Single Correct Answer Type

1. The modulus of $\frac{1-i}{3+i}+\frac{4i}{5}$ is
a) $\sqrt{5}$ unit b) $\frac{\sqrt{11}}{5}$ unit c) $\frac{\sqrt{5}}{5}$ unit d) $\frac{\sqrt{12}}{5}$ unit

2. If $\frac{\log x}{a-b}=\frac{\log y}{b-c}=\frac{\log z}{c-a}$ then xyz is equal to
a) 0 b) 1 c) -1 d) 2

3. The area of the triangle formed by the points representing $-z, iz$ and $z-iz$ in the Argand plane is
a) $\frac{1}{2}|z|^2$ b) $|z|^2$ c) $\frac{3}{2}|z|^2$ d) $\frac{1}{4}|z|^2$

4. If $\frac{(1+i)^2}{2-i}=x+iy$, then $x+y$ is equal to
a) $-\frac{2}{5}$ b) $\frac{6}{5}$ c) $\frac{2}{5}$ d) $-\frac{6}{5}$

5. Let $3-i$ and $2+i$ be affixes of two points A and B the argand plane and P represents the complex number $z=x+i\,y$. Then, the locus of P if $|z-3+i|=|z-2-i|$ is
a) Circle on AB as diameter
b) The line AB
c) The perpendicular bisector of AB
d) None of these

6. If $x^2-2\,x\cos\theta+1=0$, then $x^{2n}-2\,x^n\cos n\,\theta+1$ is equal to
a) $\cos 2\,n\,\theta$ b) $\sin 2\,n\,\theta$ c) 0 d) None of these

7. Given $z=\frac{q+ir}{1+p}$, then $\frac{p+iq}{1+r}=\frac{1+iz}{1-iz}$ if
a) $p^2+q^2+r^2=1$ b) $p^2+q^2+r^2=2$ c) $p^2+q^2-r^2=1$ d) None of these

8. The expression $(1+i)^{n_1}+(1+i^3)^{n_2}$ is real iff
a) $n_1=-n_2$ b) $n_1=4r+(-1)^r n_2$ c) $n_1=2r+(-1)^r n_2$ d) None of these

9. If p,q,r are positive and are in AP, then roots of the quadratic equation $px^2+qx+r=0$ are complex for
a) $\left|\frac{r}{p}-7\right|\geq 4\sqrt{3}$ b) $\left|\frac{p}{r}-7\right|<4\sqrt{3}$ c) All p and r d) No p and r

10. If the roots of the equation $\frac{1}{x+p}+\frac{1}{x+q}=\frac{1}{r},(x\neq -p, x\neq -q, r\neq 0)$ are equal in magnitude but opposite in sign, then $p+q$ is equal to
a) r b) $2r$ c) r^2 d) $\frac{1}{r}$

11. The solution set of the inequation $|2x-3|<|x+2|$, is
a) $(-\infty,1/3)$ b) $(1/3,5)$ c) $(5,\infty)$ d) $(-\infty,1/3)\cup(5,\infty)$

12. In writing an equation of the form $ax^2+bx+c=0$; the coefficient of x is written incorrectly and roots are found to be equal. Again in writing the same equation the constant term is written incorrectly and it is found that one root is equal to those of the previous wrong equation while the other is double of it. If α and β be the roots of correct equation, then $(\alpha-\beta)^2$ is equal to
a) 5 b) $5\,\alpha\,\beta$ c) $-4\,\alpha\,\beta$ d) -4

13. If x is complex, the expression $\frac{x^2+34x-71}{x^2+2x-7}$ takes all which lie in the interval (a,b) where
a) $a=-1, b=1$ b) $a=1, b=-1$ c) $a=5, b=9$ d) $a=9, b=5$

14. Let a,b,c be real, if $ax^2+bx+c=0$ has two real roots α and β, where $\alpha<-2$ and $\beta>2$, then
a) $4-\frac{2b}{a}+\frac{c}{a}<0$ b) $4+\frac{2b}{a}-\frac{c}{a}<0$ c) $4-\frac{2b}{a}+\frac{c}{a}=0$ d) $4+\frac{2b}{a}+\frac{c}{a}=0$

15. Two students while solving a quadratic equation in x, one copied the constant term incorrectly and got the roots 3 and 2. The other copied the constant term coefficient of x^2 correctly as -6 and 1 respectively the correct roots are

a) $3, -2$ b) $-3, 2$ c) $-6, -1$ d) $6, -1$

16. $E_1: a+b+c=0$, if l is a root of $ax^2+bx+c=0$, $E_2: b^2-a^2=2ac$, if $\sin\theta, \cos\theta$ are the roots of $ax^2+bx+c=0$.
Which of the following is true?
a) E_1 is true, E_2 is true b) E_1 is true, E_2 is false
c) E_1 is false E_2 is true d) E_1 is false, E_2 is false

17. If $\omega = \frac{-1+\sqrt{3}i}{2}$, then $(3+\omega+3\omega^2)^4$ is
a) 16 b) -16 c) 16ω d) $16\omega^2$

18. If $iz^3+z^2-z+i=0$, then $|z|$ is equal to
a) 1 b) i c) -1 d) $-i$

19. The least value of $|a|$ for which $\tan\theta$ and $\cot\theta$ are roots of the equation $x^2+ax+1=0$, is
a) 2 b) 1 c) 1/2 d) 0

20. If 1, 2, 3 and 4 are the roots of the equation $x^4+ax^3+bx^2+cx+d=0$, then $a+2b+c$ is equal to
a) -25 b) 0 c) 10 d) 24

21. The number of integral solutions of $2(x+2)>x^2+1$, is
a) 2 b) 3 c) 4 d) 5

22. If one root of the equation $(a-b)x^2+ax+1=0$ be double the other and if $a \in R$, then the greatest value of b is
a) 9/8 b) 7/8 c) 8/9 d) 8/7

23. The argument of $(1-i\sqrt{3})(1+i\sqrt{3})$ is
a) 60° b) 120° c) 210° d) 240°

24. If the area of the triangle on the complex plane formed by the points $z, z+iz$, and iz is 200, then the value of $|3z|$ must be equal to
a) 20 b) 40 c) 60 d) 80

25. If the roots of the equation $bx^2+cx+a=0$ be imaginary, then for all real values of x, the expression $3b^2x^2+6bcx+2c^2$ is
a) Greater than $4ab$ b) Less than $4ab$ c) Greater than $-4ab$ d) Less than $-4ab$

26. If $(ax^2+c)y+(d\,x^2+c')=0$ and x is a rational function of y and ac is negative, then
a) $ac'+a'c=0$ b) $\frac{a}{a'}=\frac{c}{c'}$ c) $a^2+c^2=a'^2+c'^2$ d) $aa'+cc'=1$

27. If n is a positive integer, then $(1+i\sqrt{3})^n+(1-i\sqrt{3})^n$ is equal to
a) $2^{n-1}\cos\frac{n\pi}{3}$ b) $2^n\cos\frac{n\pi}{3}$ c) $2^{n+1}\cos\frac{n\pi}{3}$ d) None of these

28. The points represented by the complex numbers $1+i, -2+3i, \frac{5}{3}i$ on the argand diagram are
a) Vertices of an equilateral triangle b) Vertices of an isosceles triangle
c) Collinear d) None of the above

29. If the amplitude of $z-2-3i$ is $\frac{\pi}{4}$, then the locus of $z=x+iy$, is
a) $x+y-1=0$ b) $x-y-1=0$ c) $x+y+1=0$ d) $x-y+1=0$

30. The value of
$\frac{[(\cos 20°+i\sin 20°)(\cos 75°+i\sin 75°)(\cos 10°+i\sin 10°)]}{\sin 15°-i\cos 15°}$ is
a) 0 b) -1 c) i d) 1

31. Let α, β be the roots of $x^2+bx+1=0$. Then the equation whose roots are $-\left(\alpha+\frac{1}{\beta}\right)$ and $-\left(\beta+\frac{1}{\alpha}\right)$, is
a) $x^2=0$ b) $x^2+2b+4=0$ c) $x^2-2bx+4=0$ d) $x^2-bx+1=0$

32. The vector $z=-4+5i$ is turned counterclockwise through an angle of 180° and stretched $1\frac{1}{2}$ times. The complex number corresponding to newly obtained vector is
a) $6-\frac{15}{2}i$ b) $-6+\frac{15}{2}i$ c) $6+\frac{15}{2}i$ d) None of these

33. If $(3-i)z=(3-i)\bar{z}$, then the complex number z is

a) $a(3-i), a \in R$ b) $\dfrac{a}{(3+i)}, a \in R$ c) $a(3+i), a \in R$ d) $a(-3+i), a \in R$

34. For real values of x, the expression $\frac{(x-b)(x-c)}{(x-a)}$ will assume all real values provided

a) $a \le c \le b$ b) $b \ge a \ge c$ c) $b \le c \le a$ d) $a \ge b \ge c$

35. If $(x-1)^3$ is a factor of $x^4 + ax^3 + bx^2 + cx - 1$, then the other factor is

a) $x - 3$ b) $x + 1$ c) $x + 2$ d) $x - 1$

36. The centre of a square is at the origin and $1 + i$ is one of its vertices. The extremities of its diagonals which does not pass through this vertex are

a) $1 - i, -1 + i$ b) $1 - i, -1 - i$ c) $-1 + i, -1 - i$ d) None of these

37. If $p(x) = ax^2 + bx + c$ and $Q(x) = -ax^2 + dx + c$, where $ac \ne 0$, then $P(x)Q(x) = 0$ has at least

a) Four real roots b) Two real roots

c) Four imaginary roots d) None of these

38. If $a = \cos\theta + i\sin\theta$, then $\frac{1+a}{1-a}$ is equal to

a) $\cot\dfrac{\theta}{2}$ b) $\cot\theta$ c) $i\cot\dfrac{\theta}{2}$ d) $i\tan\dfrac{\theta}{2}$

39. If $x^2 + 2ax + b \ge c, \forall x \in R$, then

a) $a - c \ge a^2$ b) $c - a \ge b^2$ c) $a - b \ge c^2$ d) None of these

40. Let A, B, C be three collinear points which are such that $AB.AC = 1$ and the points are represented in the Argand plane by the complex numbers $0, z_1$ and z_2 respectively, Then,

a) $z_1 z_2 = 1$ b) $z_1\bar{z}_2 = 1$ c) $|z_1||z_2| = 1$ d) None of these

41. If $z^2 + z|z| + |z|^2 = 0$, then the locus of z is

a) A circle b) A straight line

c) A pair of straight lines d) None of these

42. If $|z - i| = 1$ and $\arg(z) = \theta$, where $0 < \theta < \frac{\pi}{2}$, then $\cot\theta - \frac{2}{z}$ equals

a) $2i$ b) $-i$ c) i d) $1 + i$

43. If for complex numbers z_1 and z_2, $\arg(z_1) - \arg(z_2) = 0$, then $|z_1 - z_2|$ is equal to

a) $|z_1| + |z_2|$ b) $|z_1| - |z_2|$ c) $||z_1| - |z_2||$ d) 0

44. If x, y, z are real and distinct, then $x^2 + 4y^2 + 9z^2 - 6yz - 3zx - 2xy$ is always

a) Non-negative b) Non-positive c) Zero d) None of these

45. The locus of the centre of the circle which touches the circles $|z - z_1| = a$ and $|z - z_2| = b$ externally (z, z_1 and z_2 are complex numbers) will be

a) An ellipse b) A hyperbola c) A circle d) None of these

46. The modulus and amplitude of $\left(1 + i\sqrt{3}\right)^8$ are respectively

a) 256 and $\frac{\pi}{3}$ b) 256 and $\frac{2\pi}{3}$ c) 2 and $\frac{2\pi}{3}$ d) 256 and $\frac{8\pi}{3}$

47. The solution set of the inequation $x^2 + (a+b)x + ab < 0, a < b$, is

a) (a, b) b) $(-\infty, a) \cup (b, \infty)$ c) $(-b, -a)$ d) $(-\infty, -b) \cup (-a, \infty)$

48. If ω is an imaginary cube root of unity and $x = a + b, y = a\omega + b\omega^2,\ z = a\omega^2 + b\omega$, then $x^2 + y^2 + z^2$ is equal to

a) $6ab$ b) $3ab$ c) $6a^2b^2$ d) $3a^2b^2$

49. The square roots of $-7, -24\sqrt{-1}$ are

a) $\pm(4 + 3\sqrt{-1})$ b) $\pm(3 + 4\sqrt{-1})$ c) $\pm(3 - 4\sqrt{-1})$ d) $\pm(4 - 3\sqrt{-1})$

50. A real value of x will satisfy the equation $\left(\frac{3-4ix}{3+4ix}\right) = \alpha - i\beta$ (α, β are real), if

a) $\alpha^2 - \beta^2 = -1$ b) $\alpha^2 - \beta^2 = 1$ c) $\alpha^2 + \beta^2 = 1$ d) $\alpha^2 - \beta^2 = 2$

51. If $\omega(\ne 1)$ is a cube root of unity and $(1 + \omega)^7 = A + B\omega$, then A and B are respectively

a) 0, 1 b) 1, 1 c) 1, 0 d) −1, 1

52. If the equation $x^2 + 9y^2 - 4x + 3 = 0$ is satisfied values of x and y, then

a) $1 \le x \le 3$ b) $2 \le x \le 3$ c) $-\dfrac{1}{3} < y < 1$ d) $0 < y < \dfrac{2}{3}$

53. If the sum of the roots of the equation $(a+1)x^2+(2a+3)x+(3a+4)=0$ is -1, then the product of the roots is

a) 0 b) 1 c) 2 d) 3

54. The roots of the equation $2^{x+2}3^{3x/(x-1)}=9$ are given by

a) $1-\log_2 3, 2$ b) $\log_2\left(\frac{2}{3}\right), 1$ c) $2, -2$ d) $-2, 1-\frac{\log 3}{\log 2}$

55. If $a+b+c=0$ and $a \neq c$ then the roots of the equation $(b+c-a)x^2+(c+a-b)x+(a+b-c)=0$, are

a) Real and unequal
b) Real and equal
c) Imaginary
d) None of these

56. If α, β are the roots of the equation $x^2+\sqrt{\alpha}\,x+\beta=0$, then the values of α and β are

a) $\alpha=1, \beta=-1$ b) $\alpha=1, \beta=-2$ c) $\alpha=2, \beta=1$ d) $\alpha=2, \beta=-2$

57. If $b>a$, then the equation $(x-a)(x-b)-1=0$ has

a) Both roots in $[a,b]$
b) Both roots in $(-\infty, a)$
c) Roots in $(-\infty, a)$ and other in (b, ∞)
d) Both roots in (b, ∞)

58. The value of $\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\left(\cos\frac{\pi}{8}+\ i\sin\frac{\pi}{8}\right)\ldots\infty$ is

a) 1 b) 0 c) -1 d) None of these

59. The value of the expression
$2\left(1+\frac{1}{\omega}\right)\left(1+\frac{1}{\omega^2}\right)+3\left(2+\frac{1}{\omega}\right)\left(2+\frac{1}{\omega^2}\right)+\cdots+(n+1)\left(n+\frac{1}{\omega}\right)\left(n+\frac{1}{\omega^2}\right)$ is

a) $\left[\frac{n(n+1)}{2}\right]^2$ b) $\left[\frac{n(n+1)}{2}\right]^2-n$ c) $\left[\frac{n(n+1)}{2}\right]^2+n$ d) None of these

60. One of the square root of $6+4\sqrt{3}$ is

a) $\sqrt{3}(\sqrt{3}+1)$ b) $-\sqrt{3}(\sqrt{3}-1)$ c) $\sqrt{3}(-\sqrt{3}+1)$ d) None of these

61. The solution set of the inequation $|x|-1<1-x$, is

a) $(-1,1)$ b) $(0,\infty)$ c) $(-1,\infty)$ d) None of these

62. If $\left(\sqrt{3}+i\right)^{10}=a+ib$, then a and b are respectively

a) $128\ \&\ 128\sqrt{3}$ b) $64\ \&-64\sqrt{3}$ c) $512\ \&-512\sqrt{3}$ d) None of these

63. The number of real solutions of the equation $\left(5+2\sqrt{6}\right)^{x^2-3}+\left(5-2\sqrt{6}\right)^{x^2-3}=10$, is

a) 2 b) 4 c) 6 d) None of these

64. Number of roots of the equation $x-\frac{2}{x-1}=1-\frac{2}{x-1}$ is

a) One b) Two c) Infinite d) None of these

65. The smallest positive integer n for which $(1+i)^{2n}=(1-i)^{2n}$ is

a) 1 b) 2 c) 3 d) 4

66. If $\frac{z-1}{z+1}$ is purely imaginary number $(z \neq -1)$, then $|z|$ is equal to

a) 1 b) 2 c) 3 d) 4

67. If one vertex of a square whose diagonals intersect at the origin is $3(\cos\theta+i\sin\theta)$, then the two adjacent vertices are

a) $\pm 3\ (\sin\theta-i\cos\theta)$ b) $\pm(\sin\theta+i\cos\theta)$ c) $\pm(\cos\theta-i\sin\theta)$ d) None of these

68. If the sum of the roots of the equation $ax^2+bx+c=0$ is equal to the sum of the squares of their reciprocals of their reciprocals, then

a) c^2b, a^2c, b^2a are in A.P.
b) c^2b, a^2c, b^2a are in G.P.
c) $\frac{b}{c}, \frac{a}{b}, \frac{c}{a}$ are in G.P.

d) $\frac{a}{b}, \frac{b}{c}, \frac{c}{a}$ are in G.P.

69. In the argand plane, if O, P and Q represent respectively the origin O and the complex numbers z and $z + iz$ respectively, then $\angle OPQ$ is

a) $\frac{\pi}{4}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{2}$ d) $\frac{2\pi}{3}$

70. If $n \in Z$, then $\frac{2^n}{(1-i)^{2n}} + \frac{(1+i)^{2n}}{2^n}$ is equal to

a) 0 b) 2 c) $[1 + (-1)^n]i^n$ d) None of these

71. Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\frac{\alpha}{2}, 2\beta$ be the roots of the equation $x^2 - qx + r = 0$. Then the value of r is

a) $\frac{2}{9}(p-q)(2q-p)$ b) $\frac{2}{9}(q-p)(2p-q)$ c) $\frac{2}{9}(q-2p)(2q-p)$ d) $\frac{2}{9}(2p-q)(2q-p)$

72. If ω is an imaginary cube root of unity, then $(1 + \omega - \omega^2)^7$ equals

a) $128\,\omega$ b) $-128\,\omega$ c) $128\,\omega$ d) $-128\,\omega^2$

73. If $z + z^{-1} = 1$, then $z^{100} + z^{-100}$ is equal to

a) i b) $-i$ c) 1 d) -1

74. $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ will be purely imaginary, if θ is equal to

a) $2n\pi \pm \frac{\pi}{3}$ b) $n\pi + \frac{\pi}{3}$ c) $n\pi \pm \frac{\pi}{3}$ d) None of these

75. If $x^2 + 2ax + 10 - 3a > 0$ for all $x \in R$, then

a) $-5 < a < 2$ b) $a < -5$ c) $a > 5$ d) $2 < a < 5$

76. Let z_1, z_2 be two complex numbers such that $z_1 + z_2$ and $z_1 z_2$ both are real, then

a) $z_1 = -z_2$ b) $z_1 = \overline{z_2}$ c) $z_1 = -\overline{z_2}$ d) $z_1 = z_2$

77. If Im $\left(\frac{2z+1}{iz+1}\right) = -2$, then locus of z is

a) A circle b) A parabola c) A straight line d) None of these

78. Let $'z'$ be a complex number and $'a'$ be a real parameter such that $z^2 + az + a^2 = 0$, then

a) Locus of z is a pair of straight lines b) Locus of z is a circle

c) $\arg(z) = \pm\frac{5\pi}{3}$ d) $|z| = -2|a|$

79. The points z_1, z_2, z_3, z_4 in the complex plane are the vertices of a parallelogram taken in order, iff

a) $z_1 + z_4 = z_2 + z_3$ b) $z_1 + z_3 = z_2 + z_4$ c) $z_1 + z_2 = z_3 + z_4$ d) None of these

80. If a real valued function f of a real variable x is such that $\frac{1}{(1+x)(1+x^2)} = \frac{A}{1+x} + \frac{f(x)}{1+x^2}$, then $f(x)$ is equal to

a) $\frac{1-x}{2}$ b) $\frac{x^2+1}{2}$ c) $1 - x$ d) None of these

81. If $a + b + c = 0$, then the roots of the equation $4ax^2 + 3bx + 2c = 0$ are

a) Equal b) Imaginary c) Real d) None of these

82. For how many values of k, $x^2 + x + 1 + 2k\,(x^2 - x - 1) = 0$ is a perfect square?

a) 2 b) 0 c) 1 d) 3

83. The number of solutions of $\frac{\log 5 + \log(x^2+1)}{\log(x-2)} = 2$ is

a) 2 b) 3 c) 1 d) None of these

84. The number of real roots of the equation $|x|^2 - 3|x| + 2 = 0$ is

a) 4 b) 3 c) 2 d) 1

85. If the difference between the roots of $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ is same and $a \neq b$, then

a) $a + b + 4 = 0$ b) $a + b - 4 = 0$ c) $a - b - 4 = 0$ d) $a - b + 4 = 0$

86. The equation $\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4} = \log_x \sqrt{2}$ has

a) At least one real solutions b) Exactly three real solutions

c) Exactly one irrational solution d) Complex roots

87. If z_1, z_2, z_3 be three complex numbers such that $|z_1 + 1| \leq 1, |z_2 + 2| \leq 2$ and $|z_3 + 4| \leq 4$, then the maximum value of $|z_1| + |z_2| + |z_3|$ is

a) 7 b) 10 c) 12 d) 14

88. If $\log_{\sqrt{3}} 5 = a$ and $\log_{\sqrt{3}} 2 = b$, then $\log_{\sqrt{3}} 300$ is equal to

a) 2(a + b) b) $2(a + b + 1)$ c) $2(a + b + 2)$ d) $a + b + 4$

89. If p, q, r, s, t are numbers such that $p + q < r + s, q + r < s + t, r + s < t + p, s + t < p + q$, then the largest and smallest numbers are

a) p and q respectively b) r and t respectively c) r and q respectively d) q and p respectively

90. The number of integral solutions of $\frac{x+2}{x^2+1} > \frac{1}{2}$ is

a) 4 b) 5 c) 3 d) None of these

91. Let α, β be the roots of the equation $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$. If $\alpha, \beta, \gamma, \delta$ are in GP, then integral values of p, q are respectively

a) $-2, -32$ b) $-2, 3$ c) $-6, 3$ d) $-6, -32$

92. If the complex numbers z_1, z_2, z_3 satisfying $\frac{z_1+z_3}{z_2-z_3} = \frac{1-i\sqrt{3}}{2}$, then triangle is

a) An equilateral triangle b) A right angled triangle

c) A acute angled triangle d) An obtuse angled isosceles triangle

93. If ω is a complex cube root of unity, then $225 + (3\omega + 8\omega^2)^2 + (3\omega^2 + 8\omega)^2$ is equal to

a) 72 b) 192 c) 200 d) 248

94. The locus of z satisying the inequality $\left|\frac{z+2i}{2z+i}\right| < 1$ where $z = x + iy$, is

a) $x^2 + y^2 < 1$ b) $x^2 - y^2 < 1$ c) $x^2 + y^2 > 1$ d) $2x^2 + 3y^2 < 1$

95. If the roots of $x^3 - 12x^2 + 39x - 28 = 0$ are in A.P., then their common difference is

a) ± 1 b) ± 2 c) ± 3 d) ± 4

96. The solution set of the inequation $\frac{2}{|x-4|} > 1, x \neq 4$, is

a) $(2, 6)$ b) $(2, 4) \cup (4, 6)$ c) $(-\infty, 2) \cup (6, \infty)$ d) None of these

97. The value of sum $\sum_{n=1}^{13}(i^n + i^{n+1})$, where $i = \sqrt{-1}$, equals

a) i b) $i - 1$ c) $-i$ d) 0

98. If α and β are imaginary cube roots of unity, then $\alpha^4 + \beta^4 + \frac{1}{\alpha\beta}$ is equal to

a) 3 b) 0 c) 1 d) 2

99. If a, b, c are all positive and in H.P., then the roots of $ax^2 + 2\,bx + c = 0$ are

a) Real b) Imaginary c) Rational d) Equal

100. For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$, the minimum value of $|z_1 - z_2|$ is

a) 4 b) 3 c) 1 d) 2

101. If α and β be the roots of the equation $x^2 + x\sqrt{\alpha} + \beta = 0$, then

a) $\alpha = 1$ and $\beta = -1$ b) $\alpha = 1$ and $\beta = -2$

c) $\alpha = 2$ and $\beta = 1$ d) $\alpha = 2$ and $\beta = -2$

102. The number of real roots of the equation $(x - 1)^2 + (x - 2)^2 + (x - 3)^2 = 0$ is

a) 1 b) 2 c) 3 d) None of these

103. If $z(\neq -1)$ is a complex number such that $\frac{z-1}{z+1}$ is purely imaginary, then $|z| =$

a) 1 b) 2 c) 3 d) 5

104. If $z_{,}, z_2$ and z_3 are any three complex numbers, then the fourth vertex of the parallelogram whose three vertices are z_1, z_2 and z_3 taken in order is

a) $z_1 - z_2 + z_3$ b) $z_1 + z_2 + z_3$ c) $\frac{1}{3}(z_1 - z_2 + z_3)$ d) $\frac{1}{3}(z_1 + z_2 - z_3)$

105. If z is a complex number such that Re $(z) =$ Im (z), then

a) Re $(z^2) = 0$ b) Im $(z^2) = 0$ c) Re $(z^2) =$ Im (z^2) d) Re $(z^2) = -$Im (z^2)

106. $\sqrt{-1 - \sqrt{1 - \sqrt{1 - \ldots\infty}}}$ is equal to

a) 1 b) -1 c) ω^2 d) $-\omega$

107. Let a be a complex number such that $|a| < 1$ and $z_1, z_2, \ldots$ be vertices of a polygon such that $z_k = 1 + a + a^2 + \ldots + a^{k-1}$. Then the vertices of the polygon lie within a circle is

a) $|z - a| = a$
b) $\left|z - \frac{1}{1-a}\right| = |1 - a|$
c) $\left|z - \frac{1}{1-a}\right| = \frac{1}{|1-a|}$
d) $|z - (1 - a)| = |1 - a|$

108. If α, β, γ are the roots of the equation $x^3 - 7x + 7 = 0$, then $\frac{1}{\alpha^4} + \frac{1}{\beta^4} + \frac{1}{\gamma^4}$ is

a) 7/3
b) 3/7
c) 4/7
d) 7/4

109. If $\sin\theta + \cos\theta = h$, then the quadratic equation having $\sin\theta$ and $\cos\theta$ as its roots, is

a) $x^2 - hx + (h^2 - 1) = 0$
b) $2x^2 - 2hx + (h^2 - 1) = 0$
c) $x^2 - hx + 2(h^2 - 1) = 0$
d) $x^2 - 2hx + (h^2 - 1) = 0$

110. If α and β are the roots of the equation $ax^2 + bx + c = 0, (c \neq 0)$, then the equation whose roots are $\frac{1}{a\alpha+b}$ and $\frac{1}{a\beta+b}$ is

a) $acx^2 - bx + 1 = 0$
b) $x^2 - acx + bc + 1 = 0$
c) $acx^2 + bx - 1 = 0$
d) $x^2 + acx - bc + 11 = 0$

111. The value of $\sqrt{i}$ is

a) $1 - i$
b) $1 + i$
c) $i - 1$
d) $\pm\frac{1}{\sqrt{2}}(1 + i)$

112. If one root of the quadratic equation $ax^2 + bx + c = 0$ is equal to nth power of the other root, then the value of $(ac^n)^{1/(n+1)} + (a^n c)^{1/(n+1)}$ is equal to

a) b
b) $-b$
c) $\frac{1}{b^{n+1}}$
d) $-\frac{1}{b^{n+1}}$

113. The modulus of the complex number z such that $|z + 3 - i| = 1$ and $\arg(z) = \pi$ is equal to

a) 1
b) 2
c) 9
d) 3

114. The product of cube roots of -1 is equal to

a) -1
b) 0
c) -2
d) 4

115. If the roots of $x^3 - 3x^2 - 6x + 8 = 0$ are in arithmetic progression, then the roots of the equation are

a) 3, 4, 5
b) 4, 7, 10
c) -2, 1, 4
d) 1, 4, 7

116. The number of values of a for which $(a^2 - 3a + 2)x^2 + (a^2 - 5a + 6)x + a^2 - 4 = 0$ is an identity in x, is

a) 0
b) 2
c) 1
d) 3

117. If z_1, z_2, z_3 are vertices of an equilateral triangle inscribed in the circle $|z| = 2$ and if $z_1 = 1 + i\sqrt{3}$, then

a) $z_1 = -2, z_3 = 1 - i\sqrt{3}$
b) $z_2 = 2, z_3 = 1 - i\sqrt{3}$
c) $z_2 = -2, z_3 = -1 - i\sqrt{3}$
d) $z_2 = 1 - i\sqrt{3}, z_3 = -1 - i\sqrt{3}$

118. The solution set of the inequation $\frac{x^2-3x+4}{x+1} >, x \in R$, is

a) $(3, \infty)$
b) $(-1, 1) \cup (3, \infty)$
c) $[-1, 1] \cup [3, \infty)$
d) None of these

119. The number of real solutions of the equation $\left(\frac{9}{10}\right)^x = -3 + x - x^2$ is

a) None
b) One
c) Two
d) More than two

120. The quadratic equation whose roots are three times the roots of $3ax^2 + 3bx + c = 0$ is

a) $ax^2 + 3bx + 3c = 0$
b) $ax^2 + 3bx + c = 0$
c) $9ax^2 + 9bx + c = 0$
d) $ax^2 + bx + 3c = 0$

121. The values of x satisfying $|x^2 + 4x + 3| + (2x + 5) = 0$ are

a) $-4, -1 - \sqrt{3}$
b) $4, 1 + \sqrt{3}$
c) $-4, 1 - \sqrt{3}$
d) $-4, 1 + \sqrt{3}$

122. If $x = \sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}$, then $x^2(x - 4)^2$ is equal to

a) 7
b) 4
c) 2
d) 1

123. If $|a_k| < 1, \lambda_k \geq 0$ for $k = 1, 2, \ldots, n$ and $\lambda_1 + \lambda_2 + \ldots \lambda_n = 1$, then the value of $|\lambda_1 a_1 + \lambda_2 a_2 + \ldots + \lambda_n a_n|$ is

a) Equal to one b) Greater than one c) Zero d) Less than one

124. If $\tan\alpha$ and $\tan\beta$ are roots of the equation $x^2+px+q=0$ with $p\neq 0$, then
a) $\sin^2(\alpha+\beta)+p\sin(\alpha+\beta)\cos(\alpha+\beta)+q\cos^2(\alpha+\beta)=q$
b) $\tan(\alpha+\beta)=\frac{p}{q+1}$
c) $\cos(\alpha+\beta)=-p$
d) $\sin(\alpha+\beta)=1-q$

125. The amplitude of $\sin\frac{\pi}{5}+i\left(1-\cos\frac{\pi}{5}\right)$ is
a) $\frac{2\pi}{5}$ b) $\frac{\pi}{15}$ c) $\frac{\pi}{10}$ d) $\frac{\pi}{5}$

126. The value of sum $\sum_{n=1}^{13}(i^n+i^{n+1})$, where $i=\sqrt{-1}$, equals
a) $-i$ b) $i-1$ c) $-i$ d) 0

127. If $x>0$ and $\log_3 x+\log_3(\sqrt{3})+\log_3(\sqrt[4]{x})+\log_3(\sqrt[8]{x})+\log_3(\sqrt[16]{x})+\ldots=4$, then x equals
a) 9 b) 81 c) 1 d) 27

128. Is S is the set of all real x such that $\frac{2x}{2x^2+5x+2}>\frac{1}{x+1}$, then S is equal to
a) $(-2,-1)$
b) $(-2/3,0)$
c) $(-2/3,-1/2)$
d) $(-2,-1)\cup(-2/3,-1/2)$

129. The value of p for which the difference between the roots of the equation $x^2+px+8=0$ is 2 are
a) ± 2 b) ± 4 c) ± 6 d) ± 8

130. If $x^2+ax+10=0$ and $x^2+bx-10=0$ have a common root, then a^2-b^2 is equal to
a) 10 b) 20 c) 30 d) 40

131. If $|z_1|=|z_2|=|z_3|=1$ and z_1,z_2,z_3 represent the vertices of an equilateral triangle, then
a) $z_1+z_2+z_3=0$ and $z_1z_2z_3=1$
b) $z_1+z_2+z_3=1$ and $z_1z_2z_3=1$
c) $z_!z_2+z_2z_3+z_3z_1=0$ and $z_1+z_2+z_3=0$
d) $z_1z_2+z_2z_3+z_3z_1=0$ and $z_1z_2z_3=1$

132. If $\sqrt{x+iy}=\pm(a+ib)$, then $\sqrt{-x-iy}$ is equal to
a) $\pm(b+ia)$ b) $\pm(a-ib)$ c) $\pm(b-ia)$ d) None of these

133. If the roots of the equation $x^2+px+q=0$ are α and β and roots of the equation $x^2-xr+s=0$ are α^4,β^4, then the roots of the equation $x^2-4qx+2q^2=0$ are
a) Both negative b) Both positive
c) Both real d) One negative and one positive

134. If $a,b,c$are the sides of the triangle ABC such that $a\neq b\neq c$ and $x^2-2(a+b+c)x+3\lambda(ab+bc+ca=0$ has real roots, then
a) $\lambda<\frac{4}{3}$ b) $\lambda>\frac{5}{3}$ c) $\lambda\in\left(\frac{4}{3},\frac{5}{3}\right)$ d) $\lambda\in\left(\frac{1}{3},\frac{5}{3}\right)$

135. The centre of a regular polygon of n sides is located at the point $z=0$ and one of its vertex z_1 is known. If z_2 be the vertex adjacent to z_1, then z_2 is equal to
a) $z_1\left(\cos\frac{2\pi}{n}\pm i\sin\frac{2\pi}{n}\right)$ b) $z_1\left(\cos\frac{\pi}{n}\pm i\sin\frac{\pi}{n}\right)$
c) $z_1\left(\cos\frac{\pi}{2n}\pm i\sin\frac{\pi}{2n}\right)$ d) None of these

136. Let $z=\cos\theta+i\sin\theta$. Then, the value of $\sum_{m=1}^{15}\mathrm{Im}(z^{2m-1})$ at $\theta=2°$ is
a) $\frac{1}{\sin 2°}$ b) $\frac{1}{3\sin 2°}$ c) $\frac{1}{2\sin 2°}$ d) $\frac{1}{4\sin 2°}$

137. Let $a\in R$. If the origin and the non-real roots of $2z^2+2z+a=0$ form the three vertices of an equilateral triangle in the argand plane, then $a=$
a) 1 b) 2 c) -1 d) None of these

138. The region of the Argand diagram defined by $|z-1|+|z+1|\le 4$ is
a) Interior of an ellipse
b) Exterior of a circle
c) Interior and boundary of an ellipse
d) None of the above

139. The radius of the circle $\left|\frac{z-i}{z+i}\right|=5$ is given by
a) $\frac{13}{12}$
b) $\frac{5}{12}$
c) 5
d) 625

140. The roots of the cubic equation $(z+\alpha\beta)^3=\alpha^3, \alpha\neq 0$
a) Represent sides of an equilateral triangle
b) Represent the sides of an isosceles triangle
c) Represent the sides of a triangle whose one side is of length $\sqrt{3}\,\alpha$
d) None of these

141. If $\left(\sqrt{5}+\sqrt{3}i\right)^{33}=2^{49}\,z$, then modulus of the complex number z is equal to
a) 1
b) $\sqrt{2}$
c) $2\sqrt{2}$
d) 4

142. If centre of a regular hexagon is at origin and one of the vertex on argand diagram is $1+2i$, then its perimeter is
a) $2\sqrt{5}$
b) $6\sqrt{2}$
c) $4\sqrt{5}$
d) $6\sqrt{5}$

143. The value of $\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7}-i\cos\frac{2\pi k}{7}\right)$ is
a) -1
b) 0
c) $-i$
d) i

144. The cubic equation whose roots are thrice to each of the roots of $x^3+2x^2-4x+1=0$ is
a) $x^3-6x^2+36x+27=0$
b) $x^3+6x^2+36x+27=0$
c) $x^3-6x^2-36x+27=0$
d) $x^3+6x^2-36x+27=0$

145. Let $(\sin a)x^2+(\sin a)x+(1-\cos a=0)$. The value of a for which roots of this equation are real and distinct, is
a) $(0,2\tan^{-1}1/4)$
b) $(0,2\pi/3)$
c) $(0,\pi)$
d) $(0,2\pi)$

146. If α and β $(\alpha<\beta)$ are the roots of the equation $x^2+bx+c=0$ where $c<0<b$,then
a) $0<\alpha<\beta$
b) $\alpha<0<\beta<|\alpha|$
c) $\alpha<\beta<0$
d) $\alpha<0<|\alpha|<\beta|$

147. If $1+x^2=\sqrt{3}x$, then $\sum_{n=1}^{24}\left(x^n-\frac{1}{x^n}\right)^2$ is equal to
a) 0
b) 48
c) -24
d) -48

148. The roots of the equation $|x^2-x-6|=x+2$ are
a) $-2,1,4$
b) 0,2,4
c) 0,1,4
d) $-2,2,4$

149. Let α,β be the roots of the equation $ax^2+bx+c=0$, and let $\alpha^n+\beta^n=S_n$ for $n\ge 1$. Then, the value of the determinant

$$\begin{vmatrix} 3 & 1+S_1 & 1+S_2 \\ 1+S_1 & 1+S_2 & 1+S_3 \\ 1+S_2 & 1+S_3 & 1+S_4 \end{vmatrix}$$ is

a) $\frac{b^2-4ac}{a^4}$
b) $\frac{(a+b+c)(b^2+4ac)}{a^4}$
c) $\frac{(a+b+c)(b^2-4ac)}{a^4}$
d) $\frac{(a+b+c)^2(b^2-4ac)}{a^4}$

150. If $z_1,z_2,z_3,\dots,z_n$ are n nth roots of unity, then for $k=1,2,\dots,n$
a) $|z_k|=k|z_{n+1}|$
b) $|z_{k+1}|=k|z_k|$
c) $|z_{k+1}|=|z_k|+|z_{k+1}|$
d) $|z_k|=|z_{k+1}|$

151. If α,β are the roots of the equation $x^2-(1+n^2)x+\frac{1}{2}(1+n^2+n^4)=0$, then $\alpha^2+\beta^2$ is
a) n^2
b) $-n^2$
c) n^4
d) $-n^4$

152. If one root of equation $x^2+ax+12=0$ is 4 while the equation $x^2+ax+b=0$ has equal roots, then the value of b is

a) $\frac{4}{49}$ b) $\frac{49}{4}$ c) $\frac{7}{4}$ d) $\frac{4}{7}$

153. If $a = \log_2 3, b = \log_2 5, c = \log_7 2$, then $\log_{140} 63$ in terms of a, b, c is

a) $\frac{2ac + 1}{2c + abc + 1}$ b) $\frac{2ac + 1}{2a + c + a}$ c) $\frac{2ac + 1}{2c + ab + a}$ d) None of these

154. Number of non-zero integral solutions of the equation $(1 - i)^n = 2^n$ is

a) 1 b) 2 c) Infinite d) None of these

155. The number of non-zero solutions of the equation
$x^2 - 5x - (Sgn(x))6 = 0$, is

a) 1 b) 2 c) 3 d) 4

156. If n is a positive integer greater than unity and z is a complex number satisfying the equation $z^n = (z + 1)^n$, then

a) Re $(z) < 0$ b) Re $(z) > 0$ c) Re $(z) = 0$ d) None of these

157. If $1, \omega, \omega^2$ are the cube roots of unity, then $(1 + \omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8)$ is equal to

a) 1 b) 0 c) ω^2 d) ω

158. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then

a) $x = 3, y = 1$ b) $x = 13, y = 3$ c) $x = 0, y = 3$ d) $x = 0, y = 0$

159. If z_1, z_2, z_3 are vertices of an equilateral triangle with z_0 its centroid, then $z_1^2 + z_2^2 + z_3^2 =$

a) z_0^2 b) $9\, z_0^2$ c) $3\, z_0^2$ d) $2\, z_0^2$

160. For all `$x'$, $x^2 + 2ax + (10 - 3a) > 0$, then the interval in which `a' lies, is

a) $a < -5$ b) $-5 < a < 2$ c) $a > 5$ d) $2 < a < 5$

161. If α_1, α_2 and β_1, β_2 are the roots of the equation $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ respectively and system of equations $\alpha_1 y + \alpha_2 z = 0$ and $\beta_1 y + \beta_2 z = 0$ has a non-zero solution, then

a) $a^2 qc = p^2 br$ b) $b^2 = pr = q^2 ac$ c) $c^2 = ar = r^2 pb$ d) None of these

162. If $1, \omega, \omega^2$ are the cube roots of unity, then $(1 - \omega + \omega^2)(1 - \omega^2 + \omega^4)(1 - \omega^4 + \omega^8)(1 - \omega^8 + \omega^{16})$... upto $2n$ factors is

a) $2n$ b) 2^{2n} c) 1 d) -2^{2n}

163. If α and β are different complex numbers with $|\beta| = 1$, then $\left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right|$ is

a) 0 b) 3/2 c) 1/2 d) 1

164. In a right-angled triangle, the sides are a, b and c, with c as hypotenuse, and $c - b \neq 1, c + b \neq 1$. Then the value of $(\log_{c+b} a + \log_{c-b} a)/(2 \log_{c+b} a \times \log_{c-b} a)$ will be

a) 2 b) -1 c) $\frac{1}{2}$ d) 1

165. The set of real values of x for which $\frac{10x^2 + 17x - 34}{x^2 + 2x - 3} < 8$, is

a) $(-5/2, 2)$ b) $(-3, -5/2) \cup (1, 2)$ c) $(-3, 1)$ d) None of these

166. If $\left(\frac{1 + \cos\phi + i \sin\phi}{1 + \cos\phi - i \sin\phi}\right) = u + iv$, where u and v all real numbers, then u is

a) $n \cos\phi$ b) $\cos n\phi$ c) $\cos\left(\frac{n\phi}{2}\right)$ d) $\sin\left(\frac{n\phi}{2}\right)$

167. The number of real roots of the equation $2\, x^4 + 5\, x^2 + 3 = 0$, is

a) 4 b) 1 c) 0 d) 3

168. If α and β are the roots of $x^2 - 2x + 4 = 0$, then the value of $\alpha^6 + \beta^6$ is

a) 32 b) 64 c) 128 d) 256

169. If $|z + 4| \leq 3$, then the greatest and the least value of $|z + 1|$ are

a) $6, -6$ b) $6, 0$ c) $7, 2$ d) $0, -1$

170. If P, P' represent the complex number z_1 and its additive inverse respectively, then the equation of the circle with PP' as a diameter is

a) $\frac{z}{z_1} = \frac{\bar{z}_1}{z}$ b) $z\bar{z} = z_1\bar{z}_1 = 0$ c) $z\bar{z}_1 + \bar{z}z_1 = 0$ d) None of these

171. If $x+1$ is a factor of $x^4+(p-3)x^3-(3p-5)x^2+(2p-9)x+6$, then the value of p is

a) -4 b) 0 c) 4 d) 2

172. If $A=\{x: f(x)=0\}$ and $B=\{x: g(x)=0\}$, then $A\cap B$ will be the set of roots of the equation

a) $\{f(x)\}^2+\{g(x)\}^2=0$

b) $\frac{f(x)}{g(x)}$

c) $\frac{g(x)}{f(x)}$

d) None of these

173. If α and β are the roots of the equation $x^2+px+q=0$ and if the sum $(\alpha+\beta)x-\frac{\alpha^2+\beta^2}{2}.x^2+\frac{\alpha^3+\beta^3}{3}.x^3-...$ exists then it is equal to

a) $\log(x^2+px+q)$ b) $\log(x^2-px+q)$ c) $\log(1+px+qx^2)$ d) $\log(1-px+qx^2)$

174. Let z be a complex number satisfying $|z-5i|\leq 1$ such that amp (z) is minimum. Then z is equal to

a) $\frac{2\sqrt{6}}{5}+\frac{24i}{5}$ b) $\frac{24}{5}+\frac{2\sqrt{6}i}{5}$ c) $\frac{2\sqrt{6}}{5}-\frac{24i}{5}$ d) None of these

175. If α and β are the roots of $x^2+px+1=0$ and γ and δ are the roots of $x^2+qx+1=0$, then the value of $(\alpha-\gamma)(\beta-\gamma)(\alpha+\delta)(\beta+\delta)$, is

a) p^2-q^2 b) q^2-p^2 c) p^2 d) q^2

176. For two complex numbers z_1, z_2 the relation $|z_1+z_2|=|z_1|+|z_2|$ holds, if

a) $\arg(z_1)=\arg(z_2)$ b) $\arg(z_1)+\arg(z_2)=\frac{\pi}{2}$

c) $z_1z_2=1$ d) $|z_1|=|z_2|$

177. If ω is a complex cube root of unity, then $\sin\left\{(\omega^{10}+\omega^{23}\pi-\frac{\pi}{4})\right\}$ is equal to

a) $\frac{1}{\sqrt{2}}$ b) $\frac{1}{2}$ c) 1 d) $\frac{\sqrt{3}}{2}$

178. If the equation $x^3-3x+a=0$ has distinct roots between 0 and 1, then the value of a is

a) 2 b) 1/2 c) 3 d) None of these

179. If α, β are roots of the equation $375x^2-25x-2=0$ and $S_n=a^n+\beta^n$, then $\lim_{n\to\infty}\sum_{r=1}^{n}S_r$ is equal to

a) 7/116 b) 1/12 c) 29/348 d) None of these

180. If $y=\tan x\cot 3x, x\in R$, then

a) $\frac{1}{3}<y<1$ b) $\frac{1}{3}\leq y\leq 1$ c) $\frac{1}{3}\leq y\leq 3$ d) None of these

181. If α is a root of the equation $2x(2x+1)=1$, then the other roots is

a) $3\alpha^3-4\alpha$ b) $-2\alpha(\alpha+1)$ c) $4\alpha^3-3\alpha$ d) None of these

182. If the roots of the equation $x^2-bx+c=0$ be two consecutive integers, then b^2-4c equals

a) 1 b) 2 c) 3 d) -2

183. If x, y, z are in GP and $a^x=b^y=c^z$, then

a) $\log_a c=\log_b a$ b) $\log_b a=\log_c b$ c) $\log_c b=\log_a c$ d) None of the above

184. If the complex numbers $z_1=a+i, z_2=1++ib, z_3=0$ form the vertices of equilateral triangle (a, b are real numbers between 0 and 1), then

a) $a=\sqrt{3}-1, b=\frac{\sqrt{3}}{2}$

b) $a=2-\sqrt{3}, b=2-\sqrt{3}$

c) $a=1/2, b=3/4$

d) None of these

185. Sum of the series $\sum_{r=0}^{n}(-1)^r\,{}^nC_r\,\{i^{5r}+i^{6r}+i^{7r}+i^{8r}\}$, is

a) 2^n b) $2^{n/2+1}$ c) $n^n+2^{n/2+1}$ d) $2^n+2^{n/2+1}\cos\frac{n\pi}{4}$

186. If a, b and c are distinct positive real numbers in AP, then the roots of the equation $ax^2+2bx+c=0$ are

a) Imaginary b) Rational and equal c) Rational and distinct d) Irrational

187. Let $z(\neq 2)$ be a complex number such that $\log_{1/2}|z-2| > \log_{1/2}|z|$, then

a) Re $(z) > 1$ b) Im $(z) > 1$ c) Re $(z) = 1$ d) Im $(z) = 1$

188. The equation $z^5 + z^4 + z^3 + z^2 + z + 1 = 0$ is satisfied by

a) $z = \pm 1$ b) $z = -1$ c) $z = \pm\frac{1}{2} + \frac{i\sqrt{3}}{2}$ d) None of the above

189. The equation $x^2 - 3|x| + 2 = 0$ has

a) No real root b) One real root c) Two real roots d) Four real roots

190. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of q is

a) 4 b) 12 c) 3 d) $\frac{49}{4}$

191. If $[x]^2 = [x+2]$, where $[x]$ = the greatest integer less than or equal to x, then x must be such that

a) $x = 2, -1$ b) $[-1, 0] \cup [2, 3]$ c) $x \in [-1, 0]$ d) None of these

192. If α, β are the roots of $ax^2 + bx + c = 0$ the equation whose roots are $2 + \alpha, 2 + \beta$ is

a) $ax^2 + x(4a - b) + 4a - 2b + c = 0$

b) $ax^2 + x(4a - b) + 4a + 2b + c = 0$

c) $ax^2 + x(b - 4a) + 4a + 2b + c = 0$

d) $ax^2 + x(b - 4a) + 4a - 2b + c = 0$

193. If α, β and γ are angles such that $\tan\alpha + \tan\beta + \tan\gamma = \tan\alpha\tan\beta\tan\gamma$ and $x = \cos\alpha + i\sin\alpha$, $y = \cos\beta + i\sin\beta$ and $z = \cos\gamma + i\sin\gamma$, then xyz is equal to

a) 1, but not -1 b) -1, but not 1 c) +1 or -1 d) 0

194. If $\arg(z_1 z_2) = 0$ and $|z_1| = |z_2| = 1$, then

a) $z_1 + z_2 = 0$ b) $z_1\overline{z_2} = 1$ c) $z_1 = \overline{z_2}$ d) None of these

195. If the equation $2x^2 - 7x + 1 = 0$ and $ax^2 + bx + 2 = 0$ have a common root, then

a) $a = 2, b = -7$ b) $a = -\frac{7}{2}, b = 1$ c) $a = 4, b = -14$ d) None of these

196. The polynomial $x^{3m} + x^{3n+1} + x^{3k+2}$ is exactly divisible by $x^2 + x + 1$ if

a) m, n, k are rational

b) m, n, k are integers

c) m, n, k are positive integers

d) None of these

197. If $a, b, c \neq 0$ and belongs to the set $\{0, 1, 2, 3, \dots\dots.., 9\}$,

Then $\log_{10}\left(\frac{a + 10b + 10^2c}{10^{-4}a + 10^{-3}b + 10^{-2}c}\right)$ is equal to

a) 1 b) 2 c) 3 d) 4

198. If the roots of the equation $x^2 + px + q = 0$ are $\tan 30°$ and $\tan 15°$ respectively, then the value of $2 + q - p$ is

a) 3 b) 0 c) 1 d) 2

199. If $z = x - iy$ and $z^{1/3} = p + iq$, then $\left(\frac{x}{p} + \frac{y}{q}\right)/(p^2 + q^2)$ is equal to

a) 1 b) -1 c) 2 d) -2

200. If $\sec\alpha$ and $\operatorname{cosec}\alpha$ are the roots of the equation $x^2 - px + q = 0$, then

a) $p^2 = p + 2q$ b) $q^2 = p + 2q$ c) $p^2 = q(q+2)$ d) $q^2 = p(p+2)$

201. The number of real roots of the equation $\left(x + \frac{1}{x}\right)^3 + x + \frac{1}{x} = 0$ is

a) 0 b) 2 c) 4 d) 6

202. If $a, b, c \in R$ and $a + b + c = 0$, then the quadratic equation $4ax^2 + 3bx + 2c = 0$ has

a) One positive and one negative root

b) Imaginary roots

c) Real roots

d) None of these

203. If α, β γ and δ are the roots of the equation $x^4 - 1 = 0$, then the value of

$\frac{a\,\alpha+b\,\beta+c\,\gamma+d\,\delta}{a\,\gamma+b\,\delta+c\,\alpha+d\,\beta}+\frac{a\,\gamma+b\,\delta+c\,\alpha+d\,\beta}{a\,\alpha+b\,\beta+c\,\gamma+d\,\delta}$, is

a) 3 β b) 0 c) 2 γ d) None of these

204. If $\log_5 \log_5 \log_2 x = 0$, then the value of x is

a) 32 b) 125 c) 625 d) 25

205. $\left(\frac{1}{1+2i}+\frac{3}{1+i}\right)\left(\frac{3+4i}{2-4i}\right)$ is equal to

a) $\frac{1}{2}+\frac{9}{2}i$ b) $\frac{1}{2}-\frac{9}{2}i$ c) $\frac{1}{4}-\frac{9}{4}i$ d) $\frac{1}{4}+\frac{9}{4}i$

206. If α, β are the roots of the equation $x^2+px+q=0$ and α^4, β^4 are the roots of $x^2-rx+s=0$, then the equation $x^2-4qx+2q^2-r=0$ has always (p, q, r, s are real numbers)

a) Two real roots

b) Two negative roots

c) Two positive roots

d) One positive and one negative roots

207. If x is real, then the minimum value of $\frac{x^2-x+1}{x^2+x+1}$, is

a) $\frac{1}{3}$ b) 3 c) $\frac{1}{2}$ d) 1

208. In the equation $a_4x^4+a_3x^3+a_2x^2+a_1x+a_0=0$ roots of the equation are $\alpha_i,\ i=1,2,3,4$. Now, x is replaced by $x-1$, now roots of new equation are

a) $\frac{1}{\alpha_i+1}, i=1,2,3,4$ b) $\alpha_i+1, i=1,2,3,4$

c) $\alpha_i-1, i=1,2,3,4$ d) None of these

209. The closest distance of the origin from a curve given as $a\bar{z}+\bar{a}z+|a|^2=0$ is

a) 1 b) $\frac{\text{Re}(|a|)}{|a|}$ c) $\frac{\text{Im}(|a|)}{|a|}$ d) $\frac{|a|}{2}$

210. Let a, b and c be such that

$\frac{1}{(1-x)(1-2x)(1-3x)}=\frac{a}{1-x}+\frac{b}{1-2x}+\frac{c}{1-3x}$ then $\frac{a}{1}+\frac{b}{3}+\frac{c}{5}$ is equal to

a) 1/15 b) 1/6 c) 1/5 d) 1/3

211. The root of the equation $2(1+i)x^2-4(2-i)x-5-3i=0$, where $i=\sqrt{-1}$, which has greater modulus, is

a) $\frac{3-5i}{2}$ b) $\frac{5-3i}{2}$ c) $\frac{3+i}{2}$ d) $\frac{3i+1}{2}$

212. For any complex number z, the minimum value of $|z|+|z-2i|$, is

a) 0 b) 1 c) 2 d) 4

213. The value of expression $2(1+\omega)(1+\omega^2)+3(2+\omega)(2+\omega^2)+4(3+\omega)(3+\omega^2)+\cdots+(n+1)(n+$ $\omega n+\omega 2$, where ω is an imaginary cube root of unity is

a) $\left\{\frac{n(n+1)}{2}\right\}^2$ b) $\left\{\frac{n(n+1)}{2}\right\}^2-n$ c) $\left\{\frac{n(n+1)}{2}\right\}^2+n$ d) None of these

214. For the equation $x^{\frac{3}{4}(\log_2 x)^2+\log_2(x)-\frac{5}{4}}=\sqrt{2}$ which one of the following is not true?

a) Has at least one real solution

b) Has exactly three real solutions

c) Has exactly one irrational solution

d) All of these

215. If (x^2-3x+2) is a factor of $x^4-px^2+q=0$, then the values of p and q are

a) -5, 4 b) 5, 4 c) 5, -4 d) -5, -4

216. If the ratio of the equation $x^2+px+q=0$ be equal to the ratio of the roots of $x^2+lx+m=0$, then

a) $p^2m=q^2l$ b) $pm^2=q^2l$ c) $p^2l=q^2m$ d) $p^2m=l^2q$

217. If x_1, x_2, x_3, x_4 are roots of the equation $x^4-x^3\sin 2\beta+x^2\cos 2\beta-x\cos\beta-\sin\beta=0$, then $\tan^{-1}x_1+$ $\tan -1x2+\tan -1x3+\tan -1x4$ is equal to

a) β b) $\frac{\pi}{2}-\beta$ c) $\pi-\beta$ d) $-\beta$

218. If the roots of the equation $a(b-c)x^2+b(c-a)x+c(a-b)=0$ are equal, then a,b,c are in
a) H.P. b) G.P. c) A.P. d) None of these

219. If the sum of the squares of the roots of the equation $x^2-(a-2)x-(a+1)=0$ is least, then the value of a is
a) 0 b) 2 c) -1 d) 1

220. If $a+b+c \neq 0$, then $(a+b\omega+c\omega^2)^3+(a+b\omega^2+c\omega^2)=$
a) $(2a-b-c)(2b-c-a)(2c-a-b)$
b) abc
c) $(2a+b+c)(2b+c+a)(2c+a+b)$
d) None of these

221. The roots of the equation $\log_2(x^2-4x+5)=(x-2)$ are
a) 4, 5 b) 2, -3 c) 2, 3 d) 3, 5

222. If $\cos\alpha$ is a root of $25x^2+5x-12=0, -1<x<0$. Then, the value of $\sin 2\alpha$ is
a) $\frac{12}{25}$ b) $\frac{-12}{25}$ c) $\frac{-24}{25}$ d) $\frac{20}{25}$

223. If α,β,γ are the roots of the equation $2x^3-3x^2+6x+1=0$, then $\alpha^2+\beta^2+\gamma^2$ is equal to
a) $-\frac{15}{4}$ b) $\frac{15}{4}$ c) $\frac{9}{4}$ d) 4

224. If $2z_1-3z_2+z_3=0$, then z_1,z_2,z_3 are represented by
a) Three vertices of a triangle
b) Three collinear points
c) Three vertices of a rhombus
d) None of these

225. The condition that one root of the equation $ax^2+bx+c=0$ may be double of the other, is
a) $b^2=9ac$ b) $2b^2=9ac$ c) $2b^2=ac$ d) $b^2=ac$

226. The locus of $z=i+2\exp\left\{i\left(\theta+\frac{\pi}{4}\right)\right\}$, where θ is parameter, is
a) A circle b) An ellipse c) A parabola d) Hyperbola

227. If $\alpha\neq\beta$ and $\alpha^2=5\alpha-3, \beta^2=5\beta-3$, then the equation having α/β and β/α as its roots is
a) $3x^2+19x+3=0$ b) $3x^2-19x+3=0$ c) $3x^2-19x-3=0$ d) $x^2-16x+1=0$

228. If $2-3x-2x^2\geq 0$, then
a) $x\leq -2$ b) $-2\leq x\leq\frac{1}{2}$ c) $x\geq -2$ d) $x\leq\frac{1}{2}$

229. If $f(x)=ax^2+bx+c, g(x)=-ax^2+bx+c$ where $ac\neq 0$, then $f(x)g(x)=0$ has
a) At least three real roots
b) No real roots
c) At least two real roots
d) Two real roots and two imaginary roots

230. If $|z+4|\leq 3$, then the maximum value of $|z+1|$ is
a) 4 b) 10 c) 6 d) 0

231. If $x=\sqrt{7}-\sqrt{5}$ and $y=\sqrt{13}-\sqrt{11}$, then
a) $x>y$ b) $x<y$ c) $x=y$ d) None of these

232. If α,β are the roots of the equation $ax^2+bx+c=0$ such that $\beta<\alpha<0$, then the quadratic equation whose roots are $|\alpha|,|\beta|$, is given by
a) $|a|x^2+|b|x+|c|=0$ b) $ax^2-|b|x+c=0$
c) $|a|x^2-|b|x+|c|=0$ d) $a|x|^2+b|x|+c=0$

233. If magnitude of a complex number $4-3i$ is tripled and is rotated anti-clockwise by an angle π, then resulting complex number world be
a) $-12+9i$ b) $12+9i$ c) $7-6i$ d) $7+6i$

234. If $z=r\,e^{i\,\theta}$, then $|e^{i\,z}|$ is equal to

a) $e^{-r\sin\theta}$ b) $r\, e^{-r\sin\theta}$ c) $e^{-r\cos\theta}$ d) $r\, e^{-r\cos\theta}$

235. The roots of the equation $|2x-1|^2-3|2x-1|+2=0$ are

a) $\left\{-\frac{1}{2},0,\frac{1}{2}\right\}$ b) $\left\{-\frac{1}{2},0,\frac{3}{2}\right\}$ c) $\left\{-\frac{3}{2},\frac{1}{2},0,1\right\}$ d) $\left\{-\frac{1}{2},0,1,\frac{3}{2}\right\}$

236. If $|3x+2|<1$, then x belongs to the interval

a) $(-1,-1/3)$ b) $[-1,-1/3]$ c) $(-\infty,-1)$ d) $(-1/3,\infty)$

237. The set $C=\{z: z\bar{z}+a\bar{z}+\bar{a}z+b=0, b\in R$ and $b<|a|^2\}$ is

a) A finite set b) An infinite set c) An empty set d) None of these

238. The equation $\bar{z}=\bar{a}+\frac{r^2}{(z-a)}, r>0$ represents

a) An ellipse b) A parabola

c) A circle d) A straight line through point $\bar{a}$

239. If ω is a complex cube root of unity, then $\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}+\frac{c+a\omega+b\omega^2}{a+b\omega+c\omega^2}+\frac{b+c\omega+a\omega^2}{b+c\omega^4+a\omega^5}$ is equal to

a) 1 b) ω c) ω^2 d) 0

240. If the roots of the equation $3x^2-6x+5=0$ are α and β, then the equation whose roots are $\alpha+\beta$ and $\frac{2}{\alpha+\beta}$ will be

a) $x^2+3x-1=0$ b) $x^2+3x-2=0$ c) $x^2+3x+2=0$ d) $x^2-3x+2=0$

241. The roots of $ax^2+2\,bx+c=0$ and $bx^2-2\sqrt{ac}\,x+b=0$ are simultaneously real, then

a) $a=b, c=0$ b) $ac=b^2$ c) $4b^2=ac$ d) None of these

242. The solution set of the inequation $\left|\frac{3}{x}+1\right|>2$, is

a) $(0,3]$ b) $[-1,0)$ c) $(-1,0)\cup(0,3)$ d) None of these

243. The number of real solutions of the equation $|x^2+4x+3|+2x+5=0$ are

a) 1 b) 2 c) 3 d) 4

244. The region of the complex plane for which $\left|\frac{z-a}{z+\bar{a}}\right|=1$ [Re$(a)\neq 0$], is

a) x-axis b) y-axis

c) The straight line $x=a$ d) None of these

245. The locus of point z satisfying Re$(z^2)=0$, is

a) A pair of straight lines

b) A circle

c) A rectangular hyperbola

d) None of these

246. The co0mplex number $z=x+iy$ which satisfy the equation $\left|\frac{z-5i}{z+5i}\right|=1$ lies on

a) The axis of x b) The straight line $y=5$

c) The circle passing through the origin d) None of the above

247. If $2+i\sqrt{3}$ is a root of $x^2+px+q=0$ where $p,q\in R$, then

a) $p=-4, q=7$ b) $p=4, q=7$ c) $p=4, q=-7$ d) $p=-4, q=-7$

248. If $\sqrt{3x^2-7x-30}+\sqrt{2x^2-7x-5}=x+5$, then x is equal to

a) 2 b) 3 c) 6 d) 5

249. if $\sqrt{x+iy}=\pm(a+ib)$, then $\sqrt{-x-iy}$ is equal to

a) $\pm(b+ia)$ b) $\pm(a-ib)$ c) $(ai+b)$ d) $\pm(b-ia)$

250. If roots of $x^2-ax+b=0$ are prime numbers, then

a) `$b$' is a prime number b) `a' is a composite number

c) $1+a+b$ is a prime number d) None of the above

251. Let z_1 and z_2 be complex numbers, then $|z_1+z_2|^2+|z_1-z_2|^2$ is equal to

a) $|z_1|^2+|z_2|^2$ b) $2(|z_1|^2+|z_2|^2)$ c) $2(z_1^2+z_2^2)$ d) $4z_1z_2$

252. If $f(x)=x^2+2bx+2c^2$ and $g(x)=-x^2-2cx+b^2$ such that min $f(x)>$ max $g(x)$, then the relation between b and c is

a) $|c|<|b|\sqrt{2}$ b) $0<c<b\sqrt{2}$ c) $|c|<|b|\sqrt{2}$ d) $|c|>|b|\sqrt{2}$

253. The number of complex numbers z such that $|z-1|=|z+1|=|z-i|$ equals

a) 0 b) 1 c) 2 d) ∞

254. If $ax^2+bx+c=0$ and $2x^2+3x+4=0$ have a common root where $a,b,c\in N$ (set of natural numbers), the least value of $a+b+c$ is

a) 13 b) 11 c) 7 d) 9

255. The solution set of the inequation $\frac{x+4}{x-3}\leq 2$, is

a) $(-\infty,3)\cup(10,\infty)$ b) $(3,10]$ c) $(-\infty,3)\cup[10,\infty)$ d) None of these

256. Let $f(x)=x^2+ax+b$, where $a,b\in R$. If $f(x)=0$ has all its roots imaginary, then the roots of $f(x)+f'(x)+f''(x)=0$ are

a) Real and distinct b) Imaginary c) Equal d) Rational and equal

257. If $\log_{10}7=0.8451$, then the position of the first significant figure of 7^{-20} is

a) 16 b) 17 c) 20 d) 15

258. If z and w are two non-zero complex numbers such that $|zw|=1$ and $\arg(z)-\arg(w)=\frac{\pi}{2}$, then $\bar{z}$ is equal to

a) 1 b) -1 c) i d) $-i$

259. If the sum of the roots of the equation $ax^2+2x+3a=0$ is equal to their products, then the value of a is

a) $-\frac{2}{3}$ b) -3 c) 4 d) $-\frac{1}{2}$

260. If α,β,γ are the roots of $x^3+2x^2-3x-1=0$, then $\alpha^{-2}+\beta^{-2}+\gamma^{-2}$ is equal to

a) 12 b) 13 c) 14 d) 15

261. If $x^2+2x+2xy+my-3=0$ has two rational factors, then the values of m will be

a) $-6,-2$ b) $-6,2$ c) $6,-2$ d) $6,2$

262. If α and β are the roots of the equation $ax^2+bx+c=0$, $\alpha\beta=3$ and a,b,c are in AP, then $\alpha+\beta$ is equal to

a) -4 b) 1 c) 4 d) -2

263. If the difference between the roots of $x^2+ax-b=0$ is equal to the difference between the roots of $x^2-px+q=0$, then a^2-p^2 in terms of b and q is

a) $-4(b+q)$ b) $4(b+q)$ c) $4(b-q)$ d) $4(q-b)$

264. The value of $\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}+\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}$ will be

a) 1 b) -1 c) 2 d) -2

265. The imaginary part of $(z-1)(\cos\alpha-i\sin\alpha)+(z-1)^{-1}\times(\cos\alpha+i\sin\alpha)$ zero, if

a) $|z-1|=2$ b) $\arg(z-1)=2\alpha$ c) $\arg(z-1)=\alpha$ d) $|z|=1$

266. If $z=x+iy$ and $\left|\frac{1-iz}{z-i}\right|=1$, the locus of z is

a) x-axis b) y-axis

c) Circle with unity radius d) None of the above

267. If p and q are roots of the quadratic equation $x^2+mx+m^2+a=0$, then the value of p^2+q^2+pq is

a) 0 b) a c) $-a$ d) $\pm m^2$

268. For the equation $|x|^2+|x|-6=0$

a) There is only one root

b) There are only two distinct roots

c) There are only three distinct roots

d) There are four distinct roots

269. Let z_1 and z_2 be two complex numbers such that $\frac{z_1}{z_2}+\frac{z_2}{z_1}=1$. Then,

a) z_1,z_2 are collinear

b) z_1,z_2 and the origin from a right angled triangle

c) z_1,z_2 and the origin from an equilateral triangle

d) None of these

270. If x, a, b, c are real and $(x-a+b)^2+(x-b+c)^2=0$, then a, b, c are in
a) H.P. b) G.P. c) A.P. d) None of these
271. If $z=i\log(2-\sqrt{3})$, then $\cos z=$
a) i b) $2i$ c) 1 d) 2
272. If $x=3+i$, then $x^3-3x^2-8x+15$ is equal to
a) 45 b) -15 c) 10 d) 6
273. If z_1, z_2 are any two complex numbers, then
a) $|z_1+z_2| \geq |z_1|+|z_2|$ b) $|z_1+z_2| > |z_1|+|z_2|$
c) $|z_1+z_2| \leq |z_1|+|z_2|$ d) $|z_1+z_2| = |z_1|+|z_2|$
274. The value of a for which the equation $2x^2+2\sqrt{6}x+a=0$ has equal root, is
a) 3 b) 4 c) 2 d) $\sqrt{3}$
275. If $a=\cos\left(\frac{2\pi}{7}\right)+i\sin\left(\frac{2\pi}{7}\right)$, then the quadratic equation whose roots are $\alpha=a+a^2+a^4$ and $\beta=a^3+a^5+a^6$ is
a) $x^2-x+2=0$ b) $x^2+x-2=0$ c) $x^2-x-2=0$ d) $x^2+x+2=0$
276. If $|z_1|=|z_2|$ and $\arg(z_1)+\arg(z_2)=0$, then
a) $z_1=z_2$ b) $z_1=\overline{z_2}$ c) $z_1z_2=1$ d) None of these
277. For any complex number z, the minimum value of $|z|+|z-1|$ is
a) 1 b) 0 c) 1/2 d) 3/2
278. The equation $z\bar{z}+a\bar{z}+\bar{a}z+b=0,\ b\in R$ represents a circle, if
a) $|a|^2=b$ b) $|a|^2>b$ c) $|a|^2<b$ d) None of these
279. If z_1, z_2, z_3, z_4 are the affixes of four points in the argand plane and z is the affix of a point such that $|z-z_1|=|z-z_2|=|z-z_3|=|z-z_4|$, then z_1, z_2, z_3, z_4 are
a) Concyclic b) Vertices of a parallelogram
c) Vertices of a rhombus d) In a straight line
280. Let α, β be the roots of the equation $ax^2+2bx+c=0$ and γ, δ be the roots of the equation $px^2+2qx+r=0$. If $\alpha, \beta, \gamma, \delta$ are in GP, then
a) $q^2ac=b^2pr$ b) $qac=bpr$ c) $c^2pq=r^2ab$ d) $p^2ab=a^2qr$
281. If α, β, γ are such that $\alpha+\beta+\gamma=2, \alpha^2+\beta^2+\gamma^2=6, \alpha^3+\beta^3+\gamma^3=8$, then $\alpha^4+\beta^4+\gamma^4$ is equal to
a) 7 b) 12 c) 18 d) 36
282. If the roots of the equation $x^2-2ax+a^2+a-3=0$ are real and less than 3, then
a) $a<2$ b) $2\leq a\leq 3$ c) $3\leq a\leq 4$ d) $a>4$
283. For any two complex numbers z_1 and z_2 and any real numbers a and b, $|az_1-bz_2|^2+|bz_1+az_2|^2$ is equal to
a) $(a^2+b^2)(|z_1|+|z_2|)$ b) $(a^2+b^2)(|z_1|^2+|z_2|^2)$
c) $(a^2+b^2)(|z_1|^2-|z_2|^2)$ d) None of the above
284. If ω is the complex cube root of unity, then the value of $\omega+\omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\frac{27}{128}+\cdots\right)}$
a) -1 b) 1 c) $-i$ d) i
285. If α and β are the roots of the equation $ax^2+bx+c=0$ and, if $px^2+qx+r=0$has roots $\frac{1-\alpha}{\alpha}$ and $\frac{1-\beta}{\beta}$, then r is equal to
a) $a+2b$ b) $a+b+c$ c) $ab+bc+ca$ d) abc
286. If $1+x^2=\sqrt{3}x$, then $\sum_{n=1}^{24}\left(x^n-\frac{1}{x^n}\right)^2$ is equal to
a) 48 b) -48 c) $\pm 48(\omega-\omega^2)$ d) None of these
287. If α, β are the roots of $x^2+bx-c=0$, then the equation whose roots are b and c is
a) $x^2+\alpha x-\beta=0$
b) $x^2-x(\alpha+\beta+\alpha\beta)-\alpha\beta(\alpha+\beta)=0$
c) $x^2+(\alpha+\beta-\alpha\beta)x-\alpha\beta(\alpha+\beta)=0$
d) $x^2+x(\alpha+\beta+\alpha\beta)+\alpha\beta(\alpha+\beta)=0$
288. Let α, β be the roots of the equation $x^2-ax+b=0$ and $A_n=\alpha^n+\beta^n$. Then, $A_{n+1}-aA_n+bA_{n-1}$ is

equal to

a) $-a$ b) b c) 0 d) $a-b$

289. The quadratic equations $x^2+(a^2-2)x-2a^2=0$ and $x^2-3x+2=0$ have
a) No common root for all $a \in R$
b) Exactly one common root for all $a \in R$
c) Two common roots for some $a \in R$
d) None of these

290. If a, b, c are distinct positive numbers each being different from 1 such that
$(\log_b a . \log_c a - \log_a a) + (\log_a b . \log_c b - \log_b b) + (\log_a c . \log_b c - \log_c c) = 0$, then abc is
a) 0 b) e c) 1 d) 2

291. Suppose that two persons A and B solve the equation $x^2+ax+b=0$. While solving A commits a mistake in the coefficient of x was taken as 15 in place of -9 and finds the roots as -7 and -2. Then the equation is
a) $x^2+9x+14=0$ b) $x^2-9x+14=0$ c) $x^2+9x-14=0$ d) $x^2-9x-14=0$

292. The values of $'a'$ for which the roots of the equation $x^2+x+a=0$ are real and exceed 'a' are
a) $0<a<1/4$ b) $a<1/4$ c) $a<-2$ d) $-2<a<0$

293. If a, b, c are positive real numbers, then the number of real roots of the equation $ax^2+b|x|+c=0$ is
a) 2 b) 4 c) 0 d) None of these

294. If a and b are two distinct real roots of the polynomial $f(x)$ such that $a<b$, then there exists a real number c lying between a and b, such that
a) $f(c)=0$ b) $f'(c)=0$ c) $f''(c)=0$ d) None of these

295. If the cube root of unity are $1, \omega, \omega^2$, then the roots of the equation $(x-1)^3+8=0$, are
a) $-1, 1+2\omega, 1+2\omega^2$ b) $-1, 1-2\omega, 1-2\omega^2$
c) $-1, -1, -1$ d) $-1, -1+2\omega, -1-2\omega^2$

296. A value of k for which the quadratic equation $x^2-2x(l+3k)+7(2k+3)=0$ has equal roots, is
a) 1 b) 2 c) 3 d) 4

297. $7^{2\log_7 5}$ is equal to
a) $\log_7 35$ b) 5 c) 25 d) $\log_7 25$

298. The expression $\tan\left\{i \log\left(\frac{a-ib}{a+ib}\right)\right\}$ reduces to
a) $\dfrac{ab}{a^2+b^2}$ b) $\dfrac{2\,ab}{a^2-b^2}$ c) $\dfrac{ab}{a^2-b^2}$ d) $\dfrac{2\,ab}{a^2+b^2}$

299. The common roots of the equations $x^3+2x^2+2x+1=0$ and $1+x^{2002}+x^{2003}=0$ are (where ω is a complex cube root of unity)
a) ω, ω^2 b) $1, \omega^2$ c) $-1, -\omega$ d) $\omega, -\omega^2$

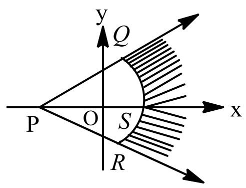
300. If α, β, γ are the cube roots of a negative number p, then for any three real numbers x, y, z the value of $\frac{x\alpha+y\beta+z\gamma}{x\beta+y\gamma+z\alpha}$ is
a) $\dfrac{1-i\sqrt{3}}{2}$ b) $\dfrac{-1-i\sqrt{3}}{2}$ c) $(x+y+z)i$ d) π

301. $\sin x+\cos x=y^2-y+a$ has no value of x for any y, if `a' belongs to
a) $(0, \sqrt{3})$ b) $(-\sqrt{3}, 0)$ c) $(-\infty, -\sqrt{3})$ d) $(\sqrt{3}, \infty)$

302. The value of $\sqrt{42+\sqrt{42+\sqrt{42+\dots}}}$ is equal to
a) 7 b) -6 c) 5 d) 4

303. The quadratic equation in x such that the arithmetic mean of its roots is 5 and geometric mean of the roots is 4, is given by
a) $x^2+20x+16=0$ b) $x^2-10x+16=0$ c) $x^2+10x+16=0$ d) $x^2-10x-16=0$

304. The shaded region, where
$P \equiv (-1, 0), Q \equiv (-1+\sqrt{2}, \sqrt{2})$
$R \equiv (-1+\sqrt{2}, -\sqrt{2}), S \equiv (1, 0)$ is represented by



a) $|z+1|>2, |\arg(z+1)<\frac{\pi}{4}|$ b) $|z+1|<2, \arg(z+1)<\frac{\pi}{2}$
c) $|z-1|>2, \arg(z+1)>\frac{\pi}{4}$ d) $|z-1|<2, |\arg(z+1)>\frac{\pi}{4}$

305. If ω and ω^2 are the two imaginary cube roots of unity, then the equation whose roots are $a\omega^{317}$ and $a\omega^{382}$, is
a) $x^2+ax+a^2=0$ b) $x^2+a^2x+a=0$ c) $x^2-ax+a^2=0$ d) $x^2-a^2x+a=0$

306. If x is real, then expression $\frac{x+2}{2x^2+3x+6}$ takes all values in the interval
a) $\left(\frac{1}{13},\frac{1}{3}\right)$ b) $\left[-\frac{1}{13},\frac{1}{3}\right]$ c) $\left(-\frac{1}{3},\frac{1}{13}\right)$ d) None of these

307. If $z(\overline{z+\alpha})+\bar{z}(z+\alpha)=0$ where α is a complex constant, then z is represented by a point on
a) A circle b) A straight line c) A parabola d) None of these

308. The value of a for which the equations $x^3+ax+1=0$ and $x^4+ax^2+1=0$ have a common root, is
a) -2 b) -1 c) 1 d) 2

309. If $2x=-1+\sqrt{3}i$, then the value of $(1-x^2+x)^6-(1-x+x^2)^6$ is equal to
a) 32 b) -64 c) 64 d) 0

310. If α,β are roots of $x^2+px-q=0$ and γ,δ are root of $x^2+px+r=0$, then the value of $(\alpha-\gamma)(\alpha-\delta)$ is
a) $p+q$ b) $q-r$ c) $r-q$ d) $q+r$

311. The roots α,β and γ of an equation
$x^3-3\,a\,x^2+3\,b\,x-c=0$ are in H.P. Then,
a) $\beta=\frac{1}{a}$ b) $\beta=b$ c) $\beta=\frac{b}{c}$ d) $\beta=\frac{c}{b}$

312. The value of $1+i^2+i^4+i^6+\dots+i^{2n}$ is
a) Positive b) Negative
c) Zero d) Cannot be determined

313. If $1,\omega,\omega^2$ are the cube roots of unity, then $\begin{vmatrix} 1+\omega & \omega^2 & -\omega \\ 1+\omega^2 & \omega & -\omega^2 \\ \omega^2+\omega & \omega & -\omega^2 \end{vmatrix}$
a) ω^2 b) 0 c) 1 d) ω

314. The value of $1+\sum_{k=0}^{14}\left\{\cos\frac{(2k+1)}{15}\pi+i\sin\frac{(2k+1)}{15}\pi\right\}$ is
a) 0 b) -1 c) 1 d) i

315. If $az_1+bz_2+cz_3=0$ for complex numbers z_1,z_2,z_3 and real numbers a,b,c, then z_1,z_2,z_3 lie on a
a) Straight line
b) Circle
c) Depends on the choice of a,b,c
d) None of these

316. If $\omega(\neq 1)$ be a cube root of unity and $(1+\omega)^7=A+B\omega$, then A and B are respectively the numbers:
a) 0, 1 b) 1, 1 c) 1, 0 d) -1, 1

317. If $x\in R$, then the expression 9^x-3^x+1 assumes
a) All real values
b) All real values greater than 0
c) All real values greater than 3/4
d) All real values greater than 1/4

318. The locus represented by the equation $|z-1|=|z-i|$ is
a) A circle of radius 1 b) An ellipse with foci at 1 and $-i$
c) A line through the origin d) A circle on the line joining 1 and $-i$ as diameter

319. The number of real roots of the equation $x^4 + \sqrt{x^4 + 20} = 20$ is

a) 4 b) 2 c) 0 d) 1

320. If the roots of the quadratic equation $x^2 - 4x - \log_3 a = 0$ are real, then the least value of a is

a) 81 b) 1/81 c) 1/64 d) None of these

321. For the equation $\frac{1}{x+a} - \frac{1}{x+b} = \frac{1}{x+c}$, if the product of the roots is zero, then the sum of the roots is

a) 0 b) $\frac{2ab}{b+c}$ c) $\frac{2bc}{b+c}$ d) $-\frac{2bc}{b+c}$

322. If $\frac{3x+2}{(x+1)(2x^2+3)} = \frac{A}{x+1} + \frac{Bx+C}{2x^2+3}$, then $A + C - B$ is equal to

a) 0 b) 2 c) 3 d) 5

323. If $z^2 + (p + iq)z + (r + is) = 0$, where, p, q, r, s are non-zero has real roots, then

a) $pqs = s^2 + q^2 r$ b) $pqr = r^2 + p^2 s$ c) $prs = q^2 + r^2 p$ d) $qrs = p^2 + s^2 q$

324. The values of p for which the difference between the roots of the equation $x^2 + px + 8 = 0$ is 2, are

a) ± 2 b) ± 4 c) ± 6 d) ± 8

325. If $\cos \alpha + 2 \cos \beta + 3 \cos \gamma = \sin \alpha + 2 \sin \beta + 3 \sin \gamma = 0$ and $\alpha + \beta + \gamma = n\pi$, then $\sin 3\alpha + 8 \sin 3\beta +$ $27 \sin 3\gamma =$

a) 0 b) 3 c) 8 d) -18

326. If $\frac{3}{2} + \frac{7}{2} i$ is a solution of the equation $ax^2 - 6x + b = 0$, where a and b are real numbers, then the value of $a + b$ is equal to

a) 10 b) 22 c) 30 d) 31

327. Re $(z^2) = 1$ is represented by

a) The circle $x^2 + y^2 = 1$ b) The hyperbola $x^2 - y^2 = 1$

c) Parabola or a circle d) All of the above

328. If $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are the roots of the equation $x^4 + (2 - \sqrt{3})x^2 + 2 + \sqrt{3} = 0$, then the value of $(1 - \alpha_1)(1 -$ $\alpha 2 1 - \alpha 3 (1 - \alpha 4)$ is

a) 1 b) 4 c) $2 + \sqrt{3}$ d) 5

329. If $x = 8 + 3\sqrt{7}$ and $xy = 1$, then the value of $\frac{1}{x^2} + \frac{1}{y^2}$ is

a) 254 b) 192 c) 292 d) 66

330. The complex numbers z having positive argument and satisfying $|2 - 3i| < |z|$, is

a) $\frac{12}{5} + \frac{16}{5} i$ b) $\frac{4}{5} + \frac{6}{5} i$ c) $\frac{6}{5} - \frac{5}{2} i$ d) None of these

331. Let S denote the set of all values of S for which the equation $2x^2 - 2(2a + 1)x + a(a + 1) = 0$ has one root less than a and other root greater than a, then S equals

a) $(0, 1)$ b) $(-1, 0)$ c) $(0, 1/2)$ d) None of these

332. If $a \le 0$, then one of the roots of $x^2 - 2a|x - a| - 3a^2 = 0$ is

a) $(-1 + \sqrt{6})a$ b) $(\sqrt{6} - 1)a$ c) a d) None of these

333. If $A(z_1), B(z_2)$ and $C(z_3)$ be the vertices of a triangle ABC in which $\angle ABC = \frac{\pi}{4}$ and $\frac{AB}{BC} = \sqrt{2}$, then the value of z_2 is equal to

a) $z_3 + i(z_1 + z_3)$ b) $z_3 - i(z_1 - z_3)$ c) $z_3 + i(z_1 - z_3)$ d) None of these

334. When $\frac{z+i}{z+2}$ is purely imaginary, the locus described by the point z in the argand diagram is a

a) Circle of radius $\frac{\sqrt{5}}{2}$ b) Circle of radius $\frac{5}{4}$ c) Straight line d) Parabola

335. Area of the triangle formed by 3 complex numbers $1 + i, i - 1, 2i$ in the Argand plane is

a) 1/2 b) 1 c) $\sqrt{2}$ d) 2

336. The value of $x^4 + 9x^3 + 35x^2 - x + 4$ for $x = -5 + 2\sqrt{-4}$ is

a) 0 b) -160 c) 160 d) -164

337. The locus of the point z satisfying $\arg\left(\frac{z-1}{z+1}\right) = k$, where k is non-zero real, is

a) A circle with centre on y-axis

b) A circle with centre on x-axis
c) A straight line parallel to x-axis
d) A straight line making an angle of 60° with the x-axis

338. The product of the real roots of the equation $|2x+3|^2 - 3|2x+3| + 2 = 0$, is
a) 5/4 b) 5/2 c) 5 d) 2

339. If α, β be the roots of the quadratic equation $x^2 + x + 1 = 0$, then the equation whose roots are α^{19}, β^7 is
a) $x^2 - x + 1 = 0$ b) $x^2 - x - 1 = 0$ c) $x^2 + x - 1 = 0$ d) $x^2 + x + 1 = 0$

340. In the argand plane, the complex number $z = 4 - 3i$ is turned in the clockwise sense through 180° and stretched three times. The complex number represented by the new number is
a) $12 + 9i$ b) $12 - 9i$ c) $-12 - 9i$ d) $-12 + 9i$

341. If A is the A.M. of the roots of the equation $x^2 - 2ax + b = 0$ and G is the G.M. of the roots of the equation $x^2 - 2bx + a^2 = 0$, then
a) $A > G$ b) $A \neq G$ c) $A = G$ d) None of these

342. The maximum value of $|z|$ where z satisfies the condition $\left|z + \frac{2}{z}\right| = 2$, is
a) $\sqrt{3} - 1$ b) $\sqrt{3} + 1$ c) $\sqrt{3}$ d) $\sqrt{2} + \sqrt{3}$

343. Let α, β are the roots of equation $2x^2 - (p+1)x + (p-1) = 0$. If $\alpha - \beta = \alpha\beta$, then what is the value of p?
a) 1 b) 2 c) 3 d) -2

344. Let $a = e^{i\frac{2\pi}{3}}$. Then, the equation whose roots are $a + a^{-2}$ and $a^2 + a^{-4}$ is
a) $x^2 - 2x + 4 + 0$ b) $x^2 - x + 1 = 0$ c) $x^2 + x + 4 = 0$ d) $x^2 + 2x + 4 = 0$

345. The coefficients of x in the quadratic equation $x^2 + bx + c = 0$ was taken as 17 in place of 13, its roots were found to be -2 and -15. The correct root of the original equation are
a) -10, -3 b) -9, -4 c) -8, -5 d) -7, -6

346. If x, y, z are real and distinct, then $u = x^2 + 4y^2 + 9z^2 - 6yz - 3zx - 2xy$ is always
a) Non-negative b) Non-positive c) Zero d) None of these

347. If α, β are the roots of the equation $6x^2 - 5x + 1 = 0$, then the value of $\tan^{-1}\alpha + \tan^{-1}\beta$ is
a) 0 b) $\pi/4$ c) 1 d) $\pi/2$

348. If $a^x = b^y = c^z = d^w$, then the value of $x\left(\frac{1}{y} + \frac{1}{z} + \frac{1}{w}\right)$ is
a) $\log_a(abc)$ b) $\log_a(bcd)$ c) $\log_b(cda)$ d) $\log_e(dab)$

349. The locus of z satisfying the inequality $\log_{1/3}|z+1| > \log_{1/3}|z-1|$ is
a) Re $(z) < 0$ b) Re $(z) > 0$ c) Im $(z) < 0$ d) None of these

350. If a, b, c are real numbers in G.P. such that a and c are positive, then the roots of the equation $ax^2 + bx + c = 0$
a) Are real and are in ratio $b : ac$
b) Are real
c) Are imaginary and are in ratio $1 : \omega$, where ω is a complex cube root of unity
d) Are imaginary and are in ratio $-1 : \omega$

351. $\tan\alpha$ and $\tan\beta$ are the roots of the equation $x^2 + ax + b = 0$, then the value of $\sin^2(\alpha+\beta) + a\sin(\alpha+$ $\beta\cos\ \alpha+\beta+b\cos 2(\alpha+\beta)$ is equal to
a) ab b) b c) $\frac{a}{b}$ d) a

352. If ω is an imaginary cube root of unity, then the value of $\sin\left\{(\omega^{10} + \omega^{23})\pi - \frac{\pi}{4}\right\}$ is
a) $\frac{1}{\sqrt{2}}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{-1}{\sqrt{3}}$ d) $-\frac{\sqrt{3}}{2}$

353. The equation formed by decreasing each root of $ax^2 + bx + c = 0$ by 1 is $2x^2 + 8x + 2 = 0$, then
a) $a = -b$ b) $b = -c$ c) $c = -a$ d) $b = a + c$

354. If $|z - 2| = \min\{|z-1|, |z-5|\}$, where z is a complex number, then
a) Re $(z) = \frac{3}{2}$ b) Re $(z) = \frac{7}{2}$ c) Re $(z) \in \left\{\frac{3}{2}, \frac{7}{2}\right\}$ d) None of these

355. If a, b are odd integers, then the roots of the equation $2\ ax^2 + (2\ a + b)x + b = 0, a \neq 0$ are

a) Rational b) Irrational c) Non-real d) None of these

356. If α, β are the roots of the equation $lx^2 + mx + n = 0$, then the equation whose roots are $\alpha^3\beta$ and $\alpha\beta^3$, is

a) $l^4x^2 - nl(m^2 - 2nl)x + n^4 = 0$
b) $l^4x^2 + nl(m^2 - 2nl)x + n^4 = 0$
c) $l^4x^2 + nl(m^2 - 2nl)x - n^4 = 0$
d) $l^4x^2 - nl(m^2 + 2nl)x + n^4 = 0$

357. If α, β, γ are the roots of the equation $x^3 - 7x + 7 = 0$, then $\frac{1}{\alpha^4} + \frac{1}{\beta^4} + \frac{1}{\gamma^4}$ is

a) 7/3 b) 3/7 c) 4/7 d) 7/4

358. If $x^2 - 2r\ a_r x + r = 0; r = 1, 2, 3$ are three quadratic equations of which each pair has exactly one root common, then the number of solutions of the triplet (a_1, a_2, a_3) is

a) 1 b) 2 c) 9 d) 27

359. Let $z = \frac{11-3i}{1+i}$. If α is a real number such that $z - i\alpha$ is real, then the value of α is

a) 4 b) -4 c) 7 d) -7

360. The coefficient of x in the equation $x^2 + px + q = 0$ was taken as 17 in place of 13 its roots were found to be -2 and -15. The roots of the original equation are

a) 3 ,10 b) $-3, -10$ c) $-5, -8$ d) None of these

361. If $z^2 + z + 1 = 0$, where z is a complex number, then the value of $\left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + \left(z^3 +\right.$
1z3+...+z6+1z62 is

a) 6 b) 12 c) 18 d) 24

362. The set of possible values of a for which $x^2 - (a^2 - 5a + 5)x + (2a^2 - 3a - 4) = 0$
has roots whose sum and product are both less than 1 is

a) $(-1, 5/2)$ b) $(1, 4)$ c) $[1, 5/2]$ d) $(1, 5/2)$

363. If $(x - 2)$ is a common factor of the expressions $x^2 + ax + b$ and $x^2 + cx + d$, then $\frac{b-d}{c-a}$ is equal to

a) -2 b) -1 c) 1 d) 2

364. The value of $\log_2 20 \log_2 80 - \log_2 5 \log_2 320$ is equal to

a) 5 b) 6 c) 7 d) 8

365. The greatest number among $\sqrt[3]{9}, \sqrt[4]{11}, \sqrt[6]{17}$ is

a) $\sqrt[3]{9}$
b) $\sqrt[4]{11}$
c) $\sqrt[6]{17}$
d) Cannot be determined

366. The value of $\left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)^{1000}$ is

a) ω^3 b) ω^2 c) $\omega^3 - \omega$ d) ω

367. If each pair of the equation $x^2 + ax + b = 0, x^2 + bx + c = 0$ and $x^2 + cx + a = 0$ has a common root, then product of all common roots is

a) $\sqrt{abc}$ b) $2\sqrt{abc}$ c) $\sqrt{ab + bc + ca}$ d) $2\sqrt{ab + bc + ca}$

368. The value of $\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11} + i\cos\frac{2k\pi}{11}\right)$ is

a) 1 b) -1 c) $-i$ d) i

369. If z is a complex number such that $z \neq 0$ and Re (z) = 0, then

a) Re $(z^2) = 0$ b) Im $(z^2) = 0$ c) Re $(z^2) =$ Im (z^2) d) None of these

370. If z_1, z_2, z_3, z_4 are four complex numbers represented by the vertices of a quadrilateral taken in order such that $z_1 - z_4 = z_2 - z_3$ and $\arg\left(\frac{z_4 - z_1}{z_2 - z_1}\right) = \frac{\pi}{2}$, then the quadrilateral is

a) A square
b) A rectangle
c) A rhombus
d) A cyclic quadrilateral

371. The real root of the equation $x^3 - 6x + 9 = 0$ is

a) -6 b) -9 c) 6 d) -3

372. The value of the expression

$1. (2 - \omega)(2 - \omega^2) + 2(3 - \omega)(3 - \omega^2) + \ldots\ldots + \ldots + (n - 1)(n - \omega)(n - \omega^2)$

Where ω is an imaginary cube root of unit is

a) $(n-1)n(n^2+3n+4)/4$
b) $(n-1)n(n^2+3n+4)/2$
c) $(n+1)n(n^2+3n+4)/2$
d) None of the above

373. If $|z_1|=|z_2|=|z_3|$ and $z_1+z_2+z_3=0$, then z_1, z_2, z_3 are vertices of
a) A right angled triangle
b) An equilateral triangle
c) Isosceles triangle
d) Scalene triangle

374. If x is real, then the value of $\frac{x+2}{2x^2+3x+6}$ is equal to
a) $\left(\frac{1}{13}, \frac{1}{3}\right)$
b) $\left(-\frac{1}{13}, \frac{1}{3}\right)$
c) $\left(-\frac{1}{3}, \frac{1}{13}\right)$
d) None of these

375. Which of the following is correct?
a) $1+i>2-i$
b) $2+i>1+i$
c) $2-i>1+i$
d) None of these

376. If n is an integer which leaves remainder one when divided by three, then $\left(1+\sqrt{3}i\right)^n+\left(1-\sqrt{3}i\right)^n$ equals
a) -2^{n+1}
b) 2^{n+1}
c) $-(-2)^n$
d) -2^n

377. If $\left(\frac{\sqrt{3}/2+(1/2)\,i}{\sqrt{3}/2-(1/2)\,i}\right)^{120}=p+iq$, then
a) $p=\cos 20°, q=\sin 20°$
b) $p=-\cos 20°, q=-\sin 20°$
c) $p=\cos 20°, q=-\sin 20°$
d) $p=1, q=0$

378. The roots of $|x-2|^2+|x-2|-6=0$ are
a) 4, 2
b) 0, 4
c) -1, 3
d) 5, 1

379. The greatest negative integer satisfying $x^2-4x-77<0$ and $x^2>4$, is
a) -4
b) -6
c) -7
d) None of these

380. The values of x and y satisfying the equation $\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i$ are
a) $x=-1, y=3$
b) $x=3, y=-1$
c) $x=0, y=1$
d) $x=1, y=0$

381. If α and β are roots of the quadratic equation $x^2+4x+3=0$, then the equation whose roots are $2\alpha+\beta$ and $\alpha+2\beta$ is
a) $x^2-12x+35=0$
b) $x^2+12x-33=0$
c) $x^2-12x-33=0$
d) $x^2+12x+35=0$

382. In a triangle PQR, $\angle R=\frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2+bx+c=0, a\neq 0$, then
a) $b=a+c$
b) $b=c$
c) $c=a+b$
d) $a=b+c$

383. The set of values of $'a'$ for which
$x^2+ax+\sin^{-1}(x^2-4x+5)+\cos^{-1}(x^2-4x+5=0)$
has at least one real root is given by
a) $\left(-\infty, -\sqrt{2}\,\pi\right]\cup[\sqrt{2}\,\pi, \infty)$
b) $(-\infty, -\sqrt{2}\,\pi)\cup(\sqrt{2}\,\pi, \infty)$
c) R
d) None of these

384. If z lies on the circle $|z-1|=1$, then $\frac{z-2}{z}$ is
a) Purely real
b) Purely imaginary
c) Positive real
d) None of these

385. If α, β be the roots of $x^2-a(x-1)+b=0$, then the value of $\frac{1}{\alpha^2-a\alpha}+\frac{1}{\beta^2+a\beta}+\frac{2}{a+b}$ is
a) $\frac{4}{a+b}$
b) $\frac{1}{a+b}$
c) 0
d) -1

386. If $x=\log_b a, y=\log_c b, z=\log_a c$, then xyz is
a) 0
b) 1
c) 3
d) None of the above

387. If $\log_4 2+\log_4 4+\log_4 x+\log_4 16=6$, then the value of x is
a) 64
b) 4
c) 8
d) 32

388. If $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$, the modulus and argument form of $(1 + \cos 2\alpha) + i \sin 2\alpha$ is

a) $-2\cos\alpha\{\cos(\pi + \alpha) + i\sin(\pi + \alpha)\}$

b) $2\cos\alpha\{\cos\alpha + i\sin\alpha\}$

c) $2\cos\alpha\{\cos(-\alpha) + i\sin(-\alpha)\}$

d) $-2\cos\alpha\{\cos(\pi - \alpha) + i\sin(\pi - \alpha)\}$

389. Let α and β be the roots of $x^2 - 2x\cos\emptyset + 1 = 0$. Then the equation whose roots are α^n, β^n is

a) $x^2 - 2x\cos n\emptyset - 1 = 0$ b) $x^2 - 2x\cos n\emptyset + 1 = 0$

c) $x^2 - 2x\sin n\emptyset + 1 = 0$ d) $x^2 + 2x\sin n\emptyset - 1 = 0$

390. $\sqrt{1 - c^2} = nc - 1$ and $z = e^{i\theta}$, then $\frac{c}{2n}(1 + nz\left(1 + \frac{n}{z}\right)$ is equal to

a) $1 - c\cos\theta$ b) $1 + 2c\cos\theta$ c) $1 + c\cos\theta$ d) $1 - 2c\cos\theta$

391. The number of solutions of the system of equations Re $(z^2) = 0$; $|z| = 2$ is

a) 4 b) 3 c) 2 d) 1

392. If $|z - 25i| \leq 15$, then
$|\text{maximum amp }(z) - \text{minimum amp }(z)| =$

a) $\cos^{-1}\left(\frac{3}{5}\right)$ b) $\pi - 2\cos^{-1}\left(-\frac{3}{5}\right)$ c) $\frac{\pi}{2} + \cos^{-1}\left(\frac{3}{5}\right)$ d) $\sin^{-1}\left(\frac{3}{5}\right) - \cos^{-1}\left(\frac{3}{5}\right)$

393. If α, β be the roots of the equation $x^2 - px + q = 0$ and α_1, β_1 be the roots of the equation $x^2 - qx + p = 0$, then the equation whose roots are $\frac{1}{\alpha_1\beta} + \frac{1}{\alpha\beta_1}$ and $\frac{1}{\alpha\alpha_1} + \frac{1}{\beta\beta_1}$, is

a) $pqx^2 - pqx + p^2 + q^2 + 4qp = 0$

b) $p^2q^2x^2 - p^2q^2x + p^3 + q^3 - 4pq = 0$

c) $p^3q^3x^2 - p^3q^3x + p^4 + q^4 - 4p^2q^2 = 0$

d) $(p + q)x^2 - (p + q)x + p^2 + q^2 = 0$

394. If $iz^4 + 1 = 0$, then z can be take the value

a) $\frac{1 + i}{\sqrt{2}}$ b) $\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}$ c) $\frac{1}{4i}$ d) i

395. If P is the point in the Agrand diagram corresponding to the complex number $\sqrt{3} + i$
And if OPQ is an isosceles right angled triangle, right angled at $'O'$ then Q represents the complex number

a) $-1i\sqrt{3}$ or $1 - i\sqrt{3}$ b) $1 \pm i\sqrt{3}$ c) $\sqrt{3} - i$ or $1 - i\sqrt{3}$ d) $-1 \pm i\sqrt{3}$

396. The solution of equation $|z| - z = 1 + 2i$ is

a) $\frac{3}{2} + 2i$ b) $\frac{3}{2} - 2i$ c) $3 - 2i$ d) None of these

397. If $\alpha + \beta = 4$ and $\alpha^3 + \beta^3 = 44$, then α, β are the roots of the equation

a) $2x^2 - 7x + 6 = 0$ b) $3x^2 + 9x + 11 = 0$ c) $9x^2 - 27x + 20 = 0$ d) $3x^2 - 12x + 5 = 0$

398. The number of non-zero integer solutions of the equation $|1 - i|^x = 2^x$ is

a) Infinite b) 1 c) 2 d) None of these

399. If α and β are the roots of the equation $x^2 - 7x + 1 = 0$, then the value of $\frac{1}{(\alpha-7)^2} + \frac{1}{(\beta-7)^2}$ is

a) 45 b) 47 c) 49 d) 50

400. If the equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root, then it is equal to

a) $\frac{p - p'}{q - q'}$ b) $\frac{p + p'}{q + q'}$ c) $\frac{q' - q}{p - p'}$ d) $\frac{q + q'}{p + p'}$

401. The area of the triangle whose vertices are i, ω and ω^2, where $i = \sqrt{-1}$ and ω, ω^2 are complex cube roots of unity, is

a) $\frac{3\sqrt{3}}{2}$ sq. units b) $\frac{3\sqrt{3}}{4}$ sq. units c) 0 d) $\frac{\sqrt{3}}{4}$

402. If n is a positive integer greater than unity and z is a complex number satisfying the equation $z^n = (z + 1)^n$, then

a) Im $(z) < 0$ b) Im $(z) > 0$ c) Im $(z) = 0$ d) None of these

403. The complex numbers z_1, z_2, z_3, z_4 taken in that order in the Argand plane represent the vertices of a

parallelogram iff

a) $z_1 + z_4 = z_2 + z_3$ b) $z_1 + z_3 = z_2 + z_4$ c) $z_1 + z_2 = z_3 + z_4$ d) None of these

404. If α, β are the roots of the equation $x^2 - 2x\cos\phi + 1 = 0$, then the equation whose roots are α^n, β^n, is

a) $x^2 - 2x\cos\ n\phi - 1 = 0$

b) $x^2 - 2x\cos\ n\phi + 1 = 0$

c) $x^2 - 2x\sin n\phi + 1 = 0$

d) $x^2 + 2x\sin n\phi - 1 = 0$

405. If $a < c < b$, then the roots of the equation $(a-b)^2x^2 + 2(a+b-2c)x + 1 = 0$ are

a) Imaginary

b) Real

c) One real and one imaginary

d) Equal and imaginary

406. If the equations $ax^2 + bx + c = 0$ and $x^3 + 3\,x^2 + 3\,x + 2 = 0$ have to common roots, then

a) $a = b \neq c$ b) $a = -b = c$ c) $a = b = c$ d) None of these

407. If roots of the equation $(a-b)x^2 + (c-a)x + (b-c) = 0$ are equal, then a, b, c are in

a) AP b) HP c) GP d) None of these

408. The smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$ is

a) 3 b) 2 c) 4 d) None of these

409. If $x^2 + px + q = 0$ is the quadratic equation whose roots are $a - 2$ and $b - 2$ where a, b are the roots of $x^2 - 3x + 1 = 0$, then

a) $p = 1, q = 5$ b) $p = 1, q = -5$ c) $p = -1, q = 1$ d) $p = 1, q = -1$

410. If $\sec\alpha$ and $\tan\alpha$ are the roots of $ax^2 + bx + c = 0$, then

a) $a^2 - b^2 + 2ac = 0$

b) $a^3 + b^3 + c^3 - 2abc = 0$

c) $a^4 + 4ab^2c = b^4$

d) None of these

411. The points represents the complex numbers z, for which $|z-a|^2 + |z+a|^2 = b^2$ lie on

a) A straight line b) A circle c) A parabola d) A hyperbola

412. The solution of $\log_{99}\{\log_2(\log_3 x)\} = 0$ is

a) 4 b) 9 c) 44 d) 99

413. If the roots of the equation $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4c$ is

a) -1 b) 0 c) 1 d) 2

414. For $a \neq b$, if the equation $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ have a common root, then the value of $a + b$ equals

a) -1 b) 0 c) 1 d) 2

415. Let $f(x)$ be a quadratic expression which is positive for all real x and $g(x) = f(x) + f'(x) + f''(x)$, then for any real x,

a) $g(x) < 0$ b) $g(x) > 0$ c) $g(x) = 0$ d) $g(x) \geq 0$

416. If $a = \cos\alpha + i\sin\alpha$, $b = \cos\beta + i\sin\beta$, $c = \cos\gamma + i\sin\gamma$ and $b/c + c/a + a/b = 1$, then $\cos(\beta-\gamma) + \cos\gamma - \alpha + \cos(\alpha - \beta)$ is equal to

a) 0 b) 1 c) -1 d) None of these

417. The value of $\frac{\cos 30° + i\sin 30°}{\cos 60° - i\sin 60°}$ is equal to

a) i b) $-i$ c) $\dfrac{1+\sqrt{3}i}{2}$ d) $\dfrac{1-\sqrt{3}i}{2}$

418. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then $\frac{\alpha}{a\beta+b} + \frac{\beta}{a\alpha+b}$ is equal to

a) $\dfrac{2}{a}$ b) $\dfrac{2}{b}$ c) $\dfrac{2}{c}$ d) $-\dfrac{2}{a}$

419. If the difference of the roots of the equation $x^2 - bx + c = 0$ be 1, then

a) $b^2 - 4c - 1 = 0$ b) $b^2 - 4c = 0$ c) $b^2 - 4c + 1 = 0$ d) $b^2 + 4c - 1 = 0$

420. The graph of the function $y = 16x^2 + 8(a+5)x - 7\,a - 5$ is strictly above the x-axis, then $'a'$ must satisfy the inequality

a) $-15 < a < -2$ b) $-2 < a < -1$ c) $5 < a < 7$ d) None of these

421. If α, β are the roots of $x^2 - 3x + a = 0, a \in R$ and $\alpha < 1 < \beta$, then

a) $a \in (-\infty, 2)$ b) $a \in (-\infty, 9/4)$ c) $a \in (2, 9/4]$ d) None of these

422. One of the values of $\left(\frac{1+i}{\sqrt{2}}\right)^{2/3}$ is

a) $\sqrt{3} + i$ b) $-i$ c) i d) $-\sqrt{3} + i$

423. If the equation $x^2 + px + q = 0$ has roots u and v where p, q are non-zero constants. Then,

a) $qx^2 + px + 1 = 0$ has roots $\frac{1}{u}$ and $\frac{1}{v}$

b) $(x-p)(x+q) = 0$ has roots $u + v$ and uv

c) $x^2 + p^2x + q^2 = 0$ has roots u^2 and v^2

d) $x^2 + qx + p = 0$ has roots $\frac{u}{v}$ and $\frac{v}{u}$

424. If a, b, c are in GP, then the equation $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in

a) AP b) HP c) GP d) None of these

425. If ω is a complex cube root of unity, then the equation $|z - \omega|^2 + |z - \omega^2|^2 = \lambda$ will represent a circle, if

a) $\lambda \in (0,3/2)$ b) $\lambda \in [3/2, \infty)$ c) $\lambda \in (0, 3)$ d) $\lambda \in [3, \infty)$

426. The real roots of the equation $x^{2/3} - x^{1/3} - 2 = 0$ are

a) 1, 8 b) -1, -8 c) -1, 8 d) 1, -8

427. Let $A(z_1), B(z_2), C(z_3)$ be the vertices of an equilateral triangle ABC in the Argand plne, then the number $\left(\frac{z_2 - z_3}{2z_1 - z_2 - z_3}\right)$ is

a) Purely real

b) Purely imaginary

c) A complex number with non-zero real and imaginary parts

d) None of these

428. If $\bar{z}$ be the conjugate of the complex number z, then which of the following relations is false?

a) $|z| = |\bar{z}|$ b) $z.\bar{z} = |\bar{z}|^2$ c) $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$ d) $\arg(z) = \arg(\bar{z})$

429. If $(a_1 + ib_1)(a_2 + ib_2) \ldots (a_n + ib_n) = A + iB$, then $\sum_{i=1}^{n} \tan^{-1}\left(\frac{b_i}{a_i}\right)$ is equal to

a) $\frac{B}{A}$ b) $\tan\left(\frac{B}{A}\right)$ c) $\tan^{-1}\left(\frac{\mathrm{B}}{\mathrm{A}}\right)$ d) $\tan^{-1}\left(\frac{A}{B}\right)$

430. The values of $'a'$ for which $(a^2 - 1)x^2 + 2(a-1)x + 2$ is positive for any x, are

a) $a \geq 1$ b) $a \leq 1$ c) $a > -3$ d) $a < -3$ or $a > 1$

431. The roots of the equation $x^4 - 2x^3 + x = 380$ are

a) $5, -4, \frac{1 \pm 5\sqrt{-3}}{2}$ b) $-5, 4, \frac{-1 \pm 5\sqrt{-3}}{2}$ c) $5, 4, \frac{-1 \pm 5\sqrt{-3}}{2}$ d) $-5, -4, \frac{1 \pm 5\sqrt{3}}{2}$

432. The value of $(2-\omega)(2-\omega^2)(2-\omega^{10})(2-\omega^{11})$, where ω is the complex cube root of unity, is

a) 49 b) 50 c) 48 d) 47

433. The number of solutions for the equations $|z-1| = |z-2| = |z-i|$ is

a) One solution b) 3 solutions c) 2 solutions d) No solution

434. If α, β and γ are the roots of $x^3 + 8 = 0$, then the equation whose roots are α^2, β^2 and γ^2 is

a) $x^3 - 8 = 0$ b) $x^3 - 16 = 0$ c) $x^3 + 64 = 0$ d) $x^3 - 64 = 0$

435. The quadratic equation whose roots are $\sin^2 18°$ and $\cos^2 36°$ is

a) $16x^2 - 12x + 1 = 0$ b) $16x^2 + 12x + 1 = 0$

c) $16x^2 - 12x - 1 = 0$ d) $16x^2 + 10x + 1 = 0$

436. If $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$, then

a) $\text{Re}\,(z) = 0$

b) $\text{IM}\,(z) = 0$

c) Re $(z) => 0$, Im $(z) > 0$

d) Re $(z) > 0$, Im $(z) < 0$

437. If one root of the equation $lx^2 + mx + n = 0$ is $\frac{9}{2}$ (l, m and n are positive integers) and $\frac{m}{4n} = \frac{1}{m}$, then $I + n$ is equal to

a) 80 b) 85 c) 90 d) 95

438. If $\frac{x^3}{(2x-1)(x+2)(x-3)} = A + \frac{B}{2x-1} + \frac{C}{x+2} + \frac{D}{x-3}$, then A is equal to

a) $\frac{1}{2}$ b) $-\frac{1}{50}$ c) $-\frac{8}{25}$ d) $\frac{27}{25}$

439. If α, β, γ are the roots of $x^3 + 4x + 1 = 0$, then the equation whose roots are $\frac{\alpha^2}{\beta+\gamma}, \frac{\beta^2}{\alpha+\gamma}, \frac{\gamma^2}{\alpha+\beta}$ is

a) $x^3 - 4x - 1 = 0$ b) $x^3 - 4x + 1 = 0$ c) $x^3 + 4x - 1 = 0$ d) $x^3 + 4x + 1 = 0$

440. The solution set of the equation $pqx^2 - (p+q)^2x + (p+q)^2 = 0$ is

a) $\left\{\frac{p}{q}, \frac{q}{p}\right\}$ b) $\left\{pq, \frac{p}{q}\right\}$ c) $\left\{\frac{q}{p}, pq\right\}$ d) $\left\{\frac{p+q}{p}, \frac{p+q}{q}\right\}$

441. The system of equation $|z + 1 - i| = \sqrt{2}$ and $|z| = 3$ has

a) No solution b) One solution c) Two solutions d) None of these

442. If $x = a + b, y + a\alpha + b\beta$ and $z = a\beta + b\alpha$, where α and β are complex cube roots of unity, then xyz is equal to

a) $a^2 + b^2$ b) $a^3 + b^3$ c) a^3b^3 d) $a^3 - b^3$

443. If α, β are the roots of equation $ax^2 + bx + c = 0$, then the value of the determinant

$\begin{vmatrix} 1 & \cos\,(\beta - \alpha) & \cos\alpha \\ \cos\,(\alpha - \beta) & 1 & \cos\beta \\ \cos\alpha & \cos\beta & 1 \end{vmatrix}$, is

a) $\sin(\alpha + \beta)$ b) $\sin\alpha \sin\beta$ c) $1 + \cos\,(\alpha + \beta)$ d) None of these

444. The least positive integer n for which $\left(\frac{1+i}{1-i}\right)^n$ is real, is

a) 2 b) 4 c) 8 d) None of these

445. Let $[x]$ denote the greatest integer less than or equal to x. Then, in $[0,3]$ the number of solutions of the equation $x^2 - 3x + [x] = 0$, is

a) 6 b) 4 c) 2 d) 0

446. If at least one root of $2x^2 + 3x + 5 = 0$ and $ax^2 + bx + c = 0, a, b, c \in N$ is common, then the maximum value of $a + b + c$ is

a) 10 b) 0 c) Does not exist d) None of these

447. If $x = \sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots \infty}}}$, then x is equal to

a) $\frac{1+\sqrt{5}}{2}$ b) $\frac{1-\sqrt{5}}{2}$ c) $\frac{1\pm\sqrt{5}}{2}$ d) None of these

448. If $a > 0$ and the equation $|z - a^2| + |z - 2a| = 3$ represents an ellipse, then a belongs to the interval

a) $(1, 3)$ b) $(\sqrt{2}, \sqrt{3})$ c) $(0, 3)$ d) $(1, \sqrt{3})$

449. If x is real, the function $\frac{(x-a)(x-b)}{(x-c)}$ will assume all real values, provided

a) $a > b > c$ b) $a \leq b \leq c$ c) $a > c > b$ d) $a \leq c \leq b$

450. The value of the expression

$1\,.\,(2-\omega)(2-\omega^2) + 2.\,(3-\omega)(3-\omega^2) + \dots\dots + (n-1).\,(n-\omega)(n-\omega^2)$

Where ω is an imaginary cube root of unity, is

a) $\frac{1}{2}(n-1)\,n\,(n^2 + 3n + 4)$ b) $\frac{1}{4}(n-1)\,n\,(n^2 + 3n + 4)$

c) $\frac{1}{2}(n+1)\,n\,(n^2 + 3n + 4)$ d) $\frac{1}{4}(n+1)\,n\,(n^2 + 3n + 4)$

451. The points representing cube roots of unity

a) Are collinear

b) Lie on a circle of radius $\sqrt{3}$

c) From an equilateral triangle

d) None of these

452. If the equations $ax^2 + bc + c = 0$ and $2x^2 + 3x + 4 = 0$ have a common root, then $a : b : c$

a) 2 : 3 : 4 b) 1 : 2 : 3 c) 4 : 3 : 2 d) None of these

453. Consider the following statements:

1. The equation $x^2 - cx + d = 0$ and $x^2 - ax + b = 0$ have common root and second equation has equal roots if $ac = 2(b + d)$.

2. If α is a root of the equation $4x^2 + 2x - 1 = 0$, then the other root is $4\alpha^3 - 3\alpha$.

3. The expression $(x - 1)(x - 3)(x - 4)(x - 6) + 10$ is positive for all real values of x.

Which of these is/are correct?

a) Only (3) b) Only (2) c) All of these d) None of these

454. The equation$x^2 - 3|x| + 2 = 0$ has

a) No real roots b) One real root c) Two real roots d) Four real roots

455. The solution set of the inequation $0 < |3x + 1| < \frac{1}{3}$, is

a) $(-4/9, -2/9)$

b) $[-4/9, -2/9]$

c) $(-4/9, -2/9) - \{-1/3\}$

d) $[-4/9, -2/9] - \{-1/3\}$

456. The solution set of the equation $x^{\log_x (1-x)^2} = 9$ is

a) $\{-2, 4\}$ b) $\{4\}$ c) $\{0, -2, 4\}$ d) None of these

457. If $x^2 + ax + 1$ is a factor of $ax^3 + bx + c$, then

a) $b + a + a^2 = 0, a = c$ b) $b - a + a^2 = 0, a = c$ c) $b + a - a^2 = 0, a = 0$ d) None of these

458. If the complex numbers z_1, z_2 and the origin form an equilateral triangle, then $z_1^2 + z_2^2$ is equal to

a) $z_1 z_2$ b) $z_1 \bar{z}_2$ c) $\bar{z}_2 z_1$ d) $|z_1|^2 = |z_2|^2$

459. If two equations $x^2 + a^2 = 1 - 2ax$ and $x^2 + b^2 = 1 - 2bx$ have only one common root, then

a) $a - b = 1$ b) $a - b = 1$ c) $a - b = 2$ d) $|a - b| = 1$

460. If α, β are the roots of $x^2 - ax + b = 0$ and if $\alpha^n + \beta^n = V_n$, then

a) $V_{n+1} = aV_n + bV_{n-1}$ b) $V_{n+1} = aV_n + aV_{n-1}$

c) $V_{n+1} = aV_n - bV_{n-1}$ d) $V_{n+1} = aV_{n-1} - bV_n$

461. $z^2 + \alpha z + \beta = 0$($\alpha, \beta$ are complex numbers) has a real root, then

a) $(\alpha + \bar{\alpha})(\alpha\bar{\beta} + \bar{\alpha}\beta) + (\beta - \bar{\beta})^2 = 0$

b) $(\alpha - \bar{\alpha})(\beta - \bar{\beta})^2 = 0$

c) $(\bar{\alpha} - \alpha)(\alpha\bar{\beta} - \bar{\alpha}\beta) = (\beta - \bar{\beta})^2$

d) None of these

462. If $2^x . 3^{x+4} = 7^x$, then x is equal to

a) $\dfrac{4 \log_e 3}{\log_e 7 - \log_e 6}$ b) $\dfrac{4 \log_e 3}{\log_e 6 - \log_e 7}$ c) $\dfrac{2 \log_e 3}{\log_e 7 - \log_e 6}$ d) $\dfrac{3 \log_e 4}{\log_e 6 - \log_e 7}$

463. If α, β and γ are the roots of the equation $x^3 - 8x + 8 = 0$, then $\sum \alpha^2$ and $\sum \frac{1}{\alpha\beta}$ are respectively

a) 0 and -16 b) 16 and 18 c) -16 and 0 d) 16 and 0

464. If x is real, then $\frac{x^2-2x+4}{x^2+2x+4}$ takes values in the interval

a) $\left[\frac{1}{3}, 3\right]$ b) $\left(\frac{1}{3}, 3\right)$ c) $(3, 3)$ d) $\left(-\frac{1}{3}, 3\right)$

465. The value of $2 + \frac{1}{2+\frac{1}{2+\dots\infty}}$ is

a) $1 - \sqrt{2}$ b) $1 + \sqrt{2}$ c) $1 \pm \sqrt{2}$ d) None of these

466. Let z_1 be a complex number with $|z_1| = 1$ and z_2 be any complex number, then $\left|\frac{z_1 - z_2}{1 - z_1 z_2}\right|$is equal to

a) 0 b) 1 c) -1 d) 2

467. If α, β are the roots of $x^2+px+q=0$ and also of $x^{2n}+p^nx^n+q^n=0$ and if $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ are the roots of $x^n+1+(x+1)^n=0$, then n is

a) An odd integer b) An even integer c) Any integer d) None of these

468. If $x=\left(\frac{1+i}{2}\right)$, (where $i=\sqrt{-1}$), then the expression $2x^4-2x^2+x+3$ equals

a) $3-\left(\frac{i}{2}\right)$ b) $3+\left(\frac{i}{2}\right)$ c) $\frac{(3+i)}{2}$ d) $\frac{(3-i)}{2}$

469. Let α, α^2 be the roots of $x^2+x+1=0$, then the equation whose roots are α^{31}, α^{62} , is

a) $x^2-x+1=0$ b) $x^2+x-1=0$ c) $x^2+x+1=0$ d) $x^{60}+x^{30}+1=0$

470. If one root of the equation $8x^2-6x-a-3=0$ is the square of the other, then the values of a are

a) $4, -24$ b) $4, 24$ c) $-4, -24$ d) $-4, 24$

471. If $x_n=\cos\frac{\pi}{3^n}+i\sin\frac{\pi}{3^n}$, then $x_1, x_2, x_3, \dots x_\infty$ is equal to

a) 1 b) -1 c) i d) $-i$

472. The centre of a regular hexagon is at the point$z=i$. If one of its vertices is at $2+i$, then the adjacent vertices of $2+i$ are at the points

a) $1\pm 2i$ b) $i+1\pm\sqrt{3}$ c) $2+i(1\pm\sqrt{3})$ d) $1+i(1\pm\sqrt{3})$

473. If the real part of $\frac{\bar{z}+2}{\bar{z}-1}$ is 4, then the locus of the point representing z in the complex plane is

a) a circle b) a parabola c) a hyperbola d) an ellipse

474. Given that $ax^2+bx+c=0$ has no real roots and $a+b+c<0$, then

a) $c=0$ b) $c>0$ c) $c<0$ d) $c=0$

475. If $2\sin^2\frac{\pi}{8}$ is a root of the equation $x^2+ax+b=0$, where a and b are rational numbers, then $a-b$ is equal to

a) $-\frac{5}{2}$ b) $-\frac{3}{2}$ c) $-\frac{1}{2}$ d) $\frac{1}{2}$

476. If α is a complex number satisfying the equation $\alpha^2+\alpha+1=0$, then α^{31} is equal to

a) α b) α^2 c) 1 d) i

477. If $z_r=\cos\left(\frac{\pi}{2^r}\right)+i\sin\left(\frac{\pi}{2^r}\right)$, then $z_1.z_2.z_3 \dots$ upto ∞ equals

a) -3 b) -2 c) 1 d) -1

478. If $\alpha_1, \alpha_2, \alpha_3$ respectively denote the moduli of the complex numbers $-i, \frac{1}{3}(1+i)$ and $-1+i$, then their increasing order is

a) $\alpha_1, \alpha_2, \alpha_3$ b) $\alpha_3, \alpha_2, \alpha_1$ c) $\alpha_2, \alpha_1, \alpha_3$ d) $\alpha_3, \alpha_1, \alpha_2$

479. The solution set of the inequation $\frac{2x+4}{x-1}\geq 5$, is

a) (1 ,3) b) (1 ,3] c) $(-\infty, 1)\cup[3,\infty)$ d) None of these

480. If the equation $\frac{a}{x-a}+\frac{b}{x-b}=1$ has roots equal in magnitude but opposite in sign, then the value of $a+b$ is

a) -1 b) 0 c) 1 d) None of these

481. The roots of the equation $x^3-3x-2=0$ are

a) -1, -1, 2 b) -1, 1, -2 c) -1, 2, -3 d) -1, -1, -2

482. If the sum of the squares of the roots of the equation $x^2-(\sin\alpha-2)x-(1+\sin\alpha)=0$ is least, then $\alpha=$

a) $\pi/4$ b) $\pi/3$ c) $\pi/2$ d) $\pi/6$

483. $\left(\frac{-1+\sqrt{-3}}{2}\right)^{100}+\left(\frac{-1-\sqrt{-3}}{2}\right)^{100}$ is equal to

a) 2 b) Zero c) -1 d) 1

484. The set of values of p for which the roots of the equation $3x^2+2x+p(p-1)=0$ are of opposite signs is

a) $(-\infty, 0)$ b) $(0, 1)$ c) $(1, \infty)$ d) $(0, \infty)$

485. The roots of $(x-a)(x-a-1)+(x-a-1)(x-a-2)+(x-a)(x-a-2)=0, a\in R$ are always

a) Equal b) Imaginary c) Real and distinct d) Rational and equal

486. If z is a complex number satisfying $z+z^{-1}=1$, then $z^n+z^{-n}, n\in N$ has the value

a) $2(-1)^n$, when n is a multiple of 3

b) $(-1)^n$ when n is not a multiple of 3
c) $(-1)^{n+1}$ when n is a multiple of 3
d) 0 when n is not a multiple of 3

487. If α, β, γ be the roots of $x^3 + a^3 = 0 (a \in R)$, then the number of equation(s) whose roots are $\left(\frac{\alpha}{\beta}\right)^2$ and $\left(\frac{\alpha}{\gamma}\right)^2$, is
a) 1 b) 2 c) 3 d) 6

488. If $\left|z - \frac{4}{z}\right| = 2$, then the maximum value of $|z|$ is equal to
a) $\sqrt{3} + 1$ b) $\sqrt{5} + 1$ c) 2 d) $2 + \sqrt{2}$

489. If $z\bar{z} = 0$, iff
a) Re $(z) = 0$ b) Im $(z) = 0$ c) $z = 0$ d) None of these

490. Let z, w be complex numbers such that $\bar{z} + \overline{iw} = 0$ and $\arg(zw) = \pi$. Then $\arg(z)$ equals
a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\frac{3\pi}{4}$ d) $\frac{5\pi}{4}$

491. If ω is an imaginary cube root of unity, n is a positive integer but not a multiple of 3, then the value of $1 + \omega^n + \omega^{2n}$ is
a) 3 b) $\omega + 2$ c) 0 d) $\omega^2 + 1$

492. The quadratic equations
$x^2 - 6x + a = 0$
And $x^2 - cx + 6 = 0$
Have one root in common. The other roots of the first and second equations are integers in the ratio 4: 3. Then the common root is
a) 2 b) 1 c) 4 d) 3

493. If $\left|\frac{z+i}{z-i}\right| = \sqrt{3}$, then radius of the circle is
a) $\frac{2}{\sqrt{21}}$ b) $\frac{1}{\sqrt{21}}$ c) $\sqrt{3}$ d) $\sqrt{21}$

494. If $\sin\alpha$ and $\cos\alpha$ are roots of the equation $px^2 + qx + r = 0$, then
a) $p^2 + q^2 + 2pr = 0$ b) $(p + r)^2 = q^2 - r^2$ c) $p^2 + q^2 - 2pr = 0$ d) $(p - r)^2 = q^2 + r^2$

495. The number of real roots of the equation $\frac{2x-3}{x-1} + 1 = \frac{6x^2-x-6}{x-1}$, is
a) 0 b) 1 c) 2 d) None of these

496. If $\alpha \neq 1$ is any nth root of unity, then $S = 1 + 3\alpha + 5\alpha^2$ ... upon n terms, is equal to
a) $\frac{2n}{1-\alpha}$ b) $-\frac{2n}{1-\alpha}$ c) $\frac{n}{1-\alpha}$ d) $-\frac{n}{1-\alpha}$

497. $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ will be real, if θ is
a) $2n\pi$ b) $n\pi + \frac{\pi}{2}$ c) $n\pi$ d) None of these

498. The number of positive integral roots of $x^4 + x^3 - 4x^2 + x + 1 = 0$ is
a) 0 b) 1 c) l2 d) 4

499. If the area of triangle on the argand place formed by the complex numbers $-z, iz, z - iz$ is 600 sq. unit, then $|z|$ is equal to
a) 10 b) 20 c) 30 d) 40

500. $\frac{3x^2+1}{x^2-6x+8}$ is equal to
a) $3 + \frac{49}{2(x-4)} - \frac{13}{2(x-2)}$ b) $\frac{49}{2(x-4)} - \frac{13}{2(x-2)}$
c) $\frac{-49}{2(x-4)} + \frac{13}{2(x-2)}$ d) $\frac{49}{2(x-4)} + \frac{13}{2(x-2)}$

501. If $x - c$ is a factor of order m of the polynomial $f(x)$ of degree $n (1 < m < n)$, then $x = c$ is a root of the polynomial

a) $f^m(x)$ b) $f^{m-1}(x)$ c) $f^n(x)$ d) None of these

502. The polynomial $(ax^2 + bx + c)(ax^2 - dx - c), ac \neq 0$ has
a) Four real roots
b) At least two real roots
c) At most two real roots
d) No real roots

503. The roots of the quadratic equation
$(a + b - 2c)x^2 - (2a - b - c)x + (a - 2b + c) = 0$ are
a) $a + b + c$ and $a - b + c$
b) $\frac{1}{2}$ and $a - 2b + c$
c) $a - 2b + c$ and $\frac{1}{a+b-c}$
d) None of these

504. The roots of the equation $|x^2 - x - 6| = x + 2$ are
a) -2, 1, 4 b) 0, 2, 4 c) 0, 1, 4 d) -2, 2, 4

505. If z_1, z_2, z_3 be vertices of an equilateral triangle occurring in the anticlockwise sense, then
a) $z_1^2 + z_2^2 + z_3^2 = 2(z_1z_2 + z_2z_3 + z_3z_1)$
b) $\dfrac{1}{z_1 + z_2} + \dfrac{1}{z_2 + z_3} + \dfrac{1}{z_3 + z_1} = 0$
c) $z_1 + \omega\, z_2 + \omega^2 z_3 = 0$
d) None of these

506. The values of k for which the equations $x^2 - kx - 21 = 0$ and $x^2 - 3kx + 35 = 0$ will have a common roots are
a) $k = \pm 4$ b) $k = \pm 1$ c) $k = \pm 3$ d) $k = 0$

507. The real part of $(1 - \cos\theta + 2i\sin\theta)^{-1}$ is
a) $\dfrac{1}{3 + 5\cos\theta}$ b) $\dfrac{1}{5 - 3\cos\theta}$ c) $\dfrac{1}{3 - 5\cos\theta}$ d) $\dfrac{1}{5 + 3\cos\theta}$

508. If $x = \frac{1}{2}\left(\sqrt{7} + \frac{1}{\sqrt{7}}\right)$, then $\frac{\sqrt{x^2-1}}{x-\sqrt{x^2-1}}$ is equal to
a) 1 b) 2 c) 3 d) 4

509. If the sum of the roots of the equation $ax^2 + bx + c = 0$ be equal to the sum of the reciprocal of their squares, then bc^2, ca^2, ab^2 will be in
a) AP b) GP c) HP d) None of these

510. The equation whose roots are reciprocal of the roots of the equation $ax^2 + bx + c = 0$ is
a) $bx^2 + cx + a = 0$ b) $bx^2 + ax + c = 0$ c) $cx^2 + ax + b = 0$ d) $cx^2 + bx + a = 0$

511. If one root of the equation $ax^2 + bx + c = 0$ is reciprocal of the one root of the equation $a_1x^2 + b_1x + c_1 = 0$, then
a) $(aa_1 - cc_1)^2 = (bc_1 - b_1a)(b_1c - a_1b)$
b) $(ab_1 - a_1b)^2 = (bc_1 - b_1c)(ca_1 - c_1a)$
c) $(bc_1 - b_1c)^2 = (ca_1 - a_1c)(ab_1 - a_1b)$
d) None of these

512. If z is a non-real 7^{th} root of -1, then $z^{86} + z^{175} + z^{289}$ is equal to
a) 0 b) -1 c) 3 d) 1

513. If α and β the roots of $x^2 - x - 1 = 0$ and $A_n = \alpha^n + \beta^n$, then AM of A_{n-1} and A_n is
a) $2A_{n+1}$ b) $\dfrac{A_{n+1}}{2}$ c) $2A_{n-2}$ d) None of these

514. If $\omega(\neq 1)$ is a cube root of unity, then
$\begin{vmatrix} 1 & 1 + i + \omega^2 & \omega^2 \\ 1 - i & -1 & \omega^2 - 1 \\ -i & -i + \omega - 1 & -1 \end{vmatrix}$ equals
a) 0 b) 1 c) i d) ω

515. If a complex number z lies in the interior or on the boundary of a circle or radius 3 and centre at $(-4, 0)$, then the greatest and least values of $|z + 1|$ are
a) 5, 0 b) 6, 1 c) 6, 0 d) None of these

516. If $\text{Im}\left(\frac{z-1}{2z+1}\right) = -4$, then locus of z is

a) An ellipse b) A parabola c) A straight line d) A circle

517. If $w = \alpha + i\beta$, where $\beta \neq 0$ and $z \neq 1$, satisfies the condition that $\left(\frac{w-\bar{w}z}{1-z}\right)$ is purely real, then the set of values of z is

a) $|z| = 1, z \neq 2$ b) $|z| = 1$ and $z \neq 1$ c) $z = \bar{z}$ d) None of these

518. If $(x-2)$ is a common factor of the expressions $x^2 + ax + b$ and $x^2 + cx + d$, then $\frac{b-d}{c-a}$ is equal to

a) -2 b) -1 c) 1 d) 2

519. If $\alpha_0, \alpha_1, \alpha_2, \dots, \alpha_{n-1}$ be the n^{th} roots of unity, then the value of $\sum_{i=0}^{n-1} \frac{\alpha_i}{3-\alpha_i}$ is equal to

a) $\frac{n}{3^n - 1}$ b) $\frac{n+1}{3^n - 1}$ c) $\frac{n-1}{3^n - 1}$ d) None of these

520. p, q, r and s are integers. If the A.M. of the roots of $x^2 - px + q^2 = 0$ and GM of the roots of $x^2 - rx + s^2 = 0$ are equal, then

a) q is an odd integer b) r is an even integer c) p is an even integer d) s is an odd integer

521. The condition that $x^3 - px^2 + qx - r = 0$ may have two of its roots equal in magnitude but of opposite sign, is

a) $r = pq$ b) $r = 2p^3 + pq$ c) $r = p^2q$ d) None of the above

522. If α and β are the solutions of the quadratic equation $ax^2 + bx + c = 0$ such that $\beta = \alpha^{1/3}$, then

a) $(ac)^{1/3} + (ab)^{1/3} + c = 0$ b) $(a^3b)^{1/4} + (ab^3)^{1/4} + c = 0$

c) $(a^3c)^{1/4} + (ac^3)^{1/4} + b = 0$ d) $(a^4c)^{1/3} + (ac^4)^{1/3} + b = 0$

523. Let a, b, c be real. If $ax^2 + bx + c = 0$ has two real roots α and β, where $\alpha < -1$ and $\beta > 1$, then $1 + \frac{c}{a} + \left|\frac{b}{a}\right|$ is

a) <0 b) >0 c) ≤ 0 d) None of these

524. If z_1, z_2, z_3, z_4 represent the vertices of a rhombus taken in the anticlockwise order, then

a) $z_1 + z_2 + z_3 + z_4 = 0$ b) $z_1 + z_2 = z_3 + z_4$ c) $\text{amp}\frac{z_2 - z_4}{z_1 - z_3} = \frac{\pi}{2}$ d) $\text{amp}\frac{z_1 - z_2}{z_3 - z_4} = \frac{\pi}{2}$

525. If $7^{\log_7(x^2 - 4x + 5)} = x - 1$, x may have values

a) $2, 3$ b) 7 c) $-2, -3$ d) $2, -3$

526. The solution set of the inequation $\frac{1}{|x|-3} < \frac{1}{2}$, is

a) $(-\infty, -5) \cup (5, \infty)$

b) $(-3, 3)$

c) $(-\infty, -5) \cup (-3, 3) \cup (5, \infty)$

d) None of these

527. If $z = \sqrt{3} + i$, then the argument of $z^2 e^{z-i}$ is equal to

a) $\frac{\pi}{2}$ b) $\frac{\pi}{6}$ c) $e^{\pi/6}$ d) $\pi/3$

528. If two equations $a_1 x^2 + b_1 x + c_1 = 0$ and, $a_2x^2 + b_2x + c_2 = 0$ have a common root, then the value of $(a_1b_2 -$ *a2b1b1c2–b2c1,* is

a) $-(a_1c_2 - a_2c_1)^2$ b) $(a_1a_2 - c_1c_2)^2$ c) $(a_1c_1 - a_2c_2)^2$ d) $(a_1c_2 - c_1a_2)^2$

529. The value of expression $\left(1 + \frac{1}{\omega}\right)\left(1 + \frac{1}{\omega^2}\right) + \left(2 + \frac{1}{\omega}\right)\left(2 + \frac{1}{\omega^2}\right) + \left(3 + \frac{1}{\omega}\right)\left(3 + \frac{1}{\omega^2}\right) + \dots + \left(n + \frac{1}{\omega}\right)\left(n + \frac{1}{\omega^2}\right)$, where ω is an imaginary cube root of unity is

a) $\frac{n(n^2+2)}{3}$ b) $\frac{n(n^2-2)}{3}$ c) $\frac{n(n^2+1)}{3}$ d) None of these

530. If $(x + iy)^{1/3} = 2 + 3i$, then $3x + 2y$ is equal to

a) -20 b) -60 c) -120 d) 60

531. If the roots of the equation $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$ are equal in magnitude but opposite in sign, then the product of the roots will be

a) $\frac{p^2+q^2}{2}$ b) $-\frac{(p^2+q^2)}{2}$ c) $\frac{p^2-q^2}{2}$ d) $-\frac{(p^2-q^2)}{2}$

532. $(1+i)^8+(1-i)^8=$

a) 2^8 b) 2^5 c) $2^4\cos\frac{\pi}{4}$ d) $z^8\cos\frac{\pi}{8}$

533. If $\frac{x^2-bx}{ax-c}=\frac{\lambda-1}{\lambda+1}$ has roots equal in magnitude and opposite in sign then the value of λ is

a) $\frac{a-b}{a+b}$ b) $\frac{a+b}{a-b}$ c) c d) $\frac{1}{c}$

534. Real roots of the equation $x^2+5|x|+4=0$ are

a) $-1,-4$ b) $1,4$ c) $-4,4$ d) None of these

535. If $z_r(r=0,1,2,\dots,6)$ be the roots of the equation $(z+1)^7+z^7=0$, then $\sum_{r=0}^{6}\text{Re}\,(z_r)=$

a) 0 b) 3/2 c) 7/2 d) $-7/2$

536. Given that the equation $z^2+(p+iq)z+r+is=0$, where p,q,r,s are real and non-zero roots, then

a) $pqr=r^2+p^2s$ b) $prs=q^2+r^2p$ c) $qrs=p^2+s^2q$ d) $pqs=s^2+q^2r$

537. The values of a for which $2x^2-2(2a+1)x+a(a+1)=0$ may have one root less than a and other root greater than a are given by

a) $1>a>0$ b) $-1<a<0$ c) $a\geq 0$ d) $a>0$ or $a<-1$

538. If $a=\cos\alpha+i\sin\alpha,\ b=\cos\beta+i\sin\beta, c=\cos\gamma+i\sin\gamma$ and $\frac{b}{c}+\frac{c}{a}+\frac{a}{b}=1$, then $\cos(\beta-\gamma)+$ cosγ−α+cos(α−β) is equal to

a) 3/2 b) $-3/2$ c) 0 d) 1

539. If α,β are the roots of the equation $(x-a)(x-b)=5$, then the roots of the equation $(x-\alpha)(x-\beta)+5=0$ are

a) $a,5$ b) $b,5$ c) a,α d) a,b

540. If α,β,γ are the roots of $x^3+bx+c=0$, then $\alpha^2\beta+\alpha\beta^2+\beta^2\gamma+\beta\gamma^2+\gamma^2\alpha+\gamma\alpha^2$ is equal to

a) c b) $-c$ c) $-3c$ d) $3c$

541. $\left|\frac{1}{2}(z_1+z_2)+\sqrt{z_1z_2}\right|+\left|\frac{1}{2}(z_1+z_2)-\sqrt{z_1z_2}\right|$ is equal to

a) $|z_1+z_2|$ b) $|z_1-z_2|$ c) $|z_1|+|z_2|$ d) $|z_1|-|z_2|$

542. Let $p,q\in\{1,2,3,4\}$.The number of equations of the form $px^2+qx+1=0$ having real roots, is

a) 15 b) 9 c) 7 d) 8

543. The locus of the points representing the complex numbers z for which $|z|-2=|z-i|-|z+5\,i|=0$ is

a) A circle with centre at the origin

b) A straight line passing through the origin

c) The single point $(0,-2)$

d) None of these

544. If $a\leq 0$, then the real values of x satisfying $x^2-2a|x-a|-3a^2=0$ are

a) $a(1-\sqrt{2}),a(-1+\sqrt{6})$

b) $a(1+\sqrt{2}),a(1-\sqrt{6})$

c) $a(1-\sqrt{2}),a(1-\sqrt{6})$

d) None of these

545. If the roots of the equation $ax^2-4x+a^2=0$ are imaginary and the sum of the roots is equal to their product, then $a=$

a) -2 b) 4 c) 2 d) None of these

546. If the roots of the equation $4x^3-12x^2+11x+k=0$ are in arithmetic progression, then k is equal to

a) -3 b) 1 c) 2 d) 3

547. If at least one value of the complex number $z=x+i\,y$ satisfy the condition $|z+\sqrt{2}|=\sqrt{a^2-3a+2}$ and the inequality $|z+i\sqrt{2}|<a$, then

a) $a>2$ b) $a=2$ c) $a<2$ d) None of these

548. If the roots of $ax^2-bx-c=0$ change by the same quantity, then the expression in a,b,c that does not change is

a) $\frac{b^2-4ac}{a^2}$ b) $\frac{b-4c}{a}$ c) $\frac{b^2+4ac}{a^2}$ d) $\frac{b^2-4ac}{a}$

549. The solution of set of the equation $x^{\log_x(1-x)^2} = 9$ is

a) $\{-2, 4\}$ b) $\{4\}$ c) $\{0, -2, 4\}$ d) None of these

550. If ω is a complex cube root of unity, then the value of $\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix}$ is

a) x^3 b) $2x^3$ c) $3x^3$ d) None of these

551. The value of $\left[\sqrt{2}\{\cos(56°15') + i\sin(56°\,15')\}\right]^8$ is

a) $4\,i$ b) $8\,i$ c) $16\,i$ d) $-16\,i$

552. The real part of $(1 - \cos\theta + 2i\sin\theta)^{-1}$ is

a) $\frac{1}{3+5\cos\theta}$ b) $\frac{1}{5-3\cos\theta}$ c) $\frac{1}{3-5\cos\theta}$ d) $\frac{1}{5+3\cos\theta}$

553. Suppose the quadratic equations $x^2 + px + q = 0$ and $x^2 + rx + s = 0$ are such that p, q, r, s are real and $pr = 2(q+s)$. Then

a) Both the equations always have real roots
b) At least one equation always has real roots
c) Both the equation always have non-real roots
d) At least one equation always has real and equal roots

554. If $i = \sqrt{-1}$, then

$4 + 5\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{334} + 3\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{365}$ is equal to

a) $1 - i\sqrt{3}$ b) $-1 + i\sqrt{3}$ c) $i\sqrt{3}$ d) $-i\sqrt{3}$

555. The values of $(16)^{1/4}$ are

a) $\pm2, \pm2i$ b) $\pm4, \pm4i$ c) $\pm1, \pm i$ d) None of these

556. Let z_1, z_2, z_3 be three vertices of an equilateral triangle circumscribing the circle $|z| = \frac{1}{2}$. If $z_1 = \frac{1}{2} + i\frac{\sqrt{3}}{2}$ and z_1, z_2, z_3 were in anticlockwise sense, then z_2 is

a) $1 + i\sqrt{3}$ b) $1 - i\sqrt{3}$ c) 1 d) -1

557. The value of $\frac{4(\cos 75° + i\sin 75°)}{0.4(\cos 30° + i\sin 30°)}$ is

a) $\frac{\sqrt{2}}{10}(1+i)$ b) $\frac{\sqrt{2}}{10}(1-i)$ c) $\frac{10}{\sqrt{2}}(1-i)$ d) $\frac{10}{\sqrt{2}}(1+i)$

558. The value of a for which the sum of the squares of the roots of the equation $x^2 - (a-2)x - a - 1 = 0$ assumes the least value, is

a) 0 b) 1 c) 2 d) 3

559. The amplitude of $\frac{1+i\sqrt{3}}{\sqrt{3}+i}$ is

a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$ c) $\frac{2\pi}{3}$ d) $\frac{\pi}{6}$

560. The value of p for which both the roots of the equation $4\,x^2 - 20\,px + (25\,p^2 + 15\,p - 66) = 0$ are less than 2, lies in

a) $4/5, 2$ b) $-1, -4/5$ c) $2, \infty$ d) $(-\infty, -1)$

561. If ω is acomplex cube root of unity, then the value of $\sin\left\{(\omega^{10} + \omega^{23})\pi - \frac{\pi}{6}\right\}$ is

a) $\frac{1}{\sqrt{2}}$ b) $\frac{\sqrt{3}}{2}$ c) $-\frac{1}{\sqrt{2}}$ d) $\frac{1}{2}$

562. The modulus and amplitude of $\frac{1+2i}{1-(1-i)^2}$ are

a) $\sqrt{2}$ and $\frac{\pi}{6}$ b) 1 and 0 c) 1 and $\frac{\pi}{3}$ d) 1 and $\frac{\pi}{4}$

563. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval

a) $[4, 5]$ b) $(-\infty, 4)$ c) $(6, \infty)$ d) $(5, 6]$

564. If $z = \frac{7-i}{3-4i}$, then z^{14} is equal to

a) 2^7 b) $2^7 i$ c) $2^{14} i$ d) $-2^7 i$

565. If the roots of the equation $x^3 + bx^2 + 3x - 1 = 0$ from a non-decreasing H.P., then

a) $b \in (-3, \infty)$ b) $b = -3$ c) $b \in (-\infty, -3)$ d) None of these

566. Rational roots of the equation $2x^4 + x^3 - 11x^2 + x + 2 = 0$ are

a) $\frac{1}{2}$ and 2 b) $\frac{1}{2}, 2, \frac{1}{4}, -2$ c) $\frac{1}{2}, 2, 3, 4$ d) $\frac{1}{2}, 2, \frac{3}{4}, -2$

567. The expression $y = ax^2 + bx + c$ has always the same sign as, c if

a) $4\,ac < b^2$ b) $4\,ac > b^2$ c) $ac < b^2$ d) $ac > b^2$

568. Let a, b, c be real number $a \neq 0$. If α is a root of $a^2x^2 + bx + c = 0$, β is a root of $a^2x^2 - bx - c = 0$ and $0 < \alpha < \beta$, then the equation $a^2x^2 + 2bx + 2c = 0$ has a root of γ that always satisfies

a) $\gamma = \frac{\alpha + \beta}{2}$ b) $\gamma = \alpha + \frac{\beta}{2}$ c) $\gamma = \alpha$ d) $\alpha < \gamma < \beta$

569. The smallest positive integer n for which $(1 + i)^{2n} = (1 - i)^{2n}$ is

a) 4 b) 8 c) 2 d) 12

570. If $2\alpha = -1 - i\sqrt{3}$ and $2\beta = -1 + i\sqrt{3}$, then $5\alpha^4 + 5\beta^4 + 7\alpha^{-1}\beta^{-1}$ is equal to

a) -1 b) -2 c) 0 d) 2

571. The solution set of the equation

$\left[4\left(1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \ldots\right)\right]^{\log_2 x} = \left[54\left(1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots\right)\right]^{\log_x 2}$ is

a) $\left\{4, \frac{1}{4}\right\}$ b) $\left\{2, \frac{1}{2}\right\}$ c) $\{1, 2\}$ d) $\left\{8, \frac{1}{8}\right\}$

572. The maximum distance from the origin of coordinates to the point z satisfying the equation $\left|z + \frac{1}{z}\right| = a$ is

a) $\frac{1}{2}\left(\sqrt{a^2 + 1} + a\right)$ b) $\frac{1}{2}\left(\sqrt{a^2 + 2} + a\right)$ c) $\frac{1}{2}\left(\sqrt{a^2 + 4} + a\right)$ d) None of these

573. The solution of $6 + x - x^2 > 0$, is

a) $-1 < x < 2$ b) $-2 < x < 3$ c) $-2 < x < -1$ d) None of these

574. If $(\cos\theta + i\sin\theta)(\cos 2\theta + i\sin 2\theta) \ldots (\cos n\theta + i\sin n\theta) = 1$, then the value of θ is

a) $4m\pi$ b) $\frac{2m\pi}{n(n+1)}$ c) $\frac{4m\pi}{n(n+1)}$ d) $\frac{m\pi}{n(n+1)}$

575. If α, β, γ are the roots of the equation $x^3 - 6x^2 + 11x + 6 = 0$, then $\sum \alpha^2\beta + \sum \alpha\beta^2$ is equal to

a) 80 b) 84 c) 90 d) -84

576. Let z_1, z_2, z_3 be three complex numbers satisfying $\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} = 0$. Let $z_k = r_k(\cos\alpha_k + i\sin\alpha_k)$ and $\omega_k = \frac{\cos 2\alpha_k + i\sin 2\,\alpha_k}{z_k}$ for $k = 1,2,3$. If ω_1, ω_2 and ω_3 are the affixes of points A_1, A_2 and A_3 respectively in the Argand plane, then $\Delta A_1A_2A_3$ has its

a) Incentre at the origin

b) Centroid at the origin

c) Circumcentre at the origin

d) Orthocentre at the origin

577. If $\left|\frac{z-25}{z-1}\right| = 5$, find the value of $|z|$

a) 3 b) 4 c) 5 d) 6

578. If α and β are the roots of $ax^2 + bx + c = 0$, then the equation $ax^2 - bx(x-1) + c(x-1)^2 = 0$ has roots

a) $\frac{\alpha}{1-\alpha}, \frac{\beta}{1-\beta}$ b) $\frac{1-\alpha}{\alpha}, \frac{1-\beta}{\beta}$ c) $\frac{\alpha}{\alpha+1}, \frac{\beta}{\beta+1}$ d) $\frac{\alpha+1}{\alpha}, \frac{\beta+1}{\beta}$

579. The argument of the complex number $\frac{13-5i}{4-9i}$ is

a) $\pi/3$ b) $\pi/4$ c) $\pi/5$ d) $\pi/6$

580. If $\sin\theta, \sin\alpha\cos\theta$ are in G.P., then the roots of $x^2 + 2x\cot\alpha + 1 = 0$ are always

a) Equal b) Real c) Imaginary d) Greater than 1

581. If $f(x)$ is a polynomial of degree n with rational coefficients and $1+2i, 2-\sqrt{3}$ and 5 are three roots of $f(x)=0$, then the least value of n is

a) 5 b) 4 c) 3 d) 6

582. If $\frac{3x}{(x-a)(x-b)}=\frac{2}{x-a}+\frac{1}{x-b'}$, then $a:b$ is equal to

a) $1:2$ b) $-2:1$ c) $1:3$ d) $3:1$

583. If $(x+iy)=\sqrt{\frac{1+2i}{3+4i'}}$, then $(x^2+y^2)^2$ is equal to

a) 5 b) 1/5 c) 2/5 d) 5/2

584. The number of real roots of $f(x)=0$, where $f(x)=(x-1)(x-2)(x-3)(x-4)$ lying in the interval (1, 3) is

a) 1 b) 2 c) 3 d) 4

585. If z is a complex number, then $|3z-1|=3|z-2|$ represents

a) y-axis b) A circle
c) x-axis d) A line parallel to y-axis

586. The triangle with vertices at the points $z_1, z_2, (1-i)z_1+i\,z_2$ is

a) Right angled but not isosceles
b) Isosceles but not right angled
c) Right angled and isosceles
d) Equilateral

587. If $x=\frac{1}{x}=2\sin\alpha, y=y+\frac{1}{y}=2\cos\beta$, then $x^3y^3+\frac{1}{x^3y^3}$ is

a) $2\cos 3(\beta-\alpha)$ b) $2\cos 3(\beta+\alpha)$ c) $2\sin 3(\beta-\alpha)$ d) $2\sin 3(\beta+\alpha)$

588. If α, β, γ are the cube roots of $p, p<0$ then for any x, y and z the values of $\frac{x\,\alpha+y\,\beta+z\,\gamma}{x\,\beta+y\,\gamma+z\,\alpha}$ are

a) ω, ω^2 b) $-\omega, -\omega^2$ c) $1, -1$ d) None of these

589. If $p^2-p+1=0$, then the value of p^{3n} can be

a) 1 b) -1 c) 0 d) None of these

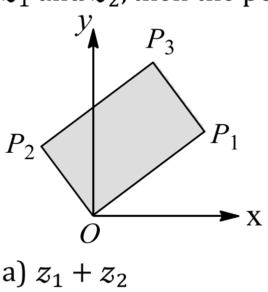
590. If $a=\cos\theta+i\sin\theta$, then $\frac{1+a}{1-a}$ is equal to

a) $\cot\frac{\theta}{2}$ b) $\cot\theta$ c) $i\cot\frac{\theta}{2}$ d) $i\tan\frac{\theta}{2}$

591. If $z=r(\cos\theta+i\sin\theta)$, then the value of $\frac{z}{\bar{z}}+\frac{\bar{z}}{z}$ is

a) $\cos 2\theta$ b) $2\cos 2\theta$ c) $2\cos\theta$ d) $2\sin\theta$

592. In a give parallelogram, if the points P_1 and P_2 represent two complex numbers z_1 and z_2, then the point P_3 represents the number



a) z_1+z_2 b) z_1-z_2 c) $z_1\times z_2$ d) $z_1\div z_2$

593. If $\arg(z)=\theta$ then $\arg(\bar{z})$ is equal to

a) $\theta-\pi$ b) $\pi-\theta$ c) θ d) $-\theta$

594. If z is a complex number, then the minimum value of $|z|+|z-1|$ is

a) 1 b) 0 c) $\frac{1}{2}$ d) None of these

595. The complex numbers $\sin x+i\cos 2x$ and $\cos x-i\sin 2x$ are conjugate to each other for

a) $x=n\pi$ b) $x=\left(n+\frac{1}{2}\right)\pi$ c) $x=0$ d) No value of x

596. If the equations $x^2+ax+b=0$ and $x^2+bx+a=0$ have a common root, then the numerical value of $a+b$ is

a) 1 b) 0 c) -1 d) None of these

597. If $S = \left\{z \in C : \arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{3}\right\}$, then S is

a) An ellipse b) A straight line c) A circle d) A parabola

598. If $b^2 \geq 4ac$ for the equation $ax^4 + bx^2 + c = 0$, then all the roots of the equation will be real positive of

a) $b > 0, a < 0, c > 0$ b) $b < 0, a > 0, c > 0$ c) $b > 0, a > 0, c > 0$ d) $b > 0, a > 0, c < 0$

599. Let z_1 and z_2 be two complex numbers with α and β as their principle arguments such that $\alpha + \beta > \pi$, then principal $\arg(z_1 z_2)$ is given by

a) $\alpha + \beta + \pi$ b) $\alpha + \beta - \pi$ c) $\alpha + \beta + 2\pi$ d) $\alpha + \beta$

600. Let ω be a complex cube root of unity. If the equation $|z - \omega|^2 + |z - \omega^2|^2 = \lambda$ represents a circle with points representing ω and ω^2 as the end points of a diameter, then $\lambda =$

a) 4 b) 3 c) 2 d) $\sqrt{2}$

601. Let $2\sin^2 x + 3\sin x - 2 > 0$ and $x^2 - x - 2 < 0$, (x is measured in radians). Then x lies in the interval

a) $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ b) $\left(-1, \frac{5\pi}{6}\right)$ c) $(-1, 2)$ d) $\left(\frac{\pi}{6}, 2\right)$

602. One root of $(1)^{1/3}$ is

a) $\frac{\sqrt{3}i}{2}$ b) $\frac{1+\sqrt{3}i}{2}$ c) $\frac{1-\sqrt{3}i}{4}$ d) $\frac{-1-\sqrt{3}i}{2}$

603. If one of the root of the equation $x^2 + ax + 3 = 0$ is 3 and one of the roots of the equation $x^2 + ax + b = 0$ is three times the other root, then the value of b is equal to

a) 3 b) 4 c) 2 d) 1

604. If $|z - 4 - 3i| \leq 1$ and m and n are the least and greatest value of $|z|$ and λ is the least value of $\frac{x^4+x^2+4}{x}$ in the interval $(0, \infty)$ then λ is equal to

a) m b) n c) $m + n$ d) None of these

605. The harmonic mean of the roots of the equation $(5 + \sqrt{2})x^2 - (4 + \sqrt{5})x + 8 + 2\sqrt{5} = 0$ is

a) 2 b) 4 c) 6 d) 8

606. If a, b, c and u, v, ω are complex numbers representing the vertices of two triangles such that $c = (1 - r)\,a + rb$ and $\omega = (1 - r)u + rv$, where r is a complex number, then the two triangles

a) Have the same area b) Are similar c) Are congruent d) None of these

607. The value of k for which the equation

$3x^2 + 2x(k^2 + 1) + k^2 - 3k + 2 = 0$

has roots of opposite signs, lies in the interval

a) $(-\infty, 0)$ b) $(-\infty, -1)$ c) $(1, 2)$ d) $(3/2, 2)$

608. The locus of point z satisfying $\text{Re}\left(\frac{1}{z}\right) = k$, where k is a non-zero real number, is

a) A straight line b) A circle c) An ellipse d) A hyperbola

609. If $z \neq 0$ be a complex number and $\arg(z) = \pi/4$, then

a) Re $(z) =$ Im (z) only b) Re $(z) =$ Im $(z) > 0$ c) Re $(z^2) =$ Im (z^2) d) None of these

610. If $z_1, z_2, \ldots, z_n$ are complex numbers such that $|z_1| = |z_2| = \ldots = |z_n| = 1$, then $|z_1 + z_2 + \ldots + z_n|$ is equal to

a) $|z_1 z_2 z_3 \ldots z_n|$ b) $|z_1| + |z_2| + \ldots + |z_n|$ c) $\left|\frac{1}{z_1} + \frac{1}{z_2} + \ldots + \frac{1}{z_n}\right|$ d) n

611. If $z_r = \cos\frac{r\alpha}{n^2} + i\sin\frac{r\alpha}{n^2}$, where $r = 1, 2, 3 \ldots, n$, then $\lim_{n\to\infty} z_1\, z_2\, z_3 \ldots z_n$ is equal to

a) $\cos\alpha + i\sin\alpha$ b) $\cos\left(\frac{\alpha}{2}\right) - i\sin\left(\frac{\alpha}{2}\right)$ c) $e^{i\alpha/2}$ d) $\sqrt[3]{e^{i\alpha}}$

612. If α, β and γ are the roots of equation $x^3 - 3x^2 + x + 5 = 0$, then $y = \sum \alpha^2 + \alpha\beta\gamma$ satisfies the equation

a) $y^3 + y + 2 = 0$ b) $y^3 - y^2 - y - 2 = 0$

c) $y^3 + 3y^2 - y - 3 = 0$ d) $y^2 + 4y^2 + 5y + 20 = 0$

613. If the roots of $(z - 1)^n = i\,(z + 1)^n$ are plotted in the Argand plane, they are

a) On a parabola

b) Concyclic

c) Collinear

d) The vertices of a triangle

614. If n is a positive integer, then $(1+i)^n+(1-i)^n$ is equal to

a) $(\sqrt{2})^{n-2}\cos\left(\frac{n\pi}{4}\right)$ b) $(\sqrt{2})^{n-2}\sin\left(\frac{n\pi}{4}\right)$ c) $(\sqrt{2})^{n+2}\cos\left(\frac{n\pi}{4}\right)$ d) $(\sqrt{2})^{n+2}\sin\left(\frac{n\pi}{4}\right)$

615. The roots of the equation $(3-x)^4+(2-x)^4=(5-2x)^4$ are

a) All real

b) All imaginary

c) Two real and two imaginary

d) None of these

616. In which quadrant of the complex plane, the point $\frac{1+2i}{1-i}$ lies?

a) Fourth b) First c) Second d) Third

617. If α,β are roots of $ax^2+bx+c=0$, then the equation $ax^2-bx(x-1)+c(x-1)^2=0$ has roots

a) $\frac{\alpha}{1-\alpha},\frac{\beta}{1-\beta}$ b) $\frac{1-\alpha}{\alpha},\frac{1-\beta}{\beta}$ c) $\frac{\alpha}{\alpha+1},\frac{\beta}{\beta+1}$ d) $\frac{\alpha+1}{\alpha},\frac{\beta+1}{\beta}$

618. If the expression $\frac{\left[\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)-i\tan(x)\right]}{\left[1+2i\sin\left(\frac{x}{2}\right)\right]}$ is real then the set of all possible value of x is

a) $n\pi+\alpha$ b) $2n\pi$ c) $\frac{n\pi}{2}+\alpha$ d) None of these

619. If x is an integer satisfying $x^2-6x+5\le 0$ and $x^2-2x>0$, then the number of positive values of x, is

a) 3 b) 4 c) 2 d) Infinite

620. For any two complex numbers z_1 and z_2 and any real numbers a and b; $|az_1-bz_2|^2+|(bz_1+az_2)|^2$ is equal to

a) $(a^2+b^2)(|z_1|+|z_2|)$ b) $(a^2+b^2)(|z_1|^2+|z_2|^2)$

c) $(a^2+b^2)(|z_1|^2-|z_2|^2)$ d) None of the above

621. If α,β are roots of the equation $6x^2-5x+1=0$, then the value of $\tan^{-1}\alpha+\tan^{-1}\beta$ is

a) 0 b) $\frac{\pi}{4}$ c) 1 d) $\frac{\pi}{2}$

622. If $|z_1|=|z_2|=\cdots=|z_n|=1$, then the value of $|z_1+z_2+z_3+\ldots+z_n|$ is

a) 1 b) $|z_1|+|z_2|+\ldots+|z_n|$

c) $\left|\frac{1}{z_1}+\frac{1}{z_2}+\ldots+\frac{1}{z_n}\right|$ d) None of these

623. The value of $\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7}-i\cos\frac{2\pi k}{7}\right)$, is

a) -1 b) 0 c) $-i$ d) i

624. The solution set of $x^2+x+|x|+1<0$, is

a) $(0,\infty)$ b) $(-\infty,0)$ c) R d) ϕ

625. Let $\omega_n=\cos\left(\frac{2\pi}{n}\right)+i\sin\left(\frac{2\pi}{n}\right)$, $i^2=-1$, then $(x+y\omega_3+z\omega_3^2)(x+y\omega_3^2+z\omega_3)$ is equal to

a) 0 b) $x^2+y^2+z^2$

c) $x^2+y^2+z^2-yz-zx-xy$ d) $x^2+y^2+z^2+yz+zx+xy$

626. The quadratic equation $x^2+15|x|+14=0$ has

a) Only positive solutions b) Only negative solutions

c) No solution d) Both positive and negative solution

627. If α,β,γ are the roots of the equation $x^3+ax^2+bx+c=0$, then $\alpha^{-1}+\beta^{-1}+\gamma^{-1}=$

a) $\frac{a}{c}$ b) $-\frac{b}{c}$ c) $\frac{b}{a}$ d) $\frac{c}{a}$

628. The solution of the quadratic equation $(3|x|-3)^2=|x|+7$ which belongs to the domain of definition of the function $y=\sqrt{x(x-3)}$ are given by

a) $\pm\frac{1}{9},\pm 2$ b) $-\frac{1}{9},2$ c) $\frac{1}{9},-2$ d) $-\frac{1}{9},-2$

629. If α is a cube root of unit and is not real, then $\alpha^{3n+1}+\alpha^{3n+3}+\alpha^{3n+5}$ has the value

a) -1 b) 0 c) 1 d) 3

630. The number of solutions of the equation $z^2 + \bar{z} = 0$ is

a) 2 b) 4 c) 6 d) 8

631. If $\log_{\sqrt{3}}\left(\frac{|z|^2-|z|+1}{2+|z|}\right) < 2$, then the locus of z is

a) $|z| = 5$ b) $|z| < 5$ c) $|z| > 5$ d) None of these

632. If $f(x) = 2x^3 + mx^2 - 13x + n$ and 2, 3 are roots of the equation $f(x) = 0$, then the values of m and n are

a) -5, -30 b) -5, 30 c) 5, 30 d) None of these

633. Let A, B and C represent the complex numbers z_1, z_2, z_3 respectively on the complex plane. If the circumcenter of the triangle ABC lies at the origin, then the orthocentre is represented by the complex number

a) $z_1 + z_2 - z_3$ b) $z_2 + z_3 - z_1$ c) $z_3 + z_1 - z_2$ d) $z_1 + z_2 + z_3$

634. Let $x = \alpha + \beta, y = \alpha\omega + \beta\omega^2, z = \alpha\omega^2 + \beta\omega, \omega$ is an imaginary cube root of unity. The value of xyz is

a) $\alpha^2 + \beta^2$ b) $\alpha^2 - \beta^2$ c) $\alpha^3 + \beta^3$ d) $\alpha^3 - \beta^3$

635. Let $n = 2006!$. Then

$\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \ldots + \frac{1}{\log_{2006} n}$ is equal to

a) 2006 b) 2005 c) 2005! d) 1

636. If $a(p+q)^2 + 2b\,pq + c = 0$ and $a(p+r)^2 + 2b\,pr + c = 0$, then $qr =$

a) $p^2 + \frac{c}{a}$ b) $p^2 + \frac{a}{c}$ c) $p^2 + \frac{a}{b}$ d) $p^2 + \frac{b}{a}$

637. If $\frac{2z_1}{3z_2}$ is purely imaginary number, then $\left|\frac{z_1-z_2}{z_1+z_2}\right|^4$ is equal to

a) 3/2 b) 1 c) 2/3 d) 4/9

638. $\sqrt{4}, \sqrt[4]{4}, \sqrt[8]{4}, \sqrt[16]{4}$, ... to ∞ are root of equation

a) $x^2 - 4 = 0$ b) $x^2 - 4x + 6 = 0$ c) $x^2 - 5x + 4 = 0$ d) $x^2 - 3x + 2 = 0$

639. Let x_1, x_2 be the roots of the equation $x^2 - 3x + p = 0$ and let x_3, x_4 be the roots of the equation $x^2 - 12x + q = 0$. If the numbers x_1, x_2, x_3, x_4 (in order) form an increasing G.P., then

a) $p = 2, q = 16$ b) $p = 2, q = 32$ c) $p = 4, q = 16$ d) $p = 4, q = 32$

640. If the equation $x^2 - 3kx + 2e^{2\log k} - 1 = 0$ has real roots such that the product of roots is 7, then the value of k is

a) ± 1 b) 2 c) ± 3 d) None of these

641. If the product of the roots of the equation $(a+1)x^2 + (2a+3)x + (3a+4) = 0$ is 2, then the sum of roots is

a) 1 b) -1 c) 2 d) -2

642. Value of $\sum_{k=1}^{6}\left[\sin\left(\frac{2k\pi}{7}\right) - i\cos\left(\frac{2k\pi}{7}\right)\right]$ is equal to

a) -1 b) 1 c) 0 d) None of these

643. If $\sin\alpha, \sin\beta$ *and* $\cos\alpha$ are in GP, then roots of $x^2 + 2x\cot\beta + 1 = 0$ are always

a) Real b) Real and negative c) Greater than one d) Non-real

644. Let z_1 and z_2 be the roots of the equation $z^2 + pz + q = 0$ where p, q are real. The points represented by z_1, z_2 and the origin form an equilateral triangle, if

a) $p^2 = 3q$ b) $p^2 > 3q$ c) $p^2 < 3q$ d) $p^2 = 2q$

645. If $1, \omega, \omega^2$ are the cube roots of unity, then $(3 + \omega^2 + \omega^4)^6$ is equal to

a) 64 b) 729 c) 2 d) 0

646. If $z = x + iy$ and $w = \frac{1-iz}{z-i}$, then $|w| = 1$ implies that in the complex plane

a) z lies on imaginary axis

b) z lies on real axis

c) z lies on unit circle

d) None of these

647. The number of real roots of the equation $(x+3)^4 + (x+5)^4 = 16$, is

a) 0 b) 2 c) 3 d) 4

648. The solution set of the inequation $\left|x+\frac{1}{x}\right|>2$, is

a) $R-\{0\}$ b) $R-\{-1,0,1\}$ c) $R-\{1\}$ d) $R-\{-1,1\}$

649. If Re $\left(\frac{z+4}{2z-i}\right)=\frac{1}{2}$, then z is represented by a point lying on

a) A circle b) An ellipse c) A straight line d) None of these

650. $\sin A, \sin B, \cos A$ are in GP. Roots of $x^2+2x\cot B+1=0$ are always

a) Real b) Imaginary c) Greater than 1 d) Equal

651. If α, β are the roots of the equation $ax^2+bx+c=0$, then the value of $\frac{1}{a\alpha+b}+\frac{1}{a\beta+b}$ is equal to

a) $\frac{ac}{b}$ b) 1 c) $\frac{ab}{c}$ d) $\frac{b}{ac}$

652. A And B are two points on the Argand plane such that the segment AB is bisected at the point$(0,0)$. If the point A, which is in the third quadrant has principle amplitude θ, then the principle amplitude of the point B is

a) $-\theta$ b) $\pi-\theta$ c) $\theta-\pi$ d) $\pi+\theta$

653. If $\frac{2z_1}{3z_2}$ is purely imaginary, then $\left|\frac{z_1-z_2}{z_1+z_2}\right|$ is

a) $\frac{2}{3}$ b) $\frac{3}{2}$ c) $\frac{4}{9}$ d) 1

654. If $(1+k)\tan^2 x-4\tan x-1+k=0$ has real roots $\tan x_1$ and $\tan x_2$, then

a) $k^2\leq 5$ b) $k^2\geq 6$ c) $k=3$ d) None of these

655. If m_1, m_2, m_3 and m_4 respectively denote the moduli of the complex numbers $1+4i, 3+i, 1-i$ and $2-3i$, then the correct one, among the following is

a) $m_1<m_2<m_3<m_4$ b) $m_4<m_3<m_2<m_1$
c) $m_3<m_2<m_4<m_1$ d) $m_3<m_1<m_2<m_4$

656. If $(\cos\theta+i\sin\theta)(\cos 2\theta+i\sin 2\theta)\ldots(\cos n\theta+i\sin n\theta)=1$, then the value of θ is

a) $\frac{2m\pi}{n(n+1)}$ b) $4m\pi$ c) $\frac{4m\pi}{n(n+1)}$ d) $\frac{m\pi}{n(n+1)}$

657. Let z_1, z_2, z_3 be the affixes of the vertices of a triangle having the circumcentre at the origin. If z is the affix of it's orthocentre, then z is equal to

a) $\frac{z_1+z_2+z_3}{3}$ b) $\frac{z_1+z_2+z_3}{2}$ c) $z_1+z_2+z_3$ d) None of these

658. If A, B, C are three points in the Argand plane representing the complex numbers z_1, z_2, z_3 such that $z_1=\frac{\lambda z_2+z_3}{\lambda+1}$, where $\lambda\in R$, then the distance of A from the line BC is

a) λ b) $\frac{\lambda}{\lambda+1}$ c) 1 d) 0

659. If the vertices of a quadrilateral be $A=1+2i, B=-3+i, C=-2-3i$ and $D=2-2i$, then the quadrilateral is

a) Parallelogram b) Rectangle c) Square d) Rhombus

660. If the roots of the equation $(p^2+q^2)x^2-2q(p+r)x+(q^2+r^2)=0$ be real and equal, then p, q, r will be in

a) AP b) GP c) HP d) None of these

661. The equation of the locus of z such that $\left|\frac{z-i}{z+i}\right|=2$, where $z=x+iy$ is a complex number, is

a) $3x^2+3y^2+10y-3=0$ b) $3x^2+3y^2+10y+3=0$
c) $3x^2-3y^2-10y-3=0$ d) $x^2+y^2-5y+3=0$

662. If $z_1=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

And $z_2=\sqrt{3}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$, then $|z_1z_2|$ is

a) 6 b) $\sqrt{2}$ c) $\sqrt{6}$ d) $\sqrt{3}$

663. If Re $(z)<0$, then the value of $|1+z+z^2+\cdots+z^n|$ cannot exceed

a) n b) $n|z|^n + 1$ c) $|z^n| - \frac{1}{|z|}$ d) $|z|^n + \frac{1}{|z|}$

664. Let a, b be the solutions of $x^2 + px + 1 = 0$ and c, d be the solutions of $x^2 + qx + 1 = 0$. If $(a-c)(b-c)$ and $(a+d)(b+d)$ are the solutions of $x^2 + ax + \beta = 0$, then β equals
a) $p+q$ b) $p-q$ c) p^2+q^2 d) q^2-p^2

665. The number of integral solutions of $x^2 - 3x - 4 < 0$, is
a) 3 b) 4 c) 6 d) None of these

666. All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than -2 but less than 4 lie in the interval
a) $m > 3$ b) $-1 < m < 3$ c) $1 < m < 4$ d) $-2 < m < 0$

667. The roots of the given equation $(p-q)x^2 + (q-r)x + (r-p) = 0$ are
a) $\frac{p-q}{r-p}, 1$ b) $\frac{q-r}{p-q}, 1$ c) $\frac{r-p}{p-q}, 1$ d) $1, \frac{q-r}{p-q}$

668. Let α, β, δ be the roots of $x^4 + x^2 + 1 = 0$. Then, the equation whose roots are $\alpha^2, \beta^2, \gamma^2, \delta^2$ is
a) $(x^2 - x + 1)^2 = 0$ b) $(x^2 + x + 1)^2 = 0$ c) $x^4 - x^2 + 1 = 0$ d) $x^2 + x + 1 = 0$

669. The solution set of $|x^2 - 10| \leq 6$, is
a) $(2, 4)$ b) $(-4, -2)$ c) $(-4, -2) \cup (2, 4)$ d) $[-4, -2] \cup [2, 4]$

670. If $x = \sqrt{3018 + \sqrt{36 + \sqrt{169}}}$, then the value of x is
a) 55 b) 44 c) 63 d) 42

671. If the roots of the given equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$ are real, then
a) $p \in (-\pi, 0)$ b) $p \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ c) $p \in (0, \pi)$ d) $p \in (0, 2\pi)$

672. Let the two numbers have arithmetic mean 9 and geometric mean 4. Then, these numbers are the roots of the quadratic equation
a) $x^2 - 18x - 16 = 0$ b) $x^2 - 18x + 16 = 0$
c) $x^2 + 18x - 16 = 0$ d) $x^2 + 18x + 16 = 0$

673. If α and β are the roots of the equation $x^2 - x + 1 = 0$ then $\alpha^{2009} + \beta^{2009}$ is equal to
a) -2 b) -1 c) 1 d) 2

674. If $x^2 + px + q = 0$ has the roots α *and* β, then the value of $(\alpha - \beta)^2$ is equal to
a) $p^2 - 4q$ b) $(p^2 - 4q)^2$ c) $p^2 + 4q$ d) $(p^2 + 4q)^2$

675. The conjugate of complex number $\frac{2-3i}{4-i}$ is
a) $\frac{3i}{4}$ b) $\frac{11+10i}{17}$ c) $\frac{11-10i}{17}$ d) $\frac{2+3i}{4i}$

676. If p, q, r are real and $p \neq q$, then the roots of the equation $(p-q)x^2 + 5(p+q)x - 2(p-q) = r$, are
a) Real and equal b) Unequal and rational
c) Unequal and irrational d) Nothing can be said

677. Let $y = \sqrt{\frac{(x+1)(x-3)}{(x-2)}}$, then all real values of x for which y takes real values, are
a) $-1 \leq x < 2$ or $x \geq 3$ b) $-1 \leq x < 3$ or $x > 2$ c) $1 \leq x < 2$ or $x \geq 3$ d) None of these

678. If $x^{2/3} - 7x^{1/3} + 10 = 0$, then the value of x is
a) $\{125\}$ b) $\{8\}$ c) $\emptyset$ d) $\{125, 8\}$

679. If one root of the equation $x^2 - \lambda x + 12 = 0$ is even prime while $x^2 + \lambda x + \mu = 0$ has equal roots, then μ is
a) 8 b) 16 c) 24 d) 32

680. The locus of the points z which satisfy the condition $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{3}$ is
a) A straight line b) A circle c) A parabola d) None of these

681. The value of $\begin{vmatrix} 1+\omega & \omega^2 & -\omega \\ 1+\omega^2 & \omega & -\omega^2 \\ \omega^2+\omega & \omega & -\omega^2 \end{vmatrix}$ is equal to (ω is an imaginary cube root of unity)
a) 0 b) 2ω c) $2\omega^2$ d) $-3\omega^2$

682. If the absolute value of the difference of the roots of the equation $x^2 + ax + 1 = 0$ exceeds $\sqrt{3a}$, then
a) $a \in (-\infty, -1) \cup (4, \infty)$
b) $a \in [0, 4)$
c) $a \in (-1, 4)$
d) $a \in [0, 4)$
683. Consider the following statements:
1. If the quadratic equation is $ax^2 + bx + c = 0$ such that $a + b + c = 0$, then roots of the equation $ax^2 + bx + c = 0$ will be $1, \frac{c}{a}$.
2. If $ax^2 + bx + c = 0$ is quadratic equation such that $a - b + c = 0$, then roots of the equation will be, $-1, \frac{c}{a}$.
Which of the statements given above are correct?
a) Only (1) b) Only (2) c) Both (1) and (2) d) Neither (1) nor (2)
684. The equation
$(x - b)(x - c) + (x - a)(x - b) + (x - a)(x - c) = 0$ has all its roots
a) Positive b) Real c) Imaginary d) Negative
685. Let p and q be real numbers such that $p \neq 0, p^3 \neq q$ and $p^3 \neq -q$. If α and β are non-zero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is
a) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$
b) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$
c) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$
d) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$
686. If α and β be the roots of the equation $2x^2 + 2(a + b)x + a^2 + b^2 = 0$, then the equation whose roots are $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$, is
a) $x^2 - 2ab\,x - (a^2 - b^2)^2 = 0$
b) $x^2 - 4abx - (a^2 - b^2)^2 = 0$
c) $x^2 - 4abx + (a^2 - b^2)^2 = 0$
d) None of these
687. The equation $\bar{b}\,z + b\,\bar{z} = c$, where b is a non-zero complex constant and c is a real number, represents
a) A circle
b) A straight line
c) A pair of straight lines
d) None of these
688. The equation $z\bar{z} + (2 - 3i)z + (2 + 3i)\bar{z} + 4 = 0$ represents a circle of radius
a) 2 b) 3 c) 4 d) 6
689. The value of $i \log(x - i) + i^2\pi + i^3 \log(x + i) + i^4(2 \tan^{-1} x)$, (where, $x > 0$ and $i = \sqrt{-1}$), is
a) 0 b) 1 c) 2 d) 3
690. If $x = \log_a bc, y = \log_b ca, z = \log_a ab$, then the value of $\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z}$ will be
a) $x + y + z$ b) 1 c) $ab + bc + ca$ d) abc
691. If z_1 and z_2 are two complex numbers such that $\left|\frac{z_1 - z_2}{1 - \overline{z_1} z_2}\right| = 1$, then which one of the following is not true?
a) $|z_1| = 1, |z_2| = 1$ b) $z_1 = e^{i\theta}, \theta \in R$ c) $z_2 = e^{i\theta}, \theta \in R$ d) All of these
692. The principle amplitude of $(\sin 40° + i \cos 40°)^5$ is
a) 70° b) −110° c) 110° d) −70°
693. If x satisfies $|x^2 - 3x + 2| + |x - 1| = x - 3$, then
a) $x \in \emptyset$ b) $x \in [1, 2]$ c) $x \in [3, \infty]$ d) $x \in (-\infty, \infty)$
694. The value of $\sum_{r=1}^{8} \left(\sin \frac{2r\pi}{9} + i \cos \frac{2r\pi}{9}\right)$, is
a) -1 b) 1 c) i d) $-i$
695. The centre and the radius of the circle
$z\bar{z} + (2 + 3i)\bar{z} + (2 - 3i)z + 12 = 0$ are respectively
a) $-(2 + 3i), (1)$ b) $(3 + 2i), (1)$ c) $(3 + 6i), (3)$ d) None of these
696. If α, β are roots of the equation $ax^2 + bx + c = 0$, then the equation whose roots are $2\alpha + 3\beta$ and $3\alpha + 2\beta$ is

a) $ab\,x^2-(a+b)cx+(a+b)^2=0$
b) $ac\,x^2-(a+c)bx+(a+c)^2=0$
c) $ac\,x^2+(a+c)bx-(a+c)bx-(a+c)^2=0$
d) None of these

697. The sum of the real roots of the equation
$|x-2|^2+|x-2|-2=0$, is
a) 4 b) 3 c) 2 d) 10

698. The values of x and y such that y satisfy the equation ($x, y \in$ real number) $x^2-xy+y^2-4x-4y+16=0$ is
a) 4, 4 b) 3, 3 c) 2, 2 d) None of these

699. If $\log_{0.3}(x-1)<\log_{0.09}(x-1)$, then x lies in the interval
a) $2, \infty$ b) $(1, 2)$ c) $-2, -1$ d) None of these

700. If $\frac{2x}{2x^2+5x+2}>\frac{1}{x+1}$, then
a) $-2>x>-1$ b) $-2\geq x\geq -1$ c) $-2<x<-1$ d) $-2<x\leq -1$

701. The approximate value of $\sqrt[3]{28}$ is
a) 3.0037 b) 3.037 c) 3.0086 d) 3.37

702. If $\left|\frac{z-2}{z+2}\right|=\frac{\pi}{6}$, then the locus of z is
a) A straight line b) A circle c) A parabola d) An ellipse

703. If $z=x+iy$, then area of the triangle whose vertices are points $z, iz, z+iz$ is
a) $\frac{1}{2}|z|^2$ b) $\frac{1}{4}|z|^2$ c) $|z|^2$ d) $\frac{3}{2}|z|^2$

704. If roots of the equation $ax^2+bx+c=0$; $(a, b, c \in N)$ are rational numbers, then which of the following cannot be true?
a) All a, b, c are even
b) All a, b and c are odd
c) b is even while `$a'$and `c' are odd
d) None of the above

705. If $|a_i|<1, \lambda_i\geq 0$ for $i=1,2,\ldots,n$ and $\lambda_1+\lambda_2+\cdots+\lambda_n=1$, then the value of $|\lambda_1a_1+\lambda_2a_2+\cdots+\lambda_na_n|$ is
a) Equal to 1 b) Less than 1 c) Greater than 1 d) None of these

706. If α, β be the roots of the quadratic equation $ax^2+bx+c=0$ and k be a real number, then the condition so that $\alpha<k<\beta$ is given by
a) $ac>0$ b) $ak^2+bk+c=0$ c) $ac<0$ d) $a^2k^2+abk+ac<0$

707. If $x=2+2^{2/3}+2^{1/3}$, then the value of x^3-6x^2+6x is
a) 3 b) 2 c) 1 d) None of these

708. If the complex numbers z_1, z_2, z_3 are in AP, then they lie on
a) A circle b) A parabola c) A straight line d) An ellipse

709. If z be a complex number, then $|z-3-4i|^2+|z+4+2i|^2=k$ represents a circle, if k is equal to
a) 30 b) 40 c) 55 d) 35

710. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix}=x+iy$, then
a) $x=3, y=1$ b) $x=1, y=3$ c) $x=0, y=3$ d) None of these

711. If the roots of the equation $ax^2+bx+c=0$ of the form $\frac{k+1}{k}$ and $\frac{k+2}{k+1}$, then $(a+b+c)^2$ is equal to
a) $2b^2-ac$ b) $\sum a^2$ c) b^2-4ac d) b^2-2ac

712. The smallest positive integral value of n such that $\left[\frac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right]^n$ is purely imaginary, is equal to
a) 4 b) 3 c) 2 d) 8

713. The locus of the point $z=x+iy$ satisfying $\left|\frac{z-2i}{z+2i}\right|=1$ is
a) x-axis b) y-axis c) $y=2$ d) $x=2$

714. If the roots of the equation $qx^2 + px + q = 0$ are complex, where p, q are real, then the roots of the equation $x^2 - 4qx + p^2 = 0$ are
a) Real and unequal b) Real and equal c) Imaginary d) None of these

715. If $e^{\cos x} - e^{-\cos x} = 4$, then the value of $\cos x$ is
a) $\log_e(2+\sqrt{5})$ b) $-\log_e(2+\sqrt{5})$ c) $\log_e(-2+\sqrt{5})$ d) None of these

716. $\sqrt{12 - \sqrt{68 + 48\sqrt{2}}}$ is equal to
a) $\sqrt{2} - 3$ b) $2 + \sqrt{2}$ c) $2 - \sqrt{2}$ d) $6 - 2\sqrt{8}$

717. The area of the triangle whose vertices are represented by the complex number $0, z, ze^{i\alpha}, (0 < \alpha < \pi)$ equals
a) $\frac{1}{2}|z|^2 \cos\alpha$ b) $\frac{1}{2}|z|^2 \sin\alpha$ c) $\frac{1}{2}|z|^2 \sin\alpha\cos\alpha$ d) $\frac{1}{2}|z|^2$

718. The general value of θ which satisfies the equation $(\cos\theta + i\sin\theta)(\cos 3\theta + i\sin 3\theta)(\cos 5\theta + i\sin 5\theta)\ldots(\cos(2n-1)\theta + i\sin(2n-1)\theta = 1)$ is
a) $\frac{r\pi}{n^2}$ b) $\frac{(r-1)\pi}{n^2}$ c) $\frac{(2r+1)}{n^3}$ d) $\frac{2r\pi}{n^2}$

719. The solution set of the inequation $|x-1| + |x-2| + |x-3| \geq 6$, is
a) $[0, 4]$ b) $(-\infty, -2) \cup [4, \infty)$ c) $(-\infty, 0] \cup [4, \infty)$ d) None of these

720. If the centre of a regular hexagon is at the origin and one of its vertices on argand diagram is $1 + 2i$, then its perimeter is
a) $2\sqrt{5}$ b) $6\sqrt{2}$ c) $4\sqrt{5}$ d) $6\sqrt{5}$

721. If α and β are different complex numbers with $|\beta| = 1$ then $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$ is equal to
a) 0 b) 1/2 c) 1 d) 2

722. If α, β, γ are the roots of the equation $x^3 + 4x - 1 = 0$, then $(\alpha+\beta)^{-1} + (\beta+\gamma)^{-1} + (\gamma+\alpha)^{-1}$ is equal to
a) 2 b) 3 c) -4 d) 5

723. Which of the following statement is true?
(i) The amplitude of the product of the two complex numbers is equal to product of their amplitudes
(ii) For any polynomial $f(x)$ with real coefficients imaginary rosts always occur in conjugate pairs
(iii) Order relation exists in complex numbers whereas it does not exist in real numbers
(iv) The values of ω used as a cube root of unity and as a fourth root of unity are different
a) (i) and (ii) only b) (i) and (iv) only c) (iii) and (ii)only d) (i), (ii) and (iv) only

724. The solution of the equation
$(3+2\sqrt{2})^{x^2-8} + (3+2\sqrt{2})^{8-x^2} = 6$ are
a) $3 \pm 2\sqrt{2}$ b) ± 1 c) $\pm 3\sqrt{3}, \pm 2\sqrt{2}$ d) $\pm 3, \pm\sqrt{7}$

725. The value of a for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a-1)x + 2 = 0$ is twice as large as the other, is
a) 2/3 b) -2/3 c) 1/3 d) -1/3

726. Given that $\tan A$ and $\tan B$ are the roots of $x^2 - px + q = 0$, then the value of $\sin^2(A+B)$ is
a) $\frac{p^2}{p^2(1-q)^2}$ b) $\frac{q^2}{p^2+q^2}$ c) $\frac{q^2}{p^2-(1-q^2)}$ d) $\frac{p^2}{p^2+q^2}$

727. If square root of $-7 + 24i$ is $x + iy$, then x is
a) ± 1 b) ± 2 c) ± 3 d) ± 4

728. If the points z_1, z_2, z_3 are the vertices of an equilateral triangle in the Argand plane, then which one of the following is not correct?
a) $\frac{1}{z_1 - z_2} + \frac{1}{z_2 - z_3} + \frac{1}{z_3 - z_1} = 0$
b) $z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$
c) $(z_1 - z_2)^2 + (z_2 - z_3)^2 + (z_3 - z_1)^2 = 0$
d) $z_1^3 + z_2^3 + z_3^3 + 3\,z_1z_2z_3 = 0$

729. If $\left(\frac{1+i}{1-i}\right)^x = 1$, then

a) $x = 4n$, where n is any positive integer
b) $x = 2n$, where n is any positive integer
c) $x = 4n + 1$, where n is any positive integer
d) $x = 2n + 1$, where n is any positive integer

730. If $z = x + iy$ is a variable complex number such that $\arg\frac{z-1}{z+1} = \frac{\pi}{4}$, then

a) $x^2 - y^2 - 2x = 1$ b) $x^2 + y^2 - 2x = 1$ c) $x^2 + y^2 - 2y = 1$ d) $x^2 + y^2 + 2x = 1$

731. If $\sin\alpha, \cos\alpha$ are the roots of the equation $ax^2 + bx + c = 0$, then

a) $a^2 - b^2 + 2ac = 0$ b) $(a - c)^2 = b^2 + c^2$ c) $a^2 + b^2 - 2ac = 0$ d) $a^2 + b^2 + 2ac = 0$

732. Argument of the complex number $\left(\frac{-1-3i}{2+i}\right)$ is

a) 45° b) 135° c) 225° d) 240°

733. If the equation $ax^2 + 2\,bx - 3\,c = 0$ has no real roots and $\frac{3\,c}{4} < a + b$, then

a) $c < 0$ b) $c > 0$ c) $c \geq 0$ d) $c = 0$

734. If the roots of the equation $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$ are equal in magnitude but opposite in sign, then their product is

a) $\frac{1}{2}(a^2 + b^2)$ b) $-\frac{1}{2}(a^2 + b^2)$ c) $\frac{1}{2}ab$ d) $-\frac{1}{2}ab$

735. The conjugate of the complex number $\frac{(1+i)^2}{1-i}$ is

a) $1 - i$ b) $1 + i$ c) $-1 + i$ d) $-1 - i$

736. The complex number $z = x + iy$, which satisfy the equation $\left|\frac{z-5i}{z+5i}\right| = 1$ lies on

a) Real axis
b) The line $y = 5$
c) A Circle passing through the origin
d) None of the above

737. The equation $2\cos^2\left(\frac{x}{2}\right)\sin^2 x = x^2 + \frac{1}{x^2}, 0 \leq x \leq \frac{\pi}{2}$ has

a) No real solution
b) One real solution
c) More than one real solution
d) None of these

738. If $z_n = \cos\left\{\frac{\pi}{n(n+1)(n+2)}\right\} + i\sin\left\{\frac{\pi}{n(n+1)(n+2)}\right\}$ for $n = 1,2,3, \dots$, then the valu of $\lim\,(z_1 z_2 \dots, z_n)$ is

a) $\frac{1-i}{\sqrt{2}}$ b) $\frac{-1+i\sqrt{3}}{\sqrt{2}}$ c) $\frac{-1-i\sqrt{3}}{\sqrt{2}}$ d) $\frac{1+i}{\sqrt{2}}$

739. If $x + iy = \sqrt{\frac{a+ib}{c+id}}$, then $x^2 + y^2$ is equal to

a) $\frac{a^2 - b^2}{c^2 + d^2}$ b) $\frac{a^2 + b^2}{c^2 + d^2}$ c) $\frac{a^2 + b^2}{c^2 - d^2}$ d) None of these

740. $\arg(\bar{z})$ is equal to

a) $\pi - \arg(z)$ b) $2\pi - \arg(z)$ c) $\pi + \arg(z)$ d) $2\pi + \arg(z)$

741. Consider the following statements :

I. The points having affixes $\square_1, \square_2, \square_3$ from an equilateral triangle, iff
$$\frac{1}{\square_1 - \square_2} + \frac{1}{\square_2 - \square_3} + \frac{1}{\square_3 - \square_1} = 0$$
II. If $\square$ is a complex number, then $\square^{\square}$ is periodic.
III. If $|\square_1| = |\square_2|$ and $\arg\left(\frac{\square_1}{\square_2}\right) = \square$, then $\square_1 + \square_2 = 0$.

Which of the statements given above are correct?

a) (1)and (2) b) (2)and (3) c) (3)and (1) d) All (1), (2) and (3)

742. The joint of $z_1 = a + i\,b$ and $z_2 = \frac{1}{-a+i\,b}$ passes through

a) Origin b) $z = 1 + i\,0$ c) $z = 0 + i$ d) $z = 1 + i$

743. The equation $(\cos p - 1)x^2 + \cos p\,x + \sin p = 0$, in variable x, has real roots. Then, p belongs to the interval

a) $(0, 2\pi)$ b) $(-\pi, 0)$ c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ d) $(0, \pi)$

744. If the roots of the equation $x^2 + a^2 = 8x + 6a$ are real, then a belongs to the interval

a) $[2, 8]$ b) $[-2, 8]$ c) $[-8, 2]$ d) None of these

745. If $z_1 = 1 + 2i$ and $z_2 = 3 + 5i$, then Re $[\bar{z}_2 z_1/z_2]$ is equal to

a) -31/17 b) 17/22 c) -17/31 d) 22/17

746. The value of $\log_2 \log_3 \dots \log_{100} 100^{99^{98^{\cdot^{\cdot^{2^1}}}}}$ is equal to

a) 0 b) 1 c) 2 d) 100!

747. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is

a) (-3, 3) b) $(-3, \infty)$ c) $(3, \infty)$ d) $(-\infty, -3)$

748. If z_1 and z_2 are two complex numbers such that $\left|\frac{z_1 - z_2}{z_1 + z_2}\right| = 1$, then

a) $z_1 = k\, z_2, k \in R$ b) $z_1 = i\, k\, z_2, k \in R$ c) $z_1 = z_2$ d) None of these

749. Let α and β be two fixed non-zero complex numbers and $'z'$ a variable complex number. If the lines $\alpha\bar{z} + \bar{\alpha}z + 1 = 0$ and $\beta\,\bar{z} + \bar{\beta}\,z - 1 = 0$ are mutually perpendicular, then

a) $\alpha\,\beta + \bar{\alpha}\,\bar{\beta} = 0$ b) $\alpha\,\beta - \bar{\alpha}\,\bar{\beta} = 0$ c) $\bar{\alpha}\,\beta - \alpha\,\bar{\beta} = 0$ d) $\alpha\,\bar{\beta} + \bar{\alpha}\,\beta = 0$

750. If b and c are odd integers, then the equation $x^2 + bx + c = 0$ has

a) Two odd roots

b) Two integer roots, one odd and one even

c) No integer roots

d) None of these

751. Consider the following statements:

1. If the ratio of roots of the quadratic equation $ax^2 + bx + c = 0$ be $p{:}\,q$, then $pqb^2 = (p + q)^2 ac$.

2. If the roots of $ax^2 + bx + c = 0$ are α and β, then the roots of $cx^2 + bx + a = 0$ will be $\frac{1}{\alpha}, \beta$.

3. The roots of the equation $ax^2 + bx + c = 0$ are reciprocal to $a'x^2 + b'x + c' = 0$, if $(cc' - aa')^2 = (ba' - cb')(ab' - bc')$.

Which of the statements given above are correct?

a) (1) and (2) b) (2) and (3) c) (1) and (3) d) All (1), (2) and (3)

752. Locus of z, if $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, is

a) A circle b) A semi circle c) A straight line d) None of these

753. If x_1, x_2, x_3 are distinct roots of the equation $ax^2 + bx + c = 0$, then

a) $a = b = 0, c \in R$ b) $a = c = 0, b \in R$ c) $b^2 - 4ac \geq 0$ d) $a = b = c = 0$

754. If $(1 - p)$ is a root of quadratic equation $x^2 + px + (1 - p) = 0$, then its roots are

a) 0, 1 b) -1, 1 c) 0, -1 d) -1, 2

755. If ω is a complex cube root of unity, then the value of $\omega^{99} + \omega^{100} + \omega^{101}$ is

a) 1 b) -1 c) 3 d) 0

756. The value of $'c'$ for which $|\alpha^2 - \beta^2| = 7/4$, where α and β are the roots of $2\,x^2 + 7\,x + c = 0$, is

a) 4 b) 0 c) 6 d) 2

757. If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$, then $\cos 3\alpha + \cos 3\beta + \cos 3\gamma$ equals

a) 0 b) $\cos(\alpha + \beta + \gamma)$ c) $3\cos(\alpha + \beta + \gamma)$ d) $3\sin(\alpha + \beta + \gamma)$

758. If z_2 and z_2 are two nth roots of unity, then $\arg\left(\frac{z_1}{z_2}\right)$ is a multiple of

a) $n\pi$ b) $\frac{3\pi}{n}$ c) $\frac{2\pi}{n}$ d) None of these

759. If the roots of $a_1x^2 + b_1x + c_1 = 0$ are α_1, β_1 and those of $a_2x^2 + b_2x + c_2 = 0$ are $\alpha_2\ \beta_2$ such that $\alpha_1\,\alpha_2 = \beta_1\,\beta_2 = 1$, then

a) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ b) $\frac{a_1}{c_2} = \frac{b_1}{b_2} = \frac{c_1}{a_2}$ c) $a_1a_2 = b_1b_2 = c_1c_2$ d) None of these

760. If $a + b + c = 0$, then the roots of the equation $4ax^2 + 3bx + 2c = 0$ are

a) Equal b) Imaginary c) Real d) None of these

761. If z is a complex number in the Argand plane such that $\arg\left(\frac{z-3\sqrt{3}}{z+3\sqrt{3}}\right)=\frac{\pi}{3}$ then the lous of z is

a) $|z-3i|=6$

b) $|z-3i|=6, \text{Im}\,(z)>0$

c) $|z-3i|=6, \text{Im}\,(z)<0$

d) None of these

762. If $\sin\alpha$ and $\cos\alpha$ are the roots of the equation $px^2+qx+r=0$, then

a) $p^2+q^2-2pr=0$ b) $p^2-q^2+2pr=0$ c) $p^2-q^2-2pr=0$ d) $p^2+q^2+2qr=0$

763. The equation $x^{\frac{3}{4}(\log_2 x)^2+(\log_2 x)-\frac{5}{4}}=\sqrt{2}$ has

a) At least one real solution b) Exactly three real solution

c) Exactly one irrational solution d) All of the above

764. If $z=x+i\,y$, then the equation $\left|\frac{2\,z-1}{z+1}\right|=m$ does not represent a circle when $m=$

a) 1/2 b) 1 c) 2 d) 3

765. Let $z=x+iy$ be a complex number where x and y are integers. Then the area of the rectangle whose vertices are the roots of the equation $z\bar{z}^3+\bar{z}z^3=350$ is

a) 48 b) 32 c) 40 d) 80

766. If $(a_1+i\,b_1)(a_2+i\,b_2)\dots(a_n+i\,b_n)=A+i\,B$, then $(a_1^2+b_1^2)(a_2^2+b_2^2)\dots(a_n^2+b_n^2)$ is equal to

a) 1 b) A^2+B^2 c) $A+B$ d) $\frac{1}{A^2}+\frac{1}{B^2}$

767. The complex numbers $\sin x+i\cos 2x$ and $\cos x-i\sin 2x$ are conjugate to each other for

a) $x=n\pi$ b) $x=\left(n+\frac{1}{2}\right)\pi$ c) $x=0$ d) No value of x

768. The number which exceeds its positive square roots by 12, is

a) 9 b) 16 c) 25 d) None of these

769. The solution set of the inequation $\frac{|x-2|}{x-2}<0$, is

a) $(2,\infty)$ b) $(-\infty,2)$ c) R d) $(-2,2)$

770. The product of all values of $(\cos\alpha+i\sin\alpha^{3/5})$ is

a) 1 b) $\cos\alpha+i\sin\alpha$

c) $\cos 3\alpha+i\sin 3\alpha$ d) $\cos 5\alpha+i\sin 5\alpha$

771. If $a=\cos\alpha+i\sin\alpha, b=\cos\beta+i\sin\beta, c=\cos\gamma+i\sin\gamma$ and $\frac{b}{c}+\frac{c}{a}+\frac{a}{b}=1$, then $\cos(\beta-\gamma)+$ cosγ−α+cos(α−β) is equal to

a) $\frac{3}{2}$ b) $-\frac{3}{2}$ c) 0 d) 1

772. If $\frac{x-4}{x^2-5x+6}$ can be expanded in the ascending powers of x, then the coeficient of x^3 is

a) $-\frac{73}{648}$ b) $\frac{73}{648}$ c) $\frac{71}{648}$ d) $-\frac{71}{648}$

773. If $a=\cos\theta+i\sin\theta$, then $\frac{1+a}{1-a}$ is equal to

a) $i\cot\frac{\theta}{2}$ b) $i\tan\frac{\theta}{2}$ c) $i\cos\frac{\theta}{2}$ d) $i\,\text{cosec}\frac{\theta}{2}$

774. The points in the set $\left\{z\in C: \arg\left(\frac{z-2}{z-6i}\right)=\frac{\pi}{2}\right\}$ (where C denotes the set of all complex numbers) lie on the curve which is a

a) Circle b) Pair of lines c) Parabola d) Hyperbola

775. The number of solution of $\log_4(x-1)=\log_2(x-3)$ is

a) 3 b) 1 c) 2 d) 0

776. If $\cos\alpha+2\cos\beta+3\cos\gamma=\sin\alpha+2\sin\beta+3\sin\gamma=0$, then the value of $\sin 3\alpha+8\sin 3\beta+27\sin 3\gamma$ is

a) $\sin(\alpha+\beta+\gamma)$ b) $3\sin(\alpha+\beta+\gamma)$ c) $18\sin(\alpha+\beta+\gamma)$ d) $\sin(\alpha+2\beta+3\gamma)$

777. If $f(x)=\sum_{k=2}^{n}\left(x-\frac{1}{k-1}\right)\left(x-\frac{1}{k}\right)$, then the product of roots of $f(x)=0$ as $n\to\infty$, is

a) -1 b) 0 c) 1 d) None of these

778. If $3p^2 = 5p + 2$ and $3q^2 = 5q + 2$ where $p \neq q$, then the equation whose roots are $3p - 2q$ and $3q - 2p$ is
a) $3x^2 - 5x - 100 = 0$
b) $5x^2 + 3x + 100 = 0$
c) $3x^2 - 5x + 100 = 0$
d) $3x^2 + 5x - 100 = 0$

779. If $a, b, c \in R$ and the equations $ax^2 + bx + c = 0$ and $x^3 + 3x^2 + 3x + 2 = 0$ have two roots in common, then
a) $a = b \neq c$ b) $a = b = -c$ c) $a = b = c$ d) None of these

780. Which of the following is a fourth root of $\frac{1}{2} + i\frac{\sqrt{3}}{2}$?
a) $\text{cis}\frac{\pi}{12}$ b) $\text{cis}\frac{\pi}{2}$ c) $\text{cis}\frac{\pi}{6}$ d) $\text{cis}\frac{\pi}{3}$

781. Number of integer roots of the equation $(x + 2)(x + 3)(x + 8)(x + 12) = 4x^2$ is
a) 0 b) 4 c) 2 d) None of these

782. If the roots of $ax^2 + bx + c = 0$ are α, β and the roots of $Ax^2 + Bx + C = 0$ are $\alpha - k, \beta - k$, then $\frac{B^2-4AC}{b^2-4ac}$ is equal to
a) 0 b) 1 c) $\left(\frac{A}{a}\right)^2$ d) $\left(\frac{a}{A}\right)^2$

783. If $\frac{5z^2}{11z_1}$ is purely imaginary, then the value of $\left|\frac{2z_1+3z_2}{2z_1-3z_2}\right|$ is
a) 37/33 b) 2 c) 1 d) 3

784. If a root of the equation $ax^2 + bx + c = 0$ be reciprocal of a root of the equation $a'x^2 + b'x + c' = 0$, then
a) $(cc' - aa')^2 = (ba' - cb')(ab' - bc')$
b) $(bb' - aa')^2 = (ca' - bc')(ab' - bc')$
c) $(cc' - aa')^2 = (ba' + cb')(ab' + bc')$
d) None of the above

785. If $|z + 8| + |z - 8| = 16$, where z is a complex number, then the point z will lie on
a) A circle b) An ellipse c) A straight line d) None of these

786. If one root of the equation $x^2 + (1 - 3i)x - 2(1 + i) = 0$ is $-1 + i$, then the other root is
a) $-1 - i$ b) $\frac{-1 - i}{2}$ c) i d) $2i$

787. The sum of non-real roots of the equation
$(x^2 + x - 2)(x^2 + x - 3) = 12$, is
a) 1 b) -1 c) -6 d) 6

788. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of q is
a) 49/4 b) 4/49 c) 4 d) None of these

789. If $\frac{3(x-2)}{5} \geq \frac{5(2-x)}{3}$, then x belongs to the interval
a) $(2, \infty)$ b) $[2, \infty)$ c) $(-\infty, 2]$ d) None of these

790. If $ax^2 + bx - c$ is divisible by $x^2 + bx + c$, then $'a'$ is a root of the equation
a) $cx^2 - bx - 1 = 0$ b) $ax^2 - bx - 1 = 0$ c) $bx^2 - ax - 1 = 0$ d) None of these

791. If p and q are the roots of the equation $x^2 + px + q = 0$, then
a) $p = 1$ b) $p = 1$ or 0 c) $p = -2$ d) $p = -2$ or 0

792. If $4 \leq x \leq 9$, then
a) $(x - 4)(x - 9) \leq 0$ b) $(x - 4)(x - 9) \geq 0$ c) $(x - 4)(x - 9) < 0$ d) $(x - 4)(x - 9) > 0$

793. The number of real solutions of the equation
$\frac{6 - x}{x^2 - 4} = 2 + \frac{x}{x + 2}$, is
a) 1 b) 2 c) 0 d) None of these

794. For any complex number z, the minimum value of $|z| + |z - 1|$ is
a) 0 b) 1 c) 2 d) -1

795. The difference between two roots of the equation $x^3 - 13x^2 + 15x + 189 = 0$ is 2. Then the roots of the equation are
a) -3, 7, 9 b) -3, -7, -9 c) 3, -5, 7 d) -3, -7, 9

796. If $|z+4| \leq 3$, then the greatest and the least value of $|z+1|$ are respectively
a) 6, -6 b) 6, 0 c) 7, 2 d) 0, -1

797. The number of roots of the equation $x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$ is
a) 1 b) 2 c) 0 d) Infinitely many

798. The equation $z\bar{z} + (4-3i)z + (4+3i)\bar{z} + 5 = 0$ represents a circle of radius
a) 5 b) $2\sqrt{5}$ c) 5/2 d) None of these

799. The number of solutions for the equation $x^2 - 5|x| + 6 = 0$ is
a) 4 b) 3 c) 2 d) 1

800. If one root is square of the root of the equation $x^2 + px + q = 0$, then the relation between p and q is
a) $p^3 - (3p-1)q + q^2 = 0$ b) $p^3 - q(3p+1) + q^2 = 0$
c) $p^3 + q(3p-1) + q^2 = 0$ d) $p^3 + q(3p+1) + q^2 = 0$

801. If the roots of the equation $8x^3 - 14x^2 + 7x - 1 = 0$ are in GP, then the roots are
a) $1, \frac{1}{2}, \frac{1}{4}$ b) 2, 4, 8 c) 3, 6, 12 d) None of these

802. The values of x and y such that y satisfy the equation ($x, y \in$ real numbers) $x^2 - xy + y^2 - 4x - 4y + 16 = 0$ is
a) 4, 4 b) 3, 3 c) 2, 2 d) None of these

803. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then the equation whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$, is
a) $ac\,x^2 + (a+c)bx + (a+c)^2 = 0$ b) $ab\,x^2 + (a+c)bx + (a+c)^2 = 0$
c) $ac\,x^2 + (a+b)cx + (a+c)^2 = 0$ d) None of these

804. The real part of $\left[1 + \cos\left(\frac{\pi}{5}\right) + i\sin\left(\frac{\pi}{5}\right)\right]$ is
a) 1 b) $\frac{1}{2}$ c) $\frac{1}{2}\cos\left(\frac{\pi}{10}\right)$ d) $\frac{1}{2}\cos\left(\frac{\pi}{5}\right)$

805. If $1, \omega, \omega^2$ are the cube roots of unity, then
$\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to
a) 0 b) 1 c) ω d) ω^2

806. If both the roots of the equation $ax^2 + bx + c = 0$ are zero, then
a) $b = c = 0$ b) $b = 0, c \neq 0$ c) $b \neq 0, c = 0$ d) $b \neq 0, c \neq 0$

807. If the roots of the equation $x^2 - px + q = 0$ differ by unity, then
a) $p^2 = 4q$ b) $p^2 = 4q + 1$ c) $p^2 = 4q - 1$ d) None of these

808. If $|z| = 1$ and $w = \frac{z-1}{z+1}$ (where $z \neq -1$), then Re(w) is
a) 0 b) $\frac{1}{|z+1|^2}$ c) $\left|\frac{1}{z+1}\right| \cdot \frac{1}{|z+1|^2}$ d) $\frac{\sqrt{2}}{|z+1|^2}$

809. The least positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = \frac{2}{\pi}\sin^{-1}\left(\frac{1+x^2}{2x}\right)$, where $x > 0$, is
a) 2 b) 4 c) 8 d) 12

810. If α, β, γ are the roots of the equation $x^3 + x + 1 = 0$, then the value of $\alpha^3 + \beta^3 + \gamma^3$ is
a) 0 b) 3 c) -3 d) -1

811. Let z be a complex number (not lying on x-axis) of maximum modulus such $\left|z + \frac{1}{z}\right| = 1$. Then,
a) Im $(z) = 0$ b) Re $(z) = 0$ c) amp $(z) = \pi$ d) None of these

812. If $x = c$ is a root of order 2 of a polynomial $f(x)$, then $x = c$ is also a root of the polynomial
a) $f'(x)$ b) $f''(x)$ c) $f'''(x)$ d) None of these

813. If $\cos\alpha + 2\cos\beta + 3\cos\gamma = \sin\alpha + 2\sin\beta + 3\sin\gamma = 0$ and $\alpha + \beta + \gamma = 0$, then $\cos 3\alpha + 8\cos 3\beta + 27\cos 3\gamma =$
a) 0 b) 3 c) 18 d) -18

814. If ω is a complex cube root of unity, then the value of $\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} + \frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}$ is

a) 1 b) 0 c) 2 d) -1

815. Let $f(x) = ax^2 + bx + c, a \neq 0$ and $\Delta = b^2 - 4ac$. If $\alpha + \beta, \alpha^2 + \beta^2$ and $\alpha^3 + \beta^3$ are in GP, then

a) $\Delta \neq 0$ b) $b\Delta = 0$ c) $c\Delta = 0$ d) $bc \neq 0$

816. If $i = \sqrt{-1}$ and n is a positive integer, then $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is equal to

a) 1 b) i c) i^n d) 0

817. The additive inverse of $1 - i$ is

a) $0 + 0i$ b) $-1 + i$ c) $-1 + i$ d) None of these

818. The equation $x^2 - 2\sqrt{2}kx + 2e^{2\log k} - 1 = 0$ has the product of roots equal to 31, then for what value of k it has real roots?

a) 1 b) 2 c) 3 d) 4

819. The locus of the point z which satisfy the condition $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{3}$, is

a) A straight line b) A circle c) A parabola d) None of these

820. The complex number $\frac{(-\sqrt{3}+3i)(1-i)}{(3+\sqrt{3}i)(i)(\sqrt{3}+\sqrt{3}i)}$ when represented in the Argand daigram is

a) In the second quadrant b) In the first quadrant
c) On the y-axis (imaginary axis) d) On the x-axis (real axis)

821. If a, b, c are in G.P., then the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in

a) A.P. b) G.P. c) H.P. d) None of these

822. The value of $\sqrt{8 + 2\sqrt{8 + 2\sqrt{8 + 2\sqrt{2}}}}$, is

a) 10 b) 6 c) 8 d) 4

823. Let α, β be the roots of $x^2 - 2x\cos\phi + 1 = 0$, then the equation whose roots are α^n, β^n, is

a) $x^2 - 2x\cos n\phi - 1 = 0$ b) $x^2 - 2x\cos n\phi + 1 = 0$
c) $x^2 - 2x\sin n\phi + 1 = 0$ d) $x^2 + 2x\sin n\phi - 1 = 0$

824. $\frac{(\cos\theta + i\sin\theta)^4}{(\sin\theta + i\cos\theta)^5}$ is equal to

a) $\cos\theta - i\sin\theta$ b) $\sin\theta - i\cos\theta$ c) $\cos 9\theta - i\sin 9\theta$ d) $\sin 9\theta - i\cos 9\theta$

825. The equations $ax^2 + bx + a = 0$ and $x^3 - 2x^2 + 2x - 1 = 0$ have 2 roots in common. Then, $a + b$ must be equal to

a) 1 b) -1 c) 0 d) None of these

826. If the roots of the equation $(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2) = 0$ are equal then a, b, c are in

a) G.P. b) A.P. c) H.P. d) None of these

827. If at least one root of the equation $x^3 + ax^2 + bx + c = 0$ remains unchanged, when a, b and c are decreased by one, then which one of the following is always a root of the given equation?

a) 1 b) -1
c) ω, an imaginary cube root of unity d) i

828. If $\text{Re}\left(\frac{z-8i}{z+6}\right) = 0$, then z lies on the curve

a) $x^2 + y^2 + 6x - 8y = 0$
b) $4x - 3y + 24 = 0$
c) $x^2 + y^2 - 8 = 0$
d) None of these

829. If $\alpha \neq \beta$ and $\alpha^2 = 5\alpha - 3, \beta^2 = 5\beta - 3$, then the equation having α/β and β/α as its roots is

a) $3x^2 + 19x + 3 = 0$ b) $3x^2 - 19x + 3 = 0$ c) $3x^2 - 19x - 3 = 0$ d) $x^2 - 16x + 1 = 0$

830. If the cube roots of unity are $1, \omega, \omega^2$, then the roots of the equation $(x - 2)^3 + 27 = 0$ are

a) -1, -1, -1 b) $-1, -\omega, -\omega^2$ c) $-1, 2 + 3\omega, 2 + 3\omega^2$ d) $-1, 2 - 3\omega, 2 - 3\omega^2$

831. If the equation $2x^2 + 3x + 5\lambda = 0$ and $x^2 + 2x + 3\lambda = 0$ have a common root, then λ is equal to

a) 0 b) -1 c) $0, -1$ d) $2, -1$

832. If z is a complex number in the Argand plane, then the equation $|z - 2|^2 + |z + 2|^2 = 8$ represents

a) A parabola b) An ellipse c) A hyperbola d) A circle

833. Let α and β be the roots of the equation $x^2 + x + 1 = 0$. The equation whose roots are α^{19}, β^7 is
a) $x^2 - x - 1 = 0$ b) $x^2 - x + 1 = 0$ c) $x^2 + x - 1 = 0$ d) $x^2 + x + 1 = 0$

834. If α and β are the roots of the equation $x^2 - 6x + a = 0$ and satisfy the relation $3\alpha + 2\beta = 16$, then the value of a is
a) -8 b) 8 c) -16 d) 9

835. The values of x satisfying $|x - 4| + |x - 9| = 5$ is
a) $x = 4, 9$ b) $4 \leq x \leq 9$ c) $x \leq 4$ or $x \geq 9$ d) None of these

836. Let $a_n = i^{(n+1)^2}$, where $i = \sqrt{-1}$ and $n = 1, 2, 3, \ldots$Then the value of $a_1 + a_3 + a_5 + \ldots + a_{25}$ is
a) 13 b) $13 + i$ c) $13 - i$ d) 12

837. For $n = 6\,k, k \in Z, \left(\frac{1-i\sqrt{3}}{2}\right)^n + \left(\frac{-1-i-\sqrt{3}}{2}\right)^n$ has the value
a) -1 b) 0 c) 1 d) 2

838. A value of n such that $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^n = 1$ is
a) 12 b) 3 c) 2 d) 1

839. The number of integral solutions of $\frac{x+1}{x^2+2} > \frac{1}{4}$, is
a) 1 b) 2 c) 5 d) None of these

840. If α and β are the roots of $x^2 + 5x + 4 = 0$, then the equation whose roots are $\frac{\alpha+2}{3}, \frac{\beta+2}{3}$ is
a) $9x^2 + 3x + 2 = 0$ b) $9x^2 - 3x - 2 = 0$ c) $9x^2 + 3x - 2 = 0$ d) $9x^2 - 3x + 2 = 0$

841. Real roots of the equation $k, x^2 + 5|x| + 4 = 0$ are
a) 1, -1 b) 2, 0 c) 0, 1 d) None of these

842. If α and β are the roots of the equation $ax^2 + bx + c = 0$, then $(1 + \alpha + \alpha^2)\,(1 + \beta + \beta^2)$ is equal to
a) Zero b) Positive c) Negative d) None of these

843. If $\alpha + i\,\beta = \tan^{-1}(z), z = x + i\,y$ and α is constant, the locus of $'z'$ is
a) $x^2 + y^2 + 2\,x \cot 2\,\alpha = 1$
b) $\cot 2\,\alpha(x^2 + y^2) = 1 + x$
c) $x^2 + y^2 + 2y \tan 2\,\alpha = 1$
d) $x^2 + y^2 + 2\,x \sin 2\,\alpha = 1$

844. Both the roots of the given equation
$(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$ are always
a) Positive b) Negative c) Real d) Imaginary

845. The roots of $4x^2 + 6px + 1 = 0$ are equal, then the value of p is
a) 4/5 b) 1/3 c) $\pm 2/3$ d) 4/3

846. The complex number z satisfies the condition $\left|z - \frac{25}{z}\right| = 24$. The maximum distance from the origin of coordinates to the point z is
a) 25 b) 30 c) 32 d) None of these

847. If $(x + 1)$ is a factor of $x^4 - (p - 3)x^3 - (3p - 5)x^2 + (2p - 7)x + 6$, then the value of p is
a) 4 b) 2 c) 1 d) None of these

848. If a, b, c are real and $x^3 - 3b^2x + 2c^3$ is divisible by $x - a$ and $x - b$, then
a) $a = -b = -c$
b) $a = 2b = 2c$
c) $a = b = c$ or $a = -2b = -2c$
d) None of these

849. If a, b, c are in A.P. and if $(b - c)x^2 + (c - a)x + a - b = 0$ and $2(c + a)x^2 + (b + c)x = 0$ have a common root then
a) a^2, b^2, c^2 are in A.P. b) a^2, c^2, b^2 are in A.P. c) a^2, c^2, b^2 are in G.P. d) None of these

850. Let z and w be two complex numbers such that $|z| \leq 1$, $|w| \leq 1$ and $|z + iw| = |z - \overline{iw}| = 2$. Then, z is equal to

a) 1 or i b) i or $-i$ c) 1or -1 d) i or -1

851. If $\log_{\tan 30°}\left(\frac{2|z|^2+2|z|-3}{|z|+1}\right) < -2$, then

a) $|z| < 3/2$ b) $|z| > 3/2$ c) $|z| > 2$ d) $|z| < 2$

852. If $x^2 + px + 1$ is a factor of the expression $ax^3 + bx + c$, then

a) $a^2 + c^2 = -ab$ b) $a^2 - c^2 = -ab$ c) $a^2 - c^2 = ab$ d) None of these

853. If z_1, z_2 are two complex numbers such that Im $(z_1 + z_2) = 0$, Im $(z_1z_2) = 0$, then

a) $z_1 = -z_2$ b) $z_1 = z_2$ c) $z_1 = \sqrt{z}_2$ d) None of these

854. The system $y^{(x^2+7x+12)} = 1$ and $x + y = 6, y > 0$ has

a) No solution
b) One solution
c) Two solution
d) More than 2 solutions

855. The set of all real values of x for which $\frac{8x^2+16x-51}{(2x-3)(x+4)} < 3$, is

a) $(3/2, 5/2)$ b) $(-4, -3)$ c) $(-4, -3) \cup (3/2, 5/2)$ d) None of these

856. If $\omega(\neq 1)$ be a cube root of unity and $(1 + \omega^2)^n = (1 + \omega^4)^n$, then the least positive value of n is

a) 2 b) 3 c) 5 d) 6

857. How many roots of the equation $x - \frac{1}{x-1} = 1 - \frac{2}{x-1}$ have?

a) One b) Two c) Infinite d) None of these

858. If g (x) and $h(x)$ are two polynomials such that the polynomial $P(x) = \text{g}(x^3) + x\, h(x^3)$ is divisible by $x^2 + x + 1$, then which one of the following is not true?

a) $\text{g}(1) = h(1) = 0$ b) $\text{g}(1) = h(1) \neq 0$ c) $\text{g}(1) = -h(1)$ d) $\text{g}(1) + h(1) = 0$

859. The maximum number of real roots of the equation $x^{2n} - 1 = 0$ is

a) 2 b) 3 c) n d) $2n$

860. Given that 'a' is a fixed complex number, and λ' is a scalar variable, the point z satisfying $z = a(1 + i\lambda)$ traces out

a) A straight line through the point $'a'$
b) A circle with centre at the point $'a'$
c) A straight line through the point $'a'$ and perpendicular to the join 0 and that point $'a'$
d) None of these

861. The complex numbers z_1, z_2, z_3 are the vertices of a triangle. Then the complex number z which makes the triangle into a parallelogram, is

a) $z_1 + z_2 - z_3$ b) $z_1 - z_2 + z_3$ c) $z_2 + z_3 - z_1$ d) All of these

862. If a and b are the non-zero distinct roots of $x^2 + ax + b = 0$, then the least value of $x^2 + ax + b$ is

a) $\frac{2}{3}$ b) $\frac{9}{4}$ c) $-\frac{9}{4}$ d) 1

863. If z_1, z_2 are two complex numbers satisfying $\left|\frac{z_1+3z_2}{3-z_1\overline{z_2}}\right| = 1, |z_1| \neq 3$, then $|z_2|$ is equal to

a) 1 b) 2 c) 3 d) 4

864. The value of the determinant $\begin{vmatrix} 1+i & 1-i & i \\ 1-i & i & 1+i \\ i & 1+i & 1-i \end{vmatrix}$, where $i = \sqrt{-1}$ is

a) $7 + 4i$ b) $7 - 4i$ c) $4 + 7i$ d) $4 - 7i$

865. If $w = \frac{z}{z-\frac{1}{3}i}$ and $|w| = 1$, then z lies on

a) A parabola b) A straight line c) A circle d) An ellipse

866. The value of $\frac{\log_3 5 \times \log_{25} 27 \times \log_{49} 7}{\log_{81} 3}$ is

a) 1 b) 6 c) $\frac{2}{3}$ d) 3

867. The value of $'k'$ for which one of the roots of $x^2 - x + 3\,k = 0$, is double of one of the roots of $x^2 - x + k = 0$ is

a) 1 b) -2 c) 2 d) None of these

868. If $a < b < c < d$, then the roots of the equation $(x-a)(x-c)+2(x-b)(x-d)=0$ are
a) Real and distinct b) Real and equal c) Imaginary d) None of these

869. If $|z| = \max\{|z-2|, |z+2|\}$, then
a) $|z+\bar{z}| = 1$ b) $|z+\bar{z}| = 4$ c) $|z+\bar{z}| = 2$ d) None of these

870. If α, β, γ are the roots of $x^3+2x^2-3x-1=0$, then $\alpha^{-2}+\beta^{-2}+\gamma^{-2}$ is equal to
a) 12 b) 13 c) 14 d) 15

871. The magnitude and amplitude of $\frac{(1+i\sqrt{3})(2+2i)}{(\sqrt{3}-i)}$ are respectively
a) $2, \frac{3\pi}{4}$ b) $2\sqrt{2}, \frac{3\pi}{4}$ c) $2\sqrt{2}, \frac{\pi}{4}$ d) $2\sqrt{2}, \frac{\pi}{2}$

872. If $m \in Z$ and the equation $m\,x^2+(2\,m-1)x+(m-2)=0$ has rational roots, then m is of the form
a) $n(n+2), n \in Z$ b) $n(n+1), n \in Z$ c) $n(n-2), n \in Z$ d) None of these

873. For three complex numbers $1-i, i, 1+i$ which of the following is true?
a) They form a right triangle b) They are collinear
c) They form an equilateral triangle d) They form an isosceles triangle

874. The triangle formed by the points $1, \frac{1+i}{\sqrt{2}}$ and i as vertices in the Argand diagrams is
a) Scalene b) Equilateral c) Isosceles d) Right-angled

875. The minimum value of $|a+b\omega+c\omega^2|$, where a, b and c are all not equal integers and $\omega(\neq 1)$ is a cube root of unity, is
a) $\sqrt{3}$ b) 1/2 c) 1 d) 0

876. If ω is a complex cube root of unity, then for positive integral value of n, the product of $\omega.\omega^2.\omega^3 \ldots \omega^n$ will be
a) $\frac{1-i\sqrt{3}}{2}$ b) $-\frac{1-i\sqrt{3}}{2}$ c) 1 d) Both (b) and (c)

877. If the equations $k(6x^2+3)+rx+2x^2-1=0$ and $6k(2x^2+1)+px+4x^2-2=0$ have both roots common, then the value of $(2r-p)$ is
a) 0 b) 1/2 c) 1 d) None of these

878. If $\frac{3}{2+\cos\theta+i\sin\theta} = a+ib$, then $[(a-2)^2+b^2]$ is equal to
a) 0 b) 1 c) -1 d) 2

879. The centre of a square $ABCD$ is at $z=0$. A is z_1, then the centroid of the triangle ABC is
a) $z_1(\cos\pi \pm i\sin\pi)$
b) $\frac{z_1}{3}(\cos\pi \pm i\sin\pi)$
c) $z_1(\cos\pi/2 \pm i\sin\pi/2)$
d) $\frac{z_1}{3}(\cos\pi/2 \pm i\sin\pi/2)$

880. If z is a complex number, then $(\bar{z}^{-1})(\bar{z})$ is equal to
a) 1 b) -1 c) 0 d) None of these

881. If $|z^2-1| = |z|^2+1$, then z lies on
a) The real axis b) The imaginary axis c) A circle d) An ellipse

882. Let S denote the set of all real values of a for which the roots of the equation $x^2-2ax+a^2-1=0$ lie between 5 and 10, then S equals
a) $(-1, 2)$ b) $(2, 9)$ c) $(4, 9)$ d) $(6, 9)$

883. If $e^{\cos x} - e^{-\cos x} = 4$ then the value of $\cos x$ is
a) $\log(2+\sqrt{5})$ b) $-\log(2+\sqrt{5})$ c) $\log(-2+\sqrt{5})$ d) None of these

884. If z is a comple number such that $z = -z$, then
a) z is purely real b) z is purely imaginary
c) z is any complex number d) Real part of z is the same as its imaginary part

885. The condition that $x^{n+1}-x^n+1$ shall be divisible by x^2-x+1 is that

a) $n = 6k + 1$ b) $n = 6k - 1$ c) $n = 3k + 1$ d) $n = 3k - 1$

886. The value of $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^6 + \left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}}\right)^6$ is

a) 2 b) -2 c) 1 d) 0

887. If α, β and γ are the roots of $x^2 + qx + r = 0$, then $\sum \frac{\alpha}{\beta+\gamma}$, is

a) 3 b) $q + r$ c) q/r d) -3

888. Let α, α^2 be the roots of $x^2 + x + 1 = 0$, then the equation whose roots are α^{31}, α^{62} is

a) $x^2 - x + 1 = 0$ b) $x^2 + x - 1 = 0$ c) $x^2 + x + 1 = 0$ d) $x^{60} + x^{30}$+1=0

889. If $-\pi < \arg(z) < -\frac{\pi}{2}$ then $\arg(\bar{z}) - \arg(-\bar{z})$ is

a) π b) $-\pi$ c) $\pi/2$ d) $-\pi/2$

890. If $A(z_1), B(z_2)$ and $C(z_3)$ be the vertices of a triangle ABC in which $\angle ABC = \frac{\pi}{4}$ and $\frac{AB}{BC} = \sqrt{2}$, then the value of z_2 is equal to

a) $z_3 + i(z_1 + z_3)$ b) $z_3 - i(z_1 - z_3)$ c) $z_3 + i(z_1 - z_3)$ d) None of these

891. The equation $z^2 = \bar{z}$ has

a) No solution b) Two solutions
c) Four solutions d) An infinite number of solutions

892. The curve represented by $\text{Im}(z^2) = k$, where k is a non-zero real number, is

a) A pair of straight lines
b) An ellipse
c) A parabola
d) A hyperbola

893. If $(3 + i)(z + \bar{z}) - (2 + i)(z - \bar{z}) + 14i = 0$, then $z\bar{z}$ is equal to

a) 5 b) 8 c) 10 d) 40

894. If $\alpha + \beta = -2$ and $\alpha^3 + \beta^3 = -56$, then the quadratic equation whose roots are α and β is

a) $x^2 + 2x - 16 = 0$ b) $x^2 + 2x + 15 = 0$ c) $x^2 + 2x - 12 = 0$ d) $x^2 + 2x - 8 = 0$

895. The set of values of x satisfying inequations $|x - 1| \le 3$ and $|x - 1| \ge 1$, is

a) $[2, 4]$ b) $(-\infty, 2] \cup [4, \infty)$ c) $[-2, 0] \cup [2, 4]$ d) None of these

896. If z_1, z_2 are two complex numbers such that $\left|\frac{z_1 - z_2}{z_1 + z_2}\right| = 1$ and $i\, z_1 = k\, z_2$ where $k \in R$, then the angle between $z_1 - z_2$ and $z_1 + z_2$ is

a) $\tan^{-1}\left(\frac{2\,k}{k^2 + 1}\right)$ b) $\tan^{-1}\left(\frac{2\,k}{1 - k^2}\right)$ c) $-2\tan^{-1} k$ d) $2\tan^{-1} k$

897. If roots of $ax^2 + bx + c = 0, a, b, c \in R, a \ne 0$ are imaginary then

a) $ac > 0$
b) $ab > 0$
c) $bc > 0$
d) Exactly two of ab, bc and ca are positive

898. If α and β be the roots of $x^2 + px + q = 0$, then $\frac{(\omega\alpha + \omega^2\beta)(\omega^2\alpha + \omega\beta)}{\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}}$ is equal to

a) $-\frac{q}{p}$ b) $\alpha\, \beta$ c) $-\frac{p}{q}$ d) ω

899. If z_1, z_2 and z_3, z_4 are two pairs of conjugate complex numbers, then $\arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right)$ equals

a) 0 b) $\pi/2$ c) $3\,\pi/2$ d) π

900. If $x = \frac{1}{2}\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)$, then $\frac{\sqrt{x^2 - 1}}{x - \sqrt{x^2 - 1}}$ is equal to

a) 1 b) 2 c) 3 d) 1/2

901. If P, Q, R, S are represented by the complex numbers $4 + i, 1 + 6i, -4 + 3i, -1 - 2i$ respectively, then $PQRS$ is a

a) A rectangle b) A square c) A rhombus d) A parallelogram

902. If $x \in R$, the least value of the expression $\frac{x^2-6x+5}{x^2+2x+1}$, is

a) -1 b) $-1/2$ c) $-1/3$ d) None of these

903. If x^2-3x+2 be a factor of x^4-px^2+q, then (p,q) is equal to

a) $(3,4)$ b) $(4,5)$ c) $(4,3)$ d) $(5,4)$

904. The number of solutions of the equation
$2\sin(e^x)=5^x+5^{-x}$, is

a) 0 b) 1 c) 2 d) Infinitely many

905. The centre of a square is at $z=0$. A is z_1, then the centroid of the triangle ABC is

a) $z_1(\cos\pi \pm i\sin\pi)$ b) $\frac{1}{3}z_1(\cos\pi \pm i\sin\pi)$ c) $z_1\left(\cos\frac{\pi}{2} \pm i\sin\frac{\pi}{2}\right)$ d) $\frac{1}{3}z_1\left(\cos\frac{\pi}{2} \pm i\sin\frac{\pi}{2}\right)$

906. If x is real, then the maximum and minimum values of the expression $\frac{x^2-3x+4}{x^2+3x+4}$ will be

a) 2, 1 b) $5, \frac{1}{5}$ c) $7, \frac{1}{7}$ d) None of these

907. For positive integers n_1, n_2 the value of the expression $(1+i)^{n_1}+(1+i^3)^{n_1}+(1+i^5)^{n_2}+(1+i^7)^{n_2}, i=\sqrt{-1}$ is a real number if and only if

a) $n_1=n_2+1$ b) $n_1=n_2-1$ c) $n_1=n_2$ d) $n_1>0, n_2>0$

908. Let $a=\cos\frac{2\pi}{7}+i\sin\frac{2\pi}{7}$, $\alpha=a+a^2+a^4$ and $\beta=a^3+a^5+a^6$. Then, the equation whose roots are α,β is

a) $x^2-x+2=0$ b) $x^2+x-2=0$ c) $x^2-x-2=0$ d) $x^2+x+2=0$

909. If the roots of the given equation $(\cos p-1)x^2+(\cos p)x+\sin p=0$ are real, then

a) $p\in(-\pi,0)$ b) $p\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ c) $p\in(0,\pi)$ d) $p\in(0,2\pi)$

910. The set of all integral values of x for which $5x-1<(x+1)^2<7x-3$, is

a) Φ b) $\{1\}$ c) $\{2\}$ d) $\{3\}$

911. If one root of the equation $5x^2+13x+k=0$ is reciprocal of other, then the value of k is

a) 0 b) 5 c) 1/6 d) 6

912. The equation $(a+2)x^2+(a-3)x=2a-1, a\neq-2$ has rational roots for

a) All rational values of a except $a=-2$
b) All real values of a except $a=-2$
c) Rational values of $a>1/2$
d) None of these

913. The value of k for which the equation $(k-2)x^2+8x+k+4=0$ has both roots real, distinct and negative, is

a) 0 b) 2 c) 3 d) -4

914. If α,β are the roots of $ax^2+c=bx$, then the equation $(a+cy)^2=b^2y$ in y has the roots

a) α^{-1},β^{-1} b) α^2,β^2 c) $\alpha\beta^{-1},\alpha^{-1}\beta$ d) $\sqrt{\alpha},\sqrt{\beta}$

915. The number of real roots of $3^{2x^2-7x+7}=9$ is

a) 0 b) 2 c) 1 d) 4

916. If one of the roots of the equation $x^2+bx+3=0$ is thrice the other, then b is equal to

a) ±3 b) ±2 c) 0 d) ±4

917. $\sqrt{2+\sqrt{5}-\sqrt{6-3\sqrt{5}+\sqrt{14-6\sqrt{5}}}}$ is equal to

a) 1 b) 2 c) 3 d) 4

918. If $\cos A+\cos B+\cos C=0$, $\sin A+\sin B+\sin C=0$ and $A+B+C=180°$, then the value of $\cos 3A+\cos 3B+\cos 3C$ is

a) 3 b) -3 c) $\sqrt{3}$ d) 0

919. The vertices B and D of a parallelogram are $1-2i$ and $4+2i$. If the diagonals are at right angles and $AC=2BD$, the complex number representing A is

a) $\frac{5}{2}$ b) $3i-\frac{3}{2}$ c) $3i-4$ d) $3i+4$

920. One lies between the roots of the equation $-x^2 + ax + a = 0, a \in R$ if and only if a lies in the interval

a) $\left(\frac{1}{2}, \infty\right)$ b) $\left[-\frac{1}{2}, \infty\right)$ c) $\left(-\infty, \frac{1}{2}\right)$ d) $\left(-\infty, \frac{1}{2}\right]$

921. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the square of their reciprocals, then $\frac{a}{c}, \frac{b}{a}$ and $\frac{c}{b}$ are in

a) Arithmetic progression
b) Geometric progression
c) Harmonic progression
d) Arithmetico-geometric progression

922. If the roots of $(a^2 + b^2)x^2 - 2(bc + ad)x + c^2 + d^2 = 0$ are equal, then

a) $\frac{a}{b} = \frac{c}{d}$ b) $\frac{a}{c} + \frac{b}{d} = 0$ c) $\frac{a}{d} = \frac{b}{c}$ d) $a + b = c + d$

923. If ω is a cube root of unity, then the value of $(1 - \omega + \omega^2)^5 + (1 + \omega - \omega^2)^5$ is

a) 30 b) 32 c) 2 d) None of these

924. The complex number satisfying $|z + 1| = |z - 1|$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$ is

a) $(\sqrt{2} + 1) + 0i$ b) $0 + (\sqrt{2} + 1)i$ c) $0 + (\sqrt{2} - 1)i$ d) $(-\sqrt{2} + 1) + 0i$

925. If α and β are the roots of the equation $x^2 - ax + b = 0$ and $A_n = \alpha^n + \beta^n$, then which one of the following is true?

a) $A_{n+1} = a\,A_n + b\,A_{n-1}$
b) $A_{n+1} = b\,A_n + a\,A_{n-1}$
c) $A_{n+1} = a\,A_n - b\,A_{n-1}$
d) $A_{n+1} = b\,A_n - aA_{n-1}$

926. If the sum of two of the roots of $x^3 + px^2 - qx + r = 0$ is zero, then pq is equal to

a) $-r$ b) r c) $2r$ d) $-2r$

927. The roots of the equation $x^4 - 8x^2 - 9 = 0$ are

a) $\pm 1, \pm i$ b) $\pm 3, \pm i$ c) $\pm 2, \pm i$ d) None of these

928. If $a = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$, then the value of $\left(\frac{1+a}{2}\right)^{3n}$ is

a) $(-1)^n$ b) $\frac{(-1)^n}{2^{3n}}$ c) $\frac{1}{2^{3n}}$ d) $(-1)^n + 1$

929. $(a^2 - 3a + 2)x^2 + (a^2 - 5a + 6)x + a - 2 = r$ for three distinct values of x for some $r \in R$, if $a + r$ is equal to

a) 1 b) 2 c) 3 d) Does not exist

930. Given that $a, b \in \{0, 1, 2, \dots, 9\}$ with $a + b \neq 0$ and that $\left(a + \frac{b}{10}\right)^x = \left(\frac{a}{10} + \frac{b}{100}\right)^y = 1000$. Then, $\frac{1}{x} - \frac{1}{y}$ is equal to

a) 1 b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) $\frac{1}{4}$

931. If z_1, z_2, z_3, z_4 are the four complex numbers represented by the vertices of a quadrilateral taken in order such that $z_1 - z_4 = z_2 - z_3$ and $\arg\left(\frac{z_4 - z_1}{z_2 - z_1}\right) = \pm\frac{\pi}{2}$, then the quadrilateral is

a) A rhombus
b) A square
c) A rectangle
d) Not a cyclic quadrilateral

932. The solution set of $x^2 + 2 \leq 3x \leq 2x^2 - 5$, is

a) Φ b) $[1, 2]$ c) $(-\infty, -1] \cup [5/2, \infty)$ d) None of these

933. The number of real solution of the equation $\left(\frac{9}{10}\right) = -3 + x - x^2$ is

a) 0 b) 1 c) 2 d) None of these

934. Solution of the equation $4.9^{x-1} = 3\sqrt{(2^{2x+1})}$ is

a) 3 b) 2 c) $\frac{3}{2}$ d) $\frac{2}{3}$

935. For what value of λ the sum of the squares of the roots of $x^2+(2+\lambda)x-\frac{1}{2}(1+\lambda)=0$ is minimum?

a) $\frac{3}{2}$ b) 1 c) $\frac{1}{2}$ d) $\frac{11}{4}$

936. If $2+i\sqrt{3}$ is a root of the equation $x^2+px+q=0$, where p and q are real, then (p,q) is equal to

a) $(-4,7)$ b) $(4,-7)$ c) $(4,7)$ d) $(-4,-7)$

937. The value of x in the given equation $4^x-3^{x-\frac{1}{2}}=3^{x+\frac{1}{2}}-2^{2x-1}$ is

a) $\frac{4}{3}$ b) $\frac{3}{2}$ c) $\frac{2}{1}$ d) $\frac{5}{3}$

938. If $x^{2/3}-7x^{1/3}+10=0$ then the set of values of x, is

a) $\{12,5\}$ b) $\{8\}$ c) Φ d) $\{8,125\}$

939. If $z_0=\frac{1-i}{z}$, then the value of the product $(1+z_0)(1+z_0^2)\left(1+z_0^{2^2}\right)\left(1+z_0^{2^2}\right)\dots\left(1+z_0^{2^2}\right)$ must be

a) $(1-i)\left(1+\frac{1}{2^{n-1}}\right)$, if $n\geq 1$

b) $(1-i)\left(1-\frac{1}{2^{2^n}}\right)$, if $n>1$

c) $(1-i)\left(1-\frac{1}{2^{n-1}}\right)$, if $n\geq 1$

d) $(1-i)\left(1+\frac{1}{2^{2^n}}\right)$, if $n>1$

940. If complex numbers z_1,z_2 and z_3 represent the vertices A,B and C respectively of an isosceles triangle ABC of which $\angle C$ is right angle, then correct statement is

a) $z_1^2+z_2^2+z_3^2=z_1\,z_2\,z_3$ b) $(z_3-z_1)^2=z_3-z_2$

c) $(z_1-z_2)^2=(z_1-z_3)(z_3-z_2)$ d) $(z_1-z_2)^2=2(z_1-z_3)(z_3-z_2)$

941. If the equation $(a+1)x^2-(a+2)x+(a+3)=0$ has roots equal in magnitude but opposite in sign, then the roots of the equation are

a) $\pm a$ b) $\pm\frac{1}{2}a$ c) $\pm\frac{3}{2}a$ d) $\pm 2a$

942. If $\log_{27}(\log_2 x)=\frac{1}{3}$, then the value of x is

a) 3 b) 6 c) 9 d) 27

943. The point of intersection of the curves $\arg(z-3i)=\frac{3\pi}{4}$ and $\arg(2z+1-2i)=\frac{\pi}{4}$ (where $i=\sqrt{-1}$) is

a) $\frac{1}{4}(3+9i)$ b) $\frac{1}{4}(3-9i)$ c) $\frac{1}{2}(3+2i)$ d) No point

944. The solution set of $x^2+2\leq 3x\leq 2x^2-5$, is

a) ϕ b) $[1,2]$ c) $(-\infty,-1]\cup[5/2,\infty)$ d) None of these

945. If a,b and c are in geometric progression and the roots of the equation $ax^2+2bx+c=0$ are α and β and those of $cx^2+2bx+a=0$ are γ and δ, then

a) $\alpha\neq\beta\neq\gamma\neq\delta$ b) $\alpha\neq\beta$ and $\gamma\neq\delta$

c) $a\alpha=a\beta=c\gamma=c\delta$ d) $\alpha=\beta$ and $\gamma\neq\delta$

946. Root(s) of the equation $9x^2-18|x|+5=0$ belonging to the domain of definition of the function $f(x)=\log(x^2-x-2)$ is (are)

a) $\frac{-5}{3},\frac{-1}{3}$ b) $\frac{5}{3},\frac{1}{3}$ c) $\frac{-5}{3}$ d) $\frac{-1}{3}$

947. If α,β are the roots of the equation $\lambda(x^2-x)+x+5=0$ and if λ_1 *and* λ_2 are two values of λ obtained from $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{4}{5}$, then $\frac{\lambda_1}{\lambda_2^2}+\frac{\lambda_2}{\lambda_1^2}$ is equal to

a) 4192 b) 4144 c) 4096 d) 4048

948. If the roots of the equation $\frac{\alpha}{x-\alpha}+\frac{\beta}{x-\beta}=1$ be equal in magnitude but opposite in sign, then $\alpha+\beta$ is equal to

a) 0 b) 1 c) 2 d) None of these

949. If $C^2+S^2=1$, then $\frac{1+C+i\,S}{1+C-i\,S}$ is equal to

a) $C+i\,S$ b) $C-i\,S$ c) $S+i\,C$ d) $S-i\,C$

950. The points representing complex number z for which $|z-3|=|z-5|$ lie on the locus given by

a) An ellipse b) A circle c) A straight line d) None of these

951. The solution set of the inequation $\frac{4x+3}{2x-5}<6$, is

a) $(5/2\,,33/8)$

b) $(-\infty,5/2)\cup(33/8,\infty)$

c) $(5/2,\infty)$

d) $(33/8\,,\infty)$

952. The number of quadratic equations which are unchanged by squaring their roots is

a) 2 b) 4 c) 6 d) None of these

953. A point P which represents a complex number z moves such that $|z-z_1|=|z-z_2|$, then its locus is

a) A circle with centre z_1 b) A circle with centre z_2

c) A circle with centre z d) Perpendicular bisector of line joining z_1 and z_2

954. The α,β are the roots of the equation $x^2+ax+b=0$, then $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$ is equal to

a) $\dfrac{a^2-2b}{b^2}$ b) $\dfrac{b^2-2a}{b^2}$ c) $\dfrac{a^2+2b}{b^2}$ d) $\dfrac{b^2+2a}{b^2}$

955. If $z=x+iy, z^{1/3}=a-ib$ and $\frac{x}{a}-\frac{y}{b}=k(a^2-b^2)$, then value of k equals

a) 2 b) 4 c) 6 d) 1

956. Let z_1 and z_2 be the non-real roots of the equation $3z^2+3z+b=0$. If the origin together with the points represented by z_1 and z_2 form an equilateral triangle, then the value of b is

a) 1 b) 2 c) 3 d) None of these

957. If correlation $n=2002$, evaluate

$$\frac{1}{\log_2 n!}+\frac{1}{\log_3 n!}+\frac{1}{\log_4 n!}+\ldots+\frac{1}{\log_{2002} n!}$$

a) 1 b) 2 c) 3 d) 4

958. If x^2+px+1 is a factor of the cubic polynomial ax^3+bx+c, then

a) $a^2+c^2=-ab$ b) $a^2-c^2=-ab$ c) $a^2-c^2=ab$ d) None of these

959. Find the complex number z satisfying the equations

$$\left|\frac{z-12}{z-8\,i}\right|=\frac{5}{3'}\ \left|\frac{z-4}{z-8}\right|=1$$

a) 6 b) $6\pm 8\,i$ c) $6\pm 8i, 6+17\,i$ d) None of these

960. The number of real roots of the equation $e^{\sin x}-e^{-\sin x}-4=0$ are

a) 1 b) 2 c) Infinite d) None of these

961. If $a=\sqrt{2\,i}$, then which of the following is correct?

a) $a=1+i$ b) $a=1-i$ c) $a=-(\sqrt{2})i$ d) None of these

962. If $2+i\sqrt{3}$ is a root of the equation $x^2+px+q=0$, then the value of (p,q) is

a) (-7, 4) b) (-4, 7) c) (4, -7) d) (7, -4)

963. The equation of a circle whose radius and centre are r and z_0 respectively, is

a) $z\bar{z}-z\bar{z}_0-\bar{z}z_0+z_0\bar{z}_0=r^2$ b) $z\bar{z}+z\bar{z}_0-\bar{z}z_0+z_0\bar{z}_0=r^2$

c) $z\bar{z}-z\bar{z}_0+\bar{z}z_0-z_0\bar{z}_0=r^2$ d) None of the above

964. The value of $\sum_{n=0}^{\infty}\left(\frac{2i}{3}\right)^n$ is

a) $\dfrac{9+6i}{13}$ b) $\dfrac{9-6i}{13}$ c) $9+6i$ d) $9-6i$

965. If $y=2^{1/\log_x(8)}$, then x is equal to

a) y b) y^2 c) y^3 d) None of these

966. If $1,\alpha,\alpha^2,\ldots,\alpha^{n-1}$ are the n, n^{th} roots of unity and z_1 and z_2 are any two complex numbers, then $\sum_{r=0}^{n-1}|z_1+a^r z_2|^2=$

a) $n(|z_1|^2+|z_2|^2)$ b) $(n-1)(|z_1|^2+|z_2|^2)$ c) $(n+1)(|z_1|^2+|z_2|^2)$ d) None of these

967. If $8, 2$ are the roots of $x^2+ax+\beta=0$, and $3,3$ are the roots of $x^2+\alpha\, x+b=0$, then the roots of $x^2+ax+b=0$ are

a) $8,-1$ b) $-9,2$ c) $-8,-2$ d) $9,1$

968. The modulus of $\sqrt{2\,i}-\sqrt{-2\,i}$ is

a) 2 b) $\sqrt{2}$ c) 0 d) $2\sqrt{2}$

969. If a and b are the roots of the equation $x^2+ax+b=0, a\neq 0, b\neq 0$, then the values of a and b are respectively

a) 2 and -2 b) 2 and -1 c) 1 and -2 d) 1 and 2

970. The trigonometric form of $z=(1-i\cot 8)^3$ (where $i=\sqrt{-1}$) is

a) $\text{cosec}^3\, 8.\, e^{i\left(24-\frac{3\pi}{2}\right)}$ b) $\text{cosec}^3\, 8.\, e^{-i\left(2u-\frac{3\pi}{2}\right)}$

c) $\text{cosec}^3\, 8.\, e^{i\left(36-\frac{\pi}{2}\right)}$ d) $\text{cosec}^2\, 8.\, \mathrm{e}^{-i24+\frac{\pi}{2}}$

971. If one root of the equation $x^2+px+q=0$ is $2+\sqrt{3}$, then the value of the p and q respectively

a) $-4,1$ b) $4,-1$ c) $2,\sqrt{3}$ d) $-2,-\sqrt{3}$

972. The value of $7\log_2\frac{16}{15}+5\log_2\frac{25}{24}+3\log_2\frac{81}{80}$ is

a) 1 b) $\log_2(105)$ c) $\log_2\left(\frac{9}{8}\right)$ d) $\log_2\left(\frac{8}{9}\right)$

973. The value of x which satisfy the equation

$\sqrt{5x^2-8x+3}-\sqrt{5x^2-9x+4}=\sqrt{2x^2-2x}-\sqrt{2x^2-3x+1}$ is

a) 3 b) 2 c) 1 d) 0

974. If the expression $\left(mx-1+\frac{1}{x}\right)$ is always non-negative, then the minimum value of m must be

a) $-\frac{1}{2}$ b) 0 c) $\frac{1}{4}$ d) $\frac{1}{2}$

975. If $\log_e\left(\frac{a+b}{2}\right)=\frac{1}{2}(\log_e a+\log_e b)$, then

a) $a=b$ b) $a=\frac{b}{2}$ c) $2a=b$ d) $a=\frac{b}{3}$

976. If the equations $ax^2+bx+c=0$ and $cx^2+bx+a=0, a\neq c$ have a negative common root, then $a-b+c=$

a) 0 b) 2 c) 1 d) None of these

977. If ω is a complex cube root of unity, then $\frac{(1+i)^{2n}-(1-i)^{2n}}{(1+\omega^4-\omega^2)(1-\omega^4+\omega^2)}$ is equal to

a) 0, if n is an even integer

b) 0 for all $n\in Z$

c) $2^{n-1}i$ for all $n\in N$

d) None of these

978. The value of m for which the equation $x^3-mx^2+3x-2=0$ has two roots equal in magnitude but opposite sign, is

a) 4/5 b) 3/4 c) 2/3 d) 1/2

979. If $\frac{(x+1)}{(2x-1)(3x+1)}=\frac{A}{(2x-1)}+\frac{B}{(3x+1)}$, then$16A+9B$ is equal to

a) 4 b) 5 c) 6 d) 8

980. If $x+iy=\frac{3}{2+\cos\theta+i\sin\theta}$, then x^2+y^2 is equal to

a) $3x-4$ b) $4\,x-3$ c) $4\,x+3$ d) None of these

981. If $|z_1|=|z_2|=\cdots=|z_n|=1$, then the value of $|z_1+z_2+\cdots+z_n|$, is

a) n b) $\left|\frac{1}{z_1}+\frac{1}{z_2}+\cdots+\frac{1}{z_n}\right|$ c) 0 d) None of these

982. If the equation $x^3+ax^2+b=0, b\neq 0$ has a root of order 2, then

a) $a^2+2b=0$ b) $a^2-2b=0$ c) $4a^3+27b+1=0$ d) $4a^3+27b=0$

983. The solution of the equation $2x^3-x^2-22x-24=0$ when two of the roots in the ration $3:4$, is

a) $3, 4, \frac{1}{2}$ b) $-\frac{3}{2}, -2, 4$ c) $-\frac{1}{2}, \frac{3}{2}, 2$ d) $\frac{3}{2}, 2, \frac{5}{2}$

984. If z_1 and z_2 be complex numbers such that $z_1 \neq z_2$ and $|z_1| = |z_2|$.If z_1 has positive real part and z_2 has negative imaginary part, then $\frac{(z_1+z_2)}{(z_1-z_2)}$ may be

a) Purely imaginary b) Real and positive c) Real and negative d) None of these

985. If $a > 0, b > 0, c > 0$, then both the roots of the equation $ax^2 + bx + c = 0$

a) Are real and negative
b) Have negative real part
c) Are rational numbers
d) None of the above

986. If α, β and γ are the roots of the equation $x^3 - 3x + 1 = 0$, then $[\alpha] + [\beta] + [\gamma]$ is ($[\cdot]$ denotes the greatest integer function)

a) -3 b) -2 c) -1 d) Does not exist

987. If $\arg(z - a) = \frac{\pi}{4}$, where $a \in R$, then the locus of $z \in C$ is a

a) Hyperbola b) Parabola c) Ellipse d) Straight line

988. Common roots of the equations $z^3 + 2z^2 + 2z + 1 = 0$ and $z^{1985} + z^{100} + 1 = 0$ are

a) ω, ω^2 b) ω, ω^3 c) ω^2, ω^3 d) None of these

989. The greatest and the least value of $|z_1 + z_2|$, if $z_1 = 24 + 7i$ and $|z_2| = 6$, are respectively

a) $31, 19$ b) $25, 19$ c) $31, 25$ d) None of these

990. If $a, c \neq 0$ and α, β are the roots of the equation $ax^2 + bx + c = 0$, then the quadratic equation with $1/\alpha$ and $1/\beta$ as its root is

a) $x^2/a + x/b + 1/c = 0$
b) $cx^2 + bx + a = 0$
c) $bx^2 + cx + a = 0$
d) $ax^2 + cx + b = 0$

991. The value of $\log_2 \log_2 \log_4 256 + 2 \log_{\sqrt{2}} 2$ is

a) 1 b) 2 c) 3 d) 5

992. If $z_1 = 1 + 2i,\ z_2 = 2 + 3i,\ z_3 = 3 + 4i$, then z_1, z_2, z_3 represents the vertices of a/an

a) Equilateral triangle b) Isosceles triangle c) Right angled triangle d) None of these

993. The roots of the quadratic equation $x^2 - 2\sqrt{3}x - 22 = 0$ are

a) Imaginary
b) Real, rational and equal
c) Real irrational and unequal
d) Real, rational and unequal

994. If α, β, γ are the roots of $x^3 - 2x^2 + 3x - 4 = 0$, then the value of $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$ is

a) -7 b) -5 c) -3 d) 0

995. If $k > 1, |z_1| < k$ and $\left|\frac{k - z_1\overline{z_2}}{z_1 - k\, z_2}\right| = 1$, then

a) $|z_2| < k$ b) $|z_2| = k$ c) $z_2 = 0$ d) $|z_2| = 1$

996. The conjugate of a complex number is $\frac{1}{i-1}$. Then, that complex number is

a) $\frac{1}{i-1}$ b) $-\frac{1}{i-1}$ c) $\frac{1}{i+1}$ d) $-\frac{1}{i+1}$

997. If the roots of the equation $ax^2 + bx + c = 0$ be α and β , then the roots of the equation $cx^2 + bx + a = 0$ are

a) $-\alpha, -\beta$ b) $\alpha, \frac{1}{\beta}$ c) $\frac{1}{\alpha}, \frac{1}{\beta}$ d) None of these

998. If the points z_1, z_2, z_3 are the vertices of an equilateral triangle in the complex plane, then the value of $z_1^2 + z_2^2 + z_3^2$ is equal to

a) $\frac{z_1}{z_2} + \frac{z_2}{z_3} + \frac{z_3}{z_1}$ b) $z_1z_2 + z_2z_3 + z_3z_1$ c) $z_1z_2 - z_2z_3 - z_3z_1$ d) $-\frac{z_1}{z_2} - \frac{z_2}{z_3} - \frac{z_3}{z_1}$

999. If the expressions $x^2 - 11x + a$ and $x^2 - 14x + 2a$ have a common root, then the values of $'a'$ is

a) $0, 24$ b) $0, -24$ c) $1, -1$ d) $-2, 1$

1000. If $|x - 1| + |x| + |x + 1| \geq 6$, then x belongs to

a) $(-\infty, 2]$ b) $(-\infty, -2] \cup [2, \infty)$ c) R d) ϕ

100 Let z_1, z_2 and z_3 be the affixes of the vertices of a triangle having the circumcentre at the origin. If z is the

1. affix of its orthocentre, then z is equal to

a) $\frac{z_1+z_2+z_3}{3}$ b) $\frac{z_1+z_2+z_3}{2}$ c) $z_1+z_2+z_3$ d) None of these

1002. If the equation $ax^2+2bx-3c=0$ has non-real roots and $(3c/4)<(a+b)$, then c is

a) <0 b) >0 c) ≥ 0 d) $=0$

1003. If $1, \omega, \omega^2$ are the cube roots of unity, then $\omega^2(1+\omega)^3-(1+\omega^2)\omega$ is equal to

a) 1 b) -1 c) i d) 0

1004. If $b_1b_2=2(c_1+c_2)$, then the least one of the equation $x^2+b_1x+c_1=0$ and $x^2+b_2x+c_2=0$ has

a) Real roots b) Purely imaginary roots
c) Imaginary roots d) None of the above

1005. The imaginary part of $\frac{(1+i)^2}{i(2i-1)}$ is

a) 4/5 b) 0 c) 2/5 d) -(4/5)

1006. The partial fraction of $\frac{3x^3-8x^2+10}{(x-1)^4}$ is

a) $\frac{3}{(x-1)}+\frac{1}{(x-1)^2}+\frac{7}{(x-1)^3}+\frac{5}{(x-1)^4}$ b) $\frac{3}{(x-1)}+\frac{1}{(x-1)^2}-\frac{7}{(x-1)^3}+\frac{5}{(x-1)^4}$
c) $\frac{3}{(x-1)}+\frac{1}{(x-1)^2}-\frac{7}{(x-1)^3}+\frac{5}{(x-1)^4}$ d) None of the above

1007. If $|z-i\text{Re}\,(z)=|z-\text{Im}\,(z)|$ (where $i=\sqrt{-1}$), then z lies on

a) $\text{Re}\,(z)=2$ b) $\text{Im}\,(z)=2$ c) $\text{Re}\,(z)+\text{Im}\,(z)=2$ d) None of the above

1008. if one of the roots of the equation $x^2+(1-3i)x-2(1+i)=0$ is $-1+i$, then the other root is

a) $-1-i$ b) $-\frac{1}{2}-\frac{i}{2}$ c) i d) $2i$

1009. If the imaginary part of the expression $\frac{z-1}{e^{i\theta}}+\frac{e^{i\theta}}{z-1}$ be zero, then the locus of z is

a) A straight line parallel to x-axis b) A parabola
c) A circle of radius 1 and centre (1, 0) d) None of the above

1010. The locus of the point $z=x+y-iy$ satisfying the equation $\left|\frac{z-1}{z+1}\right|=1$ is given by

a) $x=0$ b) $y=0$ c) $x=y$ d) $x+y=0$

1011. Number of real roots of the equation $(6-x)^4+(8-x)^4=16$ is

a) 4 b) 2 c) 0 d) None of these

1012. If $\left|\frac{x^2+6}{5x}\right|\geq 1$, then x belongs to

a) $(-\infty,-3)$
b) $(-\infty,-3)\cup(3,\infty)$
c) $(-\infty,-3]\cup[-2,0)\cup(0,2]\cup[3,\infty)$
d) R

1013. POQ is a straight line through the origin O. P and Q represent the complex numbers $a+ib$ and $c+i\,d$ respectively and $OP=OQ$. Then which one of the following is not true?

a) $|a+i\,b|=|c+i\,d|$
b) $a+b=c+d$
c) $\arg(a+i\,b)=\arg(c+i\,d)$
d) None of these

1014. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then the equation whose roots are $2\alpha + 3\beta$ and $3\alpha + 2\beta$, is

a) $acx^2 + (a + c)bx - (a + c)^2 = 0$
b) $acx^2 - (a + c)bx + (a + c)^2 = 0$
c) $abx^2 - (a + b)cx + (a + b)^2 = 0$
d) None of the above

1015. The argument of $\frac{1+i\sqrt{3}}{1-i\sqrt{3}}$ is

a) $2\pi/3$ b) $\pi/3$ c) $-\pi/3$ d) $-2\pi/3$

1016. The number of non-zero integral solutions of the equation $|1 - i|^x = 2^x$ is

a) Infinite b) 1 c) 2 d) None of these

1017. The smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$, is

a) $n = 8$ b) $n = 12$ c) $n = 16$ d) None of these

1018. If $\frac{x^2+x+1}{x^2+2x+1} = A + \frac{B}{x+1} + \frac{C}{(x+1)^2}$, then $A - B$ is equal to

a) $4C$ b) $4C + 1$ c) $3C$ d) $2C$

1019. The solution set of the inequation $\left|x + \frac{1}{x}\right| < 4$, is

a) $(2 - \sqrt{3}, 2 + \sqrt{3}) \cup (-2 - \sqrt{3}, -2 + \sqrt{3})$
b) $R - (2 - \sqrt{3}, 2 + \sqrt{3})$
c) $R - (-2 - \sqrt{3}, -2 + \sqrt{3})$
d) None of these

1020. If α is a root of the quadratic equation $x^2 + 6x - 2 = 0$, then another root β is

a) $\alpha^2 + 5\,\alpha - 8$ b) $\frac{\alpha}{3\,\alpha - 1}$ c) $\frac{2\,\alpha^2 + 12\,\alpha - 6}{\alpha}$ d) All of these

1021. If ω is a complex root of the equation $z^3 = 1$, then $\omega + \omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\frac{27}{128}+\dots\right)}$ is equal to

a) -1 b) 0 c) 9 d) i

1022. The solution set of the inequation $\left|\frac{2x-1}{x-1}\right| > 2$, is

a) $(3/4, 1) \cup (1, \infty)$ b) $(3/4, \infty)$ c) $(-\infty, 3/4)$ d) None of these

1023. If $2 - i$ is the root of the equation $ax^2 + 12x + b = 0$ (where a and b are real), then the value of ab is equal to

a) 45 b) 15 c) -15 d) -45

1024. Let $f(x) = x^2 + ax + b$; $a, b \in R$. If $f(1) + f(2) + f(3) = 0$, then the roots of the equation $f(x) = 0$

a) Are imaginary
b) Are real and equal
c) Are from the set $\{1, 2, 3\}$
d) Real and distinct

1025. Product of the real roots of the equation $t^2x^2 + |x| + 9 = 0, (t \neq 0)$

a) Is always positive b) Is always negative c) Does not exist d) None of these

1026. The centre of a square $ABCD$ is at $z = 0$. The affix of the vertex A is z_1. Then, the affix of the centroid of the triangle ABC is

a) $z_1(\cos\pi \pm i\sin\pi)$ b) $\frac{z_1}{3}(\cos\pi \pm i\sin\pi)$ c) $z_1\left(\cos\frac{\pi}{2} \pm i\sin\frac{\pi}{2}\right)$ d) $\frac{z_1}{3}\left(\cos\frac{\pi}{2} \pm i\sin\frac{\pi}{2}\right)$

1027. The point (4, 1) undergoes the following three transformations successively

(i) Refletion about the line $y = x$
(ii) Translation through a distance of 2 unit along the positive direction of x-axis
(iii) Rotation through an angle of $\frac{\pi}{4}$ about the origin in the anti-clockwise direction

The final position of the point is

a) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ b) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ c) $(-\sqrt{2}, 7\sqrt{2})$ d) $(\sqrt{2}, 7\sqrt{2})$

1028. If c and d are roots of the equation $(x-a)(x-b)-k=0$, then a, b are roots of the equation

a) $(x-c)(x-d)-k=0$

b) $(x-c)(x-d)+k=0$

c) $(x-a)(x-c)+k=0$

d) $(x-b)(x-d)+k=0$

1029. The real roots of $|x|^3 - 3x^2 + 3|x| - 2 = 0$ are

a) 0, 2 b) ± 1 c) ± 2 d) 1, 2

1030. Number of solutions of the equation $z^2 + |z|^2 = 0$, where $z \in C$ is

a) 1 b) 2 c) 3 d) Infinity many

1031. If the equation $ax^2 + 2bx + 3c = 0$ and $3x^2 + 8x + 15 = 0$ have a common root, where a, b, c are the lengths of the sides of a ΔABC, then $\sin^2 A + \sin^2 B + \sin^2 C$ is equal to

a) 1 b) $\frac{3}{2}$ c) $\sqrt{2}$ d) 2

1032. If ω is a complex cube root of unity, then $(1-\omega+\omega^2)^6 + (1-\omega^2+\omega)^6 =$

a) 0 b) 6 c) 64 d) 128

1033. If $\tan^{-1}(\alpha + i\beta) = x + iy$, then x is equal to

a) $\frac{1}{2}\tan^{-1}\left(\frac{2\alpha}{1-\alpha^2-\beta^2}\right)$ b) $\frac{1}{2}\tan^{-1}\left(\frac{2\alpha}{1+\alpha^2+\beta^2}\right)$ c) $\tan^{-1}\left(\frac{2\alpha}{1-\alpha^2-\beta^2}\right)$ d) None of these

1034. Let a, b, c be positive numbers. The following system of equations in x, y and z, $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$; $\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ and $\frac{-x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ has

a) No solution b) Unique solution

c) Infinitely many solutions d) Finitely may solutions

1035. If $\log_2[\log_3\{\log_4(\log_5 x)\}] = 0$, then the value of x is

a) 5^{24} b) 1 c) 2^{25} d) 5^{64}

1036. If $1, a_1, a_2, \dots, a_{n-1}$ are the n roots of unity, then the value of $(1-a_1)(1-a_2)(1-a_3)\dots(1-a_{n-1})$ is equal to

a) $\sqrt{3}$ b) $\frac{1}{2}$ c) n d) 0

1037. If $z = \frac{4}{1-i}$, then $\bar{z}$ is (where $\bar{z}$ is complex conjugate of z)

a) $2(1+i)$ b) $(1+i)$ c) $\frac{2}{1-i}$ d) $\frac{4}{1+i}$

1038. The roots of the equation $(q-r)x^2 + (r-p)x + (p-q) = 0$ are

a) $\frac{r-p}{q-r}, 1$ b) $\frac{p-q}{q-r}, 1$ c) $\frac{p-r}{q-r}, 2$ d) $\frac{q-r}{p-q}, 2$

: ANSWER KEY :

1)	c	2)	b	3)	c	4)	c	189)	d	190)	d	191)	b	192)	d
5)	c	6)	c	7)	a	8)	b	193)	c	194)	c	195)	c	196)	b
9)	b	10)	b	11)	b	12)	b	197)	d	198)	a	199)	d	200)	c
13)	c	14)	a	15)	d	16)	a	201)	a	202)	c	203)	d	204)	a
17)	c	18)	a	19)	a	20)	c	205)	d	206)	a	207)	a	208)	b
21)	b	22)	a	23)	d	24)	c	209)	d	210)	a	211)	a	212)	c
25)	c	26)	b	27)	c	28)	c	213)	c	214)	d	215)	b	216)	d
29)	d	30)	b	31)	c	32)	a	217)	b	218)	a	219)	d	220)	c
33)	a	34)	b	35)	b	36)	a	221)	c	222)	c	223)	a	224)	b
37)	b	38)	c	39)	a	40)	b	225)	b	226)	a	227)	b	228)	b
41)	c	42)	c	43)	c	44)	a	229)	c	230)	c	231)	a	232)	c
45)	b	46)	b	47)	c	48)	a	233)	a	234)	a	235)	d	236)	a
49)	c	50)	c	51)	b	52)	a	237)	b	238)	c	239)	d	240)	d
53)	c	54)	d	55)	a	56)	b	241)	b	242)	c	243)	b	244)	b
57)	c	58)	c	59)	c	60)	d	245)	a	246)	a	247)	a	248)	c
61)	d	62)	c	63)	b	64)	d	249)	d	250)	d	251)	b	252)	d
65)	b	66)	a	67)	a	68)	a	253)	b	254)	d	255)	c	256)	b
69)	c	70)	d	71)	d	72)	d	257)	b	258)	d	259)	a	260)	b
73)	d	74)	c	75)	a	76)	b	261)	c	262)	d	263)	a	264)	b
77)	a	78)	a	79)	b	80)	a	265)	c	266)	a	267)	c	268)	b
81)	c	82)	a	83)	d	84)	a	269)	c	270)	c	271)	d	272)	b
85)	a	86)	b	87)	d	88)	b	273)	c	274)	a	275)	d	276)	b
89)	a	90)	c	91)	a	92)	a	277)	a	278)	b	279)	a	280)	a
93)	d	94)	c	95)	c	96)	b	281)	c	282)	a	283)	b	284)	a
97)	b	98)	b	99)	b	100)	d	285)	b	286)	b	287)	d	288)	c
101)	b	102)	d	103)	a	104)	a	289)	b	290)	c	291)	b	292)	c
105)	a	106)	c	107)	c	108)	b	293)	c	294)	b	295)	b	296)	b
109)	b	110)	a	111)	d	112)	b	297)	c	298)	b	299)	a	300)	b
113)	d	114)	a	115)	c	116)	c	301)	d	302)	a	303)	b	304)	a
117)	a	118)	b	119)	a	120)	a	305)	a	306)	b	307)	a	308)	a
121)	a	122)	d	123)	d	124)	a	309)	d	310)	d	311)	d	312)	d
125)	c	126)	b	127)	a	128)	d	313)	b	314)	c	315)	c	316)	b
129)	c	130)	d	131)	a	132)	c	317)	c	318)	c	319)	b	320)	b
133)	c	134)	a	135)	a	136)	d	321)	d	322)	b	323)	a	324)	c
137)	d	138)	c	139)	b	140)	d	325)	a	326)	d	327)	b	328)	d
141)	b	142)	d	143)	d	144)	d	329)	a	330)	a	331)	d	332)	b
145)	a	146)	b	147)	d	148)	d	333)	c	334)	a	335)	b	336)	b
149)	d	150)	d	151)	a	152)	b	337)	a	338)	b	339)	d	340)	d
153)	d	154)	d	155)	a	156)	a	341)	c	342)	b	343)	b	344)	d
157)	a	158)	d	159)	c	160)	b	345)	a	346)	a	347)	b	348)	b
161)	b	162)	b	163)	d	164)	d	349)	a	350)	c	351)	b	352)	a
165)	b	166)	b	167)	c	168)	c	353)	b	354)	c	355)	a	356)	a
169)	b	170)	a	171)	c	172)	a	357)	b	358)	b	359)	d	360)	b
173)	d	174)	a	175)	b	176)	a	361)	b	362)	d	363)	d	364)	d
177)	a	178)	d	179)	c	180)	d	365)	a	366)	d	367)	a	368)	c
181)	c	182)	a	183)	b	184)	b	369)	b	370)	b	371)	d	372)	a
185)	d	186)	c	187)	a	188)	b	373)	b	374)	b	375)	d	376)	c

377)	d	378)	b	379)	d	380)	b	581)	a	582)	b	583)	b	584)	a
381)	d	382)	c	383)	a	384)	b	585)	d	586)	c	587)	c	588)	a
385)	c	386)	b	387)	d	388)	a	589)	d	590)	c	591)	b	592)	a
389)	b	390)	c	391)	a	392)	b	593)	d	594)	a	595)	d	596)	c
393)	b	394)	b	395)	a	396)	b	597)	c	598)	b	599)	b	600)	b
397)	d	398)	d	399)	b	400)	c	601)	d	602)	d	603)	a	604)	b
401)	d	402)	d	403)	b	404)	b	605)	b	606)	b	607)	c	608)	b
405)	a	406)	c	407)	a	408)	c	609)	b	610)	c	611)	c	612)	b
409)	d	410)	c	411)	b	412)	b	613)	b	614)	c	615)	c	616)	c
413)	c	414)	a	415)	b	416)	b	617)	c	618)	b	619)	a	620)	b
417)	a	418)	d	419)	a	420)	a	621)	b	622)	c	623)	d	624)	d
421)	a	422)	c	423)	a	424)	a	625)	c	626)	c	627)	b	628)	d
425)	b	426)	d	427)	b	428)	d	629)	b	630)	b	631)	b	632)	b
429)	c	430)	d	431)	a	432)	a	633)	d	634)	c	635)	d	636)	a
433)	a	434)	d	435)	a	436)	b	637)	b	638)	c	639)	b	640)	b
437)	b	438)	a	439)	c	440)	d	641)	b	642)	d	643)	a	644)	a
441)	a	442)	b	443)	d	444)	a	645)	a	646)	b	647)	b	648)	b
445)	c	446)	c	447)	a	448)	c	649)	c	650)	a	651)	d	652)	c
449)	d	450)	b	451)	c	452)	a	653)	d	654)	a	655)	c	656)	c
453)	c	454)	d	455)	c	456)	b	657)	c	658)	d	659)	c	660)	b
457)	d	458)	a	459)	c	460)	c	661)	b	662)	c	663)	d	664)	d
461)	d	462)	a	463)	d	464)	a	665)	b	666)	b	667)	c	668)	b
465)	b	466)	b	467)	b	468)	a	669)	d	670)	a	671)	c	672)	b
469)	c	470)	d	471)	c	472)	d	673)	c	674)	a	675)	b	676)	d
473)	a	474)	c	475)	a	476)	a	677)	a	678)	d	679)	b	680)	b
477)	d	478)	c	479)	b	480)	b	681)	d	682)	a	683)	a	684)	b
481)	a	482)	c	483)	c	484)	b	685)	b	686)	b	687)	b	688)	b
485)	c	486)	a	487)	a	488)	b	689)	a	690)	b	691)	d	692)	b
489)	c	490)	c	491)	c	492)	a	693)	a	694)	d	695)	a	696)	d
493)	c	494)	a	495)	b	496)	b	697)	a	698)	a	699)	a	700)	c
497)	c	498)	c	499)	b	500)	a	701)	b	702)	b	703)	a	704)	b
501)	b	502)	b	503)	d	504)	d	705)	b	706)	d	707)	b	708)	c
505)	c	506)	a	507)	d	508)	c	709)	c	710)	d	711)	c	712)	a
509)	a	510)	d	511)	a	512)	b	713)	a	714)	a	715)	d	716)	c
513)	b	514)	a	515)	c	516)	d	717)	b	718)	d	719)	c	720)	d
517)	b	518)	d	519)	c	520)	c	721)	c	722)	c	723)	b	724)	d
521)	a	522)	c	523)	a	524)	c	725)	a	726)	a	727)	c	728)	d
525)	a	526)	c	527)	d	528)	d	729)	a	730)	c	731)	a	732)	c
529)	a	530)	c	531)	b	532)	b	733)	a	734)	b	735)	d	736)	a
533)	a	534)	d	535)	d	536)	d	737)	a	738)	c	739)	d	740)	b
537)	d	538)	d	539)	d	540)	d	741)	d	742)	a	743)	d	744)	b
541)	c	542)	c	543)	c	544)	a	745)	d	746)	b	747)	a	748)	b
545)	c	546)	a	547)	a	548)	c	749)	d	750)	c	751)	c	752)	b
549)	b	550)	d	551)	c	552)	d	753)	d	754)	c	755)	d	756)	c
553)	b	554)	c	555)	a	556)	d	757)	c	758)	c	759)	b	760)	c
557)	d	558)	b	559)	d	560)	d	761)	b	762)	b	763)	c	764)	c
561)	d	562)	b	563)	b	564)	d	765)	a	766)	b	767)	d	768)	b
565)	b	566)	a	567)	b	568)	d	769)	b	770)	d	771)	d	772)	a
569)	c	570)	d	571)	a	572)	c	773)	a	774)	a	775)	b	776)	c
573)	b	574)	c	575)	b	576)	b	777)	b	778)	a	779)	c	780)	a
577)	c	578)	c	579)	b	580)	b	781)	c	782)	c	783)	c	784)	a

785)	c	786)	d	787)	b	788)	a
789)	b	790)	a	791)	b	792)	a
793)	a	794)	b	795)	a	796)	b
797)	c	798)	b	799)	a	800)	a
801)	a	802)	a	803)	a	804)	b
805)	a	806)	a	807)	b	808)	a
809)	b	810)	c	811)	b	812)	a
813)	c	814)	d	815)	c	816)	d
817)	b	818)	d	819)	b	820)	c
821)	a	822)	d	823)	b	824)	d
825)	c	826)	a	827)	c	828)	a
829)	b	830)	d	831)	c	832)	d
833)	d	834)	b	835)	a	836)	a
837)	d	838)	a	839)	c	840)	c
841)	d	842)	b	843)	a	844)	c
845)	c	846)	a	847)	a	848)	c
849)	b	850)	c	851)	c	852)	c
853)	c	854)	d	855)	c	856)	b
857)	d	858)	b	859)	a	860)	d
861)	a	862)	c	863)	a	864)	c
865)	b	866)	d	867)	b	868)	a
869)	c	870)	b	871)	b	872)	b
873)	d	874)	c	875)	c	876)	d
877)	a	878)	b	879)	d	880)	a
881)	b	882)	d	883)	a	884)	b
885)	a	886)	a	887)	d	888)	c
889)	a	890)	c	891)	c	892)	d
893)	c	894)	d	895)	c	896)	c
897)	a	898)	a	899)	a	900)	a
901)	b	902)	c	903)	d	904)	a
905)	d	906)	c	907)	d	908)	d
909)	c	910)	d	911)	b	912)	a
913)	c	914)	b	915)	b	916)	d
917)	b	918)	b	919)	b	920)	a
921)	c	922)	a	923)	b	924)	b
925)	c	926)	a	927)	b	928)	b
929)	b	930)	c	931)	c	932)	a
933)	a	934)	c	935)	c	936)	a
937)	b	938)	d	939)	b	940)	d
941)	b	942)	d	943)	d	944)	a
945)	c	946)	c	947)	d	948)	a
949)	a	950)	c	951)	b	952)	b
953)	d	954)	a	955)	b	956)	a
957)	a	958)	c	959)	c	960)	d
961)	a	962)	b	963)	a	964)	a
965)	c	966)	a	967)	d	968)	a
969)	c	970)	a	971)	a	972)	a
973)	c	974)	c	975)	a	976)	a
977)	a	978)	c	979)	c	980)	b
981)	b	982)	d	983)	b	984)	a
985)	b	986)	c	987)	d	988)	a

989)	a	990)	b	991)	d	992)	d
993)	d	994)	a	995)	d	996)	d
997)	c	998)	b	999)	a	1000)	b
1001)	c	1002)	a	1003)	d	1004)	d
1005)	d	1006)	b	1007)	c	1008)	d
1009)	c	1010)	a	1011)	b	1012)	c
1013)	a	1014)	d	1015)	a	1016)	b
1017)	d	1018)	d	1019)	a	1020)	d
1021)	a	1022)	a	1023)	a	1024)	d
1025)	d	1026)	d	1027)	b	1028)	b
1029)	c	1030)	d	1031)	d	1032)	d
1033)	a	1034)	d	1035)	d	1036)	c
1037)	d	1038)	b				

: HINTS AND SOLUTIONS :

1 **(c)**

Let $z = \frac{1-i}{3+i} + \frac{4i}{5}$

$= \frac{5 - 5i + 12i - 4}{5(3+i)} = \frac{1+7i}{5(3+i)}$

$= \frac{(1+7i)(3-i)}{5(9+1)} = \frac{10+20i}{50} = \frac{1+2i}{5}$

$\therefore \quad |z| = \sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{2}{5}\right)^2} = \frac{1}{5}\sqrt{1+4} = \frac{\sqrt{5}}{5}$

2 **(b)**

Let each ratio be k and let $A = xyz$,

Then $\log x = k(a-b)$, $\log y = k(b-c)$

And $\log z = k(c-a)$

$\therefore \quad \log A = \log x + \log y + \log z$

$= k(a-b) + k(b-c) + k(c-a)$

$= k[a - b + b - c + c - a]$

$= k[0]$

$\therefore \quad \log A = \log(xyz) = 0 \quad [\because A = xyz]$

$\Rightarrow xyz = e^0 = 1$

3 **(c)**

Let $z = x + iy$. Then, coordinates of the vertices of the triangle are $(-x, -y)$, $(-y, x)$ and $(x+y, y-x)$

$\therefore$ Area of the triangle

$= \frac{1}{2}\begin{vmatrix} -x & -y & 1 \\ -y & x & 1 \\ x+y & y-x & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} -x & -y & 1 \\ x-y & x+y & 0 \\ 2x+y & 2y-x & 0 \end{vmatrix}$ Applying $R_2 \to R_2 \to R_1$
$R_3 \to R_3 \to R_1$

$= -\frac{3}{2}(x^2 + y^2) = -\frac{3}{2}|z|^2$

Hence, Area $= \frac{3}{2}|z|^2$

4 **(c)**

Given, $\frac{(1+i)^2}{2-i} = x + iy$

$\Rightarrow \quad \frac{2i}{2-i} \times \frac{2+i}{2+i} = x + iy$

$\Rightarrow \quad \frac{4i-2}{5} = x + iy$

$\Rightarrow \quad x + iy = -\frac{2}{5} + \frac{4}{5}i$

$\therefore \quad x + y = -\frac{2}{5} + \frac{4}{5} = \frac{2}{5}$

5 **(c)**

We have,

$|z - 3 + i| = |z - 2 - i|$

$\Rightarrow |z - (3 - i)| = |z - (2 + i)|$

$\Rightarrow AP = BP$

$\Rightarrow$ locus of P is the perpendicular bisector of AB

7 **(a)**

We have, $z = \frac{1+ir}{1+p} \quad \therefore iz = \frac{-r+iq}{1+p}$

By componendo and dividendo

$\frac{1+iz}{1-iz} - \frac{1+p-r+iq}{1+p+r-iq}$

$\therefore \ \frac{p+iq}{1+r} = \frac{1+iz}{1-iz}$ if $\frac{p+iq}{1+r} = \frac{1+p-r+iq}{1+p+r-iq}$

or $p(1+p+r) + q^2 + i\{q(1+p+r) - pq\}$

$= (1+r)(1+p-r) + iq(1+r)$

$\Rightarrow \ p(1+p+r) + q^2 = (1+r)(1+p-r)$

and $q(1+p+r) - pq = q(1+r)$

[this is obviously true]

$\therefore$ The condition is

$p(1+p+r) + q^2 = (1+r)(1+p-r)$

or $p + p^2 + pr + q^2 = 1 + p - r + r + pr - r^2$

or $p^2 + q^2 + r^2 = 1$

9 **(b)**

Since, $2q = p + r$

Given that, $px^2 + qx + r = 0$ has complex roots

$\therefore \quad D < 0$

$\Rightarrow q^2 - 4pr < 0$

$\Rightarrow \left(\frac{p+r}{2}\right)^2 - 4pr < 0$

$\Rightarrow \; p^2 + r^2 - 14pr < 0$

$\Rightarrow \; \frac{p^2}{r^2} + 1 - \frac{14p}{r} < 0$

$\Rightarrow \; \left(\frac{p^2}{r^2} - \frac{14\,p}{r} + 49\right) - 48 < 0$

$\Rightarrow \left(\frac{p}{r} - 7\right)^2 < 48 \quad \Rightarrow \quad \left|\frac{p}{r} - 7\right| < 4\sqrt{3}$

10 **(b)**

Given, $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$

$\Rightarrow r(2x + p + q) = [x^2 + (p+q)x + pq]$

$\Rightarrow x^2 + (p + q - 2r)x + pq - r(p+q) = 0$

As we know, if roots are equal in magnitude but opposite in sign, then coefficient of x will be zero

$\therefore \quad p + q - 2r = 0 \;\; \Rightarrow p + q = 2r$

11 **(b)**

We have, $|2x - 3| < |x + 2|$

Following cases arise:

<u>CASE I</u> *When* $x < -2$

In this case, we have

$|2x - 3| = -(2x - 3)$ and $|x + 2| = -(x + 2)$

$\therefore |2x - 3| < |x + 2|$

$\Rightarrow -(2x - 3) < -(x + 2)$

$\Rightarrow 2x - 3 > x + 2 \Rightarrow x - 5 > 0 \Rightarrow x > 5$

But, $x < -2$. So, there is no solution in this case

<u>CASE II</u> *When* $-2 \le x < \frac{3}{2}$

In this case, we have

$|x + 2| = x + 2$ and $|2x - 3| = -(2x - 3)$

$\therefore |2x - 3| < |x + 2|$

$\Rightarrow -(2x - 3) < x + 2 \Rightarrow 3x - 1 > 0 \Rightarrow x > \frac{1}{3}$

But, $-2 \le x < \frac{3}{2}$. Therefore, $x \in \left(\frac{1}{3}, \frac{3}{2}\right)$

<u>CASE III</u> *When* $x \ge \frac{3}{2}$

In this case, we have

$|x + 2| = x + 2$ and $|2x - 3| = 2x - 3$

$\therefore |2x - 3| < |x + 2| \Rightarrow 2x - 3 < x + 2 \Rightarrow x < 5$

But, $x \ge \frac{3}{2}$. Therefore, $x \in [3/2, 5)$

Hence, the solution set is $x \in (1/3, 5)$

12 **(b)**

Let the correct equation is

$ax^2 + bx + c = 0$,

Then $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

When b is written incorrectly, then the roots are equal.

Let these are γ and γ.

$\therefore \; \gamma.\gamma = \frac{c}{a} \Rightarrow \; \gamma^2 = \alpha\beta \;$ ...(i)

When c is written icorrectly, then the roots are γ and 2γ.

$\therefore \; \gamma + 2\gamma = -\frac{b}{a} \Rightarrow 3\gamma = \alpha + \beta$

$\Rightarrow 9\gamma^2 = (\alpha + \beta)^2 \;\Rightarrow 9\alpha\beta = (\alpha - \beta)^2 + 4\alpha\beta$

[from Eq. (i)]

$\therefore (\alpha - \beta)^2 = 5\alpha\beta$

13 **(c)**

Let $y = \frac{x^2+34x-71}{x^2+2x-7}$

$\Rightarrow \;\; x^2(y-1) + x(2y - 34) + 71 - 7y = 0$

Since, x is complex number

$\therefore \quad D < 0$

$\Rightarrow (2y - 34)^2 - 4(y-1)(71 - 7y) < 0$

$\Rightarrow \;\; (y - 17)^2 - (71y - 7y^2 - 71 + 7y) < 0$

$\Rightarrow 8y^2 - 112y + 360 < 0$

$\Rightarrow y^2 - 14y + 45 < 0$

$\Rightarrow \; (y - 9)(y - 5) < 0$

$\Rightarrow \;\; 5 < y < 9$

$\therefore \;\; a = 5, b = 9$

14 **(a)**

Given, a, b, c are real, $ax^2 + bx + c = 0$ has two real roots α and β, where $\alpha < -2$ and $\beta > 2$

$\Rightarrow \;\; f(-2) < 0 \;$ and $\; f(2) > 0$

$\Rightarrow \quad 4a - 2b + c < 0 \;$ and $\; 4a + 2b + c > 0$

$\Rightarrow \quad 4 - \frac{2b}{a} + \frac{c}{a} < 0 \;$ and $\; 4 + \frac{2b}{a} + \frac{c}{a} > 0$

15 **(d)**

Let the correct equation be $ax^2 + bx + c = 0$ and the correct roots are α and β. Taking c wrong, the roots are 3 and 2.

$\therefore \;\; \alpha + \beta = 3 + 2 = 5 \;$ ...(i)

Also, $a = 1$ and $c = -6$

$\therefore \; \alpha\beta = \frac{c}{a} = -6 \;$ ...(ii)

On solving Eqs.(i) and (ii), the correct roots are 6 and -1.

16 **(a)**

Since, 1 is root of $ax^2 + bx + c = 0$

$\Rightarrow \quad a + b + c = 0$

$\therefore E_1 : a + b + c = 0$ is true

Since, $\cos\theta, \sin\theta$ are the roots of $ax^2 + bx + c = 0$

$\therefore \quad \sin\theta + \cos\theta = -\frac{b}{a}$

And $\quad \sin\theta\cos\theta = \frac{c}{a}$

$\Rightarrow \quad (\sin\theta + \cos\theta)^2 = \frac{b^2}{a^2}$

$\Rightarrow\ \sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta=\dfrac{b^2}{a^2}$

$\Rightarrow\ \ 1+2\left(\dfrac{c}{a}\right)=\dfrac{b^2}{a^2}$

$\Rightarrow b^2-a^2=2ac$

Hence, E_1 and E_2 both are true

17 **(c)**

$(3+\omega+3\omega^2)^4=[3+(1+\omega^2)+\omega]^4$

$=[-3\omega+\omega]^4$

$=(-2\omega)^4$

$=16\omega$

18 **(a)**

$z^3+\dfrac{1}{i}z^2-\dfrac{z}{i}+1=0$

$\Rightarrow\ \ z^3-iz^2+iz+1=0$

$\Rightarrow\ \ z^2(z-i)+i(z-i)=0$

$\Rightarrow\ \ (z-i)(z^2+i)\ \ \Rightarrow\ |z|=1$

19 **(a)**

Given equation $x^2+ax+1=0$.

Since, roots are $\tan\theta$ and $\cot\theta$.

$\therefore$ Product of roots, $\tan\theta\ \cdot\cot\theta=a\ \Rightarrow a=1$

Again, since roots are real.

$\therefore a^2-4\geq 0\ \Rightarrow |a|\geq 2$

Thus, the least value of $|a|$ is 2.

20 **(c)**

If 1,2, 3, 4 are the roots of given equation, then

$(x-1)(x-2)(x-3)(x-4)$

$\qquad\qquad=x^4+ax^3+bx^2+cx+d$

$\Rightarrow (x^2-3x+2)(x^2-7x+12)$

$\qquad\qquad=x^4+ax^3+bx^2+cx+d$

$\Rightarrow x^4-10x^3+35x^2-50x+24$

$=x^4+ax^3+bx^2+cx+d$

$\Rightarrow a=-10, b=35, c=-50, d=24$

$\therefore\ \ a+2b+c=-10+2\times 35-50=10$

Alternate

Since, 1, 2, 3 and 4 are the roots of the equation

$x^4+ax^3+bx^2+cx+d=0$, then

$1+a+b+c+d=0$ ...(i)

$16+8a+4b+2c+d=0$ ...(ii)

$81+27a+9b+3c+d=0$ ...(iii)

And $256+64a+16b+4c+d=0$ ...(iv)

On solving Eqs. (i), (ii), (iii) and (iv), we get

$a=-10, b=35, c=-50,\qquad d=24$

Now, $a+2b+c=-10+2\times 35+(-50)$

$=-10+70-50=10$

21 **(b)**

We have,

$2(x+2)>x^2+1$

$\Rightarrow x^2-2x-3<0\Rightarrow (x-3)(x+1)<0\Rightarrow -1$

$\qquad\qquad<x<3$

So, there are three integral values viz. 0,1, 2

22 **(a)**

Let the roots be α and $2\ \alpha$. Then,

$\alpha+2\ \alpha=-\dfrac{a}{a-b}$ and $2\ \alpha^2=\dfrac{1}{a-b}$

$\Rightarrow \alpha=-\dfrac{a}{3(a-b)}$ and $\alpha^2=\dfrac{1}{2(a-b)}$

$\Rightarrow \dfrac{a^2}{9(a-b)^2}=\dfrac{1}{2(a-b)}$

$\Rightarrow 2a^2=9a-9b$

$\Rightarrow 2a^2-9a+9b=0$

$\Rightarrow 81-72b\geq 0\qquad [\because a\in R]$

$\Rightarrow b\leq 9/8$

Hence, the greatest value of b is $\frac{9}{8}$

23 **(d)**

Let $z=\frac{1-i\sqrt{3}}{1+i\sqrt{3}}=\frac{1-i\sqrt{3}}{1+i\sqrt{3}}\times\frac{1-i\sqrt{3}}{1-i\sqrt{3}}=-\frac{1}{2}-\frac{i\sqrt{3}}{2}$

$\Rightarrow\ \ \arg(z)=\theta=\tan^{-1}\left(\dfrac{\sqrt{3}/2}{1/2}\right)=\tan^{-1}(\sqrt{3})$

$\Rightarrow\ \ \ \theta=60°$

Since, given number lies in IIIrd quadrant

$\therefore\ \ \ \theta=180°+60°=240°$

24 **(c)**

Let $z=x+iy$

Then, $z+iz=(x+iy)+i(x+iy)=(x-y)+i(x+y)$

and $iz=i(x+iy)=-y+ix$

If Δ be the area of the triangle formed by $z, z+iz$ and iz, then

$\Delta=\dfrac{1}{2}\left|\begin{vmatrix}x & y & 1\\ x-y & x+y & 1\\ -y & x & 1\end{vmatrix}\right|$

Applying $R_2\rightarrow R_2-(R_1+R_3)$

Then $\Delta=\frac{1}{2}\left\|\begin{matrix}x & y & 1\\ 0 & 0 & -1\\ -y & x & 1\end{matrix}\right\|=\frac{1}{2}(x^2+y^2)$

$=\frac{1}{2}|z|^2=200$ (given)

$\Rightarrow\ |z|^2=400$

$\Rightarrow\ |z|=20$

$\therefore\ |3z|=3|z|=60$

25 **(c)**

Given $bx+cx+a=0$ has imaginary roots

$\Rightarrow c^2-4ab<0$

$\Rightarrow c^2<4ab$

$\Rightarrow\ -c^2>-4ab$ ...(i)

Let $f(x)=3b^2x^2+6bcx+2c^2$

Here, $3b^2>0$

So, the given expression has a minimum value

$\therefore$ Minimum value$=\frac{-D}{4a}$

$=\dfrac{4ac-b^2}{4a}$

$= \dfrac{4(3b^2)(2c^2) - 36b^2c^2}{4(3b^2)}$

$= -\dfrac{12b^2c^2}{12b^2} = -c^2 > -4ab$

[from Eq. (i)]

26 **(b)**

Given, $(ax^2 + c)y + (a'\,x^2 + c') = 0$

or $x^2(ay + a') + (cy + c') = 0$

Since, x is rational, then the discriminant of the above equation must be a perfect square.

$\therefore 0 - 4\,(ay + a') + (cy + c') = 0$

$\Rightarrow\ -acy^2 - (ac' + a'c)y - a'c'$

Must be a perfect square

$\Rightarrow\ (ac' - a'c)^2 - 4aca'\,c' = 0$

$\Rightarrow\ (ac' - a'c)^2 = 0$

$\Rightarrow ac' = a'c$

$\Rightarrow \dfrac{a}{a'} = \dfrac{c}{c'}$

27 **(c)**

$\left(1 + i\sqrt{3}\right)^n + \left(1 - i\sqrt{3}\right)^n$

$= 2^n\left[\left(\dfrac{1}{2} + \dfrac{i\sqrt{3}}{2}\right)^n + \left(\dfrac{1}{2} - \dfrac{i\sqrt{3}}{2}\right)^n\right]$

$= 2^n\left[\left(\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}\right)^n + \left(\cos\dfrac{\pi}{3} - i\sin\dfrac{\pi}{3}\right)^n\right]$

$= 2^n\left[2\cos\dfrac{n\pi}{3}\right] = 2^{n+1}\cos\dfrac{n\pi}{3}$

28 **(c)**

Let $z_1 = 1 + i, z_2 = -2 + 3i$ and $z_3 = 0 + \frac{5}{3}i$

Then, $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ -2 & 3 & 1 \\ 0 & \frac{5}{3} & 1 \end{vmatrix}$

$= 1\left(3 - \dfrac{5}{3}\right) + 1(2) + 1\left(\dfrac{-10}{3}\right)$

$= \dfrac{4}{3} + 2 - \dfrac{10}{3}$

$= \dfrac{4 + 6 - 10}{3} = 0$

Hence, area of triangle is zero, therefore points are collinear

29 **(d)**

We have, $z - 2 - 3i = x + iy - 2 - 3i = (x - 2) + i(y - 3)$

Given, $\tan^{-1}\left(\frac{y-3}{x-2}\right) = \frac{\pi}{4}$

$\Rightarrow\quad y - 3 = x - 2$

$\Rightarrow\quad x - y + 1 = 0$

30 **(b)**

$\dfrac{[(\cos 20° + i\sin 20°)(\cos 75° + i\sin 75°)(\cos 10° +}{\sin 15° - i\cos 15°}$

$= \dfrac{e^{i20°}e^{i75°}.e^{i10°}}{-i(\cos 15° + i\sin 15°)}$

$= -\dfrac{e^{i105°}}{ie^{i15°}}$

$= -\dfrac{e^{i90°}}{i} = -1$

31 **(c)**

Since α, β are roots of $x^2 + b\,x + 1 = 0$

$\therefore \alpha + \beta = -b, \alpha\beta = 1$

We have,

$\left(-\alpha - \dfrac{1}{\beta}\right) + \left(-\beta - \dfrac{1}{\alpha}\right)$

$= -(\alpha + \beta) - \left(\dfrac{1}{\beta} + \dfrac{1}{\alpha}\right) = -(\alpha + \beta) - \dfrac{(\alpha + \beta)}{\alpha\,\beta}$

$= b + b = 2b$

and, $\left(-\alpha - \dfrac{1}{\beta}\right)\left(-\beta - \dfrac{1}{\alpha}\right) = \alpha\,\beta + 2 + \dfrac{1}{\alpha\,\beta}$

$= 1 + 2 + 1 = 4$

Thus, the equation whose roots are $-\alpha - \frac{1}{\beta}$ and $-\beta - \frac{1}{\alpha}$ is

$x^2 - 2bx + 4 = 0$

32 **(a)**

The required vector is given by

$\dfrac{3}{2}(z)e^{i\,\pi} = \dfrac{3}{2}(-4 + 5\,i)(-1 + 0i) = 6 - \dfrac{15}{2}i$

33 **(a)**

Given, $\frac{z}{\bar{z}} = \frac{3-i}{3+i}$ [let $z = x + iy$]

$\Rightarrow\quad \dfrac{x + iy}{x - iy} = \dfrac{3 - i}{3 + i} \quad\Rightarrow\quad x = 3a, \quad y = -a$

$\Rightarrow\quad z = a(3 - i)$, where $a \in R$

34 **(b)**

Let $m = \dfrac{(x - b)(x - c)}{x - a}$

$\Rightarrow x^2 - (b + c + m)x + (bc + am) = 0$

Since x is real, we must have

$(b + c + m)^2 - 4(bc + am) \geq 0$

$\Rightarrow m^2 + 2(b + c - 2a)m + (b - c)^2 \geq 0$ for all m

$\Rightarrow 4(b + c - 2a)^2 - 4(b - c)^2 \leq 0$

$\Rightarrow (b + c - 2a)^2 - (b - c)^2 \leq 0$

$\Rightarrow (b + c - 2a + b - c)(b + c - 2a - b + c) \leq 0$

$\Rightarrow 2(b - a)2(c - a) \leq 0$

$\Rightarrow (a - b)(a - c) \leq 0$

$\Rightarrow b \leq a \leq c$ or, $c \leq a \leq b$

35 **(b)**

Let $f(x) = x^4 + ax^3 + bx^2 + cx - 1$

Since $(x - 1)^3$ is a factor of $f(x)$. Therefore, $(x - 1)^2$ is a factor of $f'(x)$ and $(x - 1)$ is a factor of $f''(x)$

$\therefore f(1) = 0, f'(1) = 0$ and $f''(1) = 0$

$\Rightarrow a + b + c = 0, 3a + 2b + c = -4$ and $6a + 2b = -12$

$\Rightarrow a = -2, b = 0, c = 2$

$\therefore f(x) = x^4 - 2x^3 + 2x - 1$
$= (x^4 - 1) - 2x(x^2 - 1)$
$\Rightarrow f(x) = (x^2 - 1)(x^2 + 1 - 2x)$
$= (x + 1)(x - 1)^3$
Hence, $(x + 1)$ is the other factor of $f(x)$

36 **(a)**
Required vertices are given by
$z = (1 + i)\, e^{\pm i\pi/2} = (1 + i)(\pm i) = \pm(-1 + i)$

37 **(b)**
Let all four roots are imaginary. Then roots of both equation $P(x) = 0$ and $Q(x) = 0$ are imaginary.
Thus, $b^2 - 4ac < 0; d^2 - 4ac < 0$, so $b^2 + d^2 < 0$ which is impossible unless $b = 0, d = 0$.
So, if $b \neq 0$ or $d \neq 0$ at least two roots must be real, if $b = 0, d = 0$ we have the equations
$P(x) = ax^2 + c = 0$
and $Q(x) = -ax^2 + c = 0$
or $x^2 = -\frac{c}{a}; x^2 = \frac{c}{a}$ as one of $\frac{c}{a}$ and $-\frac{c}{a}$ must be positive so two roots must be real.

38 **(c)**
$$\frac{1 + a}{1 - a} = \frac{e^{\frac{-i\theta}{2}}(1 + e^{i\theta})}{e^{\frac{-i\theta}{2}}(1 - e^{i\theta})} = \frac{e^{-i\left(\frac{\theta}{2}\right)} + e^{i\frac{\theta}{2}}}{e^{-i\left(\frac{\theta}{2}\right)} - e^{-i\frac{\theta}{2}}}$$
$$= \frac{2\cos\frac{\theta}{2}}{-2i\sin\frac{\theta}{2}} = i\cot\frac{\theta}{2}$$

39 **(a)**
Let, $f(x) = x^2 + 2ax + b$
$= (x + a)^2 + b - a^2$
So, minimum value of $f(x) = b - a^2$.
Since, $f(x) \geq c, \forall x \in R$ hence $b - a^2 \geq c$
$ie, b - c \geq a^2$

41 **(c)**
We have, $z^2 + z|z| + |z|^2 = 0$
$$\Rightarrow \left(\frac{z}{|z|}\right)^2 + \frac{z}{|z|} + 1 = 0$$
This is a quadratic equation in $\frac{z}{|z|}$, therefore roots are $\frac{z}{|z|} = \omega, \omega^2 \Rightarrow z = \omega|z|$ or $z = \omega^2|z|$
Let $z = x + iy$
$$\Rightarrow x + iy = |z|\left(\frac{-1}{2} + \frac{i\sqrt{3}}{2}\right)$$
or $x + iy = |z|\left(\frac{-1}{2} - \frac{i\sqrt{3}}{2}\right)$
$$\Rightarrow x = -\frac{1}{2}|z|, y = |z|\frac{\sqrt{3}}{2}$$
or $x = -\frac{|z|}{2}, y = -\frac{|z|\sqrt{3}}{2}$
$\Rightarrow y + \sqrt{3}x = 0$
or $y - \sqrt{3}x = 0$
$\Rightarrow y^2 - 3x^2 = 0$
$\Rightarrow$ It represents a pair of straight lines

42 **(c)**
Clearly, $|z - i| = 1$ represeents a circle having centre C at $(0, 1)$ and radius 1. Let $P(z)$ be a point on the circle such that $z = r(\cos\theta + i\sin\theta)$
$$\therefore \cot\theta - \frac{2}{z} = \cot\theta - \frac{2}{r}(\cot\theta - i\sin\theta)$$
$$\Rightarrow \cot\theta - \frac{2}{z} = \cot\theta - \frac{2}{r}\cos\theta + \left(\frac{2}{r}\sin\theta\right)i$$
$$\Rightarrow \cot\theta - \frac{2}{z} = \cot\theta - \cot\theta + i \quad \left[\because \sin\theta = \frac{r}{2}\right]$$
$$\Rightarrow \cot\theta - \frac{2}{z} = i$$

43 **(c)**
We have,
$|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2|z_1||z_2|\cos(\theta_1 - \theta_2)$,
Where $\theta_1 = \arg(z_1)$ and $\theta_2 = \arg(z_2)$
$\therefore \arg(z_1 - z_2) = 0$
$\Rightarrow |z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2|z_1||z_2|$
$\Rightarrow |z_1 - z_2|^2 = (|z_1| - |z_2|)^2$
$\Rightarrow |z_1 - z_2| = \big||z_1| - |z_2|\big|$

44 **(a)**
We have,
$x^2 + 4y^2 + 9z^2 - 6yz - 3zx - 2xy$
$= x^2 + (2y)^2 + (3z)^2 - (2y)(3z) - (3z)x - x(2y)$
$$= \frac{1}{2}\{(x - 2y)^2 + (2y - 3z)^2 + (3z - x)^2\} \geq 0$$
Hence, the given expression is always non-negative

45 **(b)**
Let A, B be the centres of circles $|z - z_1| = a$ and $|z - z_2| = b$ respectively. Let $P(\alpha)$ be the centre of the variable circle $|z - \alpha| = r$ which touches the given circles externally. Then,
$AP = r + a$ and $PB = r + b$
$\Rightarrow AP - BP = (r + a) - (r + b)$
$\Rightarrow AP - BP = a - b$
$\Rightarrow$ Locus of P is a hyperbola having its foci at A and B respectively

46 **(b)**
Let $z = \left(1 + i\sqrt{3}\right)^8$
$$= (-2)^8\left(\frac{1 + i\sqrt{3}}{-2}\right) = (-2)^8(\omega^2)^8 \quad [\because \omega^3 = 1]$$
$= 2^8\omega^{16} = 2^8\omega$
$$= 2^8\left(\frac{-1 + i\sqrt{3}}{2}\right)$$
$$= 2^8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

$\therefore$ Modulus$= 2^8 = 256$ and amplitude$=\frac{2\pi}{3}$

47 **(c)**

We have,

$x^2 + (a+b)x + ab < 0$

$\Rightarrow (x+a)(x+b) < 0 \Rightarrow -b < x < -a \Rightarrow x \in (-b, -a)$

48 **(a)**

$x^2 + y^2 + z^2 = (a+b)^2 + \omega^2(a+b\omega)^2 + (a\omega^2 + b\omega)^2$

$= a^2 + b^2 + 2ab + a^2\omega^2 + b^2\omega^4 + 2ab\omega^3 + a^2\omega^4 + b^2\omega^2 + 2ab\omega^3$

$= a^2(1+\omega+\omega^2) + b^2(1+\omega+\omega^2) + 6ab \quad [\because \omega^4 = \omega]$

$= 6ab \quad [\because 1+\omega+\omega^2 = 0]$

49 **(c)**

$\sqrt{-7-24\sqrt{-1}} = \sqrt{-1}\sqrt{7+24i}$

We know

$\sqrt{a+ib} = \pm\left[\sqrt{\frac{1}{2}(\sqrt{a^2+b^2}+a)} + i\sqrt{\frac{1}{2}(\sqrt{a^2+b^2}-a)}\right]$

$\therefore i\sqrt{7+24i}$

$= i\left[\pm\left\{\sqrt{\frac{1}{2}(\sqrt{49+576}+7)} + i\sqrt{\frac{1}{2}(\sqrt{49+576}-7)}\right\}\right]$

$= i\left[\pm\left\{\sqrt{\frac{1}{2}(32)} + i\sqrt{\frac{1}{2}(18)}\right\}\right]$

$= \pm(3 - 4\sqrt{-1})$

50 **(c)**

Given, $\alpha - i\beta = \left(\frac{3+i(-4x)}{3+i(4x)}\right)$

$\Rightarrow \quad |\alpha + i(-\beta)| = \left|\frac{3+i(-4x)}{3+i(4x)}\right|$

$= \frac{|3+i(-4x)|}{|3+i(4x)|}$

$\Rightarrow \quad \alpha^2 + \beta^2 = \frac{9+16x^2}{9+16x^2}$

$\Rightarrow \quad \alpha^2 + \beta^2 = 1$

51 **(b)**

$(1+\omega)^7 = (1+\omega)(1+\omega)^6$

$= (1+\omega)(-\omega^2)^6 = (1+\omega)$

$\Rightarrow A + B\omega = 1 + \omega$

$\Rightarrow A = 1, B = 1$

52 **(a)**

Given equation is $x^2 + 9y^2 - 4x + 3 = 0$ ...(i)

or $x^2 - 4x + 9y^2 + 3 = 0$

Since x is real.

$\therefore (-4)^2 - 4(9y^2+3) \geq 0$

$\Rightarrow 16 - 4(9y^2+3) \geq 0$

$\Rightarrow 4 - 9y^2 - 3 \geq 0$

$\Rightarrow 9y^2 - 1 \leq 0$

$\Rightarrow (3y-1)(3y+1) \leq 0$

$\Rightarrow \frac{-1}{3} \leq y \leq \frac{1}{3}$

Eq. (i) can also be written as

$9y^2 + 0y + x^2 - 4x + 3 = 0$

Since y is real.

$\therefore 0^2 - 4.9(x^2 - 4x + 3)) \geq 0$

$\Rightarrow x^2 - 4x + 3 \leq 0$

$\Rightarrow (x-1)(x-3)) \leq 0$

$\Rightarrow 1 \leq x \leq 3$

53 **(c)**

Let α, β be the roots of the equation

$(a+1)x^2 + (2a+3)x + (3a+4) = 0$. Then,

$\alpha + \beta = -1 \Rightarrow -\left(\frac{2a+3}{a+1}\right) = -1 \Rightarrow a = -2$

$\therefore$Product of the roots $= \frac{3a+4}{a+1} = \frac{-6+4}{-2+1} = 2$

54 **(d)**

We have, $2^{x+2}3^{3x/(x-1)} = 9$

Taking log on both sides, we get

$(x+2)\log 2 + \left(\frac{3x}{x-1}\right)\log 3 = 2\log 3$

$\Rightarrow (x+2)\left(\log 2 + \frac{1}{x-1}\log 3\right) = 0$

$\Rightarrow x = -2$ or $\frac{1}{1-x} = \frac{\log 2}{\log 3}$

$\Rightarrow 1 - x = \frac{\log 3}{\log 2}$

$\Rightarrow x = 1 - \frac{\log 3}{\log 2}$

55 **(a)**

Using $a + b + c = 0$, the given equation reduces to $ax^2 + bx + c = 0$

Clearly, $x = 1$ is a root of this equation

Let D be its discriminant. Then,

$D = b^2 - 4ac = (-a-c)^2 - 4ac = (a-c)^2 > 0$ $[\because a \neq c]$

Hence, the roots are real and unequal

56 **(b)**

We have, $\alpha + \beta = -\sqrt{\alpha}$ and $\alpha\,\beta = \beta$

Now,

$\alpha\,\beta = \beta \Rightarrow \alpha = 1$

$\therefore \alpha + \beta = -\sqrt{\alpha} \Rightarrow \beta = -2$

57 **(c)**

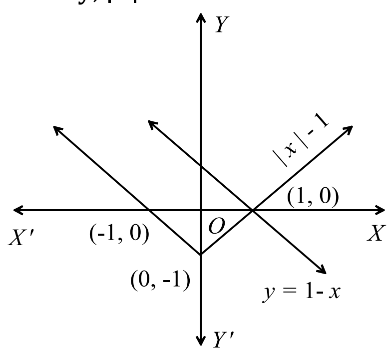
We have,
$(x-a)(x-b)-1=0$
$\Rightarrow x^2-x(a+b)+ab-1=0$
Let α,β be the roots of this equation. Then,
$\alpha+\beta=a+b$ and $\alpha\,\beta=ab-1$
$\Rightarrow$ If one root is less than a, then the other root is greater than b
$\Rightarrow$ One root lies in $(-\infty,a)$ and the other is in (b,∞)
ALITER Clearly, a and b are the roots of the equation $(x-a)(x-b)=0$
Therefore, the curve $y=(x-a)(x-b)$ opens upward and cuts x-axis at $(a,0)$ and $(b,0)$
The curve $y=(x-a)(x-b)-1$ is obtained by translating $y=(x-a)(x-b)$ through one unit in vertically downward direction. So, it will cross x-axis at two points one lying on the left of $(a,0)$ and other one the right of $(b,0)$
Hence, one of the roots lies in $(-\infty,a)$ and other in (b,∞)

58 **(c)**
$$\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\left(\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}\right)\ldots\infty$$
$$=\cos\left(\frac{\pi}{2}+\frac{\pi}{4}+\frac{\pi}{8}+\ldots\infty\right)+i\sin\left(\frac{\pi}{2}+\frac{\pi}{4}+\frac{\pi}{8}+\ldots\infty\right)$$
$$=\cos\left(\frac{\frac{\pi}{2}}{1-\frac{1}{2}}\right)+i\sin\left(\frac{\frac{\pi}{2}}{1-\frac{1}{2}}\right)$$
$\cos\pi+i\sin\pi=-1$

59 **(c)**
$$2\left(1+\frac{1}{\omega}\right)\left(1+\frac{1}{\omega^2}\right)+3\left(2+\frac{1}{\omega}\right)\left(2+\frac{1}{\omega^2}\right)+\cdots+(n+1)\left(n+\frac{1}{\omega}\right)\left(n+\frac{1}{\omega^2}\right)$$
$$=2(1+\omega)(1+\omega^2)+3(2+\omega)(2+\omega^2)+\ldots+(n+1)(n+\omega)(n+\omega^2)$$
$$=\sum_{r=1}^{n}(r+1)(r+\omega)(r+\omega^2)$$
$$=\sum_{r=1}^{n}(r+1)[r^2+(\omega+\omega^2)r+\omega^3]$$
$$=\sum_{r=1}^{n}(r+1)(r^2-r+1)$$
$$=\sum_{r=1}^{n}(r^3+1)$$
$$=\left[\frac{n(n+1)}{2}\right]^2+n$$

60 **(d)**
Let $\sqrt{6+4\sqrt{3}}=\sqrt{x}+\sqrt{y}$
$\Rightarrow 6+4\sqrt{3}=x+y+2\sqrt{xy}$
$\Rightarrow\ x+y=6,\ \sqrt{xy}=2\sqrt{3}$
Now, $(x-y)^2=(x+y)^2-4xy$
$=36-4(4\times 3)$
$=-12<0$
It is not possible
Hence, square root is not possible

61 **(d)**
We have, $|x|-1<1-x$
Two cases arise
CASE I *When* $x\geq 0$
In this case, we have $|x|=x$
$\therefore |x|-1<1-x\Rightarrow x-1<1-x\Rightarrow 2(x-1)<0\Rightarrow x<1$
But, $x\geq 0$. Therefore, $x\in[0,1)$
CASE II *When* $x<0$
In this case, we have $|x|=-x$
$\therefore |x|-1<1-x\Rightarrow -x-1<1-x\Rightarrow -1<1$
This is true for all $x<0$
Hence, $x\in(-\infty,0)\cup[0,1)$ i.e. $x\in(-\infty,1)$
ALITER Draw the graphs of $y=|x|-1$ and $y=1-x$
Clearly, $|x|-1<1-x$ for all $x\in(-\infty,1)$



62 **(c)**
We have,
$(\sqrt{3}+i)^{10}=a+ib$
$\Rightarrow i^{10}(1-i\sqrt{3})^{10}=a+i\,b$
$\Rightarrow -(-2\omega)^{10}=a+i\,b\qquad\left[\because \omega=-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right]$
$\Rightarrow -2^{10}\omega^{10}=a+i\,b$
$\Rightarrow -2^{10}\omega=a+i\,b$
$\Rightarrow -2^{10}\left(\frac{-1}{2}+i\frac{\sqrt{3}}{2}\right)=a+i\,b$
$\Rightarrow 2^9-2^9\sqrt{3}\,i=a+i\,b\ \Rightarrow a=2^9$ and $b=-2^9\sqrt{3}$

63 **(b)**

We have,

$$\left(5+2\sqrt{6}\right)^{x^2-3}+\left(5-2\sqrt{5}\right)^{x^2-3}=\left(5+2\sqrt{6}\right)+\left(5-2\sqrt{6}\right)$$

$\Rightarrow x^2-3=\pm 1 \Rightarrow x=\pm 2, \pm\sqrt{2}$

64 **(d)**

If $x \neq 1$, multiplying each term by $(x-1)$the given equation reduces to $x(x-1)=(x-1)$ or $(x-1)^2=0$ or $x=1$, which is not possible as considering $x \neq 1$, thus given equation has no roots

65 **(b)**

Given, $(1+i)^{2n}=(1-i)^{2n}$

$\Rightarrow\ 2^n i^n=2^n(-1)^n i^n \ \Rightarrow\ 1=(-1)^n$

$\therefore$ The smallest value of n is 2

66 **(a)**

Since, $\frac{z-1}{z+1}$ is purely imaginary

$\therefore\ \dfrac{z-1}{z+1}=-\overline{\left(\dfrac{z-1}{z+1}\right)}$

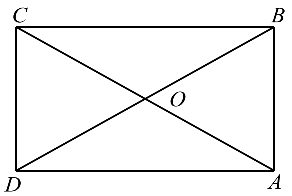
$\Rightarrow\ \dfrac{z-1}{z+1}=\dfrac{\bar{z}-1}{\bar{z}+1}$

$\Rightarrow\ \dfrac{2z}{-2}=\dfrac{2}{-2\bar{z}} \ \Rightarrow\ z\bar{z}=1$

$\Rightarrow\ |z|^2=1 \ \Rightarrow\ |z|=1$

67 **(a)**

Let the vertex A be $3(\cos\theta+i\sin\theta)$, then OB and OD can be obtained by rotating OA through $\frac{\pi}{2}$ and $-\frac{\pi}{2}$ respectively



Thus, $\overrightarrow{OB}=\left(\overrightarrow{OA}\right)e^{i\frac{\pi}{2}}$ and, $\overrightarrow{OD}=\overrightarrow{OA}\,e^{-i\frac{\pi}{2}}$

$\Rightarrow \overrightarrow{OB}=3(\cos\theta+i\sin\theta)\,i$ and, $\overrightarrow{OD}=3(\cos\theta+i\sin)(-i)$

$\Rightarrow \overrightarrow{OB}=3(-\sin\theta+i\cos\theta)$ and, $\overrightarrow{OD}=3(\sin\theta-i\cos\theta)$

Thus, vertices B and D are represented by $\pm 3(\sin\theta-i\cos\theta)$

68 **(a)**

Let α,β be the roots of the given quadratic equation. Then,

$\alpha+\beta=-b/a, \alpha\beta=c/a$

It is given that

$\alpha+\beta=\dfrac{1}{\alpha^2}+\dfrac{1}{b^2}$

$\Rightarrow \alpha^2+\beta^2=(\alpha+\beta)\alpha^2\beta^2$

$\Rightarrow (\alpha+\beta)^2-2\,\alpha\,\beta=(\alpha+\beta)(\alpha\,\beta)^2$

$\Rightarrow \dfrac{b^2}{a^2}-\dfrac{2c}{a}=\dfrac{-bc^2}{a^3}$

$\Rightarrow \dfrac{2c}{a}=\dfrac{b^2}{a^2}+\dfrac{bc^2}{a^3}$

$\Rightarrow 2a^2c=ab^2+bc^2 \Rightarrow c^2b, a^2c, b^2a$ are in A.P.

Dividing both sides of $2a^2c-ab^2+bc^2$ by abc, we get

$2\dfrac{a}{b}=\dfrac{b}{c}+\dfrac{c}{a} \Rightarrow \dfrac{b}{c},\dfrac{a}{b},\dfrac{c}{a}$ are in A. P.

69 **(c)**

Clearly, angle between z and iz is a right angle

$\therefore \angle OPQ=\dfrac{\pi}{2}$

70 **(d)**

We have,

$$\frac{2^n}{(1-i)^{2n}}+\frac{(1+i)^{2n}}{2^n}$$

$$=\frac{2^n}{\{(1-i)^2\}n}+\frac{\{(1+i)^2\}^n}{2^n}$$

$$=\frac{2^n}{(1-2i+i^2)^n}+\frac{(1+2i+i^2)^n}{2^n}$$

$$=\frac{2^n}{(-2i)^n}+\frac{(2i)^n}{2^n}=\left(-\frac{1}{i}\right)^n+i^n=i^n+i^n=2i^n$$

71 **(d)**

Since, the equation $x^2-px+r=0$ has roots (α,β) and the equation $x^2-qx+r=0$ has roots $\left(\frac{\alpha}{2},2\beta\right)$

$\therefore\ \alpha+\beta=p$ and $r=\alpha\beta$ and $\frac{\alpha}{2}+2\beta=q$

$\Rightarrow\ \beta=\frac{2q-p}{3}$ and $\alpha=\frac{2(2p-q)}{3}$

$\therefore\ \alpha\beta=r=\dfrac{2}{9}(2q-p)(2p-q)$

72 **(d)**

We have, $(1+\omega-\omega^2)^7=(-\omega^2-\omega^2)^7$

$=(-2)^7(\omega^2)^7=-128\,\omega^2$

73 **(d)**

We have,

$z+z^{-1}=1 \Rightarrow z^2-z+1=0 \Rightarrow z=-\omega$ or $-\omega^2$

For $z=-\omega$, we have

$$z^{100}+z^{-100}=(-\omega)^{100}+(-\omega)^{-100}=\omega+\frac{1}{\omega}$$
$$=\omega+\omega^2=-1$$

For $z=-\omega^2$, we have

$$z^{100}+z^{-100}=(-\omega^2)^{100}+(-\omega^2)^{-100}$$
$$=\omega^{200}+\frac{1}{\omega^{200}}$$

$\Rightarrow z^{100}+z^{-100}=\omega^2+\dfrac{1}{\omega^2}=\omega^2+\omega=-1$

74 **(c)**

Let $z=\frac{3+2i\sin\theta}{1-2i\sin\theta}$

$\Rightarrow \quad z = \frac{3+2i\sin\theta}{1-2i\sin\theta} \times \frac{(1+2i\sin\theta)}{1+2i\sin\theta}$

$= \frac{3-4\sin^2\theta + 8i\sin\theta}{1+4\sin^2\theta}$

For purely imaginary of z, put $\text{Re}(z) = 0$

ie, $\frac{3-4\sin^2\theta}{1+4\sin^2\theta} = 0$

$\Rightarrow \quad \sin\theta = \pm\frac{\sqrt{3}}{2}$

$\Rightarrow \quad \theta = n\pi + (-1)^n\left(+\frac{\pi}{3}\right) = n\pi \pm \frac{\pi}{3}$

75 **(a)**

We have,

$x^2 + 2ax + 10 - 3a > 0$ for all $x \in R$

$\Rightarrow 4a^2 - 40 + 12a < 0$ [Using: discriminant < 0]

$\Rightarrow a^2 + 3a - 10 < 0$

$\Rightarrow (a+5)(a-2) < 0 \Rightarrow -5 < a < 2$

76 **(b)**

Let $a_1 = a + ib, z_2 = c + id$. Then,

$z_1 + z_2$ is real

$\Rightarrow (a+c) + i(b+d)$ is real

$\Rightarrow b + d = 0 \Rightarrow d = -b$ ...(i)

$z_1 z_2$ is real

$\Rightarrow (ac - bd) + i(ad + bc)$ is real

$\Rightarrow ad + bc = 0$

$\Rightarrow a(-b) + bc = 0$ Using (i)

$\Rightarrow a = c$

$\therefore z_1 = a + ib = c - id = \bar{z}_2$ [$\because a = c$ and $b = -d$]

77 **(a)**

Let $z = z + i\,y$. Then,

$\frac{2\,z+1}{i\,z+1} = \frac{(2x+1)+2\,i\,y}{(1-y)+ix}$

$= \frac{(1-y+2x)+i\,(2\,y-2y^2-2x^2-x)}{(1-y)^2+x^2}$

$\text{Im}\left(\frac{2\,z+1}{i\,z+1}\right) = 3$

$\Rightarrow \frac{2\,y-2\,y^2-2\,x^2-x}{(1-y)^2+x^2} = 3$

$\Rightarrow 2\,y - 2\,y^2 - 2\,x^2 - x = 3\,x^2 + 3(1-y)^2$

$\Rightarrow 5\,x^2 + 5\,y^2 - 8\,y + x + 3 = 0$, which is a circle

78 **(a)**

$z_2 + ax + a^2 = 0 \;\Rightarrow\; z = a\omega, a\omega^2$

(where $'\omega'$ is a non-real root of unity)

$\Rightarrow$ Locus of z is a pair of straight lines

and $\arg(z) = \arg(a) + \arg(\omega)$

or $\arg(z) = \arg(a) + \arg(\omega^2)$

$\Rightarrow \arg(z) = \pm\frac{2\pi}{3}$

Also, $|z| = |a||\omega|$ or $|z| = |a||\omega^2|$

$\Rightarrow |z| = |a|$

79 **(b)**

Diagonals of parallelogram $ABCD$ are bisected each other at a point *ie*,

$\frac{z_1+z_3}{2} = \frac{z_2+z_4}{2}$

$\Rightarrow \quad z_1 + z_2 = z_2 + z_4$

80 **(a)**

Now, $\frac{1}{(1+x)(1+x^2)} = \frac{A}{1+x} + \frac{Bx+C}{1+x^2}$

Where $Bx + C = f(x)$

$\Rightarrow 1 = A(1+x^2) + (Bx+C)(1+x)$

On comparing the coefficient of x^2, x and constant terms, we get

$0 = A + B,\; 0 = B + C$ and $1 = A + C$

$\Rightarrow \quad A = C = \frac{1}{2}$ and $B = -\frac{1}{2}$

$\therefore \quad \frac{1}{(1+x)(1+x^2)} = \frac{1}{2(1+x)} + \frac{-\frac{x}{2}+\frac{1}{2}}{1+x^2}$

$\therefore \quad f(x) = -\frac{x}{2} + \frac{1}{2} = \frac{1-x}{2}$

81 **(c)**

We have, $a + b + c = 0$ ...(i)

Let $D = B^2 + 4AC$

$= 9b^2 - 4(4a)(2c) = 9b^2 - 32ac$

$= 9(a+c)^2 - 32ac$ [from Eq. (i)]

$= 9(a-c)^2 + 4ac$

Hence, roots are real.

82 **(a)**

Given, $x^2(1+2k) + x(1-2k) + 1(1-2k) = 0$ ...(i)

Given, $D = 0,\; b^2 - 4ac = 0$

$\Rightarrow \quad (1-2k)^2 - 4(1+2k)(1-2k) = 0$

$\Rightarrow 20k^2 - 4k - 3 = 0$

$\Rightarrow k = \frac{1}{2}, \frac{3}{10}$

83 **(d)**

We have, $\frac{\log 5 + \log(x^2+1)}{\log(x-2)} = 2$

$\Rightarrow \log\{5(x^2+1)\} = \log(x-2)^2$

$\Rightarrow 5(x^2+1) = (x-2)^2$

$\Rightarrow 4x^2 + 4x + 1 = 0$

$\Rightarrow x = -\frac{1}{2}$

But for $x = -\frac{1}{2}$, $\log(x-2)$ is not meaningful.

$\therefore$ It has no root.

84 **(a)**

We have,

$|x|^2 - 3|x| + 2 = 0$

$\Rightarrow (|x|-1)(|x|-2) = 0$

$\Rightarrow |x| = 1, 2 \Rightarrow x = \pm 1, \pm 2$

85 **(a)**

Let α_1, β_1 be the roots of $x^2 + ax + b = 0$ and α_2, β_2 be the roots of $x^2 + bx + a = 0$. Then,
$\alpha_1 + \beta_1 = -a, \alpha_1\beta_1 = b;\ \alpha_2 + \beta_2 = -b, \alpha_2\beta_2 = a$
It is given that
$|\alpha_1 - \beta_1| = |\alpha_2 - \beta_2|$
$\Rightarrow (\alpha_1 - \beta_1)^2 = (\alpha_2 - \beta_2)^2$
$\Rightarrow (\alpha_1 + \beta_1)^2 - 4\,\alpha_1\beta_1 = (\alpha_2 + \beta_2)^2 - 4\alpha_2\beta_2$
$\Rightarrow a^2 - 4b = b^2 - 4a$
$\Rightarrow (a^2 - b^2) + 4(a - b) = 0 \Rightarrow a + b + 4 = 0$
$[\because a \neq b]$

86 **(b)**
$\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4} = \log_x \sqrt{2}$
$\Rightarrow \frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4} = \frac{1}{2\log_2 x}$
$\Rightarrow 3(\log_2 x)^3 + 4(\log_2 x)^2 - 5(\log_2 x) - 2 = 0$
Put $\log_2 x = y$
$\therefore 3y^2 + 4y^2 - 5y - 2 = 0$
$\Rightarrow (y-1)(y+2)(3y+1) = 0$
$\Rightarrow y = 1, -2, \frac{-1}{3}$
$\Rightarrow \log_2 x = 1, -2, -\frac{1}{3}$
$\Rightarrow x = 2, \frac{1}{2^{1/3}}, \frac{1}{4}$

87 **(d)**
Since $|z + a| \leq a$ implies z lies on or inside a circle with centre $(-a, 0)$ and radius a, we have
$|z_1| + |z_2| + |z_3| \leq 14$

88 **(b)**
$\log_{\sqrt{3}} 300 = \log_{\sqrt{3}} 3 + \log_{\sqrt{3}} 100$
$= 2\log_{\sqrt{3}} \sqrt{3} + 2\log_{\sqrt{3}} 5 + 2\log_{\sqrt{3}} 2$
$= 2(1 + a + b)$ $[\because \log_{\sqrt{b}} 5 = a, \log_{\sqrt{b}} 2 = b]$

89 **(a)**
We have,
$p + q < r + s$ ...(i)
$q + r < s + t$ ...(ii)
$r + s < t + p$ ...(iii)
and, $s + t < p + q$ ...(iv)
From (i) and (iii), we have
$p + q < r + s < t + p \Rightarrow q < t$
From (ii) and (iv), we have
$q + r < s + t < p + q \Rightarrow r < p$
From (i) and (iv), we have
$s + t < p + q < r + s \Rightarrow t < r$
$\therefore q < t < r < p$
From (i), we have $p + q < r + s$
Also, $r < p$
$\therefore p + q + r < r + s + p \Rightarrow q < s$
From (iv), we have $s + t < p + q$
Also, $q < t$
$\therefore s + t + q < p + q + t \Rightarrow s < p$
$\therefore q < s < p$
Hence, the largest and the smallest numbers are p and q respectively

90 **(c)**
We have,
$\frac{x+2}{x^2+1} > \frac{1}{2}$
$\Rightarrow 2x + 4 > x^2 + 1$
$\Rightarrow x^2 - 2x - 3 < 0$
$\Rightarrow (x-3)(x+1) < 0$
$\Rightarrow -1 < x < 3 \Rightarrow x = 0,1,2$ $[\because x$ is an integer$]$

91 **(a)**
Let r be the common ratio of the GP. Since $\alpha, \beta, \gamma, \delta$ are in GP, then $\beta = \alpha r, \gamma = \alpha r^2$ and $\delta = \alpha r^3$.
For equations, $x^2 - x + p = 0$
$\therefore \alpha + \beta = 1$
$\Rightarrow \alpha + \alpha r = 1$
$\Rightarrow \alpha(1 + r) = 1$ ....(i)
and $\alpha\beta = p \Rightarrow \alpha(\alpha r) = p$
$\Rightarrow \alpha^2 r = p$ ...(ii)
For equation, $x^2 - x + q = 0$
$\gamma + \delta = 4$
$\Rightarrow \alpha r^2 + \alpha r^3 = 4$
$\Rightarrow \alpha r^2(1 + r) = 4$ ...(iii)
and $\gamma\delta = q \Rightarrow \alpha r^3 \cdot \alpha r^2 = q$
$\Rightarrow \alpha^2 r^5 = q$ ...(iv)
On dividing Eq. (iii) by Eq. (i),we get
$r^2 = 4 \Rightarrow r = \pm 2$
If we take $r = 2$, then α is not integral, so we take $r = -2$.
Substituting $r = -2$ in Eq. (i), we get
$\alpha = -1$
Now, from Eq. (ii), we have
$p = \alpha^2 r = (-1)^2(-2) = -2$
and from Eq. (iv), we have
$q = \alpha^2 r^5 = (-1)^2(-2)^5 = -32$
$\Rightarrow (p, q) = (-2, -32)$

92 **(a)**
Let the vertices of triangle be
$A(z_1), B(z_2)$ and $C(z_3)$
Given, $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$
$\Rightarrow \left|\frac{z_1 - z_3}{z_2 - z_3}\right| = \frac{|2|}{|2|} = 1$
$\therefore |z_1 - z_3| = |z_2 - z_3|$
$\Rightarrow |AC| = |BC|$
Now, $\frac{z_1 - z_3}{z_2 - z_3} = e^{-i\pi/3}$
$\Rightarrow \arg\left(\frac{z_1 - z_3}{z_2 - z_3}\right) = -\frac{\pi}{3}$

$\therefore \quad \angle BCA = \frac{\pi}{3}$

$\Rightarrow \quad |AC| = |BC| \quad$ and $\quad \angle BCA = 60°$

$\Rightarrow \quad |AB| = |BC| = |CA|$

$\Rightarrow \quad \Delta ABC$ is an equilateral triangle.

93 **(d)**

We have, $225 + (3\omega + 8\omega^2)^2 + (3\omega^2 + 8\omega)^2$

$= 225 + 9\omega^2 + 64\omega^4 + 48\omega^3 + 9\omega^4 + 64\omega^2 + 48\omega^3$

$= 225 + 9\omega^2 + 64\omega + 48 + 9\omega + 64\omega^2 + 48$

$= 225 + 73(\omega^2 + \omega) + 96 = 225 - 73 + 96$

$= 248$

94 **(c)**

Let $z = x + iy$

Given, $\left|\frac{z+2i}{2z+i}\right| < 1$

$\Rightarrow \quad \frac{\sqrt{(x)^2 + (y+2)^2}}{(2x)^2 + (2y+1)^2} < 1$

$\Rightarrow \quad x^2 + y^2 + 4 + 4y < 4x^2 + 4y^2 + 1 + 4y$

$\Rightarrow \quad 3x^2 + 3y^2 > 3$

$\Rightarrow \quad x^2 + y^2 > 1$

95 **(c)**

Let $a - d, a, a + d$ be the roots of the equation $x^3 - 12x^2 + 39\,x - 28 = 0$. Then,

$a - d + a + a + d = 12$ and, $(a - d)(a + d) = 28$

$\Rightarrow 3a = 12$ and $a(a^2 - d^2) = 28$

$\Rightarrow a = 4$ and $d = \pm 3$

96 **(b)**

We have,

$\frac{2}{|x-4|} > 1$

$\Rightarrow 2 > |x - 4|$

$\Rightarrow |x - 4| < 2 \Rightarrow -2 < x - 4 < 2 \Rightarrow 2 < x < 6$

But $\frac{2}{|x-4|} > 1$ is not defined at $x = 4$

$\therefore x \in (2,4) \cup (4,6)$

97 **(b)**

As sum of any four consecutive powers of iota is zero

$\therefore \sum_{n=1}^{13} (i^n + i^{n+1})$

$= (i + i^2 + \ldots + i^{13}) + (i^2 + i^3 + \ldots + i^{14})$

$= i + i^2 = i - 1$

98 **(b)**

The complex cube roots of unity are $1, \omega, \omega^2$

Let $\alpha = \omega, \;\; \beta = \omega^2$

Then, $\alpha^4 + \beta^4 + \alpha^{-1}\beta^{-1} = \omega^4 + (\omega^2)^4 + (\omega)^{-1}(\omega^2)^{-1}$

$= \omega + \omega^2 + 1 = 0$

99 **(b)**

Since a, b, c are in H.P.

$\therefore b = \frac{2\,ac}{a + c}$

Now,

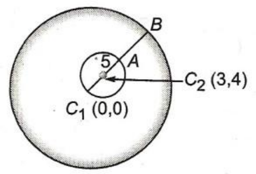
Disc $= 4\,b^2 - 4\,ac = 4\left\{\frac{4\,a^2c^2}{(a+c)^2} - ac\right\}$

$= -4\,ac\frac{(a-c)^2}{(a+c)^2} < 0$

Hence, roots of the given equation are imaginary

100 **(d)**

The two circle whose centre and radius are $C_1(0,0), r_1 = 12, C_2(3,4), r_2 = 5$ and it passes through origin *ie*, the centre of C_1



Now, $C_1C_2 = \sqrt{3^2 + 4^2} = 5$

And $r_1 - r_2 = 12 - 5 = 7$

$\therefore \quad C_1C_2 < r_1 - r_2$

Hence, circle C_2 lies inside the circle C_1

From figure the minimum distance between them, is

$AB = C_1B - C_1A$

$= r_1 - (C_1C_2 + C_2A)$

$= 12 - 10 = 2$

101 **(b)**

Since, α and β be the roots of the equation $x^2 + \sqrt{\alpha}x + \beta = 0$, therefore

$\alpha + \beta = -\sqrt{\alpha}$ and $\alpha\beta = \beta$

From second relation $\beta \neq 0$

$\therefore \;\; \alpha = 1$

$\therefore \;\; 1 + \beta = -1 \;\Rightarrow\; \beta = -2$

Hence, $\alpha = 1$ and $\beta = -2$

102 **(d)**

The equation has no real root, because LHS is always positive while RHS is zero

103 **(a)**

Let $z = x + iy$. Then,

$$\frac{z-1}{z+1} = \frac{(x^2+y^2-1)+2\,iy}{(x+1)^2+y^2}$$

Since $\frac{z-1}{z+1}$ is purely imaginary. Therefore,

$$\text{Re}\left(\frac{z-1}{z+1}\right) = 0$$

$$\Rightarrow \frac{x^2+y^2-1}{(x+1)^2+y^2} = 0$$

$\Rightarrow x^2 + y^2 = 1 \Rightarrow |z|^2 = 1 \Rightarrow |z| = 1$

ALITER We have,

$\left(\frac{z-1}{z+1}\right)$ is purely imaginary

$$\Rightarrow \arg\left(\frac{z-1}{z+1}\right) = \pm\frac{\pi}{2}$$

$\Rightarrow z$ lies on the circle $|z| = 1$

104 **(a)**

Let z be the fourth vertex of parallelogram, then

$$\frac{z_1+z_3}{2} = \frac{z_2+z}{2} \Rightarrow z = z_1 + z_3 - z_2$$

105 **(a)**

Let $z = x + iy$

$\Rightarrow zz = (x+iy)(x+iy)$

$= x^2 - y^2 + 2ixy$

$= 0 + 2ixy \quad [\because \text{Re}(z) = \text{Im}(z) \Rightarrow x = y]$

$\Rightarrow \text{Re}(z^2) = 0$

106 **(c)**

Let $x = \sqrt{-1-\sqrt{-1-\sqrt{-1-\ldots\infty}}}$

Then, $x = \sqrt{-1-x}$ or $x^2 = -1 - x$

or $x^2 + x + 1 = 0$

$$\therefore x = \frac{-1\pm\sqrt{1-4.1.1}}{2.1} = \frac{-1\pm\sqrt{-3}}{2}$$

$$= \frac{-1\pm\sqrt{3}i}{2} = \omega \text{ or } \omega^2$$

107 **(c)**

We have, $z_k = 1 + a + a^2 + \ldots + a^{k-1} = \frac{1-a^k}{1-a}$

$$\Rightarrow z_k - \frac{1}{1-a} = \frac{-a^k}{1-a}$$

$$\Rightarrow \left|z_k - \frac{1}{1-a}\right| = \frac{|a^k|}{|1-a|}$$

$$= \frac{|a^k|}{|1-a|} < \frac{1}{|1-a|} \quad (\because |a| < 1)$$

$\Rightarrow z_k$ lies within a circle $\left|z - \frac{1}{1-a}\right| = \frac{1}{|1-a|}$

108 **(b)**

Here, $\sum\alpha = 0, \sum\alpha\beta = -7, \alpha\beta\gamma = -7$

$$\therefore \frac{1}{\alpha^4} + \frac{1}{\beta^4} + \frac{1}{\gamma^4} = \frac{\alpha^4\beta^4 + \beta^4\gamma^4 + \gamma^4\alpha^4}{\alpha^4\beta^4\gamma^4}$$

$= \frac{\sum\alpha^4\beta^4}{\alpha^4\beta^4\gamma^4} \quad$ ...(i)

Now, $\sum\alpha\beta\sum\alpha\beta\sum\alpha\beta\sum\alpha\beta = (\sum\alpha\beta)^2\,(\sum\alpha\beta)^2$

$\Rightarrow (-7)^4 = [\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$
$+ 2\alpha\beta\gamma(\alpha+\beta+\gamma)]$

$[\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 + 2\alpha\beta\gamma(\alpha+\beta+\gamma)]$

$= (\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2)(\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2)$

$[\therefore \sum\alpha = \alpha + \beta + \gamma = 0]$

$= \alpha^4\beta^4 + \beta^4\gamma^4 + \gamma^4\alpha^4 + 2\alpha^4\beta^2\gamma^2 + 2\alpha^2\beta^4\gamma^2$
$+ 2\alpha^2\beta^2\gamma^4$

$$= \sum\alpha^4\beta^4 + 2\alpha^2\beta^2\gamma^2(\alpha^2+\beta^2+\gamma^4)$$

$$= \sum\alpha^4\beta^4 + 2\alpha^2\beta^2\gamma^2\left[(\sum a)^2 - 2\sum\alpha\beta\right]$$

$$= \sum\alpha^4\beta^4 + 2\alpha^2\beta^2\gamma^2[\,0 - 2\times(-7)]$$

$$= \sum\alpha^4\beta^4 + 2(-7)^2(2\times 7)$$

$$\Rightarrow \sum\alpha^4\beta^4 = (-7)^4 + 4(-7)^3$$

$$\Rightarrow \sum\alpha^4\beta^4 = (-7)^3(-7+4) = -3(-7)^3$$

On putting this value in Eq. (i) , we get

$$\frac{1}{\alpha^4} + \frac{1}{\beta^4} + \frac{1}{\gamma^4} = \frac{-3(-7)^3}{(-7)^4} = \frac{-3}{-7} = \frac{3}{7}$$

109 **(b)**

Given, $\sin\theta + \cos\theta = h$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = h^2$

[squaring]

$$\Rightarrow \sin\theta\cos\theta = \frac{h^2-1}{2}$$

The quadratic equation having the roots $\sin\theta$ and $\cos\theta$ is

$x^2 - (\sin\theta + \cos\theta)x + \sin\theta\cos\theta = 0$

$\therefore \quad 2x^2 - 2hx + (h^2 - 1) = 0$

110 **(a)**

Replacing x by $\frac{1-bx}{ax}$ we get the required equation

$$a\left(\frac{1-bx}{ax}\right)^2 + b\left(\frac{1-bx}{ax}\right) + c = 0$$

$\Rightarrow a(1 + b^2x^2 - 2bx) + ax(b - b^2x) + ca^2x^2 = 0$

$\Rightarrow a + ab^2x^2 - 2abx + abx - ab^2x^2 + a^2cx^2 = 0$

$\Rightarrow \quad acx^2 - bx + 1 = 0$

111 **(d)**

$$\sqrt{i} = \sqrt{\frac{2i}{2}} = \frac{1}{\sqrt{2}}\sqrt{2i + 1 + i^2}$$

$$= \frac{1}{\sqrt{2}}\sqrt{(1+i)^2} = \pm\frac{1}{\sqrt{2}}(1+i)$$

112 **(b)**

Let α and α^n be the roots of the equation, then

$\alpha + \alpha^n = -\frac{b}{a}$ and $\alpha.\alpha^n = \frac{c}{a}$ $\Rightarrow$ $\alpha^{n+1} = \frac{c}{a}$

On eliminating α, we get

$$\left(\frac{c}{a}\right)^{\frac{1}{n+1}} + \left(\frac{c}{a}\right)^{\frac{1}{n+1}} = -\frac{b}{a}$$

$\Rightarrow a.a^{-\frac{1}{n+1}}c^{\frac{1}{n+1}} + a.a^{-\frac{n}{n+1}}c^{\frac{n}{n+1}} = -b$

$\Rightarrow (a^n c)^{\frac{1}{n+1}} + (ac^n)^{\frac{1}{n+1}} = -b$

113 **(d)**

Let $z = x + iy$

$\therefore \quad |z + 3 - i| = |(x + 3) + i(y - 1)| = 1$

$\Rightarrow \quad \sqrt{(x+3)^2 + (y-1)^2} = 1$

...(i)

$\because \quad \arg z = \pi \qquad \Rightarrow \quad \tan^{-1}\frac{y}{x} = \pi$

$\Rightarrow \quad \frac{y}{x} = \tan \pi = 0 \quad \Rightarrow \quad y = 0$

...(ii)

from Eqs.(i)and (ii)we get

$x = -3, \;\; y = 0$

$\therefore \quad z = -3$

$\Rightarrow \quad |z| = |-3| = 3$

114 **(a)**

Let $x = (-1)^{1/3}$

$x = (\cos \pi + i \sin \pi)^{1/3}$

$x = \left[\cos\left(\frac{2n+1}{3}\right)\pi + i \sin\left(\frac{2n+1}{3}\right)\pi\right]$

$= e^{i(2n+1)\pi/3}$

Put $n = 0, 1, 2$ we get

$x = e^{i\pi/3}, e^{i\pi}, e^{5i\pi/3}$

$\therefore$ Products of roots= $e^{i\pi/3}, e^{\pi i}. e^{5\pi i/3} = e^{3\pi i}$

$= (\cos 3\pi + i \sin 3\pi) = -1$

Alternate Method

We know that the cube roots of -1 are -1, $-\omega, -\omega^2$

$\therefore$ Their product = $(-1)(-\omega)(-\omega^2) = -1$

115 **(c)**

Sum of the roots

$= -\frac{b}{a} = -\frac{(-3)}{1} = 3$

From the given options only (c) $ie, -2, 1, 4$ satisfies this condition

116 **(c)**

If $(a^2 - 3a + 2)x^2 + (a^2 - 5a + 6)x + a^2 - 4 = 0$ is an identity in x, then

$a^2 - 3a + 2 = 0, a^2 - 5a + 6 = 0$ and $a^2 - 4 = 0$ must holdgood simultaneously.

Clearly, $a = 2$ is the value of $'a'$ which satisfies these equations

117 **(a)**

Since z_2 and z_3 can be obtained by rotating vector representing z_1 through $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$ respectively

$\therefore z_2 = z_1\,\omega$ and $z_3 = z_1\,\omega^2$

$\Rightarrow z_2 = (1 + i\sqrt{3})\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$ and, z_3

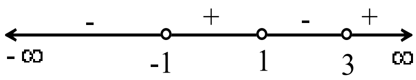
$= (1 + i\sqrt{3})\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$

$\Rightarrow z_2 = -2 + 0\,i$ and $z_3 = 1 - i\sqrt{3}$

118 **(b)**

We have,

$\frac{x^2 - 3x + 4}{x + 1} > 1$



$\Rightarrow \frac{x^2 - 4x + 3}{x + 1} > 0$

$\Rightarrow \frac{(x-1)(x-3)}{x+1} > 0 \Rightarrow x \in (-1,1) \cup (3, \infty)$

119 **(a)**

$\left(\frac{9}{10}\right)^x = -3 + x - x^2$

$\Rightarrow \left(\frac{9}{10}\right)^x = -\left\{\left(x - \frac{1}{2}\right)^2 + \frac{11}{4}\right\}$

$\Rightarrow$ LHS is always positive while RHS is always negative. Hence, the given equation has no solution.

120 **(a)**

Let root of $3ax^2 + 3bx + c = 0$ be α, then

$3a\alpha^2 + 3b\alpha + c = 0$

According to the given condition,

$\Rightarrow \quad x = 3\alpha$

$\Rightarrow \quad \alpha = \frac{x}{3}$

$\therefore \quad 3a\frac{x^2}{9} + 3b\frac{x}{3} + c = 0$

$\Rightarrow \quad ax^2 + 3bx + 3c = 0$

121 **(a)**

<u>CASE I</u> $When\ x^2 + 4x + 3 \geq 0\ i.e.\ x \leq -3$ or $x \geq -1$

In this case, we have

$|x^2 + 4x + 3| = x^2 + 4x + 3$

$\therefore |x^2 + 4x + 3| + (2x + 5) = 0$

$\Rightarrow x^2 + 4x + 3 + 2x + 5 = 0$

$\Rightarrow x = -2, -4 \Rightarrow x = -4 \qquad [\because x \leq -3$ or, $x \geq -1]$

<u>CASE II</u> $When\ x^2 + 4x + 3 < 0$ i.e. $-3 < x < -1$

In this case, we have

$|x^2 + 4x + 3| = -(x^2 + 4x + 3)$

$\therefore |x^2 + 4x + 3| + (2x + 5) = 0$

$\Rightarrow -x^2 - 4x - 3 + 2x + 5 = 0$

$\Rightarrow -x^2 - 2x + 2 = 0$

$\Rightarrow x^2 + 2x - 2 = 0$

$\Rightarrow x = \dfrac{-2 \pm 2\sqrt{3}}{2} = -1 \pm \sqrt{3}$

$\Rightarrow x = -1 - \sqrt{3} \quad [\because -3 < x < -1]$

122 **(d)**

Given, $x = \sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}} = \sqrt{\frac{(2+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}}$

$= 2 + \sqrt{3}$

$\therefore \quad x^2(x-4)^4 = \left(2+\sqrt{3}\right)^2\left(2+\sqrt{3}-4\right)^2$

$= \left(\sqrt{3}+2\right)^2\left(\sqrt{3}-2\right)^2$

$= \left[\left(\sqrt{3}\right)^2 - (2)^2\right]^2$

$= (-1)^2 = 1$

123 **(d)**

We have, $|\lambda_1 a_1 + \lambda_2 a_2 + \ldots + \lambda_n a_n|$

$\leq |\lambda_1 a_1| + |\lambda_2 a_2| + \ldots + |\lambda_n a_n|$

$= |\lambda_1||a_1| + \ldots + |\lambda_n||a_n|$

$= \lambda_1|a_1| + \ldots + \lambda_n|a_n| \quad (\because \text{each } \lambda_k \geq 0)$

$< \lambda_1 + \ldots + \lambda_n$

$(\because |a_k| < 1$ and so $\lambda_k|a_k| < \lambda_k$ for all $k = 1, 2, \ldots n)$

Hence, $|\lambda_1 a_1 + \lambda_2 a_2 + \ldots + \lambda_n a_n| < 1$

124 **(a)**

It is given that $\tan\alpha$ and $\tan\beta$ are the roots of the equation $x^2 + px + q = 0$

$\therefore \tan\alpha + \tan\beta = -p$ and $\tan\alpha\tan\beta = q$

$\Rightarrow \tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \dfrac{-p}{1-q} = \dfrac{p}{q-1}$

The LHS of choice (a) can be written as

$= \cos^2(\alpha+\beta)\{\tan^2(\alpha+\beta) + p\tan(\alpha+\beta) + q\}$

$= \dfrac{1}{1+\tan^2(\alpha+\beta)}\{\tan^2(\alpha+\beta) + p\tan(\alpha+\beta) + q\}$

$= \dfrac{1}{1+\frac{p^2}{(q-1)^2}}\left\{\dfrac{p^2}{(q-1)^2} + \dfrac{p^2}{q-1} + q\right\} = q$

So, option (a) is correct

125 **(c)**

$\sin\dfrac{\pi}{5} + i\left(1 - \cos\dfrac{\pi}{5}\right)$

$= 2\sin\dfrac{\pi}{10}.\cos\dfrac{\pi}{10} + i\,2\sin^2\dfrac{\pi}{10}$

$= 2\sin\frac{\pi}{10}\left(\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}\right)$

$\therefore \quad \tan\theta = \dfrac{\sin\frac{\pi}{10}}{\cos\frac{\pi}{10}} = \tan\dfrac{\pi}{10} \quad \Rightarrow \quad \theta = \dfrac{\pi}{10}$

126 **(b)**

We know that, sum of any four consecutive powers of i is zero

$\therefore \quad \sum_{n=1}^{13}(i^n + i^{n+1})$

$= (i + i^2 + \ldots + i^{13}) + (i^2 + i^3 + \ldots + i^{14})$

$= i^{13} + i^{14}$

$= i - 1$

127 **(a)**

$\log_3 x + \log_3\sqrt{x} + \log_3\sqrt[4]{x} + \log_3\sqrt[8]{x} + \ldots = 4$

$\Rightarrow \log_3 x + \dfrac{1}{2} + \log_3 x + \dfrac{1}{4}\log_3 x + \dfrac{1}{8}\log_3 x + \ldots = 4$

$\Rightarrow \log_3 x\left[1 + \dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \ldots\right] = 4$

$\Rightarrow \log_3 x\left[\dfrac{1}{1-\frac{1}{2}}\right] = 4$

$\Rightarrow \log_3 x = 2$

$\Rightarrow x = 3^2 = 9$

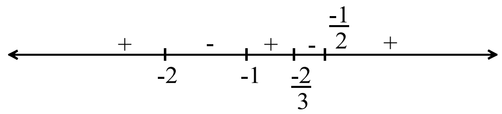
128 **(d)**

We have,

$\dfrac{2x}{2x^2+5x+2} > \dfrac{1}{x+1}$

$\Rightarrow \dfrac{2x}{2x^2+5x+2} - \dfrac{1}{x+1} > 0$

$\Rightarrow \dfrac{2x^2+2x-2x^2-5x-2}{(x+1)(2x+1)(x+2)} > 0$



$\Rightarrow \dfrac{3x+2}{(x+1)(2x+1)(x+2)} < 0$

$\Rightarrow x \in (-2, -1) \cup (-2/3, -1/2)$

129 **(c)**

Let α, β be the roots of the equation $x^2 + px + 8 = 0$

Then, $\alpha + \beta = -p$ and $\alpha\beta = 8$

Now,

$\alpha - \beta = 2$

$\Rightarrow (\alpha+\beta)^2 - 4\,\alpha\beta = (2)^2 \Rightarrow p^2 - 32 = 4 \Rightarrow p = \pm 6$

130 **(d)**

Let α be a common root of the equations $x^2 + ax + 10 = 0$ and $x^2 + bx - 10 = 0$. Then,

$\alpha^2 + a\,\alpha + 10 = 10$

and, $\alpha^2 + b\,\alpha - 10 = 0$

Adding and subtracting these two equations, we get

$2\,\alpha^2 + \alpha(a+b) = 0$ and $(a-b)\alpha + 20 = 0$

$\Rightarrow \alpha = -\dfrac{a+b}{2}$ and $\alpha = -\dfrac{20}{a-b}$

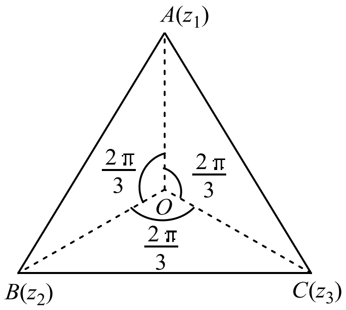
$\Rightarrow -\dfrac{a+b}{2} = -\dfrac{20}{a-b} \Rightarrow a^2 - b^2 = 40$

131 **(a)**

We have,

$|z_1| = |z_2| = |z_3|$

$\Rightarrow OA = OB = OC$, where O is the origin



$\Rightarrow$ Circumcentre of $\Delta\, ABC$ is at the origin

But, the triangle is equilateral. Therefore , its centroid coincides with the circumcentre

Thus,

$\dfrac{z_1 + z_2 + z_3}{3} = 0 \Rightarrow z_1 + z_2 + z_3 = 0$

Clearly, $z_2 = z_1 e^{i\,2\,\pi/3} = z_1\,\omega$ and $z_3 = z_1\,e^{i\,4\,\pi/3} = z_1\,\omega^2$

Let OA be along x-axis such that $OA = 1$ unit.

Then, $z_1 = 1$

$\therefore z_2 = \omega$ and $z_3 = \omega^2$

Hence, $z_1 z_2 z_3 = \omega^2 = 1$

Thus, we have

$z_1 + z_2 + z_3 = 0$ and $z_1 z_2 z_3 = 1$

132 **(c)**

We have,

$\sqrt{x+iy} = \pm\,(a + i\,b)$

$\Rightarrow x + iy = a^2 - b^2 + 2i\,ab$

$\Rightarrow x = a^2 - b^2, y = 2\,ab$

$\therefore \sqrt{-x-iy} = \sqrt{-(a^2-b^2) - 2i\,ab}$

$\Rightarrow \sqrt{-x-iy} = \sqrt{b^2 - a^2 - 2i\,ab} = \sqrt{(b-ia)^2}$

$= \pm(b - ia)$

133 **(c)**

Since, $\alpha\,,\beta$ are the roots of the equation $x^2 + px + q = 0$, then

$\alpha + \beta = p,\ \alpha\beta = q$ ...(i)

and α^4, β^4 are the roots of $x^2 - xr + s = 0$.

Then, $\alpha^4 + \beta^4 = r$ ...(ii)

and $\alpha^4\beta^4 = s$

If D is discriminant of the equation $x^2 - 4qx + 2q^2 - r = 0$,

Then $D = 16q^2 - 4(2q^2 - r) = 8q^2 + 4r$

$= 8\alpha^2\beta^2 + 4(\alpha^4 + \beta^4)$ [from Eqs. (i) and (ii)]

$= 4(\alpha^2\beta^2)^2 \geq 0$

Hence, the equation $x^2 - 4qx + 2q^2 - r = 0$ has always two real roots.

134 **(a)**

Since, a, b and c are the sides of a $\Delta\,ABC$, then

$|a-b| < |c| \Rightarrow a^2 + b^2 - 2ab < c^2$

Similarly, $b^2 + c^2 - 2bc < a^2,\ \ c^2 + a^2 - 2ca < b^2$

On adding, we get

$(a^2 + b^2 + c^2) < 2(ab + bc + ca)$

$\Rightarrow \dfrac{a^2+b^2+c^2}{ab+bc+ca} < 2 \quad \ldots..(i)$

Also, $D \geq 0, (a+b+c)^2 - 3\lambda(ab+bc+ca) \geq 0$

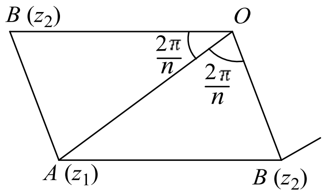
$\Rightarrow \dfrac{a^2+b^2+c^2}{ab+bc+ca} > 3\lambda - 2 \quad \ldots\ldots\ldots(ii)$

From Eqs. (i) and (ii),

$3\lambda - 2 < 2 \Rightarrow \lambda < \frac{4}{3}$

135 **(a)**

Let A be the vertex with affix z_1. There are two possibilities of z_2 ie, z_2 can be obtained by rotating z_1 through $\frac{2\pi}{n}$ either in clockwise or in anti-clockwise direction.



$\therefore \dfrac{z_2}{z_1} = \left|\dfrac{z_2}{z_1}\right| e^{\pm\frac{i2\pi}{n}}$

$\Rightarrow z_2 = z_1\left(\cos\dfrac{2\pi}{n} \pm i\sin\dfrac{2\pi}{n}\right) \quad (\because |z_2| = |z_1|)$

136 **(d)**

Given, $z = \cos\theta + i\sin\theta = e^{i\theta}$

$\therefore \sum_{m=1}^{15} \text{Im}\,(z^{2m-1}) = \sum_{m=1}^{15} \text{Im}\left(e^{i\theta}\right)^{2m-1}$

$= \sum_{m=1}^{15} \text{Im}\, e^{i(2m-1)\theta}$

$= \sin\theta + \sin 3\theta + \sin 5\theta + \ldots + \sin 29\,\theta$

$= \dfrac{\sin\left(\frac{\theta+29\theta}{2}\right)\sin\left(\frac{15\times 2\theta}{2}\right)}{\sin\left(\frac{2\theta}{2}\right)}$

$= \dfrac{\sin(15\,\theta)\sin(15\,\theta)}{\sin\theta} = \dfrac{1}{4\sin 2°}$

137 **(d)**

We have,

$2z^2 + 2z + a = 0 \Rightarrow z = \dfrac{-2 \pm \sqrt{4-8a}}{4}$

$= \dfrac{-1 \pm \sqrt{1-2a}}{2}$

For z to be non-real, we must have

$4 - 8a < 0 \Rightarrow a > \dfrac{1}{2}$

Let $z_1 = \dfrac{-1+\sqrt{1-2a}}{2}$ and $z_2 = \dfrac{-1-\sqrt{1-2a}}{2}$

Now, origin and points representing z_1 and z_2 will form an equilateral triangle in the argand plane, if

$z_1^2 + z_2^2 = z_1 z_2 \quad [\because z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1]$

$\Rightarrow (z_1 + z_2)^2 = 3\, z_1\, z_2$

$\Rightarrow 1 = \dfrac{3a}{2} \Rightarrow a = \dfrac{2}{3}$

Clearly, $a = 2/3$ satisfies the condition $a > 1/2$

Hence, $a = 2/3$

138 **(c)**

Let P, A, B represent complex numbers $z, 1+0i, -1+0i$ respectively, then

$|z-1| + |z+1| \le 4 \quad \Rightarrow \quad PA + PB \le 4$

$\Rightarrow$ P moves in such a way that the sum of its distance from two fixed points is always less than or equal to 4

$\Rightarrow$ Locus of P is the interior and boundary of ellipse having foci at (1, 0) and (-1, 0)

139 **(b)**

On comparing the given circle with $\left|\frac{z-\alpha}{z-\beta}\right| = k$, we get

$\alpha = i,\ \beta = -i, \quad k = 5$

$\therefore$ Radius$= \left|\frac{k(\alpha-\beta)}{1-k^2}\right| = \left|\frac{5(i+i)}{1-25}\right| = \frac{5}{12}$

140 **(d)**

We have,

$(z + \alpha\beta)^3 = \alpha^3 \Rightarrow z = \alpha - \alpha\beta, z = \alpha\omega - \alpha\beta, z = \alpha\omega^2 - \alpha\beta$

Thus, the vertices A, B and C of $\Delta\, ABC$ are respectively, $\alpha - \alpha\,\beta, \alpha\omega - \alpha\,\beta$ and $\alpha\,\omega^2 - \alpha\,\beta$

Clearly, $AB = BC = AC = |\alpha|\,|1-\omega| = \sqrt{3}\,|\alpha|$

141 **(b)**

Given, $\left(\sqrt{5} + \sqrt{3}i\right)^{33} = 2^{49} z$

Let $\sqrt{5} = r\cos\theta, \quad \sqrt{3} = r\sin\theta$

$\therefore \quad r^2 = 5 + 3 \quad \Rightarrow \quad r = 2\sqrt{2}$

$\therefore \quad (r\cos\theta + ir\sin\theta)^{33} = 2^{49} z$

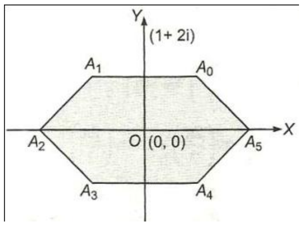
$\Rightarrow \quad |r^{33}(\cos 33\theta + i\sin 33\theta)| = |2^{49} z|$

$\Rightarrow \quad \left(2\sqrt{2}\right)^{33} |\cos 33\theta + i\sin 33\theta| = 2^{49}|z|$

$\Rightarrow \quad 2^{\frac{99}{2}}(1) = 2^{49}|z| \quad \Rightarrow \quad |z| = \sqrt{2}$

142 **(d)**

Let the vertices be $z_0, z_1, \dots, z_5$ w.r.t. centre O at origin and $|z_0| = \sqrt{5}$



$\Rightarrow \quad A_0A_1 = |z_1 - z_0|$

$= |z_0 e^{i\theta} - z_0|$

$= |z_0||\cos\theta + i\sin\theta - 1|$

$= \sqrt{5}\sqrt{(\cos\theta - 1)^2 + \sin^2\theta}$

$= \sqrt{5}\sqrt{2(1-\cos\theta)}$

$= \sqrt{5}.\,2\sin\dfrac{\theta}{2}$

$\Rightarrow \quad A_0A_1 = \sqrt{5}\,2\sin\left(\frac{\pi}{6}\right) = \sqrt{5} \quad \left(\because \theta = \frac{2\pi}{6} = \frac{\pi}{3}\right)$ ...(i)

Similarly, $A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_0 = \sqrt{5}$

Hence, the perimeter of regular hexagon

$= A_0A_1 + A_1A_2 + A_2A_3 + A_3A_4 + A_4A_5 + A_5A_0$

$= 6\sqrt{5}$

143 **(d)**

Let $z = \cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}$, then by using De Moivre's theorem

$\therefore \quad z^k = \cos\frac{2\pi k}{7} + i\sin\frac{2\pi k}{7}$ ...(i)

Let $S = \sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7} - i\cos\frac{2\pi k}{7}\right)$

$= \sum_{k=1}^{6}\left[(-i)\left(\cos\frac{2\pi k}{7} + i\sin\frac{2\pi k}{7}\right)\right]$

$= (-i)\displaystyle\sum_{k=1}^{6}\left(\cos\frac{2\pi k}{7} + i\sin\frac{2\pi k}{7}\right)$

$= (-i)\sum_{k=1}^{6} z^k$ [from Eq.(i)]

$= (-i)[z + z^2 + z^3 + \dots + z^6]$

It is GP of which the first term is z, number of terms is 6 and the common ratio is

$z = \cos\dfrac{2\pi}{7} + i\sin\dfrac{2\pi}{7} \neq 1$

$\therefore \quad S = (-i)\dfrac{z(1-z^6)}{1-z}$

$= (-i)\dfrac{z - z^7}{1-z}$

$= (-i)\dfrac{z-z^7}{1-z} = i \quad \left[\because z^7 = \left(\cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}\right)^7 = \cos 2\pi + i\sin 2\pi = 1\right]$

144 **(d)**

Let α, β *and* γ be the roots of the given equation

$\therefore\ \alpha+\beta+\gamma=-2, \alpha\beta+\beta\gamma+\gamma\alpha=-4$

And $\alpha\beta\gamma=-1$

Let the required cubic equation has the roots $3\alpha, 3\beta$ and 3γ.

$\therefore 3\alpha+3\beta+3\gamma=-6$

$3\alpha.3\beta+3\beta.3\gamma+3\gamma.3\alpha=-36$

And $3\alpha.3\beta.3\gamma=-27$

$\therefore$ Required equation is

$x^3-(-6)x^2+(-36)x-(-27)=0$

$\Rightarrow x^3+6x^2-36x+27=0$

145 **(a)**

Since, $D>0, \sin^2 a-4\sin a(1-\cos a)>0$

$\Rightarrow \sin a>0$ or $(\sin a-4+4\cos a)>0$

$\Rightarrow a\in(0,\pi)$ or $\frac{1-\cos a}{\sin a}<\frac{1}{4}$

$\Rightarrow a\in(0,\pi)$ or $a\in\left(0, 2\tan^{-1}\left(\frac{1}{4}\right)\right)$

$\Rightarrow a\in\left(0, 2\tan^{-1}\left(\frac{1}{4}\right)\right)$

146 **(b)**

Since, α , β are the roots of equation $x^2+bx+c=0$.

Here, $D=b^2-4c>0$ because $c<0<b$. So, roots are real and unequal.

Now, $\alpha+\beta=-b<0$ and $\alpha\,\beta=c<0$

$\therefore$ One root is positive and the other is negative, the negative root being numerically bigger. As $\alpha<\beta, \alpha$ is the negative root while β is the positive root. So, $|\alpha|>$ β and $\alpha<0<\beta$.

147 **(d)**

Given, $x^2-\sqrt{3}x+1=0$

$$\Rightarrow\ x=\frac{\sqrt{3}\pm\sqrt{3-4}}{2}=\frac{\sqrt{3}\pm i}{2}=\cos\frac{\pi}{6}\pm i\sin\frac{\pi}{6}$$

$$\Rightarrow\ x^n=\cos\frac{n\pi}{6}\pm i\sin\frac{n\pi}{6}$$

And $\frac{1}{x^n}=\cos\frac{n\pi}{6}\pm i\sin\frac{n\pi}{6}$

$$\therefore\ x^n-\frac{1}{xn}=\pm 2i\sin\frac{n\pi}{6}$$

$$\Rightarrow\ \left(x^n-\frac{1}{x^n}\right)^2=-4\sin^2\frac{n\pi}{6}$$

Hence, $\sum_{n=1}^{24}\left(x^n-\frac{1}{x^n}\right)^2$

$$=-4\left[\sin^2\frac{\pi}{6}+\sin^2\frac{2\pi}{6}+\ldots+\sin^2\frac{24\pi}{6}\right]$$

$=-4(12)=-48$

148 **(d)**

We have,

$$|x^2-x-6|=\begin{cases}x^2-x-6, \text{if } x\le -2 \text{ or } x\ge 3\\ -(x^2-x-6), \text{if } -2<x<3\end{cases}$$

CASE I *When* $x\le -2$ or, $x\ge 3$

In this case, we have $|x^2-x-6|=x^2-x-6$

$\therefore |x^2-x-6|=x+2$

$\Rightarrow x^2-x-6=x+2$

$\Rightarrow x^2-2x-8=0$

$\Rightarrow (x-4)(x+2)=0$

$\Rightarrow x=-2, 4$

CASE II *When* $-2<x<3$

In this case, we have $|x^2-x-6|=-(x^2-x-6)$

$|x^2-x-6|=x+2$

$\Rightarrow -(x^2-x-6)=x+2$

$\Rightarrow x^2-4=0$

$\Rightarrow x=\pm 2$

$\Rightarrow x=2 \qquad [\because 2\in(-2,3)]$

Hence, the roots are $-2, 2, 4$

149 **(d)**

We have,

$$\begin{vmatrix}3 & 1+S_1 & 1+S_2\\ 1+S_1 & 1+S_2 & 1+S_3\\ 1+S_2 & 1+S_3 & 1+S_4\end{vmatrix}$$

$$=\begin{vmatrix}1+1+1 & 1+\alpha+\beta & 1+\alpha^2+\beta^2\\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3\\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4\end{vmatrix}$$

$$=\begin{vmatrix}1 & 1 & 1\\ 1 & \alpha & \beta\\ 1 & \alpha^2 & \beta^2\end{vmatrix}\begin{vmatrix}1 & 1 & 1\\ 1 & \alpha & \beta\\ 1 & \alpha^2 & \beta^2\end{vmatrix}=\begin{vmatrix}1 & 1 & 1\\ 1 & \alpha & \beta\\ 1 & \alpha^2 & \beta^2\end{vmatrix}^2$$

Now,

$$\begin{vmatrix}1 & 1 & 1\\ 1 & \alpha & \beta\\ 1 & \alpha^2 & \beta^2\end{vmatrix}$$

$$=\begin{vmatrix}1 & 1 & 1\\ 0 & \alpha-1 & \beta-1\\ 0 & \alpha^2-1 & \beta^2-1\end{vmatrix} \quad \left[\begin{matrix}\text{Applying } R_2\to R_2-R_1\\ R_3\to R_3-R_1\end{matrix}\right.$$

$=(\alpha-1)(\beta^2-1)-(\beta-1)(\alpha^2-1)$

$=\alpha\beta^2-\alpha-\beta^2-\alpha^2\beta+\beta+\alpha^2$

$=(\alpha^2-\beta^2)-(\alpha-\beta)-\alpha\beta(\alpha-\beta)$

$=(\alpha-\beta)[\alpha+\beta-1-\alpha\beta]$

$=\sqrt{(\alpha+\beta)^2-4\,\alpha\beta}\,\{\alpha+\beta-1-\alpha\,\beta\}$

$$=\sqrt{\frac{b^2-4ac}{a^2}}\left\{-\frac{b}{a}-1-\frac{c}{a}\right\}$$

$$=-\sqrt{\frac{b^2-4ac}{a^2}}\left(\frac{a+b+c}{a}\right)$$

Hence,

$$\begin{vmatrix}3 & 1+S_1 & 1+S_2\\ 1+S_1 & 1+S_2 & 1+S_3\\ 1+S_2 & 1+S_3 & 1+S_4\end{vmatrix}$$

$$=\left\{-\sqrt{\frac{b^2-4ac}{a^2}}\left(\frac{a+b+c}{a}\right)\right\}^2$$

$$=\frac{(b^2-4ac)(a+b+c)^2}{a^4}$$

150 **(d)**

We have,

$z_k = e^{\frac{i 2\pi k}{n}}, \qquad k = 0,1,2,\dots,n-1$

$\therefore |z_k| = \left|e^{\frac{i 2\pi k}{n}}\right| = 1 \quad \text{for all} = 0,1,2,\dots n-1$

$\Rightarrow |z_k| = |z_{k+1}| \qquad \text{for all } k = 0,1,2,\dots,n-1$

151 **(a)**

Here, $\alpha + \beta = 1 + n^2$ and $\alpha\beta = \frac{1+n^2+n^4}{2}$

Now, $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (1+n^2)^2 - (1+n^2+n^4) = n^2$

152 **(b)**

Since, 4is a root of $x^2 + ax + 12 = 0$

$\therefore 16 + 4a + 12 = 0 \Rightarrow a = -7$

Let the roots of the equation $x^2 + ax + b = 0$ be α and α

$\therefore 2\alpha = -a$

$\Rightarrow \alpha = \frac{7}{2}$

And $\alpha.\alpha = b$

$\Rightarrow \left(\frac{7}{2}\right)^2 = b$

$\Rightarrow b = \frac{49}{4}$

153 **(d)**

$\log_{140} 63 = \log_{2^2\times5\times7}(3\times3\times7)$

$= \frac{\log_2(3\times3\times7)}{\log_2(2^2\times5\times7)}$

$= \frac{2\log_2 3 + \log_2 7}{2\log_2 2 + \log_2 5 + \log_2 7}$

$= \frac{2a + \frac{1}{c}}{2 + b + \frac{1}{c}} = \frac{2ac+1}{2c+bc+1}$

154 **(d)**

We have,

$(1-i)^n = 2^n$

$\Rightarrow |1-i|^n = |2|^n$

$\Rightarrow \left(\sqrt{2}\right)^n = 2^n \Rightarrow 2^{n/2} = 2^n \Rightarrow 2^{n/2} = 1 \Rightarrow n = 0$

So, there is no non-zero integral solution of the given equation

155 **(a)**

We have the following cases:

CASE I *When* $x < 0$

In this case, we have $Sgn\, x = -1$

$\therefore x^2 - 5x - (Sgn\, x)6 = 0$

$\Rightarrow x^2 - 5x + 6 = 0 \Rightarrow x = 2,3$

But, $x < 0$. So, the equation has no solution in this case.

CASE II *When* $x > 0$

In this case, we have $Sgn\, x = 1$

$\therefore x^2 - 5x - (Sgn\, x)6 = 0$

$\Rightarrow x^2 - 5x - 6 = 0$

$\Rightarrow (x-6)(x+1) = 0 \Rightarrow x = -1,6 \Rightarrow x = 6\ [\because x > 0]$

Hence, the given equation has only one solution

156 **(a)**

We have,

$z^n = (1+z)^n$

$\Rightarrow |z^n| = |(1+z)^n|$

$\Rightarrow |z|^n = |1+z|^n$

$\Rightarrow |z| = |1+z|$

$\Rightarrow |z-0| = ||z-(-1)|$

$\Rightarrow z$ lies on the perpendicular bisector of the segment joing (0,0) and (0, −1)

$\Rightarrow z = -\frac{1}{2} \Rightarrow \text{Re}\,(z) < 0$

157 **(a)**

Given, $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$

$= (1+\omega)(-\omega)(1+\omega)(1+\omega^2)$

$[\because 1+\omega+\omega^2 = 0 \text{ and } \omega^4 = \omega]$

$= (1+\omega)^2(-\omega-\omega^3)$

$= (1+\omega^2+2\omega)(-\omega-1)$

$= (\omega)(\omega^2) = 1$

158 **(d)**

We have,

$\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = \begin{vmatrix} 6i & 0 & 1 \\ 4 & 0 & -1 \\ 20 & 0 & i \end{vmatrix}$ Applying $C_2 \to C_2 + 3i\, C_3$

$= 0 = 0 + 0i$

$\therefore x = 0, y = 0$

159 **(c)**

Since z_1, z_2, z_3 are vertices of an equilateral triangle

$\therefore z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$

$\Rightarrow (z_1+z_2+z_3)^2 = 3(z_1^2+z_2^2+z_3^2)$

$\Rightarrow (3z_0)^2 = 3(z_1^2+z_2^2+z_3^2) \quad \left[\because \frac{z_1+z_2+z_3}{3} = z_0\right]$

$\Rightarrow z_1^2 + z_2^2 + z_3^2 = 3z_0^2$

160 **(b)**

As we know, $ax^2 + bx + c > 0$ for all $x \in R$, iff $a > 0$ and $D < 0$

$\therefore x^2 + 2ax + (10-3a) > 0, \forall x \in R$

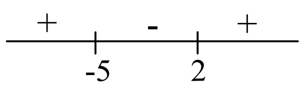
$\Rightarrow D < 0$

$\Rightarrow 4a^2 - 4(10-3a) < 0$

$\Rightarrow 4(a^2 + 3a - 10) < 0$

$\Rightarrow (a+5)(a-2) < 0$

Using number line rule



$a \in (-5, 2)$

161 **(b)**

Given that α_1, α_2 are the roots of the equation $ax^2 + bx + c = 0$, then

$\alpha_1 + \alpha_2 = -\frac{b}{a}$ and $\alpha_1\alpha_2 = \frac{c}{a}$ ...(i)

Now, β_1, β_2 are the roots of $px^2 + qx + r = 0$, then

$\beta_1 + \beta_2 = -\frac{q}{p}$ and $\beta_1\beta_2 = \frac{r}{p}$ ...(ii)

Given system is $\alpha_1 y + \alpha_2 z = 0$ and $\beta_1 y + \beta_2 z = 0$.

$\Rightarrow \frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2}$

Now, $\frac{\alpha_1\alpha_2}{\beta_1\beta_2} = \frac{\frac{c}{a}}{\frac{r}{p}}$

$\Rightarrow \frac{\alpha_1}{\beta_1} \cdot \frac{\alpha_2}{\beta_2} = \frac{cp}{ar}$ ...(iii)

Since, $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} \Rightarrow \frac{\alpha_1}{\alpha_2} = \frac{\beta_1}{\beta_2} \Rightarrow \frac{\alpha_1^2}{\alpha_2^2} = \frac{\beta_1^2}{\beta_2^2}$

$\Rightarrow \frac{\alpha_1^2+\alpha_2^2}{\alpha_2^2} = \frac{\beta_1^2+\beta_2^2}{\beta_2^2}$ (on adding 1 on both sides)

$\Rightarrow \frac{\alpha_2^2}{\beta_2^2} = \frac{\alpha_1^2 + \alpha_2^2}{\beta_1^2 + \beta_2^2}$

$= \frac{(\alpha_1 + \alpha_2)^2 - 2\alpha_1\alpha_2}{(\beta_1 + \beta_2)^2 - 2\beta_1\beta_2}$

On substituting the values from Eqs. (i), (ii) and (iii), we get

$\frac{cp}{ar} = \frac{\frac{b^2}{a^2} - 2\left(\frac{c}{a}\right)}{\frac{q^2}{p^2} - 2\left(\frac{r}{p}\right)} = \frac{(b^2 - 2ac)p^2}{(q^2 - 2pr)a^2}$

$\Rightarrow \frac{c}{r} = \frac{pb^2 - 2acp}{q^2 a - 2apr}$

$\Rightarrow b^2 rp - 2acpr = q^2 ac - pr2ac$

$\Rightarrow b^2 pr = q^2 ac$

162 **(b)**

$(1 - \omega + \omega^2)(1 - \omega^2 + \omega^3.\omega)$

$(1 - \omega^3.\omega + \omega^6.\omega^2)(1 - \omega^6.\omega^2 + \omega^{15}.\omega)$... upto $2n$

$= (1\omega + \omega^2)(1 - \omega^2 + \omega)$

$(1 - \omega + \omega^2)(1 - \omega^2 + \omega)$... upto $2n$

$= [(-2\omega)(-2\omega^2)] \times [(-2\omega)(-2\omega^2)] \times$... upto $2n$

$= (2^2\omega^3) \times (2^2\omega^3) \times$.... upto n

$=[2^2 \times 2^2 \times 2^2 \times$...upto n$]= 2^{2n}$

163 **(d)**

Given, α and β are different complex numbers and $|\beta| = 1$

$\therefore \left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right| = \frac{|\beta - \alpha|}{|\beta\bar{\beta} - \bar{\alpha}\beta|} = \frac{|\beta - \alpha|}{|\beta||\bar{\beta} - \bar{\alpha}|} = 1$

164 **(d)**

$\frac{\log_{c+b} a + \log_{c-a} a}{2\log_{c+b} a \,.\log_{c-b} a}$

$= \frac{\frac{\log a}{\log(c+b)} + \frac{\log a}{\log(c-b)}}{2\frac{\log a}{\log(c+b)}\cdot\frac{\log a}{\log(c-b)}}$

$= \frac{\log a\{\log(c-b) + \log(c+b)\}}{2(\log a)^2} = \frac{\log(c^2 - b^2)}{2\log a}$

$= \frac{\log a^2}{\log a^2}$ $(\because a^2 + b^2 = c^2)$

$= 1$

165 **(b)**

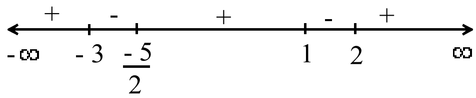
We have,

$\frac{10x^2 + 17x - 34}{x^2 + 2x - 3} < 8$

$\Rightarrow \frac{10x^2 + 17x - 34 - 8x^2 - 16x + 24}{x^2 + 2x - 3} < 0$

$\Rightarrow \frac{2x^2 + x - 10}{x^2 + 2x - 3} < 0$

$\Rightarrow \frac{(2x + 5)(x - 2)}{(x + 3)(x - 1)} < 0 \Rightarrow x \in (-3, -5/2) \cup (1,2)$



166 **(b)**

$\left(\frac{1 + \cos\phi + i\sin\phi}{1 + \cos\phi - i\sin\phi}\right)^n = u + iv$

$\Rightarrow \left(\frac{2\cos^2\frac{\phi}{2} + 2i\sin\frac{\phi}{2}\cos\frac{\phi}{2}}{2\cos^2\frac{\phi}{2} - 2\sin\frac{\phi}{2}\cos\frac{\phi}{2}}\right)^n = u + iv$

$\Rightarrow \left(\frac{e^{i\frac{\phi}{2}}}{e^{-i\frac{\phi}{2}}}\right)^n = u + iv \quad \Rightarrow \quad (e^{in\phi}) = u + iv$

$\Rightarrow \cos n\phi + i\sin n\phi = u + iv$

$\Rightarrow u = \cos n\phi, \; v = \sin n\phi$

167 **(c)**

We have,

$2x^4 + 5x^2 + 3 > 0$ for all $x \in R$

So, $2x^4 + 5x^2 + 3 = 0$ has no real root

168 **(c)**

Given, α, β are the roots of $x^2 - 2x + 4 = 0$

$\therefore \alpha + \beta = 2$ ...(i)

And $\alpha\beta = 4$ ...(ii)

Now, $\alpha - \beta = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

$= \sqrt{4 - 4 \times 4} = \sqrt{-12}$

$\Rightarrow \alpha - \beta = 2\sqrt{3}i$ ...(iii)

On solving Eqs. (i) and (ii0, we get

$\alpha = \frac{2 + 2\sqrt{3}i}{2} = -2\left(\frac{-1 - \sqrt{3}i}{2}\right) = -2\omega^2$

And $\beta = \frac{2-2\sqrt{3}i}{2} = -2\left(\frac{-1+\sqrt{3}i}{2}\right) = -2\omega$

Now, $\alpha^6 + \beta^6 = (-2\omega^2)^6 + (-2\omega)^6$

$= 64(\omega^3)^4 + 64(\omega^3)^2$

$= 128 \quad [\because \omega^3 = 1]$

169 **(b)**

We have, $|z+4| \le 3 \Rightarrow -3 \le z+4 \le 3$

$\Rightarrow -6 \le z+1 \le 0 \Rightarrow 0 \le -(z+1) \le 6$

$\Rightarrow 0 \le |z+1| \le 6$

Hence, greatest and least values of $|z+1|$ are 6 and 0 respectively

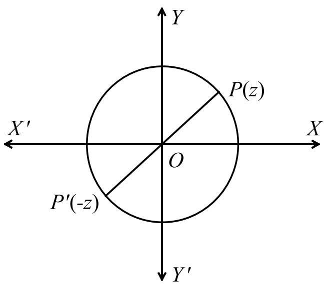
170 **(a)**

Let $P(z)$ be any point on the circle

$OP = OP''$

$\Rightarrow |z| = |z_1|$

$\Rightarrow |z|^2 = |z_1|^2 \Rightarrow z\bar{z} = z_1 \bar{z_1} \Rightarrow \frac{z}{z_1} = \frac{\bar{z}_1}{z}$



171 **(c)**

It is given that $x+1$ be a factor of $f(x)$ given by

$$f(x) = x^4 + (p-3)x^3 - (3p-5)x^2 + (2p-9)x + 6$$

$\therefore f(-1) = 0$

$\Rightarrow 1 - p + 3 - 3p + 5 - 2p + 9 + 6 = 0$

$\Rightarrow 6p = 24 \Rightarrow p = 4$

172 **(a)**

Let $\alpha \in A \cap B$. Then,

$\alpha \in A \cap B$

$\Rightarrow \alpha \in A$ and $\alpha \in B$

$\Rightarrow f(\alpha) = 0$ and $g(\alpha) = 0$

$\Rightarrow [f(\alpha)]^2 + [g(\alpha)]^2 = 0$

$\Rightarrow \alpha$ is a root of $[f(x)]^2 + [g(x)]^2 = 0$

173 **(d)**

Here, $\alpha + \beta = -p$ and $\alpha\beta = q$

Now, $(\alpha+\beta)x - \frac{\alpha^2+\beta^2}{2}x^2 + \frac{\alpha^3+\beta^3}{3}x^3 - \cdots$

$$= \left(\alpha x - \frac{\alpha^2 x^2}{2} + \frac{\alpha^3 x^3}{3} - \ldots\right) + \left(\beta x - \frac{\beta^2 x^2}{2} + \frac{\beta^3 x^3}{3} - \ldots\right)$$

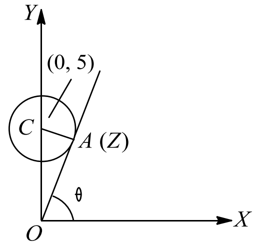
$= \log(1+\alpha x) + \log(1+\beta x)$

$= \log\{1 + (\alpha+\beta)x + \alpha\beta x^2\}$

$= \log(1 - px + qx^2)$

174 **(a)**

We have, $|z - 5i| \le 1$



Let $\theta = \angle AOX = \text{min. amp}(z)$,

$\therefore \angle AOC = 90° - \theta$

$\Rightarrow \sin(90° - \theta) = \frac{1}{5}$

$\Rightarrow \cos\theta = \frac{1}{5}$

$\therefore z = OA\cos\theta + i\, OA\sin\theta$

$$\Rightarrow z = \sqrt{5^2-1}\left(\frac{1}{5}\right) + i\sqrt{5^2-1}\sqrt{1 - \frac{1}{5^2}}$$

$$= \frac{2\sqrt{6}}{5}(1 + i\, 2\sqrt{6})$$

175 **(b)**

Since α, β are the roots of the equation $x^2 + px + 1$ and γ, δ are the roots of the equation $x^2 + qx + 1 = 0$

$\therefore \alpha^2 + p\,\alpha + 1 = 0, \beta^2 + p\,\beta + 1 = 0,$

$\gamma^2 + q\,\gamma + 1 = 0$ and $\delta^2 + q\,\delta + 1 = 0 \quad$ ...(i)

Also, $\alpha + \beta = -p, \alpha\,\beta = 1, \gamma + \delta = -q$ and $\gamma\,\delta = 1$

$\therefore (\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta)$

$= \{\alpha\,\beta - \gamma(\alpha+\beta) + \gamma^2\}\{\alpha\,\beta + \delta(\alpha+\beta) + \delta^2\}$

$= (\gamma^2 + p\,\gamma + 1)(\delta^2 - p\,\delta + 1)$

$= (p\gamma - q\gamma)(-q\,\delta - p\,\delta) \quad$ [Using (i)]

$= (p+q)(q-p)\gamma\,\delta = (q^2 - p^2)$

176 **(a)**

Since, $|z_1 + z_2| = |z_1| + |z_2|$

$\Rightarrow |z_1|^2 + |z_2|^2 + 2|z_1||z_2|\cos(\theta_1 - \theta_2)$

$= |z_1|^2 + |z_2|^2 + 2|z_1||z_2|$

$\Rightarrow \cos(\theta_1 - \theta_2) = 1 = \cos 0°$

$\Rightarrow \theta_1 - \theta_2 = 0 \Rightarrow \theta_1 = \theta_2$

$\Rightarrow \arg(z_1) = \arg(z_2)$

177 **(a)**

$$\sin\left\{(\omega^{10} + \omega^{23})\pi - \frac{\pi}{4}\right\} = \sin\left\{(\omega + \omega^2)\pi - \frac{\pi}{4}\right\}$$

$$= \sin\left(-\pi - \frac{\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

178 **(d)**

Let $f(x) = x^3 - 3x + a$

If $f(x)$ has distinct roots between 0 and 1. Then, $f'(x) = 0$ has a root between 0 and 1

But, $f'(x) = 0 \Rightarrow 3x^2 - 3 = 0 \Rightarrow x = \pm 1$

Clearly, $f'(x) = 0$ does not have any root between 0 and 1.

So, $f(x)$ does not have distinct roots between 0 and 1 for any value of α

179 **(c)**

It is given that α, β are the roots of the equation $375x^2 - 25x - 2 = 0$

$$\therefore \alpha + \beta = \frac{1}{15} \text{ and } \alpha\,\beta = \frac{-2}{375}$$

$$\therefore \lim_{n\to\infty} \sum_{r=1}^{n} S_r = \lim_{n\to\infty} \sum_{r=1}^{n} (\alpha^r + \beta^r)$$

$$\Rightarrow \lim_{n\to\infty} \sum_{r=1}^{n} S_r = (\alpha + \alpha^2 + \alpha^3 + \cdots \infty) + (\beta + \beta^2 + \beta^3 + \cdots \infty)$$

$$\Rightarrow \lim_{n\to\infty} \sum_{r=1}^{n} S_r = \frac{\alpha}{1-\alpha} + \frac{\beta}{1-\beta} \quad [\because |\alpha| < 1, |\beta| < 1]$$

$$\Rightarrow \lim_{n\to\infty} \sum_{r=1}^{n} S_r = \frac{\alpha + \beta - 2\,\alpha\,\beta}{1 - (\alpha+\beta) + \alpha\,\beta} = \frac{\frac{1}{15} + \frac{4}{375}}{1 - \frac{1}{15} - \frac{2}{375}} = \frac{29}{348}$$

180 **(d)**

We have,

$y = \tan x \cot 3x$

$$\Rightarrow y = \frac{\tan x}{\tan 3x}$$

$$\Rightarrow y = \frac{\tan x(1 - 3\tan^2 x)}{3\tan x - \tan^3 x}$$

$$\Rightarrow y = \frac{1 - 3\tan^2 x}{3 - \tan^2 x}$$

$$\Rightarrow \tan^2 x = \frac{3y-1}{y-3}$$

$$\Rightarrow \frac{3y-1}{y-3} \geq 0 \qquad [\because \tan^2 x \geq 0]$$

$$\Rightarrow y \leq \frac{1}{3} \text{ or } y > 3$$

181 **(c)**

Let α, β be the roots of the equation $2x(2x+1) = 1$. Then,

$$\alpha + \beta = -\frac{1}{2} \text{ and } \alpha\,\beta = -\frac{1}{4}$$

$\Rightarrow 4\alpha^2 + 2\,\alpha - 1 = 0$ ...(i)

Again,

$$\alpha + \beta = -\frac{1}{2}$$

$$\Rightarrow \beta = -\frac{1}{2} - \alpha$$

$$\Rightarrow \beta = -\frac{1 + 2\,\alpha}{2}$$

$$\Rightarrow \beta = -\frac{4\,\alpha^2 + 2\,\alpha + 2\,\alpha}{2} \qquad \text{[Using (i)]}$$

$\Rightarrow \beta = -2\,\alpha\,(\alpha + 1)$

$\Rightarrow \beta = -2\,\alpha^2 - 2\,\alpha$

$\Rightarrow \beta = -2\,\alpha \times \alpha - 2\,\alpha$

$\Rightarrow \beta = \alpha(4\,\alpha^2 - 1) - 2\,\alpha$ [Using (i)]

$\Rightarrow \beta = 4\,\alpha^3 - 3\,\alpha$

182 **(a)**

Let two consecutive integers n and $(n+1)$ be the roots of $x^2 - bx + c = 0$. Then, $n + (n+1) = b$ and $n(n+1) = c$

$\therefore\ b^2 - 4c = (2n+1)^2 - 4n(n+1) = 1$

183 **(b)**

Given, $a^x = b^y = c^z = m$(say)

$\Rightarrow\ x = \log_a m,\ \ y = \log_b m,\ \ z = \log_c m$

Again as, x, y, z are in GP, so

$$\frac{y}{x} = \frac{z}{y}$$

$$\Rightarrow \frac{\log_b m}{\log_a m} = \frac{\log_c m}{\log_b m}$$

$$\Rightarrow \frac{\log_m a}{\log_m b} = \frac{\log_m b}{\log_m c}$$

$\Rightarrow \log_b a = \log_c b$

184 **(b)**

Let $O, A\ (z_1)$ and $B(z_2)$ be the vertices of the triangle. The triangle is an equilateral triangle

$\therefore z_2 = z_1\, e^{\pm\, i\,\pi/3}$

$\Rightarrow 1 + i\,b = (a + i)(\cos\pi/3 \pm \sin\pi/3)$

$\Rightarrow 1 + ib = (a + i)(1/2 \pm i\,\sqrt{3}/2)$

$$\Rightarrow 1 + i\,b = \left(\frac{a}{2} \pm \frac{\sqrt{3}}{2}\right) + i\left(\frac{1}{2} \pm a\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \frac{a}{2} \pm \frac{\sqrt{3}}{2} = 1 \text{ and } b = \frac{1}{2} \pm \frac{1}{2}a\sqrt{3}$$

$$\Rightarrow \left(a = 2 - \sqrt{3} \text{ or } a = 2 + \sqrt{3}\right) \text{ and } b = \frac{1}{2} \pm \frac{a}{2}\sqrt{3}$$

$\Rightarrow a = 2 - \sqrt{3}$ and $b = 2 - \sqrt{3}$ $[\because 0 < a, b < 1]$

185 **(d)**

We have,

$$\sum_{r=0}^{n} (-1)^r\ {}^nC_r\{i^{5r} + i^{6r} + i^{7r} + i^{8r}\}$$

$$= \sum_{r=0}^{n} (-1)^r\,{}^nC_r\{i^r + i^{2r} + i^{3r} + 1\}$$

$$= \sum_{r=0}^{n} (-1)^r\,{}^nC_r\, i^r + \sum_{r=0}^{n} (-1)^r\ {}^nC_r(i^2)^r + \sum_{r=0}^{n} (-1)^r\ {}^nC_r(i^3)^r + \sum_{r=0}^{n} (-1)^r\ {}^nC_r$$

$= (1 - i)^n + (1 - i^2)^n + (1 - i^3)^n + (1 - 1)^n$

$= (1 - i)^n + 2^n + (1 + i)^n$

$= 2^n + 2^{n/2}\left\{\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right\}^n$

$+ 2^{n/2}\left\{\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right\}^n$

$= 2^n + 2^{n/2+1}\cos\frac{n\pi}{4}$

186 **(c)**

Since, $b = \frac{a+c}{2}$ ...(i)

Now, discriminant, $D = B^2 - 4AC$

$= 4b^2 - 4ac$

$= 4\left(\frac{a+c}{2}\right)^2 - 4ac$ [from Eq. (i)]

$= (a-c)^2 \geq 0$

$\therefore$ Roots of the given equation are rational and distinct

187 **(a)**

We have,

$\log_{1/2}|z-2| > \log_{1/2}|z|$

$\Rightarrow |z-2| < |z|$

$\Rightarrow z$ lies on the right side of the perpendicular bisector of the segment joining (0, 0) and (2, 0)

$\Rightarrow \text{Re}(z) > 1$

189 **(d)**

Since, $x^2 - 3|x| + 2 = 0$

$\Rightarrow (|x|-2)(|x|-1) = 0$

$\Rightarrow |x| = 2$ or $|x| = 1$

$\Rightarrow x = \pm 2$ or $x = \pm 1$

$\therefore$ The given equation has four real roots

190 **(d)**

Let 4 and α be roots of given equation

$\therefore 4\alpha = 12 \quad \Rightarrow \alpha = 3$

And $4 + 3 = -p \quad \Rightarrow \quad p = -7$

$\therefore$ Equation $x^2 + px + q = 0$ will reduce to

$x^2 - 7x + q = 0$

Let this equation have β, β as its roots

$\therefore \; 2\beta = 7 \quad \Rightarrow \quad \beta = \frac{7}{2}$ and $\beta^2 = q$

$\Rightarrow \quad q = \left(\frac{7}{2}\right)^2 = \frac{49}{4}$

191 **(b)**

$[x]^2 - [x] - 2 = 0$

$\Rightarrow ([x]-2)([x]+1) = 0$

$\Rightarrow [x] = 2, -1$

$\Rightarrow x \in [-1, 0] \cup [2, 0]$

192 **(d)**

We have,

$\alpha + \beta = -b/a$ and $\alpha\beta = c/a$

Now,

Sum of the roots $= 2 + \alpha + 2 + \beta = 4 + (\alpha + \beta) = 4 - b/a$

Product of the roots $= (2+\alpha)(2+\beta)$

$= 4 + \alpha\beta + 2(\alpha + \beta)$

$= 4 + \frac{c}{a} - \frac{2b}{a} = \frac{4a + c - 2b}{a}$

Hence, required equation is

$x^2 - x\left(4 - \frac{b}{a}\right) + \frac{4a + c - 2b}{a} = 0$

or, $ax^2 + (b - 4a)x + 4a - 2b + c = 0$

<u>ALITER</u> Required equation can be obtained by replacing x by $x + 2$ in the given equation

193 **(c)**

Given, $\tan\alpha + \tan\beta + \tan\gamma = \tan\alpha\tan\beta\tan\gamma$ ...(i)

$\therefore \; \tan(\alpha + \beta + \gamma)$

$= \frac{\tan\alpha + \tan\beta + \tan\gamma - \tan\alpha\tan\beta\tan\gamma}{1 - \tan\alpha\tan\beta - \tan\beta\tan\gamma - \tan\gamma\tan\alpha}$

$\Rightarrow \; \tan(\alpha + \beta + \gamma)$

$= \frac{0}{1 - \tan\alpha\tan\beta - \tan\beta\tan\gamma - \tan\gamma\tan\alpha}$

[From Eq. (i)]

$\Rightarrow \; \tan(\alpha + \beta + \gamma) = 0$

$\Rightarrow \; \alpha + \beta + \gamma = 0^\circ$ or π

$\therefore xyz = (\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta)(\cos\gamma + i\sin\gamma)$

$= \cos(\alpha + \beta + \gamma) + i\sin(\alpha + \beta + \gamma)$

$= \cos 0^\circ + i\sin 0^\circ = 1$

or $\quad xyz = \cos\pi + i\sin\pi = -1$

194 **(c)**

We have,

$\arg(z_1 z_2) = 0$

$\Rightarrow \arg(z_1) + \arg(z_2) = 0$

$\Rightarrow \arg(z_1) = -\arg(z_2)$

$\Rightarrow \arg(z_1) = \arg(\bar{z}_2)$

Since, $|z_1| = |z_2| = 1$. Therefore, $|z_1| = |\bar{z}_2| = 1$

Hence, $z_1 = \bar{z}_2$

195 **(c)**

Let α be a common root of the two equations. Then,

$2\alpha^2 - 7\alpha + 1 = 0$

$a\alpha^2 + b\alpha + 2 = 0$

$\Rightarrow \frac{\alpha^2}{-14 - b} = \frac{\alpha}{a - 4} = \frac{1}{2b + 7a}$

$\Rightarrow \frac{a-4}{2b+7a} = \frac{b+14}{4-a}$

$\Rightarrow (7a + 2b)(b + 14) + (a-4)^2 = 0$

Clearly, $a = 4, b = -14$ satisfy this equation

196 **(b)**

We know that ω and ω^2 are roots of $x^2 + x + 1 = 0$. Therefore, $x^{3m} + 3^{3n+1} + x^{3k+2}$ will be exactly divisible by $x^2 + x + 1$, if ω and ω^2 are its roots

For $x = \omega$, we have

$x^{3m} + x^{3n+1} + x^{3k+2} = \omega^{3m} + \omega^{3n+1} + \omega^{3k+2} = 1 + \omega + \omega^2 = 0$ provided that m, n, k are integers

Similarly, $x=\omega^2$ will be a root of $x^{3m}+x^{3n+1}+x^{3k+2}$ if m,n,k are integers

197 **(d)**

$\log_{10}\left(\frac{a+10b+10^2c}{10^{-4}a+10^{-3}b+10^{-2}c}\right)$

$=\log_{10}\left(\frac{a+10b+10^2c}{\frac{1}{10^4}(a+10b+10^2c)}\right)$

$=\log_{10}10^4=4$

198 **(a)**

Since, $\tan 30°$ and $\tan 15°$ are the roots of equation

$x^2+px+q=0$

$\therefore\ \tan 30°+\tan 15°=-p$

And $\tan 30°\tan 15°=q$

Now, $2+q-p=2+\tan 30°+\tan 15°+\tan 30° \tan 15°$

$=2+\tan 30°+\tan 15°+1-\tan 30°\tan 15°$

$\left(\because \tan 45°=\frac{\tan 30°+\tan 15°}{1-\tan 30°\tan 15°}\right)$

$\Rightarrow\ 2+q-p=3$

199 **(d)**

Given, $z^{1/3}=p+iq$

$\Rightarrow\quad (x-iy)=(p+iq)^3\quad$ [put $z=x-iy$]

$\Rightarrow\quad (x-iy)=p^3-iq^3+3p^2qi-3pq^2$

$\Rightarrow\ (x-iy)=(p^3-3pq^2)+i(3p^2q-q^3)$

$\Rightarrow\ x=(p^3-3pq^2)$ and $-y=3p^2q-q^3$

$\Rightarrow \frac{x}{p}=(p^2-3q^2)$ and $\frac{y}{q}=(q^2-3p^2)$

$\therefore\ \frac{x}{p}+\frac{y}{q}=-2p^2-2q^2$

$\Rightarrow \frac{\frac{x}{p}+\frac{y}{q}}{(p^2+q^2)}=-2$

200 **(c)**

Here, $\sec\alpha+\operatorname{cosec}\alpha=p$ and $\sec\alpha.\operatorname{cosec}\alpha=q$

$\Rightarrow \frac{\sin\alpha+\cos\alpha}{\sin\alpha\cos\alpha}=p$ and $\sin\alpha\cos\alpha=\frac{1}{q}$

$\Rightarrow(\sin\alpha+\cos\alpha)^2=\left(\frac{p}{q}\right)^2$

$\Rightarrow \sin^2\alpha+\cos^2\alpha+2\sin\alpha\cos\alpha=\frac{p^2}{q^2}$

$\Rightarrow q^2\left(1+\frac{2}{q}\right)=p^2$

$\Rightarrow q(q+2)=p^2$

201 **(a)**

$\left(x+\frac{1}{x}\right)^3+\left(x+\frac{1}{x}\right)=0$

$\Rightarrow\ \left(x+\frac{1}{x}\right)\left[\left(x+\frac{1}{x}\right)^2+1\right]=0$

$\Rightarrow x+\frac{1}{x}=0$

$\Rightarrow x^2=-1$ which is not possible

Hence, no real roots exist

202 **(c)**

Let D be the discriminant of the given quadratic. Then,

$D=9b^2-32\,ac$

$\Rightarrow D=9(-a-c)^2-32ac\quad [\because a+b+c=0]$

$\Rightarrow D=9a^2+9\,c^2-14ac$

$\Rightarrow D=c^2\left\{9\left(\frac{a}{c}\right)^2-14\left(\frac{a}{c}\right)+9\right\}$

$=c^2\left\{\left(\frac{3a}{c}-\frac{7}{3}\right)^2+\frac{32}{9}\right\}>0$

Hence, the roots are real

203 **(d)**

Let $\alpha=1,\beta=-1,\gamma=i$ and $\delta=-i$. Then,

$\frac{a\,\alpha+b\,\beta+c\,\gamma+d\,\delta}{a\,\gamma+b\,\delta+c\,\alpha+d\,\beta}+\frac{a\,\gamma+b\,\delta+c\,\alpha+d\,\beta}{a\,\alpha+b\,\beta+c\,\gamma+d\,\delta}$

$=\frac{a-b+i(c-d)}{(a-b)i+(c-d)}+\frac{(a-b)i+(c-d)}{a-b+i(c-d)}$

$=\frac{\{(a-b)+i(c-d)\}^2+\{(a-b)i+(c-d)\}^2}{i\{(a-b)+i\,(c-d)\}\,\{(a-b)-i(c-d)\}}$

$=\frac{4(a-b)(c-d)}{(a-b)^2+(c-d)^2}$

204 **(a)**

Given, $\log_5\log_5\log_2 x=0$

$\Rightarrow \log_5\log_2 x=5^0=1$

$\Rightarrow \log_2 x=5\Rightarrow x=2^5\Rightarrow x=32$

205 **(d)**

$\left(\frac{1}{1-2i}+\frac{3}{1+i}\right)\left(\frac{3+4i}{2-4i}\right)$

$=\left[\frac{1+2i}{1^2+2^2}+\frac{3-3i}{1^2+1^2}\right]\left[\frac{6-16+12i+8i}{2^2+4^2}\right]$

$=\left[\frac{2+4i+15-15i}{10}\right]\left[\frac{-1+2i}{2}\right]$

$=\frac{(17-11i)(-1+2i)}{20}$

$=\frac{5+45i}{20}=\frac{1}{4}+\frac{9}{4}i$

206 **(a)**

Let α,β be the roots of $x^2+px+q=0$

$\Rightarrow \alpha+\beta=-p,\alpha\,\beta=q$

α^4,β^4 are roots of $x^2-rx+s=0$

$\Rightarrow \alpha^4+\beta^4=r,\alpha^4\beta^4=s$

Let D be the discriminant of $x^2-4\,qx+2\,q^2-r=0$. Then,

$D=8q^2+4r$

$\Rightarrow D=8\,\alpha^2\beta^2+4(\alpha^4+\beta^4)=4(\alpha^2+\beta^2)^2>0$

So, the given equation has real roots

207 **(a)**

Let $y=\frac{x^2-x+1}{x^2+x+1}$

$\Rightarrow\quad x^2(y-1)+x(y+1)+1(y-1)=0$

Here, $D \geq 0$ as x is real

$\therefore \quad (y+1)^2 - 4(y-1)^2 \geq 0$

$\Rightarrow \quad y^2 + 2y + (1 - 4y^2 + 1 - 2y) \geq 0$

$\Rightarrow \quad -3y^2 - 10y + 3 \geq 0$

$\Rightarrow 3y^2 - 10y + 3 \leq 0$

$\Rightarrow (3y-1)(y-3) \leq 0$

$\Rightarrow \dfrac{1}{3} \leq y \leq 3$

208 **(b)**

Now, $x - 1 = \alpha_i \;\Rightarrow x = \alpha_i + 1$ for new equation, $i = 1, 2, 3, 4$

209 **(d)**

$$d = \frac{a.0 + 0.\bar{a} + |a|^2}{2|a|} = \frac{|a|}{2}$$

210 **(a)**

We have

$$1 = a(1-2x)(1-3x) + b(1-x)(1-3x) + c(1-x)(1-2x)$$

On putting $x = \frac{1}{2}$, we get

$$1 = 0 + b\left(1 - \frac{1}{2}\right)\left(1 - \frac{3}{2}\right) + 0$$

$$\Rightarrow \quad 1 = b\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)$$

$\Rightarrow \quad b = -4$

On putting $x = 1$, we get

$1 = a(1-2)(1-3) + 0 + 0$

$$\Rightarrow \quad 1 = a(-1)(-2) \quad \Rightarrow \quad a = \frac{1}{2}$$

On putting $x = \frac{1}{3}$, we get

$$1 = 0 + 0 + c\left(1 - \frac{1}{3}\right)\left(1 - \frac{2}{3}\right)$$

$$\Rightarrow \quad 1 = c\left(\frac{2}{3}\right)\left(\frac{1}{3}\right) \quad \Rightarrow \quad c = \frac{9}{2}$$

Now, $\frac{a}{1} + \frac{b}{3} + \frac{c}{5} = \frac{1}{2} + \frac{(-4)}{3} + \frac{9}{5.2} = \frac{15-40+27}{30} = \frac{1}{15}$

211 **(a)**

The given equation is

$2(1+i)x^2 - 4(2-i)x - 5 - 3i = 0$

$$\Rightarrow \; x = \frac{4(2-i) \pm \sqrt{16(2-i)^2 + 8(1+i)(5+3i)}}{4(1+i)}$$

$= \frac{i}{1+i}$ or $\frac{4-i}{1+i}$

$$= \frac{-1-i}{2} \text{ or } \frac{3-5i}{2}$$

Now, $\left|\frac{-1-i}{2}\right| = \sqrt{\frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{1}{2}}$

and $\left|\frac{3-5i}{2}\right| = \sqrt{\frac{9}{4} + \frac{25}{4}} = \sqrt{\frac{17}{2}}$

Also, $\sqrt{\frac{17}{2}} > \sqrt{\frac{1}{2}}$

Hence, required root is $\frac{3-5i}{2}$.

212 **(c)**

Using triangle inequality, we have

$|z - 2i| \geq |2i| - |z| \Rightarrow |z - 2i| + |z| \geq 2$

Hence, the minimum value of $|z - 2i| + |z|$ is 2

214 **(d)**

We have,

$$x^{\frac{3}{4}(\log_2 x)^2 + (\log_2 x) - \frac{5}{4}} = \sqrt{2}$$

$$\Rightarrow \frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4} = \log_x \sqrt{2}$$

$$\Rightarrow \frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4} = \frac{1}{2}\log_x 2 = \frac{1}{2} \times \frac{1}{\log_2 x}$$

$$\Rightarrow \frac{3}{4}(\log x)^3 + (\log_2 x)^2 - \frac{5}{4}(\log_2 x) = \frac{1}{2}$$

$\Rightarrow 3(\log_2 x)^3 + 4(\log_2 x)^2 - 5(\log_2 x) - 2 = 0$

$\Rightarrow 3\,y^3 + 4\,y^2 - 5\,y - 2 = 0$, where $y = \log_2 x$

$\Rightarrow (y-1)(3y+1)(y+2) = 0$

$$\Rightarrow y = 1, -\frac{1}{3}, -2$$

$$\Rightarrow \log_2 x = 1, -\frac{1}{3}, -2 \Rightarrow x = \frac{2{,}1}{2^{1/3}}, \frac{1}{4}$$

215 **(b)**

We have, $x^2 - 3x + 2 = 0 \quad \Rightarrow \quad (x-1)(x-$ *2=0*

$\Rightarrow \quad x = 1.2$

For $x = 1, x^4 - px^2 + q = 0 \;\Rightarrow\; 1 - p + q = 0$ ...(i)

For $x = 2, \quad 16 - 4p + q = 0 \quad$ ...(ii)

On solving Eqs. (i) and (ii), we get

$p = 5, \quad q = 4$

216 **(d)**

Let α, β be roots of $x^2 + px + q = 0$ and a, b be roots of $x^2 + lx + m = 0$. Then,

$\alpha + \beta = -p, \alpha\,\beta = q, a + b = -l$ and $ab = m$

Now,

$$\frac{\alpha}{\beta} = \frac{a}{b} \qquad \text{[Given]}$$

$$\Rightarrow \frac{\alpha+\beta}{\alpha-\beta} = \frac{a+b}{a-b}$$

$$\Rightarrow \frac{(\alpha+\beta)^2}{(\alpha-\beta)^2} = \frac{(a+b)^2}{(a-b)^2}$$

$$\Rightarrow \frac{p^2}{p^2 - 2q} = \frac{l^2}{l^2 - 2m} \Rightarrow p^2 m = l^2 q$$

217 **(b)**

We have, $\sum x_1 = \sin 2\beta, \sum x_1 x_2 = \cos 2\beta$

$\sum x_1 x_2 x_3 = \cos\beta$ and $x_1 x_2 x_3 x_4 = -\sin\beta$

$\tan^{-1} x_1 + \tan^{-1} x_2 + \tan^{-1} x_3 + \tan^{-1} x_4$

$$= \tan^{-1}\left(\frac{\sum x_1 - \sum x_1 x_2 x_3}{1 - \sum x_1 x_2 + x_1 x_2 x_3 x_4}\right)$$

$$= \tan^{-1}\left(\frac{\sin 2\beta - \cos\beta}{1 - \cos 2\beta - \sin\beta}\right)$$

$$= \tan^{-1}\left(\frac{(2\sin\beta - 1)\cos\beta}{\sin\beta(2\sin\beta - 1)}\right) = \tan^{-1}(\cot\beta)$$

$= \tan^{-1}\left[\tan\left(\frac{\pi}{2}-\beta\right) = \frac{\pi}{2}-\beta\right]$

218 **(a)**

We have,

$a(b-c)x^2 + b(c-a)x + c(a-b) = 0$

Clearly, $x = 1$ is a root of this equation. It is given that the equation has equal roots. So, both the roots are equal to 1

$\therefore$ Product of the roots = 1

$\Rightarrow \frac{c(a-b)}{a(b-c)} = 1$

$\Rightarrow 2ac = ab + bc \Rightarrow b = \frac{2ac}{a+c} \Rightarrow a, b, c$ are in H.P.

219 **(d)**

Let α, β be the roots of the equation $x^2 - (a-2)x - (a+1) = 0$. Then,

$\alpha + \beta = a - 2$ and $\alpha\,\beta = -(a+1)$

$\therefore \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\,\alpha\,\beta$

$\Rightarrow \alpha^2 + \beta^2 = (a-2)^2 + 2(a+1) = a^2 - 2a + 6$

$= (a-1)^2 + 5$

Clearly, it is least when $a = 1$

220 **(c)**

We know that

$x^3 + y^3 = (x+y)(x\,\omega + y\,\omega^2)(x\,\omega^2 + y\,\omega)$

$\therefore (a + b\,\omega + c\,\omega^2)^3 + (a + b\,\omega^2 + c\,\omega)^3$

$= (a + b\,\omega + c\,\omega^2 + a + b\,\omega^2 + c\,\omega) \times (a\,\omega + b\,\omega^2 + c + a\,\omega^2 + b\,\omega^4 + c\,\omega^3) \times (a\,\omega^2 + b\,\omega^3 + c\,\omega^4 + a\,\omega + b\,\omega^3 + c\,\omega^2)$

$= (2a - b - c)(2c - a - b)(2b - c - a)$

221 **(c)**

We have,

$\log_2(x^2 - 4x + 5) = (x-2) \Rightarrow x^2 - 4x + 5 = 2^{x-2}$

Clearly, $x = 2$ and 3 satisfy this equation

222 **(c)**

Solving the given equation, we get

$x = 3/5$ or, $x = -4/5$

$\Rightarrow x = -4/5 \qquad [\because -1 < x < 0]$

$\Rightarrow \cos\alpha = -4/5 \Rightarrow \sin\alpha = -24/25$

223 **(a)**

Since, α, β, γ are the roots of the equation $2x^3 - 3x^2 + 6x + 1 = 0$

Here, $\alpha + \beta + \gamma = \frac{3}{2}$ ...(i)

$\alpha\beta + \beta\gamma + \gamma\alpha = 3$ ...(ii)

And $\alpha\beta\gamma = -\frac{1}{2}$ ...(iii)

On squaring Eq. (i), we get

$\alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \gamma\alpha) = \frac{9}{4}$

$\Rightarrow \alpha^2 + \beta^2 + \gamma^2 = \frac{9}{4} - 2(3) = -\frac{15}{4}$ [from Eq. (ii)]

224 **(b)**

Here, $a = 2, b = -3$ and $c = \pm 1$

Clearly $a + b + c = 0$

Therefore, z_1, z_2, z_3 are collinear points

ALITER We have,

$2z_1 - 3z_2 + z_3 = 0$

$\Rightarrow z_2 = \frac{2\,z_1 + z_3}{2+1}$

$\Rightarrow z_2$ divides the segment joining z_1 and z_3 in the ratio $1 : 2$

$\Rightarrow z_1, z_2, z_3$ are collinear

225 **(b)**

Let the roots be α and $2\,\alpha$. Then,

$3\,\alpha = -\frac{b}{a}$ and $2\,\alpha^2 = \frac{c}{a}$

$\Rightarrow \alpha = -\frac{b}{3a}$ and $\alpha^2 = \frac{c}{2a} \Rightarrow \left(-\frac{b}{3a}\right)^2 = \frac{c}{2a} \Rightarrow 2b^2 = 9ac$

226 **(a)**

We have,

$z = i + 2i^{i\left(\theta + \frac{\pi}{4}\right)} \Rightarrow |z - i| = 2$

$\Rightarrow$ Locus of z is a circle

227 **(b)**

Given, $\alpha^2 - 5\alpha + 3 = 0$ and $\beta^2 - 5\beta + 3 = 0$

$\Rightarrow \alpha = \frac{5\pm\sqrt{13}}{2}$ and $\beta = \frac{5\pm\sqrt{13}}{2}$

Since, $\alpha \neq \beta$

$\therefore \alpha = \frac{5+\sqrt{13}}{2}$ and $\beta = \frac{5-\sqrt{13}}{2}$

$\alpha = \frac{5-\sqrt{13}}{2}$ and $\beta = \frac{5+\sqrt{13}}{2}$

Now, $\alpha^2 + \beta^2 = \frac{50+26}{4} = 19$

And $\alpha\beta = \frac{1}{4}(25 - 13) = 3$

$\therefore$ Required equation is

$x^2 - x\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) + \frac{\alpha\beta}{\alpha\beta} = 0$

$\Rightarrow x^2 - x\left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right) + 1 = 0$

$\Rightarrow 3x^2 - 19x + 3 = 0$

228 **(b)**

We have,

$2 - 3x - 2x^2 \geq 0$

$\Rightarrow 2x^2 + 3x - 2 \leq 0 \Rightarrow (2x-1)(x+2) \leq 0$

$\Rightarrow -2 \leq x \leq \frac{1}{2}$

229 **(c)**

Let D_1 and D_2 be discriminates of $ax^2 + bx + c = 0$ and $-ax^2 + bx + c = 0$ respectively. Then,

$D_1 = b^2 - 4ac, D_2 = b^2 + 4ac$

Now, $ac \neq 0 \Rightarrow$ either $ac > 0$ or $ac < 0$

If $ac > 0$, then $D_2 > 0$. Therefore, roots of $-ax^2 + bx + c = 0$ are real

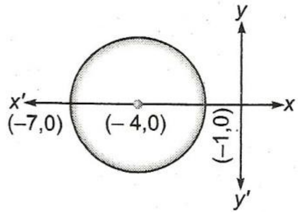
If $ac < 0$, then $D_1 > 0$. Therefore, roots of $ax^2 + bx + c = 0$ are real.

Thus, $f(x)g(x)$ has at least two real roots

230 **(c)**

$|z+4| \leq 3$ represents the interior and boundary of the circle with centre at (-4, 0) and radius=3.

As -1 is an end point of a diameter of the circle, maximum possible value of $|z+1|$ is 6



Alternate

$|z+1| = |z+4-3| \leq |z+4| + |-3| \leq 6$

Hence, maximum value of $|z+1|$ is 6

231 **(a)**

Given, $x = \sqrt{7} - \sqrt{5}$ and $y = \sqrt{13} - \sqrt{11}$

$\therefore \quad x = 2.64 - 2.23$

And $\quad y = 3.60 - 3.31$

$\Rightarrow x = 0.41$ and $y = 0.29$

$\therefore x > y$

232 **(c)**

Since, α and β be the roots of the equation $ax^2 + bx + c = 0$, then

$\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

Now, sum of roots $= |\alpha| + |\beta|$

$= -\alpha - \beta \quad (\because \beta < \alpha < 0)$

$= -\left(-\frac{b}{a}\right) = \left|\frac{b}{a}\right| \quad (\because |\alpha| + |\beta| > 0)$

and product of roots $= |\alpha||\beta| = \left|\frac{b}{a}\right|$

Hence, required equation is

$x^2 - \left|\frac{b}{a}\right| x + \left|\frac{c}{a}\right| = 0$

$\Rightarrow \; |a|x^2 - |b|x + |c| = 0$

233 **(a)**

Here, $x = 4, \quad y = -3$

Let $\; x = r\cos\theta, \; y = r\sin\theta = -3$

Now, $\; r = \sqrt{x^2 + y^2} = \sqrt{16+9} = 5$

and $\quad \theta = \tan^{-1}\left(-\frac{3}{4}\right)$

now, let R and ϕ be the magnitude and angle of resultant complex number.

$\therefore$ According to question.

$R = 3r$ and $\phi = \pi + \theta$

$\Rightarrow \quad \phi = \pi + \tan^{-1}\left(-\frac{3}{4}\right)$

$= \pi - \tan^{-1}\left(\frac{3}{4}\right) = -\tan^{-1}\left(\frac{3}{4}\right)$

$\therefore \quad \cos\phi = -\frac{4}{5}, \quad \sin\phi = \frac{3}{5} \quad$ [$\because$ ϕ lies in IInd quadrant]

Hence, new complex number will be

$R(\cos\phi + i\sin\phi) = 3.5\left(-\frac{4}{5} + i\frac{3}{5}\right)$

$= \frac{3.5}{5}(-4+3i) = -12 + 9i$

234 **(a)**

We have,

$z = r\,e^{i\theta} = r(\cos\theta + i\sin\theta)$

$\Rightarrow i\,z = r(-\sin\theta + i\cos\theta)$

$\Rightarrow e^{i\,z} = e^{-r\sin\theta}e^{i\,r\cos\theta}$

$\Rightarrow \left|e^{i\,z}\right| = e^{-r\sin\theta}\left|e^{i\,r\cos\theta}\right| = e^{-r\sin\theta}$

235 **(d)**

Given equation is

$|2x-1|^2 - 3|2x-1| + 2 = 0$

Let $\; |2x-1| = t$

$\therefore t^2 - 3t + 2 = 0$

$\Rightarrow (t-1)(t-2) = 0 \;\Rightarrow t = 1, 2$

$\Rightarrow \; |2x-1| = 1$ and $|2x-1| = 2$

$\Rightarrow 2x - 1 = \pm 1$ and $2x - 1 = \pm 2$

$\Rightarrow x = 1, 0$ and $x = \frac{3}{2}, -\frac{1}{2}$

236 **(a)**

We have,

$|3x+2| < 1$

$\Rightarrow \left|x + \frac{2}{3}\right| < \frac{1}{3} \Rightarrow -\frac{1}{3} < x + \frac{2}{3} < \frac{1}{3} \Rightarrow x \in (-1, -1/3)$

237 **(b)**

Given, $\; C = \{z: z\bar{z} + a\bar{z} + \bar{a}z + b = 0, \; b \in R$ and $b < |a|^2\}$

Since, $z\bar{z} + a\bar{z} + \bar{a}z + b = 0, \; b \in R$ represents circle having centre at $-a$ and radius $\sqrt{|a|^2 - b}$

Then, z lies on the circle having infinite points

Hence, C represents infinite sets

238 **(c)**

Given, $\; \bar{z} = \bar{a} + \frac{r^2}{z-a}, \; r > 0$

$\Rightarrow \; \bar{z}(z-a) = \bar{a}(z-a) + r^2$

$\Rightarrow \quad z\bar{z} - a\bar{z} - \bar{a}z + a\bar{a} + r^2 = 0$

This represents the equation of a circle

239 **(d)**

$$\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}+\frac{c+a\omega+b\omega^2}{a+b\omega+c\omega^2}+\frac{b+c\omega+a\omega^2}{b+c\omega^4+a\omega^5}$$
$$=\frac{\omega^2(a+b\omega+c\omega^2)}{(a+b\omega+c\omega^2)}+\frac{\omega(a\omega+b\omega^2+c)}{(a\omega+b\omega^2+c)}+\frac{(b+c\omega+a\omega^2)}{(b+c\omega+a\omega^2)}$$
$$=\omega^2+\omega+1=0$$

240 **(d)**

Here, $\alpha+\beta=2$ and $\alpha\beta=\frac{5}{3}$

Now, $\alpha+\beta+\frac{2}{\alpha+\beta}=2+\frac{2}{2}=3$

And $(\alpha+\beta)\times\frac{2}{\alpha+\beta}=2$

$\therefore$ Required equation is

$$x^2-\left((\alpha+\beta)+\frac{2}{(\alpha+\beta)}\right)x+\left((\alpha+\beta)\times\frac{2}{(\alpha+\beta)}\right)=0$$

$\Rightarrow\ x^2-3x+2=0$

241 **(b)**

It is given that the equations $ax^2+2bx+c=0$ and $bx^2-2\sqrt{ac}\,x+b=0$ have real roots

$\therefore b^2\geq ac$ and $b^2\leq ac\Rightarrow b^2=ac$

242 **(c)**

We have,

$\left|\frac{3}{x}+1\right|>2$

$\Rightarrow\frac{3}{x}+1<-2$ or, $\frac{3}{x}+1>2$

$\Rightarrow\frac{3}{x}<-3$ or, $\frac{3}{x}>1$

$\Rightarrow\frac{1}{x}<-1$ or, $\frac{3-x}{x}>0$

$\Rightarrow\frac{x+1}{x}<0$ or, $\frac{x-3}{x}<0$

$\Rightarrow x\in(-1,0)$ or, $x\in(0,3)\Rightarrow x\in(-1,0)\cup(0,3)$

243 **(b)**

We have,

$|x^2+4x+3|+2x+5=0$

Here two cases arise.

Case I When $x^2+4x+3>0$

$\Rightarrow\ x^2+4x+3+2x+5=0$

$\Rightarrow x^2+6x+8=0$

$\Rightarrow(x+2)(x+4)=0$

$\Rightarrow x=-2,-4$

$x=-2$ is not satisfying the condition $x^2+4x+3>0$. So $x=-4$ is the only solution of the given equation.

Case II When $x^2+4x+3<0$

$\Rightarrow\ -(x^2+4x+3)+2x+5=0$

$\Rightarrow\ -x^2-2x+2=0$

$\Rightarrow x^2+2x-2=0$

$\Rightarrow\left(x+1+\sqrt{3}\right)\left(x+1-\sqrt{3}\right)=0$

$\Rightarrow x=-1+\sqrt{3},-1-\sqrt{3}$

Hence, $x=-(1+\sqrt{3})$ satisfy the given condition.

Since, $x^2+4x+3<0$ while $x=-1+\sqrt{3}$ is not satisfying the condition. Thus, number of real solutions are two.

244 **(b)**

We have, $\left|\frac{z-a}{z+\bar{a}}\right|=1$

$\Rightarrow\ |z-a|=|z+\bar{a}|\ \Rightarrow\ |z+a|^2=|z+\bar{a}|^2$

$\Rightarrow\ (z-a)(\overline{z-a})=(z+\bar{a})(\overline{z+\bar{a}})$

$\Rightarrow\ (z-a)(\bar{z}-\bar{a})=(z+\bar{a})(\bar{z}+a)\quad[\because(\bar{\bar{a}})=a]$

$\Rightarrow\ z\bar{z}-z\bar{a}-a\bar{z}+a\bar{a}=z\bar{z}+za+\bar{a}\bar{z}+\bar{a}a$

$\Rightarrow\ za+z\bar{a}+\bar{a}\bar{z}+a\bar{z}=0$

$\Rightarrow\ (a+\bar{a})(z+\bar{z})=0$

$\Rightarrow\ z+\bar{z}=0\quad[\because a+\bar{a}=2\,\mathrm{Re}(a)\neq0]$

$\Rightarrow\ 2\mathrm{Re}\,(z)=0\ \Rightarrow 2x=0$

$\Rightarrow\ x=0\ \Rightarrow y$-axis

245 **(a)**

Let $z=x+iy$. Then, $z^2=x^2-y^2+2\,ixy$

$\therefore$ Re $(z^2)=0\Rightarrow x^2-y^2=0\Rightarrow y=\pm x$

Thus, $\mathrm{Re}(z^2)=0$ represents a pair of straight lines

246 **(a)**

Given, $\frac{x+iy-5i}{x+iy+5i}=1$

$\Rightarrow\quad|x+iy-5i|=|x+iy+5i|$

$\Rightarrow\quad x^2+(y-5)^2=x^2+(5+y)^2\qquad\left[\because\ \left|\frac{z_1}{z_2}\right|=\frac{|z_1|}{|z_2|}\right]$

$\Rightarrow\quad y=0$

$\Rightarrow\quad z=x$ ie, then z lies on the axis of x.

247 **(a)**

Since $2+i\sqrt{3}$ is a root of $x^2+px+q=0$.

Therefore, $2-i\sqrt{3}$ is also its root

Now,

Sum of the roots $=-p$

$\Rightarrow\left(2+i\sqrt{3}\right)+\left(2-i\sqrt{3}\right)=-p\Rightarrow p=-4$

and, Product of the roots $=q\Rightarrow 7=q$

248 **(c)**

We have,

$\sqrt{3x^2-7x-30}+\sqrt{2x^2-7x-5}=x+5$

$\Rightarrow\sqrt{3x^2-7x-30}=(x+5)-\sqrt{2x^2-7x-5}$

On squaring both sides, we get

$$3x^2-7x-30=x^2+25+10x+(2x^2-7x-5)-2(x+5)\sqrt{2x^2-7x-5}$$

$\Rightarrow\sqrt{2x^2-7x-5}=5$

Again on squaring both sides, we get
$2x^2 - 7x - 30 = 0$
$\Rightarrow x = 6$

249 **(d)**
Given, $\sqrt{x+iy} = \pm(a+ib)$
$\Rightarrow \quad x + iy = a^2 - b^2 + 2iab$
$\Rightarrow \quad x = a^2 - b^2, \; y = 2ab$
$\therefore \quad \sqrt{-x-iy} = \sqrt{-(a^2-b^2) - 2iab}$
$= \sqrt{b^2 - a^2 - 2iab} = \pm(b - ia)$

250 **(d)**
Let α , β are the roots of the equation
$x^2 - ax + b = 0.$
$\therefore \; \alpha + \beta = a \quad$ ...(i)
and $\alpha\beta = b \quad$ ...(ii)
Roots are prime numbers, so clearly b cannot be a prime number as it is product of two prime numbers [from Eq. (ii)]. Sum of two prime numbers is always an even number except in one situation when one prime number is 2. `a' can be a prime number and can be composite number.
Now, $1 + a + b = 1 + \alpha\beta + \alpha + \beta = (1+\alpha)(1+\beta)$
$(1+\alpha), (1+\beta)$ can be prime numbers, can be composite numbers, so $1 + a + b$ is not certain.
So, option (d) is correct.

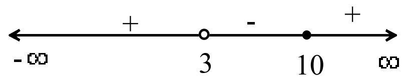
251 **(b)**
Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$
$\therefore \quad |z_1 + z_2|^2 + |z_1 - z_2|^2$
$= (x_1+x_2)^2 + (y_1+y_2)^2 + (x_1-x_2)^2 + (y_1-y_2)^2$
$= 2(x_1^2 + y_1^2 + x_2^2 + y_2^2)$
$= 2(|z_1|^2 + |z_2|^2)$

252 **(d)**
Given that, $f(x) = x^2 + 2bx + 2c^2$
and $g(x) = -x^2 - 2cx + b^2$
min $f(x) = -\frac{D}{4a} = -\frac{4b^2-8c^2}{4}$
$= -(b^2 - 2c^2)$ (upward parabola)
max $g(x) = -\frac{D}{4a} = \frac{4c^2+4b^2}{4}$
$= b^2 + c^2$ (downward parabola)
Now, $2c^2 - b^2 > b^2 + c^2$
$\Rightarrow c^2 > 2b^2 \Rightarrow |c| > |b|\sqrt{2}$

253 **(b)**
$z = 0$ is the only complex number which satisfies the given relations

254 **(d)**
Let α be the common root of the given equations
Then, $a\alpha^2 + b\alpha + c = 0$
And $2\alpha^2 + 3\alpha + 4 = 0$
$\Rightarrow \quad \alpha^2 + (a-2) + \alpha(b-3) + c - 4 = 0$
$\Rightarrow a - 2 = 0, b - 3 = 0$ and $c - 4 = 0$
$\Rightarrow a = 2, b = 3$ and $c = 4$
$\therefore \quad a + b + c = 2 + 3 + 4 = 9$

255 **(c)**
We have,
$\frac{x+4}{x-3} \leq 2$



$\Rightarrow \frac{x+4-2x+6}{x-3} \leq 0$
$\Rightarrow -\frac{(x-10)}{x-3} \leq 0 \Rightarrow \frac{x-10}{x-3} \geq 0 \Rightarrow x \in (-\infty, 3) \cup [10, \infty)$

256 **(b)**
Given, $f(x) = x^2 - ax + b$ has imaginary roots
$\therefore$ Discriminant, $D < 0 \Rightarrow a^2 - 4b < 0$
Now, $f'(x) = 2x + a$
$f''(x) = 2$
Also, $f(x) + f'(x) + f''(x) = 0$ ...(i)
$\Rightarrow \quad x^2 + ax + b + 2x + a + 2 = 0$
$\Rightarrow \quad x^2 + (a+2)x + b + a + 2 = 0$
$\therefore \quad x = \frac{-(a+2) \pm \sqrt{(a+2)^2 - 4(a+b+2)}}{2}$
$= \frac{-(a+2) \pm \sqrt{a^2 - 4b - 4}}{2}$
Since, $a^2 - 4b < 0$
$\therefore \quad a^2 - 4b - 4 < 0$
Hence, Eq. (i) has imaginary roots

257 **(b)**
Let $x = 7^{-20}$
$\log_{10} x = -20 \log_{10} 7$
$= -20(0.8451) = -16.902$
Hence, the first significant figure is 17

258 **(d)**
Let $z = r_1 e^{i\theta} \quad \Rightarrow \quad \bar{z} = r_1 e^{-i\theta}$ and $w = r_2 e^{i\emptyset}$
Given, $|zw| = 1$
$\Rightarrow \quad |r_1 e^{i\theta} . r_2 e^{i\emptyset}| = 1 \quad \Rightarrow \quad r_1 r_2 = 1$ ...(i)
And $\arg(z) - \arg(w) = \frac{\pi}{2} \quad \Rightarrow \quad \theta - \emptyset = \frac{\pi}{2}$ ...(ii)
Now, $\bar{z}w = r_1 e^{-i\theta} . r_2 e^{i\emptyset} = r_1 r_2 r^{-i(\theta - \emptyset)}$
$= 1 . e^{-i\pi/2} = \cos\frac{\pi}{2} - i \sin\frac{\pi}{2}$
[from Eqs. (i) and (ii)]
$\Rightarrow \quad \bar{z}w = -i$

259 **(a)**
Sum of roots$= \frac{-2}{a}$

And product of the roots$=\frac{3a}{a}=3$

Given, $-\frac{2}{a}=3 \Rightarrow a=-\frac{2}{3}$

260 **(b)**

Here, $\alpha+\beta+\gamma=-2$ ...(i)

$\alpha\beta+\beta\gamma+\gamma\alpha=-3$ ...(ii)

And $\alpha\beta\gamma=1$ ...(iii)

On squaring Eq. (ii), we get

$\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2+2\alpha\beta\gamma(\alpha+\beta+\gamma)=9$

$\Rightarrow \alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2=9-2(1)(-2)=13$

Now, $\alpha^{-2}+\beta^{-2}+\gamma^{-2}=\frac{\beta^2\gamma^2+\gamma^2\alpha^2+\alpha^2\beta^2}{(\alpha\beta\gamma)^2}=\frac{13}{1}=13$

261 **(c)**

Given equation $x^2+2x+2xy+my-3=0$ can be rewritten as $x^2+2x(1+y)+(my-3)=0$.

But factors are rational so discriminant b^2-4ac is a perfect square.

Now, $b^2-4ac=4\{(1+y)^2-(my-3)\}\geq 0$

$\Rightarrow 4\{y^2+1+2y-my+3\}\geq 0$

$\Rightarrow y^2+2y-my+4\geq 0$

Hence, $2y-my=\pm 4y$(as it is perfect square).

$\Rightarrow 2y-my=4y$

$\Rightarrow m=-2$

Now, taking $(-)$ve sign, we get $m=6$

262 **(d)**

Here, $\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$

But $\alpha\beta=3 \Rightarrow c=3a$

Also, $b=\frac{a+c}{2} \Rightarrow b=\frac{a+3a}{2}=2a$

Hence, $\alpha+\beta=-\frac{2a}{a}=-2$

263 **(a)**

Let α,β are the roots of the equation $x^2+ax-b=0$

$\therefore \alpha+\beta=-a, \alpha\beta=-b$

And γ,δ are the roots of the equation $x^2-px+q=0$

$\therefore \gamma+\delta=p, \gamma\delta=q$

Given, $\alpha-\beta=\gamma-\delta \Rightarrow (\alpha-\beta)^2=(\alpha-\beta)^2$

$\Rightarrow (\alpha+\beta)^2-4\alpha\beta=(\gamma+\delta)^2-4\gamma\delta$

$\Rightarrow a^2+4b=p^2-4q$

$\Rightarrow a^2-p^2=-4(b+q)$

264 **(b)**

Multiplying the numerator and denominator by ω and ω^2 respectively of I and II expression, we get

$$\frac{\omega(a+b\omega+c\omega^2)}{b\omega+c\omega^2+a}+\frac{\omega^2(a+b\omega+c\omega^2)}{c\omega^2+a+b\omega}$$

$=\omega+\omega^2=-1$ $[\because 1+\omega+\omega^2=0]$

265 **(c)**

Let $z-1=r(\cos\theta+i\sin\theta)=re^{i\theta}$

$\therefore$ Given expression$=re^{i\theta}\cdot e^{-i\alpha}+\frac{1}{re^{i\theta}}.e^{i\alpha}$

$=re^{i(\theta-\alpha)}+\frac{1}{r}e^{-i(\theta-\alpha)}$

Since, imaginary part of given expression is zero, we have

$r\sin(\theta-\alpha)-\frac{1}{r}\sin(\theta-\alpha)=0$

$r^2-1=0 \Rightarrow r^2=1$

$\Rightarrow r=1$

$\Rightarrow |z-1|=1$

or $\sin(\theta-\alpha)=0 \Rightarrow \theta-\alpha=0$

$\Rightarrow \theta=\alpha$

$\Rightarrow \arg(z-1)=\alpha$

266 **(a)**

Given, $\left|\frac{1-iz}{z-i}\right|=1$

$\Rightarrow \left|\frac{1-i(x+iy)}{x+iy-i}\right|=1 \Rightarrow \left|\frac{(1+y)-ix}{x+i(y-1)}\right|=1$

$\Rightarrow \sqrt{(1+y)^2+x^2}=\sqrt{x^2+(y-1)^2}$

$\Rightarrow (1+y)^2+x^2=x^2+(y-1)^2$

$\Rightarrow y=0$

$\therefore$ Locus of z is x-axis

267 **(c)**

We have,

$p+q=-m, pq=m^2+a$

$\therefore p^2+pq+q^2=(p+q)^2-pq=m^2-(m^2+a)=-a$

268 **(b)**

We have,

$|x|^2+|x|-6=0$

$\Rightarrow (|x|+3)(|x|-2)=0$

$\Rightarrow |x|=2$ $[\because |x|+3\neq 0]$

$\Rightarrow x=\pm 2$

269 **(c)**

We have,

$\frac{z_1}{z_2}+\frac{z_2}{z_1}=1$

$\Rightarrow z_1^2+z_2^2=z_1 z_2$

$\Rightarrow z_1^2+z_2^2+z_3^2=z_1 z_2+z_1 z_3+z_2 z_3$, where $z_3=0$

$\Rightarrow z_1, z_2$ and the origin form an equilateral triangle

270 **(c)**

We have,

$(x-a+b)^2+(x-b+c)^2=0$

$\Rightarrow x-a+b=0$ and $x-b+c=0$

$\Rightarrow x=a-b$ and $x=b-c$

$\Rightarrow a-b=b-c \Rightarrow 2b=a+c \Rightarrow a,b,c$ are in A.P.

271 **(d)**

We have,
$z = i\log(2-\sqrt{3})$
$\Rightarrow e^{iz} = e^{i^2}\log(2-\sqrt{3}) = e^{-\log(2-\sqrt{3})}$
$\Rightarrow e^{iz} = e^{\log(2-\sqrt{3})^{-1}} = e^{\log(2+\sqrt{3})} = (2+\sqrt{3})$
$\Rightarrow \cos z = \frac{e^{iz}+e^{-iz}}{2} = \frac{(2+\sqrt{3})+(2-\sqrt{3})}{2} = 2$
272 **(b)**
Given, $x = 3+i$ ...(i)
Now, $x^3-3x^2-8x+15$
$= (3+i)^3 - 3(3+i)^2 - 8(3+i) + 15$
$= (27+i^3+27i+9i^2) - 3(9+i^2+6i) - 24 - 8i + 15$
$= -15$
273 **(c)**
If z_1, z_2 are complex numbers, then
$|z_1+z_2| \le |z_1|+|z_2|$ [by triangle inequality]
274 **(a)**
Since, roots are equal
$\therefore (2\sqrt{6})^2 = 4.2.a$
$\Rightarrow 24 = 8a$
$\Rightarrow a = 3$
275 **(d)**
We have, $a = \cos\left(\frac{2\pi}{7}\right) + i\sin\left(\frac{2\pi}{7}\right)$
$\Rightarrow a^7 = \left[\cos\left(\frac{2\pi}{7}\right)+i\sin\left(\frac{2\pi}{7}\right)\right]^{-7}$
$= \cos 2\pi + i\sin 2\pi = 1$ ...(i)
Let $S = \alpha+\beta = (a+a^2+a^4)+(a^3+a^5+a^6)$
[$\because \alpha = a+a^2+a^4, \beta = a^3+a^5+a^6$]
$\Rightarrow S = a+a^2+a^3+a^4+a^5+a^6 = \frac{a(1-a^6)}{1-a}$
$\Rightarrow S = \frac{a-a^7}{1-a} = \frac{a-1}{1-a} = -1$ ...(ii)
Let $P = \alpha\beta = (a+a^2+a^4)(a^3+a^5+a^6)$
$= a^4+a^6+a^7+a^5+a^7+a^8+a^7+a^9+a^{10}$
$= a^4+a^6+1+a^5+1+a+1+a^2+a^3$ [from Eq. (i)]
$= 3+(a+a^2+a^3+a^4+a^5+a^6) = 3+S$
$= 3-1 = 2$ [from Eq.(ii)]
Required equation is, $x^2-Sx+P=0$
$\Rightarrow x^2+x+2=0$
276 **(b)**
Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$. Then,
$|z_2| = |z_1| \Rightarrow |z_2| = r_1$
And, $\arg(z_1)+\arg(z_2) = 0 \Rightarrow \arg(z_2) = -\arg z1 = -\theta 1$
$\therefore z_2 = r_1\{\cos(-\theta_1)+i\sin(\theta_1)\}$
$= r_1(\cos\theta_1 - i\sin\theta_1) = \bar{z}_1$
$\Rightarrow \bar{z}_2 = (\bar{z}_1) = z_1$
277 **(a)**
We have,
$|z-(z-1)| \le |z|+|z-1| \Rightarrow 1 \le |z|+|z-1|$
Hence, the minimum value of $|z|+|z-1|$ is 1
278 **(b)**
Given, $z\bar{z}+a\bar{z}+\bar{a}z+b=0,\ b\in R$
On adding $a\bar{a}$ on both sides in the given equation, we get
$z\bar{z}+a\bar{z}+\bar{a}z+a\bar{a}+b = a\bar{a}$
$\Rightarrow (z-a)(\bar{z}+\bar{a}) = a\bar{a}-b$
$\Rightarrow |z+a|^2 = |a|^2-b$
This equation will represent a circle, if
$|a|^2-b>0 \Rightarrow |a|^2>b$
279 **(a)**
We have, $|z-z_1| = |z-z_2| = |z-z_3| = |z-z_4|$
Therefore, the point having affix z is equidistant from the four points having affixes z_1, z_2, z_3, z_4. Thus z is the affix of either the centre of a circle or the point of intersection of diagonals of a square (or rectangle). Therefore, z_1, z_2, z_3, z_4 are either concyclic or vertices of a square (of rectangle).
Hence, z_1, z_2, z_3, z_4 are concyclic
280 **(a)**
Since, α, β and γ, δ are the roots of the equation $ax^2+2bx+c=0$ and $px^2+2qx+r=0$ respectively, then
$\alpha+\beta = -\frac{2b}{a}, \alpha\beta = \frac{c}{a}, \gamma+\delta = -\frac{2q}{p}, \gamma\delta = \frac{r}{p}$
As given α, β, γ and δ are in GP, therefore
$\frac{\alpha}{\gamma} = \frac{\beta}{\delta}$ ...(i)
But $\frac{\alpha\beta}{\gamma\delta} = \frac{pc}{ar} \Rightarrow \left(\frac{\beta}{\delta}\right)^2 = \frac{pc}{ar}$ [from Eq. (i)]
Also, $\frac{\alpha}{\beta} = \frac{\gamma}{\delta} \Rightarrow \frac{\alpha+\beta}{\beta} = \frac{\gamma+\delta}{\delta} \Rightarrow \frac{\alpha+\beta}{\gamma+\delta} = \frac{\beta}{\delta}$
$\Rightarrow \frac{bp}{aq} = \sqrt{\frac{pc}{ar}} \Rightarrow \frac{b^2p^2}{a^2q^2} = \frac{pc}{ar} \Rightarrow q^2ac = b^2pr$
281 **(c)**
Given, $\alpha+\beta+\gamma = 2, \alpha^2+\beta^2+\gamma^2 = 6,$
$\alpha^3+\beta^3+\gamma^2 = 8$
Now, $(\alpha+\beta+\gamma)^2 = 2^2$
$\Rightarrow \alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\beta\gamma+\gamma\alpha) = 4$
$\Rightarrow 2(\alpha\beta+\beta\gamma+\gamma\alpha) = 4-6 = -2$
Also, $\alpha^3+\beta^3+\gamma^3-3\alpha\beta\gamma$
$= (\alpha+\beta+\gamma)(\alpha^2+\beta^2+\gamma^2-\alpha\beta-\beta\gamma-\gamma\alpha)$
$\Rightarrow 8-3\alpha\beta\gamma = 2[6-(-1)]$
$\Rightarrow 8-3\alpha\beta\gamma = 14$
$\Rightarrow 3\alpha\beta\gamma = 8-14$
$\Rightarrow \alpha\beta\gamma = -2$
Now, $\alpha^4+\beta^4+\gamma^4 = (\alpha^2+\beta^2+\gamma^2)^2 - 2\sum\alpha^2\beta^2$
$= (\alpha^2+\beta^2+\gamma^2)^2 - 2\left[\left(\sum\beta\gamma\right)^2 - 2\alpha\beta\gamma\sum\alpha\right]$

$= (6)^2 - 2[(-1)^2 - 2(-2)2]$
$= 36 - 2[9]$
$= 36 - 18 = 18$

282 **(a)**
Given equation is $x^2 - 2ax + a^2 + a - 3 = 0$.
If roots are real then $D \geq 0$
$\Rightarrow 4a^2 - 4(a^2 + a - 3) \geq 0$
$\Rightarrow -a + 3 \geq 0$
$\Rightarrow a - 3 \leq 0 \Rightarrow a \leq 3$
As roots are less than 3, hence $f(3) > 0$
$9 - 6a + a^2 + a - 3 > 0$
$\Rightarrow a^2 - 5a + 6 > 0$
$\Rightarrow (a-2)(a-3) > 0$
$\Rightarrow$ Either $a < 2$ or $a > 3$.
Hence, only $a < 2$ satisfy.

283 **(b)**
$(|az_1 - bz_2|)^2 + |(bz_1 + az_2)|^2$
$= a^2|z_1|^2 + b^2|z_2|^2 - 2ab\,\text{Re}(\overline{z_1}z_2) + b^2|z_1|^2 + a^2|z_2|^2 + 2ab\text{Re}(\overline{z_1}z_2)$
$= (a^2 + b^2)(|z_1|^2 + |z_2|^2)$

284 **(a)**
We have,
$\frac{1}{2} + \frac{3}{8} + \frac{9}{32} + \frac{27}{128} + \cdots$
$= \frac{1}{2}\left(1 + \frac{3}{2^2} + \frac{3^2}{2^4} + 3^3 + 2^6 + \cdots\right) = \frac{1}{2}\left(\frac{1}{1 - \frac{3}{4}}\right)$
$= 2$
$\because \omega + \omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\frac{27}{128}\ldots\right)} = \omega + \omega^2 = -1$

285 **(b)**
Here, $\alpha + \beta = -\frac{b}{a}$, $\alpha\beta = \frac{c}{a}$ ...(i)
The quadratic equation whose roots are $\frac{1-\alpha}{\alpha}$ and $\frac{1-\beta}{\beta}$, is
$x^2 - \left(\frac{1-\alpha}{\alpha} + \frac{1-\beta}{\beta}\right)x + \frac{1-\alpha}{\alpha}.\frac{1-\beta}{\beta} = 0$
$\Rightarrow x^2 - \left(\frac{\beta - \alpha\beta + \alpha - \alpha\beta}{\alpha\beta}\right)x + \frac{1 - \beta - \alpha + \alpha\beta}{\alpha\beta} = 0$
$\Rightarrow x^2 - \left(\frac{-\frac{b}{a}, -2.\frac{c}{a}}{\frac{c}{a}}\right)x + \frac{1 - \left(-\frac{b}{a}\right) + \frac{c}{a}}{\frac{c}{a}} = 0$ [from Eq. (i)]
$\Rightarrow x^2 - \frac{(-b-2c)x}{c} + \frac{a+b+c}{c} = 0$
$\Rightarrow cx^2 + (b+2c)x + (a+b+c) = 0$
On comparing with $px^2 + qx + r = 0$, we get
$r = q + b + c$

286 **(b)**
We have,
$1 + x^2 = \sqrt{3}x$
$\Rightarrow x^2 - \sqrt{3}\,x + 1 = 0 \Rightarrow x = \frac{\sqrt{3}+i}{2} = -i\,\omega, i\,\omega^2$
Clearly, $-i\,\omega$ and $i\,\omega^2$ are reciprocal of each other and the given expression does not alter by replacing x by $\frac{1}{x}$. So, we will compute its value for one of these two values of x
For $x = i\,\omega^2$, we have
$\sum_{n=1}^{24}\left(x^n - \frac{1}{x^n}\right)^2 = \sum_{n=1}^{24}\{(i\,\omega^2)^n - (-i\,\omega)^n\}^2$
$\Rightarrow \sum_{n=1}^{24}\left(x^n - \frac{1}{x^n}\right)^2 = \sum_{n=1}^{24}(-1)^n\{\omega^{2n} - (-1)^n\,\omega^n\}^2$
$\Rightarrow \sum_{n=1}^{24}\left(x^n - \frac{1}{x^n}\right)^2$
$= \sum_{k=1}^{8}(-1)^{3k}\{\omega^{6k} - (-1)^{3k}\omega^{3k}\}^2$
$+ \sum_{k=0}^{7}(-1)^{3k+1}\{\omega^{6k+2} - (-1)^{3k+1}\omega^{3k+1}\}$
$+ \sum_{k=0}^{7}(-1)^{3k+2}\{\omega^{6k+4} - (-1)^{3k+2}\omega^{3k+2}\}$
$= \sum_{k=1}^{8}(-1)^{3k}\{1 - (-1)^{3k}\}^2$
$+ \sum_{k=0}^{7}(-1)^{3k+1}\{\omega^2 - (-1)^{3k+1}\omega\}^2$
$+ \sum_{k=0}^{7}\{\omega - (-1)^{3k+2}\omega^2\}^2$
$= \sum_{k=1}^{8}(-1)^{3k}\{2 - 2(-1)^{3k}\}$
$+ \sum_{k=0}^{7}(-1)^{3k+1}\{\omega + \omega^2 - 2(-1)^{3k+1}\}$
$+ \sum_{k=0}^{7}(-1)^{3k+2}\{\omega^2 + \omega - 2(-1)^{3k+2}\}$
$= (-4) \times 4 + \sum_{k=0}^{7}(-1)^{3k-1}\{-1 + 2(-1)^{3k+2}\}$
$+ \sum_{k=0}^{7}(-1)^{3k+2}\{-1 + 2(-1)^{3k+3}\}$
$= -16 + \{-1 \times 4 + (-3) \times 4\} + \{-3 \times 4 + 4 \times -1\}$

287 **(d)**
Since α, β are roots of $x^2 + bx - c = 0$

$\therefore \alpha+\beta=-b, \alpha\beta=-c$
The equation whose roots are b, c is
$x^2-x(b+c)+bc=0$
$\Rightarrow x^2-x(-\alpha-\beta-\alpha\,\beta)+\alpha\,\beta(\alpha+\beta)=0$
$\Rightarrow x^2+x(\alpha+\beta+\alpha\,\beta)+\alpha\,\beta(\alpha+\beta)=0$

288 **(c)**
Here, $\alpha^2-a\alpha+b=0$ and $\beta^2+a\beta+b=0$
Now, $A_{n+1}-aA_n+bA_{n-1}$
$=\alpha^{n+1}+\beta^{n+1}-a(\alpha^n+\beta^n)+b(\alpha^{n-1}+\beta^{n-1})$
$=\alpha^{n-1}(\alpha^2-a\alpha+b)+\beta^{n-1}(\beta^2-a\beta+b)$
$=0$

289 **(b)**
Let α be a common root of the equations
$x^2+(a^2-2)x-2a^2=0$ and $x^2-3x+2=0$
Then,
$\alpha^2+(a^2-2)\alpha-2a^2=0$ and $\alpha^2-3\,\alpha+2=0$
Now,
$\alpha^2-3\,\alpha+2=0 \Rightarrow \alpha=1,2$
Putting, $\alpha=1$ in $\alpha^2+(a^2-2)\alpha-2a^2=0$, we get
$\Rightarrow a^2+1=0$, which is not possible for any $a \in R$
Putting $\alpha=2$ in $\alpha^2+(a^2-2)\alpha-2a^2=0$, we get
$4+2(a^2-2)-2a^2=0$, which is true for all $a \in R$
Thus, the two equations have exactly one common root for all $a \in R$

290 **(c)**
$(\log_b a.\log_c a-\log_a a)+(\log_a b.\log_c b-\log_b b)+(\log_a c.\log_b c-\log_c c)=0$
$$\Rightarrow \left(\frac{\log a}{\log b}.\frac{\log a}{\log c}-\frac{\log a}{\log a}\right)+\left(\frac{\log b}{\log a}.\frac{\log b}{\log c}-\frac{\log b}{\log b}\right)+\left(\frac{\log c}{\log a}.\frac{\log c}{\log b}-\frac{\log c}{\log c}\right)=0$$
$\Rightarrow (\log a)^3+(\log b)^3+(\log c)^3-3\log a\log b\log c=0$
$\Rightarrow \quad (\log a+\log b+\log c)=0$
$\left(\because \text{if } a^3+b^3+c^3=3abc, \text{ then } a+b+c=0\right)$
$\Rightarrow abc=1$

291 **(b)**
Let the incorrect equation is
$x^2+15x+b=0$
Since, roots are -7 and -2
$\therefore$ Product of roots, $b=14$
So, correct equation is $x^2-9x+14=0$

292 **(c)**
Let $f(x)=x^2+x+a$. Both the roots of $f(x)=0$ will exceed a, if
(i) Discriminant >0
(ii) A lies outside the roots i.e. $f(a)>0$
(iii) $a<x$-coordinate of vertex
$\therefore a<\frac{1}{4}, a^2+2\,a>0$ and $a<-1/2$
$\Rightarrow a<-1/2$ and $a^2+2\,a>0$
$\Rightarrow a<-1/2$ and $a(a+2)>0$
$\Rightarrow a<-\frac{1}{2}$ and $a+2<0 \quad [\because a<0]$
$\Rightarrow a<-1/2$ and $a<-2 \Rightarrow a<-2$

293 **(c)**
Since a, b, c are positive
$\therefore ax^2+b|x|+c>0$
Hence, the equation $ax^2+b|x|+c=0$ has no real roots

294 **(b)**
By Rolle's Theorem, between any two roots of a polynomial $f(x)$, there is a root of $f'(x)$.
Therefore, $f'(c)=0$ for same $c \in (a,b)$

295 **(b)**
Given, $(x-1)^3=(-2)^3 \;\Rightarrow\; \left(\frac{x-1}{-2}\right)=(1)^{1/3}$
$\therefore$ Cube roots of $\left(\frac{x-1}{-2}\right)$ are $1, \omega$ and ω^2
$\Rightarrow$ Cube roots of $(x-1)$ are $-2, -2\omega$ and $-2\omega^2$
$\Rightarrow$ Cube roots of x are $-1, 1-2\omega$ and $1-2\omega^2$

296 **(b)**
Given equation is
$x^2-2x(1+3k)+7(2k+3)=0$
For equal roots, discriminant=0
$\therefore \quad 4(1+3k)^2=4\times 7(2k+3$
$\Rightarrow \quad 9k^2-8k-20=0 \;\Rightarrow k=2, \frac{-10}{9}$

297 **(c)**
$7^{2\log_7 5}=7^{\log_7 (5)^2}$
$=(5)^2=25 \quad [\because a^{\log_a x}=x; x>0, x\neq 0,1]$

298 **(b)**

We have,

$\log\left(\frac{a-ib}{a+ib}\right)$

$= \log(a-ib) - \log(a+ib)$

$= \left[\log\sqrt{a^2+b^2} + i\tan^{-1}\left(\frac{-b}{a}\right)\right]$

$- \left[\log\sqrt{a^2+b^2} + i\tan^{-1}\left(\frac{b}{a}\right)\right]$

$= -2i\tan^{-1}\left(\frac{b}{a}\right)$

$\therefore i\log\left(\frac{a-i\,b}{a+ib}\right)$

$= 2\tan^{-1}\left(\frac{b}{a}\right)$

$= \tan^{-1}\left(\frac{2\frac{b}{a}}{1-\frac{b^2}{a^2}}\right)$

$= \tan^{-1}\left(\frac{2\,ab}{a^2-b^2}\right)$

$\Rightarrow \tan\left\{i\log\left(\frac{a-ib}{a+ib}\right)\right\}$

$= \tan\left\{\tan^{-1}\left(\frac{2\,ab}{a^2-b^2}\right)\right\} = \frac{2\,ab}{a^2-b^2}$

299 **(a)**

We have,

$x^3+2x^2+2x+1=0$

$\Rightarrow (x^3+1)+2x(x+1)=0$

$\Rightarrow (x+1)(x^2+x+1)=0 \Rightarrow x=-1,\omega,\omega^2$

Let $f(x)=1+x^{2002}+x^{2003}$. Then,

$f(-1)=1+(-1)^{2002}+(-1)^{2003}=1+1-1$

$\neq 0$

$f(\omega)=1+(\omega)^{2002}+(\omega)^{2003}=1+\omega+\omega^2=0$

$f(\omega)=1+(\omega^2)^{2002}+(\omega^2)^{2003}=1+\omega^2+\omega$

$=0$

Hence, ω and ω^2 are common roots of the two equations

300 **(b)**

As $p<0$, therefore $p=-q$, where $q>0$

$\therefore \quad p^{1/3}=(-q)^{1/3}=q^{1/3}(-1)^{1/3}$

$\Rightarrow \quad p^{1/3}=-q^{1/3},-q^{1/3}\omega,-q^{1/3}\omega^2$

Let $\alpha=-q^{1/3}$, $\beta=q^{1/3}\omega$, and $\gamma=-q^{1/3}\omega^2$

$\therefore \quad \frac{x\alpha+y\beta+z\gamma}{x\beta+y\gamma+z\alpha}=\frac{x+y\omega+z\omega^2}{x\omega+y\omega^2+z}=\omega^2$

$=\frac{-1-i\sqrt{3}}{2}$

301 **(d)**

Since, $y^2-y+a=\left(y-\frac{1}{2}\right)^2+a-\frac{1}{4}$

and $-\sqrt{2}\le\sin x+\cos x\le\sqrt{2}$, given equation will have no real values of x for any y, if

$a-\frac{1}{4}>\sqrt{2}$

$ie, a\in\left(\sqrt{2}+\frac{1}{4},\infty\right)$

$\Rightarrow a\in(\sqrt{3},\infty)$ (as $\sqrt{2}+\frac{1}{4}<\sqrt{3}$)

302 **(a)**

Let $y=\sqrt{42+\sqrt{42+\sqrt{42}+\cdots}}$

$\Rightarrow \quad y=\sqrt{42+y}$

On squaring both sides, we get

$y^2=42+y$

$\Rightarrow y^2-y-42=0$

$\Rightarrow (y-7)(y+6)=0$

$\Rightarrow \quad y=7,6$

Since, $y=-6$ does not satisfy the given equation

$\therefore$ The required solution is $y=7$

303 **(b)**

Let α and β be the roots of the given equation, then

$\alpha+\beta=10, \qquad \alpha\beta=16$

$\therefore$ Required equation is

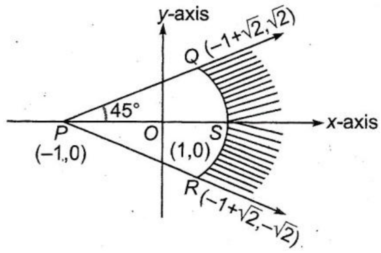
$x^2-(\alpha+\beta)x+\alpha\beta=0$

$\Rightarrow x^2-10x+16=0$

304 **(a)**

Here, $|PQ|=|PS|=|PR|=2$

$\therefore$ Shaded part represents the external part of circle having centre (-1, 0) and radius 2



As we know equation of circle having centre z_0 and radius r, is

$|z-z_0|=r$

$\therefore \quad |z—(-1+0i)|>2$

$\Rightarrow \quad |z+1|>2$ ...(i)

Also, argument of $z+1$ with represent to positive direction of x-axis is $\pi/4$

$\therefore \quad \arg(z+1)\le\frac{\pi}{4}$

And argument of $z+1$ in anti-clockwise direction is $-\pi/4$.

$\therefore \quad -\frac{\pi}{4}\le\arg(z+1)$

$\Rightarrow \quad |\arg(z+1)|\le\frac{\pi}{4}$

305 **(a)**

If ω and ω^2 are two imaginary cube roots of unity. Then, $1+\omega+\omega^2=0$

$\Rightarrow \omega + \omega^2 = -1$

Now, $a\omega^{317} + a\omega^{382} = a(\omega^{317} + \omega^{382})$

$= a(\omega^2 + \omega) = -a$

And $a\omega^{317} \times a\omega^{382} = a^2\omega^{699} = a^2$

Therefore, the required equation is

$x^2 - (a\omega^{317} + a\omega^{382}) + (a\omega^{317} \times a\omega^{382}) = 0$

$\Rightarrow x^2 + ax + a^2 = 0$

306 **(b)**

Let the given expression by y.

$\therefore y = \dfrac{x+2}{2x^2+3x+6}$

$\Rightarrow 2x^2y + (3y-1)x + (6y-2) = 0$

If $y \neq 0$, then $\Delta \geq 0$ for real x.

$ie, b^2 - 4ac \geq 0$

$\therefore (3y-1)^2 - 8y(6y-2) \geq 0$

$\Rightarrow -39y^2 + 10y + 1 \geq 0$

$\Rightarrow (13y+1)(3y-1) \leq 0$

$\Rightarrow -\dfrac{1}{13} \leq y \leq \dfrac{1}{3}$

If $y = 0$, then $x = -2$ which is real and this value of y is included in the above range.

307 **(a)**

We have,

$z(\overline{z+\alpha}) + \bar{z}(z+\alpha) = 0$

$\Rightarrow z(\bar{z}+\bar{\alpha}) + \bar{z}(z+\alpha) = 0 \Rightarrow z\bar{z} + \dfrac{1}{2}z\bar{\alpha} + \dfrac{1}{2}\bar{z}\alpha$

$= 0$

Clearly, it represents a circle having centre at $-\frac{1}{2}\alpha$ and radius $= \frac{1}{2}|\alpha|$

308 **(a)**

On multiplying first equation by x, we get

$x^4 + ax^2 + x = 0$ ....(i)

and another given equation is

$x^4 + ax^2 + 1 = 0$ ....(ii)

On subtracting Eq. (ii) from Eq. (i), we get

$x - 1 = 0 \Rightarrow x = 1$

Which is a common root.

On putting this value in Eq. (ii), we get

$1 + a + 1 = 0$

$\Rightarrow a = -2$

309 **(d)**

Given, $x = \frac{-1+\sqrt{3}i}{2} = \omega$

$\therefore \quad (1 - x^2 + x)^6 - (1 - x + x^2)^6$

$= (1 - \omega^2 + \omega)^6 - (1 - \omega + \omega^2)^6$

$= (-2\omega^2)^6 - (-2\omega)^6 \quad [\because 1 + \omega + \omega^2 = 0]$

$= 2^6\omega^{12} - 2^6\omega^6 = 0 \quad [\because \omega^3 = 1]$

310 **(d)**

We have,

$(\alpha-\gamma)(\alpha-\delta) = \alpha^2 - \alpha(\gamma+\delta) + \gamma\delta$

$\Rightarrow (\alpha-\gamma)(\alpha-\delta) = \alpha^2 + p\,\alpha + r \; [\because \gamma + \delta = -p, \gamma\delta = r]$

$\Rightarrow (\alpha-\gamma)(\alpha-\delta) = q + r \quad \begin{bmatrix}\because x^2 + px - q = 0 \\ \therefore \alpha^2 + p\,\alpha = q\end{bmatrix}$

311 **(d)**

Since the roots of the equation

$x^3 - 3\,ax^2 + 3\,bx - c = 0$ are in H.P. Therefore, the roots of the reciprocal equation i.e.

$cy^3 - 3by^2 + 3by^2 + 3ay - 1 = 0$ are in A.P.

i. e. $\dfrac{1}{\alpha}, \dfrac{1}{\beta}, \dfrac{1}{\gamma}$ are in A. P.

$\therefore \dfrac{2}{\beta} = \dfrac{1}{\alpha} + \dfrac{1}{\gamma}$

$\Rightarrow \dfrac{3}{\beta} = \dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma} \Rightarrow \dfrac{3}{\beta} = \dfrac{3b}{c} \Rightarrow \beta$

$= \dfrac{c}{b} \quad \left[\because \dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma} = \dfrac{3b}{c}\right]$

312 **(d)**

Let $S = 1 + i^2 + i^4 + i^6 + \ldots + i^{2n}$

$= 1 - 1 + 1 - 1 + 1 - \ldots + (-1)^n$

The value of S depends on n

$\therefore$ The value cannot be determined

313 **(b)**

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$\Delta = \begin{vmatrix} 1 + \omega^n + \omega^{2n} & \omega^n & \omega^{2n} \\ \omega^n + \omega^{2n} + 1 & \omega^{2n} & 1 \\ \omega^{2n} + 1 + \omega^n & 1 & \omega^n \end{vmatrix}$

$= (1 + \omega^n + \omega^{2n}) \begin{vmatrix} 1 & \omega^n & \omega^2 n \\ 1 & \omega^{2n} & 1 \\ 1 & 1 & \omega^n \end{vmatrix}$

$= (1 + \omega^n + \omega^{2n}) \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ 0 & \omega^{2n} - \omega^n & \omega^{2n} - 1 \\ 0 & \omega^n - 1 & \omega^n - \omega^{2n} \end{vmatrix}$ Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$= (1 + \omega^n + \omega^{2n})\{(\omega^{2n} - \omega^n)(\omega^n - \omega^{2n}) - (\omega^{2n} - 1)(\omega^n - 1)\}$

$= (1 + \omega^n + \omega^{2n})\{\omega^{3n} - \omega^{4n} - \omega^{2n} + \omega^{3n} - \omega^{3n} + \omega^{2n} + \omega^n - 1\}$

$= (1 + \omega^n + \omega^{2n})(1 - \omega^n - \omega^{2n} + 1 - 1 + \omega^{2n} + \omega^n - 1) = 0$

314 **(c)**

$1 + \sum_{k=0}^{14}\left\{\cos\dfrac{(2k+1)}{15}\pi + i\sin\dfrac{(2k+1)}{15}\pi\right\}$

$= 1 + \sum_{k=0}^{14} e^{i\frac{(2k+1)}{15}\pi}$

$= 1 + (\alpha + \alpha^3 + \alpha^5 + \ldots + \alpha^{29})$ (where $\alpha = e^{i\pi/15}$)

$= 1 + \alpha\left(\frac{1-\alpha^{30}}{1-\alpha^2}\right) = 1 \quad [\because \alpha^{30} = e^{i2\pi=1}]$

315 **(c)**

We know that, if $az_1 + bz_2 + cz_3 = 0$ and $a + b + c = 0$, then z_1, z_2, z_3 lie on a line

316 **(b)**

We have,

$(1+\omega)^7 = A + B\,\omega$

$\Rightarrow (-\omega^2)^7 = A + B\,\omega \qquad [\because 1+\omega+\omega^2 = 0]$

$\Rightarrow -\omega^{14} = A + B\,\omega$

$\Rightarrow -\omega^2 = A + B\,\omega \Rightarrow 1 + \omega = A + B\,\omega \Rightarrow A = B = 1$

317 **(c)**

Since the function $f(x) = 9^x - 3^x + 1$ is continuous for all x and every continuous function attains every value between its maximum and minimum values. Therefore, $f(x)$ takes every value between its minimum and maximum values.

We have,

$f(x) = 9^x - 3^x + 1 = \left(3^x - \frac{1}{2}\right)^2 + \frac{3}{4} > \frac{3}{4}$ for all x

Thus, $f(x)$ assumes all real values greater than $3/4$

318 **(c)**

Given, $|z-1| = |z-i|$

$\Rightarrow$ z lies on the perpendicular bisector of the line joining (1, 0)

And (0, 1) and it is a straight line passing through origin.

319 **(b)**

Since, $x^2 + 20 + \sqrt{x^4+20} = 22 + 20$

Let $\sqrt{x^4+20} = y$

$\therefore y^2 + y - 42 = 0$

$\Rightarrow (y-6)(y+7) = 0 \Rightarrow y = 6 \quad (\because y \neq -7)$

$\Rightarrow \sqrt{x^4+20} = 6 \quad \Rightarrow \quad x^4 + 20 = 36$

$\Rightarrow \quad x^4 = 16 \quad \Rightarrow \quad x = \pm 2$

Hence, the number of real roots of the equation is 2

320 **(b)**

Since, the roots of the given equation are real

$\therefore$ Discriminant $>0 \Rightarrow 16 + 4\log_3 a \geq 0$

$\Rightarrow \log_3 a \geq -4 \quad \Rightarrow \quad a \geq 3^{-4} \quad \Rightarrow \quad a \geq \frac{1}{81}$

Hence, the least value of a is $\frac{1}{81}$

321 **(d)**

Since, $\frac{b-a}{x^2+(a+b)x+ab} = \frac{1}{x+c}$

$\Rightarrow x^2 + 2ax + ab + ca - bc = 0$

Since, the product of roots is zero

Then, $ab + ca - bc = 0 \Rightarrow a = \frac{bc}{b+c}$

$\therefore$ Sum of roots$= -2a = \frac{-2bc}{b+c}$

322 **(b)**

Given, $\frac{3x+2}{(x+1)(2x^2+3)} = \frac{A}{(x+1)} + \frac{Bx+C}{(2x^2+3)}$

$\Rightarrow \quad 3x + 2 = A(2x^2+3) + (Bx+C)(x+1)$

On putting $x + 1 = 0$ *ie*, $x = -1$

We get $3(-1) + 2 = A[2(-1)^2 + 3]$

$\Rightarrow \quad A = -\frac{1}{5}$

Now, on comparing the coefficients of x^2 and x, we get

$0 = 2A + B$

$\Rightarrow \quad B = \frac{2}{5}$

And $3 = B + C$

$\Rightarrow \quad C = 3 - \frac{2}{5} = \frac{13}{5}$

$\therefore \quad A + C - B = -\frac{1}{5} + \frac{13}{5} - \frac{2}{5} = \frac{10}{5} = 2$

323 **(a)**

Let $z = \alpha$ be a real root of

$z^2 + (p+iq)z + (r+is) = 0$. Then,

$\alpha^2 + (p+iq)\alpha + (r+is) = 0$

$\Rightarrow \alpha^2 + p\alpha + r = 0$ and $q\alpha + s = 0$

$\Rightarrow \frac{s^2}{q^2} - \frac{ps}{q} + r = 0 \Rightarrow psq = s^2 + q^2 r$

324 **(c)**

Let α, β be the roots of the equation $x^2 + px + 8 = 0$

Then,

$\alpha + \beta = -p$ and $\alpha\beta = 8$

It is given that

$|\alpha - \beta| = 2$

$\Rightarrow |\alpha - \beta|^2 = 4$

$\Rightarrow (\alpha+\beta)^2 - 4\alpha\beta = 4 \Rightarrow p^2 - 32 = 4 \Rightarrow p = \pm 6$

326 **(d)**

Let $\alpha = \frac{3}{2} + \frac{7}{2}i$

$\beta = \frac{3}{2} - \frac{7}{2}i$

$\therefore \quad \alpha + \beta = 3, \quad \alpha\beta = \frac{9}{4} + \frac{49}{4} = \frac{29}{2}$

$\Rightarrow \frac{6}{a} = 3, \frac{b}{a} = \frac{29}{2}$

$\Rightarrow \quad a = 2, \qquad b = 29$

$\Rightarrow \quad a + b = 31$

327 **(b)**

Let $z = x + iy$

$\Rightarrow z^2 = x^2 - y^2 + 2i\,xy$

$\Rightarrow \mathrm{Re}(z^2) = \mathrm{Re}(x^2 - y^2 + 2ixy)$

$\Rightarrow 1 = □^2 - □^2 \qquad [\because \mathrm{Re}(□^2) = 1$ (given)$]$

328 **(d)**

Here, $\sum \alpha_1 = 0, \quad \sum \alpha_1\alpha_2 = (2-\sqrt{3}),$

$\sum \alpha_1\alpha_2\alpha_3 = 0, \quad \sum \alpha_1\alpha_2\alpha_3\alpha_4 = 2 + \sqrt{3}$

$(1-\alpha_1)(1-\alpha_2)(1-\alpha_3)(1-\alpha_4)$

$= (1 + \alpha_1\alpha_2 - \alpha_1 - \alpha_2)(1 - \alpha_3)(1 - \alpha_4)$
$= (1 + \alpha_1\alpha_2 - \alpha_1 - \alpha_2 - \alpha_3 - \alpha_1\alpha_2\alpha_3 + \alpha_1\alpha_3 + \alpha_2\alpha_3)(1 - \alpha_4)$
$= 1 + \sum \alpha_1\alpha_2 - \sum \alpha_1\alpha_2\alpha_3 - \sum \alpha_1 + \alpha_1\alpha_2\alpha_3\alpha_4$
$= 1 + 2 - \sqrt{3} - 0 - 0 + 2 + \sqrt{3} = 5$

329 **(a)**
$\because\ x = 8 + 3\sqrt{7}$
$\therefore y = \dfrac{1}{8 + 3\sqrt{7}} = 8 - 3\sqrt{7}$
Now, $\frac{1}{x^2} + \frac{1}{y^2} = \frac{x^2+y^2}{(xy)^2}$
$= (x + y)^2 - 2 \quad [\because xy = 1]$
$= \left(8 + 3\sqrt{7} + 8 - 3\sqrt{7}\right)^2 - 2$
$= (16)^2 - 2 = 254$

330 **(a)**
Let $z = \frac{12}{5} + \frac{16}{5}i$
$\because\ \tan\theta = \dfrac{16}{12} = \dfrac{4}{3} > 0$
$\therefore\quad \theta > 0$
And $|z| = \sqrt{\left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2} = \frac{1}{5}\sqrt{144 + 256} = 4$
Now, $|2 - 3i| = \sqrt{4 + 9} = \sqrt{13}$
$\therefore\quad |2 - 3i| < |z|$

331 **(d)**
Let $f(x) = 2x^2 - 2(2a + 1)x + a(a + 1)$
Clearly, $y = f(x)$ is a parabola opening upward. It is given that a lies between its roots
$\therefore$ Discriminant > 0 *and* $f(a) < 0$
$\Rightarrow 4(2a + 1)^2 - 8a(a + 1) > 0$ and $2a^2 - 2a(2a + 1) + a(a + 1) < 0$
$\Rightarrow 2a^2 + 2a + 1 > 0$ and $a(a + 1) > 0$
$\Rightarrow a(a + 1) > 0 \quad [\because 2a^2 + 2a + 1 > 0$ for all $a \in R]$
$\Rightarrow a < -1$ or $a > 0$

332 **(b)**
Case I When $n \geq a$
$\therefore\quad x^2 - 2a(x - a) - 3a^2 = 0$
$\Rightarrow\quad x^2 - 2ax - a^2 = 0 \ \Rightarrow\ x = a \pm \sqrt{2}a$
Now, for $x \geq a, a < 0$
$\Rightarrow\quad x = a\left(1 - \sqrt{2}\right) \quad [\because x = a\left(1 + \sqrt{2}\right) < a]$ ...(i)
Case II When $x < a$
$\therefore\quad x^2 + 2a(x - a) - 3a^2 = 0$
$\Rightarrow\quad x^2 + 2ax - 5a^2 = 0$
$\Rightarrow\quad x = -a \pm \sqrt{6}a$
Now, for $x < a,\ a < 0$
$\Rightarrow\quad x = a(\sqrt{6} - 1)$ ...(ii)
$[\because\ x = -a\left(1 + \sqrt{6}\right) > a]$
From Eqs. (i) and (ii),
$x = \left\{a\left(1 - \sqrt{2}\right), a\left(\sqrt{6} - 1\right)\right\}$

333 **(c)**
Since, $\frac{AB}{BC} = \sqrt{2}$
Considering the rotation about $'B'$, we get,
$\dfrac{z_1 - z_2}{z_3 - z_2} = \dfrac{|z_1 - z_2|}{|z_3 - z_2|}\ e^{i\pi/4}$
$= \dfrac{AB}{BC}\ e^{i\pi/4}$
$= \sqrt{2}\left(\dfrac{1}{\sqrt{2}} + \dfrac{i}{\sqrt{2}}\right) = 1 + i$
$\Rightarrow\ z_1 - z_2 = (1 + i)(z_3 - z_2)$
$\Rightarrow\ z_1 - (1 + i)z_3 = z_2(1 - 1 - i)$
$\Rightarrow\ iz_2 = -z_1 + (1 + i)z_3$
$\Rightarrow\ z_2 = iz_1 - i(1 + i)z_3$
$= z_3 + i(z_1 - z_3)$

334 **(a)**
Let $z = x + iy$
$\therefore\ \dfrac{z + i}{z + 2} = \dfrac{x + iy + i}{x + iy + 2} = \dfrac{x + i(y + 1)}{(x + 2) + iy}$
$= \dfrac{[x + i(y + 1)] \times [(x + 2) - iy]}{[(x + 2) + iy] \times [(x + 2) - iy]}$
$= \left[\dfrac{x^2 + 2x + y^2 + y}{(x + 2)^2 + y^2}\right] + i\left[\dfrac{(y + 1)(x + 2) - xy}{(x + 2)^2 + y^2}\right]$
Since, it is purely imaginary, therefore real part must be equal to zero
$\therefore\ \dfrac{x^2 + y^2 + 2x + y}{(x + 2)^2 + y^2} = 0$
$\Rightarrow\ x^2 + y^2 + 2x + y = 0$
It represents the equation of circle and its radius
$= \sqrt{1 + \dfrac{1}{4} - 0} = \dfrac{\sqrt{5}}{2}$
Therefore, locus of z in argand diagram is a circle of radius $\frac{\sqrt{5}}{2}$

335 **(b)**
The coordinates of the points representing $1 + i, i - 1$ and $2i$ are $(1,1), (-1,1)$ and $(0,2)$ respectively
$\therefore$ Required area $= \dfrac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 2 & 1 \end{vmatrix} = 1$ sq. unit.

336 **(b)**
We have,
$x = -5 + 4\,i$
$\Rightarrow (x + 5)^2 = -16\ \Rightarrow x^2 + 10\,x + 41 = 0$ ...(i)
Now,
$x^4 + 9\,x^3 + 35\,x^2 - x + 4$
$= x^2(x^2 + 10\,x + 41) - x(x^2 + 10x + 41) + 4(x^2 + 10x + 41) - 160$
$= 0\,x^2 - 0\,x + 4 \times 0 - 160 = -160$ [Using

(i)]

337 **(a)**

We have,

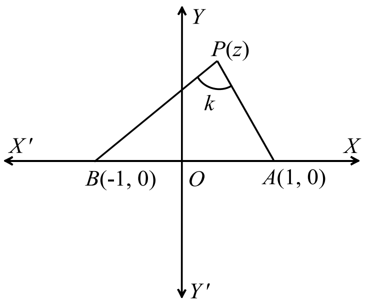
$\arg\left(\frac{z-1}{z+1}\right)=k$

$\Rightarrow \arg\left(\frac{z-1}{-z-1}\right)=k$

$\Rightarrow \angle BPA=k$

$\Rightarrow P$ lies on the circle passing through $A(1,0)$ and $B(-1,0)$. Clearly, the circle is symmetric about y-axis.

Hence, P lies on the circle having its centre of y-axis



338 **(b)**

We have,

$|2x+3|^2-3|2x+3|+2=0$

$\Rightarrow (|2x+3|-2)(|2x+3|-1)=0$

$\Rightarrow |2x+3|=1,2$

$\Rightarrow 2x+3=\pm 1,\pm 2 \Rightarrow x=-1,-2,-\frac{1}{2},-\frac{5}{2}$

$\therefore$ Product of roots $=\frac{5}{2}$

339 **(d)**

$\alpha=\omega,\beta=\omega^2$ will satisfy the given equation

Now, $\alpha^{19}=\omega^{19}=\omega$

$\beta^7=\omega^{14}=\omega^2$

$\Rightarrow$ Required equation is

$x^2-(\omega+\omega^2)x+\omega^2=0$

$\Rightarrow \quad x^2+x+1=0$

340 **(d)**

We have, $z=4-3i$

$\therefore \quad |z|=\sqrt{4^2+(-3)^2}=5$

Let z_1 be the new complex number obtained by rotating z in the clockwise sense through 180°, therefore

$z_1=-4+3i$

Therefore required complex number is

$3(-4+3i)=-12+9i$

341 **(c)**

Sum of the roots of $x^2-2ax+b^2=0$ is $2a$

$\therefore A=$ A. M. of the roots $=a$

Product of the roots of $x^2-2bx+a^2=0$ is a^2

$\therefore G=$ G. M. of the roots $=a$

Clearly, $A=G$

342 **(b)**

$\left|z+\frac{2}{z}\right|=2 \Rightarrow |z|-\frac{2}{|z|}\leq 2$

$\Rightarrow |z|^2-2|z|-2\leq 0$

This is a quadratic equation in $|z|$

$\therefore |z|\leq \frac{2\pm\sqrt{4+8}}{2}\leq 1\pm\sqrt{3}$

Hence, maximum value of $|z|$ is $1+\sqrt{3}$

343 **(b)**

Here, $\alpha+\beta=\frac{p+1}{2}$ and $\alpha\beta=\frac{p-1}{2}$

Now, $(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$

$\Rightarrow (\alpha\beta)^2=(\alpha+\beta)-4\alpha\beta \quad [\because \alpha-\beta=\alpha\beta$ given$]$

$\Rightarrow \left(\frac{p-1}{2}\right)^2=\left(\frac{p+1}{2}\right)^2-4\left(\frac{p-1}{2}\right)$

$\Rightarrow p^2+1-2p=p^2+1+2p-8p+8 \Rightarrow p=2$

344 **(d)**

Here, $a=e^{i\,2\pi/3}=\omega$

$\therefore \quad a+\frac{1}{a^2}=\omega+\frac{1}{\omega^2}=\omega+\omega=2\omega$

Similarly, $a^2+\frac{1}{a^2}=\omega^2+\frac{1}{\omega^4}=2\omega^2$

$\therefore \quad a+\frac{1}{a^2}+a^2+\frac{1}{a^4}=2\omega+2\omega^2=-2$

And $\left(a+\frac{1}{a^2}\right)\left(a^2+\frac{1}{a^4}\right)=2\omega.2\omega^2=4$

$\therefore$ required equation is $x^2+2x+4=0$

345 **(a)**

Given that, $x^2+bx+c=0$ and $b=17$ ...(i)

Since, roots of this equation are -2 and -15

$\therefore \quad (x+2)(x+15)=x^2+17x+30$ ...(ii)

From Eqs. (i) and (ii), $c=30$

If $b=13$, then

$x^2+13x+c=0 \Rightarrow x^2+13x+30=0$

$\Rightarrow \quad x=-3,-10$

346 **(a)**

Given that $x,y,z\in R$ and distinct and

$u=x^2+4y^2+9z^2-6yz-3zx-2xy$

$=\frac{1}{2}(2x^2+8y^2+18z^2-12yz-6zx-4xy)$

$=\frac{1}{2}\{(x^2-4xy+4y^2)+(x^2-6xz+9z^2)$
$+(4y^2-12yz+9z^2)\}$

$=\frac{1}{2}\{(x-2y)^2+(x-3z)^2+(2y-3z)^2\}>0$

So, u is always non-negative.

347 **(b)**

Here, $\alpha+\beta=\frac{5}{6}$ and $\alpha\beta=\frac{1}{6}$

$\therefore \tan^{-1}\alpha+\tan^{1}\beta=\tan^{-1}\left(\frac{\alpha+\beta}{1-\alpha\beta}\right)$

$= \tan^{-1}\left(\dfrac{\frac{5}{6}}{1-\frac{1}{6}}\right)$

$= \tan^{-1} 1 = \dfrac{\pi}{4}$

348 **(b)**

Given, $a^x = b^y = c^z = d^w$

$\Rightarrow x = y \log_a b = z \log_a c = w \log_a d$

$\Rightarrow y = \dfrac{x}{\log_a b}, z = \dfrac{x}{\log_a c}, w = \dfrac{x}{\log_a d}$

Now, $x\left(\frac{1}{y}+\frac{1}{z}+\frac{1}{w}\right)$

$= x\left[\dfrac{\log_a b}{x} + \dfrac{\log_a c}{x} + \dfrac{\log_a d}{x}\right]$

$= \dfrac{x}{x}[\log_a bcd] = \log_a(bcd)$

349 **(a)**

We know that, if $\log_a m > \log_a n$

$\Rightarrow m > n$ or $m < n$ according as $a > 1$ or $0 < a < 1$

$\therefore \log_{\left(\frac{1}{3}\right)}|z+1| > \log_{\left(\frac{1}{3}\right)}|z-1|$

$\Rightarrow |z+1| < |z-1| \qquad \left(\because 0 < \dfrac{1}{3} < 1\right)$

Let $z = x + iy$

$|x+iy+1| < |x+iy-1|$

$\Rightarrow (x+1)^2 + y^2 < (x-1)^2 + y^2$

$\Rightarrow 4x < 0 \Rightarrow x < 0 \Rightarrow \text{Re}(z) < 0$

350 **(c)**

We have, $b^2 = ac$

Let α, β be the roots of the equation $ax^2 + bx + c = 0$. Then,

$\alpha, \beta = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \alpha, \beta = \dfrac{-\sqrt{ac} + i\sqrt{3\,ac}}{2a} \qquad [\because b^2 = ac]$

$\Rightarrow \alpha = \left(-\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2}\right)\sqrt{\dfrac{c}{a}}$ and, $\beta = \left(-\dfrac{1}{2} - i\dfrac{\sqrt{3}}{2}\right)\sqrt{\dfrac{c}{a}}$

$\Rightarrow \alpha = \omega\sqrt{\dfrac{c}{a}}$ and, $\beta = \omega^2\sqrt{\dfrac{c}{a}} \Rightarrow \alpha : \beta = 1 : \omega$

351 **(b)**

Since, $\tan\alpha$ and $\tan\beta$ are the roots of the equation $x^2 + ax + b = 0$, then

$\tan\alpha + \tan\beta = -\dfrac{a}{1}$

and $\tan\alpha \cdot \tan\beta = b$

$\Rightarrow \dfrac{\sin\alpha}{\cos\alpha} + \dfrac{\sin\beta}{\cos\beta} = -\dfrac{a}{1}$

and $\frac{\sin\alpha \sin\beta}{\cos\alpha \, \cos\beta} = b$

$\therefore \sin^2(\alpha+\beta) + a\sin(\alpha+\beta)\cos(\alpha+\beta) + b\cos^2(\alpha+\beta)$

$= \cos^2((\alpha+\beta)\,[\tan^2(\alpha+\beta) + b + a\tan(\alpha+\beta)]$

$= \dfrac{\tan^2(\alpha+\beta) + b + a\tan(\alpha+\beta)}{1 + \tan^2(\alpha+\beta)}$

$= \dfrac{\frac{a}{b-1}\left(a + \frac{a}{b-1}\right)}{1 + \frac{a^2}{(b-1)^2}} = b$

352 **(a)**

We have, $\omega^{10} + \omega^{23} = \omega + \omega^2 = -1$

$\therefore \left\{(\omega^{10} + \omega^{23})\pi - \dfrac{\pi}{4}\right\} = \sin\left(\dfrac{-5\pi}{4}\right) = \sin\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}$

353 **(b)**

The equation formed by decreasing each root of $ax^2 + bx + c = 0$ by 1 is

$a(x+1)^2 + b(x+1) + c = 0$

$\Rightarrow ax^2 + x(2a+b) + a + b + c = 0$

This is identical to the equation $2x^2 + 8x + 2 = 0$

$\therefore \dfrac{a}{2} = \dfrac{2a+b}{8} = \dfrac{a+b+c}{2}$

$\Rightarrow 4a = 2a + b, a = a + b + c$ and $2a + b = 4a + 4b + 4c$

$\Rightarrow 2a = b, b + c = 0$ and $2a + 3b + 4c = 0$

$\Rightarrow b = 2a, b = -c$ and $c = -2a \Rightarrow 2a = b = -c$

354 **(c)**

$|z-2| = \min\{|z-1|, |z-5|\}$

ie, $|z-2| = |z-1|$, where $|z-1| < |z-5|$

$\Rightarrow \text{Re}(z) = \frac{3}{2}$ which satisfy $|z-1| < |z-5|$

Also, $|z-2| = |z-5|$, where $|z-5| < |z-1|$

$\Rightarrow \text{Re}(z) = \frac{7}{2}$ which satisfy $|z-5| < |z-1|$

355 **(a)**

We have,

$2a\,x^2 + (2a+b)x + b = 0, a \neq 0$

$\Rightarrow x = \dfrac{-(2a+b) \pm (2a-b)}{4a} \Rightarrow x = -1, -\dfrac{b}{2a}$

Hence, the roots are rational

356 **(a)**

Here, $\alpha + \beta = -\frac{m}{l}, \alpha\beta = \frac{n}{l}$

Now, $\alpha^3\beta + \alpha\beta^3 = \alpha\beta(\alpha^2 + \beta^2)$

$= \alpha\beta[(\alpha+\beta)^2 - 2\alpha\beta]$

$= \dfrac{n}{l}\left[\left(\dfrac{-m}{l}\right)^2 - \dfrac{2n}{l}\right]$

$= \dfrac{n}{l}\left(\dfrac{m^2}{l^2} - \dfrac{2n}{l}\right)$

And $\alpha^3\beta . \alpha\beta^3 = (\alpha\beta)^4 = \frac{n^4}{l^4}$

$\therefore$Required quadratic equation is

$x^2 - \dfrac{n}{l}\left(\dfrac{m^2}{l^2} - \dfrac{2n}{l}\right)x + \dfrac{n^4}{l^4} = 0$

$\Rightarrow l^4x^2 - nl(m^2 - 2nl)x + n^4 = 0$

357 **(b)**

Here, $\sum\alpha = 0, \quad \sum\alpha\beta = -7, \quad \alpha\beta\gamma = -7$

$\therefore \dfrac{1}{\alpha^4}+\dfrac{1}{\beta^4}+\dfrac{1}{\gamma^4}=\dfrac{\alpha^4\beta^4+\beta^4\gamma^4+\gamma^4\beta^4}{\alpha^4\beta^4\gamma^4}$

$=\frac{\sum\alpha^4\beta^4}{\alpha^4\beta^4\gamma^4}$ ...(i)

Now, $\sum\alpha\beta\sum\alpha\beta\sum\alpha\beta\sum\alpha\beta=(\sum\alpha\beta)^2(\sum\alpha\beta)^2$

$\Rightarrow (-7)^2=[\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2+2\alpha\beta\gamma(\alpha+\beta+\gamma)][\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2+2\alpha\beta\gamma(\alpha+\beta+\gamma)]$

$=(\alpha^4\beta^4+\beta^4\gamma^4+\gamma^4\alpha^4)(\alpha^4\beta^4+\beta^4\gamma^4+$
$\gamma 4\alpha 4\ \ \alpha=\alpha+\beta+\gamma=0$

$=\alpha^4\beta^4+\beta^4\gamma^4+\gamma^4\alpha^4+2\alpha^4\beta^2\gamma^2+2\alpha^2\beta^4\gamma^2+2\alpha^2\beta^2\gamma^4$

$=\sum\alpha^4\beta^4+2\alpha^2\beta^2\gamma^2[\left(\sum\alpha\right)^2-2\sum\alpha\beta]$

$=\sum\alpha^4\beta^4+2\alpha^2\beta^2\gamma^2[0-2\times(-7)]$

$=\sum\alpha^4\beta^4+2(-7)^2(2\times 7)$

$\Rightarrow \sum\alpha^4\beta^4=(-7)^4+4(-7)^3$

$\Rightarrow \sum\alpha^4\beta^4=(-7)^3(-7+4)=-3(-7)^3$

On putting this value in Eq. (i), we get

$\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^4}+\dfrac{1}{\gamma^4}=\dfrac{-3(-7)^3}{(-7)^4}=\dfrac{-3}{-7}=\dfrac{3}{7}$

358 **(b)**

We have,

$x^2-2\,a_1x+1=0$ ...(i)

$x^2-4a_2x+2=0$ ...(ii)

$x^2-6a_3x+3=0$ ...(iii)

Let $\alpha,\beta;\beta,\gamma$ and γ,α be the pairs of roots of equations (i),(ii) and (iii) respectively. Then,

$\alpha+\beta=2a_1, \alpha\,\beta=1$ ...(iv)

$\beta+\gamma=4a_2, \beta\,\gamma=2$ ...(v)

$\gamma+\alpha=6a_3, \gamma\,\alpha=3$ ...(vi)

Now,

$\alpha\,\beta=1, \beta\,\gamma=2$ and $\gamma\,\alpha=3$

$\Rightarrow (\alpha\,\beta)(\beta\,\gamma)(\gamma\,\alpha)=1\times 2\times 3\Rightarrow \alpha,\beta\,\gamma=\pm\sqrt{6}$

$\therefore \alpha=\pm\sqrt{\dfrac{3}{2}}, \beta=\pm\sqrt{\dfrac{2}{3}}, \gamma=\pm\sqrt{6}$

and,

$\alpha+\beta-2a_1, \beta+\gamma=4a_2, \gamma+\alpha=6a_3$

$\Rightarrow \alpha+\beta+\gamma=a_1+2a_2+3a_3$

$\therefore \alpha=a_1-2a_2+3a_3, \beta=a_1+2a_2-3a_3, \gamma=-a_1+2a_2+3a_3$

Thus, we have the following sets of simultaneous linear equations:

$a_1-2a_2+3a_3=\sqrt{\dfrac{3}{2}}$ $\quad a_1-2a_2+3a_3=-\sqrt{\dfrac{3}{2}}$

$a_1+2a_2-3a_3=\sqrt{\dfrac{2}{3}}$ and, $a_1+2a_2-3a_3=-\sqrt{\dfrac{2}{3}}$

$-a_1+2a_2+3a_3=\sqrt{6}$ $\quad -a_1+2a_2+3a_3=-\sqrt{6}$

Hence, there are two triplets (a_1,a_2,a_3)

359 **(d)**

Given,

$z=\dfrac{11-3i}{1+i}\times\dfrac{1-i}{1-i}=\dfrac{8-14i}{2}=4-7i$

Since, $z=i\alpha$ is real, therefore $4-7i-i\alpha$ is real, if $\alpha=-7$

360 **(b)**

Let the equation (incorrectly written form) be

$x^2+17x+q=0$

Since, roots are $-2,-15$.

$\therefore q=30$

So, correct equation is $x^2+13x+30=0$

$\Rightarrow x^2+10x+3x+30=0$

$\Rightarrow (x+3)(x+10)=0$

$\Rightarrow x=-3,-10$

361 **(b)**

Given, $z^2+z+1=0$

$\Rightarrow z=\omega,\omega^2$

Take $z=\omega$

$\therefore \left(z+\dfrac{1}{z}\right)^2+\left(z^2+\dfrac{1}{z^2}\right)^2+\left(z^3+\dfrac{1}{z^3}\right)^2+\left(z^4+\dfrac{1}{z^4}\right)^2+\left(z^5+\dfrac{1}{z^5}\right)^2+\left(z^6+\dfrac{1}{z^6}\right)^2$

$=\left(\omega+\dfrac{1}{\omega}\right)^2+\left(\omega^2+\dfrac{1}{\omega^2}\right)^2+\left(\omega^3+\dfrac{1}{\omega^3}\right)^2+\left(\omega^4+\dfrac{1}{\omega^4}\right)^2+\left(\omega^5+\dfrac{1}{\omega^5}\right)^5+\left(\omega^6+\dfrac{1}{\omega^6}\right)^2$

$=(\omega+\omega^2)^2+(\omega^2+\omega)^2+(1+1)^2+(\omega+\omega^2)^2+(\omega^2+\omega)^2+(1+1)^2$

$=1+1+4+1+1+4=12$

Similarly, for $z=\omega^2$, we get the same result

362 **(d)**

We have,

$a^2-5a+5<1$ and $2a^2-3a-4<1$

$\Rightarrow a^2-5a+4<0$ and $2a^2-3a-5<0$

$\Rightarrow (a-1)(a-4)<0$ and $(2a-5)(a+1)<0$

$\Rightarrow 1<a<4$ and $-1<a<\frac{5}{2}\Rightarrow 1<a<\frac{5}{2}$

363 **(d)**

Since, $(x-2)$ is a commom factor of the expressions x^2+ax+b and x^2+cx+d

$\Rightarrow\quad 4+2a+b=0\quad$ ...(i)

And $\quad 4+2c+d=0\quad$ ...(ii)

$\Rightarrow\quad 2a+b=2c+d$

$\Rightarrow\quad b-d=2(c-a)$

$\Rightarrow\quad \dfrac{b-d}{c-a}=2$

364 **(d)**

$\log_2 20\log_2 80-\log_2 5\log_2 320$

$=\log_2(2^2\times 5)\log_2(2^4\times 5)-\log_2 5\log_2(2^6\times 5)$

$=(2+\log_2 5)(4+\log_2 5)-\log_2 5(6+\log_2 5)$

$=8+6\log_2 5+(\log_2 5)^2$

$\quad -6\log_2 5-(\log_2 5)^2=8$

365 **(a)**

$\because$ LCM of 3, 4, 6 is 12.

$\therefore\quad \sqrt[3]{9}=9^{1/3}=(9^4)^{1/12}=(6561)^{1/12}$

$\sqrt[4]{11}=(11)^{1/4}=(11^3)^{1/12}=(1331)^{1/12}$

$\sqrt[6]{17}=(17)^{1/6}=(17^2)^{1/2}=(289)^{1/12}$

Hence, $\sqrt[3]{9}$ is the greatest number.

366 **(d)**

We know, $\omega=\frac{-1+\sqrt{3}i}{2}$

$\therefore\quad \left(-\dfrac{1}{2}+\dfrac{\sqrt{3}i}{2}\right)^{1000}=(\omega)^{1000}=\omega\quad [\because \omega^3=1]$

367 **(a)**

Let the roots be $\alpha,\beta,\beta,\gamma$ and γ,α, then

$\alpha\beta=b,\beta\gamma=c$ and $\gamma\alpha=a$

$\Rightarrow\ \alpha\beta\gamma=\sqrt{abc}$

368 **(c)**

of $\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11}+i\cos\frac{2k\pi}{11}\right)=i\ \sum_{k=1}^{10}\left(\cos\frac{2k\pi}{11}-\right.$

$i\sin 2k\pi 11$

$=i\displaystyle\sum_{k=1}^{10}\left(e^{-\frac{2k\pi}{11}i}\right)$

$=i\displaystyle\sum_{k=1}^{10}r^k$ where $r=e^{\frac{-2\pi i}{11}}$

$=i(r+r^2+r^3+\dots r^{10})$

$=\dfrac{i.r(r^{10}-1)}{r-1}$

$=i\left(\dfrac{r^{11}-r}{r-1}\right)$

$=i\left(\frac{1-r}{r-1}\right)\quad\because\ r^{11}=e^{-2i\pi}=1$

$=-i$

369 **(b)**

Let $z=x+iy$ be such that Re $(z)=0$. Then,

$z=iy\ \Rightarrow z^2=-y^2\Rightarrow$ Im $(z^2)=0$

370 **(b)**

$z_1-z_4=z_2-z_3$

$\Rightarrow\ \dfrac{z_1+z_3}{2}=\dfrac{z_2+z_4}{2}$

$\Rightarrow$ Diagonals bisect each other

Given that, $\arg\left(\frac{z_4-z_1}{z_2-z_1}\right)=\frac{\pi}{2}$

$\Rightarrow$ Angle at $z_1=\frac{\pi}{2}$

So, it form a rectangle

371 **(d)**

Given, $\quad x^3+6x+9=0$

$\Rightarrow\quad (x+3)(x^2-3x+3)=0$

$\Rightarrow x=-3\ $ or $\ x^2-3x+3=0$

Now, Discriminant, $D=\sqrt{9-4\times 3}=\sqrt{-3}$ imaginary

Hence, real roots of the given equation is -3

372 **(a)**

$T_r=r[(r+1)-\omega][(r+1)-\omega^2]$

$=r[(r+1)^2-(\omega+\omega^2)(r+1)+\omega^3]$

$=r[(r+1)^2-(-1)(r+1)+1]=r^3+3r^2+3r$

$\therefore\quad \displaystyle\sum_{r=1}^{n-1}T_r=\sum_{r=1}^{n-1}(r^3+3r^2+3r)$

$=\dfrac{1}{4}(n-1)^2(n)^2+3.\dfrac{1}{6}(n-1)n(2n-1)$

$\quad +3.\dfrac{1}{2}(n-1)n$

$=\dfrac{1}{4}\ (n-1)n(n^2+3n+4)$

373 **(b)**

We have,

$|z_1|=|z_2|=|z_3|=1$

$\Rightarrow$ Origin is the circumcentre of the triangle with the circum radius 1

Also, $z_1+z_2+z_3=0$

$\Rightarrow\dfrac{z_1+z_2+z_3}{3}=0$

$\Rightarrow$ Centroid coincides with the origin

Hence, the circumcenter and centroid coincides

Consequently the triangle is equilateral

374 **(b)**

Let $y=\frac{x+2}{2x^2+3x+6}$

$\Rightarrow\quad 2yx^2+(3y-1)x+x+6y-2=0$

But x is real, then

$(3y-1)^2-4(2y)(6y-2)\geq 0\quad [\because D\geq 0]$

$\Rightarrow\quad (13y+1)(3y-1)\leq 0$

$\Rightarrow\ -\dfrac{1}{13}\leq y\leq\dfrac{1}{3}$

375 **(d)**

Since the field of complex numbers is not an ordered field. In other words, the order relation is not defined on the set of all complex numbers

376 **(c)**

Now, $\left(1+\sqrt{3}i\right)^n+\left(1-\sqrt{3}i\right)^n$

$=\left[2\left(\frac{1+\sqrt{3}i}{2}\right)\right]^n+\left[2\left(\frac{1-\sqrt{3}i}{2}\right)\right]^n$

$=(-2\omega^2)^n+(-2\omega)^n$

$=(-2)^n[(\omega^2)^{3r+1}+(\omega)^{3r+1}]$

[$\because n=3r+1$, where r is an integer]

$=(-2)^n(\omega^2+\omega)=-(-2)^n$

377 **(d)**

We have,

$\left(\frac{\sqrt{3}/2+(1/2)\,i}{\sqrt{3}/2\,-(1/2)\,i}\right)^{120}=\left(\frac{1/2-i\sqrt{3}/2}{-1/2-i\sqrt{3}/2}\right)^{120}$

$=\left(\frac{-\omega}{\omega^2}\right)^{120}=\left(\frac{1}{\omega}\right)^{120}=(\omega^2)^{120}=\omega^{240}=1+oi$

Hence, $p=1, q=0.=-48$

378 **(b)**

Let $|x-2|=y$

$\therefore\ y^2+y-6=0$

$\Rightarrow\ y=-3,2$

$\Rightarrow\ |x-2|=-3,\ |x-2|=2$

$\Rightarrow\ \pm(x-2)=2$ [$\because |x-2|$ cannot be negative]

$\therefore\ x=4,0$

379 **(d)**

We have,

$x^2-4x-77<0$ and $x^2>4$

$\Rightarrow (x-11)(x+7)<0$ and $(x-2)(x+2)>0$

$\Rightarrow x\in(-7,-2)\cup(2,11)$

Clearly, the largest negative integer belonging to this set is -3

380 **(b)**

Given, $\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i$

$\Rightarrow\quad (4+2i)x+(9-7i)y-3i-3=10i$

Equating real and imaginary parts, we get

$2x-7y=13$ and $4x+9y=3$, hence $x=3$ and $y=-1$

381 **(d)**

Given, α,β are the roots of equation $x^2+4x+3=0$

$\therefore\ \alpha+\beta=-4$ and $\alpha\beta=3$

Now, $2\alpha+\beta+\alpha+2\beta=3(\alpha+\beta)=-12$

And $(2\alpha+\beta)(\alpha+2\beta)=2\alpha^2+4\alpha\beta+\alpha\beta+2\beta^2$

$=2(\alpha+\beta)^2+\alpha\beta$

$=2(-4)^2+3=35$

Hence, required equation is

x^2-(sum of roots)$x+$(product of roots)$=0$

$\Rightarrow\quad x^2+12x+35=0$

382 **(c)**

Here, $\tan\frac{P}{2}+\tan\frac{Q}{2}=-\frac{b}{a}$

And $\tan\frac{P}{2}\tan\frac{Q}{2}=\frac{c}{a}$ ... (i)

Also, $\frac{P}{2}+\frac{Q}{2}+\frac{R}{2}=\frac{\pi}{2}$ [$\because P+Q+R=\pi$]

$\Rightarrow \frac{P+Q}{2}=\frac{\pi}{4}$ $\left[\because \angle R=\frac{\pi}{2}, \text{given}\right]$

$\tan\left(\frac{P}{2}+\frac{Q}{2}\right)=1\Rightarrow\frac{\tan\frac{P}{2}+\tan\frac{Q}{2}}{1-\tan\frac{P}{2}\tan\frac{Q}{2}}=1$

$\Rightarrow\frac{-\frac{b}{a}}{1-\frac{c}{a}}=1\ \Rightarrow\ -\frac{b}{a}=1-\frac{c}{a}$ [from Eq. (i)]

$\Rightarrow c=a+b$

383 **(a)**

We have,

$x^2+ax+\sin^{-1}(x^2-4x+5)+\cos^{-1}(x^2-4x+5)=0$

$\Rightarrow x^2+ax+\frac{\pi}{2}=0$

This equation will have real roots, if

$a^2-2\pi\geq0$

$\Rightarrow (a-\sqrt{2\pi})(a+\sqrt{2\pi})\geq0$

$\Rightarrow a\in\left(-\infty,-\sqrt{2\pi}\right]\cup\left[\sqrt{2\pi},\infty\right)$

384 **(b)**

We have,

$|z-1|=1\Rightarrow z-1=e^{i\theta}\Rightarrow z=1+e^{i\theta}$

$\therefore\frac{z-2}{z}=\frac{1+e^{i\theta}-2}{1+e^{i\theta}}=\frac{(\cos\theta-1)+i\sin\theta}{(\cos\theta+1)+i\sin\theta}$

$\Rightarrow\frac{z-2}{z}=\tan\frac{\theta}{2}\left\{\frac{-\sin\frac{\theta}{2}+i\cos\frac{\theta}{2}}{\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}}\right\}=i\tan\frac{\theta}{2}$

$\Rightarrow\frac{z-2}{z}$ is purely imaginary

385 **(c)**

Since, α and β are the roots of $x^2-ax+a+b=0$, then

$\alpha+\beta=a$ and $\alpha\beta=a+b$

$\Rightarrow\ \alpha^2+\alpha\beta=a\alpha$

$\Rightarrow\ \alpha^2-a\alpha=-(a+b)$

And $\alpha\beta+\beta^2=a\beta$

$\Rightarrow\ \beta^2-a\beta=-(a+b)$

$\therefore\ \frac{1}{\alpha^2-a\alpha}+\frac{1}{\beta^2-a\beta}+\frac{1}{a+b}$

$=\frac{1}{-(a+b)}+\frac{1}{-(a+b)}+\frac{2}{(a+b)}=0$

386 **(b)**

Given, $x=\log_b a=\frac{\log_e a}{\log_e b}$

$y=\log_c b=\frac{\log_e b}{\log_e c}$

And $z=\log_a c=\frac{\log_e c}{\log_e a}$

$\therefore\quad xyz=\frac{\log_e a}{\log_e b}.\frac{\log_e b}{\log_e c}.\frac{\log_e c}{\log_e a}=1$

387 **(d)**

We have,

$\log_4 2 + \log_4 4 + \log_4 x + \log_4 16 = 6$

$\Rightarrow \log_4(2 \times 4 \times x \times 16) = 6$

$\Rightarrow 128x = 4^6$

$\Rightarrow x = \dfrac{4^3}{2} = 32$

388 **(a)**

We have,

$(1 + \cos 2\alpha) + i \sin 2\alpha$

$= 2\cos^2 \alpha + 2\, i \sin \alpha \cos \alpha$

$= 2\cos \alpha[\cos \alpha + i \sin \alpha]$

$= -2\cos \alpha[-\cos \alpha - i \sin \alpha]$

$= -2\cos \alpha[\cos(\pi + \alpha)$

$+ i \sin(\pi + \alpha)] \quad \left[\because \dfrac{\pi}{2} < \alpha < 3\pi/2\right]$

389 **(b)**

The given equation is $x^2 - 2x\cos\emptyset + 1 = 0$

$\therefore x = \dfrac{2\cos\emptyset \pm \sqrt{4\cos^2\emptyset - 4}}{2} = \cos\emptyset \pm i\sin\emptyset$

Let $\alpha = \cos\emptyset + i\sin\emptyset$, then $\beta = \cos\emptyset - i\sin\emptyset$

$\therefore \quad \alpha^n + \beta^n = (\cos\emptyset + i\sin\emptyset)^n$

$+ (\cos\emptyset - i\sin\emptyset)^n$

$= 2\cos n\emptyset$

And $\alpha^n\beta^n = (\cos n\emptyset + i\sin n\emptyset)(\cos n\emptyset -$

$i\sin n\emptyset)$

$= \cos^2 n\emptyset + \sin^2 n\emptyset = 1$

$\therefore$ Required equation is $x^2 - 2x\cos n\emptyset + 1 = 0$

390 **(c)**

Here, $\sqrt{1 - c^2} = nc - 1$

$\Rightarrow \quad 1 - c^2 = n^2c^2 - 2nc + 1$

$\therefore \ \frac{c}{2n} = \frac{1}{1+n^2} \quad$ ...(i)

or $\quad \frac{c}{2n}(1 + nz)\left(1 + \frac{n}{z}\right) = \frac{1}{1+n^2}\left\{1 + n^2 +\right.$

$nz+1z$

$= \dfrac{1}{1 + n^2}\{1 + n^2 + n(2\cos\theta)\}$

$= \dfrac{(1 + n^2) + 2n\cos\theta}{1 + n^2}$

$= 1 + \left(\frac{2n}{1+n^2}\right)\cos\theta$ [using Eq.(i)]

$= 1 + c\cos\theta$

391 **(a)**

Let $z = x + iy$. Then,

Re $(z^2) = 0$

$\Rightarrow \text{Re}(x^2 - y^2 + 2\,ixy) = 0$

$\Rightarrow x^2 - y^2 = 0 \Rightarrow y = \pm x \quad$ ...(i)

and, $|z| = 2 \Rightarrow x^2 + y^2 = 4 \quad$ ...(ii)

Solving (i) and (ii), we get $x = \pm\sqrt{2}$

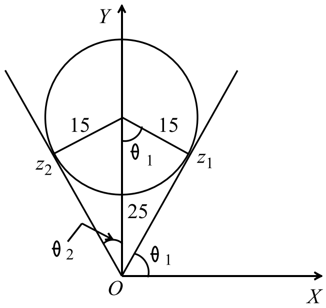
Thus, the solutions are

$(\sqrt{2}, \sqrt{2}), (-\sqrt{2}, \sqrt{2}), (\sqrt{2}, -\sqrt{2}), (-\sqrt{2}, -\sqrt{2})$

392 **(b)**

We have,

Max. amp $(z) = \text{amp}(z_2)$, and Min.amp $(z) = \text{amp}(z_1)$



Now,

amp $(z_1) = \theta_1 = \cos^{-1}\left(\dfrac{15}{25}\right) = \cos^{-1}\left(\dfrac{3}{5}\right)$

and,

amp $(z_2) = \dfrac{\pi}{2} + \theta_2$

$= \dfrac{\pi}{2} + \sin^{-1}\left(\dfrac{15}{25}\right) = \dfrac{\pi}{2} + \sin^{-1}\left(\dfrac{3}{5}\right)$

$\therefore$ |Max. amp (z) − Min. amp (z)|

$= \left|\dfrac{\pi}{2} + \sin^{-1}\dfrac{3}{5} - \cos^{-1}\dfrac{3}{5}\right|$

$= \left|\dfrac{\pi}{2} + \dfrac{\pi}{2} - \cos^{-1}\dfrac{3}{5} - \cos^{-1}\dfrac{3}{5}\right| = \pi - 2\cos^{-1}\dfrac{3}{5}$

393 **(b)**

Here, $\alpha+\beta=p$ and $\alpha\beta=q$

Also, $\alpha_1+\beta_1=q$ and $\alpha_1\beta_1=p$

$\therefore$ Sum of given roots

$$=\left(\frac{1}{\alpha_1\beta}+\frac{1}{\alpha\beta_1}\right)+\left(\frac{1}{\alpha\alpha_1}+\frac{1}{\beta\beta_1}\right)$$

$$=\frac{\alpha\beta_1+\alpha_1\beta+\beta\beta_1+\alpha\alpha_1}{\alpha\beta\alpha_1\beta_1}$$

$$=\frac{(\alpha+\beta)(\alpha_1+\beta_1)}{(\alpha\beta)(\alpha_1\beta_1)}=\frac{pq}{qp}=1$$

and product of given roots

$$=\left(\frac{1}{\alpha_1\beta}+\frac{1}{\alpha\beta_1}\right)\left(\frac{1}{\alpha\alpha_1}+\frac{1}{\beta\beta_1}\right)$$

$$=\frac{(\alpha\beta_1+\alpha_1\beta)(\alpha\alpha_1+\beta\beta_1)}{\alpha^2\beta^2\alpha_1^2\beta_1^2}$$

$$=\frac{\alpha\beta(\alpha_1^2+\beta_1^2)+\alpha_1\beta_1(\alpha^2+\beta^2)}{\alpha^2\beta^2\alpha_1^2\beta_1^2}$$

$$=\frac{\alpha\beta[(\alpha_1+\beta_1)^2-2\alpha_1\beta]+\alpha_1\beta_1[(\alpha+\beta)^2-2\alpha\beta]}{(\alpha\beta)^2(\alpha_1\beta_1)^2}$$

$$=\frac{q(q^2-2p)+p(p^2-2q)}{q^2p^2}$$

$$=\frac{p^3+q^3-4qp}{p^2q^2}$$

Hence, the required equation is given by

$x^2-(\text{sum of roots})x+(\text{product of roots})=0$

$\Rightarrow (p^2q^2)x^2-(p^2q^2)x+p^3+q^3-4qp=0$

394 **(b)**

Given, $iz^4+1=0$

$\Rightarrow \quad z^4=i$

$\Rightarrow \quad z=\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)^{1/4}$

By using De-Moivre's theorem, we get

$z=\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}$

395 **(a)**

Let $\quad z=\sqrt{3}+i$

$\therefore \quad \arg(z)=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=30°$

For making a right angled ΔOPQ, point Q either in IInd quadrant or IVth quadrant

If the point Q is in IInd quadrant, then we take $\theta=120°$

$\therefore\ \tan 120°=-\cot 30°=\frac{\sqrt{3}}{-1}$

$\therefore$ Point Q is $(-1,\sqrt{3})$ and if the point Q is in IVth quadrant then we take

$\theta=-60°$

$\therefore \quad \tan(-60°)=-\tan 60°=-\frac{1}{\sqrt{3}}$

$\therefore$ Point Q is $(1,\sqrt{3})$

396 **(b)**

Let $\ z=x+iy$

Given, $\ |z|-z=1+2i$

$\Rightarrow \quad \sqrt{x^2+y^2}-(x+iy)=1+2i$

$\Rightarrow \quad \sqrt{x^2+y^2}-x=1,\ \ y=-2$

$\Rightarrow \quad \sqrt{x^2+4}-x=1$

$\Rightarrow \quad x^2+4=(1+x)^2$

$\Rightarrow \quad 2x=3 \quad \Rightarrow \quad x=\frac{3}{2}$

$\therefore \quad z=\frac{3}{2}-2i$

397 **(d)**

Given, $\alpha+\beta=4$ and $\alpha^3+\beta^3=44$

$\Rightarrow \quad (\alpha+\beta)^2-3\alpha\beta(\alpha+\beta)=44$

$\Rightarrow 64-44=12\alpha\beta \quad \Rightarrow \quad \alpha\beta=\frac{5}{3}$

$\therefore$ Required equation is

$x^2-(\alpha+\beta)x+\alpha\beta=0 \ \Rightarrow x^2-4x+\frac{5}{3}=0$

$\Rightarrow \quad 3x^2-12x+5=0$

398 **(d)**

Given, $\ |1-i|^x=2^x$

$\Rightarrow \quad \left(\sqrt{1+1}\right)^x=2^x \quad \Rightarrow \quad 2^{x/2}=2^x$

$\Rightarrow \quad \frac{x}{2}=x \quad \Rightarrow \quad x=0$

Therefore, the number of non-zero integral solutions is zero

399 **(b)**

Here, $\alpha+\beta=7$ and $\alpha\beta=1$

$\therefore\ \alpha-7=-\beta, \quad \beta-7=-\alpha$

$\therefore \quad \frac{1}{(\alpha-7)^2}+\frac{1}{(\beta-7)^2}=\frac{1}{\beta^2}+\frac{1}{\alpha^2}=\frac{\alpha^2+\beta^2}{(\alpha\beta)^2}$

$=(\alpha+\beta)^2-2\alpha\beta$

$=49-2=47$

400 **(c)**

Let α be a common root of $x^2+px+q=0$ and $x^2+p'x+q'=0$. Then,

$\alpha^2+p\alpha+q=0$ and $\alpha^2+p'\alpha+q'=0$

$\Rightarrow \alpha=\frac{q-q'}{p-p'}$ [On subtracting]

401 **(d)**

The vertices of the triangle are

$A(0,1), B\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ and $C\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$

$$\therefore \text{Area of } \Delta ABC=\frac{1}{2}\begin{vmatrix}0 & 1 & 1\\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1\\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1\end{vmatrix}$$

$\Rightarrow$ Area of $\Delta\ ABC$

$$=\frac{1}{2}\left[-\left(-\frac{1}{2}+\frac{1}{2}\right)+1\left(\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4}\right)\right]$$

$\Rightarrow$ Area of $\Delta ABC=\frac{\sqrt{3}}{4}$ sq. units

403 **(b)**

Let $ABCD$ be a parallelogram such that affixes of A, B, C, D are z_1, z_2, z_3, z_4 respectively. Then,

$\overrightarrow{AB}=\overrightarrow{DC} \Rightarrow z_2-z_1=z_3-z_4 \Rightarrow z_2+z_4$
$=z_1+z_3$

Conversely, if $z_2+z_4=z_1+z_3$, then

$z_2-z_1=z_3-z_4$

$\Rightarrow \overrightarrow{AB}=\overrightarrow{DC}$

$\Rightarrow ABCD$ is a parallelogram

Thus, $z_2+z_4=z_1+z_3$ is a necessary and sufficient condition for the figure $ABCD$ to be a parallelogram

404 **(b)**

We have, $(x+3)^4+(x+5)^4=16$

$\Rightarrow (y-1)^4+(y+1)^4=16$, where $y=\frac{x+2+x+5}{2}=x+4$

$\Rightarrow (y^2+1-2y)^2+(y^2+1+2y)^2=16$

$\Rightarrow (y^2+1)^2-4y^2=16$

$\Rightarrow (y^2-1)^2=16$

$\Rightarrow y^2-1=\pm 4 \Rightarrow y^2=5 \Rightarrow y=\pm\sqrt{5}$

405 **(a)**

The discriminant Δ of the given equation is given by

$\Delta=4(a+b-2c)^2-4(a-b)^2$

$\Rightarrow \Delta=4(a-c+b-c)^2-4(a-c+c-b)^2$

$\Rightarrow \Delta=4[(a-c)+(b-c)]^2$
$-4[(a-c)-(b-c)]^2$

$\Rightarrow \Delta=16(a-c)(b-c)<0 \quad [\because a<c<b]$

Hence, roots of the given equation are imaginary

406 **(c)**

We have,

$x^3+3x^2+3x+2=0$

$\Rightarrow (x^3-1)+3(x^2+x+1)=0$

$\Rightarrow (x^2+x+1)(x-1+3)=0$

$\Rightarrow (x+2)(x^2+x+1)=0 \Rightarrow x=-2, \omega, \omega^2$

Since $x^3+3x^2+3x+2=0$ and $ax^2+bx+c=0$ have two common roots. Therefore, ω and ω^2 are common roots of the two equations.

Hence, $a=b=c=1$

407 **(a)**

Since, roots of the equation

$(a-b)x^2+(c-a)x+(b-c)=0$ are equal.

$\therefore$ Discriminant, $B^2-4AC=0$

$\Rightarrow (c-a)^2-4(a-b)(b-c)=0$

$\Rightarrow a^2+4b^2+c^2+2ac-4ab-4bc=0$

$\Rightarrow (a+c-2b)^2=0$

$\Rightarrow a+c=2b$

Hence, a, b, c are in AP.

408 **(c)**

We have,

$\frac{1+i}{1-i}=\frac{(1+i)^2}{(1+i)(1-i)}=\frac{2i}{2}=i$

$\therefore \left(\frac{1+i}{1-i}\right)^n=1 \Rightarrow i^n=1 \Rightarrow n$ is a multiple of 4

Hence, the least positive integer n satisfying the above condition is 4

409 **(d)**

It is given that a, b are roots of the equation

$x^2-3x+1=0$

$\therefore a+b=3$ and $ab=1$

It is also given that $a-2$ and $b-2$ are the roots of the equation $x^2+px+q=0$

$\therefore a-2+b-2=-p$ and $(a-2)(b-2)=q$

$\Rightarrow a+b-4=--p$ and $ab-2(a+b)+4=q$

$\Rightarrow 3-4=-p$ and $1-6+4=q \Rightarrow p=1$ and $q=-1$

410 **(c)**

We have,

$\sec\alpha+\tan\alpha=-\frac{b}{a}$ and $\sec\alpha\tan\alpha=\frac{c}{a}$

$\therefore 1=\sec^2\alpha-\tan^2\alpha$

$\Rightarrow 1=(\sec\alpha+\tan\alpha)(\sec\alpha-\tan\alpha)$

$\Rightarrow 1=(\sec\alpha+\tan\alpha)^2\{(\sec\alpha+\tan\alpha)^2$
$-4\sec\alpha\tan\alpha\}$

$\Rightarrow 1=\frac{b^2}{a^2}\left(\frac{b^2-4ac}{a^2}\right) \Rightarrow a^4+4ab^2c=b^4$

411 **(b)**

Given, $|(x-a)+iy|^2+|(x+a)+iy|^2=b^2$

(where $z=x+iy$)

$\Rightarrow \quad (x-a)^2+y^2+(x+a)^2+y^2=b^2$

$\Rightarrow \quad x^2+y^2=\frac{b^2-2a^2}{2}$

Hence, it represents a equation of circle

412 **(b)**

Given, $\log_{99}(\log_2(\log_3 x))=0$

$\Rightarrow \log_2(\log_3 x)=(99)^0=1$

$\Rightarrow \log_3 x=2$

$\Rightarrow x=3^2=9$

413 **(c)**

Let α and β are the roots then

$\alpha+\beta=b, \alpha\beta=c$

Given, $|\alpha-\beta|=1$

$\Rightarrow (\alpha+\beta)^2-4\alpha\beta=1$

$\Rightarrow b^2-4c=1$

414 **(a)**

Let α be the common root for both the equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$, then

$\alpha^2 + a\alpha + b = 0$

And $\alpha^2 + b\alpha + a = 0$

$\Rightarrow \dfrac{\alpha^2}{(a^2 - b^2)} = \dfrac{\alpha}{b-a} = \dfrac{1}{b-a}$

$\therefore \quad \alpha^2 = -(a+b)$ and $\alpha = 1$

Hence, $a + b = -1$

415 **(b)**

Let $f(x) = ax^2 + bx + c$ be a quadratic expression such that $f(x) > 0$ for all $x \in R$. Then,

$f(x) > 0 \Rightarrow a < 0$ and $b^2 - 4ac < 0$

Now,

$g(x) = f(x) + f'(x) + f''(x)$

$\Rightarrow g(x) = ax^2 + x(b + 2a) + (b + 2a + c)$

Let D be the discriminant of $g(x)$. Then,

$D = (b + 2a)^2 - 4a(b + 2a + c)$

$\Rightarrow D = b^2 - 4a^2 - 4ac = (b^2 - 4ac) - 4a^2 < 0[\therefore b^2 - 4ac < 0]$

Thus, we have

$D < 0$ and $a > 0 \Rightarrow g(x) > 0$ for all $x \in R$

416 **(b)**

Given, $\frac{b}{c} + \frac{c}{a} + \frac{a}{b} = 1$

$\Rightarrow \dfrac{\cos\beta + i\sin\beta}{\cos\gamma + i\sin\gamma} + \dfrac{\cos\gamma + i\sin\gamma}{\cos\alpha + i\sin\alpha} + \dfrac{\cos\alpha + i\sin\alpha}{\cos\beta + i\sin\beta} = 1$

$\Rightarrow \cos(\beta - \gamma) + i\sin(\beta - \gamma) + \cos(\gamma - \alpha) + i\sin(\gamma - \alpha) + \cos(\alpha - \beta) + i\sin(\alpha - \beta) = 1$

On equating real part on both sides, we get

$\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = 1$

417 **(a)**

$\dfrac{\cos 30° + i\sin 30°}{\cos 60° - i\sin 60°}$

$= (\cos 30° + i\sin 30°)(\cos 60° + i\sin 60°)$

$= \cos 90° + i\sin 90° = i$

418 **(d)**

Since, $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

Also $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \frac{b^2 - 2ac}{a^2}$

Now, $\frac{\alpha}{a\beta + b} + \frac{\beta}{a\alpha + b}$

$= \dfrac{a(\alpha^2 + \beta^2) + b(\alpha + \beta)}{\alpha\beta a^2 + ab(\alpha + \beta) + b^2}$

$= \dfrac{a\left(\frac{b^2 - 2ac}{a^2}\right) + b\left(-\frac{b}{a}\right)}{\left(\frac{c}{a}\right)a^2 + ab\left(-\frac{b}{a}\right) + b^2} = -\dfrac{2}{a}$

419 **(a)**

Let α and β be the roots of the equation $x^2 - bx + c = 0$.

$\Rightarrow \alpha + \beta = b$ and $\alpha\beta = c$

$\therefore \alpha - \beta = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

$\Rightarrow 1 = \sqrt{b^2 - 4c}$

$\Rightarrow b^2 - 4c - 1 = 0$

420 **(a)**

Since the graph of $y = 16x^2 + 8(a+5)x - 7a - 5$ is strictly above the x-axis

$\therefore y > 0$ for all x

$\Rightarrow 16x^2 + 8(a+5)x - 7a - 5 > 0$ for all x

$\Rightarrow 64(a+5)^2 + 64(7a+5) < 0 \qquad [\because \text{Disc} < 0]$

$\Rightarrow a^2 + 10a + 25 + 7a + 5 < 0$

$\Rightarrow a^2 + 17a + 30 < 0 \Rightarrow -15 < a < -2$

421 **(a)**

Let $f(x) = x^2 - 3x + a$

Clearly, $y = f(x)$ represents a parabola opening upward

It is given that 1 lies between the roots of $f(x) = 0$

Discriminant > 0 and $f(1) < 0$

$\Rightarrow 9 - 4a > 0$ and $1 - 3 + a < 0$

$\Rightarrow a < \dfrac{9}{4}$ and $a < 2 \Rightarrow a < 2 \Rightarrow a \in (-\infty, 2)$

422 **(c)**

Let $z = \left(\frac{1+i}{\sqrt{2}}\right)^{2/3} = \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^{2/3}$

$= \left(\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}\right)^{2/3}$

$= e^{(i\pi/4)^{2/3}} = e^{i\pi/6}$

$= \cos(2n+1)\dfrac{\pi}{6} + i\sin(2n+1)\dfrac{\pi}{6}$

Put $n = 1$,

$z = \cos\left(\dfrac{3\pi}{6}\right) + i\sin\left(\dfrac{3\pi}{6}\right) = 0 + i = i$

423 **(a)**

Since u, v are roots of $x^2 + px + q = 0$. Therefore, the equation whose roots are $1/u$ and $1/v$ is

$\dfrac{1}{x^2} + \dfrac{p}{x} + q = 0$ or, $qx^2 + px + 1 = 0$

424 **(a)**

Since, a, b and c are in GP

$\therefore \quad b^2 = ac$

Given, equation $ax^2 + 2bx + c = 0$ becomes

$ax^2 + 2\sqrt{ac}x + c = 0$

$\Rightarrow \quad \left(ax + \sqrt{c}\right)^2 = 0$

$\Rightarrow \quad x = -\sqrt{\dfrac{c}{a}}$ (respected roots)

Since, this root satisfy the second equation

$dx^2 + 2cx + f = 0$

$\therefore \quad d\dfrac{c}{a} - 2e\sqrt{\dfrac{c}{a}} + f = 0$

$\Rightarrow \frac{d}{a}+\frac{f}{c}=\frac{2e}{c}\sqrt{\frac{c}{a}}=\frac{2e}{b} \quad (\because b=\sqrt{ac})$

$\Rightarrow \frac{d}{a},\frac{e}{b},\frac{f}{c}$ are in GP

425 **(b)**

We know that for given z_1, z_2, the equation

$|z-z_1|^2+|z-z_2|^2=\lambda$ represents a circle, if

$\lambda \geq \frac{1}{2}|z_1-z_2|^2$

Therefore, the equation

$|z-\omega|^2+|z-\omega^2|^2=\lambda$ will represent a circle, if

$\lambda \geq \frac{1}{2}|\omega-\omega^2|^2$

$\Rightarrow \lambda \geq \frac{1}{2}\left|i\sqrt{3}\right|^2 \Rightarrow \lambda \geq \frac{3}{2} \Rightarrow \lambda \in [3/2, \infty)$

426 **(d)**

Put $x^{1/3=}=y$, then

$y^2+y-2=0$

$\Rightarrow y=1$ or $y=-2$

$\Rightarrow x^{1/3}=1$ or $x^{1/3}=-2$

$\therefore x=(1)^3$ or $x=(-2)^3=-8$

Hence, the real roots of the given equation are $1, -8$

427 **(b)**

Let AD be the attitude of ΔABC. Then, D is the mid-point of BC

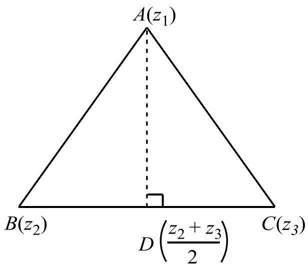
Now,

$\angle ADC=90°$

$\Rightarrow \arg\left(\frac{z_1-z_3}{z_1-\frac{z_2+z_3}{2}}\right)=\pm\frac{\pi}{2}$

$\Rightarrow \arg\left(\frac{z_2-z_3}{2z_1-z_2-z_3}\right)=\pm\frac{\pi}{2}$

$\Rightarrow \frac{z_2-z_3}{2z_1-z_2-z_3}$ is purely imaginary



428 **(d)**

Let $z=x+iy \Rightarrow \bar{z}=x-iy$

Since, $\arg(z)=\tan^{-1}\frac{y}{x}$

and $\arg(\bar{z})=\tan^{-1}\left(\frac{-y}{x}\right)$

$\Rightarrow \arg(z)\neq\arg(\bar{z})$

430 **(d)**

We know that the expression $ax^2+bx+c>0$ for all x, if $a>0$ and $b^2<4ac$.

$\therefore (a^2-1)x^2+2(a-1)x+2$ is positive for all x, if

$a^2-1>0$ and $4(a-1)^2-8(a^2-1)<0$

$\Rightarrow a^2-1>0$ and $-4(a-1)(a+3)<0$

$\Rightarrow a^2-1>0$ and $(a-1)(a+3)>0$

$\Rightarrow a^2>1$ and $a<-3$ or $a>1$

$\Rightarrow a<-3$ or $a>1$

431 **(a)**

Given equation is $x^4-2x^3+x-380=0$

$\Rightarrow (x-5)(x+4)(x^2-1+19)=0$

Now, roots of x^2-x+19 are

$\frac{1\pm\sqrt{1-4\times 19}}{2}=\frac{1\pm 5\sqrt{-3}}{2}$

$\therefore$ Roots are $5, -4, \frac{1+5\sqrt{-3}}{2}, \frac{1-5\sqrt{-3}}{2}$

432 **(a)**

$(2-\omega)(2-\omega^2)(2-\omega^{10})(2-\omega^{11})$

$=(2-\omega)(2-\omega^2)(2-\omega)(2-\omega^2)$

$=[(2-\omega)(2-\omega^2)]^2$

$=[4-2(\omega+\omega^2)+1]^2=(4+2+1)^2=49$

433 **(a)**

Let $z=x+iy$

$\therefore |z-1|=|z-2|=|z-i|$

$\Rightarrow |(x-1)\pm iy|=|(x-2)+iy|$

$=|x+i(y-1)|$ [put $z=x+iy$]

$\Rightarrow x^2-2x+1+y^2=x^2+4-4x+y^2$

$=x^2+y^2+1-2y$

Taking Ist and IInd terms

$-2x+1=4-4x \Rightarrow 2x=3$ ...(i)

Taking IInd and IIIrd terms

$4-4x=1-2y \Rightarrow 4x-2y=3$ ...(ii)

Taking Ist and IIIrd terms

$-2x+1=1-2y \Rightarrow x=y$ ...(iii)

From Eq. (i), $x=\frac{3}{2}$

From Eqs. (i) and (iii), $y=\frac{3}{2}$

On putting the value of x and y in Eq. (ii), we get

$4\left(\frac{3}{2}\right)-2\left(\frac{3}{2}\right)=3 \Rightarrow 3=3$

$\therefore$ One solution exists.

434 **(d)**

Let $y=x^2$. Then, $x=\sqrt{y}$

$\therefore x^3+8=0$

$\Rightarrow y^{3/2}+8=0 \Rightarrow y^3=64 \Rightarrow y^3-64=0$

Thus, the equation having roots α^2, β^2 and γ^2 is $x^3-64=0$

435 **(a)**

Here, $\sin 18°+\cos^2 36°$

$=\left(\frac{\sqrt{5}-1}{4}\right)^4+\left(\frac{\sqrt{5}+1}{4}\right)^2$

$=\frac{5+1-2\sqrt{5}}{16}+\frac{5+1+2\sqrt{5}}{16}$

$= \frac{12}{16} = \frac{3}{4}$

And $\sin^2 18°.\cos^2 36° = \left(\frac{\sqrt{5}-1}{4}\right)^2 \left(\frac{\sqrt{5}+1}{4}\right)^2$

$= \left(\frac{5-1}{4 \times 4}\right)^2 = \frac{1}{16}$

Required equation is

x^2 −(sum of roots)x +(products of roots)=0

$\Rightarrow \quad x^2 - \frac{3}{4}x + \frac{1}{16} = 0$

$\Rightarrow \quad 16x^2 - 12x + 1 = 0$

436 **(b)**

We have,

$z = (-i\,\omega)^5 + (i\,\omega^2)^5$

$\Rightarrow z = -i\,\omega^5 + i\,\omega^{10}$

$\Rightarrow z = -i\,\omega^2 + i\,\omega = -i(\omega^2 - \omega) = i^2\sqrt{3} = -\sqrt{3}$

437 **(b)**

Given,$lx^2 + mx + n = 0$ ...(i)

Now,

$D = m^2 - 4ln = 0$ ($\because m^2 = 4\,ln$ given)

It means roots of given equation are equal

$\therefore \left(x - \frac{9}{2}\right)^2 = 0$

$\Rightarrow 4x^2 + 81 - 36x = 0$ ...(ii)

On comparing Eqs. (i) and (ii), we get

$l = 4, m = -36, n = 81$

$\therefore l + n = 4 + 81 = 85$

438 **(a)**

Given,

$$\frac{x^3}{(2x-1)(x+2)(x-3)} = A + \frac{B}{(2x-1)} + \frac{C}{(x+2)} + \frac{D}{(x-3)}$$

Let $f(x) = \frac{x^3}{(2x-1)(x+2)(x-3)}$

$= \frac{x^3}{2x^3 - 3x^2 - 11x + 6}$

Here, the power of x are same in Nr and Dr

∴ First we divide the numerator by denominator

$2x^3 - 3x^2 - 11x + 6 \frac{1/2}{x^3}$

$x^3 - \frac{3}{2}x^2 - \frac{11}{2}x + 3$

$- \quad + \quad + \quad -$

$\frac{3}{2}x^2 + \frac{11}{2}x - 3$

$$\therefore \frac{x^3}{(2x-1)(x+2)(x-3)} = \frac{1}{2} + \frac{\frac{3}{2}x^2 + \frac{11}{2}x - 3}{(2x-1)(x+2)(x-3)}$$

$\Rightarrow \quad A = \frac{1}{2}$

439 **(c)**

Given, α, β and γ are the roots of $x^3 + 4x + 1 = 0$

$\therefore \quad \alpha + \beta + \gamma = 0, \alpha\beta + \beta\gamma + \gamma\alpha = 4, \alpha\beta\gamma = -1$

Now, $\frac{\alpha^2}{\beta+\gamma} + \frac{\beta^2}{\gamma+\alpha} + \frac{\gamma^2}{\alpha+\beta} = \frac{\alpha^2}{-\alpha} + \frac{\beta^2}{-\beta} + \frac{\gamma^2}{-\gamma}$

$= -(\alpha + \beta + \gamma) = 0$

$$\frac{\alpha^2\beta^2}{(\beta+\gamma)(\gamma+\alpha)} + \frac{\beta^2\gamma^2}{(\gamma+\alpha)(\alpha+\beta)} + \frac{\gamma^2\alpha^2}{(\beta+\gamma)(\alpha+\beta)}$$

$= \alpha\beta + \beta\gamma + \gamma\alpha = 4$

And $\frac{\alpha^2\beta^2\gamma^2}{(\beta+\gamma)(\gamma+\alpha)(\alpha+\beta)} = -\alpha\beta\gamma = 1$ ($\because \alpha + \beta + \gamma = 0$)

∴ Required equation is

$x^3 + 4x - 1 = 0$

440 **(d)**

The given equation is

$pqx^2 - (p+q)^2x + (p+q)^2 = 0$

$$\therefore \quad x = \frac{(p+q)^2 \pm \sqrt{(p+q)^4 - 4pq(p+q)^2}}{2pq}$$

$$\Rightarrow \quad x = \frac{(p+q)^2 \pm (p^2 - q^2)}{2pq}$$

$$\Rightarrow \quad x = \frac{p+q}{q}, \frac{p+q}{p}$$

442 **(b)**

$x = a + b, y = a\alpha + b\beta$ and $z = a\beta + b\alpha$

Now, $xyz = (a+b)(a\omega + b\omega^2)(a\omega^2 + b\omega)$,

Where $\alpha = \omega$ and $\beta = \omega^2$

$\therefore\ xyz = (a+b)(a^2 + ab\omega^2 + ab\omega + b^2)$,

$= (a+b)(a^2 - ab + b^2) = a^3 + b^3$

443 **(d)**

We have,

$$\begin{vmatrix} 1 & \cos(\beta-\alpha) & \cos\alpha \\ \cos(\alpha-\beta) & 1 & \cos\beta \\ \cos\alpha & \cos\beta & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \cos\alpha & \sin\alpha & 0 \\ \cos\beta & \sin\beta & 0 \\ 1 & 0 & 0 \end{vmatrix} \begin{vmatrix} \cos\alpha & \sin\alpha & 0 \\ \cos\beta & \sin\beta & 0 \\ 1 & 0 & 0 \end{vmatrix}$$

$= (0)(0) = 0$ for all values of α, β

444 **(a)**

$$\frac{1+i}{1-i} = \frac{(1+i)^2}{1-i^2} = \frac{2i}{2} = i$$

So, $\left(\frac{1+i}{1-i}\right)^n = (i)^n \Rightarrow n = 2$

445 **(c)**

We have the following cases:

CASE I *When* $x \in [0,1)$

In this cases, we have $[x] = 0$

$\therefore x^2 - 3x + [x] = 0$

$\Rightarrow x^2 - 3x = 0 \Rightarrow x = 0,3 \Rightarrow x = 0$

CASE II *When* $x \in [1,2)$

In this case, we have $[x] = 1$

$$\therefore x^3 - 3x + [x] = 0 \Rightarrow x^2 - 3x + 1 = 0 \Rightarrow x = \frac{3 \pm \sqrt{5}}{2}$$

Clearly, these values of x do not belong to [1, 2].

So, the equation has no solution in [1, 2)

CASE III *When* $x \in [2,3)$

$\therefore x^2 - 3x + [x] = 0$

$\Rightarrow x^2 - 3x + 2 = 0 \Rightarrow x = 1,2 \Rightarrow x = 2$

Hence, the given equation has two solutions only

446 **(c)**

Roots of the equation $2x^2 + 3x + 5 = 0$ are

$x = \frac{-3 \pm \sqrt{9-40}}{6}$ (imaginary roots)

Hence, both roots coincide, so on comparing

$$\frac{a}{2} = \frac{b}{3} = \frac{c}{5} = k$$

$\Rightarrow a = 2k, b = 3k, c = 5k$

$\Rightarrow a + b + c = 10k$

So, maximum value does not exist.

447 **(a)**

We have, $x = \sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots \infty}}}$

$\Rightarrow x = \sqrt{1+x}$

$\Rightarrow x^2 = 1 + x \Rightarrow x^2 - x - 1 = 0$

$$\Rightarrow x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

As $x > 0$, we take only $x = \frac{1+\sqrt{5}}{2}$.

448 **(c)**

The equation $|z - a^2| + |z - 2a| = 3$ represents an ellipse having foci at $S(a^2, 0)$ and $S'(2a, 0)$ and major axis 3. If e is the eccentricity of this ellipse, then

$$e = \frac{SS'}{\text{Major axis}} \Rightarrow e = \frac{|a^2 - 2a|}{3}$$

But, $0 < e < 1$

$$\therefore 0 < \frac{|a^2 - 2a|}{3} < 1$$

$\Rightarrow |a^2 - 2a| < 3$

$\Rightarrow -3 < a^2 - 2a < 3$

$\Rightarrow a^2 - 2a + 3 > 0$ and $a^2 - 2a - 3 < 0$

$\Rightarrow a \in R$ and $a \in (-1,3) \Rightarrow a \in (-1,3)$

But, $a > 0$. Therefore, $a \in (0,3)$

449 **(d)**

Let $y = \frac{(x-a)(x-b)}{(x-c)}$

$\Rightarrow y(x - c) = x^2 - (a+b)x + ab$

$\Rightarrow x^2 - (a + b + y)x + ab + cy = 0$

Now, discriminant

$D = (a + b + y)^2 - 4(ab + cy)$

$= y^2 + 2y(a + b - 2c) + (a - b)^2$

Since, x is real and y assumes all real values, we must have $D \geq 0$ for all real values of y.

$\Rightarrow 4(a + b - 2c)^2 - 4(a - b)^2 \leq 0$

$\Rightarrow 4(a + b - 2c + a - b)(a + b - 2c - a + b) \leq 0$

$\Rightarrow 16(a - c)(b - c) \leq 0$

$\Rightarrow (c - a)(c - b) \leq 0$

450 **(b)**

rth term of the given series

$= r[(r + 1) - \omega][(r + 1) - \omega^2]$

$= r[(r + 1)^2 - (\omega + \omega^2)(r + 1) + \omega^3]$

$= r[(r + 1)^2 - (-1)(r + 1) + 1]$

$= r(r^2 + 3r + 3) = r^3 + 3r^2 + 3r$

Thus, sum of the give series

$$= \sum_{r=1}^{(n-1)} (r^3 + 3r^2 + 3r)$$

$$= \frac{1}{4}(n-1)^2 n^2 + 3.\frac{1}{6}(n-1)(n)(2n-1) + 3.\frac{1}{2}(n-1)n$$

$$= \frac{1}{4}(n-1)n\,(n^2 + 3n + 4)$$

451 **(c)**

The cube roots or unity are $1, \omega, \omega^2$. Let P, Q and R represent $1, \omega$ and ω^2 respectively. Clearly,

$PQ = |1-\omega| = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{3}$

$QR = |\omega - \omega^2| = \sqrt{3}$, and $RP = |1-\omega^2| = \sqrt{3}$

$\therefore PQ = QR = RP$

Thus, points representing $1, \omega, \omega^2$ form an equilateral triangle.

ALITER Let $z_i = 1, z_2 = \omega$ and $z_3 = \omega^2$. Then,

$z_1^2 + z_2^2 + z_2^3 = z_1 z_2 + z_2 z_3 + z_3 z_1$

Hence, points representing $1, \omega, \omega^2$ form an equilateral triangle

452 **(a)**

The equation $2x^2 + 3x + 4 = 0$ has complex roots which always occur in pairs. So, the two equations have both roots common

$\therefore \frac{a}{2} = \frac{b}{3} = \frac{c}{4} \Rightarrow a : b : c = 2 : 3 : 4$

454 **(d)**

We have,

$x^2 - 3|x| + 2 = 0$

$\Rightarrow (|x|-2)(|x|-1) = 0 \Rightarrow |x| = 1,2 \Rightarrow x = \pm 1, \pm 2$

So, the given equation has four real roots

455 **(c)**

We have,

$0 < |3x+1| < \frac{1}{3}$

$\Rightarrow |3x+1| \neq 0$ and $|3x+1| < \frac{1}{3}$

$\Rightarrow x \neq -\frac{1}{3}$ and $-\frac{1}{3} < 3x+1 < \frac{1}{3}$

$\Rightarrow -\frac{1}{3} < 3x+1 < \frac{1}{3}$ and $x \neq -\frac{1}{3}$

$\Rightarrow -\frac{4}{3} < 3x < -\frac{2}{3}$ and $x \neq -\frac{1}{3}$

$\Rightarrow -\frac{4}{9} < x < -\frac{2}{9}$ and $x \neq -\frac{1}{3}$

$\Rightarrow x \in \left(-\frac{4}{9}, -\frac{2}{9}\right)$ and $x \neq -\frac{1}{3} \Rightarrow x \in \left(-\frac{4}{9}, -\frac{2}{9}\right) - \left\{-\frac{1}{3}\right\}$

456 **(b)**

$x^{\log_x (1-x)^2} = 9$

$\Rightarrow \; 9 = (1-x)^2$

$\Rightarrow \; x^2 - 2x - 8 = 0$

$\Rightarrow (x+2)(x-4) = 0$

$\Rightarrow x = 4 \qquad [\because x \neq -2]$

457 **(d)**

$x^2 + ax + 1$ must divide $ax^3 + bx + c$.

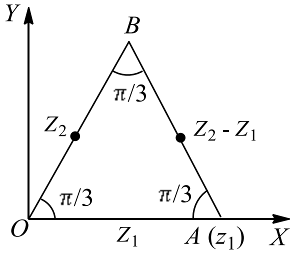
Now, $\frac{ax^3+bx+c}{x^2+ax+1} = \frac{a(x-a)+(b-a+a^3)x+c+a^2}{x^2+ax+1}$

Reminder must be zero

$\Rightarrow b - a + a^3 = 0, a^2 + c = 0$

458 **(a)**

Let OA, OB be the sides of an equilateral ΔOAB and let OA, OB represent the complex numbers z_1, z_2 respectively



From the equilateral ΔOAB,

$\overrightarrow{\mathbf{AB}} = z_2 - z_1$

$\therefore \arg\left(\frac{z_2 - z_1}{z_2}\right) = \arg(z_2 - z_1) - \arg z_2 = \frac{\pi}{3}$

and $\arg\left(\frac{z_2}{z_1}\right) = \arg(z_2) - \arg(z_1) = \frac{\pi}{3}$

Also, $\left|\frac{z_2 - z_1}{z_2}\right| = 1 = \left|\frac{z_2}{z_1}\right|$, since triangle is equilateral. Thus, the complex numbers $\frac{z_2 - z_1}{z_2}$ and $\frac{z_2}{z_1}$ have same modulus and same argument, which implies that the numbers are equal, that is

$\frac{z_2 - z_1}{z_2} = \frac{z_2}{z_1} \Rightarrow z_1 z_2 - z_1^2 = z_2^2$

$\Rightarrow z_1^2 + z_2^2 = z_1 z_2$

459 **(c)**

Given equations are comparing with $ax^2 + bx + c = 0$

And $a'x^2 + b'x + c' = 0$ respectively, we get

$a = 1, \quad b = 2a, \quad c = a^2 - 1$

And $a' = 1, \quad b' = 2b, \quad c' = b^2 - 1$

Condition for common roots is

$(ac' - a'c)^2 = (bc' - b'c)(ab' - a'b)$

$\Rightarrow \; [1(b^2-1) - 1(a^2-1)]^2 = [2a(b^2-1) - 2b(a^2-1)][1(2b) - 1(2a)]$

$\Rightarrow \; (b^2 - a^2)^2 = 4(b-a)(b-a)(ab+1)$

$\Rightarrow (b+a)^2 = 4(ab) + 4$

$\Rightarrow \; (b-a)^2 = 4$

$\Rightarrow \; a - b = 2$

460 **(c)**

Multiplying $x^2 - ax + b = 0$ by x^{n-1}

$x^{n+1} - ax^n + bx^{n-1} = 0 \quad$ ...(i)

α, β are the roots of $x^2 - ax + b = 0$, therefore they will satisfy Eq. (i)

Also, $\alpha^{n+1} - a\alpha^n + b\alpha^{n-1} = 0 \quad$ ...(ii)

and $\beta^{n+1} - \alpha\beta^n + b\beta^{n-1} = 0$

Adding Eqs. (ii) and (iii), we get

$(\alpha^{n+1} + \beta^{n+1}) - a(\alpha^n + \beta^n) + b(\alpha^{n-1} + \beta^{n-1}) = 0$

or $V_{n+1} - aV_n + bV_{n-1} = 0$

or $V_{n+1} = aV_n - bV_{n-1} = 0$ (Given $\alpha^n + \beta^n = V_n$)

461 **(d)**

Let $z = x + 0i$ be a real root of the given equation. Then,

$x^2 + \alpha x + \beta = 0$

$\Rightarrow x^2 + (a + ib)x + (c + id) = 0$, where

$\alpha = a + ib, \beta = c + id$

$\Rightarrow (x^2 + ax + c) + i(bx + d) = 0$

$\Rightarrow x^2 + ax + c = 0$ and $bx + d = 0$

$\Rightarrow x^2 + ax + c = 0$ and $x = -\frac{d}{b}$

$\Rightarrow \frac{d^2}{b^2} - \frac{ad}{b} + c = 0$

$\Rightarrow d^2 - abd + b^2c = 0$

$$\Rightarrow \left(\frac{\beta - \bar{\beta}}{2i}\right)^2 - \left(\frac{\alpha + \bar{\alpha}}{2}\right)\left(\frac{\alpha - \bar{\alpha}}{2i}\right)\left(\frac{\beta - \bar{\beta}}{2i}\right) + \left(\frac{\alpha - \bar{\alpha}}{2i}\right)^2\left(\frac{\beta + \bar{\beta}}{2}\right) = 0$$

$$\Rightarrow -2(\beta - \bar{\beta})^2 + (\alpha + \bar{\alpha})(\alpha - \bar{\alpha})(\beta - \bar{\beta}) - (\alpha - \bar{\alpha})^2(\beta + \bar{\beta}) = 0$$

$$\Rightarrow 2(\beta - \bar{\beta})^2 = (\alpha - \bar{\alpha})\{(\alpha + \bar{\alpha})(\beta - \bar{\beta}) - (\alpha - \bar{\alpha})(\beta + \bar{\beta})\} = 0$$

$\Rightarrow 2(\beta - \bar{\beta})^2 = (\alpha - \bar{\alpha})(-2\,\alpha\,\bar{\beta} + 2\,\bar{\alpha}\beta)$

$\Rightarrow (\beta - \bar{\beta})^2 = (\bar{\alpha} - \alpha)(\alpha\bar{\beta} - \bar{\alpha}\beta)$

462 **(a)**

Given, $2^x . 3^{x+4} = 7^x$

Taking log on both sides, we get

$x \log_e 2 + (x + 4) \log_e 3 = x \log_e 7$

$\Rightarrow x(\log_e 2 + \log_e 3 - \log_e 7) = -4 \log_e 3$

$\Rightarrow x = \frac{4 \log_e 3}{\log_e 7 - \log_e 6}$

463 **(d)**

Here, $\alpha + \beta + \gamma = 0, \alpha\beta + \beta\gamma + \gamma\alpha = -8, \alpha\beta\gamma = -8$ ...(i)

$\therefore (\alpha + \beta + \gamma)^2 = 0$

$\Rightarrow \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \gamma\alpha) = 0$

$\Rightarrow \sum \alpha^2 = -2(-8) = 16$ [from Eq. (i)]

And $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} = \frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}$

$\Rightarrow \frac{1}{\sum \alpha\beta} = \frac{0}{-8} = 0$ [from Eq. (i)]

464 **(a)**

Let $y = \frac{x^2-2x+4}{x^2+2x+4}$

Then, $x^2(y - 1) + 2x(y + 1) + 4(y - 1) = 0$

Since, x is real, therefore

Discriminant, $4(y + 1)^2 - 16(y - 1)^2 \geq 0$

$\Rightarrow (y + 1)^2 - [2(y - 1)]^2 \geq 0$

$\Rightarrow (3 - y)(3y - 1) \geq 0 \Rightarrow \frac{1}{3} \leq y \leq 3$

465 **(b)**

Let $x = 2 + \frac{1}{2+\frac{1}{2+...\infty}}$

$\Rightarrow x = 2 + \frac{1}{x}$

$\Rightarrow x^2 - 2x - 1 = 0$

$\Rightarrow x = \frac{2 \pm \sqrt{4 + 4}}{2}$

$\Rightarrow x = 1 \pm \sqrt{2}$

But the value of the given expression cannot be negative or less than 2, therefore $1 + \sqrt{2}$ is required answer.

466 **(b)**

We have, $|z_1| = 1$

$\therefore \left|\frac{z_1 - z_2}{1 - z_1z_2}\right| = \frac{|z_1 - z_2|}{|z_1\bar{z_1} - z_1z_2|}$ [$\because$ 1 $= z_1\bar{z_1}$]

$= \frac{|z_1 - z_2|}{|z_1||\bar{z_1} - z_2|} = \frac{1}{|z_1|} = 1$

467 **(b)**

Since α, β are roots of $x^2 + px + q = 0$

$\therefore \alpha + \beta = -p$ and $\alpha\,\beta = q$ ...(i)

Now,

α, β are roots of $x^{2n} + p^nx^n + q^n = 0$

$\Rightarrow \alpha^{2n} + p^n\alpha^n + q^n = 0$ and $\beta^{2\,n} + p^n\beta^n + q^n = 0$

$\Rightarrow \alpha^{2\,n} - \beta^{2\,n} + p^n\alpha^n - p^n\beta^n = 0$

$\Rightarrow (\alpha^n + \beta^n)(\alpha^n - \beta^n) + P^n(\alpha^n - \beta^n) = 0$

$\Rightarrow (\alpha^n - \beta^n)(\alpha^n + \beta^n + p^n) = 0$

$\Rightarrow \alpha^n + \beta^n + p^n = 0$

$\Rightarrow \alpha^n + \beta^n = -p^n$ ...(ii)

Since $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ are roots of $x^n + 1 + (x + 1)^n = 0$

$\therefore \alpha^n + \beta^n + (\alpha + \beta)^n = 0$

$\Rightarrow \alpha^n + \beta^n = -(\alpha + \beta)^n$

$\Rightarrow -p^n = -(-p)^n$ [$\because \alpha + \beta = -p$ and $\alpha^n + \beta^n = -p^n$]

$\Rightarrow p^n = (-p^n) \Rightarrow n$ is even

468 **(a)**

Given, $x = \left(\frac{1+i}{2}\right)$

$\Rightarrow 2x - 1 = i \Rightarrow 4x^2 + 1 - 4x = -1$

$\Rightarrow 2x^2 - 2x + 1 = 0$

Since, $2x^4 - 2x^2 + x + 3$

$= (2x^2 - 2x + 1)(x^2 + x) + (3 - x^2)$

$= 0 + 3 - \left(\frac{1 + i}{2}\right)^2$

$= 3 - \left(\frac{i}{2}\right)$

469 **(c)**

Since, α, α^2 be the roots of $x^2 + x + 1 = 0$.

$\therefore \alpha + \alpha^2 = -1$ ...(i)

and $\alpha^3 = 1$ ...(ii)
Now, $\alpha^{31} + \alpha^{62} = \alpha^{31}(1 + \alpha^{31})$
$\Rightarrow \alpha^{31} + \alpha^{62} = \alpha^{30}(1 + \alpha^{30} \cdot \alpha)$
$\Rightarrow \alpha^{31} + \alpha^{62} = (\alpha^3)^{10} \cdot \alpha\{1 + (\alpha^3)^{10} \cdot \alpha\}$
$\Rightarrow \alpha^{31} + \alpha^{62} = \alpha(1 + \alpha)$ [from Eq. (ii)]
$\Rightarrow \alpha^{31} + \alpha^{62} = -1$ [from Eq. (i)]
Again $\alpha^{31} \cdot \alpha^{62} = \alpha^{93}$
$\alpha^{31} \cdot \alpha^{62} = [\alpha^3]^{31} = 1$
$\therefore$ Required equation is
$x^2 - (\alpha^{31} + \alpha^{62})x + \alpha^{31} \cdot \alpha^{62} = 0$
$\Rightarrow x^2 + x + 1 = 0$

470 **(d)**
Let the roots be α, α^2. Then,
$\alpha + \alpha^2 = 6/8 \Rightarrow \alpha = 1/2, -3/2$
Now,
Product the roots $= -\dfrac{a+3}{8}$
$\Rightarrow \alpha^3 = -\dfrac{a+3}{8}$
$\Rightarrow \dfrac{1}{8} = -\dfrac{a+3}{8}$ or, $-\dfrac{27}{8} = -\dfrac{a+3}{8}$
$\Rightarrow a = -4$ or, $a = 24$

471 **(c)**
We have,
$x_n = \cos\left(\frac{\pi}{3^n}\right) + i \sin\left(\frac{\pi}{3^n}\right)$
$\therefore x_1\, x_2\, x_3 \dots x_\infty$
$= \cos\left(\frac{\pi}{3} + \frac{\pi}{3^2} + \cdots\right) + i \sin\left(\frac{\pi}{2} + \frac{\pi}{3^2} + \cdots\right)$
$= \cos\left(\frac{\pi/3}{1-1/3}\right) + i \sin\left(\frac{\pi/3}{1-1/3}\right)$
$= \cos\frac{\pi}{2} + i \sin\frac{\pi}{2} = 0 + i = i$

472 **(d)**
Given, $z = i$
Let $z_1 = 1 + i(1 \pm \sqrt{3})$ and $z_2 = 2 + i$
Now, $|z_2 - z| = |1 + i - i| = 2$
As we know that the distance from the centre to every vertices is equal
Now, $|z_1 - z| = |1 + i(1 \pm \sqrt{3}) - i|$
$= |1 \pm i\sqrt{3}|$
$= \sqrt{1^2 + (\sqrt{3})^2} = 2$

473 **(a)**
Let $z = x + iy$
$\therefore$ Re $\left(\frac{x - iy + 2}{x - iy - 1}\right) = 4$ (given)
$\therefore$ Re $\left[\frac{(x+2) - iy}{(x-1) - iy} \times \frac{(x-1) + iy}{(x-1) + iy}\right] = 4$
$\Rightarrow (x+2)(x-1) + y^2 = 4[(x-1)^2 + y^2]$
$\Rightarrow x^2 + y^2 - 3x + 2 = 0$, which represents a circle

474 **(c)**
Since the equation $a\,x^2 + b\,x + c = 0$ has no real roots. Therefore, the curve $y = ax^2 + bc + c$ does not intersect with x-axis. Consequently,
$\phi(x) = a\,x^2 + b\,x + c$ has same sign for all values of x. It is given that
$a + b + c < 0$
$\Rightarrow \phi(1) = a + b + c < 0$
$\Rightarrow \phi(x) < 0$ for all $x \Rightarrow \phi(0) < 0 \Rightarrow c < 0$

475 **(a)**
$\because\ 2\sin^2\frac{\pi}{8} = 1 - \cos\frac{\pi}{4} = 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}}$ (irrational root)
So, other root is $\frac{\sqrt{2}+1}{\sqrt{2}}$.
Sum of roots $= -a = 1 - \frac{1}{\sqrt{2}} + 1 + \frac{1}{\sqrt{2}} = 2 \Rightarrow a = -2$
Product of roots $= 1 - \frac{1}{2} = \frac{1}{2} = b$
So, $a - b = -2 - \frac{1}{2} = -\frac{5}{2}$

476 **(a)**
Given equation is
$\alpha^2 + \alpha + 1 = 0$
$\therefore\ \alpha = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$
Let $\alpha = \omega, \omega^2$
1. If $\alpha = \omega$, then

$\alpha^{31} = (\omega)^{31} = \omega = \alpha$

2. If $\alpha = \omega^2$, then

$\alpha^{31} = (\omega^2)^{31} = \omega^{62} = \omega^2 = \alpha$

Hence, α^{31} is equal to α

477 **(d)**
$z_1.z_2.z_3, \dots \infty$
$= \cos\left(\frac{\pi}{2} + \frac{\pi}{2^2} + \frac{\pi}{2^3} + \dots\right) + i \sin\left(\frac{\pi}{2} + \frac{\pi}{2^2} + \frac{\pi}{2^3} + \dots\right)$
$= \cos\left(\frac{\frac{\pi}{2}}{1 - \frac{1}{2}}\right) + i \sin\left(\frac{\frac{\pi}{2}}{1 - \frac{1}{2}}\right)$
$= \cos\pi + i \sin\pi = -1$

478 **(c)**
Given, $\alpha_1 = |-i| = 1$
$\alpha_2 = \left|\frac{1}{3}(1 + i)\right| = \frac{1}{3}\sqrt{2}$
and $\alpha_3 = |-1 + i| = \sqrt{2}$
$\therefore$ The increasing order is $\alpha_2, \alpha_1, \alpha_3$

479 **(b)**

We have,

$\frac{2x+4}{x-1} \geq 5$

$\Rightarrow \frac{2x+4-5x+5}{x-1} \geq 0 \Rightarrow \frac{x-3}{x-1} \leq 0 \Rightarrow x \in (1,3]$

480 **(b)**

We have,

$x^2 - x(a+b) + ab = ax + bx - 2ab$

$\Rightarrow x^2 - 2x(a+b) + 3ab = 0$

Since the roots are equal in magnitude but opposite in sign

$\therefore$ Sum of the roots $= 0 \Rightarrow 2(a+b) = 0 \Rightarrow a + b = 0$

481 **(a)**

Given equation is

$x^3 - 3x - 2 = 0 \;\Rightarrow\; (x+1)(x^2 - x - 2) = 0$

$\Rightarrow \;(x+1)(x+1)(x-2) = 0$

$\Rightarrow \; x = -1, -1, 2$

482 **(c)**

Let p, q be the roots of the given equation. Then,

$p^2 + q^2 = (p+q)^2 - 2pq$

$\Rightarrow p^2 + q^2 = (\sin\alpha - 2)^2 + 2(1 + \sin\alpha)$

$\Rightarrow p^2 + q^2 = \sin^2\alpha - 2\sin\alpha + 6 = (\sin\alpha - 1)^2 + 5$

Clearly, $p^2 + q^2$ is last when

$\sin\alpha - 1 = 0 \Rightarrow \sin\alpha = 1 \Rightarrow \alpha = \pi/2$

483 **(c)**

$\left(\frac{-1+\sqrt{-3}}{2}\right)^{100} + \left(\frac{-1-\sqrt{-3}}{2}\right)^{100}$

$= \omega^{100} + \omega^{200} = \omega + \omega^2 = -1$

484 **(b)**

Since roots of the given equation are of opposite signs. Therefore,

Product of roots < 0

$\Rightarrow \frac{p(p-1)}{3} < 0 \Rightarrow p(p-1) < 0 \Rightarrow p \in (0,1)$

485 **(c)**

Given, $(x-a)(x-a-1) + (x-a-1)(x-a-$ *2+x−ax−a−2=0*

Let $x - a = t$, then

$t(t-1) + (t-1)(t-2) + t(t-2) = 0$

$\Rightarrow t^2 - t + t^2 - 3t + 2 + t^2 - 2t = 0$

$\Rightarrow \; 3t^2 - 6t + 2 = 0$

$\Rightarrow t = \frac{6 \pm \sqrt{36-24}}{2(3)} = \frac{6 \pm 2\sqrt{3}}{2(3)}$

$\Rightarrow x - a = \frac{3 \pm \sqrt{3}}{3}$

$\Rightarrow \; x = a + \frac{3 \pm \sqrt{3}}{3}$

Hence, x is real and distinct

486 **(a)**

We have,

$z + z^{-1} = 1 \Rightarrow z^2 - z + 1 = 0 \Rightarrow z = -\omega, -\omega^2$

For $z = -\omega$, we have

$z^n + z^{-n} = (-\omega)^n + (-\omega)^{-n}$

$\Rightarrow z^n + z^{-n} = (-1)^n \left(\omega^n + \frac{1}{\omega^n}\right)$

$\Rightarrow z^n + z^{-n} = (-1)^n(\omega^n + \omega^{2n})$

$\Rightarrow z^n + z^{-n}$

$= \begin{cases} (-1)^n \times -1, & \text{if, } n \text{ is not a multiple of 3} \\ 2(-1)^n, & \text{if } n \text{ is a multiple of 3} \end{cases}$

$\Rightarrow z^n + z^{-n}$

$= \begin{cases} (-1)^{n-1}, & \text{if } n \text{ is not a multiple of 3} \\ 2(-1)^n, & \text{if } n \text{ is a multiple of 3} \end{cases}$

Since ω and ω^2 are reciprocal of each other and $z^n + z^{-n}$ does not change when z is replaced by $\frac{1}{z}$.

Therefore, the value of $z^n + z^{-n}$ remains same for $z = -\omega^2$

487 **(a)**

We have,

$x^3 + a^3 = 0 \Rightarrow x^3 = -a^3 \Rightarrow -a, -a\,\omega, -a\,\omega^2$,

where ω is a complex cube root of unity

Let $\alpha = -a, \beta = -a\,\omega$ and $\gamma = -a\,\omega^2$. Then,

$\left(\frac{\alpha}{\beta}\right)^2 = \left(\frac{-a}{a\,\omega}\right)^2 = \omega$ and, $\left(\frac{\alpha}{\gamma}\right)^2 = \left(\frac{-a}{-a\,\omega^2}\right)^2 = \omega^2$

The equation whose roots are $\left(\frac{\alpha}{\beta}\right)^2 = \omega$ and $\left(\frac{\alpha}{\gamma}\right)^2 = \omega^2$ is

$x^2 + x + 1 = 0$

For other combinations of α, β and γ we obtain the same equation. Hence, there is only one equation

488 **(b)**

$|z| = \left|\left(z - \frac{4}{z}\right) + \frac{4}{z}\right|$

$\Rightarrow \quad |z| \leq \left|z - \frac{4}{z}\right| + \frac{4}{|z|}$

$\Rightarrow \quad |z| \leq 2 + \frac{4}{|z|}$

$\Rightarrow \quad \left(|z|^2 - (\sqrt{5}+1)\right)\left(|z| - (1-\sqrt{5})\right) \leq 0$

$\Rightarrow \quad 1 - \sqrt{5} \leq |z| \leq \sqrt{5} + 1$

489 **(c)**

$z\bar{z} = |z|^2 = 0$ (given)

$\Rightarrow |z| = 0 \;\Rightarrow\; z = 0$

490 **(c)**

Since, $\bar{z} + i\bar{w} = 0 \quad \Rightarrow \quad \bar{z} = -i\bar{w}$

$\Rightarrow \quad z = -iw \quad \Rightarrow \quad w = -iz$

Also, $\arg(zw) = \pi \quad \Rightarrow \quad \arg(-iz^2) = \pi$

$\Rightarrow \quad \arg(-i) + 2\arg(z) = \pi$

$\Rightarrow\ -\frac{\pi}{2}$
$+2\arg(z)$
$=\pi$ $\left[\because\ \arg(-i)=-\frac{\pi}{2}\right]$

$\Rightarrow\quad 2\arg(z)=\frac{3\pi}{2}$

$\Rightarrow\quad \arg(z)=\frac{3\pi}{4}$

491 **(c)**
Since, n is not a multiple of 3, therefore$n=3m+1, n=3m+2$, where m is a positive integer.
For $n=3m+1$,
$1+\omega^n+\omega^{2n}=1+\omega^{3m+1}+\omega^{2(3m+1)}$
$=1+\omega^{3m}\omega+(\omega^3)^{2m}\omega^2=1+\omega+\omega^2=0$
Similarly, for $n=3m+2$
$\therefore\quad 1+\omega^n+\omega^{2n}=1+\omega^{3m+2}+\omega^{2(3m+2)}$
$=1+\omega^{3m}.\omega^2+(\omega^3)^{2m}.\omega^3.\omega=0$
$[\because\ \omega^3=1]$

492 **(a)**
Let the roots of $x^2-6x+a=0$ be $\alpha, 4\beta$ and that of $x^2-cx+6=0$ be $\alpha, 3\beta$
$\therefore\ \alpha+4\beta=6$ and $4\alpha\beta=a$
And $\alpha+3\beta=c$ and $3\alpha\beta=6$
$\Rightarrow\ \frac{a}{6}=\frac{4}{3}\ \Rightarrow\ a=8$
$\therefore\ x^2-6x+8=0\ \Rightarrow\ x=2,4$
And $x^2-cx+6=0\ \Rightarrow\ 2^2-2c+6=0\ \Rightarrow c=5$
$\therefore\ x^2-5x+6=0$
$\Rightarrow x=2,3$
Hence, common root is 2

493 **(c)**
Given, $\frac{|x+(y+1)i|}{|x+(y-1)i|}=\sqrt{3}$
$\Rightarrow\quad x^2+(y+1)^2=3[x^2+(y-1)^2]$
$\Rightarrow\quad x^2+y^2-4y+1=0$
On comparing with
$x^2+y^2+2gx+2fy+c=0$, we get
$g=0,\ f=-2,\ c=1$
$\therefore$ Radius of circle$=\sqrt{g^2+f^2-c}=\sqrt{(-2)^2-1}=\sqrt{3}$

494 **(a)**
We have,
$\sin\alpha+\cos\alpha=-\frac{q}{p}$ and $\sin\alpha\cos\alpha=\frac{r}{p}$
$\Rightarrow(\sin\alpha+\cos\ \alpha)^2=\frac{q^2}{p^2}$ and $\sin\alpha\cos\alpha=\frac{r}{p}$
$\Rightarrow 1+2\sin\alpha\cos\ \alpha=\frac{q^2}{p^2}$ and $\sin\alpha\cos\ \alpha=\frac{r}{p}$
$\Rightarrow 1+\frac{2r}{p}=\frac{q^2}{p^2}\quad\Rightarrow p^2-q^2+2pr=0$

495 **(b)**
The equation is meaningful for $x\neq 1$
When $x\neq 1$,we have,
$\frac{2x-3}{x-1}+1=\frac{6x^2-x-6}{x-1}$
$\Rightarrow 3x-4=6x^2-x-6$
$\Rightarrow 6x^2-4x-2=0$
$\Rightarrow 3x^2-2x-1=0$
$\Rightarrow(3x+1)(x-1)=0\Rightarrow x=-\frac{1}{3}\quad[\because x\neq 1]$

496 **(b)**
$S=1+3\alpha+5\alpha^2+\ldots+(2n-1)\alpha^{n-1}$ ...(i)
$\Rightarrow \alpha S=\alpha+3\alpha^2+5\alpha^3+\ldots+(2n-1)\alpha^n$ ...(ii)
On subtracting Eq.(ii) from Eq. (i), we get
$(1-\alpha)S=1+2\alpha+2\alpha^2+\ldots+2\alpha^{n-1}$
$-(2n-1)\alpha^n$
$=2(1+\alpha+\alpha^2+\ldots+\alpha^{n-1})-1-(2n-1)\alpha^n$
$=\frac{2(1-\alpha^n)}{1-\alpha}-2n=-2n\quad(\because a^n=1)$
$\Rightarrow\ S=\frac{-2n}{(1-\alpha)}$

497 **(c)**
We have, $\left(\frac{3+2i\sin\theta}{1-2i\sin\theta}\right)=\frac{(3+2i\sin\theta)}{(1-2i\sin\theta)}\frac{(1+2i\sin\theta)}{(1+2i\sin\theta)}$
$=\left(\frac{3-4\sin^2\theta}{1+4\sin^2\theta}\right)+i\left(\frac{8\sin\theta}{1+4\sin^2\theta}\right)$
Since, it is real therefore Im (z) should be zero
$\Rightarrow\ \frac{8\sin\theta}{1+4\sin^2\theta}=0\ \Rightarrow\ \sin\theta=0$
$\therefore\ \theta=n\pi$, where $n=0,1,2,3,\ldots$

498 **(c)**
We have,
$x^4+x^3-4x^2+x+1=0$
$\Rightarrow(x^4-2x^2+1)+x(x^2-2x+1)=0$
$\Rightarrow(x^2-1)^2+x(x-1)^2=1$
$\Rightarrow(x-1)^2((x+1)^2+x)=0$
$\Rightarrow(x-1)^2(x^2+3x+1)=0$
$\Rightarrow x=1$ (twice), $x=-\frac{3\pm\sqrt{5}}{2}$
Thus, the given equation has two integral roots

499 **(b)**
Area of the triangle on the argand palne formed by the complex numbers $-z, iz, z-iz$ is $\frac{3}{2}|z|^2$
$\therefore\quad \frac{3}{2}|z|^2=600\quad\Rightarrow\quad |z|=20$

500 **(a)**
$\frac{3x^2+1}{x^2-6x+8}=3+\frac{18x-23}{x^2-6x+8}$ [On dividing] ...(i)
Now, $\frac{18x-23}{(x-2)(x-4)}=\frac{A}{(x-2)}+\frac{B}{(x-4)}$
$\Rightarrow\quad 18x-23=A(x-4)+B(x-2)$
$\Rightarrow\quad 18x-23=(A+B)x-4A-2B$
On equating the coefficient of x and constant

term, we get

$A+B=18$

And $-4A-2B=-23$

On solving these equations, we get

$A=-\frac{13}{2}, \quad B=\frac{49}{2}$

$\therefore \quad \frac{18x-23}{(x-2)(x-4)}=-\frac{13}{2(x-2)}+\frac{49}{2(x-4)}$

Then, from Eq. (i), we get

$\frac{3x^2+1}{x^2-6x+8}$

$=3-\frac{13}{2(x-2)}+\frac{49}{2(x-4)}$

501 **(b)**

Since $x-c$ is a factor of order m of the polynomial $f(x)$

$\therefore f(x)=(x-c)^m\,\phi(x)$, where $\phi(x)$ is a polynomial of degree $n-m$

$\Rightarrow f(x), f'(x) \dots f^{m-1}(x)$ are all zero for $x=c$ but $f^m(x)\neq 0$ at $x=c$

$\Rightarrow x=c$ is root of $f(x), f'(x), \dots, f^{m-1}(x)$

502 **(b)**

Let $f(x)=(ax^2+bx+c)(ax^2-dx-c)$

$\Rightarrow D_1=b^2-4ac$ and $D_2=d^2+4ac$

$\Rightarrow D_1+D_2=b^2-4ac+d^2+4ac$

$=b^2+d^2 \geq 0$

$\therefore$ At least one of $D_1$and D_2 is positive

Hence, the polynomial has at least two real roots

503 **(d)**

One of the roots of the given equation is $x=1$, as the sum of the coefficients is zero

504 **(d)**

Given, $|x^2-x-6|=x+2$

Now, we have to consider two cases,

Case I When $x\leq -$ or $x\geq 3$

$\Rightarrow \quad x^2-x-6=x+2$

$\Rightarrow \quad x^2-2x-8=0 \quad\Rightarrow\quad x=-2,4$

Case II When $-2<x<3$

$\Rightarrow\ -(x^2-x-6)=x+2 \ \Rightarrow x^2=4\ \Rightarrow x=\pm 2$

Hence, the roots are $(-2,2,4)$

505 **(c)**

Let ABC be an equilateral triangle such that the affixes of the vertices A, B and C are $z_!, z_2$ and z_3 respectively. Let the circumcentre of $\Delta\, ABC$ be at the origin. Then, $OA=z_1, OB=z_2$ and $OC=z_3$

Now,

$OB=OA\,e^{i2\pi/3}$ and $OC=OA\,e^{i\,4\pi/3}$

$\Rightarrow z_2=z_1e^{i\,2\pi/3}$ and $z_3=z_1e^{i\,4\pi/3}$

$\Rightarrow z_2=z_1\omega$ and $z_3=z_1\omega^2$

$\therefore z_1+z_2+z_3=z_1(1+\omega+\omega^2)=z_1\times 0=0$

506 **(a)**

Let α be the common roots to the equations $x^2-kx-21=0$ and $x^2-3kx+35=0$

$\therefore\quad \alpha^2-k\alpha-21=0$ and $\alpha^2-3k\alpha+35=0$

Now, by cross multiplication method, we get

$\frac{\alpha^2}{(-35k-63k)}=\frac{\alpha}{(-21-35)}=\frac{1}{(-3k+k)}$

$\Rightarrow \quad \frac{\alpha^2}{-98k}=\frac{\alpha}{-56}=\frac{1}{-2k}$

$\Rightarrow \quad \frac{\alpha^2}{-98k}=\frac{1}{-2k} \quad\Rightarrow\quad \alpha^2=49 \qquad$ ...(i)

And $\frac{\alpha}{-56}=-\frac{1}{2k} \quad\Rightarrow\quad \alpha=\frac{28}{k} \qquad$ ...(ii)

From Eqs. (i) and (ii),

$\frac{28\times 28}{k^2}=49$

$\Rightarrow \quad k^2=16$

$\Rightarrow \quad k=\pm 4$

507 **(d)**

$\{(1-\cos\theta)+i.2\sin\theta\}^{-1}$

$=\left(2\sin^2\frac{\theta}{2}+\text{i}.4\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)^{-1}$

$=\left(2\sin\frac{\theta}{2}\right)^{-1}\left(\sin\frac{\theta}{2}+\text{i}2\cos\frac{\theta}{2}\right)^{-1}$

$=\left(2\sin\frac{\theta}{2}\right)\frac{1}{\sin\frac{\theta}{2}+i\,2\cos\frac{\theta}{2}}\times\frac{\sin\frac{\theta}{2}-i2\cos\frac{\theta}{2}}{\sin\theta-i\,2\cos\frac{\theta}{2}}$

$=\frac{\sin\frac{\theta}{2}-2\,i\cos\frac{\theta}{2}}{2\sin\frac{\theta}{2}\left(\sin^2\frac{\theta}{2}+4\cos^2\frac{\theta}{2}\right)}$

$=\frac{\sin\frac{\theta}{2}-2\,i\cos\frac{\theta}{2}}{2\sin\frac{\theta}{2}\left(1+3\cos^2\frac{\theta}{2}\right)}$

$\therefore$ It's real part is

$\frac{\sin\frac{\theta}{2}}{2\sin\frac{\theta}{2}\left(1+3\cos^2\frac{\theta}{2}\right)}=\frac{1}{2\left\{1+3\left(\frac{\cos\theta+1}{2}\right)\right\}}$

$=\frac{1}{5+3\cos\theta}$

508 **(c)**

Given, $x=\frac{1}{2}\left(\sqrt{7}+\frac{1}{\sqrt{7}}\right)$

$\Rightarrow x^2=\frac{1}{4}\left(7+\frac{1}{7}+2\right)=\frac{16}{7}$

Now, $\frac{\sqrt{x^2-1}}{x-\sqrt{x^2-1}}=\frac{\sqrt{x^2-1}}{x-\sqrt{x^2-1}}\times\frac{(x+\sqrt{x^2-1})}{(x+\sqrt{x^2-1})}$

$=\frac{x\sqrt{x^2-1}+x^2-1}{1}$

$=\frac{1}{2}\left(\sqrt{7}+\frac{1}{\sqrt{7}}\right)\sqrt{\frac{16}{7}-1}+\frac{16}{7}-1$

$=\frac{1}{2}\left(\sqrt{7}+\frac{1}{\sqrt{7}}\right)\times\frac{3}{\sqrt{7}}+\frac{9}{7}$

$=\frac{1}{2}\left(3+\frac{3}{7}\right)+\frac{9}{7}$

$= 3$

509 **(a)**

Let the roots are α, β, so $\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$

Now, $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{\alpha^2\beta^2}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$

$= \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c^2}{a^2}} = \frac{b^2 - 2ac}{c^2}$

Also, $\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$ [given]

$\Rightarrow -\frac{b}{a} = \frac{b^2 - 2ac}{c^2}$

$\Rightarrow -bc^2 = ab^2 - 2a^2c$

$\Rightarrow 2a^2c = ab^2 + bc^2$

$\Rightarrow ab^2, ca^2, bc^2$

Or bc^2, ca^2, ab^2 are in AP

510 **(d)**

Let roots of the equation $ax^2 + bx + c = 0$ are α and β

$\therefore \alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

$= \frac{-\frac{b}{a}}{\frac{c}{a}} = \frac{-b}{c}$

And $\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\frac{c}{a}} = \frac{a}{c}$

$\therefore$ Required equation is

$x^2 - \left(-\frac{b}{c}\right)x + \frac{a}{c} = 0$

$\Rightarrow cx^2 + bx + a = 0$

Alternate To find the equation of reciprocal rots, interchange the coefficients of x^2 and constant term in the given equation then required equation is $cx^2 + bx + a = 0$

511 **(a)**

Let α be a root of the equation $ax^2 + bx + c = 0$. Then, $1/\alpha$ is a root of $a_1x^2 + b_1x + c_1 = 0$

$\Rightarrow a\,\alpha^2 + b\,\alpha + c = 0$ ... (i)

and, $\frac{a_1}{\alpha^2} + \frac{b_1}{\alpha} + c_1 = 0 \Rightarrow c_1\alpha^2 + b_1\alpha + a_1 = 0$ ... (ii)

from (i) and (ii), we have

$\frac{\alpha^2}{ba_1 - b_1c} = \frac{\alpha}{cc_1 - aa_1} = \frac{1}{ab_1 - c_1b}$

$\Rightarrow \alpha^2 = \frac{ba_1 - b_1c}{ab_1 - c_1b'}, \alpha = \frac{cc_1 - aa_1}{ab_1 - c_1b}$

Now, $\alpha^2 = (\alpha)^2 \Rightarrow (ba_1 - b_1c)(ab_1 - c_1b) = (cc_1 - aa_1)^2$

512 **(b)**

We have,

$z = (-1)^{1/7}, z \neq -1 \Rightarrow z^7 = -1$

$\therefore z^{86} + z^{175} + z^{289}$

$= (z^7)^{12}z^2 + (z^7)^{25} + (z^7)^{41}z^2 = z^2 - 1 - z^2 = -1$

513 **(b)**

Since, α and β the roots of the equation $x^2 - x - 1 = 0$

$\therefore \alpha + \beta = 1$ and $\alpha\beta = -1$

Hence, AM of A_{n-1} and $A_n = \frac{A_{n-1}+A_n}{2}$

$= \frac{\alpha^{n-1} + \beta^{n-1} + \alpha^n + \beta^n}{2}$

$= \frac{\alpha^{n-1}(1+\alpha) + \beta^{n-1}(1+\beta)}{2}$

$= \frac{\alpha^{n-1}.\alpha^2 + \beta^{n-1}\beta^2}{2}$

$= \frac{1}{2}(\alpha^{n+1} + \beta^{n+1})$

$= \frac{1}{2}A^{n+1}$

515 **(c)**

It is given that

$|z + 4| \leq 3$

$\therefore |z + 1| = |z + 4 - 3|$

$\Rightarrow |z + 1| \leq |z + 4| + |3| \leq 3 + 3$ $[\because |z + 4| \leq 3]$

Hence, the greatest value of $|z + 1|$ is 6

Since the least value of the modulus of a complex number is zero

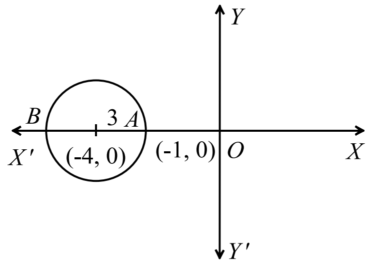
$\therefore |z + 1| = 0 \Rightarrow z = -1$

$\Rightarrow |z + 4| = |-1 + 4| = 3$

$\Rightarrow |z + 4| \leq 3$ is satisfied by $z = -1$

Therefore, the least value of $|z + 1|$ is 0

ALITER Here, we have to find the greatest and least of distances of all points lying inside or the circle from the point $A(-1, 0)$. It is evident from the Fig. S.3, that the greatest distance is 6 when P coincides with B and the least distance is 0 when P coindies with A



516 **(d)**

$\frac{z-1}{2z+1} = \frac{(x-1)+iy}{(2x+1)+2iy} \times \frac{(2x+1)-2iy}{(2x+1)-2iy}$

$$= \frac{\{(x+1)(2x+1)+2y^2\}+ iy\{-2x+2+2x+1\}}{(2x+1)^2+4y^2}$$

Given, $\text{Im}\left(\frac{z-1}{2z+1}\right) = -4$

$\therefore \quad \frac{3y}{(2x+1)^2+4y^2} = -4$

$\Rightarrow \; 16x^2+16y^2+16x+3y+4=0$

$\therefore$ The locus of z is a circle.

517 **(b)**

Let $z_1 = \frac{w-\overline{w}z}{1-z}$ be purely real

$\Rightarrow \quad z_1 = \overline{z_1}$

$\therefore \quad \frac{w-\overline{w}z}{1-z} = \frac{\overline{w}-w\bar{z}}{1-\bar{z}}$

$\Rightarrow \quad w - w\bar{z} - \overline{w}z + \overline{w}z\,\bar{z}$

$= \quad \overline{w} - z\overline{w} - w\bar{z} + wz\,\bar{z}$

$\Rightarrow \quad (w-\overline{w}) + (\overline{w}-w)|z|^2 = 0$

$\Rightarrow \quad (w-\overline{w})+(1-|z|^2)=0$

$\Rightarrow \quad |z|^2 = 1$

[as, $w-\overline{w} \neq 0$, since $\beta \neq 0$]

$\Rightarrow \quad |z| = 1 \quad$ and $\quad z \neq 1$

518 **(d)**

Since, $(x-2)$ is a common factor of the expressions x^2+ax+b and x^2+cx+d

$\Rightarrow 4+2a+b=0$ ...(i)

and $4+2c+d=0$ ...(ii)

$\Rightarrow 2a+b=2c+d$

$\Rightarrow b-d = 2(c-a)$

$\Rightarrow \frac{b-d}{c-a} = 2$

519 **(c)**

Since, $\alpha_0, \alpha_1, \alpha_2, \dots, \alpha_{n-1}$ are n^{th} roots of unity. Therefore,

$x^n - 1 = (x-\alpha_0)(x-\alpha_1)\dots(x-\alpha_n - 1)$

$\Rightarrow \log(x^n-1) = \log(x-\alpha_0) + \log(x-\alpha_1)+\cdots + \log(x-\alpha_{n-1})$

Differentiating both sides w.r.t. x, we get

$$\frac{nx^{n-1}}{x^n-1} = \frac{1}{3-\alpha_0}+\frac{1}{x-\alpha_1}+\cdots+\frac{1}{x-\alpha_{n-1}}$$

Putting $x=3$ on both sides, we get

$$\frac{n3^{n-1}}{3^n-1} = \frac{1}{3-\alpha_0}+\frac{1}{3-\alpha_1}+\cdots+\frac{1}{3-\alpha_{n-1}} \quad \dots \text{(i)}$$

Now,

$$\sum_{i=0}^{n-1}\frac{\alpha_i}{3-\alpha_i} = -\sum_{i=0}^{n-1}\frac{\{(3-\alpha_i)-3\}}{(3-\alpha_i)}$$

$$\Rightarrow \sum_{i=0}^{n-1}\frac{\alpha_i}{3-\alpha_i} = -\sum_{i=0}^{n-1}1 + 3\sum_{i=0}^{n-1}\frac{1}{3-\alpha_i}$$

$$\Rightarrow \sum_{i=0}^{n-1}\frac{\alpha_i}{3-\alpha_i} = -n+3 \times \frac{n\,3^{n-1}}{3^n-1} \quad \text{[Using (i)]}$$

$$\Rightarrow \sum_{i=0}^{n-1}\frac{\alpha_i}{3-\alpha_i} = -n+n\frac{3^n}{3^n-1} = \frac{n}{3^n-1}$$

520 **(c)**

Let α, β be the roots of $x^2-px+q^2=0$ and γ, δ be the roots of $x^2-rx+s^2=0$. Then,

$\alpha+\beta = p$ and $\gamma\,\delta = s^2 \Rightarrow \frac{\alpha+\beta}{2} = \frac{p}{2}$ and $\sqrt{\gamma\,\delta} = |s|$

It is given that $\frac{\alpha+\beta}{2} = \sqrt{\gamma\,\delta}$

$\Rightarrow \frac{p}{2} = |s| \Rightarrow p = 2|s| \Rightarrow p$ is an even integer

521 **(a)**

Let the rots of the equation be α, β, γ. Also,

$\alpha = -\beta$ [given]

$\therefore \; \alpha+\beta+\gamma = p \;\Rightarrow\; -\beta+\beta+\gamma = p$

$\Rightarrow \; \gamma = p$ ...(i)

Now, since γ is a root of the equation.

$\therefore$ It satisfies the given equation

$\Rightarrow \; \gamma^3 - p\gamma^2 + q\gamma - r = 0$

$\Rightarrow p^3 - pp^2 + pq - r = 0$ [from Eq. (i)]

$\Rightarrow \; r = pq$

522 **(c)**

Here, $\alpha+\beta = -\frac{b}{a}, \; \alpha\beta = \frac{c}{a}$

But given that $\beta = \alpha^{1/3}$

$\therefore \; \alpha + \alpha^{1/3} = -\frac{b}{a}$ and $\alpha.\alpha^{1/3} = \frac{c}{a}$

$\Rightarrow \; \alpha^{4/3} = \frac{c}{a} \;\Rightarrow\; \alpha = \left(\frac{c}{a}\right)^{3/4}$

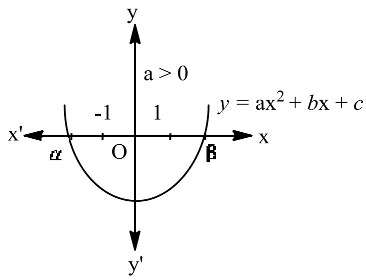
$\therefore \; \alpha + \alpha^{1/3} = -\frac{b}{a}$

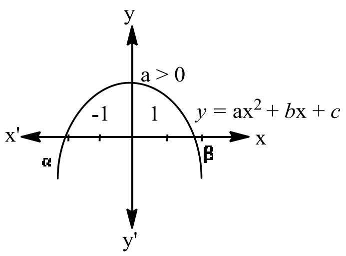
$\Rightarrow \left(\frac{c}{a}\right)^{3/4} + \left(\frac{c}{a}\right)^{1/4} = -\frac{b}{a}$

$\Rightarrow (ac^3)^{1/4} + (a^3c)^{1/4} + b = 0$

523 **(a)**

From figure it is clear that, if $a > 0$, then $f(-1) < 0$ and $f(1) < 0$ and if $a < 0, f(-1) > 0$ and $f(1) > 0$. In both cases $af(-1) < 0$ and $af(1) < 0$





$\Rightarrow \ a(a-b+c) < 0$ and $a(a+b+c) < 0$

$\Rightarrow \ 1-\frac{b}{a}+\frac{c}{a} < 0$ and $1+\frac{b}{a}+\frac{c}{a} < 0$ [divide by a^2]

$$\Rightarrow \ 1 \pm \frac{b}{a}+\frac{c}{a} < 0 \ \Rightarrow \ 1+\left|\frac{b}{a}\right|+\frac{c}{a} < 0$$

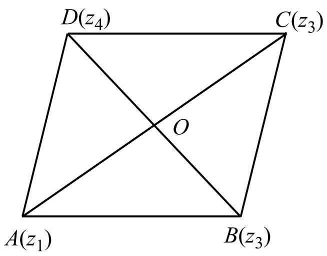
524 **(c)**

Since the diagonals of a rhombus bisect each other at right-angle

$$\therefore \frac{z_1+z_3}{2}=\frac{z_2+z_4}{2} \Rightarrow z_1+z_3=z_2+z_4$$

Also,

$$\angle AOB=\frac{\pi}{2} \Rightarrow \arg\left(\frac{z_2-z_4}{z_1-z_3}\right)=\frac{\pi}{2}$$



525 **(a)**

We have,

$$7^{\log_7(x^2-4x+5)}=x-1$$

$$\Rightarrow x^2-4x+5=x-1 \Rightarrow x^2-5x+6=x-1$$

$$\Rightarrow x=2,3$$

526 **(c)**

We have, $\frac{1}{|x|-3} < \frac{1}{2}$

Clearly, $\frac{1}{|x|-3}$ is not defined for $|x|=3$ i. e. $x=-3,3$

Now, $\frac{1}{|x|-3} < \frac{1}{2}$

$$\Rightarrow \frac{1}{|x|-3}-\frac{1}{2} < 0$$

$$\Rightarrow \frac{2-|x|+3}{|x|-3} < 0$$

$$\Rightarrow \frac{|x|-5}{|x|-3} > 0$$

$\Rightarrow |x| < 3$ or, $|x| > 5$

$\Rightarrow x \in (-3,3)$ or, $x \in (-\infty,-5) \cup (5,\infty)$

$\Rightarrow x \in (-\infty,-5) \cup (-3,3) \cup (5,\infty)$

527 **(d)**

Given, $z=\sqrt{3}+i$,

$\arg(z^2e^{z-i})=\arg[(3-1+2\sqrt{3}i)e^{\sqrt{3}}]$

$$=\arg\left[(2+2\sqrt{3}i)e^{\sqrt{3}}\right]=\tan^{-1}\left[\frac{2\sqrt{3}}{2}\right]=\frac{\pi}{3}$$

528 **(d)**

Since the equations $a_1x^2+b_1x+c_1=0$ and $a_2x^2+b_2x+c_2=0$ have a common root α (say). Therefore,

$a_1\alpha^2+b_1\alpha+c_1=0$ and $a_2\alpha^2+b_2\alpha+c_2=0$

$$\therefore \frac{\alpha^2}{b_1c_2-b_2c_1}=\frac{\alpha}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$$

$$\Rightarrow \alpha^2=\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}, \alpha=\frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}$$

Now, $\alpha^2=(\alpha)^2$

$$\Rightarrow \left(\frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}\right)^2=\left(\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}\right)$$

$$\Rightarrow (c_1a_2-c_2a_1)^2=(a_1b_2-a_2b_1)(b_1c_2-b_2c_1)$$

530 **(c)**

Given, $(x+iy)^{1/3}=2+3i$

$\Rightarrow \ x+iy=(2+3i)^3$

$=8+36i+54i^2+27i^3$

$=-46+9i$

Equating real and imaginary parts from both sides, we get

$x=-46, \ y=9$

$\therefore \ \ 3x+2y=-138+18=-120$

531 **(b)**

Given equation $\frac{1}{x+p}+\frac{1}{x+q}=\frac{1}{r}$ can be rewritten as

$x^2+x(p+q-2r)+pq-pr-qr=0$ ...(i)

Let roots are α and $-\alpha$, then the product of roots

$-\alpha^2=pq-pr-qr-r(p+q)$ ....(ii)

and sum of roots, $0=-(p+q-2r)$

$\Rightarrow r=\frac{p+q}{2}$ ...(iii)

On solving Eqs. (ii) and (iii), we get

$$-\alpha^2=pq-\frac{p+q}{2}(p+q)$$

$$=-\frac{1}{2}\{(p+q)^2-2pq\}$$

$$\Rightarrow \ \alpha^2=-\frac{(p^2+q^2)}{2}$$

532 **(b)**

We have,

$$1+i=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$$

and, $1-i=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

$\therefore (1+i)^8+(1-i)^8$

$=2^4\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)^8+2^4\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)^8$

$=2^4(\cos 2\pi+i\sin 2\pi)+2^4(\cos 2\pi-i\sin 2\pi)$

$2^4(2\cos 2\pi)=2^5$

533 **(a)**

The given equation is

$x^2(\lambda+1)-x\{b(\lambda+1)+a(\lambda-1)\}+c(\lambda-1)=0$

This equation has roots equal in magnitude but opposite in sign

$\therefore$ Sum of the roots = 0

$\Rightarrow \frac{b(\lambda+1)+a(\lambda-1)}{\lambda+1}=0\Rightarrow \lambda=\frac{a-b}{a+b}$

534 **(d)**

Since $x^2+5|x|+4>0$ for all $x\in R$

Therefore, $x^2+5|x|+4=0$ has no real roots

535 **(d)**

Let $z_r=x_r+iy_r; r=0,1,3,4,\dots,6$

We have,

$(z_r+1)^7+z_r^7=0, r=0,1,\dots,6$

$\Rightarrow (z_r+1)^7=-z_r^7$

$\Rightarrow |z_r+1|^7=|z_r|^7$

$\Rightarrow |z_r+1|=|z_r|\Rightarrow |z_r+1|^2=|z_r|^2$

$\Rightarrow (x_r+1)^2+y_r^2=x_r^2+y_r^2\Rightarrow 2x_r+1=0\Rightarrow x_r=-\frac{1}{2}$

$\therefore \sum_{r=0}^{6}x_r=-\frac{7}{2}\Rightarrow \sum_{r=0}^{6}\text{Re }(z_r)=-\frac{7}{2}$

536 **(d)**

Given that,

$z^2+(p+iq)z+r+i\,s=0$ ...(i)

Let $z=\alpha$ (where α is real) be a root of Eq. (i), then

$\alpha^2+((p+iq)\alpha+r+i\,s=0$ ...(i)

$\Rightarrow \alpha^2+p\alpha+r+i(q\alpha+s)=0$

On equating real and imaginary parts, we get

$\alpha^2+p\alpha+r=0$ ...(ii)

and $q\alpha+s=0\Rightarrow \alpha=\frac{-s}{q}$

On putting the value of α in Eq.(ii), we get

$\left(\frac{-s}{q}\right)^2+p\left(\frac{-s}{q}\right)+r=0$

$\Rightarrow s^2-pqs+q^2r=0$

$\Rightarrow pqs=s^2+q^2r$

537 **(d)**

The given condition suggest that a lies between the roots.

Let $f(x)=2x^2-2(2a+1)x+a(a+1)$

For `a' to lie between the roots we must have

Discriminant ≥ 0 and $f(a)<0$

Now, Discriminant ≥ 0

$\Rightarrow 4(2a+1)^2-8a(a+1)\geq 0$

$\Rightarrow 8\left(a^2+a+\frac{1}{2}\right))\geq 0$, which is always true.

Also, $f(a)<0$

$\Rightarrow 2a^2-2a(2a+1)+a(a+1)<0$

$\Rightarrow -a^2-a<0\Rightarrow a^2+a>0\Rightarrow a(1+a)>0$

$\Rightarrow a>0$ or $a<-1$

538 **(d)**

We have,

$a=\cos\alpha+i\sin\alpha, b=\cos\beta+i\sin\beta$ and

$c=\cos\gamma+i\sin\gamma$

$\therefore a/b=\cos(\alpha-\beta)+i\sin(\alpha-\beta)$,

$b/c=\cos(\beta-\gamma)+i\sin(\beta-\gamma)$

$c=\cos(\gamma-\alpha)+i\sin(\gamma-\alpha)$

$\therefore \frac{a}{b}+\frac{b}{c}+\frac{c}{a}=1$

$\Rightarrow [\cos(\alpha-\beta)+\cos(\beta-\gamma)+\cos(\gamma-\alpha)]$
$+i[\sin(\alpha-\beta)+\sin(\beta-\gamma)+\sin(\gamma-\alpha)]$
$=1+i\,0$

$\cos(\alpha-\beta)+\cos(\beta-\gamma)+\cos(\gamma-\alpha)=1$

539 **(d)**

Since, α and β are the roots of equation

$(x-a)(x-b)=5$

Or $x^2-(a+b)x+ab-5=0$

Then, $\alpha+\beta=(a+b)$ and $\alpha\beta=ab-5$

$\therefore (x-\alpha)(x-\beta)+5=0$ (given)

$\Rightarrow x^2-(\alpha+\beta)x+\alpha\beta+5=0$

$\Rightarrow x^2-(a+b)x+ab-5+5=0$

$\Rightarrow (x-a)(x-b)=0$

Hence, a and b are the roots of equation

$(x-\alpha)(x-\beta)+5=0$

540 **(d)**

Here, $\sum\alpha=0$, $\sum\alpha\beta=b$ and $\alpha\beta\gamma=-c$ ...(i)

Now, $\sum\alpha\sum\alpha\beta=(\alpha+\beta+\gamma).(\alpha\beta+\beta\gamma+\gamma\alpha)$

$=\sum\alpha^2\beta+3\alpha\beta\gamma$

$\Rightarrow \sum\alpha^2\beta=\sum\alpha\sum\alpha\beta-3\alpha\beta\gamma$

$=0.\sum\alpha\beta-3(-c)$

[from Eq. (i)]

$=3c$

541 **(c)**

$\left|\frac{1}{2}(z_1+z_2)+\sqrt{z_1z_2}\right|+\left|\frac{1}{2}(z_1+z_2)-\sqrt{z_1z_2}\right|$

$=\frac{1}{2}\left|(\sqrt{z_1}+\sqrt{z_2})^2\right|+\frac{1}{2}\left|(\sqrt{z_1}-\sqrt{z_2})^2\right|$

$=\frac{1}{2}\left|\sqrt{z_1}+\sqrt{z_2}\right|^2+\frac{1}{2}\left|\sqrt{z_1}-\sqrt{z_2}\right|^2$ $[\because |z^2|=|z|^2]$

$= \frac{1}{2} . 2 \left[\left| \sqrt{z_1} \right|^2 + \left| \sqrt{z_2} \right|^2 \right] = |z_1| + |z_2|$

542 **(c)**

We have, $px^2 + qx + 1 = 0$, for real roots discriminant ≥ 0

$\Rightarrow q^2 - 4p \geq 0 \ \Rightarrow q^2 \geq 4p$

For $p = 1, q^2 \geq 4 \ \Rightarrow q = 2, 3, 4$

$p = 2, q^2 \geq 8 \ \Rightarrow q = 3, 4$

$p = 3, q^2 \geq 12 \ \Rightarrow q = 4$

$p = 4, q^2 \geq 16 \ \Rightarrow q = 4$

Total seven solutions are possible.

543 **(c)**

We have,

$|z| - 2 = |z - i| - |z + 5i| = 0$

$\Rightarrow |z| = 2$ and $|z - i| = |z + 5i|$

$\Rightarrow z$ lies on the circle $|z| = 2$ and also on the perpendicular bisector of the line segment joining $(0, -5)$ and $(0,1)$ i.e., $y = -2$

Putting $y = -2$ in $|z| = 2$ i. e. $x^2 + y^2 = 4$, we get $x = 0$

Hence, the locus of z is the single point $(0, -2)$

544 **(a)**

<u>CASE I</u> $When\ x - a \geq 0\ i.e. x \geq a$:

In this case, we have $|x - a| = x - a$

$\therefore x^2 - 2a|x - a| - 3a^2 = 0$

$\Rightarrow x^2 - 2a(x - a) - 3a^2 = 0$

$\Rightarrow x^2 - 2ax - a^2 = 0 \Rightarrow x = a(1 \pm \sqrt{2})$

But, $a \leq 0$ and $x > a$. Therefore, $x = a(1 - \sqrt{2})$

<u>CASE II</u> $When\ (x - a) < 0\ i.e. x < a$

In this case, we have $|x - a| = -(x - a)$

$\therefore x^2 - 2a|x - a| - 3a^2 = 0$

$\Rightarrow x^2 + 2a(x - a) - 3a^2 = 0$

$\Rightarrow x^2 + 2ax - 5a^2 = 0 \Rightarrow x = a(-1 \pm \sqrt{6})$

But, $x < a$ and $a \leq 0$ Therefore, $x = a(-1 + \sqrt{6})$

545 **(c)**

We have,

$16 - 4a^3 < 0$ and $\frac{4}{a} = a$

$\Rightarrow 4 - a^2 < 0$ and $a^2 = 4$

$\Rightarrow a^3 - 4 > 0$ and $a = \pm 2 \Rightarrow a = 2$

546 **(a)**

Since, the roots of the equation $4x^3 - 12x + 11x + k = 0$ are in AP which are $\alpha - a, \alpha, \alpha + a$.

$\therefore$ Sum of roots, $3\alpha = \frac{12}{4} = 3 \ \Rightarrow \ \alpha = 1$

Since, α is a root, therefore it satisfies the given equation

ie, $\quad 4x^3 - 12x^2 + 11x + k = 0$

$\therefore 4 - 12 + 11 + k = 0 \quad \Rightarrow \quad k = -3$

547 **(a)**

The equations $\left|z + \sqrt{2}\right| = \sqrt{a^2 - 3a + 2}$ and $\left|z + \sqrt{2}\, i\right| = a$ represent two circles having centre $C_1(-\sqrt{2}, 0)$ and $C_2(0, -\sqrt{2})$ and radii $= \sqrt{a^2 - 3a + 2}$ and a respectively.

These two circles will intersect, if

$C_1C_2 <$ Sum of the radii

$\Rightarrow 2 < \sqrt{a^2 - 3a + 2} + a$

$\Rightarrow (2 - a)^2 < a^2 - 3a + 2 \Rightarrow -a + 2 < 0 \Rightarrow a > 2$

548 **(c)**

Let α, β be the roots of $ax^2 - bx - c = 0$ and let α', β' be the roots of $a'x^2 - b'x - c' = 0$ such that $|\alpha - \beta| = |\alpha' - \beta'|$

$\Rightarrow (\alpha - \beta)^2 = (\alpha' - \beta')^2$

$\Rightarrow (\alpha + \beta)^2 - 4\,\alpha\,\beta = (\alpha' + \beta')^2 - 4\,\alpha'\beta'$

$\Rightarrow \frac{b^2 + 4ac}{a^2} = \frac{b'^2 + 4a'c'}{a'^2}$

Hence, the expression $\frac{b^2+4ac}{a^2}$ does not vary in value

549 **(b)**

We have, $x^{\log_x (1-x)^2} = 9$

Taking log on both sides, we get

$\log_x(9) = \log_x(1 - x)^2 \ \ (\because a^x = N \Rightarrow \log_a N = x)$

$\Rightarrow 9 = (1 - x)^2$

$\Rightarrow 1 + x^2 - 2x - 9 = 0$

$\Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow x = -2, 4$

$\Rightarrow x = 4 \quad (\because x = -2)$

550 **(d)**

$$\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix}$$

$$= \begin{vmatrix} x+1+\omega+\omega^2 & \omega & \omega^2 \\ \omega+x+\omega^2+1 & x+\omega^2 & 1 \\ \omega^2+1+x+\omega & 1 & x+\omega \end{vmatrix}$$

$(C_1 \to C_1 + C_2 + C_3)$

$$= x \begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & x+\omega^2 & 1 \\ 1 & 1 & x+\omega \end{vmatrix} \quad (\because 1 + \omega + \omega^2 = 0)$$

$$= x \begin{vmatrix} 1 & \omega & \omega^2 \\ 0 & x+\omega^2-\omega & 1-\omega^2 \\ 0 & 1-\omega & x+\omega-\omega^2 \end{vmatrix}$$

$(R_2 \to R_2 - R_1, R_3 \to R_3 - R_1)$

$= x[(x + \omega^2 - \omega)(x + \omega - \omega^2) - (1 - \omega)(1 - \omega^2)]$

$= x[x + 3 - 3]$

$= x^2$

551 **(c)**

Using De-Moivre's Theorem, we have

$\left[\sqrt{2}\{\cos(56°\,15') + i \sin(56°\,15')\}\right]^8$

$= 16(\cos 450° + i \sin 450°) = 16\,i$

552 **(d)**

We have, $\{(1-\cos\theta)+i.2\sin\theta\}^{-1}$

$= \left(2\sin^2\frac{\theta}{2}+i.4\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)^{-1}$

$= \left(2\sin\frac{\theta}{2}\right)^{-1}\left(\sin\frac{\theta}{2}+i.2\cos\frac{\theta}{2}\right)^{-1}$

$= \left(2\sin\frac{\theta}{2}\right)^{-1}.\frac{1}{\sin\frac{\theta}{2}+i.2\cos\frac{\theta}{2}}\times\frac{\sin\frac{\theta}{2}-i.2\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}-i.2\cos\frac{\theta}{2}}$

$= \frac{\sin\frac{\theta}{2}-i.2\cos\frac{\theta}{2}}{2\sin\frac{\theta}{2}\left(\sin^2\frac{\theta}{2}+4\cos^2\frac{\theta}{2}\right)}$

$= \frac{\sin\frac{\theta}{2}+i.2\cos\frac{\theta}{2}}{2\sin\frac{\theta}{2}\left(1+3\cos^2\frac{\theta}{2}\right)}$

It's real part

$= \frac{\sin\frac{\theta}{2}}{2\sin\frac{\theta}{2}\left(1+3\cos^2\frac{\theta}{2}\right)} = \frac{1}{2\left(1+3\cos^2\frac{\theta}{2}\right)}$

$= \frac{1}{2+3(\cos\theta+1)} = \frac{1}{5+3\cos\theta}$

553 **(b)**

Let the discriminant of the equation $x^2+px+q=0$ is D_1, then $D_1=p^2-4q$

and the discriminant of the equation $x^2+rx+s=0$ is D_2, then $D_2=r^2-4s$

$\therefore\ D_1+D_2=p^2+r^2-4(q+s)=p^2+r^2-2pr$ (from the given relation)

$\Rightarrow\ D_1+D_2=(p-r)^2\ge 0$

Clearly, at least one of D_1 and D_2 must be non-negative, consequently at least one of the equation has real roots.

554 **(c)**

We know, $-\frac{1}{2}+\frac{i\sqrt{3}}{2}=\omega$

$\therefore\quad 4+5(\omega)^{334}+3(\omega)^{365}$

$=4+5(\omega^3)^{111}.\omega^1+3(\omega^3)^{121}.\omega^2$

$=4+5\omega+3\omega^2$

$=3(1+\omega+\omega^2)+1+2\omega$

$=1+(-1+i\sqrt{3})$

$=i\sqrt{3}$

555 **(a)**

We have,

$(16)^{1/4}=(2^4)^{1/4}=2(1)^{1/4}$

$=2(\cos 0+i\sin 0)^{1/4}$

$=2(\cos\frac{1}{4}(2k\pi+0)+i\sin\frac{1}{4}(2k\pi+0)$

$k=0,1,2,3$

$=2\times 1, 2\times i, 2\times -1, 2\times -i$

$=\pm 2,\pm 2i$

556 **(d)**

Let ABC be the equilateral triangle circumscribing the circle $|z|=\frac{1}{2}$. Let z_1,z_2,z_3 be the affixes of vertices A,B and C respectively in anti-clock wise sense. Clearly, O (origin) is the circumcentre of ΔABC

$\therefore z_2=z_1e^{i2\pi/3}=(-\omega^2)(\omega)=-\omega^3=-1$

557 **(d)**

$\frac{4(\cos 75°+i\sin 75°)}{0.4(\cos 30°+i\sin 30°)}$

$=10(\cos 75°+i\sin 75°)\,(\cos 30°+i\sin 30°)$

$=10\,e^{75i}.e^{-30i}=10e^{45i}$

$=10(\cos 45°+i\sin 45°)=\frac{10}{\sqrt{2}}(1+i)$

558 **(b)**

Let α and β be the roots, then

$\alpha+\beta=(a-2)$ and $\alpha\beta=-(a+1)$

Now, $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=(a-2)^2+2(a+1)$

$=(a-1)^2+5$

$\Rightarrow\ \alpha^2+\beta^2\ge 5$

Thus, the minimum value of $\alpha^2+\beta^2$ is 5 at $a=1$

559 **(d)**

Let $z=\frac{1-i\sqrt{3}}{\sqrt{3}+i}=\frac{1+i\sqrt{3}}{(\sqrt{3}+i)}\times\frac{(\sqrt{3}-i)}{(\sqrt{3}-i)}=\frac{\sqrt{3}+i}{2}$

$\therefore\quad \text{amp}\,(z)=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=\frac{\pi}{6}$

560 **(d)**

Let $f(x)=4x^2-20px+(25p^2+15p-66)$

Clearly, $y=f(x)$ represents a parabola opening upward.

So, roots of the equation $f(x)=0$ will be less than 2, if

(i) Discriminant ≥ 0

(ii) 2 lies outside the roots i.e. $f(2)>0$

(iii) x-coordinate of vertex <2

Now,

(i) $f(2)>0$

$\Rightarrow 16-40p+25p^2+15p-66>0$

$\Rightarrow 25p^2-25p-50>0\Rightarrow p^2-p-2>0$

$\Rightarrow p<-1$ or, $p>2$ ...(i)

(ii) Discriminant ≥ 0

$\Rightarrow 400p^2-16(25p^2+15p-66)\ge 0$

$\Rightarrow 15p-66\le 0\Rightarrow p\le 22/5$ ...(ii)

(iii) x-coordinate of vertex <2

$\Rightarrow -\left(\frac{-20p}{4}\right)<2\Rightarrow\frac{20p}{4}<4\Rightarrow p<4/5$ ... (iii)

From (i),(ii) and (iii), we have

$p<-1$ i.e., $p\in(-\infty,-1)$

561 **(d)**
Since, ω is a complex cube root of unity
Now, $\omega^{10} + \omega^{23} = (\omega^3)^3\omega + (\omega^3)^7\omega^2$
$= \omega + \omega^2 = -1$
$\therefore\ \sin\left\{(\omega^{10} + \omega^{23})\pi - \frac{\pi}{6}\right\} = \sin\left(-\pi - \frac{\pi}{6}\right)$
$= \sin\frac{\pi}{6} = \frac{1}{2}$

562 **(b)**
Let $z = \frac{1+2i}{1-(1-i)^2} = 1$
$\therefore\ |z| = 1$ and amp $(z) = \tan^{-1}\left(\frac{0}{1}\right) = 0$

563 **(b)**
Let $f(x) = x^2 - 2kx + k^2 + k - 5$
Since, both roots are less than 5
Then, $D \geq 0, -\frac{b}{2a} < 5$ and $f(5) > 0$
Now, $D = 4k^2 - 4(k^2 + k - 5) = -4k + 20 \geq 0$
$\Rightarrow\ k \leq 5$ ...(i)
$-\frac{b}{2a} < 5 \Rightarrow k < 5$ ...(ii)
And $f(5) > 0$
$\Rightarrow\ 25 - 10k + k^2 + k - 5 > 0$
$\Rightarrow\ (k-5)(k-4) > 0$
$\Rightarrow\ k > 4$ and $k > 5$ ...(iii)
From Eqs. (i), (ii) and (iii), we get
$k < 4$

564 **(d)**
$\because\ z = \frac{7-i}{3-4i} = \frac{(7-i)(3+4i)}{(3)^2 - (4i)^2} = (1+i)$
$\therefore\ z^{14} = (1+i)^{14} = \left[\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]^{14}$
$= 2^7\left(\cos\frac{7\pi}{2} + i\sin\frac{7\pi}{2}\right) = -2^7 i$

565 **(b)**
Since roots of the equation $x^3 + bx^2 + 3x - 1 = 0$ form a non-decreasing H.P. Therefore, roots of the equation
$-x^3 + 3x^2 + bx + 1 = 0$ form a non-increasing A.P.
Let the roots be $a - d, a$ and $a + d$, where $d \leq 0$
$\therefore a - d + a + a + d = 3$ ...(i)
$a(a-d) + a(a+d) + a^2 - d^2 = -b$ ...(ii)
$a(a^2 - d^2) = 1$ ...(iii)
From (i), we have $a = 1$
Putting $a = 1$ in (iii), we get $d = 0$
Subtracting the values of a and d (ii), we get
$b = -3$

566 **(a)**
Given equation can be reduced to a quadratic equation.
$\therefore 2x^2 + x - 11 + \frac{1}{x} + \frac{2}{x^2} = 0$
$\Rightarrow 2\left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right) - 11 = 0$
Put $x + \frac{1}{x} = y$
$2(y^2 - 2) + y - 11 = 0$
$\Rightarrow 2y^2 + y - 15 = 0$
$\Rightarrow y = -3$ and $\frac{5}{2}$
$\Rightarrow\ x + \frac{1}{x} = -3, x + \frac{1}{x} = \frac{5}{2}$
$\Rightarrow x^2 + 3x + 1 = 0, 2x^2 - 5x + 2 = 0$
Only 2nd equation has rational roots as $D = 9$ and roots are $\frac{1}{2}$ and 2.

567 **(b)**
Let $f(x) = a\,x^2 + b\,x + c$. Then, $f(0) = c$
Thus, the curve $y = f(x)$ meets y-axis at $(0, c)$
If $c > 0$, then by hypothesis $f(x) > 0$. This means that the curve $y = f(x)$ does not meet x-axis
If $c < 0$, then by hypothesis, $f(x) < 0$, which means that the curve $y = f(x)$ is always below x-axis and so it does not intersect with x-axis
Thus, in both the cases $y = f(x)$ does not intersect with x-axis i. e. $f(x) \neq 0$ for any real x
Hence, $f(x) = 0$ i.e. $ax^2 + b\,x + c = 0$ has imaginary roots and so we have $b^2 < 4\,ac$

568 **(d)**
Since, α and β are the roots of given equation.
Let $f(x) = a^2x^2 + 2bx + 2c = 0$
Then, $f(\alpha) = a^2\alpha^2 + 2b\alpha + 2c = 0$
$= a^2\alpha^2 + 2(b\alpha + c) = a^2\alpha^2 - 2a^2\alpha^2$
$= -a^2\alpha^2 = -$ve
and $f(\beta) = a^2\beta^2 + 2(b\beta + c) = a^2\beta^2 + 2a^2\beta^2$
$= 3a^2\beta^2 = +$ve
Since, $f(\alpha)$ and $f(\beta)$ are of opposite signs therefore by theory of equations there lies a root γ of the equation $f(x) = 0$ between
α and $\beta, ie, \alpha < \gamma < \beta$.

569 **(c)**
We have,
$(1+i)^{2n} = (1-i)^{2n}$
$\Rightarrow \left(\frac{1+i}{1-i}\right)^{2n} = 1$
$\Rightarrow \left\{\frac{(1+i)^2}{(1+i)(1-i)}\right\}^{2n} = 1$
$\Rightarrow i^{2n} = 1$
$\Rightarrow 2n$ is a multiple of 4
$\Rightarrow$ The smallest positive value of n is 2

570 **(d)**
Given, $2\alpha = -1 - i\sqrt{3}$ and $2\beta = -1 + i\sqrt{3}$
$\therefore\ \alpha + \beta = -1$ and $\alpha\beta = 1$

Now, $5\alpha^4 + 5\beta^4 + \frac{7}{\alpha\beta}$

$= 5[\{(\alpha+\beta)^2 - 2\alpha\beta\}^2 - (\alpha\beta)^2 + \frac{7}{\alpha\beta}$

$= 5[\{(-1)^2 - 2\times 1\}^2 - 2(1)^2] + \frac{7}{1}$

$= 5(1-2) + 7 = 2$

571 **(a)**

$$\left[4\left(1-\frac{1}{3}+\frac{1}{9}-\frac{1}{27}+\dots\right)\right]^{\log_2 x} = \left[54\left(1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\dots\right)\right]^{\log_x 2}$$

$\Rightarrow \left[4\left(\frac{1}{1+1/3}\right)\right]^{\log_2 x} = \left[54\left(\frac{1}{1-1/3}\right)\right]^{\log_x 2}$

$\Rightarrow \left[4\left(\frac{3}{4}\right)\right]^{\log_2 x} = \left[54\times\frac{3}{2}\right]^{\log_x 2}$

$\Rightarrow 3^{\log_2 x} = 3^{4\log_x 2}$

$\Rightarrow \log_2 x = 4\log_x 2 = \frac{4}{\log_2 x}$

$\Rightarrow (\log_2 x)^2 = 4 \Rightarrow \log_2 x = \pm 2$

If $\log_2 x = 2$

$\Rightarrow x = 2^2 = 4$

And if $\log_2 x = -2$

$\Rightarrow \quad x = 2^{-2} = \frac{1}{4}$

$\therefore$ Solution set of the equation is $\left\{4, \frac{1}{4}\right\}$

572 **(c)**

Let $z = r(\cos\theta + i\sin\theta)$

Given that $\left|z+\frac{1}{z}\right| = a \Rightarrow \left|z+\frac{1}{z}\right|^2 = a^2$

$\Rightarrow r^2 + \frac{1}{r^2} + 2\cos 2\theta = a^2 \quad$ ...(i)

On differentiating w.r.t. θ, we get

$2r\frac{dr}{d\theta} - \frac{2}{r^3}\frac{dr}{d\theta} - 4\sin 2\theta = 0$

$\Rightarrow \frac{dr}{d\theta}\left(2r - \frac{2}{r^3}\right) = 4\sin 2\theta$

For maximum or minimum, put $\frac{dr}{d\theta} = 0$, we get

$\theta = 0, \frac{\pi}{2}$

$\therefore r$ is maximum for $\theta = \frac{\pi}{2}$, therefore from Eq.(i)

$r^2 + \frac{1}{r^2} - 2 = a^2 \Rightarrow r - \frac{1}{r} = a$

$\Rightarrow r^2 - ar - 1 = 0$

$\Rightarrow r = \frac{a+\sqrt{a^2+4}}{2}$

573 **(b)**

We have,

$6 + x - x^2 > 0$

$\Rightarrow x^2 - x - 6 < 0 \Rightarrow (x-3)(x+2) < 0 \Rightarrow -2 < x < 3$

574 **(c)**

$(\cos\theta + i\sin\theta)(\cos 2\theta + i\sin 2\theta)\dots(\cos n\theta + i\sin n\theta) = 1$

$\Rightarrow \cos(\theta + 2\theta + 3\theta + \dots + n\theta) + i\sin(\theta + 2\theta + 3\theta + \dots + n\theta) = 1$

$\Rightarrow \cos\left(\frac{n(n+1)}{2}\theta\right) + i\sin\left(\frac{n(n+1)}{2}\theta\right) = 1$

On comparing the coefficients of real and imaginary on both sides, we get

$\cos\left(\frac{n(n+1)}{2}\theta\right) = 1$

and $\sin\left(\frac{n(n+1)}{2}\theta\right) = 0$

$\Rightarrow \left(\frac{n(n+1)}{2}\theta\right) = 2m\pi$

$\Rightarrow \theta = \frac{4m\pi}{n(n+1)}$, where $m \in I$

575 **(b)**

Here, $\alpha+\beta+\gamma = 6, \quad \alpha\beta+\beta\gamma+\gamma\alpha = 11$

And $\alpha\beta\gamma = -6$

Now, $\sum\alpha^2\beta + \sum\alpha\beta^2 = \alpha^2\beta + \beta^2\alpha + \gamma^2\alpha + \alpha\beta 2 + \beta\gamma 2 + \gamma\alpha 2$

$= \alpha\beta(\alpha+\beta) + \beta\gamma(\beta+\gamma) + \gamma\alpha(\gamma+\alpha)$

$= \alpha\beta(6-\gamma) + \beta\gamma(6-\alpha) + \gamma\alpha(6-\beta)$

$= 6(\alpha\beta+\beta\gamma+\gamma\alpha) - 3\alpha\beta\gamma$

$= 6(11) + 3(6) = 84$

576 **(b)**

We have,

$z_k = r_k(\cos\alpha_k + i\sin\alpha_k)$ and $\omega_k = \frac{\cos 2\alpha_k + i\sin 2\alpha_k}{z_k}$

$\Rightarrow \omega_k = \frac{z_k}{r_k^2}, k = 1,2,3$

$\Rightarrow \omega_1 = \frac{z_1}{|z_1|^2}, \omega_2 = \frac{z_2}{|z_2|^2}\omega_3 = \frac{z_3}{|z_3|^2}$

$\Rightarrow \omega_1 = \frac{1}{\bar{z}_1}, \omega_2 = \frac{1}{\bar{z}_2}, \omega_3 = \frac{1}{\bar{z}_3}$

$\Rightarrow \omega_1 + \omega_2 + \omega_3 = \frac{1}{\bar{z}_1} + \frac{1}{\bar{z}_2} + \frac{1}{\bar{z}_3}$

$\Rightarrow \omega_1 + \omega_2 + \omega_3 \Rightarrow \frac{\omega_1+\omega_2+\omega_3}{3} = 0$

Hence, the centroid of $\Delta A_1 A_2 A_3$ is at the origin

577 **(c)**

Let $z = x + iy$

$\therefore \quad \left|\frac{z-25}{z-1}\right| = 5$

$\Rightarrow \left|\frac{(x-25)+iy}{(x-1)+iy}\right| = 5$

$\Rightarrow \quad |(x-25)+iy| = 5|(x-1)+iy|$

$\Rightarrow \quad \sqrt{(x-25)^2+y^2} = 5\sqrt{(x-1)^2+y^2}$

On squaring both sides, we get

$(x-25)^2 + y^2 = 25\{(x-1)^2 + y^2\}$

$\Rightarrow \quad x^2 - 50x + 625 + y^2 = 25x^2 - 50x + 25 + 25y^2$

$\Rightarrow \quad 24x^2 + 24y^2 = 600$

$\Rightarrow \quad x^2 + y^2 = 25$

$\Rightarrow \quad \sqrt{x^2 + y^2} = 5 \qquad [\because |z| = \sqrt{(x^2 + y^2)}]$

$\Rightarrow \quad |z| = 5$

578 **(c)**

We have, $ax^2 - bx(x-1) + c(x-1)^2 = 0$ ...(i)

$\Rightarrow a\left(\frac{x}{1-x}\right)^2 + b\left(\frac{x}{1-x}\right) + c = 0$

Also, α and β be the roots of $ax^2 + bx + c = 0$.

$\therefore \alpha = \frac{x}{1-x}$ and $\beta = \frac{x}{1-x}$

$\Rightarrow x = \frac{\alpha}{\alpha+1}, x = \frac{\beta}{\beta+1}$

Hence, $\frac{\alpha}{\alpha+1}$ and $\frac{\beta}{\beta+1}$ are the required roots.

579 **(b)**

Let $z = \frac{13-5i}{4-9i} \times \frac{4+9i}{4+9i} = \frac{97+97i}{97} = 1 + i$

$\therefore \quad \arg(z) = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$

580 **(b)**

It is given that $\sin\theta, \sin\alpha, \cos\theta$ are in G.P.

$\therefore \sin^2\alpha = \sin\theta\cos\theta$

$\Rightarrow 2\sin^2\alpha = \sin 2\theta \Rightarrow 1 - \cos 2\alpha = \sin 2\theta$ ...(i)

Let D be the discriminant of the equation

$x^2 + 2x\cot\alpha + 1 = 0$

Then,

$D = 4\cot^2\alpha - 4 = 4\,\frac{\cos 2\alpha}{\sin^2\alpha}$

$= 4\frac{(1-\sin 2\theta)}{\sin^2\alpha}$ [Using (i)]

$\Rightarrow D = 4\left(\frac{\cos\theta - \sin\theta}{\sin\alpha}\right)^2 > 0$

Hence, the roots of the given equation are real

581 **(a)**

Since, $(1+2i)$, $(2-\sqrt{3})$ and 5 are the some roots of polynomial $f(x)$ of degree n. As we know that conjugate are also the roots of the polynomial. Therefore, $(1-2i)$ and $(2+\sqrt{3})$ are also the roots of the polynomial.

$\therefore$ The least value of n is 5

582 **(b)**

Given, $\frac{3x}{(x-a)(x-b)} = \frac{2}{(x-a)} + \frac{1}{(x-b)}$

$\Rightarrow \quad 3x = 2(x-b) + 1(x-a)$

On comparing the coefficient of constant term, we get

$-2b - a = 0$

$\Rightarrow \quad \frac{a}{b} = -\frac{2}{1}$ or $\quad a{:}b = -2{:}1$

583 **(b)**

Given, $x + iy = \left(\frac{1+2i}{3+4i}\right)^{\frac{1}{2}}$

$\Rightarrow \quad (x+iy)^2 = \frac{1+2i}{3+4i}$

Taking modulus from both sides we get

$|x+iy|^2 = \left|\frac{1+2i}{3+4i}\right|$

$\Rightarrow \quad x^2 + y^2 = \sqrt{\frac{1+4}{9+16}}$

$\Rightarrow \quad (x^2+y^2)^2 = \frac{5}{25} = \frac{1}{5}$

584 **(a)**

Given, $f(x) = (x-1)(x-2)(x-3)(x-4)$

The real roots are 1, 2, 3, 4

Hence, only 2 lies in the interval (1, 3)

585 **(d)**

$|3z - 1| = 3|z - 2|$

$\Rightarrow \quad \left|z - \frac{1}{3}\right| = |z - 2|$

$\Rightarrow$ z is perpendicular bisector of $\left(\frac{1}{3}, 0\right)$ and (2,0)

$\Rightarrow \quad x = \frac{7}{6}$

586 **(c)**

Let P, Q, R be the vertices of the triangle having affixes z_1, z_2 and $(1-i)z_1 + i\,z_2$ respectively.

Then,

$|PQ| = |z_2 - z_1|$,

$|QR| = |(1-i)z_1 - (1-i)z_2| = \sqrt{2}|z_1 - z_2|$

and, $|RP| = |(1-i)z_1 + i\,z_2 - z_1| = |i(z_2 - z_1)|$

$= |z_2 - z_1|$

Clearly, $|PQ| = |RP|$ and $|QR|^2 = |PQ|^2 + |RP|^2$

Hence, ΔPQR is isosceles right angled triangle

587 **(c)**

$\because \quad x + \frac{1}{x} = 2\sin\alpha$

$\Rightarrow x^2 - 2x\sin\alpha + 1 = 0$

$\therefore \quad x = \frac{2\sin\alpha \pm \sqrt{4\sin^2\alpha - 4}}{2}$

$\Rightarrow \quad x = \sin\alpha \pm i\,\cos\alpha$

Similarly, $\quad y = \cos\beta \pm i\cos\beta$

$\therefore \quad xy = (\sin\alpha \pm i\cos\alpha)(\cos\beta \pm i\sin\beta)$

$= \sin(\beta - \alpha) \pm i\cos(\beta - \alpha)$

$xy = \pm i[\cos(\beta - \alpha) - i\sin(\beta - \alpha)]$

And $\frac{1}{xy} = \pm\frac{1}{i}[\cos(\beta - \alpha) + i\sin(\beta - \alpha)]$

Now, $\quad (xy)^3 + \frac{1}{(xy)^3} = \pm i^3[\cos 3(\beta - \alpha) -$

$i\sin 3\beta - \alpha$

$\pm\frac{1}{i^3}[\cos 3(\beta - \alpha) + i\sin 3(\beta - \alpha)]$

$= \pm i\,[\cos 3(\beta - \alpha) - i\sin 3(\beta - \alpha)]$

$\pm\frac{1}{i}[\cos 3(\beta - \alpha) + i\sin 3(\beta - \alpha)]$

$= \pm\frac{1}{i}[\{\cos 3(\beta-\alpha) - i\sin 3(\beta-\alpha)\}$
$- \{\cos 3(\beta-\alpha) + i\sin 3(\beta-\alpha)\}]$
$= \pm\frac{1}{i}(-2i\sin 3(\beta-\alpha)) = 2\sin 3(\beta-\alpha)$

588 **(a)**

The three cube roots of $p(p<0)$ (i.e. solutions of $x^3 - p = 0$) are $p^{1/3}, p^{1/3}\omega, p^{1/3}\omega^2$

Let $\alpha = p^{1/3}, \beta = p^{1/3}\omega, \gamma = p^{1/3}\omega^2$. Then,

$$\frac{x\,\alpha + y\,\beta + z\,\gamma}{x\,\beta + y\,\gamma + z\,\alpha} = \frac{x + y\,\omega + z\,\omega^2}{x\,\omega + y\,\omega^2 + z} = \omega^2$$

If $\alpha = p^{1/3}, \gamma = p^{1/3}\omega^2$, then

$$\frac{x\,\alpha + y\,\beta + z\,\gamma}{x\,\beta + y\,\gamma + z\,\alpha} = \frac{x + y\,\omega^2 + z\,\omega}{x\,\omega^2 + y\,\omega + z}$$
$$= \omega\frac{(x + y\,\omega^2 + z\,\omega)}{x\,\omega^3 + y\,\omega^2 + z\,\omega} = \omega$$

Every other choice of a, β, γ will give its value as ω or ω^2

590 **(c)**

Since, $a = \cos\theta + i\sin\theta$

$$\therefore\ \frac{1+a}{1-a} = \frac{1+\cos\theta + i\sin\theta}{1-\cos\theta - i\sin\theta}$$
$$= \frac{[(1+\cos\theta) + i\sin\theta][(1-\cos\theta) + i\sin\theta]}{[(1-\cos\theta) - i\sin\theta][(1-\cos\theta) + i\sin\theta]}$$
$$= \frac{2i\sin\theta}{(1-\cos\theta)^2 + \sin^2\theta}$$
$$= \frac{i.4\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{4\sin^2\frac{\theta}{2}} = i\cot\frac{\theta}{2}$$

591 **(b)**

$$\frac{z}{\bar{z}} + \frac{\bar{z}}{z} = \frac{re^{i\theta}}{re^{-i\theta}} + \frac{re^{-i\theta}}{re^{i\theta}} = e^{i2\theta} + e^{-i2\theta} = 2\cos 2\theta$$

592 **(a)**

In a parallelogram $OP_1P_2P_3$, the mid point of P_1P_2 and OP_3 are the same. But mid point of P_1P_2 is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

So, that the coordinates of P_3 are $(x_1 + x_2, y_1 + y_2)$

Thus, the point P_3 corresponds to sum of the complex numbers z_1 and z_2

$\therefore\ \overrightarrow{\mathbf{OP}}_3 = \overrightarrow{\mathbf{OP}}_1 + \overrightarrow{\mathbf{OP}}_2 = z_1 + z_2$

593 **(d)**

Let $z = a + ib$

$\therefore\ \arg(z) = \theta = \tan^{-1}\frac{b}{a}$

$\because\ \bar{z} = a - ib$

$\therefore\ \arg(\bar{z}) = \tan^{-1}\left(-\frac{b}{a}\right) = -\tan^{-1}\left(\frac{b}{a}\right) = -\theta$

594 **(a)**

We know that, $|-z| = |z|$

and $|z_1 + z_2| \le |z_1| + |z_2|$

Now, $|z| + |z-1| = |z| + |1-z|$

$\ge |z + (1-z)| = |1| = 1$

Hence, minimum value of $|z| + |z-1|$ is 1

595 **(d)**

Given numbers are conjugate to each other,

$\therefore\ \sin x + i\cos 2x = \cos x - i\sin 2x$

$\sin x = \cos x$

And $\cos 2x = \sin 2x$

$\therefore\ \tan x = 1 \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \dots$ ...(i)

And $\tan 2x = 1 \Rightarrow 2x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \dots$

$\Rightarrow x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \dots$ ...(ii)

There exists no value of x common in Eqs. (i) and (ii)

596 **(c)**

Let α be a common root of $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$. Then,

$\alpha^2 + a\,\alpha + b = 0$ and $\alpha^2 + b\,\alpha + a = 0$

$\Rightarrow (\alpha^2 + a\,\alpha + b) - (\alpha^2 + b\,\alpha + a) = 0$

$\Rightarrow \alpha(a-b) = (a-b) \Rightarrow \alpha = 1$

Putting $\alpha = 1$, in either of these two, we get $a + b = -1$

597 **(c)**

$$\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{3}$$
$$\Rightarrow\ \arg(x - 2 + iy) - \arg(x + 2 + iy) = \frac{\pi}{3}$$
$$\Rightarrow\ \tan^{-1}\left(\frac{y}{x-2}\right) - \tan^{-1}\left(\frac{y}{x+2}\right) = \frac{\pi}{3}$$
$$\Rightarrow\ \tan^{-1}\left(\frac{\frac{y}{x-2} - \frac{y}{x+2}}{1 + \frac{y}{x-2}.\frac{y}{x+2}}\right) = \frac{\pi}{3}$$
$$\Rightarrow\ \frac{4}{x^2 + y^2 - 4} = \tan\frac{\pi}{3} = \sqrt{3}$$
$$\Rightarrow\ 4y = \sqrt{3}(x^2 + y^2 - 4)$$
$$\Rightarrow\ \sqrt{3}(x^2 + y^2) - 4\sqrt{3} - 4y = 0$$

Which represents the equation of a circle.

598 **(b)**

If $b^2 - 4ac \ge 0$, then the equation $ax^4 + bx^2 + c = 0$ has all roots positive real, if $b < 0, a > 0, c > 0$

599 **(b)**

We know that principle argument of a complex number lie between $-\pi$ and π, but $\alpha + \beta > \pi$

Therefore, principle

$\arg(z_1z_2) = \arg(z_1) + \arg(z_2) = \alpha + \beta$ is give by $\alpha + \beta - \pi$

600 **(b)**

The given equation will represent a circle with the line segment joining $P(\omega)$ and $Q(\omega^2)$ as a diameter, if

$\lambda = PQ^2 = |\omega - \omega^2|^2 \Rightarrow \lambda = 3$

602 **(d)**

Let $z = (1)^{1/3}$

$z^3 - 1 = 0$

$\Rightarrow \quad (z-1)(z^2+z+1) = 0$

$\Rightarrow \quad z = 1, \dfrac{-1 \pm \sqrt{1-4}}{2}$

$\Rightarrow \quad z = 1, \dfrac{-1 \pm \sqrt{3}i}{2}$

Hence, $\frac{-1-\sqrt{3}i}{2}$ is one of the root of $(1)^{1/3}$

603 **(a)**

Let α and 3 are the roots of the equation

$x^2 + ax + 3 = 0$

$\therefore \quad 3\alpha = 3 \quad \Rightarrow \quad \alpha = 1$

And $3 + \alpha = -a \quad \Rightarrow \quad a = -4$

Again, let β and 3β are the roots of the equation

$x^2 + ax + b = 0$

$\therefore \quad \beta + 3\beta = 4\beta = -a \quad \Rightarrow \quad \beta = 1$

And $\beta . 3\beta = b \quad \Rightarrow \quad b = 3$

604 **(b)**

We have,

$|z - 4 - 3i| \leq 1$

But, $|z - 4 - 3i| = |z - (4+3i)| \geq \big||z| -$
4+3i

$\Rightarrow 1 \geq \big||z| - 5\big|$

$\Rightarrow \big||z| - 5\big| \leq 1$

$\Rightarrow -1 \leq |z| - 5 \leq 1$

$\Rightarrow 4 \leq |z| \leq 6 \Rightarrow m = 4$ and $n = 6$

Let $y = \dfrac{x^4 + x^2 + 4}{x}$

$\Rightarrow y = x^3 + x + \dfrac{4}{x}$

$\Rightarrow y = x^3 + x + \dfrac{1}{x} + \dfrac{1}{x} + \dfrac{1}{x} + \dfrac{1}{x}$

Clearly, the product of $x^3, x, \frac{1}{x}, \frac{1}{x}, \frac{1}{x}, \frac{1}{x}$ is 1 i.e. a constant. So, their sum i.e. y will be least when they are equal i.e.

$x^3 = x = \dfrac{1}{x} \Rightarrow x = 1$

$\therefore$ Least value of $y = 1 + 1 + 4 = 6$

Hence, $\lambda = 6$

605 **(b)**

Given equation is $(5+\sqrt{2})x^2 - (4+\sqrt{5})x + 8 + 2\sqrt{5} = 0$.

Let x_1 and x_2 are the roots of the equation.

$\Rightarrow x_1 + x_2 = \frac{4+\sqrt{5}}{5+\sqrt{2}}$ ....(i)

and $x_1 x_2 = \frac{8+2\sqrt{5}}{5+\sqrt{2}} = \frac{2(4+\sqrt{5})}{5+\sqrt{2}} = 2(x_1 + x_2)$ ...(ii)

$\therefore$ Harmonic mean $= \frac{2x_1x_2}{x_1+x_2} = \frac{4(x_1+x_2)}{(x_1+x_2)} = 4$ [from Eq. (ii)]

606 **(b)**

We have,

$$\begin{vmatrix} a & u & 1 \\ b & v & 1 \\ c & w & 1 \end{vmatrix} = \begin{vmatrix} a & u & 1 \\ b & v & 1 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

Applying $R_3 \to R_3 - (1-r)R_1 - r\,R_2$

Hence, two triangle are similar

607 **(c)**

It is given that the roots are of opposite signs

$\therefore$ Product of roots < 0

$\Rightarrow \dfrac{k^2 - 3\,k + 2}{3} < 0 \Rightarrow k^2 - 3k + 2 < 0 \Rightarrow k \in (1, 2)$

608 **(b)**

Given, $\text{Re}\left(\frac{1}{z}\right) = k \quad \Rightarrow \quad \text{Re}\left(\frac{1}{x+iy}\right) = k$

$\Rightarrow \quad \text{Re}\left(\dfrac{x}{x^2+y^2} - \dfrac{iy}{x^2+y^2}\right) = k$

$\Rightarrow \quad k = \dfrac{x}{x^2+y^2}$

$\Rightarrow \quad x^2 + y^2 - \dfrac{1}{k}x = 0$

Which is an equation of circle.

609 **(b)**

Let $z = x + i\,y$. Then, $z \neq 0 \Rightarrow x \neq 0, y \neq 0$

Now,

$\arg(z) = \dfrac{\pi}{4}$

$\Rightarrow z$ lies on the line $y = x$ lying in the first quadrant

$\therefore x = y > 0 \;\Rightarrow \text{Re}\,(z) = Im(z) > 0$

610 **(c)**

Given, $|z_1| = |z_2| = \cdots |z_n| = 1$

$\Rightarrow \quad |z_1|^2 = |z_1|^2 = \cdots |z_n|^2 = 1$

$\Rightarrow \quad z_1\overline{z_1} = z_1\overline{z_2} = \cdots = z_n\overline{z_n} = 1$

$\Rightarrow \quad \overline{z_1} = \frac{1}{z_1}, \quad \overline{z_2} = \frac{1}{z_2}, \dots . \overline{z_n} = \frac{1}{z_n}$ ...(i)

Now, $|z_1 + z_2 + \ldots + z_n|$

$= |\overline{z_1 + z_2 + \ldots + z_n}| = |\overline{z_1} + \overline{z_2} + \cdots + \overline{z_n}|$

$= \left|\frac{1}{z_1} + \frac{1}{z_2} + \ldots + \frac{1}{z_n}\right|$ [using Eq. (i)]

611 **(c)**

We have, $z_r = \cos\frac{r\alpha}{n^2} + i\sin\frac{r\alpha}{n^2}$

where $r = 1, 2, 3, \dots, n$

$\therefore \quad z_1 = \cos\dfrac{\alpha}{n^2} + i\sin\dfrac{\alpha}{n^2};$

$z_2 = \cos\dfrac{2\alpha}{n^2} + i\sin\dfrac{2\alpha}{n^2}$

$\vdots \qquad \vdots \qquad \vdots$

$z_n = \cos\dfrac{n\alpha}{n^2} + i\sin\dfrac{\alpha}{n^2}$

$\therefore \quad \lim_{n\to\infty}(z_1 z_2 z_3 \dots z_n)$

$$= \lim_{n\to\infty} \left(\cos\frac{\alpha}{n^2} + i\sin\frac{\alpha}{n^2}\right)\left(\cos\frac{2\alpha}{n^2} + i\sin\frac{2\alpha}{n^2}\right)\ldots\left(\cos\frac{n\alpha}{n^2} + i\sin\frac{n\alpha}{n^2}\right)$$

$$= \lim_{n\to\infty}\left[\cos\left\{\frac{\alpha}{n^2}(1+2+3+\ldots+n)\right\} + i\sin\left\{\frac{\alpha}{n^2}(1+2+3+\ldots+n)\right\}\right]$$

$$= \lim_{n\to\infty}\left[\cos\frac{\alpha n(n+1)}{2n^2} + i\sin\frac{\alpha n(n+1)}{2n^2}\right]$$

$$= \cos\frac{\alpha}{2} + i\sin\frac{\alpha}{2}$$

$$= e^{\frac{i\alpha}{2}}$$

612 **(b)**

Here, $\alpha+\beta+\gamma = 3, \alpha\beta+\beta\gamma+\gamma\alpha = 1$ and $\alpha\beta\gamma = -5$

Now, $y = \alpha^2+\beta^2+\gamma^2+\alpha\beta\gamma$

$= (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\gamma\alpha) + \alpha\beta\gamma$

$= (3)^2 - 2(1) - 5$

$\Rightarrow y = 2$

So, $y = 2$ satisfies the equation

$y^3 - y^2 - y - 2 = 0$

614 **(c)**

$(1+i)^n + (1-i)^n$

$$= \left(\sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)\right)^n + \left(\sqrt{2}\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)\right)^n$$

$$= 2^{n/2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^n + 2^{n/2}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)^n$$

$$= 2^{n/2}\left(\cos\frac{n\pi}{4} + \sin\frac{n\pi}{4} + \cos\frac{n\pi}{4} - i\sin\frac{n\pi}{4}\right)$$

$$= 2^{\frac{n}{2}+1}\cos\left(\frac{n\pi}{4}\right) = \left(\sqrt{2}\right)^{n+2}\cos\left(\frac{n\pi}{4}\right)$$

615 **(c)**

Let $y = \frac{3-x+2-x}{2} = \frac{5-2x}{2}$. Then,

$(3-x)^4 + (2-x)^4 = (5-2\,x)^4$

$$\Rightarrow \left(\frac{2y-1}{2}\right)^4 + \left(\frac{2y+1}{2}\right)^4 = (2y)^4$$

$\Rightarrow (4y^2+1-4y)^2 + (4y^2+1+4y)^2 = 256y^4$

$\Rightarrow 112y^4 - 24y^2 - 1 = 0$

$\Rightarrow (28y^2+1)(4y^2-1) = 0$

$$\Rightarrow y = \pm\frac{1}{2} \Rightarrow x = 2.,3 \quad \left[\because x = \frac{5-2y}{2}\right]$$

The equation $7\,x^2 - 35\,x + 44 = 0$ has imaginary roots. Thus, the given equation has two real and two imaginary roots

616 **(c)**

Let $z = \frac{1+2i}{1-i} = \frac{(1+2i)(1+i)}{(1-i)(1+i)} = -\frac{1}{2} + \frac{3}{2}i$

Here, coefficient of x is negative and y is positive, therefore it lies in the second quadrant

617 **(c)**

Since α, β are roots of $a\,x^2 + b\,x + c = 0$

$\therefore \alpha+\beta = -b/a, \alpha\,\beta = c/a$

The equation $ax^2 - bx(x-1) + c(x-1)^2 = 0$ can be written as

$x^2(a-b+c) + x(b-2c) + c = 0$

Let, γ, δ be its roots. Then,

$$\gamma+\delta = -\frac{(b-2c)}{a-b+c} = \frac{-b+2\,c}{a-b+c} = \frac{-\frac{b}{a}+\frac{2\,c}{a}}{1-\frac{b}{a}+\frac{c}{a}}$$

$$\Rightarrow \gamma+\delta = \frac{\alpha+\beta+2\,\alpha\alpha\beta}{1+\alpha+\beta+\alpha\,\beta} = \frac{\alpha}{\alpha+1} + \frac{\beta}{\beta+1}$$

$$\text{and,} \gamma\,\delta = \frac{c}{a-b+c} = \frac{\frac{c}{a}}{1-\frac{b}{a}+\frac{c}{a}}$$

$$= \frac{\alpha\,\beta}{1+\alpha+\beta+\alpha\,\beta} = \frac{\alpha}{\alpha+1}\cdot\frac{\beta}{\beta+1}$$

Thus, the equation $a\,x^2 - b\,x(x-1) + c(x-1)^2 = 0$ has $\gamma = \frac{\alpha}{\alpha+1}$ and $\delta = \frac{\beta}{\beta+1}$ as its two roots

618 **(b)**

Since, $\frac{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right) - i\tan x}{1+2i\sin\frac{x}{2}} \in R$

$$\Rightarrow \frac{\left\{\sin\frac{x}{2}+\cos\frac{x}{2} - i\tan x\right\}\left\{1-2i\sin\frac{x}{2}\right\}}{1+4\sin^2\frac{x}{2}} \in R$$

It will be real, if imaginary part is zero

$$\therefore -2\sin\frac{x}{2}\left\{\sin\frac{x}{2}+\cos\frac{x}{2}\right\} - \tan x = 0$$

$$\Rightarrow 2\sin\frac{x}{2}\left\{\sin\frac{x}{2}+\cos\frac{x}{2}\right\} + \frac{\sin x}{\cos x} = 0$$

$$\Rightarrow \sin\frac{x}{2}\left[\left\{\sin\frac{x}{2}+\cos\frac{x}{2}\right\}\left\{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right\} + \cos\frac{x}{2}\right] = 0$$

$$\therefore \sin\frac{x}{2} = 0$$

$\Rightarrow x = 2n\pi$ ...(i)

or $\left\{\sin\frac{x}{2}+\cos\frac{x}{2}\right\}\left\{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right\} + \cos\frac{x}{2} = 0$

on dividing by $\cos^3\frac{x}{2}$

$$\left(\tan\frac{x}{2}+1\right)\left(1-\tan^2\frac{x}{2}\right) + \left(1+\tan^2\frac{x}{2}\right) = 0$$

$$\Rightarrow \tan^3\frac{x}{2} - \tan\frac{x}{2} - 2 = 0$$

Let $\tan\frac{x}{2} = t$, then $f(t) = t^3 - t - 2$,

Then $f(1) = -2 < 0$ and $f(2) = 4 > 0$

Thus, $f(t)$ changes sign from negative to positive in $(1, 2)$

$\therefore$ Let $t = k$ be the root for which $f(k) = 0$ and $k \in (1, 2)$

$\therefore$ $t = k$ or $\tan\frac{x}{2} = k = \tan\alpha$

Hence, $\frac{x}{2} = n\pi + \alpha$

$$\Rightarrow \begin{cases} x = 2n\pi + 2\alpha \quad \alpha = \tan^{-1}k \\ \quad \text{or } x = 2n\pi \end{cases} \quad \text{where}$$

$k \in (1, 2)$

619 **(a)**

We have,

$x^2 - 6x + 5 \leq 0$ and $x^2 - 2x > 0$

$\Rightarrow (x-1)(x-5) \leq 0$ and $x(x-2) > 0$

$\Rightarrow 1 \leq x \leq 5$ and $(x < 0$ or $x > 2)$

$\Rightarrow 2 < x \leq 5 \Rightarrow x = 3, 4, 5 \quad [\because x \in Z]$

620 **(b)**

Now, $|(az_1 - bz_2)|^2 + |(bz_1 + az_2)|^2$

$= a^2|z_1|^2 + b^2|z_2|^2 - 2ab\ \text{Re}|z_1\bar{z}_2| + b^2|z_1|^2 + a^2|z_2|^2 + 2ab\ \text{Re}|\bar{z}_1 z_2|$

$= (a^2 + b^2)(|z_1|^2 + |z_2|^2)$

621 **(b)**

It is given that α, β are roots of $6x^2 - 5x + 1 = 0$

$\therefore \alpha + \beta = \frac{5}{6}$ and $\alpha\beta = \frac{1}{6}$

$\therefore \tan^{-1}\alpha + \tan^{-1}\beta$

$= \tan^{-1}\left(\frac{\alpha + \beta}{1 - \alpha\beta}\right) = \tan^{-1}\left(\frac{\frac{5}{6}}{1 - \frac{1}{6}}\right) = \tan^{-1} 1 = \frac{\pi}{4}$

622 **(c)**

We have, $|z_k| = 1, k = 1, 2 \dots, n$

$\Rightarrow \ |z_k|^2 = 1 \ \Rightarrow \ z_k\bar{z}_k = 1 \ \Rightarrow \ \bar{z}_k = \frac{1}{z_k}$

$\therefore \ |z_1 + z_2 + \ldots + z_n| = |\overline{z_1 + z_2 + \ldots + z_n}| \quad (\because |z| = |\bar{z}|)$

$= |\bar{z}_1 + \bar{z}_2 + \ldots + \bar{z}_n|$

$= \left|\frac{1}{z_1} + \frac{1}{z_2} + \ldots + \frac{1}{z_n}\right|$

623 **(d)**

We have,

$\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7} - i\cos\frac{2\pi k}{7}\right)$

$= \sum_{k=1}^{6} -i\left(\cos\frac{2\pi k}{7} + i\sin\frac{2\pi k}{7}\right)$

$= \sum_{k=1}^{6} -i\, e^{\frac{i2\pi k}{7}} = -i\sum_{k=1}^{6} r^k$, where $r = e^{\frac{i2\pi}{7}}$

$= -i\frac{r(1-r^6)}{(1-r)} = -i\left(\frac{r - r^7}{1-r}\right) = -i\left(\frac{r-1}{1-r}\right) = i \quad [\because r^7 = 1]$

624 **(d)**

<u>CASE I</u> *When* $x \geq 0$

In this case, we have $|x| = x$

$\therefore x^2 + x + |x| + 1 < 0$

$\Rightarrow x^2 + 2x + 1 < 0 \Rightarrow (x+1)^2 < 0$, which is not true

<u>CASE II</u> *When* $x < 0$

In this case, we have $|x| = -x$

$\therefore x^2 + x + |x| + 1 \leq 0$

$\Rightarrow x^2 + 1 \leq 0$, which is not true for any $x < 0$

Hence, there is no value of x satisfying the given inequation

625 **(c)**

We have, $\omega_n = \cos\left(\frac{2\pi}{n}\right) + i\sin\left(\frac{2\pi}{n}\right)$

$\Rightarrow \ \omega_3 = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$

$= -\frac{1}{2} + \frac{i\sqrt{3}}{2} = \omega$

and $\omega_3^2 = \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)^2$

$= \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$

$= -\frac{1}{2} - \frac{i\sqrt{3}}{2} = \omega^2$

$\therefore \ (x + y\omega_3 + z\omega_3^2)(x + y\omega_3^2 + z\omega_3)$

$= (x + y\omega + z\omega^2)(x + y\omega^2 + z\omega)$

$= x^2 + y^2 + z^2 - xy - yz - zx$

626 **(c)**

$\because \ x^2 + 15|x| + 14$

$= |x^2| + 15|x| + 14 > 0$

For all real x

$\Rightarrow$ Given equation has no solution

627 **(b)**

It is given that α, β, γ are the roots of the equation $x^3 + ax^2 + bx + c = 0$

$\therefore \alpha + \beta + \gamma = -a, \alpha\beta + \beta\gamma + \gamma\alpha = b$ and, $\alpha\beta\gamma = c$

Hence,

$\alpha^{-1} + \beta^{-1} + \gamma^{-1} = \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\sum\alpha\beta}{\alpha\beta\gamma} = -\frac{b}{c}$

628 **(d)**

Domain of the function $y = \sqrt{x(x-3)}$ is

$x(x-3) \geq 0$

$\Rightarrow x \leq 0$ or $x \geq 3 \quad$ ...(i)

Given equation can be rewritten as

$9|x|^2 - 19|x| + 2 = 0$

$\Rightarrow (9|x| - 1)(|x| - 2) = 0$

$\Rightarrow |x| = 2$ or $|x| = \frac{1}{9}$

$\therefore$ Solution of the given equation are $\pm 2, \pm\frac{1}{9}$.

In the domain (i) the required solutions are $-2, -\frac{1}{9}$.

629 **(b)**

Since, α is an imaginary cube root of unity. Let it be ω, then $\alpha^{3n+1} + \alpha^{3n+3} + \alpha^{3n+5} = (\omega)^{3n+1} + (\omega)^{3n+3} + (\omega)^{3n+5}$

$= \omega + 1 + \omega^5$

$= \omega + 1 + \omega^2 = 0$

630 **(b)**

Given, $z^2 + \bar{z} = 0$
$\therefore\ (x + iy)^2 + (x - iy) = 0$
$\Rightarrow\ x^2 - y^2 + x + i(2xy - y) = 0$
$\Rightarrow\ x^2 - y^2 + x = 0$ and $2xy - y = 0$
Now, $2xy - y = 0 \Rightarrow y = 0,\ x = \frac{1}{2}$
When $y = 0,\ x^2 - 0 + x = 0 \Rightarrow x = 0, -1$
When $x = \frac{1}{2}$,
$\left(\frac{1}{2}\right)^2 - y^2 + \frac{1}{2} = 0 \Rightarrow y^2 = \frac{1}{4} + \frac{1}{2} \Rightarrow y = \pm\frac{\sqrt{3}}{2}$
$\therefore$ Solutions are $(0,0), (-1,0), \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right), \left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$

631 **(b)**
$\log_{\sqrt{3}}\left(\frac{|z|^2 - |z| + 1}{2 + |z|}\right) < 2$
$\Rightarrow \frac{|z|^2 - |z| + 1}{2 + |z|} < (\sqrt{3})^2$
$\Rightarrow |z|^2 - |z| + 1 < 3(2 + |z|)$
$\Rightarrow |z|^2 - 4|z| - 5 < 0$
$\Rightarrow (|z| + 1)(|z| - 5) < 0$
$\Rightarrow -1 < |z| < 5 \Rightarrow |z| < 5$ as $|z| > 0$
$\therefore$ Locus of z is $|z| < 5$

632 **(b)**
Since, 2 and 3 are the roots of the equation
$2x^3 + mx^2 - 13x + n = 0$
$\therefore\ f(2) = 2(2)^3 + m(2)^2 - 13(2) + n = 0$
And $f(3) = 2(3)^3 + m(3)^2 - 13(3) + n = 0$
$\Rightarrow 4m + n = 10$ and $9m + n = -15$
$\Rightarrow m = -5,\ n = 30$

633 **(d)**
The affix of the centroid G of the triangle is $(z_1 + z_2 + z_3)/3$
Since the centroid G divides the line joining the circumcentre and orthocentre in the ratio $1 : 2$.
Therefore, if z is the affix of the orthocentre, then
$\frac{z_1 + z_2 + z_3}{3} = \frac{1.z + 2 \cdot 0}{1 + 2} \Rightarrow z = z_1 + z_2 + z_3$

634 **(c)**
$xyz = (\alpha + \beta)(\alpha\omega + \beta\omega^2)(\alpha\omega^2 + \beta\omega)$
$= (\alpha + \beta)[\alpha^2 + \alpha\beta(\omega^2 + \omega) + \beta^2]$
$\left[\because 1 + \omega + \omega^2 = 0 \text{ and } \omega^3 = 1\right]$
$= (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$
$= \alpha^3 + \beta^3$

635 **(d)**
Given, $n = 2006!$
$\therefore \frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \ldots + \frac{1}{\log_{2006} n}$
$= \log_n 2 + \log_n 3 + \ldots + \log_n 2006$
$= \log_n(2.3.4. \ldots .2006)$
$= \log_n(2006!) = \log_n n = 1$

636 **(a)**
We have,
$a(p + q)^2 + 2\,bpq + c = 0$ and, $a(p + r)^2 + 2\,bpr + c = 0$
It is evident from these two equations, that q and r are roots of the equation
$a(p + x)^2 + 2bpx + c = 0$
or, $ax^2 + 2x(a + b)p + ap^2 + c = 0$
$\therefore$ Product of the roots $= \frac{ap^2 + c}{a}$
$\Rightarrow qr = \frac{ap^2 + c}{a} = p^2 + \frac{c}{a}$

637 **(b)**
It is given that $\frac{2z_1}{3z_2}$ is purely imaginary. So, let
$\frac{2z_1}{3z_2} = ki \Rightarrow \frac{z_1}{z_2} = \frac{3k}{2} i = mi$
$\therefore \left|\frac{z_1 - z_2}{z_1 + z_2}\right|^4 = \left|\frac{\frac{z_1}{z_2} - 1}{\frac{z_1}{z_2} + 1}\right|^4 = \left|\frac{mi - 1}{mi + 1}\right|^4 = \left|\frac{m + 1}{m - 1}\right|^4 = 1$

638 **(c)**
$4^{1/2}, 4^{1/4}, 4^{1/8}, 4^{1/16}, \ldots$ are given roots, then
Sum of roots$= 4^{\frac{1}{2}} + 4^{\frac{1}{4}} + 4^{\frac{1}{8}} + \cdots = 5$
Product of roots$= 4^{1/2}.4^{1/4}.4^{1/8} \ldots$
$= 4^{1/2 + 1/4 + 1/8 + \cdots}$
$= 4^{\frac{1/2}{1 - 1/2}} = 4$
$\therefore$ Required equation is $x^2 - 5x + 4 = 0$

639 **(b)**
It is given that
x_1, x_2 are roots of $x^2 - 3x + p = 0$
$\Rightarrow x_1 + x_2 = 3, x_1x_2 = p$
x_3, x_4 are roots of $x^2 - 12x + q = 0$
$\Rightarrow x_3 + x_4 = 12$ and $x_3x_4 = q$
It is given that x_1, x_2, x_3, x_4 form an increasing G.P.
Therefore, $x_1 = a, x_2 = ar, x_3 = ar^2, x_4 = ar^3$, where $r > 1$
Now,
$\left.\begin{array}{l} x_1 + x_2 = 3 \Rightarrow a(1 + r) = 3 \\ x_3 + x_4 = 12 \Rightarrow ar^2(1 + r) = 12 \end{array}\right\} \Rightarrow r = 2$ and $a = 1$
$\therefore x_1 = 1, x_2 = 2, x_3 = 4, x_4 = 8$
Thus, $p = x_1x_2 = 2$ and $q = x_3x_4 = 32$

640 **(b)**
We have,
$x^2 - 3\,kx + 2 \times e^{2\log_e k} - 1 = 0$ [$\because \log_e k$ is defined for $k > 0$]
$\Rightarrow x^2 - 3kx + (2k^2 - 1) = 0$

Now,
Product of roots $= 7 \Rightarrow 2k^2 - 1 = 7 \Rightarrow k = 2$ [$\because$ $k > 0$]

641 **(b)**
We have,
$(a+1)x^2 + (2a+3)x + (3a+4) = 0$
Let α and β be the roots of the equation.
According to the given condition
$\alpha\beta = 2$
$\Rightarrow \dfrac{3a+4}{a+1} = 2$
$\Rightarrow 3a + 4 = 2a + 2$
$\Rightarrow a = -2$
Also, $\alpha + \beta = -\frac{2a+3}{a+1} = -\frac{-4+3}{-2+1} = -1$

642 **(d)**
$$\sum_{k=1}^{6}\left[\sin\left(\frac{2k\pi}{7}\right) - i\cos\left(\frac{2k\pi}{7}\right)\right] = -i\sum_{k=1}^{6}\left(e^{\frac{2\pi i}{7}}\right)^k$$
$= -i(r^1 + r^2 + \ldots + r^6)$ $\left[\text{let } r = e^{\frac{2\pi i}{7}}\right]$
$= -ir\,\dfrac{(1-r^6)}{1-r} = \dfrac{-i(r-r^7)}{1-r}$
$= \frac{-i(r-1)}{1-r} = i$ $[\because r^7 = e^{2\pi i} = 1]$

643 **(a)**
Since, $\sin\alpha, \sin\beta$ and $\cos\alpha$ are in GP, then
$\sin^2\beta = \sin\alpha\cos\alpha$ ...(i)
Given equation is $x^2 + 2x\cot\beta + 1 = 0$.
$\therefore$ Discriminant, $D = b^2 - 4ac$
$= (2\cot\beta)^2 - 4 = 4(\text{cosec}^2\beta - 2)$
$= 4(\text{cosec}\,\alpha\sec\alpha - 2)$ [from Eq. (i)]
$= 4(2\,\text{cosec}\,2\alpha - 2) \geq 0$
$\therefore$ Roots are real.

644 **(a)**
We have, $z^2 + pz + q = 0$ and let $p^2 = 3q$
$\Rightarrow z = \dfrac{-p \pm \sqrt{p^2 - 4q}}{2}$
$= \dfrac{-p \pm \sqrt{3q - 4q}}{2}$
$= \dfrac{-p \pm i\sqrt{q}}{2}$
Let $z_1 = \frac{-p+i\sqrt{q}}{2}$
And $z_2 = \frac{-p-i\sqrt{q}}{2}$
Further, let z_1 and z_2 be the affixes of points A and B respectively. Then,
$$OA = |z_1| = \sqrt{\left(-\frac{p}{2}\right)^2 + \left(\frac{\sqrt{q}}{2}\right)^2} = \sqrt{\frac{p^2}{4} + \frac{q}{4}}$$
$= \sqrt{\dfrac{3q}{4} + \dfrac{q}{4}} = \sqrt{q}$
$$OB = |z_2| = \sqrt{\left(-\frac{p}{2}\right)^2 + \left(+\frac{\sqrt{q}}{2}\right)^2}$$
$= \sqrt{\dfrac{p^2}{4} + \dfrac{q}{4}} = \sqrt{\dfrac{3q}{4} + \dfrac{q}{4}} = \sqrt{q}$
And $AB = |z_1 - z_2| = |i\sqrt{q}| = \sqrt{0 + (\sqrt{q})^2} = \sqrt{q}$
$\therefore$ $OA = OB = AB$
$\Rightarrow$ ΔAOB is an equilateral triangle.
Thus, $p^2 = 3q$

645 **(a)**
$(3 + \omega^2 + \omega^4)^6 = (3 + \omega^2 + \omega)^6 = (3-1)^6 = 64$

646 **(b)**
We have,
$|\omega| = 1$
$\Rightarrow |1 - iz| = |z - i|$
$\Rightarrow |z + i| = |z - i|$
$\Rightarrow z$ lies on the perpendicular bisector of the segment joining $(0,1)$ and $(0,-1)$
$\Rightarrow z$ lies on x-axis

648 **(b)**
We have , $\left|x + \dfrac{1}{x}\right| > 2$
We know that
$x + \frac{1}{x} > 2$ for all $x > 0, x \neq 1$ and $x + \frac{1}{x} < -2$ for all $x < 0, x \neq -1$
$\therefore \left|x + \dfrac{1}{x}\right| > 2$ for all $x \neq 0, -1, 1$
Hence, the solution set of the given inequation is $R - \{-1, 0, 1\}$

649 **(c)**
We have,
$\text{Re}\left(\dfrac{z+4}{2z-i}\right) = \dfrac{1}{2}$
$\Rightarrow \text{Re}\left(\dfrac{z+4}{z - \frac{i}{2}}\right) = 1$
$\Rightarrow \text{Re}\left(\dfrac{(x+4) + iy}{x + i\left(y - \frac{1}{2}\right)}\right) = 1$
$\Rightarrow \text{Re}\left[\dfrac{\{(x+4) + iy\}\left\{x - i\left(y - \frac{1}{2}\right)\right\}}{x^2 + \left(y - \frac{1}{2}\right)^2}\right] = 1$
$\Rightarrow \text{Re}\left\{\dfrac{x(x+4) + y\left(y - \frac{1}{2}\right) + \{xy - (x+4)\left(y - \right.}{x^2 + \left(y - \frac{1}{2}\right)^2}\right.$
$= 1$

$\Rightarrow \dfrac{x(x+4)+y\left(y-\frac{1}{2}\right)}{x^2+\left(y-\frac{1}{2}\right)^2}=1$

$\Rightarrow x^2+4x+y^2-\dfrac{y}{2}=x^2+y^2-y+\dfrac{1}{4}$

$\Rightarrow 4x+\dfrac{y}{2}-\dfrac{1}{4}=0$

$\Rightarrow 16x+2y-1=0$, which represents a straight line

650 **(a)**

Since, $\sin A, \sin B, \cos A$ are in GP

$\therefore\ \sin^2 B=\sin A\cos A$ ...(i)

Also, $x^2+2x\cot B+1=0$ [given]

Now, $b^2-4ac=4\cot^2 B-4$

$=\dfrac{4\cos^2 B-4\sin^2 B}{\sin^2 B}$

$=\dfrac{4(1-2\sin^2 B)}{\sin^2 B}$

$=\dfrac{4(1-2\sin A\cos A)}{\sin^2 B}$

$=4\left(\frac{\sin A-\cos A}{\sin B}\right)^2$ [from Eq. (i)]

≥ 0

$\therefore$ Roots of given equation are always real

651 **(d)**

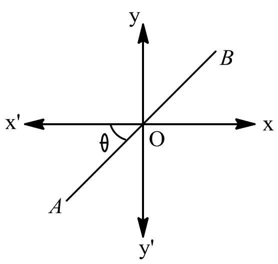
Here, $\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$

$\therefore\ \dfrac{1}{a\alpha+b}+\dfrac{1}{a\beta+b}=\dfrac{a(\alpha+\beta)+2b}{a^2\alpha\beta+ab(\alpha+\beta)+b^2}$

$=\dfrac{a\left(-\frac{b}{a}\right)+2b}{a^2\left(\frac{c}{a}\right)+ab\left(-\frac{b}{a}\right)+b^2}=\dfrac{b}{ac}$

652 **(c)**

From the figure it is clear that amplitude of point $B=\theta-\pi$



653 **(d)**

Let $\frac{2z_1}{3z_2}=ik\quad\Rightarrow\quad\frac{z_1}{z_2}=\frac{3ik}{2}$

$\therefore\quad\left|\dfrac{z_1-z_2}{z_1+z_2}\right|=\left|\dfrac{(z_1/z_2)-1}{(z_1/z_2)+1}\right|=\left|\dfrac{(3ik/2)-1}{(3ik/2)+1}\right|=1$

654 **(a)**

The given equation will have real roots iff

Disc $\geq 0\Rightarrow 16-4\,(k^2-1)\geq 0\Rightarrow k^2\leq 5$

655 **(c)**

Let $z_1=1+4i,\ z_2=3+i,\ z_3=1-i$ and $z_4=2-3i$

$\therefore\quad m_1|z_1|,\quad m_2=|z_2|,\ m_3=|z_3|$ *and* $m_4=|z_4|$

$\Rightarrow\quad m_1=\sqrt{1+4^2}=\sqrt{17},\qquad m_2=\sqrt{3^2+1^2}=\sqrt{10},$

$m_3=\sqrt{1^2+1^2}=\sqrt{2}$ and $m_4=\sqrt{2^2+3^2}=\sqrt{13}$

$\Rightarrow\quad m_3<m_2<m_4<m_1$

656 **(c)**

Given, $(\cos\theta+i\sin\theta)(\cos 2\theta+i\sin 2\theta)\ldots(\cos n\theta+i\sin n\theta)=1$

$\therefore\ \cos(\theta+2\theta+3\theta+\ldots+n\theta)$
$+\,i\sin(\theta+2\theta+3\theta+\ldots+n\theta)=1$

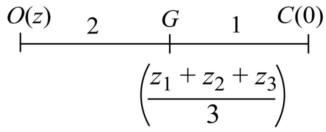
$\Rightarrow\ \cos\left(\dfrac{n(n+1)}{2}\theta\right)+i\sin\left(\dfrac{n(n+1)}{2}\theta\right)=1$

$\Rightarrow\quad\cos\left(\frac{n(n+1)}{2}\theta\right)=1$ and $\sin\left(\frac{n(n+1)}{2}\theta\right)=0$

$\therefore\quad\dfrac{n(n+1)}{2}\theta=2m\pi\ \Rightarrow\ \theta=\dfrac{4m\pi}{n(n+1)}$

657 **(c)**

Let O is orthocenter, G is centroid and C is circumcentre, then



$\dfrac{z_1+z_2+z_3}{3}=\dfrac{2\times 0+1(z)}{3}$

$\Rightarrow\ z=z_1+z_2+z_3$

658 **(d)**

We have, $z_1=\frac{\lambda z_2+z_3}{\lambda+1}$

This means that the point A divides BC internally in the ratio $1:\lambda$. So, A lies on the segment BC

Hence, distance of A from BC is zero

659 **(c)**

Given that, the vertices of quadrilateral are $A=(1+2i), B=(-3+i), C=(-2-3i)$ and $D=(2-2i)$

Now, $AB=\sqrt{16+1}=\sqrt{17},\ BC=\sqrt{1+16}=\sqrt{17}$

$CD=\sqrt{16+1}=\sqrt{17}, DA=\sqrt{1+16}=\sqrt{17}$

$AC=\sqrt{9+25}=\sqrt{34}, BD=\sqrt{25+9}=\sqrt{34}$

$\therefore$ Sides $AB=BC=CD=DA$ and diagonals $AC=BD$

Hence, it is a square

660 **(b)**

Given equation is

$(p^2+q^2)x^2-2q(p+r)x+(q^2+r^2)=0$

Since, roots are real and equal, then

$b^2-4ac=0$

$\Rightarrow 4q^2(p+r)^2-4(p^2+q^2)(q^2+r^2)=0$

$\Rightarrow q^2(p^2 + r^2 + 2pr) - (p^2q^2 + p^2r^2 + q^4 + q^2r^2) = 0$

$\Rightarrow q^2p^2 + q^2r^2 + 2pq^2r - p^2q^2 - p^2r^2 - q^4 - q^2r^2 = 0$

$\Rightarrow 2pq^2r - p^2r^2 - q^4 = 0$

$\Rightarrow (q^2 - pr)^2 = 0$

$\Rightarrow q^2 = pr$

$\therefore p, q$ and r will be in GP.

661 **(b)**

Since, $\left|\frac{z-i}{z+i}\right| = 2 \quad \Rightarrow \quad \left|\frac{x+iy-i}{x+iy+i}\right| = 2$ [where $z = x + iy$]

$\Rightarrow \quad |x + i(y-1)| = 2|x + (y+1)i|$

$\Rightarrow \quad x^2 + (y-1)^2 = 4[x^2 + (y+1)^2]$

$\Rightarrow \quad x^2 + y^2 - 2y + 1 = 4x^2 + 4y^2 + 8y + 4$

$\Rightarrow \quad 3x^2 + 3y^2 + 10y + 3 = 0$

662 **(c)**

$|z_1| = \sqrt{2}, \qquad |z_2| = \sqrt{3}$

$\therefore \quad |z_1 z_2| = |z_1||z_2 = \sqrt{6}$

663 **(d)**

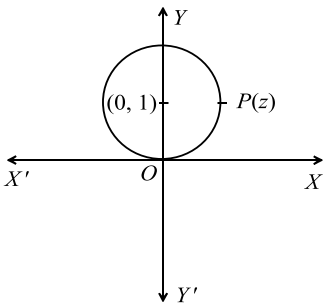
We have,

$|z_1 - z_2| \le |z_1| + |z_2|$

$\therefore |1 + z + z^2 + \cdots + z^n| = \left|\frac{z^{n+1} - 1}{z - 1}\right| \le \frac{z^{n+1} + 1}{|z - 1|}$

$\Rightarrow |1 + z + z^2 + \cdots + z^n| \le \frac{|z|^{n+1} + 1}{|z|} \quad \left[\begin{array}{l}\because \text{Re}(z) < 0 \\ \therefore |z - 1| \ge |z|\end{array}\right]$

$\Rightarrow |1 + z + z^2 + \cdots + z^n| \le |z|^n + \frac{1}{|z|}$



664 **(d)**

Since, $a + b = -p, ab = 1 \quad$ ...(i)

And $c + d = -q, cd = 1$

Now, $(a - c)(b - c)$ and $(a + d)(b + d)$ are the roots of $x^2 + ax + \beta = 0$

$\therefore \quad (a - c)(b - c)(a + d)(b + d) = \beta$

$\Rightarrow \quad (ab - ac - bc + c^2)(ab + ad + bd + d^2) = \beta$

$\Rightarrow \quad \{1 - c(a + b) + c^2\}\{1 + d(a + b) + d^2\} = \beta$

$\Rightarrow \quad (1 + pc + c^2)(1 - pd + d^2) = \beta$

$\Rightarrow \quad 1 - pd + d^2 + pc - p^2cd + pcd^2 + c^2 - pc^2d + c^2d^2 = \beta$

$\Rightarrow \quad 1 - pd + d^2 + pc - p^2 + pd + c^2 - pc + 1 = \beta$

$[\because \ cd = 1]$

$\Rightarrow \quad 2 + d^2 + c^2 - p^2 = \beta$

$\Rightarrow \quad 2cd + c^2 + d^2 - p^2 = \beta \qquad [\because 1 = cd]$

$\Rightarrow \quad (c + d)^2 - p^2 = \beta$

$\Rightarrow q^2 - p^2 = \beta \quad (\because \ c + d = -q)$

665 **(b)**

We have,

$x^2 - 3x - 4 < 0 \Rightarrow (x - 4)(x + 1) < 0 \Rightarrow -1 < x < 4$

Clearly, integers 0, 1, 2 and 3 satisfy this inequality

666 **(b)**

According to the equation,

$D \ge 0, -2 < -\frac{b}{2a} < 4, \ f(4) > 0$ and $f(-2) > 0$

Now, $D \ge 0; \quad 4m^2 - 4m^2 + 4 \ge 0$

$\Rightarrow \quad 4 > 0 \ \forall \ m \in R \quad$ ...(i)

$-2 < -\frac{b}{2a} < 4; \ -2 < \left(\frac{2m}{2.1}\right) < 4$

$\Rightarrow \ -2 < m < 4 \quad$ ...(ii)

$f(4) > 0$

$\Rightarrow 16 - 8m + m^2 - 1 > 0 \Rightarrow (m - 3)(m - 5) > 0$

$\Rightarrow \ -\infty < m < 3$ and $5 < m < \infty \quad$ ...(iii)

And $f(-2) > 0$

$\Rightarrow \quad 4 + 4m + m^2 - 1 > 0$

$\Rightarrow \quad (m + 3)(m + 1) > 0$

$\Rightarrow \ -\infty < m < -3$ and $-1 < m < \infty \quad$ ...(iv)

$\therefore$ From Eqs. (i), (ii), (iii) and (iv), we get m lie between -1 and 3

667 **(c)**

Given equation is

$(p-q)x^2+(q-r)x+(r-p)=0$

$\Rightarrow x=\dfrac{(r-q)\pm\sqrt{(q-r)^2-4(r-p)(p-q)}}{2(p-q)}$

$=\dfrac{(r-q)\pm\sqrt{q^2+r^2-2qr-4(rp-rq-p^2+pq)}}{2(p-q)}$

$\Rightarrow x=\dfrac{(r-q)\pm(q+r-2p)}{2(p-q)}$

$\Rightarrow x=\dfrac{r-p}{p-q},1$

668 **(b)**

Since $\alpha,\beta,\gamma,\delta$ are roots of $x^4+x^2+1=0$. To obtain the equation whose roots are $\alpha^2,\beta^2,\gamma^2,\delta^2$, we put $x^2=y$. Putting $x^2=y$, the given equation reduces to

$y^2+y+1=0$

Thus, the required equation is

$(y^2+y+1)^2=0$ or, $(x^2+x+1)^2=0$

669 **(d)**

We have,

$|x^2-10|\le 6\Rightarrow -6\le x^2-10\le 6\Rightarrow 4\le x^2 \le 16$

$\Rightarrow x\in[-4,-2]\cup[2,4]$

$\left[\begin{array}{c}\because a^2\le x^2\le b^2\\ \Leftrightarrow x\in[-b,-a]\cup[a,b]\end{array}\right]$

670 **(a)**

Given, $x=\sqrt{3018+\sqrt{36+\sqrt{169}}}$

$=\sqrt{3018+\sqrt{36+13}}$

$=\sqrt{3018+7}=\sqrt{3025}=55$

671 **(c)**

Given equation is $(\cos p-1)x^2+(\cos p)x+\sin p=0$

Since, roots are real, its discriminant, $D\ge 0$

$\therefore \cos^2 p-4(\cos p-1)\sin p\ge 0$

$\Rightarrow \cos^2 p-4\cos p\sin p+4\sin p\ge 0$

$\Rightarrow (\cos p-2\sin p)^2-4\sin^2 p+4\sin p\ge 0$

$\Rightarrow (\cos p-2\sin p)^2+4\sin p(1-\sin p)\ge 0$ .....(i)

Now, $(1-\sin p)\ge 0$ for all real p and $\sin p>0$ for $0<p<\pi$. Therefore, $4\sin p(1-\sin p)\ge 0$ when $0<p<\pi$ or $p\in(0,\pi)$.

672 **(b)**

Let the two numbers are x_1 and x_2

Given, $\frac{x_1+x_2}{2}=9$ and $x_1.x_2=16$

$\Rightarrow x_1+x_2=18$ and $x_1.x_2=16$

Hence, required equation is

x^2-(sum of roots)x +product of roots=0

$\Rightarrow x^2-18x+16=0$

673 **(c)**

α and β are roots of the equation

$x^2-x+1=0$

$\Rightarrow \alpha+\beta=1, \alpha\beta=1$

$\Rightarrow \alpha=-\omega,\beta=-\omega^2$

or $\alpha=-\omega^2,\beta=-\omega$

Taking $\alpha=-\omega,\beta=-\omega^2$

$\alpha^{2009}+\beta^{2009}=(-\omega)^{2009}-(-\omega^2)^{2009}$

$=-(\omega^2+\omega)$

$=1$

674 **(a)**

$\alpha+\beta=-p,\alpha\beta=q$

$\therefore\ \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=p^2-2q$

$\Rightarrow(\alpha-\beta)^2=\alpha^2+\beta^2-2\alpha\beta$

$=(p^2-2q)+2q$

$=p^2-4q$

675 **(b)**

Conjugate of $\frac{2-3i}{4-i}$ is $\frac{2+3i}{4+i}$

$\therefore\ \dfrac{2+3i}{4+i}=\dfrac{2+3i}{4+i}\times\dfrac{4-i}{4-i}$

$=\dfrac{8+3-2i+12i}{16+1}$

$=\dfrac{11+10i}{17}$

676 **(d)**

Here, $a=(p-q), b=5(p+q)$ and $c=-(2p-2q+r)$

Now, $b^2-4ac=25(p+q)^2+4(p-q)(2p-2q+r$

$=25(p+q)^2+8(p-q)^2+4r(p-q)$

Hence, it depends on the value of p,q and r

677 **(a)**

We have,

$y=\sqrt{\frac{(x+1)(x-3)}{(x-2)}}$ here x cannot be 2.

$\therefore$ Either both N^r and D^r are positive.

$x \geq -1, x \geq 3$ and $x > 2$

$\Rightarrow x \geq 3$ ....(i)

or N^r is negative and D^r is negative.

$x \geq -1$ and $x > 2$

$\Rightarrow -1 \leq x < 2$ ...(ii)

From Eqs. (i) and (ii), we get

$-1 \leq x < 2$ or $x \geq 3$

678 **(d)**

Let $\alpha = x^{1/3}$, then it reduces to

$\alpha^2 - 7\alpha + 10 = 0$

$\Rightarrow (\alpha - 5)(\alpha - 2) = 0 \Rightarrow \alpha = 5, 2$

$\therefore \alpha^3 = x \Rightarrow x = 125$ and 8

679 **(b)**

We know that only even prime is 2, then

$(2)^2 - \lambda(2) + 12 = 0 \Rightarrow \lambda = 8$ ...(i)

and $x^2 + \lambda x + \mu = 0$ has equal roots.

$\therefore \lambda^2 - 4\mu = 0$ or $(8)^2 - 4\mu = 0$ [from Eq. (i)]

$\therefore \mu = 16$

680 **(b)**

Given, $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{3}$

Let $z = x + iy$

$$\therefore \frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1} \times \frac{(x+1)-iy}{(x+1)-iy}$$

$$= \frac{x^2+y^2-1+2iy}{(x+1)^2+y^2}$$

$$\therefore \arg\left(\frac{z-1}{z+1}\right) = \tan^{-1}\frac{2y}{x^2+y^2-1} = \frac{\pi}{3}$$

$$\Rightarrow \frac{2y}{x^2+y^2-1} = \sqrt{3}$$

$$\Rightarrow x^2 + y^2 - \frac{2}{\sqrt{3}}y - 1 = 0$$

Which is the equation of a circle.

681 **(d)**

$$\Delta = \begin{vmatrix} 1+\omega & \omega^2 & -\omega \\ 1+\omega^2 & \omega & -\omega^2 \\ \omega^2+\omega & \omega & -\omega^2 \end{vmatrix} = \begin{vmatrix} -\omega^2 & \omega^2 & -\omega \\ -\omega & \omega & -\omega^2 \\ -1 & \omega & -\omega^2 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} \omega^2 & \omega^2 & \omega \\ \omega & \omega & \omega^2 \\ 1 & \omega & \omega^2 \end{vmatrix} = \omega^2 \begin{vmatrix} \omega^2 & \omega & 1 \\ \omega & 1 & \omega \\ 1 & 1 & \omega \end{vmatrix}$$

$\Rightarrow \Delta = \omega^2\{\omega^2(\omega - \omega) - \omega(\omega^2 - \omega) + (\omega - 1)\}$

$\Rightarrow \Delta = \omega^2\{0 - \omega^3 + \omega^2 + \omega - 1\} = -3\,\omega^2$

682 **(a)**

We have,

$|\alpha - \beta| > \sqrt{3a}$

$\Rightarrow |\alpha - \beta|^2 > 3a$

$\Rightarrow (\alpha + \beta)^2 - 4\,\alpha\,\beta > 3a$

$\Rightarrow a^2 - 4 > 3a$

$\Rightarrow a^2 - 3a - 4 > 0 \Rightarrow (a-4)(a+1) > 0 \Rightarrow a \in (-\infty. -1) \cup (4, \infty)$

684 **(b)**

The given equation is

$3x^2 - 2x(a+b+c) + (ab+bc+ca) = 0$

Let D be its discriminant. Then,

$D = 4(a+b+c)^2 - 12(ab+bc+ca)$

$\Rightarrow D = 4(a^2+b^2+c^2-ab-bc-ca)$

$\Rightarrow D = 2\{(a-b)^2+(b-c)^2+(c-a)^2\} \geq 0$

So, roots of the given equation are real

685 **(b)**

Sum of roots$= \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$ and product $=1$

Given, $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$

$\Rightarrow (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2) = q$

$\therefore \alpha^2 + \beta^2 - \alpha\beta = \frac{-q}{p}$ ...(i)

And $(\alpha+\beta)^2 = p^2$

$\Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = p^2$

From Eqs. (i) and (ii), we get

$$\alpha^2 + \beta^2 = \frac{p^3 - 2q}{3p}$$

And $\alpha\beta = \frac{p^3+q}{3p}$

$\therefore$ Required equation is

$$x^2 - \frac{(p^3-2q)x}{(p^3+q)} + 1 = 0$$

$\Rightarrow (p^3+q)x^2 - (p^3-2q)x + (p^3+q) = 0$

686 **(b)**

Since, α and β are the roots of the equation

$2x^2 + 2(a+b) + a^2 + b^2 = 0.$

$\therefore (\alpha+\beta)^2 = (a+b)^2$ and $\alpha\beta = \frac{a^2+b^2}{2}$

Now, $(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

$$= (a+b)^2 - 4\left(\frac{a^2+b^2}{2}\right)$$

$= -(a-b)^2$

Now, the required equation whose roots are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$ is

$x^2 - \{(\alpha+\beta)^2 + (\alpha-\beta)^2\}x + (\alpha+\beta)^2(\alpha-\beta)^2 = 0$

$\Rightarrow x^2 - \{(a+b)^2 - (a-b)^2\}x - (a+b)^2(a-b)^2 = 0$

$\Rightarrow x^2 - 4abx - (a^2-b^2)^2 = 0$

688 **(b)**

Let $z = x + iy$, therefore given equation becomes

$(x+iy)(x-iy) + (2-3i)(x+iy) + (2+3i)(x-iy) + 4 = 0$

$\Rightarrow x^2 + y^2 + 2x + 3y - 3ix + 2iy + 2x - 2iy + 3ix + 3y + 4 = 0$

$\Rightarrow x^2 + y^2 + 4x + 6y + 4 = 0$

Therefore, given equation represents a circle with radius

$= \sqrt{2^2 + 3^2 - 4}$

$= \sqrt{4 + 9 - 4} = \sqrt{9} = 3$

689 **(a)**

Here, $i\left\{\log\left(\frac{x-i}{x+i}\right)\right\}-\pi+2\tan^{-1}x=k$ (say)

$\therefore\ \log\left(\frac{x+i}{x-i}\right)=i(k+\pi-2\tan^{-1}x)$

or $\frac{x+i}{x-i}=e^{i\theta}$, where $\theta=k+\pi-2\tan^{-1}x$

$\Rightarrow\ x+i=(x\cos\theta+\sin\theta)+i(x\sin\theta-\cos\theta)$

$\Rightarrow\ x=x\cos\theta+\sin\theta$ and $1=x\sin\theta-\cos\theta$

$\Rightarrow\ x=\cot\frac{\theta}{2}\ \Rightarrow\ \theta=2\cot^{-1}x$

or $k+\pi-2\tan^{-1}x=2\cot^{-1}x$

$\Rightarrow\ k+\pi=2(\cot^{-1}x+\tan^{-1}x)=2\left(\frac{\pi}{2}\right)$

$\Rightarrow\ k+\pi=\pi$ or $k=0$

690 **(b)**

Now, $1+x=\log_a a+\log_a bc=\log_a abc$

$\Rightarrow\ \frac{1}{1+x}=\log_{abc}a$

Similarly, $\frac{1}{1+y}=\log_{abc}b$ and $\frac{1}{1+z}=\log_{abc}c$

$\therefore\ \frac{1}{1+x}+\frac{1}{1+y}+\frac{1}{1+z}$

$=\log_{abc}a+\log_{abc}b+\log_{abc}c$

$=\log_{abc}abc=1$

691 **(d)**

We have,

$\left|\frac{z_1-z_2}{1-z_1\bar{z}_2}\right|=1$

$\Rightarrow|z_1-z_2|^2=|1-z_1\bar{z}_2|^2$

$\Rightarrow|z_1|^2+|z_2|^2-2\,\text{Re}\,(z_1\bar{z}_2)|$

$=1+|z_1\bar{z}_2|^2-2\,\text{Re}\,(z_1\bar{z}_2)$

$\Rightarrow|z_1|^2+|z_2|^2=1+|z_1|^2\,|z_2|^2$

$\Rightarrow(1-|z_1|^2)(1-|z_2|^2)=0$

$\Rightarrow|z_1|=1$ or, $|z_2|=1$

$\Rightarrow z_1=e^{i\,\theta}$ or , $z_2=e^{i\,\theta}$, where $\theta\in R$

692 **(b)**

$(\sin 40°+i\cos 40\,°)^5$

$=i^5(\cos 40°-i\sin 40°)^5$

$=i\,(\cos 200°-i\sin 200°)$

$=i[\cos(180°+20°)-i\sin(180°+20°)]$

$=i(-\cos 20°-i\sin 20°)$

$=-i\cos 20°-\sin 20°$

$=\cos(-110°)+i\sin(-110°)$

$\therefore$ Principle amplitude$=-110°$

693 **(a)**

We have, $|x^2-3x+2|+|x-1|=x-3$

Therefore $x\geq 3$

$\therefore\ x^2-3x+2+x-1=x-3$

$\Rightarrow\ x^2-3x+4=0$

$\Rightarrow\ \left(x-\frac{3}{2}\right)^2=-\frac{7}{4}$

Hence, no solution exist

694 **(d)**

We have,

$\sum_{r=1}^{8}\left(\sin\frac{2\,r\,\pi}{9}+i\cos\frac{2\,r\,\pi}{9}\right)$

$=\sum_{r=1}^{8}i\left(\cos\frac{2\,r\,\pi}{9}-i\sin\frac{2\,r\,\pi}{9}\right)$

$=i\sum_{r=1}^{8}e^{-i\frac{2\,r\,\pi}{9}}$

$=i\sum_{r=1}^{8}\alpha^r$, when $\alpha=e^{-\frac{2\,\pi\,i}{9}}$

$=i\,\alpha\frac{(1-\alpha^8)}{(1-\alpha)}$

$=i\frac{(\alpha-\alpha^9)}{1-\alpha}$

$=i\left(\frac{\alpha-1}{1-\alpha}\right)$ $[\because\alpha^9=e^{-i\,2\,\pi}=\cos 2\,\pi-i\sin 2\,\pi$

$=1]$

$=-i$

695 **(a)**

Given equation of circle is

$z\bar{z}+(2+3i)\bar{z}+(2-3i)z+12=0$

Here, centre is {-(2+3i)} and radius

$=\sqrt{|2+3i|^2-12}=\sqrt{13-12}=1$

696 **(d)**

We have,

$\alpha+\beta=\frac{-b}{a},\alpha\,\beta=\frac{c}{a}$

The required equation is

$x^2-5x(\alpha+\beta)+(2\,\alpha+3\,\beta)(3\,\alpha+2\,\beta)=0$

$\Rightarrow x^2+\frac{5xb}{a}+\{6(\alpha^2+\beta^2)+13\,\alpha\,\beta\}=0$

$\Rightarrow x^2+\frac{5bx}{a}+\{6(\alpha+\beta)^2+\alpha\,\beta\}=0$

$\Rightarrow x^2+\frac{5b}{a}x+\left(\frac{6b^2}{a^2}+\frac{c}{a}\right)=0$

$\Rightarrow a^2x^2+5abx+(6b^2+ac)=0$

697 **(a)**

We have,

$|x-2|^2+|x-2|-2=0$

$\Rightarrow(|x-2|+2)(|x-2|-1)=0$

$\Rightarrow|x-2|-1=0$ $[\because|x-2|+2\neq 0]$

$\Rightarrow x-2=\pm 1\Rightarrow x=3,1$

$\therefore$ Sum of the roots = 4

698 **(a)**

Given, $x^2-xy+y^2-4x-4y+16=0$

$\Rightarrow x^2-(y+4)x+y^2-4y+16=0$

For real x, $(y+4)^2-4(y^2-4y+16)\geq 0$

$\Rightarrow\ -3y^2+24y-48=0$

$\Rightarrow y^2 - 8y + 16 = 0$
$\Rightarrow (y-4)^2 = 0$
$\Rightarrow y = 4$
$\therefore x = 4$
$\Rightarrow (x, y) = (4, 4)$

699 **(a)**
$\log_{0.3}(x-1) < \log_{0.09}(x-1)$
Here, $x - 1 > 0$
And $\quad \log_{0.3}(x-1) < \log_{(0.3)^2}(x-1)$
$\Rightarrow x > 1$ and $\log_{0.3}(x-1) < \frac{1}{2}\log_{0.3}(x-1)$
$\Rightarrow x > 1$ and $\log_{0.3}(x-1) < 0$
$\Rightarrow x > 1$ and $x - 1 > 1$
$\Rightarrow x > 1$ and $x > 2$
$\therefore \quad x \in (2, \infty)$

700 **(c)**
Given that, $\frac{2x}{2x^2+5x+2} > \frac{1}{(x+1)}$
$$\Rightarrow \frac{2x}{(2x+1)(x+2)} > \frac{1}{(x+1)}$$
$$\Rightarrow \frac{2x}{(2x+1)(x+2)} - \frac{1}{(x+1)} > 0$$
$$\Rightarrow \frac{2x(x+1) - (2x+1)(x+2)}{(x+1)(2x+1)(x+2)} > 0$$
$$\Rightarrow \frac{2x^2 + 2x - 2x^2 - 4x - x - 2}{(x+1)(2x+1)(x+2)} > 0$$
$$\Rightarrow \frac{-3x-2}{(x+1)(2x+1)(x+2)} > 0$$
Equating each factor equal to 0, we have
$$x = -2, -1, -\frac{2}{3}, -\frac{1}{2}$$
It is clear that $-\frac{2}{3} < x < -\frac{1}{2}$ or $-2 < x - 1$.

701 **(b)**
Let $y = \sqrt[3]{28}$
Taking log on both sides, we get
$$\log y = \frac{1}{3}\log 28$$
$$= \frac{1}{3} \times 1.4472$$
$= 0.4824$
$\Rightarrow y = \text{antilog}\ (0.4824)$
$= 3.037$ (approximately)

702 **(b)**
As we know, the equation of the form $\left|\frac{z-2}{z+2}\right| = n$ is a circle, if $n \neq 1$

703 **(a)**
The vertices of the triangle are $z, iz, z + iz$
or $\ x + iy, -y + ix, \ (x-y) + i(x+y)$
$$\therefore \text{ Required area} = \frac{1}{2}\begin{vmatrix} x & y & 1 \\ -y & x & 1 \\ x-y & x+y & 1 \end{vmatrix}$$
$$= \frac{1}{2}|[x(x - x - y) - y(-y - x + y) + 1(-yx - y^2 - x^2 + xy)]|$$
$$= \frac{1}{2}(x^2 + y^2) = \frac{1}{2}|z|^2$$

704 **(b)**
For rational roots $b^2 - 4ac$ must be a perfect square of a rational number and as a, b, c are natural numbers $b^2 - 4ac$ must be a perfect square of an integer.
$b^2 - 4ac = I^2 \ \Rightarrow b^2 = I^2 = 4ac$
$\Rightarrow 4ac = (b - I)(b + I)$
$$\Rightarrow ac = \frac{b-I}{2} \cdot \frac{b+I}{2}$$
$b - I, b + I$ are both odd integers or both even integers but ac is an odd integer. So, $b - I$ and $b + I$ must be even integers. b is odd I must be odd. Now, let
$b - I = 2m$, (m odd integer)
$b + I = 2n$, (n odd integer)
$I = (n - m)$, ($n - m$ is an even integer)
So, contradiction $\Rightarrow$ $b^2 - 4ac$ is not a perfect square. So, all a, b, c cannot be odd integers.

705 **(b)**
We have,
$|\lambda_1 a_1 + \cdots + \lambda_n a_n|$
$\leq |\lambda_1 a_1| + |\lambda_2 a_2| + \cdots + |\lambda_n a_n|$
$= |\lambda_1||a_1| + |\lambda_2||a_2| + \cdots + |\lambda_n||a_n|$
$= \lambda_1|a_1| + \lambda_2|a_2| + \cdots + \lambda_n|a_n| \qquad [\because \lambda_i \geq 0]$
$< \lambda_1 + \lambda_2 + \cdots + \lambda_n = 1 \qquad [\because |a_1| < 1]$
$|\lambda_1 a_1 + \cdots + \lambda_n a_n| < 1$

706 **(d)**
Since, α, β are the roots of the equation $ax^2 + bx + c = 0$.
$\therefore \ ax^2 + bx + c = a(x - \alpha)(x - \beta)$
$\Rightarrow \ \alpha, \beta$ be the roots of $ax^2 + bx + c = 0$. Also $\alpha < k < \beta$
So, $a(k - \alpha)(k - \beta) < 0$
Also, $a^2k^2 + abk + ac = a(ak^2 + bk + c) = a^2(k - \alpha)(k - \beta) < 0$
$\Rightarrow \ a^2k^2 + abk + ac < 0$

707 **(b)**
We have,
$x = 2 + 2^{2/3} + 2^{1/3}$
$\Rightarrow x - 2 = 2^{2/3} + 2^{1/3}$
$\Rightarrow (x-2)^3 = 2^2 + 2 + 3 \times 2^{2/3} \times 2^{1/3}(2^{2/3} + 2^{1/3})$
$\Rightarrow x^3 - 6x^2 + 12x - 8 = 4 + 2 + 3 \times 2 \times (x - 2)$
$\Rightarrow x^3 - 6x^2 + 6x = 2$

708 **(c)**

Since, $z_2 = \frac{z_1+z_3}{2}$ [$\because$ z_1, z_2 and z_3 are in AP]

$\Rightarrow$ B is the mid point of the line AC

$\Rightarrow$ A, B, C are collinear

$\Rightarrow$ z_1, z_2, z_3 lie on a straight line

709 **(c)**

The equation $|z-(3+4i)|^2 + |z-9-4-2i)|$^2 $= R$ will represent a circle iff

$k \geq \frac{1}{2}|(3+4i)-(-4-2i)|^2$ [Using: $k \geq$ 12z1−z22

i.e. $k \geq \frac{1}{2}|7+6i|^2 \Rightarrow k \geq \frac{85}{2}$

711 **(c)**

$\because \frac{k+1}{k} + \frac{k+2}{k+1} = -\frac{b}{a}$ ...(i)

and $\frac{k+1}{k} . \frac{k+2}{k+1} = \frac{c}{a}$

$\Rightarrow \frac{k+2}{k} = \frac{c}{a}$

$\Rightarrow \frac{2}{k} = \frac{c}{a} - 1 = \frac{c-a}{a}$

$\Rightarrow k = \frac{2a}{c-a}$

On putting the value of k in the Eq. (i), we get

$\frac{c+a}{2a} + \frac{2c}{c+a} = -\frac{b}{a}$

$\Rightarrow (c+a)^2 + 4ac = -2b(a+c)$

$\Rightarrow (a+b+c)^2 = b^2 - 4ac$

712 **(a)**

$\left[\frac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right]^n$

$= \left[\frac{1+\cos\alpha+i\sin\alpha}{1+\cos\alpha-i\sin\alpha}\right]^n$ $\left(\text{Put } \alpha = \frac{\pi}{2} - \frac{\pi}{8}\right)$

$= \left[\frac{2\cos^2\frac{\alpha}{2}+2i\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}-2i\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}\right]^n$

$= \left[\frac{\cos\frac{\alpha}{2}+i\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}-i\sin\frac{\alpha}{2}}\right]^n$

$= \left(e^{2i\frac{\alpha}{2}}\right)^n = e^{in^{\alpha}}$

$= e^{in\left(\frac{3\pi}{8}\right)} = \cos\frac{3n\pi}{8} + i\sin\frac{3n\pi}{8}$

For $n = 4$, we get imaginary part

713 **(a)**

Given, $\left|\frac{z-2i}{z+2i}\right| = 1$

$\Rightarrow$ $\left|\frac{x+iy-2i}{x+iy+2i}\right| = 1$

$\Rightarrow$ $\sqrt{x^2+(y-2)^2} = \sqrt{x^2+(y+2)^2}$

$\Rightarrow$ $x^2+y^2+4-4y = x^2+y^2+4+4y$

$\Rightarrow$ $y = 0$

Thus, the locus of z is x-axis

714 **(a)**

The given equations are

$qx^2+px+q=0$ ...(i)

and $x^2-4qx+p^2=0$ ...(ii)

Since, root of the Eq. (i) are complex, therefore

$p^2-4q^2<0$

Now, discriminant of Eq. (ii) is

$16q^2-4p^2 = -4(p^2-4q^2) > 0$

Hence, roots are real and unequal.

715 **(d)**

Let $e^{\cos x} = y$. Then,

$e^{\cos x} - e^{-\cos x} = 4$

$\Rightarrow y - \frac{1}{y} = 4$

$\Rightarrow y^2 - 4y - 1 = 0$

$\Rightarrow y = 2 \pm \sqrt{5}$

$\Rightarrow y = 2+\sqrt{5}$ as $y > 0$

$\Rightarrow e^{\cos x} = 2+\sqrt{5} \Rightarrow \cos x = \log_e(2+\sqrt{5})$

Clearly, $\log_e(2+\sqrt{5}) > 1$ and $\cos x \leq 1$

So, there is no value of $\cos x$ satisfying the given equation

716 **(c)**

$\sqrt{12-\sqrt{68+48\sqrt{2}}}$

$= \sqrt{12-\sqrt{(6)^2+(4\sqrt{2})^2+2\times6\times4\sqrt{2}}}$

$= \sqrt{12-6-4\sqrt{2}} = \sqrt{6-4\sqrt{2}}$

$= \sqrt{(2-\sqrt{2})^2} = 2-\sqrt{2}$

717 **(b)**

Vertices of the triangle are $0 = 0+i0, z = x+iy$ and $ze^{i\alpha} = (x+iy)(\cos\alpha+i\sin\alpha)$

$= (x\cos\alpha - y\sin\alpha) + i(y\cos\alpha + x\sin\alpha)$

$\therefore$ Area of triangle

$= \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ x & y & 1 \\ (x\cos\alpha - y\sin\alpha) & (y\cos\alpha + x\sin\alpha) & 1 \end{vmatrix}$

$= \frac{1}{2}[xy\cos\alpha + x^2\sin\alpha - xy\cos\alpha + y^2\sin\alpha]$

$= \frac{1}{2}(x^2+y^2)\sin\alpha = \frac{1}{2}|z|^2\sin\alpha$ $\left(\because |z| = \sqrt{x^2+y^2}\right)$

718 **(d)**

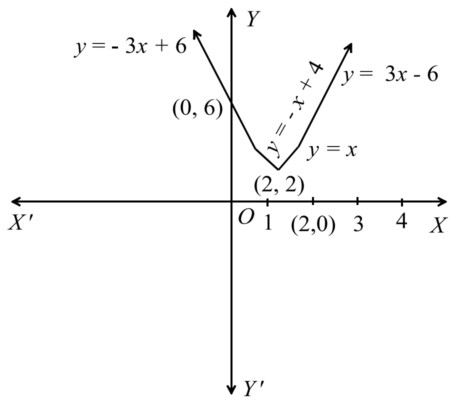
We have,

$(\cos\theta + i\sin\theta)(\cos 3\theta + i\sin 3\theta)$

$\ldots[\cos(2n-1)\theta + i\sin(2n-1)\theta] = 1+i0$

$\Rightarrow \cos[\theta + 3\,\theta + 5\,\theta + \cdots + (2\,n - 1)\theta]$
$+ i \sin[\theta + 3\,\theta + 5\,\theta + \cdots$
$+ (2n - 1)\theta] = 1 + i\,0$
$\Rightarrow \cos(n^2\theta) + i\sin(n^2\theta) = 1 + i\,0$
$\Rightarrow \cos n^2\,\theta = 1$ and $\sin n^2\theta = 0$
$\Rightarrow n^2\theta = 2\,r\,\pi \Rightarrow \theta = \dfrac{2\,r\,\pi}{n^2}$

719 **(c)**
We have,
$|x-1| + |x-2| + |x-3| \geq 6$
Following cases arise:
CASE I $When\ x < 1$
In this case, we have
$|x-1| = -(x-1), |x-2| = -(x-2)$
and $|x-3| = -(x-3)$
$\therefore |x-1| + |x-2| + |x-3| \geq 6$
$\Rightarrow -3x + 6 \geq 6 \Rightarrow x \leq 0$
But, $x < 1$. Therefore, $x \leq 0$ i.e. $x \in (-\infty, 0]$
CASE II $When\ 1 \leq x < 2$
In this case, we have
$|x-1| = x-1, |x-2| = -(x-2)$
and, $|x-3| = -(x-3)$
$\therefore |x-1| + |x-2| + |x-3| \geq 6$
$\Rightarrow x - 1 - (x-2) - (x-3) \geq 6$
$\Rightarrow -x + 4 \geq 6 \Rightarrow -x - 2 \geq 0 \Rightarrow x + 2 \leq 0 \Rightarrow x \leq -2$
But, $1 \leq x < 2$. Therefore, $x \in [1, 2)$
CASE III $When\ 2 \leq x < 3$
In this case, we have
$|x-1| = x-1, |x-2| = x-2$
and, $|x-3| = -(x-3)$
$\therefore |x-1| + |x-2| + |x-3| \geq 6$
$\Rightarrow x - 1 + x - 2 - (x-3) \geq 6 \Rightarrow x \geq 6$
But, $2 \leq x < 3$. So, there is no solution in this case
CASE IV $When\ x \geq 3$
In this case, we have
$|x-1| = x-1, |x-2| = x-2$ and $|x-3| = x-3$
$\therefore |x-1| + |x-2| + |x-3| \geq 6$
$\Rightarrow x - 1 + x - 2 + x - 3 \geq 6 \Rightarrow 3x \geq 2 \Rightarrow x \geq 4$
But, $x \geq 3$. Therefore, $x \in [4, \infty)$
Hence, $x \in (-\infty, 0] \cup [4, \infty)$



720 **(d)**
Let $ABCDEF$ be the regular hexagon having its centre at the origin O. Let $1 + 2i$ be the affix of vertex A. Then,
$OA = |1 + 2i| = \sqrt{5}$
$\therefore$ Perimeter $= 6(\text{Side}) = 6 \times OA = 6\sqrt{5}$

721 **(c)**
Given that, $|\beta| = 1$
$\therefore \left|\dfrac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right| = \left|\dfrac{\beta - \alpha}{\beta\,\bar{\beta} - \bar{\alpha}\beta}\right|$
$= \left|\dfrac{\beta - \alpha}{\beta(\bar{\beta} - \bar{\alpha})}\right| = \dfrac{1}{|\beta|}\left|\dfrac{\beta - \alpha}{(\bar{\beta} - \bar{\alpha})}\right|$
$= \dfrac{1}{|\beta|} = 1 \;\; (\because |z| = |\bar{z}|)$

722 **(c)**
Here, $\alpha + \beta + \gamma = 0, \;\; \alpha\beta + \beta\gamma + \gamma\alpha = 4$
And $\alpha\beta\gamma = 1$
$\therefore \dfrac{1}{\alpha + \beta} + \dfrac{1}{\beta + \gamma} + \dfrac{1}{\gamma + \alpha} = -\dfrac{1}{\gamma} - \dfrac{1}{\alpha} - \dfrac{1}{\beta}$
$= -\left[\dfrac{1}{\gamma} + \dfrac{1}{\alpha} + \dfrac{1}{\beta}\right] = -\left[\dfrac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma}\right] = -4$

724 **(d)**
Given, $\left(3 + 2\sqrt{2}\right)^{x^2 - 8} + \left(3 + 2\sqrt{2}\right)^{8 - x^2} = 6$
Let $\left(3 + 2\sqrt{2}\right)^{x^2 - 8} = y$
$\therefore \;\; y + y^{-1} = 6$
$\Rightarrow y^2 - 6y + 1 = 0$
$\Rightarrow \;\; y = \dfrac{6 \pm \sqrt{36 - 4}}{2 \times 1}$
$= \dfrac{6 \pm 4\sqrt{2}}{2} = 3 \pm 2\sqrt{2}$
For positive sign
$\left(3 + 2\sqrt{2}\right)^{x^2 - 8} = 3 + 2\sqrt{2}$
$\Rightarrow x^2 - 8 = 1 \;\; \Rightarrow \;\; x = \pm 3$
For negative sign
$\left[\left(3 + 2\sqrt{2}\right)^{-1}\right]^{8 - x^2} = 3 - 2\sqrt{2}$
$\Rightarrow \left(3 - 2\sqrt{2}\right)^{8 - x^2} = 3 - 2\sqrt{2}$
$\Rightarrow 8 - x^2 = 1 \;\; \Rightarrow \;\; x^2 = 7$

$\Rightarrow \quad x = \pm\sqrt{7}$

725 **(a)**

Let roots be α and 2α

$\therefore \ \alpha + 2\alpha = 3\alpha = -\dfrac{(3a-1)}{(a^2-5a+3)}$

And $\alpha.2\alpha = 2\alpha^2 = \frac{2}{(a^2-5a+3)}$

$\Rightarrow \ \dfrac{(3a-1)^2}{9(a^2-5a+3)^2} = \dfrac{1}{(a^2-5a+3)}$

$\Rightarrow (3a-1)^2 = 9(a^2-5a+3)$

$\Rightarrow 45a - 6a = 27 - 1 \quad \Rightarrow \quad a = \dfrac{2}{3}$

726 **(a)**

Here, $\tan A + \tan B = p$ and $\tan A \tan B = q$

Now, $\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A \tan B} = \frac{p}{1-q}$

$\therefore \ \sin^2(A+B) = \dfrac{1-\cos[2(A+B)]}{2}$

$= \dfrac{1}{2}\left[1 - \dfrac{1-\tan^2(A+B)}{1+\tan^2(A+B)}\right]$

$= \dfrac{1}{2}\left[1 - \dfrac{1-\left(\frac{p}{1-q}\right)^2}{1+\left(\frac{p}{1-q}\right)^2}\right]$

$= \dfrac{1}{2}\left[\dfrac{(1-q)^2+p^2-(1-q)^2+p^2}{(1-q)^2+p^2}\right]$

$= \dfrac{p^2}{p^2+(1-q)^2}$

727 **(c)**

Given, $x + iy = \sqrt{-7+24i}$

$\therefore \quad x = \pm\sqrt{\dfrac{1}{2}[(-7)^2+(24)^2-7]}$

$= \pm\sqrt{\dfrac{1}{2}[49+576-7]}$

$= \pm\sqrt{\dfrac{1}{2}[25-7]} = \pm\sqrt{9} = \pm 3$

728 **(d)**

Since the triangle is equilateral. Therefore,

$(z_2 - z_1) = e^{\frac{i\pi}{3}}(z_3 - z_1)$ and $z_1 - z_3$

$= e^{\frac{i\pi}{3}}(z_2 - z_3)$

$\Rightarrow \dfrac{z_2 - z_1}{z_1 - z_3} = \dfrac{z_3 - z_1}{z_2 - z_3}$

$\Rightarrow (z_2 - z_1)(z_2 - z_3) = (z_3 - z_1)(z_1 - z_3)$

$\Rightarrow z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$

$\Rightarrow (z_1 - z_2)^2 + (z_2 - z_3)^2 + (z_3 - z_1)^2 = 0$

Again from (i), we have

$\Rightarrow (z_2 - z_3)(z_3 - z_1) + (z_1 - z_2)(z_3 - z_1)$
$+ (z_1 - z_2)(z_2 - z_3) = 0$

$\Rightarrow \dfrac{1}{z_1 - z_2} + \dfrac{1}{z_2 - z_3} + \dfrac{1}{z_3 - z_1} = 0$

729 **(a)**

$\left(\dfrac{1+i}{1-i}\right)^x = \left[\dfrac{(1+i)(1+i)}{(1-i)(1+i)}\right]^x = \left[\dfrac{(1+i)}{1-i^2}\right]^x$

$= \left[\dfrac{1-1+2i}{2}\right]^x$

$\Rightarrow \quad \left(\frac{1+i}{1-i}\right)^x = (i)^x = 1 \qquad$ [given]

$\therefore \quad x = 4n$

730 **(c)**

Given, $z = x + iy$

$\therefore \quad \dfrac{z-1}{z+1} = \dfrac{x+iy-1}{x+iy+1}$

$\dfrac{(x-1)+iy}{(x+1)+iy} \times \dfrac{(x+1)-iy}{(x+1)-iy}$

$= \dfrac{x^2+y^2+2iy-1}{x^2+1+2x+y^2}$

$\therefore \quad \arg\left(\dfrac{z-1}{z+1}\right) = \tan^{-1}\dfrac{2y}{x^2+y^2-1}$

$\Rightarrow \quad \tan^{-1}\frac{2y}{x^2+y^2-1} = \frac{\pi}{4}$ [given]

$\Rightarrow \quad \dfrac{2y}{x^2+y^2-1} = \tan\dfrac{\pi}{4} = 1$

$\Rightarrow \quad x^2 + y^2 - 2y = 1$

731 **(a)**

Since, $\sin\alpha$ and $\cos\alpha$ are the roots of the equation $ax^2 + bx + c = 0$, then

$\sin\alpha + \cos\alpha = -\frac{b}{a}$ and $\sin\alpha \ \cos\alpha = \frac{c}{a}$

To eliminate α, we get

$1 = \sin^2\alpha + \cos^2\alpha$

$\Rightarrow 1 = (\sin\alpha + \cos\alpha)^2 - 2\sin\alpha \ \cos\alpha$

$\Rightarrow 1 = \dfrac{b^2}{a^2} = \dfrac{2c}{a}$

$\Rightarrow a^2 - b^2 + 2ac = 0$

732 **(c)**

$\left(\dfrac{-1+3i}{2+i}\right) = \dfrac{-1-3i}{2+i} \times \dfrac{2-i}{2-i}$

$= \dfrac{-2+i-6i+3i^2}{4+1} = -1-i$

$\therefore$ Argument of $\left(\frac{-1-3i}{2+i}\right) = \tan^{-1}\left(\frac{-1}{-1}\right) = 225°$

[Since, the given complex number lies in IIIrd quadrant]

733 **(a)**

Let $f(x) = ax^2 + 2bx - 3c$

We have,

$\dfrac{3c}{4} < a + b \Rightarrow 4a + 4b - 3c > 0 \Rightarrow f(2) > 0$

Now,

$f(x) = 0$ has no real root

$\Rightarrow f(x) > 0$ for all x or, $f(x) < 0$ for all x

$\Rightarrow f(x) > 0$ for all x $\quad [\because f(2) > 0]$
$\Rightarrow f(0) > 0 \Rightarrow -3c > 0 \Rightarrow c < 0$

734 **(b)**

We have,

$$\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$$

$$\Rightarrow x^2 + x(a+b-2c) + ab - ac - bc = 0$$

Let its roots be α, β. Then,

$$\alpha + \beta = 0 \text{ (given)} \Rightarrow c = \frac{a+b}{2} \quad \dots \text{(i)}$$

Now,

$$\alpha\beta = ab - ac - bc = ab - c(a+b)$$
$$= -\frac{1}{2}(a^2 + b^2) \text{ [Using (i)]}$$

735 **(d)**

Given complex number is

$$\frac{(1+i)^2}{1-i} = \frac{(1+i^2+2i)}{1-i} \times \frac{1+i}{1+i} = \frac{2i+2i^2}{1+1}$$
$$= i - 1$$

$\therefore$ Required conjugate is $-i-1$

736 **(a)**

$$\left|\frac{z-5i}{z+5i}\right| = 1 \quad \Rightarrow \quad \left|\frac{x+i(y-5)}{x+i(y+5)}\right| = 1$$

$\Rightarrow \quad |x+i(y-5)| = |x+i(y+5)|$

$\Rightarrow \quad x^2 + 25 - 10y + y^2 = x^2 + y^2 + 25 + 10y$

$\Rightarrow \quad y = 0$

737 **(a)**

Clearly,

LHS $= 2\cos^2(x/2)\sin^2 x \leq 2$ and, RHS $= x^2 + \frac{1}{x^2} \geq 2$

Thus, the equality holds when each side is equal to 2. But, RHS is equal to 2 for $x = 1$ while LHS is less than 2 for this value of x. Consequently the equation has no solution

738 **(c)**

Using partial fractions, we have

$$\frac{\pi}{n(n+1)(n+2)} = \pi\left\{\frac{1}{2n} - \frac{1}{n+1} + \frac{1}{2(n+2)}\right\}$$

$$\Rightarrow \frac{\pi}{n(n+1)(n+2)} = \frac{\pi}{2}\left\{\left(\frac{1}{n} - \frac{1}{n+1}\right) - \left(\frac{1}{n+1} - \frac{1}{n+2}\right)\right\}$$

$$\therefore z_n = \cos\frac{\pi}{2}\left\{\left(\frac{1}{n} - \frac{1}{n+1}\right) - \left(\frac{1}{n+1} - \frac{1}{n+2}\right)\right\} + i\sin\frac{\pi}{2}\left\{\left(\frac{1}{n} - \frac{1}{n+1}\right) - \left(\frac{1}{n+1} - \frac{1}{n+2}\right)\right\}$$

Now,

$z_1 z_2 \dots z_n$

$$= \cos\frac{\pi}{2}\left[\left\{\left(1-\frac{1}{2}\right) + \left(\frac{1}{2}-\frac{1}{3}\right) + \left(\frac{1}{3}-\frac{1}{4}\right) + \cdots + \left(\frac{1}{n} - \frac{1}{n+1}\right)\right\} - \left\{\left(\frac{1}{2}-\frac{1}{3}\right) + \left(\frac{1}{3}-\frac{1}{4}\right) + \cdots + \left(\frac{1}{n+1} - \frac{1}{n+2}\right)\right\}\right]$$
$$+ i\sin\frac{\pi}{2}\left[\left\{\left(1-\frac{1}{2}\right) + \left(\frac{1}{2}-\frac{1}{3}\right) + \left(\frac{1}{3}-\frac{1}{4}\right) + \cdots + \left(\frac{1}{n} - \frac{1}{n+1}\right)\right\} - \left\{\left(\frac{1}{2}-\frac{1}{3}\right) + \left(\frac{1}{3}-\frac{1}{4}\right) + \cdots + \left(\frac{1}{n+1} - \frac{1}{n+2}\right)\right\}\right]$$

$$= \cos\frac{\pi}{2}\left[\left(1 - \frac{1}{n+1}\right) - \left(\frac{1}{2} - \frac{1}{n+2}\right)\right] + i\sin\frac{\pi}{2}\left[\left(1 - \frac{1}{n+1}\right) - \left(\frac{1}{2} - \frac{1}{n+2}\right)\right]$$

$$= \cos\frac{\pi}{2}\left\{\frac{n}{n+1} - \frac{n}{2(n+2)}\right\} + i\sin\frac{\pi}{2}\left\{\frac{n}{n+1} - \frac{n}{2(n+2)}\right\}$$

$$\therefore \lim_{n\to\infty}(z_1 z_2 \dots z_n) = \cos\frac{\pi}{2}\left\{1 - \frac{1}{2}\right\} + i\sin\frac{\pi}{2}\left\{1 - \frac{1}{2}\right\}$$

$$\Rightarrow \lim_{n\to\infty}(z_1 z_2 \dots z_n) = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} = \frac{1+i}{\sqrt{2}}$$

739 **(d)**

Given, $x + iy = \left(\frac{a+ib}{c+id}\right)^{1/2}$

$$\Rightarrow \quad |x+iy| = \left|\frac{a+ib}{c+id}\right|^{1/2}$$

(Taking modulus from both side and using $|z^n| = |z|^n$)

$$\Rightarrow \quad |x+iy|^2 = \left|\frac{a+ib}{c+id}\right|$$

$$\Rightarrow \quad x^2 + y^2 = \sqrt{\frac{a^2+b^2}{c^2+d^2}}$$

740 **(b)**

Let $z = x + iy$

$\therefore \ \arg(z) = \tan^{-1}\left(\frac{y}{x}\right)$

Then, $\arg(\bar{z}) = \tan^{-1}\left(-\frac{y}{x}\right) = 2\pi - \tan^{-1}\frac{y}{x}$

$= 2\pi - \arg(z)$

Since, in argument of a conjugate of a complex, the real axis is unaltered, but imaginary axis be changed, hence it is given by $2\pi - \arg(z)$

742 **(a)**

We have,

$z_1 = a + ib, z_2 = \frac{1}{-a + ib} = \frac{-a - i\,b}{a^2 + b^2}$
$= \frac{-a}{a^2 + b^2} - \frac{i\,b}{a^2 + b^2}$

The equation of a line passing through points having affixes z_1 and z_2 is

$z(\overline{z_1} - \overline{z_2}) - \bar{z}(z_1 - z_2) + z_1\overline{z_2} - \overline{z_1}z_2 = 0$

So, the equation of the required line is

$z\left[\left(a + \frac{a}{a^2 + b^2}\right) + i\left(-b - \frac{b}{a^2 + b^2}\right)\right]$
$-\bar{z}\left[\left(a + \frac{a}{a^2 + b^2}\right) + i\left(b + \frac{b}{a^2 + b^2}\right)\right]$
$+(a + i\,b)\left(-\frac{a}{a^2 + b^2} + i\frac{b}{a^2 + b^2}\right)$
$-(a - ib)\left(-\frac{a}{a^2 + b^2} - \frac{b}{a^2 + b^2}\right)$
$= 0$

$\Rightarrow z[(a^3 + ab^2 + a) - i(a^2b + b^3 + b)]$
$-\bar{z}[(a^3 + ab^2 + a) + i(a^2b + b)] = 0$

Clearly, it passes through the origin

743 **(d)**

The discriminant D of the given equation is given by

$D = \cos^2 p - 4\sin p(\cos p - 1)$
$= \cos^2 p + 4\sin p(1 - \cos p)$

Since the equation has real roots. Therefore, $D \geq 0$

$\Rightarrow \cos^2 p + 4\sin p(1 - \cos p) \geq 0$

$\Rightarrow \sin p \geq 0$

$\Rightarrow p \in (0, \pi)$

744 **(b)**

If the roots of the equation $x^2 - 8x + a^2 - 6a = 0$ are real, then

$\Rightarrow 64 - 4(a^2 - 6a) \geq 0 \quad [\because \text{Disc} \geq 0]$

$\Rightarrow a^2 - 6a - 16 \leq a \in [-2, 8]$

745 **(d)**

$\overline{z_2}z_1 = (3 - 5i)(1 + 2i) = 13 + i$

$\therefore \frac{\overline{z_2}z_1}{z_2} = \frac{(13 + i)}{(3 + 5i)} \times \frac{(3 - 5i)}{(3 - 5i)} = \frac{44 - 62i}{34}$

$\therefore$ Real part of $\left(\frac{\overline{z_2}z_1}{z_2}\right) = \frac{44}{34} = \frac{22}{17}$

746 **(b)**

Let $S = \log_2 \log_3 \ldots \log_{99} \log_{100} 100^{99^{98^{\cdot^{\cdot^{2^1}}}}}$

$= \log_2 \log_3 \ldots \log_{99} 99^{98^{\cdot^{\cdot^{2^1}}}} \quad [\because \log_a a = 1]$

$= \log_2 2^1 = 1$

747 **(a)**

Let α and β be the roots of given equation $x^2 + ax + 1 = 0$

Then

$\alpha + \beta = -a$ and $\alpha\beta = 1$

Now, $|\alpha - \beta| = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta} = \sqrt{a^2 - 4}$

Given condition, $\sqrt{a^2 - 4} < \sqrt{5}$

$\Rightarrow a^2 - 4 < 5 \Rightarrow |a| < 3$

$\Rightarrow a \in (-3, 3)$

748 **(b)**

We have,

$\left|\frac{z_1 - z_2}{z_1 + z_2}\right| = 1$

$\Rightarrow \frac{z_1 - z_2}{z_1 + z_2} = \cos\alpha + i\sin\alpha$

$\Rightarrow \frac{2\,z_1}{-2\,z_2} = \frac{1 + \cos\alpha + i\sin\alpha}{\cos\alpha - 1 + i\sin\alpha}$

$\Rightarrow \frac{z_1}{z_2} = i\cot\frac{\alpha}{2} \Rightarrow z_1 = i\,k\,z_2$, where $k = \cot\frac{\alpha}{2}$

ALITER We have,

$\left|\frac{z_1 - z_2}{z_1 + z_2}\right| = 1$

$\Rightarrow |z_1 - z_2| = |z_1 + z_2|$

$\Rightarrow$ Diagonals of a parallelogram with sides z_1 and z_2 are equal

$\Rightarrow$ It is a rectangle $\Rightarrow z_2 = \left|\frac{z_2}{z_1}\right| e^{i\,\pi/2} = k\,i$

749 **(d)**

Since the lines are perpendicular

$\therefore \frac{-\alpha}{\alpha} + \frac{-\beta}{\beta} = 0 \Rightarrow \alpha\bar{\beta} + \bar{\alpha}\,\beta = 0$

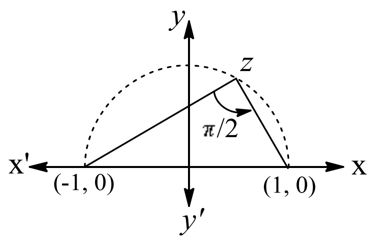
750 **(c)**

Since a quadratic equation with coefficients as odd integers cannot have rational roots.

Therefore, the given equation has no rational root

752 **(b)**

We have, $\arg(z - 1) - \arg(z + 1) = \frac{\pi}{2}$



It is clear from the figure that it a semi circle

753 **(d)**

Since, quadratic equation $ax^2 + bx + c = 0$ has three distinct roots. So, it must be identity. So, $a = b = c = 0$.

754 **(c)**

Since, $(1, -p)$ is the root of given equation so it will satisfy the given equation

$\therefore \quad (1-p)^2 + p(1-p) + (1-p) = 0$

$\Rightarrow \quad (1-p)[1-p+p+1] = 0$

$\Rightarrow p = 1$

On putting the value of p in given equation, we get

$x^2 + x = 0 \quad \Rightarrow \quad x = 0, -1$

755 **(d)**

$\omega^{99} + \omega^{100} + \omega^{101}$

$= (\omega^3)^{33} + (\omega^3)^{33}\omega + (\omega^3)^{33}\omega^2$

$= 1 + \omega + \omega^2 = 0$

756 **(c)**

We have, $\alpha + \beta = -7/2$ and $\alpha\,\beta = c/2$

Now,

$|\alpha^2 - \beta^2| = \frac{7}{4}$

$\Rightarrow \alpha^2 - \beta^2 = \pm\frac{7}{4}$

$\Rightarrow (\alpha+\beta)(\alpha-\beta) = \pm\frac{7}{4}$

$\Rightarrow -\frac{7}{2}\sqrt{\frac{49}{4} - 2\,c} = \pm\frac{7}{4}$

$\Rightarrow \sqrt{49 - 8c} = \mp 1 \Rightarrow 49 - 8\,c = 1 \Rightarrow c = 8$

757 **(c)**

We have, $\cos\alpha + \cos\beta + \cos\gamma = 0$ ...(i)

and $\sin\alpha + \sin\beta + \sin\gamma = 0$ ...(ii)

Let $a = \cos\alpha + i\sin\alpha$;

$b = \cos\beta + i\sin\beta$

and $c = \cos\gamma + i\sin\gamma$

Therefore, $a + b + c = (\cos\alpha + \cos\beta + \cos\gamma)$

$+i\,(\sin\alpha + \sin\beta + \sin\gamma)$

$= 0 + i0 = 0$ [from Eqs.(i)and (ii)]

If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

$\Rightarrow \quad (\cos\alpha + i\sin\alpha)^3 + (\cos\beta + i\sin\beta)^3 + (\cos\gamma + i\sin\gamma)^3$

$= 3(\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta)(\cos\gamma + i\sin\gamma)$

$\Rightarrow \quad (\cos 3\alpha + i\sin 3\alpha) + (\cos 3\beta + i\sin 3\beta) + (\cos 3\gamma + i\sin 3\gamma)$

$= 3[\cos(\alpha+\beta+\gamma) + i\sin(\alpha+\beta+\gamma)]$

$\Rightarrow \quad \cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha+\beta+\gamma)$

759 **(b)**

We have,

$\alpha_1\alpha_2 = \beta_1\beta_2 = 1 \Rightarrow \alpha_1 = \frac{1}{\alpha_2}$ and $\beta_1 = \frac{1}{\beta_2}$

This means that the roots of the equation $a_2x^2 + b_2x + c_2 = 0$ are reciprocal of the roots of the equation $a_1x^2 + b_1x + c_1 = 0$

Therefore, equations $a_1x^2 + b_1x + c_1 = 0$ and $c_2x^2 + b_2x + a_2 = 0$ have same roots

$\therefore \frac{a_1}{c_2} = \frac{b_1}{b_2} = \frac{c_1}{a_2}$

760 **(c)**

Given, $a + b + c = 0,\ 4ax^2 + 3bx + 2c = 0$

Now, $D = 9b^2 - 4(4a)(2c)$

$= 9(a+c)^2 - 32ac = 9(a-c)^2 + 4ac > 0$

Hence, roots are real

761 **(b)**

We have,

$\arg\left(\frac{z - 3\sqrt{3}}{z + 3\sqrt{3}}\right) = \frac{\pi}{3}$

$\arg\left(\frac{3\sqrt{3} - z}{-3\sqrt{3} - 2}\right) = \frac{\pi}{3}$

$\Rightarrow \arg\left(\frac{\overrightarrow{PA}}{\overrightarrow{PB}}\right) = \frac{\pi}{3}$

$\Rightarrow P$ moves in such a way that when PB is rotated through $\frac{\pi}{3}$ in coincides with PA

$\Rightarrow P$ lies on the segment of the circle such that $\angle BPA = \frac{\pi}{3}$ and P is above x-axis

Now, $\arg\left(\frac{z-3\sqrt{3}}{z+3\sqrt{3}}\right) = \frac{\pi}{3}$

$\Rightarrow \arg(z - 3\sqrt{3}) - \arg(z + 3\sqrt{3}) = \frac{\pi}{3}$

$\Rightarrow \tan^{-1}\frac{y}{x - 3\sqrt{3}} - \tan^{-1}\frac{y}{x + 3\sqrt{3}} = \frac{\pi}{3}$, where $z = x + iy$

$\Rightarrow \tan^{-1}\left(\frac{\frac{y}{x-3\sqrt{3}} - \frac{y}{x+3\sqrt{3}}}{1 + \frac{y^2}{x^2-27}}\right) = \frac{\pi}{3}$

$\Rightarrow \frac{6\sqrt{3}\,y}{x^2 + y^2 - 27} = \sqrt{3}$

$\Rightarrow x^2 + y^2 - 6y - 27 = 0$

$\Rightarrow x^2 + (y-3)^2 = 36$

$\Rightarrow |(x + iy) - (0 + 3i)|^2 = 36 \Rightarrow |z - 3i| = 6$

Hence, the locus of z is $|z - 3i| = 6, \mathrm{Im}\,(z) > 0$

762 **(b)**

Here, $\sin\alpha + \cos\alpha = -\frac{q}{p}$ and $\sin\alpha . \cos\alpha = \frac{r}{p}$

$\therefore (\sin\alpha + \cos\alpha)^2 = \left(-\frac{q}{p}\right)^2$

$\Rightarrow \sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha = \frac{q^2}{p^2}$

$\Rightarrow 1 + 2.\frac{r}{p} = \frac{q^2}{p^2}$

$\Rightarrow p(p + 2r) = q^2$

$\Rightarrow p^2 - q^2 + 2rp = 0$

763 **(c)**

For the given equation to be meaningful, we must have $x > 0$. For $x > 0$, the given equation can be written as

$\frac{3}{4}(\log_2 x)^2 \log_2 x - \frac{5}{4} = \log_x \sqrt{2} = \frac{1}{2}\log_x 2$

Put $t = \log_2 x$ so that $\log_x 2 = \frac{1}{t}$

$\therefore \frac{3}{4}t^2 + t - \frac{5}{4} = \frac{1}{2}\left(\frac{1}{t}\right)$

$\Rightarrow 3t^3 + 4t^2 - 5t - 2 = 0$

$\Rightarrow (t-1)(t+2)(3t+1) = 0$

$\Rightarrow \log_2 x = t = 1, -2, -\frac{1}{3}$

$\Rightarrow x = 2, 2^{-2}, 2^{-\frac{1}{3}}$

or $x = 2, \frac{1}{4}, \frac{1}{2^{1/3}}$

Thus, the given equation has exactly three real solution out of which exactly one is irrational $ie, \frac{1}{2^{1/3}}$.

765 **(a)**

Since, $z\bar{z}(z^2 + \bar{z}^2) = 350$

$\Rightarrow \quad 2(x^2 + y^2)(x^2 - y^2) = 350$

$\Rightarrow \quad (x^2 + y^2)(x^2 - y^2) = 175$

Since, $x, y \in I$, the only possible case which gives integral solution, is

$x^2 + y^2 = 25$ ...(i)

$x^2 - y^2 = 7$ ...(ii)

From Eqs. (i) and (ii), we get

$x^2 = 16, \quad y^2 = 9$

$\Rightarrow \quad x = \pm 4, \quad y = \pm 3$

$\therefore$ Area of rectangle$= 8 \times 6 = 48$

766 **(b)**

Let $a_k + ib_k = r_k(\cos\theta_k + i\sin\theta_k), k = 1, 2, .., n$.

Then, $r_k = \sqrt{a_k^2 + b_k^2}$ and $\tan\theta_k = \frac{b_k}{a_k}$

$\therefore (a_1 + ib_1)(a_2 + ib_2)\dots(a_n + ib_n) = A + iB$

$\Rightarrow r_1 r_2 \dots r_n[\cos(\theta_1 + \theta_2 + \dots + \theta_n)$
$+ i\sin(\theta_1 + \theta_2 + \dots + \theta_n)]$
$= A + iB$

$\Rightarrow r_1 r_2 r_3 \dots r_n = \sqrt{A^2 + B^2}$ and $\tan(\theta_1 + \theta_2 + \dots + \theta_n) = \frac{B}{A}$

$\Rightarrow r_1^2\, r_2^2\, r_3^2 \dots r_n^2 = A^2 + B^2$ and $\theta_1 + \theta_2 + \dots + \theta_n = \tan^{-1}\frac{B}{A}$

$\Rightarrow (a_1^2 + b_1^2)(a_2^2 + b_2^2)\dots(a_n^2 + b_n^2) = A^2 + B^2$

and, $\tan^{-1}\frac{b_1}{a_1} + \dots + \tan^{-1}\frac{b_n}{a_n} = \tan^{-1}\frac{B}{A}$

767 **(d)**

Let $(\cos x - i\sin 2x) = \sin x + i\cos 2x$

$\Rightarrow \cos x + i\sin 2x = \sin x + i\cos 2x$

$\therefore \cos x = \sin x$ and $\sin 2x = \cos 2x$

$\Rightarrow \tan x = 1$ and $\tan 2x = 1$

Which is impossible

768 **(b)**

Let the required number is x.

According to given condition

$x = \sqrt{x} + 12$

$\Rightarrow x - 12 = \sqrt{x}$

$\Rightarrow x^2 - 25x + 144 = 0$

$\Rightarrow x^2 - 16x - 9x + 144 = 0$

$\Rightarrow x = 16, 9$

Since $x = 9$ does not hold the condition.

$\therefore x = 16$

769 **(b)**

We have,

$$\frac{|x-2|}{x-2} = \begin{cases} \frac{x-2}{x-2} = 1, & \text{if } x > 2 \\ \frac{-(x-2)}{x-2} = -1, & \text{if } x < 2 \end{cases}$$

$\therefore \frac{|x-2|}{x-2} < 0$ is true for all $x < 2$

Hence, the solution set of the given inequation is $(-\infty, 2)$

770 **(d)**

$(\cos\alpha + i\sin\alpha)^{3/5} = e^{i3/5} = e^{i(2n\pi + 3\alpha)/5}$

$\therefore$ Required product $= e^{i3\alpha/5} . e^{i(2\pi+3\alpha)/5} . e^{i(4\pi+3\alpha)/5} . e^{i(6\pi+3\alpha)/5} . e^{i(8\pi+3\alpha)/5}$

$= e^{i(4\pi + 3\alpha)}$

$= \cos(4\pi + 3\alpha) + i\sin(4\pi + 3\alpha)$

$= \cos 3\alpha + i\sin 3\alpha$

771 **(d)**

We have, $a = \cos\alpha + i\sin\alpha$;

$b = \cos\beta + i\sin\beta$

and $c = \cos\gamma + i\sin\gamma$

Now, $\frac{b}{c} = \frac{\cos\beta + i\sin\beta}{\cos\gamma + i\sin\gamma} \times \frac{\cos\gamma - i\sin\gamma}{\cos\gamma - i\sin\gamma}$

$= \cos\beta\cos\gamma + \sin\beta\sin\gamma$
$+ i[\sin\beta\cos\gamma - \sin\gamma\cos\beta]$

$\Rightarrow \frac{b}{c} = \cos(\beta - \gamma) + i\sin(\beta - \gamma)$ ...(i)

Similarly, $\frac{c}{a} = \cos(\gamma - \alpha) + i\sin(\gamma - \alpha)$ ...(ii)

and $\frac{a}{b} = \cos(\alpha - \beta) + i\sin(\alpha - \beta)$ ...(iii)

On adding Eqs. (i), (ii), (iii), we get

$\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta)$
$+ i[\sin(\beta - \gamma) + \sin(\gamma - \alpha) + \sin(\alpha - \beta)] = 1$

On equating real part on both sides, we get

$\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = 1$

772 **(a)**

$$\frac{x-4}{x^2-5x+6}=\frac{x-4}{(x-2)(x-3)}$$
$$=\frac{2}{(x-2)}-\frac{1}{(x-3)}$$
$$=2(x-2)^{-1}-(x-3)^{-1}$$
$$=2(-2)^{-1}\left(1-\frac{x}{2}\right)^{-1}-(-3)^{-1}\left(1-\frac{x}{3}\right)^{-1}$$
$$=-\left[1+\left(\frac{x}{2}\right)+\left(\frac{x}{2}\right)^2+\left(\frac{x}{2}\right)^3+\dots\right]$$
$$+\frac{1}{3}\left[1+\left(\frac{x}{3}\right)+\left(\frac{x}{3}\right)^2+\left(\frac{x}{3}\right)^3+\dots\right]$$

$\therefore$ Coefficient of x^3 in $\frac{x-4}{x^2-5x+6}$

$$=-\left(\frac{1}{2}\right)^3+\frac{1}{3}\left(\frac{1}{3}\right)^3=-\frac{1}{8}+\frac{1}{81}=-\frac{73}{648}$$

773 **(a)**

Given, $a=\cos\theta+i\sin\theta$

Now, $\frac{1+a}{1-a}=\frac{1+\cos\theta+i\sin\theta}{1-\cos\theta-i\sin\theta}$

$$=\frac{(1+\cos\theta)+i\sin\theta}{(1-\cos\theta)-i\sin\theta}\times\frac{(1-\cos\theta)+i\sin\theta}{(1-\cos\theta)+i\sin\theta}$$
$$=\frac{\sin^2\theta+2i\sin\theta-\sin^2\theta}{1+\cos^2\theta-2\cos\theta+\sin^2\theta}$$
$$=\frac{i\,4\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}{4\sin^2\frac{\theta}{2}}=i\cot\frac{\theta}{2}$$

774 **(a)**

Given $\arg\left(\frac{z-2}{z-6i}\right)=\frac{\pi}{2}$

$\therefore\ \arg(z-2)-\arg(z-6i)=\frac{\pi}{2}$

$\Rightarrow\ \arg[(x-2)+iy]-\arg[x+i(y-6)]=\frac{\pi}{2}$

$\Rightarrow\ \tan^{-1}\frac{y}{x-2}-\tan^{-1}\frac{y-6}{x}=\frac{\pi}{2}$

$\Rightarrow\ \left(\frac{\frac{y}{x-2}-\frac{y-6}{x}}{1+\frac{y}{x-2}.\frac{y-6}{x}}\right)=\tan\frac{\pi}{2}$

$\Rightarrow\ 1+\frac{y}{x-2}.\frac{y-6}{x}=0$

$\Rightarrow\ x(x-2)+y(y-6)=0$

This is an equation of circle in diametric form.

775 **(b)**

$\log_4(x-1)=\log_2(x-3)$

$\Rightarrow\log_4(x-1)=2\log_4(x-3)=\log_4(x-3)^2$

$\Rightarrow x-1=x^2+9-6x$

$\Rightarrow x^2-7x+10=0$

$\Rightarrow x=5$ or 2

Hence, $x=5$ [$\because x=2$ makes $\log(x-3)$ undefined]

$\therefore$ Number of solution is 1

776 **(c)**

Let $\alpha=\cos\alpha+i\sin\alpha, b=\cos\beta+i\sin\beta$ and, $c=\cos\gamma+i\sin\gamma$ Then,

$a+2b+3c$

$=(\cos\alpha+2\cos\beta+3\cos\gamma)$
$\quad+i(\sin\alpha+2\sin\beta+3\sin\gamma)=0$

$\Rightarrow a^3+8\,b^3+27\,c^3=18\,abc$

$\Rightarrow\cos 3\,\alpha+8\cos 3\,\beta$
$\quad+27\cos 3\,\gamma=18\cos(\alpha+\beta+\gamma)$

and, $\sin 3\,\alpha+8\sin 3\,\beta+27\sin 3\,\gamma=$
$18\sin(\alpha+\beta+\gamma)$

777 **(b)**

We have,

$$\left(x-\frac{1}{k-1}\right)\left(x-\frac{1}{k}\right)$$
$$=x^2-x\left(\frac{1}{k-1}+\frac{1}{k}\right)+\frac{1}{k(k-1)}$$
$$=x^2-x\left(\frac{1}{k-1}+\frac{1}{k}\right)+\left(\frac{1}{k-1}-\frac{1}{k}\right)$$
$$\therefore f(x)=\sum_{k=2}^{n}\left(x-\frac{1}{k-1}\right)\left(x-\frac{1}{k}\right)$$
$$=\sum_{k=2}^{n}x^2-x\sum_{k=2}^{n}\left(\frac{1}{k-1}+\frac{1}{k}\right)+\sum_{k=2}^{n}\left(\frac{1}{k-1}-\frac{1}{k}\right)$$
$$=(n-1)x^2-x\left\{1+2\left(\frac{1}{2}+\frac{1}{3}+\dots+\frac{1}{n}\right)+\frac{1}{n}\right\}$$
$$+\left(1-\frac{1}{n}\right)$$

$\therefore$ Product of roots $=\frac{1}{n}$

Hence, product of roots as $n\to\infty$ is 0

778 **(a)**

Since, $3p^2=5p+2$

$\Rightarrow p=2,-\frac{1}{3}$

And, $3q^2=5q+2\ \Rightarrow\ q=2,-\frac{1}{3}$

$\because p\neq q$

Here, we assume that $p=2$ and $q=-\frac{1}{3}$

Now, the given roots of the equation are $(3p-2q)$ and $(3q-2p)ie,\left(\frac{20}{3},-5\right)$

Sum of roots$=\frac{20}{3}-5=\frac{5}{3}$

And product of roots$=\frac{20}{3}\times(-5)=-\frac{100}{3}$

$\therefore$ Required equation is

$$x^2-\frac{5}{2}x-\frac{100}{3}=0$$

$\Rightarrow 3x^2-5x-100=0$

779 **(c)**

We have,

$x^3+3x^2+3x+2=0$ ...(i)

$\Rightarrow(x+1)^3+1=0$

$\Rightarrow x+1=(-1)^{1/3}$

$\Rightarrow x+1=-1,-\omega,-1-\omega^2\Rightarrow x=-2,\omega^2,\omega$

It is given that equation (i) and $a\,x^2+bx+c=0$ have two common roots. Also, a quadratic equation has either both real roots or both non-real complex conjugate roots. Therefore, ω and ω^2 are the common roots

$\therefore \omega+\omega^2=-\frac{b}{a}$ and $\omega\times\omega^2=\frac{c}{a}\Rightarrow a=b=c$

780 **(a)**

$\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{1/4}=\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^{1/4}$

$=\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}$

781 **(c)**

$(x+2)(x+12)(x+3)(x+8)=4x^2$.

$\Rightarrow (x^2+14x+24)(x^2+11x+24)=4x^2$

$\Rightarrow \left(x+14+\frac{24}{x}\right)\left(x+11+\frac{24}{x}\right)=4$

Put $x+\frac{24}{x}=y$

$(y+14)(y+11)=4$

$\Rightarrow y^2+25y+154-4=0$

$\Rightarrow y^2+25y+150=0$

$\Rightarrow y^2+15y+10y+150=0$

$\Rightarrow y(y+15)+10(y+15)=0$

$\Rightarrow y=-10,-15$

$\Rightarrow x+\frac{24}{x}=-10, x+\frac{24}{x}=-15$

$\Rightarrow x^2+10x+24=0, x^2+15x+24=0$

$\Rightarrow x^2+6x+4x+24=0$

$\Rightarrow x(x+6)+4(x+6)=0$

$\Rightarrow x=-4,-6$

and $x^2+15x+24=0$

$\Rightarrow x=\frac{-15\pm\sqrt{225-96}}{2}$

$=\frac{-15\pm\sqrt{129}}{2}$

Number of integer root is 2.

782 **(c)**

Since, α,β and $\alpha-k,\beta-k$ are the roots of the equations $ax^2+bx+c=0$ and $Ax^2+Bx+C=0$ respectively.

$\Rightarrow \alpha+\beta=-\frac{b}{a}, \alpha\beta=\frac{c}{a}$

and $\alpha+\beta-2k=-\frac{B}{A},\ (\alpha-k)(\beta-k)=\frac{C}{A}$

Now, $(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta=\frac{(b^2-4ac)}{a^2}$ ...(i)

Also, $\{(\alpha-k)-(\beta-k)\}^2$

$=\{(\alpha-k)+(\beta-k)\}^2-4(\alpha-k)(\beta-k)$

$=\left(-\frac{B}{A}\right)^2-4\left(\frac{C}{A}\right)$

$=\frac{B^2-4AC}{A^2}$ ...(ii)

From Eqs. (i) and (ii)

$\frac{(b^2-4ac)}{a^2}=\frac{B^2-4AC}{A^2}$

$\therefore \frac{B^2-4AC}{b^2-4ac}=\left(\frac{A}{a}\right)^2$

783 **(c)**

Let $\frac{5z_2}{11z_1}=iy \quad\Rightarrow\quad \frac{z_2}{z_1}=\frac{11}{5}\,iy$

Now, $\left|\frac{2z_1+3z_2}{2z_1-3z_2}\right|=\left|\frac{2+3\frac{z_2}{z_1}}{2-3\frac{z_2}{z_1}}\right|=\left|\frac{2+\frac{33}{5}iy}{2-\frac{33}{5}iy}\right|=1$

784 **(a)**

Let α be the root of equation $ax^2+bx+c=0$ then $\frac{1}{\alpha}$ be a root of second equation, therefore

$a\alpha^2+b\alpha+c=0$ ....(i)

and $a'\frac{1}{\alpha^2}+b'\frac{1}{\alpha}+c'=0$

or $c'\alpha^2+b'\alpha+a'=0$ ...(ii)

On solving Eqs. (i) and (ii), we get

$\frac{\alpha^2}{ba'-b'c}=\frac{\alpha}{cc'-aa'}=\frac{1}{ab'-bc'}$

$\Rightarrow (cc'-aa')^2=(ba'-cb')(ab'-bc')$

785 **(c)**

Given, $|x+iy+8|+|x+iy-8|=16$

$\Rightarrow \quad |(x+8)+iy|=16-|(x-8)+iy|$

$\Rightarrow \quad \sqrt{(x+8)^2+y^2}=16-\sqrt{(x-8)^2+y^2}$

$\Rightarrow \quad x^2+64+16x+y^2=256+x^2+64$

$-16x+y^2-32\sqrt{(x-8)^2+y^2}$

$\Rightarrow \quad 32x=32[8-\sqrt{(x-8)^2+y^2}]$

$\Rightarrow \quad \sqrt{(x-8)^2+y^2}=8-x$

$\Rightarrow \quad (x-8)^2+y^2=(8-x)^2$

$\Rightarrow \quad y^2=0 \quad\Rightarrow\quad y=0$

Which, represents a straight line.

786 **(d)**

Let another root of equation

$x^2+(1-3i)x-2(1+i)=0$ is α

$\therefore \quad \alpha+(-1+i)=-(1-3i)$

$\Rightarrow \quad \alpha=2i$

787 **(b)**

The given equation is

$(x^2+x-2)(x^2+x-3)=12$

$\Rightarrow (y-2)(y-3)=12$, where $y=x^2+x$

$\Rightarrow y^2-5y-6=0$

$\Rightarrow y=6,-1$

$\Rightarrow x^2+x=6$ or $x^2+x=1$

$\Rightarrow x^2+x-6=0$ or, $x^2+x-1=0$

$\Rightarrow x=-3,2,\omega,\omega^2$

$\therefore$Sum of real roots $=-3+2=-1$

788 **(a)**

Since $x=4$ is a root of the equation x^2+px+

$12 = 0$.
$\therefore 16 + 4p + 12 = 0 \Rightarrow p = -7$
The equation $x^2 + px + q = 0$ has equal roots
$\therefore p^2 = 4q \Rightarrow 49 = 4q \Rightarrow q = 49/4$
789 **(b)**
We have,
$\frac{3(x-2)}{5} \geq \frac{5(2-x)}{3}$
$\Rightarrow 9(x-2) \geq 25(2-x)$
$\Rightarrow 34x - 68 \geq 0 \Rightarrow x - 2 \geq 0 \Rightarrow x \in [2, \infty)$
790 **(a)**
If $ax^3 + bx + c$ is divisible by $x^2 + bx + c$, then the remainder must be zero when $ax^3 + bx + c$ is divided by $x^2 + bx + c$
We have,
$ax^3 + bx + c = (x^2 + bx + c)(ax - ab)$
$+ \{x(b - ac + ab^2) + c - abc\}$
$\therefore$Remainder $= 0$
$\Rightarrow x(b - ac + ab^2) - c + abc = 0$ for all x
$\Rightarrow b - ac + ab^2 = 0$ and $-c + abc = 0$
$\Rightarrow b - ac + ab^2 = 0$ and $ab = 1$ $[\because c \neq 0]$
$\Rightarrow b - ac + a\left(\frac{1}{a}\right)^2 = 0$ $[\because ab = 1 \Rightarrow b = 1/a]$
$\Rightarrow ab - a^2c + 1 = 0$
$\Rightarrow a^2c - ab - 1 = 0$
$\Rightarrow a$ is a root of $x^2c - bx - 1 = 0$
791 **(b)**
Since p and q are roots of the equation
$x^2 + px + q = 0$
$\therefore p^2 + p^2 + q = 0$ and $q^2 + pq + q = 0$
$\Rightarrow 2p^2 + q = 0$ and $q(q + p + 1) = 0$
$\Rightarrow 2p^2 + q = 0$ and $(q = 0$ or, $q = -p - 1)$
Now,
$q = 0$ and $2p^2 + q = 0$
And
$q = -p - 1$ and $2p^2 + q = 0$
$\Rightarrow 2p^2 - p - 1 = 0$
$\Rightarrow p = 1$ or, $p = -1/2$
Hence, $p = 0, 1, -1/2$
792 **(a)**
Clearly, $(x-4)(x-9) \leq 0$ for all $x \in (4, 9)$
793 **(a)**
We have,
$\frac{6-x}{x-2} = 2 + \frac{x}{x+2}$ ... (i)
Clearly, this is meaningful when $x \neq \pm 2$
Multiplying both sides of (i) by $x + 2$, we get
$\frac{6-x}{x-2} = 2(x+2) + x$
$\Rightarrow 3x^2 - x - 14 = 0$
$\Rightarrow (x+2)(3x-7) = 0 \Rightarrow x = \frac{7}{3}$ $[\because x + 2 \neq 0]$
Hence, the given equation has only one real solution
794 **(b)**
Since, $|-z| = |z|$
And $|z_1 + z_2| \leq |z_1| + |z_2|$
Now, $|z| + |z-1| = |z| + |1-z| \geq |z +$
$1 - z = 1$
795 **(a)**
Let roots of given equation are $\alpha, \alpha + 2$ and β
$\therefore \alpha + \alpha + 2 + \beta = 13$ ...(i)
$\alpha(\alpha+2) + (\alpha+2)\beta + \alpha\beta = 15$ ...(ii)
And $\alpha(\alpha+2)\beta = -189$ ...(iii)
These three equations are satisfies by the option (a)
796 **(b)**
We have $|z + 4| \leq 3$
$-3 \leq z + 4 \leq 3$
$-6 \leq z + 1 \leq 0$
$0 \leq -(z+1) \leq 6$
$0 \leq |z+1| \leq 6$
Hence, greatest and least value of $|z+1|$ are 6 and 0 respectively
797 **(c)**
The given equation is meaningful for $x \neq 1$.
Now,
$x - \frac{2}{x-1} = 1 - \frac{2}{x-1} \Rightarrow x = 1$
But, the equation exist for $x \neq 1$
Hence, the equation has no solution
798 **(b)**
We know that the equation $z\bar{z} + a\bar{z} + \bar{a}z + b = 0$ represents a circle of radius $\sqrt{|a|^2 - b}$
Here, $a = 4 + 3i$ and $b = 5$
$\therefore$ Radius $= \sqrt{|4+3i|^2 - 5} = \sqrt{20} = 2\sqrt{5}$
799 **(a)**
$x^2 - 5|x| + 6 = 0$
$\Rightarrow |x^2| - 5|x| + 6 = 0$
$\Rightarrow (|x| - 2)(|x| - 3) = 0$
$\Rightarrow |x| = 2, \quad |x| = 3$
$\Rightarrow x = \pm 2, \quad x = \pm 3$
Hence, the given equation has four solutions
800 **(a)**
Let roots of the equation $x^2 + px + q = 0$ be α and α^2
$\therefore \alpha + \alpha^2 = -p$ and $\alpha^3 = q$
$\Rightarrow \alpha(\alpha+1) = -p$
$\Rightarrow \alpha^3[\alpha^3 + 1 + 3\alpha(\alpha+1)] = -p$
$\Rightarrow q(q + 1 - 3p) = -p^3$

$\Rightarrow p^3-(3p-1)q+q^2=0$

801 **(a)**

Since, the roots of the equation $8x^3-14x^2+7x-1=0$ are in GP. Let the roots be $\frac{\alpha}{\beta}, \alpha, \alpha\beta, \beta \neq 0$. Then, the product of roots is $\alpha^3=\frac{1}{8} \Rightarrow \alpha=\frac{1}{2}$ and hence, $\beta=\frac{1}{2}$.

So, roots are $1, \frac{1}{2}, \frac{1}{4}$.

802 **(a)**

Given, $x^2-xy+y^2-4x-4y+16=0$

$\Rightarrow \; x^2-(y+4)x+y^2-4y+16=0$

For real x, $(y+4)^2-4(y^2-4y+16)\geq 0$

$\Rightarrow \; -3y^2+24y-48=0$

$\Rightarrow \; y^2-8y+16=0$

$\Rightarrow \; (y-4)^2=0 \;\Rightarrow\; y=4$

$\therefore$ From given equation $x=4$

$\Rightarrow \; (x,y)=(4,4)$

803 **(a)**

Since, α and β are the roots of $ax^2+bx+c=0$.

$\Rightarrow \; \alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$ ...(i)

If roots are $\alpha+\frac{1}{\beta}, \beta+\frac{1}{\alpha}$, then

Sum of roots $=\left(\alpha+\frac{1}{\beta}\right)+\left(\beta+\frac{1}{\alpha}\right)=(\alpha+\beta)+\frac{\alpha+\beta}{\alpha\beta}$

$=\frac{-b}{ac}(a+c)$ [from Eq. (i)]

$=\alpha\beta+1+1+\frac{1}{\alpha\beta}=2+\frac{c}{a}+\frac{a}{c}$ [from Eq. (i)]

and product of roots $=\left(\alpha+\frac{1}{\beta}\right)+\left(\beta+\frac{1}{\alpha}\right)$

$=\dfrac{2ac+c^2+a^2}{ac}=\dfrac{(a+c)^2}{ac}$

Hence, required equation is given by

x^2-(sum of roots)$x+$(prouduct of roots)$=0$

$\Rightarrow x^2+\dfrac{b}{ac}(a+c)x+\dfrac{(a+c)^2}{ac}=0$

$\Rightarrow acx^2+(a+c)bx+(a+c)^2=0$

804 **(b)**

$\left[1+\cos\frac{\pi}{5}+i\sin\frac{\pi}{5}\right]^{-1}$

$=\dfrac{1}{2\cos^2\frac{\pi}{10}+2\,i\sin\frac{\pi}{10}\cos\frac{\pi}{10}}$

$=\dfrac{1}{2\cos\frac{\pi}{10}\left(\cos\frac{\pi}{10}+i\sin\frac{\pi}{10}\right)}\times\dfrac{\cos\frac{\pi}{10}-i\sin\frac{\pi}{10}}{\left(\cos\frac{\pi}{10}-i\sin\frac{\pi}{10}\right)}$

$=\dfrac{\cos\frac{\pi}{10}-i\sin\frac{\pi}{10}}{2\cos\frac{\pi}{10}}$

$\therefore$ Real part is $\frac{1}{2}$

805 **(a)**

$$\Delta=\begin{vmatrix}1 & \omega^n & \omega^{2n}\\ \omega^n & \omega^{2n} & 1\\ \omega^{2n} & 1 & \omega^n\end{vmatrix}$$

$=1(\omega^{3n}-1)-\omega^n(\omega^{2n}-\omega^{2n})+\omega^{2n}(\omega^n-\omega^{4n})$

$=(1-1)-0+\omega^{2n}[\omega^n-(\omega^3)^n\omega^n]$ $(\because \omega^{3n}=1)$

$=0+0+0=0$

806 **(a)**

Let α, β be the two roots of the equation $ax^2+bx+c=0$. Then,

$\alpha+\beta=-b/a$ and $\alpha\,\beta=c/a$

$\Rightarrow -\dfrac{b}{a}=0$ and $\dfrac{c}{a}=0$ $\quad[\because \alpha=\beta=0]$

$\Rightarrow b=0, c=0$

807 **(b)**

Let the roots be α and $\alpha+1$. Then,

$\alpha+\alpha+1=p \Rightarrow \alpha=\dfrac{p-1}{2}$ ... (i)

and, $\alpha(\alpha+1)=q \Rightarrow \alpha^2+\alpha=q$ ... (ii)

From (i) and (ii), we get

$\left(\dfrac{p-1}{2}\right)^2+\left(\dfrac{p-1}{2}\right)=q$ [On eliminating α]

$\Rightarrow p^2-2p+1+2p-2=4q \Rightarrow p^2=4q+1$

808 **(a)**

Since, $|z|=1$ and $w=\frac{z-1}{z+1} \;\Rightarrow\; z=\frac{1+w}{1-w}$

$\Rightarrow \; |z|=\dfrac{|1+w|}{|1-w|} \;\Rightarrow\; |1-w|=|1+w|$ $[\because |z|=1]$

$\Rightarrow \; 1+|w|^2-2\,\mathrm{Re}\,(w)=1+|w|^2+2\,\mathrm{Re}\,(w)$

$\Rightarrow \; \mathrm{Re}\,(w)=0$

809 **(b)**

We observe that $\sin^{-1}\left(\frac{1+x^2}{2x}\right)$ is defined for

$-1\leq\dfrac{1+x^2}{2x}\leq 1$

$\Rightarrow \left|\dfrac{1+x^2}{2x}\right|\leq 1$

$\Rightarrow \left|\dfrac{1+x^2}{2}\right|\leq |x|$

$\Rightarrow 1+x^2-2|x|\leq 0 \Rightarrow (|x|-1)^2\leq 0 \Rightarrow |x|=1$ $[\because x>0]$

Thus, we have,

$\left(\dfrac{1+i}{1-i}\right)^n=\dfrac{2}{\pi}\sin^{-1}(1)$

$\Rightarrow i^n=1 \Rightarrow n$ is a multiple of 4

Hence, the least positive integral value of n is 4

810 **(c)**

Here, $\alpha+\beta+\gamma=0,\; \alpha\beta+\beta\gamma+\gamma\alpha=1$

And $\alpha\beta\gamma=-1$

$\therefore\; \alpha^3+\beta^3+\gamma^3$

$= (\alpha+\beta+\gamma)[\alpha^2+\beta^2+\gamma^2-\alpha\beta-\beta\gamma-\gamma\alpha] + 3\alpha\beta\gamma$

$= 0 + 3(-1) = -3$

811 **(b)**

Let $z = r(\cos\theta + i\sin\theta)$. Then,

$\left|z+\frac{1}{z}\right| = 1$

$\Rightarrow \left|z+\frac{1}{z}\right|^2 = 1$

$\Rightarrow \left|r(\cos\theta+i\sin\theta)+\frac{1}{r}(\cos\theta-i\sin\theta)\right|^2 = 1$

$\Rightarrow \left(r+\frac{1}{r}\right)^2\cos^2\theta+\left(r-\frac{1}{r}\right)^2\sin^2\theta = 1$

$\Rightarrow r^2+\frac{1}{r^2}+2\cos 2\theta = 1$

Since $|z| = r$ is maximum. Therefore, $\frac{dr}{d\theta} = 0$

Differentiating (i)w.r.t.θ, we get

$2r\frac{dr}{d\theta}-\frac{2}{r^3}\frac{dr}{d\theta}-4\sin 2\theta = 0$

Putting $\frac{dr}{d\theta}$, we get

$\sin 2\theta = 0 \Rightarrow \theta = \frac{\pi}{2} \Rightarrow z$ is purely imaginary

$[\because \theta \neq 0]$

812 **(a)**

Since $x = c$ is a root of order 2 of the polynomial $f(x)$

$\therefore f(x) = (x-c)^2\,\phi(x)$

$\Rightarrow f'(x) = 2(x-c)\,\phi(x) + (x-c)^2\,\phi'(x)$

$\Rightarrow f'(c) = 0 \Rightarrow x = c$ is a root of $f'(x)$

814 **(d)**

We have,

$\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}+\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}$

$= \frac{\omega^2(a+b\omega+c\omega^2)}{(c\omega^2+a\omega^3+b\omega^4)}+\omega\frac{(a+b\omega+c\omega^2)}{(b\omega+c\omega^2+a\omega^3)}$

$= \omega^2+\omega = -1$

815 **(c)**

Since, $(\alpha+\beta),(\alpha^2+\beta^2)$ and $(\alpha^3+\beta^3)$ are in GP.

$(\alpha^2+\beta^2)^2 = (\alpha+\beta)(\alpha^3+\beta^3)$

$\Rightarrow \alpha^4+\beta^4+2\alpha^2\beta^2 = \alpha^4+\beta^4+\alpha\beta^3+\beta\alpha^3$

$\Rightarrow \alpha\beta(\alpha^2+\beta^2-2\alpha\beta) = 0$

$\Rightarrow \alpha\beta(\alpha-\beta)^2 = 0$

$\Rightarrow \alpha\beta = 0$ or $\alpha = \beta$

$ie, \frac{c}{a} = 0$ or $\Delta = 0$

$\Rightarrow c\Delta = 0$

816 **(d)**

$i^n(1+i+i^2+i^3) = i^n(1+i-1-i) = 0$

817 **(b)**

If $z = x+iy$ is the additive inverse of $1-i$, the

$x+iy+(1-i) = 0$

$\Rightarrow\ x+1 = 0,\ y-1 = 0$

$\Rightarrow\ x = -1,\ y = 1$

Here required additive inverse is $-1+i$

818 **(d)**

Given equation is

$x^2-2\sqrt{2}kx+2e^{2\log k}-1 = 0$

Also, product of its root $2e^{2\log k}-1 = 31$

$\Rightarrow\ 2e^{2\log k} = 32 \quad\Rightarrow\quad k^2 = 16$

$\Rightarrow\ k = \pm 4$ [Since, log is not defined for $k < 0$]

$\therefore\ k = 4$

819 **(b)**

Let $z = x+iy$

$\therefore\ \frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1}$

$= \frac{(x^2+y^2-1)+2iy}{(x+1)^2+y^2}$

$\therefore\ \arg\left(\frac{z-1}{z+1}\right) = \tan^{-1}\frac{2y}{x^2+y^2-1}$

$\Rightarrow\ \tan^{-1}\frac{2y}{x^2+y^2-1} = \frac{\pi}{3}$ (given)

$\Rightarrow\ \frac{2y}{x^2+y^2-1} = \tan\frac{\pi}{3} = \sqrt{3}$

$\Rightarrow\ x^2+y^2-1 = \frac{2}{\sqrt{3}}y$

$\Rightarrow\ x^2+y^2-\frac{2}{\sqrt{3}}y-1 = 0$

Which is an equation of a circle

820 **(c)**

Let $z = \frac{(-\sqrt{3}+3i)(1-i)}{(3i-\sqrt{3})\sqrt{3}(1+i)}$

$= \frac{1}{\sqrt{3}}\left(\frac{1-i}{1+i}\times\frac{1-i}{1-i}\right) = -\frac{i}{\sqrt{3}}$

The complex number z is represented on y-axis (imaginary axis)

821 **(a)**

It is given that a, b, c are in G.P.

$\therefore b^2 = ac$

Now,

$ax^2+2bx+c = 0$

$\Rightarrow ax^2+2\sqrt{ac}\,x+c = 0$ [Using $b^2 = ac$]

$\Rightarrow (\sqrt{a}x+\sqrt{c})^2 = 0 \Rightarrow x = -\frac{\sqrt{c}}{\sqrt{a}}$

Thus, $x = -\sqrt{\frac{c}{a}}$ is a common root

Putting $x = -\sqrt{\frac{c}{a}}$ in $dx^2+2ex+f = 0$, we get

$d\frac{c}{a}-2e\sqrt{\frac{c}{a}}+f = 0$

$\Rightarrow \frac{d}{a}-2e.\frac{1}{\sqrt{ac}}+\frac{f}{c} = 0$ [Dividing both sides by c]

$\Rightarrow \frac{d}{a} - \frac{2e}{b} + \frac{f}{c} = 0 \quad [\because b^2 = ac]$

$\Rightarrow \frac{d}{a} + \frac{f}{c} = \frac{2e}{b} \Rightarrow \frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in GP.

822 **(d)**

Let $x = \sqrt{8 + 2\sqrt{8 + 2\sqrt{8 + 2\sqrt{8}}}}$. Then,

$x = \sqrt{8 + 2x}$

$\Rightarrow x^2 = 8 + 2x \Rightarrow x^2 - 2x - 8 = 0 \Rightarrow x = 4 \quad [\because x > 0]$

823 **(b)**

The given equation is $x^2 - 2x\cos\phi + 1 = 0$.

$\therefore x = \frac{2\cos\phi \pm \sqrt{4\cos^2\phi - 4}}{2} = \cos\phi \pm i\sin\phi$

Let $\alpha = \cos\phi + i\sin\phi$, then $\beta = \cos\phi - i\sin\phi$

$\therefore \alpha^n + \beta^n = (\cos n\phi + i\sin\phi)^n + (\cos\phi - i\sin\phi)^n$

$= 2\cos n\phi$

and $\alpha^n\beta^n = (\cos n\phi + i\sin n\phi)(\cos n\phi - i\sin n\phi)$

$= \cos^2 n\phi + \sin^2 n\phi = 1$

$\therefore$ Required equation is $x^2 - 2x\cos n\phi + 1 = 0$

824 **(d)**

$\frac{(\cos\theta + i\sin\theta)^4}{(\sin\theta + i\cos\theta)^5} = \frac{(\cos\theta + i\sin\theta)^4}{i^5(\cos\theta - i\sin\theta)^5}$

$= -i(\cos\theta + i\sin\theta)^9$

$= \sin 9\theta - i\cos 9\theta$

825 **(c)**

We have,

$x^3 - 2x^2 + 2x - 1 = 0$

$\Rightarrow (x-1)(x^2 - x + 1)$

$\Rightarrow x - 1$ or $x = -\omega, -\omega^2$

Since $ax^2 + bx + a = 0$ and $x^3 - 2x^2 + 2x - 1 = 0$ have two roots in common. Therefore, $-\omega$ and $-\omega^2$ are common roots.

Now,

$-\omega$ is a root of $ax^2 + bx + a = 0$

$\Rightarrow a\omega^2 - b\omega + a = 0$

$\Rightarrow a(1 + \omega^2) - b\omega = 0 \Rightarrow -a\omega - b\omega = 0 \Rightarrow a + b = 0$

827 **(c)**

Equations $x^3 + ax^2 + bx + c = 0$

and $x^3 + (a-1)x^2 + (b-1)x + (c-1) = 0$

have at least one common root, let common root be α.

$\therefore \alpha^3 + a\alpha^2 + b\alpha + c = 0$

and $\alpha^3 + a\alpha^2 + b\alpha + c - \alpha^2 - \alpha - 1 = 0$

$\Rightarrow \alpha^2 + \alpha + 1 = 0$

$\Rightarrow \alpha = \omega, \omega^2$(where ω and ω^2 are the cube roots of unity)

828 **(a)**

Let $z = x + iy$. Then,

$\frac{z - 8i}{z + 6} = \frac{x + (y-8)i}{(x+6) + iy}$

$= \frac{\{x + (y-8)i\}\{x + 6 - iy\}}{(x+6)^2 + y^2}$

$\Rightarrow \frac{z - 8i}{z + 6} = \frac{(x^2 + 6x + y^2 - 8y) + i(xy - 8x - xy)}{(x+6)^2 + y^2}$

$\therefore \text{Re}\left(\frac{z - 8i}{z + 6}\right) = 0 \Rightarrow x^2 + y^2 + 6x - 8y = 0$

Hence, $z = x + iy$ lies on the circle

ALITER We have,

$\text{Re}\left(\frac{z - 8i}{z + 6}\right) = 0$

$\Rightarrow \arg\left(\frac{z - (0 + 8i)}{z - (-6 + 0i)}\right) = \pm\frac{\pi}{2}$

$\Rightarrow z$ lies on the circle having $(0, 8)$ and $(-6, 0)$ as the end-points of the diameter

829 **(b)**

We have,

$\alpha^2 = 5\alpha - 3 \Rightarrow \alpha^2 - 5\alpha + 3 = 0 \Rightarrow \alpha = \frac{5 \pm \sqrt{13}}{2}$

Similarly, $\beta^2 = 5\beta - 3 \Rightarrow \beta = \frac{5 \pm \sqrt{13}}{2}$

Since $\alpha \neq \beta$

$\therefore \alpha = \frac{5 + \sqrt{13}}{2}$ and $\beta = \frac{5 - \sqrt{13}}{2}$

or, $\alpha = \frac{5 - \sqrt{13}}{2}$ and $\beta = \frac{5 + \sqrt{13}}{2}$

Thus, the either case, we have

$\alpha^2 + \beta^2 = \frac{1}{4}(50 + 26) = 19,$

and, $\alpha\beta = \frac{1}{4}(25 - 13) = 3$, in both the cases

Thus, the equation having α/β and β/α as its roots is

$x^2 - x\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) + \frac{\alpha\beta}{\alpha\beta} = 0$

$\Rightarrow x^2 - x\left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right) + 1 = 0 \Rightarrow 3x^2 - 19x + 3 = 0$

830 **(d)**

Given, $(x-2)^3 = -27 = -3^3$

$\Rightarrow \quad (x-2) = -3(1)^{1/3}$

$\Rightarrow \quad (x-2) = -3, -3\omega, -3\omega^2$

$\Rightarrow \quad x = -1, 2 - 3\omega, 2 - 3\omega^2$

831 **(c)**

Given equations are $2x^2 + 3x + 5\lambda = 0$ and $x^2 + 2x + 3\lambda = 0$ have a common root, if $\frac{x^2}{(9-10)\lambda} =$

$\frac{x}{(5-6)\lambda} = \frac{1}{(4-3)}$

$\Rightarrow \frac{x^2}{-\lambda} = \frac{x}{-\lambda} = \frac{1}{1}$

$\Rightarrow x^2 = -\lambda, x = -\lambda$ or $\lambda = -1, 0$

832 **(d)**

Given, $|x+iy-2| + |x+iy+2| = 8$

$\Rightarrow (x-2)^2 + y^2 + (x+2)^2 + y^2 = 8$

$\Rightarrow x^2 - 4x + 4 + y^2 + x^2 + 4x + 4 + y^2 = 8$

$\Rightarrow x^2 + y^2 = 0$

Which represents a circle whose radius is zero.

833 **(d)**

The equation $x^2 + x + 1 = 0$ has ω and ω^2 as its roots. Let $\alpha = \omega$ and $\beta = \omega^2$. Then,

$\alpha^{19} = \omega^{19} = \omega$ and $\beta^7 = \omega^{14} = \omega^2$

Hence, α^{19} and β^7 are roots of the same equation

834 **(b)**

Given relation is

$3\alpha + 2\beta = 16 \Rightarrow 2(\alpha+\beta) + \alpha = 16$

$\Rightarrow 2 \times 6 + \alpha = 16 \Rightarrow \alpha = 4$ $[\because \alpha + \beta = 6, \alpha\beta = a]$

$\therefore \alpha^2 - 6\alpha + a = 0$

$\Rightarrow 16 - 24 + a = 0 \Rightarrow a = 8$

835 **(a)**

Given equation is $|x-4| + |x-9| = 5$

$$\Rightarrow \begin{cases} 4 - x + 9 - x = 5, x \le 4 \\ x - 4 + 9 - x = 5, 4 < x \le 9 \\ x - 4 + x - 9 = 5, x > 9 \end{cases}$$

$$\Rightarrow \begin{cases} x = 4, x \le 4 \\ \text{no solution}, 4 < x \le 9 \\ x = 9, x > 9 \end{cases}$$

So, $x = 4, 9$

836 **(a)**

Given, $a_n = i^{(n+1)^2}$

Here, $a_1 = i^{2^2} = 1, a_2 = i^{3^2} = i,$

$a_3 = i^{4^2} = 1, \quad a_4 = i^{5^2} = i,$

$a_5 = i^{6^2} = 1, \ldots$

Here, we see that for all odd values of n, we get the value of a_n is 1

$\therefore a_1 + a_3 + a_5 + \cdots + a_{25} =$

$\underbrace{1+1+1+\ldots+1}_{13} = 13$

837 **(d)**

We have,

$\left(\frac{1-i\sqrt{3}}{2}\right)^n + \left(\frac{-1-i\sqrt{3}}{2}\right)^n$

$= \omega^n + (\omega^2)^n = \omega^{6k} + \omega^{12k} = (\omega^3)^{2k} + (\omega^3)^{4k} = 2$

838 **(a)**

Given, $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^n = 1$

$\Rightarrow \left(cis\frac{\pi}{6}\right)^n = 1 \Rightarrow n = 12$

839 **(c)**

We have,

$\frac{x+1}{x^2+2} > \frac{1}{4}$

$\Rightarrow \frac{4x + 4 - x^2 - 2}{4(x^2+2)} > 0$

$\Rightarrow \frac{-x^2 + 4x + 2}{x^2 + 2} > 0$

$\Rightarrow \frac{x^2 - 4x - 2}{x^2 + 2} < 0$

$\Rightarrow x^2 - 4x - 2 < 0$ $\quad [\because x^2 + 2 > 0$ for all $x]$

$\Rightarrow 4 - \sqrt{6} < x < 4 + \sqrt{6} \Rightarrow x = 2, 3, 4, 5, 6$

$[\because x \in Z]$

840 **(c)**

Here, $\alpha + \beta = -5$ and $\alpha\beta = 4$

Now, $\frac{\alpha+2}{3} + \frac{\beta+2}{3} = \frac{\alpha+\beta+4}{3} = \frac{-5+4}{3} = \frac{-1}{3}$

And $\left(\frac{\alpha+2}{3}\right)\left(\frac{\beta+2}{3}\right) = \frac{\alpha\beta + 2(\alpha+\beta) + 4}{9}$

$= \frac{4 + 2(-5) + 4}{9} = \frac{-2}{9}$

Required equation is

x^2-(sum of roots)x +products of roots=0

$\therefore x^2 + \frac{1}{3}x - \frac{2}{9} = 0 \Rightarrow 9x^2 + 3x - 2 = 0$

841 **(d)**

$x^2 + 5|x| + 4 = 0$

$\Rightarrow |x^2| + 4|x| + |x| + 4 = 0$

$\Rightarrow |x|(|x| + 4) + 1(|x| + 4) = 0$

$\Rightarrow (|x| + 1)(|x| + 4) = 0$

$\Rightarrow |x| = -1$ and $|x| = -4$

Which is not possible

Hence, no real root exist

842 **(b)**

Here, $\alpha + \beta = \frac{-b}{a}, \alpha\beta = \frac{c}{a}$

So, $(1 + \alpha + \alpha^2)(1 + \beta + \beta^2)$

$= 1 + \beta + \beta^2 + \alpha + \alpha\beta + \alpha\beta^2 + \alpha^2 + \alpha^2\beta + \alpha^2\beta^2$

$= 1 + (\alpha + \beta) + \alpha\beta + \alpha\beta(\alpha + \beta) + (\alpha\beta)^2 + \alpha^2 + \beta^2$

$= 1 + (\alpha + \beta) - \alpha\beta + \alpha\beta(\alpha + \beta) + (\alpha\beta)^2 + (\alpha + \beta)^2$

$= 1 - \frac{b}{a} - \frac{c}{a} - \frac{bc}{a^2} + \frac{c^2}{a^2} + \frac{b^2}{a^2}$

$= \frac{a^2 + b^2 + c^2 - ab - bc - ca}{a^2}$

$= \frac{1}{2a^2}(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca)$

$$= \frac{1}{2a^2}[(a-b)^2 + (b-c)^2 + (c-a)^2] \geq 0$$

843 **(a)**

We have,

$\alpha + i\,\beta = \tan^{-1}(z)$

$\Rightarrow \alpha + i\,\beta = \tan^{-1}(x + i\,y)$ ... (i)

$\Rightarrow \alpha - i\,\beta = \tan^{-1}(x - iy)$ ... (ii)

From (i) and (ii), we get

$(\alpha + i\,\beta) + (\alpha - i\,\beta)$

$= \tan^{-1}(x + iy) + \tan^{-1}(x - iy)$

$\Rightarrow 2\,\alpha = \tan^{-1}\left(\frac{x + iy + x - iy}{1 - (x + iy)(x - iy)}\right)$

$\Rightarrow \tan 2\,\alpha = \frac{2\,x}{1 - (x^2 + y^2)}$

$\Rightarrow 1 - x^2 - y^2 = 2\,x \cot 2\,\alpha$

$\Rightarrow x^2 + y^2 + 2\,x \cot 2\,\alpha = 1$

844 **(c)**

Given, $3x^2 - 2(a + b + c)x + (ab + bc + ca) = 0$

Now, $B^2 - 4AC = 4\{(a + b + c)^2 - 3(ab + bc + ca)\}$

$= 4\{a^2 + b^2 + c^2 - ab - bc - ac\}$

$= 2\{(a-b)^2 + (b-c)^2 + (c-a)^2\} \geq 0$

Hence, both roots are always real

845 **(c)**

Here, $b^2 - 4ac = 0$

$\Rightarrow\ 36p^2 - 4(4)(1) = 0$

$\Rightarrow\ 36p^2 = 16$

$\Rightarrow\ p = \pm\frac{2}{3}$

846 **(a)**

$\left|z - \frac{25}{z}\right| \geq \left||z| - \frac{25}{|z|}\right| \Rightarrow 24 \geq \left||z| - \frac{25}{|z|}\right|$

$\Rightarrow\ -24 \leq |\,z| - \frac{25}{|z|} \leq 24$

or $-24|z| \leq |z|^2 - 25 \leq 24|z|$

$\therefore\ |z|^2 + 24|z| - 25 \geq 0$ and $|z|^2 - 24|z| - 25 \leq 0$

$\Rightarrow\ (|z| + 25)(|z| - 1) \geq 0$ and $(|z| - 25)(|z| + 1) \leq 0$

$\therefore\ |\,z| - 1 \geq 0$ and $|z| - 25 \leq 0$

Hence, $1 \leq |z| \leq 25$

or $1 \leq |z - 0| \leq 25$

847 **(a)**

If $(x + 1)$ is a factor of

$x^4 - (p-3)x^3 - (3p-5)x^2 + (2p-7)x + 6$,

then by putting $x = -1$, we get

$1 + (p-3) - (3p-5) - (2p-7) + 6 = 0$

$\Rightarrow\ -4p = -16\ \Rightarrow p = 4$

848 **(c)**

It is given that $f(x) = x^3 - 3b^2x + 2c^2$ is divisible by $x - a$ and $x - b$

$\therefore f(a) = 0$ and $f(b) = 0$

$\Rightarrow a^3 - 3b^2a + 2c^3 = 0$ ...(i)

and $b^3 - 3b^3 + 2c^3 = 0$ ...(ii)

From (ii), we get $b = c$

Putting, $b = c$ in (i), we get

$a^3 - 3ab^2 + 2b^3 = 0$

$\Rightarrow (a-b)(a^2 + ab - 2b^2) = 0$

$\Rightarrow a = b$ or $a^2 + ab = 2b^2$

Thus, $a = b = c$ or, $a^2 + ab = 2b^2$ and $b = c$

Clearly, $a^2 + ab = 2b^2$ is satisfied by $a = -2b$

$\therefore\ a^2 + ab = 2b^2$ and $b = c$

$\Rightarrow a = -2b$ and $b = c \Rightarrow a = -2b = -2c$

849 **(b)**

Since a, b, c are in A.P. Therefore,

$c - b = d$ (common difference), $b - a = d$ and $c - a = 2d$

We have,

$(b-c)x^2 + (c-a)x + a - b = 0$

$\Rightarrow -d\,x^2 + 2\,dx - d = 0$

$\Rightarrow x^2 - 2\,x + 1 = 0$

$\Rightarrow x = 1$ (twice)

Thus, $x = 1$ is a common root of the two equations

Since, $x = 1$ is a root of $2(c + a)x^2 + (b + c)x = 0$

$\therefore\ 2(c + a) + b + c = 0$

$\Rightarrow 2a + b + 3c = 0$

$\Rightarrow 2a + \frac{a + c}{2} + 3c = 0$ [$\because a, b, c$ are in A. P.]

$\Rightarrow 5a + 7c = 0 \Rightarrow c = -\frac{5a}{7}$

Now,

$2b = a + c$ and $c = -\frac{5a}{7} \Rightarrow b = \frac{a}{7}$

$\therefore\ b^2 = \frac{a^2}{49}$ and $c^2 = \frac{25a^2}{49}$

Clearly, $a^2 + b^2 = a^2 + \frac{a^2}{49} = \frac{50a^2}{49} = 2c^2$

$\therefore\ a^2, c^2, b^2$ are in A.P.

850 **(c)**

Let $z = x_1 + iy_1$ and $w = x_2 + iy_2$

As $|z| \leq 1$ and $|w| \leq 1$

$\Rightarrow \quad x_1^2 + y_1^2 < 1$ and $x_2^2 + y_2^2 \leq 1$

Now, $|z + iw| = |x_1 + iy_1 + i(x_2 + iy_2)| = 2$

$\Rightarrow \quad (x_1 - y_2)^2 + (y_1 + x_2)^2 = 4 \quad$ ....(i)

And $|z - i\overline{w}| = |x_1 + iy_1 - i(x_2 - iy_2)| = 2$

$\Rightarrow \quad (x_1 - y_2)^2 + (y_1 - x_2)^2 = 4 \quad$ ...(ii)

On solving Eqs. (i) and (ii), we get

$y_1 x_2 = 0$

$\Rightarrow$ Either $\quad y_1 = 0$ or $x_2 = 0$

When $\; y_1 = 0, \quad x_1^2 \leq 1$

$\Rightarrow \quad x = \pm 1$

$\therefore \quad z = \pm 1 + i0$

851 **(c)**

We have,

$$\log_{\tan 30°}\left(\frac{2\,|z|^2 + 2\,|z| - 3}{|z| + 1}\right) < -2$$

$$\Rightarrow \frac{2\,|z|^2 + 2\,|z| - 3}{|z| + 1} > (\tan 30°)^{-2}$$

$$\Rightarrow \frac{2\,|z|^2 + 2\,|z| - 3}{|z| + 1} > 3$$

$\Rightarrow 2\,|z|^2 - |z| - 6 > 0$

$\Rightarrow (|z| - 2)(2\,|z| + 3) > 0 \Rightarrow |z| > 2 \quad [\because 2\,|z| + 3 > 0]$

852 **(c)**

Given that $x^2 + px + 1$ is a factor of $ax^3 + bx + c = 0$, then let $ax^3 + bx + c \equiv (x^2 + px +$ *1ax+λ,* where λ is a constant. Then, equating the coefficients of like powers of x on both sides, we get

$0 = ap + \lambda, b = p\lambda + a, c = \lambda$

$$\Rightarrow p = -\frac{\lambda}{a} = -\frac{c}{a}$$

Hence, $b = \left(-\frac{c}{a}\right) c + a \;\Rightarrow ab = a^2 - c^2$

853 **(c)**

Since $\text{Im}(z_1 + z_2) = 0$, and $\text{Im}(z_1\,z_2) = 0$

$\Rightarrow z_1 + z_2$ and $z_1\,z_2$ both are real

Let $z_1 = a_1 + ib_1, z_2 = a_2 + ib_2$. Then,

$z_1 + z_2$ is real $\Rightarrow b_2 = -b_1$

$z_1 z_2$ is real

$\Rightarrow a_1\,b_2 + a_2\,b_1 = 0$

$\Rightarrow -a_1\,b_1 + a_2\,b_1 = 0 \quad [\because b_2 = -b_1]$

$\Rightarrow a_1 = a_2$

So, $z_1 = a_1 + ib_1 = a_2 - ib_2 = \bar{z}_2$

854 **(d)**

$y^{x^2+7x+12} = 1$

$\Rightarrow x^2 + 7x + 12 = 0$

$\Rightarrow x = -3, -4$

$\Rightarrow y = 9, 10 \quad$ (when $y \neq 1$)

Again when $y = 1, x = 5$.

$\therefore$ Solutions are $(-3\,,9), (-4, 10), (5, 1)$

855 **(c)**

We have,

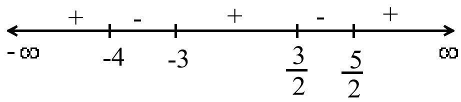
$$\frac{8x^2 + 16x - 51}{(2x - 3)(x + 4)} < 3$$

$$\Rightarrow \frac{8x^2 + 16x - 51 - 6x^2 - 15x + 36}{(2x - 3)(x + 4)} < 0$$

$$\Rightarrow \frac{2x^2 + x - 15}{(2x - 3)(x + 4)} < 0$$

$$\Rightarrow \frac{(2x - 5)(x + 3)}{(2x - 3)(x + 4)} < 0 \Rightarrow x$$

$\in (-4, -3) \cup (3/2\,, 5/2)$



856 **(b)**

We have, $(1 + \omega^2)^n = (1 + \omega^4)^n$

$\Rightarrow \quad (-\omega)^n = (-\omega^2)^n$

$\Rightarrow \quad \omega^n = 1$

$\Rightarrow \quad n = 3$ is least positive value of n

857 **(d)**

We have, $x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$. If $x \neq 1$ multiplying each term by$(x - 1)$, the given equation reduces to $x(x - 1) = (x - 1)$or $(x - 1)^2 = 0$ or $x = 1$ which is not possible as considering $x \neq 1$.

Thus, given equation has no roots.

858 **(b)**

We have,

$x^2 + x + 1 = (x - \omega)(x - \omega^2)$

Now,

$P(x) = g(x^3) + xh(x^3)$ is divisible by $x^2 + x + 1$

$\Rightarrow x = \omega$ and $x = \omega^2$ are roots of $P(x) = 0$

$\Rightarrow P(\omega) = 0, \qquad P(\omega)^2 = 0$

$\Rightarrow g(1) + \omega\,h(1) = 0$ and $g(1) + \omega^2 h(1) = 0$

$\Rightarrow g(1) = 0 = h(1)$

859 **(a)**

Let, $f(x) = x^{2n} - 1$

At $x = \pm 1, f(x) = 0$

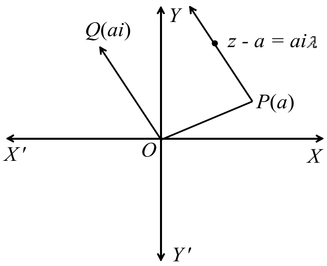
Hence, for no other real value of $x, f(x)$ is zero

860 **(d)**

We have, $z = a(1 + i\lambda) \Rightarrow z = a + ai\,\lambda$

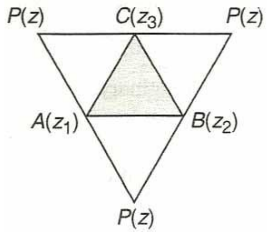
Since ai represents a point in the direction perpendicular to the join of O and a and $(ai)\lambda$ is a variable point in this direction. Therefore, $z = a + 1(a\lambda)$ is a point on a line through $''a''$ perpendicular to the join of O and the point a

ALITER $z = z_1 + \lambda\,z_2, \lambda \in R$ represents a line passing thorough z_1 and parallel to z_2. So, $z = a + ai\,\lambda$ is a line passing through a and parallel to ai



861 **(a)**

Let A, B, C be the points represented by the numbers z_1, z_2, z_3 and P be the point represented by z



Now, the four points A, B, C, P form a parallelogram in the following three orders.

(i)A, B, P, C (ii)B, C, P, A and (iii)C, A, P, B

In case (i), the condition for A, B, P, C to form a parallelogram is $\overrightarrow{\mathbf{AB}} = \overrightarrow{\mathbf{CP}}$ $ie, z_2 - z_1 = z - z_3$ or $z = z_2 + z_3 - z_1$

Similarly, in case (ii) and (iii), $\overrightarrow{\mathbf{BC}} = \overrightarrow{\mathbf{AP}}$ $ie,$

$z_3 - z_2 = z - z_1$ or $z = z_3 + z_1 - z_2$

and $\overrightarrow{\mathbf{CA}} = \overrightarrow{\mathbf{BP}}$

$ie, z_1 - z_3 = z - z_2$

or $z = z_1 + z_2 - z_3$

862 **(c)**

Since a, b are roots of $x^2 + ax + b = 0$. Therefore,

$a^2 + a^2 + b = 0$ and, $b^2 + ab + b = 0$

$\Rightarrow b = -2\,a^2$ and $b + a + 1 = 0$

$\Rightarrow -2\,a^2 + a + 1 = 0$

$\Rightarrow 2\,a^2 - a - 1 = 0 \Rightarrow a = 1$ or, $a = -1/2$

Now,

$a = 1, \Rightarrow b = -2$ $[\because b + a + 1 = 0]$

and, $a = -1/2 \Rightarrow b = -1/2$

But, $a \neq b$. Therefore, $a = 1, b = -2$

$\therefore$ Least value of $x^2 + ax + b$ is

$$-\left(\frac{a^2 - 4b}{4}\right) = -\left(\frac{1 + 8}{4}\right) = -\frac{9}{4}$$

863 **(a)**

Given, $\left|\frac{z_1 - 3z_2}{3 - z_1\bar{z}_2}\right| = 1,\ |z_1| \neq 3$

$\Rightarrow\ |z_1 - 3z_2| = |3 - z_1\bar{z}_2|$ $\left[\because \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right]$

$\Rightarrow\ |z_1 - 3z_2|^2 = |3 - z_1\bar{z}_2|^2$

$\Rightarrow\ (z_1 - 3z_2)(\bar{z}_1 - 3\bar{z}_2)$

$= (3 - z_1\bar{z}_2)(3 - \bar{z}_1 z_2)$ $[\because \bar{\bar{z}}_2 = z_2]$

$\Rightarrow\ |z_1|^2 - 3z_1\bar{z}_2 - 3z_2\bar{z}_1 + 9|z_2|^2$

$= 9 - 3\bar{z}_1 z_2 - 3z_1\bar{z}_2 + |z_1|^2|z_2|^2$

$\Rightarrow\ |z_1|^2 + 9|z_2|^2 - 9 - |z_1|^2|z_2|^2 = 0$

$\Rightarrow\ (9 - |z_1|^2)(1 - |z_2|^2) = 0$

$\Rightarrow\ |z_1|^2 = 9$ or $|z_2|^2 = 1$

$\Rightarrow\ |z_1| = 3$ or $|z_2| = 1$

$\therefore\ |z_2| = 1$ [but $|z_1| \neq 3$ given]

864 **(c)**

We have,

$$\begin{vmatrix} 1+i & 1-1 & i \\ 1-i & i & 1+i \\ i & 1+i & 1-i \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 1 & i \\ 1 & 1+2i & 1+i \\ 1+2i & 2 & 1-i \end{vmatrix}, \text{ Applying } C_1 \to C_1 + C_2, C_2 \to C_2 \text{ -}$$

$$= \begin{vmatrix} 0 & 0 & i \\ -1-4i & 3i & 1+i \\ -3+2i & 3+i & 1-i \end{vmatrix}, \text{ Applying } C_1 \to C_1 - 2\,C_2, C_2 \to C$$

$$= i\begin{vmatrix} -1-4i & 3i \\ -3+2i & 3+i \end{vmatrix} = 4 + 7i$$

865 **(b)**

Since, $\left|\frac{z}{z - i/3}\right| = 1$

$\Rightarrow\ 3|z| = |3z - i|$

$\Rightarrow\ 3|x + iy| = |3(x + iy) - i|$ [put $z = x + iy$]

$\Rightarrow\ 3\sqrt{x^2 + y^2} = \sqrt{(3x)^2 + (3y - 1)^2}$

$\Rightarrow\ 9x^2 + 9y^2 = 9x^2 + 9y^2 + 1 - 6y$

$\Rightarrow\ y = \frac{1}{6}$

Which shows that z lies on a straight line.

866 **(d)**

$$\frac{\log_3 5 . \log_{25} 27 . \log_{49} 7}{\log_{81} 3}$$

$$= \frac{\frac{\log 5}{\log 3} . \frac{3}{2} . \frac{\log 3}{\log 5} . \frac{1}{2}}{\frac{1}{4}}$$

$= 3$

867 **(b)**

Let α be a root of $x^2 - x + k = 0$. Then, $2\,\alpha$ is a root of $x^2 - x + 3\,k = 0$

$\therefore \alpha^2 - \alpha + 3\,k = 0$ and $4\,\alpha^2 - 2\,\alpha + 3\,k = 0$

$$\Rightarrow \frac{\alpha^2}{-3\,k + 2\,k} = \frac{\alpha}{4\,k - 3\,k} = \frac{1}{-2 + 4}$$

$\Rightarrow \alpha^2 = -\frac{k}{2}$ and $\alpha = \frac{k}{2}$

Now,

$\alpha^2 = (\alpha)^2 \Rightarrow -\frac{k}{2} = \left(\frac{k}{2}\right)^2 \Rightarrow k^2 + 2\,k = 0 \Rightarrow k$

$= 0$ or, -2

868 **(a)**
Given equation can be rewritten as
$3x^2-(a+c+2b+2d)x+ac+2bd=0$
$\therefore$ Discriminant, D
$=(a+c+2b+2d)^2-4\cdot 3(ac+2bd)$
$=\{(a+2d)+(c+2b)\}^2-12(ac+2bd)$
$=\{(a+2d)+(c+2b)\}^2+4(a+2d)(c+2b)-12(ac+2bd)$
$=\{(a+2d)+(c+2b)\}^2-8ac+8ab-8dc-8bd$
$=\{(a+2d)+(c+2b)\}^2+8(c-b)(d-a)$
Which is+ve, since $a<b<c<d$.
Hence, roots are real and distinct.

869 **(c)**
If $|z|=|z-2| \Rightarrow z+\bar{z}=2$
Also, $|z|=|z+2| \Rightarrow z+\bar{z}=-2$
Thus, $|z+\bar{z}|=2$

870 **(b)**
Here, $\alpha+\beta+\gamma=-2$ ...(i)
$\alpha\beta+\beta\gamma+\gamma\alpha=-3$ ...(ii)
and $\alpha\beta\gamma=1$ ...(iii)
On solving Eq. (ii), we get
$\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2+2\alpha\beta\gamma(\alpha+\beta+\gamma)=9$
$\Rightarrow \alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2=9-2(1)(-2)=13$
Now, $\alpha^{-2}+\beta^{-2}+r^{-2}=\frac{\beta^2\gamma^2+\gamma^2\alpha^2+\alpha^2\beta^2}{(\alpha\beta\gamma)^2}=\frac{13}{1}=13$

871 **(b)**
Let $z=\frac{(1-i\sqrt{3})(2+2i)}{(\sqrt{3}-i)}$
$=\frac{(2-2\sqrt{3})+2i(1+\sqrt{3})}{(\sqrt{3}-i)}\times\frac{(\sqrt{3}-i)}{(\sqrt{3}-i)}$
$=\frac{2\sqrt{3}-6+2i-2\sqrt{3}i+2\sqrt{3}i+6i-2-2\sqrt{3}}{3+1}$
$=\frac{-8+8i}{4}=-2+2i$
$\therefore$ Magnitude of $z=\sqrt{4+4}=2\sqrt{2}$
And amplitude of $z=\tan^{-1}\left(\frac{2}{-2}\right)=\frac{3\pi}{4}$

872 **(b)**
The discriminant D of the given equation is given by $D=(2m-1)^2-4\,m(m-2)=4m+1$
If the given equation has rational roots, then the discriminant should be a perfect square of a rational number, say a
i.e., $4m+1=a^2$
$\Rightarrow a^2$ is an integer [$\because$ $4m+1$ is an integer]
$\Rightarrow a$ is an integer
Now, $4m+1=a^2$
$\Rightarrow 4m=(a^2-1)$
$\Rightarrow 4m=(a-1)(a+1)$
$\Rightarrow (a-1)(a+1)$ is an even integer of the form $4m$
$\Rightarrow a-1$ and $a+1$ are even integers [$\because$ $4m$ is an even integer]
$\Rightarrow a$ is an odd integer
Let $a=2n+1$, where $n\in Z$. Then,
$a^2=4m+1$
$\Rightarrow (2n+1)^2=4m+1 \Rightarrow m=n(n+1)$, where $n\in Z$

873 **(d)**
Let $z_1=1-i$, $z_2=i$ and $z_3=1+i$
$\therefore$ $|z_1|=\sqrt{1^2+1^2}=\sqrt{2}$
$|z_2|=\sqrt{1^2}=1$
And $|z_3|=\sqrt{1^2+1^2}=\sqrt{2}$
Hence, given complex numbers form an isosceles triangle.

874 **(c)**
Let ABC be the triangle such that the affixes of its vertices A, B, C are $1, \frac{1+i}{\sqrt{2}}$ and i respectively. Then,
$AB=\left|\frac{1+i}{\sqrt{2}}-1\right|=\left|\frac{1-\sqrt{2}}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right|=\sqrt{2-\sqrt{2}}$
$BC=\left|i-\frac{1+i}{\sqrt{2}}\right|=\left|\frac{-1}{\sqrt{2}}+i\left(1-\frac{1}{\sqrt{2}}\right)\right|=\sqrt{2-\sqrt{2}}$
and, $CA=|1-i|=\sqrt{2}$
Clearly $AB=BC$. So, the triangle is isosceles

875 **(c)**
Let $x=|a+b\omega+c\omega^2|$
$\Rightarrow x^2=(a^2+b^2+c^2-ab-bc-ca)$
$\Rightarrow x^2=\frac{1}{2}\{(a-b)^2+(b-c)^2+(c-a)^2\}$ ...(i)
Since a, b, c are all integers but not all simultaneously equal
$\Rightarrow$ If $a=b$, then $a\neq c$ and $b\neq c$
As, difference of integers=integer
$\Rightarrow (b-c)^2\geq 1$
[as minimum difference of two consecutive integers is (± 1)]
Also, $(c-a)^2\geq 1$
$\therefore$ From Eq. (i),
$x^2=\frac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2]$
$\geq\frac{1}{2}[0+1+1]$
$\Rightarrow x^2\geq 1$
Hence, minimum value of x is 1

876 **(d)**
We have, $\omega^1\cdot\omega^2\cdot\omega^3\cdot\ldots\cdot\omega^n$
$=\omega^{1+2+3+\ldots+n}=\omega^{\frac{n(n+1)}{2}}=S_n$ (say)

On putting $n = 1, 2, 3, \dots, n$, we get
$S_1 = \omega^1 = \omega, S_2 = \omega^3 = 1,$
$S_3 = \omega^6 = 1, \dots, S_7 = \omega^{28} = \omega$
$\therefore$ We always get 1 and ω

877 **(a)**
The two equations can be written as
$x^2(6k+2) + rx + (3k-1) = 0$ ... (i)
and, $x^2(12k+4) + px + (6k-2) = 0$ ... (ii)
Equation (ii) can be written as
$x^2(6k+2) + \frac{p}{2}x + (3k-1) = 0$ ... (iii)
Comparing (i) and (iii), we get
$r = \frac{p}{2} \Rightarrow 2r - p = 0$

878 **(b)**
Given, $\frac{3}{2+\cos\theta+i\sin\theta} = a + ib$
$\Rightarrow \frac{3[(2+\cos\theta) - i\sin\theta]}{(2+\cos\theta)^2 + \sin^2\theta} = a + ib$
$\Rightarrow \frac{3[2+\cos\theta - i\sin\theta]}{5+4\cos\theta} = a + ib$
$\Rightarrow \quad a = \frac{3(2+\cos\theta)}{5+4\cos\theta}, \quad b = -\frac{3\sin\theta}{5+4\cos\theta}$
$\therefore \quad (a-2)^2 + b^2$
$= \left(\frac{6+3\cos\theta}{5+4\cos\theta} - 2\right)^2 + \frac{9\sin^2\theta}{(5+4\cos\theta)^2}$
$= \frac{(-4-5\cos\theta)^2 + 9\sin^2\theta}{(5+4\cos\theta)^2}$
$= \frac{16 + 25\cos^2\theta + 40\cos\theta + 9\sin^2\theta}{(5+4\cos\theta)^2}$
$= \frac{16 + 16\cos^2\theta + 40\cos\theta + 9}{(5+4\cos\theta)^2}$
$= \frac{(5+4\cos\theta)^2}{(5+4\cos\theta)^2} = 1$

880 **(a)**
Let $z = x + iy$
$\Rightarrow \bar{z} = x - iy$
and $(\bar{z}^{-1}) = \frac{1}{x-iy} = \frac{x+iy}{x^2+y^2}$
$\therefore \ (\bar{z}^{-1})\bar{z} = \frac{x+iy}{x^2+y^2} \times (x - iy) = 1$

881 **(b)**
We know that, is
$|z_1 + z_2| = |z_1| + |z_2|$, then $\arg(z_1) = \arg(z_2)$
$\because \quad |z^2 + (-1)| = |z^2| + |-1|$
$\Rightarrow \quad \arg(z^2) = \arg(-1)$
$\Rightarrow \quad 2\arg(z) = \pi \qquad [\because \ \arg(-1) = \pi]$
$\Rightarrow \arg(z) = \frac{\pi}{2}$
$\Rightarrow$ z lies on y-axis (imaginary axis).

882 **(d)**
The given equation is
$x^2 - 2ax + a^2 - 1 = 0$
$\Rightarrow (x-a)^2 - 1^2 = 0 \Rightarrow x - a = \pm 1 \Rightarrow x = a + 1, a - 1$
It is given that roots lie between 5 and 10
$\therefore 5 < a - 1 < 10$ and $5 < a + 1 < 10$
$\Rightarrow 6 < a < 11$ and $4 < a < 9 \Rightarrow 6 < a < 9$

883 **(a)**
Let $e^{\cos x} = y$
Then, $y - \frac{1}{y} = 4 \ \Rightarrow \ y^2 - 4y - 1 = 0$
$\Rightarrow \ y = \frac{-(-4) \pm \sqrt{16 - 4 \times (-1)}}{2} \Rightarrow y = \frac{4 \pm 2\sqrt{5}}{2}$
$\Rightarrow \ y = 2 + \sqrt{5} = e^{\cos x}$ [$\because$ exponential is always positive]
$\Rightarrow \ \cos x = \log(2 + \sqrt{5})$

884 **(b)**
Given, $z = -\bar{z}$
$\Rightarrow \quad x + iy = -\overline{(x+iy)}$ [Put $z = x + iy$]
$\Rightarrow \quad x + iy = -(x - iy)$
$\Rightarrow \quad x = 0$
Hence, z is a purely imaginary.

886 **(a)**
We have,
$\omega = \frac{-1+i\sqrt{3}}{2}$ and $\omega^2 = \frac{-1-i\sqrt{3}}{2}$
$\Rightarrow \frac{\omega^2}{\omega} = \frac{1+i\sqrt{3}}{1-i\sqrt{3}}$ and $\frac{\omega}{\omega^2} = \frac{1-i\sqrt{3}}{1+i\sqrt{3}}$
$\therefore \left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^6 + \left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}}\right)^6 = \left(\frac{\omega^2}{\omega}\right)^6 + \left(\frac{\omega}{\omega^2}\right)^6$
$= \omega^6 + \frac{1}{\omega^6} = 2$

887 **(d)**
It is given that α, β, γ are the roots of the equation
$x^3 + qx + r = 0$
$\therefore \alpha + \beta + \gamma = 0 \Rightarrow \alpha + \beta = -\gamma, \beta + \gamma = -\alpha, \gamma + \alpha = -\beta$
Hence,
$\sum \frac{\alpha}{\beta+\gamma} = \frac{\beta}{\gamma+\alpha} + \frac{\gamma}{\alpha+\beta} = -\frac{\alpha}{\alpha} - \frac{\beta}{\beta} - \frac{\gamma}{\gamma} = -3$

888 **(c)**
Here, $\alpha + \alpha^2 = -1$ ...(i)
And $\alpha^3 = 1$ ...(ii)
Now, $\alpha^{31} + \alpha^{62} = \alpha^{31}(1 + \alpha^{31})$
$\Rightarrow \alpha^{31} + \alpha^{62} = \alpha^{30}\alpha(1 + \alpha^{30}.\alpha)$
$\Rightarrow \alpha^{31} + \alpha^{62} = (\alpha^3)^{10}.\alpha\{1 + (\alpha^3)^{10}.\alpha\}$
$\Rightarrow \alpha^{31} + \alpha^{62} = \alpha(1 + \alpha)$ [from Eq. (ii)]
$\Rightarrow \alpha^{31} + \alpha^{62} = -1$ [from Eq. (i)]
And $\alpha^{31}.\alpha^{62} = \alpha^{93}$
$= (\alpha^3)^{31} = 1$

∴ Required equation is

$x^2 - (\alpha^{31} + \alpha^{62})x + \alpha^{31}.\alpha^{62} = 0$

$\Rightarrow x^2 + x + 1 = 0$

889 **(a)**

If $\arg(z) = -\pi + \theta$

$\Rightarrow \quad \arg(\bar{z}) = \pi - \theta$

$\arg(-\bar{z}) = -\theta$

$\arg(\bar{z}) - \arg(-\bar{z}) = \pi - \theta - (-\theta) = \pi - \theta + \theta$

$= \pi$

890 **(c)**

Given, $\frac{AB}{BC} = \sqrt{2}$

Consider the rotation about $'B'$, we get

$\frac{z_1 - z_2}{z_3 - z_2} = \frac{|z_1 - z_2|}{|z_3 - z_2|} e^{i\pi/4}$

$= \frac{AB}{BC} e^{i\pi/4}$

$= \sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) = 1 + i$

$\Rightarrow z_1 - z_2 = (1 + i)(z_3 - z_2)$

$\Rightarrow z_1 - (1 + i)z_3 = z_2(1 - 1 - i)$

$\Rightarrow iz_2 = -z_1 + (1 + i)z_3$

$\Rightarrow z_2 = iz_1 - i(1 + i)z_3$

$= z_3 + i(z_1 - z_3)$

891 **(c)**

We have, $z^2 = \bar{z}$

On multiplying by z both sides (if $z \neq 0$),

$z^3 = 1$ has three solutions and $z = 0$ is also a solution

So, total number of solution is 4

892 **(d)**

Let $z = x + iy$. Then, $z^2 = x^2 - y^2 + 2\,ixy$

$\therefore \text{Im}(z^2) = k \Rightarrow 2xy = k \Rightarrow xy = \frac{k}{2}$, which is a hyperbola

893 **(c)**

Let $z = x + iy$, then $\bar{z} = x - iy$

$\therefore \quad z + \bar{z} = 2x$ and $z - \bar{z} = 2iy$

Given, $(3 + i)(z + \bar{z}) - (2 + i)(z - \bar{z}) + 14i = 0$

$\Rightarrow \quad (3 + i)2x - (2 + i)2iy + 14i = 0$

$\Rightarrow \quad 6x + 2ix - 4yi + 2y + 14i = 0 + oi$

On comparing real and imaginary part, we get

$6x + 2y = 0$

And $2x - 4y + 14 = 0$

On solving, we get $x = -1, y = 3$

$\therefore \quad z\bar{z} = |z|^2 = \left(\sqrt{(-1)^2 + (3)^2}\right)^2 = 10$

894 **(d)**

Given that, $\alpha + \beta = -2$ and $\alpha^3 + \beta^3 = -56$

$\Rightarrow (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta) = -56$

$\Rightarrow \alpha^2 + \beta^2 - \alpha\beta = 28$

Also, $(\alpha + \beta)^2 = (-2)^2$

$\Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = 4$

$\Rightarrow 28 + 3\alpha\beta = 4$

$\Rightarrow \alpha\beta = -8$

∴ Required equation is $x^2 + 2x - 8 = 0$

895 **(c)**

We have,

$|x - 1| \leq 3$ and $|x - 1| \geq 1$

$\Rightarrow 1 - 3 \leq x \leq 1 + 3$ and $x \leq 1 - 1$ or $x \geq 1 + 1$

$\Rightarrow -2 \leq x \leq 4$ and ($x \leq 0$ or, $x \geq 2$)

$\Rightarrow x \in [-2, 0] \cup [2, 4]$

896 **(c)**

We have,

$\left|\frac{z_1 - z_2}{z_1 + z_2}\right| = 1$

$\Rightarrow \frac{z_1 - z_2}{z_1 + z_2} = \cos\alpha + is\sin\alpha$

$\Rightarrow \frac{2\,z_1}{-2\,z_2}$

$= \frac{\cos\alpha + i\sin\alpha + 1}{\cos\alpha - 1 + i\sin\alpha}$ [Applying componendo and dividendo]

$\Rightarrow \frac{z_1}{z_2} = i\cot\frac{\alpha}{2}$

$\Rightarrow i\,z_1 = -\cot\frac{\alpha}{2}z_2$

$\Rightarrow k = -\cot\frac{\alpha}{2} \qquad [\because i\,z_1 = k\,z_2]$

$\Rightarrow \tan\alpha = \frac{2\,k}{k^2 - 1} \qquad \left[\because \tan\alpha = \frac{2\tan\alpha/2}{1 - \tan^2\alpha/2}\right]$

$\Rightarrow \tan\alpha = \frac{-2\,k}{1 - k^2} \Rightarrow \alpha$

$= \tan^{-1}\left(\frac{-2\,k}{1 - k^2}\right) = -2\tan^{-1}k$

$\Rightarrow \alpha = -2\tan^{-1}k$ is the angle between $z_1 - z_2$ and $z_1 + z_2$

897 **(a)**

Let $f(x) = ax^2 + bx + c$

If the roots of $f(x) = 0$ are imaginary, then we cannot say anything about b (i.e. it can be positive, negative or zero). So, options (b),(c) and (d) are not necessarily true

Further, if $a > 0$, then the graph of $y = f(x)$ is above x-axis and hence

$f(x) > 0$ for all $x \in R \Rightarrow f(0) > 0 \Rightarrow c > 0$

$\therefore ac > 0$

Similarly, if $a < 0$, then the graph of $y = f(x)$ is below x-axis and hence

$f(x) < 0$ for all $x \in R \Rightarrow f(0) < 0 \Rightarrow c < 0$

$\therefore ac > 0$

898 **(a)**

Since, α and β are the roots of the equation $x^2 + px + q = 0$, therefore

$\alpha + \beta = -p$ and $\alpha\beta = q$

Now, $(\omega\alpha + \omega^2\beta)(\omega^2\alpha + \omega\beta)$
$= \alpha^2 + \beta^2 + (\omega^4 + \omega^2)\alpha\beta \quad (\because \omega^3 = 1)$
$= \alpha^2 + \beta^2 - \alpha\beta \quad (\because \omega + \omega^2 = -1)$
$= (\alpha + \beta)^2 - 3\alpha\beta$
$= p^2 - 3q$
Also, $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta}$
$= \dfrac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta}$
$= \dfrac{p(3q - p^2)}{q}$
$\therefore$ The given expression $= \frac{(p^2-3q)}{\frac{p(3q-p^2)}{q}} = -\frac{q}{p}$

899 **(a)**
We have,
$z_2 = \bar{z}_1$ and $z_4 = \bar{z}_3$,
$\therefore z_1 z_2 = |z_1|^2$ and $z_3 z_4 = |z_3|^2$
Now, $\arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right)$
$= \arg\left(\frac{z_1\, z_2}{z_4\, z_3}\right) = \arg\left(\frac{|z_1|^2}{|z_3|^2}\right) = \arg\left(\left|\frac{z_1}{z_3}\right|^2\right) = 0$

900 **(a)**
Given, $\quad x = \frac{1}{2}\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)$
$\therefore x^2 = \dfrac{1}{4}\left(3 + \dfrac{1}{3} + 2\right) = \dfrac{4}{3}$
Now, $\frac{\sqrt{x^2-1}}{x-\sqrt{x^2-1}} \times \frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}}$
$= \dfrac{x\sqrt{x^2-1} + (x^2-1)}{1}$
$= \dfrac{1}{2}\left(\sqrt{3} + \dfrac{1}{\sqrt{3}}\right)\sqrt{\dfrac{4}{3} - 1} + \left(\dfrac{4}{3} - 1\right)$
$= \dfrac{1}{2}\left(\dfrac{4}{\sqrt{3}}\right)\dfrac{1}{\sqrt{3}} + \dfrac{1}{3} = \dfrac{2}{3} + \dfrac{1}{3} = 1$

901 **(b)**
$|PQ| = \sqrt{(4-1)^2 + (1-6)^2} = \sqrt{9+25} = \sqrt{34}$
$|QR| = \sqrt{(1+4)^2 + (6-3)^2} = \sqrt{25+9} = \sqrt{34}$
$|RS| = \sqrt{(-4+1)^2 + (3+2)^2} = \sqrt{9+25} = \sqrt{34}$
$|SP| = \sqrt{(-1-4)^2 + (-2-1)^2} = \sqrt{25+9}$
$= \sqrt{34}$
$\Rightarrow \quad |PQ| = |QR| = |RS| = |SP|$
Now, $|PR| = \sqrt{(-8)^2 + (2)^2} = \sqrt{68}$
And, $|QS| = \sqrt{(-2)^2 + (-8)^2} = \sqrt{68}$
Hence, it is a square.

902 **(c)**
The given expression is meaningful for $x \neq -1$
Let $y = \dfrac{x^2 - 6x + 5}{x^2 + 2x + 1}$. Then,
$x^2(y-1) + 2(y+3)x + y - 5 = 0$
$\Rightarrow 4(y+3)^2 - 4(y-1)(y-5) \geq 0 \quad [\because x \in R$
$\therefore$ Disc $\geq 0]$
$\Rightarrow (y^2 + 6y + 9) - (y^2 - 6y + 5) \geq 0 \Rightarrow y$
$\geq -1/3$
Hence, the given expression last value of the is $-\frac{1}{3}$

903 **(d)**
Given that $x^2 - 3x + 2$ be a factor of $x^4 - px^2 + q = 0$ ...(i)
$\Rightarrow (x^2 - 3x + 2) = 0$
$\Rightarrow (x-2)(x-1) = 0$
$\Rightarrow x = 2, 1$
On putting these values in Eq. (i), we get
$4p - q - 16 = 0$ ...(ii)
and $p - q - 1 = 0$ ...(iii)
On solving Eqs.(ii) and (iii), we get
$p = 5$ and $q = 4$
$\Rightarrow (p, q) = (5,4)$

904 **(a)**
The RHS of the given equation is greater than or equal to 2 as it is the sum of a positive number and its reciprocal while the LHS is less than or equal to 2. Therefore, the equation holds true only when each side is equal to 2.
LHS is equal to 2 when $x = \log \pi/2$ while RHS is equal to 2 when $x = 0$
Thus, the given equation has no solution

906 **(c)**
Let $y = \frac{x^2-3x+4}{x^2+3x+4}$
$\Rightarrow (y-1)x^2 + 3(y+1)x + 4(y-1) = 0$
$\because x$ is real.
$\therefore D \geq 0$
$\Rightarrow 9(y+1)^2 - 16(y-1)^2 \geq 0$
$\Rightarrow -7y^2 + 50y - 7 \geq 0$
$\Rightarrow -7y^2 - 50y + 7 \leq 0$
$\Rightarrow (y-7)(7y-1) \leq 0$ ...(i)
$\Rightarrow y \leq 7$ and $y \geq \dfrac{1}{7} \Rightarrow \dfrac{1}{7} \leq y \leq 7$
Hence, maximum value is 7 and minimum value is $\frac{1}{7}$.

907 **(d)**
Using $i^3 = -i^5$ and $i^7 = -i$, we can write the given expression as
$(1+i)^{n_1} + (1-i)^{n_1} + (1+i)^{n_2} + (1-i)^{n_2}$
$= 2\,[{}^{n_1}C_0 + {}^{n_1}C_2 i^2 + {}^{n_1}C_{4} i^4 + \dots]$
$+ 2\,[{}^{n_2}C_0 + {}^{n_2}C_2 i^2 + {}^{n_2}C_4 i^4 + \dots]$
$= 2\,[{}^{n_1}C_0 - {}^{n_1}C_2 + {}^{n_1}C_4 - \dots]$
$+ 2\,[{}^{n_2}C_0 - {}^{n_2}C_2 + {}^{n_2}C_4 - \dots]$
This is real number, if the values of $n_1$and n_2 is greater than zero

908 **(d)**

We have,

$a = \cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}$

$\Rightarrow a^7 = \left(\cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}\right)^7$

$= \cos 2\pi + i\sin 2\pi = 1 + 0\, i = 1$

Now,

$\alpha + \beta = a + a^2 + a^3 + a^4 + a^5 + a^6$

$\Rightarrow \alpha + \beta = a\left\{\frac{1 - a^6}{1 - a}\right\} = \frac{a - a^7}{1 - a} = \frac{a - 1}{1 - a} = -1$ $[\because a^7 = 1]$

and, $\alpha\,\beta = (a + a^2 + a^4)(a^3 + a^5 + a^6)$

$\Rightarrow \alpha\,\beta = a^4(1 + a + a^3)(1 + a^2 + a^3)$

$\Rightarrow \alpha\,\beta = a^4(1 + a^2 + a^3 + a + a^3 + a^4 + a^3 + a^5 + a^6)$

$\Rightarrow \alpha\,\beta = a^4(1 + a + a^2 + 3\,a^3 + a^4 + a^5 + a^6)$

$\Rightarrow \alpha\,\beta = \alpha^4 + a^5 + a^6 + 3\,a^7 + a^8 + a^9 + a^{10}$

$\Rightarrow \alpha\,\beta = 3 + a + a^2 + a^3 + a^4 + a^5 + a^6$ $\left[\begin{matrix}\because a^7 = 1 \therefore a^8 = a^7 a = a, \\ a^9 = a^7 a^2 = a^2 \text{ and} \\ a^{10} = a^7 a^3 = a^3\end{matrix}\right]$

$\Rightarrow \alpha\,\beta = 3 + a\left(\frac{1 - a^6}{1 - a}\right) = 3 + \frac{a - a^7}{1 - a}$

$= 3 + \frac{a - 1}{1 - a}$ $[\because a^7 = 1]$

$\Rightarrow \alpha\,\beta = 3 - 1 = 2$

So, the required equation is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0 \Rightarrow x^2 + x + 2 = 0$

909 **(c)**

Here, $D \geq 0$

$\Rightarrow \cos^2 p - 4(\cos p - 1)\sin p \geq 0$

$\Rightarrow \cos^2 p - 4\cos p \sin p + 4\sin p \geq 0$

$\Rightarrow (\cos p - 2\sin p)^2 + 4\sin p(1 - \sin p) \geq 0$ ...(i)

Since, $(1 - \sin p) \geq 0$ for all real p and $\sin p > 0$ for $0 < p < \pi$

$\therefore 4\sin p(1 - \sin p) \geq 0$ when $0 < p < \pi$

910 **(d)**

We have,

$5x - 1 < (x + 1)^2 < 7x - 3$

$\Rightarrow 5x - 1 < x^2 + 2x + 1$ and $x^2 + 2x + 1 < 7x - 3$

$\Rightarrow x^2 - 3x + 2 > 0$ and $x^2 - 5x + 4 < 0$

$\Rightarrow (x - 2)(x - 1) > 0$ and $(x - 4)(x - 1) < 0$

$\Rightarrow x \in (2, 4) \Rightarrow x = 3$ $[\because x$ is an integer$]$

911 **(b)**

Let the roots be α and $1/\alpha$. Then,

Product of roots $= \frac{k}{5} \Rightarrow \alpha\left(\frac{1}{\alpha}\right) = \frac{k}{5} \Rightarrow k = 5$

912 **(a)**

We have,

Sum of the coefficients $= 0$

Therefore, $x = 1$ is a rational root of the given equation.

Let the other rational robe α. Then,

Product of the roots $= \frac{2a-1}{a+2}$

$\Rightarrow \alpha \times 1 = \frac{2a - 1}{a + 2} \Rightarrow \alpha = \frac{2a - 1}{a + 2}$

Clearly, α is rational for all rational values of a except -2

913 **(c)**

Let $f(x) = (k - 2)x^2 + 8x + k + 4$

If $f(x) = 0$ has both negative roots, then we must have

(i) Discriminant > 0

(ii) Vertex of $y = f(x)$ is on left side of y-axis

(iii) $(k - 2)f(0) > 0$

Now,

(i) Discriminant > 0

$\Rightarrow 64 - 4(k - 2)(k + 4) > 0$

$\Rightarrow k^2 + 2k - 24 < 0 \Rightarrow -6 < k < 4$ ...(i)

(ii) Vertex is on left side of y-axis

$\Rightarrow -\frac{8}{2(k-2)} < 0 \Rightarrow k - 2 > 0 \Rightarrow k > 2$ ...(ii)

(iii) $(k - 2)f(0) > 0$

$\Rightarrow (k - 2)(k + 4) > 0 \Rightarrow k < -4$ or $k > 2$ ...(iii)

From (i),(ii) and (iii), we obtain $k \in (2, 4)$

Hence, $k = 3$

914 **(b)**

We have,

$ax^2 + c = bx$

$\Rightarrow (ax^2 + c)^2 = b^2x^2 \Rightarrow (ay + c)^2 = b^2y$, where $y = x^2$

Thus, $(ay + c)^2 = b^2y$ has its root as α^2, β^2

915 **(b)**

Given that $3^{2x^2-7x+7} = 3^2 \Rightarrow 2x^2 - 7x + 7 = 2$

$\Rightarrow 2x^2 + 7x + 5 = 0$

Now, $D = b^2 - 4ac$

$= (-7)^2 - 4 \times 2 \times 5$

$= 49 - 40 = 9 > 0$

Hence, it has two real roots.

916 **(d)**

Let α and 3α be the roots of the given equation, then

$\therefore\ \alpha + 3\alpha = 4\alpha = -b$

And $\alpha.3\alpha = 3\alpha^2 = 3$

$\Rightarrow\ \alpha = \pm 1$

$\therefore\ b = \pm 4$

917 **(b)**

$$\sqrt{2+\sqrt{5}-\sqrt{6-3\sqrt{5}+\sqrt{14-6\sqrt{5}}}}$$

$$=\sqrt{2+\sqrt{5}-\sqrt{6-3\sqrt{5}+\sqrt{(9+5-6\sqrt{5})}}}$$

$$=\sqrt{2+\sqrt{5}-\sqrt{6-3\sqrt{5}+\sqrt{(3-\sqrt{5})^2}}}$$

$$=\sqrt{2+\sqrt{5}-\sqrt{9-4\sqrt{5}}}$$

$$=\sqrt{2+\sqrt{5}-\sqrt{(-2+\sqrt{5})^2}}$$

$$=\sqrt{2+\sqrt{5}+2-\sqrt{5}}=2$$

918 **(b)**

Let $x=\cos A+i\sin A, y=\cos B+i\sin B,\ z=\cos C+i\sin C$. Then,

$$x+y+z=(\cos A+\cos B+\cos C)+i(\sin A+\sin B+\sin C)$$

$\Rightarrow x+y+z=0+i\,0=0$

$\Rightarrow x^3+y^3+z^3=3\,xyz$

$$\Rightarrow(\cos 3A+i\sin 3A)+(\cos 3B+i\sin 3B)+(\cos 3C+i\sin 3C)=3[\cos(A+B+C)+i\sin(A+B+C)]$$

$\Rightarrow \cos 3A+\cos 3B+\cos 3C=3\cos(A+B+C)$

and, $\sin 3A+\sin 3B+\sin 3C=3\sin(A+B+C)$

It is given that $A+B+C=180°$

$\therefore \cos 3A+\cos 3B+\cos 3C=3\cos 180°=-3$

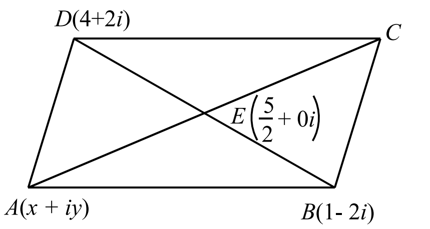
919 **(b)**

We have,

$|\overrightarrow{BD}|=|(4+2i)-(1-2i)|=\sqrt{9+16}=5$

Let the affix of A be $z=x+iy$. The affix of the mid point of BD is $(5/2,0)$.

Since the diagonals of a parallelogram bisect each other. Therefore, the affix of the point of intersection of the diagonals is $(5/2,0)$



We have,

$\overrightarrow{EA}=2\,\overrightarrow{EB}\,e^{i\,\pi/2}$

$\Rightarrow \overrightarrow{EA}=2\,\overrightarrow{EB}(-i)$

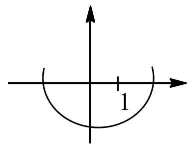
$\Rightarrow z-(5/2+0i)=2\left(-\frac{3}{2}-2i\right)(-i)=-\frac{3}{2}+3i$

920 **(a)**

$-x^2+ax+a=0$

$\Rightarrow x^2-ax-a=0$

Let $f(x)=x^2-ax-a$



$f(1)<0$

$\Rightarrow 1-a-a<0$

$\Rightarrow 1<2a$

$\Rightarrow a>\frac{1}{2}$

921 **(c)**

Let α and β are the roots of the given equation

Then, $\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$

Also given, $\alpha+\beta=\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

$$=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha^2\beta^2}$$

$$\Rightarrow\ \left(-\frac{b}{a}\right)=\left(\frac{-b/a}{c/a}\right)^2-\frac{2}{c/a}$$

$$\Rightarrow\ -\frac{b}{a}=\left(\frac{b}{c}\right)^2-\frac{2a}{c}$$

$$\Rightarrow\ \frac{2a}{c}=\frac{b}{c}\left(\frac{b}{c}+\frac{c}{a}\right)$$

$$\Rightarrow\ \frac{2a}{b}=\frac{b}{c}+\frac{c}{a}$$

$\Rightarrow\ \frac{c}{a},\frac{a}{b},\frac{b}{c}$ are in AP

$\Rightarrow\ \frac{a}{c},\frac{b}{a},\frac{c}{b}$ are in HP

922 **(a)**

Since roots are real.

$\therefore \{2(bc+ad)\}^2=4(a^2+b^2)(c^2+d^2)$

$$\Rightarrow 4b^2c^2+4a^2d^2+8abcd=4a^2c^2+4a^2d^2+4b^2c^2+4b^2d^2$$

$\Rightarrow\ 4a^2d^2+4b^2c^2-8abcd=0$

$\Rightarrow 4(ad-bc)^2=0$

$\Rightarrow ad=bc$

$\Rightarrow \frac{a}{b}=\frac{c}{d}$

923 **(b)**

$(1-\omega+\omega^2)^5+(1+\omega-\omega^2)^5$

$=(-2\omega)^5+(-2\omega^2)^5$

$=-32\omega^3\omega^2-32(\omega^3)^3\omega$

$=-32(\omega^2+\omega)=32$

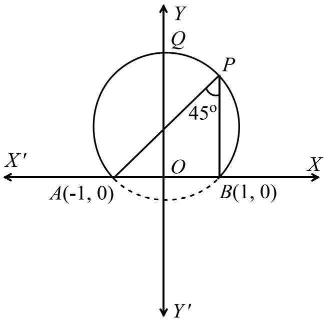
924 **(b)**

Clearly, $|z+1|=|z-1|$

Represents the perpendicular bisector of the segment joining $A(-1,0)$ and $B(1,0)$ i.e. y-axis

$\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{4}$ represents the segment of the circle passing through A and B and lying above x-axis such that angle in the segment is $\pi/4$

It is evident from the figure that point Q satisfies both the conditions



Let the affix of Q be $z=iy, y\in R$. Then,

$$\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{4}$$

$$\Rightarrow \arg\left(\frac{iy-1}{iy+1}\right)=\frac{\pi}{4}$$

$$\Rightarrow \arg\left(\frac{y+i}{y-i}\right)=\frac{\pi}{4}$$

$$\Rightarrow \arg\left(\frac{y^2-1}{y^2+1}+\frac{2iy}{y^2+1}\right)=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{2y}{y^2-1}\right)=\frac{\pi}{4}$$

$$\Rightarrow \frac{2y}{y^2-1}=1\Rightarrow y-2y-1=0\Rightarrow y=\sqrt{2}+1 \quad [\because y>0]$$

Hence, $z=\left(\sqrt{2}+1\right)i$

925 **(c)**

It is given that α,β are the roots of the equation $x^2-ax+b=0$

$\therefore \alpha+\beta=a$ and $\alpha\,\beta=b$

$\Rightarrow \alpha^2+\beta^2=a^2-2b$

$\Rightarrow \alpha^2+\beta^2=a(\alpha+\beta)-2b$

$\Rightarrow A_2$

$=a\,A_1$

$-A_0 b \quad \left[\begin{array}{c}\because A_n=\alpha^n+\beta^n \therefore A_2=\alpha^2+\beta^2\\ A_1=\alpha+\beta \text{ and } A_0=2\end{array}\right]$

Clearly, it is obtained from option (c) by replacing n by 2

Now,

$$a\,A_n-b\,A_{n-1}=(\alpha+\beta)(\alpha^n+\beta^n)-\alpha\,\beta(\alpha^{n-1}+\beta^{n-1})$$

$$\Rightarrow a\,A_n-b\,A_{n-1}=\alpha^{n+1}+\beta^{n+1}=A_{n+1}$$

926 **(a)**

Let α,β,γ are the roots of the given equation. Then,

$\alpha+\beta+\gamma=-p$

$\alpha\beta+\beta\gamma+\gamma\alpha=-q$

And $\alpha\beta\gamma=-r$

Now, $pq=(\alpha+\beta+\gamma)(\alpha\beta+\beta\gamma+\gamma\alpha)$

$=(0+\gamma)[\alpha\beta+\gamma(\alpha+\beta)]$ ($\because \alpha+\beta=0$ is given)

$=\alpha\beta\gamma$

$=-r$

927 **(b)**

$\because\ x^4-8x^2-9=0$

$\Rightarrow x^4-9x^2+x^2-9=0$

$\Rightarrow x^2(x^2-9)+1(x^2-9)=0$

$\Rightarrow (x^2+1)(x^2-9)=0$

$\Rightarrow x=\pm i,\pm 3$

928 **(b)**

$$\frac{1+a}{2}=\frac{1}{2}\left(1+\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}\right)$$

$$=\frac{1}{2}2\cos\frac{2\pi}{3}\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$$

$$=-\frac{1}{2}\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$$

$$\therefore\ \left(\frac{1+a}{2}\right)^{3n}=\left(\frac{-1}{2}\right)^{3n}\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)^{3n}$$

$$=\frac{(-1)^n}{2^{3n}}$$

929 **(b)**

Now, $a^2-3a+2=0$

$\Rightarrow a=1,2$ ...(i)

and $a^2-5a+6=0$

$\Rightarrow a=2,3$ ...(ii)

$\Rightarrow a-2-r=0$

At $a=2$ [common value from Eqs. (i) and (ii)]

$r=0$

So, $a+r=2$

930 **(c)**

Given, $\left(a+\frac{b}{10}\right)^x=\left(\frac{a}{10}+\frac{b}{100}\right)^y=1000$

Let $a=0$

And $b=1$

$$\therefore\ \left(\frac{1}{10}\right)^x=\left(\frac{1}{100}\right)^y=1000$$

$$\Rightarrow\ 10^{-x}=10^{-2y}=10^3$$

$$\Rightarrow\ x=-3,\ y=-\frac{3}{2}$$

Now, $\frac{1}{x}-\frac{1}{y}=-\frac{1}{3}+\frac{2}{3}=\frac{1}{3}$

931 **(c)**

We have,

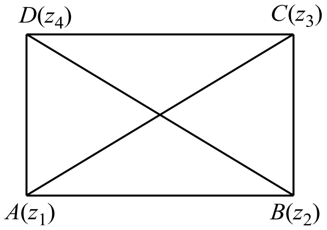
$z_1-z_4=z_2-z_3$

$\Rightarrow z_1+z_3=z_2+z_4$

$\Rightarrow \frac{z_1+z_3}{2}=\frac{z_2+z_4}{2}$

$\Rightarrow$ Affix of the mid point of AC is same as that of BD

$\Rightarrow AC$ and BD bisect each other



Also, $\arg\left(\frac{z_4 - z_1}{z_2 - z_1}\right) = \pm\frac{\pi}{2}$

$\Rightarrow \angle BAD = \frac{\pi}{2}$

Thus, $ABCD$ is a rectangle and hence a cyclic quadrilateral also

932 **(a)**

We have,

$x^2 + 2 \leq 3x \leq 2x^2 - 5$

$\Rightarrow x^2 - 3x + 2 \leq 0$ and $2x^2 - 3x - 5 \geq 0$

$\Rightarrow (x-1)(x-2) \leq 0$ and $(2x-5)(x+1) \geq 0$

$\Rightarrow 1 \leq x \leq 2$ and $x \leq -1$ or $x \geq \frac{5}{2}$

There is no value of x satisfying these conditions

933 **(a)**

Let $f(x) = -3 + x - x^2$

Now, $D = 1^2 - 4(3) = -11 < 0$

Here, coefficient of $x^2 < 0$

$\therefore\ f(x) < 0$

Thus, LHS of the given equation is always positive whereas the RHS is always less than zero

Hence, the given equation has no solution

934 **(c)**

$4.9^{x-1} = 3\sqrt{(2^{2x+1})}$

$\Rightarrow\ 3^{2x-2-1} = 2^{\frac{2x+1}{2}-2}$

$\Rightarrow\ 3^{2x-3} = 2^{\frac{2x-3}{2}}$

$\Rightarrow 2^{\frac{2x-3}{2}} = \left(3^{\frac{2x-3}{2}}\right)^2$

$\Rightarrow 2x - 3 = 0$

$\therefore\ x = \frac{3}{2}$

935 **(c)**

Given equation is $x^2 + (2+\lambda)x - \frac{1}{2}(1+\lambda) = 0$.

Let α and β are the roots of the given equation.

$\Rightarrow\ \alpha + \beta = -(2+\lambda)$ and $\alpha\beta = -\left(\frac{1+\lambda}{2}\right)$

Now, $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$\Rightarrow\ \alpha^2 + \beta^2 = [-(2+\lambda)]^2 + 2\,\frac{(1+\lambda)}{2}$

$\Rightarrow\ \alpha^2 + \beta^2 = \lambda^2 + 4 + 4\lambda + 1 + \lambda = \lambda^2 + 5\lambda + 5$

Now, we take the option simultaneously.

$\Rightarrow$ It is minimum for $\lambda = \frac{1}{2}$.

936 **(a)**

Since, $2 + i\sqrt{3}$ is a root of equation $x^2 + px + q = 0$. Therefore, $2 - i\sqrt{3}$ will be other root.

Now, Sum of the roots $= \left(2 + i\sqrt{3}\right) + (2 - i\sqrt{3}) = -p$

$\Rightarrow 4 = -p$

Product of roots $= \left(2 + i\sqrt{3}\right) + (2 - i\sqrt{3}) = q$

$\Rightarrow 7 = q$

Hence, $(p, q) = (-4, 7)$

937 **(b)**

Given equation is

$4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$

$\Rightarrow 2^{2x} + 2^{2x-1} = 3^{x+\frac{1}{2}} + 3^{x-\frac{1}{2}}$

$\Rightarrow 2^{2x}\left(1 + \frac{1}{2}\right) = 3^{x-\frac{1}{2}}(3+1)$

$\Rightarrow\ 2^{2x}.\frac{3}{2} = 3^{x-\frac{1}{2}}.4$

$\Rightarrow\ 2^{2x-3} = 3^{x-\frac{3}{2}}$

Taking log on both sides, we get

$(2x-3)\log 2 = \left(x - \frac{3}{2}\right)\log 3$

$\Rightarrow 2x\log 2 - 3\log 2 = x\log 3 - \frac{3}{2}\log 3$

$\Rightarrow x\log 4 - x\log 3 = 3\log 2 - \frac{3}{2}\log 3$

$\Rightarrow x\log\left(\frac{4}{3}\right) = \log 8 - \log 3\sqrt{3}$

$\Rightarrow \log\left(\frac{4}{3}\right)^x = \log\frac{8}{3\sqrt{3}}$

$\Rightarrow\ \left(\frac{4}{3}\right)^x = \frac{8}{3\sqrt{3}}$

$\Rightarrow\ \left(\frac{4}{3}\right)^x = \left(\frac{4}{3}\right)^{3/2}$

$\therefore x = \frac{3}{2}$

938 **(d)**

We have

$x^{1/3} - 7x^{1/3} + 10 = 0$

$\Rightarrow x^{1/3} = 2, x^{1/3} = 5 \Rightarrow x^{1/3} = 2, x^{1/3} = 5 \Rightarrow x = 8{,}125$

939 **(b)**

Let $P = (1+z_0)(1+z_0^2)(1+z_0^2 \ldots \left(1+z_0^{2^n}\right)$

Then,

$(1-z_0)P = (1 - z_0^{2^{n+1}})$

$\Rightarrow P = \frac{1 - z_0^{2^{n+1}}}{1 - z_0} = \frac{1 - (z_0^2)^{2^n}}{1 - z_0}$

$\Rightarrow P = \frac{1 - \left(-\frac{i}{2}\right)^{2n}}{\frac{1+i}{2}}\quad \left[\because z_0 = \frac{1-i}{2} \therefore z_0^2 = -\frac{i}{2}\right]$

$\Rightarrow P = \frac{2}{1+i}\left\{1 - \frac{(-1)^{2^n}(i)^{2^n}}{2^{2n}}\right\}$

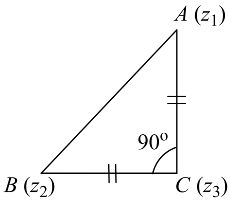
$$\Rightarrow P=\begin{cases}(1-i)\left(1-\frac{1}{2^{2^n}}\right), \text{if } n>1\\ (1-i)\left(1+\frac{1}{4}\right), \ \text{if } n=1\end{cases}$$

$$\Rightarrow P=\begin{cases}(1-i)\left(1-\frac{1}{2^{2^n}}\right), \text{if } n>1\\ \frac{5}{4}(1-i) \qquad , \text{if } n=1\end{cases}$$

940 **(d)**

Since, ABC is a right angled isosceles triangle

$\therefore \ BC=AC$ and $\angle C=\frac{\pi}{2}$



By rotation about C in anti-clockwise sense

$CB=CA\, e^{i\pi/2}$

$\Rightarrow \ (z_2-z_3)=(z_1-z_3)e^{i\pi/2}$

$=i\,(z_1-z_3) \quad (\because e^{i\pi/2}=i)$

On squaring both sides, we get

$(z_2-z_3)^2=-(z_1-z_3)^2$

$\Rightarrow \ z_2^2+z_3^2-2z_2z_3=-z_1^2-z_3^2+2z_1z_3$

$\Rightarrow \ z_1^2+z_2^2-2z_1z_2$

$=2z_1z_3+2z_2z_3-2z_3^2-2z_1z_2$

$\Rightarrow \ (z_1-z_2)^2=2[(z_1z_3-z_3^2)-(z_1z_2-z_2z_3)]$

$\Rightarrow \ (z_1-z_2)^2=2(z_1-z_3)(z_3-z_2)$

941 **(b)**

Given equation is

$(a+1)x^2-(a+2)x+(a+3)=0$

Since, roots are equal in magnitude and opposite in sign

$\therefore$ Coefficient of x is zero $ie, a+2=0$

$\Rightarrow \ a=-2$ ...(i)

$\therefore$ Equation is

$(-2+1)x^2-(-2+2)x+(-2+3)=0$

$\Rightarrow \quad -x^2+1=0$

$\Rightarrow \ x=\pm 1$ ...(ii)

Only option (b) $i.e., \pm\frac{1}{2}a$ satisfies Eqs. (i) and (ii)

942 **(d)**

Given, $\log_{27}\log_3 x=\frac{1}{3}$

$\Rightarrow (\log_3 x)=(27)^{1/3}=3$

$\Rightarrow x=(3)^3$

$\Rightarrow x=27$

943 **(d)**

$\because \ \arg(z-3i)=\arg(x+iy-3i)=\frac{3\pi}{4}$

$\Rightarrow \ x<0, y-3>0 \ \left(\because \frac{3\pi}{4} \text{ is in II quadrant}\right)$

$\frac{y-3}{x}=\tan\frac{3\pi}{4}=-1$

$\Rightarrow \ y=-x+3$ ... (i)

$\forall x<0$ and $y>3$

and $\arg(2z+1-2i)=\arg[(2x+1)+$

$i2y-2=\pi 4$

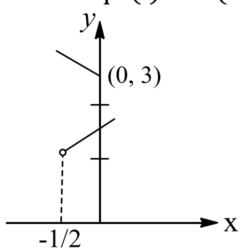
$\Rightarrow 2x+1>0, 2y-2>0 \ \left(\because \frac{\pi}{4} \text{ is in I quadrant}\right)$

$\therefore \ \frac{2y-2}{2x+1}=\tan\frac{\pi}{4}=1$

$\Rightarrow \ 2y-2=2x+1$

$\Rightarrow \ y=x+\frac{3}{2}, \forall x>\frac{-1}{2}, y>1$ ...(ii)

From Eqs.(i) and(ii)



It is clear from the graph that there is no point of intersection

944 **(a)**

We have,

$x^2+2\le 3x\le 2x^2-5$

$\Rightarrow x^2+2\le 3x$ and $3x\le 2x^2-5$

$\Rightarrow x^2-3x+2\le 0$ and $2x^2-3x-5\ge 0$

$\Rightarrow (x-1)(x-2)\le 0$ and $(2x-5)(x+1)\ge 0$

$\Rightarrow 1\le x\le 2$ and $x\in(-\infty,-1]\cup[5/2,\infty)$

But, there is no value of x satisfying these two conditions

945 **(c)**

$ax^2+2bx+c=0$

$\Rightarrow ax^2+2\sqrt{ac}x+c=0 \qquad [\because b^2=ac]$

$\Rightarrow \ \left(\sqrt{ax}+\sqrt{c}\right)^2=0 \Rightarrow x=\frac{-\sqrt{c}}{\sqrt{a}}, \frac{-\sqrt{c}}{\sqrt{a}}$

$\Rightarrow a\alpha=a\beta$

Now, $cx^2+2bx+a=0$

$\Rightarrow \ cx^2+2\sqrt{ac}x+a=0$

$\Rightarrow \left(\sqrt{c}x+\sqrt{a}\right)^2=0$

$\Rightarrow \quad x=\frac{-\sqrt{a}}{\sqrt{c}}=\frac{-\sqrt{a}}{\sqrt{c}} \Rightarrow c\gamma=c\delta$

$\therefore \ a\alpha=a\beta=c\gamma=c\delta$

946 **(c)**

The function $f(x) = \log(x^2 - x - 2)$ is defined for $x^2 - x - 2 > 0 \Rightarrow x < -1$ or $x > 2$ ...(i)

Now,

$9x^2 - 18|x| + 5 = 0$

$\Rightarrow 9|x|^2 - 18|x| + 5 = 0$

$\Rightarrow (3|x| - 1)(3|x| - 5) = 0$

$\Rightarrow |x| = 1, 5/3 \Rightarrow |x| = \pm 1, \pm 5/3$

Thus, roots of $x^2 - 18|x| + 5 = 0$ are

$\pm 5/3, \pm 1/3$.

Clearly, a root of the above equation lying in the domain of the definition of $\log(x^2 - x - 2)$ is $-5/3$

947 **(d)**

Since, α and β are the roots of

$\lambda x^2 + (1 - \lambda)x + 5 = 0$

$\therefore\ \alpha + \beta = \frac{\lambda - 1}{\lambda}, \alpha\beta = \frac{5}{\lambda}$

Since, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{4}{5}$

$\Rightarrow \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{4}{5}$

$\Rightarrow \frac{(\lambda - 1)^2 - 10\lambda}{5\lambda} = \frac{4}{5}$

$\Rightarrow \lambda^2 - 16\lambda + 1 = 0$

Now, $\lambda_1 + \lambda_2 = 16$ and $\lambda_1 . \lambda_2 = 1$

$\therefore \frac{\lambda_1}{\lambda_2^2} + \frac{\lambda_2}{\lambda_1^2} = \frac{\lambda_1^3 + \lambda_2^3}{(\lambda_1\lambda_2)^2}$

$= \frac{(\lambda_1 + \lambda_2)^3 - 3\lambda_1\lambda_2(\lambda_1 + \lambda_2)}{(1)^2}$

$= (16)^3 - 3 \times 1(16)$

$= 4048$

948 **(a)**

Given equation $\frac{\alpha}{x-\alpha} + \frac{\beta}{x-\beta} = 1$ can be rewritten as

$x^2 - 2(\alpha + \beta)x + 3\alpha\beta = 0$

Let its roots be α' and $-\alpha'$.

$\Rightarrow\ \alpha' + (-\alpha') = 2(\alpha + \beta)$

$\Rightarrow 0 = 2(\alpha + \beta)$

$\Rightarrow\ \alpha + \beta = 0$

949 **(a)**

Let $C = \cos\theta, S = \sin\theta$. Then,

$\frac{1 + C + i\,S}{1 + C - i\,S} = \frac{1 + \cos\theta + i\sin\theta}{1 + \cos\theta - i\sin\theta}$

$\Rightarrow \frac{1 + C + i\,S}{1 + C - i\,S} = \frac{2\cos^2\frac{\theta}{2} + 2\,i\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2} - 2\,i\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$

$\Rightarrow \frac{1 + C + i\,S}{1 + C - i\,S} = \frac{\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}}{\cos\frac{\theta}{2} - i\sin\frac{\theta}{2}} = \frac{e^{i\,\theta/2}}{e^{-i\,\theta/2}}$

$\Rightarrow \frac{1 + C + i\,S}{1 + C - i\,S} = e^{i\,\theta} = \cos\theta + i\sin\theta$

950 **(c)**

Given, $|z - 3| = |z - 5|$

$\Rightarrow\ (z - 3)(\bar{z} - 3) = (z - 5)(\bar{z} - 5)$

[on squaring both sides]

$\Rightarrow\ 2\bar{z} + 2z = 16 \quad \Rightarrow \quad z + \bar{z} = 8$

$\Rightarrow\ 2x = 8 \quad \Rightarrow \quad x = 4$

Hence, locus of z is a straight line parallel to y-axis

951 **(b)**

We have,

$\frac{4x + 3}{2x - 5} < 6$

$\Rightarrow \frac{4x + 3 - 12x + 30}{2x - 5} < 0 \Rightarrow \frac{-8\left(x - \frac{33}{8}\right)}{2\left(x - \frac{5}{2}\right)} < 0$

$\Rightarrow \frac{x - \frac{33}{8}}{x - \frac{5}{2}} > 0 \Rightarrow x \in \left(-\infty, \frac{5}{2}\right) \cup \left(\frac{33}{8}, \infty\right)$

952 **(b)**

Let α, β be the roots of a quadratic and α^2, β^2 be the roots of another quadratic. Since the quadratics remain same.

$\therefore \alpha + \beta = \alpha^2 + \beta^2$ ... (i)

and, $\alpha\,\beta = \alpha^2\beta^2$ ... (ii)

Now,

$\alpha\beta = \alpha^2\beta^2$

$\Rightarrow \alpha\beta(\alpha\,\beta - 1) = 0 \Rightarrow \alpha = 0$ or, $\beta = 0$ or, $\alpha\,\beta = 1$

If $\alpha = 0$ then

$\beta = \beta^2$ [Putting $\alpha = 0$ in (i)]

$\Rightarrow \beta(1 - \beta) = 0 \Rightarrow \beta = 0, \beta = 1$

Thus, we get two sets of values of α and β viz.

$\alpha = 0, \beta = 0$ and $\alpha = 0, \beta = 1$

If $\alpha\,\beta = 1$, then

$\alpha + \frac{1}{\alpha} = \alpha^2 + \frac{1}{\alpha^2}$ $\left[\text{Putting } \beta = \frac{1}{\alpha} \text{ in (i)}\right]$

$\Rightarrow \alpha + \frac{1}{\alpha} = \left(\alpha + \frac{1}{\alpha}\right)^2 - 2$

$\Rightarrow \left(\alpha + \frac{1}{\alpha}\right)^2 - \left(\alpha + \frac{1}{\alpha}\right) - 2 = 0$

$\Rightarrow \alpha + \frac{1}{\alpha} = 2$ or, $\alpha + \frac{1}{\alpha} = -1$

$\Rightarrow \alpha = 1$ or $\alpha = \omega, \omega^2$

Putting $\alpha = 1$ in $\alpha\,\beta = 1$, we get $\beta = 1$

Putting $\alpha = \omega$ in $\alpha\,\beta = 1$, we get $\beta = \omega^2$

Putting $\alpha = \omega^2$ in $\alpha\,\beta = 1$, we get $\beta = \omega$

Thus, we get four pairs of values of α and β viz. $\alpha = 0, \beta = 0; \alpha = 0, \beta = 1; \alpha = \omega, \beta = \omega^2; \alpha = 1, \beta = 1$

Hence, there are four quadratic equations which remains unchanged by squaring their roots

953 **(d)**

Given, $|z - z_1| = |z - z_2|$

It is perpendicular bisector of line joining z_1 and z_2

954 **(a)**

Here, $\alpha+\beta=-a, \quad \alpha\beta=b$

$\therefore \frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\alpha^2+\beta^2}{(\alpha\beta)^2}$

$=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}=\frac{a^2-2b}{b^2}$

955 **(b)**

Given, $(x+iy)^{1/3}=a-ib$

And $\frac{x}{a}-\frac{y}{b}=k(a^2-b^2)$

$\therefore \quad x+iy=(a-ib)^3$

$=(a^3-3ab^2)+i(b^3-3a^2b)$

$\therefore \quad x=a^3-3ab^2, \quad y=b^3-3a^2b$

$\Rightarrow \frac{x}{a}=a^2-3b^2, \qquad \frac{y}{b}=b^2-3a^2$

$\therefore \frac{x}{a}-\frac{y}{b}=a^2-3b^2-b^2+3a^2=4(a^2-b^2)$

But $\frac{x}{a}-\frac{y}{b}=k(a^2-b^2)$ [given]

$\therefore \quad k=4$

956 **(a)**

$z_1+z_2=-1$ and $z_1z_2=\frac{b}{3}$

As z_1, z_2 and origin form an equilateral triangle, we have, $z_1^2+z_2^2+z_3^2=z_1z_2+z_2z_3+z_3z_1$

$\Rightarrow \quad z_1^2+z_2^2+0=z_1z_2+0+0$

$\Rightarrow \quad (z_1+z_2)^2=3z_1z_2$

$\Rightarrow \quad 1=b$

957 **(a)**

$\frac{1}{\log_2 n!}+\frac{1}{\log_3 n!}+\frac{1}{\log_4 n!}+\ldots+\frac{1}{\log_{2002} n!}$

$=\frac{\log 2}{\log n!}+\frac{\log 3}{\log n!}+\ldots+\frac{\log 2002}{\log n!}$

$=\frac{\log(2.3.4\ldots2002)}{\log n!}$

$=\frac{\log 2002!}{\log n!}$

$=\frac{\log 2002!}{\log 2002!}=1 \qquad [\because n=2002, \text{given}]$

958 **(c)**

Since, x^2+px+1 is a factor of ax^3+bx+c

$\therefore ax^3+bx+c=(x^2+px+1)(lx+m)$

On equating the coefficients of like powers of x, we get

$l=a, m+lp=0, b=pm+l$ and $c=m$

$\Rightarrow c+ap=0$ and $b=pc+a$

$\Rightarrow b=-\frac{c^2}{a}+a \Rightarrow a^2-c^2=ab$

959 **(c)**

We have,

$\left|\frac{z-12}{z-8\,i}\right|=\frac{5}{3}$ and $\left|\frac{z-4}{z-8}\right|=1$

Let $z=x+iy$. Then,

$\left|\frac{z-12}{z-8\,i}\right|=\frac{5}{3}$

$\Rightarrow 3|z-12|=5|z-8\,i|$

$\Rightarrow 3|(x-12)+i\,y|=5|x+(y-8)\,i|$

$\Rightarrow 9(x-12)^2+9\,y^2=25\,x^2+25(y-8)^2$ ...(i)

and, $\left|\frac{z-4}{z-8}\right|=1$

$\Rightarrow |z-4|=|z-8|$

$\Rightarrow |x-4+i\,y|=|x-8+i\,y|$

$\Rightarrow (x-4)^2+y^2=(x-8)^2+y^2$

$\Rightarrow x=6$

Putting $x=6$ in (i), we get

$y^2-25\,y-136=0 \Rightarrow y=17, 8$

Hence, $z=6+17\,i$ or, $z=6+8\,i$

960 **(d)**

Given equation is $e^{\sin x}-e^{-\sin x}-4=0$

Let $e^{\sin x}=y$, then given equation can be written as

$y^2-4y-1=0 \Rightarrow y=2\pm\sqrt{5}$

But the value of $y=e^{\sin x}$ is always positive so we take only $y=2+\sqrt{5}$

$\Rightarrow \log_e y=\log_e(2+\sqrt{5})$

$\Rightarrow \sin x=\log_e\left(2+\sqrt{5}\right)>1$

Which is impossible since $\sin x$ cannot be greater than 1.

Hence, we cannot find any real value of x which satisfies each given equation.

961 **(a)**

We have,

$\sqrt{z}=\pm\left[\sqrt{\frac{1}{2}\{|z|+\text{Re}\,(z)\}}\pm i\sqrt{\frac{1}{2}\{|z|-\text{Re}\,(z)\}}\right],$

$\therefore \sqrt{i}=\pm\left(\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right) \Rightarrow \sqrt{2}i=\pm(1+i)$

Hence, $a=\sqrt{2i}=\pm(1+i)$

962 **(b)**

Since, $2+i\sqrt{3}$ is a root of the equation $x^2+px+q=0$, then the other root will be $2-i\sqrt{3}$

$\therefore \quad 2+i\sqrt{3}+2-i\sqrt{3}=-p$

$\Rightarrow \quad p=-4$

And $\left(2+i\sqrt{3}\right)\left(2-i\sqrt{3}=q\right)$

$\Rightarrow \qquad q=7$

$\therefore$ The value of (p, q) is $(-4, 7)$

963 **(a)**

Equation of circle whose centre is z_0 and radius is

r, is $|z-z_0|^2 = r^2$
$\Rightarrow (z-z_0)(\overline{z-z_0}) = r^2$
$\Rightarrow (z-z_0)(\bar{z}-\bar{z}_0) = r^2$
$\Rightarrow z\bar{z} - z\bar{z}_0 - \bar{z}z_0 + z_0\bar{z}_0 = r^2$

964 **(a)**

$$\sum_{n=0}^{\infty}\left(\frac{2i}{3}\right)^n = 1 + \left(\frac{2i}{3}\right) + \left(\frac{2i}{3}\right)^2 + \left(\frac{2i}{3}\right)^3 + \dots$$
$$= \frac{1}{1-\frac{2i}{3}} = \frac{3}{3-2i} \times \frac{3+2i}{3+2i}$$
$$= \frac{9+6i}{13}$$

965 **(c)**

Given, $y = 2^{1/\log_x(8)} = 2^{\log_8(x)}$
$\Rightarrow y = 2^{\log_2 \sqrt[3]{x}} = \sqrt[3]{x}$
$\Rightarrow x = y^3$

966 **(a)**

Since, $1, \alpha, \alpha^2, \dots, \alpha^{n-1}$ are n, n^{th} roots of unity

$$\therefore \sum_{r=0}^{n-1} \alpha^r = 0 \text{ and}, \sum_{r=0}^{n-1} (\bar{\alpha})^r = 0$$

Now,

$$\sum_{r=0}^{n-1} |z_1 + \alpha^r z_2|^2$$
$$= \sum_{r=0}^{n-1} (z_1 + \alpha^r z_2)(\bar{z}_1 + \bar{\alpha}^r \bar{z}_2)$$
$$= \sum_{r=0}^{n-1} (|z_1|^2 + |\alpha|^{2r}|z_2|^2 + z_1\bar{\alpha}^r\bar{z}_2 + \bar{z}_1 \alpha^r z_2)$$
$$= \sum_{r=0}^{n-1} \{|z_1|^2 + |z_2|^2 + z_1\bar{z}_2(\bar{\alpha})^r + \bar{z}_1 z_2 \alpha^r\}$$
$$= \sum_{r=0}^{n-1} |z_1|^2 + |z_2|^2 + z_1\bar{z}_2 \sum_{r=0}^{n-1} (\bar{\alpha})^r + \bar{z}_1 z_2 \sum_{r=0}^{n-1} \alpha^r$$

967 **(d)**

Since $8, 2$ are roots of $x^2 + ax + \beta = 0$ and $3, 3$ are roots of $x^2 + \alpha x + b = 0$. Therefore,
$8 + 2 = -a, 8 \times 2 = \beta$ and $3 + 3 = -\alpha, 3 \times 3 = b$
$\Rightarrow a = -10, \beta = 16, \alpha = -6$ and $b = 9$
Thus, $x^2 + ax + b = 0$, becomes $x^2 - 10x + 9 = 0$ whose roots are $1, 9$

968 **(a)**

We have,

$$\sqrt{z} = \left\{\sqrt{\frac{|z| + \text{Re}(z)}{2}} + i\sqrt{\frac{|z| + \text{Re}(z)}{2}}\right\}, \text{if Im}(z) > 0$$

and $$\sqrt{z} = \pm\left\{\sqrt{\frac{|z| + \text{Re}(z)}{2}} - i\sqrt{\frac{|z| - \text{Re}(z)}{2}}\right\}, \text{if Im}(z) < 0$$

$\therefore \sqrt{2i} = \pm(1+i)$ and $\sqrt{-2i} = \pm(1-i)$
$\Rightarrow \sqrt{2i} - \sqrt{-2i} = \pm 2$
$\Rightarrow |\sqrt{2i} - \sqrt{-2i}| = 2$

969 **(c)**

$a + b = -a$ ...(i)
$ab = b$ ...(ii)
From Eq. (ii)
$a = 1$ $\because b \neq 0$
From Eq. (i)
$b = -2$

970 **(a)**

$z = (1 - i\cot 8)^3$
$= \text{cosec}^3 8(\sin 8 - i\cos 8)^3$
$= \text{cosec}^3 8\left(\cos\left(\frac{\pi}{2} - 8\right) - i\sin\left(\frac{\pi}{2} - 8\right)\right)^3$
$= \text{cosec}^3 8\left(\cos\left(\frac{3\pi}{2} - 24\right) - i\sin\left(\frac{3\pi}{2} - 24\right)\right)$
$= \text{cosec}^3 8 . e^{-i\left(\frac{3\pi}{2} - 24\right)}$
$= \text{cosec}^3 8 . e^{i\left(24 - \frac{3\pi}{2}\right)}$

971 **(a)**

Since, one root of the equation $x^2 + px + q = 0$ is $2 + \sqrt{3}$, then the other root will be
$2 - \sqrt{3}$
$\therefore$ Since of roots $2 + \sqrt{3} + 2 - \sqrt{3} = -p$
$\Rightarrow p = -4$
And product of roots
$(2 + \sqrt{3})(2 - \sqrt{3}) = q$
$\Rightarrow q = 1$

972 **(a)**

$$7\log_2\frac{16}{15} + 5\log_2\frac{25}{24} + 3\log_2\frac{81}{80}$$
$$= 7[4\log_2 2 - \log_2 3 - \log_2 5] + 5[2\log_2 5 - \log_2 3 - 3\log_2 2] + 3[4\log_2 3 - 4\log_2 2 - \log_2 5]$$
$$= \log_2 2 = 1$$

973 **(c)**

$\because \sqrt{5x^2 - 8x + 3} - \sqrt{5x^2 - 9x + 4} = \sqrt{2x^2 - 2x} - \sqrt{2x^2 - 3x + 1}$
Also, $(5x^2 - 8x + 3) - (5x^2 - 9x + 4) = (2x^2 - 2x) - (2x^2 - 3x + 1)$
$\Rightarrow x - 1 = x - 1$
$\Rightarrow x = 1$ is the required value.

974 **(c)**

We know that $ax^2 + bx + c \geq 0$ if $a > 0$ and $b^2 - 4ac \leq 0$

Now, $mx - 1 + \frac{1}{x} \geq 0$

$\Rightarrow \dfrac{mx^2 - x + 1}{x} \geq 0$

$\Rightarrow mx^2 - x + 1 \geq 0$ and $x > 0$

Now, $mx^2 - x + 1 \geq 0$, if $m > 0$ and $1 - 4m \leq 0$ or if $m > 0$ and $m \geq \frac{1}{4}$.

Thus, the minimum value of m is $\frac{1}{4}$.

975 **(a)**

Given, $\log_e\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log_e a + \log_e b)$

$\Rightarrow\therefore \quad \dfrac{a+b}{2} = \sqrt{ab}$

$\Rightarrow a + b - 2\sqrt{ab} = 0$

$\Rightarrow \quad \sqrt{a} = \sqrt{b}$

$\Rightarrow a = b$

976 **(a)**

Let α be a negative common root of equations $ax^2 + bx + c = 0$ and $cx^2 + bx + a = 0$. Then,

$a\,\alpha^2 + b\,\alpha + c = 0$ and $c\,\alpha^2 + b\,\alpha + a = 0$

$\Rightarrow (a - c)\alpha^2 + (c - a) = 0 \quad$ [On subtraction]

$\Rightarrow \alpha^2 - 1 = 0 \qquad [\because a \neq c]$

$\Rightarrow \alpha = \pm 1$

$\Rightarrow \alpha = -1 \qquad [\because \alpha < 0]$

Putting $\alpha = -1$ in $a\,\alpha^2 + b\,\alpha + c = 0$, we get

$a - b + c = 0$

977 **(a)**

We have,

$\dfrac{(1+i)^{2n} - (1-i)^{2n}}{(1 + \omega^4 - \omega^2)(1 - \omega^4 + \omega^2)}$

$= \dfrac{\{(1+i)^2\}^{\mathrm{n}} - \{(1-i)^2\}^{\mathrm{n}}}{(1 + \omega^4 - \omega^2)(1 - \omega^4 + \omega^2)}$

$= \dfrac{(2i)^n - (-2i)^n}{(1 + \omega - \omega^2)(1 - \omega + \omega^2)}$

$= \dfrac{(2i)^n - (-2i)^n}{(-2\,\omega^2)(-2\omega)}$

$= 2^{n-2}\{i^n - (-i)^n\} = \begin{cases} 0 & \text{, if } n \text{ is even} \\ 2^{n-1}\, i^n, & \text{if } n \text{ is odd} \end{cases}$

978 **(c)**

Let $\alpha, -\alpha$ and β be the roots of $x^3 - mx^2 + 3x - 2 = 0$. Then,

$\alpha + (-\alpha) + \beta = m \Rightarrow \beta = m$

But, $\beta = m$ is a root of $x^3 - mx^2 + 3x - 2 = 0$

$\therefore m^3 - m^3 + 3m - 2 = 0 \Rightarrow m = \dfrac{2}{3}$

979 **(c)**

Given, $\frac{x+1}{(2x-1)(3x+1)} = \frac{A}{(2x-1)} + \frac{B}{(3x+1)}$

$\Rightarrow \quad (x + 1) = A(3x + 1) + B(2x - 1)$

$\Rightarrow \quad (x + 1) = x(3A + 2B) + A - B$

On equating the coefficient of x and constant on both sides, we get

$3A + 2B = 1 \qquad$ ...(i)

And $\quad A - B = 1 \qquad$ ...(ii)

On solving Eqs. (i) and (ii), we get

$A = \dfrac{3}{5}, \quad B = -\dfrac{2}{5}$

$\therefore 16A + 9B = 16\left(\dfrac{3}{5}\right) + 9\left(-\dfrac{2}{5}\right) = 6$

980 **(b)**

$x + iy = \dfrac{3}{2 + \cos\theta + i\sin\theta}$

$= \dfrac{3(2 + \cos\theta - i\sin\theta)}{(2 + \cos\theta)^2 + \sin^2\theta}$

$= \dfrac{6 + 3\cos\theta - 3i\sin\theta}{4 + \cos^2\theta + 4\cos\theta + \sin^2\theta}$

$= \left[\dfrac{6 + 3\cos\theta}{5 + 4\cos\theta}\right] + i\left[\dfrac{-3\sin\theta}{5 + 4\cos\theta}\right]$

On equating the real and imaginary parts on both sides, we get

$x = \dfrac{3(2 + \cos\theta)}{(5 + 4\cos\theta)}, \qquad y = \dfrac{-3\sin\theta}{5 + 4\cos\theta}$

$\therefore \quad x^2 + y^2 = \dfrac{9}{(5 + 4\cos\theta)^2}[4 + \cos^2\theta + 4\cos\theta + \sin^2\theta]$

$= \dfrac{9}{5 + 4\cos\theta} = 4\left[\dfrac{6 + 3\cos\theta}{5 + 4\cos\theta}\right] - 3$

$= 4x - 3$

981 **(b)**

We have,

$|z_1| = |z_2| = \cdots = |z_n| = 1$

$\Rightarrow z_1\bar{z}_1 = z_2\bar{z}_2 = \cdots = z_n\bar{z}_n = 1$

$\Rightarrow \bar{z}_1 = \dfrac{1}{z_1}, \bar{z}_2 = \dfrac{1}{z_2}, \ldots, \bar{z}_n = \dfrac{1}{z_n}$

Now,

$|z_1 + z_2 + \cdots + z_n| = |\overline{z_1 + z_2 + \cdots + z_n}|$

$\Rightarrow |z_1 + z_2 + \cdots + z_n| = |\bar{z}_1 + \bar{z}_2 + \cdots + \bar{z}_n|$

$\Rightarrow |z_1 + z_2 + \cdots + z_n| = \left|\dfrac{1}{z_1} + \dfrac{1}{z_2} + \cdots + \dfrac{1}{z_n}\right|$

982 **(d)**

Let $f(x) = x^3 + ax^2 + b$. If $f(x) = 0$ will have a root of order 2, then $f(x) = 0$ and $f'(x) = 0$ have a common root

We have, $f'(x) = 3x^2 + 2ax$

$\therefore f'(x) = 0 \Rightarrow x = 0, x = -\dfrac{2a}{3}$

Clearly, $x = 0$ is not a root of $f(x) = 0$. Therefore,

$x=-\frac{2a}{3}$ is a common root

Putting $x=-\frac{2a}{3}$ in $x^3+ax^2+b=0$, we get

$\left(-\frac{2a}{3}\right)^3+a\left(-\frac{2a}{3}\right)^2+b=0$

$\Rightarrow -8a^3+12a^3+27\,b=0 \Rightarrow 4a^3+27b=0$

983 **(b)**

Given equation is $2x^3-x^2-22x-24=0$

On putting $x=2,-2$ only $x=-2$ satisfies this equation

So, $x=-2$ is a root of this equation and from the given options only (b) has this root

984 **(a)**

Let $z_1=a+ib=(a,b)$

and $z_2=c-id=(c,-d)$

where $a>0$ and $d>0$

Given, $|z_1|=|z_2|$

$\Rightarrow a^2+b^2=c^2+d^2$ ...(i)

Now, $\frac{z_1+z_2}{z_1-z_2}=\frac{(a+ib)+(c-id)}{(a+ib)-(c-id)}$

$=\frac{(a+c)+i(b-d)}{(a-c)+i(b+d)}$

$=\frac{[(a+c)+i(b-d)]}{[(a-c)+i(b+d)]}\frac{[(a-c)-i(b+d)]}{[(a-c)-i(b+d)]}$

$=\frac{(a^2+b^2)-(c^2+d^2)-2(ad+bc)i}{a^2+c^2-2ac+b^2+d^2+2bd}$

$=\frac{-(ad+bc)i}{a^2+b^2-ac+bd}$ [from Eq. (i)]

$\therefore \frac{(z_1+z_2)}{(z_1-z_2)}$ is purely imaginary

Alternative , Assume any two complex number satisfying both conditions, $z_1\neq z_2$ and $|z_1|=|z_2|$

Let $z_1=2+i, z_2=1-2i$,

$\therefore \frac{z_1+z_2}{z_1-z_2}=\frac{3-i}{1+3i}\times\frac{1-3i}{1-3i}=-\frac{10i}{10}=-i$

$\therefore$ It is purely imaginary

985 **(b)**

The roots of the equation $ax^2+bx+c=0$ are given by

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

(i) Let $b^2-4ac>0, b>0$

Now, if $a>0, c>0, b^2-4ac<b^2$

$\Rightarrow$ The roots are negative.

(ii) Let $b^2-4ac<0$, then the roots are given by

$x=\frac{-b\pm i\sqrt{(4ac-b^2)}}{2a}$ $(i=\sqrt{-1})$

Which are imaginary and have negative part.

$(\because b>0)$

$\therefore$ In each case the root have negative real part.

986 **(c)**

Since, the value of function at different points are

$f(-2)<0, f(-1)>0, f(0)>0, f(1)<0, f(2)>0$

Hence, one root lie in $(-2,1)$.

$\therefore$ 2nd root lie in (0, 1) and last root lie in (1, 2).

$\therefore [\alpha]=-2, [\beta]=0, [\gamma]=1$

$\therefore [\alpha]+[\beta]+[\gamma]=-1$

987 **(d)**

Given, $\arg(x-a+iy)=\frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{y}{x-a}\right)=\frac{\pi}{4}$

$\Rightarrow \frac{y}{x-a}=\tan\frac{\pi}{4}$

$\Rightarrow y=x-a$

Which is an equation of straight line.

988 **(a)**

The given equation $z^3+2z^2+2z+1=0$ can be rewritten as $(z+1)(z^2+z+1)=0$. Its roots $-1, \omega$ and ω^2

Let $f(z)=z^{1985}+z^{100}+1$

Put $z=-1, \omega$ and ω^2 respectively, we have

$f(-1)=(-1)^{1985}+(-1)^{100}+1\neq 0$

Therefore, -1 is not a root of the equation $f(z)=0$

Again, $f(\omega)=\omega^{1985}+\omega^{100}+1$

$=(\omega^3)^{661}\omega^2+(\omega^3)^{33}\omega+1$

$=\omega^2+\omega+1=0$

Therefore, ω is a root of the equation $f(z)$=0

Similarly, $f(\omega^2)=0$

Hence, ω and ω^2 are the common roots

989 **(a)**

We know that,

$||z_1|-|z_2||\leq|z_1+z_2|\leq|z_1|+|z_2|$

So, greatest and least value of z_1+z_2, where$z_1=24+7i$ and $|z_2|=6$ are 31 and 9 respectively

990 **(b)**

Here, $\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$

Sum of the given roots= $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=-\frac{b}{c}$

And product of the given roots= $\frac{1}{\alpha}.\frac{1}{\beta}=\frac{a}{c}$

$\therefore$Required equation is

x^2 −(sum of roots)x +product of roots=0

$\Rightarrow x^2+\frac{b}{c}x+\frac{a}{c}=0$

$\Rightarrow cx^2+bx+a=0$

991 **(d)**

$\log_2\log_2\log_4 256+2\log_{\sqrt{2}}2$

$=\log_2\log_2\log_4(4)^4+2\frac{1}{\log_2\sqrt{2}}$

$= \log_2 \log_2 4 + \dfrac{4}{\log_2 2}$

$= \log_2 2 + 4 = 1 + 4 = 5$

992 **(d)**

For collinear points $\begin{vmatrix} z_1 & \bar{z_1} & 1 \\ z_2 & \bar{z_2} & 1 \\ z_3 & \bar{z_3} & 1 \end{vmatrix} = 0$

$\therefore \begin{vmatrix} 1+2i & 1-2i & 1 \\ 2+3i & 2-3i & 1 \\ 3+4i & 3-4i & 1 \end{vmatrix} = \begin{vmatrix} 4i & 1-2i & 1 \\ 6i & 2-3i & 1 \\ 8i & 3-4i & 1 \end{vmatrix}$ $[C_1 \rightarrow C_1 - C_2]$

$= \begin{vmatrix} -2i & -1+i & 0 \\ -2i & -1+i & 0 \\ 8i & 3-4i & 1 \end{vmatrix} = 0$

$[R_1 \rightarrow R_1 - R_2, \ R_2 \rightarrow R_2 - R_3]$

993 **(d)**

Discriminant $(D) = \left(-2\sqrt{3}\right)^2 + 88$

$= 100$

$= 10^2$

$\Rightarrow$ Roots are real, rational and unequal

994 **(a)**

Here, $\alpha + \beta + \gamma = \frac{2}{1} = 2, \alpha\beta + \beta\gamma + \gamma\alpha = 3$

And $\alpha\beta\gamma = 4$

We know that

$\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$

$= (\alpha\beta + \beta\gamma + \gamma\alpha)^2 - 2(\alpha\beta\gamma)(\alpha + \beta + \gamma)$

$= (3)^2 - 2(4)(2) = -7$

995 **(d)**

We have,

$\left|\dfrac{k - z_1\bar{z}_2}{z_1 - kz_2}\right| = 1$

$\Rightarrow |k - z_1\bar{z}_2| = |z_1 - kz_2|$

$\Rightarrow |k - z_1\bar{z}_2|^2 = |z_1 - kz_2|^2$

$\Rightarrow k^2 + |z_1\bar{z}_2|^2 - kz_1\bar{z}_2 - k\overline{z_1\bar{z}_2}$

$= |z_1|^2 + k^2|z_2|^2 - k\, z_1\, \bar{z}_2 - k\bar{z}_1\, z_2$

$\Rightarrow k^2 + |z_1|^2|z_2|^2 = |z_1|^2 + k^2|z_2|^2$

$\Rightarrow k^2(|z_2|^2 - 1) - |z_1|^2(|z_2|^2 - 1) = 0$

$\Rightarrow (k^2 - |z_1|^2)(|z_2|^2 - 1) = 0 \Rightarrow |z_2|^2 = 1 \Rightarrow |z_2| = 1$

996 **(d)**

Let $z = \frac{1}{i-1}$

Then, $\bar{z} = \overline{\left(\frac{1}{i-1}\right)} = \frac{1}{-i-1} = -\frac{1}{i+1}$

997 **(c)**

Since, α and β are the roots of $ax^2 + bx + c = 0$.

Then, $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

Let the roots of $cx^2 + bx + a = 0$ be α', β', then

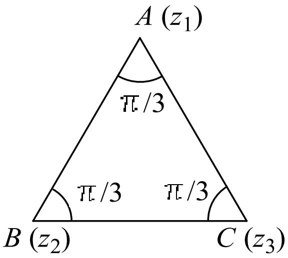
$\alpha' + \beta' = -\dfrac{b}{c}$ and $\alpha'\beta' = \dfrac{a}{c}$

Now, $\frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{-b}{a}}{\frac{c}{a}} = \frac{-b}{c}$

$\Rightarrow \dfrac{1}{\alpha} + \dfrac{1}{\beta} = \alpha' + \beta'$

Hence, $\alpha' = \frac{1}{\alpha}$ and $\beta' = \frac{1}{\beta}$

998 **(b)**



$\therefore \ \overline{AC} = \overline{AB}\, e^{i\pi/3}$

By rotating $\frac{\pi}{3}$ in clockwise sense

$\Rightarrow (z_3 - z_1) = (z_2 - z_1)e^{i\pi/3}$ ...(i)

Also, $(z_1 - z_2) = (z_3 - z_2)e^{i\pi/3}$ ...(ii)

On dividing Eq.(i) by Eq. (ii), we get

$\dfrac{(z_3 - z_1)}{(z_1 - z_2)} = \dfrac{(z_2 - z_1)}{(z_3 - z_2)}$

$\Rightarrow z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$

999 **(a)**

Let $x - \alpha$ be the common factor of $x^2 - 11x + a = 0$ and $x^2 - 14x + 2a = 0$. Then,

$\alpha^2 - 11\alpha + a = 0$ ... (i)

and, $\alpha^2 - 14\alpha + 2a = 0$ ... (ii)

Subtracting (ii) from (i), we get

$3\alpha - a = 0 \Rightarrow \alpha = a/3$

Putting $\alpha = a/3$ in (i), we get $a = 0{,}24$

1001 **(c)**

Let O, G and C be the orthocenter, centroid and circumcentre respectively, then

$\dfrac{z_1 + z_2 + z_3}{3} = \dfrac{2 \times 0 + 1(z)}{3}$

$\Rightarrow \quad z = z_1 + z_2 + z_3$

1002 **(a)**

Let $f(x) = ax^2 + 2bx - 3c$ and $f(x) = 0$ has non-real roots, $f(x)$ will have the same sign for all values of x.

Also, $\frac{3c}{4} < (a + b) \Rightarrow 4a + 4b - 3c < 0$

$\Rightarrow f(2) > 0$

$\Rightarrow f(0) > 0$

$\Rightarrow c < 0$

1003 **(d)**

$\omega^2(1 + \omega)^3 - (1 + \omega^2)\omega$

$= \omega^2(-\omega^2)^3 - (-\omega)\omega$ $[\because \ 1 + \omega + \omega^2 = 0]$

$= -\omega^2(\omega^3)^2 + \omega^2 = 0$ $[\because \ \omega^3 = 1]$

1004 **(d)**

Let D_1 and D_2 be discriminants of $x^2 + b_1x + c_1 = 0$ and $x^2 + b_2x + c_2 = 0$ respectively. Then

$D_1 + D_2 = b_1^2 - 4c_1 + b_2^2 - 4c_2$

$= (b_1^2 + b_2^2) - 4(c_1 + c_2)$

$= b_1^2 + b_2^2 - 2b_1 b_2 \quad [\because b_1 b_2 = 2(c_1 + c_2), \text{given}]$

$= (b_1 - b_2)^2 \geq 0$

$\Rightarrow D_1 \geq 0$ or $D_2 \geq 0$

$\Rightarrow D_1$ and D_2 both are positive.

100 **(d)**

5 $\dfrac{(1+i)^2}{i(2i-1)} = \dfrac{2i}{i(2i-1)} = \dfrac{2(2i+1)}{4i^2-1} - \dfrac{4}{5}i - \dfrac{2}{5}$

$\therefore$ Imaginary part is $-\frac{4}{5}$

100 **(b)**

6 $\dfrac{3x^3 - 8x^2 + 10}{(x-1)^4}$

$$= \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{D}{(x-1)^4}$$

$\Rightarrow \quad 3x^3 - 8x^2 + 10$

$$= A(x-1)^3 + B(x-1)^2 + C(x-1) + D$$

Equating coefficient of different powers of $x, 3 = A$

$-8 = -3A + B \quad \Rightarrow \quad B = 1$

$0 = 3A - 2B + C \quad \Rightarrow \quad C = -7$

$10 = -A + B - C + D \quad \Rightarrow \quad D = 5$

$\therefore$ Given expression

$$= \frac{3}{x-1} + \frac{1}{(x-1)^2} - \frac{7}{(x-1)^3} + \frac{5}{(x-1)^4}$$

100 **(c)**

7 $|z - i\text{Re}(z)| = |z - \text{Im}(z)|$

If $z = x + iy$

Then $|x + iy - ix| = |x + iy - y|$

$\sqrt{x^2 + (y-x)^2} = \sqrt{(x-y)^2 + y^2}$

or $x^2 = y^2$

$\therefore \quad x = \pm y$

$\Rightarrow \quad \text{Re}(z) = \pm \text{Im}(z)$

$\Rightarrow \quad \text{Re}(z) + \text{Im}(z) = 0$

and $\text{Re}\,(z) - \text{Im}(z) = 0$

100 **(d)**

8 It is given that $\alpha = -1 + i$ is a root of $x^2 + (1-3i)x - 2(1+i) = 0$. Let β be the orther root. Then,

$\alpha + \beta = -(1-3i) \Rightarrow \beta = -1 + 3i + 1 - i = 2i$

100 **(c)**

9 Let $z - 1 = re^{i\alpha}$

$\therefore \quad (x-1) + iy = r(\cos\alpha + i\sin\alpha)$

$\therefore \quad r^2 = (x-1)^2 + y^2$

and $\tan\alpha = \frac{y}{x-1}$

The expression

$\dfrac{z-1}{e^{i\theta}} + \dfrac{e^{i\theta}}{z-1} = re^{i(\alpha-\theta)} + \dfrac{1}{r}e^{-i(\alpha-\theta)}$

Which is given as real

$\therefore \quad r\sin(\alpha - \theta) - \dfrac{1}{r}\sin(\alpha - \theta) = 0$

$\Rightarrow \quad r - \dfrac{1}{r} = 0 \quad \Rightarrow \quad r^2 = 1$

$\Rightarrow \quad (x-1)^2 + y^2 = 1$

Which is a circle with centre (1, 0) and radius 1

101 **(a)**

0 Since, $\left|\frac{x+iy-1}{x+iy+1}\right| = 1$

$\Rightarrow \quad \sqrt{(x-1)^2 + y^2} = \sqrt{(x+1)^2 + y^2}$

$\Rightarrow \quad x^2 - 2x + 1 + y^2 = x^2 + 1 + 2x + y^2$

$\Rightarrow \quad x = 0$

101 **(b)**

1 Put $\frac{6-x+8-x}{2} = y \Rightarrow x = 7 - y$

$(y-1)^4 + (y+1)^4 = 16$

$\Rightarrow y^4 + 6y^2 + 1 = 8$

$\Rightarrow y^4 + 6y^2 - 7 = 0$

$\Rightarrow y^2 = 1, -7$

$\Rightarrow y^2 = 1 \quad (\because y^2 = -7$ is not possible$)$

$\Rightarrow y = \pm 1$

$\Rightarrow x = 6, 8$

$\therefore$ Total number of real roots are 2.

101 **(c)**

2 We have,

$\left|\dfrac{x^2+6}{5x}\right| \geq 1$

$\Rightarrow \dfrac{x^2+6}{5x} \leq -1$ or, $\dfrac{x^2+6}{5x} \geq 1$

$\Rightarrow \dfrac{x^2+5x+6}{5x} \leq 0$ or, $\dfrac{x^2-5x+6}{5x} \geq 0$

$\Rightarrow \dfrac{(x+2)(x+3)}{(x-0)} \leq 0$ or, $\dfrac{(x-2)(x-3)}{x-0} \geq 0$

$\Rightarrow x \in (-\infty, -3] \cup [-2, 0)$ or, $x \in (0, 2] \cup [3, \infty)$

$\Rightarrow x \in (-\infty, -3] \cup [-2, 0) \cup (0, 2] \cup [3, \infty)$

101 **(a)**

3 Clearly, P and Q are on the opposite side of the origin O such that $OP = OQ$. Therefore,

$OP = OQ$ and $\overrightarrow{OQ} = \overrightarrow{OP}\,e^{i\pi}$

$\Rightarrow |a + ib| = |c + id|$ and $c + id = e^{i\pi}(a + ib)$

$\Rightarrow |a + ib| = |c + id|$ and $c = -a, d = -b$

$\Rightarrow |a + ib| = |c + id|$ and $a + c = 0, b + d = 0$

$\Rightarrow |a+ib| = |c+id|$ and $a+c=b+d$

101 **(d)**

4 Here, $\alpha+\beta = -\frac{b}{a}$, $\alpha\beta = \frac{c}{a}$

The required equation is

$x^2 - 5x(\alpha+\beta) + (2\alpha+3\beta)(3\alpha+2\beta) = 0$

$\Rightarrow \quad x^2 + \frac{5b}{a}x[6(\alpha+\beta)^2] + [3\alpha\beta] = 0$

$\Rightarrow \; x^2 + \frac{5b}{a}x + \left[6\frac{b^2}{a^2} + \frac{c}{a}\right] = 0$

$\Rightarrow \; a^2x^2 + 5abx + 6b^2 + ac = 0$

101 **(a)**

5 $\frac{1+i\sqrt{3}}{1-i\sqrt{3}} = \frac{1+i\sqrt{3}}{1-i\sqrt{3}} \times \frac{1+i\sqrt{3}}{1+i\sqrt{3}}$

$= \frac{(1+i\sqrt{3})^2}{1+3} = -\frac{1}{2} + \frac{i\sqrt{3}}{2}$

$\therefore \; \tan\theta = \frac{\sqrt{3}}{2} \times \frac{2}{-1} = -\sqrt{3} = -\tan\frac{\pi}{3}$

$\Rightarrow \quad \tan\theta = \tan\left(\pi - \frac{\pi}{3}\right)$

$\Rightarrow \quad \theta = \frac{2\pi}{3}$

101 **(b)**

6 We have, $|1-i|^x = 2^x \Rightarrow (\sqrt{2})^x = 2^x$

$\Rightarrow \; 2^{x/2} = 2^x$

$\Rightarrow \; \frac{x}{2} = x \; \Rightarrow \; x = 0$

Therefore, the number of non-zero integral solution is one

101 **(d)**

7 $\frac{1+i}{1-i} = \frac{(1+i)^2}{1-i^2} = i$

Since, $\left(\frac{1+i}{1-i}\right)^n = 1 \quad \Rightarrow \quad i^n = 1$

Hence, smallest positive integer is 4

101 **(d)**

8 $\frac{x^2+x+1}{x^2+2x+1} = 1 - \frac{x}{x^2+2x+1}$ [on dividing] ...(i)

Now, $\frac{x}{x^2+2x+1} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2}$

$\Rightarrow \quad x = A(x+1) + B$

On equating the coefficient of x and constant, we get

$A = 1$ and $A + B = 0$

$\Rightarrow \quad A = 1$ and $B = -1$

From Eq. (i), we get

$\frac{x^2+x+1}{x^2+2x+1} = 1 - \frac{1}{(x+1)} + \frac{1}{(x+1)^2}$

$\Rightarrow \quad A + \frac{B}{(x+1)} + \frac{C}{(x+1)^2} = 1 - \frac{1}{(x+1)} + \frac{1}{(x+1)^2}$ [given]

$\Rightarrow \; A = 1, \; B = -1$ and $C = 1$

Now, $A - B = 1 + 1 = 2 = 2C$

101 **(a)**

9 We have,

$\left|x + \frac{1}{x}\right| < 4$

$\Rightarrow -4 < x + \frac{1}{x} < 4$

$\Rightarrow x + \frac{1}{x} + 4 > 0$ and $x + \frac{1}{x} - 4 < 0$

$\Rightarrow \frac{x^2+4x+1}{x} > 0$ and $\frac{x^2-4x+1}{x} < 0$

$\Rightarrow \frac{(x+2+\sqrt{3})(x+2-\sqrt{3})}{x-0} > 0$

and, $\frac{(x-2-\sqrt{3})(x-2+\sqrt{3})}{x-0} < 0$

$\Rightarrow x \in (-2-\sqrt{3}, -2+\sqrt{3}) \cup (0, \infty)$

and, $x \in (-\infty, 0) \cup (2-\sqrt{3}, 2+\sqrt{3})$

$\Rightarrow x \in (2-\sqrt{3}, 2+\sqrt{3}) \cup (-2-\sqrt{3}, -2+\sqrt{3})$

102 **(d)**

0 If α and β are roots of the equation $x^2 + 6x - 2 = 0$

Then,

$\alpha + \beta = -6 \; \Rightarrow \beta = -6 - \alpha$

Since α is a root of $x^2 + 6x - 2 = 0$

$\therefore \alpha^2 + 6\alpha - 2 = 0$

Now,

$\beta = -6 - \alpha$

$\Rightarrow \beta = -6 - \alpha + 0$

$\Rightarrow \beta = -6 - \alpha + \alpha^2 + 6\,\alpha - 2 \quad [\because \alpha^2 + 6\alpha - 2 = 0]$

$\Rightarrow \beta = \alpha^2 + 5\,\alpha - 8$

Now,

$\alpha\,\beta = -2$

$\Rightarrow \beta = \frac{-2}{\alpha}$

$\Rightarrow \beta = \frac{-2 + 2(\alpha^2 + 6\,\alpha - 2)}{\alpha} \quad [\because \alpha^2 + 6\,\alpha - 2 = 0]$

$\Rightarrow \beta = \frac{2\,\alpha^2 + 12\,\alpha - 6}{\alpha}$

Now,

$\alpha + \beta = -6$ and, $\alpha\beta = -2$

$\Rightarrow \frac{\alpha+\beta}{\alpha\,\beta} = 3 \Rightarrow \frac{1}{\alpha} + \frac{1}{\beta} = 3 \Rightarrow \beta = \frac{\alpha}{3\,\alpha - 1}$

102 **(a)**

1 $\omega + \omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\frac{27}{128}+\ldots\right)}$

$= \omega + \omega^{\left(\frac{1/2}{1-3/4}\right)}$

$= \omega + \omega^2 = -1 \quad (\because 1 + \omega + \omega^2 = 0)$

102 **(a)**

2 We have,

$\left|\frac{2x-1}{x-1}\right| > 2$

$\Rightarrow \dfrac{2x-1}{x-1} < -2$ or, $\dfrac{2x-1}{x-1} > 2$

$\Rightarrow \dfrac{4x-3}{x-1} < 0$ or, $\dfrac{1}{x-1} > 0$

$\Rightarrow \dfrac{4x-3}{x-1} < 0$ or, $x-1 > 0$

$\Rightarrow 3/4 < x < 1$ or, $x > 1 \Rightarrow x \in (3/4, 1) \cup (1, \infty)$

102 **(a)**

3 If one root is $2-i$, then the other root will be $2+i$

Given equation is $ax^2 + 12x + b = 0$

$\therefore 2 - i + 2 + i = \dfrac{-12}{a}$

$\Rightarrow a = -3$

And $(2-i)(2+i) = \frac{b}{a}$

$\Rightarrow 5 = \dfrac{b}{-3} \Rightarrow b = -15$

$\therefore ab = -3 \times (-15) = 45$

102 **(d)**

4 Since, $f(1) + f(2) + f(3) = 0$

$f(1), f(2), f(3)$ all cannot be of same sign.

$\Rightarrow$ Roots are real and distinct.

102 **(d)**

5 We have, $t^2x^2 + |x| + 9 > 0$ for all $x \in R$

So, given equation has no real root

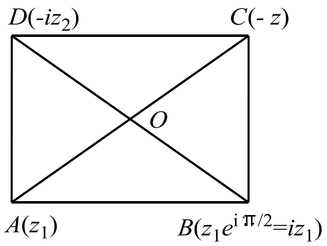
102 **(d)**

6 Affix of A is z_1 means that $\overrightarrow{OA} = z_1$ and $\overrightarrow{OB}$ and $\overrightarrow{OC}$ are obtained by rotating $\overrightarrow{OA}$ through $\frac{\pi}{2}$ and π.

Therefore, affixes of B and C are $i\,z_1$ and $-z_1$ respectively. Hence, the affix of the centroid of triangle ABC is

$\dfrac{z_1 + i\,z_1 + (-z_1)}{3} = \dfrac{i}{3}z_1 = \dfrac{1}{3}z_1\left(\cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}\right)$

If A, B, C are taken in clockwise sense, then the affix of the centroid is $\frac{1}{3}z_1\left(\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}\right)$



Thus, the affix of the centroid is $\frac{1}{3}z_1\left(\cos\frac{\pi}{2} \pm\right.$

$i\sin\pi 2$

102 **(b)**

7 Let $z = 4 + i$ when reflected along $y = x$ will become $z = 1 + 4i$

When translated by 2 unit $z = 3 + 4i$

When rotated by angle $\pi/4$ in anti-clockwise direction will give

$z = (3+4i)\left(\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}\right)$

$z = \dfrac{1}{\sqrt{2}}[3 - 4 + i(3+4)] = -\dfrac{1}{\sqrt{2}} + i\dfrac{7}{\sqrt{2}}$

102 **(b)**

8 Since c and d are roots of the equation

$(x-a)(x-b) - k = 0$

$\therefore (x-a)(x-b) - k = (x-c)(x-d)$

$\Rightarrow (x-c)(x-d) + k = (x-a)(x-b)$

Clearly, a, b are the roots of $(x-a)(x-b) = 0$ and $(x-a)(x-b) = (x-c)(x-d) + k$

$\therefore a, b$ are roots of $(x-c)(x-d) + k = 0$

102 **(c)**

9 We have,

$|x|^3 - 3\,x^2 + 3\,|x| - 2 = 0$

$\Rightarrow |x|^3 - 3|x|^2 + 3|x| - 2 = 0$

$\Rightarrow (|x| - 2)(|x|^2 - |x| + 1) = 0$

$\Rightarrow |x| = 2, |x|^2 - |x| + 1 = 0$

$\Rightarrow x = \pm\,2$ [$\because |x|^2 - |x| + 1 = 0$ has imaginary roots]

Thus, the given equation has two real roots

103 **(d)**

0 Let $z = x + i\,y$. Then,

$z^2 + |z|^2 = 0$

$\Rightarrow x^2 - y^2 + 2\,ixy + x^2 + y^2 = 0$

$\Rightarrow 2x^2 + 2ixy = 0$

$\Rightarrow x^2 = 0, 2xy = 0 \Rightarrow x = 0, y \in R$

Hence, there are infinitely many solution

103 **(d)**

1 Discriminant of the equation $3x^2 + 8x + 15 = 0$ is given by

$D = 64 - 180 = -116 < 0$

So, its roots are imaginary and therefore roots are conjugate to each other. Therefore, one common root means both the roots are common.

$\therefore \dfrac{a}{3} = \dfrac{2b}{8} = \dfrac{3c}{15}$

$\Rightarrow \frac{a}{3} = \frac{b}{4} = \frac{c}{5} = k$ (say), $k \neq 0$

$\Rightarrow a = 3k, b = 4k, c = 5k$

Now, $a^2 + b^2 = c^2$

$\Rightarrow \Delta ABC$ is right angled.

$\therefore \sin^2 A + \sin^2 B = \sin^2 C$

$\Rightarrow \sin^2 A + \sin^2 B$

$+ \sin^2 C = 2\sin^2 C$

$= 2\sin^2 90° = 2$

103 **(d)**

2 We have,

$(1 - \omega + \omega^2)^6 + (1 - \omega^2 + \omega)^6$

$= (-2\,\omega)^6 + (-2\,\omega^2)^6 = 2^6 + 2^6 = 2^7 = 128$

103 **(a)**

3 We have, $\tan^{-1}(\alpha + i\beta) = x + iy$

$\Rightarrow\ \alpha + i\beta = \tan(x + iy)$ ...(i)

Taking conjugate,

$(\alpha + i\beta = \tan(x + iy)$ ...(ii)

$\therefore\ \tan 2x = \tan[(x + iy) + (x - iy)]$

$\Rightarrow\ \tan 2x = \dfrac{(\alpha + i\beta) + (\alpha - i\beta)}{1 - (\alpha + i\beta) + (\alpha - i\beta)}$

$= \dfrac{2\alpha}{1 - (\alpha^2 + \beta^2)}$

$\therefore\ x = \dfrac{1}{2}\tan^{-1}\left(\dfrac{2\alpha}{1 - \alpha^2 - \beta^2}\right)$

103 **(d)**

4 Given system of equation is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ ...(i)

$\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ ...(ii)

and $-\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ ...(iii)

On adding all these equations, we get

$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 3$ ...(iv)

On subtracting Eq. (i) from Eq. (iv), Eq. (ii) from Eq. (iv) and Eq. (iii) from Eq. (iv),we get

$\frac{2z^2}{c^2} = 2, \frac{2y^2}{b^2}=2,\ \frac{2x^2}{a^2} = 2$

$\Rightarrow z = \pm c, y = \pm b, x = \pm a$

103 **(d)**

5 Given, $\log_2[\log_3\{\log_4(\log_5 x)\}] = 0$

$\Rightarrow \log_3\{\log_4(\log_5 x)\} = 2^0 = 1$

$\Rightarrow \log_4(\log_5 x) = 3$

$\Rightarrow \log_5 x = 4^3 = 64$

$\Rightarrow x = 5^{64}$

103 **(c)**

6 As, $1, a_1, a_2, \dots, a_{n-1}$ are nth roots unity

$\Rightarrow\ (x^n - 1) = (x - 1)(x - a_1)(x - a_2) \dots (x - a_{n-1})$

$\Rightarrow\ \dfrac{x^n - 1}{x - 1} = (x - a_1)(x - a_2) \dots (x - a_{n-1})$

$\therefore\ x^{n-1} = x^{n-2} + \dots x^2 + x + 1$

$= (x - a_1)(x - a_2) \dots (x - a_{n-1})$

$\left[as\ \dfrac{x^n - 1}{x - 1} = x^{n-1} + x^{n-2} + \dots + x + 1\right]$

Putting $x = 1$, we get

$1 + 1 + \cdots n$ times $= (1 - a_1)(1 - a_2) \dots (1 - an{-}1$

$\Rightarrow\ (1 - a_1)(1 - a_2) \dots (1 - a_{n-1}) = n$

103 **(d)**

7 $\bar{z} = \dfrac{4}{1 + i}$

103 **(b)**

8 $(q - r)x^2 + (r - p)x + (p - q) = 0$

$\Rightarrow\ (q - r)x^2 + (r - q + q - p)x + (p - q) = 0$

$\Rightarrow (q - r)x^2 - (q - r)x - (p - q)x + (p - q) = 0$

$\Rightarrow (q - r)x(x - 1) - (p - q)(x - 1) = 0$

$\Rightarrow (x - 1)\{(q - r)x - (p - q)\} = 0$

$\Rightarrow x = 1, \dfrac{p - q}{q - r}$

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