Single Correct Answer Type

1. In $\Delta\, ABC, a^2(\cos^2 B - \cos^2 C) + b^2(\cos^2 C - \cos^2 A) + c^2\,(\cos^2 A - \cos^2 B)$ is equal to
   a) 0 b) 1 c) $a^2 + b^2 + c^2$ d) $2(a^2 + b^2 + c^2)$
2. If $\sin A : \sin B : \sin C = 3:4:5$, then $\cos A : \cos B$ is equal to
   a) $4:3$ b) $5:3$ c) $3:4$ d) $3:5$
3. If A, B, C are the angles of a triangle, then $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2}$ is equal to
   a) $\frac{s}{R}$ b) $\frac{R}{s}$ c) $\frac{\Delta}{s^2}$ d) $\frac{s^2}{\Delta}$
4. Coordinates of the foot of the perpendicular drawn from $(0, 0)$ to the line joining $(a\cos\alpha\,, a\sin\alpha)$ and $(a\cos\beta,\ a\sin\beta)$ are
   a) $\left(\frac{a}{2}, \frac{b}{2}\right)$ b) $\left(\frac{a}{2}(\cos\alpha + \cos\beta), \frac{a}{2}(\sin\alpha + \sin\beta)\right)$
   c) $\left(\cos\frac{\alpha+\beta}{2}, \sin\frac{\alpha+\beta}{2}\right)$ d) $\left(0, \frac{b}{2}\right)$
5. Three points are $A(6, 3), B(-3, 5), C(4, -2)$ and $P(x, y)$ is a point, then the ratio of area of ΔPBC and ΔABC is
   a) $\left|\frac{x+y-2}{7}\right|$ b) $\left|\frac{x-y+2}{2}\right|$ c) $\left|\frac{x-y-2}{7}\right|$ d) None of these
6. Two vertical poles 20 m and 80 m stands apart on a horizontal plane. The height of the point of intersection of the lines joining the top of each pole to the foot of the other is
   a) 15 m b) 16 m c) 18 m d) 50 m
7. A person on a ship sailing north sees two lighthouses which are 6 km apart, in a line due west. After an hour's tailing one of them bears south west and the other southern south west. The ship is travelling at a rate of
   a) 12 km/hr b) 6 km/hr c) $3\sqrt{2}$ km/hr d) $(6 + 3\sqrt{2})$ km/hr
8. If α, β, γ are the real roots of the equation
   $x^3 - 3px^2 + 3qx - 1 = 0$,
   Then the centroid of the triangle whose vertices are
   $\left(\alpha, \frac{1}{\alpha}\right), \left(\beta, \frac{1}{\beta}\right)$ and $\left(\gamma, \frac{1}{\gamma}\right)$, is
   a) (p, q) b) (q, p) c) $(-p, q)$ d) $(q, -p)$
9. If two vertices of a triangle are $(-2, 3)$ and $(5, -1)$. Orthocentre lies at the origin and centroid on the line $x + y = 7$, then the third vertex lies at
   a) $(7, 4)$ b) $(8, 14)$ c) $(12, 21)$ d) None of these
10. What is the equation of the locus of a point which moves such that 4 times its distance from the x-axis is the square of its distance from the origin?
    a) $x^2 + y^2 - 4y = 0$ b) $x^2 + y^2 - 4|y| = 0$ c) $x^2 + y^2 - 4x = 0$ d) $x^2 + y^2 - 4|x| = 0$
11. If $a^2 + b^2 = c^2$, then $s(s-a)(s-b)(s-c)$ is equal to
    a) a^2b^2 b) $\frac{1}{4}a^2b^2$ c) $\frac{1}{2}a^2b^2$ d) $\frac{1}{2}ab$
12. The harmonic conjugate of $(4, -2)$ with respect to $(2, -4)$ and $(7,1)$ is
    a) $(-8, -14)$ b) $(2, 3)$ c) $(-2, -3)$ d) $(13, -5)$
13. If O is the origin and $P(x_1, y_1), Q(x_2, y_2)$ are two points, then $OP.\,OQ\sin\angle POQ =$
    a) $x_1x_2 + y_1y_2$ b) $x_1y_2 + x_2y_1$ c) $|x_1y_2 - x_2y_1|$ d) None of these
14. If $\Delta\, ABC$, if $a = 3, b = 4, c = 5$, then the value of $\sin 2B$ is
    a) 4/5 b) 3/20 c) 24/25 d) 1/50
15. From an aeroplane vertically over a straight horizontal road, the angles of depression of two consecutive milestones on opposite sides of the aeroplane are observed to be α and β. The height of the aeroplane

above the road is

a) $\dfrac{\tan\alpha+\tan\beta}{\tan\alpha\tan\beta}$ b) $\dfrac{\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}$ c) $\dfrac{\cot\alpha\cot\beta}{\cot\alpha+\cot\beta}$ d) None of these

16. In $\Delta\, ABC$, if $\angle A = 45°, \angle B = 75°$, then $a + c\sqrt{2}$ is equal to

a) 0 b) 1 c) b d) $2b$

17. Three vertical poles of heights h_1, h_2 and h_3 at the vertices A, B and C of a ΔABC subtend angles α, β and γ respectively at the circumcentre of the triangle. If $\cot\alpha$, $\cot\beta$ and $\cot\gamma$ are in AP, then h_1, h_2, h_3 are in

a) AP b) GP c) HP d) None of these

18. The area enclosed within the curve $|x| + |y| = 1$ is

a) 1 sq unit b) $2\sqrt{2}$ sq units c) $\sqrt{2}$ sq units d) 2 sq units

19. P is a point on the segment joining the feet of two vertical poles of height a and b.The angles of elevation of the top of the poles from P are 45° each. Then, the squre of the distance between the top of the poles is

a) $\dfrac{a^2+b^2}{2}$ b) a^2+b^2 c) $2(a^2+b^2)$ d) $4(a^2+b^2)$

20. By rotating the coordinates axes through 30° in anticlockwise sense the equation $x^2 + 2\sqrt{3}\,xy - y^2 = 2a^2$ changes to

a) $X^2 - Y^2 = 3a^2$ b) $X^2 - Y^2 = a^2$ c) $X^2 - Y^2 = 2a^2$ d) None of these

21. The x-coordinate of the incentre of the triangle where the mid points of the sides are (0, 1), (1, 1)and (1, 0) is

a) $2+\sqrt{2}$ b) $1+\sqrt{2}$ c) $2-\sqrt{2}$ d) $1-\sqrt{2}$

22. Let $A(2,-3)$ and $B(-2,1)$ be vertices of a triangleABC. If the centroid of this triangle moves on the line $2x + 3y = 1$, then the locus of the vertex C is the line

a) $2x+3y=9$ b) $2x-3y=7$ c) $3x+2y=5$ d) $3x-2y=3$

23. The angle of elevation of the top of a tower at a point on the ground is 30°. If on walking 20 m toward the tower the angle of elevation becomes 60°, then the height of the tower is

a) 10 m b) $\dfrac{10}{\sqrt{3}}$ m c) $10\sqrt{3}$ m d) None of these

24. In a $\Delta\, ABC$, if $2s = a + b + c$ and $(s-b)(s-c) = x\sin^2\frac{A}{2}$, then the value of x is

a) bc b) ca c) ab d) abc

25. If p_1, p_2 denote the length of the perpendiculars from the origin on the lines $x\ \sec\alpha + y\,\mathrm{cosec}\,\alpha = 2a$ and $x\ \cos\alpha + y\ \sin\alpha = a\cos 2\alpha$ respectively, then $\left(\frac{P_1}{P_2}, \frac{P_2}{P_1}\right)^2$ is equal to

a) $4\sin^2 4\alpha$ b) $4\cos^2 4\alpha$ c) $4\,\mathrm{cosec}^2\, 4\alpha$ d) $4\sec^2 4\alpha$

26. The equation $\sqrt{(x-2)^2+(y-1)^2} + \sqrt{(x+2)^2+(y-4)^2} = 5$ represents

a) Circle b) Ellipse c) Line segment d) None of these

27. The value of $\frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} + \frac{1}{r^2}$ is

a) 0 b) $\dfrac{a^2+b^2+c^2}{\Delta^2}$ c) $\dfrac{\Delta^2}{a^2+b^2+c^2}$ d) $\dfrac{a^2+b^2+c^2}{\Delta}$

28. The sides of a triangle are 4cm, 5cm and 6cm. the area of the triangle is equal to

a) $\dfrac{15}{4}$ cm^2 b) $\dfrac{15}{4}\sqrt{7}$ cm^2 c) $\dfrac{4}{15}\sqrt{7}$ cm^2 d) None of these

29. A vertical lamp-post, 6 m high, stands at a distance of 2 m from a wall, 4 m high. A 1.5 m tall man starts to walk away from the wall on the other side of the wall, in line with the lamp-post the maximum distance to which the man can walk remaining in the shadow is

a) $\dfrac{5}{2}$ m b) $\dfrac{3}{2}$ m c) 4 m d) None of these

30. A tower subtends an angle α at a point A in the plane of its base and the angle of depression of the foot of the tower at a point b feet just above A is β. Then, the height of the tower is

a) $b\tan\alpha\cot\beta$ b) $b\cot\alpha\tan\beta$ c) $b\cot\alpha\cot\beta$ d) $b\tan\alpha\tan\beta$

31. In a ABC, if $b = 2, \angle B = 30°$, then the area of the circumcircle of $\Delta\, ABC$ in square unit is

a) π b) 2π c) 4π d) 6π

32. The base of a cliff is circular. From the extremities of a diameter of the base of angle of elevation of the top of the cliff are $30°$ and $60°$. If the height of the cliff be 500 m, then the diameter of the base of the cliff is

a) $1000\sqrt{3}$ m b) $\dfrac{2000}{\sqrt{3}}$ m c) $\dfrac{1000}{\sqrt{3}}$ m d) $\dfrac{2000}{\sqrt{2}}$ m

33. If R denotes circumradius, then in $\Delta\, ABC, \frac{b^2-c^2}{2aR}$ is equal to

a) $\cos(B-C)$ b) $\sin(B-C)$ c) $\cos B - \cos C$ d) None of these

34. The area between the curve $y = 1 - |x|$ and the x-axis is equal to

a) 1 sq unit b) $\frac{1}{2}$ sq unit c) $\frac{1}{3}$ sq unit d) 2 sq units

35. Angles A, B and C of a triangle are in AP with common difference 15 degree, then angle A is equal to

a) $45°$ b) $60°$ c) $75°$ d) $30°$

36. In a triangle $\left(1-\frac{r_1}{r_2}\right)\left(1-\frac{r_1}{r_3}\right) = 2$, then the triangle is

a) Right angled b) Equilateral c) Isosceles d) None of these

37. The angle of elevation of the sun, if the length of the shadow of a tower is $\sqrt{3}$ times the height of the pole, is

a) $150°$ b) $30°$ c) $60°$ d) $45°$

38. If the equation $2x^2 + y^2 - 4x - 4y = 0$ is transformed to the equation $2X^2 + Y^2 - 8X - 8Y + 18 = 0$ by shifting the origin at a point P without rotating the coordinates axes, then the coordinates of P are

a) $(1, 2)$ b) $(1, -2)$ c) $(-1, 2)$ d) $(-1, -2)$

39. A vertical pole PS has two marks Q and R such that the portions PQ, PR and PS subtend angles α, β, γ at a point on the ground distance x from the pole. If $PQ = a, PR = b, PS = c$ and $\alpha + \beta + \gamma = 180°$ then x^2 is equal to

a) $\dfrac{a}{a+b+c}$ b) $\dfrac{b}{a+b+c}$ c) $\dfrac{c}{a+b+c}$ d) $\dfrac{abc}{a+b+c}$

40. If in a $\Delta\, ABC, (s-a)(s-b) = s(s-c)$, then angle C is equal to

a) $90°$ b) $45°$ c) $30°$ d) $75°$

41. At a point on the ground the angle of elevation of a tower is such that its cotangent is $\frac{3}{5}$. On walking 32 m towards the tower the cotangent of the angle of elevation is $\frac{2}{5}$. The height of the tower is

a) 160 m b) 120 m c) 64 m d) None of these

42. Area of quadrilateral whose vertices are $(2, 3)$, $(3, 4)$, $(4, 5)$ and $(5, 6)$, is equal to

a) 0 b) 4 c) 6 d) None of these

43. If the area of a triangle ABC is Δ, then $a^2 \sin 2B + b^2 \sin 2A$ is equal to

a) 3Δ b) 2Δ c) 4Δ d) -4Δ

44. Consider the following statements :

1. If in a $\Delta\, ABC, \frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$, then a^2, b^2, c^2 are in AP

2. If exradius r_1, r_2 and r_3 of a $\Delta\, ABC$ are in HP, then the sides a, b, c are in AP

Which of these is/are correct?

a) Only (1) b) Only (2) c) Both (1) and (2) d) None of these

45. If the sides of the triangle are $p, q, \sqrt{p^2+q^2+pq}$, then the greatest angle is

a) $\dfrac{\pi}{2}$ b) $\dfrac{5\pi}{4}$ c) $\dfrac{2\pi}{3}$ d) $\dfrac{7\pi}{4}$

46. If x, y, z are perpendicular drawn from the vertices of triangle having sides a, b and c, then the value of $\frac{bx}{c} + \frac{cy}{a} + \frac{az}{b}$ will be

a) $\dfrac{a^2+b^2+c^2}{2R}$ b) $\dfrac{a^2+b^2+c^2}{R}$ c) $\dfrac{a^2+b^2+c^2}{4R}$ d) $\dfrac{2(a^2+b^2+c^2)}{R}$

47. A balloon is observed simultaneously from three points A, B and C on a straight road directly under it. The angular elevation at B is twice and at C is thrice that of A. If the distance between A and B is 200 m and the

distance between B and C is 100 m, then the height of balloon is given by

a) 50 m b) $50\sqrt{3}$ m c) $50\sqrt{2}$ m d) None of these

48. If the distance of any point P from the points $A(a+b, a-b)$ and $B(a-b, a+b)$ are equal, then the locus of P is

a) $x-y=0$ b) $ax+by=0$ c) $bx-ay=0$ d) $x+y=0$

49. The length of altitude through A of the $\Delta\, ABC$, where $A \equiv (-3,0), B \equiv (4,-1), C \equiv (5,2)$, is

a) $\frac{2}{\sqrt{10}}$ b) $\frac{4}{\sqrt{10}}$ c) $\frac{11}{\sqrt{10}}$ d) $\frac{22}{\sqrt{10}}$

50. Triangle ABC has vertices $(0,0)$, $(11,60)$ and $(91,0)$. If the line $y=kx$ cuts the triangle into two triangles of equal area, then k is equal to

a) $\frac{30}{51}$ b) $\frac{4}{7}$ c) $\frac{7}{4}$ d) $\frac{30}{91}$

51. A pole stands at the centre of a rectangular field and it subtends angles of 15° and 45° at the mid points of the side of the field. If the length of its diagonal is 1200 m, then the height of flag staff is

a) 400 m b) 200 m c) $300\sqrt{2+\sqrt{3}}$ m d) $300\sqrt{2-\sqrt{3}}$ m

52. What is the equation of the locus a point which moves such that 4 times its distance from the x-axis is the square of its distance from the origin?

a) $x^2-y^2-4y=0$ b) $x^2+y^2-4|y|=0$ c) $x^2+y^2-4x=0$ d) $x^2+y^2-4|x|=0$

53. A person standing on the bank of a river, observe that the angle of elevation of the top of a tree on the opposite bank of the river is 60° and when he retries 40m a way from the tree the angle of elevation become 30°. The breadth of the river is

a) 20 m b) 30 m c) 40 m d) 60 m

54. There exist a $\Delta\, ABC$ satisfying

a) $\tan A + \tan B + \tan C = 0$

b) $\frac{\sin A}{2} = \frac{\sin B}{3} = \frac{\sin C}{1}$

c) $\sin A + \sin B = -\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)\cos A \cos B = \frac{\sqrt{3}}{4} = \sin A \sin B$

d) $(a+b)^2 = c^2 + ab$ and $\sqrt{2}\,(\sin A + \cos A) = \sqrt{3}$

55. From a point a meters above a lake the angle of elevation of a cloud is α and the angle of depression of its reflection is β. The height of the cloud is

a) $\frac{a\sin(\alpha+\beta)}{\sin(\alpha+\beta)}$ m b) $\frac{a\sin(\alpha+\beta)}{\sin(\beta-\alpha)}$ m c) $\frac{a\sin(\beta-\alpha)}{\sin(\alpha+\beta)}$ d) None of these

56. The orthocentre of the triangle formed by $(0,0)$, $(8,0)$, $(4,6)$ is

a) $\left(4, \frac{8}{3}\right)$ b) $(3,4)$ c) $(4,3)$ d) $(-3,4)$

57. The x-coordinate of the incentre of the triangle where the mid point of the sides are (0, 1), (1, 1) and (1, 0), is

a) $2+\sqrt{2}$ b) $1+\sqrt{2}$ c) $2-\sqrt{2}$ d) $1-\sqrt{2}$

58. The locus of the point (x,y) which is equidistant from the points $(a+b, b-a)$ and $(a-b, a+b)$ is

a) $ax=by$ b) $ax+by=0$ c) $bx+ay=0$ d) $bx-ay=0$

59. If the sum of the distances from two perpendicular lines in a plane is 1, then its locus is

a) A square b) A circle c) A straight line d) Two intersecting lines

60. A tower of x metres high, has a flagstaff at its top. The tower and the flagstaff subtend equal angles at a point distant y metres from the foot of the tower. Then the length of the flagstaff (in meters), is

a) $\frac{y(x^2-y^2)}{(x^2+y^2)}$ b) $\frac{x(y^2+x^2)}{(y^2-x^2)}$ c) $\frac{x(x^2+y^2)}{(x^2-y^2)}$ d) $\frac{x(x^2-y^2)}{(x^2+y^2)}$

61. In a $\Delta\, ABC$, $2ac \sin\frac{A-B+C}{2}$ is equal to

a) $a^2+b^2-c^2$ b) $c^2+a^2-b^2$ c) $b^2-a^2-c^2$ d) $c^2-a^2-b^2$

62. If $P=(1,0), Q=(-1,0)$ and $R=(2,0)$ are three given points, then the locus of the point $S(x,y)$ satisfying the relation $SQ^2++SR^2=2SP^2$ is
a) A straight line parallel to x-axis b) A circle through the origin
c) A circle with centre at the origin d) A straight line parallel to y-axis

63. If orthocenter and circumcentre of a triangle are respectively (1, 1) and (3, 2), then the coordinates of its centroid are
a) $\left(\frac{7}{3},\frac{5}{3}\right)$ b) $\left(\frac{5}{3},\frac{7}{3}\right)$ c) $(7,5)$ d) None of these

64. The locus of the point of intersection of the lines $x\cot\theta+y\operatorname{cosec}\theta=2$ and $x\operatorname{cosec}\theta+y\cot\theta=6$ is
a) A straight line b) Circle c) A hyperbola d) An ellipse

65. In $\Delta\, ABC$, if $\cot A, \cot B, \cot C$ be in AP, then a^2, b^2, c^2 are in
a) HP b) GP c) AP d) None of these

66. The angels of elevation of the cloud at a point 2500 m high from the lake is 15°and the angle of depression of its reflection to the lake is 45°. Then the height of cloud from the foot of lake is
a) $2500\sqrt{3}$ m b) 2500 m c) $500\sqrt{3}$ m d) None of these

67. ABC is a triangular park with $AB=AC=$100 m. A clock tower is situated at the mid point of BC. The angle of elevation, if the top of the toper at A and B are $\cot^{-1}3.2$ and $\operatorname{cosec}^{-1}2.6$ respectively. The height of the tower is
a) 16 m b) 25 m c) 50 m d) None of these

68. In $\Delta ABC, b=\sqrt{3}, c=1$ and $\angle A=30°$, then the largest angle of the triangle is
a) 60° b) 135° c) 90° d) 120°

69. In an equilateral triangle, $R:r:r_1$ is equal to
a) 1:1:1 b) 1:2:3 c) 2:1:3 d) 3:2:4

70. In a triangle, if $r_1=2r_2=3r_3$, then $\frac{a}{b}+\frac{b}{c}+\frac{c}{a}$ is equal to
a) $\frac{75}{60}$ b) $\frac{155}{60}$ c) $\frac{176}{60}$ d) $\frac{191}{60}$

71. In a triangle$ABC, a:b:c=4:5:6$. The ratio of the radius of the circumcircle to that of the incircle is
a) $\frac{15}{4}$ b) $\frac{11}{5}$ c) $\frac{16}{7}$ d) $\frac{16}{3}$

72. An aeroplane flying with uniform speed horizontally one kilometer above the ground is observed at an elevation of 60°. After 10 s, if the elevation is observed to be 30°, then the speed of the plane (in km/h) is
a) $\frac{240}{\sqrt{3}}$ b) $200\sqrt{3}$ c) $240\sqrt{3}$ d) $\frac{120}{\sqrt{3}}$

73. The angle of elevation of the top of a tower standing on a horizontal plane from a point A is α. After walking a distance a towards the foot of the tower the angle of elevation is found to be β. The height of the tower is
a) $\frac{a\sin\alpha\sin\beta}{\sin(\beta-\alpha)}$ b) $\frac{a\sin\alpha\sin\beta}{\sin(\alpha-\beta)}$ c) $\frac{a\sin(\beta-\alpha)}{\sin\alpha\sin\beta}$ d) $\frac{a\sin(\alpha-\beta)}{\sin\alpha\sin\beta}$

74. If the vertices of a triangle have integral coordinates, the triangle cannot be
a) An equilateral triangle b) A right angled triangle
c) An isosceles triangle d) None of the above

75. In a $\Delta\, ABC$, among the following which one is true?
a) $(b+c)\cos\frac{A}{2}=a\sin\left(\frac{B+C}{2}\right)$ b) $(b+c)\cos\left(\frac{B+C}{2}\right)=a\sin\frac{A}{2}$
c) $(b-c)\cos\left(\frac{B-C}{2}\right)=a\cos\left(\frac{A}{2}\right)$ d) $(b-c)\cos\frac{A}{2}=a\sin\left(\frac{B-C}{2}\right)$

76. The upper $\left(\frac{3}{4}\right)$ th portion of a vertical pole subtends an angle $\tan^{-1}\left(\frac{3}{5}\right)$ at a point in the horizontal plane through its foot and at a distance 40 m from the foot. A possible height of the vertical pole is
a) 20 m b) 40 m c) 60 m d) 80 m

77. If C and $D$are the points of internal and external division of line segment AB in the same ratio, then AC, AB, AD are in

a) AP　　b) GP　　c) HP　　d) AGP

78. A ladder rests against a vertical wall at angle α to the horizontal. If its foot is pulled away from the wall through a distance $'a'$ so that it slides a distance $'b'$ down the wall making an angle β with the horizontal, then $a =$

a) $b\tan\left(\frac{\alpha-\beta}{2}\right)$　　b) $b\tan\left(\frac{\alpha+\beta}{2}\right)$　　c) $b\cot\left(\frac{\alpha-\beta}{2}\right)$　　d) None of these

79. The angles A, B and C of a $\Delta\, ABC$ are in A.P. If $AB = 6, BC = 7$, then $AC =$

a) 5　　b) 7　　c) 8　　d) None of these

80. The locus of a point whose difference of distance from points $(3, 0)$ and $(-3, 0)$ is 4, is

a) $\frac{x^2}{4}-\frac{y^2}{5}=1$　　b) $\frac{x^2}{5}-\frac{y^2}{4}=1$　　c) $\frac{x^2}{2}-\frac{y^2}{3}=1$　　d) $\frac{x^2}{3}-\frac{y^2}{2}=1$

81. If a $\Delta\, ABC$, if $a^4+b^4+c^4=2c^2(a^2+b^2)$, then $\angle C$ is equal to

a) 60°　　b) 135°　　c) 90°　　d) 75°

82. Given the points $A(0, 4)$ and $B(0, -4)$, then the equation of the locus of the point $P(x, y)$ such that, $|AP - BP| = 6$, is

a) $\frac{x^2}{7}+\frac{y^2}{9}=1$　　b) $\frac{x^2}{9}+\frac{y^2}{7}=1$　　c) $\frac{x^2}{7}-\frac{y^2}{9}=1$　　d) $\frac{y^2}{9}-\frac{x^2}{7}=1$

83. If in ΔABC, $\sin\frac{A}{2}\sin\frac{C}{2}=\sin\frac{B}{2}$ and $2s$ is the perimeter of the triangle, then s is

a) $2b$　　b) b　　c) $3b$　　d) $4b$

84. The angle of depression of a ship from the top of a tower 30 m high is 60°.Then the distance of ship from the base of tower is

a) 30 m　　b) $30\sqrt{3}$ m　　c) $10\sqrt{3}$ m　　d) 10 m

85. At a distance $2h$ m from the foot of a tower of height h m the top of the tower and a pole at the top of the tower subtend equal angles. Height of the pole should be

a) $\frac{5h}{3}$ m　　b) $\frac{4h}{3}$ m　　c) $\frac{7h}{5}$ m　　d) $\frac{3h}{2}$ m

86. From the tower 60 m high angles of depression of the top and bottom of a house are α and β respectively. If the height of the house is $\frac{60\sin(\beta-\alpha)}{x}$, then x is equal to

a) $\sin\alpha\sin\beta$　　b) $\cos\alpha\cos\beta$　　c) $\sin\alpha\cos\beta$　　d) $\cos\alpha\sin\beta$

87. In a triangle, the lengths of the two larger sides are 10 cm and 9 cm respectively. If the angles of the triangle are in AP, then the length of the third side in cm can be

a) $5-\sqrt{6}$ only　　b) $5+\sqrt{6}$ only

c) $5-\sqrt{6}$ or $5+\sqrt{6}$　　d) Neither $5-\sqrt{6}$ nor $5+\sqrt{6}$

88. In $\Delta\, ABC$, if $\sin^2\frac{A}{2}, \sin^2\frac{B}{2}, \sin^2\frac{C}{2}$ be in HP. Then, a, b, c will be in

a) AP　　b) GP　　c) HP　　d) None of these

89. The angles of elevation of the top of a tower at the top and the foot of a pole of height 10 m are 30° and 60° respectively. The height of the tower is

a) 10 m　　b) 15 m　　c) 20 m　　d) None of these

90. If the points $(1, 1), (-1, -1), (-\sqrt{3}, \sqrt{3})$ are the vertices of a triangle, then this triangle is

a) Right angled　　b) Isosceles　　c) Equilateral　　d) None of these

91. The vertices of a family of triangles have integer coordinates. If two of the vertices of all the triangles are $(0, 0)$ and $(6, 8)$, then the least value of areas of the triangles is

a) 1　　b) $\frac{3}{2}$　　c) 2　　d) $\frac{5}{2}$

92. In a $\Delta\, ABC$, $\left(\cot\frac{A}{2}+\cot\frac{B}{2}\right)\left(a\sin^2\frac{B}{2}+b\sin^2\frac{A}{2}\right)$ is equal to

a) $\cot C$　　b) $c\cot C$　　c) $\cot\frac{C}{2}$　　d) $c\cot\frac{C}{2}$

93. The intercepts on the straight line $y = mx$ by the line $y = 2$ and $y = 6$ is less than 5, then m belongs to

a) $\left]-\frac{4}{3}, \frac{4}{3}\right[$ b) $\left]\frac{4}{3}, \frac{3}{8}\right[$ c) $\left]-\infty, -\frac{4}{3}\right[\cup\left]\frac{4}{3}, \infty\right[$ d) $\left]\frac{4}{3}, \infty\right[$

94. In ΔABC, $(b-c)\sin A + (c-a)\sin B + (a-b)\sin C$ is equal to

a) $ab + bc + ca$ b) $a^2 + b^2 + c^2$ c) 0 d) None of these

95. The inradius of the triangle whose sides are 3, 5, 6 is

a) $\sqrt{\frac{8}{7}}$ b) $\sqrt{8}$ c) $\sqrt{7}$ d) $\sqrt{\frac{7}{8}}$

96. In a ΔABC, if the sides are $a = 3, b = 5$ and $c = 4$, then $\sin\frac{B}{2} + \cos\frac{B}{2}$ is equal to

a) $\sqrt{2}$ b) $\frac{\sqrt{3}+1}{2}$ c) $\frac{\sqrt{3}-1}{2}$ d) 1

97. The elevation of an object on a hill is observed from a certain point in the horizontal plane through its base, to be 30°. After walking 120 m towards it on level ground the elevation is found to be 60°. Then the height of the object (in metres) is

a) 120 b) $60\sqrt{3}$ c) $120\sqrt{3}$ d) 60

98. If the area of the triangle with vertices $(x, 0)$, $(1, 1)$ and $(0, 2)$ is 4 sq unit, then the value of x is

a) -2 b) -4 c) -6 d) 8

99. At a distance 12 metres from the foot A of a tower AB of height 5 metres, a flagstaff BC on top of AB and the tower subtend the same angle. The, the height of flagstaff is

a) $\frac{1440}{119}$ metres b) $\frac{475}{119}$ metres c) $\frac{845}{119}$ metres d) None of these

100. A tower 50 m high, stands on top of a mount, from a point on the ground the angles of elevation of the top and bottom of the tower are found to be 75° and 60° respectively. the height of the mount is

a) 25 m b) $25(\sqrt{3}-1)$ m c) $25\sqrt{3}$ m d) $25(\sqrt{3}+1)$ m

101. Let AB is divided internally and externally at P and Q in the same ratio. Then, AP, AB, AQ are in

a) AP b) GP c) HP d) None of these

102. If the sum of the distance of a point P from two perpendicular lines in a plane is 1, then the locus of P is a

a) Rhombus b) Circle c) Straight line d) Pair of straight lines

103. A flagpole stands on a building of height 450 ft and an observer on a level ground is 300 ft from the base of the building. The angle of elevation of the bottom of the flagpole is 30°and the height of the flagpole is 50ft. If θ is the angle of elevation of the top of the flagpole, then tan θ is equal to

a) $\frac{4}{3\sqrt{3}}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{9}{2}$ d) $\frac{\sqrt{3}}{5}$

104. In ΔABC, $2\left(a\sin^2\frac{C}{2} + c\sin^2\frac{A}{2}\right)$ is equal to

a) $a + b - c$ b) $c + a - b$ c) $b + c - a$ d) $a + b + c$

105. Orthocenter of triangle with vertices (0, 0), (3, 4) and (4, 0) is

a) $\left(3, \frac{5}{4}\right)$ b) (3, 12) c) $\left(3, \frac{3}{4}\right)$ d) (3, 9)

106. Three vertices of a parallelogram taken in order are $(-1, -6)$, $(2, -5)$ and $(7, 2)$. The fourth vertex is

a) (1, 4) b) (4, 1) c) (1, 1) d) (4, 4)

107. If in a ΔABC, $\cos A \cos B + \sin A \sin B \sin C = 1$, then the triangle is

a) Isosceles b) Right angled c) Isosceles right angled d) Equilateral

108. If in a ΔABC, the sides AB and AC are perpendicular, then the true equation is

a) $\tan A + \tan B = 0$ b) $\tan B + \tan C = 0$ c) $\tan A + 2\tan C = 0$ d) $\tan B \tan C = 1$

109. The points $(1, 1)$, $(-5, 5)$ and $(13, \lambda)$ lie on the same straight line, if λ is equal to

a) 7 b) -7 c) ± 7 d) 0

110. Circumcentre of triangle whose vertices are (0, 0), (3, 0) and (0, 4) is

a) $\left(\frac{3}{2}, 2\right)$ b) $\left(2, \frac{3}{2}\right)$ c) (0, 0) d) None of these

111. The vertices of a triangle are $A(-1,-7), B(5,1)$ and $C(1,4)$. The equation of the bisector of angle ABC, is

a) $x+7y-2=0$ b) $x-7y-2=0$ c) $x-7y+2=0$ d) None of these

112. A tower subtends angles α, 2α and 3α respectively at points A, B and C, all lying on a horizontal line through the foot of the tower, then $\frac{AB}{BC}$ is equal to

a) $\frac{\sin 3\alpha}{\sin 2\alpha}$ b) $1+2\cos 2\alpha$ c) $2\cos 2\alpha$ d) $\frac{\sin 2\alpha}{\sin \alpha}$

113. A person standing on the bank of a river finds that the angle of elevation of the top of a tower on the opposite bank is 45°, then which of the following statements is correct?

a) Breadth of the river is twice the height of the tower

b) Breadth of the river and the height of the tower are the same

c) Breadth of the river is half of the height of the tower

d) None of these

114. The angular depression of the top and the foot of the chimney as seen from the top of a second chimney which is 150 m high and standing on the same level as the first are θ and ϕrespectively. The distance between their tops when $\tan\theta=\frac{4}{3}$ and $\tan\phi=\frac{5}{2}$ is equal to

a) 50 m b) 100 m c) 15 m d) None of these

115. A round balloon of radius r subtends an angle αat the eye of the observer, While the angle of elevation of its centre is β. The height of the center of balloon is

a) $r \operatorname{cosec}\alpha \sin\frac{\beta}{2}$ b) $r\sin\alpha \operatorname{cosec}\frac{\beta}{2}$ c) $r\sin\frac{\alpha}{2}\operatorname{cosec}\beta$ d) $r\operatorname{cosec}\frac{\alpha}{2}\sin\beta$

116. In a Δ ABC, a, c, A are given and b_1, b_2 are two values, if the third side b such that $b_2=2b_1$, then $\sin A$ is equal to

a) $\frac{\sqrt{9a^2-c^2}}{8a^2}$ b) $\sqrt{\frac{9a^2-c^2}{8c^2}}$ c) $\frac{\sqrt{9a^2+c^2}}{8a^2}$ d) None of these

117. If a, b, c are sides of a triangle, then

a) $\sqrt{a}+\sqrt{b}>\sqrt{c}$ b) $|\sqrt{a}-\sqrt{b}|>\sqrt{c}$ (if c is smallest)

c) $\sqrt{a}+\sqrt{b}<\sqrt{c}$ d) None of the above

118. ABC Is a triangle with $\angle A=30°, BC=10$ cm. The area of the circumcircle of the triangle is

a) 100π sq cm b) 5 sq cm c) 25 sq cm d) $\frac{100\pi}{3}$ sq cm

119. In a Δ $ABC, a:b:c=4:5:6$. The ratio of the radius of the circumcircle to that of the incircle is

a) $\frac{16}{9}$ b) $\frac{16}{7}$ c) $\frac{11}{7}$ d) $\frac{7}{16}$

120. The incentre of the triangle formed by lines $x=0, y=0$ and $3x+4y=12$, is at

a) $\left(\frac{1}{2},\frac{1}{2}\right)$ b) $(1,1)$ c) $\left(1,\frac{1}{2}\right)$ d) $\left(\frac{1}{2},1\right)$

121. Given points are $A(0,4)$ and $B(0,-4)$, the locus of $P(x,y)$ such that $|AP-BP|=6$, is

a) $9x^2-7y^2+63=0$ b) $9x^2+7y^2-63=0$ c) $9x^2+7y^2+63=0$ d) None of these

122. The angle of elevation of the top of a tower from a point A due South of the tower is α and from a point B due East of the tower is β. If $AB=d$, then the height of the tower is

a) $\frac{d}{\sqrt{\tan^2\alpha-\tan^2\beta}}$ b) $\frac{d}{\sqrt{\tan^2\alpha+\tan^2\beta}}$ c) $\frac{d}{\sqrt{\cot^2\alpha+\cot^2\beta}}$ d) $\frac{d}{\sqrt{\cot^2\alpha-\cot^2\beta}}$

123. Let P be the point $(1,0)$ and Q be the point on$y^2=8x$. The locus of mid point of PQ is

a) $x^2-4y+2=0$ b) $x^2+4y+2=0$ c) $y^2+4x+2=0$ d) $y^2-4x+2=0$

124. Let $A(k,2)$ and $B(3,5)$ are points. The point (t,t) divide $\overline{AB}$ from A's side in the ratio of k, then $k=\cdots, k\in R-\{0,-1\}$

a) -4 b) -2 c) 4 d) 2

125. If a, b, c the sides of a Δ ABC are in AP and a is the smallest side, then $\cos A$ equals

a) $\frac{3c-4b}{2c}$ b) $\frac{3c-4b}{2b}$ c) $\frac{4c-3b}{2c}$ d) None of these

126. Area of the triangle formed by the lines $y = 2x, y = 3x$ and $y = 5$ is equal to (in square unit)

a) $\frac{25}{6}$ b) $\frac{25}{12}$ c) $\frac{5}{6}$ d) $\frac{17}{12}$

127. The angles of depression of the top and the foot of a chimney as seen from the top of a second chimney, which is 150 m high and standing on the same level as the first are θ and ϕ respectively, then the distance between their tops when $\tan\theta = \frac{4}{3}$ and $\tan\phi = \frac{5}{2}$, is

a) $\frac{150}{\sqrt{3}}$ m b) $100\sqrt{3}$ m c) 150 m d) 100 m

128. If one side of a triangle is double the other and the angles opposite to these sides differ by $60°$, then the triangle is

a) Obtuse angled b) Acute angled c) Isosceles d) Right angled

129. If the three points $(3q, 0), (0, 3p)$ and $(1, 1)$ are collinear then which one is true?

a) $\frac{1}{p}+\frac{1}{q}=0$ b) $\frac{1}{p}+\frac{1}{q}=1$ c) $\frac{1}{p}+\frac{1}{q}=3$ d) $\frac{1}{p}+\frac{3}{q}=1$

130. If in a $\Delta ABC, a = 15, b = 36, c = 39$, then $\sin\frac{C}{2}$ is equal to

a) $\frac{\sqrt{3}}{2}$ b) $\frac{1}{2}$ c) $\frac{1}{\sqrt{2}}$ d) $-\frac{1}{\sqrt{2}}$

131. In a ΔABC, let $\angle C = \frac{\pi}{2}$, if r is the inradius and R is the circumradius of the $\Delta\ ABC$, then $2(r+R)$ equals

a) $c+a$ b) $a+b+c$ c) $a+b$ d) $b+c$

132. From the top of a light house 60 m high with its base at the sea level the angle of depression of a boat is $15°$. The distance of the boat from the foot of light house is

a) $\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)60$ m b) $\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)60$ m c) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ m d) None of these

133. If $\cos^2 A + \cos^2 C = \sin^2 B$, then $\Delta\ ABC$ is

a) Equilateral b) Right angled c) Isosceles d) None of these

134. The sides of triangle are in the ratio $1:\sqrt{3}:2$, then the angles of the triangle are in ratio

a) $1:3:5$ b) $2:3:1$ c) $3:2:1$ d) $1:2:3$

135. A tower stands at the top of a hill whose height is 3 times the height of the tower. The tower is found to subtend at a point 3 km away on the horizontal through the foot of the hill, an angle θ, where $\tan\theta = \frac{1}{9}$. The height of the tower is

a) 12 b) 3 c) $\frac{9\pm\sqrt{33}}{8}$ d) None of these

136. Angles A, B and C of a $\Delta\ ABC$ are in AP. If $\frac{b}{c} = \frac{\sqrt{3}}{\sqrt{2}}$, then angle A is equal to

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{5\pi}{12}$ d) $\frac{\pi}{2}$

137. The angle of depression of a boat in a river is $30°$ from the top of a tower, 87 m high and the speed of the boat is 5.8 km/h. The time taken by the boat to reach at the base of the tower is

a) 9 min b) $\frac{9\sqrt{3}}{10}$ min c) 25 min d) 15 min

138. If the centroid of the triangle formed by the points $(a, b), (b, c)$ and (c, a) is at the origin, then $a^3 + b^3 + c^3 =$

a) 0 b) abc c) $3\ abc$ d) $-3\ abc$

139. The sides of a $\Delta\ ABC$ are $BC = 5, CA = 4$ and $AB = 3$. If A is at the origin and the bisector of the internal angle A meets BC in $D(12/7, 12/7)$, then the coordinates of the incentre, are

a) $(2, 2)$ b) $(2, 3)$ c) $(3, 2)$ d) $(1, 1)$

140. If a, b and c are the sides of a triangle such that $a^4 + b^4 + c^4 = 2c^2(a^2 + b^2)$, then the angles opposite to the side C is

a) $45°$ or $90°$ b) $30°$ or $135°$ c) $45°$ or $135°$ d) $60°$ or $120°$

141. In radius of a circle which is inscribed in a isosceles triangle one of whose angle is $2\pi/3$, is $\sqrt{3}$, then area of triangle is

a) $4\sqrt{3}$ b) $12-7\sqrt{3}$ c) $12+7\sqrt{3}$ d) None of these

142. A triangular park is enclosed on two sides by a fence and on the third side by a straight river bank. The two sides having fence are of same length$x$. The maximum area enclosed by the park is

a) $\sqrt{\frac{x^3}{8}}$ b) $\frac{1}{2}x^2$ c) πx^2 d) $\frac{3}{2}x^2$

143. In a ΔABC, if $\tan\frac{A}{2}=\frac{5}{6}$, $\tan\frac{C}{2}=\frac{2}{5}$, then

a) a, b, c are in AP b) a, b, c are in GP c) $b.a.c$ are in AP d) a, b, c are in AP

144. The vertices P, Q, R of a triangle are $(2, 1)$, $(5, 2)$ and $(3, 4)$ respectively. Then, the circumcentre is

a) $\left(\frac{13}{4}, -\frac{9}{4}\right)$ b) $\left(-\frac{13}{4}, \frac{9}{4}\right)$ c) $\left(-\frac{13}{4}, -\frac{9}{4}\right)$ d) $\left(\frac{13}{4}, \frac{9}{4}\right)$

145. In a Δ ABC, $(a+b+c)(b+c-a)=kbc$, if

a) $k<0$ b) $k>6$ c) $0<k<4$ d) $k>4$

146. If A $(6,-3)$, $B(-3,5)$, $C(4,-2)$, $P(\alpha,\beta)$, then the ratio of the areas of the triangles PBC , ABC is

a) $|\alpha+\beta|$ b) $|\alpha-\beta|$ c) $|\alpha+\beta+2|$ d) $|\alpha+\beta\ -2|$

147. ABC is a triangular park with $AB=AC=100$ m. A clock tower is situated at the mid point of BC. The angles of elevation of the top of the tower at A and B are $\cot^{-1} 3.2$ and $\operatorname{cosec}^{-1} 2.6$ respectively. The height of the tower is

a) 50 m b) 25 m c) 40 m d) None of these

148. In a Δ ABC, $a\cot A+b\cot B+c\cot C$ is equal to

a) $r+R$ b) $r-R$ c) $2(r+R)$ d) $2(r-R)$

149. If $(1, a)$, $(2, b)$, and $(3, c)$; $a, b, c\ \in R$ are the vertices of a triangle, its centroid can

a) Not be on x-axis b) Not be on y-axis c) Be on $(0, 0)$ d) None of these

150. The pair of lines $\sqrt{3}x^2-4xy+\sqrt{3}y^2=0$ are rotated about the origin by $\pi/6$ in the anti-clockwise sense. The equation of the pair in the new position is

a) $\sqrt{3}y^2-xy=0$ b) $\sqrt{3}x^2-xy=0$ c) $x^2-y^2=0$ d) $\sqrt{3}x^2+xy=0$

151. In triangle ABC, $a=2$, $b=3$ and $\sin A=\frac{2}{3}$ then B is equal to

a) $30°$ b) $60°$ c) $90°$ d) $120°$

152. If the sides of a right angle triangle form an AP, the 'sin' of the acute angles are

a) $\left(\frac{3}{5}, \frac{4}{5}\right)$ b) $\left(\sqrt{3}, \frac{1}{\sqrt{3}}\right)$ c) $\left(\sqrt{\frac{\sqrt{5}-1}{2}}, \sqrt{\frac{\sqrt{5}-1}{2}}\right)$ d) $\left(\sqrt{\frac{\sqrt{3}-1}{2}}, \sqrt{\frac{\sqrt{3}-1}{2}}\right)$

153. In a Δ ABC, $2a^2+4b^2+c^2=4ab+2ac$, then $\cos B$ is equal to

a) 0 b) $\frac{1}{8}$ c) $\frac{3}{8}$ d) $\frac{7}{8}$

154. The line joining $A(b\cos\alpha, b\sin\alpha)$ and $B(a\cos\beta, a\sin\beta)$ is produced to the point $M(x, y)$ so that $AM{:}\,MB=b:a$, then
$x\cos\left(\frac{\alpha+\beta}{2}\right)+y\sin\left(\frac{\alpha+\beta}{2}\right)$ is

a) -1 b) 0 c) 1 d) a^2+b^2

155. A house of height 100 m subtends a right angle at the window of an opposite house. If the height of the window be 64m, then the distance between the two houses is

a) 48 m b) 36 m c) 54 m d) 72 m

156. A vertical tower stands on a declivity which is inclined at $15°$ to the horizon. From the foot of the tower a man ascends the declivity for 80 ft and them, finds that the tower subtends an angle of $30°$. The height of tower is

a) $20(\sqrt{6}-\sqrt{2})$ft b) $40(\sqrt{6}-\sqrt{2})$ft c) $40(\sqrt{6}+\sqrt{2})$ft d) None of these

157. $(0,-1)$ And $(0,3)$ are two opposite vertices of a square. The other two vertices are

a) $(0,1),(0,-3)$ b) $(3,-1),(0,0)$ c) $(2,1),(-2,1)$ d) $(2,2),(1,1)$

158. The points $(1,3)$ and $(5,1)$ are two opposite vertices of a rectangle. The other two vertices lie on the line $y=2x+c$, are

a) $(2,0)$ and $(4,4)$ b) $(2,0)$ and $(-4,-4)$ c) $(2,0)$ and $(-4,4)$ d) $(-2,0)$ and $(4,4)$

159. If in a $\Delta\, ABC, r_3=r_1+r_2+r$, then $\angle A+\angle B$ is equal to

a) $120°$ b) $100°$ c) $90°$ d) $80°$

160. In a triangle ABC, if $a=3, b=4, c=5$, then the distance between its incentre and circumcentre is

a) $\frac{1}{2}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{3}{2}$ d) $\frac{\sqrt{5}}{2}$

161. One side of length $3a$ of triangle of area a^2 square unit lies on the line$x=a$. Then, one of the lines on which the third vertex lies, is

a) $x=-a^2$ b) $x=a^2$ c) $x=-a$ d) $x=\frac{a}{3}$

162. In a $\Delta\, ABC$, if D is the middle point BC and AD is perpendicular to AC, then $\cos B$ is equal to

a) $\frac{2b}{a}$ b) $-\frac{b}{a}$ c) $\frac{b^2+c^2}{ca}$ d) $\frac{c^2+a^2}{ca}$

163. The angle of depression of a point situated at a distance of 70 metres from the base of a tower is $45°$. The height of the tower is

a) 70 m b) $70\sqrt{2}$ m c) $\frac{70}{\sqrt{2}}$ m d) 35 m

164. The sides of a triangle are three consecutive natural numbers and its largest angle is twice the smallest one. Then, the sides of the triangle are

a) 1, 2, 3 b) 2, 3, 4 c) 3, 4, 5 d) 4, 5, 6

165. Consider the following statements :

1. $\frac{b^2-c^2}{a\sin(B-C)}=2R$

2. $a\sin(B-C)+b\sin(C-A)+c\sin(A-B)=0$

Which of these is/are correct?

a) Only (1) b) Only (2) c) Both (1) and (2) d) None of these

166. The four distinct point $(0,0),(2,0),(0,-2)$ and $(k,-2)$ are concyclic, if k is equal to

a) -2 b) 2 c) 1 d) 0

167. If origin is shifted to $(7,-4)$, then point $(4,5)$ shifted to

a) $(-3,9)$ b) $(3,9)$ c) $(11,1)$ d) None of these

168. In $\Delta ABC, (a+b+c)\left(\tan\frac{A}{2}+\tan\frac{B}{2}\right)$ is equal to

a) $2c\cot\frac{C}{2}$ b) $2a\cot\frac{A}{2}$ c) $2b\cot\frac{B}{2}$ d) $\tan\frac{C}{2}$

169. In a $\Delta\, ABC$, sides a,b,c are in AP and $\frac{2}{1!9!}+\frac{2}{3!7!}+\frac{1}{5!5!}=\frac{8^a}{(2b)!}$, then the maximum value of $\tan A\tan B$ is equal to

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{4}$ d) $\frac{1}{4}$

170. If the angle of elevation of two towers from the middle point of the line joining their feet be $60°$ and $30°$ respectively, then the ratio of their heights is

a) $2:1$ b) $1:\sqrt{2}$ c) $3:1$ d) $1:\sqrt{3}$

171. In a $\Delta\, ABC, \angle C=60°$then $\frac{1}{a+c}+\frac{1}{b+c}$ is equal to

a) $\frac{1}{a+b+c}$ b) $\frac{2}{a+b+c}$ c) $\frac{3}{a+b+c}$ d) None of these

172. In ΔABC, if $(a+b+c)(a-b+c)=3ac$, then

a) $\angle B=60°$ b) $\angle B=30°$ c) $\angle C=60°$ d) $\angle A+\angle C=90°$

173. If a^2, b^2, c^2 are in AP, then which of the following are also in AP?

a) $\sin A, \sin B, \sin C$ b) $\tan A, \tan B, \tan C$ c) $\cot A, \cot B, \cot C$ d) None of these

174. In a triangle ABC, if $\sin A \sin B = \frac{ab}{c^2}$, then the triangle is

a) Equilateral b) Isosceles c) Right angled d) Obtuse angled

175. The perimeter of a $\Delta\, ABC$ is 6 times the arithmetic mean of the sine ratios of its angles. If $a = 1$, then A is equal to

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{2}$ d) $\frac{2\pi}{3}$

176. The centriod of the triangle ABC, where $A \equiv (2,3), B \equiv (8,10)$ and $C \equiv (5,5)$ is

a) (5, 6) b) (6, 5) c) (6, 6) d) (15, 18)

177. The angle of elevation of the top of the tower observed from each of the tree point A, B, C on the ground forming a triangle is the same angle α. If R is the circumaradias of the triangle ABC, then the height of the tower is

a) $R \sin \alpha$ b) $R \cos \alpha$ c) $R \cot \alpha$ d) $R \tan \alpha$

178. The angle of elevation of the top of a hill from a point is α. After walking b metres towards the top up a slope inclined at an angle β to the horizon, the angle of elevation of the top becomes γ. Then, the height of the hill is

a) $\frac{b \sin \alpha \sin(\gamma - \beta)}{\sin(\gamma - \alpha)}$ b) $\frac{b \sin \alpha \sin(\gamma - \alpha)}{\sin(\gamma - \beta)}$ c) $\frac{b \sin\ (\gamma - \beta)}{\sin(\gamma - \alpha)}$ d) $\frac{\sin(\gamma - \beta)}{b \sin \alpha \sin(\gamma - \alpha)}$

179. The area of the $\Delta\, ABC$, in which $a = 1, b = 2, \angle C = 60°$, is

a) 4 sq unit b) $\frac{1}{2}$sq unit c) $\frac{\sqrt{3}}{2}$sq unit d) $\sqrt{3}$ sq units

180. If t_1, t_2 and t_3 are distinct points $(t_1, 2at_1 + at_1^3), (t_2, 2at_2 + at_2^3)$ and $t_3, 2at_3 + at_3^3)$ are collinear, if

a) $t_1t_2t_3 = 1$ b) $t_1 + t_2 + t_3 = t_1t_2t_3$ c) $t_1 + t_2 + t_3 = 0$ d) $t_1 + t_2 + t_3 = -1$

181. If A and B are two points having coordinates (3, 4) and (5, −2) respectively and P is a point such that $PA = PB$ and area of triangle $PAB = 10$ sq unit, then the coordinates of P are

a) (7, 4) and (13, 2) b) (7, 2) and (1, 0) c) (2, 7) and (4, 13) d) None of these

182. In $\Delta\, ABC, \angle A = \frac{\pi}{2}, b = 4, c = 3$, then the value of $\frac{R}{r}$ is equal to

a) $\frac{5}{2}$ b) $\frac{7}{2}$ c) $\frac{9}{2}$ d) $\frac{35}{24}$

183. In the angles A, B and C of a triangular are in the arithmetic progression and if a, b and c denotes the lengths of the sides opposite to A, B and C respectively, then the value of the expression $\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A$ is

a) $\frac{1}{2}$ b) $\frac{\sqrt{3}}{2}$ c) 1 d) $\sqrt{3}$

184. Two sides of a triangle are given by the roots of the equation $x^2 - 5x + 6 = 0$ and the angle between the sides is $\frac{\pi}{3}$. Then, the perimeter of the triangle is

a) $5 + \sqrt{2}$ b) $5 + \sqrt{3}$ c) $5 + \sqrt{5}$ d) $5 + \sqrt{7}$

185. In a triangle ABC, if $\angle A = 60°, a = 5, b = 4$, then c is a root of the equation

a) $c^2 - 5c - 9 = 0$ b) $c^2 - 4c - 9 = 0$ c) $c^2 - 10c + 25 = 0$ d) $c^2 - 5c - 41 = 0$

186. The angle of elevation of the top of vertical tower from a point A on the horizontal ground is found to be $\frac{\pi}{4}$. From A, a man walks 10 m up a path sloping at a angle $\frac{\pi}{6}$. After this the slope becomes steeper and after walking up another 10 m, the man reaches the top of the tower. Distance of A from the foot of the tower is

a) $5(1 + \sqrt{3})$m b) $\frac{5}{2}(1 + \sqrt{3})$m c) $5(\sqrt{3} - 1)$m d) $\frac{5}{2}(\sqrt{3} - 1)$m

187. If the distance between the points $(a \cos \theta, a \sin \theta)$ and $(a \cos \phi, a\sin \phi)$ is $2a$, then θ is equal to

a) $2n\pi \pm \pi + \phi, n \in Z$ b) $n\pi + \frac{\pi}{2} + \phi, n \in Z$

c) $n\pi - \phi, n \in Z$ d) $2n\pi + \phi, n \in Z$

188. If $A(0,0), B(12,0), C(12,2), D(6,7)$ and $E(0,5)$ are the vertices of the pendagon $ABCDE$, then its area in

square units, is

a) 58 b) 60 c) 61 d) 63

189. A flag is standing vertically on a tower of height b. On a point at a distance a from the foot of the tower, the flag and the tower subtend equal angles. The height of the flag is

a) $b.\dfrac{a^2+b^2}{a^2-b^2}$ b) $a.\dfrac{a^2-b^2}{a^2+b^2}$ c) $b.\dfrac{a^2-b^2}{a^2+b^2}$ d) $a.\dfrac{a^2+b^2}{a^2-b^2}$

190. A kite is flying at an inclination of 60° with the horizontal. If the length of the thread is 120 m, then the height of the kite is

a) $60\sqrt{3}$ m b) 60 m c) $\dfrac{60}{\sqrt{3}}$ m d) 120 m

191. $\frac{a\cos A+b\cos B+c\cos C}{a+b+c}$ is equal to

a) $1/r$ b) r/R c) R/r d) $1/R$

192. AB is a vertical pole. The end A is on the level ground. C is the middle point of AB. P is a point on the level ground. The portion BC subtends an angle β at P. If $AP=n\,AB$, then $\tan\beta=$

a) $\dfrac{n}{2\,n^2+1}$ b) $\dfrac{n}{n^2-1}$ c) $\dfrac{n}{n^2+1}$ d) None of these

193. If $P(3,7)$ is a point on the line joining $A(1,1)$ and $B(6,16)$, then the harmonic conjugate Q of point P has the coordinates

a) $(9,29)$ b) $(-9,29)$ c) $(9,-29)$ d) $(-9,-29)$

194. The angles of a triangle are in the ratio 3:5:10. Then, the ratio of the smallest side to the greatest side is

a) 1: sin 10° b) 1: 2 sin 10° c) 1: cos 10° d) 1: 2 cos 10°

195. In $\Delta\,ABC$, if $\begin{vmatrix}1 & a & b\\ 1 & c & a\\ 1 & b & c\end{vmatrix}=0$, then

$\sin^2 A+\sin^2 B+\sin^2 C$ is equal to

a) $\dfrac{4}{9}$ b) $\dfrac{9}{4}$ c) $3\sqrt{3}$ d) 1

196. From a station A due West of a tower the angle of elevation of the top of the tower is seen to be 45°. From a station B, 10 m from A and in the direction 45° South of East of angle of elevation is 30°, the height of tower is

a) $5\sqrt{2}(\sqrt{5}+1)$m b) $\dfrac{5(\sqrt{5}+1)}{2}$m c) $\dfrac{5\sqrt{2}(\sqrt{5}+1)}{2}$m d) None of these

197. A straight line with negative slope passing through the point (1, 4) meets the coordinate axes at A and B. The minimum value of $OA+OB$ is equal to

a) 5 b) 6 c) 9 d) 8

198. An observer finds that the elevation of the top of a tower is $22\frac{1}{2}^\circ$ and after walking 150 metres towards the foot of the tower he finds that the elevation of the top has increased to $67\frac{1}{2}^\circ$. The height of the tower in metres is

a) 50 b) 75 c) 125 d) 175

199. In an isosceles $\Delta\,ABC$, $AB=AC$. If vertical angle A is 20°, then a^3+b^3 is equal to

a) $3a^2b$ b) $3b^2c$ c) $3c^2a$ d) abc

200. In a $\Delta\,ABC$, $a(\cos^2 B+\cos^2 C)+\cos A\ (c\cos C+b\cos B)$ is equal to

a) a b) b c) c d) $a+b+c$

201. ABC Is a triangle with vertices $A(-1,4)$, $B(6,-2)$ and $C(-2,4)$. D,E And F are the points which divide each AB,BC and CA respectively in the ratio 3:1 internally. Then, the centroid of the triangle DEF is

a) $(3,6)$ b) $(1,2)$ c) $(4,8)$ d) $(-3,6)$

202. If in a $\Delta\,ABC$, $\frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$, then the value of the angle C is

a) 60° b) 30° c) 45° d) None of these

203. A tower subtends an angle of 30° at a point distance d from the foot of the tower and on the same level as the foot of the tower. At a second point, h vertically above the first, the angle of depression of the foot of

the tower is 60°. The height of the tower is

a) $\frac{h}{3}$ b) $\frac{h}{3d}$ c) $3h$ d) $\frac{3h}{d}$

204. Points D, E are taken on the side BC of theΔABC, such that$BD = DE = EC$. If $\angle BAD = x, \angle DAE = y, \angle EAC = z$, then the value of $\frac{\sin(x+y)\sin(y+z)}{\sin x \sin z}$ is equal to

a) 1 b) 2 c) 4 d) None of these

205. $\sin A : \sin C = \sin(A - B) : \sin(B - C)$, then a^2, b^2, c^2 are in

a) AP b) GP c) HP d) None of these

206. The point on the line $3x + 4y = 5$, which is equidistant from (1, 2) and (3, 4) is

a) $(7, -4)$ b) $(15, -10)$ c) $\left(\frac{1}{7}, \frac{8}{7}\right)$ d) $\left(0, \frac{5}{4}\right)$

207. If A and B are two fixed points, then the locus of a point which moves in such a way that the angle, APB is a right angle is

a) A circle b) An ellipse c) A parabola d) None of these

208. If in a $\Delta\, ABC, a\cos^2\left(\frac{C}{2}\right) + c\cos^2\left(\frac{A}{2}\right) = \frac{3b}{2}$, then the sides a, b and c

a) Are in AP b) Are in GP c) Are in HP d) Satisfy $a + b = c$

209. For a regular polygon, let r and r be the radii of the inscribed and the circumscribed circles. A false statement among the following is

a) There is regular polygon with $\frac{r}{R} = \frac{1}{2}$

b) There is a regular polygon $\frac{r}{R} = \frac{1}{\sqrt{2}}$

c) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$

d) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

210. In a triangle vertex angles are A, B, C and side BC are given. The area of ΔABC is

a) $\frac{s(s-a)(s-b)(s-c)}{2}$

b) $\frac{b^2 \sin C \sin A}{\sin B}$

c) $ab \sin C$

d) $\frac{1}{2} \cdot \frac{a^2 \sin B \sin C}{\sin A}$

211. A flag staff in the centre of a rectangular field whose diagonal is 1200 m and subtends angle 15° and 45° at the mid point of the sides of the field. The height of the flag staff is

a) 200 m b) $300\sqrt{2+\sqrt{3}}$ m c) $300\sqrt{2-\sqrt{3}}$ m d) 400 m

212. On the level ground the angle of elevation of the top of a tower is 30°. On moving 20 m nearer the tower, the angle of elevation is found to be 60°. The height of the tower is

a) 10 m b) 20 m c) $10\sqrt{3}$ m d) None of these

213. The angle of elevation of the top of a tower at any point on the ground is 30° and moving 20 metres towards the tower it becomes 60°. The height of the tower is

a) 10 m b) $10\sqrt{3}$ m c) $\frac{10}{\sqrt{3}}$ m d) None of these

214. If angles A, B and C are in AP, then $\frac{a+b}{c}$ is equal to

a) $2\sin\frac{A-C}{2}$ b) $2\cos\frac{A-C}{2}$ c) $\cos\frac{A-C}{2}$ d) $\sin\frac{A-C}{2}$

215. If in a $\Delta\, ABC$, the altitude from the vertices A, B, C on opposite sides are in HP, then $\sin A, \sin B, \sin C$ are in

a) GP

b) Arithmetic-Geometric Progression

c) AP

d) HP

216. The area of an equilateral triangle that can be inscribed in the circle $x^2 + y^2 - 4x - 6y - 12 = 0$, is

a) $\frac{25\sqrt{3}}{4}$ sq units b) $\frac{35\sqrt{3}}{4}$ sq units c) $\frac{55\sqrt{3}}{4}$ sq units d) $\frac{75\sqrt{3}}{4}$ sq units

217. The area of a triangle is 5 and its two vertices are $A(2, 1)$ and $B(3, -2)$. The third vertex lies on $y = x + 3$. Then, third vertex is

a) $\left(\frac{7}{2}, \frac{13}{2}\right)$ b) $\left(\frac{5}{2}, \frac{5}{2}\right)$ c) $\left(-\frac{3}{2}, \frac{3}{2}\right)$ d) $(0, 0)$

218. The incentre of the triangle with vertices $(1, \sqrt{3}), (0, 0)$ and $(2, 0)$ is

a) $\left(1, \frac{\sqrt{3}}{2}\right)$ b) $\left(\frac{2}{3}, \frac{1}{\sqrt{3}}\right)$ c) $\left(\frac{2}{3}, \frac{\sqrt{3}}{2}\right)$ d) $\left(1, \frac{1}{\sqrt{3}}\right)$

219. The area (in square unit) of the triangle formed by the points with polar coordinates $(1, 0)$, $\left(2, \frac{\pi}{3}\right)$ and $\left(3, \frac{2\pi}{3}\right)$ is

a) $\frac{11\sqrt{3}}{4}$ b) $\frac{5\sqrt{3}}{4}$ c) $\frac{5}{4}$ d) $\frac{11}{4}$

220. The points P is equidistant from $A\ (1, 3), B\ (-3, 5)$ and $C\ (5, -1)$, then PA is equal to

a) 5 b) $5\sqrt{5}$ c) 25 d) $5\sqrt{10}$

221. A rod of length l slides with its ends on two perpendicular lines. The locus of a point which divides it in the ratio $1 : 2$, is

a) $36x^2 + 9y^2 = 4l^2$ b) $36x^2 + 9y^2 = l^2$ c) $9x^2 + 36y^2 = 4l^2$ d) $9x^2 - 36y^2 = 4l^2$

222. The circumradius of the triangle whose sides are 13, 12 and 5, is

a) 15 b) $\frac{13}{2}$ c) $\frac{15}{2}$ d) 6

223. Locus of centroid of the triangle whose vertices are $(a \cos t, a \sin t), (b \sin t, -b \cos t)$ and $(1, 0)$, where t is a parameter, is

a) $(3x - 1)^2 + (3y)^2 = a^2 - b^2$ b) $(3x - 1)^2 + (3y)^2 = a^2 + b^2$
c) $(3x + 1)^2 + (3y)^2 = a^2 + b^2$ d) $(3x + 1)^2 + (3y)^2 = a^2 - b^2$

224. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60°. He moves away from the pole along the line BC to a point D such that CD=7m. From D the angle of elevation of the point A is 45°. Then the height of the pole is

a) $\frac{7\sqrt{3}}{2}\left(\frac{1}{\sqrt{3}+1}\right)$m b) $\frac{7\sqrt{3}}{2}\left(\frac{1}{\sqrt{3}-1}\right)$m c) $\frac{7\sqrt{3}}{2}(\sqrt{3}+1)$m d) $\frac{7\sqrt{3}}{2}(\sqrt{3}-1)$m

225. If C is the reflection of $A(2, 4)$ in x-aixs and B is the reflection of C in y-axis, then $|AB|$ is

a) 20 b) $2\sqrt{5}$ c) $4\sqrt{5}$ d) 4

226. ABC is an isosceles triangle if the coordinates of the base are $B(1, 3)$ and $C(-2, 7)$, the coordinates of vertex A can be

a) $(1, 6)$ b) $\left(-\frac{1}{2}, 5\right)$ c) $\left(\frac{5}{6}, 6\right)$ d) $\left(-8, \frac{1}{8}\right)$

227. Point $\left(\frac{1}{2}, -\frac{13}{4}\right)$ divides the line joining the points $(3, -5)$ and $(-7, 2)$ in the ratio of

a) 1: 3 internally b) 3: 1 internally c) 1: 3 externally d) 3: 1 externally

228. The orthocenter of the triangle with vertices $O(0, 0), A\left(0, \frac{3}{2}\right), B(-5, 0)$ is

a) $\left(\frac{5}{2}, \frac{3}{4}\right)$ b) $\left(\frac{-5}{2}, \frac{3}{4}\right)$ c) $\left(-5, \frac{3}{2}\right)$ d) $(0, 0)$

229. Area of triangle formed by the lines $x + y = 3$ and angle bisectors of the pair of straight lines $x^2 - y^2 + 2y = 1$ is

a) 2 sq units b) 4 sq units c) 6 sq units d) 8 sq units

230. $ABCD$ is a rectangular field. A vertical lamp post of height12m stands at the corner A. If the angle of elevation of its top from B is 60° and from C is 45°, then the area of the field is

a) $48\sqrt{2}$ sq m b) $48\sqrt{3}$ sq m c) 48 sq m d) $12\sqrt{2}$ sq m

231. If two adjacent sides of a cylinder quadrilateral are 2 and 5 and the angle between them is 60°. If the third side is 3, then the remaining fourth side is

a) 2 b) 3 c) 4 d) 5

232. In $\Delta\ ABC, 2R^2 \sin A \sin B \sin C$ is equal to

a) s^2 b) $ab+bc+ca$ c) Δ d) None of these

233. If in a $\Delta\, ABC$, the altitudes from the vertices A, B, C on opposite sides are in HP, then $\sin A$, $\sin B$, $\sin C$ are in
a) HP
b) Arithmetico-Geometric Progression
c) AP
d) GP

234. The middle point of the line segment joining $(3,-1)$ and $(1,1)$ is shifted by two units (in the sense of increasing y) perpendicular to the line segment. Then the coordinates of the point in the new position are
a) $(2-\sqrt{2}, 2)$ b) $(2, 2-\sqrt{3})$ c) $(2+\sqrt{2}, \sqrt{3})$ d) None of these

235. If area of triangle with vertices $(0,0)$, $(0,6)$ and (α, β) is 15 sq unit, then
a) $\alpha = \pm 5, \beta = 5$
b) $\alpha = \pm 10, \beta = 5$
c) $\alpha = \pm 5, \beta = 2$
d) $\alpha = \pm 5, \beta$ can take any real value

236. Area of quadrilateral whose vertices are $(2,3)$, $(3,4)$, $(4,5)$ and $(5,6)$ is equal to
a) 0 b) 4 c) 6 d) None of the above

237. If the distance between $(2,3)$ and $(-5,2)$ is equal to the distance between $(x, 2)$ and $(1,3)$, then the values of x are
a) $-6, 8$ b) $6, 8$ c) $-8, 6$ d) $-7, 7$

238. A circle is inscribed in a equilateral triangle of side a. The area of the circle is
a) $3\pi a^2$sq units b) $2a^2$sq units c) a^2sq units d) None of these

239. The x-axis, y-axis and a line passing through the point $A(6,0)$ form a triangleABC. If $\angle A = 30°$ then the area of the triangle, in sq units is
a) $6\sqrt{3}$ b) $12\sqrt{3}$ c) $4\sqrt{3}$ d) $8\sqrt{3}$

240. In a $\Delta\, ABC$, if $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$ and the side $a = 2$, then area of the triangle is
a) 1 sq unit b) 2 sq unit c) $\frac{\sqrt{3}}{2}$ sq unit d) $\sqrt{3}$ sq unit

241. In an ambiguous case of solving a triangle when $a = \sqrt{5}, b = 2, \angle A = \frac{\pi}{6}$, the two possible values of third side are c_1 and c_2, then
a) $|c_1 - c_2| = 2\sqrt{6}$ b) $|c_1 - c_2| = 4\sqrt{6}$ c) $|c_1 - c_2| = 4$ d) $|c_1 - c_2| = 6$

242. In $\Delta\, ABC$, $(a-b)^2 \cos^2 \frac{C}{2} + (a+b)^2 \sin^2 \frac{C}{2}$ is equal to
a) a^2 b) b^2 c) c^2 d) None of these

243. In $\Delta\, ABC$, with usual notation, observe the two statements given below
I. $rr_1r_2r_3 = \Delta^2$
II. $r_1r_2 + r_2r_3 + r_3r_1 = s^2$
Which of the following is correct?
a) Both I and III are true b) I is true, II is false c) I is false, II is true d) Both I and II are false

244. If the angles of a triangle be in the ratio 1:2:7, then the ratio of its greatest side to the least side is
a) 1:2 b) 2:1 c) $(\sqrt{5}+1):(\sqrt{5}-1)$ d) $(\sqrt{5}-1):(\sqrt{5}+1)$

245. From the top of a cliff 300 metres high, the top of a tower was observed at an angle of depression 30° and from the foot of the tower the top of the cliff was observed at an angle of elevation 45°, the height of the tower is
a) $50(3-\sqrt{3})$ m b) $200(3-\sqrt{3})$ m c) $100(3-\sqrt{3})$ m d) None of these

246. If x_1, x_2, x_3 and y_1, y_2, y_3 are both in GP with the same common ratio, then the points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$
a) Lie on a straight line
b) Lie on an ellipse
c) One vertices of a triangle
d) Lie on a circle

247. The coordinates axes are rotated through an angle135°. If the coordinates of a point P in the new system are known to be $(4,-3)$, then the coordinates of P in the original system are
a) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ b) $\left(\frac{1}{\sqrt{2}}, -\frac{7}{\sqrt{2}}\right)$ c) $\left(-\frac{1}{\sqrt{2}}, -\frac{7}{\sqrt{2}}\right)$ d) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$

248. The coordinate axes rotated though an angle 135°. If the coordinates of a point P in the new system are

known to be $(4,-3)$, then the coordinates of P in the original system are

a) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ b) $\left(\frac{1}{\sqrt{2}}, -\frac{7}{\sqrt{2}}\right)$ c) $\left(-\frac{1}{\sqrt{2}}, -\frac{7}{\sqrt{2}}\right)$ d) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$

249. When the elevation of sun changes from 45° to 30° the shadow of a tower increases by 60 m the height of the tower is

a) $30\sqrt{3}$ m b) $30(\sqrt{2}+1)$m c) $30(\sqrt{3}-1)$m d) $30(\sqrt{3}+1)$m

250. The angle of elevation of the top of a tower from a point A $\tan^{-1} 6$ and that from B due West of it, is $\tan^{-1} 7.5$. If h is the height of the tower, then $AB = \lambda h$, where λ^2 is equal to

a) $\frac{21}{700}$ b) $\frac{42}{1300}$ c) $\frac{41}{900}$ d) None of these

251. In a triangle, if $r_1 + r_3 = kcos^2 \frac{B}{2}$, then k is equal to

a) R b) $2R$ c) $3R$ d) $4R$

252. AB is a vertical pole and C is its middle point. The end A is on the level ground and P is any point on the level ground other than A the portion CB subtends an angle β at P. If $AP : AB = 2 : 1$, then $\beta =$

a) $\tan^{-1}\frac{4}{9}$ b) $\tan^{-1}\frac{1}{9}$ c) $\tan^{-1}\frac{5}{9}$ d) $\tan^{-1}\frac{2}{9}$

253. If the points $(x+1, 2), (1, x+2), \left(\frac{1}{x+1}, \frac{2}{x+1}\right)$ are collinear, then x is

a) 4 b) 5 c) -4 d) None of these

254. If in a $\Delta\, ABC$, CD is the angular bisector of the $\angle\, ACB$, then CD is equal to

a) $\frac{a+b}{2ab}\cos\frac{C}{2}$ b) $\frac{a+b}{ab}\cos\frac{C}{2}$ c) $\frac{2ab}{a+b}\cos\frac{C}{2}$ d) None of these

255. A tower stands at the center of a circuit park. A and B are two points on the boundry of the park such that $AB(=a)$ subtends an angles of 60° at the foot of the tower and the angle of elevation of the top of the tower from A or B is 30°. The height of the tower is

a) $\frac{2a}{\sqrt{3}}$ b) $2a\sqrt{3}$ c) $\frac{a}{\sqrt{3}}$ d) $\sqrt{3}$

256. Consider three points

$P = (-\sin(\beta-\alpha), -\cos\beta)$

$Q = (\cos(\beta-\alpha), \sin\beta)$

And $R = (\cos(\beta-\alpha+\theta), \sin(\beta-\theta))$,

Where$0 < \alpha, \beta, \theta < \frac{\pi}{4}$. Then,

a) P lies on the line segment RQ b) Q lies on the line segment PR

c) R lies on the line segment QP d) P, Q, R are non-collinear

257. In $\Delta\, ABC$, if $8R^2 = a^2 + b^2 + c^2$, then the triangle is

a) Right angled b) Equilateral c) Acute angled d) Obtuse angled

258. A triangle with vertices $(4,0), (-1,-1), (3,5)$ is

a) Isosceles and right angled b) Isosceles but not right angled

c) Right angled but not isosceles d) Neither right angled nor isosceles

259. In a ΔABC, the correct formulae among the following are

III. $r = 4R \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

IV. $r_1 = (s-a)\tan\frac{A}{2}$

V. $r_3 = \frac{\Delta}{s-c}$

a) Only I, II b) Only II, III c) Only I, III d) I, II, III

260. The area (in square unit) of the triangle formed by the lines $x = 0, y = 0$ and $3x + 4y = 12$, is

a) 3 b) 4 c) 6 d) 12

261. Three distinct points A, B and C given in the two dimensional coordinate plane such that the ratio of the distance of any one of them from the point $(1, 0)$ to the distance from the point $(-1, 0)$ is equal to$\frac{1}{3}$. Then , the circumcentre of the triangle ABC is at the point

a) $\left(\frac{5}{4}, 0\right)$ b) $\left(\frac{5}{2}, 0\right)$ c) $\left(\frac{5}{3}, 0\right)$ d) $(0, 0)$

262. One possible condition for the three points $(a, b), (b, a)$ and $(a^2, -b^2)$ to be collinear, is
a) $a - b = 2$ b) $a + b = 2$ c) $a = 1 + b$ d) $a = 1 - b$

263. Three vertical towers standing at A, B, C subtends the angle $\theta_A, \theta_B, \theta_C$ respectively at the circumcentre of the ΔABC, then $\tan\theta_A$, $\tan\theta_B$ and $\tan\theta_C$ are in
a) AP b) GP c) HP d) None of these

264. The area of the triangle whose sides are $6, 5, \sqrt{13}$ (in square unit) is
a) $5\sqrt{2}$ b) 9 c) $6\sqrt{2}$ d) 11

265. If points $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are such that x_1, x_2, x_3 and y_1, y_2, y_3 are in AP, then
a) A, B and C are concyclic points
b) A, B and C are collinear points
c) A, B and C are vertices of an equilateral triangle
d) None of the above

266. Two points $P(a, 0)$ and $Q(-a, 0)$ are given, R is a variable point on one side of the line PQ such that $\angle RPQ - \angle RQP$ is 2α, then
a) Locus of R is $x^2 - y^2 + 2xy\cot 2\alpha - a^2 = 0$
b) Locus of R is $x^2 + y^2 + 2xy\cot\alpha - a^2 = 0$
c) Locus of R is a hyperbola, if $\alpha = \frac{\pi}{4}$
d) Locus of R is a circle, if $\alpha = \frac{\pi}{4}$

267. In aΔ ABC, medians, AD and BE are drawn. If $AD = 4$, $\angle DAB = \frac{\pi}{6}$ and $\angle ABE = \frac{\pi}{3}$, then the area of the ΔABC is
a) $\frac{8}{3}$ sq units b) $\frac{16}{3}$ sq units c) $\frac{32}{3\sqrt{3}}$ sq units d) $\frac{64}{3}$ sq units

268. Point Q is symmetric to $P(4, -1)$ with respect to the bisector of the first quadrant. The length of PQ is
a) $3\sqrt{2}$ b) $5\sqrt{2}$ c) $7\sqrt{2}$ d) $9\sqrt{2}$

269. The area of triangle formed by the points $(a, b + c), (b, c + a), (c, a + b)$ is equal to
a) abc b) $a^2 + b^2 + c^2$ c) $ab + bc + ca$ d) 0

270. Let $0 \le \theta \le \frac{\pi}{2}$ and $x = X\cos\theta + Y\sin\theta, y = X\sin\theta - Y\cos\theta$ such that $x^2 + 4xy + y^2 = aX^2 + bY^2$, where a, b are constants, then
a) $a = -1, b = 3, \theta = \frac{\pi}{4}$ b) $a = 1, b = -3, 2 = \frac{\pi}{3}$ c) $a = 3, b = -1, \theta = \frac{\pi}{4}$ d) $a = 3, b = -1, \theta = \frac{\pi}{3}$

271. If a point $P(4, 3)$ is shifted by a distance $\sqrt{2}$ unit parallel to the line $y = x$, then coordinates of p in new position are
a) $(5, 4)$ b) $(5 + \sqrt{2}, 4 + \sqrt{2})$ c) $(5 - \sqrt{2}, 4 - \sqrt{2})$ d) None of these

272. From the top of a cliff 50m high, the angles of depression of the top and bottom of a tower are observed to be 30° and45°. The height of tower is
a) 50m b) $50\sqrt{3}$m c) $50(\sqrt{3} - 1)$m d) $50\left(1 - \frac{\sqrt{3}}{3}\right)$m

273. The vertex of an equilateral triangle is $(2, -1)$ and the equation of its base is $x + 2y = 1$, the length of its sides is
a) $\frac{2}{\sqrt{15}}$ b) $\frac{4}{3\sqrt{3}}$ c) $\frac{1}{\sqrt{5}}$ d) $\frac{4}{\sqrt{15}}$

274. If the elevation of the sun is 30°, then the length of the shadow cast by a tower of 150 ft. height is
a) $75\sqrt{3}$ ft. b) $200\sqrt{3}$ ft. c) $150\sqrt{3}$ ft. d) None of these

275. The area of the triangle formed by the points $(2, 2), (5, 5), (6, 7)$ is equal to (in square unit)
a) $\frac{9}{2}$ b) 5 c) 10 d) $\frac{3}{2}$

276. The orthocenter of the ΔOAB, where O is the origin, $A(6, 0)$ and $B(3, 3\sqrt{3})$ is
a) $(9/2, \sqrt{3}/2)$ b) $(3, \sqrt{3})$ c) $(\sqrt{3}, 3)$ d) $(3, -\sqrt{3})$

277. In a Δ ABC, $\cos A + \cos B + \cos C$ is equal to
a) $1 + \frac{r}{R}$ b) $1 - \frac{r}{R}$ c) $1 - \frac{R}{r}$ d) $1 + \frac{R}{r}$

278. If in a $\Delta\, PQR$, $\sin P$, $\sin Q$, $\sin R$ are in AP, then

a) The altitudes are in AP
b) The altitudes are in HP
c) The medians are in GP
d) The medians are in AP

279. The angle of elevation of an object on a hill from a point on the ground is 30°. After walking 120 metres the elevation of the object is 60°. The height of the hill is

a) 120 m
b) $60\sqrt{3}$ m
c) $120\sqrt{3}$ m
d) 60 m

280. If the median AD of $\Delta\, ABC$, makes an angle θ with side AB, then $\sin(A-\theta)$ is equal to

a) $\left(\frac{b}{c}\right)$ cosec θ
b) $\left(\frac{b}{c}\right)$ sin θ
c) $\left(\frac{c}{b}\right)$ sin θ
d) $\left(\frac{c}{b}\right)$ cosec θ

281. The angle of elevation of a cliff at a point$A$ on the ground and a point B, 100 m vertically at A are α and β respectively. The height of the cliff is

a) $\frac{100\cot\alpha}{\cot\alpha-\cot\beta}$
b) $\frac{100\cot\beta}{\cot\alpha-\cot\beta}$
c) $\frac{100\cot\beta}{\cot\beta-\cot\alpha}$
d) $\frac{100\cot\beta}{\cot\beta+\cot\alpha}$

282. A quadrilateral $ABCD$ in which $AB=a, BC=b, CD=c$ and $DA=d$ is such that one circle can be inscribed in it and another circle can be circumscribed about it, then $\cos A$ is equal to

a) $\frac{ad+bc}{ad-dc}$
b) $\frac{ad-bc}{ad+bc}$
c) $\frac{ac+bd}{ac-bd}$
d) $\frac{ac-bd}{ac+bd}$

283. If λ be the perimeter of the $\Delta\, ABC$, then $b\cos^2\left(\frac{C}{2}\right)+c\cos^2\left(\frac{B}{2}\right)$ is equal to

a) λ
b) 2λ
c) $\frac{\lambda}{2}$
d) None of these

284. If in the $\Delta\, ABC, \angle B=45°$, then $a^4+b^4+c^4$ is equal to

a) $2a^2(b^2+c^2)$
b) $2c^2(a^2+b^2)$
c) $2b^2(a^2+c^2)$
d) $2(a^2b^2+b^2c^2+c^2a^2)$

285. The ratio in which the line $x+y=4$ divides the line joining the points $(1,-1)$ and $(5,7)$ is

a) 1: 2
b) 2: 1
c) 1: 3
d) 3: 1

286. The coordinates of the orthocenter of the triangle formed by (0, 0), (8, 0), (4, 6) is

a) (4, 0)
b) (6, 3)
c) (6, 0)
d) None of these

287. If $\Delta = a^2-(b-c)^2$, where Δ is the area of $\Delta\, ABC$, then $\tan A$ is equal to

a) $\frac{15}{16}$
b) $\frac{8}{15}$
c) $\frac{8}{17}$
d) $\frac{1}{2}$

288. In $\Delta\, ABC, \frac{b+c}{a}$ is equal to

a) $\frac{\cos\frac{1}{2}(B-C)}{\sin\frac{1}{2}A}$
b) $\frac{\sin\frac{1}{2}(B-C)}{\cos\frac{1}{2}A}$
c) $\frac{\cos\frac{1}{2}(B+C)}{\sin\frac{1}{2}A}$
d) $\frac{\cos\frac{1}{2}(B+C)}{\cos\frac{1}{2}A}$

289. In a cubical hall $ABCDPQRS$ with each side 10 m, G is the centre of the wall $BCRQ$ and T is the mid point of the side AB. The angle of elevation of G at the point T is

a) $\sin^{-1}(1/\sqrt{3})$
b) $\cos^{-1}(1/\sqrt{3})$
c) $\cot^{-1}(1/\sqrt{3})$
d) None of these

290. If p_1, p_2, p_3 are respectively the perpendicular from the vertices of a triangle to the opposite sides, then p_1, p_2, p_3 is equal to

a) $a^2b^2c^2$
b) $2a^2b^2c^2$
c) $\frac{4a^2b^2c^2}{R^2}$
d) $\frac{a^2b^2c^2}{8R^2}$

291. An observer standing on a 300 m high tower observes two boats in the same direction their angles of depression are 60° and 30° respectively. The distance between boats is

a) 173.2 m
b) 346.4 m
c) 25 m
d) 72 m

292. If the length of the sides of a triangle are 3, 4 and 5 unit, then R is

a) 3.5
b) 3.0
c) 2.0
d) 2.5

293. In a triangle $\left(1-\frac{r_1}{r_2}\right)\left(1-\frac{r_1}{r_3}\right)=2$, then the triangle is

a) Right angled
b) Isosceles
c) Equilateral
d) None of these

294. If in a $\Delta\, ABC, a\tan A+b\tan B=(a+b)\tan\frac{A+B}{2}$, then

a) $A = B$ b) $A = -B$ c) $A = 2B$ d) $B = 2A$

295. The coordinates of the circumcentre of the triangle with vertices $(8, 6), (8, -2)$ and $(2, -2)$ are

a) $\left(6, \frac{2}{3}\right)$ b) $(8, 2)$ c) $(5, -2)$ d) $(5, 2)$

296. Points D, E are taken on the side BC of a $\Delta\, ABC$ such that $BD = DE = EC$. If $\angle\, BAD = x, \angle\, DAE = y, \angle\, EAC = z$, then the value of $\frac{\sin(x+y)\sin(y+z)}{\sin x \sin z}$ is equal to

a) 1 b) 2 c) 4 d) None of these

297. An observer on the top of tree, finds the angle of depression of a car moving towards the tree to be 30°. After 3 min this angle becomes 60°. After how much more time, the car will reach the tree?

a) 4 min b) 4.5 min c) 1.5 min d) 2 min

298. If PQ be a vertical tower subtending angles α, β and γ at the points A, B and C respectively on the line in the horizontal plane through the foot D of tower and on the same side of it, then $BC \cot \alpha - CA \cot \alpha + AB \cot \gamma$ is equal to

a) 0 b) 1 c) 2 d) None of these

299. The equation of the three sides of a triangle are $x = 2, y + 1 = 0$ and$x + 2y = 4$. The coordinates of the circumcentre of the triangles are

a) $(4, 0)$ b) $(2, -1)$ c) $(0, 4)$ d) $(-1, 2)$

300. On one bank of river there is a tree. On another bank, an observer makes an angle of elevation of 60°at the top of the tree. The angle of elevation of the top of the tree at a distance 20 m away the bank is 30°. The width of the river is

a) 20 m b) 10 m c) 5 m d) 1 m

301. Consider the following statements :

1. In $\Delta\, ABC, a = \sqrt{3} + 1, \angle B = 30°, \angle C = 45°$, then c is equal to
2. In a triangle, if $a^2 + b^2 + c^2 = 8R^2$, then the triangle is right angled
3. In a $\Delta\, ABC, a = 2, b = 3, c = 4$, then $\cos A = \frac{7}{8}$

Which of the statements given above is/are correct?

a) Only (1) b) Only (2) c) Only (3) d) All of these

302. If A is the area and $2s$ the sum of three sides of a triangle, then

a) $A \leq \frac{s^2}{3\sqrt{3}}$ b) $A \leq \frac{s^2}{2}$ c) $A > \frac{s^2}{\sqrt{3}}$ d) None of these

303. A flag staff of 5 m high stands on a building of 25 m high. At an observer at a height of 30 m, the flag staff and the building subtend equal angles. the distance of the observer from the top of the flag staff is

a) $\frac{5\sqrt{3}}{2}$ m b) $5\sqrt{\frac{3}{2}}$ m c) $5\sqrt{\frac{2}{3}}$ m d) None of these

304. A tower of height b subtends an angle at a point O on the level of the foot of the tower and at a distance $'a'$ from the foot of the tower. If the pole mounted on the tower also subtends an equal angle at O, the height of the pole is

a) $b\left(\frac{a^2 - b^2}{a^2 + b^2}\right)$ b) $b\left(\frac{a^2 + b^2}{a^2 - b^2}\right)$ c) $a\left(\frac{a^2 - b^2}{a^2 + b^2}\right)$ d) $a\left(\frac{a^2 + b^2}{a^2 - b^2}\right)$

305. If the angles of a triangle are in the ratio 4: 1: 1, then the ratio of the longest side to the perimeter is

a) $\sqrt{3}: (2 + \sqrt{3})$ b) 1: 6 c) $1: (2 + \sqrt{3})$ d) 2: 3

306. If two angles of a triangle are 45° and $\tan^{-1}(2)$, then the third angle is

a) 60° b) 75° c) $\tan^{-1} 3$ d) 90°

307. The angle of elevation of top of a tower form a point on the ground is 30° and it is 60° when it is viewed from a point located 40m away the initial point towards the tower. The height of the tower is

a) $-20\sqrt{3}$ m b) $\frac{\sqrt{3}}{20}$ m c) $-\frac{\sqrt{3}}{20}$ m d) $20\sqrt{3}$ m

308. The orthocentre of the triangle formed by the points $(0, 0), (4, 0)$ and $(3, 4)$ is

a) $(2, 0)$ b) $\left(\frac{3}{2}, 2\right)$ c) $\left(\frac{3}{4}, 3\right)$ d) $\left(3, \frac{3}{4}\right)$

309. In order to remove xy-term from the equation $5x^2 + 4\sqrt{3}\,xy + 9y^2 - 8 = 0$ the coordinate axes must be rotated through an angle

a) $\pi/6$ b) $\pi/4$ c) $\pi/3$ d) $\pi/2$

310. If C is a point on the line segment joining $A(-3, 4)$ and $B(2, 1)$ such that $AC = 2\,BC$, then the coordinate of C is

a) $\left(\frac{1}{3}, 2\right)$ b) $\left(2, \frac{1}{3}\right)$ c) $(2, 7)$ d) $(7, 2)$

311. The image of the centre of the circle $x^2 + y^2 = a^2$ with respect to the mirror image $x + y = 1$, is

a) $\left(\frac{1}{\sqrt{2}}, \sqrt{2}\right)$ b) $(\sqrt{2}, \sqrt{2})$ c) $(\sqrt{2}, 2\sqrt{2})$ d) None of these

312. In a $\Delta\, ABC$, $\operatorname{cosec} A(\sin B \cos C + \cos B \sin C)$ is equal to

a) $\frac{c}{a}$ b) $\frac{a}{c}$ c) 1 d) $\frac{c}{ab}$

313. If $A(3, 5), B(-5, -4), C(7, 10)$ are the vertices of a parallelogram, taken in the order, then the coordinates of the fourth vertex are

a) $(10, 19)$ b) $(15, 19)$ c) $(19, 10)$ d) $(19, 15)$

314. Two pillars of equal height stand on either side of a road-way which is 60 m wide. At a point in the road-way between the pillars, the elevation of the top of pillars are 60°and 30°.The height of the pillars is

a) $15\sqrt{3}$ m b) $\frac{15}{\sqrt{3}}$ m c) 15 m d) 20 m

315. From the top of a cliff of height a, the angle of depression of the foot of a certain tower is found to be double the angle of elevation of the top of the tower of height h. If θ be the angle of elevation, then its value is

a) $\cos^{-1}\sqrt{\frac{2h}{a}}$ b) $\sin^{-1}\sqrt{\frac{2h}{a}}$ c) $\sin^{-1}\sqrt{\frac{a}{2-h}}$ d) $\tan^{-1}\sqrt{3 - \frac{2h}{a}}$

316. The points $A(2a, 4a), B(2a, 6a)$ and $C\left(2a + \sqrt{3}a, 5a\right), a > 0$ are the vertices of

a) An isosceles triangle b) A right angled triangle

c) An acute angled triangle d) None of the above

317. If a vertex of a triangle is $(1, 1)$ and the mid points of two sides through the vertex are $(-1, 2)$ and $(3, 2)$, then the centroid of the triangle is

a) $\left(1, \frac{7}{3}\right)$ b) $\left(\frac{1}{3}, \frac{7}{3}\right)$ c) $\left(-\frac{1}{3}, \frac{7}{3}\right)$ d) $\left(-1, \frac{7}{3}\right)$

318. P, Q, R and S are the points on the line joining the points $P(a, x)$ and $T(b, y)$ such that $PQ = QR = RS = ST$, then $\left(\frac{5a+3b}{8}, \frac{5x+3y}{8}\right)$ is the mid point of

a) PQ b) QR c) RS d) ST

319. Orthocenter of the triangle formed by the lines $x + y = 1$ and $xy = 0$ is

a) $(0, 0)$ b) $(0, 1)$ c) $(1, 0)$ d) $(-1, 1)$

320. The circumcentre of the triangle with vertices $(0, 30)$, $(4, 0)$ and $(30, 0)$ is

a) $(10, 10)$ b) $(10, 12)$ c) $(12, 12)$ d) $(17, 17)$

321. A vertical pole (more than 100 m high) consists of two portions, the lower being-one third of the whole, if the upper portion subtends and angle $\tan^{-1}\frac{1}{2}$ at a point in a horizontal plane through the foot of the pole and distance 40 ft from it, then the height of the pole is

a) 100 ft b) 120 ft c) 150 ft d) None of these

322. If a point $P(4, 3)$ is rotated through an angle 45° in anti-clockwise direction about origin, then coordinates of P in new position are

a) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ b) $\left(-\frac{7}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ c) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ d) $\left(\frac{1}{\sqrt{2}}, -\frac{7}{\sqrt{2}}\right)$

323. The horizontal distance between two towards is 60m and the angle of depression of the top of the first

tower as seen from the top of the second is 30°. If the height of the second tower be 150m, then the height of the first tower is

a) 90 m b) $(150 - 60\sqrt{3})$m c) $(150 + 20\sqrt{3})$m d) None of the above

324. In a $\Delta\, ABC$, if $\sin A + \sin B + \sin C)(\sin A + \sin B - \sin C) = 3 \sin A \sin B$, then the angle C is equal to

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$

325. From the bottom of a pole of height h the angle of elevation of the top of a tower is α and the pole subtends an angle β at the top of the tower. The height of the tower is

a) $\frac{h \tan(\alpha - \beta)}{\tan(\alpha - \beta) - \tan\alpha}$ b) $\frac{h \cot(\alpha - \beta)}{\cot(\alpha - \beta) - \cot\alpha}$ c) $\frac{\cot(\alpha - \beta)}{\cot(\alpha - \beta) - \cot\alpha}$ d) None of these

326. A variable line through the point$\left(\frac{1}{5}, \frac{1}{5}\right)$ cuts the coordinate axes in the points A and$B$. If the point P divides AB internally in the ratio 3:1, then the locus of P is

a) $3y + x = 20xy$ b) $y + 3x = 20xy$ c) $x + y = 20xy$ d) $3x + 3y = 20xy$

327. If the points (1, 2) and (3, 4) were to be on the same side of the line $3x - 5y + a = 0$, then

a) $1 < a < 6$ b) $7 < a < 11$ c) $a > 11$ d) $a < 7$ or $a > 11$

328. The area of the region bounded by the lines $y = |x - 2|,\ x = 1,\ x = 3$ and the x-axis is

a) 1 b) 2 c) 3 d) 4

329. In triangle ABC, the value of $\frac{\cot\frac{A}{2}\cot\frac{B}{2} - 1}{\cot\frac{A}{2}\cot\frac{B}{2}}$ is

a) $\frac{a}{a + b + c}$ b) $\frac{c}{a + b + c}$ c) $\frac{2a}{a + b + c}$ d) $\frac{2c}{a + b + c}$

330. If the coordinates of the centroid and a vertex of an equilateral triangle are (1, 1) and (1, 2) respectively, then the coordinates of another vertex, are

a) $\left(\frac{2 - \sqrt{3}}{2}, -\frac{1}{2}\right)$ b) $\left(\frac{2 + 3\sqrt{3}}{2}, -\frac{1}{2}\right)$ c) $\left(\frac{2 + \sqrt{3}}{2}, \frac{1}{2}\right)$ d) None of these

331. In any triangle ABC, $c^2 \sin 2B + b^2 \sin 2C$ is equal to

a) $\frac{\Delta}{2}$ b) Δ c) 2Δ d) 4Δ

332. A house of height 100 m subtends a right angle at the window of an opposite house. If the height of the window be 64 m, then the distance between the two houses is

a) 48 m b) 36 m c) 54 m d) 72 m

333. The angle of elevation of the top of a vertical pole when observed from each vertex of a regular hexagon is $\frac{\pi}{3}$. If the area of the circle circumscribing the hexagon be A metre^2, then the area of the hexagon is

a) $\frac{3\sqrt{3}\,A}{8}$ m^2 b) $\frac{\sqrt{3}\,A}{\pi}$ m^2 c) $\frac{3\sqrt{3}\,A}{4\pi}$ m^2 d) $\frac{3\sqrt{3}\,A}{2\pi}$ m^2

334. The area of the segment of a circle of radius a subtending an angle of 2α at the centre is

a) $a^2\left(\alpha + \frac{1}{2}\sin 2\alpha\right)$ b) $\frac{1}{2}a^2 \sin 2\alpha$ c) $a^2\left(\alpha + \frac{1}{2}\sin 2\alpha\right)$ d) $a^2\alpha$

335. A man of height 6 ft. observes the top of a tower and the foot of the tower at angles of 45° and 30° of elevation and depression respectively. The height of the tower is

a) 13.79 m b) 14.59 m c) 14.29 m d) None of these

336. If the sides of the triangles are $5k, 6k, 5k$ and radius of incircle is 6, the value of k is equal to

a) 4 b) 5 c) 6 d) 7

337. At the foot of the mountain the elevation of its summit is 45°, after ascending 100 m towards the mountain up a slope of 30° inclination, the elevation is found to be 60°. The height of the mountain is

a) $\frac{\sqrt{3} + 1}{2}$m b) $\frac{\sqrt{3} - 1}{2}$m c) $\frac{\sqrt{3} + 1}{2\sqrt{3}}$m d) None of these

338. At each end of a horizontal line of length $2a$, the angular elevation of the peak of a vertical tower is θ and that at its middle point it is ϕ. The height of the peak is

a) $a \sin \theta \sin \phi$

b) $\dfrac{a \sin \theta \sin \phi}{\sqrt{\sin(\theta+\phi)\sin(\phi-\theta)}}$

c) $\dfrac{a \cos \theta \cos \phi}{\sqrt{\cos(\phi+\theta)\cos(\phi-\theta)}}$

d) None of the above

339. In a triangleABC, $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$. if $a = \frac{1}{\sqrt{6}}$, then the area of the triangle (in square unit) is

a) 1/24 b) $\sqrt{3}/24$ c) $\dfrac{1}{8}$ d) $\dfrac{1}{\sqrt{3}}$

340. A ladder leavs again a wall at an angle α to the horizontal. Its foot is pulled away through a distance a_1 so that it slides a distance $b_1$down the wall and rests inclined at angle β with the horizontal. It foot is further pulled aways through a_2, so that it slides a further distance b_2 down the wall and is now, inclined at an angle γ. If $a_1 a_2 = b_1 b_2$, then

a) $\alpha + \beta + \gamma$ is greater than π

b) $\alpha + \beta + \gamma$ is equal to π

c) $\alpha + \beta + \gamma$ is less than π

d) Nothing can be said about $\alpha + \beta + \gamma$

341. In a ΔABC, $\frac{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}{4b^2c^2}$ equals

a) $\cos^2 A$ b) $\cos^2 B$ c) $\sin^2 A$ d) $\sin^2 B$

342. The locus of a point P which moves such that $2PA = 3PB$, where $A(0,0)$ and $B(4,-3)$ are points, is

a) $5x^2 - 5y^2 - 72x + 54y + 225 = 0$

b) $5x^2 + 5y^2 - 72x + 54y + 225 = 0$

c) $5x^2 + 5y^2 + 72x - 54y + 225 = 0$

d) $5x^2 + 5y^2 - 72x - 54y - 225 = 0$

343. The incentre of the triangle formed by (0, 0), (5, 12), (16, 12) is

a) $(7, 9)$ b) $(9, 7)$ c) $(-9, 7)$ d) $(-7, 9)$

344. In $\Delta\, ABC$, if $\tan\frac{A}{2}\tan\frac{C}{2} = \frac{1}{2}$, then a, b, c are in

a) AP b) GP c) HP d) None of these

345. In $\Delta\, ABC$, if $\sin^2\frac{A}{2}, \sin^2\frac{B}{2}, \sin^2\frac{C}{2}$ be in HP, then a, b, c will be in

a) AP b) GP c) HP d) None of these

346. If O is the origin and $P(2,3)$ and $Q(4,5)$ are two points, then $OP.OQ \cos \angle POQ =$

a) 8 b) 15 c) 22 d) 23

347. If in a $\Delta\, ABC$, $2b^2 = a^2 + c^2$, then $\frac{\sin 3B}{\sin B}$ is equal to

a) $\dfrac{c^2 - a^2}{2ca}$ b) $\dfrac{c^2 - a^2}{ca}$ c) $\left(\dfrac{c^2 - a^2}{ca}\right)^2$ d) $\left(\dfrac{c^2 - a^2}{2ca}\right)^2$

348. A spherical balloon of radius r subtends an angle α at the eye of an observer. If the angle of elevation of the centre of the balloon be β, then height of the centre of the balloon is

a) $r \operatorname{cosec}\left(\frac{\alpha}{2}\right) \sin \beta$ b) $r \operatorname{cosec} \alpha \sin\left(\frac{\beta}{2}\right)$ c) $r \sin\left(\frac{\alpha}{2}\right) \operatorname{cosec} \beta$ d) $r \sin \alpha \operatorname{cosec}\left(\frac{\beta}{2}\right)$

349. A point $P(2, 4)$ transtates to the point Q along the parallel to the positive direction of x-axis by 2 units. If O be the origin, then $\angle\, OPQ$ is

a) $\sin^{-1}\sqrt{\dfrac{399}{400}}$ b) $\cos^{-1}\left(\dfrac{1}{20}\right)$ c) $-\sin^{-1}\left(\sqrt{\dfrac{399}{400}}\right)$ d) None of these

350. From the top of a hill h meters high, the angles of depressions of the top and the bottom of a pillar are α and β respectively. The height (in meters) of the pillar is

a) $\dfrac{h(\tan\beta - \tan\alpha)}{\tan\beta}$ b) $\dfrac{h(\tan\alpha - \tan\beta)}{\tan\alpha}$ c) $\dfrac{h(\tan\beta + \tan\alpha)}{\tan\beta}$ d) $\dfrac{h(\tan\beta + \tan\alpha)}{\tan\alpha}$

351. If $G(1, 4)$ is the centroid of triangle ABC having its two vertices A and B at $(4, -3)$ and $(-9, 7)$ respectively, then area of the triangle ABC in square units, is

a) $\dfrac{138}{2}$ b) $\dfrac{319}{2}$ c) $\dfrac{183}{2}$ d) $\dfrac{381}{2}$

352. If $a = 2\sqrt{2}, b = 6, A = 45°$, then

a) No triangle is possible

b) One triangle is possible

c) Two triangles are possible
d) Either no triangle or two triangles are possible

353. Two poles of equal height stand on either side of a 100 m wide road. At a point between the poles the angles of elevation of the topes of the poles are 30° and 60°. The height of each pole is
a) 25 m
b) $25\sqrt{3}$ m
c) $\frac{100}{\sqrt{3}}$ m
d) None of these

354. Area (in sq unit) enclosed by $y = 1, 2x + y = 2$ and $x + y = 2$ is
a) $\frac{1}{2}$ sq unit
b) $\frac{1}{4}$ sq unit
c) 1 sq unit
d) 2 sq units

355. The feet of the perpendicular drawn from P to the sides of a $\Delta\, ABC$ are collinear, then P is
a) Circumcentre of $\Delta\, ABC$
b) Lies on the circumcircle of $\Delta\, ABC$
c) Excentre of $\Delta\, ABC$
d) None of the above

356. Let $O(0,0), P(3,4), Q(6,0)$ be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The coordinates of R are
a) $\left(\frac{4}{3}, 3\right)$
b) $\left(3, \frac{2}{3}\right)$
c) $\left(3, \frac{4}{3}\right)$
d) $\left(\frac{4}{3}, \frac{2}{3}\right)$

357. A tree is broken by wind, its upper part touches the ground at a point 10 metres from the foot of the tree and makes an angle of 45° with the ground. The entire length of the tree is
a) 15 metres
b) 20 metres
c) $10(1 + \sqrt{2})$ metres
d) $10\left(1 + \frac{\sqrt{3}}{2}\right)$ metres

358. Without change of axes the origin is shifted to (h, k), then from the equation $x^2 + y^2 - 4x + 6y - 7 = 0$ the terms containing linear powers are missing. The point (h, k) is
a) $(3, 2)$
b) $(-3, 2)$
c) $(2, -3)$
d) $(-2, -3)$

359. The horizontal distance between two towers is 60 m and the angle of depression of the top of the first tower as seen from the top of the second is 30°. If the height of the second tower be 150 m, then the height of the first tower is
a) $(150 - 60\sqrt{3})$m
b) 90 m
c) $(150 - 20\sqrt{3})$m
d) None of these

360. Let $a > 0, b > 0$. The sum of the distance of the point (a, b) from the lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{b} + \frac{y}{a} = 1$ is
a) $\frac{a+b}{2}$
b) $\sqrt{ab}$
c) $\frac{2ab}{a+b}$
d) $\sqrt{a^2 + b^2}$

361. In $\Delta\, ABC$, if $\frac{1}{b+c} + \frac{1}{c+a} = \frac{3}{a+b+c}$, then C is equal to
a) 90°
b) 60°
c) 45°
d) 30°

362. If in a $\Delta\, ABC$, $4 \sin A = 4 \sin B = 3 \sin C$, then $\cos C$ is equal to
a) 1/3
b) 1/9
c) 1/27
d) 1/18

363. In a $\Delta\, ABC$, $(b + c)(bc) \cos A + (a + c)(ac) \cos B + (a + b)(ab) \cos C$ is
a) $a^2 + b^2 + c^2$
b) $a^3 + b^3 + c^3$
c) $(a + b + c)(a^2 + b^2 + c^2)$
d) $(a + b + c)(ab + bc + ca)$

364. If a flag staff of 6 m high placed on the top of a tower throws a shadow of $2\sqrt{3}$ m along the ground, then the angle (in degrees) that the sun makes with the ground is
a) 60°
b) 80°
c) 75°
d) None of these

365. If the angles of a $\Delta\, ABC$ be in AP, then
a) $c^2 = a^2 + b^2 - ab$
b) $b^2 = a^2 + c^2 - ac$
c) $a^2 = b^2 + c^2 - ac$
d) $b^2 = a^2 + c^2$

366. The sides of a triangle are respectively 7 cm, $4\sqrt{3}$ cm and $\sqrt{13}$ cm, then the smallest angle of the triangle is
a) $\frac{\pi}{6}$
b) $\frac{\pi}{3}$
c) $\frac{\pi}{4}$
d) $\frac{\pi}{5}$

367. The straight lines $x + y = 0, 3x + y - 4 = 0$ and $x + 3y - 4 = 0$ form a triangle which is
a) Right angled
b) Equilateral
c) Isosceles
d) None of these

368. A tower of x metres height has flag staff at its top. The tower and the flag staff subtend equal angles at a point distant y metres from the foot of the tower. Then, the length of the flag staff in metres is
a) $y\left(\frac{x^2 - y^2}{x^2 + y^2}\right)$
b) $x\left(\frac{x^2 + y^2}{y^2 - x^2}\right)$
c) $x\left(\frac{x^2 + y^2}{x^2 - y^2}\right)$
d) $x\left(\frac{x^2 - y^2}{x^2 + y^2}\right)$

369. If $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$, then the points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) are

a) Vertices of an equilateral triangle
b) Vertices of a right angled triangle
c) Vertices of an isosceles triangle
d) None of the above

370. If two angles of $\Delta\, ABC$ are 45° and 60°, then the ratio of the smallest and the greatest sides are

a) $(\sqrt{3}-1):1$
b) $\sqrt{3}:\sqrt{2}$
c) $1:\sqrt{3}$
d) $\sqrt{3}:1$

371. An aeroplane flying horizontally 1 km above the ground is observed at an elevation of 60° and after 10 s the elevation is observed to be 30°. The uniform speed of the aeroplane (in km/h) is

a) 240
b) $240\sqrt{3}$
c) $60\sqrt{3}$
d) None of these

372. The angle of elevation of the top of a tower from the top and bottom of a building of height $'a'$ are 30° and 45° respectively. If the tower and the building stand at the same level, the height of the tower is

a) $\dfrac{a(3+\sqrt{3})}{2}$
b) $a(\sqrt{3}+1)$
c) $a\sqrt{3}$
d) $a(\sqrt{3}-1)$

373. If in a $\Delta\, ABC, 2b^2 = a^2 + c^2$, then $\frac{\sin 3B}{\sin B}$ is equal to

a) $\dfrac{c^2-a^2}{2ca}$
b) $\dfrac{c^2-a^2}{ca}$
c) $\left(\dfrac{c^2-a^2}{ca}\right)^2$
d) $\left(\dfrac{c^2-a^2}{2ca}\right)^2$

374. The angle of elevation of a cloud from a point h mt. above is $\theta°$ and the angle of depression of its reflection in the lake is ϕ. Then, the height is

a) $\dfrac{h\sin(\phi-\theta)}{\sin(\phi+\theta)}$
b) $\dfrac{h\sin(\phi+\theta)}{\sin(\phi-\theta)}$
c) $\dfrac{h\sin(\theta+\phi)}{\sin(\theta-\phi)}$
d) None of these

375. In aΔABC, $\angle B = \frac{\pi}{3}$ and $\angle C = \frac{\pi}{4}$. If D divides BC internally in ratio 1:3, then the value of $\frac{\sin\angle BAD}{\sin\angle CAD}$ is

a) $\dfrac{1}{\sqrt{3}}$
b) $\dfrac{1}{\sqrt{6}}$
c) $\sqrt{\dfrac{2}{3}}$
d) $\dfrac{1}{3}$

376. Let equation of the side BC of a $\Delta\, ABC$ be $x+y+2=0$. If coordinates of its orthocentre and circumcentre are (1, 1) and (2, 0) respectively, then radius of the circumcircle of $\Delta\, ABC$ is

a) 3
b) $\sqrt{10}$
c) $2\sqrt{2}$
d) None of these

377. In a ΔABC, if $b+c=2a$ and $\angle A = 60°$, then ΔABC is

a) Equilateral
b) Right angled
c) Isosceles
d) Scalene

378. The orthocenter of the triangle with vertices $(-2,-6), (-2,4)$ and $(1,3)$ is

a) $(3,1)$
b) $(1,1/3)$
c) $(1,3)$
d) None of these

379. If $a>0, b>0$ the maximum area of the triangle formed by the points $O(0,0), A(a\cos\theta, b\sin\theta)$ and $B(a\cos\theta, -b\sin\theta)$ is (in sq unit)

a) $\dfrac{ab}{2}$ when $\theta = \dfrac{\pi}{4}$
b) $\dfrac{3ab}{2}$ when $\theta = \dfrac{\pi}{4}$
c) $\dfrac{ab}{2}$ when $\theta = -\dfrac{\pi}{4}$
d) $a^2\, b^2$

380. If the coordinates of orthocentre O' and centroid G of a $\Delta\, ABC$ are (0,1) and (2,3) respectively, then the coordinates of the circumcentre are

a) $(3,2)$
b) $(1,0)$
c) $(4,3)$
d) $(3,4)$

381. The point P is equidistant from $A(1,3), B(-3,5)$ and $C(5,-1)$, then PA is equal to

a) 5
b) $5\sqrt{5}$
c) 25
d) $5\sqrt{10}$

382. The coordinates of the incentre of the triangle having sides
$3x-4y=0, 5x+12y=0$
and $y-15=0$ are

a) $-1,8$
b) $1,-8$
c) $2,6$
d) None of these

383. The mid point of the line joining the points $(-10,8)$ and $(-6,12)$ divides the line joining the points $(4,-2)$ and $(-2,4)$ in the ratio

a) 1: 2 internally
b) 1: 2 externally
c) 2: 1 internally
d) 2: 1 externally

384. The centre of circle inscribed in square formed by the lines $x^2-8x+12=0$ and $y^2-14y+45=0$, is

a) $(4, 7)$ b) $(7, 4)$ c) $(9, 4)$ d) $(4, 9)$

385. If A, A_1, A_2, A_3 be the areas of the incircle and excircles, then $\frac{1}{\sqrt{A_1}}+\frac{1}{\sqrt{A_2}}+\frac{1}{\sqrt{A_3}}$ is equal to

a) $\frac{1}{\sqrt{A}}$ b) $\frac{2}{\sqrt{A}}$ c) $\frac{3}{\sqrt{A}}$ d) $\frac{4}{\sqrt{A}}$

386. If $x = X\cos\theta - Y\sin\theta, y = X\sin\theta + Y\cos\theta$ and $x^2+4xy+y^2 = AX^2+BY^2, 0 \le \theta \le \frac{\pi}{2}$, then

a) $\theta = \frac{\pi}{6}$ b) $\theta = \frac{\pi}{4}$ c) $A = -6$ d) $B = 1$

387. If $P(1,0), Q = (-1,0)$ and $R = (2,0)$ are three given points, then the locus of a point S satisfying the relation $SQ^2 + SR^2 = 2SP^2$ is

a) A straight line parallel to x-axis b) A circle through origin

c) A circle with centre at the origin d) A straight line parallel to y-axis

388. The triangle formed by $x^2 - 3y^2 = 0$ and $x = 4$ is

a) Isosceles b) Equilateral c) Right angled d) None of these

389. The angle of elevation of the top of an incomplete vertical pillar at a horizontal distance of 100 m from its base is 45°. If the angle of elevation of the top of the complete pillar at the same point is to be 60°, then the height of the incomplete pillar is to be increased by

a) $50\sqrt{2}$ m b) 100 m c) $100(\sqrt{3}-1)$ m d) $100(\sqrt{3}+1)$ m

390. If $O(0,0), A(4,0)$ and $B(0,3)$ are the vertices of a triangle OAB, then the coordinates of the excentre opposite to the vertex $O(0,0)$ are

a) $(12, 12)$ b) $(6, 6)$ c) $(3, 3)$ d) None of these

391. A tower subtends an angle α at a point A in the plane of its base and angle of depression of the foot of the tower at a point l metres just above A is β. The height of the tower is

a) $l\tan\beta\cot\alpha$ b) $l\tan\alpha\cot\beta$ c) $l\tan\alpha\tan\beta$ d) $l\cot\alpha\cot\beta$

392. Observe the following statements

I. In $\Delta\, ABC\ b\cos^2\frac{C}{2} + c\cos^2\frac{B}{2} = s$

II. $\Delta\, ABC, \cot\frac{A}{2} = \frac{b+c}{2} \Rightarrow B = 90°$

Which of the following is correct?

a) Both I and II are true

b) I is true, II is false

c) I is false, II is false

d) Both I and II are false

393. If A and B are two points on one bank of a straight river and C, D are two other points on the other bank of river. If direction from A to B is same as that from C to D and $AB = a, \angle CAD = \alpha, \angle DAB = \beta, \angle CBA = \gamma$, then CD is equal to

a) $\frac{a\sin\beta\sin\gamma}{\sin\alpha\sin(\alpha+\beta+\gamma)}$ b) $\frac{a\sin\alpha\sin\gamma}{\sin\beta\sin(\alpha+\beta+\gamma)}$ c) $\frac{a\sin\alpha\sin\beta}{\sin\gamma\sin(\alpha+\beta+\gamma)}$ d) None of these

394. $ABCD$ is a square plot. The angle of elevation of the top of a pole standing at D from A or C is 30° and that from B is θ, then $\tan\theta$ is equal to

a) $\sqrt{6}$ b) $1/\sqrt{6}$ c) $\sqrt{3}/2$ d) $\sqrt{2/3}$

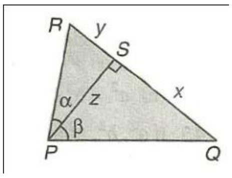
395. In a ΔABC, if $(\sqrt{3}-1)a = 2b, A = 3B$, then $\angle C$ is

a) 60° b) 120° c) 30° d) 45°

396. The angle of elevation of an object from a point on the level ground is α. Moving d meters on the ground towards the object, the angle of elevation is found to be β. Then the height (in meters) of the object is

a) $d\tan\alpha$ b) $d\cot\beta$ c) $\frac{d}{\cot\alpha+\cot\beta}$ d) $\frac{d}{\cot\alpha-\cot\beta}$

397. In a $\Delta\, PQR$ as shown in figure given that $x : y : z = 2 : 3 : 6$, then the value of $\angle QPR$ is



a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) None of these

398. If by shifting the origin at $(1, 1)$ the coordinates of a point P become $(\cos\theta, \cos\phi)$, then the original coordinates of P were
a) $(2\cos^2\theta/2, 2\cos^2\phi/2)$
b) $(2\sin^2\theta/2, 2\sin^2\phi/2)$
c) $(2\cos\theta/2, 2\cos\phi/2)$
d) $(2\sin\theta/2, 2\sin\phi/2)$

399. In a $\Delta\, ABC$, if $r_1 = 2r_2 = 3r_3$, then
a) $\frac{a}{b} = \frac{4}{5}$ b) $\frac{a}{b} = \frac{5}{4}$ c) $a + b - 2c = 0$ d) $2a = b + c$

400. In a $\Delta\, ABC$, if a, b, c are in AP, then the value of $\frac{\sin\frac{A}{2}\sin\frac{C}{2}}{\sin\frac{B}{2}}$ is
a) 1 b) $\frac{1}{2}$ c) 2 d) -1

401. The top of a hill observed from the top and bottom of a building h is at angles of elevation p and q respectively. The height of hill is
a) $\frac{h\cot q}{\cot q - \cot p}$ b) $\frac{h\cot p}{\cot p - \cot q}$ c) $\frac{h\tan p}{\tan p - \tan q}$ d) None of these

402. In a triangle, if $b = 20, c = 21$ and $\sin A = \frac{3}{5}$, then a is equal to
a) 12 b) 13 c) 14 d) 15

403. In $\Delta\, ABC, a = 2, b = 4$ and $\angle C = 60°$, then $\angle A$ and $\angle B$ are equal to
a) $90°, 30°$ b) $60°, 60°$ c) $30°, 90°$ d) $60°, 45°$

404. If in an equilateral triangle $R = \sqrt{3}$ cm, then the length of each side of the triangle is
a) 1 cm b) 2 cm c) 3 cm d) None of these

405. In a $\Delta\, ABC, \frac{b - c\cos A}{c - b\cos A}$ is equal to
a) $\frac{\sin B}{\sin C}$ b) $\frac{\cos C}{\cos B}$ c) $\frac{\cos B}{\cos C}$ d) None of these

406. In a $\Delta\, ABC$, if $2s = a + b + c$, then the value of the $\frac{s(s-a)}{bc} - \frac{(s-b)(s-c)}{bc}$ is equal to
a) $\sin A$ b) $\cos A$ c) $\tan A$ d) None of these

407. The centroid of a triangle is $(2, 7)$ and two of its vertices are $(4, 8)$ and $(-2, 6)$. The third vertex is
a) $(0, 0)$ b) $(4, 7)$ c) $(7, 4)$ d) $(7, 7)$

408. Each side of a square subtends an angle of 60° at the top of a tower h metres high standing in the centre of the square. If a is the length of each side of the square, then
a) $2\,a^2 = h^2$ b) $2\,h^2 = a^2$ c) $3\,a^2 = 2\,h^2$ d) $2\,h^2 = 3\,a^2$

409. In $\Delta\, ABC, (a - b)^2\cos^2\frac{C}{2} + (a + b)^2\sin^2\frac{C}{2}$ is equal to
a) a^2 b) b^2 c) c^2 d) None of these

410. If the area of a $\Delta\, ABC$ is given by $\Delta = a^2 - (b - c)^2$, then $\tan\left(\frac{A}{2}\right)$ is equal to
a) -1 b) 0 c) $\frac{1}{4}$ d) $\frac{1}{2}$

411. If $O'(4, 8/3)$ is the orthocentre of the triangle ABC the coordinates of whose vertices are $O(0,0), A(8,0)$ and $B(4,6)$, then the coordinates of the orthocentre of $\Delta\, O'AB$ are
a) $(0, 0)$ b) $(8, 0)$ c) $(4, 6)$ d) None of these

412. In a $\Delta\, ABC$, if $b^2 + c^2 = 3a^2$, then $\cot B + \cot C - \cot A$ is equal to

a) 1 b) $\frac{ab}{4\Delta}$ c) 0 d) $\frac{ac}{4\Delta}$

413. From the top of a tower, the angle of depression of a point on the ground is 60°.If the distance of this point from the tower is $\frac{1}{\sqrt{3}+1}m$, then the height of the tower is

a) $\frac{4\sqrt{3}}{2}$ m b) $\frac{\sqrt{3}+3}{2}$ m c) $\frac{3-\sqrt{3}}{2}$ m d) $\frac{\sqrt{3}}{2}$ m

414. If in a $\Delta ABC, a = 6$ cm, $b = 8$ cm, $c = 10$ cm, then the value of $\sin 2A$ is

a) 6/25 b) 8/25 c) 10/25 d) 24/25

415. In $\Delta\ ABC, \frac{1+\cos(A-B)\cos C}{1+\cos(A-C)\cos B}$ is equal to

a) $\frac{a-b}{a+b}$ b) $\frac{a+b}{a+c}$ c) $\frac{a^2-b^2}{a^2-c^2}$ d) $\frac{a^2+b^2}{a^2+c^2}$

416. The base of a cliff is circular. From the extremities of a diameter of the base angles of elevation of the top of the cliff are 30°and 60°. If the height of the cliff be 500 m, then the diameter of the base of the cliff is

a) $\frac{2000}{\sqrt{3}}$ m b) $\frac{1000}{\sqrt{3}}$ m c) $\frac{2000}{\sqrt{2}}$ m d) $1000\sqrt{3}$ m

417. The sides BC, CA and AB of a triangle ABC are of lengths a, b and c respectively. If D is the mid point of BC and AD is perpendicular to AC, then the value of $\cos A \cos C$ is

a) $\frac{3(a^2-c^2)}{2ac}$ b) $\frac{2(a^2-c^2)}{3bc}$ c) $\frac{(a^2-c^2)}{3ac}$ d) $\frac{2(c^2-a^2)}{3ac}$

418. The mid points of the sides of a triangle are $D(6, 1), E(3, 5)$ and $F(-1, -2)$, then the vertex opposite to D is

a) $(-4, 2)$ b) $(-4, 5)$ c) $(2, 5)$ d) $(10, 8)$

419. The locus of a points which moves such that the sum of the squares of its distance from three vertices of the triangle is constant is a/an

a) Circle b) Straight line c) Ellipse d) None of the above

420. The angles A, B and C of a $\Delta\ ABC$ are in AP. if $b: c = \sqrt{3}:\sqrt{2}$, then the angle A is

a) 30° b) 15° c) 75° d) 45°

421. If the three points $(0, 1)$, $(0, -1)$ and $(x, 0)$ are vertices of an equilateral triangle, then the values of x are

a) $\sqrt{3}, \sqrt{2}$ b) $\sqrt{3}, -\sqrt{3}$ c) $-\sqrt{5}, \sqrt{3}$ d) $\sqrt{2}, -\sqrt{2}$

422. The sides of a triangle are $\sin\alpha\cos\alpha$ and $\sqrt{1+\sin\alpha\cos\alpha}$ for some$0 < \alpha < \frac{\pi}{2}$. Then, the greatest angle of the triangle is

a) 60° b) 90° c) 120° d) 150°

423. From an aeroplane flying, vertically above a horizontal road, the angles of depression of two consecutive stones on the same side of the aeroplane are observed to be 30°and 60°respectively. The height at which the aeroplane is flying in km, is

a) $\frac{4}{\sqrt{3}}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{2}{\sqrt{3}}$ d) 2

424. A flag staff 20 m long standing on a wall 10 m high subtends an angle whose tangent is 0.5 at a point on the ground. If θ is the angle subtended by the wall at this point, then

a) $\tan\theta = 1$ b) $\tan\theta = 3$ c) $\tan\theta = \frac{1}{2}$ d) None of these

425. The angle of elevation of the top of a vertical tower from two points distance a and b from the base and in the same line with it, are complimentary. If θ is the angle subtended at the top of the tower by the line joining these points then $\sin\theta =$

a) $\frac{a-b}{\sqrt{2}\,(a+b)}$ b) $\frac{a+b}{a-b}$ c) $\frac{a-b}{a+b}$ d) None of these

426. In a triangle with one angle if 120°, the length of the sides forms an AP. If the length of the greatest sides is 7 cm, then area of triangle is

a) $\frac{3\sqrt{15}}{4}$ cm^2 b) $\frac{15\sqrt{3}}{4}$ cm^2 c) $\frac{15}{4}$ cm^2 d) $\frac{3\sqrt{3}}{4}$ cm^2

427. If in a $\Delta\, ABC, \frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}$, then $\cos A$ is equal to

a) $\frac{1}{5}$ b) $\frac{5}{7}$ c) $\frac{19}{35}$ d) None of these

428. If two vertices of an equilateral triangle are $(0, 0)$ and $(3, 3\sqrt{3})$, then the third vertex lies at

a) $(3, -3)$ b) $(-3, 3)$ c) $(-3, 3\sqrt{3})$ d) None of these

429. In an isosceles right angled $\Delta\, ABC, \angle B = 90°, AD$ is the median, then $\frac{\sin \angle BAD}{\sin \angle CAD}$ is

a) $\frac{1}{\sqrt{2}}$ b) $\sqrt{2}$ c) 1 d) None of these

430. If $P(1, 2), Q(4, 6), R(5, 7)$ and $S(a, b)$ are the vertices of a parallelogram $PQRS$, then

a) $a = 2, b = 4$ b) $a = b, b = 4$ c) $a = 2, b = 3$ d) $a = 3, b = 5$

431. Angles of a triangle are in the ratio4: 1: 1. The ratio between its greatest side and perimeter is

a) $\frac{3}{2+\sqrt{3}}$ b) $\frac{1}{2+\sqrt{3}}$ c) $\frac{\sqrt{3}}{\sqrt{3}+2}$ d) $\frac{2}{2+\sqrt{3}}$

432. From the top of a cliff h metres above sea level an observer notices that angles of depression of an object A and its image B are complementary. If the angle of depression at A is θ. The height of A above sea level is

a) $h \sin \theta$ b) $h \cos \theta$ c) $h \sin 2\theta$ d) $h \cos 2\theta$

433. The radius of the incircle of triangle when sides are 18, 24 and 30 cm is

a) 2 cm b) 4 cm c) 6 cm d) 9 cm

434. The points $(1, 3)$ and $(5, 1)$ are the opposite vertices of a rectangle. The other two vertices lie on the line $y = 2x + c$, then the value of c will be

a) 4 b) -4 c) 2 d) -2

435. If the vertices of a triangle at $O(0,0), A(a, 0)$ and $B(0, a)$. Then, the distance between its circumcentre and orthocentre is

a) $\frac{a}{2}$ b) $\frac{a}{\sqrt{2}}$ c) $\sqrt{2}\, a$ d) $\frac{a}{4}$

436. The straight lines $x = y,\ x - 2y = 3$ and $x + 2y = -3$ form a triangle, which is

a) Isosceles b) Equilateral c) Right angled d) None of these

437. The vertices of a triangle are $(6, 0), (0, 6)$ and $(6, 6)$. The distance between its circumcentre and centroid is

a) 2 b) $\sqrt{2}$ c) 1 d) $2\sqrt{2}$

438. In a $\Delta\, ABC, a = 5, b = 7$ and $\sin A = \frac{3}{4}$, then the number of possible triangles are

a) 1 b) 0 c) 2 d) Infinite

439. The points $(k, 2 - 2k), (-k + 1, 2k), (-4 - k, 6 - 2k)$ are collinear, then k is equal to

a) 2, 3 b) 1, 0 c) $\frac{1}{2}$, 1 d) 1, 2

440. If the vertices P, Q, R of a $\Delta\, PQR$ are rational points, which of the following points of the $\Delta\, PQR$ is (are) always rational points?
(A rational point is a point both of whose coordinates are rational numbers)

a) Centroid b) Incentre c) Circumcentre d) Orthocentre

441. Which of the following pieces of data does not uniquely determine acute angled $\Delta\, ABC$ (R =circumradius)?

a) $a, \sin A, \sin B$ b) a, b, c c) $a, \sin B, R$ d) $a, \sin A, R$

442. If a ΔABC, $2ca \sin \frac{A-B+C}{2}$ is equal to

a) $a^2 + b^2 - c^2$ b) $c^2 + a^2 - b^2$ c) $b^2 - c^2 - a^2$ d) $c^2 - a^2 - b^2$

443. In any ΔABC under usual notation, $a(b \cos C - c \cos B)$ is equal to

a) $b^2 - c^2$ b) $c^2 - b^2$ c) $\frac{b^2 - c^2}{2}$ d) $\frac{c^2 - b^2}{2}$

444. In ΔABC, G is he centroid, D is the mid point ofBC. If $A = (2, 3)$and $G(7, 5)$, then the point D is

a) $\left(\frac{9}{2}, 4\right)$ b) $\left(\frac{19}{2}, 6\right)$ c) $\left(\frac{11}{2}, \frac{11}{2}\right)$ d) $\left(8, \frac{13}{2}\right)$

445. The transformed equation of $3x^2 + 3y^2 + 2xy = 2$, when the coordinate axes are rotated through an angle of $45°$, is

a) $x^2 + 2y^2 = 1$ b) $2x^2 + y^2 = 1$ c) $x^2 + y^2 = 1$ d) $x^2 + 3y^2 = 1$

446. The equation of the locus of a point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$, then the value of c is

a) $\sqrt{(a_1^2 + b_1^2 + c_1^2)}$ b) $a_1^2 - b_1^2 - c_1^2$

c) $\frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$ d) None of the above

447. $ABC$is a right angled triangle with $\angle B = 90°, a = 6$ cm. If the radius of the circumcircle is 5 cm. Then the area of ΔABC is

a) 25 cm^2 b) 30 cm^2 c) 36 cm^2 d) 24 cm^2

448. The transformed equation of $x^2 + 6xy + 8y^2 = 10$ when the axes are rotated through an angle $\frac{\pi}{4}$ is

a) $15x^2 - 14xy + 3y^2 = 20$ b) $15x^2 + 14xy - 3y^2 = 20$

c) $15x^2 + 14xy + 3y^2 = 20$ d) $15x^2 - 14xy - 3y^2 = 20$

449. The angle of elevation of the top of a TV tower from three points A, B and C in a straight line through the foot of the tower are α, 2α and 3α respectively. If $AB = a$, then height of the tower is

a) $a \tan$ α b) $a \sin$ α c) a sin 2α d) a sin 3α

450. The ratio in which the x-axis divides the line segment joining (3,6) and $(4, -3)$ is

a) 2 : 1 b) 1 : 2 c) 3 : 4 d) None of these

451. AB is vertical tower. The point A is on the ground and C is the middle point of AB. The part CB subtend an angle α at a point P on the ground. If $AP = n\ AB$, then the correct relation is

a) $n = (n^2 + 1) \tan \alpha$ b) $n = (2n^2 - 1) \tan \alpha$ c) $n^2 = (2n^2 + 1) \tan \alpha$ d) $n = (2n^2 + 1) \tan \alpha$

452. If the points $(-2, -5), (2, -2), (8, a)$ are collinear, then the value of a is

a) $-\frac{5}{2}$ b) $\frac{5}{2}$ c) $\frac{3}{2}$ d) $\frac{1}{2}$

453. If in a $\Delta\ ABC, \cos A + 2 \cos B + \cos C = 2$, then a, b, c are in

a) AP b) GP c) HP d) None of these

454. In a $\Delta\ ABC, a = 5, a = 4$ and $\cos(A + B) = \frac{31}{32}$. In this triangle, c is equal to

a) $\sqrt{6}$ b) 36 c) 6 d) None of these

455. In a triangle $r_1 > r_2 > r_3$, then

a) $a > b > c$ b) $a < b < c$ c) $a > b$ and $b < c$ d) $a < b$ and $b > c$

456. In a $\Delta\ ABC$, if $a = 2x, b = 2y$ and $\angle C = 120°$, then the area of the triangle is

a) xy sq unit b) $xy\ \sqrt{3}$ sq unit c) $3xy$ sq unit d) $2xy$ sq unit

457. If the points $(a, b), (a', b')$ and $(a - a', b - b')$ are collinear, then

a) $ab' = a'b$ b) $ab = a'b'$ c) $aa' = bb'$ d) $a^2 + b^2 = 1$

458. If $A(-a, 0)$ and $B(a, 0)$ are two fixed points, then the locus of the point at which AB subtends a right angle, is

a) $x^2 + y^2 = 2a^2$ b) $x^2 - y^2 = a^2$ c) $x^2 + y^2 + a^2 = 0$ d) $x^2 + y^2 = a^2$

459. If the coordinates of two vertices of an equilateral triangle ae $(2, 4)$ and $(2, 6)$, then the coordinates of its third vertex are

a) $(\sqrt{3}, 5)$ b) $(2\sqrt{3}, 5)$ c) $(2 + \sqrt{3}, 5)$ d) $(2, 5)$

460. If point (x, y) is equidistant from $(a + b, b - a)$ and $(a - b, a + b)$, then

a) $ax + by = 0$ b) $ax - by = 0$ c) $bx + ay = 0$ d) $bx - ay = 0$

461. If A, B, C, D are the angles of a quadrilateral, then $\frac{\sum \tan A}{\sum \cot A}$ is equal to

a) $\prod \tan A$ b) $\prod \cot A$ c) $\sum \tan^2 A$ d) $\sum \cot^2 A$

462. If the angles of a triangle are in the ratio 3:4:5, then the sides are in the ratio

a) $2{:}\sqrt{6}{:}\sqrt{3} + 1$ b) $\sqrt{2}{:}\sqrt{6}{:}\sqrt{3} + 1$ c) $2{:}\sqrt{3}{:}\sqrt{3} + 1$ d) $3{:}4{:}5$

463. A line joining $A(2,0)$ and $B(3,1)$ is rotated about A in anti-clockwise direction through 15°. Find the equation of the line in the new position. If B goes to C in the new position, then coordinates of C are

a) $\left(2+\frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{\sqrt{2}}\right)$ b) $\left(2-\frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{\sqrt{2}}\right)$ c) $\left(2+\frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}\right)$ d) None of these

464. ABC is a triangle with vertices $A(-1,4), B(6,-2)$ and $C(-2,4)$. D, E and F are the points which divide each AB, BC, and CA respectively in the ratio 3 : 1 internally. Then, the centroid of ΔDEF is

a) $(3,6)$ b) $(1,2)$ c) $(4,8)$ d) $(-3,6)$

465. A house subtends a right angle at the window of an opposite house and the angle of elevation of the window from the bottom of the first house is 60°. If the distance between the two house be 6 m, then the height of the first house is

a) $8\sqrt{3}$ m b) $6\sqrt{3}$ m c) $4\sqrt{3}$ m d) None of these

466. The orthocentre of the triangle whose vertices are $\{at_1 t_2, a(t_1+t_2)\}, \{at_2 t_3, a(t_2+t_3)\}, \{at_3 t_1, a(t_3+t_1)\}$ is

a) $\{-a, a(t_1+t_2+t_3+t_1+t_2t_3)\}$ b) $\{-a, a(t_1+t_2+t_3+t_1t_2t_3)\}$

c) $\{-a, a(t_1-t_2-t_3-t_1t_2t_3)\}$ d) $\{-a, a(t_1+t_2-t_3-t_1t_2t_3)\}$

467. A flag staff is upon the top of a building. If at a distance of 40 m from the base of building the angles of elevation of the topes of the flag staff and building are 60° and 30° respectively, then the height of the flag staff is

a) 46.19 m b) 50 m c) 25 m d) None of these

468. A person observes the angle of elevation of a building as 30°. The persons proceeds towards the building with a speed of $25(\sqrt{3}-1)$ m/h. After two hours, he observes the angle of elevation as 45°.The height of the building (in metres)is

a) $50(\sqrt{3}-1)$ b) $50(\sqrt{3}+1)$ c) 50 d) 100

469. In a $\Delta ABC, \sum(b+c)\tan\frac{A}{2}\tan\left(\frac{B-C}{2}\right)$ is equal to

a) a b) b c) c d) 0

470. In a ΔABC, angle A is greater than angle B. If the measures of angles A and B satisfy the equation $3\sin x - 4\sin^3 x - k = 0,\ 0 < k < 1$, then the measure of angle C is

a) $\frac{\pi}{3}$ b) $\frac{\pi}{2}$ c) $\frac{2\pi}{3}$ d) $\frac{5\pi}{6}$

471. In a $\Delta ABC, a, c, A$ are given and b_1, b_2 are two values of third side b such that $b = 2b_1$. Then, $\sin A$ is equal to

a) $\sqrt{\frac{9a^2-c^2}{8a^2}}$ b) $\sqrt{\frac{9a^2-c^2}{8c^2}}$ c) $\sqrt{\frac{9a^2+c^2}{8a^2}}$ d) None of these

472. The incentre of a triangle with vertices $(7,1), (-1,5)$ and $(3+2\sqrt{3}, 3+4\sqrt{3})$ is

a) $\left(3+\frac{2}{\sqrt{3}}, 3+\frac{4}{\sqrt{3}}\right)$ b) $\left(1+\frac{2}{3\sqrt{3}}, 1+\frac{4}{3\sqrt{3}}\right)$ c) $(7,1)$ d) None of the above

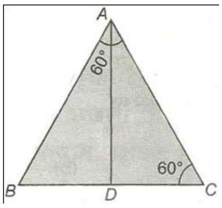
473. In any triangle $ABC, \frac{\tan\frac{A}{2}-\tan\frac{B}{2}}{\tan\frac{A}{2}+\tan\frac{B}{2}}$ is equal to

a) $\frac{a-b}{a+b}$ b) $\frac{a-b}{2}$ c) $\frac{a-b}{a+b+c}$ d) $\frac{c}{a+b}$

474. If orthocentre and circumcentre of triangle are respectively $(1,1)$ and $(3,2)$, then the coordinates of its centroid are

a) $\left(\frac{7}{3}, \frac{5}{3}\right)$ b) $\left(\frac{5}{3}, \frac{7}{3}\right)$ c) $(7,5)$ d) None of these

475. In $\Delta ABC, AD$ is median and $\angle A = 60°$, then $4\,AD^2$ is equal to



a) $b^2 + c^2 - bc$ b) $2b^2 + c^2 - 2bc$ c) $b^2 + c^2 + 2bc$ d) None of these

476. The circumcentre of the triangle formed by the lines $y = x, y = 2x$ and $y = 3x + 4$ is

a) $(6, 8)$ b) $(6, -8)$ c) $(3, 4)$ d) $(-3, -4)$

477. If in a $\Delta\, ABC, \cot\frac{A}{2} = \frac{b+c}{a}$, then the $\Delta\, ABC$ is

a) Isosceles b) Equilateral c) Right angled d) None of these

478. Area of the triangle formed by the lines $3x^2 - 4xy + y^2 = 0, 2x - y = 6$ is

a) 16 sq units b) 25 sq units c) 36 sq units d) 49 sq units

479. The shadow of a tower is found to be 60 m shorter when the sun's altitude changes from 30° to 60°. The height of the tower from the ground is approximately equal to

a) 62 m b) 301 m c) 101 m d) 52 m

480. Let $0 < \alpha < \frac{\pi}{2}$ be a fixed angle. If $P = (\cos\theta,\ \sin\theta)$ and $Q = \{\cos(\alpha - \theta),\ \sin(\alpha - \theta)\}$. Then Q is obtained from P by

a) Clockwise rotation around the origin through angle α

b) Anti-clockwise rotation around origin through angle α

c) Reflection in the line through the origin with slope $\tan\alpha$

d) Reflection in the line through the origin with slope $\tan\frac{\alpha}{2}$

481. The equation of the base of an equilateral triangle is $x + y = 2$ and the vertex is $(2, -1)$. The area of triangle is

a) $2\sqrt{3}$ b) $\frac{\sqrt{3}}{6}$ c) $\frac{1}{\sqrt{3}}$ d) $\frac{2}{\sqrt{3}}$

482. If $(0, 1)$ is the orthocentre and $(2, 3)$ is the centroid of a triangle. Then, its circumcentre is

a) $(3, 2)$ b) $(1, 0)$ c) $(4, 3)$ d) $(3, 4)$

483. The angle of elevation of the top of the tower observed from each of the three points A, B, C on the ground forming a triangle is the same angle α. If R is the circumradius of the $\Delta\, ABC$, then the height of the tower is

a) $R\sin\alpha$ b) $R\cos\alpha$ c) $R\cot\alpha$ d) $R\tan\alpha$

484. The circumcentre of a triangle formed by the lines $xy + 2x + 2y + 4 = 0$ and $x + y + 2 = 0$ is

a) $(-1, -1)$ b) $(0, -1)$ c) $(1, 1)$ d) $(-1, 0)$

485. If the distance of any point P from the points $A(a + b, a - b)$ and $B(a - b, a + b)$ are equal, then the locus of P is

a) $ax + by = 0$ b) $x - y = 0$ c) $x + y = 0$ d) $bx - ay = 0$

486. In an equilateral triangle of side $2\sqrt{3}$ cm, the circumcentre is

a) 1 cm b) $\sqrt{3}$ cm c) 2 cm d) $2\sqrt{3}$ cm

487. Let ABC be a triangle, two of whose vertices are $(15, 0)$ and $(0, 10)$. If the orthocenter is $(6, 9)$, then the third vertex is

a) $(15, 10)$ b) $(10, -15)$ c) $(0, 0)$ d) None of these

488. An aeroplane flying at a height of 300 metres above the ground passes vertically above another plane at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. The height of the lower plane from the ground (in metres) is

a) $100\sqrt{3}$ b) $\frac{100}{\sqrt{3}}$ c) 50 d) $150(\sqrt{3} + 1)$

489. If the sides of a triangle are in ratio 3:7:8, then $R{:}\,r$ is equal to

a) 2:7 b) 7:2 c) 3:7 d) 7:3

490. In a $\Delta\, ABC, (b + c - a)\tan\frac{A}{2}$ is equal to

a) $\frac{2\Delta}{s}$ b) $\frac{\Delta}{s}$ c) $\frac{\Delta s}{bc}$ d) $\frac{s}{a}R$

491. The sum of the radii of inscribed and circumscribed circles for an n sides regular polygon of side a, is

a) $a\cot\left(\frac{\pi}{n}\right)$ b) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$ c) $a\cot\left(\frac{\pi}{2n}\right)$ d) $\frac{a}{4}\cot\left(\frac{\pi}{2n}\right)$

492. If $\Delta = a^2 - (b - c)^2$, where Δ is the area of $\Delta\, ABC$, then $\tan A$ is equal to

a) $\frac{15}{16}$ b) $\frac{8}{17}$ c) $\frac{8}{15}$ d) $\frac{1}{2}$

493. The median BE and AD of a triangle with vertices $A(0,b), B(0,0), C(a,0)$ are perpendicular to each other, if

a) $a = \frac{b}{2}$ b) $b = \frac{a}{2}$ c) $ab = 1$ d) $a = \pm\sqrt{2}b$

494. If a, b, c be the sides of a $\Delta\, ABC$ and if roots of the equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ are equal, then $\sin^2\left(\frac{A}{2}\right), \sin^2\left(\frac{B}{2}\right), \sin^2\left(\frac{C}{2}\right)$ are in

a) AP b) GP c) HP d) AGP

495. In $\Delta\, ABC, \frac{1+\cos(A-B)\cos C}{1+\cos(A-C)\cos B}$ is equal to

a) $\frac{a-b}{a-c}$ b) $\frac{a+b}{a+c}$ c) $\frac{a^2-b^2}{a^2-c^2}$ d) $\frac{a^2+b^2}{a^2+c^2}$

496. In a $\Delta\, ABC, a = 13$ cm, $b = 12$ cm and $c = 5$ cm. The distance of A from BC is

a) $\frac{144}{13}$ b) $\frac{65}{12}$ c) $\frac{60}{13}$ d) $\frac{25}{13}$

497. If in a $\Delta ABC, r_1 < r_2 < r_3$, then

a) $a < b < c$ b) $a > b > c$ c) $b < a < c$ d) $a < c < b$

498. A ladder rests against a wall making an angle α with the horizontal. The foot of the ladder is pulled away from the wall through a distance x, so that it slides a distance y down the wall making an angle β with the horizontal. The correct relation is

a) $x = y\tan\left(\frac{\alpha+\beta}{2}\right)$ b) $y = x\tan\left(\frac{\alpha+\beta}{2}\right)$ c) $x = y\tan(\alpha+\beta)$ d) $y = x\tan(\alpha+\beta)$

499. Let $A(h,k), B(1,1)$ and $C(2,1)$ be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1, then the set of values which $'k'$ can take is given by

a) $\{1, 3\}$ b) $\{0, 2\}$ c) $\{-1, 3\}$ d) $\{-3, -2\}$

500. A variable line $\frac{x}{a} + \frac{y}{b} = 1$ is such that $a + b = 4$. The locus of the mid point of the portion of the line intercepted between the axes is

a) $x + y = 4$ b) $x + y = 8$ c) $x + y = 1$ d) $x + y = 2$

501. If in a $\Delta\, ABC, \tan\left(\frac{A}{2}\right), \tan\left(\frac{B}{2}\right), \tan\left(\frac{C}{2}\right)$ are in HP, then the sides a, b, c are in

a) AP b) GP c) HP d) None of these

502. The shadow of tower standing on a level ground is x metres long when the sun's altitude is 30°, while it is y metres long when the altitude is 60°. If the height of the tower is $45.\frac{\sqrt{3}}{2}$ m, then $x - y$ is

a) 45 m b) $45\sqrt{3}$ m c) $\frac{45}{\sqrt{3}}$ m d) $45.\frac{\sqrt{3}}{2}$ m

503. All points lying inside the triangle formed by the points $(1,3)$, $(5,0)$ and $(-1,2)$ satisfy

a) $3x + 2y \geq 0$ b) $2x + y - 13 < 0$ c) $2x - 3y - 12 \leq 0$ d) All of these

504. If the area of the triangle with vertices $(x,0), (1,1)$ and $(0,2)$ is 4 sq unit, then the value of x is

a) -2 b) -4 c) -6 d) 8

505. If the centroid of the triangle formed by the points $(0,0)$, $(\cos\theta, \sin\theta)$ and $(\sin\theta, -\cos\theta)$ lies on the line $y = 2x$, then θ is equal to

a) $\tan^{-1} 2$ b) $\tan^{-1} 3$ c) $\tan^{-1}(-3)$ d) $\tan^{-1}(-2)$

506. In order to remove first degree terms from the equation $2x^2 + 7y^2 + 8x - 14y + 4 = 0$, the origin is shifted at the point

a) $(-2, 1)$ b) $(1, 2)$ c) $(2, 1)$ d) $(1, -2)$

507. If $t_1 + t_2 + t_3 = -t_1t_2t_3$, then orthocentre of the triangle formed by the points $[at_1t_2, a(t_1 + t_2)], [at_2t_3, a(t_2 + t_3)]$ and $[at_3t_1, a(t_3 + t_1)]$, lies on

a) $(a, 0)$ b) $(-a, 0)$ c) $(0, a)$ d) $(0, -a)$

508. If $A(-5, 0)$ and $B(3, 0)$ are two vertices of a triangleABC. Its area is 20 sq cm. The vertex C lies on the line$x - y = 2$. The coordinates of C are

a) $(-7, -5)$ or $(3, 5)$ b) $(-3, -5)$ or $(-5, 7)$ c) $(7, 5)$ or $(3, 5)$ d) $(-3, -5)$ or $(7, 5)$

509. The locus of a point P which moves such that $2PA = 3PB$, where coordinates of points A and B are (0,0) and (4,–3), is

a) $5x^2 - 5y^2 - 72x + 54y + 225 = 0$ b) $5x^2 + 5y^2 - 72x + 54y + 225 = 0$

c) $5x^2 + 5y^2 + 72x - 54y + 225 = 0$ d) $5x^2 + 5y^2 - 72x - 54y - 225 = 0$

510. The coordinates of the centroid of a triangle having its circumcentre and orthocentre at $(7/2, 5/2)$ and (2,1) respectively, are

a) $(3, 2)$ b) $(13/6, 3/2)$ c) $(5/2, 3/2)$ d) $(3/2, 5/2)$

511. The base angle of triangle are $22\frac{1}{2}°$ and$112\frac{1}{2}°$. If b is the base and h is the height of the triangle, then

a) $b = 2h$ b) $b = 3h$ c) $b = (1 + \sqrt{3})h$ d) $b = (2 + \sqrt{3})h$

512. Let ABC be a triangle such that $\angle ACB = \frac{\pi}{6}$ and let a, b and c denote the lengths of the sides opposite to A, B and C respectively. The value (s) of x for which $a = x^2 + x + 1, b = x^2 - 1$ and $c = 2x + 1$ is (are)

a) $-(2 + \sqrt{3})$ b) $1 + \sqrt{3}$ c) $2 + \sqrt{3}$ d) $4\sqrt{3}$

513. If the distance between the points $P(a \cos 48°, 0)$ and $Q(0, a \cos 12°)$ is d, then $d^2 - a^2 =$

a) $\frac{a^2}{4}(\sqrt{5} - 1)$ b) $\frac{a^2}{4}(\sqrt{5} + 1)$ c) $\frac{a}{8}(\sqrt{5} - 1)$ d) $\frac{a^2}{8}(\sqrt{5} + 1)$

514. In a $\Delta\, ABC, a(b \cos C - c \cos B)$ is equal to

a) a^2 b) $b^2 - c^2$ c) 0 d) None of these

515. If the points $(a_1, b_1), (a_2, b_2)$ and (a_3, b_3) are collinear, then lines $a_ix + b_iy + 1 = 0$ for $i = 1, 2, 3$ are

a) Concurrent b) Identical c) Parallel d) None of these

516. In $\Delta\, ABC, (b + c) \cos A + (c + a) \cos B + (a + b) \cos C$ is equal to

a) 0 b) 1 c) $a + b + c$ d) $2(a + b + c)$

517. If in a $\Delta\, ABC$ $a = 5, b = 4, A = \frac{\pi}{2} + B$, then C

a) is $\tan^{-1}\left(\frac{1}{9}\right)$ b) is $\tan^{-1}\left(\frac{9}{40}\right)$ c) Cannot be evaluated d) is $2\tan^{-1}\left(\frac{1}{9}\right)$

518. If p_1, p_2, p_3 are altitudes of a $\Delta\, ABC$ drawn from the vertices A, B, C and Δ the area of the triangle, then $p_1^{-2} + p_2^{-2} + p_3^{-2}$ is equal to

a) $\frac{a + b + c}{\Delta}$ b) $\frac{a^2 + b^2 + c^2}{4\Delta^2}$ c) $\frac{a^2 + b^2 + c^2}{\Delta^2}$ d) None of these

: ANSWER KEY :

1)	a	2)	a	3)	d	4)	b	189)	a	190)	a	191)	b	192)	a
5)	a	6)	b	7)	d	8)	a	193)	d	194)	d	195)	b	196)	d
9)	d	10)	b	11)	b	12)	a	197)	c	198)	b	199)	c	200)	a
13)	c	14)	c	15)	b	16)	d	201)	b	202)	a	203)	a	204)	c
17)	c	18)	d	19)	c	20)	b	205)	a	206)	b	207)	a	208)	a
21)	c	22)	a	23)	c	24)	a	209)	c	210)	d	211)	c	212)	c
25)	c	26)	c	27)	b	28)	b	213)	b	214)	b	215)	c	216)	d
29)	a	30)	a	31)	c	32)	b	217)	a	218)	d	219)	b	220)	d
33)	b	34)	a	35)	b	36)	a	221)	a	222)	b	223)	b	224)	c
37)	b	38)	d	39)	d	40)	a	225)	c	226)	c	227)	a	228)	d
41)	a	42)	a	43)	c	44)	c	229)	a	230)	a	231)	a	232)	c
45)	c	46)	a	47)	d	48)	a	233)	c	234)	d	235)	d	236)	a
49)	d	50)	a	51)	d	52)	b	237)	a	238)	d	239)	a	240)	d
53)	a	54)	d	55)	b	56)	a	241)	c	242)	c	243)	a	244)	c
57)	c	58)	d	59)	a	60)	b	245)	a	246)	a	247)	d	248)	d
61)	b	62)	d	63)	a	64)	c	249)	d	250)	d	251)	d	252)	b
65)	c	66)	a	67)	b	68)	d	253)	c	254)	c	255)	c	256)	d
69)	c	70)	d	71)	c	72)	c	257)	a	258)	a	259)	c	260)	c
73)	a	74)	a	75)	d	76)	b	261)	a	262)	c	263)	d	264)	b
77)	c	78)	b	79)	d	80)	a	265)	b	266)	a	267)	c	268)	b
81)	b	82)	d	83)	a	84)	c	269)	d	270)	c	271)	a	272)	d
85)	a	86)	d	87)	c	88)	c	273)	a	274)	d	275)	d	276)	b
89)	c	90)	c	91)	a	92)	d	277)	a	278)	b	279)	b	280)	c
93)	c	94)	c	95)	a	96)	a	281)	c	282)	b	283)	c	284)	c
97)	b	98)	c	99)	c	100)	c	285)	a	286)	d	287)	b	288)	a
101)	a	102)	a	103)	a	104)	b	289)	a	290)	d	291)	b	292)	d
105)	c	106)	b	107)	c	108)	d	293)	a	294)	a	295)	d	296)	c
109)	b	110)	a	111)	c	112)	b	297)	c	298)	a	299)	a	300)	b
113)	a	114)	b	115)	d	116)	b	301)	d	302)	a	303)	b	304)	b
117)	a	118)	a	119)	b	120)	b	305)	a	306)	c	307)	d	308)	d
121)	a	122)	c	123)	d	124)	b	309)	c	310)	a	311)	d	312)	c
125)	c	126)	b	127)	d	128)	d	313)	b	314)	a	315)	d	316)	c
129)	c	130)	c	131)	c	132)	b	317)	a	318)	b	319)	a	320)	d
133)	b	134)	d	135)	c	136)	c	321)	b	322)	c	323)	c	324)	b
137)	b	138)	c	139)	d	140)	c	325)	b	326)	b	327)	d	328)	a
141)	c	142)	b	143)	d	144)	d	329)	d	330)	c	331)	d	332)	a
145)	c	146)	d	147)	b	148)	c	333)	d	334)	d	335)	a	336)	a
149)	b	150)	a	151)	c	152)	a	337)	a	338)	b	339)	b	340)	c
153)	d	154)	b	155)	a	156)	b	341)	c	342)	b	343)	a	344)	d
157)	c	158)	a	159)	c	160)	d	345)	c	346)	d	347)	d	348)	a
161)	d	162)	c	163)	d	164)	d	349)	d	350)	a	351)	c	352)	a
165)	c	166)	b	167)	a	168)	a	353)	b	354)	b	355)	b	356)	c
169)	b	170)	c	171)	c	172)	a	357)	c	358)	c	359)	d	360)	d
173)	c	174)	c	175)	a	176)	a	361)	b	362)	b	363)	b	364)	a
177)	d	178)	a	179)	c	180)	c	365)	b	366)	a	367)	c	368)	b
181)	b	182)	a	183)	d	184)	d	369)	d	370)	a	371)	b	372)	a
185)	b	186)	a	187)	a	188)	d	373)	d	374)	b	375)	b	376)	b

377)	a	378)	c	379)	a	380)	d
381)	d	382)	a	383)	d	384)	a
385)	a	386)	b	387)	d	388)	b
389)	c	390)	b	391)	b	392)	b
393)	b	394)	b	395)	b	396)	d
397)	b	398)	a	399)	b	400)	b
401)	b	402)	b	403)	c	404)	c
405)	b	406)	b	407)	b	408)	b
409)	c	410)	c	411)	a	412)	c
413)	c	414)	d	415)	d	416)	a
417)	d	418)	a	419)	a	420)	c
421)	b	422)	c	423)	b	424)	a
425)	c	426)	b	427)	a	428)	c
429)	b	430)	c	431)	c	432)	d
433)	c	434)	b	435)	b	436)	d
437)	b	438)	b	439)	c	440)	a
441)	d	442)	b	443)	a	444)	b
445)	b	446)	c	447)	d	448)	c
449)	c	450)	a	451)	d	452)	b
453)	a	454)	d	455)	a	456)	b
457)	a	458)	d	459)	c	460)	d
461)	a	462)	a	463)	a	464)	b
465)	a	466)	b	467)	a	468)	c
469)	d	470)	c	471)	b	472)	a
473)	b	474)	a	475)	d	476)	b
477)	c	478)	c	479)	d	480)	d
481)	b	482)	d	483)	d	484)	a
485)	b	486)	c	487)	c	488)	a
489)	b	490)	a	491)	b	492)	c
493)	d	494)	c	495)	d	496)	c
497)	a	498)	a	499)	c	500)	d
501)	a	502)	a	503)	d	504)	c
505)	c	506)	a	507)	b	508)	d
509)	b	510)	a	511)	a	512)	b
513)	d	514)	b	515)	a	516)	c
517)	b	518)	b				

: HINTS AND SOLUTIONS :

1 **(a)**

$a^2(\cos^2 B - \cos^2 C) + b^2(\cos^2 C - \cos^2 A) + c^2 (\cos^2 A - \cos^2 B)$

$= a^2(1 - \sin^2 B - 1 + \sin^2 C) + b^2(1 - \sin^2 C - 1 + \sin^2 A)$

$+c^2 (1 - \sin^2 A - 1 + \sin^2 B)$

$= a^2(\sin^2 C - \sin^2 B) + b^2(\sin^2 A - \sin^2 C) + c^2 (\sin^2 B - \sin^2 A)$

$= k^2a^2(c^2 - b^2) + k^2b^2(a^2 - c^2) + k^2c^2 (b^2 - c^2)$

$= 0$

2 **(a)**

Let $\sin A = 3k, \sin B = 4k, \sin C = 5k$

$\because \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = p$ [say]

$\Rightarrow \frac{3k}{a} = \frac{4k}{b} = \frac{5k}{c} = p$

$\Rightarrow a = 3\left(\frac{k}{p}\right), b = 4\left(\frac{k}{p}\right), c = 5\left(\frac{k}{p}\right)$

$\Rightarrow a = 3l, b = 4l, c = 5l \quad \left[\text{let } l = \frac{k}{p}\right]$

$\therefore \cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$= \frac{16 + 25 - 9}{2 \times 4 \times 5} = \frac{32}{40} = \frac{4}{5}$

$\therefore \cos B = \frac{c^2 + a^2 - b^2}{2ac}$

$= \frac{25 + 9 - 16}{2 \times 3 \times 5} = \frac{18}{30} = \frac{3}{5}$

Now, $\cos A : \cos B = \frac{4}{5} : \frac{3}{5} = 4:3$

4 **(b)**

Slope of perpendicular to the line joining the points

$(a\cos\alpha, a\sin\alpha)$ and $(a\cos\beta, a\sin\beta) = -\frac{\cos\alpha - \cos\beta}{\sin\alpha - \sin\beta}$

$= \tan\frac{\alpha + \beta}{2}$

Hence, equation of perpendicular is

$y = \tan\left(\frac{\alpha+\beta}{2}\right)x \quad$ ...(i)

Now, on solving the equation of line with Eq. (i), we get

$\left[\frac{a}{2}(\cos\alpha + \cos\beta), \frac{a}{2}(\sin\alpha + \sin\beta)\right]$

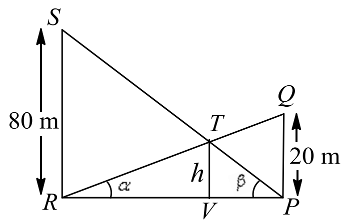
5 **(a)**

Area of $\frac{\Delta PBC}{\Delta ABC} = \left[\frac{\{-3(-2-y)+4(y-5)+x(5+2)\}}{\{6(5+2)-3(-2-3)+4(3-5)\}}\right]$

$= \left|\frac{7x + 7y - 14}{49}\right| = \left|\frac{x + y - 2}{7}\right|$

6 **(b)**

Let PQ and RS be the poles of height 20 m and 80 m subtending angles α and β at R and P respectively. Let h be the height of the point T, the intersection of QR and PS



Then, $PR = h\cot\alpha + h\cot\beta$

$= 20\cot\alpha = 80\cot\beta$

$\Rightarrow \cot\alpha = 4\cot\beta$

$\Rightarrow \frac{\cot\alpha}{\cot\beta} = 4$

Again, $h\cot\alpha + h\cot\beta = 20\cot\alpha$

$\Rightarrow (h - 20)\cot\alpha = -h\cot\beta$

$\Rightarrow \frac{\cot\alpha}{\cot\beta} = \frac{h}{20 - h} = 4$

$\Rightarrow h = 80 - 4h$

$\Rightarrow h = 16$ m

8 **(a)**

Since, α, β, γ are the roots of the equation

$x^3 - 3px^2 + 3qx - 1 = 0$

$\therefore \alpha + \beta + \gamma = 3p$

$\alpha\beta + \beta\gamma + \gamma\alpha = 3q$

and $\alpha\beta\gamma = 1$

Let $G(x, y)$ be the centroid of the given triangle

$\therefore x = \frac{\alpha + \beta + \gamma}{3} = p$

and $y = \frac{\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}}{3}$

$= \frac{\beta\gamma + \gamma\alpha + \alpha\beta}{3\alpha\beta\gamma} = q$

Hence, coordinates of the centroid of triangle are (p, q)

9 **(d)**

Let $O(0,0)$ be the orthocenter, $A(h,k)$ be the third vertex and $B(-2,3)$ and $C(5,-1)$ the other two vertices. Then, the slope of the line through A and O is $\frac{k}{h}$, while the line through B and C has the slope $\frac{(-1-3)}{(5+2)} = -\frac{4}{7}$. By the property of the orthocenter, these two lines must be perpendicular, so we have

$\left(\frac{k}{h}\right)\left(-\frac{4}{7}\right) = -1 \Rightarrow \frac{k}{h} = \frac{7}{4} \quad$ ...(i)

Also, $\frac{5-2+h}{3}+\frac{-1+3+k}{3}=7$

$\Rightarrow\ \ h+k=16\quad$ ...(ii)

Which is not satisfied by the points given in the options (a), (b) or (c)

10 **(b)**

Let (h,k) be the point

According to question,

$4\sqrt{(h-h)^2+k}=h^2+k^2$

$\Rightarrow\ \ 4|k|=h^2+k^2$

Locus of the point is

$4|y|=x^2+y^2\ \Rightarrow\ x^2+y^2-4|y|=0$

12 **(a)**

Given points are $P(4,-2), A(2,-4)$ and $B(7,1)$

Suppose P divides AB in the ratio $\lambda:1$. Then,

$\dfrac{7\lambda+2}{\lambda+1}=4\Rightarrow\lambda=\dfrac{2}{3}$

Thus, P divides AB internally in the ratio $2:3$

The coordinates of the point dividing AB externally in the ratio $2:3$ are

$\left(\dfrac{2\times7-3\times2}{2-3},\dfrac{2\times1-3\times-4}{2-3}\right)=(-8,-14)$

Hence, the harmonic conjugate of R with respect to A and B is $(-8,-14)$

13 **(c)**

If O is the origin and $P(x_1,y_1), Q(x_2,y_2)$ are two points, then

$OP\times OQ\cos\angle POQ=x_1x_2+y_1y_2$

$\therefore OP\times OQ\times\sin\angle POQ$

$=\sqrt{OP^2\times OQ^2-OP^2\times OQ^2\times\cos^2\angle POQ}$

$=\sqrt{(x_1^2+y_1^2)(x_2^2+y_2^2)-(x_1x_2+y_1y_2)^2}$

$=\sqrt{(x_1y_2-x_2y_1)^2}=|x_1y_2-x_2y_1|$

14 **(c)**

$\cos B=\dfrac{(3)^2+(5)^2-(4)^2}{2\times3\times5}=\dfrac{3}{5}$

$\Rightarrow\ \sin B=\sqrt{1-\dfrac{9}{25}}=\dfrac{4}{5}$

$\therefore\ \sin 2B=2\sin B\cos B$

$=2\times\dfrac{4}{5}\times\dfrac{3}{5}=\dfrac{24}{25}$

16 **(d)**

Given that, $\angle A=45°,\angle B=75°$

$\angle c=180°-45°-75°=60°$

$\therefore\ \ a+c\sqrt{2}=k(\sin A+\sqrt{2}\sin C)$

$=k(\sin 45°+\sqrt{2}\sin 60°)$

$=k\left(\frac{1}{\sqrt{2}}+\sqrt{2}\frac{\sqrt{3}}{2}\right)=k\left(\frac{1+\sqrt{3}}{\sqrt{2}}\right)\quad$ ...(i)

And $k=\frac{b}{\sin B}$

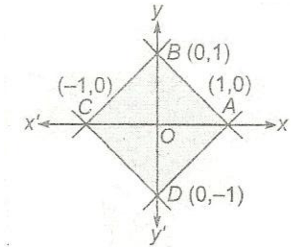
$=\dfrac{b}{\sin 75°}=\dfrac{2\sqrt{2}b}{\sqrt{3}+1}$

On putting the value of k in Eq. (i), we get

$a+c\sqrt{2}=2b$

18 **(d)**

From figure $ABCD$ is s square



Whose diagonals AC and BD are of length 2 unit

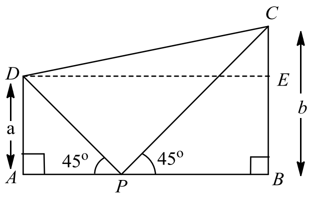
Hence, required area$=\frac{1}{2}\,AC\times BD$

$=\frac{1}{2}\times2\times2=2$ sq units

19 **(c)**

In ΔAPD,

$\tan 45°=\dfrac{a}{AP}\Rightarrow AP=a$



and in $\Delta\ BPC$,

$\tan 45°=\dfrac{b}{PB}$

$\Rightarrow PB=b$

$\therefore DE=a+b$ and $CE=b-a$

In ΔDEC,

$DC^2=DE^2+EC^2$

$=(a+b^2)+(b-a^2)$

$=2(a^2+b^2)$

20 **(b)**

If the axes are rotated through 30°, we have

$x=X\cos 30°-Y\sin 30°=\dfrac{\sqrt{3}X-4}{2}$

and, $y=X\sin 30°+Y\cos 30°=\dfrac{X+\sqrt{3}Y}{2}$

Substituting these values in $x^2+2\sqrt{3}\,xy-y^2=2a^2$, we get

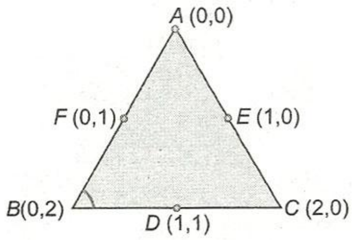
$\left(\sqrt{3}X-Y\right)^2+2\sqrt{3}\left(\sqrt{3}X-Y\right)\left(X+\sqrt{3}Y\right)$

$\qquad-\left(X+\sqrt{3}Y\right)^2=8a^2$

$\Rightarrow X^2-Y^2=a^2$

21 **(c)**

Since, F, E and D are the mid points of the sides AB, AC and BC of triangle ABC respectively, then the vertices of triangle are $A(0,0), B(0,2), C(2,0)$



Now, $AB = c = \sqrt{0^2 + 2^2} = 2$

$BC = a = \sqrt{2^2 + 2^2} = 2\sqrt{2}$

And $CA = b = \sqrt{2^2 + 0^2} = 2$

$\therefore$ x coordinates of incentre

$= \dfrac{ax_1 + bx_2 + cx_3}{a + b + c}$

$= \dfrac{2\sqrt{2}(0) + 2(0) + 2(2)}{2\sqrt{2} + 2 + 2}$

$= \dfrac{2}{2 + \sqrt{2}} = 2 - \sqrt{2}$

22 **(a)**

Let (x, y) be the coordinates of vertex C and (x_1, y_1) be the coordinates of centroid of the triangle.

$\therefore \quad x_1 = \frac{x+2-2}{3}$ and $y_1 = \frac{y-3+1}{3}$

$\Rightarrow \quad x_1 = \frac{x}{3}$ and $y_1 = \frac{y-2}{3}$

Since, the centriod lies on the line $2x + 3y = 1$

$\therefore \quad 2x_1 + 3y_1 = 1$

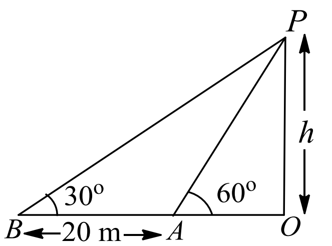
$\Rightarrow \quad \dfrac{2x}{3} + \dfrac{3(y-2)}{3} = 1$

$\Rightarrow \quad 2x + 3y = 9$

This equation represents the locus of the vertex C

23 **(c)**

Let the height of the tower be h



In $\Delta\, PAO, \tan 60° = \frac{h}{OA}$

$\Rightarrow \; OA = h \cot 60° = \frac{h}{\sqrt{3}}$ ...(i)

In $\Delta\, PBO, \tan 30° = \frac{h}{OB}$

$\Rightarrow \; OB = \dfrac{h}{\frac{1}{\sqrt{3}}}$

$\Rightarrow \; AB + AO = \sqrt{3}\, h$

$\Rightarrow \; 20 + \frac{h}{\sqrt{3}} = \sqrt{3}\, h$ [using Eq.(i)]

$\Rightarrow \; h = \dfrac{20}{\sqrt{3} - \frac{1}{\sqrt{3}}}$

$\Rightarrow \; h = \dfrac{20\sqrt{3}}{2}$

$\Rightarrow \; h = 10\sqrt{3}$ m

24 **(a)**

We have, $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$

$\Rightarrow \; bc \sin^2 \dfrac{A}{2} = (s-b)(s-c)$

On comparing with $x \sin^2 \frac{A}{2} = (s-b)(s-c)$

We get, $x = bc$

25 **(c)**

$\therefore \; p_1^2 + p_2^2 = \dfrac{4a^2}{\sec^2\alpha + \mathrm{cosec}^2\alpha} + \dfrac{a^2\cos^2 2\alpha}{\cos^2\alpha + \sin^2\alpha}$

$= a^2\left(\dfrac{4\cos^2\alpha\sin^2\alpha}{\cos^2\alpha + \sin^2\alpha} + \dfrac{\cos^2 2\alpha}{1}\right)$

$= a^2(\sin^2 2\alpha + \cos^2 2\alpha) = a^2$

and $p_1^2 p_2^2 = a^4 \sin^2 2\alpha \cos^2 2\alpha = \left(\frac{1}{4}\right) a^4 \sin^2 4\alpha$

$\therefore \; \left(\dfrac{p_1}{p_2} + \dfrac{p_2}{p_1}\right)^2 = \dfrac{(p_1^2 + p_2^2)^2}{p_1^2 p_2^2}$

$= \dfrac{4}{\sin^2 4\alpha} = 4\,\mathrm{cosec}^2 4\alpha$

26 **(c)**

Let $A(2, 1), B(-2, 4)$

$\therefore \; AB = 5$

Hence, the locus is the line segment AB

27 **(b)**

$\dfrac{1}{r^2} + \dfrac{1}{r_1^2} + \dfrac{1}{r_2^2} + \dfrac{1}{r_3^2}$

$= \dfrac{s^2 + (s-a)^2 + (s-b)^2 + (s-c)^2}{\Delta^2}$

$= \dfrac{4s^2 + a^2 + b^2 + c^2 - 2s(a+b+c)}{\Delta^2}$

$= \dfrac{a^2 + b^2 + c^2}{\Delta^2}$

28 **(b)**

Let $a = 4$ cm, b = 5cm and c = 6 cm

$\therefore \; s = \dfrac{4+5+6}{2} = \dfrac{15}{2}$

Hence, area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{\left(\dfrac{15}{2}\right)\left(\dfrac{15}{2} - 4\right)\left(\dfrac{15}{2} - 5\right)\left(\dfrac{15}{2} - 6\right)}$

$= \sqrt{\dfrac{15}{2} \times \dfrac{7}{2} \times \dfrac{5}{2} \times \dfrac{3}{2}} = \dfrac{15}{4}\sqrt{7}\text{ cm}^2$

30 **(a)**

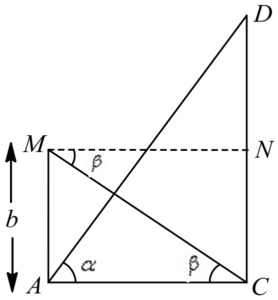
Let CD be the tower

In $\Delta ACM, \tan\beta = \frac{b}{AC}$

$\Rightarrow \; AC = b\cot\beta$

and in $\Delta ADC, \tan\alpha = \frac{CD}{AC}$

$\Rightarrow\ CD = b\cot\beta\tan\alpha$



31 **(c)**

Since, $R = \frac{b}{2\sin B} = \frac{2}{2\sin 30^\circ} = \frac{2}{1}$

Area of circumcircle$= \pi R^2$

$= \pi \times (2)^2 = 4\pi$ sq unit

33 **(b)**

$$\frac{b^2 - c^2}{2aR} = \frac{4R^2(\sin^2 B - \sin^2 C)}{4R^2 \sin A}$$

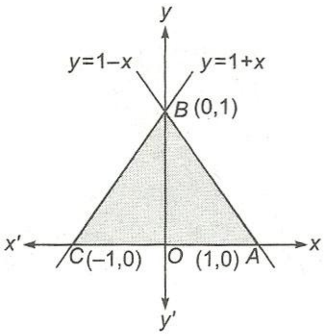
$$= \frac{\sin(B + C)\sin(B - C)}{\sin A}$$

$= \sin(B - C)$

34 **(a)**

Given curve is

$y = 1 - |x|$



$\therefore$ Area of $\Delta ABC = 2$ area of ΔAOB

$= 2 \times \frac{1}{2} \times 1 \times 1 = 1$ sq unit

35 **(b)**

Given, angles A, B, C of ΔABC are in AP with d (common difference)$= 15^\circ$

$\therefore\ B = A + 15^\circ$ and $C = A + 30^\circ$

Also, $A + B + C = 180^\circ$

$\Rightarrow A + A + 15^\circ + A + 30^\circ = 180^\circ$

$\Rightarrow\ \angle A = 45^\circ$

$\therefore\ \angle B = 45^\circ + 15^\circ = 60^\circ$

36 **(a)**

Since, $\left(1 - \frac{r_1}{r_2}\right)\left(1 - \frac{r_1}{r_3}\right) = 2$

$$\therefore\ \left(1 - \frac{s-b}{s-a}\right)\left(1 - \frac{s-c}{s-a}\right) = 2$$

$$\Rightarrow\ \frac{(b-a)(c-a)}{(s-a)^2} = 2$$

$\Rightarrow\ 2bc - 2ab - 2ac + 2a^2$

$= b^2 + c^2 + a^2 + 2bc - 2ab - 2ac$

$\Rightarrow\ a^2 = b^2 + c^2$

So, triangle is right angled

37 **(b)**

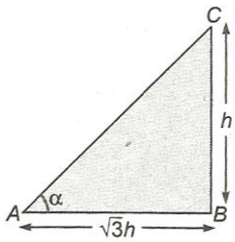
Let the height of the tower be $BC = h$,then length of shadow of tower $AB = \sqrt{3}h$.

In $\Delta ABC, \tan\alpha = \frac{BC}{AB}$

$$\Rightarrow\ \tan\alpha = \frac{h}{\sqrt{3}h}$$

$\Rightarrow\ \tan\alpha = \tan 30^\circ$

$\Rightarrow\ \alpha = 30^\circ$



38 **(d)**

Let the coordinates of P be (h, k). Then,

$x = X + h, y = Y + k$

Substituting these in $2x^2 + y^2 - 4x - 4y = 0$, we get

$2X^2 + Y^2 + 4(h-1) \times +2(k-2)Y + 2h^2 + k^2$
$- 4h - 4k = 0$

Comparing this equation with

$2\,X^2 + Y^2 - 8X - 8Y + 18 = 0$, we get

$h - 1 = -2, (k-2) = -4$ and $2h^2 + k^2 - 4h - 4k = 18$

$\Rightarrow h = -1, k = -2$

40 **(a)**

We have, $\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

$\because\ (s-a)(s-b) = s(s-c)$ (given)

$$\therefore\ \tan\frac{C}{2} = \sqrt{\frac{s(s-c)}{s(s-c)}}$$

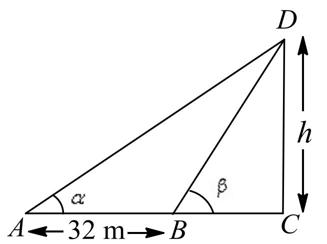
$$\Rightarrow\ \tan\frac{C}{2} = \tan\frac{\pi}{4}$$

$\Rightarrow\ \angle C = 90^\circ$

41 **(a)**

Given that, $\cot\alpha = \frac{3}{5}$ and $\cot\beta = \frac{2}{5}$

In $\Delta\ BCD$, $\tan\beta = \frac{h}{BC}$



$\Rightarrow\ BC = h\cot\beta\ \Rightarrow\ BC = \frac{2h}{5}$ ...(i)

and in $\Delta\ ACD, \tan\alpha = \frac{h}{32 + BC}$

$\Rightarrow\ h=\left(32+\frac{2h}{5}\right)\frac{5}{3}$ [using Eq.(i)]

$\Rightarrow\ 3h=160+2h$

$\Rightarrow\ h=160$ m

42 **(a)**

The vertices of quadrilateral $ABCD$ is $A(2,3), B(3,4), C(4,5)$ and $D(5,6)$

$\therefore\ AB=\sqrt{(3-2)^2+(4-3)^2}$

$=\sqrt{(1)^2+(1)^2}=\sqrt{2}$

Similarly, $BC=\sqrt{2}, CD=\sqrt{2}$, and $DA=3\sqrt{2}$

$\therefore\ a=b=c=\sqrt{2}$ and $d=3\sqrt{2}$

and $s=\frac{a+b+c+d}{2}$

$=\dfrac{\sqrt{2}+\sqrt{2}+\sqrt{2}+3\sqrt{2}}{2}$

$=\dfrac{6\sqrt{2}}{2}=3\sqrt{2}$

$\therefore$ Area of quadrilateral

$=\sqrt{(s-a)(s-b)(s-c)(s-d)}$

$=\sqrt{\begin{matrix}(3\sqrt{2}-\sqrt{2})(3\sqrt{2}-\sqrt{2})\\(3\sqrt{2}-\sqrt{2})(3\sqrt{2}-\sqrt{2})\end{matrix}}$

$=0$

43 **(c)**

We have, $\Delta=\frac{1}{2}bc\sin A$

$\Rightarrow\ \frac{1}{2}k^2\sin B\sin C\sin A=\Delta$ ...(i)

$\therefore\ a^2\sin 2B+b^2\sin 2A$

$=2(a^2\sin B\cos B+b^2\sin A\cos A)$

$=2k^2(\sin^2 A\sin B\cos B+\sin^2 B\sin A\cos A)$

$=2k^2(\sin A\sin B\sin C)=4\Delta$ [from Eq. (i)]

44 **(c)**

1. $\frac{\sin A}{\sin C}=\frac{\sin(A-B)}{\sin(B-C)}$

$\Rightarrow\ \sin(B+C)\sin(B-C)=\sin(A+B)\sin(A-B)$

$\Rightarrow \sin^2 B-\sin^2 C=\sin^2 A-\sin^2 B$

$\Rightarrow\ b^2-c^2=a^2-b^2$

$\Rightarrow\ 2b^2=a^2+c^2$

$\Rightarrow\ a^2, b^2, c^2$ are in AP

2. r_1, r_2, r_3 are in HP

$\Rightarrow \frac{1}{r_1}, \frac{1}{r_2}, \frac{1}{r_3}$ are in AP

$\Rightarrow\ \dfrac{2}{r_2}=\dfrac{1}{r_1}+\dfrac{1}{r_3}$

$\Rightarrow\ 2(s-b)=s-a+s-c$

$\Rightarrow\ 2b=(a+c)$

$\Rightarrow\ a, b, c$ are in AP

Hence, both of these statements are correct

45 **(c)**

The largest side of triangle is $\sqrt{p^2+q^2+pq}$

Greatest angle will be opposite to largest side. Let θ be greatest angle, then

$\cos\theta=\dfrac{p^2+q^2-p^2-q^2-pq}{2pq}=-\dfrac{1}{2}$

$\Rightarrow\ \theta=\dfrac{2\pi}{3}$

46 **(a)**

Let area of triangle be Δ, then according to question

$\Delta=\dfrac{1}{2}ax=\dfrac{1}{2}by=\dfrac{1}{2}cz$

$\therefore\ \dfrac{bx}{c}+\dfrac{cy}{a}+\dfrac{az}{b}=\dfrac{b}{c}\left(\dfrac{2\Delta}{a}\right)+\dfrac{c}{a}\left(\dfrac{2\Delta}{b}\right)+\dfrac{a}{b}\left(\dfrac{2\Delta}{c}\right)$

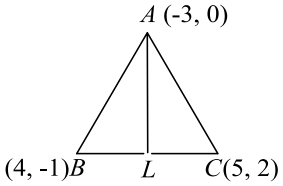
$=\dfrac{2\Delta(b^2+c^2+a^2)}{abc}$

$=\dfrac{2(a^2+b^2+c^2)}{abc}\cdot\dfrac{abc}{4R}\ \left(\because\ \Delta=\dfrac{abc}{4R}\right)$

$=\dfrac{a^2+b^2+c^2}{2R}$

49 **(d)**

In $\Delta\ ABC$ the vertices are $A(-3,0), B(4,-1)$ and $C(5,2)$



$\therefore \quad BC = \sqrt{(5-4)^2 + (2+1)^2}$

$= \sqrt{1+9} = \sqrt{10}$

Area of $\Delta\, ABC$

$= \dfrac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \dfrac{1}{2}[-3(-1-2) + 4(2-0) + 5(0+1)]$

$= \dfrac{1}{2}[9+8+5] = 11$

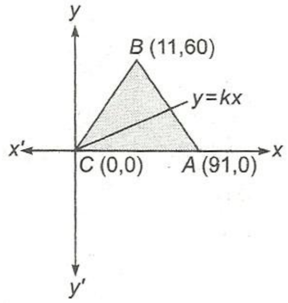
As we know that, area of $\Delta = \frac{1}{2} \times BC \times AL$

$\Rightarrow \quad 11 = \dfrac{1}{2} \times \sqrt{10} \times AL$

$\Rightarrow \quad AL = \dfrac{2 \times 11}{\sqrt{10}} = \dfrac{22}{\sqrt{10}}$

50 **(a)**

As the line divides the ΔABC in equal to area. Mid point of $AB(51, 30)$ which lies on $y = kx$



$\therefore\ 30 = 51k \quad \Rightarrow \quad k = \dfrac{30}{51}$

52 **(b)**

Let (h, k) be the point

According to question, $4\sqrt{(h-h)^2 + k^2} = h^2 + k^2$

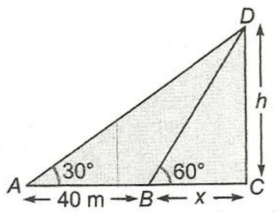
$\Rightarrow \quad 4|k| = h^2 + k^2$

Locus of the point is $4|y| = x^2 + y^2$

$\Rightarrow \quad x^2 + y^2 - 4|y| = 0$

53 **(a)**

In ΔCBD, $\tan 60° = \frac{h}{x}$



$\Rightarrow h = x\sqrt{3}$ ... (i)

and in ΔCAD, $\tan 30° = \frac{h}{40+x}$

$\Rightarrow \ h\sqrt{3} = 40 + x$

$\Rightarrow \ 3x = 40 + x$ [from Eq. (i)]

$\Rightarrow \ x = 20$ m

54 **(d)**

(a) We know, $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

Since, $\tan A + \tan B + \tan C = 0$

$\Rightarrow$ Let either of $\tan A$, $\tan B$ or $\tan C$ is zero *ie*, one angle is 0

So, it cannot be a triangle

(b) $\frac{\sin A}{2} = \frac{\sin B}{3} = \frac{\sin C}{1}$

$\Rightarrow \ a : b : c = 2 : 3 : 1$

Let $a = 2k, b = 3k, c = k, a + c = b$ (so triangle not possible)

(c) $\sin A \ \sin B = \frac{\sqrt{3}}{4} = \cos A \cos B$

Either $\sin A$, $\sin B$ are both positive or both negative but, if both are positive $\sin A + \sin B > 0$ but $\sin A + \sin B$ is negative so both negative but, if both are negative, then $\angle A$ and $\angle B$ are more than 90°, so it cannot be a triangle

(d) $(a+b)^2 = c^2 + ab$

$\Rightarrow \ a^2 + b^2 + 2ab = c^2 + ab$

$\Rightarrow \ a^2 + b^2 - c^2 = -ab$

$\Rightarrow \ \dfrac{a^2 + b^2 - c^2}{2ab} = -\dfrac{1}{2} = \cos C$

$\Rightarrow \ \angle C = 120°$

$\sin A + \cos A = \dfrac{\sqrt{3}}{\sqrt{2}}$

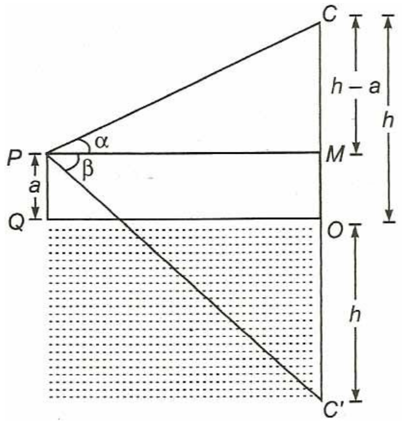
$\Rightarrow 1 + \sin 2A = \dfrac{3}{2} \ \Rightarrow \ \sin 2A = \dfrac{1}{2}$

$\Rightarrow \ 2A = 30° \ \Rightarrow \ \angle A = 15°$

So, it can form a triangle

55 **(b)**

In $\Delta\, PMC$, $\tan \alpha = \frac{h-a}{PM}$



$\Rightarrow\ PM = (h-a)\cot\alpha$ ...(i)

In $\Delta\, PMC'$, $\tan\beta = \frac{h+a}{PM}$

$\Rightarrow\ h + a = PM\tan\beta$

$\Rightarrow\ h = (h-a)\cot\alpha\tan\beta - a$

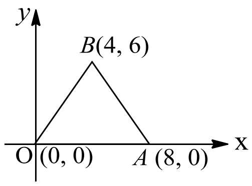
$\Rightarrow\ h(1-\cot\alpha\cot\beta) = -a(1+\cot\alpha\tan\beta)$

$$\Rightarrow\ h = \frac{a(\sin\alpha\cos\beta + \cos\alpha\sin\beta)}{\sin\beta\cos\alpha - \sin\alpha\cos\beta}$$

$$= \frac{a\sin(\alpha+\beta)}{\sin(\beta-\alpha)}\,\text{m}$$

56 **(a)**

Line perpendicular to OA passing through B is $x = 4$



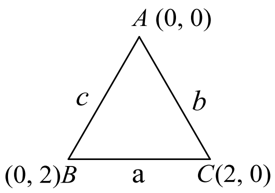
Slope of $AB = -\frac{3}{2}$

Line perpendicular to AB through origin is $y = \frac{2}{3}x$

$\therefore$ The point of intersection of a line $x = 4$ and $y = \frac{2}{3}x$ is $\left(4, \frac{8}{3}\right)$

57 **(c)**

Since, (0, 1), (1, 1) and (1, 0) are mid points of sides AB, BC and CA respectively



$\therefore$ Coordinates of A, B and C are (0, 0), (0, 2) and (2, 0) respectively

Now, $AB = 2, BC = 2\sqrt{2}, CA = 2$

$\therefore$ x-coordinate of incentre

$$= \frac{0+0+2.2}{2+2\sqrt{2}+2} \quad \left(\because\ x = \frac{ax_1+bx_2+cx_3}{a+b+c}\right)$$

$$= \frac{2}{2+\sqrt{2}} = 2-\sqrt{2}$$

58 **(d)**

Let $P(x, y)$ is equidistant from the mid points $A(a+b, b-a)$ and $(a-b, a+b)$

$\therefore\quad PA^2 = PB^2$

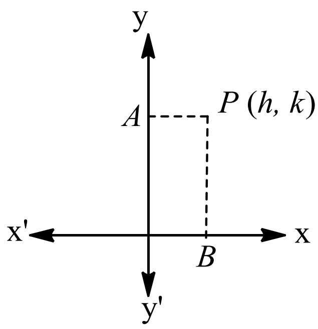
$\Rightarrow\ (a+b-x)^2 + (b-a-y)^2$

$= (a-b-x)^2 + (a+b-y)^2$

$\Rightarrow\ bx - ay = 0$

59 **(a)**

Let the locus of a point in a plane be $P(h, k)$



According to the question,

$|PA| + |PB| = 1 \quad \Rightarrow \quad |h| + |k| = 1$

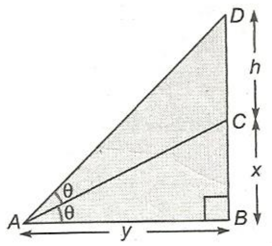
Hence, locus of a point is

$|x| + |y| = 1$

Which represents the equation of square

60 **(b)**

Let BCbe the height of tower and CD be height of the flagstaff



In ΔBAC, $\tan\theta = \frac{x}{y}$ ...(i)

In ΔDAB, $\tan 2\theta = \frac{x+h}{y}$

$\Rightarrow\ \frac{2\tan\theta}{1-\tan^2\theta} = \frac{x+h}{y} \ \Rightarrow\ \frac{2\left(\frac{x}{y}\right)}{1-\frac{x^2}{y^2}} = \frac{x+h}{y}$ [from Eq. (i)]

$\Rightarrow\ 2xy^2 - xy^2 + x^3 = (y^2 - x^2)h$

$$\Rightarrow\ h = \frac{x(x^2+y^2)}{(y^2-x^2)}$$

61 **(b)**

$$2ac\sin\left(\frac{A-B+C}{2}\right)$$

$$= 2ac\sin\left(\frac{180° - 2B}{2}\right)$$

$= 2ac\sin(90° - B) = 2ac\cos B = a^2 + c^2 - b^2$

62 **(d)**

$(x+1)^2 + y^2 + (x-2)^2 + y^2 = 2[(x-1)^2 + y^2]$

On simplification, we get $2x + 3 = 0$

63 **(a)**

We know that centroid divides the line segment joining orthocenter and circumcentre in the ratio2: 1. Since, the coordinates of orthocenter and circumcentre are (1, 1) and (3, 2) respectively

$\therefore$ The coordinates of centroid are

$$\left(\frac{2.3+1.1}{2+1}, \frac{2.2+1.1}{2+1}\right)=\left(\frac{7}{3}, \frac{5}{3}\right)$$

64 **(c)**

Given equation are

$x \cot \theta + y \operatorname{cosec} \theta = 2$ ...(i)

And $x\, cosec\, \theta + y \cot \theta = 6$ ...(ii)

On squaring and subtracting Eq. (i) from Eq. (ii), we get

$x^2(\operatorname{cosec}^2\theta - \cot^2\theta) + y^2(\cot^2\theta - \operatorname{cosec}^2\theta) = (6)^2 - (2)^2$

$\Rightarrow x^2 - y^2 = 32$

It represents an equation of hyperbola

65 **(c)**

Since, $\cot A + \cot C = 2 \cot B$

$\Rightarrow \frac{\cos A}{\sin A} + \frac{\cos C}{\sin C} = \frac{2 \cos B}{\sin B}$

$\Rightarrow \frac{b^2+c^2-a^2}{2bc(ka)} + \frac{a^2+b^2-c^2}{2ab(kc)} = 2\frac{a^2+c^2-b^2}{2ac(kb)}$

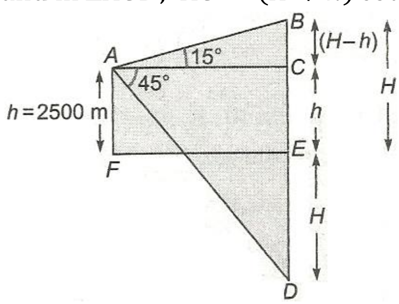
$\Rightarrow a^2 + c^2 = 2b^2$

Hence, a^2, b^2, c^2 are in AP

66 **(a)**

In $\Delta ABC, AC = (H-h)\cot 15°$ ... (i)

and in ΔACD, $AC = (H+h)\cot 45°$...(ii)



From Eqs. (i) and (ii).

$(H-h)\cot 15° = (H+h)\cot 45°$

$\Rightarrow H = \frac{h(\cot 15° + 1)}{\cot 15° - 1}$

$\therefore H = \frac{2500(2+\sqrt{3}+1)}{(2+\sqrt{3}-1)}$

$= \frac{2500(3+\sqrt{3})}{(\sqrt{3}+1)} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}-1)}$

$= 2500\sqrt{3}$ m

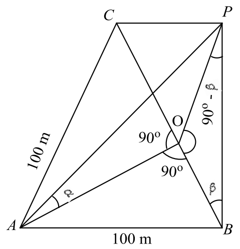
67 **(b)**

Let OP be the clock tower standing at the mid point O of side BC of ΔABC. Let $\alpha = \angle PAO = \cot^{-1} 3.2$ and $\beta = \angle PBO = \operatorname{cosec}^{-1} 2.6$

Then, $\cot \alpha = 3.2$ and $\operatorname{cosec} \beta = 2.6$

$\therefore \cot \beta = \sqrt{\operatorname{cosec}^2\beta - 1} = \sqrt{(2.6)^2 - 1} = 2.4$

In ΔPAO and ΔPBO,we have



$AO = h \cot \alpha = 3.2h$

and $BO = h \cot B = 2.4\, h$

In $\Delta ABO, AB^2 = OA^2 + OB^2$

$\Rightarrow 100^2 = (3.2h)^2 + (2.4\, h)^2$

$\Rightarrow 100^2 = 16h^2$

$\Rightarrow h^2 = 625 \Rightarrow h = 25$ m

68 **(d)**

$\because \cos 30° = \frac{3+1-a^2}{2\sqrt{3}}$ $\left[\because \cos A = \frac{b^2+c^2-a^2}{2bc}\right]$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{4-a^2}{2\sqrt{3}} \Rightarrow a^2 = 1$

$\Rightarrow a = 1$

Here, we see side b is largest, so

$\angle B$ must be greatest

$\therefore$ By sine rule, $\frac{b}{\sin B} = \frac{a}{\sin A}$

$\Rightarrow \frac{\sqrt{3}}{\sin B} = \frac{1}{\sin 30°}$

$\Rightarrow \sin B = \frac{\sqrt{3}}{2}$

$\Rightarrow \angle B = 120°$

69 **(c)**

Let each side of equilateral triangle $= a$

$\therefore \Delta = \frac{\sqrt{3}}{4}a^2, \quad S = \frac{3a}{2}$

Now, $r = \frac{\Delta}{5} = \frac{\sqrt{3}}{4}a^2 \cdot \frac{2}{3a} = \frac{a}{2\sqrt{3}}$

$R = \frac{abc}{4\Delta} = \frac{a^3}{\sqrt{3}a^2} = \frac{a}{\sqrt{3}}$

$r_1 = \frac{\Delta}{s-a} = \frac{\sqrt{3}}{4}a^2 \cdot \frac{2}{a} = \frac{\sqrt{3}}{2}a$

$\therefore R : r_1 : r_1 = \frac{a}{\sqrt{3}} : \frac{a}{2\sqrt{3}} : \frac{\sqrt{3}}{2}a$

$= 2 : 1 : 3$

70 **(d)**

Given, $r_1 = 2r_2 = 3r_3$

$\therefore \frac{\Delta}{s-a} = \frac{2\Delta}{s-b} = \frac{3\Delta}{s-c} = \frac{\Delta}{k}$ [say]

Then, $s-a = k, s-b = 2k, s-c = 3k$

$\Rightarrow 3s - (a+b+c) = 6k \Rightarrow s = 6k$

$\therefore \frac{a}{5} = \frac{b}{4} = \frac{c}{3} = k$

$\therefore \frac{a}{b} + \frac{b}{c} + \frac{c}{a} = \frac{5}{4} + \frac{4}{3} + \frac{3}{5} = \frac{191}{60}$

71 **(c)**

Let sides of the triangle are $4x, 5x, 6x$

$$s=\frac{4x+5x+6x}{2}=\frac{15}{2}x$$

$$\Delta=\sqrt{\frac{15}{2}\times\left(\frac{15}{2}x-4x\right)\left(\frac{15}{2}x-5x\right)\left(\frac{15}{2}x-6x\right)}$$

$$=\sqrt{\frac{15}{2}x\times\frac{7}{2}x\times\frac{5}{2}x\times\frac{3}{2}x}$$

$$=\frac{15\sqrt{7}x^2}{4}$$

Circumradius, $R=\frac{4x\times5x\times6x}{4\times\frac{15\sqrt{7}x^2}{4}}$

$$=\frac{8}{\sqrt{7}}x$$

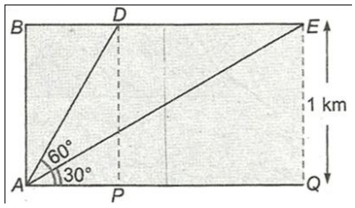
Inradius, $r=\frac{\frac{15\sqrt{7}}{4}x^2}{\frac{15}{2}x}$

$$=\frac{\sqrt{7}}{2}x$$

$$\frac{R}{r}=\frac{\frac{8x}{\sqrt{7}}}{\frac{\sqrt{7}x}{2}}=\frac{16}{7}$$

72 **(c)**

In $\Delta DAP, \tan 60^\circ=\frac{1}{AP}$ $\quad [\because EQ=DP=1]$



$$\Rightarrow \quad AP=\frac{1}{\sqrt{3}}$$

In $\Delta EAQ, \tan 30^\circ=\frac{EQ}{AP+PQ}$

$$\Rightarrow \frac{1}{\sqrt{3}}+PQ=\sqrt{3}$$

$\Rightarrow PQ=\frac{2}{\sqrt{3}}$ km

$\therefore$ Speed of plane $=\frac{\text{Distance}}{\text{Time}}$

$$=\frac{\frac{2}{\sqrt{3}}}{\frac{10}{60\times60}}=240\sqrt{3}\text{ km/h}$$

74 **(a)**

Area of triangle $=\frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}$

= A rational number if vertices have integral coordinates only

If triangle is equilateral, then area

$$=\frac{\sqrt{3}}{4}[(x_1-x_2)^2+(y_1-y_2)^2]$$

=irrational quantity

So, triangle cannot be equilateral

75 **(d)**

$$\frac{b-c}{a}=\frac{k(\sin B+\sin C)}{k\sin A}$$

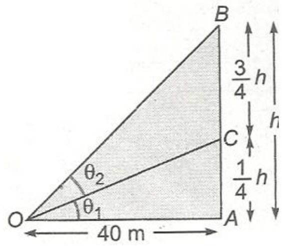
$$=\frac{2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)}{2\sin\frac{A}{2}\cos\frac{A}{2}}$$

$$\Rightarrow \frac{b+c}{a}=\frac{\cos\left(\frac{B-C}{2}\right)}{\sin\frac{A}{2}}$$

Similarly, $\frac{b-c}{a}=\frac{\sin\left(\frac{B-C}{2}\right)}{\cos\frac{A}{2}}$

76 **(b)**

Given, $\theta=\tan^{-1}\frac{3}{5} \Rightarrow \tan\theta_2=\frac{3}{5}$ ... (i)



In $\Delta AOC, \tan\theta_1=\frac{AC}{AO}=\frac{h}{160}$ ... (ii)

and in $\Delta AOB, \tan(\theta_1+\theta_2)=\frac{h}{40}$

$$\Rightarrow \frac{\tan\theta_1+\tan\theta_2}{1-\tan\theta_1\tan\theta_2}=\frac{h}{40}$$

$\Rightarrow \frac{\frac{h}{160}+\frac{3}{5}}{1-\frac{h}{160}\times\frac{3}{5}}=\frac{h}{40}$ [from Eqs. (i) and (ii)]

$$\Rightarrow \frac{5[h+96]}{800-3h}=\frac{h}{40}$$

$\Rightarrow h^2-200h+6400=0$

$\Rightarrow (h-160)(h-40)=0$

$\Rightarrow h=160$ or $h=40$

Hence, height of the vertical pole is 40 m

77 **(c)**

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two points. Let C and D be the points of internal and external division of AB in the ratio $\lambda : 1$. Then, the coordinates of C and D are

$\left(\frac{\lambda x_2 + x_1}{\lambda + 1}, \frac{\lambda y_2 + y_1}{\lambda + 1}\right)$ and $\left(\frac{\lambda x_2 - x_1}{\lambda - 1}, \frac{\lambda y_2 - y_1}{\lambda - 1}\right)$ respectively

$\therefore AC = \frac{\lambda}{\lambda + 1} AB$ and $AD = \frac{\lambda}{\lambda - 1} AB$

Clearly, $\frac{1}{AC} + \frac{1}{AD} = \frac{2}{AB} \Rightarrow AC, AB, AD$ are in H.P.

79 **(d)**

We have,

$A + B + C = 180°$

$\Rightarrow 3B = 180$ [$\because A, B, C$ are in AP]

$\Rightarrow B = 60°$

$\Rightarrow \cos B = \frac{1}{2}$

$\Rightarrow \frac{AB^2 + BC^2 - AC^2}{2AB.BC} = \frac{1}{2}$

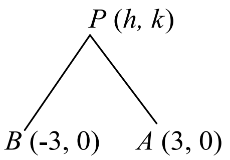
$\Rightarrow 36 + 49 - AC^2 = 6 \times 7 \Rightarrow AC^2 = 43 \Rightarrow AC = \sqrt{43}$

80 **(a)**

Let the point be $P(h, k)$

It is given that difference of the distance from points $A(3, 0)$ and $B(-3, 0)$ is 4 *ie*, $PA - PB = 4$

$\Rightarrow \sqrt{(h-3)^2 + k^2} - \sqrt{(h+3)^2 + k^2} = 4$



$\Rightarrow \sqrt{(h-3)^2 + k^2} = 4 + \sqrt{(h+3)^2 + k^2}$

On squaring both sides, we get

$(h-3)^2 + k^2 = 16 + (h+3)^2 + k^2 + 8\sqrt{(h+3)^2 + k^2}$

$\Rightarrow h^2 + 9 - 6h + k^2 = 16 + h^2 + 9 + 6h + k^2 + 8\sqrt{(h+3)^2 + k^2}$

$\Rightarrow -6h = 16 + 6h + 8\sqrt{(h+3)^2 + k^2}$

$\Rightarrow -8\sqrt{(h+3)^2 + k^2} = 12h + 16$

Again , squaring both sides, we get

$64(h+3)^2 + k^2 = (12h + 16)^2$

$\Rightarrow 64(h^2 + 9 + 6h + k^2) = 144h^2 + 256 + 2.16.12h$

$\Rightarrow 64(h^2 + 9 + 6h + k^2) = 16(9h^2 + 16 + 24h)$

$\Rightarrow 4(h^2 + 9 + 6h + k^2) = 9h^2 + 16 + 24h$

$\Rightarrow 4h^2 + 36 + 24h + 4k^2 = 9h^2 + 16 + 24h$

$\Rightarrow 5h^2 - 4k^2 = 20$

$\Rightarrow \frac{h^2}{4} - \frac{k^2}{5} = 1$

Hence, the locus of points P is $\frac{x^2}{4} - \frac{y^2}{5} = 1$

81 **(b)**

$\because \cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$\Rightarrow \cos^2 C = \left(\frac{a^2 + b^2 - c^2}{2ab}\right)^2$

$\Rightarrow \cos^2 C = \frac{[a^4 + b^4 + c^4 + 2a^2b^2 - 2c^2(a^2 + b^2)]}{4a^2b^2}$

$\Rightarrow \cos^2 C = \frac{1}{2}$ [$\because a^4 + b^4 + c^4 = 2c^2(a^2 + b^2)$ given]

$\Rightarrow \cos C = \pm \frac{1}{\sqrt{2}}$

$\Rightarrow \angle C = 45°$ or $135°$

82 **(d)**

Locus, of P is $\left|\sqrt{x^2 + y^2 - 8y + 16} -\right.$

x2+y2+8y+16=6

On squaring, we get

$x^2 + y^2 - 2 = \sqrt{x^2 + y^2 + 8y + 16}\sqrt{x^2 + y^2 - 8y + 16}$

$\Rightarrow (x^2 + y^2 - 2)^2 = (x^2 + y^2 + 16)^2 - (8y)^2$

On simplification, we get

$\frac{y^2}{9} - \frac{x^2}{7} = 1$

83 **(a)**

Given, $\sin\frac{A}{2}\sin\frac{C}{2} = \sin\frac{B}{2}$

$\Rightarrow \sqrt{\frac{(s-b)(s-c)}{bc}}\sqrt{\frac{(s-a)(s-b)}{ab}} = \sqrt{\frac{(s-a)(s-c)}{ac}}$

$\Rightarrow \frac{s-b}{b} = 1$

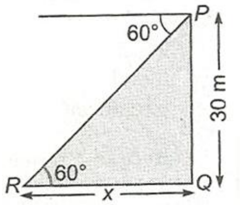
$\Rightarrow s = 2b$

84 **(c)**

In ΔPRQ

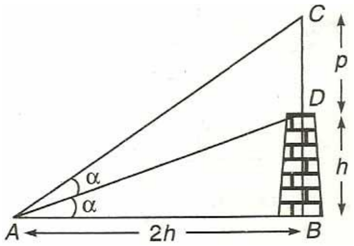
$\tan 60° = \frac{30}{x}$

$\Rightarrow x = 10\sqrt{3}$ m



85 **(a)**

In $\Delta ABD, \tan\alpha = \frac{h}{2h}$



$\Rightarrow\ \tan\alpha = \frac{1}{2}$ ...(i)

In $\Delta ABC,\ \tan 2\alpha = \frac{h+p}{2h}$

$\Rightarrow\ \dfrac{2\tan\alpha}{1-\tan^2\alpha} = \dfrac{h+p}{2h}$

$\Rightarrow\ \dfrac{2\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)^2} = \dfrac{h+p}{2h}$

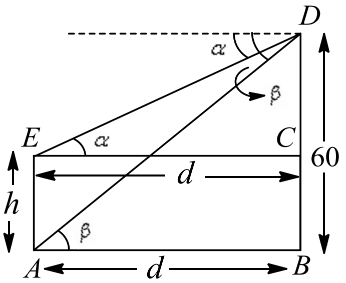
$\Rightarrow\ \dfrac{4}{3} = \dfrac{h+p}{2h}$

$\Rightarrow\ 8h = 3h + 3p$

$\Rightarrow\ 5h = 3p\ \Rightarrow\ p = \dfrac{5h}{3}$ m

86 **(d)**

In $\Delta\ ABD, \tan\beta = \frac{60}{d}$



$\Rightarrow\ d = 60\cot\beta$ ...(i)

In $\Delta\ DEC, \tan\alpha = \frac{DC}{EC}$

$\Rightarrow\ DC = d\tan\alpha$

$\Rightarrow\ 60 - h = d\tan\alpha\quad (\because BC = EA = h)$

$\Rightarrow\ 60 - h = 60\cot\beta\tan\alpha$ [from Eq.(i)]

$\Rightarrow\ h = 60\left(1 - \dfrac{\cos\beta}{\sin\beta}\cdot\dfrac{\sin\alpha}{\cos\alpha}\right)$

$\Rightarrow\ h = \dfrac{60\sin(\beta-\alpha)}{\cos\alpha\sin\beta}$

$\Rightarrow\ \frac{60\sin(\beta-\alpha)}{x} = \frac{60\sin(\beta-\alpha)}{\cos\alpha\sin\beta}$ (given)

$\Rightarrow\ x = \cos\alpha\sin\beta$

87 **(c)**

We know that, in triangle larger side has an larger angle opposite to it. Since, angles $\angle A, \angle B$ and $\angle C$ are in AP

$\Rightarrow\ 2B = A + C$

$\because\ A + B + C = \pi$

$\Rightarrow\ B = 60°$

$\therefore\ \cos B = \dfrac{a^2+c^2-b^2}{2ac}$

$\Rightarrow\ \cos 60° = \dfrac{1}{2} = \dfrac{100+a^2-81}{20a}$

$\Rightarrow\ a^2 + 19 = 10a$

$\Rightarrow\ a^2 - 10a + 19 = 0$

$\therefore\ a = \dfrac{10 \pm \sqrt{100-76}}{2} = 5 \pm \sqrt{6}$

88 **(c)**

Here, $\frac{1}{\sin^2\frac{A}{2}}, \frac{1}{\sin^2\frac{B}{2}}, \frac{1}{\sin^2\frac{C}{2}}$ are in AP

$\Rightarrow\ \dfrac{1}{\sin^2\frac{C}{2}} - \dfrac{1}{\sin^2\frac{B}{2}} = \dfrac{1}{\sin^2\frac{B}{2}} - \dfrac{1}{\sin^2\frac{A}{2}},$

$\Rightarrow\ \dfrac{ab}{(s-a)(s-b)} - \dfrac{ac}{(s-a)(s-c)}$

$= \dfrac{ac}{(s-a)(s-c)} - \dfrac{bc}{(s-b)(s-c)}$

$\Rightarrow\ \left(\dfrac{a}{s-a}\right)\left(\dfrac{b(s-c)-c(s-b)}{(s-b)(s-c)}\right)$

$= \left(\dfrac{c}{s-c}\right)\left(\dfrac{a(s-b)-b(s-a)}{(s-a)(s-b)}\right)$

$\Rightarrow\ ab + bc = 2ac\ \Rightarrow \dfrac{1}{c} + \dfrac{1}{a} = \dfrac{2}{b}$

Hence, a, b, c are in HP

90 **(c)**

Let the vertices of triangle be $P(1,1), Q(-1,-1)$ and $R(-\sqrt{3}, \sqrt{3})$

$\therefore PQ = \sqrt{(1+1)^2 + (1+1)^2} = 2\sqrt{2}$

$QR = \sqrt{\left(-\sqrt{3}+1\right)^2 + \left(\sqrt{3}+1\right)^2}$

$= \sqrt{3 + 1 - 2\sqrt{3} + 3 + 1 + 2\sqrt{3}} = 2\sqrt{2}$

and $RP = \sqrt{\left(-\sqrt{3}-1\right)^2 + \left(\sqrt{3}-1\right)^2}$

$= \sqrt{3 + 1 + 2\sqrt{3} + 3 + 1 - 2\sqrt{3}}$

$= 2\sqrt{2}$

$\Rightarrow\ PQ = QR = RP$

$\therefore$ Triangle is an equilateral triangle

91 **(a)**

Let the third vertex be (a, b)

$\therefore$ Area of $\Delta = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ a & b & 1 \\ 6 & 8 & 1 \end{vmatrix} = \frac{1}{2}|[8a - 6b]|$

As (a, b) are integers, so we take

$(0, 0), (1, 1), (1, 2)$

At $(0, 0), \Delta = 0$, it is not possible

At $(1,1)\Delta = 1$
At $(1,2), \Delta = 2$
Here, we see that minimum area is 1

92 **(d)**

$$\left(\cot\frac{A}{2}+\cot\frac{B}{2}\right)\left(a\sin^2\frac{B}{2}+b\sin^2\frac{A}{2}\right)$$

$$=\left(\frac{\cos\frac{A}{2}+\sin\frac{B}{2}+\cos\frac{B}{2}+\sin\frac{A}{2}}{\sin\frac{A}{2}\sin\frac{B}{2}}\right)\left(a\sin^2\frac{B}{2}+b\sin^2\frac{A}{2}\right)$$

$$=\frac{\sin\left(\frac{A+B}{2}\right)\left(a\sin^2\frac{B}{2}+b\sin^2\frac{A}{2}\right)}{\sin\frac{A}{2}\sin\frac{B}{2}}$$

$$=\left(\cos\frac{C}{2}\right)\left(a\frac{\sin\frac{B}{2}}{\sin\frac{A}{2}}+b\frac{\sin\frac{A}{2}}{\sin\frac{B}{2}}\right)$$

$$=\sqrt{\frac{s(s-c)}{ab}}\left(a\frac{\sqrt{\frac{(s-a)(s-c)}{ac}}}{\sqrt{\frac{(s-b)(s-c)}{bc}}}+b\frac{\sqrt{\frac{(s-b)(s-c)}{bc}}}{\sqrt{\frac{(s-a)(s-c)}{ac}}}\right)$$

$$=\sqrt{\frac{s(s-c)}{ab}}\left(\sqrt{\left(\frac{s-a}{s-b}\right)ab}+\sqrt{\left(\frac{s-b}{s-a}\right)ab}\right)$$

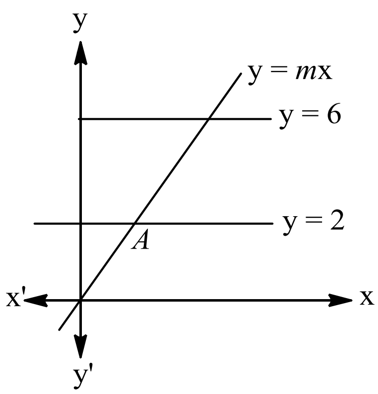
$$=\sqrt{s(s-c)}\left(\frac{s-a+s-b}{\sqrt{(s-a)(s-b)}}\right)$$

$$=\sqrt{s(s-c)}\left(\frac{2s-a-b}{\sqrt{(s-a)(s-b)}}\right)$$

$$=c\sqrt{\frac{s(s-c)}{(s-a)(s-b)}}=c\cot\frac{C}{2}$$

93 **(c)**

Given lines $y=mx, y=2, y=6$



Coordinates of points A and B are $\left(\frac{2}{m},2\right),\left(\frac{6}{m},6\right)$ respectively

$\therefore\ AB=\sqrt{\left(\frac{2}{m}-\frac{6}{m}\right)^2+(2-6)^2}<5$ [given]

$$\Rightarrow\ \left(\frac{2}{m}-\frac{6}{m}\right)^2+(4)^2<25$$

$$\Rightarrow\ \left(\frac{2}{m}-\frac{6}{m}\right)^2<9\ \Rightarrow\ -3<\frac{2}{m}-\frac{6}{m}<3$$

$$\Rightarrow\ -\frac{4}{3}>m>\frac{4}{3}$$

$$\therefore\ \ m\in\left]-\infty,-\frac{4}{3}\right[\cup\left]\frac{4}{3},\infty\right[$$

94 **(c)**

By sine rule

$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$ (say)

$\therefore\ (b-c)\sin A+(c-a)\sin B+(a-b)\sin C$
$=(b-c)ak+(c-a)bk+(a-b)kc$
$=k[ab-ac+bc-ab+ac-bc]$
$=0$

95 **(a)**

Given that, $a=3, b=5, c=6$

Now, $s=\frac{a+b+c}{2}=7$

$\therefore\ \Delta=\sqrt{s(s-a)(s-b)(s-c)}$
$=\sqrt{7(7-3)(7-5)(7-6)}$
$=\sqrt{7\cdot4\cdot2\cdot1}=2\sqrt{14}$

$\therefore\ r=\frac{\Delta}{s}=\frac{2\sqrt{14}}{7}=\sqrt{\frac{8}{7}}$

96 **(a)**

$$\cos B=\frac{3^2+4^2-5^2}{2(3)(4)}=\frac{9+16-25}{2(3)(4)}=0$$

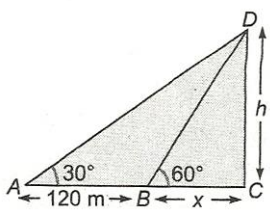
$\Rightarrow\ \angle B=90°$

$\therefore\ \sin\frac{B}{2}+\cos\frac{B}{2}=\sin 45°+\cos 45°$

$=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=\sqrt{2}$

97 **(b)**

In ΔCAD, $\tan 30°=\frac{CD}{AC}$



$\Rightarrow\frac{1}{\sqrt{3}}=\frac{h}{120+x}$

$\Rightarrow\sqrt{3}h=120+x$ ...(i)

and in ΔCBD, $\tan 60°=\frac{CD}{BC}$

$\Rightarrow\sqrt{3}=\frac{h}{x}\Rightarrow h=\sqrt{3}x$ ...(ii)

From Eqs. (i) and (ii), we get, $x=60$ m

On putting $x=60$ in Eq.(i), we get

$h=60\sqrt{3}$ m

98 **(c)**

Area of triangle$=\frac{1}{2}[x(1-2)+1(2-0)+0(0-$

1)]

$= \frac{1}{2}[-x + 2 + 0] = 4$ [given]

$\Rightarrow \ 2 - x = 8 \ \Rightarrow x = -6$

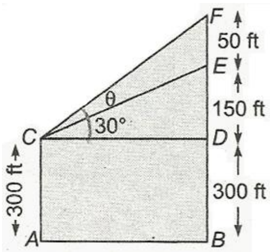
102 **(a)**

The sum of the distance of a point P from two perpendicular lines in aplane is 1, then the locus of P is a rhombus

103 **(a)**

In ΔDCE, $\tan 30° = \frac{150}{CD}$

$\Rightarrow CD = \sqrt{3} \times 150$



Now, In ΔDCF,

$$\tan\theta = \frac{DF}{CD} = \frac{200}{\sqrt{3}.150} = \frac{4}{3\sqrt{3}}$$

104 **(b)**

$$2\left(a\sin^2\frac{C}{2} + c\sin^2\frac{A}{2}\right)$$

$$= 2\left(a\,\frac{(s-a)(s-b)}{ab} + c\frac{(s-b)(s-c)}{bc}\right)$$

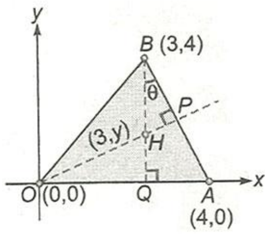
$$= 2\left(\frac{(s-b)}{b}(s-a+s-c)\right)$$

$$= \frac{2}{b}(s-b)b$$

$$= 2\,(s-b) = a - b + c$$

105 **(c)**

Let H be the orthocenter of ΔOAB



$\therefore$ (slope of OP) . (slope of BA) $= -1$

$$\Rightarrow \quad \left(\frac{y-0}{3-0}\right).\left(\frac{4-0}{3-4}\right) = -1$$

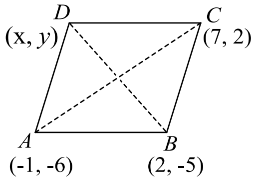
$$\Rightarrow -\frac{4}{3}y = -1$$

$$\Rightarrow \quad y = \frac{3}{4}$$

$\therefore$ Required orthocentre$= (3, y) = \left(3, \frac{3}{4}\right)$

106 **(b)**

Let the fourth vertex be $D(x, y)$



We know that two diagonals of a parallelogram are bisect each other

$$\therefore \ \frac{-1+7}{2} = \frac{2+x}{2} \ \Rightarrow \ x = 4$$

and $\frac{-6+2}{2} = \frac{-5+y}{2} \ \Rightarrow \ y = 1$

$\therefore$ Fourth vertex of D is $(4, 1)$

107 **(c)**

We have, $\cos A\cos B + \sin A\sin B\sin C = 1$

$\Rightarrow 2\cos A\cos B + 2\sin A\sin B\sin C = 2$

$\Rightarrow \ 2\cos A\cos B + 2\sin A\sin B\sin C$

$= \cos^2 A + \sin^2 A + \cos^2 B + \sin^2 B$

$\Rightarrow \ (\cos A - \cos B)^2 + (\sin A - \sin B)^2$

$+ 2\sin A\sin B\,(1 - \sin C) = 0$

$\Rightarrow \ \cos A - \cos B = 0, \qquad \sin A - \sin B = 0$

and $\ 1 - \sin C = 0$

$\Rightarrow \ \ A = B$ and $C = 90°$

$\Rightarrow \ \ a = b$ and $C = 90°$

108 **(d)**

We have, $A + B + C = 180°$

$\Rightarrow \ A = 180° - (B + C)$

$\Rightarrow \ \tan A = \tan(180° - B - C)$

$\Rightarrow \ \tan 90° = -\tan(B + C)$

$$\Rightarrow \ \infty = -\frac{\tan B + \tan C}{1 - \tan B\tan C}$$

$\Rightarrow \ 1 - \tan B\tan C = 0$

$\Rightarrow \ \tan B\tan C = 1$

109 **(b)**

Since, the given points lies on a line, then

$$\begin{vmatrix} 1 & 1 & 1 \\ -5 & 5 & 1 \\ 13 & \lambda & 1 \end{vmatrix} = 0$$

$\Rightarrow \ 1(5 - \lambda) - 1(-5 - 13) + 1(-5\lambda - 65) = 0$

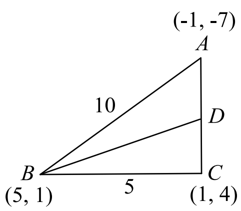
$\Rightarrow \ -6\lambda = 42 \ \Rightarrow \ \ \lambda = -7$

110 **(a)**

The vertices of triangle are $(0, 0)$, $(3, 0)$ and $(0, 4)$. It is a right angled triangle, therefore circumcentre is $\left(\frac{3}{2}, 2\right)$

111 **(c)**

$BC = 5, BA = 10$



Let D divides AC in the ratio 2:1

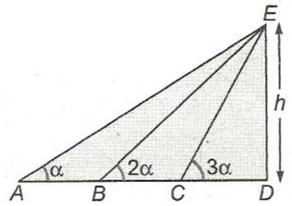
$\therefore$ Coordinate of D is $\left(\frac{1}{3}, \frac{1}{3}\right)$

The bisector is the line joining B and D is

$\frac{y-1}{x-5} = \frac{1}{7}$ or $x - 7y + 2 = 0$

112 **(b)**

In ΔECD, $\tan 3\alpha = \frac{h}{CD}$



$\Rightarrow CD = h \cot 3\alpha$ ... (i)

In ΔEBD, $\tan 2\alpha = \frac{h}{BD}$

$\Rightarrow BD = h \cot 2\alpha$ ...(ii)

In ΔEAD, $\tan \alpha = \frac{h}{AD}$

$\Rightarrow AD = h \cot \alpha$ ... (iii)

From Eqs. (ii) and (iii),

$AD - BD = h \cot \alpha - h \cot 2\alpha$

$AB = h(\cot \alpha - \cot 2\alpha)$ ... (iv)

From Eqs. (i) and (ii),

$BD - CD = h \cot 2\alpha - h \cot 3\alpha$

$\Rightarrow BC = h(\cot 2\alpha - \cot 3\alpha)$ ... (v)

From Eqs. (iv) and (v),

$$\frac{AB}{BC} = \frac{h(\cot \alpha - \cot 2\alpha)}{h(\cot 2\alpha - \cot 3\alpha)}$$

$$= \frac{\frac{\cos \alpha}{\sin \alpha} - \frac{\cos 2\alpha}{\sin 2\alpha}}{\frac{\cos 2\alpha}{\sin 2\alpha} - \frac{\cos 3\alpha}{\sin 3\alpha}}$$

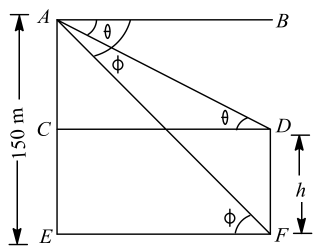
$$= \frac{\frac{\sin(2\alpha - \alpha)}{\sin \alpha \sin 2\alpha}}{\frac{\sin(3\alpha - 2\alpha)}{\sin 2\alpha \sin 3\alpha}}$$

$$= \frac{\sin 3\alpha}{\sin \alpha} = 3 - 4\sin^2 \alpha$$

$= 3 - 2(1 - \cos 2\alpha)$

$= 1 + 2\cos 2\alpha$

114 **(b)**



In ΔAEF, $\tan \phi = \frac{AE}{EF} = \frac{150}{EF}$

$\frac{5}{2} = \frac{150}{EF}$

$\Rightarrow EF = 60$ m

and in ΔACD

$\tan \theta = \frac{AC}{CD}$

$\Rightarrow \frac{4}{3} = \frac{150 - h}{60}$ $[\because CD = EF]$

$\Rightarrow 80 = 150 - h$

$\Rightarrow h = 70$ m

$\therefore AC = 80$ m and $CD = 60$ m

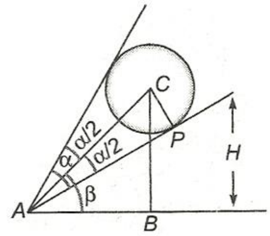
$\Rightarrow AD = \sqrt{AC^2 + CD^2}$

$= \sqrt{6400 + 3600}$

$= \sqrt{10000} = 100$ m

115 **(d)**

In ΔAPC, $\sin(\angle PAC) = \frac{CP}{AC}$



$\Rightarrow AC = \frac{r}{\sin\frac{\alpha}{2}} = r \operatorname{cosec}\frac{\alpha}{2}$ ...(i)

Again, in ΔABC, $\sin \beta = \frac{BC}{AC}$

$\Rightarrow BC = AC \sin \beta$

$\Rightarrow H = r \operatorname{cosec}\left(\frac{\alpha}{2}\right) \sin \beta$[from Eq.(i)]

116 **(b)**

Since, $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\Rightarrow b^2 - 2bc \cos A + (c^2 - a^2) = 0$

It is given that b_1 and b_2 are the roots of this equation

Therefore, $b_1 + b_2 = 2c \cos A$ and $b_1 b_2 = c^2 - a^2$

$\Rightarrow 3b_1 = 2c \cos A$ and $2b_1^2 = c^2 - a^2$

$[\because b_2 = 2b_1]$

$$\Rightarrow 2\left(\frac{2c}{3}\cos A\right)^2 = c^2 - a^2$$

$\Rightarrow 8c^2 (1 - \sin^2 A) = 9c^2 - 9a^2$

$$\Rightarrow \sin A = \sqrt{\frac{9a^2 - c^2}{8c^2}}$$

117 **(a)**

$\because (\sqrt{a} + \sqrt{b} + \sqrt{c})(\sqrt{a} + \sqrt{b} - \sqrt{c})$

$= (\sqrt{a} + \sqrt{b})^2 - c$

$= a + b - c + 2\sqrt{ab} > 0$

$\therefore \sqrt{a} + \sqrt{b} > \sqrt{c}$

118 **(a)**

In ΔABC, $\angle A = 30°$, $BC = 10$ cm

O is the centre of circle

$\therefore \angle BOC = 60°$

and OB and OC are the radius

$\therefore \angle OBC = \angle OCB = 60°$

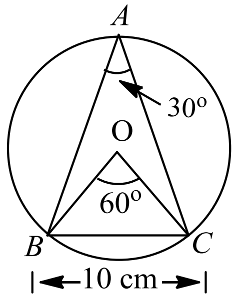
$\Rightarrow$ ΔOBC is an equilateral triangle

$\therefore$ radius of circle is

$OB = OC = BC = 10$ cm

Now, area of the circumcircle is πr^2

$= \pi(10)^2 = 100\pi$ sq cm



119 **(b)**

We have, $R = \frac{abc}{4\Delta}$ and $r = \frac{\Delta}{s}$

$\therefore \dfrac{R}{r} = \dfrac{abc}{4\Delta} \cdot \dfrac{s}{\Delta}$

$= \dfrac{abc}{4(s-a)(s-b)(s-c)}$

Since, $a:b:c = 4:5:6$

$\Rightarrow \dfrac{a}{4} = \dfrac{b}{5} = \dfrac{c}{6} = k$ (say)

Thus, $\frac{R}{r} = \frac{(4k)(5k)(6k)}{4\left(\frac{15k}{2}-4k\right)\left(\frac{15k}{2}-5k\right)\left(\frac{15k}{2}-6k\right)}$

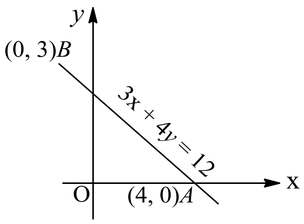
$= \dfrac{120k^3 \cdot 2}{k^3 \cdot 7 \cdot 5 \cdot 3} = \dfrac{16}{7}$

120 **(b)**

Given equation of lines are

$x = 0, y = 0$ and $3x + 4y = 12$

Incentre is on the line $y = x$ (Angled bisector of OA and OB)



Angle bisector of $y = 0$ and $3x + 4y = 12$ is

$\pm 5y = 3x + 4y - 12$

$\Rightarrow$ $3x + 9y = 12$

and $3x - y = 12$

Here, $3x + 9y = 12$ internal bisector

So, intersection point of $y = x$ and $3x + 9y = 12$ is $(1, 1)$

$\therefore$ The required point of the incentre of triangle is $(1, 1)$

121 **(a)**

$BP - AP = \pm 6$ or $BP = AP \pm 6$

$\Rightarrow \sqrt{x^2 + (y+4)^2} = \sqrt{x^2 + (y-4)^2} \pm 6$

Squaring and simplification, we get

$4y - 9 = \pm 3\sqrt{x^2 + (y-4)^2}$

Again squaring, we get

$9x^2 - 7y^2 + 63 = 0$

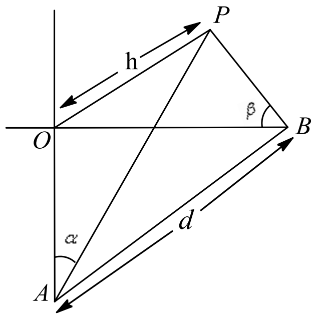
122 **(c)**

Let OP be the tower whose height is h metres

In $\Delta\ OAP, \tan\alpha = \frac{OP}{OA}$

$\Rightarrow$ $OA = h\cot\alpha$ ...(i)

In $\Delta\ OBP, \tan\beta = \frac{OP}{OB}$



$\Rightarrow$ $OB = h\cot\beta$ ...(ii)

Now, in $\Delta\ OAB$, $AB^2 = OA^2 + OB^2$

$\Rightarrow$ $d^2 = h^2(\cot^2\alpha + \cot^2\beta)$ [from Eq. (i) and (ii)]

$\Rightarrow$ $h = \dfrac{d}{\sqrt{\cot^2\alpha + \cot^2\beta}}$

123 **(d)**

Since, the coordinates of P are $(1, 0)$

Let any point Q on $y^2 = 8x$ is $(2t^2, 4t)$

Again, let mid point of PQ is (h, k), so

$h = \frac{2t^2+1}{2}$ $\Rightarrow$ $2h = 2t^2 + 1$ ...(i)

and $k = \frac{4t+0}{2}$ $\Rightarrow$ $t = \frac{k}{2}$ ...(ii)

on putting the value of t from Eq. (ii) in Eq. (i), we get

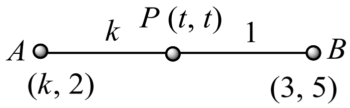
$2h = \dfrac{2k^2}{4} + 1 \quad \Rightarrow \quad 4h = k^2 + 2$

Hence, locus of (h, k) is $y^2 - 4x + 2 = 0$

124 **(b)**

Let $P(t, t)$ divides AB in the ratio $k:1$, then

$\frac{3k+k}{k+1} = t$ and $\frac{5k+2}{k+1} = t$



$\Rightarrow$ $\dfrac{3k+k}{k+1} = \dfrac{5k+2}{k+1}$

$\Rightarrow$ $4k - 5k = 2$

$\Rightarrow$ $k = -2$

125 **(c)**

Since, a, b and c, the sides of a triangle are in AP

$\therefore$ $2b = a + c$ ...(i)

We know that, $\cos A = \frac{b^2+c^2-a^2}{2bc}$

$\Rightarrow$ $\cos A = \frac{b^2+c^2-(2b-c)^2}{2bc}$ [from Eq. (i)]

$\Rightarrow \cos A = \dfrac{b^2+c^2-4b^2-c^2+4bc}{2bc}$

$\Rightarrow \cos A = \dfrac{4c-3b}{2c}$

126 **(b)**

The intersection points of given lines are

$(0,0), \left(\frac{5}{2}, 5\right), \left(\frac{5}{3}, 5\right)$

$\therefore$ Area of $\Delta = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ \frac{5}{2} & 5 & 1 \\ \frac{5}{3} & 5 & 1 \end{vmatrix}$

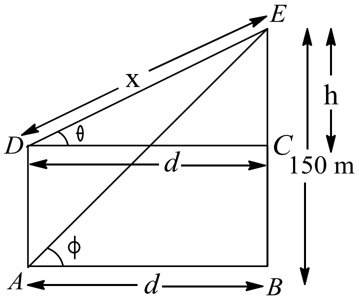
$= \dfrac{1}{2}\left[1\left(\dfrac{25}{2} - \dfrac{25}{3}\right)\right]$

$= \frac{1}{2} \times \frac{25}{6} = \frac{25}{12}$ sq units

127 **(d)**

Given that,

$\tan\theta = \frac{4}{3}$ and $\tan\phi = \frac{5}{2}$ ...(i)



In $\Delta\, ABE, \tan\phi = \frac{150}{d}$

$\Rightarrow\ d = 150\cot\phi$

$= 150 \times \frac{2}{5} = 60$ m ...(ii)

In $\Delta\, DCE, \tan\theta = \frac{h}{d}$

$\Rightarrow\ \frac{4}{3} = \frac{h}{d}$ [from Eq.(i)]

$\Rightarrow\ h = \frac{4}{3}\ (60)$ [from Eq.(ii)]

$\Rightarrow\ h = 80$ m

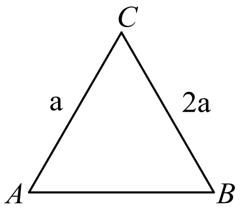
Now, in $\Delta\, DCE,\ DE^2 = DC^2 + CE^2$

$\Rightarrow\ x^2 = 60^2 + 80^2 = 10000$

$\Rightarrow\ x = 100$ m

128 **(d)**

Given, $A - B = 60°$



By sine rule,

$\dfrac{2a}{\sin A} = \dfrac{a}{\sin B}$

$\Rightarrow\ \sin A - 2\sin B = 0$

$\Rightarrow\ \sin(60° + B) - 2\sin B = 0$

$\Rightarrow \frac{\sqrt{3}}{2}\cos B + \frac{1}{2}\sin B - 2\sin B = 0$

$\Rightarrow \dfrac{\sqrt{3}}{2}\cos B - \dfrac{3}{2}\sin B = 0$

$\Rightarrow\ \sqrt{3}\left(\dfrac{1}{2}\cos B - \dfrac{\sqrt{3}}{2}\sin B\right) = 0$

$\Rightarrow\ \sqrt{3}[\cos(60° + B)] = 0$

$\Rightarrow 60° + B = 90°$

$\Rightarrow B = 30°$

$\Rightarrow A = 90°$

Hence, it is right angled triangle

129 **(c)**

$\begin{vmatrix} 3q & 0 & 1 \\ 0 & 3p & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$

$\Rightarrow\ 3q(3p-1) + 1(0-3p) = 0$

$\Rightarrow\ \ 9pq = 3p + 3q$

$\Rightarrow\ \dfrac{1}{p} + \dfrac{1}{q} = 3$

130 **(c)**

Here, $s = \frac{15+36+39}{2} = 45$

$\therefore\ \sin\dfrac{C}{2} = \sqrt{\dfrac{(s-a)(s-b)}{ab}}$

$\Rightarrow\ \sin\dfrac{C}{2} = \sqrt{\dfrac{(45-15)(45-36)}{15 \times 36}}$

$= \sqrt{\dfrac{30 \times 9}{15 \times 36}} = \dfrac{1}{\sqrt{2}}$

131 **(c)**

Since, $\frac{c}{\sin C} = 2R\ \Rightarrow c = 2R\ \ [\because C = 90°]$ ...(i)

And $\tan\frac{C}{2} = \frac{r}{s-c}$

$\Rightarrow\ \tan\dfrac{\pi}{4} = \dfrac{r}{s-c}$

$\therefore\ \ r = s - c$

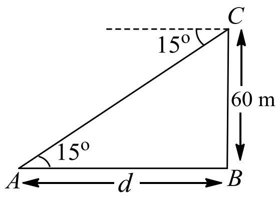
$\Rightarrow\ \ a + b - c = 2r$ ...(ii)

From Eqs. (i) and (ii), we get

$2(r+R) = a + b$

132 **(b)**

Let BC be the light house



In $\Delta\, ABC, \tan 15° = \frac{60}{d}$

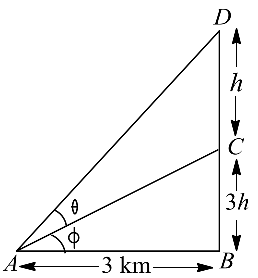
$\Rightarrow\ d = 60\cot 15° = 60\left(\dfrac{\sqrt{3}+1}{\sqrt{3}-1}\right)$ m

133 **(b)**

Given that, $\cos^2 A + \cos^2 C = \sin^2 B$
Obviously it is not an equilateral triangle because $A = B = C = 60°$ does not satisfy the given condition. But $B = 90°$, then $\sin^2 B = 1$ and $\cos^2 A + \cos^2 C = \cos^2 A + \cos^2\left(\frac{\pi}{2} - A\right)$
$= \cos^2 A + \sin^2 A = 1$
Hence, this satisfies the condition, so it is a right angled triangle but not necessary isosceles triangle

134 **(d)**
Given, $a:b:c = 1:\sqrt{3}:2$
Here, $c^2 = a^2 + b^2$
$\therefore$ Triangle is right angled at C
$\therefore \angle C = 90°$
and $\frac{a}{b} = \frac{1}{\sqrt{3}}$
$\Rightarrow \tan A = \frac{1}{\sqrt{3}}$
$\Rightarrow \angle A = 30°$ and $\angle B = 60°$ [as $\angle A + \angle B = 90°$]
$\therefore$ Ratio of angles $= \angle A : \angle B : \angle C = 90°$
$= 30° : 60° : 90° = 1 : 2 : 3$

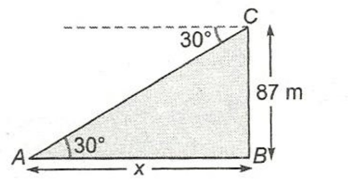
135 **(c)**
Given, $\tan\theta = \frac{1}{9}$
In ΔABC, $\tan\phi = \frac{3h}{3} = h$ ...(i)



In ΔABD, $\tan(\theta + \phi) = \frac{4h}{3}$
$$\Rightarrow \frac{\tan\theta + \tan\phi}{1 - \tan\phi\tan\theta} = \frac{4h}{3}$$
$$\Rightarrow \frac{\frac{1}{9} + h}{1 - \frac{h}{9}} = \frac{4h}{3}$$
$$\Rightarrow \frac{1 + 9h}{9 - h} = \frac{4h}{3}$$
$\Rightarrow 3 + 27h = 36h - 4h^2$
$\Rightarrow 4h^2 - 9h + 3 = 0$
$$\Rightarrow h = \frac{9 \pm \sqrt{81 - 48}}{2 \times 4} = \frac{9 \pm \sqrt{33}}{8}$$

136 **(c)**
Let $a = 60° - d$, $B = 60°$, $C = 60° + d$
$$\therefore \frac{b}{c} = \frac{\sin B}{\sin C} = \sqrt{\frac{3}{2}}$$
$$\Rightarrow \frac{\sin 60°}{\sin(60° + d)} = \sqrt{\frac{3}{2}}$$
$$\Rightarrow \frac{\sqrt{3}}{2\sin(60° + d)} = \sqrt{\frac{3}{2}}$$
$$\Rightarrow \sin(60° + d) = \frac{1}{\sqrt{2}}$$
$\Rightarrow 60° + d = 45° \Rightarrow d = -15°$
So, $\angle A = 75°$

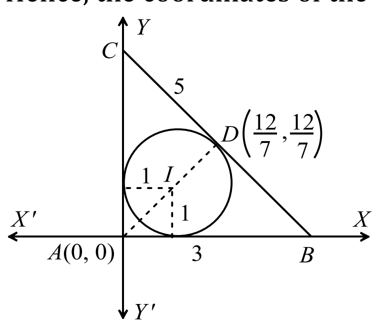
137 **(b)**
In $\Delta ABC, \tan 30° = \frac{87}{x}$
$\Rightarrow x = 87 \times \sqrt{3}$



$$\therefore \text{Speed} = \frac{\text{Diatance}}{\text{Time}}$$
$$\Rightarrow \text{Time} = \frac{87 \times \sqrt{3} \times 60}{5.8 \times 1000} = \frac{9\sqrt{3}}{10} \text{ min}$$

138 **(c)**
It is given that the centroid of the triangle formed by the points (a,b), (b,c) and (c,a) is at the origin
$$\therefore \left(\frac{a+b+c}{3}, \frac{a+b+c}{3}\right) = (0,0)$$
$\Rightarrow a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$

139 **(d)**
We have,
Δ= Area of Δ ABC
$$\Rightarrow \Delta = \frac{1}{2} \times AB \times = \frac{1}{2} \times 3 \times 4 = 6 \text{ sq. units}$$
$$s = \text{Semi} - \text{perimeter} = \frac{1}{2}(3 + 4 + 5) = 6 \text{ units}$$
$$\therefore r = \text{In} - \text{radius} = \frac{\Delta}{s} = 1$$
Hence, the coordinates of the incentre are (1, 1)



140 **(c)**
Given, $a^4 + b^4 + c^4 = 2c^2(a^2 + b^2)$ ...(i)
$$\because \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$\Rightarrow\ \cos^2 C$

$= \left[\dfrac{a^4+b^4+c^4+2a^2b^2-2c^2(a^2+b^2)}{4a^2b^2}\right]$

$= \left[\dfrac{2c^2(a^2+b^2)+2a^2b^2-2c^2(a^2+b^2)}{4a^2b^2}\right]$

[from Eq. (i)]

$\Rightarrow\ \cos^2 C = \dfrac{1}{2}$

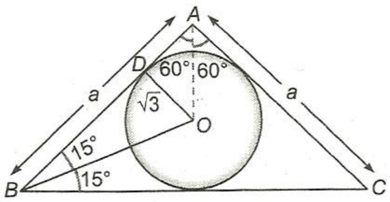
$\Rightarrow\ \cos C = \pm\dfrac{1}{\sqrt{2}}$

$\Rightarrow\ \angle C = 45°$ or $135°$

141 **(c)**

Let $AB = AC$ and $\angle A = 120°$

$\therefore$ Area of triangle$= \frac{1}{2}a^2 \sin 120°$



Where, $a = AD + BD$

$= \sqrt{3}\tan 30° + \sqrt{3}\cot 15°$

$= 1 + \sqrt{3}\left(\dfrac{1+\tan 45° \tan 30°}{\tan 45° - \tan 30°}\right)$

$= 1 + \sqrt{3}\left(\dfrac{\sqrt{3}+1}{\sqrt{3}-1}\right)$

$\therefore\ a = 4 + 2\sqrt{3}$

$\Rightarrow$ Area of triangle$= \frac{1}{2}\left(4+2\sqrt{3}\right)^2\left(\frac{\sqrt{3}}{2}\right) = 12 + 7\sqrt{3l}$

142 **(b)**

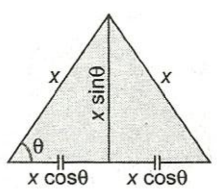
Area $= \dfrac{1}{2} \times$ base $\times$ altitude

$= \dfrac{1}{2} \times (2x\cos\theta) \times (x \sin\theta)$

$= \dfrac{1}{2}x^2 \sin 2\theta$

(Since, maximum value of $\sin 2\theta$ is 1)

$\therefore$ Maximum area $= \dfrac{1}{2}x^2$



143 **(d)**

Here, $\tan\frac{A}{2}\ \tan\frac{C}{2} = \frac{1}{3}$

$\Rightarrow \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}.\dfrac{(s-b)(s-a)}{s(s-c)}} = \dfrac{1}{3}$

$\Rightarrow\ \dfrac{s-b}{s} = \dfrac{1}{3} \Rightarrow 2s = 3b$

$\Rightarrow\ 2b = a + c$

$\Rightarrow\ a, b, c$ are in AP

144 **(d)**

Let the vertices of a triangle are $P(2,1), Q(5,2)$ and $R(3,4)$ and $A(x,y)$ be the circumcentre of ΔPQR

$\therefore\ AP^2 = AQ^2$

$\Rightarrow\ (2-x)^2 + (1-y)^2 = (5-x)^2 + (2-y)^2$

$\Rightarrow\ 4 + x^2 - 4x + 1 + y^2 - 2y$

$= 25 + x^2 - 10x + 4 + y^2 - 4y$

$\Rightarrow\ 6x + 2y = 24$

$\Rightarrow\ 3x + y = 12$ ...(i)

and $AP^2 = AR^2$

$\Rightarrow\ (2-x)^2 + (1-y)^2 = (3-x)^2 + (4-y)^2$

$\Rightarrow\ 4 + x^2 - 4x + 1 + y^2 - 2y$

$= 9 + x^2 - 6x + 16 + y^2 - 8y$

$\Rightarrow\ 2x + 6y = 20$

$\Rightarrow\ x + 3y = 10$ ...(ii)

On solving Eqs. (i) and (ii), we get

$x = \frac{13}{4}$ and $y = \frac{9}{4}$

$\therefore$ Circumcentre is $\left(\frac{13}{4}, \frac{9}{4}\right)$

145 **(c)**

$(a+b+c)(b+c-a) = kbc$

$\Rightarrow\ 2s(2s-2a) = kbc$

$\Rightarrow\ \dfrac{s(s-a)}{bc} = \dfrac{k}{4}$

$\Rightarrow\ \cos^2\left(\dfrac{A}{2}\right) = \dfrac{k}{4}$

$\because\ 0 < \cos^2\left(\dfrac{A}{2}\right) < 1$

$\therefore\ 0 < \dfrac{k}{4} < 1$

$\Rightarrow\ 0 < k < 4$

146 **(d)**

$\because$ Area of $\Delta\, PBC = \dfrac{1}{2}\left\|\begin{matrix} \alpha & \beta & 1 \\ -3 & 5 & 1 \\ 4 & -2 & 1 \end{matrix}\right\|$

$= \dfrac{1}{2}|7\alpha + 7\beta - 14|$

Also, Area of $\Delta\, ABC = \frac{1}{2}\left\|\begin{matrix} 6 & -3 & 1 \\ -3 & 5 & 1 \\ 4 & -2 & 1 \end{matrix}\right\|$

$= \dfrac{1}{2}|42 - 21 - 14| = \dfrac{7}{2}$

$\therefore\ \dfrac{\text{Area of } \Delta\, PBC}{\text{Area of } \Delta\, ABC} = \dfrac{\frac{7}{2}|\alpha + \beta - 2|}{\frac{7}{2}}$

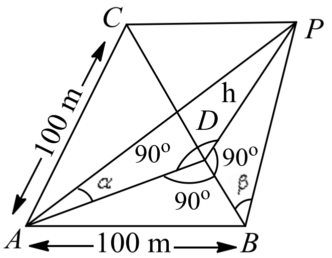
$= |\alpha + \beta - 2|$

147 **(b)**

Let DP is clock tower standing at the middle point

D of BC

Let $\angle PAD = \alpha = \cot^{-1} 3.2 \Rightarrow \cot\alpha = 3.2$



and $\angle PBD = \beta = \operatorname{cosec}^{-1} 2.6$

$\Rightarrow \operatorname{cosec}\beta = 2.6$

$\therefore \cot\beta = \sqrt{(\operatorname{cosec}^2\beta - 1)}$

$= \sqrt{5.76} = 2.4$

In ΔPAD and PBD,

$AD = h\cot\alpha = 3.2\,h$

and $BD = h\cot\beta = 2.4\,h$

In ΔABD, $AB^2 = AD^2 + BD^2$

$\Rightarrow 100^2 = [(3.2)^2 + (2.4)^2]h^2 = 16h^2$

$\Rightarrow h = \dfrac{100}{4} \Rightarrow h = 25$ m

148 **(c)**

$a\cot A + b\cot B + c\cot C$

$= \dfrac{a}{\sin A}\cos A + \dfrac{b}{\sin B}\cos B + \dfrac{c}{\sin C}\cos C$

$= 2R\,(\cos A + \cos B + \cos C)$

$= 2R\left(1 + \dfrac{r}{R}\right) = 2(r + R)$

150 **(a)**

Given pair of lines are rotated about the origin by $\pi/6$ in the anti-clockwise sense.

$\therefore x = x'\cos\dfrac{\pi}{6} - y'\sin\dfrac{\pi}{6} = \dfrac{\sqrt{3}x' - y'}{2}$

and $y = x'\sin\frac{\pi}{6} + y'\cos\frac{\pi}{6} = \frac{x' + \sqrt{3}y'}{2}$

on putting the values of x and y in given pair of lines, we get

$$\sqrt{3}\left(\frac{\sqrt{3}x' - y'}{2}\right)^2 - 4\left(\frac{\sqrt{3}x' - y'}{2}\right)\left(\frac{x' + \sqrt{3}y'}{2}\right) + \sqrt{3}\left(\frac{x' + \sqrt{3}y'}{2}\right)^2 = 0$$

$$\Rightarrow \sqrt{3}\left(3x'^2 + y'^2 - 2\sqrt{3}x'y'\right) - 4\left(\sqrt{3}x'^2 3x'y' - x'y' - \sqrt{3}y'^2\right) + \sqrt{3}\left(x'^2 + 3y'^2 + 2\sqrt{3}x'y'\right) = 0$$

$$\Rightarrow 3\sqrt{3}x'^2 + \sqrt{3}y'^2 - 6x'y' - 4\sqrt{3}x'^2 - 8x'y' + 4\sqrt{3}y'^2 + \sqrt{3}x'^2 + 3\sqrt{3}y'^2 + 6x'y' = 0$$

$\Rightarrow 8\sqrt{3}y'^2 - 8x'y' = 0$

$\Rightarrow \sqrt{3}\,y'^2 - x'y' = 0$

$\therefore$ Required equation is $\sqrt{3}y^2 - xy = 0$

151 **(c)**

Using sine rule,

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$

$\Rightarrow \dfrac{\frac{2}{3}}{2} = \dfrac{\sin B}{3}$

$\Rightarrow \sin B = 1$

$\Rightarrow B = 90°$

152 **(a)**

since, b, c and a are in AP

By sine rule, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\Rightarrow a = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $[\because \angle A = 90°]$

$\Rightarrow \sin B = \dfrac{b}{a}, \sin C = \dfrac{c}{a}$

153 **(d)**

$2a^2 + 4b^2 + c^2 = 4ab + 2ac$

$\Rightarrow a^2 + (2b)^2 - 4ab + a^2 + c^2 - 2ac = 0$

$\Rightarrow (a - 2b)^2 + (a - c)^2 = 0$

$\Rightarrow a = 2c = c$

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$= \dfrac{c^2 + c^2 - \left(\frac{c}{2}\right)^2}{2 \times c \times c} = \dfrac{2c^2 - \frac{c^2}{4}}{2c^2}$

$\Rightarrow \cos B = \dfrac{7}{8}$

154 **(b)**

Given, M divides AB in the ratio $b : a$(externally)

$\therefore x = \dfrac{ba\cos\beta - ba\cos\alpha}{b - a}$

and $y = \frac{ab\sin\beta - ab\sin\alpha}{b - a}$

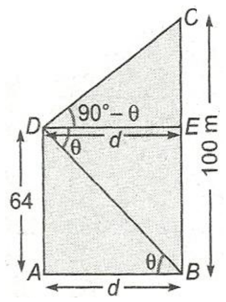
$\Rightarrow \dfrac{x}{y} = \dfrac{\cos\beta - \cos\alpha}{\sin\beta - \sin\alpha}$

$\Rightarrow \dfrac{x}{y} = \dfrac{2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}{2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\beta-\alpha}{2}\right)}$

$\Rightarrow x\cos\left(\dfrac{\alpha+\beta}{2}\right) + y\sin\left(\dfrac{\alpha+\beta}{2}\right) = 0$

155 **(a)**

In ΔDAB, $\tan\theta = \frac{64}{d}$



$\Rightarrow d = 64\cot\theta$ ... (i)

In ΔCDE, $\tan(90° - \theta) = \frac{(100-64)}{d}$

$\Rightarrow\ d = 36 \tan\theta \quad ...(ii)$

On multiplying Eqs. (i) and (ii), we get

$d^2 = 36 \times 64 \Rightarrow d = 48$

156 **(b)**

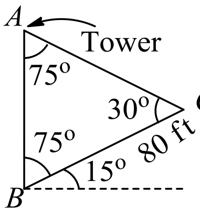
Let BC be the declivity and BA be the tower

$\therefore$ In $\Delta\ ABC$, on applying sine rule

$$\frac{BC}{\sin 75^\circ} = \frac{AB}{\sin 30^\circ}$$

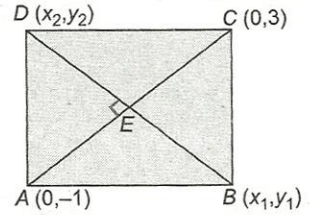
$$\Rightarrow AB = \frac{80 \sin 30^\circ}{\sin 75^\circ}$$

$$= \frac{40 \times 2\sqrt{2}}{\sqrt{3}+1} = 40(\sqrt{6}-\sqrt{2})\text{ft}$$



157 **(c)**

Let the points be $B(x_1, y_1)$ and $D(x_2, y_2)$ and coordinates of mid point of BD are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$



And coordinates of mid point of AC are (0, 1)

We know that mid point of both the diagonals lie on the same point$E$.

$\therefore\ \frac{x_1+x_2}{2} = 0$ and $\frac{y_1+y_2}{2} = 1$

$\Rightarrow\ x_1 + x_2 = 0 \quad ...(i)$

and $y_1 + y_2 = 2 \quad (ii)$

also, slope of $BD \times$ slope of $AC = -1$

$$\frac{(y_1-y_2)}{(x_1-x_2)} \times \frac{(3+1)}{(0-0)} = -1$$

$\Rightarrow\ y_1 - y_2 = 0 \quad ...(iii)$

On solving Eqs. (ii) and (iii), we get

$y_1 = 1, y_2 = 1$

Now, slope of $AB \times$ slope of $BC = -1$

$$\Rightarrow \frac{(y_1+1)}{(x_1-0)} \times \frac{(y_1-3)}{(x_1-0)} = -1$$

$\Rightarrow\ (y_1+1)(y_1-3) = -x_1^2$

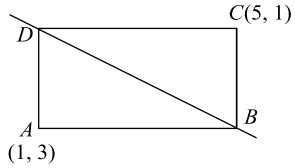
$\Rightarrow\ 2(-2) = -x_1^2 \quad [\because\ y_1 = 1]$

$\Rightarrow\ x_1 = \pm 2$

$\therefore$ The required points are (2, 1) and (−2, 1)

158 **(a)**

The diagonals meet at the mid point of AC, *ie* at (3, 2) which lies on $y = 2x + c$



$\therefore\ c = -4$

Let $B = (\alpha, 2\alpha - 4)$

$\because\ AB \perp BC$

$$\Rightarrow \left(\frac{2\alpha-7}{\alpha-1}\right)\left(\frac{2\alpha-5}{\alpha-5}\right) = -1$$

$\therefore\ \alpha^2 - 6\alpha + 8 = 0$

$\Rightarrow\ \alpha = 2, 4$

The other two vertices are (2, 0) and (4, 4)

159 **(c)**

Given, $r_3 - r = r_1 + r_2$

$$\Rightarrow 4R \sin\frac{C}{2}\left(\cos\frac{A}{2}\cos\frac{B}{2} - \sin\frac{B}{2}\sin\frac{A}{2}\right)$$

$$= 4R\cos\frac{C}{2}\left[\sin\frac{A}{2}\cos\frac{B}{2} + \cos\frac{A}{2}\sin\frac{B}{2}\right]$$

$$\Rightarrow \sin\frac{C}{2}\left[\cos\left(\frac{A+B}{2}\right)\right] = \cos\frac{C}{2}\left[\sin\left(\frac{A+B}{2}\right)\right]$$

$$\Rightarrow \sin\frac{C}{2}\left[\cos\left(\frac{\pi}{2}-\frac{C}{2}\right)\right] = \cos\frac{C}{2}\left[\sin\left(\frac{\pi}{2}-\frac{C}{2}\right)\right]$$

$$\left[\because\ A+B+C = \pi \Rightarrow \frac{A}{2}+\frac{B}{2} = \frac{\pi}{2}-\frac{C}{2}\right]$$

$$\Rightarrow \sin^2\frac{C}{2} = \cos^2\frac{C}{2} \Rightarrow \tan\frac{C}{2} = 1$$

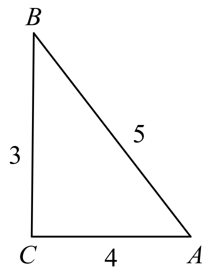
$$\Rightarrow \angle C = \frac{\pi}{2}$$

We know, $A + B + C = \pi \Rightarrow A + B = \frac{\pi}{2}$

160 **(d)**

Given $a = 3, b = 4, c = 5$

$\Rightarrow c^2 = a^2 + b^2$



Therefore, it is a right angled triangle at C

$$\therefore\ R = \frac{1}{2}c = \frac{5}{2}$$

and $r = \frac{\Delta}{s} = \frac{\frac{1}{2}\times 3\times 4}{\frac{12}{2}} = 1$

$\therefore$ Distance between incentre and circumcentre

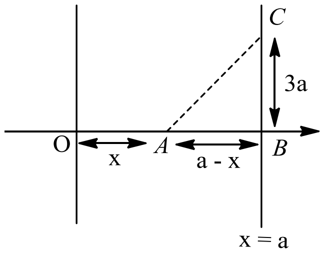
$$= \sqrt{R^2 - 2Rr}$$

$$= \sqrt{\left(\frac{5}{2}\right)^2 - 2.\frac{5}{2}.1}$$

$$= \sqrt{\frac{5}{2}}\sqrt{\frac{5}{2} - 2} = \frac{\sqrt{5}}{2}$$

161 **(d)**

Area of $\Delta ABC = a^2$



$\Rightarrow\ \frac{1}{2}\ (a-x)3a = a^2$

$\Rightarrow\ a - x = \frac{2}{3}\ a$

$\Rightarrow\ \ x = \frac{a}{3}$

Hence, one of the line on which third vertex lies is $x = \frac{a}{3}$

162 **(c)**

Draw BE perpendicular to CA produced, then $BD = DC = \frac{a}{2}$ and $EA = AC = b$

In $\Delta\ AEB$,

$\cos(\pi - A) = \frac{b}{c}$

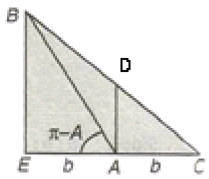
$\Rightarrow\ \cos A = -\frac{b}{c}$

$\Rightarrow\ \frac{b^2+c^2-a^2}{2bc} = -\frac{b}{c}$

$\Rightarrow\ \ a^2 = 3b^2 + c^2$

$\therefore\ \cos B = \frac{c^2+a^2-b^2}{2ac}$

$= \frac{c^2+3b^2+c^2-b^2}{2ca} = \frac{b^2+c^2}{ca}$



164 **(d)**

Let the sides of $\Delta\ ABC$ be $a = n, b = n+1, c = n+2$, where n is a natural number. Then, C is the greatest and A is the least angle

As given $C = 2A$

$\therefore\ \sin C = \sin 2A = 2\sin A\cos A$

$\therefore\ kc = 2ka\frac{b^2+c^2-a^2}{2bc}$

$\Rightarrow\ \ bc^2 = a(b^2+c^2-a^2)$

On substituting the values of a, b, c, we get

$(n+1)(n+2)^2 = n[(n+1)^2 + (n+2)^2 - n^2]$

$= n(n^2+6n+5)$

$= n(n+1)(n+5)$

$\Rightarrow\ (n+1)[(n+2)^2 - n(n+5)] = 0$

Since, $n \neq 1$

Thus, $(n+2)^2 = n(n+5)$

$\Rightarrow\ \ n^2+4n+4 = n^2+5n$

$\Rightarrow\ \ n = 4$

Hence, the sides of the triangle are 4, 5 and 6

165 **(c)**

3. $\frac{b^2-c^2}{a\sin(B-C)} = \frac{2R^2(\sin^2 B - \sin^2 C)}{2R\sin A\sin(B-C)}$

$= \frac{2R\sin(B+C)\sin(B-C)}{\sin(B+C)\sin(B-C)} = 2R$

4. $a\sin(B-C) + b\sin(C-A) + c\sin(A-B) = 0$

$= 2R[\sin A\sin(B-C) + \sin B\sin(C-A) + \sin C\sin(A-B)]$

$= 2R[\sin(B+C)\sin(B-C) + \sin(C+A)\sin(C-A) + \sin(A+B)\sin(A-B)]$

$= 2R[\sin^2 B - \sin^2 C + \sin^2 C - \sin^2 A + \sin^2 A - \sin^2 B]$

$= 2R(0) = 0$

Hence, both of statements are correct

166 **(b)**

Let the general equation of the circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$

The equation of circle passing through (0, 0), (2, 0) and $(0, -2)$

$c = 0$ ...(i)

$4 + 4g + c = 0$ ...(ii)

and $4 - 4f + c = 0$ ...(iii)

On solving Eqs. (i), (ii) and (iii), we get

$c = 0,\ \ g = -1,\ \ f = 1$

$\therefore$ The equation of circle becomes $x^2 + y^2 - 2x + 2y = 0$

Since, it is passes through $(k, -2)$, we get

$k^2 + 4 - 2k - 4 = 0\ \Rightarrow\ \ k = 0, 2$

We have already take a point $(0, -2)$, so we take only $k = 2$

167 **(a)**

Let $X = x - h,\ \ Y = y - k$

$\Rightarrow\ \ 0 = 7 - h,\ \ \ \ \ 0 = -4 - k$

$\Rightarrow\ \ h = 7,\ \ \ \ \ k = -4$

Hence, $X = x - 7$ and $Y = y + 4$, then the point (4, 5) shifted to $(-3, 9)$

168 **(a)**

$(a+b+c)\left(\tan\frac{A}{2}+\tan\frac{B}{2}\right)$

$$= 2\left(s\tan\frac{A}{2}+s\tan\frac{B}{2}\right)$$

$= 2\left(\frac{\Delta}{s-a}+\frac{\Delta}{s-b}\right)$

$= 2\Delta\frac{2s-(a+b)}{(s-a)(s-b)}$

$= 2\Delta\left(\frac{c}{(s-a)(s-b)}\right)$

$= 2c\cot\frac{C}{2}$

169 **(b)**

$\because\ \frac{2}{1!\,9!}+\frac{2}{3!\,7!}+\frac{1}{5!\,5!}=\frac{8^a}{(2b)!}$

$\Rightarrow\ \frac{1}{1!\,9!}+\frac{1}{3!\,7!}+\frac{1}{5!\,5!}+\frac{1}{3!\,7!}+\frac{1}{9!\,1!}=\frac{8^a}{(2b)!}$

$\Rightarrow\ \frac{1}{10!}\left(\frac{10!}{1!\,9!}+\frac{10!}{3!\,7!}+\frac{10!}{5!\,5!}+\frac{10!}{7!\,3!}+\frac{10!}{9!\,1!}\right)$

$$=\frac{8^a}{(2b)!}$$

$\Rightarrow\ \frac{1}{10!}\left({}^{10}C_1+{}^{10}C_3+{}^{10}C_5+{}^{10}C_7+{}^{10}C_9\right)$

$$=\frac{8^a}{(2b)!}$$

$\Rightarrow\ \frac{2^9}{10!}=\frac{8^a}{(2b)!}=\frac{2^{3a}}{(2b)!}$

$\Rightarrow\ a=3,\quad b=5$

Also, $2b=a+c\ \Rightarrow\ 10=3+c\ \Rightarrow c=7$

$\therefore\ a=3,\quad b=5,\quad c=7$

$\because\ \frac{\tan A+\tan B}{2}\geq\sqrt{\tan A\tan B}$ ... (i)

Also, $\cos C=\frac{a^2+b^2-c^2}{2ab}$

$=\frac{9+25-49}{30}=-\frac{1}{2}$

$\Rightarrow \angle C=120°$ and $A,B<60°$

$\tan A+\tan B+\tan C=\tan A\tan B\tan C$

$\Rightarrow\ \tan A+\tan B-\sqrt{3}=-\sqrt{3}\tan A\tan B$

$\therefore\ \tan A+\tan B=\sqrt{3}(1-\tan A\tan B)$ ...(ii)

Also, $\tan A+\tan B>0$

$\Rightarrow\ \sqrt{3}(1-\tan A\tan B)>0$

$\Rightarrow \tan A\tan B<1$ ... (iii)

From Eq. (i) and (ii),

$\frac{\sqrt{3}(1-\tan A\tan B)}{2}\geq\sqrt{(\tan A\tan B)}$

Let $\tan A\tan B=\lambda$

$\therefore\ \sqrt{3}(1-\lambda)\geq 2\sqrt{\lambda}$

$\Rightarrow\ 3\lambda^2-10\lambda+3\geq 0$

$\Rightarrow\ (3\lambda-1)(\lambda-3)\geq 0$

$\because\ \lambda-3<0$ [from Eq. (iii)]

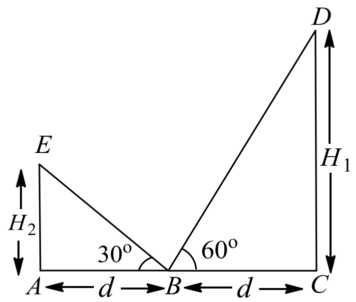
$\therefore\ 3\lambda-1\leq 0$

$\Rightarrow\ \lambda\leq\frac{1}{3}$

$\Rightarrow\ \tan A\tan B\leq\frac{1}{3}$

170 **(c)**

In $\Delta BCD, \tan 60°=\frac{H_1}{d}$



$\Rightarrow\ H_1=d\tan 60°$

and in ΔABE, $\tan 30°=\frac{H_2}{d}$

$\Rightarrow\ H_2=d\tan 30°$

$\therefore\ \frac{H_1}{H_2}=\frac{\tan 60°}{\tan 30°}=\frac{\sqrt{3}}{1/\sqrt{3}}=\frac{3}{1}$

171 **(c)**

We have, $\cos C=\frac{a^2+b^2-c^2}{2ab}$

$\Rightarrow\ \cos 60°=\frac{a^2+b^2-c^2}{2ab}$

$\Rightarrow\ a^2+b^2-c^2=ab$

$\Rightarrow\ b^2+bc+a^2+ac=ab+ac+bc+c^2$

On dividing by $(a+c)(b+c)$ and add 2 on both sides, we get

$1+\frac{b}{a+c}+1+\frac{a}{b+c}=3$

$\Rightarrow\ \frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$

172 **(a)**

$(a+c)^2-b^2=3ac\ \Rightarrow\ a^2+c^2-b^2=ac$

But $\cos B=\frac{a^2+c^2-b^2}{2ac}=\frac{1}{2}\ \Rightarrow\ \angle B=\frac{\pi}{3}=60°$

173 **(c)**

Given, a^2,b^2,c^2 are in AP

$\Rightarrow\ \sin^2 B-\sin^2 A=\sin^2 C-\sin^2 B$

$\Rightarrow\ \sin(B+A)\sin(B-A)=\sin(C+B)\sin(C-B)$

$\Rightarrow\ \sin C\,(\sin B\cos A-\cos B\sin A)$

$=\sin A\,(\sin C\cos B-\cos C\sin B)$

On dividing by $\sin A\sin B\sin C$, we get

$2\cot B=\cot A+\cot C$

$\Rightarrow\ \cot A, \cot B, \cot C$ are in AP

174 **(c)**

Given, $\sin A\sin B=\frac{ab}{c^2}$

$\Rightarrow c^2=\frac{ab}{\sin A\sin B}=\left(\frac{a}{\sin A}\right)\left(\frac{b}{\sin B}\right)$

$\Rightarrow c^2=\left(\frac{c}{\sin C}\right)^2$

$\because \left(\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}\right)$

$\Rightarrow \sin^2 C = 1$

$\Rightarrow \ c = 90°$

Hence, ΔABC is a right angled triangle

175 **(a)**

Let a, b, c be the sides of triangle, then

$a + b + c = \frac{6}{3}(\sin A + \sin B + \sin C)$

$\Rightarrow \ a + b + c = 2(\sin A + \sin B + \sin C)$

$\Rightarrow \frac{a}{2} = \sin A$

But $a = 1$

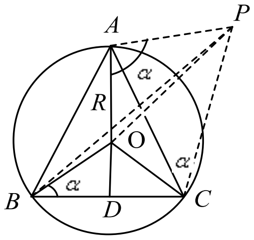
$\therefore \ \sin A = \frac{1}{2} \Rightarrow \angle A = \frac{\pi}{6}$

176 **(a)**

The centroid of $\Delta ABC = \left(\frac{2+8+5}{3}, \frac{3+10+5}{3}\right) = (5, 6)$

177 **(d)**

Since, the tower OP makes equal angle at the vertices of the triangle, therefore foot of the tower is the circumcentre



In ΔOAP, $\tan\alpha = \frac{OP}{OA}$

$\Rightarrow \ OP = OA\tan\alpha$

$\Rightarrow \ OP = R\tan\alpha$

178 **(a)**

In $\Delta\ ABE$,

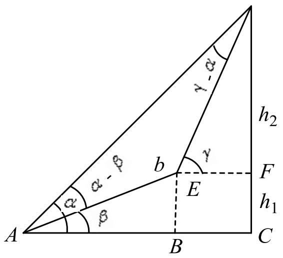
$\sin\beta = \frac{BE}{b}$

$\Rightarrow BE = h_1 = b\sin\beta$

Using sine rule in ΔAED,

$\frac{\sin(\alpha-\beta)}{ED} = \frac{\sin(\gamma-\alpha)}{b}$

$\Rightarrow ED = \frac{b\sin(\alpha-\beta)}{\sin(\gamma-\alpha)}$



Now, in ΔFED,

$\sin\gamma = \frac{h_2}{ED}$

$\Rightarrow \ h_2 = \frac{b\sin(\alpha-\beta)\sin\gamma}{\sin(\gamma-\alpha)}$

$\therefore$Total height, CD

$= h_1 + h_2 = b\sin\beta + \frac{b\sin(\alpha-\beta)\sin\gamma}{\sin(\gamma-\alpha)}$

$= \frac{b[\sin\beta\sin(\gamma-\alpha) + \sin(\alpha-\beta)\sin\gamma]}{\sin(\gamma-\alpha)}$

$= \frac{b[\sin\beta\{\sin\gamma\cos\alpha - \cos y\sin\alpha\} + \sin\gamma\{\sin\alpha\cos\beta\cos\alpha\}]}{\sin(\gamma-\alpha)}$

$= \frac{b[\sin\beta\sin\gamma\cos\alpha - \sin\beta\cos\gamma\sin\alpha + \{\sin\gamma\sin\alpha\cos\beta - \sin\gamma\sin\beta\cos\alpha\}]}{\sin(\gamma-\alpha)}$

$= \frac{b\sin\alpha\sin(\gamma-\beta)}{\sin(\gamma-\alpha)}$

179 **(c)**

Given, $a = 1, b = 2, \angle C = 60°$

$\therefore$ Area of triangle $= \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 1 \times 2 \times \sin 60°$

$= \frac{\sqrt{3}}{2}$ sq unit

180 **(c)**

The given points are collinear

If $\begin{vmatrix} t_1 & 2at_1 + at_1^3 & 1 \\ t_2 & 2at_2 + at_2^3 & 1 \\ t_3 & 2at_3 + at_3^3 & 1 \end{vmatrix} = 0$

$\Rightarrow \quad a\begin{vmatrix} t_1 & 2t_1 + t_1^3 & 1 \\ t_2 & 2t_2 + t_2^3 & 1 \\ t_3 & 2t_3 + t_3^3 & 1 \end{vmatrix} = 0$

Applying $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$, we get

$\begin{vmatrix} t_1 & 2t_1 + t_1^3 & 1 \\ t_2 - t_1 & 2(t_2 - t_1) + (t_2^3 - t_1^3) & 0 \\ t_3 - t_1 & 2(t_3 - t_1) + (t_3^3 - t_1^3) & 0 \end{vmatrix} = 0$

$\Rightarrow \quad (t_2 - t_1)(t_3 - t_1)\begin{vmatrix} t_1 & 2t_1 + t_1^3 & 1 \\ 1 & 2 + t_2^2 + t_1^2 + t_1t_2 & 0 \\ 1 & 2 + t_3^2 + t_1^2 + t_3t_1 & 0 \end{vmatrix}$

$= 0$

$\Rightarrow \quad (t_2 - t_1)(t_3 - t_1)(t_3 - t_2)(t_3 + t_2 + t_1) = 0$

$\Rightarrow \ t_1 + t_2 + t_3 = 0$

$[\because t_1 \neq t_2 \neq t_3]$

181 **(b)**

Let P is a point on the perpendicular bisector of AB, its equation is

$(y - 1) = \frac{1}{3}(x - 4) \Rightarrow x - 3y - 1 = 0$

So, general point is $P(3h + 1, h)$

Area $= \frac{1}{2}\begin{bmatrix} 3h+1 & h & 1 \\ 3 & 4 & 1 \\ 5 & -2 & 1 \end{bmatrix} = \pm 10$

$\Rightarrow \ h = 2, 0$

Or position of points is (7, 2) and (1, 0)

182 **(a)**

Given, $\angle A = \frac{\pi}{2}$, then $a^2 = b^2 + c^2 = 4^2 + 3^2 = 25$

and $\frac{a}{\sin A} = 2R \Rightarrow a = 2R$ and $a = 5$

also, $r = \frac{\Delta}{s} = \frac{bc}{a+b+c}$ $\left(\because \Delta = \frac{bc}{2}\right)$

$$\therefore \frac{R}{r} = \frac{a(a+b+c)}{2bc} = \frac{5 \times 12}{2 \times 4 \times 3} = \frac{5}{2}$$

183 **(d)**

Since A, B, C are in AP

$\Rightarrow 2B = A + C \Rightarrow \angle B = 60°$

$\therefore \frac{a}{2}(2\sin C\cos C) + \frac{c}{a}(2\sin A\cos A)$

$= 2k\,(a\cos C + c\cos A)$

$= 2k(b)$

$= 2\sin B$ [using, $b = a\cos C + c\cos A$]

$= \sqrt{3}$

184 **(d)**

Given equation is

$x^2 - 5x + 6 = 0$

$\Rightarrow (x-3)(x-2) = 0$

$\Rightarrow x = 3, 2$

These are the sides of a triangle

Let $a = 3, b = 2, \angle C = \frac{\pi}{3}$

$$\Rightarrow \cos\left(\frac{\pi}{3}\right) = \frac{3^2 + 2^2 - c^2}{2.3.2} \quad \left[\because \cos C = \frac{a^2 + b^2 - c^2}{2ab}\right]$$

$$\Rightarrow \frac{1}{2} = \frac{13 - c^2}{12} \Rightarrow c^2 = 7$$

$\Rightarrow c = \sqrt{7}$ [sides cannot be negative]

$\therefore$ Perimeter of a triangle $= a + b + c$

$= 3 + 2 + \sqrt{7} = 5 + \sqrt{7}$

185 **(b)**

Given, $\angle A = 60°, a = 5, b = 4$

$$\therefore \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

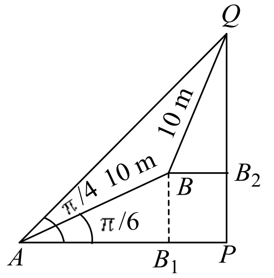
$$\Rightarrow \cos 60° = \frac{1}{2} = \frac{16 + c^2 - 25}{8c}$$

$\Rightarrow 4c = c^2 - 9$

$\Rightarrow c^2 - 4c - 9 = 0$

186 **(a)**

Let PQ be the height h of the tower and A, B are the points of observations



We have, $\angle QAP = \frac{\pi}{4}, \angle BAP = \frac{\pi}{6}$,

$AB = 10$ m, $BQ = 10$ m

$$\therefore \angle QAB = \frac{\pi}{12} = \angle AQB$$

$$\Rightarrow \angle ABQ = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

On applying cosine rule in ΔABQ, we get

$$AQ^2 = AB^2 + BQ^2 - 2AB \cdot BQ\cos\frac{5\pi}{6}$$

$$= 100 + 100 + 200 \cdot \frac{\sqrt{3}}{2}$$

$= 100(2 + \sqrt{2})$

$\Rightarrow AQ = 10\sqrt{2 + \sqrt{3}}$

In ΔAPQ, $AP = AQ\cos\frac{\pi}{4} = \frac{10\sqrt{2+\sqrt{3}}}{\sqrt{2}}$

$= 5\sqrt{4 + 2\sqrt{3}} = 5\sqrt{(\sqrt{3}+1)^2}$

$\Rightarrow AP = 5(1 + \sqrt{3})$m

187 **(a)**

Let the points be $A = (a\cos\theta, a\sin\theta)$ and $B = (a\cos\phi, a\sin\phi)$

$\therefore AB$

$= \sqrt{(a\cos\theta - a\cos\phi)^2 + (a\sin\theta - a\sin\phi)^2}$

$= \sqrt{a^2\cos^2\theta + a^2\cos^2\phi - 2a^2\cos\theta\cos\phi + a^2\sin^2\theta + a^2\sin^2\phi - 2a^2\sin\theta\sin\phi}$

$= \sqrt{2a^2 - 2a^2(\cos\theta\cos\phi + \sin\theta\sin\phi)}$

$= \sqrt{2}a(\sqrt{1 - \cos(\theta - \phi)})$

$$\Rightarrow 2a = \sqrt{2}a\,\sqrt{2}\sin\left(\frac{\theta - \phi}{2}\right)$$

$$\Rightarrow \sin\left(\frac{\theta - \phi}{2}\right) = 1$$

$$\Rightarrow \frac{\theta - \phi}{2} = n\pi \pm \frac{\pi}{2}$$

$\Rightarrow \theta - \phi = 2n\pi \pm \pi$

$\Rightarrow \theta = 2n\pi \pm \pi - \phi$

Where $n \in Z$

188 **(d)**

Area of pentagon

$$= \frac{1}{2}\begin{bmatrix} x_1y_2 + x_2y_3 + x_3y_4 + x_4y_5 \\ +x_5y_1 - (y_1x_2 + y_2x_3 + y_3x_4 \\ +y_4x_5 + y_5x_1) \end{bmatrix}$$

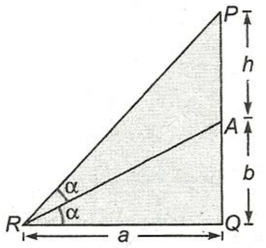
$$= \frac{1}{2}[0(0) + 12(2) + 12(7) + 6(5) + 0(0) - \{0 + 0 + 2(6) + 7(0) + 5(0)\}]$$

$$= \frac{1}{2}[(24 + 84 + 30 - 12)]$$

$= 63$ sq unit

189 **(a)**

Let the height of the flag be h



In $\Delta ARQ, \tan\alpha = \frac{b}{a}$ .... (i)

and in ΔPRQ, $\tan 2\alpha = \frac{h+b}{a}$ ... (ii)

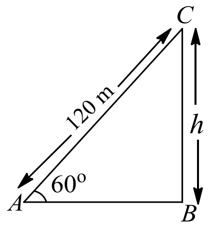
$$\Rightarrow \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{h+b}{a}$$

$$\Rightarrow \frac{2\times\frac{b}{a}}{1-\frac{b^2}{a^2}} = \frac{h+b}{2} \quad \text{[from Eq. (i)]}$$

$$\Rightarrow \frac{2ab}{a^2-b^2} = \frac{h+b}{a} \Rightarrow h = \frac{b(a^2+b^2)}{(a^2-b^2)}$$

190 **(a)**

Let BC be the height of kite



In ΔABC, $\sin 60° = \frac{h}{120}$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{h}{120}$$

$$\Rightarrow \quad h = 60\sqrt{3} \text{ m}$$

The height of the kite is $60\sqrt{3}$ m

191 **(b)**

$$\frac{a\cos A + b\cos B + c\cos C}{a+b+c}$$

$$= \frac{(2R\sin A)\cos A + (2R\sin B)\cos B + 2R\sin C\,(\cos C)}{2R\sin A + 2R\sin B + 2R\sin C}$$

$$\left(\because R = \frac{a}{2\sin A} = \frac{b}{2\sin B} = \frac{c}{2\sin C} = \frac{abc}{4\Delta}\right)$$

$$= \frac{R[2\sin A\cos A + 2\sin B\cos B + 2\sin C\cos C]}{2R[\sin A + \sin B + \sin C]}$$

$$= \frac{1}{2}.\frac{(\sin 2A + \sin 2B + \sin 2C)}{(\sin A + \sin B + \sin C)}$$

$$= \frac{4\sin A\sin B\sin C}{2[4\cos(A/2)\cos(B/2)\cos(C/2)]}$$

$$\left(\begin{array}{l}\because\ \sin 2\alpha + \sin 2\beta + \sin 2\gamma = 4\sin\alpha\sin\beta\sin\gamma \\ \text{and}\ \ \sin\alpha + \sin\beta + \sin\gamma = 4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}\end{array}\right)$$

$$= \frac{4\left[2\sin\frac{A}{2}\cos\frac{A}{2}\times 2\sin\frac{B}{2}\cos\frac{B}{2}\times 2\sin\frac{C}{2}\times\cos\frac{C}{2}\right]}{2\times 4\left[\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}\right]}$$

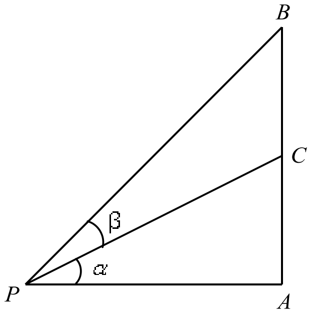
$$= 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

$$= \frac{r}{R} \quad \left[\because r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right]$$

192 **(a)**

Let $\angle APC = \alpha$. Then,

$$\tan\alpha = \frac{AC}{AP} = \frac{AC}{n\,AB} = \frac{AB}{2n\,AB} = \frac{1}{2n}$$



In $\Delta\, APB$, we have

$\tan(\alpha+\beta)=\frac{AB}{AP}=\frac{AB}{n\,AB}=\frac{1}{n}$

Now, $\beta=\alpha+\beta-\alpha$

$\Rightarrow \tan\beta=\frac{\tan(\alpha+\beta)-\tan\alpha}{1+\tan(\alpha+\beta)\tan\alpha}$

$\Rightarrow \tan\beta=\frac{1/n-1/2n}{1+1/n\cdot 1/2n}=\frac{n}{2\,n^2+1}$

193 **(d)**

Suppose $P(3,7)$ divides the segment joining $A(1,1)$ and $B(6,16)$ in the ratio $\lambda:1$. Then,

$\frac{6\lambda+1}{\lambda+1}=3$ and $\frac{16\,\lambda+1}{\lambda+1}=7$

$\Rightarrow \lambda=\frac{2}{3}$

$\Rightarrow P$ divides AB internally in the ratio $2:3$

Thus, Q divides AB externally in the ratio $2:3$ and hence its coordinates are

$\left(\frac{2\times 6-3\times 1}{2-3},\frac{2\times 16-3\times 1}{2-3}\right)\equiv(-9,-29)$

194 **(d)**

Let the angles of a triangle are $3x, 5x$ and $10x$

$\therefore 3x+5x+10x=180° \Rightarrow x=10°$

$\therefore$ Smallest angle of a triangle $=30°$

And the greatest angle$=100°$

Required ratio$=\sin 30°:\sin 100°$

$=\frac{1}{2}:\cos 10°=1:2\cos 10°$

195 **(b)**

Given, $\begin{vmatrix}1 & a & b\\ 1 & c & a\\ 1 & b & c\end{vmatrix}=0$

$\Rightarrow c^2-ab-a(c-a)+b(b-c)=0$

$\Rightarrow a^2+b^2+c^2-ab-bc-ca=0$

$\Rightarrow \frac{1}{2}[2a^2+2b^2+2c^2-2ab-2bc-2ca]=0$

$\Rightarrow \frac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2=0$

$\Rightarrow a=b=c$

$\Rightarrow \angle A=60°, \angle B=60°, \angle C=60°$

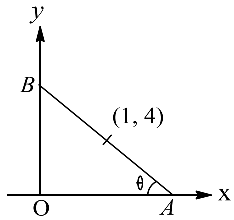
$\therefore \sin^2 A+\sin^2 B+\sin^2 C$

$=\sin^2 60°+\sin^2 60°+\sin^2 60°$

$=\left(\frac{\sqrt{3}}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2=\frac{9}{4}$

197 **(c)**

Let $\angle\, OAB=\theta$



Then, $OA+AB=1+4\cot\theta+4+\tan\theta$

$=5+4\cot\theta+\tan\theta\geq 5+4=9$

(using AM $\geq$ GM)

199 **(c)**

Given $\angle A=20°$

$\therefore \angle B=\angle C=80°$

Then, $b=c$

$\therefore \frac{a}{\sin 20°}=\frac{b}{\sin 80°}=\frac{c}{\sin 80°}$

$\Rightarrow \frac{a}{\sin 20°}=\frac{b}{\cos 10°}$

$\Rightarrow a=2b\sin 10°$ ...(i)

$\therefore a^3+b^3=8b^3\sin^3 10°+b^3$

$=b^3\{2(4\sin^3 10°)+1\}$

$=b^3\{2(3\sin 10°-\sin 30°)+1\}$

$=b^3\{6\sin 10°\}$

$=3b^2\{2b\sin 10°\}$

$=3b^2a$ [from Eq. (i)]

$=3ac^2$ $(\because b=c)$

200 **(a)**

$a(\cos^2 B+\cos^2 C)+\cos A\,(c\cos C+b\cos B)$

$=\cos B\,(a\cos B+b\cos A)$

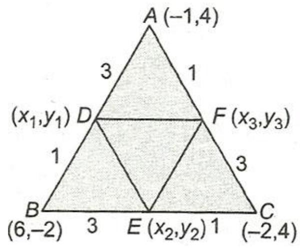
$+\cos C\,(a\cos C+c\cos A)$

$=(\cos B)c+(\cos C)b$

$=a$

201 **(b)**

Let $(x_1,y_1),(x_2,y_2)$ and (x_3,y_3) are the coordinates of the points D, E and F



$\therefore x_1=\frac{3\times 6-1\times 1}{4}=\frac{17}{4}$

and $y_1=\frac{-2\times 3+4}{4}=-\frac{2}{4}=-\frac{1}{2}$

similarly, $x_2=0,\ y_2=\frac{5}{2}$

and $x_3=-\frac{5}{4},\ y_3=4$

let (x,y) be the coordinates of centroid of ΔDEF

$\therefore x=\frac{1}{3}\left(\frac{17}{4}+0-\frac{5}{4}\right)=1$

and $y = \left(-\frac{1}{2} + \frac{5}{2} + 4\right)\frac{1}{3} = 2$

$\therefore$ Coordinates of centroid are (1, 2)

202 **(a)**

We have, $\frac{1}{a+c} = \frac{1}{b+c} = \frac{3}{a+b+c}$

$\Rightarrow \dfrac{a+b+2c}{(a+c)(b+c)} = \dfrac{3}{a+b+c}$

$\Rightarrow a^2 + ab + ac + ab + b^2 + bc + 2ca + 2bc + 2c^2$

$= 3(ab + ac + bc + c^2)$

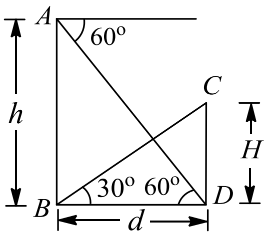
$\Rightarrow a^2 + b^2 - c^2 = ab$

$\Rightarrow \dfrac{a^2 + b^2 - c^2}{2ab} = \dfrac{1}{2} = \cos C$

$\Rightarrow \angle C = 60°$

203 **(a)**

Let CD be the tower of height H metre



From $\Delta\, BCD, \frac{H}{d} = \tan 30°$

and from $\Delta\, ABD$,

$\dfrac{h}{d} = \tan 60°$

$\therefore \dfrac{H/d}{h/d} = \dfrac{\tan 30°}{\tan 60°} \Rightarrow H = \dfrac{h}{3}$

204 **(c)**

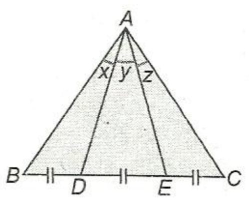
Using sine rule in ΔADC,

$\dfrac{\sin(y+z)}{DC} = \dfrac{\sin C}{AD}$

In $\Delta ABD, \frac{\sin x}{BD} = \frac{\sin B}{AD}$

In $\Delta\, AEC, \frac{\sin z}{EC} = \frac{\sin C}{AE}$

In $\Delta\, ABE, \frac{\sin(x+y)}{BE} = \frac{\sin B}{AE}$



$\therefore \dfrac{\sin(x+y)\sin(y+z)}{\sin x \sin z} = \dfrac{BE}{AE} \times \dfrac{DC}{AD} \times \dfrac{AD}{BD} \times \dfrac{AE}{EC}$

$= \dfrac{2BD \times 2EC}{BD \times EC} = 4$

205 **(a)**

$\because \dfrac{\sin A}{\sin C} = \dfrac{\sin(A-B)}{\sin(B-C)}$

$\Rightarrow \dfrac{\sin A}{\sin C} = \dfrac{\sin A \cos B - \cos A \sin B}{\sin B \cos C - \cos B \sin C}$

$\Rightarrow \dfrac{a}{c} = \dfrac{a \cos B - b \cos A}{b \cos C - c \cos B}$

$\Rightarrow ab \cos C + bc \cos A = 2ac \cos B$

$\Rightarrow \dfrac{a^2 2 + b^2 - c^2}{2} + \dfrac{b^2 + c^2 - a^2}{2} = 2\dfrac{c^2 + a^2 - b^2}{2}$

$\Rightarrow a^2 + c^2 = 2b^2$

$\Rightarrow a^2, b^2, c^2$ are in AP

206 **(b)**

Let point (x_1, y_1) be on the line $3x + 4y = 5$

$\therefore 3x_1 + 4y_1 = 5$ ...(I)

Also, $(x_1 - 1)^2 + (y_1 - 2)^2 = (x_1 - 3)^2 + (y_1 - 4)^2$

$\Rightarrow x_1^2 + y_1^2 - 2x_1 - 4y_1 + 5$
$= x_1^2 + y_1^2 - 6x_1 - 8y_1 + 25$

$\Rightarrow 4x_1 + 4y_1 = 20$ ...(ii)

On solving Eqs. (i) and (ii), we get

$x_1 = 15, y_1 = -10$

207 **(a)**

Let $A(x_1, y_1), B(x_2, y_2)$ be two fixed points and let $P(h, k)$ be a variable point such that

$\angle APB = \dfrac{\pi}{2}$

Then, slope of $AP \times$ slope of $BP = -1$

$\Rightarrow \dfrac{k - y_1}{h - x_1} \cdot \dfrac{k - y_1}{h - x_2} = -1$

$\Rightarrow (h - x_1)(h - x_2) + (k - y_1)(k - y_2) = 0$

Hence, locus of (h, k) is

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Which is a circle having AB as diameter

208 **(a)**

Given, $a \cos^2 \frac{C}{2} + c \cos^2 \frac{A}{2} = \frac{3b}{c}$

$\Rightarrow a\left[\dfrac{s(s-c)}{ab}\right] + c\left[\dfrac{s(s-a)}{bc}\right] = \dfrac{3b}{2}$

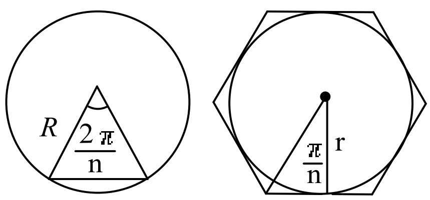
$\Rightarrow \dfrac{s(s-c+s-a)}{b} = \dfrac{3b}{2}$

$\Rightarrow 2s(2s - c - a) = 3b^2$

$\Rightarrow (a + b + c)b = 3b^2$

$\Rightarrow 2b = a + c$

209 **(c)**



$\because \dfrac{a}{2R} = \sin \dfrac{\pi}{n}$

and $\frac{a}{2r} = \tan \frac{\pi}{n}$ $\therefore \frac{r}{R} = \cos \frac{\pi}{n}$

For $n = 3$ gives $\frac{r}{R} = \frac{1}{2}$

For $n = 4$ gives $\frac{r}{R} = \frac{1}{\sqrt{2}}$

For $n = 6$ gives $\frac{r}{R} = \frac{\sqrt{3}}{2}$

210 **(d)**

Given, the vertex angles are A, B, C are side BC

$\therefore \ Area = \frac{1}{2} a(k \sin C) \sin B$

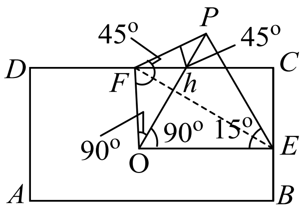
$= \frac{1}{2} . \frac{a^2 k \sin C \sin B}{a}$

$= \frac{1}{2} . \frac{a^2 k \sin C \sin B}{k \sin A}$

$= \frac{1}{2} a^2 . \frac{\sin B \sin C}{\sin A}$

211 **(c)**

Let OP be the flag staff of height h standing at the centre O of the field



In $\Delta \ OEP$, $OE = h \cot 15° = h(2 + \sqrt{3})$

and in $\Delta \ OFP$, $OF = h \cot 45° = h$

$\therefore \ EF = h\sqrt{1 + \left(2 + \sqrt{3}\right)^2}$

$= 2h\sqrt{2 + \sqrt{3}}$

Since, $AC = 1200$

$\Rightarrow \ 2EF = 4h\sqrt{(2 + \sqrt{3})}$

$\Rightarrow \ h = \frac{300}{\sqrt{2 + \sqrt{3}}}$

$= 300\sqrt{2 - \sqrt{3}}$ m

214 **(b)**

$A + B + C = \pi$

$2B = A + C$

$\Rightarrow 3B = \pi$

$\Rightarrow B = \frac{\pi}{3}$

$\therefore \frac{a + c}{b} = \frac{\sin A + \sin C}{\sin B}$

$= \frac{2 \sin \frac{A+C}{2} \cos \frac{A-C}{2}}{\sin \frac{\pi}{3}}$

$= \frac{2 \sin \frac{\pi}{3} \cos \frac{A-C}{2}}{\sin \frac{\pi}{3}}$

$= 2 \cos \frac{A - C}{2}$

215 **(c)**

The altitudes from the vertices A, B and C are $\frac{2\Delta}{a}, \frac{2\Delta}{b}$ and $\frac{2\Delta}{c}$ respectively

Also, these are in HP

$\Rightarrow \ \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in HP

$\Rightarrow \ a, b, c$ are in AP

$\Rightarrow \ \sin A , \sin B , \sin C$ are in AP

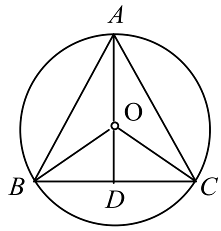
216 **(d)**

Given, the equation of circle is

$x^2 + y^2 - 4x - 6y - 12 = 0$

Radius of this circle= $\sqrt{4 + 9 + 12}$

=5 unit



In ΔBOD,

$\cos 30° = \frac{BD}{OB}$

$\Rightarrow \ BD = \frac{\sqrt{3}}{2} \times 5 = \frac{5\sqrt{3}}{2}$ unit

$\therefore \ BC = 2BD = 5\sqrt{3}$

Hence, area of $\Delta ABC = \frac{\sqrt{3}}{4}\left(5\sqrt{3}\right)^2 = \frac{75}{4}\sqrt{3}$ sq units

218 **(d)**

Let $A\left(1, \sqrt{3}\right), B(0, 0), C(2, 0)$ be the given points

$\therefore \ BC = \sqrt{(2 - 0)^2 + (0 - 0)^2} = 2$

$CA = \sqrt{(2 - 1)^2 + \left(0 - \sqrt{3}\right)^2} = 2$

and $AB = \sqrt{1 + 3} = 2$

$\therefore$ Triangle is equilateral

We know that in equilateral triangle incentre is the same as Centroid of the triangle

$\therefore$ Incentre is $\left(\frac{1+0+2}{3}, \frac{\sqrt{3}+0+0}{3}\right) = \left(1, \frac{1}{\sqrt{3}}\right)$

219 **(b)**

We know that area of the triangle with polar coordinates

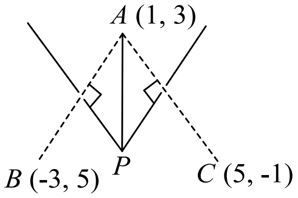
$= \frac{1}{2} | \sum r_1 r_2 \sin(\theta_1 - \theta_2) |$

$= \frac{1}{2} |2.1 \sin\left(0 - \frac{\pi}{3}\right) + 2.3 \sin\left(\frac{\pi}{3} - \frac{2\pi}{3}\right) + 3.1 \sin\left(\frac{2\pi}{3} - 0\right)|$

$= \frac{1}{2}\left|-2.\frac{\sqrt{3}}{2} - \frac{6\sqrt{3}}{2} + \frac{3\sqrt{3}}{2}\right| = \frac{5\sqrt{3}}{4}$ sq units

220 **(d)**

For finding the distance AP, first we find out the perpendicular bisector of AB and AC



Perpendicular bisector of $A(1,3)$ and $B(-3,5)$ is

$2x(x_1 - x_2) + 2y(y_1 - y_2)$
$= (x_1^2 + y_1^2) - (x_2^2 + y_2^2)$

$\Rightarrow\ 2x(1+3) + 2y(3-5) = (1-9) - (9-25)$

$\Rightarrow\ 2x - y + 6$ ...(i)

Similarly, perpendicular bisector of $A(1,3)$ and $C(-3,5)$ is

$2x(1-5) + 2y(3+1) = (1+9) - (25+1)$

$\Rightarrow\ -8x + 8y = -16$

$\Rightarrow\ x - y - 2 = 0$ ...(ii)

Point of intersection of Eqs. (i) and (ii) is

$P = (-8, -10)$

Then, the distance between P and A is

$PA = \sqrt{(1+8)^2 + (3+10)^2}$

$= \sqrt{81 + 169}$

$= \sqrt{250} = \sqrt{25 \times 10} = 5\sqrt{10}$

222 **(b)**

Let sides are $a = 13, b = 12, c = 5$

Now, $a^2 = b^2 + c^2$

$\Rightarrow\ (13)^2 = (12)^2 + 5^2$

$\Rightarrow\ 169 = 169$

$\therefore\ \angle A = 90°$

Since, $R = \frac{a}{2\sin A} = \frac{13}{2\sin 90°} = \frac{13}{2}$

223 **(b)**

Since, the vertices of the triangle are $(a\cos t, a\sin t), (b\sin t, -\cos t)$ and $(1, 0)$

Let the coordinate of centroid be

$x = \dfrac{a\cos t + b\sin t + 1}{3}$

$\Rightarrow\quad 3x - 1 = a\cos t + b\sin t$ ...(i)

and $y = \frac{a\sin t - b\cos t + 0}{3}$

$3y = a\sin t - b\cos t$ ...(ii)

On squaring and adding Eqs. (i) and (ii), we get

$(3x-1)^2 + (3y)^2$
$= a^2(\cos^2 t + \sin^2 t) + b^2(sin^2 t$
$+ \cos^2 t)$

$\Rightarrow\ (3x-1)^2 + (3y)^2 = a^2 + b^2$

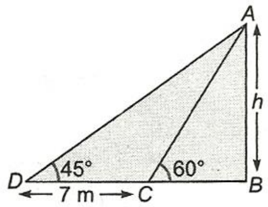
224 **(c)**

In $\Delta ABC, BC = h\cot 60°$

and in $\Delta ABD, BD = h\cot 45°$

Since, $BD - BC = DC$

$\Rightarrow h\cot 45° - h\cot 60° = 7$

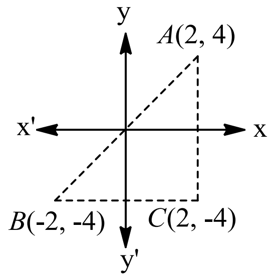


$\Rightarrow\ h = \dfrac{7}{\cot 45° - \cot 60°} = \dfrac{7}{\left(1 - \frac{1}{\sqrt{3}}\right)}$

$= \dfrac{7\sqrt{3}(\sqrt{3}+1)}{2}$ m

225 **(c)**

Here, coordinates are $A(2,4), B(-2,-4)$ and $C(2,-4)$



Now, $|AB| = \sqrt{(-2-2)^2 + (-4-4)^2}$

$= \sqrt{16 + 64} = \sqrt{80} = 4\sqrt{5}$

226 **(c)**

Let the vertex $A(x, y)$ is equidistant from B and C

$\therefore\ (x-1)^2 + (y-3)^2 = (x+2)^2 + (y-7)^2$

$\Rightarrow\ 6x - 8y + 43 = 0$

Only the option (c) satisfy it

227 **(a)**

Let the ratio be $k:1$

$\therefore \dfrac{-7k+3}{k+1} = \dfrac{1}{2} \Rightarrow\ k = \dfrac{1}{3}$

Hence, ratio is 1: 3 internally

228 **(d)**

We know that orthocenter of the right angled triangle ABC, right angled at A is A

Here, triangle is right angled at $O(0,0)$

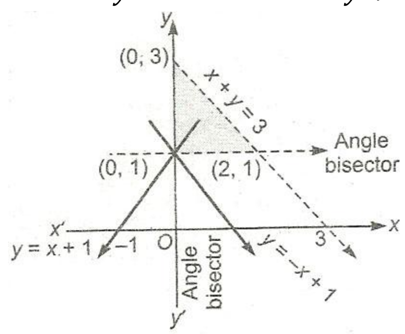
$\therefore$ Orthocentre $=(0,0)$

229 **(a)**

Here, $x^2 - y^2 + 2y = 1$

$\Rightarrow\ x^2 = (y-1)^2$

$\Rightarrow\ x = y - 1$ and $x = -y + 1$



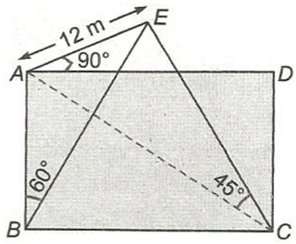
Which could be graphically as shown in figure

Which gives angle bisector as $y = 1$ and $x = 0$

$\therefore$ Required area$= \frac{1}{2} \times 2 \times 2 = 2$ sq units

230 **(a)**

In ΔABE, $\tan 60° = \frac{12}{AB} \Rightarrow AB = 4\sqrt{3}$ m



and in ΔACE

$\tan 45° = \dfrac{12}{AC} \Rightarrow AC = 12$ m

In ΔABC, $BC = \sqrt{AC^2 - AB^2} = \sqrt{144 - 48} = 4\sqrt{6}$ m

$\therefore$ Area of rectangular field $= AB \times BC$

$= 4\sqrt{3} \times 4\sqrt{6} = 48\sqrt{2}$ sqm

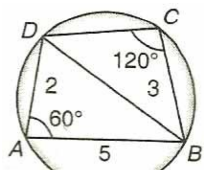
231 **(a)**

In $\Delta\, ABD$,

$$\cos 60° = \frac{2^2 + 5^2 - BD^2}{2(2)(5)}$$

$\Rightarrow \quad BD^2 = 19$

Now, in $\Delta\, BCD$



$$\cos 120° = \frac{CD^2 + 9 - 19}{(2)(3)(CD)}$$

$\Rightarrow \quad CD^2 + 3CD - 10 = 0$

$\Rightarrow \quad CD = -5, 2$

$\Rightarrow \quad CD = 2 \quad (\because CD \neq -5)$

232 **(c)**

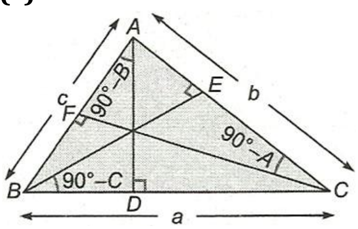
We have, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{2R}$

$\Rightarrow \sin A = \dfrac{a}{2R}$

$\therefore \; 2R^2 \sin A \sin B \sin C = 2R^2 \dfrac{a}{2R} \cdot \dfrac{b}{2R} \cdot \dfrac{c}{2R}$

$= \dfrac{abc}{4R} = \Delta$

233 **(c)**



In $\Delta\, BAD$,

$\cos(90° - B) = \dfrac{AD}{c}$

$\Rightarrow AD = c \sin B$

Similarly, $BE = a \sin C$ and $CF = b \sin A$

Since, AD, BE, CF are in HP

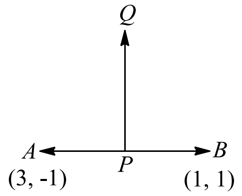
$\therefore \; c \sin B$, $a \sin C$, $b \sin A$ are in HP

$\Rightarrow \; \frac{1}{\sin C \sin B}, \frac{1}{\sin A \sin C}, \frac{1}{\sin B \sin A}$ are in AP

$\Rightarrow \; \sin A$, $\sin B$, $\sin C$ are in AP

234 **(d)**

Let P be the middle point of the line segment joining $A(3, -1)$ and $B(1, 1)$ is $(2, 0)$



Let P be shifted to Q by 2 unit and y-coordinate of Q is greater than that of P

Now, slope of $AB = \frac{1-(-1)}{1-3} = 1$

then slope of $PQ = -1$

The coordinate of Q becomes $(2 \pm 2\cos\theta, 0 \pm 2\sin\theta$, where $\tan\theta = 1$

ie, $(2 \pm \sqrt{2}, \pm\sqrt{2})$

As, y-coordinates of Q is greater than that of P

$\therefore$ We take $Q = (2 + \sqrt{2}, \sqrt{2})$

235 **(d)**

Area of triangle $= \frac{1}{2} \times 6 \times |\alpha| = 15$

$\Rightarrow \quad |\alpha| = 5 \quad \Rightarrow \quad \alpha = \pm 5$

and β can take any real value

236 **(a)**

Let points are $A(2, 3)$, $B(3, 4)$, $C(4, 5)$, $D(5, 6)$

$\therefore \quad AB = a = \sqrt{(3-2)^2 + (4-3)^2} = \sqrt{2}$

Similarly, $BC = b = \sqrt{2}$, $CD = c = \sqrt{2}$

And $\; DA = d = 3\sqrt{2}$

Now, $s = \frac{a+b+c+d}{2}$

$= \dfrac{\sqrt{2} + \sqrt{2} + \sqrt{2} + 3\sqrt{2}}{2} = 3\sqrt{2}$

$\therefore \;$ Area $= \sqrt{(s-a)(s-b)(s-c)(s-d)}$

$= \sqrt{(3\sqrt{2} - \sqrt{2})(3\sqrt{2} - \sqrt{2})(3\sqrt{2} - \sqrt{2})(3\sqrt{2} - 3\surd}$

$= 0$

237 **(a)**

$\sqrt{(2+5)^2 + (3-2)^2} = \sqrt{(x-1)^2 + (2-3)^2}$

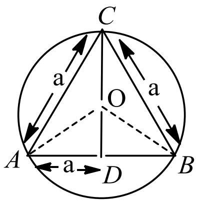
$\Rightarrow \quad 49 + 1 = (x-1)^2 + 1$

$\Rightarrow \quad x - 1 = \pm 7 \; \Rightarrow x = -6, 8$

238 **(d)**

In an equilateral triangle length of median

$CD = \frac{\sqrt{3}}{2} a$



Also, centroid of triangle lies on the centre of circle

$\therefore\ OC = \frac{2}{3} \times \frac{\sqrt{3}}{2} a = \frac{a}{\sqrt{3}}$

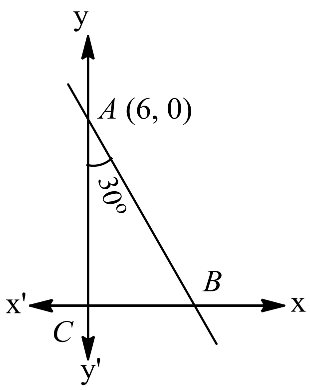
$\therefore$ Area of circle $= \pi(OC)^2 = \frac{\pi a^2}{3}$ sq units

239 **(a)**

In ΔABC, $\tan 30° = \frac{CB}{CA}$

$\Rightarrow\ \frac{1}{\sqrt{3}} = \frac{CB}{6}$

$\Rightarrow\ CB = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$



$\therefore$ Area of $\Delta ACB = \frac{1}{2} \times CA \times CB$

$= \frac{1}{2} \times 6 \times 2\sqrt{3} = 6\sqrt{3}$ sq units

240 **(d)**

We have, $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$

$\Rightarrow\ \frac{\cos A}{k \sin A} = \frac{\cos B}{k \sin B} = \frac{\cos C}{k \sin C}$

$\Rightarrow\ \cot A = \cot B = \cot C$

$\Rightarrow\ A = B = C = 60°$

ΔABC is an equilateral triangle

$\therefore\ \Delta = \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} (2)^2 \quad [\because a = 2 \text{ (given)}]$

$= \sqrt{3}$

241 **(c)**

We have, $\cos A = \frac{b^2+c^2-a^2}{2bc}$

or $c^2 - 2bc \cos A + b^2 - a^2 = 0$

$\therefore\ c_1 + c_2 = 2b \cos A = 2 \times 2 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$

and $c_1 c_2 = b^2 - a^2 = 4 - 5 = -1$

$\therefore\ |c_1 - c_2| = \sqrt{(c_1 + c_2)^2 - 4c_1 c_2}$

$= \sqrt{12 + 4} = \sqrt{16} = 4$

242 **(c)**

$(a-b)^2 \cos^2 \frac{C}{2} + (a+b)^2 \sin^2 \frac{C}{2}$

$= (a^2 + b^2 - 2ab) \cos^2 \frac{C}{2}$

$\qquad + (a^2 + b^2 + 2ab) \sin^2 \frac{C}{2}$

$= a^2 + b^2 + 2ab \left(\sin^2 \frac{C}{2} - \cos^2 \frac{C}{2}\right)$

$= a^2 + b^2 - 2ab \cos C$

$= a^2 + b^2 - (a^2 + b^2 - c^2) = c^2$

243 **(a)**

1. $rr_1r_2r_3 = \frac{\Delta}{s} \cdot \frac{\Delta}{(s-a)} \cdot \frac{\Delta}{(s-b)} \cdot \frac{\Delta}{(s-c)} = \frac{\Delta^4}{\Delta^2} = \Delta^2$

2. Now, $r_1r_2 + r_2r_3 + r_3r_1$

$= \frac{\Delta\Delta}{(s-a)(s-b)} + \frac{\Delta\Delta}{(s-b)(s-c)}$

$\qquad + \frac{\Delta\Delta}{(s-c)(s-a)}$

$= \Delta^2 \left[\frac{(s-c) + (s-a) + (s-b)}{(s-a)(s-b)(s-c)}\right]$

$= \frac{\Delta^2[3s - (a+b+c)]}{\Delta^2/s} = s^2$

244 **(c)**

Let angles of a triangle are x, $2x$ abd $7x$ respectively

$\therefore\ x + 2x + 7x = 180° \ \Rightarrow\ x = 18°$

Hence, the angles are $18°, 36°, 126°$

Greatest side $\propto \sin 18°$

$\therefore$ Required ratio $= \frac{\sin 126°}{\sin 18°}$

$= \frac{\sin(90° + 36°)}{\sin 18°}$

$= \frac{\cos 36°}{\sin 18°} = \frac{\sqrt{5}+1}{\sqrt{5}-1}$

246 **(a)**

Area of triangle $= \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ rx_1 & ry_1 & 1 \\ r^2x_1 & r^2y_1 & 1 \end{vmatrix} = 0$

$\therefore$ Points are collinear

247 **(d)**

Given, new coordinates$= (4, -3)$ and $\theta = 135°$

$\therefore\ 4 = x \cos 135° + y \sin 135°$

$\Rightarrow\ 4 = -\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}$ ...(i)

and $-3 = -x \sin 135° + y \cos 135°$

$\Rightarrow\ -3 = -\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}}$ ...(ii)

On adding Eqs. (i) and (ii), we get

$1 = -\frac{2x}{\sqrt{2}} \ \Rightarrow\ x = -\frac{1}{\sqrt{2}}$

On subtracting Eqs. (i) and (ii), we get

$7 = \frac{2y}{\sqrt{2}} \quad \Rightarrow \quad y = \frac{7}{\sqrt{2}}$

Thus $(x, y) = \left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$

248 **(d)**

We know, if coordinate axes are rotated, then

$p = (x\cos\theta - y\sin\theta, x\sin\theta + y\cos\theta)$

It is rotated at an angle 135° $ie, \theta = 135°$ and the new point be

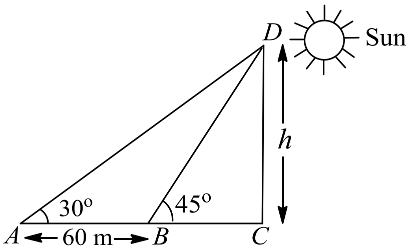
$p = [4\cos(90° + 45°) + 3\sin(90° + 45°), 4\sin(90° + 45°) - 3\cos(90° + 45°)]$

$= [-4\sin 45° + 3\cos 45°, 4\cos 45° + 3\sin 45°]$

$= \left[4.\left(\frac{-1}{\sqrt{2}}\right) + 3.\frac{1}{\sqrt{2}}, 4.\frac{1}{\sqrt{2}} + 3.\frac{1}{\sqrt{2}}\right] = \left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$

249 **(d)**

Let the height of the tower be h meters



In $\Delta\, BCD, \tan 45° = \frac{h}{BC}$

$\Rightarrow \quad BC = h \quad$ ...(i)

In $\Delta\, ACD, \tan 30° = \frac{h}{AC}$

$\Rightarrow \quad AC = \frac{h}{\tan 30°}$

$\Rightarrow \quad AB + BC = \sqrt{3}\, h$

$\Rightarrow \quad 60 + h = \sqrt{3}\, h \quad$ [from Eq.(i)]

$\Rightarrow \quad h = \frac{60}{\sqrt{3} - 1}$

$\Rightarrow \quad h = \frac{60(\sqrt{3} + 1)}{2}$

$\Rightarrow \quad h = 30(\sqrt{3} + 1)\text{m}$

251 **(d)**

Given, $r_1 + r_3 = k\cos^2\frac{B}{2}$

$ie, \qquad s\tan\frac{A}{2} + s\tan\frac{C}{2} = k\cos^2\frac{B}{2}$

$$\Rightarrow \quad s\left[\sqrt{\frac{(s-b)(s-c)}{s(s-a)}} + \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}\right] = k\frac{s(s-b)}{ac}$$

$$\Rightarrow \quad k = \frac{ac}{s-b}\left[\sqrt{\frac{(s-b)(s-c)}{s(s-a)}} + \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}\right]$$

$$= \frac{ac}{s-b} \times \frac{\sqrt{s-b}}{\sqrt{s}}\left[\frac{\sqrt{(s-c)}}{\sqrt{(s-a)}} + \frac{\sqrt{(s-a)}}{\sqrt{(s-c)}}\right]$$

$$= \frac{ac(2s - a - c)}{\sqrt{s(s-a)(s-b)(s-c)}}$$

$$= \frac{ac(b)}{\Delta} = 4R$$

254 **(c)**

Area of $\Delta CAB =$ area of $\Delta CAD +$ area of ΔCDB

$\Rightarrow \frac{1}{2}ba\sin C = \frac{1}{2}b \cdot CD\sin\frac{C}{2} + \frac{1}{2}a \cdot CD\sin\frac{C}{2}$

$\Rightarrow \; CD + \frac{2ab}{a+b}\cos\frac{C}{2}$

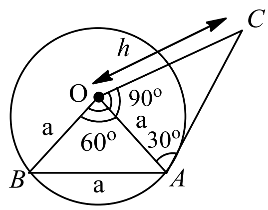
255 **(c)**

Let h be the height of a tower.

Since, $\angle AOB = 60°$

$\therefore \Delta OAB$ is an equilateral triangle.

$\therefore \; OA = OB = AB = a$



In $\Delta OAC, \tan 30° = \frac{h}{a} \Rightarrow h = \frac{a}{\sqrt{3}}$

256 **(d)**

$$\Delta = \begin{vmatrix} -\sin(\beta - \alpha) & -\cos\beta & 1 \\ \cos(\beta - \alpha) & \sin\beta & 1 \\ \cos(\beta - \alpha + \theta) & \sin(\beta - \theta) & 1 \end{vmatrix}$$

Clearly, $\Delta \neq 0$ for any value of α, β, θ, Hence, points are non-collinear

257 **(a)**

Given, $8R^2 = a^2 + b^2 + c^2$

$= 4R^2(\sin^2 A + \sin^2 B + \sin^2 C)$

$\Rightarrow \; \sin^2 A + \sin^2 B + \sin^2 C = 2$

$\Rightarrow \; (\cos^2 A - \sin^2 C) + \cos^2 B = 0$

$\Rightarrow \; \cos(A - C)\cos(A + C) + \cos^2 B = 0$

$\Rightarrow 2\cos B\cos A\cos C = 0$

$\Rightarrow \; \cos A = 0 \;$ or $\; \cos B = 0 \;$ or $\; \cos C = 0$

$\Rightarrow \quad A = \frac{\pi}{2} \;$ or $\; B = \frac{\pi}{2} \;$ or $\; C = \frac{\pi}{2}$

258 **(a)**

$\because \quad AB + \sqrt{(4+1)^2 + (0+1)^2} = \sqrt{26}$

$BC = \sqrt{(3+1)^2 + (5+1)^2} = \sqrt{52}$

$CA = \sqrt{(4-3)^2 + (0+5)^2} = \sqrt{26}$

So, in isosceles triangle side $AB = CA$

Also, $(\sqrt{26})^2 + (\sqrt{26})^2 = 52$

So, triangle is right angled and also is isosceles triangle

259 **(c)**

By Standard results,

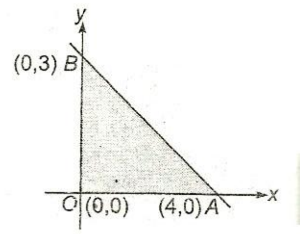
3. $r = 4R \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$

4. $r_1 = s \tan\frac{A}{2}$

5. $r_3 = \frac{\Delta}{s-c}$

260 **(c)**

On solving given equation of lines, we get the points $A(4,0)$, $(0,3)B(0,3)$ and $O(0,0)$



$\therefore$ Area of ΔOAB

$= \dfrac{1}{2} \times OA \times OB$

$= \dfrac{1}{2} \times 4 \times 3$

$= 6$ sq units

261 **(a)**

Let (x, y) denotes the coordinates in A, B and C plane

Then, $\frac{(x-1)^2+y^2}{(x+1)^2+y^2} = \frac{1}{9}$

$\Rightarrow\ 9x^2 + 9y^2 - 18x + 9 = x^2 + y^2 + 2x + 1$

$\Rightarrow\ 8x^2 + 8y^2 - 20x + 8 = 0$

$\Rightarrow\ x^2 + y^2 - \dfrac{5}{2}x + 1 = 0$

$\therefore\ A, B, C$ lie on a circle with $C\left(\frac{5}{4}, 0\right)$

262 **(c)**

Since, points $(a, b), (b, a)$ and $(a^2, -b^2)$ are collinear

$$\therefore \begin{vmatrix} a & b & 1 \\ b & a & 1 \\ a^2 & -b^2 & 1 \end{vmatrix} = 0$$

Applying, $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\begin{vmatrix} a & b & 1 \\ b-a & a-b & 0 \\ a^2-a & -b^2-b & 0 \end{vmatrix} = 0$$

$\Rightarrow (a-b)(b^2 + b - a^2 + a) = 0$

$\Rightarrow (a-b)(a+b)(b+1-a) = 0$

$\Rightarrow$ Either $a - b = 0$ or $(a+b) = 0$ or $(b+1-a) = 0$

$\Rightarrow\ a = b + 1$

263 **(d)**

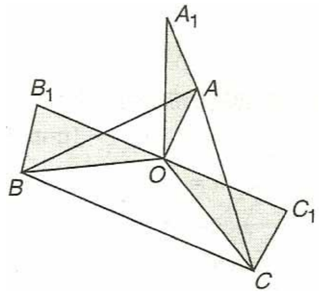
Let AA_1, BB_1 and CC_1 be the towers and O be the circumcentre of $\Delta\, ABC$

$\angle A_1OA = \theta_A, \angle B_1OB = \theta_B, \angle C_1OC = \theta_C$

Now, $OA = AA_1 \cot\theta_A$

$OB = BB_1 \cot\theta_B$

and $OC = CC_1 \cot\theta_C$



Since, O is the circumcentre of a triangle

$\therefore\ OA = OB = OC$

$\Rightarrow\ AA_1 \cot\theta_A = BB_1 \cot\theta_B = CC_1 \cot\theta_c$

$\Rightarrow\ \dfrac{\tan\theta_A}{AA_1} = \dfrac{\tan\theta_B}{BB_1} = \dfrac{\tan\theta_C}{CC_1}$

Any other relationship between $\tan\theta_A$, $\tan\theta_B$ and $\tan\theta_C$ cannot be establish

264 **(b)**

Let $a = 6, b = 5, c = \sqrt{13}$

$\therefore\ \cos C = \dfrac{6^2 + 5^2 - 13}{2 \times 6 \times 5} = \dfrac{4}{5}$

Now, $\sin C = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

$\therefore$ Area of $\Delta\, ABC = \frac{1}{2} ab \sin C$

$= \dfrac{1}{2} \times 6 \times 5 \times \dfrac{3}{5} = 9$ sq unit

265 **(b)**

Since, y_1, y_2, y_3 and x_1, x_2, x_3 are in AP

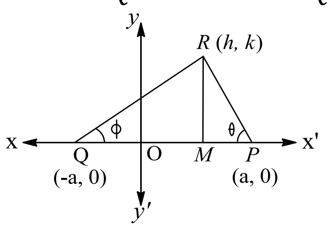
$\therefore\ y_2 - y_1 = y_3 - y_2$ and $x_2 - x_1 = x_3 - x_2$

So, $\frac{y_3 - y_2}{x_3 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$

So, points are collinear

266 **(a)**

Let $\angle RPQ = \theta$ and $\angle RQP = \phi$



$\therefore\ \theta - \phi = 2\alpha$

Let $RM \perp PQ$, so that $RM = k$,

$MP = a - h$ and $MQ = a + h$

Then, $\tan\theta = \frac{RM}{MP} = \frac{k}{a-h}$

and $\tan\phi = \frac{RM}{MQ} = \frac{k}{a+h}$

Again now, $2\alpha = \theta - \phi$

$\therefore\ \tan 2\alpha = \tan(\theta - \phi)$

$= \dfrac{\tan\theta - \tan\phi}{1 + \tan\theta\tan\phi}$

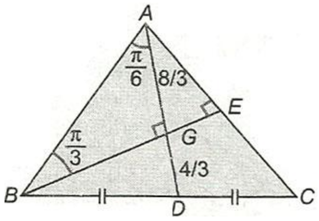
$= \dfrac{k(a+h) - k(a-h)}{a^2 - h^2 + k^2}$

$\Rightarrow\ a^2 - h^2 + k^2 = 2hk\cot 2\alpha$

Hence, the locus is $x^2 - y^2 + 2xy\cot 2\alpha - a^2 = 0$

267 **(c)**

Since, the centroid G divide the line AD in the ratio 2:1



$\therefore AG = \dfrac{8}{3}$ *and* $DG = \dfrac{4}{3}$

In ΔABG, $\tan\frac{\pi}{3} = \frac{AG}{BG}$

$\Rightarrow\ BG = AG\cot\dfrac{\pi}{3}$

$\Rightarrow\ BG = \dfrac{8}{3} \times \dfrac{1}{\sqrt{3}} = \dfrac{8}{3\sqrt{3}}$

Area of $\Delta ADB = \dfrac{1}{2} \times AD \times BG$

$= \dfrac{1}{2} \times 4 \times \dfrac{8}{3\sqrt{3}} = \dfrac{16}{3\sqrt{3}}$

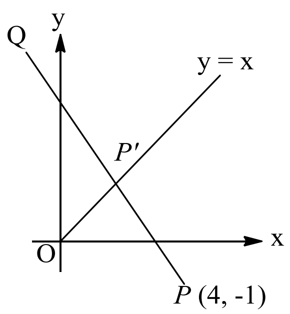
Since, median divides a triangle into two triangle of equal area. Therefore,

Area of $\Delta ABC = 2 \times$ area of ΔADB

$= 2 \times \dfrac{16}{3\sqrt{3}} = \dfrac{32}{3\sqrt{3}}$ sq units

268 **(b)**

Since, Q is the image of P



$\therefore PQ = 2PP''$

$= \dfrac{2|4-(-1)|}{\sqrt{1^2+1^2}} = 5\sqrt{2}$

269 **(d)**

Area $= \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} a & a+b+c & 1 \\ b & a+b+c & 1 \\ c & a+b+c & 1 \end{vmatrix}$

$= \dfrac{(a+b+c)}{2}\begin{vmatrix} a & 1 & 1 \\ b & 1 & 1 \\ c & 1 & 1 \end{vmatrix} = 0$

270 **(c)**

We have,

$x^2 + 4xy + y^2 = aX^2 + bY^2$

$\Rightarrow (X\cos\theta + Y\sin\theta)^2$
$\quad + 14(X\cos\theta + Y\sin\theta)(X\sin\theta$
$\quad - Y\cos\theta) + (X\sin\theta - Y\cos\theta)^2$
$\quad = aX^2 + bY^2$

$\Rightarrow a = 1 + 4\sin\theta\cos\theta, b = 1 - 4\sin\theta\cos\theta$ and $\sin^2\theta - \cos^2\theta = 0$

$\Rightarrow a = 1 + 4\sin\theta\cos\theta, b = 1 - 4\sin\theta\cos\theta$ and $\theta = \frac{\pi}{4}$

$\Rightarrow a = 3, b = -1$

271 **(a)**

Given, distance $r = \sqrt{2}$ and $\theta = 45°$

$\therefore\quad x = 4 + \sqrt{2}\cos 45° = 4 + \sqrt{2}.\dfrac{1}{\sqrt{2}} = 5$

and $\ y = 3 + \sqrt{2}\sin 45° = 3 + \sqrt{2}.\frac{1}{\sqrt{2}} = 4$

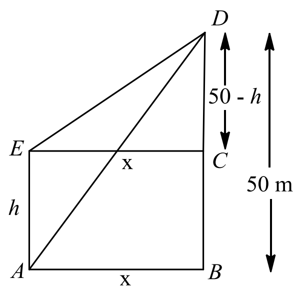
272 **(d)**

Let the height of the cliff be $BD = 50$ m and height of the tower be $AE = h$ metre.

In $\Delta\ DEC$,

$\tan 30° = \dfrac{50-h}{x}$

$\Rightarrow x = \frac{50-h}{\frac{1}{\sqrt{3}}} = \sqrt{3}(50-h)$ ...(i)



and in ΔBAD,

$\tan 45° = \dfrac{50}{x} \Rightarrow x = 50$ m

From Eq. (i),

$50 = \sqrt{3}(50-h)$

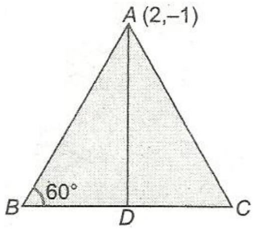
$\Rightarrow h = \dfrac{50(\sqrt{3}-1)}{\sqrt{3}}$ m

$\Rightarrow h = 50\left(1 - \dfrac{\sqrt{3}}{3}\right)$ m

273 **(a)**

Let the vertex of triangle be $A(2,-1)$ and equation of BC is

$x + 2y = 1$



Since, the triangle is equilateral triangle

$\therefore\ \angle ABC = 60°$

and $AD = \left|\frac{2(1)+2(-1)-1}{\sqrt{1+2^2}}\right| = \frac{1}{\sqrt{5}}$

In ΔABD, $\frac{AD}{AB} = \sin 60°$

$$\Rightarrow AB = \frac{\frac{1}{\sqrt{5}}}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{15}}$$

$$\Rightarrow \quad AB = BC = AC = \frac{2}{\sqrt{15}}$$

275 **(d)**

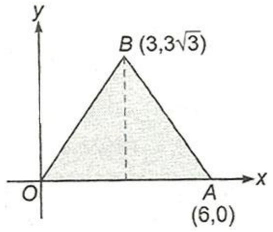
Area of $\Delta = \frac{1}{2}\begin{vmatrix} 2 & 2 & 1 \\ 5 & 5 & 1 \\ 6 & 7 & 1 \end{vmatrix}$

$= \frac{1}{2}[2(5-7) - 2(5-6) + 1(35-30)]$

$= \frac{1}{2}(-4+2+5) = \frac{3}{2}$ sq units

276 **(b)**

Line perpendicular to OA passing through B is $x = 3$



Slope of $AB + \frac{3\sqrt{3}-0}{3-6} = -\sqrt{3}$

Line perpendicular to AB through origin is

$y = \frac{1}{\sqrt{3}}x$

$\therefore$ The point of intersection of a line $x = 3$ and $y = \frac{1}{\sqrt{3}}x$ is $(3, \sqrt{3})$, which is the required orthocenter

277 **(a)**

We know that, in any ΔABC

$$\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

and $r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

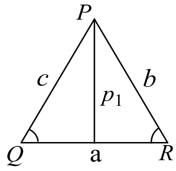
$$\therefore\ \cos A + \cos B + \cos C = 1 + \frac{r}{R}$$

278 **(b)**

Given, $\sin P$, $\sin Q$, $\sin R$ are in AP

$\Rightarrow\quad a, b, c$ are in AP

$$\because \frac{\sin P}{a} = \frac{\sin Q}{b} = \frac{\sin R}{c} = \lambda \quad \text{(say)}$$



Let p_1, p_2, p_3 be altitudes from P, Q, R

$\therefore\ p_1 = c\sin Q = \lambda bc$,

$p_2 = a\sin R = \lambda ac$,

$p_3 = b\sin P = \lambda ab$

Since, a, b, c are in AP,

$\therefore\ \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in HP

$\Rightarrow\ \frac{abc}{a}, \frac{abc}{b}, \frac{abc}{c}$ are in HP

$\Rightarrow\ bc, ac, ab$ are in HP

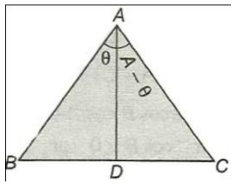
$\Rightarrow\ \lambda bc, \lambda ac, \lambda ab$ are in HP

$\Rightarrow\ p_1, p_2, p_3$ are in HP

280 **(c)**

In $\Delta\, ABD$, $\frac{BD}{\sin\theta} = \frac{AD}{\sin B}$

$\Rightarrow\quad BD = AD = \frac{\sin\theta}{\sin B}$ ...(i)



In $\Delta\, ACD$, $\frac{CD}{\sin(A-\theta)} = \frac{AD}{\sin C}$

$$\Rightarrow\quad CD = AD\frac{\sin(A-\theta)}{\sin C}$$

$\therefore\ BD = CD$

$$\Rightarrow\ AD\frac{\sin\theta}{\sin B} = AD\frac{\sin(A-\theta)}{\sin C}$$

$$\Rightarrow\ \sin(A-\theta) = \frac{\sin C}{\sin B}\sin\theta = \frac{c}{b}\sin\theta$$

282 **(b)**

Since, circle is inscribed in the quadrilateral $ABCD$, then

$a + c = b + d$ ...(i)

And quadrilateral is cyclic, then

$A + C = \pi$

or $C = \pi - A$ ...(ii)

now, in $\Delta\, ABD$

$$\cos A = \frac{a^2 + d^2 - (BD)^2}{2ad}$$

$\Rightarrow\ BD^2 = a^2 + d^2 - 2ad\cos A$ ...(iii)

And in $\Delta\, BCD$,

$$\cos C = \frac{b^2 + c^2 - (BD)^2}{2bc}$$

$\Rightarrow\ (BD)^2 = b^2 + c^2 - 2bc\cos C$

$= b^2 + c^2 - 2bc\cos(\pi - A)$ [from Eq.(ii)]

$\Rightarrow (BD)^2 = b^2 + c^2 + 2bc\cos A$ ... (iv)

From Eqs. (iii) and (iv), we get

$\cos A = \frac{a^2+d^2-b^2-c^2}{2(bc+ad)}$ ...(v)

Now, from Eq. (i),

$(a-d)^2 = (b-c)^2$

$\Rightarrow a^2+d^2-b^2-c^2 = 2(ad-bc)$ ...(vi)

$\therefore$ From Eqs. (v) and (vi),

$\cos A = \left(\frac{ad-bc}{ad+bc}\right)$

283 **(c)**

We have, $a+b+c=\lambda=2s$

$\therefore s = \frac{\lambda}{2}$

$b\cos^2\frac{C}{2} + c\cos^2\frac{B}{2} = b\frac{s(s-c)}{ab} + c\frac{s(s-b)}{ac}$

$= \frac{s}{a}\{s-c+s-b\}$

$= \frac{s}{a}\cdot a = s = \frac{\lambda}{2}$

284 **(c)**

$\cos B = \frac{1}{\sqrt{2}} = \frac{c^2+a^2-b^2}{2ac}$

$\Rightarrow \sqrt{2}ac = c^2+a^2-b^2$

$\Rightarrow 2a^2c^2 = c^4+a^4+b^4+2c^2a^2 - 2(c^2+a^2)b^2$

$\Rightarrow c^4+a^4+b^4 = 2(c^2+a^2)b^2$

285 **(a)**

Let the point of the line divides the line in the ratio $m:1$

$\therefore$ Coordinates of point are $\left(\frac{5m+1}{m+1}, \frac{7m-1}{m+1}\right)$

Which lies on $y+x=4$

$\Rightarrow \frac{7m-1+5m+1}{m+1} = 4$

$\Rightarrow \frac{12m}{m+1} = 4 \Rightarrow 12m = 4m+4$

$\Rightarrow 8m = 4 \Rightarrow m = \frac{1}{2}$

$\therefore$ Required ratio is $1:2$

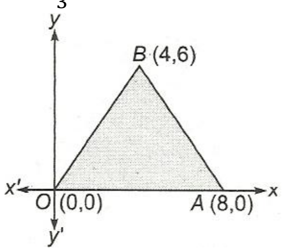
286 **(d)**

Since, line perpendicular to OA passing through B is $x=4$

Slope of $AB = -\frac{3}{2}$

Line perpendicular to AB through origin is,

$y = \frac{2}{3}x$



$\therefore$ The point of intersection of lines $x=4$ And $y=\frac{2}{3}x$ is $\left(4,\frac{8}{3}\right)$. Which is the required orthocenter

288 **(a)**

We have, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\therefore \frac{b+c}{a} = \frac{\sin B+\sin C}{\sin A}$

$= \frac{\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)}{\sin\frac{A}{2}\cos\frac{A}{2}} = \frac{\cos\left(\frac{B-C}{2}\right)}{\sin\frac{A}{2}}$

$[A+B+C=\pi \Rightarrow A = \pi-(B+C)]$

289 **(a)**

Let h be the mid point of BC. Since, $\angle TBH = 90°$, then $TH^2 = BT^2+BH^2 = 5^2+5^2 = 50$

Also since,

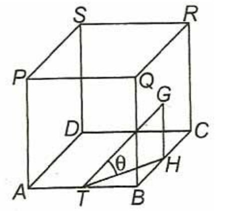
$\angle THG = 90°, TG^2 = TH^2+GH^2 = 50+25 = 75$

Let θ be the required angle of elevation of G at T

Then, $\sin\theta = \frac{GH}{TG}$

$= \frac{5}{5\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = \sin^{-1}(1/\sqrt{3})$



290 **(d)**

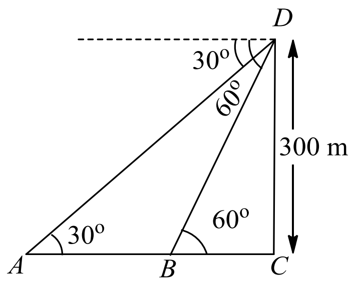
Given, $\Delta = \frac{1}{2}ap_1 = \frac{1}{2}bp_2 = \frac{1}{2}cp_3$

$\therefore p_1 = \frac{2\Delta}{a}, p_2 = \frac{2\Delta}{b}, p_3 = \frac{2\Delta}{c}$

$\therefore p_1p_2p_3 = \frac{8\Delta^3}{abc} = \frac{8}{abc}\left(\frac{abc}{4R}\right)^3 = \frac{a^2b^2c^2}{8R^2}$

291 **(b)**

In ΔBCD, $\cot 60° = \frac{BC}{300}$



$\Rightarrow BC = 300\times\frac{1}{\sqrt{3}}$ ...(i)

In ΔACD, $\cot 30° = \frac{AC}{300}$

$\Rightarrow AC = 300\sqrt{3}$ ...(ii)

$\therefore$ Distance between two boats $= AB$

$= AC - BC = 300\sqrt{3} - \frac{300}{\sqrt{3}}$ [using Eqs.(i)and (ii)]

$$= 300\frac{(3-1)}{\sqrt{3}} = \frac{600 \times \sqrt{3}}{3} = 346.4 \text{ m}$$

292 **(d)**

$\because\ 3^2 + 4^2 = 5^2$

$\Rightarrow$ Given triangle is a right angled triangle whose length of hypotenuse is 5 unit

$\therefore\ R = \frac{5}{2} = 2.5$

293 **(a)**

Given, $\left(1 - \frac{r_1}{r_2}\right)\left(1 - \frac{r_1}{r_3}\right) = 2$

$$\therefore\ \left(1 - \frac{s-b}{s-a}\right)\left(1 - \frac{s-c}{s-a}\right) = 2$$

$$\Rightarrow\ \frac{(b-a)(c-a)}{(s-a)^2} = 2$$

$\Rightarrow\ a^2 = b^2 + c^2$

Hence, triangle is a right angled triangle

294 **(a)**

From given, we get

$$a\left(\tan A - \tan\frac{A+B}{2}\right) = b\left(\tan\frac{A+B}{2} - \tan B\right)$$

$$\Rightarrow\ a \cdot \frac{\sin\left(A - \frac{A+B}{2}\right)}{\cos A \cdot \cos\frac{A+B}{2}} = b \cdot \frac{\sin\left(\frac{A+B}{2} - B\right)}{\cos\frac{A+B}{2}\cos B}$$

$$\Rightarrow\ \frac{a\sin\left(\frac{A-B}{2}\right)}{\cos A} = \frac{b\sin\left(\frac{A-B}{2}\right)}{\cos B}$$

$$\Rightarrow\ \sin\left(\frac{A-B}{2}\right)\{a\cos B - b\cos A\} = 0$$

$\Rightarrow$ Either $\sin\frac{A-B}{2} = 0$ or $a\cos B = b\cos A$

When, $\sin\frac{A-B}{2} = 0 \quad \Rightarrow \quad A = B$

or when $a\cos B = b\cos A$

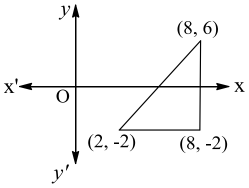
$\Rightarrow\ \sin A\cos B - \cos A\sin B = 0$

$\Rightarrow\ \sin(A - B) = 0$

$\Rightarrow\ A = B$

295 **(d)**

Triangle is right angled triangle. In right angled triangle mid point of hypotenuse is circumcentre So, coordinates of the circumcentre are (5, 2)



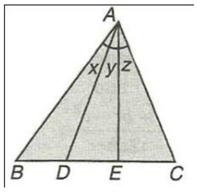
296 **(c)**

From $\Delta\, ABC$, $\frac{\sin(y+z)}{DC} = \frac{\sin C}{AD}$

From $\Delta\, ABD$, $\frac{\sin x}{BD} = \frac{\sin B}{AD}$

From $\Delta\, AEC$, $\frac{\sin z}{EC} = \frac{\sin C}{AE}$

From $\Delta\, ABE$, $\frac{\sin(x+y)}{BE} = \frac{\sin B}{AE}$



$$\therefore\ \frac{\sin(x+y)\sin(y+z)}{\sin x \sin z} = \frac{\frac{BE\sin B}{AE} \times \frac{DC\sin C}{AD}}{\frac{BD\sin B}{AD} \times \frac{EC\sin C}{AE}}$$

$$= \frac{BE}{AE} \times \frac{DC}{AD} \times \frac{AD}{BD} \times \frac{AE}{EC}$$

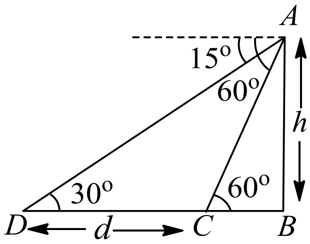
$$= \frac{2BD \times 2EC}{BD \times EC} = 4$$

297 **(c)**

In $\Delta\, ABC$, $\tan 60° = \frac{h}{BC}$

$\Rightarrow\ BC = h\cot 60°$

In $\Delta\, ABD$, $\tan 30° = \frac{h}{BD}$



$\Rightarrow\ BD = h\cot 30°$

$\Rightarrow\ BC + CD = h\cot 30°$

$\Rightarrow\ CD = h\cot 30° - BC$

$\Rightarrow\ d = h\cot 30° - h\cot 60°$ [from Eq. (i)]

$$\therefore\ \text{Speed of car} = \frac{\text{distance from D to C}}{\text{time taken}}$$

$$= \frac{d}{3} = \frac{h\cot 30° - h\cot 60°}{3}$$

$$\therefore\ \text{Time taken from } C \text{ to } B = \frac{\text{distance}}{\text{speed}}$$

$$= \frac{h\cot 60°}{h(\cot 30° - \cot 60°)}$$

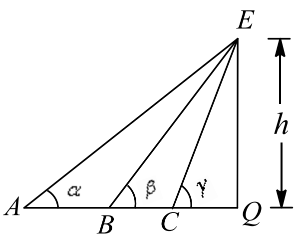
$$= \frac{\frac{1}{\sqrt{3}} \times 3}{\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)}$$

$$= \frac{3}{2} = 1.5 \text{ min}$$

298 **(a)**

Let h be the height of the tower, then

$h = AQ\tan\alpha = BQ\tan\beta = CQ\tan\gamma$



$\Rightarrow\ BC = BQ - CQ = h(\cot\beta - \cot\gamma)$,

$CA = h(\cot\alpha - \cot\gamma)$

and $AB = h(\cot\alpha - \cot\beta)$

Now, $BC\cot\alpha - CA\cot\beta + AB\cot\gamma$

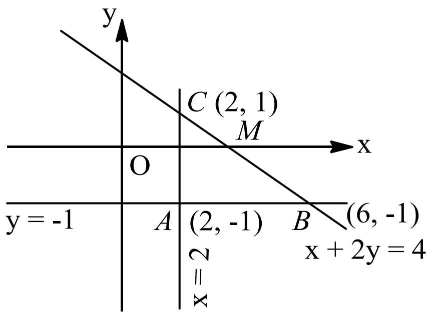
$= h[\cot\alpha(\cot\beta - \cot\gamma) - \cot\beta(\cot\alpha - \cot\gamma) + \cot\gamma(\cot\alpha - \cot\beta)]$

$= 0$

299 **(a)**

On solving the given equations of sides, we get the coordinates of the vertices of the triangle,

$A(2,-1), B(6,-1)$ and $C(2,1)$

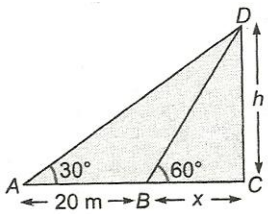


The circumcentre of ΔABC is the mid point of BC

ie, $M \equiv \left(\frac{8}{2}, \frac{0}{2}\right) = (4,0)$

300 **(b)**

Let h metres be the height of tree CD and x meters be the width of river



In ΔBCD, $\tan 60° = \frac{h}{x} \Rightarrow h = \sqrt{3}x$ …..(i)

and in ΔACD, $\tan 30° = \frac{h}{x+20}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+20}$

$\Rightarrow 3x = x + 20$ [from Eq. (i)]

$x = 10$ m

302 **(a)**

We have,

$2s = a + b + c, A^2 = s(s-a)(s-b)(s-c)$

$\because$ AM $\geq$ GM

$\Rightarrow \frac{s-a+s-b+s-c}{3} \geq \sqrt[3]{(s-a)(s-b)(s-c)}$

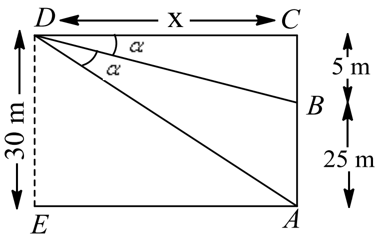
$\Rightarrow \frac{3s-2s}{3} \geq \frac{(A^2)^{1/3}}{s^{1/3}}$

$\Rightarrow \frac{s^3}{27} \geq \frac{A^2}{s} \Rightarrow A \leq \frac{s^2}{3\sqrt{3}}$

303 **(b)**

In ΔDBC, $\tan\alpha = \frac{5}{x}$ ...(i)

and in ΔDAC, $\tan 2\alpha = \frac{30}{x}$



$\Rightarrow \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{30}{x}$

$\Rightarrow \frac{2\left(\frac{5}{x}\right)}{1-\frac{25}{x^2}} = \frac{30}{x}$

$\Rightarrow \frac{10x}{x^2-25} = \frac{30}{x}$

$\Rightarrow 10x^2 = 30x^2 - 750$

$\Rightarrow 20x^2 = 750$

$\Rightarrow x^2 = \frac{75}{2}$

$\Rightarrow x = 5\sqrt{\frac{3}{2}}$ m

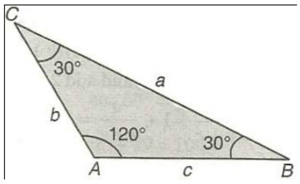
305 **(a)**

Here, ratio of angles are 4:1:1

$\Rightarrow 4x + x + x = 180°$

$\Rightarrow = 30°$

$\therefore \angle A = 120°, \quad \angle B = \angle C = 30°$



Thus, the ratio of longest side to the perimeter $= \frac{a}{a+b+c}$

Let $b = c = x$

$\therefore a^2 = b^2 + c^2 - 2bc\cos A$

$\Rightarrow a^2 = 2x^2 - 2x^2\cos A = 2x^2(1-\cos A)$

$\Rightarrow a^2 = 4x^2\sin^2\frac{A}{2}$

$\Rightarrow a = 2x\sin\frac{A}{2}$

$\Rightarrow a = 2x\sin 60° = \sqrt{3}x$

Thus, required ratio is

$\frac{a}{a+b+c} = \frac{\sqrt{3}x}{x+x+\sqrt{3}x} = \frac{\sqrt{3}}{2+\sqrt{3}}$

306 **(c)**

Let the third angle is θ

In a triangle, $\frac{\pi}{4} + \tan^{-1}2 + \theta = \pi$

$\Rightarrow \theta = \pi - \frac{\pi}{4} - \tan^{-1}2$

$\Rightarrow \tan\theta = \tan\left[\pi - \left(\frac{\pi}{4} + \tan^{-1}2\right)\right]$

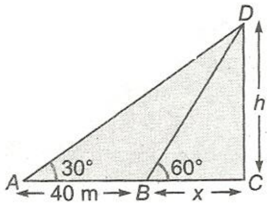
$= -\tan\left(\frac{\pi}{4} + \tan^{-1}2\right)$

$\Rightarrow \tan\theta = -\dfrac{1+2}{1-2} = 3$

$\Rightarrow \theta = \tan^{-1} 3$

307 **(d)**

In $\Delta BCD, \tan 60° = \frac{CD}{BC} = \frac{h}{x} \Rightarrow h = \sqrt{3x}$ ...(i)



and in ΔACD,

$\tan 30° = \dfrac{CD}{AC} = \dfrac{h}{x+40} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{h}{40+x}$

$40 + x = h\sqrt{3}$

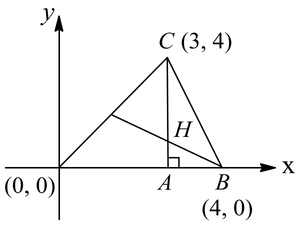
$\Rightarrow 40 + x = 3x$ [using Eq.(i)]

$\Rightarrow x = 20$

On putting this value is Eq.(i), we get $h = 20\sqrt{3}$ m

308 **(d)**

Let $H(3, \alpha)$ is the orthocenter



$\therefore$ Slope of $BH \times$ Slope of $AC = -1$

$\Rightarrow -\alpha.\dfrac{4}{3} = -1$

$\Rightarrow \alpha = \dfrac{3}{4}$

Hence, orthocenter of a triangle is $\left(3, \frac{3}{4}\right)$

309 **(c)**

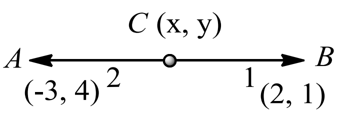
Let the axes be rotated through an angle θ. Then,

$\tan 2\theta = \dfrac{2h}{a-b} = \dfrac{4\sqrt{3}}{5-9} = -\sqrt{3} \Rightarrow 2\theta = \dfrac{2\pi}{3} \Rightarrow \theta = \dfrac{\pi}{3}$

310 **(a)**

Since, $AC = 2BC$

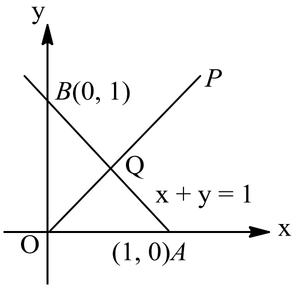
$\therefore$ Coordinates of C are



$\left(\dfrac{4-3}{2+1}, \dfrac{2+4}{2+1}\right)$ *ie*, $\left(\dfrac{1}{3}, 2\right)$

311 **(d)**

Let P be the image of the origin about the line $x + y = 1$. Since, $OA = OB$, therfore Q is mid point of AB



$\therefore$ Coordinates of Q are $\left(\frac{1}{2}, \frac{1}{2}\right)$

Let the coordinates of P are (x_1, y_1)

Also, Q is the mid point of OP

$\therefore \frac{0+x_1}{2} = \frac{1}{2}$ and $\frac{0+y_1}{2} = \frac{1}{2}$

$\Rightarrow x_1 = 1, \quad y_1 = 1$

$\therefore$ The cordiantes of P are (1, 1)

312 **(c)**

We have, cosec $A(\sin B \cos C + \cos B \sin C)$

$= \left(\dfrac{\sin B}{\sin A}\cos C + \dfrac{\sin C}{\sin A}\cos B\right)$

$= \left(\dfrac{b}{a}\cos C + \dfrac{c}{a}\cos B\right) = 1$

$(\because a = b\cos C + c\cos B)$

313 **(b)**

Let coordinates of fourth vertex are (x, y)

$\therefore \dfrac{x-5}{2} = \dfrac{7+3}{2}$

$\Rightarrow x = 15$

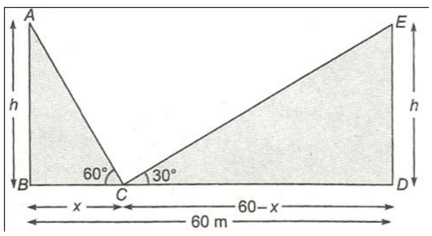
and $\frac{y-4}{2} = \frac{10+5}{2}$

$\Rightarrow y = 19$

$\therefore$ Coordinates of fourth vertex are (15, 19)

314 **(a)**

Let AB and DE be two towers of equal height 'h'



$\therefore$ In $\Delta ABC, \tan 60° = \dfrac{h}{x} \Rightarrow x = \dfrac{h}{\sqrt{3}}$ ... (i)

Again, in $\Delta CDE, \tan 30° = \frac{h}{60-x}$

$\Rightarrow 60 - x = h\sqrt{3}$

$\Rightarrow 60 = h\sqrt{3} + \frac{h}{\sqrt{3}}$ [from Eq. (i)]

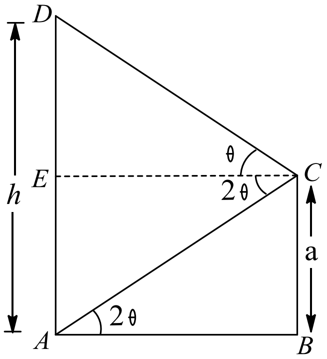
$\Rightarrow h = 15\sqrt{3}$ m

315 **(d)**

In $\Delta\ DEC, \tan\theta = \frac{DE}{CE}$

$\Rightarrow \tan\theta = \dfrac{h-a}{CE}$

$\Rightarrow CE = \dfrac{h-a}{\tan\theta}$



In $\Delta\, ABC$,

$$\tan 2\theta = \frac{a}{AB} = \frac{a}{CE} \quad (\because CE = AB)$$

$$\Rightarrow \frac{2\tan\theta}{1-\tan^2\theta} = \frac{a}{\frac{h-a}{\tan\theta}} \quad \text{[from Eq.(i)]}$$

$$\Rightarrow a\tan\theta = (h-a) \times \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\Rightarrow a\tan\theta\,(1-\tan^2\theta) - 2(h-a)\tan\theta = 0$$

$$\Rightarrow \tan\theta\,(a - a\tan^2\theta - 2h + 2a) = 0$$

$\because \tan\theta \neq 0$

$$\therefore \tan^2\theta = \frac{-2h+3a}{a}$$

$$\Rightarrow \theta = \tan^{-1}\sqrt{\frac{-2h+3a}{a}}$$

$$= \tan^{-1}\sqrt{3 - \frac{2h}{a}}$$

316 **(c)**

Given vertices are $A(2a, 4a), B(2a, 6a)$ and $C(2a + \sqrt{3}a, 5a), a > 0$

Now, $AB = \sqrt{(2a+2a)^2 + (4a-6a)^2} = 2a$

$$BC = \sqrt{\left(\sqrt{3}a\right)^2 + a^2} = 2a$$

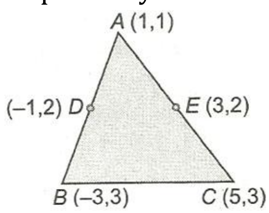
and $CA = \sqrt{\left(\sqrt{3}a\right)^2 + (-a)^2} = 2a$

$\therefore AB = BC = CA$

Hence, triangle is an equilateral triangle, therefore it is an acute angled triangle

317 **(a)**

Let D and E are the mid points of AB and AC. So, coordinates of B and C are $(-3, 3)$ and $(5, 3)$ respectively

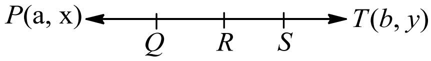


Centroid of triangle $= \left(\frac{1-3+5}{3}, \frac{1+3+3}{3}\right)$

$$= \left(1, \frac{7}{3}\right)$$

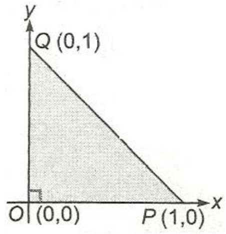
318 **(b)**

R is mid point of PT



Point $\left(\frac{5a+3b}{8}, \frac{5x+3y}{8}\right)$ divides PT in ratio 3 : 5 and that is mid point of QR

319 **(a)**



On solving the given equations of lines, we get the coordinates of the vertices of a ΔOPQ which are $O(0,0), P(1,0)$ and $Q(0,1)$. Since, the triangle is right angled at $O(0,0)$, therefore $O(0,0)$ is its orthocenter

320 **(d)**

Let the circumcentre of triangle be $P(x, y)$ and let the vertices of a ΔABC be $A(0,30), B(4,0)$ and $C(30,0)$

$\therefore \quad PA^2 = PB^2 = PC^2$

$\Rightarrow \quad (x-0)^2 + (y-30)^2 = (x-4)^2 + (y-0)^2$

$= (x-30)^2 + (y-0)^2$

From Ist and IInd terms,

$x^2 + y^2 - 60y + 900 = x^2 + y^2 - 8x + 16$

$\Rightarrow \quad 8x - 60y + 884 = 0 \quad$ ...(i)

From IInd and IIIrd terms,

$x^2 - 8x + 16 + y^2 = x^2 - 60x + 900 + y^2$

$\Rightarrow \quad 52x = 884 \quad \Rightarrow \quad x = 17$

On putting $x = 17$ in Eq. (i), we get

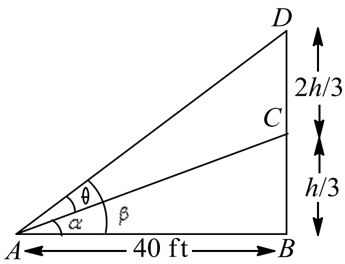
$y = 17$

Hence, required point is (17, 17)

321 **(b)**

Since, $\tan\theta = \frac{1}{2}$

In $\Delta\, ABC$, $\tan\alpha = \frac{\frac{h}{3}}{AB} = \frac{h}{120} \quad$ ...(i)



In $\Delta\, ADB$, $\tan\beta = \frac{3h}{120} \quad$ ...(ii)

$\therefore \tan\theta = \tan(\beta - \alpha)$

$$\Rightarrow \tan\theta = \frac{\tan\beta - \tan\alpha}{1 - \tan\beta\tan\alpha}$$

$$\Rightarrow \frac{1}{2} = \frac{\frac{3h}{120} - \frac{h}{120}}{1 + \frac{3h^2}{14400}} \quad \left(\because \tan\theta = \frac{1}{2}, \text{given}\right)$$

$$\Rightarrow \frac{1}{2} = \frac{\frac{2h}{120}}{\frac{14400+3h^2}{14400}}$$

$$\Rightarrow \frac{1}{2} = \frac{240h}{14400 + 3h^2}$$

$\Rightarrow 14400 + 3h^2 = 480\,h$

$\Rightarrow 4800 + h^2 - 160h = 0$

$\Rightarrow (h-40)(h-120) = 0$

Since, the height of the pole is more than 100 m

$\therefore h = 120$ ft

322 **(c)**

Slope of line $OP = \frac{3}{4}$, let new position is $Q(x, y)$

slope of $OQ = \frac{y}{a}$ also $x^2 + y^2 = OQ^2 = 25 = (OP^2)$

$$\tan 45° = \left|\frac{\frac{y}{x} - \frac{3}{4}}{1 + \frac{3y}{4x}}\right| \Rightarrow \pm 1 = \frac{4y - 3x}{4x + 3y}$$

Straight lines and pair of straight lines

$4x + 3y = 4y - 3x$

or $-4x - 3y = 4y - 3x$

$x = \frac{1}{7}y$ ...(i)

$-x = 7y$ ...(ii)

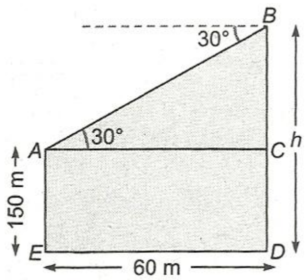
Correct relation is $x = \frac{1}{7}y$ as new point must lies in Ist quadrant

$x^2 + 49x^2 = 25$

$$\Rightarrow x = -\frac{1}{\sqrt{2}},\ y = \frac{7}{\sqrt{2}}$$

323 **(c)**

In ΔABC, $\tan 30° = \frac{BC}{AC}$



$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h - 150}{60}$$

$\Rightarrow h = (150 + 20\sqrt{3})$ m

324 **(b)**

Since, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$ (say)

$\therefore (a+b+c)(a+b-c) = 3ab$

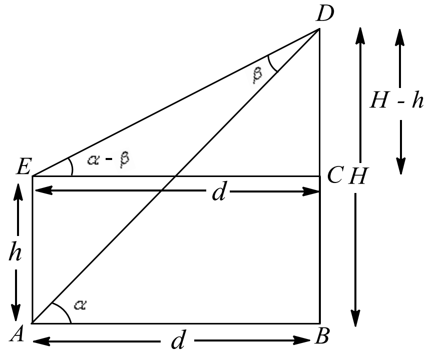
$\Rightarrow a^2 + b^2 + 2ab - c^2 = 3ab$

$$\Rightarrow \frac{a^2 + b^2 - c^2}{2ab} = \frac{1}{2} \Rightarrow \cos C = \cos\frac{\pi}{3}$$

$$\Rightarrow \angle C = \frac{\pi}{3}$$

325 **(b)**

In $\Delta\, ABD$, $\tan\alpha = \frac{H}{d}$



$\Rightarrow d = H\cot\alpha$ ...(i)

In $\Delta\, ECD$, $\tan(\alpha - \beta) = \frac{H-h}{d}$

$\Rightarrow \tan(\alpha - \beta) = \frac{H-h}{H\cot\alpha}$ [from Eq.(i)]

$\Rightarrow H[1 - \cot\alpha\tan(\alpha - \beta)] = h$

$$\Rightarrow H = \frac{h\cot(\alpha - \beta)}{\cot(\alpha - \beta) - \cot\alpha}$$

326 **(b)**

Let the equation of line be

$$\frac{x}{a} + \frac{y}{b} = 1$$

Since, it passes through $\left(\frac{1}{5}, \frac{1}{5}\right)$

$$\therefore \frac{1}{5a} + \frac{1}{5b} = 1$$

$\Rightarrow a + b = 5ab$ ...(i)

Since, the point $P(x, y)$ divides AB joining $A(a, 0)$ and $B(0, b)$ internally in ratio 3:1

$\therefore x = \frac{a}{4},\ y = \frac{3b}{4} \Rightarrow a = 4x$ and $b = \frac{4y}{3}$

On putting the value of a and b in Eq. (i), we get

$$4x + \frac{4y}{3} = 5(4x)\left(\frac{4y}{3}\right)$$

$\Rightarrow 3x + y = 20\,xy$

327 **(d)**

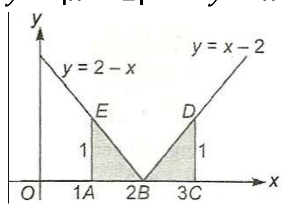
$(3 - 10 + a)(9 - 20 + a) > 0$

or $(a-7)(a-11) > 0$

$\therefore a \in (-\infty, 7) \cup (11, \infty)$

328 **(a)**

$y = |x - 2| \Rightarrow y = x - 2$ and $y = 2 - x$



Area of shaded region $= 2\,ABC = 2.\frac{1}{2}.1.1 = 1$

329 **(d)**

$$\frac{\cot\frac{A}{2}\cot\frac{B}{2}-1}{\cot\frac{a}{2}\cot\frac{B}{2}}=1-\tan\frac{A}{2}\tan\frac{B}{2}$$

$$=1-\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}\sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$$

$$=1-\frac{s-c}{s}=\frac{c}{s}$$

$$=\frac{2c}{a+b+c}$$

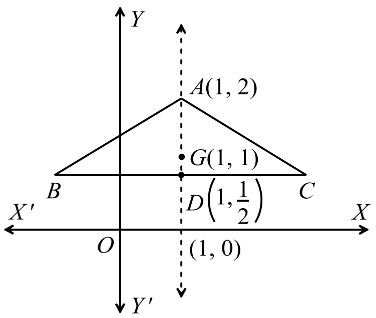
330 **(c)**

We have, $AG=1$

$\therefore GD=\frac{1}{2}AG=\frac{1}{2}$

Hence, the coordinates of D are $(1,1/2)$

Clearly, AG is parallel to y-axis and the triangle ABC is equilateral. Therefore, BC is parallel to x-axis at a distance of $\frac{1}{2}$ unit from it



Let a be the length of each side of $\Delta\, ABC$. Then,

$$AD=\frac{\sqrt{3}}{2}a\Rightarrow\frac{\sqrt{3}}{2}a=\frac{3}{2}\Rightarrow a=\sqrt{3}$$

Since D is the mid-point of BC and $BC=\sqrt{3}$

$$\therefore BD=CD=\frac{\sqrt{3}}{2}$$

Hence, the coordinates of B and C are $\left(1-\frac{\sqrt{3}}{2},\frac{1}{2}\right)$ and $\left(1+\frac{\sqrt{3}}{2},\frac{1}{2}\right)$ respectively

331 **(d)**

$c^2\sin 2B+b^2\sin 2C$

$=c^2(2\sin B\cos B)+b^2(2\sin C\cos C)$

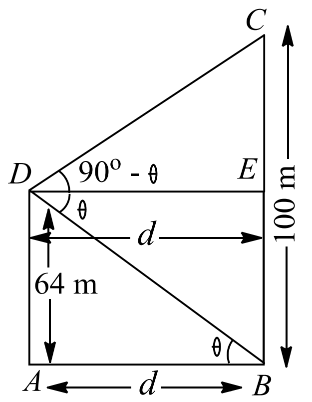
$=2c^2\left(\frac{2\Delta}{ac}\cos B\right)+2b^2\left(\frac{2\Delta}{ab}\cos C\right)$

$=4\Delta\left(\frac{c\cos B+b\cos C}{a}\right)$

$=4\Delta\left(\frac{a}{a}\right)=4\Delta$

332 **(a)**

In $\Delta\, DAB, \tan\theta=\frac{64}{d}$



$\Rightarrow\; d=64\cot\theta$ ...(i)

In $\Delta CDE, \tan(90°-\theta)=\frac{(100-64)}{d}$

$\Rightarrow\; d=36\tan\theta$ ...(ii)

From Eqs. (i) and (ii), we get

$d^2=36\times 64\;\Rightarrow\; d=48$ m

334 **(d)**

We know that area of circle is πr^2

If radius, $r=a$, then $A=\pi a^2$

And the area of the segment of angle $2\pi=\pi a^2$

$\therefore$ Area of 1 angle$=\frac{\pi a^2}{2\pi}$

$\therefore$ Area of 2α angle $=\frac{2\alpha\pi a^2}{2\pi}=\alpha a^2$

336 **(a)**

Let $a=5k, b=6k$ and $c=5k$

$$s=\frac{5k+6k+5k}{2}=8k$$

$$\therefore\; r=\frac{\Delta}{s}$$

$$=\sqrt{\frac{8k(8k-5k)(8k-6k)(8k-5k)}{8k}}$$

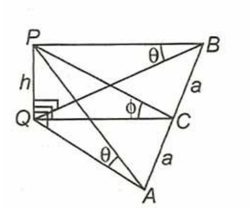
$$=\sqrt{\frac{8k.3k.2k.3k}{8k}}=\frac{3k}{2}$$

$$\Rightarrow\; k=\frac{2r}{3}=\frac{2\times 6}{3}=4$$

338 **(b)**

Let the height of the vertical tower $PQ=h, C$ is the middle point of line segment AB. Since, PQ is perpendicular to the plane QAB,

$\therefore\; \angle PQA=\angle PQC=\angle PQB=90°$, we get



$$\frac{PQ}{QA}=\tan\theta\Rightarrow\; QA=h\cot\theta$$

Similarly, $QB=h\cot\theta$ and $QC=h\cot\phi$

Since, $QA=QB$, the $\Delta\, QAB$ is an isosceles triangle

Here, QCA is a right angled triangle in which $\angle QCA = 90°$

$\therefore\ OC^2 + AC^2 = QA^2$

$\Rightarrow\ h^2 \cot^2 \phi + a^2 = h^2 \cot^2 \theta$

or $h^2 = \frac{a^2}{\cot^2\theta - \cot^2\phi}$

$$\Rightarrow\ h^2 = \frac{a^2}{(\operatorname{cosec}^2\theta - 1) - (\operatorname{cosec}^2\phi - 1)}$$

$$= \frac{a^2}{\operatorname{cosec}^2\theta - \operatorname{cosec}^2\phi}$$

$$= \frac{a^2}{\frac{1}{\sin^2\theta} - \frac{1}{\sin^2\phi}}$$

$$\Rightarrow\ h^2 = \frac{a^2 \sin^2\theta \sin^2\phi}{\sin^2\phi - \sin^2\theta} = \frac{a^2 \sin^2\theta \sin^2\phi}{\sin(\phi-\theta)\sin(\phi-\theta)}$$

Hence, the required height h of the peak

$$= \frac{a\sin\theta\sin\phi}{\sqrt{\sin(\phi+\theta)\sin(\phi-\theta)}}$$

339 **(b)**

Given, $\frac{\cos A}{K\sin A} = \frac{\cos B}{K\sin B} = \frac{\cos C}{K\sin C}$

$\Rightarrow\ \cot a = \cot B = \cot C$

$\Rightarrow\ A = B = C = 60°$

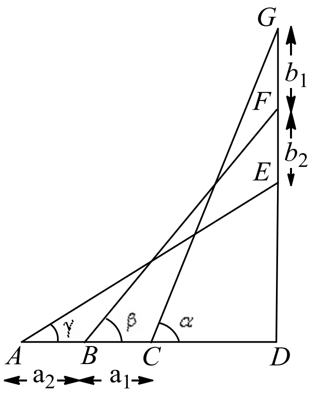
$\Rightarrow\ \Delta ABC$ is an equilateral triangle.

$\therefore\ \Delta \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times \frac{1}{6} = \frac{\sqrt{3}}{24}$ sq unit

340 **(c)**

$\because\ \frac{a_1}{b_1} = \tan\left(\frac{\alpha+\beta}{2}\right)$ ...(i)

and $\frac{a_2}{b_2} = \tan\left(\frac{\beta+\gamma}{2}\right)$ ...(ii)



Since, $a_1 a_2 = b_1 b_2$

$$\Rightarrow\ \frac{a_1}{b_1} = \frac{b_2}{a_2}$$

$$\Rightarrow\ \tan\left(\frac{\alpha+\beta}{2}\right) = \frac{1}{\tan\left(\frac{\beta+\gamma}{2}\right)}$$

$$\Rightarrow\ \tan\left(\frac{\alpha+\beta}{2}\right)\tan\left(\frac{\beta+\gamma}{2}\right) = 1$$

$$\therefore\ \frac{\alpha+\beta}{2} + \frac{\beta+\gamma}{2} = \frac{\pi}{2}$$

$\Rightarrow\ \alpha+\beta+\gamma = \pi - \beta < \pi$

341 **(c)**

We know that, $2s = a + b + c$

$$\therefore\ \frac{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}{4b^2c^2}$$

$$= \frac{2s(2s-2a)(2s-2b)(2s-2c)}{4b^2c^2}$$

$$= 4\frac{s(s-a)}{bc} \times \frac{(s-b)(s-c)}{bc}$$

$$= 4\cos^2\frac{A}{2} \times \sin^2\frac{A}{2} = \sin^2 A$$

342 **(b)**

Let $P(h,k)$ be the required point, then $2PA = 3PB$

$\Rightarrow\ 4PA^2 = 9PB^2$

$\Rightarrow\ 4[(h-0)^2 + (k-0)^2]$

$= 9[(h-4)^4 + (k+3)^2]$

$\Rightarrow\ 4(h^2+k^2) = 9[h^2 + 16 - 8h + k^2 + 9 + 6k]$

$\Rightarrow\ 5h^2 + 5k^2 - 72h + 54k + 225 = 0$

$\therefore$ Required locus of $P(h,k)$ is

$5x^2 + 5y^2 - 72x + 54y + 225 = 0$

343 **(a)**

Here, $a = \sqrt{(16-5)^2 + (12-12)^2} = 11$

$b = \sqrt{(16-0)^2 + (12-0)^2} = 20$

And $c = \sqrt{(5-0)^2 + (12-0)^2} = 13$

$\therefore$

Incentre= $\left(\frac{11\times0+20\times5+13\times16}{11+20+13}, \frac{11\times0+20\times12+13\times12}{11+20+13}\right) = (7,9)$

344 **(d)**

We have, $\tan\frac{A}{2}\tan\frac{C}{2} = \frac{1}{2}$

$$\Rightarrow\ \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}\sqrt{\frac{(s-a)(s-b)}{s(s-c)}} = \frac{1}{2}$$

$$\Rightarrow\ \frac{s-b}{s} = \frac{1}{2}\ \Rightarrow\ 2s - 2b - s = 0$$

$\Rightarrow\ a + c - 3b = 0$

345 **(c)**

Since, $\sin^2\frac{A}{2}, \sin^2\frac{B}{2}, \sin^2\frac{C}{2}$ be in HP

$\Rightarrow\ \frac{1}{\sin^2\frac{A}{2}}, \frac{1}{\sin^2\frac{B}{2}}, \frac{1}{\sin^2\frac{C}{2}}$ are in AP

$$\Rightarrow\ \frac{1}{\sin^2\frac{C}{2}} - \frac{1}{\sin^2\frac{B}{2}} = \frac{1}{\sin^2\frac{B}{2}} - \frac{1}{\sin^2\frac{A}{2}}$$

$$\Rightarrow\ \frac{ab}{(s-a)(s-b)} - \frac{ac}{(s-a)(s-c)}$$

$$= \frac{ac}{(s-a)(s-c)} - \frac{bc}{(s-b)(s-c)}$$

$$\Rightarrow\ \left(\frac{a}{s-a}\right)\left(\frac{b(s-c)-c(s-b)}{(s-b)(s-c)}\right)$$

$$= \left(\frac{c}{s-c}\right)\left(\frac{a(s-b)-b(s-a)}{(s-a)(s-b)}\right)$$

$\Rightarrow\ abs - abc - acs + abc$

$= acs - abc - bcs + abc$

$\Rightarrow \; ab - ac = ac - bc \;\; \Rightarrow \;\; ab + bc = 2ac$

$\Rightarrow \; \frac{1}{c} + \frac{1}{a} = \frac{2}{b}$

$\Rightarrow \; a, b, c$ are in HP

346 **(d)**

It is given that O is the origin and $P(2,3)$ and $Q(4,5)$ are two points

$\therefore \cos \angle POQ = \frac{OP^2 + OQ^2 - PQ^2}{2(OP)(OQ)}$

$\Rightarrow OP \times OQ \cos \angle POQ = \frac{1}{2}\{OP^2 + OQ^2 - PQ^2\}$

$\Rightarrow OP \times OQ \cos \angle POQ = \frac{1}{2}\{13 + 41 - 8\} = 23$

ALITER If O is the origin and $P(x_1, y_1), Q(x_2, y_2)$ are two points, then

$OP \times OQ \times \cos \angle POQ = x_1 x_2 + y_1 y_2$

Here, $x_1 = 2, y_1 = 3, x_2 = 4$ and $y_2 = 5$

$\therefore OP \times OQ \times \cos \angle POQ = 8 + 15 = 23$

347 **(d)**

$\frac{\sin 3B}{\sin B} = \frac{3 \sin B - 4 \sin^3 B}{\sin B} = 3 - 4 \sin^2 B$

$= 3 - 4(1 - \cos^2 B)$

$= -1 + \frac{4(a^2 + c^2 - b^2)^2}{4(ac)^2}$

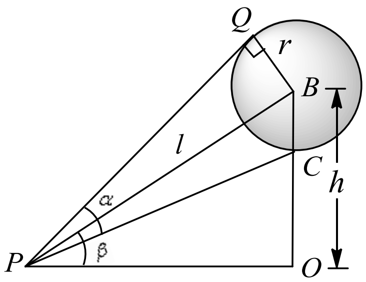
$= -1 + \frac{\left(\frac{a^2+c^2}{2}\right)^2}{(ac)^2}$

$= -1 + \frac{(a^2 + c^2)^2}{4(ac)^2} \quad [\because 2b^2 = a^2 + c^2 \text{ (given)}]$

$= \frac{(a^2 + c^2)^2 - 4a^2c^2}{4(ac)^2} = \left(\frac{c^2 - a^2}{2ac}\right)^2$

348 **(a)**

Since, $\angle QPC = \alpha$



$\therefore \; \angle QPB = \angle BPC = \frac{\alpha}{2}$

In $\Delta \, PQB, \sin \frac{\alpha}{2} = \frac{r}{l}$

$\Rightarrow \;\; l = r \sec \frac{\alpha}{2} \quad ...(i)$

and in $\Delta \, POB, \sin \beta = \frac{h}{l}$

$\Rightarrow \;\; h = l \sin \beta$

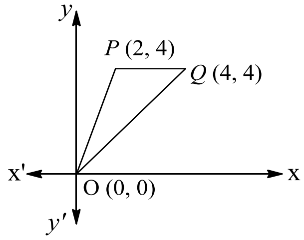
$\Rightarrow \;\; h = r \operatorname{cosec} \frac{\alpha}{2} \sin \beta \quad$ [from Eq.(i)]

349 **(d)**

$\cos OPQ = \frac{OP^2 + PQ^2 - OQ^2}{2OP.PQ}$

$= \frac{(4^2 + 2^2) + 2^2 - (4^2 + 4^2)}{2\sqrt{4^2 + 2^2}.\sqrt{2^2}}$

$= \frac{24 - 32}{42\sqrt{5}}$



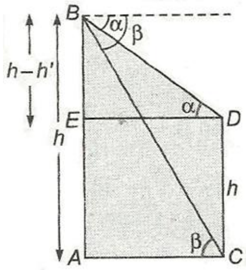
$\Rightarrow \; \angle \, OPQ = \cos^{-1}\left(-\frac{1}{\sqrt{5}}\right)$

350 **(a)**

Let AB be a hill whose height is h metres and CD be a pillar of height h' meters.

In $\Delta EDB, \tan \alpha = \frac{h-h'}{ED}$...(i)

and in $\Delta ACB, \tan \beta = \frac{h}{AC} = \frac{h}{ED}$ ... (ii)



$\therefore$ From Eqs. (i)and (ii),

$\frac{\tan \alpha}{\tan \beta} = \frac{h - h'}{h}$

$\Rightarrow h.\frac{\tan \alpha}{\tan \beta} = h - h' \Rightarrow h' = \frac{h(\tan \beta - \tan \alpha)}{\tan \beta}$

351 **(c)**

We have,

Area of $\Delta \, ABC = 3$ Area of $\Delta \, GAB$

Now,

Area of $\Delta \, GAB = \frac{1}{2}$

$\times$ Absolute value of $\begin{vmatrix} 1 & 4 & 1 \\ 4 & -3 & 1 \\ -9 & 7 & 1 \end{vmatrix}$

$\Rightarrow$ Area of $\Delta GAB = \frac{1}{2}|-10 - 52 + 1| = \frac{61}{2}$ sq. units

Hence, Area of $\Delta ABC = \frac{183}{2}$ sq. units

352 **(a)**

Using sine rule,

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\Rightarrow \; \frac{2\sqrt{2}}{\sin 45°} = \frac{6}{\sin B}$

$$\Rightarrow \sin B = \frac{6}{2\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

$$= \frac{6}{4} = \frac{3}{2} > 1$$

Which is not possible

Hence, no triangle is possible

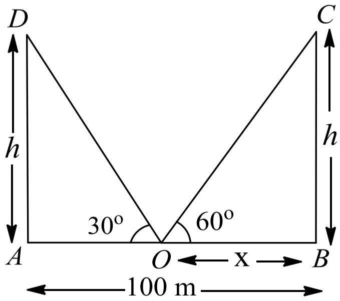
353 **(b)**

Let the height of pole $AD = BC = h$

In $\Delta\, OBC, \tan 60° = \frac{h}{x}$

$$\Rightarrow \quad x = \frac{h}{\sqrt{3}}$$

In $\Delta\, AOD, \tan 30° = \frac{h}{100-x}$



$$\Rightarrow \quad h = (100 - x)\frac{1}{\sqrt{3}}$$

$$= \left(100 - \frac{h}{\sqrt{3}}\right)\frac{1}{\sqrt{3}}$$

$\Rightarrow \quad 3h = 100\sqrt{3} - h$ [from Eq.(i)]

$$\Rightarrow \quad h = \frac{100\sqrt{3}}{4}$$

$$\Rightarrow \quad h = 25\sqrt{3}\text{ m}$$

354 **(b)**

The vertices of a triangle are $(0,2), \left(\frac{1}{2}, 1\right), (1,1)$

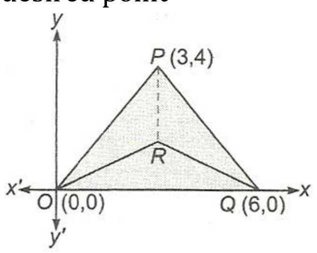
$$\therefore \text{ Area of triangle} = \frac{1}{2}\begin{vmatrix} 0 & 2 & 1 \\ \frac{1}{2} & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[-2\left(\frac{1}{2} - 1\right) + 1\left(\frac{1}{2} - 1\right)\right]$$

$= \frac{1}{4}$ sq unit

356 **(c)**

Since, triangle is isosceles, hence centriod is the desired point



$\therefore$ Coordinates of $R\left(3, \frac{4}{3}\right)$

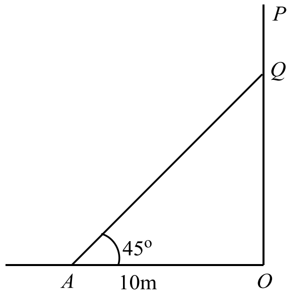
357 **(c)**

Let $AQ(= PQ)$ be the broken part of the tree OP.

It is given that $OA = 10$ m and $\angle OAQ = 45°$

In $\Delta\, OAQ$, we have

$$\tan 45° = \frac{OQ}{OA} \Rightarrow OQ = 10°$$



Also, $AQ^2 = OA^2 + OQ^2$

$$\Rightarrow AQ = \sqrt{100 + 100} = 10\sqrt{2}$$

$$\therefore OP = OQ + PQ = OQ + AQ = 10 + 10\sqrt{2}$$

$$= 10(\sqrt{2} + 1)\text{ mts}$$

358 **(c)**

Let the new coordinates be $P(x', y')$ after shifting origin to $P(x', y')ie, x = x' + h$ and $y = y' + k$

$$\therefore \quad (x' + h)^2 + (y' + k)^2 - 4(x' + h) + 6(y' + k) - 7 = 0$$

$$\Rightarrow (x')^2 + (y')^2 + 2(h - 2)x' + 2(k + 3)y' + (h^2 + k^2 - 4h + 6k - 7) = 0$$

According to the question,

$h - 2 = 0$ and $k + 3 = 0$

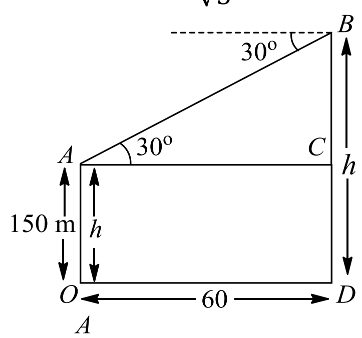
$\Rightarrow \quad (h, k) = (2, -3)$

359 **(d)**

In $\Delta\, ABC, \tan 30° = \frac{BC}{AC}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h - 150}{60}$$

$$\Rightarrow \quad h - 150 = \frac{60}{\sqrt{3}}$$



$$\Rightarrow \quad h = (150 + 20\sqrt{3})\text{m}$$

360 **(d)**

The sum of the distance

$$= \frac{\frac{a}{a} + \frac{b}{a} - 1}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}} + \frac{\frac{a}{b} + \frac{b}{a} - 1}{\sqrt{\left(\frac{1}{b}\right)^2 + \left(\frac{1}{a}\right)^2}}$$

$$= \left(\frac{a}{b} + \frac{b}{a}\right)\frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}$$

$= \sqrt{a^2 + b^2}$

361 **(b)**

Given, $\frac{1}{b+c} + \frac{1}{c+a} = \frac{3}{a+b+c}$

$\Rightarrow\ 1 + \frac{b}{a+c} + 1 + \frac{a}{b+c} = 3$

$\Rightarrow\ b(b+c) + a(a+c) = (a+c)(b+c)$

$\Rightarrow\ a^2 + b^2 - c^2 = ab$

$\therefore\ \cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{ab}{2ab} \Rightarrow \angle C = 60°$

362 **(b)**

Given, $\frac{\sin A}{1/4} = \frac{\sin B}{1/4} = \frac{\sin C}{1/3}$

$\therefore\ \frac{\sin A}{3} = \frac{\sin B}{3} = \frac{\sin C}{4}$

Here, $a = 3k, b = 3k$ and $c = 4k$, where k is a proportionality constant

$\therefore\ \cos C = \frac{9k^2 + 9k^2 - 16k^2}{2 \times 3k \times 3k} = \frac{1}{9}$

363 **(b)**

$(b+c)(bc)\cos A + (a+c)(ac)\cos B + (a+b)(ab)\cos C$

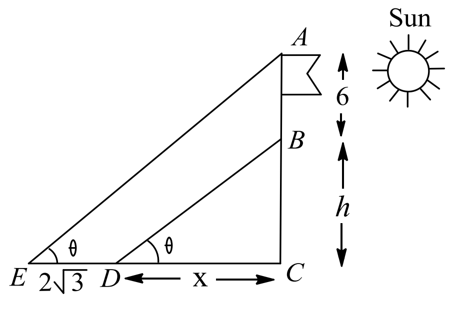
$= (b+c)(bc)\left(\frac{b^2+c^2-a^2}{2bc}\right) + (a+c)(ac) \times \left(\frac{a^2+c^2-b^2}{2ca}\right) + (a+b)(ab)\left(\frac{a^2+b^2-c^2}{2ab}\right)$

$= \frac{1}{2}\{(b+c)(b^2+c^2-a^2) + (a+c) \times (a^2+c^2-b^2) + (a+b)(a^2 + b^2 - c^2)\}$

$= \frac{1}{2}\{2a^3 + 2b^3 + 2c^3\} = a^3 + b^3 + c^3$

364 **(a)**

In $\Delta\, DCB, \tan\theta = \frac{h}{x}$ ...(i)



and in $\Delta\, ECA, \tan\theta = \frac{h+6}{2\sqrt{3}+x}$

$\Rightarrow \frac{h}{x} = \frac{h+6}{2\sqrt{3}+x}$ [from Eq.(i)]

$\Rightarrow\ 2\sqrt{3}\,h + hx = hx + 6x$

$\Rightarrow\ 2\sqrt{3}\,h = 6x$

$\Rightarrow\ h = \frac{6x}{2\sqrt{3}}$

From Eq. (i), we get

$\tan\theta = \frac{6x}{2\sqrt{3}x} = \sqrt{3}$

$\Rightarrow\ \theta = 60°$

365 **(b)**

Since, angles A, B, C are in AP

$\therefore\ 2B = A + C$

$\because\ A + B + C = 180°$

$\Rightarrow\ B = 60°$

Now, $\cos B = \frac{a^2+c^2-b^2}{2ac}$

$\Rightarrow\ \frac{1}{2} = \frac{a^2+c^2-b^2}{2ac}$

$\Rightarrow\ a^2 + c^2 - b^2 = ac$

$\Rightarrow\ b^2 = a^2 + c^2 - ac$

366 **(a)**

Let $a = 7$ cm, $b = 4\sqrt{3}$ cm and $c = \sqrt{13}$ cm

Here, we see that the smallest side is c

Therefore, the smallest angle will be C

$\therefore\ \cos C = \frac{(7)^2 + \left(4\sqrt{3}\right)^2 - \left(\sqrt{13}\right)^2}{2 \times 7 \times 4\sqrt{3}} = \frac{\sqrt{3}}{2}$

$\Rightarrow\ \angle C = \frac{\pi}{6}$

367 **(c)**

On solving the given straight lines, we get vertices of ΔABC

Which are $A(2,-2), B(1,1), C(-2,2)$

$\therefore\ AB = \sqrt{(1-2)^2 + (1+2)^2} = \sqrt{10}$

$BC = \sqrt{(-2-1)^2 + (2-1)^2} = \sqrt{10}$

And $CA = \sqrt{(2+2)^2 + (-2-2)^2} = \sqrt{32}$

Here, $AB = Bc$

$\Rightarrow$ Triangle is isosceles triangle

369 **(d)**

If $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$, then the points are collinear

370 **(a)**

Let $\angle A = 45°$ and $\angle B = 60°$

$\therefore\ \angle C = 75°$

Let smallest and greatest sides are a and c

$\therefore\ a : c = \sin 45° : \sin 75°$

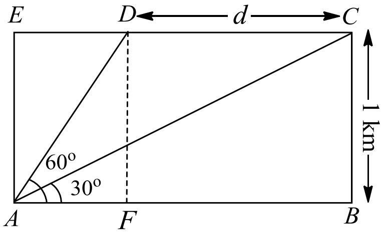
$= \frac{1}{\sqrt{2}} : \frac{\sqrt{3}+1}{2\sqrt{2}} = 2 : \sqrt{3}+1$

$= 2\left(\sqrt{3}-1\right) : \left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)$

$= \left(\sqrt{3}-1\right) : 1$

371 **(b)**

In $\Delta\, ADF, \tan 60° = \frac{1}{AF}$



$\Rightarrow\ AF = \cot 60° = \frac{1}{\sqrt{3}}$ ...(i)

In $\Delta ABC, \tan 30° = \frac{1}{AB}$

$\Rightarrow\ AB = \cot 30°$

$\Rightarrow\ AF + FB = \sqrt{3}$

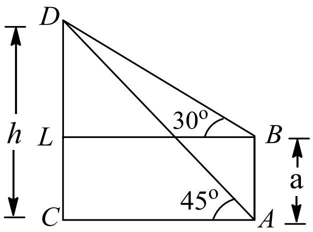
$\Rightarrow\ d = \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$ [using Eq.(i)]

$\therefore$ Speed of aeroplane $= \dfrac{\text{distance from } D \text{ to } C}{\text{time taken}}$

$= \dfrac{\frac{2}{\sqrt{3}}}{10} \times 60 \times 60 = 240\sqrt{3}$ km/h

372 **(a)**

Let CD is a tower of height h. AB is building of height a



In $\Delta BLD, \tan 30° = \frac{h-a}{LB}$

$\therefore\ LB = \dfrac{(h-a)}{\tan 30°} = \sqrt{3}(h-a)$

In $\Delta ACD, \tan 45° = \frac{h}{LB}$

or $h(\sqrt{3}-1) = \sqrt{3}\,a$

$\therefore\ h = \dfrac{\sqrt{3}a}{\sqrt{3}-1} = \dfrac{\sqrt{3}(\sqrt{3}+1)a}{2}$

$\therefore\ h = \left(\dfrac{3+\sqrt{3}}{2}\right)a$

373 **(d)**

$\dfrac{\sin 3B}{\sin B} = \dfrac{3\sin B - 4\sin^3 B}{\sin B}$

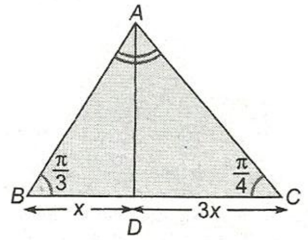
$= 3 - 4(1-\cos^2 B)$

$= -1 + 4\left(\dfrac{a^2+c^2-b^2}{2ac}\right)^2 = -1 + \dfrac{\left(\frac{a^2+c^2}{2}\right)^2}{(ac)^2}$

$= \dfrac{(a^2+c^2)^2 - 4a^2c^2}{4(ac)^2} = \left(\dfrac{c^2-a^2}{2ac}\right)^2$

375 **(b)**

Since, $\frac{BD}{DC} = \frac{1}{3}$



In ΔABD, by sine rule

$AD = \dfrac{\sin\frac{\pi}{3}}{\sin\angle BAD}.BD$ ...(ii)

In ΔADC, by sine rule

$AD = \dfrac{\sin\frac{\pi}{4}}{\sin\angle DAC}.DC$ ...(iii)

From Eqs. (ii) and (iii),

$\dfrac{\sin\angle BAD}{\sin\angle DAC} = \dfrac{\sin\frac{\pi}{3}}{\sin\frac{\pi}{4}}.\dfrac{BD}{DC} = \dfrac{\sqrt{3}}{2}.\sqrt{2}.\dfrac{1}{3} = \dfrac{1}{\sqrt{6}}$

376 **(b)**

Since, reflection of the orthocenter of $\Delta\, ABC$ in base BC will always lie on the circumcircle of the triangle ABC, therefore coordinate of a point lying on the circumcircle is

$\left(1 - \frac{1\times4}{2}, 1 - \frac{1\times4}{2}\right)$ *ie*, $(-1,-1)$ and coordinates of the circumcentre is $(2, 0)$

$\therefore$ Radius of the circumcentre of $\Delta\, ABC$

$= \sqrt{(2+1)^2 + (1)^2} = \sqrt{10}$

377 **(a)**

Given, $b + c = 2a, \angle A = 60°$

Since, $\angle A = 60°\ \ \angle B + \angle C = 120°$

Also, $b + c = 2a$ [given]

$\sin B + \sin C = 2\sin 60°$

$\Rightarrow \left[\because \dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c} = k\right]$

$\Rightarrow 2\sin\left(\dfrac{B+C}{2}\right)\cos\left(\dfrac{B-C}{2}\right) = \dfrac{2\sqrt{3}}{2}$

$\Rightarrow\ 2\sin 60°\cos\left(\dfrac{B-C}{2}\right) = \sqrt{3}\ \left[\because \sin 60° = \dfrac{\sqrt{3}}{2}\right]$

$\Rightarrow\ \cos\left(\dfrac{B-C}{2}\right) = 1\ \Rightarrow\ \dfrac{B-C}{2} = 0$

$\Rightarrow\ \angle B = \angle C$

Hence, triangle is an equilateral triangle

378 **(c)**

Let the vertices of the triangle are $A(-2,-6), B(-2,4)$ and $C(1,3)$

Now, $AB = \sqrt{(-2+2)^2 + (-6-4)^2} = 10$

$BC = \sqrt{(-2-1)^2 + (4-3)^2} = \sqrt{10}$

and $CA = \sqrt{(1+2)^2 + (3+6)^2} = \sqrt{90}$

Here, $AB^2 = BC^2 + CA^2$

$\therefore$ Triangle is right angled triangle at C

$\therefore\ \ \angle C = 90°$

So, orthocenter is $(1, 3)$

379 **(a)**

Area of $\Delta = \frac{1}{2}\left\|\begin{vmatrix} 0 & 0 & 1 \\ a\cos\theta & b\sin\theta & 1 \\ a\cos\theta & -b\sin\theta & 1 \end{vmatrix}\right\|$

$\Rightarrow \ \Delta = \dfrac{1}{2}|[1(-ab\sin\theta\cos\theta - ab\sin\theta\cos\theta)]|$

$= \dfrac{ab\sin 2\theta}{2}$

Since, maximum value of $\sin 2\theta$ is 1, when $\theta = \frac{\pi}{4}$

$\therefore \quad \Delta_{\max} = \dfrac{ab}{2}$

380 **(d)**

Let $O(x, y)$ be the circumcentre. Then, G divides $O'O$ in the ratio $2:1$

$\therefore \dfrac{2x+0}{2+1} = 2$ and $\dfrac{2y+1}{2+1} = 3 \Rightarrow x = 3$ and $y = 4$

Hence, the coordinates of O are $(3, 4)$

381 **(d)**

Perpendicular bisector of $A(1,3)$ and $B(-3,5)$ is

$2x - y + 6 = 0 \quad$ ...(i)

And perpendicular bisector of $A(1,3)$ and $C(5,-1)$ is

$x - y - 2 = 0 \quad$ ...(ii)

On solving Eqs. (i) and (ii), we get

$x = -8, y = -10$

$\therefore$ Coordinates of P are $(-8, -10)$

Thus, $PA = \sqrt{(1+8)^2 + (3+10)^2}$

$= \sqrt{81 + 169} = 5\sqrt{10}$

382 **(a)**

Given sides are $3x - 4y = 0, 5x + 12y = 0$ and$y - 15 = 0$. The vertices of a triangle are $A(0,0), B(20,15), C(-36,15)$

Now, $AB = c = \sqrt{(20-0)^2 + (15-0)^2}$

$= \sqrt{400 + 225}$

$= 25$

$BC = a = \sqrt{(20+36)^2 + (15-15)^2}$

$= 56$

$CA = b = \sqrt{(-36-0)^2 + (15-0)^2}$

$= \sqrt{1296 + 225} = 39$

$\therefore$ Incentre

$= \left(\dfrac{ax_1 + bx_2 + cx_3}{a+b+c}, \dfrac{ay_1 + by_2 + cy_2}{a+b+c}\right)$

$= \left(\dfrac{56\times 0 + 39\times 20 - 36\times 25}{56+39+25}, \dfrac{56\times 0 + 39\times 15 + 25\times 15}{56+39+25}\right)$

$= \left(\dfrac{-120}{120}, \dfrac{960}{120}\right)$

$= (-1, 8)$

383 **(d)**

The mid point of the line joining the points $(-10, 8)$ and $(-6, -12)$ is$\left(\frac{-10-6}{2}, \frac{8+12}{2}\right)$ *ie*, $(-8, 10)$. Let $(-8, 10)$ divides the line joining the points $(4, .-2)$ and $(-2, 4)$ in the ratio $m:n$

Then, $\frac{m(-2)+n(4)}{m+n} = -8$

$\Rightarrow \ -2m + 4n = -8m - 8n$

$\Rightarrow \ 6m = -12n \ \Rightarrow \dfrac{m}{n} = \dfrac{-2}{1}$

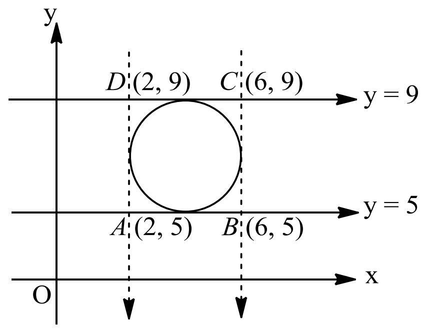
$\therefore$ Required ratio $2:1$ externally.

384 **(a)**

Given the circle is inscribed in square formed by the lines

$x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$

$\Rightarrow \quad x = 6$ and $x = 2$, $y = 5$ and $y = 9$



$ABCD$ clearly forms a square

$\therefore$ Centre of inscribed circle

=point of intersection of diagonals

=mid point of AC or BD

$= \left(\dfrac{2+6}{2}, \dfrac{5+9}{2}\right)$

$\Rightarrow$ Centre of inscribed circle $= (4, 7)$

385 **(a)**

Area of a circle $= \pi \times (\text{radius})^2$

$\therefore \quad A = \pi r^3, A_1 = \pi r_1^2, A_2 = \pi r_2^2, A_3 = \pi r_3^2$

$\therefore \dfrac{1}{\sqrt{A_1}} + \dfrac{1}{\sqrt{A_2}} + \dfrac{1}{\sqrt{A_3}}$

$= \dfrac{1}{r_1\sqrt{\pi}} + \dfrac{1}{r_2\sqrt{\pi}} + \dfrac{1}{r_3\sqrt{\pi}}$

$= \dfrac{1}{\sqrt{\pi}}\left(\dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3}\right)$

$= \dfrac{1}{\sqrt{\pi}}\left(\dfrac{s-a}{\Delta} + \dfrac{s-b}{\Delta} + \dfrac{s-c}{\Delta}\right)$

$= \dfrac{1}{\sqrt{\pi}}\left(\dfrac{3s - (a+b+c)}{\Delta}\right)$

$= \dfrac{1}{\sqrt{\pi}} . \dfrac{3s - 2s}{\Delta} = \dfrac{1}{\sqrt{\pi}} . \dfrac{s}{\Delta}$

$= \dfrac{1}{r\sqrt{\pi}} = \dfrac{1}{\sqrt{A}}$

386 **(b)**

We have,

$x^2 + 4xy + y^2 = aX^2 + bY^2$

$\Rightarrow (X\cos\theta + Y\sin\theta)^2 + 14(X\cos\theta + Y\sin\theta)(X\sin\theta - Y\cos\theta) + (X\sin\theta - Y\cos\theta)^2 = aX^2 + bY^2$

$\Rightarrow a = 1 + 4\sin\theta\cos\theta, b = 1 - 4\sin\theta\cos\theta$ and $\sin^2\theta - \cos^2\theta = 0$

$\Rightarrow a = 1 + 4\sin\theta\cos\theta, b = 1 - 4\sin\theta\cos\theta$ and $\theta = \frac{\pi}{4}$

$\Rightarrow a = 3, b = -1$

387 **(d)**

Let $S(x, y)$, then

$(x+1)^2 + y^2 + (x-2)^2 + y^2 = 2[(x-1)^2 + y^2]$

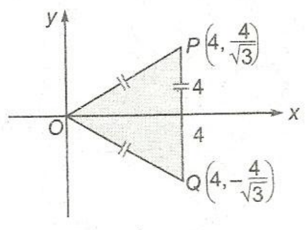
$\Rightarrow\ 2x + 1 + 4 - 4x = -4x + 2$

$\Rightarrow\ x = \frac{-3}{2}$

Hence, it is a straight line parallel to y-axis

388 **(b)**

Given lines are



$x^2 - 3y^2 = 0$

$x^2 - 3y^2 = 0$ ...(i)

And $x = 4$ ...(ii)

Put $x = 4$ from Eq. (ii) in Eq. (i), we get

$16 - 3y^2 = 0$

$\Rightarrow\ y^2 = \frac{16}{3}\ \Rightarrow\ y = \pm\frac{4}{\sqrt{3}}$

The vertices of the triangle are

$O(0,0), P\left(4, \frac{4}{\sqrt{3}}\right), Q\left(4, -\frac{4}{\sqrt{3}}\right)$

$$OP = \sqrt{4^2 + \left(\frac{4}{\sqrt{3}}\right)^2} = \frac{8}{\sqrt{3}}$$

$$OQ = \sqrt{4^2 + \left(\frac{4}{\sqrt{3}}\right)^2} = \frac{8}{\sqrt{3}}$$

$$PQ = \sqrt{\left(\frac{4}{\sqrt{3}} + \frac{4}{\sqrt{3}}\right)^2} = \frac{8}{\sqrt{3}}$$

$\Rightarrow\ OP = QP = PQ = \frac{8}{\sqrt{3}}$

$\therefore\ \Delta OPQ$ is an equilateral triangle

389 **(c)**

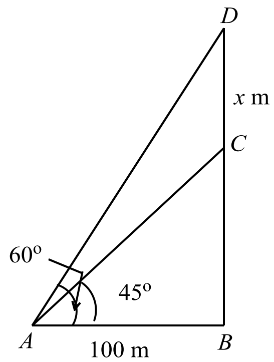
Let BC be the incomplete and BD be the complete pillar. In $\Delta^s ABC$ and ABD, we have

$\tan 45° = \frac{BC}{AB}$ and $\tan 60° = \frac{BD}{AB}$

$\Rightarrow BC = 100$ m and $BD = 100\sqrt{3}$

$\Rightarrow BC + CD = 100\sqrt{3}$

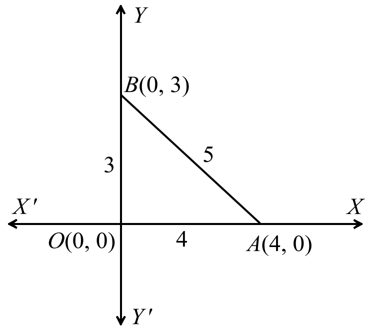
$\Rightarrow 100 + x = 100\sqrt{3} \Rightarrow x = 100(\sqrt{3} - 1)$ m



390 **(b)**

The coordinates of the vertices of $\Delta\ OAB$ are $O(0,0), A(4,0)$ and $B(0,3)$

$\therefore OA = 4, OB = 3$ and $AB = 5$



Hence, the coordinates of the excentre opposite to the vertex O are

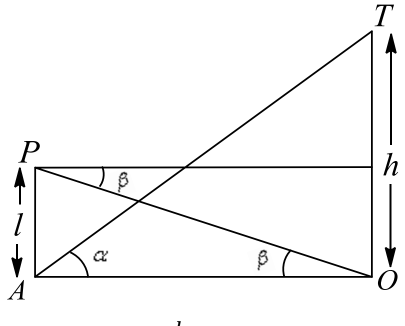
$$\left(\frac{-5\times0+3\times4+4\times0}{-5+3+4}, \frac{-5\times0+3\times0+4\times3}{-5+3+4}\right) = (6,6)$$

391 **(b)**

In $\Delta\ AOT, \tan\alpha = \frac{h}{OA}$

$\Rightarrow\ OA = h\cot\alpha$

In $\Delta\ AOP, \tan\beta = \frac{l}{AO}$



$\Rightarrow\ \tan\beta = \frac{l}{h\cot\alpha}$ [from Eq.(i)]

$\Rightarrow\ h = l\tan\alpha\cot\beta$

392 **(b)**

6. $b\cos^2\frac{C}{2}+c\cos^2\frac{B}{2}=b.\frac{s(s-c)}{ab}+c.\frac{s(s-b)}{ac}$

$$=\frac{s}{a}[2s-(b+c)]=s$$

Hence, statement I is true

7. Let $\cot\frac{A}{2}=\frac{b+c}{2}$

$$\Rightarrow \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}}=\frac{\sin B+\sin C}{\sin A}$$

$$\Rightarrow \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}}=\frac{2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)}{2\sin\frac{A}{2}\cos\frac{A}{2}}$$

$$\Rightarrow \cos\frac{A}{2}=\cos\left(\frac{B-C}{2}\right)$$

$$\Rightarrow \frac{A}{2}=\frac{B-C}{2} \quad \Rightarrow \quad A+C=B$$

But $A+B+C=\pi$, therefore $B=\frac{\pi}{2}$

But given statement is

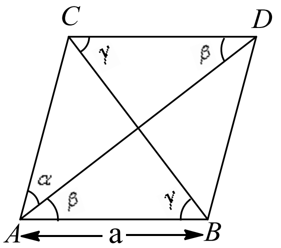
$$\cot\frac{A}{2}=\frac{b+c}{2} \Rightarrow \angle B=90°$$

Hence, statement II is not true

393 **(b)**

$\because \angle CDA=\beta, \angle DCB=\gamma$,

and $\angle ACB=\pi-(\alpha+\beta+\gamma)$



In $\Delta\, ABC$, on applying sine rule, we get

$$\frac{AB}{\sin[\pi-(\alpha+\beta+\gamma)]}=\frac{CA}{\sin\gamma}$$

$$\Rightarrow \; CA=\frac{a\sin\gamma}{\sin(\alpha+\beta+\gamma)}$$

Now, in $\Delta\, CAD$, on applying sine rule

$$\frac{CA}{\sin\beta}=\frac{CD}{\sin\alpha}$$

$$\Rightarrow \; CD=\frac{CA\cdot\sin\alpha}{\sin\beta}$$

$$=\frac{a\sin\gamma\sin\alpha}{\sin\beta\sin(\alpha+\beta+\gamma)}$$

394 **(b)**

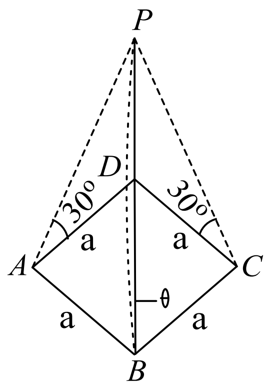
Let PD be a pole

In $\Delta\, DAP$, $\tan 30°=\frac{DP}{AD}$

$$\Rightarrow \; DP=\frac{a}{\sqrt{3}}$$

In $\Delta\, PDB$, $\tan\theta=\frac{DP}{BD}$

$$\Rightarrow \; \tan\theta=\frac{a/\sqrt{3}}{\sqrt{2}a}=\frac{1}{\sqrt{6}}$$



395 **(b)**

Since, $\frac{a}{b}=\frac{2}{\sqrt{3}-1}$ and $A=3B$

By sine rule, $\frac{\sin A}{\sin B}=\frac{a}{b}$

$$\Rightarrow \frac{\sin 3B}{\sin B}=\frac{2}{\sqrt{3}-1}$$

$$\Rightarrow \frac{3\sin B-4\sin^3 B}{\sin B}=\frac{2}{\sqrt{3}-1}$$

$$\Rightarrow \; 3-4\sin^2 B=\frac{2(\sqrt{3}+1)}{2}$$

$$\Rightarrow \frac{2-\sqrt{3}}{4}=\sin^2 B$$

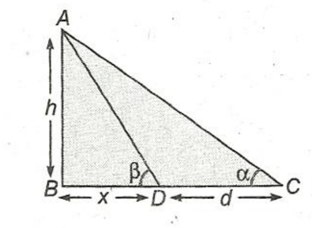
$$\Rightarrow \; \sin B=\frac{\sqrt{3}-1}{2\sqrt{2}}$$

$\Rightarrow \angle B=15°, \angle A=45°$ and $\angle C=120°$

396 **(d)**

In ΔABC, $\tan\alpha=\frac{h}{x+d}$

$\Rightarrow x+d=h\cot\alpha$



and in ΔABD,

$$\tan\beta=\frac{h}{x}\Rightarrow x=h\cot\beta$$

On putting this value in Eq. (i), we get

$h\cot\beta+d=h\cot\alpha$

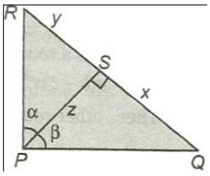
$$\Rightarrow \; h=\frac{d}{\cot\alpha-\cot\beta}$$

397 **(b)**

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$= \frac{\frac{y}{z} + \frac{x}{z}}{1 - \frac{y}{z} \times \frac{x}{z}}$$

$$= \frac{\frac{3}{6} + \frac{2}{6}}{1 - \frac{3}{6} \times \frac{2}{6}} = 1$$



$\Rightarrow\ \alpha + \beta = \angle QPR = \frac{\pi}{4}$

398 **(a)**

Let (x, y) be the original coordinates of P. Then,
$x = 1 + \cos\theta$ and $y = 1 + \cos\phi$

$\Rightarrow x = 2\cos^2\frac{\theta}{2}, y = 2\cos^2\frac{\phi}{2}$

399 **(b)**

Given, $r_1 = 2r_2 = 3r_3$

$\Rightarrow \frac{\Delta}{(s-a)} = \frac{2\Delta}{(s-b)} = \frac{3\Delta}{(s-c)}$

$\Rightarrow\ (s-b) = 2(s-a)$ and $(s-c) = 3(s-a)$

$\Rightarrow\ \frac{a+b+c}{2} - b = 2\left(\frac{a+b+c}{2} - a\right)$

and $\frac{a+b+c}{2} - c = 3\left(\frac{a+b+c}{2} - a\right)$

$\Rightarrow\ a + c - b = 2(-a + b + c)$

and $a + b - c = 3(-a + b + c)$

$\Rightarrow\ 3a = 3b + c$ and $2a = b + 2c$

$\Rightarrow\ 4a = 5b\ \Rightarrow\ \frac{a}{b} = \frac{5}{4}$

400 **(b)**

Since a, b and x are in AP

$\Rightarrow\ 2b = a + c$

$\Rightarrow\ 3b = 2s\ \Rightarrow\ s = \frac{3b}{2}$

Now, $\frac{\sin\frac{A}{2}\sin\frac{C}{2}}{\sin\frac{B}{2}} = \sqrt{\frac{ac(s-b)(s-c)(s-b)(s-a)}{(s-a)(s-c)\times bc\times ab}}$

$= \frac{s-b}{b} = \frac{\frac{3b}{2} - b}{b} = \frac{1}{2}$

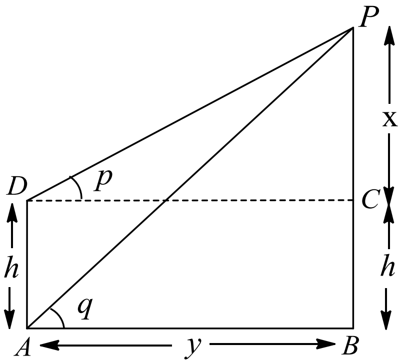
401 **(b)**

Let AD be the building of height h and BP be the hill. Then,

$\tan q = \frac{h+x}{y}$ ...(i)

and $\tan p = \frac{x}{y}$

$\Rightarrow\ y = x\cot p$ ...(ii)



From Eq. (i) and (ii), we get

$\tan q = \frac{h+x}{x\cot p}$

$\Rightarrow\ x\cot p = (h+x)\cot q$

$\Rightarrow\ x = \frac{h\cot q}{\cot p - \cot q}$

$\Rightarrow\ h + x = \frac{h\cot q}{\cot p - \cot q} + h$

$\therefore$ Height of hill $= \frac{h\cot p}{\cot p - \cot q}$

402 **(b)**

$\cos^2 A = 1 - \sin^2 A = 1 - \left(\frac{3}{5}\right)^2 = \frac{16}{25}$

$\Rightarrow\ \cos A = \frac{4}{5}$

$\Rightarrow\ \frac{(20)^2 + (21)^2 - a^2}{2.20.21} = \frac{4}{5}$

$\Rightarrow\ a^2 = 169\ \Rightarrow a = 13$

403 **(c)**

Given, $\angle C = 60°, a = 2, b = 4$

$\therefore\ \cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$\Rightarrow\ 2ab\cos 60° = a^2 + b^2 - c^2$

$\Rightarrow\ ab = a^2 + b^2 - c^2$

$\Rightarrow\ 8 = 4 + 16 - c^2\ \Rightarrow\ c^2 = 12$

$\Rightarrow\ c = \sqrt{12} = 2\sqrt{3}$

We have, $\sin A = \frac{a\sin C}{c} = \frac{2\frac{\sqrt{3}}{2}}{2\sqrt{3}} = \frac{1}{2}$

$\Rightarrow\ A = 30°$

and $\sin B = \frac{b\sin C}{c} = \frac{4\frac{\sqrt{3}}{2}}{2\sqrt{3}} = 1 \Rightarrow B = 90°$

405 **(b)**

$\frac{b - c\cos A}{c - b\cos A} = \frac{b - \frac{b^2+c^2-a^2}{2b}}{c - \frac{b^2+c^2-a^2}{2c}}$

$= \left(\frac{b^2 + a^2 - c^2}{c^2 + a^2 - b^2}\right) = \frac{c}{b}$

$= \frac{b^2 + a^2 - c^2}{2ab} \cdot \frac{2ac}{c^2 + a^2 - b^2} = \frac{\cos C}{\cos B}$

406 **(b)**

We have, $\frac{s(s-a)}{bc} - \frac{(s-b)(s-c)}{bc}$

$= \cos^2\frac{A}{2} - \sin^2\frac{A}{2}$

$= \cos\frac{2A}{2} = \cos A$

407 **(b)**

Let the third vertex of the triangle be (x, y), then

$\frac{x+4+(-2)}{3} = 2 \Rightarrow x = 4$

and $\frac{y+8+6}{3} = 7 \Rightarrow y = 7$

$\therefore$ The coordinates of the third vertex are $(4, 7)$

408 **(b)**

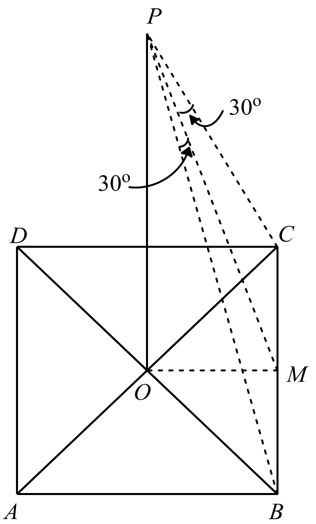
Let $ABCD$ be a square of each side of length a. It is given that $\angle BPC = 60°$. Let M be the midpoint of BC. Then $\angle BPM = \angle CPM = 30°$

In $\Delta\, BPM$, we have

$\tan\angle BPM = \frac{BM}{PM}$

$\Rightarrow PM = \sqrt{3}\, BM = \frac{\sqrt{3}}{2}a$

In $\Delta\, OPM$, we have



$PM^2 = OM^2 + OP^2 \Rightarrow \frac{3\,a^2}{4} = \frac{a^2}{4} + h^2 \Rightarrow a^2 = 2\,h^2$

409 **(c)**

$(a-b)^2\cos^2\frac{C}{2} + (a+b)^2\sin^2\frac{C}{2}$

$= (a^2+b^2-2ab)\cos^2\frac{C}{2} + (a^2+b^2+2ab)\sin^2\frac{C}{2}$

$= a^2+b^2+2ab\left(\sin^2\frac{C}{2} - \cos^2\frac{C}{2}\right)$

$= a^2+b^2-2ab\cos C$

$= a^2+b^2-(a^2+b^2-c^2)$

$= c^2 \quad \left(\because \cos C = \frac{a^2+b^2-c^2}{2ab}\right)$

410 **(c)**

Given, $\Delta = a^2-(b-c)^2$

$= (a+b-c)(a-b+c)$

$= 2(s-c)\cdot 2(s-b)$

$\sqrt{s(s-a)(s-b)(s-c)} = 4(s-b)(s-c)$

$\Rightarrow \frac{1}{4} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \tan\frac{A}{2}$

$\therefore \tan\frac{A}{2} = \frac{1}{4}$

411 **(a)**

If O' is the orthocentre of a $\Delta\, ABC$, then the points O', A, B, C are such that each point is the orthocentre of the triangle formed by the remaining three points. So, the coordinates of the orthocentre of $\Delta\, O'AB$ are $(0, 0)$

412 **(c)**

$\cot B + \cot C - \cot A$

$= \frac{\cos B}{\sin B} + \frac{\cos C}{\sin C} - \cot A$

$= \frac{\sin C\cos B + \cos C\sin B}{\sin B\sin C} - \cot A$

$= \frac{\sin(B+C)}{\sin B\sin C} - \frac{\cos A}{\sin A}$

$= \frac{\sin^2 A - \sin B\sin C\cos A}{\sin A\sin B\sin C} \quad (\because A+B+C=\pi)$

$= \frac{a^2 - bc\cos A}{k(abc)}$

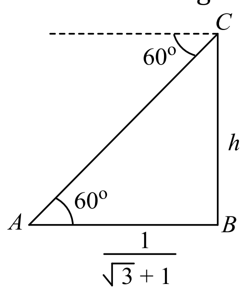
$= \frac{a^2 - bc\frac{(b^2+c^2-a^2)}{2bc}}{(abc)k}$

$= \frac{2a^2-(3a^2-a^2)}{2(abc)k} \quad (\because\ b^2+c^2+= 3a^2\ \text{given})$

$= \frac{(a^2-a^2)}{abck} = 0$

413 **(c)**

Let h be the height of the tower.



$= \frac{h}{1}$

$\Rightarrow h = \frac{\sqrt{3}(\sqrt{3}-1)}{3-1}$

$= \frac{3-\sqrt{3}}{2}$ m

414 **(d)**

$\because\ \cos A = \frac{b^2+c^2-a^2}{2bc}$

$$= \frac{(8)^2 + (10)^2 - (6)^2}{2 \times 8 \times 10} = \frac{4}{5}$$

$$\Rightarrow \sin A = \frac{3}{5}$$

$$\therefore \sin 2A = 2 \sin A \cos A$$

$$= 2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$$

415 **(d)**

$$\frac{1 + \cos(A + B) \cos C}{1 + \cos(A - C) \cos B}$$

$$= \frac{1 + \cos[\pi - (A + B)] \cos(A - B)}{1 + \cos[\pi - (A + C)] \cos(A - C)}$$

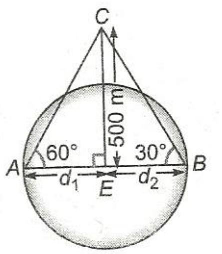
$$= \frac{1 - \cos(A + B) \cos(A - B)}{1 - \cos(A - C) \cos(A + C)}$$

$$= \frac{1 - \cos^2 A + \sin^2 B}{1 - \cos^2 A + \sin^2 C} = \frac{\sin^2 A + \sin^2 B}{\sin^2 A + \sin^2 C}$$

$\therefore \frac{1+\cos(A-B) \cos C}{1+\cos(A-C) \cos B} = \frac{a^2+b^2}{a^2+c^2}$ [by sine rule]

416 **(a)**

InΔAEC, $\tan 60° = \frac{500}{d_1} \Rightarrow d_1 = \frac{500}{\sqrt{3}} m$ ...(i)



and in ΔBEC, $\tan 30° = \frac{500}{d_2}$

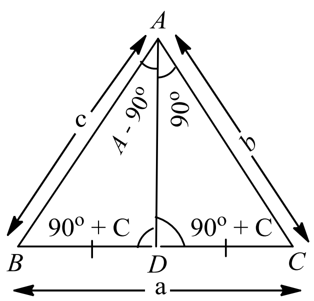
$\Rightarrow d_2 = 500\sqrt{3}$ m ... (ii)

$\therefore$ Required diameter,

$$AB = d_1 + d_2 = \frac{500}{\sqrt{3}} + 500\sqrt{3} = \frac{2000}{\sqrt{3}} \text{m}$$

417 **(d)**

Using sine rule in Δ ACD,



$$\frac{\sin(90° - C)}{b} = \frac{\sin 90°}{\frac{a}{c}} \Rightarrow \cos C = \frac{2b}{a}$$

And in ΔABD,

$$\frac{\sin(A - 90°)}{\frac{a}{2}} = \frac{\sin(90° + C)}{c}$$

$$\Rightarrow -\frac{\cos A}{\frac{a}{2}} = \frac{\cos C}{c} \Rightarrow \cos A = -\frac{b}{c}$$

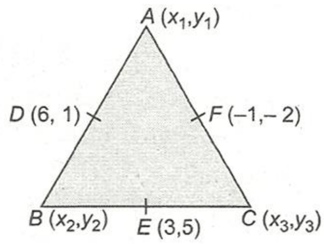
$$\Rightarrow \frac{b^2 + c^2 - a^2}{2bc} = -\frac{b}{c} \Rightarrow c^2 - a^2 = 3b^2$$

$$\therefore \cos A \cos C = -\frac{2b^2}{ac} = \frac{2}{3ac}(c^2 - a^2)$$

418 **(a)**

Let the vertices of triangle of A, B, C are $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ respectively

Given mid points of the sides AB, BC and CA of ΔABC are $D(6, 1), E(3, 5)$ and $F(-1, -2)$ respectively



$$\therefore \frac{x_1 + x_2}{2} = 6, \frac{y_1 + y_2}{2} = 1$$

$$\Rightarrow \left.\begin{array}{l} x_1 + x_2 = 12, \\ y_1 + y_2 = 2 \end{array}\right\} \quad \text{...(i)}$$

Similarly, $\left.\begin{array}{l} x_2 + x_3 = 6, \\ y_2 + y_3 = 10 \end{array}\right\}$ ...(ii)

And $\left.\begin{array}{l} x_1 + x_3 = -2, \\ y_1 + y_3 = -4 \end{array}\right\}$ ...(iii)

On solving Eqs. (i), (ii) and (iii), we get

$x_1 = 2, \quad x_2 = 10, \quad x_3 = -4$

And $y_1 = -6, y_2 = 8, y_3 = 2$

Now, the vertex opposite to D is C ie, $(-4, 2)$

419 **(a)**

Let $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of the triangle ABC and let $P(h, k)$ be any point on the locus,

Then, $PA^2 + PB^2 + PC^2 = c$ (constant)

$$\Rightarrow \sum_{i=1}^{3} [(h - x_i)^2 + (k - y_i)^2] = c$$

$$\Rightarrow h^2 + k^2 - \frac{2h}{3}(x_1 + x_2 + x_3) - \frac{2k}{3}(y_1 + y_2 + y_3) + \sum_{i=1}^{3} (x_i^2 + y_i^2) - c = 0$$

So, locus of (h, k) is

$$x^2 + y^2 - \frac{2x}{3}(x_1 + x_2 + x_3) - \frac{2y}{3}(y_1 + y_2 + y_3) + \lambda = 0$$

Where $\lambda = \sum_{i=1}^{3} (x_i^2 + y_i^2) - c = 0$ (constant)

Clearly, the locus of a point is a circle with, centre at

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

420 **(c)**

Since, $B = \frac{A+C}{2}$

$\Rightarrow \; B = 90° - \dfrac{B}{2} \; \Rightarrow \; B = 60°$ $[\because A + B + C = 180°]$

$\therefore \frac{\sin 60°}{\sqrt{3}} = \frac{\sin C}{\sqrt{2}}$ [by sine rule]

$\Rightarrow \; \sin C = \dfrac{1}{\sqrt{2}} \; \Rightarrow \; C = 45°$

$\therefore \; \angle A = 180° - (60° + 45°) = 75°$

421 **(b)**

Since, $A(0,1), B(0,-1)$ and $C(x,0)$ are the vertices of an equilateralΔABC.

$\therefore \quad AB = BC$

$\Rightarrow \quad \sqrt{0+4} = \sqrt{x^2+1}$

$\Rightarrow \quad x^2 = 3$

$\Rightarrow \quad x = \pm\sqrt{3}$

422 **(c)**

Let $a = \sin\alpha\,, b = \cos\alpha\,, c = \sqrt{1 + \sin\alpha\cos\alpha}$

Here, we see that the greatest side is c

$\therefore \; \cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

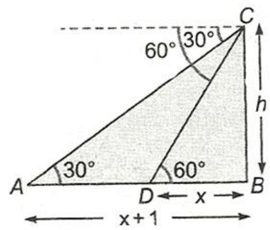
$\Rightarrow \; \cos C = \dfrac{\sin^2\alpha\cos^2\alpha - 1 - \sin\alpha\cos\alpha}{2\sin\alpha\cos\alpha}$

$\Rightarrow \; \cos C = -\dfrac{\sin\alpha\cos\alpha}{2\sin\alpha\cos\alpha}$

$\Rightarrow \; \cos C = -\dfrac{1}{2} \; \Rightarrow \; \angle C = 120°$

423 **(b)**

Let the distance of two consecutive stones are $x, x+1$



In ΔBCD,

$\tan 60° = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}}$ ...(i)

In $\Delta ABC, \tan 30° = \frac{h}{x+1}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+1} \Rightarrow \frac{h}{\sqrt{3}} + 1 = \sqrt{3}h$ [from Eq. (i)]

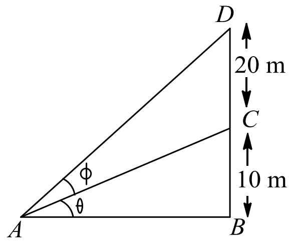
$\Rightarrow h = \dfrac{\sqrt{3}}{2}$km

424 **(a)**

Given, $\tan\phi = 0.5 = \frac{1}{2}$

In $\Delta ABC, \tan\theta = \frac{10}{AB}$

$\Rightarrow \; AB = \dfrac{10}{\tan\theta}$



In ΔABD,

$\tan(\theta + \phi) = \dfrac{30}{AB}$

$\Rightarrow \; \dfrac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi} = \dfrac{30\tan\theta}{10}$

$\Rightarrow \; \tan\theta + \dfrac{1}{2} = 3\tan\theta - \dfrac{3}{2}\tan^2\theta$

$\Rightarrow \; 3\tan^2\theta - 4\tan\theta + 1 = 0$

$\Rightarrow \; (3\tan\theta - 1)(\tan\theta - 1) = 0$

$\Rightarrow \; \tan\theta = \dfrac{1}{3}, 1$

$\therefore \; \tan\theta = 1$

426 **(b)**

Since, the sides of a triangle are in AP

$\therefore \; 2b = a + c \quad$ and $\quad c = 7$ cm

$\because \; \cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$\Rightarrow \; \cos 120° = -\dfrac{1}{2} = \dfrac{a^2 + \frac{a^2+c^2+2ac}{4} - c^2}{2a\frac{(a+c)}{2}}$

$\Rightarrow \; -2a(a+c) = 4a^2 + a^2 + c^2 + 2ac - 4c^2$

$\Rightarrow \; -2a^2 - 2ac = 5a^2 - 3c^2 + 2ac$

$\Rightarrow \; 7a^2 + 4ac - 3c^2 = 0$

$\Rightarrow \; 7a^2 + 28a - 147 = 0 \quad (\because \; c = 7)$

$\Rightarrow \; a^2 + 4a - 21 = 0$

$\Rightarrow \; (a+7)(a-3) = 0$

$\Rightarrow \; a = 3$ and $a \neq -7$

$\because \; b = 5$

Now, $s = \frac{a+b+c}{2} = \frac{3+5+7}{2} = \frac{15}{2}$

$\therefore$ Area of triangle$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{\dfrac{15}{2}\left(\dfrac{15}{2} - 3\right)\left(\dfrac{15}{2} - 5\right)\left(\dfrac{15}{2} - 7\right)}$

$= \sqrt{\dfrac{15}{2}\cdot\dfrac{9}{2}\cdot\dfrac{5}{2}\cdot\dfrac{1}{2}} = \dfrac{15}{4}\sqrt{3}\,\text{cm}^2$

427 **(a)**

Given, $\frac{b+c}{11} = \frac{a+c}{12} = \frac{a+b}{13} = k$ [say]

$\Rightarrow \; b + c = 11k, c + a = 12\,k, a + b = 13k$

$\Rightarrow \; 2(a+b+c) = 36k$

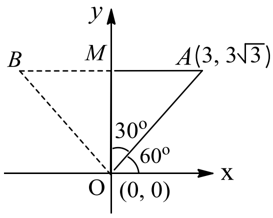
$\Rightarrow \; a + b + c = 18k$

$\Rightarrow \; a = 7k, b = 6k, c = 5k$

$\therefore \; \cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$$= \frac{36 + 25 - 49}{2 \times 6 \times 5} = \frac{1}{5}$$

428 **(c)**

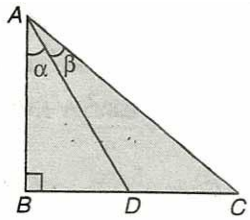


Angle AOM is 30°. Hence, required point of B is $(-3, 3\sqrt{3})$

429 **(b)**

Let $AB = BC = 2x$,

Then $BD = DC = x$ and $AD = (\sqrt{5})x$



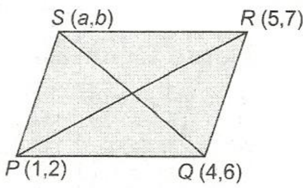
Applying sine rule, $\frac{x}{\sin\beta} = \frac{AD}{\sin 45°}$

$$\Rightarrow \sin\beta = \frac{\frac{x}{\sqrt{2}}}{(\sqrt{5})x} = \frac{1}{\sqrt{10}}$$

and $\sin\alpha = \frac{x}{\sqrt{5}x} = \frac{1}{\sqrt{5}} \Rightarrow \frac{\sin\alpha}{\sin\beta} = \frac{\sqrt{2}}{1}$

430 **(c)**

We know that, in a parallelogram diagonals cut each other at middle point



$$\therefore \frac{a+4}{2} = \frac{1+5}{2} \Rightarrow a = 2$$

and $\frac{b+6}{2} = \frac{2+7}{2} \Rightarrow b = 3$

431 **(c)**

Assume that angles are $4x, x$ and x

As $4x + x + x = 180°$ [$\therefore \angle A + \angle B + \angle C = 180°$]

$\Rightarrow x = 30°$

$\therefore$ Angles are $120°, 30°$ and $30°$

Ratio of sides $= \sin A : \sin B : \sin C$

$= \sin 120° : \sin 30° : \sin 30°$

$= \sqrt{3} : 1 : 1$

$\therefore$ Required ratio $= \frac{\sqrt{3}}{1+1+\sqrt{3}} = \frac{\sqrt{3}}{2+\sqrt{3}}$

433 **(c)**

Here, $s = \frac{18+24+30}{2} = 36$

Now, $\Delta = \sqrt{36(36-18)(36-24)(36-30)}$

$\Delta = \sqrt{36 \times 18 \times 12 \times 6} = 216$

So, radius of the incircle

$$r = \frac{\Delta}{s} = \frac{216}{36} = 6 \text{ cm}$$

434 **(b)**

We know that the mid point of diagonals lies on line $y = 2x + c$, here mid point is (3, 2), hence $c = -4$

435 **(b)**

Clearly, $\Delta\, OAB$ is an isosceles right angled triangle. So, its orthocentre is at $O(0,0)$ and the circumcentre is the mid-point of AB having coordinates $(a/2, a/2)$

Hence, required distance $= \sqrt{\frac{a^2}{4} + \frac{a^2}{4}} = \frac{a}{\sqrt{2}}$

436 **(d)**

The lines of a triangle are $x = y, x - 2y = 3$ and $x + 2y = -3$. Intersection points at sides are $A(-3,-3), B(-1,-1)$ and $C\left(0, -\frac{3}{2}\right)$

$\therefore AB = \sqrt{4+4} = 2\sqrt{2}$

$$AC = \sqrt{9 + \frac{9}{4}} = \frac{3\sqrt{5}}{2}$$

and $BC = \sqrt{1 + \frac{1}{4}} = \frac{\sqrt{5}}{2}$

437 **(b)**

Let the vertices of a triangle be $A(6,0), B(0,6)$ and $C(6,6)$

Now, $AB = \sqrt{6^2 + 6^2} = 6\sqrt{2}$

$BC + \sqrt{6^2 + 0} = 6$

And $CA = \sqrt{0 + 6^2} = 6$

Also, $AB^2 = BC^2 + CA^2$

Therefore, ΔABC is right angled at$C$. So, mid point of AB is the circumcentre of ΔABC

$\therefore$ Coordinate of circumcentre are (3, 3)

Coordinates of centroid are,

$G\left(\frac{6+0+6}{3}, \frac{0+6+6}{3}\right)$, *ie*, (4, 4)

$\therefore$ Required distance $= \sqrt{(4-3)^2 + (4-3)^2} = \sqrt{2}$

438 **(b)**

Given that, $a = 5, b = 7$ and $\sin A = \frac{3}{4}$

As we know, $\frac{\sin A}{a} = \frac{\sin B}{b}$

$$\Rightarrow \frac{3}{4 \times 5} = \frac{\sin B}{7} \Rightarrow \sin B = \frac{21}{20}$$

Which is not possible because its value is greater than one

440 **(a)**

Since, coordinates of the centroid are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$, the centroid is always a rational point

441 **(d)**

$\because$ In a $\Delta\, ABC$,

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin\{\pi-(A+B)\}}=2R$$

$$\Rightarrow\ \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin(A+B)}=2R$$

8. If we know $a, \sin A$, $\sin B$, then we can find b, c, A, B and C

9. We can find A, B, C by using cosine rule

10. $\because$ $a, \sin B$, R are given, then we can find $\sin A$, b

11. $a, \sin A$, R are given, then we know only the ratio $\frac{b}{\sin B}$

or $\frac{c}{\sin(A+B)}$; we cannot determine the values of $b, c, \sin B$, $\sin C$ separately

$\therefore$ $\Delta\, ABC$ cannot be determined in this case

442 **(b)**

Since, $A+C=\pi-B\ \Rightarrow\ \frac{A-B+C}{2}=\frac{\pi}{2}-B$

$$\therefore\ 2ac\sin\left(\frac{A-B+C}{2}\right)=2ac\cos B$$

$$=2ca.\frac{c^2+a^2-b^2}{2ca}$$

$$=a^2+c^2-b^2$$

443 **(a)**

$$a(b\cos C-c\cos B)$$

$$=ab\left(\frac{a^2+b^2-c^2}{2ab}\right)-ac\left(\frac{a^2+c^2-b^2}{2ac}\right)$$

$$=\frac{a^2+b^2-c^2}{2}-\frac{a^2+c^2-b^2}{2}$$

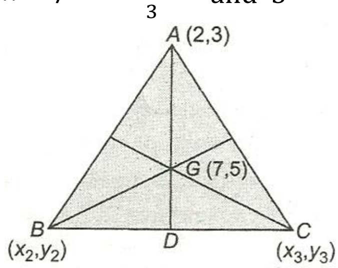
$$=b^2-c^2$$

444 **(b)**

Since, D is the midpoint of BC. So, coordinate of D are $\left(\frac{x_2+x_3}{2},\frac{y_2+y_3}{2}\right)$

Given, $G(7,5)$ is the centroid of ΔABC

$\therefore\ \ 7=\frac{2+x_2+x_3}{3}$ and $5=\frac{3+y_2+y_3}{3}$



$\Rightarrow\ \ x_2+x_3=21-2$ and $y_2+y_3=15-3$

$\Rightarrow\ \ \frac{x_2+x_3}{2}=\frac{19}{2}$ and $\frac{y_2+y_3}{2}=6$

$\therefore$ Coordinates of $D\left(\frac{19}{2},6\right)$

445 **(b)**

Since, the axes are rotated through an angle 45°, then we replace (x,y) by ($x\cos 45°-$ *y*sin *45°, x*sin *45°+y*cos *45° ie, x2−y2,x2+y2* in the given equation

$$3x^2+3y^2+2xy=2$$

$$\therefore\ 3\left(\frac{x}{\sqrt{2}}-\frac{y}{\sqrt{2}}\right)^2+3\left(\frac{x+y}{\sqrt{2}}\right)^2$$

$$+2\left(\frac{x-y}{\sqrt{2}}\right)\left(\frac{x+y}{\sqrt{2}}\right)=2$$

$$\Rightarrow \frac{3}{2}(x^2-y^2-2xy)+\frac{3}{2}(x^2+y^2+2xy)$$

$$+\frac{2}{2}(x^2-y^2)=2$$

$$\Rightarrow\ \ 4x^2+2y^2=2$$

$$\Rightarrow\ \ 2x^2+y^2=1$$

446 **(c)**

Let $P(x,y)$ be any point on the line

Also, $(a_1-a_2)x+(b_1-b_2)y+c=0$ ...(i)

Since, $(x-a_1)^2+(y-b_1)^2=(x-a_2)^2+(y-b_2)^2$

$$\Rightarrow\ \ x^2+a_1^2-2a_1x+y^2+b_1^2-2b_1y$$

$$=x^2+a_2^2-2a_2x+y^2+b_2^2-2b_2y$$

$$\Rightarrow\ \ 2(a_2-a_1)x+2(b_2-b_1)y$$

$$=a_2^2+b_2^2+b_2^2-a_1^2-b_1^2$$

$\Rightarrow\ \ (a_1-a_2)x+(b_1-b_2)y+\frac{(a_2^2+b_2^2-a_1^2-b_1^2)}{2}=0$ ...(ii)

Since, Eqs. (i) and (ii) represents the same equation of line

$$\therefore\ \ c=\frac{a_2^2+b_2^2-a_1^2-b_1^2}{2}$$

447 **(d)**

Let D be the centre of circumcircle

$\therefore\ \ BD=5$ cm

In ΔABC,

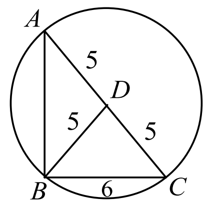
$AC^2+AB^2+BC^2$

$\Rightarrow\ \ 100=AB^2+36$

$\Rightarrow\ AB^2=64\ \ \Rightarrow\ \ AB=8$

$\therefore$ Area of $\Delta ABC=\frac{1}{2}\times AB\times BC$

$$=\frac{1}{2}\times 8\times 6=24\text{ cm}^2$$



448 **(c)**

The given equation is

$x^2+6xy+8y^2=10$ ...(i)

Since, axes are rotated through an angle $\frac{\pi}{4}$

$\therefore\ x=x_1\cos\frac{\pi}{4}-y_1\sin\frac{\pi}{4}=\frac{x_1-y_1}{\sqrt{2}}$

and $y=x_1\sin\frac{\pi}{4}+y_1\cos\frac{\pi}{4}=\frac{x_1+y_1}{\sqrt{2}}$

on putting the value of x and y in Eq. (i)

$$\left(\frac{x_1-y_1}{\sqrt{2}}\right)^2+6\left(\frac{x_1+y_1}{\sqrt{2}}\right)\left(\frac{x_1+y_1}{\sqrt{2}}\right)+8\left(\frac{x_1+y_1}{\sqrt{2}}\right)=10$$

$$\Rightarrow\ x_1^2+y_1^2-2x_1y_1+6x_1^2-6y_1^2+8x_1^2+8y_1^2+16x_1y_1=20$$

$\Rightarrow\ 15x_1^2+3y_1^2+14x_1y_1=20$

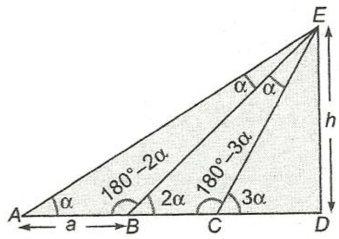
$\therefore$ Required equation is

$15x^2+14xy+3y^2=20$

449 **(c)**

In $\Delta ABE, \angle BAE=\angle AEB$

$\therefore\ AB=BE$



In ΔBCE,using sine rule,

$$\frac{BE}{\sin(180°-3\alpha)}=\frac{CE}{\sin 2\alpha}$$

$$\Rightarrow CE=\frac{a\sin 2\alpha}{\sin 3\alpha}\ ...\text{(i)}$$

Now, In ΔDCE, $\sin 3\alpha=\frac{h}{CE}$

$$\Rightarrow \sin 3\alpha=\frac{h}{a\sin 2\alpha/\sin 3\alpha}\text{[from Eq. (i)]}$$

$\Rightarrow h=a\sin 2\alpha$

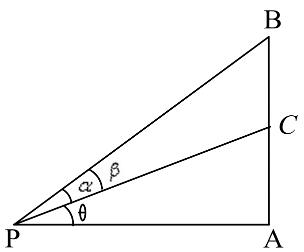
450 **(a)**

We know that the x-axis divides the segment joining $P(x_1,y_1)$ and $(x_2\ ,y_2)$ in the ratio $-y_1:y_2$.

So, the required ratio is $-6:-3$ i.e. $2:1$

451 **(d)**

In $\Delta\ PAB, \tan\beta=\frac{AB}{AP}$



In $\Delta\ PAC, \tan\theta=\frac{AC}{AP}$

$\therefore\ \tan\alpha=\tan(\beta-\theta)$

$$=\frac{\tan\beta-\tan\theta}{1+\tan\beta\tan\theta}$$

$$=\frac{\frac{AB}{AP}-\frac{AC}{AP}}{1+\frac{AB}{AP}\cdot\frac{AC}{AP}}\quad ...\text{(i)}$$

$\because\ AP=n\ (AB)=n(2AC)$ ($\because C$ is the mid point of BA)

From Eq. (i), we get

$$\tan\alpha=\frac{\frac{1}{n}-\frac{1}{2n}}{1+\frac{1}{2n^2}}=\frac{n}{2n^2+1}$$

$\Rightarrow\ n=(2n^2+1)\tan\alpha$

452 **(b)**

Let the three points be $A(-2,-5), B(2,-2)$ and $C(8,a)$

If three points are collinear, then

sope of AB = slope of BC

$$\Rightarrow\ \frac{-2+5}{2+2}=\frac{a+2}{8-2}$$

$$\Rightarrow\ \frac{3}{4}=\frac{a+2}{6}\ \Rightarrow\ 9=2a+4\ \Rightarrow\ a=\frac{5}{2}$$

453 **(a)**

From given relation, we can write

$\cos A+\cos C=2(1-\cos B)$

$$\Rightarrow\ 2\cos\frac{A+C}{2}\cos\frac{A-C}{2}=2\cdot 2\sin^2\frac{B}{2}$$

$$\Rightarrow\ \cos\frac{A-C}{2}=2\sin\frac{B}{2}$$

$$\Rightarrow\ 2\sin\frac{A+c}{2}\cdot\cos\frac{A-C}{2}=4\sin\frac{B}{2}\cos\frac{B}{2}$$

$\Rightarrow\ \sin A+\sin C=2\sin B$

$\Rightarrow\ a,b,c$ are in AP

454 **(d)**

Given, $\cos(A+B)=\frac{31}{32}\ \Rightarrow \cos(\pi-C)=\frac{31}{32}$

$$\Rightarrow\ -\cos C=-\frac{a^2+b^2-c^2}{2ab}=\frac{31}{32}$$

$$\Rightarrow\ \frac{(5)^2+(4)^2-c^2}{2\times 5\times 4}=-\frac{31}{32}$$

$$\Rightarrow\ 41-c^2=-\frac{155}{4}$$

$$\Rightarrow\ c^2=\frac{319}{4}$$

$$\Rightarrow\ c=\frac{\sqrt{319}}{2}$$

455 **(a)**

We know, $r_1=\frac{\Delta}{s-a}, r_2=\frac{\Delta}{s-b}, r_3=\frac{\Delta}{s-c}$

Given that, $r_1>r_2>r_3$

$$\Rightarrow\ \frac{\Delta}{s-a}>\frac{\Delta}{s-b}>\frac{\Delta}{s-c}$$

$$\Rightarrow\ \frac{1}{s-a}>\frac{1}{s-b}>\frac{1}{s-c}$$

$\Rightarrow\ s-a<s-b<s-c$

($s-a, s-b, s-c$ are positive)

$\Rightarrow \ -a < -b < -c$

$\Rightarrow \ a > b > c$

456 **(b)**

We have, $a = 2x$, $b = 2y$ and $\angle C = 120°$

Area of triangle, $\Delta = \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 2x \times 2y \sin 120°$

$= xy\sqrt{3}$ sq unit

457 **(a)**

$$\frac{a - a' - a'}{a' - a} = \frac{b - b' - b'}{b' - b} \quad \left(\because \ \frac{x_3 - x_2}{x_2 - x_1} = \frac{y_3 - y_2}{y_2 - y_1} \right)$$

$$\Rightarrow \ \frac{a - 2a'}{a' - a} = \frac{b - 2b'}{b' - b}$$

$$\Rightarrow \ \frac{a}{a'} = \frac{b}{b'}$$

$\Rightarrow \ ab' = a'b$

459 **(c)**

If two vertices of an equilateral triangle have the coordinates (x_1, y_1) and (x_2, y_2), then the coordinates of its third vertex are

$$\left(\frac{x_1 + x_2 \pm \sqrt{3}\,(y_1 - y_2)}{2}, \frac{y_1 + y_2 \pm \sqrt{3}(x_1 - x_2)}{2} \right)$$

Here, we have

$x_1 = 2, y_1 = 4, x_2 = 2$ and $y_2 = 6$

Hence, the coordinates of the third vertex are

$$\left(\frac{4 \pm \sqrt{3}(-2)}{2}, \frac{10 \pm \sqrt{3} \times 0}{2} \right) = \left(2 \mp \sqrt{3}, 5\right)$$

461 **(a)**

$\because \ A + B + C + D = 2\pi$

$\Rightarrow \ \tan(A + B + C + D) = 0$

$$\Rightarrow \ \frac{\sum \tan A - \sum \tan A \tan B \tan C}{1 - \sum \tan A \tan B + \tan A \tan B \tan C \tan D} = 0$$

$$\Rightarrow \ \sum \tan A - \sum \tan A \tan B \tan C = 0$$

$$\Rightarrow \sum \tan A = \tan A \tan B \tan C \tan D \sum \cot A$$

$$\Rightarrow \frac{\sum \tan A}{\sum \cot A} = \prod \tan A$$

462 **(a)**

Let the angles of a triangle are $3\theta, 4\theta, 5\theta$

$\therefore \ 3\theta + 4\theta + 5\theta = 180° \ \Rightarrow \ \theta = 15°$

$\therefore$ Angles of a triangle are $45°, 60°, 75°$

Now, $\sin A = \sin 45° = \frac{1}{\sqrt{2}}$

$\sin B = \sin 60° = \frac{\sqrt{3}}{2}$

and $\sin C = \sin 75° = \frac{\sqrt{3}+1}{2\sqrt{2}}$

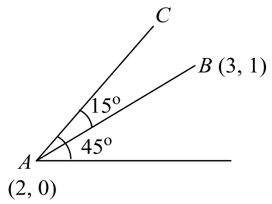
$\therefore a : b : c = \sin A : \sin B : \sin C$

$$= \frac{1}{\sqrt{2}} : \frac{\sqrt{3}}{2} : \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

$= 2 : \sqrt{6} : \sqrt{3} + 1$

463 **(a)**

Since, $AB = AC = \sqrt{2}$



The slope AB is 1. Hence, AB is inclined at $45°$ with the x-axis and AC is inclined at $60°$ with the x-axis. Equation of AC is

$y = \sqrt{3}(x - 2)$

The coordinates of C is $(2 + \sqrt{2} \cos 60°, 0 + \sqrt{2} \sin 60°)$

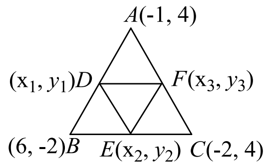
or $\left(2 + \frac{1}{\sqrt{2}}, \sqrt{\frac{3}{2}}\right)$

464 **(b)**

Let $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) are coordinates of the points D, E and F, which divide each AB, BC and CA respectively in the ratio 3:1 (internally)

$$\therefore \ x_1 = \frac{3 \times 6 - 1 \times 1}{4} = \frac{17}{4}$$

$$y_1 = \frac{-2 \times 3 + 4 \times 1}{4} = -\frac{2}{4} = -\frac{1}{2}$$



Similarly, $x_2 = 0, y_2 = \frac{5}{2}$

and $x_3 = -\frac{5}{4}, y_3 = 4$

Let (x, y) be the coordinates of centroid of $\Delta\, DEF$

$$\therefore \ x = \frac{1}{3}\left(\frac{17}{4} + 0 - \frac{5}{4}\right) = 1$$

and $y = \frac{1}{3}\left(-\frac{1}{2} + \frac{5}{2} + 4\right) = 2$

$\therefore$ Coordinates of Centroid are (1, 2)

465 **(a)**

Let D be the position of window of second house and BC be the position of the first house.

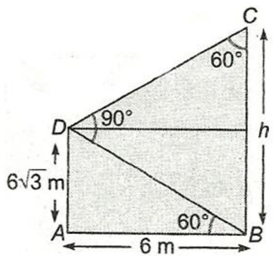
In ΔADB, $\tan 60° = \frac{AD}{AB}$

$\Rightarrow AD = 6\sqrt{3}$

and $DB^2 = \left(6\sqrt{3}\right)^2 + (6)^2$

$\Rightarrow DB = 12m$

In ΔDCB, $\sin 60° = \frac{12}{h}$



$$h = \frac{12}{\sqrt{3}/2} = 8\sqrt{3}\text{ m}$$

466 **(b)**

Let the vertices be C, A, B respectively. The altitude from A is

$$\frac{y - a(t_2 + t_3)}{x - at_2t_3} = -t_1$$

$\Rightarrow\ xt_1 + y = at_1t_2t_3 + a(t_2 + t_3)$ ...(i)

The altitude from B is

$xt_2 + y = at_1t_2t_3 + a(t_3 + t_1)$ ...(ii)

Subtracting Eq. (ii) from Eq. (i), $x = -a$

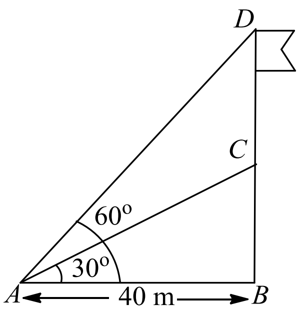
Hence, $y = a(t_1 + t_2 + t_3 + t_1t_2t_3)$

$\therefore$ The orthocenter is $\{-a, a(t_1 + t_2 + t_3 + t_1t_2t_3)\}$

467 **(a)**

In $\Delta\, ABC, \tan 30° = \frac{BC}{40}$

$\Rightarrow\ BC = \frac{40}{\sqrt{3}}$ ...(i)



In $\Delta\, DAB, \tan 60° = \frac{BD}{40}$

$\Rightarrow\ BD = 40\sqrt{3}$

$\Rightarrow\ DC + BC = 40\sqrt{3}$

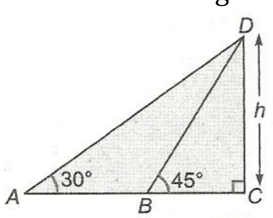
$\Rightarrow\ DC + \frac{40}{\sqrt{3}} = 40\sqrt{3}$ [from Eq.(i)]

$\Rightarrow\ DC = 40\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = \frac{80}{\sqrt{3}}$

$\Rightarrow\ DC = 46.19$ m

468 **(c)**

Let DC be the height of building



$\therefore AB = \text{Speed} \times \text{Time}$

$= 25(\sqrt{3} - 1).2$

$= 50(\sqrt{3} - 1)$

$\therefore$ In $\Delta DBC, \tan 45° = \frac{DC}{BC}\ \Rightarrow\ BC = h$

In $\Delta DAC, \tan 30° = \frac{h}{50(\sqrt{3}-1)+h}$

$\Rightarrow\ 50(\sqrt{3} - 1) + h = \sqrt{3}h$

$\Rightarrow\ h = 50$ m

469 **(d)**

$(b + c)\tan\frac{A}{2}\tan\left(\frac{B - C}{2}\right)$

$= (b + c)\tan\frac{A}{2}.\frac{(b - c)}{(b + c)}\cot\frac{A}{2}$

$= b - c$

$\therefore\ \sum(b + c)\tan\left(\frac{B - C}{2}\right)\tan\frac{A}{2}$

$= b - c + c - a + a - b = 0$

470 **(c)**

As, $A > B$ and $3\sin x - 4\sin^3 x - k = 0, 0 < k < 1$

$\Rightarrow\ \sin 3x = k$

As A and B satisfy given equation

$\therefore\ \sin 3A = k, \quad \sin 3B = k$

$\Rightarrow\ \sin 3A - \sin 3B = k$

$\Rightarrow\ 2\cos\left(\frac{3A + 3B}{2}\right)\sin\left(\frac{3A - 3B}{2}\right) = 0$

$\Rightarrow\ \cos\left(\frac{3A + 3B}{2}\right) = 0$ or $\sin\left(\frac{3A - 3B}{2}\right) = 0$

$\Rightarrow\ \frac{3A + 3B}{2} = 90°$ or $\frac{3A - 3B}{2} = 0$

$\Rightarrow\ A + B = 60°$ or $A = B$

But given, $A > B$ ($\therefore$ neglectring $A = B$)

Thus, $A + B = 60°$

and $A + B + C = 180°\ \Rightarrow\ \angle C = 120°$

471 **(b)**

We have, $\cos A = \frac{b^2+c^2-a^2}{2bc}$

$\Rightarrow\ b^2 - 2bc\cos A + (c^2 - a^2) = 0$

It is given that b_1 and b_2 are roots of this equation

$\therefore\ b_1 + b_2 = 2\,c\cos A$ and $b_1b_2 = c^2 - a^2$

$\Rightarrow\ 3b_1 = 2c\cos A$ and $2b_1^2 = c^2 - a^2$ [$\because\ b_2 = 2b_1$ (given)]

$\Rightarrow\ 2\left(\frac{2c}{3}\cos A\right)^2 = c^2 - a^2$

$\Rightarrow\ 8c^2(1 - \sin^2 A) = 9c^2 - 9a^2$

$\Rightarrow\ \sin^2 A = \left(1 - \frac{9c^2 - 9a^2}{8c^2}\right)$

$\Rightarrow\ \sin A = \sqrt{\frac{9a^2 - c^2}{8c^2}}$

472 **(a)**

Let the vertices of the ΔABC are $A(7, 1), B(-1, 5)$ and $C(3 + 2\sqrt{3}, 3 + 4\sqrt{3})$

Now, $AB = \sqrt{(7 + 1)^2 + (1 - 5)^2}$

$= \sqrt{64+16} = \sqrt{80}$

$BC = \sqrt{\left(-1-3-2\sqrt{3}\right)^2 + \left(5-3-4\sqrt{3}\right)^2}$

$= \sqrt{16+12+16\sqrt{3}+4+48-16\sqrt{3}}$

$= \sqrt{80}$

and $CA = \sqrt{\left(3+2\sqrt{3}-7\right)^2 + \left(3+4\sqrt{3}-1\right)^2}$

$= \sqrt{16+12-16\sqrt{3}+4+48+16\sqrt{3}}$

$= \sqrt{80}$

Here, $AB = BC = CA = \sqrt{80}$

$\therefore$ Hence, it is an equilateral triangle, so incentre and centriod coincides

So, incentre $= \left(\frac{7-1+3+2\sqrt{3}}{3}, \frac{1+5+3+4\sqrt{3}}{3}\right)$

$= \left(\frac{9+2\sqrt{3}}{3}, \frac{9+4\sqrt{3}}{3}\right)$

$= \left(3+\frac{2}{\sqrt{3}}, 3+\frac{4}{\sqrt{3}}\right)$

473 **(b)**

$\frac{\tan\frac{A}{2} - \tan\frac{B}{2}}{\tan\frac{A}{2} + \tan\frac{B}{2}} = \frac{\sqrt{\frac{(s-b)(s-c)}{s(s-a)}} - \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}}{\sqrt{\frac{(s-b)(s-c)}{s(s-a)}} + \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}}$

$= \frac{(s-b)\sqrt{s(s-c)} - (s-a)\sqrt{s(s-c)}}{(s-b)\sqrt{s(s-c)} + (s-a)\sqrt{s(s-c)}}$

$= \frac{\sqrt{s(s-c)}(s-b-s+a)}{\sqrt{s(s-c)}(s-b-s+a)} = \frac{a-b}{c}$

474 **(a)**

As we know that orthocenter, centroid and circumcentre are collinear and the centroid divides the line segment joining orthocenter and circumcentre in the ratio 2:1. If the coordinates of orthocentre and circumcentre are (1, 1) and (3, 2) respectively, the coordinate of centroid is

$\left(\frac{2.3+1.1}{2+1}, \frac{2.2+1.1}{2+1}\right) = \left(\frac{7}{3}, \frac{5}{3}\right)$

475 **(d)**

$\because \angle A = 60°, \cos 60° = \frac{b^2+c^2-a^2}{2bc}$

$\Rightarrow\ b^2+c^2-a^2 = bc$

Now, $AB^2 + AC^2 = 2(AD^2 + BD^2)$

$\Rightarrow\ c^2+b^2 = 2AD^2 + 2\left(\frac{a}{2}\right)^2$

$\Rightarrow\ 2b^2+2c^2-a^2 = 4AD^2$

$\Rightarrow\ b^2+c^2+bc = 4AD^2\ \ (\because\ b^2+c^2-a^2 = bc)$

476 **(b)**

The intersection of lines are $(0,0), (-4,-8)$ and $(-2,-2)$

Let circumcentre is (x_1, y_1)

$\therefore\ \ x_1^2 + y_1^2 = (x_1+4)^2 + (y_1+8)^2$

$\Rightarrow\ \ 8x_1 + 16y_1 + 80 = 0\ \ ...(i)$

And $\ x_1^2 + y_1^2 = (x_1+2)^2 + (y_1+2)^2$

$\Rightarrow\ \ 4x_1 + 4y_1 + 8 = 0\ \ ...(ii)$

On solving Eqs. (i) and (ii), we get

$x_1 = 6$ and $y_1 = -8$

477 **(c)**

Given, $\cot\frac{A}{2} = \frac{b+c}{a}$

$\Rightarrow\ \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} = \frac{\sin B + \sin C}{\sin A}$

$= \frac{2\sin\frac{B+C}{2}\cos\frac{B-C}{2}}{2\sin\frac{A}{2}\cos\frac{A}{2}}$

$\Rightarrow\ \cos\frac{A}{2} = \cos\frac{B-C}{2}$

$\therefore\ \frac{A}{2} = \frac{B-C}{2}$

$\Rightarrow\ A + C = B\ \ \Rightarrow \angle B = \frac{\pi}{2}\ \ (\because A+B+C = \pi)$

478 **(c)**

Given lines are

$3x^2 - 4xy + y^2 = 0$

$\Rightarrow\ \ (3x-y)(x-y) = 0$

$\Rightarrow\ \ 3x-y=0,\ x-y=0$ and $2x-y=6$

The points of intersection of these lines are $(0,0), (-6,-18)$ and $(6,6)$

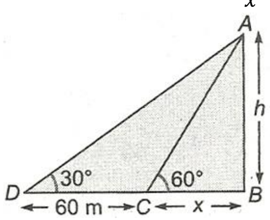
$\therefore$ Area of triangle $= \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ -6 & -18 & 1 \\ 6 & 6 & 1 \end{vmatrix}$

$= \frac{1}{2}(-36+108) = \frac{1}{2}(72)$

$= 36$ sq units

479 **(d)**

In $\Delta ABC, \tan 60° = \frac{h}{x} \Rightarrow h\sqrt{3}x$ ...(i)



and in ΔABD,

$\tan 30° = \frac{h}{x+60} \Rightarrow x + 60 = \sqrt{3}h$ ... (ii)

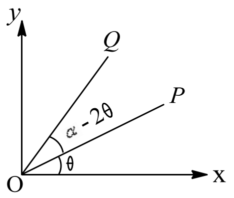
From Eqs. (i) and (ii),

$h = \sqrt{3} \times 30 = 51.96$ m

=52m (approx)

480 **(d)**

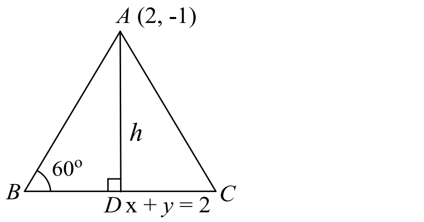
OP is inclined at angle θ with x-axis and OQ is inclined at angle $\alpha - \theta$ with x-axis



The bisector of angle POQ is inclined at angle $\frac{\theta+\alpha-\theta}{2}=\frac{\alpha}{2}$ with x-axis

481 **(b)**

The altitude h is $\frac{|2-1-2|}{\sqrt{2}}=\frac{1}{\sqrt{2}}$



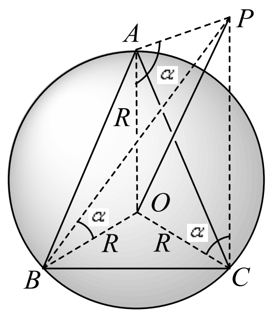
In $\Delta\, ABC,\ BC=2h\cot 60°=\sqrt{\frac{2}{3}}$

$\therefore$ Area of $\Delta=\frac{h}{2}BC$

$=\frac{1}{2\sqrt{2}}\sqrt{\frac{2}{3}}=\frac{\sqrt{3}}{6}$

483 **(d)**

Let OP be the tower. Since, the tower make equal angles at the vertices of the triangle, therefore foot of tower is at the circumcentre



In $\Delta\, OAP, \tan\alpha=\frac{OP}{OA}$

$\Rightarrow\ OP=OA\tan\alpha$

$\Rightarrow\ OP=\mathrm{R}\tan\alpha\quad (\because OA=R,\text{given})$

484 **(a)**

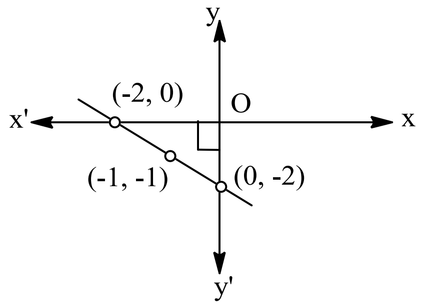
Given, $xy+2x+2y+4=0$ ...(i)

and $x+y+2=0$ ...(ii)

From Eqs. (i) and (ii),

$xy=0$

$\Rightarrow\ x=y=0$



$\therefore$ vertices of triangle are $(-2,0),(0,0),(0,-2)$

Since, this triangle is right angled triangle and in a right angled triangle circumcentre is mid point of hypotenuse.

$\therefore$ $(-1,-1)$ is the circumcentre

485 **(b)**

Let P be (x,y) and we have

$A=(a+b,a-b),\qquad B=(a-b,a+b)$

Here, $PA=PB\quad\Rightarrow\quad PA^2=PB^2$

$\Rightarrow\ [x-(a+b)^2]+[y-(a-b)]^2$

$=[x-(a-b)]^2+[y-(a-b)]^2$

$\Rightarrow\ [x-(a+b)]^2-[x-(a-b)]^2$

$=[y-(a+b)]^2-[y-(a-b)]^2$

$\Rightarrow\ [x-(a+b)+x-(a-b)][x-(a+b)-x+a-b]$

$=[y-(a+b)+y-(a-b)][y-(a+b)-y+a-b]$

$\Rightarrow\ (2x-2a)(-2b)=(2y-2a)(-2b)$

$\Rightarrow\ x-y=0$

486 **(c)**

Since, $R=\frac{a}{2\sin A}$

$\Rightarrow\ R=\frac{2\sqrt{3}}{2\sin 60°}=2\text{ cm}$

487 **(c)**

Let two of vertices of a triangle are $A(15,0)$ and $B(0,10)$ and third vertex is $C(h,k)$

We know that line passing through A and B should be perpendicular to line through C and orthocenter O

$\therefore\ \left(\frac{-2}{3}\right)\left(\frac{9-k}{6-h}\right)=-1$

$\Rightarrow\ 2k=3h$

Which is satisfied by (0, 0). Hence, coordinates of third vertex are (0, 0)

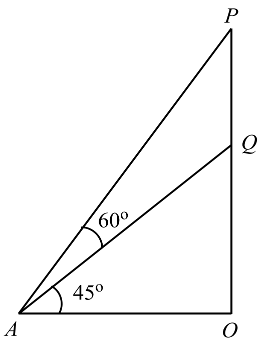
488 **(a)**

Let P and Q be the positions of two planes. It is given that $OP=300$ m. From triangle OAQ, we have $OA=OQ$.

From $\Delta\, OAP$, we have

$\tan 60°=\frac{OP}{OA}\Rightarrow\sqrt{3}=\frac{300}{OQ}\Rightarrow OQ=\frac{300}{\sqrt{3}}$

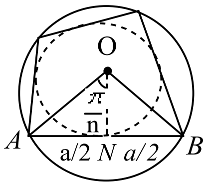
$=100\sqrt{3}$ mts



489 **(b)**
Let the sides be $a = 3x, b = 7x, c = -8x$. Then,
$2s = a + b + c \Rightarrow s = 9x$
$\therefore \Delta = \sqrt{s(s-a)(s-b)(s-c)}$
$= \sqrt{9x \times 6x \times 2x \times x} = 6\sqrt{3}x^2$
Now, $R = \frac{abc}{4\Delta}$ and $r = \frac{\Delta}{s}$
$\therefore \frac{R}{r} = \frac{sabc}{4\Delta^2} = \frac{9x \times 3x \times 7x \times 8x}{4 \times 108 \times x^2} = \frac{7}{2}$

490 **(a)**
$(b+c-a)\tan\frac{A}{2} = (2s-2a)\tan\frac{A}{2}$
$= 2(s-a)\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$
$= 2\frac{\sqrt{s(s-a)(s-b)(s-c)}}{s} = \frac{2\Delta}{s}$

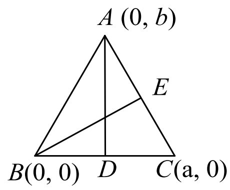
491 **(b)**
Let $AB = a$
$ON \perp AB$
and $AN = BN$



In ΔAON, $\tan\frac{\pi}{n} = \frac{AN}{ON}$
$\Rightarrow ON = AN\cot\frac{\pi}{n} = \frac{a}{2}\cot\frac{\pi}{n}$
and $\sin\frac{\pi}{n} = \frac{AN}{OA}$
$\Rightarrow OA = \frac{a}{2}\text{cosec}\frac{\pi}{n}$ ...(ii)
Now, sum of the radii $= ON + OA$
$= \frac{a}{2}\cot\frac{\pi}{n} + \frac{a}{2}cosec\frac{\pi}{n}$
[from Eqs. (i) and (ii)]
$= \frac{a}{2}\left[\frac{1+\cos\frac{\pi}{n}}{\sin\frac{\pi}{n}}\right]$
$= \frac{a}{n}\left[\frac{2\cos^2\frac{\pi}{2n}}{2\sin\frac{\pi}{2n}\cos\frac{\pi}{2n}}\right]$
$= \frac{a}{2}\cot\frac{\pi}{2n}$

492 **(c)**
$\Delta = 2bc - (b^2 + c^2 - a^2) = 2bc(1 - \cos A)$
$= 2bc.2\sin^2\frac{A}{2}$ ...(i)
But $\Delta = \frac{1}{2}bc\sin A = \frac{1}{2}bc.2\sin\frac{A}{2}\cos\frac{A}{2}$
$\Rightarrow \Delta = bc\sin\frac{A}{2}\cos\frac{A}{2}$ ...(ii)
On dividing Eq. (ii) by Eq. (i), we get
$\tan\frac{A}{2} = \frac{1}{4}$
$\therefore \tan A = \frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}} = \frac{\frac{1}{2}}{1-\frac{1}{16}} = \frac{8}{15}$

493 **(d)**
The vertices of a ΔABC are $A(0,b), B(0,0)$ and $C(a,0)$



Mid point of E and D are $\left(\frac{a}{2},\frac{b}{2}\right)$ and $\left(\frac{a}{2},0\right)$
The slope of median BE, $m_1 = \frac{b}{a}$
and slope of AD, $m_2 = -\frac{2b}{a}$
Since, the medians are perpendicular to each other
$\therefore m_1m_2 = -1$
$\Rightarrow \frac{b}{a} \times \left(\frac{-2b}{a}\right) = -1$
$\Rightarrow -2b^2 = -a^2 \Rightarrow a = \pm\sqrt{2}b$

494 **(c)**
$\because a(b-c) + b(c-a) + c(a-b) = 0$
$\therefore x = 1$ is a root of the equation
$a(b-c)x^2 + b(c-a)x + c(a-b) = 0$
Then, other root =1 ($\because$ roots are equal)
$\therefore$ Product of roots $= 1 = \frac{c(a-b)}{a(b-c)}$
$\Rightarrow ab - ac = ca - bc$
$\Rightarrow b = \frac{2ac}{a+c}$
$\therefore a, b, c$ are in HP
Then, $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP
$\Rightarrow \frac{s}{a}, \frac{s}{b}, \frac{s}{c}$ are in AP
$\Rightarrow \frac{s}{a} - 1, \frac{s}{b} - 1, \frac{s}{c} - 1$ are in AP
$\Rightarrow \frac{(s-a)}{a}, \frac{(s-b)}{b}, \frac{(s-c)}{c}$ are in AP
Multiplying each by $\frac{abc}{(s-a)(s-b)(s-c)}$
Then $\frac{bc}{(s-b)(s-c)}, \frac{ca}{(s-c)(s-a)}, \frac{ab}{(s-a)(s-b)}$ are in AP
$\Rightarrow \frac{(s-b)(s-c)}{bc}, \frac{(s-c)(s-a)}{ca}, \frac{(s-a)(s-b)}{ab}$ are in HP

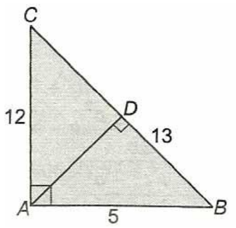
$\Rightarrow \sin^2\left(\frac{A}{2}\right), \sin^2\left(\frac{B}{2}\right), \sin^2\left(\frac{C}{2}\right)$ are in HP

495 **(d)**

$$\frac{1+\cos C\cos(A-B)}{1+\cos(A-C)\cos B}$$
$$=\frac{1-\cos(A+B)\cos(A-B)}{1-\cos(A-C)\cos(A+C)} \quad (\because A+B+C=\pi)$$
$$\Rightarrow \frac{1-\cos^2 A+\sin^2 B}{1-\cos^2 A+\sin^2 C}=\frac{\sin^2 A+\sin^2 B}{\sin^2 A+\sin^2 C}$$
$$=\frac{a^2+b^2}{a^2+c^2}$$

496 **(c)**

$\because\ b^2+c^2=a^2$



$\therefore\ \Delta ABC$ is right angled triangle and right angled at A

Clearly, from figure,

$$\frac{1}{2}(AB)(AC)=\frac{1}{2}(AD)(BC)$$
$$\Rightarrow\ AD=\frac{(AB)(AC)}{BC}$$
$$=\frac{5\cdot 12}{13}=\frac{60}{13}$$

497 **(a)**

Since, $r_1<r_2<r_3$

$$\Rightarrow \frac{1}{r_1}>\frac{1}{r_2}>\frac{1}{r_3}$$
$$\Rightarrow \frac{s-a}{\Delta}>\frac{s-b}{\Delta}>\frac{s-c}{\Delta}$$
$$\Rightarrow\ (s-a)>(s-b)>(s-c)$$
$$\Rightarrow\ -a>-b>-c$$
$$\Rightarrow\ a<b<c$$

498 **(a)**

Let l be the length of the ladder

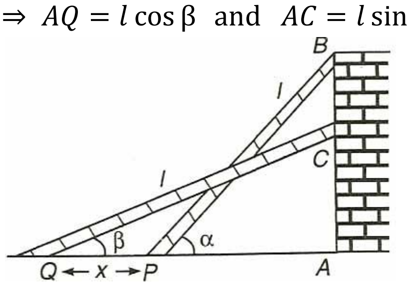
$ie, BP=CQ=1$

In $\Delta PAB, \cos\alpha=\frac{PA}{PB}$ and $\sin\alpha=\frac{AB}{PB}$

$\Rightarrow\ PA=l\cos\alpha$ and $AB=l\sin\alpha$ ...(i)

In $\Delta QAC,\ \cos\beta=\frac{AQ}{QC}$

$\Rightarrow AQ=l\cos\beta$ and $AC=l\sin\beta$ ...(ii)



Now, $CB=AB-CA$

$=l\sin\alpha-l\sin\beta$ [from Eqs. (i) and (ii)]

$=l(\sin\alpha-\sin\beta)$

and $QP=AQ-PA$

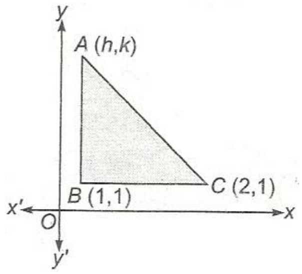
$=l\cos\beta-l\cos\alpha$

$=l(\cos\beta-\cos\alpha)$

$$\therefore\ \frac{CB}{QP}=\frac{l(\sin\alpha-\sin\beta)}{l(\cos\beta-\cos\alpha)}$$
$$\Rightarrow\ \frac{y}{x}=\frac{2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)}{2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}$$
$$\Rightarrow\ \frac{y}{x}=\cot\left(\frac{\alpha+\beta}{2}\right)$$
$$\Rightarrow\ x=y\tan\left(\frac{\alpha+\beta}{2}\right)$$

499 **(c)**

$\because\ A(h.k), B(1,1)$ and $C(2,1)$ are the vertices of a right angled triangle ABC



Now, $AB=\sqrt{(1-h)^2+(1-k)^2}$

Or $BC=\sqrt{(2-1)^2+(1-1)^2}=1$

Or $CA=\sqrt{(h-2)^2+(k-1)^2}$

Now, Pythagorus theorem

$AC^2=AB^2+BC^2$

$$\Rightarrow\ 4+h^2-4h+k^2+1-2k$$
$$=h^2+1-2h+k^2+1-2k+1$$
$$\Rightarrow\ 5-4h=3-2h$$
$$\Rightarrow\ h=1 \quad ...(i)$$

Now, given that area of the triangle is 1

Then, area $(\Delta ABC)=\frac{1}{2}\times AB\times BC$

$$\Rightarrow\ 1=\frac{1}{2}\times\sqrt{(1-h)^2+(1-k)^2}\times 1$$
$$\Rightarrow\ 2=\sqrt{(1-h)^2+(1-k)^2} \quad ...(ii)$$

On putting $h=1$ from Eq. (i), we get

$2=\sqrt{(k-1)^2}$

On squaring both the sides, we get

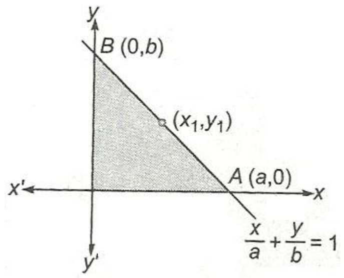
$4=k^2+1-2k$

$\Rightarrow\ k^2-2k-3=0\ \Rightarrow\ k=-1,3$

Thus, the set of values of k is $\{-1,3\}$

500 **(d)**

Let the coordinate of mid point of AB is (x_1,y_1)



$\therefore\ x_1 = \dfrac{a+0}{2}, y_1 = \dfrac{0+b}{2}$

$\Rightarrow\quad a = 2x_1, b = 2y_1$

Given, $a + b = 4\ \Rightarrow\ \ x_1 + y_1 = 2$

Hence, the locus of the mid point is $x + y = 2$

501 **(a)**

$\because\ \tan\frac{A}{2}, \tan\frac{B}{2}, \tan\frac{C}{2}$ are in HP

$\therefore\ \cot\frac{A}{2}, \cot\frac{B}{2}, \cot\frac{C}{2}$ are in AP

$\Rightarrow\ \cot\dfrac{B}{2} - \cot\dfrac{A}{2} = \cot\dfrac{C}{2} - \cot\dfrac{B}{2}$

$\Rightarrow\ \dfrac{s(s-b)}{\Delta} - \dfrac{s(s-a)}{\Delta} = \dfrac{s(s-c)}{\Delta} - \dfrac{s(s-b)}{\Delta}$

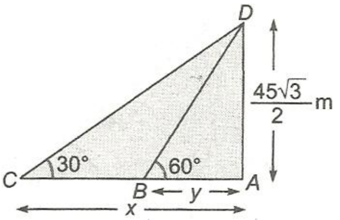
$\Rightarrow\ s - b - s + a = s - c - s + b$

$\Rightarrow\ 2b = a + c$

$\Rightarrow\ a, b, c$ are in AP

502 **(a)**

In $\Delta ACD, \tan 30° = \frac{45\sqrt{3}/2}{x}$



$\Rightarrow x = \dfrac{45\sqrt{3}}{2 \times \frac{1}{\sqrt{3}}} = \dfrac{135}{2}$ m

and in ΔABD,

$\tan 60° = \dfrac{45\sqrt{3/2}}{y}\ \Rightarrow\ y = \dfrac{45}{2}$

$\therefore x - y = \dfrac{135}{2} - \dfrac{45}{2} 45$ m

503 **(d)**

For point $(1, 3), 3x + 2y = 3 + 6 > 0$

For point $(5, 0), 3 \times 5 + 0 > 0$

and for point $(-1, 2), -3 + 4 > 0$

Similarly, other inequalities also hold

Hence, option (d) is correct

504 **(c)**

Given that, $x_1 = x,\ x_2 = 1,\ x_3 = 0$

and $y_1 = 0,\ y_2 = 1,\ y_3 = 2$

$\therefore$ Area of triangle

$= \dfrac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \dfrac{1}{2}[x(1-2) + 1(2-0) + 0(0-1)]$

$= \dfrac{1}{2}[-x + 2 + 0] = \dfrac{1}{2}(2 - x)$

But area of triangle is 4 sq unit

$\therefore\ \dfrac{1}{2}(2 - x) = 4$

$\Rightarrow\ 2 - x = 8\ \Rightarrow\ x = -6$

505 **(c)**

Given vertices of triangle are

$O(0,0), A(\cos\theta, \sin\theta)$ and $B(\sin\theta, -\cos\theta)$, then coordinates of Centroid are

$\left(\dfrac{\cos\theta + \sin\theta}{3}, \dfrac{\sin\theta - \cos\theta}{3}\right)$

Since, Centroid lies on the line $y = 2x$

$\therefore\ \dfrac{\sin\theta - \cos\theta}{3} = \dfrac{2\cos\theta + 2\sin\theta}{3}$

$\Rightarrow\ \ \sin\theta = -3\cos\theta$

$\Rightarrow\ \theta = \tan^{-1}(-3)$

506 **(a)**

In order to remove first degree terms from the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ the origin is shifted at $(-g/a, -f/a)$

In the equation $2x^2 + 7y^2 + 8x - 14y + 4 = 0$, we have

$a = 2, b = 7, g = 4$ and $f = -7$

Hence, the coordinates of the required point are $(-4/2, -7/7) = (-2,1)$

507 **(b)**

Let $A[at_1t_2, a(t_1 + t_2)], B[at_2t_3, a(t_2 + t_3)], C[at_3t_1, a(t_3 + t_1)]$

Slope of $AB(m_{AB}) = \frac{a(t_3 - t_1)}{at_2(t_3 - t_1)} = \frac{1}{t_2}$

Equation of line through C perpendicular to AB is

$y - a(t_3 + t_1) = -t_2(x - at_3t_1)$

$\Rightarrow\ y - a(t_3 + t_1) = -t_2x + at_1t_2t_3$ ...(i)

Similarly, equation of line through B perpendicular to CA is

$y - a(t_2 + t_3) = -t_1(x - at_2t_3)$

$\Rightarrow\ y - a(t_2 + t_3) = -t_1x + at_1t_2t_3$ ...(ii)

Using $t_1t_2t_3 = -(t_1 + t_2 + t_3)$ in Eqs. (i) and (ii), we get

$y = -t_2x - at_2$

and $y = -t_1x - at_1$

$\Rightarrow\ t_2(x + a) = t_1(x + a)$

$\Rightarrow\ x = -a, y = 0$

508 **(d)**

Let the coordinates of the third vertex C be (h, k). Then,

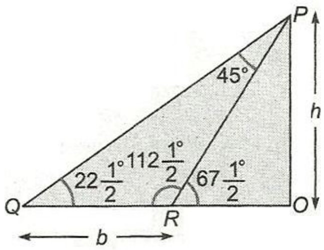
$\dfrac{1}{2}\begin{vmatrix} h & k & 1 \\ -5 & 0 & 1 \\ 3 & 0 & 1 \end{vmatrix} = \pm 20\ \Rightarrow\ k = \pm 5$

Since, (h,k) lies on $x-y=2$. Therefore,
$h-k=2$
For $k=5, h=7$. For $k=-5, h=-3$
Hence, the coordinates of the third vertex C are $(-3,-5)$ or $(7,5)$

509 **(b)**
Let $P(h,k)$ be the required point, then
$4PA^2 = 9PB^2$
$\Rightarrow\ 4(h^2+k^2) = 9(h-4)^2 + 9(k+3)^2$
$\Rightarrow\ 4h^2+4k^2 = 9(h^2+16-8h) + 9(k^2+9+6k)$
$\Rightarrow\ 5h^2+5k^2-72h+54k+225=0$
$\therefore$ Required locus of $P(h,k)$ is
$5x^2+5y^2-72x+54y+225=0$

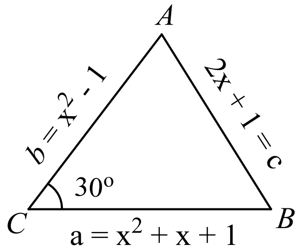
510 **(a)**
The coordinates of the orthocentre O' and circumcentre O are $(2,1)$ and $\left(\frac{7}{2},\frac{5}{2}\right)$ respectively
We know that the centroid G divides OO' in the ratio $1:2$
So, the coordinates of G are
$$\left(\frac{1\times 2+2\times\frac{7}{2}}{1+2},\frac{1\times 1+2\times\frac{5}{2}}{1+2}\right)\equiv(3,2)$$

511 **(a)**
In $\Delta POR, \frac{PR}{\sin 90°} = \frac{h}{\sin 67\frac{1}{2}°}$ ...(i)



And in ΔPQR
$$\frac{PR}{\sin 22\frac{1}{2}°} = \frac{b}{\sin 45°} \quad \text{... (ii)}$$
From Eqs. (i) and (ii)
$$\frac{\sin 45°\, h}{\sin 67\frac{1}{2}°\, b} = \frac{\sin 22\frac{1}{2}°}{\sin 90°} \Rightarrow \frac{h}{b} = \frac{\frac{1}{2}\sin 45°}{\sin 45°}$$
$\Rightarrow\ 2h = b$

512 **(b)**
Using, $\cos C = \frac{a^2+b^2-c^2}{2ab}$



$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{(x^2+x+1)^2+(x^2-1)^2-(2x+1)^2}{2(x^2+x+1)(x^2-1)}$$
$\Rightarrow (x+2)(x+1)(x-1)x + (x^2-1)^2$
$= \sqrt{3}(x^2+x+1)(x^2-1)$
$\Rightarrow\ x^2+2x+(x^2-1) = \sqrt{3}\,(x^2+x+1)$
$\Rightarrow\ 2\left(2-\sqrt{3}\right)x^2 + \left(2-\sqrt{3}\right)x - \left(\sqrt{3}+1\right) = 0$
$\Rightarrow\ x = -\left(2+\sqrt{3}\right)$ and $x = 1+\sqrt{3}$
But, $x = -\left(2+\sqrt{3}\right) \Rightarrow c$ is negative
$\therefore\ x = 1+\sqrt{3}$ is the only solution

513 **(d)**
It is given that the distance between the points $P(a\cos 48°, 0)$ and $Q(0, a\cos 12°)$ is d
i.e., $PQ = d$
$\Rightarrow PQ^2 = d^2$
$\Rightarrow a^2\cos^2 48° + a^2\cos^2 12° = d^2$
$\Rightarrow a^2(1+\cos 96°) + a^2(1+\cos 24°) = 2d^2$
$\Rightarrow 2a^2 + a^2(\cos 96° + \cos 24°) = 2d^2$
$\Rightarrow 2a^2 + 2a^2\cos 60°\cos 36° = 2d^2$
$$\Rightarrow 2a^2 + a^2\left(\frac{\sqrt{5}+1}{4}\right) = 2d^2 \Rightarrow d^2 - a^2$$
$$= \frac{a^2}{8}(\sqrt{5}+1)$$

514 **(b)**
$ab\cos C - ac\cos B$
$$= \frac{a^2+b^2-c^2}{2} - \frac{c^2+a^2-b^2}{2}$$
$$= \frac{a^2+b^2-c^2-c^2-a^2+b^2}{2}$$
$= b^2 - c^2$

515 **(a)**
For collinearity, $\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix} = 0$
For concurrency to lines $a_ix + b_iy + 1 = 0, i = 1, 2, 3$ we have
$\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix} = 0$, so lines are concurrent

516 **(c)**
$(b+c)\cos A + (c+a)\cos B + (a+b)\cos C$
$= b\cos A + c\cos A + c\cos B + a\cos B + a\cos C + b\cos C$
$= (b\cos A + a\cos B) + (c\cos A + a\cos C) + (c\cos B + b\cos C)$
$= c + b + a$

517 **(b)**
By sine rule, $\frac{\sin A}{a} = \frac{\sin B}{b}$
Here, $\frac{\sin\left(\frac{\pi}{2}+B\right)}{5} = \frac{\sin B}{4}$ [by sine rule]

$\Rightarrow \tan B = \frac{4}{5}$

Also, $\angle A + \angle B + \angle C = \pi$

$\Rightarrow \frac{\pi}{2} + 2\angle B + \angle C = \pi$

$\Rightarrow 2\tan^{-1}\left(\frac{4}{5}\right) + \angle C = \frac{\pi}{2}$

$\Rightarrow \angle C = \frac{\pi}{2} - 2\tan^{-1}\left(\frac{4}{5}\right)$

$\Rightarrow \angle C = \frac{\pi}{2} - \tan^{-1}\left(\frac{\frac{8}{5}}{1 - \frac{16}{25}}\right)$

$\Rightarrow = \frac{\pi}{2} - \tan^{-1}\left(\frac{40}{9}\right) = \cot^{-1}\left(\frac{40}{9}\right)$

$\Rightarrow \angle C = \tan^{-1}\left(\frac{9}{40}\right)$

518 **(b)**

We have, $\frac{1}{2}ap_1 = \Delta, \frac{1}{2}bp_2 = \Delta, \frac{1}{2}cp_3 = \Delta$

Where a, b, c are the sides of a triangle

$\Rightarrow p_1 = \frac{2\Delta}{a}, \quad p_2 = \frac{2\Delta}{b}, \quad p_3 = \frac{2\Delta}{c}$

$\therefore \frac{1}{p_1^2} + \frac{1}{p_2^2} + \frac{1}{p_3^2}$

$= \frac{a^2}{4\Delta^2} + \frac{b^2}{4\Delta^2} + \frac{c^2}{4\Delta^2} + \frac{a^2 + b^2 + c^2}{4\Delta^2}$