Single Correct Answer Type

1. If $(1+x)^{15}=C_0+C_1x+C_2x^2+\ldots+C_{15}x^{15}$, then $C_2+2C_3+3C_4+\ldots+14C_{15}$ is equal to
a) 14.2^{14} b) $13.2^{14}+1$ c) $13.2^{14}-1$ d) None of these
2. If the coefficients of second, third and fourth terms in the expansion of $(1+x)^{2n}$ are in A.P., then
a) $2n^2+9n+7=0$ b) $2n^2-9n+7=0$ c) $2n^2-9n-7=0$ d) None of these
3. If $|x|<\frac{1}{2}$, then the coefficient of x^r in the expansion of $\frac{1+2x}{(1-2x)^2}$, is
a) $r2^r$ b) $(2r-1)2^r$ c) $r2^{2r+1}$ d) $(2r+1)2^r$
4. $\binom{30}{0}\binom{30}{10}-\binom{30}{1}\binom{30}{11}+\ldots\binom{30}{20}\binom{30}{30}$ is equal to
a) ${}^{30}C_{11}$ b) ${}^{60}C_{10}$ c) ${}^{30}C_{10}$ d) ${}^{65}C_{55}$
5. If $(1-x+x^2)^n=a_0+a_1x+a_2x^2+\cdots+a_{2n}x^{2n}$, then $a_0+a_2+a_4+\cdots+a_{2n}$ is equal to
a) $\frac{3^n+1}{2}$ b) $\frac{3^n-1}{2}$ c) $\frac{3^{n-1}+1}{2}$ d) $\frac{3^{n-1}-1}{2}$
6. If $C_0, C_1, C_2, \ldots\ldots C_n$ denote the binomial coefficient in the expansion of $(1+x)^n$, then $C_0\frac{C_1}{2}+\frac{C_2}{3}+\ldots+\frac{C_n}{n+1}$ is equal to
a) $\frac{2^{n+1}-1}{n+1}$ b) $\frac{2^n-1}{n}$ c) $\frac{2^{n-1}-1}{n-1}$ d) $\frac{2^{n+1}-1}{n+2}$
7. If the ratio of the 7th term from the beginning to the seventh term from the end in the expansion of $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^x$ is $\frac{1}{6}$ then x, is
a) 9 b) 6, 15 c) 12, 9 d) None of these
8. If in the expansion of $\left(x^3-\frac{1}{x^2}\right)^n$, $n\in N$, sum of the coefficients of x^5 and x^{10} is zero, then $n=$
a) 5 b) 10 c) 15 d) 20
9. The range of the values of the term independent of x in the expansion of $\left(x\sin^{-1}\alpha+\frac{\cos^{-1}\alpha}{x}\right)^{10}$, $\alpha\in[-1,1]$ is
a) $\left[\frac{{}^{10}C_5\cdot\pi^{10}}{2^5}, -\frac{{}^{10}C_5\pi^{10}}{2^{20}}\right]$ b) $\left[-\frac{{}^{10}C_5\cdot\pi^{10}}{2^5}, \frac{{}^{10}C_5\cdot\pi^{10}}{2^{20}}\right]$
c) $\left[\frac{{}^{10}C_5\cdot\pi^5}{2^5}, \frac{{}^{10}C_5\cdot\pi^5}{2^{20}}\right]$ d) $\left[-\frac{{}^{10}C_5\cdot\pi^5}{2^5}, \frac{{}^{10}C_5\cdot\pi^5}{2^{20}}\right]$
10. $\binom{30}{0}\binom{30}{10}-\binom{30}{1}\binom{30}{11}+\ldots+\binom{30}{20}\binom{30}{30}$ is equal to
a) ${}^{30}C_{11}$ b) ${}^{60}C_{10}$ c) ${}^{30}C_{10}$ d) ${}^{65}C_{55}$
11. The rth terms in the expansion of $(a+2n)^n$ is
a) $\frac{n(n+1)\ldots(n-r+1)}{r!}a^{n-r+1}(2x)^r$ b) $\frac{n(n-1)\ldots(n-r+2)}{(r-1)!}a^{n-r+1}(2x)^{r-1}$
c) $\frac{n(n+1)\ldots(n-r)}{r+1}a^{n-r}(x)^r$ d) None of the above
12. The coefficient of t^{24} in the expansion of $(1+t^2)^{12}(1+t^{12})(1+t^{24})$ is
a) ${}^{12}C_6+2$ b) ${}^{12}C_5$ c) ${}^{12}C_6$ d) ${}^{12}C_7$
13. The coefficient of x^n in the expansion of $\frac{(1+x)^2}{(1-x)^3}$, is
a) n^2+2n+1 b) $2n^2+n+1$ c) $2n^2+2n+1$ d) n^2+2n+2
14. The sum of the coefficient in the expansion of $(1+x-3x^2)^{3148}$ is
a) 8 b) 7 c) 1 d) -1
15. If C_r stands for ${}^nC_r$, then the sum of first $(n+1)$ terms of the series $a\,C_0-(a+d)C_1+(a+2d)C_2-(a+3d)C_3+\cdots$, is

a) $\frac{a}{2^n}$ b) $n\ a$ c) 0 d) None of these

16. The value of $\frac{18^3+7^3+3\cdot18\cdot7\cdot25}{3^6+6\cdot243\cdot2+15\cdot181\cdot4+20\cdot27\cdot8+15\cdot9\cdot16+6\cdot3\cdot32+64}$, is
a) 10 b) 1 c) 2 d) 20
17. The coefficient of x^5 in the expansion of $(1+x^2)^5(1+x)^4$, is
a) 30 b) 60 c) 40 d) None of these
18. If $n=5$, then
$({}^nC_0)^2+({}^nC_1)^2+({}^nC_2)^2+....+({}^nC_5)^2$ is equal to
a) 250 b) 254 c) 245 d) 252
19. The coefficient of x^{50} in the expression $(1+x)^{1000}+2x(1+x)^{999}+3x^2(1+x)^{998}+\cdots+1001\,x^{1000}$ is
a) ${}^{1000}C_{50}$ b) ${}^{1001}C_{50}$ c) ${}^{1002}C_{50}$ d) ${}^{1000}C_{51}$
20. For $|x|<1$, the constant term in the expansion of
$\frac{1}{(x-1)^2(x-2)}$ is
a) 2 b) 1 c) 0 d) $-\frac{1}{2}$
21. $1+\frac{2\cdot1}{3\cdot2}+\frac{2\cdot5}{3\cdot6}\left(\frac{1}{2}\right)^2+\frac{2\cdot5\cdot8}{3\cdot6\cdot9}\cdot\left(\frac{1}{2}\right)^3+...$ is equal to
a) $2^{1/3}$ b) $3^{1/4}$ c) $4^{1/3}$ d) $3^{1/3}$
22. If in the expansion of $\left(3x-\frac{2}{x^2}\right)^{15}$ rth term is independent of x, then value of r is
a) 6 b) 10 c) 9 d) 12
23. If $(1+x)^n=C_0+C_1x+C_2x^2+...+C_nx^n$, then the value of $\sum_{0\le r<s\le n}\sum(r+s)(C_r+C_s)$ is
a) $n^2.2^n$ b) $n.2^n$ c) $n^2.2^{2n}$ d) None of these
24. If $C_0, C_1, C_2, \dots, C_n$ denote the binomial coefficient in the expansion of $(1+x)^n$, then the value of $a\,C_0+(a+b)C_1+(a+2\,b)C_2+\cdots+(a+n\,b)C_n$, is
a) $(a+nb)2^n$ b) $(a+nb)2^{n-1}$ c) $(2\,a+nb)2^{n-1}$ d) $(2\,a+nb)2^n$
25. $C_0C_r+C_1C_{r+1}+C_2C_{r+2}+...+C_{n-r}C_n$ is equal to
a) $\frac{(2n)!}{(n-r)!\,(n+r)!}$
b) $\frac{n!}{r!\,(n+r)!}$
c) $\frac{n!}{(n-r)!}$
d) None of these
26. If the coefficients of x^2 and x^3 in the expansion of $(3+ax)^9$ are the same, then the value of a, is
a) $-\frac{7}{9}$ b) $-\frac{9}{7}$ c) $\frac{7}{9}$ d) $\frac{9}{7}$
27. The total number of terms in the expansion of $(x+a)^{100}+(x-a)^{100}$after simplification will be
a) 202 b) 51 c) 50 d) None of these
28. Coefficient of x^{19} in the polynomial $(x-1)(x-2)\ .....(x-20)$is equal to
a) 210 b) -210 c) 20! d) None of these
29. The sum of the last eight coefficient in the expansion of $(1+x)^{15}$ is
a) 2^{16} b) 2^{15} c) 2^{14} d) None of these
30. The number of terms in the expansion of $(a+b+c)^n$ will be
a) $n+1$
b) $n+3$
c) $\frac{(n+1)(n+2)}{2}$
d) None of these
31. The coefficient of y in the expansion of $(y^2+c/y)^5$, is

a) $29\,c$ b) $10\,c$ c) $10\,c^3$ d) $20\,c^2$

32. The value of $(0.99)^{15}$ is
a) 0.8432 b) 0.8601 c) 0.8502 d) None of these

33. The sum of the coefficients in the expansion of $(x+y)^n$ is 4096. The greatest coefficient in the expansion is
a) 1024 b) 924 c) 824 d) 724

34. If in the expansion of $(1+x)^n$, the coefficient of rth and $(r+2)$th term be equal, then r is equal to
a) $2n$ b) $\frac{2n+1}{2}$ c) $\frac{n}{2}$ d) $\frac{2n-1}{2}$

35. If the second, third and fourth term in the expansion of $(x+a)^n$ are 240,720 and 1080 respectively, then the value of n is
a) 15 b) 20 c) 10 d) 5

36. The value of $\frac{1}{81^n}-\frac{10}{81^n}\ {}^{2n}C_1\ \frac{10^2}{81^n}\ {}^{2n}C_2-\frac{10^3}{81^n}2\ {}^{2n}C_3+\ldots+\frac{10^{2n}}{81^n}$ is
a) 2 b) 0 c) $\frac{1}{2}$ d) 1

37. If $(1+x+x^2)^n=\sum_{r=0}^{2n} a_r x^r$
then, $a_1-2a_2+3a_3\ldots-2na_{2n}$ is equal to
a) n b) $-n$ c) 0 d) $2n$

38. The coefficient of the middle term in the expansion of $(1+x)^{2n}$, is
a) $\frac{1\cdot3\cdot5\ldots(2n-1)}{n\,!}2^n$ b) $\frac{1\cdot3\cdot5\ldots(2n-1)}{(n\,!)^2}2^n$ c) $\frac{(2\,n)\,!}{(n\,!)^2}2^{2n}$ d) None of these

39. The constant term in the expansion of $(1+x)^{10}\left(1+\frac{1}{x}\right)^{12}$ is
a) ${}^{22}C_{10}$ b) 0 c) ${}^{22}C_{11}$ d) None of these

40. If $a_1=1$ and $a_n=na_{n-1}$ for all positive integer $n\geq 2$, then a_5 is equal to
a) 125 b) 120 c) 100 d) 24

41. If $(1+x)^n=C_0+C_1x+C_2x^2+\ldots+C_nx^n$, then the value of $C_0+2C_1+3C_2+\ldots+(n+1)C_n$ will be
a) $(n+2)2^{n-1}$
b) $(n+1)2^n$
c) $(n+1)2^{n-1}$
d) $(n+2)2^n$

42. In the expansion of $\left(x^3-\frac{1}{x^2}\right)^n$, $n\in N$, if the sum of the coefficients of x^5 and x^{10} is 0, then $n=$
a) 25 b) 20 c) 15 d) None of these

43. In the expansion of $(1+x+x^2+x^3)^6$, then coefficient of x^{14} is
a) 130 b) 120 c) 128 d) 125

44. The 14th term from the end in the expansion of $\left(\sqrt{x}-\sqrt{y}\right)^{17}$ is
a) ${}^{17}C_5x^6\left(-\sqrt{y}\right)^5$ b) ${}^{17}C_6\left(\sqrt{x}\right)^{11}y^3$ c) ${}^{17}C_4x^{13/2}y^2$ d) None of these

45. The sum of the coefficients in the expansion of $(1+2x+3x^2+\ldots+nx^n)^2$ is
a) $\sum 1$ b) $\sum n$ c) $\sum n^2$ d) $\sum n^3$

46. If a_k is the coefficient of x^k in the expansion of $(1+x+x^2)^n$ for $k=0,1,2,\ldots.,2n$ then
a) $-a_0$ b) 3^n c) $n\cdot3^{n+1}$ d) $n\cdot3^n$

47. The coefficient of x^n in the polynomial $(x+\ {}^nC_0)(x+\ 3\ {}^nC_1)(x+\ 5\ {}^nC_2)\ldots[x+(2n+1)\ {}^nC_n]$
a) $n.\,2^n$ b) $n.\,2^{n+1}$ c) $(n+1)2^n$ d) $n.\,2^n+1$

48. ${}^{n-2}C_r+2^{n-2}C_{r-1}+{}^{n-2}C_{r-2}$ equals
a) ${}^{n+1}C_r$ b) ${}^nC_r$ c) ${}^nC_{r+1}$ d) ${}^{n-1}C_r$

49. For $|x|<1$, the constant term in the expansion of $\frac{1}{(x-1)^2(x-2)}$ is
a) 2 b) 1 c) 0 d) $-\frac{1}{2}$

50. Coefficient of x in the expansion of $\left(x^2 + \frac{a}{x}\right)^5$ is

a) $9a^2$ b) $10a^3$ c) $10a^2$ d) $10a$

51. $\frac{1}{n!} + \frac{1}{2!(n-2)!} + \frac{1}{4!(n-4)!} + \cdots$ is equal to

a) $\frac{2^{n-1}}{n!}$ b) $\frac{2^n}{(n+1)!}$ c) $\frac{2^n}{n!}$ d) $\frac{2^{n-2}}{(n-1)!}$

52. The greatest coefficient in the expansion of $(1+x)^{10}$, is

a) $\frac{10!}{5!6!}$ b) $\frac{10!}{(5!)^2}$ c) $\frac{10!}{5!7!}$ d) None of these

53. In the expansion of $\left(\frac{a}{x} + bx\right)^{12}$,the coefficient of x^{-10} will be

a) $12a^{11}$ b) $12b^{11}a$ c) $12a^{11}b$ d) $12a^{11}b^{11}$

54. The coefficient of x^{10} in the expansion of $(1 + x^2 - x^3)^8$, is

a) 476 b) 496 c) 506 d) 528

55. If the $(r+1)^{\text{th}}$ term in the expansion of $\left\{\sqrt[3]{\frac{a}{\sqrt{b}}} + \sqrt{\frac{b}{\sqrt[3]{a}}}\right\}^{21}$ contains a and b to one and the same power, then the value of r, is

a) 9 b) 10 c) 8 d) 6

56. The $(r+1)$th term in the expansion of $(1-x)^{-4}$ will be

a) $\frac{x^r}{r!}$ b) $\frac{(r+1)(r+2)(r+3)}{6}x^r$

c) $\frac{(r+2)(r+3)}{2}x^r$ d) None of these

57. If $y = \frac{1}{3} + \frac{1\cdot3}{3\cdot6} + \frac{1\cdot3\cdot5}{3\cdot6\cdot9} + \cdots$, then the value of $y^2 + 2\,y$ is

a) 2 b) -2 c) 0 d) None of these

58. Let $S(k) = 1 + 3 + 5 + \ldots + (2k-1) = 3 + k^2$. Then, which of the following is true?

a) $S(1)$ is correct

b) $S(k) \Longrightarrow S(k+1)$

c) $S(k) \not\Rightarrow S(k+1)$

d) Principle of mathematical induction can be used to prove the formula

59. The number of irrational terms in the expansion of $(5^{1/6} + 2^{1/8})^{100}$ is

a) 96 b) 97 c) 98 d) 99

60. If the rth term in the expansion of $(x/3 - 2/x^2)^{10}$ contains x^4, then r is equal to

a) 2 b) 3 c) 4 d) 5

61. When $32^{(32)^{(32)}}$ is divided by 7, then the remainder is

a) 2

b) 8

c) 4

d) None of these

62. The value of x, for which the 6th term in the expansion of $\left\{2^{\log_2 \sqrt{(9^{x-1}+7)}} + \frac{1}{2^{(1/5)\log_2(3^{x-1}+1)}}\right\}^7$ is 84, is equal to

a) 4 b) 3 c) 2 d) 5

63. If $P(n)$: $2 + 4 + 6 + \cdots + (2n), n \in N$, then

$P(k) = k(k+1) + 2$implies

$P(k+1) = (k+1)(k+2) + 2$

is true for all $k \in N$. So, statement $P(n) = n(n+1) + 2$ is true for

a) $n \geq 1$ b) $n \geq 2$ c) $n \geq 3$ d) None of these

64. The number of terms in the expansion of $(1 + 2\,x + x^2)^{20}$, when expanded in descending powers of x, is

a) 20 b) 21 c) 40 d) 41

65. The binomial coefficients which are in decreasing order are

a) ${}^{15}C_5,\ {}^{15}C_6,\ {}^{15}C_7$ b) ${}^{15}C_{10},\ {}^{15}C_9,\ {}^{15}C_8$ c) ${}^{15}C_6,\ {}^{15}C_7,\ {}^{15}C_8$ d) ${}^{15}C_7,\ {}^{15}C_6,\ {}^{15}C_5$

66. $10^n + 3(4^{n+2}) + 5$ is divisible by $(n \in N)$

a) 7 b) 5 c) 9 d) 17

67. $\frac{{}^8C_0}{6} = {}^8C_1 + {}^8C_2 \cdot 6 - {}^8C_3 \cdot 6^2 + \dots + {}^8C_8 \cdot 6^7$ is equal to

a) 0 b) 6^7 c) 6^8 d) $\frac{5^8}{6}$

68. If the coefficients of the second, third and fourth terms in the expansion of $(1+x)^n$ are in AP, then n is equal to

a) 7

b) 2

c) 6

d) None of these

69. The expansion of $(8-3x)^{3/2}$ in terms of powers of x is valid only if

a) $x > \frac{8}{3}$ b) $|x| < \frac{8}{3}$ c) $x < \frac{3}{8}$ d) $x < \frac{8}{3}$

70. If ${}^nC_0,\ {}^nC_1,\ {}^nC_2 \dots .\ {}^nC_n$ denote the coefficient of the binomial expansion $(1+x)^n$, then the value of $C_1 + 3C_3 + 5C_5 + \dots$is

a) $n2^{n-2}$ b) $n2^{n-1}$ c) $(n+1)2^n$ d) $(n+2)2^{n-1}$

71. The value of x in the expansion $[x + x^{\log_{10} x}]^5$, if the third term in the expansion is 1000000, is

a) 10 b) 11 c) 12 d) None of these

72. ${}^nC_0 - \frac{1}{2}\ {}^nC_1 + \frac{1}{3}\ {}^nC_2 - \dots + (-1)^n \frac{{}^nC_n}{n+1}$ is equal to

a) n b) $\frac{1}{n}$ c) $\frac{1}{n+1}$ d) $\frac{1}{n-1}$

73. The remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9, is

a) 0 b) 2 c) 7 d) 8

74. The number of rational terms in the expansion of $\left(\sqrt[3]{4} + \frac{1}{\sqrt[4]{6}}\right)^{20}$, is

a) 3 b) 18 c) 4 d) 16

75. The number of terms in the expansion of $\left(x^2 + 1 + \frac{1}{x^2}\right)^n, n \in N$, is

a) $2\,n$ b) $3\,n$ c) $2n+1$ d) $3n+1$

76. The digit at the unit place in the number $19^{2005} + 11^{2005} - 9^{2005}$ is

a) 2 b) 1 c) 0 d) 8

77. The coefficient of the middle term in the expansion of $(x+2y)^6$ is

a) ${}^6C_3$ b) $8({}^6C_3)$ c) $8({}^6C_5)$ d) ${}^6C_4$

78. The coefficient of x^{-17} in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$ is

a) ${}^{15}C_{11}$ b) ${}^{15}C_{12}$ c) $-{}^{15}C_{11}$ d) $-{}^{15}C_3$

79. If $\frac{(1-3x)^{1/2} + (1-x)^{5/3}}{\sqrt{4-x}}$ is approximately equal to $a + bx$ for small values of x, then (a, b) is equal to

a) $\left(1, \frac{35}{24}\right)$ b) $\left(1, -\frac{35}{24}\right)$ c) $\left(2, \frac{35}{12}\right)$ d) $\left(2, -\frac{35}{12}\right)$

80. If ${}^{18}C_{15} + 2({}^{18}C_{16}) + {}^{17}C_{16} + 1 = {}^nC_3$, then n is equal to

a) 19 b) 20 c) 18 d) 24

81. $49^n + 16n - 1$ is divisible by

a) 3 b) 19 c) 64 d) 29

82. In the expansion of $(1+x)^{50}$, the sum of the coefficients of odd powers of x is

a) 0 b) 2^{49} c) 2^{50} d) 2^{51}

83. The number of terms in the expansion of $(x+y+z)^{10}$, is

a) 11 b) 33 c) 66 d) 1000

84. If $\alpha = \frac{5}{2!\,3} + \frac{5\cdot 7}{3!\,3^2} + \frac{5\cdot 7\cdot 9}{4!\,3^2} + \dots\dots$, then $\alpha^2 + 4\alpha$ equal to

a) 21 b) 23 c) 25 d) 27

85. If $|x| < \frac{1}{2}$, then the coefficient of x^r in the expansion of $\frac{1+2x}{(1-2x)^2}$, is

a) $r2^r$ b) $(2r-1)2^r$ c) $r2^{2r+1}$ d) $(2r+1)2^r$

86. The coefficient of $x^n\ y^n$ in the expansion of $\{(1+x)(1+y)(x+y)\}^n$, is

a) $\sum_{r=0}^{n} C_r^2$ b) $\sum_{r=0}^{n} C_{r+2}^2$ c) $\sum_{r=0}^{n} C_{r+3}^2$ d) $\sum_{r=0}^{n} C_r^3$

87. If $(1+x+x^2)^n = C_0 + C_1x + C_2x^2 + \cdots$, then the value of $C_0C_1 - C_1C_2 + C_2C_3 - \cdots$, is

a) 3^n b) $(-1)^n$ c) 2^n d) None of these

88. If a, b, c are in AP, then the sum of the coefficients of $\{1 + (ax^2 - 2bx + c)^2\}^{1973}$ is

a) -2 b) -1 c) 0 d) 1

89. If the second term in the expansion $\left[\sqrt[13]{a} + \frac{a}{\sqrt{a^{-1}}}\right]^n$ is $14a^{5/2}$, then the value of $\frac{^nC_3}{^nC_2}$ is

a) 4 b) 3 c) 12 d) 6

90. If $n > 1$, then $(1+x)^n - nx - 1$ is divisible by

a) $2\,x$ b) x^2 c) x^3 d) x^4

91. The coefficient of $x^6\ a^{-2}$ in the expansion of $\left(\frac{x^2}{a} - \frac{a}{x}\right)^{12}$, is

a) $^{12}C_6$ b) $-\,^{12}C_5$ c) 0 d) None of these

92. If $(5+2\sqrt{6})^n = I + f; n, I \in N$ and $0 \le f < 1$, then I equals

a) $\frac{1}{f} - f$ b) $\frac{1}{1+f} - f$ c) $\frac{1}{1+f} + f$ d) $\frac{1}{1-f} - f$

93. If $n \in N, n > 1$, then value of $E = a - {}^nC_1(a-1) + {}^nC_2(a-2) + \ldots + (-1)^n(a-n)\,{}^nC_n$ is

a) a

b) 0

c) a^2

d) 2^n

94. If a_r is the coefficient of x^{r-1} in $(1+x)^n + (1+x)^{n+1} + \ldots + (1+x)^{n+k}$ $(n < r-1 \le n+k)$, then $\sum_{r=0}^{n+k+1} (-1)^r a_r$ is equal to

a) 0

b) $n+k+1$

c) $(n+k+1)!$

d) $^{n+k+1}C_r$

95. The sum of $1 + n\left(1-\frac{1}{x}\right) + \frac{n(n+1)}{2!}\left(1-\frac{1}{x}\right)^2 + \cdots \infty$, will be

a) x^n b) x^{-n} c) $\left(1-\frac{1}{x}\right)^n$ d) None of these

96. If $T_0, T_1, T_2, \dots, T_n$ represents the terms in the expansion of $(x+a)^n$, then $(T_0 - T_2 + T_4 - \ldots)^2 + (T_1 - T_3 + T_5 - \ldots)^2$ is equal to

a) $(x^2 + a^2)$ b) $(x^2+a^2)^n$

c) $(x^2+a^2)^{1/n}$ d) $(x^2+a^2)^{-1/n}$

97. If the coefficient of $(2r+1)$th term and $(r+2)$th term in the expansion of $(1+x)^{43}$ are equal, then r is equal to

a) 12 b) 14 c) 16 d) 18

98. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots C_nx^n$, then $C_0^2 + C_1^2 + C_2^2 + C_3^2 + \ldots + C_n^2$ is equal to

a) $\frac{n!}{n!\,n!}$ b) $\frac{(2n)!}{n!\,n!}$ c) $\frac{(2n)!}{n!}$ d) None of these

99. If n is a positive integer and $C_k = {}^nC_k$, then $\sum_{k=1}^{n} k^3 \left(\frac{C_k}{C_{k-1}}\right)^2$ equals

a) $\frac{n(n+1)(n+2)}{12}$ b) $\frac{n(n+1)^2(n+2)}{12}$ c) $\frac{n(n+1)(n+2)^2}{12}$ d) None of these

100. The value of
$1 \times 2 \times 3 \times 4 + 2 \times 3 \times 4 \times 5 + 3 \times 4 \times 5 \times 6 + \cdots$
$+n(n+1)(n+2)(n+3)$, is

a) $\frac{1}{5}(n+1)(n+2)(n+3)(n+4)(n+5)$

b) $\frac{1}{5}n(n+1)(n+2)(n+3)(n+4)$

c) $\frac{1}{5}n(n+1)(n+2)(n+3)(n+4)$

d) ${}^{n+4}C_5$

101. The coefficient of $x^8y^6z^4$ in the expansion of $(x+y+z)^{18}$, is not equal to

a) ${}^{18}C_{14} \times {}^{14}C_8$ b) ${}^{18}C_{10} \times {}^{10}C_6$ c) ${}^{18}C_6 \times {}^{12}C_8$ d) ${}^{18}C_6 \times {}^{14}C_6$

102. The coefficient of x^4 in the expansion of $(1+x+x^2+x^3)^{11}$, is

a) 900 b) 909 c) 990 d) 999

103. If the sum of the coefficients in the expansion of $(1-3\,x+10\,x^2)^n$ is a and if the sum of the coefficients in the expansion of $(1+x^2)^n$ is b, then

a) $a = 3\,b$ b) $a = b^3$ c) $b = a^3$ d) None of these

104. For $n \in N, 10^{n-2} \geq 81n$ is

a) $n > 5$ b) $n \geq 5$ c) $n < 5$ d) $n > 8$

105. The first 3 terms in the expansion of $(1+ax)^n$ $(n \neq 0)$ are $1, 6x$, and $16x^2$. Then, the value of a and n are respectively

a) 2 and 9 b) 3 and 2 c) $\frac{2}{3}$ and 9 d) $\frac{3}{2}$ and 6

106. If the binomial expansion of $(a+b\,x)^{-2}$ is $\frac{1}{4} - 3\,x + \cdots$, then $(a, b) =$

a) $(2, 12)$ b) $(2, 8)$ c) $(-2, -12)$ d) None of these

107. In the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$, the coefficient of x^{39}, is

a) 1365 b) -1365 c) 455 d) -455

108. For natural numbers m, n if $(1-y)^m(1+y)^n = 1 + a_1y^2 + \ldots$and $a_1 = a_2 = 10$, then $(m, n)$is

a) (35,20) b) (45,35) c) (35,45) d) (20,45)

109. If a_1, a_2, a_3, a_4 are the coefficients of any four consecutive terms in the expansion of $(1+x)^n$, then $\frac{a_1}{a_1+a_2} + \frac{a_3}{a_3+a_4}$ is equal to

a) $\frac{a_2}{a_2+a_3}$ b) $\frac{1}{2}\frac{a_2}{(a_2+a_3)}$ c) $\frac{2a_2}{a_2+a_3}$ d) $\frac{2a_3}{a_2+a_3}$

110. If the sum of the coefficient in the expansion of $(a^2x^2 - 6ax + 11)^{10}$, where a is constant is 1024, then the value of a is

a) 5 b) 1 c) 2 d) 3

111. If x^{2r} occurs in $\left(x + \frac{2}{x^2}\right)^n$, then $n - 2r$ must be of the form

a) $3k - 1$ b) $3k$ c) $3k + 1$ d) $3k + 2$

112. $(2^{3n} - 1)$ will be divisible by $(\forall n \in N)$

a) 25 b) 8 c) 7 d) 3

113. If the sum of the coefficients in the expansion of $(\alpha\,x^2 - 2x + 1)^{35}$ is equal to the sum of the coefficient in the expansion of $(x - \alpha\,y)^{35}$, then $\alpha =$

a) 0 b) 1 c) Any real number d) None of these

114. If the ninth term in the expansion of $\left\{3^{\log_3 \sqrt{25^{x-1}+7}} + 3^{-1/8 \log_3 (5^{x-1}+1)}\right\}^{10}$ is equal to 180 and $x > 1$, then x equals

a) $\log_{10} 15$ b) $\log_5 15$ c) $\log_e 15$ d) None of these

115. The coefficient of x^{53} in the following expansion $\sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m} \cdot 2^m$ is

a) ${}^{100}C_{47}$ b) ${}^{100}C_{53}$ c) $-{}^{100}C_{53}$ d) $-{}^{100}C_{100}$

116. The coefficient of the middle term in the binomial expansion in powers of x of $(1+\alpha x)^4$ and of $(1-\alpha x)^6$ is the same, if α equals

a) $-\frac{5}{3}$ b) $\frac{10}{3}$ c) $-\frac{3}{10}$ d) $\frac{3}{5}$

117. The term independent of x in the expansion of $\left(\sqrt{\frac{x}{3}} + \frac{3}{2x^2}\right)^{10}$ will be

a) $\frac{3}{2}$

b) $\frac{5}{4}$

c) $\frac{5}{2}$

d) None of these

118. If $(1+x-2x^2)^6 = 1 + a_1 x + a_2 x^2 + \ldots + a_{12} x^{12}$, then the value of $a_2 + a_4 + \ldots + a_{12}$ Is

a) 31 b) 32 c) 64 d) 1024

119. If $(1+x-3x^2)^{10} = 1 + a_1 x + a_2 x^2 + \ldots + a_{20} x^{20}$, then $a_2 + a_4 + a_6 + \ldots + a_{20}$is equal to

a) $\frac{3^{10}+1}{2}$ b) $\frac{3^9+1}{2}$ c) $\frac{3^{10}-1}{2}$ d) $\frac{3^9-1}{2}$

120. If $(1+x)^{2n} = a_0 + a_1 x + a_2 x^2 + \cdots + a_{2n} x^{2n}$, then

$(a_0 - a_2 + a_4 - a_6 + \cdots - a_{2n})^2 + (a_1 - a_3 + a_5 - a_7 + \cdots + a_{2n-1})^2$ is equal to

a) 2^n b) 4^n c) 0 d) None of these

121. The coefficient of $a^5 b^6 c^7$ in the expansion of $(bc + ca + ab)^9$ is

a) 100 b) 120 c) 720 d) 1260

122. In the polynomial $(x-1)(x-2)(x-3) \ldots (x-100)$. The coefficient of $x^{99}$is

a) 5050 b) -5050 c) 100 d) 99

123. Let n be an odd integer. If $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r \theta$ for every value of θ, then

a) $b_0 = 1, b_1 = 3$ b) $b_0 = 0, b_1 = n$

c) $b_0 = -1, b_1 = n$ d) $b_0 = 0, b_1 = n^2 - 3n + 3$

124. In the expansion of $\left(x + \sqrt{x^2-1}\right)^6 + \left(x - \sqrt{x^2-1}\right)^6$, the number of terms, is

a) 7 b) 14 c) 6 d) 4

125. If n is odd, then $C_0^2 - C_1^2 + C_2^2 - C_3^2 + \ldots + (-1)^n C_n^2$ is equal to

a) 0 b) 1 c) ∞ d) $\frac{n!}{\left(\frac{n}{2}\right)^2 !}$

126. If $(1-x+x^2)^n = a_0 + a_1 x + \ldots a_{2n} x^{2n}$ then the value of $a_0 + a_2 + a_4 + \ldots + a_{2n}$ is

a) $3^n + \frac{1}{2}$ b) $3^n - \frac{1}{2}$ c) $\frac{3^n - 1}{2}$ d) $\frac{3^n + 1}{2}$

127. ${}^{15}C_0 \cdot {}^5C_5 + {}^{15}C_1 \cdot {}^5C_4 + {}^{15}C_2 \cdot {}^5C_3 + {}^{15}C_3 \cdot {}^5C_2 + {}^{15}C_4 \cdot {}^5C_1$ is equal to

a) $2^{20} - 2^5$ b) $\frac{20!}{5!\,15!} - 1$ c) $\frac{20!}{5!\,15!} - 1$ d) $\frac{20!}{5!\,15!} - \frac{15!}{5!\,10!}$

128. In the expansion of the following expression $1 + (1+x) + (1+x)^2 + \ldots + (1+x)^n$, the coefficient of $x^4 (0 \le k \le n)$ is

a) ${}^{n+1}C_{k+1}$ b) ${}^nC_k$ c) ${}^nC_{n-k-1}$ d) None of these

129. In the binomial expansion of $(a-b)^n, n \ge 5$, the sum of 5th and 6th terms is zero,

then $\frac{a}{b}$ equals

a) $\frac{5}{n-4}$ b) $\frac{6}{n-5}$ c) $\frac{n-5}{6}$ d) $\frac{n-4}{5}$

130. The middle term in the expansion of $\left(1-\frac{1}{x}\right)^n (1-x)^n$, is

a) ${}^{2n}C_n$ b) $-{}^{2n}C_n$ c) $-{}^{2n}C_{n-1}$ d) None of these

131. In the expansion of $\left(x^3-\frac{1}{x^2}\right)^{15}$, the constant term, is

a) ${}^{15}C_6$ b) 0 c) $-{}^{15}C_6$ d) 1

132. The number of terms in the expansion of $(a+b+c)^{10}$ is

a) 11 b) 21 c) 55 d) 66

133. The expansion of $\frac{1}{(4-3x)^{1/2}}$ by Only One Correct Option will be valid, if

a) $x<1$

b) $|x|<1$

c) $-\frac{2}{\sqrt{3}}<x<\frac{2}{\sqrt{3}}$

d) None of these

134. The largest term in the expansion of $(3+2x)^{50}$, where $x=\frac{1}{5}$ is

a) 5th b) 3rd c) 7th d) 6th

135. If $(1+ax)^n = 1+8x+24x^2+\ldots$, then the values of a and n are

a) 2, 4 b) 2, 3 c) 3, 6 d) 1, 2

136. The value of $(0.99)^{15}$ is

a) 0.8432 b) 0.8601 c) 0.8502 d) None of these

137. $\frac{1}{1!(n-1)!}+\frac{1}{3!(n-3)!}+\frac{1}{5!(n-5)!}+\ldots$ is equal to

a) $\frac{2^n}{n!}$ b) $\frac{2^{n-1}}{n!}$ c) 0 d) None of these

138. If x is so small that x^3 and higher powers of x may be neglected, then

$\frac{(1+x)^{3/2}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{1/2}}$ may be approximated as

a) $\frac{x}{2}-\frac{3}{8}x^2$ b) $-\frac{3}{8}x^2$ c) $3x+\frac{3}{8}x^2$ d) $1-\frac{3}{8}x^2$

139. The number of terms in the expansion of $(2x+3y-4z)^n$, is

a) $n+1$ b) $n+3$ c) $\frac{(n+1)(n+2)}{2}$ d) None of these

140. If m, n, r are positive integers such that $r<m, n$, then ${}^mC_r + {}^mC_{r-1}\,{}^nC_1 + {}^mC_{r-2}\,{}^nC_2 + \cdots + {}^mC_1\,{}^nC_{r-1} + {}^nC_r$ equals

a) $({}^nC_r)^2$ b) ${}^{m+n}C_r$ c) ${}^{m+n}C_r + {}^mC_r + {}^nC_r$ d) None of these

141. If the expansion in power of x of the function

$\frac{1}{(1-ax)(1-bx)}$ is $a_0+a_1x+a_2x^2+a_3x^3+\ldots$, then a_n is

a) $\frac{a_n-b^n}{b-a}$ b) $\frac{a^{n+1}-b^{n+1}}{b-a}$ c) $\frac{b^{n+1}-a^{n+1}}{b-a}$ d) $\frac{b^n-a^n}{b-a}$

142. If $(1+2x+x^2)^5=\sum_{k=0}^{15} a_kx^k$, then $\sum_{k=0}^{7} = a_{2k}$ is equal to

a) 128 b) 156 c) 512 d) 1024

143. If n is even, then the middle term in the expansion of $\left(x^2+\frac{1}{x}\right)^n$ is $924x^6$, then n is equal to

a) 10 b) 12 c) 14 d) None of these

144. The coefficient of x^5 in the expansion of $(1+x^2)^5(1+x)^4$ is

a) 30
b) 60
c) 40
d) None of these

145. The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^n$ is
a) ${}^nC_4$
b) ${}^nC_4 + {}^nC_2$
c) ${}^nC_4 + {}^nC_2 + {}^nC_2$
d) ${}^nC_4 + {}^nC_2 + {}^nC_1 \cdot {}^nC_2$

146. If a, b, c, d be four consecutive coefficients in the binomial expansion of $(1 + x)^n$, then the value of the expression $\left\{\left(\frac{b}{b+c}\right)^2 - \frac{ac}{(a+b)(c+d)}\right\}$ (where $x > 0$) is
a) < 0
b) > 0
c) $= 0$
d) 2

147. The coefficient of x^3 in $\left(\sqrt{x^5} + \frac{3}{\sqrt{x^3}}\right)^6$, is
a) 0
b) 120
c) 420
d) 540

148. The coefficient of x^{-7} in the expansion of $\left[ax - \frac{1}{bx^2}\right]^{11}$ will be
a) $\frac{462a^6}{b^5}$
b) $\frac{462a^5}{b^6}$
c) $-\frac{462a^5}{b^6}$
d) $-\frac{462a^6}{b^5}$

149. The coefficient of x^5 in the expansion of $(x + 3)^6$ is
a) 18
b) 6
c) 12
d) 10

150. For $r = 0, \ldots, 10$ let A_r, B_r and C_r denotes, respectively, the coefficient of x^r in the $(1 + x)^{10}, (1 + x)^{20}$, and $(1 + x)^{30}$. Then

$$\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$$

is equal to
a) $B_{10} - C_{10}$
b) $A_{10}(B_{10}^2 - C_{10}A_{10})$
c) 0
d) $C_{10} - B_{10}$

151. If p and q be positive, then the coefficients of x^p and x^q in the expansion of $(1 + x)^{p+q}$ will be
a) Equal
b) Equal in magnitude but opposite in sign
c) Reciprocal to each other
d) None of the above

152. If for positive integers $r > 1, n > 2$, the coefficient of the $(3r)$th and $(r + 2)$th powers of x in the expansion of $(1 + x)^{2n}$ are equal, then
a) $n = 2r$
b) $n = 3r$
c) $n = 2r + 1$
d) None of these

153. The range of values of the term independent of x in the expansion of $\left(x \sin^{-1}\alpha + \frac{\cos^{-1}\alpha}{x}\right)^{10}, \alpha \in [-1,1]$, is
a) $\left[-\frac{{}^{10}C_5\, \pi^{10}}{2^5}, \frac{{}^{10}C_5\, \pi^{10}}{2^{20}}\right]$
b) $\left[\frac{{}^{10}C_5\, \pi^2}{2^{20}}, \frac{{}^{10}C_5\, \pi^2}{2^5}\right]$
c) $[1, 2]$
d) $(1, 2)$

154. If the coefficient of rth and $(r + 1)$th terms in the expansion of $(3 + 7x)^{29}$ are equal, then r equals
a) 15
b) 21
c) 14
d) None of these

155. If the third term in the expansion $\left[x + x^{\log_{10} x}\right]^5$ is 10^6, then $x(> 1)$ may be
a) 1
b) 10
c) $10^{-5/2}$
d) 10^2

156. In the expansion of $(1 + x)^{50}$, the sum of the coefficient of add power of x is
a) Zero
b) 2^{49}
c) 2^{50}
d) 2^{51}

157. If the coefficients of r^{th} and $(r + 1)^{th}$ terms in the expansion of $(3 + 7x)^{29}$ are equal, then $r =$
a) 15
b) 21
c) 14
d) None of these

158. In the expansion of $(1 + x)^{2n} (n \in N)$, the coefficients of $(p + 1)^{th}$ and $(p + 3)^{th}$ terms are equal, then

a) $p = n - 2$ b) $p = n - 1$ c) $p = n + 1$ d) $p = 2\,n - 2$

159. Let $(1+x)^n = \sum_{r=0}^{n} a_r\, x^r$. Then,
$\left(1+\frac{a_1}{a_0}\right)\left(1+\frac{a_2}{a_1}\right)\ldots\left(1+\frac{a_n}{a_{n-1}}\right)$ *is equal to*
a) $\frac{(n+1)^{n+1}}{n\,!}$ b) $\frac{(n+1)^{n}}{n\,!}$ c) $\frac{n^{n-1}}{(n-1)\,!}$ d) $\frac{(n+1)^{n-1}}{(n-1)\,!}$

160. If $C_0, C_1, C_2, \ldots, C_n$ denote the binomial coefficients in the expansion of $(1+x)^n$, then the value of $\sum_{r=0}^{n}(r+1)C_r$, is
a) $n\,2^n$ b) $(n+1)2^{n-1}$ c) $(n+2)2^{n-1}$ d) $(n+2)2^{n-2}$

161. If $(2x^2 - x - 1)^5 = a_0 + a_1x + a_2x^2 + \ldots + a_{10}x^{10}$, then $a_2 + a_4 + a_6 + a_8 + a_{10}$ is equal to
a) 15 b) 30 c) 16 d) 32

162. If the coefficient of $(r+1)^{\text{th}}$ term in the expansion of $(1+x)^{2n}$ be equal to that of $(r+3)^{th}$ term, then
a) $n - r + 1 = 0$ b) $n - r - 1 = 0$ c) $n + r + 1 = 0$ d) None of these

163. The coefficient of x^{100} in the expansion of
$\sum_{j=0}^{200}(1+x)^j$ is
a) $\binom{200}{100}$ b) $\binom{201}{102}$ c) $\binom{200}{101}$ d) $\binom{201}{100}$

164. The value of $\frac{1}{n\,!} + \frac{1}{2\,!(n-2)\,!} + \frac{1}{4\,!(n-4)\,!} + \cdots$, is
a) $\frac{2^{n-2}}{(n-1)\,!}$ b) $\frac{2^{n-1}}{n\,!}$ c) $\frac{2^n}{n\,!}$ d) $\frac{2^n}{(n-1)\,!}$

165. The coefficient of x^4 in the expansion of $\left(\frac{x}{2} - \frac{3}{x^2}\right)^{10}$ is
a) $\frac{504}{259}$ b) $\frac{450}{263}$ c) $\frac{405}{256}$ d) None of these

166. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$.Then , $C_0C_1 + C_1C_2 + \ldots + C_{n-1}C_n$ is equal to
a) $\frac{(2n)!}{(n-1)!\,(n+1)!}$ b) $\frac{(2n-1)!}{(n-1)!\,(n+1)!}$ c) $\frac{2n!}{(n+2)!\,(n+1)!}$ d) None of these

167. $7^9 + 9^7$ is divided by
a) 128 b) 24 c) 64 d) 72

168. If $n > \left(8+3\sqrt{7}\right)^{10}, n \in N$, then the last value of n is
a) $\left(8+3\sqrt{7}\right)^{10} - \left(8-3\sqrt{7}\right)^{10}$ b) $\left(8+3\sqrt{7}\right)^{10} + \left(8-3\sqrt{7}\right)^{10}$
c) $\left(8+3\sqrt{7}\right)^{10} - \left(8-3\sqrt{7}\right)^{10} + 1$ d) $\left(8+3\sqrt{7}\right)^{10} - \left(8-3\sqrt{7}\right)^{10} - 1$

169. The ninth term of the expansion $\left(3x - \frac{1}{2x}\right)^8$ is
a) $\frac{1}{512x^9}$ b) $\frac{-1}{512x^9}$ c) $\frac{-1}{256x^8}$ d) $\frac{1}{256x^8}$

170. If x^{2k} occurs in the expansion of $\left(x + \frac{1}{x^2}\right)^{n-3}$, then
a) $n - 2k$ is a multiple of 2 b) $n - 2k$ is a multiple of 3
c) $k = 0$ d) None of the above

171. The number of terms with integral coefficients in the expansion of $\left(7^{1/3} + 5^{1/2}x\right)^{600}$, is
a) 100 b) 50 c) 101 d) None of these

172. The coefficient of $x^3y^4z^5$ in the expansion of $(xy + yz + xz)^6$ is
a) 70 b) 60 c) 50 d) None of these

173. If $(1 + 2\,x + x^2)^n = \sum_{r=0}^{2\,n} a, x^r$, then $a_r =$
a) $({}^nC_r)^2$ b) ${}^nC_r \cdot {}^nC_{r+1}$ c) ${}^{2n}C_r$ d) ${}^{2n}C_{r+1}$

174. ${}^{20}C_4 + 2 \cdot {}^{20}C_3 + {}^{20}C_2 - {}^{22}C_{18}$ is equal to
a) 0 b) 1242 c) 7315 d) 6345

175. If $y = 3\,x + 6\,x^2 + 10\,x^3 + \cdots$, then $x =$

a) $\frac{4}{3}-\frac{1\cdot 4}{3^2\cdot 2}y^2+\frac{1\cdot 4\cdot 7}{3^2\cdot 3}y^3 \dots$
b) $-\frac{4}{3}+\frac{1\cdot 4}{3^2\cdot 2}y^2-\frac{1\cdot 4\cdot 7}{3^2\cdot 3}y^3+\cdots$
c) $\frac{4}{3}+\frac{1\cdot 4}{3^2\cdot 2}y^2+\frac{1\cdot 4\cdot 7}{3^2\cdot 3}y^3+\cdots$
d) None of these

176. The expression $\{x+(x^2-1)^{1/2}\}^5+\{x-(x^2-1)^{1/2}\}^5$ is a polynomial of degree
a) 5 b) 6 c) 7 d) 8

177. The value of $C_0^2+3\cdot C_1^2+5\cdot C_2^2+\cdots$ to $(n+1)$ terms, is
a) ${}^{2n-1}C_{n-1}$
b) $(2n+1)^{2n-1}C_n$
c) $2(n+1)\cdot {}^{2n-1}C_{n-1}$
d) ${}^{2n-1}C_n+(2\,n+1)\,{}^{2n-1}C_{n-1}$

178. If $n-{}^1C_r=(k^2-3)\,{}^nC_{r+1}$, then $k\in$
a) $(-\infty,-2)$ b) $[2,\infty)$ c) $[-\sqrt{3},\sqrt{3}]$ d) $(\sqrt{3},2]$

179. The total number of terms in the expansion of $(x+y)^{100}+(x-y)^{100}$ after simplification is
a) 51 b) 202 c) 100 d) 50

180. If $(1+x)^n=C_0+C_1\,x+C_2\,x^2+\cdots+C_n\,x^n$, thn for n odd, $C_1^2+C_3^2+C_5^2+\cdots+C_n^2$ is equal to
a) 2^{2n-2} b) 2^n c) $\frac{(2\,n)\,!}{2\,(n\,!)^2}$ d) $\frac{(2\,n)\,!}{(n\,!)^2}$

181. $\sum_{k=0}^{10}\ {}^{20}C_k$ is equal to
a) $2^{19}+\frac{1}{2}\,{}^{20}C_{10}$ b) 2^{19} c) ${}^{20}C_{10}$ d) None of these

182. The approximate value of $(7.995)^{1/3}$ correct to four decimal places is
a) 1.9995 b) 1.9996 c) 1.9990 d) 1.9991

183. If the binomial coefficients of 2nd , 3rd and 4th terms in the expansion of $\left\{\sqrt{2^{\log_{10}(10-3^x)}}+\sqrt[5]{2^{(x-2)\log_{10}3}}\right\}^m$ are in A.P and the 6th term is 21, then the value(s) of x, is (are)
a) 1, 3 b) 0, 2 c) 4 d) -1

184. If ${}^nC_{12}={}^nC_6$, then ${}^nC_2$ is equal to
a) 72 b) 153 c) 306 d) 2556

185. In the expansion of $\left(x-\frac{1}{x}\right)^6$, the coefficient of x^0 is
a) 20 b) -20 c) 30 d) -30

186. The term independent of x in the expansion of $\left(x^3+\frac{2}{x^2}\right)^{15}$ is
a) T_7 b) T_8 c) T_9 d) T_{10}

187. If the $(r+1)$th term in the expansion of $\left(\frac{a^{1/3}}{b^{1/6}}+\frac{b^{1/2}}{a^{1/6}}\right)^{21}$ has equal exponents of both a and b, then value of r is
a) 8 b) 9 c) 10 d) 11

188. The coefficient of $1/x$ in the expansion of $\left(\frac{1}{x}+1\right)^n(1+x)^n$ is
a) ${}^{2n}C_n$ b) ${}^{2n}C_{n-1}$ c) ${}^{2n}C_1$ d) ${}^nC_{n-1}$

189. Let $[x]$ denote the greatest integer less than or equal to x. If $x=\left(\sqrt{3}+1\right)^5$, then $[x]$ is equal to
a) 75 b) 50 c) 76 d) 152

190. The value of $2\,C_0+\frac{2^2}{2}C_1+\frac{2^3}{3}C_2+\frac{2^4}{4}C_3+\cdots+\frac{2^{11}}{11}C_{10}$, is
a) $\frac{3^{11}-1}{11}$ b) $\frac{2^{11}-1}{11}$ c) $\frac{11^3-1}{11}$ d) $\frac{11^2-1}{11}$

191. The sum of coefficients of the expansion $\left(\frac{1}{x}+2x\right)^n$ in 6561. The coefficient of term independent of x is

a) $16\ {}^{8}C_4$ b) ${}^{8}C_4$ c) ${}^{8}C_5$ d) None of these

192. In the expansion of $(1+x)^{30}$, the sum of the coefficients of odd powers of x is

a) 2^{30} b) 2^{31} c) 0 d) 2^{29}

193. The 6th term in the expansion of $\left(2x^2-\frac{1}{3x^2}\right)^{10}$ is

a) $\frac{4580}{17}$ b) $-\frac{896}{27}$ c) $\frac{5580}{17}$ d) None of these

194. ${}^{47}C_4+\sum_{r=1}^{5}\ {}^{52-r}C_3$ is equal to

a) ${}^{45}C_6$ b) ${}^{52}C_5$ c) ${}^{52}C_4$ d) None of these

195. The coefficient of x^n in the expansion of $(1-2\,x+3\,x^2-4\,x^3+\cdots)^{-n}$, is

a) $\frac{(2\,n)\,!}{n\,!}$ b) $\frac{(2\,n)\,!}{(n\,!)^2}$ c) $\frac{1}{2}\frac{(2\,n)\,!}{(n\,!)^2}$ d) None of these

196. The term independent of x in the expansion of $(1+x)^n(1+1/x)^n$, is

a) $C_0^2+2\,C_1^2+3\cdot C_2^2+\cdots+(n+1)C_n^2$

b) $(C_0+C_1+\cdots+C_n)^2$

c) $C_0^2+C_1^2+\cdots+C_n^2$

d) None of these

197. If A and B are coefficients of x^r and x^{n-r} respectively in the expansion of $(1+x)^n$, then

a) $A=B$ b) $A+B=0$ c) $A=r\,B$ d) $A=nB$

198. If $x=\frac{\begin{bmatrix}729+6(2)(243)+15(4)(81)\\+20(8)(27)+15(16)(9)\\+6(32)3+64\end{bmatrix}}{1+4(4)+6(16)+4(64)+256}$, then $\sqrt{x}-\frac{1}{\sqrt{x}}$ isequal to

a) 0.2 b) 4.8 c) 1.02 d) 5.2

199. If the coefficients of pth, $(p+1)$th and $(p+2)$th terms in the expansion of $(1+x)^n$ are in AP, then

a) $n^2-2np+4p^2=0$

b) $n^2-n(4p+1)+4p^2-2=0$

c) $n^2-n(4p+1)+4p^2=0$

d) None of the above

200. The sum of the rational terms in the expansion of $(\sqrt{2}+3^{1/5})^{10}$ is

a) 41 b) 32 c) 18 d) 9

201. Let T_n denotes the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1}-T_n=21$, then n equals

a) 5 b) 7 c) 6 d) 4

202. Sum of the last 30 coefficients in the expansion of $(1+x)^{59}$, when expanded in ascending powers of x is

a) 2^{59} b) 2^{58} c) 2^{30} d) 2^{29}

203. If $|x|<1$, then $1+n\left(\frac{2x}{1+x}\right)+\frac{n(n+1)}{2!}\left(\frac{2x}{1+x}\right)^2+\ldots.$ is equal to

a) $\left(\frac{2x}{1+x}\right)^n$ b) $\left(\frac{1+x}{2x}\right)^n$ c) $\left(\frac{1-x}{1+x}\right)^n$ d) $\left(\frac{1+x}{1-x}\right)^n$

204. In the expansion of $(1+3x+2x^2)^6$ the coefficient of x^{11} is

a) 144 b) 288 c) 216 d) 576

205. The term independent of x in the expansion of $(1-x)^2\left(x+\frac{1}{x}\right)^{10}$, is

a) ${}^{11}C_5$ b) ${}^{10}C_5$ c) ${}^{10}C_4$ d) None of these

206. If the sum of the coefficient in the expansion of $(x-2y+3z)^n$ is 128, then the greatest coefficient in the expansion of $(1+x)^n$ is

a) 35 b) 20 c) 10 d) None of these

207. The digit at the unit place in the number $19^{2005} + 11^{2005} - 9^{2005}$ is

a) 2 b) 1 c) 0 d) 8

208. The sum of the series

$\sum_{r=0}^{n}(-1)^r\, {}^nC_r\left(\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\frac{15^r}{2^{4r}}+\ldots+m \text{ terms}\right)$ is

a) $\frac{2^{mn}-1}{2^{mn}(2^n-1)}$

b) $\frac{2^{mn}-1}{2^n-1}$

c) $\frac{2^{mn}+1}{2^n+1}$

d) None of these

209. r and n are positive integers such that $r > 1, n > 2$ and coefficients of $(r+2)^{\text{th}}$ term and $(3r)^{\text{th}}$ term in the expansion of $(1+x)^{2n}$ are equal, then n equals

a) $3r$ b) $3r+1$ c) $2r$ d) $2r+1$

210. Assuming to be small that x^2 and higher powers of x can be neglected, then $\frac{\left(1+\frac{3}{4}x\right)^{-4}(16-3x)^{1/2}}{(8+x)^{2/3}}$ is approximately equal to

a) $1+\frac{305}{96}x$ b) $1-\frac{305}{96}x$ c) $1+\frac{96}{305}x$ d) $1-\frac{96}{305}x$

211. The coefficient of x^5 in $(1+x^2)^5(1+x)^4$ is

a) 20 b) 30 c) 60 d) 55

212. The number of terms in the expansion of $\left(1+5\sqrt{2}\,x\right)^9+\left(1-5\sqrt{2}\,x\right)^9$, is

a) 5 b) 7 c) 9 d) 10

213. If $x=\frac{1}{3}$, then the greatest term in the expansion of $(1+4x)^8$ is the

a) 3^{rd} term b) 6^{th} term c) 5^{th} term d) 4^{th} term

214. The coefficient of x^n in the expansion of $(1+2x+3x^2+\ldots)^{1/2}$ is

a) -1 b) 0 c) 2 d) 1

215. The coefficient of $\lambda^n\mu^n$ in the expansion of $[(1+\lambda)(1+\mu)(\lambda+\mu)]^n$ is

a) $\sum_{r=0}^{n} C_r^2$ b) $\sum_{r=0}^{n} C_{r+2}^2$ c) $\sum_{r=0}^{n} C_{r+3}^2$ d) $\sum_{r=0}^{n} C_r^3$

216. If $(1+x-2x^2)^6 = 1 + C_1x + C_2x^2 + C_3x^3 + \cdots + C_{12}x^{12}$, then the value of $C_2 + C_4 + C_6 + \cdots + C_{12}$, is

a) 30 b) 32 c) 31 d) None of these

217. If in the expansion of $(1+ax)^n, n \in N$, the coefficients of x and x^2 are 8 and 24 respectively, then

a) $a=2, n=4$ b) $a=4, n=2$ c) $a=2, n=6$ d) $a=-2, n=4$

218. The greatest coefficient in the expansion of $(1+x)^{2n}$ is

a) ${}^{2n}C_n$ b) ${}^{2n}C_{n+1}$ c) ${}^{2n}C_{n-1}$ d) ${}^{2n}C_{2n-1}$

219. The coefficient of x^4 in $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$ is

a) $\frac{405}{256}$ b) $\frac{504}{259}$ c) $\frac{450}{263}$ d) None of these

220. If n is an odd natural number and ${}^nC_0 < {}^nC_1 < {}^nC_2 < \cdots < {}^nC_r > {}^nC_{r+1} > {}^nC_{r+2} > \cdots > {}^nC_n$, then $r=$

a) $\frac{n}{2}$ b) $\frac{n-1}{2}$ c) $\frac{n-2}{2}$ d) Does not exist

221. If $[x]$ denotes the greatest integer less than or equal to x, and $F = R - [R]$ where $R = \left(5\sqrt{5}+11\right)^{2n+1}$, then $R\,F$ is equal to

a) 4^{2n+1} b) 4^{2n} c) 4^{2n-1} d) None of these

222. If the coefficients of 5th, 6th and 7th terms in the expansion of $(1+x)^n$ be in AP, then the value of n is

a) 7 only b) 14 only c) 7 or 14 d) None of these

223. Let $R = (2+\sqrt{3})^{2n}$ and $f = R - [R]$ where $[\cdot]$ denotes the greatest integer function, then $R(1-f)$ is equal

to

a) 1 b) 2^{2n} c) $2^{2n}-1$ d) ${}^{2n}C_n$

224. The value of

$({}^7C_0+{}^7C_1)+({}^7C_1+{}^7C_2)+\cdots+({}^7C_6+{}^7C_7)$is

a) 2^8-1 b) 2^8+1 c) 2^8 d) 2^8-2

225. The value of ${}^{4n}C_0+{}^{4n}C_4+{}^{4n}C_8+\ldots+{}^{4n}C_{4n}$ is

a) $2^{4n-2}+(-1)^n 2^{2n-1}$

b) $2^{4n-2}+2^{2n-1}$

c) $2^{2n-1}+(-1)^n 2^{4n-2}$

d) None of these

226. If there is a term containing x^{2r} in $\left(x+\frac{1}{x^2}\right)^{n-3}$, then

a) $n-2r$ is a positive integral multiple of 3

b) $n-2r$ is even

c) $n-2r$ is odd

d) None of these

227. Last two digit of the number 19^{9^4} is

a) 19 b) 29 c) 39 d) 81

228. $1+\frac{1}{3}x+\frac{1\cdot4}{3\cdot6}x^2+\frac{1\cdot4\cdot7}{3\cdot6\cdot9}x^3+\ldots$ is equal to

a) x b) $(1+x)^{1/3}$ c) $(1-x)^{1/3}$ d) $(1-x)^{-1/3}$

229. What is the sum of the coefficient of $(x^2-x-1)^{99}$?

a) 1 b) 0 c) -1 d) None of these

230. If n is even positive integer, then the condition that the greatest term in the expansion of $(1+x)^n$ may have the greatest coefficient also, is

a) $\dfrac{n}{n+2}<x<\dfrac{n+2}{n}$ b) $\dfrac{n+1}{n}<x<\dfrac{n}{n+1}$ c) $\dfrac{n}{n+4}<x<\dfrac{n+4}{4}$ d) None of these

231. The value of ${}^{15}C_8+{}^{15}C_9-{}^{15}C_6-{}^{15}C_7$ is

a) -1 b) 0 c) 1 d) None of these

232. If $(1-x+x^2)^n=a_0+a_1x+a_2x^2+\ldots+a_{2n}x^{2n}$, then the $a_0+a_2+a_4+\ldots+a_{2n}$ is equal to

a) $\dfrac{3^n+1}{2}$

b) $\dfrac{3^n-1}{2}$

c) $\dfrac{1-3^n}{2}$

d) $3^n+\dfrac{1}{2}$

233. If a_r is the coefficient of x^r , in the expansion of $(1+x+x^2)^n$, then $a_1-2a_2+3a_3-\ldots-2n\,a_{2n}$ is equal to

a) 0 b) n c) $-n$ d) $2n$

234. If ${}^{n-1}C_r=(k^2-3).\ {}^nC_{r+1}$, then k is belongs to

a) $(-\infty,-2]$ b) $[2,\infty)$ c) $[-\sqrt{3},\sqrt{3}]$ d) $(\sqrt{3},2]$

235. The greatest value of the term independent of x, as α varies over R, in the expansion of $\left(x\cos\alpha+\frac{\sin\alpha}{x}\right)^{20}$ is

a) ${}^{20}C_{10}$ b) ${}^{20}C_{15}$ c) ${}^{20}C_{19}$ d) None of these

236. If the coefficient of the middle term in the expansion of $(1+x)^{2n+2}$ is p and the coefficients of middle term in the expansion of $(1+x)^{2n+1}$ are q and r, then

a) $p+q=r$ b) $p+r=q$ c) $p=q+r$ d) $p+q+r=0$

237. If sum of the coefficients of the first, second and third terms of the expansion of $\left(x^2+\frac{1}{x}\right)^m$ is 46, then the coefficient of the term that does not contain x is

a) 84

b) 92
c) 98
d) 106

238. If the r^{th} $(r+1)^{\text{th}}$ and $(r+2)^{\text{th}}$ coefficients of $(1+x)^n$ are in A.P., then n is a root of the equation
a) $x^2 - x(4\,r+1) + 4\,r^2 - 2 = 0$
b) $x^2 + x(4\,r+1) + 4\,r^2 - 2 = 0$
c) $x^2 + x(4\,r+1) + 4\,r^2 + 2 = 0$
d) None of these

239. If $|x| < 1$, then the coefficient of x^6 in the expansion of $(1+x+x^2)^{-3}$ is
a) 3 b) 6 c) 9 d) 12

240. $\left(1+\frac{C_1}{C_0}\right)\left(1+\frac{C_2}{C_1}\right)\left(1+\frac{C_3}{C_2}\right)...\left(1+\frac{C_n}{C_{n-1}}\right)$ is equal to
a) $\frac{n+1}{n!}$ b) $\frac{(n+1)^n}{(n-1)!}$ c) $\frac{(n-1)^n}{n!}$ d) $\frac{(n+1)^n}{n!}$

241. The coefficient of x^{20} in the expansion of $(1+3x+3x^2+x^3)^{20}$ is
a) ${}^{60}C_{40}$ b) ${}^{30}C_{20}$ c) ${}^{15}C_2$ d) None of these

242. The greatest value of the term independent of x in the expansion of $(x\sin\alpha + x^{-1}\cos\alpha)^{10}, \alpha \in R$ is
a) 2^5 b) ${}^{10}C_5$ c) $\frac{1}{2^5}\,({}^{10}C_5)$ d) None of these

243. If $(1+2x+3x^2)^{10} = a_0 + a_1\,x + a_2\,x^2 + \cdots + a_{20}\,x^{20}$, then a_1 equals
a) 10 b) 20 c) 210 d) None of these

244. If the coefficient of rth, $(r+1)$th and $(r+2)$ th terms in the binomial expansion of $(1+y)^m$ are in AP, then m and r satisfy the equation
a) $m^2 - m(4r-1) + 4r^2 + 2 = 0$ b) $m^2 - m(4r+1) + 4r^2 - 2 = 0$
c) $m^2 - m(4r+1) + 4r^2 + 2 = 0$ d) $m^2 - n(4r-1) + 4r^2 - 2 = 0$

245. Coefficient of the term independent of x in the expansion
$\left(x+\frac{1}{x^2}\right)^6$ is equal to
a) 10 b) 15 c) 16 d) None of the above

246. The last term in the binomial expansion of $\left(\sqrt[3]{2}-\frac{1}{\sqrt{2}}\right)^n$ is $\left(\frac{1}{3\cdot\sqrt[3]{9}}\right)^{\log_3 8}$. Then, the 5th term from the beginning is
a) ${}^{10}C_6$ b) $2\times{}^{10}C_4$ c) $\frac{1}{2}\times{}^{10}C_4$ d) None of these

247. Using mathematical induction, then numbers $a_n's$ are defined by
a_0=1, $a_{n+1} = 3n^2 + n + a_n, (n \geq 0)$ Then, a_n is equal to
a) n^3+n^2+1 b) n^3-n^2+1 c) n^3-n^2 d) n^3+n^2

248. The coefficient of x^{-10} in $\left(x^2-\frac{1}{x^3}\right)^{10}$ is
a) -252 b) 210 c) $-(5!)$ d) -120

249. The value of $C_0 + 3\,C_1 + 5\,C_2 + 7\,C_3 \ldots + (2\,n+1)\,C_n$ is equal to
a) 2^n b) $2^n + n\cdot 2^{n-1}$ c) $2^n\cdot(n+1)$ d) None of these

250. If p is nearly equal to q and $n > 1$, such that $\frac{(n+1)p+(n-1)q}{(n-1)p+(n+1)q} = \left(\frac{p}{q}\right)^k$, then the value of k, is
a) n b) $\frac{1}{n}$ c) $n+1$ d) $\frac{1}{n+1}$

251. If the sum of the coefficients in the expansion of $(a^2x^2 - 2ax + 1)^{51}$ vanishes, then the value of a is
a) 2 b) -1 c) 1 d) -2

252. $1+\frac{2}{4}+\frac{2\cdot 5}{4\cdot 8}+\frac{2\cdot 5\cdot 8}{4\cdot 8\cdot 12}+\frac{2\cdot 5\cdot 8\cdot 11}{4\cdot 8\cdot 12\cdot 16}+\ldots$ is
a) $4^{-2/3}$ b) $\sqrt[3]{16}$ c) $\sqrt[3]{4}$ d) $4^{3/2}$

253. The coefficient of $x^r (0 \le r \le (n-1))$ in the expansion of $(x+3)^{n-1} + (x+3)^{n-2}(x+2) + (x+$ *3n−3 x+22+...+x+2n−1,* is

a) ${}^nC_r(3^r - 2^n)$ b) ${}^nC_r(3^{n-r} - 2^{n-r})$ c) ${}^nC_r(3^r + 2^{n-r})$ d) None of these

254. Let C_1, C_2, C_3 are the usual binomial coefficients. Let $S = C_1 + 2C_2 + 3C_3 + \ldots + nC_n$, then S equals

a) $n2^n$ b) 2^{n-1} c) $n2^{n-1}$ d) 2^{n+1}

255. If the value of x is so small that $x^2$and greater powers can be neglected, then $\frac{\sqrt{1+x}+\sqrt[3]{(1-x)^2}}{1+x+\sqrt{1+x}}$ is equal to

a) $1+\frac{5}{6}x$ b) $1-\frac{5}{6}x$ c) $1+\frac{2}{3}x$ d) $1-\frac{2}{3}x$

256. The coefficient of x^n in the expansion of $(1+x)(1-x)^n$ is

a) $(n-1)$ b) $(-1)^n(1-n)$ c) $(-1)^{n-1}(n-1)^2$ d) $(-1)^{n-1}n$

257. The middle term in the expansion of $(x-a)^8$ is

a) ${}^{-8}C_4x^4a^4$ b) ${}^8C_4x^4a^4$ c) ${}^8C_3x^5a^3$ d) ${}^{-8}C_5x^2a^5$

258. If the expansion of $(1+x)^{20}$, the coefficients of r^{th} and $(r+4)^{th}$ terms are equal, then the value of r, is

a) 7 b) 8 c) 9 d) 10

259. If $(1+x-2x^2)^6 = 1 + a_1\,x + a_2\,x^2 + \cdots + a_{12}\,x^{12}$, then $a_2 + a_4 + a_6 + \cdots + a_{12} =$

a) 30 b) 65 c) 31 d) 63

260. The value of $\frac{1}{81^n} = \frac{10}{81^n}\ {}^{2n}C_1 + \frac{10^2}{81^n}\ {}^{2n}C_2 - \frac{10^3}{81^n}\ {}^{2n}C_3 + \cdots + \frac{10^{2n}}{81^n}$, is

a) 2 b) 0 c) 1/2 d) 1

261. The value of $\left(\frac{{}^{50}C_0}{1} + \frac{{}^{50}C_2}{3} + \frac{{}^{50}C_4}{5} + \ldots + \frac{{}^{50}C_{50}}{51}\right)$ is

a) $\frac{2^{50}}{51}$ b) $\frac{2^{50}-1}{51}$ c) $\frac{2^{50}-1}{50}$ d) $\frac{2^{51}-1}{51}$

262. Matrix A is such that $A^2 = 2A - I$ where I is the identity matrix, then for $n \ge 2$, A^n is equal to

a) $n\,A - (n-1)I$ b) $n\,A - I$ c) $2^{n-1}A - (n-1)I$ d) $2^{n-1}A - I$

263. If $\sum_{r=0}^{2n} a_r(x-100)^r = \sum_{r=0}^{2n} b_r(x-101)^r$ and

$a_k = \frac{2^k}{{}^kC_n}$ for all $k \ge n$, then b_n equals

a) $2^n(2^{n+1}-1)$ b) $2^n(2^n+1)$ c) $2^n(2^n-1)$ d) $2^{n+1}(2^n-1)$

264. The coefficient of $x^r[0 \le r \le (n-1)]$ in the expansion of $(x+3)^{n-1} + (x+3)^{n-2}(x+2) + (x+$ *3n−3x+22+...+x+2n−1* is

a) ${}^nC_r(3^r - 2^n)$

b) ${}^nC_r(3^{n-r} - 2^{n-r})$

c) ${}^nC_r(3^r + 2^{n-1})$

d) None of these

265. The middle term in the expansion of $\left(x+\frac{1}{x}\right)^{10}$, is

a) ${}^{10}C_1\frac{1}{x}$ b) ${}^{10}C_5$ c) ${}^{10}C_6$ d) ${}^{10}C_7x$

266. The sum of the magnitudes of the coefficients in the expansion of $(1-x+x^2-x^3)^n$, is

a) 0 b) 2^n c) 3^n d) 4^n

267. The coefficient of x^7 in the expansion of $(x-2\,x^2)^{-3}$, is

a) 67485 b) 67548 c) 67584 d) 67845

268. If $n > 3$, then

$xyz\,C_0 - (x-1)(y-1)(z-1)C_1 + (x-2)(y-2)(z-2)C_2$
$-(x-3)(y-3)(z-3)C_3 + \cdots + (-1)^n(x-n)(y-n)(z-n)C_n$ equals

a) xyz b) $nxyz$ c) $-xyz$ d) 0

269. $\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + \ldots + 15\frac{C_{15}}{C_{14}}$ is equal to

a) 100 b) 120 c) -120 d) None of these

270. The digit at unit's place in the number $17^{1995} + 11^{1995} - 7^{1995}$, is

a) 0 b) 1 c) 2 d) 3

271. The value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is

a) ${}^{56}C_4$ b) ${}^{56}C_3$ c) ${}^{55}C_3$ d) ${}^{55}C_4$

272. The term independent of x in the expansion of

$\left(x - \frac{1}{x}\right)^4 \left(x + \frac{1}{x}\right)^3$ is

a) -3 b) 0 c) 3 d) 1

273. The coefficients of x^2y^2, yzt^2 and $xyzt$ in the expansion of $(x+y+z+t)^4$ are in the ratio

a) $4:2:1$ b) $1:2:4$ c) $2:4:1$ d) $1:4:2$

274. If $(1 + 2x + 3x^2)^{10} = a_0 + a_1x + a_2x^2 + \ldots + a_{20}x^{20}$, then a_1 equals

a) 10 b) 20 c) 210 d) None of these

275. The coefficient of x^5 in the expansion of

$(1 + x^2)^5(1 + x)^4$ is

a) 30 b) 60 c) 40 d) None of these

276. The sum of the series $\sum_{r=0}^{10} {}^{20}C_r$, is

a) 2^{20} b) 2^{19} c) $2^{19} + \frac{1}{2}\ {}^{20}C_{10}$ d) $2^{19} - \frac{1}{2}\ {}^{20}C_{10}$

277. If the last term in the binomial expansion of $\left(2^{1/3} - \frac{1}{\sqrt{2}}\right)^n$ is $\left(\frac{1}{3^{5/3}}\right)^{\log_3 8}$, then the 5th term from the beginning is

a) 210 b) 420 c) 105 d) None of these

278. The coefficient of x^{-10} in $\left(x^2 - \frac{1}{x^3}\right)^{10}$, is

a) -252 b) 210 c) $-5!$ d) -120

279. The coefficient of x^n in the binomial expansion of $(1 - x)^{-2}$, is

a) $\frac{2^n}{2!}$ b) $n + 1$ c) n d) $2n$

280. Let $(1 + x)^n = \sum_{r=0}^{n} C_r\, x^r$ and, $\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + \cdots + n\frac{C_n}{C_{n-1}} = \frac{1}{k}n(n+1)$, then the value of k, is

a) 1/2 b) 2 c) 1/3 d) 3

281. The coefficient of x^m in $(1+x)^p + (1+x)^{p+1} + \cdots + (1+x)^n, p \le m \le n$ is

a) ${}^{n+1}C_{m+1}$ b) ${}^{n-1}C_{m-1}$ c) ${}^{n}C_{m}$ d) ${}^{n}C_{m+1}$

282. If $(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \ldots + {}^nC_nx^n$, then $\frac{{}^nC_1}{{}^nC_0} + \frac{2\,{}^nC_2}{{}^nC_1} + \frac{3\,{}^nC_3}{{}^nC_2} + \ldots + \frac{n\,{}^nC_n}{{}^nC_{n-1}}$ is equal to

a) $\frac{n(n-1)}{2}$

b) $\frac{n(n+2)}{2}$

c) $\frac{n(n+1)}{2}$

d) $\frac{(n-1)(n-2)}{2}$

283. If the magnitude of the coefficient of x^7 in the expansion of

$\left(x^2 + \frac{1}{bx}\right)^8$, where a, b, are positive numbers, is equal to the magnitude of the

coefficient of x^{-7} in the of $\left(ax - \frac{1}{bx^2}\right)^8$, then a, and b are connected by the relation

a) $ab = 1$ b) $ab = 2$ c) $a^2b = 1$ d) $ab^2 = 2$

284. If $T_0, T_1, T_2, \ldots, T_n$ represent the term in the expansion of $(x + a)^n$, then the value of $(T_0 - T_2 + T_4 -$ *T6…2+T1−T3+T5+…2,* is

a) $(x^2 - a^2)^n$ b) $(x^2 + a^2)^n$ c) $(a^2 - x^2)^n$ d) None of these

285. The sum $\sum_{i=0}^{m} \binom{10}{i}\binom{20}{m-i}$, $\left(\text{where}, \binom{p}{q} = 0 \text{ if } p < q\right)$ is maximum, when m is

a) 5 b) 10 c) 15 d) 20

286. Let $(1+x)^n = 1 + a_1x + a_2x^2 + \ldots + a_nx^n$. If a_1, a_2 and a_3 are in AP, then the value of n is

a) 4 b) 5 c) 6 d) 7

287. If $(1+x)^{15} = a_0 + a_1x + \ldots + a_{15}x^{15}$, then $\sum_{r=1}^{15} r\frac{a_r}{a_{r-1}}$ is equal to

a) 110 b) 115 c) 120 d) 135

288. The coefficient of x^5 in the expansion of $(1+x^2)^5(1+x)^4$, is

a) 30 b) 60 c) 40 d) None of these

289. The coefficient of x^r in the expansion of $(1-x)^{-2}$ is

a) r b) $r+1$ c) $r+3$ d) $r-1$

290. The expression $\frac{1}{\sqrt[3]{6-3x}}$ is equal to

a) $6^{1/3}\left[1+\frac{x}{6}+\frac{2x^2}{6^2}+\ldots\right]$

b) $6^{-1/3}\left[1+\frac{x}{6}+\frac{2x^2}{6^2}+\ldots\right]$

c) $6^{1/3}\left[1-\frac{x}{6}+\frac{2x^2}{6^2}-\ldots\right]$

d) $6^{-1/3}\left[1-\frac{x}{6}+\frac{2x^2}{6^2}-\ldots\right]$

291. If the coefficient of r^{th}, $(r+1)^{\text{th}}$ and $(r+2)^{\text{th}}$ terms in the expansion of $(1+x)^{14}$ are in A.P., then the value of r, is

a) 5,9 b) 6,9 c) 7,9 d) None of these

292. The interval in which x must lie so that the numerically greatest term in the expansion of $(1-x)^{21}$ has the numerically greatest coefficient, is

a) $\left[\frac{5}{6},\frac{6}{5}\right]$ b) $\left(\frac{5}{6},\frac{6}{5}\right)$ c) $\left(\frac{4}{5},\frac{5}{4}\right)$ d) $\left[\frac{4}{5},\frac{5}{4}\right]$

293. If the 6th term in the expansion of $\left(\frac{1}{x^{8/3}} + x^2\log_{10}x\right)^8$ is 5600, then value of x is

a) 2 b) $\sqrt{5}$ c) $\sqrt{10}$ d) 10

294. The coefficient of x^{20} in the expansion of $(1+x^2)^{40}\left(x^2+2+\frac{1}{x^2}\right)^{-5}$, is

a) ${}^{30}C_{10}$ b) ${}^{30}C_{25}$ c) 1 d) None of these

295. If n is a positive integer, then n^3+2n is divisible by

a) 2 b) 6 c) 15 d) 3

296. If in the expansion of $(1+x)^m(1-x)^n$, the coefficient of x and x^2 are 3 and -6 respectively, then m is

a) 6 b) 9 c) 12 d) 24

297. The coefficient of x^4 in the expansion of $(1+x+x^2+x^3)^n$, is

a) ${}^nC_4$

b) ${}^nC_4 + {}^nC_2$

c) ${}^nC_4 + {}^nC_1 + {}^nC_4 \times {}^nC_2$

d) ${}^nC_4 + {}^nC_2 + {}^nC_1 \times {}^nC_2$

298. Sum of the infinite series

$1+\frac{1}{3}\cdot\frac{1}{2}+\frac{2}{3}\cdot\frac{5}{6}\cdot\frac{1}{2^2}+\frac{2}{3}\cdot\frac{5}{6}\cdot\frac{8}{9}\cdot\frac{1}{2^3}+\cdots\infty$ is

a) $2^{1/3}$ b) $4^{1/3}$ c) $8^{1/3}$ d) $2^{1/5}$

299. If in the expansion of $(a-2b)^n$.The sum of the 5th and 6th term is zero, then the value of $\frac{a}{b}$ is

a) $\frac{n-4}{5}$ b) $\frac{2(n-4)}{5}$ c) $\frac{5}{n-4}$ d) $\frac{5}{2(n-4)}$

300. If in the expansion of $(1+x)^m(1-x)^n$, the coefficient of x and x^2 are 3 and -6 respectively, then m is

a) 6 b) 9 c) 12 d) 24

301. If the sum of the coefficient in the expansion of $(x+y)^n$ is 1024, then the value of the greatest coefficient in the expansion is

a) 356 b) 252 c) 210 d) 120

302. The remainder when 5^{99} is divided by 13, is

a) 6 b) 8 c) 9 d) 10

303. The two consecutive terms in the expansion of $(3+2x)^{74}$ whose coefficient are equal, are

a) 11.12 b) 7,8 c) 30,31 d) None of these

304. If the 4th term in the expansion of $\left(\frac{2}{3}x-\frac{3}{2x}\right)^n$ is independent of x, then n is equal to

a) 5 b) 6 c) 9 d) None of these

305. The coefficient of x^{32} in the expansion of $\left(x^4-\frac{1}{x^3}\right)^{15}$ is

a) $^{-15}C_3$ b) $^{15}C_4$ c) $^{-15}C_5$ d) $^{15}C_2$

306. The number of dissimilar terms in the expansion of $(a+b)^n$is $n+1$therefore no of dissimilar terms of the expansion $(a+b+c)^{12}$ is

a) 13 b) 39 c) 78 d) 91

307. The coefficient of x^7 in $(1+3x-2x^3)^{10}$ is equal to

a) 62640 b) 26240 c) 64620 d) None of these

308. The middel term in the expansion of $\left(x-\frac{1}{x}\right)^{18}$ is

a) $^{18}C_9$ b) $-^{18}C_9$ c) $^{18}C_{10}$ d) $-^{18}C_{10}$

309. The term independent of x in the expansion of

$\left(\frac{2\sqrt{x}}{5}-\frac{1}{2x\sqrt{x}}\right)^{11}$ is

a) 5th term b) 6th term c) 11th term d) No term

310. If n is an integer greater than 1, then

$a-{}^nC_1(a-1)+{}^nC_2(a-2)+\ldots+(-1)^n(a-n)$ is equal to

a) a b) 0 c) a^2 d) 2^n

311. The sum of the rational terms in the expansion of $\left(2^{1/5}+\sqrt{3}\right)^{20}$, is

a) 71 b) 85 c) 97 d) None of these

312. If $[x]$ denotes the gretest integer less than or equal to x, then $[(6\sqrt{6}+14)^{2n+1}]$

a) Is an even integer b) Is an odd integer c) Depends on n d) None of these

313. If $n\in N$, then the sum of the coefficients in the expansion of the binomial $(5\,x-4\,y)^n$, is

a) 1 b) -1 c) n d) 0

314. The coefficient of the term independent of x in the expansion of

$\left[\frac{(x+1)}{x^{2/3}-x^{1/3}+1}-\frac{(x-1)}{x-x^{1/2}}\right]^{10}$ is

a) 210 b) 105 c) 70 d) 112

315. $\frac{C_0}{1}+\frac{C_2}{3}+\frac{C_4}{5}+\frac{C_6}{7}+\ldots$ is equal to

a) $\frac{2^{n+1}}{n+1}$ b) $\frac{2^{n+1}-1}{n+1}$ c) $\frac{2^n}{n+1}$ d) None of these

316. Coefficient of x^n in the expansion of $\frac{(1+x)^n}{1-x}$

a) $4n$ b) 2^n c) n^2 d) $\frac{n(n+1)}{2}$

317. If$(1+x-2x^2)^6=1+a_1x+a_2x^2+\ldots+a_{12}x^{12}$, then the expression $a_2+a_4+a_6+\ldots+a_{12}$ has the value

a) 32 b) 63 c) 64 d) None of these

318. If $(1+x)^n=\sum_{r=0}^{n}a_rx^r$ and $b_r=1+\frac{a_r}{a_{r-1}}$ and $\prod_{r=1}^{n}b_r=\frac{(101)^{100}}{100!}$, then n is

a) 99 b) 100 c) 101 d) 102

319. Coefficient of $x^2y^3z^4$ in $(ax+by+cz)^9$ is
a) $1060a^2b^3c^4$ b) $1160a^2b^3c^4$ c) $1260a^2b^3c^4$ d) $960a^2b^3c^4$

320. The constant term in the expansion of
$\left(x^2-\frac{1}{x}\right)^9$ is
a) 80 b) 72 c) 84 d) 82

321. $\sum_{k=1}^{\infty} k\left(1+\frac{1}{n}\right)^{k-1} =$
a) $n(n-1)$ b) $n(n+1)$ c) n^2 d) $(n+1)^2$

322. If the coefficients of three consecutive terms in the expansion of $(1+x)^n$ are in the ratio 1: 7: 42, then the value of n is
a) 60 b) 70 c) 55 d) None of these

323. The number of non-zero terms in the expansion of $\left(1+3\sqrt{2}\,x\right)^9+\left(1-3\sqrt{2}\,x\right)^9$, is
a) 9 b) 0 c) 5 d) 10

324. If the coefficient of 7th and 13th term in the expansion of $(1+x)^n$ are equal, then n is equal to
a) 10 b) 15 c) 18 d) 20

325. In the expansion of $\left(2x^2-\frac{1}{x}\right)^{12}$, the term independent of x is
a) 8th b) 7th c) 9th d) 10th

326. If $C_0, C_1, C_2, \dots, C_n$ are coefficients in the binomial expansion of $(1+x)^n$, then $C_0C_2+C_1C_3+C_2C_4+\cdots+C_{n-2}C_n$ is equal to
a) $\frac{(2\,n)\,!}{(n-2)\,!\,(n+2)\,!}$ b) $\frac{(2\,n)\,!}{((n-2)!)^2}$ c) $\frac{(2\,n)\,!}{((n+2)!)^2}$ d) None of these

327. For all integers $n \geq 1$, which of the following is divisible by 9?
a) 8^n+1 b) 4^n-3n-1 c) $3^{2n}+3n+1$ d) 10^n+1

328. The sum of the series
${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots + {}^{20}C_{10}$ is
a) $-{}^{20}C_{10}$ b) $\frac{1}{2}\,{}^{20}C_{10}$ c) 0 d) ${}^{20}C_{10}$

329. Let $[x]$ donate the greatest integer less than or equal to x. If $x=(\sqrt{3}+1)^5$, then $[x]$ is equal to
a) 75 b) 50 c) 76 d) 152

330. The coefficient of x^{53} in the expansion of
$\sum_{m=0}^{100} {}^{100}C_m\,(x-3)^{100-m}\cdot 2^m$
is
a) ${}^{100}C_{47}$ b) ${}^{100}C_{53}$ c) ${}^{-100}C_{53}$ d) ${}^{-100}C_{100}$

331. If the fourth term in the expansion of $\left(ax+\frac{1}{x}\right)^n$ is $\frac{5}{2}$, then
a) $a=1/2$ and n = 6 b) $a=1/3$ and n = 5 c) $a=2$ and n = 3 d) $a=1/4$ and n = 1

332. If $(1+x)^n = C_0+C_1x+C_2x^2+\ldots+C_nx^n$, then
$\sum_{0\le r<s\le n} \sum (r+s)C_rC_s$ is equal to
a) $n[2^{2n-1} - {}^{2n-1}C_{n-1}]$
b) $n[2^{2n-1} + {}^{2n-1}C_{n-1}]$
c) $2n[2^{2n-1} - {}^{2n-1}C_{n-1}]$
d) None of these

333. If the coefficient of x^7 in the expansion of $(ax^2+b^{-1}x^{-1})^{11}$ is equal to the coefficient of x^{-7} in $(ax-b^{-1}x^{-2})^{11}$, then $ab=$

a) 1 b) 2 c) 3 d) 4

334. The coefficient of x^n in the expansion of $\frac{1}{(1-x)(3-x)}$, is

a) $\dfrac{3^{n+1}-1}{2\cdot 3^{n+1}}$ b) $\dfrac{3^{n+1}-1}{3^{n+1}}$ c) $2\left(\dfrac{3^{n+1}-1}{3^{n+1}}\right)$ d) None of these

335. The greatest term in the expansion of $(1+3x)^{54}$, where $x = 1/3$ is

a) T_{28} b) T_{25} c) T_{26} d) T_{24}

336. If in the expansion of $\left(\frac{1}{x}+x\tan x\right)^5$ the ratio of 4th term to the 2nd term is $\frac{2}{27}\pi^4$, then the value of x can be

a) $-\dfrac{\pi}{6}$ b) $-\dfrac{\pi}{3}$ c) $\dfrac{\pi}{6}$ d) $\dfrac{\pi}{12}$

337. The remainder when $32^{(32)^{(32)}}$ is divided by 7, is

a) 1 b) 2 c) 3 d) 4

338. If $A = 1000^{1000}$ and $B = (1001)^{999}$, then

a) $A > B$

b) $A = B$

c) $A < B$

d) None of these

339. If the r th term in the expansion of $\left(\dfrac{x}{3}-\dfrac{2}{x^2}\right)^{10}$ contains x^4, then r is equal to

a) 3 b) 0 c) -3 d) 5

340. The coefficient of term independent of x in

$\left[\sqrt{\left(\dfrac{x}{3}\right)}+\dfrac{\sqrt{3}}{x^2}\right]^{10}$ is

a) $\dfrac{5}{3}$ b) $\dfrac{4}{5}$ c) 6 d) $\dfrac{1}{2}$

341. The ratio of the coefficient of x^{15} to the term independent of x in $\left(x^2+\frac{2}{x}\right)^{15}$, is

a) 1/4 b) 1/16 c) 1/32 d) 1/64

342. The sum ${}^{40}C_0 + {}^{40}C_1 + {}^{40}C_2 + \cdots + {}^{40}C_{20}$ is equal to

a) $2^{40}+\dfrac{40!}{(20!)^2}$ b) $2^{39}-\dfrac{1}{2}\times\dfrac{40!}{(20!)^2}$ c) $2^{39} + {}^{40}C_{20}$ d) None of these

343. The coefficient of x^5 in the expansion of $\frac{1+x^2}{1+x}$, $|x| < 1$, is

a) -1 b) 2 c) 0 d) -2

344. The coefficient of x^2 term in the binomial expansion of $\left(\dfrac{1}{3}x^{1/2}+x^{-1/4}\right)^{10}$ is

a) $\dfrac{70}{243}$ b) $\dfrac{60}{423}$ c) $\dfrac{50}{13}$ d) None of these

345. If the expansion of $\left(\dfrac{3\sqrt{x}}{7}-\dfrac{5}{2x\sqrt{x}}\right)^{13n}$ contains a term independent of x in 14th term, then n should be

a) 10 b) 5 c) 6 d) 4

346. The interval in which x must lie so that the greatest term in the expansion of $(1+x)^{2n}$ has the greatest coefficient, is

a) $\left(\dfrac{n-1}{n},\dfrac{n}{n-1}\right)$

b) $\left(\dfrac{n}{n+1},\dfrac{n+1}{n}\right)$

c) $\left(\dfrac{n}{n+2},\dfrac{n+2}{n}\right)$

d) None of these

347. The coefficient of a^3b^4c in the expansion of $(1+a-b+c)^9$is equal to

a) $\frac{9!}{3!\,6!}$ b) $\frac{9!}{4!\,5!}$ c) $\frac{9!}{3!\,5!}$ d) $\frac{9!}{3!\,4!}$

348. If $(1+x+x^2)^n=\sum_{r=0}^{2n} a_r x^r$, then $a_1-2a_2+3a_3 \dots -2na_{2n}$ is equal to

a) n b) $-n$ c) 0 d) $2n$

349. In the expansion of $(1+x)^{30}$, the sum of the coefficient of odd powers of x, is

a) 2^{30} b) 2^{31} c) 0 d) 29

350. If the fourth term in the expansion of $\left\{\sqrt{x^{\left(\frac{1}{\log x+1}\right)}}+x^{1/12}\right\}^6$ is equal to 200 and $x>1$, then x is equal to

a) $10^{\sqrt{2}}$ b) 10 c) 10^4 d) None of these

351. The coefficient of x^6 in $\{(1+x)^6+(1+x)^7+\cdots+(1+x)^{15}\}$ is

a) ${}^{16}C_9$ b) ${}^{16}C_5-{}^{6}C_5$ c) ${}^{16}C_6-1$ d) None of these

352. The sum of the series $1+\frac{1}{5}+\frac{1\cdot 3}{5\cdot 10}+\frac{1\cdot 3\cdot 5}{5\cdot 10\cdot 15}+.\dots$ is equal to

a) $\frac{1}{\sqrt{5}}$ b) $\frac{1}{\sqrt{2}}$ c) $\sqrt{3}$ d) $\sqrt{\frac{5}{3}}$

353. If $A=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, Then which one of the following holds for all $n\geq 1$, by the principle of mathematical induction?

a) $A^n=2^{n-1}A+(n-1)I$ b) $A^n=nA+(n-1)I$

c) $A^n=2^{n-1}A-(n-1)I$ d) $A^n=nA-(n-1)I$

354. The greatest term in the expansion of $\sqrt{3}\left(1+\frac{1}{\sqrt{3}}\right)^{20}$ is

a) $\frac{26840}{9}$ b) $\frac{24840}{9}$ c) $\frac{25840}{9}$ d) None of these

355. If $(1+x)^n=\sum_{r=0}^{n} C_r x^r$, then $\left(1+\frac{C_1}{C_0}\right)\left(1+\frac{C_2}{C_1}\right)\dots\left(1+\frac{C_n}{C_{n-1}}\right)$ is equal to

a) $\frac{n^{n-1}}{(n-1)}!$ b) $\frac{(n+1)^{n-1}}{(n-1)!}$ c) $\frac{(n+1)^n}{n!}$ d) $\frac{(n+1)^{n+1}}{n!}$

356. The expression $\left[x+(x^3-1)^{1/2}\right]^5+\left[x-(x^3-1)^{1/2}\right]^5$is a polynomial of degree

a) 5 b) 6 c) 7 d) 8

357. The sum of the coefficients of the polynomial $(1+x-3\,x^2)^{2143}$ is

a) -1 b) 1 c) 0 d) None of these

358. If C_r stands for ${}^nC_r$, the sum of the given series $\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}\cdot[C_0^2-2C_1^2+3C_2^2-\dots+(-1)^n\,(n+1)C_n^2]$, where n is an even positive integer, is

a) 0 b) $(-1)^{n/2}(n+1)$

c) $(-1)^n(n+2)$ d) $(-1)^{n/2}(n+2)$

359. The value of $(1.002)^{12}$ upto fourth place of decimal is

a) 1.0242 b) 1.0245 c) 1.0004 d) 1.0254

360. The coefficient of the term independent of x in the expansion of $(1+x+2x^3)\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$ is

a) $\frac{1}{3}$ b) $\frac{19}{54}$ c) $\frac{17}{54}$ d) $\frac{1}{4}$

361. If a and d are two complex numbers, then the sum to $(n+1)$ terms of the following series $aC_0-(a+d)C_1+(a+2d)C_2-\dots+\dots$ is

a) $\frac{a}{2^n}$ b) na c) 0 d) None of these

362. $2^{3n}-7n-1$ is divisible by

a) 64 b) 36 c) 49 d) 25

363. If n is a positive integer, then $5^{2n+2}-24n-25$ isdivisible by

a) 574 b) 575 c) 675 d) 576

364. If $a_n = \sum_{r=0}^{n} \frac{1}{^nC_r}$, then $\sum_{r=0}^{n} \frac{r}{^nC_r}$ equals

a) $(n-1)a_n$ b) na_n c) $\frac{1}{2}na_n$ d) None of these

365. The expression $\frac{1}{\sqrt{4x+1}}\left\{\left(1+\frac{\sqrt{4x+1}}{2}\right)^7-\left(1-\frac{\sqrt{4x+1}}{2}\right)^7\right\}$ is a polynomial in x of degree

a) 7 b) 5 c) 4 d) 3

366. If the coefficient of x^7 in $\left(ax^2+\frac{1}{bx}\right)^{11}$ equal the coefficient of x^{-7} in $\left(ax-\frac{1}{bx^2}\right)^{11}$, then a and b satisfy the relation

a) $ab=1$ b) $\frac{a}{b}=1$ c) $a+b=1$ d) $a-b=1$

367. The middle term in the expansion of $\left(x+\frac{1}{2x}\right)^{2n}$ is

a) $\frac{1\cdot3\cdot5\ldots(2n-3)}{n!}$

b) $\frac{1\cdot3\cdot5\ldots(2n-1)}{n!}$

c) $\frac{1\cdot3\cdot5\ldots(2n+1)}{n!}$

d) None of these

368. The coefficient of x^5 in the expansion of $(1+x)^{21}+(1+x)^{22}+\ldots+(1+x)^{30}$ is

a) $^{51}C_5$ b) $^{9}C_5$ c) $^{31}C_6-{}^{21}C_6$ d) $^{30}C_5+{}^{20}C_5$

369. If $(r+1)^{th}$ term is the first negative term in the expansion of $(1+x)^{7/2}$, then the value of r, is

a) 5 b) 6 c) 4 d) 7

370. If in expansion of $(1+x)^{21}$, the coefficient of x^r and x^{r+1} be equal, then $r$is equal to

a) 9 b) 10 c) 11 d) 12

371. The greatest term in the expansion of $\sqrt{3}\left(1+\frac{1}{\sqrt{3}}\right)^{20}$, is

a) $\frac{25840}{9}$ b) $\frac{24840}{9}$ c) $\frac{26840}{9}$ d) None of these

372. The coefficient of x^n in the expansion of $(1+x+x^2+\cdots)^{-n}$, is

a) 1 b) $(-1)^n$ c) n d) $n+1$

373. The coefficient of t^{24} in the expansion of $(1+t^2)^{12}(1+t^{12})(1+t^{24})$ is

a) $^{12}C_6$ +2 b) $^{12}C_5$ c) $^{12}C_6$ d) $^{12}C_7$

374. If the coefficients of T_r, T_{r+1}, T_{r+2} terms of $(1+x)^{14}$ are in AP, then the value of r is

a) 6 b) 7 c) 8 d) 9

375. The larger of $99^{50}+100^{50}$ and 101^{50} is

a) $99^{50}+100^{50}$ b) Both are equal c) 101^{50} d) None of these

376. The value of the sum of the series

$3\cdot{}^nC_0-8\cdot{}^nC_1+13\;{}^nC_2-18\cdot{}^nC_3+$ ...upto $(n+1)$ terms is

a) 0 b) 3^n c) 5^n d) None of these

377. The last positive integer n such that $\binom{n-3}{3}+\binom{n-1}{4}>\binom{n}{3}$ is

a) 6 b) 7 c) 8 d) 9

378. If S be the sum of coefficients in the expansion of $(\alpha x+\beta y-\gamma z)^n$, where $(\alpha,\beta,\gamma>0)$, then the value of $\lim_{n\to\infty}\frac{S}{\{S^{1/n}+1\}^n}$ is

a) $e^{\left(\frac{\alpha\beta}{\gamma}\right)}$ b) $e^{\left(\frac{\alpha+\beta-\gamma}{\alpha+\beta-\gamma+1}\right)}$ c) $\frac{\alpha\beta}{\gamma}$ d) 0

379. If the sum of the numerical coefficients in the binomial expansion of $\left(\frac{1}{x}+2x\right)^n$ is equal to 6561, then the term independent of x, is

a) ${}^{8}C_4$ b) ${}^{8}C_4 \times 2^4$ c) ${}^{6}C_4 \times 2^4$ d) None of these

380. $\sum_{0 \le i} \sum_{< j \le 10} {}^{10}C_j \; {}^{j}C_i$ is equal to

a) $2^{10} - 1$ b) 2^{10} c) $3^{10} - 1$ d) 3^{10}

381. The coefficient of $x^{2^{m+1}}$ in the expansion of $E = \frac{1}{(1+x)(1+x^2)(1+x^4)(1+x^8)\ldots(1+x^{2^m})}$, $|x| < 1$ is

a) 3 b) 2 c) 1 d) 0

382. If $n \in N$ such that $(7 + 4\sqrt{3})^n = I + F$, where $I \in N$ and $0 < F < 1$. Then, the value of $(I + F)(1 - F)$ is

a) 0 b) 1 c) 7^{2n} d) 2^{2n}

383. The largest coefficient in the expansion of $(1 + x)^{24}$ is

a) ${}^{24}C_{24}$ b) ${}^{24}C_{13}$ c) ${}^{24}C_{12}$ d) ${}^{24}C_{11}$

384. The coefficient of x^4 in the expansion of $(1 + x + x^3 + x^4)^{10}$, is

a) ${}^{40}C_4$ b) ${}^{10}C_4$ c) 210 d) 310

385. If the third term in the binomial expansion of $(1 + x)^m$ is $-\frac{1}{8}x^2$, then the rational value of m, is

a) 2 b) 1/2 c) 3 d) 4

386. If $C_r = {}^{n}C_r$ and $(C_0 + C_1)(C_1 + C_2) \ldots (C_{n-1} + C_n) = k\frac{(n+1)^n}{n\,!}$, then the value of k, is

a) $C_0\, C_1\, C_2 \ldots C_n$ b) $C_1^2 C_2^2 \ldots C_n^2$ c) $C_1 + C_2 + \cdots + C_n$ d) None of these

387. The coefficient of x^{53} in the expansion

$\sum_{m=0}^{100} {}^{100}C_m\,(x - 3)^{100-m} \cdot 2^m$, is

a) ${}^{100}C_{47}$ b) ${}^{100}C_{53}$ c) $-\,{}^{100}C_{53}$ d) $-\,{}^{100}C_{100}$

388. If the ratio of the coefficient of third and fourth term in the expansion of $\left(x - \frac{1}{2x}\right)^n$ is 1:2, then the value of n will be

a) 18 b) 16 c) 12 d) -10

389. The coefficient of x^n in the expansion of $(1 + x + x^2 + \cdots)^{-n}$ is

a) 1 b) $(-1)^n$ c) n d) $n + 1$

390. If in the expansion of $(a - 2b)^n$, the sum of 4th and 5th term is zero, then the value of $\frac{a}{b}$ is

a) $\frac{n-4}{5}$ b) $\frac{n-3}{2}$ c) $\frac{5}{n-4}$ d) $\frac{5}{2(n-4)}$

391. The coefficient of x^5 in the expansion of $(2 - x + 3x^2)^6$, is

a) -4692 b) 4692 c) 2346 d) -5052

392. The number of terms whose values depend on x in the expansion of $\left(x^2 - 2 + \frac{1}{x^2}\right)^n$, is

a) $2n + 1$ b) $2n$ c) n d) None of these

393. If the coefficients of x^7 and x^8 in $(2 + x/3)^n$ are equal, then n is equal to

a) 56 b) 55 c) 45 d) 15

394. The term independent of x in $\left\{\sqrt{\frac{x}{3}} + \frac{3}{2\,x^2}\right\}^{10}$, is

a) $\frac{9}{4}$ b) $\frac{3}{4}$ c) $\frac{5}{4}$ d) $\frac{7}{4}$

395. Coefficient of x^{-4} in $\left(\frac{3}{2} - \frac{3}{x^2}\right)^{10}$ is

a) $\frac{405}{226}$ b) $\frac{504}{289}$ c) $\frac{450}{263}$ d) None of these

396. If the coefficients of $(2\,r + 4)^{\text{th}}$ and $(r - 2)^{\text{th}}$ terms in the expansion of $(1 + x)^{18}$ are equal, then the value of r, is

a) 5 b) 6 c) 7 d) 9

397. The coefficient of x^6 in the expansion of $(1 + x + x^2)^{-3}$, is

a) 6 b) 5 c) 4 d) 3

398. $49^n + 16n - 1$ is divisible by

a) 3 b) 29 c) 19 d) 64

399. The value of $1^2 \cdot C_1 + 3^2 \cdot C_3 + 5^2 \cdot C_5 + \ldots$is

a) $n(n-1)^{n-2} + n \cdot 2^{n-1}$ b) $n(n-1)^{n-2}$

c) $n(n-1) \cdot 2^{n-3}$ d) None of these

400. If the coefficient of x^2 and x^3 in the expansion of $(3 + ax)^9$ be same, then the value of a is

a) 3/7 b) 7/3 c) 7/9 d) 9/7

401. The coefficient of x in the expansion of $(1+x)(1+2x)(1+3x) \ldots \ldots (1+100x)$ is

a) 5050 b) 10100 c) 5151 d) 4950

402. The positive value of a so that the coefficients of x^5 and x^{15} are equal in the expansion of $\left(x^2 + \frac{a}{x^3}\right)^{10}$

a) $\frac{1}{2\sqrt{3}}$ b) $\frac{1}{\sqrt{3}}$ c) 1 d) $2\sqrt{3}$

403. In the expansion of $(2 - 3x^3)^{20}$, if the ratio of 10th term to 11th term is 45/22, then x is equal to

a) $-\frac{2}{3}$ b) $\frac{-3}{2}$ c) $-\sqrt[3]{\frac{2}{3}}$ d) $-\sqrt[3]{\frac{3}{2}}$

: ANSWER KEY :

1)	b	2)	b	3)	d	4)	c	189)	d	190)	a	191)	a	192)	d
5)	a	6)	a	7)	a	8)	c	193)	b	194)	c	195)	b	196)	c
9)	b	10)	c	11)	b	12)	a	197)	a	198)	b	199)	b	200)	a
13)	c	14)	c	15)	c	16)	b	201)	b	202)	b	203)	d	204)	d
17)	b	18)	d	19)	c	20)	d	205)	a	206)	a	207)	b	208)	a
21)	c	22)	a	23)	a	24)	c	209)	c	210)	b	211)	c	212)	a
25)	a	26)	d	27)	b	28)	b	213)	b	214)	d	215)	d	216)	c
29)	c	30)	c	31)	c	32)	b	217)	a	218)	a	219)	a	220)	b
33)	b	34)	c	35)	d	36)	d	221)	a	222)	c	223)	a	224)	d
37)	b	38)	a	39)	a	40)	b	225)	a	226)	a	227)	a	228)	d
41)	a	42)	c	43)	b	44)	c	229)	c	230)	a	231)	b	232)	a
45)	d	46)	d	47)	c	48)	b	233)	c	234)	d	235)	d	236)	c
49)	d	50)	b	51)	a	52)	b	237)	a	238)	a	239)	a	240)	d
53)	c	54)	a	55)	a	56)	b	241)	a	242)	c	243)	b	244)	b
57)	a	58)	b	59)	b	60)	b	245)	b	246)	a	247)	b	248)	b
61)	c	62)	c	63)	d	64)	b	249)	c	250)	b	251)	c	252)	b
65)	d	66)	c	67)	d	68)	a	253)	b	254)	c	255)	b	256)	b
69)	d	70)	a	71)	a	72)	c	257)	b	258)	c	259)	c	260)	d
73)	b	74)	a	75)	c	76)	b	261)	a	262)	a	263)	a	264)	b
77)	b	78)	c	79)	b	80)	b	265)	b	266)	d	267)	c	268)	d
81)	c	82)	b	83)	c	84)	b	269)	b	270)	b	271)	a	272)	b
85)	d	86)	d	87)	d	88)	d	273)	b	274)	b	275)	b	276)	c
89)	a	90)	b	91)	c	92)	d	277)	a	278)	b	279)	b	280)	b
93)	b	94)	a	95)	a	96)	b	281)	a	282)	c	283)	a	284)	b
97)	b	98)	b	99)	b	100)	b	285)	c	286)	d	287)	c	288)	b
101)	c	102)	c	103)	b	104)	b	289)	b	290)	b	291)	a	292)	b
105)	c	106)	a	107)	d	108)	c	293)	d	294)	b	295)	d	296)	c
109)	c	110)	d	111)	b	112)	c	297)	d	298)	b	299)	b	300)	c
113)	b	114)	b	115)	c	116)	c	301)	b	302)	b	303)	c	304)	b
117)	b	118)	a	119)	c	120)	b	305)	b	306)	d	307)	a	308)	b
121)	d	122)	b	123)	b	124)	d	309)	d	310)	b	311)	d	312)	a
125)	a	126)	d	127)	d	128)	a	313)	a	314)	a	315)	c	316)	b
129)	d	130)	a	131)	c	132)	d	317)	d	318)	b	319)	c	320)	c
133)	d	134)	c	135)	a	136)	b	321)	c	322)	c	323)	c	324)	c
137)	b	138)	b	139)	c	140)	b	325)	c	326)	a	327)	b	328)	b
141)	c	142)	c	143)	b	144)	b	329)	d	330)	c	331)	a	332)	a
145)	d	146)	b	147)	d	148)	b	333)	a	334)	a	335)	a	336)	b
149)	a	150)	d	151)	a	152)	c	337)	d	338)	a	339)	a	340)	a
153)	a	154)	b	155)	b	156)	b	341)	c	342)	d	343)	d	344)	a
157)	b	158)	b	159)	b	160)	c	345)	d	346)	b	347)	d	348)	b
161)	a	162)	b	163)	d	164)	b	349)	d	350)	b	351)	a	352)	d
165)	c	166)	a	167)	c	168)	d	353)	d	354)	c	355)	c	356)	c
169)	d	170)	b	171)	c	172)	b	357)	a	358)	d	359)	a	360)	c
173)	c	174)	a	175)	d	176)	c	361)	c	362)	c	363)	d	364)	c
177)	c	178)	d	179)	a	180)	c	365)	d	366)	a	367)	b	368)	c
181)	a	182)	b	183)	b	184)	b	369)	a	370)	b	371)	a	372)	b
185)	b	186)	d	187)	b	188)	b	373)	a	374)	d	375)	c	376)	a

377)	**d**	**378)**	**d**	**379)**	**b**	**380)**	**c**	**393)**	**b**	**394)**	**c**	**395)**	**d**	**396)**	**b**
381)	**c**	**382)**	**b**	**383)**	**c**	**384)**	**d**	**397)**	**d**	**398)**	**d**	**399)**	**d**	**400)**	**d**
385)	**b**	**386)**	**a**	**387)**	**c**	**388)**	**d**	**401)**	**a**	**402)**	**a**	**403)**	**a**		
389)	**b**	**390)**	**b**	**391)**	**d**	**392)**	**b**								

: HINTS AND SOLUTIONS :

1 **(b)**

We have,

$(1+x)^{15} = C_0 + C_1x + C_2x^2 + \ldots + C_{15}x^{15}$

$\Rightarrow \frac{(1+x)^{15}-1}{x} = C_1 + C_2x + \ldots C_{15}x^{14}$

On differentiating both sides w.r.t. x, we get

$\frac{x \cdot 15(1+x)^{14} - (1+x)^{15} + 1}{x^2}$

$= C_2 + 2C_3x + \ldots + 14C_{15}x^{13}$

On putting $x = 1$, we get

$C_2 + 2C_3 + \ldots + 14C_{15} = 15.2^{14} - 2^{15} + 1$

$= 13.2^{14} + 1$

2 **(b)**

It is given that

${}^{2n}C_1$, ${}^{2n}C_2$ and ${}^{2n}C_3$ are A.P.

$\therefore 2 \cdot {}^{2n}C_2 = {}^{2n}C_1 + {}^{2n}C_3$

$2 \cdot \frac{(2n)!}{(2n-2)!\,2!} = \frac{(2n)!}{(2n-1)!} + \frac{(2n)!}{(2n-3!\,3!)}$

$\Rightarrow 2\frac{(2n)(2n-1)}{2}$

$= 2n + \frac{(2n)(2n-1)(2n-2)}{3!}$

$\Rightarrow 6(2n-1) = 6 + (2n-1)(2n-2)$

$\Rightarrow 12n - 6 = 6 + 4n^2 - 6n + 2$

$\Rightarrow 4n^2 - 18n + 14 = 0 \Rightarrow 2n^2 - 9n + 7 = 0$

3 **(d)**

$\frac{1+2x}{(1-2x)^2} = (1+2x)(1-2x)^{-2}$

$= (1+2x)\left(1 + \frac{2}{1!}(2x) + \frac{2\cdot 3}{2!}(2x)^2 + \ldots + \frac{2\cdot 3 \ldots r}{(r-1)!}(2x)^{r-1} + \frac{2\cdot 3\cdot 4 \ldots (r+1)(2x)^r}{r!}\right)$

The coefficient of $x^r = 2\frac{r!}{(r-1)!}2^{r-1} + \frac{(r-1)!}{r!}2^r$

$= r2^r + (r+1)2^r = 2^r(2r+1)$

4 **(c)**

Given, $A = {}^{30}C_0 \cdot {}^{30}C_{10} - {}^{30}C_1 \cdot {}^{30}C_{11} + {}^{30}C_2 \cdot {}^{30}C_{12} + \cdots + {}^{30}C_{20} \cdot {}^{30}C_{30}$

$=$ coefficient of x^{20} in $(1+x)^{30}(1-x)^{30}$

$=$ coefficient of x^{20} in $(1+x^2)^{30}$

$=$ coefficient of x^{20} in $\sum_{r=0}^{30} (-1)^r \; {}^{30}C_r(x^2)^r$

$= (-1)^{10} \cdot {}^{30}C_{10}$ {for coefficient of x^{20}, let $r = 10$}

$= {}^{30}C_{10}$

5 **(a)**

We have,

$a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \cdots + a_{2n}x^{2n}$

$= (1 - x + x^2)^n$

Putting $x = 1$ and -1, we get

$(a_0 + a_2 + a_4 + \cdots) + (a_1 + a_3 + a_5 + \cdots)$

$= 1 \quad \ldots$ (i)

And,

$(a_0 + a_2 + a_4 + \cdots) - (a_1 + a_3 + a_5 \ldots)$

$= 3^n \quad \ldots$ (ii)

Adding (i) and (ii), we get

$a_0 + a_2 + a_4 + \cdots = \frac{3^n + 1}{2}$

6 **(a)**

We know that ,

$(1+x)^n = C_0 + C_1x + C_2x^2 + \cdots + C_nx^n$

On integrating both sides, from 0 to 1, we get

$\left[\frac{(1+x)^{n+1}}{n+1}\right]_0^1 = \left[C_0x + \frac{C_1x^2}{2} + \frac{C_2x^3}{3} + \cdots + \frac{C_nx^{n+1}}{n+1}\right]_0^1$

$\Rightarrow \frac{2^{n+1}-1}{n+1} = C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots + \frac{C_n}{n+1}$

7 **(a)**

7th term from the beginning in the expansion of $\left(2^{1/3} + \frac{1}{3^{1/3}}\right)^x$ is given by

$T_7 = {}^xC_6\left(2^{1/3}\right)^{x-6}\left(\frac{1}{3^{1/3}}\right)^6$

7th term from the end in the expansion of $\left(2^{1/3} + \frac{1}{3^{1/3}}\right)^x$ is the $(x+1-7+1)^{\text{th}} = (x-5)^{\text{th}}$ term from the beginning. Therefore,

$T_{x-5} = {}^xC_{x-6}\left(2^{1/3}\right)^6\left(\frac{1}{3^{1/3}}\right)^{x-6}$

We have,

$\frac{T_7}{T_{x-5}} = \frac{1}{6}$

$\Rightarrow 6\,T_7 = T_{x-5}$

$\Rightarrow 6\; {}^xC_6\, 2^{\frac{x-6}{3}}3^{-2} = {}^xC_{x-6}\, 2^2 3^{-\left(\frac{x-6}{3}\right)}$

$\Rightarrow 2^{\frac{x-9}{3}} = 3^{-\left(\frac{x-9}{3}\right)}$

$\Rightarrow 6^{\frac{x-9}{3}} = 1 \Rightarrow x - 9 = 0 \Rightarrow x = 9$

9 **(b)**

$\because T_{r+1} = {}^{10}C_r(x\sin^{-1}\alpha)^{10-r}\left(\frac{\cos^{-1}\alpha}{x}\right)^r$

$= {}^{10}C_r(\sin^{-1}\alpha)^{10-r}(\cos^{-1}\alpha)^r x^{10-2r}$

$\therefore$ For the term independent of x,

$10 - 2r = 0 \Rightarrow r = 5$

$T_{5+1} = {}^{10}C_5(\sin^{-1}\alpha)^5(\cos^{-1}\alpha)^5$

$= {}^{10}C_5(\sin^{-1}\alpha\cos^{-1}\alpha)^5$

Let $f(\alpha) = \sin^{-1}\alpha.\cos^{-1}\alpha$

$= \sin^{-1}\alpha\left(\frac{\pi}{2} - \sin^{-1}\alpha\right)$

Put $\sin^{-1}\alpha = t$

$\therefore f(\alpha) = t\left(\frac{\pi}{2} - t\right)$

$= -\left\{t^2 - \frac{\pi}{2}t\right\}$

$= -\left\{\left(t - \frac{\pi}{4}\right)^2 - \frac{\pi^2}{16}\right\}$

$= \frac{\pi^2}{16} - \left(t - \frac{\pi}{4}\right)^2$

$\therefore f(a) = \frac{\pi^2}{16} - \left(\sin^{-1}\alpha - \frac{\pi}{4}\right)^2$

Maximum value of $f(\alpha)$ is $\frac{\pi^2}{16}$, when $\sin^{-1}\alpha = \frac{\pi}{4}$

Also, $-1 \le \alpha \le 1$

$\therefore -\frac{\pi}{2} \le \sin^{-1}\alpha \le \frac{\pi}{2}$

Minimum value $f(\alpha) = \frac{\pi^2}{16} - \left(-\frac{\pi}{2} - \frac{\pi}{4}\right)^2 = -\frac{\pi^2}{2}$

$\therefore$ Range is $\left[{}^{10}C_5\left(-\frac{\pi^2}{2}\right)^5, {}^{10}C_5\left(\frac{\pi^2}{16}\right)^5\right]$

ie, $\left[-\frac{{}^{10}C_5\pi^{10}}{2^5}, \frac{{}^{10}C_5\pi^{10}}{2^{20}}\right]$

10 **(c)**

Let $A = \binom{30}{0}\binom{30}{10} - \binom{30}{1}\binom{30}{11}x$

$+ \binom{30}{2}\binom{30}{12} - \ldots + \binom{30}{20}\binom{30}{30}$

or $A = {}^{30}C_0.\ {}^{30}C_{10} - {}^{30}C_1.\ {}^{30}C_{11}$

$+ {}^{30}C_2.\ {}^{30}C_{12} - \ldots + {}^{30}C_{20}.\ {}^{30}C_{30}$

= coefficient of x^{20} in $(1+x)^{30}(1-x)^{30}$

= coefficient of x^{20} in $(1-x^2)^{30}$

= coefficient of x^{20} in $\sum_{r=0}^{30}(-1)^r\ {}^{30}C_r(x^2)^r$

$= (-1)^{10}\ {}^{30}C_{10}$ (for coefficient of x^{20}, let $r = 10$)

$= {}^{30}C_{10}$

11 **(b)**

The rth term of $(a + 2n)^n$ is

${}^nC_{r-1}(a)^{n-r+1}(2x)^{r-1}$

$= \frac{n!}{(n-r+1)!\,(r-1)!}a^{n-r+1}(2x)^{r-1}$

$= \frac{n(n-1)\ldots(n-r+2)}{(r-1)!}a^{n-r+1}(2x)^{r-1}$

12 **(a)**

We have, $(1+t^2)^{12}(1+t^{12})(1+t^{24})$

$= (1 + {}^{12}C_1t^2$

$+ {}^{12}C_2t^4 + \ldots + {}^{12}C_6t^{12} + \ldots + {}^{12}C_{12}t^{24} + \ldots)(1$

$+ t^{12} + t^{24} + t^{36})$

$\therefore$ Coefficient of t^{24} in $(1+t^2)^{12}(1+t^{12})(1+t^{24})$

$= {}^{12}C_6 + {}^{12}C_{12} + 1 = {}^{12}C_6 + 2$

13 **(c)**

We have,

$\frac{(1+x)^2}{(1-x)^3} = (x^{2v} + 2x + 1)(1-x)^{-3}$

$\Rightarrow \frac{(1+x)^2}{(1+x)^3} = x^2(1-x)^{-3} + 2x(1-x)^{-3}$

$+ (1-x)^{-3}$

$\therefore$ Coeff. of x^n in $\frac{(1+x)^2}{(1-x)^3}$

= Coeff. of x^n in

$x^2(1-x)^{-3}$ + Coeff. of x^n in $2x(1-x)^{-3}$ +

Coeff. of x^n in $(1-x)^{-3}$

= Coeff. of x^{n-2} in $(1-x)^{-3}$

+ 2 . Coeff. of x^{n-1} in $(1-x)^{-3}$

+ Coeff. of x^n in $(1-x)^{-3}$

$= {}^{n-2+3-1}C_{3-1} + 2\cdot{}^{n-1+3-1}C_{3-1} + {}^{n+3-1}C_{3-1}$

$= {}^nC_2 + 2\cdot{}^{n+1}C_2 + {}^{n+2}C_2$

$= \frac{n(n-1)}{2} + 2\frac{(n+1)\,n}{2} + (n+2)\frac{(n+1)}{2}$

$= \frac{1}{2}(n^2 - n + 2^2 + 2n + n^2 + 3n + 2)$

$= 2n^2 + 2n + 1$

14 **(c)**

On substituting $x = 1$ in $(1 + x - 3x^2)^{3148}$, then sum of coefficient

$= (1 + 1 - 3)^{3148} = (-1)^{3148} = 1$

15 **(c)**

$aC_0 - (a+d)C_1 + (a+2d)C_2 - (a+3d)C_3 + \cdots$

$+ (-1)^n(a+nd)C_n$

$= \sum_{r=0}^{n}(a+rd)(-1)^r\ {}^nC_r$

$= a\sum_{r=0}^{n}(-1)^r\ {}^nC_r - dn\sum_{r=1}^{n-1}{}^{n-1}C_{r-1}(-1)^{r-1}$

$= a \times 0 - dn \times 0 = 0$

16 **(b)**

We have,

$\frac{18^3 + 7^3 + 3\cdot18\cdot7\cdot25}{3^6 + 6\cdot243\cdot2 + 15\cdot81\cdot4 + 20\cdot27\cdot8 + 15\cdot9\cdot}$

$= \frac{18^3 + 7^3 + 3\cdot18\cdot7\cdot}{{}^6C_0\,3^6 + {}^6C_1\,3^5\cdot2^1 + {}^6C_2\,3^4\cdot2^2 + {}^6C_3\,3^3\,2^3}$

$= \frac{(18+7)^3}{(3+2)^6} = \frac{5^6}{5^6} = 1$

17 **(b)**

We have,

$(1+x^2)^5(1+x)^4$

$= ({}^5C_0 + {}^5C_1\,x^2 + {}^5C_2\,x^4 + \cdots)$

$\times ({}^4C_0 + {}^4C_1\,x + {}^4C_2\,x^2 + {}^4C_3\,x^3 + {}^4C_4\,x^4)$

$\therefore$ Coefficient of x^5 in $\{(1+x^2)^5(1+x)^4\}$

$= {}^5C_2 \times {}^5C_1 + {}^4C_3 \times {}^5C_1 = 60$

18 **(d)**

$({}^nC_0)^2 + ({}^nC_1)^2 + ({}^nC_2)^2 + \cdots + ({}^nC_5)^2$

$= ({}^5C_0)^2 + ({}^5C_1)^2 + ({}^5C_2)^2 + ({}^5C_3)^2 + ({}^5C_4)^2 + ({}^5C_5)^2$

$= 1 + 25 + 100 + 100 + 25 + 1 = 252$

19 **(c)**

Let

$S = (1+x)^{1000} + 2\,x(1+x)^{999} + 3\,x^2(1+x)^{998} + \cdots + 1000\,x^{999}(1+x) + 1001\,x^{1000}$ ... (i)

$\therefore \dfrac{x}{1+x}S = x(1+x)^{999} + 2\,x^2(1+x)^{998} + \cdots + 1000\,x^{1000} + \dfrac{1001\,x^{1001}}{1+x}$ ... (ii)

Subtracting (ii) from (i), we get

$\left(1-\dfrac{x}{x+1}\right)S = (1+x)^{1000} + x(1+x)^{999} + x^2(1+x)^{998} + \cdots + x^{1000} - \dfrac{1001\,x^{1001}}{1+x}$

$\Rightarrow S = (1+x)^{1001} + x\,(1+x)^{1000} + x^2(1+x)^{999} + \cdots + x^{1000}(1+x) - 1001\,x^{1001}$

$\Rightarrow S = (1+x)^{1001}\dfrac{\left\{1-\left(\frac{x}{1+x}\right)^{1001}\right\}}{\left\{1-\frac{x}{1+x}\right\}} - 1001\,x^{1001}$

$\Rightarrow S = (1+x)^{1002}\left\{1-\left(\dfrac{x}{1+x}\right)^{1001}\right\} - 1001\,x^{1001}$

$\Rightarrow S = (1+x)^{1002} - x^{1001}(1+x) - 1001\,x^{1001}$

$\therefore$ Coefficient of x^{50} in S is ${}^{1002}C_{50}$

20 **(d)**

$\dfrac{1}{(x-1)^2(x-2)} = \dfrac{1}{-2(1-x)^2\left(1-\frac{x}{2}\right)}$

$= -\dfrac{1}{2}\left[(1-x)^{-2}\left(1-\dfrac{x}{2}\right)^{-1}\right]$

$= -\dfrac{1}{2}\left[(1+2x+\cdots)\left(1+\dfrac{x}{2}+\cdots\right)\right]$

$\therefore$ Coefficient of constant term is $-\frac{1}{2}$.

21 **(c)**

Let $S = 1 + \frac{2\cdot1}{3\cdot2} + \frac{2\cdot5}{3\cdot6}\left(\frac{1}{2}\right)^2 + \frac{2\cdot5\cdot8}{3\cdot6\cdot9}\left(\frac{1}{2}\right)^3 + \ldots$

$= 1 + \dfrac{\frac{2}{3}}{1}\left(\dfrac{1}{2}\right) + \dfrac{\left(\frac{2}{3}\right)\left(\frac{5}{3}\right)}{2!}\left(\dfrac{1}{2}\right)^2 + \dfrac{\left(\frac{2}{3}\right)\left(\frac{5}{3}\right)\left(\frac{8}{3}\right)}{3!}\left(\dfrac{1}{2}\right)^3 + \ldots$

$= \left(1-\dfrac{1}{2}\right)^{-2/3} = \left(\dfrac{1}{2}\right)^{-2/3} = 2^{2/3} = 4^{1/3}$

$\left[\because (1-x)^{-n} = 1 + nx + \dfrac{n(n+1)}{2!}x^2 + \ldots\right]$

22 **(a)**

rth term in the expansion of $\left(3x - \dfrac{2}{x^2}\right)^{15}$ is

$T_r = {}^{15}C_{r-1}(3x)^{15-r+1}\left(\dfrac{-2}{x^2}\right)^{r-1}$

$= {}^{15}C_{r-1}(3)^{15-r+1}(-2)^{r-1}(x)^{15-3r+3}$

For the term independent of x, put

$15 - 3r + 3 = 0 \Rightarrow r = 6$

23 **(a)**

We have, $\sum_{r=0}^{n}\sum_{s=0}^{n}(r+s)(C_r+C_s)$

$= \sum_{r=0}^{n}\sum_{s=0}^{n}(rC_r + rC_s + sC_r + sC_s)$

$= \sum_{r=0}^{n}\left[\sum_{s=0}^{n} rC_r + r\sum_{s=0}^{n} C_s + C_s\sum_{s=0}^{n} s + \sum_{s=0}^{n} sC_s\right]$

$= \sum_{r=0}^{n}\left[(n+1)r\cdot C_r + r2^n + \dfrac{n(n+1)}{2}C_r + n \cdot 2^{n-1}\right]$

$= (n+1)n\cdot 2^{n-1} + (2^n)\dfrac{n(n+1)}{2} + \dfrac{n(n+1)}{2}2^n + n2^{n-1}(n+1)$

$= n(n+1)2^n + n(n+1)2^n$

$= 2n(n+1)2^n$ ...(i)

Also, $\sum_{\mathrm{r}=0}^{n}\sum_{s=0}^{n}(\mathrm{r}+\mathrm{s})(C_{\mathrm{r}}+C_s)$

$= \sum_{r=0}^{n} 4rC_r + 2\sum_{0\le r<s\le n}\sum(r+s)(C_r+C_s)$

$\therefore\ 2n(n+1)2^n = 4n\cdot 2^{n-1} + 2\sum_{0\le r<s\le n}\sum(r+s)(C_r+C_s)$

$\Rightarrow \sum_{0\le r<s\le n}\sum(r+s)(C_r+C_s) = n^2\cdot 2^n$

25 **(a)**

We know,

$(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_rx^r + \ldots$ ...(i)

and $\left(1+\frac{1}{x}\right)^n = C_0 + C_1\frac{1}{x} + C_2\frac{1}{x^2} + \ldots + C_r\frac{1}{x^r} + C_{r+1}\frac{1}{x^{r+1}} + C_{r+2}\frac{1}{x^{r+2}}\ldots C_n\frac{1}{x^n}$ ...(ii)

On multiplying Eqs. (i) and (ii), equation coefficient of x^r in $\frac{1}{x^n}(1+x)^{2n}$ or the coefficient of x^{n+r} in $(1+x)^{2n}$, we get the value of required expression which is

${}^{2n}C_{n+r} = \dfrac{(2n)!}{(n-r)!\,(n+r)!}$

27 **(b)**

In $(x+a)^{100} + (x-a)^{100}$ n is even

$\therefore$ Total number of terms $= \frac{n}{2} + 1 = \frac{100}{2} + 1 = 51$

28 **(b)**

Given polynomial is

$(x-1)(x-2)(x-3)\ldots(x-19)(x-20)$

$= x^{20} - (1+2+3+\cdots+19+20)x^{19}$

$+(1\times2+2\times3+\cdots+19\times20)x^{18}$

$-\cdots+(1\times2\times3\times4\times\ldots\times19\times20)$

$\therefore$ Coefficient of $x^{19} = -(1+2+3+\ldots+19+20)$

$= -\left[\frac{20}{2}(1+20)\right]$

$= -10\times21 = -210$

29 **(c)**

We know that,

${}^{15}C_0 + {}^{15}C_1 + {}^{15}C_2 + \cdots + {}^{15}C_{15} = 2^{15}$

$\Longrightarrow 2({}^{15}C_8 + {}^{15}C_9 + \cdots + {}^{15}C_{15})2^{15}$ $[\because {}^nC_r = {}^nC_{n-r}]$

$\Longrightarrow {}^{15}C_8 + {}^{15}C_9 + \cdots + {}^{15}C_{15} = 2^{14}$

30 **(c)**

The number of terms in the expansion of $(a+b+c)^n$

$= \frac{(n+1)(n+2)}{2}$

31 **(c)**

We have,

$T_{r+1} = {}^5C_r(y^2)^{5-r}\left(\frac{c}{y}\right)^r = {}^5C_r y^{10-3r} c^r$

This will contain y, if $10-3r = 1 \Rightarrow r = 3$

$\therefore$ Coefficient of $y = {}^5C_3\, c^3 = 10\, c^3$

32 **(b)**

$\because (0.99)^{15} = (1-0.01)^{15}$

$= 1 - {}^{15}C_1(0.01) + {}^{15}C_2(0.01)^2 - {}^{15}C_3(0.01)^3 + \ldots$

We want to answer correct upto 4 decimal places and as such, we have left further expansion.

$= 1 - 15(0.01) + \frac{15\cdot14}{1\cdot2}(0.0001) - \frac{15\cdot14\cdot13}{1\cdot2\cdot3}(0.000001) + \ldots$

$= 1 - 0.15 + 0.0105 - 0.000455 + \ldots$

$= 1.0105 - 0.150455$

$= 0.8601$

33 **(b)**

By hypothesis, $2^n = 4096 = 2^{12} \Rightarrow n = 12$

Since, n is even, hence greatest coefficient

$= {}^nC_{n/2} = {}^{12}C_6 = \frac{12\cdot11\cdot10\cdot9\cdot8\cdot7}{1\cdot2\cdot3\cdot4\cdot5\cdot6} = 924$

34 **(c)**

Given that, ${}^nC_{r-1} = {}^nC_{r+1}$

$\Rightarrow \frac{n!}{(n-r+1)(n-r)(n-r-1)!\,(r-1)!}$

$= \frac{n!}{(n-r-1)!\,(r+1)(r)(r-1)!}$

$\Rightarrow r^2 + r = n^2 - nr + n - nr + r^2 - r$

$\Rightarrow n^2 - 2nr - 2r + n = 0$

$\Rightarrow (n-2r)(n+1) = 0 \Rightarrow r = \frac{n}{2}$

35 **(d)**

It is given that

${}^nC_1\, x^{n-1}a^1 = 240$ ... (i)

${}^nC_2\, x^{n-2}\, a^2 = 720$ ... (ii)

${}^nC_3\, x^{n-3}\, a^3 = 1080$ ... (iii)

From (i), (ii) and (iii)

$\frac{({}^nC_2)^2\, x^{2n-4}a^4}{{}^nC_1\,{}^nC_3\, x^{2n-4}\, a^4} = \frac{720\times720}{240\times1080}$

$\Rightarrow \frac{6\,n^2(n-1)^2}{4\,n^2(n-1)(n-2)} = 2$

$\Rightarrow \frac{3\,(n-1)}{2\,(n-2)} = 2$

$\Rightarrow 3n - 3 = 4n - 8 \Rightarrow n = 5$

36 **(d)**

$\frac{1}{81^n}(1 - 10\cdot{}^{2n}C_1 + 10^2\cdot{}^{2n}C_2 - 10^3\cdot{}^{2n}C_3 + \cdots + 10^{2n})$

$= \frac{1}{(81)^n}(1-10)^{2n} = 1$

37 **(b)**

We have,

$(1+x+x^2)^n =$

$a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots + a_{2n}x^{2n}$

On differentiating both sides, we get

$n(1-1+1)^{n-1}(1+2x) = a_1 + 2a_2x + 3a_3x^2 + \ldots + 2na_{2n}x^{2n-1}$

On putting $x = -1$ we get

$n(1-1+1)^{n-1}(1-2) = a_1 - 2a_2 + 3a_3 - \ldots - 2na_{2n}$

$\Longrightarrow a_1 - 2a_2 + 3a_3 - \ldots - 2na_{2n} = -n$

38 **(a)**

Since $(n+1)^{\text{th}}$ term is the middle term in the expansion of $(1+x)^{2n}$

$\therefore$ Coefficient of the middle term

$= {}^{2n}C_n = \frac{(2n)!}{n!\,n!}$

$= \frac{(1\cdot3\cdot5\ldots(2n-1)(2\cdot4\cdot6\ldots(2n-2)(2n))}{n!\,n!}$

$= \frac{1\cdot3\cdot5\ldots.(2n-1)2^n\,n!}{n!\,n!}$

$= \frac{1\cdot3\cdot5\ldots(2n-1)2^n}{n!}$

39 **(a)**

We have,

$(1+x)^{10}\left(1+\frac{1}{x}\right)^{12}=\frac{(1+x)^{22}}{x^{12}}$

$\therefore$ Constant term in $(1+x)^{10}\left(1+\frac{1}{x}\right)^{12}$

$=$ Coefficient of x^{12} in $(1+x)^{22}$

$= {}^{22}C_{12} = {}^{22}C_{10}$

40 **(b)**

Given, $a_n = na_{n-1}$

For $n=2$

$a_2 = 2a_1 = 2 \quad (\because a_1 = 1\text{given})$

$a_3 = 3a_2 = 3(2) = 6$

$a_4 = 4(a_3) = 4(6) = 24$

$a_5 = 5(a_4) = 5(24) = 120$

41 **(a)**

Since,

$x(1+x)^n = xC_0 + C_1x^2 + C_2x^3 + \ldots + C_nx^{n+1}$

On differentiating w.r.t. x, we get

$(1+x)^n + nx(1+x)^{n-1}$

$= C_0 + 2C_1x$

$+ 3C_2x^2 + \ldots + (n+1)C_nx^n$

Put $x=1$, we get

$C_0 + 2C_1 + 3C_2 + \ldots + (n+1)C_n = 2^n + n2^{n-1}$

$= 2^{n-1}(n+2)$

42 **(c)**

Let T_{r+1} denote the $(r+1)^{th}$ term in the expansion of $\left(x^3-\frac{1}{x^2}\right)^n$. Then,

$T_{r+1} = {}^nC_r\, x^{3n-5r}(-1)^r$

For this term to contain x^5, we must have

$3n-5r = 5 \Rightarrow r = \frac{3n-5}{5}$

$\therefore$ Coefficient of $x^5 = {}^nC_{\frac{3n-5}{5}}(-1)^{\frac{3n-5}{5}}$

Similarly,

Coefficient of $x^{10} = {}^nC_{\frac{3n-10}{5}}(-1)^{\frac{3n-10}{5}}$

Now,

Coefficient of x^5 + Coefficient of $x^{10} = 0$

$\Rightarrow {}^nC_{\frac{3n-5}{5}}(-1)^{\frac{3n-5}{5}} + {}^nC_{\frac{3n-10}{5}}(-1)^{\frac{3n-10}{5}} = 0$

$\Rightarrow {}^nC_{\frac{3n-5}{5}} = {}^nC_{\frac{3n-10}{5}}$

$\Rightarrow \frac{3n-5}{5} + \frac{3n-10}{5} = n$

$\Rightarrow 6n - 15 = 5n$

$\Rightarrow n = 15$

43 **(b)**

$(1+x+x^2+x^3)^6 = (1+x)^6(1+x^2)^6$

$= ({}^6C_0 + {}^6C_1x + {}^6C_2x^2 + {}^6C_3x^3 + {}^6C_4x^4$
$+ {}^6C_5x^5 + {}^6C_6x^6) \times ({}^6C_0$
$+ {}^6C_1x^2 + {}^6C_2x^4 +$
${}^6C_3x^6 + {}^6C_4x^8 + {}^6C_5x^{10} + {}^6C_6x^{12})$

$\therefore$ Coefficient of x^{14} in $(1+x+x^2+x^3)^6$

$= {}^6C_2 \cdot {}^6C_6 + {}^6C_4 \cdot {}^6C_5 + {}^6C_6 \cdot {}^6C_4$

$= 15 + 90 + 15 = 120$

44 **(c)**

The 14th term from the end in the expansion of $\left(\sqrt{x}-\sqrt{y}\right)^{17}$ is the $(18-14+1)^{\text{th}}$ i.e. 5th term from the beginning and is given by

${}^{17}C_4(\sqrt{x})^{13}(-\sqrt{y})^4 = {}^{17}C_4\, x^{13/2}y^2$

45 **(d)**

Put $x=1$, we get

$(1+2+3+\cdots+n)^2 = \sum n^3$

46 **(d)**

We have,

$(1+x+x^2)^n = a_0 + a_1x + a_2x^3 + \ldots. + a_{2n}x^{2n}$

On differentiating both sides, we get

$n(1+x+x^2)^{n-1}(1+2x) = a_1 + 2a_2x + 3a_3x^2$
$+ \ldots + 2na_{2n}x^{2n-1}$

Now, on putting $x=1$, we get

$n(3)^{n-1}\cdot(3) = a_1 + 2a_2 + 3a_3 + \ldots + 2na_{2n}$

$\Rightarrow a_1 + 2a_2 + 3a_3 + \ldots. + 2na_{2n} = n\cdot 3^n$

47 **(c)**

There are total $(n+1)$ factors, let $P(x)=0$

Let $(x+{}^nC_0)(x+3\,{}^nC_1)(x+5\,{}^nC_2)\ldots[x+$
2n+1 nCn

$= a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$

Clearly, $a_n = 1$

and roots of the equation $P(x)=0$ are

$-{}^nC_0, -3\,{}^nC_1, \ldots$

Sum of roots $= -a_{n-1}/a_n$

$= -{}^nC_0 - 3\,{}^nC_1 - 5\,{}^nC_2 \ldots$

$\Rightarrow a_{n-1} = (n+1)2^n$

48 **(b)**

${}^{n-2}C_r + 2\cdot {}^{n-2}C_{r-1} + {}^{n-2}C_{r-2}$

$= ({}^{n-2}C_r + {}^{n-2}C_{r-1}) + ({}^{n-2}C_{r-1} + {}^{n-2}C_{r-2})$

$= {}^{n-1}C_r + {}^{n-1}C_{r-1} \;(\therefore {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r)$

$= {}^nC_r$

49 **(d)**

$\because \frac{1}{(x-1)^2(x-2)} = \frac{1}{-2(1-x)^2\left(1-\frac{x}{2}\right)}$

$= -\frac{1}{2}\left[(1-x)^{-2}\left(1-\frac{x}{2}\right)^{-1}\right]$

$= -\frac{1}{2}\left[(1+2x+\ldots)\left(1+\frac{x}{2}+\ldots\right)\right]$

$\therefore$ Coefficient of constant term is $-\frac{1}{2}$

50 **(b)**

In the expansion of $\left(x^2+\frac{a}{x}\right)^5$, the general term is

$T_{r+1}= {}^5C_r(x^2)^{5-r}\left(\frac{a}{x}\right)^r = {}^5C_r\cdot a^r\cdot x^{10-3r}$

For the coefficient of x, put

$10-3r=1 \Rightarrow r=3$

$\therefore$ Coefficient of $x= {}^5C_3a^3=10a^3$

52 **(b)**

Coefficient of x^r in the expansion of $(1+x)^{10}$ is ${}^{10}C_r$ and it is maximum for $r=\frac{10}{2}=5$

Hence, Greatest coefficient= ${}^{10}C_5=\frac{10\,!}{(5\,!)^2}$

53 **(c)**

Given expansion is $\left(\frac{a}{x}+bx\right)^{12}$

$\therefore$ General term, $T_{r+1}= {}^{12}C_r\left(\frac{a}{x}\right)^{12-r}(bx)^r$

$= {}^{12}C_r(a)^{12-r}b^rx^{-12+2r}$

For coefficient of x^{-10}, put

$-12+2r=-10$

$\Rightarrow r=1$

Now, the coefficient of x^{-10} is

${}^{12}C_1(a)^{11}(b)^1=12a^{11}b$

55 **(a)**

We have,

$$T_{r+1}= {}^{21}C_r\left(\sqrt[3]{\frac{a}{\sqrt{b}}}\right)^{21-r}\left(\sqrt{\frac{b}{\sqrt[3]{a}}}\right)^r$$

$\Rightarrow T_{r+1}= {}^{21}C_r\,a^{7-\frac{r}{2}}b^{\frac{2}{3}r-\frac{7}{2}}$

Since the powers of a and b are the same

$\therefore 7-\frac{r}{2}=\frac{2}{3}r-\frac{7}{2}\Rightarrow r=9$

56 **(b)**

$(1-x)^{-4}=1.x^0+4x^1+\frac{4.5}{2}x^2+\ldots$

$$=\left[\frac{1.2.3}{6}x^0+\frac{2.3.4}{6}x+\frac{3.4.5}{6}x^2+\frac{4.5.6}{6}x^3+\ldots+\frac{(r+1)(r+2)(r+3)}{6}x^r+\ldots\right]$$

Therefore, $T_{r+1}=\frac{(r+1)(r+2)(r+3)}{6}x^r$

57 **(a)**

We have,

$$y=\frac{1}{3}+\frac{1\cdot3}{3\cdot6}+\frac{1\cdot3\cdot5}{3\cdot6\cdot9}+\cdots$$

$$\Rightarrow y+1=1+\frac{1}{3}+\frac{1\cdot3}{3\cdot6}+\frac{1\cdot3\cdot5}{3\cdot6\cdot9}+\cdots$$

Comparing the series on RHS with

$1+n\,x+\frac{n(n-1)}{2\,!}x^2+\cdots$, we get

$n\,x=\frac{1}{3}$ ... (i)

and, $\frac{n(n-1)}{2}x^2=\frac{1}{6}$ ... (ii)

Dividing (ii) by square of (i), we get

$\frac{n-1}{2\,n}=\frac{9}{6}\Rightarrow n=-\frac{1}{2}$

$\Rightarrow x=-\frac{2}{3}$ [putting $n=-\frac{1}{2}$ in (i)]

$\therefore y+1=(1+x)^n$

$\Rightarrow y+1=\left(1-\frac{2}{3}\right)^{-1/2}$

$\Rightarrow y+1=\left(\frac{1}{3}\right)^{-1/2}$

$\Rightarrow (y+1)^2=\left(\frac{1}{3}\right)^{-1}\Rightarrow y^2+2\,y+1=3$

$\Rightarrow y^2+2\,y=2$

58 **(b)**

$S(k)=1+3+5\ldots.+(2k-1)=3+k^2$

Put $k=1$ in both sides, we get

LHS $=1$ and RHS$=3+1=4$

$\Rightarrow$ LHS $\neq$ RHS

Put $(k+1)$ in both sides in the place of k, we get

LHS $=1+3+5\ldots.+(2k-1)+(2k+1)$

RHS $=3+(k+1)^2=3+k^2+2k+1$

Let LHS = RHS

Then, $1+3+5\ldots.+(2k-1)+(2k+1)$

$=3+k^2+2k+1$

$\Rightarrow 1+3+5+\ldots+(2k-1)=3+k^2$

If $S(k)$ is true, then $S(k+1)$ is also true.

Hence, $S(k)\Rightarrow S(k+1)$

59 **(b)**

The general term in the expansion of $(5^{1/6}+$ *218)100* is given by

$T_{r+1}= {}^{100}C_r\left(5^{1/6}\right)^{100-r}(2^{1/8})^r$

As 5 and 2 are relatively prime, T_{r+1} will be rational, if

$\frac{100-r}{6}$ and $\frac{r}{8}$ are both integers *ie*, if $100-r$ is a multiple of 6 and r is a multiple of 8. As $0\le r\le 100$, multiples of 8 upto 100 and corresponding value of $100-r$ are

$r=0,8,16,24,\ldots.,88,96$

ie, $100-r=100,92,84,76,\ldots,12,4$

Out of $100-r$, multiples of 6 are 84, 60, 36, 12

$\therefore$ There are four rational terms

Hence, number of irrational terms is $101-4=97$

60 **(b)**

We have,

$T_r= {}^{10}C_{r-1}\left(\frac{x}{3}\right)^{10-r+1}\left(-\frac{2}{x^2}\right)^{r-1}$

$\Rightarrow T_r = {}^{10}C_{r-1}\left(\frac{1}{3}\right)^{11-r} (-2)^{r-1} x^{13-3r}$

For this term to contain x^4, we must have

$13 - 2r = 4 \Rightarrow r = 3$

61 **(c)**

We have, $32^{32} = (2^5)^{32} = 2^{160} = (3-1)^{160}$

$= {}^{160}C_0 3^{160} - {}^{160}C_1 \cdot 3^{159} + \ldots - {}^{160}C_{159} \cdot 3 + {}^{160}C_{160} 3^{\circ}$

$= 3m + 1$, where $m \in N$

$32^{(32)^{(32)}} = (32)^{3m+1}$

$= (2^5)^{3m+1} = 2^{15m+5}$

$= 2^{3(5m+1)} \cdot 2^2 = (2^3)^{5m+1} \cdot 2^2$

$= (7+1)^{5m=1} \times 4$

$= \{ {}^{5m+1}C_0 7^{5m+1} + {}^{5m+1}C_1 7^{5m} + \ldots + {}^{5m+1}C_{5m+1} 7 + {}^{5m+1}C_{5m+1} . 7^0 \} \times 4$

$= (7n + 1) \times 4$,

where $n = {}^{5m+1}C_0 7^{5m+1} + \ldots + {}^{5m+1}C_{5m} \cdot 7$

$28n + 4$

Thus, when $32^{(32)^{(32)}}$ is divided by 7, the remainder is 4

62 **(c)**

We have,

$\left[2^{\log_2 \sqrt{9^{x-1}+7}} + \frac{1}{2^{(1/5)\log_2(3^{x-1}+1)}}\right]^7$

$= \left[\sqrt{9^{x-1}+7} + \frac{1}{(3^{x-1}+1)^{1/5}}\right]^7$

$\therefore T_6 = {}^7C_5 \left(\sqrt{9^{x-1}+7}\right)^{7-5} \left[\frac{1}{(3^{x-1}+1)^{1/5}}\right]^5$

$= {}^7C_5 (9^{x-1}+7) \frac{1}{(3^{x-1}+1)}$

$\Rightarrow 84 = {}^7C_5 \frac{(9^{x-1}+7)}{(3^{x-1}+1)}$

$\Rightarrow 9^{x-1} + 7 = 4(3^{x-1}+1)$

$\Rightarrow \frac{3^{2x}}{9} + 7 = 4\left(\frac{3^x}{3} + 1\right)$

$\Rightarrow 3^{2x} - 12(3^x) + 27 = 0$

$\Rightarrow y^2 - 12y + 27 = 0$ (put $y = 3^x$)

$\Rightarrow (y-3)(y-9) = 0$

$\Rightarrow y = 3, 9 \Rightarrow 3^x = 3, 9 \Rightarrow x = 1, 2$

63 **(d)**

Here, $P(1) = 2$ and from the equation

$P(k) = k(k+1) + 2$

$\Longrightarrow P(1) = 4$

So, $P(1)$ is not true

Hence, mathematical induction is not applicable.

64 **(b)**

We have,

$(1 + 2x + x^2)^{20} = \{(1+x)^2\}^{20} = (1+x)^{40}$

Clearly, $(1+x)^{40}$ contains 41 terms

Hence, $(1 + 2x + x^2)^{20}$ contains 41 terms

65 **(d)**

The series of binomial coefficient is

${}^{15}C_8$

${}^{15}C_0, {}^{15}C_1, {}^{15}C_2, \ldots, {}^{15}C_7$ $\longleftarrow$ decreasing value $\quad\downarrow\quad$ ${}^{15}C_9, \ldots, {}^{15}C_9, {}^{15}C_{15}$ $\longrightarrow$ decreasing value

From the above discussion, we can say that decreasing series is ${}^{15}C_7$, ${}^{15}C_6$, ${}^{15}C_5$.

66 **(c)**

For $n = 1, 10^n + 3 \cdot 4^{n+2} + 5$

$= 10 + 3 \cdot 4^3 + 5 = 207$ This is divisible by 9.

$\therefore$ By induction, the result is divisible by 9.

67 **(d)**

$\frac{{}^8C_0}{6} - {}^8C_1 + {}^8C_2 \cdot 6 - {}^8C_3 \cdot 6^2 + \cdots + {}^8C_8 \cdot 6^7$

$= \frac{1}{6}[{}^8C_0 - 6\,{}^8C_1 + 6^2\,{}^8C_2 - 6^3\,{}^8C_3 + \cdots + 6^8\,{}^8C_8]$

$= \frac{1}{6}[(1-6)^8] = \frac{5^8}{6}$

68 **(a)**

In the expansion of $(1+x)^n$,it is given that ${}^nC_1, {}^nC_2, {}^nC_3$ are in AP

$\Rightarrow 2\,{}^nC_2 = {}^nC_1 + {}^nC_3$

$\Rightarrow 2 \cdot \frac{n(n-1)}{1 \cdot 2} = \frac{n}{1} + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}$

$\Rightarrow 6(n-1) = 6 + (n-2)(n-1)$

$\Rightarrow 6n - 6 = 6 + n^2 - 3n + 2$

$\Rightarrow n^2 - 9n + 14 = 0$

$\Rightarrow (n-2)(n-7) = 0$

$\Rightarrow n = 2, 7$

But $n = 2$ is not acceptable because, when $n = 2$, there are only three terms in the expansion of $(1+x)^2$

$\therefore n = 7$

70 **(a)**

$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots + {}^nC_n x^n$ ...(i)

On differentiating both sides w. r. t. x, we get

$n(1+x)^{n-1} = {}^nC_1 + 2\,{}^nC_2 x + \ldots + n\,{}^nC_n x^{n-1}$ ...(ii)

On putting $x = 1$ in Eq. (ii), we get

$n(2)^{n-1} = {}^nC_1 + 2\,{}^nC_2 + \ldots + n\,{}^nC_n$ ...(iii)

On putting $x = -1$ in Eq. (ii) we get

$0 = {}^nC_1 - 2\,{}^nC_2 + 3\,{}^nC_3 - \ldots (-1)^{n-1} \cdot {}^nC_n$ ...(iv)

On adding Eqs. (iii) and (iv), we get

$n2^{n-1} = 2({}^nC_1 + 3\,{}^nC_3 + \ldots)$

$\Rightarrow\ ^nC_1 + 3\ ^nC_3 + 5\ ^nC_5 + \ldots = \frac{n}{2}\cdot 2^{n-1} = n2^{n-2}$

71 **(a)**

Given expression is $\left(x + x^{\log 10^x}\right)^5$

$\therefore T_3 = \ ^5C_2 \cdot x^3\left(x^{\log 10^x}\right)^2 = 10^6$ (given)

Put $x = 10$, then $10^4 . 10^2 = 10^6$ is satisfied.

Hence, $x = 10$.

72 **(c)**

Given, $^nC_0 - \frac{1}{2}\ ^nC_1 + \frac{1}{3}\ ^nC_2 - \ldots + (-1)^n \frac{^nC_n}{n+1}$

At $n = 1$, $^1C_0 - \frac{1}{2}\ ^1C_1 = 1 - \frac{1}{2} = \frac{1}{2}$

At $n = 2$, $^2C_0 - \frac{1}{2}\ ^2C_1 + \frac{1}{3}\ ^2C_2 = 1 - 1 + \frac{1}{3} = \frac{1}{3}$

Which is satisfied only in option (c)

73 **(b)**

$8^{2n} - (62)^{2n+1} = (1+63)^n - (63-1)^{2n+1}$

$= (1+63)^n + (1-63)^{2n+1}$

$= (1 + \ ^nC_1 63 + \ ^nC_2(63)^2 + \cdots + (63)^n)$

$+ (1 - ^{(2n+1)}C_1 63 + ^{(2n+1)}C_2(63)^2 + \cdots$

$+ (-1)\left(63)^{(2n+1)}\right)$

$= 2 + 63[\ ^nC_1 + \ ^nC_2(63) + \ldots + (63)^{n-1}\ _-{}^{(2n+1)}C_1$

$+ ^{(2n+1)}C_2(63) - \ldots - (63)^{(2n)}]$

$\therefore$ Remainder is 2.

74 **(a)**

We have,

$T_{r+1} = \ ^{20}C_r \times 4^{\frac{20-r}{3}} \times 6^{-\frac{r}{4}}$

$\Rightarrow T_{r+1} = \ ^{20}C_r 2^{\frac{160-11r}{12}} 3^{-\frac{r}{4}}, r = 0,1,2,\ldots,20$

This term will be rational if $\frac{160-11r}{12}$ and $\frac{r}{4}$ are rational numbers

Now, $\frac{r}{4}$ is rational if $r = 0,4,8,12,16,20$

Clearly, $\frac{160-11r}{12}$ is rational for $= 8,16$ and 20

Hence, there are only 3 rational terms

75 **(c)**

We have,

$\left(x^2 + 1 + \frac{1}{x^2}\right)^n = \frac{1}{x^{2n}}(1 + x^2 + x^4)^n$

$= \frac{1}{t^n}(1 + t + t^2)^n$, where $t = x^2$

Clearly, $(1 + t + t^2)^n$ is a polynomial of degree $2n$

Hence, there are $(2n + 1)$ terms

76 **(b)**

$(19)^{2005} + (11)^{2005} - (9)^{2005}$

$= (10+9)^{2005} + (10+1)^{2005} - (9)^{2005}$

$= (9^{2005} + \ ^{2005}C_1(9)^{2004} \times 10 + \ldots) + (\ ^{2005}C_0 +$

2005C110+...−92005

$= (\ ^{2005}C_1 9^{2005} \times 10 +$multipale of 10)+(1+multipal of 10)

$\therefore$ Unit digit=1

77 **(b)**

In the expansion of $(x + 2y)^6$,

$\left(\frac{6}{2} + 1\right)$ th term is the middle term.

$\therefore \quad T_4 = T_{3+1} = \ ^6C_3 x^{6-3}(2y)^3$

$= \quad 8(\ ^6C_3)(xy)^3$

$\therefore$ Coefficient of middle term

$= 8(\ ^6C_3)$

78 **(c)**

General terms, $T_{r+1} = (1)^r\ ^{15}C_r\ (x^4)^{15-r} \cdot \left(\frac{1}{x^3}\right)^r$

$= (-1)^r\ ^{15}C_r \cdot x^{60-7r}$

For the coefficient of x^{-17}, put $60 - 7r = -17$

$\Rightarrow \quad 60 + 17 = 7r \Rightarrow r = 11$

Now, coefficient of $x^{-17} = (-1)^{11}\,{}^{15}C_{11} = -^{15}C_{11}$

79 **(b)**

$\frac{(1-3x)^{1/2} + (1-x)^{5/3}}{2\left(1 - \frac{x}{4}\right)^{1/2}}$

$= \frac{\left[\left[1 + \frac{1}{2}(-3x) + \frac{1}{2}\left(-\frac{1}{2}\right)\frac{1}{2}(-3x)^2 + \ldots\right] + \left[1 + \frac{5}{3}(-x) + \frac{5}{3}\cdot\frac{2}{3}\cdot\frac{1}{2}(-x)^2 + \ldots\right]\right]}{2\left[1 + \frac{1}{2}\left(-\frac{x}{4}\right) + \frac{1}{2}\left(-\frac{1}{2}\right)\frac{1}{2}\left(-\frac{x}{4}\right)^2 + \ldots\right]}$

$= \frac{2\left[1 - \frac{19}{12}x - \frac{41}{144}x^2 - \ldots\right]}{2\left[1 - \frac{x}{8} - \frac{1}{128}x^2 - \ldots\right]}$

$= \left[1 - \frac{19}{12}x - \frac{41}{144}x^2 - \ldots\right]\left[1 - \frac{x}{8} - \frac{1}{128}x^2 \ldots\right]^{-1}$

$= 1 - \frac{35}{24}x + \ldots$

On neglecting higher powers of x, we get

$a + bx = 1 - \frac{35}{24}x$

$\Rightarrow a = 1, b = -\frac{35}{24}$

80 **(b)**

$^{18}C_{15} + 2(\ ^{18}C_{16}) + \ ^{17}C_{16} + 1 = \ ^nC_3$

$\Rightarrow \ ^{18}C_{15} + \ ^{18}C_{16} + \ ^{18}C_{16} + \ ^{17}C_{16} + ^{17}C_{17} = \ ^nC_3$

$\Rightarrow \quad ^{19}C_{16} + \ ^{18}C_{16} + \ ^{18}C_{17} = \ ^nC_3$

$\Rightarrow \quad ^{19}C_{16} + \ ^{19}C_{17} = \ ^nC_3$

$\Rightarrow \ ^{20}C_{17} = \ ^nC_3 \Rightarrow \ ^{20}C_3 = \ ^nC_3 \Rightarrow n = 20$

81 **(c)**

We have, $49^n + 16n - 1 = (1+48)^n + 16n - 1$

$= 1 + \ ^nC_1(48) + \ ^nC_2(48)^2 + \ldots + \ ^nC_n(48)^n$

$+ 16n - 1$

$= (48n + 16n) + \ ^nC_2(48)^2$

$+ \ ^nC_3(48)^3 + \ldots + \ ^nC_n(48)^n$

$= 64n + 8^2[\ ^nC_2 \cdot 6^2 + \ ^nC_3 \cdot 6^3 \cdot 8 + \ ^nC_4 \cdot 6^4$

$\cdot 8^2 + \ldots + \ ^nC_n \cdot 6^n \cdot 8^{n-2}]$

Hence, $49^n + 16n - 1$ is divisible by 64

82 **(b)**

We have, $(1+x)^{50} = \sum_{r=0}^{50}\ ^{50}C_r x^r$. (The sum of

coefficients of odd powers of x)
$= {}^{50}C_1 + {}^{50}C_3 + \ldots + {}^{50}C_{49}$
$= 2^{50-1} = 2^{49}$

84 **(b)**

Given, $\alpha = \frac{5}{2!\,3} + \frac{5\cdot 7}{3!\,3^2} + \frac{5\cdot 7\cdot 9}{4!\,3^2} + \cdots$ ... (i)

On comparing

$$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots \quad \text{... (ii)}$$

With respect to factorial, we get

$$n(n-1)x^2 = \frac{5}{3} \quad \text{.... (iii)}$$

$$n(n-1)(n-2)x^3 = \frac{5\cdot 7}{3^2} \quad \text{... (iv)}$$

and $n(n-1)(n-2)(n-3)x^4 = \frac{5\cdot 7\cdot 9}{3^3}$ .. (v)

on dividing Eq. (iv) by (iii) and Eq. (v) by Eq. (iv), we get

$$(n-2)x = \frac{7}{3} \quad \text{... (vi)}$$

and $(n-3)x = 3$ ...(vii)

Again, dividing Eq. (vi) by Eq. (vii), we get

$$\frac{n-2}{n-3} = \frac{7}{9}$$

$\Rightarrow \quad 9n - 18 = 7n - 21$

$\Rightarrow \quad 2n = -3 \Rightarrow n = -\frac{3}{2}$

On putting the value of n in Eq. (vi), we get

$$\left(-\frac{3}{2} - 2\right)x = \frac{7}{3} \Rightarrow x = -\frac{2}{3}$$

$\therefore$ From Eq. (ii),

$$\left(1 - \frac{2}{3}\right)^{-3/2} = 1 + 1 + \frac{5}{2!\,3} + \frac{5\cdot 7}{3!\,3^2} + \cdots$$

$$\Rightarrow 3^{3/2} - 2 = \frac{5}{2!\,3} + \frac{5\cdot 7}{3!\,3^2} + \ldots$$

$\Rightarrow \quad \alpha = 3^{3/2} - 2$ [from Eq. (i)]

Now, $\alpha^2 + 4\alpha = (3^{3/2} - 2)^2 + 4(3^{3/2} - 2)$
$= 27 + 4 - 4\cdot 3^{3/2} + 4\cdot 3^{3/2} - 8$
$= 23$

85 **(d)**

$$\frac{1+2x}{(1-2x)^2} = (1+2x)(1-2x)^{-2}$$

$$= (1+2x)\left(1 + \frac{2}{1!}(2x) + \frac{2\cdot 3}{2!}(2x)^2 + \cdots + \frac{2\cdot 3\ldots r}{(r-1)!}(2x)^r + \frac{2\cdot 3\cdot 4\ldots(r+1)(2x)^r}{r!}\right)$$

The coefficient of x^r

$$= 2\frac{r!}{(r-1)!}2^{r-1} + \frac{(r+1)!}{r!}2^r$$
$$= r2^r + (r+1)r^2$$
$$= 2^r(2r+1)$$

86 **(d)**

We have,

$\{(1+x)(1+y)(x+y)\}^n$
$= (1+x)^n(1+y)^n(x+y)^n$

$\therefore$ Coefficient of $x^n\,y^n$ in $\{(1+x)(1+y)(x+y)\}^n$

$$= \sum_{r=0}^{n} ({}^nC_r)^3$$

87 **(d)**

We have,

$(1+x+x^2)^n = C_0 + C_1x + C_2\,x^2 + \cdots + C_{2n}x^{2\,n}$

Replacing x by $-\frac{1}{x}$, we get

$$\left(1 - \frac{1}{x} + \frac{1}{x^2}\right)^n = C_0 - C_1\frac{1}{x} + C_2\frac{1}{x^2} + \cdots + C_{2n}\frac{1}{x^{2\,n}}$$

Now,

$C_0\,C_1 - C_1\,C_2 + C_2\,C_3 - \cdots$

$= \text{Coeff. of } x \text{ in } \{C_0 + C_1\,x + C_2\,x^2 + \cdots\}\left\{C_0 - C_1\frac{1}{x} + C_2\frac{1}{x^2} - \cdots\right\}$

$= \text{Coeff. of } x \text{ in } (1+x+x^2)^n\left(1 - \frac{1}{x} + \frac{1}{x^2}\right)^n$

$= \text{Coeff. of } x^{2\,n+1} \text{ in } (1+x+x^2)^n(x^2-x+1)^n$

$= \text{Coeff. of } x^{2\,n+1} \text{ in } [(1+x^2)^2 - x^2]^2$

$= \text{Coeff. of } x^{2\,n+1} \text{ in } [1+x^2+x^4]^n = 0$

88 **(d)**

$\because a, b, c$ are in AP

$\Rightarrow\ 2b = a + c$

$\Rightarrow\ a - 2b + c = 0$

On putting $x = 1$, we get

Required sum $= (1 + (a-2b+c)^2)^{1973} =$ $(1+0)^{1973} = 1$

89 **(a)**

We have , $T_2 = 14a^{5/2}$

$\Rightarrow\ {}^nC_1(a^{1/13})^{n-1}(a^{3/2})^1 = 14a^{5/2}$

$\Rightarrow\ na^{\frac{n-1}{13}+\frac{3}{2}} = 14a^{5/2}$

$\Rightarrow \quad n = 14$

$$\therefore \frac{{}^nC_3}{{}^nC_2} = \frac{{}^{14}C_3}{{}^{14}C_2} = 4$$

90 **(b)**

For $n > 1$, we have

$(1+x)^n = {}^nC_0 + {}^nC_1\,x + {}^nC_2\,x^2 + {}^nC_3\,x^3 + \cdots + {}^nC_n\,x^n$

$\Rightarrow (1+x)^n = 1 + nx + ({}^nC_2\,x^2 + {}^nC_3\,x^3 + \cdots + {}^nC_n\,x^n)$

$\Rightarrow (1+x)^n - 1 - nx$
$= x^2({}^nC_2 + {}^nC_3\,x + {}^nC_4\,x^2 + \cdots + {}^nC_n\,x^{n-2})$

Clearly, RHS is divisible by x^2 and x. So, LHS is also divisible by x as well as x^2

91 **(c)**

Let T_{r+1} be the $(r+1)^{th}$ terms in the expansion of $\left(\frac{x^2}{a} - \frac{a}{x}\right)^{12}$. Then,

$$T_{r+1} = {}^{12}C_r \left(\frac{x^2}{a}\right)^{12-r} \left(-\frac{a}{x}\right)^r$$
$$= {}^{12}C_r\, x^{24-3r}(-1)^r\, a^{2r-12}$$

For the coefficient of $x^6\, y^{-2}$, we must have

$24 - 3r = 6$ and $2r - 12 = -2$

These two equations are inconsistent

Hence, there is no term containing $x^6\, a^{-2}$

So, its coefficient is 0

92 **(d)**

$\because\ I + f + f' = (5+2\sqrt{6})^n + (5-2\sqrt{6})^n$

$= 2k$(even integer)

$\therefore\ f + f' = 1$

Now, $(I+f)f' = (5+2\sqrt{6})^n(5-2\sqrt{6})^n = (1)^n = 1$

$\Rightarrow\ (I+f)(1-f) = 1$

or $I = \frac{1}{(1-f)} - f$

93 **(b)**

Given equation can be rewritten as

$E = a[{}^nC_0 - {}^nC_1 + {}^nC_2 - \ldots + (-1)^n\,{}^nC_n]$
$+ [{}^nC_1 - (2)({}^nC_2)$
$+ (3)({}^nC_3) - \ldots + (-1)^n(n)\,({}^nC_n)]$

$\Rightarrow E = 0 + 0 = 0$ (by properties)

94 **(a)**

Coefficient of x^{r-1} in

$(1+x)^n + (1+x)^{n+1} + \ldots + (1+x)^{n+k}$

$= {}^nC_{r-1} + {}^{n+1}C_{r-1} + \ldots + {}^{n+k}C_{r-1}$

$= {}^nC_r + {}^nC_{r-1} + {}^{n+1}C_{r-1} + \ldots + {}^{n+k}C_{r-1} - {}^nC_r$

$= {}^{n+k+1}C_r - {}^nC_r$

Now, $\sum_{r=0}^{n+k+1}(-1)^r a_r = \sum_{r=0}^{n+k+1}(-1)^r\ {}^{n+k-1}C_r - \sum_{r=0}^{n+k+1}(-1)^r\ {}^nC_r = 0$

95 **(a)**

We have, $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots$

If x is replace by $-\left(1-\frac{1}{x}\right)$ and n is $-n$, then expression

becomes $\left[1 - \left(1 - \frac{1}{x}\right)\right]^{-n}$

$$= 1 + (-n)\left[-\left(1-\frac{1}{x}\right)\right] + \frac{(-n)(-n-1)}{2!}\left[-\left(1-\frac{1}{x}\right)\right]^2 + \ldots$$

$$\Longrightarrow x^n = 1 + n\left(1-\frac{1}{x}\right) + \frac{n(n+1)}{2!}\left(1-\frac{1}{x}\right)^2 + \ldots$$

96 **(b)**

Given expansion is$(x+a)^n$

On replacing a by ai and $-ai$ respectively, we get

$(x+ai)^n = (T_0 - T_2 + T_4 - \ldots) + i(T_1 - T_3 + T_5 - \ldots)$ ...(i)

and $(x-ai)^n = (T_0 - T_2 + T_4 - \ldots) + i(T_1 - T_3 + T_5 - \ldots)$ ...(ii)

On multiplying Eqs. (ii) and (i), we get required result

$(x^2+a^2)^n = (T_0 - T_2 + T_4 - \ldots)^2$
$+ (T_1 - T_3 + T_5 - \ldots)^2$

97 **(b)**

Given coefficient of $(2x+1)$th term=coefficient of $(r+2)$th term

$\Longrightarrow\ {}^{43}C_{2r} = {}^{43}C_{r+1}$

$\Longrightarrow 2r + (r+1) = 43$ or $2r = r+1$

$\Longrightarrow r = 14$ or $r = 1$

98 **(b)**

We have,

$(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$ ...(i)

and $\left(1+\frac{1}{x}\right)^n = C_0 + C_1\frac{1}{x} + C_2\left(\frac{1}{x}\right)^2 + \ldots + C_n\left(\frac{1}{x}\right)^n$ ...(ii)

On multiplying Eqs. (i) and (ii) and taking the coefficient of constant terms in right hand side

$= C_0^2 + C_1^2 + C_2^2 + \ldots + C_n^2$

In right hand side $(1+x)^n\left(1+\frac{1}{x}\right)^n$ or in $\frac{1}{x^n}(1+x)^{2n}$ or term containing x^n in $(1+x)^{2n}$.

Clearly the coefficient of x^n in $(1+x)^{2n}$ is equal to ${}^{2n}C_n = \frac{(2n)!}{n!n!}$

99 **(b)**

We have,

$$\frac{C_k}{C_{k-1}} = \frac{{}^nC_k}{{}^nC_{k-1}} = \frac{n-k+1}{k}$$

$$\therefore \sum_{k=1}^{n} k^3\left(\frac{C_k}{C_{k-1}}\right)^2$$

$$= \sum_{k=1}^{n} k^3\frac{(n-k+1)^2}{k^2} = \sum_{k=1}^{n} k(n-k+1)^2$$

$$= (n+1)^2\left(\sum_{k=1}^{n} k\right) - 2(n+1)\left(\sum_{k=1}^{n} k^2\right) + \left(\sum_{k=1}^{n} k^3\right)$$

$$= (n+1)^2 \frac{n(n+1)}{2} - \frac{2(n+1)n(n+1)(2n+1)}{6} + \left\{\frac{n(n+1)}{2}\right\}^2$$

$$= \frac{n(n+1)^2}{12}\{6(n+1) - 4(2n+1) + 3n\}$$

$$= \frac{n(n+1)^2(n+2)}{12}$$

100 **(b)**

Let

$$S = 1 \times 2 \times 3 \times 4 + 2 \times 3 \times 4 \times 5 + 3 \times 4 \times 5 \times 6 + \cdots + n(n+1)(n+2)(n+3)$$

$$\Rightarrow S = \sum_{r=1}^{n} r(r+1)(r+2)(r+3)$$

$$\Rightarrow S = \sum_{r=1}^{n} \frac{(r+3)!}{(r-1)!}$$

$$\Rightarrow S = 4! \sum_{r=1}^{n} \frac{(r+3)!}{(r-1)!\,4!}$$

$$\Rightarrow S = 4! \sum_{r=1}^{n} \frac{(r+3)!}{(r-1)!\,4!}$$

$$\Rightarrow S = 4! \sum_{r=1}^{n} {}^{r+3}C_4$$

$$\Rightarrow S = 4! \sum_{r=12}^{n} \text{Coefficient of } x^4 \text{in } (1+x)^{r+3}$$

$$\Rightarrow S = 4! \times \text{Coefficient of } x^4 \text{ in } \sum_{r=1}^{n} (1+x)^{r+3}$$

$$\Rightarrow S = 4! \times \text{Coefficient of } x^4 \text{ in } (1+x)^4 \left\{\frac{(1+x)^n - 1}{(1+x) - 1}\right\}$$

$$\Rightarrow S = 4! \times \text{Coefficient of } x^5 \text{ in } \{(1+x)^{\mathrm{n}+4} - (1+x)^4\}$$

$$\Rightarrow S = 4! \times \text{Coefficient of } x^5 \text{ in } (1+x)^{n+4}$$

$$\Rightarrow S = 4! \times {}^{n+4}C_5$$

$$= \frac{1}{5} n(n+1)(n+2)(n+3)(n+4)$$

101 **(c)**

We have,

$$(x+y+z)^{18} = \sum_{r+s+t+=18} \frac{18!}{r!\,s!\,t!} x^r\, y^s\, z^t$$

$$\therefore \text{Coefficient of } x^8 y^6 z^4 = \frac{18!}{8!\,6!\,4!} = \frac{18!}{10!\,8!} \times \frac{10!}{6!\,4!}$$

$$= {}^{18}C_{10} \times {}^{10}C_6$$

Also, Coefficient of $x^8y^6z^4 = \frac{18!}{8!\ 6!\ 4!}$

$$= \frac{18!}{4!\ 14!} \times \frac{14!}{8!\ 6!}$$

$$= {}^{18}C_{14} \times {}^{14}C_8 = {}^{18}C_4 \times {}^{14}C_6$$

Again,

Coefficient of $x^8y^6z^4 = \frac{18!}{8!\,6!\,4!} = \frac{18!}{12!\,6!} \times \frac{12!}{8!\,4!}$

$$= {}^{18}C_6 \times {}^{12}C_8$$

102 **(c)**

We have,

$$1 + x + x^2 + x^3 = (1+x)(1+x^2)$$

$$\therefore (1 + x + x^2 + x^3)^{11} = (1+x)^{11}(1+x^2)^{11}$$

$$= ({}^{11}C_0 + {}^{11}C_1\, x + {}^{11}C_2\, x^2 + {}^{11}C_3\, x^3 + {}^{11}C_4\, x^4 + \cdots)$$

$$\times ({}^{11}C_0 + {}^{11}C_1\, x^2 + {}^{11}C_2\, x^4 + \cdots)$$

$\Rightarrow$ Coefficient of x^4 in $(1 + x + x^2 + x^3)^{11}$

= Coefficient of x^4 in

$$\{({}^{11}C_0 + {}^{11}C_1\, x + {}^{11}C_2\, x^2 + \cdots)({}^{11}C_0 + {}^{11}C_1\, x^2 + {}^{11}C_2\, x^4 + \cdots)\}$$

$$= {}^{11}C_0 \times {}^{11}C_2 + {}^{11}C_2 \times {}^{11}C_1 + {}^{11}C_4 \times {}^{11}C_0 = 990$$

103 **(b)**

We have,

a = Sum of the coefficients in the expansion of $(1 - 3x + 10\,x^2)^n$

$\Rightarrow a = (1 - 3 + 10)n = 8^n = 2^{3n}$

b = Sum of the coefficients in the expansion of $(1 + x^2)^n$

$\Rightarrow b = (1+1)^n = 2^n$

Clearly, $a = b^3$

104 **(b)**

Let $P(n)$: $10^{n-2} \geq 81n$

For $n = 4, 10^2 \ngeq 81 \times 4$

For $n = 5, 10^3 \geq 81 \times 5$

Hence, by mathematical induction for $n \geq 5$, the proposition is true.

105 **(c)**

Given that, $T_1 = {}^nC_0 = 1$ ...(i)

$T_2 = {}^nC_1 ax = 6x$

$\Rightarrow \frac{n!}{(n-1!)} a = 6 \Rightarrow na = 6$ ...(ii)

and $T_3 = {}^nC_2 (ax)^2 = 6x^2$

$\Rightarrow \frac{n(n-1)}{2} a^2 = 16$ ...(iii)

Only option (c) is satisfying Eqs. (ii) and (iii)

106 **(a)**

It is given that

$$(a + b\,x)^{-2} = \frac{1}{4} - 3\,x$$

$$\Rightarrow a^{-2}\left(1 + \frac{b}{a}x\right)^{-2} = \frac{1}{4} - 3\,x$$

$$\Rightarrow a^{-2}\left(1 - \frac{2}{a} b\, x\right) = \frac{1}{4} - 3\,x \quad \left[\begin{array}{l}\text{Neglecting } x^2 \text{ and} \\ \text{higher powers of } x\end{array}\right]$$

$\Rightarrow a^{-2} = \frac{1}{4}, \frac{-2\,b}{a^3} = -3$

Now, $a^{-2} = \frac{1}{4} \Rightarrow a^2 = 4 \Rightarrow a = 2 \quad [\because a > 0]$

Putting $a = 2$ in $-\frac{2\,b}{a^3} = -3$, we get $-\frac{2\,b}{8} = -3 \Rightarrow b = 12$

107 **(d)**

We have,

$T_{r+1} = {}^{15}C_r (x^4)^{15-r} \left(-\frac{1}{x^3}\right)^r$

$= {}^{15}C_r\, x^{60-7r}\, (-1)^r$

If x^{39} occurs in T_{r+1}, then

$60 - 7r = 39 \Rightarrow r = 3$

$\therefore$ Coefficient of $x^{39} = {}^{15}C_3 (-1)^3 = -455$

108 **(c)**

$(1-y)^m (1+y)^n = 1 + a_1 y + a_2 y^2 + a_3 y^3 + \ldots$

On differentiating w. r. t. y, we get

$-m(1-y)^{m-1}(1+y)^n + (1-y)^m\, n(1+y)^{n-1}$

$= a_1 + 2a_2 y + 3a_3 y^2 + \ldots \quad \ldots(\text{i})$

On putting $y = 0$ in Eq. (i), we get

$-m + n = a_1 = 10 \quad [\because a_1 = 10 \text{ given}] \quad \ldots(\text{ii})$

Again on differentiating Eq. (i) w. r. t. y, we get

$-m[-(m-1)(1-y)^{m-2}(1+y)^n + (1+y)^{m-1} n(1+y)^{n-1}]$

$+n[-m(1-y)^{m-1}(1+y)^{n-1} + (1-y)^m (n-1)(1+y)^{n-2}]$

$= 2a_2 + 6a_3 y + \ldots \quad \ldots(\text{iii})$

On putting $y = 0$ in Eq. (iii), we get

$-m[-(m-1) + n] + n[-m + (n-1)] = 2a_2 = 20$

$\Rightarrow m(m-1) - mn - mn + n(n-1) = 20$

$\Rightarrow \quad m^2 + n^2 - m - n - 2mn = 20$

$\Rightarrow \quad (m-n)^2 - (m+n) = 20$

$\Rightarrow \quad 100 - (m+n) = 20$

[using Eq. (iii)]

$\Rightarrow \quad m + n = 80 \quad \ldots(\text{iv})$

On solving Eqs. (ii) and (iv), we get

$m = 35$ and $n = 45$

109 **(c)**

Let a_1, a_2, a_3, a_4 be respectively the coefficients of $(r+1)$th, $(r+2)$th, $(r+3)$th and $(r+4)$th terms in the expansion of $(1+x)^n$. Then,

$a_1 = {}^nC_r, a_2 = {}^nC_{r+1}, a_3 = {}^nC_{r+2}, a_4 = {}^nC_{r+3}$

Now, $\frac{a_1}{a_1+a_2} + \frac{a_3}{a_3+a_4} = \frac{{}^nC_r}{{}^nC_r + {}^nC_{r+1}} + \frac{{}^nC_{r+2}}{{}^nC_{r+2} + {}^nC_{r+3}}$

$= \frac{{}^nC_r}{{}^{n+1}C_{r+1}} + \frac{{}^nC_{r+2}}{{}^{n+1}C_{r+3}} \quad (\because {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1})$

$= \frac{{}^nC_r}{\frac{n+1}{r+1}\,{}^nC_r} + \frac{{}^nC_{r+2}}{\frac{n+1}{r+3}\,{}^nC_{r+2}} \quad \left(\because {}^nC_r = \frac{n}{r}\,{}^{n-1}C_{r-2}\right)$

$= \frac{r+1}{n+1} + \frac{r+3}{n+1} = \frac{2(r+2)}{n+1}$

$= 2\frac{{}^nC_{r+1}}{{}^{n+1}C_{r+2}} = 2\frac{{}^nC_{r+1}}{{}^nC_{r+1} + {}^nC_{r+2}}$

$= \frac{2a_2}{a_2 + a_3}$

110 **(d)**

$(a^2 - 6a + 11)^{10} = 1024$

$\Longrightarrow \quad (a^2 - 6a + 11)^{10} = 2^{10}$

$\Longrightarrow \quad a^2 - 6a + 11 = 2$

$\Longrightarrow \quad a^2 - 6a + 9 = 0$

$\Longrightarrow \quad (a-3)^2 = 0$

$\Longrightarrow \quad a = 3$

111 **(b)**

The general termof $\left(x + \frac{2}{x^2}\right)^n$ is

$T_{R+1} = {}^nC_R (x)^{n-R} \left(\frac{2}{x^2}\right)^R$

$= {}^nC_R x^{n-3R} 2^R$

For x^{2r} occurs, it means

$n - 3R = 2r$

$\Longrightarrow \quad n - 2r = 3R$

Hence, $n - 2r$ is of the form $3k$

112 **(c)**

$2^{3n} - 1 = (2^3)^n - 1$

$= 8^n - 1 = (1+7)^n - 1$

$= 1 + {}^nC_1 7 + {}^nC_2 7^2 + \ldots + {}^nC_n 7^n - 1$

$= 7[{}^nC_1 + {}^nC_2 7 + \ldots + {}^nC_n 7^{n-1}]$

$\therefore 2^{3n} - 1$ is divisible by 7

113 **(b)**

We have,

$(\alpha - 2 + 1)^{35} = (1 - \alpha)^{35}$

$\Rightarrow (\alpha - 1)^{35} = -(\alpha - 1)^{35}$

$\Rightarrow 2(\alpha - 1)^{35} = 0 \Rightarrow \alpha = 1$

114 **(b)**

We have,

$\therefore 3^{\log_3 \sqrt{25^{x-1}+7}} \quad [\because a^{\log_a n} = n]$

$= \sqrt{25^{x-1} + 7} = \sqrt{(5^{x-1}) + 7} = \sqrt{y^2 + 7}$, where $y = 5^{x-1}$

and,

$3^{-(1/8)\log_3(5^{x-1}+1)}$

$= 3^{\log_3 (5^{x-1}+1)^{-1/8}} = (5^{x-1}+1)^{-1/8} = (y+1)^{-1/8}$

$\therefore \left\{3^{\log_3 \sqrt{25^{x-1}}+7} + 3^{-1/8 \log_3(5^{x-1}+1)}\right\}^{10}$

$= \left[\sqrt{y^2+7} + (y+1)^{-1/8}\right]^{10}$

Now,

$T_9 = 180$

$\Rightarrow {}^{10}C_8 \left\{\left(\sqrt{y^2+7}\right)^{10-8} [(y+1)^{-1/8}\right\}^8 = 180$

$\Rightarrow\ {}^{10}C_8(y^2+7)(y+1)^{-1}=180$

$\Rightarrow 45\left(\frac{y^2+7}{y+1}\right)=180$

$\Rightarrow y^2+7=4y+4 \Rightarrow y^2-4y+3=0$

$\Rightarrow y=1, y=3$

$\Rightarrow 5^{x-1}=1$ or, $5^{x-1}=3$

$\Rightarrow 5^x=5$ or, $5^x=15$

$\Rightarrow x=1$ or, $x=\log_5 15$

$\Rightarrow x=\log_5 15 \qquad [\because x>1]$

115 **(c)**

The given sigma expansion

$\sum_{m=0}^{100} {}^{100}C_m(x-3)^{100-m}.2^m$ can be written as

$[(x-3)+2]^{100}=(x-1)^{100}=(x-1)^{100}$

$\therefore$ Coefficient of x^{53} in

$(1-x)^{100}=(-1)^{53}\,{}^{100}C_{53}=-\,{}^{100}C_{53}$

116 **(c)**

The coefficient of x in the middle term of expansion of

$(1+\alpha x)^4 = {}^4C_2\alpha^2$

The coefficient of x in the middle term of expansion of

$(1-\alpha x)^6 = {}^6C_3(-\alpha)^3$

Given, ${}^4C_2\alpha^2 = {}^6C_3(-\alpha)^3$

$\Longrightarrow \quad 6\alpha^2=-20\alpha^3$

$\Longrightarrow \quad \alpha=\frac{-6}{20}=\frac{-3}{10}$

117 **(b)**

The general term in the expansion of $\left(\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right)^{10}$ is

$T_{r+1} = {}^{10}C_r\left(\frac{x}{3}\right)^{\frac{10-r}{2}}\left(\frac{3}{2x^2}\right)^r$

$= {}^{10}C_r 3^{\frac{-10+3r}{2}}.2^{-r}.x^{\frac{10-5r}{2}}$

For independent of x,

$\frac{10-5r}{2}=0 \Rightarrow r=2$

$\therefore\ T_3 = {}^{10}C_2\times\left(\frac{1}{3}\right)^4\left(\frac{3}{2}\right)^2$

$=\frac{10\times 9}{2\times 1}\times\frac{1}{3\times 3\times 2\times 2}=\frac{5}{4}$

118 **(a)**

Given ,

$(1+x-2x^2)^6=1+a_1x+a_2x^2+\ldots+a_{12}x^{12}$ ...(i)

On putting $x=1$ in Eq. (i), we get

$(1+1-2)^6=1+a_1+a_2+\cdots+a_{12}$

$\Longrightarrow (0)^6=1+a_1+a_2+\ldots+a_{12}$ ....(ii)

On putting $x=-1$ in Eq. (i), we get

$(1-1-2)^6=1-a_1+a_2-a_3+\cdots+a_{12}$

$\Longrightarrow (-2)^6=1+a_1+a_2-a_3+\ldots+a_{12}$ ...(iii)

On adding Eqs. (ii) and (iii) we get

$(-2)^6=2(1+a_2+a_4+\cdots+a_{12})$

$\Longrightarrow \frac{64}{2}-1=a_2+a_4+\cdots+a_{12}$

$\therefore a_2+a_4+\cdots+a_{12}=31$

119 **(c)**

Since, $(1+x-3x^2)^{10}=1+a_1x+a_2x^2+\cdots+a_{20}x^{20}$

On putting $x=-1$, we get

$(1-1-3)^{10}=1-a_1+a_2-\cdots+a_{20}$

$=3^{10}$ ... (i)

Again putting $x=1$, we get

$(1+1-3)^{10}=1+a_1+a_2-\cdots+a_{20}=1$ ... (ii)

On adding Eqs. (i) and (ii), we get

$2(1+a_2+a_4+\cdots+a_{20})=3^{10}+1$

$\Longrightarrow a_2+a_4+\ldots+a_{20}=\frac{3^{10}+1}{2}-1=\frac{3^{10}-1}{2}$

120 **(b)**

We have,

$(1+x)^{2n}=(a_0+a_2x^2+a_4x^4+\cdots)+x(a_1+a_3x^2+a_5x^4+\cdots)$

Replacing x by i and $-i$ respectively and multiplying, we get

$(a_0-a_2+a_4\ldots)^2+(a_1-a_3+a_5\ldots)^2$

$=(1+i)^{2n}(1-i)^{2n}$

$\Rightarrow (a_0-a_2+a_4-\cdots)^2+(a_1-a_3+a_5\ldots)^2$

$=2^{2n}=4^n$

121 **(d)**

$(bc+ca+ab)^9=[bc+a(b+c)]^9$

$\therefore$ Coefficient of $a^5b^6c^7$

= coefficient of $a^5b^6c^7$ in ${}^9C_5(bc)^4a^5(b+c)^5$

=coefficient of b^2c^3 *in* ${}^9C_5(b+c)^5$

$= {}^9C_5\times {}^5C_3=1260$

122 **(b)**

We have, $(x-1)(x-2)(x-3)\ldots(x-100)$

Number of terms = 100

$\therefore$ Coefficient of x^{99} in $(x-1)(x-2)(x-3\ldots(x-100)$

$=(-1-2-3-\ldots-100)$

$=-(1+2+\ldots+100)$

$=-\frac{100\times 101}{2}=-5050$

123 **(b)**

Given, $\sin n\theta=\sum_{r=0}^{n}b_r\sin^r\theta$

$\Rightarrow\ \sin n\theta=b_0\sin^0\theta+b_1\sin^1\theta+b_2\sin^2\theta+b_3\sin^3\theta+\ldots+b_n\sin^n\theta$

$\Rightarrow\ \sin n\theta=b_0+b_1\sin\theta+b_2\sin^2\theta+\ldots+b_n\sin^n\theta$

(n is an odd integer)

$\because \sin n\theta = {}^nC_1\sin\theta\cos^{n-1}\theta - {}^nC_3\sin^3\theta\cos^{n-3}\theta+\ldots$

$= {}^nC_1 \sin\theta(1-\sin^2\theta)^{(n-1)/2} -$
${}^nC_3 \sin^3\theta\,(1-\sin^2\theta)^{(n-3)/2} + \ldots$
$\therefore b_0 = 0, b_1 =$ coefficient of $\sin\theta = {}^nC_1 = n$
($\because n-1, n-3$ are all even integers)

124 **(d)**
We have,
$$\left(x+\sqrt{x^2-1}\right)^6 + \left(x-\sqrt{x^2-1}\right)^6$$
$$= 2\left\{{}^6C_0\,x^6 + {}^6C_2\,x^4\left(\sqrt{x^2-1}\right)^2 + {}^6C_4\,x^2\left(\sqrt{x^2-1}\right)^4 + {}^6C_6\left(\sqrt{x^2-1}\right)^6\right\}$$
$$= 2\{x^6 + {}^6C_2\,x^4(x^2-1) + {}^6C_2\,x^2(x^2-1)^2 + (x^2-1)^3\}$$
$$= [\{2\ {}^6C_2+1\}x^6 - 3\{{}^6C_2+1\}x^4 + 4\ {}^6C_2\,x^2 - 1]$$
Clearly, it contains 4 terms

125 **(a)**
We know that,
$(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$
$(x-1)^n = C_0x^n - C_1x^{n-1} + C_2x^{n-2} - \ldots + (-1)^nC_n$
On multiplying both equations and equating the coefficient of x^n, we get
$C_0^2 - C_1^2 + C_2^2 - \ldots + (-1)^nC_n^2 = {}^nC_{n/2}(-1)^{n/2}(x^2)^{n/2}$
Above is possible only when $\frac{n}{2}$ is an integer *ie*, n is even and in case n is odd, then term x^n will not occur

126 **(d)**
$(1-x+x^2)^n = a_0 + a_1x + \ldots + a_{2n}x^{2n}$
Putting $x = -1$ and 1
Successively and adding, we get
$$a_0 + a_2 + a_4 + \ldots + a_{2n} = \frac{3^n+1}{2}$$

127 **(d)**
Now, coefficient of x^{15} in $(1+x)^{20}$
= coefficient of x^{15} in $(1+x)^{15}(1+x)^5$
$\Rightarrow\ {}^{20}C_{15}$ =coefficient of x^{15} in (${}^{15}C_0x^{15} +$
15C1x14+ 15C2x13+ 15C3x12+ 15C4x11+ 15C5x10
$({}^5C_0x^5 + {}^5C_1x^4 + {}^5C_2x^3 + {}^5C_3x^2 + {}^5C_4x + {}^5C_5)$
$= {}^{20}C_{15} = {}^{15}C_0 \cdot {}^5C_5 + {}^{15}C_1 \cdot {}^5C_4 + {}^{15}C_2 \cdot {}^5C_3 + {}^{15}C_3 \cdot {}^5C_2 + {}^{15}C_4 \cdot {}^5C_1 + {}^{15}C_5 \cdot {}^5C_0$
$\Rightarrow {}^{15}C_0 \cdot {}^5C_5 + {}^{15}C_1 \cdot {}^5C_4 + {}^{15}C_2 \cdot {}^5C_3 + {}^{15}C_3 \cdot {}^5C_2 + {}^{15}C_4 \cdot {}^5C_1 = {}^{20}C_{15} - {}^{15}C_5 \cdot {}^5C_0$
$= \frac{20!}{5!15!} - \frac{15!}{5!10!}$

128 **(a)**
The given expression is
$1 + (1+x) + (1+x)^2 + \ldots + (1+x)^n$ being in GP
Let $S = 1 + (1+x) + (1+x)^2 + \ldots + (1+x)^n$
$$= \frac{(1+x)^{n+1}-1}{(1+x)-1} = x^{-1}[(1+x)^{n+1}-1]$$
$\therefore$ The coefficient of x^k in S
= The coefficient of x^k in $[(1+x)^{n+1}-1]$
$= {}^{n+1}C_{k+1}$

129 **(d)**
Since, in a binomial expansion of $(a-b)^n, n \geq 5$, then sum of 5th and 6th terms is equal to zero.
$\therefore\quad {}^nC_4a^{n-4}(-b)^4 + {}^nC_5a^{n-5}(-b)^5 = 0$
$$\Rightarrow \frac{n!}{(n-4)!\,4!}a^{n-4}b^4 - \frac{n!}{(n-5)!\,5!}a^{n-5}b^5 = 0$$
$$\Rightarrow \frac{n!}{(n-5)!\,4!}a^{n-5}\cdot b^4\left(\frac{a}{n-4} - \frac{b}{5}\right) = 0$$
$$\Rightarrow \quad \frac{a}{b} = \frac{n-4}{5}$$

130 **(a)**
We have,
$$\left(1-\frac{1}{x}\right)^n(1-x)^n = (1-x)^{2n}\frac{(-1)^n}{x^n} = \frac{(-1)^n(1-x)^{2n}}{x^n}$$
$\therefore$ Middle term in $\left(1-\frac{1}{x}\right)^n(1-x)^n$
$= \frac{(-1)^n}{x^n}$ middle term in $(1-x)^{2n}$
$= \frac{(-1)^n}{x^n} \times (n+1)^{th}$ term in $(1-x)^{2n}$
$= \frac{(-1)^n}{x^n} \times {}^{2n}C_n(-x)^n = {}^{2n}C_n$

132 **(d)**
The number of terms in the expansion of $(a+b+c)^{10}$
$$= {}^{12}C_2 = \frac{11\cdot12}{2} = 66$$

133 **(d)**
The given expression of $\frac{1}{(4-3x)^{1/2}}$ can be rewritten as
$4^{-1/2}\left(1-\frac{3}{4}x\right)^{-1/2}$ and it is valid only when
$\left|\frac{3}{4}x\right| < 1$
$$\Rightarrow -\frac{4}{3} < x < \frac{4}{3}$$

134 **(c)**
$$\because (3+2x)^{50} = 3^{50}\left(1+\frac{2x}{3}\right)^{50}$$

Here, $T_{r+1} = 3^{50}\ {}^{50}C_r \left(\frac{2x}{3}\right)^r$

and $T_r = 3^{50}\ {}^{50}C_{r-1} \left(\frac{2x}{3}\right)^{r-1}$

But $x = \frac{1}{5}$

$\therefore \frac{T_{r+1}}{T_r} \geq 1 \Rightarrow \frac{{}^{50}C_r}{{}^{50}C_{r-1}} \frac{2}{3} \cdot \frac{1}{5} \geq 1$

$\Rightarrow\ 102 - 2r \geq 15r\ \Rightarrow\ r \leq 6$

135 **(a)**

Given that, $(1 + ax)^n = 1 + 8x + 24x^2 + \ldots$

$\Rightarrow 1 + \frac{n}{1} ax + \frac{n(n-1)}{1.2} a^2x^2 + \ldots$

$= 1 + 8x + 24x^2 + \ldots$

On comparing the coefficients of x, x^2, we get

$na = 8, \frac{n(n-1)}{1.2} a^2 = 24$

$\Rightarrow\ na(n-1)a = 48$

$\Rightarrow\ 8(8-a) = 48$

$\Rightarrow\ 8 - a = 6$

$\Rightarrow\ a = 2\ \Rightarrow\ n = 4$

136 **(b)**

$\because\ (0.99)^{15} = (1 - 0.01)^{15}$

$= 1 - {}^{15}C_1(0.01) + {}^{15}C_2(0.01)^2$

$- {}^{15}C_3(0.01)^3 + \ldots$

We want to answer correct upto 4 decimal places and as such, we have left further expansion.

$= 1 - 15(0.01) + \frac{15 \cdot 14}{1 \cdot 2}(0.0001)$

$- \frac{15 \cdot 14 \cdot 13}{1 \cdot 2 \cdot 3}(0.000001) + \ldots$

$= 1 - 0.15 + 0.0105 - 0.000455 + \ldots$

$= 1.0105 - 0.150455$

$= 0.8601$

137 **(b)**

Given that,

$\frac{1}{1!\,(n-1)!} + \frac{1}{3!\,(n-3)!} + \frac{1}{5!\,(n-5)!} + \ldots$

$= \frac{1}{n!}\left[\frac{n!}{1!\,(n-1)!} + \frac{n!}{3!\,(n-3)!} + \frac{n!}{5!\,(n-5)!} + \ldots\right]$

$= \frac{1}{n!}\left[{}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots\right]$

$= \frac{2^{n-1}}{n!}$

138 **(b)**

$\frac{(1+x)^{3/2} - \left(1 + \frac{1}{2}x\right)^3}{(1-x)^{\frac{1}{2}}}$

$= \frac{\left(1 + \frac{3}{2}x + \frac{\frac{3}{2}\cdot\frac{1}{2}}{2}x^2\right) - \left(1 + \frac{3x}{2} + \frac{3\cdot 2}{2}\cdot\frac{x^2}{4}\right)}{(1-x)^{1/2}}$

[neglecting higher powers of x]

$= -\frac{3x^2}{8}(1-x)^{-1/2}$

$= -\frac{3x^2}{8}\left(1 + \frac{1}{2}x + \frac{\frac{1}{2}\cdot\frac{3}{2}}{2}\cdot x^2\right) = -\frac{3x^2}{8}$

[neglecting higher powers of x]

139 **(c)**

Total number of terms in the expansion of $(2x + 3y - 4z)^n$, is

${}^{n+3-1}C_{3-1} = {}^{n+2}C_2 = \frac{(n+2)(n+1)}{2}$

140 **(b)**

We have,

$(1+x)^m(1+x)^n = \left(\sum_{r=0}^{m} {}^mC_r\, x^r\right) \cdot \left(\sum_{r=0}^{n} {}^nC_r\, x^r\right)$

Equation coefficients of x^r on both sides, we get

${}^mC_r + {}^mC_{r-1}\, {}^nC_1 + {}^mC_{r-2}\, {}^nC_2 + \cdots + {}^mC_1\, {}^nC_{r-1}$

$+ \cdots + {}^mC_0\, {}^nC_r = {}^{m+n}C_r$

141 **(c)**

$(1 - a\,x)^{-1}(1 - bx)^{-1}$

$= (a^0 + a\,x + a^2x^2 + \ldots(b^0 + bx + b^2x^2 + \ldots.)$

Hence, $a_n =$coefficient of x^n in $(1 - ax)^{-1}(1 -$

bx−1

$a^0b^n + ab^{n-1} + \ldots.. + a^nb^0$

$= a^0b^n\left(1 + \frac{a}{b} + \left(\frac{a}{b}\right)^2 + \ldots + \left(\frac{a}{b}\right)^n\right)$

$= a^0b^n\left(\frac{\left(\frac{a}{b}\right)^{n+1} - 1}{\frac{a}{b} - 1}\right)$

$= \frac{a^{n+1} - b^{n+1}}{a - b} = \frac{b^{n+1} - a^{n+1}}{b - a}$

142 **(c)**

Given, $(1 + 2x + x^2)^5 = \sum_{k=0}^{15}\ a_kx^k$

$\Longrightarrow (1+x)^{10} = a_0x^0 + a_1x + a_2x^2 + \cdots + a_{15}x^{15}$

$\Longrightarrow {}^{10}C_0 + {}^{10}C_1x + {}^{10}C_2x^2 + \cdots + {}^{10}C_{10}x^{10}$

$= a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + a_{15}x^{15}$

On equating the coefficient of constant and even power of x, we get

$a_0 = 10_{C_0}, a_2 = 10_{C_2},$

$a_4 = {}^{10}C_4, \ldots.. a_{10} = {}^{10}C_{10}, a_{12} = a_{14} = 0$

$\therefore \sum_{k=0}^{7}\ a_{2k} = {}^{10}C_0 + {}^{10}C_2 + {}^{10}C_4 + {}^{10}C_6$

$+ {}^{10}C_8 + {}^{10}C_{10} + 0 + 0$

$= 2^{10-1} = 2^9 = 512$

143 **(b)**

Since, n is even, therefore $\left(\frac{n}{2} + 1\right)$th term is the middle term.

$\therefore T_{\frac{n}{2}+1} = {}^nC_{n/2}(x^2)^{n/2}\left(\frac{1}{x}\right)^{n/2}$

$= 924x^6$ (given)

$\Rightarrow\ x^{n/2} = x^6 \Rightarrow n = 12$

144 **(b)**

We have, $(1+x^2)^5(1+x)^4$

$= ({}^5C_0 + {}^5C_1x^2 + {}^5C_2x^4 + \ldots)({}^4C_0 + {}^4C_1x + {}^4C_2x^2 + {}^4C_3x^3 + {}^4C_4x^4)$

The coefficient of x^5 in $[(1+x^2)^5(1+x)^4]$

$= {}^5C_2 . {}^4C_1 + {}^4C_3 . {}^5C_1 = 10.4 + 4.5 = 60$

145 **(d)**

$(1+x+x^2+x^3)^n = \{(1+x)^n(1+x^2)^n\}$

$= (1 + {}^nC_1x + {}^nC_2x^2 + \ldots + {}^nC_nx^n)(1 + {}^nC_1x^2 + {}^nC_2x^4 + \ldots + {}^nC_nx^{2n})$

Therefore the coefficient of $x^4 = {}^nC_2 + {}^nC_2\,{}^nC_1 + {}^nC_4$

$= {}^nC_4 + {}^nC_2 + {}^nC_1\,{}^nC_2$

146 **(b)**

Let $a = {}^nC_{r-1}, b = {}^nC_r, c = {}^nC_{r+1}$

and $d = {}^nC_{r+2}$

$\therefore\ a+b = {}^{n+1}C_r, b+c = {}^{n+1}C_{r+1}, c+d = {}^{n+1}C_{r+2}$

$$\Rightarrow \frac{a+b}{a} = \frac{{}^{n+1}C_r}{{}^nC_{r-1}} = \frac{n+1}{r} \Rightarrow \frac{a}{a+b} = \frac{r}{n+1}$$

and $\frac{b+c}{b} = \frac{{}^{n+1}C_{r+1}}{{}^nC_r} = \frac{n+1}{r+1} \Rightarrow \frac{b}{b+c} = \frac{r+1}{n+1}$

and $\frac{c+d}{c} = \frac{{}^{n+1}C_{r+2}}{{}^nC_{r+1}} = \frac{n+1}{r+2} \Rightarrow \frac{c}{c+d} = \frac{r+2}{n+1}$

$\therefore\ \frac{a}{a+b}, \frac{b}{b+c}, \frac{c}{c+d}$ are in AP

$\because$ AM $> GM$

$$\Rightarrow \frac{b}{b+c} > \sqrt{\frac{ac}{(a+b)(c+d)}}$$

or $\left\{\left(\frac{b}{b+c}\right)^2 - \frac{ac}{(a+b)(c+d)}\right\} > 0$

147 **(d)**

We have,

$$T_{r+1} = {}^6C_r\left(\sqrt{x^5}\right)^{6-r}\left(\frac{3}{\sqrt{x^3}}\right)^r$$

$\Rightarrow T_{r+1} = {}^6C_r\,x^{15-\frac{5}{2}r-\frac{3}{2}r}\,3^r = {}^6C_r\,x^{15-4r}\,3^r$

This will contain x^3, if $15 - 4r = 3 \Rightarrow r = 3$

$\therefore$ Coefficient of $x^3 = {}^6C_3 \cdot 3^3 = 540$

148 **(b)**

General term, $T_{r+1} = {}^{11}C_r\dfrac{a^{11-r}}{b^r}(-1)^r x^{11-3r}$

For the coefficient of x^{-7}, put

$11 - 3r = -7 \Longrightarrow r = 6$

$\therefore$ Coefficient of $x^{-7} = {}^{11}C_6\dfrac{a^5}{b^6} = \dfrac{462a^5}{b^6}$

149 **(a)**

We have,

Coefficient of x^5 in $(x+3)^6 = {}^6C_1 \times 3^1 = 18$

150 **(d)**

$A_r =$ Coefficient of x^r in $(1+x)^{10} = {}^{10}C_r$

$B_r =$ Coefficient of x^r in $(1+x)^{20} = {}^{20}C_r$

$C_r =$ Coefficient of x^r in $(1+x)^{30} = {}^{30}C_r$

$$\therefore \sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$$

$$= \sum_{r=1}^{10} A_rB_{10}B_r, - \sum_{r=1}^{10} A_rC_{10}A_r$$

$$= \sum_{r=1}^{10} {}^{10}C_r\,{}^{20}C_{10}\,{}^{20}C_r \sum_{r=1}^{10} {}^{10}C_r\,{}^{30}C_{10}\,{}^{10}C_r\text{l}$$

$$\sum_{r=1}^{10} {}^{10}C_{10-rl}\,{}^{20}C_{10}\,{}^{20}C_r - \sum_{r=1}^{10} {}^{10}C_{10-r}\,{}^{30}C_{10}\,{}^{10}C_rl$$

$$= {}^{20}C_{10}\sum_{r=1}^{10} {}^{10}C_{10-r}\,{}^{20}C_r$$

$$- {}^{30}C_{10}\sum_{r=1}^{10} {}^{10}C_{10-r}\,{}^{10}C_r$$

$= {}^{20}C_{10}({}^{30}C_{10} - 1) - {}^{30}C_{10}({}^{20}C_{10} - 1)$

$= {}^{20}C_{10}({}^{30}C_{10} - 1) - {}^{30}C_{10}({}^{20}C_{10} - 1)$

$= {}^{30}C_{10} - {}^{20}C_{10} = C_{10} - B_{10}$

151 **(a)**

$\because$Coefficient of $x^p$is ${}^{p+q}C_p$ and coefficient of x^q is ${}^{(p+q)}C_q$

$\therefore$ Both the coefficients are equal

152 **(c)**

In the expansion of $(1+x)^{2n}$, the general term

$= {}^{2n}C_kx^k, 0 \le k \le 2n$

As given for $r > 1, n > 2$, ${}^{2n}C_{3r} = {}^{2n}C_{r+2}$

$\Rightarrow$Either $3r = r+2$ or $3r = 2n - (r+2)$ ($\because$ $nCr = nCn-r$

$\Rightarrow\ r = 1$ or $n = 2r+1$

We take the relation only

$n = 2r+1$ ($\because r > 1$)

153 **(a)**

The general term in the expansion of $\left(x\sin^{-1}\alpha + \cos{-1\alpha x 10}\right.$ is given by

$$T_{r+1} = {}^{10}C_r(x\sin^{-1}\alpha)^{10-r}\left(\frac{\cos^{-1}\alpha}{x}\right)^r$$

$\Rightarrow T_{r+1}$

$= {}^{10}C_r(\sin^{-1}\alpha)^{10-r}(\cos^{-1}\alpha)^r\,x^{10-2r}$ ... (i)

This will be independent of x, if

$10 - 2r = 0 \Rightarrow r = 5$

Putting $r = 5$ in (i), we get

$T_6 = {}^{10}C_5(\sin^{-1}\alpha\cos^{-1}\alpha)^5$

$\Rightarrow T_6 = {}^{10}C_5\left\{\sin^{-1}\alpha\left(\frac{\pi}{2} - \sin^{-1}\alpha\right)\right\}^5$

$\Rightarrow T_6 = {}^{10}C_5\left\{\frac{\pi}{2}\sin^{-1}\alpha - (\sin^{-1}\alpha)^2\right\}^5$

$\Rightarrow T_6 = {}^{10}C_5\left\{\frac{\pi^2}{16} - \left(\frac{\pi}{4} - \sin^{-1}\alpha\right)^2\right\}^5$

Now,

$-\frac{\pi}{2} \le \sin^{-1}\alpha \le \frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{2} \le -\sin^{-1}\alpha \le \frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{4} \le \left(\frac{\pi}{4} - \sin^{-1}\alpha\right) \le \frac{3\pi}{4}$

$\Rightarrow 0 \le \left(\frac{\pi}{4} - \sin^{-1}\alpha\right)^2 \le \frac{9\pi^2}{16}$

$\Rightarrow -\frac{9\pi^2}{16} \le -\left(\frac{\pi}{4} - \sin^{-1}\alpha\right)^2 \le 0$

$\Rightarrow -\frac{\pi^2}{2} \le \frac{\pi^2}{16} - \left(\frac{\pi}{4} - \sin^{-1}\alpha\right)^2 \le \frac{\pi^2}{16}$

$\Rightarrow -{}^{10}C_5\left(\frac{\pi^2}{2}\right)^5 \le {}^{10}C_5\left\{\frac{\pi^2}{16} - \left(\frac{\pi}{4} - \sin^{-1}\alpha\right)^2\right\}^5$

$\le {}^{10}C_5\left(\frac{\pi^2}{16}\right)^5$

$\Rightarrow -\frac{{}^{10}C_5\,\pi^{10}}{2^5} \le T_6 \le {}^{10}C_5\frac{\pi^{10}}{2^{20}}$

154 **(b)**

In the expansion of $(3 + 7x)^{29}$

$T_{r+1} = {}^{29}C_r \cdot 3^{29-r} \cdot (7x)^r$

$= ({}^{29}C_r \times 3^{29-r} \times 7^r)x^r$

Let a_r= coefficient of $(r + 1)$th term

$= {}^{29}C_r \times 3^{29-r} \times 7^r$

and a_{r-1} =coefficient of rth term

$= {}^{29}C_{r-1} \times 3^{30-r} \times 7^{r-1}$

According to question $a_r = a_{r-1}$

$\Longrightarrow {}^{29}C_r \times 3^{29-r} \times 7^r = {}^{29}C_{r-1} \times 3^{30-r} \times 7^{r-1}$

$\Longrightarrow \frac{{}^{29}C_r}{{}^{29}C_{r-1}} = \frac{3}{7} \Longrightarrow \frac{30 - r}{r} = \frac{3}{7}$

$\Longrightarrow 210 - 7r = 3r \Longrightarrow r = 21$

156 **(b)**

In the expansion of $(1 + x)^{50}$ the sum of the coefficient of odd powers

$= C_1 + C_3 + C_5 + \ldots = 2^{50-1} = 2^{49}$

157 **(b)**

It is given that the coefficients of r^{th} and $(r + 1)^{\text{th}}$ term in the expansion of $(3 + 7x)^{29}$ are equal

$\therefore {}^{29}C_{r-1} \times 3^{30-r} \times 7^{r-1} = {}^{29}C_r \times 3^{29-r} \times 7^r$

$\Rightarrow {}^{29}C_{r-1} \times 3 = {}^{29}C_r \times 7$

$\Rightarrow \frac{3}{30 - r} = \frac{7}{r} \Rightarrow r = 21$

158 **(b)**

We have,

${}^{2n}C_p = {}^{2n}C_{p+2} \Rightarrow p + p + 2 = 2n \Rightarrow p = n - 1$

159 **(b)**

We have

$(1 + x)^n = \sum_{r=0}^{n} a_r x^r \Rightarrow a_r = {}^nC_r$

Now,

$\left(1 + \frac{a_1}{a_0}\right)\left(1 + \frac{a_2}{a_1}\right)\ldots\left(1 + \frac{a_n}{a_{n-1}}\right)$

$= \prod_{r=1}^{n}\left(1 + \frac{a_r}{a_{r-1}}\right)$

$= \prod_{r=1}^{n}\left(\frac{a_{r-1} + a_r}{a_{r-1}}\right)$

$= \prod_{r=1}^{n}\left(\frac{{}^nC_r + {}^nC_{r-1}}{{}^nC_{r-1}}\right)$

$= \prod_{r=1}^{n}\frac{{}^{n+1}C_r}{{}^nC_{r-1}}$

$= \prod_{r=1}^{n}\frac{n + 1}{r} \qquad \left[\because {}^{n+1}C_r = \frac{n + 1}{r}\,{}^nC_{r-1}\right]$

$= (n + 1)^n\left(\frac{1}{1} \times \frac{1}{2} \times \frac{1}{3} \times \ldots \times \frac{1}{n}\right) = \frac{(n + 1)^n}{n\,!}$

161 **(a)**

$(2x^2 - x - 1)^5 = a_0 + a_1x + a_2x^2 + \cdots + a_{10}x^{10}$

On putting $x = 0$, we get

$-1 = a_0$

On putting $x = 1$, we get

$0 = a_0 + a_1 + a_2 + \ldots + a_{10}$ ...(i)

On putting $x = -1$, we get

$(2 + 1 - 1)^5 = a_0 - a_1 + a_2 - \ldots + a_{10}$ ...(ii)

On adding Eqs. (i) and (ii), we get

$0 + (2)^5 = 2(a_0 + a_2 + \cdots + a_{10})$

$\Longrightarrow \quad 16 - 1 = a_2 + \cdots + a_{10}$

$\Longrightarrow a_2 + a_3 + \ldots + a_{10} = 15$

162 **(b)**

We have,

${}^{2n}C_r = {}^{2n}C_{r+2} \Rightarrow r + r + 2 = 2n \Rightarrow n = r + 1$

163 **(d)**

$\therefore$ coefficient of x^{100} in the expansion of $\sum_{j=0}^{200}(1 + x)^j$ will be $\sum_{j=0}^{200}\ {}^{j}C_{100}$

$= [{}^{100}C_{100} + {}^{101}C_{100} + {}^{102}C_{100} + \cdots + {}^{200}C_{100}]$

$[\because {}^nC_n + {}^{n+1}C_n + {}^{n+2}C_n + \cdots + {}^{2n-1}C_n = {}^{2n}C_{n+1}]$

$= \binom{201}{100}$

164 **(b)**

We have,

$$\frac{1}{n\,!}+\frac{1}{2\,!\,(n-2)\,!}+\frac{1}{4\,!\,(n-4)\,!}+\cdots$$
$$=\frac{1}{n\,!}\left\{\frac{n\,!}{n\,!}+\frac{n\,!}{2\,!\,(n-2)\,!}+\frac{n\,!}{4'!\,(n-4)\,!}+\cdots\right\}$$
$$=\frac{1}{n\,!}\{\,^nC_0+\,^nC_2+\,^nC_4+\cdots\}=\frac{2^{n-1}}{n\,!}$$

165 **(c)**

General term $T_{r+1}=\,^{10}C_r\left(\frac{x}{2}\right)^{10-r}\left(-\frac{3}{x^2}\right)^r$

$=\,^{10}C_r\cdot\frac{x^{10-3r}\cdot(-1)^r\cdot 3^r}{2^{10-r}}$

For the coefficient of x^4put

$10-3r=4$

$\Longrightarrow\quad r=2$

Hence, coefficient of x^4 is

$^{10}C_2\cdot\frac{3^2}{2^8}=\frac{405}{256}$

166 **(a)**

Given, $(1+x)^n=C_0+C_1x+C_2x^2+\cdots+C_nx^n$

Also, $(x+1)^n=C_n+C_{n-1}x+C_{n-2}x^2+\cdots+C_0x^n$

On multiplying both equations and comparing coefficient of x^{n-1} on both sides, we get

$C_0C_2+C_1C_2+C_2C_3+\cdots+C_{n-1}C_n=\,^{2n}C_{n-1}$

$=\frac{(2n)!}{(n-1)!\,(n+1)!}$

167 **(c)**

Now, $7^9=(8-1)^9=-1(1-8)^9$

$=-1+\,^9C_18-\,^9C_28^2+\ldots+\,^9C_98^9$ and

$9^7=(1+8)^7$

$=1+\,^7C_18+\,^7C_28^2+\,^7C_38^3+\ldots+\,^7C_78^7$

$\therefore\ 7^9+9^7=8(\,^9C_1+\,^7C_1)+8^2(\,^7C_2-\,^9C_2)+\cdots$

$=8(9+7)+8^2(21-36)+\cdots$

$=64\times 2+64(-15)+\cdots$

Hence, it is divisible by 64

168 **(d)**

Let $f=\left(8-3\sqrt{7}\right)^{10}$, here $0<f<1$

$\therefore\ \left(8+3\sqrt{7}\right)^{10}+\left(8-3\sqrt{7}\right)^{10}$ is an integer hence, this is the value of n

169 **(d)**

We have, $\left(3x-\frac{1}{2x}\right)^8$

$\therefore$ Ninth term $T_9=8_{c_8}(3x)^{8-8}\left(\frac{-1}{2x}\right)^8$

$=\frac{1}{256x^8}$

170 **(b)**

The general term in the expansion of $\left(x+\frac{1}{x^2}\right)^{n-3}$ is given by

$T_{r+1}=\,^{n-3}C_r(x)^{n-3-r}\left(\frac{1}{x^2}\right)^r$

$=\,^{n-3}C_rx^{n-3-3r}$

As x^{2k} occurs in the expansion of $\left(x+\frac{1}{x^2}\right)^{n-3}$, we must have $n-3-3r=2k$ for some non-negative integer r

$\Rightarrow\ 3(1+r)=n-2k$

$\Rightarrow\ n-2k$ is a multiple of 3

171 **(c)**

Let T_{r+1} denote the $(r+1)^{\text{th}}$ term in the expansion of $\left(7^{1/3}+5^{1/2}x\right)^{600}$. Then,

$T_{r+1}=\,^{600}C_r\,(7^1)^{600-r}\left(5^{1/3}x\right)^r$

$=\,^{600}C_r7^{200-\frac{r}{3}}\times 5^{\frac{r}{2}}\times x^r$

Here, $0\le r\le 600$

For $200-\frac{r}{3}$ and $\frac{r}{2}$ to be integers, we must have $\frac{r}{3}$ and $\frac{r}{2}$ as integers, and $0\le r\le 600$

$\Rightarrow r$ is multiple of 2 and 3 both and $0\le r\le 600$

$\Rightarrow r$ is a multiple of 6 and $0\le r\le 600$

$\Rightarrow r=0,6,12,\ldots,600$

Hence, there are 101 terms with integral coefficients

172 **(b)**

We have,

$$(xy+yz+zx)^6=\sum_{r+s+t=6}\frac{6!}{r!\,s!\,t!}(xy)^r(yz)^s(zx)^t$$
$$=\sum_{r+s+t=6}\frac{6!}{r!\,s!\,t!}x^{r+t}y^{r+s}z^{s+t}$$

If the general term in the above expansion contains $x^3y^4z^5$, then

$r+t=3, r+s=4$ and $s+t=5$

Also, $r+s+t=6$

On solving these equations, we get

$r=1, s=3, t=2$

$\therefore$ Coefficient of $x^3y^4z^5=\frac{6!}{1!\,3!\,2!}=60$

173 **(c)**

We have,

$$(1+2x+x^2)^n=\sum_{r=0}^{2n}a_r\,x^r$$
$$\Rightarrow\{(1+x)^2\}^n=\sum_{r=0}^{2n}a_r\,x^r$$
$$\Rightarrow(1+x)^{2n}=\sum_{r=0}^{2n}a_r\,x^r$$
$$\Rightarrow\sum_{r=0}^{2n}\,^{2n}C_r\,x^r=\sum_{r=0}^{2n}a_r\,x^r\ \Rightarrow a_r=\,^{2n}C_r$$

174 **(a)**

Given, ${}^{20}C_4 + {}^{20}C_3 + {}^{20}C_3 + {}^{20}C_2 - {}^{22}C_{18}$
$= {}^{21}C_4 + {}^{20}C_3 + {}^{20}C_2 - {}^{22}C_{18}$
$= {}^{22}C_4 - {}^{22}C_{18} = {}^{22}C_{18} - {}^{22}C_{18} = 0$

175 **(d)**
We have,
$y = 3x + 6x^2 + 10x^3 + \cdots$
$\Rightarrow 1 + y = (1 + 3x + 6x^2 + 10x^3 + \cdots)$
$\Rightarrow 1 + y = (1 - x)^{-3}$
$\Rightarrow (1 - x) = (1 + y)^{-1/3}$
$\Rightarrow x = 1 - (1 + y)^{-1/3}$
$\Rightarrow x = \frac{1}{3}y - \frac{1 \cdot 4}{3^2 \cdot 2}y^2 + \frac{1 \cdot 4 \cdot 7 \cdot}{3^3\ 3\,!}y^3 \ldots$

176 **(c)**
We know that,
$(x + a)^n + (x - a)^n = 2[{}^nC_0x^n + {}^nC_2x^{n-2}a^2 + \ldots.]$
Here, $n = 5, x = x$ and $a = (x^3 - 1)^{1/2}$
$\therefore [x + (x^3 - 1)^{1/2}]^5[x - (x^3 - 1)^{1/2}]^5$
$= 2[{}^5C_0x^5 + {}^5C_2x^3(x^3 - 1) + {}^5C_4x(x^3 - 1)^2]$
$= 2[x^5 + 10x^3(x^3 - 1) + 5x(x^3 - 1)^2]$
$\therefore$ Given expression is a polynomial of degree 7.

177 **(c)**
We have,
$C_0^2 + 3 \cdot C_1^2 + 5 \cdot C_2^2 + \cdots + (2n + 1)C_n^2$
$= \{C_0^2 + C_1^2 + C_2^2 + \cdots + C_n^2\}$
$+\{2\,C_1^2 + 4 \cdot C_2^2 + 6 \cdot C_3^2 + \cdots + 2\,n\,C_n^2\}$ ... (i)
We have,
$(1 + x)^{2n} = (1 + x)^n(1 + x)^n$
$\Rightarrow (1 + x)^{2n} = (C_0 + C_1x + C_2x^2 + \cdots + C_nx^n) \times (C_0x^n + C_1x^{n-1} + \cdots + C_{n-1}x + C_n)$
On equating the coefficient of x^n on both sides, we get
${}^{2n}C_n = C_0^2 + C_1^2 + C_2^2 + \cdots C_n^2$ ... (ii)
Also,
$n(1 + x)^{n-1}(1 + x)^n = (C_1 + 2\,C_2\,x + 3\,C_3\,x^2 + \cdots + n\,C_n\,x^{n-1})$
$\times (C_0\,x^n + C_1\,x^{n-1} + C_2\,x^{n-2} + \cdots + C_n)$
On equating the coefficient of x^{n-1} on both sides, we get
$n \cdot {}^{2n-1}C_{n-1} = (C_1^2 + 2\,C_2^2 + 3\,C_3^2 + \cdots + n\,C_n^2)$
$\Rightarrow 2n \cdot {}^{2n-1}C_{n-1} = 2\,C_1^2 + 4\,C_2^2 + 6\,C_3^2 + \cdots + +2\,n\,C_n^2$ ... (iii)
From (i),(ii) and (iii), we obtain
$C_0^2 + 3 \cdot C_1^2 + 5\,C_2^2 + \cdots + (2n + 1)C_n^2$
$= \frac{2n}{n}\,{}^{2n-1}C_{n-1} + 2n \cdot {}^{2n-1}C_{n-1}$
$= 2(n + 1)\,{}^{2n-1}C_{n-1}$

178 **(d)**
Here ${}^{n-1}C_r = (k^2 - 3)\,{}^nC_{r+1}$
$\Longrightarrow {}^{n-1}C_r = (k^2 - 3)\frac{n}{r + 1}\,{}^{n-1}C_r$
$\Longrightarrow k^2 - 3 = \frac{r + 1}{n}$
$\left[\text{since}, n - 1 \geq r \Longrightarrow \frac{r + 1}{n} \leq 1 \text{and } n, r \geq 0\right]$
$\Longrightarrow 0 < k^2 - 3 \leq 1 \Longrightarrow 3 < k^2 \leq 4$
$\Longrightarrow k \in [-2, -\sqrt{3}) \cup (\sqrt{3}, 2]$

179 **(a)**
Let $f(x) = (x + y)^{100} + (x - y)^{100}$
Here, $n = 100$, which is even.
$\therefore$ Total number of terms
$= \frac{n + 2}{2} = \frac{100 + 2}{2}$
$= 51$

180 **(c)**
We know that
$C_0^2 + C_1^2 + C_2^2 + C_3^2 + \cdots + C_n^2 = \frac{(2\,n)\,!}{n\,!\,n\,!}$ ... (i)
and, $C_0^2 - C_1^2 + C_2^2 - C_3^2 + \cdots - C_n^2 = 0$, when n is odd ...(ii)
subtracting (ii) from (i), we get
$2(C_1^2 + C_3^2 + C_5^2 + \cdots + C_n^2) = \frac{(2\,n)\,!}{(n\,!)^2}$
$\Rightarrow C_1^2 + C_3^2 + C_5^2 + \cdots + C_n^2 = \frac{(2n)\,!}{2(n\,!)^2}$

181 **(a)**
$\sum_{k=0}^{10} {}^{20}C_k = {}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \cdots + {}^{20}C_{10}$
On putting $x = 1$and $n = 20$in $(1 + x)^n$
$= {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \cdots + {}^nC_nx^n$
We get
$2^{20} = 2({}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \cdots + {}^{20}C_9) + {}^{20}C_{10}$
$\Longrightarrow 2^{19} = ({}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \cdots + {}^{20}C_9) + \frac{1}{2}\,{}^{20}C_{10}$
$\Longrightarrow 2^{19} = {}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \cdots + {}^{20}C_{10} - \frac{1}{2}\,{}^{20}C_{10}$
$\Longrightarrow {}^{20}C_0 + {}^{20}C_1 + \ldots + {}^{20}C_{10} = 2^{19} + \frac{1}{2}\,{}^{20}C_{10}$

182 **(b)**
$(7.995)^{1/3} = (8 - 0.005)^{1/3}$
$= (8)^{1/3}\left[1 - \frac{0.005}{8}\right]^{1/3}$
$= 2\left[1 - \frac{1}{3} \times \frac{0.005}{8} + \frac{\frac{1}{3}\left(\frac{1}{3} - 1\right)}{2 \cdot 1}\left(\frac{0.005}{8}\right)^2 + \ldots\right]$

$= 2\left[1 - \frac{0.005}{24} - \frac{\frac{1}{3}\times\frac{1}{3}}{1}\times\frac{(0.005)^2}{64} + \ldots\right]$
$= 2(1 - 0.000208)$ (neglecting other terms)
$= 2 \times 0.999792$
$= 1.9996$

183 **(b)**
It is given that ${}^mC_1$, ${}^mC_2$ and ${}^mC_3$ are in A.P.
$\therefore 2\ {}^mC_2 = {}^mC_1 + {}^mC_3$
$\Rightarrow m^2 - 9m + 14 = 0$
$\Rightarrow m = 2, 7$
For $m = 2$, there are only three terms. Therefore, $m = 7$.
Now,
$\Rightarrow 21 = {}^7C_5\left\{\sqrt{2^{\log_{10}(10-3^x)}}\right\}^{7-5}\left\{\sqrt[5]{2^{(x-2)\log_{10}3}}\right\}^5$
$\Rightarrow 21 = 21 \cdot 2^{\log_{10}(10-3^x)} \cdot 2^{(x-2)\log_{10}3}$
$\Rightarrow 1 - 2^{\log_{10}(10-3^x)+(x-2)\log_{10}3}$
$\Rightarrow 2^0 = 2^{\log_{10}[(10-3^x)\cdot 3^{x-2}]}$
$\Rightarrow (10 - 3^x)3^{x-2} = 1$
$\Rightarrow 3^{2x-2} - 10.3^{x-2} + 1 = 0$
$\Rightarrow 3^{2x} - 10.3^x + 9 = 0$
$\Rightarrow (3^x - 1)(3^x - 9) = 0$
$\Rightarrow 3^x = 1, 3^x = 9 \Rightarrow x = 0, 2$

184 **(b)**
Given, ${}^nC_{12} = {}^nC_6$
or ${}^nC_{n-12} = {}^nC_6$
$\Longrightarrow n - 12 = 6 \Longrightarrow n = 18$
$\therefore {}^nC_2 = {}^{18}C_2 = 153$

185 **(b)**
Let $(r + 1)$th term be the coefficient of x^0 in the expansion of
$\left(x - \frac{1}{x}\right)^6$.
$\therefore T_{r+1} = {}^6C_r x^{6-r}\left(-\frac{1}{x}\right)^r$
$= (-1)^r\ {}^6C_r x^{6-2r}$
Since, this term is a constant term.
$\therefore 6 - 2r = 0 \Longrightarrow r = 3$
$\therefore T_4 = (-1)^3\ {}^6C_3 = -20$

186 **(d)**
General term, $T_{r+1} = {}^{15}C_r(x^3)^{15-r}\left(\frac{2}{x^2}\right)^r$
$= {}^{15}C_r x^{45-5r}(2)^r$
For term independent of x, put $45 - 5r = 0 \Longrightarrow r = 9$
$\therefore$ Independent term $= T_{9+1} = T_{10}$

187 **(b)**
We have, $T_{r+1} = {}^{21}C_r\left(\frac{a^{1/3}}{b^{1/6}}\right)^{21-r}\left(\frac{b^{1/2}}{a^{1/6}}\right)^r$
$= {}^{21}C_r \frac{a^{7-(r/3)}}{b^{7/2-r/6}}\cdot\frac{b^{r/2}}{a^{r/6}}$
$= {}^{21}C_r a^{7-(r/2)} b^{2r/3-7/2}$
Since, exponents of a and b in the $(r + 1)$th term are equal
$\therefore 7 - \frac{r}{2} = \frac{2r}{3} - \frac{7}{2}$
$\Rightarrow \frac{21}{2} = \frac{7}{6}r \Rightarrow r = 9$

188 **(b)**
$\left(\frac{1}{x} + 1\right)^n (1 + x)^n = \frac{1}{x^n}(1 + x)^{2n}$
$= \frac{1}{x^n}(1 + {}^{2n}C_1 x + {}^{2n}C_2 x^2 + \cdots$
$+ {}^{2n}C_{n-1}x^{n-1} + \cdots + {}^{2n}C_{2n}x^{2n})$
The coefficient of $\frac{1}{x}$ is ${}^{2n}C_{n-1}$.

189 **(d)**
$x = (\sqrt{3} + 1)^5 = (\sqrt{3})^5 + {}^5C_1(\sqrt{3})^4 + {}^5C_2(\sqrt{3})^3$
$+ {}^5C_3(\sqrt{3})^2 + {}^5C_4(\sqrt{3}) + {}^5C_5$
$= 9\sqrt{3} + 45 + 30\sqrt{3} + 30 + 5\sqrt{3} + 1$
$= 76 + 44\sqrt{3}$
$\therefore [x] = \left[(\sqrt{3} + 1)^5\right] = [76 + 44\sqrt{3}]$
$= [76] + [44 \times 1.732]$
$= 76 + [76.2]$
$= 76 + 76 = 152$

190 **(a)**
We have,
$2\ C_0 + \frac{2^2}{2}C_1 + \frac{2^3}{2}C_2 + \cdots + \frac{2^{11}}{11}C_{10}$
$= \sum_{r=0}^{10} {}^{10}C_r \frac{2^{r+1}}{r+1}$
$= \frac{1}{11}\sum_{r=0}^{10}\frac{11}{r+1}\ {}^{10}C_r\ 2^{r+1}$
$= \frac{1}{11}\sum_{r=0}^{10} {}^{11}C_{r+1}\cdot 2^{r+1}$
$= \frac{1}{11}({}^{11}C_1\ 2^1 + \cdots + {}^{11}C_{11}\cdot 2^{11})$
$= \frac{1}{11}({}^{11}C_0\cdot 2^0 + {}^{11}C_1 2^1 + \cdots + {}^{11}C_{11}\cdot 2^{11}$
$- {}^{11}C_0\cdot 2^0)$
$= \frac{1}{11}[(1 + 2)^{11} - 1] = \frac{3^{11} - 1}{11}$

191 **(a)**

Sum of coefficients of the expansion $\left(\frac{1}{x}+2x\right)^n$

$= 6561$

$\therefore (1+2)^n = 3^8 \Rightarrow 3^n = 3^8 \Rightarrow n = 8$

Now, $T_{r+1} = {}^8C_r 2^{8-r} x^{-8+2r}$

Since, this term is independent of x, then

$-8+2r = 0 \Rightarrow r = 4$

$\therefore$ Coefficient of independent term, $T_5 = {}^8C_4 \cdot 2^4 = 16 \cdot {}^8C_4$

192 **(d)**

Sum of coefficient of odd powers of x in $(1+x)^{30}$

$= C_1 + C_3 + \ldots + C_{29} = 2^{30-1} = 2^{29}$

193 **(b)**

6th term in the expansion of $\left(2x^2 - \frac{1}{3x^2}\right)^{10}$ is

$T_6 = {}^{10}C_5 (2x^2)^5 \left(-\frac{1}{3x^2}\right)^5$

$= -\frac{10!}{5!\,5!} \times 32 \times \frac{1}{243}$

$= -\frac{896}{27}$

194 **(c)**

${}^{47}C_4 \sum_{r=1}^{5} {}^{52-r}C_3$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{47}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{48}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{49}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{50}C_4$

$= {}^{51}C_3 + {}^{51}C_4 + {}^{52}C_4$

195 **(b)**

We have,

$(1-2\,x+3\,x^2-4\,x^3+\cdots)^{-n} = \{(1+x)^{-2}\}^{-n}$

$= (1+x)^{2\,n}$

$\therefore$ Coefficient of x^n in $(1-2x+3x^2-4x^3+\cdots)^{-\mathrm{n}}$

$=$ Coefficient of x^n in $(1+x)^{2n} = {}^{2n}C_n = \frac{(2\,n)\,!}{(n\,!)^2}$

196 **(c)**

We have,

$(1+x)^n\left(1+\frac{1}{x}\right)^n$

$= (C_0 + C_1 x + \cdots + C_n x^n)\left\{C_0 + \frac{C_1}{x} + \frac{C_2}{x^2} + \cdots + \frac{C_n}{x^n}\right\}$

$\therefore$ Term independent of $x = C_0^2 + C_1^2 + C_2^2 + \cdots + C_n^2$

197 **(a)**

We have,

$A =$ Coeff. of x^r in the expansion of $(1+x)^n$ ${}^nC_r$

$B =$ Coeff. of x^{n-r} in the expansion of $(1+x)^n = {}^nC_{n-r}$

$\because {}^nC_r = {}^nC_{n-r} \quad \therefore A = B$

198 **(b)**

We have,

$$x = \frac{\begin{bmatrix} 729 + 6(2)(243) + 15(4)(81) \\ +20(8)(27) + 15(16)(9) + 6(32)(3) + 64 \end{bmatrix}}{1 + 4(4)6(16) + 4(64) + 256}$$

$$= \frac{\begin{bmatrix} {}^6C_0(3)^6 + {}^6C_1 3^5 2 + {}^6C_2 3^4 2^2 \\ + {}^6C_3 3^3\, 2^3 + {}^6C_4 3^2 2^4 + {}^6C_5 3\, 2^5 + {}^6C_6 2^6 \end{bmatrix}}{{}^4C_0 + {}^4C_1 4 + {}^4C_2 4^2 + {}^4C_3 4^3 + {}^4C_4 4^4}$$

$\Rightarrow x = \frac{(3+2)^6}{(1+4)^4} = \frac{5^6}{5^4}$

$\Rightarrow x = 5^2$

$\therefore \sqrt{x} - \frac{1}{\sqrt{x}} = 5 - \frac{1}{5} = 4.8$

199 **(b)**

Coefficient of pth, $(p+1)$th and $(p+2)$th terms in the expansion $(1+x)^n$ are ${}^nC_{p-1}$, ${}^nC_p$, ${}^nC_{p+1}$ respectively

Since, these are in AP

$\therefore 2\,{}^nC_p = {}^nC_{p-1} + {}^nC_{p+1}$

$\Rightarrow 2\frac{n!}{(n-p)!\,p!}$

$= \frac{n!}{(n-p+1)!\,(p-1)!} + \frac{n!}{(n-p-1)!\,(p+1)!}$

$\Rightarrow \frac{2}{(n-p)!\,p!} = \frac{p}{(n-p+1)(n-p)!\,p!} + \frac{n-p}{(n-p)!\,(p+1)p!}$

$\Rightarrow \frac{2}{1} = \frac{p}{(n-p+1)} + \frac{n-p}{p+1}$

$\Rightarrow n^2 - n(4p+1) + 4p^2 - 2 = 0$

200 **(a)**

$(2^{1/2} + 3^{1/5})^{10} = {}^{10}C_0 2^5 + {}^{10}C_1 2^{9/2} \cdot 3^{1/5} + \ldots\ldots + {}^{10}C_{10} \cdot 3^2$

Thus, sum of rational terms of above expansion $= 2^5 + 3^2 = 41$

201 **(b)**

According to given condition, $T_n = {}^nC_3$

and $T_{n+1} - T_n = 21$

$\Rightarrow {}^{n+1}C_3 - {}^nC_3 = 21$

$\Rightarrow \frac{1}{6}(n+1)(n)(n-1) - \frac{1}{6}n(n-1)(n-2) = 21$

$\Rightarrow \frac{n(n-1)}{6}[(n+1)-(n-2)] = 21$

$\Rightarrow \frac{n(n-1)\cdot 3}{6} = 21$

$\Rightarrow n(n-1) = 42$

$\Rightarrow\ n = 7$

202 **(b)**

Since, total number of terms $= 59 + 1 = 60$

$\therefore$ Required sum $= \frac{2^{59}}{2} = 2^{58}$

203 **(d)**

Since, $(1-x)^{-n} = 1 + \frac{n}{1!}x + \frac{n(n+1)}{2!}x^2 + \cdots$

On putting $x = \frac{2x}{1+x}$ on both sides, we get

$$\left(1 - \frac{2x}{1+x}\right)^{-n} = 1 + \frac{n}{1!}\left(\frac{2x}{1+x}\right) + \frac{n(n+1)}{2!}\left(\frac{2x}{1+x}\right)^2 + \cdots$$

$$\Rightarrow 1 + \frac{n}{1!}\left(\frac{2x}{1+x}\right) + \frac{n(n+1)}{2!}\left(\frac{2x}{1+x}\right)^2 + \ldots = \left(\frac{1-x}{1+x}\right)^{-n} = \left(\frac{1+x}{1-x}\right)^n$$

204 **(d)**

General term in the expansion of $(1+3x+2x^2)^6$

$$= \sum \frac{6!}{r_1!\,r_2!\,r_3!}(1)^{r_1}(3x)^{r_2}(2x^2)^{r_3}$$

Where $r_1 + r_2 + r_3 = 6$ ....(i)

For coefficient of x^{11}, we have

$r_2 + 2\,r_3 = 11$ ...(ii)

Now, from Eqs, (i) and (ii), we get

$r_1 = r_3 - 5$

For $r_3 = 5,\ r_1 = 0$

And $r_2 = 1$

$\therefore$ Coefficient of $x^{11} = \frac{6!}{0!\,1!\,5!}(1)^0(3)^1(2)^5$

$= 6 \times 3 \times 2^5 = 18 \times 32 = 576$

205 **(a)**

We have,

$$(1-x)^2\left(x + \frac{1}{x}\right)^{10}$$

$$= (1 - 2x + x^2)\sum_{r=0}^{10} {}^{10}C_r\, x^{10-2r}$$

$$= \sum_{r=0}^{10} {}^{10}C_r\, x^{10-2r} - 2\sum_{r=0}^{10} {}^{10}C_r\, x^{11-2x} + \sum_{r=0}^{10} {}^{10}C_r\, x^{12-2\,x}$$

Hence, the term independent of x is

${}^{10}C_5 - 2 \times 0 + {}^{10}C_6 = {}^{10}C_5 + {}^{10}C_6 = {}^{11}C_6 = {}^{11}C_5$

206 **(a)**

Sum of the coefficients in the expansion of $(x - 2y + 3z)^n$ is $(1 - 2 + 3)^n = 2^n$

(Put $x = y = z = 1$)

$\therefore\ 2^n = 128$

$\Rightarrow\ n = 7$

Therefore, the greatest coefficient in the expansion of $(1+x)^7$ is ${}^7C_3$ or ${}^7C_4$ because both are equal to 35

207 **(b)**

$19^{2005} + 11^{2005} - 9^{2005}$

$= (10+9)^{2005} + (10+1)^{2005} - (9)^{2005}$

$= (9^{2005} + {}^{2005}C_1(9)^{2004} \times 10 + \ldots) + ({}^{2005}C_0 + {}^{2005}C_1 10 + \ldots) - (9)^{2005}$

$= ({}^{2005}C_1 9^{2004} \times 10 +$ multiple of 10$) + (1 +$ multiple of 10$)$

$\therefore$ Unit digit = 1

208 **(a)**

$$\sum_{r=0}^{n}(-1)^r\, {}^nC_r\left(\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + \ldots \text{upto } m \text{ terms}\right)$$

$$= \sum_{r=0}^{n}(-1)^r\, {}^nC_r \cdot \frac{1}{2^r} + \sum_{r=0}^{n}(-1)^r \cdot {}^nC_r \frac{3^r}{2^{2r}} + \sum_{r=0}^{n}(-1)^r\, {}^nC_r \frac{7^r}{2^{3r}} + \ldots$$

$$= \left(1 - \frac{1}{2}\right)^n + \left(1 - \frac{3}{4}\right)^n + \left(1 - \frac{7}{8}\right)^n + \ldots \text{upto } m \text{ terms}$$

$$= \frac{1}{2^n} + \frac{1}{2^{2n}} + \frac{1}{2^{3n}} + \cdots \text{ upto m terms}$$

$$= \frac{\frac{1}{2^n}\left(1 - \left(\frac{1}{2^n}\right)^m\right)}{\left(1 - \frac{1}{2^n}\right)}$$

$$= \frac{2^{mn} - 1}{2^{mn}(2^n - 1)}$$

209 **(c)**

We have,

Coeff. of $(r+2)^{th}$ term in $(1+x)^{2n} =$ Coeff. of $(3r)^{th}$ term

$\Rightarrow {}^{2n}C_{r+1} = {}^{2n}C_{3r-1}$

$\Rightarrow r + 1 + 3r - 1 = 2n \Rightarrow 4r = 2n \Rightarrow n = 2r$

210 **(b)**

$$\frac{\left(1 + \frac{3}{4}x\right)^{-4}(16)^{1/2}\left(1 - \frac{3}{16}x\right)^{1/2}}{(8)^{2/3}\left(1 + \frac{x}{8}\right)^{2/3}}$$

$= \left(1+\frac{3x}{4}\right)^{-4}\left(1-\frac{3x}{16}\right)^{\frac{1}{2}}\left(1+\frac{x}{8}\right)^{-\frac{2}{3}}$

$= \left(1+(-4)\frac{3}{4}x\right)\left(1-\left(\frac{1}{2}\right)\frac{3x}{16}\right)\left(1+\left(-\frac{2}{3}\right)\frac{x}{8}\right)$

$= (1-3x)\left(1-\frac{3}{32}x\right)\left(1-\frac{x}{12}\right)$

$= 1-\frac{305}{96}x$ (On neglecting x^2 and higher powers of x)

211 **(c)**

We have,

$(1+x^2)^5(1+x)^4$

$= ({}^5C_0 + {}^5C_1x^2 + {}^5C_2\,x^4 + {}^5C_3\,x^6 + \cdots) \times ({}^4C_0 + {}^4C_1\,x + {}^4C_2\,x^2 + {}^4C_3\,x^3 + {}^4C_4\,x^4$

$\therefore$ Coefficient of $x^5 = {}^5C_1 \times {}^4C_3 + {}^5C_2 \times {}^4C_1$

$= 20 + 40 = 60$

212 **(a)**

We have,

$\left(1+5\sqrt{2}\,x\right)^9 + \left(1-5\sqrt{2}\,x\right)^9$

$= 2\left\{{}^9C_0 + {}^9C_2\left(5\sqrt{2}\,x\right)^2 + \cdots + {}^9C_8\left(5\sqrt{2}\,x\right)^8\right\}$

Clearly, it has 5 terms

214 **(d)**

$(1+2x+3x^2+\ldots)^{1/2} = [(1-x)^{-2}]^{1/2}$

$= (1-x)^{-1}$

$= 1+x+x^2+\cdots+x^n+\cdots\infty$

$\therefore$The coefficient of $x^n = 1$

215 **(d)**

Coefficient of $\lambda^n\mu^n$ in $(1+\lambda)^n(1+\mu)^n(\lambda+\mu)^n$

=coefficient of $\lambda^n\mu^n$in

$\sum_{r=0}^{n} {}^nC_r\lambda^r \sum_{s=0}^{n} {}^nC_s\mu^s \sum_{t=0}^{n} {}^nC_t\lambda^{n-t}\mu^t$

$= ({}^nC_0)^3 + ({}^nC_1)^3 + ({}^nC_2)^3 + \ldots = \sum_{r=0}^{n}({}^nC_r)^3$

216 **(c)**

We have,

$(1+x-2\,x^2)^6$

$= 1 + C_1x + C_2x^2 + C_3x^3 + C_4x^4 + 9\ldots + C_{12}\,x^{12}$

Putting $x = 1, -1$, we get

$0 = 1 + C_1 + C_2 + C_3 + C_4 + \cdots + C_{12}$ ... (i)

$64 = 1 - C_1 + C_2 - C_3 + C_4 - \cdots + C_{12}$ ... (ii)

Adding (i) and (ii), we get

$64 = 2(1 + C_2 + C_4 + \cdots + C_{12})$

$\Rightarrow C_2 + C_4 + \cdots + C_{12} = 31$

217 **(a)**

We have,

Coeff. Of x in $(1+ax)^n = 8$ and, Coeff. Of x^2 in $(1+ax)^n = 24$

$\Rightarrow {}^nC_1a = 8$ and ${}^nC_2a^2 = 24$

$\Rightarrow na = 8$ and $n(n-1)a^2 = 48$

$\Rightarrow 64 - 8a = 48 \Rightarrow a = 2$

$\therefore na = 8 \Rightarrow n = 4$

218 **(a)**

Since, $(1+x)^{2n} = {}^{2n}C_0 + {}^{2n}C_0 + {}^{2n}C_1x + {}^{2n}C_2x^2$

$+ \ldots + {}^{2n}C_nx^n + \ldots {}^{2n}C_{2n}x^{2n}$

Total number of terms in the expansion $= 2n+1$

$\therefore (n+1)$th term is middle term. This term has greatest coefficient.

Hence, required greatest coefficient $= {}^{2n}C_n$

219 **(a)**

The general term in $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$ is

$T_{r+1} = (-1)^r\,{}^{10}C_r\left(\frac{x}{2}\right)^{10-r}\left(\frac{3}{x^2}\right)^r$

$= (-1)^r\,{}^{10}C_r.\frac{3^r}{2^{10-r}}.x^{10-3r}$

For coefficient of x^4 , we have to take $10-3r = 4$

$\Rightarrow 3r = 6 \Rightarrow r = 2$

$\therefore$ Coefficient of x^4 in $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$

$= (-1)^2.\,{}^{10}C_2.\frac{3^2}{2^8} = \frac{9\times45}{256} = \frac{405}{256}$

220 **(b)**

Clearly, ${}^nC_r$ is the greatest and n is odd

$\therefore r = \frac{n+1}{2}$ or, $\frac{n-1}{2}$

221 **(a)**

We have, $R = [R] + F$

Let $G = \left(5\sqrt{5}-11\right)^{2n+1}$. Then, $0 < G < 1$ as $0 < 5\sqrt{5}-11 < 1$

Now,

$R - G = \left(5\sqrt{5}+11\right)^{2n+1} - \left(5\sqrt{5}-11\right)^{2n+1}$

$\Rightarrow R - G = 2\{{}^{2n+1}C_1\left(5\sqrt{5}\right)^{2n}(11)^1 + {}^{2\,n+1}C_3\left(5\sqrt{5}\right)^{2\,n-2}(11)^3 + \cdots + {}^{2\,n+1}C_{2\,n+1}\,(11)^{2\,n+1}\}$

$\Rightarrow R - G =$ an even integer

$\Rightarrow [R] + F - G =$ an even integer

$\Rightarrow F - G$ is an integer

$\Rightarrow F - G = 0$

$\Rightarrow F = G$

$\Rightarrow RF = RG = \left(5\sqrt{5}+11\right)^{2\,n+1}\left(5\sqrt{5}-11\right)^{2\,n+1}$

$= 4^{2\,n+1}$

222 **(c)**

Coefficients of $T_5 = {}^nC_4, T_6 = {}^nC_5$ and $T_7 = {}^nC_6$

According to the given condition,

$2\,{}^nC_5 = {}^nC_4 + {}^nC_6$

$\Rightarrow 2\left[\frac{n!}{(n-5)!\,5!}\right]=\left[\frac{n!}{(n-4)!\,4!}+\frac{n!}{(n-6)!\,6!}\right]$

$\Rightarrow 2\left[\frac{6}{(n-5)}\right]=\left[\frac{5.6}{(n-4)(n-5)}+1\right]$

$\Rightarrow \frac{12}{(n-5)}=\frac{30+n^2-9n+20}{(n-4)(n-5)}$

$\Rightarrow n^2-21n+98=0$

$\Rightarrow (n-7)(n-14)=0$

$\Rightarrow n=7$ or 14

223 **(a)**

Given that, $R=(2+\sqrt{3})^{2n}$ and $f=R-[R]$

As $0<2-\sqrt{3}<1$, we get $0<F=(2-\sqrt{3})^{2n}<1$

We have, $R+F=(2+\sqrt{3})^{2n}+(2-\sqrt{3})^{2n}$

$=2\left[{}^{2n}C_0 2^{2n}+{}^{2n}C_2 2^{2n-2}(\sqrt{3})^2\right.$

$\left.+{}^{2n}C_4(2^{2n-4})(\sqrt{3})^4+\ldots+{}^{2n}C_{2n}(\sqrt{3})^{2n}\right]$

$\Rightarrow R+F$ is an even integer

$\Rightarrow [R]+f+F$ is an even integer

$\Rightarrow f+F$ is an integer

But, $0\le f<1$ and $0<F<1$

$\Rightarrow 0<f+F<2$

But the only integer between 0 and 2 is 1. Thus,

$f+F=1 \Rightarrow 1-f=F$

Now, $R(1-f)=RF=(2+\sqrt{3})^{2n}(2-\sqrt{3})^{2n}$

$=(4-3)^{2n}=1^{2n}=1$

224 **(d)**

$({}^7C_0+{}^7C_1)+({}^7C_1+{}^7C_2)+\cdots+({}^7C_6+{}^7C_7)$

$={}^8C_1+{}^8C_2+\cdots+{}^8C_7+({}^8C_0+{}^8C_8)$

$-({}^8C_0+{}^8C_8)$

$=2^8-2$

225 **(a)**

We have,

${}^{4n}C_0+{}^{4n}C_2x^2+{}^{4n}C_4x^4+\ldots+{}^{4n}C_{4n}x^{4n}$

$=\frac{1}{2}[(1+x)^{4n}+(1-x)^{4n}]$

On putting $x=1$ and $x=i$, we get

${}^{4n}C_0+{}^{4n}C_2+\ldots+{}^{4n}C_{4n}=\frac{1}{2}[2^{4n}]$ ...(i)

and ${}^{4n}C_0+{}^{4n}C_2+\ldots+{}^{4n}C_{4n}=\frac{1}{2}[(1+i)^{4n}+$

$1-i4n$ ...(ii)

On adding Eqs. (i) and (ii), we get

$2[{}^{4n}C_0+{}^{4n}C_4+\ldots+{}^{4n}C_{4n}]$

$=2^{4n-1}+\frac{1}{2}[(1+i)^{4n}$

$+(1-i)^{4n}]$

Now,$(1+i)^{4n}+(1-i)^{4n}$

$=\left[\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right]^{4n}$

$+\left[\sqrt{2}\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)\right]^{4n}$

$=2^{2n}(\cos n\pi+i\sin n\pi)+2^{2n}(\cos n\pi-i\sin n\pi)$

$=2^{2n+1}\cos n\pi=2^{2n+1}(-1)^n$

$\therefore 2[{}^{4n}C_0+{}^{4n}C_4+\ldots+{}^{4n}C_{4n}]$

$=2^{4n-1}+\frac{1}{2}2^{2n+1}(-1)^n$

$\Rightarrow {}^{4n}C_0+{}^{4n}C_4+\ldots+{}^{4n}C_{4n}$

$=2^{4n-1}+(-1)^n 2^{2n-1}$

226 **(a)**

Suppose $(s+1)^{\text{th}}$ term contains x^{2r}

We have,

$T_{s+1}={}^{n-3}C_s\,x^{n-3-s}\left(\frac{1}{x^2}\right)^s={}^{n-3}C_s\,x^{n-3-3s}$

This will contain x^{2r}, if

$n-3-3s=2r$

$\Rightarrow s=\frac{n-3-2r}{3}$

$\Rightarrow s=\frac{n-2r}{3}-1$

$\Rightarrow s+1=\frac{n-2r}{3}$

$\Rightarrow n-2r=3(s+1)$

$\Rightarrow n-2r$ is a positive integral multiple of 3

228 **(d)**

Let $1+\frac{1}{3}x+\frac{1\cdot4}{3\cdot6}x^2+\frac{1\cdot4\cdot7}{3\cdot6\cdot9}x^3+\ldots=(1+y)^n$

$=1+ny+\frac{n(n-1)}{2!}y^2+\ldots$

On comparing the terms, we get

$ny=\frac{1}{3}x, \frac{n(n-1)}{2!}\ y^2=\frac{1\cdot4}{3\cdot6}x^2$

On solving, we get

$n=-\frac{1}{3}, \quad y=-x$

$\therefore$ Required expansion is $(1-x)^{-1/3}$

229 **(c)**

On putting $x=1$, we get the sum of coefficient of $(x^2-x-1)^{99}$

$=(1-1-1)^{99}=(-1)^{99}=-1$

231 **(b)**

${}^{15}C_8+{}^{15}C_9-{}^{15}C_6-{}^{15}C_7$

$={}^{15}C_8+{}^{15}C_9-{}^{15}C_9-{}^{15}C_8$

$=0$

232 **(a)**

$(1-x+x^2)^n=a_0+a_1x+a_2x^2+\ldots+a_{2n}x^{2n}$

On putting $x=1$, we get

$(1-1+1)^n=a_0+a_1+a_2+\ldots+a_{2n}$

$\Rightarrow 1=a_0+a_1+a_2+\ldots+a_{2n}$ ...(i)

Again, putting $x=-1$, we get

$3^n=a_0-a_1+a_2-\ldots+a_{2n}$ ...(ii)

On adding Eqs. (i) and (ii), we get

$\frac{3^2+1}{2}=a_0+a_2+a_4+\ldots+a_{2n}$

233 **(c)**

Let us take

$a_0 + a_1x + a_2x^2 + \ldots + a_{2n}x^{2n} = (1+x+x^2)^n$

On differentiating both sides w.r.t. x, we get

$a_1 + 2a_2x + \ldots + 2na_{2n}x^{2n-1}$

$= n(1+x+x^2)^{n-1}(2x+1)$

Put $x = -1$

$\Rightarrow a_1 - 2a_2 + 3a_3 - \ldots - 2na_{2n} = -n$

234 **(d)**

$\because (k^2-3) = \frac{^{n-1}C_r}{^{n-1}C_{r+1}} = \frac{^{n-1}C_r}{\left(\frac{n}{r+1}\right)^{n-1}C_r} = \left(\frac{r+1}{n}\right)$ ...(i)

$\because 0 \le r \le n-1$

$\Rightarrow 1 \le r+1 \le n$

$\Rightarrow \frac{1}{n} \le \frac{r+1}{n} \le 1$

$\Rightarrow \frac{1}{n} \le k^2 - 3 \le 1$ [from Eq. (i)]

$\Rightarrow 3 + \frac{1}{n} \le k^2 \le 4$

When $n \to \infty, 3 \le k^2 \le 4$

or $k \in [-2, -\sqrt{3}] \cup (\sqrt{3}, 2]$

235 **(d)**

The general term in the expansion of $\big(x\cos\alpha +$

sinα*x20* is *20Crx*cos*α20−r*sinα*xr*=
20Crx20−2r(cosα*)20−r(*sinα*)r*

For this term to be independent of x, we get

$20 - 2r = 0 \Rightarrow r = 10$

Let β =Term independent of x

$= {^{20}C_{10}}(\cos\alpha)^{10}(\sin\alpha)^{10}$

$= {^{20}C_{10}}(\cos\alpha\sin\alpha)^{10}$

$= {^{20}C_{10}}\left(\frac{\sin 2a}{2}\right)^{10}$

Thus, the greatest possible value of β is

${^{20}C_{10}}\left(\frac{1}{2}\right)^{10}$

236 **(c)**

Since $(n+2)^{th}$ term is the middle term in the expansion of $(1+x)^{2n+2}$. Therefore,

$p = {^{2n+2}C_{n+1}}$

Since $(n+1)^{th}$ and $(n+2)^{th}$ terms are two middle terms in the expansion of $(1+x)^{2n+1}$. Therefore,

$q = {^{2n+1}C_n}$ and $r = {^{2n+1}C_{n+1}}$

But, ${^{2n+1}C_n} + {^{2n+1}C_{n+1}} = {^{2n+2}C_{n+1}}$

$\Rightarrow q + r = p$

237 **(a)**

Given

${^mC_0} + {^mC_1} + {^mC_2} = 46$

$\Rightarrow 2m + m(m-1) = 90$

$\Rightarrow m^2 + m - 90 = 0 \Rightarrow m = 9$ as $m > 0$

Now, $(r+1)$th term of $\left(x^2 + \frac{1}{x}\right)^m$ is

${^mC_r}(x^2)^{m-r}\left(\frac{1}{x}\right)^r = {^mC_r}x^{2m-3r}$

For this to be independent of x put

$2m - 3r = 0 \Rightarrow r = 6$

$\therefore$ Coefficient of the term independent of x is

${^9C_6} = 84$

238 **(a)**

Since ${^nC_{r-1}}$, ${^nC_r}$ and ${^nC_{r+1}}$ are in A. P.

$\therefore 2\,{^nC_r} = {^nC_{r-1}} + {^nC_{r+1}}$

$\Rightarrow 2\frac{n!}{(n-r)!\,r!}$

$= \frac{n!}{(n-r+1)!\,(r-1)!}$

$+ \frac{n!}{(n-r-1)!\,(r+1)!}$

$\Rightarrow n^2 - n(4r+1) + 4r^2 - 2 = 0$

$\Rightarrow n$ is a root of the equation $x^2 - x(4r+1) \pm$ $4r^2 - 2 = 0$

239 **(a)**

$(1+x+x^2)^{-3} = \left[\frac{1}{(1+x+x^2)}\right]^3$

$= \left[\frac{1-x}{1-x^3}\right]^3$

$= (1-x)^3(1-x^3)^{-3}$

$= (1 - x^3 - 3x^2 + 3x)(1 + 3x^3 + 6x^6 + \cdots)$

$\therefore$ Coefficient of x^6 in $(1+x+x^2)^{-3}$

$= 6 - 3 = 3$

240 **(d)**

$\left(1+\frac{C_1}{C_0}\right)\left(1+\frac{C_2}{C_1}\right)\left(1+\frac{C_3}{C_2}\right)\ldots\left(1+\frac{C_n}{C_{n-1}}\right)$

$= \left(1+\frac{n}{1}\right)\left(1+\frac{n(n-1)}{2n}\right)\ldots\left(1+\frac{1}{n}\right)$

$= \left(\frac{1+n}{1}\right)\left(\frac{1+n}{2}\right)\ldots\left(\frac{1+n}{n}\right) = \frac{(1+n)^n}{n!}$

241 **(a)**

$(1+3x+3x^2+x^3)^{20} = (1+x)^{60}$

$\therefore$ Coefficent of x^{20} in $(1+x)^{60}$ is ${^{60}C_{20}}$ or ${^{60}C_{40}}$.

242 **(c)**

The general term in the expansion of $(x\sin\alpha +$

*x−1*cosα*)10* is

$T_{r+1} = {^{10}C_r}(x\sin\alpha)^{10-r}(x^{-1}\cos\alpha)^r$

$= {^{10}C_r}(\sin\alpha)^{10-r}(\cos\alpha)^r x^{10-2r}$

For the term independent of x, put

$10 - 2r = 0 \Rightarrow r = 5$

$\therefore$ Coefficient of term independent of x, is

${^{10}C_5}(\sin\alpha)^5(\cos\alpha)^5 = {^{10}C_5}\left(\frac{1}{2^5}\right)(\sin 2\alpha)^5$

$\le \frac{1}{2^5}({^{10}C_5})$ [$\because \sin(2\alpha) \le 1$]

243 **(b)**

The general term in the expansion of (1 + 2x + 3x210 is

$\frac{10!}{r!\,s!\,t!}1^r(2x)^s(3x^2)^t$, where $r+s+t=10$

$=\frac{10!}{r!\,s!\,t!}2^s\times 3^t\times x^{s+2t}$

We have to find a_1 i.e. the coefficient of x

For the coefficient of x^1, we must have

$s+2t=1$

But, $r+s+t=10$

$\therefore s=1-2t$ and $r=9+t$, where $0\le r,s,t\le 10$

Now, $t=0\Rightarrow s=1, r=9$

For other values of t, we get negative values of s.

So, there is only one term containing x and its coefficient is

$\frac{10!}{9!\,1!\,0!}\times 2^1\times 3^0=20$

Hence, $a_1=20$

<u>ALITER</u> we have,

$(1+2x+3x^2)^{10}$

$={}^{10}C_0+{}^{10}C_1(2x+3x^2)+{}^{10}C_2(2x+3x^2)^2+\dots+{}^{10}C_0(2x+3x^2)^{10}$

$\therefore a_1=$ Coeff. of $x=20$

244 **(b)**

Since, the coefficient of given terms are ${}^mC_{r-1}, {}^mC_r, {}^mC_{r+1}$ respectively and they are in AP.

$\therefore \quad {}^mC_{r-1}+{}^mC_{r+1}=2\,{}^mC_r$

$\Longrightarrow \frac{m!}{(r-1)!\,(m-r+1)!}+\frac{m!}{(r+1)!\,(m-r-1)!}=2\frac{m!}{r!\,(m-r)!}$

$\Longrightarrow \frac{1}{(m-r+1)(m-r)}+\frac{1}{(r+1)r}=\frac{2}{r!\,(m-r)}$

$\Longrightarrow \frac{r(r+1)+(m-r+1)(m-r)}{r(r+1)(m-r+1)(m-r)}=\frac{2}{r(m-r)}$

$\Longrightarrow r^2+r+m^2+r^2-2mr+m-r$

$=2(mr-r^2+r+m-r+1)$

$\Longrightarrow 4r^2-4mr-m-2+m^2=0$

$\Longrightarrow m^2-m(4r+1)+4r^2-2=0$

245 **(b)**

General term, $T_{r+1}={}^6C_rx^{6-r}\left(\frac{1}{x^2}\right)^r$

$\Longrightarrow T_{r+1}={}^6C_rx^{6-3r}$

For term independent of x, put

$6-3r=0$

$\Longrightarrow r=2$

$\therefore$ Coefficient of independent term $={}^6C_2=15$

246 **(a)**

We have,

$T_{n+1}=\left(\frac{1}{3\times\sqrt[3]{9}}\right)^{\log_3 8}$

$\Rightarrow {}^nC_n\left(\sqrt[3]{2}\right)^0\left(-\frac{1}{\sqrt{2}}\right)^n=\left(\frac{1}{3\times\sqrt[3]{9}}\right)^{\log_3 8}$

$\Rightarrow \left(-\frac{1}{\sqrt{2}}\right)^n=\left(\frac{1}{3^{5/3}}\right)^{\log_3 8}$

$\Rightarrow (-1)^n2^{-n/2}=\left(3^{-5/3}\right)^{\log_3 2^3}$

$\Rightarrow (-1)^n2^{-n/2}=\left(3^{\log_3^{2^{-5}}}\right)$

$\Rightarrow (-1)^n2^{-n/2}=2^{-5}\Rightarrow\frac{n}{2}=5\Rightarrow n=10$

247 **(b)**

Given, $a_0=1, a_{n+1}=3n^2+n+a_n$

$\Longrightarrow a_1=3(0)+0+a_0=1$

$\Longrightarrow a_2=3(1)^2+1+a_1=3+1+1=5$

From option (b),

Let $P(n)=n^3-n^2+1$

$\therefore \quad P(0)=0-0+1=1=a_0$

$P(1)=1^3-1^2+1=1=a_1$

and $P(2)=(2)^3-(2)^2+1=5=a_2$

248 **(b)**

General term , $T_{r+1}={}^{10}C_r(x^2)^{10-r}\left(-\frac{1}{x^3}\right)^r$

$={}^{10}C_rx^{20-5r}(-1)^r$

Since, this term condition x^{-10}

$\therefore \qquad 20-5r=-10\Longrightarrow r=6$

$\therefore$ Coefficient of $x^{-10}={}^{10}C_6(-1)^6=210$

251 **(c)**

The sum of the coefficients of the polynomial $(a^2x^2-2ax+1)^{51}$ is obtained by putting $x=1$

Therefore, by given condition $(a^2-2a+1)^{51}=0$

$\Rightarrow a=1$

252 **(b)**

Let $S=1+\frac{2}{4}+\frac{2\cdot5}{4\cdot8}+\frac{2\cdot5\cdot8}{4\cdot8\cdot12}+\cdots$

On comparing with

$(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+\cdots$

we get $\qquad nx=\frac{2}{4} \qquad \dots.(i)$

and $\quad \frac{n(n-1)}{2!}x^2=\frac{2\cdot5}{4\cdot8} \qquad \dots(ii)$

From Eqs, (i) and (ii)

$\frac{\frac{n(n-1)}{2!}x^2}{x^2x^2}=\frac{\frac{2\cdot5}{4\cdot8}}{\frac{2\cdot2}{4\cdot4}}$

$\Longrightarrow \frac{n-1}{n}=\frac{5}{2}\Longrightarrow n=-\frac{2}{3}$

On putting the value of n in Eq. (i) we get

$-\frac{2}{3}x=\frac{2}{4}\Longrightarrow x=-\frac{3}{4}$

$\therefore S = (1+x)^n = \left(1-\frac{3}{4}\right)^{-2/3} = \left(\frac{1}{4}\right)^{-2/3} = \sqrt[3]{16}$

253 **(b)**

We have,

$(x+3)^{n-1} + (x+3)^{n-2}(x+2) + \cdots + (x+2)^{n-1}$

$= \frac{(x+3)^n - (x+2)^n}{(x+3)-(x+2)} = (x+3)^n - (x+2)^n$

$\therefore$ Coefficient of x^r in the given expression

= Coeff. of x^r in $\{(x+3)^n - (x+2)^n\}$

$= {}^nC_r\, 3^{n-r} - {}^nC_r\, 2^{n-r} = {}^nC_r(3^{n-r} - 2^{n-r})$

254 **(c)**

Let $S = C_1 + 2C_2 + 3C_3 + \cdots + nC_n = \sum_{r=1}^{n} r \cdot {}^nC_r$

$= \sum_{r=1}^{n} r \cdot \frac{n}{r}\, {}^{n-1}C_{r-1} \quad \left[\because {}^nC_r = \frac{n}{r}\, {}^{n-1}C_{r-1}\right]$

$= n \sum_{r=1}^{n} {}^{n-1}C_{r-1}$

$= n[{}^{n-1}C_0 + {}^{n-1}C_1 + {}^{n-1}C_2 + \cdots + {}^{n-1}C_{n-1}]$

$= n2^{n-1}$

255 **(b)**

Given expression $\frac{\sqrt{1+x}+\sqrt[3]{(1-x)^2}}{1+x+\sqrt{1+x}}$ can be rewritten as

$\frac{(1+x)^{1/2} + (1-x)^{2/3}}{1+x+(1+x)^{1/2}}$

$= \frac{\left[1+\frac{1}{2}x-\frac{1}{8}x^2+\ldots\right] + \left[1-\frac{2}{3}x-\frac{1}{9}x^2-\ldots\right]}{1+x+\left[1+\frac{1}{2}x-\frac{1}{8}x^2+\ldots\right]}$

$= \frac{2-\frac{1}{6}x-\frac{17}{72}x^2+\ldots}{2+\frac{3}{2}x-\frac{1}{8}x^2+\ldots} = \frac{\left[1-\frac{1}{12}x-\frac{17}{144}x^2+\ldots\right]}{\left[1+\frac{3}{4}x-\frac{1}{16}x^2+\ldots\right]}$

$= \left[1-\frac{1}{12}x-\frac{17}{144}x^2+\ldots\right]\left[1+\frac{3}{4}x - \frac{1}{16}x^2+\ldots\right]^{-1}$

$= 1-\frac{5}{6}x+\ldots$

$= 1-\frac{5}{6}x$ (On neglecting the higher powers of x)

256 **(b)**

The coefficient of x^n in the expansion of $(1+x)(1-x)^n$

= coefficient of x^n in $(1-x)^n$ + The coefficient of x^{n-1} in $(1-x)^n$

$= (-1)^n \frac{n!}{n!\,0!} + (-1)^{n-1} \frac{n!}{1!\,(n-1)!}$

$= (-1)^n(1-n)$

257 **(b)**

Middle term of $(x-a)^8$ is

$T_5 = {}^8C_4 x^4 (-a)^4 = {}^8C_4 x^4 a^4$

258 **(c)**

We have,

Coeff. of r^{th} term in $(1+x)^{20}$ = Coeff. of $(r+4)^{th}$ term in $(1+x)^{20}$

$\Rightarrow {}^{20}C_{r-1} = {}^{20}C_{r+3}$

$\Rightarrow (r-1)+(r+3) = 20 \Rightarrow 2r+2 = 20 \Rightarrow r = 9$

259 **(c)**

Putting $x = 1$ and $x = -1$ in the given expansion and adding, we get

$2[1 + a_2 + a_4 + \cdots + a_{12}] = (-2)^6$

$\Rightarrow a_2 + a_4 + \cdots + a_{12} = 31$

260 **(d)**

We have,

$\frac{1}{81^n} - \frac{10}{81^n}\, {}^{2n}C_1 + \frac{10^2}{81^n}\, {}^{2n}C_2 - \frac{10^3}{81^n}\, {}^{2n}C_3 + \cdots + \frac{10^{2n}}{81^n}$

$= \frac{1}{81^n}\{{}^{2n}C_0 - {}^{2n}C_1\, 10^1 + {}^{2n}C_2\, 10^2 - {}^{2n}C_3\, 10^3 + \cdots + {}^{2n}C_{2n} 10^{2n}\}$

$= \frac{1}{81^n}(1-10)^{2n} = \frac{(-9)^{2n}}{81^n} = \frac{81^n}{81^n} = 1$

261 **(a)**

$\left(\frac{{}^{50}C_0}{1} + \frac{{}^{50}C_2}{3} + \frac{{}^{50}C_4}{5} + \cdots + \frac{{}^{50}C_{50}}{51}\right)$

$= \frac{1}{1} + \frac{50 \times 49}{3 \times 2! \times} + \frac{50 \times 49 \times 48 \times 47}{5 \times 4!} + \cdots$

$= \frac{1}{51}\left(51 + \frac{51 \times 50 \times 49}{3!} + \frac{51 \times 50 \times 49 \times 48 \times 47}{5!} + \cdots\right)$

$= \frac{1}{51}({}^{51}C_1 + {}^{51}C_3 + {}^{51}C_5 + \cdots) = \frac{1}{51} \cdot 2^{51-1}$

$= \frac{2^{50}}{51}$

262 **(a)**

As we have $A^2 = 2A - I$

$\Longrightarrow A^2A = (2A-I)A = 2A^2 - IA$

$\Longrightarrow A^3 = 2(2A-I) - IA = 3A - 2I$

Similarly, $A^4 = 4A - 3I$

$A^5 = 5A - AI$

$A^n = 5A - 4I$

… … … … … …

… … … … … …

$A^n = nA - (n-1)I$

263 **(a)**

We have,

$\sum_{r=0}^{2n} a_r(x-100)^r = \sum_{r=0}^{2n} b_r(x-101)^r$

$$\Rightarrow \sum_{r=0}^{2n} b_r\, t^r = \sum_{r=0}^{2n} a_r\,(1+t)^r,\ \text{where } t = x - 101$$

On equating the coefficients of t^n on both sides, we get

$$b_n = a_n\ {}^nC_n + a_{n+1}\ {}^{n+1}C_n + a_{n+2}\ {}^{n+2}C_n + \cdots + a_{2n}\ {}^{2n}C_n$$

$$\Rightarrow b_n = \sum_{r=n}^{2n} a_r\ {}^rC_n$$

$$= \sum_{r=n}^{2n} 2^r$$

$$= 2^n \sum_{r=n}^{2n} 2^{r-n} = 2^n(2^{n+1} - 1)$$

264 **(b)**

We have,

$$(x+3)^{n-1} + (x+3)^{n-2}(x+2) + (x+3)^{n-3}(x+2)^2 + \ldots + (x+2)^{n-1}$$

$$= \frac{(x+3)^n - (x+2)^n}{(x+3)-(x+2)} = (x+3)^n - (x+2)^n$$

$$\left(\because \frac{x^n - a^n}{x-a} = x^{n-1} + x^{n-2}a^1 + x^{n-3}a^2 + \ldots + a^{n-1}\right)$$

Therefore, the coefficient of x^r in the given expression

=coefficient of x^r in $[(x+3)^n - (x+2)^n]$

$= {}^nC_r 3^{n-r} - {}^nC_r 2^{n-r}$

$= {}^nC_r(3^{n-r} - 2^{n-r})$

266 **(d)**

The sum of the magnitudes of the coefficients is obtained by replacing x by -1 in $(1 - x + x^2 - x3n$

Hence, required sum $= (1+1+1+1)^n = 4^n$

267 **(c)**

Let x^7 occur in $(r+1)^{\text{th}}$ term

Now,

$$T_{r+1} = \frac{x^{-3}(-3)(-3-1)(-3-2)\ldots(-3-r+1)}{r\,!}\left(-\frac{2}{\cdot}\right.$$

$$\Rightarrow T_{r+1} = \frac{3\cdot 4\cdot 5\ldots(r+2)}{r\,!}2^r\, x^{r-3}$$

This will contain x^7, if

$\therefore r - 3 = 7 \Rightarrow r = 10$

$$\therefore \text{Coefficient of } x^7 = \frac{3\cdot 4\cdot 5\ldots(10+2)}{10\,!}\cdot 2^{10}$$

$$= 66 \times 2^{10} = 67584$$

268 **(d)**

The given expansion

$$= \sum_{r=0}^{n}(x-r)(y-r)(z-r)(-1)^r\, C_r$$

$$= \sum_{r=0}^{n}(-1)^r\, xyz\, C_r \sum_{r=0}^{n}(-1)^r\, r(x+y+z)C_r$$

$$+ \sum_{r=0}^{n}(-1)^r\, r^2\,(xy+yz+zx)C_r$$

$$- \sum_{r=0}^{n}(-1)^r\, xyz\, r^3\, C_r$$

$$= xyz\sum_{r=0}^{n}(-1)^r\, C_r - (x+y+z)\sum_{r=0}^{n}(-1)^r\, r\, C_r$$

$$+(xy+yz+zx)\sum_{r=0}^{n}(-1)^r\, r^2\, C_r$$

$$- xyz\sum_{r=0}^{n}(-1)^r\, r^3\, C_r$$

$$= xyz\times 0 - (x+y+z)\times 0 + (xy+yz+zx)\times 0 - xyz\times 0 = 0$$

269 **(b)**

We know that,

$$\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + \ldots + n\frac{C_n}{C_{n-1}} = \frac{n(n+1)}{2}$$

On putting $n = 15$, then $\frac{15\times(15+1)}{2} = 15\times 8 = 120$

270 **(b)**

We have,

$17^{1995} + 11^{1995} - 7^{1995}$

$= (7+10)^{1995} + (1+10)^{1995} - 7^{1995}$

$$= \{7^{1995} + {}^{1995}C_1 7^{1994}\cdot 10^1 + {}^{1995}C_2\cdot 7^{1993}10^2 + \cdots + {}^{1995}C_{1995}\cdot 10^{1995}\}$$
$$+ \{{}^{1995}C_0 + {}^{1995}C_1\, 10^1 + {}^{1995}C_2\, 10^2 + \cdots + {}^{1995}C_{1995}10^{1995}\} - 7^{1995}$$

$$= \{{}^{1995}C_1 7^{1994}\cdot 10^1 + \cdots + 10^{1995}\} + \{{}^{1995}C_1 10^1 + \cdots + {}^{1995}C_{1995}10^{1995}\} + 1$$

= (a multiple of 10) +1

Thus, the unit's digit is 1

271 **(a)**

$${}^{50}C_4 + {}^{55}C_3 + {}^{54}C_3 + {}^{53}C_3 + {}^{52}C_3 + {}^{51}C_3 + {}^{50}C_3$$

$$= {}^{51}C_4 + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$$

$$[\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r]$$

$$= {}^{52}C_4 + {}^{52}C_3 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$$

$$= {}^{53}C_4 + {}^{53}C_3 + {}^{54}C_3 + {}^{55}C_3$$

$$= {}^{54}C_4 + {}^{54}C_3 + {}^{55}C_3 = {}^{55}C_4 + {}^{55}C_3 = {}^{56}C_4$$

272 **(b)**

$$\left(x - \frac{1}{x}\right)^4\left(x + \frac{1}{x}\right)^3$$

$= \left({}^4C_0x^4 - {}^4C_1x^2 + {}^4C_2 - {}^4C_3 \frac{1}{x^2} + {}^4C_4 \frac{1}{x^4} \right)$
$\times \left({}^3C_0x^3 + {}^3C_1x + {}^3C_2\frac{1}{x} + {}^3C_3\frac{1}{x^3} \right)$
Clearly, there is no term from x on RHS, therefore the term independent of x on LHS is zero.

273 **(b)**
Coefficient of x^2y^2 in $(x+y+z+t)^4 = \frac{4!}{2!2!} = 6$
and
coefficient of yzt^2 in $(x+y+z+t)^4$
$= \frac{4!}{1!\,1!\,1!\,2!} = 12$
Also, coefficients of $xyzt$ in
$(x+y+z+t)^4 = \frac{4!}{1!\,1!\,1!\,1!} = 24$
$\therefore$ Required ratio is $6:12:24 = 1:2:4$

274 **(b)**
The general term in the expansion of
$(1+2x+3x^2)^{10}$
is $\sum \frac{10!}{r!\,s!\,t!} 1^r (2x)^5 (3x^2)^t$
$= \frac{10!}{r!\,s!\,t!} 2^s \times 3^t \times x^{s+2t}$
Where $r+s+t=10$
We have to find $a_1 ie$, coefficient of x
For the coefficient of x^1, we must have
$s+2t=1$
But $r+s+t=10$
$\therefore s = 1-2t$ and $r = 9+t$
Where $0 \le r,s,t \le 10$
Now, $t=0 \Longrightarrow s=1, r=9$
For other, values of t, we get negative value s. So, there is only one term containing x and its coefficient is
$\frac{10!}{9!\,1!\,0!} 2^1 \times 3^0 = 20$
Hence, $a_1 = 20$
Alternate On differentiating given equation w. r. t. x, we get
$10(1+2x+3x^2)^9 = a_1 + 2a_2\,x + \ldots + 20a_{20}x^{19}$
Put $x=0$, we get
$20 = a_1$

275 **(b)**
$(1+x^2)^5(1+x)^4$
$= ({}^5C_0 + {}^5C_1x^2 + {}^5C_2x^4 + \cdots)({}^4C_0 + {}^4C_1x$
$+ {}^4C_2x^2 + {}^4C_3x^3 + {}^4C_4x^4$
The coefficient of x^5 in $[1+x^2)^5(1+x)^4]$
$= {}^5C_2 \cdot {}^4C_1 + {}^5C_1 \cdot {}^4C_3$
$= 10 \cdot 4 + 4 \cdot 5 = 60$

276 **(c)**
We have,
${}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + {}^{20}C_3 + \cdots + {}^{20}C_{10} + {}^{20}C_{11}$
$+ \cdots + {}^{20}C_{20} = 2^{20}$
$\Rightarrow \{ {}^{20}C_0 + {}^{20}C_{20}\} + \{ {}^{20}C_1 + {}^{20}C_{19}\} + \cdots$
$+ \{ {}^{20}C_9 + {}^{20}C_{11}\} + {}^{20}C_{10} = 2^{20}$
$\Rightarrow 2\{ {}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \cdots + {}^{20}C_9\} + {}^{20}C_{10}$
$= 2^{20}$
$\Rightarrow 2\{ {}^{20}C_0 + {}^{20}C_1 + \cdots + {}^{20}C_{10}\} = 2^{20} + {}^{20}C_{10}$
$\Rightarrow {}^{20}C_0 + {}^{20}C_1 + \cdots + {}^{20}C_{10} = 2^{19} + \frac{1}{2}\,{}^{20}C_{10}$

277 **(a)**
Last term of $\left(2^{1/3} - \frac{1}{\sqrt{2}}\right)^n$ is
$T_{n+1} = {}^nC_n (2^{1/3})^{n-n} \left(-\frac{1}{\sqrt{2}}\right)^n$
$= {}^nC_n (-1)^n \frac{1}{2^{n/2}} = \frac{(-1)^n}{2^{n/2}}$
Also, we have
$\left(\frac{1}{3^{5/3}}\right)^{\log_3 8} = 3^{-(5/3)\log_3 2^3} = 2^{-5}$
Thus, $\frac{(-1)^n}{2^{n/2}} = 2^{-5} \Rightarrow \frac{(-1)^n}{2^{n/2}} = \frac{(-1)^{10}}{2^5}$
$\Rightarrow \frac{n}{2} = 5 \Rightarrow n = 10$
Now, $T_5 = T_{4+1} = {}^{10}C_4 (2^{1/3})^{10-4} \left(-\frac{1}{\sqrt{2}}\right)^4$
$= \frac{10!}{4!\,6!} (2^{1/3})^6 (-1)^4 (2^{-1/2})^4$
$= 210(2)^2(1)(2^{-2}) = 210$

278 **(b)**
Let T_{r+1} be the $(r+1)^{th}$ term in the expansion of $\left(x^2 - \frac{1}{x^3}\right)^{10}$. Then, $T_{r+1} = {}^{10}C_r\, x^{20-5r}(-1)^r$
This will contain x^{-10}, if $20 - 5r = -10 \Rightarrow r = 6$
$\therefore$ Coefficient of $x^{-10} = {}^{10}C_6(-1)^6 = {}^{10}C_6 = 210$

280 **(b)**
$\frac{C_1}{C_0} + 2 \cdot \frac{C_2}{C_1} + 3 \cdot \frac{C_3}{C_2} + \cdots + n \cdot \frac{C_n}{C_{n-1}} = \frac{1}{k}\, n(n+1)$
$\Rightarrow \sum_{r=1}^{n} r \cdot \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{1}{k}\, n(n+1)$
$\Rightarrow \sum_{r=1}^{n} (n-r+1) = \frac{1}{k} n(n+1)$
$\Rightarrow n + (n-1) + (n-2) + \cdots + 1. = \frac{1}{k} n(n+1)$
$\Rightarrow \frac{n(n+1)}{2} = \frac{1}{k}\, n(n+1)$
$\Rightarrow k = 2$

281 **(a)**
We have,
$(1+x)^p + (1+x)^{p+1} + \cdots + (1+x)^n$

$$= \frac{(1+x)^p\{(1+x)^{n-p+1}-1\}}{(1+x)-1}$$

$$= \frac{1}{x}\{(1+x)^{n+1}-(1+x)^p\}$$

$\therefore$ Coefficient of x^m

= Coefficient of x^{m+1} in $(1+x)^{n+1}$

$= {}^{n+1}C_{m+1}$

282 **(c)**

Given that,

$(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \ldots + {}^nC_nx^n$

Let $S_n = \frac{{}^nC_1}{{}^nC_0} + \frac{2\,{}^nC_2}{{}^nC_1} + \frac{3\,{}^nC_3}{{}^nC_2} + \ldots + \frac{n\,{}^nC_n}{{}^nC_{n-1}}$

Put $n = 1,2,3, \ldots$, then

$S_1 = \frac{{}^1C_1}{{}^1C_0} = 1,$

$S_2 = \frac{{}^2C_1}{{}^2C_0} + 2\frac{{}^2C_2}{{}^2C_1}$

$= \frac{2}{1} + 2 \cdot \frac{1}{2} = 2 + 1 = 3$

By taking option, (put $n = 1,2, \ldots$) (a) and (b) does not hold condition, but option (c) satisfies.

283 **(a)**

Let the term containing x^7 in the expansion of

$$\left(ax^2 + \frac{1}{bx}\right)^8 \text{ is } T_{r+1}.$$

$$\therefore \quad T_{r+1} = {}^8C_r(ax^2)^{8-r}\left(\frac{1}{bx}\right)^r$$

$$= {}^8C_r\frac{a^{8-r}}{b^r}x^{16-3r}$$

Since, this term contains x^7.

$\therefore \quad 16 - 3r = 7$

$\Rightarrow \quad r = 3$

$\therefore$ Coefficient of x^7 in the expansion of $\left(ax^2 + \frac{1}{bx}\right)^8$

$$= {}^8C_3 \cdot \frac{a^5}{b^3}$$

Also, the term containing x^{-7} in the expansion of $\left(ax - \frac{1}{bx^2}\right)^8$ is T_{R+1}

$$T_{R+1} = {}^8C_R(a\,x)^{8-R}\left(-\frac{1}{bx^2}\right)^R$$

$$= (-1)^R\,{}^8C_R\frac{a^{8-R}}{b^R}x^{8-3R}$$

Since, this term contains x^{-7}

$\therefore \quad 8 - 3R = -7$

$\Rightarrow \quad R = 5$

$\therefore$ Coefficient of x^{-7} in the expansion of $\left(a\,x - \frac{1}{bx^2}\right)^8$

$$= (-1)^5\,{}^8C_5 \cdot \frac{a^3}{b^5}$$

According to the given condition,

$$\left|{}^8C_3 \cdot \frac{a^5}{b^3}\right| = \left|{}^8C_5 \cdot \frac{a^3}{b^5}\right|$$

$\Rightarrow \quad a^2b^2 = 1 \Rightarrow ab = 1$

284 **(b)**

We have,

$(x+a)^n = T_0 + T_1 + T_2 + \cdots + T_n$ ... (i)

Replacing a by ai and $-ai$ respectively in (i), we get

$(a+ai)^n = (T_0 - T_2 + T_4 - T_6 + \cdots)$
$+ i(T_1 - T_3 + T_5 - \cdots)$ ... (ii)

And,

$(x-ia)^n = (T_0 - T_2 + T_4 - T_6 + \cdots)$
$- i(T_1 - T_3 + T_5 - \cdots)$ ... (iii)

Multiplying (ii) and (iii), we get

$(x+ai)^n(x-ia)^n = (T_0 - T_2 + T_4 - T_6 - \cdots)^2 + (T_1 - T_3 + T_5 - \cdots)^2$

$\Rightarrow (x^2+a^2)^n = (T_0 - T_2 + T_4 - T_6 \ldots)^2 + (T_1 - T_3 + T_5 - \cdots)^2$

285 **(c)**

$$\because \sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i} = \sum_{i=0}^{m}{}^{10}C_i \cdot {}^{20}C_{m-1}$$

= Coefficient of x^m in the expansion of (1 + *x)10(1+x)20*

$= {}^{30}C_m$

It is maximum, when

$m = \frac{30}{2} = 15$

286 **(d)**

$a_1 = {}^nC_1, a_2 = {}^nC_2$

$a_3 = {}^nC_3$

$\because a_1, a_2$ and a_3 are in AP

$\Rightarrow \quad 2a_2 = a_1 + a_3$

$\Rightarrow 2 \cdot {}^nC_2 = {}^nC_1 + {}^nC_3$

$\Rightarrow 2 \cdot \frac{n(n-1)}{2!} = n + \frac{n(n-1)(n-2)}{3!}$

$\Rightarrow n^2 - 9n + 14 = 0$

$\Rightarrow (n-2)(n-7) = 0$

$\Rightarrow n = 7$ ($\because n = 2$is not Possible)

287 **(c)**

$(1+x)^{15} = a_0 + a_1x + a_2x^2 + \ldots + a_{15}x^{15}$

$\Rightarrow {}^{15}C_0 + {}^{15}C_1x + {}^{15}C_2x^2 + \ldots + {}^{15}C_{15}x^{15}$
$= a_0 + a_1x + a_2x^2 + \ldots + a_{15}x^{15}$

Equating the coefficient of various powers of x, we get

$a_0 = {}^{15}C_0, a_1 = {}^{15}C_1, a_2 = {}^{15}C_2, ...a_{15} = {}^{15}C_{15}$

$$\therefore \sum_{r=1}^{15} r\frac{a_r}{a_{r-1}} = \sum_{r=1}^{15} r\frac{{}^{15}C_r}{{}^{15}C_{r-1}}$$

$$= \sum_{r=1}^{15} r\frac{\frac{15!}{r!(15-r)!}}{\frac{15}{(r-1)(15-r+1)!}}$$

$$= \sum_{r=1}^{15} \frac{r(r-1)!\,(15-r+1)!}{r!\,(15-r)!}$$

$$= \sum_{r=1}^{15} 15-r+1$$

$= 15 + 14 + 13 + ...+2 + 1$

$$= \frac{15(15+1)}{2} = 120$$

288 **(b)**

Coefficient of x^5 in $(1+x^2)^5(1+x)^4$

= Coefficient of x^5 in $({}^5C_0 + {}^5C_1\, x^2 + {}^5C_2\, x^4 +$ *...1+x4*

$= {}^5C_1 \times$ Coefficient of x^3 in $(1+x)^4 + {}^5C_2 \times$ Coefficient of x in $(1+x)^4$

$= {}^5C_1 \times {}^4C_3 + {}^5C_2 \times {}^4C_1 = 20 + 40 = 60$

289 **(b)**

Since, $(1-x)^{-2} = 1 + 2x + 3x^2 + ...+(r+1)x^r$

$\therefore$ Coefficient of x^r in $(1-x)^{-2}$is $(r+1)$.

290 **(b)**

$$\frac{1}{(6-3x)^{1/3}} = (6-3x)^{-1/3} = 6^{-1/3}\left[1-\frac{x}{2}\right]^{-1/3}$$

$$= 6^{-1/3}\left[1 + \left(-\frac{1}{3}\right)\left(-\frac{x}{2}\right) + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{2.1}\left(-\frac{x}{2}\right)^2 + \ldots\right]$$

$$= 6^{-1/3}\left[1 + \frac{x}{6} + \frac{2x^2}{6^2} + \ldots\right]$$

291 **(a)**

The coefficient of $(r+1)^{th}$ term in the expansion of $(1+x)^{14}$ is ${}^{14}C_r$

It is given that

${}^{14}C_{r-1}, {}^{14}C_r, {}^{14}C_{r+1}$ are in A.P.

$\Rightarrow 2\, {}^{14}C_r = {}^{14}C_{r-1} + {}^{14}C_{r+1}$

$$\Rightarrow 2 = \frac{{}^{14}C_{r-1}}{{}^{14}C_r} + \frac{{}^{14}C_{r+1}}{{}^{14}C_r}$$

$$\Rightarrow 2 - \frac{r}{15-r} + \frac{14-r}{r+1}$$

$\Rightarrow 2(15-r)(r+1) = r^2 + r + 210 - 29\,r + r^2$

$\Rightarrow 4\,r^2 - 56\,r + 180 = 0$

$\Rightarrow r^2 - 14\,r + 45 = 0 \;\Rightarrow r = 5, 9$

292 **(b)**

If n id odd, then numerically the greatest coefficient in the expansion of $(1-x)^n$ is ${}^nC_{\frac{n-1}{2}}$ or, ${}^nC_{\frac{n+1}{2}}$

Therefore, in case of $(1-x)^{21}$ the numerically greatest coefficient is ${}^{21}C_{10}$ or, ${}^{21}C_{11}$

Numerically greatest term

$= {}^{21}C_{11}\, x^{11}$ or, ${}^{21}C_{10}x^{10}$

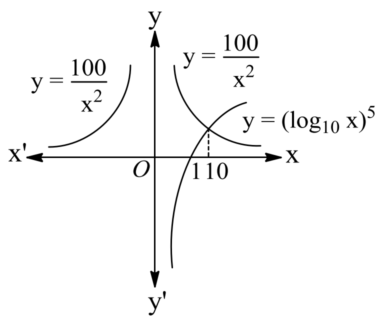
$\therefore {}^{21}C_{11}x^{11} > {}^{21}C_{12}\, x^{12}$ and ${}^{21}C_{10}\, x^{10} > {}^{21}C_9\, x^9$

$$\Rightarrow \frac{21\,!}{10\,!\,11\,!} > \frac{21\,!}{9\,!\,12\,!}\,x \text{ and } \frac{21\,!}{11\,!\,10\,!}\,x > \frac{21\,!}{9\,!\,12\,!}$$

$$\Rightarrow \frac{6}{5} > x \text{ and } x < \frac{5}{6} \Rightarrow x \in (5/6, 6/5)$$

293 **(d)**

Note that for $\log_{10} x$ to be defined, $x > 0$



We have, $T_6 = T_{5+1} = {}^8C_5\left(\frac{1}{x^{8/3}}\right)^{8-5}(x^2\log_{10} x)^5$

$$\Rightarrow\; 5600 = \frac{8!}{5!\,3!}\left(\frac{1}{x^8}\right)x^{10}(\log_{10} x)^5$$

$\Rightarrow\; 5600 = 56x^2(\log_{10} x)^5$

$\Rightarrow\; 100 = x^2(\log_{10} x)^5$

$$\Rightarrow\; \frac{100}{x^2} = (\log_{10} x)^5$$

Let $y = \frac{100}{x^2}$

$\therefore\; y = (\log_{10} x)^5$

From the figure it is clear that curves intersect in just one point.

This point is $(10, 1)$

Therefore, $x = 10$

294 **(b)**

We have,

$$(1+x^2)^{40}\left(x^2 + 2 + \frac{1}{x^2}\right)^{-5} = (1+x^2)^{40}(x^2+1)^{-10}x^{10}$$

$$\Rightarrow (1+x^2)^{40}\left(x^2 + 2 + \frac{1}{x^2}\right)^{-5} = (1+x^2)^{30}x^{10}$$

$\therefore$ Coeff of x^{20} in the expansion of $(1+x^2)^{40}\left(x^2 + 2 + \frac{1}{x^2}\right)^{-5}$

= Coefficient of x^{20} in $(1+x^2)^{30}\cdot x^{10}$

= Coefficient of x^{10} in $(1+x^2)^{30} = {}^{30}C_5 = {}^{30}C_{25}$

295 **(d)**

Let $P(n)=n^3+2n$

$\Rightarrow P(1)=1+2=3$

$\Rightarrow P(2)=8+4=12$

$\Rightarrow P(3)=27+6=33$

Here, we see that all these number are divisible by 3

296 **(c)**

$(1+x)^m(1-x)^n$

$$=\left(1+mx+\frac{m(m-1)x^2}{2!}+\ldots\right)\left(1-nx+\frac{n(n-1)}{2!}x^2-\ldots\right)$$

$$=1+(m-n)x+\left[\frac{n^2-n}{2}-mn+\frac{(m^2-m)}{2}\right]x^2\ldots$$

Given, $m-n=3 \Rightarrow n=m-3$

and $\frac{n^2-n}{2}-mn+\frac{m^2-m}{2}=-6$

$$\Rightarrow \frac{(m-3)(m-4)}{2}-m(m-3)+\frac{m^2-m}{2}=-6$$

$$\Rightarrow m^2-7m+12-2m^2+6m+m^2-m+12=0$$

$\Rightarrow -2m+24=0 \Rightarrow m=12$

297 **(d)**

We have,

$(1+x+x^2+x^3)^n$

$=(1+x)^n(1+x^2)^n$

$=(C_0+C_1\,x+C_2\,x^2+\cdots+C_n\,x^n)(C_0+C_1x^2+\cdots+C_n\,x^{2n})$

$\therefore$ Coefficient of $x^4=C_0C_2+C_2C_1+C_4C_0$

$={}^nC_2+{}^nC_2\cdot{}^nC_1+{}^nC_4$

298 **(b)**

Let, $S=1+\frac{1}{3}\cdot\frac{1}{2}+\frac{2}{3}\cdot\frac{5}{6}\cdot\frac{1}{2^2}+\frac{2}{3}\cdot\frac{5}{6}\cdot\frac{8}{9}\cdot\frac{1}{2^3}+\ldots\ldots\infty$

and we know that

$$(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+\frac{(n(n-1)(n-2)}{3!}x^2+\cdots\infty$$

On comparing these two, we get

$$nx=\frac{2}{3}\cdot\frac{1}{2} \quad \ldots\text{(i)}$$

and $\frac{n(n-1)}{2\cdot1}x^2=\frac{2}{3}\cdot\frac{5}{6}\cdot\frac{1}{2^2}$ ... (ii)

from Eqs. (i) and (ii),

$$\Rightarrow \frac{\frac{n(n-1)}{2\cdot1}}{n^2}=\frac{\frac{2}{3}\times\frac{5}{6}\times\frac{1}{4}}{\frac{2}{3}\times\frac{1}{2}\times\frac{2}{3}\times\frac{1}{2}}$$

$$\Rightarrow \frac{n-1}{2n}=\frac{5}{4}$$

$$\Rightarrow 5n=2n-2$$

$$\Rightarrow n=-\frac{2}{3}$$

On putting value of n in Eq. (i), we get

$$x=-\frac{1}{2}$$

$$\therefore \text{Sum of series}=\left(1-\frac{1}{2}\right)^{-\frac{2}{3}}=(4)^{1/3}$$

299 **(b)**

Here, $T_5+T_6=0$

$\Rightarrow {}^nC_4a^{n-4}(-2b)^4+{}^nC_5a^{n-5}(-2b)^5=0$

$\Rightarrow 16\cdot{}^nC_4a^{n-4}b^4=32\,{}^nC_5a^{n-5}b^5$

$$\Rightarrow \frac{{}^nC_5}{{}^nC_4}\cdot\frac{a^{n-5}b^5}{a^{n-4}b^4}=\frac{1}{2}$$

$$\Rightarrow \frac{b}{a}=\frac{1}{2}\cdot\frac{{}^nC_4}{{}^nC_5}$$

$$\Rightarrow \frac{a}{b}=\frac{2\cdot\frac{n!}{5!(n-5)!}}{\frac{n!}{4!(n-4)!}}$$

$$=\frac{4!\,(n-4)!}{5!\,(n-5)!}\times2=\frac{2(n-4)}{5}$$

300 **(c)**

Given , $(1+x)^m(1-x)^n$

$$=\left(1+mx+m\frac{(m-1)}{2!}x^2+\cdots\right)\left(1-nx+\frac{n(n-1)}{2!}x^2-\cdots\right)$$

$$=1+(m-n)x+\left[\frac{n^2-n}{2}-mn+\frac{m^2-m}{2}\right]x^2+\cdots$$

Also, given $m-n=3 \Rightarrow n=m-3$

and $\frac{\text{n}^2-\text{n}}{2}-mn+\frac{m^2-m}{2}=-6$

$$\Rightarrow \frac{(m-3)(m-4)}{2}-m(m-3)+\frac{m^2-m}{2}-6$$

$$\Rightarrow m^2-7m+12-2m^2+6m+m^2-m+12=0$$

$\Rightarrow -2m+24=0 \Rightarrow m=12$

301 **(b)**

Given sum of the coefficient $=1024$

ie, $2^n=1024=2^{10}$

$\Rightarrow n=10$

Since, n is even, so greatest coefficient

$={}^nC_{n/2}={}^{10}C_5=252$

302 **(b)**

We have,

$5^{99}=5^3\times5^{96}$

$=(13\times9+8)\{1+{}^{24}C_1(13\times48)+\cdots+{}^{24}C_{24}(13\times48)^{24}\}$

$= (13 \times 9 + 8) + (13 \times 9 + 8)\{ {}^{24}C_1(13 \times 48) + \cdots + {}^{24}C_{24}(13 \times 48)^{24}\}$

$= 8 + 13 \times$ An integer

Hence, remainder $= 8$

303 **(c)**

General term of $(3 + 2x)^{74}$ is

$T_{r+1} = {}^{74}C_r(3)^{74-r}2^r x^r$

Let two consecutive terms are T_{r+1}th and T_{r+2}th terms

According to the given condition,

Coefficient of T_{r+1} =Coefficient of T_{r+2}

$\Rightarrow {}^{74}C_r 3^{74-r}2^r = {}^{74}C_{r+1}3^{74-(r+1)}2^{r+1}$

$\Rightarrow \frac{{}^{74}C_{r+1}}{{}^{74}C_r} = \frac{3}{2} \Rightarrow \frac{74-r}{r+1} = \frac{3}{2}$

$\Rightarrow 148 - 2r = 3r + 3 \Rightarrow r = 29$

Hence, two consecutive terms are 30 and 31.

304 **(b)**

Given expansion is $\left(\frac{2}{3}x - \frac{3}{2x}\right)^n$

$\therefore T_4 = {}^nC_3\left(\frac{2}{3}x\right)^{n-3}\left(-\frac{3}{2x}\right)^3$

$= {}^nC_3\left(\frac{2}{3}\right)^{n-6} x^{n-6}(-1)^3$

Since, it is independent of x

$\therefore n - 6 = 0 \Rightarrow n = 6$

305 **(b)**

General term, $T_{r+1} = {}^{15}C_r(x^4)^{15-r}\left(-\frac{1}{x^3}\right)^r$

$= {}^{15}C_r x^{60-7r}(-1)^r$

For the coefficient of x^{3r} put

$60 - 7r = 32 \Rightarrow r = 4$

Now, coefficient of x^{32} in $\left(x^4 - \frac{1}{x^3}\right)^{15} =$

${}^{15}C_4(-1)^4 = {}^{15}C_4$

306 **(d)**

The number of terms in$(a + b + c)^{12}$

$= {}^{12+2}C_2 = {}^{14}C_2 = 91$

307 **(a)**

Coefficient of x^7 in $(1 + 3x - 2x^3)^{10}$

$= \sum \frac{10!}{n_1!\, n_2!\, n_3!}(1)^{n_1}(3)^{n_2}(-2)^{n_3}$

Where, $n_1 + n_2 + n_3 = 10,\ \ n_2 + 3n_3 = 7$

Different possibilities are as follows

n_1	n_2	n_3
3	7	0
5	4	1
7	1	2

$\therefore$ Coefficient of $x^7 = \frac{10!}{3!7!}(1)^3(3)^7(-2)^0$

$+ \frac{10!}{5!\ 4!\ 1!}(1)^5(3)^4(-2)^1$

$+ \frac{10!}{7!\ 1!\ 2!}(1)^7(3)^1(-2)^2$

$= 62640$

308 **(b)**

The general term in the expansion of $\left(x - \frac{1}{x}\right)^{18}$ is

$T_{r+1} = {}^{18}C_r(x)^{18-r}\left(-\frac{1}{x}\right)^r$

Here, $n = 18$

$\therefore$ the middle term is T_{9+1}, where $r = 9$

$\therefore \quad T_{9+1} = {}^{18}C_9(-1)^9 x^{18-2r}$

$= -{}^{18}C_9 x^{18-18} = -{}^{18}C_9$

309 **(d)**

General term

$T_{r+1} = (-1)^r\, {}^{11}C_r\left(\frac{2\sqrt{x}}{5}\right)^{11-r}\left(\frac{1}{2x^{3/2}}\right)^r$

$= \frac{2^{11-2r}}{5^{11-r}}(-1)^r\, {}^{11}C_r x^{\frac{11-r}{2} - \frac{3r}{2}}$

For term independent of x, put $\frac{11-r}{2} - \frac{3r}{2} = 0$

$\Rightarrow \frac{11-4r}{2} = 0 \Rightarrow r = \frac{11}{4} \notin N$

$\therefore$ There is no term which is independent of x.

310 **(b)**

$a[C_0 - C_1 + C_2 - C_3 + \cdots (-1)^n \cdot C_n]$

$+[C_1 - 2C_2 + 3C_3 - \cdots + (-1)^{n-1}nC_n]$

$= a0 + 0 = 0$

312 **(a)**

Let $\left(6\sqrt{6} + 14\right)^{2n+1} = I + F$, where $I \in N$ and $0 < F < 1$

Also, let $G = \left(6\sqrt{6} - 14\right)^{2n+1}$. Then, $0 < G < 1$

Clearly,

$I + F - G$ is an even integer

$\Rightarrow F = G$

$\Rightarrow I$ is an even integer

313 **(a)**

The sum of the coefficients is obtained by putting $x = y = 1$ in $(5x - 4y)^n$. So, required sum $= 1$

314 **(a)**

Now, $\left[\frac{(x+1)}{x^{2/3} - x^{1/3} + 1} - \frac{(x-1)}{x - x^{1/2}}\right]^{10}$

$= \left[\frac{(x^{1/3})^3 + 1^3}{x^{2/3} - x^{1/3} + 1} - \frac{\left((\sqrt{x})^2 - 1\right)}{\sqrt{x}(\sqrt{x} - 1)}\right]^{10}$

$= [x^{1/3} + 1 - (x^{-1/2} + 1]^{10}$

$= [x^{1/3} + x^{-1/2}]^{10}$

$\therefore T_{r+1} = {}^{10}C_r(x)^{\frac{10-r}{3}}(-x^{-1/2})^r$

$= {}^{10}C_r(-1)^r x^{\frac{20-5r}{6}}$

For the term independent of x

Put $\frac{20-5r}{6} = 0$

$\Rightarrow r = 4$

$\therefore$ Required coefficient = ${}^{10}C_4 = 210$

315 **(c)**

Putting the values of C_0, C_2, C_4 ...., we get

$$1 + \frac{n(n-1)}{3 \cdot 2!} + \frac{n(n-1)(n-2)(n-3)}{5 \cdot 4!} + \cdots$$

$$= \frac{1}{n+1}[(n+1) + \frac{(n+1)n(n-1)}{3!} + \frac{(n+1)n(n-1)(n-2)(n-3)}{5!} + \cdots]$$

Put $n + 1 = N$

$$= \frac{1}{N}[N + \frac{N(N-1)(N-2)}{3!}] + \frac{N(N-1)(N-2)(N-3)(N-4)}{5!} + \cdots$$

$$= \frac{1}{N}[\,{}^NC_1 + {}^NC_3 + {}^NC_5 + \cdots]$$

$$= \frac{1}{N}[2^{N-1}] = \frac{2^n}{n+1} \ [\because N = n+1]$$

316 **(b)**

We have,

$$\frac{(1+x)^n}{1-x} = (1+x)^n(1-x)^{-1}$$

$$\Rightarrow \frac{(1+x)^n}{1-x} = ({}^nC_0\,x^n + {}^nC_1\,x^{n-1} + \cdots + {}^nC_{n-1}x + {}^nC_nx^0) \times (1 + x + x^2 + x^3 + \cdots + x^n + \cdots)$$

$\therefore$ Coefficient of x^n in $\frac{(1+x)^n}{1-x}$

$= {}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n = 2^n$

317 **(d)**

Given that, $(1 + x - 2x^2)^6 = 1 + a_1x + a_2x^2 + \ldots + a_{12}x^{12}$

On putting $x = 1$ and $x = -1$ and adding the results, we get

$64 = 2(1 + a_2 + a_4 + \ldots + a_{12})$

$\therefore\ a_2 + a_4 + a_6 + \ldots + a_{12} = 31$

318 **(b)**

$\because\ (1+x)^n = \sum_{r=0}^{n} {}^nC_rx^r = \sum_{r=0}^{n} a_rx^r$ (given)

$\therefore\ a_r = {}^nC_r$

Also, $b_r = 1 + \frac{a_r}{a_{r-1}} = 1 + \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{{}^{n+1}C_r}{{}^nC_{r-1}}$

$$b_r = \left(\frac{n+1}{r}\right)$$

$$\because \prod_{r=1}^{n} b_r = \prod_{r=1}^{n}\left(\frac{n+1}{r}\right) = \frac{(n+1)^n}{n!}$$

$= \frac{(101)^{100}}{100!}$ (given)

$\therefore\ n = 100$

319 **(c)**

Coefficient of $x^2y^3z^4 = \frac{9!}{2!\,3!\,4!}a^2b^3c^4 = 1260a^2b^3c^4$

320 **(c)**

$$T_{r+1} = {}^9C_r(x^2)^{9-r}\left(-\frac{1}{x}\right)^r$$

$= {}^9C_rx^{18-2r-r}(-1)^r$

For term independent of x, put $18 - 2r - r = 0 \Longrightarrow r = 0$

$\therefore$ Constant term, $T_7 = {}^9C_6(-1)^6 = 84$

321 **(c)**

We have,

$$\sum_{k=1}^{\infty} k\left(1 + \frac{1}{n}\right)^{k-1}$$

$$= \sum_{k=1}^{\infty} k\,x^{k-1}, \text{where } x = 1 + \frac{1}{n}$$

$= 1 + 2x + 3x^2 + 4x^3 + \cdots$ to ∞

$$= (1-x)^{-2} = \left(-\frac{1}{n}\right)^{-2} = n^2$$

322 **(c)**

Let $(r+1)$th, $(r+2)$th and $(r+3)$th be three consecutive terms.

Then, ${}^nC_r : {}^nC_{r+1} : {}^nC_{r+2} = 1:7:42$

Now, $\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{1}{7} \Rightarrow \frac{r+1}{n-r} = \frac{1}{7} \Rightarrow n - 8r = 7$

and $\frac{{}^nC_{r+1}}{{}^nC_{r+2}} = \frac{7}{42}$

$$\Rightarrow \frac{r+2}{n-r-1} = \frac{1}{6}$$

$\Rightarrow n - 7r = 13$ ...(ii)

On solving Eqs. (i) and (ii), we get $n = 55$

323 **(c)**

We have,

$$\left(1 + 3\sqrt{2}\,x\right)^9 + \left(1 - 3\sqrt{2}\,x\right)^9$$

$$= 2\left\{{}^9C_0 + {}^9C_2\left(3\sqrt{2}\,x\right)^2 + \cdots + {}^9C_8\left(3\sqrt{2}\,x\right)^8\right\}$$

Clearly, there are 5 terms in the above expansion

324 **(c)**

Given that, ${}^nC_6 = {}^nC_{12} \Rightarrow {}^nC_{n-6} = {}^nC_{12}$

$\Rightarrow n - 6 = 12 \Rightarrow n = 18$

325 **(c)**

The general term in the expansion of $\left(2x^2 - \frac{1}{x}\right)^{12}$ is

$T_{r+1} = (-1)^r\,{}^{12}C_r \cdot 2^{12-r} \cdot x^{24-3r}$

The term independent of x, put $24 - 3r = 0$

$\Longrightarrow \quad r = 8$

$\therefore$ In the expansion of $\left(2x^2 - \frac{1}{x}\right)^{12}$, the term independent of x is 9th term.

327 **(b)**

$\because 4^n = (1+3)^n$

$= 1 + 3n + \frac{n(n-1)}{2!}3^2 + \cdots$

$\Rightarrow\ 4^n - 3n - 1 = 3^2\left[\frac{n(n-1)}{2!} + ..\right]$

It is clear from above that $4^n - 3n - 1$is divisible by 9.

328 **(b)**

On putting $x = -1$in

$(1+x)^{20} = {}^{20}C_0 + {}^{20}C_1x + \ldots + {}^{20}C_{10}x^{10} + \cdots + {}^{20}C_{10}x^{10}$,

we get

$0 = {}^{20}C_0 - {}^{20}C_1 + \cdots - {}^{20}C_9 + {}^{20}C_{10} - {}^{20}C_{11} + \cdots + {}^{20}C_{20}$

$\Rightarrow 0 = {}^{20}C_0 - {}^{20}C_1 + \cdots - {}^{20}C_9 + {}^{20}C_{10} - {}^{20}C_9 + \cdots + {}^{20}C_0$

$\Rightarrow 0 = 2({}^{20}C_0 - {}^{20}C_1 + \cdots - {}^{20}C_9) + {}^{20}C_{10}$

$\Rightarrow\ {}^{20}C_{10} = 2({}^{20}C_0 - {}^{20}C_1 + \cdots + {}^{20}C_{10})$

$\Rightarrow\ {}^{20}C_0 - {}^{20}C_1 + \ldots + {}^{20}C_{10} = \frac{1}{2}\ {}^{20}C_{10}$

329 **(d)**

Now, $(\sqrt{3}+1)^5 = (\sqrt{3})^5 + {}^5C_1(\sqrt{3})^4 + {}^5C_2(\sqrt{3})^3 + {}^5C_3(\sqrt{3})^2 + {}^5C_4(\sqrt{3}) + {}^5C_5$

$= 9\sqrt{3} + 45 + 30\sqrt{3} + 30 + 5\sqrt{3} + 1$

$= 76 + 44\sqrt{3}$

$\therefore \left[(\sqrt{3}+1)^5\right] = [76 + 44\sqrt{3}]$

$= [76] + [44 \times 1.732]$

$= 76 + 76 = 152$

330 **(c)**

The given sigma expansion

$\sum_{m=0}^{100} {}^{100}C_m\,(x-3)^{100-m} \cdot 2^m$ can be rewritten as

$[(x-3)+2]^{100} = (x-1)^{100} = (1-x)^{100}$

$\therefore x^{53}$ will occur in T_{54}

$\Rightarrow\ T_{54} = {}^{100}C_{53}(-x)^{53}$

$\therefore$ Required coefficient is$-\ {}^{100}C_{53}$.

331 **(a)**

$T_4 = {}^nC_3(ax)^{n-3}\left(\frac{1}{x}\right)^3 = \frac{5}{2}$ [given]

$\Rightarrow\ {}^nC_3a^{n-3}x^{n-6} = \frac{5}{2}$

$\Rightarrow n - 6 = 0$ [$\because$ RHS is independent of x]

$\Rightarrow n = 6$

On putting $n = 6$ in Eq. (i), we get

${}^6C_3a^3 = \frac{5}{2} \Rightarrow a^3 = \frac{1}{8} \Rightarrow a = \frac{1}{2}$

332 **(a)**

We have,

$\sum_{r=0}^{n}\sum_{s=0}^{n}(r+s)C_rC_s = \sum_{r=0}^{n} 2rC_r^2 + 2\sum_{0\le r<s\le n}\sum(r+s)\,C_rC_s$

$\Rightarrow \sum_{r=0}^{n}\sum_{s=0}^{n}(r+s)C_rC_s = 2\sum_{r=0}^{n} r\cdot C_r^2 + 2\sum_{0\le r<s\le n}\sum(r+s)\,C_rC_s$

$\Rightarrow n\cdot 2^{2n} = 2\cdot\left(\frac{n}{2}\ {}^{2n}C_n\right) + 2\sum_{0\le r<s\le n}\sum(r+s)\,C_rC_s$

$\Rightarrow \sum_{0\le r<s\le n}\sum(r+s)\,C_rC_s = \frac{1}{2}[n\cdot 2^{2n} - n\cdot\ {}^{2n}C_n]$

$= \frac{n}{2}\left[2^{2n} - \frac{2n}{n}\ {}^{2n-1}C_{n-1}\right]$

$= n[2^{2n-1} - {}^{2n-1}C_{n-1}]$

333 **(a)**

Suppose x^7 occurs in $(r+1)^{th}$ term in the expansion of $\left(ax^2 + \frac{1}{bx}\right)^{11}$

We have,

$T_{r+1} = {}^{11}C_r(a\,x^2)^{11-r}\left(\frac{1}{b\,x}\right)^r$

$= {}^{11}C_r\,a^{11-r}\,b^{-r}x^{22-3r}$

This will contain x^7, if

$22 - 3r = 7 \Rightarrow r = 5$

$\therefore$ Coefficient of x^7 in $(ax^2 + b^{-1}\,x^{-1})^{11}$

$= {}^{11}C_5a^6b^{-5}$

Let x^{-7} occur in $(s+1)^{th}$ term of the expansion of $\left(ax - \frac{1}{b\,x^2}\right)^{11}$

We have,

$T_{s+1} = {}^{11}C_s\,(a\,x)^{11-s}\left(-\frac{1}{b\,x^2}\right)^s$

$\Rightarrow T_{s+1} = {}^{11}C_s\,a^{11-s}b^{-s}(-1)^s\,x^{11-3s}$

This is contain x^{-7}, if

$11 - 3s = -7 \Rightarrow s = 6$

$\therefore$ Coefficient of x^{-7} in $(ax - b^{-1}\,x^{-2})^{11}$

$= {}^{11}C_6\,a^5b^{-6}$

It is given that

${}^{11}C_6\,a^5b^{-6} = {}^{11}C_5a^6b^{-5} \Rightarrow ab = 1$

334 **(a)**

We have,

$\frac{1}{(1-x)(3-x)} = \frac{1}{2}\left(\frac{1}{1-x} - \frac{1}{3-x}\right)$

$\Rightarrow \frac{1}{(1-x)(3-x)} = \frac{1}{2}\{(1-x)^{-1} - (3-x)^{-1}\}$

$\Rightarrow \frac{1}{(1-x)(3-x)} = \frac{1}{2}\left\{(1-x)^{-1} - \frac{1}{3}\left(1 - \frac{x}{3}\right)^{-1}\right\}$

$\therefore$ Coefficient $x^n = \frac{1}{2}\left\{1 - \frac{1}{3}\cdot\frac{1}{3^n}\right\} = \frac{1}{2}\frac{(3^{n+1}-1)}{3^{n+1}}$

335 **(a)**

For greatest term in $(x+a)^a$is

$$\frac{n-r+1}{r}\left|\frac{a}{x}\right| \geq 1$$

$$\Rightarrow \quad \frac{54-r+1}{r}\left|\frac{3x}{1}\right| \geq 1$$

$\Rightarrow 55 - r \geq r \Rightarrow r = 27 \quad \left[\because x = \frac{1}{3}\right]$

$\therefore$ Greatest term in the expansion of $(1+3x)^{54}$ is T_{28}.

336 **(b)**

We have,

$T_4 = {}^5C_3\left(\frac{1}{x}\right)^{5-3}(x\tan x^3) = 10x\tan^3 x$

and $T_2 = {}^5C_2\left(\frac{1}{x}\right)^{5-1}(x\tan x) = \frac{5\tan x}{x^3}$

Given $\frac{T_4}{T_2} = \frac{2}{27}\pi^4 \Rightarrow 2x^4\tan^2 x = \frac{2}{27}\pi^4$

$\Rightarrow x^2\tan x = \pm\frac{1}{3\sqrt{3}}\pi^2$

It is possible (from among the answers) when $x = \pm\frac{\pi}{3}$

337 **(d)**

We have,

$32^{32} = (2^5)^{32} = 2^{160} = (3-1)^{160}$

$\Rightarrow 32^{32} = {}^{160}C_0\, 3^{160} - {}^{160}C_1\cdot 3^{159} + \cdots - {}^{160}C_{159}\cdot 3 + {}^{160}C_{160}3^0$

$\Rightarrow 32^{32} = ({}^{160}C_0\cdot 3^{160} - {}^{160}C_1\cdot 3^{159} + \cdots - {}^{160}C_{159}\cdot 3) + 1$

$\Rightarrow 32^{32} = 3m+1$, where $m \in N$

$\therefore 32^{(32)^{(32)}} = (32)^{3m+1} = (2^5)^{3m+1} = 2^{15m+5} = 2^{3(5m+1)}\cdot 2^2$

$\Rightarrow 32^{(32)^{(32)}} = (2^3)^{5m+1}\cdot 2^2 = (7+1)^{5m+1}\times 4$

$\Rightarrow 32^{(32)^{(32)}} = \{{}^{5m+1}C_0\, 7^{5m+1} + {}^{5m+1}C_1\, 7^{5m} + \cdots + {}^{5m+1}C_{5m}\, 7^1 + {}^{5m+1}C_{5m+1}\cdot 7^0\} \times 4$

$\Rightarrow 32^{(32)^{(32)}} = (7n+1)\times 4$,

where $n = {}^{5m+1}C_0 7^{5m+1} + \cdots + {}^{5m+1}C_{5m}7$

$\Rightarrow 32^{(32)^{(32)}} = 28n + 4$

Thus, when $32^{(32)^{(32)}}$ is divided by 7, the remainder is 4

338 **(a)**

Since, $\left(1+\frac{1}{n}\right)^n < 3$ for $\forall n \in N$

Now, $\frac{(1001)^{999}}{(1000)^{1000}} = \frac{1}{1001}\cdot\left(\frac{1001}{1000}\right)^{1000}$

$= \frac{1}{1001}\left(1+\frac{1}{1000}\right)^{1000} < \frac{1}{1001}\cdot 3 < 1$

$(1001)^{999} < (1000)^{1000}$

$\therefore B < A$

339 **(a)**

$\therefore T_r = 10_{C_{r-1}}\left(\frac{x}{3}\right)^{11-r}\left(\frac{-2}{x^2}\right)^{r-1}$

$= 10_{C_{r-1}}(x)^{13-3r}(3)^{-11+r}(-1)^{r-1}(2)^{r-1}$

For x^4, we put $13 - 3r = 4 \Rightarrow r = 3$

340 **(a)**

Given, $\left[\sqrt{\frac{x}{3}} + \frac{\sqrt{3}}{x^2}\right]^{10}$

General term, $T_{r+1} = {}^{10}C_r\left(\frac{x}{3}\right)^{\frac{1}{2}(10-r)}\left(\frac{\sqrt{3}}{x^2}\right)^r$

$\Rightarrow T_{r+1} = {}^{10}C_r\left(\frac{1}{3}\right)^{\frac{10-r}{2}}(\sqrt{3})^r x^{\frac{1}{2}(10-r)-2r}$

For term independent of x put

$$\frac{1}{2}(10-r) - 2r = 0$$

$\Rightarrow \quad r = 2$

$\therefore T_{2+1} = T_3 = {}^{10}C_2\left(\frac{1}{3}\right)^{\frac{8}{2}}(\sqrt{3})^2$

$\Rightarrow = 45 \times \frac{1\times 3}{81} = \frac{5}{3}$

341 **(c)**

We have,

$T_{r+1} = {}^{15}C_r(x^2)^{15-r}\left(\frac{2}{x}\right)^r = {}^{15}C_r\, x^{30-3r}\cdot 2^r$

If T_{r+1} contains x^{15}, then

$30 - 3r = 15 \Rightarrow r = 5$

$\therefore$ Coefficient of $x^{15} = {}^{15}C_5(2^5)$

If T_{r+1} does not contain x, then

$30 - 3r = 0 \Rightarrow r = 10$

$\therefore$ Coefficient of $x^0 = {}^{15}C_{10}(2^{10})$

Hence, required ratio $= \frac{{}^{15}C_5(2^5)}{{}^{15}C_{10}(2^{10})} = \frac{1}{32}$

342 **(d)**

We have,

${}^{40}C_0 + {}^{40}C_1 + {}^{40}C_2 + \cdots + {}^{40}C_{20}$

$= \frac{1}{2}[2\cdot{}^{40}C_0 + 2\cdot{}^{40}C_1 + 2\cdot{}^{40}C_2 + \cdots + 2\cdot{}^{40}C_{20}]$

$= \frac{1}{2}[({}^{40}C_0 + {}^{40}C_{40}) + ({}^{40}C_1 + {}^{40}C_{0=39}) + \cdots + ({}^{40}C_{19} + {}^{40}C_{21}) + 2\,{}^{40}C_{20}]$

$= \frac{1}{2}[\{{}^{40}C_0 + {}^{40}C_1 + {}^{40}C_2 + \cdots + {}^{40}C_{19} + {}^{40}C_{20} + {}^{40}C_{21}\} + {}^{40}C_{20}]$

$= \frac{1}{2}\left[2^{40} + \frac{40!}{(20\,!)^2}\right] = 2^{39} + \frac{1}{2}\frac{40!}{(20!)^2}$

343 **(d)**

We have,

$$\frac{1+x^2}{1+x} = (1+x^2)(1+x)^{-1}$$
$$= (1+x^2)(1-x+x^2-x^3+x^4-x^5+\cdots)$$
$\therefore$ Coefficient of x^5 in $\left(\frac{1+x^2}{1+x}\right) = -1-1 = -2$

344 **(a)**

Genaral term $T_{r+1} = {}^{10}C_r \left(\frac{1}{3}x^{1/2}\right)^{10-r} (x^{-1/4})^r$
$$= {}^{10}C_r \frac{1}{3^{10-r}} x^{5-3r/4}$$
For the coefficient of x^2 put
$$5 - \frac{3r}{4} = 2$$
$\Longrightarrow \quad r = 4$

$\therefore$ Coefficient of $x^2 = {}^{10}C_4 \frac{1}{3^{10-4}} = \frac{70}{243}$

345 **(d)**

The 14th term in the expansion of $\left(\frac{3\sqrt{x}}{7} - \frac{5}{2x\sqrt{x}}\right)^{13n}$ is

$$T_{14} = {}^{13n}C_{13}\left(\frac{3}{7}x^{1/2}\right)^{13n-13}(-1)^{13}\left(\frac{5}{2}x^{-3/2}\right)^{13}$$
$$= {}^{13n}C_{13}\left(\frac{3}{7}\right)^{13n-13}(-1)^{13}\left(\frac{5}{2}\right)^{13} x^{\frac{13n-13}{2}-\frac{39}{2}}$$
For this term to be independent of x, we put
$$13n - 52 = 0$$
$\Longrightarrow \quad n = 4$

346 **(b)**

Here, the greatest coefficient is ${}^{2n}C_n$

$\therefore\ {}^{2n}C_n x^n > {}^{2n}C_{n+1}x^{n-1} \Rightarrow x > \frac{n}{n+1}$

and ${}^{2n}C_n x^n > {}^{2n}C_{n-1}x^{n-1} \Rightarrow x < \frac{n+1}{n}$

$\therefore x$ must lie in the interval $\left(\frac{n}{n+1}, \frac{n+1}{n}\right)$

347 **(d)**

$(1+a-b+c)^9$
$$= \sum \frac{9!}{x_1!\,x_2!\,x_3!\,x_4!} \cdot (1)^{x_1}(a)^{x_2}(-b)^{x_3}(c)^{x_4}$$
$\Longrightarrow$ Coefficient of $a^3b^4c = \frac{9!}{1!\,3!\,4!\,1!} = \frac{9!}{3!\,4!}$

348 **(b)**

We have, $(1+x+x^2)^n = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots + a_{2n}x^{2n}$

On differentiating both sides, we get

$n(1+x+x^2)^{n-1}(1+2x)$
$$= a_1 + 2a_2x + 3a_3x^2 + \ldots + 2na_{2n}x^{2n-1}$$
On putting $x = -1$, we get

$n(1-1+1)^{n-1}(1-2)$
$$= a_1 - 2a_2 + 3a_3 - \ldots - 2na_{2n}$$
$\Rightarrow a_1 - 2a_2 + 3a_3 - \ldots - 2na_{2n} = -n$

350 **(b)**

We have,

$T_4 = 200$

$\Rightarrow {}^6C_3 \left\{\sqrt{x^{\frac{1}{\log x+1}}}\right\}^3 \left(x^{\frac{1}{12}}\right)^3 = 200$

$\Rightarrow 20\, x^{\frac{3}{2(\log x+1)}+\frac{1}{4}} = 200$

$\Rightarrow x^{\frac{3}{2(\log x+1)}+\frac{1}{4}} = 10$

$\Rightarrow \frac{3}{2(\log x+1)} + \frac{1}{4} = \log_x 10$

$\Rightarrow \frac{3}{2(y+1)} + \frac{1}{4} = \frac{1}{y}$, where $y = \log_{10} x$

$\Rightarrow \frac{6+y+1}{4(y+1)} = \frac{1}{y}$

$\Rightarrow 7\,y + y^2 = 4\,y + 4$

$\Rightarrow y^2 + 3\,y - 4 = 0$

$\Rightarrow (y+4)(y-1) = 0$

$\Rightarrow y = -4, y = 1$

$\Rightarrow \log_{10} x = -4$ or, $\log_{10} x = 1$

$\Rightarrow x = 10^{-4}$ or, $x = 10^1 \Rightarrow x = 10 \quad [\because x > 1]$

351 **(a)**

We have,

$\{(1+x)^6 + (1+x)^7 + \cdots + (1+x)^{15}\}$

$= (1+x)^6 \left\{\frac{1-(1+x)^{10}}{1-(1+x)}\right\}$

$= (1+x)^6 \left\{\frac{1-(1+x)^{10}}{-x}\right\}$

$= \frac{1}{x}\{(1+x)^{16} - (1+x)^6\}$

$\therefore$ Coefficient of x^6 in $\{(1+x)^6 + (1+x)^7 + \cdots + (1+x)^{15}\}$

$=$ Coeff. of x^7 in $\{(1+x)^{16} - (1+x)^6\} = {}^{16}C_7 = {}^{16}C_9$

352 **(d)**

Let $S = 1 + \frac{1}{5} + \frac{1\cdot 3}{5\cdot 10} + \frac{1\cdot 3\cdot 5}{5\cdot 10\cdot 15} + \ldots$

On comparing with

$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$, we get

$\Rightarrow nx = \frac{1}{5}$ and $\frac{n(n-1)x^2}{2!} = \frac{1\cdot 3}{5\cdot 10}$

$\Rightarrow \quad n = -\frac{1}{2}$

and $\quad x = -\frac{2}{5}$

$\therefore$ sum $= \left(1-\frac{2}{5}\right)^{-1/2} = \left(\frac{3}{5}\right)^{-1/2} = \sqrt{\frac{5}{3}}$

353 **(d)**

$A^2 = \begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$

$A^3 = \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\3 & 1\end{bmatrix}$

… … … … … …

… … … … … …

$A^n = \begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}$ Can be verified by induction. Now, taking option

(b) $\begin{bmatrix}1 & 0\\n & 1\end{bmatrix} = \begin{bmatrix}n & 0\\n & n\end{bmatrix} + \begin{bmatrix}n-1 & 0\\0 & n-1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}1 & 0\\n & 1\end{bmatrix} \neq \begin{bmatrix}2n-1 & 0\\1 & 2n-1\end{bmatrix}$

(d)$nA-(n-1)I = \begin{bmatrix}n & 0\\n & n\end{bmatrix} - \begin{bmatrix}n-1 & 0\\1 & n-1\end{bmatrix}$

$= \begin{bmatrix}1 & 0\\n & 1\end{bmatrix} = A^n$

354 **(c)**

$T_{r+1} = \sqrt{3}\cdot\ {}^{20}C_r\left(\frac{1}{\sqrt{3}}\right)^r$

and $T_r = \sqrt{3}\cdot\ {}^{20}C_{r-1}\left(\frac{1}{\sqrt{3}}\right)^{r-1}$

Now, $\frac{T_{r+1}}{T_r} = \frac{20-r+1}{r}\left(\frac{1}{\sqrt{3}}\right)$

Since, $T_{r+1} \geq T_r \Rightarrow 20-r+1 \geq \sqrt{3}r$

$\Rightarrow r \leq \frac{21}{\sqrt{3}+1} = \frac{21}{2\cdot 73} \Rightarrow r \leq 7.692$

$\Rightarrow r = 7$

$\therefore$ The greatest term is

$$T_3 = \sqrt{3}\cdot\ {}^{20}C_7\left(\frac{1}{\sqrt{3}}\right)^7 = \frac{25840}{9}$$

355 **(c)**

We have, $\left(1+\frac{C_1}{C_0}\right)\left(1+\frac{C_2}{C_1}\right)\dots\left(1+\frac{C_n}{C_{n-1}}\right)$

$= \left(1+\frac{n}{1}\right)\left(1+\frac{\frac{n(n-1)}{2!}}{n}\right)\dots\left(1+\frac{1}{n}\right)$

$= \frac{(1+n)}{1}\cdot\frac{(1+n)}{2}\cdot\frac{(1+n)}{3}\dots\frac{(1+n)}{n} = \frac{(n+1)^n}{n!}$

356 **(c)**

Given expression

$= 2[\ {}^5C_0x^5 +\ {}^5C_2x^3(x^3-1) +\ {}^5C_4x(x^3-1)^2]$

$= 2[x^5 + 10\,x^3(x^3-1) + 5\,x(x^3-1)^2]$

$= 5\,x^7 + 10\,x^6 + x^5 - 10\,x^4 - 10x^3 + 5x,$

Which is a polynomial of degree 7

357 **(a)**

The sum of the coefficients is obtained by putting $x = 1$ in $(1+x-3\,x^2)^{2143}$

$\therefore$ required sum $= (1+1-3)^{2143} = -1$

358 **(d)**

We have, $C_0^2 - 2C_1^2 + 3C_2^2 - \dots + (-1)^n(n+1)C_n^2$

$= [C_0^2 - C_1^2 + C_2^2 - \dots + (-1)^nC_n^2]$

$- [C_1^2 - 2C_2^2$

$+ 3C_3^2 - \dots + (-1)^n nC_n^2]$

$= (-1)^{n/2}\frac{n!}{\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}\cdot -(-1)^{n/2-1}\cdot\frac{1}{2}n\frac{n!}{\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}$

$= (-1)^{n/2}\cdot\frac{n!}{\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}\cdot\left(1+\frac{n}{2}\right)$

Therefore, the value of the given expression is

$\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!} \times (-1)^{n/2}\cdot\frac{(n)!}{\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}\left(1+\frac{n}{2}\right)$

$= (-1)^{n/2}(2+n)$

359 **(a)**

We have, $(1.002)^{12}$ or it can be rewritten as $(1+0.002)^{12}$

$\Rightarrow$

$(1.002)^{12} = 1 +\ {}^{12}C_1(0.002) +\ {}^{12}C_2(0.002)^2 +\ {}^{12}C_3(0.002)^3 + \dots$

We want the answer upto 4 decimal places and as such, we have left further expansion.

$\therefore\ (1.002)^{12} = 1 + 12(0.002) + \frac{12\cdot 11}{1\cdot 2}(0.002)^2$

$+ \frac{12\cdot 11\cdot 10}{1\cdot 2\cdot 3}(0.002)^3 + \dots$

$= 1 + 0.024 + 2.64\times 10^{-4} + 1.76\times 10^{-6} + \dots$

$= 1.0242$

360 **(c)**

The general term in the expansion of $\left(\frac{3}{2}x^2 -\right.$

*13x9*is

$T_{r+1} =\ {}^9C_r\left(\frac{3}{2}x^2\right)^{9-r}\left(-\frac{1}{3x}\right)^r$

$=\ {}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{18-3r}$ …(i)

Now, the coefficients of the terms x^0, x^{-1} and x^{-3} in $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is

For x^0, $18-3r = 0 \Rightarrow r = 6$

For x^{-1}, there exists no integer value of r

For x^{-3}, $18-3r = -3 \Rightarrow r = 7$

Now, the coefficient of the term independent of x in the expansion of $(1+x+2x^3)\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$

$= 1.\ {}^9C_6(-1)^6\left(\frac{3}{2}\right)^{9-6}\left(\frac{1}{3}\right)^6 + 0$

$+ 2.\ {}^9C_7(-1)^7\left(\frac{3}{2}\right)^{9-7}\left(\frac{1}{3}\right)^7$

$= \frac{9.8.7}{1.2.3}\cdot\frac{3^3}{2^3}\cdot\frac{1}{3^6} + 2.\frac{9.8}{1.2}(-1)\frac{3^2}{2^2}\cdot\frac{1}{3^7}$

$= \frac{7}{18} - \frac{2}{27} = \frac{17}{54}$

361 **(c)**

We can write,

$aC_0 - (a+d)C_1 + (a+2d)C_2 - \ldots$ upto $(n+1)$ terms

$= a(C_0 - C_1 + C_2 - \ldots) + d(-C_1 + 2C_2 - 3C_3 + \ldots)$ ...(i)

We know,

$(1-x)^n = C_0 - C_1x + C_2x^2 - \ldots + (-1)^n C_n x^n$ ...(ii)

On differentiating Eq. (ii) w.r.t. x, we get

$-n(1-x)^{n-1} = -C_1 + 2C_2x - \ldots (-1)^n C_n n x^{n-1}$ ...(iii)

On putting $x = 1$ in Eqs. (ii) and (iii), we get

$C_0 - C_1 + C_2 - \ldots + (-1)^n C_n = 0$ ...(iv)

and $-C_1 + 2C_2 - \ldots + (-1)^n nC_n = 0$ ...(v)

From Eq. (i),

$aC_0 - (a+d)C_1 + (a+2d)C_2 - \ldots$ upto $(n+1)$ terms

$= a \cdot 0 + d \cdot 0 = 0$ [from Eqs. (iv) and (v)]

362 **(c)**

Let $P(n) = 2^{3n} - 7n - 1$

$\therefore \quad P(1) = 0, P(2) = 49$

$P(1)$ and $P(2)$ are divisible by 49.

Let $P(k) \equiv 2^{3k} - 7k - 1 = 49I$

$\therefore P(k+1) \equiv 2^{3k+3} - 7k - 8$

$= 8(49I + 7k + 1) - 7k - 8$

$= 49(8I) + 49k = 49I_1$

Alternate

$P(n) = (1+7)^n - 7n - 1$

$= 1 + 7n + 7^2 \frac{n(n-1)}{2!} + \cdots - 7n - 1$

$= 7^2 \left(\frac{n(n-1)}{2!} + \cdots \right)$

363 **(d)**

Let $P(n) = 5^{2n+2} - 24n - 25$

For $\quad n = 1$

$P(1) = 5^4 - 24 - 25 = 576$

$P(2) = 5^6 - 24(2) - 25 = 15552$

$= 576 \times 27$

Here, we see that $P(n)$ is divisible by 576

364 **(c)**

Let $b = \sum_{r=0}^{n} \frac{r}{^nC_r} = \sum_{r=0}^{n} \frac{n-(n-r)}{^nC_r}$

$= n \sum_{r=0}^{n} \frac{1}{^nC_r} - \sum_{r=0}^{n} \frac{n-r}{^nC_r}$

$= na_n - \sum_{r=0}^{n} \frac{n-r}{^nC_{n-r}} \quad (\because\ ^nC_r = {}^nC_{n-1})$

$= na_n - b$

$\Rightarrow 2b = na_n \Rightarrow b = \frac{n}{2} a_n$

365 **(d)**

We have,

$\frac{1}{\sqrt{4x+1}} \left\{ \left(1 + \frac{\sqrt{4x+1}}{2}\right)^2 - \left(1 - \frac{\sqrt{4x+1}}{2}\right)^2 \right\}$

$= \frac{1}{2^7 \sqrt{4x+1}} \left[2\left\{ {}^7C_1\sqrt{4x+1} + {}^7C_3\left(\sqrt{4x+1}\right)^3 + {}^7C_5\left(\sqrt{4x+1}\right)^5 + {}^7C_7\left(\sqrt{4x+1}\right)^7 \right\} \right]$

$= \frac{1}{2^6} \{ {}^7C_1 + {}^7C_3(4x+1) + {}^7C_5(4x+1)^2 + {}^7C_7(4x+1)^3 \}$

Clearly, it is a polynomial of degree 3

366 **(a)**

In the expansion of $\left(ax^2 + \frac{1}{bx}\right)^{11}$,

$T_{r+1} = {}^{11}C_r (ax^2)^{11-r} \left(\frac{1}{bx}\right)^r$

$= {}^{11}C_r \frac{a^{11-r}}{b^r} \cdot x^{22-3r}$

For coefficient of x^7, put $22 - 3r = 7$

$\Longrightarrow \quad r = 5$

$\therefore \quad T_6 = {}^{11}C_5 \frac{a^6}{b^5} \cdot x^7$

$\therefore$ Coefficient of x^7 in the expansion of $\left(ax^2 + \frac{1}{bx}\right)^{11}$ is

$11_{C_5} \frac{a^6}{b^5}$

Similarly, coefficient of x^{-7} in the expansion of $\left(ax + \frac{1}{bx}\right)^{11}$ is

${}^{11}C_5 \frac{a^5}{b^6}$

Now, $\quad {}^{11}C_5 \frac{a^6}{b^5} = {}^{11}C_6 \frac{a^5}{b^6}$

$\Longrightarrow \quad ab = 1$

367 **(b)**

Given expansion is $\left(x + \frac{1}{2x}\right)^{2n}$

$\therefore$ Middle term $= {}^{2n}C_n (x)^n \left(\frac{1}{2x}\right)^n$

$= \frac{2n!}{n!\,n!\,2^n} = \frac{1 \cdot 3 \cdot 5 \ldots (2n-1)}{n!}$

368 **(c)**

$(1+x)^{21} + (1+x)^{22} + \ldots + (1+x)^{30}$

$= (1+x)^{21}[1 + (1+x)^1 + \ldots + (1+x)^9]$

$= (1+x)^{21} \left[\frac{(1+x)^{10} - 1}{(1+x) - 1} \right]$

$= \frac{1}{x}[(1+x)^{31} - (1+x)^{21}]$

$\therefore$ Coefficient of x^5 in the given expression

$=$ Coefficient of x^5 in $\frac{1}{x}[(1+x)^{31} - (1+x)^{21}]$

$=$ Coefficient of x^6 in $[(1+x)^{31} - (1+x)^{21}]$

$= {}^{31}C_6 - {}^{21}C_6$

369 **(a)**

We have,

$$T_{r+1} = \frac{\frac{7}{2}\left(\frac{7}{2}-1\right)\left(\frac{7}{2}-2\right)\dots\left(\frac{7}{2}-r+1\right)x^r}{r\,!}$$

This will be the first negative term if

$\frac{7}{2} - r + 1 < 0 \Rightarrow r > \frac{9}{2}$

Hence, $r = 5$

370 **(b)**

According to question, coefficient of x^r =coefficient of x^{r+1}

$\Longrightarrow {}^{21}C_r = {}^{21}C_{r+1}$ ...(i)

But ${}^{21}C_r = {}^{21}C_{21-r}$ ...(ii)

On comparing Eqs. (i) and (ii), we get

$r + 1 = 21 - r \Longrightarrow r = \frac{21-1}{2} = 10$

371 **(a)**

Let $(r+1)^{th}$ term be the greatest term

We have,

$T_{r+1} = \sqrt{3}\cdot {}^{20}C_r\left(\frac{1}{\sqrt{3}}\right)^r$ and T_r

$= \sqrt{3}\ {}^{20}C_{r-1}\left(\frac{1}{\sqrt{3}}\right)^{r-1}$

$\therefore \frac{T_{r+1}}{T_r} = \frac{20-r+1}{r}\left(\frac{1}{\sqrt{3}}\right)$

$\therefore T_{r+1} \geq T_r$

$\Rightarrow 20 - r + 1 \geq \sqrt{3}\,r$

$\Rightarrow 21 \geq r(\sqrt{3}+1)$

$\Rightarrow r \leq \frac{21}{\sqrt{3}+1} \Rightarrow r \leq 7.686 \Rightarrow r = 7$

Hence, greatest term $T_8 = \sqrt{3}\,{}^{20}C_7\left(\frac{1}{\sqrt{3}}\right)^7 = \frac{25840}{9}$

372 **(b)**

We have,

$(1 + x + x^2 + \cdots)^{-n} = [(1-x)^{-1}]^{-n} = (1-x)^n$

$\therefore$ Coefficient of $x^n = (-1)^n\ {}^nC_n = (-1)^n$

373 **(a)**

We have, $(1+t^2)^{12}(1+t^{12})(1+t^{24})$

$= 1 +^{12} C_1t^2 + {}^{12}C_2t^4 + {}^{12}C_3t^6 + \dots + {}^{12}C_6t^{12} + \dots)(1 + t^{12} + t^{24} + t^{36})$

$\therefore$ Coefficient of t^{24} in $(1+t^2)^{12}(1+t^{12})(1+$ *t24=*${}^{12}C6+2$

374 **(d)**

$T_r = {}^{14}C_{r-1}x^{r-1};\ T_{r+1} = {}^{14}C_rx^r;\ T_{r+2} = {}^{14}C_{r+1}x^{r+1}$

Since, these terms are in AP

$\therefore\ 2T_{r+1} = T_r + T_{r+2}$

$\Rightarrow 2\,{}^{14}C_r = {}^{14}C_{r-1} + {}^{14}C_{r+1}$ ...(i)

$\Rightarrow 2\cdot\frac{14!}{r!\,(14-r)!} = \frac{14!}{(r-1)!\,(15-r)!} + \frac{14!}{(r+1)!\,(13-r)!}$

$\Rightarrow \frac{2}{r\cdot(r-1)!\,(14-r)\cdot(13-r)!}$

$= \frac{1}{(r-1)!\cdot(15-r)\cdot(14-r)\cdot(13-r)!}$

$+ \frac{1}{(r+1)r(r-1)!\,(13-r)!}$

$\Rightarrow \frac{2}{r(14-r)} = \frac{1}{(15-r)(14-r)} + \frac{1}{(r+1)r}$

$\Rightarrow \frac{1}{r(14-r)} = \frac{1}{(15-r)(14-r)} + \frac{1}{(r+1)r} - \frac{1}{r(14-r)}$

$\Rightarrow \frac{(15-r)-r}{r(15-r)(14-r)} = \frac{(14-r)-(r+1)}{(r+1)r(14-r)}$

$\Rightarrow (15-r) - r = (13-2r)(15-r)$

$\Rightarrow 15r + 15 - 2r^2 - 2r = 195 - 30r - 13r + 2r^2$

$\Rightarrow 4r^2 - 56r + 180 = 0$

$\Rightarrow r^2 - 14r + 45 = 0$

$\Rightarrow (r-5)(r-9) = 0 \Rightarrow r = 5,9$

But 5 is not given.

Hence, $r = 9$

375 **(c)**

We have, $(101)^{50} = (100+1)^{50}$

$= (100)^{50} + {}^{50}C_1(100)^{49} + {}^{50}C_2(100)^{48} + \cdots$ ....(i)

and $(99)^{50} = (100-1)^{50}$

$= (100)^{50} - {}^{50}C_1(100)^{49} + {}^{50}C_2(100)^{48} + \cdots$ ...(ii)

On subtracting Eq. (ii) from Eq. (i), we get

$(101)^{50} - (99)^{50}$

$= 2\{{}^{50}C_1(100)^{49} + {}^{50}C_3(100)^{47} + \cdots\}$

$= 2\times{}^{50}C_1(100)^{49} + \{2\times{}^{50}C_3(100)^{47} + \cdots\}$

$= (100(100)^{49} +$ a positive number$)$

$> (100)^{50}$

$\therefore (101)^{50} > (100)^{50} + (99)^{50}$

376 **(a)**

The general term of the given series is

$T_r = (-1)^r(3+5r)\ {}^nC_r$

$\therefore$ Sum $\sum_{r=0}^{n}(1)^r\,(3+5r)\,{}^nC_r$

$= 3 \sum_{r=0}^{n} (-1)^r\,{}^nC_r + 5\sum_{r=0}^{n}(1)^r\; r\,{}^nC_r$

$= 3(C_0 - C_1 + C_2 - C_3 + C_4 - ... + (-1)^n \cdot C_n)$

$+5(-C_1 + 2C_2 - 3C_3 + 4C_4 - \cdots + (-1)^n \cdot n \cdot C_n)$

$\Longrightarrow S = 0 + 0 = 0$

378 **(d)**

$S = (\alpha \cdot 1 + \beta \cdot 1 + \gamma \cdot 1)^n = (\alpha + \beta - \gamma)^n$

$\therefore \lim_{n\to\infty} \frac{S}{\{S^{1/n}+1\}^n} = \lim_{n\to\infty}\left(\frac{\alpha+\beta-\gamma}{\alpha+\beta-\gamma+1}\right)^n = 0$

$(\because\ \alpha+\beta-\gamma+1 > \alpha+\beta-\gamma)$

379 **(b)**

It is given that the sum of the numerical coefficients in the binomial expansion of

$\left(\frac{1}{x}+2x\right)^n$ is 6561

$\therefore (1+2)^n = 6561$ [Putting $x = 1$]

$\Rightarrow 3^n = 3^8 \Rightarrow n = 8$

The general term in the expansion of $\left(\frac{1}{x}+2x\right)^n$ is given by

$T_{r+1} = {}^nC_r\left(\frac{1}{x}\right)^{n-r}(2x)^r = {}^nC_r 2^r x^{-n+2r}$

$= {}^8C_r 2^r\, x^{2r-8}$

This will be independent of x id $r = 4$

Hence, the constant term = ${}^8C_4 2^4$

381 **(c)**

Multiplying the numerator and denominator by $1-x$, we have $E = \frac{1-x}{(1-x)(1+x)(1+x^2)(1+x^4)...(1+x^{2^m})}$

$= \frac{1-x}{(1-x^2)(1+x^2)(1+x^4)\dots(1-x^{2^m})}$

$= \frac{1-x}{(1-x^4)(1+x^4)\dots(1-x^{2^m})}$

$= \frac{1-x}{\left(1-x^{2^{m+1}}\right)} = (1-x)(1-x^{2^{m+1}})^{-1}$

$= (1-x)(1 + x^{2^{m+1}} + x^{2^{m+2}} + \dots)$

$\therefore$ Coefficient of $x^{2^{m+1}}$ is 1

382 **(b)**

Let $G = \left(7-4\sqrt{3}\right)^n$. Then,

$0 < G < 1$ as $0 < 7-4\sqrt{3} < 1$

Now,

$I + F + G = \left(7+4\sqrt{3}\right)^n + \left(7-4\sqrt{3}\right)^n$

$\Rightarrow I + F + G = 2({}^nC_0\, 7^n + {}^nC_2\, 7^{n-2}\left(4\sqrt{3}\right)^2 + \cdots)$

$\Rightarrow I + F + G =$ an integer

$\Rightarrow F + G = 1$

$\Rightarrow G = 1 - F$

$\therefore (I+F)(1-F) = (I+F)G$

$= \left(7+4\sqrt{3}\right)^n\left(7-4\sqrt{3}\right)^n = 1$

383 **(c)**

Since, number of terms in the expansion of $(1+x)^{24}$ is 25.

Therefore, the middle term is 13th term.

$\therefore$ Required greatest coefficient = ${}^{24}C_{12}$.

384 **(d)**

We have,

$(1+x+x^3+x^4)^{10}$

$= (1+x)^{10}(1+x^3)^{10}$

$= ({}^{10}C_0 + {}^{10}C_1\, x + {}^{10}C_2\, x^2 + {}^{10}C_3\, x^3 + {}^{10}C_4\, x^4 + \cdots + {}^{10}C_{10}\, x^{10}) \times ({}^{10}C_0 + {}^{10}C_1\, x^3 + {}^{10}C_2\, x^6 + \cdots + {}^{10}C_{10}\, x^{30})$

$\therefore$ Coefficient of $x^4 = {}^{10}C_0 \times {}^{10}C_4 + {}^{10}C_1 \times {}^{10}C_1$

$= 310$

385 **(b)**

We have,

$(1+x)^m = 1 + m\,x + \frac{m(m-1)}{2\,!}x^2 + \cdots$

It is given that the third term is $-\frac{1}{8}\,x^2$

$\therefore \frac{m(m-1)}{2}x^2 = -\frac{1}{8}x^2$

$\Rightarrow 4\,m^2 - 4\,m = -1 \Rightarrow (2\,m-1)^2 = 0 \Rightarrow m = \frac{1}{2}$

386 **(a)**

We have,

$(C_0 + C_1)(C_1 + C_2)(C_2 + C_3)\dots(C_{n-1} + C_n)$

$= C_1\,C_2 \dots C_{n-1}C_n\left(1+\frac{C_0}{C_1}\right)\left(1+\frac{C_1}{C_2}\right)\left(1+\frac{C_2}{C_3}\right)\dots\left(1+\frac{C_{n-1}}{C_n}\right)$

$= C_1\,C_2 \dots C_{n-1}\,C_n\left(1+\frac{C_0}{C_1}\right)\left(1+\frac{2}{n-1}\right)\left(1+\frac{3}{n-2}\right)\dots\left(1+\frac{n}{1}\right)$

$= C_1\,C_2 \dots C_{n-1}\,C_n\frac{(n+1)^n}{n\,!}$

$\therefore k = C_1\,C_2\,C_3 \dots C_{n-1}\,C_n = C_0C_1C_2 \dots C_{n-1}\,C_n$

387 **(c)**

We have,

$\sum_{m=0}^{100} {}^{100}C_m\,(x-3)^{100-2}2^m = [(x-3)+2]^{100}$

$= (1-x)^{100}$

$\therefore$ Coefficient of $x^{53} = {}^{100}C_{53}(-1)^{53} = -{}^{100}C_{53}$

388 **(d)**

Given expansion is $\left(x-\frac{1}{2x}\right)^n$

$\therefore\ T_3 = {}^nC_2(x)^{n-2}\left(-\frac{1}{2x}\right)^2$

and $T_4 = {}^nC_3(x)^{n-3}\left(-\frac{1}{2x}\right)^3$

But according to the given condition,

$\frac{T_3}{T_4} = -\frac{n(n-1)\times3\times2\times1\times8}{n(n-1)(n-2)\times2\times1\times4\times} = \frac{1}{2}$ (given)

$\Rightarrow\ -n+2 = 12 \Rightarrow n = -10$

389 **(b)**

We have, $(1+x+x^2+\dots)^{-n} = \left(\frac{1}{1-x}\right)^{-n} = (1-x)^n$

$\therefore$ The coefficient of x^n is $(-1)^n$

390 **(b)**

Here, $T_4 = {}^nC_3(a)^{n-3}(-2b)^3$

and $T_5 = {}^nC_4(a)^{n-4}(-2b)^4$

Given, $T_4+T_5 = 0$

$\Rightarrow\ {}^nC_3(a)^{n-3}(-2b)^3 + {}^nC_4(a)^{n-4}(-2b)^4 = 0$

$\Rightarrow\ (a)^{n-4}(-2b)^3[a\,{}^nC_3 + {}^nC_4(-2b)] = 0$

$\Rightarrow\ \frac{a}{b} = \frac{2\,{}^nC_4}{{}^nC_3}$

$= \frac{2.n(n-1)(n-2)(n-3)}{4.3.2.1}\times\frac{3.2.1}{n(n-1)(n-2)}$

$= \frac{n-3}{2}$

391 **(d)**

The general term in the expansion of $(2-x+$ *3x26* is given by

$\frac{6!}{r!\,s!\,t!}2^r(-x)^s(3x^2)^t$, where $r+s+t=6$

$= \frac{6!}{r!\,s!\,t!}2^r\times(-1)^s\times3^t\times x^{s+2t}$, where $r+s+t = 6$

For the coefficient of x^5, we must have $s+2t=5$

But, $r+s+t=6$

$\therefore s = 5-2t$ and $r = 1+t$, where $0\le r,s,t\le 6$

Now,

$t=0 \Rightarrow r=1, s=5$

$t=1 \Rightarrow r=2, s=3$

$t=2 \Rightarrow r=3, s=1$

Thus, there are three terms containing x^5 and hence

Coefficient of x^5

$= \frac{6!}{1!\,5!\,0!}\times2^1\times(-1)^5\times3$

$+\frac{6!}{2!\,3!\,1!}\times2^2\times(-1)^3\times3^1+\frac{6!}{3!\,1!\,2!}\times2^3\times(-1)^1\times3^2$

$= -12-720-4320 = -5052$

392 **(b)**

We have,

$\left(x^2-2+\frac{1}{x^2}\right)^n = \left(x-\frac{1}{x}\right)^{2n}$

$\therefore T_{r+1} = {}^{2n}C_r\,x^{2n-r}\left(-\frac{1}{x}\right)^r = {}^{2n}C_r\,x^{2n-2r}(-1)^r$

This term will be independent of x if $2n-2r = 0$ i.e. $r=n$

$\therefore$ Number of terms dependent on $x = (2n+1)-1 = 2n$

393 **(b)**

We have,

Coeff. of x^7 = Coeff. of x^8

$\Rightarrow\ {}^nC_7\times2^{n-7}\times\left(\frac{1}{3}\right)^7 = {}^nC_8\times2^{n-8}\times\left(\frac{1}{3}\right)^8$

$\Rightarrow 6({}^nC_7) = {}^nC_8 \Rightarrow 48 = n-7 \Rightarrow n = 55$

394 **(c)**

We have,

$\therefore T_{r+1} = {}^{10}C_r\left(\sqrt{\frac{x}{3}}\right)^{10-r}\left(\frac{3}{2x^2}\right)^r$

$\Rightarrow T_{r+1} = {}^{10}C_r\left(\frac{1}{3}\right)^{5-\frac{r}{2}}\left(\frac{3}{2}\right)^r x^{5-\frac{5r}{2}}$

For this term to be independent of x, we must have

$5-\frac{5r}{2} = 0 \Rightarrow r = 2$, which is an integer

Hence, third term is independent of x

Also, $T_3 = {}^{10}C_2\left(\frac{1}{3}\right)^4\left(\frac{3}{2}\right)^2 = 45\times\frac{1}{81}\times\frac{9}{4} = \frac{5}{4}$

395 **(d)**

Suppose x^{-4} occurs in $(r+1)^{\text{th}}$ term

We have,

$T_{r+1} = {}^{10}C_r\left(\frac{3}{2}\right)^{10-r}\left(\frac{-3}{x^2}\right)^r$

$= {}^{10}C_r\left(\frac{3}{2}\right)^{10-r}(-3)^r\,x^{-2r}$

This will contain x^4, if $-2r = -4 \Rightarrow r = 2$

$\therefore$ Coefficient of $x^{-4} = {}^{10}C_2\left(\frac{3}{2}\right)^{10-2}(-3)^2$

$= \frac{3^{12}\times5}{2^8}$

396 **(b)**

By hypothesis, we have

${}^{18}C_{2r+3} = {}^{18}C_{r-3} \Rightarrow 2r+3 = r-3 \Rightarrow r = 6$

398 **(d)**

$49^n+16n-1 = (1+48)^n+16nn-1$

$= 1+n_{C_1}(48)+n_{C_2}(48)^2+\dots+n_{C_n}(48)^n+16n-1$

$=$

$(48n+16n)+n_{C_2}(48)^2+$
$n_{C_3}(48)^3+...+n_{C_n}(48)^n$
$64n+8^2(n_{C_2}\cdot 6^2+n_{C_3}\cdot 6^3\cdot 8+n_{C_4}\cdot 6^4\cdot$
$8^2+......+n_{C_n}\cdot 6^n\cdot 8^{n-2})$
Hence, $49^n+16n-1$ is divisible by 64
Alternate Let $P(n)=49^n+16n-1$
For $n=1$
$P(1)=49+16-1=64$

399 **(d)**
We have

$$\sum_{r=1}^{n} r^2\cdot {}^nC_r = n(n-1)2^{n-2}+n\cdot 2^{n-1} \quad ...(i)$$

and $\sum_{r=1}^{n}(-1)^{r-1}r^2\,{}^nC_r=0$ ...(ii)
On adding Eqs. (i) and (ii), we get

$$2[1\,{}^2C_1+3^2C_3+5^2C_5+\cdots] = n(n-1)2^{n-2}+n\cdot 2^{n-1}$$

$$\Rightarrow 1^2C_1+3^2C_3+5^2C_5+\ldots = n(n-1)2^{n-3}+n\cdot 2^{n-2}$$

400 **(d)**
Since, $(3+ax)^9={}^9C_0 3^9+{}^9C_1 3^8(ax)+{}^9C_2 3^7(ax)^2+{}^9C_3 3^6(ax)^3+\cdots$
Since, coefficient of x^2 = coefficient of x^3

$$\Rightarrow \quad {}^9C_2 3^7a^2={}^9C_3 3^6a^3$$

$$\Rightarrow \quad \frac{{}^9C_2}{{}^9C_3}\cdot 3=a$$

$$\Rightarrow \quad \frac{\frac{9\times 8}{2\times 1}}{\frac{9\times 8\times 7}{3\times 2}}\times 3=a$$

$$\Rightarrow \quad a=\frac{9}{7}$$

401 **(a)**
The coefficient of x in the expansion of
$(1+x)(1+2x)(1+3x)...(1+100x)$
$=1+2+3+....+100$

$$\frac{100(100+1)}{2}=50\times 101=5050$$

402 **(a)**
Suppose x^5 occurs in $(r+1)^{\text{th}}$ term of the expansion of $\left(x^2+\frac{a}{x^3}\right)^{10}$
We have,

$$T_{r+1}={}^{10}C_r\,(x^2)^{10-r}\left(\frac{a}{x^3}\right)^r={}^{10}C_r\,x^{20-5r}\,a^r$$

$\therefore 20-5r=5\Rightarrow r=3$
$\therefore$ Coefficient of $x^5={}^{10}C_3\,a^3$
Similarly, Coefficient of $x^{15}={}^{10}C_1\,a^1$
Now,
Coeff. of x^5 = Coeff. of x^{15}
$\Rightarrow {}^{10}C_3\,a^3={}^{10}C_1\,a$

$$\Rightarrow 120\,a^3=10\,a\Rightarrow a^2=\frac{1}{12}\Rightarrow a=\frac{1}{2\sqrt{3}}$$

403 **(a)**
$\because$ 10th term in the expansion of $(2-3x^3)^{20}$ is ${}^{20}C_9 2^9(-1)^9(2)^{11}(3)^9x^{27}$ and 11th term is ${}^{20}C_{10}2^{10}3^{10}x^{30}$

$$\therefore \quad \frac{{}^{20}C_9(-1)^9(2)^{11}(3)^9x^{27}}{{}^{20}C_{10}\cdot 2^{10}\cdot 3^{10}\cdot x^{30}}=\frac{45}{22}$$

$$\Rightarrow \quad -\frac{10}{11}\cdot\frac{2}{3x^3}=\frac{45}{22}$$

$$\Rightarrow \quad x^3=-\frac{8}{27}\Rightarrow x=-\frac{2}{3}$$