Single Correct Answer Type

1. Area bounded by the curve $y=(x-1)(x-2)(x-3)$ and x-axis lying between the ordinates $x=0$ and $x=3$ is equal to
a) 9/4 b) 11/4 c) 11/2 d) 7/4
2. The area of the region bounded by the curves $y=e^x, y=\log_e x$ and lines $x=1, x=2$ is
a) $(e-1)^2$ b) e^2-e+1 c) $e^2-e+1-2\log_e 2$ d) $e^2+e-2\log_e 2$
3. The value of k for which the area of the figure bounded by the curve $y=8x^2-x^5$, the straight line $x=1$ and $x=k$ and the x-axis is equal to 16/3
a) 2 b) $\sqrt[3]{8-\sqrt{17}}$ c) 3 d) -1
4. The area bounded by the curve $y=x$, x-axis and ordinates $x=-1$ to $x=2$, is
a) 0 sq unit b) 1/2 sq unit c) 3/2 sq unit d) 5/2 sq unit
5. The area (in square unit) of the region bounded by the curves $2x=y^2-1$ and $x=0$ is
a) $\frac{1}{3}$ sq unit b) $\frac{2}{3}$ sq unit c) 1 sq unit d) 2 sq units
6. The area bounded by the curve $y=4x-x^2$ and the x-axis, is
a) $\frac{30}{7}$ sq. units b) $\frac{31}{7}$ sq. units c) $\frac{32}{3}$ sq. units d) $\frac{34}{3}$ sq. units
7. The volume of the solid generated by revolving the region bounded by $y=x^2+1$ and $y=2x+1$ about x-axis is
a) $\frac{104\pi}{15}$ cu units b) $\frac{42\pi}{15}$ cu units c) $\frac{52\pi}{15}$ cu units d) None of these
8. The area bounded by the curves $|x|+|y|\geq 1$ and $x^2+y^2\leq 1$ is
a) 2 sq unit b) π sq unit c) $(\pi-2)$ sq unit d) $(\pi+2)$ sq unit
9. The area bounded by the curves $y=\cos x$ and $y=\sin x$ between the ordinance $x=0$ and $x=\frac{3\pi}{2}$ is
a) $(4\sqrt{2}-2)$sq units b) $(4\sqrt{2}+2)$sq units c) $(4\sqrt{2}-1)$sq units d) $(4\sqrt{2}+1)$sq units
10. Area bounded by the curves $y=\left[\frac{x^2}{64}+2\right]$, $y=x-1$ and $x=0$ above x-axis is ([.] denotes the greatest integer function)
a) 2 sq unit b) 3 sq unit c) 4 sq unit d) None of these
11. The area bounded by the curve $y^2=8x$ and $x^2=8y$, is
a) $\frac{16}{3}$ sq. units b) $\frac{3}{16}$ sq. units c) $\frac{14}{3}$ sq. units d) $\frac{3}{14}$ sq. units
12. The area enclosed between the curve $y=\log_e(x+e)$ and the coordinate axis is
a) 4 sq units b) 3 sq units c) 2 sq units d) 1 sq unit
13. If area bounded by the curves $y^2=4ax$ and $y=mx$ is $a^2/3$, then the value of m is
a) 2 b) -2 c) 1/2 d) 1
14. The area of the figure bounded by the curves $y=|x-1|$ and $y=3-|x|$ is
a) 2 b) 3 c) 4 d) 1
15. The area bounded by the curves $y=\sqrt{5-x^2}$ and $y=|x-1|$ is
a) $\left(\frac{5\pi}{4}-2\right)$ sq units b) $\frac{(5\pi-2)}{4}$ sq units c) $\frac{(5\pi-2)}{2}$ sq units d) $\left(\frac{\pi}{2}-5\right)$ sq units
16. Area bounded by the curve $xy^2=a^2(a-x)$ and y-axis, is
a) $\pi a^2/2$ b) πa^2 c) $3\pi a^2$ d) $2\pi a^2$
17. The area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, is

a) πab b) $\frac{\pi}{4}(a^2+b^2)$ c) $\pi(a+b)$ d) $\pi a^2 b^2$

18. The area bounded by the curve $y = x^6(\pi - x)^8$ is

a) $\frac{\pi^{15}\times 3!\times 4!}{15!}$ sq unit b) $\frac{\pi^{6}\times 6!\times 8!}{15!}$ sq unit c) $\frac{\pi^{15}\times 6!\times 8!}{15!}$ sq unit d) $\frac{\pi^{8}\times 6!\times 8!}{15!}$ sq unit

19. The part of circle $x^2 + y^2 = 9$ in between $y = 0$ and $y = 2$ is revolved about y-axis. The volume of generating solid will be

a) $\frac{46}{3}\pi$ cu units b) 12 π cu jnits c) 16 π cu units d) 28 π cu units

20. The area of the region by curves $y = x\log x$ and $y = 2x - 2x^2$ is

a) $\frac{1}{2}$ sq units b) $\frac{3}{12}$ sq units c) $\frac{7}{12}$ sq units d) None of these

21. The area of the region formed by $x^2 + y^2 - 6x - 4y + 12 \le 0, y \le x$ and $x \le 5/2$ is

a) $\frac{\pi}{6} - \frac{\sqrt{3}+1}{8}$ b) $\frac{\pi}{6} + \frac{\sqrt{3}+1}{8}$ c) $\frac{\pi}{6} - \frac{\sqrt{3}-1}{8}$ d) None of these

22. Area bounded by the curve $y = \log_e x, x = 0, y \le 0$ and x-axis is

a) 1 sq unit b) 1/2 sq unit c) 2 sq unit d) None of these

23. Area bounded by the curves $y = |x-1|, y = 0$ and $|x| = 2$, is

a) 4 b) 5 c) 3 d) 6

24. The area included between the parabolas $y^2 = 4x$ and $x^2 = 4y$ is (in square units)

a) 4/3 b) 1/3 c) 16/3 d) 8/3

25. The area of region bounded by the curves $y = |x-1|$ and $y = 3 - |x|$ is

a) 2 sq units b) 3 sq units c) 4 sq units d) 6 sq units

26. The area bounded by the curves $y = x^3, y = x^2$ and the ordinates $x = 1, x = 2$ is

a) $\frac{17}{12}$ b) $\frac{12}{13}$ c) $\frac{2}{7}$ d) $\frac{7}{2}$

27. The area bounded by the graph $y = |[x-3]|$, the x-axis and the lines $x = -2$ and $x = 3$ is([.] denotes the greatest integer function)

a) 7 sq unit b) 15 sq unit c) 21 sq unit d) 28 sq unit

28. Area bounded by the curve $y^2 = 16x$ and line $y = mx$ is $\frac{2}{3}$ then m is equal to

a) 3 b) 4 c) 1 d) 2

29. The area enclosed by $y = 3x - 5, y = 0, x = 3$ and $x = 5$ is

a) 12 sq units b) 13 sq unit c) $13\frac{1}{2}$ sq unit d) 14 sq unit

30. The area of the region bounded by the curves $y = |x-2|, x = 1, x = 3$ and the x-axis is

a) 1 b) 2 c) 3 d) 4

31. The area common to the circle $x^2 + y^2 = 64$ and the parabola $y^2 = 4x$ is

a) $\frac{16}{3}(4\pi + \sqrt{3})$ sq unit b) $\frac{16}{3}(8\pi - \sqrt{3})$ sq unit c) $\frac{16}{3}(4\pi - \sqrt{3})$ sq unit d) None of these

32. The ratio of the areas between the curves $y = \cos x$ and $y = \cos 2x$ and x-axis from $x = 0$ to $x = \pi/3$ is

a) 1 : 2 b) 2 : 1 c) $\sqrt{3} : 1$ d) None of these

33. The slope of tangent to a curve $y = f(x)$ at $(x, f(x))$ is $2x + 1$. If the curve passes through the point (1, 2), then the area of the region bounded by the curve, the x-axis and the line $x = 1$ is

a) $\frac{5}{6}$ sq unit b) $\frac{6}{5}$ sq unit c) $\frac{1}{6}$ sq unit d) 6 sq unit

34. The area bounded by the curves $y = |x| - 1$ and $y = -|x| + 1$ is

a) 1 sq unit b) 2 sq unit c) $2\sqrt{2}$ sq unit d) 4 sq unit

35. The area of smaller portion bounded by $|y| = -x + 1$ and $y^2 = 4x$ is

a) 1 sq unit b) 2 sq unit c) 3 sq unit d) None of these

36. If A_1 is the area enclosed by the curve $xy = 1$, x-axis and the ordinates $x = 1, x = 2$; and A_2 is the area enclosed by the curve $xy = 1$, x-axis and the ordinates $x = 2, x = 4$, then

a) $A_1 = 2A_2$ b) $A_2 = 2A_1$ c) $A_2 = 3A_1$ d) $A_1 = A_2$

37. The area of the region bounded by the parabola $(y-2)^2 = x-1$, the tangent to the parabola at the point (2,3) and the x-axis is
a) 6 sq units b) 9 sq units c) 12 sq units d) 3 sq units

38. The area of the region $\{(x,y): x^2+y^2 \le 1 \le x+y\}$, is
a) $\frac{\pi}{5}$ b) $\frac{\pi}{4}$ c) $\frac{\pi^2}{3}$ d) $\frac{\pi}{4}-\frac{1}{2}$

39. The length of the parabola $y^2 = 12x$ cut off by the latusretum is
a) $6[\sqrt{2}+\log(1+\sqrt{2})]$ b) $3[\sqrt{2}+\log(1+\sqrt{2})]$ c) $6[\sqrt{2}-\log(1+\sqrt{2})]$ d) $3[\sqrt{2}-\log(1+\sqrt{2})]$

40. The area bounded by $y=\sin^{-1} x = \frac{1}{\sqrt{2}}$ and x-axis is
a) $\left(\frac{1}{\sqrt{2}}+1\right)$ sq unit
b) $\left(1-\frac{1}{\sqrt{2}}\right)$ sq unit
c) $\frac{\pi}{4\sqrt{2}}$ sq unit
d) $\left(\frac{\pi}{4\sqrt{2}}+\frac{1}{\sqrt{2}}-1\right)$ sq unit

41. The area of the smaller segment cut off from the circle $x^2+y^2=9$ by $x=1$ is
a) $\frac{1}{2}(9\sec^{-1}3-\sqrt{8})$ sq unit
b) $(9\sec^{-1}(3)-\sqrt{8})$ sq unit
c) $(\sqrt{8}-9\sec^{-1}3)$ sq unit
d) None of these

42. The area of the region bounded by $1-y^2=|x|$ and $|x|+|y|=1$ is
a) 1/3 sq unit b) 2/3 sq unit c) 4/3 sq unit d) 1 sq unit

43. The area between the parabola $y^2=4ax$ and the line $y=mx$ in square units is
a) $\frac{5a^2}{3m}$ b) $\frac{8a^2}{3m^3}$ c) $\frac{7a^2}{4m^2}$ d) $\frac{3a^2}{5m}$

44. The area bounded by the curves $y=\sin x$ between the ordinates $x=0, x=\pi$ and the x-axis, is
a) 2 sq. units b) 4 sq. units c) 3 sq. units d) 1 sq. units

45. The area bounded by $|x-1| \le 2$ and $x^2-y^2=1$, is
a) $6\sqrt{2}+\frac{1}{2}\log|3+2\sqrt{2}|$
b) $6\sqrt{2}+\frac{1}{2}\log|3-2\sqrt{2}|$
c) $6\sqrt{2}-\log|3+2\sqrt{2}|$
d) None of these

46. The area bounded by $y=\log x$, x-axis and ordinates $x=1, x=2$ is
a) $\frac{1}{2}(\log 2)^2$ b) $\log(2/e)$ c) $\log(4/e)$ d) $\log 4$

47. The area bounded by $y=x^2+1$ and the tangents to it drawn from the origin, is
a) 8/3 sq. units b) 1/3 sq. units c) 2/3 sq. units d) None of these

48. The area bounded by the x-axis, the curve $y=f(x)$ and the lines $x=1$ and $x=b$ is equal to $(\sqrt{(b^2+1)}-\sqrt{2})$ for all $b>1$, then $f(x)$ is
a) $\sqrt{(x-1)}$ b) $\sqrt{(x+1)}$ c) $\sqrt{(x^2+1)}$ d) $\frac{x}{\sqrt{(1+x^2)}}$

49. The area enclosed between the curves $y=\sin^2 x$ and $y=\cos^2 x$ in the interval $0 \le x \le \pi$ is
a) 2 sq unit b) $\frac{1}{2}$ sq unit c) 1 sq unit d) None of these

50. The area bounded by $y=\sin^{-1}x, x=\frac{1}{\sqrt{2}}$ and x-axis is
a) $\left(\frac{1}{\sqrt{2}}+1\right)$ sq units
b) $\left(1-\frac{1}{\sqrt{2}}\right)$ sq uints
c) $\frac{\pi}{4\sqrt{2}}$ sq units
d) $\left(\frac{\pi}{4\sqrt{2}}+\frac{1}{\sqrt{2}}-1\right)$ sq units

51. The area between the curves $x=-2y^2$ and $x=1-3y^2$, is
a) 4/3 b) 3/4 c) 3/2 d) 2/3

52. The area of the region bounded by $y=|x-1|$ and $y=3-|x|$, is
a) 2 b) 3 c) 4 d) 1

53. The area bounded by $y = [x]$ and the two ordinates $x = 1$ and $x = 1.7$ is

a) $\frac{17}{10}$ b) 1 c) $\frac{17}{5}$ d) $\frac{7}{10}$

54. Line $x = 1$ divides A enclosed by circle $x^2 + y^2 = 16$ in two portions A_1 and $A_2 (A_1 > A_2)$, then $\frac{A_1}{A_2}$ is

a) 4 b) 3 c) 2 d) None of these

55. The area enclosed by the curve $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is

a) 10π sq unit b) 20π sq unit c) 5π sq unit d) 4π sq unit

56. The area of the figure bounded by the curve $|y| = 1 - x^2$ is

a) 2/3 b) 4/3 c) 8/3 d) $-5/3$

57. The area enclosed within the curve $|x| + |y| = 1$ is

a) 1 sq unit b) $2\sqrt{2}$ sq units c) $\sqrt{2}$ sq units d) 2 sq units

58. The area bounded by the parabola $y^2 = 4ax$ and $x^2 = 4ay$, is

a) $\frac{8a^3}{3}$ b) $\frac{16a^2}{3}$ c) $\frac{32a^2}{3}$ d) $\frac{64a^2}{3}$

59. The area enclosed between the curves $y = ax^2$ and $x = ay^2 (a > 0)$ is 1 sq unit. Then value of a is

a) $\frac{1}{\sqrt{3}}$ b) $\frac{1}{2}$ c) 1 d) $\frac{1}{3}$

60. The area bounded by the curves $y = x^3$ and $y = x$ is

a) 1/2 sq units b) 1/4 sq units c) 1/8 sq units d) 1/16 sq units

61. The area bounded between the parabola $y^2 = 4x$ and the line $y = 2x - 4$ is equal to

a) $\frac{17}{3}$ sq units b) $\frac{19}{3}$ sq units c) 9 sq units d) 15 sq units

62. The area in square units bounded by the curves $y = x^3, y = x^2$ and the ordinates $x = 1, x = 2$ is

a) 17/12 b) 12/13 c) 2/7 d) 7/2

63. The area bounded by the curve $y = \sin^2 x$ and lines $x = \frac{\pi}{2}, x = \pi$ and x-axis is

a) $\frac{\pi}{2}$ sq unit b) $\frac{\pi}{4}$ sq unit c) $\frac{\pi}{8}$ sq unit d) None of these

64. Maximum area of rectangle whose two vertices lies on the x-axis and two on the curve $y = 3 - |x|, \forall |x| < 3$, is

a) 9 sq unit b) $\frac{9}{4}$ sq unit c) 3 sq unit d) None of these

65. The area between the curve $y = x \sin x$ and x-axis where $0 \leq x \leq 2\pi$, is

a) 2π b) 3π c) 4π d) π

66. The area common to the parabola $y = 2x^2$ and $y = x^2 + 4$, is

a) $\frac{2}{3}$ sq. units b) $\frac{3}{2}$ sq. units c) $\frac{32}{3}$ sq. units d) $\frac{3}{32}$ sq. units

67. If a curve $y = a\sqrt{x} + bx$ passes through the point (1, 2) and the area bounded by the curves, line $x = 4$ and x-axis is 8 sq unit, then

a) $a = 3, b = -1$ b) $a = 3, b = 1$ c) $a = -3, b = 1$ d) $a = -3, b = -1$

68. If the area above the x-axis bounded by the curves $y = 2^{kx}$ and $x = 0$ and 2 is $\frac{3}{\log 2}$ then the value of k is

a) 1/2 b) 1 c) -1 d) 2

69. The area included between the curves $y = \frac{1}{x^2+1}$ and x-axis is

a) $\frac{\pi}{2}$ sq unit b) π sq unit c) 2π sq unit d) None of these

70. The area enclosed between the parabola $y = x^2 - x + 2$ and the line $y = x + 2$ in square unit equals

a) 8/3 b) 1/3 c) 2/3 d) 4/3

71. Area of region satisfying $x \leq 2, y \leq |x|$ and $x \geq 0$ is

a) 1 sq unit b) 4 sq unit c) 2 sq unit d) None of these

72. The area bounded by the curves $y = \sqrt{x}, 2y + 3 = x$ and x-axis in the first quadrant is

a) 9 b) 27/4 c) 36 d) 18

73. Area enclosed by the curve $\pi[4(x-\sqrt{2})^2+y^2]=8$ is

a) π sq units b) 2 sq units c) 3π sq units d) 4 sq units

74. The area in square units of the region bounded by the curve $x^2=4y$, the line $x=2$ and the x-axis, is

a) 1 b) 2/3 c) 4/3 d) 8/3

75. The parabola $y^2=4x$ and $x^2=4\,y$ divide the square region bounded by the lines $x=4, y=4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom, then $S_1{:}\,S_2{:}\,S_3$ is

a) 1:1:1 b) 2:1:2 c) 1:2:3 d) 1:2:1

76. The area bounded by the curve $y^2=16x$ and line $y=mx$ is $\frac{2}{3}$, then m is equal to

a) 3 b) 4 c) 1 d) 2

77. The value of c for which the area of the figure bounded by the curve $y=8x^2-x^5$, the straight lines $x=1$ and $x=c$ and the x-axis is equal to $\frac{16}{3}$ is

a) 2 b) $\sqrt{8-\sqrt{17}}$ c) 3 d) -1

78. The area bounded by $y=2-x^2$ and $x+y=0$ is

a) $\frac{7}{2}$ sq. units b) $\frac{9}{2}$ sq. units c) 9 sq. units d) None of these

79. The area bounded by the curve $x=a\cos^3 t,\ \ y=a\sin^3 t$, is

a) $\frac{3\pi a^2}{8}$ b) $\frac{3\pi a^2}{16}$ c) $\frac{3\pi a^2}{32}$ d) $3\pi a^2$

80. Area bounded by the parabola $x^2=4y$ and the line $x=4y-2$, is

a) 9/8 b) 9/4 c) 9/2 d) 9/7

81. The area formed by triangular shared region bounded by the curves $y=\sin x, y=\cos x$ and $x=0$ is

a) $(\sqrt{2}-1)$ sq unit b) 1 sq unit c) $\sqrt{2}$ sq unit d) $(1+\sqrt{2})$ sq unit

82. The area of the region bounded by the curve $y=2x-x^2$ and the line $y=x$ is

a) 1/2 b) 1/3 c) 1/4 d) 1/6

83. The area bounded by the curves $y=e^x, y=e^{-x}$ and $y=2$, is

a) $\log(16/e)$ b) $\log(4/e)$ c) $2\log(4/e)$ d) $\log(8/e)$

84. The area bounded by $y=4-x^2$ and $y=\left[3+\frac{x^2}{4}\right]$, where $[\cdot]$ denotes greatest integer function, is

a) 1 sq unit b) $\frac{1}{3}$ sq unit c) $\frac{2}{3}$ sq unit d) $\frac{4}{3}$ sq unit

85. The value of m for which the area included between the curves $y^2=4ax$ and $y=mx$ equals, $a^2/3$ is

a) 1 b) 2 c) 3 d) $\sqrt{3}$

86. The area bounded by $y=2-|2-x|$ and $y=\frac{3}{|x|}$ is

a) $\frac{4+3\ln 3}{2}$ b) $\frac{4-3\ln 3}{2}$ c) $\frac{3}{2}\ln 3$ d) $\frac{1}{2}+\ln 3$

87. The area of the region bounded by the curve $9x^2+4y^2-36=0$ is

a) 9 π sq units b) 4 π sq units c) 36π sq units d) 6 π sq unit

88. The area of the plane region bounded by the curves $x+2\,y^2=0$ and $x+3y^2=1$ is equal to

a) $\frac{4}{3}$ sq uints b) $\frac{5}{3}$ sq units c) $\frac{1}{3}$ sq units d) $\frac{2}{3}$ sq units

89. The area included between curves $y=x^2-3x+2$ and $y=-x^2+3x-2$ is

a) $\frac{1}{6}$ sq unit b) $\frac{1}{2}$ sq unit c) 1 sq unit d) $\frac{1}{3}$ sq unit

90. The area bounded by the curve $y^2=x$ and the ordinate $x=36$ is divided in the ratio 1 : 7 by the ordinate $x=a$. Then $a=$

a) 8 b) 9 c) 7 d) 0

91. Area of the region bounded by the curve $y^2=4x$, y-axis and the line $y=3$ is

a) 2 sq. units b) 9/4 sq. units c) $6\sqrt{3}$ sq. units d) None of these

92. The area bounded by the curve $y = x + \sin x$ and its inverse function between the ordinates $x = 0$ and $x = 2\pi$, is
a) 8π sq unit b) 4π sq unit c) 8 sq unit d) None of these

93. The area of the region bounded by $y = 2x - x^2$ and the x-axis is
a) $\frac{8}{3}$ sq units b) $\frac{4}{3}$ sq units c) $\frac{7}{3}$ sq units d) $\frac{2}{3}$ sq units

94. The area of the closed figure bounded by $y = 1/\cos^2 x, x = 0, y = 0$ and $x = \pi/4$, is
a) $\pi/4$ b) $1 + \pi/4$ c) 1 d) 2

95. Area bounded by the curve $y = x \sin x$ and x-axis between $x = 0$ and $x = 2\pi$ is
a) 2π sq unit b) 3π sq unit c) 4π sq unit d) 5π sq unit

96. The line $y = mx$ bisects the area enclosed by the lines $x = 0, y = 0, x = 3/2$ and the curve $y = 1 + 4x - x^2$. The value of m, is
a) 13/8 b) 13/32 c) 13/16 d) 13/4

97. Area lying between the curves $y^2 = 4x$ and $y = 2x$ is equal to
a) 2/3 b) 1/3 c) 1/4 d) 1/2

98. The area contained between the x-axis and one arc of the curve $y = \cos 3x$, is
a) 1/3 b) 2/3 c) 2/7 d) 2/5

99. The area bounded by the curve $y = \sec x$, the x-axis and the lines $x = 0$ and $x = \pi/4$, is
a) $\log(\sqrt{2} + 1)$ b) $\log(\sqrt{2} - 1)$ c) $\frac{1}{2}\log 2$ d) $\sqrt{2}$

100. The area of the region bounded by the parabola $y = x^2 + 1$ and the straight line $x + y = 3$ is given by
a) $\frac{45}{7}$ b) $\frac{25}{4}$ c) $\frac{\pi}{18}$ d) $\frac{9}{2}$

101. The area bounded by the x-axis and the curve $y = 4x - x^2 - 3$ is
a) 4/3 b) 3/4 c) 7 d) 3/2

102. The area bounded by the curves
$y^2 = 4a^2(x - 1)$ and lines $x = 1$ and $y = 4a$ is
a) $4a^2$ sq units b) $\frac{16a}{3}$ sq units c) $\frac{16a^2}{3}$ sq units d) None of these

103. The area between the curves
$y = xe^x$ and $y = xe^{-x}$ and line $x = 1$, in square unit, is
a) $2\left(e + \frac{1}{e}\right)$ sq units b) 0 sq unit c) $2e$ sq units d) $\frac{2}{e}$ sq unit

104. The area (in square unit) bounded by the curves $4y = x^2$ and $2y = 6 - x^2$ is
a) 8 b) 6 c) 4 d) 10

105. The area (in square unit)bounded by the curves $y^2 = 4x$ and $x^2 = 4y$ in the plane is
a) $\frac{8}{3}$ b) $\frac{16}{3}$ c) $\frac{32}{3}$ d) $\frac{64}{3}$

106. The positive value of the parameter $'a'$ for which the area of the figure bounded by $y = \sin a\, x, y = 0, x = \pi a$ and $x{=}\pi 3a$ is 3, is equal to
a) 2 b) 1/2 c) $\frac{2 + \sqrt{3}}{3}$ d) 3/2

107. Area bounded by the curves $y = x^2$ and $y = 2 - x^2$ is
a) 8/3 sq units b) 3/8 sq units c) 3/2 sq units d) None of these

108. The positive value of the parameter 'a' for which the area of the figure founded by $y = \sin ax, y = 0, x = \pi/a$ and $x{=}\pi/3a$ is 3, is equal to
a) 2 b) 1/2 c) $\frac{2 + \sqrt{3}}{3}$ d) $\sqrt{3}$

109. The area between the curve $y = 2x^4 - x^2$, the x-axis and the ordinates of two minima of the curve is
a) $\frac{7}{120}$ sq unit b) $\frac{9}{120}$ sq unit c) $\frac{11}{120}$ sq unit d) $\frac{13}{120}$ sq unit

110. If the ordinate $x = a$ divides the area bounded by x-axis part of the curve $y = 1 + \frac{8}{x^2}$ and the ordinates $x = 2, x = 4$ into two equal parts, then a is equal

a) $\sqrt{2}$ sq unit b) $2\sqrt{2}$ sq unit c) $3\sqrt{2}$ sq unit d) None of these

111. The volume of the solid obtained by revolving about y-axis the area enclosed between the ellipse $x^2 + 9y^2 = 9$ and the straight line $x + 3y = 3$, in the first quadrant is

a) 3π b) 4π c) 6π d) 9π

112. The area of the plane region bounded by the curve $x = y^2 - 2$ and the line $y = -x$ is (in square units)

a) $\frac{13}{3}$ b) $\frac{2}{5}$ c) $\frac{9}{2}$ d) $\frac{5}{2}$

113. The area bounded by $y = x^2 + 2$, x-axis , $x = 1$ and $x = 2$ is

a) $\frac{16}{3}$ sq units b) $\frac{17}{3}$ sq units c) $\frac{13}{3}$ sq units d) $\frac{20}{3}$ sq units

114. Area of the region bounded by the curves $y = 2^x, y = 2x - x^2, x = 0$ and $x = 2$ is given by

a) $\frac{3}{\log 2} - \frac{4}{3}$ b) $\frac{3}{\log 2} + \frac{4}{3}$ c) $3 \log 2 - \frac{4}{3}$ d) $3 \log^2 - \frac{4}{3}$

115. The area of the quadrilateral formed by the tangents at the end points of latusrectum to ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$, is

a) 27/4 sq unit b) 9 sq unit c) 27/2 sq unit d) 27 sq unit

116. The area bounded by the loop of the curve $ay^2 = x^2(a - x)$ is equal to

a) $\frac{4}{15}a^2$ sq unit b) $\frac{8}{15}a^2$ sq unit c) $\frac{16}{15}a^2$ sq unit d) None of these

117. The area of the closed figure bounded by the curves $y = \sqrt{x}, y = \sqrt{4 - 3x}$ and $y = 0$, is

a) 4/9 b) 8/9 c) 16/9 d) 5/9

118. The area bounded by the curves $y = 3x$ and $y = x^2$ is (in square unit)

a) 10 b) 5 c) 4.5 d) 9

119. The area of the figure bounded by the parabolas $x = -2y^2$ and $x = 1 - 3y^2$ is

a) 8/3 b) 6/3 c) 4/3 d) 2/3

120. Area bounded by the liens $y = x, x = -1, x = 2$ and x-axis is

a) 5/2 sq units b) 3/2 sq units c) 1/2 sq unit d) None of these

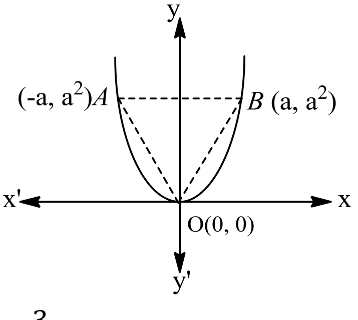
121. The part of straight line $y = x + 1$ between $x = 2$ and $x = 3$ is revolved about x-axis, then the curved surface of the solid thus generated is

a) $\frac{37\pi}{3}$ b) $7\pi\sqrt{2}$ c) 37π d) $7\pi/\sqrt{2}$

122. Area bounded by $y^2 = x, y = 0, x = 1, x = 4$ is

a) $\frac{28}{3}$ sq units b) $\frac{3}{28}$ sq units c) $\frac{8}{3}$ sq units d) $\frac{4}{3}$ sq units

123. The figure shows a ΔAOB and the parabola $y = x^2$. The ratio of the area of the ΔAOB to the area of the region AOB of the parabola $y = x^2$ is equal to



a) $\frac{3}{5}$ b) $\frac{3}{4}$ c) $\frac{7}{8}$ d) $\frac{5}{6}$

124. If the area above x-axis, bounded by the curves $y = 2^{kx}$ and $x = 0$ and $x = 2$ is $\frac{3}{\log 2}$, then the value of k is

a) 1/2 b) 1 c) -1 d) 2

125. The area between the curves $y = \cos x, x$-axis and the line $y = x + 1$, is
a) 1/2 b) 1 c) 3 d) 2

126. The area bounded by the parabola $x = 4 - y^2$ and y −axis, in square units, is
a) $\frac{3}{32}$ b) $\frac{32}{3}$ c) $\frac{33}{2}$ d) $\frac{16}{3}$

127. The volume of the solid formed by rotating the area enclosed between the curve $y = x^2$ and the line $y = 1$ about $y = 1$ is (in cubic unit)
a) $\frac{9\pi}{5}$ b) $\frac{2\pi}{5}$ c) $\frac{8\pi}{3}$ d) $\frac{7\pi}{5}$

128. The volume of spherical cap of height h cut off from a sphere of radius a is equal to
a) $\frac{\pi}{3}h^2(3a - h)$ b) $\pi(a - h)(2a^2 - h^2 - ah)$
c) $\frac{4\pi}{3}h^3$ d) None of these above

129. The area of the region bounded by the straight lines $x = 0$ and $x = 2$ and the curves $y = 2^x$ and $y = 2x - x^2$ is equal to
a) $\frac{2}{\log 2} - \frac{4}{3}$ b) $\frac{3}{\log 2} - \frac{4}{3}$ c) $\frac{1}{\log 2} - \frac{4}{3}$ d) $\frac{4}{\log 2} - \frac{3}{2}$

130. The area bounded by the curves $f(x) = ce^x (c > 0)$, the x-axis and the two ordinates $x = p$ and $x = q$, is proportional to
a) $f(p)f(q)$ b) $|f(p) - f(q)|$ c) $f(p) + f(q)$ d) $\sqrt{f(p)f(q)}$

131. The area between x-axis and curve $y = \cos x$ when $0 \le x \le 2\pi$, is
a) 0 b) 2 c) 3 d) 4

132. Area enclosed between the curves $y^2(2a - x) = x^3$ and line $x = 2a$ above x-axis is
a) πa^2 sq unit b) $\frac{3\pi a^2}{2}$ sq unit c) $2\pi a^2$ sq unit d) $3\pi a^2$ sq unit

133. The area lying between parabola $y^2 = 4ax$ and it's latusrectum is
a) $\frac{4}{3}a^2$ sq unit b) $\frac{16}{3}a^2$ sq unit c) $\frac{8}{3}a^2$ sq unit d) None of these

134. Ratio of the area cut off a parabola by any double ordinate is that corresponding rectangle contained by that double ordinate and its distance from the vertex is
a) 1/2 b) 1/3 c) 2/3 d) 1

135. The area cut off the parabola $4y = 3x^2$ by the straight line $2y = 3x + 12$ in square units is
a) 16 b) 21 c) 27 d) 36

136. The area bounded by the curve $y^2(2a - x) = x^3$ and the line $x = 2a$ is
a) $3\pi a^2$ sq units b) $\frac{3\pi a^2}{2}$ sq units c) $\frac{3\pi a^2}{4}$ sq units d) $\frac{6\pi a^2}{5}$ sq units

137. The area bounded by $y = -x^2 + 2x + 3$ and $y = 0$ is
a) 32 sq units b) 32/3 sq units c) 1/32 sq unit d) 1/3 sq unit

138. The area of the region bounded by the curve $a^4y^2 = (2a - x)x^5$ is to that of the circle whose radius is a, is given by the ratio
a) 4: 5 b) 5: 8 c) 2: 3 d) 3: 2

139. The area bounded by the curves $y^2 = x$ and $y = x^2$ is
a) $\frac{2}{3}$ sq unit b) 1 sq unit c) $\frac{1}{2}$ sq unit d) None of these

140. Area common to the curves $y = \sqrt{x}$ and $x = \sqrt{y}$ is
a) 1 b) 2/3 c) 1/3 d) 4/3

141. The area bounded by the parabola $y^2 = 4ax$, latusrectum and x-axis, is
a) 0 b) $\frac{4}{3}a^2$ c) $\frac{2}{3}a^2$ d) $\frac{a^2}{3}$

142. If A is the area between the curve $y = \sin x$ and x-axis in the interval $[0, \pi/4]$, then in the same interval, area between the curve $y = \cos x$ and x-axis is
a) A b) $\pi/2 - A$ c) $1 - A$ d) $A - 1$

143. The area bounded by $y = \tan^{-1} x$, $x = 1$ and x-axis is

a) $\left(\frac{\pi}{4} + \log\sqrt{2}\right)$ sq unit
b) $\left(\frac{\pi}{4} - \log\sqrt{2}\right)$ sq unit
c) $\left(\frac{\pi}{4} - \log\sqrt{2} + 1\right)$ sq unit
d) None of these

144. The area of the smaller segment cut off from the circle $x^2 + y^2 = 9$ by $x = 1$ is

a) $\frac{1}{2}(9\sec^{-1}3 - \sqrt{8})$sq unit
b) $(9\sec^{-1}3 - \sqrt{8})$sq unit
c) $(\sqrt{8} - 9\sec^{-1}3)$sq unit
d) None of the above

145. Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$, the line $x = \sqrt{3}y$ and x-axis, is

a) π sq units
b) $\frac{\pi}{2}$ sq units
c) $\frac{\pi}{3}$ sq units
d) None of these

146. The area of the figure bounded by $y = e^{x-1}, y = 0, x = 0$ and $x = 2$, is

a) < 2
b) > 2
c) $= 2$
d) None of these

147. Area bounded by the curves $y = x\sin x$ and x-axis between $x = 0$ and $x = 2\pi$ is

a) 2π
b) 3π
c) 4π
d) 5π

148. The area of region $\{(x, y): x^2 + y^2 \leq 1 \leq x + y\}$ is

a) $\frac{\pi^2}{5}$ sq unit
b) $\frac{\pi^2}{2}$ sq unit
c) $\frac{\pi^2}{4}$ sq unit
d) $\left(\frac{\pi}{4} - \frac{1}{2}\right)$ sq unit

149. The area bounded by the curves $y = f(x)$, the x-axis and the ordinates $x = 1$ and $x = b$ is $(b - 1)\sin(3b + 4)$. Then, $f(x)$ is

a) $(x - 1)\cos(3x + 4)$
b) $\sin(3x + 4)$
c) $\sin(3x + 4) + 3(x - 1)\cos(3x + 4)$
d) None of the above

150. AOB is the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in which $OA = a, OB = b$. The area between the arc AB and the chord AB of the ellipse is

a) $\frac{1}{2}ab(\pi + 2)$
b) $\frac{1}{4}ab(\pi - 4)$
c) $\frac{1}{4}ab(\pi - 2)$
d) None of these

151. Area bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is equal to

a) $\frac{8}{9}$ sq unit
b) $\frac{9}{8}$ sq unit
c) $\frac{4}{3}$ sq unit
d) None of these

152. The area of the region bounded by the curve $y = \tan x$, a line parallel to y-axis at $x = \frac{\pi}{4}$ and the x-axis is

a) $\frac{1}{4}$ sq unit
b) $\log\sqrt{2} + \frac{1}{4}$ sq unit
c) $\log\sqrt{2} - \frac{1}{4}$ sq unit
d) None of these

153. Let A_1 be the area of the parabola $y^2 = 4ax$ lying between vertex and latusrectum and A_2 be the area between latusrectum and double ordinate $x = 2a$. Then, $A_1/A_2 =$

a) $2\sqrt{2} - 1$
b) $(2\sqrt{2} + 1)/7$
c) $(2\sqrt{2} - 1)/7$
d) None of these

154. The area of the closed igure bounded by $x = -1, x = 2$ and $y = \begin{cases} -x^2 + 2, x \leq 1 \\ 2x - 1, \ x > 1 \end{cases}$ and the x-axis is

a) $\frac{16}{3}$ sq unit
b) $\frac{10}{3}$ sq unit
c) $\frac{13}{3}$ sq unit
d) $\frac{7}{3}$ sq unit

155. The area bounded by the curve $y = \log_e x$ and x-axis and the straight line $x = e$ is

a) e sq. units
b) 1 sq. units
c) $1 - \frac{1}{e}$ sq. units
d) $1 + \frac{1}{e}$ sq. units

156. The area bounded by the curves $\sqrt{x} + \sqrt{y} = 1$ and $x + y = 1$ is

a) 1/3 sq unit
b) 1/6 sq unit
c) 1/2 sq unit
d) None of these

157. If A is the area of the region bounded by the curve $y = \sqrt{3x + 4}$, x-axis and the lines $x = -1$ and $x = 4$ and B is that area bounded by curve $y^2 = 3x + 4$, x-axis and the liens $x = -1$ and $x = 4$, then $A: B$ is equal to

a) 1:1
b) 2:1
c) 1:2
d) None of these

158. The area bounded by the curves $y = \sqrt{x}, 2y + 3 = x$ and x-axis in the Ist quadrant is

a) 9 sq unit
b) 27/4 sq unit
c) 36 sq unit
d) 18 sq unit

159. The sine and cosine meet each other at number of points and develop the symmetrical area number of times, area of one such region is

a) $4\sqrt{2}$ b) $3\sqrt{2}$ c) $2\sqrt{2}$ d) $\sqrt{2}$

160. Let $f(x)$ be a non-negative continuous function such that the area bounded by the curve $y = f(x)$, x-axis and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is π $\left(\beta \sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\,\beta\right)$ the $f\left(\frac{\pi}{2}\right)$ s

a) $\left(1 - \frac{\pi}{4} + \sqrt{2}\right)$ b) $\left(1 - \frac{\pi}{4} - \sqrt{2}\right)$ c) $\left(\frac{\pi}{4} - \sqrt{2} + 1\right)$ d) $\left(\frac{\pi}{4} + \sqrt{2} - 1\right)$

161. The area bounded by the curves $y = |x|$ and $y = 4 - |x|$ is

a) 4 sq unit b) 16 sq unit c) 2 sq unit d) 8 sq unit

162. The smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$ is equal to

a) $2\,(\pi - 2)$ b) $\pi - 2$ c) $2\,\pi - 1$ d) $\pi - 1$

163. The area bounded by the curve $y = \sec^2 x, y = 0$ and $|x| = \frac{\pi}{3}$ is

a) $\sqrt{3}$ sq unit b) $\sqrt{2}$ sq unit c) $2\sqrt{3}$ sq unit d) None of these

164. The area bounded by the curve $x = 4 - y^2$ and the y-axis is

a) 16 sq units b) 32 sq units c) $\frac{32}{3}$ sq units d) $\frac{16}{3}$ sq units

165. The area bounded by the curve $y = x|x|$, x-axis and the ordinates $x = 1, x = -1$ is given by

a) 0 b) $\frac{1}{3}$ c) $\frac{2}{3}$ d) None of these

166. The area of the region bounded by $x^2 + y^2 - 2y - 3 = 0$ and $y = |x| + 1$, is

a) π b) 2π c) 4π d) $\pi/2$

167. The area of the region (in square units) bounded by the curve $x^2 = 4y$, line $x = 2$ and x-axis, is

a) 1 b) 2/3 c) 4/3 d) 8/3

168. The area bounded by $x = 1, x = 2, xy = 1$ and x-axis is

a) $(\log 2)$ sq unit b) 2 sq unit c) 1 sq unit d) None of these

169. The area of the region for which $0 < y < 3 - 2\,x - x^2$ and $x > 0$, is

a) $\int_1^3 (3 - 2\,x - x^2)\,dx$ b) $\int_0^3 (3 - 2\,x - x^2)\,dx$ c) $\int_0^1 (3 - 2\,x - x^2)\,dx$ d) $\int_{-1}^3 (3 - 2\,x - x^2)\,dx$

170. Area bounded by parabola $y^2 = x$ and straight line $2y = x$, is

a) 4/3 b) 1 c) 2/3 d) 1/3

171. The area of the triangle formed by the positive x-axis and the normal and tangent to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$, is

a) $\sqrt{3}$ b) $1/\sqrt{3}$ c) $2\sqrt{3}$ d) $3\sqrt{3}$

172. The line $x = \frac{\pi}{4}$ divides the area of the region bounded by $y = \sin x,\ y = \cos x$ and x-axis $\left(0 \le x \le \frac{x}{2}\right)$ into two regions of areas A_1 and A_2. Then $A_1 : A_2$ equals

a) 4:1 b) 3:1 c) 2:1 d) 1:1

173. Area of the region bounded by the curve $y = \begin{cases} x^2, x < 0 \\ x,\ x \ge 0 \end{cases}$ and the line $y = 4$ is

a) $\frac{10}{3}$ sq unit b) $\frac{20}{3}$ sq unit c) $\frac{40}{3}$ sq unit d) None of these

174. The area of the closed figure bounded by the curves $y = \cos x, y = 1 + \frac{2}{\pi}x$ and $x = \pi/2$, is

a) $\frac{\pi + 4}{4}$ b) $\frac{3\pi - 4}{4}$ c) $\frac{3\pi}{4}$ d) $\frac{\pi}{4}$

175. The area enclosed between the curves $y = x$ and $y = 2x - x^2$ is (in square unit)

a) $\frac{1}{2}$ b) $\frac{1}{6}$ c) $\frac{1}{3}$ d) $\frac{1}{4}$

176. If A_n be the area bounded by the curve $y = (\tan x)^n$ and the lines $x = 0, y = 0$ and $x = \pi/4$, then for $x > 2$

a) $A_n + A_{n-2} = \frac{1}{n-1}$ b) $A_n + A_{n-2} < \frac{1}{n-1}$ c) $A_n - A_{n-2} = \frac{1}{n-1}$ d) None of these

177. The area cut off from a parabola by any double ordinate is k times the corresponding rectangle contained by that double ordinate and its distance from the vertex, then k is

a) $\frac{2}{3}$ b) $\frac{1}{3}$ c) $\frac{3}{2}$ d) 3

178. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is

a) $\frac{2}{3}$ sq unit b) 1 sq unit c) $\frac{1}{6}$ sq unit d) $\frac{1}{3}$ sq unit

179. The area of the loop between the curve $y = a \sin x$ and x-axis is

a) a b) $2a$ c) $3a$ d) $4a$

180. The area of the region bounced by $y^2 = x$ and $y = |x|$ is

a) $\frac{1}{3}$ sq unit b) $\frac{1}{6}$ sq unit c) $\frac{2}{3}$ sq unit d) 1 sq unit

181. Area bounded by the curve $y = (x-1)(x-2)(x-3)$ and x-axis lying between the ordinates $x = 0$ and $x = 3$ is equal to

a) $\frac{9}{4}$ sq unit b) $\frac{11}{4}$ sq unit c) $\frac{13}{4}$ sq unit d) $\frac{15}{4}$ sq unit

182. The area included between the parabolas $y^2 = 4ax$ and $x^2 = 4$ by is

a) $(8/3)ab$ b) $(16/3)ab$ c) $(4/3)ab$ d) $(5/3)ab$

183. Area include between curves $y = x^2 - 3x + 2$ and $y = -x^2 + 3x - 2$ is

a) $\frac{1}{6}$ sq unit b) $\frac{1}{2}$ sq unit c) 1 sq unit d) $\frac{1}{3}$ sq unit

184. The area bounded by the curve $y = x^3$, the x-axis and the ordinates $x = -2$ and $x = 1$ is

a) $17/2$ b) $15/2$ c) $15/4$ d) $17/4$

185. The area of the region lying between the line $x - y + 2 = 0$ and the curve $x = \sqrt{y}$ is

a) 9 b) $9/2$ c) $10/3$ d) $5/2$

186. Area lying in the first quadrant and bounded by the curve $y = x^3$ and the line $y = 4x$, is

a) 2 b) 3 c) 4 d) 5

187. The area between the parabola $y = x^2$ and the line $y = x$ is

a) $\frac{1}{6}$ sq unit b) $\frac{1}{3}$ sq unit c) $\frac{1}{2}$ sq unit d) None of these

188. The area enclosed between the curves $y = x^3$ and $y = \sqrt{x}$ is (in square unit)

a) $5/3$ b) $5/4$ c) $5/12$ d) $12/5$

189. If $f(x)$ be continuous function such that the area bounded by the curve $y = f(x)$, the x-axis and the lines $x = a$ and $x = 0$ is $\frac{a^2}{2} + \frac{a}{2}\sin a + \frac{\pi}{2}\cos a$.

Value of $f\left(\frac{\pi}{2}\right)$ is

a) $\frac{1}{2}$ b) $\frac{a}{2}$ c) $\frac{a^2}{2}$ d) $\frac{\pi}{2}$

190. The area of the figure bounded by the curves $y = e^x, y = e^{-x}$ and the straight line $x = 1$ is

a) $e + \frac{1}{e}$ b) $e - \frac{1}{e}$ c) $e + \frac{1}{e} - 2$ d) None of these

191. The area bounded by $y = xe^{|x|}$ and lines $|x| = 1, y = 0$ is

a) 4 sq unit b) 6 sq unit c) 1 sq unit d) 2 sq unit

192. The area bounded by the parabola $y^2 = 8x$ and its latusretum in square unit is

a) 16/3 sq units b) 32/3 sq units c) 8/3 sq units d) 64/3 sq units

193. The areas of the figure into which curve $y^2 = 6x$ divides the circle $x^2 + y^2 = 16$ are in the ratio

a) $\frac{2}{3}$ b) $\frac{4\pi - \sqrt{3}}{8\pi + \sqrt{3}}$ c) $\frac{4\pi + \sqrt{3}}{8\pi - \sqrt{3}}$ d) None of these

194. If A is the area lying between the curve $y = \sin x$ and x-axis between $x = 0$ and $x = \pi/2$. Area of the region between the curve $y = \sin 2x$ and x-axis in the same interval is given by

a) $A/2$ b) A c) $2A$ d) $3/2\ A$

195. If the ordinate $x = a$ divides the areaby the curve $y = \left(1 + \frac{8}{x^2}\right)$ x-axis and the ordinates $x = 2, x = 4$ into two equal parts, then the value of a is

a) $2a$ b) $2\sqrt{2}$ c) $\frac{a}{2}$ d) None of these

196. The area of the region bounded by $y = |x-1|$ and $y = 1$ is
a) 1 b) 2 c) 1/2 d) 3/2

197. If the area bounded by the curve $y = f(x)$, the coordinate axes, and the line $x = x_1$ is given by $x_1 e^{x_1}$. Then, $f(x)$ equals
a) e^x b) $x\,e^x$ c) $x\,e^x - e^x$ d) $x\,e^x + e^x$

198. The area bounded by the curve $y = \frac{1}{2}x^2$, the x-axis and the ordinate $x = 2$ is
a) $\frac{1}{3}$ sq units b) $\frac{2}{3}$ sq units c) 1 sq units d) $\frac{4}{3}$ sq units

199. The area bounded by $y = x^2, y = [x+1], x \le 1$ and the y-axis is
a) 1/3 b) 2/3 c) 1 d) 7/3

200. The area between the curve $y = 4 + 3x - x^2$ and x-axis is
a) 125/6 sq unit b) 125/3 sq unit c) 125/2 sq unit d) None of these

201. In the interval $[0, \pi/2]$, area lying between the curves $y = \tan x, y = \cot x$ and x-axis is
a) $\log 2$ b) $\frac{1}{2}\log 2$ c) $2\log\left(\frac{1}{\sqrt{2}}\right)$ d) $\frac{3}{2}\log 2$

202. The area bounded by the curve $y = f(x) = x^4 - 2x^3 + x^2 + 3$, x-axis and ordinates corresponding to minimum of the function $f(x)$, isf
a) 1 sq unit b) $\frac{91}{30}$ sq unit c) $\frac{30}{9}$ sq unit d) 4 sq unit

203. The area enclosed between the curves $y = x^3$ and $y = \sqrt{x}$ is
a) $\frac{5}{3}$ sq units b) $\frac{5}{4}$ sq units c) $\frac{5}{12}$ sq units d) $\frac{12}{5}$ sq units

204. The area of the figure bounded by $|y| = 1 - x^2$ is in square units,
a) 4/3 b) 8/3 c) 16/3 d) 5/3

205. The area bounded by the x-axis, part of the curve $y = 1 + \frac{8}{x^2}$ and the ordinates $x = 2$ and $x = 4$, is divided into two equal parts by the ordinate $x = a$, then the value of $'a'$ is
a) $2\sqrt{2}$ b) $\pm 2\sqrt{2}$ c) $\pm\sqrt{2}$ d) ± 2

206. Area of the region bounded by the curve $y = \tan x$, tangent drawn to the curve at $x = \frac{\pi}{4}$ and the x– axis is
a) $\log\sqrt{2}$ b) $\log\sqrt{2} + \frac{1}{4}$ c) $\log\sqrt{2} - \frac{1}{4}$ d) $\frac{1}{4}$

207. The area bounded by the curve $y = 2x - x^2$ and the line $y = -x$ is
a) $\frac{3}{2}$ sq units b) $\frac{9}{3}$ sq units c) $\frac{9}{2}$ sq units d) None of these

208. The area out off by latusrectum form the parabola $y^2 = 4ax$ is
a) $(8/3)\,a$ sq units b) $(8/3)\sqrt{a}$ sq units c) $(3/8)\,a^2$ sq units d) $(8/3)a^2$ sq units

209. The volume of the solid is generated by revolving about the y-axis. The figure bounded by the parabola $y = x^2$ and $x = y^2$ is
a) $\frac{21}{5}\pi$ b) $\frac{24}{5}\pi$ c) $\frac{3\pi}{10}$ d) $\frac{5}{24}\pi$

210. The area bounded by the curves $y = (x-1)^2, y = (x+1)^2$ and $y = \frac{1}{4}$ is
a) $\frac{1}{3}$ sq unit b) $\frac{2}{3}$ sq unit c) $\frac{1}{4}$ sq unit d) $\frac{1}{5}$ sq unit

211. The area of the region between the curves
$y = \sqrt{\frac{1+\sin x}{\cos x}}$ and $y = \sqrt{\frac{1-\sin x}{\cos x}}$

Bounded by the line $x = 0$ and $x = \frac{\pi}{4}$

a) $\int_0^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}} dt$
b) $\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}} dt$
c) $\int_0^{\sqrt{2}+1} \frac{4t}{(1+t^2)\sqrt{1-t^2}} dt$
d) $\int_0^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}} dt$

212. The area induced between the curves $y = \frac{x^2}{4a}$ and $y = \frac{8\,a^3}{x^2+4\,a^2}$ is given by

a) $a^2\left(2\,\pi - \frac{4}{3}\right)$
b) $a^2\left(\pi - \frac{4}{3}\right)$
c) $a^2\left(2\,\pi + \frac{1}{3}\right)$
d) $a^2\left(\pi + \frac{4}{3}\right)$

213. The area between $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the straight line $\frac{x}{a} + \frac{y}{b} = 1$, is

a) $\frac{1}{2}ab$
b) $\frac{1}{2}\pi\, ab$
c) $\frac{1}{4}ab$
d) $\frac{1}{4}\pi\, ab - \frac{1}{2}ab$

214. The area bounded by the parabola $y^2 = 4ax$ and the line $x = a$ and $x = 4a$ is

a) $\frac{35a^2}{3}$
b) $\frac{4a^2}{3}$
c) $\frac{7a^2}{3}$
d) $\frac{56a^2}{3}$

215. Let $f(x) = \min\{x+1, \sqrt{(1-x)}\}$, then area bounded by $f(x)$ and x-axis is

a) $\frac{1}{6}$ sq unit
b) $\frac{5}{6}$ sq unit
c) $\frac{7}{6}$ sq unit
d) $\frac{11}{6}$ sq unit

216. The area bounded by the curve $y = \sin 2x$, y –axis and $y = 1$, is

a) 1
b) 1/4
c) $\pi/4$
d) $\pi/4 - 1/2$

217. The area common to the circle $x^2 + y^2 = 16a^2$ and the parabola $y^2 = 6ax$ is

a) $\frac{4a^2}{3}(4\pi - \sqrt{3})$ sq unit
b) $\frac{4a^2}{3}(8\pi - 3)$ sq unit
c) $\frac{4a^2}{3}(4\pi + \sqrt{3})$ sq unit
d) None of these

218. The area bounded by the parabolas

$y = 4x^2, y = \frac{x^2}{9}$ and the line $y = 2$ is

a) $\frac{5\sqrt{2}}{3}$ sq units
b) $\frac{10\sqrt{2}}{3}$ sq units
c) $\frac{15\sqrt{2}}{3}$ sq units
d) $\frac{20\sqrt{2}}{3}$ sq units

219. If the area bounded by the x-axis, the curve $y = f(x)$ and lines $x = a$ and $x = b$ is independent of $b, \forall\, b > a$ (a is a constant), then f is

a) The zero function
b) The identity function
c) A non-zero constant function
d) None of the above

220. The area bounded by curve

$x^2 + y^2 = 25, 4y = |4 - x^2|$ and $x = 0$ above the x-axis is

a) $24\sin^{-1}\left(\frac{4}{5}\right)$
b) $25\sin^{-1}\left(\frac{4}{5}\right)$
c) $4 + 25\sin^{-1}\left(\frac{4}{5}\right)$
d) None of these

221. The area bounded by the curve $x = 3y^2 - 9$ and the line $x = 0, y = 0$ and $y = 1$ is

a) 8 sq unit
b) 8/3 sq unit
c) 3/8 sq unit
d) 3 sq unit

222. The area of the figure bounded by the curves $y^2 = 2x + 1$ and $x - y - 1 = 0$ is

a) 2/3
b) 4/3
c) 8/3
d) 16/3

223. The value of a for which the area between the curves $y^2 = 4ax$ and $x^2 = 4ay$ is 1 unit, is

a) $\sqrt{3}$
b) 4
c) $4\sqrt{3}$
d) $\sqrt{3}/4$

224. The area bounded by $y = |\sin x|$, x-axis and the lines $|x| = \pi$ is

a) 2 sq units
b) 3 sq units
c) 4 sq units
d) None of these

225. The area out of the region bounded by $y^2 = 4ax$ and $x^2 = 4ay, a > 0$ in square unit is

a) $\frac{16a^2}{3}$ sq units
b) $\frac{14a^2}{3}$ sq units
c) $\frac{13a^2}{3}$ sq units
d) $16a^2$ sq units

226. The area enclosed between the curve $y = 1 + x^2$, the x-axis and the line $y = 5$ is given by

a) $\frac{14}{3}$ sq units
b) $\frac{7}{3}$ sq units
c) 5 sq units
d) $\frac{16}{3}$ sq units

227. The volume of the solid generated by the revolving of the curve

$y = \dfrac{a^3}{a^2 + x^2}$ about x– axis is

a) $\frac{1}{2}\pi^3 a^2$ cu units b) $\pi^3 a^2$ cu units c) $\frac{1}{2}\pi^2 a^3$ cu units d) $\pi^2 a^3$ cu units

228. Area of the region satisfying $x \leq 2, y \geq |x|$ and $x \geq 0$ is

a) 4 sq units b) 1 sq units c) 2 sq units d) None of these

229. The area of the figure bounded by $y^2 = 2x + 1$ and $x - y = 1$ is

a) $\frac{2}{3}$ b) $\frac{4}{3}$ c) $\frac{8}{3}$ d) $\frac{16}{3}$

230. The area bounded by the curve $y = x^4 - 2x^3 + x^2 + 3$ with x-axis and ordinates corresponding to the minima of y, is

a) 1 b) $\frac{91}{30}$ c) $\frac{30}{9}$ d) 4

231. The area bounded by curves $y^2 = 8x$ and $x^2 = 8y$ is

a) 64 sq units b) $\frac{64}{3}$ sq units c) $\frac{8}{3}$ sq units d) None of these

232. The area (in square unit) of the region enclosed by the curves $y = x^2$ and $y = x^3$ is

a) $\frac{1}{12}$ b) $\frac{1}{6}$ c) $\frac{1}{3}$ d) 1

233. The area bounded by the y-axis, $y = \cos x$ and $y = \sin x, 0 \leq x \leq \pi/4$ is

a) $2(\sqrt{2} - 1)$ b) $\sqrt{2} - 1$ c) $\sqrt{2} + 1$ d) $\sqrt{2}$

234. The area bounded by $y = 2 - |2 - x|$ and $y = \frac{3}{|x|}$ is

a) $\frac{4 + 3\log 3}{2}$ sq unit b) $\frac{4 - 3\log 3}{2}$ sq unit c) $\frac{3}{2}\log 3$ sq unit d) $\frac{1}{2} + \log 3$ sq unit

235. The area of the figure bounded by $y = \sin x, y = \cos x$ in the first quadrant, is

a) $2(\sqrt{2} - 1)$ b) $\sqrt{3} + 1$ c) $2(\sqrt{3} - 1)$ d) None of these

236. The area between the curve $y = x\,e^x$ and $y = x\,e^{-x}$ and the line $x = 1$ in square unit, is

a) $2\left(e + \frac{1}{e}\right)$ sq unit b) 0 sq unit c) $2e$ sq unit d) $\frac{2}{e}$ sq unit

: ANSWER KEY :

1)	b	2)	c	3)	b	4)	d
5)	b	6)	c	7)	a	8)	c
9)	a	10)	c	11)	a	12)	d
13)	a	14)	c	15)	b	16)	b
17)	a	18)	c	19)	a	20)	c
21)	c	22)	a	23)	b	24)	c
25)	c	26)	a	27)	b	28)	b
29)	d	30)	a	31)	b	32)	b
33)	a	34)	b	35)	d	36)	d
37)	b	38)	d	39)	a	40)	d
41)	b	42)	b	43)	b	44)	a
45)	c	46)	c	47)	c	48)	d
49)	c	50)	d	51)	a	52)	c
53)	d	54)	d	55)	b	56)	c
57)	d	58)	b	59)	a	60)	a
61)	c	62)	a	63)	b	64)	d
65)	c	66)	c	67)	a	68)	b
69)	b	70)	d	71)	c	72)	a
73)	d	74)	b	75)	a	76)	b
77)	d	78)	b	79)	a	80)	a
81)	a	82)	d	83)	c	84)	d
85)	b	86)	b	87)	d	88)	a
89)	d	90)	b	91)	b	92)	c
93)	b	94)	c	95)	c	96)	c
97)	b	98)	b	99)	a	100)	d
101)	a	102)	b	103)	d	104)	a
105)	b	106)	b	107)	a	108)	b
109)	a	110)	b	111)	a	112)	c
113)	c	114)	d	115)	d	116)	b
117)	b	118)	c	119)	c	120)	a
121)	b	122)	a	123)	b	124)	b
125)	a	126)	b	127)	b	128)	a
129)	b	130)	b	131)	d	132)	b
133)	c	134)	c	135)	c	136)	b
137)	b	138)	b	139)	d	140)	c
141)	b	142)	c	143)	b	144)	b
145)	c	146)	b	147)	c	148)	d
149)	c	150)	c	151)	b	152)	d
153)	b	154)	a	155)	b	156)	a
157)	c	158)	a	159)	b	160)	a
161)	d	162)	b	163)	c	164)	c
165)	c	166)	a	167)	b	168)	a
169)	c	170)	a	171)	c	172)	d
173)	c	174)	b	175)	b	176)	a
177)	a	178)	c	179)	b	180)	b
181)	b	182)	b	183)	d	184)	d
185)	c	186)	c	187)	a	188)	c

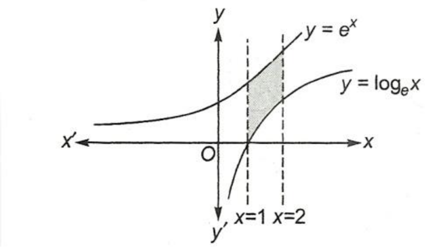
189)	a	190)	a	191)	d	192)	b
193)	c	194)	b	195)	b	196)	a
197)	c	198)	d	199)	b	200)	a
201)	a	202)	b	203)	c	204)	b
205)	b	206)	c	207)	c	208)	d
209)	c	210)	a	211)	b	212)	a
213)	d	214)	d	215)	c	216)	d
217)	c	218)	d	219)	a	220)	c
221)	b	222)	d	223)	d	224)	c
225)	a	226)	d	227)	c	228)	c
229)	d	230)	b	231)	b	232)	a
233)	b	234)	b	235)	a	236)	d

: HINTS AND SOLUTIONS :

2 **(c)**

Required area

$$A=\int_1^2 (e^x-\log_e x)dx$$



$$=[e^x]_1^2-\left[x\log_e x-\int 1dx\right]_1^2$$
$$=e^2-e-[x\log_e x-x]_1^2$$
$$=e^2-e-[2\log_e 2-2-(0-1)]$$
$$=e^2-e-2\log_e 2+1$$

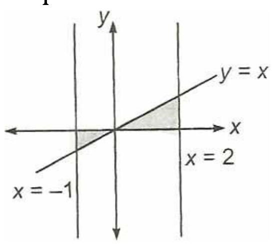
3 **(b)**

We have,

$$\int_1^k (8\,x^2-x^5)\,dx=\frac{16}{3}$$
$$\Rightarrow \left[\frac{8\,x^3}{3}-\frac{x^6}{6}\right]_1^k=\frac{16}{3}$$
$$\Rightarrow \left(\frac{8\,k^3}{3}-\frac{k^6}{6}\right)-\left(\frac{8}{3}-\frac{1}{6}\right)=\frac{16}{3}$$
$$\Rightarrow 16\,k^3-k^6-16+1=32$$
$$\Rightarrow k^6-16k^3+47=0\Rightarrow k^3=8\pm\sqrt{17}\Rightarrow k=\left(8\pm\sqrt{17}\right)^{1/3}$$

4 **(d)**

Required area

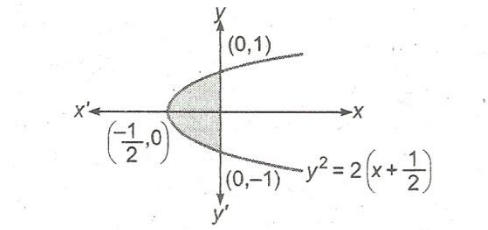


$$\left|\int_{-1}^0 x\,dx\right|+\left|\int_0^2 x\,dx\right|$$
$$=\left|\left[\frac{x^2}{2}\right]_{-1}^0\right|+\left|\left[\frac{x^2}{2}\right]_0^2\right|$$
$$=\left|-\frac{1}{2}\right|+|2|$$
$$=2+\tfrac{1}{2}=\tfrac{5}{2}\text{ sq unit}$$

5 **(b)**

Given curve can be rewritten as

$$y^2=2\left(x+\frac{1}{2}\right)$$



$$\therefore \text{ Required area}=\int_{-1}^1 x\ dy$$
$$=2\int_0^1 \frac{y^2-1}{2}dy$$
$$=\left|\left[\frac{y^3}{3}-y\right]_0^1\right|$$
$$=\frac{2}{3}\text{ sq unit}$$

7 **(a)**

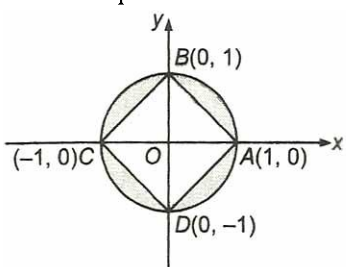
On the solving the given equations of curves, we get $x=0,2$

$\therefore$ Required volume

$$=\pi\int_0^2 [(2x+1)^2-(x^2+1)^2]dx$$
$$=\pi\int_0^2 (-x^4+2x^2+4x)dx$$
$$=\pi\left[-\frac{x^5}{5}+\frac{2x^3}{3}+\frac{4x^2}{2}\right]_0^2=\frac{104\pi}{15}\text{ sq units}$$

8 **(c)**

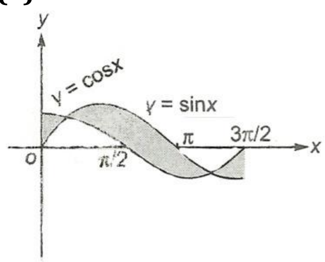
Area of square $ABCD=2$ sq unit



Area of circle $=\pi$ sq unit

$\Rightarrow$ Required area $=(\pi-2)$ sq unit

9 **(a)**



Required area

$= \int_0^{x/4} (\cos x - \sin x)dx$

$+ \int_{\pi/4}^{5\pi/4} (\sin x$

$- \cos x)dx$

$+ \int_{5\pi/4}^{3\pi/2} (\cos x - \sin x)dx$

$= (\sin x + \cos x)_0^{\pi/4}$

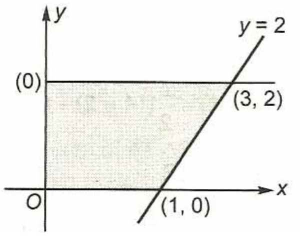
$+ (-\cos x$

$- \sin x)_{\pi/4}^{5\pi/4} + (\sin x + \cos x)_{5\pi/4}^{3\pi/2}$

$= (4\sqrt{2} - 2)$sq units

10 **(c)**

$-8 < x < 8 \Rightarrow y = 2$



$\therefore$ Required area $= \frac{1}{2}(1+3) \times 2$

$= 4$ sq unit

12 **(d)**

Required area, $A = \int_{1-e}^{0} \log_e(x+e)dx$

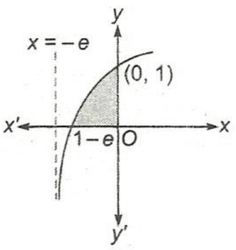
Put $x + e = t \Longrightarrow dx = dt$

$\therefore A = \int_1^e \log_e t\, dt$

$= [t \log_e t - t]_1^e$

$= (e - e - 0 + 1)$

$= 1$ sq unit



13 **(a)**

The two curves $y^2 = 4\,ax$ and $y = mx$ intersect at $(4a/m^2, 4a/m)$ and the area enclosed by the two curves is given by $\int_0^{4a/m^2} (\sqrt{4\,ax} - mx)\,dx$

$$\therefore \int_{300}^{4a/m^2} \left(\sqrt{4\,ax} - mx\right) dx = \frac{a^2}{3} \Rightarrow \frac{8}{3}\frac{a^2}{m^3} = \frac{a^2}{3} \Rightarrow m^3$$

$= 8 \Rightarrow m = 2$

14 **(c)**

Let A be the required area. Then,

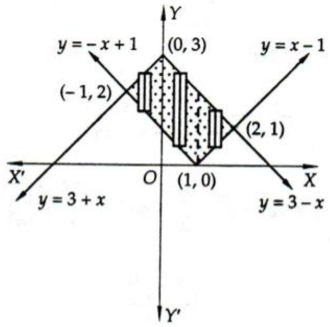
$$A = \int_{-1}^{0} \{(3+x) - (-x+1)\}$$

$$+ \int_0^1 \{(3-x) - (-x+1)\}\,dx$$

$$+ \int_1^2 \{(3-x) - (x-1)\}\,dx$$

$$\Rightarrow A = \int_{-1}^{0} (2+2x)dx + \int_0^1 2dx + \int_1^2 (4-2x)\,dx$$

$$\Rightarrow A = [2x + x^2]_{-1}^0 + [2x]_0^1 + [4x - x^2]_1^2 = 4$$

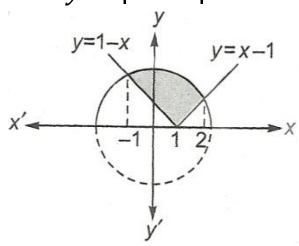


15 **(b)**

Given, $\quad y = \sqrt{5 - x^2}$ and $y = |x - 1|$

or $y^2 + x^2 = 5$

and $y = |x - 1|$



$\therefore$ Required area

$$= \int_{-1}^{2} \sqrt{5 - x^2}\,dx - \int_{-1}^{1} (1-x)\,dx - \int_1^2 (x-1)\,dx$$

$$= \left[\frac{x}{2}\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_{-1}^{2} - \left[x - \frac{x^2}{2}\right]_{-1}^{1}$$

$$- \left[\frac{x^2}{2} - x\right]_1^2$$

$$= \left[1 + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}} + 1 + \frac{5}{2}\sin^{-1}\frac{1}{\sqrt{5}}\right]$$

$$- \left[1 - \frac{1}{2} - \left(-1 - \frac{1}{2}\right)\right]$$

$$- \left[2 - 2 - \left(\frac{1}{2} - 1\right)\right]$$

$$= 2 + \frac{5}{2}\sin^{-1}\left(\frac{2}{\sqrt{5}}\sqrt{1 - \frac{1}{5}} + \frac{1}{\sqrt{5}}\sqrt{1 - \frac{4}{5}}\right) - \frac{5}{2}$$

$$= \frac{5}{2}\sin^{-1}(1) = \frac{5\pi}{4} - \frac{1}{2} = \left(\frac{5\pi - 2}{4}\right) sq\ unit$$

19 **(a)**

Volume of generated solid

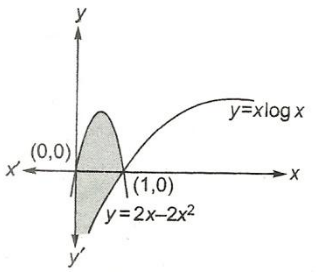
$$= \pi \int_0^2 x^2 \ dy = \pi \int_0^2 (9 - y^2) dy = \pi \left[9y - \frac{1}{3}y^3\right]_0^2$$

$$\pi = \left[18 - \frac{8}{3}\right] = \frac{46}{3}\pi \text{ cu units}$$

20 **(c)**

∴ Required area

$$= \int_0^1 [(2x - 2x^2) - (x \log x)] dx$$

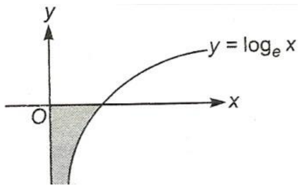


$$= \left[x^2 - \frac{2x^3}{3} - \left(\frac{x^2}{2}\log x - \frac{x^2}{4}\right)\right]_0^1$$

$$= \left[1 - \frac{2}{3} - \left(0 - \frac{1}{4}\right)\right] = \frac{7}{12} \text{ sq unit}$$

22 **(a)**

$$\text{Required area} = \left|\int_0^{-\infty} e^y \ dy\right|$$

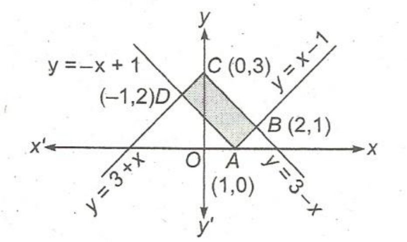


$$= |[e^y]|_0^{-\infty} = 1 \text{ sq unit}$$

25 **(c)**

Given, $y = |x - 1| = \begin{cases} x - 1, x > 1 \\ -x + 1, x \le 1 \end{cases}$

and $y = 3 - |x| = \begin{cases} 3 + x, x \le 0 \\ 3 - x, x > 0 \end{cases}$



On solving $y = x - 1$ and $y = 3 - x$, we get $x = 2, y = 1$

Now, $AB^2 = (2 - 1)^2 + (1 - 0)^2 = 2$

$\Rightarrow \ AB = \sqrt{2}$

and $BC^2 = (0 - 2)^2 + (3 - 1)^2 = 8$

$\Rightarrow \ BC = 2\sqrt{2}$

∴ Area of rectangle $ABCD = AB \times BC$

$= \sqrt{2} \times 2\sqrt{2}$=4 sq units

26 **(a)**

Required area $= \int_1^2 (x^3 - x^2) dx$

$$= \left(\frac{x^4}{4} - \frac{x^3}{3}\right)_1^2$$

$$= \left(4 - \frac{8}{3}\right) - \left(\frac{1}{4} - \frac{1}{3}\right) = \frac{17}{22}$$

27 **(b)**

Required area $= \int_{-2}^3 |\ [x - 3]| dx$

$$= \int_{-2}^{-1} |\ [x - 3]| dx + \int_1^0 |[x - 3]| dx$$

$$+ \int_0^1 |[x - 3]| dx + \int_1^2 |[x - 3]| dx + \int_2^3 |[x - 3]| dx$$

$$= \int_{-2}^{-1} 5 \cdot dx + \int_{-1}^0 4 \cdot dx$$

$$+ \int_0^{-1} 3 \cdot dx$$

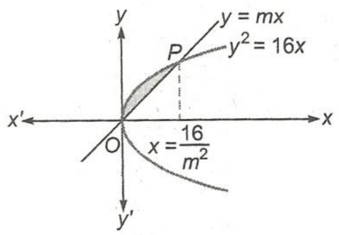
$$+ \int_1^2 2 \cdot dx + \int_2^3 1 \cdot dx$$

$= 5(1) + 4(1) + 3(1) + 2(1) + 1(1)$

$= 15$ sq unit

28 **(b)**

$$\text{Area} = \int_0^{16/m^2} \left(\sqrt{16x} - mx\right) dx = \frac{2}{3}$$



$$\Rightarrow \left[4 \cdot \frac{2}{3} x^{3/2} - \frac{mx^2}{2}\right]_0^{16/m^2} = \frac{2}{3}$$

$$\Rightarrow \frac{1}{m^3}\left[\frac{512}{3} - \frac{256}{2}\right] = \frac{2}{3}$$

$$\Rightarrow \ m^3 = \frac{128}{3} \times \frac{3}{2} = 64$$

$\Rightarrow \ m = 4$

29 **(d)**

$$\text{Requred area} = \int_3^5 (3x - 5) dx$$

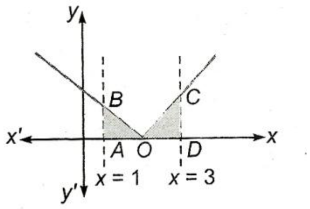
$$= \left(\frac{3x^2}{2} - 5x\right)_3^5 = \left(\frac{75}{2} - 25\right) - \left(\frac{27}{2} - 15\right)$$

$$= \frac{75}{2} - 25 - \frac{27}{2} + 15 = \frac{48}{2} - 10 = 14 \text{ sq units}$$

30 **(a)**

$$\text{Required area} = \int_1^3 |x - 2| dx$$

$$= \int_1^2 (2 - x) dx + \int_2^3 (x - 2) dx$$



$= \left[2x - \frac{x^2}{2}\right]_1^2 + \left[\frac{x^2}{2} - 2x\right]_2^3$

$= 2 - \frac{3}{2} - \frac{3}{2} + 2 = 1$sq unit

Alternate

Area= Area of ΔAOB + Area of ΔODC

$= \frac{1}{2} \times 1 \times 1 + \frac{1}{2} \times 1 \times 1$

$= 1$ sq unit

33 **(a)**

We have, $\frac{dy}{dx} = 2x + 1$

$\Rightarrow\ y = x^2 + x + c$, it passes through (1, 2)

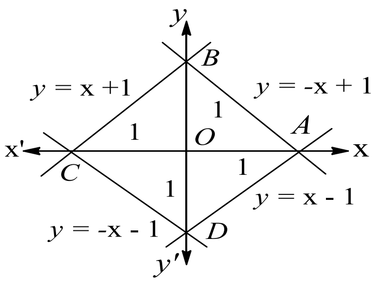
$\therefore\ c = 0$

Then, $y = x^2 + x$

$\therefore$ Required area $= \int_0^1 (x^2 + x)dx = \frac{5}{6}$ sq unit

34 **(b)**

The lines are $y = x - 1, x \geq 0$



$y = -x - 1, x < 0 = -x + 1, x \geq 0$ and

$y = x + 1, x < 0$

Required area $= (4 \times$ area of $\Delta\ AOB)$

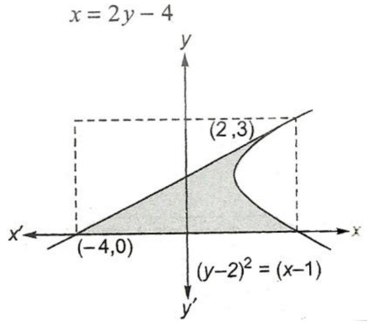
$= 4 \times \left(\frac{1}{2} \times 1 \times 1\right)$

$= 2$ sq unit

37 **(b)**

The equation of tangent at (2,3) to the given parabola is

$x = 2y - 4$



$\therefore$ Required area

$= \int_0^3 \{(y-2)^2 + 1 - 2y + 4\}dy$

$= \left[\frac{(y-2)^3}{3} - y^2 + 5y\right]_0^3$

$= \frac{1}{3} - 9 + 15 + \frac{8}{3}$

$= 9$ sq units

39 **(a)**

The equation of latusrectum of the parabola $y^2 = 12x$ is $x = 3$

Coordinates of end points of latuserectum are (3,6) and (3,-6)

Required length $= 2\int_0^3 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$= 2\int_0^3 \sqrt{1 + \left(\frac{6}{y}\right)^2}$

$= 2\int_0^3 \sqrt{\frac{12x + 36}{12x}}\, dx$

$= 2\int_0^3 \frac{x+3}{\sqrt{x^2 + 3x}}\, dx$

$= 2\left[\int_0^3 \frac{2x+3}{2\sqrt{x^2+3x}}\, dx + \frac{3}{2}\int_0^3 \frac{1}{\sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2}}\, dx\right]$

$= 2\left[\sqrt{x^2 + 3x} + \frac{3}{2}\log\left|\left(x + \frac{3}{2}\right) + \sqrt{x^2 + 3x}\right|\right]_0^3$

$= 2\left[3\sqrt{2} + \frac{3}{2}\log\left(\frac{9}{2} + 3\sqrt{2}\right) - \frac{3}{2}\log\left(\frac{3}{2}\right)\right]$

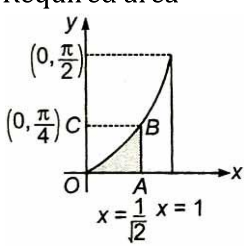
$= 2\left[3\sqrt{2} + 3\log\left(3 + 2\sqrt{2}\right)^{\frac{1}{2}}\right]$

$= 2\left[3\sqrt{2} + 3\log(\sqrt{2} + 1)\right]$

$= 6\left[\sqrt{2} + \log(1 + \sqrt{2})\right]$

40 **(d)**

Required area



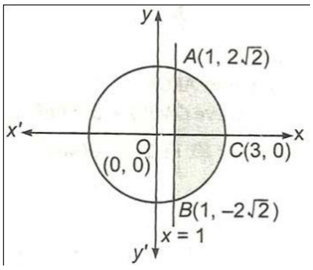
= Area of recangle $OABC$ − Area of curve $OABO$

$= \frac{\pi}{4\sqrt{2}} - \int_0^{\pi/4} \sin y\, dy$

$= \frac{\pi}{4\sqrt{2}} + [\cos y]_0^{\pi/4} = \left[\frac{\pi}{4\sqrt{2}} + \left\{\frac{1}{\sqrt{2}} - 1\right\}\right]$ sq unit

41 **(b)**

The equation of circle is $x^2 + y^2 = 9$



∴ Area of the smaller segment cut off from the circle

$x^2 + y^2 = 9$ by $x = 1$, given by

∴ Required area, $A = 2\int_1^3 \sqrt{9 - x^2}dx$

$= 2 \cdot \frac{1}{2}\left[x\sqrt{9 - x^2} + 9\sin^{-1}\frac{x}{3}\right]_1^3$

$= \left[\frac{9\pi}{2} - \sqrt{8} - 9\sin^{-1}\left(\frac{1}{3}\right)\right]$

$= \left[9\left(\frac{\pi}{2} - \sin^{-1}\left(\frac{1}{3}\right)\right) - \sqrt{8}\right]$

$= \left[9\cos^{-1}\left(\frac{1}{3}\right) - \sqrt{8}\right]$

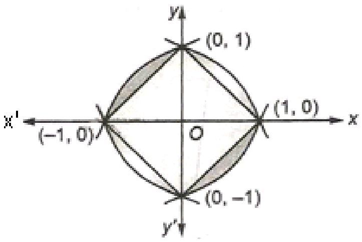
$= [9\sec^{-1}(3) - \sqrt{8}]$ sq unit

42 **(b)**

Since, $|x| + |y| = 1$

$$\Rightarrow \begin{cases} x + y = 1, x > 0, y > 0 \\ x - y = 1, x > 0, y < 0 \\ -x + y = 1, x < 0, y > 0 \\ -x - y = 1, x < 0, y < 0 \end{cases}$$

and $1 - y^2 = |x|$



$$\Rightarrow \begin{cases} 1 - y^2 = x, x \geq 0 \\ 1 - y^2 = -x, x < 0 \end{cases}$$

∴ Required area $= \left|2\int_0^1 \sqrt{(1 - x)}dx\right| + \left|2\int_{-1}^0 \sqrt{(x + 1)}dx\right| - 4\left(\frac{1}{2} \cdot 1 \cdot 1\right)$

$= \frac{2}{3}$ sq unit

45 **(c)**

Required area

$= 2\int_{-1}^3 \sqrt{x^2 - 1}\,dx$

$= 2\left[\frac{x\sqrt{x^2 - 1}}{2} - \frac{1}{2}\log|x + \sqrt{x^2 - 1}|\right]_{-1}^3$

$= \left(x\sqrt{x^2 - 1} - \log|x + \sqrt{x^2 - 1}|\right)_{-1}^3$

$= 6\sqrt{2} - \log|3 + 2\sqrt{2}|$

46 **(c)**

Requred area $= \int_1^2 \log x \; dx$

$= [x\log x - x]_1^2 = 2\log 2 - 1$

$= \log 4 - \log e = \log\left(\frac{4}{e}\right)$

48 **(d)**

$\int_1^b f(x)dx = \sqrt{(b^2 + 1)} - \sqrt{2}$

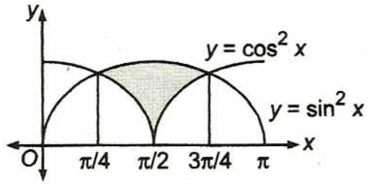
On differentiating both sides w. r. t. b, we get

$f(b) = \frac{b}{\sqrt{(b^2 + 1)}}$

Hence, $f(x) = \frac{x}{\sqrt{(x^2+1)}}$

49 **(c)**

Required area $= \int_{\pi/4}^{3\pi/4}(\sin^2 x - \cos^2 x)dx$



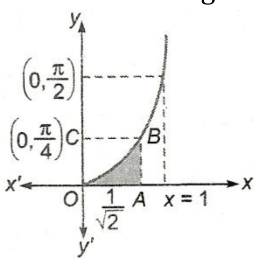
$= -\int_{\pi/4}^{3\pi/4} \cos(2x)dx = -\left[\frac{\sin 2x}{2}\right]_{\pi/4}^{3\pi/4}$

$= -\frac{1}{2}\left(\sin\frac{3\pi}{2} - \sin\frac{\pi}{2}\right) = 1$ sq unit

50 **(d)**

Required area

= area of rectangle $OABC$ − area of curve $OBCO$



$= \frac{\pi}{4\sqrt{2}} - \int_0^{\frac{\pi}{4}} \sin y\,dy$

$= \frac{\pi}{4\sqrt{2}} + [\cos y]_0^{\frac{\pi}{4}}$

$= \left[\frac{\pi}{4\sqrt{2}} + \left(\frac{1}{\sqrt{2}} - 1\right)\right]$ sq unit

53 **(d)**

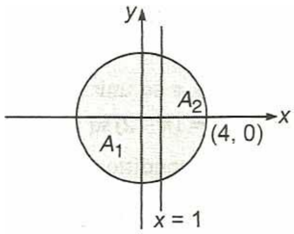
Required area $= \int_1^{1.7}[x]dx$

$= \int_1^{1.7} dx = 1.7 - 1 = 0.7 = \frac{7}{10}$

54 **(d)**

Equation of circle is $x^2 + y^2 = 16$

∴ Total area of circle $= A_1 + A_2 = 16\pi$ ...(i)



$\frac{A_1}{A_2}=\frac{16\pi}{A_2}-1$ [on dividing Eq. (i) by A_2]

and $A_2=2\int_1^4\sqrt{16-x^2}dx$

$$A_2=2\left\{\frac{x}{2}\sqrt{16-x^2}+\frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right)\right\}_1^4$$

$$=2\left\{4\pi-\frac{\sqrt{15}}{2}-8\sin^{-1}\left(\frac{1}{4}\right)\right\}$$

$$=8\pi-\sqrt{15}-16\sin^{-1}\left(\frac{1}{4}\right)$$

$$\therefore\ \frac{A_1}{A_2}=\frac{16\pi}{8\pi-\sqrt{15}-16\sin^{-1}\left(\frac{1}{4}\right)}-1$$

55 **(b)**

Given equation of ellipse is $\frac{x^2}{25}+\frac{y^2}{16}=1$

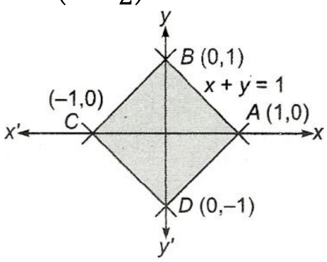
Here, $a=5, b=4$

We know that the area of an ellipse $\frac{x^4}{a^2}+\frac{y^2}{b^2}=1$ is $\pi ab=\pi(5)(4)=20\pi$ sq unit

57 **(d)**

$$\text{Area}=4\int_0^1(1-x)dx=4\left[x-\frac{x^2}{2}\right]_0^1$$

$$=4\left(1-\frac{1}{2}\right)=2\text{ sq units}$$



Alternate

From figure $ABCD$ is square, whose diagonals AC and BD are of length 2 unit.

Hence, Required area $=\frac{1}{2}\times AC\times BD$

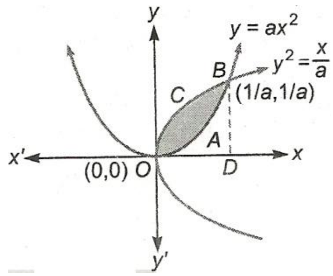
$=\frac{1}{2}\times 2\times 2$

$=2$ sq units

59 **(a)**

The points of intersection of given curves are (0,0) and

$\left(\frac{1}{a},\frac{1}{a}\right)$



$\therefore$ Required area $OABCO$

= area of $OCBDO$

− area of $OABDO$

$$\Rightarrow\ \int_0^{1/a}\left(\sqrt{\frac{x}{a}}-ax^2\right)dx=1\quad\text{[given]}$$

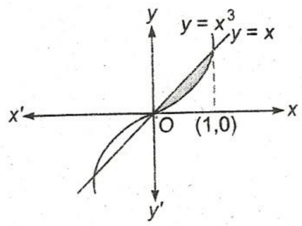
$$\Rightarrow\ \left(\frac{1}{\sqrt{a}}\cdot\frac{x^{3/2}}{3/2}-\frac{ax^3}{3}\right)_0^{1/a}=1$$

$$\Rightarrow\ \frac{2}{3a^2}-\frac{1}{3a^2}=1$$

$$\Rightarrow\ a^2=\frac{1}{3}\Rightarrow a=\frac{1}{\sqrt{3}}\qquad[\text{as } a>0]$$

60 **(a)**

$$\text{Required area}=2\int_0^1(x-x^3)dx$$

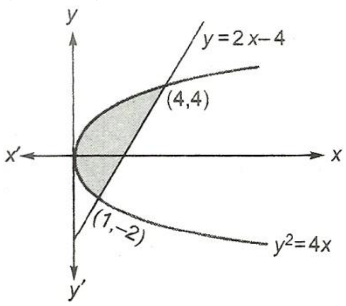


$$=2\left[\frac{x^2}{2}-\frac{x^4}{4}\right]_0^1=2\left[\frac{1}{2}-\frac{1}{4}\right]=\frac{1}{2}\text{sq unit}$$

61 **(c)**

The point of instruction of $y^2=4x$ and $y=2x-4$ is

$(2x-4)^2=4x$



$\Rightarrow\ x^2-5x+4=0$

$\Rightarrow (x-1)(x-4)=0$

$\Rightarrow x=1,4$

$\Rightarrow y=-2,4$

$\therefore$ Required area

$$\int_{-2}^4\left(\frac{y+4}{2}\right)dy-\int_{-2}^4\frac{y^2}{4}dy$$

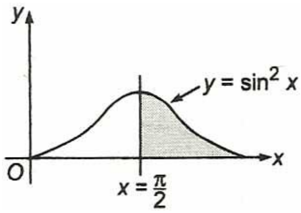
$= \frac{1}{2}\left[\frac{y^2}{2} + 4y\right]_{-2}^{4} - \frac{1}{4}\left[\frac{y^3}{3}\right]_{-2}^{4}$

$= \frac{1}{2}[8 + 16 - (2 - 8)] - \frac{1}{12}[64 + 8]$

$= 15 - 6 = 9$ sq units

63 **(b)**

Required area $A = \int_{\pi/2}^{\pi} \sin^2 x\, dx$



$= \frac{1}{2}\int_{\pi/2}^{\pi} (1 - \cos 2x)dx$

$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_{\pi/2}^{\pi}$

$= \frac{\pi}{4}$ sq unit

67 **(a)**

Given equation of curve is $y = a\sqrt{x} + bx$. This curve passes through (1, 2)

$\therefore 2 = a + b$ ...(i)

and area bounded by the curve and line $x = 4$ and x-axis is 8 sq unit, then

$\int_0^4 (a\sqrt{x} + bx)dx = 8$

$\Rightarrow \frac{2a}{3}\left[x^{3/2}\right]_0^4 + \frac{b}{2}\left[x^2\right]_0^4 = 8$

$\Rightarrow \frac{2a}{3} \cdot 8 + 8b = 8 \Rightarrow 2a + 3b = 3$ ...(ii)

On solving Eqs. (i) and (ii), we get

$a = 3$ and $b = -1$

68 **(b)**

Given. area $= \int_0^2 2^{kx}\ dx = \frac{3}{\log 2}$

$\Rightarrow \left[\frac{2^{kx}}{\log_e 2}\right]_0^2 = \frac{3}{\log 2}$

$\Rightarrow \frac{2^{2k}}{\log_e 2} - \frac{1}{\log_e 2} = \frac{3}{\log 2}$

$\Rightarrow 2^{2k} - 1 = 3$

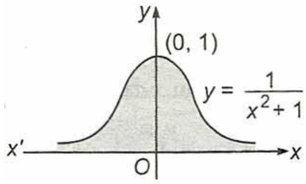
$\Rightarrow 2^{2k} = 2^2$

$\Rightarrow 2k = 2$

$\Rightarrow k = 1$

69 **(b)**

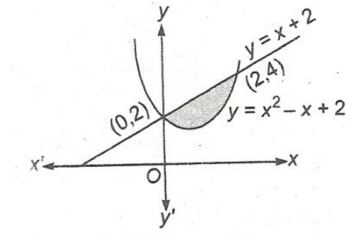
Required area $= \int_{-\infty}^{\infty} \frac{1}{x^2+1} dx$



$= 2[\tan^{-1} x]_0^{\infty} = \pi$ sq unit

70 **(d)**

Given the equation of parabola can be rewritten as



$\left(x - \frac{1}{2}\right)^2 = y - \frac{7}{4}$

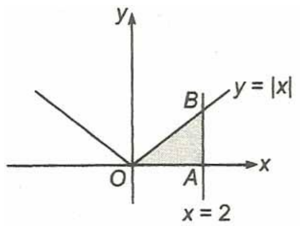
$\therefore$ Required area $= \int_0^2 [(x + 2) - (x^2 - x + 2)]dx$

$\int_0^2 (-x^2 + 2x)dx$

$= \left[-\frac{x^3}{3} + x^2\right]_0^2 = -\frac{8}{3} + 4 = \frac{4}{3}$ sq units

71 **(c)**

Required area = area of ΔOAB

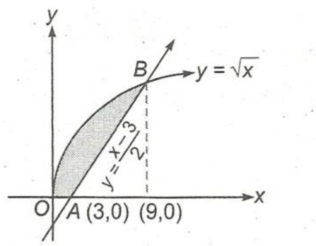


$= \frac{1}{2} \times 2 \times 2 = 2$ sq unit

72 **(a)**

Required area $OABO = \int_0^9 \sqrt{x}\, dx - \int_3^9 \left(\frac{x - 3}{2}\right) dx$

$= \left(\frac{x^{3/2}}{3/2}\right)_0^9 - \frac{1}{2}\left(\frac{x^2}{2} - 3x\right)_3^9$



$= \left(\frac{2}{3} \cdot 27\right) - \frac{1}{2}\left\{\left(\frac{81}{2} - 27\right) - \left(\frac{9}{2} - 9\right)\right\}$

$= 9$ sq units

73 **(d)**

The given equation can be rewritten as

$$\frac{(x-\sqrt{2})^2}{2/\pi}+\frac{y^2}{8/\pi}=1$$

Which represent an ellipse.

Here, $a=\sqrt{\frac{2}{\pi}}$

and $b=\sqrt{\frac{8}{\pi}}$

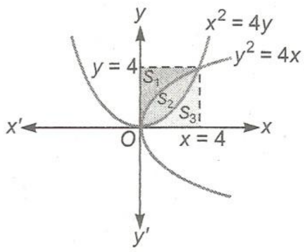
Area enclosed by an ellipse= πab

$=\pi\sqrt{\frac{2}{\pi}}\sqrt{\frac{8}{\pi}}$

$=4$ sq units

75 **(a)**

$S_1=S_3=\int_0^4 \frac{x^2}{4}dx$



$=\frac{1}{4}\left[\frac{x^3}{3}\right]_0^4=\frac{16}{3}$ sq units

Now, $S_2+S_3=\int_0^4\sqrt{4x}\,dx=2\times\left[\frac{x^{3/2}}{3/2}\right]_0^4$

$=\frac{32}{3}$ sq units

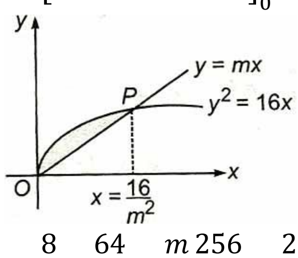
$\Longrightarrow\ s_2=\frac{16}{3}$ sq units

$\therefore\ S_1:S_2:S_3=\frac{16}{3}:\frac{16}{3}:\frac{16}{3}=1:1:1$

76 **(b)**

Required area $=\int_0^{16/m^2}\left(\sqrt{16x}-mx\right)dx=\frac{2}{3}$ (given)

$\Rightarrow\left[4\times\frac{2}{3}x^{\frac{3}{2}}-m\frac{x^2}{2}\right]_0^{16/m^2}=\frac{2}{3}$



$\Rightarrow\ \frac{8}{3}\times\frac{64}{m^3}-\frac{m}{2}\frac{256}{m^4}=\frac{2}{3}$

$\Rightarrow\ \frac{1}{m^3}\left[\frac{512}{3}-128\right]=\frac{2}{3}$

$\Rightarrow\ m=4$

77 **(d)**

For $c<1$, $\int_c^1(8x^2-x^5)dx=\frac{16}{3}$

$\Rightarrow\ \frac{8}{3}-\frac{1}{6}-\frac{8c^3}{3}+\frac{c^6}{6}=\frac{16}{3}$

$\Rightarrow\ c^3\left[-\frac{8}{3}+\frac{c^3}{6}\right]=\frac{16}{3}-\frac{8}{3}+\frac{1}{6}=\frac{17}{6}$

$\Rightarrow\ c=-1$ satisfy the above equation

For $c\geq 1$, none of the values of c satisfy the required condition that

$\int_1^c(8x^2-x^5)dx=\frac{16}{3}$

81 **(a)**

The given equation of curves are

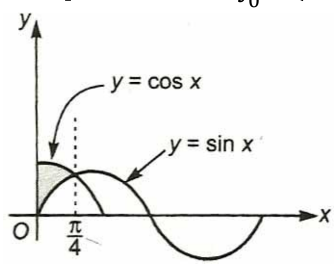
$y=\sin x$ ...(i)

and $y=\cos x$ ...(ii)

From Eqs. (i) and (ii), we get

$\sin x=\cos x\ \Rightarrow\ x=\frac{\pi}{4}$

$\therefore$ Required area $=\int_0^{\pi/4}(\cos x-\sin x)dx$



$=[\sin x+\cos x]_0^{\pi/4}$

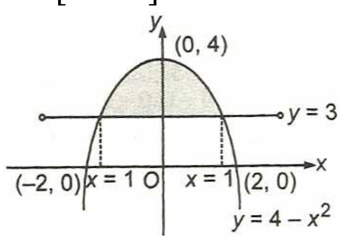
$=\left(\sin\frac{\pi}{4}+\cos\frac{\pi}{4}-\cos 0\right)$

$=\left[\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}-1\right]$

$=\frac{2}{\sqrt{2}}-1=(\sqrt{2}-1)$ sq unit

84 **(d)**

$y=\left[3+\frac{x^2}{4}\right]=3,-2<x<2$



Required area $=2\left\{\int_0^1(4-x^2)dx-3\right\}$

$=2\left\{\left[4x-\frac{x^3}{3}\right]_0^1-3\right\}$

$=2\left\{4-\frac{1}{3}-3\right\}$

$=\frac{4}{3}$ sq unit

87 **(d)**

The given eqution of curve can be written as $\frac{x^2}{4}$

$$+\frac{y^2}{9}=1$$

Here, $a=2, b=3$

$\therefore$ Required area$=\pi\ ab$

$=\pi\times 2\times 3$

$=6\pi$ sq units

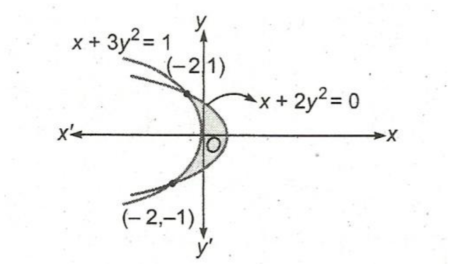
88 **(a)**

On solving the given curves, we get

$y=\pm 1$ and $x=-2$

$\therefore$ Required aerea $=\left|\int_{-1}^{1}(x_1-x_2)dy\right|$

$=\left|\int_{-1}^{1}(1-3y^2+2y^2)dy\right|$



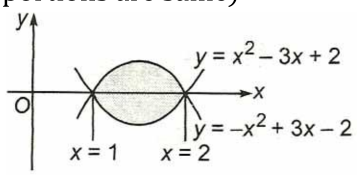
$=\left|2\int_{0}^{1}(1-y^2)dy\right|$

$=\left|2\left[y-\frac{y^3}{3}\right]_0^1\right|$

$=\frac{4}{3}$ sq units

89 **(d)**

Required area $=2\int_1^2(-x^2+3x-2)dx$ ($\because$ both portions are same)



$=2\left[-\frac{x^3}{3}+\frac{3x^2}{2}-2x\right]_1^2$

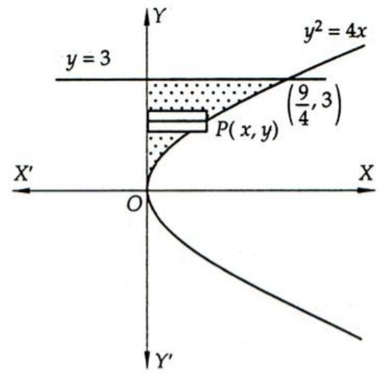
$=2\left[-\frac{8}{3}+6-4-\left(-\frac{1}{3}+\frac{3}{2}-2\right)\right]$

$=2\left[-\frac{8}{3}+2+\frac{5}{6}\right]=\frac{1}{3}$ sq unit

91 **(b)**

Let A be the required area. Then,

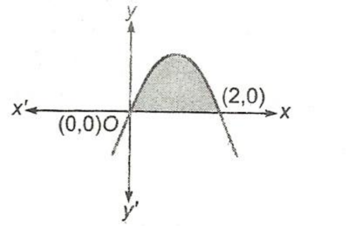
$$A=\int_0^3 x\,dy=\int_0^3\frac{y^2}{4}dy=\left[\frac{y^3}{12}\right]_0^3=\frac{27}{12}=\frac{9}{4}$$



93 **(b)**

Given curve can be rewritten as

$(x-1)^2=-(y-1)$



The curve cut the x-axis at (0,0) and (2,0)

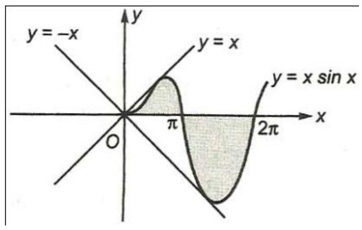
$\therefore$ Required area $=\int_0^2(2x-x^2)dx$

$=\left[x^2-\frac{x^3}{3}\right]_0^2$

$=\frac{4}{3}$ sq units

95 **(c)**

Required area $=\int_0^{\pi}x\sin x\,dx+\left|\int_{\pi}^{2\pi}x\sin x\,dx\right|$



$=\{-x\cos x+\sin x\}_0^{\pi}+|\{-x\cos x+\sin x\}_{\pi}^{2\pi}|$

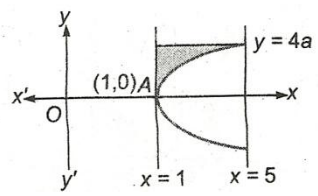
$=(\pi+0)-(0+0)+|(-2\pi+0)-(\pi+0)$

$=\pi+3\pi$

$=4\pi$ sq unit

102 **(b)**

On solving $y^2=4a^2(x-1)$and $y=4a$, we get $x=5$



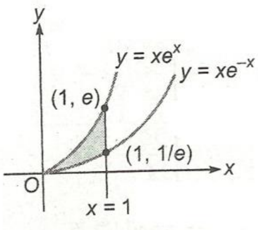
$\therefore$ Required area $=\int_1^5\left(4a-2a\sqrt{x-1}\right)dx$

$=\left[4ax-2a\frac{(x-1)^{3/2}}{3/2}\right]_1^5$

$= \dfrac{16a}{3}$ sq units

103 **(d)**

Required area $= \int_0^1 xe^x \, dx - \int_0^1 xe^{-x} \, dx$

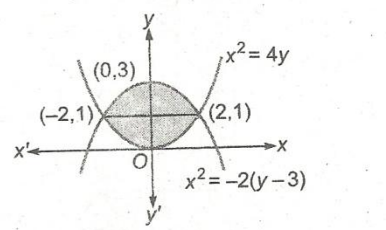


$= [xe^x - e^x]_0^1 - [-xe^{-x}e^{-x}]_0^1 = \dfrac{2}{e}$ sq unit

104 **(a)**

The point intersections of given curves are (2,1) and (−2,1).

$\therefore$ Required area $= 2\int_0^1 x \, dy + 2\int_1^3 x \, dy$



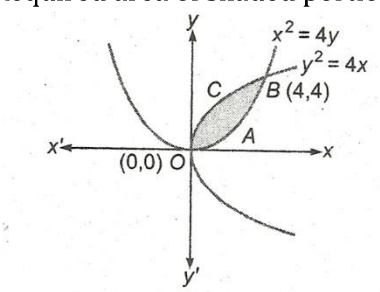
$= 2\int_0^1 \sqrt{4y} \, dy + 2\int_1^3 \sqrt{6-2y} \, dy$

$= 4\left[\dfrac{y^{3/2}}{3/2}\right]_0^1 + 2\left[\dfrac{(6-2y)^{3/2}}{3/2} \times \dfrac{1}{-2}\right]_1^3$

$= \dfrac{8}{3} + \dfrac{16}{3} = 8$ sq units

105 **(b)**

Required area of shaded portion $OABCO$



$= \int_0^4 \left(\sqrt{4x} - \dfrac{x^2}{4}\right) dx$

$= \left[\dfrac{2x^{3/2}}{3/2} - \dfrac{x^3}{12}\right]_0^4$

$= \left[\dfrac{32}{3} - \dfrac{16}{3}\right]$

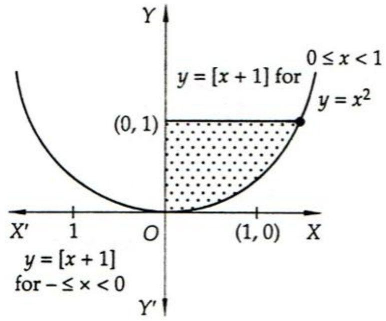
$= \dfrac{16}{3}$ sq units

106 **(b)**

We have,

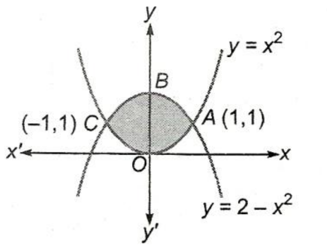
$$\int_{\pi/3a}^{\pi/a} \sin ax \, dx = 3 \Rightarrow \frac{1}{a}\left[1 + \frac{1}{2}\right] = 3 \Rightarrow 2a = 1$$

$$\Rightarrow a = \frac{1}{2}$$



107 **(a)**

Required area = 2 area of curve $OABO$



$= 2\int_0^1 [(2-x^2) - (x^2)]dx$

$= 2\int_0^1 (2-2x^2)dx$

$= 4\left[x - \dfrac{x^3}{3}\right]_0^1 = \dfrac{8}{3}$ sq units

109 **(a)**

$\because y = 2x^4 - x^2$

$\therefore \dfrac{dy}{dx} = 8x^3 - 2x$

For maxima or minima, put $\frac{dy}{dx} = 0$, we get

$x = -\dfrac{1}{2}, 0, \dfrac{1}{2}$

Then, $\left(\frac{d^2y}{dx^2}\right)_{x=\frac{1}{2}} > 0, \left(\frac{d^2y}{dx^2}\right)_{x=0} < 0$

and $\left(\frac{d^2y}{dx^2}\right)_{x=\frac{1}{2}} > 0$

$\therefore$ Required area $= \left|\int_{-1/2}^{1/2}(2x^4 - x^2)dx\right| = \frac{7}{120}$ sq unit

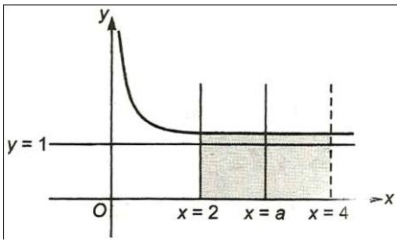
110 **(b)**

$\therefore$ Area $= \int_2^4 \left(1 + \frac{8}{x^2}\right) dx$

Since, the ordinate $x = a$ divides area into two equal parts, therefore,

$$\int_2^a \left(1 + \frac{8}{x^2}\right) dx = \frac{1}{2}\int_2^4 \left(1 + \frac{8}{x^2}\right) dx$$

$$\Rightarrow \left[x - \frac{8}{x}\right]_2^a = \frac{1}{2}\left[x - \frac{8}{x}\right]_2^4$$



$\Rightarrow \left(a-\frac{8}{a}\right)-(2-4)=\frac{1}{2}[(4-2)-(2-4)]$

$\Rightarrow a-\frac{8}{a}+2=2$

$\Rightarrow a=\sqrt{8}=2\sqrt{2}$ sq unit

111 **(a)**

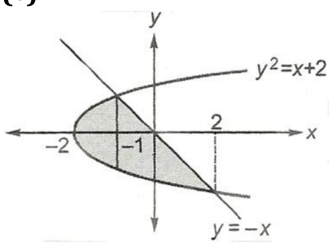
Volume $=\int_0^1 \pi\, x^2\, dy-\int_0^1 \pi\, x^2\, dy$

$=\pi\int_0^1 9(1-x^2)dy-\pi\int_0^1 9(1-y)^2\, dy$

$=9\pi\left[\left(y-\frac{y^3}{3}\right)+\frac{(1-y)^3}{3}\right]_0^1$

$=9\pi\left[\left(1-\frac{1}{3}\right)+\left(0-\frac{1}{3}\right)\right]=3\pi$

112 **(c)**



Required area

$=2\int_{-2}^{-1}\sqrt{x+2}\,dx+\int_{-1}^{2}(-x+\sqrt{x+2})dx$

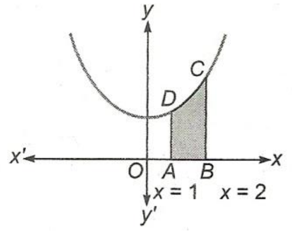
$=\frac{4}{3}[(x+2)^{3/2}]_{-2}^{-1}+\left[\frac{-x^2}{2}+\frac{2}{3}(x+2)^{3/2}\right]_{-1}^{2}$

$=\frac{9}{2}$ sq units

113 **(c)**

Required area= area of curve $ABCD$

$=\int_1^2(x^2+2)dx=\left[\frac{x^3}{3}+2x\right]_1^2$



$=\left(\frac{8}{3}+4\right)-\left(\frac{1}{3}+2\right)$

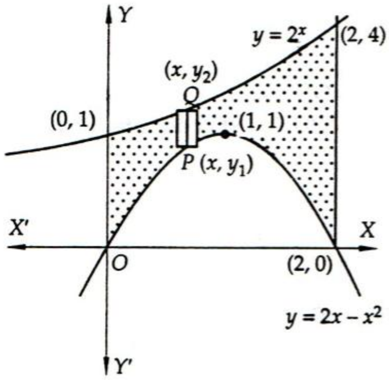
$=\frac{13}{3}$ sq units

114 **(d)**

Let the required area be A sq. units. Then,

$A=\int_0^2 (y_2-y_1)\,dx$

$\Rightarrow A=\int_0^2\{2^x-(2x-x^2)\}\,dx$



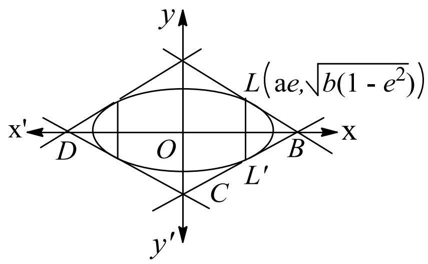
$\Rightarrow A=\left[\frac{2^x}{\log 2}-x^2+\frac{x^3}{3}\right]_0^2$

$\Rightarrow A=\frac{4}{\log 2}-4+\frac{8}{3}-\frac{1}{\log 2}$

$\Rightarrow A=\frac{3}{\log 2}-\frac{4}{3}$

115 **(d)**

Given equation of ellipse is $\frac{x^2}{9}+\frac{y^2}{5}=1$



To find tangents at the end points of latusrectum we find ae,

$ie, ae=\sqrt{a^2-b^2}=\sqrt{4}=2$

By symmetry the quadrilateral is rhombus

So, area of rhombus is four times the area of the right angled formed by the tangent and axes in the first quadrant

$\Rightarrow$ Equation of tangent at $\left[ae,\sqrt{b(1-e^2)}\right]=\left(2,\frac{5}{3}\right)$ is

$\frac{2}{9}x+\frac{5}{3}\cdot\frac{y}{5}=1$

$\Rightarrow \frac{x}{9/2}+\frac{y}{3}=1$

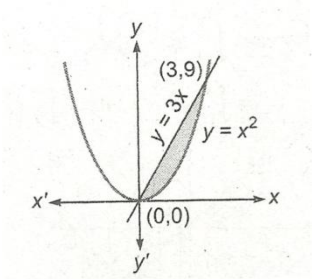
$\therefore$ Area of quadrilateral $ABCD=4(\text{area of }\Delta\, AOB)$

$=4\left(\frac{1}{2}\cdot\frac{9}{2}\cdot 3\right)=27$ sq unit

118 **(c)**

The intersection points of given curves are (0,0) and (3,9)

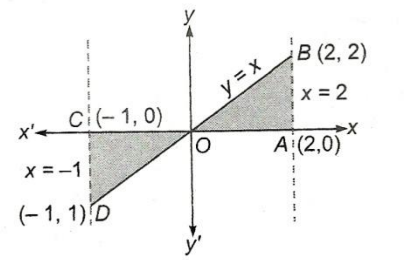
$\therefore$ Required area $= \int_0^3 (3x - x^2)dx$



$= \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3 = \frac{27}{6} = 4.5$ sq units

120 **(a)**

Required area $= \int_{-1}^{2} y\,dx = \left|\int_{-1}^{0} y\,dx\right| + \int_0^2 y\,dx$



$= \left|\int_{-1}^{0} x dx\right| + \int_0^2 x\,dx$

$= \left|\left[\frac{x^2}{2}\right]_{-1}^{0}\right| + \left[\frac{x^2}{2}\right]_0^2 = \frac{5}{2}$ sq unit

Alternate

Required area = Area of ΔOAB + Area of ΔOCD

$\frac{1}{2} \times 2 \times 2 + \frac{1}{2} \times 1 \times 1$

$= \frac{5}{2}$ sq units

121 **(b)**

Curved surface $= \int_a^b 2\ \pi y \sqrt{\left[1 + \left(\frac{dy}{dx}\right)^2\right]}\,dx$

Given that, $a = 2b = 3$ and $y = x + 1$

$\therefore \frac{dy}{dx} = 1 + 0 \Rightarrow \frac{dy}{dx} = 1$

Therefore, curved surface

$= \int_2^3 2\pi(x+1)\sqrt{[1 + (1)^2]}dx$

$= 2\sqrt{2}\ \pi \int_2^3 (x+1)dx$

$= 2\sqrt{2}\pi \left[\frac{(x+1)^2}{2}\right]_2^3 = \sqrt{2}\pi(16 - 9) = 7\pi\sqrt{2}$

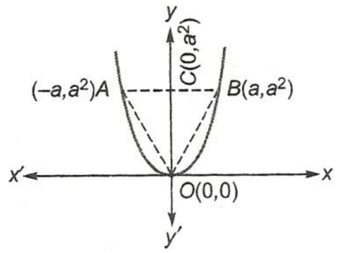
122 **(a)**

Required area $= 2\int_1^4 \sqrt{x}\,dx$

$= 2\left[\frac{2}{3}x^{3/2}\right]_1^4 = \frac{4}{3}[8 - 1] = \frac{28}{3}$ sq units

123 **(b)**

Area of curve $OAB = 2\int_0^{a^2} x\,dy$



$= 2\int_0^{a^2} \sqrt{y}\,dy = 2\left[\frac{y^{3/2}}{3/2}\right]_0^{a^2}$

$= \frac{4}{3}[a^3]$

Now, Area of $\Delta OAB = \frac{1}{2} \times AB \times OC$

$= \frac{1}{2} \times 2a \times a^2 = a^3$

$\therefore \frac{\text{Area of } \Delta\ AOB}{\text{Area of curve } AOB} = \frac{a^3}{\frac{4}{3}a^3} = \frac{3}{4}$

124 **(b)**

Area bounded by curves $y = 2^{kx}$ and $x = 0$ and $x = 2$ is given by

$A = \int_0^2 2^{kx}dx$

$= \left[\frac{2^{kx}}{k\log 2}\right]_0^2 = \left[\frac{2^{2k} - 1}{k\log 2}\right]$

But $A = \frac{3}{\log 2}$

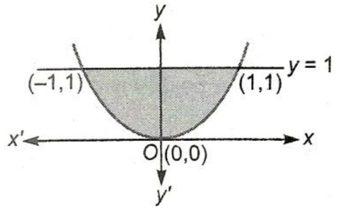
$\therefore \frac{2^{2k} - 1}{k\log 2} = \frac{3}{\log 2} \Rightarrow 2^{2k} - 1 = 3k$

This, relation is satisfied by only option (b)

127 **(b)**

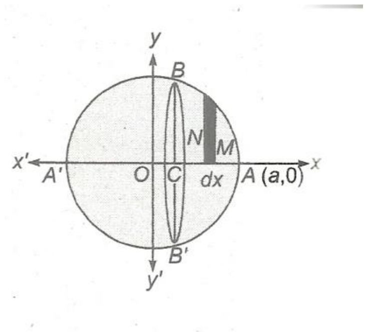
Required volume $= \pi \int_{-1}^{1} y^2 dx = 2\pi \int_0^1 x^4 dx$

$= 2\pi \left[\frac{x^5}{5}\right]_0^1 = \frac{2\pi}{5}$ cu unit



128 **(a)**

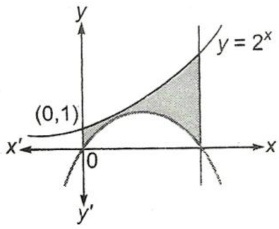
The required volume of the segment is generated by revolving the area $ABCA$ of the circle $x^2 + y^2 = a^2$ about the x-axis and for the arc BA.



Here, $CA = h$
and $OA = a$ [given]
$\therefore OC = OA - CA = a - h$
$\therefore x$ varies from $a - h$ to a

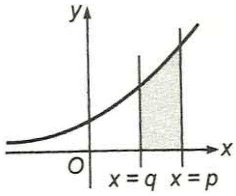
$$\therefore \text{The required volume} = \int_{a-h}^{a} \pi y^2\, dx$$
$$= \pi \int_{a-h}^{a} (a^2 - x^2) dx = \pi \left[a^2 x - \frac{1}{3}x^3\right]_{a-h}^{a}$$
$$= \pi \left[\left(a^3 - \frac{1}{3}a^3\right) - \left\{a^3 - a^2 h - \frac{1}{3}(a^3 - 3a^2 h + 3ah^2 - h^3)\right\}\right]$$
$$= \pi \left[a^2 h - a^2 h + ah^2 - \frac{1}{3}h^3\right]$$
$$= \frac{1}{3}\pi h^2 (3a - h)$$

129 **(b)**
Required area
$$= \int_0^2 [2^x - (2x - 2^2)] dx$$

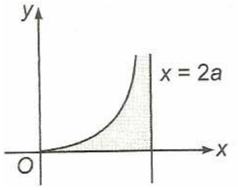


$$= \left[\frac{2^x}{\log 2} - x^2 + \frac{x^3}{3}\right]_0^2$$
$$= \frac{4}{\log 2} - 4 + \frac{8}{3} - \frac{1}{\log 2}$$
$$= \left(\frac{3}{\log 2} - \frac{4}{3}\right) \text{ sq unit}$$

130 **(b)**
Required area $= \int_q^p ce^x dx$
$= [ce^x]_q^p$
$= c[e^p - e^q]$
$= f(p) - f(q)$



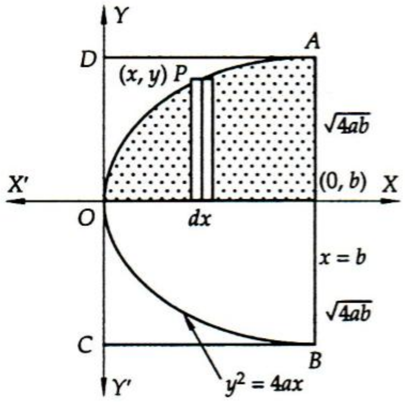
132 **(b)**
Given equation of curve is
$y^2(2a - x) = x^3$



Which is symmetrical about x-axis and passes through origin
Also, $\frac{x^3}{2a-x} < 0$
For $x > 2a$ or $x < 0$
So, curve does not lie in $x > 2a$ and $x < 0$, therefore curve lies wholly on $0 \le x \le 2a$
$\therefore$ Required area $= \int_0^{2a} \frac{x^{3/2}}{\sqrt{2a-x}} dx$
Put $x = 2\, a \sin^2 \theta$
$\Rightarrow dx = 2a \cdot 2 \sin\theta + \cos\theta d\theta$
$\therefore$ Required area $= \int_0^{\pi/2} 8a^2 \sin^4 \theta d\theta$
$= 8a^2 \left[\frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}\right]$ (using gamma function)
$= \frac{3\pi a^2}{2}$ sq unit

133 **(c)**
Required area $= 2\int_0^a \sqrt{4ax} dx$
$= 4\sqrt{a} \times \frac{2}{3}\left[x^{3/2}\right]_0^a = \frac{8}{3}a^2$ sq unit

134 **(c)**
Let $y^2 = 4ax$ be a parabola and let $x = b$ be a double ordinate. Then,
$A_1 =$ Area enclosed by the parabola $y^2 = 4ax$ and the double ordinate $x = b$
$$\Rightarrow A_1 = 2\int_0^b y\, dx = 2\int_0^b \sqrt{4ax}\, dx = 4\sqrt{a}\int_0^b \sqrt{x^3}\, dx$$
$$\Rightarrow A_1 = 4\sqrt{a}\left[\frac{2}{3}x^{3/2}\right]_0^b = 4\sqrt{a} \times \frac{2}{3} b^{3/2}$$
$$= \frac{8}{3} a^{1/2} b^{3/2}$$



And, $A_2 =$ Area of the rectangle $ABCD$

$\Rightarrow A_2 = AB \times AD = 2\sqrt{4ab} \times b = 4\, a^{1/2} b^{3/2}$

$\therefore A_1 : A_2 = 8/3\, a^{1/2} b^{3/2} : 4\, a^{1/2} b^{3/2} = 2/3 : 1$

$= 2 : 3$

136 **(b)**

The curve $y^2(2a - x) = x^3$ is symmetrical about x-axis and passes through origin.

Also, $\frac{x^3}{2a - x} < 0\ for\ x > 2a$ and $x < 0$

So, curve does not lie in $x > 2a$ and $x < 0$, therefore curves lies wholly on $0 \le x \le 2a$

$\therefore$ Requried area $= \int_0^{2a} \frac{x^{3/2}}{\sqrt{2a - x}} dx$

Put $x = 2a\sin^2\theta$

$\Longrightarrow$ $0\ dx = 4a \sin\theta \cos\theta\ d\ \theta$

$\therefore$ Requried area $= \int_0^{\frac{\pi}{2}} 8a^2 \sin^4\theta\ d\ \theta$

$= 8a^2 \left[\frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}\right]$

$= \frac{3\pi a^2}{2}$ sq unit

137 **(b)**

Intersection points of given curves are $(-1,0)$ and $(3,0)$

Required area $= \int_{-1}^{3} (-x^2 + 2x + 3) dx$

$= \left[\frac{x^3}{3} + \frac{2x^2}{2} + 3x\right]_{-1}^{3}$

$= \left[-9 + 9 + 9 - \left(\frac{1}{3} + 1 - 3\right)\right]$

$= \frac{32}{3}$ sq units

138 **(b)**

Given curve $a^4 y^2 = (2a - x)x^5$

Cut off x-axis, when $y = 0$

$0 = (2a - x)x^5$

$\therefore x = 0, 2a$

Hence, the area bounded by the curve $a^4 y^2 = (2a - x)x^5$ is

$A_1 = \int_0^{2a} \frac{\sqrt{(2a - x)} x^{5/2}}{a^2} dx$

Put $x = 2a \sin^2\theta$

$\therefore dx = 4a \sin\theta \cos\theta d\theta$

$\therefore A_1$

$= \int_0^{\pi/2} \frac{\sqrt{2a} \cos\theta (2a)^{5/2} \sin^5\theta\ 4a \sin\theta \cos\theta}{a^2} d\theta$

$= 32a^2 \int_0^{\pi/2} \sin^6\theta \cos^2\theta\ d\theta$

$= 32a^2 \cdot \frac{(5\cdot3\cdot1)(1)}{8\cdot6\cdot4\cdot2} \cdot \frac{\pi}{2}$ (by walli's formula)

$= \frac{5\pi a^2}{8}$

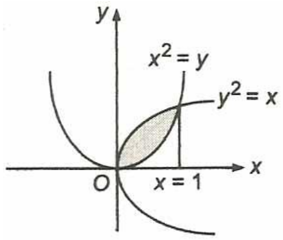
Area of circle, $A_2 = \pi a^2$

$\therefore \frac{A_1}{A_2} = \frac{5}{8}$

$\Rightarrow A_1 : A_2 = 5:8$

139 **(d)**

Required area $= \int_0^1 (\sqrt{x} - x^2) dx$



$= \left[\frac{2x^{3/2}}{3} - \frac{x^3}{3}\right]_0^1 = \left(\frac{2}{3} - \frac{1}{3}\right) = \frac{1}{3}$ sq unit

142 **(c)**

We have,

$A = \int_0^{\pi/4} \sin x\, dx = [-\cos x]_0^{\pi/4} = \frac{\sqrt{2} - 1}{\sqrt{2}}$

$= 1 - \frac{1}{\sqrt{2}}$ ... (i)

Let A_1 be the required area. Then,

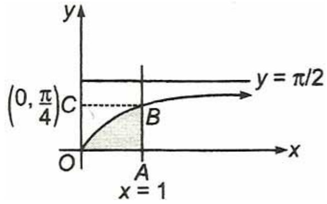
$A_1 = \int_0^{\pi/4} \cos x\, dx = [\sin x]_0^{\pi/4} = \frac{1}{\sqrt{2}}$ ... (ii)

From (i) and (ii), we have

Required area $A_1 = 1 - A$

143 **(b)**

Required area = Area of rectangle $OABC$ −Area of curve $OABO$



$\frac{\pi}{4} - \int_0^{\pi/4} \tan y \, dy$

$= \frac{\pi}{4} + [\log \cos y]_0^{\pi/4}$

$= \frac{\pi}{4} + \log \cos \frac{\pi}{4} - \log \cos(0)$

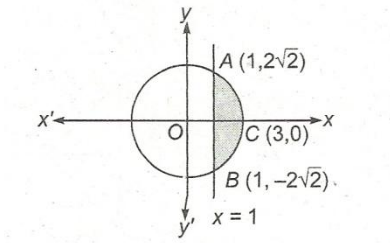
$= \frac{\pi}{4} + \log 1 - \log \sqrt{2} - \log 1$

$= \left(\frac{\pi}{4} - \log \sqrt{2}\right)$ sq unit

144 **(b)**

Required area $= 2 \int_1^3 \sqrt{9 - x^2}\, dx$

$= 2 \cdot \frac{1}{2} \left[x\sqrt{9 - x^2} + 9 \sin^{-1} \frac{x}{3}\right]_1^3$



$= \left[9 \sin^{-1}(1) - \sqrt{8} - 9 \sin^{-1}\left(\frac{1}{3}\right)\right]$

$= \left[9\left\{\cos^{-1}\left(\frac{1}{3}\right)\right\} - \sqrt{8}\right] \left[\because \ \cos^{-1}\theta = \frac{\pi}{2} - \sin^{-1}\theta\right]$

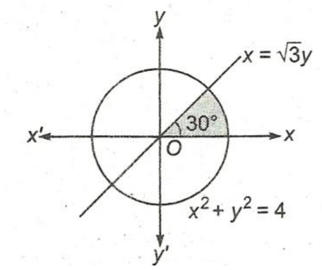
$= [9\sec^{-1}(3) - \sqrt{8}]$sq unit

145 **(c)**

Required area $= \int_0^1 (x_2 - x_1) dy$

$= \int_0^1 \left(\sqrt{4 - y^2} - \sqrt{3}\, y\right) dy$

$= \left[\frac{1}{2} y\sqrt{4 - y^2} + \frac{1}{2}(4)\sin^{-1}\frac{y}{2} - \frac{\sqrt{3}\, y^2}{2}\right]_0^1$



$= \frac{\sqrt{3}}{2} + \sin^{-1}\left(\frac{1}{2}\right) - \frac{\sqrt{3}}{2} - 2 \sin^{-1} 0$

$= \frac{\pi}{3}$sq units

Alternate

$=$ Area $= \frac{\theta}{360°} \times \pi r^2$

$= \frac{30}{360} \times \pi(2)^2$

$= \frac{\pi}{3}$sq units

148 **(d)**

Given equation of circle and line are

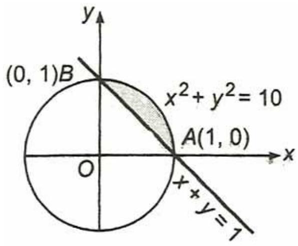
$x^2 + y^2 = 1$ ...(i)

and $x + y = 1$ ...(ii)

From Eqs. (i) and (ii),

$x^2 + (1 - x)^2 = 1$

$\Rightarrow \ x^2 + 1 + x^2 - 2x = 1$



$\Rightarrow \ 2x^2 - 2x = 0 \ \Rightarrow \ 2x(x - 1) = 0$

$\Rightarrow \ x = 0, x = 1 \ \Rightarrow \ y = 1, y = 0$

∴ Point of intersection of circle and line are $A(1, 0)$ and $B(0, 1)$

∴ Required area $= \int_0^1 \left[\sqrt{1 - x^2} - (1 - x)\right] dx$

$= \left[\frac{x\sqrt{1 - x^2}}{2} + \frac{1}{2}\sin^{-1} x - x + \frac{x^2}{2}\right]_0^1$

$= \frac{1}{2} \cdot \frac{\pi}{2} - 1 + \frac{1}{2}$

$= \left(\frac{\pi}{4} - \frac{1}{2}\right)$ sq unit

149 **(c)**

$\because \ \int_1^b f(x) dx = (b - 1) \sin(3b + 4)$

∴ On differentiating both sides with respect to b, we get

$f(b) = 3(b - 1) \cos(3b + 4) + \sin(3b + 4)$

$\therefore \ f(x) = 3(x - 1) \cos(3x + 4) + \sin(3x + 4)$

150 **(c)**

The required area A is given by

$$A = \int_0^a (y_1 - y_2)\, dx$$

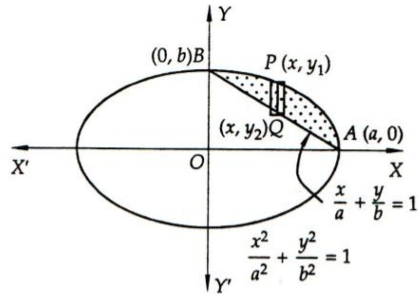
$$\Rightarrow A = \int_0^a \left\{\frac{b}{a}\sqrt{a^2 - x^2} - \frac{b}{a}(a - x)\right\} dx$$

$\Rightarrow A = \frac{b}{a}\left[\frac{1}{2}x\sqrt{a^2-x^2}+\frac{1}{2}a^2\sin^{-1}\frac{x}{a}\right]^a$

$+\frac{b}{2a}[(a-x)^2]_0^a$

$\Rightarrow A = \frac{b}{a}\left\{\frac{1}{2}a^2\sin^{-1}(1)\right\}+\frac{b}{2a}(0-a^2)$

$\Rightarrow A = \frac{\pi}{4}ab-\frac{1}{2}ab=\frac{ab}{4}(\pi-2)$



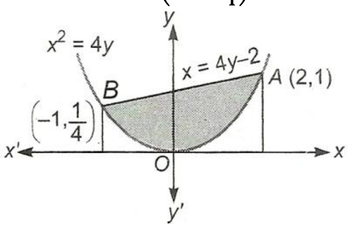
ALITER A = Area of the ellipse in first quadrant – Area of ΔOAB

$\Rightarrow A = \frac{\pi ab}{4}-\frac{1}{2}ab=\frac{ab}{4}(\pi-2)$

151 **(b)**

The point of intersection of the parabola and the line are

$A(2,1)$and $B\left(1-\frac{1}{4}\right)$



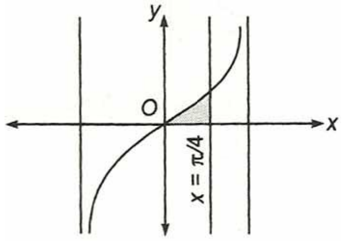
$\therefore$ The requerd area $=\left[\left[\int_{-1}^{2}y\,dx\right]-\left[\int_{-1}^{2}y\,dx\right]\right]$

$=\int_{-1}^{2}\frac{1}{4}(x+2)dx-\int_{-1}^{2}\frac{1}{4}x^2\,dx$

$=\frac{1}{4}\left[\frac{x^2}{2}+2x\right]_{-1}^{2}-\frac{1}{4}\left[\frac{x^3}{3}\right]_{-1}^{2}=\frac{9}{8}$ sq units

152 **(d)**

Required area $=\int_0^{\pi/4}\tan x dx=[\log\sec x]_0^{\pi/4}$



$=\log\sec\left(\frac{\pi}{4}\right)-\log\sec 0$

$=\log\sqrt{2}-\log 1=\log\sqrt{2}$ sq unit

153 **(b)**

We have,

$A_1 = 2\int_0^a\sqrt{4ax}\,dx$ and, A_2

$=2\int_0^{2a}\sqrt{4ax}-2\int_0^a\sqrt{4ax}\,dx$

$\Rightarrow A_1=\frac{8a^2}{3}$ and, $A_2=\frac{16}{3}\sqrt{2}a^2-\frac{8}{3}a^2$

$\Rightarrow \frac{A_1}{A_2}=\frac{1}{2\sqrt{2}-1}-\frac{2\sqrt{2}+1}{7}$

154 **(a)**

Required area

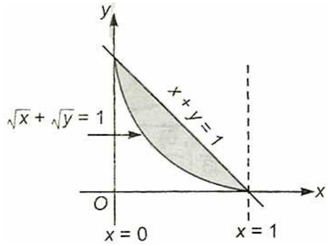
$A=\int_{-1}^{1}(-x^2+2)dx+\int_1^2(2x-1)dx$

$=\left[-\frac{x^3}{3}+2x\right]_{-1}^{1}+[x^2-x]_1^2$

$=\frac{10}{3}+2=\frac{16}{3}$ sq unit

156 **(a)**

Required area = area of $\Delta\ AOB-\int_0^1\left(1-\sqrt{x}\right)^2dx$



$=\frac{1}{2}\times 1\times 1-\left[x-\frac{2x^{3/2}}{3}\right]_0^1$

$=\frac{1}{3}$ sq unit

157 **(c)**

Given area bounded by the curve, $y=\sqrt{3x+4}$, x-axis and the line $x=-1$ and $x=4$ is A and area bounded by the curve $y=\sqrt{3x+4}$

$ie, y=\pm(3x+4)^{1/2}$ x-axis and the line $x=-1$ and $x=4$ is B

$\therefore B=2A$

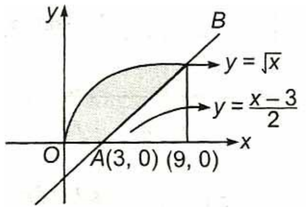
[Since, it is the area of both sides about x-axis]

Now, $A:B=A:2\,A=1:2$

158 **(a)**

Required area $=\int_0^9\sqrt{x}dx-\int_3^9\left(\frac{x-3}{2}\right)dx$

$=\left(\frac{x^{3/2}}{3/2}\right)_0^9-\frac{1}{2}\left(\frac{x^2}{2}-3x\right)_3^9$



$= \left(\frac{2}{7}\cdot 27\right) - \frac{1}{2}\left\{\left(\frac{81}{2} - 27\right) - \left(\frac{9}{2} - 9\right)\right\}$

$= 18 - 9 = 9$ sq unit

160 **(a)**

Given, $\int_{\pi/4}^{\beta} f(x)dx = \beta \sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\,\beta$

On differentiating w.r.t. β on both sides, we get

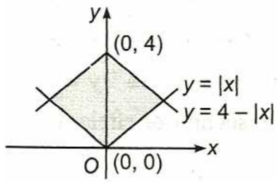
$f(\beta) = \sin\beta + \beta\cos\beta - \frac{\pi}{4}\sin\beta + \sqrt{2}$

Put $\beta = \frac{\pi}{2}$

Then, $f\left(\frac{\pi}{2}\right) = \sin\frac{\pi}{2} + \frac{\pi}{2}\cos\frac{\pi}{2} - \frac{\pi}{4}\sin\frac{\pi}{2} + \sqrt{2} = 1 - \frac{\pi}{4} + \sqrt{2}$

161 **(d)**

It is a square of diagonal of length 4 unit and sides is $2\sqrt{2}$



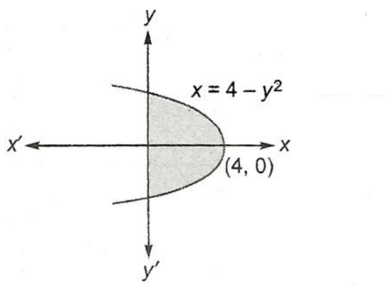
$\therefore$ Required area, $A = \left(2\sqrt{2}\right)^2 = 8$ sq unit

163 **(c)**

Required area $= \int_{-\pi/3}^{\pi/3} \sec^2 x\, dx$

$= [\tan x]_{-\pi/3}^{\pi/3} = 2\sqrt{3}$ sq unit

164 **(c)**



The required area

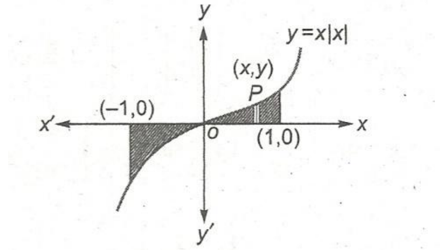
$= 2 \times \int_0^4 \sqrt{4 - x}\; dx$

$= 2\left[\frac{(4-x)^{3/2}}{3/2}\right]_0^4 = 2\left[-\frac{2}{3}\times 0 + \frac{2}{3}(4)^{3/2}\right]$

$= \frac{32}{2}$ sq units

165 **(c)**

Required area $= \left|\int_{-1}^{1} x|x|\, dx\right|$



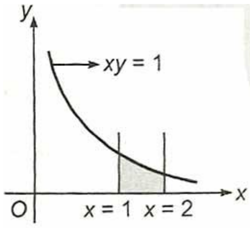
$= \left|\int_{-1}^{0} x|x|\, dx\right| + \int_0^1 x|x|dx$

$= \left|\int_{-1}^{0} -x^2\, dx\right| + \int_0^1 x^2 dx$

$= \left|\left[\frac{-x^3}{3}\right]_{-1}^{0}\right| + \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

168 **(a)**

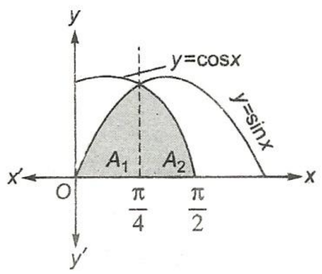
Required area $= \int_1^2 \frac{1}{x}dx$



$= [\log|x|]_1^2 = \log 2$ sq unit

172 **(d)**

Area, $A_1 = \int_0^{\frac{\pi}{4}} \sin x\, dx$



$= -[\cos x]_0^{\pi/4}$

$= 1 - \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{2} - 1}{\sqrt{2}}$

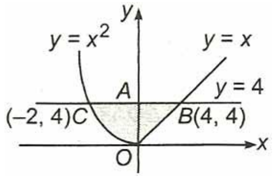
and area, $A_2 = \int_{\pi/4}^{\pi/2} \cos x\, dx$

$= [\sin x]_{\pi/4}^{\pi/2} = \left[1 - \frac{1}{\sqrt{2}}\right] = \frac{\sqrt{2}-1}{\sqrt{2}}$

$\therefore\ A_1 : A_2 = \frac{\sqrt{2}-1}{\sqrt{2}} : \frac{\sqrt{2}-1}{\sqrt{2}} = 1:1$

173 **(c)**

Required area $= \int_{-2}^{0} (4 - x^2)dx + \int_0^4 (4 - x)dx$

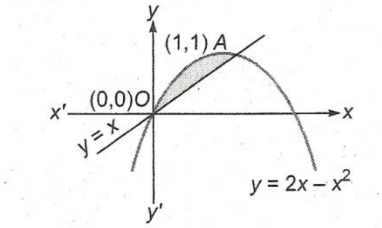


$= \left[4x - \frac{x^3}{3}\right]_{-2}^{0} + \left[4x - \frac{x^2}{2}\right]_{0}^{4}$

$= 8 - \frac{8}{3} + 8 = \frac{40}{3}$ sq unit

175 **(b)**

The intersection points of given curves are (0,0) and (1,1)



$\therefore$ Required area $= \int_0^1 [(2x - x^2) - x]dx$

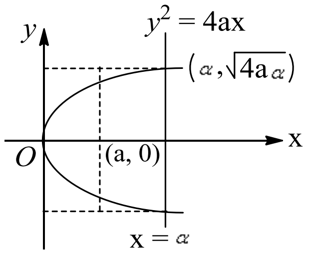
$= \int_0^1 (x - x^2)dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$

$= \frac{1}{6}$ sq unit

177 **(a)**

Required area $= 2\int_0^{\alpha} \sqrt{4ax}dx$

$= k(\alpha)(2\sqrt{4a\alpha})$

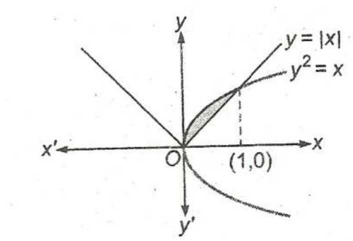


$\frac{8\sqrt{a}}{3}\alpha^{3/2} = 4\sqrt{a}k\alpha^{3/2}$

$\Rightarrow k = \frac{2}{3}$

178 **(c)**

Required area $= \int_0^1 (\sqrt{x} - x)dx$



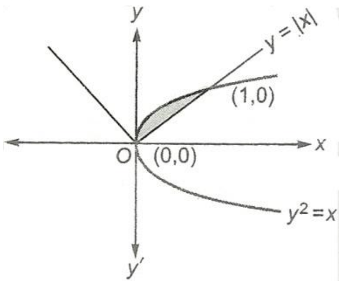
$= \left[\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_0^1 = \frac{1}{6}$ sq unit

179 **(b)**

Clearly,

Required are $= \int_0^{\pi} a \sin x \, dx = 2a$

180 **(b)**



Required area $= \int_0^1 (\sqrt{x} - x)dx = \left(\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right)_0^1$

$= \frac{2}{3} - \frac{1}{2} = \frac{4-3}{6} = \frac{1}{6}$ sq unit

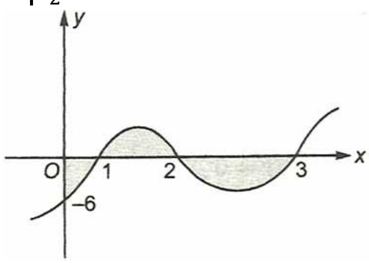
181 **(b)**

Required area

$= \left|\int_0^1 (x-1)(x-2)(x-3)dx\right|$

$+ \int_1^2 (x-1)(x-2)(x-3)dx$

$+ \left|\int_2^3 (x-1)(x-2)(x-3)dx\right|$

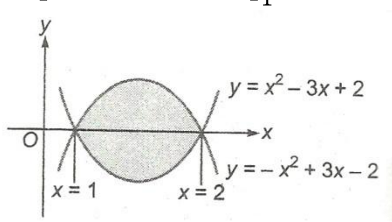


$= \frac{9}{4} + \frac{1}{4} + \frac{1}{4} = \frac{11}{4}$ sq unit

183 **(d)**

Required area $= 2\int_1^2 (-x^2 + 3x - 2)dx$

$= 2\left[\frac{-x^3}{3} + \frac{3x^2}{2} - 2x\right]_1^2$

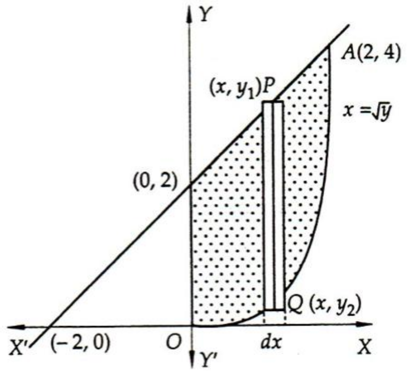


$= 2\left[-\frac{8}{3} + 4 - \frac{7}{6}\right] = \frac{1}{3}$ sq unit

185 **(c)**

Let A be the n,

$$A = \int_0^2 (y_1 - y_2)\, dx$$



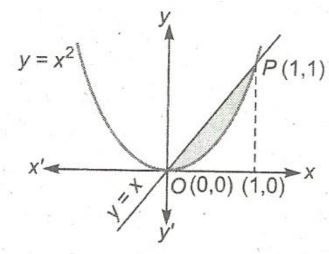
$$\Rightarrow A = \int_0^2 [(x+2) - (x^2)]\, dx = \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_0^2$$

$$= 2 + 4 - \frac{8}{3} = \frac{10}{3}$$

187 **(a)**

The points of intersection of given curves are O (0,0) and $P(1,1)$.

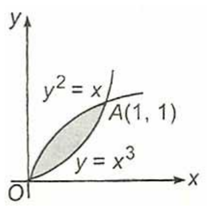
$\therefore$ Required area $= \int_0^1 x\ dx - \int_0^1 x^2\ dx$



$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{6}$ sq unit

188 **(c)**

On solving $y = \sqrt{x}$ or $y^2 = x, (y \geq 0)$ and $y = x^3$



We get points of intersection which are (0, 0) and (1, 1)

$\therefore$ Required area $= \int_0^1 (\sqrt{x} - x^3) dx$

$= \left[\frac{x^{3/2}}{3/2} - \frac{x^4}{4}\right]_0^1 = \frac{5}{12}$ sq unit

189 **(a)**

According to the given condition,

Area of curve $= \int_0^a f(x) dx$

$\Longrightarrow \frac{a^2}{2} + \frac{a}{2}\sin a + \frac{\pi}{2}\cos a$

$= \int_0^a f(x) dx$

On differentiating both sides w.r.t. a, we get

$a + \frac{1}{2}\sin a + \frac{a}{2}\cos a - \frac{\pi}{2}\sin a = f(a)$

$\Longrightarrow\ f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + \frac{1}{2}\sin\frac{\pi}{2} + \frac{\pi}{4}\cos\frac{\pi}{2} - \frac{\pi}{2}\sin\frac{\pi}{2}$

$\Longrightarrow\ f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + \frac{1}{2} - \frac{\pi}{2}$

$\Longrightarrow\ f\left(\frac{\pi}{2}\right) = \frac{1}{2}$

190 **(a)**

The required area A is given by

$$A = \int_0^1 (e^x - e^{-x})\, dx = e + \frac{1}{e}$$

191 **(d)**

Since, $|x| = 1$

$\therefore\ x = \pm 1$

$y = xe^{|-x|} = \begin{cases} xe^{-x}, -1 < x < 0 \\ xe^{x}, 0 \leq x < 1 \end{cases}$

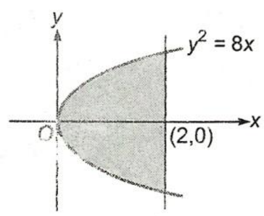
$\therefore$ Required area $= \left|\int_{-1}^0 xe^{-x} dx\right| + \left|\int_0^1 xe^x dx\right|$

$= \left|\{-xe^{-x} - e^{-x}\}_{-1}^0\right| + \left|\{xe^x - e^x\}_0^1\right|$

$= 2$ sq unit

192 **(b)**

Required area $= 2\int_0^2 \sqrt{8x}\ dx = 4\sqrt{2}\left[\frac{x^{3/2}}{3/2}\right]_0^2$



$= 4\sqrt{2}\left[\frac{2\sqrt{2}}{3/2}\right]$

$= \frac{32}{3}$ sq units

193 **(c)**

We have,

A_1 = Area bounded by the two curves

$\Rightarrow A_1 = \int_0^2 \sqrt{6\,x}\, dx + \int_2^4 \sqrt{16 - x^2}\, dx$

$$= \frac{4\sqrt{3} + 16\,\pi}{3}$$

A_2 = Area bounded by $x^2 + y^2 = 16$ and outside $y^2 = 6\,x$

$\Rightarrow A_2 = 16\,\pi - \frac{4\sqrt{3} + 16\,\pi}{3} = \frac{32\,\pi - 4\sqrt{3}}{3}$

$\therefore$ Required ratio $= A_1 : A_2 = 4\,\pi + \sqrt{3} : 8\,\pi - \sqrt{3}$

194 **(b)**

We have,

$$A = \int_{0}^{\pi/2} \sin x \, dx = 1$$

Let A_1 be the required area. Then,

$$A_1 = \int_{0}^{\pi/2} \sin 2x \, dx \Rightarrow A_1 = -\frac{1}{2}[\cos 2x]_0^{\pi/2}$$

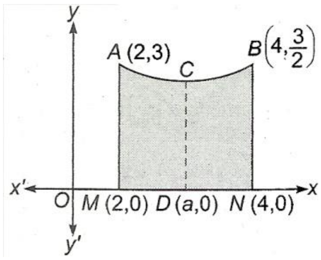
$$= -\frac{1}{2}[\cos \pi - 1] = 1 = A$$

Clearly, $A_1 = A$

195 **(b)**

Area of curve $MNBA = \int_2^4 \left(1 + \frac{8}{x^2}\right) dx$

$= \left[x - \frac{8}{x}\right]_2^{-4} = 4$ ….(i)



Area of curve $ACDM = \int_2^a \left(1 + \frac{8}{x^2}\right) dx$

$= \left[x - \frac{8}{x}\right]_2^{a} = a - \frac{8}{a} - [2 - 4] = a - \frac{8}{a} + 2$ ….(ii)

Form Eqs. (i) and (ii), we get

$$a - \frac{8}{a} + 2 = \frac{1}{4}(4)$$

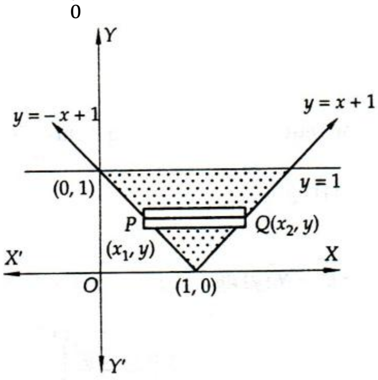
$\Rightarrow \ a^2 - 8 = 0 \ \Rightarrow a = 2\sqrt{2} \quad [\because a > 0]$

196 **(a)**

Let A denote the required area. Then,

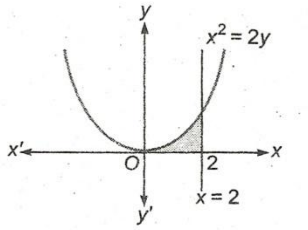
$$A = \int_0^1 (x_2 - x_1)\, dy = \int_0^1 \{(x+1) - (-x+1)\}\, dx$$

$$\Rightarrow A = \int_0^1 2x \, dx = [x^2]_0^1 = 1$$



198 **(d)**

Required area $= \int_0^2 y \ dx$



$= \int_0^2 \frac{x^2}{2} dx = \left[\frac{x^3}{6}\right]_0^2 = \frac{4}{3}$ sq units

200 **(a)**

Equation of curve are $y = 0$ …(i)

and $y = 4 + 3x - x^2$ …(ii)

On solving Eqs. (i) and (ii), we get

$x = -1, 4$

$\therefore$ Curve does not intersect x-axis between $x = -1$ and $x = 4$

$\therefore$ Required area $= \int_{-1}^4 (4 + 3x - x^2) dx$

$= \left[4x + \frac{3x^2}{2} - \frac{x^3}{3}\right]_{-1}^4$

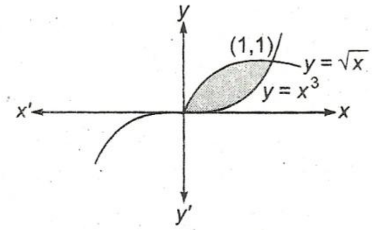
$= \left[16 + 24 - \frac{64}{3} + 4 - \frac{3}{2} - \frac{1}{3}\right]$

$= 44 - \frac{65}{3} - \frac{3}{2}$

$= \frac{264 - 130 - 9}{6} = \frac{125}{6}$ sq unit

203 **(c)**

The point of intersection of given curves are (0,0) and (1,1).



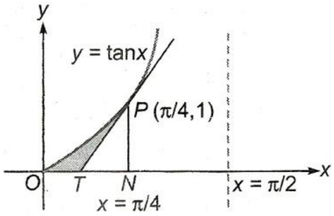
$\therefore$ Required area $= \int_0^1 \left(\sqrt{x} - x^3\right) dx$

$= \left[\frac{x^{3/2}}{3/2} - \frac{x^4}{4}\right]_0^1$

$= \frac{5}{12}$ sq unit

206 **(c)**

When $x = \frac{\pi}{4}$



$y = \tan\frac{\pi}{4} = 1$

$\frac{dy}{dx} = \sec^2 x \quad [\because y = \tan x]$

$\Rightarrow \left[\frac{dy}{dx}\right]_{x=\pi/4} = 2$

Equation of tangent at $P\left(\frac{\pi}{4}, 1\right)$ is

$y - 1 = 2\left(x - \frac{\pi}{4}\right) \Rightarrow y = 2x + 1 - \frac{\pi}{2}$

It meets $x-$ axis at $T\left(\frac{\pi - 2}{4}, 0\right)$

Required area $= \int_0^{\pi/4} \tan x\, dx - \frac{1}{2} TN \cdot PN$

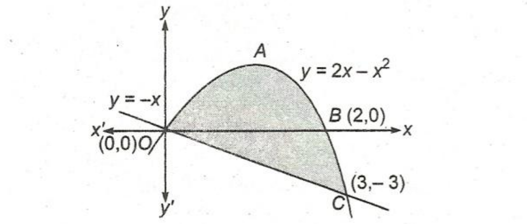
$= [\log \sec x]_0^{\pi/4} - \frac{1}{2} \cdot \frac{1}{2} \cdot 1$

$\left[\because TN = ON - OT = \frac{\pi}{4} - \frac{\pi - 2}{4} = \frac{1}{2}\right]$

$= \log\sqrt{2} - 0 - \frac{1}{4} = \left(\log\sqrt{2} - \frac{1}{4}\right)$ sq unit

207 **(c)**

The point of intersection of given curves are $(0,0)$and $(3, -3)$



$\therefore$ Required area

$= \text{area of curve } OAB$

$+ \text{area of curve } OCB$

$= \int_0^2 (2x - x^2)dx + \left|\int_0^3 (-x)dx\right|$

$- \left|\int_2^3 (2x - x^2)dx\right|$

$= \left[x^2 - \frac{x^3}{3}\right]_0^2 + \left|\left[-\frac{x^2}{2}\right]_0^3\right| - \left|\left[x^2 - \frac{x^3}{3}\right]_2^3\right|$

$= \frac{4}{3} + \frac{9}{2} - \frac{4}{3} = \frac{9}{2}$ sq units

Alternate

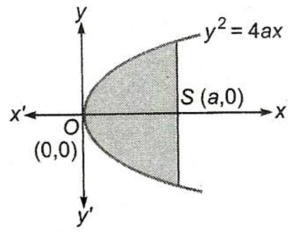
Area $= \int_0^3 [(2x - x^2) - (-x)]dx$

$= \int_0^3 (3x - x^2)dx = \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3$

$= \frac{27}{2} - \frac{27}{3} = \frac{9}{2}$ sq units

208 **(d)**

Required area $= 2\int_0^a \sqrt{4ax}\, dx$



$= 2.2\sqrt{a}\left[\frac{x^{3/2}}{3/2}\right]_0^a = \frac{8}{3}a^2$ sq units

209 **(c)**

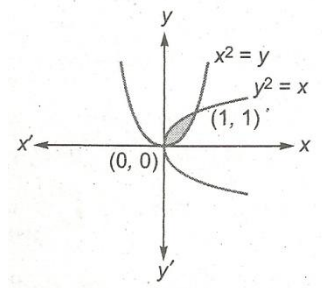
$\therefore$ Required volume

$V = \left|\int_0^1 \pi\, x^2\ dy\right|$

$= \left|\pi\int_0^1 (\,y^4 - \,y)dy\right|$

$= \left|\pi\left[\frac{y^5}{5} - \frac{y^2}{2}\right]_0^1\right|$

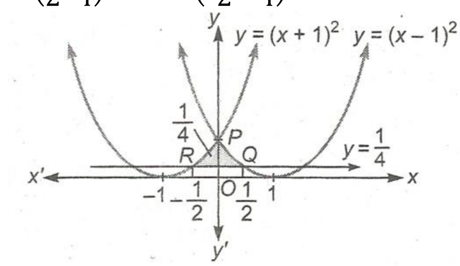
$= \left|\pi\left[\frac{1}{5} - \frac{1}{2}\right]\right| = \frac{3\pi}{10}$



210 **(a)**

The points of intersection of given curves and line are

$Q\left(\frac{1}{2}, \frac{1}{4}\right)$ and $R\left(\frac{-1}{2}, \frac{1}{4}\right)$



Required area $= 2\int_0^{1/2} \left\{(x-1)^2 - \frac{1}{4}\right\} dx$

$= 2\left\{\frac{(x-1)^3}{3} - \frac{1}{4}x\right\}_0^{1/2}$

$= 2\left\{\frac{(-1/2)^3}{3} - \frac{1}{8} - \left(-\frac{1}{3} - 0\right)\right\}$

$= \frac{1}{3}$ sq unit

211 **(b)**

Required area $= \int_0^{\pi/4} \left(\sqrt{\frac{1+\sin x}{\cos x}} - \sqrt{\frac{1-\sin x}{\cos x}} \right) dx$

$\because \left[\frac{1+\sin x}{\cos x} > \frac{1-\sin x}{\cos x} > 0 \right]$

$= \int_0^{\pi/4} \left(\sqrt{\frac{1+\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}{\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}}} - \sqrt{\frac{1-\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}{\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}}} \right) dx$

$= \int_0^{\pi/4} \frac{1+\tan\frac{x}{2} - 1 + \tan\frac{x}{2}}{\sqrt{1-\tan^2\frac{x}{2}}} dx$

$= \int_0^{\pi/4} \frac{2\tan\frac{x}{2}}{\sqrt{1-\tan^2\frac{x}{2}}} dx$

put $\tan\frac{x}{2} = t \Rightarrow \frac{1}{2}\sec^2\frac{x}{2} dx = dt$

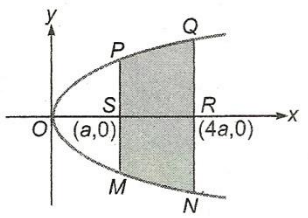
$\therefore$ Required area $= \int_0^{\tan\frac{\pi}{8}} \frac{4t\ dt}{(1+t^2)\sqrt{1-t^2}}$

$= \int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}} dt$

$\left[\because \tan\frac{\pi}{8} = \sqrt{2}-1\right]$

214 **(d)**

Required area $=$ 2 area of curve $PSRQP$



$= 2\int_a^{4a} \sqrt{4\,ax}\, dx = 4\sqrt{a}\left[\frac{x^{3/2}}{3/2}\right]_a^{4a}$

$= \frac{8}{3}\sqrt{a}\left(8a^{\frac{3}{2}} - a^{\frac{3}{2}}\right) = \frac{56a^2}{3}$ squnits

215 **(c)**

$f(x) = \min\{x+1, \sqrt{(1-x)}\}$

$= \begin{cases} x+1, -1 \le x < 0 \\ \sqrt{1-x}, 0 < x \le 1 \end{cases}$

$\therefore$ Required area

$= \left|\int_{-1}^{0}(x+1)dx\right| + \left|\int_0^1 \sqrt{(1-x)}dx\right|$

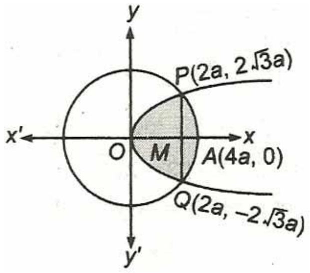
$= 7/6$ sq unit

217 **(c)**

Given equation of curves are

$x^2 + y^2 = 16a^2$ and $y^2 = 6ax$

The point of intersection are $x = 2a, y = \pm 2\sqrt{3}a$



$\therefore$ Required area,

$A = 2$ area of curve $APOP$

$= 2$[area of curve $OMPO$ +area of curve $MAPM$]

$= 2\left[\int_0^{2a} \sqrt{6a}\sqrt{x}dx\right] + 2\left[\int_{2a}^{4a} \sqrt{(4a)^2 - x^2}dx\right]$

$= 2 \cdot \frac{2}{3}\sqrt{6a}\left[x^{3/2}\right]_0^{2a}$

$+ 2\left[\frac{x}{2}\sqrt{(4a)^2 - x^2} + \frac{1}{2}(4a)^2 \sin^{-1}\frac{x}{4a}\right]_{2a}^{4a}$

$= \frac{4}{3}\sqrt{6a}(2a)^{3/2}$

$+ 2\left[\left(0 - 2a\sqrt{3a}\right) + 8a^2\left(\sin^{-1} 1 - \sin^{-1}\frac{1}{2}\right)\right]$

$= \frac{16}{3}\sqrt{3}a^2 - 4\sqrt{3}a^2 + 16a^2\frac{\pi}{3}$

$\Rightarrow A = \frac{4\sqrt{3}a^2}{3} + \frac{16\pi a^2}{3}$

$= \frac{4a^2}{3}(4\pi + \sqrt{3})$ sq unit

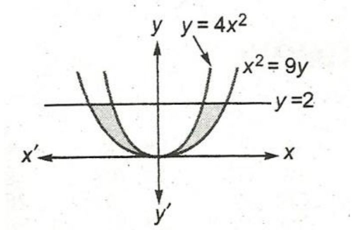
218 **(d)**

Required area $= 2\int_0^2 \left(3\sqrt{y} - \frac{\sqrt{y}}{2}\right) dy$

$= 2\int_0^2 \left(\frac{5\sqrt{y}}{2}\right) dy = 5(y^{3/2})_0^2\ \frac{2}{3}$

$= \frac{10}{3}(\sqrt{8} - 0)$

$= \frac{20\sqrt{2}}{3}$ sq units



219 **(a)**

According to the given condition

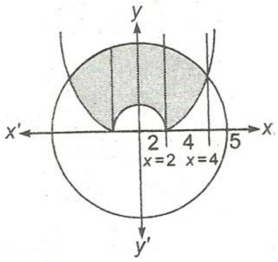
$\int_a^b f(x)dx = c$

On differentiating w.r.t. b, we get

$f(b) = 0 \Rightarrow f(x) = 0$

220 **(c)**

Required area $= 2\left[\int_0^4 \sqrt{(25) - x^2}\,dx - \int_0^2 \frac{4 - x^2}{4}\,dx - \int_2^4 \frac{x^2 - 4}{4}\,dx\right]$



$= 2\left[\left[\frac{x}{2}\sqrt{25 - x^2} + \frac{25}{2}\sin^{-1}\frac{x}{5}\right]_0^4 - \frac{1}{4}\left[4x - \frac{x^3}{3}\right]_0^2 - \frac{1}{4}\left[\frac{x^3}{3} - 4x\right]_2^4\right]$

$= 2\left[\left[2 \times 3 + \frac{25}{5}\sin^{-1}\left(\frac{4}{5}\right) - \frac{1}{4}\left(\frac{64}{3} - 16\right) + \frac{1}{4}\left(\frac{8}{3} - 8\right)\right]\right] = 4 + \sin^{-1}\left(\frac{4}{5}\right)$

221 **(b)**

Required area $= \left|\int_0^1 (3y^2 - 9)dy\right|$

$= \left|[y^3 - 9y]_0^1\right|$

$= |1 - 9| = 8$ sq unit

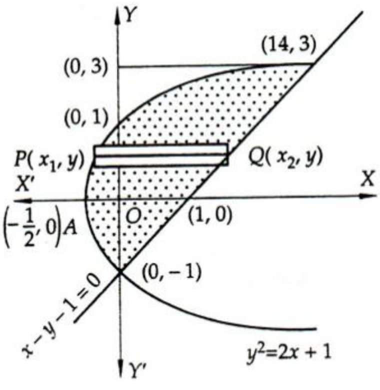
222 **(d)**

Let A be the required area. Then,

$A = \int_{-1}^{3} (x_2 - x_1)\,dy = \int_{-1}^{3} \left\{(y + 1) - \left(\frac{y^2 - 1}{2}\right)\right\} dy$

$\Rightarrow A = \frac{1}{2}\int_{-1}^{3} (2y + 3 - y^2)\,dy$

$= \frac{1}{2}\left[y^2 + 3y - \frac{y^3}{3}\right]_{-1}^{3}$

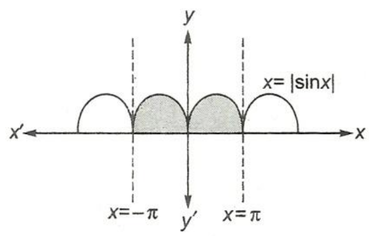


$\Rightarrow A = \frac{1}{2}\left[(9 + 9 - 9) - \left(1 - 3 + \frac{1}{3}\right)\right] = \frac{1}{2}\left[9 + \frac{5}{3}\right]$

$= \frac{16}{3}$

224 **(c)**

Required area $= 2\int_0^{\pi} \sin x\,dx$



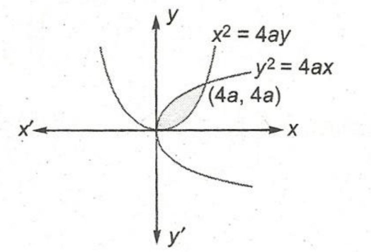
$= 2[-\cos x]_0^{\pi} = 2[1 + 1] = 4$ sq units

225 **(a)**

The point of intersection of given curves are (0,0) and $(4a, 4a)$.

Required area $= \int_0^{4a}\left(2\sqrt{a}\sqrt{x} - \frac{x^2}{4a}\right)dx$

$= \left[2\sqrt{a}\cdot\frac{x^{3/2}}{3/2} - \frac{x^3}{12a}\right]_0^{4a}$

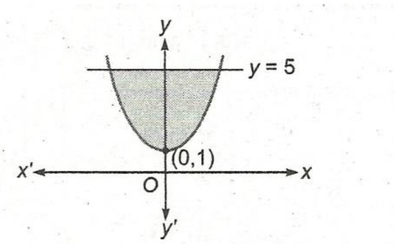


$= \frac{32a^2}{3} - \frac{16a^2}{3}$

$= \frac{16a^2}{3}$ sq units

226 **(d)**

Required area $= \int_1^5 x\,dy = \int_1^5 \sqrt{y - 1}\,dx$



$= \left[\frac{(y - 1)^{3/2}}{3/2}\right]_1^5 = \frac{2}{3}[(4)^{\frac{3}{2}} - 0]$

$= \frac{16}{3}$ sq units

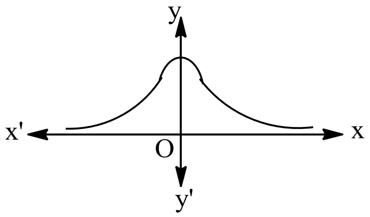
227 **(c)**

The figure of the given curve $y = \frac{a^3}{a^2 + x^2}$ is

∴ Required volume

$V = 2\int_0^{\infty} \pi y^2\, dx$

$= 2\pi a^6 \int_0^{\infty} \frac{1}{(a^2+x^2)^2}\, dx$



Put $x = a\tan\theta$

$\Rightarrow \quad dx = a\sec^2\theta\, d\theta$

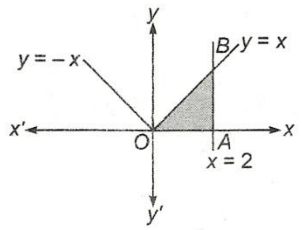
$\therefore V = 2\pi a^6 \int_0^{\pi/2} \frac{a\sec^2\theta}{(a^2+a^2\tan^2\theta)^2} d\theta$

$= \frac{2\pi a^6}{a^3}\int_0^{\pi/2} \cos^2\theta d\theta = 2\pi a^3\left[\frac{1}{2}\cdot\frac{\pi}{2}\right]$

$= \frac{\pi^2 a^3}{2}$ cu units

228 **(c)**

Required area $= \int_0^2 x\, dx = \left[\frac{x^2}{2}\right]_0^2 = 2$sq units

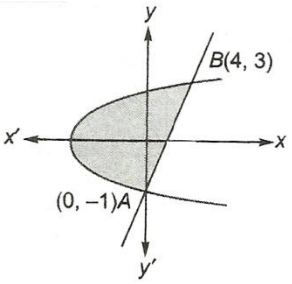


229 **(d)**

Given curves are

$y^2 = 2x+1$

and $x - y = 1$



Points of intersection are $A(0,-1)$and $B(4,3)$

Area $= \int_{-1}^{3}(1+y)dy - \int_{-1}^{3}\left(\frac{y^2-1}{2}\right)dy$

$= \left[y+\frac{y^2}{2}\right]_1^3 - \left[\frac{1}{2}\left(\frac{y^3}{3}-y\right)\right]_{-1}^3$

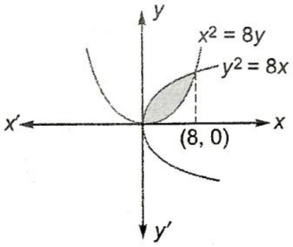
$= \left[3+\frac{9}{2}-\left(-1+\frac{1}{2}\right)\right] - \frac{1}{2}\left[9-3-\left(-\frac{1}{3}+1\right)\right]$

$= 8 - \frac{8}{3} = \frac{16}{3}$

231 **(b)**

Given, curves are $y^2 = 8x \quad \Rightarrow y = \sqrt{8x}$

and $x^2 = 8y \Rightarrow y = \frac{x^2}{8}$



The points of intersection of two curves are (0,0),(8,8)

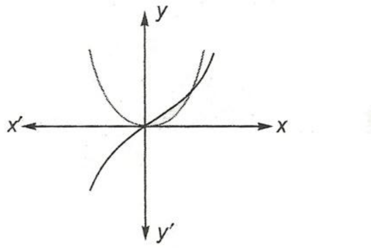
Now, required area $= \int_0^8\left(\sqrt{8x} - \frac{x^2}{8}\right)dx$

$= \left[\frac{\sqrt{8}x^{3/2}}{3/2} - \frac{x^3}{8.3}\right]_0^8$

$= \frac{64}{3}$ sq units.

232 **(a)**

Intersection point of given curves is (1,1)



$\therefore$ Area $= \int_0^1 (x^2 - x^3)dx$

$= \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$

$= \frac{1}{12}$ sq unit

233 **(b)**

The required area A is given by

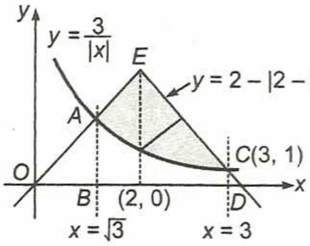
$A = \int_0^{\pi/4} (\cos x - \sin x)\, dx = \sqrt{2} - 1$

234 **(b)**

Required area $= \int_{\sqrt{3}}^{3}(2-12-x)dx - \int_{\sqrt{3}}^{3}\frac{3}{|x|}dx$

$= \int_{\sqrt{3}}^{2} x\, dx + \int_2^3 (4-x)dx - \int_{\sqrt{3}}^{3}\frac{3}{x}dx$

$= \left[\frac{x^2}{2}\right]_{\sqrt{3}}^2 + \left[4x - \frac{x^2}{2}\right]_2^3 - [3\log x]_{\sqrt{3}}^3$



$$= \frac{1}{2}[4-3] + \left[12 - \frac{9}{2} - (8-2)\right] - 3[\log 3 - \log \sqrt{3}]$$

$$= \frac{1}{2} + \frac{3}{2} - 3\log\frac{3}{\sqrt{3}} = \frac{4}{2} - \frac{3}{2}\log 3$$

$$= \frac{4 - 3\log 3}{2} \text{ sq unit}$$

235 **(a)**

Let A denote the required area. Then,

$$A = \int_0^{\pi/4} (\cos x - \sin x)\, dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)\, dx$$

$$= 2(\sqrt{2} - 1)$$

236 **(d)**

Required area $= \int_0^1 (xe^x - xe^{-x}) dx$

$= [xe^x - e^x + xe^{-x} + e^{-x}]_0^1$

$= e - e + \frac{1}{e} + \frac{1}{e} = \frac{2}{e}$ sq unit

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