,\beta$

Since (h,k) lies on $px + qy - r = 0$

$\therefore ph + qk - r = 0 \Rightarrow 2p\,\alpha + 2q\,\beta - r = 0$



Hence, the locus of (α,β) is

$2\,px + 2qy - r = 0 \Rightarrow px + qy - \frac{r}{2} = 0$

295 **(b)**

The centre and radius of given circle are $C_1\left(-\frac{3}{2},3\right)$ and $r_1 = \frac{9}{2}$

Let the centre and radius of required circle are $C_2(g,f)$ and $r_2 = 2$

Since, the required circle is rolled outside the given circle.

$\therefore\ C_1C_2 = r_1 + r_2$

$\Rightarrow \sqrt{\left(g+\frac{3}{2}\right)^2 + (f-3)^2} = 2 + \frac{9}{2}$

$\Rightarrow\ g^2 + \frac{9}{4} + 3g + f^2 + 9 - 6f = \left(\frac{13}{2}\right)^2$

$\Rightarrow g^2 + f^2 + 3g - 6f = 31$

Hence, locus of the centre is

$x^2 + y^2 + 3x - 6y - 31 = 0$

296 **(a)**

Given, $x + 1 = \sec t,\ \frac{y-2}{3} = \tan t$

Since $\sec^2 t - \tan^2 = 1$

$\therefore\ \frac{(x+1)^2}{1} - \frac{(y-2)^2}{9} = 1$

Now, $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{9}{1}} = \sqrt{10}$

$\therefore$ Foci $= (-1 \pm ae, 2)$

$= \left(-1-\sqrt{10},2\right)$ and $\left(-1+\sqrt{10},2\right)$

297 **(b)**

We have,

$PA + PB = 4$

$\Rightarrow P$ lies on the ellipse having its foci at A and B and length of the major axis $= 4$

298 **(c)**

It is given that the vertices of an ellipse are at $A'(-12,4)6$ and $A(14,4)$. So, its centre is at (1,4) and

$2a =$ Length of major axis $= 26 \Rightarrow a = 13$

Clearly, major axis is parallel to x-axis

$\therefore b^2 = a^2(1-e^2) = 169\left(1 - \frac{144}{169}\right) = 25$

Hence, the equation of the ellipse is $\frac{(x-1)^2}{169} + \frac{(y-4)^2}{25} = 1$

299 **(c)**

Let $L\left(ae, \frac{b^2}{a}\right)$ be an end of latusrectum

The equation of normal at L is

$\frac{a}{e}x - ay = a^2 - b^2$ or, $\frac{a}{e}x - ay = a^2e^2$

It cuts major axis at $G(ae^3, 0)$

$\therefore CG = ae^3$

300 **(a)**

We know that the locus of the point of intersection of perpendicular tangents to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is a circle $x^2 + y^2 = a^2 - b^2$.

Thus, locus of the point of intersection of perpendicular tangents to the hyperbola $\frac{x^2}{3} - \frac{y^2}{1} = 1$ is a circle

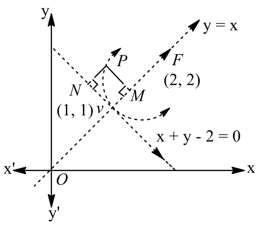
$x^2 + y^2 = 3 - 1$

$\Rightarrow\ x^2 + y^2 = 2$

301 **(b)**

The equation of the ellipse is
$25(x^2 - 6x) + 16(y^2) = 175$
$\Rightarrow 25(x-3)^2 + 16(y-0)^2 = 400$
$\Rightarrow \frac{(x-3)^2}{16} + \frac{(y-0)^2}{25} = 1$
The major axis of this ellipse is on a line parallel to y-axis i.e. $x = 3$. Therefore, its eccentricity e is given by
$e = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

302 **(a)**
As distance of vertex from origin is $\sqrt{2}$ and focus is $2\sqrt{2}$



$\therefore V(1,1)$ and $F(2,2)$ (ie, lying on $y = x$)

Length of latusrectum $= 4a = 4\sqrt{2}$ [where $a = \sqrt{2}$]

$\therefore$ By definition of parabola

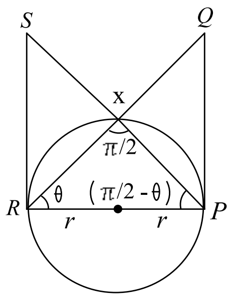
$PM^2 = (4a)(PN)$

where, PN is length of perpendicular upon $x + y - 2 = 0$ (ie, tangent at vertex)

$\Rightarrow \frac{(x-y)^2}{2} = 4\sqrt{2}\left(\frac{x+y-2}{\sqrt{2}}\right)$

$\Rightarrow (x-y)^2 = 8(x+y-2)$

303 **(a)**
Let RS and PQ are the tangents at the extremities of diameter of circle



In $\Delta RQP, \tan\theta = \frac{PQ}{PR} = \frac{PQ}{2r}$ ... (i)
Also, in ΔSRP,
$\tan\left(\frac{\pi}{2} - \theta\right) = \frac{RS}{RP} = \frac{RS}{2r}$
$\Rightarrow \cot\theta = \frac{RS}{2r}$ ... (ii)
From Eqs. (i) and (ii), we get
$\tan\theta \,.\, \cot\theta = \frac{PQ.RS}{4r^2}$
$\Rightarrow 4r = PQ.RS$
$\Rightarrow 2r = \sqrt{(PQ)(RS)}$

304 **(a)**
Since the line passing through the focus and perpendicular to the directrix is x-axis. Therefore, axis of the required parabola is x-axis. Let the coordinates of the focus S be $(a, 0)$.
Since the vertex is the mid point of the line joining the focus and the point $(-5,0)$ where the directrix $x + 5 = 0$ meets the axis.
$\therefore -3 = \frac{a-5}{2} \Rightarrow a = -1$
Thus, the coordinates of the focus are $(-1,0)$.
Let $P(x, y)$ be a point on the parabola. then, by definition, we have
$\sqrt{(x+1)^2 + y^2} = (x+5)^2 \Rightarrow y^2 = 8(x+3)$

305 **(c)**
Given equation of parabola is rewritten as
$169\{(x-1)^2 + (y-3)^2\}$
$= (13)^2\left\{\left(\frac{5x - 12y + 17}{13}\right)^2\right\}$
$\Rightarrow (x-1)^2 + (y-3)^2 \left(\frac{5x - 12y + 17}{13}\right)^2$
$\Rightarrow SP = PM$
$\therefore$ Focus is $(1, 3)$ and equation of directrix is
$5x - 12y + 17 = 0$
The distance of the focus from directrix $=$
$\left|\frac{5 - 36 + 17}{\sqrt{25 + 144}}\right| = \frac{14}{13}$
$\therefore$ Length of latusrectum $= 2 \times \frac{14}{13} = \frac{28}{13}$

306 **(a)**
Since the length of the subtangent at a point on the parabola is twice the abscissa of the point and the length of the subnormal is equal to semi-latusrectum. Therefore, if $P(x, y)$ is the required point, then
$2x = 2a \Rightarrow x = a$
Since (x, y) lies on the parabola $y^2 = 4ax$
$\therefore y^2 = 4ax$
$\Rightarrow 4a^2 = y^2 \Rightarrow y = \pm 2a$
Thus, the required points are $(a, 2a)$ and $(a, -2a)$

307 **(b)**

Normal at $\left(ae, \frac{b^2}{a}\right)$ of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

$$\frac{x-ae}{\frac{ae}{a^2}}=\frac{y-\frac{b^2}{a}}{\left(\frac{b^2}{a}/b^2\right)}$$

Since, it passes through $(0,-b)$, then

$$\frac{0-ae}{\frac{ae}{a^2}}=\frac{-b-\frac{b^2}{a}}{\frac{1}{a}}$$

$$\Rightarrow -a^2=-a\left(b+\frac{b^2}{a}\right)$$

$\Rightarrow a^2=ab+b^2$

$\Rightarrow a^2=ab+a^2-a^2e^2$ $(\because b^2=a^2-a^2e^2)$

$\Rightarrow b=ae^2$

$\Rightarrow b^2=a^2e^4$

$\Rightarrow a^2(1-e^2)=a^2e^4$

$\Rightarrow 1-e^2=e^4$

$\Rightarrow e^2(e^2+1)=1$

308 **(b)**

Let (α,β) be the pole of the given straight line with respect to the circle $x^2+y^2=a^2$.then, the equation of the polar is

$\alpha\, x+\beta\, y-a^2=0$ ...(i)

It is given that (α,β) lies on the circle $x^2+y^2=9\,a^2$

$\therefore \alpha^2+\beta^2=9\,a^2$ ...(ii)

Since line in (i) touches the circle $x^2+y^2=r^2$

$$\therefore \left|\frac{-a^2}{\sqrt{\alpha^2+\beta^2}}\right|=r \Rightarrow \frac{a^2}{\sqrt{9\,a^2}}=r \Rightarrow 9\,r^2=a^2$$

309 **(c)**

Given equation can be rewritten as

$(x+2)^2=-2(y-2)$

Equation of latusrectum is

$$y-2=-\frac{1}{2} \Rightarrow y=\frac{3}{2} \Rightarrow 2y=3$$

311 **(a)**

Given, $y=mx+\frac{25\sqrt{3}}{3}$ ... (i)

and $\frac{x^2}{16}+\frac{y^2}{9}=1$ ... (ii)

Here, Eq. (i) is normal to Eq. (ii), then

$$\frac{(a^2+b^2)^2}{c^2}=\frac{a^2}{m^2}-\frac{b^2}{1}$$

$$\Rightarrow \frac{(16+9)^2\times 9}{625\times 3}=\frac{16}{m^2}-\frac{9}{1}$$

$$\Rightarrow \frac{16}{m^2}=12 \Rightarrow m=\pm\frac{2}{\sqrt{3}}$$

$$\left[\begin{array}{c}\therefore \text{condition for } lx+my+n \\ =0 \text{ to be a hyperbola is } \frac{a^2}{l^2}-\frac{b^2}{m^2}=\frac{(a^2+b^2)^2}{n^2}\end{array}\right]$$

312 **(c)**

Given equation can be rewritten as

$$\Rightarrow \frac{(x-1)^2}{4}+\frac{(y-1)^2}{9}=1$$

Also $e=\sqrt{1-\frac{4}{9}}=\frac{\sqrt{5}}{3}$ $[\because a<b]$

$\therefore$ Equations of latusrectum are

$y-1=\pm 3.\frac{\sqrt{5}}{3}$ [using $y=\pm be$]

$\Rightarrow y=1\pm\sqrt{5}$

313 **(a)**

The equations of the circles are

$$x^2+y^2+\frac{\lambda}{2}x-\left(\frac{1+\lambda^2}{2}\right)y-5=0 \quad \text{... (i)}$$

And,

$x^2+y^2+4x+6y+3=0$ ... (ii)

These circles will be orthogonal, if

$2(g_1g_2+f_1f_2)=c_1+c_2$

$$\Rightarrow 2\left\{2\times\frac{\lambda}{4}+3\times\left(\frac{1+\lambda^2}{-4}\right)\right\}=-5+3$$

$$\Rightarrow \lambda-\frac{3}{2}(1+\lambda^2)=-2$$

$\Rightarrow 2\,\lambda-3-3\,\lambda^2=-4 \Rightarrow 3\,\lambda^2-2\,\lambda-1=0 \Rightarrow \lambda=1,-1/3$

Hence, there are two circles

314 **(d)**

The equation of the hyperbola $x^2-y^2=a^2$ referred to its asymptotes as the coordinates axes is $xy=\frac{a^2}{2}$

Comparing $xy=32$ with $xy=\frac{a^2}{2}$, we get $a=8$

$\therefore$Length of semi-transverse axis = 8

315 **(c)**

The equation of a normal to the parabola $y^2=24x$ is

$y=mx-12\,m-6\,m^3$,

Where m is the slope of the normal

But, it is parallel to $y=2\,x+3$. Therefore, $m=2$

Thus, the equation of the parallel normal is

$y = 2x - 24 - 48 \Rightarrow y = 2x - 72$

The distance $'d'$ between $y = 2x + 3$ and $y = 2x - 72$ is given by

$d = \left|\frac{72+3}{\sqrt{4+1}}\right| = 15\sqrt{5}$

316 **(d)**

We have,

Area of $\Delta SPS' = \frac{1}{2}$(Base × Height)

$\Rightarrow$ Area of $\Delta SPS' = \frac{1}{2}(2ae) \times \beta$

$\Rightarrow$ Area of $\Delta SPS' = ae\,\beta$

$= ae \times \frac{b}{a}\sqrt{a^2 - \alpha^2} \left[\because \frac{\alpha^2}{a^2} + \frac{\beta^2}{b^2} = 1\right]$

$\Rightarrow$ Area of $\Delta SPS' = be\sqrt{a^2 - \alpha^2}$

317 **(c)**

Let the equation of the circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$

Since, this passes through (1, 2)

$\therefore\ 1^2 + 2^2 + 2g(1) + 2f(2) + c = 0$

$\Rightarrow 5 + 2g + 4f + c = 0$ ...(i)

Also, the circle $x^2 + y^2 = 4$ intersects the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ orthologonally

$\therefore\ 2(g.0 + f.0) = c - 4 \Rightarrow c = 4$

On putting the value of c in Eq. (i), we get

$2g + 4f + 9 = 0$

Hence, the locus of centre $(-g, -f)$ is

$-2x - 4y + 9 = 0 \Rightarrow 2x + 4y - 9 = 0$

318 **(c)**

We have,

$\lambda L_2 L_3 + \mu L_3 L_1 + v L_1 L_2 = 0$

$\Rightarrow \lambda(y - m_2x - c_2)(y - m_3x - c_3)$

$+\mu(y - m_3x - c_3)(y - m_1x - c_1)$

$+v(y - m_1x - c_1)(y - m_2x - c_2) = 0$

This equation will represent a circle, if

Coefficient of x^2 = Coefficient of y^2 and

Coefficient of $xy = 0$

$\Rightarrow \lambda(m_2m_3 - 1) + \mu(m_3m_1 - 1) + v(m_1m_2 - 1) = 0$

and,

$\lambda(m_2 + m_3) + \mu(m_3 + m_1) + v(m_1 + m_2) = 0$

319 **(a)**

Since, B and C are the ends of diameter as $\angle BAC$ is 90°

$\therefore$ Equation of circle is

$x(x-1) + y(y-1) = 0$

$\Rightarrow x^2 + y^2 - x - y = 0$

Now, point D satisfies this equation

$\Rightarrow 4a^2 + 9a^2 - 5a = 0$

$\Rightarrow a(13a - 5) = 0$

$\Rightarrow a = 0, a = \frac{5}{13}$

320 **(d)**

The centres of given circles are $C_1(3, 1)$ and $C_2(-1, 4)$ and corresponding radii are

$r_1 = \sqrt{3^2 + 1^2 - 1} = 3$

and $r_2 = \sqrt{(-1)^2 + 4^2 - 13} = 2$

Now, $C_1C_2 = \sqrt{(-1-3)^2 + (4-1)^2} = 5$

$\therefore\ C_1C_2 = r_1 + r_2$

Hence, two circles touch externally

321 **(b)**

Given equation can be rewritten as

$9(x-1)^2 + 5(y-2)^2 = 45$

$\Rightarrow \frac{(x-1)^2}{5} + \frac{(y-2)^2}{9} = 1$

$\therefore$ Eccentricity, $e = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{5}{9}} = \frac{2}{3}$

322 **(d)**

Given that equation of parabola is $y^2 = 8x$

$\Rightarrow a = 2$

We know, if the normal at point $(at_1^2, 2at_1)$ is passing through the point on the parabola $(at_2^2, 2at_2)$, then $t_2 = -t_1 - \frac{2}{t_1}$

Given point is (2, 4)

$\Rightarrow at_1^2 = 2$

$\Rightarrow t_1 = 1$

$\therefore t_2 = -1 - \frac{2}{1} = -3$

The other end will be $(at_2^2, 2at_2) ie$, (18, −12)

323 **(d)**

The normal to a circle passes through the centre of the circle and centre of circles in (a) and (d) satisfy the equation of the normal.

But, the point $\left(3 + \frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right)$ does not lie on circle given in option (a)

Hence, the required circle is as given in option (d)

324 **(b)**

Given equation of curve is $3x^2 - 4y^2 = 72$

Since, the points (6, 3) and (6-3) lies on the curve.

At point (6, 3)

$d_1 = \frac{3(6) + 2(3) - 1}{\sqrt{3^3 + 2^2}} = \frac{23}{\sqrt{13}}$

At point (6 − 3)

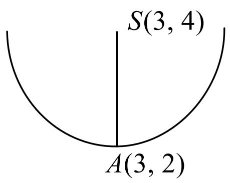
$$d_2 = \frac{3(6)+2(-3)-1}{\sqrt{3^3+2^2}} = \frac{11}{\sqrt{13}}$$

Here, d_2 is minimum.

Hence, the point $(6, -3)$ is on the curve which is nearest to the given line

325 **(a)**

The equation of such mirror is an equation of the parabola whose axis is y-axis and whose focus is $(0, 0)$



∴ Required equation is $x^2 = 4a(y + a)$

326 **(d)**

The centres and radii of given circles are $C_1(0,0), r_1 = 4$ and $C_2(0,1)$,

$r_2 = \sqrt{0+1} = 1$

Now, $C_1C_2 = \sqrt{0 + (0-1)^2} = 1$

and $r_1 - r_2 = 4 - 1 = 3$ ∴ $C_1C_2 < r_1 - r_2$

Hence second circle lies inside the first circle, so no common tangent is possible

327 **(c)**

The equation of any tangent to the parabola $y^2 = 4\,ax$ in terms of its slope m is $y = mx + \frac{a}{m}$ and the coordinates of the point of contact are $(a/m^2, 2a/m)$

Therefore, the equation of any tangent to $y^2 = ax$ is

$$y = mx + \frac{a}{4\,m}$$

and the coordinates of the point of contact are $\left(\frac{a}{4\,m^2}, \frac{a}{2\,m}\right)$

It is given that $m = \tan 45° = 1$

So, the coordinates of the point of contact are $(a/4, a/2)$

328 **(b)**

Given, eccentricity, $e = \frac{4}{3}$

Distance between foci $= 4 = 2\,ae \Rightarrow a^2 = \frac{9}{4}$

∴ $b^2 = a^2(e^2 - 1) = \frac{9}{4}\left(\frac{16}{9} - 1\right) = \frac{7}{4}$ and centre is $(0, 4)$.

∴ Equation of hyperbola is $\frac{x^2}{9} - \frac{(y-4)^2}{7} = \frac{1}{4}$

329 **(a)**

It is given that the circle with PQ as a diameter passes through the origin. This means that $\angle POQ = 90°$ i.e. the lines joining the origin to the points of intersection of $ax^2 + 2hxy + by^2 = 1$ and $lx + my + n = 0$ are at right angle.

The combined equation of OP and OQ is given by

$$ax^2 + 2hxy + by^2 = \left(\frac{lx + my}{-n}\right)^2$$

This represents a pair of perpendicular lines

∴ Coeff. of x^2 + Coeff. of $y^2 = 0$

$\Rightarrow an^2 - l^2 + bn^2 - m^2 = 0 \Rightarrow l^2 + m^2 = (a + b)n^2$

330 **(b)**

The coordinates of the centre of given circle are$(6, -2)$. Clearly the line $x + 3y = 0$ passes through this point. Hence, $x + 3y = 0$ is a diameter of the given circle.

331 **(c)**

The given equation of parabola is

$x^2 - 4x - 8y + 12 = 0$

$\Rightarrow x^2 - 4x = 8y - 12$

$\Rightarrow x^2 - 4x + 4 = 8y - 12 + 4$

$\Rightarrow (x-2)^2 = 8(y-1)$

∴ The length of latusrectum $= 4a = 8$

332 **(c)**

If the coordinates of a point on the parabola $y^2 = 4\,ax$ are $P(x, y)$, then its focal distance is $SP = x + a$.

Here, a = 2 and $SP = 4$

∴ $4 = x + 2 \Rightarrow x = 2$

∴ $y^2 = 8x \Rightarrow y^2 = 8 \times 2 \Rightarrow y = \pm 4$

Thus, the coordinates of the required point are $(2, \pm 4)$

333 **(a)**

Given equation can be rewritten as

$$36\left(x^2 - x + \frac{1}{4}\right) + 144\left(y^2 - \frac{2}{3}y + \frac{1}{9}\right) = 144$$

$$\Rightarrow \frac{\left(x - \frac{1}{2}\right)^2}{4} + \frac{\left(y - \frac{1}{3}\right)^2}{1} = 1$$

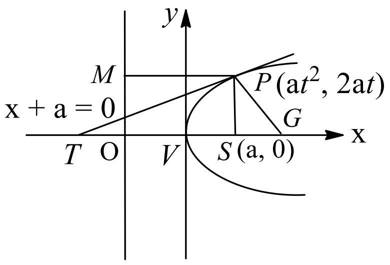
$$\therefore\; e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$$

334 **(c)**

Let $P(at^2, 2at)$ be any point on the parabola $y^2 = 4ax$, then equation of tangent and normal at $P(at^2, 2at)$ are $ty = x + at^2$ and $y = -tx + 2at + at^3$ respectively

Since, tangent and normal meet its axis at T and G

∴ Coordinates of T and G are $(-at^2, 0)$ and $(2a + at^2, 0)$ respectively



From definition of parabola

$SP = PM = a + at^2$

Now, $SG = VG - VS = 2a + at^2 - a = a + at^2$

And $ST = VS + VT = a + at^2$

Hence, $SP = SG = ST$

335 **(b)**

The coordinates of the points of contact of tangents of slope m to the hyperbola $x^2 - y^2 = a^2$ are

$$\left(\pm\frac{am}{\sqrt{m^2-1}}, \pm\frac{a}{\sqrt{m^2-1}}\right)$$

Here, we have $a = \sqrt{3}$ and $m = -2$

So, the required points are $(-2,1)$ and $(2,-1)$

336 **(b)**

Given, equation of ellipse is $\frac{x^2}{5/4} + \frac{y^2}{5/3} = 1$

Here, $a^2 = \frac{5}{4}, b^2 = \frac{5}{3}$

Given, line is $y = 3x + 7$, whose slope is 3, therefore slopes of the parallel line is also 3

Now, equations of tangent are

$$\Rightarrow\ y = mx \pm \sqrt{a^2m^2 + b^2}$$

$$\Rightarrow y = 3x \pm \sqrt{\frac{5}{4}(3)^2 + \frac{5}{3}}$$

$$\Rightarrow\ y = 3x \pm \sqrt{\frac{155}{12}}$$

337 **(c)**

Equation of tangent with slope $-\frac{3}{4}$ is

$$y = -\frac{3}{4}x + c$$

According to condition of tangency

$$c = \sqrt{32 \times \left(\frac{-3}{4}\right)^2 + 18}$$

$$= \sqrt{18 + 18} = 6$$

$$\therefore\ y = -\frac{3}{4}x + 6$$

$$\Rightarrow\ 4y + 3x = 24$$

It meets the coordinate axes in A and B

$\therefore A \equiv (8,0)$ and $B \equiv (0,6)$

Required area $= \frac{1}{2} \times 8 \times 6 = 24$ sq unit

338 **(b)**

Given equation of line is $lx + my + n = 0$ or $y = -\frac{lx}{m} - \frac{n}{m}$ and equation of parabola $y^2 = 4ax$

Condition for tangency

$$\left(-\frac{n}{m}\right) = \frac{a}{-l/m}$$

$$\Rightarrow\ nl = am^2$$

339 **(c)**

Given lines is $y = -3x - k$

And equation of circle is $x^2 + y^2 = 10$

Here, $a^2 = 10, m = -3, c = -k$

For tangency, $c^2 = a^2(1 + m^2)$

For tangency, $c^2 = a^2(1 + m^2)$

$\Rightarrow\ k^2 = 10(1 + 9)\ \Rightarrow k = \pm 10$

340 **(d)**

Let $P(a\sec\theta, a\tan\theta)$ be a point on the hyperbola $x^2 - y^2 = a^2$. The equation of tangent at P is

$x\sec\theta - y\tan\theta = a$

The coordinates of the vertices of triangle formed by the above tangent and the lines $x + y = 0$ and $x - y = 0$ are

$O(0,0), A\big(a(\sec\theta + \tan\theta), a(\sec\theta + \tan\theta)\big)$

and $B\big(a(\sec\theta - \tan\theta), -a(\sec\theta - \tan\theta)\big)$

Clearly, ΔAOB is right angled at O

$\therefore$ Area of $\Delta\ AOB = \frac{1}{2}OA \times OB$

$\Rightarrow$ Area of $\Delta\ AOB$

$$= \frac{1}{2} \times a\sqrt{2}(\sec\theta + \tan\theta) \times a\sqrt{2}(\sec\theta - \tan\theta)$$

$\Rightarrow$ Area of $\Delta AOB = a^2$ sq. units

341 **(a)**

The centre of circle is $(2, 4)$

Radius$= \sqrt{4 + 16 + 5} = 5$

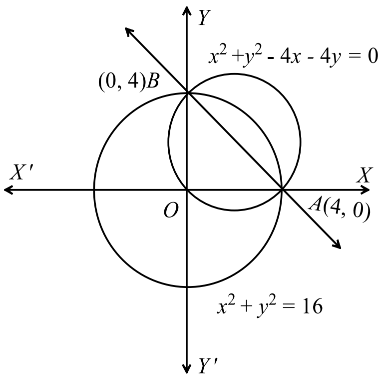
$\because$ Perpendicular distance of $3x - 4y - \lambda = 0$ from $(2, 4)$ is equal to the readius of circle

$\therefore \quad \left|\dfrac{6-16-\lambda}{\sqrt{9+16}}\right| = 5$

$\Rightarrow \; -10-\lambda = \pm 25 \quad \lambda = -35, 15$

342 **(d)**

The equation of the common chord of the circles $x^2+y^2-4x-4y=0$ and $x^2+y^2=16$ is $x+y=4$ which meets $x^2+y^2=16$ at $A(4,0)$ and $B(-4,0)$



Obviously $OA \perp OB$. Hence the common chord AB makes a right angle at the centre of the circle $x^2+y^2=16$

343 **(d)**

Given the distance between the foci $= 2ae = 16$ and eccentricity of ellipse $(e) = \frac{1}{2}$

$\therefore$ Length of the major axis of the ellipse

$= 2a = \dfrac{2ae}{e} = \dfrac{16}{\frac{1}{2}} = 32$

344 **(a)**

The centre of given circles are $C_1(0,0), C_2(-3,1)$ and $C_3(6,-2)$

Now, $\begin{vmatrix} 0 & 0 & 1 \\ -3 & 1 & 1 \\ 6 & -2 & 1 \end{vmatrix} = 1(6-6) = 0$

Hence, centres are collinear

345 **(a)**

Normal at the extremity of latusrectum in the first quadrant $(ae, b^2/a)$ is

$\dfrac{x-ae}{ae/a^2} = \dfrac{y-b^2/a}{b^2/ab^2}$

As it passes through $(0,-b)$

$\dfrac{-ae}{ae/a^2} = \dfrac{-b-b^2/a}{1/a}$

$\Rightarrow -a^2 = -ab-b^2$

$\Rightarrow a^2-b^2 = ab$

$\Rightarrow a^2e^2 = ab$

Or $e^2 = b/a$

$\therefore e^4 = \dfrac{b^2}{a^2} = 1-e^2$

$\Rightarrow e^4+e^2 = 1$

347 **(b)**

Here, $a^2 = -\frac{1}{4}, b^2, = \frac{1}{9}, m = \frac{8}{9}$

$\therefore$ Point of contact is

$\left(\pm\dfrac{a^2m}{\sqrt{a^2m^2+b^2}}, \mp\dfrac{b^2}{\sqrt{a^2m^2+b^2}}\right)$

$= \left(\pm\dfrac{\frac{1}{4}.\frac{8}{9}}{\sqrt{\frac{1}{4}\times\frac{64}{81}+\frac{1}{9}}}, \pm\dfrac{\frac{1}{9}}{\sqrt{\frac{1}{4}\times\frac{64}{81}+\frac{1}{9}}}\right)$

$= \left(\pm\dfrac{2}{5}, \pm\dfrac{1}{5}\right)$

348 **(a)**

Given equation is $ax^2+2bx+c=0$. Since, roots are not real

$\therefore \; b^2 < ac$

$\Rightarrow ax^2+2bxy+cy^2+dx+ey+f=0$

Can represent an ellipse

349 **(a)**

Given vertices are $(5,0), (-5,)$

$\therefore a = 5$

Also, one of the directrix let $x = \frac{a}{e}$ is

Given as $x = \frac{25}{7} \Rightarrow \; e = \frac{7}{5}$

$\therefore b^2 = a^2(e^2-1) = 25\left(\dfrac{49}{25}-1\right) = 24$

Equation hyperbola is $\frac{x^2}{25} - \frac{y^2}{24} = 1$

350 **(a)**

Given equation of ellipse is $\frac{x^2}{4} + \frac{y^2}{\frac{7}{4}} = 1$

Here, $a^2 = 4, b^2 = \frac{7}{4}$

$\therefore b^2 = a^2(1-e^2)$

$\Rightarrow \dfrac{7}{4} = 4(1-e^2)$

$\Rightarrow e^2 = 1 - \dfrac{7}{16} = \dfrac{9}{16}$

$\Rightarrow e = \dfrac{3}{4}$

Thus, the foci are $\left(\pm\frac{3}{2}, 0\right)$

The radius of required circle $=$

$\sqrt{\left(\frac{3}{2}-\frac{1}{2}\right)^2 + (2-0)^2}$

$= \sqrt{1+4} = \sqrt{5}$

351 **(d)**

Given hyperbola is $\frac{x^2}{9} - \frac{y^2}{4} = 1$

$$\therefore\ e = \sqrt{\frac{a^2 + b^2}{a^2}} = \sqrt{\frac{9+4}{9}} = \frac{\sqrt{13}}{3}$$

$\therefore$ Directrices are $x = -\frac{9}{\sqrt{13}}$ and $x = \frac{9}{\sqrt{13}}$

352 **(c)**

The intersection point of line $y = 7x - 25$ and circle $x^2 + y^2 = 25$ is $x^2 + (7x - 25)^2 = 25$

$\Rightarrow\ \ 50x^2 - 350x + 600 = 0$

$\Rightarrow\ \ (x-3)(x-4) = 0$

$\Rightarrow\ \ x = 3, \qquad x = 4\ \ \Rightarrow y = -4, 3$

$\therefore$ Coordinates of $A(3, -4)$ and $B(4, 3)$

$\therefore$ Distance between A and B =

$\sqrt{(4-3)^2 + (3+4)^2}$

$= 5\sqrt{2}$

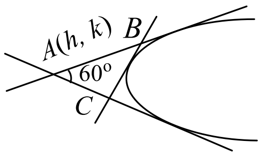
Alternate Required distance= $2\sqrt{\frac{a^2(1+m^2)-c^2}{1+m^2}}$

$$= 2\sqrt{\frac{25(1+49) - 625}{1+49}} = 5\sqrt{2}$$

353 **(c)**

Equation of any tangent to the parabola is

$y = mx + \frac{a}{m}$



It passes through $A(h, k)$

$\therefore k = mh + \frac{a}{m}$

$\Rightarrow m^2h - mk + a = 0$

Let m_1 and m_2 be the roots

$\Rightarrow m_1 + m_2 = \frac{k}{h}, m_1m_2 = \frac{a}{h}$

$\therefore \tan 60° = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$

$$\Rightarrow 3 = \frac{(m_1 - m_2)^2}{(1 + m_1m_2)^2} \Rightarrow 3 = \frac{\frac{k^2}{h^2} - \frac{4a}{h}}{\left(1 + \frac{a}{h}\right)^2}$$

$\Rightarrow 3(h + a)^2 = k^2 - 4ah$

$\therefore$ Locus of a point is

$y^2 = 3(x + a)^2 + 4ax$

354 **(a)**

The equation of a tangent to $x^2 + 4y^2 = 4$ at $(2\cos\theta, \sin\theta)$ is

$2x\cos\theta + 4y\sin\theta = 4$ or, $x\cos\theta + 2y\sin\theta = 2$ ... (i)

This cuts the ellipse $x^2 + 2y^2 = 6$ at P and Q

Let $R(h, k)$ be the point of intersection of tangents at P and Q.

Then, PQ is the chord of contact of tangents drawn from $R(h, k)$ to the ellipse $x^2 + 2y^2 = 6$.

Therefore, the equation of PQ is

$hx + 2ky = 6$ ...(ii)

Clearly, (i) and (ii) represent the same line

$$\therefore \frac{h}{\cos\theta} = \frac{2k}{2\sin\theta} = \frac{6}{2}$$

$\Rightarrow h = 3\cos\theta, k = 3\sin\theta$

$\Rightarrow h^2 + k^2 = 9$

$\Rightarrow (h, k)$ lies on $x^2 + y^2 = 9$, which is the director of circle of the ellipse $x^2 + 2y^2 = 6$

Hence, the angle between the tangents is a right angle

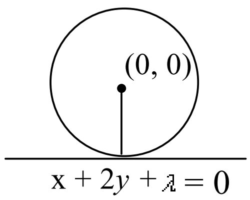
355 **(c)**

Centre of circle is (0, 0)

Equation of tangent which is parallel to $x + 2y + 3 = 0$ is

$x + 2y + \lambda = 0$ ...(i)

As we know perpendicular distance from centre (0, 0) to $x + 2y + \lambda = 0$ should be equal to radius



$$\therefore \frac{0 + 2\times 0 + \lambda}{\sqrt{1^2 + 2^2}} = \pm 2$$

$\Rightarrow\ \lambda = \pm 2\sqrt{5}$

On putting the value of λ in Eq. (i), we get

$x + 2y = \pm 2\sqrt{5}$

Which represents the required equation of tangents

357 **(b)**

Let (h, k) be the pole. Then, the equation of the polar is

$\frac{hx}{a^2} + \frac{ky}{b^2} = 1$

It is at a distance d from the centre $C(0,0)$ of the ellipse

$$\therefore \left|\frac{1}{\sqrt{\frac{h^2}{a^4} + \frac{k^2}{b^4}}}\right| = d \Rightarrow \frac{h^2}{a^4} + \frac{k^2}{b^4} = \frac{1}{d^2}$$

Hence, the locus of (h, k) is $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{d^2}$

358 **(a)**

The equation of the normal to $x^2 = 4\,ay$ is of the form $x = my - 2\,am - am^3$. Therefore, $k =$

$-2\ am - am^3$

359 **(c)**

If $y = 2x + \lambda$ is tangent to given hyperbola,

then $\lambda = \pm\sqrt{a^2m^2 - b^2}$

$= \pm\sqrt{(100)(4) - 144} = \pm 16$ [$\because a^2 = 100, b^2 = 14$]

360 **(d)**

The centres and radii of the circles are:

Centres : $C_1(1/2, 0)$ $C_2(-1/2, 0)$

Radii : $r_1 = \frac{1}{2}$ $r_2 = \frac{1}{2}$

Clearly, $C_1C_2 = r_1 + r_2$

Therefore, the circles touch each other externally

Hence, there are 3 common tangents

361 **(b)**

The circle passes through (0,0), (3,0) and (0,4).

So, its equation is $x^2 + y^2 - 3\,x - 4\,y = 0$

362 **(c)**

If the straight line $y = mx + c$ cuts the circle $x^2 + y^2 = a^2$ in real points, then the equation

$x^2 + (mx + c)^2 = a^2$ must have real roots

i.e. $x^2(1 + m^2) + 2\,mcx + c^2 - a^2 = 0$ must have real roots

$\Rightarrow 4m^2c^2 - 4(1 + m^2)(c^2 - a^2) \geq$

$\Rightarrow -c^2 + a^2(1 + m^2) \geq 0$

$\Rightarrow a^2(1 + m^2) \geq c^2 \Rightarrow \sqrt{a^2(1 + m^2)} \geq c$

363 **(d)**

Given, $\frac{x^2}{9} - \frac{y^2}{4} = 1$ ... (i)

The equation of the chord of contact of tangents from (x_1, y_1) and (x_2, y_2) to the given hyperbola are

$\frac{xx_1}{9} - \frac{yy_1}{4} = 1$ ... (ii)

and $\frac{xx_2}{9} - \frac{yy_2}{4} = 1$ ... (iii)

Since, lines (ii) and (iii) are at right angles

$\frac{4}{9} . \times \frac{x_1}{y_1} \times \frac{4}{9} . \frac{x_2}{y_2} = -1$

$\Rightarrow \frac{x_1x_2}{y_1y_2} = -\frac{81}{16}$

364 **(d)**

Centre and radius of given circle are $(-\lambda, 0)$ and $r = \sqrt{\lambda^2 - c}$

For limiting, point $r = 0, \lambda = \pm\sqrt{c}$

Thus, we get two limiting points of the given coaxial system as $(\pm\sqrt{c}, 0)$

For real and distinct $c > 0$

365 **(b)**

We have, $a^2 = \frac{1}{2}, b^2 = \frac{1}{3}, m = \frac{4}{3}$

The required points are

$$\left(\pm\frac{a^2m}{\sqrt{a^2m^2 + b^2}}, \pm\frac{b^2}{\sqrt{a^2m^2 + b^2}}\right)$$

$$= \left(\pm\frac{\frac{1}{2} \times \frac{4}{3}}{\sqrt{\frac{1}{2} \times \frac{16}{9} + \frac{1}{3}}}, \pm\frac{\frac{1}{3}}{\sqrt{\frac{1}{2} \times \frac{16}{9} + \frac{1}{3}}}\right)$$

$$= \left(\pm\frac{2}{\sqrt{11}}, \pm\frac{1}{\sqrt{11}}\right)$$

366 **(b)**

Let mid point be (h, k)

$\therefore$ Equation of chord is

$T = S_1$

$yy_1 - 2a(x + x_1) = y_1^2 - 4ax_1$

Since, it passes through origin

$\therefore\ -2ax_1 = y_1^2 - 4ax_1$

$\Rightarrow\ y_1^2 = 2ax_1$

$\therefore$ Locus is $y^2 = 2ax$

367 **(a)**

We have,

$\frac{2\,b^2}{a} = a \Rightarrow 2\,b^2 = a^2 \Rightarrow 2\,a^2(1 - e^2) = a^2 \Rightarrow e = \frac{1}{\sqrt{2}}$

368 **(a)**

Given equation of parabola is $y = x^2$ ...(i)

Equation of straight line is $y = 2x - 4$ ...(ii)

On solving Eqs. (i) and (ii), we get

$x^2 - 2x + 4 = 0$

Let $z = x^2 - 2x + 4$

$\therefore z' = 2x - 2$

For least value, $z' = 0 \Rightarrow 2x - 2 = 0 \Rightarrow x = 1$

z'' is positive at $x = 1$

$\therefore$ It is minimum, putting $x = 1$ in Eq. (i), we get $y = 1$

So, the required point at the least distance from the line is (1, 1)

370 **(b)**

The eccentricity of $\frac{x^2}{16} + \frac{y^2}{25} = 1$ is $e_1 = \sqrt{1 - \frac{16}{25}} =$

$\frac{3}{5}$

$\therefore\ e_2 = \frac{5}{3}\ (\because e_1 e_2 = 1)$

and foci of given ellipse $(0, \pm 3)$

$\therefore 2b = 3 + 3 = 6 \Rightarrow b = 3 \Rightarrow b^2 = 9$

$\Rightarrow a^2 = 16$

$\Rightarrow$equation of hyperbola is $\frac{x^2}{16} - \frac{y^2}{9} = -1$

Hence, (b) is the correct answer

371 **(a)**

Given , $y^2 = 16x$, then $a = 4$

Let line perpendicular to given line $y - 3x - 1 = 0$ is

$x + 3y = \lambda$

$\Rightarrow y = -\frac{1}{3}x + \frac{\lambda}{3}$

Here, $c = \frac{\lambda}{3}, m = -\frac{1}{3}$

$\therefore$ Condition of tangency is, $c = \frac{a}{m}$

$\Rightarrow \frac{\lambda}{3} = \frac{4}{-1/3} \Rightarrow \lambda = -36$

$\therefore$ Required tangent is $x + 3y + 36 = 0$

372 **(d)**

Since, $\frac{\sqrt{S_1}}{\sqrt{S_2}} = \frac{2}{3}$

$\therefore \frac{\sqrt{x_1^2 + y_1^2 + 4x_1 + 3}}{\sqrt{x_1^2 + y_1^2 - 6x_1 + 5}} = \frac{2}{3}$

$\Rightarrow 9x_1^2 + 9y_1^2 + 36x_1 + 27 - 4x_1^2 - 4y_1^2 + 24x_1 - 20 = 0$

$\Rightarrow 5x_1^2 + 5y_1^2 + 60x_1 + 7 = 0$

$\therefore$ Locus of point is

$5x^2 + 5y^2 + 60x + 7 = 0$

373 **(a)**

Centre of circle is $(1, -2)$

$\therefore$ Required equation of normal=equation of straight line passing through $(1, -2)$ and $(2, 1)$

$ie, y + 2 = \frac{-2-1}{1-2}(x - 1)$

$\Rightarrow y + 2 = 3x - 3$

$\Rightarrow 3x - y - 5 = 0$

374 **(a)**

Let x_1 and x are the roots of the equation

$x^2 + 2ax - b^2 = 0$

$\therefore x_1 + x_2 = -2a$ and $x_{1x_2} = -b^2$

Also, y_1 and y_2 are roots of the equation

$y^2 + 2py - q^2 = 0$

$\therefore\ y_1 + y_2 = -2p$ and $y_1 y_2 = -q^2$

The equation of the circle with $P(x_1, y_1)$ and $Q(x_2, y_2)$ as then end points of diameter is

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

$\Rightarrow x^2 + y^2 - x(x_1 + x_2) - y(y_1 + y_2) + x_1 x_2 + y_1 y_2 = 0$

$\Rightarrow x^2 + y^2 + 2ax + 2py - b^2 - q^2 = 0$

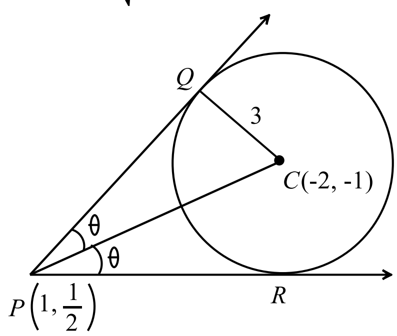
375 **(b)**

We have,

$x^2 + y^2 + 4x + 2y - 4 = 0$

$PQ =$ Length of the tangent drawn from $P(1, 1/2)$ to the circle (i)

$\Rightarrow PQ = \sqrt{1 + \frac{1}{4} + 4 + 1 - 4} = \frac{3}{2}$



In $\Delta\, CPQ$, we have,

$\tan\theta = \frac{CQ}{PQ} = \frac{3}{3/2} = 2$

$\therefore$ Required angle $= 2\,\theta = 2\tan^{-1} 2 = \sin^{-1}\frac{4}{5}$

376 **(b)**

Given , $a^2 = 25$ and $b^2 = 16$

$\therefore\ e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

So, the coordinates of foci S and S' are (3, 0) and $(-3, 0)$ respectively. Let $P(5\cos\theta, 4\sin\theta)$ be a variable point on the ellipse.

Then, $\Delta =$ area of $\Delta PSS' = \begin{vmatrix} 3 & 0 & 1 \\ -3 & 0 & 1 \\ 5\cos\theta & 4\sin\theta & 1 \end{vmatrix} = 12\sin\theta$

[Since, value of $\sin\theta$ lies between -1 and 1]

So, maximum value of area of $\Delta PSS'$ is 12

377 **(d)**

We have,

$3x^2+y^2=12 \Rightarrow \frac{x^2}{4}+\frac{y^2}{12}=1$

This is of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where $b^2>a^2$

$\therefore$ Length of the $L.R.=\frac{2a^2}{b}=\frac{2(4)}{\sqrt{12}}=\frac{4}{\sqrt{3}}$

378 **(a)**

Given equation can be rewritten as

$x(x-2y)-3(x-2y)=0 \Rightarrow x=3$ And $x=2y$ are two normals. Their intersection point is the centre $\left(3,\frac{3}{2}\right)$

379 **(c)**

The given equation of parabola is

$y=2x^2+x \Rightarrow x^2+\frac{x}{2}=\frac{y}{2}$

$\Rightarrow x^2+\frac{x}{2}+\frac{1}{16}=\frac{y}{2}+\frac{1}{16}$

$\Rightarrow \left(x+\frac{1}{4}\right)^2=\frac{1}{2}\left(y+\frac{1}{8}\right)$

It can be rewritten as $X^2=\frac{1}{2}Y$ ...(i)

Where $x+\frac{1}{4}=X$ and $y+\frac{1}{8}=Y$

On comparing with $X^2=4AY$, we get

$A=\frac{1}{8}$, focus of Eq. (i) is $\left(0,\frac{1}{8}\right)$ *ie*,

$X=0, Y=\frac{1}{8}$

$\Rightarrow x+\frac{1}{4}=0, y+\frac{1}{8}=\frac{1}{8}$

$\Rightarrow x=-\frac{1}{4}, y=0$

$\therefore$ Focus of given parabola is $\left(-\frac{1}{4},0\right)$

380 **(a)**

Let general equation of a circle is

$x^2+y^2+2gx+2fy+c=0$ ...(i)

If the circle (i) cuts orthogonally each of the given three circles

Then, condition is

$2g_1g_2=2f_1f_2=c_1+c_2$

Applying the condition one by one, we get

$2g+17f=c+4$ ...(ii)

$7g+6f=c+11$ ...(iii)

And $-g+22f=c+3$ ...(iv)

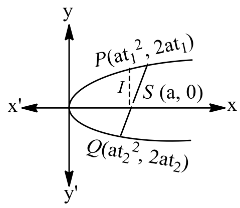
On solving Eqs. (ii), (iii) and (iv), we get

$g=-3, f=-2$

Therefore, the centre of the circle is (3, 2)

381 **(c)**

Let two points on the parabola are $p(at_1^2, 2at_1)$ and $Q\ (at_2^2, 2at_2)$



Now, $SP=\sqrt{(a-at_1^2)^2+(0-2at_1)^2}$

$=a+at_1^2$

$SQ=\sqrt{(a-at_2^2)^2+(0-2at_2)^2}$

$=a+at_2^2=a+\frac{a}{t_1^2} \quad (\because t_1t_2=-1)$

Now, $\frac{2\times SP\times SQ}{SP+SQ}=\frac{2\times a(1+t_1^2)\times a\left(1+\frac{1}{t_1^2}\right)}{(a+at_1^2)+\left(a+\frac{a}{t_1^2}\right)}$

$=\frac{2a\left(2+\frac{1}{t_1^2}+t_1^2\right)}{\left(2+\frac{1}{t_1^2}+t_1^2\right)}=2a=l$ (given)

Hence, SP, l, SQ are in HP

382 **(a)**

The point (1,2) lies on the circle $x^2+y^2=5$

Hence, there is only one tangent

383 **(a)**

In the given equation of hyperbola

$a=4$ and $b=3$

We know that the difference of focal distance of any point of the hyperbola $=2a$

$=2\times 4=8$

384 **(a)**

Since, the circle touching both the coordinates axes in fourth quadrant, so equation is

$(x-3)^2+(y+3)^2=3^2$

$\Rightarrow x^2+y^2-6x+6y+9=0$

385 **(a)**

The value of the parameter for the other end of the focal chord is $-1/t$.Therfore, the coordinates of the end points of the focal chord are $(at^2, 2\,at)$ and $\left(\frac{a}{t^2},-\frac{2a}{t}\right)$ and hence the length of the focal chord is

$\sqrt{\left(\frac{a}{t^2}-at^2\right)^2+\left(-\frac{2\,a}{t}-2\,at\right)^2}$

$=a\left(t+\frac{1}{t}\right)\sqrt{\left(t-\frac{1}{t}\right)^2+4}=a\left(t+\frac{1}{t}\right)^2$

386 **(b)**

The point $(2a, a-1)$ will lie in the interior of the larger segment of the circle $x^2+y^2=25$ cut off by $x^2+4y=0$, if it is in the interior of the circle and exterior of the parabola.

$\therefore 4a^2+(a-1)^2-25<0$ and $4a^2+4(a-1)>0$

$$\Rightarrow (5a-12)(a+2)<0 \text{ and } \left(a+\frac{1+\sqrt{5}}{2}\right)\left(a+\frac{1-\sqrt{5}}{2}\right)>0$$

$$\Rightarrow -2<a<\frac{12}{5} \text{ and } \left(a<\frac{-1-\sqrt{5}}{2} \text{ or, } a>\frac{\sqrt{5}-1}{2}\right)$$

$\Rightarrow a=1,2$

387 **(b)**

Let the equation of the circle be

$(x-a)^2+(y-a)^2=a^2, a>0$

It touches $4x+3y-12=0$

$\therefore \left|\frac{4a+3a-12}{5}\right|=a \Rightarrow 7a-12=5a \Rightarrow a=6$

388 **(c)**

Here, $a=4, b=5$ and $e=\sqrt{1-\frac{16}{25}}=\frac{3}{5}$

$\therefore$ Equation of directrix is $y=\pm\left(\frac{5}{3/5}\right)$

$\Rightarrow 3y=\pm 25$

389 **(a)**

Let the equation of the circle through (a, b) be

$x^2+y^2+2gx+2fy+c=0$ ...(i)

Where, $a^2+b^2+2ag+2fb+c=0$ ...(ii)

Since the circle $x^2+y^2=p^2$ cut the circle (i) orthogonally.

$\therefore 2g\times 0+2f\times 0=c-p^2 \Rightarrow c=p^2$ ...(iii)

Substituting the value of c in (ii), we obtain

$a^2+b^2+2ag+2fb+p^2=0$

Hence, the locus of $(-g,-f)$ is $a^2+b^2-2ax-2by+p^2=0$

390 **(d)**

Given that, foci are $(3,0)$ and $(-1,0)$ and $e=\frac{2}{3}$

$\therefore 2ae=4 \Rightarrow a=3$

Also, $e^2=1-\frac{b^2}{a^2}$

$\Rightarrow \frac{4}{9}=1-\frac{b^2}{9}$

$\Rightarrow b=\sqrt{5}$

Since, centre of the ellipse is the mid point of the line joining the two foci, therefore the coordinates of the centre are $(1, 0)$

$\therefore$ Equation of ellipse is

$$\frac{(x-1)^2}{9}+\frac{(y-0)^2}{5}=1$$

Hence, the parametric coordinates are $(1+3\cos\theta, \sqrt{5}\sin\theta)$

391 **(a)**

Now, taking option (a)

$r=2\sin\theta$

Let $x=r\cos\theta,\ y=r\sin\theta$

$\therefore\ r^2=2r\sin\theta$

$\Rightarrow\ x^2+y^2=2y$

Which represents a equation of circle

392 **(d)**

Required equation of chord is

$T=S_1$

$\Rightarrow -2x+3y-81=4+9-81$

$\Rightarrow 2x-3y=-13$

393 **(d)**

The angle of intersection of two circles is given by

$$\cos\theta=\frac{r_1^2+r_2^2-C_1C_2^2}{2\,r_1r_2}$$

Where r_1, r_2 are radii of two circles and C_1C_2 is the distance between their centres.

Here, $r_1=\sqrt{\frac{1}{4}+\frac{1}{4}}=\sqrt{\frac{1}{2}}=r_2$ and $C_1C_2=1$

$\therefore \cos\theta=0 \Rightarrow \theta=\frac{\pi}{2}$

394 **(c)**

Let θ be the eccentric angle of the point of contact. Then, the equation of the tangent is

$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$

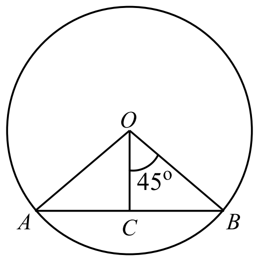
It is same as $\frac{x}{a}+\frac{y}{b}=\sqrt{2}$

$\therefore \cos\theta=\sin\theta=\frac{1}{\sqrt{2}} \Rightarrow \theta=\frac{\pi}{4}$

395 **(c)**

Let AB be the chord of length $\sqrt{2}$, O be the centre of the circle and let OC be the perpendicular from O on AB. Then,

$$AC=BC=\frac{\sqrt{2}}{2}=\frac{1}{\sqrt{2}}$$



$AB = \sqrt{2}$

In $\Delta\ OBC$, we have

$OB = BC \text{ cosec } 45° = \frac{1}{\sqrt{2}} \times \sqrt{2} = 1$

$\therefore$Area of the circle $= \pi(OB)^2 = \pi$. sq. units

396 **(d)**

Let $P(h, k)$ be a point. Then, the chord of contact of tangents from P to $y^2 = 4ax$ is

$ky = 2a(x + h)$ ...(i)

This touches the parabola $x^2 = 4by$. So, it should be of the form

$x = my + \frac{b}{m}$ ... (ii)

Equation (i) can be re-written as

$x = \frac{k}{2a}y - h$ ... (iii)

Since (ii) and (iii) represent the same line

$\therefore m = \frac{k}{2a}$ and $\frac{b}{m} = -h$

Eliminating m from these two equations, we get

$2ab = -hk$

Hence, the locus of $P(h, k)$ is $xy = -2ab$, which is a hyperbola

397 **(d)**

The equation of any normal to $y^2 = -8\,x$ is

$y = mx + 4\,m + 2\,m^2$ ... (i) [Using $y = mx - 2\,am - am^3$]

The equation of the given line is $2\,x + y + k = 0$

$\Rightarrow y = -2\,x - k$

Comparing (i) and (ii), we get

$m = -2$ and $k = -4\,m - 2\,m^3 \Rightarrow k = 8 + 16 = 24$

398 **(b)**

Let the equation of circle passing through origin is

$x^2 + y^2 + 2gx + 2fy = 0$

It also passes through (2, 1)

$\therefore\ 4 + 1 + 4g + 2f = 0$

$\Rightarrow\ 4g + 2f = -5$ ...(i)

Also, circle touches the line $y = x$

$\therefore$ Perpendicular from centre$(-g, -f)$ to the tangent=radius

$\Rightarrow\ \frac{|-f + g|}{\sqrt{1^2 + 1^2}} = \sqrt{g^2 + f^2} \Rightarrow\ f^2 + g^2 - 2fg$

$= 2(g^2 + f^2)$

$\Rightarrow\ (g + f)^2 = 0\ \Rightarrow g = -f$

$\therefore$ From Eq. (i), $4(-f) + 2f = -5$

$\Rightarrow\ f = \frac{5}{2}$ and $g = -\frac{5}{2}$

$\therefore\ x^2 + y^2 - 5x + 5y = 0$

On comparing with $x^2 + y^2 + px + qy = 0$

$\therefore\ p = -5, q = 5$

399 **(a)**

Since, $x + y - 1 = 0$ is a tangent to the parabola $y^2 - y + x = 0$, then the point of contact is (0, 1)

400 **(b)**

Let (h, k) be the coordinates of the centre of circle of which the given chord is the diameter. Then, (h, k) be mid point of the chord, so, its equation is $S' = T$

$h^2 + k^2 - 2ah = hx + ky - a(x + h)$

$\Rightarrow\ x(h - a) + ky = h^2 + k^2 - ah$

If it passes through (0, 0), therefore $h^2 + k^2 - ah = 0$ and the locus of (h, k) is $x^2 + y^2 - ax = 0$

401 **(b)**

The equations of the axes of the ellipse are $x + y - 2 = 0$ and $x - y = 0$. The centre of the given ellipse is the point of intersection of the axes $x + y - 2 = 0$ and $x - y = 0$ i.e. the point (1,1)

402 **(d)**

Equation of directrix of $(x - 1)^2 = 2(y - 2)$ is

$y - 2 = -\frac{1}{2}$

$\Rightarrow\ 2y - 3 = 0$

403 **(a)**

We have,

$y^2 = 2(x - 3) \Rightarrow (y - 0)^2 = 2(x - 3)$ ...(i)

As the equation of the parabola

The equation of the tangent is

$x - 2y - 1 = 0 \Rightarrow y = \frac{x}{2} - \frac{1}{2} \Rightarrow y - 0$

$= \frac{1}{2}(x - 3) + 1$

So, the coordinates of the point of contact are given by

$x - 3 = \frac{1/2}{(1/2)^2}, y - 0$

$= \frac{2 \times 1/2}{1/2}$ $\left[\text{Using} : x = \frac{a}{m^2} \text{ and } y = \frac{2a}{m}\right]$

$\Rightarrow x = 5$ and $y = 2$

404 **(b)**

Let $P(x, y)$ be any point on the hyperbola, then by definition, we have

$SP = e\,PM$

$\Rightarrow SP^2 = e^2\,PM^2$

$\Rightarrow (x-2)^2 + (y-1)^2 = 4\left|\frac{x+2\,y-1}{\sqrt{5}}\right|^2$

$\Rightarrow x^2 - 16\,xy - 11y^2 - 12\,x + 6\,y + 21 = 0$

This is the required equation of the hyperbola

405 **(c)**

Let the mid point be $P(h, k)$. Equation of this chord is

$T = S_1$ *ie*, $\quad ky - 2a(x+h) = k^2 - 4ah$

It must passes through $(a, 0)$

$(-2a)(a+h) = k^2 - 4ah$

Hence, the locus is $y^2 = 2ax - 2a^2$

406 **(c)**

The combined equation of the pair of tangents drawn from $(1, 2)$ to the ellipse $3x^2 + 2y^2 = 5$ is

$SS' = T^2$

$\Rightarrow (3x^2 + 2y^2 - 5)[3(1)^2 + 2(2)^2 - 5] = [3x(1) + 2y(2) - 5]^2$

$\Rightarrow (3x^2 + 2y^2 - 5)(3 + 8 - 5) = (3x + 4y - 5)^2$

$\Rightarrow 9x^2 - 24xy - 4y^2 + 40y + 30x - 55 = 0$

This is the equation of pair of straight lines,

Where, $a = 9, h = -12, b = -4$

The angle between these lines is given by

$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$

$\Rightarrow \tan\theta = \frac{2\sqrt{144+36}}{9-4} = \frac{2\sqrt{180}}{5} = \frac{12}{\sqrt{5}}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{12}{\sqrt{5}}\right)$

407 **(b)**

Given, eccentricity $e = \frac{\sqrt{5}}{3}$ and foci $= \left(\pm\sqrt{5}, o\right)$

$\Rightarrow\ ae = \sqrt{5}\ \Rightarrow\ a = 3$

$\therefore\ b^2 = a^2\,(1 - e^2\_ = 9\left(1 - \frac{5}{9}\right)$

$\Rightarrow\ b^2 = 4$

The equation of ellipse is

$\frac{x^2}{9} + \frac{y^2}{4} = 1$

$\Rightarrow\ 4x^2 + 9y^2 = 36$

408 **(b)**

Let $P(h, k)$ be the mid-point of a chord. Then, the equation of the chord is

$\frac{hx}{a^2} + \frac{ky}{b^2} = \frac{h^2}{a^2} + \frac{k^2}{b^2}$ or, y

$= \left(-\frac{b^2}{a^2}\frac{h}{k}\right)x + \frac{b^2}{k}\left(\frac{h^2}{a^2} + \frac{k^2}{b^2}\right)$

This touches the circle $x^2 + y^2 = b^2$

$\therefore \frac{b^4}{k^2}\left(\frac{h^2}{a^2} + \frac{k^2}{b^2}\right)^2 = b^2\left(1 + \frac{b^4h^2}{a^4k^2}\right)$

$\Rightarrow \left(\frac{h^2}{a^2} + \frac{k^2}{b^2}\right)^2 = b^2\left(\frac{h^2}{a^4} + \frac{k^2}{b^4}\right)$

Hence, the locus of (h, k) is $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^2 = b^2\left(\frac{x^2}{a^4} + \frac{y^2}{b^4}\right)$

409 **(a)**

The coordinates of the ends of the latusrectum of the parabola $y^2 = 4\,ax$ are $(a, 2a)$ and $(a, -2a)$ respectively.

The equations of the normal at $(a, 2a)$ and $(a, -2a)$ to $y^2 = 4\,ax$ are

$x + y - 3a = 0$ and $x - y - 3a = 0$ respectively.

The combined equation of these two normal is

$x^2 - y^2 - 6\,ax + 9\,a^2 = 0$

410 **(c)**

Given , $\frac{x^2}{3} + \frac{y^2}{2} = 1$

Polar of $P(1,2)$ with respect to ellipse is $S_1 = 0$

$\Rightarrow\ x + 3y - 3 = 0$

Since, (1,2) and $(k, -1)$ are conjugates, therefore one passes through the polar of the other.

$k - 3 - 3 = 0$

$\Rightarrow\ k = 6$

411 **(a)**

Asymptotes of the given hyperbola are $y = \pm\frac{b}{a}x$

$\therefore$Angle between the asymptotes $= 2\theta$, where $\tan\theta = \frac{b}{a}$

$\Rightarrow$ Angle between the asymptotes $= 2\tan^{-1}\left(\frac{b}{a}\right)$

412 **(d)**

Given, $\frac{(x+2)^2}{7\times 14} + \frac{(y-1)^2}{14} = 1$

Here, $\ a^2 = 7 \times 14$ and $b^2 = 14$

We know, $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{14}{7 \times 14}} = \sqrt{\frac{6}{7}}$

414 **(a)**

We know, that, the angle between the two tangents from (α, β)

To the circle $x^2 + y^2 = r^2$ is $2 \tan^{-1} \frac{r}{\sqrt{S_1}}$

Let $S = x^2 + y^2 - 5x + 4y - 2$

Here, $r = \sqrt{\left(-\frac{5}{2}\right)^2 + (2)^2 + 2} = \frac{7}{2}$

At point $(-1, 0)$

$S_1 = (-1)^2 + (0)^2 - 5(-1) + 4(0) - 2 = 4$

$\therefore$ Required angle, $\theta = 2 \tan^{-1} \frac{7/2}{\sqrt{4}}$

$= 2 \tan^{-1} \left(\frac{7}{4}\right)$

415 **(a)**

Since the circle passes through the origin, has centre on x-axis and has radius a. So, its centre is at $(a, 0)$. The equation of the circle is

$(x - a)^2 + (y - 0)^2 = a^2 \Rightarrow x^2 + y^2 - 2ax = 0$ ...(i)

The circle passing through the intersection of (i) and the line $y = mx$ is

$x^2 + y^2 - 2ax + \lambda(y - mx) = 0$

$\Rightarrow x^2 + y^2 - x(2a + \lambda m) + \lambda y = 0$ ...(ii)

Since $y = mx$ is a diameter of this circle.

Therefore, centre $\left(\frac{2a + \lambda m}{2}, -\frac{\lambda}{2}\right)$ lies on it.

i. e. $-\frac{\lambda}{2} = m\left(\frac{2a + \lambda m}{2}\right) \Rightarrow \lambda = -\frac{2am}{1 + m^2}$

putting the value of λ in (ii), we get

$(1 + m^2)(x^2 + y^2) - 2a(x + my) = 0$

This is the equation of the required circle

416 **(a)**

Let $B(at_1^2, 2at_1)$ and $C(at_2^2, 2at_2)$ be the coordinates of the end-points of focal chord BC. Then,

$\Delta =$ Area of ΔABC

$\Rightarrow \Delta = \frac{1}{2}$ Absolute value of $\begin{vmatrix} 0 & 0 & 1 \\ at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \end{vmatrix}$

$\Rightarrow \Delta = |a^2 t_1 t_2 (t_1 - t_2)|$

$\Rightarrow \Delta = a^2 |t_1 - t_2|$ $[\because t_1 t_2 = -1]$

$\Rightarrow |2at_1 - 2at_2| = \frac{2\,\Delta}{a}$

417 **(b)**

We have, $\angle BSS' = \theta$

$\therefore$ Slope of $BS = \tan(180° - \theta)$

$\Rightarrow \frac{-b}{ae} = -\tan\theta$

$\Rightarrow b = ae \tan\theta$

$\Rightarrow b^2 = a^2 e^2 \tan^2\theta$

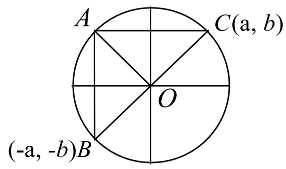
$\Rightarrow a^2(1 - e^2) = a^2 e^2 \tan^2\theta$

$\Rightarrow 1 - e^2 = e^2 \tan^2\theta$

$\Rightarrow 1 = e^2 \sec^2\theta \Rightarrow \cos^2\theta = e^2 \Rightarrow \cos\theta = e$

418 **(b)**

Since, the hypocenter of a right angled triangle inscribed in a circle is a diameter of the circle. If the coordinates of the end C of the hypotenuse BC are (a, b), the coordinates of B



are$(-a, -b)$. Equation of BC is$\frac{y}{x} = \frac{b}{a}$. If A is the vertex of the isosceles triangle, then OA is perpendicular to BC and the equation of AO is $\frac{y}{x} = -\frac{a}{b}$ which meets the circle $x^2 + y^2 = r^2$ at points for which

$\left(\frac{a^2}{b^2} + 1\right)x^2 = r^2 = a^2 + b^2$

$[\because (a, b)$ lies on $x^2 + y^2 = r^2]$

$\Rightarrow x^2 = b^2 \Rightarrow x = \pm b$

$\Rightarrow y = \pm a$

$\therefore$ Coordinates of A are $(-b, a)$ or $(b, -a)$

419 **(b)**

Equation of family of concentric circles to the circle $x^2 + y^2 + 6x + 8y - 5 = 0$ is

$x^2 + y^2 + 6x + 8y + \lambda = 0$

Which is similar to $x^2 + y^2 + 2gx + 2fy + c = 0$

Thus, the point $(-3, 2)$ lies on the circle

$x^2 + y^2 + 6x + 8y + c = 0$

$\therefore$ $(-3)^2 + (2)^2 + 6(-3) + 8(2) + c = 0 \Rightarrow c = -11$

420 **(c)**

Proceeding as in Example 39, we have

$x^2 = a^2\left(\frac{2e^2 - 1}{e^2}\right)$

This will give exactly one value of x if $2e^2 - 1 = 0$ i.e. $e = \frac{1}{\sqrt{2}}$

421 **(d)**

The equations of chords of contact of the tangents drawn from the origin and the point (g, f) to the given circle are respectively

$gx + fy + c = 0$ ...(i)

and $2\,gx + 2\,fy + g^2 + f^2 + c = 0$ ...(ii)

Clearly, (i) and (ii) are parallel. Therefore, the distance $'d'$ between them is given by

$$d=\frac{g^2+f^2+c}{\sqrt{4\,g^2+4\,f^2}}-\frac{c}{\sqrt{g^2+f^2}}=\frac{g^2+f^2-c}{2\sqrt{g^2+f^2}}$$

422 **(b)**

Clearly, $P(-1,-3)$ is the external centre of similitude. Thus,

Required length of the common tangent = $|l_1-l_2|$,

where l_1 and l_2 are the lengths of the tangents to the given circles drawn from point $P(-1,-3)$

Now,

l_1 = Length of the tangent from $P(-1,-3)$ to $x^2+y^2=1$

$\Rightarrow l_1=\sqrt{1+9-1}=3$

And,

l_2 = Length of the tangent from $P(-1,-3)$ to $x^2+y^2-2x-6y+6=0$

$\Rightarrow l_2=\sqrt{1+9+2+18+6}=6$

$\therefore$ Length of the common tangent $=|l_1-l_2|=|3-6|=3$

423 **(b)**

We have,

$x^2+y^2-2x-2\,\lambda\,y-8=0$

$\Rightarrow (x^2+y^2-2x-8)-2\,\lambda\,y=0$ ...(i)

This equation represents a family of circles passing through the points P and Q which are points of intersection of the circle $x^2+y^2-2x-8=0$ and $y=0$

The coordinates of the centre of (i) are $(1,\lambda)$

Equation of PQ is $y=0$

If PQ is a diameter of (i). Then, $\lambda=0$

Putting $\lambda=0$ in (i), we get

$x^2+y^2-2x-8=0$ as the equation of the required circle

424 **(d)**

Given equation of circle can be rewritten as

$(x+2)^2+(y+3)^2=0$

$\therefore$ Radius of circle is 0

425 **(c)**

The coordinates of the focus are

$\left(\frac{-6+6}{2},\frac{4+4}{2}\right)=(0,4)$

$\therefore$ Distance between focus and vertex = 2

Clearly, parabola opens upward, has its axis along y-axis. So, its equation is

$(x-0)^2=4\times 2(y-2)\Rightarrow x^2-8y+16=0$

426 **(b)**

The coordinates of the centre of the circle $x^2+y^2-12x+4\,y+6=0$ are $(6,-2)$.

Clearly, the line $x+3\,y=0$ passes through this point

Hence, $x+3y=0$ is a diameter of the given circle

427 **(a)**

Given equation can be rewritten as

$$\frac{x^2}{16}-\frac{y^2}{9}=1$$

$\therefore$ Required equation of director circle is

$x^2+y^2=16-9$

$\Rightarrow\ x^2+y^2=7$

428 **(b)**

The equation of any line through $P(\alpha,\beta)$ is

$\frac{x-\alpha}{\cos\theta}=\frac{y-\beta}{\sin\theta}=k$ (say)

Any point on this line is $(\alpha+k\cos\theta,\beta+k\sin\theta)$.

This point lies on the given circle if

$(\alpha+k\cos\theta)^2+(\beta+k\sin\theta)^2=r^2$

$\Rightarrow k^2+2\,k(\alpha\cos\theta+\beta\sin\theta)+\alpha^2+\beta^2-r^2=0$ ...(i)

This equation, being quadratic in k, gives two values of k and hence the distances of two points A and B on the circle from the point P.

Let $PA=k_1, PB=k_2$, where k_1,k_2 are the roots of equation (i)

Then,

$PAPB=k_1k_2=\alpha^2+\beta^2-r^2$

ALITER $PAPB$ is the power of the point $P(\alpha,\beta)$ with respect to the circle $x^2+y^2=r^2$. Therefore, $PAPB=\alpha^2+\beta^2-r^2$

429 **(a)**

If (α,β) is a point on the chord PQ, then the either it is the interior point or one of the end-points of the chord PQ.

$\therefore 3\le\alpha\le 4$ and $-4\le\beta\le 3$

430 **(a)**

Let the equation $y=mx+c$ be the common tangents so the curve $y^2=8x$ and $x^2+y^2=2$

Then, $c=\frac{2}{m}$ and $c^2=2(1+m^2)$

If $m^2=t$, then

$\frac{4}{t}=2(1+t)\Rightarrow t^2+t-2=0$

$\Rightarrow (t+2)(t-1)=0\Rightarrow t=1,-2$

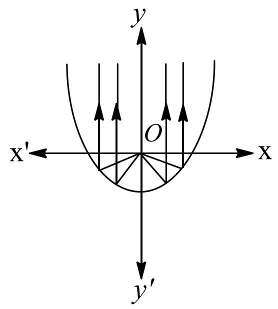
Thus, $m=\pm 1$ $(\because t\neq -2)$

Hence, tangents are $y=x+c$ and $y=-x+c$ which are perpendicular to each other

431 **(a)**

$\because y^2-12x-4y+4=0$

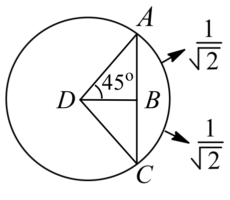
$\Rightarrow (y-2)^2=12x$



Its vertex is $(0, 2)$and $a = 3$,
Its focus is $(3, 2)$
Hence, for the required parabola; focus is $(3, 4)$,
Vertex is $(3, 2)$and $a = 2$
Hence, for the required parabola is
$(x - 3)^2 = 4(2)(y - 2)$
Or $x^2 - 6x - 8y + 25 = 0$

432 **(a)**
The given equation can be written as
$$\frac{(x+1)^2}{9} + \frac{(y+2)^2}{25} = 1$$
Clearly, it represents an ellipse whose centre $(-1, -2)$ and semi-major and minor axes 5 and 3 respectively.
The eccentricity e of the ellipse is given by
$9 = 25(1 - e)^2 \Rightarrow e = \dfrac{4}{5}$
The coordinates of the foci of the ellipse are given by
$x + 1 = 0$ and $y + 2 = \pm\left(5 \times \frac{4}{5}\right)$
$\Rightarrow x = -1, y = 2$ or, $x = -1$ and $y = -6$
Hence, the coordinates of the foci are $(-1,2)$ and $(-1, -6)$ respectively

433 **(a)**
In $\Delta ADB, AD = \frac{1}{\sqrt{2}}$ cosec $45° = 1$

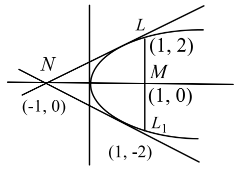


434 **(c)**
Since the focus and vertex of the parabola are on y-axis. Therefore, its directrix is parallel to x-axis and axis of the parabola is y-axis. Let the equation of the directrix be $y = k$. The directrix meets the axis of the parabola at $(0, k)$. But, vertex is the mid point of the line segment joining the focus to the point where directrix meets axis of the parabola.
$\therefore \dfrac{k+2}{2} = 4 \Rightarrow k = 6$
Thus, the equation of the directrix is $y = 6$
Let (x, y) be a point on the parabola. then, by definition
$(x - 0)^2 + (y - 2)^2 = (y - 6)^2 \Rightarrow x^2 + 8\,y = 32$

435 **(d)**
Let the equation of the ellipse to be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
It is given that be $b = 6$ and $\frac{b}{e} = 9$
$\therefore b^2 = 36$ and $e = \frac{2}{3}$
Now, $a^2 = b^2(1 - e^2) \Rightarrow a^2 = 36\left(1 - \frac{4}{9}\right) = 20$
So, the equation of the ellipse of $\frac{x^2}{20} + \frac{y^2}{36} = 1$ or,
$9x^2 + 5y^2 = 180$

436 **(b)**
The coordinates at the ends of the latusrectum of the parabola $y^2 = 4x$ are $L(1,2)$ and $L_1(1 - 2)$

Equation of tangent at L and l_1 are $2y = 2(x + 1)$ and $-2y = 2(x + 1)$, which gives $x = -1, y = 0$



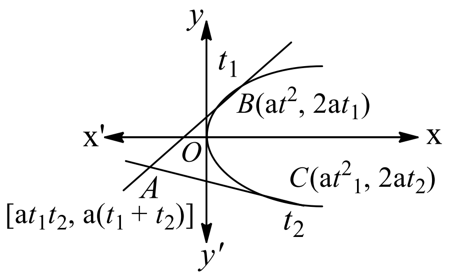
438 **(c)**
Given equation is
$$(x - 1)^2 + (y - 2)^2 = 3 \times \left(\frac{2x + 3y - 2}{\sqrt{13}}\right)^2$$
On comparing with $PS = ePM$

$\therefore\ e = 3$

Hence, it represents a hyperbola

439 **(b)**
Let the centre of circle be $(g, 5)$
$\therefore \quad \frac{3(g) - 4(5)}{\sqrt{3^2 + 4^2}} = 5 \quad$ [radius]
$\Rightarrow \quad 3g = 25 + 20 \quad \Rightarrow \quad g = 15$
$\therefore$ Equation of circle whose centre (15, 5) and radius 5 is
$(x - 15)^2 + (y - 5)^2 = 5^2$
$\Rightarrow \quad x^2 - 30x + y^2 - 10y + 225 = 0$

440 **(b)**
Let third tangent is tangent at vertices, then
$p_1 = |at_1 t_2|, P_2 = at_1^2, P_3 = at_2^2$ clearly p_2, p_1, p_3 are in GP



441 **(a)**
Let the equation of circle is
$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)
Given that, $xy = 1$ ...(ii)
From Eq.(i) and (ii), we get
$x^4 + 2gx^3 + cx^2 + 2fx + 1 = 0$
∴ Product of roots $x_1x_2x_3x_4 = 1$ and similarly
$y_1y_2y_3y_4 = 1$

442 **(c)**
We know that the line $y = mx + c$ touches the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, if $c^2 = a^2m^2 - b^2$.
The given hyperbola and the line are $\frac{x^2}{5} - \frac{y^2}{9} = 1$ and $y = 3x + \lambda$
Here, $a^2 = 5, b^2 = 9, m = 3$
$\therefore \lambda\sqrt{a^2m^2 - b^2} = \sqrt{45-9} = \sqrt{36} = 6$

443 **(d)**
Since, the required circle touch $x = 0, y = 0$ and $x = 4$
Centre is (2, 2) and radius=2
∴ Required circle is
$(x-2)^2 + (y-2)^2 = (2)^2$
$\Rightarrow x^2 + y^2 - 4x - 4y + 4 = 0$

445 **(b)**
Here, $c = c, m = 2, a^2 = 16$
$\because \quad c^2 = a^2(1+m^2) \quad \therefore \quad c^2 = 16(1+4)$
$\Rightarrow \quad c^2 = 80$

446 **(a)**
Given equation of tangent is $\frac{x}{a}\frac{\sqrt{3}}{2} + \frac{y}{b}\frac{1}{2} = 1$ and equation of tangent at the point $(a\cos\phi, b\sin\phi)$ on the ellipse is $\frac{x}{a}\cos\phi + \frac{y}{b}\sin\phi = 1$
Both are same
$\therefore \cos\phi = \frac{\sqrt{3}}{2}, \sin\phi = \frac{1}{2} \Rightarrow \phi = \frac{\pi}{6}$

447 **(a)**
Given , foci $(\pm ae, 0) = (\pm 2, 0)$ and $e = \frac{1}{2}$

$\therefore \ ae = 2 \Rightarrow a = 4$

Now, $b = a\sqrt{1 - \frac{1}{4}} = 2\sqrt{3}$

$\therefore \ a^2 = 16$ and $b^2 = 12$

448 **(c)**
In the standard form of an ellipse sum of the focal distances of a point is $2a$

449 **(b)**
Given circles are $x^2 + y^2 - y = 0$ and $x^2 + y^2 + y = 0$ centres and radii of these circles are
$C_1\left(0, \frac{1}{2}\right), C_2\left(0, -\frac{1}{2}\right)$
And $r_1 = \frac{1}{2}, r_2 = \frac{1}{2}$
Now, $C_1C_2 = \sqrt{0 + \left(\frac{1}{2} + \frac{1}{2}\right)^2} = 1$
And $r_1 + r_2 = \frac{1}{2} + \frac{1}{2} = 1$
$\because \quad C_1C_2 = r_1 + r_2$
It means that two circles touch each other externally
Hence, number of common tangents are 3

450 **(a)**
Since, asymptotes are at90°, it means that it is a rectangular hyperbola.
∴ Eccentricity is $\sqrt{2}$.

451 **(c)**
It is given, centre is $(2, -3)$ and circumference of circle $= 10\pi$
$\Rightarrow \quad 2\pi r = 10\pi \Rightarrow r = 5$
The equation of circle, if centre is $(2, -3)$ and radius is 5, is
$(x-2)^2 + (y+3)^2 = 5^2$
$\Rightarrow \quad x^2 + y^2 - 4x + 6y + 13 = 25$
$\Rightarrow x^2 + y^2 - 4x + 6y - 12 = 0$

453 **(b)**
Given equation is

$y^2 - 8y - x + 19 = 0$

$\Rightarrow \ (y-4)^2 = x - 19 + 16$

$\Rightarrow \ (y-4)^2 = (x-3)$

$\Rightarrow \qquad Y^2 = 4AX$

Where, $Y = \ y - 4, A = \frac{1}{4}$ and $X = x - 3$

$\therefore$ Focus $= (A, 0) = \left(\frac{1}{4}, 0\right) = \left(\frac{13}{4}, 4\right)$

Vertex= (3,4)

Directrix $X = -\frac{1}{4}$

$\Rightarrow\ x-3=-\dfrac{1}{4}$

$\Rightarrow\ x=\dfrac{11}{4}$

454 **(a)**

Given equation can be rewritten as

$x(x-2y)-3(x-2y)=0$

$\Rightarrow\ (x-3)(x-2y)=0$

$\Rightarrow\ x=3, x=2y$

$\Rightarrow\ x=3,\quad y=\dfrac{3}{2}$

$\therefore$ Centre of circle is $\left(3,\frac{3}{2}\right)$

455 **(c)**

If the normal at t_1 meets the parabola at t_2, then

$t_2=-t_1-\dfrac{2}{t_1}$

Here, $t_1=1$ and $t_2=t$. Therefore, $t=-3$

456 **(d)**

Eliminating λ from the two given equations, we get

$\left(\dfrac{x}{a}+\dfrac{y}{b}\right)\left(\dfrac{x}{a}-\dfrac{y}{b}\right)=1$

$\Rightarrow \frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, which is the equation of a hyperbola

457 **(a)**

Centre of the hyperbola is the mid-point of the line segment joining two foci. Therefore, coordinates od the centre are (1,5).

Now, Distance between the foci = 10

$\Rightarrow 2a\,e=10 \Rightarrow ae=5 \Rightarrow a=4 \quad [\because e=5/4]$

$\therefore b^2=a^2(e^2-1) \Rightarrow b=3$

Hence, the equation of the hyperbola is

$\dfrac{(x-1)^2}{16}-\dfrac{(y-5)^2}{9}=1$

458 **(c)**

Given equation can be rewritten as

$(x-2)^2=8(y-1)$

$\Rightarrow\ X^2=4AY$

Where $X=x-2, Y=y-1$ and $A=2$

So, directrix is given by

$Y=-A \Rightarrow\ y-1=-2$

$\Rightarrow\ y=-1$

459 **(d)**

Given, $y+2a=\frac{a^2}{3}\left(x^2+\frac{3}{2a}x\right)$

$\Rightarrow\ y+2a=\dfrac{a^3}{3}\left(x+\dfrac{3}{4a}\right)^2-\dfrac{9}{16^2}\times\dfrac{a^3}{3}$

$\Rightarrow\left(y+\dfrac{35a}{16}\right)=\dfrac{a^3}{3}\left(x+\dfrac{3}{4a}\right)^2$

Thus, the vertices of parabola is $\left(-\frac{3}{4a},-\frac{35a}{16}\right)$

Let $h=-\frac{3}{4a}$ and $k=-\frac{35a}{16}$

$\therefore hk=\dfrac{105}{64}$

Hence, the locus of vertices of a parabola is $xy=\frac{105}{64}$

460 **(b)**

Equation of the coaxial system of circle is

$S_1+\lambda S_2=0$

$\therefore\ (x^2+y^2+5x+y+4)+\lambda(x^2+y^2+10x-4y-1)=0$

$\Rightarrow\ x^2+y^2+\dfrac{5(1+2\lambda)}{(1+\lambda)}x+\dfrac{(1-4\lambda)}{(1+\lambda)}y+\dfrac{4-\lambda}{1+\lambda}=0$

$\therefore$ The centre of the circle is

$\left(-\frac{5(1+2\lambda)}{2(1+\lambda)},-\frac{(1-4\lambda)}{2(1+\lambda)}\right)$...(i)

For limiting point, $r=0$

$\therefore\ \sqrt{\dfrac{25(1+2\lambda)^2}{4(1+\lambda)^2}+\dfrac{(1-4\lambda)^2}{4(1+\lambda)^2}-\dfrac{(4-\lambda)}{(1+\lambda)}}=0$

$\Rightarrow\ 25(1+2\lambda)^2+(1-4\lambda)^2-4(4-\lambda)(1+\lambda)=0$

$\Rightarrow\ 120\lambda^2+80\lambda+10=0 \Rightarrow\ (6\lambda+1)(2\lambda+1)=0$

$\Rightarrow\ \lambda=-\frac{1}{6}$ and $-\frac{1}{2}$

On substituting the values of λ in Eq. (i), we get $(-2,-1)$ and $(0,-3)$

461 **(a)**

The equation tangent to $\frac{x^2}{a^2}+\frac{y^2}{b}=1$ at any point $P(a\cos\theta, b\sin\theta)$ is

$\dfrac{x}{a}\cos\theta+\dfrac{y}{b}\sin\theta=1 \qquad \ldots$ (i)

The equations of tangents at $B(0,b)$ and $B'(0,-b)$ are $y=b$ and $y=-b$ respectively. These two tangents intersect (i) at $L\left(\frac{a(1-\sin\theta)}{\cos\theta},b\right)$ and $L'\left(\frac{a(1+\sin\theta)}{\cos\theta},-b\right)$ respectively

$\therefore BL = \left|\frac{a(1-\sin\theta)}{\cos\theta}\right|$ and $B'L' = \left|\frac{a(1+\sin\theta)}{\cos\theta}\right|$

$\Rightarrow BL \times B'L' = \left|\frac{a^2(1-\sin^2\theta)}{\cos\theta}\right| = a^2$

462 **(c)**

Let the equation to the required ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. It passes through $(-3,1)$

$\therefore \frac{9}{a^2} + \frac{1}{b^2} = 1$

$\Rightarrow 9\,b^2 + a^2 = a^2b^2$

$\Rightarrow 9\,a^2(1-e^2) + a^2 = a^4(1-e^2) \quad [\because b^2 = a^2(1-e^2)]$

$\Rightarrow 9\,a^2\left(1-\frac{2}{5}\right) + a^2 = a^4\left(1-\frac{2}{5}\right) \Rightarrow a^2 = \frac{32}{3}$

Now, $b^2 = a^2(1-e^2) \Rightarrow b^2 = \frac{32}{3}\left(1-\frac{2}{5}\right) = \frac{32}{5}$

Hence, the equation of the required ellipse is

$\frac{x^2}{\frac{32}{3}} + \frac{y^2}{\frac{32}{5}} = 1$ or, $3x^2 + 5\,y^2 = 32$

463 **(b)**

The locus of the point which moves such that the ratio of its distance from two fixed point in the plane is always a constant $k(k < 1)$ is an ellipse.

464 **(c)**

We have, $\frac{x^2}{12-k} + \frac{y^2}{8-k} = 1$

This equation will represent a hyperbola, if $(12-k)$ and $(8-k)$ are of opposite signs

$\Rightarrow (12-k)(8-k) < 0 \Rightarrow (k-12)(k-8) < 0$

$\Rightarrow 8 < k < 12$

465 **(b)**

The given line is a diameter of the circle and the origin lies on the circle. So, required angle is the angle in a semi-circle, which is a right angle

466 **(c)**

Here, $g_1 = \frac{k}{2}$, $f_1 = 2$, $c_1 = 2$

And $g_2 = -1$, $f_2 = -\frac{3}{4}$, $c_2 = \frac{k}{2}$

$\because$ Given circles cut orthogonally

$\therefore \quad 2 \times \frac{k}{2} \times (-1) + 2 \times 2 \times \left(-\frac{3}{4}\right) = 2 + \frac{k}{2}$

$\Rightarrow \quad -k - 3 = 2 + \frac{k}{2} \quad \Rightarrow \quad k = \frac{-10}{3}$

467 **(b)**

We know that the diameters $y = m_1x$ and $y = m_2x$ are conjugate diameters of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, if $m_1m_2 = \frac{b^2}{a^2}$

Here, $a^2 = 9, b^2 = 16$ and $m_1 = 1/2$

$\therefore m_1m_2 = \frac{b^2}{a^2} \Rightarrow \frac{1}{2}(m_2) = \frac{16}{9} \Rightarrow m_2 = \frac{32}{9}$

Hence, the required diameter is $y = \frac{32\,x}{9}$

468 **(d)**

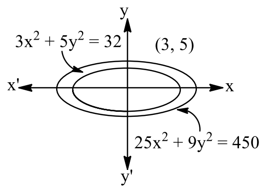
Centre is point of intersection of two diameter *ie*, the point is $C(8,-2)$

$\therefore\ r = CP = \sqrt{4+16} = \sqrt{20}$

469 **(c)**

Let $S_1 = 3x^2 + 5y^2 - 32$

and $S_2 = 25x^2 + 9y^2 - 450$



At point (3, 5)

$S_1 = 3(3)^2 + 5(5)^2 - 32 = 120 > 0$

and $S_2 = 25(3)^2 + 9(5)^2 - 450$

$= 225 + 225 - 450 = 0$

$\therefore$ Point $(3,5)$ lies outside the first ellipse and for second ellipse lies on the ellipse.

Hence, two tangents for first ellipse and one tangent for second ellipse can be drawn

470 **(c)**

Put the value of

$(x,y) \equiv (\tan\theta + \sin\theta, \tan\theta - \sin\theta)$ In the given option, we get the required result.

On putting the value of x and y in option (c), we get

$[(\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta^2]^2 = 16(\tan\theta + \sin\theta) \times (\tan\theta - \sin\theta)$

$\Rightarrow [\tan^2\theta + \sin^2\theta - \tan^2\theta - \sin^2\theta + 4\tan\theta\sin\theta]^2 = 16(\tan^2\theta - \sin^2\theta)$

$\Rightarrow (4\tan\theta \cdot \sin\theta)^2 = 16(\tan^2\theta - \sin^2\theta)$

$\Rightarrow 16\tan^2\theta\sin^2\theta = 16\tan^2\theta\,(1-\cos^2\theta)$

$\Rightarrow \quad 16\tan^{2\cdot}\theta\sin^2\theta = 16\tan^2\theta\sin^2\theta$

Hence, the option (c) satisfies

471 **(d)**

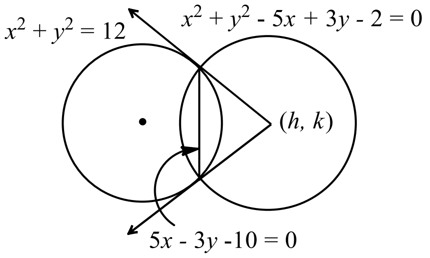
Given, $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Length of latusrectum

$= \frac{2b^2}{a} = \frac{2 \times 9}{4} = \frac{9}{2}$

472 **(b)**

Let (h, k) be the point of intersection of the tangents. Then, the chord of contact of tangents is the coomon chord of the circles $x^2 + y^2 = 12$ and $x^2 + y^2 - 5\,x + 3\,y - 2 = 0$



The equation of the common chord is

$5x - 3y - 10 = 0$ ...(i)

Also, the equation of the chord of contact is

$hx + ky - 12 = 0$ ...(ii)

Equations (i) and (ii) represent the same line. Therefore,

$\frac{h}{5} = \frac{k}{-3} = \frac{-12}{-10} \Rightarrow h = 6, k = -18/5$

Hence, the required point is $(6, -18/5)$

473 **(c)**

If S and S' are two foci of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $P(x, y)$ is any point on it. Then,

$S'P - SP = 2a =$ Transverse axis

474 **(a)**

Let the equation of the hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

It is given that $a = 3$ and $ae = 5$

$\therefore e = \frac{5}{3}$ and $b^2 = a^2(e^2 - 1) \Rightarrow b^2 = 9\left(\frac{25}{9} - 1\right)$

$= 16$

So, the equation of the hyperbola is $\frac{x^2}{9} - \frac{y^2}{16} = 1$

475 **(a)**

The coordinates of Q and R are $(0, b \operatorname{cosec} \theta)$ and $(0, b \sin \theta)$

$\therefore CQ = b \operatorname{cosec} \theta$ and $CR = b \sin \theta$

$\Rightarrow CQ \times CR = b^2$

476 **(c)**

Given, equation of hyperbola

$(10x - 5)^2 + (10y - 4)^2 = \lambda^2(3x + 4y - 1)^2$ can be rewritten as

$$\frac{\sqrt{\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{2}{5}\right)^2}}{\left|\frac{3x+4y-1}{5}\right|} = \left|\frac{\lambda}{2}\right|$$

This is of the form of $\frac{PS}{PM} = e$

Where, P is any point on the hyperbola and S is a focus and M is the point of directrix.

Here, $\left|\frac{\lambda}{2}\right| > 1 \Rightarrow |\lambda| > 2 \quad (\because e > 1)$

$\Rightarrow \lambda < -2$ or $\lambda > 2$

478 **(c)**

On homogenising $y^2 - x^2 = 4$ with the help of the line $\sqrt{3}x + y = 2$, we get

$y^2 - x^2 = 4\,\frac{\left(\sqrt{3}x + y\right)^2}{4}$

$\Rightarrow y^2 - x^2 = 3x^2 + y^2 + 2\sqrt{3}xy$

$\Rightarrow 4x^2 + 2\sqrt{3}xy = 0$

$\therefore \tan \theta = 2\frac{\sqrt{h^2 - ab}}{a + b}$

$\Rightarrow \tan \theta = \frac{2\sqrt{3 - 0}}{4}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

479 **(b)**

Let the standard equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\ (a > b)$

Minor axis $= 2b = 8 \Rightarrow b = 4$

And eccentricity $= e = \frac{\sqrt{5}}{3}$

Now, $b^2 = a^2(1 - e^2)$

$\Rightarrow (4)^2 = a^2\left(1 - \frac{5}{9}\right)$

$\Rightarrow 16 = a^2\left(\frac{4}{9}\right)$

$\Rightarrow a^2 = 36 \Rightarrow a = 6$

Length of major axis $= 2a = 12$

480 **(c)**

We know that, if three normals to the parabola $y^2 = 4ax$ through point (h, k), then $h > 2a$

Here, $h = a$ and $a = \frac{1}{4}$

$\therefore a > 2.\frac{1}{4} \Rightarrow a > \frac{1}{2}$

481 **(c)**

Obviously it is an ellipse, because the normal and tangent at point P of an ellipse bisect the internal and external angles between the focal distance of the point

482 **(a)**

Since $3\,x + y = 0$ is a tangent to the circle with centre at $(2, -1)$

$\therefore$Radius = Length of the $\perp$ from $(2, -1)$ on $3\,x + y = 0$

$\Rightarrow$ Radius $= \frac{6 - 1}{\sqrt{9 + 1}} = \frac{5}{\sqrt{10}} = \sqrt{\frac{5}{2}}$

So, the equation of the circle is

$$(x-2)^2+(y+1)^2=\frac{5}{2}$$

$$\Rightarrow x^2+y^2-4x+2y+\frac{5}{2}=0$$

The combined equation of the tangents drawn from the origin to this circle is

$$\left(x^2+y^2-4x+2y+\frac{5}{2}\right)\left(\frac{5}{2}\right)=\left(-2x+y+\frac{5}{2}\right)^2$$

$$\Rightarrow 3x^2-8xy-3y^2=0 \Rightarrow 3x+y=0, x-3y=0$$

483 **(d)**

Given equation is $2x^2+5xy+2y^2+4x+5y=0$ and equation of its asymptotes is

$2x^2+5xy+2y^2+4x+5y+\lambda=0$ ...(i)

Which is the equation of pair of straight lines

Eq.(i) is compared by the standard equation of pair of straight lines.

$\Rightarrow a=2, b=2, h=\frac{5}{2}, g=2, f=\frac{5}{2}$ and $c=\lambda$

The condition for a pair of straight lines is

$abc+2fgh-af^2-bg^2-ch^2=0$

$$\therefore 2(2)(\lambda)+2\left(\frac{5}{2}\right)2\left(\frac{5}{2}\right)-2\left(\frac{5}{2}\right)^2-2(2)^2-\lambda\left(\frac{5}{2}\right)^2=0$$

$$\Rightarrow 4\lambda+25-\frac{25}{2}-8-\frac{25\lambda}{4}=0$$

$$\Rightarrow \frac{25\lambda}{4}-4\lambda=\frac{25}{2}-8 \Rightarrow \lambda=2$$

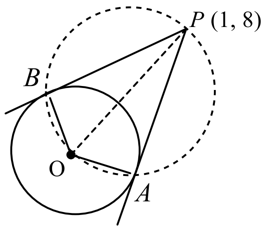
On putting the value of λ in Eq.(i), we get

$2x^2+5xy+2y^2+4x+5y+2=0$

Which is the required equation.

484 **(b)**

The centre of the given circle is $O(3,2)$



Since, OA and OB are perpendicular to PA and PB. Also, OP is the diameter of the circumcircle of $\triangle PAB$

Its equation is

$(x-3)(x-1)+(y-2)(y-8)=0$

$\Rightarrow \quad x^2+y^2-4x-10y+19=0$

485 **(b)**

Condition for tangency to the ellipse is

$c^2=a^2m^2\pm b^2$

$\Rightarrow \ c^2=9(-1)^2\pm 16$

$c^2=25$

$\Rightarrow \ c=\pm 5$

486 **(d)**

The given equation can be written as

$(x-4)^2=y-(c-16)$

Therefore, the vertex of the parabola is $(4, c+16)$. This point lies on x-axis

$\therefore c-16=0 \Rightarrow c=16$

487 **(d)**

The centre and radius of given circle are (r,h) and r

Thus, $x=0$ is one of tangent

Let another tangent is $y=mx$ to the circ le. This line will be tangent, if

$$\frac{h-mr}{\sqrt{1+m^2}}=r$$

$$\Rightarrow \quad m=\left(\frac{h^2-r^2}{2hr}\right)$$

Therefore, equation of tangent is

$$y=\frac{(h^2-r^2)}{2hr}x$$

$\Rightarrow \quad (h^2-r^2)x-2hr\,y=0$

Required tangents are $x=0$ and $(h^2-r^2)x-2hry=0$

488 **(b)**

Equation of tangent to hyperbola having slope m is

$y=mx+\sqrt{9m^2-4}$ ...(i)

Equation of tangent to circle is

$y=m(x-4)+\sqrt{16m^2+16}$ ...(ii)

Eqs. (i)and (ii)will be identical for $m=\frac{2}{\sqrt{5}}$ satisfy

$\therefore$ Equation of common tangent is

$2x-\sqrt{5}y+4=0$

489 **(d)**

The given ellipse is $\frac{x^2}{9}+\frac{y^2}{4}=1$. The value of the expression $\frac{x^2}{9}+\frac{y^2}{4}-1$ is positive for $x=1, y=2$ and negative for $x=2, y=1$. Therefore, P lies outside E and Q lies inside E. The value of the expression x^2+y^2-9 is $-$ive for both the points P and Q. Therefore, P and Q both lie inside C. Hence, P lies inside C but outside E

490 **(b)**

Let the equation of the ellipse be

$$\frac{(x-1)^2}{a^2}+\frac{(y+1)^2}{b^2}=1 \quad \ldots\text{(i)}$$

It is given that $a=8$ and ellipse (i) passes through (1,3)

$\therefore b^2=16$

Hence, the equation of the ellipse is $\frac{(x-1)^2}{64}+\frac{(y+1)^2}{16}=1$

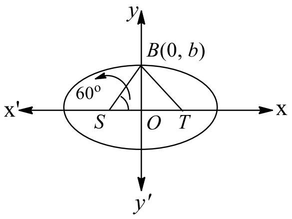
491 **(a)**

Equation of AB is

493 **(c)**

Let the equation of ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$



In ΔSOB,

$$\tan 60^\circ=\frac{OB}{OS}$$

$$\Rightarrow \sqrt{3}=\frac{b}{ae}$$

$$\Rightarrow \frac{b}{a}=e\sqrt{3}$$

Now, $e^2=1-\frac{b^2}{a^2}\Rightarrow e^2=1-3e^2$

$$\Rightarrow 4e^2=1\Rightarrow e=\frac{1}{2}$$

494 **(d)**

The locus of the point of intersection of the perpendicular tangents to ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, is a director circle and whose equation is given by

$$x^2+y^2=a^2+b^2$$

$\therefore$ Here, the equation of director circle is

$$x^2+y^2=9+4\ \Rightarrow\ x^2+\ y^2=13$$

495 **(b)**

Any tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

$y=mx\pm\sqrt{a^2m^2-b^2}$

or, $y=mx+x$, where $c=\pm\sqrt{a^2m^2-b^2}$ ... (i)

This will touch the hyperbola $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$,

$c^2=a^2-b^2m^2$

$\Rightarrow a^2m^2-b^2=a^2-b^2m^2$ [Using (i)]

$\Rightarrow m^2(a^2+b^2)=a^2+b^2\Rightarrow m=\pm 1$

Hence, the equations of the common tangents are

$y=\pm x\pm\ \sqrt{a^2-b^2}$

497 **(b)**

The required equation is

$4x+5y-20=8+5-20$ [Using : $T=S'$]

$\Rightarrow 4x+5y=13$

498 **(c)**

Let $P(x_1,y_2)$ be any point outside the circle.

Length of tangent to the circle $x^2+y^2+4x+3=0$ is

$$\sqrt{x_1^2+y_1^2+4x_1+3}$$

And length of tangent of the circle $x^2+y^2-6x+5=0$ is

$$\sqrt{x_1^2+y_1^2-6x_1+5}$$

$\therefore$ According to question,

$$\frac{\sqrt{x_1^2+y_1^2+4x_1+3}}{\sqrt{x_1^2+y_1^2-6x_1+5}}=\frac{2}{3}$$

$$\Rightarrow 9x_1^2+9y_1^2+36x_1+27-4x_1^2-4y_1^2+24x_1-20=0$$

$$\Rightarrow\ 5x_1^2+5y_1^2+60x_1+7=0$$

$\therefore$ Locus of point is $5x^2+5y^2+60x+7=0$

499 **(c)**

Let $S=x^2+y^2-2x$

At $P(-1,0), S_1=(-1)^2+0-2(-1)=3>0$

This point $P(-1,0)$ lies outside the circle

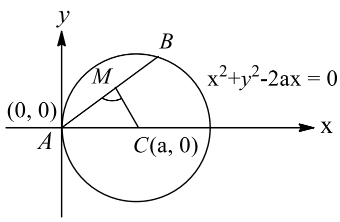
500 **(c)**

Here, $r=\sqrt{(4-2)^2+(6-3)^2}=13$

$\therefore$ area of circle$=\pi r^2=\pi\times 13=13\pi$ sq units

501 **(d)**

Let AB is a chord and its equation is $y=mx$ ...(i)



Equation of CM which is perpendicular to AB, is

$x+my=\lambda$

It passes through the centre $(a,0)$

$\Rightarrow x+my=a$ ...(ii)

On eliminating m from Eqs. (i) and (ii), we get

$x^2+y^2=ax$

$\Rightarrow x^2+y^2-ax=0$ is the locus of the centre of the required circle

502 **(d)**

Since, distance between directrices, $\frac{2a}{e}=10$

$$\Rightarrow\ a=\frac{10\times\sqrt{2}}{2}=5\sqrt{2}$$

∴Distance between foci, $2ae = 2 \times 5\sqrt{2} \times \sqrt{2}$

$= 20$

503 **(d)**

The given equation of ellipse can be rewritten as

$\frac{x^2}{16} + \frac{y^2}{9} = 1$

Here, $a = 4, b = 3$

∴ Length of latusrectum $= \frac{2b^2}{a} = \frac{2\times 9}{4} = \frac{9}{2}$

504 **(d)**

It is given that

$x^2 + y^2 = a^2$ ..(i)

and $xy = c^2$ ...(ii)

From Eq.(i) and (ii),

$x^2 + \frac{c^4}{x^2} = a^2$

$\Rightarrow x^4 - a^2x^2 + c^4 = 0$ ...(iii)

Now, x_1, x_2, x_3, x_4 will be roots of Eq.(iii)

$\therefore x_1 + x_2 + x_3 + x_4 = 0$

and $x_1x_2x_3x_4 = c^4$

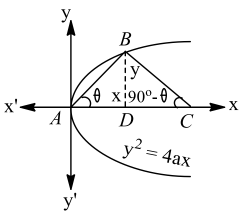
Similarly, $y_1 + y_2 + y_3 + y_4 = 0$

and $y_1y_2y_3y_4 = c^4$

505 **(c)**

In $\Delta\, ABD$, we have

$\tan\theta = \frac{y}{x}$ ... (i)



In ΔBCD, we have

$\tan(90° - \theta) = \frac{y}{CD}$

$\Rightarrow CD = y\tan\theta = \frac{y^2}{x}$ [using Eq. (i)]

$\Rightarrow CD = \frac{4ax}{x} = 4a$

506 **(c)**

The two circles are

$x^2 + y^2 - 4\,x - 6\,y - 3 = 0$ and, $x^2 + y^2 + 2\,x + 2\,y + 1 = 0$

The coordinates of the centres and radii are :

Centres: $C_1(2,3)$ $C_2(-1,-1)$

Radii: $r_1 = 4$ $r_2 = 1$

Clearly, $C_1C_2 = 5 = r_1 + r_2$

Therefore, there are 3 common tangents to the given circles

507 **(a)**

Let $P\ (at_1, 2at_1), Q\ (at_2^2, 2at_2)$ be a focal chord of the parabola $y^2 = 4ax$

Therefore, the tangents at P and Q meet at $[at_1t_2, a(t_1 + t_2)]$

Since, $t_1t_2 = -1$

$x_1 = -a$

and $y_1 = a(t_1 + t_2)$

and normal at P and Q, meet at

$[2a + a(t_1^2 + t_2^2 - 1), a(t_1 + t_2)]$

$\therefore\ x_2 = 2a + a(t_1^2 + t_2^2 - 1)$

and $y_2 = a(t_1 + t_2)$

$\therefore\ x_1x_2 + y_1y_2 = -a\,[2a + a(t_1^2 + t_2^2 - 1)] + a^2(t_1 + t_2)^2$

$= -3a^2$

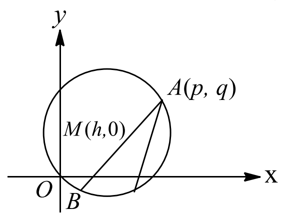
Now, $x_1x_2 + y_1y_2 = at_1^2 . at_2^2 + 2at_1 . 2at_2$

$= a^2(t_1t_2)^2 + 4a^2(t_1t_2)$

$= a^2 - 4a^2 = -3a^2$ $(\because\ t_1t_2 = -1)$

508 **(d)**

Suppose AB is a chord of the circle through $A(p,q)$ having $M(h,0)$ as its mid point. The coordinates of B are $(-p + 2h, -q)$



As B lies on the circle

$x^2 + y^2 = px + qy$, we have

$(-p + 2h)^2 + (-q)^2 = p(-p + 2h) + q(-q)$

$\Rightarrow 2p^2 + 2q^2 - 6ph + 4h^2 = 0$

$\Rightarrow 2h^2 - 3ph + p^2 + q^2 = 0$ ...(i)

As there are two distinct chords from $A(p,q)$ which are bisected on x-axis, there must be two distinct values of h satisfying Eq. (i)

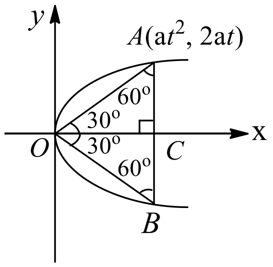
$D = 9p^2 - (4)(2)(p^2 + q^2) > 0$

$\Rightarrow\ p^2 > 8q^2$

509 **(d)**

In $\Delta\, OCA$, $\tan 30° = \frac{AC}{OC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{2at}{at^2} \Rightarrow t = 2\sqrt{3}$



Again , in $\Delta\, OCA$

$OA = \sqrt{OC^2 + AC^2}$

$= \sqrt{(at^2)^2 + (2at)^2}$

$= \sqrt{[(2\sqrt{3})^2]^2 a^2 + 4a^2(2\sqrt{3})^2}$

$= \sqrt{144a^2 + 48a^2} = \sqrt{192a^2}$

$\Rightarrow OA = 8\sqrt{3}a$

510 **(b)**

The coordinates of centres C_1 and C_2 of two circles are (1,0) and (2,3) respectively. Let r_1 and r_2 be the radii of two circles. Then, $r_1 = 2$ and $r_2 = \sqrt{21}$

Clearly, $r_1 - r_2 < C_1C_2 < r_1 + r_2$

Hence, the two circles intersect each other

511 **(b)**

Given equation of an ellipse can be rewritten as

$$\frac{(x-1)^2}{1/8} + \frac{(y+1)^2}{1/6} = 1$$

Here, $b > a$

Now, $e = \sqrt{1 - \frac{1/8}{1/6}} = \frac{1}{2}$

$\therefore$ Directrix, $y + 1 = \pm\left(\frac{\sqrt{1/6}}{1/2}\right)$ $\left[\because y = \pm\frac{b}{e}\right]$

$\Rightarrow y + 1 = \pm\frac{2}{\sqrt{6}} \Rightarrow 3y + 3 = \pm\sqrt{6}$

512 **(a)**

The line $y = mx + c$ touches the circle $x^2 + y^2 = r^2$, if and only if $c = \pm r\sqrt{1 + m^2}$ here we have line $3x - 2y = k$

$\Rightarrow y = \frac{3}{2}x - \frac{1}{2}k$

and circle $x^2 + y^2 = 4r^2$

$\therefore$ By condition $c = \pm a\sqrt{1 + m^2}$, we have

$-\frac{1}{2}k = \pm 2r\sqrt{1 + \frac{9}{4}}$

On squaring both sides, we get

$\frac{1}{4}k^2 = 4r^2\left(\frac{13}{4}\right)$

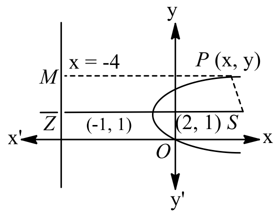
$\Rightarrow k^2 = 52r^2$

513 **(a)**

By the condition of parabola

$PM^2 = PS^2$

$\Rightarrow (x+4)^2 = (x-2)^2 + (y-1)^2$



$\Rightarrow y^2 - 2y - 12x - 11 = 0$

514 **(a)**

Let $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ be a tangent to the ellipse.

It is given that

$p = a\sec\theta$ and $q = b\,\text{cosec}\,\theta$

$\therefore \frac{a^2}{p^2} + \frac{b^2}{q^2} = 1$

515 **(d)**

Since, given that foci of an ellipse are (2, 2)and (4, 2) major axis is of length 10

$\Rightarrow 2ae = 2$ ...(i)

and $2a = 10 \Rightarrow a = 5$ ...(ii)

From Eqs. (i) and (ii),

$2 \times 5 \times e = 2$

$\Rightarrow e = \frac{1}{5}$

$\because b^2 = a^2(1 - e^2) \therefore b^2 = 25\left(1 - \frac{1}{25}\right) = 24$

and centre of an ellipse =mid point of foci =(3, 2)

Equation of an ellipse is

$$\frac{(x-3)^2}{25} + \frac{(y-2)^2}{24} = 1$$

516 **(a)**

Given tangents $5x - 12y + 10 = 0$ and $5x -$

$12y - 16 = 0$ are parallel

$\therefore$ Radius $= \left|\frac{c_1 - c_2}{2\sqrt{a^2 + b^2}}\right|$

$= \left|\frac{10 - (-16)}{2\sqrt{5^2 + (-12)^2}}\right| = \left|\frac{26}{2.13}\right| = 1$

517 **(b)**

As we know, if P is any point on the ellipse, then sum of focal distances of any point on the ellipse is equal to the length of major axis, $ie, PS + Ps' = 2a = 2 \cdot \sqrt{20} = 4\sqrt{5}$

518 **(b)**

Sum of ordinates of feet of normals drawn from a point is zero

So, there arithmetic mean is zero

519 **(b)**

Eliminating t from $x = t^2 + 1, y = 2t$, we obtain $y^2 = 4\,x - 4$

Substituting $x = 2s, y = \frac{2}{s}$ in $y^2 = 4x - 4$, we obtain

$2s^3 - s^2 - 1 = 0 \Rightarrow (s - 1)(2\,s^2 + s + 1) = 0$

$\Rightarrow s = 1$

Putting $s = 1$ in $x = 2s, y = \frac{2}{s}$, we obtain $x = 2, y = 2$

Hence, the required point is (2,2)

520 **(b)**

Transverse and conjugate axes of a rectangular hyperbola are equal i.e. $b = a$

$\therefore e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + 1} = \sqrt{2}$

521 **(c)**

Let the equation of circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

Since, circle (i) cuts the given circle orthogonally

$\therefore\ 2(-g)(3) + 2(-f)(-2) = c - 3$

$\Rightarrow\ -6g + 4f = c - 3$ ...(ii)

Also, Eq. (i) passes through (3, 0)

$\therefore\ 3^2 + 0^2 + 2g(3) + 2f(0) + c = 0$

$\Rightarrow\ 6g + c + 9 = 0$ ...(iii)

As Eq. (i) touches y-axis

$\therefore\ |-f| = \sqrt{g^2 + f^2 - c}$

$\Rightarrow\ g^2 = c$ ...(iv)

From Eqs. (iii) and (iv), we get

$g = -3$ and $c = 9$

$\therefore$ From Eq. (ii),

$-6(-3) + 4f = 9 - 3 \Rightarrow f = -3$

$\therefore$ Required equation of circle is

$x^2 + y^2 - 6x - 6y + 9 = 0$

522 **(a)**

Radius $= \sqrt{(a - \pi)^2 + (b - e)^2}$

=irrational$= k$

$\therefore$ Circle $(x - \pi)^2 + (y - e)^2 = k^2$

523 **(c)**

Given length of latusrectum $= \frac{2b^2}{a} = 9$

$\Rightarrow b^2 = \frac{9a}{2}$ ...(i)

and $e = \frac{5}{4}$

$\frac{25}{16} = 1 + \frac{b^2}{a^2}$

$\Rightarrow\ 1 + \frac{9a}{2a^2} = \frac{25}{16}$ [form eq.(i)]

$\Rightarrow \frac{9}{2a} = \frac{9}{16} \Rightarrow a = 8$

On putting the value of a in Eq. (i), we get

$b^2 = \frac{9 \times 8}{2} \Rightarrow b = 6$

$\therefore$ Equation of hyperbola is

$\frac{x^2}{8^2} - \frac{y^2}{6^2} = 1 \Rightarrow \frac{x^2}{64} - \frac{y^2}{36} = 1$

524 **(d)**

Given, S (6, 4) and $S'(-4, 4)$ and eccentricity, $e = 2$

$\therefore\ SS' = \sqrt{(6 + 4)^2 + (4 - 4)^2} = 10$

But $SS' = 2\,ae$

$\therefore\ 2a \times 2 = 10$

$\Rightarrow a = \frac{5}{2}$

And we know that,

$b^2 = a^2(e^2 - 1)$

$\Rightarrow\ b^2 = \frac{25}{4}(4 - 1) = \frac{75}{4}$

Centre of hyperbola is $\left(\frac{6+(-4)}{2}, \frac{4+4}{2}\right) = (1, 4)$

$\therefore$ Equation of hyperbola is $\frac{(x-1)^2}{\frac{25}{4}} - \frac{(y-4)^2}{\frac{75}{4}} = 1$

$\Rightarrow\ \frac{4(x - 1)^2}{25} - \frac{4(y - 4)^2}{75} = 1$

525 **(b)**

We have,

$y^2 = 4\,ax \Rightarrow 2y\frac{dy}{dx} = 4a \Rightarrow \frac{dy}{dx} = \frac{2a}{y}$

At (x_1, y_1), we have

Subtangent $= \frac{y_1}{(dy/dx)} = \frac{y_1}{2a/y_1} = \frac{y_1^2}{2a}$

Subnormal $= y_1 \dfrac{dy}{dx} = 2a$

Clearly, $y_1^2 = \left(\dfrac{y_1^2}{2a}\right) \times 2a$

i.e. $(\text{Ordinate})^2 = \text{Subtangent} \times \text{Subnormal}$

Hence, subtangent, ordinate and subnormal are in G.P.

527 **(a)**

Let the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

It passes through (2,1)

$\therefore \dfrac{4}{a^2} + \dfrac{1}{b^2} = 1$

$\Rightarrow \dfrac{4}{a^2} + \dfrac{1}{a^2(1-e^2)} = 1 \Rightarrow \dfrac{4}{a^2} + \dfrac{1}{a^2\left(1-\frac{1}{4}\right)} = 1$

$\Rightarrow \dfrac{4}{a^2} + \dfrac{4}{3\,a^2} = 1 \Rightarrow \dfrac{16}{3\,a^2} = 1 \Rightarrow a^2 = \dfrac{16}{3}$

$\therefore \dfrac{4}{a^2} + \dfrac{1}{b^2} = 1 \Rightarrow \dfrac{1}{b^2} = \dfrac{1}{4} \Rightarrow b^2 = 4$

Hence, the equation of the ellipse is

$\dfrac{3x^2}{16} + \dfrac{y^2}{4} = 1$ or, $3x^2 + 4y^2 = 16$

528 **(c)**

Given equation can be rewritten as

$\frac{(x-3)^2}{9} + \frac{(y-5)^2}{25} = 1 \quad [b > a]$

$\therefore\ e = \sqrt{1 - \dfrac{a^2}{b^2}} = \sqrt{1 - \dfrac{9}{25}} = \dfrac{4}{5}$

529 **(c)**

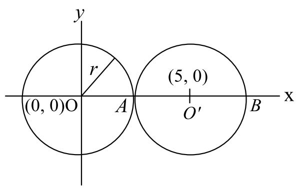
The equations of the given circles are

$x^2 + y^2 - 10x + 16 = 0$

$\Rightarrow (x-5)^2 + y^2 = 3^2 \quad$ ...(i)

Whose centre is $(5, 0)$ and radius=3

And $x^2 + y^2 = r^2 \quad$ ...(ii)



Whose centre is $(0, 0)$ and radius $= r$

Clearly, these two circles will intersect each other at two distinct points, if $r > OA$

$\Rightarrow r > 5 - 3 \ \Rightarrow r > 2$ and $r < OB$

$\Rightarrow r < 2 + 3 + 3 \ \Rightarrow r < B$

$\therefore\ 2 < r < 8$

530 **(c)**

Equation of normal is $y = mx - \frac{m}{2} - \frac{m^3}{4}\left(a = \frac{1}{4}\right)$. It passes through $(c, 0)$

$\therefore 0 = cm - \dfrac{m}{2} - \dfrac{m^3}{2} \Rightarrow m = 0$

And $\frac{m^2}{4} = c - \frac{1}{2} \Rightarrow c > \frac{1}{2}$

Then, all values of m are real

531 **(b)**

$2a = \dfrac{17}{8}.2b$

$\Rightarrow\ a = \dfrac{17}{8}b$

$\because b^2 = a^2(1-e^2)$

$\Rightarrow b^2 = \dfrac{289}{64}b^2(1-e^2)$

$\Rightarrow 1 - e^2 = \dfrac{64}{289}$

$\Rightarrow\ e^2 = \dfrac{225}{289}$

$\Rightarrow\ e = \dfrac{15}{17}$

532 **(c)**

Given equation can be rewritten as

$\frac{(x-1)^2}{9} - \frac{y^2}{3} = 1$ ...(i)

Then, equation of its conjugate hyperbola will be

$\frac{y^2}{3} - \frac{(x-1)^2}{9} = 1$ ...(ii)

Here, $a^2 = 9,\ b^2 = 3$

$\therefore\quad a^2 = b^2(e^2 - 1) \ \Rightarrow\ 9 = 3(e^2 - 1)$

$\Rightarrow e^2 - 1 = 3 \ \Rightarrow\ e = 2$

533 **(c)**

Let $P(h, k)$ be the pole of a focal chord of the parabola $y^2 = 4ax$. Then, the equation of the chord is

$ky - 2a(x + h) = 0$

It passes through $(a, 0)$

$\therefore a + h = 0$

Hence, the locus of (h, k) is $x + a = 0$ i.e. $x = -a$

Clearly, it is the directrix of the parabola

534 **(c)**

The equation of the given conic is

$4(x^2 - 6x + 9) + 16(y^2 - 2y + 1) = 53$

or, $4(x-3)^2 + 16(y-1)^2 = 53$

or, $\frac{(x-3)^2}{\frac{53}{4}} + \frac{(y-1)^2}{\frac{53}{16}} = 1$

Let e be the eccentricity of the above ellipse. Then,

$$e = \sqrt{1 - \frac{53/16}{53/14}} = \frac{\sqrt{3}}{2}$$

536 **(d)**

Let the coordinates of A and B be (x_1, y_1) and (x_2, y_2) respectively. Then, x_1, x_2 are roots of $x^2 + 2ax - b^2 = 0$ and y_1, y_2 are roots of $x^2 + 2px - q^2 = 0$

$\therefore x_1 + x_2 = -2a, x_1 x_2 = -b^2$

and $y_1 + y_2 = -2p, y_1 y_2 = -q^2$

Now,

$$\text{Radius} = \frac{1}{2} AB$$

$$\Rightarrow \text{Radius} = \frac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\Rightarrow \text{Radius} = \frac{1}{2}\sqrt{(x_1 + x_2)^2 + (y_1 + y_2)^2 - 4x_1x_2 - 4\,y_1y_2}$$

$$\Rightarrow \text{Radius} = \frac{1}{2}\sqrt{4a^2 + 4p^2 + 4b^2 + 4q^2} = \sqrt{a^2 + b^2 + p^2 + q^2}$$

537 **(a)**

The point $(a, 4)$ lies out side the circles $x^2 + y^2 + 10\,x = 0$ and $x^2 + y^2 - 12x + 20 = 0$.

Therefore,

$a^2 + 16 + 10a > 0$ and $a^2 + 16 - 12a + 20 > 0$

$\Rightarrow (a + 2)(a + 8) > 0$ and $(a - 6)^2 > 0$

$\Rightarrow (a + 2)(a + 8) > 0$ and $a \neq 6$ $\quad [\because (a - 6)^2 > 0$ for all $a \neq 6]$

$\Rightarrow a \in (-\infty, -8) \cup (-2, 6) \cup (6, \infty)$

538 **(a)**

Major axis $= 6 = 2a$

$\Rightarrow a = 3$

Also, $e = \frac{1}{2} \Rightarrow b = \frac{3\sqrt{3}}{2}$

Thus required equation is

$$\frac{(x - 7)^2}{9} + \frac{y^2}{\frac{27}{4}} = 1$$

$$\Rightarrow 3x^2 + 4y^2 - 42x + 120 = 0$$

539 **(d)**

Let point $P(x_1, y_1)$ be any point on the circle, therefore it satisfy the circle

$(x_1 - 3)^2 + (y_1 + 2)^2 = 5r^2$ ...(i)

The length of the tangent drawn from point $P(x_1, y_1)$ to the circle $(x - 3)^2 + (y + 2)^2 = r^2$ is

$\sqrt{(x_1 - 3)^2 + (y_1 + 2)^2 - r^2} = \sqrt{5r^2 - r^2}$ [from Eq. (i)]

$\Rightarrow 16 = 2r \Rightarrow r = 8$

$\therefore$ The area between two circles

$= \pi 5r^2 - \pi r^2 = 4\pi r^2 = 4\pi \times 8^2 = 256\pi$ sq units

540 **(b)**

Clearly, PA is the length of the tangent drawn from $P(2,1)$ to the circle $2(x^2 + y^2) - 3x + 4y = 0$

$$\Delta PA = \sqrt{4 + 1 - \frac{3}{2} \times 2 + 2 \times 1} = 2$$

541 **(c)**

Let equation of circle touching x-axis is

$x^2 - 2hx + h^2 + y^2 - 2ky = 0$

It passes through (2, 2) and (9, 9)

$\Rightarrow 4 - 4h + h^2 + 4 - 4k = 0$ ...(i)

and $81 - 18h + 81 + h^2 - 18k = 0$ ...(ii)

On solving Eqs. (i) and (ii), we get

$7h^2 - 252 = 0 \Rightarrow h = 6$

542 **(b)**

Since, the tangent to the parabola at point t_1 and t_2 are $t_1 y = x + at_1^2$ and $t_2 y = at_2^2$

Also, tangents are perpendicular to the parabola therefore, $\frac{1}{t_1} \cdot \frac{1}{t_2} = -1$ or $t_2 t_2 = -1$

We also know that their point of intersection is $[at_1 t_2, a(t_1 + t_2)]$ or $[-a, a(t_1 + t_2)]$

$\therefore$ Point of intersection lie on directrix $x = -a$ or $x + a = 0$

543 **(d)**

If y_1, y_2 and y_3 are the ordinates of three points on the parabola $y^2 = 4ax$, then the area of the triangle formed by them is given by

$$\Delta = \frac{1}{8a} |(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$$

Here, $a = 1, y_1 = 1, y_2 = 2$ and $y_3 = 4$

$\therefore$ Required area

$$= \frac{1}{8} |(1 - 2)(2 - 4)(4 - 1)| \text{sq. units}$$

$\Rightarrow$ Required area $= \frac{3}{4}$ sq. units

544 **(a)**

Let $P(c \cos \theta, c \sin \theta)$ be a point on $x^2 + y^2 = c^2$. Then, the chord of contact of tangents drawn from P to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

$$\left(\frac{c \cos \theta}{a^2}\right) x + \left(\frac{c \sin \theta}{b^2}\right) y = 1 \quad \text{... (i)}$$

Let $Q(h, k)$ be the mid-point of the chord of contact of tangents to the ellipse drawn from point P. Then, its equation is

$$\frac{hx}{a^2}+\frac{ky}{b^2}=\frac{h^2}{a^2}+\frac{k^2}{b^2} \quad \text{... (ii)}$$

Clearly, (i) and (ii) represent the same line

$$\therefore \frac{c\cos\theta}{h}=\frac{c\sin\theta}{k}=\frac{1}{\frac{h^2}{a^2}+\frac{k^2}{b^2}}$$

$$\Rightarrow \cos\theta=\frac{h}{c\left(\frac{h^2}{a^2}+\frac{k^2}{b^2}\right)} \text{ and } \sin\theta=\frac{k}{c\left(\frac{h^2}{a^2}+\frac{k^2}{b^2}\right)}$$

$$\Rightarrow \cos^2\theta+\sin^2\theta=\left(\frac{h^2}{c^2}+\frac{k^2}{c^2}\right)\cdot\frac{1}{\left(\frac{h^2}{a^2}+\frac{k^2}{b^2}\right)^2}$$

$$\Rightarrow c^2\left(\frac{h^2}{a^2}+\frac{k^2}{b^2}\right)^2=h^2+k^2$$

Hence, the locus of (h,k) is $c^2\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^2=x^2+y^2$

545 **(a)**

The combined equation of the lines joining the origin to the points of intersection of $x\cos\alpha+y\sin\alpha=p$ and $x^2+y^2-a^2=0$ is a homogeneous equation of second degree given by

$$x^2+y^2-a^2\left(\frac{a\cos\alpha+y\sin\alpha}{p}\right)^2=0$$

$$\Rightarrow [x^2(p^2-a^2\cos^2\alpha)+y^2(p^2-a^2\sin^2\alpha) - 2xya^2\sin\alpha\cos\alpha=0]$$

The lines given by this equation are at right angle, if

$(p^2-a^2\cos^2\alpha)+(p^2-a^2\sin^2\alpha)=0$

$\Rightarrow 2p^2=a^2(\sin^2\alpha+\cos^2\alpha)$

$\Rightarrow a^2=2p^2$

546 **(b)**

The equation $x^2-y^2=0$, is an equation of rectangular hyperbola. Therefore, the locus of the equation $x^2-y^2=0$ is a hyperbola

547 **(d)**

Since, the given hyperbola is a rectangular hyperbola, therefore the eccentricity of given hyperbola is $\sqrt{2}$

548 **(b)**

Centre of circle is (2, 4) and radius is 5. The line will intersect the circle at two distinct points, if the distance of (2, 4) from $3x-4y=m$ is less than radius of the circle.

ie, $\left|\frac{6-16-m}{5}\right|<5$

$\Rightarrow -25<10+m<25$

$\Rightarrow -35<m<15$

549 **(c)**

The length of the tangent drawn to the circle $x^2+y^2-2x+4y-11=0$ from the point (1, 3)

$=\sqrt{1^2+3^2-2.1+12-11}=\sqrt{22-13}=3$

550 **(b)**

Given , $\frac{x^2}{2}-\frac{y^2}{1}=1$

Here, $a^2=2, b^2=1$

Equation of asymptotes to the given hyperbola is

$\frac{x}{\sqrt{2}}-\frac{y}{1}=0$ and $\frac{x}{\sqrt{2}}+\frac{y}{1}=0$

Let $P(\sqrt{2}\sec\theta,\tan\theta)$ be any point, then product of length of perpendicular.

$$=\frac{\left[\frac{\sqrt{2}\sec\theta}{\sqrt{2}}-\frac{\tan\theta}{1}\right]\left[\frac{\sqrt{2}\sec\theta}{\sqrt{2}}+\frac{\tan\theta}{1}\right]}{\sqrt{\frac{1}{2}+\frac{1}{1}}\sqrt{\frac{1}{2}+\frac{1}{1}}}$$

$$=\frac{\sec^2\theta-\tan^2\theta}{\frac{3}{2}}$$

$$=\frac{2}{3}$$

552 **(b)**

We have, $x^2+y^2-8x+4y+4=0$

Here, centre=(4, -2)

And radius $=\sqrt{(4)^2+(-2)^2-4}=4$

Here, radius of circle is equal to x-coordinates of the centre

$\therefore$ Circle touches y-axis

553 **(c)**

Chord through intersection points P and Q of the given circles is $S_1-S_2=0$

$$\therefore (x^2+y^2+2ax+cy+a) - (x^2+y^2-3ax+dy-1)=0$$

$\Rightarrow 5ax+(c-d)y+a+1=0$

On comparing it with $5x+by-a=0$, we get

$\frac{5a}{5}=\frac{c-d}{b}=\frac{a+1}{-a}$

$\Rightarrow a(-a)=a+1$

$\Rightarrow a^2+a+1=0$

Which gives no real value of a

Hence, the line will passes through P and Q for no value of a

554 **(b)**

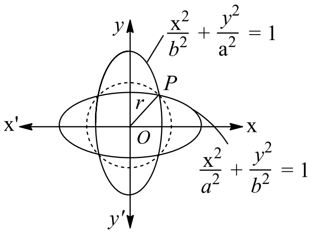
Focus is the mid-point of latusrectum

So, its coordinates are $\left(\frac{3-5}{2},\frac{6+6}{2}\right)=(-1,6)$

555 **(b)**

Given equation of hyperbola is $25x^2 - 16y^2 = 400$. If (6,2) is the mid point of the chord, then equation of chord is $T = S_1$.
$\Rightarrow 25(6x) - 16(2y) = (25)(36) - 16(4)$
$\Rightarrow 75x - 16y = 418$

556 **(d)**
Given equation of ellipse are
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$



The point of the intersection of these ellipse are

$$\left(\pm\frac{ab}{\sqrt{a^2+b^2}}, \pm\frac{ab}{\sqrt{a^2+b^2}}\right) ie,$$

$$P\left(\frac{ab}{\sqrt{a^2+b^2}}, \frac{ab}{\sqrt{a^2+b^2}}\right)$$

$\therefore$ The distance between $OP = r$

$$= \sqrt{\left(\frac{ab}{\sqrt{a^2+b^2}} - 0\right)^2 + \left(\frac{ab}{\sqrt{a^2+b^2}} - 0\right)^2}$$

$$= \frac{ab}{\sqrt{a^2+b^2}}\sqrt{2}$$

$\therefore$ Equation of circle is

$x^2 + y^2 = r^2$

$$\Rightarrow x^2 + y^2 = \frac{2a^2b^2}{a^2+b^2}$$

557 **(b)**
Let $(f, g)$and $(h, k)$are $(4t_1^2, 8t_1)$ and $(4t_2^2, 8t_2)$

respectively. Since, they are end points of a focal chord.

$\therefore t_1 t_2 = -1$

Now, $fh = 4t_1^2 . 4t_2^2 = 16(t_1 t_2)^2 = 16$

558 **(a)**
Since, the line $y - 3x = 0$ touches the circle
$\therefore$ radius =perpendicular distance from the centre (1, 1) to the tangent
$= \frac{|1-3|}{\sqrt{1+9}} = \frac{2}{\sqrt{10}}$ ...(i)
Let the other equation of tangent which is passing through origin is $y = mx$
Radius $= \frac{|1-m|}{\sqrt{1+m^2}}$
$\Rightarrow \frac{4}{10} = \frac{(1-m)^2}{(1+m^2)}$
$\Rightarrow 3m^2 - 10m + 3 = 0$
$\Rightarrow (3m-1)(m-3) = 0$
$\Rightarrow m = 3, \frac{1}{3}$
At $m = 3, y = 3x$ it is already given
At $m = \frac{1}{3}, 3y = x$

560 **(b)**
Let (h, k) be the mid-point of a chord of $y^2 = 4x$.
Then, its equation is
$ky - 2(x + h) = k^2 - 4h$ [Using : $T = S'$]
or, $ky - 2x - k^2 + 2h = 0$
This passes through the vertex (0,0)
$\therefore -k^2 + 2h = 0$
Hence, the locus of (h, k) is $-y^2 + 2x = 0$ or, $y^2 = 2x$

562 **(d)**
Let coordinates of O and A (0, 0) and $(at^2, 2at)$ respectively

$\therefore$ Coordinates of mid point of OA are

$$\left(\frac{0+at^2}{2}, \frac{0+2at}{2}\right) = \left(\frac{at^2}{2}, at\right)$$

Since , $(at^2) = 4\left(\frac{at^2}{2}\right)$

Hence, that locus of required point is $y^2 = 2x$

563 **(b)**
Let the equation of circle is
$x^2 + y^2 + 2gx + 2fy + c = 0$
It cuts the circle $x^2 + y^2 = 4$ orthogonally, if
$2g.0 + 2f.0 = c - 4 \Rightarrow c = 4$
$\therefore$ equation of circle is
$x^2 + y^2 + 2gx + 2fy + 4 = 0$
$\because$ It passes through the points (a, b)
$\therefore a^2 + b^2 + 2ag + 2fb + 4 = 0$
Locus of centre $(-g, -f)$ will be
$a^2 + b^2 - 2xa - 2yb + 4 = 0$
$\Rightarrow 2ax + 2by - (a^2 + b^2 + 4) = 0$

564 **(a)**
Let $P(t_1^2, 2t_1)$ be a point on $y^2 = 4x$ such that the normal to P cuts the parabola at $Q(t_2^2, 2t_2^2)$ and PQ subtends a right angle at the vertex. Then,
$t_2 = -t_1 - \frac{2}{t_1}$ and $t_1^2 = 2 \Rightarrow t_2 = -2\sqrt{2}$ and $t_1 = \sqrt{2}$

$\therefore PQ = \sqrt{(t_2^2 - t_1^2)^2 + 4(t_2 - t_1)^2} = \sqrt{36 + 18}$
$= 6\sqrt{3}$

565 **(a)**
The given equation can be written as
$-\frac{x^2}{4} + \frac{y^2}{3} = 1$
The eccentricity of this hyperbola is given by
$e = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{1 + \frac{4}{3}} = \sqrt{\frac{7}{3}}$

566 **(a)**
Clearly, centre of the circle is equidistant from the point (2,0) and y-axis.
Hence, the locus of the centre of the circle is a parabola having its focus at (2, 0) and directrix y-axis

567 **(c)**
The intersection of diameter lines is the centre of the circle, *ie*, $C(1, -1)$
$\therefore$ Required equation of circle is
$(x-1)^2 + (y+1)^2 = 7^2$
$\Rightarrow x^2 + y^2 - 2x + 2y - 47 = 0$

568 **(c)**
Let the coordinates of P are (x, y) according to given condition

$(x-1)^2 + (y-1)^2 = \frac{(x+y+2)^2}{2}$

$\Rightarrow x^2 + y^2 - 2xy - 8x - 8y = 0$

Here, $a = 1, b = 1, h = -1, g = -4, f = -4, c = 0$

Now , $abc + 2fgh - af^2 - bg^2 - ch^2$

$= 1.1.0 + 2(-4)(-4)(-1) - 1(-4)^2 - 1(-4)^2 - 0$

$= -64 \neq 0$

and $h^2 - ab = 1 - 1 = 0$

Since, $\Delta \neq 0$ and $h^2 = ab$

Hence, locus of P is a parabola

569 **(b)**
For the points of intersection of the two given curves

$C_1: y^2 = 4x$ and $C_2: x^2 + y^2 - 6x + 1 = 0$

We have, $x^2 + 4x - 6x + 1 = 0 \Rightarrow (x-1)^2 = 0$

$\Rightarrow x = 1, 1 \Rightarrow y = 2, -2$

Thus, the given curves touch each other at exactly two points (1, 2) and $(a, -2)$

570 **(d)**
Given, $x^2 + y^2 + 2gx + 2fy + c = 0$
$\therefore$ Radius of circle $= \sqrt{g^2 + f^2 - c}$
$= \sqrt{c - c} = 0$ [given$g^2 + f^2 = c$]

571 **(b)**
We know that the length $'l'$ of the chord intercepted by the circle $x^2 + y^2 = a^2$ on the straight line $y = mx + c$ is
$l = 2\sqrt{\frac{a^2(1+m^2) - c^2}{1+m^2}}$
Here, $a = 5$ and $y = mx + c$ passes through (2,3)
$\therefore 3 = 2m + c \Rightarrow c = 3 - 2m$
$\therefore 2\sqrt{\frac{a^2(1+m^2) - c^2}{1+m^2}} = 8$
$\Rightarrow 2\sqrt{\frac{25(1+m^2) - (3-2m)^2}{1+m^2}} = 8$
$\Rightarrow 5m^2 + 12m = 0 \Rightarrow m = 0, -12/5$
$\Rightarrow c = 3$ or, $39/5$ [Using $c = 3 - 2m$]
Hence, the equations of the required lines are
$y = 3$ and $12x + 5y = 39$

572 **(a)**
Normal at a point $(m^2, -2m)$ on the parabola $y^2 = 4x$ is given by $y = mx - 2m - m^3$. If this is normal to the circle also, then it will passes through centre $(-3, 6)$ of the circle
$\therefore 6 = -3m - 2m - m^3 \Rightarrow m = -1$
Since, shortest distance between parabola and circle will occurs along common normal
$\therefore$ Shortest distance = distance between $(m^2, -2m)$
And centre $(-3, 6)$ − radius of circle $= 4\sqrt{2} - 5$

573 **(b)**
Given, $\frac{x^2}{5} + \frac{y^2}{9} = 1$

Here, $a^2 = 5, \quad b^2 = 9$

Equation of normal to the ellipse at the point (0, 3) is

$\frac{x-0}{0/5} = \frac{y-3}{3/9} \left[\because \frac{x-x_1}{x_1/a^2} = \frac{y-y_1}{y_1/b^2}\right]$

$\Rightarrow \quad x = 0$

Which is the equation of y –axis

574 **(d)**

The equation of any normal to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

$ax\sec\phi - by\operatorname{cosec}\phi = a^2-b^2$ ... (i)

Given straight line $x\cos\alpha + y\sin\alpha = p$ will be a normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, if Eq.(i)and $x\cos\alpha + y\sin\alpha = p$ represent the same line.

$$\frac{a\sec\phi}{\cos\alpha} = -\frac{b\operatorname{cosec}\phi}{\sin\alpha} = \frac{a^2-b^2}{p}$$

$$\Rightarrow \cos\phi = \frac{ap}{(a^2-b^2)\cos\alpha},$$

$$\sin\phi = \frac{-bp}{(a^2-b^2)\sin\alpha}$$

$$\therefore \sin^2\phi + \cos^2\phi = 1$$

$$\Rightarrow \frac{b^2p^2}{(a^2-b^2)^2\sin^2\alpha} + \frac{a^2p^2}{(a^2-b^2)^2\cos^2\alpha} = 1$$

$$\Rightarrow p^2(b^2\operatorname{cosec}^2\alpha + a^2\sec^2\alpha) = (a^2-b^2)^2$$

575 **(c)**

Let the coordinates of P be (x_1, y_1). The equation of the chord of contact of tangents drawn from (x_1, y_1) to the parabola $y^2=4ax$ is

$yy_1 = 2a(x+x_1) \Rightarrow y = \frac{2a}{y_1}x + \frac{2a\,x_1}{y_1}$ ... (i)

It touches the hyperbola $x^2-y^2=a^2$

$\therefore 4a^2\frac{x_1^2}{y_1^2} = a^2 \times \frac{4a^2}{y_1^2} - a^2$

$\Rightarrow 4\,x_1^2 = 4a^2 - y_1^2 \Rightarrow 4x_1^2 + y_1^2 = 4a^2$

Hence, (x_1, y_1) lies on $4x^2+y^2=4a^2$

576 **(b)**

We know that the locus of the point of intersection of perpendicular tangents to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is the director circle given by $x^2+y^2=a^2+b^2$

Hence, the perpendicular tangents drawn to $\frac{x^2}{25}+\frac{y^2}{16}=1$ intersect on the curve $x^2+y^2=25+16$

i.e. $x^2+y^2=41$

577 **(a)**

Since, $PA.PB = PT^2$, where PT is length of tangent

Here, $PT = \sqrt{S_1} = \sqrt{1^2+4^2-16} = 1$

$\therefore PA.PB = 1$

578 **(b)**

Given that, circle $S_1 \equiv x^2+y^2+4x+22y+c=0$ bisects the circumference of the circle

$S_2 = x^2+y^2-2x+8y-d=0$

The common chord of the given circles is

$S_1 - S_2 = 0$

$\Rightarrow x^2+y^2+4x+22y+c-x^2-y^2+2x-8y+d=0$

$\Rightarrow 6x+14y+c+d=0$ ...(i)

So, Eq. (i) passes through the centre of the second circle, *ie*, $(1,-4)$

$\therefore 6-56+c+d=0$

$\Rightarrow c+d=50$

579 **(b)**

Equation of tangent to parabola $y^2 16x$ at $P(3,6)$ is

$6y = 8(x+3)$

$\Rightarrow 3y = 4x+12$

$\Rightarrow 3y-4x-12=0$

580 **(a)**

(α,β) lies on the director circle of the ellipse *ie*, on $x^2+y^2=9$

So, we can assume

$\alpha = 3\cos\theta$, $\beta = 3\sin\theta$

$\therefore F = 12\cos\theta + 9\sin\theta = 3(4\cos\theta + 3\sin\theta)$

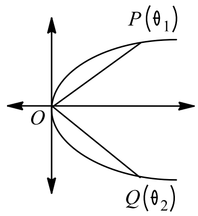
$\Rightarrow -15 \le F \le 15$

581 **(b)**

Let $P(a\cos\theta_1, b\sin\theta_1)$ and $Q(a\cos\theta_2, b\sin\theta_2)$ be two point on the ellipse. Then,

$m_1 =$ slope of $OP = \frac{b}{a}\tan\theta_1$

and $m_2 =$ slope of $OQ = \frac{b}{a}\tan\theta_2$



$$\therefore m_1m_2 = \frac{b^2}{a^2}\tan\theta_1\tan\theta_2$$

$$= \frac{b^2}{a^2} \times \frac{-a^2}{b^2}$$

$$\left[\because \tan\theta_1\ \tan\theta_2 = -\frac{a^2}{b^2}(\text{given})\right]$$

$= -1$

$\therefore < POQ = \frac{\pi}{2}$

Hence, PQ makes a right angle at the centre of the ellipse

582 **(b)**
Let the equation of circles be
$S_1 \equiv x^2 + y^2 + 13x - 3y = 0$ ...(i)
And $S_2 = 2x^2 + 2y^2 + 4x - 7y - 25 = 0$ ...(ii)
The equation of intersecting circle is $\lambda S_1 + S_2 = 0$
$\Rightarrow \lambda(x^2 + y^2 + 13x - 3y) + \left(x^2 + y^2 + 2x - \frac{7y}{2} - \frac{25}{2}\right) = 0$
$\Rightarrow \left[x^2(1+\lambda) + y^2(1+\lambda) + x(2+13\lambda) - y\left(\frac{7}{2} + 3\lambda\right) - \frac{25}{2}\right] = 0$ ...(iii)
$\therefore$ Centre $= \left(-\frac{(2+13\lambda)}{2(1+\lambda)}, \frac{(7/2)+3\lambda}{2(1+\lambda)}\right)$
$\because$ Centre lies on $13x + 30y = 0$
$\Rightarrow -13\left(\frac{2+13\lambda}{2}\right) + 30\left(\frac{(7/2)+3\lambda}{2}\right) = 0$
$\Rightarrow -26 - 169\lambda + 105 + 90\lambda = 0 \Rightarrow \lambda = 1$
Hence, putting the value of x in Eq. (iii), then required equation of circle is
$4x^2 + 4y^2 + 30x - 13y - 25 = 0$

583 **(b)**
Let $S_1 + \lambda S_2 = 0$
Since, it passes through (1, 1), then
$(1 + 1 + 3 + 7 + 2p - 5) + \lambda(1 + 1 + 2 + 2 - p^2) = 0$
$\Rightarrow \lambda = -\frac{7+2p}{6-p^2}$
But $p^2 \neq 6 \Rightarrow p \neq \pm\sqrt{6}$
But the other circle $x^2 + y^2 + 2x + 2y - 6 = 0$ at $p = \pm\sqrt{6}$ also satisfy the point (1, 1)
So, $p = \pm\sqrt{6}$ is valid
Now, $\lambda \neq -1 \Rightarrow \frac{7+2p}{6-p^2} \neq 1$
$\Rightarrow 7 + 2p \neq 6 - p^2$
$\Rightarrow p^2 + 2p + 1 \neq 0 \Rightarrow p \neq -1$

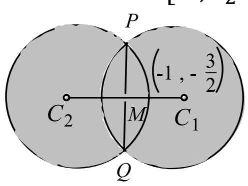
584 **(a)**
The equation of normal is
$y = mx - 8m - 4m^3$ $(\because y = mx - 2am - am^3)$
Since, it is passing through (2, 0)
$\therefore 0 = 2m - 8m - 4m^3$
$\Rightarrow m = 0$ and $2m^2 = -3$ (no real value exist)
Only one real value of m exist
$\therefore$ One normal can be drawn

585 **(b)**
Given, $\frac{x^2}{24} + \frac{y^2}{13.5} = 1$
$\therefore SP + S'P = 2a = 4\sqrt{6}$

586 **(b)**
Since, tangent at P and Q on the parabola meet in T
If the coordinates of P and Q are $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ respectively, then coordinates of T are $\{at_1t_2, a(t_1 + t_2)\}$
$\therefore SP = a(1 + t_1^2)$
$SP = a(1 + t_2^2)$
$ST^2 = a^2(1 - t_1t_2)^2 + a^2(t_1 + t_2)^2$
$= a^2(1 + t_1^2 + t_2^2 + t_1^2t_2^2)$
$= a(1 + t_1^2)a(1 + t_2^2) = SP \cdot SQ$
Thus, SP, ST, SQ are in GP

587 **(b)**
Equation of tangent at $(3\sqrt{3}\cos\theta, \sin\theta)$ to the ellipse $\frac{x^2}{27} + y^2 = 1$ is $\frac{x\cos\theta}{3\sqrt{3}} + y\sin\theta = 1$
It cuts intercepts on the coordinate axes.
$\therefore$ Sum of intercepts on axes is
$3\sqrt{3}\sec\theta + \text{cosec}\theta = f(\theta)$ (say)
On differentiating w. r. t. θ
$f'(\theta) = \frac{3\sqrt{3}\sin^3\theta - \cos^3\theta}{\sin^2\theta\cos^2\theta}$
For maxima and minima, put $f'(\theta) = 0$
$\Rightarrow 3\sqrt{3}\sin^3\theta - \cos^3\theta = 0$
$\Rightarrow \tan^3\theta = \frac{1}{3\sqrt{3}}$
$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$
At $\theta = \frac{\pi}{3}, f''(\theta) > 0$
$\therefore f(\theta)$ is minimum at $\theta = \frac{\pi}{6}$

588 **(b)**
The equation of common chord PQ is
$2x + 1 = 0$ $[ie, S_2 - S_1 = 0]$



Here, $C_1 = \left(-1, \frac{-3}{2}\right)$, $r_1 = \frac{3}{2} = C_1P$
and $C_2 = \left(-2, \frac{-3}{2}\right)$, $r_2 = \frac{\sqrt{17}}{2}$
$C_1M =$ Perpendicular distance from C_1 to the common chord
$\therefore C_1M = \frac{|-2+1|}{\sqrt{2^2}} = \frac{1}{2}$

Now, $PQ = 2PM = 2\sqrt{(C_1M)^2 - (C_1M)^2} = 2\sqrt{\frac{9}{4} - \frac{1}{4}} = 2\sqrt{2}$

589 **(c)**
The centres and radii of given circles are
$C_1(5, 0), C_2(0, 0)$ and
$r_1 = \sqrt{25 + 0 - 16} = 3$
$r_2 = r$
Now, $C_1C_2 = 5$
For intersection of two circle,
$r_2 - r_1 < C_1C_2 < r_1 + r_2$
$\Rightarrow \ r - 3 < 5 < 3 + r$
$\Rightarrow \ r < 8$ and $r > 2$
$\Rightarrow \ 2 < r < 8$

590 **(d)**
Given straight lines form a triangle. So, there will be an in-circle and three ex-circles touching all the sides

591 **(a)**
We know that $y = m_1x$ and $y = m_2x$ are conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if
$m_1m_2 = -\frac{b^2}{a^2}$
Here, $m_1 = \frac{b}{a}$. Therefore, $m_2 = -\frac{b}{a}$
Hence, $y = -\frac{bx}{a}$ is the required diameter

592 **(a)**
Given equation of parabola can be rewritten as
$(y - 1)^2 = 4(x - 1)$
Axis of parabola is $y = 1$, equation of normal is
$(y - 1) = m(x - 1) - 2m - m^3$
Let $(h, 1)$ is a point on it's axis, then
$0 = m(h - 1) - 2m - m^3$
$\Rightarrow m^2 = h - 3$
$\Rightarrow h \geq 3$ for real values of m

593 **(a)**
Let the variable circle be
$x^2 + y^2 + 2gx + c = 0$ ...(i)
It passes through (2,0) and touches y-axis
$\therefore 4 + 4g + c = 0$ and $c = f^2$
$\Rightarrow 4 + 4g + f^2 = 0$
Hence, the locus of the centre $(-g, -f)$ of circle (i) is
$4 - 4x + y^2 = 0$
$\Rightarrow y^2 = 4(x - 1)$, which is a parabola

594 **(c)**
Let the equation of the centric circles be $x^2 + y^2 - 2x - 4y + \lambda = 0$, it passes through (3, 4)
$\therefore 3^2 + 4^2 - 2(3) - 4(4) + \lambda = 0$
$\Rightarrow \ \lambda = -3$
Thus, the equation of concentric circle is
$x^2 + y^2 - 2x - 4y - 3 = 0$

595 **(b)**
Clearly, required point is the point of intersection of the line $y = 2x + 11$ and the line perpendicular to it passing through the centre of the circle.
The coordinates of the centre are $(-1, 1/4)$
The equation of the line through $(-1, 1/4)$ and perpendicular to $y = 2x + 11$ is
$y - \frac{1}{4} = -\frac{1}{2}(x + 1) \Rightarrow 2x + 4y + 1 = 0$
Clearly, $(-9/2, 2)$ is the point of intersection of $y = 2x + 11$ and $2x + 4y + 1 = 0$
So, the coordinates of the required point are (9/2,2)

596 **(d)**
Equation of ellipse is $\frac{x^2}{2} + \frac{y^2}{1} = 1$. General equation of tangent to the ellipse of slope m is
$y = mx \pm \sqrt{2m^2 + 1}$
Since, this is equally inclined to axes, so $m = \pm 1$.
Thus, tangents are
$y = \pm x \pm \sqrt{2 + 1} = \pm x \pm \sqrt{3}$
Distance of any tangent from origin
$= \frac{|0 + 0 \pm \sqrt{3}|}{\sqrt{1^2 + 1^2}} = \frac{\sqrt{3}}{2}$

598 **(c)**
The centres of given circles are $C_1(1, 0), C_2(2, 3)$ and
$r_1 = \sqrt{1^2 + 0 + 3} = 2, \quad r_2 = \sqrt{4 + 9 + 8} = \sqrt{21}$
Now, $C_1C_2 = \sqrt{(2 - 1)^2 + (3 - 0)^2} = \sqrt{10} = 3.16$
and $r_1 + r_2 = 2 + \sqrt{21} = 6.58$
Hence, two circles intersect, each other at two points

599 **(c)**
Let e and e' are the eccentricities of a hyperbola and its conjugate hyperbola.

Then, $\frac{1}{e^2} + \frac{1}{(e')^2} = 1 \Rightarrow \frac{1}{3} + \frac{1}{(e')^2} = 1 \Rightarrow e' = \sqrt{\frac{3}{2}}$

600 **(a)**
We know that the normal drawn at a point $P(at_1^2, 2at_1)$ to the parabola $y^2 = 4ax$ meets again the parabola at $Q(at_2^2, 2at_2)$, then

$t_2 = -t_1 - \frac{2}{t_1}$

Here, $t_1 = P$ and $t_2 = Q$

$\therefore q = -p - \dfrac{2}{p}$

$\Rightarrow\ p^2 + pq + 2 = 0$

601 **(c)**

Let $P(x, y)$ be any point on the conic. Then,

$\sqrt{(x-1)^2 + (y+1)^2} = \sqrt{2}\left|\dfrac{x-y+1}{\sqrt{2}}\right|$ [Using: $SP = e\,PM$]

$\Rightarrow 2\,xy - 4x + 4y + 1 = 0$

602 **(b)**

The equation of the chord of the circle $x^2 + y^2 - 2x - 8 = 0$ having (2,2) as its mid-point is

$2x + 2y - (x+2) - 8 = 4 + 4 - 4 - 8$ [Using : $S' = T$]

$\Rightarrow x + 2y - 6 = 0$

The equation of a circle passing through P and Q is

$x^2 + y^2 - 2x - 8 + \lambda(x + 2y - 6) = 0$ ...(i)

It passes through (1,2)

$\therefore 1 + 4 - 2 - 8 + \lambda(1 + 4 - 6) = 0 \Rightarrow \lambda = -5$

Putting the value of λ in (i), we obtain

$x^2 + y^2 - 7x - 10y + 22 = 0$

As the equation of the required circle

603 **(d)**

Given equation of circle is $x^2 + y^2 - 3x - 4y + 2 = 0$ and it cuts the x-axis

$\therefore y = 0$

The equation of the circle becomes

$x^2 + 0 - 3x - 4(0) + 2 = 0$

$\Rightarrow x^2 - 3x + 2 = 0 \ \Rightarrow x^2 - 2x - x + 2 = 0$

$\Rightarrow (x-1)(x-2) = 0 \ \Rightarrow x = 1, 2$

Therefore, the points are (1, 0), (2, 0)

604 **(a)**

Since the normal at $(ap^2, 2\,ap)$ on $y^2 = 4\,ax$ meets the curve again at $(aq^2, 2\,aq)$. Therefore, $px + y = 2\,ap + ap^3$ passes through $(aq^2, 2\,aq)$

$\Rightarrow paq^2 + 2\,aq = 2\,ap + ap^3$

$\Rightarrow p(q^2 - p^2) = 2(p - q)$

$\Rightarrow p(q + p) = -2$ $[\because p \neq q]$

$\Rightarrow p^2 + pq + 2 = 0$

605 **(b)**

Let the equation of L_1 be$y = mx$. Since, the intercepts made by the circle on L_1 and L_2 are equal, their distance from the centre of the circle are also equal

The centre of the given circle is $\left(\frac{1}{2}, -\frac{3}{2}\right)$

$\therefore \left|\dfrac{\frac{1}{2} - \frac{3}{2} - 1}{\sqrt{1+1}}\right| = \left|\dfrac{m \times \frac{1}{2} + \frac{3}{2}}{\sqrt{m^2+1}}\right|$

$\Rightarrow \dfrac{2}{\sqrt{2}} = \dfrac{|m+3|}{2\sqrt{m^2+1}}$

$\Rightarrow 8(m^2 + 1) = (m+3)^2$

$\Rightarrow 7m^2 - 6m - 1 = 0$

$\Rightarrow (m-1)(7m+1) = 0$

$\Rightarrow m = 1$ or $m = -\dfrac{1}{7}$

So, the equations representing L_1 are

$y = x$ or $y = \left(-\dfrac{1}{7}\right)x$

$\Rightarrow x - y = 0$ or $x + 7y = 0$

606 **(d)**

Given hyperbola is a rectangular hyperbola. So, its asymptotes are at right angle

607 **(b)**

Since, $\frac{a}{e} - ae = 4$ and $e = \frac{1}{2}$

$\therefore\ 2a - \dfrac{a}{2} = 4$

$\Rightarrow a = \dfrac{8}{3}$

608 **(a)**

Let $y = mx$ be a tangent drawn from the origin to the circle having its centre at $(2, -1)$ and touching $3x + y = 0$.

Then,

$\left|\dfrac{2m+1}{\sqrt{m^2+1}}\right| = \left|\dfrac{6-1}{\sqrt{9+1}}\right|$

$\Rightarrow 2(2m+1)^2 = 5(m^2+1)$

$\Rightarrow 3m^2 + 8m - 3 = 0 \Rightarrow (3m-1)(m+3) = 0$

$\Rightarrow m = -3, \dfrac{1}{3}$

Thus, the equation of the tangents drawn from the origin are $y = -3x$ and $y = x/3$

609 **(d)**

The centre of the required circle is the image of the centre $(-8,12)$ with respect to the line mirror $4\,x + 7\,y + 13 = 0$ and radius equal to the radius of the given circle. Let (h, k) be the image of the point $(-8,12)$ with respect to the line mirror.

$4x + 7y + 13 = 0$. Then,

$\dfrac{h - (-8)}{4} = \dfrac{k - 12}{7} = -2\left(\dfrac{4 \times -8 + 7 \times 12 + 13}{4^2 + 7^2}\right)$

$\Rightarrow h = -16, k = -2$

Thus, the centre of the image circle is $(-16, -2)$.

The radius of the image circle is same as that of the given circle i.e.5.

Hence, the equation of the required circle is

$(x+16)^2 + (y+2)^2 = 5^2$

$\Rightarrow x^2 + y^2 + 32\,x + 4\,y + 235 = 0$

610 **(d)**

We have,

$x^2 + y^2 + 4x + 6y + 13 = 0$

$\Rightarrow (x+2)^2 + (y+3)^2 = 0$

$\Rightarrow x + 2 = 0, y + 3 = 0 \Rightarrow x = -2, y = -3$

Hence, the given equation represents the point $(-2, -3)$

611 **(d)**

The radical axis of circle Ist and IInd is

$S_1 - S_2 = 0$

$\Rightarrow \quad (x^2 + y^2 - 16x + 60) - (x^2 + y^2 - 12x + 27) = 0$

$\Rightarrow \quad -4x + 33 = 0 \quad \Rightarrow \quad x = \frac{33}{4}$ ...(i)

The radical axis of circle IInd and IIIrd is

$S_2 - S_3 = 0$

$\Rightarrow \quad (x^2 + y^2 - 12x + 27) - (x^2 + y^2 - 12y + 8) = 0$

$\Rightarrow \quad -12x + 12y + 19 = 0$ ...(ii)

$\therefore$ From Eqs. (i) and (ii), we get radical centre $\left(\frac{33}{4}, \frac{20}{3}\right)$

612 **(d)**

Equation of the tangent at (x_1, y_1) is

$xx_1 - yy_1 - 4(x + x_1) + (y + y_1) + 11 = 0$

Put $x_1 = 2$ and $y_1 = 1$, we get

$2x - y - 4(x + 2) + (y + 1) + 11 = 0$

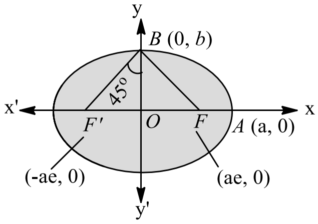
$\Rightarrow -2x - 8 + 12 = 0$

$\Rightarrow \quad x - 2 = 0$

613 **(d)**

Since, $\angle FBF'' = 90°$, then

$\angle OBF'' = 45°$ and $\angle BF''O = 45°$



$\Rightarrow ae = b$

[$\because \Delta BOF''$ is an isosceles traiangle]

and $\quad e^2 = 1 - \frac{b^2}{a^2}$

$\Rightarrow \quad e^2 = 1 - \frac{a^2e^2}{a^2}$

$\Rightarrow \quad e = \frac{1}{\sqrt{2}}$ [$\because e$ cannot be negative]

614 **(c)**

Let θ be the eccentric angle of the point of contact P(say)

Then, the coordinates of P are $(a\cos\theta, b\sin\theta)$

The equation of tangent at P is $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$

But, $\frac{x}{a} + \frac{y}{b} = \sqrt{2}$ is tangent at P

$\therefore \cos\theta = \frac{1}{\sqrt{2}}$ and $\sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$

615 **(b)**

We have,

$5x^2 + 9y^2 = 45 \Rightarrow \frac{x^2}{9} + \frac{y^2}{5} = 1$

Here $a^2 = 9, b^2 = 5$ and the major axis is along x −axis

$\therefore L.R. = \frac{2b^2}{a} = \frac{2(5)}{3} = \frac{10}{3}$

616 **(c)**

$\therefore$ The intersection of two diameter is the centre of circle, is $(1, -1)$

Let r be the radius of circle, then

$\Rightarrow$ Area of circle $\pi r^2 = 49\pi \quad \Rightarrow \quad r = 7$ unit

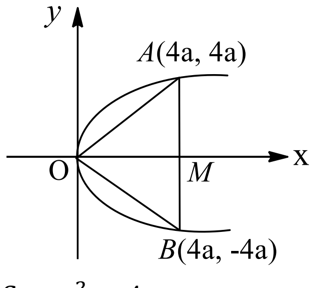
$\therefore$ Equation of required circle is

$(x-1)^2 + (y+1)^2 = 49$

$\Rightarrow \quad x^2 + y^2 - 2x + 2y - 47 = 0$

617 **(b)**

Given equation of parabola is $y^2 = 4ax$. Since, $AB = 8a$, it means ordinate of A and B respectively $4a$ and $-4a$. General point on this parabola is $(at^2, 2at) \Rightarrow t = \pm 2$



So, $at^2 = 4a$

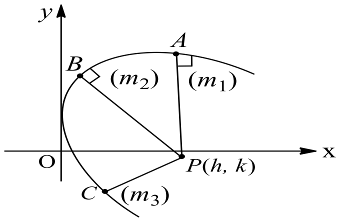
$\therefore OM = 4a, AM = 4a$

So, $\angle AOM = 45°$

$\therefore$ The angle AOB is $90°$

618 **(d)**

It is clear from the figure, that only one common tangent is possible



620 **(a)**

The chord of contact of (α, β) is $\frac{x\alpha}{a^2}+\frac{y\beta}{b^2}=1$. It touches the circle $x^2+y^2=c^2$

$\therefore \frac{1}{\sqrt{\frac{\alpha^2}{a^4}+\frac{\beta^2}{b^4}}}=c$

$\Rightarrow \frac{\alpha^2}{a^4}+\frac{\beta^2}{b^4}=\frac{1}{c^2}$

Thus, the locus of (α, β) is

$\frac{x^2}{a^4}+\frac{y^2}{b^4}=\frac{1}{c^2}$

621 **(c)**

Since, asymptotes $3x+4y=2$ and $4x-3y+5=0$ are perpendicular to each other.

Hence, hyperbola is rectangular hyperbola but we know that the eccentricity of rectangular hyperbola is $\sqrt{2}$.

622 **(d)**

If the normal at $(at_1^2, 2\,at_1)$ on $y^2=4\,ax$ meets the curve again at $(at_2^2, 2\,at_2)$, then

$t_2=-t_1-\frac{2}{t_1}$

The values of parameter t_1 for the point $(a, 2a)$ is given by $a\,t_1^2=a$ and $2\,at_1=2\,a$

$\Rightarrow t_1=1$

$\therefore t_2=-t_1-\frac{2}{t_1}$

$\Rightarrow t_2=-1-\frac{2}{-1}=-3$

Hence, $t=-3$

623 **(a)**

We have,

$\frac{x}{a}\times\frac{y}{b}=(\cosh\theta+\sinh\theta)(\cosh\theta-\sinh\theta)$

$\Rightarrow \frac{xy}{ab}=\cosh^2\theta-\sinh^2\theta=1$

$\Rightarrow xy=ab$, which is a hyperbola

624 **(c)**

Clearly, $x-2y-a=0$ is a focal chord of slope 1/2

$\therefore$ Length of the chord $=4a\operatorname{cosec}^2\theta=4a(1+4)=20\,a$

625 **(c)**

The equation of the normal to $y^2=4\,ax$ at (x_1, y_1) is

$y-y_1=-\frac{y_1}{2\,a}(x-x_1)$

So, the equation of the normal at $\left(\frac{a}{m^2}, \frac{2\,a}{m}\right)$ is

$y-\frac{2\,a}{m}=-\frac{1}{m}\left(x-\frac{a}{m^2}\right)$

$\Rightarrow m^3y-2\,am^2=-m^2x+a \Rightarrow m^3y$
$=2\,am^2-m^2x+a$

626 **(c)**

The tangent at the point of shortest distance from the line $x+y=7$ parallel to the given line

Any point on the given ellipse is

$(\sqrt{6}\cos\theta, \sqrt{3}\sin\theta)$

Equation of the tangent is

$\frac{x\cos\theta}{\sqrt{6}}+\frac{y\sin\theta}{\sqrt{3}}=1$. It is parallel to $x+y=7$

$\Rightarrow \frac{\cos\theta}{\sqrt{6}}=\frac{\sin\theta}{\sqrt{3}}$

$\Rightarrow \frac{\cos\theta}{\sqrt{2}}=\frac{\sin\theta}{1}=\frac{1}{\sqrt{3}}$

The required point is $(2, 1)$

627 **(a)**

We have,

$x^2+y^2-2x+4y-4=0$

$\Rightarrow (x^2-2x+1)+(y^2+4y+4)=3^2$

$\Rightarrow (x-1)^2+(y+2)^2=3^2$

The equation of any tangent of slope m is given by

$y+2=m(x-1)\pm 3\sqrt{1+m^2}$

628 **(b)**

We know that the limit points other than the origin of the coaxial syatem of circles $x^2+y^2+2\,gx+2\,fy+c=0$ are given by

$\left(-\frac{gc}{g^2+f^2}, -\frac{fc}{g^2+f^2}\right)$

Here, $g=-3, f=-4, c=1$

Hence, other limiting point is $(3/25, 4/25)$

629 **(b)**

We have,

$m=$ Slope of the tangent $=-\frac{5}{12}$

If a line of slope m is tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, then the coordinates of the point of contact are

$\left(\pm\frac{a^2m}{\sqrt{a^2m^2-b^2}}, \frac{b^2}{\sqrt{a^2m^2-b^2}}\right)$

Here, $a^2=9, b^2=1$ and $m=-5/12$

So, points of contact are $(\mp 5, \pm 4/3)$ $i.e$ $(-5, 4/3)$ and $(5, -4/3)$. Out of these two points $(5, -4/3)$ lies on the line $5x+12y=9$.

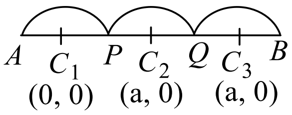
Hence, $(5, -4/3)$ is the required point

630 **(a)**

We know that, if two perpendicular tangents to

the circle $x^2 + y^2 = a^2$ meet at P, then the point P lies on a director circle
Thus, the equation of director circle to the circle $x^2 + y^2 = a^2$ is $x^2 + y^2 = 2a^2$
Which is the required locus of point P

631 **(b)**
Since, $AP = PQ = QB$. The coordinates of P are $(a, 0)$ and of Q are $(2a, 0)$ the centre of the circles on AP, pQ and QB asdiameters are respectively $C_1\left(\frac{a}{2}, 0\right), C_2\left(\frac{3a}{2}, 0\right)$ and $C_3\left(\frac{5a}{2}, 0\right)$ and the radius of each one of them is $\left(\frac{a}{2}\right)$



Hence, the equations of the circles with centre C_1, C_2 and C_3 are respectively
$$\left(x - \frac{a}{2}\right)^2 + y^2 = \frac{a^2}{4};\ \left(x - \frac{3a}{2}\right)^2 + y^2 = \frac{a^2}{4}$$
and $\left(x - \frac{5a}{2}\right)^2 + y^2 = \frac{a^2}{4}$
So that, if $S(h, k)$ be any point on the locus, then
$$\left(h - \frac{a}{2}\right)^2 + \left(h - \frac{3a}{2}\right)^2 + \left(h - \frac{5a}{2}\right)^2 + 3\left(k^2 - \frac{a^2}{4}\right) = b^2$$
$\Rightarrow\ 3(h^2 + k^2) - 9ah + 8a^2 = b^2$
Hence, the locus of $S(h, k)$ is
$3(x^2 + y^2) - 9ax + 8a^2 - b^2 = 0$

632 **(d)**
Since, The intersection of a line $y = 2x + c$ and a parabola $y^2 = 4ax + 4a^2$ is

$(2x + c)^2 = 4ax + 4a^2$

$\Rightarrow 4x^2 + 4(c - a)x + (c^2 - 4a^2) = 0$

Since, it is a tangent line

$\therefore 16(c - a)^2 - 4 \times 4(c^2 - 4a^2) = 0\ \ [\because D = 0]$

$\Rightarrow\ c^2 + a^2 - 2ac - c^2 + 4a^2 = 0$

$\Rightarrow\ c = \dfrac{5a}{2}$

633 **(c)**
The interesetion of line and circle is (0, 0) and (2, -2). Now, taking option (c),

ie $y^2 = 2x$

At point (0,0) $\Rightarrow 0 = 0$

and at point $(2, -2) \Rightarrow (-2)^2 = 2(2)\ \Rightarrow\ 4 = 4$

Hence, option (c) is the correct answer

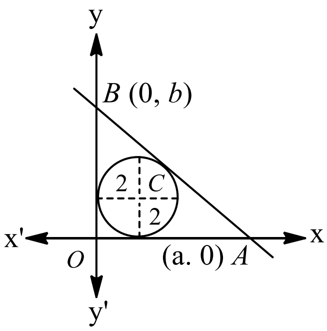
634 **(c)**
We know, if $lx + my + n = 0$ is normal to the hyperbola

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

Then, $\frac{a^2}{l^2} - \frac{b^2}{m^2} = \frac{(a^2+b^2)^2}{n^2}$

Put, $n = -1$, therefore, $\frac{a^2}{l^2} - \frac{b^2}{m^2} = (a^2 + b^2)^2$

635 **(b)**
Let the equation of AB be $\frac{x}{a} + \frac{x}{b} = 1$
Since, the line AB touches the circle
$x^2 + y^2 - 4x - 4y + 4 = 0$
$$\therefore\ \frac{\left|\frac{2}{a} + \frac{2}{b} - 1\right|}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} = 2$$



[Since, $O(0, 0)$ and $C(2, 2)$ lie on the same side of AB, therefore $\frac{2}{a} + \frac{2}{b} - 1 < 0$]
$$\Rightarrow\ \frac{-(2b) + 2a - ab}{\sqrt{a^2 + b^2}} = 2$$
$\Rightarrow 2a + 2b - ab + 2\sqrt{a^2 + b^2} = 0$ ...(i)
Since, ΔOAB is a right angled triangle. So, its circumcentre is the mid point of AB
$\therefore\ h = \frac{a}{2}$ and $k = \frac{b}{2}\ \Rightarrow\ a = 2h$ and $b = 2k$ ...(ii)
From Eqs. (i) and (ii), we get
$4h + 4k - 4hk + 2\sqrt{4h^2 + 4k^2} = 0$
$\Rightarrow\ h + k - hk + \sqrt{h^2 + k^2} = 0$
So, the locus of $P(h, k)$ is
$x + y - xy + \sqrt{x^2 + y^2} = 0\ \ \therefore k = 1$

636 **(b)**
Let $P(a\cos\theta\, b\sin\theta), Q(a\cos\theta, -b\sin\theta)$

Let a point $R(h, k)$ divides the line joining the points P and Q internally in the ration 1:2

Then , $PR:RQ = 1:2$

Now, by division formula

$h = a\cos\theta \Rightarrow \cos\theta = \frac{h}{a}$ ...(i)

and $k = \frac{b}{3}\sin\theta$

$\Rightarrow \sin\theta = \frac{3k}{b}$ ... (ii)

On squaring and adding Eqs. (i) and (ii), we get

$\frac{h^2}{a^2} + \frac{9k^2}{b^2} = 1$

Hence locus of R is

$\frac{x^2}{a^2} + \frac{9y^2}{b^2} = 1$

637 **(c)**
Given vertex of parabola $(h, k) \equiv (1, 1)$
And its focus is $(a + h, k) \equiv (3, 1)$
Or $a + h = 3$
$\Rightarrow a = 2$
Since, y-coordinate of vertex and focus are same, therefore axis of parabola to x-axis. Thus, equation of parabola is
$(y - k)^2 = 4a(x - h)$
$\Rightarrow (y - 1)^2 = 8(x - 1)$

638 **(b)**
Obviously, point (5, 5) lies only on the circle $x^2 + y^2 - 18x - 16y + 120 = 0$, also radius of this circle is 5
Hence, option (b) is correct

639 **(b)**
The equation of the chord of contact of tangent drawn from a point $P(x_1, y_1)$ to $x^2 + y^2 = a^2$ is
$x x_1 + y y_1 = a^2$
It will touch $(x - a)^2 + y^2 = a^2$, if
$\left|\frac{ax_1 + 0\,y_1 - a^2}{\sqrt{x_1^2 + y_1^2}}\right| = a$
$\Rightarrow x_1 - a = \pm\sqrt{x_1^2 + y_1^2}$
$\Rightarrow (x_1 - a)^2 = x_1^2 + y_1^2 \Rightarrow y_1^2 = a^2 - 2ax_1$
Hence, the locus of (x_1, y_1) is $y^2 = a^2 - 2ax$, which is a parabola

640 **(d)**
Let the equation of the line be $\frac{x}{a} + \frac{y}{b} = 1$. This meets the coordinate axes at $A(a, 0)$ and $B(0, b)$
Also, it is at a distance c from the origin
$\therefore \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} = c \Rightarrow \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$ ... (i)
The equation of the circle passing through OAB is
$x^2 + y^2 - ax - by = 0$
Let $P(h, k)$be the co-ordinates of its centre. Then,
$h = \frac{a}{2}$ and $k = \frac{b}{2} \Rightarrow a = 2h$ and $b = 2k$
Substituting the values of a and b in (i), we get
$h^{-2} + k^{-2} = 4c^{-2}$
Hence, the locus of $P(h, k)$ is $x^{-2} + y^{-2} = 4c^{-2}$

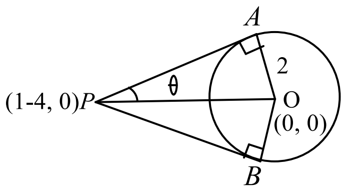
642 **(d)**
Given , $ae = 1$ and $a = 2 \Rightarrow e = \frac{1}{2}$

$\therefore b = \sqrt{4\left(1 - \frac{1}{4}\right)} \Rightarrow b = \sqrt{3}$

Hence, minor axis is $2\sqrt{3}$

643 **(b)**
The equation of the common chord AB of the two circles is
$2x + 1 = 0$ [Using :$S_1 - S_2 = 0$]
The equation of the required circle is
$(x^2 + y^2 + 2x + 3y + 1) + \lambda(2x + 1) = 0$
[Using : $S_1 + \lambda(S_2 - S_1) = 0$]
$\Rightarrow x^2 + y^2 + 2x(\lambda + 1) + 3y + \lambda + 1 = 0$
Since AB is a diameter of this circle. Therefore, centre of this circle lies on AB
So, $-2\lambda - 2 + 1 = 0 \Rightarrow \lambda = -1/2$
So, the equation of the required circle is
$x^2 + y^2 + x + 3y + 1/2 = 0$
$\Rightarrow 2x^2 + 2y^2 + 2x + 6y + 1 = 0$

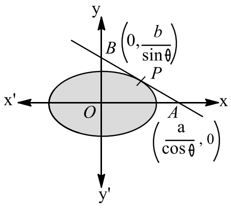
644 **(a)**
In ΔPOB,



$\sin\theta = \frac{2}{4} = \frac{1}{2}$
$\Rightarrow \theta = 30°$
$\therefore$ area$(\Delta POA) = \frac{1}{2} \times 2 \times 4 \times \sin 30° = 2$
Hence, area (quad $PAOB$)=2 area (ΔPOA)
$= 2 \times 2 = 4$ sq units

645 **(a)**
Equation of tangent at $P(a\cos\theta, b\sin\theta)$ is

$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$



Whose point of intersection of axes are

$A\left(\frac{a}{\cos\theta},0\right)$ and $B\left(0,\frac{b}{\sin\theta}\right)$

$\therefore$ Area of $\Delta AOB=\frac{1}{2}\left|\frac{a}{\cos\theta}.\frac{b}{\sin\theta}\right|$

$\Delta=\frac{ab}{|\sin 2\theta|}$

Now area is minimum when $|\sin 2\theta|$ is maximum *ie*, $|\sin 2\theta|=1$

$\therefore\ \Delta_{\text{minimum}}=ab$

646 **(b)**

Given equation can be rewritten as

$(x-4)^2=-2\left(y-\frac{9}{2}\right)$

$\therefore$ Vertex of the parabola is $\left(4,\frac{9}{2}\right)$

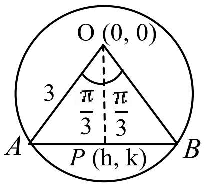
647 **(c)**

Let the coordinates of a point P be (h,k) which is mid point of the chord AB

Now, $OP=\sqrt{(h-0)^2+(k-0)^2}$

$=\sqrt{h^2+k^2}$

In $\Delta AOP, \cos\frac{\pi}{3}=\frac{OP}{OA}$



$\Rightarrow\ \frac{1}{2}=\frac{\sqrt{h^2+k^2}}{3}$

$\Rightarrow\ h^2+k^2=\frac{9}{4}$

Hence, the required locus is

$x^2+y^2=\frac{9}{4}$

648 **(b)**

The required equation of circle is

$(x^2+y^2+13x-3y)+\lambda\left(11x+\frac{1}{2}y+\frac{25}{2}\right)=0$ ...(i)

It passes through (1, 1)

$\therefore\ 12+\lambda(24)=0$

$\Rightarrow\ \lambda=-\frac{1}{2}$

On putting in Eq. (i), we get

$x^2+y^2+13x-3y-\frac{11}{2}x-\frac{1}{4}y-\frac{25}{4}=0$

$\Rightarrow\ 4x^2+4y^2+52x-12y-22x-y-25=0$

$\Rightarrow\ 4x^2+4y^2+30x-13y-25=0$

649 **(d)**

The equation of any normal $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

$ax\ \sec\phi-by\ \text{cosec}\ \phi=a^2-b^2$...(i)

The straight line $x\cos\alpha+y\ \sin\alpha=p$ will be a normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then Eq. (i) and $x\cos\alpha+y\ \sin\alpha=p$ will represent the same line

$\therefore\ \frac{a\sec\phi}{\cos\alpha}=\frac{-b\ \text{cosec}\ \phi}{\sin\alpha}=\frac{a^2-b^2}{p}$

$\Rightarrow\cos\phi=\frac{ap}{(a^2-b^2)\cos\alpha}$

And $\sin\phi=\frac{-bp}{(a^2-b^2)\sin\alpha}$

$\because\ \sin^2\phi+\cos^2\phi=1$

$\Rightarrow\frac{b^2p^2}{(a^2-b^2)^2\sin^2\alpha}+\frac{a^2p^2}{(a^2-b^2)^2\cos^2\alpha}=1$

$\Rightarrow p^2(b^2\text{cosec}^2\alpha+a^2\sec^2\alpha)=(a^2-b^2)^2$

650 **(d)**

Given hyperbola is

$\frac{x^2}{25}-\frac{y^2}{16}=1$

Here, $a=5$ and $b=4$

Asymptotes are $y=\pm\frac{4}{5}x$

651 **(b)**

If the line $y=mx+c$ touches the parbola $y^2=4ax$, then

$y=mx+\frac{a}{m}$

$\Rightarrow\ \ my=m^2x+a$

$\Rightarrow\ my=x+am^2\ \ \left[\text{replacing } m \text{ by } \frac{1}{m}\right]$

$\Rightarrow\ x-my+am^2=0$

652 **(a)**

The director circle of $16x^2-25y^2=400$ is x^2+

$y^2 = 9$

Clearly, $(2\sqrt{2}, 1)$ lies on it. So, angle between tangents drawn from $(2\sqrt{2}, 1)$ is a right angle

653 **(d)**

The centre of given circle is $(0, -\lambda)$

$\therefore \quad r = \sqrt{0 + \lambda^2 - 4} = 0$

$\Rightarrow \quad \lambda = \pm 2$

So, limiting points are $(0, \pm 2)$

654 **(b)**

Let the required equation of circle be $x^2 + y^2 + 2gx + 2fy = 0$. Since, the above circle cuts the given circles orthogonally

$\therefore \; 2(-3g) + 2f(0) = 8 \quad \Rightarrow \quad 2g = -\dfrac{8}{3}$

And $\quad -2g - 2f = -7$

$\Rightarrow \; 2f = -7 + \dfrac{8}{3} = \dfrac{29}{3}$

$\therefore$ required equation of circle is

$x^2 + y^2 - \dfrac{8}{3}x + \dfrac{29}{3}y = 0$

or $\quad 3x^2 + 3y^2 - 8x + 29y = 0$

655 **(d)**

The equation of normal of slope m to the parabola $y^2 = 4ax$ is $y = mx - 2am - am^3$

This will touch the hyperbola $x^2 - y^2 = a^2$, if $(-2am - am^3)^2 = a^2m^2 - a^2$

$\Rightarrow 4m^2 + m^6 + 4m^4 = m^2 - 1$

$\Rightarrow m^6 + 4m^4 + 3m^2 + 1 = 0$

656 **(b)**

Let $P(h, k)$ be the point from which two tangents are drawn to $y^2 = 4x$. Any tangent to the parabola $y^2 = 4x$ is

$y = mx + \dfrac{1}{m}$

If it passes through $P(h, k)$, then

$k = mh + \dfrac{1}{m} \Rightarrow m^2h - mk + 1 = 0$

Let m_1, m_2 be the roots of this equation. Then,

$m_1 + m_2 = \dfrac{k}{h}$ and $m_1m_2 = \dfrac{1}{h}$

$\Rightarrow 3m_2 = \dfrac{k}{h}$ and $2m_2^2 = \dfrac{1}{h} \qquad [\because m_1 = 2\,m_2 \text{(given)}]$

$\Rightarrow 2\left(\dfrac{k}{3h}\right)^2 = \dfrac{1}{h} \Rightarrow 2k^2 = 9h$

Hence, $P(h, k)$ lies on $2y^2 = 9x$

657 **(c)**

Given circle is $x^2 + y^2 - 6x + 4y - 12 = 0$

Centre of this circle is $(3, -2)$

Let other end of the diameter is (α, β)

$\therefore \quad \dfrac{\alpha - 1}{2} = 3, \qquad \dfrac{\beta + 1}{2} = -2$

$\Rightarrow \quad \alpha = 7, \beta = -5$

$\therefore$ Other end of the diameter is $(7, -5)$

658 **(b)**

Equation of director circle of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is

$x^2 + y^2 = 25 + 16$

$\Rightarrow x^2 + y^2 = 41$

$\because$ The given point $(5, 4)$ lies on the director circle, therefore the tangents are drawn from this points to the ellipse makes an angle $90°$

659 **(d)**

Let the equation of tangent to the circle $x^2 + y^2 = 16$ is

$y = mx + 4\sqrt{1 + m^2} \quad (\because y = mx + a\sqrt{1 + m^2})$

And let the equation of tangent to the ellipse $\frac{x^2}{49} + \frac{y^2}{4} = 1$ is

$y = mx + \sqrt{49m^2 + 4} \quad (\because y = mx + \sqrt{a^2m^2 + b^2})$

For common tangent

$4\sqrt{1 + m^2} = \sqrt{49m^2 + 4}$

$\Rightarrow 16 + 16m^2 = 49m^2 + 4$

$\Rightarrow 12 = 33m^2$

$\Rightarrow m^2 = \dfrac{12}{33} \Rightarrow m = \dfrac{2}{\sqrt{11}}$

$\therefore y = \dfrac{2}{\sqrt{11}}x + 4\sqrt{1 + \dfrac{4}{11}}$

$= \dfrac{2}{\sqrt{11}}x + 4\sqrt{\dfrac{15}{11}}$

661 **(a)**

The coordinate of the point of intersection of line $y = x$ and circle $x^2 + y^2 - 2x = 0$ is $A(0, 0)$ and $B(1, 1)$

$\therefore$ Equation of circle with AB as its diameter is

$x(x - 1) + y(y - 1) = 0$

$\Rightarrow \quad x^2 + y^2 - x - y = 0$

662 **(c)**

Given, $(x + 5)^2 = 16y$

$\Rightarrow X^2 = 4AY$ where $X = x + 5, A = 4, Y = y$.

The ends of the latusrectum are

$(2A, A)$ and $(-2A, A)$

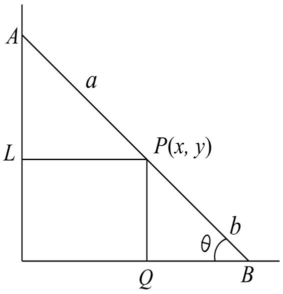
$\Rightarrow x + 5 = 2(4), y = 4 \Rightarrow x = 3, y = 4$

and $x + 5 = -2(4), y = 4 \Rightarrow x = -13, y = 4$

Here required points are $(3, 4)$ and $(-13, 4)$

663 **(d)**

The coordinates of points A and B are (3, 0) and (0, 4) respectively



Let the equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

Since, this circle passes through (0, 0), (3, 0) and (0, 4) respectively, then

$c = 0, g = -\frac{3}{2}$ and $f = -2$

On putting these value in Eq. (i), we get

$x^2 + y^2 - 3x - 4y = 0$

Which is required equation of circle

664 **(b)**

Diameter of circle is diagonal of square

Radius of the circle =5

or diameter of the circle=10

$\therefore$ Area of square $= \frac{(10)^2}{2} = 50$ sq unit

665 **(b)**

The given equation of rectangular hyperbola is

$xy = 18$ ...(i)

On comparing Eq.(i), with general equation of rectangular hyperbola

$xy = \frac{a^2}{2}$

We get, $\frac{a^2}{2} = 18 \Rightarrow a^2 = 36$

$\Rightarrow a = 6$

$\therefore$ Length of the transverse axis of rectangular hyperbola is $2a = 2 \times 6 = 12$

666 **(b)**

Clearly, circle $15x^2 + 15y^2 - 94x + 18y + 55 = 0$ passes through $(1, -2)$ and $(4, -3)$

Also, it touches $3x + 4y = 7$

667 **(b)**

The equation of tangent to the given ellipse in parametric form is

$\frac{x}{5}\cos\theta + \frac{y}{3}\sin\theta = 1$ ... (i)

But, the given equation of tangent is $\frac{3x}{15\sqrt{2}} + \frac{3y}{15\sqrt{2}} = 1$ ... (ii)

Since, Eqs. (i) and (ii) represent the same line

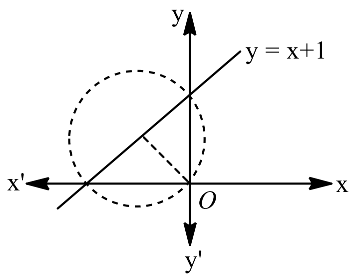
$\therefore \frac{\cos\theta}{5} = \frac{3}{15\sqrt{2}}$ and $\frac{\sin\theta}{3} = \frac{5}{15\sqrt{2}}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$ and $\sin\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi}{4}$

668 **(d)**

The smallest circle means that its radial is, distance from origin to the diameter is smallest.



Let equation of line perpendicular to $y - x = 1$ is $x + y = \lambda$

Also, it passes through (0, 0)

$\therefore \lambda = 0$

$\therefore$ Perpendicular line is $x + y = 0$

The intersection point of lines are $\left(-\frac{1}{2}, \frac{1}{2}\right)$

Which is the centre of circle.

Alternate It is clear from the figure that centre lies on IInd quadrant.

Hence, option (d) is correct

669 **(c)**

The equation of the tangent at $(a\sec\alpha, b\tan\alpha)$ to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

$\frac{x\sec\alpha}{a} - \frac{y\tan\alpha}{b} = 1$

This meets the transverse axis at $T(a\cos\alpha, 0)$

Let $S'(-ae, 0)$ be the focus of the hyperbola. Then,

$S'T = ae + a\cos\alpha = a(e + \cos\alpha)$

670 **(b)**

Given, $e = \frac{1}{2}$ and $\frac{a}{e} = 4 \Rightarrow a = 2$

$\therefore b^2 = a^2(1 - e^2)$

$\Rightarrow b^2 = 4\left(1 - \frac{1}{4}\right) = 3$

$\therefore$ Equation of ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1 \Rightarrow 3x^2 + 4y^2 = 12$

671 **(b)**

Let the equation of the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, and let e be the eccentricity of the ellipse. It is given that distance between foci $= 2h$

$\therefore 2\,ae = 2\,h \Rightarrow ae = h$ ....(i)

Focal distance of the one end of minor axis, say $(0, b)$ is k

$\therefore a + e(0) = k \Rightarrow a = k$ ....(ii)

From (i) and (ii), we have

$b^2 = a^2(1-e^2) = a^2-(ae)^2 = k^2-h^2$

Hence, the equation of the ellipse is $\frac{x^2}{k^2}+\frac{y^2}{k^2-h^2}=1$

672 **(a)**

Required equation of circle is

$x^2+y^2-6x-8y+\lambda(x+y-1)=0$

$\Rightarrow x^2+y^2-(6-\lambda)x-(8-\lambda)y-\lambda)=0$

Whose centre is $\left(3-\frac{\lambda}{2}, 4-\frac{\lambda}{2}\right)$

Which lies on the line $x+y-1=0$

$\Rightarrow 3-\frac{\lambda}{2}+4-\frac{\lambda}{2}-1=0$

$\Rightarrow \lambda = 6$

Hence, required equation is

$x^2+y^2-6x-8y+6x+6y-6=0$

$\Rightarrow x^2+y^2-2y-6=0$

673 **(a)**

Given, $x = 3(\cos t+\sin t), y = 4(\cos t-\sin t)$

$\Rightarrow \frac{x}{3}=\cos t+\sin t, \frac{y}{4}=\cos t-\sin t$

$\therefore \left(\frac{x}{3}\right)^2+\left(\frac{y}{4}\right)^2=(\cos t+\sin t)^2+(\cos t-\sin t)^2$

$\Rightarrow \frac{x^2}{9}+\frac{y^2}{16}=2$

$\Rightarrow \frac{x^2}{18}+\frac{y^2}{32}=1$, which is an ellipse

674 **(b)**

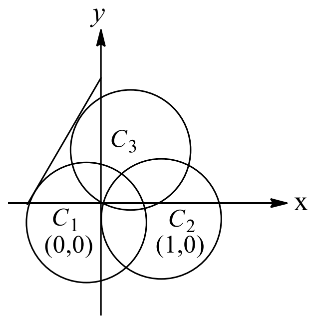
Equation of any circle through (0, 0) and (1, 0) is

$$(x-0)(x-1)+(y-0)(y-0)+\lambda\begin{vmatrix}x & y & 1\\0 & 0 & 1\\1 & 0 & 1\end{vmatrix}=0$$

$\Rightarrow x^2+y^2-x+\lambda y=0$

If it represents C_3, its radius=1

$\Rightarrow 1=\left(\frac{1}{4}\right)+\left(\frac{\lambda^2}{4}\right) \Rightarrow \lambda=\pm\sqrt{3}$



As the centre of C_3, lies above the x-axis, we take $\lambda=-\sqrt{3}$ and thus, an equation of C_3 is $x^2+y^2-x-\sqrt{3}y=0$

Since, C_1 and C_3 intersect and are of unit radius, their common tangents are parallel to the line joining their centres (0, 0) and $\left(\frac{1}{2},\frac{\sqrt{3}}{1}\right)$

So, let the equation of a common tangent be

$\sqrt{3}x-y+k=0$

It will touch C_1, if

$\left|\frac{k}{\sqrt{3+1}}\right|=1 \Rightarrow k=\pm 2$

From the figure, we observe that the required tangent makes positive intercept on the y-axis and negative on the x-axis and hence, its equation is $\sqrt{3}x-y+2=0$

675 **(c)**

Given, $\frac{x^2}{16}+\frac{y^2}{4}=1$

Here, $a=4, b=2$

Equation of normal

$4x\sec\theta-2\,y\,\mathrm{cosec}\,\theta=12$

Since, it passes through Q on x-axis

So, put $y=0$, we get

$x=3\cos\theta$

$\therefore Q(3\cos\theta, 0)$

Now, mid point of PQ,

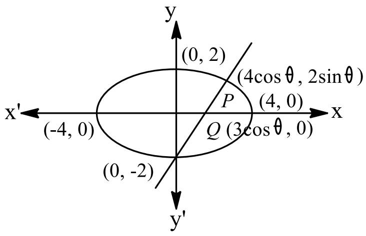
$M\left(\frac{7\cos\theta}{2}, \sin\theta\right)=(h,k)$ (say)

$\therefore h=\frac{7\cos\theta}{2} \Rightarrow \cos\theta=\frac{2h}{7}$

and $k=\sin\theta$

$\Rightarrow \frac{4h^2}{49}=k^2=1$ $(\because \cos^2\theta+\sin^2\theta=1)$

Here, locus of M is $\frac{4x^2}{49} + y^2 = 1$ ...(i)



For given ellipse $e^2 = 1 - \frac{4}{16} = \frac{3}{4}$

$\therefore\ e = \frac{\sqrt{3}}{2}$

$\therefore$ Abscissa of focus is

$x = \pm 4 \times \frac{\sqrt{3}}{2} = \pm 2\sqrt{3}$ $(\because x = \pm ae)$ ...(iii)

On solving Eqs. (i) and (ii), we get

$\frac{4}{49} \times 12 + y^2 = 1$

$\Rightarrow\ y^2 = 1 - \frac{48}{49} = \frac{1}{49}$

$\therefore$ Required points $\left(\pm 2\sqrt{3}, \pm \frac{1}{7}\right)$

676 **(a)**

Since, the focus and vertex of the parabola are on y-axis, therefore its axis of the parabola is $y -$ axis

Let the equation of the directrix be $y = k$ the directrix meets the axis of the parabola at $(0, k)$. But vertex is the mid point of the line segment joining the focus to the point where directrix meets axis of the parabola

$k + \frac{3}{2} = 6\ \Rightarrow\ k = 9$

Thus, the equation of directrix is $y = 9$

Equation of parabola is

$(x-0)^2 + (y-3)^2 = (y-9)^2$

$\Rightarrow\ x^2 + 12y - 72 = 0$

677 **(a)**

The equation of tangent of slope m to the parabola $y^2 = 4x$ is $y = mx + \frac{1}{m}$

This will be a chord of the circle $x^2 + y^2 = 4$, if Length of the perpendicular from the centre $(0,0)$ is less than the radius

i. e. $\left|\frac{1}{m\sqrt{m^2+1}}\right| < 2$

$\Rightarrow 4m^4 + 4m^2 - 1 > 0$

$\Rightarrow \left(m^2 - \frac{\sqrt{2}-1}{2}\right)\left(m^2 + \frac{1+\sqrt{2}}{2}\right) > 0$

$\Rightarrow \left(m^2 - \frac{\sqrt{2}-1}{2}\right) > 0$

$\Rightarrow \left(m - \sqrt{\frac{\sqrt{2}-1}{2}}\right)\left(m + \sqrt{\frac{\sqrt{2}-1}{2}}\right) > 0$

$\Rightarrow m \in \left(-\infty, -\sqrt{\frac{\sqrt{2}-1}{2}}\right) \cup \left(\sqrt{\frac{\sqrt{2}-1}{2}}, \infty\right)$

678 **(a)**

It is given that $y^2 = 4ax$ passes through $(2, -6)$

$\therefore 36 = 8a \Rightarrow a = \frac{9}{2}$

Hence, L. R. $= 4a = 4 \times \frac{9}{2} = 18$

679 **(c)**

The equation of the line $y = x$ in distance form is

$\frac{x}{\cos\theta} = \frac{y}{\sin\theta} = r$, where $\theta = \frac{\pi}{4}$

For point P, we have $r = 6\sqrt{2}$

Therefore, coordinates of P are given by

$\frac{x}{\cos\frac{\pi}{4}} = \frac{y}{\sin\frac{\pi}{4}} = 6\sqrt{2} \Rightarrow x = 6, y = 6$

Since $P(6,6)$ lies on $x^2 + y^2 + 2\,gx + 2\,fy + c = 0$

$\therefore 72 + 12(g+f) + c = 0$ ...(i)

Since $y = x$ touches the circle. Therefore, the equation

$2\,x^2 + 2\,x(g+f) + c = 0$ has equal roots

$\Rightarrow 4(g+f)^2 = 8\,c \Rightarrow (g+f)^2 = 2\,c$ ...(ii)

From (i), we have

$[12(g+f)]^2 = [-(c+72)]^2$

$\Rightarrow 144(g+f)^2 = (c+72)^2$

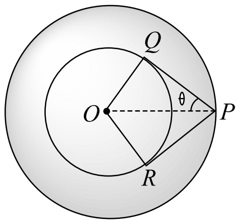
$\Rightarrow 144\,(2\,c) = (c+72)^2$ [Using (ii)]

$\Rightarrow (c-72)^2 = 0 \Rightarrow c = 72$

680 **(c)**

Let PQ and RP be the two tangents and P be the point on the circle $x^2 + y^2 = a^2$ whose coordinates are $(a\cos t, a\sin t)$ and $\angle OPQ = \theta$

Now, PQ =length of tanget from P on the circle $x^2 + y^2 = a^2\sin^2\alpha$



$\therefore PQ = \sqrt{a^2 \cos^2 t + a^2 \sin^2 t - a^2 \sin^2 \alpha}$

$= \sqrt{a^2(\cos^2 t + \sin^2 t) - a^2 \sin^2 \alpha}$

$= a \cos \alpha \quad (\because \cos^2 t + \sin^2 t = 1)$

and OQ =radius of the circle $(x^2 + y^2 = a^2 \sin^2 \alpha)$

$\Rightarrow OQ = a \sin \alpha$

$\therefore \tan \theta = \frac{OQ}{PQ} = \frac{a \sin \alpha}{a \cos \alpha} = \tan \alpha \;\; \Rightarrow \;\; \theta = \alpha$

$\therefore$ Angle between tangents $= \angle QPR = 2\theta = 2\alpha$

681 **(a)**

$\because e^2 = 1 - \frac{b^2}{a^2}$

$\therefore a^2 - b^2 = a^2 e^2$

So, the points on the minor axis at a distance $\sqrt{a^2 - b^2}$ from the centre $(0, 0)$ of the ellipse are $(0, \pm ae)$

The equation of tangent at any point $(a \cos \theta, b \sin \theta)$ on the ellipse is

$\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$

$\therefore$ Required sum

$$= \left[\frac{\frac{ae \sin \theta}{b} - 1}{\sqrt{\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2}}}\right]^2 + \left[\frac{\frac{-ae \sin \theta}{b} - 1}{\sqrt{\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2}}}\right]^2$$

$$= \frac{(ae \sin \theta - b)^2 + (ae \sin \theta + b)^2}{(b^2 \cos^2 \theta + a^2 \sin^2 \theta)} \times a^2$$

$$= \frac{2a^2[(a^2 - b^2) \sin^2 \theta + b^2]}{b^2 \cos^2 \theta + a^2 \sin^2 \theta} = 2a^2$$

682 **(b)**

Since, two chords bisect each other, it means both the chords passes through the centre of circle.

$\therefore$ Length of chords are equal

ie, $a^2 - 1 = 3(a + 1)$

$\Rightarrow \;\; a^2 - 3a - 4 = 0$

$\Rightarrow \;\; (a - 4)(a + 1) = 0$

$\Rightarrow \;\; a = 4 \quad (\because \;\; a = -1$ is not possible)

$\therefore$ Radius of circle$= \frac{a^2 - 1}{2} = \frac{16 - 1}{2}$

$= \frac{15}{2}$

683 **(a)**

Let point is $(\sqrt{6} \cos \theta, \sqrt{2} \sin \theta)$ and let it's distance d from origin

$\therefore d = \sqrt{\sqrt{6 \cos^2 \theta + 2 \sin^2 \theta)}}$

$\Rightarrow 2 = \sqrt{2 + 4 \cos^2 \theta}$

$\Rightarrow 2 + 4 \cos^2 \theta = 4$

$\Rightarrow \cos^2 \theta = \frac{1}{2}$

$\Rightarrow \cos \theta = \pm \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$

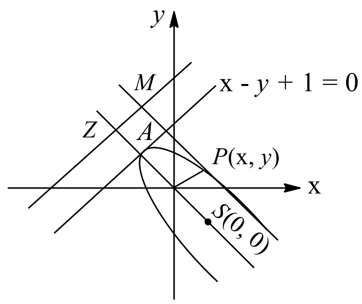
684 **(c)**

Given that focus is $S(0, 0)$

Let A is the vertex of parabola. Take any point Z on the directrix such that $AS = AZ$. Since, the given tangent $x - y + 1 = 0$ is parallel to the directrix

Equation of directrix is $x - y + \lambda = 0$

$\because A$ is the mid point of SZ



$\therefore SZ = 2SA$

$\Rightarrow \frac{|0 - 0 + \lambda|}{\sqrt{1^2 + 1^2}} = 2 \times \frac{|0 - 0 + 1|}{\sqrt{1^2 + 1^2}}$

$\Rightarrow |\lambda| = 2 \Rightarrow \lambda = 2$

$\therefore$ Equation of directrix is $x - y + 2 = 0$

Now, P be any point on the parabola

$\therefore SP = PM \Rightarrow SP^2 = PM^2$

$\Rightarrow (x - 0)^2 + (y - 0)^2 = \left(\frac{|x - y + 2|}{\sqrt{2}}\right)^2$

$\Rightarrow x^2 + y^2 + 2xy - 4x + 4y - 4 = 0$

686 **(c)**

Given equation can be rewritten as

$$\frac{(x - 3)^2}{16} + \frac{y^2}{25} = 1$$

Here, $a^2 = 16$ and $b^2 = 25$

$$\therefore e = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$$

Hence, the foci Basic Terms of Conics are $(0, \pm be) ie, (3, \pm 3)$

687 **(c)**

The equation of tangent at point $(1, 2)$ to the circle $x^2 + y^2 - 4x - 6y + 9 = 0$, is

$x + 2y - 2(x + 1) - 3(y + 2) + 9 = 0$

$\Rightarrow \quad x+y-1=0$

Since, the inverse of the point (1, 2) is the foot (α, β) of the perpendicular from the point (1, 2) to the line $x+y-1$

$\therefore \quad \frac{\alpha-1}{1}=\frac{\beta-2}{1}=-\frac{(1.1+1.2-1)}{1^2+1^2}$

$\Rightarrow \quad \alpha-1=\beta-2=-1$

$\Rightarrow \ \alpha=0, \beta=1$

Hence, required point is (0, 1)

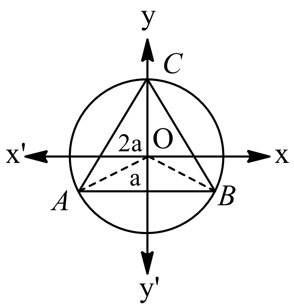
688 **(c)**

Comparing $ax^2+2hxy+by^2+2gx+2fy+c=0$ with $x^2+y^2+2gx+2fy+c=0$, we find that the given equation will represent a circle if $a=b$ and $h=0$

689 **(c)**

In an equilateral triangle, the circumcentre of a circle lies on the centroid of the triangle

Here, radius of circle is $2a$



$\therefore$ Required equation of circle is

$x^2+y^2=4a^2$

690 **(a)**

The given ellipse is $\frac{x^2}{4}+\frac{y^2}{3}=1 \Rightarrow a=2, b=\sqrt{3}$

$\therefore \ b^2=a^2(1-e^2)$

$\Rightarrow \ 3=4(1-e^2) \Rightarrow e=\frac{1}{2}$

$\therefore \ ae=1$

Hence, the eccentricity e_1, of the required hyperbola is given by

$1=e_1 \sin\theta \Rightarrow e_1=\operatorname{cosec}\theta$

$\Rightarrow \ b^2=\sin^2\theta(\operatorname{cosec}^2\theta-1)=\cos^2\theta$

Hence , the required hyperbola is

$$\frac{x^2}{\sin^2\theta}-\frac{y^2}{\cos^2\theta}=1$$

or $x^2\operatorname{cosec}^2\theta-y^2\sec^2\theta=1$

691 **(c)**

Two given tangents are parallel to each other. Therefore, the distance between them is equal to the diameter of the circle

$\therefore$ Radius

$=\frac{1}{2}\times\left\{\begin{matrix}\text{Distance between } 3x-4y+4=0 \\ \text{and } 6x-8y-7=0\end{matrix}\right\}$

$\Rightarrow$ Radius $=\frac{1}{2}\left|\frac{4+\frac{7}{2}}{\sqrt{9+16}}\right|=\frac{3}{4}$

692 **(d)**

For the circle to lie inside the square of unit side length, we must have

Radius $\leq \frac{1}{2}$

$\Rightarrow \sqrt{\sin^2\alpha+\cos^2\alpha-\sin^2\theta} \leq \frac{1}{2}$

$\Rightarrow |\cos\theta| \leq \frac{1}{2}$

$\Rightarrow -\frac{1}{2} \leq \cos\theta \frac{1}{2} \Rightarrow \theta \in [\pi/3, 2\pi/3]$

$\cup [4\pi/3, 5\pi/3]$

693 **(c)**

The equation of any tangent to $y^2=4(x+1)$ is

$y=m(x+1)+\frac{1}{m}$ ... (i)

The equation of any tangent to $y^2=8(x+2)$ is

$y=m'(x+2)+\frac{2}{m'}$ ....(ii)

It is given that (i) and (ii) are perpendicular. Therefore,

$mm'=-1 \Rightarrow m'=-\frac{1}{m}$

Putting $m'=-\frac{1}{m}$ in (ii), we get

$y=-\frac{1}{m}(x+2)-2m$ ... (iii)

The point of intersection of (i) and (iii) is given by solving (i) and (ii)

On subtracting (iii) from (i), we get

$0=\left(m+\frac{1}{m}\right)x+3\left(m+\frac{1}{m}\right) \Rightarrow x+3=0$ [$\because m+\frac{1}{m} \neq 0$]

694 **(a)**

If $P(x,y)$ b ea point on a parabola, then by the definition of parabola

$(PS)^2=(PM)^2$

$\Rightarrow (x-3)^2+(y+4)^2=\left(\frac{6x-7y+5}{\sqrt{6^2+7^2}}\right)^2$

$\Rightarrow \ 85(x^2-6x+9+y^2+8y+16)$

$\Rightarrow\ 36x^2+49y^2+25-84xy-70y+60x$

$\Rightarrow\ (7x+6y)^2-570x+750y+2100=0$

695 **(c)**

Since every diameter of an ellipse passes through the centre and is bisected by it.

Therefore, the coordinates of the other end are $(-\sqrt{3},-2)$

696 **(b)**

The angle between the asymptotes of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $2\tan^{-1}\frac{b}{a}$

So, the angle between the asymptotes of $27x^2-9y^2=24$ is

$2\tan^{-1}(\sqrt{3})=\frac{2\pi}{3}$ $\left[\because a=\frac{2\sqrt{2}}{3} \text{ and } b=\frac{2\sqrt{2}}{\sqrt{3}}\right]$

697 **(c)**

If the point $(\sin\theta,\cos\theta)$ lies inside the circle $x^2+y^2-2x-2y+\lambda=0$, for all θ. Then,

$1-2(\sin\theta+\cos\theta)+\lambda<0$ for all θ

$\Rightarrow 1+\lambda<2(\sin\theta+\cos\theta)$ for all θ

$\Rightarrow 1+\lambda<-2\sqrt{2}$

$\left[\begin{array}{l}\because -\sqrt{2}\le\sin\theta+\cos\theta\le\sqrt{2}\\ \therefore \text{Min. value of } \sin\theta+\cos\theta \text{ is } -\sqrt{2}\end{array}\right]$

$\Rightarrow \lambda<-1-2\sqrt{2}$

698 **(c)**

Equation of tangent at (1,2) is

$3x+4y=5$

Joint equation of tangent is

$(3x^2+2y^2-5)(3+8-5)=(3x+4y-5)^2$

$\Rightarrow 9x^2-4y^2-24xy+30x+40y-30=0$

Here, $a=9, b=-4, h=-12, g=15$

(Comparing it with $ax^2+by^2+2hxy+2gx+2fy+c=0$)

$\theta=\tan^{-1}\left(\frac{2\sqrt{144+36}}{5}\right)$

$=\tan^{-1}\left(\frac{2.2.3\sqrt{5}}{5}\right)$

$\Rightarrow\ \theta=\tan^{-1}\left(\frac{12}{\sqrt{5}}\right)$

699 **(b)**

Let APQ be an isosceles triangle of area Δ. Then,

$\Delta=\frac{1}{2}(PQ\times AL)$

$\Rightarrow \Delta=\frac{1}{2}\times 2\,b\sin\theta\times(a-a\cos\theta)$

$\Rightarrow \Delta=b\sin\theta(a-a\cos\theta)$

$\Rightarrow \Delta=\frac{ab}{2}(2\sin\theta-\sin 2\theta)$

$\Rightarrow \frac{d\,\Delta}{d\,\theta}=ab(\cos\theta-\cos 2\theta)$ and $\frac{d^2\Delta}{d\,\theta^2}=ab(-\sin\theta+2\sin^2\theta)$

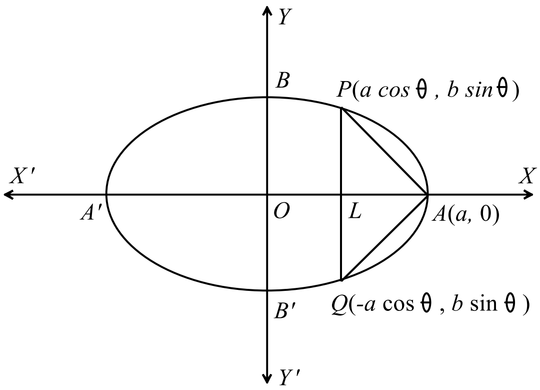
For maximum or minimum, we must have

$\frac{d\,\Delta}{d\,\theta}=0\Rightarrow\cos\theta=\cos 2\theta$

$\Rightarrow\theta=2\pi-2\theta\Rightarrow\theta=\frac{2\pi}{3}$

Clearly, $\frac{d^2\Delta}{d\,\theta^2}<0$ for $\theta=2\pi/3$

Hence, Δ is maximum for $\theta=2\pi/3$



Putting $\theta=2\pi/3$, we have

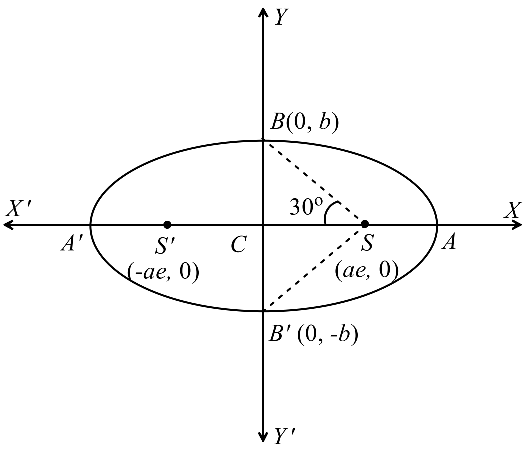
$\Delta_{\max}=\frac{ab}{2}\left(2\sin\frac{2\pi}{3}-\sin\frac{4\pi}{3}\right)=\frac{3\sqrt{3}}{4}ab$

700 **(a)**

Let BB'' be the minor axis of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Let $S(ae,o)$ and $S''(-ae,0)$ be two foci of the ellipse. Then,

$m_1=$ Slope of $SB=\frac{-b}{ae}$, $m_2=$ Slope of $SB''=\frac{b}{ae}$



Now,

$\angle BSB = 60°$

$\Rightarrow \tan 60° = \dfrac{m_1 - m_2}{1 + m_1 m_2}$

$\Rightarrow \sqrt{3} = \dfrac{-2b/ae}{1 - b^2/a^2e^2}$

$\Rightarrow \sqrt{3}(a^2e^2 - b^2) = -2abe$

$\Rightarrow \sqrt{3}(2a^2e^2 - a^2) = 2a^2e\sqrt{1-e^2}$

$\Rightarrow 3(2e^2-1)^2 = 4e^2(1-e^2)$

$\Rightarrow 16e^4 - 16e^2 + 3 = 0 \Rightarrow (4e^2-3)(4e^2-1)$

$= 0 \Rightarrow e = \dfrac{\sqrt{3}}{2}, \dfrac{1}{2}$

<u>ALITER</u> In ΔSOB, we have

$\tan 30° = \dfrac{CB}{CS}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{b}{ae}$

$\Rightarrow \sqrt{3}\, b = ae$

$\Rightarrow 3b^2 = a^2e^2 \Rightarrow 3a^2(1-e^2) = a^2e^2 \Rightarrow 4e^2 = 3$

$\Rightarrow e = \dfrac{\sqrt{3}}{2}$

701 **(a)**

Let P is the position of man and S, S' are position of flags, then

$SP + S'P = 10 = 2a \Rightarrow a = 5$

$\therefore SS' = 2ae = 8 \Rightarrow e = \dfrac{4}{5}$

Now, $e^2 = 1 - \frac{b^2}{a^2} \Rightarrow \frac{16}{25} = 1 - \frac{b^2}{25}$

$\Rightarrow b^2 = 9 \Rightarrow b = 3$

Area of ellipse$= \pi ab = 15\pi$ sq m

702 **(d)**

The point of intersection of $x + 3y = 2$ and $x^2 + xy - y^2 = 1$ is given by

$(2-3y)^2 + (2-3y)y - y^2 = 1$

$\Rightarrow\ 4 + 9y^2 - 12y + 2y - 3y^2 - y^2 = 1$

$\Rightarrow\ 5y^2 - 10y + 3 = 0$

$\therefore\ y = \dfrac{10 \pm \sqrt{100-60}}{2 \times 5}$

$= \dfrac{10 \pm \sqrt{40}}{10}$

$\therefore\ x = 2 - 3\left(\dfrac{10 \pm \sqrt{40}}{10}\right)$

$= \dfrac{-10 \mp \sqrt{40}}{10}$

$\therefore$ Points of intersection are

$A\left(-1 - \dfrac{\sqrt{40}}{10}, 1 + \dfrac{\sqrt{40}}{10}\right)$

and $B\left(-1 + \frac{\sqrt{40}}{10}, 1 - \frac{\sqrt{40}}{10}\right)$

$\therefore$ Mid point of AB is $(-1,1)$

703 **(c)**

The centres and radii of given circles are

$C_1(2,3), C_2(-2,-3)$

and $r_1 = \sqrt{4+9+12} = 5,\ r_2 = \sqrt{4+9-4} = 3$

Now, $C_1C_2 = \sqrt{(2+2)^2 + (3+3)^2} = \sqrt{52}$

Here, $C_1C_2 < r_1 + r_2$

Hence, given circles intersect at two points

704 **(d)**

Equation of director circle of the parabola

$\dfrac{x^2}{16} - \dfrac{y^2}{4} = 1$

$x^2 + y^2 = 16 - 4$

$\Rightarrow x^2 + y^2 = 12$

705 **(a)**

The intersection point of two given lines is the centre of circle *ie*, $(1,-1)$

Circumference of circle$= 10\pi$ (given)

$\Rightarrow\ 2\pi r = 10\pi\ \Rightarrow r = 5$

$\therefore$ equation of circle having centre $(1,-1)$ and radius 5 is

$(x-1)^2 + (y+1)^2 = 5^2$

$\Rightarrow\ \ x^2 + y^2 - 2x + 2y - 23 = 0$

706 **(c)**

Here, $g_1 = \lambda,\ f_1 = 3,\ c_1 = 1$

and $g_2 = 2,\ f_2 = 1,\ c_2 = 0$

since, they intersect orthogonally

$\therefore\ \ 2g_1g_2 + 2f_1f_2 = c_1 + c_2$

$\Rightarrow\ \ 2\lambda \times 2 + 6 \times 1 = 1 + 0$

$\Rightarrow\ \ 4\lambda + 6 = 1$

$\Rightarrow\ \ \lambda = -\dfrac{5}{4}$

707 **(d)**

The equation of any tangent to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

$\dfrac{x}{a}\sec\theta - \dfrac{y}{b}\tan\theta = 1$ ... (i)

The equations of the asymptotes of the hyperbola are

$\dfrac{x}{a} - \dfrac{y}{b} = 0$ ... (ii)

and, $\dfrac{x}{a} + \dfrac{y}{b} = 0$ ... (iii)

The coordinates of the vertices of the triangle formed by the lines (i),(ii) and (iii) are

$O(0,0), P\left(\frac{a}{\sec\theta-\tan\theta}, \frac{b}{\sec\theta-\tan\theta}\right)$

And,

$Q\left(\frac{a}{\sec\theta+\tan\theta}, \frac{-b}{\sec\theta+\tan\theta}\right)$

$\therefore$ Area of $\Delta\, OPQ$

$$=\frac{1}{2}\left|\frac{-ab}{\sec^2\theta-\tan^2\theta} - \frac{ab}{\sec^2\theta-\tan^2\theta}\right| = ab$$

708 **(c)**

Let the coordinates of P be (h,k).

The equations of the chords of contact of tangents drawn from P to the hyperbola $x^2-y^2=a^2$ and the circle $x^2+y^2=a^2$ are $hx-ky=a^2$ and $hx+ky=a^2$ respectively.

These two are at right angle.

$\therefore -\frac{h}{k}\times\frac{h}{k}=-1 \Rightarrow h^2-k^2=0$

Hence, $P(h,k)$ lies on $x^2-y^2=0$

709 **(c)**

Let $P(a\cos\theta, b\sin\theta)$ be a point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ of eccentricity e. Then, the coordinance of A, A', S and S' are $(a,0), (-a,0), (ae,0)$ and $(-ae,0)$ respectively.

then,

$$\text{Area of } \Delta\, PSS' = \frac{1}{2}\begin{vmatrix} a\cos\theta & b\sin\theta & 1 \\ ae & 0 & 1 \\ -ae & 0 & 1 \end{vmatrix}$$

$$= abe\sin\theta$$

$$\text{and, Area of } \Delta APA' = \frac{1}{2}\begin{vmatrix} a\cos\theta & b\sin\theta & 1 \\ a & 0 & 1 \\ -a & 0 & 1 \end{vmatrix}$$

$$= ab\sin\theta$$

$\therefore$ Area of $\Delta\, PSS'$: Area of $\Delta APA' = e:1$

710 **(d)**

Given equation of ellipse can be rewritten as

$(3x-3)^2+(5y-10)^2=225$

$$\Rightarrow \frac{9(x-1)^2}{225}+\frac{25(y-2)^2}{225}=1$$

$\therefore$ Centre of ellipse is $(1, 2)$

711 **(c)**

Given, $\frac{x^2}{7}+\frac{y^2}{9}=1$

Here, $a^2=7,\ b^2=9$

Since, $a<b$

Length of major axis $=2b=6$

712 **(d)**

Equation of given ellipse is $\frac{(x-1)^2}{9}+\frac{(y-2)^2}{4}=1$

Equation of normal ellipse $\frac{X^2}{9}+\frac{Y^2}{4}=1$ is

$3X\ \sec\theta - 2Y\ \text{cosec}\,\theta = 5$

Slope of normal is $\frac{3}{2}\tan\theta$

Which is parallel to $3x-y=1$, then $\frac{3}{2}\tan\theta=3$

$\Rightarrow \tan\theta=2$

$\therefore \sin\theta=\frac{2}{\sqrt{5}}, \cos\theta=\frac{1}{\sqrt{5}}$

So, equation of normal is $3\sqrt{5}X-\sqrt{5}Y=5$

$\because X=x-1, Y=y-2$

$\therefore 3\sqrt{5}(x-1)-\sqrt{5}(y-2)=5$

$\Rightarrow \sqrt{5}(3x-y)=5(\sqrt{5}+1)$

$\Rightarrow 3x-y=\sqrt{5}(\sqrt{5}+1)$

713 **(b)**

Let the equation of hyperbola and conjugate hyperbola be

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$

Then, the eccentricities are

$e^2=\frac{a^2+b^2}{a^2}$ and $e'^2=\frac{a^2+b^2}{b^2}$

$$\therefore \frac{1}{e^2}+\frac{1}{e'^2}=\frac{a^2}{a^2+b^2}+\frac{b^2}{a^2+b^2}=1$$

714 **(a)**

Equation of line which is inclined to the axis at $\frac{\pi}{4}$ is $y=x$

The point of intersection of above line and given parabola is $(0,0), (4a,4a)$

Length of the chord is $= \sqrt{(4a-0)^2+(4a-0)^2}=4a\sqrt{2}$

715 **(c)**

$e^2=\frac{a^2-b^2}{a^2}$ and $e'^2=\frac{b^2-a^2}{b^2}$

$\Rightarrow \frac{1}{e^2}+\frac{1}{e'^2}=1$

716 **(d)**

Since, given lines are parallel.

$\therefore\quad d=\frac{15-5}{\sqrt{4^2+3^2}}=\frac{10}{5}$

$\Rightarrow\ d=2=$diameter of the circle

$\therefore$ Radius of circle$=1$

$\therefore$ Area of circle$= \pi r^2 = \pi$ sq unit

717 **(a)**

Sum of the coefficients in the expansion of $(\alpha^2 x^2 - 2\,\alpha\, x + 1)^{51}$ is zero

$\therefore (\alpha^2 - 2\,\alpha + 1)^{51} = 0 \Rightarrow \alpha = 1$

$\therefore (\alpha, 2\,\alpha^2) = (1,2)$

Now, $S_1 = 1 + 4 - 4 > 0$

So, the point $(\alpha, 2\,\alpha^2)$ lies outside the circle

718 **(c)**

The equation of the auxiliary circle of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $x^2 + y^2 = a^2$

The equation of a tangent to the auxiliary circle is

$x\cos\theta + y\sin\theta = a$ ... (i)

Let (h, k) be the pole of (i) with respect to the ellipse. Then,

$\frac{hx}{a^2} + \frac{ky}{b^2} = 1$ ... (ii)

Clearly, (i) and (ii) represent the same line

$\therefore \frac{\cos\theta}{h/a^2} = \frac{\sin\theta}{k/b^2} = \frac{a}{1}$

$\Rightarrow \cos\theta = \frac{h}{a}$ and $\sin\theta = \frac{ka}{b^2}$

$\Rightarrow \frac{h^2}{a^2} + \frac{k^2 a^2}{b^4} = 1$

Hence, the locus of (h, k) is $\frac{x^2}{a^2} + \frac{y^2 a^2}{b^4} = 1$

or, $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{a^2}$

719 **(a)**

Let $S = x^2 + y^2 - 8x$

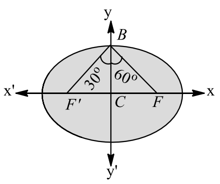
At point $(5, -7)$

$S = 5^2 + (-7)^2 - 8(5) = 34 > 0$

So, point lies outside the circle

720 **(b)**

In ΔCBF; $\tan 30° = \frac{F''C}{b} \Rightarrow F''C = \frac{b}{\sqrt{3}}$



$\Rightarrow ae = \frac{b}{\sqrt{3}} \Rightarrow a^2 e^2 = \frac{1}{3}[a^2(1 - e^2)]$

$\Rightarrow 4e^2 = 1 \Rightarrow e = \frac{1}{2}$

721 **(c)**

Given equation of parabola are

$x^2 = 4y$ and $y^2 = 4s$ ...(i)

$\therefore \left(\frac{x^2}{4}\right)^2 = 4x \Rightarrow x^2 - 64x = 0$

$\Rightarrow x = 0, x = 4$

On putting the values of x in Eq. (i), we get

$y = 0$ and $y = 4$

Hence, points of intersection are (0, 0) and (4, 4)

722 **(d)**

As both the circles pass through the origin and so they must have the same tangent at (0, 0). The general equation of tangent of the given circles are

$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) = 0$

$xx_1 + yy_1 + g'(x + x_1) + f'(y + y_1) = 0$

On substituting $x_1 = 0$ and $y_1 = 0$, we get

$gx + fy = 0 \quad \Rightarrow \quad g'x + f'y = 0$

or $\frac{f}{g} = \frac{f'}{g}$ or $g'f = gf'$

723 **(d)**

Let the coordinates of P and Q are $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ respectively. Then the coordinates of R are $\{2a + a(t_1^2 + t_2^2 + t_1 t_2), -at_1 t_2(t_1 + t_2)\}$

Since, R lies on the parabola

$\therefore a^2 t_1^2 t_2^2 (t_1 + t_2)^2$

$= 4a[2a + a\{(t_1 + t_2)^2 - t_1 t_2\}]$

$\Rightarrow (t_1 + t_2)^2\{t_1^2 t_2^2 - 4\} + 4(t_1 t_2 - 2) = 0$

$\Rightarrow t_1 t_2 = 2$

$\Rightarrow y_1 y_2 = (2at_1)(2at_2) = 4a^2 t_1 t_2$

$\therefore y_1 y_2 = 8a^2$

724 **(a)**

Let the equation of line be $y = mx + c$. Since, this is the tangent to the circle $x^2 + y^2 = 5$

$\therefore \;\; c = \pm a\sqrt{1 + m^2}$

$= \pm\sqrt{5}\sqrt{1 + m^2}$ ...(i)

Also, the above line is tangent to the parabola $y^2 = 40x$

$\therefore \; c = \frac{a}{m} = \frac{10}{m}$

From Eqs.(i) and (ii), we get

$\frac{10}{m} = \pm\sqrt{5}\sqrt{1 + m^2}$

$\Rightarrow \quad m^4 + m^2 - 20 = 0$

$\Rightarrow (m^2 + 5)(m^2 - 4) = 0$

$\Rightarrow m^2 = 4, m^2 \neq -5$

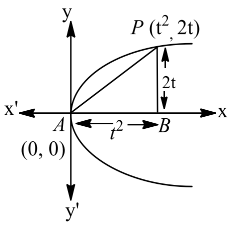
$\Rightarrow m = \pm 2$

$\Rightarrow c = \pm 5$

$\therefore \ y = \pm 2x \pm 5$

725 **(b)**

Let A be the vertex of the parabola and AP is chord of parabola such that slope of AP is $\cot \alpha$. Let coordinates of P be $(t^2, 2t)$, which is a point on the parabola.



Slope of $AP = \frac{2t}{t^2}$

$\Rightarrow \tan \alpha = \frac{2}{t}$

$t = 2 \cot \alpha$

In $\Delta\, APB, AP = \sqrt{4t^2 + t^4}$

$= t\sqrt{4 + t^2}$

$= 2 \cot \alpha \sqrt{4(1 + \cot^2 \alpha)}$

$= 4 \cot \alpha \ \operatorname{cosec} \alpha$

$= 4 \cos \alpha \operatorname{cosec}^2 \alpha$

726 **(b)**

We observe that the circle $x^2 + y^2 = 4$ is orthogonal to the circles given in options (a) and (b). The radical axis of this circle with the circle in option (a) is $x = 1/2$ where as with the circle in option (b) is $x = 1$

727 **(b)**

Given, $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$
Whose extremities of diameter are (x_1, y_1) and (x_2, y_2)

$\therefore$ Coordinates of centre of circle is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

728 **(d)**

Given, $\cos \theta = \frac{x}{4} - 1$ and $\sin \theta = \frac{y}{3} - 1$

$\therefore \ \cos^2 \theta + \sin^2 \theta = 1$

$$\Rightarrow \left(\frac{x}{4} - 1\right)^2 + \left(\frac{y}{3} - 1\right)^2 = 1$$

$$\Rightarrow \frac{(x-4)^2}{16} + \frac{(y-3)^2}{9} = 1$$

729 **(c)**

Chord of contact of tangents at any point $P(x_1, y_1)$ on the circle $x^2 + y^2 = r_1^2$ to the circle $x^2 + y^2 = r_2^2$ is $xx_1 + yy_1 = r_2^2$ which touches the circle $x^2 + y^2 = r_3^2$

$$\therefore \quad \frac{|0.x_1 + 0.y_1 - r_2^2|}{\sqrt{x_1^2 + y_1^2}} = r_3$$

$$\Rightarrow \quad r_2^2 = r_3.\sqrt{x_1^2 + y_1^2} = r_3.r_1 \ \ [\because r_1^2 = x_1^2 + y_1^2]$$

So, r_1, r_2, r_3 are in GP

730 **(b)**

Let $P(h, k)$ be the pole of a tangent to the director circle $x^2 + y^2 = a^2 + b^2$ with respect to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Then the equation of the polar is

$$\frac{hx}{a^2} + \frac{ky}{b^2} = 1 \Rightarrow y = \left(-\frac{b^2h}{a^2k}\right)x + \frac{b^2}{k}$$

This touches $x^2 + y^2 = a^2 + b^2$

$$\therefore \frac{b^4}{k^2} = (a^2 + b^2)\left(1 + \frac{b^4h^2}{a^4k^2}\right) \Rightarrow \frac{1}{a^2 + b^2} = \frac{h^2}{a^4} + \frac{k^2}{b^4}$$

Hence, the locus of (h, k) is $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{a^2+b^2}$

731 **(b)**

Equation of chord of hyperbola $x^2 - y^2 = a^2$ with mid point as (h, k) is give by

$xh - yk = h^2 - k^2$

$$\Rightarrow y = \frac{h}{k} \times \frac{(h^2 - k^2)}{k}$$

This will touch the parabola $y^2 = 4ax$, if

$$-\left(\frac{h^2 - k^2}{k}\right) = \frac{a}{h/k}$$

$\Rightarrow \ ak^2 = -h^3 + k^2h$

$\therefore$ Locus of the mid point is $x^3 = y^2(x - a)$

732 **(b)**

Since, point $(1 - 2)$ lies on curve $y^2 = 4ax$

$4 = 4a \ \Rightarrow \ a = 1$

Equation of any tangent to the parabola is

$y = mx + \frac{1}{m}$

It also passes through $(1, -2)$.

$-2 = m + \frac{1}{m}$

$\Rightarrow\ m^2 + 2m + 1 = 0$

$(m + 1)^2 = 0\ \Rightarrow\ m = -1$

$\therefore\ y = -x - 1\ \Rightarrow\ x + y + 1 = 0$

733 **(a)**

Given equation of circles are $x^2 + y^2 + 2x + 2ky + 6 = 0$ and $x^2 + y^2 + 2ky + k = 0$. They intersect each other orthogonally

$\therefore 2 \cdot 1 \cdot 0 + 2 \cdot k \cdot k = 6 + k$

$\Rightarrow\ 2k^2 - k - 6 = 0$

$\Rightarrow\ (2k + 3)(k - 2) = 0$

$\Rightarrow\ k = 2, -\frac{3}{2}$

734 **(a)**

The given equation is $2x^2 - 3y^2 = 5$, it can be rewritten as $\frac{x^2}{\frac{5}{2}} - \frac{y^2}{\frac{5}{3}} = 1$

Now, $b^2 = a^2(e^2 - 1)$

$\Rightarrow \frac{5}{3} = \frac{5}{2}(e^2 - 1)$

$e^2 = \frac{2}{5}\left(\frac{5}{3} + \frac{5}{2}\right) = \frac{2}{5}\left(\frac{25}{6}\right)$

$\Rightarrow\ e = \sqrt{\frac{5}{3}}$

$\therefore$ Foci of hyperbola $= (\pm ae, 0)$

$= \left(\pm\sqrt{\frac{5}{2}} \cdot \sqrt{\frac{5}{3}}, 0\right) = \left(\pm\frac{5}{\sqrt{6}}, 0\right)$

735 **(b)**

Centres and radii of the given circles are :

Centres : $C_1(1,0)$ $C_2(1,0)$

Radii : $r_1 = 3$ $r_2 = 5$

Clearly, $C_1 C_2 = \sqrt{2} < r_2 - r_1$

Therefore, one circle lies entirely inside the other

736 **(a)**

The given circles cuts orthogonally, if

$2g_1g_2 + 2f_1f_2 = c_1 + c_2$

$\therefore\ 2 \times \frac{g}{2} \times 0 + 2 \times 4 \times 0 = 4 - 4$

This is true for any real value of g.

737 **(d)**

Here, $a^2 = 36, b^2 = 16$

Since, $a >$ b, so the sum of the focal distance of any point P on the ellipse is $PS + PS' = 2a$

$\Rightarrow\ PS + PS' = 2 \times 6 = 12$

738 **(b)**

Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be the ellipse with centre C and eccentricity e. Then,

$CS = ae, CA = a$ and $CZ = \frac{a}{e}$

Clearly, $CA^2 = CS \times CZ$

So, CS, CA and CZ are in G.P.

740 **(c)**

Equation of common chord is

$S_1 - S_2 = 0$

$\Rightarrow\ 6y + 6 = 0$

$\Rightarrow\ y = -1$

On putting $y = -1$ in first circle,

$\therefore\ x^2 + 1 + 2x - 3 + 2 = 0$

$\Rightarrow\ x^2 + 2x = 0\ \Rightarrow\ x = 0, -2$

$\therefore$ End points of diameter are $(0, -1)$ and $(-2, -1)$

Equation of circle is

$(x - 0)(x + 2) + (y + 1)(y + 1) = 0$

$\Rightarrow\ x^2 + 2x + y^2 + 2y + 1 = 0$

741 **(a)**

The equation of a system of circle with its centre on the axis of x is $x^2 + y^2 + 2gx + c = 0$. Any point on the radical axis is $(0, y_1)$

Putting $x = 0, y = \pm\sqrt{-c}$. If c is negative we have two real points on radical axis, then the circles are said to be intersecting circles

742 **(d)**

Condition for line $lx + my + n = 0$ is normal to the ellipse is

$$\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2 - b^2)^2}{n^2}$$

Here, $l = 2, m = -\frac{8}{3}\lambda, n = 3, a^2 = 1, b^2 = 4$

$$\therefore\ \frac{1}{2^2} + \frac{4}{\left(-\frac{8}{3}\lambda\right)^2} = \frac{(1 - 4)^2}{(3)^2}$$

$$\Rightarrow \frac{1}{4} + \frac{36}{64\lambda^2} = 1$$

$$\Rightarrow\ \lambda^2 = \frac{9 \times 4}{16 \times 3}, \lambda = \pm\frac{\sqrt{3}}{2}$$

743 **(a)**

We have,

$x^2 \tan^2\theta + y^2 \sec^2\theta = 1$

$$\Rightarrow \frac{x^2}{\cot^2\theta}+\frac{y^2}{\cos^2\theta}=1$$

Now, length of the latusrectum $=\frac{1}{2}$

$$\Rightarrow 2\frac{\cos^2\theta}{\cot\theta}=\frac{1}{2} \text{ and } \cot\theta>\cos\theta$$

$$\Rightarrow \sin 2\theta=\frac{1}{2} \text{ and } \cot\theta>\cos\theta$$

$$\Rightarrow 2\theta=\frac{\pi}{6},\frac{5\pi}{6} \text{ and } \cot\theta>\cos\theta$$

$$\Rightarrow \theta=\frac{\pi}{12},\frac{5\pi}{12} \text{ and } \cot\theta>\cos\theta \Rightarrow \theta=\frac{\pi}{12},\frac{5\pi}{12}$$

744 **(c)**

Slope of $2y=x$ is $\frac{1}{2}(m_1,\text{say})$

and slope of $3y+4x=0$ is $-\frac{4}{3}(m_2,\text{say})$

$$m_1m_2=-\frac{b^2}{a^2}$$

$$\Rightarrow \left(\frac{1}{2}\right)\left(-\frac{4}{3}\right)=-\frac{b^2}{a^2}$$

$$\Rightarrow \frac{b^2}{a^2}=\frac{2}{3}$$

Eccentricity, $e=\sqrt{1-\frac{b^2}{a^2}}$

$$=\sqrt{1-\frac{2}{3}}=\frac{1}{\sqrt{3}}$$

745 **(b)**

Give equation is $\frac{1}{r}=\frac{1}{8}+\frac{3}{8}\cos\theta$, it can be rewriter as $\frac{8}{r}=1+3\cos\theta$, which is the form of $\frac{1}{r}=1+e\cos\theta$

On comparing, we get

$e=3>1$

$\therefore$ Given equation represents a hyperbola.

746 **(b)**

It is given that

$$L.R.=\frac{1}{3}(\text{Major axis}) \Rightarrow \frac{2b^2}{a}=\frac{2a}{3}$$

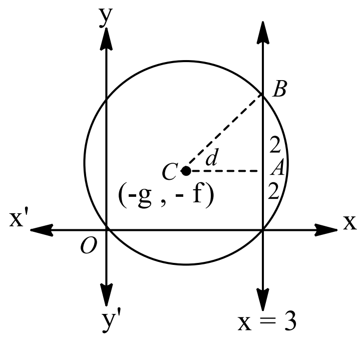
$$\Rightarrow 3b^2=a^2 \Rightarrow 3a^2(1-e^2)=a^2 \Rightarrow 3-3e^2=1$$

$$\Rightarrow e=\sqrt{\frac{2}{3}}$$

747 **(b)**

Let centre of circle be $C(-g,-f)$, then equation of circle passing through origin be

$x^2+y^2+2gx+2fy=0$



$\therefore$ Distance, $d=|-g-3|=g+3$

In ΔABC, $(BC)^2=AC^2+BA^2$

$\Rightarrow \quad g^2+f^2=(g+3)^2+2^2$

$\Rightarrow \quad g^2+f^2=g^2+6g+9+4$

$\Rightarrow \quad f^2=6g+13$

Hence, required locus is $y^2+6x=13$

748 **(d)**

Since, the equation of latusrectum and equation of tangent both are parallel and they lie in the same side of the origin

$$\therefore a=\left|\frac{-8+12}{\sqrt{1^2+1^2}}\right|=\frac{4}{\sqrt{2}}=2\sqrt{2}$$

$\therefore$ Length of latusrectum $=4a=4(2\sqrt{2})=8\sqrt{2}$

749 **(b)**

Given, $2ae=16$ and $e=\sqrt{2}$

$\Rightarrow 2a\sqrt{2}=16 \Rightarrow a=4\sqrt{2}$

$\because \quad e=\sqrt{2}$, it means it is a rectangular hyperbola, where $a=b=4\sqrt{2}$

$\therefore$ The equation of the hyperbola is $x^2-y^2=32$.

750 **(a)**

Let the equation of the required circle be

$x^2+y^2+2gx+2fy+c=0 \quad$ ...(i)

It touches y-axis at $(0,2)$. Therefore,

$4+4f+c=0$ and $c=f^2$

$\Rightarrow f^2+4f+2=0 \Rightarrow f=-2$

$\therefore c=4$ and $f=-2$

Circle (i) cuts intercept of 4 units on x-axis

$\therefore 2\sqrt{g^2-c}=4 \Rightarrow g^2-c=4 \Rightarrow g=\pm 2\sqrt{2}$. $[\because c=4]$

But, the circle cuts intercept with positive side of x-axis

$\therefore g=-2\sqrt{2}$

Substituting the values of g,f and c in (i), we obtain

$x^2+y^2-4\sqrt{2}\,x-4y+4=0$

As the equation of the required circle

751 **(d)**

Given, $\frac{x}{a}=\left(t+\frac{1}{t}\right)$ and $\frac{y}{b}=\left(t-\frac{1}{t}\right)$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=\left(t+\frac{1}{t}\right)^2-\left(t-\frac{1}{t}\right)^2=4$$

$$\Rightarrow \frac{x^2}{4a^2}-\frac{y^2}{4b^2}=1$$

752 **(b)**

Given curves are $9x^2-16y^2=144$

and $x^2+y^2=9$

Let the equation of common tangent be

$y=mx+c$

Since, $y=mx+c$ is a tangent to $\frac{x^2}{16}-\frac{y^2}{9}=1$

$\therefore c^2=16m^2-9 (\because c^2=a^2m^2-b^2)$ ... (i)

Similarly, $y=mx+c$, is a tangent to $x^2+y^2=9$

$c=3\sqrt{m^2+1} \Rightarrow c^2=9(1+m^2)$ ...(ii)

From Eqs. (i) and (ii), we get

$$16m^2-9=9+9m^2 \Rightarrow m^2=\frac{18}{7} \Rightarrow m=3\sqrt{\frac{2}{7}}$$

From Eq. (ii), $c^2=9\left(1+\frac{18}{7}\right) \Rightarrow c^2=9\left(\frac{25}{7}\right)$

$$\Rightarrow c=\frac{\pm 5}{\sqrt{7}}$$

Hence, $y=3\sqrt{\frac{2}{7}}x+\frac{15}{\sqrt{7}}$

753 **(b)**

Let a be the radius of the circle. Since the centre is on y-axis and passes through the origin.

Therefore, coordinates of the centre are $(0, a)$ and so the equation of the circle is

$(x-0)^2+(y-a)^2=a^2 \Rightarrow x^2+y^2-2ay=0$

This passes through (2,3)

$\therefore 4+9-6a=0 \Rightarrow a=13/6$

Hence, the required circle is $3x^2+3y^2-13y=0$

754 **(a)**

We know that the angles between the asymptotes of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ are given by

$\theta=2\tan^{-1}\frac{b}{a}$ and $\pi-\theta=\pi-2\tan^{-1}\frac{b}{a}$

Here, $a=4$ and $b=3$

So, required angles are $2\tan^{-1}\frac{3}{4}$ and $\pi-2\tan^{-1}\frac{3}{4}$

755 **(b)**

The point $\left(-5+\frac{\lambda}{\sqrt{2}},-3+\frac{\lambda}{\sqrt{2}}\right)$ will be an interior point of the larger segment of the circle $x^2+y^2=16$ cut off by the line $x+y=2$, if

(i) it is an interior point of the circle

(ii) the centre of the circle and the point lies on the same side of $x+y=2$

$$\therefore \left(-5+\frac{\lambda}{\sqrt{2}}\right)^2+\left(-3+\frac{\lambda}{\sqrt{2}}\right)^2-16<0$$

and,

$$(0+0-2)\left(-5+\frac{\lambda}{\sqrt{2}}-3+\frac{\lambda}{\sqrt{2}}-2\right)>0$$

$\Rightarrow 18-8\sqrt{2}\lambda+\lambda^2<0$ and $\sqrt{2}\lambda-10<0$

$\Rightarrow 4\sqrt{2}-\sqrt{14}<\lambda<4\sqrt{2}+\sqrt{14}$ and $\lambda<5\sqrt{2}$

$\Rightarrow 4\sqrt{2}-\sqrt{14}<\lambda<5\sqrt{2}$

$\Rightarrow \lambda \in (4\sqrt{2}-\sqrt{14}, 5\sqrt{2})$ $3(x^2+y^2)-25x=0$

756 **(d)**

Equation of the common chord is $S_1-S_2=0$

$\therefore (x^2+y^2+6x-2y+k)$
$-(x^2+y^2+2x-6y-15)=0$

$\Rightarrow 4x+4y+k+15=0$

Centre of second circle is $C_2(-1,3)$

Since, equation of the chord passes through the centre $(-1,3)$ of second circle

$\therefore 4(-1)+4(3)+k+15=0 \Rightarrow k=-23$

757 **(b)**

Let (h,k) be the point whose chord of contact w.r.t. hyperbola $x^2-y^2=9$ is $x=9$. We know that chord of (h,k) w.r.t. hyperbola $x^2-y^2=9$ is $T=0$.

$\Rightarrow hx-ky-9=0$

But it is the equation of line $x=9$. This is possible only when $h=1, k=0$.

Again equation of pair of tangents is

$T^2=SS_1$

$\Rightarrow (x-9)^2=(x^2-y^2-9)(1-9)$

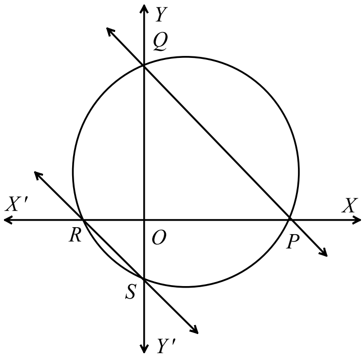
$\Rightarrow x^2-18x+81=(x^2-y^2-9)(-8)$

$\Rightarrow 9x^2-8y^2-18x+9=0$

758 **(a)**

Let the given lines be $L_1=a_1x+b_1y+c_1=0$ and $L_2=a_2x+b_2y+c_2=0$. Suppose L_1 meets the coordinate axes at P and Q and L_2 meets at R and S. Then, coordinates of P, Q, R and S are respectively.

$P(-c_1/a_1, 0), Q(0,-c_1/b_1), R(-c_2/a_2, 0)$ and $S(0,-c_2/b_2)$



Since, P, Q, R, S are concyclic

$\therefore OP \times OR = OQ \times OS$

$\Rightarrow \left|-\frac{c_1}{a_1}\right|\left|-\frac{c_2}{a_2}\right| = \left|-\frac{c_1}{b_1}\right|\left|-\frac{c_2}{b_2}\right| \Rightarrow |a_1a_2| = |b_1b_2|$

759 **(c)**

Let the equation of the circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$

This passes through $(-3,4)$ and $(5,4)$

$\therefore -6\,g + 8\,f + c + 25 = 0$ ...(i)

and, $10\,g + 8\,f + c + 41 = 0$ ...(ii)

Subtracting (ii) from (i), we get $g = -1$

Since the centre $(-g, -f)$ lies on $4\,y = x + 7$

$\therefore -4f = -g + 7 \Rightarrow$ $[\because g = -1]$

So, centre of the circle is at $(1,2)$

Now, $AD = 2\,GM$

$\Rightarrow AD = 2$ (Length of the $\perp$ from G on AB whose eqn. is $y = 4$)

$\Rightarrow AD = 2 \times 2 = 4$

Also, $AB = 8$

Hence, area of rectangle $ABCD = 4 \times 8 = 32$ sq. units

760 **(b)**

The equation of the circle passing through the intersection of the circle $x^2 + y^2 - 2x = 0$ and the line AB (whose equation is $y = x$), is

$x^2 + y^2 - 2x + \lambda(y - x) = 0$

$\Rightarrow x^2 + y^2 - x(2 + \lambda) + \lambda y = 0$ ...(i)

Line $y = x$ will be a diameter of this circle, if it passes through the centre $\left(\frac{2+\lambda}{2}, -\frac{\lambda}{2}\right)$

$\therefore -\frac{\lambda}{2} = \frac{2+\lambda}{2} \Rightarrow \lambda = -1.$

Putting $\lambda = -1$ in (i), we get

$x^2 + y^2 - x - y = 0$ as the equation of the required circle

761 **(b)**

Let e be the eccentricity of the ellipse. It is given that $\Delta SLL'$ is equilateral

$\therefore SL = SL' = LL'$

$\Rightarrow a + e \times ae = \frac{2b^2}{a}$ $\left[\begin{array}{l}\because SL = \text{Focal distance of} \\ L(e, b^2/a) = a + e \times ae\end{array}\right]$

$\Rightarrow a^2(1 + e^2) = 2\,a^2(1 - e^2) \Rightarrow e = \frac{1}{3}$

762 **(c)**

We have,

$r^2 - 2\sqrt{2}\,r(\cos\theta + \sin\theta) - 5 = 0$

$\Rightarrow x^2 + y^2 - 2\sqrt{2}(x + y) - 5 = 0$

$[\because x = r\cos\theta, y = r\sin\theta$ and $x^2 + y^2 = r^2]$

Clearly, radius of this circle is $R = \sqrt{2 + 2 + 5} = 3$

763 **(c)**

We have,

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{dy}{dx} = -\frac{b^2x}{a^2y}$

And,

$x^2 - y^2 = c^2 \Rightarrow \frac{dy}{dx} = \frac{x}{y}$

The two curves will cut at right angles, if

$\left(\frac{dy}{dx}\right)_{C_1} \times \left(\frac{dy}{dx}\right)_{C_2} = -1$

$\Rightarrow -\frac{b^2x}{a^2y} \times \frac{x}{y} = -1$

$\Rightarrow \frac{x^2}{a^2} = \frac{y^2}{b^2} \Rightarrow \frac{x^2}{a^2} = \frac{y^2}{b^2}$

$= \frac{1}{2}$ $\left[\text{Using} : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\right]$

Substituting these values in $x^2 - y^2 = c^2$, we get

$\frac{a^2}{2} - \frac{b^2}{2} = c^2 \Rightarrow a^2 - b^2 = 2\,c^2$

764 **(c)**

Chord of contact are

$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$ and $\frac{xx_2}{a^2} + \frac{yy_2}{b^2} = 1$

Product of slopes $= -1$

$\Rightarrow \left(-\frac{x_1}{a^2}\cdot\frac{b^2}{y_1}\right)\left(-\frac{x_2}{a^2}\cdot\frac{b^2}{y_2}\right) = -1$

$\Rightarrow \frac{x_1x_2}{y_1y_2} = -\frac{a^4}{b^4}$

765 **(b)**

Let the equation of the hyperbola be

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ ... (i)

The coordinates of its centre C, vertex A and the corresponding focus S are $(0,0)(a, 0)$ and $(ae, 0)$ respectively.

It is given that A is mid-way between C and S

$\therefore a = \frac{ae + 0}{2} \Rightarrow e = 2$

$\therefore b^2 = a^2(4 - 1) = 3a^2$

Hence, the equation of the hyperbola is

$\frac{x^2}{a^2} - \frac{y^2}{3a^2} = 1$ or, $3x^2 - y^2 = 3a^2$

767 **(d)**

The equation of a tangent of slope $(-4/3)$ to the ellipse $\frac{x^2}{18}+\frac{y^2}{32}=1$ is

$$y=-\frac{4}{3}x+\sqrt{18\times\frac{16}{9}+32}\ \left[\text{Using}: y = mx+\sqrt{a^2m^2+b^2}\right]$$

$\Rightarrow 4x+3y=24$

This cuts the co-ordinate axes at $A(6,0)$ and $B(0,8)$ respectively

$\therefore$ Area of $\Delta\, OAB=\frac{1}{2}\times OA\times OB$

$\Rightarrow$ Area of $\Delta\, OAB=\frac{1}{2}\times 6\times 8$ sq. units

$= 24$ sq. units

768 **(c)**

The point of intersection between the curves $x^2=4(y+1)$ and $x^2=-4(y+1)$ is $(0,1)$

The slopes of curve first and curve second at the point $(0,-1)$ are respectively

$m_1=\frac{2x}{4}=0$ and $m_2=\frac{-2x}{4}=0$

$\therefore\ \tan\theta=\frac{m_1-m_2}{1+m_1m_2}=0\Rightarrow\theta=0^\circ$

769 **(a)**

$\sqrt{x^2+(y-1)^2}=k\sqrt{x^2+(y+1)^2}$

$x^2+y^2-2y+1=k^2+(x^2+y^2+2y+1)$

$+2k\sqrt{x^2+(y+1)^2}$

$\Rightarrow\ -4y-k^2=2k\sqrt{x^2+(y+1)^2}$

$\Rightarrow\ 16y^2+k^4+8yk^2=4k^2(x^2+y^2+2k+1)$ [squaring]

$\Rightarrow\ 4x^2k^2+(4k^2-16)\,y^2=k^4-4k^2$

To represent an equation of hyperbola, the coefficient of either x^2 or y^2 is negative

But coefficient of x^2 cannot be negative, so we take the coefficient of y^2

$4k^2-16<0$

$\Rightarrow\ k^2\leq 4$

$\Rightarrow\ -2<k<2$

As the given equation k cannot be negative

$0<k<2$

770 **(a)**

Let $S=x^2+y^2-20$

At point $(6,2)$; $S_1=6^2+2^2-20=20$

$\therefore\ \theta=2\tan^{-1}\frac{r}{\sqrt{S_1}}=2\tan^{-1}\frac{\sqrt{20}}{\sqrt{20}}=\frac{\pi}{2}$

771 **(c)**

Given equation of hyperbola can be rewritten as

$$\frac{x^2}{\left(\frac{12}{5}\right)^2}-\frac{y^2}{\left(\frac{9}{5}\right)^2}=1$$

$\therefore$ Eccentricity given by $e'^2=1+\frac{b'^2}{a'^2}$

$\Rightarrow e'^2=1+\frac{9}{16}=\frac{25}{16}\Rightarrow e'=\frac{5}{4}$

The foci of a hyperbola are

$(\pm a'e',0)=\left(\pm\frac{12}{5}\times\frac{5}{4},0\right)=(\pm 3,0)$

Given equation of ellipse is $\frac{x^2}{16}+\frac{y^2}{b^2}=1$

Foci of an ellipse are $(\pm ae,o)=(\pm 4e,o)$. But given focus of ellipse and

hyperbola coincide, then

$4e=3\ \Rightarrow\ e=\frac{3}{4}$

Also, $b^2=a^2(1-e^2)$

$=16\left(1-\frac{9}{16}\right)=16-9=7$

772 **(c)**

Given, $\frac{x^2}{36}-\frac{y^2}{k^2}=1\ \Rightarrow\ k^2=\frac{36y^2}{x^2-36}$

$k^2>0$ if $x^2-36>0$

$\Rightarrow\ x^2>36$

This is true only for point (10, 4). So, (10,4) lies on the hyperbola

773 **(b)**

Since the locus of the point of intersection of perpendicular tangents to a parabola is its directrix. Therefore, the required locus is $y=-a$

774 **(c)**

The equation of any normal to $y^2=4\,ax$ is

$y=mx-2\,am-am^3$ ....(i)

The combined equation of the lines joining the origin (vertex) to the points of intersection of (i) and $y^2 = 4\,ax = 0$

$y^2 = 4\,ax\left(\frac{y - mx}{-2\,am - am^3}\right)$

$\Rightarrow y^2(2\,am + am^3) + 4\,axy - 4\,amx^2 = 0$

This represents a pair of perpendicular lines

$\therefore$ Coeff. of x^2 + Coeff. of $y^2 = 0$

$\Rightarrow 2\,am + am^3 - 4\,am = 0$

$\Rightarrow m^2 = 2 \Rightarrow m = \sqrt{2}$

775 **(c)**

The equation of the common chord of the circles $(x-a)^2 + y^2 = a^2$ and $x^2 + (y+b)^2 = b^2$ is

$I \equiv S_1 - S_2 = 0$

$\Rightarrow x^2 + a^2 - 2ax + y^2 - a^2 - x^2 - y^2 - b^2 - 2by + b^2 = 0$

$\Rightarrow ax + by = 0$ ...(i)

Now, the equation of required circle is

$S_1 + \lambda L = 0$

$\therefore \{(x-a)^2 + y^2 - a^2\} + \lambda\{ax + by\} = 0$

$\Rightarrow x^2 + y^2 + x(a\lambda - 2a) + \lambda by = 0$ ...(ii)

Since, Eq. (i) is a diameter of Eq. (ii), then

$a\left(-\frac{a\lambda - 2a}{2}\right) + b\left(-\frac{\lambda b}{2}\right) = 0$

$\Rightarrow \lambda = \frac{2a^2}{a^2 + b^2}$

On putting the value of λ in Eq. (ii), we get

$(a^2 + b^2)(x^2 + y^2) = 2ab(bx - ay)$

Which is the required equation of circle

776 **(a)**

We know that,

$\frac{a}{2\sin A} = R(\text{circum} - \text{radius of } \Delta\, ABC)$

$\therefore a \leq \sin A \Rightarrow 2R\sin A \leq \sin A \Rightarrow R \leq \frac{1}{2}$

The equation of the circum-circle is $x^2 + y^2 = R^2$. Therefore, for any point (x, y) inside the circum-circle, we have

$x^2 + y^2 < R^2 < \frac{1}{4}$ $\left[\because R \leq \frac{1}{2}\right]$

Now,

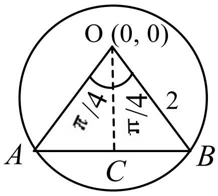
$\frac{1}{4} > x^2 + y^2 \geq 2\sqrt{x^2y^2}$ [Using : $A.M. \geq G.M.$]

$\Rightarrow |xy| < \frac{1}{8}$

777 **(b)**

Let mid point of the chord AB is $C(x_1, y_1)$

In ΔCOB, $\sin\frac{\pi}{4} = \frac{BC}{OB}$



$\Rightarrow \frac{1}{\sqrt{2}} = \frac{BC}{2}$

$\Rightarrow BC = \sqrt{2}$

Using Pythagoras theorem,

$OB^2 = OC^2 + CB^2$

$\Rightarrow (2)^2 = x_1^2 + y_1^2 + (\sqrt{2})^2$

$\Rightarrow x_1^2 + y_1^2 = 2$

Hence, locus of mid point of chord is

$x^2 + y^2 = 2$

778 **(d)**

Given $y^2 = -4\left(x + \frac{3}{4}\right)$

$\Rightarrow Y^2 = -4\,X$, where $X = x + \frac{3}{4}$ and $Y = y$

The equation of directrix of parabola is

$X = 1 \Rightarrow x + \frac{3}{4} = 1$

$\Rightarrow x - \frac{1}{4} = 0$

779 **(d)**

Any point on the parabola $y^2 = 4\,ax$ is $(at^2, 2at)$

$\therefore at^2 = \frac{9}{2}$

and $2at = 6 \Rightarrow t = \frac{3}{a}$ ...(i)

$\therefore a\left(\frac{3}{a}\right)^2 = \frac{9}{2} \Rightarrow a = 2$

On putting the value of a in Eq. (i), we get

$t = \frac{3}{2}$

$\therefore$ Parameter of the point P is $\frac{3}{2}$

780 **(a)**

Since the distance between the focus and directrix of a parabola is half of the length of the latusrectum.

$\therefore L.R. = 2$(Length of the $\perp$ from (3,3) on $3\,x - 4\,y - 2 = 0$)

$\Rightarrow L.R. = 2\left|\frac{9 - 12 - 2}{\sqrt{9 + 16}}\right| = 2$

781 **(c)**

We know that the general equation of second degree represents a rectangular hyperbola, if $\Delta \neq 0, h^2 > ab$ and Coeff. of x^2 + Coeff. of $y^2 = 0$

Therefore, the given equation represents a rectangular hyperbola, if

$\lambda + 5 = 0$ i.e. $\lambda = -5$

782 **(c)**

The coordinates of R are $(at_1 t_2, a(t_1 + t_2))$

As it lies on x-axis.

$\therefore a(t_1 + t_2) = 0 \Rightarrow t_2 = -t_1$

Now,

Area of $\Delta\, PQR$

$$= \text{Absolute value of} \frac{1}{2}\begin{vmatrix} at_1t_2 & 0 & 1 \\ at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}|2a^2t_1t_2(t_1 - t_2) + 2a^2t_1t_2(t_1 - t_2)|$$

$$= 2a^2|t_1t_2(t_1 - t_2)|$$

$$= 2a^2|t_1t_2(-t_2 - t_2)| \qquad [\because t_2 = -t_1]$$

$$= 4a^2t_1t_2^2$$

783 **(a)**

In a circle AB is as a diameter where the coordinates of A are (p, q) and let the coordinates of B are (x_1, y_1)

Equation of circle in diameter form is

$(x - p)(x - x_1) + (y - q)(y - y_1) = 0$

$\Rightarrow x^2 - (p + x_1)x + y^2 - (y_1 - q)y + px_1 + qy_1 = 0$

Since, the circle touches x-axis

$\therefore \quad y = 0$

$\Rightarrow \quad x^2 - (p + x_1)x + px_1 + qy_1 = 0$

Also, the discriminant of above equation will be equal to zero because circle touches x-axis

$\therefore \quad (p + x_1)^2 = 4(px_1 + qy_1)$

$\Rightarrow \quad (x_1 - p)^2 = 4qy_1$

Therefore, the locus of point B is $(x - p)^2 = 4qy$

784 **(b)**

The given equation can be written as

$(x - 2)^2 = 3(y - 2)$

The directrix of this parabola is given by

$y - 2 = -3/4 \Rightarrow y = 5/4$

785 **(c)**

We know that angle between two asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $2\tan^{-1}\left(\frac{b}{a}\right)$.

Equation of given hyperbola is $\frac{x^2}{16} - \frac{y^2}{9} = 1$.

Here, $a = 4$ and $b = 3$

$\therefore$ Required angle $= 2\tan^{-1}\left(\frac{3}{4}\right)$

786 **(b)**

It is given that $ae = 4$ and $e = \frac{4}{5}$

$\therefore a = 5$

Now, $b^2 = a^2(1 - e^2) \Rightarrow b^2 = 25\left(1 - \frac{16}{25}\right) = 9$

Hence, the equation of the ellipse is $\frac{x^2}{25} + \frac{y^2}{9} = 1$

787 **(d)**

Let the family of circles passing through origin be

$x^2 + y^2 + 2gx + 2fy = 0$

They intersect circle $x^2 + y^2 + 4x - 6y - 13 = 0$ Orthogonally

So, $2g(2) - 2f(3) = -13$

Hence, locus of $(-g, -f)$ is

$-4x + 6y + 13 = 0$

$\Rightarrow 4x - 6y - 13 = 0$

788 **(d)**

We know that the angle of intersection of two circles of radii r_1 and r_2 is given by

$\cos\theta = \frac{r_1^2 + r_2^2 - d^2}{2r_1r_2}$, where d is the distance between their centres.

Here, $r_1 = 2, r_2 = \sqrt{2}$ and $d = \sqrt{2}$

$$\therefore \cos\theta = \frac{4 + 2 - 2}{2 \times 2 \times \sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$$

789 **(d)**

Equation of the tangents at $P(a\sec\theta, b\tan\theta)$ is

$\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$

$\therefore$ Equation of the normal at P is

$ax + b\,\mathrm{cosec}\,\theta\, y = (a^2 + b^2)\sec\theta$ ...(i)

Similarly, the equation of normal at $Q(a\sec\phi, b\tan\phi)$ is

$ax + b\,\mathrm{cosec}\,\phi\, y = (a^2 + b^2)\sec\phi$ ...(ii)

On subtracting Eq. (ii) from Eq. (i), we get

$$y = \frac{a^2 + b^2}{b} \cdot \frac{\sec\theta - \sec\phi}{\mathrm{cosec}\,\theta - \mathrm{cosec}\,\phi}$$

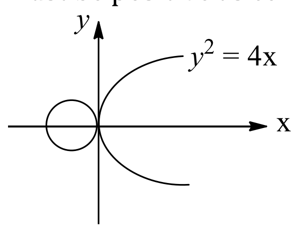
So that $k = y = \frac{a^2 + b^2}{b} \cdot \frac{\sec\theta - \sec\left(\frac{\pi}{2} - \theta\right)}{\mathrm{cosec}\,\theta - \mathrm{cosec}\left(\frac{\pi}{2} - \theta\right)}$

$$= \frac{a^2 + b^2}{b} \cdot \frac{\sec\theta - \mathrm{cosec}\,\theta}{\mathrm{cosec}\,\theta - \sec\theta}$$

$$= -\left[\frac{a^2 + b^2}{b}\right]$$

790 **(a)**

Centre of circle must on negative x-axis for that λ must be positive as centre of circle is $(-\lambda, 0)$



∴ Option (a) is correct

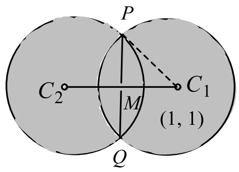
791 **(a)**

Let $S_1 \equiv x^2 + y^2 - 2x - 2y - 7 = 0$

and $S_2 \equiv x^2 + y^2 + 4x + 2y + k = 0$

here, $g_1 = -1,\ f_1 = -1,\ c_1 = -7,\ r_1 = 3$

$g_2 = 2,\quad f_2 = 1,\ c_2 = k$



Equation of common chord is $S_1 - S_2 = 0$

$\Rightarrow\ 6x + 4y + 7 + k = 0$ ...(i)

$\because\ 2(g_1g_2 + f_1f_2) = c_1 + c_2$

$\therefore\ 2(-2-1) = -7 + k\ \Rightarrow\ k = 1$

∴ From Eq. (i), $6x + 4y + 8 = 0$

Let C_1M = Perpendicular distance from centre$C_1(1,1)$ to the common chord $6x + 4y + 8 = 0$.

$$\therefore\ C_1M = \frac{|6+4+8|}{\sqrt{6^2+4^2}} = \frac{9}{\sqrt{13}}$$

Now, $PQ = 2PM = 2\sqrt{(C_1P)^2 - (C_1M)^2}$

$$= 2\sqrt{9 - \left(\frac{9}{\sqrt{13}}\right)^2} = \frac{12}{\sqrt{13}}$$

792 **(b)**

Given limiting points are $(1,2), (-2,1)$

The mid point is $\left(-\frac{1}{2}, \frac{3}{2}\right)$

Now, slope$= \frac{1-2}{-2-1} = \frac{1}{3}$

∴ required equation, $y - \frac{3}{2} = -3\left(x + \frac{1}{2}\right)$

$\Rightarrow\ 3x + y = 0$

793 **(c)**

Clearly, $P(1, 1/2)$ is the internal centre of similitude. Thus, if PT_1 and PT_2 are the lengths of tangents drawn from P to the given circles, then

Length of the common tangent $= PT_1 + PT_2 = \frac{3}{2} + \frac{1}{2} = 2$

794 **(c)**

Let $S \equiv \frac{x^2}{16} + \frac{y^2}{25} - 1 = 0$

At point $(7,6), S_1 > 0$. So two tangents can be drawn from this point

795 **(c)**

Let $S_1 \equiv x^2 + y^2 - 6x - 12y + 37 = 0$

and $S_2 \equiv x^2 + y^2 - 6y + 7 = 0$

the equation of common tangent of the two circles is $S_1 - S_2 = 0$

$\Rightarrow\ x^2 + y^2 - 6x - 12y + 37 - (x^2 + y^2 - 6y + 7) = 0$

$\Rightarrow\ x - y - 5 = 0$

796 **(d)**

The position of the points $(1,2)$ and $(2,1)$ with respect to the circle $x^2 + y^2 = 9$ is given by $1^2 + 2^2 = 5 < 9$ and $2^2 + 1^2 = 5 < 9$. Thus, both P and Q lie incide C

The position of the points $(1,2)$ and $(2,1)$ with respect to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is given by

$$\because \frac{1^2}{9} + \frac{2^2}{4} = \frac{1}{9} + 1 > 1$$

And $\frac{2^2}{9} + \frac{1}{4} = \frac{16+9}{36} = \frac{25}{36} < 1$,

P lies outside E and Q lies inside E. Thus, P lies inside C but outside E

798 **(a)**

Let $P(x_1, y_1)$ be the point, then the chord of contact of tangents drawn from p to the circle $x^2 + y^2 = a^2$ is $xx_1 + yy_1 = a^2$

$$\therefore x^2 + y^2 = a^2\left(\frac{xx_1 + yy_1}{a^2}\right)$$

$\Rightarrow x^2 + y^2 - xx_1 - yy_1 = 0$

Which is the equation of required locus

799 **(c)**

The equation of the tangent at $P(a\cos\theta, b\sin\theta)$ to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$

Length of perpendicular from the focus $(ae, 0)$ on the ellipse $= p$

$$= \left|\frac{e\cos\theta - 1}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}}\right|$$

$$= \left|\frac{ab(e\cos\theta - 1)}{\sqrt{b^2\cos^2\theta + a^2(1-\cos^2\theta)}}\right|$$

$$= \left|\frac{ab(e\cos\theta - 1)}{\sqrt{a^2 - a^2e^2\cos^2\theta}}\right|$$

$$\Rightarrow b\sqrt{\frac{1 - e\cos\theta}{1 + e\cos\theta}} = p$$

$$\Rightarrow \frac{b^2}{p^2} = \frac{1 + e\cos\theta}{1 - e\cos\theta}$$

Now, $r^2 = (ae - a\cos\theta)^2 + b^2\sin^2\theta$

$= a^2[(e - \cos\theta)^2 + (1-e)^2\sin^2\theta]$

$= a^2[e^2\cos^2\theta - 2e\cos\theta + 1] = a^2(1 - e\cos\theta)^2$

$\Rightarrow r = a(1 - e\cos\theta)$

$$\therefore \frac{2a}{r} - \frac{b^2}{p^2} = \frac{2}{1 - e\cos\theta} - \frac{1 + e\cos\theta}{1 - e\cos\theta} = 1$$

800 **(b)**

Given, $e = \frac{1}{2}$ and foci is $(\pm 1, 0)$

$$\Rightarrow ae = 1 \Rightarrow a = \frac{1}{\frac{1}{2}} = 2$$

Now, $b^2 = a^2(1-e^2) = 2^2\left(1-\frac{1}{4}\right) = 4\left(\frac{3}{4}\right) = 3$

∴ The equation of ellipse is

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \Rightarrow \frac{x^2}{4}+\frac{y^2}{3}=1$

801 **(d)**

The equation of a tangent to $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at $P(a\sec\theta, b\tan\theta)$ is $\frac{x}{a}\sec\theta-\frac{y}{b}\tan\theta=1$

It cuts the directrix $x = \frac{a}{e}$ at $Q\left(\frac{a}{e}, b\left(\frac{\operatorname{cosec}\theta - e\cot\theta}{e}\right)\right)$

$\therefore m_1 = \text{Slope of } SP = \frac{b\tan\theta - 0}{a\sec\theta - ae}$

$$= \frac{b\sin\theta}{a(1-e\cos\theta)}$$

and, $m_2 = \text{Slope of } SQ = \frac{b(\operatorname{cosec}\theta - e\cot\theta)}{e(a/e - ae)}$

Clearly, $m_1 m_2 = -1$

Hence, PQ subtends a right angle at the focus S.

802 **(a)**

The equation of any tangent to $y^2 = 4\,ax$ is $y = mx + \frac{a}{m}$.

If it touches $x^2 = 4\,ay$, then the equation

$x^2 = 4\,a\left(mx+\frac{a}{m}\right)$ must have equal roots

$\Rightarrow mx^2 - 4\,am^2x - 4\,a^2 = 0$ must have equal roots

$\Rightarrow 16\,a^2m^4 = -16\,a^2m \Rightarrow m = -1 \quad [\because m \neq 0]$

Putting $m=-1$ in $y = mx + \frac{a}{m}$, we get

$y = -x - a$ or, $x + y + a = 0$ as the common tangent

803 **(c)**

The given equation can be written as

$(x+2)^2 = -2(y-2)$

The equation of the tangent at the vertex is

$y - 2 = 0 \quad [\because y = 0 \text{ is tangent to } x^2 = -4ay]$

804 **(b)**

Equation of ellipse $\frac{x^2}{14}+\frac{y^2}{5}=1$

Any point on the ellipse is $(\sqrt{14}\cos\theta, \sqrt{5}\sin\theta)$

∴ Equation of normal at $(\sqrt{14}\cos\theta, \sqrt{5}\sin\theta)$ is

$\sqrt{14}x\sec\theta - \sqrt{5}y\operatorname{cosec}\theta = 9$

It passes through $(a\cos 2\theta, b\sin 2\theta)$

$\Rightarrow \sqrt{14}\sqrt{14}\cos 2\theta\sec\theta - \sqrt{5}\sqrt{5}\sin 2\theta\operatorname{cosec}\theta = 9$

$\Rightarrow 14\frac{\cos 2\theta}{\cos\theta} - 5\frac{\sin 2\theta}{\sin\theta} = 9$

$\Rightarrow 14(2\cos^2\theta - 1) - 10\cos^2\theta = 9\cos\theta$

$\Rightarrow 18\cos^2\theta - 9\cos\theta - 14 = 0$

$\Rightarrow 18\cos^2\theta - 21\cos\theta + 12\cos\theta - 14 = 0$

$\Rightarrow (3\cos\theta + 2)(6\cos\theta - 7) = 0$

$\Rightarrow \cos\theta = -\frac{2}{3}, \cos\theta \neq -\frac{7}{6}$

805 **(b)**

We have,

$y^2 + 6x - 2y + 13 = 0$

$\Rightarrow y^2 - 2y = -6x - 13 \Rightarrow (y-1)^2 = -6(x+2)$

Clearly, the vertex of this parabola is at $(-2,1)$

806 **(b)**

Given, $e = 2,\ 2ae = 8$

$ae = 4 \Rightarrow a = 2$

$b^2 = a^2(e^2-1) \Rightarrow b^2 = 4(4-1)$

$\Rightarrow b^2 = 12$

∴ Equation of hyperbola is

$\frac{x^2}{4}-\frac{y^2}{12}=1$

807 **(b)**

Since, $\frac{S_1}{S_2} = \frac{x_1^2+y_1^2+2x_1-4y_1-20}{x_1^2+y_1^2-4x_1+2y_1-44} = \frac{2}{3}$

$\Rightarrow \quad x_1^2 + y_1^2 + 14x_1 - 16y_1 + 28 = 0$

∴ Locus of point P is

$x^2 + y^2 + 14x - 16y + 28 = 0$

Centre of the circle is $(-7, 8)$

808 **(c)**

The coordinates of the centres and radii of the circles are:

Centre	$C_1(3,4)$	$C_2(1/2,4)$
Radius	$r_1 = 6$	$r_2 = \frac{1}{2}\sqrt{65}$

We observe that $r_1 - r_2 < C_1C_2 < r_1 + r_2$

So, the circles intersect at two points

809 **(a)**

The given equation may be written as

$\frac{x^2}{\frac{32}{3}}-\frac{y^2}{8}=1$

$\Rightarrow \frac{x^2}{\left(\frac{4\sqrt{2}}{\sqrt{3}}\right)^2}-\frac{y^2}{(2\sqrt{2})^2}=1$

On comparing the given equation with the standard equation $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, we get

$a^2 = \left(\frac{4\sqrt{2}}{\sqrt{3}}\right)^2$ and $b^2 = (2\sqrt{2})^2$

∴ Length of transverse axis of a hyperbola

$= 2a = 2 \times \frac{4\sqrt{2}}{\sqrt{3}} = \frac{8\sqrt{2}}{\sqrt{3}}$

810 **(a)**

Given, $(3x-1)^2 = -4(9y+2)$

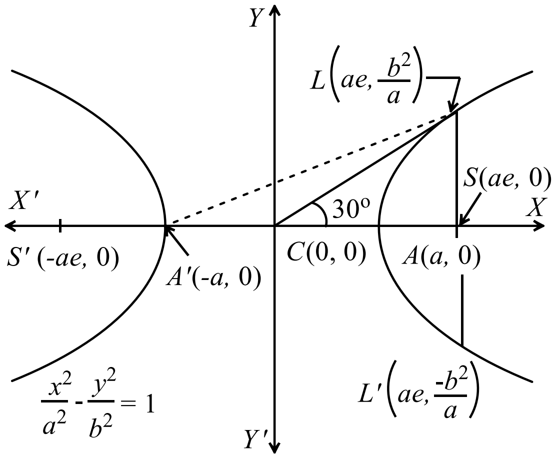
Hence, the vertex is $\left(\frac{1}{3}, \frac{-2}{9}\right)$

811 **(b)**

Let LSL'' be a latusrectum through the focus $S(ae, 0)$ of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. It subtends $60°$ angle at the other focus $S''(-ae, 0)$

We have, $\angle LS''L'' = 60°$

$\therefore \angle LS''S = 30°$



In $\Delta LS''L$, we have

$\tan 30° = \frac{LS}{S''S}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{b^2/a}{2ae}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{b^2}{2a^2e}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{e^2-1}{2e}$

$\Rightarrow \sqrt{3}e^2 - 2e - \sqrt{3} = 0 \Rightarrow (e-\sqrt{3})(\sqrt{2}\,e+1)$

$= 0 \Rightarrow e = \sqrt{3}$

812 **(a)**

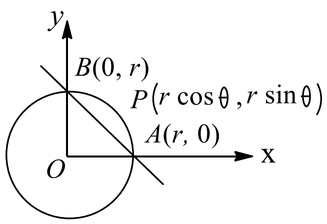
Using the result $a_1a_2 = b_1b_2$, we get

$\lambda.1 = -1.-2$

$\Rightarrow \lambda = 2$

813 **(b)**

Given equation of circle is$x^2 + y^2 = r^2$. Let any point on the circle is $P(r\cos\theta, r\sin\theta)$ and let the coordinates of centriod of the triangle be (α, β)



Then, $\alpha = \frac{r + r\cos\theta}{3}$

$\Rightarrow \frac{r}{3}\cos\theta = \alpha - \frac{r}{3}$

and $\beta = \frac{r + r\sin\theta}{3}$

$\Rightarrow \frac{r}{3}\sin\theta = \beta - \frac{r}{3}$

Now, $\left(\alpha - \frac{r}{3}\right)^2 + \left(\beta - \frac{r}{3}\right)^2 = \frac{r^2}{9}$

$\therefore$ The locus is $\left(x - \frac{r}{3}\right)^2 + \left(y - \frac{r}{3}\right)^2 = \left(\frac{r}{3}\right)^2$ which is a circle

814 **(d)**

Let the general equation of circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

It cuts the circle $x^2 + y^2 - 20x + 4 = 0$ orthogonally, then

By the condition, $2(g_1g_2 + f_1f_2) = c_1 + c_2$

$2(-10g + 0 \times f) = c + 4 \Rightarrow -20g = c + 4$ ...(ii)

$\because$ Circle (i) touches the line $x = 2$ or $x + 0y - 2 = 0$

$\therefore$ Perpendicular distance from centre to the tangent=radius

$\Rightarrow \left|\frac{-g + 0 - 2}{\sqrt{1^2 + 0^2}}\right| = \sqrt{g^2 + f^2 - c}$

$\Rightarrow (g+2)^2 = g^2 + f^2 - c$

$\Rightarrow g^2 + 4 + 4g = g^2 + f^2 - c$

$\Rightarrow 4g + 4 = f^2 - c$ ...(iii)

On eliminating c from Eqs. (ii) and (iii), we get

$-16g + 4 = f^2 + 4 \Rightarrow f^2 + 16g = 0$

Hence, the locus of $(-g, -f)$ is $y^2 - 16x = 0$

(replacing $-f$ and $-g$ by x and y)

815 **(b)**

Let $P(x_1, y_2)$ be a point on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$.

Then, the length of the tangents drawn from $P(x_1, y_1)$ to the circle

$x^2 + y^2 + 2gx + 2fy + c\sin^2\alpha + (g^2 + f^2)\cos^2\alpha = 0$ is given by

$PQ = PR$

$\Rightarrow PQ$

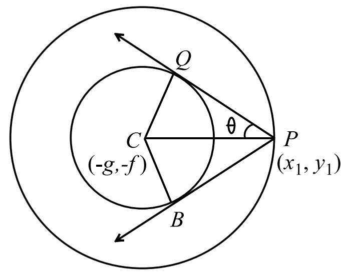
$= \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c\sin^2\alpha + (g^2 + f^2}$

$\Rightarrow PQ = \sqrt{-c + c\sin^2\alpha + (g^2 + f^2)\cos^2\alpha}$

$\Rightarrow PQ = \sqrt{g^2 + f^2 - c}\cos\alpha$

The radius of the circle

$x^2 + y^2 + 2gx + 2fy + c\sin^2\alpha + (g^2 + f^2)\cos^2\alpha = 0$, is



$CQ = CR$

$\Rightarrow CQ = \sqrt{g^2 + f^2 - c\sin^2\alpha - (g^2+f^2)\cos^2\alpha}$

$\Rightarrow CQ = \left\{\sqrt{g^2+f^2-c}\right\}\sin\alpha$

In $\Delta\, CPQ$, we have

$\tan\theta = \frac{CQ}{PQ} = \frac{\left\{\sqrt{g^2+f^2-c}\right\}\sin\alpha}{\left\{\sqrt{g^2+f^2-c}\right\}\cos\alpha}$

$= \tan\alpha \Rightarrow \theta = \alpha$

Hence, $\angle QPR = 2\,\alpha$

816 **(a)**

The mid point of the chord is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$. The equation of the chord in terms of its mid point is

$x\left(\frac{y_1+y_2}{2}\right) + y\left(\frac{x_1+x_2}{2}\right) = 2\left(\frac{x_1+x_2}{2}\right)\left(\frac{y_1+y_2}{2}\right)$

$[\because T = S_1]$

$\Rightarrow \frac{x}{x_1+x_2} + \frac{y}{y_{1+y_2}} = 1$

817 **(b)**

Given, $y = a\tan\alpha + a$

Condition for tangency is $a^2 = a^2(1+\tan^2\alpha)$

$[\because \; c^2 = a^2(1+m^2)]$

$\Rightarrow \; \sec^2\alpha = 1$

$\Rightarrow \; \cos^2\alpha = 1$

818 **(c)**

Let $P(x_1, y_1)$ be a point on $x^2 + y^2 = 4$. Then, the equation of the tangent at P is $x\,x_1 + y\,y_1 = 4$

This meets the coordinate axes at $A(4/x_1, 0)$ and $B(0, 4/y_1)$

Obviously (a) and (b) are not true

Let (h, k) be the mid-point of AB. Then,

$h = \frac{2}{x_1}, k = \frac{2}{y_1} \Rightarrow x_1 = \frac{2}{5}, y_1 = \frac{2}{k}$

since, (x_1, y_1) lies on $x^2 + y^2 = 4$

$\therefore \frac{4}{h^2} + \frac{4}{k^2} = 4 \Rightarrow \frac{1}{h^2} + \frac{1}{k^2} = 1$

Hence, the locus of (h, k) is $\frac{1}{x^2} + \frac{1}{y^2} = 1$, i.e. $x^2 + y^2 = x^2y^2$

819 **(a)**

Since the circle touches both the axes and the straight line $4\,x + 3\,y = 6$ in first quadrant. Therefore, coordinates of its centre are (a, a) and radius $= a$, where $a > 0$

Since $4\,x + 3\,y - 6 = 0$ touches the circle

$\therefore \frac{7\,a - 6}{\sqrt{16+9}} = \pm a \Rightarrow 7\,a - 6 = \pm 5\,a \Rightarrow a = 3, \frac{1}{2}$

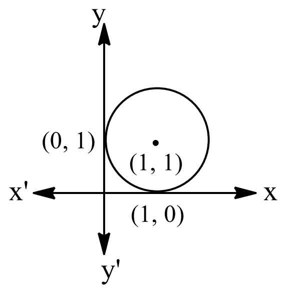
Since $(0,0)$ and $(1/2, 1/2)$ lie on the same side of the line $4\,x + 3\,y = 6$ whereas $(0,0)$ and $(3,3)$ lie on the opposite side of the origin. Therefore, for the required circle, we have $a = 1/2$. Hence, equation of the required circle is

$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2$

$= \left(\frac{1}{2}\right)^2$ or, $4\,x^2 + 4\,y^2 - 4\,x - 4\,y + 1 = 0$

820 **(d)**

Since circle touches the x-axis and y-axis at points $(1, 0)$ respectively.So, centre of circle is $(1, 1)$and radius is 1



Hence, equation of circle is

$(x-1)^2 + (y-1)^2 = 1^2$

$\Rightarrow \; x^2 + y^2 - 2x - 2y + 1 = 0$

821 **(b)**

Shortest distance between two curves occured along the common normal

Normal to $y^2 = 4x$ at $(m^2, 2m)$ is

$y + mx - 2m - m^3 = 0$

Normal to $y^2 = 2(x-3)$ at $\left(\frac{m^2}{2} + 3, m\right)$ is

$y + m(x-3) - m - \frac{m^3}{2} = 0$

Both normals are same, if $-2m - m^3 = -4m - \frac{1}{2}m^3$

$\Rightarrow m = 0, \pm 2$

So, points will be $(4, 4)$ and $(5, 2)$ or $(4, -4)$ and $(5, -2)$

Hence, shortest distance will be

$\sqrt{1+4} = \sqrt{5}$

822 **(b)**

Given equation can be rewritten as

$\frac{x^2}{5} + \frac{(y-3)^2}{9} = 1$

$\therefore \; e = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{5}{9}}$

$\Rightarrow \; e = \frac{2}{3}$

823 **(c)**

Given equation of circle can be rewritten as

$x^2+y^2-\frac{3}{2}x+3y+1=0$

Whose centre is $\left(\frac{3}{4},-\frac{3}{2}\right)$

and radius, $r=\sqrt{\frac{9}{16}+\frac{9}{4}-1}=\sqrt{\frac{29}{16}}$

Area of circle $=\pi r^2=\frac{29\pi}{16}$

$\Rightarrow$ Area of required circle$=2\times\frac{29\pi}{16}=\frac{29\pi}{8}$

Let R be the radius of required circle

$\therefore\quad R^2=\frac{29}{8}$

Now, equation of circle is $\left(x-\frac{3}{4}\right)^2+\left(y+\frac{3}{2}\right)^2=\frac{29}{8}$

$\Rightarrow\ 16x^2+16y^2-24x+48y-13=0$

824 **(a)**

Equation of normal to the hyperbola at the point $(5\sec\theta,4\tan\theta)$ is

$5x\cos\theta+4y\cot\theta=25+16$ ...(i)

This line is perpendicular to the lie $2x+y=1$.

$\therefore\ m_1m_2=-1$

$\Rightarrow\left(\frac{-5\cos\theta}{4\cot\theta}\right)(-2)=-1$

$\Rightarrow\ \sin\theta=-\frac{2}{5}$

$\therefore\cos\theta=\sqrt{1-\frac{4}{25}}=\mp\frac{\sqrt{21}}{5}$

and $\cot\theta=\mp\frac{\sqrt{21}}{2}$

from Eq.(i)

$5x\frac{\sqrt{21}}{5}-\frac{4y\sqrt{21}}{2}=41$

$\Rightarrow\ \sqrt{21}(x-2y)=41$

825 **(d)**

Given, $y^2=18x$ ...(i)

According to the given condition

$y=3x$

From Eqs, (i) and (ii),

$(3x)^2=18x$ [form Eq.(i)]

$\Rightarrow\ x^2=2x\ \Rightarrow\ x=0,2$

$\Rightarrow\ y=0,\pm6$

827 **(c)**

Using the result $(C_1C_2)^2=r_1^2+r_2^2$, we get

$(2-5)^2+(3-6)^2=r^2+r^2$

$\Rightarrow 2r^2=18$

$\Rightarrow r^2=9$

$\Rightarrow r=3$

828 **(c)**

Let $R(h,k)$ be the point of intersection of tangents drawn at P and Q to the given circle. Then, PQ is the chord of contact of tangents drawn from R to $x^2+y^2=25$. So its equation is

$hx+ky-25=0$ ...(i)

it is given that the equation of PQ is

$x-2y+1=0$ ...(ii)

Since (i) and (ii) represent the same line

$\therefore\frac{h}{1}=\frac{k}{-2}=\frac{-25}{1}\Rightarrow h=-25,k=50$

Hence, the required point is $(-25,50)$

830 **(b)**

Since, the coordinates of foci of hyperbola are $(-5,3)$ and $(7,3)$

$\therefore\ 2ae=7-(-5)=12$

$\Rightarrow\ a=\frac{12\times2}{3\times2}=4\quad[\because e=3/2]$

$e=\sqrt{1+\frac{b^2}{a^2}}\Rightarrow\frac{9}{4}-1=\frac{b^2}{16}$

$\Rightarrow\ b^2=20$

Hence, length of latusrectum $=\frac{2b^2}{a}=\frac{2\times20}{4}=10$

831 **(d)**

$A\equiv(a\cos\theta,b\sin\theta)$

$B\equiv(a\cos(\theta+\alpha),b\sin(\theta+\alpha))$

$C\equiv(a\cos(\theta+2\alpha),b\sin(\theta+2\alpha))$

$\Delta\equiv$ Area of ΔABC

$=\frac{1}{2}\begin{vmatrix}1 & a\cos\theta & b\sin\theta\\ 1 & a\cos(\theta+\alpha) & b\sin(\theta+\alpha)\\ 1 & a\cos(\theta+2\alpha) & b\sin(\theta+2\alpha)\end{vmatrix}$

$=2ab\sin^2\left(\frac{\alpha}{2}\right)\sin\alpha$

$\Delta(\alpha)=ab\sin\alpha(1-\cos\alpha)$

$=\frac{ab}{2}(2\sin\alpha-\sin2\alpha)$

$\Delta'(\alpha)=0$

$\Rightarrow\cos\alpha=1$

Or $\cos\alpha=-\frac{1}{2}$

$\cos\alpha=1$ gives $\Delta=0$

$\cos\alpha=-\frac{1}{2}$ gives maximum value of $\Delta=\frac{3\sqrt{3}}{4}ab$

832 **(c)**

Given, vertex of parabola $(h,k)=(1,1)$ and its focus $(a+h,k)=(3,1)$ or $a+h=3$

$\Rightarrow\ a = 2$

Since, y-coordinate of vertex and focus are same, therefore axis of parabola is parallel to x-axis. Thus, equation of parabola is $(y-k)^2 = 4a(x-h) \Rightarrow (y-1)^2 = 8(x-1)$

833 **(c)**

The equation of the tangent at $\left(4\cos\phi, \frac{16}{\sqrt{11}}\sin\phi\right)$ to the ellipse $16x^2 + 11y^2 = 256$ is

$16(4\cos\phi)x + 11\left(\frac{16}{\sqrt{11}}\sin\phi\right)y = 256$

$\Rightarrow 4x\cos\phi + \sqrt{11}y\sin\phi = 16$

This touches the circle $(x-1)^2 + y^2 = 4^2$

$\therefore \left|\frac{4\cos\phi - 16}{\sqrt{16\cos^2\phi + 11\sin^2\phi}}\right| = 4$

$\Rightarrow (\cos\phi - 4)^2 = 16\cos^2\phi + 11\sin^2\phi$

$\Rightarrow 4\cos^2\phi + 8\cos\phi - 5 = 0$

$\Rightarrow (2\cos\phi - 1)(2\cos\phi + 5) = 0$

$\Rightarrow \cos\phi = \frac{1}{2} \Rightarrow \phi = \pm\frac{\pi}{3} \qquad \left(\because \cos\phi \neq \frac{5}{2}\right)$

834 **(c)**

Here, $g_1 = 1,\ f_1 = k,\ c_1 = 6$

and $\ g_2 = 0,\ f_2 = k,\ c_2 = k$

Since, circles intersects orthogonally

$\therefore\ 2g_1g_2 + 2f_1f_2 = c_1 + c_2$

$\Rightarrow\ 0 + 2k^2 = 6 + k$

$\Rightarrow\ 2k^2 - k - 6 = 0$

$\Rightarrow\ k = 2, -\frac{3}{2}$

835 **(a)**

The equation of the asymptotes of the hyperbola

$3x^2 + 4y^2 + 8xy - 8x - 4y - 6 = 0$

is $3x^2 + 4y^2 + 8xy - 8x - 4y + \lambda = 0$.

It should represent a pair of straight lines.

$\therefore abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$3\cdot 4\cdot \lambda + 2\cdot(-2)(-4)4 - 3(-2)^2 - 4(-4)^2 - \lambda(4)^2 = 0$

$\Rightarrow 12\lambda + 56 - 12 - 56 - 16\lambda = 0$

$\Rightarrow -4\lambda - 12 = 0$

$\Rightarrow\ \lambda = -3$

$\therefore$ Required equation is

$3x^2 + 4y^2 + 8xy - 8x - 4y - 3 = 0$

836 **(d)**

If (x_1, y_1) is the mid point of the chord of the circle, $x^2 + y^2 - 4x = 0$, then its equation is

$xx_1 + yy_1 - 2(x + x_1) = x_1^2 + y_1^2 - 4x_1$

Put $x_1 = 1, y_1 = 0$, we get

$x + 0 - 2(x+1) = 1^2 + 0 - 4$

$\Rightarrow\ x = 1$

837 **(c)**

The required circle is

$x^2 + y^2 - a^2 + \lambda\left(x - \frac{a}{2}\right) = 0 \qquad$ [Using: $S + \lambda L = 0$]

This passes through $(2a, 0)$

$\therefore 4a^2 - a^2 + \left(\frac{3a}{2}\right)\lambda = 0 \Rightarrow \lambda = -2a$

Hence, the required circle is

$x^2 + y^2 - a^2 - 2a\left(x - \frac{a}{2}\right) = 0$

$\Rightarrow x^2 + y^2 - a^2 - 2ax + a^2 = 0$

$\Rightarrow x^2 + y^2 - 2ax = 0$

838 **(d)**

The point $(1 + \cos\theta, \sin\theta)$ is an interior point of the circle $x^2 + y^2 = 1$

$\therefore (1 + \cos\theta)^2 + (\sin\theta)^2 - 1 < 0$

$\Rightarrow 1 + 2\cos\theta < 0$

$\Rightarrow \cos\theta < -\frac{1}{2} \Rightarrow \theta \in (2\pi/3, 4\pi/3)$

839 **(a)**

(x, y) is the set of points equidistant from point $(2, 3)$ and the line $3x + 4y - 2 = 0$. So the given equation represents a parabola

840 **(a)**

Given, $x^2 + y^2 - 2x - 6y - \frac{7}{3} = 0$

The centre of this circle is (1, 3)

Also, two diameter of this circle are along the lines $3x + y = c_1$ and $x - 3y = c_2$

These two diameters should be passed from (1, 3)

$\therefore\ c_1 = 6$ and $c_2 = -8$

Hence, $c_1c_2 = 6 \times (-8) = -48$

841 **(c)**

We have,

$e_1^2 = 1 - \frac{4}{18} = \frac{14}{18} = \frac{7}{9}$ and $e_2^2 = 1 + \frac{4}{9} = \frac{13}{9}$

$\therefore 2e_1^2 + e_2^2 = 3$

842 **(c)**

Now, radical axis of circles S_1 and S_2 is

$S_1 - S_2 = 0$

$\Rightarrow\ x^2 + y^2 - 6x - 6y + 4 - x^2 - y^2 + 2x + 4y - 3 = 0$

$\Rightarrow\ 4x + 2y - 1 = 0$ ...(i)

Radical axis of circle S_2 and S_3 is

$S_2 - S_3 = 0$

$\Rightarrow\ x^2 + y^2 - 2x - 4y + 3 - x^2 - y^2 - 2ky - 2y - 1 = 0$

$\Rightarrow\ (2 + 2k)x + 6y - 2 = 0$ ...(ii)

For existence of radical centre

$$\begin{vmatrix} 4 & 2 \\ 2+2k & 6 \end{vmatrix} \neq 0$$

$\Rightarrow\ 24 - 2(2+2k) \neq 0\ \Rightarrow\ k \neq 5$

843 **(d)**

Given equation of ellipse is

$$\frac{x^2}{\frac{5}{3}} + \frac{y^2}{\frac{5}{4}} = 1$$

The equation of tangents in slope from is

$$y = mx \pm \sqrt{\frac{5}{3}m^2 + \frac{5}{4}}$$

Slope of tangents are $\frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$

$$\therefore y = \pm\frac{1}{\sqrt{3}}x \pm \sqrt{\frac{5}{9} + \frac{5}{4}}$$

$$\Rightarrow y = \pm\frac{1}{\sqrt{3}}x \pm \frac{\sqrt{65}}{6}$$

844 **(c)**

Given circle is $x^2 + y^2 = a^2$ and point is (h, h)

$\therefore$ Equation of tangent at (h, h) is

$$xh + yh = a^2\ \Rightarrow\ y = -x + \frac{a^2}{h}$$

$\therefore$ Slope of the tangent is -1

845 **(a)**

The equation of ellipse can be rewritten as

$$\frac{(x+1)^2}{9} + \frac{(y-2)^2}{5} = 1$$

$$\therefore\ e = \sqrt{1 - \frac{5}{9}} = \frac{2}{3}\quad [\because a > b]$$

Foci are $\left\{\left(-1 \pm 3.\frac{2}{3}\right), 2\right\}$ *ie*, $(1,2)$ and $(-3,2)$

846 **(c)**

If $x = a\sin^2 t \Rightarrow y^2 = 4a(a\sin^2 t)$

$\Rightarrow y = \pm 2a\sin t$

$\therefore$ Option (c) is correct

847 **(a)**

Since, the semi-latusrectum of a parabola is HM of segments of a focal chord

$\therefore$ Semi-latusrectum $= \frac{2SP\cdot SQ}{SP+SQ} = \frac{2\times3\times2}{3+2} = \frac{12}{5}$

$\Rightarrow$ Latusrectum of the parabola $=2\times$ semi-latusrectum

$$= \frac{24}{5}$$

848 **(d)**

In given options $x^2 - y^2 = 0$, does not represent a hyperbola

849 **(a)**

Given parabola is

$$2x^2 = 14y$$

$$\Rightarrow x^2 = 7y$$

Here, $a = \frac{7}{4}$

$\therefore$ Equation of dierctrix is

$$y = -\frac{7}{4}$$

850 **(c)**

We know that the angle between the asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $2\tan^{-1}\left(\frac{b}{a}\right)$

Here, $a = 1$ and $b = \sqrt{3}$

$\therefore$ Required angle $= 2\tan^{-1}(\sqrt{3}) = 2\pi/3$

851 **(b)**

It is given that $e = \frac{1}{2}ae = 2$

Therefore, $a = 4$

$\because b^2 = a^2(1 - e^2)$

$\Rightarrow\ b^2 = 12$

Thus, the required ellipse is $\frac{x^2}{16} + \frac{y^2}{12} = 1$

852 **(c)**

The equation of the tangent at $P(3,4)$ to the circle $x^2 + y^2 = 25$is$3x + 4y = 25$, which meets the coordinate axes at $A\left(\frac{25}{3}, 0\right)$ and$B\left(0, \frac{25}{4}\right)$. If O be the origin, then the ΔOAB is a right angled triangle with $OA = \frac{25}{3}$ and $OB = \frac{25}{4}$

Area of the $\Delta OAB = \frac{1}{2} \times OA \times OB = \frac{1}{2} \times \frac{25}{3} \times \frac{25}{4} = \frac{625}{24}$

853 **(d)**

The equation of the circle passing through the points of intersection of the lines $2x + 3y - 6 = 0$ and $9x + 6y - 18 = 0$ with the coordinate axes is

$$(2x + 3y - 6)(9x + 6y - 18) - (2 \times 6 + 9 \times 3)xy = 0$$

$\Rightarrow x^2 + y^2 - 5x - 5y + 6 = 0$

The coordinates of the centre are $(5/2, 5/2)$

854 **(c)**

Let the slopes of the two tangents to the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be cm and $\frac{c}{m}$.

The equations of tangents are
$y = cmx + \sqrt{a^2c^2m^2 - b^2}$ ...(i)
And $my - cx = \sqrt{a^2c^2 - b^2m^2}$ ...(ii)
On squaring and subtracting Eq. (ii) from Eq. (i), we get
$(y - cmx)^2 - (my - cx)^2$
$= a^2c^2m^2 - b^2 - a^2c^2 + b^2m^2$
$\Rightarrow (1 - m^2)(y^2 - c^2x^2) = -(1 - m^2)(a^2c^2 + b^2)$
$\Rightarrow\ y^2 + b^2 = c^2(x^2 - a^2)$

855 **(c)**
Given equation can be rewritten as.

$(y - 2)^2 = 12x$

Here, vertex and focus are $(0, 2)$and $(3, 2)$

$\therefore$ Vertex of the required parabola is $(3, 2)$ and focus is $(3, 4)$. The axis of symmetry is $x = 3$ and latusrectum$= 4 \times 2 = 8$

Hence, required equation is

$(x - 3)^2 = 8(y - 2)$

$\Rightarrow\ x^2 - 6x - 8y + 25 = 0$

856 **(d)**
The equation of tangent to the parabola

$y^2 = 4x$ is $y = mx + \dfrac{1}{m}$

This is also the tangent to parabola $x^2 = -8y$

$\therefore\ x^2 = -8\left(mx + \dfrac{1}{m}\right)$

$\Rightarrow\ mx^2 + 8m^2x + 8 = 0$ has equal roots

$\Rightarrow\ 64\,m^4 = 32m\ \ [\because D = 0]$

$\Rightarrow\ m = \dfrac{1}{\sqrt[3]{2}}$

$\therefore$ Equation of tangent is $y = \frac{1}{\sqrt[3]{2}}x + \sqrt[3]{2}$

857 **(c)**
Here centre $(-2, 2)$ and radius is 2
Hence, both coordinates and radius is same, so it touches both axes

858 **(c)**
The centre of the circle is the point of intersection of the given diameters $2x - 3y = 5$ and $3x - 4y = 7$
Which is $(1, -1)$ and the radius is r, where $\pi r^2 = 154$
$\Rightarrow\ r^2 = 154 \times \dfrac{7}{22}\ \ \Rightarrow\ r = 7$
and hence, the required equation of the circle is
$(x - 1)^2 + (y + 1)^2 = 7^2$
$\Rightarrow\ x^2 + y^2 - 2x + 2y = 47$

859 **(c)**
Equation of an ellipse is
$16x^2 + 25y^2 = 400$
$\Rightarrow \dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$
Here, $a^2 = 25$ and $b^2 = 16$
But $b^2 = a^2(1 - e^2)$
$\Rightarrow 16 = 25(1 - e^2) \Rightarrow \dfrac{16}{25} = 1 - e^2$
$\Rightarrow e^2 = \dfrac{9}{25} \Rightarrow e = \dfrac{3}{5}$
Now, foci of the ellipse are $(3, 0)$
Now, $PF_1 + PF_2 = 2a = 2 \times 5 = 10$

860 **(c)**
Given equation of ellipse is $\frac{x^2}{4} + \frac{y^2}{9} = 1$
Here, $a^2 = 4, b^2 = 9 \Rightarrow b > a$
$\therefore 4 = 9(1 - e^2) \Rightarrow e = \dfrac{\sqrt{5}}{3}$
Distance between the directrices $= \frac{2b}{e}$
$= \dfrac{2 \times 3 \times 3}{\sqrt{5}} = \dfrac{18}{\sqrt{5}}$

861 **(b)**
Any point on the ellipse is $P(3\cos\theta, 2\sin\theta)$
Equation of the tangent at P is
$\dfrac{x}{3}\cos\theta + \dfrac{y}{2}\sin\theta = 1$
Which meets the tangents $x = 3$ and $y = -3$ at the extremities of the major axis at $T\left(3, \frac{2(1-\cos\theta)}{\sin\theta}\right)$ and $T'\left(-3, \frac{2(1+\cos\theta)}{\sin\theta}\right)$
Equation of circle on TT' as diameter is
$$(x - 3)(x + 3) + \left(y - \frac{2(1 - \cos\theta)}{\sin\theta}\right)\left(y - \frac{2(1 + \cos\theta)}{\sin\theta}\right) = 0$$
$\Rightarrow x^2 + y^2 - \dfrac{4}{\sin\theta}y - 5 = 0$
Which passes through $(\sqrt{5}, 0)$

862 **(a)**
Let $y = m_1x$ and $y = m_2x$ be a pair of conjugate diameters of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and let $P(a\cos\theta, b\sin\theta)$ and $Q(a\cos\phi, b\sin\phi)$ be ends of these two diameters. Then,

$$m_1 m_2 = -\frac{b^2}{a^2}$$

$$\Rightarrow \frac{b\sin\theta - 0}{a\cos\theta - 0} \times \frac{b\sin\phi - 0}{a\cos\phi - 0} = -\frac{b^2}{a^2}$$

$$\Rightarrow \sin\theta\sin\phi = -\cos\theta\cos\phi$$

$$\Rightarrow \cos(\theta - \phi) = 0$$

$$\Rightarrow \theta - \phi = \pm\frac{\pi}{2}$$

863 **(b)**

Given lines are $3x - 4y + 5 = 0$ and $3x - 4y - \frac{9}{2} = 0$, which are parallel to each other

$\therefore$ Perpendicular distance, $d = \left|\frac{5+\frac{9}{2}}{\sqrt{3^2+4^2}}\right| = \frac{19}{10}$

$\therefore$ Radius of circle$= \frac{d}{2} = \frac{19}{20} = 0.95$

864 **(c)**

The equation of the tangent at $(2\sec\theta, 3\tan\theta)$ is

$$\frac{x}{2}\sec\theta - \frac{y}{3}\tan\theta = 1$$

It is parallel to the line $3x - y + 4 = 0$

$$\therefore \frac{3\sec\theta}{2\tan\theta} = 3 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$$

865 **(b)**

Circle through the points $(0,0)$, $(a,0)$ and $(0,b)$ is

$x^2 + y^2 - ax - by = 0$

Its centre is $\left(\frac{a}{2}, \frac{b}{2}\right)$

866 **(c)**

In centre of circle is (c, α) and radius is a, then equation of circle is

$r^2 - 2cr\cos(\theta - \alpha) = a^2 - c^2$

Here, centre $\left(2, \frac{\pi}{2}\right)$ and radius 3

$\therefore$ Equation of circle is $r^2 - 2 \times 2r\cos\left(\theta - \frac{\pi}{2}\right) = 3^2 - 2^2$

$\Rightarrow r^2 - 4r\sin\theta = 5$

867 **(c)**

Let the equation of circle be

$x^2 + y^2 + 2hx + 2ky + c = 0$ ...(i)

The locus of whose centre is to be obtained, since the circle cuts

$x^2 + y^2 + 4x - 6y + 9 = 0$ ...(ii)

And $x^2 + y^2 - 4x + 6y + 4 = 0$ ...(iii)

Orthogonally, then

$2h(2) + 2k(-3) = c + 9$

$\Rightarrow 4h - 6k = c + 9$

And $2h(-2) + 2k(3) = c + 4$

$\Rightarrow -4h + 6k = c + 4$

On solving Eqs. (iv) and (v), we get

$c + 9 = -c - 4$

$\Rightarrow 2c = -13$ ...(vi)

On putting the value of c in Eq. (iv)

$\Rightarrow 8h - 12k = 5$ ...(vii)

Centre of the given circle is $(-h, -k)$

$\therefore$ Locus of $(-h, -k)$ from Eq. (vii) is

$8(-x) - 12(-y) = 5$

$\Rightarrow 8x - 12y + 5 = 0$

868 **(b)**

Given common tangents are $2x - 4y - 9 = 0$ and $2x - 4y + \frac{7}{3} = 0$ which are parallel

$\therefore$ Diameter=distance between tangents

= distance between parallel lines

$$= \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}} = \frac{\left|-9 - \frac{7}{3}\right|}{\sqrt{2^2 + (4)^2}}$$

$$\Rightarrow \quad d = \frac{34}{3.2\sqrt{5}}$$

$\therefore$ Radius$= \frac{17}{6\sqrt{5}}$

869 **(a)**

Given, $x^2 + 2x - 3 = 0$

$\Rightarrow \quad x_1 = -3, x_2 = 1$

and $y^2 + 4y - 12 = 0$

$\Rightarrow \quad y_1 = -6, \quad y_2 = 2$

$\therefore$ Points are $P(-3, -6)$ and $Q(1, 2)$

Since, P and Q are end points of a diameter

$\therefore$ Centre=mid point of PQ

$$= \left(\frac{-3+1}{2}, \frac{-6+2}{2}\right) = (-1, -2)$$

870 **(d)**

Since, the line $y = x - 1$ passes through focus $(1, 0)$

$\Rightarrow y = x - 1$ is a focal chord

So, angle between tangent is $\frac{\pi}{2}$

871 **(b)**

We have,

$x^2 - 4x - 8y - 4$

$\Rightarrow (x - 2)^2 = 8(y + 1)$

Thus, the coordinates of the focus are given by

$x - 2 = 0, y + 1 = 2$ $\left[\because x^2 = 4ay \text{ has its focus at } (0, a). \text{Here, } a = 2\right]$

$\Rightarrow x = 2, y = 1$

Hence, the coordinates of the focus are (2,1)

872 **(b)**

Let the equation of line be $y = mx + c$.If this line touch the parabola $y^2 = 8x$, then

$$y = mx + \frac{2}{m}$$

This line also touches the circle $x^2 + y^2 = 2$, then radius = perpendicular distance from centre (0,0) to the line

$\Rightarrow \sqrt{2} = \left|\frac{0+0-\frac{2}{m}}{\sqrt{1+m^2}}\right|$

$\Rightarrow m^2(1+m^2) = 2 \Rightarrow m = 1$

$\therefore$ This required equation of tangent be

$y = x + 2$

873 **(a)**

Let (h, k) is mid point of chord.

Then, its equation is

$3hx - 2ky + 2(x+h) - 3(y+k)$
$= 3h^2 - 2k^2 + 4h - 6k$

$\Rightarrow x(3h+2) + y(-2k-3)$
$= 3h^2 - 2k^2 + 2h - 3k$

Since, this line is parallel to $y = 2x$.

$\therefore \frac{3h+2}{2k+3} = 2$

$\Rightarrow 3h + 2 = 4k + 6 \Rightarrow 3h - 4k = 4$

Thus, locus of mid point is $3x - 4y = 4$.

874 **(b)**

The centre of given circle is $(-g, -f)$

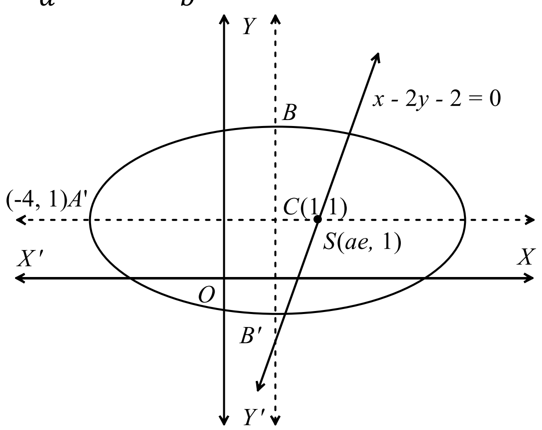
If the given line $ax + by + c = 0$ is normal to the circle, then it passes through the centre of circle.

$\therefore a(-g) + b(-f) + c = 0 \Rightarrow ag + bf - c = 0$

875 **(a)**

Clearly, C, being the mid-point of AA'', has the coordinates $(1,1)$. Also, slope of AA'' is 0. So, AA'' is parallel to x-axis. Thus, the axes of the ellipse are parallel to the coordinate axes. Let the equation of the ellipse be

$\frac{(x-1)^2}{a^2} + \frac{(y-1)^2}{b^2} = 1$ ... (i)



Now,

$AA'' = 10 \Rightarrow 2a = 10 \Rightarrow a = 5$

Since $x - 2y - 2 = 0$ is a focal chord. Therefore,

$ae - 2 - 2 = 0 \Rightarrow ae = 4$

Now,

$b^2 = a^2(1-e^2) = 25 - 16 = 9$

Hence, the equation of the ellipse is $\frac{(x-1)^2}{25} + \frac{(y-1)^2}{9} = 1$

876 **(c)**

Given equation can be written as

$24\left(x^2 - 5x + \frac{25}{4}\right) + 9(y^2 - 10y + 25)$

$+225 - 150 - 225 = 0$

$\Rightarrow \frac{\left(x-\frac{5}{2}\right)^2}{\frac{150}{24}} + \frac{(y-5)^2}{\frac{150}{9}} = 1$

$\therefore e = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{9}{24}} = \sqrt{\frac{15}{24}}$

877 **(b)**

Any normal to the parabola $y^2 = 12x$ is $y + tx = 6t + 3t^3$. It is similar to the line $y + x = k$

$\Rightarrow t = 1, 6t + 3t^3 = k$

$\therefore 6(1) + 3(1)^3 = k$ or $k \Rightarrow 9$

878 **(b)**

Given equation of circles are

$x^2 + y^2 + 4x + 8y = 0$ ...(i)

and $x^2 + y^2 + 8x + 2ky = 0$ ...(ii)

for circle (i),

centre, $C_1 = (-2, -4)$

and radius $r_1 = \sqrt{4 + 16 - 0} = \sqrt{20} = 2\sqrt{5}$

for circle (ii),

centre, $C_2 = (-4, -k)$

and radius, $r_2 = \sqrt{16 + k^2 - 0} = \sqrt{16 + k^2}$

given circles touch each other externally

$\therefore |C_1C_2| = r_1 + r_2$

$\Rightarrow \sqrt{(-2+4)^2 + (-4+k)^2} = 2\sqrt{5} + \sqrt{16+k^2}$

$\Rightarrow 4 + 16 + k^2 - 8k$
$= 20 + 16 + k^2 + 4\sqrt{5}\sqrt{16+k^2}$

$\Rightarrow -16 - 8k = 4\sqrt{5}\sqrt{16+k^2}$

$\Rightarrow -4 - 2k = \sqrt{5}\sqrt{16+k^2}$

$\Rightarrow (4+2k) = \left(\sqrt{5}\sqrt{16+k^2}\right)^2$

$\Rightarrow 16 + 4k^2 + 16k = 5(16 + k^2)$

$\Rightarrow k^2 - 16k + 64 = 0$

$\Rightarrow (k-8)^2 = 0$

$\Rightarrow k = 8$

879 **(b)**

The circles will touch each other if the length of the common chord is zero i.e.

$\sqrt{4c^2 - 2(a-b)^2} = 0 \Rightarrow 2c^2 = (a-b)^2$

$$\Rightarrow a - b = \pm\sqrt{2}\,c$$

880 **(a)**

We know , if P is any point on the curve, then sum of focal distances=length of major axis

$= SP + S'P = 2a = 2(5) = 10$

881 **(a)**

If $y = mx + 1$, touches the parabola $y =^2 = 4x$,

then $c = \frac{a}{m} \Rightarrow 1 = \frac{1}{m} \Rightarrow m = 1$

882 **(b)**

Given, $2a = 8$ and $2ae = 10$

$\Rightarrow e = \frac{10}{8} = \frac{5}{4}$

Now, $b^2 = a^2(e^2 - 1) = 16\left(\frac{25}{16} - 1\right) = 9$

$\Rightarrow b = \pm 3$

Hence, length of latusrectum $= \frac{2b^2}{a} = \frac{2\times 9}{4} = \frac{9}{2}$

883 **(b)**

The equation of the circumcircle of the rectangle is

$x(x-4) + y(y-3) = 0$

$\Rightarrow x^2 + y^2 - 4x - 3y = 0 \Rightarrow (x-2)^2 + \left(y - \frac{3}{2}\right)^2$

$$= \left(\frac{5}{2}\right)^2$$

The equations of the tangents to this circle which are parallel to the diagonal joining (0,0) and (4,3) are

$y - \frac{3}{2} = \frac{3}{4}(x-2)$

$$\pm \frac{5}{2}\sqrt{1 + \frac{9}{16}} \qquad \left[\begin{array}{c}\because \text{Slope the} \\ \text{tangent} = 3/4\end{array}\right]$$

i.e. $6x - 8y \pm 25 = 0$

884 **(b)**

Given, $y = 4x + c$ and $\frac{x^2}{4} + y^2 = 1$

Condition for tangency,

$c^2 = a^2m^2 + b^2$

$\therefore c^2 = 4(4)^2 + 1^2$

$\Rightarrow c^2 = 65$

$\Rightarrow c = \pm\sqrt{65}$

Hence, for two values, of c, the line touches the curve

885 **(a)**

The equation of a tangent to the parabola $y^2 = 4x$ is $= mx + \frac{1}{m}$. If it passes through $(-2, -1)$, then

$-1 = -2m + \frac{1}{m} \Rightarrow 2m^2 - m - 1 = 0$

$m_1 + m_2 = \frac{1}{2}, m_1 + m_2 = -\frac{1}{2}$

Now, $\tan\alpha = \pm\frac{m_1 - m_2}{1 + m_1 m_2}$

$= \pm\frac{\sqrt{(m_1 + m_2)^2 - 4m_1m_2}}{1 + m_1m_2}$

$= \pm\frac{\sqrt{1/4 + 4/2}}{1 - 1/2} = 3$

886 **(c)**

We know that, the locus of point of intersection of two perpendicular tangents drawn on the ellipse is $x^2 + y^2 = a^2 + b^2$, which is called director circle

Given equation of ellipse is $\frac{x^2}{9} + \frac{y^2}{4} = 1$

Here, $a^2 = 9, b^2 = 4$

$\therefore$ Locus is $x^2 + y^2 = a^2 + b^2$

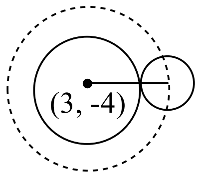
$\Rightarrow x^2 + y^2 = 9 + 4$

$\Rightarrow x^2 + y^2 = 13$

887 **(c)**

Centre of required circle $= (3, -4)$

Radius of required circle $= 5 + 1 = 6$



$\therefore$ Locus of circle is

$(x-3)^2 + (y+4)^2 = 36$

$\Rightarrow x^2 - 6x + 9 + y^2 + 16 + 8y = 36$

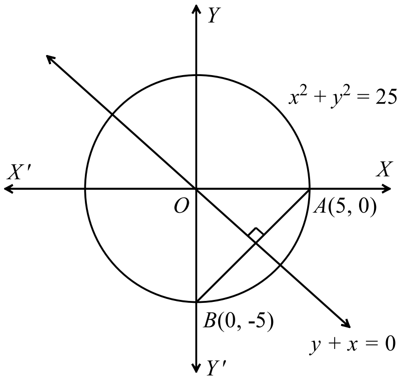
$\Rightarrow x^2 + y^2 - 6x + 8y - 11 = 0$

888 **(a)**

The equation of a line passing through (5,0) and perpendicular to $x + y = 0$, is $x - y = 5$

Clearly, it cuts y-axis at $B(0, -5)$

$\therefore AB = \sqrt{5^2 + 5^2} = 5\sqrt{2}$



889 **(a)**

Let $y = x + c$ is parallel to the given line. Since, it is a tangent to the given hyperbola

$c^2 = 3 - 2 \Rightarrow c = \pm 1$

So, required tangents are $y = x \pm 1$

890 **(c)**

Any tangent to $x^2 = 4y$ is of the form

$x = my + \frac{1}{m}$

Therefore, $y = 2x + k$ or, $x = \frac{1}{2}y - \frac{k}{2}$ will be a tangent to $x^2 = 4y$,

if

$m = \frac{1}{2}$ and $\frac{1}{m} = -\frac{k}{2} \Rightarrow 1 = -\frac{k}{4} \Rightarrow k = -4$

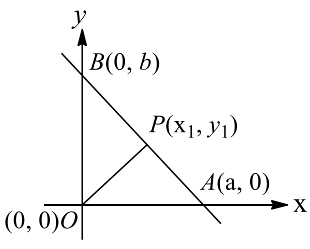
891 **(b)**

Equation of line is

$\frac{x}{a} + \frac{y}{b} = 1$ ...(i)

Let P be the foot of the perpendicular drawn from the origin to the line whose coordinates are (x_1, y_1)

Since, $OP \perp AB$



$\therefore$ Slope of OP $\times$ Slope of $AB = -1$

$\Rightarrow \left(\frac{y_1}{x_1}\right)\left(\frac{b}{-a}\right) = -1$

$\Rightarrow by_1 = ax_1$ ...(ii)

Since, P lies on the line AB, then

$\frac{x_1}{a} + \frac{y_1}{b} = 1$

$\Rightarrow bx_1 + ay_1 = ab$ ...(iii)

From Eq. (ii) and (iii), we get

$x_1 = \frac{ab^2}{a^2 + b^2}$ and $y_1 = \frac{a^2 b}{a^2 + b^2}$

Now, $x_1^2 + y_1^2 = \left(\frac{ab^2}{a^2+b^2}\right)^2 + \left(\frac{a^2 b}{a^2+b^2}\right)^2$

$\Rightarrow x_1^2 + y_1^2 = \frac{a^2 b^4}{(a^2 + b^2)^2} + \frac{a^4 b^2}{(a^2 + b^2)^2}$

$\Rightarrow x_1^2 + y_1^2 = \frac{a^2 b^2 (a^2 + b^2)}{(a^2 + b^2)^2}$

$\Rightarrow x_1^2 + y_1^2 = \frac{a^2 b^2}{(a^2 + b^2)}$

$\Rightarrow x_1^2 + y_1^2 = \frac{1}{\frac{1}{a^2} + \frac{1}{b^2}}$

But $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$ (given)

$\therefore x_1^2 + y_1^2 = c^2$

Thus, the locus of $P(x_1, y_1)$ is

$x^2 + y^2 = c^2$

Which is the equation of circle

892 **(d)**

Let $P(a\cos\theta, b\sin\theta)$ be any point on the ellipse. The equation of the tangent at P is

$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$

It cuts the lines $x = a$ and $x = -a$ at

$L\left(a, \frac{b(1-\cos\theta)}{\sin\theta}\right)$ and $L'\left(-a, \frac{b(1+\cos\theta)}{\sin\theta}\right)$ respectively

$\therefore AL = \frac{b(1-\cos\theta)}{\sin\theta}$ and $AL' = \frac{b(1+\cos\theta)}{\sin\theta}$

$\Rightarrow AL \cdot AL' = b^2$

893 **(c)**

The equation of the normal at $(at^2, 2\,at)$ is

$tx + y = 2\,at + at^3$

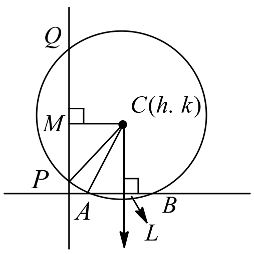
Clearly, its slope is $-t$

894 **(c)**

Let $C(h,k)$ be the centre of the circle passing through the end points of the rod AB and PQ of lengths a and b respectively. CL and CM be perpendicular drawn from C on AB and PQ respectively. Then, $CA = CP$ (radii of the same circle)

$\Rightarrow k + \frac{a^2}{4} = h^2 + \frac{b^2}{4} \quad \left(\because AL = \frac{a}{2} \text{ and } MP = \frac{b}{2}\right)$

$\Rightarrow 4(h^2 - k^2) = a^2 - b^2$



Hence, locus of (h,k) is $4(x^2 - y^2) = a^2 - b^2$

895 **(d)**

Given circles are $x^2 + y^2 - 2x + 8y + 13 = 0$ and $x^2 + y^2 - 4x + 6y + 11 = 0$

Here, $C_1 = (1,-4), C_2 = (2,-3)$

$\Rightarrow \quad r_1 = \sqrt{1 + 16 - 13} = 2$

And $r_2 = \sqrt{4 + 9 - 11} = \sqrt{2}$

Now, $d = C_1C_2 = \sqrt{(2-1)^2 + (-3+4)^2} = \sqrt{2}$

$\therefore \cos\theta = \frac{|d^2 - r_1^2 - r_2^2|}{2r_1r_2} = \frac{|2-4-2|}{2 \times 2 \times \sqrt{2}} = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = 45°$

896 **(b)**

We know that the equation of the normal at (x_1, y_1) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is given by

$\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$

The equation of the ellipse is

$9x^2 + 16y^2 = 180 \Rightarrow \frac{x^2}{20} + \frac{y^2}{\frac{45}{4}} = 1$

The equation of the normal to this ellipse at (2,3) is

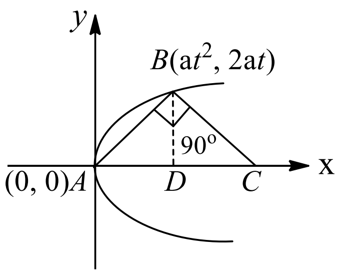
$\frac{20}{2}x - \frac{45}{12}y = 20 - \frac{45}{4} \quad \left[\text{Using : } \frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2\right]$

$\Rightarrow 10x - \frac{15}{4}y = \frac{35}{4} \Rightarrow 8x - 3y = 7$

897 **(c)**

Given equation of parabola is $y^2 = 4ax$

Let the coordinates of B are $(at^2, 2at)$



Slope of $AB = \frac{2}{t}$

Since, BC is perpendicular to AB

So, slope of $BC = -\frac{t}{2}$

Equation of BC is $y - 2at = -\frac{t}{2}(x - at^2)$

This line meets to the x-axis at point C

Put $y = 0 \Rightarrow x = 4a + at^2$

So, distance $CD = 4a + at^2 - at^2 = 4a$

898 **(c)**

Focal distance of any point $P(x,y)$ on the ellipse is equal to $a + ex$

Here, $x = a\cos\theta$. Hence, $SP = a + ae\cos\theta = a(1 + e\cos\theta)$

899 **(c)**

Given equation of curve is $y^2 + 2xy + x^2 + 2x + 3y + 1 = 0$

Here $h^2 = ab$, therefore the given curve is a parabola. The position of the point $(1,-2)$ with respect to the parabola is obtained as $(-2)^2 + 2(1)(-2) + (1)^2 + 2(1) + 3(-2) + 1 = -2 < 0$

Since, point is inside the parabola therefore no tangent can be drawn to the parabola

900 **(c)**

Now taking option (c).

Let $x = a\frac{e^t + e^{-t}}{2} \Rightarrow \frac{2x}{a} = e^t + e^{-t}$ ...(i)

And $\frac{2y}{a} = e^t - e^{-t}$ ...(ii)

On squaring and subtracting Eq. (ii) from Eq. (i), we get

$$\frac{4x^2}{a^2} - \frac{4y^2}{b^2} = 4$$

$$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

901 **(c)**

Let the other end be $(t, 3-t)$

So, the equation of the variable circle is

$(x-1)(x-t)+(y-1)(y-3+t)=0$

$\Rightarrow \quad x^2+y^2-(1+t)x-(4-t)y+3=0$

$\therefore$ The centre (α, β) is given by

$\alpha = \frac{1+t}{2}, \beta = \frac{4-t}{2} \quad \Rightarrow \quad 2\alpha + 2\beta = 5$

Hence, the locus is $2x+2y=5$

902 **(b)**

Let the points be $A=(2,2)$and$B=(3,3)$. Since the circle passing through these points, so they satisfy the equation of the circle.

Now, taking option (b),

Let $\ S \equiv x^2+y^2-5x-5y+12=0$

At $\ A=(2,2)$

$2^2+2^2-5(2)-5(2)+12=0$

At $\ B=(3,3)$

$3^2+3^3-5(3)-5(3)+12=0$

903 **(d)**

The equation of parabola can be written as

$$(y+2)^2 = -4\left(x-\frac{1}{2}\right)$$

$\Rightarrow \quad Y^2=-4X \ $ where $X = x-\frac{1}{2}, Y = y+2$

An equation of its directrix is $X=1$

$\therefore$ Required directrix is $x=\frac{3}{2}$

904 **(b)**

Here, $a=5, b=4$

$\therefore$Required sum$= a+b$

$= \ 9$

905 **(d)**

Given parabola $y^2=ax$

$ie, y^2 = 4\left(\frac{a}{4}\right)x \quad$ ...(i)

let point of contact is (x_1, y_1), then equation of tangent is

$$yy_1 = \frac{a}{2}(x+x_1)$$

Here, $m = \frac{a}{2y_1} = \tan 45^\circ$

$$\Rightarrow \frac{a}{2y_1} = 1 \Rightarrow y_1 = \frac{a}{2}$$

From Eq. (i), $x_1 = \frac{a}{4}$

$\therefore$ Point of contact is $\left(\frac{a}{4}, \frac{a}{2}\right)$

906 **(b)**

$y=mx+c$ is tangent to $x^2+y^2=a^2$, if

$c = \pm\sqrt{1+m^2}$

Since, $\ y = -\frac{lx}{m}+\frac{1}{m}$ is tangent to $x^2+y^2=a^2$, if

$$\frac{1}{m} = \pm\frac{a}{m}\sqrt{l^2+m^2}\left[\because \quad c = a\sqrt{(1+m^2)}\right]$$

$$\Rightarrow \quad l^2 = m^2 = \frac{1}{a^2}$$

Hence, locus of point (l,m) is $x^2+y^2 = \frac{1}{a^2}$

907 **(b)**

Given equation can be rewritten as

$$\frac{(x-1)^2}{16} - \frac{(y+2)^2}{9} = 1$$

$$\therefore \ e = \sqrt{\frac{16+9}{16}} = \frac{5}{4}$$

908 **(d)**

Since, the semi latusrectum of a parabola is the harmonic mean between the segments of any focal chord of the parabola.

$\therefore$ l is the harmonic mean between b and c.

Hence, $l = \frac{2bc}{b+c}$

909 **(b)**

Given ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose are is $\pi\ ab$. The auxiliary circle to the given ellipse is $x^2+y^2=a^2$ whose area is πa^2

Given that, $\pi a^2 = 2\pi ab \Rightarrow \ a = 2b$

Now, eccentricity of ellipse

$$= \sqrt{1-\frac{b^2}{a^2}}$$

$$= \sqrt{1-\frac{b^2}{4b^2}} = \frac{\sqrt{3}}{2}$$

910 **(b)**

Mid point of (4, 0) and (0, 4) is (2, 2)

Required distance$= \sqrt{(2-0)^2+(2-0)^2}$

$= \sqrt{8}$
$= 2\sqrt{2}$

911 **(c)**

If mid point is given, then equation of chord is $T = S_1$

$\therefore\ xx_1 + yy_1 - a^2 = x_1^2 + y_1^2 - a^2$

$\Rightarrow\ xx_1 + yy_1 = x_1^2 + y_1^2$

912 **(c)**

The equation of any tangent to $y^2 = 8x$ is

$y = mx + \frac{2}{m}$

If it passes through (1,3), then

$3 = m + \frac{2}{m} \Rightarrow m^2 - 3m + 2 = 0 \Rightarrow m = 1,2$

Let θ be the angle between the tangents drawn from (1,3). Then,

$\tan\theta = \left|\frac{2-1}{1+2\times 1}\right| = \frac{1}{3} \Rightarrow \theta = \tan^{-1}\frac{1}{3}$

913 **(c)**

The centre and radius of given circle are $(3,-2)$ and 5 respectively

The equation of a line parallel to $4x + 3y + 5 = 0$ is $4x + 3y + \lambda = 0$

$\therefore\ \left|\frac{4\times 3 + 3\times(-2) + \lambda}{\sqrt{4^2+3^2}}\right| = 5$

$\Rightarrow\ \lambda = 19, -31$

$\therefore$ Equation of tangents are

$4x + 3y + 19 = 0$ and $4x + 3y - 31 = 0$

914 **(a)**

Since, asymptotes $3x - 4y = 7$ and $4x + 3y = 8$ are perpendicular, therefore it is a rectangular hyperbola, so eccentricity is $\sqrt{2}$.

915 **(d)**

The length of tangent from the point (1, 2) to the circle $x^2 + y^2 + x + y - 4 = 0$ is

$\sqrt{1+4+1+2-4}$, *ie*, 2

And the length of tangent from the point (1, 2) to the circle

$3x^2 + 3y^2 - x - y - k = 0$ is

$\sqrt{3+12-1-2-k}$ *ie*, $\sqrt{12-k}$

$\therefore\ \frac{2}{\sqrt{12-k}} = \frac{4}{3}$

$\Rightarrow\ \frac{3}{2} = \sqrt{12-k}$

$\Rightarrow \frac{9}{4} = 12 - k\ \Rightarrow\ k = \frac{39}{4}$

916 **(a)**

The coordinates of P and Q are $(a\cos\theta, b\sin\theta)$ and $(-a\sin\theta, b\cos\theta)$ respectively. Let (h,k) be the co-ordinates of the mid-point of PQ. Then, $2h = a(\cos\theta - \sin\theta)$ and $2k = b(\sin\theta + \cos\theta)$

$\Rightarrow \frac{4h^2}{a^2} + \frac{4k^2}{b^2} = 2$

Hence, the locus of (h,k) is

$\frac{4x^2}{a^2} + \frac{4y^2}{b^2} = 2$ or, $\frac{2x^2}{a^2} + \frac{2y^2}{b^2} = 1$

917 **(b)**

Given circles can be rewritten as

$x^2 + y^2 + \frac{2g_1}{a} + \frac{2f_1}{a}y + \frac{c_1}{a} = 0$

And $\ x^2 + y^2 + \frac{2g_2}{b}x + \frac{2f_2}{b}y + \frac{c_2}{b} = 0$

Centres of circles are $C_1\left(-\frac{g_1}{a}, -\frac{f_1}{a}\right)$ and $C_2\left(-\frac{g_2}{b}, -\frac{f_2}{b}\right)$

respectively,

We know, if two circles cut orthogonally, then

$2(G_1 + G_2 + F_1F_2) = C_1 + C_2$

$\therefore\ 2\left(\frac{g_1g_2}{ab} + \frac{f_1f_2}{ab}\right) = \frac{c_1}{a} + \frac{c_2}{b}$

$\Rightarrow\ 2(g_1g_2 + f_1f_2) = bc_1 + ac_2$

918 **(d)**

Given equation can be rewritten as

$$\frac{(x-3)^2}{16} + \frac{(y-1)^2}{4} = 1$$

This represents an ellipse

$$e = \sqrt{1 - \frac{4}{16}} = \frac{\sqrt{3}}{2}$$

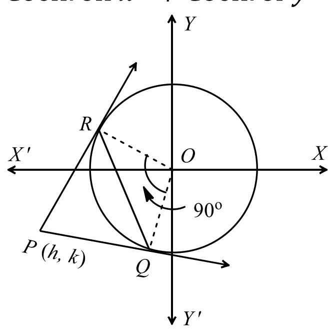
919 **(d)**

The equation of the chord of contact of tangents drawn from the point (h,k) to the circle $x^2 + y^2 = a^2$ is $hx + ky = a^2$. The combined equation of OQ and OR is

$x^2 + y^2 = a^2\left(\frac{hx+ky}{a^2}\right)^2$

Since OQ is perpendicular to OR. Therefore,

Coeff. off x^2 + Coeff. of $y^2 = 0 \Rightarrow 2\,a^2 = h^2 + k^2$



920 **(a)**

The equation of a tangent parallel to y-axis is $x = c$.

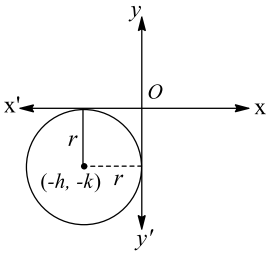
This touches $x^2 + y^2 = 9$. Therefore $c = \pm 3$

Thus, the equation of the tangents are $x = \pm 3$

Clearly, $x = 3$ is the tangent not lying in the third quadrant and it meets the circle at (3,0)

921 **(d)**

Let $(-h, -k)$ be the centre of the circle



Circle touches the coordinate axes in IIIrd quadrant

$\therefore$ Radius $= -h = -k$

$\Rightarrow h = k = -5$

$\therefore$ The required equation of circle is

$(x+5)^2 + (y+5)^2 = 25$

922 **(d)**

Given equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{4} = 1$

Here, $a^2 = 9, b^2 = 4$

and equation of line is $y = -x + \sqrt{2}p$ ...(i)

If the line $y = mx + c$ touches the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $c^2 = a^2m^2 - b^2$ ...(ii)

From Eq.(i), we get

$m = -1, c = \sqrt{2}p$

On putting these values in Eq. (ii), we get

$\left(\sqrt{2}p\right)^2 = 9(1) - 4$

$\Rightarrow 2p^2 = 5$

923 **(b)**

Let $P(x_1, y_2)$ be the point outside the circle. From the given condition

$$\frac{x_1^2 + y_1^2 + 2x_1 - 4y_1 - 20}{x_1^2 + y_1^2 - 4x_1 + 2y_1 - 44} = \frac{2}{3}$$

$\Rightarrow 3x_1^2 + 3y_1^2 + 6x_1 - 12y_1 - 60$

$= 2x_1^2 + 2y_1^2 - 8x_1 + 4y_1 - 88$

$\Rightarrow x_1^2 + y_1^2 + 14x_1 - 16y_1 + 28 = 0$

Thus, the locus of point is

$x^2 + y^2 + 14x - 16y + 28 = 0$

$\therefore$ Coordinates of centre of circle are $(-7, 8)$

924 **(b)**

We have, $a^2 = 36, b^2 = 49$

$\therefore$ Length of the latusrectum $= \dfrac{2a^2}{b} = 2 \times \dfrac{36}{7} = \dfrac{72}{7}$

925 **(b)**

Given equation can be rewritten as

$$\frac{x^2}{2-\lambda} + \frac{y^2}{-\lambda+5} = 1$$

To represent an ellipse,

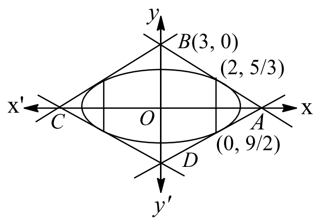
$2 - \lambda > 0$ and $-\lambda + 5 > 0$

$\Rightarrow \lambda < 2$ and $\lambda < 5$

$\Rightarrow \lambda < 2$

927 **(d)**

The quadrilateral formed by the tangents at the end points of latusrectum is a rhombus. It is symmetrical about the axes.



So, total area is four times the area of the right angled triangle formed by the tangent and axes in the Ist quadrant

Now, $ae = \sqrt{a^2 - b^2} \Rightarrow ae = 2$

$\therefore$ Coordinates of one end point of latusrectum are $\left(2, \frac{5}{3}\right)$

The equation of tangent at that point is $\frac{x}{\frac{9}{2}} + \frac{y}{3} = 1$.

This equation meets the coordinate axes at a point $A\left(0, \frac{9}{2}\right)$ and $B(3, 0)$

In ΔAOB, Area$= \frac{1}{2} \times \frac{9}{2} \times 3 = \frac{27}{4}$

Total area of rhombus $ABCD = 4 \times$ area of ΔAOB

$= 4 \times \frac{27}{4} = 27$ sq unit

928 **(a)**

Let (h, k) be the mid-point of the chord $2x + y - 4 = 0$ of the parabola $y^2 = 4x$. Then, its equation is

$ky - 2(x+h) = k^2 - 4h$ [Using $T = S'$]

$\Rightarrow 2x - ky + k^2 - 2h = 0$ ...(i)

Equations (i) and $2x + y - 4 = 0$ represent the same line

$\therefore -k = 1$ and $k^2 - 2h = -4 \Rightarrow k = -1, h = 5/2$

Hence, the required point is $(5/2, -1)$

929 **(c)**

Distance from centre (2, 1) to the line $3x + 4y - 5 =$ radius of circle

$\Rightarrow \dfrac{|3(2) + 4(1) - 5|}{\sqrt{3^2 + 4^2}} = r \quad \Rightarrow \quad r = 1$

$\therefore$ Equation of circle is

$(x-2)^2 + (y-1)^2 = 1^2$

$\Rightarrow x^2 + y^2 - 4x - 2y + 4 = 0$

930 **(c)**

Given $y^2 = -8x$

Here, $a = -2$

We know that if one end of a focal chord is $(at^2, 2at)$, then the other end will be $\left(\frac{a}{t^2}, -\frac{2a}{t}\right)$

Here, one end is $(-1, 2\sqrt{2})$

$\therefore\ at^2 = -1 \Rightarrow t = \frac{1}{\sqrt{2}}$ $\quad [\because a = -2]$

So, other end$= \left(\frac{-2}{1/2}, \frac{-2\times-2}{1/\sqrt{2}}\right) = (-4, 4\sqrt{2})$

931 **(a)**

Length of the chord

$= \sqrt{[4\cos(\theta+60°) - 4\cos\theta]^2 + [4\sin(\theta+60°) -}$

$= 4\sqrt{\begin{matrix}\cos^2(\theta+60°) + \cos^2\theta + \\ \sin^2(\theta+60°) + \sin^2\theta - 2\cos(\theta+60°)\end{matrix}}$

$\cos\theta - 2\sin(\theta+60°)\sin\theta$

$= 4\sqrt{1+1-2\cos 60°} = 4$

932 **(c)**

Given equation can be rewritten as

$$\frac{(x-2)^2}{12} - \frac{(y-1)^2}{4} = 1$$

Now, $e = \sqrt{1 + \frac{4}{14}} = \frac{2}{\sqrt{3}}$

$\therefore$ Distance between foci $= 2ae = 2 \times \sqrt{12} \times \frac{2}{\sqrt{3}} = 8$

933 **(b)**

Let the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$

This cuts the two given circles orthogonally

$\therefore 2(gg_1 - ff_1) = c + c_1$ ...(i)

and, $2(gg_2 + ff_2) = c + c_2$ ...(ii)

Subtracting (ii) from (i), we get

$2g(g_1 - g_2) + 2f(f_1 - f_2) = c_1 - c_2$

Hence, the locus of $(-g, -f)$ is

$-2x(g_1 - g_2) - 2y(f_1 - f_2) = c_1 - c_2$

$\Rightarrow 2x(g_1 - g_2) - 2y(f_1 - f_2) + c_1 - c_2 = 0,$

Which is the radical axis of the given circles

934 **(b)**

Given, $r^2 - 8(\sqrt{3}\cos\theta + \sin\theta) + 15 = 0$

Where $r\cos\theta = x$ and $y = r\sin\theta$

It can be rewritten in Cartesian form as

$x^2 + y^2 - (8\sqrt{3x} + y) + 15 = 0$

$\Rightarrow x^2 + y^2 - 8\sqrt{3} + 8y + 15 = 0$

Now, radius$= \sqrt{(4\sqrt{3})^2 + (4)^2 - 15} = 7$

935 **(b)**

The equation of a circle passing through $(0,0), (a, 0)$ and $(0, b)$ is $x^2 + y^2 - ax - by = 0$.

So, the coordinates its centre are $(a/2, b/2)$

ALITER The circle passing through $O(0,0), A(a, 0)$ and $B(0, b)$ is the circumcentre of right triangle OAB with AB as diagonal. So, its centre is the mid-point of diagonal AB

936 **(b)**

Let (x_1, y_1) be the mid-point of the line joining the common points of the given line and the given parabola. Then, the equation of the line is

$yy_1 - 4(x + x_1) = y_1^2 - 8x_1$ $\quad$ [Using $T = S'$]

$\Rightarrow 4x - yy_1 + y_1^2 - 4x_1 = 0$ ...(i)

Clearly, equation (i) and $2x - 3y + 8 = 0$ represent the same line.

$\therefore \frac{4}{2} = \frac{-y_1}{-3} = \frac{y_1^2 - 4x_1}{8}$

$\Rightarrow y_1 = 6$ and $y_1^2 - 4x_1 = 16$

$\Rightarrow y_1 = 6$ and $36 - 4x_1 = 16 \Rightarrow y_1 = 6$ and $x_1 = 5$

Hence, the required point is $(5, 6)$

937 **(a)**

We have,

$m =$ Slope of the tangent $= -3$

So, the equation of the tangent is

$y = -3x + \left(\frac{2}{-3}\right) \Rightarrow 9x + 3y + 2 = 0$ $\left[\text{Using} : y = mx + \frac{a}{m}\right]$

938 **(b)**

The equation of a chord passing through the vertex (0,0) of the parabola $y^2 = 4ax$ and making an angle θ with x-axis, is $y = x\tan\theta$. This meets the parabola $y^2 = 4ax$ at a point whose abscissa is given by

$x^2\tan^2\theta = 4ax \Rightarrow x = 4a\cot^2\theta$

$\therefore y = x\tan\theta \Rightarrow y = 4a\cot^2\theta\tan\theta = 4a\cot\theta$

Hence,

Length of the chord

$= \sqrt{16a^2\cot^2\theta + 16a^2\cot^4\theta}$

$= 4a\cot\theta\,\mathrm{cosec}\,\theta = 4a\cos\theta\,\mathrm{cosec}^2\,\theta$

ALITER Let $P(at^2, 2at)$ be one end of the chord OP of the parabola $y^2 = 4ax$, where $O(0,0)$ is the vertex of the parabola.

Then,

$OP = \sqrt{a^2t^4 + 4a^2t^2} = at\sqrt{t^2 + 4}$

Since OP makes an angle θ with the axis of the parabola

$\therefore \tan\theta = \text{Slope of } OP = \frac{2at}{at^2} = \frac{2}{t} \Rightarrow t = 2\cot\theta$

$\therefore OP = 2a\cot\theta\ \sqrt{4\cot^2\theta + 4}$

$= 4\,a\cos\theta\,\text{cosec}^2\theta = 4a\cos\theta\ \text{cosec}^2\theta$

939 **(b)**

The equation of the ellipse is $\frac{x^2}{9} + \frac{y^2}{5} = 1$

Let e be the eccentricity of the ellipse. Then,

$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{5}{9}} = \frac{2}{3}$

Hence, distance between foci $= 2ae = 4$

940 **(a)**

We know, $SP = PM \Rightarrow SP^2 = PM^2$

$\therefore\ (x-0)^2 + (y-0)^2 = \left(\frac{x+y-4}{\sqrt{1^2+1^2}}\right)^2$

$\Rightarrow x^2 + y^2 = \left(\frac{x+y-4}{\sqrt{2}}\right)^2$

$\Rightarrow 2x^2 + 2y^2 = x^2 + y^2 + 16 + 2xy - 8y - 8x$

$\Rightarrow x^2 + y^2 - 2xy + 8x + 8y - 16 = 0$

941 **(c)**

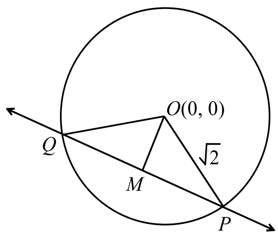
We have,

$OM =$ Length of the perpendicular from (0,0) on $y = 2x + 1$

$\Rightarrow OM = \frac{1}{\sqrt{5}}$

and, $OP =$ Radius of the given circle $= \sqrt{2}$

$\therefore PQ = 2\,PM = 2\sqrt{OP^2 - OM^2} = 2\sqrt{2 - \frac{1}{5}} = \frac{6}{\sqrt{5}}$



942 **(b)**

The centre and radii of two circles are

$C_1(1,-3), C_2\left(\frac{5}{2},-3\right)$

And $r_1 = \sqrt{1+9-6} = 2,$

$r_2 = \sqrt{\frac{25}{4} + 9 - 15} = \frac{1}{2}$

Now, $C_1C_2 = \sqrt{\left(1-\frac{5}{2}\right) + (-3+3)^2} = \frac{3}{2}$

And different of radii$= 2 - \frac{1}{2} = \frac{3}{2}$

Since, the distance between their centres is equal to the difference of their radii.

$\therefore$ The circles touch each other internally.

943 **(d)**

Here, $a^2 = 16, b^2 = 9$

The equation of normal at the point $(-4, 0)$ is

$\frac{16x}{-4} + \frac{9y}{0} = 16 + 9\left[\because \frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2\right]$

$\Rightarrow\ \frac{9y}{0} = 25 + \frac{16x}{4}\ \Rightarrow\ y = 0$

944 **(c)**

Let the centre of circle be(g, f). If one end of a diameter is (1, 1), then the other end of a diameter is $(2g - 1, 2f - 1)$

Since, this end is lie on the line $x + y = 3$

$\Rightarrow\ \ 2g - 1 + 2f - 1 = 30$

$\Rightarrow\ \ 2g + 2f = 5$

$\therefore$ Locus of centre of circle is $2x + 2y = 5$

945 **(d)**

Let (x_1, y_1) be the mid-point of the chord intercepted by the hyperbola $9x^2 - 16y^2 = 144$ on the line $9x - 8y - 10 = 0$. Then, the equation of the chord is

$9xx_1 - 16yy_1 = 9x_1^2 - 16y_1^2$

This equation and $9x - 8y - 10 = 0$ represent the same line

$\therefore \frac{x_1}{1} = \frac{-2y_1}{-1} = \frac{9x_1^2 - 16y_1^2}{10} = \lambda$ (say)

$\Rightarrow x_1 = \lambda, y_1 = \frac{\lambda}{2}$ and $9x_1^2 - 16y_1^2 = 10\,\lambda$

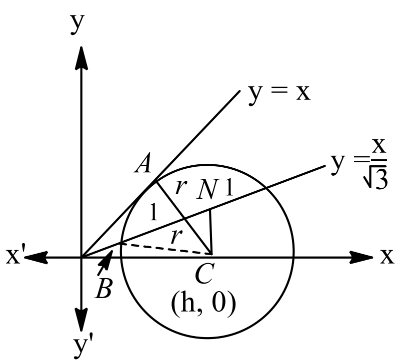
$\Rightarrow 9\,\lambda^2 - 4\,\lambda^2 = 10\,\lambda \Rightarrow \lambda = 2$

$\therefore x_1 = 2, y_1 = 1$

Hence, the mid-point is (2,1)

946 **(c)**

Since, CA is perpendicular to the tangent



$\therefore\ \ r = \frac{|h - 0|}{\sqrt{1^2 + 1^2}}$

$\Rightarrow\ \ h^2 = 2r\ \ \ldots$(i)

Since, CN is perpendicular to the chord of line,

$y = \frac{x}{\sqrt{3}}$

$\therefore\ CN = \frac{\left|\frac{h}{\sqrt{3}} - 0\right|}{\sqrt{\frac{1}{3}+1}} = \frac{h}{2}$

In $\Delta BNC,\ r^2 = 1^2 + \left(\frac{h}{2}\right)^2$

$r^2 = 1 + \frac{h^2}{4}$ ...(ii)

From Eqs. (i) and (ii), we get

$r^2 = 1 + \frac{2r^2}{4}$

$\Rightarrow\ r = \sqrt{2}$

947 **(a)**

Given equation of parabola is $y^2 = ax$,

Whose focus is $\left(\frac{a}{4}, 0\right)$

Since, the equation of focal chord $2x - y - 8 = 0$ is passes through the focus $\left(\frac{a}{4}, 0\right)$

$2\left(\frac{a}{4}\right) - 0 - 8 = 0$

$\Rightarrow\ a = 16$

$\therefore$ Equation of directrix is $x = -\frac{a}{4}$

$\Rightarrow\ x = -4$

948 **(a)**

Given equation can be rewritten as

$(x+3)^2 = -2\left(y - \frac{9}{2}\right)$

$\Rightarrow\ \ X^2 = 4AY$

where $X = x + 3, A = -\frac{1}{2}, Y = y - \frac{9}{2}$

$\therefore$ Focus is $\left(0, \frac{-1}{2}\right)$

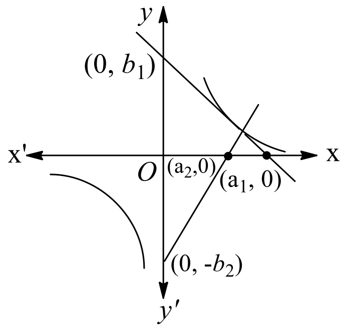
But $X = x + 3 = 0$ and $Y = y - \frac{9}{2} = -\frac{1}{2}$

$x = -3, y = 4$

$\therefore$ Required focus is $(-3, 4)$

950 **(d)**

Tangent and normal are at90°.



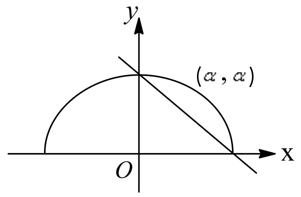
$\therefore$ Product of slopes is-1.

$\Rightarrow \left(-\frac{b_1}{a_1}\right)\left(-\frac{b_2}{a_2}\right) = -1$

$\Rightarrow a_1a_2 + b_1b_2 = 0$

951 **(a)**

The point should lies on the opposite side of the origin of the line $x + y - 1 = 0$



Then, $\alpha + \alpha - 1 > 0$

$\Rightarrow 2\alpha > 1\ \Rightarrow\ \alpha > \frac{1}{2}$ ...(i)

Also, $(\alpha^2 + \alpha^2 < 1$

$\Rightarrow\ \left(-\frac{1}{\sqrt{2}}\right) < \alpha < \left(\frac{1}{\sqrt{2}}\right)$

From relation (i) and (ii), we get

$\frac{1}{2} < \alpha < \frac{1}{\sqrt{2}}$

952 **(b)**

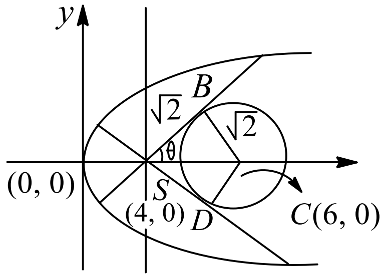
The normal at $P(3,4)$ cuts the circle again at $Q(-1,-2)$. Therefore, PQ is a diameter of the circle. Hence, its equation is

$(x-3)(x+1) + (y-4)(y+2) = 0$

or, $x^2 + y^2 - 2x - 2y - 11 = 0$

953 **(a)**

Given equation of circle is $(x-6)^2 + y^2 = \left(\sqrt{2}\right)^2$



BC = radius = $\sqrt{2}$

The length of the tangent from S to B

$\therefore SB = \sqrt{(4-6)^2 + 0 - 2} = \sqrt{2^2 - 2} = \sqrt{2}$

From figure, ΔCBS is an isosceles triangle

$\Rightarrow \theta = 45° \Rightarrow m = 1\quad (\because BC = BS)$

Similarly, for $\Delta\ CSD, m = -1$

954 **(c)**

Given that, the axis of parabola is y-axis and vertex is origin

$\therefore$ Equation of parabola is $x^2 = 4ay$

Since, it passes through $(6, -3)$

$\therefore (6)^2 = 4a(-3)$

$\Rightarrow 36 = -12a \Rightarrow a = -3$

$\therefore$ Equation of parabola is $x^2 = -12y$

955 **(a)**

We know that sum of focal distance of any point on the ellipse always equal to the length of major axis, *ie*, it is equal to $2a$

956 **(b)**

Let there be three points on the circle with rational coordinates. Then, centre of the circle will be the circumcentre of the triangle formed by the points. The coordinates of the circumcentre will be rational as the same are obtained by solving two linear equations with rational coeficients. But, the point $(\sqrt{3}, 0)$ does not have rational coordinates. So, there cannot be three points on the circle with rational coordinates. Let r be the radius of the circle. Then, its equation is

$(x - \sqrt{3})^2 + y^2 = r^2 \Rightarrow x = \sqrt{3} \pm \sqrt{r^2 - y^2}$

We observe that $x = 0, r = 2, y = \pm 1$ satisfy this equation. Thus, $(0, \pm 1)$ are two points with rational coordinates on the circle

957 **(b)**

Given, $2b = 10, 2a = 8$

$\Rightarrow\ b = 5$ and $a = 4$

Required equation of ellipse is

$$\frac{x^2}{16} + \frac{y^2}{25} = 1$$

958 **(a)**

The equation of a circle touching the coordinate axes is

$(x - a)^2 + (y - a)^2 = a^2$

This touches $\frac{x}{3} + \frac{y}{4} = 1$. i.e. $4\,x + 3\,y - 12 = 0$

$$\therefore \left|\frac{4\,a + 3\,a - 12}{\sqrt{4^2 + 3^2}}\right| = a \Rightarrow |7\,a - 12| = 5a \Rightarrow a = 6, 1$$

Thus, the equation of the required circle is

$x^2 + y^2 - 2\,ax - 2\,ay + a^2 = 0$, where $a = 1, 6$

959 **(c)**

We have,

$$\text{Required area} = \frac{1}{2}\begin{vmatrix} a\cos\alpha & b\sin\alpha & 1 \\ a\cos\beta & b\sin\beta & 1 \\ a\cos\gamma & b\sin\gamma & 1 \end{vmatrix}$$

$$= \frac{1}{2}ab\begin{vmatrix} \cos\alpha - \cos\gamma & \sin\alpha - \sin\gamma & 0 \\ \cos\beta - \cos\gamma & \sin\beta - \sin\gamma & 0 \\ \cos\gamma & \sin\gamma & 1 \end{vmatrix}$$

$$= 2ab\sin\frac{\alpha-\gamma}{2}\sin\frac{\beta-\gamma}{2}\begin{vmatrix} -\sin\frac{\alpha+\gamma}{2} & \cos\frac{\alpha+\gamma}{2} \\ -\sin\frac{\beta+\gamma}{2} & \cos\frac{\beta+\gamma}{2} \\ \cos\gamma & \sin\gamma \end{vmatrix}$$

$$= 2ab\sin\left(\frac{\alpha-\gamma}{2}\right)\sin\left(\frac{\beta-\gamma}{2}\right)\sin\left(\frac{\beta-\alpha}{2}\right)$$

$$= 2ab\sin\left(\frac{\alpha-\beta}{2}\right)\sin\left(\frac{\beta-\gamma}{2}\right)\sin\left(\frac{\gamma-\alpha}{2}\right)$$

960 **(c)**

We have, $4x^2 + 16y^2 - 24x - 32y = 1$

$\Rightarrow 4(x^2 - 6x) + 16(y^2 - 2y) = 1$

$\Rightarrow 4(x^2 - 6x + 9) + 16(y^2 - 2y + 1) - 36 - 16 = 1$

$\Rightarrow 4(x - 3)^2 + 16(y - 1)^2 = 53$

$$\Rightarrow \frac{(x-3)^2}{\frac{53}{4}} + \frac{(y-1)^2}{\frac{53}{16}} = 1$$

On comparing with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$a^2 = \frac{53}{4}$ and $b^2 = \frac{53}{16}$

$\therefore$ Eccentricity of ellipse is $e = \sqrt{\frac{a^2 - b^2}{a^2}}$

$$\Rightarrow e = \sqrt{\frac{(53/4) - (53/16)}{(53/4)}}$$

$$\Rightarrow e = \frac{\sqrt{3}}{2}$$

961 **(b)**

The vertices and foci of an ellipse are $(\pm 5, 0)$ and $(\pm 4, 0)$ respectively

$\therefore a = 5$ and $ae = 4$

$$\Rightarrow\ e = \frac{4}{5}$$

$$\because\ e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$\Rightarrow\ \frac{16}{25} = 1 - \frac{b^2}{25}$$

$$\Rightarrow\ b^2 = 9$$

Hence, equation of an ellipse is

$$\frac{x^2}{25} + \frac{y^2}{9} = 1\ \Rightarrow\ 9x^2 + 25y^2 = 225$$

962 **(b)**

The two circle are

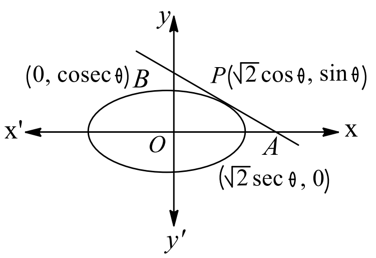
$S_1 = (x-a_1)^2 + (y-b_1)^2 = r_1^2$ ...(i)
$S_2 = (x-a_2)^2 + (y-b_2)^2 = r_2^2$ ...(ii)
The equation of the common tangent of these two circles is given by $S_1 - S_2 = 0$ i.e.
$$2x(a_1-a_2) + 2y(b_1-b_2) + (a_2^2+b_2^2) - (a_1^2+b_1^2) + r_1^2 - r_2^2 = 0$$
If this passes through the origin, then
$(a_2^2+b_2^2) - (a_1^2+b_1^2) + r_1^2 - r_2^2 = 0$
$\Rightarrow (a_2^2-a_1^2) + (b_2^2-b_1^2) = r_2^2 - r_1^2$

963 **(c)**
Given equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and equation of conjugate hyperbola is $\frac{x^2}{b^2} - \frac{y^2}{a^2} = 1$.
Since, e and e' are the eccentricities of the respective hyperbola, then
$$e^2 = 1 + \frac{b^2}{a^2}, (e')^2 = 1 + \frac{a^2}{b^2}$$
$$\therefore \frac{1}{e^2} + \frac{1}{e'^2} = \frac{a^2}{a^2+b^2} + \frac{b^2}{a^2+b^2}.$$

964 **(a)**
The centre and radius of given circle are $(1,-3)$ and 4 respectively
$\therefore$ Length of perpendicular from centre $(1,-3)$ to $3x-4y+k=0$ is equal to radius 4
$$\Rightarrow \left|\frac{3+12+k}{\sqrt{9+16}}\right| = 4$$
$\Rightarrow 15 + k = \pm 20$
$\Rightarrow k = 5, -35$

965 **(b)**
Given equation of circle is
$x^2 + y^2 + 2gx + 2fy + k = 0$ ...(i)
And equation of hyperbola is $xy = c^2$ ...(ii)
From Eqs.(i) and (ii), we get
$$x^2 + \left(\frac{c^2}{x}\right) + 2gx + 2f\left(\frac{c^2}{x}\right) + k = 0$$
$\Rightarrow x^4 + 2gx^3 + kx^2 + 2fc^2x + c^4 = 0$
$\therefore$ Sum of roots $= x_1 + x_2 + x_3 + x_4 = -\frac{2g}{1} = -2g$

966 **(a)**
Let the point be $P(\sqrt{2}\cos\theta, \sin\theta)$ on $\frac{x^2}{2} + \frac{y^2}{1} = 1$



$\therefore$ Equation of tangent at P is $\frac{x\sqrt{2}}{2}\cos\theta + y\sin\theta = 1$
Whose intercept on coordinate axes are
$A(\sqrt{2}\sec\theta, 0)$ and $B(0, \text{cosec}\,\theta)$
$\therefore$ Mid point of its intercept between axes is
$$\left(\frac{\sqrt{2}}{2}\sec\theta, \frac{1}{2}\text{cosec}\,\theta\right) = (h,k)$$
$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}h}$ and $\sin\theta = \frac{1}{2k}$
Now, $\cos^2\theta + \sin^2\theta = 1 \Rightarrow \frac{1}{2h^2} + \frac{1}{4k^2} = 1$
The locus of mid point M is $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$

967 **(d)**
The given equation can be rewritten as
$$4x^2 - 24x + 36 + 16y^2 - 32y + 16 - 36 - 16 - 12 = 0$$
$\Rightarrow (2x-6)^2 + (4y-4)^2 = 64$
$$\Rightarrow \frac{(x-3)^2}{16} + \frac{(y-1)^2}{4} = 1$$
The represents an ellipse and $a^2 = 16, b^2 = 4$
$$\therefore e = \sqrt{1 - \frac{4}{16}} = \frac{\sqrt{3}}{2}$$

968 **(c)**
Equation of the ellipse are $\frac{x^2}{13^2} + \frac{y^2}{5^2} = 1$ and $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and their eccentricity are
$e = \sqrt{1 - \frac{25}{169}}$ and $e' = \sqrt{1 - \frac{b^2}{a^2}}$
According to given condition, $e' = e$
$$\Rightarrow \sqrt{1 - \left(\frac{b^2}{a^2}\right)} = \sqrt{1 - \left(\frac{25}{169}\right)}$$
$\Rightarrow \frac{b}{a} = \frac{5}{13}$ $(\because a > 0, b > 0)$
$$\Rightarrow \frac{a}{b} = \frac{13}{5}$$

969 **(a)**
Given, $x^2 = 64\sec^2\theta, y^2 = 64\tan^2\theta$
$\therefore x^2 - y^2 = 64(\sec^2\theta - \tan^2\theta)$
$\Rightarrow x^2 - y^2 = 64$
$\therefore$ It is a rectangular hyperbola whose eccentricity is $\sqrt{2}$
The distance between directrices $= \frac{2a}{e} = \frac{2\times 8}{\sqrt{2}} = 8\sqrt{2}$

970 **(d)**
The equation of any tangent to the parabola $y^2 = 4x$ is
$$y = mx + \frac{1}{m} \quad \text{... (i)}$$

This touches the parabola $x^2 = -32y$, therefore the equation $x^2 = -32\left(mx + \frac{1}{m}\right)$ has equal roots.

$$\therefore (32\,m)^2 = 4\left(\frac{32}{m}\right) \quad [\because\ D^2 = 4ac]$$

$$\Rightarrow\ 8m^3 = 1\ \Rightarrow\ m = \frac{1}{2}$$

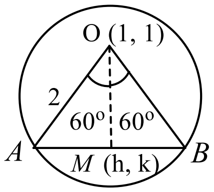
On putting the value of m is Eq. (i). we get

$$x - 2y + 4 = 0$$

971 **(c)**

The coordinates of the centre and radius of given circle are (1, 1) and 2 respectively. Let AB be the chord subtending an angle of 120° at the centre. Let M be the mid point of AB and let its coordinates be (h, k)

In ΔOAM,



$AM = OA \sin 60°$

$= 2.\frac{\sqrt{3}}{2} = \sqrt{3}$

$\therefore\ OM^2 = OA^2 - AM^2$

$= 4 - \left(\sqrt{3}\right)^2 = 1$

But $OM^2 = (h-1)^2 + (k-1)^2$

$\therefore\ (h-1)^2 + (k-1)^2 = 1$

Hence, locus of (h, k) is $(x-1)^2 + (y-1)^2 = 1$

or $x^2 + y^2 - 2x - 2y + 1 = 0$

972 **(b)**

Since, the perpendicular distance from centre (0, 0) to be tangent =radius of the circle

$$\Rightarrow\ \frac{|-1|}{\sqrt{\frac{1}{\alpha^2} + \frac{1}{\beta^2}}} = a$$

$$\Rightarrow \frac{1}{a^2} = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

The locus of $\left(\frac{1}{\alpha}, \frac{1}{\beta}\right)$ is $\frac{1}{a^2} = \frac{1}{x^2} + \frac{1}{y^2}$

973 **(b)**

Eliminating t from $y = 2\cos t$ and $x = \sin^2 t$, we get

$y^2 + 4x = 4$, which is a parabola

974 **(d)**

Equation of tangent to the ellipse $x^2 + 4y^2 = 5$ at the point $(-1,1)$ is

$$-x + 4y = 5$$

$$\Rightarrow\ x - 4y + 5 = 0$$

975 **(c)**

Clearly, (1,1) is the mid-point of the line segment joining the centres of the circles and centres lie on the line passing through (1,1) and perpendicular to $3x + 4y - 7 = 0$ i.e. $4x - 3y - 1 = 0$

Clearly, coordinates of points given in option (c) satisfy these two conditions

976 **(a)**

The equation of a circle passing through the intersection of the two given circles is

$$(x^2 + y^2 - 2\,x - 4\,y + 1) + \lambda(x^2 + y^2 - 4\,x - 2\,y + 4) = 0$$

$$\Rightarrow x^2 + y^2 - 2\,x\left(\frac{1+2\lambda}{1+\lambda}\right) - 2\,y\frac{(2+\lambda)}{1+\lambda} + \left(\frac{1+4\,\lambda}{1+\lambda}\right) = 0 \quad \text{... (i)}$$

The co-ordinates of its centre are $\left(\frac{1+2\,\lambda}{1+\lambda}, \frac{2+\lambda}{1+\lambda}\right)$

Since the centre lies on $x + 2\,y - 3 = 0$

$\therefore 1 + 2\,\lambda + 4 + 2\,\lambda - 3 - 3\,\lambda = 0 \Rightarrow \lambda = -2$

Putting $\lambda = -2$ in (i), we obtain that the required circle is

$x^2 + y^2 - 6\,x + 7 = 0$

977 **(b)**

Let $y = mx + \frac{1}{m}$ is a tangent to $y^2 = 4ax$

Equation of normal to the parabola $x^2 = 4by$ at (x_1, y_1) is

$y - y_1 = -\frac{2b}{x_1}(x - x_1)$ and $x_1^2 = 4by_1$

$$\Rightarrow y - \frac{x_1^2}{4b} = -\frac{2b}{x_1}(x - x_1)$$

$$\Rightarrow y = -\frac{2b}{x_1}x + \frac{x_1^2}{4b} + 2b$$

On comparing with $y = mx + \frac{1}{m}$, we get

$m = -\frac{2b}{x_1}$ ...(i)

$\frac{x_1^2}{4b} + 2b = \frac{1}{m}$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{4b^2}{m^2 4b} + 2b = \frac{1}{m}$$

$\Rightarrow b + 2bm^2 = m$

$\Rightarrow 2bm^2 - m + b = 0$

For real values of $m, D > 0$

$\Rightarrow 1 - 8b^2 > 0 \Rightarrow b^2 < \frac{1}{8} \Rightarrow |b| < \frac{1}{2\sqrt{2}}$

978 **(b)**

Given equation of line is

$3x - 2y = k$ ...(i)

And equation of circle is

$x^2 + y^2 = 4r^2$ ...(ii)

Eq. (i) can be rewritten as $y = \frac{3}{2}x - \frac{k}{2}$

$\Rightarrow m = \frac{3}{2}, c = -\frac{k}{2}$

The line will meet the circle in one point, if

$c = a\sqrt{1 + m^2}$

$\Rightarrow -\frac{k}{2} = (2r)\sqrt{1 + \left(\frac{3}{2}\right)^2}$

On squaring, we get

$\frac{k^2}{4} = 4r^2 \times \frac{13}{4}$

$\Rightarrow k^2 = 52\, r^2$

979 **(a)**

Given equation of hyperbola is

$\frac{x^2}{9} - \frac{y^2}{7} = 1$

Distance between foci $= 2ae = 2\sqrt{2a^2 + b^2}$

$= 2\sqrt{9 + 7}$

$= 8$

980 **(a)**

Given, $x^2y^2 = c^4$

$\Rightarrow y^2(a^2 - y^2) = c^4$

$\Rightarrow y^4 - a^2y^2 + c^4 = 0$

Let y_1, y_2, y_3 and y_4 are the roots

$\therefore y_1 + y_2 + y_3 + y_4 = 0$

981 **(c)**

(1) Given equation of parabola is

$y^2 = 4a(x + a)$ or $Y^2 = 4aX$

$\therefore$ focus is $(a, 0)$

$\Rightarrow x + a = a, y = 0$

$\Rightarrow x = 0, y = 0$

$\therefore$ Focus is at origin $(0, 0)$

(2) Given equation of line is $y = -\frac{lx}{m} - \frac{n}{m}$

It will touch the parabola $y^2 = 4ax$, if

$-\frac{n}{m} = \frac{a}{-\frac{l}{m}} \Rightarrow nl = am^2$

$\therefore$ Both statements are true

982 **(a)**

We know that the product of perpendiculars drawn from two foci S_1 and S_2 of an ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$ on the tangent at any point P on the ellipse is equal to the square of the semiminor axis.

$\therefore (S_1M_1) \cdot (S_2M_2) = 16$

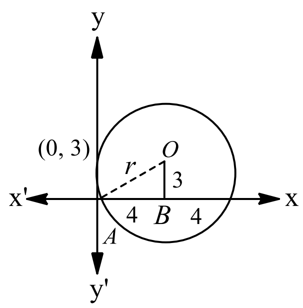
983 **(a)**

Given, equation of circle touch the y-axis at $(0, 3)$.

In ΔOAB, $r = \sqrt{3^2 + 4^2} = 5$

The point $(0, 3)$ and radius 5 satisfies the equation

$x^2 + y^2 \pm 10x - 6y + 9 = 0$



984 **(d)**

There is no xy term so we can make perfect square in x and y, from there it is clear that it's axes are parallel to coordinates axes, but whether major axis is parallel to x axis or parallel to y-axis depend on values of coefficients

985 **(c)**

Let $S_1 = x^2 + y^2 + 2x - 3y + 6 = 0$

$S_2 = x^2 + y^2 + x - 8y - 13 = 0$

So, the common chord is given by

$S_1 - S_2 = 0$

$\therefore$ Common chord is

$x + 5y + 19 = 0$

and this equation of common chord is satisfied by $(1, -4)$

only

986 **(b)**

Let the eccentric angle of B be θ. The co-ordinates of A and B are $\left(5\cos\frac{\pi}{6}, \frac{5}{3}\sin\frac{\pi}{6}\right)$ and

$\left(5\cos\theta, \frac{5}{3}\sin\theta\right)$

The mid-point of AB is at the origin

$\therefore \frac{5\cos\frac{\pi}{6} + 5\cos\theta}{2} = 0$ and $\frac{\frac{5}{3}\sin\frac{\pi}{6} + \frac{5}{3}\sin\theta}{2} = 0$

$\Rightarrow \cos\theta = -\cos\frac{\pi}{6}$ and $\sin\theta = -\sin\frac{\pi}{6}$

$\Rightarrow \theta = \frac{7\pi}{6}$ or, $\theta = -\frac{5\pi}{6}$

987 **(c)**

Any point on the given parabola is $(t^2, 2t)$. The equation of the tangent at $(1,2)$ is $x - y + 1 = 0$

The image (h, k) of the point $(t^2, 2t)$ in $x - y + 1 = 0$ is given by $\frac{h-t^2}{1} = \frac{k-2t}{-1} = \frac{-2(t^2-2t+1)}{1+1}$

$\therefore h = t^2 - t^2 + 2t - 1 = 2t - 1$

And $k = 2t + t^2 - 2t + 1 = t^2 + 1$

On eliminating t from $h = 2t - 1$ and $k = t^2 + 1$

We get, $(h+1)^2 = 4(k-1)$

The required equation of reflection is

$(x+1)^2 = 4(y-1)$

988 **(c)**

Given, $x^2 + y^2 = \frac{1}{5}$

Centre of the circle is (0, 0)

Let equation of tangent which are parallel to $3x + 4y - 1 = 0$ is

$3x + 4y + \lambda = 0$ ...(i)

$\therefore \frac{3 \times 0 + 4 \times 0 + \lambda}{\sqrt{(3)^2 + (4)^2}} = \pm\frac{1}{\sqrt{5}}$

$\Rightarrow \lambda = \pm\sqrt{5}$

On putting the value of λ in Eq. (i), we get

$3x + 4y = \pm\sqrt{5}$

989 **(c)**

The intersection point of diameter lines is (2, 3) which is the centre of circle

Now, radius= $\sqrt{(5-2)^2 + (7-3)^2}$

$= \sqrt{9 + 16} = 5$

$\therefore$Required equation of circle is

$(x-2)^2 + (y-3)^2 = 5^2$

$\Rightarrow x^2 + y^2 - 4x - 6y - 12 = 0$

991 **(a)**

If $y = mx + c$ tpuches $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $c^2 = a^2m^2 - b^2$

Here, $c = 6, a^2 = 100, b^2 = 49$

$\therefore 36 = 100\,m^2 - 49 \Rightarrow 100\,m^2 = 85 \Rightarrow m = \sqrt{\frac{17}{20}}$

992 **(a)**

Given parametric equation of parabola is

$x = t^2 + 1, y = 2t + 1$

$\Rightarrow x = \left(\frac{y-1}{2}\right)^2 + 1$

$\Rightarrow (y-1)^2 = 4(x-1)$

$\Rightarrow Y^2 = 4X$

Vertex is (1, 1), length of latusrectum = 4

Clearly, equation of directrix is

$X = -1 \Rightarrow x - 1 = -1 \Rightarrow x = 0$

993 **(c)**

The length of the subtangent at a point to the parabola is twice the abscissa of the point. Therefore, the required length is 8

994 **(a)**

Equation of asymptotes of the hyperbola are

$x^2 + 2xy - 3y^2 = 0$

The angle between asymptotes is

$\theta = \tan^{-1}\left(\frac{1-1(-3)}{1-3}\right)$

$= \tan^{-1}\left(\frac{1+3}{-2}\right) = \tan^{-1}(\pm 2)$

995 **(b)**

The equation of the circle passing through (2, 0), (0, 1) and (4, 5) is

$3(x^2 + y^2) - 13x - 17y + 14 = 0$

This passes through $(0, c)$

$\therefore 3c^2 - 17c + 14 = 0 \Rightarrow c = 1, \frac{14}{3}$

Since, $c = 1$ is already there, for point (0, 1)

Therefore, we take $c = \frac{14}{3}$

996 **(a)**

Since, $x + y - 1 = 0$ is a tangent to the parabola $y^2 - y + x = 0$, then point of contact is satisfied by both of these equations. The point (0, 1) satisfies it

997 **(a)**

If line $4x - 3y + k = 0$ touches the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$, then

$\frac{k}{3} = \sqrt{9 \times \left(\frac{4}{3}\right)^2 + 5} = \pm\sqrt{21}$

$\Rightarrow k = \pm 3\sqrt{21}$

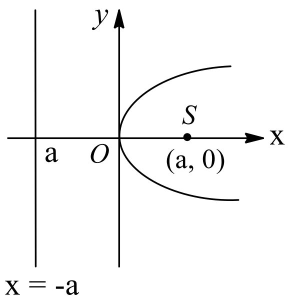
998 **(d)**

Let x be any point on the parabola, then $y = 3x$, putting this value in the given equation $y^2 = 18x$, we get

$(3x)^2 = 18x \Rightarrow x = 2$ and $y = 6$

999 **(a)**

As we know that distance from vertex to the parabola is equal to the focus and directrix



∴ The tangent at the vertex divide in the ratio 1: 1

100 **(c)**
0 Let $S \equiv 4x^2 + 5y^2 - 1 = 0$

At $(4, -3)$,

$S_1 = 4(4)^2 + 5(-3)^2 - 1 = 108 > 0$

Hence, point lies outside the curve

100 **(a)**
1 The intersection points of line and circle are $A\left(-\frac{1}{2}, \frac{1}{2}\right)$ and $B(-1, 0)$

These are the end points of a diameter

∴ The equation of circle is

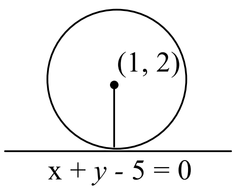
$\left(x + \frac{1}{2}\right)(x + 1) + \left(y - \frac{1}{2}\right)(y - 0) = 0$

$\Rightarrow \quad (2x + 1)(x + 1) + (2y - 1)y = 0$

$\Rightarrow \quad 2(x^2 + y^2) + 3x - y + 1 = 0$

100 **(b)**
2 ∵ Radius of circle =perpendicular distance of tangent $x + y - 5 = 0$ from the centre $(1, 2)$

$\therefore r = \frac{|1 + 2 - 5|}{\sqrt{1 + 1}} = \sqrt{2}$



Hence, the required equation of the circle is

$(x - 1)^2 + (y - 2)^2 = \left(\sqrt{2}\right)^2$

$\Rightarrow x^2 + 1 - 2x + y^2 + 4 - 4y = 2$

$\Rightarrow x^2 + y^2 - 2x - 4y + 3 = 0$

100 **(b)**
3 We know that, if (x_1, y_1) is the mid point of the chord, then equation of chord is

$T = S_1 \Rightarrow \frac{xx_1}{25} + \frac{yy_1}{9} = \frac{x_1^2}{25} + \frac{y_1^2}{9}$

∵ Point is $(1, 1)$, then

$\frac{x}{25} + \frac{y}{9} = \frac{1}{25} + \frac{1}{9}$

$\Rightarrow 9x + 25y = 34$

100 **(a)**
4 Since, the semi latusrectum of a parabola is the HM of segments of a focal chord.

$\therefore$ Semilatusrectum $= \frac{2SP \,.SQ}{SP + SQ}$

$= \frac{2 \times 3 \times 2}{3 + 2} = \frac{12}{5}$

∴ Latusrectum of the parabola= $\frac{24}{5}$

100 **(d)**
6 The condition for a circle bisecting the circumference of the second circle is

$2g_2(g_1 - g_2) + 2f_2(f_1 - f_2) = c_1 - c_2$

$\Rightarrow 2(1)(3 - 1) + 2(-3)(-1 + 3) = k + 15$

$\Rightarrow 2(2) + (-6)(2) = k + 15$

$\Rightarrow 4 - 12 = k + 15$

$\Rightarrow \; -8 = k + 15$

$\Rightarrow k = -23$

100 **(c)**
7 The given equation can be rewritten as

$\frac{x^2}{20} + \frac{y^2}{\frac{45}{4}} = 1$

On comparing the given equation with the standard equation, we get

$a^2 = 20, b^2 = \frac{45}{4}$

∴ The equation of normal at the point $(2, 3)$ is

$\frac{x - 2}{\frac{2}{20}} = \frac{y - 3}{\left(\frac{12}{45}\right)}$

$\Rightarrow 40(x - 2) = 15(y - 3)$

$\Rightarrow 8x - 3y = 7 \Rightarrow 3y - 8x + 7 = 0$

100 **(d)**
9 It is given that $\angle PAQ = \pi/2$

$\therefore \frac{b \sin \alpha}{a \cos \alpha - a} \times \frac{b \sin \beta}{a \cos \beta - a} = -1$

$\Rightarrow \frac{\sin \alpha \sin \beta}{(\cos \alpha - 1)(\cos \beta - 1)} = -\frac{a^2}{b^2}$

$\Rightarrow \frac{4 \sin \alpha/2 \sin \beta/2 \cos \alpha/2 \;\cos \beta/2}{4 \sin^2 \alpha/2 \sin^2 \beta/2} = -\frac{a^2}{b^2}$

$\Rightarrow \tan \frac{\alpha}{2} \tan \frac{\beta}{2} = -\frac{b^2}{a^2}$

101 **(b)**
0 We have, $xy = 7x + 5y$

$x(y - 7) - 5y = 0$

$x(y - 7) = 5(y - 7) + 35$

$(x - 5)(y - 7) = 35$

Now, asymptotes of $xy = c$ are $x = 0, y = 0$

$\therefore x - 5 = 0, y \Rightarrow -7 = 0$

ie, $x = 5, y = 7$ are asymptotes

101 **(d)**

1 Given equation can be rewritten as
$(x+1)^2 + (y+2)^2 = \left(2\sqrt{2}\right)^2$
Let required point be $Q(\alpha, \beta)$
Then. Mid point of $P(1, 0)$ and $Q(\alpha, \beta)$ is the centre of the circle.
ie, $\frac{\alpha+1}{2} = -1$ and $\frac{\beta+0}{2} = -2$
$\Rightarrow \alpha = -3$ and $\beta = -4$
$\therefore$ required point is $(-3, -4)$

101 **(b)**

2 Circles having (3,1) and $(-1,5)$ as limiting points are
$S_1 \equiv (x-3)^2 + (y-1)^2 = 0$
and, $S_2 \equiv (x+1)^2 + (y-5)^2 = 0$
The equation of the family of circles is
$S_1 + \lambda(S_1 - S_2) = 0$
$\Rightarrow (x-3)^2 + (y-1)^2 + \lambda(-8x + 8y - 16) = 0$ ...(i)
It passes through (0,0)
$\therefore 10 - 16\,\lambda = 0 \Rightarrow \lambda = \frac{5}{8}$
Substituting the value of λ in (i), we get
$x^2 + y^2 - 11x + 3y = 0$ as the equation of the required circle

101 **(c)**

3 Equation of tangent to $y^2 = 4x$ is $y = mx + \frac{1}{m}$

Since, tangent passes through (1, 4)

$\therefore 4 = m + \frac{1}{m} \Rightarrow m^2 - 4m + 1 = 0$

$\therefore m_1 + m_2 = 4$ and $m_1 m_2 = 1$

Now, $|m_1 - m_2| = \sqrt{(m_1 + m_2)^2 - 4m_1 m_2}$

$= \sqrt{16 - 4} = 2\sqrt{3}$

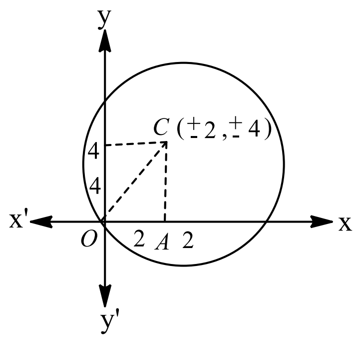
Thus, the angle between tangent

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{2\sqrt{3}}{1+1}\right| = \sqrt{3}$$

$\Rightarrow \theta = \frac{\pi}{3}$

101 **(a)**

4 In $\Delta OAC, OC^2 = 2^2 + 4^2 = 20$



$\therefore$ Required equation of circle is
$(x \pm 2)^2 + (y \pm 4)^2 = 20$
$\Rightarrow x^2 + y^2 \pm 4x \pm 8y = 0$

101 **(a)**

5 The equation of the tangent to $4y^2 = x^2 - 1$ at (1,0) is
$4(y \times 0) = x \times 1 - 1 \Rightarrow x - 1 = 0 \Rightarrow x = 1$

101 **(a)**

6 Distance between two foci, $2ae = 7 + 1 = 8$

$\therefore\ ae = 4 \Rightarrow a = 8 \ \left[\because e = \frac{1}{2}, \text{given}\right]$

$\because b^2 = a^2(1 - e^2) = 64\left(1 - \frac{1}{4}\right)$

$\Rightarrow b = 4\sqrt{3}$

Since, the centre of the ellipse is the mid point of the line joining two foci, therefore the coordinates of the centre the (3, 0)

$\therefore$ *I*ts equation is

$\frac{(x-3)^2}{8^2} + \frac{(y-0)^2}{(4\sqrt{3})^2} = 1$ ...(i)

Hence, the parametric coordinates of a point on Eq.(i) are $(3 + 8\cos\theta, 4\sqrt{3}\sin\theta)$

101 **(a)**

7 Since, focal chord of parabola $y^2 = ax$ is $2x - y - 8 = 0$
$\because$ This chord passes through focus *ie*, $\left(\frac{a}{4}, 0\right)$
$\therefore 2 \cdot \frac{a}{4} - 0 - 8 = 0$
$\Rightarrow a = 16$
$\therefore$ Directrix is $x = -4 \Rightarrow x + 4 = 0$

101 **(c)**

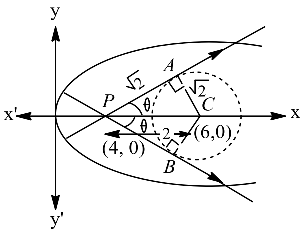
8 Here $a = 2,\ m = -1$

$\therefore$ Required point is $(am^2, -2am) = (2,4)$

101 **(a)**

9 Here, the focal chord to $y^2 = 16x$ is tangent to circle $(x-6)^2 + y^2 = 2$

$\Rightarrow$ focus of parabola is $(4, 0)$



Now, tangent are drawn from $(4, 0)$ to $(x-6)^2 + y^2 = 2$

Since, PA is tangent to circle

$$\tan\theta = \text{slope of tangent} = \frac{AC}{AP} = \frac{\sqrt{2}}{\sqrt{2}} = 1$$

or $\tan\theta = \frac{BC}{BP} = -1$

$\therefore$ Slope of focal chord as tangent to circle$= \pm 1$

1020 **(c)**
Let (h, k) be the mid point of the chord drawn through the origin. Then the equation of the chord is
$hx + ky - (x+h) = h^2 + k^2 - 2h$ [Using : $T = S'$]
This passes through (0,0)
$\therefore -h = h^2 + k^2 - 2h \Rightarrow h^2 + k^2 - h = 0$
Hence, the locus of (h, k) is $x^2 + y^2 - x = 0$

1021 **(d)**
The equation of the circle passing through the point (1,0), (0,1) and (0,0) is $x^2 + y^2 - x - y = 0$.
This passes through $(2k, 3k)$
$4k^2 + 9k^2 - 2k - 3k = 0 \Rightarrow k = 0, k = 5/13$

1022 **(a)**
Given focus for parabola is $S(0,0)$ and equation of directrix is $x + y = 4$

Let $P(x, y)$ be any point on the parabola

Then, $SP^2 = PM^2$

$$(x-0)^2 + (y-0)^2 = \left[\frac{x+y-4}{\sqrt{1+1}}\right]^2$$

$$\Rightarrow x^2 + y^2 = \frac{x^2 + y^2 + 16 + 2xy - 8y - 8x}{2}$$

$$\Rightarrow x^2 + y^2 - 2xy + 8x + 8y - 16 = 0$$

1023 **(b)**
Let the point is

$$(3t^2, 6t)$$

$\therefore$ Focal distance $= 3t^2 + 3$

$\Rightarrow 3t^2 + 3 = 12$

$\Rightarrow 3t^2 = 9$

$\Rightarrow t^2 = 3$

$\Rightarrow t = \sqrt{3}$

Hence , the required point is

$(9, 6\sqrt{3})$

1024 **(b)**
Required equation is
$(x-h)^2 + (y-k)^2 = k^2$
$\Rightarrow x^2 + y^2 - 2hx - 2ky + h^2 = 0$

1025 **(a)**
Let $P(x, y)$ be any point on the parabola. By definition of parabola $PM = PS$

$$\frac{x + 2y - 1}{\sqrt{1+4}} = \sqrt{(x-1)^2 + y^2}$$

$$\Rightarrow x^2 + 4y^2 + 1 + 4xy - 4y - 2x = 5(x^2 + 1 - 2x + y^2)$$

$$\Rightarrow 4x^2 + y^2 - 8x + 4y - 4xy + 4 = 0$$

1026 **(c)**
Required length of tangent from the point $(3, -4)$ to the circle $x^2 + y^2 - 4x - 6y + 3 = 0$
$= \sqrt{3^2 + 4^2 - 4(3) - 6(-4) + 3} = \sqrt{40}$
$\therefore$ Square of length of tangent $=40$

1027 **(c)**
Let $P(t_1^2, 2t_1), Q(t_2^2, 2t_2)$ and $R(t_3^3, 2t_3)$ be three points on $y^2 = 4x$ such that normal at P and R intersect at Q.
Then,
$t_1 t_3 = 2$
Let $S(h, k)$ be the mid-point of PR. Then,
$2h = t_1^2 + t_3^2$ and $k = t_1 + t_3$
Now,
$2h = t_1^2 + t_3^2$
$\Rightarrow 2h = (t_1 + t_3)^2 - 2t_1 t_3 \Rightarrow 2h = k^2 - 4$
So, the locus of (h, k) is
$2x = y^2 - 4$ or, $y^2 = 2(x+2)$
Clearly, it represents a parabola having vertex at

$(-2,0)$

102 **(a)**

8 The equation of the ellipse is

$4(x-3)^2+9(y+2)^2=144$ or, $\frac{(x-3)^2}{36}+\frac{(y+2)^2}{16}=1$

Let e be its eccentricity. Then,

$$e=\sqrt{1-\frac{16}{36}}=\frac{\sqrt{5}}{3}$$

So, equations of the directrices are

$$x-3=\pm\frac{6\times 3}{\sqrt{5}}=\pm\frac{18}{\sqrt{5}} \text{ or, } 5x-15\pm 18\sqrt{5}=0$$

102 **(b)**

9 We have, $a^2=25, b^2=16$

$$\therefore e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{16}{25}}=\frac{3}{5}$$

So, the coordinates of foci S and S' are $(3,0)$ and $(-3,0)$ respectively.

Let $P(5\cos\theta, 4\sin\theta)$ be a variable point on the ellipse. Then,

$A=$ Area of $\Delta\, PSS'=12\sin\theta$

Clearly, maximum value of A is 12 sq. units

103 **(b)**

0 Given curve is $y^2=16x$ Let any point be (h,k) But $2h=k$, then $k^2=16h$

$\Rightarrow 4h^2=16h$

$\Rightarrow h=0, h=4$

$\Rightarrow k=0, k=8$

$\therefore$ Points are$(0,0)$, $(4,8)$

Hence, focal distance are respectively

$0+4=4, 4+4=8$ [$\because$ focal distance $=h+a$]

103 **(a)**

1 We have,

$x=a(\sin\theta+\cos\theta), y=b(\sin\theta-\cos\theta)$

$\Rightarrow \frac{x^2}{a^2}=\frac{y^2}{b^2}=2$, which represents an ellipse

103 **(b)**

3 Here $C_1(-7,3), r_1=6$

and $C_2(5,-2), r_2=7$

$\therefore$ Required point of contact is

$$\left(\frac{r_1x_2+r_2x_1}{r_1+r_2}, \frac{r_1y_2+r_2y_1}{r_1+r_2}\right)$$

$$\equiv\left(\frac{6\times 5+7\times -7}{6+7}, \frac{6\times -2+7\times 3}{6+7}\right)$$

$$\equiv\left(-\frac{19}{13}, \frac{9}{13}\right)$$

103 **(b)**

4 Let the coordinates of P and Q be $(at_1^2, 2\,at_1)$ and $(at_2^2, 2\,at_2)$ respectively. Then, $y_1=2\,at_1$ and $y_2=2\,at_2$.

The coordinates of the point of intersection of the tangents at P and Q are $\left(at_1\,t_2, a\,(t_1+t_2)\right)$

$\therefore y_3=a(t_1+t_2)$

$\Rightarrow y_3=\frac{y_1+y_2}{2} \Rightarrow y_1, y_3, y_2$ are in A.P.

103 **(a)**

5 Equation of circle is $x^2+y^2-2x+4y-4=0$

$\therefore$ Centre is $(1,-2)$

As we know the equation of diameter is passing through centre

Now, taking option (a)

ie, $x-y-3=0$

$\Rightarrow 1+2-3=0 \Rightarrow 0=0$

$\therefore$ It is a required equation of diameter

103 **(a)**

6 Given that, $5x-12y+10=0$ ...(i)

And $-5x+12y+16=0$ ...(ii)

Slope of Eq. (i), $=\frac{5}{12}$

Slope of Eq. (ii)$=\frac{5}{12}$

Thus, Eqs. (i) and (ii) are parallel

Therefore, distance between parallel lines = diameter of the circle

$\Rightarrow \frac{|10+16|}{\sqrt{25+144}}=2\times$ radius of the circle

$\Rightarrow$ 2 radius of circle $=\frac{26}{13}$

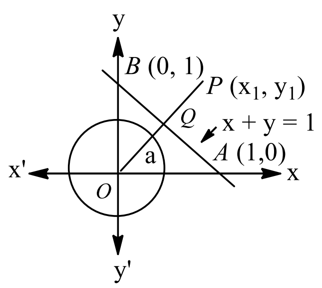
$\Rightarrow$ Radius of circle=1

103 **(d)**

7 Let P be image of the origin in the line $x+y=1$

Since, $OA=OB$, therefore Q is the mid point of AB

$\therefore$ Coordinates of Q are $\left(\frac{1}{2}, \frac{1}{2}\right)$



Let the coordinates of P be (x_1, y_1)

Since, Q is the mid point of OP

$\therefore \frac{0+x_1}{2}=\frac{1}{2}$ and $\frac{0+y_1}{2}=\frac{1}{2} \Rightarrow x_1=1, y_1=1$

$\therefore$ The coordinates of P are $(1,1)$

103 **(d)**

8

Given equation of ellipse can be rewritten as

$$\frac{(x-3)^2}{16}+\frac{y^2}{25}=0$$

The major axis of ellipse is a line parallel to y-axis therefore eccentricity of ellipse is given by

$$e=\sqrt{1-\frac{16}{25}}=\frac{3}{5}$$

103 **(c)**
9 Let $k=64$

$\therefore y=x^2-2\times 8x+64$

$\Rightarrow\ y=(x-8)^2$

$\Rightarrow$ It has vertex on x –axis

104 **(a)**
0 Given equation can be rewritten as

$9[(x+4)^2-16]-16[(y+1)^2-1]-16=0$

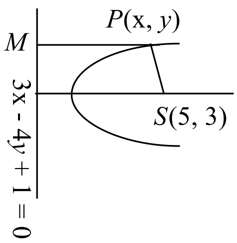
$$\Rightarrow\ \frac{(x+4)^2}{16}-\frac{(y+1)^2}{9}=1$$

Length of latusrectum $=\frac{2b^2}{a}=\frac{2\times 9}{4}=\frac{9}{2}$

104 **(d)**
2 Given that, $x^2+y^2+2gx+2fy+c=0$
and $g^2+f^2=c$
$\therefore$ Radius of circle $=\sqrt{g^2+f^2-c}$
$\Rightarrow$ Radius $=0$ $(\because g^2+f^2=c)$
Thus given equation represents a circles of radius 0

104 **(a)**
3 By definition of parabola $PM^2=PS^2$

$$\left[\frac{3x-4y+1}{\sqrt{3^2+(-4)^2}}\right]^2=(x-5)^2+(y-3)^2$$



$\Rightarrow 9x^2+16y^2+1^2-24xy-8y+6x$
$=25(x^2+25-10x+y^2+9-6y)$
$\Rightarrow 16x^2+9y^2-256x-142y+24xy+849=0$
$\Rightarrow (4x+3y)^2-256x-142y+849=0$

104 **(c)**
4 The radial axis of the Two given circles is

$2x\left(g-\frac{3}{4}\right)+2y(f-2)=0\quad[\because S_1-S_2=0]$

$\Rightarrow\ x\left(g-\frac{3}{4}\right)+y(f-2)=0$

It touches the circle
$x^2+y^2+2x+2y+1=0$

$$\therefore\ \left|\frac{-\left(g-\frac{3}{4}\right)-(f-2)}{\sqrt{\left(g-\frac{3}{4}\right)^2+(f-2)^2}}\right|=1$$

$$\Rightarrow\ \left(g-\frac{3}{4}\right)^2+(f-2)^2+2\left(g-\frac{3}{4}\right)(f-2)$$
$$=\left(g-\frac{3}{4}\right)^2+(f-2)^2$$

$\Rightarrow\ \left(g-\frac{3}{4}\right)(f-2)=0$

$\Rightarrow\ g=\frac{3}{4}$ or $f=2$

104 **(c)**
5 The equation of normal at point $P(a\cos\theta, b\sin\theta)$ is
$ax\sec\theta-by\text{ coese}\theta=a^2-b^2$
The point of intersection with coordinate axes are
$R\left(\frac{a^2-b^2}{a}\cos\theta, 0\right)$ and $\left(0,\frac{a^2-b^2}{a}\sin\theta\right)$

Now, $RP^2=\left[a\cos\theta-\left(\frac{a^2-b^2}{a}\right)\cos\theta\right]^2+b^2\sin^2\theta$

$=\frac{b^2}{a^2}(b^2\cos^2\theta+a^2\sin^2\theta)$

And $RS^2=\frac{a^2}{b^2}(b^2\cos^2\theta+a^2\sin^2\theta)$
$\therefore RP^2:RS^2=b^4:a^4$
$\Rightarrow RP:PS=b^2:a^2$

104 **(b)**
6 Let mid point of chord of the hyperbola

$\frac{x^2}{6}-\frac{y^2}{4}=1$ is

(x_1,y_1). Therefore equation of chord is

$T=S_1$

$$\Rightarrow\ \frac{xx_1}{6}-\frac{yy_1}{4}-1$$

$$=\frac{x_1^2}{6}-\frac{y_1^2}{4}-1$$

$$\Rightarrow\frac{x_1}{6}x-\frac{y_1}{4}y$$

$$=\frac{x_1^2}{6}-\frac{y_1^2}{4}$$

Comparing it with $4x - 3y = 5$, we get

$x_1 = 2, y_1 = 1$

104 **(c)**
7 Let $P(at^2, 2at)$ be a point on the parabola $y^2 = 4ax$ having $S(a, 0)$ as focus. The equation of the circle described on PQ as diameter is
$(x - at^2)(x - a) + (y - 2at)(y - 0) = 0$
Clearly, it touches y-axis i.e. $x = 0$ as $(y - 2at)y + a^2t^2 = 0$ has equal roots

104 **(c)**
8 We have,
Distance of the point from y-axis = 3(Distance of P from (4,0))
$\Rightarrow \frac{\text{Distance of } P \text{ from } (4,0)}{\text{Distance of } P \text{ from } y - \text{axis}} = \frac{1}{3}$
$\Rightarrow$ Locus of P is an ellipse with eccentricity $e = 1/3$

104 **(b)**
9 The centers and radii of given circles are
$C_1(4, -1), C_2(1, 8)$
and $r_1 = \sqrt{16 + 1 + 0} = \sqrt{17}$
$r_2 = \sqrt{1 + 64 - 25} = \sqrt{40}$
Now, $C_1C_2 = \sqrt{(1-4)^2 + (8+1)^2} = \sqrt{90}$
and $r_1 + r_2 = \sqrt{17} + \sqrt{40}$
$\therefore \quad C_1C_2 < r_1 + r_2$
Hence, the number of common tangents are2

105 **(b)**
0 Since, transverse and conjugate axes are equal

$ie, a = b$

$\therefore \frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$

Hence , $e = \sqrt{1 + \frac{a^2}{a^2}} = \sqrt{1+1} = \sqrt{2}$

105 **(b)**
1 Given, $2ae = 8$ and $\frac{2a}{e} = 6 \Rightarrow 2a = 6e$

$\therefore e^2 = \frac{8}{6} \Rightarrow e = \frac{2}{\sqrt{3}}$

$\Rightarrow \frac{4}{\sqrt{3}}a = 8 \Rightarrow a = 2\sqrt{3}$

and $b^2 = a^2(e^2 - 1) = 12\left(\frac{4}{3} - 1\right) = 4$

$\therefore$ Length of latusrectum $= \frac{2b^2}{a} = \frac{2 \times 4}{2\sqrt{3}} = \frac{4}{\sqrt{3}}$

105 **(b)**
2 Given hyperbola can be rewritten as

$$\frac{(x+2)^2}{4} - \frac{(y-1)^2}{5} = 1$$

$$\therefore e = \sqrt{\frac{4+5}{4}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

105 **(b)**
3 Let $S_1 \equiv x^2 + y^2 = 9$

$PA.PB = \left(\sqrt{S_1}\right)^2$

$= (\sqrt{(3)^2 + (11)^2 - 9})^2 = 121$

105 **(b)**
5 $\because xy = 4$ ...(i)
and $xy = 9$ ...(ii)
Eqs.(i) and (ii) are the equations of rectangular hyperbolas.
$\therefore \quad e_1 = \sqrt{2}$ and $e_2 = \sqrt{2}$,
then $e_1 - e_2 = 0$

105 **(b)**
6 Let equation of tangent to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is
$y = mx + \sqrt{a^2m^2 - b^2}$
$ie, mx - y + \sqrt{a^2m^2 - b^2} = 0$
$\therefore$ Required product
$= \left|\frac{mae + \sqrt{a^2m^2 - b^2}}{\sqrt{m^2+1}}\right| \left|\frac{-mae + \sqrt{a^2m^2 - b^2}}{\sqrt{m^2+1}}\right|$
$= \left|\frac{a^2m^2 - b^2 - m^2a^2e^2}{m^2+1}\right|$
$= \left|\frac{m^2a^2(1-e^2) - b^2}{m^2+1}\right|$
$= \left|\frac{-m^2b^2 - b^2}{m^2+1}\right| \quad [\because b^2 = a^2(e^2 - 1)]$
$= b^2$

105 **(a)**
7 Let the point be $(2, y_1)$, then
$2^2 + y_1^2 = 13$
$\Rightarrow \quad y_1 = \pm 3$
Hence, the required tangents are $2x \pm 3y = 13$

105 **(c)**
8 Let the equation of tangent parallel to $x + 2y + 3 = 0$ be $x + 2y + \lambda = 0$
Condition for tangency
$\left(-\frac{\lambda}{2}\right)^2 = 4\left(1 + \frac{1}{4}\right) \quad [\because \quad c^2 = a^2(1 + m^2)]$
$\Rightarrow \quad \lambda^2 = 20 \quad \Rightarrow \quad \lambda = \pm 2\sqrt{5}$

∴Required equation of tangent is

$x+2y=\pm 2\sqrt{5}$

1059 **(c)**

The given equation of circle is

$x^2+y^2-6x+4y-12=0$

The centre and radius of circle are $(3,-2)$ and 5 respectively

∴ Length of perpendicular from $(3,-2)$ to $4x+3y+\lambda=0$ is equal to radius 5

∴ $\left|\frac{12-6+\lambda}{\sqrt{16+9}}\right|=5$

$\Rightarrow 6+\lambda=\pm 25$

$\Rightarrow \lambda=19,-31$

Then, equations of tangents are $4x+3y+19=0$ and $4x+3y-31=0$

1060 **(d)**

The tangent at $(1,7)$ to the parabola $x^2=y-6$ is

$x(1)=\frac{1}{2}(y+7)-6$

[replacing $x^2\to xx_1$ and $2y\to y+y_1$]

$\Rightarrow 2x=y+7-12$

$\Rightarrow y=2x+5$ ...(i)

Which is also tangent to the circle

$x^2+y^2+16x+12y+c=0$

ie, $x^2+(2x+5)^2+16x+12(2x+5)+c=0$

or $5x^2+60x+85+c=0$

Must have equal roots

$\Rightarrow$ $\alpha=\beta$ for above equation *ie*,

$\Rightarrow \alpha+\beta=-\frac{60}{5}$

or $\alpha=-6$ (as $\alpha=\beta$)

∴ $x=-6$

and $y=2x+5=-7$

$\Rightarrow$ Point of contact is $(-6,-7)$

1061 **(b)**

Let the equations of the required tangent be $x+y=a$, then length of the perpendicular from centre=radius

∴ $\left|\frac{-2+2-a}{\sqrt{a}}\right|=2 \Rightarrow a=2\sqrt{2}$

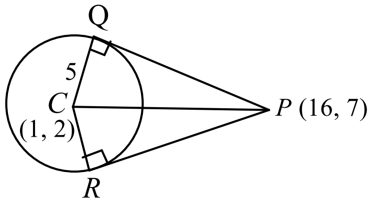
Hence, the equation of tangent is

$x+y=2\sqrt{2}$

1062 **(d)**

Given equation of circle is

$x^2+y^2-2x-4y-20=0$



∴ centre is $(1,2)$ and

Radius, $r=\sqrt{1^2+2^2+20}=5$

Now, $PC=\sqrt{(16-1)^2+(7-2)^2}=\sqrt{250}$

In ΔPCQ,

$PQ=\sqrt{PC^2-QC^2}$

$=\sqrt{\left(\sqrt{250}\right)^2-(5)^2}=15$

∴ area of quadrilateral $PQCR$

$=2$ area of ΔPCQ

$=\frac{2.1}{2}PQ.QC$

$=1.15.5=75$ sq unit

1063 **(b)**

Any point on hyperbola $\frac{(x+1)^2}{16}-\frac{(y-2)^2}{4}=1$ is of the form $(4\sec\theta-1, 2\tan\theta+2)$.

1064 **(b)**

Given parameter equation are

$\cos\theta=\frac{x-2}{3}$ and $\sin\theta=\frac{y+1}{3}$

Since, $\cos^2\theta+\sin^2\theta=1$

$\Rightarrow \left(\frac{x-2}{3}\right)^2+\left(\frac{y+1}{3}\right)^2=1$

$\Rightarrow (x-2)^2+(y+1)^2=3^2$

∴ Centre of circle is $(2,-1)$

1065 **(a)**

Given that, $xy=hx+ky$

$\Rightarrow (x-k)(y-h)=hk$

On shifting origin to (k,h) the above equation reduces to

$XY=hk=c^2$ (say)

Where, $x=X+k$ and $y=Y+h$

Then, the equation of the asymptotes are $X=0$ and $Y=0$ *ie*, $x=k, y=h$

1066 **(c)**

Given equation is

$\lambda x^2+(2\lambda-3)y^2-4x-1=0$

Here, $a=\lambda, b=(2\lambda-3)$

It represents a circle, if $a=b$

$\Rightarrow \lambda=2\lambda-3$

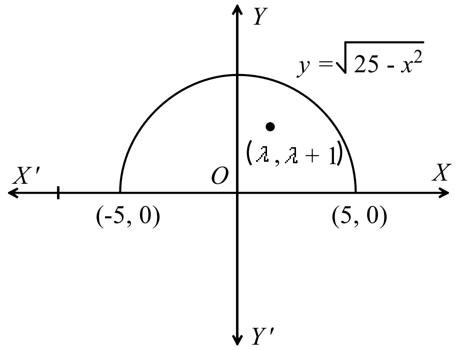
$\Rightarrow \lambda = 3$
Also, $h = 0$
Then, equation becomes
$3x^2 + 3y^2 - 4x - 1 = 0$
$\Rightarrow x^2 + y^2 - \frac{4}{3}x - \frac{1}{3} = 0$
Here, $g = -\frac{2}{3}, c = -\frac{1}{3}, f = 0$
$\therefore$ Radius= $\sqrt{\left(-\frac{2}{3}\right)^2 + 0 - \left(-\frac{1}{3}\right)} = \sqrt{\frac{4}{9} + \frac{1}{3}}$
$= \frac{\sqrt{7}}{3}$

106 **(d)**
7 Since, y-axis is major axis
$\Rightarrow f(4a) < f(a^2 - 5)$
$\Rightarrow 4a > a^2 - 5$ ($\because f$ is decreasing)
$\Rightarrow a^2 - 4a - 5 < 0$
$\Rightarrow a \in (-1, 5)$

106 **(a)**
8 The centres and radii of two circles are
$C_1(1, 3), C_2(4, -1)$
and $r_1 = r$, $r_2 = \sqrt{16 + 1 - 8} = 3$
two circles intersect in two distinct points, then
$r_1 - r_2 < C_1C_2 < r_1 + r_2$
$\Rightarrow r - 3 < \sqrt{(4-1)^2 + (-1-3)^2} < r + 3$
$\Rightarrow r - 3 < 5 < r + 3$
$\Rightarrow r < 8$ and $2 < r$
$\Rightarrow 2 < r < 8$

106 **(d)**
9 Since, both the points lie on the circle. At (5, 12), equation of tangent is
$5x + 12y = 169$ ...(i)
At $(12, -5)$, equation of tangent is
$12x - 5y = 169$ ...(ii)
It is clear that Eqs. (i) and (ii) are perpendicular to each other.
Hence, angle between them is 90°

107 **(c)**
0 If the point $(\lambda, \lambda + 1)$ lies in the interior of the region bounded by $y = \sqrt{25 - x^2}$ and x-axis, then $\lambda + 1 > 0$ and the point $(\lambda, \lambda + 1)$ must be an interior point of the circle $x^2 + y^2 = 25$
$\therefore \lambda + (\lambda + 1)^2 < 25$
$\Rightarrow 2\lambda^2 + 2\lambda + 1 < 25$
$\Rightarrow \lambda^2 + \lambda - 12 < 0 \Rightarrow (\lambda + 4)(\lambda - 3) < 0 \Rightarrow -4 < \lambda < 3$
Also, $\lambda + 1 > 0$ i. e. $\lambda > -1$
$\therefore -1 < \lambda < 3$ i. e. $\lambda \in (-1, 3)$



107 **(d)**
1 Any point on parabola $y^2 = 8x$ is $(2t^2, 4t)$. The equation of tangent at that point is
$yt = x + 2t^2$ ...(i)
Given that, $xy = -1$ ...(ii)
On solving Eqs.(i) and (ii), we get
$y(yt - 2t^2) = -1$
$\Rightarrow ty^2 - 2t^2y + 1 = 0$
$\because$ It is common tangent. It means they are intersect only at one point and the value of discriminant is equal to zero.
ie, $4t^4 - 4t = 0$
$\Rightarrow t = 0, 1$
$\therefore$ The common tangent is $y = x + 2$, (when $t = 0$, it is $x = 0$ which can touch $xy = -1$ at infinity only)

107 **(a)**
2 Given points lie on a circle $x^2 + y^2 = a^2$ and in case of an equilateral triangle centroid is same as circumcentre. Circumcentre of given triangle is at origin or centroid is at origin
$\frac{a\cos\theta_1 + a\cos\theta_2 + a\cos\theta_3}{3} = 0$,
and $\frac{a\sin\theta_1 + a\sin\theta_2 + a\sin\theta_3}{3} = 0$
$\sum \cos\theta_1 = 0, \sum \sin\theta_1 = 0$

107 **(b)**
3 We have, equation of circle is
$x^2 + y^2 - 8x + 4y + 4 = 0$
On comparing with standard equation of circle
$x^2 + y^2 + 2gx + 2fy + c = 0$, we get
$g = -4, f = 2$ and $c = 4$
$\therefore$ Coordinates of the centre $= (-g, -f)$
$= (4, -2)$
$\therefore$ Radius of the circle $= \sqrt{g^2 + f^2 - c}$
$= \sqrt{(4)^2 + (-2)^2 - 4}$
$= \sqrt{16 + 4 - 4} = 4$
Here, radius of circle is equal to x-coordinate of the centre
$\therefore$ Circle touches y-axis

107 **(a)**

4 We know that maximum four normals can be drawn from a point to the ellipse

107 **(d)**
5 Directrix of parabola is $y = 2$

$\Rightarrow\ a = -2$

$\therefore$ Required equation of parabola is

$x^2 = -4.2.y \Rightarrow\ x^2 = -8y$

107 **(c)**
6 The required equation is

$x - 2y - 9 = 1 + 4 - 9 \Rightarrow x - 2y - 5 = 0$ [Using $S' = T$]

107 **(a)**
7 Let (t, t) be the coordinates of the centre of the circle. Then, its equation is

$(x - t)^2 + (y - t)^2 = (2\sqrt{2})^2$ ... (i)

In touches the line $x + y = 4$. Therefore,

$\left|\frac{t + t - 4}{\sqrt{2}}\right| = 2\sqrt{2} \Rightarrow |t - 2| = 2 \Rightarrow t - 2 = \pm 2$

$\Rightarrow t = 0,4$

So, the coordinates of the centre are $(0,0)$ and $(4,4)$

Clearly, $(4,4)$ satisfies the inequation $x + y > 4$

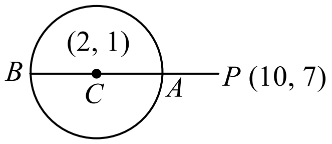
Hence, the equation of the circle is

$(x - 4)^2 + (y - 4)^2 = (2\sqrt{2})^2$

$\Rightarrow x^2 + y^2 - 8x - 8y + 24 = 0$

107 **(b)**
8 Let $S = x^2 + y^2 - 4x - 2y - 20$,

$S_1 = 10^2 + 7^2 - 4 \times 10 - 2 \times 7 - 20 > 0$



So, P lies outside the circle

Now, $PC = \sqrt{(10 - 2)^2 + (7 - 1)^2} = 10$

Radius $BC = \sqrt{4 + 1 + 20} = 5$

$\therefore$ Greatest distance, $PB = PC + CB = 10 + 5 = 15$

107 **(c)**
9 We have,

$g_1 = -1, f_1 = -1, c_1 = -7$

and, $g_2 = -4/3, f_2 = 29/6, c_2 = 0$

Clearly, $2(g_1 g_2 + f_1 f_2) = c_1 + c_2$

Hence, the two circles intersect orthogonally

108 **(d)**
0 Equation of the common chord of the given circles is

$2x - 2y = 0 \Rightarrow x - y = 0$ [Using : $S_1 - S_2 = 0$]

The equation of any circle passing through the intersection of the given circles

$x^2 + y^2 + 2x + \lambda(2x - 2y) = 0$ [Using : $S_1 + \lambda(S_1 - S_2) = 0$]

$\Rightarrow x^2 + y^2 + 2x(1 + \lambda) - 2\lambda y = 0$ ...(i)

Centre circle (i) is $(-\lambda - 1, \lambda)$

If $x - y = 0$ is a diameter of circle (i), then centre of (i) lies on $x - y = 0$

$\therefore -\lambda - 1 - \lambda = 0 \Rightarrow \lambda = -1/2$

Putting $\lambda = -1/2$ in equation (i), we obtain

$x^2 + y^2 + x + y = 0$

108 **(a)**
1 Let the equation of hyperbola be $\frac{x^2}{a} - \frac{y^2}{b^2} = 1$

Let (x_1, y_1)be any point on the hyperbola.

$\frac{x_1^2}{a_1^2} - \frac{y_1^2}{b^2} = 1 \Rightarrow\ b^2 x_1^2 - a^2 y_1^2 = a^2 b^2$ ... (i)

The asymptotes of given hyperbola are

$\frac{X^2}{a^2} - \frac{y^2}{b^2} = 0$

$\therefore$ Product of perpendicular form (x_1, y_1) to pair of lines $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$ is

$\frac{|Ax_1^2 + 2Hx_1 y_1 + By_1^2|}{\sqrt{(A - B)^2 + 4H^2}} = \frac{b^2 x_1^2 - a^2 y_1^2}{\sqrt{(b^2 + a^2)^2}}$

$= \frac{a^2 b^2}{a^2 + b^2}$ [from Eq. (i)]

108 **(c)**
2 Director circle is set of points from where drawn tangents are perpendicular, in this case $x^2 + y^2 = a^2 - b^2$ (equation of director circle) *ie*, $x^2 + y^2 = -9$ is not a real circle, so there is no point from where tangents are perpendicular.

108 **(b)**
4 The equation of the ellipse is

$3(x^2 + 2x) + 4(y^2 - 2y) = 5$

$\Rightarrow 3(x + 1)^2 + 4(y - 1)^2 = 12$

$\Rightarrow \frac{(x + 1)^2}{4} + \frac{(y - 1)^2}{3} = 1$

$\therefore a^2 = 4$ and $b^2 = 3$

Clearly, $a > b$

So, the eccentricity e is given by

$$e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{3}{4}}=\frac{1}{2}$$

108 **(c)**

5 The equation of any normal to the ellipse is

$ax\sec\theta-by\operatorname{cosec}\theta=a^2-b^2$ ... (i)

Let $P(h,k)$ be the pole of this normal chord of the ellipse. Then, the equation of the polar is

$\frac{hx}{a^2}+\frac{ky}{b^2}=1$ ... (ii)

Clearly, (i) and (ii) represent the same line

$$\therefore \frac{h}{a^3\sec\theta}=\frac{k}{-b^3\operatorname{cosec}\theta}=\frac{1}{a^2-b^2}$$

$$\Rightarrow \cos\theta=\frac{a^3}{h(a^2-b^2)}\text{ and }\sin\theta=-\frac{b^3}{k(a^2-b^2)}$$

$$\Rightarrow \cos^2\theta+\sin^2\theta=\frac{a^6}{h^2(a^2-b^2)}+\frac{b^6}{k^2(a^2-b^2)^2}$$

$$\Rightarrow \frac{a^6}{h^2}+\frac{b^6}{k^2}=(a^2-b^2)^2$$

Hence, the locus of the (h,k) is $\frac{a^6}{x^2}+\frac{b^6}{y^2}=(a^2-b^2)^2$

108 **(b)**

6 Given equation is $x^2-2y^2-2=0$, it can be rewritten as $\frac{x^2}{2}-\frac{y^2}{1}=1$

Here, $a^2=2, b^2=1$

We know that equation of hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, then the product of length of perpendicular drawn from any point on the hyperbola to the asymptotes is

$$\frac{a^2b^2}{a^2+b^2}=\frac{2(1)}{2+1}=\frac{2}{3}$$

108 **(b)**

7 Given, $x^2-y^2=\frac{25}{3}$

$$\therefore\ e_1=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{1+1}=\sqrt{2}$$

The equation of conjugate hyperbola is

$$-x^2+y^2=\frac{25}{3}$$

$$\therefore\ e_2=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{1+1}=\sqrt{2}$$

$$\therefore\ e_1^2+e_2^2=\left(\sqrt{2}\right)^2+\left(\sqrt{2}\right)^2=4$$

108 **(c)**

8 For the given line to touch the given parabola, the roots of the equation $k-x=x-x^2$ i.e. of $x^2-2x+k=0$ must be equal

$\therefore 4-4k=0\Rightarrow k=1$

108 **(c)**

9 Equation of asymptotes are

$x+2y=3$ ...(i)

and $x-y=0$

On solving Eqs.(i) and (ii), we get

$x=1, y=1$

$\therefore$ Centre of hyperbola is $(1,1)$ because asymptotes passes through the centre of the hyperbola.

109 **(c)**

0 Centre is $(7,-1)$ and radius is 5

Let $y=mx$ be the tangent on the circle

$\therefore$ length of perpendicular from centre is equal to the radius of circle

$$\Rightarrow\ \frac{7m+1}{\sqrt{1+m^2}}=\pm 5$$

$$\Rightarrow\ 49m^2+1+14m=25(1+m^2)$$

$$\Rightarrow\ 12m^2+7m-12=0$$

$$\Rightarrow\ (3m+4)(4m-3)=0$$

$$\Rightarrow\ m_1=-\frac{4}{3}\ \text{ and }\ m_2=\frac{3}{4}$$

$$\therefore\ m_1m_2=-\frac{4}{3}\cdot\frac{3}{4}=-1$$

Hence, tangents are perpendicular to each other

Alternate $\theta=2\tan^{-1}\frac{r}{\sqrt{S_1}}$

$$=2\tan^{-1}\frac{5}{5}=\frac{\pi}{2}$$

109 **(b)**

1 Given equation of hyperbola can be rewritten as

$x(y-3)-3(y-3)=2\Rightarrow(x-3)(y-3)=2$

Let $x-3=X$ and $y-3=Y$

Equation of hyperbola is of the form $XY=2$ (rectangular hyperbola). In rectangular hyperbola $a=b$, so length of latusrectum $=\frac{2b^2}{a}=2a$ (distance between vertices)

and $xy=c^2\Rightarrow 2=\frac{a^2}{2}\Rightarrow a=2$

$\therefore$ Length of latusrectum is $2a=4$

109 **(d)**

2 The circle $x^2+y^2=4$ cuts the circle $x^2+y^2-2x-4=0$ at $A(0,2)$ and $B(0,-2)$.

The circle $x^2+y^2-4x-k=0$ passes through A and B. Therefore,

$0+4-0-k=0\Rightarrow k=4$

109 **(d)**

3 Given, $2ae=8$ and $\frac{2a}{e}=18$

$$\Rightarrow\ a=\sqrt{4\times 9}=6$$

$\therefore\ e = \frac{2}{3}$

Therefore, $b = 6\sqrt{\left(1 - \frac{4}{9}\right)} = 2\sqrt{5}$

Hence, the required equation is $\frac{x^2}{36} + \frac{y^2}{20} = 1$

$\Rightarrow\ 5x^2 + 9y^2 = 180$

109 **(a)**

4 In $\Delta\, ABC$, we have

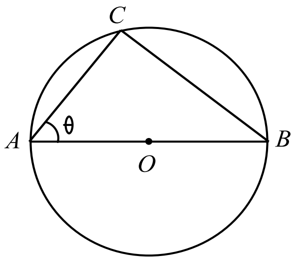
$\sin\theta = \frac{BC}{AB}$ and $\cos\theta = \frac{AC}{AB}$

$\Rightarrow BC = AB\sin\theta$ and $AC = AB\cos\theta$

Let Δ be the area of $\Delta\, ABC$. Then,

$\Delta = \frac{1}{2}BC \times AC = \frac{1}{2}(AB)^2 \sin\theta\cos\theta$

$= \frac{1}{4}(AB)^2 \sin 2\,\theta$



Clearly, it is maximum when $\sin 2\,\theta$ is maximum i.e. $\sin 2\,\theta = 1$. In that case, $\theta = \pi/4$

$\therefore BC = AC = \frac{AB}{\sqrt{2}}$

Hence, the triangle is isosceles

109 **(c)**

5 Since, the centre of circle is (1, 2) and this circle passes through (4, 6)

$\therefore$ Radius of circle=Distance between (1, 2) and (4, 6)

$= \sqrt{(4-1)^2 + (6-2)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$

Hence area of circle= πr^2

$= \pi 5^2 = 25\pi$ sq units

109 **(c)**

6 Radius of circle=Perpendicular distance from $(3, -2)$ to the line

$4x + 3y + 19 = 0$

$= \frac{4(3) + 3(-2) + 19}{\sqrt{16+9}} = 5$

$\therefore$ Required equation of circle is

$(x-3)^2 + (y+2)^2 = 5^2$

$\Rightarrow\ x^2 + y^2 - 6x + 4y - 12 = 0$